The Technological Pedagogical Content Knowledge of Mathematics Teachers Teaching Grade 12 Transformation

By

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Declaration

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I, Rets`elisitsoe Eszekiel Mokone, declare that the dissertation titled "The Technological Pedagogical Content Knowledge of Mathematics Teachers teaching Grade 12 transformation" submitted for the qualification of the Master's degree in Mathematics Education at the University of the Free State is my own independent work. All the references that I have used have been indicated and acknowledged by means of complete references. I further declare that this work has not previously been submitted by me at another university or faculty for the purpose of obtaining a qualification.

Signature

E.R.Mokone DATE

Dedication

I dedicate this project to the Almighty God who has been my strongest pillar by providing me with wisdom, knowledge and understanding. I also dedicate this work to my wife, `Makhotso Mokone, who supported me throughout this hard journey. To my children Paballo and Khotso who were not given enough time they needed from me, thank you.

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Abstract

This study reports on an investigation conducted with three Grade 12 mathematics teachers who taught transformation to learners using technological pedagogical content knowledge. The aim of the study was to explore the technological pedagogical content knowledge of mathematics teachers who teach Grade 12 transformation. A qualitative research methodology with a multiple case study research design was used to examine the technological pedagogical content knowledge of the Grade 12 mathematics teachers in the in-depth teaching and learning of transformation. A purposive sampling technique was used to select three mathematics teachers with more than five years' experience of teaching Grade 12 transformation in mathematics and who participated in a five-year program on The New Partnership for Africa`s Development (NEPAD) e-Schools Demo Project. Lesson observations and individual semi-structured interviews were used to collect data from the three teachers in different schools. Data collected through both the observations and the individual interviews was transcribed and read thoroughly to code it. Codes were categorised, and after that themes emerged. The emergent themes from individual interviews and observations were merged and triangulated to obtain the findings. The findings of the study revealed that the selected mathematics teachers who participated in the NEPAD e-school project have a limited technological pedagogical content knowledge when teaching transformation to Grade 12 learners. The findings also revealed that the same teachers use relevant technology to address learners` challenges in the teaching of transformation, but are unable to engage the learners with the use of technology to help them to improve their knowledge on transformation. The study recommends short courses for mathematics teachers focusing on pedagogical aspects, which include the use of technology to enhance their technological pedagogical content knowledge.

Keywords: Content knowledge, pedagogical knowledge, technological knowledge

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List of Acronyms

CK	Content Knowledge
DEL	Department of Education Lesotho
DGS	Dynamic Geometry Software
GSP	Geometer`s Sketchpad
ISTE	International Society of Technology Education
JMC	Joint Mathematics Council
LGCSE	Lesotho General Certificate Secondary Education
MOESS	Ministry of Education, Science and Sports
MOET	Ministry of Education and Training
NCTM	National Council of Teachers of Mathematics
NEPAD	New Partnership for Africa`s Development
PCK	Pedagogical Content Knowledge
PK	Pedagogical Knowledge
TCK	Technological Content Knowledge
TK	Technological Knowledge
TPACK	Technological Pedagogical Content Knowledge
TPK	Technological Pedagogical Knowledge

CHAPTER 1:
INTRODUCTION, BACKGROUND AND OVERVIEW OF THE STUDY

1.1 INTRODUCTION

Technological Pedagogical Content Knowledge (TPACK, previously TPCK) is knowledge which is necessary when technology is used to teach (Thompson & Mishra, 2007). Hernawati and Jailani (2019) point out TPACK can help mathematics teachers when teaching more advanced level like Grade 12 learners in order to enhance their learning experience. To meaningfully teach a specific topic of mathematics, especially transformation, teachers need to make use of TPACK (Kim, 2018). Transformation in Lesotho schools is taught from Grade 8 to Grade 12, and the content develops in each grade. The teaching of this topic in Grade 12 demands highly specific strategies because it combines different types of transformations. In the pursuit of developing the e-schools project, the Lesotho Ministry of Education, in partnership with the New Partnership for Africa`s Development, require teachers to teach mathematics using technology (Kalanda, 2012). Moreover, during the last few years, secondary schools in Lesotho took part in technology-based projects such as School NET Lesotho, NEPAD e-Schools Projects and Microsoft STIC (Kalanda, 2012). Despite a number of projects focusing on teaching with technology, especially when teaching mathematics, Grade 12 teachers` TPACK has not been evaluated at any point, in any of the projects. This study will therefore explore the TPACK of Grade 12 mathematics teachers, especially regarding the teaching of transformation.

Mathematics teachers do not usually integrate technology when they teach, so it is important to explore their TPACK (Graham, 2011). Transformation is a topic in mathematics. It is taught to Grade 12 learners in Lesotho together with interrelated concepts such as mathematical reasoning and different representations of shapes. Learners continue to struggle with the learning of transformation geometry, although it is an important area in mathematics because of its applications (Battista, 2007). According to Kilpatrick, Swafford and Findell (2001), the use of technology, which involves computer software, can easily help learners to understand the properties of shapes in transformation. Therefore, the use of technology could be a good strategy for both the teaching and learning of transformation (Battista, 2007).

1.2 BACKGROUND

Mishra and Koehler (2006) adopted Shulman's (1986) idea, and explain Pedagogical Content Knowledge (PCK) as a teacher's ability to interpret content and to use different ways of presenting it with teaching aids which help leaners to build new knowledge. Therefore Mishra and Koehler (2006) brought a new concept into PCK, namely technology, and developed TPACK. They (Mishra & Koehler, 2006) further state that TPACK is the interconnection of bodies of knowledge which comprise content knowledge, pedagogical knowledge and technological knowledge. All of these are necessary when delivering content to learners using technology.

It is necessary to examine TPACK among mathematics teachers as it is needed when teaching mathematics using technology. Roberts and Stephens (1999) explain that teaching strategies using technology can have an impact on the quality of instruction and student learning. The TPACK of mathematics teachers can help them to teach transformation in a manner that engages learners, because transformation involves the moving of shapes. Due to the increased global requirement to use technological tools, it is necessary to explore teachers` specialised knowledge for them to effectively integrate technology, which is referred to as TPACK by Koehler and Mishra (2009).

Teachers' knowledge of mathematics has broadened over the years to include aspects such as technology. In addition, the integration of the domains of knowledge such as content, technology and pedagogy, requires teachers to be familiar with the content, and be adept at presenting it to leaners using technology (Koehler & Mishra, 2009). Teachers should therefore become comfortable with different tools for the purpose of teaching using technology.

Many countries such as Spain and Ghana have introduced an educational policy which requires the integration of technology when teaching mathematics, in which case teachers have to be competent with TPACK. Teachers with relevant and adequate technological, pedagogical and content knowledge are able to integrate technology effectively into their teaching (Tajudin & Kadir, 2014). As mentioned previously, Mishra and Koehler (2006) classify TPACK into content knowledge, pedagogical knowledge, technological knowledge, pedagogical content knowledge, technological content knowledge and technological pedagogical knowledge. The constructs will be further explained in the chapter that deals with the theoretical framework. This study uses the constructs of the TPACK to identify the adequate knowledge of Lesotho mathematics teachers when they utilise technology in teaching transformation in Grade 12.

The study by Ramatlapana (2017) examined the TPACK of mathematics teachers when they integrated technology to teach circle geometry using GeoGebra. The results of her study revealed that mathematics teachers lack geometry content knowledge, pedagogical content knowledge and technological knowledge when they teach geometry. Although Ramatlapana`s (2017) investigation focused on how mathematics teachers integrate technology when teaching geometry, transformation is a sub-topic under geometry. Ramatlapana (2017) further revealed that mathematics teachers lack the knowledge for selecting relevant technologies which are applicable for teaching geometry. This study is similar to Ramatlapana`s (2017) study in that they both investigated the TPACK of mathematics teachers when teaching geometry. The difference between the two studies is that Ramatlapana (2017) was focusing on the teaching of circle geometry while this study was focusing on the transformation geometry.

A number of studies have focused on the TPACK of mathematics teachers making use of technology example (Ramatlapana, 2017; Tajudin & Kadir, 2014).The study conducted by Tajudin and Kadir (2014) examined the level of TPACK of mathematics teachers. Their findings revealed that use of technology by mathematics teachers is at an acceptable level. It can however be improved if they are given training, as this would equip them with strategies involving the use of technology. The study conducted by Tajudin and Kadir (2014) differs from that of Ramatlapana (2017), as they examined the level of mathematics teachers' TPACK. In contrast, Ramatlapana (2017) investigated the mathematics teachers` TPACK when teaching the properties of a circle, specifically using GeoGebra.

The study by Tajudin and Kadir (2014) investigated mathematics teachers` level of TPACK and their integration of technology in teaching. Their study found that in terms of pedagogical content knowledge, the teachers were using discussion as a method in teaching geometry for almost the entire lesson. They also found that the technology used by all the teachers was PowerPoint, assisted by the use of the scientific calculator. These teachers were found to be at acceptable level of TPACK in their teaching regarding their use of technology. This study did not explore the level of TPACK for mathematics teachers, but rather their TPACK, in order for them to teach transformation using technology.

According to Ramatlapana (2017), mathematics teachers can make use of technology integration when teaching geometry by designing written or technology-based tasks for the learners. An example is the use of dragging, which allows the movement of shapes from one position to another in transformation. This helps to represent such movements differently. Ramatlapana (2017) examined the integration of technology when teaching geometry using GeoGebra without considering other possible technological devices that could also be relevant. This study explored the TPACK of mathematics teachers when teaching transformation in Grade 12 using technology. It differs from that of Ramatlapana (2017) in that it was focusing on whether mathematics teachers have sufficient knowledge to teach transformation using technology. Ramatlapana (2017), on the other hand, investigated the knowledge which mathematics teachers should have in order to teach geometry using GeoGebra.

1.2.1 Evolution leading to TPACK

There has been an increased awareness regarding the importance of technology in education for the past few decades. This has led to the addition of technology to Shulman`s (1986) model of PCK by Mishra and Kohler (2006). The use of PCK would involve the decision of a teacher to design activities for leaners which are specific to the content to be studied (Harris & Hofer, 2009). The use of TPACK will therefore include the use of technology in those designed activities (Mishra & Kohler, 2006). The use of TPACK by mathematics teachers will therefore include the designed activities which will allow learners to engage with technology to move shapes on the x-y plane form one position to another while studying transformation.

Shulman`s (1986) theory of PCK was expanded with the purpose of combining technological knowledge within content knowledge and pedagogical knowledge. This means that TPACK has been generated with the purpose of understanding teacher knowledge, which is necessary when integrating technology in teaching. Moreover, TPACK has been introduced in educational research as a theoretical framework to understand the knowledge of teachers, which is necessary for the integration of technology (Mishra & Koehler, 2006; Thompson & Mishra, 2007).

Mishra & Koehler (2006) developed TPCK with the purpose of the integration of technology when teaching. Thompson & Mishra (2007) however reported a name change from Technological Pedagogical Content Knowledge (TPCK) to TPACK after a consideration that TPACK should emphasise the idea of Total PACKage of every technology which can support teaching and learning. This framework has seven domains of knowledge, namely Technological Knowledge (TK), Pedagogical Knowledge (PK), Content Knowledge (CK), Technological Content

Knowledge (TCK), Technological Pedagogical Knowledge (TPK), Pedagogical Content Knowledge (PCK), and Technological Pedagogical Content Knowledge (TPACK). The fully detailed explanation of the constructs of the framework will be dealt with in the theoretical framework section.

The exploration on the TPACK for mathematics teachers when teaching transformation using technology was done in a number of countries, including Spain, Ghana and Lesotho. Globally there is still a challenge when teaching mathematics using different approaches which involve technology for the effective use of TPACK, as agreed to by Roig-Vila, Mengual-Andrés and Quinto-Medrano (2015). Roig-Vila *et al.,* (2015) analysed the content, pedagogical and technological knowledge of mathematics teachers in Spain who integrated technology in teaching. The results from their study revealed that the knowledge of mathematics teachers tend to be stronger on content and pedagogy, than on technology, meaning that their TPACK does not sufficiently integrate technology in their lesson presentations. Their findings further revealed that there is a need to train mathematics teachers on the use of technology, specifically with regard to the content to be taught and the pedagogical approach.

Guerrero (2010) examined the context of TPACK in the mathematics classroom and developed four necessary components of knowledge when mathematics is taught while making use of technology. The four components are conception and use of technology, technology-based mathematics instruction, management, and depth and breadth of mathematics content. Guerrero (2010) argues that TPACK is characterised by the four components hence they are related to the TPACK by Mishra and Koehler (2006). He continues to show that the first component, the teacher`s conception and use of technology, brings a relationship between technology and PCK. The next two components are associated with general pedagogical content knowledge. The last component of TPACK, which is depth and breadth of mathematics content, is related to the teacher`s mathematics content knowledge.

The four components of TPACK as stated by Guerrero (2010) do not correspond directly to the seven constructs of TPACK by Mishra and Koehler (2006). That means one cannot claim that technology-based mathematics instruction corresponds to technological content knowledge. Furthermore, the number of components of TPACK does not display a one-to-one correspondence with the subdomains of knowledge, as presented in the TPACK model. For example PCK, TCK and TPK fall within the intersection of the seven constructs which is TPACK (Guerrero, 2010). Therefore this study made use of the seven constructs of TPACK by Mishra and Koehler (2006), and the four components of TPACK by Guerrero (2010) to explore the TPACK of mathematics teachers teaching transformation to Grade 12 learners. The seven constructs of the TPACK were used to explore mathematics teachers` knowledge which is relevant to integrate technology when teaching transformation. Furthermore, discussions on the teachers` TPACK were framed around the TPACK framework and Guerrero`s (2010) four components, which characterise TPACK in mathematics.

According to Guerrero (2010), teachers' conception and use of the technology component of TPACK is positively related to PCK, to the extent that technology can easily be used by a teacher to make the content of mathematics more understandable and accessible to learners. In simple terms this component includes the belief of a teacher about how best the teaching of mathematics can be addressed using technology, and not just as a tool used in teaching (Guerrero, 2005). When a teacher is at this level he/she can use technological tools in teaching such as involving learners to discover how to perform different types of transformation. Finally, teachers must not only possess the knowledge of using different features of technology, but they must know exactly how to make this fit well with their instruction.

The second component of TPACK, which is technology-based mathematics instruction, involves the teacher`s knowledge and the ability to use different instructional approaches, particularly when using technology in support of the teaching of mathematics (Guerrero, 2010). This means that technology is not used to replace a teacher, but rather should be considered as another aspect of a teacher`s instructional skill. Moreover, a teacher must use technology to perform different types of transformation as a way of helping learners to gain a better understanding. This component involves the use of technology to achieve the objectives of the instruction. Finally, this instructional component of TPACK includes the teacher`s ability to make use of technology so that it can accommodate individual learners’ abilities in learning

Guerrero (2010) asserts that management as the third component of TPACK in mathematics deals with issues of classroom management which are specifically related to teaching and learning with technology. This is because there are other types of management strategies which are seldom encountered by teachers when they are not using technology during instruction. This management involves, but is not limited to, managing learners when sending games and messages via graphing calculators, and using computer as physical shields. This management in the teaching of mathematics using technology helps teachers to take screenshots of learners` work, which facilitates the teaching of transformation.

The fourth component of TPACK involves the teacher`s responsibility to fully understand mathematical content and theorems (Guerrero, 2010). This component mainly focuses on teachers’ mathematical content knowledge, which should be more than that of the learners. This may become an issue because while learners are learning mathematics using technology, they may discover concepts which are unfamiliar to their teacher (Goss, Renshaw, Galbraith & Geiger, 2000). Consequently, teachers have to be confident in handling learners` investigations.

1.2.2 Spain

According to Roig-Vila, Mengual-Andrés and Quinto-Medrano (2015), mathematics teachers in Spain were trained on the integration of technology when teaching with the hope that the training would help them to develop their TPACK. The results of their study revealed that although mathematics teachers possess pedagogical knowledge and content knowledge, they lack technological knowledge for them to be competent with TPACK. Stols and Kriek (2011) assert that the competency of a mathematics teacher with TPACK goes along with the teacher`s knowledge on how learners think when it comes to different types of transformation. Furthermore, such a teacher must have pedagogical knowledge, which is relevant in teaching different types of transformation to the learners.

The traditional model of teaching was ineffective, so the mathematics teachers in Spain have to make use of technology (Cabero, 2003). According to Niess (2005), the development of specific knowledge when technological knowledge is integrated with content knowledge, as well as knowledge of how to teach, can be regarded as TPACK. Therefore, when teachers integrate these different domains of knowledge then they are in a position to teach their subject matter using technology effectively.

1.2.3 Ghana

In his analysis of educational policies with the focus on their developments in Sab-Saharan Africa, Tilya (2008) released a report stating that most countries, including Ghana, have an educational policy on the integration of technology in educational instruction. He further reported that Ghana introduced a new curriculum for Senior High Schools in September 2007 for teaching mathematics. This curriculum encourages the use of technology, but he pointed out that many mathematics teachers lack technological knowledge. Agyei and Voogt (2012) assert that the integration of technology into the mathematics curriculum is recommended by the Policy of Education in Ghana for all school level, but yet there is no effective use of technology when teaching mathematics. In the same way the Lesotho Government has a policy to include technology in education (Kalanda, 2012), although most teachers of mathematics do not use technology when teaching.

The Department of Education in Ghana introduced in-service training of mathematics teachers to use technology when teaching with the purpose of helping them to develop their TPACK (Ministry of Education, Science and Sports (MOESS), 2007). The introduction of in-service training was meant to help teachers to gain experience and knowledge on the use of relevant technologies, which would encourage a learner-centred approach. According to Schmidt, Baran, Thompson, Mishra, Koehler and Shin (2009), one of the PCK aspects is the assessment of learners and the integration of this knowledge with TCK. This enables teachers to assess leaners using the GeoGebra software, which helps learners to construct accurate shapes in transformations. According to Agyei and Voogt (2012), mathematics teachers who happened to be trained in Ghana on integrating technology in their teaching with the purpose to develop their TPACK, were later found to be in a position to help their leaners to use technology to easily find the centre of rotation in transformation, as well as locating the image after rotation.

According to Voogt (2008), integrating technology when teaching was a complex innovation for mathematics teachers in Ghana, as they had not yet developed their TPACK. Mathematics teachers in Ghana did not integrate technology into their teaching in spite of government’s efforts of establishing computer labs in most senior high schools (Agyei & Voogt, 2012). This was due to a lack of technological pedagogical content knowledge, which would enable them to effectively integrate technology in instruction. It was therefore important for this study to explore the TPACK of mathematics teachers in order for them to teach transformation using technology effectively.

According to Benning, Linsell and Ingram (2018), mathematics teachers in Ghana received support from experts on how to use the GeoGebra software. This allowed them the opportunity to put their technological pedagogical content knowledge into practice. They were for example able to use GeoGebra to create stimulating objects which helped learners to conceptualise the orientation of objects in transformation when they are rotated clockwise or anticlockwise. This approach helped them to move to a learner-centred approach in which learners cooperatively constructed transformation concepts. The opportunity to use GeoGebra also helped in-service mathematics teachers to effectively integrate technology in the teaching of mathematics to such a degree that their teaching in the technological environment became interesting and also enhanced learners` concentration and participation.

1.2.4 Lesotho

Technology influences current society to such a degree that most countries have educational policies which encourage teaching strategies which include the use of technology involving computer hardware and software. The Lesotho government also appreciated the existence of technology by introducing projects which could facilitate teaching and learning using technology at both primary and high school levels (Isaacs, 2007). The Ghanaian educational policy only introduced the use of technology at senior high school level, (Tilya, 2008) while Lesotho introduced it at all levels from primary to high school. In both Lesotho and Ghana, the educational policy on the use of technology in instruction has not been fully implemented.

Lesotho mathematics teachers need to possess specific knowledge in order to access different relevant technological teaching strategies. Tajudin and Kadir (2014) are of the view that relevant teaching strategies which involve the use of technology mostly help learners to benefit from teaching. In addition, Suharwoto and Lee (2005) point out that teaching strategy with technology require teachers to have content knowledge, pedagogical knowledge and technological knowledge. When these are integrated, they enable teachers to teach mathematics using TPACK.

Lesotho was among the 16 countries in which the NEPAD e-Schools Project was initiated. Certain projects were introduced in Lesotho with the purpose of facilitating teaching strategies which involve the use of technology at schools. The NEPAD e-schools were provided with the guidelines for their training, example that they are to teach specific grades, and in their training they will be assessed on how to select relevant technology to teach a specific topic on mathematics to a specific grade. Kalanda (2012) points out that Lesotho introduced three major projects, namely SchoolNet Lesotho, NEPAD e-Schools Projects and Microsoft STIC to support teaching with the use of technology. Lesotho mathematics teachers can therefore develop their TPACK through these projects with technological teaching resources.

Currently high schools in Lesotho offer the Junior Certificate and Lesotho General Certificate Secondary Education (LGCSE), in which transformation is taught. The teaching of transformation in Grade 12 is demanding as it involves many different concepts such as matrix transformations. According to Isaacs (2007), regardless of the introduced projects which are aimed at the use of technology in education, mathematics teachers are not effectively integrating technology when they teach. This is the reason for this study to explore the TPACK in order to teach transformation using technology. In teacher education in Lesotho little has been done regarding research which focuses on mathematics teachers` TPACK.

1.3 PROBLEM STATEMENT

The researcher, with more than ten years' experience as the head of a mathematics and science department at a secondary school, realised that mathematics teachers in the schools in which the New Partnership for Africa`s Development (NEPAD) e-Schools Project were piloted are not effectively using technology for the teaching and learning of transformation in Grade 12. Technological pedagogical content knowledge is knowledge about using different technologies in order to teach the content. Using the knowledge of technology and being aware of it, teachers are expected to transform their instruction into better representations using technology for those concepts which are difficult to teach using traditional ways (Jang & Tsai, 2013).The fact that teachers have themselves not learnt mathematics using technology, has the effect that they do not have the necessary knowledge to use technology in their teaching (Ozudogru & Ozudogru, 2019). This is the major reason for conducting the current study, namely to explore whether mathematics teachers have the specific knowledge to effectively integrate technology in their teaching process.

Some schools in Lesotho were selected to pilot the New Partnership for Africa`s Development (NEPAD) e-Schools Project so as to facilitate teaching and learning using technology for a period of ten years. This time period has now lapsed. The major concern is whether mathematics teachers were able to develop technological pedagogical content knowledge in the teaching of transformation in Grade 12 during this period of ten years. It is emphasised by Sampaio (2013) that, in general, mathematics teachers have limited technological content knowledge regarding the choice of relevant software or technology to teach transformation. They also lack the knowledge of integrating pedagogy and technology with content of transformation in teaching and learning. This study explored the TPACK of three teachers in a selected Demo project on the teaching and learning of transformation.

1.4 RESEARCH QUESTIONS

1.4.1 Main research question

What is the technological pedagogical content knowledge of mathematics teachers teaching transformation in Grade 12?

1.4.2 Sub-research questions

1. How do Grade 12 mathematics teachers teach transformation using technology?

2. How do Grade 12 mathematics teachers use technology to address the challenges faced by learners in learning transformation?

3. How do mathematics teachers improve learners' knowledge of transformation using technological pedagogical content knowledge?

1.5 PURPOSE OF THE STUDY

This research was aimed at investigating the technological pedagogical content knowledge of mathematics teachers teaching transformation in Grade 12 using technology.

1.6 OBJECTIVES OF THE STUDY

- To examine how mathematics teachers use technology to teach transformation to Grade 12 learners.
- To examine how Grade 12 mathematics teachers use technology to address the challenges that are faced by learners in learning transformation.
- To explore how mathematics teachers can improve learners` knowledge of transformation using TPACK.

1.7 THEORETICAL FRAMEWORK

Mishra and Koehler (2006) conducted a study which serves as theoretical framework for (TPACK). This theoretical framework is a relevant model for this study, and is used in this study purposely for the use of technology when teaching transformation in mathematics. It is further emphasised by Mishra and Koehler (2006) that when the researcher uses the TPACK theoretical framework, the researcher will be able to understand the effective use of teaching using technology, and will also be able to analyse how teachers make use of the relationship between content, technology and pedagogy in their teaching. The TPACK framework is relevant to this study, because as Landry (2010) highlights, this framework can be used to investigate the technological, pedagogical and content knowledge purposely and develop teaching approaches which involve the use of technology in the teaching of mathematics.

1.8 RESEARCH METHODOLOGY AND DESIGN

The following research methodology was adopted in order to enable the investigation into the main and sub research questions.

1.8.1 Research methodology

This study used a qualitative research approach. As McMillan and Schumacher (2010) emphasise, it focuses on description and exploration of the phenomena of interest in order to answer the research questions of the study.

1.8.2 Research paradigm

Maree (2016) explains that a research paradigm may be catergorised as postpositivist, interpretivist, critical theory and so on. Moreover, Bertram and Christiansen (2017) point out that a paradigm can determine the choices which the researcher make, such as the type of questions to be asked, what to observe or investigate, how to collect data and finally how to interpret the findings. This study is informed by the interpretivism paradigm. With interpretivism, the researcher makes interpretations with the purpose of understanding human perceptions. In this study teachers develop their TPACK as they integrate technology in the teaching of transformation. Bertram and Christiansen (2017) emphasise that interpretivism can be used to inform a multiple case study approach, in which case observations and interview instruments can be used to collect data. This study used observations and interviews to collect data which was later interpreted. Interpretive paradigm is underpinned by observation and interpretation of collected information about events (Maree, 2016).

1.8.3 Research design

This study made use of a multiple case study research design to explore the Grade 12 mathematics teachers' technological pedagogical content knowledge in the teaching and learning of transformation. Three case studies were used in this study to explore the technological pedagogical content knowledge of three mathematics teachers teaching transformation using technology. Maree (2016) points out that based on the philosophy which underpins the research, a case study can either be exploratory, descriptive or explanatory. This study made use of an exploratory case study.

1.8.4 Data collection

The data was collected from three teachers using observations and semi-structured individual interviews. The research questions of this study were used to design the two instruments as well as the constructs of the TPACK framework. The researcher himself developed the interview protocol. The observation schedule was designed by the researcher. The constructs of the TPACK theoretical framework were used as reference to check for the validity and reliability of the instruments, and they were finally checked by the supervisor of this study.

Three teachers were observed and interviewed in order to collect data. Teachers were observed while teaching learners to move shapes from one position to another on the x-y plane, performing transformations using technology in order to examine the TPACK in the teaching of transformation. Making use of multiple data collection techniques has the benefit of enabling triangulation, which improves the validity of the data (Maree, 2016). The interviews were conducted after the observations of the lessons, as this helped the researcher to further explore the teachers` TPACK displayed during the teaching and learning of transformation.

1.8.5 Data analysis

The recordings of the observations and interviews were transcribed, read several times and coded based on the sub-research questions of the study. The codes were then categorised, after which certain themes emerged.

1.9 VALUE OF RESEARCH

The Lesotho Ministry of Education will benefit from this study to help mathematics teachers and teacher trainers to develop professionally. This research will provide an important description about the connection between educational technologies and mathematics education. Teacher trainers can also benefit from this research as they may gain insight to where they should focus for the training of their student teachers on the teaching of transformation with the involvement of technology. This present study could contribute to understanding the Lesotho mathematics teachers` level of knowledge on content, pedagogy and technology. Other countries whose teachers are not yet effectively utilising technology in teaching can also benefit from this research.

1.10 DELIMITATIONS

This study focused on the TPACK of mathematics teachers when teaching transformation using technology to Grade 12 learners from selected secondary schools in Lesotho. Data was collected from two secondary schools which represented a sample of some schools in Lesotho in which technology is used when teaching mathematics. This study used observations and semi-structured individual interviews as data collection instruments. The use of the multiple case study qualitative research methodology was used because Maree (2016) highlights its importance as being research-centred. The participants would be able to provide a complete understanding of the research setting.

1.11 DEFINITION OF TERMS

The following terms are used throughout this research report and they are defined here to establish a clear and shared meaning.

Transformation: A part of geometry in which learners learn to identify the type of movements of shapes on the x-y plane, and can describe these fully (Boulter & Kirby, 1999).

Technology: Facilitating the teaching and learning to improve performance by using relevant technological processes and resources (Richey, 2008).

Content Knowledge (CK): Teachers` content knowledge of the subject (Koehler & Mishra, 2009).

Pedagogical Knowledge (PK): Teachers` knowledge about different teaching strategies which may also be informed by educational theories (Koehler & Mishra, 2009).

Pedagogical Content Knowledge (PCK): Knowledge of teaching methods which are specific to the teaching of the specific content of a subject (Shulman, 1986).

Technological Pedagogical Knowledge (TPK): Knowledge about the use of relevant technology appropriately with the teaching methods (Mishra & Koehler, 2009).

Technological Content Knowledge (TCK): Knowledge about relevant technology which is specific for a particular content of a subject (Schmidt *et al.,* 2009).

Technological Pedagogical Content Knowledge (TPACK): Professional knowledge that teachers need to meaningfully incorporate pedagogy and technology within the content they teach (Koehler & Mishra, 2009).

1.12 LAYOUT OF THE CHAPTERS

Chapter 1- Introduction and Background

The background on the use of technological pedagogical content knowledge by mathematics teachers is provided in this chapter. The focus for the background is on the TPACK of mathematics teachers on the teaching of transformation in Spain, Ghana and Lesotho. Moreover, it also provides the problem statement, aim and the research questions related to the research.

Chapter 2 – Literature Review

This chapter reviews the relevant literature related to the use of technological pedagogical content knowledge of mathematics teachers when teaching transformation in Grade 12. The literature was also reviewed in relation to the research questions of this study.

Chapter 3 – Theoretical Framework

The TPACK theoretical framework is discussed in this chapter. The constructs of the (TPACK) framework are discussed in this chapter in relation to the use of technology when teaching transformation in Grade 12.

Chapter 4 – Research Methodology

The details of the research methodology which was used in this study is discussed in this chapter. The chapter fully describes the methodology used, the selection of the participants, the instruments used to collect data and finally the analysis of the data. The research design is a multiple qualitative case study.

Chapter 5 – Data Analysis and Discussion

The collected data is presented and discussed in this chapter in relation to the emerged themes for each research instrument. The emergent themes from individual interviews and observations were merged and triangulated to obtain the findings. Multiple data collection techniques have the benefit of enabling triangulation for improving validity. The findings can also be validated by making use of triangulation (Maree, 2016).

Chapter 6 – Summary of the Findings and Conclusion

This chapter sums up the findings pertaining to the aspects of Lesotho mathematics teachers` technological pedagogical content knowledge on transformation. The conclusion is made based on the research questions in relation to the presented findings.

1.13 SUMMARY

In order to explore the technological pedagogical content knowledge of mathematics teachers teaching Grade12 transformation, the study was conducted in two secondary schools in Maseru, Lesotho, with mathematics teachers using technology. This chapter provides the summary of the study by providing and discussing issues relating to the background and overview of the study, the research questions, purpose of the study and the significance of the study.

CHAPTER 2:
LITERATURE REVIEW

2.1 INTRODUCTION

In this chapter the results of the literature relevant to this study are reviewed, specifically relation to the sub research questions. The literature is discussed keeping in mind its relevance to the integration of the three knowledge domains, namely content knowledge, pedagogical knowledge and technological knowledge when transformation is taught to learners. The three knowledge domains are considered simple because they are necessary in order for mathematics teachers to use technology to teach transformation effectively. The research questions of this study and the TPACK theoretical framework were fully taken into account when discussing the reviewed literature. The technological pedagogical content knowledge of mathematics teachers is explored for the teaching of transformation. Finally, literature on technological pedagogical content knowledge is reviewed, based on the integration of TPACK of mathematics teachers, for the effective use of technology in the teaching of transformation.

2.2 VIEWS ABOUT MATHEMATICS

Mathematics as a discipline is viewed differently by philosophers. Some view mathematics as fallible, while others view it as absolute. These different philosophical perspectives of mathematics may sometimes be termed 'absolutist'. However, what these perspectives have in common is that mathematics is viewed as absolute, objective and certain as a body of knowledge which relies on concrete foundations and deductive logic (Ernest, 1996). In accordance with this school of thought, the absolutist view of mathematics is that it is a fundamentally 'true' form of knowledge (Crowe & Zand, 2000). The recent popular philosophy of mathematics is known as fallibilism. Humans invented mathematics and it was not “discovered”, as postulated by the fallibilists. Its truth can therefore be challenged, tested and discussed (Ernest, 1996). Baron (1972) states that absolutists view mathematics as a method which can be used to discover certain “truths”,while fallibilists view mathematics as the truth to be discovered.

Behavioural learning theory encourages teachers to help leaners to discover the concepts themselves (Chunk, 2012). The teaching method where a teacher is a transmitter of knowledge is considered as teacher-centred approach (Hackman, 2004). In some cases, however, some teachers make use of a teacher-centred approach, where they just feed leaners with concepts of mathematics which is philosophically absolute. This study uses the fallibilsts view of mathematics, in which mathematics teachers use technology to help Grade 12 learners to discover mathematics concepts. Mathematics teachers help leaners to discover the concepts of transformation rather than relying only on what they are told by their teachers, which is the absolute view of mathematics.

Ernest (1994) classifies mathematics teachers into three categories, namely Explainer, Instructor and Facilitator. He states that Instructor has the perspective that mathematics should involve rules, facts and skills to be used. Explainer believes that mathematics is a body of knowledge “truth” which, learners see it as it is. Both Instructor and Explainer are categorised as absolutists in their view of mathematics. Mathematics as Facilitator Davis (1976), Cooney (1988), Ernest (1988b), Ernest (1989c), Ernest (1994) means that mathematics is regarded as dynamic, and the teacher`s role is to make learners more confident problem solvers. Philosophically, the Facilitator is fallibilist. The fallibilist philosophical stance is supported in this study, as this study uses the concept that teachers’ TPACK help them to become facilitators in the teaching of mathematics using technology.

2.3 MATHEMATICS AND TECHNOLOGY

Rogers (1995) defines technology as the instrumental action which is designed in such a manner that it relates the input with the output in order to achieve a desired outcome. Huang, Spector and Yang (2019) explain that technology may have two components, which are hardware, consisting of a physical object, or software, consisting of information. Huang et al. (2019) further clarify that there is also educational technology, which involves the use of hardware and software, as well as pedagogical approaches in support of learning and instruction. The fallibilist philosophical view of mathematics can be enhanced when using technology to help learners discover the concepts of transformation. This study places a greater emphasis on the use of educational technology when transformation is taught using technology.

There is a connection between technology and mathematics because there are some resources which can be accessed by teachers and learners from the internet. These include simulation software and graphing calculators (Roblyer & Edwards, 2000). The software can be used by teachers to explore geometric programs where they can use mathematical formulas, represent data graphically and construct shapes in transformations (Isman & Yaratan, 2004). Isman and Yaratan (2004) further explain that computer software can be used to represent data graphically in order to teach transformational geometry such as tessellation. In the same way technology can be used to manipulate algebraic expressions, compute functions and statistical probability, and be used in geometry.

2.4 MATHEMATICS AND PEDAGOGICAL CONTENT KNOWLEDGE

Pedagogical content knowledge can be used by mathematics teachers to help leaners discover the concepts of mathematics. Shulman (1986) asserts that the integration of content knowledge and pedagogical knowledge contributes to the effective teaching of the content. Mathematical knowledge as well as pedagogical knowledge is needed to help learners to construct new knowledge on mathematical concepts in their minds. According to An, Kulm, and Wu (2004), pedagogical content knowledge consists of two bodies of knowledge, which are content knowledge and knowledge of teaching Stols and Kriek (2011) are of the view that the pedagogical knowledge of a mathematics teacher in the teaching of transformation must involve the way learners think about the representation of different types of transformation. Teachers must use computers to teach so that they can improve their skills in using dynamic geometry software like GeoGebra, which is relevant in teaching transformation.

The ability of mathematics teachers to design activities which help learners to discover the concepts of different types of transformation makes a good display of PCK in the teaching of mathematics. Hill, Rowan and Ball (2005) point out that PCK in mathematics enable mathematics teachers to use shapes to represent transformation concepts and be able to analyse learners` solutions and explanations. In addition, Ball, Thames and Phelps (2008) believe that the required mathematical knowledge to teach mathematics and further establish the pedagogical content knowledge of a teacher is important when predicting the performance of a learner in the learning of mathematics. Therefore pedagogical content knowledge is important in this study, as it is one of the TPACK constructs to be explored in the teaching of transformation.

2.5 MATHEMATICS AND TPACK

TPACK can help mathematics teachers to use technology to transform shapes more easily in the teaching of mathematics (Guerrero, 2010). Guerrero (2010) relates TPACK with mathematics by creating a situation of TPACK in the mathematics classroom for which he develops four central components of knowledge necessary when technology is used to teach mathematics by teachers. The four components are conception and use of technology, technology-based mathematics instruction, management, and the depth and breadth of mathematics content. He further explains that the relation between TPACK and mathematics is more than the knowledge to make use of a specific technology, but deals with how to use a particular technology to improve mathematics teaching and learning. This means that technology can be used to represent mathematics concepts to make them clearer to the learners. Becta (2009) points out that the technological pedagogical knowledge of mathematics teachers helps them to teach transformation using Dynamic Geometry Software (DGS), because DGS helps to identify the corresponding sides of the object and image in the teaching of enlargement. It also allows them to compare their ration in relation to the scale factor.

2.6 MATHEMATICS TEACHERS TEACHING TRANSFORMATION USING TECHNOLOGY

Ramatlapana (2017) asserts that the knowledge which can be used to connect relevant technology to teach geometry transformation using appropriate pedagogical approaches can be referred to as TPACK. Content knowledge in transformation involves different movements of shapes from one position to another on the x-y plane and also the identification of each type of transformation by describing it fully. Kim (2018) argues that mathematics teachers, with their TPACK, can design tasks which involve the use of Dynamic Geometry Software in order to help learners to differentiate the types of transformations. These may include reflection, rotation and translation, which challenge learners with objects and images which are congruent. Moving shapes on the x-y plane using different types of transformation with the use of technology is regarded as a display of technological content knowledge (Ramatlapana, 2017). In addition, DGS can be used by mathematics teachers because it can be used to draw and move shapes involving transformation on the computer screen, and also to accurately measure distances. In support of this, knowing different types of technologies will assist mathematics teachers to make a good selection in order to effectively teach transformation using technology (Steketee, 2005:107).

Transformation involves two areas, which are isometry and non-isometry. In isometry transformations the object and image are congruent, whereas in non-isometry transformations the object and image are not congruent. Isometries include rotation, translation and reflection, while non-isometries include stretch, shear and enlargement. Technology can be used by mathematics teachers to teach both isometry and non- isometry. The effective teaching of transformation must involve hands-on activities for leaners so that they can acquire a conceptual understanding as opposed to a procedural understanding only (Mashingaidze, 2012:198). In addition, NCTM (2015) state that teaching and learning of mathematics should involve the use of digital and physical tools so that technology can enhance teaching and learning of mathematics even to the concepts which do not employ the aid of technology.

Dikovic (2009) points out that teaching transformation using technology encourages interaction between teachers and learners. Learners can also explore and visualise the movement of shapes in relation to their properties, which could be challenging if they were constructing these on paper using pencils (Lu, 2008). One can "drag" the point or change the angle of rotation to see how the corresponding image would change. In addition, Hollebrands (2003) states that DGS helps in the movement of shapes from one position to another using dragging when teaching reflection, translation and rotation. The teaching of transformation using technology does not only motivate learners but enhances teaching methods, which helps learners to gain knowledge and skills (Roblyer & Doering, 2010).

NCTM (2015) emphasise the teaching strategies which involve the use of technology in the teaching and learning of mathematics which relates the specific content of mathematics with technology for the learners to explore mathematics concepts. According to the International Society for Technology in Education (ISTE, 2008) there are prepared National Educational Technology Standard and Performance Indicators which stipulate relevant teaching strategies that incorporate digital tools to promote learners' creativity and use of emerging digital tools to analyse and evaluate learning in the teaching of mathematics. In support of this the National Council of Teachers of Mathematics (NCTM) (2015) advocates for the use of technology in the teaching of mathematics.

2.7 MATHEMATICS TEACHERS USING TECHNOLOGY TO ADDRESS LEARNER CHALLENGES IN LEARNING TRANSFORMATION

Idris (2006) claims that most learners experience challenges in learning transformation and that these challenges relate to the use of language in transformation and visualisation ability. She also highlights that there is a connection between spatial visualisation and geometric achievement because of geometry being visual in nature. Karakus (2008) points out that the use of GeoGebra software by mathematics teachers in teaching transformation can help them to address learners' challenges in learning transformation. Learners can for example reflect objects using the software to have a clear understanding of reflection. Karakus (2008) further emphasises that by using GeoGebra, learners are able to draw an accurate shape which they can reflect with full satisfaction of all the properties under reflection to help them understand all the concepts of transformation. In actual fact, technology can be used by mathematics teachers to address these challenges which are faced by learners when learning transformation, because the designed class activities can be saved and used later for revision of the work that was done in class.

Ustun and Ubuz (2005) assert that the programme called Geometer`s Sketchpad can be used by mathematics teachers to address the challenges faced by learners in learning transformation, because learners can use it to explore transformational shapes to see possible connections. Most learners are challenged when using a traditional approach to identify transformational shapes to see the possible connections. They are also not able to visualise spatial figures in their minds. Since visualisation is very important in studying geometry transformation, dynamic geometry software is able to provide learners with the opportunity to learn transformation concepts and easily explore the relationships when using technology (Ustun & Ubuz, 2005).

Identifying and visualising the type of transformation performed on the object to obtain the image, are the common challenges which learners face when learning transformation. In order to address these challenges, Karakus (2008) asserts that visualisation is important when learning transformation. Teachers can therefore use dynamic geometry software to help learners visualise spatial figures and explore relationships easily in order to develop a better understanding and a positive attitude. In addition, Karakus (2008) also emphasises that learners are able to achieve a higher attainment level in learning reflection and rotation when using software such as Geometer`s Sketchpad.

Geometer's Sketchpad can be used to replace the use of constructing shapes on paper using pencil, where learners tend to make mistakes. The software can help them to improve their measurements. McBroom (2012) emphasises that Geometer's Sketchpad is efficient in the teaching of transformation as it can quickly find the angle of rotation. Bu and Schoen (2011) explain that mathematics teachers should identify the challenges faced by learners when learning transformation. They can make use of software programmes such as GeoGebra to address these challenges, such as identifying the corresponding sides on the object and image, and visualising the movement of the shapes.

Koehler and Mishra (2005) point out that TCK is the knowledge that a teacher has to be able to select relevant technology to teach the subject matter so that the content and technology are related. Leong and Lim-Teo (2003) point out that Geometer's Sketchpad (GSP) is relevant software when teaching transformation because it can be used to draw accurate shapes, measure the corresponding sides in enlargement and measure the angle of rotation. In addition, Hoong & Khoh (2003) emphasise that designing activities for learners which involve the use of GSP helps them to address the challenge of describing the transformations fully. An example is in reflection, where learners are supposed to give the mirror line, and in rotation where they have to find the centre and angle and direction.

There are dynamic geometry software programmes such as GeoGebra, Cabri, Dr Geo and Geometer`s Sketchpad which have been designed with the purpose of teaching geometry in secondary schools. Mathematics teachers can engage learners with activities to use dynamic geometry software so as to help them to address the challenge of constructing accurate shapes. This can also help them to easily discover the properties of shapes in transformations (Mariotti, 2002). In addition, Clements, Sarama, Yelland and Glass (2008) are of the view that using dynamic geometry software when teaching transformation in secondary school can help learners to improve visualisation skills to relate the objects and their images in transformation. Furthermore, Sanders (1998) states that dynamic geometry software can be used by mathematics teachers in addressing learner challenges because it helps learners to develop the concepts. It also and enriches visualisation which is often challenging when learning transformation.

2.8 MATHEMATICS TEACHERS IMPROVING LEARNERS` KNOWLEDGE OF TRANSFORMATION USING TECHNOLOGICAL PEDAGOGICAL CONTENT KNOWLEDGE

Tanner and Jones (2000) assert that mathematics teachers can use their TPACK to design learning activities which involve the use of technological devices in such a way that the teacher becomes a facilitator who guides learners to perform a specific type of transformation. Moreover, the use of TPACK by mathematics teachers helps them to apply a learner-centred approach as opposed to a teacher-centred approach in teaching transformation. The designed teaching and learning activities makes it easy for the teacher to ask questions based on the observations in order for the learners to determine the centre of rotation, translation vector and line of reflection.

Jones (2011) points out that Dynamic Geometry Software (DGS) as technology in education is very appropriate when learners are attempting to learn transformation on their own. The use of technological pedagogical content knowledge by mathematics teachers enables them to help learners on the use DGS and further build new knowledge in learning transformation (Becta, 2009). Ruthven, Hennessy and Deaney (2008) assert that DGS helps to quickly create a number of different examples when performing different types of transformations, which would take longer when constructed on the blackboard or on paper. The enlargement of a shape using a positive, negative and fractional scale factor would for example take too long to do on paper, specifically with the joining of the corresponding points. With the use of DGS it is matter of choosing the centre, scale factor, and the object is enlarged.

According to the Joint Mathematical Council of the United Kingdom (JMC) (2011), mathematics teachers do not often engage learners with the use of technology when learning. Becta (2009), however, asserts that the use of DGS makes it easy to engage learners in order for them to build new knowledge regarding transformation. In support of this, Erez and Yerushalmy (2006) point out that DGS helps learners to identify the corresponding sides of the object and image in transformation. It also enables them to notice which parts of the shape remain the same and which ones changes after performing the transformation.

Schmidt *et al.* (2009) are of the view that by integrating pedagogical content knowledge with technological content knowledge, mathematics teachers can use GeoGebra software as technology to help learners to construct and move shapes accurately. In addition, Kim (2016) asserts that for the effective teaching of mathematics with technology, teachers must develop their knowledge on technology and pedagogy. Furthermore, Ittigson and Zewe (2003) point out that the use of technology to assess leaners, supports constructive pedagogy in which case leaners are able to develop high order thinking skills and therefore use technology better in order to understand transformation.

Considering the constructs of TPACK, content knowledge is knowledge about the facts, theories and principles of a subject matter (Mishra & Koehler, 2006). In transformation, content knowledge is considered as the ability to move shapes from one position to another on the x-y plane, and to further describe each type of transformation fully. The competence of a mathematics teacher with TPACK can be observed when such a teacher chooses relevant computer software to move a shape from one position to another using different types of transformation (Drijvers & Trouchè, 2008). Furthermore, Hoong and Khoh (2003) highlight that the use of software in the teaching of different types of transformation aids the visualisation and as a result has a positive effect on the learners.

Technological pedagogical content knowledge can help mathematics teachers to make use of Geogebra in the teaching of geometry transformation because it greatly increases learners` motivation and engagement in the learning process. McBroom (2012) emphasises that GeoGebra helps learners to be actively involved to discover reflecting, translating and rotating figures, and help each other to discover patterns to draw their own conclusions. Furthermore, the use of GeoGebra helps learners to have a long-term retention of transformation concepts. Mathematics teachers use their TPACK to design guided activity sheets which the learners can use it to make observations about figures when they are rotated $90°, 180°$ and $270°$ about their origin. In this way learners are able to see the object in blue while the image will be in red (McBroom, 2012). McBroom (2012) emphasises that the colouring of the object and image serves as a rich visualisation which can help in finding the corresponding image points from the pre-image points.

2.9 SUMMARY

This chapter explored literature related to the teaching of transformation using technology, improving leaners' knowledge of transformation using technological pedagogical content knowledge, as well as addressing leaners` challenges in learning transformation. The philosophy of mathematics teaching was also discussed in relation to the current study. The views obtained in the literature will be compared with the data obtained in this study and discussed in the final chapter in order to compare the results.

CHAPTER 3: THEORETICAL FRAMEWORK

3.1 INTRODUCTION

The previous chapter focused on relevant literature on the use of TPACK in teaching transformation. This chapter will focus on the TPACK as theoretical framework in investigating the specific knowledge relevant to the use of technology when teaching transformation. This chapter discusses the constructs of the technological pedagogical content knowledge (TPACK) framework in relation to the use of technology when teaching transformation. Mishra and Koehler`s (2006) TPACK model will be used as the theoretical framework. The use of this theoretical framework is a very relevant model when using technology while teaching mathematics. Teachers' competence for effective teaching with technology in the context of education needs to be examined (Mishra & Koehler, 2006). This competence can be determined by the three domains of knowledge, which are content knowledge, pedagogical knowledge and technological knowledge (Mishra & Koehler, 2006).

3.2 THEORETICAL FRAMEWORK

Mishra & Koehler`s (2006) developed technological pedagogical content knowledge (TPACK) model which is used as the theoretical framework. They explain that the TPACK is the overall intersection of technological content knowledge, pedagogical content knowledge and technological pedagogical knowledge. These are all necessary for teachers to effectively integrate technology in their teaching of specific content of a subject (see Figure 3.1). The relevance of the TPACK framework in this study is aligned on the basis that Landry (2010) highlights that is appropriate when investigating the technological knowledge, pedagogical knowledge and content knowledge so as to understand the technological teaching approaches in mathematics.

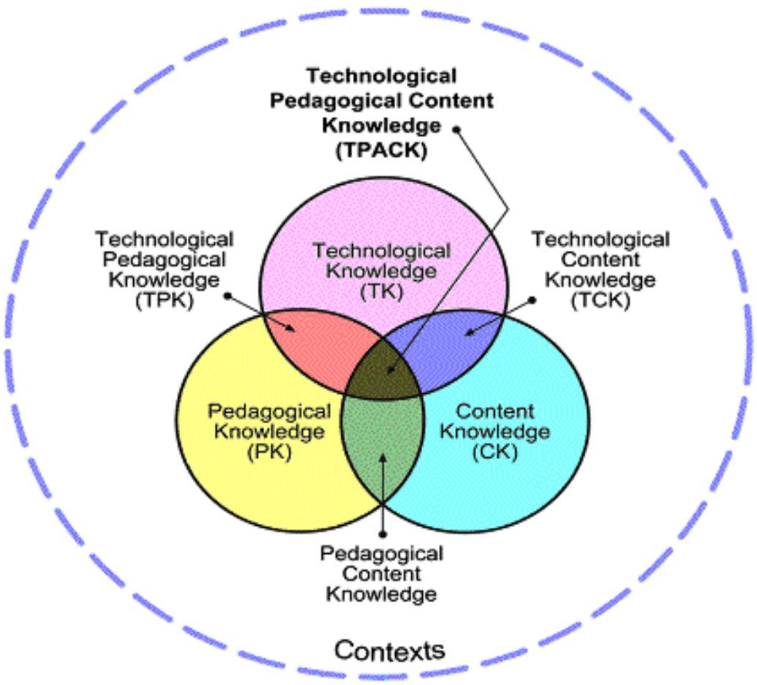

Figure 3.1: Framework of TPACK: (Koehler & Mishra, 2009:5)

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Mishra and Koehler (2006) describe the TPACK constructs as follows:

- *Content Knowledge* (CK) can be referred to as teacher knowledge of subject matter that involves concepts, theories, proofs and practices which were established during the development of that content.
- *Pedagogical Knowledge* (PK) is the knowledge of teachers, which includes different teaching methods and strategies with the use of theories in education.
- *Technological Knowledge* (TK) is the knowledge which enables mastering the use of technology as well as its applications.
- *Pedagogical Content Knowledge* (PCK) is the knowledge of teachers which involves the methods of teaching which are specific to the teaching of a particular content of a subject.
- *Technological Content Knowledge* (TCK) is the knowledge to select the relevant technology for teaching particular content.
- *Technological Pedagogical Knowledge* (TPK) is the knowledge necessary for using technology with different teaching methods and strategies.
- *Technological Pedagogical Content Knowledge* (TPACK) is the basic knowledge for using technology effectively when teaching. It involves the integration of the three knowledge domains, namely content knowledge, pedagogical knowledge and technological knowledge.

The three mathematics teachers who were the participants in this study, were expected to use technology to teach one of the types of transformation namely reflection, translation, enlargement, rotation, shear or stretch. Koehler and Mishra (2009:9) point out that the TPACK of professional knowledge is better developed when teachers effectively use technology when teaching concepts of subjects. The appropriate application of the TPACK framework in this study is on the basis that mathematics teachers were once trained on the use of technology to teach mathematics. Mishra and Koehler (2006) are of the view that for the appropriate use of the TPACK framework, teachers must have been trained in the use of technology relevant to the teaching of specific content.

They further show that the relevance of this framework in research is its guidance on the development of the instruments, choice of methodologies for collecting data, and type of data analysis to be used. In addition, Mishra and Koehler (2006) emphasise that the TPACK framework does not only help a researcher to understand the effective use of technology in teaching when teachers integrate content, pedagogy and technology in their teaching, but also helps in the analysis of the collected data. These mentioned reasons all led to selecting TPACK as the theoretical framework for this study.

The TPACK model has been used by many researchers in the integration of technology in education and as a theoretical framework (Cox & Graham, 2009). In terms of measurement, most of the instruments focused on teachers` self-report. TPACK comprises many components and measuring its effectiveness depends on the relationships of these components with each other (Koehler & Mishra, 2009). The TPACK framework emphasises integrated knowledge as opposed to focusing on knowledge in isolation. Teachers who use TPACK have an understanding of presenting specific content with relevant pedagogy and technology to effectively support teaching and learning (Koehler & Mishra, 2009).

Table 3.1: Linking the TPACK framework with the teaching of transformation

TPACK FRAMEWORK	IN THE CLASSROOM
Content Knowledge According to Mishra & Koehler (2009), content knowledge is actually the content of the subject to be taught or learned. This includes knowledge of proofs, scientific facts and theories. In this study, content knowledge is mathematics teachers' knowledge on moving shapes from one position to another on the x-y plane, as well as fully describing the type of transformation.	Knowledge about transformation relates to performing and describing different types of transformation. (a) Knowledge on how to find the line of reflection, translation vector, centre of enlargement, scale factor in enlargement, centre of rotation, angle of rotation and direction, invariant line and scale factor of stretch and shear. (b) Knowledge on how to perform different types of transformation: (i) Use of the given mirror line to reflect a shape (ii) Use of the given translation vector to translate a shape (iii) Use of the given centre, angle and direction to rotate shape (iv) Use of the given centre and scale factor to enlarge a shape (v) Use of the given invariant line and scale factor to apply either shear or stretch
Technological Knowledge Technological knowledge is knowledge about different technologies such as digital technology, the internet, interactive whiteboard and digital video (Schmidt *et al.,* 2009). Koehler and Mishra (2009) point out that technological knowledge involves a teacher's ability to use hardware and software to enhance learners' learning experiences. However, they also argue that technology keeps changing, meaning that new technology will become old technology in few years.	Knowledge about different technologies, including hardware and software which the teachers have been trained to use. (a) Knowledge of using ICT tools such as Google Docs and YouTube videos internet to teach transformation (b) Knowledge on how to use digital technologies, computer software, cell phone applications and IPads in transformation
Pedagogical Knowledge In general, this knowledge can be referred to as knowledge of different teaching methods and assessment, which may involve teaching theories (Koehler & Mishra, 2005; 2009).	Teaching methods in transformation (a) Knowledge of involving learners to construct new knowledge and understanding (b) Knowledge of helping learners to discover concepts of transformation

Shulman (1987) is of the view that pedagogical knowledge includes methods of teaching, curriculum objectives, learner characteristics, classroom management, lesson plan development and implementation, and methods of evaluating learners` understanding.	
Pedagogical Content Knowledge Pedagogical Content Knowledge consists of content knowledge and pedagogical knowledge for the purpose of teaching specific content (Schmidt *et al*., 2009). Koehler and Mishra (2009) adopted Shulman's (1987) idea and describe pedagogical content knowledge as the ability of a teacher to interpret subject matter and find different ways of presenting it with the use of teaching aids, while taking into consideration the learners' prerequisite knowledge. In this study, Pedagogical Content Knowledge refers to the ability of a teacher to integrate teaching methods with the content knowledge of teaching transformation in order to effectively meet learners` needs (Schmidt *et al.,* 2009).	The knowledge of a mathematics teacher to present the content of transformation in ways that learners successfully learn transformation concepts. (a) Knowledge on how to use specific strategies or approaches for teaching learners how to perform different types of transformation, and how to describe each type of transformation fully. (b) Knowledge on how to address the way that learners think about how to perform different types of transformation and how to describe each type of transformation. (c) Knowledge on how to identify and address learners` misconceptions in the learning of transformation. (d) Knowledge on how to meet the curriculum objectives in the teaching of transformation. (e) Identifies that the learners are able to notice that for reflection, translation and rotation the object and image are always congruent.
Technological Pedagogical Knowledge Technological Pedagogical Knowledge (TPK) is knowledge about how different technologies can be used to teach with the understanding that technology can enhance teaching (Schmidt *et al*., 2009). A teacher should be in a position to choose relevant technology for teaching any given content (Koehler & Mishra, 2009). TPK is the knowledge of general pedagogical activities which can be used with the engagement of emerging technologies (Cox & Graham, 2009). It is important for a teacher to have TPK simply because there are many technological devices	Knowledge of how different technologies can be used appropriately in teaching. (a) Knowledge on how to use ICT tools or digital technologies to help learners to perform different types of transformations and to describe them fully (b) Knowledge of helping learners to construct shapes accurately and to transform them using technology (c) Knowledge of creating more than one example to teach learners to use digital technologies to perform different types of transformation.

and software which have not been designed for educational purposes (Koehler & Mishra, 2009).	
Technological Content Knowledge Technological Content Knowledge (TCK) is the knowledge and understanding of teachers on how to use relevant technology to present specific content (Koehler & Mishra, 2009). Mishra & Koehler (2006) explain that teachers who possess TCK do not only understand the content of a subject, but also how to use technology to present that content. On the one hand technology can be used to present content, while on the other hand content may be used to teach technology (Mishra & Koehler, 2006). For this study, TCK is a component of TPACK, and mathematics teachers are expected to display the competency of choosing a relevant technology that enhances student learning of transformation (Mishra & Koehler, 2006).	Knowledge on how to select a specific technology relevant for teaching a subject matter which they have been trained for. (a) Knowledge on how to use basic skills of computer software to drag figures, label points, measure lengths and construct different shapes. (b) Knowledge on how to use ICT tools or computer software to perform reflection, translation, rotation, enlargement, shear and stretch. (c) Knowledge of selecting relevant digital technologies to find (i) Centre, angle and direction in rotation (ii) Centre and scale factor of enlargement (iii) Line of reflection (iv) Translation vector (v) Invariant line and scale factor of shear and stretch. (d) Knowledge of accurately constructing different shapes using technological devices.
Technological Pedagogical Content Knowledge Technological Pedagogical Content Knowledge is the basis. The effective use of technology in teaching involves the understanding of concepts using technology, the use of pedagogical techniques which include technology for teaching content, and the knowledge of how technology can be helpful to learners for building new information onto their existing knowledge (Mishra & Koehler, 2006). According to Mishra and Koehler (2006), the TPACK model requires teachers to have developed a detailed and flexible	Knowledge which enables teachers to integrate transformation content, pedagogy and technology in the teaching and learning of transformation. (a) Knowledge of integrating technological content knowledge, pedagogical content knowledge and technological pedagogical knowledge in the teaching of transformation (b) Can designs an appropriate activity with the technology to assist learners in learning concepts of transformation.

knowledge of all the components and be in a position to use appropriate technology in the teaching processes.	

3.3 SUMMARY

My main focus on this chapter was on the TPACK framework used in this study. The purpose of this study was to explore the TPACK of mathematics teachers when teaching transformation with the involvement of technology. This made the TPACK framework very useful for this study. The TPACK framework has seven constructs and these were defined and described in this chapter, with an emphasis on their relevance when transformation is taught in Grade 12 using technology.

CHAPTER 4:
RESEARCH METHODOLOGY

4.1 INTRODUCTION

The methodology used for this study to explore the technological pedagogical content knowledge of mathematics teachers when teaching transformation in Grade 12 is provided in this chapter. It also outlines the research design and approaches, the selection of the participants, the instruments used for the study, the procedure for collecting data as well as how the data was anlysed. Ethical considerations pertaining to this study is also presented.

4.2 PARADIGM

Maree (2016) explains that a research paradigm may be catergorised as postpositivist, interpretivist, critical theory and so on. Moreover, Bertram and Christiansen (2017) point out that a paradigm can determine the choices which the researcher make, such as the type of questions to be asked, what to observe or investigate, how to collect data and finally how to interpret the findings. An interpretivist paradigm is different from a positivist paradigm, for which knowledge is considered to be objective, and also different from a critical paradigm, in which a researcher holds the belief that the world is characterised by unequal power relations (Fraenkel, Wallen & Hyun, 2012; Gray, 2009).

This study is informed by the interpretivism paradigm. With interpretivism, the researcher makes interpretations with the purpose of understanding human perceptions. In addition, in an interpretivist paradigm, knowledge is constructed (Gray, 2009). In this study teachers show their TPACK into practice as they integrate technology in the teaching of transformation. This is also based on view of Mishra and Koehler (2006) that TPACK is socially developed, and therefore flexible when used. Bertram and Christiansen (2017) emphasise that interpretivism can be used to inform a multiple case study approach, in which case observations and interview instruments can be used to collect data. This study used observations and interviews to collect data which was later interpreted. Interpretive paradigm is underpinned by observation and interpretation of collected information about events (Maree, 2016). For these reasons an interpretive paradigm is relevant in this study.

4.3 RESEARCH APPROACH

A large quantity of literature distinguishes the qualitative and quantitative approaches. A researcher has access to this literature and can make a decision regarding the approach to be used with consideration of the phenomenon under study. According to McMillan & Schumacher (2010), qualitative research is usually based on description and exploration of the phenomenon of interest. The main focus is how people perceive and understand the way things are done in the world. The use of a qualitative approach was relevant in this study as the researcher was examining the knowledge of mathematics teachers when using technology for the teaching and learning of transformation in mathematics. The rationale to adopt a qualitative approach was based on the main purpose of the study, namely to explore the way in which mathematics teachers display their TPACK when they teach transformation to Grade 12 learners. Qualitative research helps a researcher to come up with the embedded meaning from the NEPAD e-school mathematics teachers in their natural setting when they display their TPACK into practice.

4.4 RESEARCH DESIGN

Based on the philosophy which underpins a research project, a case study may either be exploratory, descriptive or explanatory (Maree, 2016). An exploratory case study was used in this study because it helped to understand teachers` views and perceptions on the teaching of transformation using technology. In order to explore Grade 12 mathematics teachers` technological pedagogical content knowledge when teaching transformation with the use of technology, a multiple case study research design was used. This multiple case study focused on schools in which the NEPAD e-Schools project was piloted. The cases of this study were the NEPAD e-school project with their experienced mathematics teachers on the use of technology on teaching. The use of multiple case studies was helpful in indicating similar or contracting results. In multiple case studies as methodology it is possible to replicate a case under review in one study. Stake (2005) asserts that multiple case study as methodology helps a researcher to contextualise experiences and the systematic analysis of data. Multiple case studies allow the researcher to identify, describe and examine each participant's TPACK.

4.5 DATA COLLECTION INSTRUMENTS

4.5.1 Observation

The observation schedule was designed and developed by the researcher (see Appendix G).The observation schedule was catergorised using the seven constructs as set out by Koehler and Mishra (2005), which are as follows: content knowledge, pedagogical knowledge, technological knowledge, technological content knowledge, pedagogical content knowledge, technological pedagogical knowledge, and technological pedagogical content knowledge. On the observation manual there was a box next to each construct, and the researcher could make a check mark next to it when the construct was displayed during the observation. The observation schedule had three categories which are bad, good and excellent next to each construct of the TPACK. The researcher would then tick bad if the teacher cannot integrate technology with the teaching of transformation or good when the teacher is able to integrate some of the seven constructs of the TPACK in teaching transformation. The researcher would tick excellent when the teacher is able to integrate all the seven constructs of TPACK in the teaching of transformation. There was a space below each construct where the researcher could make notes regarding (a) how mathematics teachers use technology to teach transformation, (b) how mathematics teachers use technology to address leaners` challenges in learning transformation, and (c) how mathematics teachers use TPACK to improve learners` knowledge on transformation. The instruments were piloted in order to ensure their trustworthiness. The individual semi-structured interviews were used to further explore the TPACK displayed during the observations. A camera was used after the lesson observations to capture the activities used during the lesson, after which these images were saved to a computer. The camera was only used to capture the activities, and no photos were taken of the learners or the participants.

Observations and individual semi-structured interviews were used as data collection instruments in this study. Three teachers were observed when teaching learners to perform different types of transformation using technology in order to examine the TPACK in the teaching of transformation.The observation schedule was designed by the researcher (see Appendix G). Multiple data collection techniques have the benefit of enabling triangulation which improves the validity of the data. Maree (2016) explains that triangulation can be used to validate the findings. The researcher designed the interview protocol and conducted the interviews after observing the lessons, as this helped to further explore the teachers' TPACK as displayed when they were teaching transformation using technology.

Qualitative research may consist of four types of observations, which are complete participant, complete observer, observer as a participant and participant as observer (Maree, 2016). This study used the observer as participant type of observation, because as Nieuwenhuis (2007) points out, this allows the researcher to focus on his/her observation without disturbing or influencing the situation. Moreover, the semi-structured observation was used suitably in this study. As Maree (2016) points out, it uses the predetermined categories of behaviour which the researcher had set out to observe.

Observations of the lessons took place, and these were also audio recorded. Screen captures were also done of what was displayed on the computer when different types of transformation were performed. The images of every activity which were displayed on the white board were copied from the teachers' computers after the class, as these had already been saved there by the teachers. The audio recordings from the classroom observations were supplemented with notes from the classroom observation schedule. The structured observation protocol for this study was designed by the researcher, and it was used to record all the observed lessons. The audio was used to back up the notes taken during the observation to be transcribed later while the camera was used take the pictures of the displayed activities. Doing this at the same time would distract the teaching as the camera was used at the end of the lesson. After the observations, the interviews were conducted, focusing on teachers' knowledge and their experiences when using technology to teach transformation.

In order to collect data, teachers were observed while they were using technology when teaching transformation. Screen captures of the images used in class was also done. The use of screen capturing in this study is relevant, because it allows the researcher to track the actual activity when teaching (McDougall & Karadag, 2008). The observation protocol was designed in such a way that it would cater for mathematics teachers when teaching leaners how to perform the six types of transformation which are shear, stretch, enlargement, rotation, translation and reflection using technology.

4.5.2 Individual semi-structured interviews

The second data collection tool was individual semi-structured interviews, conducted with three teachers. During the interviews, the conversations were recorded and transcribed later. The researcher developed the interview protocol used in this study (see Appendix H). The research questions of the study were used to inform the design of the instruments, both the semi-structured interviews and observations. They were also aligned with the constructs of the TPACK framework. The interview schedule consisted of ten questions, were developed on the basis of Mishra and Koehler's (2006) seven constructs.The interviews were conducted after the lesson observations had taken place. This helped the researcher to further explore the teachers` TPACK as displayed during the teaching and learning of transformation. The observations and interviews instruments helped the researcher to explore the teachers` TPACK as displayed during the teaching and learning of transformation. The instruments were appropriate because they were aligned with the TPACK model.

4.6 POPULATION AND SAMPLING

Maree (2016) points out that a researcher usually has a specific purpose in mind, and will use non-probability sampling to select the participants to answer the research questions of the study. The target population for this study was Grade 12 mathematics teachers from the NEPAD e-Schools Project in Maseru, Lesotho. The population for this study consisted of 20 mathematics teachers in the NEPAD e-Schools Project. Three Grade 12 mathematics teachers were selected (two teachers from one high school and one from another high school) on the basis that they had experience in using technology to teach transformation.

The three participants of this study were selected using purposive sampling.
MacMillan (2012) points out that with sampling method a researcher select participants based on their perceived knowledge on a particular topic. They were selected on the basis of their experience of more than five years of teaching mathematics using technology in the NEPAD e-School project. They were also selected on the basis that they have been trained on how to use technology to teach mathematics. Their names (pseudonyms) are Tima and Molao. The researcher selected one mathematics teacher from another school, whose expertise shows not only a high quality of teaching mathematics but also a high

degree of integrating technology in mathematics. Her name (pseudonym) is Mpona. The two schools have the needed resources for teaching with technology.

4.7 RESEARCH SITE

For the purpose of this study the schools have been labeled as school A and school B. School A is a high school situated in the south of Maseru District with a learner number of approximately 340. All of its learners are day scholars. The school offers mathematics from Grade 8 to Grade 12. The school consists of four buildings - one for the school office and staff room, another for six classrooms, one for a science laboratory and the last for five classrooms. The observations were done in the classroom and the teacher was expected to bring his/her own technology devices along if planning to use them. The teachers were observed while teaching, and a recording was also made of the lesson. Screen captures were also made of every activity on the transformation of shapes, which was transferred to the researcher's computer after the class.

School B is a high school situated in town with a learner population of approximately 1 200. It has a boarding school for both genders and also accommodates day scholars. This school has a number of classrooms, a mathematics lab, science labs, staff rooms, offices, a computer lab and a library. The school also has a hall which is used for school purposes and can also be rented from the school for external events and functions. The school offers classes from Grade 8 to Grade 12. There are seven granted teachers in this school who teach mathematics. Most of them teach two subjects - either mathematics and physics, or mathematics and chemistry, or mathematics and geography. This school was one of the piloting schools in the NEPAD e-Schools project in Lesotho. The observation at this school was done in the mathematics lab, which was already equipped with a data projector connected to a laptop with mathematical software designed for teaching mathematics.

4.8 ETHICAL CONSIDERATIONS

4.8.1 Ethical clearance

It is important that the researcher fully understands his legal and ethical responsibility for conducting research, especially when humans are involved. This study adhered to ethical guidelines to protect and respect the participants. The researcher observed the following ethical aspects while conducting the research: permission, informed consent, confidentiality and anonymity, and protection from harm.

4.8.2 Permission

Before any data was collected for this study, approval of ethical clearance was obtained from the Ethics Committee of the Faculty of Education at the University of the Free State and the Ethical Clearance number was UFS-IISD2019/2507 (see Appendix A). The proposal, information letters and consent forms were also checked by this committee according to the standard regulations of the University. The Ethics Committee ensures that the participants of the study have relevant and detailed information before data is collected, and also that participants will be safe, treated with respect and dealt with in the spirit of justice and truthfulness. Immediately after receiving the ethical clearance (see Appendix B), a letter was sent to the Department of Education in Lesotho (DEL) to ask for permission to conduct research in schools. After permission was granted by the DEL (see Appendix D), permission was also sought from the principals of the identified high schools (see Appendix C).

4.8.3 Informed consent

Prior to the commencement of data collection, the participants were informed about their rights, the objective of the study and the benefits of participation (see Appendix E). They were given informed consent letter (see Appendix F) and were asked to voluntarily participate in this study. They were further made aware of their right to withdraw at any time without any fear. The participants were asked to sign a consent form which emphasised that the study was not evaluating them, and that the findings and conclusions would not be used against them. The participants were informed that screen captures would be done, and that no photos would be taken of them or their learners.

4.8.4 Right for privacy

It was anticipated that sensitive information might become known during the classroom observations and the interviews. Example of sensitive information could be the marks of learners, misbehavior of learners and teachers` weakness on the management of learners. The participants were fully informed that any such information revealed by the research project would only be used for the research project, and not for anything else. This gave the participants the assurance that they could be free to disclose any information about themselves regarding the teaching of transformation using technology. Furthermore, this was also to ensure that the participants would teach as usual without fear of embarrassment.

4.8.5 Protection and harm

After the data was collected, the findings were reviewed with the purpose of ensuring that the teaching and human experience was not undervalued. The researcher tried to make sure that the participants were protected from harm, such as physical harm, social disadvantage and psychological distress and discomfort. This was achieved by obtaining informed consent from the participants, maintaining honesty, avoiding deceptive practices, and maintaining sympathy and sensitivity and respect throughout the study.

4.8.6 Confidentiality and anonymity

The researcher maintained confidentiality and anonymity before and throughout the study. For confidentiality and anonymity, codes and pseudonyms were used.

In order to maintain the confidentiality of the participating schools, they were labeled School A and School B. Pseudonyms were used for all three participants, namely Tima, Molao and Mpona.

4.9 CHAPTER SUMMARY

This chapter presented the methodological approach for investigating the mathematics teachers` technological pedagogical content knowledge for teaching transformation in Grade 12. The chapter also elaborated on the research design, data collection procedures and data analysis. This explanatory multiple case study was conducted in an urban area of Maseru, Lesotho.

CHAPTER 5:
DATA ANALYSIS, INTERPRETATION AND PRESENTATION

5.1 INTODUCTION

The main focus of this study was to explore the technological pedagogical content knowledge of mathematics teachers teaching Grade 12 transformation. Data was collected from the participants in their natural setting, using observations and interviews as collection instruments. The collected data was then analysed and organised to develop themes. This chapter analyses, interprets and presents the results from the class observations and interview data.

5.2 DATA ANALYSIS

The data was collected using classroom observations and interviews as instruments. This use of a variety of instruments for data collection made it possible to enhance the trustworthiness of the findings. Triangulation was employed to cross check the data in order to draw reliable conclusions. The most important triangulation was between classroom observations and teacher interviews. The audio recordings of the classroom observations were supplemented with notes from the classroom observations. The process of transcription focused on conveying the verbal content of speech in order to capture the information shared during the conversations (Oliver, Serovich & Mason, 2005), while also recording nuance of accentuation.

The audio recordings of classroom observations and interviews were transcribed. After listening to the recordings and reading the transcriptions several times, relevant extracts were coded based on the sub research questions. Each participant teacher's set of classroom observation notes and interview transcript were analysed separately. The process of thematic analysis was used to analyse the data. This process goes through the six phases, namely familiarisation with the data, the generation of initial codes, searching of themes, reviewing of potential themes, defining and naming themes, and finally producing a report (Mortensen, 2019).

This study used thematic analysis when analysing the qualitative data. Braun and Clarke (2006) point out that thematic analysis has the benefit of helping to examine data from different participants in order to come up with similarities and differences. Data from each participant was coded as use of technology to teach transformation, use of technology to address learners` challenges in transformation, and improving learners' knowledge of transformation using TPACK. Each code was compared with the one under each participant in the same category in order to find common patterns and differences in the data. The codes were used to categorise all the collected data. The themes then emerged from the categories.

Thematic analysis was used to find the relationship between the collected data from observations and individual semi-structured interviews. Mortensen (2019) explains that the emerged themes can be developed from the patterns of the coded data across different interviews .First, data was coded using three sub-research questions. Secondly, the codes were then used to categorise all the collected data so at to develop the themes. The themes emerged from a group of categories which were under the same code. This process was done as per the collected data from the observations and interviews.

5.3 DATA PRESENTATION

Data is presented in the order that it was collected as per the instrument used. It starts with data collected from the observations, followed by data collected through the individual interviews. The themes are used to present the data in a well-organised manner. The pseudonyms the researcher has assigned to the three participants are Tima, Molao and Mpona.

5.4 OBSERVATIONS

Data from the observations was coded, catergorised and themes emerged as show in tables 5.1, for Tima, Molao and Mpona.

Table 5.1 : Emergent themes and categories 1

Codes	Categories	Emergent themes
Use of technology to teach transformation	• Technological teaching resources used in the lesson • Ability to use technology and pedagogy for content teaching	Use of technological pedagogical content knowledge when teaching transformation to Grade 12 learners
Use of technology to address learners` challenges in transformation	• The use of the selected software in the teaching and learning of transformation • The use of software to enhance visualisation	Use of relevant technology to address learners` challenges in the teaching and learning of transformation in Grade 12
Improving learners` knowledge of transformation using TPACK	• Teachers' and learners' activities • Engagement of learners with technology	Inability to use technology to improve learners` knowledge of transformation in Grade 12

Tima`s profile

Tima is a male mathematics teacher from School A, aged between 30 and 35. At the time of data collection, Tima was teaching mathematics and physics from Grade 8 to Grade 12. He had seven years of teaching experience teaching mathematics using technology at School A. Tima had taught Grade 12 learners for seven years in the NEPAD e-Schools. The researcher observed Tima during his class while he was teaching transformation to Grade 12 learners, specifically the sub-section reflection.His highest teaching qualification is a B.Sc.Ed. in Mathematics Education and Physics Education.

5.4.1 Theme 1: Use of technological pedagogical content knowledge when teaching transformation to Grade 12 leaners

The collected data from the observations revealed that Tima uses technological teaching resources when teaching reflection to Grade 12 learners. The observed use of technological pedagogical content knowledge is discussed below.

(a) Technological teaching resources used in the lesson

Tima was observed using a laptop and overhead projector as his technological teaching resources in teaching reflection to Grade 12 learners. He used no other teaching resources. He used the software GeoGebra to teach the concept of reflection by reflecting the shape he had drawn along $x - axis$ and also along $x = -2$.

The overhead projector was used to project the activities on the reflection of triangles using GeoGebra. Tima did three activities on the reflection of triangles. The first activity was the reflection of a triangle along the mirror line $x = -2$, then $x - axis$ and finally along the line $y = 2x + 3$. The activities of reflecting a triangle along $x - axis$ and also along $x = -2$ were projected on the screen.

Tima`s teaching activities with the use of GeoGebra were as follows:

He first used the GeoGebra software to draw a triangle which the software labeled ABC and then reflected it along the $x - axis$ and the software labeled the image triangle $A'\ B'C'$.
After that he explained to the learners:
"The type of transformation I have performed is reflection and the triangle $A'\ B'C'$ is the image of triangle ABC under reflection".

Figure 5.1 shows the triangle ABC and another triangle $A'\ B'C'$ as its image after the reflection along $x - axis$.This activity was done on the laptop and at the same time projected onto the screen using the overhead projector so that the learners could see what the teacher was doing.

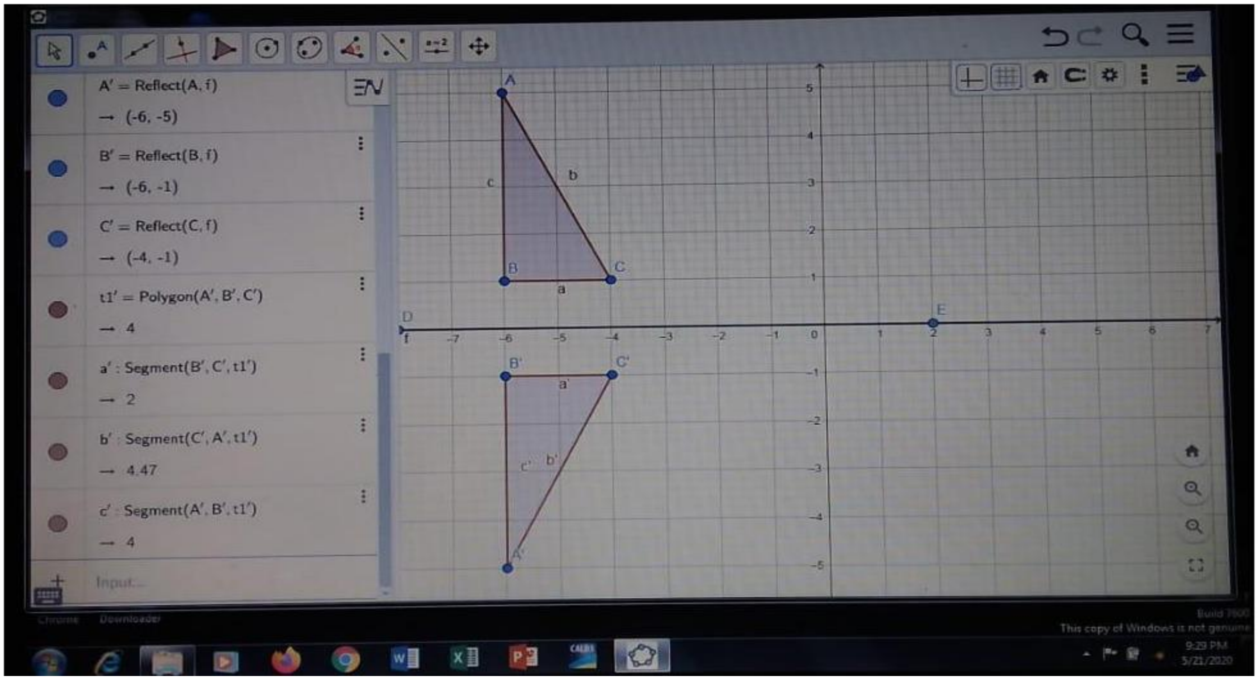

Figure 5.1: Reflection of triangle ABC along x-axis

Tima further used GeoGebra to reflect the same triangle ABC along the line $x = -2$ as shown on figure 5.2. The triangle ABC was further reflected along the line $x = -2$ as to show leaners that the same shape can be reflected to different positions using different lines of reflection. This activity was also projected for the leaners to observe the teacher`s actions. The learners were observing and listen when Tima was reflecting these shapes along different mirror lines.

The use of GeoGebra software to teach reflection helped Tima with the labeling of the points/vertices of the object at the same time with the corresponding points and sides of the image for the learners to identify the properties of shapes in reflection.

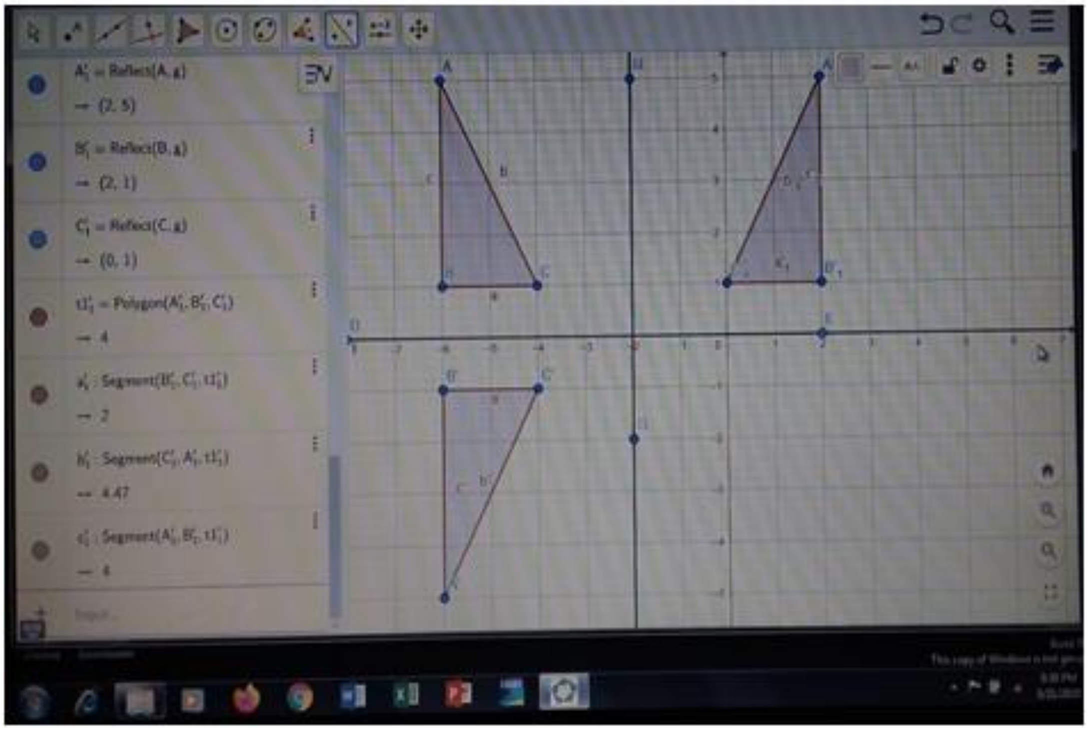

Figure 5.2: Reflection of triangle ABC along x-axis and x=-2.

Tima used GeoGebra installed on the laptop to teach combined transformation of the same type of transformation. He first reflected triangle ABC along $x - axis$ and further performed a combined transformation by reflecting the image $A'\,B'C'$ along the line $y = 2x + 3$ as shown in Figure 5.3. The overhead projector was used to project these actions on the screen.

The use of GeoGebra in Tima`s lesson in the teaching of reflection helped him to reflect the same triangle using different mirror lines faster than would have been able using the traditional way of teaching. This approach helped the learners to observe and identify the properties of objects and images which do not change, although the lines of reflection are different.

Figure 5.3 shows the reflection of the image triangle $A'\,B'C'$ along the line $y = 2x + 3$.

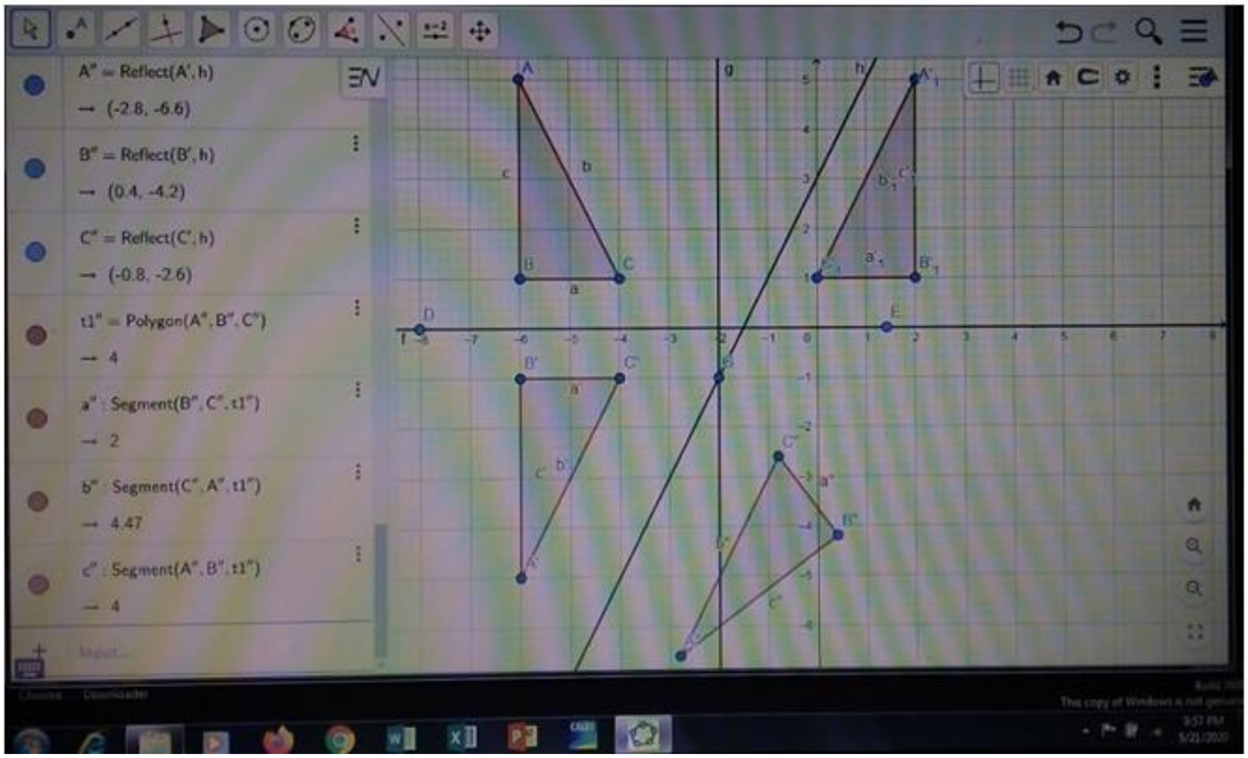

Figure 5.3: Reflection of triangle ABC along x-axis, x=-2 and y=2x+3.

Tima displayed content knowledge, technological knowledge, technological pedagogical knowledge and technological content knowledge in the presentation of this lesson. Tima reflected the triangles along the lines $x - axis, x = -2$ and $y = 2x + 3$ and this is the correct reflection under transformation. Tima used GeoGebra to reflect the triangles in the lines $x - axis, x = -2$ and $y = 2x + 3$, and this was a good display of technological content knowledge. The aspect of pedagogical content knowledge was, however, missing, because Tima used a teacher-centred approach as opposed to a learner-centred approach in his lesson presentation. The inference is that Tima had limited technological pedagogical content knowledge when teaching reflection.

(b) Ability to use technology and pedagogy for content teaching

Tima was observed linking the features of GeoGebra with the concepts of reflection when teaching Grade 12 learners. When Tima used GeoGebra to draw a triangle, the software automatically labeled the triangle ABC. He then used GeoGebra to select the line of reflection as the $x - axis$ and the type of transformation he was going to do - in this case reflection. After selecting reflection he highlighted triangle ABC and also clicked on the line of reflection $x - axis$ and the triangle was reflected. When triangle ABC was reflected its image was labeled as $A'\ B'C'$ by GeoGebra.

Tima then asked the learners what they noticed about the object and its image.
One of the learners responded:
The object and image are congruent.
Tima further explained to the learners that:
They are not only congruent but they also have the same distance from the mirror line.
He continued to use GeoGebra to reflect the same triangle ABC about the line $x = -2$ and asked the learners to identify and give the mirror line.

One of the learners responded: *It is a vertical line*.

This answer was not correct and he gave another learner the opportunity to respond.

The second learner gave the answer as $x = -2$ and she was correct.

He further asked the learners what they noticed about the object and its image.

One of the learners responded:

The object and the image have the same distance from the mirror line.

This learner was able to provide the correct answer.

Tima then asked learners to choose a shape they would like to reflect, and one of the learners chose triangle $A'\ B'C'$ as a shape to be reflected.

Tima then used GeoGebra to reflect triangle $A'\ B'C'$ along the line $y = 2x + 3$.

Tima used the GeoGebra software as a teaching strategy to quickly reflect the same shape using different mirror lines. This made it easy for the learners to notice that with reflection the object and image are congruent and have the same distance from the mirror line.

The use of technology and pedagogy for teaching reflection enabled Tima to display content knowledge, technological knowledge, technological content knowledge and technological pedagogical knowledge. The reflection of triangles in the lines $x - axis, x = -2$ and $y = 2x + 3$ was a display of content knowledge, together with the explanation that in reflection the object and the image are congruent. The use of GeoGebra to reflect the triangles along different mirror lines was enough evidence of technological content knowledge. Tima's use of GeoGebra to teach learners how to reflect shapes in the given mirror lines which was a display of technology pedagogical knowledge. The aspect of pedagogical knowledge was however missing, as he did not involve learners in using GeoGebra themselves to learn how to reflect shapes on their own.

5.4.2 Theme 2: Use of relevant technology to address learners` challenges in the teaching of transformation in Grade 12

The data from the observations revealed that teachers use relevant technology to address learners` challenges when teaching transformation to Grade 12 learners. This is discussed below.

(a) **The use of the selected software in the teaching and learning of transformation**

Tima chose to use GeoGebra as software relevant to the teaching of reflection as type of transformation to address learners` challenges. After opening GeoGebra from his laptop, Tima clicked one of the icons from GeoGebra and then a list of different types of transformation appeared. Reflection, translation and rotation were some of the listed options. He then selected the type of transformation, in this case reflection. He used GeoGebra to draw a triangle, which the software labeled ABC. He projected this activity so that all learners could see. The specific software was relevant as it had features which were already built in for the teaching of different types of transformation.

Tima used GeoGebra to teach his learners how to reflect triangle ABCalong $x - axis$,$x = -2$, and also how to reflect triangle $A'\ B'C'$ along $y = 2x + 3$. He further used GeoGebra to reflect triangle ABC along the line $x - axis$ and GeoGebra labeled the image triangle $A'\ B'C'$.

Tima asked the learners: *What do you notice about the object and the image?*

One of the learners responded: *The object and the image are the same*.

Tima further asked: *Besides being the same what else can you say?*

One of the learners responded: *They are congruent.*

This leaner was able to provide with a correct answer.

Challenge: Then one of the learners raised a hand and said: *I cannot understand how they are congruent.*

Tima then asked the learners to look at the sides of the object and image which were labeled with the same letters.

Then all the learners looked at the screen to identify the corresponding sides.

One of the learners raised a hand to give the answer.

This learner responded: *The sides which are labeled with the same letters are equal*.

Tima explained that those sides which GeoGebra had labeled with the same letters were the corresponding sides of the object and image, and since they were all equal the two shapes were congruent.

Addressing the challenge with the help of GeoGebra:

Since GeoGebra had labeled the corresponding sides of the object and image with the same letters, it was easy for the learners to identify the corresponding sides. They were also able to find that those corresponding sides were, equal which proved that they were congruent.

Tima explained: *They are not only congruent but they are the same distance from the mirror line which is $x - axis$ in this case.*

He emphasised this concept by helping learners to count the distance of point A from $x - axis$ and that of the corresponding point A' from $x - axis$. They found it to be 5 and it was the same for both points. He further asked the learners to see if they could find the same distance for the other points.

One of the learners responded: *The distance for points B and B' from the mirror line $x - axis$ is the same and it is 1.*

The answer provided by the learner was correct.

Again the use of GeoGebra helped to **address a challenge** of identifying the corresponding vertices as they were labeled.

Tima then saved this first activity, as GeoGebra allows this.

Tima then reflected triangle *ABC* along $x = -2$.

He then asked the learners to give the mirror line used to reflect the first shape.

One of the learners responded correctly and said: The line of reflection is $x = -2$.

Tima was also observed saving this second activity.

From here Tima asked the learners to choose a shape to be reflected.

One of the leaners chose triangle $A'\,B'C'$.

Then Tima reflected triangle $A'\,B'C'$ along the line $y = 2x + 3$.

Figure 5.4 shows the reflection of triangle *ABC* along $x - axis$ and also along $x = -2.$ It also shows the reflection of triangle $A'\ B'C'$ along $y = 2x + 3.$

Tima was able to reflect many shapes using GeoGebra within a short time. This allowed the learners to observe different ways of reflecting a shape. The relevance of the software is that it has features which are already built in for the teaching of different types of transformation. Tima displayed technological content knowledge by choosing relevant technology in this particular lesson for the teaching of reflection.

The challenges that learners faced in Tima`s lesson on the reflection of shapes, as explained above, were the identification of congruent shapes, identification of the corresponding sides and finding the mirror line. Tima addressed these challenges by making the learners aware that the sides of the object and image, labeled with the same letters by GeoGebra, are the corresponding sides of the object and image. Tima further addressed the challenge by explaining that those corresponding sides are equal. The learners verified this by counting their distances. The use of GeoGebra also helped learners to quickly identify the corresponding points of the object and image, because they were also labeled with the same letters (A corresponding to A'). Tima was able to address leaners` challenge of identifying the corresponding points and sides of the object and image because GeoGebra was able to label the points of reflected shape immediately after being reflected. This helped learners to notice that the two shapes under reflection were congruent, and they were the same distance from the mirror line.

Tima used GeoGebra to teach reflection. This software has features which enable the reflection of objects to obtain their images, which makes it relevant to the teaching of mathematics. The concepts of reflection were presented in different ways using GeoGebra. This allowed the learners to understand the lesson better. By doing this he displayed the knowledge of selecting a specific technology relevant to teach reflection, which is technological content knowledge.

He was able to interpret the concepts of reflection, and presented them in different ways in order for the learners to understand it better, which was a fair display of technological pedagogical knowledge. The choice of relevant technology, which was GeoGebra, also helped Tima to display technological pedagogical knowledge by involving learners thorough oral questions in which learners were able to prove that the reflected shape was congruent to the original object.

***(b)* The use of software to enhance visualisation**

Tima used GeoGebra for the visualisaton of the processes of reflection in transformation. He drew a triangle which he then reflected. GeoGebra labeled the vertices with capital letters (A, B, C) and the sides with small letters (a, b, c). Tima further used the software to draw the line of reflection, which was $x = -2$. When performing reflection, Tima selected reflection from a list of options in GeoGebra, highlighted the triangle and the line of reflection, and the triangle was reflected. From this activity learners were able to see the reflected object.

After performing reflection, GeoGebra labeled the corresponding vertices of the image triangle $A'B'C'$ and the corresponding sides $a'b'c'$.

This helped learners to observe that the following vertices and sides were corresponding:

Vertices		**Sides**	
A	$\longrightarrow A'$	a	$\longrightarrow a'$
B	$\longrightarrow B'$	b	$\longrightarrow b'$
C	$\longrightarrow C'$	c	$\longrightarrow c'$

Tima also used GeoGebra to reflect triangle $A'\,B'C'$ along the line $y = 2x + 3$.

After that Tima asked the learners:

What do you notice about the object and the image?

One of the learners responded: *The object and the image are the same*.

Tima further asked: *Besides being the same what else can you say?*

One of the learners said that they were congruent.

Challenge of visualising congruent shapes:

Some of the learners (about seven of them) then raised their hands and said: *We cannot understand how they are congruent.*

Addressing the challenge of visualising congruent shapes:

Tima then asked the learners to look at the sides of the object and image which were labeled with the same letters.

Then all the learners looked at the screen to identify the corresponding sides.

One of the learners raised his hand to give the answer, and said: *The sides which are labeled with the same letters are equal*.

Tima explained that the sides which GeoGebra has labeled with the same letters, were the corresponding sides of the object and image, and since they were all equal, the two shapes were congruent.

Use of GeoGebra for enhancing visualisation:

Since GeoGebra labeled the corresponding sides of the object and image with the same letters, it was easy for the learners to identify the corresponding sides. They were also able to determine that those corresponding sides were equal, which proved that the two shapes were congruent.

From this observation it can be noticed that learners were able to realise that the two shapes were the same, but struggled to identify that the two shapes were congruent. The visualisation of congruent shapes was enhanced by the labeling of the corresponding sides of the object and image by GeoGebra. The use of GeoGebra helped to enhance the visualisation of the learners as it is normally challenging to identify the corresponding sides and angles when they are not fully labeled. The process of reflecting the triangle ABC using GeoGbra enhanced the visualisation of the learners because they were able to see how it was reflected.

Tima used GeoGebra to address leaners` challenge of visualisation. This was achieved because leaners were able to observe when the shape was reflected. Moreover, leaners were later able to identify the corresponding sides and points as GeoGebra labeled both the corresponding sides and points of the object and image after reflection.

The labeling of the corresponding sides and angles of the object and image by GeoGebra helped the learners to visualise that the object and image were congruent after the reflection. The use of GeoGebra to address the learners` challenge of visualisation shows that Tima was familiar with the background knowledge of the learners, and decided to address them. This is an indication of pedagogical content knowledge. Since Tima used GeoGebra as technology to address learners` challenges in learning reflection, it means he displayed technological pedagogical knowledge as well.

5.4.3 Theme 3: Inability to use technology to improve learners` knowledge of transformation in Grade 12

The data from the observations revealed that mathematics teachers were unable to use technology to improve learners` knowledge of transformation in Grade 12. The teaching strategies of using technology to improve learners` knowledge of transformation are discussed below.

(a) Teachers’ and learners’ activities

Most of the teaching and learning activities in Tima`s lesson were limited to a demonstration of using GeoGebra to perform reflection. The learners were observing while Tima used GeoGebra to reflect shapes on the laptop. There were no activities for the learners to get a better understanding of the concept of reflection.

Tima demonstrated to the learners how to use GeoGebra to draw a polygon, which in this case it was a triangle. Tima further demonstrated to the learners how to reflect a triangle along the lines $x = -2$, $x - axis$ and $y = 2x + 3$ with the aid of GeoGebra.

These activities were projected on the screen using the overhead projector so that all the leaners could see what was happening. The learners only participated in the demonstration by answering oral questions. He further involved the learners in the lesson by asking them about the relationship between the object and its image, which was drawn using technology. The learners were able to tell that the two shapes were congruent, which is one of the properties of reflected shapes. In this way Tima used a technological tool in the discussion of his lesson. The teaching and learning activities demonstrated by Tima are shown in Figure 5.3., but did not give learners the opportunity to gain more knowledge on the learning of reflection.

Most of the activities from this lesson were focused on the teacher, as most of the activities were done by Tima while the learners were observing. Tima used GeoGebra to reflect triangles and also to help him explain the concepts of reflection of triangles along the lines $x - axis$,$x = -2$ and $y = 2x + 3$. Learners were only involved with the answering of oral questions. There was no opportunity for learners to use GeoGebra themselves, except for answering oral questions. Other learning activities which could have enhanced the interaction between the teacher and learners were not used. The limited use of interactive methods is sufficient evidence that Tima had limited technological pedagogical content knowledge to help learners to improve their knowledge on reflection.

***(b)* Engagement of learners with technology**

The engagement of learners in Tima`s class was limited to the answering of oral questions while Tima was using GeoGebra to perform reflection. While Tima was using GeoGebra to reflect a triangle along the lines $x = -2$, $x - axis$ and $y = 2x + 3$, the learners were watching what he was doing.

When Tima was using GeoGebra to reflect triangle ABC using the mirror line as $x - axis$, the image was labeled $A' B'C'$ by the software. Tima then asked the learners what they noticed about the object and its image.

One of the learners responded:

The object and image are congruent.

This response from the learner was correct.

Tima further explained to the learners that:

They are not only congruent but they also had the same distance from the mirror line.

While Tima was still using GeoGebra to reflect triangle ABC along $x - axis$, he invited the learners to count the distance of point A from $x - axis,$ which was the mirror line, to the corresponding point A' of the image. He did the counting together with the learners. They found the distance from point A to the mirror line to be 5 units, and the same as from the mirror line to A'.

Tima then asked learners to find the distance from point B to the mirror line, and then from the mirror line to the corresponding point of the image B'.

One of the leaners responded that the distance for points B and B' from the mirror line $x - axis$ was the same, and it was 1 unit.

This learner was able to provide the correct answer.

Tima continued to use GeoGebra to reflect the same triangle ABC using the line of reflection as $x = -2$, and asked the learners to identify and give the mirror line.

One of the learners responded: *It is a vertical line.*

This answer was not correct.

Tima did not engage with the learner and continued with the hope that the learner would know what was correct. Tima then gave another learner the opportunity to respond.

That learner gave the answer as $x = -2$ and she was correct.

He further asked the learners what they noticed about the object and its image.

One of the learners responded:

The object and the image have the same distance from the mirror line.

This response from the learner was correct.

This implied that Tima was able to use GeoGebra to improve the knowledge of learners on the reflection of objects and on finding the mirror line.

Tima finally used GeoGebra to reflect triangle $A'\ B'C'$ along the line $y = 2x + 3$.

After that Tima asked the learners*:*

What do you notice about the object and the image?

One of the learners responded: *They are congruent.*
This was the correct answer.

Tima did not engage his learners in the use of GeoGebra to reflect shapes during the lesson. There was no engagement of learners in using GeoGebra to reflect shapes in the lesson presentation. The engagement with the use of GeoGebra by learners was limited to the answering of oral questions. The teaching strategy was limited to the demonstration method of teaching, and no other interactive methods were used in the lesson presentation. The limitation of the demonstration method to support the engagement of learners in using GeoGebra showed that Tima had limited technological pedagogical content knowledge of using technology to improve learners` knowledge in the learning of reflection.

Molao`s profile

Molao is a male mathematics teacher at School A, and teaches mathematics and chemistry from Grade 8 to Grade 12. He is aged between 35 and 40 years, and has been in the teaching profession for the last 10 years, which includes nine years of teaching mathematics using technology. His highest teaching qualification is a B.Sc.Ed., majoring with Mathematics Education and Chemistry Education. He has nine years' experience of teaching Grade 12 learners at a NEPAD e-School. I observed Molao while he was teaching translation as a subtopic of transformation to Grade 12 learners using technology.

The codes, categories and emergent themes for Molao are as presented in table 5.1.

5.4.4 Theme 1: Use of technological pedagogical content knowledge when teaching transformation to Grade 12 learners

The data collected during the lesson observation of Molao revealed that he was using technological teaching aids when teaching translation in transformation to Grade 12 learners. The observed use of technological pedagogical content knowledge is discussed below.

(a) Technological teaching resources used in the lesson

Molao was using a laptop and overhead projector as his technological teaching resources to teach translation to his Grade 12 learners. The overhead projector which he connected to the laptop was used to display the concepts of translation on the screen. He used GeoGebra to draw a triangle and translated it to different positions using different translation vectors. There was only one laptop in the classroom which he used, and there were no others that the learners could use. The learners were sitting, listening and watching while the teacher used the devices to present the lesson. The projector helped the teacher to display every activity he was doing so that every learner could see him performing translation.

The overhead projector was used for the learners to observe when the object is translated to different positions using different translation vectors. All the class activities for the lesson were done on the laptop and displayed on the screen using the overhead projector. The projector was very helpful in teaching translation using technology, because the school does not have enough computers for every learner to use.

The use of laptop and overhead projector by Molao in his lesson presentation was as follows:

Molao opened the laptop and started the programme GeoGebra. He projected his activity on the screen using the overhead projector. He used GeoGebra to draw a triangle which the software labeled ABC. He continued to use GeoGebra to translate the triangle ABC using translation vector $\binom{7}{-3}$, and its image was labeled $A'B'C'$ by GeoGebra, as shown in Figure 5.4.The selected vector was displayed at the bottom left of the screen, as show in Fig 5.4.

Molao explained that *the triangle has moved 7 steps to the right and 3 steps downwards hence the vector at the bottom.*

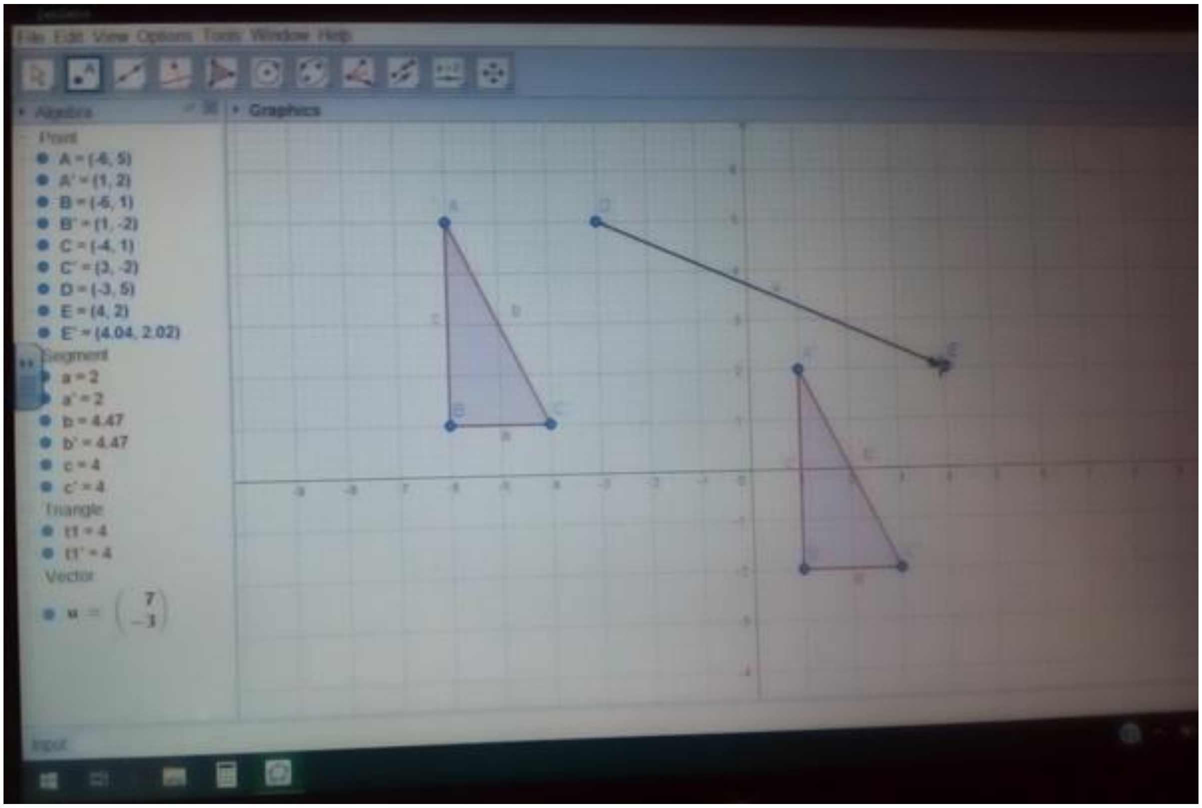

Figure 5.4: The translation of triangle ABC using vector.

After doing the first activity, shown in Figure 5.4, Molao saved it and opened GeoGebra to do another activity. He then used GeoGebra to draw a triangle, which was again labeled ABC. This activity was also displayed on the screen using overhead project for the learners to observe. He decrease the screen so that the vectors were hidden. He then involved his leaners in translating triangle ABC using a translation vector $\binom{0}{-5}$**.** He did this by asking learners to find the coordinates of the image triangle using $\binom{0}{-5}$**.** Learners looked at the triangle on the screen and used the given vector to find the coordinates, and wrote their answers in their exercise books. When the learners were done, he used GeoGebra to translate the triangle, and the image was automatically labeled $A'B'C'$ by the software. He invited the leaners to compare their coordinates with his.

After the discussion he again translated the image $A'B'C'$ using the translation vector $\binom{8}{4}$. In the same manner, Molao discussed with the learners how to obtain the translation vector by counting the number of horizontal and vertical steps from point A' to the image point A''. In the discussion he also allowed the learners to count the number of horizontal and vertical steps from point B' to B'' to verify that they would obtain the same translation vector. The activities of translating triangle ABC using vector $\binom{0}{-5}$ and triangle $A'B'C'$ using vector $\binom{8}{4}$ are shown in Figure 5.5.

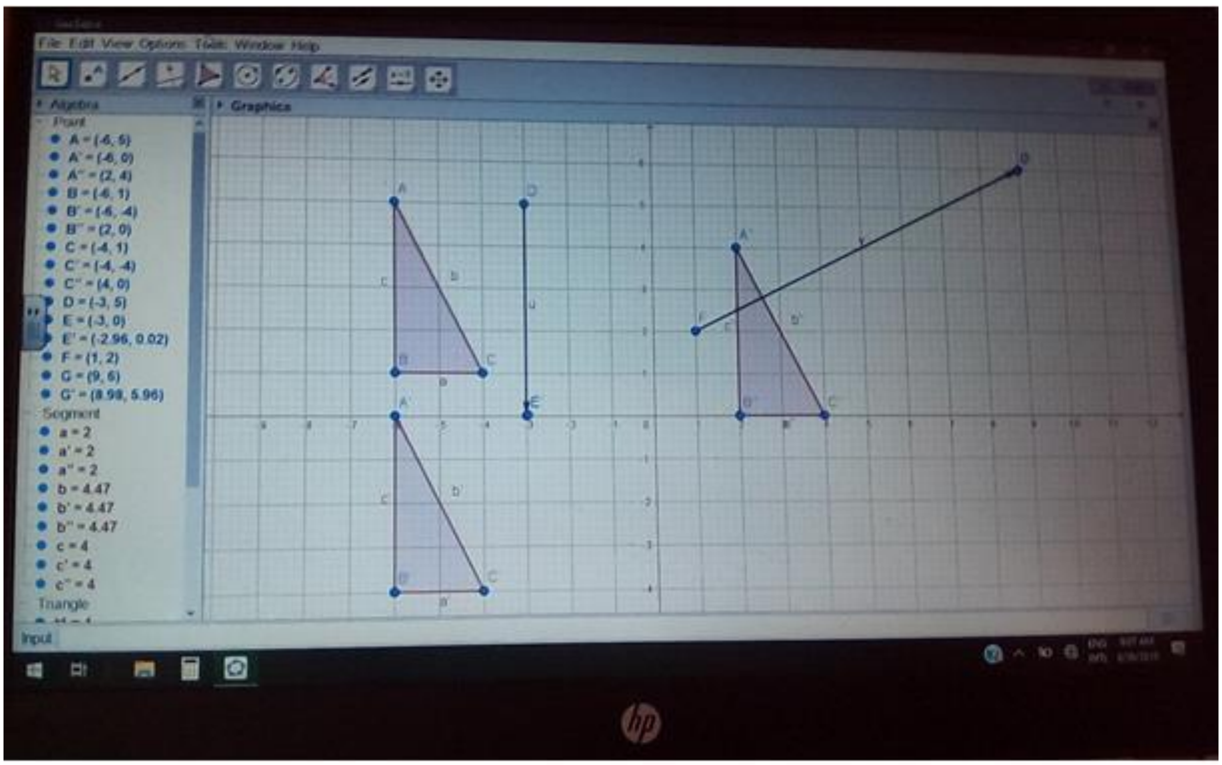

Figure 5.5: Translation of triangles ABC and A' B' C' using vectors

The use of GeoGebra in Molao`s lesson presentation on the translation of shapes was only a demonstration by the teacher. Molao was able to use GeoGebra, which was installed on his laptop, as a technology to translate a shape using different translation vectors to teach translation such as $\binom{0}{-5}$,

$\binom{8}{4}$ and $\binom{7}{-3}$. This indicates technological content knowledge. The use of GeoGebra helped the learners to find the translation vector easily, because of the line drawn on the $x - y$ plane which represented the translation vector. The labeling of the corresponding points by GeoGebra also helped learners to identify the corresponding points, which helped them to find the translation vector as well. This was a display of technological pedagogical knowledge by Molao. However there was a limited pedagogical content knowledge from Molao, because he only made use of demonstration. The learners were not involved by using technology themselves, or by any other activity, to discover the concepts of translation on their own.

***(b)* Ability to use technology and pedagogy for content teaching**

Molao was teaching translation to his learners, specifically how to translate a shape given a translation vector. He first explained to the learners that he would use the software GeoGebra to teach translation, and also told them that the use of GeoGebra would be faster than using the blackboard. Molao used GeoGebra to draw triangle, which was labeled ABC by the software. He then selected the type of transformation from the software, which in this case was translation. He further selected the vector $\binom{7}{-3}$ to translate triangle ABC, highlighted the triangle and clicked the line representing $\binom{7}{-3}$. The translation was then completed.

He also explained: *"Remember that when the shape is translated, the object and the image face the same direction".*

He then asked the learners: *"What is the other factor which shows that the type of transformation used is a translation?"*

One of the learners answered: *"The image and the object are congruent."*

He also asked the learners to give the meaning of 7 and -3 in the column vector.

The response from of the learners was that:

The 7 meant that the object moved 7 steps to the right and the -3 meant the object moved 3 steps downwards.

These learners were able to give the correct answers.

He then translated triangle ABC using a translation vector $\binom{0}{-5}$**.** The image was automatically labeled $A'B'C'$ by the software.

The response from the learners showed that they had background knowledge of vector space. Molao was able to effectively link the learners' background knowledge with the content of the lesson.

He continued with his lesson using different translation vectors such as $\binom{8}{4}$.

Molao emphasised to the learners that:

"In translation, a shift in the figure does not change the shape, size, and angle measure of the figure."

The lesson presentation from Molao did not display all the constructs of technological pedagogical content knowledge. He effectively displayed the content of transformation when translating shapes to different positions using translation vectors $\binom{7}{-3}$, $\binom{0}{-5}$ and $\binom{8}{4}$. He also displayed technological content knowledge when using GeoGebra to translate the shapes on the x-y plane. The ability to use different pedagogical approaches was however not displayed effectively, as he was mostly utilising a teacher-centred approach instead of a learner-centred approach. Therefore Molao had limited technological pedagogical content knowledge when teaching translation.

5.4.5 Theme 2: Use of relevant technology to address learners` challenges in the teaching of transformation in Grade 12

The collected data from the observations revealed that teachers use relevant technology to address learners` challenges when teaching transformation to Grade 12 learners. The use of relevant technology to address learners` challenges when teaching transformation to Grade 12 learners is discussed below.

***(a)* The use of the selected software in the teaching and learning of transformation**

Molao used the GeoGebra software to teach translation to Grade 12 learners. He used a laptop on which GeoGebra was installed for his class. He used the software to draw a triangle which was labeled ABC by GeoGebra. After drawing the triangle he selected translation as the type of transformation to perform. He then selected a line to represent the translation vector which he was going to use to translate triangle ABC. The translation vector he chose was $\binom{7}{-3}$. This indicates that GeoGebra is a relevant technology in the teaching of transformation.

After Molao had translated triangle ABC using the translation vector $\binom{7}{-3}$, the line representing the vector was still displayed on the screen, together with the image triangle $A'B'C'$ as shown in Figure 5.5.

Molao asked the learners to explain:

"What do the 7 *and* -3 *in the vector* $\binom{7}{-3}$ *represent?"*

One of the learners responded that 7 represented the movement of 7 horizontal steps to the right, and -3 represented the movement of 3 vertical steps downwards from triangle ABC to triangle $A'B'C'$.

Molao then asked the same learner to share with the rest of the class how she came to that conclusion.

The learner explained:

"I looked at the line which represents the vector as you have mentioned to us and tried to find the vectors it represents and I noticed that it is $\binom{7}{-3}$*."*

Molao commented that she was correct.

Molao used GeoGebra to draw triangle ABC in his second activity. After he drew triangle ABC he asked leaners to look at the triangle on the screen and use the grid on the screen to find the corresponding coordinates of the image triangle after the translation vector $\binom{0}{-5}$.

Challenge in finding the coordinates of the image after translation of triangle with vector $\binom{\mathbf{0}}{\mathbf{-5}}$

As the leaners were writing in their exercise books, Molao was moving around marking. When the learners were done he used GeoGebra to translate the drawn triangle and asked learners to compare their coordinates with those on the screen. The correct coordinates of the image were $A'(-6{,}0),\ B'(-6,-4)$and $C'(-4,-4)$.

Challenge: Some learners claimed that they could not obtain the correct coordinates.

Use of GeoGebra to address the challenge:

Molao just clicked Control Z on the laptop to undo the activity of translating the triangle using the vector $\binom{0}{-5}$.

Molao then helped the learners who struggled to find the correct coordinates of the image triangle by counting the horizontal and vertical movement steps. He helped the learners to find the coordinates of every image point $A'(-6{,}0),\ B'(-6,-4)$ and $C'(-4,-4)$.

After helping the learners to find the coordinates Molao explained to the learners:

When translating the object, first move horizontally and then followed by a vertical movement - always in that order.

Based on the vector $\binom{0}{-5}$ *there is no horizontal movement because of* 0 *in the vector.*

This was a display of content knowledge on translation by Molao. It was pedagogical content knowledge as he involved learners in finding the translation vector.

The learners were satisfied with his explanation.

Then Molao used GeoGebra to redo the translation of the triangle for the learners to confirm the coordinates of the image triangle.

Molao saved every activity he did while teaching.

After his demonstration the learners struggled to find the coordinates of the image triangle after translation of a triangle using vector $\binom{0}{-5}$.

This challenge was addressed with GeoGebra's undo function.

In this activity GeoGebra was able to perform functions that would be difficult to do on blackboard, or using paper and pencil. It helped Molao to quickly create a diagram, and also to correct mistakes by clicking **Control Z**. This could be awkward and very challenging to do on paper. Molao also continuously saved every activity he was doing and labeled it. This would help him in subsequent lessons to address the challenges faced by learners when confronted with new concepts on transformation.

Molao effectively displayed technological content knowledge and pedagogical content knowledge. He chose to use GeoGebra, which is relevant software in the teaching of translation. The software could perform all the functions of transformation and this was a display of technological content knowledge. Moreover, Molao involved his learners with an activity in finding the translation vectors, which was a display of pedagogical content knowledge. The constructs technological pedagogical content knowledge was however missing in Molao`s lesson, because the use of technology was limited to a demonstration which did not involve the learners in using technology.

(b) The use of software to enhance visualisation

Molao used GeoGebra to help learners to visualise translation. He used the software to draw a triangle which he would translate. GeoGebra automatically labeled the vertices with the capital letters $(\mathrm{A}, \mathrm{B}, \mathrm{C})$, and the sides with the small letters $(\mathrm{a}, \mathrm{b}, \mathrm{c})$. Molao further used GeoGebra to select a line which he drew to represent a translation vector $\binom{7}{-3}$ that he was going to use to translate triangle ABC. Molao selected translation from the list of types of transformation on GeoGebra, highlighted triangle ABC and clicked on the line representing a translation vector. This translated the triangle. The learners were able to observe when the triangle ABC was translated.

After performing translation, GeoGebra labeled the corresponding vertices of the image triangle $\mathrm{A'B'C'}$ and the corresponding sides $\mathrm{a'b'c'}$.

This helped learners to observe that the following vertices and sides were corresponding:

Vertices	***Sides***
$A \longrightarrow A'$	$a \longrightarrow a'$
$B \longrightarrow B'$	$b \longrightarrow b'$
$C \longrightarrow C'$	$c \longrightarrow c'$

Molao explained to the learners that the type of transformation he performed is called translation and it is described by giving a translation vector.
He then asked the learners to find a translation vector which mapped the object onto the image.

Challenge:
The learners then started to write in their exercise books and raised their hands one by one for Molao to come and check their answers. Molao then said that most of the translation vectors as done by the learners were not correct.

He then explained to the learners that they should start from the point of the object and move to the corresponding point of the image when finding the translation vector.

The learners again tried to find the translation but very few were able to get it.

Use of GeoGebra to address the challenge of visualisation:

Molao then asked the learners to look at the line on the screen, drawn with GeoGebra. Molao explained that the line represented the translation vector, and that they could therefore find the translation vector represented by that line.

The learners then found the translation vector represented by the line.

Visualisation: The line drawn with GeoGebra enhanced the visualisation of the translation vector since the line also had arrow facing in the direction of the movement.

This observation enhanced the learners' visualisation, as it is very challenging to identify the corresponding sides and angles when they are not fully labeled. The process of translating the triangle ABC using GeoGbra enhanced the visualisation of the learners because they were able to see it when it was translated. Another aspect which enhanced the learners' visualisation was that the line representing the translation vector drawn on the grid, which helped the learners to easily identify the translation vector.

After Molao had translated triangle ABC using the translation vector $\binom{7}{-3}$, the line representing the vector was still displayed on the screen, along with the image triangle $A'B'C'$ as shown in Figure 5.5.

Molao asked the learners to explain:

" What do the 7 and -3 in the vector $\binom{7}{-3}$ represent?"

One of the learners responded that 7 represents the movement of 7 horizontal steps to the right and 3 represents the movement of 3 vertical steps downwards from triangle ABC to triangle $A'B'C'$.

Molao then asked that learner to share with the rest of the class how she came to make that observation.

The learner explained: *"I looked at the line which represents the vector as you have mentioned to us and tried to find the vectors it represents and I noticed that it is* $\binom{7}{-3}$*."*

The line drawn helped the learner to visualise the translation vector, and the learner had given the correct answer.

Then Molao commented: *You are correct.*

The correct answer from the learner, that the translation vector is $\binom{7}{-3}$, is evidence that GeoGebra enhanced the visualisation of the translation. GeoGebra enhanced the visualisation because it could display the translation vector. This is usually challenging for learners to find when that line is not drawn. Learners were able to identify the corresponding vertices and sides. The use of GeoGbra was able to enhance their visualisation.

5.4.6 Theme 3: Inability to use technology to improve learners` knowledge of transformation in Grade 12

The data from the observations revealed that mathematics teachers were unable to use technology to improve learners` knowledge of transformation in Grade 12. The teaching strategies of using technology to improve learners` knowledge of transformation are discussed below.

***(a)* Teachers' and learners' activities**

The activities involving teaching and learning in Molao`s lesson were limited to a demonstration on GeoGebra on how to perform translation. Molao asked his learners to watch and observe while he translated shapes using GeoGebra. The activities Molao was doing on the laptop were projected to the screen using the overhead projector. Molao first demonstrated to his learners how to draw a polygon, which was a triangle, using GeoGebra. Molao demonstrated many activities to his learners on how to use GeoGebra to translate a shape. For example Molao demonstrated to the learners how to use GeoGebra to translate a shape using the following vectors: $\binom{7}{-3}$, $\binom{0}{-5}$, $\binom{2}{-2}$ and $\binom{8}{4}$.

Molalo asked all his learners to find the coordinates of the image triangle after applying the translation vector $\binom{0}{-5}$ on the triangle which was displayed on the screen. As the leaners wrote in their exercise books he was moving around checking their answers. When the learners were done he used GeoGebra to translate the drawn triangle and asked learners to compare their exercise answers with those on the screen. The correct coordinates of the image were$A'(-6,0)$, $B'(-6,-4)$ and $C'(-4,-4)$. Some learners got the correct answer, while others could not manage. Molao then used GeoGebra to help those who still experienced challenges. Some of the teaching and learning activities for Molao`s lesson are shown in Figure 5.5 and Figure 5.6.

The demonstration method was used together with an explanation of the concepts of translation. This was done with the help of the GeoGebra software, which was installed on the laptop. There were no hands-on activities for the learners to use the software themselves, and the only interaction was them answering oral questions. Other learning activities which could have aided in the teacher-learner engagement were not used. The failure to make use of interactive methods which would encourage learner engagement is enough evidence that Molao had limited technological pedagogical content knowledge to help learners to improve their knowledge about translation in transformation.

(b) **Engagement of learners with technology**

The learners' engagement in the lesson was limited to them answering oral questions and completing certain exercises while Molao was using GeoGebra to translate shapes. Molao was translating triangles using the vectors $\binom{7}{-3}$, $\binom{0}{-5}$, $\binom{2}{-2}$ and $\binom{8}{4}$, while learners were observing and listening attentively to his explanations.

When Molao had translated triangle ABC using the translation vector $\binom{7}{-3}$, he asked the learners to explain:

"What do the 7 and -3 in the vector $\binom{7}{-3}$ represent?"

One of the learners responded:

7 represents the movement of 7 horizontal steps to the right and 3 represents the movement of 3 vertical steps downwards from triangle ABC to triangle $A'B'C'$.

Molao then asked that learner to share with the rest of the class how she had come to that conclusion.

The learner explained:

"I looked at the line which represents the vector as you have mentioned to us and tried to find the vectors it represents and I noticed that it is $\binom{7}{-3}$."

Then Molao commented that the learner's answer was correct.

Molao again used GeoGebra to draw triangle ABC in his second activity. After drawing triangle ABC he asked the leaners to look at the triangle on the screen and use the grid on the screen to find the corresponding coordinates of the image triangle after the translation vector $\binom{0}{-5}$**.** While the leaners were doing the activity, Molao moved around the class checking the answers. . After the learners had finished he went to the front of the class and used GeoGebra to translate the drawn triangle. He then asked the learners to compare their coordinates with those on the screen. The correct coordinates of the image were $\mathrm{A}'(-6{,}0),\ \mathrm{B}'(-6,-4)\mathrm{and}\ \mathrm{C}'(-4,-4)$. Molao was able to use GeoGebra to involve the learners in finding the translation.

Molao did not engage the learners in the use of GeoGebra to translate shapes in order to construct new knowledge on their own in learning translation. He mainly used a teacher-centred approach instead of a learner-centred approach. There was no engagement of learners in using GeoGebra to reflect shapes in the lesson presentation. Learners' engagement with the use of GeoGebra was limited to the answering of oral questions. The teaching strategy was limited to the demonstration method of teaching and, no other interactive methods were used in the lesson presentation. The limitations of the demonstration method to support learner engagement using GeoGebra showed that Molao had limited technological pedagogical content knowledge of using technology to improve learners` knowledge in the learning of transformation.

Mpona`s profile

Mpona is a female mathematics teacher at School B, teaching mathematics and physics from Grade 8 to Grade 12. She has seven years of experience in teaching and using technology to teach mathematics to Grade 12 learners at a NEPAD e-School. Mpona has a graduate degree in education, majoring in Mathematics Education and Physics Education. She is aged between 30-35 years and I observed her while she was teaching enlargement under transformation to Grade 12 learners.

The codes, categories and emergent themes for Mpona are as presented in table 5.1.

5.4.7 Theme 1: Use of technological pedagogical content knowledge when teaching transformation to Grade 12 learners

The data collected from the observations revealed that Mpona was using technology when teaching transformation to Grade 12 learners. The use of pedagogical content knowledge by Mpona is discussed below.

***(a)* Technological teaching resources used in the lesson**

Mpona was observed presenting her lesson using a laptop, interactive white board, overhead projector and the Cartesian-coordinate framework as technological teaching resources. The laptop was already connected to the overhead projector. She handed out Cartesian-coordinate frameworks to the learners.

Mpona started her lesson by playing a video on the laptop. The video was projected to the smart board using the overhead projector so that all the learners could see. The video was on the enlargement of triangles. The first example was on the enlargement of a triangle using centre $(-5,2)$ and scale factor 2. The teacher on the video counted the number of steps from the centre to each point of the object and multiplied every distance by the scale factor 2 to obtain the position of each corresponding point of the image. After that the points of the image were joined to form an enlarged triangle.

The second example was the enlargement of another triangle using centre $(1, 2)$ and scale factor 3, but Mpona used this one as an activity for the learners. Mpona paused the video before the second example could be completed. She gave her learners who were already seated in five groups of seven members a Cartesian-coordinate frame work. The learners had to use the Cartesian-coordinate framework to make a triangle, which they then enlarged using the scale factor 3 and centre $(1, 2)$. After completion of the task she resumed the video on the enlargement of the same triangle using the same centre and scale factor. She asked the learners to watch the video so that they could use it to compare what they had done to obtain the image of their triangle with one on the video. The learners were highly motivated by the way the video presented the example, and those who had made mistakes were able to rectify them.

The smart board, laptop and overhead projector were in the front of the classroom. The concepts of transformation which were taught were displayed on the smart board. The shapes displayed from the laptop to the smart board were also modified using a pen which was able to write on the smart board. The shapes were also erasable on the smart board. Mpona involved the learners by allowing them to come to the smart board to draw the shapes of enlargement.

The use of technological teaching resources in Mpona`s lesson presentation were limited to the lecturing method. Although the learners were involved through the use of the Cartesian-coordinate framework with an activity, they were not given an opportunity to use technological teaching resources to learn translation. This showed a limited technological content knowledge and technological pedagogical knowledge from Mpona. Mpona did involve her learners with an activity on the enlargement of a triangle, which was a display of pedagogical knowledge.

(b) **Ability to use technology and pedagogy for content teaching**

Mpona made use of a video and also of the Cartesian-coordinate framework to teach enlargement. She played a video on the laptop which was projected onto the interactive white board. The first example was made from the video was on the teaching of enlargement using scale factor 2 and centre $(-5,2)$. The second example was the enlargement of another triangle using centre $(1,2)$ and scale factor 3, but Mpona used this as an activity for the learners.

Mpona paused the second video before the example could be completed. The learners were seated in groups, and each received the Cartesian-coordinate framework. The learners used this framework to draw a triangle, which they then enlarged using the scale factor 3 and centre $(1,2)$. After they were done she continued the video on the enlargement of the same triangle, using the same centre and scale factor.

She helped her learners to discover that in order to enlarge the shape they had to multiply the distance of the point from the centre by the scale factor in order to get the corresponding point of the image. Mpona's pedagogical approach was that she engaged her learners by using the discovery method of teaching. The Cartesian-coordinate framework helped the learners because it has holes which the learners are able to count. In this case the learners enlarged a triangle using centre as $(1,2)$ and scale factor 3.

Mpona helped some of the groups who could not manage to enlarge the triangle using scale factor 3 and centre $(1,2)$. After the video on the enlargement of a triangle was finished, some of the group members indicated that they could not manage to enlarge the triangle the way it had been enlarged on the video. Mpona then asked them to identify the centre $(1,2)$ on the Cartesian-coordinate framework. She then asked them to find the distance of each vertex of the triangle from the centre, and multiply this by scale factor 3.She explained to the learners that the new distances they have obtained should give them the position of the new vertices of the image triangle. The group members followed the instructions and drew the new triangle. They were encouraged when they realised that they had obtained the same shape as the one which was shown on the video.

The technology that Mpona used in this lesson was a laptop connected to an overhead projector which projected onto the interactive white board. She further made use of videos, which she could play back or pause whenever she wanted to explain something, or when the learners needed clarification of some concepts. The display of pedagogy was when Mpona split her learners into groups to help them to construct new knowledge on the enlargement of a shape. Technological knowledge was also displayed when Mpona played the video on the teaching of enlargement. There was a limited display of technological pedagogical content knowledge as the learners did not themselves use technology to learn enlargement.

5.4.8 Theme 2: Teachers use relevant technology to address learners` challenges in the teaching of transformation in Grade 12

The observational data revealed that teachers use relevant technology to address learners` challenges when teaching transformation in Grade 12. The use of relevant technology to address learners` challenges when teaching transformation in Grade 12 is discussed below.

(a) The use of the selected technology in the teaching and learning of transformation

Mpona opted to facilitate her teaching of transformation with the use of a video. Mpona opened the video from a site called “Maths Doctor”. She let the learners watch a video on the teaching of enlargement. The teacher on the video was teaching enlargement and three examples were done. The first example was the enlargement of a triangle using scale factor 2 and centre $(-5,2)$, while the second example was enlarging a triangle using scale factor 3 and centre $(1,2)$. The learners watched the video attentively. The teacher on the video explained every step taken when enlarging an object very well. She multiplied the distance of every vertex of the object from the centre by the scale factor 2 to obtain the distance of the vertex image from the same centre. After that she drew the image of the enlarged object.

Mpona used the second example from the video to give her learners an activity. The second example was the enlargement of another triangle using centre $(1,2)$ and scale factor 3. The teacher on the video drew a triangle which was going to be enlarged using scale factor 3 and centre (1,2). Mpona paused the video just when the drawing of the triangle was complete. She gave her learners who were seated in five groups of seven members a Cartesian-coordinate framework. The learners used this framework to draw a triangle which they enlarged using scale factor 3 and centre $(1,2)$. When they were done she continued the video, in which the same triangle was enlarged using the same scale factor and centre.. She asked the learners to compare their steps and images with that of the video.

After watching the video some learners indicated that two vertices of their image triangle were incorrect. They asked Mpona to replay a section of the video so that they could see where they made their mistake. After they had re-watched the video they corrected their answer and told Mpona that they struggled to find the distance of the image point from the centre.

One of the learners from the group said:

"Our mistake was that we counted the distance of the image point from the object and not from the centre."

Mpona said: *"Thanks that you are able to realise your mistake and also managed to correct it."*

For this activity Mpona was able to address learners' challenges in using the centre and scale factor to enlarge the object. She used video and an activity to help learners to identify and correct their mistakes. While teaching transformation, Mpona selected a video relevant to the teaching of enlargement as type of transformation. When teaching enlargement she played a video on enlargement. The advantage of the video in Mpona`s class was that sections of it could be replayed to address the challenge of learners who might have not grasped the concept well. The video also assisted the learners to compare the steps they used in enlarging the shape with the way it was enlarged in the video. As a result their challenges were addressed, and their knowledge on enlargement was improved. This approach showed that Mpona had pedagogical content knowledge by involving learners in the activity to enlarge a shape. The construct technological pedagogical knowledge was however missing as she did not involve the learners in the use of technology to construct new knowledge in the enlargement of shapes.

(b) The use of video and Cartesian-coordinate framework to enhance visiualisation

Mpona used a video for the teaching of enlargement to enhance visualisation for the learners. Mpona also played a video on enlargement. The video was projected onto the interactive white board where every learner could see. The teacher on the video enlarged a triangle using scale factor 2 and centre $(-5,2)$. The vertices of the triangle to be enlarged were coloured with different colours.

The first vertex was coloured red, the second green and the third blue, that is ABC. The corresponding vertices of the image triangle were also labeled with the same colours.

One of the learners raised his hand and asked Mpona to explain how the lines were extended from the centre to the corresponding points of the image triangle.

Mpona paused the video and explained: "*The distance of the point of the object from the centre is multiplied by 2 which is the scale factor. Then the number that is obtained is the distance of the image point starting from the centre."*

Mpona asked the learners to watch the video carefully and look at the colours of the letters used to label the vertices.

Mpona then continued the video.

From the video the corresponding points were labeled with the same colours.

When the enlargement of the shape was complete, the learners stated that the colouring of the vertices helped them to identify the corresponding points. This colouring of vertices enhanced the visualisation of the learners as it is very challenging for them to identify the corresponding points under enlargement.

After watching the video, Mpona gave the learners an activity to do. The learners were already seated in five groups of seven members. Each group had a Cartesian-coordinate framework as well as plastic pins and rubber bands of different colours. Mpona used the Cartesian-coordinate framework to help the learners form a triangle using these. The learners took three pins and fixed them on the Cartesian-coordinate framework where the coordinates were required. The three pins were used as way a of forming a triangle, and a red rubber band was put around them to form a triangle. The learners were asked to enlarge their triangle, and they used a green rubber band for the enlarged triangle. The learners used the Cartesian-coordinate framework to form a triangle, which they enlarged using the scale factor 3 and centre $(1,2)$.

5.4.9 Theme 3: Inability to use technology to improve learners` knowledge of transformation in Grade 12

The observational data revealed that mathematics teachers were unable to use technology to improve learners` knowledge of transformation in Grade 12. The teaching strategies of using technology to improve learners` knowledge of transformation are discussed below.

***(a)* Teachers' and learners' activities**

The teaching and learning activities for Mpona`s lesson involved the use of a Cartesian-coordinate framework, and laptop linked to an overhead projector which projected onto a white board. Mpona played a video on the teaching of enlargement. The video was projected onto the interactive white board so that every learner could see. The teacher on the video enlarged a triangle using scale factor 2 and centre $(-5,2)$. Mpona paused the video after drawing a triangle which was going to be enlarged. She then went to the interactive white board and showed the learners how to find the distance of each corner of the triangle from the centre of enlargement. She continued by multiplying every distance for each corner of the triangle by 2, which was the scale factor, in order to find the distance of the corresponding image point from the centre. After that she continued the video, which indicated the same steps as she had demonstrated. This activity helped the learners to gain more knowledge on the enlargement of a shape given a centre and scale factor.

Mpona used the second example from the video to design an activity for her learners to gain more knowledge on the enlargement of a given shape. The video continued with another example on the enlargement of a triangle, using scale factor 3 and centre (1, 2). Immediately after the drawing of the triangle was complete, Mpona paused the video. She gave her learners who were already seated in five groups of seven members a Cartesian-coordinate framework, which they hat to use to make a triangle. They then enlarged this using the scale factor 3 and centre $(1,2)$. After completion of this activity she continued the video on the enlargement of the same triangle, using the same centre and scale factor. She asked the learners to compare their actions and the image of their triangle with one on the video. While the learners were involved in the activity of enlarging the triangle, Mpona was moving around in the class trying to guide the learners. The activity helped the learners to gain more knowledge and skills on the enlargement of a shape.

Mpona designed an activity which helped learners to gain more knowledge about transformation. The activity however made use of the Cartesian-coordinate framework and not technology. The only technology used in Mpona`s lesson was the video on enlargement that the learners watched. There were no learner activities based on technology, apart from watching the video. Since Mpona designed the activity to enlarge a triangle, it shows that Mpona possesses pedagogical content knowledge. The limited use of interactive methods which involve the use of technology is, however, enough evidence that Mpona has limited technological pedagogical content knowledge to help learners to improve their knowledge in enlargement through the use of technology.

(b) **Engagement of learners with technology**

Mpona made use of the video and the Cartesian-coordinate framework to engage leaners in her teaching of enlargement. She played a video which was projected onto interactive white board. The second video was on the enlargement of a triangle using scale factor 3 and centre $(1,2)$. Mpona paused the video once the drawing of the triangle was complete, but before it was enlarged. She used the second video to engage her leaners in an activity to enlarge a triangle using the Cartesian-coordinate framework. She gave her learners who were already seated in five groups of seven members a Cartesian-coordinate framework.

The learners used this framework to make a triangle which they enlarged using scale factor 3 and centre $(1,2)$. After they did this she continued the video on the enlargement of the same triangle using the same centre and scale factor. She asked the learners to watch the video so that they could compare their actions and triangles with that on the video. Mpona helped her learners to construct knowledge on enlargement by giving them an activity. The activity was however not on the use of technology, but on the Cartesian-coordinate framework.

The learners` engagement in Mpona`s lesson was only through the use of the Cartesian-coordinate frame work, and not technology. Mpona did not engage her learners in the use of technology to enlarge shapes in her lesson. The engagement with the use of technology by learners was limited to watching a video on enlargement. There was however interaction between Mpona and the learners through the use of the Cartesian-coordinate framework to enlarge the shapes. This indicates that Mpona indeed had pedagogical content knowledge. The limitation of the demonstration method of watching a video to support the engagement of learners in using technology showed that Mpona had limited technological pedagogical content knowledge of using technology to improve learners` knowledge in the learning of enlargement in transformation.

5.5 INTERVIEWS

The interviews were conducted after the lesson observations had taken place, with the purpose of helping the researcher to further explore the teachers` TPACK displayed during the teaching and learning of transformation. The questions asked during the interviews were informed by the research questions of the study and characterised by the constructs of the TPACK framework.

Tima`s case

Table 5.2: Emergent themes and categories 2

Codes	Categories	Emergent Themes
Use of technology to teach transformation	• Technological teaching resources to address LGCSE syllabus • Ability to use technology to present a lesson and make explanation.	Use of technological pedagogical content knowledge when teaching transformation to Grade 12 learners
Use of technology to address learners` challenges in transformation	• The use of the selected software in the teaching and learning of transformation • The use of software to enhance visualisation	Use of relevant technology to address learners` challenges in the teaching and learning of transformation in Grade 12
Improving learners` knowledge of transformation using TPACK	• Teachers' and learners' activities • Engagement of learners with technology	Ability to use technology to improve learners` knowledge of transformation in Grade 12

5.5.1 Theme 1: Use of technological pedagogical content knowledge when teaching transformation to Grade 12 leaners

During the interviews the mathematics teachers were asked questions which focused on the use of technology and pedagogical content knowledge during the lesson observations. The questions which were asked are discussed below.

(a) **Technological teaching resources to address the LGCSE syllabus**

During the lesson presentation, Tima was observed using a laptop and overhead projector as his technological teaching resources to the teaching of reflection to Grade 12 learners. No additional resources were used to present the lesson. One of the questions asked during the interview was on the use of selected technology to address the LGCSE syllabus.

Tima was asked the following question:

How does the selected technology in the teaching of transformation address the LGCSE mathematics syllabus?

Tima replied: *"It simplifies the content itself. The way it is used is different from using textbooks because learners are able to see when you perform an activity on the laptop projected on the white board. They see what happens."*

Tima revealed that the use of a laptop and overhead projector as technological teaching resources to address the LGCSE syllabus is different from using textbooks because in the textbooks learners see shapes already drawn, the object and its image, while the use of technology helps learners to see what happens when object is reflected.

Tima used technological teaching resources to help learners to visualise the reflection of shapes as opposed to the use of a textbook with shapes already drawn. The response suggest that she understood how teaching using technology enhance teaching and learning process which indicates technological pedagogical knowledge as well as technological content knowledge.

(b) **Ability to use technology to present a lesson and make explanations**

During the lesson, Tima used GeoGebra to perform reflection while at the same time explaining the work to the learners. One of the questions asked during the interview was on the use of relevant technology as a teaching strategy in transformation.
The following question was asked:

How do you manage to get the chosen sub-topic of transformation to fit in well with the instructional strategy using relevant technology?

Tima responded: *"When I perform reflection I display the diagram on the screen as well as the mirror line so that when I click the diagram and the mirror line the object is reflected and this makes it easy for me to explain what is happening. It automatically reflects the object*."

Tima's response reveals that the use of GeoGebra helps to create an activity on the reflection of object, which is easy to explain as the learners see when the shape is reflected. This indicates that Tima chose technology in such a way that it helps him to explain concepts of transformation easily, which is a display of technological content knowledge.

5.5.2 Theme 2: Teachers use relevant technology to address learners` challenges in the teaching of transformation in Grade 12

During the interviews, the mathematics teachers were asked about the use of relevant technology to address learners` challenges when teaching transformation, as observed during lessons. The questions posed to the teachers with regard to their lesson observations are discussed below.

(a) The use of the selected software in the teaching and learning of transformation

Tima was observed as specifically using GeoGebra as technology to construct shapes which he reflected using different lines of reflection. One of the questions he was asked during the interview was based on the relevance of selected software in constructing accurate shapes. The question asked was as follows:

How is technology helpful in constructing accurate shapes to be transformed?

Tima replied*: "What I have realised is that GeoGebra has graphs and features of polygons which can help to construct them. It also has types of lines to choose or angle or point which are all there to be used."*

Tima revealed that he used GeoGebra because it has all the features relevant for the teaching of transformation. This indicates that Tima had technological content knowledge.

***(b)* The use of software to enhance visualisation**

The observations revealed that Tima used GeoGebra to address learners' challenge of visualisation in the learning of reflection. One of the questions asked was based on how the selected technology addresses the LGCSE syllabus.

How does the selected technology in the teaching of transformation address the LGCSE mathematics syllabus?

The response to this question was: *"It simplifies the content itself."*

A follow-up question was posed to Tima regarding his response to the previous question, in order to obtain more information on how the selected technology addresses learners` challenges in transformation.

How is this GeoGebra helpful in addressing your learners` challenges?

The response to this question was:

"The way it is used is different from using textbooks because learners are able to see when you perform an activity on the laptop projected on the white board. They see what happens."

This response from Tima agrees with what had been observed, namely that the use of GeoGebra enhances visualisation because learners are able to see when the shape is reflected, as opposed to what they see in the textbooks where the object and image have already being drawn for them. Moreover, this also indicates that Tima used the demonstration method of teaching, and did not opt for other teaching strategies which would encourage interaction between the teacher and learners - hence limited knowledge of technological pedagogical knowledge and pedagogical content knowledge. However Tima possesses technological content knowledge.

5.5.3 Theme 3: Ability to use technology to improve learners` knowledge of transformation in Grade 12

During the interviews the mathematics teachers were asked questions related to their ability to use technology to improve learners` knowledge of transformation in Grade 12, based on their observed lessons. The questions posed to the teachers during the interviews are discussed below.

***(a)* Teachers` and learners` activities**

The observation revealed that the activities in Tima`s lesson were limited to a demonstration using the GeoGebra software to perform reflection. There were no activities for the leaners. One of the questions asked during the interview was on teaching strategies using technology. Tima was asked the following question:

How do you manage to get the chosen sub-topic of transformation to fit in well with the instructional strategy using relevant technology?

The response to the above question was:

“When I perform reflection I display the diagram on the screen as well as the mirror line so that when I click the diagram and the mirror line the object is reflected and this makes it easy for me to explain what is happening. It automatically reflects the object.”

This response from Tima reveals that he only uses technology for his demonstrative activities, and does not give learners activities to improve their knowledge of transformation. This is an indication that Tima has limited technological pedagogical knowledge and pedagogical content knowledge, as he only used the demonstration method of teaching. No other methods were used which would enhance the interaction between learners and teachers.

(b) Engagement of Learners with technology

During the lesson observation, Tima did not engage learners with the use of technology. The engagement was limited to the answering of oral questions while Tima was using GeoGebra to perform reflection. One of the questions asked during the interview was on the use of technology in the teaching strategies of reflection. The following question was asked:

How do you select the technology so that it fits well in the teaching strategy for the teaching of reflection?

Tima replied: *"The features of technology help us to see which topics to teach or will be easy for learners to see. The features help us to select the topic to teach."*

The response reveals that Tima uses technology to present the content of transformation to the learners, and not so that he can engage them in order to improve their knowledge of transformation. This shows that Tima has limited technological pedagogical knowledge and pedagogical content knowledge, but that he has technological content knowledge.

Molao`s case

Table 5.3: Emergent themes and categories 3

Codes	Categories	Emergent Themes
Use of technology to teach transformation	• Technological teaching resources used in the lesson • Teaching strategies with technology	Use of technological pedagogical content knowledge when teaching transformation to Grade 12 learners
Use of technology to address learners` challenges in transformation	• The use of the selected software in the teaching and learning of transformation • The use of software to enhance visualisation	Use of relevant technology to address learners` challenges in the teaching and learning of transformation in Grade 12
Improving learners` knowledge of transformation using TPACK	• Ability to use technology to teach transformation • Engagement of learners with technology	Ability to use technology to improve learners` knowledge of transformation in Grade 12

5.5.4 Theme 1: Use of technological pedagogical content knowledge when teaching transformation to Grade 12

The mathematics teachers were asked questions during the interviews which focused on the use of technology when teaching transformation during the lesson observations. The questions which the teachers were asked with regard to the use of technology and pedagogical content knowledge are discussed below.

(a) **Technological teaching resources used in the lesson**

Molao was observed using a laptop and overhead projector as his technological teaching resources during his lesson presentation to the Grade 12 learners. He did not use any additional resource for presenting the lesson. One of the questions asked during the interviews was on the selection of the technology so that it fits in well with the teaching of translation. The following question was asked to Molao during the interview:

How do you select the technology so that it fits in well with the teaching strategy for the teaching of translation?

The response from the above question is stated below:

"*I selected GeoGebra because it is cheaper. As long as it is in the laptop it does not charge anything. It is cheaper than any other software I have seen.*"

The response from Molao reveals that his use of GeoGebra is based on its affordability. Once it has been downloaded it can be used while offline without the cost of paying for an internet connection. This also justifies the use of a laptop. The use of technology, in this case GeoGebra, was limited to the demonstration method, which did not involve the learners to learn the concepts of transformation. This shows that Molao has a limited technological pedagogical knowledge and pedagogical content knowledge. He does however have technological content knowledge.

(b) **Teaching strategies with technology**

During the observation Molao used GeoGebra, installed on the laptop, as technology to teach translation to his learners. One of the questions posed to Molao during the interview was on the strategy of using relevant technology to teach translation.
The following question was asked:

How do you manage to get the chosen sub-topic of transformation to fit in well with the instructional strategy using relevant technology?

Molao responded:

"It is very easy though this technology plays a bigger part but still I as a teacher I still ask learners to do certain activities using the same technology. I still ask them some questions to reply on. I do not allow myself to do the whole part alone."

Molao revealed his knowledge of the teaching strategies using technology helps him to design activities for the learners to use technology during the teaching and learning of transformation. His teaching strategies involve learners by asking oral questions, but he does not give them activities to do using technology. The use of technology in Molao`s lesson does not involve learners in learning the concepts of translation.

5.5.5 Theme 2: Use of relevant technology to address learners` challenges in the teaching of transformation in Grade 12

During the interviews, the mathematics teachers were asked about the use of relevant technology to address learners` challenges when teaching transformation, as seen during the observations. The questions posed to the teachers with regard to their lesson observations are discussed below.

(a) The use of the selected software in the teaching and learning of transformation

During the observation, Molao mainly used GeoGebra to teach translation to the Grade 12 learners. He was observed using GeoGebra to translate shapes using different translation vectors. One of the questions he was asked during the interviews was on the use of GeoGebra to address learners` challenges in the learning of translation. The following question was asked:

How is GeoGebra helpful in addressing your learners` challenges?

Molao responded: *"It addresses them very well because when you use a chalkboard and a piece of paper and pencil, students sometimes miss… they miss some of ideas to or some calculations when they translate an object but when using GeoGebra it is very easy because there is a grid which is scaled on the GeoGebra therefore to identify a column vector it becomes easier to them because they already know the negative sign and positive sign so they know that when moving to downwards it is negative and when moving to the right it is positive."*

This response from Molao shows that the use of GeoGebra in the teaching of transformation addresses the challenge of drawing a scale on the axis because GeoGebra can do that for the learners. Furthermore, the use of GeoGebra helps to address the challenge of identifying the correct signs when writing column vectors because of the correctly labeled scale which shows the negative and positive numbers. This indicates that Molao has technological content knowledge and technological pedagogical knowledge. He however has limited pedagogical content knowledge because he did not involve learners in using GeoGebra.

(b) The use of software to enhance visualisation

Molao was observed using GeoGebra to translate objects and also to explain to the learners how to find the translation vectors and on how to translate shapes.

One of the questions posed to Molao during the interview was on the selection of technology to address the LGCSE syllabus. Molao was asked the following question:

How does the selected technology in the teaching of transformation address the LGCSE mathematics syllabus?

Molao answered as follows:

"This technology addresses the transformation translation well, as the software brings motivation to learners to be more vigilant when you are using this technology. They become more interested in the LGCSE syllabus."

This response from Molao reveals that the use of GeoGebra motivates the learners and this makes them more vigilant. This helps them to be very observant of all activities, so that they are able to see when a shape is translated – this enhances their visualisation. However there was limited technological pedagogical knowledge as the learners were not involved in the use of GeoGebra while learning of translation.

5.5.6 Theme 3: Ability to use technology to improve learners` knowledge of transformation in Grade 12

The mathematics teachers were asked questions during the interviews which focused on their ability to use technology to improve learners` knowledge of transformation in Grade 12, based on their observed lessons. The questions posed to the teachers during the interviews are discussed below.

(a) **Ability to use technology to teach transformation**

The observations reveal that the activities involving teaching and learning in Molao`s lesson were limited to a demonstration on GeoGebra on how to perform translation. The relevant choice of technology in the teaching of translation did not help learners to construct new knowledge. A question was asked to Molao on the selection of technology and its relevance to the teaching of translation. Molao was asked the following question.

How do you select technology that is relevant to the teaching of translation?

The response from Molao on the above question was as follows:

"This software GeoGebra it is technology as it is and there are fitted applications. There is translation, reflection, rotation, enlargement…. they are all there then you just pick one that you are going to use at the moment with all the strategies which you are going to use there. If you want a line to reflect a shape or to translate the object on which to select a translation vector, they are still there. It depends to you as a teacher what you want. You just pick and select the one and then you use it for what you want."

Molao's response reveal that the use of selected technology in his lesson was to help him to present the concepts of translation, without involving the learners to construct new knowledge of transformation using technology, so that their knowledge on translation could improve. This showed that Molao has limited technological pedagogical content knowledge and pedagogical content knowledge. It was however seen that he has technological content knowledge, as he was able to use GeoGebra to effectively demonstrate the teaching of translation.

***(b)* Engagement of Learners with technology**

The observations revealed that for Molao`s teaching strategy using technology, learners` engagement was limited to the learners answering oral questions and working on their exercises while Molao was using GeoGebra to translate shapes.

Molao was therefore asked about the use of relevant technology in relation to teaching strategies. The following question was asked:

How do you manage to get the chosen sub-topic of transformation to fit in well with the instructional strategy using relevant technology?

Molao responded as follows:

"It is very easy though this technology plays a bigger part but still I as a teacher I still ask students to do certain activities using the same technology. I still ask them some questions to reply on. I do not allow myself to do the whole part alone."

This response from Moalo reveals that he engages learners by asking them oral questions. There is no engagement of the learners with the technology to improve their knowledge on transformation. This shows that Molao has limited technological pedagogical knowledge and pedagogical content knowledge.

Mpona`s case

Table 5.4: Emergent themes and categories 4

Codes	Categories	Emergent Themes
Use of technology to teach transformation	• Technological teaching resources used in the lesson • Teaching strategies with technology	Use of technological pedagogical content knowledge when teaching transformation to Grade 12 learners
Use of technology to address learners` challenges in transformation	• The use of the selected software in the teaching and learning of transformation • The use of video to enhance visualisation and use of the Cartesian-coordinate framework to construct accurate shapes.	Use of relevant technology to address learners` challenges in the teaching and learning of transformation in Grade 12
Improving learners` knowledge of transformation using TPACK	• Teachers and learners activities • Learners` engagement with technology	Ability to use technology to improve learners` knowledge of transformation in Grade 12

5.5.7 Theme 1: Use of technological pedagogical content knowledge when teaching transformation to Grade 12 learners

During the interviews the mathematics teachers were asked questions which focused on the use of technology and pedagogy during the lesson observations. The questions which the teachers were asked with regard to the use of technology and pedagogical content knowledge are discussed below.

(a) **Technological teaching resources used in the lesson**

Mpona was observed presenting her lesson using a laptop, interactive white board, overhead projector and the Cartesian-coordinate framework as technological teaching resources. One of the questions posed to Mpona was based on the use of the selected technology to address the LGCSE mathematics syllabus. The following question was asked Mpona:

How does the selected technology in the teaching of transformation address the LGCSE mathematics syllabus?

"I chose to use a laptop connected to the interactive white board simply because I can play a video from the laptop on the teaching of transformation and it is easy to pause it and explain some concepts to my learners. This technology helps me to teach faster at the same time helping the slow learners in order to cover all the concepts of performing and describing types of transformation as required by the LGCSE syllabus."

This response from Mpona reveals that she uses technological teaching resources which help her to teach how to perform and describe types of transformation. This shows that Mpona has technological content knowledge. She however has limited technological pedagogical knowledge since she uses technological teaching resources only for demonstration.

(b) **Ability to use technology and pedagogy for content teaching**

Mpona was observed using a video on the teaching of enlargement, and then involving her learners by given them an activity on the enlargement of a shape using the Cartesian-coordinate framework. A question was posed to Mpona on the choice of technology that fits in well in the teaching of transformation.

The following question was asked:

How do you choose technology so that it fits in well in the teaching strategies for the teaching of transformation to your learners?

The following was the response from Mpona:

"I chose to use a video because it can be easily reversed or paused for the learners to ask questions for clarifications. I am also able to give my learners a similar activity as the one on the video and later on play the video on the same activity so that they can observe different approaches on how to enlarge a shape either by multiplying the sides of the shape by the scale factor or by multiplying the distance of the object point from the centre to find the position of the image point."

This response from Mpona reveals that she uses technology to engage learners in the teaching of transformation for the learners to perform transformation meaningfully. This shows that Mpona has pedagogical content knowledge, as she was able to involve learners with an activity to enlarge shapes. She however has limited technological pedagogical knowledge because the given activity did not involve the use of technology.

5.5.8 Theme 2: Teachers use relevant technology to address learners` challenges in the teaching of transformation in Grade 12

During the interviews, the mathematics teachers were asked about the use of relevant technology to address learners` challenges when teaching transformation. The questions posed to the teachers with regard to their lesson observations are discussed below.

(a) The use of the selected technology in the teaching and learning of transformation

During the observation, Mpona played a video on the enlargement of shapes from the laptop connected to the interactive white board. One of the questions posed to Mpona during the interview was on the choice of a sub-topic of transformation so that she could use a relevant technology.

Mpona was asked the flowing question:

How do you manage to make the chosen sub-topic on transformation fit in well with the instructional strategies using relevant technology?

This was Mpona's result:

"There are different applications installed in this laptop which I am using in this Maths Lab therefore I chose the video specifically on the teaching of enlargement because the teacher on the video multiplies the distance of the object point from the centre of enlargement by scale factor so then I can pause it and enlarge the same shape by writing on the interactive white board using a different approach of multiplying each side of object by scale factor to obtain the image in order to help learners who have a challenge on the enlargement of a shape."

This response reveals that Mpona uses technology to help her with different ways of enlarging a shape. This is helpful, because if her learners are challenged with a particular approach, she can quickly use another approach to help them to understand. This shows pedagogical content knowledge, which is the use of different teaching approaches to help all the learners in the class. She has, however, limited technological pedagogical knowledge because the learners are not involved in the use of technology.

(b) The use of video to enhance visualisation and the use of the Cartesian-coordinate framework to construct accurate shapes

Mpona was observed playing a video on the teaching of enlargement. In the same lesson she let her learners use the Cartesian-coordinate framework to construct a triangle to be enlarged, as well as enlarging the triangle. One of the questions posed to Mpona during the interviews was on the use of technology to construct accurate shapes in transformation. The following question was asked:

How is technology helpful in constructing accurate shapes to be transformed?

The response from Mpona was as follows:

"I chose video on the enlargement of shapes because for many years of teaching transformation I have learnt that the shapes are very accurate on the video and it also helps learners to visualise when the shape is enlarged. In the same way the rubber bands which learners attach to the pins on the Cartesian-coordinate framework are very accurate because pins are fixed at the required coordinates."

The response reveals that the use of video on the enlargement of shapes addresses the challenge of visualisation on the side of learners. In the same way the Cartesian-coordinate framework helps learners to construct an accurate shape, which can be a challenge when using mathematical instruments. The learners did not use the technology themselves, they were only given an activity on enlargement. This is indicative of Mpona's pedagogical content knowledge.

5.5.9 Theme 3: Ability to use technology to improve learners` knowledge of transformation

During the interviews the mathematics teachers were asked questions which focused on their ability to use technology to improve learners` knowledge of transformation in Grade 12. The questions posed to the teachers during the interviews are discussed below.

***(a)* Teachers' and learners' activities**

During the observation, Mpona gave her learners an activity on the enlargement of a triangle after they had watched a video on the topic. Mpona was asked a question on the choice of technology and if it fits in well with the teaching strategies of transformation.

How do you choose technology so that it fits well with the teaching strategies for the teaching of transformation?

This was Mpona's response:

"I chose to use video because it can be easily reversed or paused for the learners to ask questions for clarifications. I am also able to give my learners a similar activity as the one on the video. Later on I play the video on the same activity so that they can observe different approaches on how to enlarge a shape either by multiplying the sides of the shape by the scale factor or by multiplying the distance of the object point from the centre to find the position of the image point."

This response reveals that Mpona uses technology to give her learners activities so that they can get more knowledge on the enlargement of a shape using different approaches. From her response it can be concluded that she has pedagogical content knowledge as she was able to help her learners in the given activity to enlarge a shape. She however has limited knowledge of technological pedagogical knowledge because the lesson presentation was from a video.

***(b)* Engaging learners with technology**

Mpona was observed using a video and Cartesian-coordinate framework to engage leaners in her teaching of enlargement. One of the questions posed to Mpona during the interviews was on the engagement of learners while teaching transformation using technology.

Why do you involve learners while teaching transformation using technology?

"I engaged my leaners by asking them to enlarge a similar shape, which would be enlarged on the video, so that after they had finished their activity then they can observe the same activity but this time done on the video so that they can gain more knowledge on the enlargement of a shape."

This response from Mpona reveals that she engaged the learners in the teaching of enlargement so that they could improve their knowledge on the enlargement of a shape. However the learners` engagement was not with use of technology. The learners rather engaged in the use of the Cartesian-coordinate framework. This shows that Mpona has pedagogical content knowledge, but also that she only has limited technological pedagogical knowledge, as she did not engage her learners with the use of technology.

5.6 TRIANGULATING AND EMERGING THEMES FROM THE OBSERVATIONS AND INTERVIEWS

The results that emerged from the observations and interviews were used to develop a set of themes to be discussed as the findings. Three similar themes were revealed from the observations and interviews. These three themes are the use of technology and pedagogical content knowledge when teaching transformation to Grade 12 learners, the use of relevant technology to address learners` challenges in the teaching of transformation in Grade 12, and teachers' inability to use technology to improve learners` knowledge of transformation. These three themes were found to be specific to the research questions and they are discussed in Chapter 6 as the findings.

The specific findings identified from the emerged themes from the observations and interviews which were related to the research questions of the study are as follows:

- Use of technological pedagogical content knowledge when teaching transformation to Grade 12 learners.
- Use of relevant technology to address learners` challenges in the teaching of transformation in Grade 12.
- Ability to use technology to improve learners` knowledge of transformation.

5.7 SUMMARY

Data was presented in an orderly manner, starting with the observations which were followed by the interviews. The observations and interviews revealed three themes which were merged and triangulated as the findings to be discussed in chapter 6.

CHAPTER 6: DISCUSSION OF FINDINGS, RECOMMENDATIONS, AND CONCLUSION OF THE STUDY

6.1 INTRODUCTION

The previous chapter presented the data which was collected from the observations and interviews. This chapter presents a discussion of the findings, implications, recommendations and conclusion.

6.2 REVIEWING THE RESEARCH QUESTIONS

This study was investigating the TPACK of mathematics teachers teaching transformation in Grade 12.Therefore the study aimed to answer the following research questions:

6.2.1 Main research question

What is the technological pedagogical content knowledge of mathematics teachers teaching transformation in Grade 12?

6.2.2 Sub-research questions

1. How do Grade 12 mathematics teachers teach transformation using technology?

2. How do Grade 12 mathematics teachers use technology to address the challenges faced by learners in learning transformation?

3. How do mathematics teachers improve learners` knowledge of transformation using technology technological pedagogical content knowledge?

6.3 THE FINDINGS OF THE STUDY

The three findings which are specific to the research questions are discussed in this chapter. These findings are ability to use technological pedagogical content knowledge when teaching transformation, use of relevant technology to address learners` challenges in the teaching of transformation in Grade 12, and the ability to use technology to improve learners` knowledge of transformation. The findings are discussed in this order.

6.3.1 Ability to use TPACK when teaching transformation

The study found that all the three mathematics teachers selected from the NEPAD e-Schools project were unable to use technological pedagogical content knowledge when teaching transformation to Grade 12 learners. This finding is similar to Voog`s (2008), that integrating technology was a complex innovation for mathematics teachers in Ghana, as they had not yet developed their TPACK. Agyei and Voogt (2012) also confirms that mathematics teachers in Ghana did not integrate technology into their teaching due to their lack of technological pedagogical content knowledge, which would enable them to effectively integrate technology in instruction.

Tima and Molao used laptops and overhead projectors to display the concepts of transformations on the screen. They were able to perform transformations using technology, as was indicated by their use of GeoGebra to perform different transformations when teaching. The use of GeoGebra to transform shapes along different mirror lines and using different translation vectors was enough evidence of technological content knowledge. Moving shapes on the x-y plane using different types of transformation with the use of technology is regarded as a display of technological content knowledge (Ramatlapana, 2017).

The three mathematics teachers only used technological teaching resources to help them explain the concepts of transformation, without actually involving the learners. Two of them, Tima and Molao, used GeoGebra to perform transformations and explain the concepts of transformation to the learners. The other participant, Mpona, used a video to teach. They all used technology to teach, which technological content knowledge is. The aspect of pedagogical content knowledge was missing, however, because they used a teacher-centred approach as opposed to a learner-centred approach in their lesson presentation. Mashingaidze (2012:198) argues that the effective teaching of transformation must involve hands-on activities for the leaners so that they can acquire conceptual understanding as opposed to procedural understanding only. The teachers' explanations on the concepts of transformation corresponded to the way they performed the transformations, and this was a display of content knowledge on transformation.

The use of technological teaching resources in the teachers` lesson presentations was limited to lecturing and demonstrating. The learners were not given an opportunity to use technological teaching resources themselves to learn transformations. This pointed to limited technological pedagogical knowledge and pedagogical content knowledge from the teachers. Guerrero (2010) explains that the relation between TPACK and mathematics is beyond the knowledge of operating a specific technology, but deals with how to use a particular technology to improve mathematics teaching and learning. Therefore there was a limited pedagogical content knowledge and technological pedagogical content knowledge from the three mathematics teachers because they used only demonstrating and lecturing methods, which did not involve learners through the use of technology or any activity to discover the concepts of transformation on their own.

The mathematics teachers in the NEPAD e-school project displayed some constructs of the TPACK while some were missing. The displayed constructs of TPACK are content knowledge, technological knowledge, and technological content knowledge. On the other side the following constructs of TPACK were missing from the three mathematics teachers; pedagogical knowledge, technological pedagogical knowledge and pedagogical content knowledge.

According to Mishra and Koehler (2006), for the effective use of technology in teaching, a teacher should possess all the constructs of the TPACK, namely content knowledge, pedagogical knowledge, technological knowledge, pedagogical content knowledge, technological content knowledge and technological pedagogical knowledge. Since the three mathematics teachers do not possess all the knowledge domains of TPACK, it can be concluded that they have a limited technological pedagogical content knowledge in the teaching of transformation in Grade 12.

6.3.2 Use of relevant technology to address learners` challenges

From the three mathematics teachers observed, two of them, Tima and Molao, used GeoGebra to teach transformation while the other teacher, Mpona, used a video. Both the GeoGebra and video were relevant to the teaching of transformation more especially to address learners` challenges. The two participants here, Tima and Molao, align with the view of Bu and Schoen (2011:9), that mathematics teachers can identify the challenges faced by learners when learning transformation and then make use of software programmes such as GeoGebra to address these challenges, such as identifying the corresponding sides on the object and image, and visualising the movement of the shapes. Schmidt *et al.,* (2009) confirms that mathematics teacher, with their technological knowledge can use digital video as relevant technology in their teaching.

This software, GeoGebra, is very relevant because it has features which can generate images from the transformation of objects. The concepts of transformation were presented in different ways using GeoGebra to help the learners to understand better. This displayed the teachers' knowledge of selecting a specific and relevant technology, which is technological content knowledge. Mishra and Koehler (2006) point out that TCK is the teacher's knowledge to select a relevant technology for teaching the subject matter so that content of the subject and technology are related. The teachers were able to interpret the concepts of transformation, and presented them in different ways in order for the learners to better understand the concepts, which is a fair display of technological pedagogical knowledge.

By using the software GeoGebra the teachers were able to perform transformations in a manner that would be difficult to do on the blackboard or using paper and pencil. It assisted the teachers to quickly create diagrams and also to correct mistakes faster by using the “undo” function on the laptop – something which is awkward and challenging to do on the paper. Teachers used GeoGebra to save the activities they were doing and labeled them so that they could use these in future lessons to link the concepts. This would assist learners when learning new concepts on transformation. Karakus (2008) is of the view that GeoGebra as technology can be used by mathematics teachers to address the challenges faced by learners when learning transformation, because the designed class activities can be saved and be used later for revision of what might have been done in class. In this way the participants were able to display technological content knowledge. The construct technological pedagogical content knowledge was however missing from the teachers` lessons, because the use of technology was limited to demonstration only, so the learners were not involved in using the technology.

One of the participants, Mpona, chose to use video as the relevant technology in the teaching of transformation. She used video and activity to help learners to identify and correct their mistakes while playing back the video. The advantage of video in the classroom is that it can be paused or replayed to address learners’ challenges with the concept. This approach indicated that Mpona had technological knowledge by operating the laptop and the software on it, but the construct technological pedagogical knowledge was missing as the she did not involve learners in the use of technology to construct new knowledge in the transformation of shapes. Mathematics teachers can design activities for learners which involve the use of dynamic geometry software in order to help learners to address the challenge of constructing accurate shapes. Such software can also help learners to easily discover the properties of shapes in transformations (Mariotti, 2002).

Some mathematics teachers, Tima and Molao, used GeoGebra as a relevant technology to enhance visualisation of shapes for learners when learning transformation. It helped the leaners to identify the corresponding sides of the object and image, and to identify the congruent shapes. Moreover, visualisation was useful for identifying the corresponding sides and points, as GeoGebra labeled these on both the object and image after performing transformations. The labeling of the corresponding sides and vertices of the object and image by GeoGebra helped the learners to visualise that the object and image were congruent under the isometry transformations. Identifying and visualising the type of transformation performed on the object to obtain the image are the common challenges which learners face when learning transformation. In order to address these challenges, Karakus (2008) asserts that visualisation is important when learning transformation. Teachers can therefore use dynamic geometry software to help learners easily visualise spatial figures and explore relationships in order to develop a better understanding of and a positive attitude towards transformation.

The use of GeoGebra to address learners` challenge of visualisation shows that the teachers were familiar with their learners' background knowledge, and decided to address this – an indication of pedagogical knowledge. Bu and Schoen (2011:9) also point out that mathematics teachers have to identify the challenges faced by their learners when learning transformation. Technology such as certain software programmes can address these challenges such as identifying the corresponding sides on the object and image, and visualising the movement of the shapes. When teachers use software such as GeoGebra as technology to address learners` challenges in learning transformation, they display technological pedagogical knowledge.

6.3.3 Teachers` ability to use technology to improve learners` knowledge of transformation

The study found that all the three mathematics teachers in the NEPAD e-school project were unable to use technology to engage learners with some activities to learn transformation. They used inappropriate teaching and learning activities for their lessons, and did not engage learners with the use of technology. This is in line with the Joint Mathematical Council of the United Kingdom (JMC) (2011), that mathematics teachers do not often engage learners with the use of technology when learning. On the other hand, Tanner and Jones (2000) assert that mathematics teachers can use their technological pedagogical content knowledge to design learning activities which involve the use of technological devices in such a way that the teacher becomes a facilitator who guides learners to perform a specific type of transformation.

The three mathematics teachers did not involve the learners with activities on the use of technology to learn transformation. The teaching and learning activities in their lessons did not give learners the opportunity to gain more knowledge on the learning of transformation. The engagement with the use of technology by learners was limited to watching the video on transformation. Learners' participation was limited to answering oral questions. Teachers did not use other learning activities which could enhance the interaction between them and the learners. Mashingaidze (2012:198) argues that the effective teaching of transformation must involve hands-on activities for the leaners so that they can acquire a conceptual understanding as opposed to procedural understanding only.

The technological teaching resources were only used to demonstrate the concepts of transformation without giving learners activities to do. The lesson observation revealed that the teachers were mainly using the lecturing method to teach transformation using technology. The main lecture method was the explanation of transformation concepts with the use of GeoGebra. The use of

PCK would involve the decision of a teacher to design the type of learner activities which are specific to the content to be studied (Harris & Hofer, 2009: 101). The use of TPACK will therefore include the use technology in those designed activities (Mishra & Kohler, 2006). The limited use of interactive methods is enough evidence that the teachers had limited technological pedagogical content knowledge to help learners to improve their knowledge regarding transformation.

In this study the participants' teaching strategy was limited to demonstrating the act of transformation. No other interactive methods were used in the lesson presentation. The second component of TPACK, which is technology-based mathematics instruction, involves teacher`s knowledge and the ability to use different instructional approaches, particularly when using technology in support of teaching mathematics (Guerrero, 2010). The teachers mainly used a teacher-centred approach instead of a learner-centred approach in their teaching. The limitation of demonstration method to support the engagement of learners in using GeoGebra showed that the mathematics teachers had limited technological pedagogical content knowledge of using technology to improve learners` knowledge in the learning of transformation.

6.4 IMPLICATIONS OF THE STUDY

The findings of this study revealed that the three mathematics teachers of the schools at which the NEPAD e-school project had limited technological pedagogical content knowledge when teaching transformation in Grade 12.The implication could be that these teachers received inadequate training in TPACK when they received in-service training on the use of technology to teach transformation. There seem to be lack of support with the use of technology during the piloting of the project.

The findings revealed that the three mathematics teachers used relevant technology to address learners` challenges in learning transformation. This will motivate the learners in learning transformation because GeoGebra is able to construct accurate shapes which is usually challenging when using paper and pencil. Moreover, learners will like to learn mathematics with technology when they further their studies. This may have a result of producing mathematics teachers who like to teach using technology more often.

The three teachers in the NEPAD e-school project were not able to engage learners with some activities using technology. This leads to the delay in the development of the technological pedagogical content knowledge of the mathematics teachers. This contributes to the lack of integrating content, technology and pedagogical knowledge. The result of this is the poor performance of learners in their final exam as they will not be able to perform transformations on their own.

6.5 RECOMMENDATIONS OF THE STUDY

The recommendations with regard to this study are presented in relation to the three findings which are the use of technology and pedagogical content knowledge when teaching transformation to Grade 12 learners, the use of relevant technology to address learners` challenges when teaching transformation in Grade 12, and the use of technology to improve learners` knowledge of transformation.

6.5.1 Ability to use TPACK when teaching transformation

The findings revealed that all the three mathematics teachers have limited technological pedagogical content knowledge when teaching transformation. This study recommends that besides in-service training, these teachers should be provided with short courses which include the use of technology relevant to the teaching of geometry transformation. This will help these teachers to be competent to integrate technology into their teaching and learning to develop their TPACK. This study further recommends that teacher trainers should start training on teaching and learning of mathematics using technology. This study should be extended to explore the TPACK of all mathematics teachers in Lesotho focusing on different areas of mathematics such as circle theorems, trigonometry and graphs. Research on the effective use of technology in teacher preparation programmes in Lesotho is under-researched and therefore it is time do so.

6.5.2 Use of relevant technology to address learners` challenges

This study found that the three mathematics teachers selected in NEPAD e-school project used relevant technology to address learners` challenges when teaching transformation without engaging their learners. The study recommends workshops which can focus on the pedagogical aspects, which involve the use of technology in order to address the challenges which learners face when learning transformation in Grade 12.Teachers should also look for other opportunities such as meetings with their colleagues and experts to have discussions and micro teachings on the use of relevant technologies to engage learners.

6.5.3 Teachers` ability to use technology to improve learners` knowledge of transformation

The study found that the selected mathematics teachers who were trained in NEPAD e-school project are unable to use technology to improve learners` knowledge of transformation in Grade 12.They could not improve learners` knowledge by engaging them with some activities which involve the use of technology. Therefore this study recommends Lesotho Ministry of Education to provide workshops which focus on the learners` engagement with the use of technology in learning transformation. More professional development or mentoring opportunities need to be provided in order to support instructors and teachers with incorporating technology during instruction.

6.6 CONCLUSION OF THE STUDY

The findings of this study are related to objectives of the research. The study identified the use of technological pedagogical content knowledge when teaching transformation to Grade 12 learners, the use of relevant technology to address learners` challenges in the teaching of transformation in Grade 12, and the use of technology to improve learners` knowledge of transformation. The study found that the selected mathematics teachers at NEPAD e-school project lack the technological pedagogical content knowledge when teaching transformation. It is recommended that a similar study be conducted nationally which should focus on the challenges faced by mathematics teachers when teaching transformation in Grade 12. Such a study will assist in helping teachers to overcome the challenges they face when teaching mathematics using technology.

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APPENDICES

Appendix A: Approval of ethical clearance

UNIVERSITY OF THE FREE STATE
UNIVERSITEIT VAN DIE VRYSTAAT
YUNIVESITHI YA FREISTATA
UFS UV

GENERAL/HUMAN RESEARCH ETHICS COMMITTEE (GHREC)

29-Jul-2019

Dear Mr Mokone, Retselisitsoe RE

Application Approved

Research Project Title:
The Technological Pedagogical Content Knowledge of Mathematics Teachers teaching Grade 12 transformation

Ethical Clearance number:
UFS-HSD2019/0080/2507

We are pleased to inform you that your application for ethical clearance has been approved. Your ethical clearance is valid for twelve (12) months from the date of issue. We request that any changes that may take place during the course of your study/research project be submitted to the ethics office to ensure ethical transparency. furthermore, you are requested to submit the final report of your study/research project to the ethics office. Should you require more time to complete this research, please apply for an extension. Thank you for submitting your proposal for ethical clearance; we wish you the best of luck and success with your research.

Yours sincerely

Prof Derek Litthauer
Chairperson: General/Human Research Ethics Committee

Digitally signed by Derek Litthauer
Date: 2019.07.30 11:24:19 +02'00'

205 Nelson Mandela Drive/Rylaan
Park West/Parkwes
Bloemfontein 9301
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www.ufs.ac.za

Appendix B: Application letter to conduct research: Department of Education Lesotho

UNIVERSITY OF THE FREE STATE
UNIVERSITEIT VAN DIE VRYSTAAT
YUNIVESITHI YA FREISTATA
UFS UV

Letter to the Ministry of Education

The Ministry of Education and Training
Senior Education Officer
P.O. Box 14
Maseru 100
Lesotho

February 2019

Dear Sir/Madam

RE: REQUEST FOR THE PERMISSION TO CONDUCT RESEARCH IN SOME HIGH SCHOOLS IN THE MASERU DISTRICT

My name is Retselisitsoe Mokone. I am currently enrolled for my master`s degree in the faculty of education (student number: 2015271709) at the University of the Free State under the school of Mathematics, Science and Technology Education. As part of my Master's degree, I am conducting research within the field of mathematics education. The study is titled: **The Technological Pedagogical Content Knowledge of Mathematics Teachers teaching Grade 12 transformation.** This research is aimed at investigating the technological pedagogical content knowledge of mathematics teachers from selected high schools in Maseru Lesotho teaching transformation in Grade 12 using technology.

For this reason, I wish to request your permission to collect data from two High schools in the Maseru district. The reason for choosing these schools it is because they are already using technology to teach mathematics.

Participation will require that four teachers be observed while teaching mathematics using technology in grade 12. There will also be video-capturing for the lesson observations. The four teachers will also be interviewed in semi-structured interviews which are estimated to take 40 minutes to an hour per participant. The interview process will take place in the school premises, at a time that will not interfere with teaching. The research participants will not be advantaged or disadvantaged in any way. They will be reassured that they can withdraw their consent to participate at any time during this project without any penalty. There are no foreseeable risks in participating in this study. The participation in this study is voluntary and so participants are not going to be paid for taking part in this study. The data that will result from this study will be documented in a research report and it is envisaged that the research findings be used for academic purposes including academic conference presentations and publications.

All research data will be stored in a safe place and it will be destroyed between 3-5 years after the research has been completed. The research is proposed to take place in March, and the research process is estimated to be conducted over a period of two weeks.

Yours Faithfully,

…………………..

Retselisitsoe Mokone

If you have any queries or questions you would like to ask, please don't hesitate to contact me or my supervisor at the contact details below:

Name of the student: Mokone ER	Name of the supervisor: Dr.Rabaza M
Cell-phone : +266 58776703	Tell number : +2751 401 2307
Email of the student mokoneer@yahoo.com	Email of the supervisor RabazaM@ufs.ac.za

Appendix C: Application letter to the school Principals

Letter to the Principal

……………………………..

……………………………..

February 2019

Dear Principal

RE: REQUEST FOR PERMISSION TO CONDUCT RESEARCH AT YOUR SCHOOL

My name is Retselisitsoe Mokone. I am currently enrolled for my master`s degree in the faculty of education (student number: 2015271709) at the University of the Free State. As part of my Master's degree, I am conducting research within the field of mathematics education. The study is titled: **The Technological Pedagogical Content Knowledge of Mathematics Teachers teaching Grade 12 transformation.** This research is aimed at investigating the technological pedagogical content knowledge of mathematics teachers from selected high schools in Maseru Lesotho teaching transformation in Grade 12 using technology.For this reason, I wish to request your permission to collect data from your school.

My research involves conducting semi-structured interviews with two teachers. In order to participate in the study interested teachers need to be mathematics teachers who have experience of teaching mathematics using technology and must be teaching grade 12 currently. In order to collect data mathematics teachers will be observed while teaching transformation in grade 12 using technology and they will also be interviewed. The interviews will be held at school at the time convenient to you. The interviews will be audio-recorded with teachers consent after I presented them with the consent form to sign. The interviews will be held for approximately an hour depending on teachers' responses. I therefore request your permission to interview and observe two teachers from your school. Teachers' names and the name of the school will be kept confidential at all times and in all academic writing about the study. Neither the school nor the teachers will be advantaged or disadvantaged in any way. There are no foreseeable risks in participating and no one will be paid for this study.

All research data will be stored in a safe place and it will be destroyed between 3-5 years after the research has been completed. The research is proposed to take place in March, and the research process is estimated to be conducted over a period of two weeks.

Your participation will be highly appreciated.

Yours Faithfully,

…………………………..

Retselisitsoe Mokone

If you have any queries or questions you would like to ask, please don't hesitate to contact me or my supervisor at the contact details below:

Name of the student: Mokone ER	Name of the supervisor: Dr.Rabaza M
Cell-phone : +266 58776703	Tell number : +2751 401 2307
Email of the student mokoneer@yahoo.com	Email of the supervisor RabazaM@ufs.ac.za

Appendix D: Approval to conduct research in some high schools in the Maseru district

THE KINGDOM OF LESOTHO
MINISTRY OF EDUCATION AND TRAINING
MASERU DISTRICT EDUCATION OFFICE
P.O. BOX 47. MASERU 100.
28810001 / 22 322 755

7 May 2019

The Principal
Lesotho High School
Maseru 100

Dear Sir/Madam

RE: RESEARCH

"The Technological Pedagogical Content Knowledge of Mathematics Teachers teaching Grade 12"

Mr. Retšelisitsoe Mokone is a student who is conducting a research on the above stated topic. He therefore wishes to carry out a research at Lesotho High Schools.

You are kindly requested to provide him with the information that he may require.

Thanking you in advance for your usual support.

Yours Faithfully

MASERU DISTRICT
EDUCATION OFFICE
07 MAY 2019
P.O. BOX 47, MASERU - LESOTHO
TEL.: 22322755/22313709

MPITI LEMENA (MR)
DISTRICT EDUCATION MANAGER (ai) - MASERU

Appendix E: Information leaflet for teachers

UNIVERSITY OF THE FREE STATE
UNIVERSITEIT VAN DIE VRYSTAAT
YUNIVESITHI YA FREISTATA
UFS UV

LETTER TO TEACHERS

……………………………

February 2019

Dear Teacher

REQUEST FOR PERMISSION TO CONDUCT RESEARCH

My name is Retselisitsoe Mokone. I am a Master`s student (student number: 2015271709) in the Faculty of Education at the University of the Free State. As part of my Master's degree, I am conducting research within the field of mathematics education. The study is titled: **The Technological Pedagogical Content Knowledge of Mathematics Teachers teaching Grade 12 transformation.** This research is aimed at investigating the technological pedagogical content knowledge of mathematics teachers from selected high schools in Maseru Lesotho teaching transformation in Grade 12 using technology.

If you would like to voluntarily participate in my study, I would like to interview you for approximately an hour. In order to participate in the study you need to be a mathematics teacher and also currently be teaching in grade 12.The interviews will be held at school at the time convenient to you. The interviews will be audio-recorded with your consent after I presented you with the consent form to sign. I therefore request your participation in the study and I request to observe you while teaching transformation in grade 12 using technology. The lesson will be both audio taped and video recorded with your permission. The reason why I need to video record the lesson is to make sure that I capture the whole lesson including the non verbal interaction. Your name and identity will also be kept confidential at all times and in all academic writing about the study. Your individual privacy will be maintained in all published and written data resulting from the study.

You will not be advantaged or disadvantaged in any way. Your participation is voluntary, so you can withdraw your permission at any time during this project without any penalty. There are no foreseeable risks in participating and you will not be paid for this study.

Your participation will be highly appreciated

Yours faithfully,

……………………….

Retselisitsoe Mokone

Appendix F: Teacher`s consent form for teachers

CONSENT TO PARTICIPATE IN THIS STUDY

I, ____________________________________ (participant name), confirm that the person asking my consent to take part in this research has told me about the nature, procedure, potential benefits and anticipated inconvenience of participation.

I have read (or had explained to me) and understood the study as explained in the information sheet. I have had sufficient opportunity to ask questions and am prepared to participate in the study. I understand that my participation is voluntary and that I am free to withdraw at any time without penalty . I am aware that the findings of this study will be anonymously processed into a research report, journal publications and/or conference proceedings.
I agree that research be conducted in my school.

I, _______________________________________give my consent for the following:

Permission to be audio taped

I agree to be audio taped during the interview or observation lesson YES/NO

I know that the audiotapes will be used for this project only. YES/NO

Permission to be video taped

I agree to be videotaped as I teach. YES/NO

I know that the video tape will be used for this project only. YES/NO

Permission to be interviewed

I would like to be interviewed for this study. YES/NO

I know that I can stop the interview at any time and don't have to answer all the questions asked. YES/NO

Informed Consent

I understand that: my name and information will be kept confidential and safe and that my name and the name of my school will not be revealed. I do not have to answer every question and can withdraw from the study at any time. I can ask not to be audio taped, photographed and/or videotaped.

Sign______________________________ Date____________________________

I have received a signed copy of the informed consent agreement.

Full Name of Participant: ____________________________________

Signature of Participant: __________________ Date: _____________________

Full Name(s) of Researcher(s): ________________________________

Signature of Researcher: __________________ Date: _____________________

Appendix G: Classroom observation schedule

Classroom observation checklist

A. **CONTENT KNOWLEDGE (CK):** Knowledge about transformation being how to perform and describe different types of transformation.

(c) Knowledge on how to find	**Bad**	**Good**	**Excellent**
(i) Line of reflection	☐	☐	☐
(ii) Translation vector	☐	☐	☐
(iii) Centre of enlargement, scale factor in enlargement	☐	☐	☐
(iv) Centre of rotation, angle of rotation and direction	☐	☐	☐
(v) Invariant line and scale factor of stretch	☐	☐	☐
(vi) Invariant line and scale factor of shear	☐	☐	☐
(vii) Matrix of transformation	☐	☐	☐

Comments:

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(d) Knowledge on how to perform different types of transformation

(vi) Use of the given mirror line to reflect an object	☐	☐	☐
(vii) Use of the given translation vector to translate an object	☐	☐	☐
(viii) Use of the given centre, angle and direction to rotate the object.	☐	☐	☐
(ix) Use of the given centre and scale factor to enlarge an object.	☐	☐	☐

(x) Use of the given invariant line and scale factor to apply either shear or stretch. ☐ ☐ ☐

(xi) Use of the given matrix to transform an object. ☐ ☐ ☐

Comments:

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…………………………………………………………………………………………………

B. PEDAGOGICAL KNOWLEDGE (PK): Methods and processes of teaching transformation.

	Bad	Good	Excellent
(c) Knowledge of helping students to be actively involved in constructing their own understanding.	☐	☐	☐
(d) Knowledge of helping students to discover concepts of transformation.	☐	☐	☐

Comments:

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C. TECHNOLOGICAL KNOWLEDGE (TK): Knowledge about different technologies, including both low-tech and high-tech technologies.

	Bad	Good	Excellent
(c) Knowledge of using ICT tools such as google documents and U-tubes from internet to teach transformation.	☐	☐	☐
(d) Knowledge on how to use digital technologies, computer software, cell phones applications and IPads for the teaching and learning of transformation.	☐	☐	☐

Comments:

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D. TECHNOLOGICAL CONTENT KNOWLEDGE (TCK): Knowledge on how to select a specific technology relevant for teaching a subject matter.

	Bad	Good	Excellent
(e) Knowledge on how to use basic skills of computer software to drag figures, label points, measure lengths and to construct different shapes.	☐	☐	☐
(f) Knowledge on how to use ICT tools to perform different types of transformation.	☐	☐	☐
(g) Knowledge of selecting relevant digital technologies to find			
(vi) Centre, angle and direction in rotation	☐	☐	☐
(vii) Centre and scale factor of enlargement	☐	☐	☐
(viii) Line of reflection	☐	☐	☐
(ix) Translation vector	☐	☐	☐
(x) Invariant line and scale factor of shear	☐	☐	☐
(xi) Invariant line and scale factor of stretch	☐	☐	☐
(h) Knowledge of constructing accurately different shapes using technological devices.	☐	☐	☐

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E. PEDAGOGICAL CONTENT KNOWLEDGE (PCK): The content knowledge that deals with the teaching processes.

	Bad	Good	Excellent
(f) Knowledge on how to use specific strategies or approaches for teaching how to perform different types of transformation and how to describe each type of transformation fully.	☐	☐	☐

(g) Knowledge on how to address the way learners think about to perform different types of transformation and how to describe each type of transformation. ☐ ☐ ☐

(h) Knowledge on how to identify and address learners` misconception in the learning of transformation. ☐ ☐ ☐

(i) Knowledge on how to meet the curriculum objectives in the teaching of transformation. ☐ ☐ ☐

Comments:

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F. TECHNOLOGICAL PEDAGOGICAL KNOWLEDGE (TPK): Knowledge of how different technologies can be used in teaching and the understanding that using technology can affect how to tech.

Bad Good Excellent

(d) Knowledge on how to use ICT tools, IPads, or digital technologies to help students to perform different types of transformation and to describe them fully. ☐ ☐ ☐

(e) Knowledge of helping students to construct shapes accurately and transform them using technology. ☐ ☐ ☐

(f) Knowledge of creating more than one example to teach students to use digital technologies to perform different types of transformation. ☐ ☐ ☐

Comments:

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TECHNOLOGICAL PEDAGOGICAL CONTENT KNOWLDEGE (TPCK): Knowledge required by teachers when integrating technology into their teaching the content of a subject area.

Bad Good Excellent

(c) Knowledge of integrating technological content knowledge, pedagogical content knowledge and technological pedagogical knowledge in the teaching of transformation. ☐ ☐ ☐

(d) Designs an appropriate activity with the technology that assists students in learning concepts of transformation. ☐ ☐ ☐

Comments:

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END

Appendix H: Interview schedule

INSTRUMENTS ON THE SEMI-STRUCTURED INTERVIEWS

1. How do you perform different types of transformation?

2. How do you teach students to perform different types of transformation?

3. Which type of software is more relevant in the teaching of transformation?

4. How is the selected technology in the teaching of transformation address the LGCSE mathematics syllabus?

5. How do you use technology to teach transformation to the grade 12 students?

6. How do you make the chosen sub-topic on transformation to fit well in the instructional strategies using relevant technology?

7. How do you choose technology so that it fits well in the teaching strategies for the teaching of transformation to your learners?

8. Why do you involve learner's learners while teaching transformation using technology?

9. Why do you use technology to address learners` misconceptions in the learning of transformation?

10. How is technology helpful in constructing accurate shapes to be transformed?

Appendix I: Proofread letter

To whom it may concern

This is to state that the Master’s degree dissertation titled *The Technological Pedagogical Content Knowledge of Mathematics Teachers Teaching Grade 12 Transformation* by Rets`elisitsoe Eszekiel Mokone has been language edited by me, according to the tenets of academic discourse.

B.Bibl.; B.A. Hons. (English)
25-08-2020

Appendix J: Turnitin Originality Report

- Processed on: 09-Aug-2020 21:53 SAST
- ID: 1331541522
- Word Count: 36340
- Submitted: 11

DISSERTATION SUBMISSION By Retselisitsoe Mokone

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Submitted to University of Central Lancashire on 2016-09-01
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http://eprints.ioe.ac.uk/19941/2/CLARK-WILSON%2C%20A_Redacted.pdf
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