# ENHANCING THE TEACHING AND LEARNING OF ALGEBRAIC EXPRESSIONS AND EQUATIONS THROUGH REASONING IN GRADE 9

By

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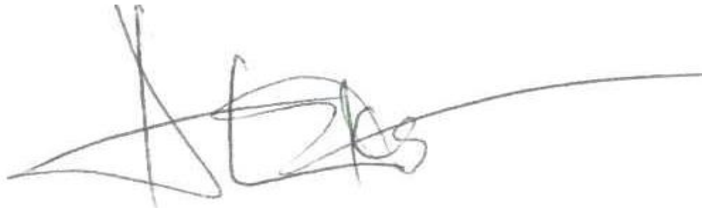
# DECLARATION

I declare that the study hereby submitted for the qualification of Doctor of Philosophy degree at the Faculty of Education, University of the Free State, is my own independent work, and that I have not previously submitted it for a qualification at any other University.





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# DEDICATION

This work is dedicated to my late mom, ‘M’e ‘Malesang Florina Lika, from whom I have inherited this big, but a good heart!

I couldn’t have asked for more!

# SUMMARY

This study sought to enhance the teaching and learning of algebraic expressions and equations using reasoning in grade 9. It is a participatory action research study underpinned by bricolage paradigm and construction theory. The underpinnings guide and enable learners to use what is at their disposal to construct reasoning constructs. The constructs help learners to forge rich algebraic and mathematical conceptual connections and interrelations. In this manner, the constructs help the instruction (teaching and learning) to achieve deep conceptual understanding (DBE 2011:8) rather than limiting it to procedural orientation. The evidence presented in literature about international best practices shows that procedure-oriented instruction and procedure fluency are, and should be, nested in conceptual knowledge. This study initiated the instruction that ensures that the nesting does not manifest in nature only, but throughout the teaching and learning processes as well. The initiative draws from the South African curriculum policy, CAPS, which requires that learners should achieve a deep conceptual understanding of the subject matter (DBE 2011:8).

The operationalisation of reasoning to conceptualise procedural instruction draws from the underutilisation of the reasoning skill, despite it being the curriculum policy imperative. Reasoning attaches sensible meaning (Yackel 2001:1) to algebraic content matter and provides direction and cushion for logical arguments aimed at attaining high order cognition. Pursuant to the study underpinnings, the reasoning-based instruction deploys learners' own reasoning constructs to ensure participatory and contextual conceptualisation. In the process, learners develop critical thinking and high order cognitive skills. These are the skills that the learners are expected to attain from the meaningful learning (Pramesti & Retnawati 2019:3) of algebra and mathematics inspired in the reasoning-based instruction.

The study has come up with the components of solution and strategies to address the research question and challenges underlying the research. The primary challenge that guides the study, namely the non-alignment between the instruction and requirements of the curriculum policy, manifests under procedure-oriented instruction; assessment; teachers' competences and curriculum-time contestation. In addition, the abstraction and complexity of algebra amidst insufficient basic mathematics competency, escalate

the supremacy of algebra in the teaching and learning of mathematics. The net resultant thereto is an inherent sifting nature of algebra.

The data analysis and interpretation presented enough evidence that the reasoning-based instruction is couched in multi-layered components of solution and strategies that respond adequately to the research question. The instruction proved the potential to break through the integrated challenges underlying algebraic instruction. The process of conceptualisation is an encompassing component of the solution. It entails contextualising and concretising textual representations of algebra in a manner that the representations make meaningful sense to learners; so much that the learners can apply algebra purposefully in advanced mathematics and related learning. Contextualisation often involves refocusing, integration and re-organisation of content matter in a manner that meets the subject and learners' needs. Concretisation includes the use of materials and examples within learners' reach to explain algebraic concepts.

The analysis of the conditions necessary for successful implementation and that of the risks and threats likely to impede the implementation reaffirmed the sustainability of the reasoning-based instruction. The indicators of success confirmed that the study has succeeded in the reform, transformation and enhancement of the teaching and learning of algebra as sought and anticipated. The study has further empowered co-researchers to use what is at their disposal to develop sustainable solutions. It can then be concluded that the research empowered the initiatives to overcome the 'lock-ins' to existing protocols and approaches, which have not been effective for the majority of teachers' and learners' populations in South Africa and the world.

# OPSOMMING

Hierdie studie mik daarop om 'n strategie te ontwerp om die onderrig en leerproses van algebraïese uitdrukkings en vergelykings in graad 9 te verbeter. Dir towrdsommer ook deur gebruik te maak van die redenasieproses. Dit is 'n deelnemende proses wat ondersteun word deur die opbou van die redenasieproses as bricoleur; gemik op ‘n beter begrip; eerder as om dit tot prosedurele beginsels te beperk. Die stawende literatuur oor internasionale beste praktyke, toon dat prosedureel-georiënteerde onderrig en die vloeibaarheid van prosedures gabasseer moet word in konseptuele kennis.

Die gebruik van redenasies om prosedurele instruksies te konseptualiseer, spruit uit die onderbenutting daarvan, alhoewel dit 'n noodsaaklike vaardigheid is waarvoor die kurrikulumbeleid, CAPS, voorsiening maak. Daar is gevind dat die redenasieproses sinvolle betekenis aan algebraïese inhoud heg en rigting en ondersteuning bied vir logiese argumente, wat op hul beurt daarop gemik is om hoër kognisie te bereik. Ingevolge die grondvlak van hierdie studie, gebruik die strategie leerders hul eie redenasievermoëns om deelnemende en kontekstuele konseptualisering te verseker. In hierdie proses ontwikkel leerders kritiese denke en hoër-orde kognitiewe vaardighede, wat nodig is vir die betekenisvolle begrip van algebra.

Die studie het met strategieë en oplossings vorendag gekom om die navorsingsvraag en die uitdagings onderligend daaraan aan te spreek. Die primêre uitdaging van die studie, naamlik die ontwrigting tussen die opdrag en vereistes van die beleid, manifesteer onder prosedure-georiënteerde onderrig, assessering, die vaardighede van onderwysers en die stryd vir kurrikulumtyd. Daarbenewens verhoog die abstraksie en kompleksiteit van algebra, te midde van onvoldoende vaardighede in basiese wiskunde, die belangrikheid van algebra in die onderrig en leer van wiskunde, en word die inherente siftigsaard daarvan blootgelê.

Die navorsingsanalise en -interpretasie het genoegsame bewyse gelewer dat die redenasie-gebaseerde strategie gevestig is in strategieë en oplossings met veelvuldige lae wat voldoende reageer het op die navorsingsvraag, en die potensiaal het om die kontekstueel geïntegreerde uitdagings van hierdie studie te staaf. Die konseptualiseringsproses is 'n omvattende komponent van die oplossing. Dit behels

om die kontekstualisering en konkretisering van die teksvoorstellings van algebra op 'n manier sinvol te maak vir leerders; in so ‘n mate dat leerders dit kan toepas in verwante vakrigtings soos tydens die navorsingsproses. Kontekstualisering behels dikwels die herfokus, integrasie en herorganisasie van inhoud op 'n manier wat voldoen aan die behoeftes van die leerders. Konkretisering behels die gebruik van items en voorbeelde binne leerders se bereik om algebraïese konsepte te verklaar.

Die analise van die voorwaardes wat benodig word vir suksesvolle implementering, en die risiko's wat die implementering kan belemer, het die volhoubaarheid van die strategie bevestig. Die sukses het bevestig dat die studie daarin geslaag het om die onderrig- en leerproses van algebra te hervorm, te transformeer en te verbeter soos verwag is. Die studie het mede-navorsers verder bemagtig om dit wat tot hul beskikking is, te gebruik om volhoubare oplossings te ontwikkel. Daar kan dan tot die gevolgtrekking gekom word dat die navorsing die inisiatiewe bemagtig het om die houvas van bestaande protokolle en benaderings te oorkom, wat nie effektief was vir die meerderheid van die onderwysers en leerders in Suid-Afrika en die wêreld nie.

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# ABBREVIATIONS AND ACRONYMS

| | |
|---|---|
| AMESA | Association for Mathematics Education of South Africa |
| ANA | Annual National Assessments |
| CAPS | Curriculum and Assessment Policy Statement |
| CDA | Critical discourse analysis |
| DBE | Department of Basic Education |
| DH | Departmental Head |
| DHET | Department of Higher Education and Training |
| FAI | Free attitude interviews |
| IQMS | Integrated Quality Management System |
| JLS | Japanese Lesson Study |
| PAR | Participatory action research |
| PLC | Professional Learning Communities |
| SAGM | Subject Assessment Guidelines for Mathematics |
| UFS | University of the Free State |

# CHAPTER 1
# OVERVIEW OF THE STUDY

## 1.1. INTRODUCTION

This study sought to enhance the teaching and learning of algebraic expressions and equations using reasoning in grade 9. This chapter outlines the study overview. The overview includes the background surrounding the teaching and learning of algebra and mathematics in grade 9, problem statement, framework, operational concepts, literature overview, research design and methodology, data analysis, implementation of the reasoning-based instruction, findings and recommendations, value of research and ethical considerations.

## 1.2. BACKGROUND

The researchers conceptualised (see 4.5.2) that it could have been appropriate to evolve or enhance the traditional ways of teaching and learning the theory-heavy disciplines like mathematics when the new curriculum policy, Curriculum and Assessment Policy Statement (CAPS), was introduced (Pramesti & Retnawati 2019:1). The theory-heavy disciplines are those in which learning is traditionally centred on teacher-structured environments (Vos 2018:2). Traditional teaching is coined by mathematics education researchers as a bad, teacher-centred and inefficient when contrasted with methods that are considered to be relatively good (Osborne 2021:2). The initiative to evolve the instruction should have picked and guarded against glaring repercussions of the omission thereof. The curriculum policy expectations in respect to mathematics performance, for example, have changed substantially and requires relatively more than the curriculum policy statements that came before it. Over and above the heightened application and high order cognition demand (DBE 2011:9; Osborne 2021:6), the minimum performance level was adjusted upward from 30% to 40% for promotion (South Africa, Department of Basic Education (DBE) 2011:154). In general, the curriculum policy has high expectations, as discussed in 1.2.1, 1.2.2 and 1.2.3. It requires an instruction that embraces active and critical learning other than uncritical and rote learning (DBE 2011:4) among other high-order principles and skills. Meanwhile, there is enough evidence to show that the instruction of algebra and mathematics has always been overshadowed by procedure-oriented instruction. Procedural aspects of knowledge relate to uncritical and rote learning, while active and critical learning is associated with deep conceptual understanding (see 2.3.2). The curriculum policy (DBE 2011:8) is specific about learners attaining the latter. Further, Long (2005:61) affirms that procedural knowledge (fluency) is and should be nested in conceptual knowledge.

Though perceived abstract and complex, algebra constitutes a higher weighting (35%) of grade 9 mathematics curriculum (DBE 2011:11) and plays a pivotal role between the basic mathematics upon which it builds, and the learning of high order (advanced) mathematics and related subjects (see 3.2.1.2). As a result, the repercussions of the procedure-oriented instruction (of algebraic expressions and equations) replicate further in high order (advanced) mathematics and related subjects. This is because the application of the algebraic principles thereof is not conceptually rooted (i.e. it is not based on conceptual understanding). In the worst but most scenarios, many learners lack the grounding basic mathematics competency and rich algebraic conceptual connection and interrelation(s). Hence they cannot apply algebra to conceptualise high order mathematics on their own.

It is assumed that the deficiency results from the glaring non-alignment between the curriculum demands (imperatives) and instruction. The non-alignment poses a grave threat to the meaningful teaching and learning of algebra (Pramesti & Retnawati 2019:3), and the functionality (application) thereof. This study is aimed at promoting learners' active participation in constructing their own reasoning (constructs). The constructs promote an instruction that aligns with the curriculum policy imperatives. The study is based on the presupposition that it is only when the majority of learners could make sense of algebra and access its conceptual network (connection and interrelations) that they could meet the set performance standards. The following sub-sections discuss the curriculum policy expectations and implications that inform the need to evolve or enhance the instruction.

### 1.2.1. Curriculum principles

The principles of CAPS outline the expectations of mathematics curriculum instruction. The expectations include to achieve the country's critical education outcomes (Mosia 2016:135–136). The following principles, that are relevant to this study, draw from the

Constitution of South Africa, in particular, the provision about seeking to improve the quality of life of all citizens and free the potential of each person (DBE 2011:foreword):

#### ***1.2.1.1. Active and critical learning: encouraging an active and critical approach to learning, rather than rote and uncritical learning of given truths***

This study anchors on the philosophy, theories and methodologies that support active participation of learners (see 4.2). It aims to achieve high order cognition and critical learning of which learners' reasoning constructs form a basis. The reasoning-based instruction promotes the construction of viable (logical) arguments and critiquing of reasons amongst learners (Mahlomaholo 2014:173; Rumsey & Langrall 2016:413). The reasoning constructs promote systematic connection of different aspects of knowledge to attain (proficient knowledge) improved product (Barker 2004:43; Kaomea 2016:10). It can therefore be argued that the instruction adheres to the principle and maintains legitimation (Kincheloe 2004:687) in this regard.

#### ***1.2.1.2. High knowledge and high skills: the minimum standards of knowledge and skills to be achieved at each grade are specified and set high, achievable standards in all subjects***

Different taxonomies rank reasoning as a high order skill (see 2.3.4), and confirm its portrayal as an agent of high knowledge and skill development in the curriculum policy (DBE 2011:9). Reasoning measures learners' ability to analyse, evaluate and synthesise provided information to conceptualise the relationship(s) among parts (Pearson n.d.:1). Learners that have acquired reasoning skills develop the habits of analysing a problem, implementing a strategy, seeking and using connections and reflecting on a solution (National Council of Teachers of Mathematics [NCTM] 2000). It is further confirmed that the high knowledge and high skill (cognition) inherent of reasoning are accessible to every (learner) child (Dowden 2017:1; NCTM 2000).

#### ***1.2.1.3. Progression: content and context of each grade shows progression from simple to complex***

The reasoning-based instruction subscribes to Katehi's (2009:12) view of developing knowledge from known to unknown, from less to more complex and from concrete to abstract content for the systematic and contextualised conceptualisation of content matter (see 5.3.1.1). The systematic progression (development) is embedded within the instruction's characteristic of placing basic mathematics central to the teaching and learning of algebraic expressions and equations. The instruction advocates for assessing if learners are firmly founded with applicable basic mathematics before introducing each curriculum component of algebra. The advocacy is consistent with the study framework in examining (assessing) the socio-historical dynamics of the (traditional instruction) problem (Rogers 2012:10). The framework frames the solution within bricolage connection of scattered information to produce (construct) an improved product (knowledge).

#### *1.2.1.4. Credibility, quality and efficiency: providing an education that is comparable in quality, breadth and depth to those of other countries*

The framework underpinnings of the reasoning-based instruction namely, the bricolage and constructivism theories assure high order cognition instruction, and provision of quality education offered in accordance with the curriculum policy imperatives (legitimation). Bricolage is a legitimating mechanism that maintains the imperatives amid structural changes (Desa 2012:727), while the constructivist theory is renowned for its credibility and efficiency in ensuring learners' active participation. The use of reasoning constructs (rich connections) in view of achieving deep conceptual understanding is inherent to quality knowledge (Star 2005:407). The advocacy for recruitment of qualified mathematics teachers (Association for Mathematics Education of South Africa [AMESA] 2018:2) and teachers' professional development and support (see 3.2.2.1(c)) guarantees the befitting abilities (competence) and effective instruction. The recruitment and development are guided by the want to achieve at least the minimum requirements for Teacher Education Qualifications for a newly qualified teacher (South Africa, Department of Higher Education and Training [DHET] 2015:62). The study further reveals and shares sentiments of the Department of Education to review algebra and mathematics curricula to provide globally competitive education within the (South African) authentic context (Vos 2018:1).

### 1.2.2. Curriculum skills

Among the specific skills the curriculum policy requires the teaching and learning of mathematics to consider, the following has direct relevance to this study:

> *"…to listen, communicate, think, reason logically and apply the mathematical knowledge gained"* (DBE 2011:9)

The active and collaborative participation central to the execution of the reasoning-based instruction develops listening and communication skills. The methodology underpinning the study, participatory action research (PAR), promotes active participation of the researched and learners' collaborations (see 4.2). It shapes the approach thereof through the method or technique of free attitude interviews (FAI), also embedded in the study. The communicative participation provides necessary platform for shaping and sharpening learners' reasoning skills. That way, the reasoning-based instruction becomes a complete key in driving logical arguments (constructs) aimed at developing conceptual understanding (DBE 2011:8; Pramesti & Retnawati 2019:3) and critical thinking. It becomes productive in shaping (and optimising) reality (Kollosche 2021:473). Critical thinking is embraced in active and critical learning of high order cognition (see 1.2.1.1), and manifests in the application of algebra (knowledge gained) in advanced mathematics and related subjects, for example.

### 1.2.3. Content matter and assessment

According to the curriculum policy document, the content area of patterns, functions and algebra collectively account for approximately 35% of the grade 9 mathematics curriculum (DBE 2011:11). It is, however, noticeable (common) that patterns, functions and other advanced topics in mathematics apply algebraic expressions and equations. The application extends further to other related subjects. It can then be deduced that the net weighting of algebraic expressions and equations content and application considered along with their pivotal role between arithmetic and advanced mathematics, escalates them to a determinant bedrock. The minimum promotion performance level of mathematics set at 40% (DBE 2011:154) further explains the determination. The teaching and learning of algebra have however rendered the subject (algebra) very difficult for the majority of learners (Annual National

Assessments [ANA] Diagnostic Report 2014a:56-59; O'Brien n.d.:9). The abstraction and complexity (difficulty) embedded in algebra are confirmed by a repeated trend of below-average performance in grade 9 mathematics since the implementation of the curriculum policy in 2014. The statistical representations of the national average percentage marks in 2012-2014 ANA (Figure 6.1) provides a comparative analysis of grade 9 mathematics performance.

The problem of below-average achievement in algebra and mathematics is noticeable in many countries across the globe. Trends in International Mathematics and Science Study (TIMSS 2015) reports on below-average performance in Botswana, South Africa, Egypt, New Zealand and other countries by eighth-grade learners. Mbugua, Komen, Muthaa et al. (2012:87) confirms that the mathematics performance of secondary schools in Kenya has remained poor over the years, while Törnroos (2002:1) reports about low performance for grades 7 to 9 in algebra in Finland. Canadians recorded the recent dissatisfaction about dropping to 13th position in mathematics rankings (Chernoff 2019:74). In South Africa, the situation has compelled the Department of Basic Education to reconsider the minimum mathematics pass mark for promotion. The basis for reconsideration includes the impact of underperformance on socio-economic issues among other factors. For a couple of years now (2015–2019), the department has reduced the mathematics pass mark set at 40% to 30% and below (eNCA 2016). I need not overemphasise that the Department of Education directed that a pass in mathematics was no longer mandatory for promotion in 2021 (DBE 2021 promotion circular). This curriculum policy waiver that came as an aftermath of Covid 19 pandemic cannot be looked at in isolation of the algebra instruction (see 3.2.1.1(a)).

Besides the promotional deviation, the inspection and moderation reports also show that the school-based assessment does not adhere to the prescribed standards. As a result, the school-based assessment fails to prepare learners for standardised (common) papers set and moderated at the provincial Department of Education level. The efforts by the department to upgrade school-based assessment standards are crippled by: (i) insufficient supervision and/or moderation to ensure that school-based assessment papers conform to the set standards; (ii) some teachers' formations do not support the standardised assessment system (Annexure B). There is enough evidence to prove that efforts taken to improve grade 9 mathematics performance have not succeeded to bring significant change. Scores of learners could not make an overall 30% mark to qualify for grade 10 mathematics enrolment, for example. To that effect, the number of matric learners who sat for mathematical literacy papers in the years 2015–2019 surpassed those who sat for mathematics.

### 1.2.4. Appropriateness of the reasoning-based instruction

The framing and operationalisation of reasoning derive from the curriculum policy imperatives (see 1.2.2). It is assumed that reasoning is not substantially utilised, if at all. The underutilisation limits learners from invoking logical argument(s) and connections leading to deep conceptual understanding required by the curriculum policy (DBE 2011:8; see 5.2.3.3). The other discovery, related to the underutilisation of reasoning, was a disregard of learners' involvement in the creation of knowledge against the imperative of active and critical learning (see 1.2.1.1). The ability of reasoning to weave (connect) information (procedural knowledge) into conceptualised knowledge became instrumental in the choice of the philosophical framework underpinning the study. The reasoning constructs use the building blocks of basic mathematics and skills strategically to guide logical and high order learning. The construction of constructs takes cognisance of Piaget's development growth. It can therefore be deduced in context that while the reasoning-based instruction may not necessarily be ground-breaking or change the world, it lends itself as an alternative instructional approach among many (Herbel-Eisenmann 2016:105), capable to shape reality (Kollosche 2021:473). It is roped in this study to enhance the teaching and learning of algebra. It is capable of empowering learners to freely manoeuvre through their own algebraic concept network and explore multiple dimensions to a solution, and to further apply the knowledge effectively in related learning.

## 1.3. PROBLEM STATEMENT

It is a contemporary interest that mathematics learning should focus on critical thinking, reasoning, sense-making and conceptual understanding (AMESA 2018:2; DBE 2011:8-9; Muchoko et. al 2019:1; NCTM 2000; Pramesti & Retnawati 2019:3; Rumsey & Langrall 2016:413; Star 2005:406; Vos 2018:2; Yackel 2001:1). Star and Stylianides (2013:178) write that for many school teachers, conceptual knowledge is preferable over procedural knowledge. This is because they see procedural knowledge as simply recalling facts or applying algorithms “without significant thought.” In Canadian math war, the teaching technique that leaves learners without a solid foundation was vigorously debated and classified as an instruction which does not make sense (Chernoff 2019:73). Long (2005:61) writes that the instruction oriented in the learning of procedures only, proved not to help learners attain high order cognition and skills. This, because he argues that conceptual knowledge is intricately linked to procedural knowledge, and that the latter is and should be nested in conceptual knowledge. Meanwhile, the South African curriculum policy in trying to meet the imperative promises, to offer education envisaging high order cognition and skills, and  quality education that compares with that of other countries (DBE 2011:4-5); is also specific about learners (including grade 9’s) attaining deep conceptual understanding (DBE 2011:8).

However the moderation and inspection reports (Chake & Msomi 2018) and diagnostic reports (ANA Diagnostic Report 2014a:56–59; National Senior Certificate: 2015 diagnostic report; National Senior Certificate: 2018 diagnostic report) confirm the prevalence and domination of procedure-oriented instruction in classrooms. Even Mosia’s (2016:135–136) sentiment that the teaching and learning of mathematics should not only regard procedural aspects as important, but the conceptual understanding as well, suggests otherwise. The sentiment resonates with the curriculum policy imperative (principle) in support of active critical learning against rote learning (DBE 2011:4). In essence, both the principle and Mosia’s sentiment intricately reiterate the problem of the procedure-oriented instruction.

The procedure-oriented instruction encompasses many if not all traditional teaching features. It is bad, teacher-centred and inefficient when contrasted with methods that are considered to be relatively good (Osborne 2021:2). Being teacher-centred means the instruction still reflects the traits of traditional period and modernist moments (see 2.2.1.1 (a) and (b)). Teachers occupy the centre stage (i.e. usurp all powers) because they are considered as the only reliable sources of knowledge creation. The traditional ways (approaches) of teaching disregard the global epistemic view that mathematics is about reasoning, creativity and collaboration (Vos 2018:2). In algebra, the approaches fail to provide learners with textual interpretation, yet the abstraction and learning difficulty experienced by learners relate primarily to the interpretation of algebraic texts (Matsolo 2006:12; Pramesti & Retnawati 2019:1; Sengul 2011:2305; Ying et al. 2020:5405). Hence, Muchoko et. al (2019:1) and Pramesti and Retnawati (2019:8) call for the development of instructional strategies that can help (empower) learners overcome algebraic learning challenges.

The trend of repeated underperformance in grade 9 algebra and mathematics at Leru secondary school and other previously disadvantaged schools in South Africa (Matsolo 2006:1) sustains the foregoing literature in regard to the challenge of traditional (procedure-oriented) instruction. Also, Turner (2016:78) writes about the majority of learners and parents lying in a mathematically-terrified extreme on the expertise-avoidance-terror spectrum. The majority in the worst (terror) spectrum is attributed to the poor preparation to apply mathematics meaningfully in a world in which the ability to do so is of increasing importance (Turner 2016:77). Failure in algebra, in particular, has become a big barrier (Muchoko et. al 2019:1) for the grade 9's (see 3.2.1.2). It adversely affects harmonious progression to FET phase and the choice of subjects therein. Hence, it is gatekeeping (blocking) learners from choosing their desirable career paths.

The procedure-oriented instruction fails to achieve the most fundamental goal of learning mathematics (Foster 2008:30; Byrd 2011:3; Pramesti & Retnawati 2019:3): inducing in learners' minds a meaningful understanding of what they learn. Research has also revealed that being procedurally fluent does not indicate a learner can conceptualise and comprehend the knowledge of algebra meaningfully (Bergeson, Fitton, Bylsma et al. 2000:29). It then surfaces out that an instruction that emphasises the knowledge of procedures rather than striving to achieve conceptual understanding, is inefficient. The instruction may be perceived as that which is limited or incomplete. Hence, it suffices to phrase the problem statement as:

> *"The inefficient teaching and learning of algebraic expressions and equations in grade 9".*

The inefficiency results in learners who depend entirely on teachers for short-lived, shallow (limited) and procedure-oriented knowledge. The limited algebraic knowledge affects the learning of advanced mathematics and related disciplines (Hamami 2020:4;

Muchoko et. al 2019:1; Pramesti & Retnawati 2019:1; Ying et al. 2020:5406). Hence, it impacts negatively on learners' overall performance and progression. The performance in grade 9 affects learners' choice of subjects in grade 10 (FET), which in turn affects the learners' career paths (see 3.2.1.2). The impact is thus both direct and far-reaching.

The tried alternatives (other didactic practices), some of which have become school culture (Vos 2018:2), have failed to produce learners who can freely manoeuvre through their own algebraic concept network and explore multiple dimensions to a solution, and to further apply the knowledge effectively in related learning areas. It is assumed that some of the alternatives are perpetual continuation of procedure-oriented instruction in rather a slightly different form. Vos (2018:2) regrets that the practices which have become school culture are not easy to change.

The complex nature of the problem and the diversity of the people affected (e.g. learners and teachers) in this case, warranted the theoretical/paradigmatic position amenable to accommodate diversity and multiple perspectives. Meanwhile, very little emphasis if any, is made in the literature read about the contribution of bricolage theory/paradigm (with its amenability to accommodation of diversity through participatory action to address complex problems) (see 2.2.1), and reasoning (with its multiple contextual traits) (see 2.3.3) to respond to the call to develop the appropriate instructional strategies (Muchoko et. al 2019:1 & Pramesti and Retnawati 2019:8). Yet, bricolage, from its origin, has been used to solve problems in entrepreneurship undertakings and many other fields (Gbadegeshin 2018:110-111) but not in mathematics education.

The call for the development of strategies confirms my own experience of the limited extent to which reasoning is used in the teaching and learning of algebra. The presupposition (based on the experience) that the underutilised remnant, reasoning, can connect the procedural knowledge into conceptual knowledge, prompted this study. The proposed reasoning-based instruction invests in a more practicable and empowering approach in which learners collaborate to reason-out their own (constructs) creativity (Rumsey & Langrall 2016:413; Vos 2018:2). It invests more in contextualised algebraic texts' explanation and interpretation (Matsolo 2006:12; Pramesti & Retnawati 2019:1; Sengul 2011:2305; Ying et al. 2020:5405). That way, learners become constructors of conceptually connected and interrelated network of algebraic and mathematics knowledge. Reason(ing) was analytically accounted for as a power of knowing (Hamami 2020:2). Hence, it is assumed that the approach that strives for meaningful learning (Pramesti & Retnawati 2019:3) can yield quality education and free learners from the setbacks of the procedure-oriented instruction. Henceforth, assist them realise the ‘deep conceptual understanding’ imperative (DBE 2011:8).

Over and above gaining the power to create knowledge, the other resultant benefits to learners include lessened learning strain (Sawyer & Alder 2001:1) as opposed to memorising and regurgitating procedures without attaching meaning to them (Muchoko et. al 2019:5), guaranteed retention (Major & Mangope 2012:144) of self-made knowledge and improved performance. It is therefore assumed that the reasoning-based instruction has a potential to reverse the assessment negative outcomes observed as a result of (inefficient instruction) poor learner preparation with respect to the “use” of mathematics (Turner 2016:77). The reasoning-based instruction also sets a platform for further participatory action research, investigative research into procedural and conceptual aspects of (algebraic) knowledge, and the functionality of reasoning capital. Furthermore, the instruction also has a potential to influence the curriculum policy review.

### 1.3.1. Research question

The key variables (concepts) constituting the conceptual framework of this study are procedural and conceptual aspects of knowledge and reasoning (see 2.1). Conceptual knowledge is intricately (complexly) linked to procedural knowledge in that the latter is nested in conceptual knowledge (Long 2005:61). Bricolage underpins the reasoning constructs to simplify the complex relationship. The study frameworks weave both aspects into meaningful and functional knowledge. In other words, this study is conceptually framed on a presupposition (assumption) that when the algebraic steps or procedures (procedural knowledge) are connected with reasoning constructs to clarify the conceptual relations in-between the procedural steps, a more enhanced and coherent (conceptual) knowledge is achieved. The presupposition augers well with Miles and Huberman (1994:18) sentiment that the conceptual framework lays out the key factors, constructs, or variables, and presumes relationships among them in pursuit to explain a phenomenon the study undertakes to achieve.

Based on the problem statement, the study seeks to address this question:

*How can we enhance the teaching and learning of algebraic expressions and equations using reasoning in grade 9?*

### 1.3.2. Aim and objectives of the study

The research aims to enhance the teaching and learning of algebraic expressions and equations using reasoning in grade 9. The research objectives are:

- to justify the need to enhance the teaching and learning of algebraic expressions and equations in grade 9;
- to derive possible components of solution to enhance the teaching and learning of algebraic expressions and equations using reasoning in grade 9;
- to suggest conditions conducive to successful teaching and learning of algebraic expressions and equations using reasoning in grade 9;
- to identify the risks and threats that may hinder the teaching and learning of algebraic expressions and equations using reasoning in grade 9, and suggest ways to prevent them; and
- to provide evidence that shows that the reasoning-based instruction enhances the teaching and learning of algebraic expressions and equations in grade 9.

## 1.4. BRICOLAGE AND CONSTRUCTIVISM AS FRAMEWORK

This study is framed in bricolage and constructivism theories (see 2.2). The theories underpin the usage of reasoning (see 2.3.3) to connect (conceptualise) procedures (procedural knowledge) into conceptual knowledge. Henceforth, the procedural and conceptual aspects of knowledge together with reasoning constitute the study conceptual framework. The operation (functionality) of or within the conceptual framework adopts and adapts the theoretical framework.

Bricolage is a post-positivism theory that embodies the researched (teachers and learners) re-organising, re-configuring and reconstructing the remnants of information in observance of legitimation (Kincheloe 2004:687). It is roped in to cushion the production of meaningful and functional knowledge (Tlali 2017:85). The envisioned enhancement in which learners work under controlled supervision of teachers (Weegar & Pacis 2012:7) to re-organise or encode existing content into new juxtapositions to attain conceptual and meaningful outcomes is bricolage in nature (Klages 2012:45). Bricolage involves activities whereby materials at hand are restructured to build more meaningful knowledge (Watson & Hill 2015:44). It is achieved by connecting the unconnected material to realise an improved product (Barker 2004:43; Kaomea 2016:10).

Constructivism is also a post-positivism theory that supports the active participation of the researched. It augers well with the bricolage principle that offers substantial possibilities to the researched. It (the principle) advocates for recognising the value of researching social contexts with participants (Renwick 2014:6). The "processing" of procedural knowledge into conceptual knowledge using reasoning attach sensible meaning (Yackel 2001:1) to algebraic content matter, and provide direction and cushion (defence) for logical arguments (Kollosche 2021:471; Mahlomaholo 2014:173; Osborne 2021:6; Westaby 2005:97). The processing aims at attaining high order (and conceptualised) cognition (Kincheloe, Hayes, Steinberg et al. 2011:4). The want for conceptualised cognition in South African schools is inspired by the curriculum policy imperatives as discussed in sections 1.2.1 and 1.2.2. However, the prevailing disintegration of procedural and conceptual aspects of knowledge fails the aspiration. The instruction leans more onto the former (see 3.2.1.1(a)). Hence, a drive behind the instruction based on reasoning theory, in this study, could further be looked at from Mosia's (2016:135–136) point of view that the teaching and learning of mathematics should not only regard procedural aspects as important, but the conceptual understanding as well.

Contextually, reasoning involves the construction of conceptual connections and interrelations, generally referred to as conceptualisation. It is aimed at providing meaningful and purposeful understanding (Pramesti & Retnawati 2019:3) of content matter; that emanates from concretised or customised explanation of algebraic symbolism (Byrd 2011:3; DBE 2011:4; Foster 2008:30; Miller & Koesling 2009:67). Reasoning also encompasses refocusing, integration and/or re-organisation (DBE 2020:16) of learning material in a way that it responds to subject (curriculum policy) and learners' needs (DHET 2015:62). In the context of this study, it (the reasoning process) specifically deploys learners' own reasoning constructs to ensure both participatory and contextual conceptualisation.

## 1.5. OPERATIONAL CONCEPTS

This section explains the meaning of operationalised concept(s) and key words or phrases as used in the study. It endeavours to afford the readership more insight and contextualised access to the script:

**Algebraic expressions and equations–** Algebraic expressions and equations are exclusive sub-branches of mathematics that use symbols and letters to represent real numbers or unknown quantities (Bolt and Hobbs 1998:5; Olivier 2012:48; Tapson 2008:6). $\boldsymbol{y = ax^n + bx + c}$ is a general algebraic equation and $\boldsymbol{ax^n + bx + c}$ is a general expression, where **a, b** and **c** represent real numbers, $\boldsymbol{x}$ and $\boldsymbol{y}$ are variables and $\boldsymbol{n}$ is an exponent (Bowie, Campbell, Heany et al. 2013:58). The operation on the equation seeks to isolate, hence reveal the numeric value(s) of unknown(s), while the computation on an expression seeks to present it in a (conventionally) simplified form. The words 'algebra' or 'topics' may be used when a general reference is made to algebraic expressions and equations. It is important to note that the topics are integral, but not the only components of algebra (DBE 2011:130-133,142-144).

**Reasoning–** a higher-order cognitive skill (behaviour) through which learners can logically realise and defend (Westaby 2005:97) what they are saying, writing or doing beyond just using a formula or calculator (Osborne 2021:6). It is a means of justifying mathematical assertions (Kollosche 2021:471). In this study reasoning is used to form necessary interrelated connections within and between algebraic concepts and procedures (O'Brien n.d.:8). The phrases 'reasoning theory' or 'reasoning skill' or 'reasoning (-based) instruction' or 'reason(ing) out' may be used interchangeably with reasoning.

**Procedural knowledge–** knowledge of some prescribed syntax, routine steps, conventions and rules that may be followed when manipulating symbols towards the solution of a problem, without necessarily understanding how and why they work (Star 2005:406). A procedure is a set of instructions for computing a function or property (Math 300 2003:1). The study intends to conceptualise the procedural aspect of knowledge into conceptual knowledge. The intention is grounded on the conviction that the former is underpinned by the latter. Algorithms are another form of procedures.

**Conceptual knowledge–** the knowledge that is attained as a result of deducing specific rules and procedures from more general mathematical relations, achieved by constructing relationships between unconnected information (Long 2005:59). An explanation that gives reasons for believing a statement is true (Math 300 2003:2). Reasoning is a cognitive skill used to construct relationships between procedures and concepts to create conceptual knowledge in the teaching and learning of mathematics.

## 1.6. LITERATURE OVERVIEW

This section reviews literature related to the teaching and learning of algebraic expressions and equations in line with the objectives of the study. The review embraces literature and good practices from South Africa, the Southern African Development Community region, other African countries and abroad.

### 1.6.1. Justification of a need to enhance the teaching and learning of algebraic expressions and equations

Several challenges that beset the teaching and learning of algebra in grade 9 justify the need for enhancing the instruction. This study is confined to the challenges categorised into four main headings namely: the non-alignment between the curriculum policy and classroom instruction (DBE 2011:4,8–9); the sifting nature of algebra (AMESA 2018:2); the abstraction and complexity of algebra (ANA Diagnostic Report 2014a:56–59; Banerjee & Subramaniam 2011:352; Borenson 2011:24; DBE 2014:9–10,43; Hewitt 2012:140–141; Matsolo 2006:62; O'Brien n.d.:9) and lack of befitting basic mathematics competency. The non-alignment between the curriculum policy and classroom instruction manifests under the procedure-oriented instruction, assessment, teachers' competence and curriculum-time contestation.

The teaching and learning of algebra are still dominated by procedure-oriented instruction (see 3.2.1.1(a)) associated with the traditional teaching methods (Osborne 2021:2). The instruction fails to meet the curriculum policy requirement: “to develop deep conceptual understanding in order to make sense of mathematics” (DBE 2011:8). The perpetuation of the instruction exacerbates the challenges of learning. For instance, learners who adopt or rely on the procedure-oriented approach tend to learn mathematics (through memorisation) by heart (Muchoko et. al 2019:5; Sawyer & Alder 2001:1). Memorisation does not benefit learners, but rather cause them unnecessary strain. It gives them neither an understanding of the subject, nor the power to apply mathematics in ordinary life (Sawyer & Alder 2001:1). Canadian parents who are mathematics professionals argued against the curriculum that encourages memorisation (Herbel-Eisenmann 2016:105). The basis of the argument was that the curriculum lacks focus towards conceptualised knowledge as opposed to discovery learning.

The challenge of non-aligned instruction impacts the school-based assessment (SBA) too. This is because the level of assessment or performance thereto, may not surpass the level of instruction. The repeated trend of below-average performance bears evidence that the SBA fails to prepare adequately for standardised (common) papers set and moderated according to the curriculum policy standards. Yet, the final assessment policy puts more weight on common papers (DBE 2011:155).

Another obstacle that contributes to the non-alignment is a want for teachers’ competence for teaching high school mathematics, especially algebra (Lempp 2008:abstract; Thornburg 2009:2). In addition, most teachers are unaware of mathematical misconceptions their learners hold (Luneta & Makonye 2010:44). The claims of limited competence are augmented by the want for effective instruction and legitimation (alignment with curriculum policy). The want for competencies benchmarked by the minimum requirements of a newly qualified teacher (DHET 2015:62) is evidenced by lack of quality in learners’ work (AMESA 2018:2; Haas 2003:31; O’Brien n.d.:9), low self-efficacy (Sengul 2011:2305) and a repeated trend of below-standard performance in algebra and mathematics.

The unsatisfactory performance in algebra and its effect in determining performance (Sawyer & Alder 2001:3) in advanced mathematics has coerced the department to reduce the mathematics minimum promotion mark from the standard 40% (level three) to 30% and below. The reduction, however, comes with conditions that have adverse effects on careers (McNeil, Weinberg, Hattikudur et al. 2010:625; Tall & Razali 1993:2), and the socio-economic development of South Africa and her citizens (see 3.2.1.2). The situation has led to progress stagnation and/or an alternative circular-based progression (eNCA 2016). The latter seeks to incorporate and prioritise the cross-cutting socio-economic considerations over set promotion standards. The end result includes stringent sifting of learners whose mathematics average mark falls below 30% to mathematical literacy in grade 10. Most of those who narrowly manage to make it into mathematics streams in grade 10 translate to a challenge postponement (DBE 2016:6). The challenge reveals in grade 12 subject enrolment whereby the enrolment in mathematical literacy increases at the expense of many learners dropping mathematics along the Further Education and Training (FET) phase.

Literature review further augments the Department of Education (DBE 2019:3) assenting view to the glaring contestation between algebra curriculum and allocated time (see 3.2.1.1(d)). To this effect, this study pays more attention to Little's (2009:3-4) warning about the curriculum that includes too many superficially taught topics in a given year (time-unit). While moderation reports confirm that the voluminous curriculum per limited time period has an adverse effect on the quality of instruction, it becomes crucial to review the curriculum in relation to allocated time and other related aspects.

The foregoing narration confirms the sentiments that the abstraction embedded within algebra and the complexity of teaching and learning remain unmitigated (Borenson 2011:24). Algebra has, for various reasons, proved very difficult and abstract for the majority of learners in grade 9 (Annual National Assessment [ANA] Diagnostic Report 2014a:56–59; Matsolo 2006:12; Rojano, Filloy & Puig 2014:390; Sengul 2011:2305).

Other crucial, perhaps intertwined factors this study has placed central to other challenges, are insufficient competency in basic mathematics and skills, and glaring cognitive (formal operation) underdevelopment (see 3.2.1.4). The deficiency glares (is observable) with the majority of learners at the senior phase exit grade (grade 9). The deficiency and underdevelopment starve the introduction of algebra of the necessary (mandatory) foundation to build on (Banerjee & Subramaniam 2011:351; Chernoff 2019:73; Fuchs & Fuchs 2005:45; Matsolo 2006:5; McNeil et al. 2010:625–626).

### 1.6.2. Possible components of solution

The reasoning-based instruction rely on activities inherent of the theories and methodologies underpinning the instruction namely bricolage, constructivism theory and PAR. Levi-Strauss (1966:17) assumed bricolage as a creation of objects with materials at hand, re-using existing artefacts and incorporating bits and pieces. Bricoleurs weave bits and pieces of scattered remnants to produce an improved product (Barker 2004:43; Kaomea 2016:10). Likewise, this study promotes the learning process in which learners work as bricoleurs to construct reasoning constructs that connect procedural steps to develop conceptual knowledge (Foster 2008:85, 89). Reason(ing) was analytically accounted for as a power of knowing (Hamami 2020:2). It allows for demonstration of what is true or at least reasonable to assume as true (Kollosche 2021:473).

Learners cope well with the use of reasoning in the process of conceptualising knowledge (Star 2005:406). Conceptualisation provides sensible meaning and purpose (Matsolo 2006:62; Tlali 2017:85; Yackel 2001:1) to the learning and application of gained knowledge (DBE 2011:9). Seeking sense-making knowledge is also bricolage (Gbadegeshin 2018:106,113). The process of conceptualising procedures supports the alteration or tinkering (Gbadegeshin 2018:115) of an instruction so that it highlights the meaning of content and symbolic structures (Pierce & Stacey 2007:12). In context, the tinkering (bricolage in nature) promotes the formation (construction) of conceptual connections and interrelations within learners' context (see 3.2.2.1(a)). That way, the learning of algebra becomes more accessible, less strenuous and sustained than when it is dominated by a procedure-oriented approach, and overshadowed by symbolic representation (Matsolo 2006:v). Conceptualisation aligns with Miller and Koesling's (2009:65–66) thought of "marrying" mathematical reasoning with reading skills to help learners understand the real-world context and mathematical concepts of a problem. The alignment augers well with Mosia's (2016:135–136) sentiments that not only quantitative skills or content-specific knowledge are important in teaching and learning mathematics, but the critical mathematics knowledge as well.

In the context of this study, an aligned instruction cascades into aligned assessment in which different cognitive level questioning is couched and observed. It sets a pace for asking questions that require learners to explain their thinking and justify their understanding (Kollosche 2021:471; O'Brien n.d.:8; Osborne 2021:6; Rumsey & Langrall 2016:419). Aligned assessment uses feedback to develop learners and improve the process of learning and teaching (DBE 2011:154). The feedback adopts a two-way dialogue between teachers and learners, driven by a notion of 'assessment as learning' (Evans 2013:82; Rhind 2017:3,5). That way, the classroom assessment becomes a proper preparation for term papers.

The study aligns with an advocacy for the provision of professional development and support (Pramesti & Retnawati 2019:1) programmes. The programmes empower teachers to close competency gaps and afford them an opportunity to share good practices (Dickey 1997:5; Foster 2008:6). The study adopts the vision of simplifying algebraic abstraction thereby using common language rather than unexplained technical language. The common language connects algebra, a theory-heavy discipline, to real life (Vos 2018:2). The reasoning-based instruction also provide language support in developing the discourse of mathematical (reasoning) argumentation (Rumsey & Langrall 2016:415). Further, the study puts emphasis on learners' cognitive development and basic mathematics competency. The competency strengthens, per requirement, the foundation upon which the learning of abstract algebra should build. The paradigmatic shift embedded in collective components of solution discussed in this section gives hope for envisioned instructional enhancement, aligned assessment, optimal performance and minimal career sifting.

### 1.6.3. Conditions for successful implementation of the reasoning-based instruction

The conditions that need consideration for successful implementation of the reasoning-based instruction include emphasising the demonstration of conceptual understanding (DBE 2011:8). The demonstration is achievable if learners engage actively and construct knowledge in their own context (Bauer & Perciful 2009:1). For example, using contextual (common) language to explain algebraic content before relating and/or adopting it to symbolic representation.

The explanation requires teachers to create space and time (Banerjee & Subramaniam 2011:352) for learners to respond to probing questions and prompts such as how?, why?, explain your answer, elaborate more, and so forth, in an enabling environment. The environment should conform to the principles and values of participatory engagement that include mutual respect (Chilisa 2012:48) between teachers and learners, and among learners; teamwork collaborations; and not being judgemental (Mathreasoning09).

Another condition involves the use of concrete materials to contextualise the instruction for the benefit of learners (Kincheloe et al. 2011:4). Creative refocusing, integration and re-organisation of learning content (DBE 2020:16) to establish rich connection (Star 2005:407) lead to conceptualised knowledge. The processes also help strike a balance between the contesting algebra curriculum and time allocation while maintaining legitimation (the instructional quality). They also dig deep into teachers' competencies.

Teachers should therefore be developed and supported to cope with the demands of the reasoning-based instruction in terms of content and instructional approach (Dickey 1997:5). This is to harmonise the instructional context, namely the teacher, the content and the instructional strategy (Kolobe & Hobden 2019:1). Teachers should, for example, provide tasks that develop and improve learners' understanding and adhere to multi-cognitive level assessment (DBE 2011:154), and use the feedback thereto to inform and improve subsequent teaching and learning. The benefits that come with the condition to develop (strengthen or nurture) learners' cognitive growth and basic mathematics competency before the introduction of algebra (see 3.2.1.4) cannot be overemphasised.

### 1.6.4. Possible risks and threats against the implementation of the reasoning-based instruction

Teachers' resistance to change is the primary threat to the successful implementation of the reasoning-based instruction. It is suspected that teachers may continue to teach the kind of algebra they have always taught in ways they have always taught (Dickey 1997:4). The instruction encompasses creative refocusing, integration and re-organising content matter as exemplary means of conceptualisation process. As a result, resistance makes it practically impossible to realise the anticipated reforms and transformation. The status quo regarding the (unmitigated) abstraction and complexity (Borenson 2011:24; Major & Mangope 2012:144), for example, would then continue to haunt (threaten) the teaching and learning of algebra.

Resistance may degenerate into teachers who continue to usurp the power and control, and leave learners passive (Major & Mangope 2012:140). Hence it will continue to limit the instruction to uncritical and rote learning (DBE 2011:4), and low cognitive assessment. Rote learning is procedural and lacks conceptual connection (Star 2005:407), and does not develop learners in high order and/or critical learning (DBE 2011:4,8). Rather, it deprives learners an opportunity to exercise their reasoning powers (Major & Mangope 2012:144), and a platform to demonstrate their full potential (DBE 2011:155) in the subject.

Some of the teachers' unions reservations towards standardised (common) papers administered by the Department of Education suppress the multi-perspectival enrichment to the instruction and assessment (Abercrombie, Hill and Turner 2006:47; Kellner 1999:xii; Rogers 2012:1). That way, they (reservations) pose a threat to the instruction that aligns with the curriculum policy. It can then be argued that they perpetuate the perceived algebraic abstraction and complexity. Lastly, the insufficient basic mathematics competency, which also confirms the underdevelopment of Piaget's formal operational growth (see 3.2.1.4), threatens the sustainability of logical reasoning constructs (NCTM 2000) integral to the reasoning-based instruction.

### 1.6.5. Indicators of success

Success is demonstrable when learners can freely explain the meaning behind procedural steps (Pierce & Stacey 2007:12). It is observable when learners display formal operational development and conceptualisation skills. The skills may include contextualising and concretisation of content matter to attain meaningful and purposeful learning (Pramesti & Retnawati 2019:3). Success shows when learners can sustain logical arguments (Mahlomaholo 2014:173) and connect procedures to attain conceptual knowledge. Learners demonstrate success of the reasoning-based instruction when they accurately use (apply) the knowledge gained in assessment activities set at different cognitive levels. The accuracy and sustainability of logical reasoning constructs rely on basic mathematics competency. The competency also affirms the formal preparedness of a learner to reason out multiple dimensions (conventions and operations) of algebra at the same time (Lynn 2006:19).

Conceptualisation shows when learners can correlate and apply the knowledge in related disciplines (Hamami 2020:4; Muchoko et. al 2019:1; Pramesti & Retnawati 2019:1; Ying et al. 2020:5406). The correlation signals attainment of bricolage skills of connecting unconnected information (knowledge) to create a useful and functional product (Given 2008:68–69; Kincheloe et al. 2011:1; Rogers 2012:1). The flexibility to connect and interrelate (navigate through) algebraic concepts and procedures points to a breakthrough towards the perceived abstraction and complexity. It also signals knowledge endurance (Long 2005:61) and manifests in reduced errors and misconceptions (Luneta & Makonye 2010:44; Pramesti & Retnawati 2019:7,8). In fact, success also shows when learners are able to diagnose and correct their own errors and misconceptions. It eventually manifests in improved performance in algebra, advanced mathematics and related subjects.

Teachers that develop competencies to nurture learners to achieve the knowledge skills ascribed to in the foregoing paragraphs indicate successful implementation of the reasoning-based instruction. The development of teachers is also noticeable when they become resourceful agents of conceptual instruction. It manifests in lifelong learning, subscription to improved (enhanced) instructional practice and the results thereto. The developmental success is remarkable when teachers can refocus, integrate and re-organise the learning material (DBE 2020:16), determine the befitting sequence and pace of content cognisant to subject and learner needs (DHET 2015:62), and manage to complete the curriculum without compromising standards.

## 1.7. METHODOLOGY AND RESEARCH DESIGN

This study is a participatory project that was executed in collaboration with the research team comprising two mathematics teachers, technology departmental head, mathematics departmental head and retired mathematics teacher. The team members acted (emerged) as co-researchers (Mahlomaholo 2014:9) that spearheaded all research activities: conceptualising the problem, planning, implementation and reflection in an iterative manner. To that effect, the study employed PAR methodology and used FAI technique to generate qualitative data. It further used quantitative research tools to collect empirical data (Figure 5.1 & 5.2).

PAR empowers and enables those who are directly affected to collaborate in action to understand their practices, and to change the situation in which they find themselves (Baum 2016:405; Bennett 2019:109; Boyle 2012:7; Crane & Richardson 2000:7; Openjuru, Jaitli, Tandon et al. 2015:219). The action and engagements are directly linked to the socio-historical and cultural contexts of the affected. In the context of this study, the principles of PAR align with the presupposition that the learner-oriented approaches to break through mathematical abstract are possible without compromising (legitimation) mathematical validity (Pinkernell 2019:2). It is a useful and effective epistemological tool for reviewing and improving practices. It has a track record of developing positive outcomes for clients (Boyle 2012:8). PAR acknowledges the research team defined by a shared problem (Israel, Shulz, Parker et al. 1998) within the same geographic location like in this study. The clients in this study are the participants of the research based at Leru secondary school. They have the best vantage point to evaluate the challenges and solutions critically because they are best placed to understand what can work (Bennett 2019: 109; Watters & Comeau 2010:6) at Leru secondary school. Their inclusion resonates with PAR's principle that the lived experience of the affected brings the value that should influence the study against the positivist view about the realities of the world (Langhout & Thomas 2010:61). Even bricoleurs understand that rigorous research involves connecting the making of meaning to human experience (Kincheloe 2005:342). Hence, bricolage theory underpins PAR in incorporating the knowledge and expertise of community members to develop research protocols that aim to benefit the affected. Thus PAR creates an environment in which the research team work together and advocate freely for what they believe in (Baum 2016:405; Macaulay, Commanda, Freeman et al. 1999:774). Therefore, it has the potential to improve and develop appropriate resources, strategies and policies (Watters & Corneau 2010:8).

It can be concluded therefore that PAR creates a situation in which action and reflection go together and ensures a more accurate analysis of social reality (Bennett 2019: 109). Co-researchers have control and transformative power over the instruction. PAR pays careful attention to power-sharing within the research team

(Baum 2016:405; Mahlomaholo 2014:9). That way, the researched as per bricolage principles, cease to be treated as objects, but rather become equal partners in the process.

The principles of the FAI technique complements PAR in many ways. The FAI is a qualitative approach that allows researchers to interact closely with the researched in a humane manner to observe and interpret their world (Dexter 1970:136; Merriam-Webster 1998:8; Patton 1990:278). It directs the data generation process in a manner that it does not alienate nor undermine the researched (Netshandama & Mahlomaholo 2010:11). It uses open-ended questions and leave a lot of communicative space and respect for the researched to express their feelings freely about the inquiry.

## 1.8. DATA ANALYSIS

The study deployed critical discourse analysis (CDA) of Van Dijk to analyse data. CDA is an interdisciplinary (Van Dijk 2015:468) form of critical social science that helps scholars to better understand the nature and sources of social problems, the challenges to address the problems, and possible components of solution to address those challenges (Fairclough 2013:13). It can then be deduced that CDA is problem-oriented and emancipatory. It uses three levels of analysis, namely textual analysis, discursive practices and social structures to analyse and support the victims of oppression (complexity) to resist and transform their lives (Wodak 2009:13). Therefore, it resonates with the frameworks and theories underpinning this study in critiquing and explaining the participatory, multi-disciplinary and multi-perspectival approaches towards enhancing the teaching and learning of algebraic expressions and equations. The transformative enhancement is depicted in using the reasoning-based instruction to promote conceptual understanding over procedural approach. The former free learners from rote and uncritical learning of the given truths (DBE 2011:4) that has proven to be unproductive and complex. The procedure-oriented instruction associates with (or results in) uncritical learning.

CDA is couched in collaborative and interdisciplinary interventions (Fairclough 2013:19; Wodak 2009:3) aimed at establishing an in-depth understanding of how language (textual representation) functions in constituting and transmitting knowledge (Van Dijk 2015:470; Wodak 2009:9). That is, it arouses a critical look into the meaning of words in the social context and strives to give an explicit explanation of the content and structures of teaching and learning. It could be deduced then that it forges a harmonised relationship between texts, instructional interactions and social structures. For example, in this study, it is intended to remove emphasis from meaningless manipulation of procedures, and focus it on attaining meaningful and functional knowledge evolving from precisely written and oral texts (reasoning constructs). It is therefore instrumental for describing, interpreting, analysing and critiquing the challenges reflected in algebraic written and oral texts (Mogashoa 2014:104). It helps scholars to analyse the discursive practices (notions) of power, dominance, discrimination and bias embedded within the texts and how they are reproduced (Van Dijk 2015:468).

## 1.9. IMPLEMENTATION OF THE REASONING-BASED INSTRUCTION

The research team used the reasoning-based instruction to teach the simplification of algebraic expressions and solving of equations to one grade 9 class at Leru secondary school. The implementation adopted PAR's reflexivity cycles of planning, acting (teaching) and observation, and reflection (Baum 2016:406). Thus generating the data responsive to research question.

## 1.10. FINDINGS AND RECOMMENDATIONS

The findings were drawn from data analysis and interpretation in Chapter 5, in cognisance of the research question and objectives of the study. They provided a basis for the recommendations made.

## 1.11. VALUE OF RESEARCH

The research draws from spotting shallow implementation of the curriculum policy as the main challenge besetting the teaching and learning of algebra. That is why the strategies and components of solution are based on using available remnants and reasoning provided for in the curriculum policy. The formulation of the reasoning-based instruction underpinned by the bricolage and constructivism theories helps nurture learners to self-propel their learning (of algebra) in a constructive, meaningful and purposeful way. In the process, they develop a skill of connecting unconnected information to create an improved (knowledge) product (Barker 2004:43; Kaomea

2016:10). The logically hybridised (richly conceptualised) knowledge (Booi and Khuzwayo 2019:1-2) is beneficial to learning. It boosts performance in algebra, advanced mathematics and related subjects. The skill to use basics to sustain logical arguments for high order cognition strikes balance on a scale of knowledge creation and demonstrates formal operational development. It forms a basis upon which learning in high school and beyond successfully build (NCTM 2000). The reasoning-based instruction, underpinned by bricolage, nurtures openness to self-evaluation, self-critique and acceptance of self-weaknesses (Luitel & Taylor 2011:8–9). That is, it practically enables the self-checking of error, thus making it easier for learners to diagnose and unlearn misconceptions and errors on their own. That way, the instruction reduces the teaching load and empowers self-driven learning capability.

The study developed co-researchers' mode of thinking and empowered them to act like bricoleurs. They (co-researchers) examined the socio-historical dynamics of procedural instruction and how they (dynamics) influence and shape (the teaching and learning of algebra) complexity (Rogers 2012:10). The examination helped them formulate appropriate connection of legitimation and cognition, algebra, and pedagogy (instruction). It further empowered them to use what is at their disposal to develop sustainable solutions. Hereby, sustainable solutions are anticipated which might serve as the basis for more innovations. That is, the research empowered initiatives to overcome the 'lock-ins' to existing solutions, which have not been effective for a considerable population of teachers and learners. It then suffices to conclude that the research has also tapped into what could have remained dormant potential in a coordinated approach to address societal challenges.

## 1.12. ETHICAL CONSIDERATIONS

The Ethics Committee of the Faculty of Education granted approval for ethical clearance and the members of the research team undertook to abide by it. The team also noted the conditions of approval given by the Free State Department of Education and Leru secondary school management team. Parents' permission was sought to keep learners in school for a maximum of one hour after school. These conditions were observed to ensure the effective and sustainable participation of learners throughout the research period.

The team undertook not to disclose confidential information from any source. A high level of professionalism (including the protection of participants' identities), and the role of co-researchers were agreed upon. The study uses pseudonyms. The team further undertook to share the research outcomes with participants and stakeholders.

## 1.13. CONCLUSION

This chapter has given an overview of the study. It explained the object of the study: to develop the bricolage and constructivism underpinned instruction that uses a remnant or underutilised reasoning to enhance the teaching and learning of algebra in accordance with the imperatives of CAPS. The background revealed that the instruction is still dominated by approaches that do not meet the requirements of CAPS, hence fail to fulfil the promises made therein. It highlighted the appropriateness of this study in attempting to fulfil the imperatives specific to meaningful and functional instruction. The fulfilment addresses the challenges that beset the teaching and learning of algebra. Challenges include the unsatisfactory performance trend at the tail of many daunting concerns. Hence, the problem statement, research question and study objectives.

The chapter explained the operational concepts and highlighted the appropriateness of framing the reasoning-based instruction within bricolage and constructivism theories. The multi-disciplinary and multi-perspectival characteristics of the frameworks provide necessary philosophical (paradigmatic) support to learners' (free) construction of reasoning connections (constructs). The constructs explain (conceptualise) procedural steps through refocusing, integration and re-organisation of subject matter.

The chapter discussed the correlation and corroboration between the objectives of the study and literature. The objectives include the challenges besetting the teaching and learning of algebra in grade 9, components of solution, conditions for successful implementation, risks and threats, and indicators of success.

The chapter explained the participatory nature of the research project underpinned by PAR methodology. PAR enables the research team members to become co-researchers and assume powers and control over research activities. The chapter explained how the PAR and FAI techniques ensure and facilitate the participation of the researched (teachers and learners) within their natural settings, in a respectful and humanely manner. It discussed the relevance of CDA theory to the study. CDA provides an in-depth analysis of texts to establish the impact of discursive practices and their implications within the social structures. The chapter further discussed the benefits and values of the research that: (i) the resultant algebraic conceptual knowledge is beneficial to learning advanced mathematics, related subjects at grade 9, FET and beyond; (ii) the study empowers co-researchers' mode of thinking to develop sustainable solutions that overcome the "lock-ins" to existing inefficient practices. The chapter then discussed the project ethical considerations before concluding.

# CHAPTER 2
# STUDY FRAMEWORK FOR ENHANCING THE TEACHING AND LEARNING OF ALGEBRA IN GRADE 9

## 2.1 INTRODUCTION

This study seeks to enhance the teaching and learning of algebraic expressions and equations using reasoning in grade 9. This chapter reviews the existing frameworks namely bricolage paradigm and constructivism theory within which the researcher intends to frame and guide the study. The review explains how and why the theories are presumed appropriate in shaping and directing the reasoning-based instruction research. The chapter also explains the key words that are related or used collectively with the major operational concepts (see section 1.5). It then narrates comprehensively the major operational concepts underlying the study, namely procedural and conceptual aspects of knowledge, and reasoning (theory). Finally, the chapter synthesises the study framework before concluding.

## 2.2 APPROPRIATENESS OF FRAMEWORKS

The theoretical framework is a foundational review of existing theories that serve as a roadmap for developing the arguments (hypothesis) of the study (Vinz 2015:7). Grant and Osanloo (2014:26) refer to the theoretical and conceptual frameworks as blueprints of the study. They are central aspects of the research process. Their overriding importance manifests in the selection of a topic, development of research questions, conceptualisation of literature review, design approach, and analysis of the study (Grant & Osanloo 2014:26). They are responsible for a corroborated, coherent and logical flow of ideas in different sectors of the research, from one chapter to another. They form a grounding base upon which the activities of the reasoning-based instruction develop, and keep the research team and participants focused on achieving the aim and objectives of the study. That way, they become structural and rational pillars anchoring the study in its entirety.

In the next session a review of bricolage theory as an appropriate paradigm to guide this study will be presented. Gbadegeshin (2018:101,102) writes that bricolage theory can be used as philosophy, paradigm or methodology. This study, underpinned by bricolage, embraces the bricoleur formats. For example, it is critical in the sense that it questions, learns and strives for the knowledge of the learners ('the silenced' and 'the excluded') (Rogers 2012). Theoretical in that it asserts the research participants work through and between multiple paradigms. Hence, the reinforcement (rigour) and harmonisation of the study framework through the engagement of bricolage and constructivism theories. Consequently, a review of constructivism theory will also be presented.

The novelty (unusual practice or perhaps the first establishment) of constituting the framework of mathematics education research with bricolage theory confirms the originality of the study, and appreciation of bricolage usage across different fields of study to combine and reuse resources for new applications, different from those for which they were originally intended (Baker & Nelson 2005:335). Constituting the framework with bricolage and constructivism theory reflects the researcher's philosophy (belief and understanding) of the ontology and epistemology underpinning their research study. Further, the combination of multiple theories and perspectives in a single study adds rigour, richness, complexity and depth to the inquiry (Denzin & Lincoln 1999:4). The choice of the theories is driven by the presupposition that the complexity of teaching and learning algebraic expressions and equations in grade 9 can be reduced through operationalizing the remnant, reasoning in this case, to conceptualise algebraic procedures (procedural knowledge) into conceptual knowledge. Henceforth, the procedural and conceptual knowledge together with reasoning constitute the conceptual framework of the study. The operation (functionality) of or within the conceptual framework adopts and adapts to the principles of bricolage and constructivism theories. The frameworks are inspired in a bricolage 'do it yourself' (see 2.2.1) notion. The notion is couched in constructivists' advocacy to relinquish the power to generate knowledge to learners. The advocacy limits teacher's role to controlled supervision (Weegar & Pacis 2012:7) to optimise learner-participatory conceptualisation. That way, the supervision comes in handy per need.

The study frameworks define the programme and guide the choice and deployment of participants, their roles and functions, and employable tools. In short, the frameworks (theoretical and conceptual) are a lens through which the researcher views the entire study in search of a solution to the problem. They are anchors without which the study could be likened to a house constructed without a plan (Grant & Osanloo 2014:16).

### 2.2.1 Bricolage as an appropriate paradigm

Bricolage theory originates from the work of Levi-Strauss (Gbadegeshin 2018:101). It is a post-positivism paradigm that evolves from the blurred genres moment (Rogers 2012:8). It is founded on a French word, bricoleur. Bricoleur in simple terms means "do it yourself" (Dictionary.com n.d.). Thus, a person who applies the principles of bricolage is referred to as a bricoleur.

Rogers (2012:1) writes that bricolage is a critical, multi-perspectival, multi-theoretical and multi-methodological approach to inquiry. Perspective refers to particular ways and attitudes of looking at reality (a point of view). Therefore, the multi-perspectival trait refers to many points of view or attitudes of looking at reality. A similar argument can be advanced about multi-theoretical trait etc. While the theoretical usage of theories change over time (Gbadegeshin 2018:101), the theoretical bricolage (reconceptualised critical theory) has always supported the use of multiple theories to underpin the work of bricoleurs to reduce human suffering (Rogers 2012; Steinberg & Kincheloe 2010:140). The two theories in this study namely bricolage and constructivism constitute the study framework. The twofold framework adds depth, rigour and multiplicity to the inquiry. It critically questions, learns and strives for the knowledge of the learners ('the silenced' and 'the excluded') (Rogers 2012).

A bricoleur researcher looks around for remnants that are not used or are underutilised and re-organise, re-configure and reconstruct them into a new useful and functional product using various resources and/or modes of orientation towards the world (Gbadegeshin 2018:101-102; Given 2008:68–69; Kincheloe et al. 2011:1; Rogers 2012:1). This suggests that a bricoleur becomes actively involved in analysing the situation for the identification of possibilities and opportunities through which to solve an existing social problem. The underutilised and unused skills, knowledge and resources can then be deconstructed to construct meaningful and useful components of solution. This study relies on what would be an underutilised learning skill, reasoning, untapped experience and knowledge of internal and external stakeholders to conceptualise the problem relating to the teaching and learning of algebraic expressions and equations in grade 9. The stakeholders include a retired mathematics teacher, Departmental head- Mathematics, mathematics cluster coordinator, science and technology teacher and a newly qualified mathematics teacher. The latter brings fresh ideas and new innovations from the university. The study conducts a situation analysis for possible opportunities and strengths available for addressing the weaknesses and threats that are experienced.

In this study, bricolage also serve to draw researchers' attention to issues of power differentials and transformation (social justice and equity) (Rogers 2012). Further, to be considerate of the multiple perspectives (beliefs, attitudes and points of view) of participants from diverse backgrounds. The complex nature of the problem in this study namely, the inefficient teaching and learning of algebraic expressions and equations in grade 9, warranted the consideration of multiple methods informed by multiple theories governing the content. To this extend, bricolage may not be limited to one aspect of its characteristic traits. It is further worth noting the comments expressed by Rogers (2012:7-8) that “the theories that underlie bricolage make it far more complex than a simple eclectic approach”. Gbadegeshin (2018:101) and Rogers (2012:7-8) add that bricolage articulates “onto the next level” by adopting and extending the five categories of bricoleurs.

The reasoning-based instruction draws from the ontology and epistemology of bricolage embracing even the subjugated sources (Rogers 2012:12–13) for improvement and/or benefit of the affected. One such benefit provided by utilisation of reasoning in context would be a resultant network of proficient knowledge through which the learners can navigate at ease. That is, knowing many related algebraic concepts while keeping very little information in mind (Ertmer & Newby 2013:43–44). The conceptualisation process aligns with other bricolage research. It is critical, multi-paradigmatic, multi-perspectival, multi-theoretical and multi-methodological approach to inquiries (Rogers 2012:1). The involvement of diverse stakeholders and learners in the construction of knowledge brings in an approach that is different from that of engineers. The latter follows a set of procedures and utilise a list of specific tools to construct. The approach caters for learners' needs in their own different contextual backgrounds. Hence, the contextual, but arbitrary connection of the existing algebraic procedures with reasoning constructs in this study.

The multi-perspectival aspect of bricolage that defies a notion of working in silos or concentrating the powers and control of knowledge on individuals inspired this study. It embraces Kellner's (1999:xii) view that "the more perspectives one can bring to their analysis and critique, the better grasp of the phenomena one will have", hence the indulgence of a research team as opposed to the monological approach (Mosia 2016:142; Rogers 2012:8) for knowledge production. It implores bricolage support to allow people to interact and filter the content matter into their socio-cultural context to make sense of it. The renowned practitioners of bricolage profoundly influenced researchers from a plethora of disciplines to operate in original and concrete settings to share ideas, theories, techniques, and experiential knowledge for the benefit of the affected (Kincheloe et al. 2011:4) as in this study, where the research participants interact to share experiences and devise contextual solution(s). The deployment of learners' reasoning constructs to conceptualise procedures is a new avenue envisaged for creating enhanced knowledge, of a high order cognition in compliance with the curriculum policy (DBE 2011:4,8–9; Kincheloe et al 2011:4).

Bricolage is therefore recommended by social movements for considering partners as knowledge producers in their own right, and not as objects to be manipulated and regulated in a setting detached from the real world of their lived experiences and practices (McGregor 2008:199). In this study participants engage actively in line with the ethics of engagement (see 4.3), own up the project and emerge as co-researchers (Mahlomaholo 2014:9). That way, the project ensures that the components of solution are original and dependent on the direct and democratic participation of the affected (Bush & Silk 2010:abstract). In essence, the study framework creates room for participants to provide solutions in a natural setting (Barker 2004:20; McGregor et al. 2010:424) where the participants' opinions and objections are treated with respect (Chilisa 2012:48; Bush & Silk 2010:abstract), as opposed to the positivists' experimental settings in which the participants are manipulated for the individuals' (researchers') benefit (McGregor et al. 2010:422-423).

#### 2.2.1.1 *Moments of qualitative research*

Bricolage is the first phase of the seventh moment of qualitative research (Dictionary.com n.d.). The next sections will unpack the historical evolution (moments) of qualitative research in relation to the teaching and learning of algebra. It will also forge the ontological and epistemological relevance and significance of bricolage to the aim and objectives of this study. It will highlight with the help of examples some research developments each moment has brought to the teaching and learning of algebra and mathematics in general. It will pay more attention to the prints of bricolage within the developments. It is however worth noting that the moments of qualitative research are not fixed, but overlap in time (Mosia 2016:145).

### *(a) First moment*

The first moment was called the **Traditional Period.** It covers the period from around 1900 to about 1945 (Denzin & Lincoln 2005:3; Kincheloe 2004; Lewis 2009:3; Mahlomaholo 2014:174; Mosia 2016:137; Willis, Jost & Nilakanta 2007:151). During the traditional period, the researchers used the natural scientific procedures. It did not study a subject of research in totality. It concentrated on one aspect, driven by one person who treated the researched as objects from which they could extract information in their own terms (McGregor & Murnane 2010:424; Penco 2010:2; Vilela 2010:344). The epistemological traces of the one-aspect research that disregarded the researched and their contextual background, are still observable within the teaching and learning of algebra even today. For instance, the concentration of algebraic computations in most classrooms is to find a correct answer without questioning the conceptual connection or reasoning constructs behind the procedural computations (Bergeson et al. 2000:29; Pierce & Stacey 2007:12). The computations are led by a sole knowledgeable and powerful constructor, a teacher, who dictates the kind of knowledge consumers (learners) should passively consume and reproduce (Major & Mangope 2012:140). The teacher also usurps all the powers of control as to how learners should acquire knowledge.

It was during the traditional period that the classical representation of algebra, which did not use symbols to represent unknowns (FreeMATHhelp.com 2018) was developed into the modern or abstract structures of algebra (Corry 1996:26). The development left the computations of algebra in its current abstract and complex form. The underlying causal behaviour of manipulations and outcomes thereof was not exposed to teachers and learners. The research aimed at determining the relations between cause and outcome and to objectively deduce the patterns of behaviour to expect in a similar situation (Mosia 2016:137). Algebraic algorithms, mnemonics and procedures were and are still used in the same fashion, to guide a learner to arrive at the correct answer (outcome). The correctness and quality of the algorithms, mnemonics and procedures created were measured by reliability, validity and predictability and consolidated into general rules regarded as universal truths. As such, the resultant computational products could hardly relate or be interpreted, nor conceptually understood by both teachers and learners. It could therefore be argued that research of the traditional period left the teaching and learning of algebra in a rote and abstract state that we still experience even today.

This line of research and thinking directed towards obtaining valid, reliable and general rules (Onwuegbuzie, Leech & Collins 2010: 696) was later challenged by ethnographers who detected the confusion caused by the facts that emerged from the data they had collected (Denzin & Lincoln 2005:25). The data failed to yield the results they could relate to or interpret in terms of the rhetoric criteria of scientific procedures as alluded to above. In the same vein, the products of the teaching and learning based on algebraic algorithms, mnemonics and procedures, fail to display the anticipated results. The empirical studies of that period and subsequent times revealed that the majority of learners could not conceptualise and comprehend the knowledge and application of algebra meaningfully (Bergeson et al. 2000:29; Pierce & Stacey 2007:12).

Contrary to the traditional period's rhetoric, the rigour or quality standards of bricolage articulated in the letter of Luitel & Taylor to Kincheloe namely multi-textuality, praxis dimension, humility, incisiveness and illumination, suggest otherwise. Bricoleurs are researchers who are textually mindful and hold onto a view that the research text cannot be independent of a person, time and space of the researched (Luitel & Taylor 2011:8). Their research should avail itself practically and critically to readers or the audience for whom it is epistemologically, methodologically or theoretically targeted (Luitel & Taylor 2011:8–9). Such critical reflexivity can be determined by the ease readers interpret the research, the critical reflection on assumptions, and the critical and constant reflection on evolving subjectivities of the inquiry and the textual constructions that are not prone to scrutiny. Bricoleurs are humble and less ostensible about argumentative issues and are open to self-evaluation, self-critique and acceptance of self-weaknesses (Luitel & Taylor 2011:8–9). They further exhibit epistemic tolerance towards adversarial traditions and perspectives associated with issues under study. The standard of incisiveness ensures that the research is focal on integral issues surrounding the field of inquiry (Luitel & Taylor 2011:8–9). For instance, focusing on the fundamentals of each challenge besetting the inefficient teaching and learning of algebraic expressions and equations in this study is incisive. Further, seeking every solution possible to counteract any risk thought to threaten the implementation of the reasoning-based instruction also proves the incisiveness of the research. Lastly, the standard of illumination has to do with enriching, deepening and making the integral issues under investigation vivid and more complex (Luitel & Taylor 2011:8–9). For instance, the research team working as bricoleurs in this study collaborated with other participants to illuminate and operationalise the underutilised reasoning skill for the conceptualisation of algebraic procedures.

***(b) Second moment***

The second moment was named the **Modernist or golden Age Phase.** It covers the period from around 1950 to 1970 (Denzin & Lincoln 2005:3; Kincheloe 2004; Lewis 2009:3; Mahlomaholo 2014:174; Mosia 2016:137; Willis et al. 2007:151)**.** It deepened the principles of the traditional period (Denzin & Lincoln 2005:3). The emphasis was to determine the relationship between the cause and outcome to predict the future using behavioural patterns. Researchers believed that subjecting human beings and their experiences to experimentation just like the objects in the science laboratory could yield reliable results. Mosia (2016:140) writes that researchers of the golden age phase sought to formalise qualitative research to liken it with quantitative research. It was only around the 1970s that researchers began questioning the reliability of the positivism procedures on human beings. Hence the advent of many new research avenues under the ambit of blurred Genres (Denzin &

Lincoln 2005:3). It can be argued that the research and classroom instructions on algebra also remained positivistic, focusing on reliable results as a means to validate the findings and observations of the previous undertakings.

### *(c) Third moment*

The third moment is **Blurred Genres (1970–1986)** (Denzin & Lincoln 2005:3; Kincheloe 2004; Lewis 2009:6; Mahlomaholo 2014:174–175; Mosia 2016:142; Willis et al. 2007:151). It is the moment during which the ever-lasting spectrum of researchable truths was revealed using a wide range of theories, paradigms, methods and strategies (Mosia 2016:142). During this period, the research participants had a significant contribution to the research process. The moment placed more emphasis on arriving at rich descriptions of the research process, interpretations and findings emanating from the multi-perspectival and multi-disciplinary nature of participatory research. Hence the emergence of bricolage to counteract the limiting monological approach to research (Mosia 2016:142; Rogers 2012:8). Bricolage, like other theories of blurred genres, accounts for the networked relationships between reality and human perception, hence its multi-disciplined, multi-theoretical, multi-perspectival and multi-methodological nature. The rebirth of bricolage marked a considerable development from the positivism perspective of concentrating the transformative beliefs and powers on a researcher (Denzin & Lincoln 2005:3). The gap between social scientists' and human scientists' approaches towards research closed. The two approaches interacted more. This study emulates the interaction whereby the positivist approach tools such as observations and empirical test results will be used to explain the qualitative factors responsible for poor performance in algebra. It will also capitalise on the interaction to determine the success of the enhancement using both empirical and qualitative indicators.

The example was set when the researchers used both the positivists' and qualitative research approaches to reveal some major complexities learners were experiencing with introductory algebra (Bergeson et al. 2000:26). Booth (1984 cited in Bergeson et al. 2000:26) found that learners were not able to write correct algebraic symbolic representations of a verbally presented procedure and vice versa and that the inability implied an ontological abstraction and complexity embedded within the instructional texts and contexts thereof. Kieran (1983) and Wagner et al.'s (1984) empirical tests found that learners were confusing algebraic expressions with equations (Bergeson et al. 2000:26). Most learners were found forcing algebraic expressions into equalities by adding "=0" when asked to simplify or evaluate (Bergeson et al. 2000:26). This is yet another ontologically inclined misconception that continues to haunt the learning of algebra, suggesting a flawed or incomplete instruction. The epistemological act of a bricoleur of the time would have been to incorporate the contributions of participants to weave the teaching and learning mechanisms precise enough to help learners realise the conceptual difference. This study also aims at addressing or reducing the perpetuation of algebraic common errors (see 3.2.1.3(a)).

***(d) Fourth moment***

Then came the **Crises in or of Representation** (1986–1990) moment (Denzin & Lincoln 2005:3; Kincheloe 2004; Lewis 2009:6; Mahlomaholo 2014:174–175; Mosia 2016:142; Willis et al. 2007:153). The significance of participants' voice manifested more during this moment. Participants demanded central attention and space in the discussions of research so that their experiential truths could be told as such (Mahlomaholo 2014:175). One such truth was a finding from Dreyfus and Eisenberg (1987) and DuFour et al.'s (1987) research that learners could procedurally solve traditional algebraic problems using graphical representation with a minimal or no understanding of the relationships between the problem and the representation (Bergeson et al. 2000:27).

As a result of the demand, research on qualitative perspectives emerged to counteract the reliance on a positivist sole truth centred on an expert (Mosia 2016:143). The new research could not be qualified through objectivity, validity and reliability rhetoric. It then became apparent that research participants had different ways of conceptualising the research processes and interpreting the findings emerging from them (Mahlomaholo 2014:175; Mosia 2016:143).

It can be argued that an appeal by Bergeson et al. (2000:1) to teachers and other mathematics education stakeholders to interpret and reflect the content of the research-based overview in their book primarily based on Washington contexts in their own structural situation and context was driven by this historical evolution. The overview on algebra included *inter alia* the work of Schoenfeld and Arcavi (1988) who argued for conceptual understanding of a variable as a basis for a smooth transition from arithmetic to algebra and meaningful application of algebra in advanced mathematics (Bergeson et al. 2000:27). It also acknowledged the works of Booth (1988), Leitzel (1989) and Wagner and Kieren (1989) admitting the complication of the algebraic variable (Bergeson et al 2000:27). Leitzel's (1989) research revealed that the concept of the variable at an introductory phase was deep and more sophisticated than teachers could expect, and it was often becoming a barrier to algebraic content understanding (Grosser & Lombard 2009). It can be arguably correct to claim based on the foregoing research findings that the notion to teach and learn mathematics concepts, including algebra for understanding over procedural rote learning (Leung, Park, Holton et al. 2014:6–10), became central to research during the moment of Crises of Representation. The epistemological expectations of the moment were to infuse the raw truths discovered through participatory research (representation) into the creation or formulation of new approaches consistent with algebraic conceptual understanding. Legitimation of the research results' interpretation and evaluation became another crisis: participants demanded the contextual criteria and standards to prevail over the positivists' standards of objectivity, reliability and validity. The crisis of representation and legitimation led to a crisis of praxis – research were not practically affecting the lives of the researched for lack of textual and contextual representation and legitimation. It can therefore be concluded that the demand for new interpretative qualitative research that could not be subjected to the standards of positivists brought about the crises ascribed to, hence the emergence of the postmodern moment.

#### *(e) Fifth moment*

The crises of representation, legitimation and praxis that plagued the fourth moment led to the fifth moment, namely **Postmodern** covering approximately the period 1990 to 1995 (Denzin & Lincoln 2005:3; Kincheloe 2004; Mahlomaholo 2014:174–175; Mosia 2016:142; Willis et al. 2007:153). During this moment, the localised (contextualised) narratives were preferred over the generalised narratives of the positivist and post-positivist period (Mahlomaholo 2014:175). The researcher had to work closely with the research participants to understand their narratives and make sense from them (Rogers 2012:12). For example, Luitel (2009) capitalised on the multi-paradigm dimension of bricolage and used postmodernism, interpretivism and criticalism paradigms to strengthen his ethnographic and philosophical inquiry aimed at transforming the philosophy of mathematics education in Nepal (Kincheloe 2011:191). He worked with the ethnic participants according to the postmodernism paradigm to construct different forms of knowledge and employed multiple genres to represent such knowledge. He even conceived a range of ethnically befitting quality standards to regulate the epistemic aspects of the inquiry (Luitel & Taylor 2011:4).

The participants had to become co-researchers who could influence the flow of research processes: conceptualisation, data collection, data analysis and findings. During this moment the research of Kieran (1990); English and Halford (1995); Wagner and Parker (1993) among many others made the following findings respectively: operational differences between arithmetic and algebra; learners perceived learning algebra as an exercise of manipulating symbolic expressions using a set of procedural rules without attempting to acquire the meaning behind the expressions or the rules; learners overgeneralised arithmetic and algebra analogies when simplifying expressions (Bergeson et al. 2000:29).

It was during this moment that Denzin (1994:44) highlighted the importance of localised understanding and solution of problems within the symbolic interaction theory. The other epistemological characteristics of the theory discouraged general theories, objectivity and quantification, and theories that ignore biographies and lived experiences of interacting individuals.

#### *(f) Sixth moment*

Then came the **Post-Experimental** (1995–2000) moment (Denzin & Lincoln 2005:3; Kincheloe 2004; Mahlomaholo 2014:175; Mosia 2016:144; Willis et al.

2007:155). It is defined by the deepening of participants' contribution to the research. It is a moment of storytelling and composing ethnographies in a manner that escalate the contribution of the participants in research. Participants' voices became loud and their experiences, fears and aspirations informed the research processes. They collaboratively participated in an enabling environment with the aid of user-friendly devices and resources where appropriate. The collaborative work of Greenes and Findell (1999) found that learning algebra could be less difficult if the introductory curriculum included experiences with algebraic reasoning problems that stress representation, balance, variable, proportionality, function, and inductive or deductive reasoning (Bergeson et al. 2000:26). The work of Whitin and Whitin (1999) confirmed the stance of this moment by finding that mathematical reasoning is nurtured naturally within collaborative communities (Bergeson et al. 2000:26). The study of Moloi (2015) also confirmed qualitative research appreciating the bricolage of community capitals through participatory action to produce knowledge. The study, based on solving problems using indigenous games, constitutes an epistemological evolution from the traditional and modernity moments of positivism to a collaborative approach. The approach borrows from different capitals in the community to enhance the teaching and learning of mathematics and algebra in schools.

#### *(g) Seventh moment*

The seventh moment is **Methodological Contestation** (2000–2004) (Denzin & Lincoln 2005:3; Kincheloe 2004; Mahlomaholo 2014:175; Mosia 2016:144; Willis et al. 2007:155)**.** The studies of this moment emerged to counteract the minute traces of positivism regarding the criteria epitomising good qualitative research (Mosia 2016:145). The criteria for evaluating qualitative research ought to concentrate on morals, ethics and epistemologies other than validity and reliability (Denzin & Lincoln 2005:3). A determination of what is morally and ethically acceptable however lies in the hands of those who have power (Denzin & Lincoln 2005:3). It renders the criteria a threat to the transformative principles of bricolage. Bricolage detests the single-handed and fixed epistemological power in all forms. Bricoleurs' counter-hegemonic work, embraced for transforming educational research (Luitel & Taylor 2011:10) lingered around decentralising power and control to the researched. As such, researchers had to ensure active participation and duly recognition of participants, and lived experiences (Kincheloe 2005:342).

It was during this moment that the research by the NCTM (2000) made a finding underpinning the conceptual framework of this study. Their research contested and found it true that the teaching and learning of algebra without logical connections between algebraic procedures and concepts weakens the possibilities of acquiring conceptual understanding (NCTM 2000:20). It can further be argued that such an instruction detracts from learning to regurgitation. Bricolage supports the use of basic mathematics (arithmetic) to connect the procedural computations for enhanced learning. The connection constitutes the reasoning framework entailed in this study. The research also highlighted the need for learners to endeavour to strike a balance between procedural fluency and conceptual understanding (NCTM 2000:35). However, the research was silent about the criteria or measure of the balance (Katz 2007:15). The criteria have since been declared unknown. Furthermore, the research findings embraced the bricolage principle of learning through collaborations: (i) that teachers needed structured time to engage in formations and collaborations geared towards a deep understanding of mathematics and sustained professional development (NCTM 2000:370; Garet, Porter, Desimone et al. 2001:936); (ii) learners' collaborations in which they interact to share competence and skills and most importantly support or expand their inputs with reasoning to consolidate their learning (NCTM 2000:60–63).

#### *(h) Eighth moment*

**Fractured Future** (2005–date). Other than reiterating the direct involvement and participation of the researched, the eighth moment directs that the social sciences and humanities should spearhead robust debates on critical issues. The featuring issues for debate include, but are not limited to, democracy, race, gender, class, nation-states, globalisation, freedom and community, equity, social justice, freedom, and peace (Denzin & Lincoln 2005:20). Willis et al. (2007:157) add that a fractured future is a moment of discovery and rediscovery during which research involves debate and discussions about new ways of collecting data, interpreting it, arguing about it and writing it. Qualitative research is no longer seen as a neutral, objective or positivist act, but rather as a multicultural process in which communities discuss and debate socio-political issues affecting their lives within a localised context.

Bricolage portrays the aspects of this moment. It displays the multi-disciplinary nature by borrowing from multiple disciplines and methods. In this study, bricolage supports an open debate geared towards articulating the benefits contained within the conceptual framework of connecting the procedural algebraic expressions and equations with reasoning to attain conceptual knowledge. The research team members work like bricoleur. They use and accept ideas and concepts that are sometimes contradictory at first and create a new contextualised solution accommodative of teachers and learners from different background and cognition. It is driven by epistemological theories that draw from Denzin & Lincoln's (2005:3) view that the true meanings arise from the process of social interaction of people who have lived experiences of the subject of research (Mahlomaholo 2014:388; Willis et al. 2007:177).

#### *2.2.1.2 Appropriateness of bricolage*

Bricolage has proved to be an appropriate paradigm for guiding the envisaged reasoning-based instruction. The instruction entails learner-participatory construction of reasoning constructs. Bricolage mergers well with constructivism theory to promote learners' active participation in the construction targeted at conceptualising algebraic procedures. The construction uses basic mathematics to connect and establish necessary interrelations within the algebraic concepts and procedures. The connection results in an improved product (Barker 2004:43; Kaomea 2016:10). In context, the product is conceptualised algebraic knowledge. The following characteristics provide more insight into the appropriateness of bricolage:

##### *(a) Bricolage simplifies complexity*

Bricolage is couched in the simplification of complexity (Renwick 2014:6). It assists the researchers to grasp the area of complexity (Kincheloe 2004:687), hence be in a position to articulate the simplified components of solution. The simplification characteristic in this study is framed within conceptualisation. Conceptualisation entails attempts to contextualise and concretise the content matter. Contextualisation involves refocusing, integrating and reorganising content matter to suit both subject and learners' needs (DHET 2015:62); connecting and inter-linking (interrelating) algebraic concepts and procedures into conceptual knowledge. The simplification and contextualisation are assured through the learners' active participation in the construction of meaningful knowledge (Pramesti & Retnawati 2019:3) that makes sense (Yackel 2001:1) to them. Bricolage anchors this collaborative initiation of simplified discourses aimed at improving instruction (Klages 2012:45; Luneta & Makonye 2010:35). The simplified content matter endures for a longer period (Long 2005:61). It spares learners unnecessary strain of having to learn procedural steps by heart (Sawyer & Alder 2001:1; Star 2005:406). Instead, it enables them to navigate flexibly through their own constructed network of knowledge.

***(b) Bricolage supports connection formations of change***

Bricolage supports the connection of the unconnected learning material to attain an improved product (Barker 2004:43; Kaomea 2016:10). It is about a selective collection of odd or underutilised bits of information and/or materials, and connected usage of those in the context of a repair job (enhancement) or new approach. The collection, correlation and usage of information escalate the process of connection to critical thinking. That way, bricolage anchors the concept of tapping from the underutilised remnant of reasoning skill to connect the algebraic procedures into conceptual knowledge. It recognises the dialectic nature of abstraction and complexity between the procedural and conceptual aspects of content matter, and forge necessary connections aimed at enhancing the former into the latter. The net result of the connection is the formation of an improved product (conceptual knowledge and understanding). Furthermore, learners become critical thinkers who can independently analyse and solve routine and non-routine problems on algebraic expressions and equations, as well as applying algebra in advanced mathematics and related disciplines (Ying et al. 2020:5406). Mathematics is virtually connected to all branches of social and natural sciences (Hamami 2020:4).

***(c) Bricolage is multi-perspectival and multi-theoretical***

The multi-perspectival and multi-theoretical characteristic of bricolage resonates with the requirement of this study to accommodate freely adduced opinions from diverse stakeholders of different backgrounds (Kellner 1999:xii). Multiple perspectives help researchers to get a better grasp and understanding of their research (Kellner 1999:xii). In this study, the collaboration of participants from different backgrounds helps to unpack the areas of complexity from different perspectives (Kincheloe 2004:687) to articulate appropriate components of solution. The different opinions guarantee a more coherent ensemble of content matter within the proposed conceptual framework (Abercrombie et al. 2006:47). It could then be argued that the characteristic also empowers teachers and learners to invent an innovative ensemble of content matter, and the teaching and learning approaches befitting their context.

***(d) Bricolage promotes a socio-cultural, transformative and democratic approach***

The interdisciplinary nature of bricolage promotes a democratic field upon which participants communicate, convene, work collaboratively and create working structures (Bush & Silk 2010:abstract). It connects the teaching and learning to a broader notion of cultural politics designed to further a multiracial, economic and political democracy in which participants are treated with respect (Bush & Silk 2010:abstract). That way, the working structures are informed, free and more rigorous to achieve the objectives of the study. They are empowered to transform the teaching and learning approaches of algebraic expressions and equations. The transformation is focused on conceptualising the procedures for the benefit of learners. As such, the research team and participants from different educational backgrounds, age and race work harmoniously in a democratic environment in which the academic inquiry is put in the public domain. The environment in which no one feels subjugated, ensures full participation and guarantees good results (Moloi 2015:32).

Bricolage, therefore, guides a directional purview for inviting critical dialogue and debate towards new epistemologies and axiological approaches set to free teachers and learners from the abstractness and complexity of algebra. It supports the notion of engaging teachers and learners as people who have the best vantage point to critically evaluate the challenges embedded within the teaching and learning of algebra. In the same vein, it backs up the phenomenon that the researched or affected are best placed to figure out the components of solution to the problem within their socio-cultural context (Bennett 2019: 109; Watters & Comeau 2010:6).

The democratic transformation of an instruction envisaged by this study can be likened to the inquiry in developing a transformative philosophy of mathematics education for Nepal (Kincheloe et al. 2011:191). The Nepalese utilised bricolage to upgrade mathematics education to include multiple knowledge systems arising from learners' multiple lifeworlds (Kincheloe et al. 2011:191). The transformation in this study conscientises teachers and learners about the gains of understanding the algebraic concepts over memorising procedures. It makes the teaching and learning of algebra to differentiate between the knower and known, perception and the lived world, and a discourse and representation (Kincheloe et al. 2011:191). It also ensures that the educational imbalances of the past are redressed, and that equal educational opportunities are provided for all sections of the population, in line with the general aims of the South African curriculum (DBE 2011:4).

#### *(e) Bricolage ensures harmonious change management*

Bricolage supports a well-managed change. It creates an enabling platform and environment in which participants are orientated of the anticipated. An example of such platforms could be conceptualisation and reflective meetings held for research team members to freely discuss alternative perspectives on the subject of change (McMillan 2015:abstract). Research participants also air their views within structures of their own setting about the changes that come with the proposed instruction. They also suggest approaches they deem appropriate to instruct the algebraic expressions and equations procedures with the new strategic discourses to effect a desirable product. This is because bricolage supports the analyses of the assumptions and institutionalised practices that have been ignored, like reasoning in this study, to seek and provide alternative perspectives and sources of knowledge (Kincheloe 2004:9).

### *(f) Bricolage provides accountability for alternative routes*

Bricolage enables the researcher to account for alternative knowledge initiatives, be it a new initiative or initiatives using alternatives that have been underutilised or neglected (Kincheloe 2011:192). One such example of an underutilised alternative is reasoning, despite being provided for in the curriculum policy (DBE 2011:8–9). Its potential has not been fully used in the teaching and learning of algebraic expressions and equations in grade 9. Rather, the procedural computations aiming at the unaccounted for answers has been instructionally preferred. The limitation of teaching and learning to algebraic procedural computations leads to rote learning (Long 2005:61; Sawyer & Alder 2001:1). The effects of rote learning are *inter alia* unenduring knowledge that causes the learners a strain of having to learn myriad procedural steps by heart (Sawyer & Alder 2001:1; Star 2005:406). It does not consider, nor encourage, proficient connection and conceptual interrelation.

To this effect, Kincheloe (2011:191) acknowledges bricolage for unfolding an enabling research space that allows accountable alternative initiatives such as connecting algebraic procedures using reasoning, to conquer some of the world’s complexities. Rogers (2012:12) shares Kincheloe’s supporting sentiments to critical bricoleurs who question and learn from the excluded. The team of researchers in this study emulate bricoleurs. They critically dig deep to solicit the knowledge silenced by the dominant practices (Rogers 2012:12). The confinement of algebraic computations to procedural steps leading to correct answers without reasonable justification accessible to learners and laymen is a typical example of such practices. Bricoleurs take over the responsibility of knowledge production and ensure that it is contextually original. In the process, the power and control over the teaching and learning of algebraic expressions and equations in a meaningful way are removed from elite groups. The transformation (process) becomes an element of the public domain for which all subjects can account.

### (g) Bricolage maintains quality standards

Bricolage acts as a legitimating mechanism for institutional change (Desa 2012:727) to align the changes brought by the reasoning-based instruction with the curriculum policy imperatives (standards). It embraces visionary changes that maintain the set standards (Kincheloe 2011:191). This study endeavours to achieve the instruction that "engages learners into active and critical learning other than rote and uncritical learning of the given truths; achieve minimum standards of knowledge and skills, and set high, achievable standards (DBE 2011:4) and develops a deep conceptual understanding" of algebra as required by the curriculum policy (DBE 2011:8). It caters for simplified learning of algebraic expressions and equations while maintaining the standards. By maintaining the standards, this study ensures that the research focuses on addressing complexity within the parameters of the curriculum policy. As a result, the search for a solution and knowledge production is intertwined to legitimation or authenticity (Kincheloe 2004:687).

### 2.2.1.3 Formats of bricolage

The formats of bricolage that include being interpretive, political, methodological, theoretical, narrative and critical (Rogers 2012), explain its philosophical stance relative to other philosophies and theories. Bricolage is part of critical post-positivism theories that amplify critical theory by embracing freedom for all, democratic values and principles, transformation of pedagogical practices, equity and equality (Lynn 2004:162; Stinson et al. 2012:46). They challenge injustices and inequalities embedded in socio-political structures, practices and discourses. It is critical, multi-theoretical, multi-methodological, multi-disciplinary and multi-perspectival (Rogers 2012:13). It recognises and pays more attention to already existing historic, social and cultural structures of the society. The deployment of bricolage in an attempt to connect critical pedagogy to the teaching of algebra and mathematics for social justice (Stinson, Bidwell & Powell 2012:77) within the socio-cultural context of teachers and learners resonates with the dictates of critical theories.

Bricolage frees its practitioners in education to unleash potentials and approaches that would have otherwise remained concealed (Long 2005:60), had the era of positivism and its dictates been sustained. The research team and participants in this study freely emulate bricoleurs by connecting what could be considered scattered bits and pieces of information (procedures) and materials at hand (reasoning) (Long 2005:60), to create an enhanced conceptual knowledge. Learners, 'the silenced' and 'the excluded' (Rogers 2012), construct the knowledge without any scientific or behavioural control. Teachers guide learners towards the latter's own reasoning constructs rather than dictating unexplained procedural steps (Bergeson et al. 2000:29; Pierce & Stacey 2007:12). That way, the construction of knowledge becomes a true product of conceptual understanding. The foregoing formats sustain the socio-cultural, transformative and democratic nature of bricolage and ensure active participation of the affected in solving their own societal challenges.

#### *2.2.1.4 Ontology of bricolage*

Ontology is a study of the nature of reality and truth (Noorderhaven 2004:91). It clarifies the researcher's position about the nature of existence in science, the implicit and/or explicit presuppositions (assumptions) from which the research would be undertaken (Noorderhaven 2004:91). Bricoleurs normally examine the socio-historical dynamics of the problem and how they influence and shape the complexity (Rogers 2012:10). The examination helps them to question the origin and nature of the complexity, hence be able to connect complexity, cognition and pedagogy. That is, the epistemological and ontological deliberations of bricolage help researchers to gain insight into new modes of thinking (Kincheloe et al. 2011:22).

The envisaged reasoning-based instruction is meant to bridge the ontological divide between the objectivists and bricolage constructivists, and discursive accounts. It seeks to reveal some hidden realities behind the perceived abstractness and complexity of algebraic procedures. It is therefore notable that bricolage supports reasoning productivity in shaping reality (Kollosche 2021:473). The complexity that is objectivist will be articulated, addressed or reduced through constructivists' actions of bricolage envisioned in the study. The study, therefore, regards the perpetuation of procedural computations without meaning embedded within rote learning as discursive. It, therefore, presupposes (assumes) that learners can achieve conceptual understanding if the teaching and learning can be directed towards meeting the curriculum requirement to "communicate, think, reason logically and apply the mathematical knowledge gained" (DBE 2011:9). That is, framing and operationalising reasoning as a means of connecting procedures will close the cognition and pedagogical gaps, hence enhance the instruction. In this regard, the study is set to explore and capitalise on new ways of thinking, seeing, being, and researching that defies the objectivists' procedure-bound science or belief that confines the pedagogy (teaching and learning) to algorithmic procedures. Algorithmic procedures hold back and fail many learners to meet the cognitive standards of the curriculum policy. It is however noted that there could be serious challenges and risks (see 3.2.1 & 3.2.4) that may threaten the presupposition.

#### *2.2.1.5 Epistemology of bricolage*

Epistemology clarifies the researcher's approach towards knowledge generation or production. The approach, like in this study, can build on the foundations of an accepted and rationally defensible theory of confirmation and inference, or set aside existing knowledge and replace it with a new discovery built from internal coherence (Noorderhaven 2004:91). Epistemology involves the exploration of how researchers come to know about the concepts they study, how this knowledge is structured, and the grounds on which these knowledge claims are based (Kincheloe et al. 2011:12–13). Epistemological understanding of the research is therefore central to the rigour of the theoretical framework as the latter is to the research processes. It is only when the epistemological understanding has been conceptualised that the research team functioning as bricoleurs, can perform activities of knowledge production. Therefore, it could be substantially argued that the longer time spent in inculcating the epistemological understanding, is a research investment and holds some far-reaching benefits in the success of the study. Epistemology should be explained comprehensively along with the frameworks of the study during conceptualisation session(s) and along the research process. As a way of demonstrating epistemological understanding, the participants may need to freely ask questions, develop concepts, construct reasoning behind procedures, and provide their own interpretations of the data they generate. Kincheloe et al. (2011:12) warn that the dimension of epistemological understanding involves taking difficult epistemological decisions that go together with the plan and are sustained throughout all the implementation phases.

In other words, the process should be devoid of tyrannical pre-specified intractable research procedures.

Bricolage examines the disempowering of socio-cultural forces embedded in the normal practice of meaningless procedural computations (Bergeson et al. 2000:29; Pierce & Stacey 2007:12) and supports inclusive social perspectives aimed at altering the normal practice towards emancipatory praxis (Kincheloe et al. 2011:192). The praxis is for learners to achieve a conceptual understanding. Epistemologically, the research team emulate bricoleurs in exploring and analysing the foundations of knowledge surrounding the teaching and learning of algebra in grade 9, assess its historical interpretation and its effects on the current situation (Rogers 2012:10). It is then that the team can understand the influence of the historical and dominant rationalities of the subject hence be in a position to articulate the possible solutions in the socio-cultural context of the affected. This study is set to sequentially flow according to the said inquiry procedures to locate and agree on the appropriate epistemological activities befitting the challenge. The role of the team of researchers emulating bricoleurs, is to attack the complexity within the subject of inquiry (instruction), uncover the hidden or invisible artefacts of power, tradition and culture, and document the nature of their influence on general scholarship (Kincheloe et al. 2011:2).

#### *2.2.1.6 Relationship between researcher and co-researchers*

The multi-theoretical and multi-perspectival nature of bricolage (see 2.2.1.2(c)) is derived from diverse experiences, knowledge, competence and skills of the participants. The diversity is harmoniously accommodated if there are common unifying goals and values that will sustain the unity of purpose among the participants (Mahlomaholo & Netshandama, 2012:43) for the duration of the study and beyond. This is in line with the different qualitative moments that are representative of different views about reality and knowledge creating processes that must be accommodated if the study is underpinned by bricolage. To this end, the choice of participants in this study was considerate of the participants' diversity that presented itself to the study. The basis for participation was guided by the feeling of obligation on the part of participants to be involved in solving the problem brought by inefficient teaching and learning of algebraic expressions and equations in grade 9, and their availability.

The relationship between the researcher and co-researchers is fundamentally built on trust, respect and humility, and balancing power issues from engagements (Mahlomaholo & Netshandama 2012:37,38). The researcher was guided by study frameworks, the aim and objectives of the research, as well as research ethics and conditions, to select the research team. The researcher assumes the role of a convenor and creates spaces in which people can work on a solution for the problem (Bungane 2014:33). The space provides room for the team to discuss the ethics of engagement, work programme, and rules and norms of conducting research assignments (Chesters 2012:abstract). The ongoing engagements within the team ensure a harmonised setting in which each member is equally important and free to contribute in all stages of the research (Grant & Osanloo 2014:26; McMillan 2015:abstract). Members' multi-perspectival contributions are treated with respect, and are used to link the existing material with new strategic discourses.

Further, the team of researchers also extend mutual respect to participants. They collaborate with participants to ensure the maximum contribution of lived experiences and resources to the research. Reports of members are independent and valuable. They are however subjected to rigorous discussions to ensure compliance with the research objectives and study frameworks. The principles of bricolage guard against collusions amongst the team of researchers. The team should critically and constantly reflect on study assumptions, evolving subjectivities of the inquiry and textual constructions (Luitel & Taylor 2011:8–9).

Generally, the overriding relationships within the research team and participants are guided by respect, and upholds the principles and values of democracy and the historic, traditional, cultural and social structures and practices of members as espoused by the study framework. As a result, both research team members and participants emerge from the research process as co-researchers (Mahlomaholo 2014:9) and play a leading role to share the outcomes of the research with other stakeholders.

### 2.2.2 Constructivism theory

Constructivism is a post-positivism theory roped into this study to complement bricolage. It is by nature a branch of cognitivism because it conceives learning as a mental activity (Ertmer & Newby 2013:55). The conviction augers well with the relativist perspective about reasoning. That, it is a mind-dependent discourse, productive in shaping reality (Kollosche 2021:473). Constructivism couches the placement of reasoning as a critical thinking skill that enables learners' potentials to attain high order cognition. However constructivism distinguishes itself from traditional cognitive theories in several ways: whereas most cognitive psychologists think of the mind as a reference tool to the real world, constructivists believe that the mind filters input from the world to produce its own unique reality (Ertmer & Newby 2013:55). The belief augers well with learners' contextual, but arbitrary connection of the existing algebraic procedures executed under the participatory action research (PAR) guidelines. Constructivism like other cognitive theories resonates with reasoning theory in making knowledge meaningful and functional (Ertmer & Newby 2013:55; Matsolo 2006:62; Pramesti & Retnawati 2019:3; Tlali 2017:85). It organises and relates new information to existing knowledge in memory. Hence, it being an ideal theory to support and analyse the reasoning behind learners' connecting and interrelating constructs. The utilisation of the remnant, reasoning, to enhance (improve) algebraic instruction is supported by bricolage. Bricolage backs an improvement of remnants (Given 2008:68–69; Kincheloe et al. 2011:1; Rogers 2012:1), and constructivism theory acknowledges the importance of building upon the capital(s) at the disposal of the researched (Moloi 2015:16, 26).

Constructivism promotes learning in which the construction of new concepts is based upon prior knowledge (Weegar & Pacis 2012:13). The proposed instruction envisions learners selecting and processing basic mathematics to construct hypotheses, making decisions, giving meaning and/or reorganising algebraic procedures into meaningful constructs (conceptual knowledge). Constructivism is defined as a philosophy, or belief, that supports learners to create their own knowledge based on interactions with their environment, including their interactions with other people (Weegar & Pacis 2012:11–12). The definition strengthens the theoretical relationship between constructivism theory and bricolage in the context of this study. Whereas bricolage advocates for active involvement of the affected (Kincheloe et al. 2011:4), constructivism advocates for an instruction in which learners work on learner-centred activities within collaborated groups (Moloi 2015:37). It involves active learning and has relevance to what constitutes an effective mathematical classroom environment (Haylock & Thangata 2007:5).

Constructivism supports the taxonomy complexities that engage learners in higher level thinking to develop them beyond the simple factual knowledge (Major & Mangope 2012:144). This study presupposes that when the algebraic steps or procedures are connected with learners' reasoning constructs to clarify the conceptual relations in-between the procedural steps, a more enhanced and coherent (conceptual) knowledge is achieved. It is emphatic about learners engaging in analysing, predicting, justifying and defending their ideas (Rumsey & Langrall 2016:419). It infers therefore that constructivism supports reasoning and involves navigation through learners' own concept map, networked by prior knowledge and experiences. It then holds true that constructivism has proven effective in assisting teachers in meeting the challenge of improving learner achievement (Weegar & Pacis 2012:11). Major and Mangope (2012:144) recommended the use of constructivism for the educational system of Botswana in support of the National Vision 2016: "to provide quality education that would enable Batswana to adapt to the changing needs of the country as well as the global changes". It is a theory renowned for the production of quality knowledge (see 2.3.2.2). In this study, the quality is related to conceptual understanding of which reasoning in the form of conceptual connections and interrelations is a derivative.

Weegar and Pacis (2012:13) write that Kumar (2006) developed a constructivism oriented instructional framework similar to the one envisaged by this study, to bridge the gap between theory and practice. This study bridges the gap between procedural knowledge and conceptual knowledge using basic mathematics for reasoning mathematical facts, connections and interrelations. The engagement of learners' active participation in acquiring knowledge is key in both frameworks. The notion of participation is driven by the constructivists' belief that learners impose meaning on the world through the construction of their own understanding based on their unique experiences. The belief to connect the making of meaning to human experience (Kincheloe 2005:342) is bricolage. The sentiment augers well with the reasoning-based instruction intended of this study: creating space and time for learners to analyse the content matter and re-arranging it to fit within their own learning schemas. That way, the instruction allows for contextual or individualised conceptualisation.

#### *2.2.2.1 Epistemology of constructivism theory*

Constructivism, which is sometimes referred to as an epistemology itself, is a learning theory that describes the process of knowledge construction (Major & Mangope 2012:139) in which learners are the main constructors of knowledge. Constructivists view knowledge production as an active rather than a passive process (Weegar & Pacis 2012:6). They believe that learners develop knowledge through active participation in their learning (Weegar & Pacis 2012:6). Moloi 2015:32 reiterates the sentiment further that the principal content of what is to be learned should not just be deposited into learners' minds, but rather be constructed by learners themselves. Constructivism is a theory that upholds the notion that knowledge and learning should inform practice but not prescribe practice (Weegar & Pacis 2012:6). The phenomena of contextualisation and concretisation embedded in this qualitative study leaves room for differentiated approach to knowledge production. By its very nature, constructivism emphasises the importance of teaching context, learners' prior knowledge, and active interaction between the learner and the content to be learned (Haylock & Thangata 2007:5). It supports the perception that people construct knowledge relative to their context (Bauer & Perciful 2009:1). It is therefore expected that the instructional framework of the study emulating the foregoing constructivists' perceptions will be inherent to learners' socio-cultural context. Further, constructivism has shifted to a more radical concept by which learners' fresh ideas are brought to class, acknowledged, and enhanced through a variety of teaching and learning techniques that actively engage them (Major & Mangope 2012:140). The characteristic of lending the responsibility of active construction of knowledge to learners' experiences, has resulted in increased popularity for the constructivist approach when designing instructional technologies (Major & Mangope 2012:140).

#### *2.2.2.2 Socio-constructivism and pragmatism*

Harries and Spooner (2000:28) and Orton (2004:198) bring in the socio-constructivism in support of learner-centred approaches. They purport that socio-constructivism has always worked wonders in building up shared understanding among discussion groups in which the meaning or understanding of concepts is actively negotiated. The group discussions lead to learners building up the shared understanding. Socio-constructivism also demarcates the roles of teachers to seeing learners interact with learning material in groups and deducing ways of contextualising knowledge on their own. That is, teachers only play a supervisory role of acting as guides, facilitators and co-explorers. In this process, learners are encouraged to question, challenge, and formulate their own ideas, opinions and conclusions within the correct mathematical context (Weegar & Pacis 2012:7).

Weegar and Pacis (2012:8) compares constructivism with pragmatism. Pragmatism depicts the difference between idealism and realism. Idealists believe that the teacher is central to learning and emphasise the lecture, discussion and imitation as means of transferring knowledge. The realists see the role of the teacher as a person who presents content in a systematic and organised way and utilises standardised tests, serialised textbooks, and specialised curricula for each discipline (Weegar & Pacis 2012:8). Pragmatists are realists who prefer an interdisciplinary curriculum that sets a stage for learners to discover knowledge themselves like constructivists (Weegar & Pacis 2012:8). It can then be argued that the active involvement of learners through the participatory action research methodology is augmented and couched in the theories of constructivism and pragmatism. It is however notable that though realists differ in approach, they concur to the idea of using reasoning to demonstrate what is true or at least reasonable to assume as true (Kollosche 2021:473).

#### *2.2.2.3 Constructivism supports problem-solving initiatives*

Problem-solving is a primary characteristic of the constructivism theory (Weegar & Pacis 2012:7). Berger et al. (2010:39) confirm the role of reasoning skills in solving problems that involve complex calculations and/or higher-order skills. The role of constructivism in cementing the relationship between reasoning and problem-solving manifests in the development of learners' mind-set through projects and interactive problem-based learning, cooperative or collaborative learning, experimentation and open-ended problems. The relationship enables learners to interact with the outside world for first-hand feel and application of concepts on their own (Weegar & Pacis 2012:8). It can then be concluded that constructivism guarantees the development of high cognitive learning and critical thinking central to the reasoning-based instruction. It equips them with the knowledge and skills commensurate to technical and technological issues in the fast-growing industrial world of work (Katehi et al. 2009:1–2).

The support of an early introduction of constructivism towards problem-solving is further emphasised in the statement, "in order for Botswana education to produce learners who are creative, analytic, problem solvers, constructivism skills should be promoted at the school level" by Major and Mangope (2012:140–141). Teachers who use the constructivist theory, concentrate on engaging learners to establish relevance and meaningfulness to what they are learning (Weegar & Pacis 2012:11). An achievement of meaningful and functional (applicable) knowledge (Pramesti & Retnawati 2019:3) is integral to the objectives of this study.

#### *2.2.2.4 Benefits of constructivism-based instruction*

Major and Mangope (2012:140–141) confirm that several studies have shown the effectiveness of the constructivist approach in general teaching and learning, as well as positive gains in mathematics learning. The teaching of algebraic expressions and equations can rely on constructivism for enhanced learning. The constructivist learning theory predicts that knowledge encoded from data by learners themselves will be more flexible, transferable and useful than knowledge encoded for them (Major & Mangope 2012:140–141). Further, Cobb (1999:15) emphasises that learners should embark more on knowledge construction than becoming knowledge reproducers. The knowledge construction facilitates conceptual understanding and ensures prolonged retention because the retention of knowledge is greatly influenced by how the knowledge was conceived (Major & Mangope 2012:144).

On the contrary, Major and Mangope (2012:140) reports that Botswana learners who were subjected to the swallowing and regurgitating style of teaching and learning, confirmed constructivists' claim that learners taught that way remain uneducated and can only reproduce the content taught. Regurgitating and reproducing cause strain (Sawyer & Alder 2001:1). Their research found that in most cases teachers who have the insufficient content knowledge and skills commonly resort to shallow non-constructivism approaches (Lempp 2008:abstract). Such approaches lead to learners becoming consumers of how algebraic procedures flow. To them, mathematics is more of a computational subject than being applicative in live experiences. The practice does not give Botswana learners sufficient wisdom to survive independently in this world of socio-political and economic unrest (Major & Mangope 2012:144). It is therefore suggested that teachers need to change their instructional techniques to constructivist-oriented approaches to optimise the results and benefits.

#### *2.2.2.5 Factors that may encourage the use of the non-constructivism approach*

Factors behind the prevalence and domination of teacher-centred approaches in classrooms include: teachers considering themselves as the main transmitters (experts) of knowledge whom learners should attentively listen to lest they miss the information they are expected to reproduce during examinations, which they have no privilege of creating themselves (Major & Mangope 2012:146); examination driven curriculum; and curriculum-time contestation (mismatch). Arthur and Martin (2006) acknowledge the fact that since examination success provides access to further education, parents and learners expect teachers to rush through a congested curriculum (Little 2009:3–4) and focus on examination drilling. The drilling (repetitiveness) has become a school didactic culture (Vos 2018:2) that works against constructing conceptual knowledge. Consequently, the priorities that may not necessarily be the goals of schooling have a direct influence on the way teachers teach (Major & Mangope 2012:146). Another factor is that most teachers find constructivists' methods that emphasise the need for conceptual learning time-consuming. To them, the constructivists' methods are considered to require relatively more time than the procedure-oriented approach. Hence, weighing the former against the latter within the centralised assessment driven systems focused on target times, they would settle for the latter (procedure-oriented approach).

## 2.3 CONCEPTUAL FRAMEWORK OVERVIEW

A conceptual framework is the researcher's understanding of how the research problem will be explored (Grant & Osanloo 2014:16). It operates within (adopts and adapts) the theoretical framework to clarify a direction the research will have to take, and the relationship between different variables (key factors and concepts) in the study. According to Miles and Huberman (1994:18), the conceptual framework is a system of concepts, assumptions, and beliefs that support and guide the research plan. This study assumes and envisions the instruction in which learners use reasoning to construct conceptual knowledge. The construction that manifests in connecting and interrelating algebraic procedures and concepts is directed and couched within the bricolage and constructivism theories (see 2.2).

The key variables (concepts) that constitute the conceptual framework of this study are therefore procedural and conceptual aspects of knowledge, and reasoning (see 2.1). Conceptual knowledge is intricately (complexly) linked to procedural knowledge in that the latter is nested in conceptual knowledge (Long 2005:61). The application of learners' own reasoning (constructs) (see 2.3.3) on (procedures) procedural knowledge (see 2.3.2), simplifies the said complex relationship, in that it weaves both aspects of knowledge into meaningful and functional instruction (knowledge). In other words, this study is conceptually framed on a presupposition (assumption) that when the algebraic expressions and equations procedural flow of steps (procedural knowledge) are connected with reasoning constructs to clarify the conceptual relations in-between the procedural steps, a more enhanced and coherent (conceptual) knowledge is achieved. The presupposition augers well with the understanding that the conceptual framework lays out the key factors, constructs, or variables, and presumes relationships among them in pursuit to explain a phenomenon the study undertakes to achieve (Miles & Huberman 1994:18). It can be drawn from the foregoing that the conceptual framework explains the researcher's ontological and epistemological view regarding the problem. It also lends itself to specifying and defining concepts within the problem statement (Luse, Mennecke and Townsend 2012:145).

### 2.3.1 Operational concepts

This section explains the key words that are related or used collectively with the operational concepts in section 1.5. The study's operational concepts and key words are derived from the processes of topic selection, research question development, conceptualising literature review, design approach, and analysis of the study (Grant & Osanloo 2014:26). In most cases, they are derived from the title where they are collectively used to highlight the thematic promise of the research. The explanations are important in guiding the readership about (i) the contextual meaning of the key words as used in the research, and (ii) the confinement of research activities with the contextual meaning of operational concepts.

**Arithmetic–** is a term used to generalise basic mathematics. It deals with numbers, their properties and numerical computation (OnlinemathLearning.com n.d.; Tapson 2008:14). It is taught at the entry-level of schooling and forms a foundation for learning algebra. The instruction of using reasoning to connect algebraic procedures is based on a deep understanding of arithmetic. Arithmetic is synonymous with the basic rules, skills or concepts of mathematics or just basic mathematics.

**Algorithms–** are another form of procedures. An algorithm is a step-by-step procedure that guarantees solutions only if followed in a predetermined order and without error (Long 2005:59). The most familiar algorithms are the elementary school procedures for adding, subtracting, multiplying and dividing, but there are many other algorithms in mathematics (Long 2005:59). According to Soicher and Vivaldi (2004:1), it is a finite sequence of unambiguous instructions to perform a specific task.

**Learning–** is a lasting change in behaviour, or in the capacity to behave in a given fashion, which results from practise or other forms of experience (Ertmer & Newby 2013:43). However, it should be understood that learning is a very complex process that has generated numerous interpretations and theories of how it is effectively accomplished. In the context of this study, learning is understood as an acquisition of deep conceptual knowledge and the ability to apply it (DBE 2011:9), being the behavioural change or skill that individuals (learners) develop upon utilising the principles of reasoning (see 2.3.3).

### 2.3.2 Procedural and conceptual aspects of knowledge

The procedural and conceptual aspects of knowledge together with reasoning, constitute the conceptual framework of this study. Their contextual definitions, roles, functions and relationships provide clear comprehension of the objectives of the study within the parameters of the study framework (Grant & Osanloo 2014:16).

#### *2.3.2.1 Definitions*

Hiebert and Lefevre (1986) cited in Star 2005:406 define procedural knowledge (or procedural fluency) in two ways: first, it is a familiarity with the individual symbols of the system and acceptable conventions; second, it is a set of rules or procedures for solving mathematical problems. Most of the algebraic procedures taught in grade 9 are rules for manipulating symbols. Procedural knowledge includes knowing the formal language, or the symbol representation system; knowing algorithms and rules for computing tasks and procedures, and knowing strategies for solving problems (Star 2005:406).

Conceptual knowledge is the knowledge that is rich in relationships (Star 2005:406). It can be thought of as a connected web of knowledge, a network in which the linking relationships are as prominent as the discrete pieces of information. Relationships pervade the individual facts and propositions so that all pieces of information are linked to some network (Star 2005:406). It then follows that though the two aspects of knowledge are per definitions different, conceptual knowledge is intricately linked to procedures, and procedural knowledge is nested in conceptual knowledge. The understanding of many concepts embedded in procedures or algorithms through conceptual connections and interrelationships, sought by this study, enriches the network and quality of conceptual knowledge (Long 2005:61).

The ontological and epistemological presupposition of the study benchmarked on attaining a skill to communicate, think, reason logically and apply mathematical knowledge gained (DBE 2011:9) is inherent or dependent on conceptual knowledge. It stems from the understanding that the procedural knowledge (fluency) alone does not help learners achieve the epistemic skills, hence the need to enhance the teaching and learning of procedures. Conversely, the attainment of the proficient command of networked conceptual knowledge gives hope for the achievement of the objectives of the study.

#### *2.3.2.2 Quality of knowledge*

While psychologists reiterate the view of seeing procedural and conceptual knowledge as two different types of knowledge, mathematics educationists regard the two as of different quality (Star and Stylianides (2013:178). Star (2005:407) embraces the definitions of procedural and conceptual knowledge and adds that the latter is defined in terms of the quality of one’s knowledge. He argues that the quality of knowledge depends on the richness of conceptual connections and interrelations inherent of such knowledge. According to them, the quality-based definition is a critical departure from which the connection relationships within the conceptual knowledge can be assessed

(Star 2005:407). The levels of quality or richness of knowledge should however be linked to development because learners go through a procedurally oriented phase before they can effectively integrate their conceptual knowledge. Even Rittleston-Johnson and Siegler (1998:109) echoed the sentiments that there is a positive correlation between children's understanding of mathematical concepts and their ability to execute procedures. In the context of this study, it is imperative to recollect that the majority of grade 9 learners in South Africa find algebra very difficult to cope with (Matsolo 2006:5). It is assumed the sense of difficulty has to do with the failure to illustrate conceptual connection and interrelation (quality knowledge) sought by the curriculum policy (DBE 2011:5). According to TIMSS, SACMEQ and ANA diagnostic reports, the learners fall way below most of their counterparts in other parts of the world in exhibiting the correlation (conceptual connection) within and between algebra and other topics. Matsolo (2006:1) writes that the challenge is more severe in townships or previously disadvantaged schools.

Comparatively, the relationships present in procedural knowledge are either sequential or heuristic (Star 2005:407). They are superficial, fully compiled or rote, and not rich in connections. It can be concluded then that the procedural knowledge is relatively lacking in quality. Algorithmic procedures guarantee solutions only if followed in a predetermined order and without error (Long 2005:59). Therefore, the execution of unconnected heuristics requires inerratic choices. That is, it takes a lot of strain for learners to guard against committing errors.

#### *2.3.2.3 Conceptual first or procedural first (math wars)*

Although most mathematics teachers agree on the difference between conceptual and procedural knowledge, there is a disagreement about which one should precede the other. Some are for developing basic skills with symbols and conventions as a foundation upon which conceptual knowledge will build while some vouch for basic understanding preceding symbolic representation and skill practice (Long 2005:59; Star 2005:404). The disagreement has since been dubbed the "math wars". Reformers, that is those who advocate for conceptual first, argue that procedural knowledge should come as a secondary and supporting tool to conceptual knowledge. Long (2005:59) cites an example of a Western Cape departmental initiative discouraging teachers from teaching procedures on a claim that with sound conceptual understanding learners would develop their own algorithms. He argues that learning this way has the advantage of retaining information for a longer time. This is because the knowledge is achieved by the construction of relationships between pieces of information or by the creation of a relationship between existing knowledge and new information that is just entering the system (Long 2005:60). It is emphasised that the approach encourages the development of own algorithms based on conceptual understanding, has value and elicits varied responses, often insightful, from learners who have grasped the concept (Long 2005:60). The approach is assumed in some cases as the logical starting point for learning the more compacted algorithms (Long 2005:60). Reformers argue that an instructional focus on procedural knowledge, rather than conceptual knowledge, leads to the development of isolated skills and rote knowledge (Long 2005:60). Hence a rush for procedural skill does more harm than good to learners. Sawyer (2001:1) holds rote learning accountable for short-lived knowledge that causes learners a strain of learning procedural steps by heart.

On the contrary, Long (2005:63) argues against the perception that rote learning is defined as learning inherent of habitual repetition and devoid of conceptual understanding. He writes against the implication that rote learning does not create a building block on which knowledge can be built, and does not provide a skill or knowledge that can be connected with any other skill or knowledge. He stands strong that a view that the execution of standard algorithms is devoid of conceptual understanding and therefore aligned with rote learning is questionable and elusive. He rather concurs with Rittleston-Johnson and Siegler's (1998:109) findings that learning is a complex process in which both the conceptual understanding underpinning the skill and the scaffolding function of the procedure play a part in establishing proficiency. This study also shares the sentiment and seeks to cement the relationship between these aspects of knowledge through reasoning.

Long further raises another issue about the misrepresentation of algorithms and procedures. He argues that a fluent execution of algorithms represents an aspect of procedural fluency. However, algorithms in themselves cannot be said to be devoid of mathematical concepts. More to the point is that they represent compressed conceptual understanding; that is mathematical concepts that have been developed to a high level of abstraction. He says an answer at any level to the question of teaching algorithms and procedures is to analyse the concepts underpinning the parts to enable more complex mathematical thinking. The approach of this study seeking to use reasoning to enhance the conceptual understanding through underpinning (nesting) procedural knowledge within conceptual knowledge resonates with the latter perception. It seeks to form necessary correlation (synergies) between the conceptual knowledge and procedural computations by tapping from both orders ("concepts preceding procedures" and "procedures preceding concepts") depending on the demands of different concepts and the background knowledge upon which the algebraic concept is built (Long 2005:62).

Meanwhile, the university teachers view procedural knowledge as a necessary basis for following or performing symbolic mathematical reasoning (Pinkernell 2019:2.). To them, the process of learning abstract mathematical concepts can be described as a progression from procedures to concepts. The procedural-conceptual dichotomy (separation) does not quite grasp the nature of knowledge in mathematics. The concepts and processes are seen as part of the same knowledge entities. In fact, Star and Stylianides (2013:179) suggest that the conceptual-procedural framework should be abandoned and replaced with the new words or phrases that describe the knowledge outcomes of interest.

#### *2.3.2.4 Conceptualising procedural knowledge*

Star (2005:405) argues that reconceptualising procedural knowledge and making it a renewed focus of research would have important implications for both research and practice. That is, knowing more about it and perhaps affording it a rightful place in the teaching and learning of mathematics will enhance the instruction. He avers that research shows that there is very little written about the procedural knowledge as opposed to conceptual knowledge yet its application in classrooms dominates the latter. He adds that disagreements on the role of procedural knowledge in mathematics learning are primarily ideological rather than empirical, and that researchers have not paid enough attention to procedural knowledge and its development. The object of this study to conceptualise procedural knowledge is in observance of its instructional dominance resulting in underrated (below-average) performance. In addition, the envisaged enhancement is intended to develop its application in the production of conceptual knowledge, associated with success (Star 2005:406).

#### *2.3.2.5 Superficial and deep knowledge*

The procedural and conceptual knowledge can be either superficial or deep (Star 2005:405). Deep procedural knowledge would be knowledge of procedures associated with comprehension, flexibility and critical judgment and that is distinct yet possibly related to knowledge of concepts (Star 2005:405). Long (2005:59) reiterates the sentiment that the two are distinct but somehow related in complex ways. Star (2005:409) then suggests that if the slight distinction between the two can be identified, it will be easy to bridge the procedural fluency through appropriate conceptual connections to conceptual knowledge in terms of quality as sought by this study. The intention is to create a proficient understanding of algebraic expressions and equations by connecting or forming necessary interrelations between procedural steps. The connections and interrelations clothe the procedures with the meaning that makes sense (Yackel 2001:1) to consumers (learners). Star (2005:410) argues that this is possible if the existence of deep procedural knowledge can be recognised and the research of what it is, how it develops, and what its relationship is to other types of desired mathematical knowledge can be undertaken. In essence, he is suggesting the widening of the shallow definition of superficial procedural knowledge to factor in the comprehension, flexibility, and critical judgment to qualify as deep. He notes that what learners know and to what extent and means of assessing procedural knowledge still need to be researched. Echoing the sentiments, Long (2005:59) uses the “relational” and “instrumental” understanding to distinguish between conceptual and procedural types of knowledge respectively. He refers to relational understanding as the ability to deduce specific rules and procedures from more general mathematical relations, and instrumental understanding as the ability to apply a rule to the solution of a problem without understanding how it works. The latter uses steps as instruments that may not function as expected should any one single component miss or be in disarray. The missing or disarranged component resembles procedural error in computations. Likewise, algorithms are step-by-step procedures that guarantee solutions only if followed in a predetermined order and without error (Long 2005:59).

#### 2.3.2.6 Additional aspects of knowledge

Another interesting exposition about the aspects of knowledge is that of Findell (2001). He adds three other strands of knowledge onto procedural and conceptual knowledge and claims that they are all necessary for mathematical proficiency (Groves 2012:121). The addition includes the strategic competence, adaptive reasoning and productive disposition. Strategic competence is the ability to formulate, represent and solve mathematical problems; adaptive reasoning is the capacity for logical thought, reflection, explanation and justification (Rumsey & Langrall 2016:419); while productive disposition refers to the habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy (Groves 2012:121). It is important to note that a holistic analysis of Findell's strands of knowledge embraces the aims and skills envisioned for achievement by the senior phase mathematics curriculum policy (DBE 2011:4,8).

Another noteworthy expansion on Hiebert and Lefevre's work is what Long (2005:62) refers to as the application of the theory on procedural and conceptual aspects of knowledge. It maintains that there is a third aspect of knowledge: that of learners who acquire concepts without procedures. He writes that these are learners who focus their operations on a question "how to?" without forming necessary conceptual links to the procedure. This group differs from those that are conceptually sound in that their conceptual knowledge is not grounded on procedures. Consequently, integration of the procedure with the conceptual knowledge appears to be the most fruitful approach in which the procedure provides the scaffolding for the conceptual learning while the conceptual knowledge underpins the procedure. Long (2005:62) further maintains that the second group (procedure-oriented learners) is slower at getting to the flexible understanding afforded by a grasp of conceptual knowledge, which it only reaches through the application of the procedures in several examples. As for the third group, he claims that the links from the conceptual to procedural knowledge and vice versa need to be made much more explicit.

### 2.3.3 Reasoning theory

The development of learners' (mathematical) reasoning is a goal of several curricula. South African curriculum is no exception (DBE 2011:9). Jeannotte and Kieran (2017:1) write that the development is also an essential element of mathematics education research. Hamami (2020:2) cited Macbeth's (2014) analyses of various mathematical practices in her general recognition of **reason** as a power of knowing, to augment his writing about the philosophy of mathematical practice. Several research studies acknowledge that reasoning can be discussed from different perspectives and different cultural backgrounds (Kollosche 2021:471). The multi-perspectival (bricolage) characteristic (nature) of reasoning augments its qualitative usage in this study. The study operationalises reasoning within the framework of bricolage to enhance the teaching and learning of algebraic expressions and equations in grade 9.

Reasoning is operationalised to forge necessary correlation between procedural and conceptual aspects of knowledge (Westaby 2005:97). Thus, the reasoning theory and these aspects of knowledge constitute the conceptual framework. According to Westaby (2005:97), the behavioural reasoning theory proposes that reasons serve as important linkages between beliefs, global motives, intentions and behaviour. This means that among many other attributes, the theory helps individuals to justify and defend their intentions and actions. Thus, the theory assumes, as in this study, that reasons can also help individuals make sense of their world by providing them with causal explanations and meaning of (what they do) their behaviour and causal relationships in their (operations) environment (Westaby 2005:100). Even realists share the same sentiment in this regard. They perceive that reasoning allows for demonstration (justification and defence) of what is true or at least reasonable to assume as true (Kollosche 2021:473; Rumsey & Langrall 2016:419).

Westaby's (2005:97) explanation of the reasoning theory further synchronises with the curriculum policy requirement in relation to learners' acquisition of the skill "to listen, communicate, think, *reason logically* and apply knowledge gained" (DBE 2011:9). The acquisition is associated with deep conceptual understanding (DBE 2011:8). In context, the understanding incorporates making sense (DBE 2011:8; Yackel 2001:1) of what one learns rather than following procedures that cannot be explained. O'Brien (n.d.:8) sees it the same in his demand that learners should be able to explain their thinking and justify (Rumsey & Langrall 2016:419) their understanding. The stance is also supported in the demand for conceptual meaning behind procedural assertions (Kollosche 2021:471; Pierce & Stacey 2007:12).

Reasoning (theory) therefore forms an integral part of inducing and deducing meaningful conclusions in the process of conceptualising algebraic procedures (procedural knowledge). It provides direction for systematically harmonised arguments aimed at attaining deep conceptual understanding (DBE 2011:8), high order cognition, and improved performance in algebra, advanced mathematics and related disciplines. Mahlomaholo (2014:173) adds that analysis backed up by reasoning and (logical) viable arguments (Rumsey & Langrall 2016:413) reduces errors and flaws in the subject of argument. The NCTM holds strong that the curriculum organised and developed around the phenomenon of reasoning brings about coherence of concepts. It turns the teaching and learning of algebraic expressions and equations into a joint chain of related knowledge (concepts) through which learners can navigate at ease. Miller and Koesling (2009:65-66) add that marrying mathematical reasoning with reading helps learners understand and form necessary coherence between the real-world context and mathematical concepts. Different groups of authors in the United States recommended inclusion of reasoning as a focal point of high school mathematics curriculum (NCTM 2000). The following sections discuss the contextual traits of reasoning in relation to this study:

#### *2.3.3.1 Reasoning for drawing logical conclusions*

Reasoning involves drawing logical conclusions based on mathematical evidence (Reid 2018:8) or stated assumptions to develop an understanding of a situation, context or concept (Martin & Speer 2009). This study operationalises reasoning to connect and provide logical meaning to algebraic procedural knowledge. The reasoning constructs (connection) ensures that the procedural knowledge makes sense (Yackel 2001:1) to learners. Dowden (2017:1) regards reasoning as the most effective requirement for learning mathematics and that it should be evident in instructions. He says it provides a cushion for learners to substantiate effective decisions. Thus, it helps them to detect and avoid fallacies and misconceptions (Dowden 2017:1). It then suffices to conclude that reason(ing) as an analytical power of knowing (Hamami 2020:2), empowers learners to take charge of their learning (mathreasoning09). The NCTM (2000) says logical reasoning is a source of usable knowledge needed for innovation and creativity. The technical-economic growth of most successful economies is built on this foundation because it has a long and far-reaching effect in improving performance in science, technology, engineering and mathematics (Katehi et al. 2009:1–2). That is, reasoning-based instruction has a high potential to sustain countries' economic competitiveness in the global markets.

#### *2.3.3.2 Reasoning for enduring knowledge*

The NCTM (2000) argues based on research that knowledge assumed through reasoning is conceptual and likely to remain with learners for a longer period than superficial procedural knowledge. The research affirms that reasoning guarantees a better performance in tertiary studies. Eccles 1997:ix also reiterates that reasoning bridges gaps between school and university mathematics and affords learners an opportunity to acquire skills required to write mathematics logically. Even this study relies on reasoning to develop a foundation upon which Further Education and Training (FET) and higher institutional mathematics can build (AMESA 2018:2; Haas 2003:31; O'Brien n.d.:9).

#### *2.3.3.3 Reasoning for analysis and conceptualisation*

Pearson (n.d.:1) says reasoning measures candidates' ability to analyse, evaluate and synthesize provided information to conceptualise the relationship(s) among parts. The expectation of this study is for learners to construct conceptual connections and interrelations between procedural steps. The construction involves analysis and other factors of reasoning. The factors differ in format and representation from one learning area to another. The instruction envisaged by this study uses basic concepts to refocus, integrate and re-organise procedures. In the process the definitions, concise notes and comparisons, conventions and conjectures are strategically interrelated (weaved) into a network of conceptual knowledge. NCTM (2000) classifies reasoning habits into four main categories namely, analysing a problem, implementing a strategy, seeking and using connections and reflecting on a solution. It should be noted however that reasoning habits fit in more than one category, and they are applied in no defined order (Martin & Speer 2009). Learners operate flexibly among the habits when reasoning out different algebraic and mathematical situations.

#### *2.3.3.4 Reasoning for mathematical instruction enhancement*

More recently, Jahnke and Krömer (2020) called for a focus on the "justification" of axioms and definitions in mathematics education and argued that the activity of testing the consequences of different hypotheses will be more authentic and insightful for the understanding of mathematical reasoning than learning proof alone. Kollosche (2021:473) and NCTM (2000) reported that reasoning and proving skills were central in the considerations the working group suggested to bring greater depth and coherence to the K-12 Mathematics Programme in Ontario Province. The group was created by the Ontario Ministry of Education Learner Achievement Division in response to the feedback from the field indicating a need for a closer look at and alignment of K-12 Mathematics. At stake was a considerable number of learners who were not doing well in mathematics. The South African curriculum policy is another example of policies that consider reasoning as a required skill for improved performance (DBE 2011:9). Long (2005:61) reiterates that though there are much gains embedded in procedural fluency when manipulating high school mathematics, the starting point before introducing the procedural steps should always be to seek reasoning behind the procedures. In other words, high school mathematics instruction should emulate bricoleurs, question the nature and behaviour behind concepts to forge necessary interrelations between the existing or prior knowledge, experiences and new content matter (Rogers 2012:10).

#### *2.3.3.5 Reasoning accessibility to all*

The behavioural reasoning theory supports Dowden's (2017:1) sentiments that reasoning is an accessible and needed skill for every child. Dowden says it is a fallacy to believe that there are people who are born naturally good at it. Moloi (2015:33) writes that though some learners may be intuitively good problem-solvers, most learners still need to be taught how to think and how to reason for them to (make sense of what they learn) solve problems successfully. Dowden (2017:1) supports teaching and nurturing reasoning from the early stages of schooling to discourage uncritical thinking. Uncritical thinking leads to reliance on procedural knowledge (Dowden 2017:1). The NCTM (2000) is particular about reasoning forming the core of high school mathematics.

### 2.3.4 Reasoning and taxonomies

Reasoning skill is a high order component of educational cognitive taxonomy (see 2.3.4.1). Taxonomies are developed to classify educational goals and objectives with greater precision, to build a curriculum commensurate with the objectives, to elaborate explicitly on expected behaviour (and skills) going through the curriculum, and to plan learning experiences and prepare appropriate evaluation tools (Paul 1985:36). In many taxonomies, reasoning is ranked as a critical thinking skill (see 2.3.4.1; 2.3.4.2; 2.3.4.5). The following sections shall discuss the orientation of reasoning skills in different taxonomies.

#### *2.3.4.1 Bloom's taxonomy*

Bloom's taxonomy provides teachers with an important framework to focus on critical thinking (Paul 1985:36). It affirms that learners who have attained analysis skills can develop reasonable solutions on their own. Analysis is a habit of reasoning and requisite for unpacking assessment items that require higher-order thinking skills (Paul 1985:36). The higher-order thinking skills are three-way and placed above the skills of remembering, understanding and applying in Bloom's taxonomy. The lower-order skills serve as fundamental parts or components of reasoning constructs. In this study, reasoning encompasses basic mathematics competency to connect and interrelate algebraic procedures and concepts. It is implied therefore that the reasoning skills (habits) namely analysing, evaluating and creating (synthesis) in Bloom's taxonomy build on lower-order skills. Conversely, learners who cannot remember, understand and apply basic mathematics competently cannot attain a skill of reasoning for lack of the (building blocks) primary need (NCTM 2000).

Bloom taxonomy describes analysing as a skill that involves very close and critical examination of the given information and breaking it down into manageable units (Paul 1985:36). Analysing, for example, could be seen as defining, comprehending and/or re-organising the existing information into equivalent formats or arrays to effect relatively easy computational steps. Evaluation involves the formation of decisions based on the merits of ideas, materials and/or phenomena (Paul 1985:36). Therefore, it may be viewed as a point where a provisional understanding of a concept that may need to be verified through several trials or backwards proof (inductive reasoning) is reached. Creating involves the consolidation of ideas to form a new and improved understanding (Paul 1985:36). It is the stage where a learner demonstrates the understanding by applying the concept in novel or non-routine situations (expansion) with confidence and can self-check the accuracy of the application (reflection). Their level of thinking and reasoning has become creative and innovative. It is worth noting that there is a complementary link between Bloom's higher-order thinking skills and the habits of reasoning (NCTM 2000) alluded to earlier (see 2.3.3.3).

#### *2.3.4.2 Subject Assessment Guidelines for Mathematics*

Subject Assessment Guidelines for Mathematics (SAGM) taxonomy, adopted for guiding the South African assessment policy (DBE 2011:156), classifies the complexity of assessment items into knowledge, routine procedure, complex procedures and solving problems (Berger, Bowie & Nyaumwe 2010:39). It has been adopted to guide assessment policies of the national curriculum statement of the Department of Education (Berger et al. 2010:31; DBE 2011:157). The reasoning skill surfaces in both complex procedures and solving problems categories where it is set to deal with problems involving complex calculations and/or higher-order skills, and those that require conceptual understanding, and to solve non-routine items (Berger et al. 2010:39) respectively. The placement and discussion of reasoning in this taxonomy largely resemble that in Bloom's taxonomy. It is a requirement for high order cognitive skills.

#### *2.3.4.3 Porter's taxonomy*

In this taxonomy, which is fivefold, the reasoning skill fits by implication within the third, fourth and fifth cognitive levels, namely, demonstrating understanding of mathematical ideas, solving non-routine problems and proving or generalisation (Berger et al. 2010:39). It again fits in levels demanding higher-order skills. It is important to note that deductive reasoning generalises based on evidence (Martin & Speer 2009). It (reasoning) provides that mathematics evidence (Reid 2018:8) through the formation of conceptual connections and interrelations. It provides explanation and defence (Kollosche 2021:471; Osborne 2021:6) behind learners' conceptualising constructs.

#### *2.3.4.4 Stein's taxonomy*

Stein's taxonomy comprises four cognitive levels namely, memorisation, procedures without connections, procedures with connections and doing mathematics (Berger et al. 2010:40). Reasoning skill becomes instrumental in the third and fourth levels where it connects procedural practices to underlying conceptual facts to form a concretised knowledge applicable in the fourth level. It unpacks the higher-order mathematics.

#### *2.3.4.5 National Assessment of Educational Progress*

National Assessment of Educational Progress classifies cognitive skills into low, moderate and high complexity levels. It applies reasoning alongside planning, analysis, judgment and creative thought in high complexity demanding items (Berger et al. 2010:40). It can be argued therefore that it is similar to Bloom's taxonomy (see 2.3.4.1) and resemble the reasoning habits (see 2.3.3.3).

### 2.3.5 Principles and values underlying the reasoning theory

For reasoning to yield positive results, there are principles and values derived from the behavioural reasoning theory that ought to be borne in mind. For example, teachers must not be judgemental, but rather create an enabling environment for learners to air their views without fear (mathreasoning09). They should concentrate on supporting learners to develop from uncritical to critical thinking and logical reasoning. This study envisages learner-oriented reasoning constructs in which the teacher's role is limited to that of a supervisor (Weegar & Pacis 2012:7) who merely ensures correct mathematical encoding and participation by all. Most of the powers to learn, assess and correct should be relinquished to learners (Weegar & Pacis 2012:7). On the contrary, the reasoning skill does not develop well, or at all, in the behaviourists' setting where teachers take centre stage and become the only dispensers of knowledge (Moloi 2015:37; Star 2005:410). The other principles and values that support reasoning include re-focusing from one learner to another, promoting team work (collaborations) within the class, motivating learners, awarding correct reasoning and creating space and time for learners' reasoning constructs.

## 2.4 SYNTHESIS

Figure 2.1 attempts to bring together (synthesise) different theories for the purpose of justifying their inevitability based on their intricate connection. It is basically intended to illustrate the summary of the study framework as discussed in the previous sections.

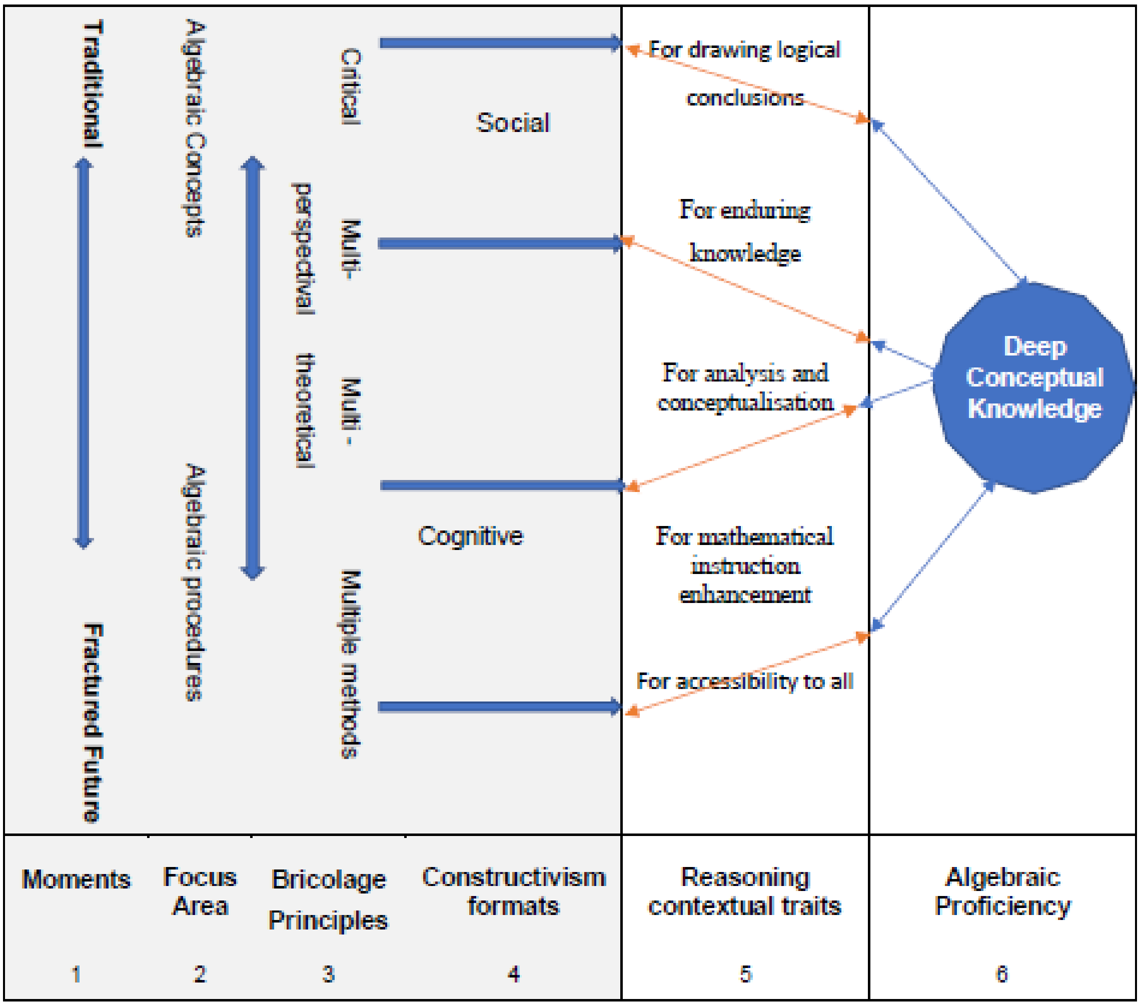



Figure 2.1 Study framework

The diagram depicts on the first level (column), bricolage moments as a continuum showing only the traditional period moment on one end and the fractured future on the other. The continuum serves to emphasise the importance of each moment in recognition of its views of the truth/reality regarding the conceptual and procedural algebraic knowledge, represented by the second level (column). These are annotated as the focus area of the study. The double arrow that follows on the focus area emphasises the integration of the two aspects of knowledge (the focus area) based on the principles of bricolage indicated in level (column) three. These principles also serve to consider mathematics learning theory proffered broadly through/by the formats of constructivism as indicated in the fourth level (column). The critical principle of bricolage, for instance, guides the constructive learning to also consider the power differential realities such as knowledge domination, cultural etc. in pursuance of the next level of reasoning in its diverse contextual conceptions. At the reasoning level, learning become more reflective, represented by the double arrows representing forward and backward movement of thoughts that simultaneously change the course/direction (dissonance) as it converges and transforms towards the deep conceptual knowledge inherent of algebraic proficiency.

The complexity of the bricoleur ontological underpinnings, consistent with bricolage principles, in respect of the focus area, namely algebraic expressions would be represented by arrows (messy) pointing in all directions and or lines entangling the reality (focus area). The learning theory (constructivist space) serves to 'filter' through the learning process to some extent, the messy and complex points of view of reality as expressed through critical participatory actions of those affected by the problem situation and who feel obliged to be engaged in solving the problem. To this extent, the sketch depicts the points of view flowing in the desired direction as parallel arrows pointing to the reasoning context. The constructivist learning 'filtered' points of view space is also messy though to a less extent. The reasoning context space (level 5) 'refracts' and/or 'diffracts' the filtered points of view by subjecting them to reasoning contextual traits. Essentially, reasoning becomes reflective and creates dissonance that leads to thoughtful critical engagement of knowledge being created. Thus, reasoning converges and/or centres the reflectively learned concepts and procedures to deep conceptual knowledge (algebraic proficiency).

## 2.5 CONCLUSION

This chapter described the roles of study frameworks as structural and rational pillars of the study that corroborate different parts of the study, and keep researchers and participants focused on the objectives of the study. It reviewed the literature of the study frameworks namely bricolage and constructivism theory. It narrated the philosophical significance of interweaving the study presupposition to the origin and historical evolution (qualitative moments) of bricolage. It discussed the extent to which the traits, formats, ontology and epistemology underpinning the work of bricoleurs inspire the researcher's presupposition to enhance (conceptualise) the teaching and learning of algebraic expressions and equations using reasoning. In the context of this study, conceptualisation refers to the process of connecting scattered bits and pieces of information. To be specific, the underutilised reasoning skill is used to connect procedures and concepts into conceptual knowledge.

The literature review on the constructivism theory explained how and why it works hand-in-glove with bricolage to support the epistemology underlying the study presupposition, study design and methodology. Constructivism is characterised by promoting the learning in which learners construct knowledge of new concepts using prior knowledge (Weegar & Pacis 2012:13).

The chapter provided contextual explanations of the key words as used in this study, and clarified relationship(s) between the components of the framework and operational concepts. It narrated the relationship between reasoning and, procedural and conceptual aspects of knowledge; and how their collective operation (functioning) and relationship constitute a conceptual framework underpinning this study. Reasoning is the (underutilised) skill that attaches sensible meaning (Yackel 2001:1) to content matter and provides direction and defence (Kollosche 2021:471; Osborne 2021:6) for systematically harmonised arguments aimed at arousing critical thinking and attaining high order cognition (Mahlomaholo 2014:173). Hence, the promise (presupposition) that it has a potential to improve the instruction and performance in algebra, mathematics and related disciplines.

Lastly, the chapter synthesised the study framework with the help of a diagram, and explained what the diagram portrays. The next chapter will review literature in relation to the objectives of the study.

# CHAPTER 3
# LITERATURE REVIEW FOR ENHANCING THE TEACHING AND LEARNING OF ALGEBRA IN GRADE 9

## 3.1 INTRODUCTION

This study seeks to enhance the teaching and learning of algebraic expressions and equations using reasoning in grade 9. This chapter is intended to review literature in relation to the objectives of the study. It discusses the extent to which literature provides the scholarly cushion for the study's presupposition (hypothesis), objectives of the study, study frameworks, methodology, and design. It is presupposed that it is only when the majority of learners could make sense of algebra and access its conceptual network (connection and interrelations) that they could meet the set performance standards. The reasoning-based instruction, framed in bricolage and constructivism theories, engages the principles of PAR to ensure the accessibility of the algebraic conceptual network.

## 3.2 LITERATURE AND OBJECTIVES OF THE STUDY

### 3.2.1 Justification of a need to enhance the teaching and learning of algebraic expressions and equations

Many challenges besetting the teaching and learning of algebra in grade 9 justify the need for instructional enhancement. The challenges include the non-alignment between the curriculum policy and classroom instruction, the sifting nature of algebra, the abstraction and complexity posed by algebra to learners and the lack of basic mathematics competency. The following sections will discuss the challenges.

#### *3.2.1.1 Alignment between the instruction and curriculum policy*

The alignment between the curriculum policy and classroom instruction ensures that the teaching and learning maintain legitimation (see 2.2.1.2(g)). Legitimation checks for adherence to set standards. It is however assumed that the teaching and learning of algebraic expressions and equations in grade 9 do not align with the curriculum and assessment requirements outlined in the curriculum policy (DBE 2011:4,8–9). The non-alignment manifests under the procedure-oriented instruction, assessment, teachers' competence and curriculum-time contestation.

*(a) Procedure-oriented instruction*

The teaching and learning of algebraic expressions and equations are still dominated by the use of procedure-oriented instruction as was used during the golden age moment (see 2.2.1.1(b)). The procedure-oriented instruction encompasses many if not all traditional teaching features (Osborne 2021:2). The emphasis is to determine the relationship between the cause and outcome to predict the future using the behavioural patterns (Mosia 2016:137), and not to engage in active and critical learning. Critical learning developing deep conceptual understanding digs deeper than mere establishment that a theorem (procedure) works, but shows why it works (Hamami 2020:12). It is more explanatory than just being 'understood'. It helps learners make sense of (the content matter) mathematics (DBE 2011:8).

The procedure-oriented instruction focuses on symbolic and algorithmic procedures (Matsolo 2006:v) by simplifying algebraic expressions and solving mathematical equations, without necessarily explaining the conceptual meaning behind the procedures (Pierce & Stacey 2007:12). It is limited in providing reasons and/or connections for the steps and procedures undertaken when simplifying algebraic expressions and solving mathematical equations. It can be argued that it promotes rote and uncritical learning of the given truths (DBE, 2011:8) against the prescripts of the curriculum policy. It, therefore, fails to achieve minimum standards of high knowledge and skills and to provide education comparable in quality, breadth and depth to those of other countries (DBE 2011:4). It also fails to develop skills that include "to listen, communicate, think, reason logically and apply the mathematical knowledge gained" (DBE 2011:9), thus failing to align with the requirements of the curriculum policy.

*(b) Assessment*

The classroom assessment does not align with the curriculum policy. The policy provides for different forms of assessment to run continuously with the instruction to collect information that can help in the development of learners and improve the process of learning and teaching (DBE 2011:154). To achieve this, the curriculum policy directs that the questions should be carefully spread to cater for different cognitive levels of learners (DBE 2011:154). It also assumes that teachers use the assessment feedback regularly to discuss the learning progress with learners to enhance their learning and experience (DBE 2011:154). A well-synchronised instruction uses the feedback to identify the sources of errors and misconceptions (Luneta & Makonye 2010:44; Pramesti & Retnawati 2019:7-8) for articulation and remedy. The Free State Department of Education issues the annual teaching plans and pacesetters aligned with the curriculum policy to guide and regulate the curriculum content coverage within the set timeframes. It has also provided sample lesson plans to guide teachers with the depth of content coverage. It leads the setting and moderation of the quarterly common papers (tests) and examinations to facilitate the standards legitimation (see 2.2.1.2(g)). However, the internal moderation and inspection reports (Chake & Msomi 2018) confirm the findings of Major & Mangope (2012:144) that most teachers limit learners to low cognitive level instruction and assessment. The instruction deprives the learners an opportunity to exercise their reasoning powers and imaginations (Major & Mangope 2012:144) in high order cognitive tasks against the spirit of the curriculum policy (DBE 2011:4,8). The grounding reason behind the status quo is affirmed by Turner's (2016:77) observation of how poorly learners are prepared to apply mathematics meaningfully in a world in which the ability to do so is increasingly imperative.

*(c) Teachers' competencies*

According to the minimum Requirements for Teacher Education Qualifications, a newly qualified teacher should at least demonstrate or display minimum competence tabulated in the document. The following competencies were considered relevant to this study:

i. newly qualified teachers must have sound subject knowledge;

ii. newly qualified teachers must know how to teach their subject(s) and how to select, determine the sequence and pace of content cognisant to both subject and learner needs;

iii. newly qualified teachers must know who their learners are and how they learn; they must understand their individual needs and tailor their teaching accordingly;

iv. newly qualified teachers must know how to communicate effectively in general, as well as in relation to their subject(s), to mediate learning;

v. newly qualified teachers must be knowledgeable about the school curriculum and be able to unpack its specialised content, as well as being able to use available resources appropriately, to plan and design suitable learning programmes;

vi. newly qualified teachers must be able to assess learners in reliable and varied ways, as well as being able to use the results of assessment to improve teaching and learning (DBE 2011:153); and

vii. newly qualified teachers must be able to reflect critically on their own practice, in theoretically informed ways and in conjunction with their professional community of colleagues to constantly improve and adapt to evolving circumstances (DHET 2015:62).

It has however been proven that most mathematics teachers fall short of some of these minimal competencies. Thornburg (2009:2) writes about the shortage of qualified mathematics teachers. The knowledge of unqualified teachers is expectedly limited, hence threatens the delivery of quality education that will see learners achieve success in algebra and mathematics at large (AMESA 2018:2). For example, the alleged shortage augers well with the reports that most teachers fail to complete curriculum, are unaware of mathematical misconceptions held by their learners (Luneta & Makonye 2010:44) and that teachers limit learners to low cognitive tasks (Major & Mangope 2012:144) against the requirements of the curriculum policy (DBE 2011:4,8–9). The failure to complete the curriculum or resorting to quick-fix approaches such as procedure-oriented instruction (see 3.2.1.1(a)) to complete it reflects on lack of competence to select, determine the sequence and pace of content cognisant to both subject and learner needs. The teacher who is not aware of learners' common misconceptions, for example, exhibits the signs that they do not know how their learners learn nor understand their individual needs, and cannot be able to tailor their teaching in consideration of the needs. Limiting learners to low cognitive items relate to a lack of competence to assess learners in reliable and varied ways (DHET 2015:62), and deprives them an opportunity to exercise their capabilities in reasoning and other high cognitive skills. The practice leaves learners behind their counterparts in other countries (TIMSS 2015; South Africa, Department of Education [DoE] 2010; ANA diagnostic reports 2014). Amidst the shortage, the curriculum policy is silent about teachers' professional development and support (Pramesti & Retnawati 2019:1) in the advent of curriculum changes.

In an attempt to address the shortcoming, in 2015 the Free State Department of Education commenced the teachers' professional learning communities (PLCs) (see 3.2.2.3), being a programme intended to develop teachers in relation to content and instructional gaps (Pramesti & Retnawati 2019:1). The initiative is in line with the requirement (want) for competent teachers who can deliver quality education (AMESA 2018:preamble). It is also responding to an international call to deepen mathematics instruction (Katehi et al. 2009:11) as evidenced in our curriculum policy (see 1.2). To that effect, Katehi et al. advise that more time should be spent on developing teachers to align the instructional materials, assessment tools and items with the curriculum requirements. However, the teachers' reports of the PLCs programme show that the programme attendance is not reciprocating the rationale behind the initiative in that the inexperienced and unqualified teachers are the ones who bunk the sessions most. The reports further indicate a lack of support from school management teams in this regard. The implication is that the majority of teachers in need of development and support are not able to reflect critically on their own practice, in conjunction with their colleagues to improve and adapt to evolving circumstances (DHET 2015:62). It is therefore not surprising that mathematics educators are not creating opportunities to dialogue with the public about issues relating to the teaching and learning of mathematics (Chernoff 2019:76). The situation further implies that the authority is also failing its responsibility to facilitate and maintain teachers' development and support programmes in the areas of content knowledge and instruction (Mosia 2016:abstract).

Again, the PLCs programme in its current format seems to be focusing on developing teachers in the areas of knowledge and imaginary (ideal) instruction, in isolation of real classroom practise, as opposed to Japanese lesson study (JLS) format (see 3.2.2.1(c)), for example. It is therefore not surprising that algebraic classroom instruction is not aligned with the curriculum policy (see 3.2.1.1(a)), and results in low performance in mathematics and related subjects (see 3.2.1.2). The performance has compelled the Department of Education to reduce the mathematics minimum performance level from 40% to 30% and below for promotion or progression purposes (eNCA 2016). It can therefore be concluded that lack of competence and dysfunctional teacher development programmes are some of the reasons why the components of the instructional context, namely the teacher, content and instructional strategy are not in harmony, but rather in a state of (disarray) 'instructional contextual contestation' (Kolobe & Hobden 2019:1) whereby the components are competing other than complementing each other.

*(d)* Curriculum-time contestation

The glaring contestation between (algebra) curriculum content and time allocation (Chake & Msomi 2018; DBE 2019:3) makes it difficult for learners to acquire and apply knowledge meaningfully (Matsolo 2006:62). The curriculum policy emphasises provision of quality education inherent of conceptual understanding, other than rote and uncritical learning (DBE 2011:4). The Department of Education has taken a stance on the support of school departmental inspection and internal moderation reports indicative of the purported imbalance between the algebraic curriculum content and time allocation. It has effected amendments downsizing the number of tasks per term on the curriculum assessment policy for the years 2020 to 2022 (DBE 2019:59–73), while the promise to revise the size of the curriculum content among others remains in the pipeline save for the implementation requiring more time (DBE 2019:3). The proposed amendment is basically a brain child of stakeholders. It is a by-product of their reports received by the department about the curriculum congestion in the main, and other concerns (DBE 2019:3). The contestation between the algebraic curriculum content and time allocation always results in superficial instruction deficient of quality (Little 2009:3–4; Star 2005:407) against the standards of the curriculum policy. The level of performance defeating the minimal requirements of the policy confirms the reports that the curriculum is not instructed according to the curriculum policy requirements.

It is presupposed that the contestation is one of the factors responsible for the status quo, more especially when it affects the core (pivotal or fundamental topic) of the curriculum (see 3.2.1.2). The eight-and-a-half and nine hours allocated for both algebraic expressions and equations in terms one and three respectively (DBE 2011:130–133, 142–143) have proven not to be realistic in the light of other time-related challenges that affect the teaching and learning of the topics. The challenges include learners' insufficient basic mathematics (pre-requisite) knowledge (see 3.2.1.4; 5.2.4), too many concepts and skills to learn, scattered components of the curriculum (see 5.3.1.4), and high-quality knowledge expectation as required by the curriculum policy (see 1.2). In the previously disadvantaged schools, the challenge of overcrowded classrooms also exacerbates the contestation. Also, the contestation disadvantages a larger percentage of learners who cannot cope with congested teaching plans and pacesetters. The NCTM (2000:3) writes that an attempt to cover many topics per time-unit impedes the learning of core concepts such as algebraic expressions and equations in depth. Little (2009:3–4) echoes the sentiment and advises against a practice of covering too many superficially taught topics in a given time-unit. This study vouches for the advice and supports a focused curriculum to ensure quality education (see 3.2.2.4). It reiterates Little's (2009:3–4) advice that the curriculum policymakers should adopt successful Asian countries' systems that focus teaching and learning on a few topics per time-unit. Asian countries rank high in the international mathematics performance ranking tables (TIMSS 2015:1).

#### 3.2.1.2 Algebra as a career sifter rather than an enabler

The pivotal roles and functions of algebra in the teaching and learning of mathematics in conjunction with its own curriculum weighting have made it more of a learners' sifter than an enabler for the scarce mathematics-based careers (AMESA 2018:2). Muchoko et. al 2019:1 writes that the failure in algebra has become a big barrier that prevents learners to follow their desirable career paths. Algebra virtually forms a pivotal link between the basic mathematics upon which it builds, and the learning of high order mathematics and related subjects that are founded on and/or apply its principles (Luneta & Makonye 2010:42, 44; McNeil et al. 2010:625). For example, it is a fundamental language for investing and communicating geometry, calculus and calculations in science and technology (Hamami 2020:4; Matsolo 2006:5). It has proven to be the most important instrument of modern life (Sawyer & Alder 2001:3). It is essential for the middle school curriculum to develop the learners' critical thinking (AMESA 2018:2; Haas 2003:31; and O'Brien n.d.:9).

According to CAPS, learners have to achieve the minimum performance level of 40%–49% in mathematics to be promoted (DBE 2011:154). The patterns, functions and algebra collectively, constitute 35% of the grade 9 mathematics curriculum (DBE 2011:11). Over and above constituting a bigger part of that 35%, the algebraic expressions and equations' pivotal role applies in the teaching and learning of patterns, functions and graphs (DBE 2011:11). The role extends further to the learning of high order mathematics such as geometry (30%) and measurement (10%). The algebraic expressions and equations have a significant weighting on their own and by application, to determine the promotion and progression of learners. However, a considerable number of learners fail to score the minimum 40% promotion mark. The majority of those who are condoned for progression fails to achieve the 30% benchmark for enrolling in mathematics in grade 10 (DBE 2014), and end up enrolling in mathematical literacy (eNCA 2016). To this end, the school subject report confirmed that for the years 2014–2018, more matriculates enrolled in mathematical literacy than in mathematics (DBE 2019:11). The learners who enrol in mathematical literacy cannot apply for mathematics-oriented courses in the higher learning institutions (Katehi et al. 2009:12; Tall & Razali 1993:2), hence are cut off the scarce careers in the sciences, health sciences, engineering, commerce or other key professions (University of the Free State [UFS] 2019:2) the country needs.

It is however important to set the record straight that along with the purported inefficiency of algebraic instruction, most schools have unwittingly heightened enrolments for mathematical literacy to improve their overall pass rates (AMESA 2018:2). They register matriculates who are considered weak in mathematics in mathematical literacy, increasing the number of learners who are directly or indirectly sifted because of the teaching and learning of algebra. This also confirms that the challenge of longing for high overall pass rates in the centralised assessment systems (Major & Mangope 2012:146) continues to haunt the teaching and learning of mathematics despite advices such as:

> "mathematics or mathematical literacy…You will have more options with mathematics than with mathematical literacy…" (UFS 2019:2).

The extract is intended to guide learners' choice of subjects in grade 10. This study envisages reversing the siftings by enhancing the teaching and learning of algebraic expressions and equations using reasoning in grade 9. The nature, positioning and application of algebra are hoped to cascade the presupposed conceptual understanding and performance improvement to other mathematics topics and related subjects.

#### *3.2.1.3 Abstraction and complexity of algebra*

Most learners are challenged by the abstract nature of solving equations and manipulating algebraic expressions (ANA Diagnostic Report 2014a:56–59; Banerjee & Subramaniam 2011:352; DBE 2014:9–10, 43; Hewitt 2012:140–141; O'Brien n.d.:9). Matsolo (2006:62) adds that many learners fail to make sense of algebra and see it as lacking in meaning and purpose. Hence, they fail to establish correct interpretation (Matsolo 2006:12; Pramesti & Retnawati 2019:1; Sengul 2011:2305; Ying et al. 2020:5405) and relate to the representations thereon. This study assumes the stance of Borenson (2011:24) that associates the abstraction to the instructional approach and holds it accountable for the adverse situation yet to see meaningful and lasting mitigation. A vast majority of experts agree that algebraic instruction has remained mostly procedural rather than conceptual. The situation makes it difficult for learners to make sense of the symbolic language and procedural computations (Fuchs & Fuchs 2005:45; Matsolo 2006:v; McNeil et al. 2010:625–626) embedded therein. This study presupposes that the use of learners' reasoning constructs can bestow the meaning and purpose of algebraic procedures, and develop them into conceptual knowledge. Learners make sense and cope well with conceptual knowledge (Star 2005:406). The abstraction manifests in technical language and notation, overlapping principles and usage of strenuous (non-constructivist) teaching and learning methods.

*(a) Technical language and notation*

Algebraic language and notation constitute linguistic limitations (Rojano et al. 2014:390) responsible for the perceived abstraction and complexity. The abstraction and learning difficulty experienced by learners in algebra relate primarily to the interpretation of algebraic texts (Matsolo 2006:12; Pramesti & Retnawati 2019:1; Sengul 2011:2305; Ying et al. 2020:5405), transition from arithmetic conventions and operations to algebraic notation, the meaning of literal symbols, and structural operations (Fuchs & Fuchs 2005:45; Matsolo 2006:v; McNeil et al. 2010:625–626). Leitzel's (1989) research revealed that a concept of the variable at an introductory phase was deep and more sophisticated than teachers could expect (Bergeson et al. 2000:26), and it often becomes a barrier to algebraic content understanding (Grosser & Lombard 2009). Research asserts that the most difficult part of solving any problem in symbolic form is the interpretation of the problem. For example, most learners matched the statement *two less than three times a number with the symbolic* representation $\mathbf{2-3x}$ instead of $\mathbf{3x-2}$, due to statement misinterpretation in context. The worst contextual scenario was observed when some learners preferred the arithmetic convention $\mathbf{2<3x}$ for the same algebraic statement. The misinterpretation that results from the language barrier and confusion leads to wrong mathematization (Pramesti & Retnawati 2019:3). The clarification notes and teaching guidelines of the curriculum policy (DBE 2011:130) alert teachers to look out for common misconceptions that include the following:

- $x + x = 2x$ **and NOT $\boldsymbol{x^2}$**. Note the convention is to write $2x$ rather than $\boldsymbol{x^2}$
- $\boldsymbol{x^2 + x^2 = 2x^2}$ **and NOT $\boldsymbol{2x^4}$**

- $a + b = a + b$ **and NOT** $ab$
- $(-2x^2)^3 = -8x^6$ **and NOT** $-6x^5$
- $-x(3x + 1) = -3\boldsymbol{x^2} - x$ **and NOT** $-3\boldsymbol{x^2} + 1$
- $\frac{6\boldsymbol{x^2}+1}{x^2} = 6 + \frac{1}{x^2}$ **and NOT** $6 + 1$
- If $x = 2$ then $-3\boldsymbol{x^2} = \mathbf{-3(2)^2 = -3 \times 4 = -12}$ **and NOT** $(-6)^2$
- If $x = -2$ then $-\boldsymbol{x^2 - x} = -(-\mathbf{2})^\mathbf{2} — \mathbf{2 = -4 + 2 = -2}$ **and NOT** $4 + 2 = 6$
- $\sqrt{25x^2 - 9x^2} = \sqrt{16x^2} = 4x$ **and NOT** $5x - 3x = 2x$
- $(x + 2)^2 = \boldsymbol{x^2 + 4x + 4}$ **and NOT** $\boldsymbol{x^2 + 4}$

Most of the misconceptions related to basic mathematics incompetence (see 3.2.1.4). Luneta and Makonye (2010:44) report that most teachers are unaware of mathematical misconceptions held by their learners, the observation that may auger well with the report on the shortage of qualified and experienced mathematics teachers (Thornburg 2009:2). Misconceptions that remain unidentified are likely not to be addressed, and have a potential of growing within the learning process, translating further into an incorrect (inaccurate) or misplaced application.

The differing order of operations and use of mnemonics add to the confusion (Banerjee & Subramaniam 2011:352; Hewitt 2012:140–141). The difficulty is worsened by the fact that despite the incompatibility in some instances, the development throughout different moments the ages shows that algebra had long been practised as advanced arithmetic, which utilises arithmetic questions to introduce algebraic reasoning (Matsolo 2006:13). That is, the operations between the two can be consistent in some cases and different in others. This study relies on learners' reasoning constructs to explain the difference, and to connect the procedural computations. The connection simplifies and attaches meaning and purpose to the procedures. That way, the perceived abstraction and complexity are replaced with lasting conceptual knowledge (Star 2005:406).

*(b) Overlapping principles*

Most principles and procedures of algebra are so close that they induce confusion (ambiguity) when the instruction and knowledge are not conceptualised (Star 2005:406). The confusion often leads to operational errors and misconceptions (Banerjee & Subramaniam 2011:353; Luneta & Makonye 2010:44; Pramesti & Retnawati 2019:7-8). Long (2005:59) writes that the algorithmic procedures guarantee correct solutions only if followed in a predetermined order and without error. As a result, the superficial instruction that is poor in connections and conceptual interrelations sets learners to commit errors and misconceptions related to overlapping principles and procedures embedded in the structures and operations of algebraic expressions and equations. It has proven quite strenuous to know (and recollect) the procedures in a predetermined order by heart (Sawyer & Alder 2001:1), more especially when learning the complicated concepts of algebra (Bergeson et al. 2000:27). This study presupposes that the use of reasoning constructs to connect procedural steps will conceptualise the content and minimise the confusion, errors and misconceptions.

*(c) Teaching and learning methods*

The teaching and learning methods used in classrooms are responsible for the unmitigated abstractness of algebra (Major & Mangope 2012:144). They encourage uncritical and rote learning, and memorisation (Muchoko et. al 2019:5) for regurgitation. Rote learning employs fully compiled algorithmic procedures (Star 2005: 407) driven by one person (McGregor & Murnane 2010:424), a teacher in this case, who unilaterally decides and pronounces the fixed or compiled (algorithmic) procedures the learners have to use. The previous section has shown that the algorithmic procedures and principles guarantee the desired results only when followed in the predetermined order, otherwise, learners who have not conceptualised the procedural steps are likely to be found wanting in the event of order disarray and/or forgetfulness. This study envisions the reasoning-based instruction in which learners lead the construction of connections between the procedural steps to achieve conceptual knowledge and understanding.

The teaching and learning of algebraic procedures without integrating the meaning (Pierce & Stacey 2007:12; Star 2005:406) magnifies the abstraction and lead to learners finding algebra very difficult (see 3.2.1.3). Sawyer & Alder (2001:1) express concern about the strain suffered by learners who learn mathematics by heart. They work hard for little or no gain. On the contrary, Ertmer and Newby (2013:43) write that conceptual knowledge provides necessary cushion for learners to know more while keeping very little in mind. This study envisions deep conceptual understanding (DBE 2011:8; Pramesti & Retnawati 2019:3) of algebraic expressions and equations using a reasoning-based instruction. The reasoning-based instruction supports a view that the rush for procedural skill without conceptual scaffolding (understanding) do more harm than good to learners (Ertmer & Newby 2013:43). Low self-efficacy and negative attitude towards algebra and mathematics are examples of the harmful effects. Sengul (2011:2305) confirms that most learners develop a negative attitude towards mathematics, and that bars their development in the subject.

The abstraction and learning difficulty experienced by learners in algebra relate primarily to the interpretation of algebraic texts (Matsolo 2006:12; Pramesti & Retnawati 2019:1; Sengul 2011:2305; Ying et al. 2020:5405), transition from arithmetic conventions and operations to algebraic notation, the meaning of literal symbols, and structural operations (Fuchs & Fuchs 2005:45; Matsolo 2006:v; McNeil et al 2010:625–626).

#### *3.2.1.4 Basic mathematics competence*

Insufficient knowledge of basic mathematics impedes the correlated teaching and learning of algebra (Banerjee & Subramaniam 2011:351) because algebra builds on arithmetic (Fuchs & Fuchs 2005:45; McNeil et al. 2010:625–626). It is deduced from Piaget’s theory of cognitive development (Simatwa 2010:366) that the learning of algebra requires systematic and gradual modification (accommodation) of basic mathematics knowledge. Therefore, learners who are promoted to grades in which algebra is introduced without knowledge of appropriate basic mathematics struggle

(Simatwa 2010:366) and find algebra very difficult to cope with (Matsolo 2006:5). The instruction (procedure-oriented) that fails to ensure proper adaptation (Rocha 2018:1) to basic knowledge and skills runs a risk of a failure from the onset.

The presupposition of this study that teachers inconsiderably assume that learners in grade 9 have acquired sufficient knowledge of basic mathematics and Piaget's concrete and formal operation skills (Woolfolk 2013:50–51), to bridge them to the effective learning of algebra, is backed in the assertion that the middle school teachers may assume that learners can (always) think logically in the abstract, yet this is often not the case (Bergeson et al. 2000:26). The operation skills coupled with the proper basic foundation manifest in the development of a logical system of thinking, and the ability to reason out hypothetical and abstract problems. The manifestation involves coordination of many factors or principles of algebra (see 3.2.1.3 (b)) at the same time (Ojose 2008:27; Simatwa 2010:368–370; Woolfolk 2013:50–51). On the contrary, it is implied that the algebraic instruction pitched above learners' cognitive development and on shallow basic knowledge foundation, is set for a failure from the onset.

This study draws from the NCTM (2000) that basic mathematics competency is an essential need for reasoning out the abstract concepts of algebra. The assertion reiterates the understanding that mathematics education is all about learning to reason with the help of mathematics (Kollosche 2021:473). In other words, the reasoning constructs (or abilities to reason out algebra) are founded on conceptual understanding of basic mathematics. Piaget's theory classifies reasoning as an adaptive development skill through which learners can accommodate and assimilate information (Simatwa 2010:367). Learners have to use basic mathematics to back up the reasoning and logical (system of thinking) argumentation (Mahlomaholo 2014:173; Rumsey & Langrall 2016:413). That is, basic mathematics is necessary to build (assimilate) new algebraic concepts and formulate coherent algebraic schemas. That way, learners' reasoning constructs demonstrate acquisition of concrete and formal operation skills and are likely to reduce operational (algebraic) errors and misconceptions (Mahlomaholo 2014:173; Pramesti & Retnawati 2019:7-8). This is why the instruction envisaged in this study is emphatic about checking and closing the arithmetic gaps before introducing algebra to heal the glaring operational (basic skills) underdevelopment experienced with the majority of learners (Chake & Msomi 2018).

The reason-based instruction nurtures or ready learners' cognitive development. Learners who have acquired basic mathematics competency and Piaget's formal cognitive skills (Ojose 2008:28; Simatwa 2010:369-370; Woolfolk 2013:50–51) to build upon, display the necessary development, enthusiasm and self-efficacy in the learning of algebra (Fuchs & Fuchs 2005:45; Matsolo 2006:v; McNeil et al. 2010:625–626).

Further, the emphasis for basic mathematics competency need to achieve the effective learning of algebra in grade 9 aligns with the curriculum policy content that makes it mandatory for learners to learn mental calculations before the introduction of algebra in both grade 8 (DBE 2011:75–84) and grade 9 (DBE 2011:119–126). The alignment consolidates a need to couple mental knowledge with cognitive skills development. There is a provision that warns teachers to guard against learners' dependence on calculators. The use of calculators is per the provision limited to big and unwieldy calculations, as well as in solution checking only (DBE 2011:76,119). There is however no suggested measure to guarantee the guard. In addition, the Free State Department of Education promotes and appreciates the importance of mental mathematics proficiency and exhibition of appropriate cognitive development skills through the provincial competition programme. The competition bars the use of calculators (Figure 3.1). However, the competitions are limited to a few outstanding performers. Yet most of the learners lack and need the competency and skills (Chake & Msomi 2018).

> You must use a pencil. Rough work paper, a ruler and an eraser are permitted. **Calculators and geometry instruments are not permitted.**

Figure 3.1 Extract adopted from the competition question paper

### 3.2.2 Possible components of solution to enhance the teaching and learning of algebraic expressions and equations in grade 9

The challenges discussed in the previous sections (see 3.2.1) warrant the formulation of strategies to bring about sustainable solutions in the teaching and learning of algebraic expressions and equations in grade 9. The possible solutions to the challenges include aligning the classroom instruction with the curriculum policy, and constructing conceptual knowledge, using learner-centred approaches and advocating for basic mathematics competency prior to the introduction of algebra. The following sections will discuss the components of solution.

#### *3.2.2.1 Alignment between the instruction and curriculum policy*

The components of solution as to the alignment between the curriculum policy and classroom instruction manifests under the procedure-oriented instruction, assessment, teachers' competence and curriculum-time contestation.

*(a) Procedure-oriented instruction*

This study is rooted in an instruction that supports the conceptualisation of algebraic procedures in line with the curriculum policy (DBE 2011:4). The process of conceptualisation is anchored by the frameworks underpinning this study namely bricolage, constructivism theory and PAR. The underpinnings support the process of nurturing learners to construct knowledge using their own learning styles and according to how they uniquely perceive realities surrounding the instructional material. The approach has the potential to mitigate the challenge of abstraction (Major & Mangope 2012:144) and complexity associated with the teaching and learning of algebra. It emphasises making knowledge meaningful and purposeful (Bergeson et al. 2000:29; Pierce & Stacey 2007:12; Pramesti & Retnawati 2019:3), and accessible (Ertmer & Newby 2013:43). It develops learners in critical thinking and application of high order cognitive skills. The skills are attained when learners are exposed to multi-cognitive tasks that develop their reasoning powers and imaginations (Major & Mangope 2012:144). The power to construct knowledge is relinquished to learners under controlled supervision (Weegar & Pacis 2012:7).

Conceptualisation also involves building a concrete foundation upon which algebra can develop. This study appreciates the importance of basic mathematics or arithmetic in that regard. It adds to many authors that emphasise the need to ensure that arithmetic gaps are closed before the introduction of algebra (Fuchs & Fuchs 2005:45; Matsolo 2006:5; McNeil et al. 2010:625-626). Competence in arithmetic lays a firm foundation and ensures little or no struggle in algebra.

The activity of conceptualising the procedures also reduces the challenge of language barrier (Matsolo 2006:12; Pramesti & Retnawati 2019:1; Sengul 2011:2305; Ying et al. 2020:5405) and misconceptions. It is the role of supervising teachers to clarify the ambiguous texts, technical encoding, language interpretations and misconceptions (Weegar & Pacis 2012:7). Further, the reasoning-based instruction emphasises the importance of providing necessary language support within the conceptualisation discourse (Rumsey & Langrall 2016:415). It is therefore conclusive that conceptualisation diagnoses learners' misconceptions for ease of intervention. Conceptualisation also generates constructs inherent to the learners' linguistic and socio-cultural background (Moloi 2015:42). Constructs learnt that way endure for a longer period (Long 2005:61).

Conceptualisation eases or reduces the strain of using strenuous learning approaches like memorisation (Muchoko et. al 2019:5) and rote learning (DBE 2011:4; Sawyer & Alder 2001:1). It provides the necessary cushion for learners to know more while keeping very little in mind (Ertmer & Newby 2013:43). That is, when they have conceptualised, learners may just need to refer and practice to refresh and reinforce the conceptual knowledge within their memories (Major & Mangope 2012:144). That way, the conceptual knowledge ascertains the strain-free learning investment, very useful at school level as well as in the industry (Katehi et al. 2009:1–2). In conclusion, it is projected that the process of conceptualisation within the reasoning-based instruction crosscuts several challenges besetting the instruction of algebra, and has the potential to turn algebra into a career enabler rather than a sifter (see 3.2.1.2).

*(b) Assessment*

Aligning the classroom assessment with the curriculum policy involves using different forms of assessment provided for in the policy, and collecting information that can help in the development of learners, and improvement of the teaching and learning process (DBE 2011:154). It involves spreading questions across different cognitive levels of SAGM taxonomy (see 2.3.4.2) and using the assessment feedback to discuss the learning progress with learners to enhance their learning and experience (DBE 2011:154), and to identify the sources of errors and misconceptions (Luneta & Makonye 2010:44; Pramesti & Retnawati 2019:7,8). It involves providing tasks that require learners to demonstrate conceptual understanding (DBE 2011:156) in preparation for standardised tests and examinations (see 3.2.1.1(b)). The tasks should require learners to explain their thinking and justify their understanding (O'Brien n.d.:8; Rumsey & Langrall 2016:419). The NCTM (2000) also reiterates that the assessment items should require learners to reason and that there should be more items of higher order in regular assessments in high school.

*(c) Teachers' competencies*

The possible solution suggested by this study in relation to the challenge of inadequate (insufficient) basic teachers' competence, is the provision of functional teachers' professional development and support (Pramesti & Retnawati 2019:1) programmes. In 2015, the Free State Department of Education introduced the professional learning communities (PLCs) for senior phase (Grades 8 and 9) mathematics teachers to address the shortcomings in the areas of content knowledge and instruction (see 3.2.1.1(c)). The conventional explanation of PLCs refers to them as a group of people (teachers) sharing and critically interrogating their practice (teaching) in an ongoing, reflective, collaborative, inclusive, learning-oriented and growth-promoting way, formed to enhance their (teachers') collective effectiveness as professionals (Hattie n.d.; Mahimuang 2018:229; Stoll, Bolam, McMahon et al. 2006:222–223). Though PLCs structures or formats may vary depending on each group's circumstances, most authors believe that they should be characterised by the following common features or descriptors:

- shared values, vision and norms;
- shared learning or deprivatisation of practice;
- collective responsibility (collaboration) focused on students' learning;
- supportive and shared leadership;
- reflective dialogue; and

- cared relationship or supportive conditions (DuFour 2004:8–10; Jones, Stall and Yarbrough 2013:357; Kruse, Louis and Bryk 1995:25; Mahimuang 2018:229–230; Stoll et al. 2006:225).

In the case of the department, the development in content knowledge and instruction also cut across sharing new curriculum development and materials. The instructional development may include competence that relate to classroom experiences such as writing correct, clear and logical mathematics, diagnosing and addressing learners' misconceptions (Luneta & Makonye 2010:44), the ability to select and determine the sequence and pace of content cognisant to both subject and learner needs (DHET 2015:62), and maintaining the assessment standards and using feedback to improve teaching and learning (DBE 2011:154).

However, the provision of a functional development and support programme for teachers (Pramesti & Retnawati 2019:1) in the context of this study extends way beyond the coverage of the PLCs in their current structure. The PLCs envisioned by this study likens or resembles the JLS. JLS, dubbed Teacher Research Groups in China, is a successful and internationally-recognised (Baba 2007:1; Doig and Groves 2011:78) teachers' development programme (practice) through which teachers research (develop and study) their own practice in school-based communities of inquiry. It aims at improving their teaching methods by working with other teachers to examine and critique one another's teaching techniques (Baba 2007:1; Doig and Groves 2011:77). JLS provides a model for large-scale, sustainable professional development that extends and renews teachers' practice, skills and beliefs in respect to curriculum changes (and pedagogical requirements) (see 1.2). It improves understanding (knowledge), competence and skills, attitudes and engagement of learners (Doig and Groves 2011:78). It draws from the assertion that professional development is reciprocal to learners' performance: "the more successfully (learners) learn, the more likely it is that the teacher will adopt practices that encourage further successful learning" (Doig and Groves 2011:78). Also, teachers like learners, learn best by teaching and building their own understanding rather than being told. It is therefore substantially arguable that JLS is underpinned by the principles of constructivist pragmatism (see

4.5.2.2) and PAR (see 4.2.2); because its improvements or strategies for enhancement of learner output (performance), like the ones envisioned by this study, result from changes teachers have made in their classroom practices (Doig and Groves 2011:78). That is, JLS is more practical with learners' involvement than our PLCs in their current format. The JLS cycle comprises (focused group) goal setting and planning (preparation) of a research lesson, teaching the research lesson under observation, post-lesson discussion (lesson review or reflection session) and consolidation of learning (Baba 2007:1; Doig and Groves 2011:79–80), just like the reflective cycles of PAR (see 4.2.2).

This study envisages an instruction by which the teaching and learning of algebraic procedures are simplified along with classroom practice and analysis of the details thereof. The detailed analysis that is carried out in reflection discussions with lesson observers, is an important feature (a secret) behind the success of JLS (Doig and Groves 2011:79). Further, JLS unlike PLCs cherishes and applies the skills of veteran (experienced) teachers with track records of helping learners to have different and better strategies or processes to learn (algebra and mathematics) the subject (Doig and Groves 2011:79). Moreover, the emphasis of the JLS is to develop a teacher within a classroom environment in which a major part of the lessons consist of learners sharing, polishing and refining their solution strategies (Doig and Groves 2011:79). It is driven by clear, well-defined and effective classroom learning and teaching, and provides teachers with opportunities to develop knowledge and skills that broaden their teaching approaches so that they can create better (direct and relevant) learning opportunities for learners (Doig and Groves 2011:79).

*(d) Curriculum-time contestation.*

In search of a solution against the glaring curriculum-time contestation, this study has adopted Little's (2009:3–4) view to focus the curriculum, rather than insisting on superficial teaching of many topics in one year (Star 2005: 407). The focused curriculum system as practised in successful (TIMSS 2015:1) Asian countries' systems (Little 2009:3–4), affords learners an opportunity to learn algebra and mathematics qualitatively and ensure a deep conceptual grasp of a few topics. It is assumed that thorough and thoughtful planning of lessons can mitigate the inherent risks of focusing more on mathematics examples (quantity) than principles (quality learning).

The superficial instruction based on drilling examples (quantity) runs short of conceptual connections and interrelations (see 2.3.2.2). The inherent importance of algebraic expressions and equations highlighted in the grade 9 curriculum and assessment plan, for example, warrants deserving (focused) attention and time guaranteed in the focused curriculum system. The focused curriculum affords teachers and learners sufficient space and time to complete the curriculum meaningfully and to practise and revise because it provides for them in its setting or development. To that effect, Little's (2009:3–4) sentiments of a focused curriculum are intricately augmented by Katehi et al.'s (2009:12) view that certain experiences can support the understanding of abstract content like algebra and skill development when instructed under enabling conditions: (i) affording learners sufficient classroom time; and (ii) creating opportunities for iterative and purposeful revisions of learnt content. The moderation and inspection reports (Chake & Msomi 2018) on the other hand, augmented the Department of Education's (DBE 2019:3) acknowledgement of the curriculum-time contestation.

#### *3.2.2.2 Turning algebra into a career enabler*

This study assumes that algebra can be a career enabler if, and only if, learners can access it meaningfully, hence perform well in it. It associates performance in algebra with the conceptual aspect of knowledge nesting procedural knowledge (see 2.3.2.1). It relies on learners' attainment of skills to communicate, think, reason logically and apply mathematical knowledge (DBE 2011:9) to construct rich conceptual connections and interrelations. It suffices then that learners demonstrate the accessibility to algebraic knowledge if they can explain or justify (Kollosche 2021:471; O'Brien n.d.:8; Rumsey & Langrall 2016:419) procedures using their own (reasoning) constructs. The NCTM (2000) writes that basic mathematics (competency) is a primary need for formulating logical reasoning constructs. The competency is therefore a foundation to turn algebra into a career enabler. Further, the NCTM (2000) confirms that involving learners in the production of knowledge (own reasoning constructs) guarantees high self-efficacy (Sengul 2011:2305) and relatively better-expected performance in mathematics and related subjects. It can be summarised that the components of the envisioned instruction encompass a process of accumulating a network of related, meaningful and functional knowledge (Pramesti & Retnawati 2019:3; Tlali 2017:85). That is, the knowledge learners can navigate through and apply at ease over a longer period (Miller & Koesling 2009:65-66), because they created (constructed) it (Moloi 2015:32). An effective application of knowledge in the learning of advanced mathematics and related disciplines (Hamami 2020:4; Pramesti & Retnawati 2019:1; Ying et al. 2020:5406) confirms freedom to manoeuvre and explore multiple dimensions (Lynn 2006:19), and results in improved performance.

#### *3.2.2.3 Abstraction and complexity of algebra*

A model of the solution adopted by this study towards algebraic abstraction and complexity draws from Moore-Harris's (2005:14) view that mathematics (algebra) is not limited to performing computations in isolation of understanding the instructional texts in a mathematical context. The instructional texts and presentations must be precise and accessible to learners. This study emphatically emulates the American standards of mathematical practice (SMP 3) in stating that learners should construct viable (procedure connecting) arguments and critique the reasoning of others (Rumsey & Langrall 2016:413). The connection (construction) adopts the principles of bricolage (see 2.2.1.2(a); 2.2.1.2(b)) to simplify the abstraction embedded within algebraic texts. It uses underutilised (subjugated) reasoning to weave (connect) scattered or unarranged information of basic mathematics and algorithmic procedures (symbolism) into conceptualised (algebraic) knowledge. The process of conceptualising procedures and provision of textual support reduces misinterpretations and language barriers (Matsolo 2006:12; Rumsey & Langrall 2016:415; Sengul 2011:2305). It utilises common language to connect algebra, a theory-heavy discipline, to real life (Vos 2018:2). Hence, the resultant algebraic constructs are developed from understanding the meaning and making sense (Yackel 2001:1) of underlying terminology. The supervisors (teachers) clarify ambiguous texts and ensure correct technical encoding (Weegar & Pacis 2012:7) before learners can engage in reasoning constructs. The conceptualisation of symbolism depicts concretisation and contextualisation. It clears the ambiguity (confusion) that goes with explaining overlapping procedures using technical (formal) language or the symbol representation system (Star 2005:406). The ambiguities are integral to procedure-oriented instruction (see 3.2.1.1(a)). The process of conceptualisation includes bringing learners' informal language and learning styles to the classroom to ease the textual and contextual challenges of symbolism (see 2.2.1.2(c)).

#### *3.2.2.4 Basic mathematics competence*

Basic mathematics competency is a foundational knowledge without which the teaching and learning of algebra cannot succeed (Fuchs & Fuchs 2005:45; Matsolo 2006:5; McNeil et al. 2010:625–626). It is also a primary need for using the reasoning-based instruction (NCTM 2000). The instruction borrows from Winter's (1995) view that mathematics education involves learning to reason with the help of mathematics, and promotes Hamami's (2020:2) analytical sentiment that reason(ing) is the power of knowing. Basic mathematics competency is further considered a tool by which learners' cognitive development (Simatwa 2010:367) to learn algebra is effectively nurtured. It is on the basis of the foregoing that this study emphasises the necessity (component) of checking learners' competency in applying basic knowledge and filling glaring gaps before introducing any algebraic topic. The component, which is ensures proper adaptation (Rocha 2018:1), guarantees successful conceptualisation. It draws from Dowden's (2017:1) view of nurturing reasoning from the early stages of schooling to discourage uncritical thinking. That way, learners would be adequately equipped and cognitively developed to reason and advance logical (viable) arguments (Mahlomaholo 2014:173; Rumsey & Langrall 2016:413). The argumentation is necessary for constructing conceptual connections and interrelations aimed at achieving high order cognition. This component of solution, which promotes the competency of basic mathematics alongside learners' cognitive development, should also embrace the warning that teachers should guard against calculator dependency (DBE 2011:76,119). This study embraces one interviewee's view that though procedures, formulas and calculators can always be available, there has to be more than that, in this case-reasoning (defence or explanation), to check and account for correctness of (procedural) steps (Osborne 2021:6). Perhaps, algebraic tasks and activities without big and wieldy calculations to discourage the use of calculators could be ideal.

### 3.2.3 Conditions for the successful implementation of the reasoning-based instruction

For the components of solution articulated in the previous section to be realised, some conditions must be considered. The conditions are factors that, if adhered to, optimises the implementation success.

#### *3.2.3.1 Alignment between the instruction and curriculum policy*

The alignment between the curriculum policy and classroom instruction manifests under the procedure-oriented instruction, assessment, teachers' competence and curriculum-time contestation.

*(a) Procedure-oriented instruction*

The condition(s) for conceptualising algebraic procedures ensure the instruction that emphasises the explanation of algebraic notation and procedures (Pierce & Stacey 2007:12) other than procedure-oriented instruction (see 3.2.1.1(a)). They focus the instruction into achieving deep conceptual understanding (DBE 2011:8; Pramesti & Retnawati 2019:3). The instruction should therefore encourage an active and critical approach to learning, rather than rote and uncritical learning (DBE 2011:4). Classrooms should become democratic (platforms) structures where learners freely air their views, and their reasoning constructs are treated with respect (Chilisa 2012:48; Bush & Silk 2010:abstract). The constructs are not subjected to judgement (mathreasoning09), but rather to professional probing that seeks to strengthen and/or clarify the construct(s) for better understanding. The instruction should concretise and contextualise learning using techniques and experiential knowledge that assist (benefit) learners (Kincheloe et al. 2011:4) in enriching conceptual connections and interrelations, and in making sense (Yackel 2001:1) of what they learn. It should therefore minimise the use of symbolism and/or technical language to explain algorithmic procedures (Matsolo 2006:v), but rather be open to learners' informal language. It should also include providing language support within the conceptualisation discourse (Rumsey & Langrall 2016:415). It should promote the habit of constructing reasoning using contextual texts before relating and adopting it to algebraic symbolic representation.

*(b) Assessment*

The condition(s) for the successful implementation of the component(s) of the solution related to assessment should ensure that the assessment aligns with the provisions of the assessment standards (DBE 2011:154–158). Teachers should use assessment and feedback to enhance learning and experience, develop learners to their full potential and improve the process of learning and teaching (see 3.2.2.1(b)). Feedback should have quality and draw from the notion of ‘assessment as learning’ in which learners become active, engaged and critical assessors (Rhind 2017:3,5).The research of Beaumont, O’Doherty and Shannon (2011:19) found that the learners considered quality feedback as that which is timely, and provides detailed explanatory comments discussed in an equitable dialogue (conversation) between teachers and learners (Evans 2013:82; Rhind 2017:3,5).

The questions should be spread across different cognitive levels (DBE 2011:157) to ensure even distribution in line with SAGM taxonomy (see 2.3.4.2). Well-distributed questions present learners with an opportunity to demonstrate their full potential (DBE 2011:155) in the subject and prepare them for the standardised tests and examinations (see 3.2.1.1(b)). Teachers should provide tasks that require learners to analyse, evaluate and synthesize (Pearson n.d.:1) as well as explaining their thoughts (NCTM 2000; O’Brien n.d.:8).

*(c) Teachers’ competencies*

It is evident from the previous sections that the conditions for successful implementation of the reasoning-based instruction are dependent on teachers’ competencies. Thus, a shortfall of any minimal competence required from a newly qualified teacher (DHET 2015:62) is not an option. Teachers are a primary driving force behind any positive classroom activity (Dickey 1997:5; and Foster 2008:6). Hence they are central pillars of the reasoning-based instruction success. They should therefore endeavour to develop and seek support (see 5.2.1.3) particularly in algebraic instruction (Lempp 2008:abstract; NCTM 2000:3) and other grey areas. Otherwise, the promise of the instruction to deliver quality education (AMESA 2018:2) aligned with the curriculum policy requirements could suffer at the expense of glaring sub-competence besetting many teachers in this regard.

The authorities should reciprocate the condition by facilitating and maintaining functional teachers' professional development programmes. They should further monitor the participation therein (see 3.2.2.1(c)), especially by inexperienced and unqualified teachers (Thornburg 2009:2). It, therefore, surfaces out that the school management teams' support is critical in overseeing the success of the development programme. The support may include creating a conducive environment and providing (securing) necessary resources (Ontario. Ministry of Education 2011) such as space, training materials, transport and meals for the advancement of the programme activities. As a backdrop for the PLCs model (format) couched in the instruction envisaged by this study (see 3.2.2.1(c)), a condition to spend more time developing teachers in a practical (classroom) setup should be prioritised.

*(d) Curriculum-time contestation*

In the context of this study whereby the volume of the curriculum and curriculum policy requirement in relation to the level of instruction are not subjects of scrutiny, the conditions for clearing the contestation between curriculum and instruction-time have to be based on one variable, namely time. The instruction should guarantee ample time for an average learner from the previously disadvantaged school (see 5.3.1.2) to acquire and apply knowledge in a meaningful way (Matsolo 2006:62; Pramesti & Retnawati 2019:3). Thorough and thoughtful planning should be aimed at focusing lessons on algebraic principles rather than drilling many examples on one concept as a time-bound condition. Further, the time allocation should consider other contextual factors such as overcrowding. Whilst the review of the curriculum (DBE 2019:3) is still in the pipeline, and it is glaringly inevitable for teachers to use the reasoning-based instruction and complete the curriculum within the allocated time, teachers could perhaps consider creating extra time as a substitute in the meantime. Otherwise, the inspection should rather question shallow instruction rather than reporting about incomplete work. Quality learning is the curriculum policy requirement that should be prioritised over rote and uncritical learning

(DBE 2011:4). NCTM (2000:3) writes that an attempt to cover all topics impedes the learning of core concepts in depth.

#### *3.2.3.2 Turning algebra into a career enabler*

The conditions for turning algebra into a career enabler should support the implementation that improves accessibility and application of algebra, hence performance thereto. The implementation should be partial to operationalise reasoning for (conceptualised) accessibility. It should nurture learners to adopt the principles of bricolage and take charge of constructing knowledge relative to their learning context (Bauer & Perciful 2009:1). That way, the instructional framework inherits learners' socio-cultural context and bridges the gap between theory and practice (Weegar & Pacis 2012:13). The practice (condition) simplifies algebraic complexity (Renwick 2014:6). Further, the condition to always seek for constructs' (explanation) assertions (Kollosche 2021:471; O'Brien n.d.:8) plays an integral role in demonstrating deep conceptual understanding (DBE 2011:8; Pramesti & Retnawati 2019:3) because it reveals how knowledge is created (Ertmer & Newby 2013:55). The foregoing conditions guarantee quality education and improved performance, hence pave way for a widened career choice.

#### *3.2.3.3 Abstraction and complexity of algebra*

The conditions for simplifying the ontological abstraction and complexity of algebra have to unpack the conceptual understanding (Pramesti & Retnawati 2019:3) compressed within its instructional texts (symbolism) and contexts (Rittleston-Johnson and Siegler 1998:109). This study has adopted a stringent solution (condition) not to explain procedures using technical language (see 3.2.2.3). It encourages the usage of a contextualised approach capitalising on learners' lived experiences against relying solely on literal symbols. Further, language support while developing the discourse of mathematical (reasoning) argumentation (Rumsey & Langrall 2016:415) should be emphasised. Emphasis on concretising symbolic representations is another foreseeable condition to diffuse the ambiguity (and confusion) embedded within the abstraction. Another condition develops from Rittleston-Johnson and Siegler's (1998:9) view to analyse the concepts underpinning the algorithms and procedures to enable mathematical (critical) thinking based on reasoning and viable arguments

(Mahlomaholo 2014:173; Rumsey & Langrall 2016:413). That is why this study advocates for systematically harmonised arguments aimed at attaining high order cognition (Long 2005:63) of algebra, founded on basic mathematics competency and monitored transition (see 3.2.2.4).

#### *3.2.3.4 Basic mathematics competence*

Lack of competence in basic mathematics impedes correlated teaching and learning of algebra (Banerjee & Subramaniam 2011:351) because the learning of algebra builds on it (Fuchs & Fuchs 2005:45; Matsolo 2006:5; McNeil et al 2010:625-626). Mosia (2016:190) reveals that teachers do not assess learners' prerequisite knowledge and skills before the introduction of new concepts. Yet the NCTM (2000) writes that basic mathematics is a primary need for using (formulating) reasoning (logical constructs). The activities that nurture competency in basic mathematics and Piaget's formal cognitive development (see 3.2.1.4) are therefore a need (condition) without which a learner could struggle to accommodate and assimilate the abstract concepts of algebra. Henceforth, it is strategically imperative (conditional) for teachers to assess and close basic mathematics gaps before introducing algebraic expressions and equations in grade 9. The condition of assessment relates to the requirement that teachers should at least have the competence in determining the sequence and pace of content cognisant to both subject and learner needs, and knowing who their learners are and how they learn (DHET 2015:62), for example.

### 3.2.4 Possible risks and threats against the implementation of the reasoning-based instruction

This section discusses the possible risks and threats that may adversely affect the implementation of the components of solution.

#### *3.2.4.1 Alignment between the instruction and curriculum policy*

The alignment of classroom instruction with curriculum policy manifests under procedure-oriented instruction, assessment, teachers' competence, and curriculum-time contestation.

*(a) Procedure-oriented instruction*

The risks of conceptualising algebraic procedures are all factors that may threaten an instruction supporting the explanation of algebraic procedures (see 3.2.1.1(a)) for conceptual understanding. Teachers who are not qualified and/or experienced to teach algebra in high schools (Lempp 2008:abstract) are a primary threat to the reasoning-based instruction. It takes a qualified teacher in the subject (mathematics) to offer quality education with success (AMESA 2018:2). The risk magnifies in the event where the teachers do not access development and support (see 3.2.2.1(c)). Teachers may still find the teacher-centred approaches in which they are sole transmitters of knowledge (Major & Mangope 2012:146) relatively time-efficient. For example, relinquishing powers to construct knowledge to learners could be seen as time-consuming (Dickey 1997:5; Miller & Koesling 2009:67) and cumbersome, hence be met with resistance. Dickey (1997:4) says "other teachers may continue to teach the kind of algebra they have always taught in ways they have always taught". The instruction that ignores lack of basic mathematics poses a major threat (see 3.2.3.4).

*(b) Assessment*

The risks and threats of assessment emanate primarily from compromised instruction (see 3.2.4.1(a)). The level of classroom assessment may not surpass the level of instruction thereto. The contestation between curriculum and time allocated for teaching and learning algebraic expressions and equations (see 3.2.1.1(d)) also influences (threatens) substandard assessment. It takes a competent (qualified) teacher (AMESA 2018:2) to assess learners in reliable and varied ways (DHET 2015:62) and to use the feedback to improve teaching and learning (DBE 2011:154). Further, Major and Mangope (2012:144) add that teachers who limit learners to low cognitive level items deprive learners an opportunity to exercise their reasoning powers and imaginations (critical thinking). The curriculum policy emphasises the need for active and critical learning. Some teachers may continue to ask many questions on the same concept rather than promoting cognitive progression by spreading questions evenly from lower to higher levels. The practice renders assessment more quantitative than qualitative. In addition, the unions' reservations (Annexure B) towards standardised (or common) papers administered by the

Free State Department of Education also pose a threat to the alignment and suppresses the multi-perspectival (Abercrombie et al. 2006:47; Kellner 1999:xii; Rogers 2012:1) enrichment. Hence they affect the assessment quality adversely. Consequently, the discursive practices and omissions risk the failure to attain minimal performance in standardised assessment (DBE 2011:154).

*(c) Teachers' competencies*

Unqualified teachers (Thornburg 2009:2) who do not receive professional development and support (Pramesti & Retnawati 2019:1) are threats to the implementation of the reasoning-based instruction. As is the authority that does not facilitate and support programmes intended to develop teachers' (competence) knowledge and instruction (Mosia 2016:abstract). Teachers who intentionally and deliberately bunk professional development initiatives and/or programmes risk to have limited knowledge (DHET 2015:62) and to continue to teach the kind of algebra they have always taught in ways they have always taught (Dickey 1997:4). They miss an opportunity to reflect critically on their own practice in conjunction with their professional colleagues and to constantly improve and adapt to evolving circumstances (DHET 2015:62). That way, they risk sticking to shallow non-constructivist (instructional) approaches (Lempp 2008:abstract) for lack of development in contemporary knowledge, competence and skills. Such approaches lead to learners becoming consumers of algebraic procedural computations rather than constructors of their own knowledge, as envisioned by the instruction underlying this study. The study borrows from the JLS formats underpinned by constructivist pragmatism to ensure that teachers prepare thorough lessons and thought-revealing tasks (Perry and Lewis 2009:376–377) informed by self-made strategies built from classroom practice with the help of veteran professionals.

*(d) Curriculum-time contestation*

The curriculum-time contestation as confirmed by the Department of Education (DBE 2019:3), brings several risks in the teaching and learning of algebraic expressions and equations. On one hand, reports show that in the current situation whereby teachers are mandated to complete curriculum content, the system runs at risk of either the non-conformity to the prescribed instruction or an instruction extending beyond the allocated time, or at the worst, both. The situation makes it difficult for teachers to drop the procedure-oriented instruction (see 3.2.1.1(a)) for participatory conceptualisation required by the curriculum policy (DBE 2011:8). This results in a state of ‘instructional contextual contestation’ (Kolobe & Hobden 2019:1). It threatens the substandard assessment evolving from the non-constructivists’ and shallow instruction (see 3.2.4.1(c)). The separate sets of algebra curricula in one class does not support correlated conceptual network in one go. The re-organisation and integration (DBE 2020:16) in this regard could save a lot of time, and strengthen instructional effectiveness (conceptual connections and interrelations). On the other hand, the interruption of teaching time adds to the threats even in an event of cleared curriculum-time contestation. The interruption can come in different forms, for instance, the disrespect for teaching time and/or the use of teaching time for other purposes.

#### *3.2.4.2 Algebra as a career sifter rather than an enabler*

Algebra and its application in advanced mathematics threaten the sifting if performed unsatisfactorily (see 3.2.1.2). AMESA (2018:2) considers mathematics teachers who are not qualified in the subject as a risk. It is suspected that they cannot deliver quality education that will see learners achieve success in algebra (Lempp 2008:abstract) and mathematics (see 3.2.1.1(c)). Quality education and performance are threatened when the instruction is leaning towards procedural orientation (see 3.2.1.1(a)) rather than being conceptual and critical. The threat widens more when the instruction does not create space and time to engage learners actively (DBE 2011:4). The framework of this study supports active and critical learning in which the powers and control of instruction should be learner-centred (Major & Mangope 2012:144). Further, since careers do not build on algebra only, it could still be risky if the instruction that integrates and enhances conceptual and procedural learning through reasoning is limited to the teaching and learning of algebra. That is, the instruction (reasoning-based) should also prevail in the application and beyond, to instil and meet the requirement for high order cognition across a wide learning spectrum. It is therefore implied that even the assessment should also be escalated to integrate and enhance conceptual and procedural learning beyond the teaching and learning of algebra, to keep up with the expected standard (see 3.2.1.1(b)) across the board. Consequently, the risk of concentrating the instructional powers in the teacher within procedure-oriented instruction dynamics, and subsequent threats thereto become destructive.

#### *3.2.4.3 Abstraction and complexity of algebra*

The abstraction and complexity embedded in the teaching and learning of algebra owe it first to the use of a foreign (second) language in most cases, and then the use of procedure-oriented instruction (see 3.2.1.1(a)). The reality (description) of algebra is by its nature complex and may be inconsistent with teachers and learners' prior knowledge that foregoes the content to be learned. Therefore, the use of technical words in a foreign language and symbolic representations (symbolism) without explanation makes it difficult for learners to make sense of the instruction. That way, the instruction does not help to simplify and contextualise the complexity (see 2.2.1.2(a)) and the approach runs parallel to the epistemological expectation of this study. The expectation is to infuse raw truths (conceptual knowledge) discovered through meaningful learners' participatory activities (Leung et al. 2014:6-10). Rather, the instruction promotes uncritical and rote learning against legitimation (Kincheloe 2004:687), while perpetuating the abstraction (threat) of ambiguity, confusion, errors and misconceptions (Luneta & Makonye 2010:44; Pramesti & Retnawati 2019:7,8). On the contrary, it is advisable to ensure that learners have prior knowledge and understanding of the technical language and symbolism before using them in algebraic operations. Lastly, insufficient or no language support while developing the discourse of mathematical (reasoning) argumentation (Rumsey & Langrall 2016:415) poses a threat too.

#### *3.2.4.4 Basic mathematics competence*

The fundamental need of basic mathematics competency for teaching and learning algebra as expressed in the previous sections cannot be overemphasised (see 3.2.1.4). It has however been found that the competency is not always readily available for the majority of learners in grade 9. It therefore becomes very risky for teachers to make assumptions about learners' readiness to accommodate and assimilate (Piaget 1952 cited in Simatwa 2010:367) algebraic content without using basic mathematics activities (Mosia 2016:190) to check their level of cognitive development and mental readiness to engage in a logical system of thinking. The teachers should also check learners' ability to reason out abstract (algebraic) problems that involve coordination of multiple principles at the same time (Ojose 2008:27; Simatwa 2010:368–370; Woolfolk 2013:50–51). In addition, McNeil et al. (2010:626) warns against (a risk of) introducing algebraic expressions and equations without checking and closing basic mathematics competency gaps. In the context of this study, lack of basics as foundation to the learning of grade 9 algebra is regarded as cognitive underdevelopment that is likely to impede (threaten) correlated (successful) learning of algebra (Banerjee & Subramaniam 2011:351; Fuchs & Fuchs 2005:45; Matsolo 2006:5; McNeil et al. 2010:625–626). Henceforth, it is very risky to assume that success to attain critical, high order reasoning skills (NCTM 2000) can be achieved without laying a proper foundation of basic mathematics competency and nurturing learners' cognitive development. The reasoning constructs envisaged from the instruction envisioned by this study are founded and logically (Mahlomaholo, 2014:173) sustained through the conceptual knowledge of basic mathematics.

### 3.2.5 Indicators of success

This section discusses the indicators of success.

#### *3.2.5.1 Alignment between the instruction and curriculum policy*

The alignment between the curriculum policy and classroom instruction manifests under procedure-oriented instruction, assessment, teachers' competence and curriculum-time contestation.

*(a) Procedure-oriented instruction*

The success in relation to conceptualising procedure-oriented instruction is determined by learners' ability to connect procedures to attain (conceptual knowledge) an improved product (Barker 2004:43; Kaomea 2016:10). Conceptualisation relies on using reasoning (constructs), being it concrete and/or contextual, to explain the meaning behind procedural steps (Pierce & Stacey 2007:12). Therefore, the ability is demonstrable when learners succeed to sustain reasoning and logical (viable) argumentation (Mahlomaholo 2014:173; Rumsey & Langrall 2016:413) in backing up their constructs. The success of conceptualisation is noticeable when learners can collect, correlate and apply algebraic knowledge to analyse, interpret (articulate) and solve problems in the related disciplines (Hamami 2020:4; Muchoko et. al 2019:1; Pramesti & Retnawati 2019:1; Ying et al. 2020:5406) of study. It is also evidenced by enduring content matter (Long 2005:61). It is also observed when learners participate freely and express their different learning styles in their constructs. It can therefore be deduced as a backdrop of the foregoing that the activities central to conceptualisation build learners' confidence and self-esteem.

*(b) Assessment*

Assessment is successful if it develops learners and improves (enhances) the process of learning and teaching (DBE 2011:154). The successful development is indicated (evidenced) by the quality of learners' responses to questions of different cognitive levels, especially the high order items (Arthur & Martin 2006). The latter requires conceptual understanding (Pramesti & Retnawati 2019:3). Success is when assessment feedback initiates an opportunity for learning. The emphasis is not on finding the correct answer but an opportunity to re-teach and re-learn the how and why issues surrounding the concepts and procedures. This happens when the classroom assessment prepares learners for standardised (common) papers using conceptual aspects of learning rather than procedural.

*(c) Teachers' competencies*

The success in relation to teachers' competence comes from (professional) changes brought by the knowledge and instructional skills teachers gain from research activities. It is an achievement emanating from the changes, for teachers to voluntarily collaborate in a focused group of experts aimed at developing solutions to their own problems. The willingness to research their own practices (see 3.2.2.1(c)) and subject the practices to critical scrutiny indicates an eagerness to learn from others. It indicates an appreciation of lifelong learning principles of social inclusion and personal development (Elfert 2020) in pursuit of knowledge and desire to improve one's own practice. The inclusion indicates the willingness to change ways that have always been used to teach and learn algebra and mathematics (Dickey 1997:4). Roping in or an invitation of veteran teacher(s) and experts with track records of helping learners to have different and better strategies or processes to learn the subject (algebra) (Doig and Groves 2011:79), guarantees successful development of competence in this regard.

The instructional practices indicative of success include logical connection of bits and pieces of algebraic concepts and procedures using reasoning (constructs) to create (generate) hybridised (richly conceptualised) knowledge (Booi & Khuzwayo 2019:2). The conceptual connection is driven by reasoning artefacts (see 2.3.3) and constructs to refocus; integrate; and/or re-organise (DBE 2020:16) the learning material in a way that it responds to learners' contextual learning needs (DHET 2015:62). The artefacts and constructs ensure effective use of time, which is one of the indicators of success depicting teachers' instructional competence. An effective application of the reasoning-based instruction in manoeuvring concepts and procedures for improved understanding in related disciplines (Hamami 2020:4; Muchoko et. al 2019:1; Pramesti & Retnawati 2019:1; Ying et al. 2020:5406) would be an added advantage to learners. It is also a display of success on teachers' competence development.

*(d) Curriculum-time contestation*

The indication of success in as far as curriculum content and time allocation are concerned resides primarily in the teacher's competence to complete the curriculum whilst conforming to instructional requirements of the curriculum policy amidst reports about glaring curriculum-time contestation. The accomplishment thereof requires the teacher to know how to teach their subject and how to select, determine the sequence and pace the content cognisant to both subject and learner needs (DHET 2015:62). The sequence and pace-setting involve curriculum refocusing, integrating and/or re-organising (DBE 2020:16) of learning material and/or curriculum aimed at creating extended time for teaching and learning algebra in a meaningful and purposeful way (Pramesti & Retnawati 2019:3). The success is even greater when the majority of learners in the previously disadvantaged schools claimed to be at more risk (Matsolo 2006:1) complete the curriculum without compromising instructional standards.

#### *3.2.5.2 Algebra as a career sifter rather than an enabler*

The success, in this case, is determined by improved performance in algebra, mathematics and related disciplines (Hamami 2020:4; Muchoko et. al 2019:1; Pramesti & Retnawati 2019:1; Ying et al. 2020:5406). The performance sets learners free to choose whether to enrol for mathematics or mathematical literacy in grade 10. In the long run, the performance is sustained by continual use of constructivists' approaches of learning resembled in a reasoning-based instruction rather than procedure-oriented instruction. NCTM (2000) research proved that knowledge assumed through reasoning is conceptual and likely to remain with learners for a longer period, hence develops a foundation upon which Further Education and Training and higher institutional mathematics can build (AMESA 2018:2; Eccles 1997:ix; Haas 2003:31; O'Brien n.d.:9). It can then be deduced that the success of the reasoning-based instruction incorporates attaining the bricolage skill(s) of connecting unconnected bits and pieces of information to create a useful and functional product using various modes of orientation (Given 2008:68–69; Kincheloe et al. 2011:1; Rogers 2012:1). The skills then serve as a scaffold or eye opener for broader career opportunities. Thus, the success qualifies the instruction(s) thereto into a socio-economic investment that equips learners with the knowledge and skills commensurate to technical and technological issues in the fast growing industrial world of work (Katehi et al. 2009:1–2).

#### *3.2.5.3 Abstraction and complexity of algebra*

The success against abstraction and complexity of algebra shows when learners find the teaching and learning of algebraic expressions and equations accessible, meaningful and purposeful (Pramesti & Retnawati 2019:3; Tlali 2017:85). This study offers reasoning within the frameworks of bricolage and constructivism theories to break through the complexity. The effects thereof are indicated when learners can interpret (Sengul 2011:2305; Matsolo 2006:12) the technical language, symbolic notation and representation. The success (ability) is demonstrated when learners construct their own reasoning connection(s) in-between concepts and procedures to create simplified (conceptual) knowledge. It manifests in reduced technical errors and misconceptions. It is also detectable when learners communicate algebraic constructs meaningfully. The communication proves the development in the discourse of algebraic and mathematical (reasoning) argumentation (Rumsey & Langrall 2016:415). It is further demonstrated in the application of knowledge gained. The application of algebra in advanced mathematics and related disciplines (Hamami 2020:4; Muchoko et. al 2019:1; Pramesti & Retnawati 2019:1; Ying et al. 2020:5406) proves that learners have established meaningfulness and functionality of what they learn (DBE 2011:8). For example, the application of algebra in projects and interactive problem-based learning, collaborative learning, experimentation and open-ended problems in which learners interact with the outside world for first-hand feel and application of concepts on their own (Weegar & Pacis 2012:8). The success in the application demonstrates creative, analytic and problem solving skills (Major & Mangope 2012:140–141) learners acquire along the process of simplifying abstraction (see 3.2.2.3). The skills prove critical thinking and attainment of high order cognition.

#### *3.2.5.4 Basic mathematics competence*

The indication of success in relation to integrating basic mathematics to the teaching and learning of algebra commences with the structuring of lesson plans. The success shows when lesson plans are (structured) tailored to nurture, promote and challenge learners' formal operational development (Woolfolk 2013:50–51) while closing applicable arithmetic gaps before introducing algebraic topics (see 3.2.1.4; 2.5.2.4). The success to balance the development and arithmetic competency indicates or guarantees preparedness to learn algebra. It can then be argued that the success aligns with both subject and learner needs (DHET 2015:62), and ensures a firm foundation (preparation) for learners to construct their own (reasoning) constructs (NCTM 2000). The reasonable and logical arguments (Mahlomaholo 2014:173; Rumsey & Langrall 2016:413) of algebraic constructs backed up by proficient application of basic mathematics competency indicate success. The constructs' argumentation and logical application prove that learners have attained meaningful and purposeful learning (Pramesti & Retnawati 2019:3) associated with deep conceptual understanding (DBE 2011:8). A well-structured foundation of basic mathematics shows when learners can communicate (interpret) transitions from arithmetic to algebraic notation (Banerjee & Subramaniam 2011:351). It also shows when they argue (reason out) logical connections of procedures (and other forms of algebraic abstraction) into conceptual knowledge (Pramesti & Retnawati 2019:3).

Success shows when learners apply basic mathematics competently to sustain conceptual connection with necessary accuracy. As a result it can be concluded that success in basic mathematics integration (nurturing) provides meaning behind computations (Pierce & Stacey 2007:12). The foregoing also reflects success in the coordination of multiple conceptual interrelations, which is further an indication of formal operational (growth) development (Woolfolk 2013:50-51).

## 3.3 CONCLUSION

This chapter critically reviewed literature surrounding the study objectives. The review has provided enough evidence to justify a need to enhance the teaching and learning of algebraic expressions and equations, depicting in the main, a need to conceptualise the instruction to align with the curriculum policy. The alignment manifests under procedure-oriented instruction, assessment, teachers' competence and curriculum-time contestation. The other challenges that came under literature review included "algebra as a sifter rather than an enabler", abstraction and complexity of algebra, and insufficient basic mathematics competence.

The chapter also reviewed literature in support of the components of solution. The literature further backed up the conditions necessary for successful implementation of the solution. It provided evidence for possible implementation risks and threats, and highlighted factors indicative of success.

The next chapter will unpack the details of the study methodology and research design.

# CHAPTER 4
# METHODOLOGY AND RESEARCH DESIGN FOR ENHANCING THE TEACHING AND LEARNING OF ALGEBRA IN GRADE 9

## 4.1 INTRODUCTION

This study seeks to enhance the teaching and learning of algebraic expressions and equations using reasoning in grade 9. This chapter will explain the study methodology and research design. Research methodology, on one hand, is a theory that explains how an enquiry will proceed (Schwandt 2007:195). The explanation provides clarity behind the choice of data generating method(s) or technique(s) in connection to the study ontological and epistemological assumptions and desired results (Openjuru et al. 2015:220). The research design, on the other hand, is a plan of action that maps out the order in which the processes of data generation namely, the conceptualisation of the problem, planning, implementation (action) and reflection will be carried out. Lastly, the chapter will discuss the theory of data analysis underpinning the study.

## 4.2 PARTICIPATORY ACTION RESEARCH

PAR is a methodological theory through which people (participants) act as researchers in pursuit of answers to the questions of their daily struggle and survival (Park 1993). It is central to collective, self-reflective inquiry in which researchers and participants collaborate in action to understand their practices to improve the situation in which they find themselves (Openjuru et al. 2015:219; Baum 2016:405). Researchers' reflexivity (engagements) is directly linked to action influenced by the socio-historical and cultural contexts of the researched (affected). The collaboration may be defined by a common geographic area or a shared problem and challenges (Israel et al. 1998) as in this study. This study focuses on an inefficient teaching and learning (problem) impact on learners in the previously disadvantaged schools (geographic area). It utilises PAR to reach out to them against the observation that research are often done for the researched but not with them, more especially the learners (Langhout & Thomas 2010:61). It assumes PAR's position that the lived experience of the researched brings values that should influence the study against the positivist view about the realities of the world (Langhout & Thomas 2010:61). It can be deduced that PAR works against approaches or practices that remove data from its context.

### 4.2.1 Historical origin of participatory action research

Action research has a long history, dating as far back as the early twentieth century (Openjuru et al. 2015:221; Baum 2016:406). It was developed by Jacob L. Moreno (1892–1974), the father of action research, in Germany in the 1920s and 1930s, and not by Kurt Lewin (1946) as it appears in other literature (Kemmis, McTaggart & Nixon 2014:18). Different ways to undertake action research have evolved across different fields of studies for political, epistemological and many other reasons (Baum 2016:406). PAR was developed as a member of the action research methodologies to support participatory learning in research, whereby the poor and deprived communities were politically encouraged to examine and analyse the structural reasons for their oppression, and the formulation of a plan for own transformation (Baum 2016:405). It came along with his idea of praxis "reflection and action on the world in order to transform it" aimed at a radical change in societal power structures. The design of this research draws from the idea- the research team usurps power and control to see the objectives of the study through. It facilitates the team's active participation in all phases (Israel et al. 1998) namely, problem conceptualisation, research planning, data collection and data analysis to ensure socio-political relevance (Willis et al. 2007:157) and ownership of the solution(s) thereto. It allows the researched community (teachers and learners) to customise the borrowed skills to fit within their structural context. That way, the borrowed skills focus on problematic issues identified by the community and it levels the playing field for meaningful participation by the researched. PAR has therefore been promoted as a means of taking action to reduce inequities (Baum 2016:406) between researchers and the researched because it creates spaces where those who have been historically deprived a voice in democracy can be empowered to critically engage in development (Langhout & Thomas 2010:61). It opens communicative space for engaging in multi-disciplinary and critical perspectives among participants (Baum 2016:406). Research shows that it has been applied in different fields of life such as agriculture, education, health and engineering (Minkler & Wallerstein 2003:4).

In this study, PAR is epistemologically (see 4.2.4) intended to guide the transformation of how knowledge of algebraic expressions and equations is created through active participation of learners. It could then be argued that PAR is not only transformative (Baum 2016:405), but decolonising as well. It enables the shifting of historical roles and relationships between adults and children – it supports the shifting from the dominant perspective that children (learners) are not able to participate in making important decisions that affect them to the possibility of meaningful partnerships (Langhout & Thomas 2010:61) that consider learners as social actors and collaborators. The study envisions learners as collaborative change (enhancement) agents in the settings and contexts of their lived experiences, and value their cognition in this regard. It also values the research team (PAR's) reflective cycle activities and engagements (see 4.2.2) geared towards facilitating the instructional transformation in line with the intended change in the creation of knowledge.

### 4.2.2 Cyclical mode in participatory action research

PAR involves cyclically related and iterative phases (Israel et al. 1998). The cyclical mode (reflective cycles) is the most important aspect upon which it relies (Baum 2016:406), and which differentiates it from other forms of research. Ball (2009:121) writes that it produces very sincere findings that retain the outline of the researcher and the experiences that emerge along with the activities of research. It is systematically implemented in collaboration with those who are directly affected by the problem of inquiry, for purposes of taking action to bring about change (Green, George & Daniel 2003:419). Participants assume power and authority to engage in the processes of planning the change, putting the plan into action and observing the process and consequences of the change, reflecting on the processes and consequences, and re-formulating the plan in the light of what had transpired (Kemmis et al. 2014:18; Baum 2016:405). That way, the results become more real than theoretical. The processes repeat successively forming a spiral of reflective cycles. The iterative reflective cycles enable participants to generate and analyse data to decide on the new (next) plan of action during reflection meetings. This reflexivity is central and deeply relational in that researchers and the other actors (community members or service or curriculum policy players) are engaging together in these processes (Sanit 2016:405). However, Kemmis et al. (2016:18) acknowledge that in practice this ‘spiral of action research’ is rarely as neat and orderly as contained in the cycles. The processes could overlap, changing the initial plans (mechanical sequence of steps) in the light of learning from experience (Kemmis et al. 2016:18). Instead, the processes are maintained in an order that tends to respond to the question and objectives of research amid the experience. The criterion of success is not necessarily dependent on following the steps in a fixed (scientific) order, but on sticking to activities geared towards achieving transformation and development through the understanding of practices, evolution of practices and the situations in which the practices have put the researched (Kemmis et al. 2016:18–19). It can therefore be argued that the cyclical mode clears the research activities from the traces of the traditional period (moment) scientific procedures (see 2.2.1.1(a)).

### 4.2.3 Objectives of participatory action research

The primary objective of PAR (as a research methodology) is to bring about change through a specific action (MacDonald 2012:34). This study operationalised reasoning (action) to change the teaching and learning strategies of algebraic expressions and equations. PAR provides for the bridging of evidence between knowledge and action (Khan, Bawani and Aziz 2013:157) in support of anticipated changes within the structural settings of the researched (Langhout & Thomas 2010:64). That is, it integrates knowledge and action for the mutual benefit of all (Israel et al. 1998) and encourages the affected to apply the research results and solution(s) to effect change. In the context of this study, it involved the researched (teachers and learners) into bricolage-induced participation. A strategically chosen group of teachers willingly collaborated into a research team that planned a change (transformation) of algebraic instruction in grade 9 and put the plan into action. The action involved classroom teaching by one teacher while other(s) were observing the consequences of the new (or enhanced) instruction. Later, the research team reflected on (the feedback) what had transpired to inform the second cycle of planning, acting and observing, and reflection (Kemmis et al. 2014:18; Baum 2016:405). Learners were guided to self-propel themselves into connecting the procedural steps using own contextual reasoning constructs. The active engagement (Langhout & Thomas 2010:62) of learners in the construction constituted and in turn informed planning, action and reflection. In the process of reflexivity cycles (see 4.2.2), the research team generated data. That way, the objective of PAR to mend the separation between knowledge and action synchronised with the objective(s) of the study based on participation for change (enhancement). The cycles engaged the researched (research team and learners) to understand their practices and benefit the affected communities (Langhout & Thomas 2010:61) because it drew from the development of participants' critical consciousness (Khan et al. 2013:157).

Another objective of PAR is to shape the choice and use of the method(s), in connection to desired outcomes (Openjuru et al. 2015:220). The method(s) should be able to harmonise the professional (teachers) and learners' perspectives (Openjuru et al. 2015:221) in search of a solution (enhanced instruction). The method should complement the principle of PAR in breaking down the practices of traditional period and modernist moments. It should discourage any form of alienation between researchers and the researched, or the subjects and objects of knowledge production (Gaventa 1988:19). It should create an environment in which people who are affected by the same problem work together and advocate freely for what they believe. It should support individual inputs to improve and develop resources, strategies and policies (Watters & Comeau 2010:8). This study uses the FAI technique (see 4.2.5.1) to educate and develop consciousness and mobilise for action.

The other objective of PAR is to promote co-learning and empower the marginalised communities to take action to redeem themselves (Baum 2016:405; Bennett 2019: 109; Boyle 2012:7; Crane & Richardson 2000:7). In this study, PAR supports emancipation of learners in the previously disadvantaged (marginalised) schools (Matsolo 2006:1) from the complexity of algebra. PAR helps researchers to pick and address the inherent inequalities between marginalised communities and researchers. It creates an environment in which the community's (or researched) knowledge is considered irrespective of how little valued it could be.

PAR is a useful and effective tool for reviewing and improving practice (Boyle 2012:8). It builds on strengths and resources already existing in the community (Israel 1998). It identifies, supports and reinforces existing social structures, processes and appropriate knowledge within the community. For instance, this study employs PAR to engage teachers and learners as resourceful members of the community who have the best vantage point to suggest what can work (Bennett 2019: 109; Watters &

Comeau 2010:6). In that way, PAR resonates with bricoleurs' principle of using scattered bits and pieces of resources to create an improved product. PAR is used in context to enhance the teaching and learning of algebra using the underutilised reasoning constructs. The enhancement creates improved (meaningful and functional) knowledge.

### 4.2.4 Ontology and epistemology of PAR

Research methodology is a system of ontological and epistemological assumptions, principles and procedures on which research is to be based (Noorderhaven 2004:91). Schwandt (2007:195) defines it as a theory of how an enquiry should proceed. He further writes that methodologies explicate and define the kind of problems worth investigating, what constitutes a researchable problem, testable hypothesis, how to frame a problem in a manner that it can be investigated using particular designs and procedures. And how to select and develop appropriate means of generating data.

#### *4.2.4.1 Ontology*

PAR has several antecedents (Openjuru et al. 2015:219). It questions the nature of knowledge and the extent to which the knowledge can represent the interests of the powerful and serve to reinforce their positions in society (Openjuru et al. 2015:219). It affirms that experience can be a basis of knowledge and that experiential learning can lead to legitimate forms of knowledge that influence practice (Kawulich 2012:9). For instance, this study utilises PAR to profile Mama's long-lived experience, knowledge and skills in algebra and mathematics as a resourceful base upon which the research team and learners can develop new ways (approaches) to enhance the teaching and learning of algebra. It empowers co-researchers, working as bricoleurs, to specifically enhance the procedure-oriented instruction (see 3.2.1.1(a); 5.2.1.1). It further supports learners' engagements on iterative planning, action and reflection to bring about positive change in the learning of algebra. It can then be concluded that PAR empowers the researched to transform and decolonise the instruction.

#### *4.2.4.2 Epistemology*

This study is couched by assumptions (presumptions) based on PAR epistemology (Langhout & Thomas 2010:63), the choice of which was done in consideration of its consistency with the study framework. It works in collaboration with bricolage paradigm and constructivism theory to guide the activities of data and knowledge generation. It is a collaborative, democratic, equitable, liberating, emancipatory and life-enhancing qualitative inquiry that reveals original feelings, views and patterns of the researched and recognises their inputs in the research process (Minkler & Wallerstein 2003:4). In the context of this study, it guided active participation of those who had the vantage point to critique and shape (Bennett 2019: 109; Watters & Comeau 2010:6) the ontology and epistemology emerging from operationalising reasoning as a strategy to enhance algebraic instruction. The observation of teacher-learner classroom interactions (actions) and informal engagements (interviews) gave rise to epistemological issues the co-researchers had to critically consider (during reflection meetings) to screen their influence on strategic ontology. As a result, a more refined approach embracing the idea of reasoning to construct algebraic connections (constructs) was developed. This proves that PAR promotes marginalised or underutilised scholarships (Openjuru et al. 2015:219) in which the oppressed (teachers and learners) lead their emancipation from bondage (of algebraic complexity). It can then be argued that epistemology of PAR has a decolonising effect that positions the teaching and learning power and control in the hands of the affected.

PAR breaks down the distinction between researchers and the researched, or the subjects and objects of knowledge creation (Gaventa 1988:19). It impresses upon the principle of people-for-themselves in the process of gaining and creating knowledge. The principle augers well with the “do it yourself” nature of bricolage (see 2.2.1). The impression to learn from the community lays important groundwork for epistemological innovation. The innovation reveals how learners generate and understand knowledge (Langhout & Thomas 2010:62). It incorporates the knowledge and expertise of the community members to improve research protocols (Baum 2016:405; Macaulay et al. 1999:774). PAR is therefore catalytic to generating knowledge that empowers learners to assume power and control of their learning (Openjuru et al. 2015:219). The knowledge generated that way is inherent of critical (thinking) theory (Openjuru et al. 2015:219) and endures for a longer period (Long 2005:61). It can be concluded that PAR breaks down discriminatory walls about knowledge production for the benefit of the researched community and strengthens the instruction output. Further, the ownership of knowledge production is attributed to the community (Kemmis et al. 2014:127–128).

### 4.2.5 Research method

The research methodology, PAR in this case, also provides clarity about the choice of the method (technique) in connection to the desired results (Openjuru et al. 2015:220). The research method is a technique used for generating and analysing data (Noorderhaven 2004:91). Kincheloe et al. 2011:3 write that in bricolage the research method is a concept that receives more respect than in more rationalistic articulations of the term because bricoleurs view the research method as much more than just a procedure. Bricoleurs view it as a technology of justification, a means by which researchers can defend and assert what they claim to know, and the process by which they have acquired such knowledge (see 4.2.4.2). Accordingly, the education and orientation of researchers (study conceptualisation) should demand that each one of them forms a critical consciousness (Khan et al. 2013:157) and whether the method to be undertaken will yield contextualised knowledge useful to the affected (Denzin & Lincoln 2005:3). In this study, the method used should assert that learners' reasoning constructs enhance algebraic instruction in their own context.

MacDonald (2012:34) writes that PAR describes and understands rather than predict and control. Hence, the methods that work together with PAR are natural and participatory modes of inquiry that target the free disclosure of lived experiences. Such methods should therefore be open to multiple realities based on different experiences and circumstances surrounding the researched individuals. They should desist from generalising. This study considered that the involvement of the research team in all phases (Israel et al. 1998) ensures that each member understands the objectives of the research and its underpinnings. The understanding, in turn, is used to develop learners to freely describe reasons behind procedural steps, in demonstration of conceptualised understanding.

When designing a research study, it is necessary that we recognise the type of evidence required to answer the research question in a reasonable way (Akhtar 2016:68). For instance, the activities (qualitative and empirical) for generating data (evidence) in this study have to help researchers respond to the research question:

*How to enhance the teaching and learning of algebraic expressions and equations using reasoning in grade 9?* The enhancement should be detectable (or measurable).

#### *4.2.5.1 Principles of the free attitude interview technique*

The FAI technique is a qualitative approach that allows researchers to interact closely with the researched in a humane manner to observe and interpret their world (Dexter 1970:136; Merriam-Webster 1998:8; Patton 1990:278). It directs the data generation process in a manner that the process does not alienate or undermine the researched (Netshandama & Mahlomaholo 2010:11). Tlali (2013:139) adds that the quality of the FAI technique in recognising the researched gives people peace of mind and hope that they will be treated with equity. It is noted in this regard that FAI technique does not only align with PAR principles, but also promotes critical, democratic and legislative frameworks of the republic.

The FAI technique prefers non-directive (Tlali 2013:139) but open-ended questions that leave a lot of communicative space and respect for the researched (an interviewee) to express their feelings about the inquiry in a manner not controlled or limited by close-ended, restricting, guiding or objective questions. The actions between the researcher and the researched flow freely, in an iterative manner. Not even at any single instance should the researcher feel superior over the researched, nor the latter feel inferior to the former because of the conduct of the researcher. Further, FAI corroborates with the PAR theory where it regards the researched own views of the character or nature of their problem(s) as fundamental (Tlali 2013:139).

The FAI technique fitted well in this study, which sought to elicit a freely advanced critical input (evidence) by the researched in their own natural setting. The enabling teacher-learner classroom interaction guided by the FAI techniques created an environment where learners spontaneously responded to interview questions. The questions were asked in an informal way along with the implementation activities, particularly during lesson presentations. The responses led to instant, but still informal probing at times. In the process, teacher(s) and observer(s) were able to pick responses addressing research questions for further discussions during reflection meetings. These discussions informed the informal follow-up questions to clarify the responses that could have otherwise remained ambiguous. In both instances, the interviewers maintained the respect the researcher has had to have towards the researched. The technique prompted learners to share the challenges surrounding the teaching and learning of algebraic expressions and equations, to suggest possible components of solution, to air views about the conditions under which the solution could work, to alert of some practices or adverse situations that could render the solution not working and to pre-empt the indicators of enhancement (success).

It can be concluded that the interviews carried out under the guidance of the FAI technique guaranteed the free disclosure of individual lived experiences. That way, the researched as per bricolage principles, ceased to be objects, but active co-researchers in the activities meant to transform their lives (Mahlomaholo 2014:9). This was achieved by sharing the information, resources and decision-making power. In this process, researchers learnt from the community and the community acquired skills of conducting research as well as learning some new developments about the enquiry. It can then be generalised that the use of FAI technique guaranteed the free positivism moments' treatment during research.

#### *4.2.5.2 Empirical data*

The multi-disciplinary nature of PAR aligns with the development of blurred genres moment (see 2.2.1.1(c)) and lends it to guiding the empirical data generation. The empirical data is however roped in as a complementary (secondary) evidence to qualitative proofs, deductions and findings about the proposed reasoning-based instruction. It (empirical evidence) is peculiarly used to assert the epistemological assumptions about the instruction. A single reflective cycle of empirical assessment involved the planning of a lesson, teaching and observing (action) and reflecting (see 4.2.2). The lessons culminated in assessment (exercise) worksheets. The accumulation of learners' worksheets (formative tasks) and summative tasks generated along successive cycles provided post-test data. When analysed against the pre-test data (baseline test results), the post-test data confirmed the precision of the assumptions. That is, the data indicated the enhancement capability of the reasoning-based approach.

The formative tasks checked the impact of the reasoning-based lessons on specific sub-topic(s) covered within a particular lesson, and the development of sub-topical integration along with the main topics' content coverage. The summative tasks tested the potential of the approach to enhance conceptual network proficiency over several algebraic concepts and riders. They also tested the approach's potential towards knowledge retention. They were conducted in the form of formal test (about mid-research) and final summative (towards the end of teacher-learner contact lessons). The conceptual understanding and knowledge retention are therefore considered valuable enhancement indicators in this regard. The patterns of performance (qualitative and empirical) guided the reflection discussions and subsequent lessons. The marks recording sheet in Table 5.1 presents raw marks and percentages for all tasks. The marks were converted to a percentage for statistical comparison (Figures 5.1 and 5.2). The observations, empirical results together with interview responses informed the analysis in Chapter 5.

## 4.3 ETHICAL CONSIDERATION

The research was carried out by the research team members (participants) coordinated by one of their own, the researcher. To ensure that the study was ethical and posing no harm to anyone, participants agreed to adhere to the guidelines of the ethics committee of the University of the Free State, the entity from which the proposal to conduct research was granted ethical clearance and approval (Annexure C). Participants also undertook to abide by the conditions of approval given by the Free State Department of Education (Annexure J) in response to the application the researcher had tendered. The application sought permission to conduct the study at Leru secondary school. Further, the participants observed the conditions of approval by the school management team (Annexure I). In the case of the learners, permission was sought from the parents to allow the teachers to keep learners in school for one hour after school (Annexure G).

In addition, participants received the study information sheet (Annexure F) that outlined the conditions of participation. It stated *inter alia* that participation in the study was voluntary and participants had the right to withdraw at any time without giving a reason. Participants agreed on issues of data confidentiality and anonymity before the start of research (Ball 2009:115). Also, at the end of the conceptualisation session, each participant submitted signed consent forms committing their free will participation in the study. Every attempt was made to conduct an unbiased and unscrupulous study:

data generated were routinely checked among co-researchers (Ball 2009:115) and appropriate measures were taken to complete the research within the stipulated time.

## 4.4 RESEARCH PROFILE

In this study, the research profile constitutes the processes that include identification and security of the research site, recruitment of participants, participants' profiles and relationship of the researcher with participants (co-researchers).

### 4.4.1 Identification and security of research site

The research was carried out at Leru Secondary School, located within Marung township. It is a public school registered under the Free State Department of Education. The qualifying rationale behind the choice included the following factors:

i. Dinaledi School status – the school was among the Dinaledi schools; earmarked, supported and resourced for deep teaching and learning of mathematics and science. Hence an expectation for it to perform relatively well in the learning areas.

ii. Mathematics-based progression stagnation – the number of learners repeating grade 9 and not doing well in mathematics was significantly high and worrisome.

iii. Cost-effectiveness – the school offered easy geographical access to the researcher and research team members, hence reduced research costs.

iv. Favourable catchment area – the school drew learners from townships' feeder schools, who were at a greater disadvantage with algebra and mathematics than their counterparts in the suburban schools (Matsolo 2006:1).

v. Enrolment – the school enrolled learners in grades 8–12, thus presented a suitable opportunity for the research to relate the pre-and-post grade 9 mathematics prospects to the teaching and learning of algebra in the grade of interest.

Therefore, the school represented an ideal situation in regard to problem statement, research question and objectives of the study (Baum 2016:405). The researcher applied for permission to register and conduct research in the Free State Department of Education upon receiving the clearance letter from the UFS Ethics Committee (Annexure D). When the prospects of application success were noticeable, the researcher started negotiations with Leru secondary school management. He hand-delivered a letter requesting permission to conduct research (Annexure E) and related attachments (Annexures F, G and H) to the principal. The principal and the researcher agreed to meet for discussions after the principal have acquainted them with the contents of the letter and attachments. Subsequently, the meeting materialised and the researcher explained the research. The principal deferred the response until the School Management Team have convened and discussed the request. Thereafter, permission was granted (Annexure I). During the data generation, the researcher received the letter of approval from the Research Department to conduct research at the Free State Department of Education (Annexure J).

### 4.4.2 Recruitment of participants (research team)

Tlali (2013:100) affirms the importance of recruiting the research team to mediate and facilitate participatory actions geared towards the execution of the research activities as detailed in the action plan. However, the recruitment should be subjected to screening criteria. The composition had to include individuals who could form a more critical, analytical and multi-perspectival team capable of arriving at an enhanced grasp and achievement of the study objectives (Ertmer & Newby 2013:43; Kellner 1999:xii). Then, the team had to comprise critical activists of mathematics education with credible qualifications, knowledge, experience, competence and skills in mathematics and/or related subjects instruction. They should show enthusiasm to work in a team (in collaboration(s)), in line with the study ethics and underpinnings and have time for active (action) participation in the research.

The researcher utilised the experience they gathered from formal and informal interactions with colleagues at workplace, in meetings, in workshops and elsewhere over a period of time to recruit individuals whose credentials befitted the qualities alluded to above. Eventually, the research team comprising the researcher (coordinator), retired mathematics teacher, mathematics teacher, mathematics departmental head (DH) and technology DH was established. The team members were working at Leru secondary school (research site) except for Mama, the veteran (most experienced and skilled) teacher who had recently retired from the same school (see 4.4.3.2).

### 4.4.3 Participants' (research team) profile

Different definitions and theories of learning and different beliefs about the way learning occurs have important implications for situations in which we want to facilitate changes (Ertmer & Newby 2013:43). The implications are determined by what people know and can do to bring about such changes. The paradigm and theories underpinning this study consider researchers and the participants as equals who should be treated with respect and without alienation because the experiences, knowledge, competence and skills each is bringing to the study is invaluable. The study is conceived from the belief that the knowledge and experience from different perspectives ensure a better grasp of the theme and phenomena underlying the study (Kellner 1999:xii). The following sections discuss the credentials of the research team members.

#### *4.4.3.1 Researcher (team coordinator)*

The researcher is a Bachelor of Science Education (B.Sc.Ed) graduate of the university, obtained in 1995. They studied mathematics and chemistry as major subjects along with education courses. In his final year at the university, they tutored the first year mathematics students. They spent approximately thirteen years teaching and lecturing mathematics and integrated science in different institutions in Lesotho. It may be relevant to mention that they also had an opportunity to serve the management and leadership positions as departmental head and deputy principal. During most of the years that they have been teaching in Lesotho, they were also deployed to mark external examinations in mathematics and science. They joined the Leru Secondary School in 2009 to teach mathematics and natural sciences. In 2011, they were deployed to mark matric physical sciences paper 2.

In 2015, they were appointed as the lead teacher in senior phase mathematics for the Marung region cluster. This is the cluster where Leru Secondary School, the research site, is located. The responsibilities of a lead teacher include (i) the organisation and leadership of the cluster professional learning communities (PLCs) for grades 8 and 9 mathematics teachers; (ii) liaising, sharing and explaining information on curriculum and tuition developments between the subject advisor and the cluster; and (iii) leading the setting of district common tasks. PLC sessions provide a platform in which mathematics teachers share good practices, experiences, knowledge, competence and skills, and challenges. The sessions have the traits of an induction programme, in-service training and refresher course workshops. They cater for both new and experienced teachers. Their experience in senior phase mathematics qualified them for provincial appointment to set grade 9 mathematics paper 2 for the November examination in 2019.

It has been through this uninterrupted, more than two-decade experience in mathematics education, that they considered themselves well-positioned and having a vantage point to spearhead (coordinate) the collaboration of experts (teachers) endeavoured to enhance the teaching and learning of algebraic expressions and equations in grade 9 through academic research. The research that draws from the experience and passionate belief they have for conceptualised mathematics instruction presented an opportunity to extend what they have always shared with colleagues at school, cluster, district and provincial platforms to a wide domain of teachers, learners, experts, researchers and policymakers. It has given them a closer, critical and analytic look into the curriculum policy. The idea of enhancing the teaching and learning of algebraic expressions and equations using reasoning emanated from the curriculum policy (DBE 2011:4,8-9). To this extent, it may be relevant to mention that the researcher studied Education Policy Studies for PGDE and obtained a Master’s degree at the same university. They are currently pursuing doctoral research in mathematics-based curriculum studies.

#### *4.4.3.2 Retired teacher*

The retired teacher taught mathematics for grades 8–12, and mathematical literacy in grades 10–12 at Leru secondary school for more than twenty years, before they retired in 2014. They retired as a departmental head for mathematics and mathematical literacy. They were part of a group of mathematics and science teachers that earned the school the Dinaledi status, awarded in recognition of the school performance in the subjects. After retirement, the school continued to employ their services in matric extra classes and camps. Other than that, they are conducting community-induced mathematics tutorials at their home. Their participation in this study confirms their unwavering support towards mathematics teaching and learning developmental initiatives and research. Their enthusiasm and participation in this research cost them the cancellation or postponement of the tutorials. The intake of their tutorials in both mathematics and mathematical literacy includes grades 5–12 learners from urban and township schools within the Marung region.

She matriculated in 1971 and started teaching as a temporary teacher in 1972. In 1973, they trained to become a qualified teacher at the Training College where they obtained a Junior Secondary Teachers Certificate in 1974. Their major subjects were mathematics and physical sciences. In 1975, they resumed their teaching career. They taught mathematics and other subjects in different grades (grades 7–12) and schools. In 1984, they joined Leru Secondary School to teach both mathematics and physical sciences. In 1986, they enrolled in a two year part-time Secondary Education Diploma Programme at university. In 1994, they enrolled for a Higher Education Diploma with the same university and graduated in 1995. They have attended a series of in-service training and workshops based on different education systems and policies. They experienced the Bantu education system as a student and a teacher. They taught outcomes based education (OBE) and different versions of national school curriculum (NSC) that include the current curriculum and assessment policy statement (CAPS).

It was against the backdrop of this remarkable experience that the coordinator recruited them for core membership status in the research team. Their experience, rich knowledge, competence and skills as well as their unique professional conduct underpin the success of this study. Their long serving time at Leru secondary school warranted their understanding of the challenges besetting the teaching and learning of algebra and mathematics, not only in the school but also in the entire Marung region. Over and above their experience and knowledge, their leadership skills and outstanding professional conduct became useful to the study. They flexibly adapted to the reasoning-based instruction and reached out to learners at ease.

#### *4.4.3.3 Mathematics departmental head*

The mathematics departmental head was heading the mathematics and mathematical literacy department at Leru secondary school since 2017. They graduated with a Bachelor of Education (B.Ed) in 2004. Their major subjects are mathematics and chemistry. They furthered their education through part-time study at the same university to obtain M.Ed in 2016. At the time of this study, they were aspiring to propose a PhD study in the same university. Their outstanding competence, managerial and leadership skills started to show during their scholarship at university. During their second year, they were appointed as the first year students' chemistry tutor. During their third year, they became an academic advisor and assistant chemistry lecturer for first-year students.

Upon completion of their first degree, they registered for a teaching career in 2005 and started teaching at a high school in the same district as Leru Secondary School. During their tenure as a matric mathematics teacher in the school, they achieved provincial record results over several years. They achieved a 100% pass record in mathematical literacy in 2008 and 2010. In 2012 and 2014 they achieved the gold award (between 90 and 100 per cent) pass record in mathematics. In 2015, they were elected to represent teachers in the school governing body. The foregoing extraordinary milestones viewed along with their maturity at a very young age compelled the school management and governing body to nominate their name for the 2015 national teacher award competition. They competed in the category of 'Excellence in teaching mathematics'. They won at the district and provincial levels paving their way to represent the province at the national level where they obtained the fourth position. In the same year, 2015, the provincial Department of Education appointed them as a mathematics lead teacher for grades 8 and 9 teachers in the cluster where their school is located. It was during the lead teachers' meetings and other interactions that they developed a close interpersonal and professional relationship with the researcher. The relationship continued to manifest even during this research.

Her dedicated participation in the study shaped the programme, curriculum coverage and data generation approach. They advised of the appropriate slots the research team could utilise for data generation and reflection sessions. They managed and controlled the research content material and assessment to ensure compliance with requirements of the curriculum policy. They participated actively throughout the study, sharing their subject experience, knowledge and competence, managerial skills, research opportunities and documents sought for analysis (see 4.5.3). Their constructive views complemented the enhancement envisaged in the reasoning-based instruction. They further continued to monitor and evaluate the improvement and enhancement brought by the instruction. They followed up on the summative assessment performance of the sampled group of learners and shared positive pointers of the instruction in their departmental meetings.

#### *4.4.3.4 Mathematics teacher*

He was a newly qualified mathematics teacher, strategically recruited on the merit of teaching mathematics in two grade 9 classes while the researcher was teaching three during the research year. As such, they were an integrated quality and management system (IQMS) peer for the researcher whom the latter had to work with in curriculum and administration related matters. During the research year, they had two years teaching experience in mathematics and physical sciences. They had the unique advantage of recent university training hence the knowledge of recent teaching and learning techniques from which the research tapped.

He is a university graduate with B.Ed in mathematics and physical sciences. The other advantage of this teacher is the fact that they had been a passionate mathematics student in the classes of the retired teacher and researcher in 2010 (grade 8) and 2011 (grade 9) respectively. This confirms the fact that the research was carried out under sound interpersonal and professional relations amongst researchers. His knowledge of new instructional innovations as to the teaching and learning of mathematics and how best they could be enhanced within the envisioned reasoning-based instruction became very instrumental in ascertaining high-quality work.

#### *4.4.3.5 Technology departmental head*

He was the departmental head of technology at Leru secondary school since 2009. They have also acted as the deputy principal in 2014, 2015 and 2018. They taught technology in grades 8 and 9. They matriculated at Leru secondary school in 1994. They then proceeded to train as a mathematics and science teacher at university, where they graduated with a B.A.Ed. in 1998. They started teaching mathematics and geography at one secondary school in the same district as Leru Secondary School in 1999 to 2001. In 2002, they joined Leru secondary school to teach mathematics and physical sciences.

In 2004, they enrolled for a part-time Accredited Certificate of Education programme in technology with one university in Kwazulu Natal and graduated in 2006. In 2006 and 2007, the university employed them on a part-time basis during school holidays to assist technology student-teachers under the auspices of 'Project for all'. They were further assigned to train technology teachers in Free State during school holidays under the same project. The training lasted for three years, 2008-2010.

He is a very active and resourceful expert in technology education in the province whom the subject advisors for technology in the department refer to from time-to-time for assistance. His participation drawing from knowledge and leadership skills became very informative to the study. They clearly articulated the areas in science and technology that are related and dependent on conceptual understanding of algebra. They further articulated the adverse effects the inefficient teaching and learning of algebra and mathematics in grade 9 could have in the learning of technology. They embraced the reasoning-based instruction as both helpful and necessary for an improved (enhanced) learning of technology. They had a unique advantage of an intertwined knowledge of mathematics, science and technology, hence being able to relate and explain the connections thereto with the envisioned reasoning-based instruction at ease.

### 4.4.4 Relationship between researcher and co-researchers

PAR pays careful attention to relationships between researchers and the researched (Openjuru et al. 2015:220). Proper relationships safeguard against power conflicts. Conflicts are capable of relapsing the project. PAR advocates for systematic power-sharing between those involved (Openjuru et al. 2015:221). In this study, the research team decided to chair the reflection meetings on a rotational basis. Learners were also encouraged to rotate the team leadership to reduce or avoid unnecessary power contestation. The research team agreed upon limiting the researcher's role to guiding and ensuring adherence to the research question and objectives of the study.

## 4.5 RESEARCH DESIGN

The design of this study is strategically guided by the principles of PAR (see 4.2). PAR aligns with the third moment (blurred genres) evolution (development) that saw the rebirth of bricolage. The development forms a basis for the networked relationships between reality and human perception. Hence, the recognition of experience and active participation of the researched (see 2.2.1.1(c)). The sections that follow will therefore provide the detailed narrations as to the participation of the research team and learners (the researched) in different stages of the research. The stages include the research launch meeting, conceptualisation of the study and data generation.

### 4.5.1 Research launch meeting

The formal invitation to the research launch meeting (Annexure K) was a culmination of informal conversations the researcher have had with suitable (qualifying) individuals (see 4.4.2). It was extended to mathematics, science and technology departmental heads and teachers, a mathematics subject advisor and retired mathematics teachers. Present in the meeting were Mfetho, Shana, Nono and Mama (Annexure L). The aim of the meeting was to introduce the study and adopt the research team. It was therefore kept open and sociable as much as possible. The agenda included introductions, the purpose of the meeting, the adoption of the research team and charting a way forward.

#### *4.5.1.1 Introductions and purpose of the meeting*

The researcher introduced themselves, welcomed and thanked all present. They then opened the floor for other participants to introduce themselves. The introduction involved mentioning full names, workplace, work history and designation, personal attributes, and anticipated expectations and contributions. The exercise helped participants to open up and realise personal commonalities and discords amongst themselves, and to relate those to the work at hand. The researcher then briefly explained the purpose of the meeting and the expected role(s) of the participants. The explanation was backed up by some references to the participant information leaflet (Annexure F), which was attached to participants' invitation letters for preparation.

#### *4.5.1.2 Adoption of the research team and way forward*

All participants present in the launch meeting were readily available and eager to constitute the research team. They had already signed consent forms attached to the participant information leaflet confirming their willingness to partake (be adopted) as research team members. The team then charted the way forward and concluded that it should hold an exclusive workshop intended for a deeper conceptualisation of the study before setting out (drawing) a plan of action.

The meeting also agreed on ground rules, principles and values that would govern the team's conduct in issues that included attendance and timekeeping, chairing and duration, recording of minutes, reporting, respect of individual opinion and viable communication channels and timeframes. As a result, the following principles and values were agreed upon to define and govern the team's conduct and approach during the entire period of research:

- Punctuality – participants agreed to honour set timelines in activities and assignments.
- Teamwork – participants agreed to work as a team in all activities, and that the spirit of teamwork should further be encouraged and cultivated among learners.
- Self-discipline, honesty, patience and determination – participants agreed to carry out an unbiased and scrupulous study (Ball, 2009:115) to ascertain authentic data.
- Mutual respect – participants agreed to maintain mutual respect among themselves and extend the same to learners.

### 4.5.2 Conceptualisation of the study

Conceptualisation of the study involves a deep understanding of what the study entails. The process ensures a high rate of research success. Tlali (2013:100) asserts the importance thereof that participants (research team) need to conceptualise the study to mediate and facilitate the participatory activities in line with the frameworks and objectives of the study. The workshop to conceptualise the study, as agreed amongst the research team presented an opportunity for the research team to discuss the frameworks and methodologies underpinning the study. It also presented an opportunity for the research team to add, modify and consolidate the concept (vision) underlying the study. All attendants of the launch meeting were present in the workshop, with an additional (fifth) member of the research team, Buti.

#### 4.5.2.1 *Study overview*

The researcher presented the study overview with the help of the projected concept map illustrated below:

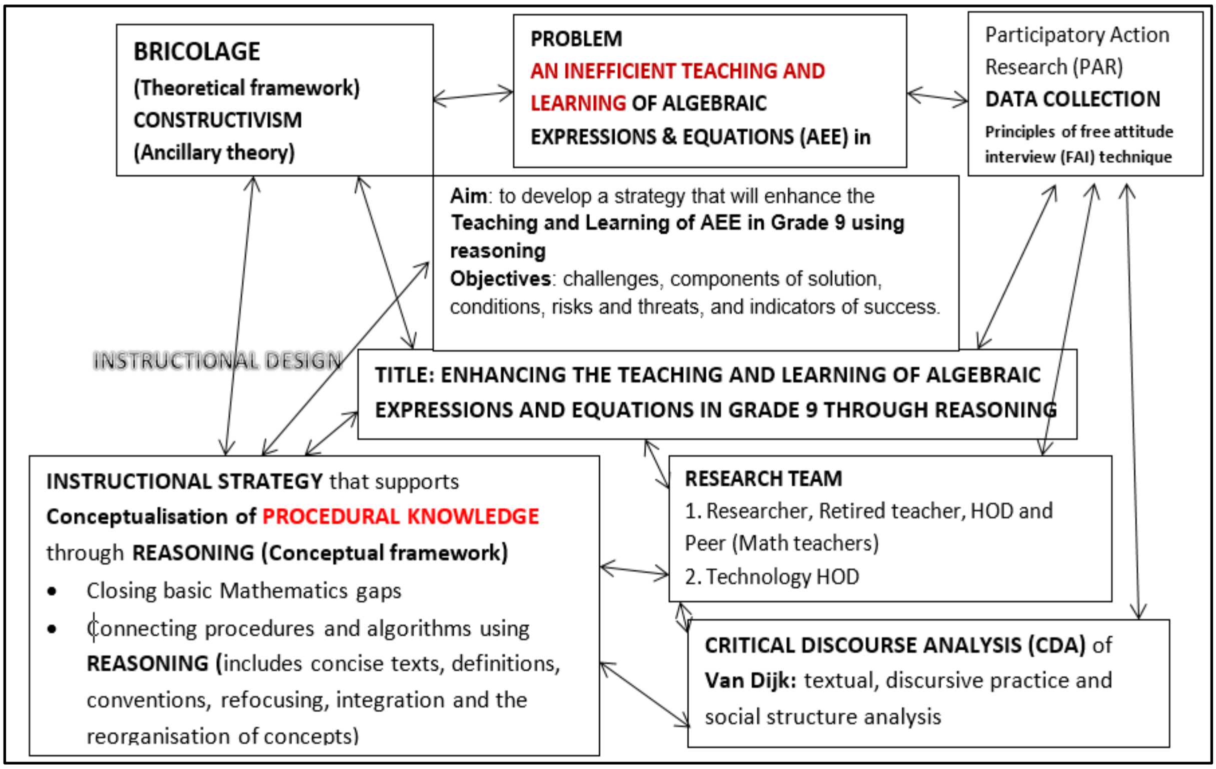



Figure 4.1 Study concept map

The presentation directed participants' interactions towards common comprehension. The first template (Figure 3.1) illustrated the meaning behind the research title. It goes with the research problem statement and the aim and objectives of the study, whereby the participants agreed that the research is about enhancing the teaching and learning of algebra in grade 9 using reasoning within the frameworks of bricolage paradigm and constructivism theory, and PAR. The data generated would be analysed using three levels of CDA. The discussion about the anticipated outcomes was presented with the help of the template below:

1. **PROFICIENT (DEEP) CONCEPTUAL UNDERSTANDING**

**(Rich, deep and flexible formation and navigation through own concept web or map)**

- Conceptualising reasoning-based instruction
- Improved classroom instruction (teaching and learning achieves conceptual knowledge)
- Curriculum and assessment policy (legitimation) oriented instruction
- Differentiated learning in which learners play integral role
- Increased parents and societies participation in mathematics learning
- Improved learners' efficacy and performance
- Increased number of thinking tanks (learners enrolling for Mathematics in Grade 10)

2. **RESEARCHERS' EMPOWERMENT**
   - To solve problems using what is available in their disposal

Figure 4.2 Conceptualisation of study outcomes

The researcher further handed out a paper detailing how the study was conceived:

**Shadows behind the study conception**

The researcher in his over ten years' experience in teaching mathematics at senior phase has observed with keen interest several problems underlying the instruction of mathematics, the result of which is repeated low or below-average performance in the subject year in and year out. They streamlined the main problem besetting mathematics instruction and realised that there are core components of the subject matter, namely basic mathematics (arithmetic) and algebra that are integral to most if not all the problems behind the complexity. The former lays the foundation for learning the latter. So, if a learner is not conversant with basic mathematics, it becomes very difficult if not impossible to form connections necessary to comprehend algebraic concepts. On the other hand, algebra is an important branch of mathematics that plays a pivotal role within mathematics on its own, relating and forming a basis upon which other topics build. For example, geometric instruction applies algebraic concepts. Also, that algebra is applicable in other subjects such as natural sciences, technology and commerce. Based on the foregoing exposition, the researcher formed an informed opinion that algebra was central to the complexity (the problem).

It is a common phenomenon in education that performance is associated with the teaching and learning, so the researcher could not be an exception. They conceived an opinion that the teaching and learning of algebra in schools should be inefficient. It became logically evident that the inefficient teaching and learning of algebra has a significant bearing on the performance of mathematics, which in turn influences negatively on learners' promotion, progression and career prospects. Having established the source of complexity the researcher created a statement: 'the inefficient teaching and learning of algebra in grade 9' as that they believed could to a larger extent summarise and guide the formation of the problem statement regarding the complexity at hand. Having figured out how to slice the problem into a manageable unit, they then pondered different means that could at least reduce the impact of the problem. It was at this point they referred to among other relevant documents, the driving document behind the teaching and learning, the curriculum policy (CAPS).

According to the curriculum policy, some of many specific and striking aims of the teaching and learning of mathematics at senior phase is to develop:

- deep conceptual understanding to make sense of mathematics;
- acquisition of specific knowledge and skills necessary for:
  - the application of mathematics to physical, social and mathematical problems;
  - the study of related subject matter (for example other subjects); and
  - further study in mathematics. (DBE 2011:8)

The curriculum policy further advises that to develop essential mathematics skills a learner should:

- develop the correct use of the language of mathematics;
- develop number vocabulary, number concept and calculation and application skills;
- learn to listen, communicate, think, reason logically and apply the mathematical knowledge gained;
- learn to investigate, analyse, represent and interpret information;
- learn to pose and solve problems; and
- build an awareness of the important role that mathematics plays in real-life situations including the personal development of the learner. (DBE 2011:8–9)

Out of many words used to describe the expectations of the policymakers, the researcher critically thought about the phrases: 'deep conceptual understanding…' and '…, think, reason logically and apply the mathematical knowledge…' (DBE 2011:8–9). It was from this logical thought and connection of ideas that the title of this study was conceived. The researcher recollected that it is not common in many mathematics classrooms to observe teachers and learners engaging on reasons underlying different concepts for conceptual understanding as required by the curriculum policy. The critical thought of whether framing reasoning as a means to curb the scourge can respond positively to the problem or not led to only one response: " you wouldn't know for real that introducing reasoning to enhance the teaching and learning of algebra in grade 9 can or cannot work unless and until you have researched about it". In sum, this is how we (the participants) find ourselves facing the uphill of having to establish the reality of the problem, and whether reasoning can or cannot be an epistemological solution. The activities of the research should therefore target to solicit the evidence that responds to the study problem and the aim and objectives of the study while maintaining the requirements of the curriculum policy document in terms of instruction and assessment, as a basis upon which the study was conceived (founded).

#### *4.5.2.2 Study philosophical frameworks, epistemology, methodology and method*

The frameworks underpinning this study are couched in the post-positivist philosophy that knowledge is authentic if it transforms into practice that empowers and transform the lives (Chilisa 2012:36) of the affected. Epistemology clarifies the researcher's approach towards knowledge generation. It (epistemology) builds on the foundations of an accepted and rationally defensible theory of confirmation and inference (Kincheloe et al. 2011:12–13; Noorderhaven 2004:91). For example, this study is inherent of constructivists' conviction that the truth relies on human experiential knowledge and that it is socially constructed in different contexts (Kawulich 2012:9). The conviction is open to accept (legitimise) participants' (communities') multiple perspectives informed by their various lived experiences. Conceptualisation of the study epistemologies turns co-researchers into true study agents who emerge out of the research as co-researchers (Mahlomaholo 2014:9). Kincheloe et al. (2011:12) alert that at times the dimensions of epistemological understanding escalate to a level where co-researchers have to individually take difficult epistemological decisions to maintain the knowledge production within the study's framework. That, to ensure that the status quo is sustained throughout all the implementation phases. Epistemological understanding of the research is therefore central to the rigour of the theoretical framework as the latter is to the research processes (Kincheloe et al. 2011:12).

The principles of PAR are also central to the study framework. They characterise collaborative emancipation of the affected through their participation in all research processes (Israel et al. 1998). That way, it (PAR) reveals the original feelings, views and patterns (Minkler & Wallerstein 2003:4) of the researched, and strategically turns it into legitimate knowledge that influences practice (Openjuru et al. 2015:219). The principles of the FAI technique constitute a method that promotes the principles (methodology) of PAR. The technique allows for natural inquiry and targets free disclosure of lived experiences by the researched communities in their own natural settings (see 4.2.5.1). It creates an enabling environment in which there is mutual respect between the researcher and the researched. Its open-ended questioning technique gives room to the disclosure of multiple realities (and perspectives) and discourages overgeneralising. Further, the research team discussed the following note that clarifies an integrated relationship between the frameworks in the context of the study.

**Study frameworks**

The study framework is a philosophical reflection of the researcher. It explains his line of thought about the nature of reality in as far as the problem at hand is concerned. It works together with the conceptual framework to link and corroborate different sectors of the study (Grant & Osanloo 2014:26), as well as ensuring focused data generation. The generated data should respond to the research question and assist researchers to achieve the study objectives. In this study, the researcher's thoughts are couched within the principles of bricolage which they conceive as portraying the same conviction as a well-known Sesotho proverb, '*thebe e sehellwa holim'a e nngwe'.* The meaning, "enhancement is best built on existence", practically equivalent to denouncing the re-invention of the wheel. The participants (researchers) and learners are therefore expected to work as bricoleurs (practitioners of bricolage), weave (connect) the scattered bits and pieces of teaching and learning remnants into (reasoning) constructs to generate conceptual knowledge. In essence, the study aims to operationalise the underutilised reasoning skill prescribed by the curriculum policy to deepen logical (conceptual) understanding (DBE 2011:4,8–9; NCTM 2000) of algebraic concepts and procedures.

Complementary to bricolage is the constructivism theory that support learners' active participation and leadership in the construction of reasoning constructs. Both bricolage and constructivism complement the principles of the methodology, PAR, in many aspects. They also underpin the CDA of Van Dijk. Constructivism is inspired by a theory that defines learning as active participation in a socially organised activity, in which the primary content of what is to be learned is not given but discovered by the learners themselves (Moloi 2015:36). That way, learning becomes challenging, meaningful, and interesting (Hamami 2020:4) to learners. It augments bricolage in supporting communities' collaborative initiatives that challenge inequalities and any form of subordination embedded in social structures, practices and discourses (Stinson et al. 2012:46) of algebraic instruction. It recognises and pays more attention to already existing historic, social and cultural structures of the society. The approach of this study through which the instruction is enhanced using underutilised (subjugated) reasoning (sources) (Rogers 2012:12) on existing procedures within the socio-cultural context of teachers and learners, operates in line with the dictates of these critical theories. It is worth noting that the reasoning constructs derive from learners' prior knowledge, mostly in a form of basic properties and operations of mathematics.

#### *4.5.2.3 Participants' viewpoint and opinion*

Participants expressed satisfaction about the content and discussions of the conceptualisation workshop. They confirmed to have understood and conceptualised the extent and nature of the problem under study (Tlali 2013:126), the basis of the problem and what the study is hoped to achieve. They also expressed conceptualisation of the framework confinements (do's and don'ts). They were however puzzled by the fact that reasoning was a mandatory requirement of the curriculum policy that is in most cases ignored or not given the attention it deserves. The puzzlement confirmed the presupposition of non-aligned (non-compliant) instruction. It also backed their opinion that the study was relevant to address the glaring discursive injustices and oppression embedded in the teaching and learning of algebra. The study, though long overdue, was inducing hope for the subjugated voices to be heard and bring about contextual transformation. It is worth noting that some said the study conceptualisation had ignited and inspired their desires to enrol for research studies of their own. They were optimistic that the study would set a pace for the areas of research they would like to pursue.

### 4.5.3 Data generation

Data generation derives from a plan of action. The plan of action maps out the order in which the activities of data generation will unfold in consideration of ethics (see 4.3) and the PAR principles guiding this study. PAR guides that the plan of action should provide a tailored approach (Ball 2009:116) to accommodate the needs of the researched (Minkler & Wallerstein 2003:4; Openjuru et al. 2015:219; Baum 2016:405). The plan of action should therefore exhibit the reflective cycles (steps) of PAR (Kemmis et al. 2014:276) and active participation of co-researchers (the research team) in situation analysis and planning, action (teaching and observation), and reflection stages. The activity narrations specify the researchers' participatory actions during research (data generation). Consequently, the plan of action (Table 4.1) comprises the reflective cycles of PAR, activities, roles, activity narrations and time frames.

**Table 4.1 Plan of action**

| Reflective cycles | Activities | Roles | Activity narrations | Time frames |
|---|---|---|---|---|
| Situation analysis and planning | (a) Document analysis<br>(i) Learners' activity/classwork books<br>(ii) Assessment tasks/tests (informal & formal)<br>(iii) Subject improvement plan<br>(iv) School improvement plan<br>(v) IQMS and other teachers' performance evaluation reports<br>(vi) Departmental guiding ATP and pacesetters<br>(vii) Sample lesson plans<br>(viii) PLCs reports<br>(ix) Mathematics curriculum policy document (CAPS)<br>• Assessment standards<br>• Cognitive levels and types of questions | Research team (RT) | (a) Research team will source the listed (relevant) documents for analysis and reflection of their impact on the inefficient teaching and learning of algebra in grade 9. | Within a week |
| | (b) Reflections<br>(i) The number of practice activities on algebraic expressions and equations | Research team (RT) | (b) Research team will reflect on document analysis in respect to the quantity and quality of activities on algebraic expressions and equations; and the use/application of relevant learners' prior knowledge | Within a week |

| | | | | |
|---|---|---|---|---|
| | (ii) The quality and cognitive level progression within the activities.<br>(iii) Introductory basis and application/relevance of learners' prior knowledge | | | |
| | (c) Focused discussions<br>(i) grades 8 and 9 mathematics teachers<br>(ii) Perspectives of other stakeholders | Research team (RT) | (c) Research team will engage in focused (group) discussions with grades 8 and 9 mathematics teachers and other stakeholders to generate multi-perspectival views about the instruction of algebra and how it can be enhanced. | Throughout the research period |
| Action (Research team activities) | (a) Baseline assessment | Research team (RT) | (a) Research team will set, moderate, administer and mark baseline assessment. The assessment will be set in a way that it tests learners' ability to think, reason logically and apply basic knowledge; to connect and interrelate basic concepts and procedures to demonstrate conceptual knowledge and understanding; in items of different cognitive levels. | 2 days |
| | (b) Classroom teaching and observation | Research team (RT) | (b) Research team members will plan lessons and assign one member per lesson to teach, while other member(s) observe. | Throughout the research period |
| Reflection and planning | (a) Reflection meetings | Research team (RT) | (a) Research team will reflect on each lesson taught and observed, and plan and teach subsequent lessons based on experience learnt from reflection discussions (meetings). | Throughout the research period |

#### *4.5.3.1 Situation analysis*

During the research launch meeting, the research team planned to engage in several activities to discuss the situation besetting the teaching and learning of algebra in grade 9. The activities included document analysis; reflections on document analysis and focused (group) discussions with stakeholders.

*(a) Document analysis*

The document analysis involved deducing the mode of instruction normally used in classrooms from classroom and homework activities. A sample of grade 8 and 9 learners' activity and classwork books for different academic years were collected and analysed. The following conversation from the research team meeting bears evidence of the document analysis:

> **Mama** (retired teacher asking Shana, one of the practising grade 9 mathematics teacher whose learners' classwork books were analysed): Why is it that your learners' books did not show comments or reasons next to the steps they undertook when simplifying expressions and when solving mathematical equations? I thought that would have assisted us to identify learners' problems with more certainty.
>
> **Nono** (in support, added): It is not only in his case, teachers generally do not teach learners to state reasons next to steps as suggested (by Mama). I have observed it for some time when moderating learners' books. Mfetho tried it with his previous classes, but I did not see it (showing reasons next to steps) this year.
>
> **Shana** (responding to the question asked): I cannot remember providing or asking learners to provide reasons or computational connections when simplifying algebraic expressions and solving mathematical equations, it is always about following the procedures as taught.

The response (Shana's) was further augmented by the sample lesson plans on algebraic expressions and equations analysed. It was however noted that the lesson plans were published as guidelines open to modification for optimal results. The analysis of activities was further looked into in relation (comparison) to the departmental annual teaching plan and pacesetters, to establish the impact of those on the mode of instruction. The analysis also established whether there is a connection between the mode of instruction as deduced from classwork and homework and the informal and formal assessment tasks provided during and after teaching algebraic expressions and equations as part of the school-based assessment. Further, the analysis of assessment mapped the question items against SAGM cognitive levels as prescribed by the curriculum policy document (CAPS). The curriculum coverage and tasks were also analysed in consideration of content weightings.

The analysis also considered the subject and school improvement plans in conjunction with the challenges and shortcomings observed from the classroom activities, curriculum coverage, mode of instruction, informal and formal assessment to learners' average performance in terminal (common/standardised) papers. Furthermore, the team analysed teachers' IQMS, internal moderation and external inspection reports.

*(b) Reflection on document analysis*

The reflection on document analysis revealed that the instruction of algebra was overshadowed by a procedural approach as confirmed in Shana's response. Activities in learners' classwork books revealed consistent reliance on textbooks' exercises. There was no evidence of the use of the guiding sample lesson plans, hence it was impossible to determine if there was any modification made on them to optimise results as recommended (see 4.5.3.1(a)). Almost all activities were found to be those that drill procedures with little or no inculcation of deep conceptual understanding. For example, a transposing procedure when solving equations was introduced and used as such. It could be deduced from learners' books and interviews that the concept was never founded on learners' knowledge of inverse operations, and additive and multiplicative identities, hence there was observable gaps of conceptual and logical connection and interrelation. Moreover, learners were to simplify numerous algebraic expressions and solve many mathematical equations based on one procedure or concept. Most of the assessment items were limited to low cognitive level(s). Items on cognitive levels three to four, if any, were not analysed to reveal reason(s) and/or meaning behind algebraic procedures and algorithms as depicted in the foregoing example. There was no evidence of assessment feedback (mostly classroom corrections) being used to improve learning (DBE 2011:154). It was therefore inferred that the standard of school-based assessment was not aligned with the CAPS prescripts. Hence, it was concluded based on facts gathered from documents alone that the activities and assessment tasks given to learners were mostly dominated by quantified or drilling procedural items than those that embrace (support) quality (conceptual) learning. Further, there was little or no evidence to prove the conceptual connection of the knowledge gained being consciously (knowledgeably) applied in the learning of advanced mathematics or any other related subject. Even the riders were in most cases treated as stand-alone sub topics. Consequently, learners' performance in standardised (common) papers averaged way below the minimum performance level of 40% (DBE 2011:154). This, in spite well written subject and school improvement plans that seemed to focus more on curriculum coverage than to address the presupposed inefficient instruction of the perceivably abstract algebra. It is worth noting, however, that the abstraction of algebra was central to the problems associated with learners' underperformance in mathematics. The teachers' IQMS, internal moderation and external inspection reports did not expose the complexity of the algebraic instruction explicitly. IQMS reports concentrated on the competence of teachers and scored them way above average, while the internal moderation and external inspection concentrated on curriculum coverage.

*(c) Focused discussions*

A focused (group) discussion is a research technique used to collect data by gathering together people from similar backgrounds or experiences to discuss a topic of interest (Morgan 2001:141). In the case of this study, the team considered the workshop (Annexure M) as one form of a focused group discussion. The challenges and issues relating to the instruction of algebra discussed therein have therefore been assumed as part of data worth being analysed to inform the findings and recommendations of this study.

Further, the team capitalised on grade 8 and 9 mathematics teachers' support meeting hosted by the subject advisor. Three members of the research team participated in the discussion in which the instruction (teaching and learning) of algebra emerged integral (see 3.2.1.2). The discussions revolved around agreements and disagreements on the source of problems. Then, followed by suggestions on different ways by which the algebraic instruction enhancement could be achieved. The discussions were held in a conducive environment and setting that embraced the principles of bricolage and PAR in respect to the multi-perspectival approach. The opinion (different perspectives) of practising teachers represented a wide range of classroom experience of the researched (teachers and learners). The subject advisor's input drew from a wide experience and perspective attached to their work in many schools and across districts and provinces. In the end, the results of a focused discussion concurred with the view that experiential learning can lead to a legitimate form of knowledge that influences practice (Kawulich 2012:9; Openjuru et al. 2015:219). It can be argued that the focused discussions further revealed the intention of this study to place the epistemic power and control in the hands of those who have best vantage point to discuss and suggest solution(s) to the problem.

#### *4.5.3.2 Action (research team activities)*

Another series of activities where the research team applied hands-on participation was in setting, moderating, administering and critical-marking of baseline assessment. The subsequent script analysis enhanced (added value to) the gains of document analysis; and set pace and sequence for subsequent activities. Both the document and baseline analyses guided the planning of research classroom lessons, teaching and observation activities.

*(a) Baseline assessment*

The research team set and moderated the baseline (Annexure N). The baseline assessment was among other factors motivated in the envisioned approach's requirement and also inspired by the discoveries of the situation analysis process, to check (diagnose) if learners were adequately equipped with basic knowledge and skills upon which the teaching and learning of grade 9 algebra could build (Fuchs & Fuchs 2005:45; Matsolo 2006:5; McNeil et al. 2010:625–626). The satisfaction of the requirement was eased by the fact that the topics of mental mathematics and algebraic expressions and equations are inherent (spirally developing) from grades 7 and 8 (DBE 2011:23–25). Therefore, the baseline was strategically based on grades 7 and 8 mental mathematics and algebraic content. It is assumed that the topics' spiral development seeks to lay a foundation for harmonised teaching and learning at the phase exit grade (grade 9). The reasoning-based instruction helped to check whether learners could successfully apply the knowledge of mental (basic) mathematics to interpret (reason out) and connect the operations of algebra. For example, an equation item: 8.3 $7^{x+1} = 1$, tested (checked) if learners could apply the law of exponents (basic mathematics) to figure out that $1 = 7^0$ to form and solve a linear equation $x + 1 = 0$ derived from a *fair* comparison of equivalent powers. A *fair* comparison is based on the fact that for the two sides to be equal, the power bases and exponents on both sides should be equal respectively.

Second, the assessment items were framed (constructed) in a manner that they could reveal learners' level of knowledge and understanding in items of different cognitive levels. The emphasis was however put more on items that could challenge learners' ability to think, reason logically (to analyse the problem) and apply gained (basic) knowledge (DBE 2011:9). The items checked if learners could form necessary conceptual connections and interrelations geared towards a sensible solution. The foregoing example is typical of the emphasis. Thus, it can be argued that the baseline also checked if the instruction by which learners were taught mental mathematics and algebra in the previous classes could sustain the knowledge acquired and application thereto for a longer period.

After administering the baseline strictly on an individual work basis, the research team further marked and script-analysed learners' performance in the baseline before engaging in critical reflection. The reflection on both situation analysis and baseline performance guided the team to plan (pitch) research lessons at the appropriate level.

*(b) Classroom teaching and observation*

Research team members planned lessons and assigned members took turns to teach learners in the classroom while other member(s) observed. The process (activity) of lesson planning focused on creating sufficient space and time for learners' active participation in line with the PAR principles, and operationalising (weaving) reasoning into the instruction. The existing lesson plans (see 4.5.3.1(a); 4.5.3.1(b)) were customised (modified) to meet the reasoning-based instruction guidelines. As part of reasoning-based instruction, the lesson plans and subsequent assessment tasks (exercises) incorporated a review of relevant basic mathematics as an introductory part to the learning and responding to algebraic items (Annexure N). As such, the team grafted the sample lesson plans into a bricolage of remnants (scattered bits and pieces) modified to trigger learners' reasoning constructs. The lesson plan included the development of the constructs into conceptually connected and interrelated knowledge. That way, the existing lesson plans became a medium upon which the operationalisation of reasoning (modification) was built.

In the classroom, the lesson presentations focussed on promoting learners' reasoning constructs in line with bricolage and PAR's principles of simplifying complexity (see 2.2.1.2(a)) through active participation of the researched (see 4.2). Teachers' guidance to activate learners' oral participation and subsequent probing need be, observed the principles of the FAI technique. There was observable mutual respect and cooperation between teachers (co-researchers) and learners (the researched) throughout the lessons. Research team members, when appropriate, also invoked team-teaching in a few instances in the classroom. The team-teaching was encouraged by learners' feedback (oral or written). When realising some common errors and misconceptions, the observing member(s) could professionally interject to highlight glaring shortcomings, emphasise strong and weak points, and perhaps re-teach the concept using an alternative method(s). The practice (team-teaching) enhanced learners' conceptual understanding as well as highlighting key points for reflection discussions and subsequent lesson planning. When satisfied that most if not all learners could demonstrate conceptual understanding, the teacher and observer(s) distributed exercise worksheets (Annexure N) to check the learners' individual cognitive development and to generate empirical performance data (Table 5.1). The marked worksheets, critical analysis thereof, and observed key points informed the reflection meeting discussions and subsequent lesson plans.

In regard to the standardised assessment, the formative and summative tests were used. The assessment effectively tested the potential of the reasoning-based instruction. The formal test items were taken from the district paper, while the summative used items resembling those asked in June 2019 and previous examination papers, set and moderated in line with the curriculum policy standards. It can therefore be concluded that the performance in both the formal test and summative assessment was a true reflection of the impact of PAR reflective cycles of planning, classroom teaching and observation, and reflections, in promoting reasoning-based instruction. The performance demonstrated if the approach (reasoning-based instruction) could simplify the perceived abstraction and complexity of algebra and its instruction (see 3.2.1.3).

#### *4.5.3.3 Reflections and planning*

The reflection meetings are integral to reflective cycles (reflexivity) of PAR (see 4.2.2), hence to the plan of action couched in the theory as well. They are relational sessions (meetings) in which co-researchers engage together to reflect on the observation and consequences of change to guide the next cycle (of planning, action and observation, then reflection) (Baum 2016:405). In the context of this study, the sessions were described as platforms in which the research team took stock to determine whether the research activities were aligned with the plan and achieving the objectives of the study. That way, the sessions help to verify activities and possibilities of success. That is, they help to assess the level of enhancement, and how best to optimise it. It can therefore be argued that the meetings presented an opportunity for reporting and discussing research activities' feedback amongst the team as well as adjusting the plan where and when necessary.

The research team reflected on each lesson taught and observed. The reflection discussions and experience informed the planning of subsequent lessons. The discussions assisted co-researchers to determine if the instruction's components of solution were enabling learners to demonstrate conceptual understanding of algebra. They (discussions) helped the researchers identify and confirm conditions (factors) that constitute a conducive environment for meaningful learning (Pramesti & Retnawati 2019:3), as well as risks that may threaten successful implementation of the components. The reflection meetings further assisted co-researchers to realise and discuss the indicators of success and to plan lessons cognisant of the factors that may affect the instruction both positively and negatively. It can therefore be concluded that the inferences drawn out of evidence presented, discussed and analysed in reflection meetings together with generated data, vindicated the strengths and weaknesses of the reasoning-based instruction with the objectives of the study. The following sections will highlight matters of discussion that arose from reflection meetings.

*(a) First reflection meeting*

The minutes of the meeting reflect a positive report about lesson one, taught by Mfetho, and observed by Shana and Nono. The lesson introduced the language and notations used in algebraic expressions and equations. The reflection concentrated on the strengths and weaknesses of the presenter (Mfetho) in relation to operationalising action and reasoning within the lesson. Some of the strengths include: (i) precise articulation of the language used in algebraic expressions and equations, as an introductory lesson that draws from the study's epistemological approach of building or ensuring basic knowledge upon which the subsequent (advanced) instruction was to found; (ii) creating space and time for learners to debate (construct reasoning) around the classification of algebraic expressions reflected elements of constructivist pragmatism and the principles of PAR; and (iii) creating an enabling (learning) environment in which learners' responses were treated with respect in line with the FAI technique. The debate triggered the construction of reasoning constructs by learners as contemplated and supported by this study. It emulated the emphasis within the American standards of mathematical practice (SMP 3) which states that learners should construct viable arguments and critique the reasoning of others (Rumsey & Langrall 2016:413). The argumentation (debate) also helped the teacher to diagnose the areas in which they needed to intervene and close gaps. It created an opportunity for learners to learn from each other and gave hope on knowledge conceptualisation and retention. It was further noticeable that the debate substantially relinquished the power, control and pace of learning to learners.

Mfetho (lesson presenter) reported about the weakness of explaining for a longer time without involving learners. However, the co-researchers said they did not observe (consider) explaining for a longer time a weakness in the reported circumstances. They argued that some lessons like the one in question, based on terminology may require uninterrupted explanation for precise and logical articulation, as was the case. Further, they had the following facts to consolidate the argument: (i) the reasoning facts learners raised to support their responses during the expressions' classification debate proved that the explanation was necessary in the circumstance; and (ii) the acceptable results in part one of exercise one based on classification of expressions and an overall average of 77% on the whole task proves contrary to the weakness of long explanation. As a result, all were content that the reports and performance thereto indicated positive prospects about the reasoning-based instruction.

*(b) Second reflection meeting*

The minutes of the meeting reflect a positive report about lesson two, taught by Mama and observed by Mfetho. The lesson dealt with some shortcomings noted from learners' feedback on exercise one, and focused on 'collecting like terms'.

The reflection concentrated on teaching or presentation. The teacher embraced the reasoning-based instruction when they guided learners using concrete examples to explain the difference between like terms in addition and subtraction of expressions, and factors of multiplication and division. As a result, learners were able to factually reason out considerations they have to make before they could identify and operate like terms. The meeting was hopeful that the instruction through which learners use reasoning to differentiate between like terms and factors would minimise the common errors and misconceptions that in most cases have to do with confusion learners experience between the two.

The meeting further learnt that Mama explained the reasoning behind the additive and multiplicative inverses in equations. They explained that when eliminating one of the like terms bearing the unknown variable, the additive inverse applies the same way as when eliminating a constant value because the operation still strives towards the sum of the inverses being *zero*, *the identity of addition*. They refreshed the concept of integer operations (basic mathematics) and number line to remind learners how the result of zero comes about. They further emphasised that when collecting like terms on different sides of the equation the inverse is collected to the like terms to ensure one term with an unknown (variable). Because of knowing the cause (reason(s)) behind operations, most learners were able to successfully operate like terms in an expression differently from like terms on different sides of an equation.

*(c) Third reflection meeting*

The report from lesson three's teaching and observation by Mama and Mfetho respectively, recorded another tone about the notion of founding algebraic knowledge on basic mathematics as couched in the proposed enhancement approach. The introduction of 'binomial times binomial' building from vertical multiplication of two-digit numbers established an analogy (concrete foundation) learners were able to relate with and apply at ease when multiplying two binomials. The analogy (comparison) clarified a reason for the procedural or computational requirement to multiply each term of the multiplier binomial, $x + 2$ by each term of the multiplicand, $x - 3$, for example. That is, learners could easily interrelate and apply the property of distribution in 'binomial times binomial' as they confidently do when multiplying two-digit numeric values. Further, the re-arrangement of the products in a way that collects like terms displayed an explicit connection and interrelation between the multiplication of binomials and the concept of 'collecting like terms'. Thus $(x + 2)(x - 3)$ were re-arranged and computed as follows:

$$\begin{array}{l} \quad x + 2 \\ \quad x - 3 \\ \hline x^2 + 2x \\ \qquad -3x - 6 \\ \hline x^2 - x - 6 \end{array}$$

The precision of an instruction utilising the operational interrelation between basic mathematics and algebraic concept to connect the procedural computation drove the conceptual understanding of 'binomial times binomial' home.

Learners' performance in this particular topic in the summative tasks further proved that the knowledge attained in a reasoning order of known to unknown (Katehi et al. 2009:12) could endure the test of time. The endurance was attributed to the precise articulation of the similarity between the known basic operation and the new algebraic concept. The operational connection from the basic multiplication of two-digit numerical values to 'binomial times binomial' became instrumental reasoning that enhanced deep conceptual understanding. The meeting also discussed the confusion embedded within integers operation. That is, it impedes a synchronised flow of the concept (binomial times binomial) for a considerable number of learners. The meeting emphasised that teachers should always revisit the basic knowledge upon which the simplification of algebraic expressions and solving of mathematical equations build for gradual reduction of the challenge. That is, the meeting reiterated the fact that the mastery of basic mathematics relies mostly on regular practice and multiple trials (Hewitt 2012:142; and Banerjee & Subramaniam 2011:351).

*(d) Fourth reflection meeting*

The meeting interrogated the possibilities of success by reflecting on whether the team was succeeding to sell (instil) the notion of reasoning specifically to enhance the instruction, and to sway the instruction in a conceptual other than procedural aspect of knowledge, or not. The comparison between learners' performance in the baseline assessment and classroom exercises formed a basis of in-depth (qualitative) analysis. The analysis reflected a remarkable improvement in the quality of learners' responses in classroom exercises (Figures 5.1 and 5.2). It reflected that lack of basic mathematics competency in baseline was a fundamental source of the downfall for the majority of learners. It was also noted that learners could have forgotten what they had previously learnt in grades 7 and 8. This, because since there were traces within their computations that the concepts were not new. The team speculated that the procedural instruction could have been the reason for the glaring shortcomings in the baseline assessment. It associated the positive performance in classroom exercises with the approach emphatic on knowing the reason(s) behind procedural computations. The performance further became a beacon of hope for positive results in the then forthcoming formal test and June examination. Improved performance in the latter tasks would be an indication of knowledge endurance (retention) (see 4.2.5.2) presumably inherent of the reasoning-based instruction. The challenge with basic mathematics was specifically identified in operations of integers and associated properties, inverses and identities, the distributive rule when removing brackets, dividing a polynomial by a monomial or integer and in applying laws of exponents. The meeting noted that the language barrier (Matsolo 2006:12; Pramesti & Retnawati 2019:1; Sengul 2011:2305; Ying et al. 2020:5405) and operation confusion between expressions and equations also added to the downfall.

The meeting revisited ways by which the reasoning-based instruction could be complemented by the application of basic concepts such as solving simple equations by inspection before involving computations. The team appreciated that the practice helps learners to strengthen their reasoning capacity with anticipation and that the anticipation checks the logic behind reasoning and develops critical thinking.

*(e) Fifth reflection meeting*

The reflection concentrated on assessing the level of enhancement the reasoning-based instruction had thus achieved. The discussion of individuals' work on exercises one to five indicated a considerable improvement and so did the class average. The meeting also noted that the improvement had varied from one sub-topic to another (Figure 5.1 & 5.2). Learners had relatively recorded low performance in exponential expressions and equations. The meeting planned for assessment learning with the aid of the feedback. It stressed that the re-learning should be dominated by slow learners' reasoning constructs for maximum (conceptual) understanding assurance. It noted that the results of exercises five, six and seven showed a great improvement as compared to performance in previous exercises. While there could have been other factors that boosted the results, the improvement was vastly attributed to more learners getting used and appreciating the reasoning approach. The factors included writing exercises as classwork and in parts because of time constraint, as opposed to exercises one to four that were written in a test format. The meeting also noted, as a reason for improved performance, the impact of basing (connecting and interrelating) the work in exercises five, six and seven on rich basic knowledge built or acquired in previous exercises. It also commended the teacher's competence and consistency in operationalising reasoning-based instruction. That, the instruction has developed the critical thinking and confidence of learners, the observation of which was glaringly notable in a way learners were analysing and responding to questions. Hence the meeting deduced that there was enough evidence indicative of success. However, the meeting noted that the approach require more time for interaction, practice and revision for optimal results. The issue of learners' fatigue during the last classes held amidst the examination period was also noted, and factored within the challenges that include the time constraint alluded to earlier.

*(f) Sixth reflection meeting*

The meeting reflected on how the teacher (Mfetho) used learners' reasoning constructs to establish the similarity between the operation of numeric (arithmetic) and algebraic fractions. Mama reported that the teacher insisted that learners should explain the meaning of common multiple (common denominator or least common multiple), and that the explanation formed a firm conceptual bridge for the transition from numeric to algebraic fractions. They further commended the teacher for asking learners to retrieve lesson one's knowledge to work out fractions with multi-term numerator(s). That, the numerator needs to be bracketed into a monomial upon which the integer (scalar) multiplication should observe the distributive property. They also commended the teacher's continual warning for learners to always check (mind) accurate integer operations. The meeting noted with satisfaction the considerable improvement of the slow learners (see 4.5.3.3(e)). It however recommended revision on factorisation before administering a summative test on account of the report about it and time constraint.

*(g) Seventh reflection meeting*

The team reflected on the impact of the reasoning approach based primarily on the use of the learners' reasoning constructs as a means of learning, and on the performance in summative tasks. The summative tasks comprised mostly of algebraic expressions and equations extracts (items) from the district formal test and 2019 June examinations. The common test was written after the research has advanced up to lesson four. The summative assessment was written after all sub-topics were taught (Annexure O). The meeting strategized to compare learners' performance in summative tasks with that of the baseline. The notes captured from the marking of the baseline and summative assessments revealed the challenges that beset the learners before using the reasoning-based instruction, and the improvement the approach had brought. Over and above the notable improvement in empirical results, there was an observable qualitative improvement in the way learners had responded to questions that required conceptual understanding. There was a noticeable positive change even in the case of those learners who were let down by basic skills inaccuracy (semi-competence). Matters (reports) arising from (informal) interviews also indicated positive prospects about the reasoning approach in the teaching and learning of algebra. The next section will discuss some of the common responses researchers elicited from the interviews.

#### *4.5.3.4 Interview questions and responses*

The study adopted open-ended questioning inspired by the free attitude interview technique (see 4.2.5.1). All participants (research team and learners) were afforded time and space to freely share their views. The questions were arranged in line with the aim and objectives of the study against which they were expected to elicit responses addressing the research question (Akhtar 2016:68). The responses from different platforms of engagement were therefore classified according to the objective-inherent questions:

*(a) Why would there be a need to enhance (improve) the teaching and learning of algebraic expressions and equations in grade 9?*

The responses generally appreciated that there were notable challenges to the teaching and learning of algebra in the senior phase and beyond. The repeated below-standard performance in grade 9 mathematics and related disciplines was sufficient evidence to prove the challenge. Most learners' career options are blocked by the performance. They classified algebraic instruction as that which is abstract and complex. Yet its weighting strength together with that of its application escalate it to a major determinant of progress and career pathway. They associated the glaring attitude, laziness, loss of interest, forgetfulness and low self-efficacy towards the learning of algebra and mathematics with the abstraction and complexity. They also raised the lack of mathematics education support at home as another challenge: They openly conceded that other than lack of knowledge, parents are less or not active in their children's education. They were therefore not getting sufficient space and time for meaningful practice at home.

*(b) What could be possible components of solution to enhance the teaching and learning of algebraic expressions and equations in grade 9?*

The participants (research team and learners) responded that the teaching and learning leaning towards conceptual rather than procedural aspect of knowledge deemed an ideal and primary component of solution. The component draws from the requirement of mathematics curriculum policy to develop and offer quality (deeply conceptualised) education based on logical reasoning and critical thinking (DBE 2011:8–9) in an environment that encourages active (action) and critical learning (DBE 2011:4). Learners' responses complemented their active involvement in support of the component. For example, learner two responded:

> **Learner two**: This new approach by which we are encouraged to connect the procedural steps with reasons helps us to understand and remember more of what we learn.

Another learner added that reasoning reduces or removes the impression of abstraction and complexity that they always had about algebra in previous grade(s). The other suggestions insisted on a need for increased practice items, space and time to enhance the conceptualisation process to function as expected. Most learners acceded to working in collaborative groups as a solution to the challenges.

*(c) Under which conditions can the components of solution work?*

Many participants suggested condition(s) related to the establishment of teachers' development and support programmes (Pramesti & Retnawati 2019:1). This, to address the need for competence (DHET 2015:62) to teach algebra effectively. The other common set of responses reiterated the need to afford learners sufficient time for interactive practice and revision (Katehi et al. 2009:12) for the idea of conceptualised learning to sink in. When probed about interactive practice, the learners indicated the importance of utilising the learner-centred approach, "it creates an environment in which we become responsible of constructing and understanding knowledge in our context". Participants were further specific about the condition to always refresh the founding basic mathematics to the particular algebraic content before introducing the latter. The responses showed that the approach is chronological and provides a firm foundation upon which the teaching and learning of algebra build at ease.

(d) *What could be possible risks and threats to the implementation of the reasoning-based instruction?*

Most participants mentioned learners' insufficient knowledge of basic mathematics and stereotyped (procedure-oriented) instruction as threats. That insufficient basic knowledge threatens the foundation of proper (conceptualised) teaching and learning of algebra. And that the procedure-oriented instruction follows a set of ordered (scientific) procedures, hence limits meaningful and functional learning. Comments also considered resistant teachers who would still stick to teacher-centred approaches and resist professional development and support (Pramesti & Retnawati 2019:1) as high risk. The solutions raised for enhancement are underpinned by the principles of constructivism and PAR in support of the learner-centred approach (instruction). Some responses were direct and elaborating "the lesser we get involved, the little we learn." A considerable number of learners raised the concern about classroom assessment complacency (routine) that fails to prepare them for standardised (external or common) assessments (DBE 2011:155) that normally carry high mark weightings. In essence, it fails to re-teach or perhaps close the procedure-oriented instruction gaps. The contestation between the time allocated for teaching and learning algebra and the volume (amount) of curriculum content was mentioned as a risk that threatens the envisioned learner-centred and reasoning-based instruction.

(e) *What evidence do we have to prove that the reasoning-based instruction improve (enhance) the teaching and learning of algebraic expressions and equations?*

Participant teachers mentioned that most learners were able to communicate logical reasoning and apply knowledge gained (DBE 2011:9) to connect concepts and procedures. The skill was particularly recommended by learner two as that which helps them to understand and remember more of what they learn (see 4.5.3.4(b)). Logical reasoning and applied knowledge implied improved basic mathematics competency. They also commented about the fruitful rapport of collaboration between teachers and learners. The rapport was indicative of the effectiveness of the learner-centred approach upon which the reasoning-based instruction anchors. Teachers (co-researchers) commended the approach for helping them realize and adapt to different learning styles of learners. This helped teachers to afford each learner assistance with their own learning needs. As a result, there was a remarkable qualitative and quantitative improvement in the performance of learners (Table 5.1; Figure 5.1 & 5.2). The performance indicated a continuation of knowledge with less or no strain. As a result, the perception of algebra as being difficult and abstract changed, and was replaced with confidence. The confidence could be detected from learners' eagerness and enthusiasm when constructing and navigating through their own algebraic conceptual network. The teachers (co-researchers) acknowledged that the component of solution had transformed (developed) their practice in a positive way and that they have been empowered to generate their own solutions using what is at their disposal.

## 4.6 DATA ANALYSIS

This study uses CDA to present, interpret and analyse the generated data. The choice was based on its (CDA) interdisciplinary characteristic (Fairclough 2013:19; Van Dijk 2015:468) rendering it compatible with the study frameworks. CDA allows for various ways of arriving at the truth (Wodak and Meyer 2009:3); and can monitor the coherence of frameworks, theories, methodologies and procedures of empirical research (Van Dijk 2015:253) using its three levels of analysis, namely textual analysis, discursive practices and social structures. Fairclough (2013:13) writes that CDA helps scholars to better understand the nature and sources of social problems, the challenges to address the problems, and possible components of solution to address those challenges (Fairclough 2013:13). It can then be deduced that CDA analyses the textual data in a manner that the reader can clearly relate to (discursive) practices within the given social structures and the impact therein.

In this study CDA is particularly intended to forge necessary relationships between the algebraic texts, instructional interactions and social structures (McGregor & Murnane 2010:3; Mogashoa 2014:105; Rogers 2012:371) to give a clearer analysis (explanation) of how a presupposed discourse (reasoning-based instruction) can expose and address the discursiveness embedded within algebraic texts. Hence, enhance the teaching and learning of algebraic expressions and equations in grade 9.

### 4.6.1 Textual analysis

CDA is a textual analysis that ensures texts are produced, read and heard in consideration of some real-world context (Hucken 1997:78). It is a type of discourse analytical research that mainly examines the way social power, dominance and hegemony are sanctioned, reproduced, and opposed by text and talk in the social and political context (Van Dijk 2015:468,478). Mogashoa (2014:105) writes that CDA determines the relationship between the text and the processes involved in listening, speaking, reading and writing. It describes, interprets, analyses and critiques the challenges reflected in the text. It arouses a critical look into the meaning of words in the social context of teaching and learning in various institutional structures, and reckons the non-neutrality of words as used in different contexts. It, therefore, attempts to reveal the meaning of the content in relation to its contextual use (Hucken 2002:4; Mogashoa 2014:109).

In this study, the textual analysis is intended to reveal and expand the meaning within the generated data. The intention resonates with Mogashoa's (2014:105) understanding that, "Human subjects use texts to make sense of their world". The analysis will help the researchers understand the generic production of deep conceptual knowledge and its benefits to the researched when contrasted with the predominant procedural knowledge. The issues of analysis within the data include the instructional power imbalances, social inequities and non-democratic practices (Hucken 1997:79) that render the teaching and learning of algebra in grade 9 inefficient. The impact of learners' reasoning constructs (presupposed social constructivist discourse) in addressing the textual challenges ascribed to in foregoing statement and others will also be text-analysed. It can then be argued that the textual analysis is essentially an epistemological requirement for this study. It is a useful tool by which teachers as leading members of profession will have a basis for self-inspection among other benefits.

### 4.6.2 Discursive practices

CDA denotes a form of resistance to unethical and biased social power relations (Van Dijk 1993:352). It helps scholars to analyse the discursive practices (notions) of power, dominance, discrimination and bias embedded within the texts and how they are initiated, maintained and reproduced (Van Dijk 2015:468). It reflects on the discursive interactions (practices) at the local, institutional and societal organisations (Mogashoa 2014:105). In this study, CDA analyses the extent to which learners are disempowered by instructional approaches that deprive them an opportunity to engage actively in knowledge generation. The proposed reasoning-based instruction dispels the perception of considering the words of those in power (teachers) as “self-evident truths” (McGregor 2010:2). It disapproves and challenges any dismissal of the words of those not in power (the researched in this case) as irrelevant, inappropriate or without substance. For example, the instruction dominated by a teacher-centred approach in which teachers consider themselves as sole transmitters of knowledge (Major & Mangope 2012:146) is discursive. The discursive practices treat learners as objects that could be controlled and regulated (McGregor & Murnane 2010:314) to learn strictly according to the teacher’s method(s). This leaves no surprise why the instruction (practice) has always resulted in a myriad of occurrences of below-standard performance in algebra and other mathematical topics. The practice focuses the power of creating knowledge onto an expert (teacher) whose words and procedures are regarded as truth that cannot be questioned or critiqued (Tlali 2013:166).

The proponents still live in the traditional period and modernist moments whereby the instruction is concentrated on one aspect, driven by one person who treated the researched as objects from which they could extract information in their own terms (McGregor & Murnane 2010:424; Penco 2010:2; Vilela 2010:344). This study discourages power dominance and encourages the notion of holding discussions and conversations with learners to understand the reality of the situation (in context) (Locke 2004:6). It draws from the analysis of how the use of technical (symbolic) language in constituting and transmitting procedural knowledge (Van Dijk 2015:470; Wodak 2009:9) derail the conceptual understanding at the institutions similar to Leru secondary school. The corresponding characteristics may include discursive cultural and societal practices that has a negative influence on the teaching and learning of algebraic expressions and equations in grade 9. A good example, in this case, is that of the authorities rushing teachers to complete the curriculum at the expense of quality education. The reasoning-based instruction puts emphasis on deep conceptual understanding and discourages a practice of covering too many superficially taught topics in a given time unit.

### 4.6.3 Social structures and practices

CDA deals with social structures and practices that are perceived as fundamental causes and consequences of unacceptable situations within societies and institutions (Mogashoa 2014:105). It likens PAR in that it is problem-oriented and aims at supporting the oppressed to resist and transform their lives (Baum 2016:405; Wodak 2009:13). It, therefore, focuses on analysing structures of power and dominance, production and justification of inequality embodied within text and talk (Van Dijk 2015:259). This study perceives the approaches for teaching and learning algebra as inefficient and discursive in that they are inconsiderate (ignorant) about the (abstract) texts used for instruction. The teaching and learning approaches are better understood when looked at in relation to learners' social issues and, language and type of texts used by learners (Mogashoa 2014:105). The practice facilitates effective communication between teachers and learners. Mogashoa (2014:107) adds that the practice expands teachers' personal horizons and help them realise their shortcomings. In other words, CDA invokes the principles of bricolage in teachers. According to Luitel & Taylor (2011:9) bricoleurs emulated by teachers, are humble and less ostensible about argumentative issues. They are open to self-evaluation, self-critique and acceptance of self-weaknesses. CDA is, therefore, an important and necessary tool to shape a culture of (striving towards) conceptual understanding through pedagogical openness whereby learners' different learning styles, lived experiences and common language are brought into the class to simplify the complexity of algebraic instruction.

## 4.7 CONCLUSION

This chapter discussed the methodological theory underpinning the study, and research design. The relevance of methodology, PAR, manifested under expositions of its history, objectives, ontology and epistemology. In summary, the relevance of PAR is best clarified in the following definition:

> PAR is a post-positivist self-reflective inquiry in which the researchers and researched collaborate in planning, action and reflection to understand the practices of the researched (Openjuru et al. 2015:219; Baum 2016:405) and pursue solutions to challenges of their struggle (Park 1993).

The chapter further explained the FAI technique consistency with the principles of PAR and how it guided participants' iterative discussions and interviews and how the empirical research adopted the reflective cycles of PAR to generate quantitative data.

The research design mapped out the order in which the processes of data generation were to unfold in consideration of the study objectives and ethics. It narrated the preliminary procedures and considerations that relate to identification and security of the research field, participants' recruitment and proceedings that led to the conceptualisation of the study and data generation activities. The activities depicted situation analysis and planning, action and reflection in line with the study's PAR orientation.

Furthermore, the chapter discussed three levels of CDA of Van Dijk earmarked for analysing the generated data in Chapter 5. It appreciated CDA's multi-disciplinary nature as a characteristic or qualification rendering it relevant to the study. The analysis is hoped to reveal the instruction review and research protocols that are regarded as fundamental, and to ensure that the people who has the best vantage point to evaluate the problem freely share their views about how reasoning-based instruction could work in their own context (Bennett 2019: 109; Watters & Comeau 2010:6). The reasoning-based instruction envisions legitimation-based transformation (enhancement).

# CHAPTER 5
# PRESENTING, INTERPRETING AND ANALYSING RESEARCH DATA FOR ENHANCING THE TEACHING AND LEARNING OF ALGEBRA IN GRADE 9

## 5.1 INTRODUCTION

This study seeks to enhance the teaching and learning of algebraic expressions and equations using reasoning in grade 9. This chapter serves to present, interpret and analyse the research data generated according to the theories and activities described in Chapter 4. The three levels of critical discourse analysis (see 4.6) will be employed to interpret and analyse data to gain a deeper meaning behind the textual data (written and spoken words) in the context of the study.

When presenting, interpreting and analysing the generated data, reference will be made to the frameworks, theories, policies, legislative imperatives and literature review. The reference helps the analysis to establish the contrasting, correlating and corroborating issues between the generated data, literature and other study underpinnings. The analysis will attempt to follow the same order as the presentation of the study objectives. The word ‘participants’ is used to generalise the research team members and learners. The captions of the textual and contextual formats will strictly limit the focus on the extracts relevant to the subject of discussion.

## 5.2 JUSTIFICATION OF A NEED TO ENHANCE THE TEACHING AND LEARNING OF ALGEBRAIC EXPRESSIONS AND EQUATIONS IN GRADE 9

The analysis of data based on the challenges besetting the teaching and learning of algebraic expressions and equations will give a deeper justification why it is necessary to enhance the teaching and learning of algebraic expressions and equations in grade 9.

### 5.2.1 Alignment between the instruction and curriculum policy

The challenge of glaring non-alignment between the curriculum policy and classroom instruction manifests under procedure-oriented instruction, assessment, teachers' competence and curriculum-time contestation.

#### *5.2.1.1 Procedure-oriented instruction*

Procedure-oriented instruction focuses on symbolic and algorithmic procedures (Matsolo 2006:v) of simplifying algebraic expressions and solving mathematical equations without necessarily explaining the procedures (Pierce & Stacey 2007:12). It is limited in giving connecting reasons for the steps and procedures undertaken when simplifying algebraic expressions and solving mathematical equations. The limitation deprives learners of conceptual learning and limits their reasoning capabilities. During the conceptualisation workshop and situation analysis process, the research team confirmed that more often than not, teachers use the procedure-oriented instruction in classrooms. The confirmation was based on their own practice and that of their colleagues. Meanwhile, the curriculum policy impresses upon developing conceptual understanding and logical reasoning (see 3.2.1.1). The conversation below depicts the situation:

> **Mama** (retired teacher asking Shana, one of the practising grade 9 mathematics teacher whose learners' classwork books were analysed): Why is it that your learners' books did not show comments or reasons next to the steps they undertook when simplifying expressions and when solving mathematical equations? I thought that would have assisted us to identify learners' problems with more certainty.
>
> **Nono** (in support, added): It is not only in his case, teachers generally do not teach learners to state reasons next to steps as suggested (by Mama). I have observed it for some time when moderating learners' books. Mfetho tried it (showing reasons next to steps) with his previous classes, but I did not see it this year.
>
> **Shana** (responding to the question asked): I cannot remember providing or asking learners to provide reasons or computational connections when simplifying algebraic expressions and solving mathematical equations, it is always about following the procedures as taught.

The conversation confirmed that the teaching and learning of algebraic expressions and equations have always concentrated on procedure-oriented instruction rather than seeking conceptual connections and interrelations underlying procedural steps. Shana corroborated Nono's observation that the instruction excluded the explanation of procedures and concepts. They also agreed on the limitation of the instruction to low cognitive skills. On the other hand, Mfetho acceded to Nono's observation that they exempted his learners from showing reasons next to the steps because of limited time (see 5.2.1.4). They however reiterated Mama's thought that though the procedure-oriented instruction requires relatively less time than conceptualised instruction, it is limited in assisting us (teachers) identify learners' problems with more certainty. Westaby (2005:97) argues that reasoning influences and defends one's (learner's) actions and intentions and helps the reader (teacher) predict the intention and perhaps the attitude or behaviour (level of understanding) of the writer (learner).

When interviewed about the reasoning-based instruction, learners also confirmed the domination or excess use of procedure-oriented instruction:

> **Learner 1**: It is for my first time to experience emphasis on reasons when learning algebra; we are used to reproducing steps similar to those given by the teacher in examples.

> **Learner 2**: This new approach by which we are encouraged to connect the procedural steps with reasons helps us to understand and remember what we learn.

The phrases "first time" and "new approach" within the learners' comments confirmed the use of procedure-oriented instruction as a norm in the previous grades and lessons. The learners' classwork books and baseline scripts' analysis confirmed an instruction that leads to learners "reproducing steps similar to those given by the teacher in examples." The discursiveness (disempowerment) of the elsewhere traditional teaching (Osborne 2021:2) was reflected (exposed) in the baseline assessment whereby most learners were found wanting and struggling with the introductory algebraic concepts learnt in grade 8. The script error analysis conducted by the team proved that although learners could hardly respond well to questions in all cognitive levels (Annexure P), the problem with high cognitive level questions (that test for conceptual understanding) was most severe. There were no signs of learners applying habits of reasoning namely, analysing a problem, implementing a strategy, seeking and using connections and reflecting on a solution (see 2.3.3.3) in their operational steps. Neither were signs on the scrap papers too. For example, in the question: 8.3 Solve the equation $7^{x+1} = 1$, learners failed to analyse that the solution could be reached if both sides of the equation were exponential expressions bearing the same base to allow for *fair* comparison of exponents. As a result, they could not implement the basic knowledge (laws of exponents) to construct the conceptual connection, $1 = 7^0$. The connection (substitution) would re-construct an equation into $7^{x+1} = 7^0$, mathematically prompting them to solve a linear equation $x + 1 =$ 0. The equation derives from the comparison of equivalent powers; that, if bases on each side are equal, so should the exponents. Finally, the reflection (substitution) of the solution $x = -1$ would verify that $7^{x+1} = 7^0 = 1$.

The high level of discrepancies (indicated by error percentages) revealed the short-lived characteristic of procedure-oriented instruction. It also revealed that learners were not able to relate and adapt the procedural knowledge to appropriate simplification of expressions and solving of mathematics equations. The analysis of responses exposed learners' reliance on rote learning and memorisation (Little 2009:3–4; Muchoko et. al 2019:1), associated with procedural knowledge. The procedural knowledge lacks chronological connection and conceptual interrelation (Star 2005:407).

The phrase "following the procedures as taught" by Shana attested to the traces of positivism moments beliefs and practices. It confirmed the teacher-centred approach whereby teachers consider themselves as sole transmitters of knowledge (Major & Mangope 2012:146). It depicted the teachers' usurpation of power and control to offer algebraic expressions and equations lessons with little or no room for learners' critique (Tlali 2013:166). It can further be argued from the phrase that the instruction assumed it normal to treat learners as objects that could be controlled and regulated (McGregor & Murnane 2010:314) to learn strictly according to the teacher's method(s). The status quo is further evidenced in learner one's concurrence to "reproducing steps similar to those given by the teacher in examples". This, to the disregard of learners' diverse learning styles and deprivation of the learning environment in which multiple perspectives and diverse experiences are considered. The consideration enhances learning. The practice also deprives teachers an opportunity to improve their pedagogical practice relating to dealing with different learning styles and handling multiple perspectives as informed by diverse experiences. As a bricoleur, the teacher should use what is available, learners' different learning styles, multiple perspectives and experiences to improve the teaching and learning of algebra. The teacher-centred approach canvassed by Shana is denounced by the transformative principles of bricolage. The principles detest the single-handed and fixed epistemological power in all forms (Denzin & Lincoln 2005:3).

On the other hand, it can be deduced that learner two's comment intricately highlighted the challenges or shortcomings of procedure-oriented instruction. That it does not embrace conceptual understanding (knowledge) in which it must be nested (Long 2005:61). Thus, the reliance on procedure-oriented instruction deprives learners an opportunity to exercise their reasoning powers (Major & Mangope (2012:144). Hence, an access to high order cognition. It deactivates or freezes a display of learners' different learning styles. It also deprives teachers an opportunity to identify learners' problems with more certainty. Hence, does not align with the curriculum policy.

#### *5.2.1.2 Assessment*

Assessment is a continuous process that runs with the instruction to collect information that can help in the development of learners and improve the process of learning and teaching (DBE 2011:154). It is best practised when questions are strategically spread across different cognitive levels (DBE 2011:154). The practice allows learners to acquire conceptual understanding (DBE 2011:8; Pramesti & Retnawati 2019:3) and computational skills at all levels. The assessment feedback is used to discuss the learning progress with learners, to enhance their learning and experience (DBE 2011:153–154), to identify sources of errors and misconceptions (Luneta & Makonye 2010:44; Pramesti & Retnawati 2019:7,8) and to guide the subsequent lessons. That is, assessment offers learners an extended opportunity to re-learn. However, during the conceptualisation workshop and situation analysis process, the research team alluded to the fact that the classroom assessment leans more towards procedure-oriented instruction. This was summed up in Nono's response to the question(s) "to what extent does our classrooms' assessment assist learners to acquire or reinforce algebraic conceptual understanding? Do we ask questions that help them to demonstrate understanding?" asked by Mfetho.

> **Nono**: Very little. Generally, we do not. We ask questions that are consistent with the level or scope of our instruction. Most of the time we rely solely on textbook questions, most of which are procedural and drilling. We seldom probe for reasons if at all.

The phrase by Nono "seldom probe for reasons" suggests that teachers (we) rarely ask questions that prompt learners to demonstrate conceptual understanding. Similarly, the phrase "consistent with the scope of our instruction" means that the assessment did not extend beyond testing procedural knowledge and where it did, it was "very little." These views are corroborated by the data presented in the script error analysis (Annexure P) that portrays the learners' common misconceptions and errors associated with the procedure-oriented instruction (see 5.2.1.1). The learners' classwork books also presented no evidence that their conceptual misunderstanding of algebraic expressions and equations was taken cognisance of, in terms of developing and improving the learning process. The cognitive levels at which the questions were pitched did not seem to have been considerate of the assessment standard requirements. The activities that learners had written indicated a significant number of items asked at the same (low) cognitive levels (Major & Mangope 2012:144), giving an impression of drilling other than instilling and nurturing conceptual knowledge.

We further interviewed the learners about the general assessment and performance in algebraic expressions and equations. The interviewees' responses implicated the non-aligned classroom assessment:

> **Learner 2**: We always do well in class works and homework (in algebraic equations and expressions), but fail quarterly common papers (tests) and examinations from the department.
>
> **Learner 3**: We also do well in assignments and investigations… (She laughs) because we help each other.

The responses corroborated the alleged consistency between the classroom assessment and the procedure-oriented instruction in Nono's response "questions consistent with the level or scope of our instruction… most of which are procedural." The analysis associated the procedure-oriented instruction with rote learning and memorisation for regurgitation and classified it as curriculum policy non-compliant (see 5.2.1.1). The corroboration also connected the classwork assessment limitation to regurgitation of procedural knowledge and low cognitive level questions. That way, the assessment deprives the learners an opportunity to develop critical thinking and high order skills (Major & Mangope 2012:144).

Learner three's remark, "because we help each other." preceded by laughter is more informing. When probed further, the learner confirmed that the statement referred to copying amongst learners in the school-based assessment (SBA) namely classwork, homework, assignments, investigation and projects. The SBA is meant among other reasons, to develop learners' knowledge and experience in preparation for the tests and examinations (DBE 2011:155). An investigative checking on sampled school-based assessment work aroused by the remark affirmed copying. Copying adds to the challenges of the cognitively non-aligned instruction and assessment. More often than not, it excludes the majority of learners from the learning process. It can be argued that copying signals a lack of self-efficacy and that it is grounded in procedure-oriented instruction. The epistemic nature of the instruction does not promote bricolage. Bricolage anchors on a principle of 'do it yourself' and hands-on activities.

It turned out that the classroom assessment preparation does not match the level of quarterly common papers (tests) and examinations, set and moderated according to the curriculum policy assessment standards. The latter is set to allow individual learners to demonstrate their full potential (DBE 2011:155). It suffices to analyse that based on the non-aligned and below standard classroom preparation, learners cannot perform well in the standardised papers. It is therefore not surprising that the majority of learners find it very difficult to cope with algebra (ANA Diagnostic Report 2014a:56–59; DBE 2014:9–10, 43) in term (quarterly) assessments. The learners' perceptions (Bauer & Perciful 2009:1) about their performance in algebraic expressions and equations juxtaposed with Nono's perspective of the teachers' "scope of instruction" (Kellner 1999:xii; Rogers 2012:1) provide clarity why learners cannot perform well in the standardised papers. They confirm the inadequacies portrayed in the script error analysis table.

#### *5.2.1.3 Teachers' competencies*

The competencies of a newly qualified teacher should include having sound subject knowledge (DHET 2015:62). A knowledgeable teacher gives hope of aligning the instruction with the curriculum policy and delivering quality education with success (AMESA 2018:2). They should also be able to teach and communicate the instruction of algebra and mathematics effectively to mediate the learning of content comparable to that of other countries (DBE 2011:4). They should be able to select, determine the sequence and pace of content cognisant to both subject and learner needs (DHET 2015:62). They should engage learners into active and critical learning. The engagement helps learners to understand and make sense of what they learn (Matsolo 2006:62; Pierce & Stacey 2007:12). It also helps teachers to know their learners and how they learn (DHET 2015:62). They should be able to assess learners in reliable and varied ways (DHET 2015:62), as well as being able to use the results of assessment to improve teaching and learning (DBE 2011:153). They should subject their practice to critical reflection in conjunction (collaboration) with professional colleagues to keep abreast with evolving circumstances (DHET 2015:62) that surround the teaching and learning of algebra and mathematics in their immediate context.

However, Dickey (1997:7), Thornburg (2009:2) and PLCs lead teachers' reports confirmed the shortage of qualified mathematics teachers. Though the authorities are not specific about the numbers, it is evident that (most) mathematics teachers may not have the required (minimal) competence. They would essentially need support to teach senior phase mathematics, especially algebra (Lempp 2008:abstract) according to the curriculum policy guidelines. Mosia (2016:184) warns about teachers who are still faced with pedagogical difficulties to improve their teaching in a manner that enables learners to access higher-order reasoning (cognition). During the conceptualisation workshop and situation analysis process, the research team engaged on the need for teachers' competence in algebraic instruction:

> **Nono** (Departmental head and former cluster lead teacher): Another major problem I have detected concerns teachers with insufficient algebraic knowledge. These are the teachers I alluded to earlier that their instruction is mostly limited to procedures and adopts the order of textbook presentations since they too cannot expand, integrate or relate the concepts for lack of sufficient knowledge.
>
> **Mfetho** (a cluster lead teacher, in addition): It is true that some mathematics teachers in many schools are not qualified to teach the subject, and that renders them incompetent in many aspects. For example, they limit learners to low cognitive skills activities. Very unfortunately, most of them do not attend PLCs and other workshops for development and support.

> **Mama**: Some teachers do not teach according to the requirements of the curriculum policy. More often than rare, they do not manage to teach all the components of the topics (algebraic expressions and equations). I have since realised the need for teachers' support and development, more especially, for new inexperienced teachers from the training institutions.

The discussions confirmed that some teachers were not conversant with some content of algebra. When probed about the alleged "(incompetency) in many aspects." in his statement, Mfetho explained that other than his observation, the document analysis has also exposed the fact that the teaching and assessment of sub-competent teachers were reliant and limited to procedures guided by the order in which the topics were presented in textbooks, hence often missing on reliability and curriculum content coverage (Mama). The reliability of questioning is determined on a basis of the strategic (varied) spread of questions across the different cognitive levels (DBE 2011:153). The feedback focused on providing learners with correct answers rather than developing and improving their conceptual learning (DBE 2011:153; DHET 2015:62). As a result, they were not able to deliver quality education and to complete the curriculum within the scheduled period if at all. Regarding the strategic instruction, Nono's comment "since they too cannot expand, integrate" related to a typical example that most teachers could not even tap into integrating (connecting) the simplification of algebraic expressions with solving related mathematical equations so that learners could realise and conceptualise the operational similarities and differences on their own. The integration skill draws from bricolage connection formations and conceptual interrelations aimed at producing hybridised content knowledge (Booi and Khuzwayo 2019:2). The skill enables learners to link the simplification of algebraic expressions to solving of mathematical equations. It is renowned for saving time and strengthening instructional effectiveness.

The lack of competence is also evidenced by teachers' incapability to teach and communicate instructions effectively (DHET 2015:62). The effectiveness is measured, among other factors, by the competence to determine the sequence and pace of content cognisant to the subject and learner needs (DHET 2015:62). One such need is to complete the scheduled work effectively in time. Mama's remark "do not manage to teach all the components" corroborated the moderation and inspection reports about incomplete curriculum coverage (instruction). Limiting learners to the procedural orientation of knowledge as alluded to in Nono's remarks proves a lack of competence to engage learners in knowledge generation and tasks that involve critical thinking, reasoning and high order skills (Major & Mangope 2012:145). The limitation is contrary to the requirements of the curriculum policy. It also fails to prepare learners for FET mathematics learning (DBE 2011:10). The limited engagement of learners deprives teachers an opportunity to know how their learners learn and how best they could address their challenges. Further, the unfortunate tendency of not attending the developmental workshops and programmes deprives teachers of the necessary support and opportunity to reflect on their own practice as compared to the experiences of their professional counterparts. They can therefore not improve and adapt to evolving contextual circumstances. As a result, their professional growth, if any, retards.

It can then be concluded that the lack of minimal teacher competence, dysfunctional teachers' professional development and support programmes (see 3.2.2.1(c)), and teachers' inconsistent attendance impede the effective teaching and learning of algebra.

#### *5.2.1.4 Curriculum-time contestation*

The curriculum (of algebraic expressions and equations) gives expression to the knowledge, skills and values worth learning in South African schools (DBE 2011:4). It aims to ensure that learners acquire and apply knowledge and skills in ways that are meaningful to their own lives. CAPS (curriculum policy) is part of the culmination of the citizens' efforts to transform the curriculum bequeathed to them by the apartheid regime (DBE 2011:foreword). It draws from the values of the constitution that include: to improve the quality of life and free the potential of each citizen to transform in a manner suitable to their needs. It, therefore, impresses upon providing quality education inherent in conceptual understanding (Pramesti & Retnawati 2019:3), other than rote and uncritical learning (DBE 2011:4). However the time allocated to teach and learn algebraic expressions and equations content seems not to be sufficient to guarantee the provision of quality education, especially for the learners in the previously disadvantaged schools (Matsolo 2006:1). During the conceptualisation workshop and situation analysis process and learner interviews, the participants unanimously agreed about the glaring curriculum-time contestation (see 3.2.1.1(d)). That, the contestation hampers the provision of quality education:

> **Shana**: Even if I knew about the curriculum policy requirement to use an instructional strategy that strives towards achieving deep conceptual understanding and logical reasoning, the time allocated for teaching and learning algebraic expressions and equations proves not to be enough. It would require relatively more time to teach and learn the curriculum content (of algebraic expressions and equations) in a manner prescribed by the curriculum policy.
> **Nono**: Our learners (from the township schools) will need relatively more time to practice on account of their background knowledge and learning conditions such as overcrowded classrooms.

Learners too, saw it the same, much work within a short space of time:

> **Learner 1** (summarising the class opinion): We need more practice to digest and revise what we have learnt. But we seem not to be getting enough time for it. It is always one topic after the other until we write tests and examinations that we do not even revise after failing dismally.

The participants' comments showed that the time allocated for teaching and learning was not reciprocal to the curriculum content (of algebraic expressions and equations) and that the mismatch made it difficult to provide quality education. In the context of this study and the curriculum policy, quality education is attained when the algebraic knowledge, skills and values are acquired and applied in ways that are meaningful and purposeful to learners (Matsolo 2006:62; Pramesti & Retnawati 2019:3; Tlali 2017:85). Shana's acknowledgement of not always teaching according to the requirements of the curriculum policy and a comment about the allocated time not being enough to complete the curriculum confirmed the non-compliance to providing knowledge, skills and values of algebra in a meaningful and purposeful way. It may be important to recollect and factor in the confirmation in section 5.2.1.1 contexts "following the procedures as taught." by the same participant (co-researcher). The confirmation highlights the relationship between the instruction-policy oversight and curriculum-time contestation. Another analysis based on a relationship between the contexts could lead to the understanding that the procedural teaching and learning were considered and used as a quick fix to complete the curriculum within the scheduled time. It is evident therefore that there were no efforts taken to engage the instructional strategies (approaches) aimed at achieving conceptual knowledge. In addition, Nono became very specific about the background underlying the contestation and resultant quick fix: the contextual challenges besetting the previously disadvantaged schools such as overcrowded classrooms still impede the realisation of the transformed curriculum. As a result, the curriculum does not improve the quality of learning. Neither does it free learners' potential as intended. An instruction offered during a fractured moment resembled that which would be expectedly legitimate during the traditional and modernist qualitative moments. Their argument was further corroborated by the class opinion summarised in learner one's comment that learners were not afforded enough time to practically interact, analyse, question and revise (Katehi et al. 2009:12) what they have learnt on their own for deeper understanding. They could not use the assessment feedback for development and improved learning as recommended (DBE 2011:153; DHET 2015:62). Henceforth, they could not reduce what the learner referred to as dismal failure in algebra.

To this end, the Department of Education has also assented to the curriculum-time contestation (mismatch). It has already implemented amendments on account of insufficient time in the curriculum assessment policy for the years 2020–2022. Meanwhile, the review of the entire curriculum remains in the pipeline since it requires more time (DBE 2019:3). The amendment document came after endless reports to the department about the curriculum-time mismatch and other concerns from the stakeholders (DBE 2019:3). The concerns reiterated the participants' view about the algebraic curriculum-time challenge surpassing teachers' competence of selecting and determining the sequence and pace of content cognisant to both subject and learner needs (DHET 2015:62). The challenge impedes the proper implementation of the curriculum jelled for lives' transformation.

It can therefore be concluded that the curriculum-time mismatch has become an impediment to the realisation of the transformed curriculum, hence to the intended transformation. The transformation was intended, among other things, to engage learners into active and critical learning (of algebraic expressions and equations) projected towards conceptual understanding, other than rote and uncritical (procedural) learning. The latter is implicated as an easy way out of the curriculum-time conflict. It can further be analysed that the Department of Education undertaking to review or heal the conflict in due time implies perpetuation of the unmitigated disaster the teaching and learning of algebra have become amidst the contestation.

### 5.2.2 Algebra as a career sifter rather than an enabler

Algebra is a branch of mathematics that has become more of a career sifter than an enabler. This, is because of poor performance in algebra (ANA Diagnostic Report 2014a:56–59; DBE 2014:9–10, 43) and other topics whose teaching and learning success depends on conceptual understanding and application of algebra. Algebra plays a pivotal role in the teaching and learning of advanced mathematics, natural and social sciences (Hamami 2020:4). The instruction of algebra in the middle schools is targeted to develop learners' critical thinking (AMESA 2018:2; Haas 2003:31; O'Brien n.d.:9) so that they can cope with high order mathematics and science (Luneta & Makonye 2010:42, 44; McNeil et al. 2010:625). It is a fundamental language and gateway for communicating geometry, calculus and carrying out calculations in science and technology (Hamami 2020:4; Matsolo 2006:5). However, the research have confirmed that algebra has become a career sifter to many learners:

> **Nono**: Learners who do well in algebra are those who are having sound basic mathematics competency. They also perform well in other branches of mathematics that apply algebra such as geometry of straight lines.
>
> **Buti**: Yes. But then, those who find it difficult to cope with algebra do not do well in topics like mechanical systems, in technology. They always struggle with conversions from algebraic word problems to symbolic representations and vice versa.
>
> **Mfetho**: Even worse, the grade 9's who do not perform well in it always score low marks in mathematics and end up enrolling in mathematical literacy against their will in grade 10.
>
> **Learner 4**: For a good number of us, algebra is the main reason we are going to follow in the steps of our brothers and sisters who could not be allowed to enrol in pure mathematics in grade 10. We perform better in other topics, but algebra.
>
> **Learner 7**: I would like to be an engineer but the struggle with mathematics, especially algebra, has forced me to consider alternative careers.

The participants' engagements confirmed the pivotal role played by algebra in the teaching and learning of mathematics and related subjects. Pivotal in the sense that it is founded on basic mathematics (see 5.2.4) and it, in turn, form the foundation for advanced learning as shown in Nono's comment. Thus, algebra also reflects on a learner's basic mathematics foundation (knowledge). The learners who are well-founded in basic mathematics cope well with the simplification of algebraic expressions and solving mathematical equations (Matsolo 2006:5). They are also able to apply the knowledge thereto in related disciplines (Hamami 2020:4; Muchoko et. al 2019:1; Pramesti & Retnawati 2019:1; Ying et al. 2020:5406) such as mechanical systems, in technology. The application (of basic knowledge and algebra) could be seen as a way by which the development of learners' critical thinking and high order skills manifest. It also appreciates the communicative role of algebra in mathematics and science.

Conversely, those who for different reasons are not well founded in basic mathematics, finds algebra difficult to cope with (Matsolo 2006:5), and always struggle with the application of algebra in advanced mathematics and related subjects. Buti cited an example of the struggle with the conversion of algebraic word problems to symbolic representations and vice versa, while Nono mentioned the difficulty with analysing, simplifying and solving the expressions and equations borne out of geometry. The struggle with algebra affects the performance in mathematics under its pivotal role in the subject. Low performance leads to the sifting of learners into other careers as many require a pass in pure mathematics (UFS 2019:2). Low performers in grade 9 mathematics are sifted into mathematical literacy stream in grade 10 (see 3.2.1.2). It can then be argued that algebra became a gatekeeper for mathematics-oriented courses and careers (AMESA 2018:2; Muchoko et. al 2019:1). On the other hand, a learner who succeeds in algebra, hence in mathematics, has a wide choice of careers including those in the sciences, health sciences, engineering, commerce and other scarce professions (Osborne 2021:6; UFS 2019:2).

The supremacy of algebra in grade 9 mathematics partly observed in Nono's comment that "Learners who do well in algebra…, also perform well in other branches of mathematics" is magnified in the phrase " those who do not perform well in it always score low marks in mathematics" uttered by Mfetho. Upon probing the remarks closely in respect to the curriculum, the participants realised that, the role of algebra in grade 9 mathematics extends its content weighting beyond the 25% curriculum policy allocation

(DBE 2011:154). This is because its inevitable application in many other topics adds significantly to its own content weighting. It, therefore, plays a significant role in determining learners' promotion and/or progression. It is conditional for learners to obtain at least 40% pass in mathematics to be promoted to grade 10 (DBE 2011:154), and at least a 30% mark to be progressed into the pure mathematics stream in grade 10 (DBE 2014; eNCA 2016).

Under this background, the study discovered a considerable number of learners who, like learner seven, have given up on careers they would have wanted to pursue had it not been for the mathematics-related conditions. Most of them disclosed that they were looking forward to being condoned and recommended for mathematical literacy streams in grade 10 just like those who came before them because of their performance in algebra. The inference is that learners were already noticing the impact of algebra in their failure to obtain at least 30% benchmark for continuing with pure mathematics. Their disclosure was backed by the school subject report confirmation that for the years 2014-2018, more matriculates enrolled in mathematical literacy than in mathematics (DBE 2019:11). The report implied that the majority could not enrol in mathematics-oriented courses in higher learning institutions (Katehi et al. 2009:12; Tall & Razali 1993:2). It is however important to note that the other factor for the dropping numbers of matriculates enrolling in mathematics is for schools to improve their overall pass rates (AMESA 2018:2) at the expense of the scarce careers the country so desperately needs. Learners who drop mathematics for literacy attain high scores in the centralised assessment (Major & Mangope 2012:146).

The research also revealed that the below standard performance in mathematics makes it more of a determining factor for promotion and/or progression than any other subject. However, it is imperative to mention that condonation for progression is also based on the number of years a learner has been in a phase and the learner's age. Be it as it may, the comments around the condition like the one below did not spare the gatekeeping characteristic of grade 9 algebra and mathematics:

> **Learner 9**: Even if you are repeating, the condonation is not automatic. It is dependent on having done all the tasks. I am not surprised that Semonti and Pontjho (not their real names) have dropped out.

Textually analysed, the comment meant that mathematics withheld or caused the learners who would have otherwise been condoned, to drop out because they were somehow found wanting in as far as algebra and/or mathematics tasks were concerned. It can therefore be argued that the instruction devoid of focus towards conceptual knowledge and understanding, fails to develop the critical thinking for learners to cope with algebra. The resultant sifting of the majority of learners out of mathematics-oriented careers has therefore become a complexity calling for immediate attention. This study implores bricolage couching in the simplification of this complexity (Renwick 2014:6). The simplification gives hope about sifting reduction. The details of the components of solution this study has undertaken to address the complexity (algebra-related sifting) will be discussed in section 5.3.2.

### 5.2.3 Abstraction and complexity of algebra

The majority of learners still find algebra abstract and complex (very difficult to cope with (Matsolo 2006:5)). The abstraction and complexity of algebra manifest under technical language and notation, overlapping principles and teaching and learning approaches.

#### *5.2.3.1 Technical language and notation*

The use of algebraic symbolic (technical) language and notation without explanation and conceptual connections subjects learners to text interpretation difficulty (Matsolo 2006:12; Pramesti & Retnawati 2019:1; Sengul 2011:2305; Ying et al. 2020:5405). It limits learners' conceptual understanding. Rumsey & Langrall (2016:415) encourages teachers to provide necessary language support while developing, in this case, the discourse of reasoning-out procedure conceptualisation. This is because technical words have different meanings from the ones used in English language. For example, the meaning of a phrase *like terms* in algebra is different from the single or joint use of the words in English language. The symbolic notation and operations also deviate from those used in arithmetic procedures (Matsolo 2006:v; McNeil et al. 2010:625–626; Fuchs & Fuchs

2005:45). For example, $4x + 3$ cannot be simplified, as opposed to the arithmetic $4 + 3 = 7$. The difficulty (of unclear or no explanation) reduces the teaching and learning of algebra to superficial and procedure-oriented instruction (see 5.2.1.1). During different research engagements, the participants confirmed the use of symbolism without explanation in the teaching and learning of algebraic expressions and equations. This was proven in their responses to the question asked by Mfetho "What makes the teaching and learning of algebra abstract as opposed to other topics in mathematics?

> **Mama**: The teaching of algebra is abstracted by a number of factors, one of which is the lack of concrete examples, referrals or manipulatives to help teachers explain the symbolic notation in an ordinary language.
>
> **Buti** (in agreement): The use of meaningless symbols (to learners) makes it even worse. Most learners struggle with the operation of variables. They operate them like arithmetic numbers.
> **Learner 9**: The language and operation is all imaginary. Algebra is very difficult. It is about unknown $x$-values we cannot even touch or see.
>
> **Learner 7**: I do not know why we are studying $x$-values. Where are we going to apply those in life?

The responses confirmed the abstraction embedded within the symbolism through which algebraic concepts and procedures are communicated. The abstraction brought by "lack of concrete examples" articulated in Mama's response was corroborated in learner nine'' comment that the language and operations of algebra were imaginary (not concretised), hence difficult to access. Buti's point about most learners struggling to cope with the variables resonated with the complication surrounding the transition from arithmetic conventions and operations to algebraic notation, structures and operations (Banerjee & Subramaniam 2011:351). It also reveals that the abstract challenge of depth and sophistication of the variable concept (Leitzel 1989) discovered during the fourth moment still haunts the instruction of algebra in this fractured future moment. Most learners find it difficult to realise the instances of operational difference between arithmetic and algebra. For example, it takes an effort for most learners to understand why $4x + 3$ cannot be simplified further, as opposed to the arithmetic $4 + 3 = 7$ . The common misconceptions learners commit is to treat variables in algebra like numerals, e.g. $4x + 3 = 7x$. Meanwhile, learner seven'' comment was very informative that learners were still finding algebra meaningless and without purpose (see 3.2.1.3), affirming the challenge of algebraic abstraction regarding its technical language and notation.

This study resolves to address the challenge through insistence on providing contextual explanations of symbolic notation, and conceptual connections in-between procedures. The approach infuses sensible meaning (Yackel 2001:1) and purpose within algebraic symbolism (see 5.3.3.1).

#### *5.2.3.2 Overlapping principles*

The operations of algebra involve many different, but overlapping principles and algorithmic procedures. An instruction deficient of explanation and conceptual connection exposes learners to the confusion of structural and operational principles and forgetfulness. The confusion (and forgetfulness) often misdirect learners into operational errors and misconceptions (Banerjee & Subramaniam 2011:353; DBE 2011:130; Luneta & Makonye 2010:44; Pramesti & Retnawati 2019:7,8). The lack of conceptual knowledge backup leaves learners no choice, but to rely on algorithmic procedures. The latter guarantee solutions only if followed in a predetermined order and without error (Long 2005:59). The participants explained the confusion caused by overlapping principles of algebra to the learning process:

> **Mama** (in despair): Learners confuse the expressions with equations. For example, they compute algebraic fractions as if they were equations, and in the process, they destroy the fractional structure of an expression.
>
> **Learner 5** (summing up the majority opinion): We sometimes fail to make a difference between the operations of expressions and equations. I confuse them.

The foregoing responses confirmed the confusion caused by the overlapping principles of algebra. The structural resemblance between the expressions and equations lead to learners confusing the operations. The converse is also true, the operational resemblance between the fractional expressions and fractional equations, for example, lead to confusion of structures.

This study counts on enthusing the ‘do it yourself’ principle (see 2.2.1) together with the principles of PAR (see 4.2) to address the challenge. This will guide learners working as bricoleurs to create their explanations (or constructs) that will empower them to logically argue (Mahlomaholo 2014:173) the difference(s) within the overlapping principles. The principles (of bricolage and PAR) auger well with the constructivists' individualism theory in support of bringing in the learners' different learning styles to simplify the complexity (see 5.3.3.2).

#### *5.2.3.3 Teaching and learning methods*

The teaching and learning methods that subscribe to uncritical and rote learning (DBE 2011:4) used in classrooms account for the unmitigated abstractness of algebra (Major & Mangope 2012:144). Rote learning employs fully compiled methods and approaches that were used since the traditional period (see 2.2.1.1(a)). The approaches pay little or no attention to explaining procedures and forging conceptual connection and interrelationship. Learning concentrates on the algorithmic procedure(s) driven by one person (McGregor & Murnane 2010:424; Moloi 2015:6), a teacher in this case, who unilaterally decides on the procedure the learners have to use with little or no room for learners' critique (Tlali 2013:166). The uncritical and rote learning limits learners' thinking, logical reasoning and disempowers them from understanding and accounting for what they learn. During the conceptualisation workshop and situation analysis process, the participants discussed the current teaching and learning methods in relation to the reasoning-based instruction:

> **Mfetho**: How many of us infuse reasoning in our lessons to explain the algorithmic procedures when teaching algebraic expressions and equations?
>
> **Nono**: None. They (pointing at Shana) were correct when they said they cannot remember providing or asking learners to provide reasons…, our teaching is more procedure-oriented.
>
> **Mama**: We normally use reasoning in a few instances to explain the flow or operation of numbers, not the procedures. The algorithmic procedures ought to be followed as they are (she laughs).

The conversation confirmed that the teaching methods are dominated by the use of procedural algorithms presented solely by teachers against the thought embedded in the reasoning-based approach. The approach stresses on infusing reasoning and/or conceptual connections to explain procedures. Mama confirmed that even in a few instances they (teachers) deployed reasoning, it was for clarifying the flow of numbers, and not necessarily to explain the procedural steps for conceptual understanding. Upon probing Mama's comment about the occasional use of reasoning, it was established that they contextually meant using reasoning to respond to questions like 'how did you get this term or number?' than explaining the development of procedural steps from their conceptual derivatives. Learner one's interview response below corroborated the opinion of the research team about rote learning:

> **Learner 1**: It is for my first time to experience emphasis on reasons when learning algebra; we are used to regurgitating the steps similar to the ones given by the teacher in examples.

In analysis, it can be conclusive that the teaching methods used in the classroom are devoid of the bricolage multi-methodological approach (Rogers 2012:1). This study inscribes the reasoning-based instruction within the multi-methodological and multi-perspectival principles of bricolage for a widened inclusivity in search of a solution (see 5.3.3). Hence, the disapproval of the instructional limitation to procedure-oriented instruction. The instruction also depicts the positivism approaches in which the power and control of teaching and learning rest in one person.

### 5.2.4 Basic mathematics competence

Basic mathematics (arithmetic) competence is a basic foundation upon which the teaching and learning of algebra build (Fuchs & Fuchs 2005:45; Matsolo 2006:5; McNeil et al. 2010:625–626). It is a primary need for using reasoning-based instruction (NCTM 2000) to enhance the teaching and learning of algebra. As a result, insufficient basic mathematics impedes the successful teaching and learning of algebra (see 3.2.1.4). However, the research team confirmed through different discussions that most learners in grade 9 are introduced to algebra without sufficient arithmetic (competence) knowledge (Banerjee & Subramaniam 2011:351; and McNeil et al 2010:625):

> **Mfetho**: Please, let us share the common challenges we face when teaching algebraic expressions and equations in our classrooms.

> **Shana** (responding): Lack of pre-knowledge sir. Learners lack basic mathematics. I always have to remind them about the things they have done in grade 8 and lower classes.

> **Nono** (in agreement): The problem of basic mathematics is very serious and inherent. Most grade 12 learners are still haunted by the problem (lack of basic mathematics), and it affects their performance.
>
> **Mama** (in another meeting): My analysis on the baseline performance has made me realise that the wrong arithmetic operations and applications of basic mathematics in integers, application of distributive laws and laws of exponents, for example, adversely affect the flow of steps even when the learners are conversant with the procedure(s).
>
> **Mfetho** (in another different platform): I have also realised that the excessive use of calculators deprives learners an opportunity to understand and relate to the properties of numbers.

The data proved learners' insufficient competence in basic mathematics. It also disclosed the consequences of introducing algebra amidst insufficient basic mathematics background. The incompetence impedes successful learning. Nono's response corroborated Mama's observation to that effect and went further to show that the consequences are inherent (on going) and always disadvantage learners in assessment tasks. According to Mama's analysis report, many learners who could still recall procedural steps were found wanting in basic mathematics operations. The analysis confirmed Shana's response "lack of pre-knowledge" echoed by all during the conceptualisation workshop and situation analysis process. It can be deduced from the foregoing facts that the challenge brought by the glaring procedure-oriented instruction in the learning of algebra (see 5.2.1.1) could be the continuation of the same approach adopted in the learning of basic mathematics earlier on. The short-lived and superficial arithmetic knowledge displayed by learners in the baseline assessment revealed as such. The knowledge proved not to be reliably accessible to support (or reason out) the learners' solutions to the algebraic expressions and equations. It can therefore be concluded that lack of basic mathematics competence impedes enhanced teaching and learning of algebraic expressions and equations.

The research team also concurred to Mfetho's observation regarding the impact of excessive use of calculators on basic mathematics incompetency. Helpful as calculators are in advanced calculations, their excessive usage overshadows the traditional ways of conceptualising arithmetic fluency at the early stages of development. It deprives learners an opportunity to understand and relate the properties of numbers. In the process of naturally forging numerical connections and interrelations, learners gain an advantage of attaining natural arithmetic competence and develop critical thinking (AMESA 2018:2; DBE 2011:4; Haas 2003:31; and O'Brien n.d.:9). Complementary to the latter, the curriculum policy warns teachers, though seemingly in vain, to guard against excessive use of calculators in mental calculations, but rather use them for big and unwieldy calculations and when checking solutions (DBE 2011:76,119). The team agreed unanimously that it is practically difficult to denounce the use of calculators in the circumstance. It also denounced the Free State Department of Education's provincial competition programme in which the use of calculators is barred to promote the mental mathematics proficiency. Though the competition was commendable, it was not very helpful in this context, because it is confined to a few mathematically gifted learners whose arithmetic skills were better than those of their counterparts. The counterparts that constitute the majority rely on the procedural use of calculators for even basic arithmetic calculations.

To this end, the study will explore the bricolage support for connection formations and alternative routes for improved change (see 2.2.1.2(b)). It will persuade teachers to follow the route in which the introduction of algebraic expressions and equations is strictly preceded by checking and closing arithmetic gaps. It will also discourage the use of calculators in the learning of algebra. The proposed practice hopes to strike a synchronised connection between arithmetic and learning of algebra (see 5.3.4).

## 5.3 POSSIBLE COMPONENTS OF SOLUTION TO ENHANCE THE TEACHING AND LEARNING OF ALGEBRAIC EXPRESSIONS AND EQUATIONS IN GRADE 9

The analysis of data suggestive of the components of solution to enhance the teaching and learning of algebraic expressions and equations in grade 9 will give more insight into the reasoning-based instruction. The components manifest under alignment between the instruction and curriculum policy, turning algebra into a career enabler, abstraction and complexity of algebra and basic mathematics competence.

### 5.3.1 Alignment between the instruction and curriculum policy

The alignment between the curriculum policy and classroom instruction manifests under procedure-oriented instruction, assessment, teachers' competence and curriculum-time contestation.

#### *5.3.1.1 Procedure-oriented instruction*

The instruction that embraces the process of conceptualisation over a procedural approach requires learners to explain the meaning behind the algebraic notation and procedural steps (Pierce & Stacey 2007:12). The explanation helps learners make sense of what they learn (Matsolo 2006:62). It infuses deep conceptual understanding (DBE 2011:8; Pramesti & Retnawati 2019:3) to learning. In this context, the study conceives the conceptual aspect as intrinsic within the procedure. In other words, the procedural knowledge is intricately nested and underpinned by conceptual knowledge (Long 2005:61). The process involves the learner-centred instruction (see 2.2.2) rich in conceptual connections (Star 2005:407). During the conceptualisation workshop and situation analysis process, the research team agreed upon conceptualising algebraic procedures:

> **Shana** (seeking for clarity from Mama): Ma'am, are you suggesting that we (teachers) must always ask learners to give reasons or account for procedural steps?
>
> **Nono** (intervening): Yes. It is important to keep on checking if learners can account for what they say and/or write… you can use expressions like why; give a reason; can you explain that etc., during the lesson and encourage them to tag some brief explanatory texts next to the procedural steps. It helps them understand what they are learning.
>
> **Mfetho** (in addition): Yes. Learners should be able to construct and explain connecting constructs in-between procedural steps to demonstrate deep conceptual understanding required by the curriculum policy.

The responses to Shana's question emphasised the need to conceptualise procedure-oriented instruction. They are particular about explaining procedural steps as a means of demonstrating conceptual understanding (DBE 2011:8). In the context of this study, the explanation(s) constitute(s) learners' reasoning constructs that induce contextual (accessible) meaning to algebraic notations and procedural steps. The constructs can be in a form of concise texts, descriptions or statement re-arrangements. Their focus should be to explain or conceptualise procedures. The initiative draws from the bricolage trait of using the remnants (reasoning) to simplify complexity (2.2.3.1). It also borrows from the trait of intertwining the search for a solution and knowledge production to legitimation (Kincheloe 2004:687).

Under conceptualising procedure-oriented instruction, the research team modified the research lesson plans to infuse the instructional presentation that would require learners to explain the procedures using their own reasoning constructs. The instructional presentation also ensured the flow from known to unknown, from less to more complex and from concrete to abstract content (Katehi et al. 2009:12) for systematic conceptualisation. The conceptualisation relied on learners' own reasoning constructs other than (bare) symbolism and procedures (Matsolo 2006:v; Miller & Koesling 2009:65–66). The construction of reasoning constructs deemed an emancipatory and self-sustainable project that empowered the affected to overcome the challenge posed by procedure-oriented instruction. The project utilised a "do it yourself" principle of bricolage augmented by alternative or underutilised reasoning constructs (see 2.2.1.2(f)). The alternative route brought positive change to classroom activities and captured the learners' interest (Moloi 2015:32). It can further be argued that the approach concretised the instruction using examples and texts learners could easily relate to rather than relying solely on abstract symbols, mnemonics and/or unexplained procedural algorithms. Concretising the abstraction complimented the reasoning connections and interrelations between the algebraic procedural steps.

The success of conceptualisation was proven by the empirical results (Table 5.1; Figure 5.1 & 5.2). The results sustained the truth behind the comment "This new approach by which we are encouraged to connect the procedural steps with reasons helps us to understand and remember what we learn" (see 5.2.1.1). The evidence also resonated with many authors' findings that the procedure-oriented instruction complicates the learning of algebra (Fuchs & Fuchs 2005:45; Matsolo 2006:v; McNeil et al. 2010:625–626). It can then be deduced that the conceptualised instruction undertaken by the study eased accessibility. Furthermore, the analysis has shown that a design of the curriculum policy embodies conceptualisation (DBE 2011:4). Henceforth, it can be concluded that the procedure-oriented instruction does not align with the curriculum policy.

#### *5.3.1.2 Assessment*

Assessment is best practised if it aligns with the curriculum policy. The policy indicates that assessment should be a continuous process that informs the development of learners and improves the process of learning and teaching (see 5.2.1.2). The questions should align with specific requirements for different forms of assessment and be spread strategically across different cognitive levels (DBE 2011:154). The assessment feedback should be used to reinforce the conceptual understanding of the content matter, and to improve the instruction (see 5.2.1.2). During the conceptualisation workshop and situation analysis process, the research team agreed upon aligning the classroom assessment with the requirements of the curriculum policy (DBE 2011:153–157):

> **Mfetho**: Then, how best can we improve on our assessment strategies in a manner that they will enhance conceptual learning, and perhaps help reduce the rate at which our learners are failing the standardised tests and examinations?
> **Mama**: Assessment starts with the instruction. I think we should prepare our learners to answer the tasks at the same level of difficulty as those asked in tests and examinations.
>
> **Nono** (nodding a head in agreement): Yes. We should raise the bar from the classroom assessment. We should ask questions that require learners to demonstrate conceptual understanding on different cognitive levels.

The responses suggested the need to align classroom assessment with the curriculum policy if it were to develop conceptual learning and prepare learners adequately for the standardised tests and examinations. The statement "Assessment starts with the instruction" in Mama's response was more telling. When probed, they vigorously explained that it would not auger well to expect learners to respond to questions asked at the level beyond that at which they have been prepared during classroom instruction. The questions that require learners to demonstrate conceptual understanding first require teachers to have instructed in a manner that conceptualises algebraic notation and procedures (see 5.3.1.2). In other words, the responses acknowledged the cognition mismatch (contestation) between the classroom instruction (see 5.2.1.1) and assessment, and standardised items set and moderated in consideration of the curriculum policy requirements. The emphasis on the need to upgrade the classroom instruction and assessment to meet the assessment policy standards is bricolage. Bricolage is couched in intertwining the search for a solution with legitimation (maintaining standards). The instructional upgrading further implied or cascaded into a need for basic mathematics competence re-check (see 5.2.1.4). Basic mathematics is a primary need for using reasoning to enhance the teaching and learning of algebra (NCTM 2000).

The foregoing background suffices to explain that the expectation to see learners pass the standardised test and examination papers in the prevailing circumstances was farfetched. That is why during the research, the team ensured that the exercise worksheet items (Annexure N) checked applicable basic mathematics before focusing on the simplification of algebraic expressions and solving mathematical equations at different levels. Engaging learners on basic mathematics items presented an opportunity to establish conceptual connections and interrelationships from the start. The competency enriched the algebraic reasoning constructs and guided critical thinking and logical argumentation (Mahlomaholo 2014:173). The questions were developed to grow with the developing instruction, from low to high cognitive skills. This, to ensure unlimited development across the multi-cognitive level tasks (DBE 2011:154). The teachers used the assessment feedback qualitatively (see 3.2.3.1 (b)) to improve the learning and teaching, and to address the sources of errors and misconceptions (see 3.2.2.1 (b)). The benefit of this component of the solution manifested first in meaningful classwork responses. Learners were able to justify their responses (Kollosche 2021:471; O'Brien n.d.:8; Rumsey & Langrall 2016:419). The retention of knowledge also attributable to the component was later evidenced by above-average performance in standardised (summative) assessment. The empirical results of activities done during and after the intervention showed great improvement in performance when compared with those of the baseline. The statistical representations in Table 5.1 and Figures 5.1 and 5.2 illustrate the improvement.

**Table 5.1 Learners' marks in research activities**

RECORDING SHEET GRADE 9

ALGEBRA — GRADE : 9

Algebraic expressions and equations tasks

| | TASK | | baseline | | Ex 1 | | Ex 2 | | Ex 3 | | Ex 4 | | Ex 5 | | Ex 6 | | Ex 7 | | Ex 8 | | Test | | Summative | | overall% | REMARKS |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | DATE | | 7-May | | 9-May | | 13-May | | 15-May | | 20-May | | 27-May | | 28-May | | 29-May | | 19-Jul | | 23-May | | 31-Jul | | | |
| | | | expressions and equation | | ns naming and basic equations | | like terms | | laws and binomial x binomial | | laws of exponents | | out common factors | | the difference of two | | factorising trinomials | | algebraic fractions | | | | expresions and equations | | | |
| No | LEARN | KS CLASS | 66 | % | 22 | % | 34 | % | 33 | % | 40 | % | 22 | % | 28 | % | 20 | % | 17 | % | 40 | % | 24 | % | | |
| 1 | L1 | 9B | 13 | 20 | 15 | 68 | 22 | 65 | 16 | 48 | 25 | 63 | 13 | 59 | 24 | 86 | 16 | 80 | 13 | 76 | **20** | 50 | 15 | 63 | * | |
| 2 | L2 | 9B | 10 | 15 | 19 | 86 | 17 | 50 | 19 | 58 | 13 | 33 | 21 | 95 | 27 | 96 | 17 | 85 | 11 | 65 | **14** | 35 | 13 | 54 | * | |
| 3 | L3 | 9B | 7 | 11 | 17 | 77 | 19 | 56 | 11 | 33 | 27 | 68 | 9 | 41 | 18 | 64 | 13 | 65 | 12 | 71 | **13** | 33 | 14 | 58 | * | |
| 4 | L4 | 9B | 17 | 26 | 19 | 86 | 31 | 91 | 21 | 64 | 25 | 63 | 19 | 86 | 27 | 96 | 20 | 100 | 15 | 88 | **21** | 53 | 22 | 92 | *** | |
| 5 | L5 | 9B | 4 | 6 | 19 | 86 | 23 | 68 | 19 | 58 | 14 | 35 | | | | | 18 | 90 | 8 | 47 | **14** | 35 | 9 | 38 | * | |
| 6 | L6 | 9B | 16 | 24 | 16 | 73 | 16 | 47 | 13 | 39 | 11 | 28 | 11 | 50 | 28 | 100 | 16 | 80 | 4 | 24 | **18** | 45 | 3 | 13 | | |
| 7 | L7 | 9B | 27 | 41 | 19 | 86 | 30 | 88 | 20 | 61 | 29 | 73 | 16 | 73 | 25 | 89 | 17 | 85 | 17 | 100 | **26** | 65 | 18 | 75 | ** | |
| 8 | L8 | 9B | 14 | 21 | 19 | 86 | 17 | 50 | 13 | 39 | 12 | 30 | 4 | 18 | 24 | 86 | 17 | 85 | 9 | 53 | **10** | 25 | 10 | 42 | * | |
| 9 | L9 | 9B | 10 | 15 | 18 | 82 | 13 | 38 | 16 | 48 | 24 | 60 | 22 | 100 | 26 | 93 | 20 | 100 | 15 | 88 | **34** | 85 | 14 | 58 | * | |
| 10 | L10 | 9B | 6 | 9 | 12 | 55 | 11 | 32 | 9 | 27 | 3 | 7.5 | | | 20 | 71 | 14 | 70 | 7 | 41 | **10** | 25 | 9 | 38 | * | |
| 11 | L11 | 9B | 8 | 12 | 20 | 91 | 28 | 82 | 19 | 58 | 17 | 43 | 22 | 100 | 27 | 96 | 17 | 85 | 12 | 71 | **26** | 65 | 15 | 63 | ** | |
| 12 | L12 | 9B | 34 | 52 | 20 | 91 | 29 | 85 | 22 | 67 | 26 | 65 | 21 | 95 | 22 | 79 | 20 | 100 | 17 | 100 | **30** | 75 | 16 | 67 | * | |
| 13 | L13 | 9B | 18 | 27 | 18 | 82 | 22 | 65 | 15 | 45 | 24 | 60 | 15 | 68 | 20 | 71 | 18 | 90 | 13 | 76 | **14** | 35 | 8 | 33 | * | |
| 14 | L14 | 9B | 31 | 47 | 20 | 91 | 24 | 71 | 17 | 52 | 28 | 70 | 16 | 73 | 28 | 100 | 15 | 75 | 16 | 94 | **23** | 58 | 17 | 71 | ** | |
| 15 | L15 | 9B | 4 | 6 | 16 | 73 | 16 | 47 | 12 | 36 | 14 | 35 | 14 | 64 | 24 | 86 | 15 | 75 | 2 | 12 | **16** | 40 | 5 | 21 | * | |
| 16 | L16 | 9B | 10 | 15 | 19 | 86 | 20 | 59 | 9 | 27 | 13 | 33 | 11 | 50 | 27 | 96 | 20 | 100 | 11 | 65 | **11** | 28 | | | | |
| 17 | L17 | 9B | 17 | 26 | 17 | 77 | 25 | 74 | 12 | 36 | 16 | 40 | 14 | 64 | 28 | 100 | 17 | 85 | 14 | 82 | **14** | 35 | 11 | 46 | * | |
| 18 | L18 | 9B | 7 | 11 | 17 | 77 | 25 | 74 | 21 | 64 | 31 | 78 | 16 | 73 | 23 | 82 | 15 | 75 | 11 | 65 | **15** | 38 | 17 | 71 | *** | |
| 19 | L19 | 9B | 4 | 6 | 15 | 68 | 17 | 50 | 11 | 33 | 12 | 30 | 7 | 32 | 19 | 68 | 11 | 55 | 12 | 71 | **18** | 45 | 12 | 50 | ** | |
| 20 | L20 | 9B | 14 | 21 | 21 | 95 | 20 | 59 | 15 | 45 | 30 | 75 | 19 | 86 | 25 | 89 | 19 | 95 | 14 | 82 | **26** | 65 | 13 | 54 | * | |
| 21 | L21 | 9B | 4 | 6 | 18 | 82 | 26 | 76 | 14 | 42 | 21 | 53 | 21 | 95 | 23 | 82 | 10 | 50 | 9 | 53 | **9** | 23 | 13 | 54 | ** | |
| 22 | L22 | 9B | 3 | 5 | 16 | 73 | 17 | 50 | 7 | 21 | 12 | 30 | 7 | 32 | 18 | 64 | 16 | 80 | 9 | 53 | **11** | 28 | 4 | 17 | * | |
| 23 | L23 | 9B | 11 | 17 | 19 | 86 | 22 | 65 | 16 | 48 | 29 | 73 | 17 | 77 | 27 | 96 | 18 | 90 | 16 | 94 | **27** | 68 | 17 | 71 | ** | |
| 24 | L24 | 9C | 25 | 38 | 18 | 82 | 10 | 29 | 17 | 52 | 26 | 65 | 13 | 59 | 17 | 61 | 18 | 90 | 9 | 53 | **19** | 48 | 14 | 58 | * | |
| 25 | L25 | 9C | 10 | 15 | 6 | 27 | 1 | 3 | 22 | 67 | 32 | 80 | 6 | 27 | 26 | 93 | 6 | 30 | 3 | 18 | **6** | 15 | 2 | 8 | | |
| 26 | L26 | 9C | 9 | 14 | 15 | 68 | 7 | 21 | 27 | 82 | 34 | 85 | 17 | 77 | 23 | 82 | 20 | 100 | 6 | 35 | **13** | 33 | 11 | 46 | * | |
| 27 | L27 | 9B | 9 | 14 | 13 | 59 | 27 | 79 | 12 | 36 | 21 | 53 | 22 | 100 | 20 | 71 | 14 | 70 | 14 | 82 | **9** | 23 | 8 | 33 | * | |
| 28 | L28 | 9C | 6 | 9 | 12 | 55 | 8 | 24 | 19 | 58 | 18 | 45 | 17 | 77 | 25 | 89 | 18 | 90 | 5 | 29 | **5** | 13 | 10 | 42 | * | |
| 29 | L29 | 9C | 11 | 17 | 12 | 55 | 2 | 6 | 22 | 67 | 24 | 60 | 18 | 82 | 22 | 79 | 15 | 75 | 3 | 18 | **1** | 3 | 7 | 29 | * | |
| 30 | L30 | 9C | 38 | 58 | 19 | 86 | 24 | 71 | 26 | 79 | 31 | 78 | 19 | 86 | 22 | 79 | 20 | 100 | 13 | 76 | **32** | 80 | 24 | 100 | *** | |
| 31 | L31 | 9C | 33 | 50 | 13 | 59 | 17 | 50 | 30 | 91 | 31 | 78 | 17 | 77 | 27 | 96 | 20 | 100 | 13 | 76 | **30** | 75 | 22 | 92 | *** | |
| 32 | L32 | 9C | 9 | 14 | 13 | 59 | 6 | 18 | 22 | 67 | 24 | 60 | 17 | 77 | 18 | 64 | 17 | 85 | 3 | 18 | **7** | 18 | 15 | 63 | ** | |
| WROTE | | | 32 | | 32 | | 32 | | 32 | | 32 | | 30 | | 31 | | 32 | | 32 | | **64** | | 31 | 32 | | |
| AVERAGE | | | 13 | 20 | 17 | 77 | 19 | 56 | 17 | 52 | 22 | 55 | 15 | 68 | 23 | 82 | 17 | 85 | 11 | 65 | **16** | | 13 | 54 | | |

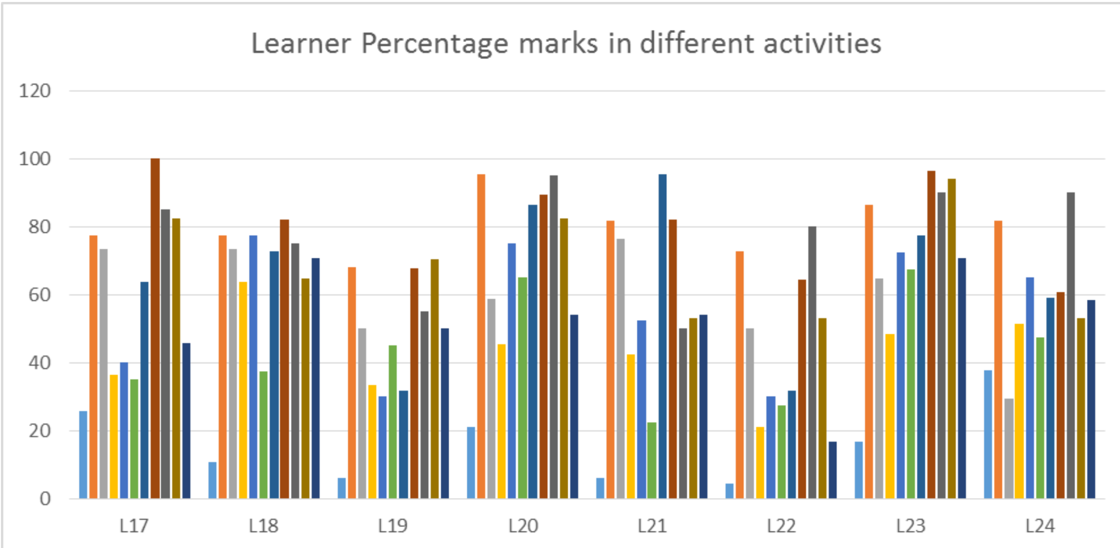
**comparison between baseline and summative**

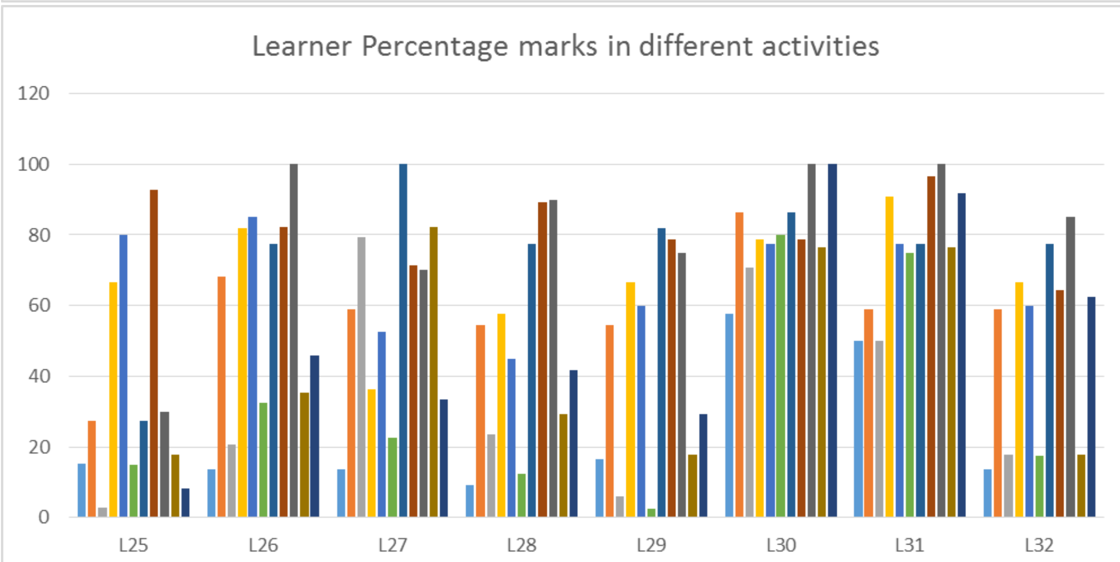
| | |
|---|---|
| * | has improved |
| ** | more improvement |
| *** | much improvement |
| (dark green cell) | no comment |
| | no improvement |

**colour codes**

| | |
|---|---|
| (yellow) | low marks |
| (dark green) | absent |
| (light green) | percent |
| (white) | raw marks |


Figure 5.1 Learners' performance in research activities




| key |
|---|
| baseline |
| expressions naming and basic equations |
| like terms |
| distributive laws and binomial x binomial |
| laws of exponents |
| Test |
| factoring out common factors |
| factorising the difference of two squares |
| factorising trinomials |
| algebraic fractions |
| summative test |

Figure 5.2 Learners' performance in research activities (continued)

It can be deduced from the representations that the use of the reasoning remnant to upgrade the classroom instruction and assessment yielded a lasting conceptual understanding required by the curriculum policy for the majority of learners. As a result, the team concluded that the approach that requires learners to explain or justify their responses (Kollosche 2021:471; O'Brien n.d.:8; Rumsey & Langrall 2016:419) should also be rolled out onto other forms of assessment namely, assignments, investigations and projects, for enhancement of the teaching and learning of algebra, mathematics and related subjects.

#### *5.3.1.3 Teachers' competencies*

This study critically discussed mathematics teachers' competencies (capabilities) espoused during the teaching and learning of algebraic expressions and equations. The capabilities were checked against the benchmarked competencies for newly qualified teachers (see 2.2.1.1(c)). The study applied the bricolage "do-it-yourself" principle to promote teachers' self-researched strategies and skills. The research then found that the current PLCs structure needs to be transformed into a functional school-based programme that borrows from the principles of JLS (see 3.2.2.1(c)). JLS is a successful, renowned and globally recognised teachers' development programme. It is a programme whose model (design) presents teachers with an opportunity to research their own practices with the help of observers (professional colleagues). They reflect on practical (school-based) rather than imaginary classroom activities to effect learner-centred changes. The model creates an environment in which the solution strategies are driven by the learners' participation (sharing, polishing and refining of the instruction) and responses (DHET 2015:62). Further, the school-based PLCs model implies frequent, cost-effective and more sustainable development and support sessions than the current structure (see 3.2.2.1(c)).

In the event where most of the mathematics teachers could not display some of the competencies, the appropriate solution would have been to develop and support the teachers to acquire the competencies. This view was proved in the responses of the research team members to the question(s) "What could be the impact of teachers' competencies towards the non-aligned instruction and assessment? And how can it be addressed?" asked by Mfetho:

> **Mama**: When I was a departmental head, I realised that most teachers needed development and support to acquire competencies worth teaching mathematics in high school, more especially, new inexperienced teachers from the training institutions. Their learners' work was way below the required standard.
>
> **Nono**: Most of them do not finish the curriculum, and they do not teach and assess the high cognitive level skills. They actually need support to pitch at the required competence (and skills).
>
> **Shana**: I have also realised that the practice in a classroom situation reveals more contextual reality than the theorised knowledge teachers acquire in training institutions. Guidance from experienced teachers plays a very important role in developing inexperienced (and newly qualified) teachers.

The responses echoed the component of a solution to develop and support teachers in practical classroom activities to acquire the competence necessary for teaching and learning algebra in consideration of "contextual reality". Further, the team bought into Shana's idea "Guidance from experienced (veteran) teachers plays a very important role in developing us (newly qualified teachers)." They reiterated the fact that school-based collaborations such as the reviewed PLCs model (see 3.2.2.1(c)) presents teachers with an opportunity to research their practices with the help of professional colleagues, who bring bricolage's multi-perspectival aspect (Kellner 1999:xii) into the development. Over and above considering learners' inputs, the PLCs model provides a platform in which teachers from different backgrounds of knowledge work together in a democratic setting (see 2.2.1.2(d)) to develop and support one another to acquire befitting competence. It is worth noting that the development model is framed in a bricolage format of freeing bricoleurs (teachers) to unleash potentials (competence) that would have otherwise remained localised or concealed (see 2.2.1.3). It avails a platform for teachers to share good practices (Annexure A; Eccles 1997:ix; Thornburg 2009:2). The programme can also become a good platform to discuss social issues such as classroom discipline and teacher-learner relationships in context. It is carried out in an environment in which no one feels subjugated, hence it ensures full participation and guarantees optimal results (Moloi 2015:32). It enhances the sequence and pace of activities in mathematics classrooms (Dickey 1997:1), and complements the institutional training skills (Lempp

2008:abstract). It can then be argued that the programme capacitates teachers to deliver quality education (AMESA 2018:2) in respect to subject knowledge, instruction and assessment skills.

The classroom observation reports and empirical results (Table 5.1; Figure 5.1 & 5.2) corroborated learner two's (see 5.2.1.1) appreciation of the instruction and competence displayed by the research teachers. For example, the learners' classwork books and empirical results revealed that the reasoning-based lessons upon which the instruction relied were carried out in a manner that could guide (mediate) learners to construct their own reasoning constructs. It can be concluded based on literature and generated data that teachers' competence complements the reasoning-based instruction in meeting the requirements of the curriculum policy. Hence, the functional PLCs that draw from the features of JLS (see 3.2.2.1(c)) are ideal for harmonising the relationship between teachers' competence, content and instructional strategy (Kolobe & Hobden, 2019:1).

#### *5.3.1.4 Curriculum-time contestation*

The curriculum (of algebraic expressions and equations) requires more time to express knowledge, skills and values worth learning in a manner described in the curriculum policy document. The policy and literature prescribe that the instruction through which learners acquire and apply knowledge, skills and values should make sense and have a purpose (DBE 2011:4; Matsolo 2006:62). During the conceptualisation workshop and situation analysis process, the research team agreed upon the need to consider the glaring curriculum-time contestation in respect to providing the instruction prescribed by the curriculum policy:

> **Mama** (follow-up question to Shana): Are you then suggesting that the time allocated for teaching algebraic expressions and equations is not enough?
>
> **Shana**: Yes Ma'am. That is why I am saying if I have to use the instruction that requires me to allow learners an ample time to construct reasoning connections in order to achieve deep conceptual understanding, then the curriculum content and the inherent annual teaching plan would have to be reviewed. The time allocation does not match the content volume.

> **Nono**: I support them (Shana). I reiterate that the time allocation for meaningful teaching and learning of algebra in grade 9 is simply not enough given the mathematical background of our learners. It needs revision.

The responses of Shana and Nono are corroborating on the suggestion that for the teaching and learning of algebraic expressions and equations to meet the knowledge, skills and values envisaged by the curriculum policy, the glaring curriculum-time contestation has to be resolved. Also, that the solution in this regard, should pay attention to other contextual factors embedded in the contestation. Further, the Department of Education has reiterated the curriculum-time contestation in mathematics and other subjects, hence a need for revision (DBE 2019:3). The team suggested for the revision to consider integrating the first and third terms' curriculum components of algebra to optimise conceptual networking. The team also recommended integrated (concurrent) teaching and learning between simplifying algebraic expressions and solving mathematical equations to mediate meaningful learning (Pramesti & Retnawati 2019:3) of their structural and operational differences. Another foreseeable option to address the contestation could be to revise the alignment of the algebra curriculum across the phases.

The suggestion for curriculum review was done in cognisance of the impact the checking and closing of basic mathematics competence gaps, and the process of conceptualisation have had on time during research. However, the gains reflected the worth of investing in the reasoning-based instruction albeit it consuming relatively more time.

### 5.3.2 Turning algebra into a career enabler

Algebra turns into a career enabler when it is accessible to learners. In the context of this study, learners demonstrate the accessibility to algebraic knowledge if they can explain procedures using their own reasoning constructs. That way, the constructs develop learners' critical thinking skills (AMESA 2018:2; Haas 2003:31; O'Brien n.d.:9). The skills empower them to process algebraic content meaningfully and purposefully. As a result, they become navigators of the proficient network of conceptual knowledge, rich in connections and interrelations. That is, the resultant network of algebraic proficiency becomes part of their common knowledge to which they relate and refer without strain. An effective application of the knowledge in the learning of advanced mathematics and related subjects points to freedom to manoeuvre and explore multiple dimensions (Lynn 2006:19) and results in improved performance. During the conceptualisation workshop and situation analysis process, the research team suggested (agreed on) the means of turning algebra into an enabler for many learners. This was proved in the responses to the question posed by Shana “Then, how can we teach algebraic expressions and equations in a manner that we reverse the challenge of low performance, hence reduce the number of learners enrolling in mathematical literacy in grade 10?

> **Mfetho**: I think the strategy of conceptualising algebraic instruction we talked about earlier can help learners improve their performance not only in algebra but in mathematics as a whole because most of the topics in the subject apply it (algebra).
>
> **Mama**: They (Mfetho) have said the mouthful. Once the learners can understand the flow of algebra through conceptualisation, their performance will improve. Certainly, they will be motivated to enrol for mathematics in grade 10.
>
> **Nono**: I also agree. For the fact that most of the learners aspire mathematics-oriented careers, the numbers can reduce naturally when the instructions like the one proposed in this study replace the feeling of algebraic complexity with an esteemed confidence.

The responses suggested the instruction (approach) of conceptualising procedures (see 5.3.1.1) as a means of turning algebra into an enabler for many learners. The research proved the suggestions true and established that this was because conceptualisation employed the bricolage multi-perspectival approach (see 2.2.1.2(c)) to bring different ideas into the construction of accessible (simplified) knowledge. The learners’ reasoning constructs inherent of their different experiences and learning styles guided knowledge generation within controlled supervision of the teacher (Weegar & Pacis 2012:7). The simplification characteristic upon which bricolage is couched (Kincheloe 2004:687; Renwick 2014:6) manifested within the re-organisation, connection formation and inter-linking of algebraic procedures and concepts to generate conceptual knowledge (see 2.2.1.2(a)). The approach helped learners to make sense of what they were learning (Matsolo 2006:62) and to pay attention. The evidence as to the accessibility of the simplification of algebraic expressions and solving mathematical equations manifested in performance (Table 5.1; Figure 5.1 & 5.2) and successful application of algebra in other topics. The resultant above benchmark performance in mathematics and related subjects motivated the desires to pursue mathematics-based careers. This was proven through informal participants' follow-up interactions where it was discovered that most of the learner participants had enrolled for mathematics in grade 10.

It can therefore be argued that the reasoning-based instruction characterised in simplification and nested in the process of conceptualisation and legitimation, has the potential to enhance the teaching and learning of algebraic expressions and equations. The enhancement manifested in the above benchmark performance in algebra by a majority of the learners and appropriate application of algebra in mathematics and related subjects. An increased number of learners enrolling in mathematics other than literacy in grade 10 highlighted the appreciation of algebra as an enabler rather than a career sifter.

### 5.3.3 Abstraction and complexity of algebra

This study has adopted an approach of conceptualising the algebraic technical language and notation (symbolism). The approach connects algebra as a theory-heavy discipline to real life (Vos 2018:2). It simplifies the abstraction and complexity embedded in the teaching and learning of algebra. It guides learners to create meaningful knowledge that enables accessibility and reduces confusion associated with the overlapping principles of algebra. It enables learners' reasoning constructs to break through the abstraction of algebraic symbolism. Further, the study urged about language support within the conceptualisation discourse (Rumsey & Langrall 2016:415). The approach was suggested by participants in response to the question(s) asked by Buti, "Then how best can we simplify the algebraic technical language and notation, and clear the confusion brought by overlapping principles for learners to access and apply algebra with understanding?"

> **Mfetho**: Simplifying in this context means conceptualising. I reckon if we can make efforts to use concrete or visualised materials to explain symbolism and use examples learners can easily relate to.
>
> **Nono**: I recommend that we involve learners as much as possible in soliciting the simplified explanation. That would help them explain symbolism using the language they understand.
>
> **Mama** (nodding head in agreement): mmh…I agree. The best way to beat abstraction of algebra is to explain the symbolism using learners' leads and constructs. Explaining symbolism this way also clears the confusion brought by overlapping principles.

The responses were in agreement about simplifying (conceptualising) the technical language (symbolism) to clear the confusion inherent in overlapping principles. In this context, conceptualisation involved contextualising and concretising the symbolic notation using learners' reasoning constructs (and leads) and common language. This cleared the confusion of overlapping principles. The processes of contextualisation and concretisation deepened the conceptualisation and subjected learning to the multi-perspectival characteristic of bricolage (see 2.2.1.2(c)). The underutilised reasoning constructs, learners' leads and common language were brought into the classroom in search of simplified and accessible knowledge. Further, necessary language support (Rumsey & Langrall 2016:415) was provided during conceptualisation. The approach that connected the theory-heavy algebra to real life (Vos 2018:2) proved considerate to both the subject (curriculum policy) and diverse learner needs (DHET 2015:62). This was proven, for example, during class debate wherein learners were able to freely express their perceptions, construct viable arguments and critique the reasoning of others (Rumsey & Langrall 2016:413). The debate aligned with the bricolage fractured future moment aspects (see 2.2.1.1(h)). Learners debated, explained and tackled issues surrounding algebraic symbolism in a localised context. As a result, the perceived complexity (and abstraction) manifesting in the use of technical language was replaced with the simplified and accessible content (Star 2005:406). The portrayal of the bricolage trait to simplify complexity (Renwick 2014:6) along the debating session was proven in the assessment thereof where most learners demonstrated conceptual understanding of transitional conventions from arithmetic to algebraic notation, symbols, structures and operations (Table 5.1 – Exercise 1). It can be argued that the framing of learners' reasoning constructs in a form of debate changed the sequence of classroom activities (Dickey 1997:1) and aroused learners' interest and full participation (Moloi 2015:32). Henceforth, learners were empowered to unlock the language-related abstraction and diffuse overlapping confusion on their own.

The encounter corroborated learner two's interview about the use of reasoning-based instruction during data generation that it was commendable for helping learners to understand and remember more of what they were learning (see 5.2.1.1). It magnified the perceptions of the research team about learners' reasoning constructs being conceptual and accessible. The accessibility further manifested in the empirical results (Table 5.1; Figure 5.1 & 5.2) where the performance of learners indicated an improvement both qualitatively and quantitatively during and after implementing the reasoning-based instruction as compared to the performance in the baseline assessment. Furthermore, the analysis based on the two sets of scripts proved the potential of the reasoning-based instruction to break through the algebraic abstraction and complexity. The proof is consistent with Ertmer and Newby's (2013:43) sentiments that own reasoning constructs develop learners to actively navigate through a wide range of conceptualised content matter while keeping very little in mind.

### 5.3.4 Basic mathematics competence

The accuracy of algebraic reasoning constructs depends on basic mathematics competence (see 3.2.2.3). The application of basic mathematics guides reasoning and logical arguments (Mahlomaholo 2014:173; Osborne 2021:6; Rumsey & Langrall 2016:413) during conceptualisation. Conceptualisation of procedures clothes the teaching and learning of algebra with meaning and purpose (Matsolo 2006:62). Therefore, learners who are competent in basic mathematics are likely to cope well with algebra (Banerjee & Subramaniam 2011:351; Fuchs & Fuchs 2005:45; Matsolo 2006:5; McNeil et al. 2010:625–626), and the reasoning-based instruction (NCTM 2000). It was against this backdrop that the research team emphasised the need for learners to develop basic mathematics competency before the introduction of algebraic expressions and equations:

> **Mfetho**: Then how will we ensure that our learners are having sufficient basic mathematics knowledge (pre-knowledge) upon which the teaching and learning of algebraic expressions and equations should build?
>
> **Shana** (responding): Like I have said, we need to check and close pre-knowledge gaps before introducing algebra.
>
> **Nono** (in agreement): I always prepare a few basic mathematics questions upon which the algebraic topic builds and use them as an introductory activity. It helps to build the foundation for correct application.

The research team agreed on checking and closing basic mathematics competence gaps before introducing algebra to build the foundation upon which the teaching and learning of algebraic expressions and equations should build. The closure of gaps constitutes an adaptation for understanding that needs to be levelled with learners' age (Rocha 2018:1) and cognitive development (Ojose 2008:27; Simatwa 2010:368–370; Woolfolk 2013:50–51). In this context, competence included demonstrating the capability to operate and apply basic mathematics in algebra without the help of a calculator. The team held that excessive usage of calculators was leaning towards procedural (see 5.2.1.1) rather than conceptual instruction. The usage was also held responsible for distorting the flow of basic computational fluency that used to come naturally with early mental development. Their argument anchored on the fact that most learners displayed serious arithmetic incapability despite the provision of the curriculum policy to revive the competence thereof before the introduction of algebra. The learning of algebra succeeds a five-week tuition on arithmetic topics encompassing the properties of numbers, multiples and factors, ratio and rate, direct and indirect proportion, properties of integers, common fractions, decimal fractions and exponents (DBE 2011:75–84).

It was against this background that the team capitalised on checking and closing gap sessions to emphasise the benefits of conceptualising how and why numbers operate in a manner they do other than focusing on correct answers. This study embraces the view that though procedures, formulas and calculators can always be available, there has to explanation (reasoning constructs) to check and account for correctness of (procedural) steps (Osborne 2021:6). The correct answers should come as a by-product of the conceptualisation interactions. The emphasis paved way for the conceptualisation of algebra as opposed to a procedure-oriented approach. Henceforth, the research lesson plans adopted the no-calculator approach and used some competition programme items (see 5.2.4) as remnants for assessing arithmetic-based activities. The resultant competency and critical thinking successfully backed up the algebraic reasoning and logical argumentation (Mahlomaholo 2014:173; Rumsey & Langrall 2016:413). It can then be concluded that the reasoning constructs based on competent basics resulted in an ongoing conceptual knowledge, and that limiting the use of calculators improves the natural arithmetic flow associated with the conceptual understanding of basic mathematics.

## 5.4 CONDITIONS FOR A SUCCESSFUL IMPLEMENTATION OF THE REASONING-BASED INSTRUCTION

The analysis of data relating to conditions for a successful implementation of the reasoning-based instruction to enhance the teaching and learning of algebraic expressions and equations in grade 9 will explain the conditions in more depth.

### 5.4.1 Alignment between the instruction and curriculum policy

The alignment between the curriculum policy and classroom instruction manifests under procedure-oriented instruction, assessment, teachers' competence and curriculum-time contestation.

#### *5.4.1.1 Procedure-oriented instruction*

The condition(s) for conceptualising algebraic procedures should ensure (enforce) an instruction that focuses on the explanation of algebraic notation and procedures (see 5.3.1.1). They should discourage teacher-centred instruction focused on symbolic representation of procedures (see 5.2.1.1). In other words, the condition(s) should reinforce strict adherence to the following imperatives underlying the study:

> "based on … Active and critical learning: encouraging **an active and critical approach to learning**, rather than rote and uncritical learning" (DBE 2011:4).
>
> "The teaching and learning of mathematics aims to develop… **deep conceptual understandings** in order to **make sense of mathematics**" (DBE 2011:8).
>
> "To develop essential mathematical skills the learner should… learn to… **think, reason logically and apply the mathematical knowledge gained**" (DBE 2011:9).

Teachers should therefore insist on learners' explanation for procedural steps. The explanation ensures rich conceptual connection and interrelationships. In many of its meetings, the research team discussed the conditions for successful conceptualisation of procedures:

> **Mfetho**: What could be the conditions for ensuring or reinforcing the conceptualisation of procedures in classrooms?
>
> **Mama**: Teachers should encourage learners to explain the procedural steps from time to time. The instruction should guard against, and discourage rote learning.

> **Nono**: I think teachers should be competent enough to guide learners generate conceptual knowledge and to use the assessment feedback for furthering the process of conceptualisation.
>
> **Mfetho**: I think the instruction should also be precise and focused enough (he laughs) to guide and limit learners' explanations and reasons towards the conceptualisation of the subject matter at hand. The assessment tasks should always embrace explanation of procedures.
> **Shana**: Teachers should build a good rapport with their learners and be able to create an environment in which learners can freely and actively participate.

Most responses are for the condition to enforce an instruction that emphasises learners' capability to explain the algebraic notation and procedures as a way of demonstrating a conceptual understanding (Pramesti & Retnawati 2019:3) of what they learn. Learners who are conditioned to this type of learning develop reasoning strategies as a way of addressing simple and complex challenges which translates into high order cognition and dispels the thought of mathematics as a quantity (procedural) subject that has nothing to do with critical issues (Mosia 2016:136). The condition is emphasised in Mama's words "guard against, and discourage rote learning." The other conditions include the recruitment of competent teachers qualified to teach mathematics (AMESA 2018:2). It takes a knowledgeable teacher (DHET 2015:62) to ensure correct mathematical encoding (reasoning constructs) (Weegar & Pacis 2012:7). The teachers' competence to assess and "use the assessment feedback for furthering the process of conceptualisation" by Nono is corroborated in Mama's response "The instruction should *test* learners to explain procedures". Shana's emphasis on the ability "to create an environment in which learners can freely and actively participate." considers the standards (conditions) consistent with the multi-perspectival characteristic of bricolage, and pursuant of "encouraging an active and critical approach to learning*"* requirement of the curriculum policy*.* This research aligned with the conditions and realised successful implementation (conceptualisation) and results (simplified complexity) through the reasoning-based instruction.

#### *5.4.1.2 Assessment*

The condition(s) for the assessment of algebra should guarantee that questions are spread across different cognitive levels, and that feedback is given timeously and used to enhance the learning and teaching (see 5.2.1.2). That way, the feedback should be different from the correction activity in which the focus is to get the correct answer. It should focus on re-learning whereby teachers and learners embark on a conversation to respond to ‘how’ and ‘why’ questions regarding the procedural steps used. In other words, the condition(s) should promote an adherence to instruction and assessment standards of the curriculum policy as proven in the following conversation:

> **Mfetho**: And what could be necessary conditions to reinforce the assessment standard prescribed in the curriculum policy?
>
> **Nono**: Teachers should not rely solely on textbook questions. They should ensure that questions cover all cognitive levels.
>
> **Mama**: I reiterate my comment about preparing learners on tasks at the same level of difficulty as those asked in tests and examinations. Teachers should mark learners’ work in time and use the feedback to re-teach the concepts that need reinforcement. And re-assess need be.
>
> **Mfetho**: I think the assessment should also demand learners’ to integrate and apply daily life experiences. The ability to do so proves conceptualisation skills.

The responses confirmed the condition (competence) of teachers who can assess learners in reliable and varied ways (DHET 2015:62), as well as being able to use the feedback to improve teaching and learning (DBE 2011:153). The condition is strengthened by the bricolage principle wherein a teacher (bricoleur) uses what is available, the feedback in this case, to enhance the understanding. The principle collaborates with PAR’s advocacy for learners’ active involvement (see 4.2.3) In addition, Mama’s comment about the timeous availability of the feedback resonates with Beaumont et al.’s (2011:19) research that the timely feedback affects the learning of algebra and mathematics positively. Mfetho’s response suggested the competence to provide tasks that demand learners to analyse, evaluate and synthesize (Pearson n.d.:1) the rational (reason(s)) behind their daily life experiences. That is, being able to provide tasks that promote integration of mathematics with learners’ socio-cultural structures and activities (AMESA 2018:preamble). Nono’s persistence on multi-cognitive level questions in classroom activities reiterates their view for using questioning “why; give a reason; can you explain” to improve learning and understanding (see 5.3.1.1). It is assumed that the compliance with the said conditions during research is another factor for resultant success.

#### *5.4.1.3 Teachers' competencies*

As a backdrop of many mathematics teachers lacking in some minimum competencies expected of a newly qualified teacher (DHET 2015:62), the condition(s) for successful implementation of solutions dependent on the competencies should guarantee teachers' development and support (see 5.2.1.3) in this regard. To that effect, the condition(s) of active participation in functional (and own self-developing) PLCs and other developmental programmes were unanimously taken on board during the conceptualisation workshop. Also, taken was the regular inspection and moderation support for mathematics teachers. Learners' responses to the interview question "Under which conditions can the solutions work?" were also suggestive and informative about the want and conditions they anticipate in relation to teachers' competencies:

> **Learner 13:** We need teachers who care for us, who understand that some of us are slow and need time to understand. And we do not want a teacher who gets angry when we ask questions.

> **Learner 8:** Teachers should be able to explain themselves in a way that we can understand, and not rush us to regurgitate (reproduce) the procedures we do not understand.

The responses confirmed the need for competent teachers. The phrases of learner thirteen "who care for us …need time to understand" and learner eight "and not rush us" relate to the competence of knowing learners and how each one of them learns (DHET 2015:62). The need for teachers' ability to "explain themselves in a way that we (learners) can understand" rather than being "rush(ed) to reproduce" without understanding, points to the competence of communicating the instruction of algebra effectively to mediate (meaningful) learning (DHET 2015:62) other than memorising (algorithmic) procedures.

There were also positive remarks about the reasoning-based instruction development of assessment. Learners found the chronological approach applied in the approach emphasising refreshing basic (applicable) mathematics before introducing algebra essential, helpful and cognitively productive. The approach subscribes to Katehi et al.'s

(2009:12) view of developing knowledge from known to unknown, from less to more complex and from concrete to abstract content for systematic conceptualisation (see 5.3.1.1).

Meanwhile Mama's comment "I realised that most teachers needed development and support to acquire competencies" (see 5.3.1.1) was suggestive of a condition capable of curing the deficiency. However, the administration of the development and support programmes should anchor on bricolage collaborative initiation of simplified discourses aimed at improving (classroom) instruction (Klages 2012:45; Luneta & Makonye 2010:35) rather than focusing only on the teacher. The PLCs model preferred by this study (see 3.2.2.1(c)) honours the foregoing condition. It insists on the consolidated field (classroom) development rather than developing teachers in isolation of the classroom (practical) environment. The classroom activities and live (learners') responses thereto provide teachers and observers with necessary feedback for qualitative and dynamic development. The model accommodates freely adduced opinions of stakeholders (teachers) from different backgrounds, hence taps from bricolage multiple perspectives (see 2.2.1.2(c)), while maintaining legitimation. It draws from the PAR principles that characterise the collaborative emancipation of the affected through their active involvement and participation in all processes (see 4.2).

#### *5.4.1.4 Curriculum-time contestation*

The conditions for striking a balance between algebraic expressions and equations curriculum and instruction time ought to be time-bound if it is assumed that the volume of the curriculum and instructional policy expectations were to remain unchanged. The balance should guarantee that an average learner from the previously disadvantaged school (see 5.3.1.2) is afforded ample time to acquire and apply knowledge and skills in a meaningful way (Matsolo 2006:62). That is, the policymakers should afford learners sufficient classroom time to cater for conceptual learning and assessment as well as creating opportunities for iterative and purposeful revisions of learnt content (Katehi et al. 2009:12). They should also be cognisant of contextual factors such as the number of learners per classroom, pre-knowledge background (more especially basic mathematics competency) and teachers' competence. It then becomes a condition for most successful implementation of the reasoning-based instruction to place prescribed number of learners in classrooms. Overcrowded classrooms affect time adversely. The other built-in condition that also has time ripple factor involves closing basic mathematics gaps before introducing algebra (see 5.3.4), and integrating algebra curriculum components for optimal instructional benefits (see 5.3.1.4).

### 5.4.2 Turning algebra into a career enabler

The conditions for turning algebra into a career enabler have to ensure learner accessibility to algebra, hence improved performance in it and related disciplines (Hamami 2020:4; Muchoko et. al 2019:1; Pramesti & Retnawati 2019:1; Ying et al. 2020:5406). In the context of this study, learners would easily access the simplification of algebraic expressions and solving of mathematical equations if the underutilised remnants of learners' reasoning constructs and different learning styles were brought to the classroom to simplify the complexity (Renwick 2014:6) of algebra. The condition should therefore be to infuse the multiple-perspectival idea of bricolage and the principle of 'do it yourself' (see 2.2.1) into the process of learning (conceptualising) algebra. The infusion is couched in the principles of PAR (see 4.2) that emphasises the involvement of the affected in search of the solution. This could imply the need to develop and support teachers to acquire the knowledge, skills and values consistent with the use of the reasoning-based instruction in which learners lead the process of conceptualisation (see 5.3.1.1). The conditions of conceptualising algebra include checking and closing basic mathematics gaps before introducing algebra (see 5.3.4). The latter has a ripple effect of increased instruction time (see 5.2.1.4).

### 5.4.3 Abstraction and complexity of algebra

The conditions for simplifying the abstraction and complexity of algebra should promote the explanation of algebraic symbolism through contextualisation. Contextualisation involves using common language and concrete objects that learners can easily relate to. Language support within conceptualisation when necessary is also commendable (see 3.2.2.3). That is, efforts need to be taken to establish accessible meaning and purpose behind algebraic symbolism and its operation in the portrayal of the bricolage characteristic of simplifying complexity (see 2.2.1.2(a)). Since this study largely subscribes to the notion “conceptual first” of the math wars (see 2.3.2.3), learners should be guided to develop procedural algorithms and conjectures from conceptual understanding and not the other way round. In other words, teachers should discourage the deployment of fully compiled methods (rote learning) (see 2.2.1.1(a)). The notion empowers its subscribers to logically argue (Mahlomaholo 2014:173) the usage of symbolism and flow of procedural steps. It screens them from becoming objects (victims) of algorithmic procedures that guarantee solutions only if followed in a predetermined order and without error (Long 2005:59). Teachers should also consider using the integration of algebraic expressions and equations instruction (see 5.3.3), and streamlining reasoning-based instruction for the learning of basic mathematics when closing gaps (see 5.3.4).

### 5.4.4 Basic mathematics competence

In the context of this study, the condition for ascertaining learners’ arithmetic competence before introducing algebra is inspired by the facts that the former is a foundation upon which the latter builds, and a primary need for using reasoning to enhance the teaching and learning of algebra (see 5.2.4). That is why the research team deemed it mandatory necessity to adopt Nono’s strategy of building basic mathematics foundation (see 5.3.4) before the introduction of each algebraic topic. The building had to employ a conceptual rather than a procedural approach. It had to be done without the use of calculators (see 5.3.4). To effect the foregoing conditions, the research team implored bricolage support for the analyses of the assumptions and institutionalised practices that have been ignored to seek and provide alternative perspectives and sources of knowledge (Kincheloe 2004:9). Bricolage provided a cushion or accountability for the assumption that the excessive use of calculators distort the intrinsic flow of basic computational fluency. The fluency used to develop naturally with early mental nurturing. The practice (mental nurturing) has since been underutilised or neglected in the advent of calculators (and programmed gadgets). Gadgets are programmed to follow algorithmic approach. They guarantee solutions only if followed in a predetermined order and without error (Long

2005:59).Teachers therefore need to ensure that activities do not include unwieldy figures that will require learners to use calculators.

## 5.5 POSSIBLE RISKS AND THREATS TOWARDS THEIMPLEMENTATION OF THE REASONING-BASED INSTRUCTION

This section analyses the risks and threats that may have a negative impact on the implementation of the reasoning-based instruction.

### 5.5.1 Alignment between the instruction and curriculum policy

The analysis related to aligning the classroom instruction with the curriculum policy manifests under procedure-oriented instruction, assessment, teachers' competence and curriculum-time contestation.

#### *5.5.1.1 Procedure-oriented instruction*

The risks that are likely to threaten the process of conceptualising algebraic procedures are factors that may undermine the condition to insist on an explanation of symbolic notation and procedures (see 5.3.1.1) using learners' reasoning constructs. The factors include teachers who may still consider themselves as sole transmitters of knowledge (Major & Mangope 2012:146) to the detriment of learners' reasoning power and high order cognition development. The principles of constructivism and PAR support the instructions characterised by learner-centred approaches. The approaches are also couched in bricolage maintenance of legitimation (Desa 2012:727). Further, the resistance to change deprives both the teacher and learners an opportunity to gain from multiple perspectives of learners informed by different backgrounds, experiences and learning styles. The other threat depicts the authorities that insist on completion of curriculum content irrespective of unforeseeable circumstances such as that which revealed through Shana's response "Lack of pre-knowledge sir..." (see 5.2.1.1). The insistence resonates with yet another risk of introducing algebra without ascertaining oneself of learners' basic mathematics competence (see 5.3.4). An interview response below articulates in clear terms how does the risk threaten a failure from the onset:

> **Learner 11:** An attempt to learn algebra without basic mathematics competency is like building the house without foundation. There is simply no way it can sustain nor survive the test of questions demanding conceptual understanding.

Limited or lack of professional development and support (see 5.3.1.3) is yet another threat posing lack of teachers' competence. Lack of competence has a ripple effect in many aspects (see 5.2.1.3). The team's implementation relied on conformity with the factors whose omission could have risked the success. It can then be deduced that the anticipation to overcome possible risks and threats optimised the success rate.

#### *5.5.1.2 Assessment*

Similarly, the risks that threaten the assessment-policy alignment are factors that undermine the dictates of the curriculum policy in respect to assessment (see 5.2.1.2). Mama's comment about the instruction that falls short to prepare learners to answer questions set across the multi-cognitive levels (see 5.3.1.2) implied the teachers who offer such instruction as risks to the alignment. For example, teachers who limit learners to low cognitive level items (Major & Mangope 2012:144), and those who are not competent to assess learners in reliable and varied ways (DHET 2015:62), and to use the feedback to improve teaching and learning (DBE 2011:153). Juxtaposing upon the said observable incompetence among teachers, the research team concluded that the union's directive for its members not to cooperate with the standardised (or common) papers administered by the Free State Department of Education threatens the alignment. It (the directive) suppresses the multi-perspectival enrichment, hence quality to assessment. The team also agreed that the department and unions needed to engage and reach an agreement in this regard. The need for standardised assessment is implied in a learner's response below:

> **Learner 2**: We always do well in class works and homework (in algebraic equations and expressions), but fail quarterly common papers (tests) and examinations from the department.

On the other hand, the analysis depicted that the prescribed SAGM cognitive weightings (DBE 2011:156) did not reciprocate the instruction and perhaps the assessment standards of the curriculum policy. The contradiction between the cognitive weightings and legitimation inherent of deep conceptual understanding, reveals when lower cognitive skills that usually sustain procedural instruction in classrooms as per Shana and learner one's responses (see 5.2.1.1) are allocated assessment weightings way higher than those that require high order cognitive skills (conceptual understanding) (DBE 2011:156). The picture (contradiction) is portrayed in the ratio of knowledge (25%) and routine procedures (45%) totalling 70%, against complex procedures (20%) and problem-solving (10%) totalling 30%, hence the aggregated ratio 70%:30%. The glaring contestation between the assessment weightings and the spirit of the curriculum policy defeats and threatens the assessment-policy alignment. Despite the anomaly within the contestation, the majority of learners has to date failed to meet the minimum requirement of the assessment policy. It could be argued that the perceived inefficient teaching and learning of algebra and other topics cannot even surpass the reduced standard of assessment. The scenario proves the risk of preferring approaches that are not conceptual. They fail to sustain learners' performance even in low cognitive level items.

#### *5.5.1.3 Teachers' competencies*

It is contextual to use the minimum set of competencies required of a newly qualified teacher (see 5.2.1.3) to benchmark the risks and threats related to teachers' competencies. The inaccessibility and/or bunking of professional development programmes, in particular, the JLS model recommended by this study, threaten(s) an opportunity for teachers to reflect their own practices with a help of professional colleagues (DHET 2015:62). Hence, retards their professional growth. There has been enough evidence to prove that the algebraic instruction of sub-competent (undeveloped) teachers is limited (Lempp 2008:abstract), to the detriment of learners. In the case of developed teachers, a threat of mismanagement of time that resulted in resorting to quick-fix (mostly procedural) approaches as opposed to seeking thought-revealing (conceptual) knowledge was detected. It can be argued that teachers who are short of minimal competence and/or do not access development and support translate into liabilities (risks) to the implementation of the reasoning-based instruction. The success of implementation relies summarily on competence of content knowledge and appropriate instructional approach (pedagogy). The approach should be based on learners' needs. It is further implied that lack of regular inspection and timeous moderation for developmental support is yet another risk threatening continued sub-competence and curriculum policy non-compliance.

#### *5.5.1.4 Curriculum-time contestation*

The risks and threats that relate to algebraic expressions and equations curriculum and time allocation revolve around the contestation (see 5.2.1.4). In an event the curriculum content is assumed essential in its entirety which is the case here, we risk either the non-conformity to the prescribed instruction (see 5.3.1.1) or an instruction extending beyond the allocated time, or at the worst, both. The risk scenario was well articulated in Shana's reiteration:

> "… That is why I am saying if I have to use the instruction that requires me to allow learners an ample time to construct reasoning connections in order to achieve deep conceptual understanding, then the curriculum content and the inherent annual teaching plan would have to be reviewed. The time allocation does not match the content."

Nono's support to the reiteration highlighted two other threats related to the contestation namely, inadequate mathematical background associated with learners from the previously disadvantaged schools (Matsolo 2006:1), and overcrowded classes in the schools. Another time-threatening risk was identified in separate sets of algebra curricula. The separation was held responsible for the suboptimal conceptual network.

### 5.5.2 Algebra as a career sifter rather than an enabler

Algebra and its application in related disciplines (Hamami 2020:4; Muchoko et. al 2019:1; Pramesti & Retnawati 2019:1; Ying et al. 2020:5406) threaten to sift than to enable if it is performed unsatisfactorily (see 5.2.2). Any factor or omission liable for unsatisfactory performance is therefore risk threatening. It threatens the efforts embedded in the reasoning-based instruction to ensure that learners access and turn algebra into a career enabler it is (see 5.3.2). An instruction leaning towards procedural rather than the conceptual approach (see 5.2.1.1) risks underperformance (see 5.5.1.2). The approaches that are not conceptual have failed to sustain learners' classroom performance even in less demanding (low cognitive level) items (see 5.5.1.2). The curriculum which does not afford an average learner in the previously disadvantaged school system (see 5.3.1.2) ample time to learn meaningfully (Matsolo 2006:62) and revise (Katehi et al. 2009:12) risks unwarranted underperformance. Nono's comment(s) about the background of these learners (see 5.2.1.4; 5.3.1.4) suggested that they did not get the same support in terms of study time and assistance as their counterparts in urban schools. The other risk is teachers who would not check and close basic mathematics gaps before introducing algebra (see 5.3.4). Also, there are those who would not consider simplifying the curriculum through basic means like re-organising, refocusing and integrating topics for optimal benefits over manageable content (DBE 2020:16). They would often not prioritise an effective (conceptual) instruction and time on algebra despite its supremacy in determining learners' promotion and progression (see 5.2.2).

### 5.5.3 Abstraction and complexity of algebra

The attempts to simplify abstraction and complexity embedded in the teaching and learning of algebra could be threatened by instructional practices that promote technical symbolism and procedures with limited or no explanation. The practices make it difficult for learners to make meaningful sense (Pramesti & Retnawati 2019:3; Yackel 2001:1) of the instruction. An example of such practices could be deduced from Mama's comment about an instruction devoid of concrete examples and the use of technical over common language (and learners' leads or constructs) when explaining symbolic notation and procedures (see 5.2.3.1). Insufficient language support may also threaten the gains of the instruction (see 3.2.4.3). The practice risks leaving learners with little or no alternative but to resort to rote (procedural) learning (Star 2005:407) and regurgitating. Despite learner one's claim that learners are used to regurgitating (see 5.2.3.3), there was more evidence to prove that approaches that are not conceptual have to date failed to surpass even below-standard assessment (see 5.5.1.2). The evidence corroborated learner five's comment about confusing (overlapping) principles (see 5.2.3.2) of algebra amidst the fact that algorithmic procedures guarantee solutions only if followed in a predetermined order and without error (Long 2005:59). It proved that instructions deficient of explanation and conceptual connection expose learners' confusion. Confusion is caused by disempowerment (non-involvement) to understand and account for what they learn (see 5.2.3.3). Rote learning is also experienced when learners are rushed through lessons under the pretext of covering the curriculum within the scheduled time (see 5.2.1.4). Enough evidence has been presented to prove the curriculum-time mismatch (see 5.2.1.4), and that the process of conceptualising algebraic procedures in context requires time (see 5.3.1.1). The inference is that it becomes difficult or impossible for an average learner in the previously disadvantaged school system to conceptually conceive the curriculum within the scheduled time. Nono's recommendation to involve learners as much as possible in soliciting the simplified explanation(s) of symbolism is also time consuming. However, avoiding the recommendation may lead to other threats namely the non-involvement and rushing learners over the curriculum.

### 5.5.4 Basic mathematics competence

Most of the risks and threats about arithmetic competence lie primarily in the assumption that all learners in grade 9 are well-founded for the introduction of algebra (see 5.2.4). Shana and Nono's comments (see 5.2.4; 5.3.4) are suggestive that the introduction without revising basic mathematics applicable to a particular algebraic topic threatens the successful teaching and learning of the topic. The omission may lead to a need for re-teaching (of basic mathematics) that could have otherwise been avoided. The omission of revision makes it difficult for most learners to form necessary transitions from arithmetic to algebraic operations (Fuchs & Fuchs 2005:45; Matsolo 2006:v; McNeil et al. 2010:625–626). In the context of this study, the omission further risks missing a primary need (basic mathematics) for using reasoning (NCTM 2000) to conceptualise algebraic procedures. The validity (accuracy) of learners' reasoning constructs depend on their basic mathematics competency. The competency sustains the validity of reasoning and logical argumentation (Mahlomaholo 2014:173; Osborne 2020:6; Rumsey & Langrall 2016:413). Another common risk lies with reliance on the use of calculators for basic (wieldy) calculations. The usage threatens the natural development of mental fluency through practice and retards critical thinking (see 5.3.4).

## 5.6 INDICATORS OF SUCCESS

This section analyses the indicators of success. It gives a detailed report of the gains of the reasoning-based instruction in enhancing the teaching and learning of algebraic expressions and equations in grade 9.

### 5.6.1 Alignment between the instruction and curriculum policy

The alignment between the curriculum policy and classroom instruction manifests under procedure-oriented instruction, assessment, teachers' competence, and curriculum-time contestation.

#### *5.6.1.1 Procedure-oriented instruction*

The indicators of success in connection to conceptualising procedure-oriented instruction should confirm learners' ability to connect the (unconnected) remnants of learning material to attain enhanced (conceptual) knowledge (see 3.2.5.1) for which they can account. Being able to account demonstrates conceptual understanding. During the reflection meetings, learners' interviews and after the completion of the research, the participants shared several observations indicative of success in relation to conceptualising procedure-oriented instruction. The following report conversation depicts an example of success observed from the lesson that set the tone of the reasoning-based instuction (lesson one):

> **Nono** (reporting about lesson one classroom observation): The lesson went on very well; the presenter (teacher) successfully managed to embrace the use of learners' reasoning constructs in the form of debate to achieve conceptual understanding of expressions' classification.
>
> **Mfetho**: Although the teacher took relatively a longer time to explain the fundamental bases for classifying expressions, at the end, the explanation paid off. It became an investment upon which the learners' expansion (reasoning constructs) relied for conceptualisation. This became evident during the subsequent expressions' classification debate. Further, the feedback I have received from the sampled worksheets reflected mostly conceptualised knowledge than not.

The report about lesson one confirmed the learners' ability to connect bits and pieces of information (including symbolic) to justify their classification(s) of different expressions.

The remark by Mfetho that the ability to explain (account for) reasoning constructs during the debate was further translated to conceptualised individual responses in the worksheets, is indicative of success (Table 5.1 – Exercise 1). The success behind conceptualising procedure-oriented instruction using the reasoning-based instruction was later confirmed in an exclusive interview with learner two:

> **Learner 2**: This new approach by which we are encouraged to connect the procedural steps with reasons helps us to understand and remember more of what we learn.

Further, the test for conceptual knowledge endurance over procedural knowledge was proven by the performance of learners in the summative test (Table 5.1; Figure 5.1 & 5.2). The performance showed noticeable improvement when compared to that of the baseline assessment. The analysis in other subsequent sections will continue to indicate the prospects of success in conceptualising the procedures using the reasoning-based instruction.

#### *5.6.1.2 Assessment*

The measure of success as far as assessment is concerned can be determined by the responses to the question of whether the assessment was able to develop learners, and improve (enhance) the process of learning and teaching (see 5.2.1.2). A remarkable level of development and improvement was detectable from learners' responses to questions of different cognitive levels, in different assessment tasks (Annexure N). Below are participants' (teachers') comments about improved assessment towards the end of research:

> **Mama**: Of late, I have noticed that learners are using many different, but correct approaches. That flexibility in navigation is reflective of conceptual understanding, and development brought by how we use the feedback (assessment for learning).

> **Mfetho**: I have also noticed that learners' reasoning constructs and questions develop (direct) the teaching approach. The focus moves from procedural to achieving conceptual knowledge.

The observations were evident in learners' performance in a summative test. The test was sourced from moderated common papers to assess individual learners' fullest potential (DBE 2011:154). The analysis of scripts reflected qualitative development, hence improved results for the majority of learners as compared to the performance in the baseline assessment (see 5.6.1.2). It confirmed Mama and Nono's views that asking questions that require learners to demonstrate conceptual understanding in classrooms and spreading them strategically over different cognitive levels, prepare (develop) learners to answer questions at the same level of difficulty as those asked in tests and examinations (see 5.3.1.2). It can therefore be concluded that aligning the classroom instruction and assessment with the requirements of the curriculum policy develops learners' critical thinking (see 2.2.1.2(b)). The development was proven in their navigation through a network of concepts to arrive at "different, but correct" responses.

#### *5.6.1.3 Teachers' competencies*

The success in relation to competence of participants, all of which were practising teachers, manifested in designing or weaving new (original) bricolage-based epistemic solutions (discourses) through collaborated and participatory cyclic efforts (acts). The cycle spanned planning, analysing documents and reflecting on them, engaging on focused group discussions, teaching and observing, and reflecting. The efforts portrayed participants' appreciation of lifelong learning principles of social inclusion and personal development (Elfert 2020) in pursuit of knowledge (production) and desire to improve one's own practice. The inclusion indicated the willingness to change ways that have always been used to teach algebra and mathematics (Dickey 1997:4) with minimal success. As a result, the participants emerged as empowered co-researchers (Mahlomaholo 2014:9) capable of mediating critical reflections (DHET 2015:62) on algebraic and mathematical instruction. For instance, the preparation of lesson plans that focus on conceptual connections driven by reasoning artefacts (see 2.3.3) and constructs to refocus; integrate; and/or re-organise (DBE 2020:16) the learning material (content) in a way that the content responds to learners' contextual learning needs (DHET 2015:62). The example indicates one of the successes that emanated from the reasoning-based mediation.

The team (teachers) also realised through the deployment of PAR that learners were useful resources for simplifying algebraic complexities that needed to be developed, rather than problems to be managed (Shinn and Yoshikawa 2008 in Langhout & Thomas 2010:65). That is, the team appreciated the importance (benefits) of knowing learners (DHET 2015:62) and collaborating with them in search of a solution towards meaningful and purposeful knowledge production. As a result, an effective application of the reasoning-based instruction in manoeuvring concepts and procedures for improved understanding displayed by learners in algebra, advanced mathematics and related disciplines (Hamami 2020:4; Muchoko et. al 2019:1; Pramesti & Retnawati 2019:1; Ying et al. 2020:5406) indicated success on teachers' competence to reach out to learners. The competence could be deduced from Mama's response "Learners have adopted reasoning strategy of responding to questions…I have also observed that they incorporate own lived experiences to clarify concepts when discussing approaches to different questions among themselves" (see 5.6.2). It can be argued therefore that one major goal of PAR, to build capacity (competence) within the community involved in research (Grant, Nelson & Mitchell 2008:591), was achieved.

Further, the eventuality of improved performance as a result of the team's (teachers') own designed instructional approach bear enough evidence of a harmonised relationship between the teachers' competence (see 5.2.1.3), content and instructional strategy (Kolobe & Hobden 2019:1). The development of pedagogical competence was confirmed in Shana's comment "Guidance from experienced teachers plays a very important role in developing inexperienced teachers". The research proved them correct not only for inexperienced teachers but also for the entire team. The rigorous engagements to which the team subjected members' reports and experiences to ensure compliance with the research objectives and frameworks were competently enriching.

#### *5.6.1.4 Curriculum-time contestation*

The indicators of success with the curriculum content and time allocation could be determined based on the efforts the research team took to deliver quality (conceptualised) algebraic instruction (education) despite the glaring contestation between the curriculum content and time allocation (Chake & Msomi 2018; DBE 2019:3). Under normal circumstances, the contestation subjects teachers to unfavourable options of having to teach the whole algebraic curriculum content and risk either the non-conformity to the prescribed instruction or an instruction extending beyond the allocated time, or at the worst, both (see 5.5.1.4). But the team considered the algebraic expressions and equations curriculum essentially appropriate in its entirety and opted to refocus, integrate and reorganise (DBE 2020:16) the learning content as well as extending the teacher-learner contact time to afford learners ample time to acquire and apply knowledge and skills in a meaningful way (Matsolo 2006:62). That is, it took the team's sacrifices to forge a harmonised relationship between teachers, content and instructional strategy (Kolobe & Hobden 2019:1), for remarkable success despite the contestation. On the other hand, the team was hopeful that the presentation of this study to the Department of Education will drive a message home for the review of algebra curriculum, in respect of time and topical integration and reorganisation (see 5.5.2). The review would make the content more manageable and accessible (DBE 2020:16).

### 5.6.2 Algebra as a career sifter rather than an enabler

Improved performance in algebra and related disciplines (Hamami 2020:4; Muchoko et. al 2019:1; Pramesti & Retnawati 2019:1; Ying et al. 2020:5406) indicates success in turning algebra into a career enabler. The performance is guaranteed when learners embark on productive learning activities. In this study, the activities were mapped along with using reasoning (constructs) to enhance the teaching and learning of algebraic expressions and equations. The success of the mapping (instruction) was evident in the responses of the team to the question asked by Nono towards the end of the study "What evidence do we have to safely claim that we are succeeding to tame (conceptualise) algebra for prolonged (lasting) benefit of learners?"

> **Mama**: Learners have adopted reasoning strategy of responding to questions. Most of them attach reasoning constructs/texts next to their workings. I have also observed that they incorporate own lived experiences to clarify concepts when discussing approaches to different questions among themselves. The individual work on worksheets confirm the success.
>
> **Mfetho** (in addition): They are now able to connect related concepts when responding even to oral questions during lessons. And the amount of confidence by which they voluntarily respond to questions like why? How? Please explain etc. is convincing. Even

the questions they ask, are hinting conceptual element. The performance in the test also indicated a lot of improvement.

The responses presented evidence of success in utilising the learning patterns (activities) inherent of improved performance. Attaching reasoning constructs to operation and ability to connect related concepts when responding to questions characterised the bricolage connection principle and assured conceptualisation for prolonged (enduring) educational and socio-economic benefit (see 3.2.5.2). The critical thinking expressed in the application of knowledge (see 5.2.2) in the summative test and the resultant performance thereto indicated successful and enduring conceptualisation. The ultimate success was recorded when the majority of the research class (learner participants) enrolled in mathematics in grade 10. The enrolment proved the success reports about improved class performance in related disciplines. The foregoing analysis could also be seen as a response to Shana's question "how can we teach algebraic expressions and equations in a manner that we reverse the challenge of low performance, hence reduce the number of learners enrolling in mathematical literacy in grade 10?" (see 5.3.2). It defuses the impression in learners four and seven's remarks (see 5.2.2) that algebra has become an obstacle for the majority of learners who would like to pursue mathematics-oriented careers. Henceforth, it can be concluded that the strategic usage of the underutilised remnants (reasoning) and connection of scattered bits and pieces of information invoked the learning instruction and activities that succeeded to turn algebra into a career enabler.

### 5.6.3 Abstraction and complexity of algebra

A claim of success in relation to abstraction and complexity of algebra should be based on whether the reasoning-based instruction was able to simplify the technical language and notation (symbolism) and defuse the confusion brought by overlapping principles among other factors. It should reveal that the instruction has overcome the dominant (discursive) practices of power in the creation of simplified knowledge (see 4.5.2). For example, the learner-oriented explanation of algebraic technical terminology in lesson one was per observation report an investment (see 5.6.1.1) upon which the subsequent interpretation of algebraic texts (Matsolo 2006:12; Pramesti & Retnawati 2019:1; Sengul 2011:2305; Ying et al. 2020:5405) and formulation of reasoning constructs relied. The use of correct texts (written or oral) proved a success in relation to the language support in the conceptualisation discourse of reasoning (see 3.2.5.3). The power of learning was relinquished to the learners. The empowerment was evident when they were able to gradually consolidate the explanation with other bits and pieces of related texts (remnants) to create contextual (simplified) and conceptualised knowledge. The main remnant, reasoning constructs, derived from the bricolage (connection) of interpretation of symbolic texts, use of common language and lived experiences (and examples). As a result, the process of conceptualising (concretising and contextualising) symbolic notation and procedures resulted in simplifying the complexities (see 2.2.1.2(a)). It also reduced errors and misconceptions associated with symbolism and overlapping principles. The simplification was detected when the summative test scripts were analysed and compared with those of the baseline assessment. Most of the common errors and misconceptions discussed during the conceptualisation workshop, identified during the situation analysis process and spotted in the baseline assessment (Annexure P) were significantly reduced.

### 5.6.4 Basic mathematics competence

The success in relation to basic mathematics competency in this study derives from the notion of placing it central to the teaching and learning of algebraic expressions and equations. The notion is a product of examining the socio-historical dynamics of the problem (instruction) (Rogers 2012:10) and framing the solution within bricolage. The examination led to a presupposition that learners can achieve conceptual understanding (Pramesti & Retnawati 2019:3) if the instruction is directed, per curriculum policy, towards empowering them to communicate, think, reason logically and apply mathematical knowledge gained (DBE 2011:9). The basic mathematics competency is a primary need (foundation) for using reasoning to enhance the teaching and learning of algebra (NCTM 2000). It became instrumental in communicating transitions from arithmetic to algebraic notation, constructing reasoning connections between procedural steps and mediating critical thinking. For example, the operational interrelation (analogy) between basic mathematics and algebra became instrumental in the multiplication of binomials (see 4.5.3.3). It, therefore, augers well to conclude that the success inherent of basic mathematics have been indicated through critical thinking (see 5.6.2), sustainable reasoning constructs and logical argumentation (Mahlomaholo 2014:173; Rumsey & Langrall 2016:413) learners displayed in various activities. Moreover, the empirical evidence also indicated considerable improvement (Table 5.1; Figures 5.1 and 5.2). Further, the improvement accounted for (indicated) a gain in checking the correctness of the procedural steps (Osborne 2021:6). It can then be deduced that the conceptualised revision of appropriate basic mathematics (see 5.3.4) before introducing algebraic topics succeeded to close arithmetic gaps and bridged some cognition shortcomings.

## 5.7 CONCLUSION

This chapter has dealt with the analysis and interpretation of presented (generated) data in relation to the study underpinnings namely the frameworks, theories, policies, legislative imperatives and literature review. The analysis adopted the three levels of Van Dijk critical discourse analysis (CDA) namely the textual, discursive practice and social structure analysis. The textual analysis of generated data was contrasted, correlated and corroborated with relevant literature to gain a better socio-historical background (Rogers 2012:10) and a deeper understanding of the discursive practices embedded within the teaching and learning of algebraic expressions and equations. It further detailed the social structure implications informed by the practices. Thus revealing the *real meaning* of why teachers and learners held the perception that algebra and algebraic instruction were problematic (abstract and complex). It also hinted at possible routes of solution in the context of the reasoning-based instruction and other underpinnings.

Different sets of textual captions based on study objectives were categorically subjected to an analytic process to obtain better clarity as to the justification of challenges besetting algebraic instruction; possible components of solution, conditions under which the components could work, implementation risks and threats, and the indicators of success.

The next chapter will present the findings and make recommendations.

# CHAPTER 6
# FINDINGS AND RECOMMENDATIONS FOR ENHANCING THE TEACHING AND LEARNING OF ALGEBRA IN GRADE 9

## 6.1 INTRODUCTION

This study sought to enhance the teaching and learning of algebraic expressions and equations using reasoning in grade 9. This chapter gives a summary of the study overview for readers' recollection why this thesis. The overview includes the problem statement, the research question, aim and objectives of the study. The chapter then presents the findings. The findings are guided by data analysis and interpretation in Chapter 5. It (the chapter) further makes recommendations in relation to the findings. The presentation of the findings and recommendations follow the order of the study objectives.

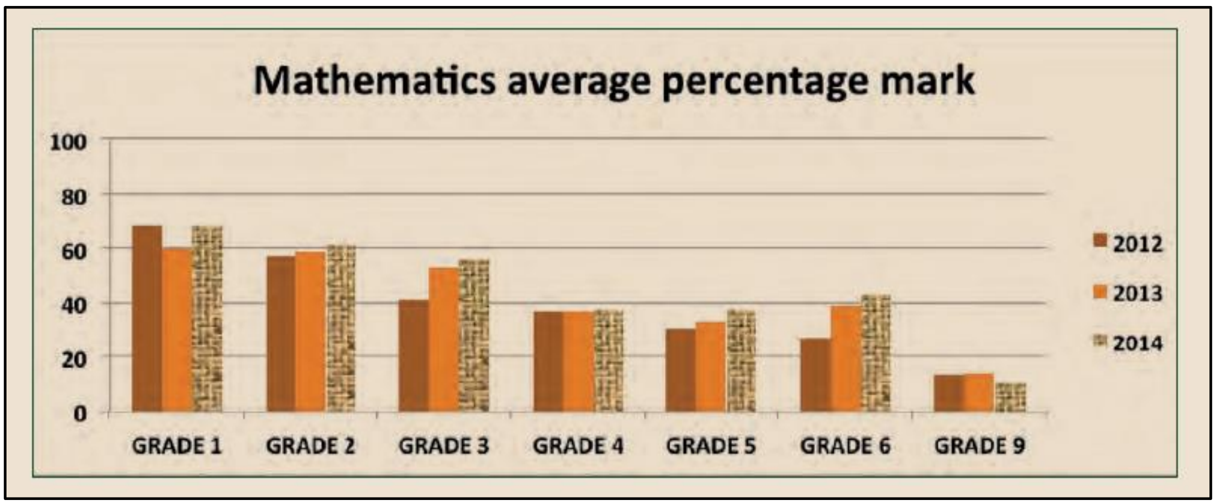
## 6.2 STUDY OVERVIEW

This study has used the reasoning-based instruction (approach) to enhance the teaching and learning of algebraic expressions and equations. The approach, couched in bricolage, involves the use of reasoning constructs to transform the procedural representation of algebra into conceptual knowledge. Learners make sense of, and apply conceptual knowledge in related disciplines (Hamami 2020:4; Muchoko et. al 2019:1; Pramesti & Retnawati 2019:1; Ying et al. 2020:5406) with understanding as required by the curriculum policy. The glaring fragmentation between the procedural and conceptual aspects of knowledge escalated the aspects to operational concepts. Investigations into the dialectic nature of these knowledge aspects (perspectives) indicated that procedural knowledge is nested in conceptual knowledge (Long 2005:61), and the instruction that relies on it without the latter is deemed incomplete and abstract to meaningful learning (Pramesti & Retnawati 2019:3). The implementation of the reasoning-based instruction proved it an alternative approach by which the (stakeholders) clients (Boyle 2012:8) can enhance the teaching and learning of algebra.

The design of the study was motivated by challenges underlying the recurrence of below-average performance in grade 9 algebra and mathematics in South Africa and many other countries across the globe (see 1.2.3). The national statistics of South African learners' performance in ANA in the years 2012–2014 is illustrated in Table 6.1 and Figure 6.1:

**Table 6.2 National average percentage marks for mathematics in 2012–2014**

| GRADE | MATHEMATICS AVERAGE PERCENTAGE MARK | | |
|---|---|---|---|
| | 2012 | 2013 | 2014 |
| 1 | 68 | 60 | 68 |
| 2 | 57 | 59 | 62 |
| 3 | 41 | 53 | 56 |
| 4 | 37 | 37 | 37 |
| 5 | 30 | 33 | 37 |
| 6 | 27 | 39 | 43 |
| 9 | 13 | 14 | 11 |




Source: Adapted from the report of the annual national assessment of 2014

Figure 6.3 National average percentage marks for mathematics in 2012–2014

The representations indicate that the recurrence of deteriorating performance from the intermediate phase (grades 4–6) continued to haunt the system taking peak at grade 9. In grade 9, there was a notable decrease in the pattern between the performance before and during the implementation of CAPS. CAPS is the current curriculum policy statement upon which the analysis and interpretation of the reasoning-based instruction for enhancement are based. The situation (deteriorating or below-standard performance) has compelled the Department of Basic Education to always reduce (or condone) mathematics minimum promotion mark from the standard 40% (level three) to 30% (level two) and below. While the condonation seeks to address the socio-economic issues attached to the prevalence of progression stagnation (see 4.4.1), it imposes conditions that has an adverse influence on the choice of careers. According to promotion circulars governing the condonation, learners who score below 30% should not be allowed to enrol for mathematics in grade 10 (eNCA 2016). The situation was conceived as that which contradicts the Minister's statement in the CAPS preamble. She categorically stated that education and the curriculum have an important role to play in realising the aims of the constitution (DBE 2011:foreword), and particularly in relation to:

- *improving the quality of life of all citizens and freeing the potential of each person*

The efforts that include a series of workshops and PLC programmes seemed not to address the challenges behind the status quo.

The impact of algebra in the teaching, learning and performance in mathematics is prime and determinant. The impact is proven in literature and the curriculum policy (see 3.2.1.2; 4.2.2). It (algebra) plays a pivotal role between basic and high order mathematics. However, the low order instruction inherent in the populist procedural perspective of knowledge continues to limit learners' potential to demonstrate their reasoning power and high order skills. The skills and ability to reason are key to the attainment of deep conceptual knowledge required by the policy. Consequently, the key challenges that necessitated a need for instructional enhancement through this study can be summarised as follows:

- The depth of teaching and learning was not in accordance with the curriculum and policy imperatives in terms of the instruction and content cognitive level. Teachers were not aware of the extent to which they should stretch and pace the instruction. That is, there was no harmonised instructional context between teacher, instruction and content (Kolobe & Hobden 2019:1).

- There was inadequate supervision. The traditional-oriented supervision focused on checking curriculum coverage over the quality thereto;
- Hence, the instruction ran short of the necessary ability to break through the abstraction and complexity of algebra;
- Teachers were not aware of the disintegration between the procedural and conceptual aspects of knowledge. Hence, they could not determine the instructional equilibrium (balance) between them;
- As a result, the instruction was dominated by a thought of mathematics as a quantitative skill or content-specific knowledge subject, not that which promotes critical knowledge;
- Very little, if any concrete effort was taken to capacitate teachers to offer connected and integrated content knowledge (DBE 2020:16).
- The repeated below-average performance resulted in learners' progress stagnation, school dropouts and career deviations and uncertainties.

The success of the reasoning-based instruction was realised (indicated) when:

- Learners were able to freely explain the meaning behind symbolic notation and procedural steps (Pierce & Stacey 2007:12).
- Learners were able to use own reasoning constructs to sustain logical arguments (Kollosche 2021:471; Mahlomaholo 2014:173; Osborne 2021:6; Rumsey & Langrall 2016:413; Westaby 2005:97) and connect procedures to attain high order cognition.
- Learners were able to correlate and apply the knowledge attained (gained) adequately when responding to algebraic questions set at different cognitive levels. The success is obtained in the attainment of ability and skills to connect unconnected information to solve problems (to simplify the algebraic complexity).

- Learners exhibited accurate and sustainable application of the skills and constructs in novel and advanced situations. The application characterises the integrated development of learners' critical thinking and high order cognitive skills.
- Learners developed the metacognitive capacity to critically assess and diagnose errors and misconceptions within their own work and that of others. The success points to the attainment of critical thinking.
- Learners' errors and misconceptions (Pramesti & Retnawati 2019:7-8) reduced significantly. This success is indebted among other factors to learners' basic mathematics competency central to the reasoning-based instruction.
- Learners were able to retain knowledge meaningfully for a longer period.
- Learners' free will participation in logical (mathematical) arguments that require reasoning-out algebraic concepts, increased substantially. This transformation further manifested in improved self-confidence, self-esteem and efficacy.
- Co-researchers became resourceful agents of the reasoning-based instruction for deep conceptual understanding (DBE 2011:8; Pramesti & Retnawati 2019:3) and learning.
- The performance in algebra and related disciplines (Hamami 2020:4; Muchoko et. al 2019:1; Pramesti & Retnawati 2019:1; Ying et al. 2020:5406) improved significantly.

### 6.2.1 Problem statement

The inefficient teaching and learning of algebraic expressions and equations in grade 9.

### 6.2.2 Research question

How can we enhance the teaching and learning of algebraic expressions and equations using reasoning in grade 9?

### 6.2.3 Aim and objectives of the study

The research aimed to enhance the teaching and learning of algebraic expressions and equations using reasoning in grade 9. The research objectives were:

- to justify the need to enhance the teaching and learning of algebraic expressions and equations in grade 9;
- to derive possible components of solution to enhance the teaching and learning of algebraic expressions and equations using reasoning in grade 9;
- to suggest conditions conducive to successful teaching and learning of algebraic expressions and equations using reasoning in grade 9;
- to identify the risks and threats that may hinder the teaching and learning of algebraic expressions and equations using reasoning in grade 9, and suggest ways to prevent them; and
- to provide evidence that shows that the reasoning-based instruction enhances the teaching and learning of algebraic expressions and equations in grade 9.

## 6.3 FINDINGS AND RECOMMENDATIONS

This section presents the findings and makes recommendations based on data analysis and literature. The presentation of findings pays more attention to establishing whether the objectives of the research were achieved or not. It discusses the findings on tested components of solution and makes recommendations guided by the conditions preferred for sustainable and successful implementation. The recommendations are further guided by the risks and threats embedded in the reasoning-based instruction.

### 6.3.1 Alignment between the instruction and curriculum policy

The challenge of non-alignment between the instruction and curriculum policy was sustained on the presupposition that the instruction largely dominated by procedural orientation (Vos 2018:2; Osborne 2021:2) could not achieve deep conceptual understanding (DBE 2011:8; Pramesti & Retnawati 2019:3). The presupposition implied a lack of instructional context harmonisation between the teacher, instruction and content (Kolobe & Hobden 2019:1). The findings of the study about the presupposed non-alignment were based on factors that were assumed to contribute more to the challenge. They are procedure-oriented instruction, assessment, teachers' competence and curriculum-time contestation.

The analysis of data on procedure-oriented instruction (see 5.2.1.1) confirmed the presupposition about the non-alignment between the curriculum policy and instruction. It became evident from participants' contextual experiences and observations that prior to the intervention brought by this study, the instruction concentrated on procedural knowledge underpinned by the traits of traditional period and modernist moments (2.2.1.1(a) and (b)). Hence, the instruction of algebra paid little or no attention to the attainment of learning focused on critical thinking, reasoning, sense-making and conceptual understanding (AMESA 2018:2; Chernoff 2019:73; DBE 2011:8-9; Mosia 2016:135–136; Muchoko et. al 2019:1; NCTM 2000; Pramesti & Retnawati 2019:3; Rumsey & Langrall 2016:413; Star 2005:406; Star & Stylianides 2013:178; Yackel 2001:1). The analysis further confirmed the limitation of the procedural instruction to low order cognition (Major & Mangope 2012:144). The cognition disadvantages learners' development to realise their full potential (DBE 2011:155). The finding resonated with literature about the instruction that falls short of the explanation of the meaning behind procedural steps (Pierce & Stacey 2007:12) and retards learners' development (DBE 2011:155). Furthermore, it was revealed that it is strenuous to recollect procedural knowledge (Sawyer & Alder 2001:1). The strain results from lack of (or limited) chronological connection and conceptual interrelation.

The study proved that assessment cannot surpass the level of instruction. The instruction set at lower level leads to assessment activities set or limited at a lower cognitive level. The analysis of responses in the baseline assessment, for example, confirmed attempts of arbitrary reproducing and regurgitating, and traces of limitation to low order cognition. The confirmation provided a reason for an enormous disparity between the school-based assessment and common papers performance (see 5.2.1.2). Classroom assessment did not prepare learners effectively for common papers (tests and examinations). Common papers are set and moderated according to the curriculum policy assessment standards. Classroom copying had also taken toll adding to the adverse effects of the instruction. The finding augers well with the explanation in literature about the short-lived characteristic of the knowledge acquired procedurally (Vos 2018:2; Osborne 2021:2) and/or through copying (see 5.2.1.2). Copying signals lack of self-efficacy and works against bricolage ‘do it yourself’ principle.

About teachers’ competence, the research has revealed that the system is still hosting a significant number of teachers who need developmental support to teach algebra and mathematics with legitimation (see 5.2.1.3). The finding is supported in literature, which further confirms that the want for teachers’ competence in the teaching of algebra limits the instruction to low-quality education (AMESA 2018:2; Chernoff 2019:76; Mosia 2016:abstract; Pramesti & Retnawati 2019:1). The limited instruction impedes learners’ mathematical development (see 3.2.1.1(c)). The curriculum policy assumes high-quality education pitched at international standard. This, to enable South Africans claim their rightful position in the competitive world of economy. Another striking finding related to the challenge of teachers’ sub-competence is the relatively dysfunctional professional development programmes coupled with attendance bunking thereto (see 3.2.2.1(c); 5.3.1.3). In essence, the finding is insinuating that most teachers are not receiving adequate development and support (see 5.3.1.3) to mediate fruitful instruction of algebra and mathematics.

Furthermore, there is a glaring contestation between the algebra curriculum and allocated instructional time (see 5.2.1.4). This challenge is compounded by the want for appropriate teachers’ competence, the abstraction embedded in algebraic content and the complexity of the instruction thereto. The accumulation of challenges on the pivotal topics necessitates a concerted effort of teachers and learners and sufficient time as espoused within the reasoning-based instruction. Also, the background knowledge and learning conditions for the majority of South African learners in townships call for a curriculum review in terms of instructional time (see 3.2.1.1(d)). The finding aligns with the literature that dispels superficial and shallow teaching of many topics in one year (Star 2005: 407).

The Department of Education is also on record acceding to the contestation and marking it for review (DBE 2019:3).

#### *6.3.1.1 Recommended strategies to align instruction with curriculum policy*

The strategies to address the non-alignment between the instruction and curriculum policy are presented according to factors under which the challenge was investigated namely, the procedure-oriented instruction, assessment, teachers' competence and curriculum-time contestation.

Conceptualisation (or the process of) is the umbrella word or phrase that describes the possible components of solution recommended by this study to enhance the teaching and learning of algebra in grade 9. Further, it augers well as a derivative from the imperative of achieving deep **conceptual** understanding (DBE 2011:8). Conceptualisation addresses the challenges that include procedure-oriented instruction as analysed in section 5.2.1.1 and summarised for findings determination in section 6.3.1. Ideally, conceptualisation in the context of this study involves:

i. concretised and/or contextualised explanation of algebraic symbolic notation and procedures;

ii. connection of procedural steps (knowledge) using reasoning constructs; and

iii. active learners' participation in the construction of reasoning constructs (see 1.4).

Hence, it (conceptualisation) adopts and adapts to the principles of the study underpinnings. It explains the algebraic language (symbolic notation and procedures) using simple or contextual words and examples learners can relate to. It provides the necessary language support within conceptualisation discourse as a key to gaining a meaningful (conceptual) understanding (Pramesti & Retnawati 2019:3) of algebraic (technical) terminology in context. The simplified language opens mutual communication (textual) channels moving forward. Concretising examples where possible is equally beneficial. For example, the following illustration depicts the reason why unlike terms cannot be computed (added) like numerals:

> When you go shopping to buy one packet of foodstuff for each animal, for 2 dogs and 3 cats, mathematically represented as $2\mathrm{d} + 3\mathrm{c}$, you cannot buy 5 packets of dog foodstuff $(5\mathrm{d})$, nor 5 packets of cat foodstuff $(5c)$. Rather you buy 2 packets of dog foodstuff $(2\mathrm{d})$ and 3 packets of cat foodstuff $(3\mathrm{c})$.

The concretisation in the example helps learners to weave or forge conceptual relationships within algebraic concepts and procedures, hence understand the symbolic notation $2d$ and $3c$ and why it is said that they are *unlike terms* that cannot be added together like in the case of the numeric representation (relation) whereby,

> 2 packets of dog foodstuff (2) plus 3 packets of cat foodstuff (3) adds to 5 packets (2+3=5).

Concretising the contrast between algebraic conceptual relationships and numeric representations helped learners to transit from arithmetic to algebraic representations and operations with demonstrable understanding. It provided a basis for the construction of reasoning constructs. Learners' own constructs in the connection of procedural steps indicated understanding and/or highlighted areas that needed attention. Explanations, contextualisation and concretisation levelled the grounds for learners to develop critical thinking, access abstract learning at ease and gain high order cognition.

Primarily the recommended strategies to address the challenges of non-aligned assessment couple with a recommendation to implement an aligned instruction in a manner ascribed to in the foregoing paragraphs. An aligned instruction provides a cushion for recommending the spread of questions across different cognitive levels and a reason to discourage the consumption of more time on drilling lower cognitive level questions (see 5.2.1.2). During the research, the instruction incorporated for the use of activities' feedback to reinforce conceptual understanding. Learners explained how they had arrived at their different answers with an aid of reasoning constructs (written for the whole class to see). Feedback discussion was used, as recommended herein, as an extended opportunity to re-learn, and not as a correct answer providing session. It is therefore recommended based on literature (see 3.2.2.1(b)) and the gains of this study (see 5.3.1.2) that the feedback process should always adopt a two-way dialogue among teachers and learners. It should be, driven by a notion of "assessment as learning" (Evans 2013:82; Rhind 2017:3,5).

Regarding the finding on a want for competence to teach algebra effectively and following the requirements of the curriculum policy, it is recommended that mathematics teachers, as the primary driving force behind any positive classroom activity (Dickey 1997:5; Foster 2008:6), need to be developed and supported (AMESA 2018:2; Chernoff 2019:76; Mosia 2016:abstract; Pramesti & Retnawati 2019:1) through lifelong learning programmes (Elfert 2020) that include PLCs. It is however recommended that the current structure of PLCs needs to be reviewed to liken the JLS format that transforms the practice (instruction) through classroom activity (practical) research. Functional (reviewed) PLCs (see 3.2.2.1(c)) are therefore recommended with conditions of monitored participation, and consistent stakeholders' consultations (reflections) to ensure contextual appropriateness and relevance, and programme sustainability.

There is a glaring contestation between the algebra curriculum and the time allocated for teaching and learning. A large amount of content is expected to be covered within a short space of time irrespective of the abstraction embedded in the content and the complexity of the instruction thereto. The curriculum volume, the abstract nature of algebra and a pivotal role played by algebra warrant provision for sufficient teaching and learning time. It is therefore recommended to focus algebra curriculum (Little's 2009:3,4) by investing slightly more time than allocated to attain deep and enduring conceptual understanding. Also, the background knowledge and learning conditions for the majority of South African learners warrant curriculum review in this regard. The finding aligns with the literature underpinning the focused curriculum (see 3.2.2.1(d)). The focused curriculum system is practised in countries whose mathematics performance trends resonate with the expectations of our (South African) curriculum policy. The Department of Education (DBE 2019:3) has already acceded to revision of the curriculum content in response to the glaring contestation reports.

It is further recommended that the review regarding the contestation between the algebra curriculum and time allocated for teaching and learning should also consider basic mathematics revision incorporated in the reasoning-based instruction. This study has found that though the incorporation has time implication, it is instrumental for the rooted (conceptualised) acquisition of algebraic knowledge. Basic mathematics forms a foundation upon which the learning of algebra build (Banerjee & Subramaniam 2011:351; Chernoff 2019:73; Fuchs & Fuchs 2005:45; Matsolo 2006:5; McNeil et al. 2010:625–626).

#### *6.3.1.2 Recommended conditions for aligning instruction with curriculum policy*

The recommended conditions to ensure an alignment between the instruction and curriculum policy are presented in cognisance of the components of solution inherent of the alignment namely, the procedure-oriented instruction, assessment, teachers' competence and curriculum-time contestation.

The conditions for aligning instruction with the curriculum policy with respect to the procedure-oriented instruction ensure inter alia that learners always demonstrate an understanding of algebraic computations. Being it the simplification of expressions, or solving of mathematical equations introduced in a classroom, teachers should guide learners to give an oral explanation(s) along with the computations and write them in a form of concise notes strategically tagged next to the appropriate step. The example below illustrates the condition:

Solve the equation

$5x + 4 = 2x - 8$ Q: Do we have like terms? L: yes;*

Q: how many groups of like terms? L: 2; **

Q: and they are? L: $5x$ *and* $2x, 4$ *and* $-8$)***

Remember to always ask 'why' question to help slow and/or weak learners catch up - when you know why you do things, it's better!

Step 1 $5x + 4 = 2x - 8$

$-2x \quad -2x$ (additive inverse of $2x$, *integers operation, equation balancing and additive identity*)

Step 2 $3x + 4 = -8$

$-4 \quad -4$ (additive inverse of $+4$, *integers operation, equation balancing and additive identity*)

Step 3 $\frac{3x}{3} = \frac{-12}{3}$ (multiplicative inverse of 3 and multiplicative identity)

$x = -4$

Checking

$5(-4) + 4 = 2(-4) - 8$

$-20 + 4 = -8 - 8$

$-16 = -16$ (LHS=RHS, so we are correct).

Source: Adopted from a research lesson presentation

The practice is more productive if it is done repeatedly in questions of different cognitive levels. That way, learners get to realise and appreciate the diversity of algebraic principles through diversified reasoning constructs per different items. The oral explanation is solicited through pedagogical probing into learners' responses. Computations that are limited to symbolic representation, that is, without connecting constructs, should be regarded as incomplete and be discouraged.

The analysis revealed that the assessment given to learners is not reliable because it is not varied across different cognitive levels (see 5.2.1.2). It is always portrayed as a drilling exercise concentrating on low cognitive level activities (Major & Mangope 2012:144). The practice or portrayal does not help learners develop critical thinking and high order cognitive skills. It is prone to procedural learning effects. It is therefore recommended that questions should be asked strategically across different cognitive levels in a manner that induce conceptual and critical learning. They should also integrate the application of algebra in live experiences to test learners' application skills based on analysing, evaluating and synthesizing (Pearson n.d.:1). It is further recommended on the basis of the finding, analysis and literature that learners should be provided with timely and quality feedback for optimal benefit (see 4.4.1.2; 2.5.3.1(b)). Quality feedback is provided timeously and discussed through explanatory (concise) notes (reasoning constructs) between teachers and learners whereby the former offer controlled (minimal) guidance.

The analysis about the want for teachers' competence to deliver algebraic instruction with legitimation revealed that most teachers rush learners over lessons and assessment activities underpinned by procedural orientation (see 5.4.1.3). The analysis revealed that the practice leaves the majority of (slow) learners who find it difficult to cope with algebra (ANA Diagnostic Report 2014a:56–59; DBE 2014:9–10,43; Matsolo 2006:5) behind. The analysis further showed that there is very little chance that the learners left behind can be offered a second chance or tutorial lessons to catch up, given the scale of the curriculum-time contestation (see 5.4.1.4). This warrants the recommendation for the system to review the current structure of PLC programmes to liken the JLS whereby teachers are developed through practical classroom activities. The practice proved to develop teachers' competence in a manner that the instruction strikes an inclusive balance between the needs of the subject and those of different learners (DHET 2015:62).

It is therefore recommended that the reviewed programme should insist on professionals' collaborated class visits to develop, support and monitor progress in practice (DHET 2015:62). It is further reiterated, as analysed in section 5.4.1.4, that the abstract nature of algebra requires the instruction that invests in quality rather than quantity. The analysis and interpretation of the glaring contestation observed between the curriculum and allocated time during research, amidst the want (condition) for quality education (high cognition), revealed the need to maintain the prescribed number of learners or less in classrooms. The recommendation to maintain the number of learners ensures the balance between quality instruction and time, and optimises the enhancement. The study has further proven that the quality education in this regard is guaranteed if, and only if learners' competency in basic mathematics is cleared before the introduction of algebra as recommended in section 6.3.1.1.

#### *6.3.1.3 Risks and threats inherent in aligning instruction with curriculum policy*

This study has identified teachers' reliance on teacher-centred approaches as a possible risk threatening the alignment of the instruction to the curriculum policy. In addition the analysis in section 5.6 and the summary in section 6.2 showed that the success behind the reasoning-based instruction anchors on learners' hands-on participation, underpinned by constructivism theory (see 2.2.2). It is therefore recommended that learner-oriented reasoning constructs should always be prioritised over excessive teachers' indulgence. Teachers should limit their role to controlled supervision (Weegar & Pacis 2012:7). The reasoning constructs keep learners active and help them assume the responsibility to guide their learning. They also open channels for individualistic learning styles in line with the curriculum policy imperative. The condition to actively involve learners counteracts the tendencies that develop along with teacher-centred instruction (Osborne 2021:2). It reduces learners' dependency on teachers and promotes the "do it yourself" or self-reliant principle couched in bricolage (see 2.2.1). The principle develops learners' skills and confidence towards high order cognition activities.

The teacher-centred approach computations are channelled towards finding answers rather than prompting ‘explained’ responses from learners. The study has revealed that teachers focus towards answers without necessarily explaining the derivatives for such answers. It is therefore recommended based on the finding that teachers’ should be discouraged from relying on activity answers found at the end of most mathematics textbooks. The analysis (see 5.3.1.2) has shown the importance of using assessment feedback to improve and develop a conceptual understanding of what was learnt. The feedback that is limited to providing correct answers only, deprives learners of these benefits.

It is also recommended that the classroom assessment should be maintained at the same level of difficulty as the common papers. The practice keeps up with legitimation and improves performance. The common papers are set and moderated according to the curriculum policy assessment standards. The recommendation draws from the study multi-perspectival underpinning. It counteracts the threat of not cooperating with the standardised (or common) papers.

The study further found that the contestation between assessment weightings and curriculum policy requirement as analysed in section 5.5.1.2 threatens the strength of the reasoning-based instruction to filter learners’ potentials in line with legitimation. SAGM taxonomy puts more weighting on a low cognitive level items against the spirit of the curriculum policy to achieve high standards. Thus, a review to strike a balance is recommended.

The analysis confirmed the finding that the authorities often pressurise teachers to complete the curriculum content within scheduled times (see 5.5.1.1). It is reiterated that teachers should invest in quality over quantity as far as algebra instruction is concerned. The investment executed in line with the requisites of reasoning-based instruction has the potential of easing the recovery of the would-be lost time. The time can be recovered during the teaching and learning of advanced mathematics founded on deep conceptual understanding of algebra. The firm foundation (understanding) of algebra eases and cuts the time short for the learning of advanced mathematics and related disciplines (Hamami

2020:4; Muchoko et. al 2019:1; Pramesti & Retnawati 2019:1; Ying et al. 2020:5406). However, the recommendation for the review of allocated time for teaching algebra to effect meaningful and purposeful learning still holds. It is further recommended that the review should also focus on other contextual factors that have a bearing on the curriculum-time contestation as analysed in section 5.5.1.4. For example, it is recommended to refocus the curriculum to integrate separate components of the algebra curriculum to optimise the conceptual network (see 5.3.1.4).

The study confirmed the finding held in literature that most teachers are sub-competent to teach high school mathematics in a manner prescribed by the curriculum policy, and would need support to cope with the instruction of algebra (Lempp 2008:abstract; Thornburg 2009:2). Functional (reviewed) PLCs (see 3.2.3.1(c); 5.3.1.3) are therefore recommended to promote the reasoning-based instruction within the ambit of legitimation. It is further recommended that the participation in reviewed PLC programmes' is regularly monitored, and that the programme review is done in consultation with teachers to ensure contextual appropriateness, relevance and developmental growth.

### 6.3.2 Turning algebra into a career enabler

The analysis has shown that the challenge of algebra becoming a career sifter rather than an enabler commences in the early stages of our education system and materialises (manifests) in phases. The finding of the challenge commencing at early stages of development draws from the emphasis in many writings that algebra instruction founds well on sound basic skills (Banerjee & Subramaniam 2011:351; Chernoff 2019:73; Fuchs & Fuchs 2005:45; Matsolo 2006:5; McNeil et al. 2010:625–626), and the evidence adduced during research that most learners in grade 9 lack the basic skills (pre-knowledge). In other words, the sifting commences with an inappropriate foundation. It then materialises from when the decision is made whether learners can or cannot be promoted and/or progressed to the next class based on mathematics performance (see 5.2.2). The analysis and interpretation thereto forged the relationship between the performance in algebra and the determining effect of mathematics performance in grade 9. The finding was that a learner who cannot average 30% in mathematics in grade 9 is discouraged from enrolling for mathematics in grade 10. It has thus been found that most learners perform below average in algebra, hence in mathematics, and as a result, have to bow out of careers they would have otherwise wished to pursue. For example, many learners who enrol in mathematical literacy in grade 12 cannot enrol in mathematics-based careers such as engineering, medicine and commerce in tertiary institutions (Osborne 2021:6). It can then be concluded that the unsatisfactory performance in algebra and mathematics narrows learners' choice of careers and participation in the field of work.

The analysis proved the presupposition that when benchmarked against the imperative of deep conceptual understanding and the targeted development of learners' critical thinking encapsulated in literature, the current teaching and learning (instruction) of algebra falls way below the set standard. The instruction leans more towards procedural orientation that lacks proper foundation and conceptual connection. The orientation falls short of developing learners to perform algebra in a manner that could turn it into a career enabler. The procedural (limited) knowledge of algebra adversely affects the teaching, learning and performance in advanced mathematics and related disciplines (Hamami 2020:4; Muchoko et. al 2019:1; Pramesti & Retnawati 2019:1; Ying et al. 2020:5406). It can be deduced from the finding that algebra becomes an instrumental communicative link, applicable in advanced mathematics, science and commerce if it is logically conceptualised in critical thinking and high order skills.

#### *6.3.2.1 Recommended strategies to turn algebra into a career enabler*

The analysis and interpretation of the (vast) contribution of algebra to career sifting showed that the sifting commences with the foundation upon which the teaching and learning of algebra builds (see 5.2.4). That is, the performance expected of grade 9 learners in algebra and mathematics to qualify for promotion or progression and enrolment in grade 10 mathematics is dependent on a good start. According to this study, a good start translates to a recommendation for teachers to ascertain that learners are competent in basic skills before introducing algebra. The other finding couched in the analysis relates to the accessibility of knowledge (algebra) to learners. The recommendation in this regard draws from the finding central to this study that learners can access algebra and perform it well when it is instructed in line with the requirements of the curriculum policy (see 5.3.1.1). The policy subscribes to the principles of constructivism theory. The theory asserts that learners should be guided to lead the construction of logical arguments (constructs) that connect and explain procedural steps. The constructs help them attain meaningful knowledge that becomes demonstrable when learners freely manoeuvre and explore multiple dimensions towards a solution. The exploration and effective application of gained knowledge prove understanding.

#### *6.3.2.2 Recommended conditions for turning algebra into a career enabler*

The reasoning-based instruction integrates the competency of basic mathematics as a requirement that needs to be confirmed before the introduction of algebra. The finding of the positive impact behind the integration cannot be overemphasised (see 5.3.4; 6.2). It is therefore recommended that teachers should ensure that learners have the necessary pre-knowledge (basic skills) upon which different topics of algebra can build before introducing each curriculum component of algebra. For example, the study proved it vital and beneficial to assess learners' knowledge of the derivatives of the laws of exponents before introducing the simplification of exponential expressions and solving mathematical equations with exponents. Another finding in which the study complemented literature showed that learners perform well when they are hands-on and work in collaborations. This warrants the recommendation for teachers and parents to initiate and support formations of functional study groups formed by and among learners themselves. The study further revealed as analysed in section 5.2.1.4 that learners in the previously disadvantaged schools need the same support their counterparts in urban schools receive, both in schools and at home, if they were to learn algebra and mathematics comparatively.

#### *6.3.2.3 Risks and threats inherent of turning algebra into a career enabler*

The analysis on risks that threaten efforts to turn algebra into a career enabler found that the deviation from the instruction prescribed by the curriculum policy is the primary cause of the repeated unsatisfactory performance. The inefficient instruction (deviation) manifests in basic mathematics, algebra and advanced mathematics, all of which form a respective conceptual chain of logical learning recommended in this study. It is thus recommended that teachers should be discouraged from using teaching approaches that focus on procedural learning other than logical conceptual orientation depicted in the chain. The chain is inherent to a rich connection. It frees learners from learning dependency and empowers them to self-sustain their learning. It further improves learners' motivation and self-efficacies, and guarantees optimal performance.

### 6.3.3 Abstraction and complexity of algebra

This study has found that the abstraction and complexity embedded within algebra and its instruction stem from a difficulty with which learners interpret algebraic symbolic and technical texts (Matsolo 2006:12; Pramesti & Retnawati 2019:1; Sengul 2011:2305; Ying et al. 2020:5405). The abstraction is amplified by limited or lack of contextual explanation, teacher-centred approaches (Osborne 2021:2) and procedure-oriented instruction (see 5.2.3). As a result, learners find the learning of algebra meaningless and without purpose (see 5.2.3.1). The situation compels learners to resort to rote learning inherent in memorising and regurgitating (Major & Mangope 2012:140). The study and literature have largely canvassed the shortfall of this line of learning. It falls short of meeting the requirements of the curriculum policy, improving learners' critical thinking and promoting high cognitive skills (see 2.2.2.4). This means that the instruction cannot break through the abstraction and complexity of algebra (see 5.2.3.3). The textual analysis backed up by literature confirmed that the difficulty encountered by learners to differentiate between operations of numbers and algebra bars or impedes the anticipated transition from the former to the latter, and adds to textual complexity.

The analysis and interpretation of generated data support literature that the textual complexity embedded in algebra instruction is responsible for the confusion instigated by overlapping principles and concepts. Learners fall victim to the confusion for lack of appropriate symbolic interpretation, logical connection and conceptual interrelations. The confusion manifests in structural misrepresentations and erratic operations. It is common that the confusion results from algebraic misconceptions (see 3.2.1.3(a)).

#### 6.3.3.1 Recommended strategies to simplify abstraction and complexity

The reasoning-based instruction underpinnings (see 2.2; 2.3) support the participatory mode of learning to simplify abstraction. Learners are guided to explain symbolic texts and forge conceptual interrelations using contextual language. The approach brings learners' different learning styles and lived experiences into the classroom. The use of common (contextual) language, concrete examples, referrals or manipulatives inherent of learners' constructs to explain the symbolic representations, improves the level of understanding. It incorporates an element of meaningful and purposeful learning (Matsolo 2006:62; Pramesti & Retnawati 2019:3; Tlali 2017:85) of algebra. The effectiveness of the approach is analysed and interpreted in section 5.3.3. Streamlining the reasoning-based instruction to the foundation when closing basic mathematics gaps readied learners' minds to the notion of logically connected arguments (constructs) to create conceptual knowledge. The instruction is accordingly recommended for the learning of algebra.

#### 6.3.3.2 Recommended conditions for simplifying abstraction and complexity

The reasoning-based instruction implored in this study is underpinned by a "conceptual first" aspect (condition) of learning to simplify algebraic abstraction and complexity. The study confirmed the statement in literature that the condition, as opposed to that of "procedural first", ensures that the teaching and learning of procedures are underpinned by conceptual understanding. Learners were empowered to logically develop procedural algorithms and conjectures from conceptual understanding and not the other way round. That way, learners get to have freedom of choice over multiple dimensions to meaningful solution and application (see 5.3.2). The use of algorithmic procedures is monitored by the conceptual understanding of how it works and applies in different situations.

The study has found that the confusion that was caused by overlapping principles of algebra was addressed by teaching and learning merging concepts (algebraic expressions and equations) concurrently. The approach is commendable. The research exercises (Annexure N – Exercise 8) where algebraic fractional expressions and fractional equations were taught and assessed concurrently are exemplary in this regard.

The approach articulates the difference and clears the structural and operational confusion. In the same vein, the approach can also use the comparison to integrate operations where appropriate.

#### *6.3.3.3 Risks and threats inherent of simplifying abstraction and complexity*

The result in this study is consistent with literature about the risks and threats inherent in the attempt to simplify abstraction and complexity of learning algebra. The risks and threats that perpetuate the abstraction are primarily centred in teachers’ resistance to unlearn the instructional practices that concentrate on using technical symbolism and procedures. The perpetuation is influenced by insufficient time among other factors. It has been established through analysis in section 5.2.1.4 that most teachers find procedure-oriented instruction a quick fix to take them through the curriculum within the stipulated time. The practice pays little or no attention to the strategies meant to simplify the abstraction of algebra for the majority of learners. Hence, the reiteration of the recommendation to prioritise deep conceptual understanding over uncritical and rote learning because of time. The analysis has shown that investing in conceptual knowledge of algebra has much positive effects in grade 9 mathematics and beyond. It has also clarified the finding of the curriculum-time mismatch.

### 6.3.4 Basic mathematics competence

This study has confirmed that learners that enrol in grade 9 with limited competence in basic mathematics become impediments to successful teaching and learning of algebra (see 5.2.4). The challenge is cushioned in literature findings that the teaching and learning of algebra build on basic mathematics, and basic mathematics is a primary need for using reasoning in the teaching and learning of algebra (NCTM 2000). It was also found that excessive use of a calculator impacts adversely on learners’ mental flow of basic skills, hence deprives them an opportunity to understand and relate naturally to the properties of numbers. The natural formation of a relationship between numbers is associated (linked) with critical thinking development. Critical thinking is an imperative for high order cognition sought by the curriculum policy.

#### *6.3.4.1 Recommended strategies to cultivate basic mathematics competence*

The finding that most learners in grade 9 do not have sufficient basic mathematics competence to form a base upon which the learning of algebra can build warrants a reiteration of the recommendation to check and close basic mathematics gaps before introducing algebra. It also warrants a need to streamline the reasoning-based instruction in the process of checking. This study has proven beyond reasonable doubt that the process is an investment in terms of building a chain of logical, coherent and conceptually interrelated (algebraic) knowledge. The competence in basic mathematics underpinned the rich conceptual connections and supported (sustained) the effective construction of logical reasoning (constructs).

#### *6.3.4.2 Recommended conditions for cultivating basic mathematics competence*

Based on the strategies in section 6.3.4.1, it is recommended that teachers' plans about the introduction of algebra should not be overshadowed by assumptions and/or expectations that learners in grade 9 were supposed to be competent in basic skills. Instead, they should assess the competence of basic mathematics for each component of algebra taught and close the gaps per need. The research activities (Annexure N) illustrate the recommended form of assessment. It is also recommended that teachers should ensure that activities do not include unwieldy figures that will require learners to use calculators. The suspension of the use of calculators in algebra helps learners to refresh their basic mathematics knowledge (skills) and systematically weave it within algebraic (reasoning) constructs. The accurate and sustainable application of the skills and constructs in novel and advanced situations characterises the integrated development of learners' critical thinking and high order cognitive skills. The development is optimised through the adherence to the foregoing conditions.

#### *6.3.4.3 Risks and threats inherent of cultivating basic mathematics competence*

This study has expressly warned teachers against the likelihood of a risk, not to ascertain themselves of learners' competency in relevant basic mathematics before introducing each curriculum component of algebra. The risk is likened to building a house without a foundation. It is simply setting learners for a failure from the onset (see 5.5.4). Without the proper foundation, most learners are likely to struggle to form necessary transitions from arithmetic to algebraic operations, and cannot sustain the construction of reasoning connections (NCTM 2000) envisaged for conceptualising procedures. The study has also analysed a threat posed by excessive use of calculators to the curriculum policy imperative to develop active and critical learning.

## 6.4 STUDY LIMITATIONS

The study encountered a number of limitations. But of the most recognisable, is the exclusion of parents. The exclusion limited the enquiry about learners' glaring underdevelopment to think and reason logically in the abstract, to the teacher and learner participants. Yet the parents are more central to the monitoring of their children's (learners') development in almost all aspects. Their participation would have given the insight as to the learners' formal operational (developmental) growth (see 3.2.1.4). Relating the insight with relevant literature and views from other participants would have consolidated the generated evidence for or against the analysis, hence the finding that the majority of learners in grade 9 are generally underdeveloped to cope with the abstractness of algebra. The parents' involvement would have also attested for or against the qualitative evidence that the reasoning-based instruction enhances the development and critical thinking, hence the learning of algebra. Therefore, an investigative article (research) centred on soliciting parents' indulgence on the foregoing issues of limitation is recommended.

The other notable limitation had to do with time. The research activities had to be confined within the curriculum allocated time in spite the glaring curriculum-time contestation. It is assumed that the impact of the reasoning-based instruction would have been realised with more clarity and certainty had it not been for the time limitation. Perhaps, the study could have gone to an extent of advising about the reasonable time necessary for the reasoning-based teaching and learning of algebra in an ideal grade 9 classroom. Further, the benefits (enhancement) and shortcomings of the approach could have also been determined with more clarity. To this end, it can be argued that the reasoning-based instruction also sets a platform for further research into the relationship of procedural and conceptual aspects of (algebraic) knowledge, and the functionality of reasoning capital on them over a reasonable time.

This study has also shown that in spite the existence of many reasoning styles, there is no specific reasoning framework that addresses all problems encountered by learners in the learning of algebra and other parts of mathematics. It restricted its activities and resultant findings to the use of reasoning constructs to conceptualise algebraic instruction in line with the curriculum policy. It confined the conceptualisation to contextualised explanation of algebraic symbolic convention and representation, refocusing, integration and content matter re-organisation. The limitation leaves a room for mathematics education researchers from different scholarly perspectives to research how reasoning can further improve the learning of algebra and advanced mathematics at school. Literature has revealed mutual relationship between reasoning and mathematical proofs (Rocha 2018:1). It could be an interesting research to investigate the impact of the reasoning-based instruction on reducing the difficulty encountered by learners in advanced mathematical (algebraic and geometrical) proofs.

## 6.5 CONCLUSION

This chapter has summarised the study overview for a recollection of the project founding purpose. The overview explained the key elements of the presupposition upon which the study was hypothetically sustained for research. It provided a brief of the challenges that underpinned the research question and objectives. It shed light about the key indicators of success the reasoning-based instruction has brought. The chapter further presented the findings that informed the interwoven recommendations. The findings revealed that the reasoning-based instruction underpinned by bricolage and constructivism theories emerged an innovative approach that uses resources at hand to create new forms and order of enhancement. Lastly, the chapter discussed the research limitations and avenues of new research instigated by this study.

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# ANNEXURE A:
# Professional Learning Communities:
# Advocacy and training Programme

education
Department of Education
FREE STATE PROVINCE

**Enquiries: Mrs. NH Msomi - 073 695 1154**
**: Mr. FR Madikgetla - 082 671 9728**

**12 JANUARY 2016**

**To: Principals**
**: HOD – Mathematics**
**: Grade 8 & 9 Mathematics Teachers**

**ONE PLUS NINE INTERVENTION MODEL: ADVOCACY AND GRADE 8 & 9 MATHEMATICS PLC TRAININGS PROGRAMME**

The Department of Basic Education plans to implement a rigorous national MST teacher development programme. The programme will focus on improving mastery of Mathematics curriculum content and instructional management. This programme will make use of appropriate and effective training interventions and techniques.

It aims to improve subject advisory services by strengthening district capacity, resources and training. Coaching and mentoring will be made effective by: focusing more on subject support than on administration and prioritizing classroom based support, coaching and mentoring. (Ministerial Task Team Draft Report, 2013)

The '1+4 Model' is based on and supports the concept of the Professional Learning Communities (PLCs) which the Minister of Basic Education, launched on 07 August 2014. The benefit of the '1+4 Model' is that teachers will meet on pre-determined working day. This methodology works on the assumption that teachers need assistance with the entire curriculum and not just certain sections of the curriculum which they presumably have difficulties teaching. The FSDoE has however, after considering its capacity to implement the strategy effectively, decided to implement the model as a 1 plus nine model, in which the PLCs will meet every fortnight and the cluster coordinators meeting a week before that. This will ensure that they are well prepared for the cluster meetings. The FSDoE model also makes use of quarterly provincial review meetings in which quarterly monitoring of improvement in performance and implementation of the 1 + 9 will be discussed.

In order to implement the model effectively and smoothly in Free State, the Grade 8 & 9 sub-directorate will therefore embark on advocacy road shows to the schools as well as conducting initial one day workshops with the various PLCs to kick start the process in the clusters and ensure that all PLC clusters are clear of their roles and responsibilities.

The training venues will be in one of the schools which will be accessible to most teachers as well as Teacher Development centres as indicated in the programme attached.

**INVITATION TO 1+9 PLC MEETING**

**Grade 8 & 9 Mathematics teachers are invited to attend 1+9 PLC meetings Scheduled as indicated in the attachment. In those meetings teachers will also be setting common formal tasks for this term. Gr 8&9. Deputy Principals who are responsible for Gr8&9 must make sure that their teachers attend these PLCs.**
**Teachers are requested to bring along:**

1. **Policy document (Senior phase)**
2. **Pace setters**
3. **Empty DVD for lesson plans / laptop**
4. **Mathematics Textbooks; Heymath! etc**

**Lunch will not be served so teachers are requested to bring their own lunch boxes.**

**Thanking you in advance for your co-operation in this matter.**

**Yours in learner development**

**NH MSOMI & FR MADIKGETLA**
**Mathematics Subject Advisors**

# ANNEXURE B:
# Union's concern:
# Provincial common papers (Examination)

**South African Democratic Teachers Union**
**Free State Province**

23 Brill Street
2nd Avenue; Westdene
BLOEMFONTEIN
9301

P.O. Box 6785
BLOEMFONTEIN; 9300
Tel: (051)430 1257/1825/3043/3062
Fax: (051) 430 1405
E-Mail: florencec@sadtu.org.za

Tuesday, 29 January 2019

To : **All Regions**
Att : **Regional Secretaries**
Cc : **Branch Secretaries**

Dear Comrades

RE : **PROVINCIAL COMMON EXAMINATIONS**

We have noted with concern the continuing arrogance of the Free State Department of Education of issuing circulars "encouraging" schools to participate in the writing of the common examinations for Grades 3, 6 and 7. This time around, they have also issued timetable for the June and November examinations.

We once more reminding members about our standing resolution that teachers should set own Question Papers. Based on our resolution we urge members to start preparing themselves to set own papers in March, June, September and November. Schools should develop management plans outlining the submission dates and the writing of different subjects.

Whilst there are some members who continue to ignore their own resolution, the majority are commended for their organisation discipline. We remain convinced that teachers are professionals who are well trained to set quality papers and that has to be respected by those in positions of authority.

Kind regards

**MOKHOLOANE MOLOI**
**PROVINCIAL SECRETARY**

"Restore the Character of SADTU as a Union of Revolutionary Professionals, Agents of change and Champions of People's Education for People's Power in Pursuit of Socialism"
ALL COMMUNICATION SHOULD BE ADDRESSED TO THE PROVINCIAL SECRETARY

# ANNEXURE C:
# Ethics Committee's clearance letter

UNIVERSITY OF THE FREE STATE
UNIVERSITEIT VAN DIE VRYSTAAT
YUNIVESITHI YA FREISTATA

UFS·UV
EDUCATION
OPVOEDKUNDE

**Faculty of Education**

11-Dec-2018

Dear Mr **Mohau Lika**

Ethics Clearance: **Enhancing the Teaching and Learning of Algebraic Expressions and Equations through Reasoning in Grade 9**

Principal Investigator: **Mr Mohau Lika**

Department: **School of Education Studies Department (Bloemfontein Campus)**

**APPLICATION APPROVED**

With reference to you application for ethical clearance with the Faculty of Education, I am pleased to inform you on behalf of the Ethics Board of the faculty that you have been granted ethical clearance for your research.

Your ethical clearance number, to be used in all correspondence is: **UFS-HSD2018/0644**

This ethical clearance number is valid for research conducted for one year from issuance. Should you require more time to complete this research, please apply for an extension.

We request that any changes that may take place during the course of your research project be submitted to the ethics office to ensure we are kept up to date with your progress and any ethical implications that may arise.

Thank you for submitting this proposal for ethical clearance and we wish you every success with your research.

Yours faithfully

Prof. MM Mokhele Makgalwa
Chairperson: Ethics Committee

**Education Ethics Committee**
**Office of the Dean: Education**
T: +27 (0)51 401 3777| F: +27 (0)86 546 1113 | E: MokheleML@ufs.ac.za
Winkie Direko Building | P.O. Box/Posbus 339 | Bloemfontein 9300 | South Africa
www.ufs.ac.za

# ANNEXURE D:
# Research application template form

Ref: Research Application

APPLICATION TO REGISTER AND CONDUCT RESEARCH IN THE Free State DEPARTMENT OF EDUCATION

- Please complete all the sections of this form that are applicable to you. If any section is not applicable please indicate this by writing N/A.

- If there are too few lines in any of the sections please attach the additional information as an addendum.

- Attach all the required documentation so that your application can be processed.

- Send the completed application to:

DIRECTOR: STRATEGIC PLANNING, POLICY AND RESEARCH

Room 319, 3rd Floor
Old CNA Building
Bloem Plaza
Charlotte Maxeke Street
BLOEMFONTEIN, 9300

OR

Free State Department of Education
Private Bag X20565
BLOEMFONTEIN, 9300

Email: berthakitching@gmail.com and B.Kitching@fseducation.gov.za

PLEASE DO NOT EMAIL ANYTHING IN PICTURE FORMAT

Tel: 051 404 9283 /9211 / 082 454 1519

1. **TITLE** (eg Ms, Mrs, Mr, Dr, Prof, etc):

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2. **INITIALS**

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3. **SURNAME**

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4. **TELEPHONE HOME:**

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7. **FAX:**

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8. **E-MAIL**

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10. **ADDRESS WORK:**

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**11. POSTAL ADDRESS**

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| L | a | d | y | b | r | a | n | d | | | | | | |
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| | | | | | | | | | | | | | | |
| Postal Code | | | | | | | | | | | 9 | 7 | 4 | 5 |

**12. NAME OF TERTIARY INSTITUTION/RESEARCH INSTITUTE AND STUDENT NUMBER**

| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| U | F | S | | 2 | 0 | 0 | 0 | 0 | 2 | 9 | 9 | 0 | 6 | | |
| | | | | | | | | | | | | | | | |

**13. OCCUPATION**

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| E | D | U | C | A | T | O | R | | | | | | | | |
| | | | | | | | | | | | | | | | |

**14. PLACE OF EMPLOYMENT**

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|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| D | O | E | | F | S | | L | E | R | E | N | G | | | |
| S | E | C | O | N | D | A | R | Y | | S | C | H | O | O | L |

**15. NAME OF COURSE**

| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| P | h | D | | E | D | U | C | A | T | I | O | N | | | |
| C | U | R | R | I | C | U | L | U | M | | | | | | |

**16. NAME OF SUPERVISOR / PROMOTER**

| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| D | R | | M | O | E | K | E | T | S | I | | | | | |
| | | | T | L | A | L | I | | | | | | | | |

**17. TITLE OF RESEARCH PROJECT**

ENHANCING THE TEACHING AND LEARNING OF ALGEBRAIC EXPRESSIONS AND EQUATIONS IN GRADE 9

## 18. CONCISE EXPLANATION OF THE RESEARCH TOPIC

This study seeks to develop the strategy that will enhance the teaching and learning of algebraic expressions and equations (algebra) through ‘reasoning’ strategy. The strategy deepens the logical algebraic knowledge and prepares learners to acquire and demonstrate the conceptual understanding. The strategy conceptualizes the normal procedural computations into conceptual knowledge through which learners can navigate with necessary proficiency and confidence to solve problems.

## 19. APPLICATION VALUE THAT THE RESEARCH MAY HAVE FOR THE FREE STATE EDUCATION DEPARTMENT

The research seeks to enhance the teaching and learning of algebraic expressions and equations in Grade 9 using reasoning. The strategy aims at enriching the algebraic procedures with appropriate connections in order to develop the procedures into conceptual knowledge learners can comprehend at ease. It is therefore hoped to cut down on errors and misconceptions learners’ commit, hence improve their performance. The research also has a cross-cutting benefits to other related topics and subjects such as technology and natural sciences as well as other grades. The co-researchers (localised) will be empowered to develop, implement and reflect on a contextualised strategy of teaching and learning algebraic expressions. As a result the more sustainable solutions, which may serve as the basis for more innovations, are anticipated. That is, the research will help overcome the ‘lock-ins’ to existing strategies, some of which have not been effective for a considerable population of teachers and learners. The research will further tap into erstwhile dormant potential in a coordinated approach to address societal challenges.

## 20. LIST OF SCHOOLS AND DISTRICTS INVOLVED IN THE RESEARCH (If not enough space, please add more rows)

| | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| L | E | R | E | N | G | | | | | | | | |
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| | | | | | | | | | | | | | |
| M | O | T | H | E | O | | D | I | S | T | R | I | CT |
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## 21. LIST OF DIRECTORATES/OFFICIALS IN THE DEPARTMENT INVOVLED IN THE RESEARCH

| | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| M | A | T | H | E | M | A | T | I | C | S | H | O | D |
| M | A | T | H | E | M | A | T | I | C | S, | T | E | C |
| H | N | O | L | O | G | Y, | S | C | I | E | N | C | E |
| T | E | A | C | H | E | R | S. | | | | | | |
| | | | | | | | | | | | | | |
| | | | | | | | | | | | | | |

## 22. DETAILS OF TARGET GROUP WITH WHOM THE RESEARCH IS TO BE UNDERTAKEN

| Target group | Number | Grade | Subject | Age | Gender | Language |
|---|---|---|---|---|---|---|
| Departmental head | 1 | | Mathematics | Any | Any | Sesotho/ English |
| Teacher(s) | 1 | 8 or 9 | Mathematics | Any | Any | Sesotho/ English |
| Teacher(s) | 1 | 8,9 | Technology | Any | Any | Sesotho/ English |
| Teacher(s) | 1 | 8,9 | Natural sciences | Any | Any | Sesotho/ English |
| Retired teachers (optional) | 1 | | Mathematics | Any | Any | Sesotho/ English |
| Learners | +/-40 | 9 | Mathematics | 14-18 | all | Sesotho/ English |

**23. FULL PARTICULARS OF HOW INFORMATION WILL BE OBTAINED, EG QUESTIONAIRES, INTERVIEWS, STANDARDIZED TESTS, ETC.**

Please attach copies of questionnaires, questions that will be asked during interviews, tests that will be completed or any other relevant documents regarding the acquisition of information.

The research team of 6 people (3 core participants- Departmental head Mathematics, researcher, retired Mathematics teacher); and practicing Mathematics teacher, technology teacher and natural science teacher will interact in meetings held after school to effect the research processes (initial meetings, conceptualization, planning, data collection and reflective meetings).

The aim of the research is to develop the strategy that will enhance the teaching and learning of algebraic expressions and equations (algebra) through reasoning.

The objectives (and inherent questions) of the research are therefore as follows:

To justify the need to enhance the teaching and learning of algebraic expressions and equations in Grade 9. (Why would there be a need to improve the teaching and learning of algebraic expressions and equations in Grade 9?)

To derive the components of the solution in the teaching and learning of algebra in Grade 9 using the reasoning approach. (What would be the components (i.e. defining aspects / exclusive features) of the measures and or strategy that would help improve the teaching and learning of algebraic expressions and equations in Grade 9?)

To determine the conditions conducive to the teaching and learning of algebra, using reasoning in Grade 9. (Under which conditions will the suggested measures / strategy be successful in improving the teaching and learning of algebraic expressions and equations?)

To identify the threats hindering the teaching and learning of algebra in Grade 9 and suggest ways to prevent them. (How can we develop an implementable strategy capable of circumventing possible hindrances (inherent threats and risks) to the successful improvement of teaching and learning of algebraic expressions to grade 9 class?)

To provide evidence that shows that the strategy enhances the teaching and learning of algebra in Grade 9? (What evidence is there to attest that the developed strategy has the potential to improve the teaching and learning of algebraic expressions and equations to grade 9 class?)

## 24. STARTING AND COMPLETION DATES OF THE RESEARCH PROJECT

| Target Group | Activity (i.e. interview, questionnaire, etc) | Time Needed |
|---|---|---|
| Departmental head - Mathematics | The departmental head will observe classroom activities. They will respond to open-ended questions asked through free attitude interview technique to elicit responses responsive to research question and objectives of the study. Further, the departmental head shares knowledge and experience in the teaching of Mathematics more particularly in the topics of the study. They offer guidance and management to ensure appropriate subject scope and content coverage. They control and monitor the technical aspects of the subject matter and ensures that the study is aligned with school programmes. | Avails themselves from time-to-time when their schedule permits from the beginning to the end of research activities. The after school classes will commence at 14h30 and end at or before 15h30 on Mondays and Wednesdays for about 3 weeks and the reflection sessions will commence at 14h30 and end at or before 15h30 on Tuesdays and Thursdays. There will further be about 2-3 extended reflection sessions based on learners' performance in summative assessments. |
| Teachers | Teachers will attend reflection sessions to interact with the core research team reports about the classroom teaching and observation and to plan the next improved lessons. They share their knowledge, experiences, competence and skills relevant to the study from their respective subject disciplines, and respond to open-ended questions asked through free attitude interview technique to elicit responses responsive to research question and objectives of the study. | Avail themselves from time-to-time when their schedules permit from the beginning to the end of research activities. The reflection sessions will take place after school at 14h30 and end at or before 15h30 on Tuesdays and Thursdays. |
| Learners | Learners are taught by the core research team members. They will interact freely with teachers for deep understanding and do activities as per work schedule/programme of the Department of Education (DBE) in regard to the topics of the study and as planned by the research team. They will be expected to respond to open-ended questions asked through free attitude interview technique to elicit the learning challenges. The results of learners' work and classroom interactive responses (written and oral) will be used as part of data. | The after school classes will run from 14h30 to 15h30 on Mondays and Wednesdays for about 3 weeks during which the topics informing this study will be taught. |

Please bear in mind that research is usually not allowed to be conducted in schools during the fourth academic term (October to December).

| |
|---|
| May–July 2019 |

## 25. WILL THE RESEARCH BE CONDUCTED DURING OR AFTER SCHOOL HOURS?

Please bear in mind that research is usually not allowed to be conducted in schools during normal teaching time.

| |
|---|
| All research activities will take place after school. Classroom teaching and assessment, and observations will be conducted after school between 14h30 and 15h30 on Mondays and Wednesdays. Parents' permission will be sought to keep learners at school after school hours. Reflection and planning meetings will also be held after school between 14h30 and 15h30 on Tuesdays and Thursdays. |

## 26. HOW MUCH TIME IS NEEDED WITH THE TARGET GROUP/S TO CONDUCT THE RESEARCH?

## 27. HAVE YOU INCLUDED/ATTACHED?

27.1 A letter from your supervisor confirming your registration for the course you are following?

| Yes | No |
|---|---|
| x | |

27.1 A draft letter / specimen that will be sent to principals requesting permission to conduct research in their schools?

| Yes | No |
|---|---|
| x | |

27.2 A draft letter / specimen that will be sent to parents requesting permission for their children to participate in the research project?

| Yes | No |
|---|---|
| x | |

27.3 A draft letter / specimen that will be sent to research participants to give their consent to take part in the research project?

| Yes | No |
|---|---|
| x | |

27.4 A copy of the questionnaires that you wish to distribute to the target group/s?

| Yes | No |
|---|---|
| | x |

27.5 A list of questions that will be asked during interviews with the target group/s?

| Yes | No |
|---|---|
| | x |

27.6 Ethical clearance certificate from higher education institution

| Yes | No |
|---|---|
| x | |

28 I **Lika Mohau Armstrong** herewith confirm that all the information in this application form is correct and that I will abide by the ethical code and the conditions under which the research may be undertaken, i.e.:

28.1 I will abide by the ethical research conditions in the discourse of my study in the FSDoE.

28.2 I will abide by the period in which the research has to be done.

28.3 I will apply for extension if I cannot complete the research within the specified period.

28.4 If I fall behind with my schedule by three months to complete my research project in the approved period, I will apply for an extension.

28.5 I will not conduct research during the fourth quarter of the academic year.

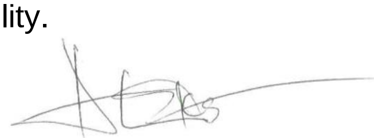
28.6 I will not disrupt normal learning and teaching times at schools to undertake my research.

28.7 I will submit a bound copy or CD of the research document to the Free State Department of Education, Room 319, 3rd Floor, Old CNA Building, Charlotte Maxeke Street, Bloemfontein, upon completion of the research.

28.8 I will upon completion of my research study make a presentation to the relevant stakeholders in the Department as per the arrangements of the Department.

28.9 The ethics documents (attached) will be adhered to in the discourse of my study in your department.

28.10 The costs relating to all the conditions mentioned above are for my own responsibility.

SIGNATURE:  DATE: 11/12/2018

## ETHICAL REQUIREMENTS: FREE-STATE DEPARTMENT OF EDUCATION

The scientific research enterprise is built on a foundation of trust and that the reports by others are valid. The reports should reflect an honest attempt by the researcher to describe the world accurately and without bias; this trust will endure only if the researcher devotes himself or herself to exemplifying and transmitting the values associated with ethical research conduct.
There are many ethical issues to be taken into serious consideration when conducting research. The Free State Department of Education believes that the researchers conducting research in this department would, amongst others, adhere to the following ethical conduct:

### ETHICS GENERAL APPLICATION

1. Be aware of having the responsibility to secure the actual permission and interests of all those involved in the study;
2. Not misuse any of the information discovered;
3. Moral responsibility maintained towards the participants;
4. Embracing corporate social responsibility;
5. Protecting the rights of people in the study as well as their privacy and sensitivity;
6. Confidentiality of those involved in the observation must be carried out, keeping their anonymity and privacy secure;
7. Follow the ethical clearance guideline of the institution that granted such.

Amplifying the voice of the participants

Enhancing collective plurality.

### ETHICS: INHERENT PRINCIPLES

8. Reliability.
9. Informing the participants about the importance of the research.
10. Values of trust, fairness and integrity are maintained in the study.

### ETHICS

11. The value of transparency is considered.
12. The research is committed to delivering the intended promise as informed by the objectives.
13. The research accentuate the values of reputation and respect.

**RESEARCHER: INITIALS AND SURNAME MA Lika SIGNATURE:**

**DATE: 11/12/2018**

# ANNEXURE E:
# Request for permission to conduct research

<u>REQUEST FOR PERMISSION TO CONDUCT RESEARCH</u>

The Principal
Leru secondary school
Private Bag X10
Ladybrand
9745

Dear Mr Mokose

I am doing research and would like to request permission to conduct the research at Leru secondary school.

DATE

May 2019–July 2019

TITLE OF THE RESEARCH PROJECT

Enhancing the Teaching and Learning of Algebraic Expressions and Equations through Reasoning in Grade 9.

RESEARCHER'S NAME AND CONTACT NUMBER(S):

*Lika MA student number 2000029906*
*Contact number 0738570570*

FACULTY AND DEPARTMENT: *Education*

STUDY LEADER'S NAME AND CONTACT NUMBER:

*Dr Tlali M.F.*
*Contact number 0833956691*

WHAT IS THE AIM / PURPOSE OF THE STUDY?

*To enhance the teaching and learning of algebraic expressions and equations in grade 9.*

WHO IS DOING THE RESEARCH?

*1. Researcher - Mathematics teacher in grade 9. 2. Retired Mathematics teacher – this teacher worked at the school where this study is based. They taught Mathematics in grades 8-12 for a period of over 20 years. Their experience and knowledge of teaching and Mathematics will be educative. 3. Departmental head for Mathematics at the school - in addition to their knowledge and experience in the teaching of Mathematics, their role will mainly be for guidance, management (work control and monitoring) because this study will be aligned with normal school programmes. 4. Natural sciences and Technology teacher (s) - to afford them an opportunity to extend learning to other subjects. Natural sciences and Technology are closely related to Mathematics. 5. Mathematics teacher – a peer in the same phase who would also serve as Integrated Quality Management System (IQMS) peer.*

HAS THE STUDY RECEIVED ETHICAL APPROVAL?

This study has received approval from the Research Ethics Committee of UFS. A copy of the approval letter is attached.

Approval number: *UFS-HSD2018/0644*

WHY IS YOUR INSTITUTION INVITED TO TAKE PART IN THIS RESEARCH PROJECT?

*Leru secondary (Dinaledi School) earmarked for good results in Mathematics and science. The number of learners repeating grade 9 and not doing well in Mathematics is significantly high and worrisome. The school offers easy access to the researcher and research team members, hence the cost effective research. The school catchment area draws learners from the townships whom literature reports as those enduring more problems with Mathematics than their counterparts in the suburban schools.*

WHAT IS THE NATURE OF PARTICIPATION IN THIS STUDY?

*The prospective participants are a researcher, Departmental head-Mathematics, a retired Mathematics teacher, Mathematics teacher, technology teacher and a natural sciences teacher, all collaborating as a research team. The team will lead the research processes and activities from initial meetings to the end of data collection and analysis. The tentative schedule of meetings is attached. The core research team members (namely a researcher, Departmental head-Mathematics and a retired Mathematics teacher) will be actively involved in actual teaching, classroom observations and assessment. The other members of will attend reflection sessions to interact with the core research team reports about the classroom teaching and observation and to plan the next improved lessons. They will also share their knowledge, experiences, competences and skills relevant to the study from their respective subject disciplines, and respond to open-ended questions asked through free attitude interview technique to elicit responses responsive to research question and objectives of the study. The after school classes will commence at 14h30 and end at or before 15h30 on Mondays and Wednesdays for about 3 weeks and the reflection sessions will commence at 14h30* *and end at or before 15h30 on Tuesdays and Thursdays. There will further be about 2-3 extended reflection sessions based on learners' performance in summative assessments. The results of learners' work and classroom interactive responses (written and oral) will be used as part of data. There is no foreseeable possible risks to willing participants because participants are free to opt out of the research any time they wish to do so and to not participate when sick or have other commitments.*

WHAT ARE THE POTENTIAL BENEFITS OF TAKING PART IN THIS STUDY?

*An enhanced teaching and learning of algebraic expressions and equations in grade 9; an improved performance in Mathematics and related subjects; empowerment of participants to develop a contextualized strategy enhancing the teaching and learning of algebraic expressions and equations and an opportunity to learn from each other especially from experienced teachers. Improved teaching practices on the part of teachers.*

WHAT IS THE POTENTIAL RISKS TAKING PART IN THIS STUDY?

*There are no foreseeable risks and/or inconvenience and/or discomfort to the participant.*

WILL THE INFORMATION BE KEPT CONFIDENTIAL?

*Names of participants will not be recorded anywhere and no one will be able to connect the participants with the answers they give. The answers will be given a fictitious code number or a pseudonym and participants will be referred to in this way in the data, any publications, or other research reporting methods such as conference proceedings. Only the researcher will have access to the data for transcribing using pseudonyms and storage. Information will strictly be used for research purposes and under strict confidentiality in other purposes, e.g. research report, journal articles, conference presentation, etc.*

HOW WILL THE INFORMATION BE STORED AND ULTIMATELY DESTROYED?

*The records (written, video and voice) will be stored by the researcher for a period of five years in a locked cupboard/filing cabinet for future research or academic purposes; electronic information will be stored on a password protected computer. Future use of the stored data will be subject to further Research Ethics Review and approval if applicable. There's no reasonably foreseeable risks of harm or side-effects to the potential participants.*

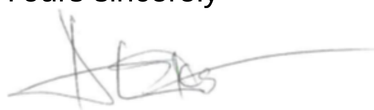
WILL THERE BE PAYMENT OR ANY INCENTIVES FOR PARTICPATING IN THIS STUDY?

*There will be no payment or reward offered, financial or otherwise for participation.*

HOW WILL THE INSTITUTION BE INFORMED OF THE FINDINGS / RESULTS OF THE STUDY?

*Report to school and presentations to interested parties (school, district office); copies of manuscripts; thesis published.*

Yours sincerely



Mohau Lika

# ANNEXURE F:
# Research study information leaflet and participation consent form

**<u>RESEARCH STUDY INFORMATION LEAFLET AND CONSENT FORM</u>**

**DATE**

**May 2019–July 2019**

**TITLE OF THE RESEARCH PROJECT**

*Enhancing the Teaching and Learning of Algebraic Expressions and Equations through Reasoning in Grade 9*

**RESEARCHER'S NAME AND CONTACT NUMBER(S):**

*Mohau Lika 2000029906 0738570570*

**FACULTY AND DEPARTMENT:** Education

**STUDYLEADER'S NAME AND CONTACT NUMBER:**

**Dr Moeketsi Tlali**

0833956691

**WHAT IS THE AIM/PURPOSE OF THE STUDY?**

*To enhance the teaching and learning of algebraic expressions and equations in grade 9.*

**WHO IS DOING THE RESEARCH?**

*1. Researcher - Mathematics teacher in grade 9. 2. Retired Mathematics teacher – this teacher worked at the school where this study is based. They taught Mathematics in grades 8-12 for a period of over 20 years. Their experience and knowledge of teaching and Mathematics will be educative. 3. Departmental head for Mathematics at the school - in addition to their knowledge and experience in the teaching of Mathematics, their role will mainly be for guidance, management (work control and monitoring) because this study will be aligned with normal school programmes. 4. Natural sciences and Technology teacher (s) - to afford them an opportunity to extend learning to other subjects. Natural sciences and Technology are closely related to Mathematics. 5. Mathematics teacher – a peer in the same phase who would also serve as Integrated Quality Management System (IQMS) peer.*

## HAS THE STUDY RECEIVED ETHICAL APPROVAL?

This study has received approval from the Research Ethics Committee of UFS. A copy of the approval letter can be obtained from the researcher.

**Approval number:** UFS-HSD2018/0644

## WHY ARE YOU INVITED TO TAKE PART IN THIS RESEARCH PROJECT?

*Your knowledge, experience, competence and skills in the teaching and learning of Mathematics, natural sciences and/or technology plus the vested interest you have shown in the subject and improvements thereto.*

## WHAT IS THE NATURE OF PARTICIPATION IN THIS STUDY?

*The prospective participants are a researcher, Departmental head-Mathematics, a retired Mathematics teacher, Mathematics teacher, technology teacher and a natural sciences teacher, all collaborating as a research team. The team will lead the research processes and activities from initial meetings to the end of data collection and analysis. The tentative schedule of meetings is attached. The core research team members (namely a researcher, Departmental head-Mathematics and a retired Mathematics teacher) will be actively involved in actual teaching, classroom observations and assessment. The other members will attend reflection sessions to interact with the core research team reports about the classroom teaching and observation and to plan the next improved lessons. They will also share their knowledge, experiences, competences and skills relevant to the study from their respective subject disciplines, and respond to open-ended questions asked through free attitude interview technique to elicit responses responsive to research question and objectives of the study. The after school classes will commence at 14h30 and end at or before 15h30 on Mondays and Wednesdays for about 3 weeks and the reflection sessions will commence at 14h30 and end at or before 15h30 on Tuesdays and Thursdays. There will further be about 2-3 extended reflection sessions based on learners' performance in summative assessments. The results of learners' work and classroom interactive responses (written and oral) will be used as part of data. There is no foreseeable possible risks to willing participants because participants are free to opt out of the research any time they wish to do so and to not participate when sick or have other commitments.*

## CAN THE PARTICIPANT WITHDRAW FROM THE STUDY?

*The participation is voluntary and there is no penalty or loss of benefit for non-participation. Being in this study is voluntary and participants are under no obligation to consent to participation. If a participant decides to take part, s/he will be given this information sheet to keep and be asked to sign a written consent form. S/he is free to withdraw at any time and without giving a reason.*

**WHAT ARE THE POTENTIAL BENEFITS OF TAKING PART IN THIS STUDY?**

*An enhanced teaching and learning of algebraic expressions and equations in grade 9; an improved performance in Mathematics and related subjects; empowerment of participants to develop a contextualized strategy enhancing the teaching and learning of algebraic expressions and equations and an opportunity to learn from each other especially from experienced teachers. Improved teaching practices on the part of teachers.*

**WHAT IS THE ANTICIPATED INCONVENIENCE OF TAKING PART IN THIS STUDY?**

*There are no foreseeable risks and/or inconvenience and/or discomfort to the participant.*

**WILL WHAT I SAY BE KEPT CONFIDENTIAL?**

*Names of participants will not be recorded anywhere and no one will be able to connect the participants with the answers they give. The answers will be given a fictitious code number or a pseudonym and participants will be referred to in this way in the data, any publications, or other research reporting methods such as conference proceedings. Only the researcher will have access to the data for transcribing using pseudonyms and storage. Information will strictly be used for research purposes and under strict confidentiality in other purposes, e.g. research report, journal articles, conference presentation, etc.*

**HOW WILL THE INFORMATION BE STORED AND ULTIMATELY DESTROYED?**

*The records (written, video and voice) will be stored by the researcher for a period of five years in a locked cupboard/filing cabinet for future research or academic purposes; electronic information will be stored on a password protected computer. Future use of the stored data will be subject to further Research Ethics Review and approval if applicable. There's no reasonably foreseeable risks of harm or side-effects to the potential participants.*

**WILL I RECEIVE PAYMENT OR ANY INCENTIVES FOR PARTICPATING IN THIS STUDY?**

*There will be no payment or reward offered, financial or otherwise for participation.*

**HOW WILL THE PARTICIPANT BE INFORMED OF THE FINDINGS / RESULTS OF THE STUDY?**

*Report to school and presentations to interested parties (participants, school, district office); copies of manuscripts; thesis published.*

**Thank you for taking time to read this information sheet and for participating in this study.**

**CONSENT TO PARTICIPATE IN THIS STUDY**

I, ____________________________________ (participant name), confirm that the person asking my consent to take part in this research has told me about the nature, procedure, potential benefits and anticipated inconvenience of participation.

I have read (or had explained to me) and understood the study as explained in the information sheet. I have had sufficient opportunity to ask questions and am prepared to participate in the study. I understand that my participation is voluntary and that I am free to withdraw at any time without penalty (if applicable). I am aware that the findings of this study will be anonymously processed into a research report, journal publications and/or conference proceedings.

I agree to the recording of the insert specific data collection method.

I have received a signed copy of the informed consent agreement.

Full Name of Participant:

_________________________________________________________________

Signature of Participant: ______________________ Date: ___________________

Full Name(s) of Researcher(s): Lika Mohau

Signature of Researcher: Date: 6/5/2019

# ANNEXURE G:
# Participant consent form – Children

**<u>RESEARCH STUDY INFORMATION LEAFLET AND CONSENT FORM</u>**

**DATE**

**May 2019–July 2019**

**TITLE OF THE RESEARCH PROJECT**

*Enhancing the Teaching and Learning of Algebraic Expressions and Equations through Reasoning in Grade 9*

**RESEARCHER'S NAME AND CONTACT NUMBER(S):**

*Mohau Lika 2000029906 0738570570*

**FACULTY AND DEPARTMENT:** Education

**STUDYLEADER'S NAME AND CONTACT NUMBER:**

**Dr Moeketsi Tlali**

0833956691

**WHAT IS THE AIM/PURPOSE OF THE STUDY?**

*To enhance the teaching and learning of algebraic expressions and equations in grade 9.*

**WHO IS DOING THE RESEARCH?**

*1. Researcher - Mathematics teacher in grade 9. 2. Retired Mathematics teacher – this teacher worked at the school where this study is based. They taught Mathematics in grades 8-12 for a period of over 20 years. Their experience and knowledge of teaching and Mathematics will be educative. 3. Departmental head for Mathematics at the school - in addition to their knowledge and experience in the teaching of Mathematics, their role will mainly be for guidance, management (work control and monitoring) because this study will be aligned with normal school programmes. 4. Natural sciences and Technology teacher (s) - to afford them an opportunity to extend learning to other subjects. Natural sciences and Technology are closely related to Mathematics. 5. Mathematics teacher – a peer in the same phase who would also serve as Integrated Quality Management System (IQMS) peer.*

## HAS THE STUDY RECEIVED ETHICAL APPROVAL?

This study has received approval from the Research Ethics Committee of UFS. A copy of the approval letter can be obtained from the researcher.

Approval number: *UFS-HSD2018/0644*

## WHY ARE YOU INVITED TO TAKE PART IN THIS RESEARCH PROJECT?

*Your knowledge, experience, competence and skills in the teaching and learning of Mathematics, natural sciences and/or technology plus the vested interest you have shown in the subject and improvements thereto.*

## WHAT IS THE NATURE OF PARTICIPATION IN THIS STUDY?

*The prospective participants are a researcher, Departmental head-Mathematics, a retired Mathematics teacher, Mathematics teacher, technology teacher and a natural sciences teacher, all collaborating as a research team. The team will lead the research processes and activities from initial meetings to the end of data collection and analysis. The tentative schedule of meetings is attached. The core research team members (namely a researcher, Departmental head-Mathematics and a retired Mathematics teacher) will be actively involved in actual teaching, classroom observations and assessment. The other members will attend reflection sessions to interact with the core research team reports about the classroom teaching and observation and to plan the next improved lessons. They will also share their knowledge, experiences, competences and skills relevant to the study from their respective subject disciplines, and respond to open-ended questions asked through free attitude interview technique to elicit responses responsive to research question and objectives of the study. The after school classes will commence at 14h30 and end at or before 15h30 on Mondays and Wednesdays for about 3 weeks and the reflection sessions will commence at 14h30 and end at or before 15h30 on Tuesdays and Thursdays. There will further be about 2-3 extended reflection sessions based on learners' performance in summative assessments. The results of learners' work and classroom interactive responses (written and oral) will be used as part of data. There is no foreseeable possible risks to willing participants because participants are free to opt out of the research any time they wish to do so and to not participate when sick or have other commitments.*

## CAN THE PARTICIPANT WITHDRAW FROM THE STUDY?

*The participation is voluntary and there is no penalty or loss of benefit for non-participation. Being in this study is voluntary and participants are under no obligation to consent to participation. If a participant decides to take part, s/he will be given this information sheet to keep and be asked to sign a written consent form. S/he is free to withdraw at any time and without giving a reason.*

## WHAT ARE THE POTENTIAL BENEFITS OF TAKING PART IN THIS STUDY?

*An enhanced teaching and learning of algebraic expressions and equations in grade 9; an improved performance in Mathematics and related subjects; empowerment of participants to develop a contextualized strategy enhancing the teaching and learning of algebraic expressions and equations and an opportunity to learn from each other especially from experienced teachers. Improved teaching practices on the part of teachers.*

## WHAT IS THE ANTICIPATED INCONVENIENCE OF TAKING PART IN THIS STUDY?

*There are no foreseeable risks and/or inconvenience and/or discomfort to the participant.*

## WILL WHAT I SAY BE KEPT CONFIDENTIAL?

*Names of participants will not be recorded anywhere and no one will be able to connect the participants with the answers they give. The answers will be given a fictitious code number or a pseudonym and participants will be referred to in this way in the data, any publications, or other research reporting methods such as conference proceedings. Only the researcher will have access to the data for transcribing using pseudonyms and storage. Information will strictly be used for research purposes and under strict confidentiality in other purposes, e.g. research report, journal articles, conference presentation, etc.*

## HOW WILL THE INFORMATION BE STORED AND ULTIMATELY DESTROYED?

*The records (written, video and voice) will be stored by the researcher for a period of five years in a locked cupboard/filing cabinet for future research or academic purposes; electronic information will be stored on a password protected computer. Future use of the stored data will be subject to further Research Ethics Review and approval if applicable. There's no reasonably foreseeable risks of harm or side-effects to the potential participants.*

## WILL I RECEIVE PAYMENT OR ANY INCENTIVES FOR PARTICPATING IN THIS STUDY?

*There will be no payment or reward offered, financial or otherwise for participation.*

## HOW WILL THE PARTICIPANT BE INFORMED OF THE FINDINGS / RESULTS OF THE STUDY?

*Report to school and presentations to interested parties (participants, school, district office); copies of manuscripts; thesis published.*

**Thank you for taking time to read this information sheet and for participating in this study.**

**CONSENT TO PARTICIPATE IN THIS STUDY**

I, ____________________________________ (participant name), confirm that the person asking my consent to take part in this research has told me about the nature, procedure, potential benefits and anticipated inconvenience of participation.

I have read (or had explained to me) and understood the study as explained in the information sheet. I have had sufficient opportunity to ask questions and am prepared to participate in the study. I understand that my participation is voluntary and that I am free to withdraw at any time without penalty (if applicable). I am aware that the findings of this study will be anonymously processed into a research report, journal publications and/or conference proceedings.

I agree to the recording of the insert specific data collection method.

I have received a signed copy of the informed consent agreement.

Full Name of Participant:

_________________________________________________________________

Signature of Participant: _____________________ Date: ___________________

Full Name(s) of Researcher(s): Lika Mohau

Signature of Researcher: Date: 6/5/2019

# ANNEXURE H:
# Participant assent form – Children

**<u>RESEARCH STUDY INFORMATION LEAFLET AND PARENTAL CONSENT FORM</u>**

**DATE**

**May 2019–July 2019**

**TITLE OF THE RESEARCH PROJECT**

*Enhancing the Teaching and Learning of Algebraic Expressions and Equations through Reasoning in Grade 9*

**RESEARCHERS NAME(S) AND CONTACT NUMBER:**

*Mohau Lika 2000029906 0738570570*

**FACULTY AND DEPARTMENT:** Education

**STUDYLEADER(S) NAME AND CONTACT NUMBER:**

Tlali MF
0833956691

**WHAT IS THIS RESEARCH PROJECT ALL ABOUT?**

*Improving the Learning of Mathematics (algebraic Expressions and Equations) through Reasoning in Grade 9. Finding ways through which grade 9 learners performance in algebraic expressions and equations can be improved. The proposition/assumption is that reasoning skills has the potential to enable the envisioned improvement of teaching for better results.*

**WHY HAVE YOU BEEN INVITED TO TAKE PART IN THIS RESEARCH PROJECT?**

*Learners' assessment tasks/learning activities and classroom interactive responses and questions based on the algebraic expressions and equations will serve as part of data. Learners' participation will be limited to them learning, not actual data collection, analysis etc.*

**WHO IS DOING THE RESEARCH?**

*Researcher - Mathematics teacher in grade 9. 2. Retired Mathematics teacher – this teacher worked at the school where this study is based. They taught Mathematics in grades 8-12 for a period of over 20 years. Their experience and knowledge of teaching* *and Mathematics will be educative. 3. Departmental head for Mathematics at the school - in addition to their knowledge and experience in the teaching of Mathematics, their role will mainly be for guidance, management (work control and monitoring) because this study will be aligned with normal school programmes. 4. Natural sciences and Technology teacher (s) - to afford them an opportunity to extend learning to other subjects. Natural sciences and Technology are closely related to Mathematics. 5. Mathematics teacher – a peer in the same phase who would also serve as Integrated Quality Management System (IQMS) peer.*

**HAS THE STUDY RECEIVED ETHICAL APPROVAL?**

This study has received approval from the Research Ethics Committee of UFS. A copy of the approval letter can be obtained from the researcher.

Approval number: *UFS-HSD2018/0644*

**WHAT WILL HAPPEN TO YOU IN THIS STUDY?**

*Learners will attend the after school classes from 14h30 to 15h30 on Mondays and Wednesdays for about 3 weeks during which the topics informing this study will be taught. They will be taught and observed by research team members. They will interact with teachers for deep understanding and do activities as per work schedule/programme of the Department of Education (DBE) in regard to the topics of the study and as planned by the research team. The results of learners' work and classroom interactive responses (written and oral) will be used as part of data. 2. The core research team members share labour: teach, observe, provide feedback, and support the actual teaching. They also assess and control the work collectively. They will be expected to participate during lessons as normal: perform class activities, answer questions asked on the topics informing the study, work with other learners, ask questions for deep understanding etc, as it normally happens during the teaching and learning processes.*

**CAN ANYTHING BAD HAPPEN TO YOU?**

*There are no risks or anything scary associated with the research. Learner participants will be advised to report to parents and the research team if possible should they fall sick or have reasons/commitments preventing them from carrying out the class activities during the course of the study.*

**CAN ANYTHING GOOD HAPPEN TO YOU?**

*Yes. The learner will have a hands on experience of how reasoning improves Mathematics learning and be able to share ideas with knowledgeable people on how conceptual Knowledge is achieved. An enhanced learning of algebraic expressions and equations; an improved performance in Mathematics and related subjects; an opportunity to be taught by a team of experienced and knowledgeable teachers.*

**WILL ANYONE KNOW YOU ARE PART OF THE STUDY?**

*No. The study is carried out on confidentiality on issues that require so.*

**WHO CAN YOU TALK TO ABOUT THE STUDY?**

Dr Moeketsi Tlali 0833956691

**WHAT IF YOU DO NOT WANT TO DO THIS?**

*You are free not to participate in the study or withdraw during the study when you are sick, have other commitments or are no longer willing to continue.*

**PLEASE RETURN**

**Name of child:**

____________________________________________________________

- Do you understand this research study and are you willing to take part in it? Yes ☐ No ☐
- Has the researcher answered all your questions? Yes ☐ No ☐
- Do you understand that you can withdraw from the study at any time? Yes ☐ No ☐
- I give the researcher permission to make use of the data gathered from my participation Yes ☐ No ☐

______________________________ ________________________

**Signature of Learner** **Date**

# ANNEXURE I:
# Letter of approval and conditions: School Management Team

**LE RENG SECONDARY SCHOOL**

**"WORK TO RISE"**

ENQ : THE PRINCIPAL
TEL : 051 924 2885

LE RENG SECONDARY SCHOOL
WORK TO RISE

PRIVATE BAG X 10
LADYBRAND
9745

02 MAY 2019

Dear Mr Lika MA

With reference to your letter requesting permission to conduct research at Le Reng Secondary school, the principal and the SMT have agreed to grant you the permission to conduct your research **after working hours, that is from 14H30**. We must remember that 7 working hours, that is learners' contact time for teaching and learning, must take priority and cannot be compromised.

We do understand the purpose of your research and we hope that the research you will embark on will help the school and learners especially when coming to Mathematics. Good luck in your studies.

We hope you shall find this in order.

Regards

MOKUNUTLU JJL

Signature

03/05/2019
Date

SCHOOL STAMP

LE-RENG SECONDARY SCHOOL
0 3 MAY 2018
DEPUTY PRINCIPAL I

# ANNEXURE J:
# Letter of approval and conditions:
# Free State Department of Education

Enquiries: KK Motshumi
Ref: Research Permission: MA Lika
Tel. 051 404 9283 / 9221 / 079 503 4943
Email: K.Motshumi@fseducation.gov.za

education
Department of Education
FREE STATE PROVINCE

4042 Manyatseng
Ladybrand
9745

Dear Mr. Lika

**APPROVAL TO CONDUCT RESEARCH IN THE FREE STATE DEPARTMENT OF EDUCATION**

This letter serves as an acknowledgement of receipt of your request to conduct research in the Free State Department of Education.

1. **Research Topic:** Enhancing the teaching and learning of Algebraic expressions and equations in grade 9.

   **Schools:** Lereng Secondary School in Motheo District.

   **Target Population**: 1 HOD responsible for mathematics, Senior phase mathematics subject advisor, 1 teacher(s) teaching grade 8/9 mathematics, 1 teacher(s) teaching technology in grade 8, 9. 1 teacher(s) teaching natural sciences in grade 8, 9. 1 retired teacher who use to teach mathematics and +- 40 grade 9 learners age 14 to 18.

2. **Period**: From date of signature until the 30th September 2019. Please note the department does not allow any research to be conducted during the fourth / academic quarter of the year nor during normal school hours.

3. Should you fall behind your schedule by three months to complete your research project in the approved period, you will need to apply for an extension.

4. The approval is subject to the following conditions:

   4.1 The collection of data should not interfere with the normal tuition time or teaching process.

   4.2 A bound copy of the research document or a CD, should be submitted to the Free State Department of Education, Room 319, 3rd Floor, Old CNA Building, Charlotte Maxeke Street, Bloemfontein.

   4.3 You will be expected, on completion of your research study to make a presentation to the relevant stakeholders in the Department.

   4.4 The attached ethics documents must be adhered to in the discourse of your study in our department.

5 Please note that costs relating to all the conditions mentioned above are your own responsibility.

Yours sincerely

DR JEM SEKOLANYANE
CHIEF FINANCIAL OFFICER

DATE: 12/06/2019

RESEARCH APPLICATION MA LIKA PERMISSION 21 MAY 2019 MOTHEO DISTRICT
Strategic Planning, Policy & Research Directorate
Private Bag X20565, Bloemfontein, 9300 - Room 318, Old CNA Building, 3rd Floor, Charlotte Maxeke Street, Bloemfontein
**Tel:** (051) 404 9283 / 9221 **Fax:** (086) 6678 678

Enquiries: KK Motshumi
Ref: Notification of research: MA Lika
Tel. 051 404 9221 / 079 503 4943
Email: K. Motshumi@fseducation.gov.za

education
Department of Education
FREE STATE PROVINCE

The Director: Motheo district

Dear Mr. Moloi

**NOTIFICATION TO CONDUCT RESEARCH PROJECT IN YOUR DISTRICT BY MA LIKA**

1. The above mentioned candidate was granted permission to conduct research in your district as follows:
2. **Research Topic:** Enhancing the teaching and learning of Algebraic expressions and equations in grade 9.

   **Schools:** Lereng Secondary School in Motheo District.

   **Target Population**: 1 HOD responsible for mathematics, Senior phase mathematics subject advisor, 1 teacher(s) teaching grade 8/9 mathematics, 1 teacher(s) teaching technology in grade 8, 9. 1 teacher(s) teaching natural sciences in grade 8, 9. 1 retired teacher who use to teach mathematics and +- 40 grade 9 learners age 14 to 18.
3. **Period**: From date of signature until the 30th September 2019. Please note the department does not allow any research to be conducted during the fourth / academic quarter of the year nor during normal school hours.
4. **Research benefits:** The research seeks to design a strategy that will enhance the teaching and learning of algebraic expressions and equations in Grade 9. The strategy aims at enriching the algebraic procedures with appropriate connections in order to develop the procedures into conceptual knowledge learners can comprehend at ease. It is therefore hoped to cut down on errors and misconceptions learners commit, hence improve their performance. The research also has a cross-cutting benefits to other related topics and subjects such as technology and natural sciences as well as other grades. Logistical procedures were met, in particular ethical considerations for conducting research in the Free State Department of Education.
5. Strategic Planning, Policy and Research Directorate will make the necessary arrangements for the researchers to present the findings and recommendations to the relevant officials in the district.

Yours sincerely
DR JEM SEKOLANYANE
Chief Financial Officer

DATE: 12/06/2019

RESEARCH APPLICATION FOR MA LIKA NOTIFICATION 21 MAY 2019, MOTHEO DISTRICT
Strategic Planning, Research & Policy Directorate
Private Bag X20565, Bloemfontein, 9300 - Old CNA Building, Room 319, 3rd Floor, Charlotte Maxeke Street, Bloemfontein
Tel: (051) 404 9283 / 9221 Fax: (086) 6678 678

# ANNEXURE K:
# Invitation letter to launch meeting

May $3^{rd}$ 2019

Dear Mr/ Mrs/Ms ________________________

**RESEARCH LAUNCH MEETING**

You are cordially invited to the Research launch meeting arranged as follows:

**Date: Monday $6^{th}$ May 2019**

**Venue: Leru Secondary School- Mathematics Laboratory**

**Time: 14h30**

I enclose herewith the research study information leaflet for details.

Please confirm your attendance: 0738570570

I look forward to meeting you.

Yours faithfully

Mohau Lika

# ANNEXURE L:
# Minutes of launch meeting

**The minutes of the launch meeting**
**6th May 2019**
**Venue: Leru Secondary School**
**Time 14h30**

**Present**: Mfetho (recording), Nono, Shana, Mama

**Agenda items**

1. Opening prayer and attendance register
2. Attendants’ introductions
3. Purpose of the meeting
4. Adoption of the research team
5. Charting a way forward
6. Closing prayer

### 1. Opening prayer and attendance register

The meeting was opened in the word of prayer led by the researcher. And attendance register.

### 2. Attendants’ introductions

The researcher introduced themselves as such, a Mathematics teacher who has been teaching the subject in Grade 8 and 9 for many years, 2009 to date, at Leru Secondary School. They then invited other attendants to introduce themselves detailing more particulars like them so that the team could know each other better than it did before the meeting. The other attendants introduced themselves in detail.

### 3. Purpose of the meeting

The researcher explained the purpose of the meeting as to seek the participation of the attendants in the research study, as explained in the Participant information leaflet (as attached to the research launch invitation letter). They also briefed the attendants about the role(s) the participants were expected to play.

### 4. Adoption of the research team

All attendants present in the launch meeting were readily available and eager to constitute the research team. They had already signed consent forms attached to the Participant information leaflet confirming their willingness to participate actively in the research. The researcher collected and filed the forms.

**5. Charting a way forward**

The team agreed on the dates for an exclusive workshop intended for conceptualisation of the study. The workshop was also earmarked to draw a plan of action to guide data generation activities. The meeting also agreed on ground rules, principles and values that would govern the team’s conduct in issues that included meetings’ attendance and time keeping, meetings’ chairing and duration, minutes recording, reporting, respect of individual opinion and viable communication channels and timeframes. The following principles and values were agreed to define and govern the team’s conduct and approach during the entire period of research: Punctuality, teamwork, self-discipline, honest, patience and determination and mutual respect.

**6. Closure**

The meeting was closed in a word of prayer led by Nono.

**Meeting adjourned at 16H05**

# ANNEXURE M:
# Minutes of the study conceptualisation workshop

**Minutes of the study conceptualisation workshop**
**Saturday 11th May 2019**
**Venue: Leru Secondary School**
**Time 09h00**

**Present**: Mfetho (recording), Nono (chair), Shana, Mama, Buti

**Agenda items**

1. Opening prayer and attendance register
2. Study overview (Conceptualisation of the study)
3. Plan of action
4. Closing prayer

### 1. Opening prayer and attendance register

The meeting was opened in the word of prayer led by Mama. And attendance register.

### 2. Study overview (Conceptualisation of the study)

Mfetho presented the study overview with the help of the projected concept map to ensure that the whole team could be on the same footing before engaging in iterative discussions for further clarity, and mapping out the plan of action as to how the activities of data generation were going to unfold. They started by reading the title and asked the other participants to explain what they were making of it- say explain the key words in it. Nono said enhancing had to do with "ho ntlafatsa" (to improve) on what is already existing and the explanation was taken on board by the team. They continued that reasoning in the title meant that there must be 'mabaka', something upon which our answer or computation is based upon. Mfetho pointed at the theoretical framework and explained the meaning of bricolage - improving what is already existing. In our case connecting the procedural knowledge with reasoning (constructs) in order to enhance conceptual learning and knowledge. They emphasised that helping learners to reason out the steps in procedural computations has in his case produced improved and enduring results. And that the major challenge is that only a few learners are able to keep pace with the instruction, maybe because the majority deems not to have basic requirements to utilise the approach (strategy). They shared that literature also supports conceptual understanding over procedural fluency. They then pointed at the methodology, that the study is framed in Participatory Action Research (PAR) whereby the team and learners (the researched) have to be seen participating actively (i.e. putting in detectable contribution in a form of actions) in the research. In our case, I think the implication is that we will need to plan together or take turns teaching, observing and marking in class, then convene as a team to reflect. They then pointed onto the method. And explained the Free Attitude Interview

(FAI) technique as a method that treats an interviewee as an equal to the interviewer, and that affords an interviewee an opportunity to respond freely to open-ended questions. This, implies that we should avoid to ask learners questions that channel their thoughts, and that we should respect and protect them to ensure that they freely give their best responses. Need be, we have to probe their responses systematically until we get explicit responses that will help us understand their challenges and how they want to solve them. The presenter, Mfetho, paused for participants' questions. There was none. they pointed onto the research team box and elaborated that the team envisaged for collecting relevant data for this study comprises practising mathematics teachers, retired mathematics teacher(s) for their highly valued experience, departmental head-mathematics for moderating, managing and controlling the content coverage- content coverage and scope should be aligned with the curriculum guidelines, technology and/or science teacher(s)-to tap from them the problems they are facing in their subjects due to lack or insufficient command of algebraic expressions and equations, and perhaps how they solve such challenges. Then after collecting data, it is hereby suggested that we analyse it using Critical Discourse Analysis (CDA) of professor Van Dijk that subdivide into analysis of text, analysis of discursive practices and analysis of social structures. The text refers to writings and language (oral) used amidst the instruction. Discursive practices are dysfunctional or bad practices that have become norms in our daily classroom conduct and instruction that need to be transformed. We will detail these subdivisions more when we get to analysis. The analysis will us recommend possible solution out of the findings, and those will inform the instructional strategy envisioned by this study that seeks to enhance the teaching and learning of algebra. Such instruction that shouldn't be fixed, should enable learners to achieve conceptual understanding, be proficient and able to navigate through own algebraic concept map utilising clear command of basic mathematics to connect, hence reason out the map network. This study draws from a presupposition that the strategy can be achieved if learners are taught to reason out computational steps, if assessment calls for reasoning, teachers' discussing new developments including reasoning of teaching algebra, teachers letting learners to be central in their own learning and would result in improved attitude towards algebra and maths at large, improved classroom instruction, differentiated learning in which teachers play supervisory role to guide learners' self-propelled learning, hence creation of more thinking tanks. Then the presenter thanked participants for their attentiveness and asked for further clarity questions. None.

After refreshment break, the participants (co-researchers) then discussed the contents of the concept map together with those of the papers titled "**An overview behind the conception of the study" and "Study frameworks"** that were handed-out during the launch meeting for attendants' familiarisation of how the study for which they have endeavoured to be part and collect data was conceived in the light of the underpinning frameworks. The participants then expressed their views about the study itself and how well they have conceptualised its objectives and frameworks.

## 3. Plan of action

After lunch break, the participants embarked on the plan of action whereby they agreed upon framing it into PAR's reflective steps, namely planning, action and observation, and reflection. The workshop (participants) also endorsed the suggestion that planning should be informed, hence go hand-in-hand with situation analysis. It was then planned that the participants working as bricolage research team, will source the listed (relevant) documents for analysis and reflection of their impact on glaringly inefficient teaching and learning of algebra in Grade 9. On their part, the participants (co-researchers) shared the challenges and issues they always encounter with the teaching and learning of algebra. Shana said his grade 9 learners lack required pre-knowledge so much that they often have to revert to grade 7 and 8 content to remind them basic knowledge they were supposed to be knowing. Nono said that algebra is a challenging topic not only to grade 9 learners, but the challenge starts from grade 8 through grade 12. Grade 12s too struggle with algebra. First, learners are not able to apply introductory/basic concepts taught in lower classes e.g. like terms. For instance, they fail to identify and differentiate between the like and unlike terms e.g. learners work out $2x^2 + 2x$ as the sum of like terms since they both have $x$. Learners cannot realise that the different exponents make $x$ and $x^2$ unlike terms. Every time it's like they are doing algebra for the first time. As a result, algebra consumes a lot of teaching time and makes us to lag behind curriculum schedule. Learners cannot take out common factors in expressions like $12x + x^2 + 6x$. They cannot multiply and divide expressions. Addition and subtraction is even worse because they cannot manipulate integers correctly. You have to explain over and over again. In my case they do equations better than expressions, maybe because of the right hand side and left hand side. They can at least realise that what is done on one side is done on the other side. But expressions are a serious problem. They cannot even name expressions, they cannot identify the monomials, binomials etc. They do not know the laws of exponents. Mama reiterated Nono's observation that equations are performed better than expressions. They have also realised the problem in fractional expressions. Learners compute fractional expressions as if they are solving equations. For example, they multiply and cancel terms in an expression with the lowest common multiple killing the fractional structure of the expression.

Mfetho also welcomed and reiterated the challenges and thanked participants for not shifting the blame. They also welcomed the fact that participants are agreeing with them that there are problems associated with the teaching and learning of algebraic expressions and equations as they have proposed. They jokingly said if participants could have said there were no problems/challenges associated with the topics, they could have packed and left the room since there would have not been a reason for continuing with the research- research commences with problem(s). It is always triggered by a problem. They said now that there were problems/challenges, the team will along the research confirm the challenges and probably notice other more challenges.

He said the team will then suggest solutions capable of addressing the challenges as the research progresses. When we have suggested the components of solution, the next task will be to determine the conditions under which the solution can work. That will be followed by looking out for the risks and threats that may undermine the success or achievement of the objectives in spite the execution of the components of solution within acceptable conditions. Then lastly we will have to suggest the indicators of success- i.e. the tangible evidence we hope to see after teaching learners using the strategy we are proposing. The evidence should confirm achievement of the research objectives. They also alerted the team that the study was confined within Leru secondary school. However it may be important to find from primary teachers of our feeder schools during reflections or otherwise where they suspect the challenges emanate. This will help us find the contextualised/localised solution for Maru region children.

To achieve this, we are going to work as this team to plan lessons that will enhance the teaching and learning of algebraic expressions and equations using reasoning. We will go to class as a team of say four of us if possible, three or at least two. One of us teaches while other(s) observe(s), team-teach, mark and make necessary interventions. In essence, we work as a team when in and outside class. After class observation, we will arrange time to sit in this form, reflect on the lesson, what went well and what did not go well. Then we plan another lesson taking cognisance of the strengths and weaknesses of the previous lesson. We agree on parts we need to strengthen. The cycle of teaching and observing followed by reflective sessions will continue for the next two-three weeks before June examinations commence. Reflections will also continue after marking summative (test and examination) work from which we will also be checking if our strategy is succeeding to enhance an enduring comprehension of algebra. Then the researcher asked the participants to ask questions for further clarity. The participants confirmed there was none.

The participants considered the workshop as one form of a focused discussion, the verbal contributions of which should inform the challenges and possible components of solution. They further set to utilise other platforms in which mathematics stakeholders meet to continue with the focused discussions on the problem (inefficient teaching and learning of algebra in Grade 9). The workshop also agreed on the baseline assessment before classroom teaching and observation since the topics were continuing from Grades 7 and 8; that the baseline will add value to document analysis and reflection.

It was finally planned that participants (co-researchers) should use different platforms on which they interact with learners and other stakeholders for informal interviewing, so as to elicit responses responding to the objectives of the study throughout the research period and beyond.

### 4. Closing prayer

The workshop was closed in a word of prayer led by Shana at 16h00.

# ANNEXURE N:
# Baseline test and exercise worksheets

**Name:**_____________________________

**Grade 9__** **May 2019**

## Algebraic Expressions and Equations Baseline Test

**INSTRUCTIONS**

1. Read all questions carefully before answering
2. Show all your workings
3. DO NOT use a calculator

1. Circle the correct answer between answers in column A and column B in the table below.

| | **A** | **B** |
|---|---|---|
| 1.1 a x a | $a^2$ | 2a |
| 1.2 b + b | $b^2$ | 2b |
| 1.3 $2^2 + 2^3$ | $2^5$ | $2^2 + 2^3$ |
| 1.4 $2^2 \times 2^3$ | $2^5$ | $2^6$ |
| 1.5 $(2^3)^2$ | $2^5$ | $2^6$ |
| 1.6 4-x | 3x | 4-x |
| 1.7 a + b | ab | b + a |
| 1.8 $y^1$ | y | 1 |
| 1.9 d | 1d | 1 |

**[9]**

2. Circle the correct options in the following multiple choice questions:

2.1 When a number $n$ is multiplied by 7, and then 6 are added to it, the result is 41. Which of these equations represents this relation?

A $7n \pm 6 = 41$
B $7n \times 6 = 41$
C $7n + 6 = 41$
D $7(n + 6) = 41$

2.2 Which of the following is true when $a$,$b$ and $c$ are different real numbers?

A $a - b = b - a$
B $a(b - c) = b(c - a)$
C $b - c = c - b$
D $ab = ba$

2.3 Which of the statements below represents the expression $3x-2$?

A Three less than twice a number
B Three times less twice a number
C Two less than three times a number
D Three times a number less than two

2.4 Complete: $(-3xy^2)^2 =$

A $-6x^2y^2$
B $-9x^2y^4$
C $9x^2y^4$
D $6x^2y^2$

2.5 Identify the statement that is NOT a law of exponents

A $a^x$ x $a^y$ = $a^{x+y}$
B $(a^x)^y$ = $a^{x+y}$
C $a^0$ = 1 (a≠0)
D $\frac{a^x}{a^y}$ = $a^{x-y}$

2.6 Determine the CORRECT match in the table below

| | Expression | Name |
|---|---|---|
| A | $x^2 + (45 + 2)$ | trinomial |
| B | $(x^2 + 45)$ | binomial |
| C | $2(x^2 + 45)$ | monomial |
| D | $2 + (x^2 + 45)$ | trinomial |

2.7 If $y = 3x^2 - 4x + 5$, calculate the value of $y$ when $x = -2$

A 9 B -15 C 1 D 25

**[7]**

3. Simplify and find the values of the expressions below where possible

3.1 $(m^3)^2$ x $(m^{-3})^2$ x $(m^3)^0$

___

___

___

(3)

3.2 $2a^2b$ x $2ab^3$ x $2ab$

___

___

___

(3)

3.3 $\frac{21xy^6}{7(xy^2)^3}$

(3)
**[9]**

4. Simplify the following expressions

4.1 xz + xy – 3xz + 5xy

(2)

4.2 $-2(x^2-2x-2+2x)$

(3)

4.3 $3y^2 - 4y + 3y - 2y^2$

(2)

4.4 3(a + 2b) – 4(b - 2a)

(3)
**[10]**

5.1 Simplify $\frac{2x^2+2x}{x}$

(2)

5.2 Divide $\mathbf{24x^3y^4 - 16x^3y^2 + 4xy}$ by **-4xy**

(3)

5.3 Subtract $\mathbf{(2x^2 -3x + 4)}$ from $\mathbf{(x^2 + 7x -9)}$

(3)
**[8]**

6. Consider the algebraic expression: **$4x - 9 + 5x^3 - 3x^2$**

6.1 What is the variable in this expression?

(1)

6.2 What is the coefficient of $x^2$?

(1)

6.3 What is the constant?

(1)

6.4 What is the degree of the expression?

(1)

6.5 Write the expression in descending order of powers of the variable.

(1)
**[5]**

7. Calculate the value of the following expression if **a =-2; b=3; c=-1**
**a +b -2c**

**[2]**

8. Solve for x

8.1 $5^x = 125$ (2)

8.2 $2^x = \frac{1}{64}$ (2)

8.3 $7^{x+1} = 1$ (2)

8.4 $3x - 1 = 5$ (2)

8.5 $2(x + 1) = 10$ (2)

8.6 $8x + 3 = 3x - 22$ (2)

8.7 $3(x + 6) = 12$ (2)

8.8 $2x - 5 = 5x + 16$ (2)

**[16]**

The end.

**ALGEBRAIC EXPRESSIONS AND EQUATIONS**

**Exercise 1**

1. Determine if the following expressions are monomials, binomials or trinomials

1.1 $-2x$ ________________________________________

1.2 $3(x-1)$ ________________________________________

1.3 $x-1$ ________________________________________

1.4 $x^2+4x-12$ ________________________________________

1.5 $2(x^2+4x-12)$ ________________________________________

1.6 $\frac{x}{4}+3$ ________________________________________

2. Solve the equations

2.1 $4x=20$

2.2 $3p=12$

2.3 $\frac{a}{4}$=7

2.4 $4x+7=1$

2.5 $4x - 2 = 3$

2.6 $\frac{y}{7} + 3 = 4$

**GRADE 9** **May 13th, 2019**

**ALGEBRAIC EXPRESSIONS AND EQUATIONS- Like terms**

**Exercise 2**

1. Simplify the following expressions by collecting the like terms

1.1 $-2x + 3 + 4x - 1$

1.2 $3(x - 1) + 2(x + 1)$

1.3 $4(x - 1) - x(2x + 3)$

1.4 $x(2x + 3) + x^2 + 4x - 12$

2. Solve the equations

2.1 $4x + 6 = 22$

2.2 $3(x - 1) = 12$

2.3 $2x + 4x = 18$

2.4 $3x - 5 - x = 7$

2.5 $4x - 2 = 3(x + 1)$

**GRADE 9** **May 15th, 2019**

**ALGEBRAIC EXPRESSIONS AND EQUATIONS**

**Exercise 3**

1. Simplify the following expressions by collecting the like terms

   1.1 $-2yx + 3y + 4yx - 2y$

   1.2 $a(a + 1) + 2(a - 3)$

   1.3 $(-3xy^2)^2 \times (-3x^{-2})^3$

   1.4 $\frac{21xy^6}{7(xy^2)^3}$

   1.5 $\frac{2x^2+4x}{2x}$

1.6 $(x-3)(x+4)$

1.7 $(y-5)^2$

1.8 $(5n+2)(n+4)$

2. Solve the equations

2.1 $5a+6=21$

2.2 $4(x-2)=2(x+1)$

**GRADE 9** **May 20th, 2019**

**ALGEBRAIC EXPRESSIONS AND EQUATIONS**

**Exercise 4**

0. Complete the following laws of exponents

A $a^x$ x $a^y$ = $a$——

B $(a^x)^y$ = $a$——

C $a$—— = 1 (a≠0)

D $\frac{a^x}{a^y}$ = $a$——

3. Simplify the following expressions

3.1 $(-3x^3y^2)^2$

3.2 $2a(a-2) - a(a-3)$

3.3 $\frac{2x^2y-6xy+4xy^2}{2xy}$

3.4 $(2y+3)^2$

4. Solve the equations

4.1 $3^x = 81$

4.2 $4^{x+1} = 64$

4.3 $4^{2x-1} = 64$

4.4 $2^x = \frac{1}{32}$

4.5 $7^{x-2} = 1$

4.6 $2(x+3) = -3(x-2)$

**GRADE 9** **May 27th, 2019**

**ALGEBRAIC EXPRESSIONS AND EQUATIONS**

**Exercise 5**

1. Basic knowledge

   Define and give examples of the factors of 8

   ______________________________________________

   ______________________________________________

   Define and give examples of the factors of 12

   ______________________________________________

   ______________________________________________

   Determine the Highest Common Factors (HCFs) of the following pairs:

   $6 \text{ and } 12$ __________

   $6 \text{ and } 15$ __________

   $8x \text{ and } 12$ __________

   $8x \text{ and } 12x$ __________

   $x^2 \text{ and } x^5$ __________

   $a^4 \text{ and } a^3$ __________

   $b^3 \text{ and } b^2$ __________

   $a^4b^3 \text{ and } a^5b^2$ __________

   $6a^4b^3 \text{ and } 15a^5b^2$ __________

5. Factorise the following expressions

   5.1 $8x + 12$

5.2 $6x^2 - 4x$

5.3 $2a - 2a^2$

5.4 $10x^2 + 5x - 15$

5.5 $3ab^2 - 6a^2b - 15ab$

5.6 $2(a + b) + c(a + b)$

5.7 $2a(a - 2) - a(a - 2)$

6. Simplify the following expressions

6.1 $2(a - 1) - a$

6.2 $3(a + b) - 2(a + b)$

7. Solve the equation $2(a - 1) - a = 5$

**GRADE 9** **May 28th, 2019**

**ALGEBRAIC EXPRESSIONS AND EQUATIONS- the difference of two squares**

**Exercise 6**

2. Basic knowledge

   List ALL perfect squares between 1 and 150

   ______________________________________________________________

   Expand and simplify the following expressions

   $(x-3)(x+3)$

   $(x-4)(x+4)$

   $(x+5)(x-5)$

   $(a-b)(a+b)$

   $(2x+5)(2x-5)$

8. Factorise the following expressions

   8.1 $x^2-9$

8.2 $4a^2 - 25$

8.3 $36y^2 - 49z^2$

8.4 $b^4 - 16$

8.5 $-m^2 + 9$

8.6 $2x^2 - 18$

9. Use the difference of two squares to **find the values** of the following (Do not use the calculator)

9.1 $11^2 - 9^2$

9.2 $80^2 - 70^2$

10. Solve the equations

10.1 $(x + 3)(x - 3) = 0$

10.2 $x^2 - 9 = 0$

**GRADE 9** **May 29th, 2019**

**ALGEBRAIC EXPRESSIONS AND EQUATIONS-factorising trinomials**

**Exercise 7**

3. Basic knowledge

Write down the pairs of factors for the following numbers

e.g. $\mathbf{12} = 1 \times 12 = 2 \times 6 = 3 \times 4 = -1 \times -12 = -2 \times -6 = -3 \times -4$

$6 =$ ______________________________________________

$8 =$ ______________________________________________

$-14 =$ ______________________________________________

$10 =$ ______________________________________________

$-24 =$ ______________________________________________

Add each pair of factors

e.g. $1 + 12 = \mathbf{13}$ ; $2 + 6 = \mathbf{8}$ ; $3 + 4 = \mathbf{7}$ ; $-1 + -12 = \mathbf{-13}$ ; $-2 + -6 = \mathbf{-8}$ ; $-3 + -4 = \mathbf{-7}$

______________________________________________

______________________________________________

______________________________________________

______________________________________________

______________________________________________

Expand and simplify the following expressions

$(x + 2)(x + 3)$

$(x-4)(x+6)$

$(x-7)(x+2)$

$(x-5)(x-2)$

11. Factorise the following expressions fully

11.1 $x^2+5x+6$

11.2 $x^2+2x-2$

11.3 $x^2-5x-14$

11.4 $x^2 - 7x + 10$

11.5 $3x^2 + 15x + 18$

11.6 $7x^2 - 28$

12. Work out the value of $3x^2 + 15x + 18$ when $x = -2$

13. Solve the equations

13.1 $x^2 + 7x + 12 = 0$

13.2 $2x^2 - 14x + 20 = 0$

13.3 $5x^2 - 45 = 0$

**GRADE 9** **July 19th, 2019**

**ALGEBRAIC EXPRESSIONS AND EQUATIONS-algebraic fractions**

**Exercise 8**

0. Basic knowledge

   Work out the following:

   $\frac{2}{5}+\frac{1}{2}$

   $\frac{6}{7}-\frac{3}{4}$

1. Simplify the following algebraic fractions

a. $\frac{5a}{7}-\frac{a}{4}$

b. $\frac{2y}{5}+\frac{y}{2}$

c. $\frac{2}{3}x+\frac{x+3}{4}$

d. $\frac{2x-1}{5}-\frac{x+2}{3}$

e. $\frac{4x}{3}-\frac{x-2}{2}$

2. Solve the equations

a. $\frac{2a}{5}+\frac{a}{2}=\frac{1}{2}$

b. $\frac{4x+2}{5}-\frac{3x-1}{2}=7$

c. $\frac{x-2}{8}=\frac{3x}{4}$

# ANNEXURE O:
# Formal test and summative assessment

**NAME:** ____________________________________________

**GRADE 9___**

**TERM2 TEST**

**MARKS: 60**

**MAY 2019**

**Question 1**

Solve the following equations

1.1 $3x + 5 = 14$ (3)

_______________________________________________________________________________

_______________________________________________________________________________

_______________________________________________________________________________

_______________________________________________________________________________

1.2 $5x - 14 = 2x + 1$ (3)

_______________________________________________________________________________

_______________________________________________________________________________

_______________________________________________________________________________

_______________________________________________________________________________

1.3 $2(p - 5) = 4(2p + 3)$ (4)

_______________________________________________________________________________

_______________________________________________________________________________

_______________________________________________________________________________

_______________________________________________________________________________

_______________________________________________________________________________

1.4 $\frac{a}{2}+ \frac{a}{3}=1$ (4)

1.5 $5^x+8=33$ (3)

**[17]**

**Question 2**

Simplify the following expressions

2.1 $-3a+3+4a-5$ (3)

2.2 $x(x-1)+2(x+1)$ (4)

2.3 $(-3xy^2)^2 \times x^2y^{-4}$ (4)

2.4 $\frac{-2x^2+4x}{-2x}$ (3)

______________________________________________________________________________

______________________________________________________________________________

______________________________________________________________________________

2.5 $\sqrt{5a^6 + 4a^6}$ (3)

______________________________________________________________________________

______________________________________________________________________________

______________________________________________________________________________

______________________________________________________________________________

2.6 $(x-5)(x+4)$ (3)

______________________________________________________________________________

______________________________________________________________________________

______________________________________________________________________________

______________________________________________________________________________

2.7 $(y-3)^2$ (3)

______________________________________________________________________________

______________________________________________________________________________

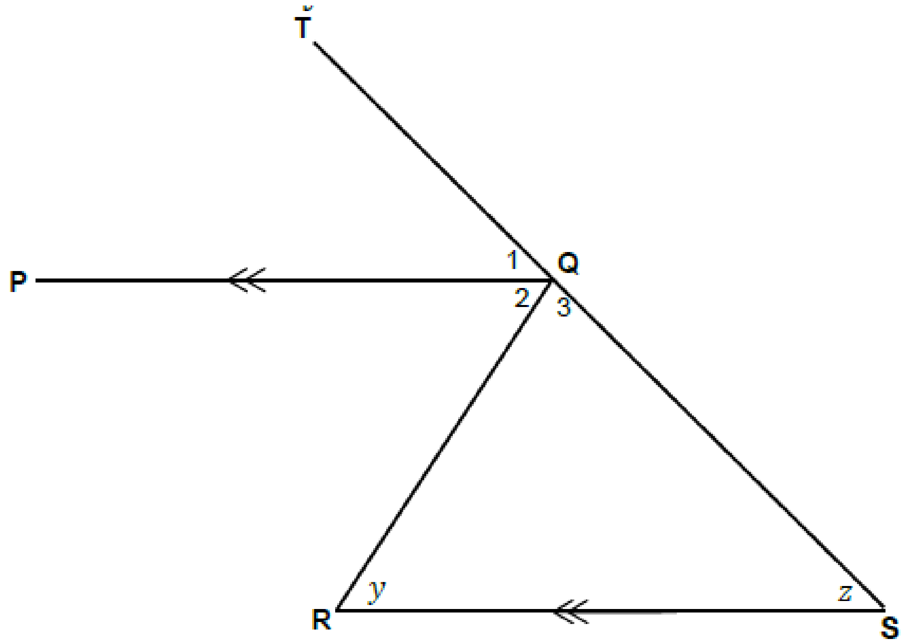
______________________________________________________________________________

______________________________________________________________________________

**[23]**

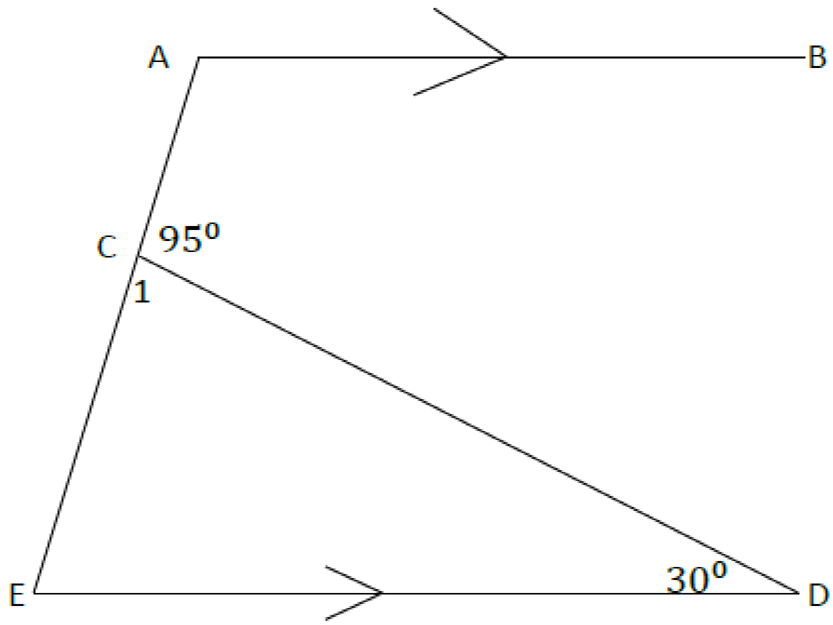
**Question 3**

3.1 In the figure below PQ || RS. Q̂1, Q̂2 and Q̂3 are equal to $2x$, $3x$ and $4x$ respectively. R̂=$y$ and Ŝ=$z$. Calculate the values of $x, y\ and\ z$. Give reasons for your statements. (11)




| Statement | Reason |
|---|---|
| | |
| | |
| | |

3.2 In the figure below AB || ED, $A\hat{C}D$=95° $and$ $\angle D$=30°. Determine the sizes of Ê and Â. Give reasons for your statements. (9)




| Statement | Reason |
|---|---|
| | |
| | |

**[20]**

**GRADE 9** **July 31st, 2019**

**SUMMATIVE ASSESSMENT** **MARKS: 24**

ALGEBRAIC EXPRESSIONS AND EQUATIONS

0. Briefly describe the difference between an expression and an equation.

______________________________________________________________________

______________________________________________________________________

**[2]**

1. Simplify the following expressions

a. $3(x-1)+2(x+1)$ (2)

b. $(x-3)(x+4)$ (3)

c. $\frac{2x-1}{5}-\frac{x+2}{3}$ (3)

**[8]**

2. Factorise the following expressions completely

    a. $4x^2 - 25$ (2)

    b. $2x^2 + 2x - 12$ (3)

**[5]**

3. Solve the equations

    a. $3x = -6$ (1)

    b. $4^{2x+1} = 64$ (3)

c. $(x-11)(x+3)=0$ (3)

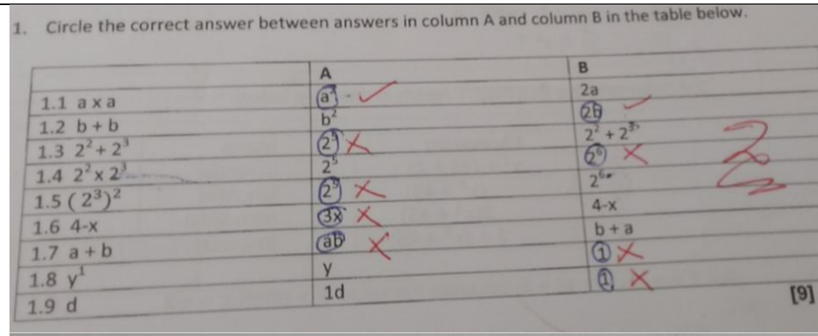
**[7]**

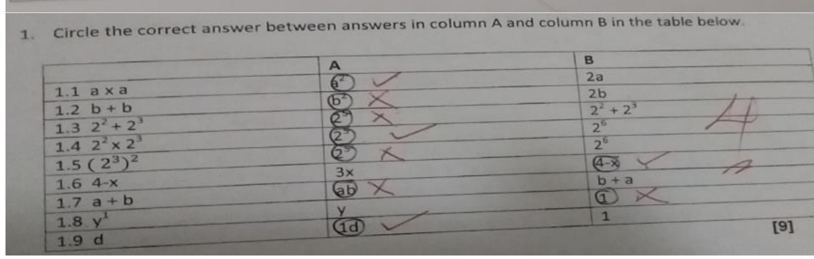
4. Use the difference of two squares to **find the value** of $80^2-70^2$ (Do NOT use the calculator)

**[2]**

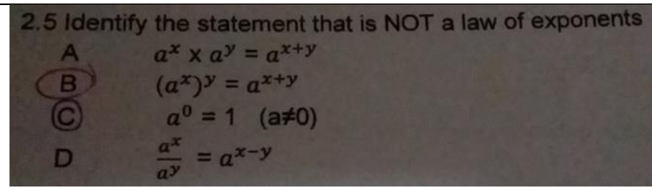
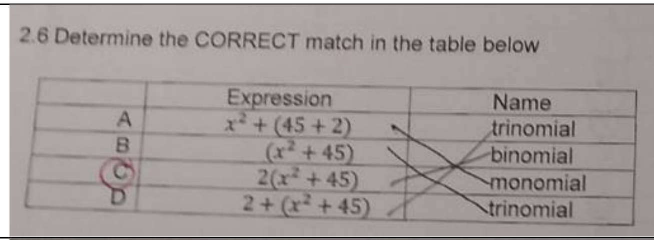
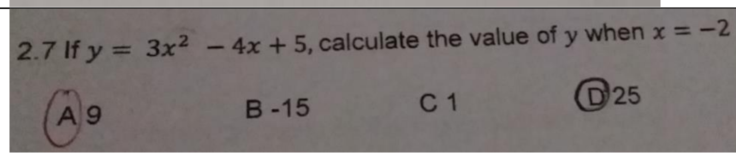
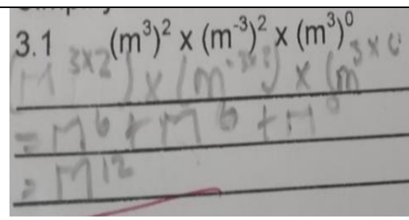
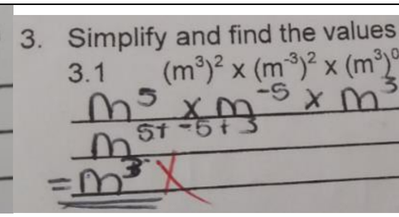
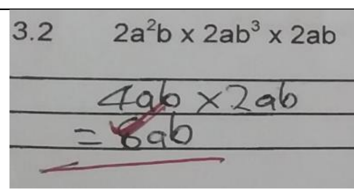
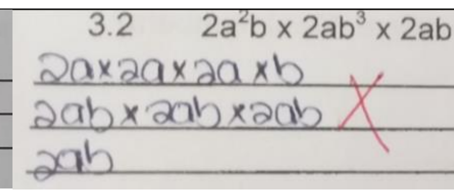
**THE END**

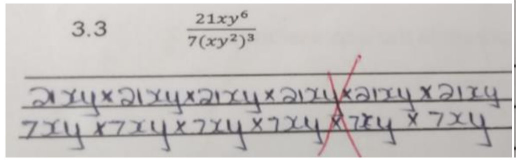
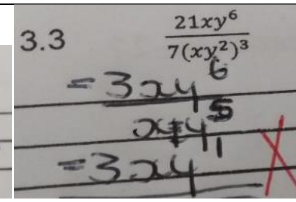
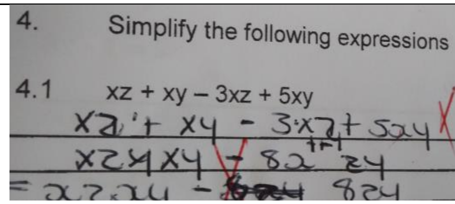
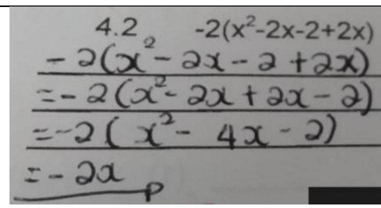
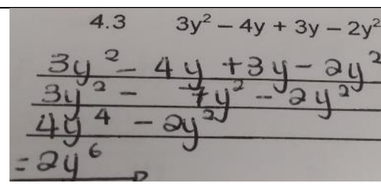
# ANNEXURE P:
# Baseline test script error analysis

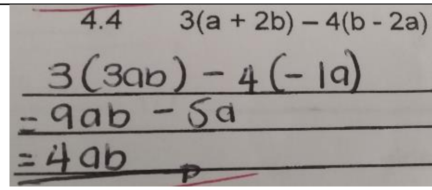
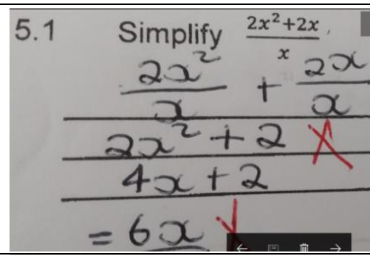
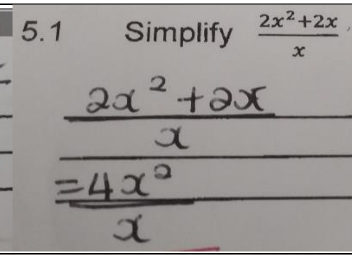
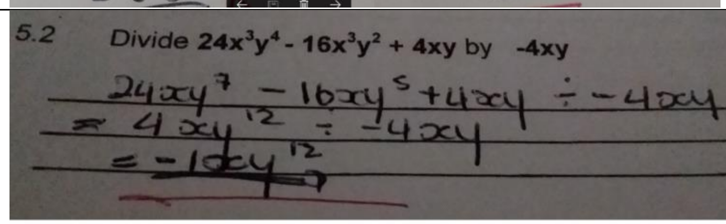
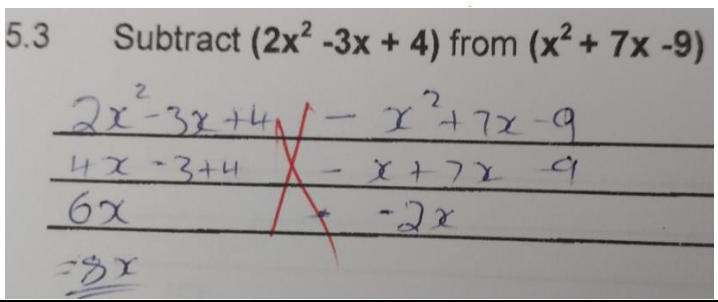
<table>
<tr><td colspan="7">Population = 37</td></tr>
<tr><td colspan="7">n = 32</td></tr>
<tr><th>Item</th><th>Learners codes</th><th>Concept and skill</th><th>Response samples</th><th>Common errors</th><th>Number</th><th>%</th></tr>
<tr><td>1.1<br>1.2</td><td>1;3;5;7;10;11;12;<br>15;17;18;19;21;<br>22;24;25;26;27;<br>28;29;32</td><td>Variable operations</td><td rowspan="3">

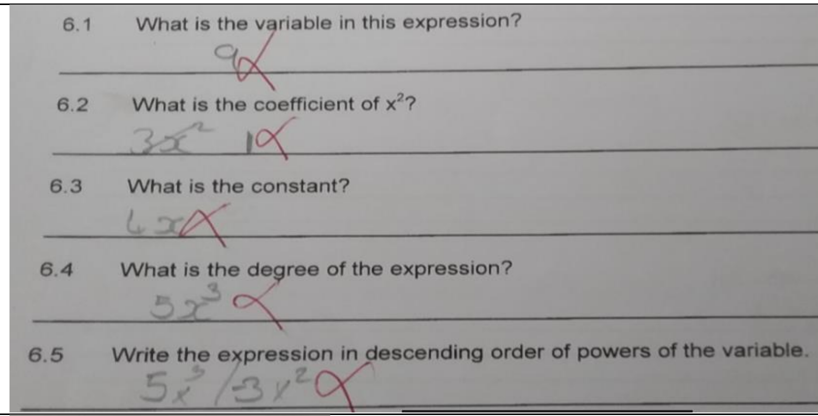
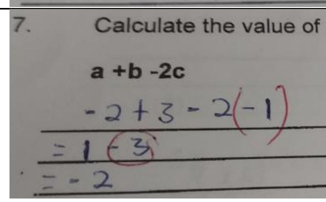
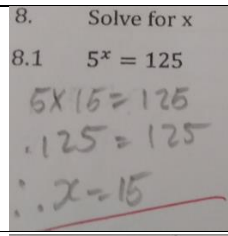
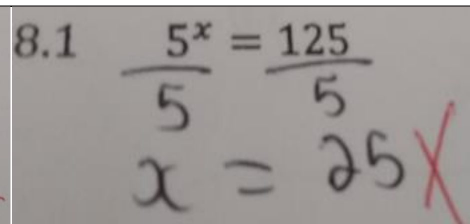
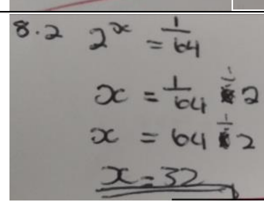
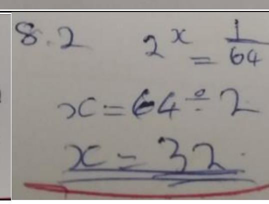
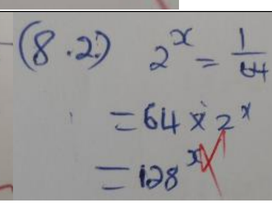

</td><td>Could not differentiate between additive and multiplicative operations of numbers and variables</td><td>20</td><td>63%</td></tr>
<tr><td>1.3<br>1.4<br>1.5</td><td>1;2;3;5;8;9;11;12<br>;13;14;15;16;17;<br>18;19;20;21;22;<br>23;24;25;27;28;<br>29;32</td><td>Application of laws of exponents and variable operations</td><td>Could not apply the laws of exponents consistently; confusion of the laws esp. the application of distributive rule</td><td>26</td><td>81%</td></tr>
<tr><td>1.6<br>1.7</td><td>1;2;3;4;5;6;7;8;9;<br>10;11;12;13;14;<br>15;16;17;18;19;<br>20;21;22;23;24;<br>25;26;27;28;29;<br>32</td><td>Identifying and collecting like terms</td><td>Could not identify and collect like terms; confuse like terms (additive operations) with factors (multiplicative operations)</td><td>30</td><td>94%</td></tr>
</table>

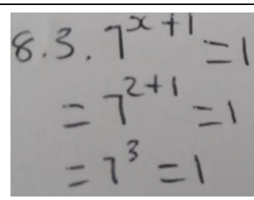
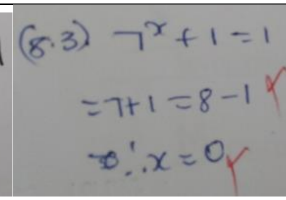
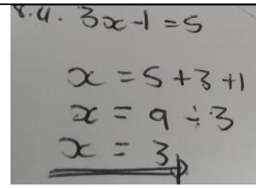
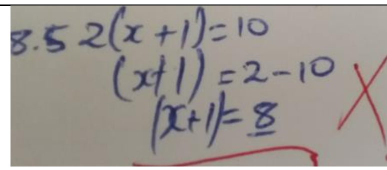
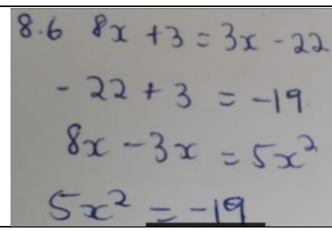
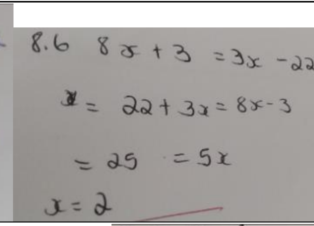
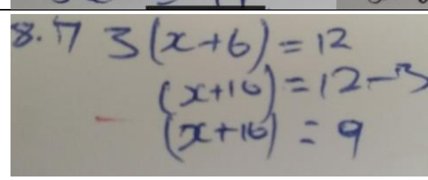
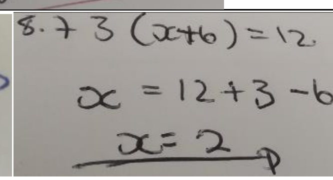
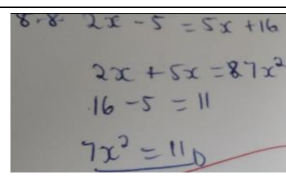
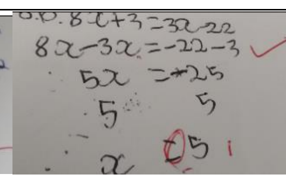
| | | | | | | |
|---|---|---|---|---|---|---|
| 1.8<br>1.9 | 1;2;3;4;5;7;9;10;<br>11;12;13;15;17;<br>18;19;20;21;22;<br>24;25;26;27;28;<br>29;32 | Symbolic notation | | Lack of consistency and confusion | 25 | 78% |
| 2.1 | 1;2;3;4;5;7;8;9;1<br>0;11;12;13;14;15<br>;16;17;18;19;20;<br>21;22;23;24;25;<br>26;27;28;29;30;<br>31;32 | Algebraic language | 2.1 When a number $n$ is multiplied by 7, and then 6 are added to it, the result is 41. Which of these equations represents this relation?<br>A $7n \pm 6 = 41$<br>B $7n \times 6 = 41$<br>C $7n + 6 = 41$<br>D $7(n + 6) = 41$ | Could not interpret the texts and symbolic notation for correct conversions (symbols ↔word expressions) | 21 | 66% |
| 2.2 | 1;2;3;5;7;9;10;11<br>;12;13;14;15;16;<br>17;18;19;20;22;<br>23;24;25;26;27;<br>28;29; 30;31;32 | Properties of numbers and variables | 2.2 Which of the following is true when $a$,$b$ and $c$ are different real numbers?<br>A $a - b = b - a$<br>B $a(b - c) = b(c - a)$<br>C $b - c = c - b$<br>D $ab = ba$ | Could not apply commutative and associative properties of numbers using variables | 28 | 88% |
| 2.3 | 1;2;3;4;5;7;8;9;1<br>0;11;12;13;14;15<br>;16;17;18;19;20;<br>21;22;23;24;25;<br>26;27;28;29;30;<br>31;32 | Algebraic language | 2.3 Which of the statements below represents the expression $3x-2$?<br>A Three less than twice a number<br>B Three times less twice a number<br>C Two less than three times a number<br>D Three times a number less than two | Could not interpret the texts and symbolic notation for correct conversions (symbols ↔word expressions) | 21 | 66% |
| 2.4 | 1;2;3;5;8;9;11;12<br>;13;14;15;16;17;<br>18;19;20;21;22;<br>23;24;25;27;28;<br>29;32 | Application of laws of exponents and variable operations | 2.4 Complete: $(-3xy^2)^2 =$<br>A $-6x^2y^2$<br>B $-9x^2y^4$<br>C $9x^2y^4$<br>D $6x^2y^2$ | Could not apply the laws of exponents consistently; confusion of the laws esp. the application of distributive rule | 26 | 81% |

| | | | | | | |
|---|---|---|---|---|---|---|
| 2.5 | 1;2;3;5;8;9;11;12;13;14;15;16;17;18;19;20;21;22;23;24;25;27;28;29;32 | Application of laws of exponents and variable operations | <br>2.5 Identify the statement that is NOT a law of exponents<br>A $a^x \times a^y = a^{x+y}$<br>B $(a^x)^y = a^{x+y}$<br>C $a^0 = 1$ (a≠0)<br>D $\frac{a^x}{a^y} = a^{x-y}$ | Could not apply the laws of exponents consistently; confusion of the laws esp. the application of distributive rule | 26 | 81% |
| 2.6 | 2;3;5;7;8;9;10;11;12;13;14;15;17;18;19;21;22;23;24;25;27;28;29;32 | Expressions naming rules | <br>2.6 Determine the CORRECT match in the table below<br>Expression / Name<br>A $x^2 + (45 + 2)$ — trinomial<br>B $(x^2 + 45)$ — binomial<br>C $2(x^2 + 45)$ — monomial<br>D $2 + (x^2 + 45)$ — trinomial | Could not interpret the question stem; could not use rules to establish the number of terms in expressions | 24 | 75% |
| 2.7 | 1;2;3;4;5;6;12;13;15;16;17;18;19;20;21;22;23;24;25;26;27;28;29;32 | Substitution and numeracy | <br>2.7 If $y = 3x^2 - 4x + 5$, calculate the value of $y$ when $x = -2$<br>A 9 B -15 C 1 D 25 | Could not use brackets to maintain values of expressions; could not operate integers correctly | 24 | 75% |
| 3.1 | 1;2;3;5;8;9;11;12;13;14;15;16;17;18;19;20;21;22;23;24;25;27;28;29;32 | Application of laws of exponents and variable operations | <br>3.1 $(m^3)^2 \times (m^{-3})^2 \times (m^3)^0$<br><br>3. Simplify and find the values<br>3.1 $(m^3)^2 \times (m^{-3})^2 \times (m^3)^0$ | Could not apply the laws of exponents consistently; confusion of the laws especially the application of distributive rule | 26 | 81% |
| 3.2 | 1;2;3;5;8;9;11;12;13;14;15;16;17;18;19;20;21;22;23;24;25;27;28;29;32 | Application of laws of exponents and variable operations | <br>3.2 $2a^2b \times 2ab^3 \times 2ab$<br><br>3.2 $2a^2b \times 2ab^3 \times 2ab$ | Could not apply the laws of exponents consistently; confusion of the laws especially the application of distributive rule | 26 | 81% |

| | | | | | | |
|---|---|---|---|---|---|---|
| 3.3 | 1;2;3;5;8;9;11;12;13;14;15;16;17;18;19;20;21;22;23;24;25;27;28;29;32 | Application of laws of exponents and variable operations |   | Could not apply the laws of exponents consistently; confusion of the laws especially the application of distributive rule | 26 | 81% |
| 4.1 | 1;2;3;4;5;6;7;8;9;10;11;12;13;14;15;16;17;18;19;20;21;22;23;24;25;26;27;28;29;32 | Identifying and collecting like terms |  | Could not identify and collect like terms; confuse like terms (additive operations) with factors (multiplicative operations) | 30 | 94% |
| 4.2 | 1;2;3;4;5;6;7;8;9;10;11;12;13;14;15;16;17;18;19;20;21;22;23;24;25;26;27;28;29;32 | Distributive rule and like terms |  | Could not apply the distributive rule; Could not identify and collect like terms; could not operate integers correctly | 30 | 94% |
| 4.3 | 1;2;3;4;5;6;7;8;9;10;11;12;13;14;15;16;17;18;19;20;21;22;23;24;25;26;27;28;29;32 | Identifying and collecting like terms |  | Could not identify and collect like terms; confuse like terms (additive operations) with factors (multiplicative operations) | 30 | 94% |

| | | | | | | |
|---|---|---|---|---|---|---|
| 4.4 | 1;2;3;4;5;6;7;8;9; 10;11;12;13;14; 15;16;17;18;19; 20;21;22;23;24; 25;26;27;28;29; 32 | Distributive rule and like terms |  | Could not apply the distributive rule; Could not identify and collect like terms; could not operate integers correctly | 30 | 94% |
| 5.1 | 1;2;3;4;5;6;7;8;9; 10;11;13;14;15; 16;17;19;20;21; 22;23;24;25;26; 27;28;29;30;31; 32 | Distributive rule and Application of laws of exponents |   | Could not apply the distributive rule and laws of exponents | 30 | 94% |
| 5.2 | 1;2;3;4;5;6;7;8;9; 10;11;13;14;15; 16;17;19;20;21; 22;23;24;25;26; 27;28;29;30;31; 32 | Distributive rule and Application of laws of exponents |  | Could not apply the distributive rule and laws of exponents | 30 | 94% |
| 5.3 | 1;2;3;4;5;7;8;9; 10;11;12;13;14; 15;16;17;18;19; 20;21;22;23;24; 25;26;27;28;29; 30;31;32 | Algebraic language and like terms |  | Could not interpret the texts and symbolic notation for correct conversions (symbols ↔word expressions) | 21 | 66% |

| | | | | | | |
|---|---|---|---|---|---|---|
| 6.1<br>6.2<br>6.3<br>6.4<br>6.5 | 1;2;3;4;5;7;8;9;<br>10;11;12;13;14;<br>15;16;17;18;19;<br>20;21;22;23;24;<br>25;26;27;28;29;<br>30;31;32 | Algebraic language |  | Could not interpret the texts and symbolic notation for correct conversions (symbols ↔word expressions) | 21 | 66% |
| 7 | 1;2;3;4;5;6;12;13<br>;15;16;17;18;19;<br>20;21;22;23;24;<br>25;26;27;28;29;<br>32 | Substitution and numeracy |  | Could not use brackets to maintain values of expressions; could not operate integers correctly | 24 | 75% |
| 8.1 | 1;2;3;5;6;8;9;10;<br>11;13;15;16;17;<br>18;19;20;21;22;<br>23;24;25;26;27;<br>28;29;32 | Numeric factoring (mental maths) and exponents comparison |  | Could not apply mental Mathematics to factor multiples for exponents comparison | 26 | 81% |
| 8.2 | 1;2;3;5;6;8;9;10;<br>11;13;15;16;17;<br>18;19;20;21;22;<br>23;24;25;26;27;<br>28;29;32 | Numeric factoring (mental maths) and exponents comparison |  | Could not apply mental Mathematics to factor multiples for exponents comparison | 26 | 81% |

| | | | | | | |
|---|---|---|---|---|---|---|
| 8.3 | 1;2;3;5;6;8;9;10; 11;13;15;16;17; 18;19;20;21;22; 23;24;25;26;27; 28;29;32 | Numeric factoring (mental maths) and exponents comparison |   | Could not apply mental Mathematics to factor multiples for exponents comparison | 26 | 81% |
| 8.4 | 2;3;4;5;6;7;8;9; 10;11;15;16;17; 18;19;20;21;22; 23;25;26;27;28; 29;32 | Inverse operations |  | Could not operate the additive and multiplicative inverses for elimination (purposeful usage of identities) | 25 | 78% |
| 8.5 | 1;2;3;4;5;6;7;8;9; 10;11;15;16;17; 18;19;20;21;22; 23;25;26;27;28; 29;31;32 | Distributive rule and inverse operations |  | Could not apply the distributive rule and operate the inverses purposefully | 27 | 84% |
| 8.6 | 1;2;3;4;5;6;7;8;9; 10;11;15;16;17; 18;19;20;21;22; 23;24;25;26;27; 28;29;32 | Inverse operations and like terms |   | Could not identify and operate like terms on different sides of equations | 27 | 84% |
| 8.7 | 1;2;3;4;5;6;7;8;9; 10;11;15;16;17; 18;19;20;21;22; 23;25;26;27;28; 29;31;32 | Distributive rule and inverse operations |   | Could not apply the distributive rule and operate the inverses purposefully | 27 | 84% |
| 8.8 | 1;2;3;4;5;6;7;8;9; 10;11;15;16;17; 18;19;20;21;22; 23;24;25;26;27; 28;29;32 | Inverse operations and like terms |   | Could not identify and operate like terms on different sides of equations | 27 | 84% |

# ANNEXURE Q:
# Confirmation of editing and proofreading


## Nicolene Barnard
## Proofreading and Technical Editing

PO Box 26959, Langenhovenpark, 9330 | 073 339 7739 |
Nicolene.Barnard1@gmail.com


29 June 2021

CONFIRMATION OF EDITING

I hereby confirm that I have done the technical layout and editing for the following thesis:

| | |
|---|---|
| Student: | Mohau Armstrong Lika |
| Title: | Enhancing the teaching and learning of algebraic expressions and equations through reasoning in grade 9 |
| Degree: | Philosophiae Doctor in Education |
| Department: | Faculty of Education, University of the Free State |

My work for the student included the technical layout of the document, as well as language editing for grammar, punctuation, spelling, and sentence structure. I tried to keep as much as possible of the student's own writing style while making sure that the student's intended meaning was not altered in the editing process. I also checked the list of references making sure that dates, spelling, and names used in the text are consistent with those listed in the reference list.

I have a B.Bibl. (Hons.) Degree and have been working as a cataloguer and librarian for 29 years. I am an expert in the field of bibliographic information and resources. I have also completed a 10-week Copy-Editing course at the University of Cape Town.

Disclaimer: The ultimate responsibility for accepting or rejecting the changes and recommendations rests with the student and I cannot be held responsible for any layout or language issues that might have emerged as a result of subsequent amendments to the text.

Yours sincerely,

Nicolene Barnard

## ANNEXURE R:
## Confirmation of academic formatting and language check

02 December 2022

To whom it may concern

RE: Academic Formatting and basic language check for Doctoral Degree candidate, Mohau Armstrong Lika, from the University of the Free State.

This letter serves as confirmation that I, Cindy Schoeman of CS Language Solutions, completed part of the formatting (table of contents and page layout) and performed a very basic language check of the PhD Research Report; 'ENHANCING THE TEACHING AND LEARNING OF ALGEBRAIC EXPRESSIONS AND EQUATIONS THROUGH REASONING IN GRADE NINE' and that it was done so without any outside assistance.

Please feel free to get in touch with me at 076 381 8999 or at cslanguagesolutions@gmail.com regarding any queries or concerns. Kind Regards,

Cindy Schoeman

CS Language Solutions