

# Perspectives on pre-service teacher knowledge for teaching early algebra

**Sharon McAuliffe & Fred Lubben**

*This paper examines a pre-service teacher's content knowledge for teaching early algebra from two perspectives, i.e. using Rowland's Knowledge Quartet theory and Ball's framework for Mathematical Knowledge for Teaching (MKfT). The study intends to examine the differences between the inferences using each framework and to reflect on how they may help us to better understand the knowledge needed for teaching early algebra. Both perspectives are used to interpret selected episodes from a videoed Grade 3 patterns lesson. The findings show that the two perspectives are simultaneously complementary and distinct in both purpose and outcomes. The Knowledge Quartet identifies and describes ways in which the teacher's content knowledge for teaching is enacted in the classroom including those aspects that need reinforcement. The MKfT scheme identifies situations and tasks in teaching where content knowledge emerges including instances where a broader base of knowledge for teaching is required. The results of the analysis show how each perspective emphasises different aspects of teacher content knowledge and, together, provide a more holistic account of what happens in developing knowledge for teaching patterns. They highlight the difficulty for teachers in helping learners to shift from focusing solely on number pattern to a simultaneous focus on function, a transition central to the teaching of early algebra.*

**Keywords:** Pre-service teacher; Knowledge Quartet theory, Framework for Mathematical Knowledge for Teaching; Early algebra

## Introduction and rationale

Mathematics results at both primary and secondary level in South Africa continue to reflect poorly on the system, learners and teachers. A study by Carnoy and Chisholm (2008), which investigates factors contributing to low levels of mathematics achievement in South African primary schools, found some evidence to suggest that high-quality mathematics teaching is related positively to learner achievement. Although the sample of 40 schools is relatively small, the study results support claims

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that larger gains in learner achievement are related to teaching by those who know more about the subject and how to teach it. The results of this study highlight teacher knowledge as an important factor in learner achievement, in addition to the learner- and curriculum-related factors focused on previously.

The work of pre-service teacher education is to prepare effective teachers of mathematics with strong content knowledge, which includes both subject matter knowledge and pedagogical content knowledge. This research is part of a more extensive study on understanding the links between pre-service teachers' developing mathematical content knowledge and the teaching and learning of early algebra. In this study, the expression 'early algebra' includes algebraic reasoning and algebra-related instruction with learners in the primary school (Carraher & Schliemann, 2007). The inclusion of algebra thinking in the early grades, or early algebra as it is also called, gives learners the opportunity to experience and develop conceptual understanding of algebra from the outset. Early algebra can help to prepare learners for more complex mathematics, to cultivate habits of mind that attend to the 'deeper underlying structure of mathematics' and to improve their algebra understanding (Blanton & Kaput, 2005: 412).

This paper looks at one pre-service teacher's knowledge demonstrated in teaching early algebra. Teacher knowledge can be defined in different ways and this research focuses on two conceptual perspectives: the Knowledge Quartet (KQ) (Rowland & Turner, 2009) and Mathematical Knowledge for Teaching (MKfT) frameworks (Ball, Thames & Phelps, 2008). Both draw from the work of Shulman (1986) who identified content-specific knowledge as subject matter knowledge, pedagogical content knowledge and curriculum knowledge.

The use of different perspectives to understand mathematics and the work of teaching is not new and has been elaborated in different ways (Ball, Charalambous, Thames & Lewis, 2009; Even, 2009; Rowland & Turner, 2009). However, the purpose of this paper is to consider one lesson through these two different theoretical lenses which lead to different interpretations of the same lesson (Even, 2009). Thus, the study intends to examine the differences between the inferences using each framework, and to reflect on how they may help us to make better sense of the knowledge needed for teaching early algebra.

## **Theoretical perspectives**

### **The Knowledge Quartet (KQ)**

Turner and Rowland (2008) describe the Knowledge Quartet as an empirically based conceptual framework for classifying ways in which pre-service student teachers' subject matter knowledge and pedagogical content knowledge come into play in the classroom. It can be used to identify and analyse mathematical content knowledge 'enacted' in teaching and to provide a structure for reflection and discussions of lessons (Turner & Rowland, 2008). On the basis of an investigation of mathematical

content knowledge of pre-service teachers in England and Wales, Rowland, Thwaites, Huckstep and Turner (2009) suggest that mathematical content knowledge is a complex combination of different types of knowledge that interact with one another and can be seen more easily in the act of teaching.

The KQ model is an elaboration of the earlier work of Shulman (1986) and responds and updates the work of Fennema and Franke (1992) by suggesting ways in which teachers' subject matter knowledge relates to their pedagogical content knowledge, and how their actions in the classroom are informed by their knowledge (Goulding & Petrou, 2008). The model does not see SMK and PCK as unique entities, but rather as inter-related and dynamic constructs. It is a link between theory and practice through the study of the act of teaching. The framework has been used by pre-service teachers, experienced teachers, tutors, mentors and teacher educators to give constructive feedback on how content knowledge affects teaching and offer suggestions on how to develop it.

The KQ consists of four dimensions, each with its own distinctive nature: foundation, transformation, connection and contingency (Rowland et al., 2009).

## **Foundation**

This member of the quadrant refers to the teacher's theoretical background and beliefs, which constitute the teacher knowledge acquired at school and at university, including initial teacher education. There are three key components to this dimension: knowledge and understanding of mathematics; knowledge of mathematics pedagogy acquired through systematic enquiry into the teaching and learning of mathematics; and beliefs about mathematics, including beliefs about why and how mathematics is learnt. There are seven codes used to identify and describe how foundation knowledge 'plays out' in the classroom: overt subject knowledge; theoretical underpinning of pedagogy; awareness of purpose; identifying errors; use of terminology; use of textbook; and reliance on procedures.

Foundation knowledge is the basis for the other three dimensions which refer to the ways and contexts in which knowledge is used in the preparation and act of teaching: they focus on knowledge-in-action as manifested in lesson planning and actual classroom teaching (Rowland et al., 2009).

## **Transformation**

Transformation knowledge looks at how the content knowledge of the teacher is transformed to enable someone else to learn it. It is what Shulman (1986) defines as the pedagogical content knowledge base for teaching and is demonstrated in the planning of the lesson and in the act of teaching through the choices of representations, illustrations and explanations given and the questions asked from learners. The codes used to classify transformation knowledge are choice and use

of examples; choice and use of representations; use of instructional materials; and teacher demonstration.

## Connection

This category concerns the coherence of the planning and teaching of the mathematics across a lesson or series of lessons. The codes include recognising the conceptual appropriateness and the cognitive demands of a task, making connections between concepts and procedures, and making decisions about the sequencing of materials for instruction.

## Contingency

The final category deals with the teacher’s response to unexpected events which occur within a lesson. It requires the teacher to take contingent action (think on one’s feet) when something unanticipated happens. The codes include deviating from the planned lesson, responding to learners’ ideas, and making use of opportunities and teacher insight during instruction.

## Mathematical Knowledge for Teaching (MKfT)

The second perspective, the Mathematical Knowledge for Teaching (MKfT) model, interrogates the mathematical knowledge needed to carry out the work of teaching (Ball et al., 2008). Resulting from a decade of research, Ball and colleagues developed a model of teacher knowledge from the examination of actual mathematics teaching in primary schools. It is a practice-based theory which focuses on the kind of professional knowledge of mathematics different from that demanded by other mathematically intensive occupations. Furthermore, it moves beyond the limiting boundaries of knowledge to include skills, reasoning, habits of mind and sensitivities needed to teach mathematics effectively (Ball et al., 2008).

The framework of MKfT makes use of Shulman’s (1986) subject matter knowledge, pedagogical content knowledge and curriculum knowledge categories, but organises and defines them in a different way (see table 1). There are six elements to MKfT: common content knowledge, specialised content knowledge, horizon content knowledge, knowledge of content and teaching, knowledge of content and learners, and knowledge of content and the curriculum.

**Table 1: Comparison of areas of teacher knowledge in the Shulman and the MKfT frameworks**

<i>Shulman</i>	<i>MKfT</i>
Subject matter knowledge	Common content knowledge Specialised content knowledge Horizon content knowledge
Pedagogical content knowledge	Knowledge of content and teaching
Curricular knowledge	Knowledge of content and students Knowledge of content and curriculum

Subject matter knowledge is composed of common content knowledge, the knowledge needed in other mathematically intensive professions, specialised content knowledge, and horizon content knowledge. Pedagogical content knowledge requires knowledge of students, pedagogy and the curriculum.

## **Specialised content knowledge**

The domain of specialised content knowledge is a central concept within the MKfT model and will be the focus of this study. Specialised content knowledge is the mathematical knowledge and skill unique to teaching, requiring teachers to be able to do a kind of mathematical work that others do not do (Ball et al., 2008). It requires unique mathematical understanding and reasoning and entails knowledge beyond that being taught to learners. It involves everyday mathematical tasks of teaching such as giving explanations, choosing examples and representations, working with learners' questions and responses, selecting and modifying tasks, and posing questions.

## **Early algebra**

Historically, school mathematics has been seen as a process of developing arithmetic and computational fluency, emphasising skills and procedures followed by a procedural approach to algebra. It is rarely related to the deeper, underlying structure of mathematics and has largely been unsuccessful in drawing large numbers of learners towards mathematics (Blanton, Schifter, Inge, Lofgren, Willis, Davis & Confrey, 2007). It is disconcerting that of the total number of candidates (511 152) who wrote the National Senior Certificate for Grade 12 in 2012, approximately 24% of learners wrote and passed mathematics at the Grade 12 level (DBE, 2012). A large proportion of mathematics at Grade 12 level requires knowledge and application of algebraic principles. The results indicate that learners have developed insufficient knowledge and skills, which suggests that an alternative approach to the development of mathematical understanding, especially algebraic reasoning, needs to be investigated.

Early algebra is a different approach to algebra teaching that highlights children's ability to think and reason algebraically in the foundational grades and cultivates habits of mind for understanding mathematics that will prevail through to the higher grades and other areas of mathematics. Lins and Kaput (2004) argue for an early start to algebra education which provides a special opportunity to develop a particular kind of generality in learners' thinking, i.e. an immersion in the 'culture of algebra' (Lins & Kaput, 2004).

Early algebra is not to be seen as additional work to the current curriculum requirements. It is not a topic that is taught after children acquire arithmetic skills and procedures, but it is developed parallel to the development of arithmetic knowledge instead. In this respect, Schliemann, Carraher and Brizuela (2007) suggest a rethink

of algebra teaching based on the notion of arithmetic having an inherently algebraic character.

The fundamental purpose of early algebra is to provide learners with a set of experiences that enables them to see mathematics – sometimes called the science of patterns – as something they can make sense of, and to provide them with the habits of mind that will support the use of the specific mathematical tools they will encounter when they study algebra (Schoenfeld, 2008). It addresses the five competencies needed for mathematical proficiency (Kilpatrick, Swafford & Findell, 2001): conceptual understanding, procedural fluency, strategic competence, adaptive reasoning and productive disposition.

The term ‘early algebra’ is used in this study to refer to algebraic thinking and reasoning in the elementary grades and is designed to help children ‘see and describe mathematical structures and relationships for which they can construct meaning’ (Blanton, 2008: 6). The structure of early algebra examples are usually conceived in two ways, i.e. iteratively and globally. The iterative approach focuses on the move from one step to the next step, while the global approach helps to establish a rule that can work for any step in the pattern. Generality makes global rules more powerful, but there is evidence to suggest that learners and teachers find these harder to produce (Warren, 2005). Generating the global function rule is sometimes established through guessing or by providing a procedure rather than through logical reasoning.

## **Methodology**

The subject of this study was a third-year education student (Dani) specialising in primary school mathematics education. She had a strong mathematical record, having achieved an A in mathematics (SG) at matriculation level. She had come straight from school to teacher education and performed well in mathematics during the first two years of her degree. She studied to be a Foundation Phase (FP) teacher and had elected to continue with further studies in mathematics education. She had completed two years of an education degree which includes mathematics education (content knowledge and pedagogical content knowledge) as compulsory courses. The third-year mathematics education course (Maths 2) is an elective, and the algebra module, which makes up a large part of the course, focuses on the development of algebraic thinking in the early grades to build a foundation for the study of formal algebra later.

At the time of data collection, Dani had completed nine weeks (with three 45-minute lectures per week) of algebraic content knowledge and pedagogical content knowledge input and was beginning her first teaching practicum block of four weeks.

The Maths 2 class had been given the task to plan, prepare and teach an early algebra lesson (30–45 minutes) based on the types of activities discussed in class.

This task was included to give the pre-service teachers an experience of designing and teaching an early algebra lesson and was used later to further develop knowledge for teaching early algebra.

One of Dani’s lessons on patterns in Grade 3 was video-recorded, and this study uses two selected episodes from this lesson. All names in the paper, of teachers and learners, are pseudonyms.

### Dani’s lesson: Patterns and function – Grade 3

#### Lesson outcomes (as given in lesson plan)

The learning outcome of the lesson requires learners to be able to recognise, describe and represent patterns and relationships, as well as to solve problems using algebraic language and skills. The assessment standard, selected by Dani, requires learners to describe observed patterns, to find the relationship between the numbers (pattern), and to generate a formula to describe the relationship.

#### Lesson description

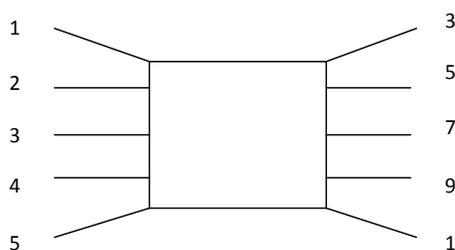
The lesson starts with making sound patterns using hands and identifying different examples of patterns: number and geometric.

The next part of the lesson investigates a particular pattern using a folded length of string and cuts. Dani has prepared two diagrams side by side on the board: first, a vertical table with two columns with headings ‘Number of cuts’ and ‘Total number of pieces of string’ and, secondly, a horizontal flow diagram labelled ‘The magic box’.

*Vertical table*

<i>Number of cuts</i>	<i>Total number of pieces of string</i>
1	3
2	5
3	7
4	9
5	11

*The magic box*



She reminds the learners that this is a similar lesson to the dog pattern lesson of the previous day in which the learners had to generalise about the relationship between the number of dogs and their total number of eyes. The pattern task in this lesson requires learners to make cuts using different pieces of the same length string generating the following data: 1 cut = 3 pieces of string; 2 cuts = 5 pieces of string, 3 cuts = 7 pieces and 4 cuts = 9 pieces. The information is recorded systematically in a vertical table and the learners are asked to describe what is happening.

Dani and her learners focus on the iterative or recursive pattern  $+2$  generated from the dependent variables after which the learners are reminded again of the work from the previous day, which involved trying to find a magic formula to describe the change from independent variables to dependent variables. Dani then moves to the other section of the chalkboard and recorded the same data in the horizontal flow diagram ('The magic box'). The learners are required to describe the relationship between the independent and dependent variables in the form of a magic formula (function rule) using a guess and check technique.

The lesson concludes with an opportunity for the learners to design their own number patterns which are discussed and evaluated by the teacher as she moves around the classroom.

## Findings and interpretation

Two different theoretical frameworks were used to analyse the video recording of the Grade 3 patterns lesson: the knowledge quartet (KQ) and the mathematical knowledge for teaching (MKfT) model. The KQ model helps to identify and analyse the mathematical content knowledge (subject matter content and/or pedagogical content knowledge) that Dani privileges in her teaching, while the MKfT model looks at the nature of the specialised content knowledge being mobilised in Dani's lesson.

Two episodes have been selected for analysis based on the interactions between the pre-service teacher and learners and the mathematical intent of the segment.

### Episode 1

This episode occurs some way into the lesson when the learners are discussing the possible patterns that emerge for the data recorded in the table below.

<i>Number of cuts</i>	<i>Total number of pieces of string</i>
1	3
2	5
3	7

Dani has created a record of the independent and dependent variables and is now looking for patterns within the numbers (Smith, 2008):

*Teacher: Okay, Aziza what's happening?*

*Learner: The number of pieces are all the odd numbers.*

*Teacher: Ah, Aziza says these are all the odd numbers, okay so 3, 5, 7, 9.*

*So, what would come next?*

*Learner: 11*

*Teacher: and then?*

*Learner: 13.*

*Teacher: Well done Aziza, you said these are all the odd numbers.*

*Teacher: Who can see a different pattern? What else is happening with those numbers?*

*Learner: The one side is increasing in ones.*

*Teacher: What are we doing from this number to get to that number (pointing to two successive dependent values), to get that number, to get that number? Aqeel.*

*Learner: Plus two.*

*Teacher: We are plussing two each time.*

Dani tries to move the learners towards describing how two quantities vary in relation to each other (Blanton, 2008). She works with the learners to identify and analyse regularities and change in the patterns and acknowledges and works with the learner contributions. She encourages the learners to make the difference quantity between the dependent values explicit and welcomes learner responses while choosing to ignore the increase in the independent variables.

## **Knowledge Quartet (KQ)**

The KQ helps us to look at the how Dani's content knowledge (subject matter content and/or pedagogical content knowledge) affects her teaching. Dani uses a physical task to generate independent and dependent variables and represents the data in the form of a vertical function table. She then encourages the learners to analyse the variables and proceeds to work with learners' responses (foundation knowledge). The learners focus on the dependent variables and the classification of the numbers while Dani helps them to focus on the difference between the numbers instead (iterative approach). Through guided discovery, the learners begin to identify the difference between the independent variables and then the dependent variables with the help of the teacher. Dani's knowledge of the different approaches to patterning helps her to adapt her teaching and to work with the learners' responses, showing knowledge of connections within the teaching episode.

## **Mathematical Knowledge for Teaching (MKfT)**

Using the MKfT perspective to interpret this teaching episode, we can begin to identify Dani's chosen specialised content knowledge for teaching patterns. Dani demonstrates knowledge of different representations for teaching patterns and makes use of a piece of string and cuts to generate and represent data for the function table. She then uses the data captured in the table to analyse the independent and

dependent variables by following an iterative approach. She works with learners' responses and tries to scaffold their emerging knowledge of patterns. This teaching episode highlights key teaching tasks related to the specialised knowledge needed for teaching patterns, i.e. selecting and using representations, working with learners' responses, and posing questions that help develop relational understanding of variables within pattern activities.

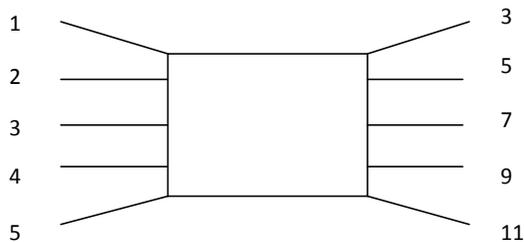
## Episode 2

The second episode, which comes after the vertical function table episode, reflects Dani's attempt to move learners from looking down the table (iterative pattern) to looking across and help learners understand how quantities change in relation to one another (global patterning). Dani decides to change the representation from a function table to a magic box (flow diagram) to focus on a horizontal analysis of the quantities, i.e. the functional relationship between the independent and dependent variables.

*Teacher: But remember what we did with the dogs. I told you. But what if I saw 700 dogs, how many eyes would there be? Remember, we got a formula and a formula was like a recipe and we could use it for every number to see how many eyes we could get. Do you remember? Right, so now we need to do the same thing. What are we doing to these numbers to get to these numbers and how did we do it? What did we use? We used our ...?*

*Learner: A magic box.*

*Teacher: We used our magic box. So we know that when we cut once we got 3, when we cut twice we got 5, when we cut three times we got 7, when we cut four times we got 9. Right, what on earth is happening inside my magic box. Ryan, what's happening in there?*



*Learner: +2*

*Teacher: Okay, so Ryan says we must +2. Okay, let's see +2. What is 1 + 2?*

*Learner: 3.*

*Teacher: What is 2 + 2?*

*Learner: 4.*

*Teacher: Okay, so that's not working. Let's see the next one. What is 3 + 2?*

Dani spends a long time working with learners' ideas and realises that the guess and check method to finding the relationship between the independent and dependent variables is not productive. She decides to scaffold the learning by providing a clue while pointing at the flow diagram:

*Teacher: We'll stop with Kim and I want you to look here [Kim has suggested  $x^2$ ]. We're getting 2 but we need 3. We're getting 4 but we need 5. We're getting 6 but we need 7. So what is the difference each time? We need to do something else. We need to keep the  $x^2$ , we need to do something else.*

*Learner: +1.*

*Teacher: So what must I do?*

*Learner:  $x^2 + 1$ .*

*Teacher: What do you think?*

*Learner: Yes.*

*Teacher: You think it's right? How can we test it? Put it in our box. Go put it in the box for us.*

## Knowledge Quartet

In looking at this episode from the KQ perspective, it seems that Dani has a number of tools available for building, analysing and representing functions. When the learners are unable to describe the relationship between the quantities, she decides to change her choice of representation (transformation knowledge). She decides to introduce 'The magic box' to help focus learner attention on the global or corresponding relationship between the quantities.

Unfortunately, this does not help learners and she is forced to scaffold the learning by highlighting the  $x^2$  suggestion given by Kim. As in the previous episode, it is apparent that Dani has foundation knowledge of the function concept in moving learners from iterative thinking to global thinking, but struggles to mediate this for learners. She resorts to giving the learners help in generating a numerical representation (possibly contingency) to describe the relationship between the number of cuts and the number of pieces of string. She does not attempt to link this to the physical act of cutting the string and does not create the need for generalising the pattern in the first instance (does not display connection).

## Mathematical Knowledge for Teaching (MKfT)

As mentioned previously, the MKfT framework does not focus on what the teacher accomplishes, but rather on the specialised knowledge needed for teaching through an analysis of the teaching tasks featured in the teaching episode. Dani engages in several tasks, some of which involve using representations to make ideas explicit, managing multiple answers given by the learners in discussion, and assessing learners understanding and moving forward. The use of different representations provides a

helpful example of the mathematical demands confronted by Dani and gives two different perspectives on this lesson episode.

First, Dani uses different representations of functions, moving from the function table to the flow diagram, to help learners move from looking at relationship in a sequence of values to understanding that quantities change in relation to one another (Blanton, 2008). However, she does not provide a reason for the need to generalise and does not ask the learners to think about the number of pieces of string generated from a large number of cuts. The learners are forced to think about describing the pattern in general terms without providing a need for doing so.

Secondly, the function concept is represented in different ways, mapping between a physical model, function table to a flow diagram towards generalisation, but there is no explanation of the inter-connectedness of these models, or of the meaning of functions in different representational modes. It is difficult for learners to avoid developing a procedural approach to understanding patterns and relationships while connections remain implicit.

## **Discussion**

The study set out to explore how different perspectives are useful in helping us understand teacher knowledge and its role in the teaching of early algebra. The KQ classifies situations in which mathematical knowledge surfaces in teaching and uses these situations to identify and analyse the subject matter knowledge and pedagogical knowledge of the teacher. The MKfT model distinguishes between different kinds of knowledge needed in the work of teaching and is applied in this context to analyse the specialised content knowledge needed by this teacher in the teaching of patterns.

The KQ is useful in this context in helping to identify and discuss the different aspects of mathematical content knowledge observed in Dani's teaching. She demonstrates knowledge of patterns and functions, and pedagogical understanding of the teaching of patterns to young learners (foundation knowledge). She is aware of the representations available to teach patterns demonstrated in her use of the concrete manipulatives, the function table and flow diagram (transformation). However, she struggles to help learners make connections between the different representations and does not give learners enough time to work with the pattern activity and reflect on the emergent relationships between the independent and dependent variables (connections). Dani recognises that learners are struggling with the pattern activity to move from iterative to global thinking and tries to scaffold learning through guided teaching (contingency).

Mason (2008) supports the notion of scaffolding and fading as a way to improve pedagogical effectiveness and present learners with a varied diet of rich mathematical experiences. He promotes a flexible view of patterns, which helps learners to recognise patterns that lead to algebraic generalisation and symbolisation. However, while

Dani tries to scaffold learning by helping learners to notice and verbally describe the pattern in the flow diagram, she misses the opportunity to demonstrate how patterns and structure are inherently linked to mathematical activity through the recognition of regularity and the description of the relationship through generalisation (Mulligan, Mitchelmore & Prescott, 2006).

The MKfT model provides a different perspective of Dani's content knowledge through the investigation of the two teaching episodes within the patterns lesson. It focuses on the specialised content knowledge needed for the teaching patterns and provides a useful guide for teachers and teacher educators in developing an understanding of the knowledge needed for teaching patterns. The analysis of the teaching episodes provides useful insight into the specialised knowledge needed for teaching patterns in this lesson. Dani needs to be able to work with learners' responses and help them to move towards generalisations. She has to ask questions that encourage learners to recognise relationships and to make links between different variables. Learners need to be helped to move beyond analysing single variable data to two (or more) quantities simultaneously and to use patterns to make connections and describe relationships (Blanton, 2008). Furthermore, she needs to have knowledge of representations and make decisions about the selection and appropriate use of mathematical tools for patterning. There is also the need to provide explanations that help learners to move from iterative patterning towards analysing and describing relationships that lead to generalisations. Dani attempts to engage with these tasks on different levels and with varying degrees of success. The MKfT model helps to focus the analysis of the teaching episodes and highlights important aspects of developing content knowledge for teaching.

## Conclusion

While the two theoretical perspectives help elaborate teacher content knowledge and its relation to the teaching of mathematics, they also provide different and productive interpretations of the same lesson on early algebra. The KQ helps to identify the content knowledge used in teaching patterns and the MKfT helps to elaborate a description of the knowledge needed for teaching patterns. The two frameworks assist in highlighting some of the issues and demands that teaching algebra in the primary grades present for this pre-service teacher. They also show that, with the correct kind of tasks and instruction, children can learn to think in sophisticated ways about how quantities relate to one another.

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