

THE EFFECTS OF PEDAGOGICAL CONTENT KNOWLEDGE IN LEARNING GRADE 12 MATHEMATICS USING PROBLEM-BASED LEARNING

By

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DECLARATION

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I, **Yudvir Bhagwonparsadh**, declare that the dissertation submitted for the Philosophiae Doctor Thesis at the University of the Free State is my independent work and has not previously been submitted to another university/faculty. Furthermore, I now cede the copyright to the University of the Free State.

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DEDICATION

This dissertation is dedicated to my sons, Siam-vir Bhagwonparsadh and Sai-vir Bhagwonparsadh, for the unconditional love and support that they have bestowed upon me throughout my studies.

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ABSTRACT

This study investigated the effects of teacher's pedagogical content knowledge in learning Grade 12 Mathematics using problem-based learning (PBL) in two selected secondary schools in the Amajuba district in KwaZulu-Natal. The problem behind the poor performance of Grade 12 Mathematics learners in the National Senior Certificate (NSC) for 2019, 2020, and 2021 is that most learners find it difficult to analyse and evaluate problem-based questions because they may be taught by some unqualified Mathematics teachers with insufficient MPCK and their low knowledge of subject content knowledge and low pedagogy. This study used Vygotsky's social constructivist theory and an inductive, exploratory qualitative case study approach. Data was collected by posing semi-structured interview questions to Mathematics teachers and open-ended questions to focus groups on learners and observing how Mathematics teachers interact with learners in the classroom. This was done to answer the "what" and "how" questions to generate theoretical insights into the phenomena under review. The findings suggest that teachers' MPCK equipped them to use inclusive educational strategies for teaching and learning Grade 12 mathematics, and had a positive outcome on learners' performances. Furthermore, the study's findings proposed that Mathematics teachers should be masters of the content, producing learning content to cognitively stimulate learners to work independently and strive for improved achievements in their current learning. More so, Mathematics teachers should be skilled at applying various teaching and learning methodologies appropriate to the learning abilities of all learners in the classroom so that learners can successfully acquire the content. The success of Grade 12 Mathematics learners adequately learning Mathematics depends on, amongst others, the teachers' ability to facilitate lessons by concretising abstract content using interactive activities in collaborative group learning. The study recommends that Grade 12 Mathematics teachers need training in using PBL as an instructional approach to teaching Mathematics. DBE needs to review the scope of the Grade 12 CAPS document so that PBL, as the preferred instructional strategy, can be adopted in teaching and learning Mathematics.

Keywords: Mathematics pedagogical content knowledge (MPCK); Problem-based learning (PBL), Specialised mathematical knowledge (SMK), Instructional strategy, Learning pedagogy, Scaffolding knowledge.

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LIST OF ABBREVIATIONS

Abbreviation	Word
NSC	National Senior Certificate
CAPS	Curriculum And Assessment Policy Statement
FET	Further Education and Training
DBE	Department Of Basic Education
MPCK	Mathematics Pedagogical Content Knowledge
SMT	School Management Team
PBL	Problem-Based Learning
MkiT	Mathematical Knowledge in Learning
ZPD	Zone of proximal development
MKft	Mathematical Knowledge for Teaching

CHAPTER 1: INTRODUCTION AND BACKGROUND OF THE STUDY

1.1 Introduction

International scholars avow that Mathematics pedagogical content knowledge (MPCK), or the "how and what to teach" in Mathematics, is paramount in enhancing learners' knowledge to improve their problem-solving skills (Baier et al., 2019). On the same note, scholars from Rwanda, Burundi, Kenya and South Africa (SA) suggest that teachers' lack of MPCK may result in ineffective teaching and learning, resulting in poor performance in Mathematics (Burrough et al., 2019; Moh'd et al., 2021). Nikmah (2018) opines that learners who learn Mathematics using the problem-based learning (PBL) rationale is the most effective learning strategy for improving learners problem-solving skills and performance in Mathematics.

PBL is a learner-focused learning strategy where learners critically present creative ideas and communicate mathematically with peers. It (PBL) can help Grade 12 Mathematics learners solve problem-type questions and improve learners' Mathematics performance (Nikmah, 2018). In addition, teachers with higher MPCK who use learner-centred teaching strategies can successfully facilitate learners' learning of new knowledge (Makofane & Maile, 2019).

Mathematics learners who use PBL to master the content often build new knowledge and develop higher-order thinking skills and inquiry abilities (Nurlaily et al., 2019). In other words, such learners, in this case, Grade 12 learners, are collectively involved in a series of learning projects involving scientifically solving problems (Sanjaya, 2008). More specifically, the learning activities allow learners to construct meaningful knowledge, skills, and attitudes in Mathematics and simultaneously develop cooperative relationships under the counsel of their subject teachers.

Mathematics teachers should be competent in content and use innovative instructional strategies to bring out learners' critical learning and problem-solving skills for effective learning (Koh & Chapman, 2019). Therefore, Mathematics teachers need to be proficient in content-based and learner-centred teaching, or MPCK, for effective teaching and learning of Mathematics (DeLuca et al., 2013). Thus, as averred by Idris and Madugu (2020), PBL is an effective model for constructive learning and producing gains in Grade 12 Mathematics.

The constructive learning of Grade 12 Mathematics depends on the teachers' Mathematics pedagogical content knowledge (MPCK), which affects learners' mastering the Grade 12 Mathematics (Baumert et al., 2018). In other words, the MPCK of teachers can influence the quality of Mathematics learning and its outcomes (Ko et al., 2016) because teachers disseminate content knowledge to learners and are responsible for effective learning (Taber, 2018).

Mathematics teachers with a strong grip on MPCK, favour a productive learning achievement from their senior secondary Mathematics learners (Burroughs et al., 2019). Why? Mathematics teacher's understanding of the different types of learning challenges that various learners experience according to their diverse learning capabilities, can accommodate these special needs in their lesson delivery, to ensure all Mathematics learners successfully acquire the new content (Shulman, 1986). Therefore, PBL is an effective learning strategy that can influence learners' Mathematics learning in Grade 12 (Ramli et al., 2020).

Effective Mathematics teachers need to facilitate learning among Grade 12 learners by collectively constructing new knowledge in Mathematics and by actively investigating and contrasting their approaches and their differences of opinions (Taber, 2018). Thus, Ball et al. (2008) put forward the theory of mathematical knowledge for teaching (MKfT), which discusses mathematics knowledge or functions utilised in the teaching and learning of Mathematics knowledge to learners. These functions entail lesson planning, evaluation of assessments of learners, issuing homework and entertaining discussions with learners with contrasting viewpoints on the Mathematics syllabus. For effective learning, Grade 12 teachers must employ communicative pedagogical practices (MPCK) to teach content (Ankomah, 2021).

Mathematics teachers need appropriate content knowledge to teach the subject. In other words, Mathematics teachers should possess a well-defined understanding of the content that they need to transfer to their learners. This distinct body of knowledge, also known as Mathematical Knowledge in Teaching (MKiT), refers to the Mathematics knowledge needed by the subject teachers to enact the requirements of teaching and learning (Baumert et al., 2018). Mathematics teachers must be able to communicate the knowledge content by comprehending their learners' errors, misunderstandings, and learning challenges they may encounter in the Mathematics lesson so that

constructive teaching and learning occur (Burroughs et al., 2019). According to Chua (2018), Mathematics teachers having a deficit in MKiT generally have lower learning achievement levels in Grade 12 Mathematics.

In most cases, teaching and learning of Grade 12 Mathematics are ineffective because unqualified Mathematics teachers lack teaching methodology and content (Anney, & Hume, 2014); and use teacher-centred learning activities; consequently, learners become passive, leading to underperformance in problem-based questions (Bature, 2020). As suggested by Magabvu (2020), learners' poor performance is due to teachers' lack of MPCK. This point is taken further by Makofane and Maile (2019) who state that the impact of MPCK in teaching and learning Grade 12 Mathematics using PBL shows that learners may critically assimilate the content in small groups.

South Africa's (Grade 12) Mathematics learners' performance is poor (Oosthuizen, 2021). For instance, learners cannot analyse and evaluate problem-based questions because they lack proficiency in Grade 12 Mathematics (Awuah, 2018). Grade 12 Mathematics learners' performance depends on how effectively PBL is used as a teaching and learning strategy (Lee et al., 2018). How can this be achieved? The teacher's comprehension of what challenges different Grade 12 Mathematics learners experience in learning can be accommodated in the lesson presentation to facilitate learning (Shulman, 1986). More importantly, teachers' MKfT or MPCK will assist in implementing appropriate PBL strategies tailored to match the various learning requirements of Grade 12 Mathematics learners to gain mathematical understanding and improve performance. Based on the above, the study seeks to highlight the effects of MPCK in learning Grade 12 Mathematics using PBL.

1.2 Background of the study

Following an empirical study conducted in recognition of the attempts to enhance the performance of senior secondary Mathematics education in the countries found in Sub-Saharan Africa (SSA), there is a need to focus on how learners learn Mathematics (Madaki, 2021). The focus should also be on improving teachers' content knowledge because their mastery of content, influences the quality of teaching and learning and this affects the learning outcomes of Mathematics learners (Rahman et, al. 2021). In other words, teachers' MPCK directly influences how learners learn Grade 12 Mathematics and how well they achieve in NSC.

In South Africa, the National Development Plan (NDP) 2030 is a blueprint for building capabilities and enhancing the capacity of its state agencies, including teachers' education, through compelling content and pedagogy support (South African Government, 2012). Furthermore, the NDP believes that developing Mathematics teachers' content and pedagogy may improve education delivery and learning outcomes in the Grade 12 classroom.

However, in South Africa, the levels of learners' achievements in their Grade 12 Mathematics is a particular concern and subsequently, there is an urgency to enhance the education targets as set out in the NDP (South African Government, 2012). The achievement levels of Mathematics education in most South African schools are the worst (Jojo, 2019).

Furthermore, DBE presented a diagnostic report on the NSC examination 2019, 2020 and 2021 and flagged several concerns about Grade 12 Mathematics learners. The report indicated that Grade 12 learners did not have independent or creative thoughts and could not manage analytical, evaluative and problem-solving questions in Mathematics by the time they exited school (McGhie et al., 2019). This was attributed to the lack of qualified teachers to teach Mathematics with insufficient MPCK and resulted in learners being unable to analyse and evaluate problem-based questions (Burroughs et al., 2019). In other words, learners' inability to analyse and evaluate problem-based questions significantly results from teachers' poor Mathematics content knowledge and low professional content (DBE, 2021), resulting in unproductive teaching and learning and lower achievement levels in Grade 12 Mathematics (Idris & Madugu, 2020).

PBL is one learning model succeeding in teaching and learning Mathematics to learners and improving performance (Li & Tsai, 2017). PBL can facilitate Mathematics learning around learners-centred activities in smaller groups, collaboratively blending knowledge and skills to solve real-life problems (Ahdhianto & Istiq'faroh, 2022). Furthermore, the teacher's role is to construct new knowledge on their existing knowledge by actively analysing and comparing the learners' arguments (Li & Tsai, 2017).

Teachers' lack of content and technique of presenting (MPCK) contributes to learners' poor performance in problem-based Mathematics questions (Makofane & Maile,

2019). This is supported by a report from Mathematics subject advisors which suggests that much work is required to improve teachers' content delivery and subject knowledge to improve learners' learning of Grade 12 Mathematics (DBE, 2017b). Therefore, teachers of Mathematics should possess an outstanding comprehension of content knowledge and academic knowledge so that learners can effectively learn in the Mathematics classroom (Danisman & Tanisli, 2017). Against this backdrop, this study investigates the effects of MPCK in learning Grade 12 Mathematics using PBL. This leads to the problem statement of this empirical review.

1.3 Statement of the problem

One of the challenges contributing to the poor performance levels of Grade 12 learners in Mathematics for 2019, 2020, and 2021 is that most of them find it difficult to analyse and evaluate problem-based questions (DBE, 2021). This may be due to Grade 12 learners learning Mathematics under the supervision of teachers with low knowledge levels of content and low professional content (Anney, & Hume, 2014; Bature, 2020; Lee et al., 2018,) leading to lower rates of learning and achievement levels (Magabvu, 2020).

Worldwide academics like Baier et al. (2019) and Walshaw (2012) believe that subject teachers need a strong command of MPCK for learners to achieve positively, which they presently do not have, whilst their geographical intellects also agree with them (Moh'd et al., 2021). Carnoy and Arends (2012) suggest that teachers with a higher level of MPCK are more effective in learners learning Mathematics and gain higher achievement levels in school-based assessments (SBAs).

Improving learning performance in Mathematics depends on the teachers' ability to transfer the content to learners to ensure effective learning (Leithwood & Jantzi, 2006) and this is dependent upon the teachers' MPCK (Neumann, et al., 2021). Ko et al. (2016) postulate that teachers with appropriate MPCK can ensure effective learning does take place when they:

- Understand their learners, modify instructions to the needs of learners, and predict misconceptions in learners' current knowledge content;
- Know the syllabus content and instructional strategies for teaching and learning it; and
- Combine Mathematics instructions with those in other subjects.

Opposing this, most Mathematics teachers are of the view that Grade 12 learners positively learn Mathematics when they know the content well and simply transfer it to their learners (Jalbani, 2014; Hill et al., 2008). Here lies the problem with teachers; they tend to lose the importance of their teaching techniques and think that their learners are now facing challenges in learning the new content (what needs to be taught and delivered) is complicated or uninteresting, rather than identifying that those instructional learning strategies (how to teach and deliver) are insufficient to meet the learning needs of various learners. More importantly, how and what is to be inculcated in learners is similar but unique. Therefore, MPCK could be the solution for learners learning Grade 12 Mathematics using PBL (Pournara et al., 2015).

Studies on PBL (Ajai et al., 2013; Malan & Ndlovu, 2014; Rui et al. 2015) and MPCK (Ankomah, 2021; Danisman & Tanisli, 2017; Gess-Newsome et al., 2017) exist, however, studies focusing on the effects of MPCK in learning Grade 12 Mathematics using PBL in a South African context are sparse.

1.4 Purpose of the study

The study sought to investigate the effects of pedagogical content knowledge in learning Grade 12 Mathematics using PBL. The concerns that poor instructional strategies experienced by Grade 12 Mathematics learners have contributed to their low levels of learning and achievement have placed DBE under tremendous pressure to act (DBE, 2021; Oosthuizen, 2021). The poor performance of Grade 12 Mathematics learners in 2019, 2020 and 2021 NSC may be attributed to teachers lacking in MPCK (Mahlaba, 2020), while some of the same teachers also become subject advisors (Makofane & Maile, 2019). Discussions around the poor performance of learners in Mathematics tend to blame Mathematics teachers' lack of effective teaching and learning strategies and not being content-equipped to teach successfully. A successful teacher has both the 'intention' and the 'effect' to assist Mathematics learners in making meaning of the content (Jaworski, 2004).

This study may assist Mathematics teachers in becoming effective content disseminators and improving learners' performance at the NSC level. This study will further contribute to the existing knowledge/literature on effective Mathematics instructional strategies and teachers' MPCK to attain quality in Mathematics education at the NSC level. The findings of the empirical review may provide a grounding for

further policy development. Therefore, the results could support Mathematics teachers in improving the quality of learning Mathematics and its outputs at the NSC level. The author envisages that DBE would take cognisance of the findings and implement them to complement their national support programme on improving Mathematics performance at the NSC level.

1.5. Research question

1.5.1 Main research question

- How does teachers' MPCK affect learning Grade 12 Mathematics when using PBL?

1.5.2 The Sub-research questions were formulated as follows:

1. What are the effects of teachers' MPCK on Grade 12 learners learning Mathematics?
2. How does the PBL instructional approach in teaching and learning Mathematics impact the Grade 12 Mathematics learners' performance?
3. What is the teachers' role in teaching Grade 12 Mathematics using PBL?
4. What framework can be suggested for the use of pedagogical content knowledge in learning Grade 12 Mathematics using PBL?

1.6 Research aim and objectives.

1.6.1 Research aim

This empirical review investigated the effects of pedagogical content knowledge in learning Grade 12 Mathematics using PBL to improve learners' learning and understanding of Mathematics in the classroom.

1.6.2 Research objectives

1. To examine the effects of teachers' MPCK on Grade 12 learners learning Mathematics.
2. To determine the impact of the PBL instructional approach in teaching and learning Mathematics on Grade 12 learners' Mathematics performance.
3. To examine the teachers' role in teaching Grade 12 Mathematics using PBL.
4. To suggest a framework for the use of pedagogical content knowledge in learning Grade 12 Mathematics using PBL.

1.7. Theoretical framework

A study's theoretical framework is a specified body of ideas and theories related to the incident that investigators wish to study (Etikan et al., 2016). It is a composition of a theory that a specific research investigation falls into (Creswell & Creswell, 2018). It is a 'blueprint' for an investigation. Thus, the study's theoretical framework describes how a distinct study is structured following a specific theory. A study's theoretical framework chaperons research and helps to determine what factor to measure, and the location of statistical correlations (Kiger & Varpio, 2020).

Several authors (Alanazi, 2019; Cohen et al., 2019; Creswell & Creswell, 2018) define a theoretical concept as a description of how and why phenomena operate. Alanazi, (2019) states that a theory should specify its subject. Therefore, a theory assists in formulating and identifying causal interconnections using strict criteria that match reasoning and empirical data (Cohen et al., 2019), supporting the notion that theories are conjectural road charts for how things operate together (Creswell, 2016).

This study is grounded in the social constructivist theory. The empirical review drew on the following theory previously utilised in school effectiveness reviews and teachers' PCK in Mathematics on Grade 12 learners' achievement levels. I framed the study based on the social constructivist theory that brought out the various aspects of the investigation. This study discusses the effects of pedagogical content knowledge in learning Grade 12 Mathematics using PBL.

1.7.1 Social constructivist theory

Passionate preachers of the social constructivist's theory (e.g., Creswell & Creswell, 2018; Alzahran & Woollard, 2013) put forward that learners grow their knowledge and expertise by mediating their encounters in a communal climate (or groups) concerning their time and space. The social constructivist learning theory is an approach to learning new knowledge within a group of learners by constructing their new knowledge through partnerships, active participation and collaborating with others through a shared experience (Alzahran & Woollard, 2013). I believe that semi-structured interviews with Mathematics teachers, focus group interviews and classroom observations with Mathematics teachers and learners, may reveal the effects of pedagogical content knowledge in learning Grade 12 Mathematics using PBL.

According to Schreiber and Valle (2015), Grade 12 Mathematics learners encounter new learning content by being exposed to group learning activities designed and implemented by the Mathematics facilitator. Their minds are transformed after they acquire the new learning content. Mathematics learners are now influenced by each others' historical, aesthetic, and social backgrounds (Badie, 2016). Amongst these factors, Mathematics learners should cooperate in smaller groups in the lesson while gaining effective teaching and learning from the subject facilitators. The more well-informed learners can now assist their grappling peers in mastering the Mathematics learning content. Henceforth, the investigation used the social constructivist theory to investigate the effects of pedagogical content knowledge in learning Grade 12 Mathematics using PBL in two secondary schools in the Amajuba District in Newcastle in KZN.

1.8 Delimitations

The term 'delimitations' refers to the borders that identify the territory that is the subject of the research (Cohen et al., 2019; Dimitrios & Antigoni, 2019) as well as the way the inquiry has been scaled down to a more manageable size (Creswell & Creswell, 2018). Because researchers must specify what aspects of a research project are excluded (Creswell, 2016), the delimitations of this study will now be discussed directly below.

1.8.1 Conceptual sphere: Most of this study is qualitative in its approach. Therefore, this empirical review has a prejudice towards the qualitative research framework.

1.8.2 Focus: Despite the truth that other factors contribute to Mathematics achievements, this investigation was only concerned with exploring the effects of pedagogical content knowledge in learning Grade 12 Mathematics using PBL in two secondary schools in the Amajuba District in Newcastle in KZN.

1.8.3 Demarcation faculty: This study only dealt with the faculty of Education at the UFS.

1.8.4 Sample boundaries: In this investigation, participants were Mathematics teachers and learners from two secondary schools in the Amajuba District.

1.8.5 Geographical boundaries: This investigation was limited to the Kwazulu-Natal province, which is one of the nine provinces in the Republic of RSA (RSA).

1.9 Definition of key terms

The terms below are used consistently in this investigation.

- **Problem-Based Learning (PBL):** This is a learner-centred educational activity in which learners learn about a subject content through a problem-solving process.
- **Lecture Method:** This is a teaching process in which the presenter or the instructor (subject teacher) preaches verbally to a class of participants.
- **MPCK:** It is a distinct body of knowledge a Mathematics teacher must possess, enabling the subject teacher to connect with different types of expertise and employ various examples to demonstrate content to learners to make sense of it
- **MKft:** Mathematical knowledge needed to teach Mathematics considers the learners' viewpoints, mindsets, and understandings for effective teaching. In other words, MKfT involves tasks or "what to teach."
- **MKiT:** refers to the teachers' pedagogical knowledge or the "how to teach," which guides the teacher in identifying suitable mathematical examples to ensure effective content teaching.
- **Effective teaching:** Refers to the teachers' adroitness to deliver the conceptual information and the relationship between understanding and flexible reasoning of the Mathematics content to learners.

1.10 Layout of chapters

The empirical review is presented across seven chapters as follows:

Chapter 1: Introduction: This chapter consists of an introduction and background of the problem, a statement of the problem, the purpose of the study, the significance of the study, the research question with secondary questions, research design, theoretical framework, delimitations, the definition of terms and chapter summary.

Chapter 2: Theoretical framework: This chapter presents the study's theoretical framework.

Chapter 3: Literature review: This chapter appraises information regarded as relevant and important to the study. The review of the literature provides a theoretical basis for the study.

Chapter 4: Design and Methodology: This chapter shows how the empirical review was conducted. It covers research designs and methodologies, paradigm, instrumentation, validity and reliability, measures to ensure trustworthiness, data collection and processing.

Chapter 5: Empirical results: This chapter summarises the results of this study. The chapter also discusses data examination and elucidation. It also covers the analysis of focus group interviews, teachers' interview questions, and classroom observations.

Chapter 6: Findings of the study: This chapter presents the outcomes of this empirical study. This study's main findings, which originate from the literature and empirical data, are encapsulated.

Chapter 7: Contributions from the study, conclusion and recommendations: I make recommendations for supplementary research based on the outcomes. Last, I discuss the limitations of this study and make recommendations for further research.

1.11 Summary of Chapter 1

This chapter provides the introduction, the study's background, the problem statement, the study's purpose, the research and sub-research questions, the research objectives, an outline of the research design and methodology, the importance of the analysis, ethical debates, and the arrangement of the chapters in this thesis. The next chapter critiques the literature of assumptions on the effects of pedagogical content knowledge in learning Grade 12 Mathematics using PBL.

CHAPTER 2: THEORETICAL PERSPECTIVE

2.1 Introduction

The preceding chapter set the effect for this empirical study by giving background information. This chapter presents the study's theoretical framework. The study adopts Vygotsky's social constructivist theory to investigate the effects of Mathematics pedagogical content knowledge in learning Grade 12 Mathematics using problem-based learning (PBL) strategy. The social constructivist perspective provides me with a lens through which the knowledge obtained from the study will be explicated (Creswell & Creswell, 2018). Furthermore, the social constructivist approach is a theoretical perspective which explores how learners generate their meaning of knowledge through group interaction (Amineh & Asl, 2015).

Through group interaction, learners come with their level of language and own culture. This forms the basis through which Grade 12 Mathematics learners experience, interface, and comprehend the content knowledge in a collaborative learning environment under the watchful eye of the facilitator (Akpan et al., 2020). After Mathematics learners internalise the new subject content; their intelligence undergoes a transition and is influenced by each mathematical learner's past historical, aesthetic, and societal background (Badie, 2016).

2.2 A social constructivist theory lesson plan

Mathematics lesson plans are generated to support teachers in implementing learning activities that are cognitively demanding and help teachers forecast learners' learning difficulties (Courtney et al., 2015). Vygotsky (1978) reasons that gaining knowledge occurs through learning activities among learners within themselves. Lesson plans cannot be rigid, but must be flexible in teaching the content, to match the varying requirements of different learners in an inclusive Mathematics classroom.

Since learning in groups is collaborative, Mathematics lesson plans should cater for those learners who offer contrasting answers to challenges they may experience in their real-life world, each specifically with its robustness and weaknesses (Bay et al., 2012). Mathematics lessons should create opportunities for learners to put forward open-ended and thought-provoking questions while encouraging them to pose questions to their peer group members (Schreiber & Brielle, 2013). Also, learners may make mistakes in mathematical problem-solving; their peers would lead them to the

correct solution to a mathematical problem, which is an example of collaborative learning.

Mathematics learners construct new knowledge from their prior knowledge (Dagar & Yadav, 2016). The lesson plans should consider different learners' pre-existing knowledge and establish appropriate learning opportunities to build new knowledge on their existing knowledge. Mathematics teachers may incorporate teaching media to assist Grade 12 learners move from the familiar (previous knowledge) to the unfamiliar (new knowledge). Teaching media must be consolidated into the Mathematics lesson plan to provide a solid object to build abstract information (Amineh & Asl, 2015).

2.3 Learning of Grade 12 Mathematics in a social constructivist learning environment

Mathematics education in Grade 12 demands teachers and learners to engage with complex mathematical content (DBE, 2018). Mathematics teachers are the disseminators of the specialised content knowledge (SCK) in the classroom and ensure effective teaching and learning (Ko et al., 2016). Adequate SCK and Mathematics pedagogical content knowledge (MPCK) can influence learners' performance in Mathematics (Ko et al., 2016). Effective teaching and learning require teachers to have a strong command of the subject content and transfer it to learners using various teaching skills using child-centred teaching strategies in a disciplined and safe environment (Kyriacou, 2009).

PBL is a practical pedagogical child-centred approach to expose Grade 12 Mathematics learners to real-life situations whereby they are now critical thinkers and creative solvers of complex problems, and are self-motivated to learn and collaborate mathematically (Koh & Chapman, 2019). Building on this, some scholars (Widyatiningtyas et al., 2015) postulate that Grade 12 learners can achieve success in their performance by learning Mathematics through the PBL approach.

Widyatiningtyas et al. (2015) opine that Mathematics teachers need to apply innovative instructional strategies that elicit critical thinking and problem-solving in Mathematics learners. PBL is the preferred approach that involves structured collaboration in a Mathematics classroom (Gillies, 2016). This means learners need to know that they depend on each other for learning and that learners responsible for

their learning will receive explicit instructions from their teachers and evaluate their work together. According to Capar and Tarim (2015) and Idris and Madugu (2020), group collaboration in Mathematics learning can lead to gains in Grade 12 Mathematics. This means that Mathematics teachers must master the disciplinary knowledge and become professionally competent to ensure Grade 12 learners successfully cooperate to learn the content. In other words, Grade 12 Mathematics teachers must display exemplary pedagogical practices and content mastery in the classroom for effective group learning.

Mathematics teachers who teach Grade 12 Mathematics are supposed to inculcate the subject matter but also must know how to teach (MPCK) (Ektina, 2010). However, the teachers' MPCK is the most critical component in successfully teaching and learning the content of Grade 12 learners because the teacher must cater to the various learning needs of every learner (Drie et al., 2001).

PBL is a social constructivist learning approach that allows different learners with varying bits of intelligence to display their uniqueness while learning Mathematics from each other (Sari, 2018). The scholar further reported that learners showed improvement in their Mathematics assessments from group learning.

Mathematics teachers need the capacity and content to facilitate PBL as an instructional learning strategy in teaching Grade 12 Mathematics. MPCK is a distinct body of knowledge a Mathematics teacher must possess. The subject teacher can connect with different types of knowledge and use different examples or demonstrations so different learners can understand it (Shulman, 1986). In other words, Mathematics teachers should grasp the new learning content's challenges on learners and their varying learning abilities. After that, the Mathematics teacher will use age-appropriate teaching strategies to ensure the assimilation of the content. Similarly, the teacher will blend the Mathematics content and pedagogy to meet learners' diverse learning needs so they can grasp the content (Shulman, 1986).

Other scholars (Chikiwa et al., 2019; Diaz, 2017; Pournara et al., 2015) have attempted to re-invent Shulman's (1986) theoretical perspective because there was a connection between knowledge and practice (Chua, 2018). The most distinguished MPCK model is the mathematical knowledge for teaching (MKfT) (Ball et al., 2008). Ball et al.'s. (2008) Domain map for mathematical knowledge for teaching model refers

to the mathematical knowledge needed to conduct the ongoing function of teaching Mathematics. The teacher emphasises effective teaching and learning of mathematical content, and the model classifies teacher knowledge into two strands: SMK and PCK (Chua, 2018).

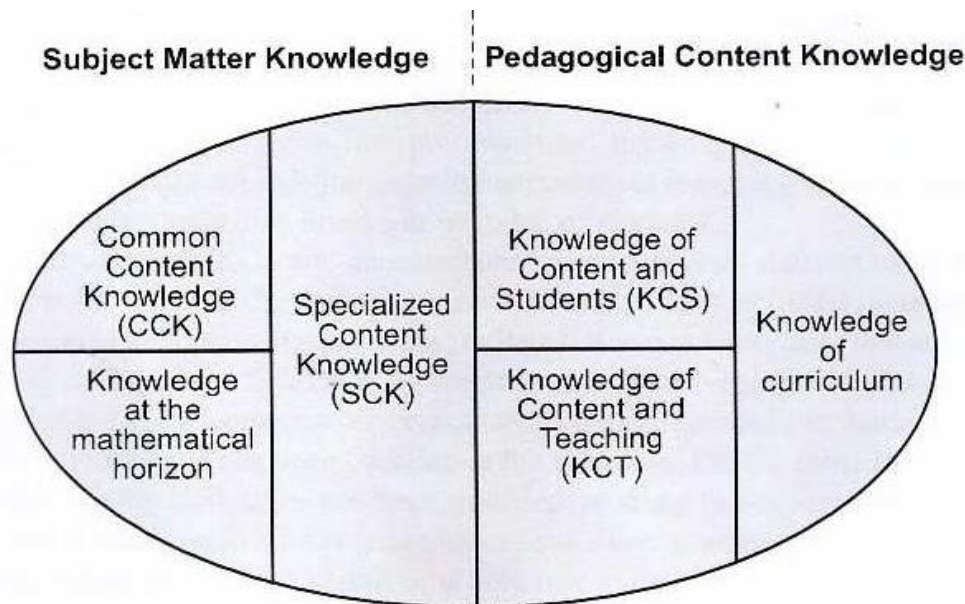


Figure 2.1: Domain map for mathematical knowledge for teaching (Hill et al., 2008)

Based on the model in Figure 2.1 above, there are two broad knowledge substructures: subject matter knowledge; and pedagogical content knowledge. Each knowledge substructure is broken down into three sub-categories. The subject matter knowledge comprises Common Content Knowledge (CCK). This is the Mathematics knowledge and abilities utilised in settings outside the limits of Mathematics teaching. The second sub-domain is Specialised Content Knowledge (SCK), Mathematics knowledge peculiar to the teaching of mathematics. It includes knowledge and skills that are not typically needed for purposes beyond teaching, such as understanding how mistakes are made. The third sub-domain is the horizon content knowledge which shows an appreciation of the association of Mathematics content throughout the scope of Grade 12 Mathematics.

The other broad knowledge category is pedagogical content knowledge. It comprises three sub-structures. The first sub-domain is the Knowledge of Content and Students (KCS), which comprises the knowledge about learners and Mathematics. The second

sub-structure is Knowledge of Content and Teaching (KCT); this consists of the teacher's understanding of Mathematics and its pedagogy. The third sub-structure is Knowledge of Content and Curriculum (KCC). It involves the teachers' knowledge of the Mathematics content alternatives and their comprehension of how a particular topic is located among all the topics inside the same subject content area and across other spheres of knowledge learners are exposed to. This sub-domain resembles Shulman's (1986) conception of curricular knowledge (Chua, 2018).

2.4 The impact of teachers' MPCK on learners' mastery of content

MPCK indicate the specialised knowledge for creating conditions for productive teaching and learning environments that every Mathematics teacher must possess (Guerriero, 2017). In other words, Mathematics teachers are responsible for facilitating the learning of Mathematics that will best meet the learning needs of their audience (Niess, 2005).

MPCK includes the Mathematics teacher's understanding of what makes the content difficult for learners to grasp, the understanding and misunderstandings that are age-related to learners, and learners' home background that may lead to their learning difficulties (Shulman, 1986). In other words, Mathematics teachers must have good knowledge of the learning difficulties the new content will impose on learners and teach according to their varying learning abilities.

To cater for learners' needs, Mathematics teachers use age-related teaching methods to ensure learners grasp the new content. The subject teacher can connect with different types of knowledge and use different examples or demonstrations so learners can make sense of it (Ankomah, 2021). Similarly, the teachers' lessons will show how teachers modify the Mathematics content and pedagogy to meet the different learning needs of learners to ensure mastery of the content (Bature, 2020). Teachers should have a good grounding of preconceptions and conceptions that Mathematics learners will experience in learning specific concepts and be able to overcome them in the lesson plans.

MPCK needs the collaboration of other aspects for effective teaching and learning of Mathematics in Grade 12, such as mathematical learners' prior knowledge, the curriculum's scope, educational objectives, and teaching presentations (Sunday & Nkopodi, 2013). Furthermore, for Grade 12 learners to effectively learn mathematics,

they need to assimilate the new content by building on their prior knowledge (Ball et al., 2008). Taber (2018) stated that effective Mathematics learning can occur in a social constructivist classroom when the scaffolding of new knowledge is based on existing knowledge. In other words, learners' grounding knowledge (intermediate and senior phase) in Mathematics forms the foundation for Grade 12 learners to learn Mathematics.

2.5 Principles of the social constructivist approach

The social constructivist approach is a theory that emphasises learners' building their knowledge of their existing schemas through their active participation in a social learning environment (Taufik et al., 2020). In tangent to this, Mathematics learners comprehend that they are co-constructors of content within the boundary of their social learning environment (Amineh & Asl, 2015).

As co-constructors of knowledge, Mathematics learners place the importance of learning on activities that involve peer-on-peer tutoring and group activities like reading and writing, to gain new information and strategies to improve achievement levels (Alzahran & Woollard, 2013). This means that Grade 12 Mathematics learners can achieve better results if they engage in smaller group work activities, thereby collectively developing a comprehension of the Mathematics content. More importantly, Mathematics learners' awareness of learning, language usage, and intercultural knowledge are critical elements in their cooperative nature of constructing new content (Akpan et al., 2020).

As learners collectively construct new knowledge on their existing knowledge, their intellectual structures, that is, their schemata and heuristics, undergo development (Palincsar, 1998). According to Kroflič (2019), learners' schemata and heuristics are generally accepted these structures are responsible for problem-solving activities This allows the learners to directly engage with the mathematical subject content; they actively learn and build the new content on their existing knowledge, which may enhance their performance levels in Grade 12 Mathematics.

When effective teaching takes place by scaffolding new content based on prior knowledge, Taufik et al. (2020) mention that Mathematics teachers play a mediatory function in developing learners' comprehension by putting forward challenging questions and presenting contrasting evidence while promoting new ideas and

thoughts in their minds. The Mathematics teacher is the more content-endowed person who mediates learners' construction of new knowledge. A teacher's facilitation increases the poor-performing learners' knowledge reservoirs and may improve their mathematical skills in Grade 12 (Mavuso, 2014).

2.6 Social constructivism as a subjective creation of reality

The constructivist learning theory is a concept that entails a group of Mathematics learners creating new knowledge through group participation, and mutual collaboration based on common and diverse experiences (Alzahrani & Woollard, 2013). Mathematics learners accept the learning concepts through small-based group learning encounters orchestrated by the Mathematics facilitator (Schreiber & Valle, 2015).

Galbin (2014) avers that social constructionism contests that absolute objectivity is not present in human studies because all methods need a set of subjective humans to evaluate other human elements. Subjective human beings are Mathematics learners with identities informed by their cultural reference to social and linguistic practices (Taufik et al., 2020). In other words, after learners digest the new subject matter, their intellectual minds transform and are affected by each mathematical learner's previous historical, cultural beliefs, and social experiences (Amineh & Asl, 2015; Badie, 2016). Similarly, learners of different social backgrounds merge over the learning content, and there are reciprocal exchanges. Eventually, the learners become habituated to other learners' historical, cultural, and social characteristics (Cojocar, 2010; Liu & Matthews, 2005).

Proponents of the social constructivists' theory believe that independent people are looking to comprehend the world in which they find themselves (Creswell & Creswell, 2018). Mathematics learners create their frame of reference on their encounters and the phenomena that surround them. Their understanding is built on their notions and is not factually related; therefore, knowledge gained by using a social constructivist concept in the teaching and learning of Grade 12 Mathematics will earn diverse views and not lead to a narrow understanding of an issue or a single possible truth (Fouché et al., 2022). This is a propitious moment in the Mathematics classroom for learners to discuss and negotiate the correct answers, mediated by the Mathematics teachers. Through these activities, Grade 12 learners will be able to resolve that, although their

solution is correct, other possible solutions do exist to the same Mathematics problem and, may be produced by their peers.

2.7 Collaborative Learning in the Mathematics Classroom

Social constructivism in a Mathematics classroom is a shared approach to acquiring the content, which requires group interaction accompanied by knowledge exchanges (Akpan et al., 2020). The teacher mediates the Mathematics content based on learner-centred and collaborative approaches (Taufik et al., 2020). The emphasis is on group work, finding new solutions to mathematical problems using creative operations, and building on existing knowledge.

Collaborative Mathematics learning opens up learners to possible authentic situations in the classroom whereby they practice problem-solving and intellectual skills application (Amineh & Asl, 2015). As Shulman (1986) claims, teachers must have pedagogical knowledge to plan and forecast the possible learners' problems with Grade 12 Mathematics and present the content to the varying needs of learners. Furthermore, Steenbrugge (2012) believes that Mathematics teachers must have expert knowledge of the content so learners can successfully acquire the content to produce improved outputs in Grade 12.

Collaborative learning is mediated through different teaching strategies employed by Mathematics teachers. These strategies include different types of groupings of learners using interactive methods (Akpan et al., 2020). Also, group discussion may include discussions with the entire Mathematics class or in small or learners working in pairs on projects and assignments. Alanazi (2019) mentions that group work allows learners to share or brainstorm ideas, creating something new or adding to existing knowledge.

2.8 The impact of effective collaborative learning on learner's performance in Grade 12 Mathematics

The pedagogical skills and mastery of content knowledge of the Mathematics teacher are crucial to creating adequate opportunities for Grade 12 learners to learn content (Akkutay & Cakmak, 2016). Effective learning in Grade 12 Mathematics is founded along with learner-centred and collaborative group work based on the learning content (Amalia et al., 2017). Similarly, learners have their knowledge backgrounds and diverse learning needs.

Complimentary to this, learners' learning needs differ, therefore positive learner outcomes in Grade 12 Mathematics can be attributed to diverse instructional resources, teachers' knowledge of Mathematics, qualification of teachers, parental support and effective teaching of learners in groups (Makofane & Maile, 2019). Moreso, effective learning is achieved by teachers who can differentiate the teaching of the content, teaching for higher-order thinking skills and flexible grouping and this culminated in a significant increase in Grade 12 Mathematics outcomes (Hunter, 2019).

Grade 12 Mathematics learners seem to have a more favourable outcome in their assessments when engaged in learning activities consistent with the regimes of group work. In other words, learners should be actively involved with the learning content by doing group projects, peer tutoring and fieldwork, role play, assignments, and short reflections (Amalia et al., 2017). Furthermore, effective Mathematics teachers should 'prepare a mental set through rapport' with Mathematics learners before they start learning Mathematics (Masitoha & Fitriyanib, 2018). This should translate into positive outcomes in Grade 12 Mathematics.

The findings by Mosimege and Winnaar (2021) demonstrated that teacher-learner-group interaction and problem-solving techniques amongst learners, facilitated by the planning of lessons incorporating different approaches to learners' varying needs, are associated with improved learners' performance in Grade 12 Mathematics. In other words, effective Mathematics learning demands that teachers know the content and how to facilitate learning to ensure positive achievement levels in Grade 12 Mathematics.

2.9 The effects of social constructivist theory on learning

The social constructivist theory in Mathematics learning can also be viewed as a philosophy of Mathematics because Mathematics is intrinsically fallible, no truths exist, and it redefines "objective" as socially agreed (Akpan et al., 2020). Effective Mathematics learning means learners understand what content they know and need to learn and be self-motivated to learn (Anwar & Rahmawati, 2017).

Shulman (1986) mentions that for learners to acquire Mathematics content effectively, quality Mathematics teachers should possess an in-depth knowledge of representing the content matters to them. Similarly, Turnuklu and Yesildere (2007) stated that

teachers' Mathematics content and MPCK are integrated parts of effective Mathematics instruction and learning. Teachers must comprehend and anticipate the learning difficulties learners may experience in understanding the new content and therefore be innovative and creative in presenting the lesson for successful mastery.

To ensure effective learning of Grade 12 Mathematics, teachers' content and MPCK are requirements. Effective teaching and learning require teachers to blend the knowledge of what to teach (specialised content knowledge (SCK) with how to teach the subject matter (MPCK). Both components constitute Mathematics pedagogical content knowledge (MPCK) (Turnuklu & Yesildere, 2007).

DBE (2021) defines effective teaching and learning of Mathematics as individuals or group members collectively building knowledge based on their perceptions of past and present knowledge which are interconnected. Therefore, the effects of social constructivist theory in learning mean that knowledge is not static but dynamic, depending on the individual who constructs it. During group work activities, learners learn better from their peers in a socially participative environment (Mavuso, 2014).

Mathematics teachers disseminate the content to learners by understanding their preconceptions, misconceptions, and challenges they may have in the Mathematics lesson so that constructive teaching and learning occur (Burroughs et al., 2019). In other words, Mathematics teachers must be able to foresee learning challenges Grade 12 learners may experience in their different groups and also identify their varying learning abilities. The teacher would be able to conceptualise the abstract content that learners may find difficult to internalise and now base their new knowledge on their existing knowledge.

Furthermore, social constructivism places the teacher as the mediator of Mathematics content learning (Kapur, 2018). At the same time, learners in smaller groups actively exchange information with their peers and gain new knowledge (Amineh & Asl, 2015). Conversely, the old teacher-centred approach or 'the expert of knowledge,' while learners were passive absorbers, has fallen out of favour with the proponents of PBL (Widyatiningtyas et al., 2015).

The Mathematics teacher is responsible for teaching the content in an enabling learning environment to achieve learners' outcomes in Grade 12 Mathematics. The social constructivist learning principles are summarised by some of the more

prominent connoisseurs of learning theories (Díaz, 2017; Vygotsky, 1978). Below is a summary of the principles of learning Mathematics:

- Learning is now facilitating the lesson through group learning;
- Facilitating learning is a constructive process for Mathematics learners;
- Mediating new content is based on previous knowledge;
- Knowledge assimilation involves a process of an internal reorganisation of schemes; and
- Facilitating learning new content is a process of (re) constructing cultural knowledge.

Effective learning consists of learners mastering the Mathematics content through group discussions while the Mathematics facilitator guides them using appropriate questions, introducing, and clarifying new knowledge, or referencing previously gained content (Kar, 2017). Furthermore, social constructivism entails teaching and learning concepts represented by authentic life experiences (Doolittle, 2014).

Learning Mathematics is demanding for learners because it involves learning complex mathematical content (DBE, 2017a). According to Shulman (1986), teachers need specialised Mathematics knowledge (SMK) and MPCK to facilitate Mathematics learning and, therefore, are trained explicitly (Steenbrugge, 2012). Mathematics teachers are the disseminators of the content knowledge in the classroom and ensure effective teaching and learning (Ko et al., 2016).

How do Mathematics teachers effectively communicate the content to learners so that they successfully master it? Mathematics teachers must have a good command of MPCK to successfully facilitate Grade 12 Mathematics learning. MPCK describes how the Mathematics teacher would teach the content while being aware of learners' misunderstandings or assessing the different concepts of knowledge that learners must acquire and appropriate for Grade 12 Mathematics (Pompea & Walker, 2017).

2.10 The role of the Social constructivist teacher in the learning of Mathematics

Social constructivism in Mathematics postulates that the teacher should teach content, emphasising solving problems through social exchanges between learners and learners or learners and teachers to create their knowledge (Rytilä, 2021). In other words, the teacher is responsible for planning and creating a conducive learning

environment by simulating real-life situations and exposing learners to practically solve authentic life problems in Mathematics (Badie, 2016).

Effective teaching of Mathematics by teachers (who are the more knowledgeable) requires facilitating the learning content between the teacher and peer support through scaffolding, peer tutoring, and cooperative learning opportunities (Akpan et al., 2020). Learners are recognised for their opinions in making sense of the content, whether they are correct or not (Amineh & Asl, 2015). In other words, the Mathematics teacher must generate a social-collaborative learning situation. Learners are free to ask questions, discuss and interact using necessary resources while learning from each other.

Learners' mindset changes when they learn from each other (Badie, 2016). The teacher's delivery of the Mathematics content must be flexible (Kroflič, 2019). Therefore, the Mathematics teacher must be knowledgeable about content delivery pedagogy and use various methodologies to present the Mathematics lesson. Successfully delivering Mathematics lessons should include posing inquiring questions, placing learners in the middle of learning, and advising them to reach their sensible understandings (Rytilä, 2021).

Traditional, teacher-centred pedagogies of learning Mathematics have fallen out of favour with most scholars like Freiberg (1999) and Mpho (2018). Effective Mathematics learning translates into learner-centred activities; learners may interact with their peers on the content or seek guidance from their teachers to direct their learning. In other words, Mathematics learners are protagonists of their learning while being guided in acquiring the learning content (Diaz, 2017).

For the successful internalisation of mathematical content and positive outcomes in Grade 12 Mathematics, Vygotsky (1978) thinks that intellectual achievement and learning are determined by the differences in Mathematics learners' zone of proximal development (ZPD). The Mathematics teacher is one mediating the learning content by tweaking the content to one level above the current level of learners and within their grasp (Sarker et al., 2019). The newfound responsibility of the Mathematics teacher in constructivist teaching should report improvements in Grade 12 Mathematics (Akpan et al., 2020).

2.11 Assumptions of the social constructivist approach

The social constructivist approach assumes that Mathematics learners actively engage with the learning content, resulting in their basic knowledge through inquiry and discovery (Akpan et al., 2020). The inquiry and discovery of new knowledge and mastery are based on understanding Grade 12 learners' culture and language (Amineh & Asl., 2015). This means that knowledge construction is transmitted by language and comprehended by Mathematics learning experiences.

Social constructivism is a learning theory that emphasises the collaborative aspect of constructing new knowledge based on prior under the guidance of a knowledgeable facilitator (Taufik et al., 2020). In other words, Mathematics learners mentally and physically generate meaning or knowledge (based on their current knowledge) as an achievement of the co-construction of meaning and reciprocal exchanges in their groups. Learners can improve their knowledge banks by receiving practical instructions from their teachers or peers and gaining higher achievement levels in Mathematics (Hyslop-Margison & Strobel, 2008).

When learners collaboratively learn, there are learning tasks they share and actively engage with it. Kapur (2018) observed that Mathematics teachers create different learning environments with tasks like group discussions, teamwork, or any learning interaction in an academic venue or multi-media forums. Through their group interaction, Mathematics learners build new knowledge within their place of learning.

2.12 Criticism of the social constructivist theory

The most widely accepted defenders of the social constructivist concept are scholars like Kapur (2018) and Vygotsky (1978). While opposing their views, scholars like Kirschner et al. (2006) and Alanazi (2019) heavily criticise the use thereof. Contemporary scholars like Amineh and Asl (2015) are advocates of social constructivism, and researchers like Liu and Matthews (2005) have contrasting views. These scholars put forward that the social constructivist theory places importance on learners actively involved in classroom activities and disregards all traditional didactic presentations, but it does not always produce effective teaching and learning.

There are different opinions on using the social constructivist learning theory in research. Therefore, the study will critique two crucial factors (out of many) that add

to a social constructivist environment: the teacher is a facilitator and mediator of learning Mathematics and learners as collaborative constructors of new knowledge.

2.12.1 The teacher as a facilitator and mediator of learning Mathematics

The teachers' content and MPCK guide their role as facilitators and negotiators of the learning content. This is directly associated with the learners' positive outcomes in Grade 12 Mathematics (Akkutay & Cakmak, 2016; Alanazi, 2019). Therefore the Mathematics teachers' approach to teaching is now a mediator and no longer giving explicit instructions while learners passively listen to the content delivered through the "chalk and talk" method. Moreso, teachers' MPCK allows them to successfully execute meaningful lessons by offering a different approach to the delivery of the content in a less rigid environment.

In their newfound role, the teacher creates social learning activities that involve hands-on project-based methods and discipline-based cognitive tools situated in groups (Kim, 2014). Now, learners collectively produce projects and gain knowledge through social learning. Mathematics teachers now mediate the learning experience in the classroom, which represents actual life occurrences using various teaching methods to ensure lifelong learning and critical skills development (Akkutay & Cakmak, 2016).

The role of the Mathematics teacher is to develop learners' mathematical knowledge through scaffolding (Sarker et al., 2019). Vygotsky opines that intellectual growth and learning amongst Mathematics learners are determined by the differences in their zone of proximal development. Congruent to this, Mathematics teachers encourage learning by providing a safe environment while paying attention to the discussions from the learners and responding with effective feedback (Alzahran & Woollard, 2013). In other words, a learner's accurate level of development and achievement is reached with Mathematics teachers mediating the learning content to them.

The Mathematics teacher creates learning opportunities for learners to engage with content knowledge (Kar, 2017). This brings out the creative designing abilities of the Mathematics teachers. These learning opportunities for group interaction allow Mathematics learners to construct new meanings in a social environment (Liu & Matthews, 2005). Vygotsky (1978) points out that teachers use tools or instruments to help learners gain knowledge. Similarly, learners effectively assimilating the content need teachers to use supportive educational structures to design situations for higher

mental processes, resulting in improved performance levels in Grade 12 Mathematics outcomes.

2.12.2 Learners as collaborative constructors of new knowledge

Scholars supporting social constructivist theory in Mathematics teaching suggest that learners collaboratively construct content knowledge by actively engaging in the instruction process and influencing their real-life encounters (Amineh & Asl, 2015). During group interaction, learners communicate with each other, exchanging information and negotiating meaning under the guidance of their more knowledgeable peers through group dynamism (Akpan et al., 2020).

Opponents of the social constructivist approach may counter-argue that since information is brokered, facilitated, and informed by individual learners' encounters in life, the understanding of meaning may vary amongst them (Turnuklu & Yesildere, 2007). Also, learning with a fixed scope is ineffective because each Mathematics learner differs in their comprehension and learning abilities (Alanazi, 2019). Therefore, having one curriculum for all learners leads to ineffective learning of Grade 12 Mathematics and poor learner outcomes. This is contrary to the goals of teaching Mathematics, that is, to develop meaningful understandings about their spheres of study and to mediate the growth of critical thinking (Akkutay & Cakmak, 2016).

In social constructivism, teachers facilitate Mathematics learning amongst learners with minimal instructions, which is likely to cause the teachers to encounter difficulties in achieving positive learner outcomes in Grade 12 Mathematics (Alanazi, 2019). Consequently, there is a lack of adequate instructions from the teacher (Ma'rufi et al., 2018).

Along the same lines, learners with ineffective instructional learning cannot rationalise the new content or physically integrate or internalise the new knowledge through smaller-group exchanges (Alanazi, 2019). As a result, learners become bored and may be unable to actively develop new subject knowledge. Scholars such as Jalbani (2014), Kirschner et al., (2006) and Kyriacou (2009) postulate that insufficient guidance from Mathematics teachers leads to ineffective teaching and poor learner outputs in Mathematics assessments.

2.13 Summary of Chapter 2

This chapter showed that using PBL in the social constructivism framework guided me to investigate the effects of pedagogical content knowledge in learning Grade 12 Mathematics. Badie (2016) describes social constructivism amongst Mathematics learners as inculcating new knowledge through mutual group construction in life-like experiences facilitated by the teacher in the classroom.

The multiplex nature of the social constructivist approach was discussed by critiquing various academics (Akkutay & Cakmak, 2016; Alanazi, 2019; Amineh & Asl, 2015; Liu & Matthews, 2005; Sarker et al., 2019) by contrasting their viewpoints. I focused on discussing two aspects contributing to a social constructivist environment in the classroom environment: the teacher is a facilitator and mediator of learning Mathematics and learners as collaborative constructors of new knowledge.

This chapter also showcased that collaboration in the Grade 12 classroom, guided by the various roles of the teacher, should create effective teaching and positive outcomes among learners in Grade 12 Mathematics. Turnkulu and Yesildere (2007) concede that Mathematics cognition is negotiated, facilitated, and informed by Mathematics learners' encounters. Therefore, it is counter-reproductive for Mathematics learners to experience a fixed curriculum because it does not cater for their individual learning needs (Alanazi, 2019). Therefore, this can lead to ineffective teaching and learning of Mathematics and poor learners' achievement levels.

Some scholars (Badie, 2016; Rytälä, 2021) emphasise that Mathematics teachers are responsible for strategising and designing a friendly learning environment by generating real-life experiences in the classroom. Therefore, Mathematics teachers need to endeavour to display life-like problems so that learners gain experience in the real world of Mathematics applications.

CHAPTER 3: LITERATURE REVIEW

3.1 Introduction

The previous chapter dealt with this study's theoretical perspective. This chapter investigates various literature sources on the effects of pedagogical content knowledge in learning Grade 12 Mathematics using PBL.

To evaluate the status of knowledge, a literature review involves critically reading, assessing, and arranging relevant material (Struwig & Stead, 2017). This research reviewed the literature. This study's literature review searched and studied current publications on the subject matter to comprehend what is known, identify gaps in content knowledge that need research (Oztemel & Gursev, 2020), and provide a conceptual, theoretical, or methodological structure (Struwig & Stead, 2017). The review gave the study a solid foundation by reviewing and analysing the literature, including those listed in the background (Creswell & Creswell, 2018). This research conducted a review of the literature to better comprehend the topic (Fouché et al., 2022).

To incorporate previous research and identify key difficulties in a subject, Oztemel & Gursev, (2020) conducted a secondary literature study (Cohen et al, 2019). Walliman (2018) believes the investigator should be able to retrieve the facts needed to complete the study. Mohajan (2021) declared that before completing a study subject that requires secondary source data, the researcher should confirm that the material is accessible in the proper format. The researcher considered both authors' opinions before starting this investigation. Gathering as much information as possible from many sources did this. This study employed only reputable sources to help me develop conclusions on the research topic and question (Fouché et al., 2022).

After exhausting the literature review sources and gathering all relevant references as suggested by Fouché et al. (2022), the researcher organised the information by theme or construct and by the most important priority for organising and writing the literature review - organizing one's thoughts. This was done to answer the study's core question and identify any sub-themes. To comprehend the topic's research, this study's review of the literature included distributed work, enacted laws, theses, dissertations, and journal articles. A literature review will also provide a better understanding of the

current practices of the effects of pedagogical content knowledge in learning Grade 12 Mathematics using PBL.

3.2 The objectives of the literature review

The first objective of this study was to examine the effects of teachers' MPCK on Grade 12 learners learning Mathematics. What follows hereunder is a literature review that covers objective 1. By reviewing the literature, I will be answering question 1 of this study, which is: *What are the effects of teachers' MPCK on Grade 12 learners learning Mathematics?*

3.2.1 Effects of teachers' MPCK on Grade 12 learners learning Mathematics

Much impact on the learning of Grade 12 Mathematics is associated with identifying the teachers' specific knowledge and competencies or the "how" to teach Mathematics for learners' understanding. (Gess-Newsome et al., 2017; Hurrell, 2013; Baumert, et al., 2018). Mathematics education is demanding for a teacher because it involves teaching complex mathematical content (DBE, 2018). Mathematics teachers are the mediators of the content knowledge in the classroom and ensure effective teaching (Ko et al., 2016). Adequate content knowledge (CK) and Mathematics pedagogical content knowledge (MPCK) can influence learners' performance in Mathematics (Ko et al., 2016; Turnuklu & Yesildere, 2007). Effective teaching and learning require teachers to have a strong command of the subject content and transfer it to the learners using various teaching skills, like child-centred instructional strategies in a disciplined and safe environment (Kyriacou, 2009).

Shulman's (1986) Mathematics knowledge for teaching and learning Mathematics consists of two categories: Mathematics pedagogical content knowledge (MPCK) and specialised Mathematics knowledge (SMK). MPCK as a construct entails merging content and pedagogy unique to Mathematics teachers' education. The scholar further suggested a correlation between pedagogical content knowledge (PCK) and subject matter knowledge (SMK) to capacitate teachers' development in instructing learners on mathematical content to improve their performance and understanding. However, the connection between cognition and teaching application is still not fully understood (Ball et al., 2008).

Teachers need specialised mathematical knowledge (SMK) to teach Mathematics and, therefore, are trained explicitly (Steenbrugge, 2012). Shulman's (1986) SMK is a

distinct body of knowledge a Mathematics teacher must possess. Guerriero (2017) concurs with Shulman (1986) that SMK is a form of knowledge that refers to the specialised expertise needed to create conditions for effective teaching and learning environments. Therefore, the Mathematics teacher's mastery of the learning content is essential for effectively communicating the learning content to Mathematics learners.

Ball et al. (2008) suggested a model of teachers' knowledge, Mathematics Knowledge for Teaching (MKfT); which refers to the mathematical knowledge needed to teach Mathematics and considers the learners' viewpoints, mindsets, and understandings for effective teaching and learning. In other words, MKfT involves tasks or "what to teach," which is needed for effective teaching and learning (Chikiwa et al, 2019).

Shulman (1986) and Ball et al. (2008) referred to mathematical content knowledge as a specialised or technical form of content knowledge that entails teaching learners. Therefore, teachers' communication of the content demands high-quality instruction for effective teaching and learning in Grade 12 Mathematics (Ball et al., 2008). Other scholars (Danisman & Tanisli, 2017; Simon, 1993) aver that a good command of content knowledge is critical to Mathematics instruction and quality outputs in learning.

Some scholars (Bowers, 2021; Chikiwa et al., 2019) have shown that teachers who are less knowledgeable in Mathematics content knowledge demonstrate a pattern of errors in teaching the content to learners and this negatively affects their learning abilities. Furthermore, a lack of Mathematics content knowledge incapacitates the teacher from growing learners' understanding of the content, fails to encourage learners to verbalise the content and therefore fails to produce new mathematical knowledge (Ball et al., 2008). Similarly, Mathematics teachers cannot actively engage learners meaningfully in the content or are unable to inspire lateral thinking to reach new levels of knowledge.

Mathematics teachers with higher levels of specialised content knowledge can effectively teach because teachers know how learners think, focus on "how" to solve problems and be exposed to various problems (Hill et al., 2008). Similarly, highly professional content knowledge guides Mathematics teachers in classroom discussions and a multi-dimensional approach to solving problems and reaching an understanding. Therefore, the teacher can teach beyond the scope of content and use

open-ended questions and appropriate instructional language to reach conceptual understanding amongst learners in Grade 12. Therefore, Mathematics teachers with a comprehensive and well-organised conception of content knowledge may generate a meaningful learning environment for Mathematics learners and could lead to positive outcomes (Lloyd & Wilson, 1998).

Hill et al. (2008) emphasised an incident where a Mathematics teacher was well-prepared in the lesson delivery and substantial in content knowledge but failed to successfully explain answers in response to a learner's question. On the other hand, a teacher with a solid conceptual understanding had difficulty expressing himself. Hill et al. (2008) conclude that content knowledge is more than just formal knowledge of the content matter, a concept similar to Shulman's (1986) MPCK. In other words, content knowledge refers to the "how" to teach or the mathematical knowledge in Teaching (MKiT) Mathematics in secondary schools. Similarly, MKiT as suggested by scholars like Rowland and Zazkis (2013), was of the view that teachers' pedagogical knowledge or the "how to teach" guides the teacher in identifying good suitable mathematical examples to ensure effective teaching and learning of the content.

MKiT or MPCK is a concept that describes the subject teacher's knowledge of how to mediate mathematical concepts successfully and is critical for effective teaching (Walshaw, 2012). Chua (2018) further explains that MKiT affects the interaction between the teacher, subject content and Grade 12 learners and their learning. More importantly, MPCK allows Mathematics teachers to make immediate decisions on varying teaching strategies to teach the content effectively (Ma'rufi et al., 2018).

MPCK equips the teacher to use different teaching methods to resolve Mathematics learners' difficulties, preconceptions, and misconceptions they may experience in acquiring the content (Ma'rufi et al., 2018). In this case, Mathematics teachers can change and skilfully integrate the knowledge according to the learning needs of its audience and make sense of it (Moh'd et al., 2021).

According to Kilic (2011), MPCK is divided into four components, namely:

1. Knowledge of the subject matter.
2. Knowledge of pedagogy.
3. Knowledge of students.
4. Knowledge of the curriculum.

Mathematics teachers are responsible for mediating the knowledge of the subject matter by using various resources and teaching media so learners can assimilate it. In other words, the Mathematics teacher leads the Grade 12 learners to discover the solutions while observing learners' incorrect thinking and responding with assistance (Ma'rufi et al., 2018).

MPCK is a unique knowledge discipline which combines knowledge of Mathematics content, knowledge of pedagogic, and the knowledge content of Mathematics learners acquiring the content (Moh'd et al., 2021). Mathematics teachers are empowered to immediately vary their lesson presentation once they discover learners are experiencing learning difficulties in making meaning of the content. For Mathematics learners to process the new content successfully, Danisman and Tanisli (2017) divided MPCK into four different categories:

1. Building learners' ideas in Mathematics.
2. Overcoming learners' misconceptions.
3. The teaching and learning of Mathematics in groups.
4. Supporting learners' thoughts on Mathematics.

MPCK provides the Mathematics teacher with different mental resources to use examples or demonstrations so learners of varying learning abilities can understand it (Shulman, 1986). Similarly, Mathematics teachers are fully aware of the potential challenges of the new learning content, and they would use age-appropriate teaching strategies to ensure the assimilation of the content. Shulman (1986) postulates that Mathematics teachers incorporate Mathematics content and pedagogy to meet the diverse learning needs of learners so that they can grasp the content and produce learning in Mathematics.

MPCK is a distinct body of knowledge a Mathematics teacher must possess, enabling the subject teacher to connect with different types of expertise and employ various examples to demonstrate Mathematics content to learners to make sense of it (Ankomah, 2021). In other words, knowledgeable Mathematics teachers show quality in their pedagogical knowledge and versatility.

MPCK also includes the Mathematics teachers' understanding of what makes the content difficult for learners to grasp, the insights and misunderstandings that are age-related to learners, and learners' home backgrounds that may lead to learning

difficulties in Mathematics (Shulman, 1986). In other words, MPCK is an essential contributing factor to effective lesson delivery in the classroom and, therefore, an integral aspect of effective Mathematics learning and teaching. MPCK equips teachers to plan and conduct flexible lesson plans using appropriate instructional strategies for Grade 12 learners and their diverse learning needs (Saad et al., 2015).

Teachers' MPCK helps direct their approach to addressing learners' misunderstandings and learning difficulties in comprehending the new content (Ma'rufi et al., 2018). Similarly, Mathematics teachers are professionally competent in transforming complex tasks by breaking them down into easily understood information units (Moh'd et al., 2021). As Shulman (1986) describes MPCK, it is the infusing of subject content and pedagogy into comprehending knowledge that is represented and adapted to the diverse learning needs of Mathematics learners. Therefore, MPCK is essential in determining the unique composition of the Mathematics lessons in the Grade 12 Mathematics classroom practice and this impacts the quality of learning the content. For instance, there is a difference between an algebraist and a Mathematics teacher teaching Mathematics in Grade 12, and the algebraist is a specialist. In contrast, the teacher knows how to teach the content to learners to ensure effective learning (Moh'd et al., 2021).

This shows a fundamental difference between the trades mentioned earlier, though related because of the complementary relationship. Turnuklu and Yesildere (2007) concur that Mathematics teachers need knowledge of how to teach rather than just knowing the factual content. In other words, Mathematics teachers must be versatile in applying learning procedures and methods to successfully convey the metaphysical message into concrete blocks of information appropriate to Grade 12 learners' learning needs.

Mathematics teachers' unified command of MPCK impacts learners' mastery of the content since it influences learners' learning and leads to improved learning outputs (Ma'rufi et al., 2018). Similarly, MPCK is a specific body of knowledge consisting of content knowledge, knowledge of pedagogies, and learners' knowledge (Kılıç, 2011). The strength of the teachers' MPCK directs them to utilise different learning theories, inclusive of estimating learners' prior knowledge and their possible learning difficulties while applying "fit for purpose" instructional strategies, like PBL. More so, MPCK can

assist Mathematics teachers in predicting learning misconceptions and immediately provide more accessible explanations to help learners learn Mathematics.

Through MPCK, the teachers can implement various theories of learning Mathematics, including forecasting previous knowledge content and misunderstandings of the Grade 12 Mathematics learners, thinking of suitable teaching and learning strategies that can fit certain Mathematics content, and also utilising different ways of presenting the Mathematics concepts to produce effective learning (Ma'rufi et al., 2018). Teachers with higher levels of MPCK are likely to have Grade 12 learners with more outstanding scores in their Mathematics assessments (Nadas, 2019). Therefore, Mathematics teachers who possess a more significant amount of MPCK are more capable of effectively communicating the knowledge and skills to Grade 12 learners and achieving higher learning outputs.

MPCK has an immediate impact on the effectiveness of a Mathematics lesson because it comprises teachers' knowledge of the content matter, pedagogical information, knowledge of learners and curriculum knowledge (Kilic, 2011). Ma'rufi et al. (2018) aver that teachers' knowledge of how to teach the content should be transformed by using different resources so that Mathematics learners can assimilate conceptual information. In other words, for learners to acquire the content, it should be done using various representations to solve learners' Mathematics problems, locate their mistakes, and further mediate the lesson to achieve more incredible performance.

Teachers with a good command of MPCK can create a synergy between the teachers' Mathematics content knowledge and the pedagogy for effective learning (Ma'rufi et al., 2018). However, when Grade 12 Mathematics teachers deliver the content, they often lack collaboration between pedagogy and content knowledge, and this negatively impacts learners' learning abilities. Therefore, MPCK can be regarded as an appropriate tool for closing the gap between mathematicians and Mathematics teachers (Moh'd et al., 2021).

Teachers are aware that MPCK determines the effectiveness of their lesson presentations and therefore how successfully learners learn the content (Stol et al., 2015). In other words, the effective learning and teaching of Mathematics involve teachers' adroitness to deliver the conceptual information, or more importantly, show

the relationship between comprehension and flexible reasoning of the Mathematics content (Kathirveloo et al., 2014). Furthermore, teachers' MPCK enables them to be reflective of their teaching practices so that they can improve their teaching. This translates into Mathematics learners receiving an improved quality of tuition due to the teachers' natural inclination to produce better learning experiences (Rahman et al., 2021).

Similarly, the teachers' MPCK would determine how excellent and structured the lesson plans are delivered because MPCK influences the content delivery process to meet varying learners' abilities (Kathirveloo et al., 2014). This is the art of teaching Mathematics effectively, by considering learners' needs, home backgrounds, and interests to ensure mastery of the content. In other words, a teacher confident in MPCK can successfully remediate a lesson without any misconceptions, despite the possible learning challenges posed by learners (Ma'rufi et al., 2018).

Shulman (1986) categorised MPCK as knowledge a teacher has to interpret the content so that Mathematics learners gain mastery and achieve higher outcomes. Similarly, teachers were also able to use Mathematics information to render precise reasoning to learners who found challenges in acquiring the content, resulting in higher outcomes in their assessments. MPCK entails knowledge of concepts and procedures in which Mathematics learners progressively acquire new content by building on existing knowledge (Puteh, 2014), also known as scaffolding (Taber, 2018).

For Grade 12 Mathematics learners to successfully learn the content through scaffolding, the teachers' MPCK affects the teachers' abilities to produce a variety of suitable teaching strategies to meet the learning needs of its audience. These may include the teacher orchestrating real-life problems in class, group work dynamics, and collaboratively making sense of the content (Stol et al., 2015). Teaching Mathematics by emulating authentic problem situations translates into learners learning by modelling situations and socially negotiating their solutions while building new knowledge on their existing knowledge. This way, Mathematics learners can graduate from factual content to more abstract concepts (Moh'd et al., 2021).

MPCK is the skilful integration of Mathematics knowledge by teachers, together with the knowledge of how to produce the content knowledge by meeting the learning needs of learners and this affects their learning (Moh'd et al., 2021). Shulman (1986)

first conceptualised this sphere of knowledge of teaching Mathematics, as MPCK as the melting of pedagogy and content into a comprehension of how topics or problems are arranged, represented, and modified to meet learners' various interests and abilities. Therefore, MPCK is an essential determinant in the Mathematics lesson presentation for learners to grasp the content (Moh'd et al., 2021). Mathematics teachers must be competent in the content and use appropriate instructional strategies like PBL, to successfully illustrate mathematical knowledge to diverse groups of learners (Turnuklu & Yesildere, 2007).

Mathematics teachers must make informed decisions on their instructional strategies to transfer the content to Grade 12 learners. Therefore, teachers need to know the diverse learning needs of their learners. MPCK is the faculty that a resourceful teacher will use to understand Mathematics learners' age-related preconceptions and misconceptions and their backgrounds that may influence their learning (Moh'd et al., 2021). This enables the teacher to construct lessons using various instructional strategies that will result in reorganising the content so that Grade 12 learners can grasp it. In other words, the teachers' MPCK will influence the choice of learning strategies used, especially when they adopt pedagogical representations of content, use of questioning strategies, knowledge of assessment, evaluation of research, and provoke them into self-directed learning (Martin, 2022).

Teachers' MPCK influences meaningful Mathematics learning because Danişman and Tanişli (2017) mentioned that teachers with insufficient MPCK taught Mathematics using traditional methods while they could not express the content creatively and had little knowledge of learners' challenges in learning. Since MPCK addresses the "how to teach Mathematics," teachers depend on MPCK to present particular topics and problems appropriately to learners to accommodate their diverse learning abilities and backgrounds (Kar, 2017).

Malan and Ndlovu (2014) postulate that most Mathematics learners are diverse in their knowledge and home background and differ in their learning abilities. Therefore, a learner-centred approach like PBL should be used to teach Mathematics to all learners with varied learning skills (Masitoha & Fitriyanib, 2018). PBL is focused on what and how learners are learning and whether they retain it or use it in their daily lives (Weimer, 2002). Therefore, Mathematics teachers must modify their pedagogic

approach to complement the practical learning styles of Grade 12 learners. By doing this, teachers depend on their MPCK to implement the different practical learning instructions or PBL to cater to the various learning styles of Grade 12 learners and actively involve them in Mathematics activities (Rahman et al., 2021). Furthermore, the critical instruction and the quality of the Grade 12 outputs given by the PBL teacher are determined by the teachers' command of content knowledge or their MPCK (Li & Tsai, 2017).

Teachers with a higher MPCK have higher content knowledge and more skills in adapting their instructional abilities to meet a heterogeneous group of Mathematics learners and their various learning abilities (Rahman et al., 2021). Mathematics teachers with a solid MPCK base are also confident content facilitators of Mathematics knowledge (Burrough et al., 2019).

Confident Mathematics instructors pose higher-order questions to learners to gain an understanding of their current knowledge base so that learners' thoughts are provoked to discover the new content in their group without freely giving it to them (Martin, 2022). Therefore, under the teachers' facilitation in the PBL classroom, Grade 12 learners can increase their analytical and problem-solving skills accompanied by collective decisions and reasoning, and this should result in improved gains in the NSC (Lee et al., 2018).

The second objective of this empirical study was to determine the impact of the PBL approach in teaching and learning Mathematics on Grade 12 learners' Mathematics performance. What follows hereunder is a literature review that covers objective 2. By reviewing the literature, I will be answering question 2 of this study, which is: *How does the PBL instructional approach in teaching and learning Mathematics impact the Grade 12 Mathematics learners' performance?*

3.2.2 The PLB approach in teaching and learning Mathematics impacts Grade 12 learners' performance

The low achievement levels in Grade 12 Mathematics earned in the NSC for 2019, 2020 and 2021 could be associated with the teachers' limited MPCK, leading to less effective Mathematics teaching and learning strategies being used. Makondo and Makondo (2020) concur with the sentiment, that ineffective instructional learning strategies learners have resulted in Grade 12 learners being unable to analyse and

evaluate problem-based questions for 2019, 2020, and 2021 because they are learning Mathematics under the supervision of unqualified teachers with insufficient MPCK (Anney, & Hume, 2014; Lee et al., 2018).

DBE's (2020) diagnostic report on the NSC examination (2020), flagged several concerns about Grade 12 Mathematics learners, more importantly, the report indicated that Grade 12 learners did not have independent or creative thoughts; and were unable to manage analytical, evaluative, and problem-solving type questions in Mathematics; and most of them had poor language skills by the time they exited school (McGhie et al., 2020). Therefore, this shows there exists a learning and teaching problem in Grade 12 Mathematics education.

Furthermore, DBE's (2020) diagnostic report also showed that Grade 12 learners had difficulties in learning because the number of learners who passed Mathematics had declined in the 2020 NSC examination compared to 2019, from 54.6% to 53.8% (DBE, 2021). In 2020, DBE could only record a completion rate of 59% of learners in Grade 12, which is an indictment of the learning challenges Grade 12 learners face in their learning of Mathematics in schools in South Africa (DBE, 2021).

Oddly, the percentage pass rate of learners with distinction in Mathematics was 3.2% in the 2021 NSC examination (DBE, 2021). Shockingly, the NSC (2021) examinations recorded that Mathematics learners achieved a national pass percentage of 53.8% in the 30% to 39% achievement bracket (DBE, 2021), which translates into most Mathematics learners passing an "Elementary achievement" (DBE, 2021). Mathematics learners achieved a national pass percentage of 35.6% in the 40% to 49% achievement bracket, while at 50% and above, learners' national achievement level was a low 22.3% (DBE, 2021) I believe this low achievement output level at the Grade 12 level indicates that South African schools' low-quality of learning Mathematics education because Grade 12 learners struggle with analytical, evaluative and problem-solving type questions and most of them were unable to answer lower-order questions however, performed poorly in questions that required problem-solving skills, analytical and evaluative skills.

High-quality Mathematics education means learners must learn Mathematics for conceptual understanding, procedural fluency, and mathematical reasoning skills and develop learners' strategic competencies. Reflecting on the "Elementary achievement"

in the 2021 NSC and the continued low achievement levels in Mathematics by Grade 12 learners in 2022, I believe the current rote-learning style is ineffective. Therefore, there is a need for an instructional approach, like PBL to promote effective learning of Grade 12 Mathematics under the guidance of competent teachers (Klang et al., 2021).

PBL is a learner-focused learning strategy where learners critically present creative ideas and communicate mathematically with peers. Li and Tsai (2017) aver that PBL can help Grade 12 Mathematics learners solve problem-type questions and improve learners' Mathematics performance. In addition, teachers with higher MPCK who use learner-centred teaching strategies can successfully facilitate learning new knowledge amongst Grade 12 learners (Makofane & Maile, 2019).

PBL is an enquiry-based instructional learning strategy designed by teachers for learners to collaborate and acquire new knowledge through self-directed learning for life-size encounters, which can lead to achievement gains in Mathematics (Mceleli, 2019). Mathematics learners are now actively posing questions and participating in investigative processes while learning new concepts and developing processing and thinking skills enabling them to transfer knowledge to their peers (Klang et al., 2021).

A study by Mabena et al. (2021) also made similar remarks that poor Mathematics achievements in the higher grades can be ascribed to poor teaching and learning practices in the classroom. Conversely, a relationship exists between small-group instructional group engagements and learners' positive performance in secondary school problem-solving in Mathematics (Klang et al., 2021). PBL is an effective instructional method that enables Mathematics learners to develop their critical thinking and problem-solving abilities.

PBL, as an effective instructional method, is focused on learner-centeredness with educational activities that are complementary to the learning needs of various learners (Kyriacou, 2009; Stols et al., 2015; Ubah, 2021). Learners studying Mathematics using PBL demand that they are actively involved, take on challenging mathematical problems, make interdisciplinary connections and communicate mathematically with real-life issues (Makondo & Makondo, 2020).

Contrary to PBL, learning methods like question and answer, exposition, lecture method, work from the textbook, and teacher demonstration encourage poor learner performance levels (Makondo & Makondo, 2020). Learners are passive absorbers of

the content, handed down by the Mathematics teacher. This does not require learners to actively assimilate and make sense of the content knowledge and this may negatively impact Grade 12 Mathematics learners' performance at the NSC level.

The rise of PBL as the modern choice of teaching Grade 12 Mathematics as opposed to traditional means of teacher-centred instruction creates more opportunities for learners to construct knowledge meaningfully and this impacts their performance (Li & Tsai, 2017). PBL has gained prominence in various disciplines, including Mathematics (Issa & Khataibeh, 2021). PBL, as an instructional learning tool, was also adopted by other education fields (Malan & Ndlovu, 2014). Where did PBL originate from?

The origins of PBL can be traced back to the medical field as a substitute for conventional means of instruction because medical students were deemed to possess prior knowledge but did not have the problem-solving abilities to use this understanding (Batdi, 2016). Barrows, a professor of the peripheral nervous system, suggested PBL as a replacement for conservative instructions (Rui et al., 2015). PBL was then used by McMaster University in Canada, for the first time and thereafter became popular in the United States of America (USA) and Europe (Rui et al., 2015). During the same time, the study of Don Woods in conceptualising PBL during his tenure at McMaster University in Canada (with his chemistry students) also gained popularity (Dirckinck-Holmfeld, 2009).

In the transformational time teaching medical students, the focus of learning favoured the problem. In this scenario, it referred to the patients and their complaints (Dirckinck-Holmfeld, 2009). In other words, it was deemed that while medical students were busy analysing the (medical) problem, they would pose questions to their patients to gain more information, and in the process of solving the problem.

This new form of instructional learning resulted in other tertiary institutions adopting PBL as a new learning model, for example, Aalborg University was established as the centre for PBL in the city of Denmark (Dirckinck-Holmfeld, 2009). The Illinois Mathematics and Science Academy (IMSA) was responsible for implementing PBL in its learning styles and soon, the middle and elementary schools also implemented PBL (Inman, 2011). After that, the use of PBL also gained popularity in secondary schools, universities, and professional schools (Savery, 2006). Based on the claimed

successes, this study investigated the effects of pedagogical content knowledge in learning Grade 12 Mathematics using PBL.

The third objective of this empirical study was to examine the role of teachers in teaching Grade 12 Mathematics using PBL. What follows hereunder is a literature review that covers objective 3. By reviewing the literature, I will be answering question 3 of this study, *which is: What is the teachers' role in teaching Grade 12 Mathematics using PBL?*

3.2.3 Teachers' role in teaching Grade 12 Mathematics using PBL

The challenge of instituting the PBL approach is to transform the role of the Mathematics teacher from a traditional knowledge disseminator to one of a facilitator of learning Mathematics (Rahman et al., 2021). For teachers to effectively facilitate Mathematics lessons, they need to present a wide range of Mathematics content to Grade 12 learners, therefore they need to use instructional strategies like PBL. Mathematics teachers use MPCK to collectively plan and facilitate their lessons for learners to adequately learn the new content (Ljeh & Nkopodi, 2013).

International scholars have shown that MPCK, or the teachers' knowledge of teaching the subject matter, is crucial for effective learning and positive outcomes in Grade 12 Mathematics (Baier et al., 2019). Shulman (1986) MPCK refers to Mathematics teachers' understanding of learners' age-related difficulties in grasping and overcoming content. Similarly, Lee et al. (2018) understanding of MPCK is the knowledge of teaching and learning learners with grade-appropriate misconceptions. Therefore, MPCK is defined as the knowledge of teaching and learning Mathematics, which includes the curriculum's scope, educational objectives, and adequate teaching resources (Moh'd et al., 2021).

The critical instruction and the quality of the Grade 12 Mathematics outputs given by the PBL teacher are determined by the teachers' command of content knowledge or their MPCK (Li & Tsai, 2017). Carnoy and Arends (2012) agree with their international and regional peers that teachers with a robust knowledge of mathematical content, could help learners to overcome their learning difficulties and improve their academic performance.

Shulman (1986) MPCK is a concept that merges the teachers' Mathematics content knowledge and pedagogy (how to teach) specific to Mathematics education. MPCK is a distinguished body of knowledge a Grade 12 Mathematics teacher must possess, empowering the teacher to create learning opportunities by using various skills and knowledge to facilitate Mathematics learning amongst groups of learners to make sense of it (Shulman 1986). In other words, MPCK enables Mathematics teachers to use instructional learning strategies (PBL) to mediate the Grade 12 Mathematics learning content to learners (Miller & Krajcik, 2019).

Mathematics mediators must present a rich and balanced lesson in Mathematics (Lovitt & Clarke, 2011). The scholars highlighted some of the following features of a Mathematics lesson as adequate learning opportunities for learners:

- Practical lessons cater for learners of varying educational needs;
- Learners are actively involved in teaching and learning-centred activities; and
- Mathematical tasks discuss a range of crucial mathematical content.

The role of PBL teachers is different from the conventional teacher, as it is known to many. The PBL teacher must create learning opportunities for Mathematics learners to develop their achievements, mathematical connection ability and self-esteem in learning Mathematics (Sari, 2018). To do this, the PBL teacher can stimulate Mathematics learners to increase their active participation and interest in learning (Nurlaily et al., 2019). In other words, with positive results, Mathematics learners have been facilitated to identify the relationship between the material world and reality.

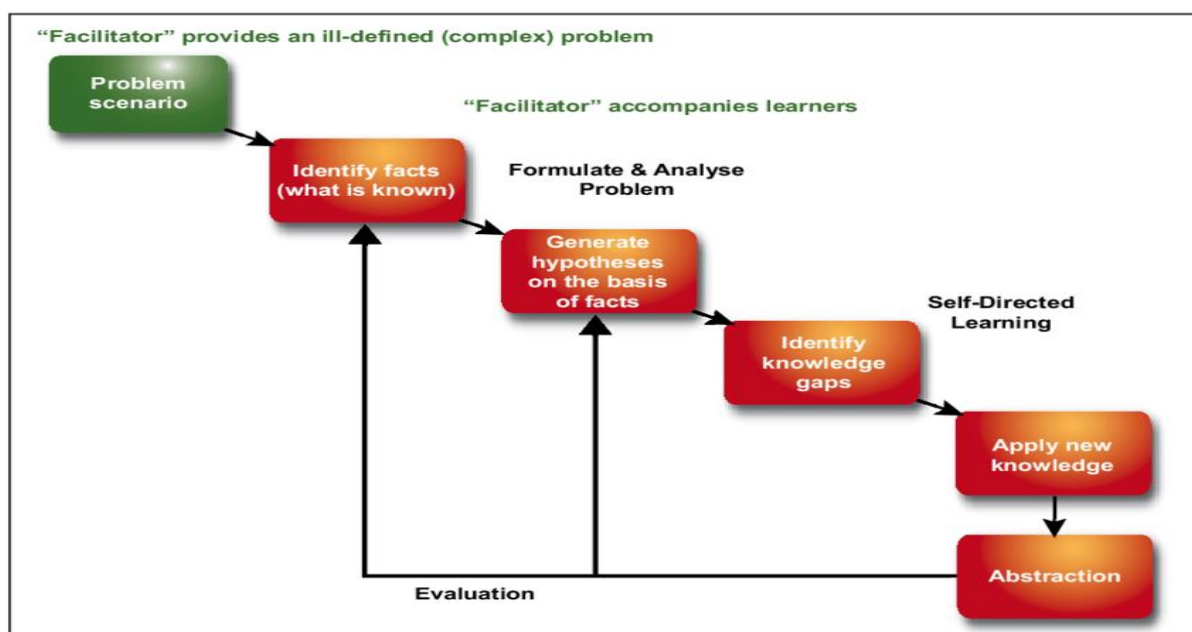
PBL is a learning pedagogy that prepares Mathematics learners for the real world. PBL enhances Mathematics learners' skills by practising their higher-order thinking abilities, introspecting their learning, and reflecting on the simulated events in the Mathematics classroom. Therefore, PBL allows Grade 12 Mathematics learners to apply their knowledge and skills to build on their schema, under the guidance of the subject teacher, and simply not reproduce the solutions for learners to passively read (Kokotsaki et al., 2016). Contrary to this, PBL is only successful if Mathematics teachers can effectively scaffold learning by facilitating and supporting learners with guidance (Kokotsaki et al., 2016).

Hmelo-Silver (2004) postulates that the scaffolding of the learning content depends on the quality of high instruction within exceptional encounters in the Mathematics

classroom. This will enable learners to gain small achievements and ultimately achieve cognitive growth just beyond their grasp (Bell, 2010). In scaffolding Mathematics learning, teachers should produce insight from their good feedback from learners engaging with PBL activities, so that they can recognise and continue with the opportunities of learning designed for the classroom (Kokotsaki et al., 2016).

Hmelo-Silver (2004) proposed below, that the PBL process is a cycle that may loop many times until learners successfully learn the content.

Figure 3.1: Problem-Based Learning (PBL) cycle



PBL emphasises learners-centredness, constructing a large and flexible knowledge bank to effectively enhance learners' problem-solving skills, active collaborators of lifelong learning, and intrinsically inspired to learn (Hmelo-Silver, 2004). The role of the PBL teacher in Mathematics is to mediate learning instead of disseminating information to learners and posing open-ended questions to trigger their thoughts to supply creative answers to problems (Klang et al., 2021). Furthermore, the PBL teachers provide methodical guidance in conceptual, well-thought-out reasoning in the Mathematics problem sums that will culminate in improved academic performances among the groups of learners (Muntari & Ahmed, 2020).

Learning Mathematics in doing group work assists Grade 12 learners in developing learning communities. In this space, learners feel comfortable exploring new thoughts and raising questions to each other or their teacher about the new content (Muntari & Ahmed, 2020). More importantly, group work develops learners' communication skills and group dynamics because group work motivates learners to be actively involved and accountable for their actions and can lead to gains in Grade 12 Mathematics, while the subject teacher monitors the learning activities and reciprocating information where necessary (Koh & Chapman, 2019).

Nurlaily et al. (2019) mention the advantages of PBL, as the preferred learning method are that it exposes learners to problems that resemble authentic real life, learners are self-motivated to learn actively, exposes them to various learning situations, and allows learners to collaboratively find solutions and improve learners' achievements in Grade 12 Mathematics, under the safe eye of the teacher. Furthermore, Mathematics learners can develop new information and expertise through their encounters in the classroom while solving life-like problems (Koh & Chapman, 2019).

Mathematics teachers using PBL to effectively instruct Grade 12 learners on the content, should appeal to their cognitive demand and age-appropriate understanding. In other words, the simulated problems posed to learners in the classroom, representing authentic life challenges, must be relevant to their context, learners-focused, collaborate on new knowledge and be self-directed in their learning (Lopes et al., 2019).

Grade 12 Mathematics learners need to be exposed to everyday contextual problems which learners need to solve. The Mathematics teachers are the designers and implementors of the learning scenarios, focused on learner-centred conditions to transform low Mathematics levels into higher performance gains (Koh & Chapman, 2019). Teachers are facilitators, guiding the resources to learners to develop their problem-solving skills. Mathematics teachers must create learning challenges that appeal to learners' cognitive levels and enable them to identify new solutions using newly acquired skills and knowledge through creative dialogic spaces (Klang et al., 2021). If this does not happen, learners become passive in Mathematics learning, and this leads to poor performance levels in Grade 12 (Aidoo et al., 2016).

PBL is a learners-focused engagement activity in which learners construct new knowledge on their previous knowledge, and the teacher guides learners by posing probing questions that will enable them to keep focused on the learning objectives (Aidoo et al., 2016). Essentially, this means that for successful instruction in mathematical problem-solving, scaffolding for knowledge must be accompanied by scaffolding for dialogue (Klang et al., 2021). Therefore, the Mathematics teacher guides Grade 12 learners on innovative ways of thinking, new understandings, and arguments for their problems. Furthermore, focusing on learners-focused activities allows learners to practice the newly acquired content under the caretakers of more knowledgeable Mathematics teachers, increasing their thinking ability (Lopes et al., 2019).

Grade 12 learners learning Mathematics through the PBL strategies acquire new information through self-directed learning (Koh & Chapman, 2019). Some of these self-directed activities allow learners to take charge of learning Mathematics to acquire new knowledge and skills through their own experiences while solving problems with new innovative solutions (Koh & Chapman, 2019). Also, Grade 12 learners get to choose a purposeful activity, thereby this is a self-imposed learning opportunity, rather than one unilaterally forced by teachers onto learners (Burlbaw et al., 2013). However, the critical role of teachers to plan and assist learners in managing their time in the learning process cannot be ignored. Furthermore, PBL teachers must induce a Mathematics classroom whereby learners receive systematic instruction in conceptual, strategic, and reflective reasoning on the content (Ceker & Ozdamli, 2016). An arbitrary learning environment encourages learners to take control of their learning and suggests positive improvements in Grade 12 Mathematics assessments (Lee et al., 2018).

PBL is an instructional learning method through which smaller groups of Mathematics learners develop their upper-level problem-solving and critical-thinking skills while eliciting from their personal experiences through cooperative learning, mediated by the subject teacher (Klang et al., 2021). The approach to PBL lies in the strength of learners striving to find solutions to simulated real-life problems in the Mathematics classroom under the guidance of competent Mathematics teachers (Abubakar et al., 2020). Moreso, PBL is an instructional learning approach based on the theory of social constructivism (Mulyanto et al., 2018). Learners in a social constructivist Mathematics

scenario are actively in the process of encountering a personal representation of the world and consciously construct new knowledge, skills and attitudes from their experiences under the observant radar of their Mathematics teachers (Amineh & Asl, 2015).

PBL is a model of learning that will make Grade 12 learners interested in learning Mathematics and become active participants in their group while establishing direct relationships and collaborative skills (Mulyanto et al., 2018). Individuals in a group, learning Mathematics often bring different attributes that lead to successfully solving real problems with a shared solution. Mathematics learners working in groups of 4 or 5 showed they could research, review, critique, and communicate mathematically to solve problems and were engaged in self-directed learning, under the knowledgeable view of skilled Mathematics teachers (Koh & Chapman, 2019).

Knowledge transfer will ensure effective Mathematics learning under the teacher's facilitation. This means that Grade 12 Mathematics teachers must be versatile in the content; that is, they must hold good classroom discipline, actively engage learners and use differentiated teaching approaches to achieve positive learners' achievements (Protheroe, 2007). Effective Mathematics instruction consists of problem-solving strategies to develop learners' knowledge concepts at appropriate developmental levels, otherwise, there is no meaningful learning. Therefore, it is incumbent upon the Mathematics teacher to deliver Mathematics lessons that appeal to their target audience (Ahdhianto & Istiq'faroh, 2022).

Making sense of the new Mathematics content using PBL allows learners to successfully acquire problem-solving skills by scaffolding content combined with group dialogue (Taber, 2018). Group discussions may allow learners to build new ways of thinking, involving questioning their understandings and verbalising and negotiating on different answers. Therefore, effective learning of Grade 12 Mathematics requires the teacher to understand learners' current knowledge and produce meaningful learning by building new knowledge on their existing content (Rytilä, 2021). Therefore, the role of effective Mathematics teachers in the classroom means that teachers are the mediators of the Mathematics content, facilitating the learning occurrence for learners or building up (scaffolding) their understandings, in an environment reminiscent of their authentic real-life scenarios (Amineh & Asl, 2015).

3.3 Summary of Chapter 3

This chapter analysed the literature on the effects of pedagogical content knowledge in learning Grade 12 mathematics using PBL. References were made to scholars that produced international, regional and local information that support the concept of collaborative teaching and learning of PBL and its positive effects on learners' achievements in Grade 12 mathematics. Furthermore, the literature deliberated on the impact of MPCK on the learning of Grade 12 Mathematics, the relationship between learning mathematics and teachers' MPCK, and the impact of MPCK on PBL. Also, the effects of MPCK on SMK in teaching and learning Grade 12 mathematics were debated in detail with references to scholarly work.

The following chapter discusses the research Design and Methodology and how the data collection process was conducted. The data research instruments and the ethical implications of the empirical study will also be debated.

CHAPTER 4: RESEARCH METHODOLOGY

4.1 Introduction

The preceding chapter dealt with this empirical study's literature analysis. This chapter discusses this investigation's research design and methodology. The chapter is guided by modified research steps by Johnson and Christensen (2020).

4.2 Research steps guiding this study

The research entails several processes, such as inquiry, investigation, inspection, and even experimentation on occasion, as stated by Bougie and Selkaran (2020). The process is both scientific and rigorous (Creswell & Creswell, 2018), and it proceeds according to a sequence of steps that has been set in advance (Cohen et al, 2019). According to Bougie and Selkaran (2020), each stage of the process must be carried out with the highest care, deliberation, objectivity, and logic. It is not necessary for these processes to be linear or to go in a single direction; rather, they involve a process that is cyclical, or more accurately helical (Cohen et al, 2019), recursive, and interactional (Creswell & Creswell, 2018).

4.2.1 Step 1: Select the research topic

This has been done.

4.2.2 Step 2: Determine research questions

Please refer to Chapter 1 (cf. 1.5)

4.2.3 Step 3: Design the study

According to Walliman (2018), there are a wide variety of research designs that may be used for a variety of various sorts of research projects. Therefore, it follows that definitions of research design are quite vague, and as a result, numerous terminologies are employed to characterise research design across different types of studies (Fouché et al., 2022).

One's investigation is brought into appropriate perspective by the use of a series of actions that are referred to as a design. These steps are taken in the order in which they are performed in the process of limiting one's investigation. In addition to this, the study design entails several different sets of decisions concerning the subject matter that is being investigated, as well as which population, which research methodologies,

and whose goals are being pursued (Creswell, 2016). This is very similar to how Ugwuanyi (2022) defines a research design. He adds that a research blueprint is a diagram for doing research, which indicates that it describes the strategies and processes for the collection of data, as well as how the collected data would be examined. This definition of a research design is very similar to how Ugwuanyi (2022) defines a research design. According to Hennink et al. (2020) definition, a research design is an approach used to solve a well-defined research problem. Because the purpose of research design is to translate a research problem into data for analysis (Creswell & Creswell, 2018) and to provide relevant answers to research questions and results at the least possible cost and resources (Bougie & Selkaran, 2020), it always determines the kinds of analysis that are to be carried out to get the desired results (Marczyk et al., 2021). Ugwuanyi (2022) defines the research design that is used in this study, as a blueprint, framework, or plan that determines the purpose of the research as well as how relevant data will be collected, analysed, and interpreted to conclude. In deduction, different authors have different perspectives on the concept of research design. However, the definition of research design that is used in this study comes from Ugwuanyi (2022).

In this study, I engaged a general and a specific research approach. Under this particular research approach, I engaged in an interactive research design (Dimitrios & Antigoni, 2019). I engaged in an interactive, descriptive case study research design (Ugwuanyi, 2022) using an exploratory, qualitative case study approach.

According to Etikan et al. (2016) doing research via a case study entails conducting an in-depth investigation of a single instance. This concept is taken a step further by Creswell and Creswell (2018), who define a case study as a form of research approach that generates an in-depth, multidimensional knowledge of a challenging topic in its actual environment. Bloomberg and Volpe (2022) aver that a case study is an investigation into individuals or groups of people that is conducted in great detail. Furthermore, a case study is necessary to get insight into a certain circumstance (Etikan et al., 2016). For Creswell and Creswell (2018), case study research is a method for analysing a research study by placing it in one or more real-world situations. Bloomberg and Volpe (2022) demonstrate how there may be a blurring of the borders between the phenomena that are being explored and the setting in which it is being studied when case study research is conducted. Therefore, if a researcher

intends to obtain an in-depth grasp of the subject's history and the processes involved, a case study methodology is required to be utilised. This method can result in solutions that address the "what," "why," and "how" of a particular problem (Cohen et al, 2019). Bloomberg and Volpe (2022) highlight how research utilising case studies may be done using a wide variety of methods, some of which include but are not limited to single or multiple, holistic or embedded case study procedures. For Creswell (2016) a case study may be conducted, either in a qualitative or quantitative research method, which can be used and this is dependent on the type of empirical study that is being carried out.

Considering the above, the research conducted here applied a case study approach. In this particular research, an intrinsic case study was employed, and the major focus of my attention was on gaining knowledge of a specific scenario. The design of this investigation is what Fouché et al., (2022) refer to as a traditional single-case design, in which I discuss in great detail various aspects of the case and shed light on it.

Thus, under the specific research design, I employed a case study research approach based on the case study of the phenomenon of PBL (Creswell, 2016). I chose a case study because I could investigate a case that was reasonable and achievable and because I could choose an example that was acquainted with me and I could also access (Rule & John, 2022).

A case study was chosen as the favoured research approach because I wanted to examine the effects of pedagogical content knowledge in learning Grade 12 Mathematics using problem-based learning (PBL) in two identified secondary schools in the Amajuba district of Newcastle, in KwaZulu-Natal. I purposely chose the two secondary schools in the Amajuba district in KwaZulu-Natal because these schools will be the best for me to investigate the problem and the research question.

The major advantage of using a case study approach for this study is that a case study research design does not generalise beyond the case, it provides an in-depth study (Walliman, 2018). and in this case, of the two selected secondary schools in the Amajuba district in KwaZulu-Natal in their existing context. In this investigation, the case study approach allows for the planning and defining of the elements, their interrelationship and methods (e.g., sampling, measurement) that constitute this research (Johnson & Christensen, 2020).

4.2.3.1 Research approach

Considering the numerous available methodological options, I chose a qualitative approach conducted with a singular technique. From the three types of research designs, namely, qualitative, quantitative, and mixed methods proposed by several authors (e.g., Creswell, 2016; Johnson & Christensen, 2020; Rule & John, 2022). I have chosen to use a qualitative research approach due to the reasons given below.

Although a quantitative research approach enables researchers to rely on rich, reliable data (Nardi, 2018; Walliman, 2018; Salkind, 2019) and to analyse a study's numerical information through statistical procedures (Nardi, 2018), this approach is largely researcher-driven, with researchers acting as detached, objective observers (Cohen et al., 2019). In addition, quantitative research requires valid and reliable measurement instruments for statistical analysis (Salkind, 2019). It employs numbers, relies on the perspective of a distant researcher, focuses on conjecture testing, is fixed, structured, and preoccupied with generalisation, and depends on concrete, numeric data (Nardi, 2018). Since it is utilised in providing answers to queries regarding the inter-relationships between measured objects to explain, foretell and direct incidents, the purpose is to initiate, sanction, or authenticate associations and to establish generalisations (Cohen et al, 2019).

In an exploratory mixed methods design, two-phase mixed method designs are used whereby researchers first explore an occurrence by initially using qualitative data before an attempt is made to test it quantitatively (Fouché et al., 2022). As a two-phase design, the results of the first phase (qualitative data) inform the second phase (quantitative data). Here, researchers explore the study's objectives in depth and then measure their prevalence (Creswell & Creswell, 2018) by identifying qualitative themes and then using that information to guide the subsequent quantitative examination of the initial qualitative results (Mohajan, 2021). Researchers conduct the two methods in separate phases by collecting only one type of data at a time.

Johnson and Christensen, (2020) identified the key factors that researchers should consider when choosing a mixed-method design for their studies. These are:

- **Completeness:** A researcher can fully address a research problem and its subproblems only by collecting, analysing and interpreting both quantitative and qualitative data.
- **Complementarity:** Quantitative aspects of the study can compensate for weaknesses in qualitative research, and vice versa. For example, the results of unstructured interviews with only a small number of individuals (which might raise concerns about generalisability to a larger population) can be replicated by administering a questionnaire to a larger, more representative sample of the population.
- **Hypothesis generation and testing:** Qualitative data often provide insights that help a researcher form hypotheses about cause-and-effect relationships—hypotheses that the researcher can subsequently test through controlled, quantitative research.
- **Development of appropriate research tools and strategies:** One type of data can inform and guide the subsequent collection of another type of data. For example, unstructured interviews (yielding qualitative data) can guide the construction of appropriate questions for a survey (which will yield quantitative data).
- **Resolution of puzzling findings:** In a quantitative study, various results can sometimes seem inconsistent or contradictory; qualitative data may reveal underlying nuances and meanings that can help the researcher make sense of the numbers.
- **Triangulation:** A researcher can make a more convincing case for particular conclusions if both quantitative and qualitative data lead to those conclusions.

Taking consideration of the above, I chose a qualitative research approach because of the following reasons.

With its epistemological roots in phenomenology (Creswell, 2016) and its use of a social constructivist paradigm (Sürücü & Maslakçı, 2020), a qualitative research approach is based on explaining a phenomenon in an encompassing, contextual, and deeply all-inclusive manner ((Sürücü & Maslakçı, 2020). Additionally, qualitative research is the general label applied to research that deals with lived experiences. According to Nardi (2018), the term "lived experiences" refers to the process of

attempting to comprehend occurrences from the perspectives of those participants who were involved in the events and to comprehend the experiences that they went through, rather than imposing the viewpoint of another person. In contrast to a quantitative strategy, which involves pre-set categories and a more organised scientific approach (Creswell, 2016), the purpose of a qualitative research approach is to comprehend the perspective of individuals who are involved in the interviews.

As a result, I decided to use a qualitative research method for this research because it was defined by the nature of its research, which was based on prejudiced relationships (Gaudet & Robert, 2018; Wild & Diggins, 2015). These relationships were biased in the manner that although I was personally involved with the subjects (Creswell, 2016), I gained the data subjectively (Cohen et al., 2019). The study unearthed the importance and seriousness of human etiquette as a qualitative study (Cohen et al., 2019; Creswell & Creswell, 2018; Devlin, 2018). Additionally, the study looked at the deep understanding of actions (Cohen et al., 2019; Morgan, 2014) and events. It, therefore, emphasized human beings' lived encounters (Creswell, 2016) to gain an understanding of their experiences (Creswell & Creswell, 2018; Devlin, 2018), it pursued an inquiry-based approach in which I entered into an authentic-world setting to uncover, interconnect, and gain an understanding of what surfaced in a naturalistic manner, and it was dependent primarily on various techniques of collecting data such as interviews and observations (Creswell, 2016).

I used qualitative research because I desired to gain in-depth information (Devlin, 2018) as it sought to comprehend the empirical study's variables (Creswell & Creswell, 2018). The study participants' intrinsic language was utilised to arrive at a genuine appreciation of their world (Fouché et al., 2022) hence the investigation adopted a case study approach (Devlin, 2018; Nardi, 2018) to narrate and explain, to investigate, to decipher and to construct theory (Cohen et al, 2019).

A good characteristic of qualitative data "is their richness and holism, with strong potential for revealing complexity such as providing thick descriptions" (Creswell, 2016) that are authentic, found in a real-life context and have a ring of veracity that has a strong effect on the reader (Creswell & Crewell, 2018).

Accordingly, the data sources in my study were regulated by the information richness of settings (Fouché et al., 2022) for the affluent nature of the qualitative (Aliyu &

Adamu, 2015; Creswell & Creswell, 2018; Gaudet & Robert, 2022), deep (Sürücü & Maslakçı, 2020) and rich data (Rule & John, 2022) which was educational and manuscript (Creswell & Creswell, 2018) to enrich the understandings (Fouché et al., 2022). The data collected was spoken and facial expressions of thoughts, feelings, etc. (Sürücü & Maslakçı, 2020) seeking to appreciate the useful demonstrations of the phenomenon of PBL in two selected secondary schools in the Amajuba district in KwaZulu-Natal to construct detailed descriptions of social reality (Cohen et al., 2019, Johnson & Christensen, 2020; Rule & John, 2022).

Quantitative research needs a big sample for generalisation (Rule & John, 2022) while the qualitative research approach can be on minor representation (Sürücü & Maslakçı, 2020) following Bougie and Selkaran (2020) “qualitative research does not generalise over a large audience”. Sampling is then aimed at uncovering information about the phenomena under review, not a scientific generalisation from a representation to a population (Johnson & Christensen, 2020). Therefore, as qualitative research, my study utilised a small sample of participants whom I purposively designated (Devlin, 2018; Salkind, 2019) to record the in-depth empirical review. Since my research was qualitative, I observed human beings: what did they do and say; the next step was data collection, data analysis and data interpretation. I looked for dissimilarity and researched themes, groupings (Johnson & Christensen, 2022) and patterns (Devlin, 2018; Salkind, 2019) using a repetition of themes in the data analysis (Creswell & Creswell, 2018). This was followed by coding and script analysis (Braun et al., 2016) and was presented in words (Cohen et al., 2019), chronicles (Cresswell, 2016), quotations, personal voice notes and writing styles (Cohen et al, 2019), using an inductive analysis (Braun et al., 2016). This study also utilised literature studies (Braun et al., 2016) to investigate and understand the facts (Creswell, 2016). The literature study was done concurrently (Salkind, 2019) and sometimes, delayed until the raw data was gained (Braun et al., 2016).

4.2.3.2 Research paradigm

A research paradigm is at times referred to as a research philosophy. It is a theoretical framework comprised of a set of beliefs and values, which guides how research is conducted and knowledge conceptualised within scientific communities (Ugwuanyi, 2022). It (a research paradigm) represents a particular worldview that defines for the researchers who hold this view, what is acceptable to research and how this should

be done (Creswell, 2016). A paradigm is a way of viewing, understanding, or making sense of a phenomenon (Ugwuanyi, 2022). It represents a specific way of interpreting one's comprehension of the world (Creswell & Creswell, 2018). Since research paradigms are a set of presuppositions or beliefs that are concerned with fundamental aspects of reality which give rise to a worldview (Dimitrios & Antigoni, 2019). This implies that paradigms serve as the lens or organising principles by which reality is interpreted (Nieuwenhuis, 2022). Al-Ababneh (2020) reveals how research paradigms should be characterised through their ontology, epistemology, and methodology, that is, what is reality, how do you know something and how do you go about finding it out? There are various kinds of paradigms upon which various researchers base their respective scientific investigations. These include positivism, realism, interpretivism, objectivism, subjectivism, pragmatism, functionalism, radical humanism, and radical structuralism (Bloomberg & Volpe, 2022). This study used an interpretivism paradigm due to the following reasons.

I reject positivism as a paradigm for this study.

The positivism paradigm study is marked by uncovering objective and evidence-based truth and posits the truth is objective (Creswell & Creswell, 2018). It (positivism) studies human action through mathematical and logical reasoning (Bloomberg & Volpe, 2022) and scientific methods (Creswell & Creswell, 2018). In positivism, the reason to do research is to learn more about how our world which is built upon the natural world works (Creswell & Creswell, 2018). As described by some scholars (Argyres et al 2020; Bezuidenhout & Davis, 2021), positivism accepts that community phenomena are most suited when they are explored through natural disciplines.

Positivists place more importance on it being well-grounded, as the most characteristic of the scientific discipline of research (Rule & John, 2022). Positivism is employed to build up data necessary to gain research studies' aims and objectives in quantitative studies (Johnson, & Christensen, 2020).

The positivism paradigm entails quantitative measurement whereby knowledge is generated objectively, independent of the researchers' or the respondents' values (Creswell & Creswell, 2018; Johnson, & Christensen, L. 2020) and where human action is studied through mathematical, logical reasoning and scientific methods (Creswell, 2016).

The primary purpose of a positivistic line of inquiry is to make accurate forecasts, exert authority over, and influence the outcomes of natural and social occurrences. According to Adhabi and Anozie (2017), the foundation of research concerns and questions in this domain is the causal (cause and effect) relationships that exist between variables that are dependent on one another and variables that are independent of one another. Positivist researchers strive to find, measure, and assess events and provide a logical explanation by establishing causal linkages between the many features (or variables) of the study topic and relating them to a certain theory or practice (Creswell & Creswell, 2018).

Qualitative studies are interpretive and inductive (Creswell, 2016) and depend mostly on multiplexed realities, in which various types of data are earned from the participants (Aliyu & Adamu, 2015; Müller & Klein, 2019; Johnson & Christensen, 2020). Its interpretative approaches are focused on the process of constructing understanding from the actions of the participants (Creswell & Creswell, 2018) in their environments (Bloomberg & Volpe, 2022).

I was able to collect qualitative information thanks to interpretivism, and interacting with individuals helped me understand how they make sense of the environment in which they live (Creswell & Creswell, 2018). As a result, one of the motivations for using interpretivism was the need to place a significant amount of importance on the information obtained from the participants of the research via interviews. I used an interpretative design approach to discover the human conception of the human phenomenon of PBL to explain and understand the participants' experiences (Creswell & Creswell, 2018; Yin, 2018), and to develop and create information through discussions, interaction, and practice throughout the process (Fouché et al., 2022). This was accomplished by using an inductive approach to social reality.

4.2.3.3 Population and sampling

I used a 5-step model for sampling which was adopted from Masha (2021) who modified sampling stages from different scholars. They are listed below and explained.

Table 4.1: A 5-step model for sampling

STEP	SAMPLING STEP	AUTHORS
Step 1	Define the target population	From Step 2 of Bradley (2013), step 1 of Cant (2018), step 1 of Iacobucci and Churchill (2015), step 1 of Struwig and Stead (2017), step 1 of Wild and Diggines (2015), step 1 of Quinlan et al. (2019), step 1 of Bougie and Sekaran (2020) and step 1 of Cant (2018)
Step 2	Select a sampling frame	Step 2 by Struwig and Stead (2017), step 2 by Wild and Diggines (2015), step 2 by Quinlan et al. (2019) and Step 2 by Cant (2018).
Step 3	Determine the sampling method (Probability or non-probability)	From Step 4 of Bradley (2013), step 4 of Cant (2018), step 4 of Struwig and Stead (2017), step 3 of Wild and Diggines (2015), step 3 of Quinlan et al. (2019), step 3 of Bougie and Sekaran (2020) and step 3 of Cant (2018).
Step 4	Determine sample size	From Step 5 of Bradley (2013), step 5 of Cant (2018), step 4 of Iacobucci and Churchill (2015), step 5 of Struwig and Stead (2017), step 4 of Wild and Diggines (2015), step 5 of Quinlan et al. (2019), step 5 of Bougie and Sekaran (2020) and step 4 of Cant (2018).
Step 5	Conduct fieldwork	From Step 6 of Bradley (2013), step 6 of Iacobucci and Churchill (2015), step 5 of Wild and Diggines (2015), step 7 of Quinlan et al. (2019), step 5 of Bougie and Sekaran (2020) and step 5 of Cant (2018).

Source: Masha (2021)

4.2.3.3.1 Step 1 of sampling: Define the target population

A community contains the complete components (units) of a specific kind of species in a fixed area at a particular time frame according to the interest of the scholars (Etikan & Alkassim (2016). Whereas, a target population is a grouping of facets or cases, whether singularly, objects, or occasions, that have the characteristics the investigator aims to review and which members of the study conform to specific criteria (Casteel & Bridier, 2021; Cohen et al., 2019; Fouché et al., 2022). This study's target population was Grade 12 mathematics learners and teachers. It was from the target population that this study sample was chosen.

4.2.3.3.2 Step 2 of sampling: Specify a sampling frame

According to scholars like Etikan and Alkassim (2016), the definition of a sampling frame is a collection of items from a population from which a sample is selected. The process of selecting a small number from a larger group to conserve time and other resources is referred to as sampling (Casteel & Bridier, 2021). The advantages of this method include but are not limited to, the fact that it is used more often, that it is less costly, and that it does not call for a list of all the qualities that are associated with the population.

4.2.3.3.3 Step 3 of sampling: Determine the sampling method and procedure

A purposive sampling technique was utilised to select two secondary schools in Newcastle, in the Amajuba district in the province of KwaZulu-Natal. The criteria used to identify the schools were one school consistently performing highly (above 80% average), and the other consistently underperforming (below 65% average) in the National Senior Certificate (NSC) for 2019, 2020 and 2021 (DBE, 2021). Year-end district reports were used to identify the two secondary schools. I envisaged these schools would assemble rich-endowed data on the elements under investigation.

Sampling encompassed the selection of respondents and the research site (Creswell & Creswell, 2018). The groups selected for sampling were Grade 12 learners and their Mathematics teachers because they not only conformed to specific criteria (Cohen et al., 2019) but also possessed the characteristics that the researcher aimed to investigate (Creswell, 2016). Thus, I employed a homogeneous purposive sampling technique amongst the Grade 12 learners because they were alike in years, culture, position, or life encounter. The participants were selected because I assumed them to be rich in information (Casteel, & Bridier, 2021; Fouché et al., 2022; Johnson, & Christensen, 2020) and informative; that is, they offered useful experiences of the elements under evaluation (Rule & John, 2022). In this manner, the participants were able to provide answers to the interview questions (Adhabi & Anozie, 2017).

I used a homogeneous purposive sampling technique amongst the Grade 12 learners because they were similar in description, background, or life experiences (Etikan et al., 2016). The number of people in focus groups differs from author to author, as seen in Appendix 7. I used six to eight participants in the focus groups, as suggested by various authors (Creswell & Creswell, 2018; Devlin, 2018; Fouché et al., 2022).

4.2.3.3.4 Step 4 of sampling: Determine the sampling size

After I determined the sampling process, the next step involved was to predict the size of the sample (Casteel & Bridier, 2021). This study's sample was a representation of the entire population of interest (Walliman, 2018).

I selected six learners for each focus group for their respective secondary schools. The Grade 12 learners' performance in the first term of 2023 was used to select two high, middle, and low-performing learners. In other words, high-performing learners achieved between 80%-100%, middle-performing learners between 50%-59%, and low-performing learners achieved 35%-39% (DBE, 2021). Gender was also applied as a criterion; therefore, the number of male learners had to equal the number of female learners. The Mathematics teacher teaching Grade 12 Mathematics was responsible for availing the learners for each focus group.

I interviewed Grade 12 Mathematics teachers from the study sites to derive meaning from their responses on the effects of pedagogical content knowledge in learning Grade 12 Mathematics using PBL. As for educators, I purposively selected two Mathematics teachers who were teaching Grade 12 Mathematics. The teachers selected were both rich in their comprehension, skills, encounters and attitude because of their present teaching experience in Grade 12 Mathematics. The sample was drawn through a specific sampling strategy from the accessible population, consisting of those elements from whom or about whom data are collected (Creswell & Creswell, 2018).

4.2.3.3.5 Step 5 of sampling: Conduct fieldwork

This was done.

4.2.4 Step 4: Collect data

This was done.

4.2.4.1 Research instruments

An overview of the data collection strategies used in the empirical review is presented. These strategies include:

- Semi-structured one-to-one interview questions to collect in-depth data from Mathematics teachers to answer Question 1;

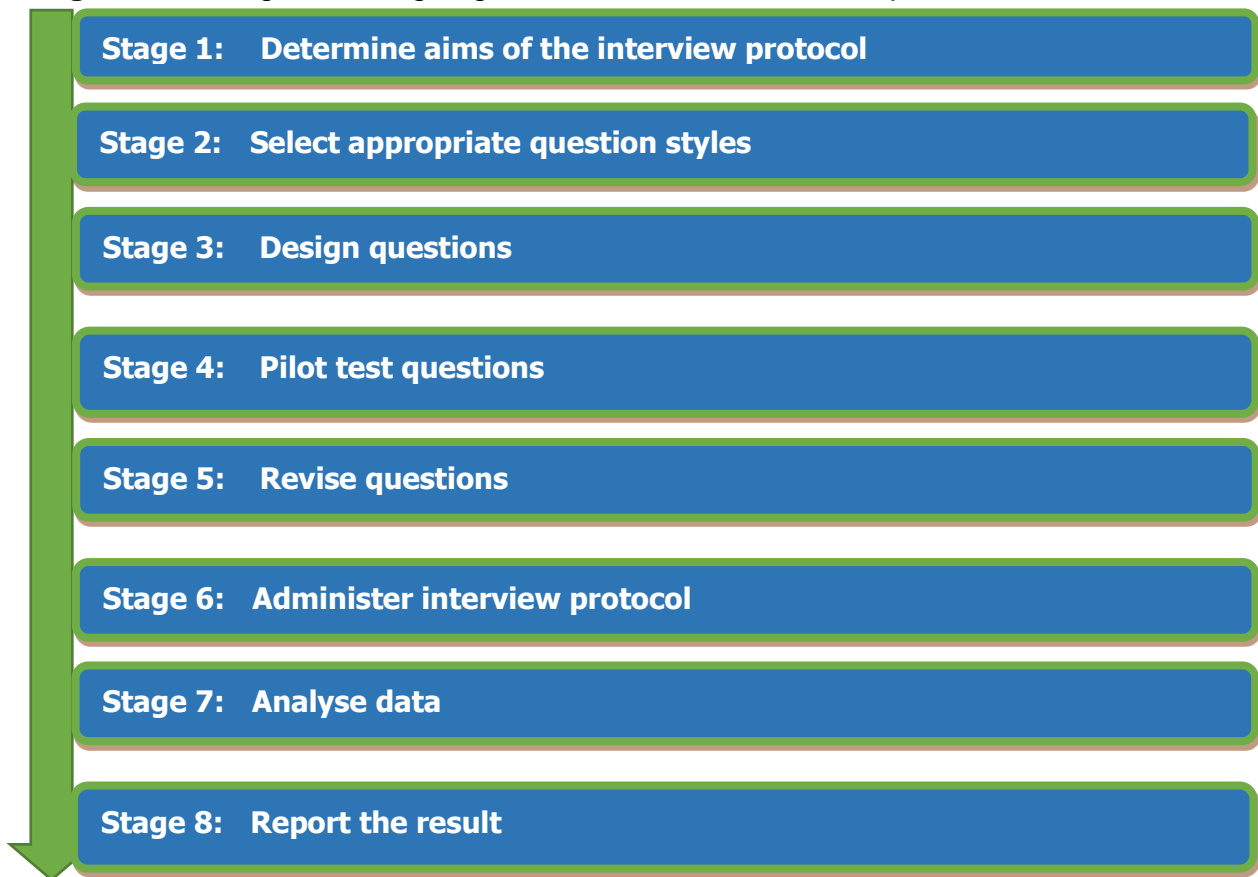
- Focus-group interviews to gather data from Grade 12 Mathematics learners by posing open-ended questions to answer Question 2; and
- Observe how Mathematics teachers interact with learners in the classroom and make field notes into detailed reports to answer Questions 3.

Research instruments are tools that are often used in the fields of Social Science, Health Science and Education to collect, measure and analyse data related to one's research interests or relevant to the subject of one's research topic. These tools are mostly used to gather information from research or participants. In other words, research instruments include all measurement tools used in data collection for research studies (Adhabi & Anozie, 2017). I used an interview protocol as a research instrument to collect data during this study's interviews. For observations, I used an observation guide to collect observation data.

I used a semi-structured interview protocol (also known as an interview schedule or interview guide) in this study as a data collection instrument. According to Cohen et al. (2019), an interview protocol is a data-collection instrument or a script prepared by the researcher and the interviewer reads it and also interprets it, if necessary to the interviewees.

I used an interview guide containing semi-structured questions to guide the interviews and to ensure that the discussions remained on the topic under study (Creswell & Creswell, 2018). Since the interview guide was created beforehand, it encouraged me to think clearly about what I intended to examine during the interview process (Bougie & Sekaran, 2020). This led me to generate a transcript from the interview process. Eventually, the rich descriptive data from the interviews helped me to understand the participants' construction of knowledge and reality (Nieuwenhuis, 2022). The stages of designing this study's interview protocol are illustrated in Figure 4.1 below.

Figure 4.1: Stages in designing a semi-structured interview protocol



Source: Naz et al. (2016)

An observation guide to recording lesson observations.

Creswell and Creswell (2018) postulate that semi-structured questions in an empirical review are posed to elicit the intended participants' opinions and experiences on a specific experience they had undergone. This study sought to review the effects of pedagogical content knowledge in learning Grade 12 Mathematics using PBL.

In a research study, open-ended questions are intended to earn the respondents' viewpoints on a social phenomenon they experienced since the objective of any research is dependent on the participants' feedback (Creswell & Creswell 2018). Fouché et al. (2022) postulate that the questions raised to the participants should be substantial so that the respondents can build their understanding from the discussions in their group. While answering the questions, the researcher can listen carefully to the participants' feedback and actively construct their meaning through conversation with the universe they interpret.

In qualitative case studies, usually, open-ended questions are exploratory in nature. Creswell and Creswell (2018) declare that open-ended questions demand a response from the participants whereby they need to articulate their experiences verbally and not be coerced into producing a predetermined response. The empirical questions put forward to the respondents sought to investigate the phenomenon under review and its meaning.

Cohen et al. (2019) notify us that researchers must pre-test the research instruments (questions) to improve the success and efficiency of the study. This is referred to as piloting. A pilot study or pre-test assisted me in the design of the data-gathering instrument designed for the research (Quinlan et al., 2019). I piloted this study's research instrument with participants from a different school similar in characteristics to the sample population.

Based on this, I can conclude that the data measuring tools were tested for their appropriateness to be defined so they may be effectively used in the actual research to gain the data needed for the three research sub-questions of this study.

The feedback from the feasibility study mitigated the possibility of the research tools failing to draw out the data on the effects of pedagogical content knowledge in learning Grade 12 Mathematics using PBL. Piloting the research tools opportune me to evaluate the measuring instruments to see whether there was any possibility to enhance the style of the questions or reconfigure them.

After piloting, I sharpened some questions, making sure they were appropriate to draw out the information needed to answer the study's research questions. Fouché et al. (2022) suggest that piloting the empirical study was important to enhance the effectiveness of the research tools. I realised that pilot testing the research instruments was important because it allowed me to "practice" performing the focus group interviews, semi-structured teacher interviews, and classroom observations.

I used different types of research instruments to get data from multiple angles, a process known as data crystallisation.

Crystallisation consists of multi-modes of data being analysed to produce a meaningful report based on evidence of the phenomenon under review and showing the researchers' bias and objectivity (Borkan, 2022). Furthermore, crystallisation refers to

validating the findings of data by using various data-collection methods (Borkan, 2022).

Crystallisation posits that the truth is constructed subjectively, and no one truth exists. Therefore, different understandings by various individuals can collaboratively work together to formulate alternative interpretations of the effects of pedagogical content knowledge in learning Grade 12 Mathematics using PBL (Borkan, 2022). According to Ellingston (2009), the principles of crystallisation include paying attention to the complex nature of explanations, the use of multi-modes of data collection and reflexivity, as well as thick and deep explanations of data.

Creswell (2016) describes thick and deep explanations as the researcher's recording their experiences in the field in which the investigator reaches patterns of social and cultural relationships and puts them in a context. In other words, thick description involves the researcher providing the background information necessary to understand the relevance, meanings and intentions underpinning social interactions in the Grade 12 Mathematics classroom. To achieve thick description and understanding, I observed, described, interpreted, and analysed the situation in which the participants were involved.

I paid attention to the complex nature of explanations providing an in-depth understanding of the topic under review. Furthermore, qualitative research aims to provide insight into people's understanding and their construction of reality, which is not a fixed entity (Creswell, 2016). I reported on the findings of the data and the different perspectives shared by the participants by generating detailed explanations or thick descriptions forms of the data collection and analysis process (Ellingston, 2009).

The investigation used multiple modes of data collection to investigate the effects of pedagogical content knowledge in learning Grade 12 Mathematics using PBL because I wanted to examine the phenomenon from more than one perspective rather than a single frame of mind (Creswell, 2016). The multi-methods included interviews with the Mathematics teachers and focus group interviews, together with classroom observations. These methods would lead to multiple forms of data collected on the same phenomenon under review. However, Richardson and St. Pierre (2018) shared a contrasting opinion that crystallisation is preferred in qualitative research because it

caters to an infinite interpretation of data rather than data triangulation (data is viewed as fixed and must be triangulated).

In qualitative research, reflexivity refers to the researcher's reflection on their role in the empirical review and their culture, background, and experiences that could influence the data interpretations and analysis (Creswell & Creswell, 2018). By exercising reflexivity in the data collection and interpretation process, the researcher could limit bias and his experiences having a disproportionate effect on the study's conclusions. Therefore, I engaged reflexivity by making short notes about participants' statements and my feelings during the interview processes, compiling memos immediately after interviews, and continuously creating and improving my subjectivity declarations. This allowed me to assess my judgements, belief systems and practices during every stage of the study and produce a fair report on the effects of pedagogical content knowledge in learning Grade 12 Mathematics using PBL.

4.2.4.2.1 Semi-structured interviews

Semi-structured interviews are a two-way dialogue between an interviewer, posing questions to participants to understand their views, beliefs, behaviours, and opinions of an occurrence of common interest (Cohen et al., 2019). Interviews are good for more appropriate and complex situations, as well as they are useful for collecting in-depth information (Mohajan, 2021) to understand the views (Creswell & Creswell, 2018) and have a wider application (Mohajan, 2021).

Semi-structured interviews aimed to collect rich data on the effects of pedagogical content knowledge in learning Grade 12 Mathematics PBL. As a prominent means of data retrieval in qualitative research, semi-structured individual interviews (Bougie & Sekaran, 2020; Nieuwenhuis, 2022; Creswell & Creswell, 2018) or partially structured interviews (Creswell, 2016) find themselves somewhere between structured and unstructured modes of interviews (Brinkmann & Kvale, 2019) were used in this study for crystallisation purposes (Johnson & Christensen, 2020).

In data collection using semi-structured interviews, I took cognisance of interview guiding tips from various authors (Bougie & Sekaran, 2020; Creswell & Creswell, 2018; Gaudet & Robert, 2018; Salkind, 2019) while conducting interviews through a 16-step model of interviewing steps formulated from various authors (Brinkmann & Kvale, 2019; Cohen et al., 2019; Creswell & Poth, 2018) as seen below.

Table 4.2: A 16-step model for conducting interviews

STEP	AUTHOR
Step 1: Determining the research questions that will be answered by interviews	From step 1 of Creswell and Poth (2018)
Step 2: Identifying interviewees who can best answer these questions based on one of the purposeful sampling procedures mentioned in the preceding discussion	From step 2 of Creswell and Poth (2018)
Step 3: Distinguishing the type of interview by determining what mode is practical and what interactions will net the most useful information to answer research questions	From step 3 of Creswell and Poth (2018)
Step 4: Thematising	From step 1 of Brinkmann and Kvale (2019), step 1 of Cohen et al. (2019)
Step 5: Designing	From step 2 of Brinkmann and Kvale (2019), step 2 of Cohen et al. (2019) and step 5 of Creswell and Poth (2018)
Step 6: Construction of schedules	Step 3 Cohen et al. (2019)
Step 7: Question formats	Step 4 Cohen et al. (2019)
Step 8: Response modes and pilot testing	From step 5 of Cohen et al. (2019) and step 6 of Creswell and Poth (2018)
Step 9: Locating a distraction-free place for conducting the interview	From step 7 of Creswell and Poth (2018)
Step 10: Obtaining consent from the interviewee to participate in the study by completing a consent form approved by the human relations review board.	From step 8 of Creswell and Poth (2018)
Step 11: Following good interview procedures	From step 9 of Creswell and Poth (2018)
Step 12: Interviewing	From step 3 of Brinkmann and Kvale (2019), step 6 of Cohen et al. (2019) and step 4 of Creswell and Poth (2018)
Step 13: Transcribing	From step 4 of Brinkmann and Kvale (2019), step 7 of Cohen et al. (2019) and step 9 of Creswell and Poth (2018)
Step 14: Analysing	From step 5 of Brinkmann and Kvale (2019), step 8 of Cohen et al. (2019)
Step 15: Verifying	From step 6 of Brinkmann and Kvale (2019) and step 9 of Cohen et al. (2019)
Step 16: Reporting	From step 7 of Brinkmann and Kvale (2019) and step 10 of Cohen et al. (2019)

Source: Prepared by the researcher

In this research project, interviews not only offered the benefit of being able to ask questions and listen (Creswell & Creswell, 2018), but they also made it possible to conduct probes (Bougie & Sekaran, 2020; Mohajan, 2021) and questions to be clarified (Mohajan, 2021), "detail-oriented probes, elaboration probes, and clarification probes" were the several types of probes that were used (Nieuwenhuis, 2022). In addition, supplemental questions were provided to me to explain various parts of the data that was obtained about the impacts of instructors' MPCK on the education of learners in Mathematics at the Grade 12 level.

The other advantages of using semi-structured interviews were that they were easy to use (Creswell & Creswell, 2018), were able to answer more specific concerns, the results were easily understood and had a costing advantage over an in-depth interview (Quinlan et al., 2019). They further allowed the participants some latitude and freedom when answering questions, making room for the conversation to go in unexpected directions (Brinkmann & Kvale, 2019), while still allowing considerable flexibility in scope and depth (Creswell, 2016).

I conducted the interviews in the participants' natural vernacular (Creswell & Creswell, 2018). The rich information gained from using qualitative interviews was without any doubt the main reason for using them (Gaudet & Robert, 2018). Beyond information earned (characteristics, routine), the interviews were utilised to gain individual as well as group practices, customs, group dynamics, thoughts, values, and representations. More importantly, interviews seemed interesting in their content and structure as well (Gaudet & Robert, 2018). According to Creswell (2016), interviews enable the investigators to earn good listening and observational abilities because they watch the actions and feelings of interviewees thus enabling them to comprehend what is happening and the reasons thereof. By doing the interviews, I was able to understand the body language of the participants. Interviews also lowered the number of 'I don't know' and 'I have no answers' questions as I was present to further explain any questions and push for responses (Naz et al., 2022).

According to Creswell and Creswell (2018), interviews consist of reasonings given by the respondents to the researcher about the elements with which they are challenged. Therefore, by employing interviews, I heard of the participants' biased encounters and

attitudes that would otherwise remain unheard of. The interviews proved to be a very nice means of overcoming distances both in space and in time (Creswell, 2016).

An Interview Protocol provided me with a set of preestablished questions to use with the respondents and designate the narrative terrain (Bougie & Sekaran, 2020). Since the Interview schedule was designed beforehand, it encouraged me to think about what I wanted to investigate during the interview process (Bougie & Sekaran, 2020). Eventually, rich descriptive data from the interviews helped me to understand the participants' construction of knowledge and reality (Nieuwenhuis, 2022).

Adhabi and Anozie (2017) define semi-structured interviews as a summary of topics and questions engineered by the researcher. Furthermore, these scholars declare that semi-structured interviews enable the investigator to pose more enriched questions than those initially drafted. In other words, semi-structured interviews gave me flexibility in the interview process which means I was able to discuss a problem in a specific manner once it arose, allowing the respondents to be more expressive (Fouché et al., 2022). I put forward open-ended questions based on the topic under examination to gain data from participants (Cohen et al., 2019). The objective of open-ended questions was to gain descriptive feedback and explanations and demonstrate knowledge on the effects of pedagogical content knowledge in learning Grade 12 Mathematics using PBL, from the Mathematics teachers.

I considered semi-structured interviews with individual Mathematics teachers the most appropriate means for the study as it would allow in-depth communication between me and the interviewee (Creswell & Creswell, 2018). More so, I sought to gain the individual experiences and perspectives of the Grade 12 teachers on the phenomenon under review.

Sometimes, the objective of qualitative semi-structured interviews (SSIs) is not achieved due to limitations. The shortcomings of SSIs are that they are not easy to employ in sensitive discussions, may become expensive (Quinlan et al., 2019), and as a qualitative research approach, they may be not necessarily considered to represent the population as they usually use a minor sample size (Wild & Diggins, 2015).

Naz et al., (2022) reassure us that semi-structured interviews are costly to conduct, prevent unauthenticated, and carry the possibility of interviewer prejudice. They are

time-consuming, and cumbersome and require the interviewer to be sophisticated. In semi-structured interviews, interviewers need to be intelligent, thoughtful, composed, and nimble-footed, as well as know relevant significant issues. The investigator must be able to build a rapport with the interview candidates and allow sufficient space to explore related thoughts while still in focus, the procedure of preparing for the interviews and setting them up, as well as carrying out the interviews and analysing the feedback is not easy or quick. The semi-structured interviews consist of the laborious task of critiquing a large volume of transcripts and information from semi-structured interviews is much more difficult to organise and analyse (Brinkmann & Kvale, 2019; Naz et al., 2022; Rule & John, 2022).

Brinkmann and Kvale (2019) offer the following shortcomings of interviews:

- The asymmetrical power relation of the interview is illusory to think of the research interview as a dominance-free dialogue between equal partners; the interviewer's research project and knowledge interest set the agenda and rule the conversation.
- The interview is a one-way dialogue. An interview is normally a one-directional questioning. The role of the interviewer is to ask, and the role of the interviewee is to answer. It is considered bad taste if interview subjects break with the ascribed role and start to question the interviewer.
- The interview is an instrumental dialogue. Unlike a good conversation, the research interview is no longer a goal in itself or a joint search for truth but a means serving the researcher's ends. The interview is an instrument in providing the researcher with descriptions, narratives, and texts, which the researcher then interprets and reports according to his or her research interests.
- The interview may be a manipulative dialogue. A research interview often follows a hidden agenda. The interviewer may want to obtain information without the interviewee knowing what the interviewer is after, attempting to.
- The interviewer's monopoly of interpretation. The interviewer generally upholds a monopoly of interpretation over the interviewee's statements. The research interviewer, as the 'big interpreter', maintains an exclusive privilege to interpret and report what the interviewee meant.

The employ of semi-structured questions for the interviews coincided with the scientific demands of my review, including the research paradigm (constructive interpretivism), research design (case study) and the research approach (qualitative research). I put forward open-ended questions based on the topic under examination to gain participant data (Cohen et al., 2019).

4.2.4.2.2 Focus group interviews

Focus groups are a type of group interview but they tend to concentrate in-depth on a particular theme or topic with an element of interaction (Walliman, 2018). Focus group discussion is frequently used as a qualitative approach to gain an in-depth understanding of social issues (Nieuwenhuis, 2022; Salkind, 2019; Walliman, 2018). The method aims to obtain data from a purposely selected group of individuals rather than from a statistically representative sample of a broader population (Salkind, 2019).

Nieuwenhuis's (2022, p. 111) revelation that "inexperienced researchers often confuse focus group interviews with a group interview" is true for most qualitative researchers and, as a result, leads to many methodological wrongs when used in academic research. Focus-group procedures have evolved in recent years and include a set of characteristics that distinguishes them from other group techniques (Creswell & Creswell, 2018).

Focus groups are semi-structured discussions by small groups of participants about a common topic or experience. They are useful in obtaining information that is difficult to obtain with other methods. Investigators use a focus-group interview to get in-depth information about and reactions to a relatively small number of topics or questions - typically fewer than 10 questions and often around five or six (Creswell & Creswell, 2018). Focus group interviews occur in an unbiased environment where the researcher moderates the interview process with a small group of 6 participants, seeking their viewpoints. (Fouché et al., 2022). By using focus groups, I managed to earn data on the effects of pedagogical content matter in learning Grade 12 Mathematics using PBL by interviewing learners from each secondary school.

The reason for using focus group interviews was to use the data to supplement the data already gained from the semi-structured interviews with the teachers, an approach known as crystallisation. Crystallisation is a way of achieving quality assurance in qualitative data (Borkan, 2022) and of validating results using many

different qualitative methods of collecting and analysing data (Creswell & Creswell, 2018).

In the focus group interviews, group dynamics were used to effectively collect data on the effects of pedagogical content knowledge in learning Grade 12 Mathematics using PBL because data that came out would not necessarily reveal itself in individual interviews. Also, group interaction on the topic allowed learners to expose their personal experiences and add their feelings to reach a greater understanding and further explain statistical data. The data collected from the focus group interviews were utilised to validate the information obtained in the qualitative approach.

The advantages of using the focus group method to elicit the data are as follows (Creswell & Creswell, 2018).

- Data is generated through group interaction in a social environment.
- It is useful when different views are required for a specific topic.
- It has high face validity.
- Data obtained is over a shorter period.
- Data generated can be complex at minimal costs.

Focus groups were also employed because they were in comparison easier to administer and at a minimal cost (Bougie & Sekaran, 2020; Creswell & Creswell, 2018; Mohajan, 2021) and within a short time (Creswell, 2016) with other methods. Also, focus groups showed a heterogeneous cross-section of interests (Walliman, 2018), opinions (Bougie & Sekaran, 2020; Nardi, 2018) and attitudes from participants (Nardi, 2018) and therefore gain that place that other data collection methods cannot achieve. This showed a dimension of knowing that often remains untouched by the more popular one-to-one interview or questionnaire. A clear benefit of using focus groups was that the method revealed what Creswell and Creswell (2018) refer to as the evolution of perceptions in a social context. They were ideal for insights into the complex issues of this study, they were suitable for studying opinions and attitudes the group interaction generated new ideas as respondents built on others' comments (Nardi, 2018). This way, the perceptions of individuals in isolation were different when they developed in a social context (Creswell & Creswell, 2018).

Some of the shortcomings of focus group interviews are that results cannot be generalisable to a bigger population, they are a challenge to use for emotional topics, they can be costly, the facilitator/moderator may have to work very hard at facilitating and maintaining interaction, participants may be easily distracted (Quinlan et al., 2019), it can be difficult to establish and/or maintain rapport with participants (Quinlan et al., 2019), there might be domination by some participants (Gumbo & Maphalala, 2015; Nieuwenhuis, 2022) as well as discomfort among some individuals when intimate details have to be revealed, and that focus groups are time-consuming as well as difficult to organise (Nardi, 2018). This is because participants must be able to congregate at the same place and at the same time, which can be difficult (Creswell, 2016). A key limitation of this technique is a small sample, which may not represent the population in general (Rule & John, 2022).

Each focus group member was kept at six because the suitable group composition would allow a free flow of discussions containing helpful information without intimidation (Fouché et al., 2022). Creswell (2016) avers that the composition of each focus group from the two chosen secondary schools should be homogeneous because members would spend more time discussing the topic under review rather than explaining themselves. In other words, members in a focus group with similar experiences and feelings would be able to engage on the topic whilst other group members openly will understand, complement, or reject the notions, thereby building new knowledge in the matter.

The duration of the focus group interviews was about an hour each, which depended on the total number of questions posed and the responses from the participants. Although the questions posed were deliberate and orchestrated before the day of the interviews, the atmosphere at the sitting was relaxed, while I posed questions from prior reactions where possible.

The Grade 12 Mathematics learners comprised one focus group from both schools and consisted of 6 learners in each. I used the year-end schedules and selected two learners based on their Mathematics performance in Grade 12. Therefore, two learners from the high, middle, and low performers were interviewed about their potential. I also selected an equal number of males to females for each focus group; one male and one female were selected for each high, middle, and low performer. This

was done to ensure the focus group learners were representative of the actual proportion of the population under review regarding their characteristics and experiences (Creswell & Creswell, 2018). In other words, the focus group learners selected were the most productive selection to provide information-rich feedback to the research questions adequately. The focus group interviews contained a variety of semi-structured questions, as well as group discussions.

The objective of using focus group interviews is based on the premise that group dynamics within focus group interviews would yield a wide range of responses reflective of their experiences, beliefs, and attitudes (Creswell, 2016). I built familiarity and rapport with the learners, by spending time with them before the commencement of the interviews so that the learners would be comfortable to answer the questions.

Observation is a qualitative data-gathering technique used, to enable me to gain a deeper understanding of the phenomenon being observed (Nieuwenhuis, 2022). According to Nieuwenhuis (2022), observation is the systematic process of recording the behavioural patterns of participants, objects and occurrences without necessarily questioning or communicating with them. Nieuwenhuis (2022) further mentions that observation is an everyday activity whereby we use our senses (seeing, hearing, smelling, tasting) and our intuition to gather bits of data.

4.2.4.2.3 Lessons observations

Observational studies can be of either the non-participant-observer or the participant-observer type. Both, again, can be either structured or unstructured (Bougie & Selkaran, 2020). For this reason, an observational study can be quantitative research in which a particular aspect of behaviour is observed systematically, with as much objectivity as possible (Cohen et al, 2019) and is more concerned with the frequency of actions (what, rather than why) or it can be a qualitative investigation in which a particular element of their behaviour is intentionally unstructured (Cohen et al, 2019). As a qualitative data-gathering method (Bloomberg & Volpe, 2022), observation is used, to enable me to gain a deeper insight into and understanding of the phenomenon being observed (Nieuwenhuis, 2022). In observations, categories of behaviour, usually pre-determined, are used to guide the recording of observed behaviour (Devlin, 2018). Devlin (2018) goes on to say that a behaviour-coding system is an

approach in observational research in which there is a checklist to code targeted behaviours, such as sitting or standing.

To help researchers decide on what to observe in the focused and selective observation stages, Creswell, and Creswell (2018) suggest that they:

- Observe events, actions and behaviour and look for a storyline;
- Sort out the regular from the irregular activities;
- Look for variation in the storyline;
- Look for negative cases or exceptions; and
- In case the observation is structured, develop a plan for systematic observation, including an estimate of how many observations will be enough.

Four types of participant observation are distinguished by two separate dimensions. These four types of observation lead to roles that are (Bloomberg & Volpe, 2022).

1. Complete participant
2. Observer-as-participant
3. Participant-as-observer
4. Complete observer

(1) Complete participant: Also known as participation in the normal setting (Johnson & Christensen, 2020), the researcher becomes completely immersed in the setting, hiding the real purpose of his presence to the extent that those being observed do not know that they are being observed (Johnson & Christensen, 2020; Nieuwenhuis, 2022).

(2) Observer as a participant: Participant observation is a special mode of observation in which the researcher is not merely a passive observer. Instead, the researcher may assume a variety of roles within a case study situation and may participate in the events being studied (Nieuwenhuis, 2022).

(3) Participant as an observer: Participant observation is an approach that has frequently been used in case studies, ethnographic studies, and grounded theory studies (Bougie & Selkaran, 2020).

In participant observation, the researcher gathers data by participating in the daily life of the group or organisation under study. It (participant observation) combines the

processes of participation and observation (Bougie & Selkaran, 2020). This type of observation is typically found in action research where the researcher becomes part of the research process and works with the participants in the situation to design and develop strategies (Bougie & Selkaran, 2020; Johnson & Christensen, 2020; Nieuwenhuis, 2022). According to Bougie and Selkaran (2020), this enables the researcher to learn about the activities of the group under study in a natural setting. Participant observation is qualitative, and its emphasis is on discovering the meaning that people attach to their actions (Bloomberg & Volpe, 2022).

(4) Complete observer: Also known as total observation, the researcher fully takes the role of an outside observer (Johnson & Christensen, 2020). The researcher's role is kept secret in the sense that he or she does not inform the people in the group being studied that they are being observed and they usually do not know that they are being observed (Nieuwenhuis, 2022).

Lesson observations entail the direct observation of a classroom lesson in practice in the classroom as it happens in real-time (Cohen et al., 2019). It also systematically observes the subject's behaviours and settings in their natural environments (Mathematics classroom) while recording data. Creswell and Creswell (2018) maintain that in lesson observations, the researcher is informed well in advance about what needs to be observed because a structured lesson observation tool is generated (refer to Appendix 9). An observation schedule was adapted from the ELRC, Collective Agreement 2 of 2020 and Stearn et al., (2012).

Using the observation schedule, I noticed both Mathematics teachers from each secondary school in their natural environment. My role in the lesson observations was that of a non-participant (complete) observer (Fouché et al., 2022). The participants saw me as an "outsider" as this research was overt (Creswell & Creswell, 2018). This assisted me in not having any undue influence on the lesson practices but could observe the Mathematics classroom activities and record the data accordingly.

I generated data on the effects of pedagogical content knowledge in learning Grade 12 Mathematics using PBL. Furthermore, I was able to identify peculiar information like the behaviour of learners and teachers in concert with each other, which is impossible to identify during interviews or focus groups (Creswell, 2016).

The selected secondary schools were observed based on the characteristics below:

- Classroom climate and practice;
- Teaching and learning practice in the classroom;
- Teachers' SMK and MPCK practices, pedagogical content knowledge;
- Learning Mathematics using PBL;
- Mathematics teachers as the facilitator; and
- Conduct of teachers and learners on problem-solving.

After completing the lesson observations of both secondary school teachers, the data was analysed. I was vigilant of my biases and suppositions and how they may affect what data was recorded next to the characteristics mentioned in the observation schedule.

4.2.4.3 Instruments trustworthiness

Trustworthiness refers to the extent to which qualitative data and analysis are credible, transferable, dependable, and confirmable (Creswell, 2016). It (trustworthiness) is an everyday term used, which is related to the 'truth value' of a research investigation (Struwig & Stead, 2017). Creswell and Creswell (2018) aver that trustworthiness focuses on an internal verification of data by participants (also known as credibility) and external verification of data by the reader (also known as dependability, confirmability, and transferability).

In an attempt to meet the standards, set by conventional dominating qualitative research, various scholars have attempted to develop criteria for trustworthiness after Creswell and Creswell (2018) who originally introduced four sub-categories for evaluating rigour and the standard of qualitative research, namely: (i) credibility; (ii) dependability; (iii) confirmability; and (iv) transferability.

These are elaborated hereunder.

4.2.4.3.1 Credibility

The term credibility is employed mainly in interpretivist and critical paradigms (Creswell, 2016), 'credibility' refers to belief in the truth of the data obtained and the clarification thereof (Brinkmann & Kvale, 2019). In other words, credibility describes how accurately the researcher defines the data that is obtained from the respondents (Stahl & King, 2020). The term credibility, or the true value of data and data analysis (Stahl & King, 2020), poses the question: How do you know if the findings are

truthful? (Struwig & Stead, 2017). In this study, the investigation was done in such a way that the findings demonstrate credibility; in other words, the reader will believe them. Confidence in the truth in this study was established through member checks and prolonged engagement (Creswell, 2016; Stahl & King, 2020).

Member checking refers to returning transcripts to participants, providing them with the opportunity to verify the meaning you have given to their words (Creswell & Creswell, 2018; Stahl & King, 2020).

Member checks are performed by testing ideas from the data, analytic categories, interpretations, and conclusions (Creswell, 2016). In this study, I did this throughout the research process formally and informally during the course of observation and conversation (Creswell, 2016). In collaboration with the participants, I also asked other individuals who are close to the participants and others who have had the same experiences, to read and comment on the final script (Creswell, 2016). I went back to the participants and did member reflections to provide additional insight and credibility to the research (Stahl & King, 2020).

Prolonged engagement refers to immersion in the lifeworld and culture of research participants over a long time. The objective of my study was to gain an understanding of the behaviour, values, and social relationships of participants in their social contexts (Stahl & King, 2020). The persistent engagement brought rigour to the qualitative study. This included keeping comprehensive field notes and a journal systematically and rigorously that enabled an audit trail of all data collected and analysed (Creswell & Creswell, 2018).

4.2.4.3.2 Dependability

Dependability in qualitative research can also be described as audibility (Stahl & King, 2020). The term dependability relates to the availability of evidence such that if the investigation were to be repeated with the same or similar respondents in the same or similar context, its findings would be similar (Brinkmann & Kvale, 2019). The term dependability raises the question: Are the findings of the investigation consistent? (Struwig & Stead, 2017).

I consulted with my promoter who was familiar with qualitative research, just to confirm whether I was keeping to the correct process in conducting semi-structured interviews, focus-group interviews, and observations (Bougie, & Sekaran, 2020). Furthermore, I

thoroughly explained and precisely followed a well-thought-out research plan and described each step was completed carefully. Last, I described exactly how data was collected, recorded, coded, and analysed and presented good examples to illustrate this process, paving the way for trusting that the results are dependable (Braun et al., 2016).

4.2.4.3.3 Confirmability

The term confirmability refers to the degree to which the analysis of the researcher can be confirmed by someone else, either a different researcher or the reader (Creswell & Creswell, 2018). It refers to the potential for congruency of data in terms of accuracy, relevance or meaning (Riazi et al., 2023). Confirmability puts forward the question: Are the findings of my research study confirmed by other sources of information? (Struwig & Stead, 2017). Therefore, ensuring that data analyses are transparent is one means of heightening the confirmability of a research project (Creswell, 2016).

Confirmability was seen in this study by establishing that the data represents the information provided by the participants and that the interpretations of the data were not fuelled by the researcher's imagination (Brinkmann & Kvale, 2019). The data, therefore, reflected the voice of the participants through quotations. Creswell & Creswell (2018) add that by including many direct quotations from the original data in research reports, the researcher allows the reader to hear exactly what participants said and how the researcher interpreted that information. In so doing, the researcher amplifies the voices of participants and illustrates some details of the process of data analysis making the use of quotations an important characteristic of rigorous qualitative research (Rule & John, 2022). Thus, where necessary, direct quotations were used (Creswell & Creswell, 2018). My study used short, eye-catching quotes. These quotations "are usually easy to read and take up little space but stand out from the text" (Creswell & Creswell, 2018).

4.2.4.3.4 Transferability

The term transferability asks the question: Could the findings be useful in similar environments to that of the study? (Struwig & Stead, 2017). Transferability of the research findings relates to the extent to which an empirical study's findings can be used in other situations, environments, and studies (Nieuwenhuis, 2022).

In this study, the qualitative section aimed at enhancing a study's generalisability through multiple informants (Ellingson, 2009) and by ensuring that the qualitative study's findings or output be recognisable not only by other scholars or readers in the field but by the context within which the study has been conducted. This was done by using quotations and contextual descriptions in the analysis that link the findings to the context. According to the reader-response theory, the readers' empathy stems from their identification with the story and reliving their emotions relating to similar events in their own lives (Creswell, 2016).

I followed Creswell and Creswell's (2018) suggestion of making a case for transferability to be the construct used in qualitative research. To do this, I focused on (a) how typical the participants were to the context being studied and (b) the context to which the findings apply. In the first consideration, the participants were typical of the phenomenon being studied while the other consideration was concerned with providing a complete understanding of the context being studied. It is from here that future readers can explore the research document and determine if the findings can be transferred to their setting or environment. It was my responsibility to paint a full picture of the context and then allow the reader to determine if the research is transferable to their context (Creswell & Creswell, 2018).

4.2.4.4 Ethical considerations

4.2.4.4.1 The participant

The research participants were my first concern. All measures towards practising sound ethical science and research were aimed at maintaining the self-respect and dignity of respondents. The relevant ethical principles used for respondents were the principles of non-maleficence, confidentiality and anonymity, informed consent, right to withdraw, beneficence and scientific integrity. I adhered to ethical issues, which are under international protocols such as the 1947 Nuremberg Code, the 1964 Declaration of Helsinki, the Belmont Report's Ethical Principles, and the Helsinki Declaration of 1972.

4.2.4.4.2 Principle of non-maleficence (Potential harm)

Studies conducted by educational researchers seldom if ever run the risk of inflicting such severe mental and physical harm on participants (Fouché et al., 2022). Non-maleficence involves subtle considerations rather than physically and psychologically

harming participants (Creswell & Creswell, 2018). I identified all risks and, through the principle of non-maleficence, did not harm the respondents physically or otherwise (Allen et al., 2023; Creswell & Creswell, 2018; Struwig & Stead, 2017).

4.2.4.4.3 Respondents' right to anonymity and confidentiality

While preparing candidates to give informed consent, I ensured the confidentiality of any individual participant (Creswell & Creswell, 2018). While confidentiality and privacy are closely related to veracity and truth-telling (Fouché et al, 2022), anonymity and confidentiality are related but differ in some important respects (Bos, 2020). Anonymity can be defined as the degree to which the source of a message can be identified (Creswell, 2016). It ranges from very high (source is nearly impossible to identify) to none (source is easily identifiable or already identified). Confidentiality, on the other hand, relates to an agreement between the researcher and the participant (Bos, 2020). It (confidentiality) means that no information provided by a candidate should be divulged in any way except for research purposes (Creswell & Creswell, 2018). The former concerns the initial collection of data, and the latter makes promises to not disclose specific personal information (Bos, 2020).

I reassured all participants of their prudence and anonymity in my investigative study by not providing their contact details to anyone (Creswell & Creswell, 2018; Fouché et al., 2022; Struwig & Stead, 2017). Since I dealt with human beings' personal information, the Protection of Personal Information (POPI) (Act no 4 of 2013) requires that personal information should be treated ethically (Walliman, 2018). Such information, as described by the POPI Act (Act no 4 of 2013) includes contact details: email, telephone, address, etc; demographic information: age, sex, race, birth date, ethnicity, etc. I noted some of the obligations of responsible parties under the POPI Act (2013), including applying reasonable security measures to protect it, only collecting information that is needed for a specific purpose and only holding as much as is needed, and only for as long as the information is needed.

4.2.4.4.4 Informed consent

The most important tool for gaining confidentiality for my study was the informed consent process (Bos, 2020). Here, I gained informed consent from all the participants by provoking them to sign an informed consent form (see Appendices 4 and 5) before the collection of data commenced (Allen et al., 2023; Struwig & Stead, 2017). Informed

consent is rooted in the idea that involvement in research should have no detrimental effects on the respondents, honour the individual's fundamental rights, and respectful relationships, bonds and promises. Accordingly, certain conditions and arrangements were designed to guarantee safe participation in research. These procedures assumed the shape of a contract with respondents who actively and knowingly agreed with the conditions (Bos, 2020). Informed consent notified this study's respondents of the following items (Bos, 2020).

- Name(s) and affiliation of researcher(s)
- Goal or aim of the research (in comprehensible language)
- Research techniques or procedures to which the participant is subjected
- Risks involved (if any)
- Estimate of time investment
- Agreement on compensation (if any)
- Conditions of confidentiality (anonymisation or pseudonymisation)
- Storage, usage and access to data
- Rights of the participant
 - ✓ to withdraw at any moment
 - ✓ to review/correct erroneous data (if possible)
 - ✓ to receive/be informed about the results (if interested)

4.2.4.4.5 Participants' right to withdraw.

I informed participants of their right to make an informed decision about whether or not they wanted to participate in the study (Creswell & Creswell, 2018) of the right to terminate their participation in the study at any time without a penalty (Bos, 2020; Creswell & Creswell, 2018; Fouché et al., 2022).

4.2.5 Step 5: Analyse data

4.2.5.1 Data Procedure Plan

The data procedure and plan outline the events I followed to collect participants' data. First, I gained authorisation from the provincial education department, secondly from the district education department, and lastly from the school principals before commencing the data collection from the two secondary schools.

I had to consider the closing date for Term 1, 24 March 2023. The first term-controlled tests will be conducted from 13 March 2023 to 17 March 2023. Therefore, I chose to conduct the research from 1 February 2023 to 14 February 2023. I developed a management plan in consultation with both schools' principals. Tables 4.3 and 4.4 represent my management plan to investigate each school secondary school, respectively.

Table 4.3: The secondary school performing above 80% in Mathematics

DATE	TIME	ACTIVITY	AGENDA
2023 02 13	14:00-15:00	Meet the focus group learners	<ul style="list-style-type: none"> • Explain the reason for the study and the criteria for their selection • Hand out and discuss consent and indemnity forms to learners for parents • Hand out and discuss assent forms to learners
2023 02 13	15:30-16:00	Meet the Grade 12 Mathematics teachers	<ul style="list-style-type: none"> • Explain the reason for the study and the criteria for their selection • Hand out and discuss consent and indemnity forms to teachers.
2023 02 20	14:00-15:00	Meet the focus group learners	<ul style="list-style-type: none"> • Conduct focus group interviews
2023 02 20	15:30-16:00	Meet the Grade 12 Mathematics teachers	<ul style="list-style-type: none"> • Conduct semi-structured individual interviews
2023 02 27	14:00-15:00	Meet the focus group learners and the Grade 12 Mathematics teachers.	<ul style="list-style-type: none"> • Conduct classroom observation

Source: Researcher's work

Table 4.4: The secondary schools performing below 65% in Mathematics

DATE	TIME	ACTIVITY	AGENDA
2023 02 14	14:00-15:00	Meet the Focus group learners	<ul style="list-style-type: none"> • Explain the reason for the study and the criteria for their selection • Hand out and discuss consent and indemnity forms to learners for parents • Hand out and discuss assent forms to learners
2023 02 14	15:30-16:00	Meet the Grade 12 Mathematics teachers	<ul style="list-style-type: none"> • Explain the reason for the study and the criteria for their selection • Hand out and discuss consent and indemnity forms to teachers.
2023 02 21	14:00-15:00	Meet the focus group learners	<ul style="list-style-type: none"> • Conduct focus group interviews
2023 02 21	15:30-16:00	Meet the Grade 12 Mathematics teachers	<ul style="list-style-type: none"> • Conduct semi-structured individual interviews
2023 02 28	14:00-15:00	Meet the focus group learners and the Grade 12 Mathematics teachers.	<ul style="list-style-type: none"> • Conduct the classroom observation

Source: Researcher's work

4.2.5.2 Data analysis

Restricted scholarship has been dedicated to unpacking the presentations of qualitative research results (Adhabi & Anozie 2017), resulting in a contestation by various scholars on the consistency of presentations of qualitative research results (Johnson & Christensen, 2020) or how these can be measured (Stahl & King, 2022). As informed by Bougue and Sekaran (2020), the analysis of qualitative data is not easy in that in comparison with quantitative data analysis, there are relatively few well-established and commonly accepted rules and guidelines for analysing qualitative data. This point is taken further by Cohen et al. (2019) who state that there is no single way of doing qualitative data analysis. It is for this reason that the arrival of the findings in a qualitative analysis depends on the inquirer (Creswell & Creswell, 2018).

The analysis of qualitative data is primarily an inductive process that was used to guide the data analysis in this study (Riazi, et al., 2023). Data analysis is a process of bringing order to the data by organising it into categories, patterns, and trends. It takes time and requires good data management techniques creativity, intellect, rigour/vigour, and hard and thoughtful work (Kiger & Varpio, 2020). Data analysis in my empirical study had me organising, describing, and explicating the data; in other words, I was making sense of the data in terms of respondents' understanding of their current situation, noting categories, themes and regularities (Cohen et al., 2019).

Once I completed the data collection process, it was time for me to compile and interpret the data to explain them in a meaningful context. This phase was completed using a qualitative data analysis technique (Stahl & King, 2020), as explained hereunder.

In this study, data analysis entailed the familiarisation and coding of data, followed by developing themes, refining, and then defining the themes and, after that linking them with the existing body of knowledge (Braun et al., 2016). I analysed data using a thematic analysis procedure. I used this procedure to recognise, analyse and identify patterns (themes) within the data sets gained (Kiger & Varpio, 2020).

I used Braun et al's. (2016) thematic data analysis process, a process used to recognise, analyse, and report categories (themes) within the data collected by me. I used thematic data analysis, scrutinised the data obtained from the respondents, and then completed the data analysis meaningfully.

Thematic data analysis requires identifying familiar themes from the different data sets. Kiger and Varpio (2020) have designed six phases in analysing data: becoming familiar with the data, creating initial codes, looking for themes, analysing themes, interpreting, and writing the themes, and generating a final report. The thematic analysis allows me to be immersed in the evidence gained by studying the transcripts of the interviews and observation schedules used for the lesson observation. The six stages of thematic data analysis were followed as set out below:

1. Familiarising yourself with the data: I wrote out the data and then actively read the transcripts repeatedly, noting down initial ideas.
2. Creating initial codes: This entailed systematically coding data items of interest, connections, and questions relevant to each code.
3. Searching for emergent themes: Examining the codes and collated into possible themes, appropriately associating codes to each potential theme.
4. Evaluating the themes: This process verifies whether the themes match the coded extracts (Level 1) and the data sets (Level 2), thereby generating a thematic table of the analysis and interpretation of the data sets.
5. Define and name the themes: Themes were further scrutinised to develop and name each theme.
6. Writing the final report: The final analysis required the selection of actual extract examples, the final analysis of the selected extracts and then associating them to the research questions and literature, producing an academic report.

I asked all the interviewees to elicit data for every code that was generated. In the study, the feedback from the respondents to the research sub-questions was utilised to create codes. These codes gave rise to the themes recognised throughout the investigation processes to reach a recurring pattern of knowledge. I arranged the themes with the raw data earned from the participants' answers and had a prevalence of meaning from all the information provided by the respondents. This is factual from the respondents who shared a similarity in the same categories (see Braun & Clarke, 2006). The categories were obtained from questions coded as follows:

- The effects of teachers' MPCK on Grade 12 learners learning Mathematics;

- The impact of the PBL approach in teaching and learning Mathematics on Grade 12 Mathematics Learners' performance; and
- The teachers' role in teaching Grade 12 Mathematics using PBL.

I used codes to arrange all the collected information to generate themes. When I categorised all the data collected, I generated themes. I used an individualistic coder to guide me in the data analysis process. This guaranteed my accuracy in the transcription of the voice recordings to texts, the interrelationship between the research questions and the data sets and gathering the raw data into a meaningful and final report (Kiger & Varpio, 2020).

4.2.6 Step 6: Write a research report

The key purpose of any research report is to offer a clear description of what has been done in the various stages of the research process (Bougie & Selkaran, 2020). Once the data were analysed and the findings assessed, I was ready to write the final report for the study. Please refer to Chapter 5 for this study's research report.

4.3 Summary of Chapter 4

In this chapter, I explained the research design and methodology of the study. I used a constructive interpretivism paradigm which paved the way for me to explore the effects of pedagogical content knowledge in learning Grade 12 Mathematics using PBL. It further argued the methodology and theoretical framework used for data analysis and its meaning. I also described the ethical considerations, delimitations and limitations and their relevance to the study.

The study was conducted at two secondary schools, and the participants were purposively selected. Therefore, the study was based on a case study research design, and the interpretive paradigm was chosen. I collected data from Grade 12 Mathematics teachers by exposing them to semi-structured individual interviews; the learners were placed in focus and interviewed and then observed in the Mathematics classroom with an observation schedule. Multi modes of data collection enabled me to triangulate the data, which added to the trustworthiness of the research findings. Furthermore, I ensured that the study met the standards of credibility, transferability, dependability, and confirmability. I validated the information contained in the study by piloting the interview questions and observation schedule on a pilot school with similar

characteristics to another secondary school. Ethical clearance was earned from the host institutions. Chapter five will critically discuss the presentation, discussion, and data analysis.

CHAPTER 5: EMPIRICAL RESULTS

5.1 Introduction

The preceding chapter discussed the research methodology of this study. This chapter offers the study's results, which include the data presentation and analysis. I collected data through semi-structured interviews with Mathematics teachers, focus group interviews with learners who were doing Grade 12 Mathematics in 2023, and observing the participants part-taking in two Mathematics lessons. The data was presented and analysed to answer the research questions of this study in Chapter 1.

I used thematic data analysis to critique and synthesise the reporting of data. Thematic data analysis generates codes and themes from the research questions in a study (Braun et al., 2016). I developed themes initially from the raw data collected. After that, I divided each theme into sub-categories. The analysis starts with semi-structured interviews, followed by focus-group interviews, and ends with classroom observations.

5.2 Results of thematic analysis

The thematic analysis results are embedded in the philosophical principle that views the participants as individuals in their world and are not devoid of each other. Creswell and Creswell (2018) postulate that thematic analysis, from the perspective of teaching and learning, is a method rather than a methodology, which means it is not associated with any theoretical perspective or epistemological belief. The diversity of work in learning and teaching Mathematics enabled the researcher to be very flexible in the data analysis and a considerable advantage. Therefore, I did not exclude the possibility of overstressing or minimising some viewpoints. Subsequently, I employed all reasonable efforts to collect and document data effectively.

Data collection from the respondents gave insight into their experiences with the phenomenon under review in their natural world. This allowed for the development of new theoretical knowledge on the effects of pedagogical content knowledge in learning Grade 12 Mathematics using PBL.

In this investigation, I used a social constructivist approach; therefore, new theoretical knowledge was derived from thematic data analysis, emphasising the cultural, social, and surroundings influenced by the respondents' individual experiences (Kiger & Varpio, 2020). I believe that the phenomenon under review pertained exclusively to

the Grade 12 Mathematics teachers' and learners' experiences in learning Mathematics and that they were in the best position to make meaning of it. Therefore, I considered their involvement and feelings and recognised the *bona fide* information they produced during the data collection process. I captured to understand the social complexities experienced by the participants. I was not inclined to know how the events would naturally develop during the data collection processes or if any other significant variables may affect the data collection process. I sought the participants' experiences on the effects of pedagogical content knowledge in learning Grade 12 Mathematics using PBL.

I arrived at the point of data saturation when sufficient information was attained during the data collection process to duplicate the study, and no additional information could be gained; therefore, no further coding was feasible (Daher, 2023). Guest and Flemming (2015) noted that a study might achieve data saturation with at least six interviews, depending on its population and sample size. I interviewed four Mathematics educators and two focus groups of six Grade 12 learners each and conducted two classroom observations. Bowen (2008) and Kerr (2010) believe that if data saturation were not accomplished, this would harm the study's validity, which was not the case in the study undertaken.

A set of pre-planned questions was used to gain a comprehension of the attitudes towards, viewpoints and thoughts of the effects of pedagogical content knowledge in learning Grade 12 Mathematics using PBL. I fast-tracked the procedure by asking a question at a specific point interval. The data was collected under the sub-categories that have come up during the data analysis and comprehension of the results. The responses of the respondents were presented rigorously, as their spoken words.

5.2.1 Semi-structured interviews

From the first research question, "*What are the effects of teachers' MPCK on Grade 12 learners learning Mathematics?*" Theme 1 emerged, "*Teachers' MPCK affects Grade 12 learners' Mathematics learning.*"

In response to the question, "*Explain what you understand about PBL within the Grade 12 Mathematics curriculum?*" The participants commented as follows:

Teacher 1: *Okay, so I understand that they find their work very abstract... So, for them, problem-solving is where you try to break it down and make it more simple. For measurements, we actually take them to the school grounds with measuring tape and get them to measure... Sometimes we talk about a cylinder. They don't know what it is. You have to physically show them around, like the bins, or ask them to take out things like the Pritt. So, it helps them visualise what it is.*

Teacher 2: *The Grade 12 curriculum is more abstract... So, whenever you teach them, you have to give them a lot of manipulatives, that is, worksheets, especially right now; that is when you are going to involve them a lot, where they will now be guided to discover.*

Both Teacher 1 and Teacher 2 asserted that Grade 12 Mathematics is abstract to the extent that learners are challenged in learning the new content. Therefore, both teachers suggest that creating real-life situations in the Mathematics classroom could assist learners in learning Mathematics. Their responses suggested that teachers must be able to relate authentic life scenarios to Mathematics concepts so learners can grasp the content nicely. This aligns with Anthony and Walshaw's (2009) sentiments that effective learning of Grade 12 Mathematics begins in the classroom when the teachers duplicate real-life scenarios with group work collaboration and learners actively understand the content together. For more on this topic, refer to Chapter 3 (cf. 3.2.1).

Teacher 3 gave the following response:

Problem-based learning involves a group of learners coming together, looking at different problems, and trying to figure them out themselves. If there's someone from there that understands, they're able to explain it to those that don't understand.

Collectively, they will come up with a solution and present it to the class to be criticised constructively. I move from group to group, but I'm not the sole speaker anymore. I now listen to how they come up with their ideas in them. I move from group to group to control and also to prevent chaos. That is the role that I play. (Teacher 4)

Both teachers asserted that group work produced positive learning achievements in Grade 12. Their responses suggested that learners collaboratively learn Mathematics while the teacher facilitates learning by supporting their learning needs. This finding corroborates Klang et al.'s (2021) ideas that Mathematics learners involved in group work actively learn new concepts and thinking skills that enable knowledge transfer to their peers. Therefore, Mathematics teachers must possess MPCK or knowledge and skills to facilitate Mathematics learning amongst groups of learners to make sense of it (Shulman, 1986). For more on this topic, refer to Chapter 3 (cf. 3.2.2).

In response to the question, "*What skills and knowledge do you think a Grade 12 Mathematics teacher must have to implement PBL in the Mathematics classroom successfully?*" The participants commented as follows:

I think content knowledge is very important for us because if we don't understand the content, it's very difficult to explain it to the learners. So, I think content knowledge is one of the skills that we must have. And also another skill I feel is very important is communication because everyone understands the content, but not everyone can teach the subject. Okay, so I feel like communication is a considerable skill, the way that you, you relate it to the learners, that's really important. I think the biggest problem is communication.
(Teacher 1)

I would say the teacher must be well endowed with subject matter content so that whatever worksheets or problems you give to the learners, you must have test-proven it. And that it is achievable for them... (Teacher 2)

Both teachers believed that Mathematics teachers must be masters of the content to comfortably guide learners on Mathematics questions of any level. The interviewees contended that mastery of the content allows teachers to implement PBL successfully in the classroom. Teacher 1 also believed that apart from content mastery, Mathematics teachers should be able to use different teaching strategies to transfer the content to learners successfully. This finding supports previous research by Kar (2017) that MPCK addresses how-to-teach Mathematics; the teacher's level of MPCK determines how successfully the teacher can present particular topics and problems

appropriately to learners to cater to their needs varying learning abilities, and backgrounds. For more on this topic, refer to Chapter 3 (cf. 3.2.1).

However, Teacher 3 suggested the following:

This is personal from my side. It doesn't matter that you have a Ph.D. Or you have just a degree or a college education to teach Grade 12... the way he expresses himself, and the way he teaches learners is more important than mastery of the content, and that is my personal opinion... Then you know, you get level one, level two, level three, level four questions. We don't go to level four questions with learners who failed (less than 30%).

They (Mathematics teachers) have to be masters in the content... if the teacher doesn't understand a specific topic... they're not going to explain or even teach the class. And then not even considering problem-based learning... the teacher doesn't know the content and cannot even use different strategies to help them (learners)... depending on how the learner can process the information or understand information, that's where you look at the different types of strategies. (Teacher 4)

Teacher 3 postulated that Mathematics teachers must know how to apply various teaching strategies is more important than mastering the content so that learners in a heterogeneous class can successfully learn the content. Teacher 4 believed that mastery of the Mathematics content could assist teachers in implementing PBL and using differentiated learning strategies to match the learning needs of learners. Both interviewees contended that varying learning support strategies utilised by the Mathematics teacher are essential for successfully learning the content in a mixed-ability Mathematics environment. Similarly, scholars (Saad et al., 2015) put forward a suggestion that Mathematics teachers should plan and produce flexible lessons using differentiated instructional strategies to indulge learners with diverse learning needs. For more on this topic, refer to Chapter 3 (cf. 3.2.3).

In response to the question, "*How do teachers' Mathematics Pedagogical Content Knowledge (MPCK) affect the learning of Grade 12 Mathematics using PBL?*" the participants commented as follows:

I think if you don't know the content properly, the learners are not going to understand... If you don't know the answer, it becomes a problem. (Teacher 1)

... Sometimes you get teachers who don't know the content, and it will negatively affect their learning of the content... (Teacher 3)

Both Mathematics teachers contended that a lack of content knowledge would translate into learners unable to comprehend the content disseminated and would result in no learning of the new content. Similarly, Ball et al. (2008) opine that teachers lacking content knowledge cannot grow the learners' capacity to assimilate the new knowledge, which becomes a learning challenge. For more on this topic, refer to Chapter 3 (cf. 3.2.1).

However, Teacher 2 suggested the following:

Without very good pedagogical strategies, the lesson will not work... A teacher must have varied methodologies for heterogeneous classes with different performance levels. So, once you (the teacher) know how you will be more persuasive or going to guide, be assertive, or be more practical? I must know how I am going to integrate all these strategies. In a nutshell, the pedagogical strategies of the teacher must be varied to accommodate every learner and every possibility because not every learner is good at learning Mathematics. (Teacher 2)

Teacher 2 further elaborated that Mathematics teachers should be knowledgeable about different learning support strategies and be able to implement and integrate them in their lesson presentations so that Grade 12 learners can learn Mathematics successfully. The teacher's response suggested that Mathematics teachers must know the content and successfully use various communication strategies in the classroom. This aligns with the sentiments shared by Stols et al. (2015) and Ubah (2021) that Mathematics teachers should use PBL in the classroom because it emphasises learner-centeredness with learning activities that complement the various learning demands and their diverse home backgrounds.

The categories that emerged for Theme 1:

- PBL is learner-focused, learner-driven learning of Mathematics;
- Grade 12 Mathematics content is abstract;

- Teachers' mastery of content enables teachers to emulate using real-life situations in teaching; and
- Teacher proficiency in content enables teachers to use different learning academic strategies to assist learners with blended learning abilities.

Research Question 2 was "How does the PBL instructional approach in teaching and learning Mathematics impact the Grade 12 Mathematics learners' performance? Theme 2 emerged: "The PBL approach in the teaching and learning Grade 12 Mathematics positively impacts learners' performance.

In response to the question, "Which PBL examples did you use to teach Grade 12 math?

For Grade 12, we don't do a lot of practical things for problem-based learning... Okay, I'll be very honest, the syllabus is so long, you are just there to finish the syllabus. So, you don't get much time to really do anything extra. There's a lot of work to cover in a short period ...I will stand, and I will explain to the learners a concept in detail.... The time the lesson is almost over... and the learners are given very little time in the class to do classwork. Most of it ends up being homework. So, the next day they come to class, you take like maybe 15 minutes to mark in detail, and then you have to move on to the next aspect. (Teacher 1)

There is this chapter... Euclidean Geometry... How do I do it via problem-solving? I want to give them a lot of different triangles, for instance, when they have to measure the angles themselves and arrive at the answer. My responsibility is to control the class and give them guidance in the beginning, and I don't give answers to learners... they know. So, they must figure it out themselves. (Teacher 2)

Teacher 1 responded that she did not implement PBL in the classroom because the syllabus is too long and has limited time; therefore, she used teacher-centred learning. As a result, there was very little time for learners to complete in-class activities. Teacher 2 reported that he used PBL in teaching Mathematics. This enabled learners to be actively engaged in the lesson by investigating and communicating the results while the teacher guided them to find out. This finding is consistent with that of Klang et al., (2021). For more on this topic, refer to Chapter 3 (cf. 3.2.2).

From the interviews with the teachers, the data revealed that PBL is not practised extensively because of limited teaching time. In contrast, PBL is an effective learning tool for Grade 12 Mathematics learners. The data further disclosed that PBL-Mathematics classroom activities are child-centred learning while teachers monitor and guide learners to learn the content rather than lecturing to their audience. This seems to align with Koh and Chapman's (2019) findings that Mathematics lessons should be centred around learners actively engaged in the content.

In response to the question, "Explain the PBL strategies you have used in teaching Grade 12 Mathematics and how these strategies have impacted learning.", the participants commented as follows:

For Grade 12, you are always just pushed for time. Maybe we have that, that odd lesson or two. When you're doing the angle of inclination, you would take them (learners) outside and show the goalpost and show them the angle outside. But it's very limited, maybe two lessons for the year because of the time you... We only have groups after the third term; we separate the highflyers from the weaker learners... we had extra classes for weaker learners. We found it's much better because you just have them amongst learners (weaker learners), with the same ability as they... they will answer and ask you questions in the class. They are not afraid to ask questions.... (Teacher 1)

I am a learner-centred teacher... almost all the chapters learners will discover. In groups, learners must also have the dialogue; they must now come up and present a chapter (which is very manageable for them.) I have to now polish; whatever the learners have discovered. If a learner is good at presenting something, I think they (peers) grasp it very well. At the end of every chapter, I give homework. I am not going to give them the solution. They know nobody will call you; you come and present the answers... I don't write corrections, and you don't tell anyone you are wrong; nobody shouts because nobody is perfect. So, the error method is that they write their solutions. They find their solutions. Some solutions are not even part of the three answers to the question; we have to analyse if it is correct and then adopt them. (Teacher 2)

Teacher 1 stated that group learning occurred only towards the end of the third term due to the need to complete the syllabus and the limited time to learn Mathematics.

The participant further suggested that learners were grouped according to similar learning challenges and were given extra classes. The lower-performing learners felt comfortable posing questions to the Mathematics teacher and communicating with each other without being teased by the higher-performing learners. Teacher 2 had a contrasting view; the teacher used PBL in his lesson presentation, where learners became self-directed presenters of their work after collaborative dialogue with their peers, often presenting new ways of answering the questions. Both teachers conceded that learners felt safe and comfortable, positively impacting Mathematics learning. This is in tandem with Kyriacou (2009), who believes that effective Mathematics teaching and learning can only occur in a disciplined and safe environment. For more on this topic, refer to Chapter 3 (cf. 3.2.1).

Their responses suggested that learners must be grouped according to their homogeneous learning abilities, where learners feel safe to pose questions and communicate with each other for effective teaching and learning of the content. This contrasts with scholars Gardner et al., (2019) findings that Mathematic teachers using PBL can create learning opportunities for learners to transfer knowledge to their peers in a heterogeneous group of Mathematics learners with various learning abilities. The interviewees contended that poor-performing Mathematics learners required additional classes to their regular timetable to assist in their Mathematics learning.

Teacher 3 gave the following response:

I haven't used PBL because if you put learners together in a group of five, two learners will sit down and try and work. The other three (weaker learners) will just sit there and wait to get the answer. They are not putting in that extra effort. So when it comes to a group, not everyone will be participating. As a weak learner, I prefer individual attention compared to the group.

Teacher 3 shared similar sentiments with Teacher 1, that lower-performing learners tend not to actively participate in the Mathematics lesson and become passive receptors of the content, and therefore no learning can take place. Teacher 3 further revealed that individual teaching and learning with underperforming learners is preferred. The data from the respondents showed Grade 12 Mathematics learners need to be active in the learning process to enjoy the content appealing to their cognitive levels and to generate new knowledge through ingenious communicative

sessions (Klang et al., 2021). Otherwise, passive learning in Mathematics, as Aidoo et al. (2016) believe, could lead to poor performance levels in Grade 12. For more on this topic, refer to Chapter 3 (cf. 3.2.2).

In response to the question, "Which of these learning strategies did you use in the teaching and learning of Grade 12 Mathematics positively affected learners' performance?" The participants commented as follows:

With the weaker learners only focusing on level one and two questions has made a very big difference, and I found that when you just focus on levels 3 and 4 (questions), the weaker learners struggle. They want to try and answer those level 3 and 4 questions, but they can't. So, focusing on level 1 and 2 questions made a very big difference. The enthusiasm is there, but as a teacher, you say, let us focus on levels 1 and 2 because that will help them pass Mathematics. (Teacher 1)

I didn't use any problem-based learning, but these are the strategies I have used that positively impacted the learning of Mathematics... For example, if a formula is given and explained to them, what was the purpose, what does the symbol represent, and why use it? From the beginning, learners understand the formula's purpose and use. Then it gets challenging because, for level two and level three questions, they need to think or change the formula, depending on the question. To address this problem, I've used the past year's (examination) papers, exposing them to different questions and methods of answering. If a person has a different method, they must show me, and if it earns them full marks, no problem, learners can use it. (Teacher 3)

Teacher 1 argued that focusing on the learning content appropriate to the learners' learning abilities seemed to address the problem of weaker learners not passing Mathematics. Teacher 3 had a similar experience; the focus was on poor-performing learners understanding the concept before applying it to Mathematics sums. Teacher 3 added that he used previous years' examinations so that learners could practice the application of the content, which positively assisted learners in assimilating the content. This finding corroborates the ideas of DBE (2020) that exposing learners to previous examination papers is a recommended support strategy for improving Grade 12 learners' abilities to learn Mathematics.

Both teachers asserted that focusing on the lower-performing learners in Grade 12 Mathematics and providing academic support with previous years' assessments may improve the learning of the content. Their responses suggested that creating a class of under-achieving Mathematics learners supplemented with learning content appropriate to their learning abilities and individual support for teachers may assist learners in improving their performance in Grade 12 Mathematics. The findings seem consistent with Kar's (2017) assertions that teachers depend on MPCK to present specific mathematical concepts appropriate to learners to accommodate their diverse learning abilities and backgrounds. For more on this topic, refer to Chapter 3 (cf. 3.2.1).

Responding to the question, "Would you consider PBL an effective learning strategy for improving learners' problem-solving skills and performance in Grade 12 Mathematics? Explain?" The participants commented as follows:

I think it's a very good idea... it would make it more interesting for the learners because math is not exciting because there's so much syllabus to cover. So, it would be nice to have problem-based learning, but it just requires a lot of extra time. So, you don't have any extra time in the class because it is about trying to complete the syllabus needed to write a common paper. It would be nice to incorporate other methods as well. However, just that time is very limited. (Teacher 1)

Yes, most definitely. Learners work in groups. They are busy learning from each other. They learn on their own in groups. If they struggle, they know I can get help from my teacher. Some learners will work better with their peers, while others will play around. I must monitor and control classroom learning and help those who struggle. It's more fun than me standing in the front and talking the whole period. (Teacher 2)

Both teachers declared that PBL would expose learners to different learning strategies, e.g., learner-centred activities that would be more stimulating to learn Mathematics. Both participants also claimed that learners would find the content more appealing by working in groups and listening to their peers teach Mathematics, while teachers played an advisory and supervisory role in learning Mathematics. Their responses proposed that Mathematics learning should use group work accompanied by peer tutoring while teachers mediated the lesson. This view is supported by Chua

(2018) that a teacher's MPCK enables the teacher to mediate mathematical concepts successfully for effective learning. Furthermore, Hmelo-Silver (2004) advocated that group work amongst Mathematics peers creates effective co-collaborators of lifelong learning and intuitively encourages Mathematics learning.

Teacher 3 gave the following response:

Grade 12 learners can work in groups of two each, just the person sitting next to them. Furthermore, if I instruct them on a specific question, they will try and answer it together.

Teacher 3 argued that group work could only be possible after instructing learners on the concept. The data from the interviewees revealed that learning Mathematics could be successful if the Mathematics teacher facilitates group work, including peer tutoring, and learners as active co-constructors of the learning content. This finding supports the research by Mulyanto et al. (2018) that peer-centred learning activities augur well for responsible and self-driven learners learning Mathematics at their peril. For more on this topic, refer to Chapter 3 (cf. 3.2.3).

The respondents conceded that learning Mathematics amongst Grade 12 learners could be successful if teachers guided groups of learners to discover the content amongst their equals in a social climate rather than dishing out mathematical concepts to passive learners (Kathirveloo et al., 2014).

The categories that emerged for Theme 2:

- The PBL teacher facilitates the Mathematics lessons;
- The PBL approach places learners in the centre of learning activities;
- The PBL lesson used concretised explanations to explain abstract concepts;
- The PBL approach caters to the level of the content proportionate to the learners' learning abilities; and
- The PBL lesson requires differentiated learning strategies for successful learning of the content.

Research Question 3 was "What is the teachers' role in teaching Grade 12 Mathematics using PBL? Theme 3 emerged: " The teachers' role in teaching Grade 12 Mathematics using PBL."

In response to the question, "How does your mastery of the content influence the teaching styles in teaching Grade 12 Mathematics?" The participants commented as follows:

Teachers need to master the content because whenever a child has a problem, they can come to you and know exactly how to solve it. You can't stand there and say, I don't know... you need to have a very good knowledge to explain to the children. That also shows your confidence as a teacher and positively affects the children's Mathematics learning. They feel confident knowing that they have a teacher who knows the content. I do not teach all classes the same way. Those learners were finding Mathematics challenging in the Commerce class... I have to explain the content much slower than in the Physics class, which has better-performing learners. Unfortunately, many don't understand in the Commerce class, so you have to take extra time to re-explain differently. I will now use real-life situations and explain abstract concepts to them, for example, in Trigonometry, so it's very different in the two classes. (Teacher 1)

If I want to present a topic, it helps me dive deeper into the various tactics (learning strategies) I have been discussing. If you are content endowed, you use a teaching strategy and know the solution. With the mastery of the content, a learner is doing Method A; I know the results because I can follow through, and if there is something to amend, it helps me to do so. Knowing the content also helps me to guide them and makes learners believe in me. They will openly come out with what they know, knowing that in the end, if there is a problem with it, you will still help them. A teacher with limited content knowledge would not be able to facilitate group learning because it will be difficult, and with the limited content matter, it means teachers cannot motivate learners to learn Mathematics. (Teacher 2)

Both teachers asserted that they needed to be masters of the content because it influenced their teaching of Grade 12 Mathematics. They further stated that mastery of the content inspired confidence in the learners to learn Mathematics. The interviewees' responses suggested that knowing the Mathematics content well enabled them to answer any Mathematics questions and provide appropriate learning support to meet learners' various learning needs. The interviewees contended that

they were masters of Grade 12 content; they could facilitate group work, and content endowed permitted learners' inspiration to learn. These results are consistent with those of Ma'rufi et al. (2018) that the Mathematics teachers' strong command of MPCK influenced their learners' content acquisition. The scholars added that since teachers use methodologies to provide learning in direct proportion to learners' learning demands, this translates into improved performance outputs. For more on this topic, refer to Chapter 3 (cf. 3.2.1).

Teacher 4 gave the following response:

If you know your content well and you have already explained an example in one way, and the learners do not understand you, then you take a different example and explain it differently with different steps. Sometimes, I use steps, and sometimes I just explain it outright. If you know your content, you'll be able to teach differently with different methods or steps. (Teacher 4)

Teacher 4 argued that with a strong command of the Mathematics content, he could use different learning strategies for learners with varying learning demands to communicate the content successfully.

The data from the interviewees revealed that Mathematics teachers with in-depth knowledge of the content would know how to expose the content to learners with different learning needs and succeed in it. Similarly, Shulman (1986) classified MPCK as the teachers' understanding of mathematical concepts and how to comprehend them so that learners assimilate the content according to their learning needs.

In response to the question, "In your experience teaching Grade 12 learners, how have your teaching styles affected the learners' performance in Mathematics?" The participants commented as follows:

My teaching strategy has always been good. I'm yet to have a particular year I didn't even get a distinction in my class. Using problem-solving is the best because I can achieve distinctions in my class. The learners can become independent. Learners can even borrow a textbook, study a chapter, and present it to the class. I've had learners who do it almost every year. So, it helps a lot. (Teacher 2)

I found group work difficult, and it doesn't work for me. Learners in a group of six... one person knows the others write down the answers. So, can I say the other learners understand the work? No, I can't. Unfortunately, some people believe in group work, but I don't know how they do it. (Teacher 3)

Teacher 2 attested that using PBL in the classroom has always produced learners with distinction passes in Grade 12 Mathematics yearly. The respondent further argued that PBL had taught learners to be independent and could peer-teach the class with high-quality passes.

Teacher 3 maintained that using group work as a learning strategy is challenging to implement for two reasons; he did not know how to conduct group-learning, and secondly, one learner dominated the learning while others became passive receivers of the content. For more on this topic, refer to Chapter 3 (cf. 3.2.2).

Both teachers asserted that teachers must be knowledgeable in applying PBL in the classroom to effect a positive change in the Grade 12 Mathematics learners' performance. Likewise, Protheroe (2007) stated that Mathematics teachers must be flexible in their lesson presentations. This means that Grade 12 Mathematics teachers must be capable of adapting the content to the cognitive levels of heterogeneous classes in a socially collaborative atmosphere to achieve positive learners' achievements.

Teacher 4 gave the following response:

When I teach, I like to bring back the basics, for example, "functions". In Grade 11 (talking about the weaker half of the learners), three-quarters forgot the Grade 10 work. So, you have to reteach the basics (Grade 10 work) and then build up (new knowledge), then it assists them. You are just adding to what they have learned. The learner understands why you're doing a certain thing or why a certain thing is done in this way; it's easier for the learner to accept it and try it out themselves. But when the learners learn to do the graph without knowing why they're doing it, they have questions because they're just doing it by not understanding... they put the wrong answer. When the learners understand why they're doing it, it helps improve their performance levels.

Teacher 4 asserted that he used the teacher-centred method to teach learners by teaching the foundational knowledge (Grade 10 and Grade 11) and proceeded incrementally with the new content (Grade 12), which resulted in positive learners' achievements. Teacher 4 also conceded that teaching Mathematics for understanding translated into improved learner achievements in Grade 12 Mathematics. The data suggest that learners' performance in Mathematics could improve if teachers were to minister the content, moving from the known to the unknown so that learners understood it well. This finding supports previous research by Hmelo-Silver (2004) that scaffolding of learning content depends on the teachers' high calibre of guidance within the realms of the Mathematics classroom that would translate into achievement gains. Similarly, in a constructivist environment, learners build knowledge on their prior schemas through direct involvement with the new content in a social learning environment (Taufik et al., 2020). From the interviews with the teachers, it seems that teaching styles positively impacted their learners' Mathematics performances in Grade 12. For more on this topic, refer to Chapter 3 (cf. 3.2.1).

Responding to the question, "How do you think knowing Grade 12 learners' preconceptions, misconceptions, and difficulties they may experience when taught a specific topic will ensure effective learning of Grade 12 Mathematics?" The participants commented as follows:

I need to change the example. So, this one (mathematical sum) will not work. Or else, there is another easier way that I can explain the problem sum to learners. So, in the next lesson, you, I don't use the example or teaching method that challenged the learners. You will use different methods in one class, but in the other class, you can't. (Teacher 1)

Knowing their misconceptions, I am expecting them. I already have the solutions. Those misconceptions keep popping up... new problems might emerge, but most of you have the idea because of your past teaching experience. E.g., teaching exponential functions and laws of exponents, where most of the learners' misconceptions arise because of multiplying the bases. It helps me to present and remedy their errors before they make them. (Teacher 2)

Both teachers asserted that pre-knowing the challenges learners may experience in learning the content allowed them to adapt their lesson delivery from the initial presentation to meet the learning needs of learners. Their responses suggested that teachers who understood learners' challenges of learning a particular concept can effectively pre-plan their lesson delivery, thereby minimising learners' struggles in acquiring the content. For more on this topic, refer to Chapter 3 (cf. 3.2.1).

The interviewees contended that Mathematics teachers should know the potential struggles Grade 12 Mathematics learners might experience with learning new concepts and should use age-appropriate teaching strategies or adaptive methodologies to ensure effective learning. This study confirms Ma'rufi et al. (2018) study that Mathematics teachers skilled in MPCK could foresee learners' challenges in learning new content or apply multiple ways of presenting the content to produce meaningful learning.

In response to the question "*Discuss an effective teaching strategy you used in teaching Grade 12 Mathematics after discovering that learners did not grasp the content initially teaching the concept?*" The participants commented as follows:

The weaker learners are separated from the highflyers. I give them individual attention as they work through the textbook and some past year examination papers, although it is not enough. I would walk around... If the child has a problem, I will try to guide them by posing direct or sometimes leading questions, which depends on the concept I am presenting... Geometry is always leading questions, while Algebra is direct questions, most of the time.
(Teacher 1)

Previously, I talked first; then I gave them the formula, then the problem questions. I realised it didn't work because most don't understand it. So yesterday, I got them to discover their formula. Today I'm going to guide them on how to find the number of times. My teaching strategy changed. Now, I give them an investigation. They go and measure whatever and compare and arrive at an acquired decision. With learners having barriers to learning, I motivate the lower, middle, and higher-pace learners. So, what I present should motivate those who are slow, plus in a way that I'm careful not to make those who are good, not bored. Therefore, I incorporate all assessment levels, from the

knowledge to the routine, problem-solving, and even the complex. Everyone is motivated; otherwise, people will get bored. (Teacher 2)

Both teachers declared that after discovering that Grade 12 Mathematics learners did not initially grasp the content through teacher-centred learning methodologies, the participants changed their teaching strategy to PBL. Their responses suggested that facilitating Mathematics learning was more effective in learners actively experiencing the content appropriate to their level rather than their initial passive encounters. These results are consistent with the study conducted by Amineh and Asl (2015) that effective Mathematics education is mediated by knowledgeable teachers and in keeping with the appropriate developmental stages of their target group. The interviewees contend that employing a differentiated approach to cater to a heterogeneous group of active learners encouraged them to learn. Similarly, Nurlaily et al. (2019) opine that the inclusivity of learners with various learning needs is satisfied, and learners are instinct-motivated to participate in groups to make sense of the content actively. For more on this topic, refer to Chapter 3 (cf. 3.2.1).

From the interviews with the teachers, the data revealed that Mathematics learning is more effective if all learners with differentiated learning, need to incrementally experience the content. The data further disclosed that Mathematics teachers must mediate the conceptual information by creating active participants in the learning experience.

The categories that emerged for Theme 3 are as follows:

- Teachers' MPCK assists in designing and implementing flexible PBL mathematical lessons;
- Teachers implement PBL to develop independent and responsible content innovators;
- Teachers' mastery of content exposes learners to deep and varied concepts;
- Teachers mediate the learning content by scaffolding the new content in proportion to learners' varying educational needs; and
- The teacher facilitates practical investigative group work representing reality.

Table 5.1: Emergent themes and categories of semi-structured interviews

Codes	Categories	Emergent Themes
The effects of teachers' MPCK on Grade 12 learners learning Mathematics	<ul style="list-style-type: none"> • PBL is learner-focused, learner-driven learning of Mathematics • Grade 12 Mathematics content is abstract • Teachers' mastery of content enables teachers to emulate using real-life situations in teaching • Teachers' mastery of content enables teachers to use various learning support strategies to assist learners with mixed learning abilities 	Teachers' MPCK positively affects Grade 12 learners' Mathematics learning
The PBL approach in teaching and learning Mathematics impacts the Grade 12 Mathematics learners' performance	<ul style="list-style-type: none"> • The PBL teacher facilitates the Mathematics lessons • The PBL approach places learners in the centre of learning activities • The PBL lesson used concretised explanations to explain abstract concepts • The PBL approach caters to the level of the content proportionate to the learners' learning abilities • The PBL lesson requires differentiated learning strategies for successful learning of the content 	The PBL facilitator uses inclusive educational strategies for teaching and learning to improve Grade 12 Mathematics learners' performance
The teachers' role in teaching Grade 12 Mathematics using PBL	<ul style="list-style-type: none"> • Teachers' MPCK assists in designing and implementing flexible PBL mathematical lessons • Teachers implement PBL to develop independent and responsible content innovators • Teachers' mastery of content exposes learners to deep and varied concepts • The teacher mediates the learning content by scaffolding the new content in proportion to learners' varying educational needs • The teacher facilitates effective investigative group work representing reality 	The teachers' role as mediators of Grade 12 Mathematics enables content discovery through practical Scenarios

5.2.2 Focus group interviews

From Research Question 1, "What are the effects of teachers' MPCK on Grade 12 learners learning Mathematics?", Theme 1 emerged: "Teachers' MPCK positively affects Grade 12 learners' Mathematics learning."

The question posed to both cohorts of learners was, "You are currently doing Grade 12 Mathematics. Describe a regular teaching and learning experience in your Mathematics classroom."

The responses from the cohort that consistently performs above 80% in the National Senior Certificate were as follows:

He walks around and asks us as individuals and gives us attention. You can go to him; he is open. (Learner 3)

He is a loud person, as they say. He comes to you and accommodates you the way you want to be accommodated. And yeah, just that he's a very, very fast person in terms of teaching. (Learner 5)

He has many strategies for teaching; if you don't understand them, he will give you another strategy you want, and he won't just leave you behind. (Learner 6)

He has every strategy to do his Math. He can help. When there's a child that doesn't understand something, you can say they can put the one who can't understand to come to the child that understands and teach him or her alone. So, they can help me understand further. (Learner 3)

The responses from the cohort that consistently underperforms below 65% of the National Senior Certificate were as follows:

The teacher will explain the problem and then ask if no one understands the problem. Just make one understand, and then we'll say we don't understand and then further explain it. Then we will get homework or exercise to try so that we can do better. So, it's a very educational experience. (Learner D)

Our teacher will use a projector on the board. He will put the textbook example and explain it to us. And then he could ask us if we understand; if we don't understand, he will explain... Give us an exercise to do. (Learner B)

He will explain the section of the example. (Learner C)

It's the same as what Learners B and C said, but I wish he would explain some of the more difficult ones because they do these in the exercise that just messes you up. So, I wish he would do some of those also. (Learner D)

The teacher usually uses examples from the textbook that have already been explained and reads them again. (Learner F)

The respondents from the cohort that consistently performed above 80% asserted that their Mathematics teachers applied child-centred learning, accompanied by different learning approaches in an inclusive classroom, and ensured their successful learning of the content. Their responses suggest that their Mathematics teachers placed them in the centre of learning Mathematics by adapting the lessons appropriately to meet the learning needs of the heterogeneous class.

The responses from the cohort that consistently underperformed below 65% suggested that the Mathematics teacher did not apply learner-centred learning in the classroom. Most participants postulated that their Mathematics teacher simply reads from the textbook without discussing the high-level questions, sometimes marking their homework in class. For more on this topic, refer to Chapter 3 (cf. 3.2.2).

Their responses suggested that the learners were passive receptors of the content while their teacher related the content with no flexibility when delivering it. The feedback further suggested that the teacher lacked mastery of content because he mainly taught from the textbook; therefore, learners lacked stimulation in learning new content.

From the interviewee responses, the data revealed that teachers with a mastery of content were versatile in content production, positively impacting learners learning of the content. On the other hand, teachers with limited content skills were teacher-centred and textbook-driven, with little or no variation in teaching methodologies. The data also showed that teachers who mastered the content used multi-dimensional strategies to establish an inclusive learning environment for different learners to achieve positive learning outcomes. This finding is shared by Kılıç (2011), who said

that the firmness of the teachers' MPCK guides them to employ a mix-assortment of learning strategies to accommodate learners with learning barriers to Mathematics. Similarly, Shulman (1986) categorised MPCK as knowledge a teacher needs to interpret the content and know how to deliver it to learners in the classroom, irrespective of their learning challenges. For more on this topic, refer to Chapter 3 (cf. 3.2.2).

Responding to the question, ". Describe a lesson your Mathematics teacher conducted in the classroom that you found most interesting." The cohort that consistently performed above 80% commented as follows:

Algebra... There is no theory. All you need to do is calculate. You can use your calculator. Algebra is not like statistics, where they will ask you for the theory.
(Learner 3)

Trigonometry because it has shaped. Well, Sir can describe something like graphs by using the buildings. He will tell us how to measure. He will show us how to work them by turning the pages and going further. Learner 1

I just enjoyed the lesson on "functions" because of the way he explained it to us. (Learner 2)

The responses from the cohort that consistently underperformed below 65 % in the National Senior Certificate were as follows:

I once didn't understand something, and the teacher made a joke by explaining it to me as apples. I enjoyed that because I understood it way better; although it was a joke, I understood it a lot better. I remember it because he made me laugh and helped me visualise the problem sum. But most of the time, we don't necessarily take simpler objects and use them to explain mathematical problems. So, it's apples that seem like a Grade R explanation, but I understood it better than regular information, which is for higher grades. (Learner D)

I enjoyed Math now and then. But most of the time, it's quite confusing. But the thing is, when the teacher explains it, it feels good because I finally understand it. (Learner A)

When they (teachers), simplify it because most teachers just take math seriously, you will remember it better when they make it enjoyable. (Learner E)

What I find interesting is when we have to use our phones and do activities because the phone is not as depressing as the math textbook. (Learner F)

The responses from the cohort that consistently performed above 80% enjoyed Grade 12 learning Mathematics because the concepts did not need a recollection of theory. After all, learning requires the practical investigation of content. The interviewees also added that the style of the teacher's discussion of the content excited them to learn. Their responses suggest that Mathematics teachers should create active learning opportunities (using ICT) for learners to experience the content.

The responses from the cohort that consistently underperformed below 65% asserted that the teacher communicated the content appropriate to their cognitive understanding, enabling learning to occur. The responding learners further suggested that their teacher's humorous display in the Mathematics classroom and the ability to use teaching aids to communicate the abstract content stimulated them to learn.

From the interviewees' responses, the data revealed that teachers should be innovative developers of the content, and practice exposing it to learners to captivate their learning spirit and become independent scholars. The data also showed that teachers who communicated the content used an inclusive approach with relevant ICT that could positively impact Mathematics learning. This finding is shared by Moh'd et al. (2022) and Ma'rufi et al. (2018); Mathematics teachers must know the content and different delivery styles to skilfully meet the various learning needs of all learners. For more on this topic, refer to Chapter 3 (cf. 3.2.1).

The question posed to both cohorts of learners was, "Describe the personality and relationship you have with your Mathematics teacher?" The responses from the cohort that consistently performed above 80% of the National Senior Certificate were as follows:

Yeah, it's a good relationship because he doesn't shout at you if you don't understand. He has time for you. Maybe he will say, "Come and attend extra classes." (Learner 3)

I take him as my uncle because when he sees you are scared of talking to him or you're having a problem, he comes to you and accommodates you in a way. He accommodates you as a child, and you seem to get very comfortable, so my relationship with him is very good. (Learner 2)

All I can say is that he is a good person to me. (Learner 5)

He is almost like a father. He cares about you and motivates you. Sometimes, when we get our reports, he will sit down with us and ask us what we think. He cares...I am motivated to learn on my own. (Learner 3)

Most participants asserted that their Mathematics teacher showed a genuine interest in their academic performance, often inviting them to discuss their learning challenges and providing support complimentary to their learning challenges. The respondents further argued that the teacher displayed the features of a "caring father," which made them comfortable discussing their learning difficulties and provided appropriate academic support. Their responses suggest that the mathematics teacher should be deeply involved in learners acquiring the content to overcome their learning obstacles. For more on this topic, refer to Chapter 3 (cf. 3.2.3).

From the interviewee responses, the data revealed that Grade 12 learners learning Mathematics depend on the nature of their relationship with the teacher and the teacher's ability to use various learning strategies congruous to the academic needs of learners. This finding corroborates the ideas of Carnoy and Arends (2012) and Li and Tsai (2017) that teachers with concrete relationships with their learners often produce lessons knowledgeable of their differentiated learning needs, and this could improve the quality of their academic performance.

The responses from the cohort that consistently underperformed below 65%:

I don't even know how to describe it. Hear the sound of this aircon? Does it ever change? No, now imagine being taught like that... (Learner F)

The lesson is boring. Now and then, he will make a few jokes. But because he reads from the textbook, it's very difficult to learn because he sits in his place. (Learner A)

Yes! He just sits at his table and reads from the textbook, so it's not like, for example, if it was another teacher who would use hand movements and make the lesson interesting. But for him, he just sits at his table and reads from his textbook. (Learner E)

He just sits by his laptop; his laptop is connected to the projector projecting on the board. (Learner C)

We redo the example already explained in the textbook; he makes the examples with the given solutions. The examples he does are from the textbooks. (Learner B)

Most participants declared their Mathematics learning was boring because their teacher was not versatile in the lesson presentation. The respondents further articulated that the teacher taught strictly from the textbook. Their responses suggested that the teacher produced text-book-driven lessons that did not captivate learners into learning.

From the interviewee feedback, the data revealed that the teacher used the teacher-centred strategy to teach the class while they passively listened to the delivery of the textbook. The data also showed that the learners needed their teacher to use age-related learning strategies relevant to the cognitive needs of divergent learners and be robust in content delivery. These results match those observed in Shulman's (1986) study that Mathematics teachers should be powerful content disseminators based on their ability to know and transfer it, ideally meeting the academic learning demands of mixed-ability learners. For more on this topic, refer to Chapter 3 (cf. 3.2.1).

Responding to the question, "Please describe your current performance levels in Grade 12 Mathematics and your feelings about it." The cohort that consistently performed above 80% commented as follows:

Mine is always at the same level, level 5 (60%) ... But I want to go higher. (Learner 2)

I feel good about my performance but want to go higher and stay consistent in the 80% and 90% s. (Learner 5)

I don't feel bad about my marks because when I drop, I drop to level 5, maybe (60%) or 6 (70%), but not lower. I'm not happy with your current performance... because I want 80% and above. (Learner 6)

I am unhappy simply because I'm always on the verge of passing or failing. I want to reach at least level 6 (70%). (Learner 5)

Most participants asserted that they were content with their current performances but were also ambitious to improve on them. Their responses suggest that their current performance in Mathematics is to their satisfaction, but they were motivated to do better.

From the interviewee responses, the data revealed that Mathematics teachers instructed learners using PBL had achieved according to their expectations but were also self-motivated to improve their current performance levels. The data also showed that Mathematics learners instructed through the medium of PBL were independently responsible for their performance levels and were achieving good passes in Grade 12 Mathematics. This finding corroborates Koh and Chapman's (2019) and Nurlaily et al. (2019) ideas that learners instructed through PBL take charge of their learning and are self-motivated to improve their achievement levels. For more on this topic, refer to Chapter 3 (cf. 3.2.1).

The responses from the cohort that consistently underperformed below 65% were as follows:

I had a terrible performance. No matter how much I tried to improve, it got more complicated, and I lost interest after a certain point. And then I go do something else. There's too much stuff that doesn't make sense. Nothing makes sense. So just so I don't feel a specific way about it because I'm used to failing its kind... But I don't know how to improve. That's the difficult issue. (Learner D)

My math marks, okay, yes, I do not hate my teacher guys. When I tell you, I rely on my teacher as I'm supposed to, I flunk because I do not see effort from him. But if I do it on my own, I just ignore him. Like, get him out of my mind. Like he doesn't exist, I do better. When I work alone, I check YouTube for different

videos, ask my classmates, and ask other teachers. And I get a better response from them than from my teacher... (Learner F)

Mine is the same as Learner D because we've been trying; I've been trying and putting in some effort and it just didn't work out. (Learner E)

Most participants asserted that their performance was poor despite their efforts to achieve good grades in Grade 12 Mathematics. The respondents further suggested that their teacher seemed unmotivated to teach them, resulting in their poor performance. The interviewee's feedback declared that collectively working with peers and other teachers, they received the support they sought, which helped improve their performance levels. Their responses suggested that the teacher was not inspired by the teaching and learning process.

From the interviewees' responses, the data revealed that the teacher should be able to provoke learning in the classroom by implementing various learning styles for achievement gains. The data also showed that learners were overwhelmed with the workload, which resulted in them not passing. These results are consistent with those of Ball et al. (2008), that if Mathematics teachers cannot inspire learners' lateral thinking, they will not gain new levels of meaning. To build up learners' knowledge capacity, Rahman et al. (2021) recommended that teachers administer various practical learning instructions or PBL to meet the demands of different learning styles of Mathematics learners. For more on this topic, refer to Chapter 3 (cf. 3.2.3).

In response to the question, "Problem-based learning or PBL is a learner-focused instructional strategy. Learners are placed in smaller groups while the teacher helps learners to learn rather than instructing while everyone is seated and listening. How many of you favour being taught Mathematics in smaller groups rather than the teacher instructing? Explain why?" The cohort that consistently performs above 80% commented as follows:

I prefer sitting on my own and the teachers instructing because some of them in groups do not concentrate. They just don't even give you some advice to help you. (Learner 4)

I prefer the teacher to be in the front because, in groups, some of us are here to distract others while others are focused on the work. That is why I end up not

learning anything. But I know I need to focus when I am on my own. (Learner 1)

They like talking about other topics... like TikTok... They are just not focused. Maybe two or three am. But the majority are not focused on learning Maths. (Learner 3)

And like Learner 3 said, we can work together, and they can talk, but when Sir comes to check, they take their books and copy. (Learner 1)

Most respondents declared they preferred the teacher-centred method and receiving the instruction. They also asserted that in their group activities, some learners lose focus, disrupt other group members, or copy the answers just before the teacher moderated their work.

Their responses suggested that group learning is impractical and therefore opted for direct instruction from their teacher. From the interviewee responses, the data revealed that learners favoured a traditional approach to learning Mathematics because other learners in the group were not responsible for their learning. The data also showed that the teacher did not seem to control the group work activities, which could have negatively impacted the learning process and their achievement gains. This finding is shared by Mulyanto et al. (2018) who reveal that learners needed to be in groups overseen by the teacher to gain information from their world, and this would positively impact learners' performance levels. For more on this topic, refer to Chapter 3 (cf. 3.2.3).

The responses from the cohort that consistently underperformed below 65% were as follows:

Working in groups will be more beneficial because different personalities are constantly working on the same problem. We are going to come up with different solutions to one problem. So, when someone explains to me, it will appear in a way that will make it way simpler to understand than a teacher teaching from a textbook. (Learner D)

In the whole class, you are afraid to pick up your hand because you may ask a question that everyone thinks is stupid. In a group of four, it's fine. Those four

people heard you, but now in a class of 40 children, they will think you are stupid. (Learner A)

Most participants asserted that group learning would help improve their learning standards because, amongst their four peers, they felt more comfortable making errors and learning from them. The interviewees further stated that collaboratively working with their peers and discussing the content could lead to better understanding and improved performance in Grade 12 Mathematics.

Their responses suggested that fellow learners socially exchange information could result in improved achievement gain in Grade 12 because they could present a variety of solutions to the same problem in a much more appealing manner than their textbook-dependent teacher.

From the respondents' feedback, the data revealed that Mathematics learners favoured learner-centred learning instead of the teacher-centred approach. The data also showed that constructive learning could only occur when learners actively participate in activities to produce varied answers to a mathematical problem under the teacher's facilitation. Similarly, Ubah (2021) posits an association between small-group instructional engagements and learners' positive achievements in secondary school Mathematics guided by a knowledgeable teacher. Furthermore, these findings also match those of Klang et al. (2021) who state that Mathematics learners' positive achievements could be attributed to learning as a socially engaged event, enabling knowledge transfer between peers. For more on this topic, refer to Chapter 3 (cf. 3.2.1).

In response to the question, "Do you think if your Mathematics teacher uses PBL to teach you Mathematics, what effect would it have on your ability to learn Mathematics?" The cohort that consistently performed above 80% commented as follows:

It'll depend on the group. Which learners do we have? It depends on those learners, maybe if they are willing to write and help each other and they're willing to help them to concentrate. (Learner 3)

It has an effect in a negative way more than a positive way because by helping others, can you explain this to them, you are learning. It can affect me negatively because I'll be writing, and you will not be asking me questions like

how this is done, all of that, so there would be copying, so my marks will drop, but yours will go higher because you are focused, and I am not. (Learner 5)

I think things would be better. If the teacher asks you to select people you think have the same mentality as you, the group will work together. (Learner 1)

As you're saying if you choose your friends, you might not concentrate. But if the teacher selects some people, he thinks could help you succeed or proceed forward, group work might work for you. (Learner 2)

I think group work is the best way because some of us cannot understand the teacher, but we can understand our friends, so group work is the best for some of us. Learner 4

The responses from the cohort that consistently underperformed below 65% were as follows:

I think it will help a lot because learners are scared to pick up their hands and tell the teacher they don't understand. (Learner D)

Major increase in our performance results. If we know we are going to class and will have fun and learn at the same time, we are all kids no matter how old we are, so if we know we will have fun, we learn. It lessens the pressure on each person. (Learner F).

Most participants asserted that learning Grade 12 Mathematics in groups would improve their achievements. The respondents also confirmed that group dynamics are essential when constituting a group for learning Mathematics.

Their responses suggested that learners operating in groups need to consider the work ethic of each person, as that could determine whether group learning would be successful. The feedback from the respondents also stated that, while some learners who led the group would benefit in learning, the passive learners might not because they could copy the work done.

From the respondents' responses, the data showed that group work would positively affect the learning of Grade 12 Mathematics. The data also revealed that Mathematics teachers would ensure Grade 12 learners actively engaged with the content collaboratively. This would mean the teacher needs to observe each group's

interaction by moving around in the classroom. This finding corroborates the findings of Mulyanto et al. (2018) study that the Mathematics teacher is now a facilitator of the learning content in a PBL classroom, often guiding learners to develop their Mathematics problem-solving skills with relevant resources. Complimentary to these outcomes, Akpan et al. (2020) believe that a social constructivist lesson facilitated by the teacher would create a collaborative learning situation for Grade 12 Mathematics learners. Furthermore, Lee et al. (2018) added that Mathematics facilitation in the PBL classroom could improve Grade 12 learners' investigative and problem-deciphering skills, as much as their collaborative decision-making skills, which may appear in the NSC, as a positive result. For more on this topic, refer to Chapter 3 (cf. 3.2.1).

The reaction to the question, "Do you think that a Mathematics teacher who knows the content well can help you learn Mathematics much better than someone who knows very little?" The cohort that consistently performed above 80% commented as follows:

The teacher can't teach something they know little about because we want to ask questions, which will require a teacher's knowledge. So, we will not be able to answer them because we know little... (Learner 6)

There is a teacher who doesn't know the concepts; when we ask questions beyond the textbook, he doesn't know. He is just reading from the textbook, so; the teacher needs to know. Everything. He does not know the answer when we ask him a question and goes off the topic. The teacher needs to know the content so he or she can explain it to us. (Learner 4)

The responses from the cohort that consistently underperformed below 65% were as follows:

Depending on the teaching method, it still varies because you could be Einstein teaching us, but if you don't know how to teach us because you understand it, your understanding is not the same as ours. (Learner D)

I'm going with what Learner D said, in the words of Einstein, no matter how well you know something, if he can't teach it to a grade R child, you do not know it yourself. (Learner F)

Yes, the teacher can help me learn, but for example, in 2021, you had a teacher; I felt understood the work but didn't know how to make us understand. (Learner C)

Most participants declared that the Mathematics teacher should know the content well to ensure effective Mathematics learning. The interviewees further stated that the Grade 12 Mathematics teachers should teach the content appropriate to their learning needs.

Their responses suggest that the Mathematics teacher should have a rich knowledge base of the content to conduct lessons beyond the textbook. Furthermore, their respondents recommended that teachers should be versatile in content delivery as they pose various questions in the learning session.

From the interviewee feedback, the data gained revealed that the Mathematics teacher should be resourceful in content so that the teacher could supply information applicable to their learning needs. The data also showed that the Mathematics teacher should be skilled with content knowledge and be able to produce practical learning situations, which would help learners master the content. The present findings seem consistent with Shulman (1986) that a teacher's MPCK is the actual consolidation of the lesson delivery when content compliments pedagogy (how to teach) and successfully exposes learners to Mathematics education. Similarly, Turnuklu and Yesildere (2007) agree that a teacher's MPCK or the knowledge of how to teach is equally essential, just as mastering the factual content so that constructive learning could take place in the classroom. For more on this topic, refer to Chapter 3 (cf. 3.2.1).

Responding to the question, "What are some of the learning difficulties you experience in Grade 12 Mathematics?" The cohort that consistently performed above 80% commented as follows:

Sir, we need to do more practical work that must bring past papers to us for revision, so we can know what to expect in the exam... the teacher can explain it in class like it's easy. But in the exam, it is not like he or she explained, so it's good to show us how they will ask the question in the exam. So, then we must understand. (Learner 2)

Sir, the challenge that I found is that you might be nailing math from the textbook in the classroom. You think that you know the topic. But now, when it comes to the exam, it's still the same thing, but more than a different way, and I found it challenging to get to know what to do... (Learner 2)

The problem is when the teachers don't advise you on how to answer specific questions, and you get them wrong. (Learner 1)

The responses from the cohort that consistently performs below 65% were as follows:

Our teacher will use a projector on the board. He will put the textbook example and explain it to us. And then, he would ask us if we understood; if we didn't understand, he would explain it again and give us an exercise. (Learner B)

He uses examples from the textbook to explain the section. And then, he will ask the class if they understand or if he will explain it and give classwork. Sometimes he will mark it there. (Learner C)

It's the same as what Learners B and C said, but I wish he would explain some of the more difficult ones because these sums just mess you up in the exam. So, I wish he would do some of those, also. (Learner D)

The teacher usually uses examples from the textbook that have already been explained and reads them again. (Learner F)

Most participants asserted that their teachers teach Mathematics from the textbook. The respondents further declared that their teachers did not discuss how to respond to high-end mathematical questions that typically appeared in the examinations.

Their responses suggest that their teachers strictly depended on the textbook as their source of information in the teaching and learning of Mathematics. The interviewees' feedback corroborated each other answers that the teachers read out answers from the textbook and could not discuss questions that were featured in the final year examinations.

From the respondents' feedback, the data revealed that teachers seemed to possess limited content knowledge and could not lead the learners with high-order

examination-type questions. The data also showed that the teachers mostly used teacher-centred teaching methods and were not flexible in content knowledge or delivery. This contributed to Grade 12 Mathematics learners' learning difficulties. These results match those Saad et al. (2015) observed that teachers need a wealth of conceptual information and Mathematical didactics to expose learners to the content via malleable lessons with various appropriate instructional styles. Therefore, Courtney et al. (2015) suggested that in a constructivist learning situation, the facilitator should forecast learning activities complementary to the varying needs of different learners. For more on this topic, refer to Chapter 3 (cf. 3.2.3).

Responding to the question, "Why do you think you experience such learning difficulties in Mathematics?" The cohort that consistently performed above 80% commented as follows:

But sometimes, the teacher doesn't give us the past exam papers, so we do not have the opportunity to work with those types of questions. He always gives us from the textbook... from the textbook, it's much easier. (Learner 6)

But he doesn't have time.... Because he teaches many people and tries to rush through the work. (Learner 3)

He does his best, but it's up to you to go and download past papers, I think, and try and do them, and go look for the memos and mark your work, but if you do not understand that specific question, go to the teacher don't go with the whole paper because he doesn't have time. He works with time. (Learner 1)

The responses from the cohort that consistently underperformed below 65% were as follows:

My learning difficulties in Mathematics, I do not understand his teaching. I think I will better understand my teacher if he uses more visuals. I am a visual learner. I think everyone remembers things better when they can picture them in their minds. I think this will make it simpler for me to understand my teacher. (Learner D)

Most participants asserted that the teacher's source of information in teaching Mathematics is the school textbook and did not provide past year exam question papers to practice for the examination. The respondents further stated that the Mathematics teacher has limited time and teaches many classes; therefore, the content presented is rushed. However, Learner 1 argued that it was their responsibility to acquire past year exam papers with memos and work on their own, and when necessary, they should discuss specific problem sums with the teacher. Furthermore, Learner D postulated that the teacher needed to concretise the content instead of conveying abstract information he did not understand.

Their responses suggest that teachers teaching Mathematics from textbooks contributed to their poor understanding, as learners were not exposed to Grade 12 exam-type questions and therefore found it difficult to learn. From the respondents' feedback, the data suggested that Mathematics teachers must be masters of the content to visually introduce previous years' examination papers so learners could comprehend the content more effectively. The data also showed that Mathematics teachers should be content endowed to use academic support strategies equivalent to their learning needs for effective learning. For more on this topic, refer to Chapter 3 (cf. 3.2.1).

Responding to the question, "How have your Mathematics learning difficulties/challenges/fears/attitude affected your performance levels in Grade 12?" The cohort that consistently performed above 80% commented as follows:

If you don't like maths. You will definitely fail. Even if you don't like the subject, you must give your full attention and focus to it. You must practice it almost every day until you know it very well. (Learner 6)

Math is fun, you just have to put your mind to it. You have to be strategic and open your mind. When solving a problem, don't look at one sum and say ncah... cannot do it. You must try using different methods to solve the problem. (Learner 5)

Most of the participants (Learner 5 and Learner 6) asserted that their positive attitude had a direct influence on their performance level in Grade 12 Mathematics. The

respondents also confirmed that a poor attitude towards learning Mathematics might result in poor achievement gains.

Their responses suggest that their attitude towards the subject determined their performance levels in Grade 12. From the interviewee responses, the data revealed a direct association between learners' attitudes toward Mathematics and their achievement gains in the subject. This finding is shared by Zan and Martino (2007) that learners' attitudes towards Mathematics play a huge role in learning it. Similarly, Kundu and Ghose (2016) found positive attitudes toward Mathematics improved their learning abilities and performance levels. For more on this topic, refer to Chapter 3 (cf. 3.2.3).

In response to the question, "*Suggest ways your Mathematics teacher can assist you to improve your performance levels into Grade 12.*" The cohort that consistently performed above 80% commented as follows:

Put more attention on the ones that need it and compare the levels. Separate the lower-performing learners from the higher-performing learners and give extra classes to the low-level performing learners. (Learner 3)

The responses from the cohort that consistently underperformed below 65 % were as follows:

My teacher tries to explain the work, but if we do not understand, he must see who is getting high marks and ask that person (peer learner) to go and explain to the class. (Learner B)

The teacher needs us to work in groups solving Mathematics problem sums. The top learners must be in each group with the lower-performing learners; instead of segregating them into two opposite classes. If you put the lower learner class with more high performers into those groups, the high performers can explain it to their peers. (Learner C)

Some of the participants (Learner C and Learner B) articulated that learners must be arranged in groups and should be peer-taught. The respondents believed that learners should be placed in a mixed-ability group with contrasting performance levels so that the more knowledgeable learners could lead the learning process.

Their responses suggest that teachers should create opportunities for learners to take on an active role in the teaching and should experience an improvement in their achievement levels in Grade 12.

From the respondents' feedback, the data revealed that interactive social learning of learners in groups could positively affect learners' performance in Grade 12. The data also showed that Mathematics teachers were not the centre of teaching and learning; however, learners were now responsible for sharing content with their peers. This finding is supported by Klang et al. (2021) that functioning Mathematics learners participating in investigative lessons could successfully transfer new theories to their peers and improve their performance standards. Similarly, Li and Tsai (2017) declared that PBL is a learner-focused research method that requires peer communication on the Mathematics content. For more on this topic, refer to Chapter 3 (cf. 3.2.2).

Responding to the question, *"Is there any other experience you would like to share regarding your teacher's content knowledge of mathematical topics and how it affects your learning and performance levels?"* The cohort that consistently performed above 80% commented as follows:

I like how, when we were in Grade 11, he referred to Grade 12 questions and helped us solve those questions so we could have more knowledge. (Learner 1)

Yes, he knows his work very well. Sometimes he says to test us, to fetch a past paper of matric and do. Then come to him, and he will mark it. My teacher wants to know if I managed the objective work. He knows his work well because if I have a problem with a particular sequence or a sum, he will just solve it and show me how he did it. (Learner 6)

The responses from the cohort that consistently underperformed below 65% were as follows:

You must also motivate and inspire others when you choose to teach as a career. But if you just choose to teach math because you're good at it and come

to the class and just teach, you don't have any fun in the Mathematics classroom. (Learner F)

Most participants (Learner 1, Learner 6, and Learner F) asserted that their teacher was rich in content and posed challenging Mathematics questions, which they found stimulating. The interviewees further stated that their teacher showed an interest in their performance abilities, often approaching them to assist with the correct information.

Their responses suggest that the Mathematics teacher seemed passionate about their learners' performance and challenged their learning by giving past year examination question papers for learners to attempt.

From the participants' responses, the data revealed that teachers with a strong command of content were personally involved in their learners' achievement levels. The data also highlighted that progressive teachers placed mathematics learners in the middle of teaching and learning and could present content appropriate to the learning needs. These findings further support the idea of Lee et al. (2018) that teachers with a good comprehension of MPCK can transfer content to learners by appealing to their grade-appropriate learning needs and challenging their abilities. Furthermore, the study's outcomes agree with Vygotsky's (1978) empirical review that teachers skilled with MPCK could stimulate learners to learn by adjusting the content to a notch above their present level and within their capability. Furthermore, Turnuklu and Yesildere (2007) believe that teachers who are experts in the Mathematics field could implement relevant teaching strategies to demonstrate content to a diverse group of learners successfully. For more on this topic, refer to Chapter 3 (cf. 3.2.1).

The following categories emerged for Theme 3:

- Mathematics teachers should have mastered the content;
- Mastery of content allows learners exposure to high-order questions;
- Mathematics teachers design and implement lessons relevant to the learners' learning needs;
- Teachers mediate abstract content by concretising it, appropriate to learners' understanding; and
- Teachers should be socially involved in learners' performance.

Table 5.2: Emergent themes and categories of focus group interviews

Codes	Categories	Emergent Themes
<p>The effects of teachers' MPCK on Grade 12 learners learning Mathematics</p>	<ul style="list-style-type: none"> • Teacher guides group-centred learning of Mathematics • Teachers are masters of the content • Teachers' mastery of content enables teachers to use various teaching methodologies for inclusive education • Teachers' mastery of content enables the innovative delivery of lessons • Teacher lack of content knowledge leads to teacher-centred textbook delivery of lessons 	<p>Teachers' MPCK positively affects Grade 12 learners' Mathematics learning</p>
<p>The PBL approach in teaching and learning Mathematics impacts the Grade 12 Mathematics learners' performance</p>	<ul style="list-style-type: none"> • The PBL teacher motivates learners to improve their achievement levels. • Learners become independently responsible for their performance in Mathematics • The PBL teacher facilitates group interaction for effective learning. • The PBL approach places learners in the centre of learning activities • The PBL lesson focuses on collaborative group learning interactive activities. • Group learning allows for peer tutoring. 	<p>The PBL facilitator uses inclusive educational strategies for teaching and learning to improve Grade 12 Mathematics learners' performance</p>
<p>The teachers' role in teaching Grade 12 Mathematics using PBL</p>	<ul style="list-style-type: none"> • Teachers guide Mathematics learners in a group on knowledge acquisition • Teachers facilitate peer tutoring in learner-group sessions • Teachers must possess content and various instructional skills in transferring content to learners • Teachers' mastery of content exposes learners to deep and varied concepts 	<p>The teachers' role as mediators of Grade 12 Mathematics enables content discovery through practical scenarios</p>

	<ul style="list-style-type: none"> • Teacher mediates the learning content proportion to learners' varying educational needs • Teachers facilitate effective lessons by concretising abstract content to learner-groups 	
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5.2.3 Classroom observations

The objective of presenting and analysing the data gained from the observation schedule from the respective schools was to make sense of the phenomena under review.

5.2.3.1 Time management

All learners were seated for the lesson; however, Teacher 1 was absent. Learners took out their worksheets and calculators and then continued working independently with the following sections from their worksheets. Fifteen minutes into the period, Teacher 1 telephoned his colleague to say that he would be late and that learners must continue working independently. Teacher 1 arrived at the start of the second lesson. He greeted the class and then sat on his chair. This observation contrasts the views expressed by Burlbaw and Ortwein (2013) that Mathematics teachers' role function is essential to learners in managing their time in the learning situation. Teacher 3 was on time in the classroom; however, the learners arrived 15 minutes after the start of the period. The lesson lasted for 30 minutes.

5.2.3.2 Classroom teaching and learning environment

Both Teachers were classroom-based, in other words, the learners came to the teacher. The Grade 12 Mathematics learners were streamlined into two classes based on their final mark achieved in Grade 11; that is, learners who earned 40% and above were placed in the "A" class, while learners who did not achieve a pass mark (0-39%) were placed in the "B" class. This finding contrasts the views of Koh and Chapman (2019) that more knowledgeable learners introduce the content to their poor-

performing peers through negotiated learning group-work scenarios, which should improve the quality of learning Mathematics.

Teacher 1's class used the old home economics room, which was converted into a Mathematics classroom; therefore, the classroom is twice the size of a standard classroom. However, only half the classroom was used. Teacher 3's class was a standard-size classroom, resulting in overcrowding. Both teachers' classes had their learners' desks arranged in cinema style, with their tables facing the teacher. This suggests that teaching and learning are teacher-centred. This contrasts with earlier findings of Hmelo-Silver (2004) that the learning of Grade 12 Mathematics depends on learner-centredness and on constructing new and flexible knowledge banks to enhance collaborative learning. During the lesson observation, most of the learners were seated and worked independently as the teachers merely explained the memo to the previous day's class work and gave the answers to the learners. Though most teachers were masters of the content, apart from their detailed explanation, they also raised possible errors learners could make in an examination and how to avoid them. The tone of the teacher was ordinary and monotone. The teacher then opened the front door and went out to smoke. Learners began to mark their work using their pencils. The learners in the front of the class raised their hands to ask the teacher questions. The teacher wrote out the explanation on the board. The teachers located themselves at the front of the classroom. Learners at the back of the classroom were playing with their cell phones and had no learning material on their desks. This continued throughout the lesson because the teachers did not recognise the misconduct committed by the learners.

Figure 5.1: The cohort of **learners** that underperforms below 65% of the National Senior Certificate, copied the answers that were projected on the whiteboard



Source: Researcher's work

5.2.3.3 The effects of teachers' MPCK on Grade 12 learners learning Mathematics

The researcher observed that while the teachers showed mastery of the content through the explanations rendered to learners in the classroom, most learners did not seem involved in the lesson. The learners in the front were engaged because the teachers mainly focused on them. The lesson presented was teacher-centred, as most of the answers to the questions were given via the overhead projector (OHP) or chalkboard. The teachers did not implement different teaching strategies to cater to the learners who seemed uninterested in the lesson. This is in contrast with the views of Widyatiningtyas et al. (2015) that the teacher-centred style of lesson presentation and learners as passive absorbers are not the characteristics of PBL. However, the teacher-focused approach favoured the learners seated in the front of the classroom but neglected the poor performers at the rear end of the classroom. Also, the content was abstract because the content resembled examination-type questions. Learners were silent and corrected their work.

Learners were unable to participate in group work. The teachers did not pose inquiring, thought-provoking, or leading questions to the learners to stimulate their thoughts to solve their mathematical problems. This contrasts with Hyslop-Margison and Strobel (2008) that Mathematics learners could improve their knowledge from their rough operational guidance from their peers and achieve better in Mathematics.

The teacher's answers seemed to be the only acceptable answer because, as he had explained, each step is a mark earned in the examination. This is contrary to the characteristics of a PBL lesson. As Mceleli (2019) explained, PBL is a learning strategy engineered by Mathematics teachers to create a collaborative learning environment that would enable learners to be self-directed in learning the new content.

Figure 5.2: Teacher one used the teacher-centred method during the observed lesson



Source: Researcher's work

5.2.3.4 Effects of the PBL approach in teaching and learning Mathematics on the Grade 12 Mathematics learners' performance

I cannot report on using PBL because there was no evidence of group work or teacher facilitation of the lessons observed. As mentioned earlier, the lessons were teacher-centred and revolved around learners correcting their homework by exposing them to the answers.

Figure 5.3: Teacher three also used the teacher-centred method to teach during the observed lesson



Source: Researcher's work

5.2.3.5 The role of teachers in teaching Grade 12 Mathematics using PBL

Klang et al. (2021) postulate that the teachers' role in teaching Grade 12 Mathematics using PBL Mathematics is facilitating learning instead of disseminating information to learners and posing open-ended questions to trigger their thoughts to supply creative answers to real-life problems. This did not feature in the classroom observations. The role of both teachers seemed to take prominence in the lesson delivery while learners passively and independently engaged with work from the OHP and chalkboard. The teacher instructed the answers to the learners while explaining an answer with more details, as a response to a learner's question.

There was little learning of the content because the teachers did not facilitate the lesson. This observation contrasts with Protheroe (2007), who believes effective Mathematics learning may only happen under the teacher's facilitation. More importantly, the scholar believes that Mathematics teachers should actively engage learners and use differentiated teaching approaches to achieve positive learners' achievements.

Most learners were disciplined in learning because they were high performers, while other learners showed no interest in the lesson because the teachers seemed to focus on the learners who produced the correct answers. The lesson did not appeal to all the learners because the teachers did not use a differentiated teaching and learning approach (Protheroe, 2007).

Figure 5.4: The cohort that underperforms below 65% copied the answers that were projected on the whiteboard



Source: Researcher's work

5.2.3.6 Teaching and learning Grade 12 Mathematics in the classroom

The teachers attempted to effectively explain the content and reflect on their prior knowledge to clarify learners' misunderstanding of the answers put forward; however, there were no further discussions on the answers given. This contrasts Taber's (2018) view that scaffolding new content based on the prior content, combined with group dialogue, would allow learners to acquire new content successfully in a constructivist learning environment.

The teachers were the sources of information, leading the teacher-centred lessons throughout the lesson presentation. This is opposed to the role of a PBL teacher, who is supposed to be a facilitator and mediator of the learning experience encapsulated in authentic environments (Amineh & Asl, 2015). The lesson content was meaningful to those learners seated in the front as they passively listened to their teachers. In contrast, learners at the back of the classroom showed no interest. They whispered to each other or played games on their cell phones. The teachers were supposed to intrinsically inspire learners to learn (Hmelo-Silver, 2004); however, those who participated were the high-performing learners, while the lower performers did not seem interested in the lesson.

There is a need for teachers to deliver Mathematics content more effectively. As suggested by Kathirveloo et al. (2016), for effective teaching of Grade 12 Mathematics, teachers must possess the knowledge and skills to transfer the conceptual information and the relationship between understanding and flexible reasoning of the Mathematics content in a communal environment. To achieve this, Moh'd et al. (2021) state that learners learn by modelling authentic life situations and socially negotiating their solutions together and can graduate from factual content to more abstract concepts. However, in the observation, the learners passively received the content while the teacher explained the answers. Teacher 3 attempted to focus on the learners learning of the content by requesting learners to approach the chalkboard and write out the answer while the teacher monitored and corrected any wrong answers; again, learners passively absorbed the content.

Figure 5.5: Teacher three teaching from the chalkboard



Source: Researcher's work

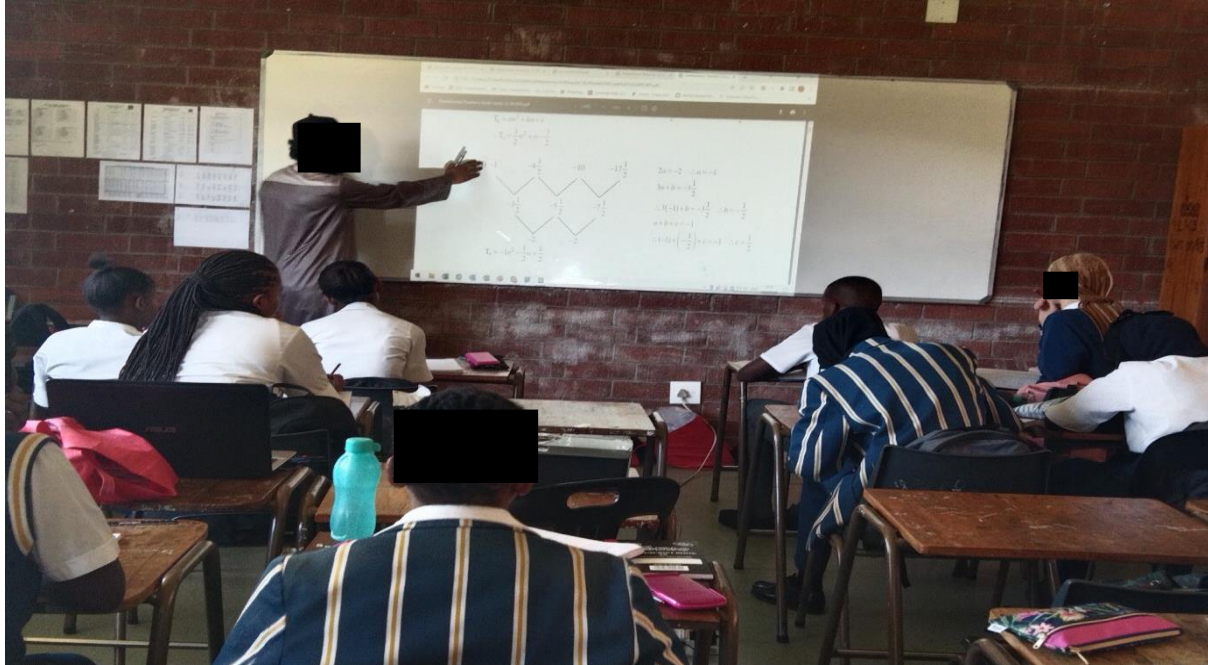
5.2.3.7 The teachers' MPCK on Grade 12 learners learning Mathematics

The Mathematics teachers from both secondary schools showed excellent command of the content but lacked the skills to use various strategies to appeal to all learners to learn the content successfully. Gess-Newsome et al. (2017) believe the most successful strategy for learning Mathematics is for teachers to know how to teach to learners' understanding. More importantly, Ektina (2010) argued that in a social constructivist environment, Mathematics teachers should teach with a solid knowledge base of the subject matter and comprehend the knowledge of teaching the subject (MPCK) levelled at their understanding. Therefore, it was evident that the teachers did not plan and could not use appropriate instructional strategies to meet learners' diverse learning needs.

The teachers were able to explain the answer to the learners and obviate the possible learning challenges learners could experience in their learning, but they could not vary their teaching styles to inspire all learners to learn. Apart from the OHP and the chalkboard, both teachers depended on worksheets and learners' WhatsApp posts of the learning activities that would be regarded as homework. This finding contradicts the views of Turnuklu and Yesildere (2007) that Mathematics teachers should possess information to know how to teach and not just know the factual content. In the teachers' observations, both teachers were not versatile in employing different learning strategies to the different learners to successfully convey the conceptual message into concrete building blocks of information. The content presented to the learners was grade-appropriate because the teachers followed the CAPS document strictly, which

guided the teachers on the curriculum's scope, educational objectives, and adequate teaching resources; therefore, in this regard, the teachers displayed the appropriate MPCK (Moh'd et al., 2018).

Figure 5.6: Teacher Two teaching using an overhead projector



Source: Researcher's work

5.2.3.8 Evaluative skills

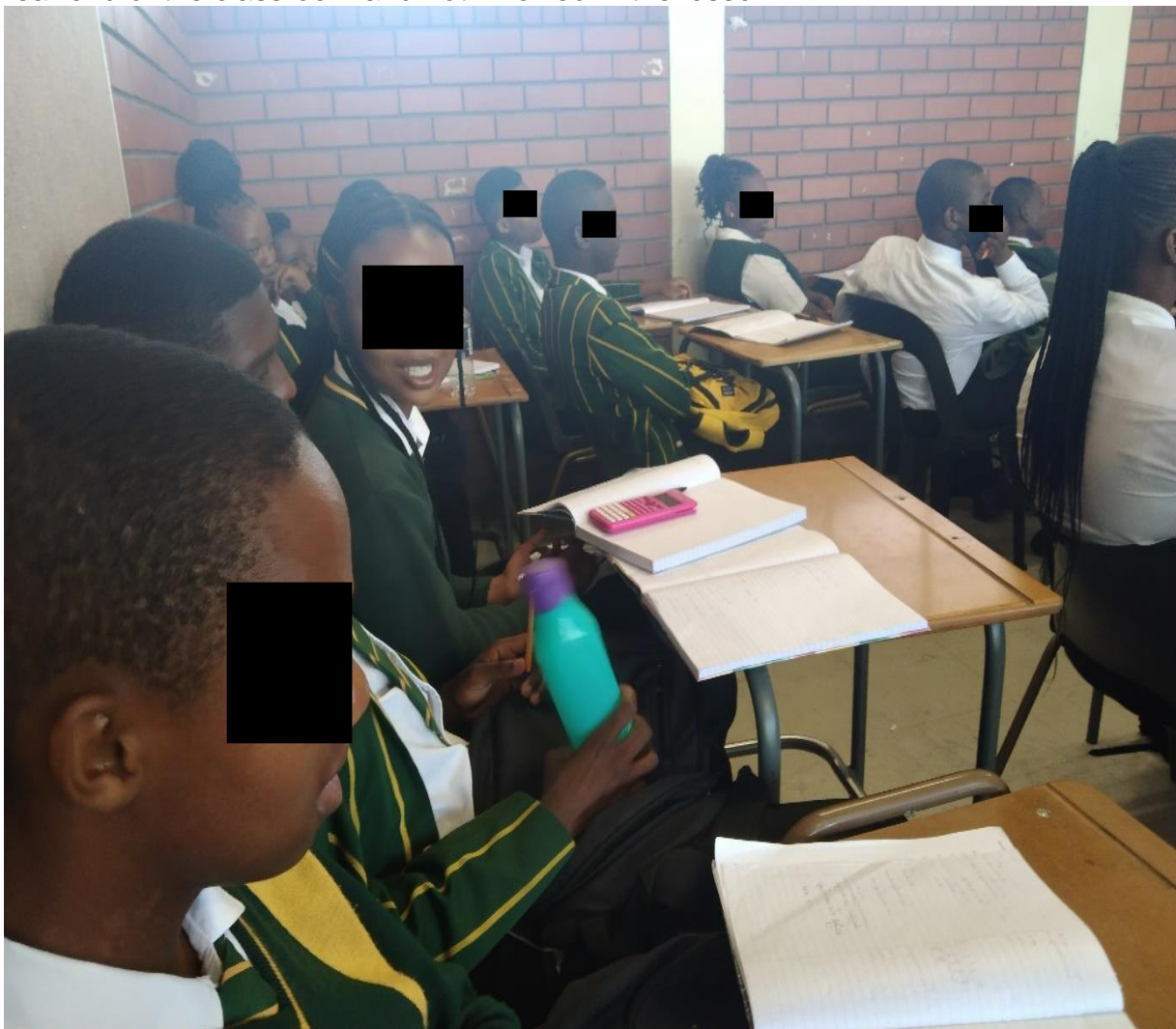
Martin (2022) advanced that teachers' MPCK will inform their choice of learning strategies in the lesson presentation. Martin (2022) scholar further stated that Mathematics teachers should use different assessment strategies, like questioning strategies and knowledge of different assessments, to provoke self-directed learning in learners. However, in the lesson observation, the only form of evaluation both teachers implemented in the classroom was placing the answers on the board for learners to correct their homework. Teacher 3 went further in this and requested individual learners to write the answers on the board while the rest of the class corrected their homework. Teacher 3 monitored the progress of the answers and appropriately corrected the learners.

Most of the learners silently worked on correcting their homework, while the few who sat at the back of the classroom were not working, and therefore there was no self-assessment. Some learners could do self-assessment tasks on the concept learned

during the lesson, as some seated in the front raised their hands to pose inquiry-type questions to their teachers, who then explained the answers in further detail.

The teachers showed evidence of formal and informal assessments recorded on SA-SAMS. DBE's (2010) policy on traditional assessment guides Mathematics teachers with a standardised means of assessing learners' progress in a standard and determining whether learners should be advanced to the following grade. The formal assessment is categorised into school-based assessment (SBA) and internal end-of-year examination. The SBA tasks were used to measure learners' progress in attaining skills and knowledge within a grade and to diagnose learners' abilities to facilitate further learning. The internal end-of-year examination confirmed that Mathematics learners had mastered the skills and knowledge needed to proceed to the next grade.

Figure 5.7: The cohort that consistently performed above 80% were seated at the rear end of the classroom and not involved in the lesson



Source: Researcher's work

5.3 Corroboration and merging of themes from the semi-structured interviews, focus group interviews and classroom observations

Data analysis of a particular study often includes more than one source of data to suit the standards of triangulation and therefore enhance the degree of trust in the validity of the empirical findings. I used triangulation of data sources (Creswell & Creswell, 2018; Johnson & Christensen, 2020; Riazi et al., 2023) which involved data from semi-structured interviews, focus group interviews and observations as well as triangulation of sources (Cohen et al, 2019; Delvin, 2018) which involved getting data from teachers and learners. Analysing various data sources was done to seek convergence or corroboration of evidence (Creswell & Creswell, 2018). More importantly, converging the different data sources aims to agree with the study's conclusion.

The different data sources employed in the study were semi-structured interviews with teachers, focus group interviews with the Grade 12 learners, and classroom observations. Therefore, the data from the focus group interviews and classroom observation corroborated the data earned from the three research questions put forward to the focus group interviews. The data from the focus group interviews and classroom observation corroborated or gave credence to the evidence that emerged from the questions asked to Mathematics teachers teaching Grade 12 Mathematics.

The initial research question that chaperoned the study was *"What are the effects of teachers' MPCK on Grade 12 learners learning Mathematics?"*

Table 5.3: Merging of themes for research question 1

Codes	Categories for semi-structured interviews	Categories for focus group interviews	Categories for classroom observation	Emergent Themes
The effects of teachers' MPCK on Grade 12 learners learning Mathematics	<ul style="list-style-type: none"> • PBL is learner-focused, learner-driven learning of Mathematics • Grade 12 Mathematics content is abstract • Teachers' mastery of content enables teachers to emulate using real-life situations in teaching • Teachers' mastery of content enables teachers to use various learning support strategies to assist learners with mixed learning abilities 	<ul style="list-style-type: none"> • Teacher guides group-centred learning of Mathematics • Teachers are masters of the content • Teachers' mastery of content enables teachers to use various teaching methodologies for inclusive education • Teachers' mastery of content enables the innovative delivery of lessons • Teacher lack of content knowledge leads to teacher-centred textbook delivery of lessons 	<ul style="list-style-type: none"> • Teachers are masters of content delivery • Teachers lack mastery of content conducted teacher-centred lessons • Teachers' lack of mastery of content did not conduct lessons for different learning needs 	Teachers' MPCK positively affects Grade 12 learners' Mathematics learning

Source: Researcher's work

Table 5.3 shows the three instruments' merged themes for the first question. The amalgamation of the various themes resulted in the closing theme. The unique finding (related to the objectives of this study) identified after combining the themes produced from the semi-structured interviews, focus group interviews, and classroom observations focused on creating answers to the research questions can be structured as follows: Teachers' MPCK positively affects Grade 12 learners' Mathematics learning.

The second research question that chaperoned the study was *“How does the PBL instructional approach in teaching and learning Mathematics impact the Grade 12 Mathematics learners' performance?”*

Table 5.4: Merging of themes for research question 2

Codes	Categories for semi-structured interviews	Categories for focus group interviews	Categories for classroom observation	Emerg ed Theme
The PBL approach in teaching and learning Mathematics impacts the Grade 12 Mathematics learners' performance	<ul style="list-style-type: none"> • The PBL teacher facilitates the Mathematics lessons • The PBL approach places learners in the centre of learning activities • The PBL lesson used concretised explanations to explain abstract concepts 	<ul style="list-style-type: none"> • The PBL teacher motivates learners to improve their achievement levels. • Learners become independently responsible for their performance in Mathematics • The PBL teacher facilitates group 	<ul style="list-style-type: none"> • The teacher motivates learners to improve their performance levels • The teacher uses learner-centred instructional strategies for teaching • A teacher used peer tutoring in the Mathematics lesson 	The PBL facilitator uses inclusive educational strategies for teaching and learning to improve Grade 12 Mathematics learners' performance

	<ul style="list-style-type: none"> • The PBL approach caters to the level of the content proportionate to the learners' learning abilities • The PBL lesson requires differentiated learning strategies for successful learning of the content 	<p>interaction for effective learning.</p> <ul style="list-style-type: none"> • The PBL approach places learners in the centre of learning activities • The PBL lesson focuses on collaborative group learning interactive activities. • Group learning allows for peer- tutoring. 		
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Source: Researcher's work

Table 5.4 above displays the consolidated themes of the three instruments in research question 2. The integration of the different themes gave rise to the final theme. The unique finding (related to the objectives of the study) identified after combining the themes identified from the semi-structured interviews and focus group interviews, which were used to answer the research questions, can be structured as follows: The PBL facilitator uses inclusive educational strategies for teaching and learning to improve Grade 12 Mathematics learners' performance. The third question that guided the study was “*What is the teachers' role in teaching Grade 12 Mathematics using PBL?*”

Table 5.5: Merging of themes for research question 3

Codes	Categories for semi-structured interviews	Categories for focus group interviews	Categories for classroom observation	Emerged Theme
<p>The teachers' role in teaching Grade 12 Mathematics using PBL</p>	<ul style="list-style-type: none"> • Teachers' MPCK assists in designing and implementing flexible PBL mathematical lessons • Teachers implement PBL to develop independent and responsible content innovators • Teachers' mastery of content exposes learners to deep and varied concepts • The teacher mediates the learning content by scaffolding the new content in proportion to 	<ul style="list-style-type: none"> • Teachers guide Mathematics learners in a group on knowledge acquisition • Teachers facilitate peer tutoring in learner-group sessions • Teachers must possess content and various instructional skills in transferring content to learners • Teachers' mastery of content exposes learners to deep and varied concepts • Teacher mediates the learning content proportion to learners' 	<ul style="list-style-type: none"> • Teachers' mastery of content exposes learners to deep and varied concepts • Teachers facilitate effective lessons by concretising abstract content • Teacher mediates the learning content proportion to learners' varying educational needs 	<p>The teachers' role as mediators of Grade 12 Mathematics enables content discovery through practical scenarios</p>

	learners' varying educational needs <ul style="list-style-type: none"> • The teacher facilitates effective investigative group work representing reality 	varying educational needs <ul style="list-style-type: none"> • Teachers facilitate effective lessons by concretising abstract content 		
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Source: Researcher's work

Table 5.5 above shows the amalgamated themes for the three instruments in Research Question 3. The fusion of the different themes resulted in the finalised themes. The unique finding (related to the objectives of the study) identified after integrating the themes from the semi-structured interviews and focus group interviews, which provided answers to the research questions, can be engineered as follows: The teachers' role as mediators of Grade 12 Mathematics enables content discovery through practical scenarios.

5.4 Triangulation of data

Triangulation means examining the research issue or phenomenon from more than one perspective (Creswell & Creswell, 2018). Quinlan et al. (2019) reveal how using triangulation, a research question might be explored using a series of interviews or using a series of observations and a focus group. Multiple sources of data are collected with the hope that they will all converge to support a particular hypothesis or theory (Cohen et al, 2019). I used triangulation of different methods such as semi-structured interviews, focus group interviews, and classroom observations, from which recurrent themes evolved that added to the validity of the deductions. I also used triangulation of relevant data sources by conducting interviews with Mathematics teachers and Grade 12 Mathematics learners in selected schools in KZN.

For this study, the measuring tools were created for the data collection process, on the phenomenon under review and were based on the literature review and informed by the objectives of the study.

I triangulated the data sources from the three research instruments to gain information on the study's research questions. After that, themes emerged within each data collection tool and were tabled and deliberated earlier. This was done by recognising and fusing similar and favoured themes from the three data collection instruments.

5.5 Summary of Chapter 5

I retrieved data from three sources and applied thematic data analysis. I was able to organise the new data and themes that emerged from the semi-structured interviews, focus group interviews, and classroom observations. I consolidated the various themes that materialised into unique themes and triangulated them.

The next chapter is on the discussion of findings.

CHAPTER 6: FINDINGS

6.1 Introduction

The preceding chapter presented the primary results of this study. This chapter reports on the study's findings. The chapter presents the discussions, recommendations, and conclusion, which sought to identify the effects of pedagogical content knowledge in learning Grade 12 Mathematics using PBL. In this chapter, I offer an analytical discourse of the qualitative outcomes discussed in Chapter 5 to provide answers to the research questions. Data were collected from semi-structured interviews, focus group interviews, and lesson observations.

This chapter is separated into themed sections. First, the chapter begins with discussing the teachers' MPCK and how it positively affects Grade 12 learners' Mathematics learning. Secondly, this chapter focuses on the PBL facilitator using inclusive educational strategies for teaching and learning to enhance Grade 12 Mathematics learners' achievement levels. Last, the chapter on the teachers' role as mediators of Grade 12 Mathematics enables content discovery through practical scenarios. These three themes have been designed around the research questions mediating the empirical study and have consistently been utilised for constructing Chapters 2, 3, 4, and 5.

Also included in this segment are essential proposals related to the effects of pedagogical content knowledge in learning Grade 12 Mathematics using PBL. The results of the empirical review are explicated in this section. I further configure the study's limitations and how they could be discussed in future research undertakings.

6.2 Findings of the study

The key findings of my research are based on the overall aims of the study, that is, the effects of pedagogical content knowledge in learning Grade 12 Mathematics using PBL.

The study's research questions below guided the empirical study:

1. What are the effects of teachers' MPCK on Grade 12 learners learning Mathematics?
2. How does the PBL instructional approach in teaching and learning Mathematics impact the Grade 12 Mathematics learners' performance?

3. What is the teachers' role in teaching Grade 12 Mathematics using PBL?

The findings are based on the study's three research questions considered below.

6.2.1 Findings for Research Question 1

The first probing question was: *What are the effects of teachers' MPCK on Grade 12 learners learning Mathematics?* Responding to the first research question, the findings advocate that the teachers' MPCK positively affects Grade 12 learners' Mathematics learning. These include the reality that PBL is learner-focused and learner-driven in Mathematics, Grade 12 Mathematics content is abstract, and teachers' mastery of content enables them to emulate real-life situations in teaching using various learning support strategies which assist learners with mixed learning abilities.

Findings for the first research question suggest that teachers highly skilled in MPCK can use various teaching methodologies in their Grade 12 Mathematics lessons to cater to the appropriate learning needs of different learners. Therefore, the teacher can design Mathematics lessons to successfully overcome learners' age-related difficulties in learning Mathematics (Shulman, 1986). To achieve this, teachers need additional information, like the curriculum's scope, educational objectives, and adequate teaching resources or MPCK (Moh'd et al., 2018). Furthermore, the participants contend that the mastery of content and delivery (MPCK) enables them to produce differentiated learning strategies for the various Grade 12 learners in the classroom. The view such as "*In a nutshell, the pedagogical strategies of the teacher must be varied to accommodate every learner and every possibility because not every learner is good at learning Mathematics.*" This finding is supported by Sari (2018), that the teacher's mastery of the content enables learners in a social constructivist environment to effectively learn the content. Therefore, teachers with a powerful command of MPCK can implement various methodologies to practically represent the theoretical concepts, appealing to Grade 12 Mathematics learners. When the content appeals to the cognitive-developmental levels of each learner, then only positive learning of Mathematics can occur.

Conversely, the findings also demonstrated that teachers with low levels of MPCK negatively affected the lesson presentation. This is evident in Grade 12 Mathematics teachers with limited MPCK used teacher-centred activities in teaching and learning. The view is, "*I think if you don't know the content properly, the learners are not going*

to understand... If you don't know the answer, it becomes a problem. Sometimes you get teachers that don't know the content, and it will negatively affect their learning of the content..." This finding is in contrast with the principles of PBL that social constructivist learners need to mutually engage with each other to collaboratively build new knowledge on existing knowledge (Mpho, 2018). The learners mentioned that they passively received instructions from the teacher using the textbook as the primary source of information.

Therefore, teachers competent in MPCK have the skills and knowledge to comprehend Grade 12 learners' age-related learning challenges and even pre-empt their learning barriers. With this knowledge, Mathematics teachers can present the Grade 12 learning content that would appeal to the collaborative learning needs of their learners. These findings relate to the studies conducted by Baier et al. (2019) that the teachers' understanding of teaching Grade 12 Mathematics is crucial to the learning challenges the target audience may experience.

Furthermore, the findings suggest that teachers rich in MPCK used PBL in Mathematics lessons, focusing on learners actively learning, and such learners drive their learning. The view such as "*There is this chapter... Euclidean Geometry... How do I do it via problem-solving? I want to give them a lot of different triangles, for instance, when they have to measure the angles themselves and arrive at the answer. My responsibility is to control the class and give them guidance in the beginning, and I don't give answers to learners... they know. So, they must figure it out themselves.*" This complements Mulyanto et al. (2018) view that, learner-centred learning develops learners' accountability in their Mathematics learning. Therefore, learners gain confidence in their group interaction and become responsible for learning.

Similarly, the teachers mentioned that learners are motivated to learn; on their own, they approach the chalkboard to write the answers down. The teacher now takes on the role of the facilitator and mediator of learning in Grade 12 and employs collaborative group work in a socially orchestrated learning environment. Similarly, in the social constructivist design of teaching and learning Mathematics, learners create meaning in knowledge through group interaction (Amineh & Asl, 2015).

The study's findings showed that Grade 12 Mathematics content is abstract and is a problem for learners learning Mathematics. Most participants agreed that Grade 12

content is too theoretical for learners to comprehend; they need Mathematics problem sums to represent real-life scenarios to facilitate their learning. The view, such as "*The Grade 12 curriculum is more abstract... So, whenever you teach them, you have to give them a lot of manipulatives, that is, worksheets. I will now use real-life situations and explain abstract concepts to them, for example, in Trigonometry,*" Similarly, Idris and Madugu (2020) agreed that Mathematics content is too abstract to arouse cognitive-developmental level for some learners; therefore, subject teachers should facilitate Mathematics learning through a more concretised approach using realistic scenarios. Therefore, teachers need to concretise the abstract content that learners may find challenging by scaffolding the content. This is complementary to the social constructivist Mathematics classroom in that constructive teaching and learning is through building new knowledge on learners existing knowledge through cooperative real-life learning opportunities (Akpan et al., 2020). Teachers need to facilitate Mathematics learning through a more concretised approach. Mathematics teachers should be well-versed in differentiated teaching approaches to cater to learners with heterogeneous learning abilities in the classroom.

The results of the empirical study suggest that Grade 12 learners doing Mathematics are individuals at various stages of knowing the content and that PBL is the preferred method for employing differentiated learning strategies to match the learning needs of learners. Most respondents agreed that different learning support structures are needed in their Mathematics classroom for learners with various learning abilities. The view, such as "*A teacher must have varied methodologies for heterogeneous classes with different performance levels*", supports this finding. Similarly, scholars Turnuklu and Yesildere (2007) postulate that teachers rich in MPCK can orchestrate flexible lessons using differentiated instructional strategies that appeal to the diverse learning needs of learners. This is related to Kılıç (2011) view that the teachers' MPCK is knowledge-based and assists in implementing a mix-assortment of learning strategies to accommodate learners with learning barriers to Mathematics in a social environment. This corroborates the study of Kyriacou (2009) that PBL is a social constructivist learning approach that caters to different learners with different levels of information.

6.2.2 Findings for Research Question 2

The second empirical research question was: *How does the PBL instructional approach in teaching and learning Mathematics impact the Grade 12 Mathematics learners' performance?* The response to this research question, the findings advocate that the PBL facilitator uses inclusive educational strategies for teaching and learning to improve Grade 12 Mathematics learners' performance. This finding includes concretising explanations to explain abstract concepts appropriate to the learners' learning abilities. Furthermore, the findings suggest that Mathematics learning is centred on learners collaboratively engaged in the learning activities.

The findings advocate that Mathematics concepts are abstract, and therefore the teacher should mediate the theory into concrete blocks of information relatable to their immediate environment. The view is *"I understand that they find their work very abstract... So, for them, problem-solving is where you try to break it down and make it simpler. E.g., For measurements, we take them to the school grounds with measuring tape and get them to measure... Sometimes we talk about a cylinder. They don't know what it is. You have to physically show them around, like the bins, or ask them to take out things like the Pritt. So, it helps them visualise what it is,"* supports this finding. Teachers used practical examples of real-life situations; worksheets were given to learners to measure objects and record their findings. This assisted learners in mastering the content. This aligns with the constructivist learning theory that teaching media must be incorporated into the lesson presentation to assist learners in receiving the abstract content in a visual form (Amineh & Asl, 2015).

Findings for research question 2 showed that teachers were responsible for planning and producing lessons proportionate to the learning needs of Grade 12 learners. To achieve this, both participants believed that knowing learners' difficulties before the lesson presentation enabled them to deliver the lesson according to learners' learning needs. The view such as *"I do not teach all classes the same way. Those learners were finding Mathematics challenging in the Commerce class... I have to explain the content much slower than in the Physics class, which has better-performing learners. Unfortunately, many don't understand in the Commerce class, so you have to take extra time to re-explain differently. I will now use real-life situations and explain abstract concepts to them, for example, in Trigonometry, so it's very different in the*

two classes," supports this finding. Teachers must know learners' misconceptions and preconceptions of a related topic to facilitate learning. These findings corroborate Courtney et al. (2015) view on the theory of social constructivism that teachers should produce learning activities to meet the learning needs of learners to overcome their academic difficulties in Mathematics.

Similarly, Amalia et al. (2017) contend that instructional strategies used by teachers for Mathematics education should be more practical for effective learning. In other words, Mathematics teachers are directly responsible for taking learners' knowledge from the abstract to the concrete through manipulation because they should be skilled in the Mathematics content matter, its pedagogy, knowledge of learners, and the curriculum. According to Nurlaily et al. (2019), PBL is the preferred instructional learning strategy for Grade 12 Mathematics teaching and learning.

The findings suggest that the PBL facilitator should be responsible for mediating the Grade 12 Mathematics content to groups of learners aimed at their learning needs for successful content acquisition. Most participants contend that collaboratively engaging in Mathematics problem sums with their peers led to a much higher comprehension because they learnt better from their peers. The view was, *"Previously, I talked first; then I gave them the formula, then the problem questions. I realised it didn't work because most don't understand it. So yesterday, I got them (learners) to discover their formulas. Today I'm going to guide them (learners) on how to find the number of times. My teaching strategy changed. Now, I give them an investigation. They go and measure whatever and compare and arrive at an acquired decision. With learners having barriers to learning, I motivate the lower, middle, and higher-pace learners. So, what I present should motivate those who are slow, plus in a way that I'm careful not to make those who are good, not bored,"* supports this finding. This finding coincides with Klang et al. (2021) views that learners, unique in their experiences, collaboratively learn by raising questions when exposed to new concepts and develop reasoning skills through content transfer by and to their peers. Nonetheless, the above issue contrasted with what was evident in the observation schedule; learners were in cinema-style seating and listened to the Mathematics teacher, who continuously explained the content. Widyatiningtyas et al. (2015) opine that the teacher-centred approach is not in keeping with the practices of PBL and that teachers are not the only ones who can explain the content.

The finding also suggests collaborative group work benefits Mathematics learners in smaller groups. Most respondents felt small groups of learners allowed them to be fully involved in the learning while the Mathematics teacher could move around the classroom and ensure learners engaged with learning. The view is such that *"Grade 12 learners can work in groups of two each, just the person sitting next to them. Furthermore, if I instruct them on a specific question, they will try and answer it together. Learners work in groups. They are busy learning from each other. They learn on their own in groups. If they struggle, they know I can get help from my teacher. Some learners will work better with their peers, while others will play around. ..."* support this finding. This aligns with a social constructivist theory; Mathematics learners can pose thought-provoking questions to their peer-group members and teachers, thereby collectively arriving at the correct answer (Charles et al., 2019; Vintage, 2018). However, this finding was not evident in the classroom observation. Learners independently worked while referring to the problem sums projected on the OHP. Learners were passive absorbers in their own space, which according to Aidoo et al. (2016), results in poor learning of Grade 12 Mathematics.

6.2.3 Findings for Research Question 3

The last research question was: *What is the teachers' role in teaching Grade 12 Mathematics using PBL?* The response to this research question, the findings recommend that the teachers' role as mediators of Grade 12 Mathematics enables content discovery by scaffolding content. The findings also suggest that the teachers' MPCK assists in designing and implementing flexible PBL mathematical lessons through practical investigative group work.

Findings for the third research question suggest that teachers should be able to mediate the Grade 12 Mathematics content to learners by moving from what content they know to the new content. This transforms the traditional role of the teacher into one of now posing a series of questions, using demonstration and feedback to help learners grasp the new content. Most participants believed scaffolding new content based on the old content in groups of learners would improve Mathematics learning. The view such as, *"When I teach, I like to bring back the basics, for example, "functions". In Grade 11 (talking about the weaker half of the learners), three-quarters forgot the Grade 10 work. So, you have to reteach the basics (Grade 10 work) and then build up (new knowledge), then it assist them. You are just adding to what they*

have learned. The learner understands why you're doing a certain thing or why a certain thing is done in this way; it's easier for the learner to accept it and try it out themselves. But when the learners learn to do the graph without knowing why they're doing it, they have questions because they're just doing it by not understanding..." supports this view. This view coincides with the study by Bay et al. (2012) that Mathematics teachers learning content representing authentic scenarios to smaller groups of learners would lead to effective learning. In a social constructivist learning environment, the Mathematics teacher's responsibility is to mediate learning new mathematical knowledge through scaffolding (Sarker et al., 2019; Vintage, 2018). However, the observation tool showed that the Mathematics lessons were teacher-centred, lacked mediation and had no thought-provoking questions posed to learners. The teachers verbally conveyed the learning content to learners without having groups or varying their presentation to meet the learning abilities of different learners or with very little feedback. Therefore, very little learning of content could have occurred because, according to Kokotsaki et al. (2016), learning Mathematics in a PBL environment can only be successful through mediating the content in groups by moving from the known events to the unknown areas of knowledge.

During scaffolding in a social constructivist classroom, learners operate in groups. Mathematics teaching employs PBL as an instructional means in established groups of 4 to 5 learners each, often engaging with the content and producing a shared answer. These learning communities generate content by researching, reviewing, critiquing, and communicating mathematically to solve problems (Koh & Olive, 2019). The role of the Mathematics mediator is scaffolding the new content proportional to learners' varying educational needs in Grade 12. To achieve this, the teacher must ascertain the required learning opportunities to assist Mathematics learners from their prior knowledge to new banks of knowledge (Amineh & Asl, 2015).

The findings recommend that lessons presented in the Mathematics classroom must be flexible. Flexible lessons cater to Mathematics classroom learners with various learning abilities. Most of the respondents suggest that their classes had learners at various stages of learning Mathematics, and PBL would allow them to utilise differentiated learning strategies to match the learning needs of their learners. The view such as *"I need to change the example. So, this one (mathematical sum) will not work. Or else, there is another easier way that I can explain the problem sum to*

learners. So, in the next lesson, you, I don't use the example or teaching method that challenged the learners. You will use different methods in one class, but in the other class, you can't..." supports this finding. Furthermore, Turnuklu and Yesildere (2007) opine that teachers with a strong MPCK can design and implement lessons catering to learners with various learning challenges to grasp the content. This finding is similar to Akkutay and Cakmak's (2016) study that Mathematics teachers mediate learning by representing actual life occurrences using various teaching methods to meet the learning demands of all learners.

Mathematics teachers can execute lessons that are inclusive of learning activities that are cognitively challenging and match the varying learning needs of each learner in the Mathematics classroom. Since individual learners vary in thought processes, Mathematics lessons should be flexible and cater to different answers to Mathematics problems. In this way, learners socially interact with the learning content and learn from their peers. Likewise, in a social constructivist environment, collaborative learning in a Grade 12 Mathematics classroom should cater to learners' activities with various possible answers, each with strengths and weaknesses discussed (Bay et al. 2012; Vintage, 2018). Given this opportunity, peers would lead the group discussion more effectively than most teachers because learners learn better from their peers and achieve positive knowledge construction (Mavuso, 2014).

The findings suggest that learning Grade 12 Mathematics should focus more on practical activities and learners' investigative group work. When Mathematics lessons are practical, learners actively learn the content. Most respondents believe that Mathematics consists of theories, making it difficult for Grade 12 learners to understand; therefore, they need practical-investigative scenarios to make learning much easier. The view such as, "*Sometimes we talk about a cylinder. They don't know what it is. You have to physically show them around, like the bins, or ask them to take out things like the Pritt. So, it helps them visualise what it is...*" supports this finding. Rahman et al. (2021) postulate that exposing learners to conceptual knowledge through practical tasks creates opportunities for learners to engage actively with the content. These engagements create opportunities for learners to pose deep-seated questions on the learning content and allow a transfer of thoughts between peers in the group under the guidance of skilled and content-rich Mathematics teachers (Klang

et al., 2021). Similarly, in a socially constructive classroom, Liu and Matthews (2005) opine that these group learning engagements lead to new knowledge construction.

Contrary to this, evidence from the observation report showed that Mathematics lessons were not practical and not investigative. Most respondents suggest that Mathematics lessons focused on the textbook and work displayed on the OHP. This contrasts with Lovitt and Clarke's (2011) study suggests that Mathematics lessons should involve learners actively in teaching and learning-centred activities for adequate learning. Furthermore, Badie (2016) and Vintage, (2018). stated that Mathematics teachers should simulate real-life problems in a social constructivist environment so that learners could practically solve these Mathematics problem sums.

6.3 Summary of Chapter 6

This chapter discussed the findings of the study. The discussion led to the answering of the three research questions of this study. The next chapter will present the conclusions and recommendations of the empirical investigation.

CHAPTER 7: CONTRIBUTIONS FROM THE STUDY, CONCLUSION AND RECOMMENDATIONS

7.1 Introduction

The previous chapter discussed the findings of the study. This chapter focuses on contributions from the study, conclusion, and recommendations. I have also created a framework and recommendations are made concerning the findings.

I begin this chapter by presenting an outline of the entire thesis, describing each chapter before moving on to the evaluation of the study. In this chapter, I account for the reasons for the study and evaluate the research problem, research questions, research objectives and research design. Thereafter I, describe the rationale of my study regarding how it makes an authentic addition to the current body of content knowledge on the effects of pedagogical content knowledge in learning Grade 12 Mathematics using problem-based learning, particularly in policy development and curriculum review.

In this segment, I investigate the contributions and achievements of my empirical study by breaking this down into methodological contribution, contribution to the meaning of the research design and its contribution to the universal meaning of research design. Further, the chapter highlights the conceptual contribution, the contextual contribution, and the contribution to the field of study.

The study contributes to the field of Mathematics by introducing a Framework for teachers' MPCK and how it affects learners learning Grade 12 Mathematics using PBL. The framework, in Figure 7.1 is based on factors that affect the teachers' MPCK and how it affects the learning of Mathematics, namely, DBE, SMT, and the Instructional Strategy: PBL.

After the framework for teachers' MPCK and how it affects learners learning Grade 12 Mathematics using PBL, the chapter makes recommendations for the study and recommendations for future research, followed by the study's limitations. The chapter closes with the conclusion of the study.

7.2 Overview of the study chapters

I developed the thesis based on the following framework:

Chapter 1 contained the introduction and background of the problem, a statement of the problem, the purpose of the study, the significance of the study, the research question with secondary questions, research design, theoretical framework, delimitations, the definition of terms and chapter summary.

Chapter 2 discussed the theoretical framework that was employed to offer a perspective on the research study. This empirical review utilised the social constructivist design.

Chapter 3 reviewed the written literature regarded as applicable and important to this study. The appraisal of the literature constituted a theoretical framework for my study and by doing just that, this chapter met the three objectives described in the study, which were:

1. To examine the effects of teachers' MPCK on Grade 12 learners learning Mathematics.
2. To determine the impact of the PBL approach in teaching and learning Mathematics on Grade 12 learners' Mathematics performance.
3. To examine the teachers' role in teaching Grade 12 Mathematics using PBL.

Chapter 4 described the design and methodology of this study. The chapter displayed how the empirical investigation was conducted. This chapter also covers research designs and methodologies, paradigm, instrumentation, validity and reliability, techniques to ensure trustworthiness, the collection of data and its processing.

Chapter 5 presents the empirical findings. This chapter encapsulated the critiquing of the data and its interpretation. It is the most exhaustive research study and contains a built-in literature analysis to substantiate the study's thesis. It covered the analysis of focus group interview questions, teachers' interview questions, and the classroom observations used to gather data for this empirical study.

Chapter 6 describes the findings of the study. The findings of the study answered the study's three probing questions, which were:

1. What are the effects of teachers' MPCK on Grade 12 learners learning Mathematics?
2. How does the PBL instructional approach in teaching and learning Mathematics impact the Grade 12 Mathematics learners' performance?
3. What is the teachers' role in teaching Grade 12 Mathematics using PBL?

Chapter 7 presents the conclusion and recommendations of this study, based on the empirical review's results. In this chapter, I also make proposals for additional investigation and discuss the constraints of the study. In the process of constructing the various chapters in this investigation, a research design guided me as seen in segment 7.2. below.

7.3 Research plan

For any practised-based research to be successful, it is dependent on a well-formulated and developed research plan (Devlin, 2018; Tahan, 2022). Research would be a waste of time and effort in the absence of a well-structured and carefully thought-out plan because it provides an opportunity for the researcher to correct mistakes and identify possible exclusions in the study (Creswell & Creswell, 2018). Riol and Thuillier (2015) postulate that for research to be more effective and worthy, traditional project management techniques may be adapted for research projects. Riol and Thuillier (2015) add that the utilisation of project management skills in qualitative research studies increases the effectiveness of the empirical study and heightens its success rate. Tahan (2022) also agrees that the success of qualitative research is dependent on the employ of project management methods because of its well-defined guidelines.

Most projects, whether similar or different in nature, share a common aspect, they all undergo the same cycle, known as the project life cycle (Mandona & Mundia, 2020). The life cycle serves as a guide to managing the research project to its logical conclusion within a time frame. The project life cycle consists of several distinct phases the project undergoes, from conception and initiation to completion (Müller & Klein, 2019).

These phases are associated with each other and generally follow a sequential pattern or overlap each other and are all dependent on one another (Müller & Klein, 2019). The phases in the project management cycle provide a life cycle of events, often

demonstrating the start and the finish of the project. The activities that will be conducted during the project will vary, depending on the project-specific requirements.

The Project Management Body of Knowledge (PMBOK®) maintains that the name and number of project phases are determined by the institution conducting the project, therefore I chose to use a typical project life cycle as suggested by Mandona and Mundia's (2020) study because the project cycle is simple and easy to implement for this study. The five-phase life is described below.

- 1) Conceptualisation stage:** The exploration of a working title and generated the research aim, objectives, and research questions.
- 2) Design stage:** The review of relevant literature and analyses of the research design and methodology.
- 3) Implementation stage:** Administration of the research tools and data analysis.
- 4) Monitoring stage:** Ensure that the investigation's objectives are met by estimating progress frequently to recognise discrepancies from the research plan so that remedial measures can be undertaken.
- 5) Completion and handover stage:** Findings from data, conclusions, recommendations, and implications of the research.

At the initiation point of the study or the conceptualisation stage, it was necessary to establish the needs of the study and the viability of the study. To do this, I used the prescribed research proposal form from the UFS. In this form, I completed the information that was necessary for the:

1. Introduction
2. Research interest, focus or problem
3. Research questions
4. Research design and methodology
5. Value of the proposed research
6. List of sources

I completed the prescribed research proposal form with detailed information, explaining how I would achieve a successful study and in the implementation phase, I used the research data tools to achieve the desired outcomes as described in the design and project management plan. In the completion and handover stage, I was confident that the research study met the criteria as set out in the design plan and that

the research study would meet the study’s aims and objectives. Once I confirmed the research findings and conclusions, recommendations, and implications, I then submitted the proposal to my supervisor for his input.

7.4 Evaluation of the research

The motivation behind the evaluation of the research in a qualitative study is through, interviews, focus groups, and classroom observations; data gained from these research instruments need to be investigated, compared, interpreted, and explained. In this section, I appraise the problem of the study, the research questions, and the objectives, together with the design of the research. Then, I completed the justification of the study by explaining its originality in terms of its contribution to the existing body of knowledge on the effects of pedagogical content knowledge in learning Grade 12 Mathematics using problem-based learning Table 7.1 outlines the study’s research objective against the product.

Table 7.1: Derivations from research objectives

OBJECTIVE	OUTCOME
1. To examine the effects of teachers' MPCK on Grade 12 learners learning Mathematics.	Teachers' MPCK positively affects Grade 12 learners' Mathematics learning.
2. To determine the impact of the PBL approach in teaching and learning Mathematics on Grade 12 learners' Mathematics performance.	The PBL facilitator uses inclusive educational strategies for teaching and learning to improve Grade 12 Mathematics learners' performance.
3. To examine the teachers' role in teaching Grade 12 Mathematics using PBL.	The teachers' role as mediators of Grade 12 Mathematics enables content discovery through practical scenarios.

Source: Prepared by the investigator

From Table 7.1 above, the following additional points were derived from the research:

- I. Mathematics teachers require more effective support from their Departmental heads (DHs) to render quality Mathematics teaching and learning using PBL.
- II. More stringent policies need to be developed by DBE on the use of PBL in the teaching and learning of Grade 12 Mathematics.

- III. DBE needs to review the scope of the Grade 12 CAPS document so that PBL, as the preferred instructional strategy, can be adopted in teaching and learning Mathematics.

Other implications derived from the objectives include the following:

The problem with Grade 12 learners' performance in Mathematics is very low, especially for 2019, 2020, and 2021, which may be attributed to learners' inability to analyse and evaluate problem-based questions (DBE, 2021). The cause of this originates from learning Mathematics from teachers with insufficient MPCK (Anney, & Hume, 2014). Similarly, learners' poor performance in Grade 12 Mathematics primarily emanates from the teachers' modest content knowledge and shallow pedagogical content. Firstly, there needs to be an understanding as to why Grade 12 learners are taught by unqualified Mathematics teachers which leads them to be unable to comprehend problem-based questions at the Grade 12 level. Is there no attractive incentive to draw student teachers to study Mathematics at the tertiary level? Or has DBE failed to provide support to Mathematics teachers in terms of academic and curriculum development? Maybe the current Mathematics CAPS policy and other education policies are not "fit for purpose?" Schirmer and Visser (2023) reported that the South African education system is dysfunctional, and DBE failed to meaningfully transform the current state of education, more especially for the disadvantaged majority in the country. Table 7.2 has recommendations for improvement of teachers' MPCK on Grade 12 learners learning Mathematics in South Africa.

Table 7.2: Recommendations for the improvement of teachers' MPCK on Grade 12 learners learning Mathematics in South Africa

CATEGORY	RECOMMENDATIONS
Occupational development, Education and training	<ul style="list-style-type: none"> • PBL, as the preferred instructional approach to teaching and learning Grade 12 Mathematics, should be encouraged among Mathematics teachers by attending workshops and seminars hosted by DBE. • Tertiary institutions should promote appropriate instructional approaches for the delivery of the curriculum, such as critical thinking, MPCK, classroom discipline, group work engagements and the use of various teaching strategies to cater for individual learners' learning needs. • Create incentives and opportunities for Grade 12 Mathematics teachers to enhance their qualifications in teaching and learning Mathematics.
DBE policies	<ul style="list-style-type: none"> • The Grade 12 Mathematics CAPS policy needs to be revised and streamlined for a more subject-specific curriculum. • DBE needs to sharpen its policy on the teaching and learning of Grade 12 Mathematics, using PBL as the preferred mode of instruction. • Curriculum planners may want to include PBL as a more popular teaching strategy in the publication of new textbooks.
Research and development	<ul style="list-style-type: none"> • Tertiary institutions and other related organisations should encourage further research on the impact of MPCK in learning Grade 12 Mathematics using PBL and make them known to policymakers so that effective policy development would address the problem of Grade 12 Mathematics learners unable to think creatively and solve problems.

Source: Prepared by the researcher

7.5 Contributions and Achievements of this Study

Shaheen (2021) describes the originality of the study as the result of a research study that adds new knowledge to the current body of knowledge. To achieve this, I made it a point to demonstrate the study's functionality in three broad sections, namely, the contribution to knowledge, the relevance to the field/practice and the usefulness to the intended target group. In this study, I have shown how the research study has contributed to the body of current education using the following sub-headings: (i) Methodologically; (ii) Contextually and (iii) Discipline.

7.5.1 Methodological contribution

Methodological contributions are significant because they enable researchers to provide answers to new questions related to how individuals, groups of people, and

organisations relate to each other and to revisit current questions with more rigour (Creswell & Creswell, 2018).

First, I designed the questions for the focus group interviews for the Grade 12 learners and individual teacher interviews for the Mathematics teachers in consultation with my supervisor. I then had a peer who reviewed the questions and after robust discussions on the effectiveness of the draft questions, it was then adopted. Thereafter, I piloted the interview questions with Grade 12 Mathematics teachers and a focus group of learners with a school similar in characteristics to the sample schools selected for the study. I regarded this form of testing as very important because it allowed me to ascertain the content validity of responses to the research questions, improve questions, format, and review the instructions to the participants and assess how long the study will take (Cresswell & Creswell, 2018). Taking the data from here, I consulted with my supervisor and after much deliberation, we unanimously agreed on the questions for the teachers, which were Appendix 8: Teachers and, Appendix 7 Focus Group (Learners) questions.

Second, in the design of the observation tool, I used Collective Agreement 2 of 2020, and Stearn, et. al.'s (2012) tool. The tool was also piloted in the same school that was used to pilot the research questions for the teachers and focus groups. The data was recorded and then it was discussed with my supervisor. Thereafter we both mutually agreed on the finalised version of the observation tool. After these processes, I was confident and ready to conduct the research in the two sample schools.

Third, I utilised three different data collection methods, to enhance the study's trustworthiness and internal validity. Accordingly, this study used a constructive interpretivism paradigm and analysed the ontological, epistemological, methodological, and axiological positions of social constructivism in a policy evaluation-based study.

Fourth, the proposed benefaction to a research investigation may be in the methodology, in which case the methodological significance must be discussed. This study used three different methods to collect data, that is; semi-structured interviews, focus group interviews and classroom observation. Therefore, this study has a substantive methodological contribution made by my research undertaking.

Fifth, this study used a qualitative approach. According to Creswell and Creswell (2018), a qualitative study is often recognised as an inquiry process formulated on a holistic and complex appreciation of a social phenomenon. Data is often gained in its natural environment and generally inductively analysed by the researcher to make meaning of it. Data was collected using three data collection modes, which included teacher interviews, focus group interviews and classroom observations. This allowed for the triangulation of data; hence, internal validity was achieved for the study (Yin, 2018).

■ **Qualitative approach design:** For the study, I employed an inductive, exploratory qualitative case study approach. An explorative case study approach allows me to perform an in-depth empirical study and maintain my focus on a single occurrence, which is the impact of pedagogical content knowledge in learning Grade 12 Mathematics using problem-based learning and making sense out of it (Creswell, 2016). More importantly, the data earned from the research study was critiqued using thematic data analysis; codes and themes emerged based on the data collected from the study's research questions and the observation tool (Braun & Clarke, 2006).

■ **Triangulation of data:** I analysed each mode of data separately, and then I triangulated the multi-method data sources so that I could generate convergent themes and codes (Creswell & Creswell, 2018). Also, the triangulation of data avoided any bias from me or the use of a single methodology. I was able to cross-validate the data that had arisen from the three different research instruments. Therefore, the degree of trust in the validity of the empirical findings was heightened.

■ **Contribution to the specific meaning of research design**

Various types of investigative designs suit different types of research studies (Aliyu & Adamu, 2015). This study used a case study research approach under a specific research design.

The particular definition of a research design relates to the category of empirical study (Yin, 2018). The other types of research designs are grounded in a theory study (Fouché et al., 2022), ethnography study (Creswell & Creswell, 2018), experimental; narrative inquiry (Yin, 2018) and phenomenology (Heuristic inquiry, ethnomethodology, symbolic interaction, semiotics, hermeneutics, systems theory, complexity theory, pragmatism, and generic qualitative inquiry) (Fouché et al., 2022).

These various types of designs command how data will be collected and the amount of control the researcher will have over the research project (Fouché et al., 2022).

■ **Contribution to the common meaning of research design**

This study also used a general research design. The liberal meaning of research design relates to the process by which data will be collated and analysed. A research design should highlight the reason for the empirical review and show the road map that will provide answers to the research questions with consistency. According to Khanday and Khanam (2023), a research design is a conceptual blueprint found in a research project that is being carried out. Other scholars like Creswell (2016), Creswell & Creswell (2018), Fouché et al. (2022) and Yin (2018) have defined the research design which can be summarised below:

A research design refers to an approach (Aliyu & Adamu, 2015), or a blueprint (Khanday and Khanam, 2023) that an investigator employs to initiate a research study, by identifying a problem that needs empirical attention (Fouché et al., 2022) and goes through the phases of planning, implementation and analysis (Creswell, 2016) to the point of data exploration, analysis, interpretation and lastly reporting (Creswell & Creswell, 2018).

Therefore, a generic research design is a strategic pathway that researchers journey on to reach answers to research questions in their research study as valid, objective, and accurate as possible (Creswell & Creswell, 2018), to produce credible results (Fouché et al., 2022).

7.5.2 Conceptual contribution

The term conceptualisation is an analytical tool used to investigate important concepts in a research study and how investigators infuse their research with current theory and research (Fouché et al., 2022). Conceptualisation is used to make theoretical distinctions and organise ideas that belong to important theories (Creswell & Creswell, 2018). In other words, conceptualisation can also refer to the underlying theoretical framework that guides and directs the research.

To develop a conceptual framework in this study, I used the theoretical framework as the source of origin and therefore the conceptual framework could be related to the specific research problem. Accordingly, the conceptual framework constituted a

theoretical framework of beliefs, principles and regulations that carry together the thoughts that encompass a wide theory.

A theoretical framework details the theoretical support of a study based on available research studies, while a conceptual framework in research allows the research investigator to arrive at their deductions, plotting out the variables the researcher may engage within the study and the flexibility amongst these variables. In other words, a conceptual framework is a less well-developed description of experiences in which important theories are associated.

7.5.3 Contextual contribution

As a researcher, I had to demonstrate the usefulness of the study, for its intended audience. Accordingly, the study has displayed its usefulness in developing an awareness of the effects of educational content knowledge in learning Mathematics using problem-based learning at the Grade 12 level.

Concerning society, my study has revealed the real-life experiences of Mathematics teachers and learners (at Grade 12 level) and their engagements in learning and teaching the subject. It further illustrates with evidence the powerful influence the teachers' MPCK (or lack of it) has on the learners learning Mathematics. This is a contribution not only to DBE and its stakeholders but also to society and non-profit organisations (NPOs) who may be affected by this phenomenon.

My research into the effects of pedagogical subject-content knowledge in learning Grade 12 Mathematics using problem-based learning; using the research design of a case study created an extensive, multi-dimensional understanding of this phenomenon in its authentic surroundings encapsulated by a specific date and time (Creswell 2016). The findings of this case study are sufficient to warrant further investigations into this phenomenon and possibly initiate a pilot programme in the Amajuba district, to enhance the assessment levels of Mathematics learners (in Grade 12) in their NSC.

7.5.4 Contribution of the Study

A research study investigates a problem aimed at solving it, to make the world where it exists, a better place or improve the quality of people's lives. The study has reviewed several different sources of literature on the effects of pedagogical content knowledge

in learning Grade 12 Mathematics using problem-based learning, which hopefully contributed to a detailed understanding and clarification of these variables and how when in concert, affect Mathematics teaching and learning. From the findings, I identified the determinants of quality MPCK in Mathematics teachers and how this impacts the learning of Mathematics in secondary schools.

The study has offered SMTs in the Amajuba District evidence findings in areas where Grade 12 Mathematics teachers and learners may need specific support in developing teachers' MPCK to improve lesson facilitation using various inclusive instructional strategies to cater for learners with various learning abilities.

The results of the study show that the effectiveness of teaching and learning Grade 12 Mathematics originates from the classroom teacher and their levels of MPCK impact their crafty ability to implement PBL, as an instructional strategy in the Mathematics classroom. Furthermore, the observation tool used in the research study may be used as a diagnostic tool to inform relevant decision-making in developing and implementing appropriate interventions to improve Mathematics learning in Grade 12.

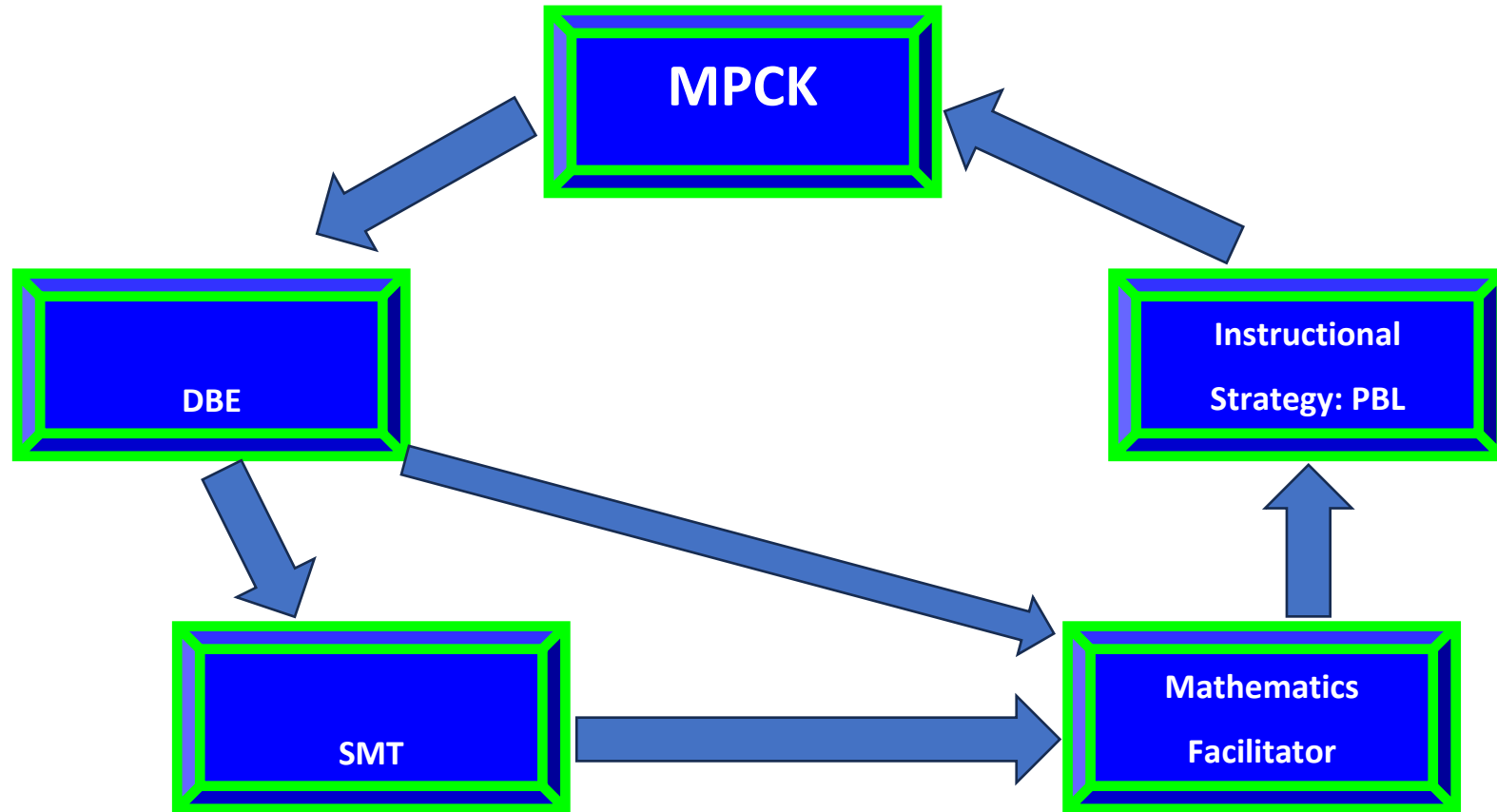
My study will make an additional benefaction to the area of MPCK in learning Grade 12 Mathematics using PBL. The knowledge obtained from the study will encourage further studies into the phenomenon reviewed. The next section presents the framework for teachers' MPCK and how it affects learners learning Grade 12 Mathematics using PBL as another great contribution of this study.

7.5.5 A Framework for teachers' MPCK and how it affects learners learning Grade 12 Mathematics using PBL.

This study entails six domains; teachers' MPCK, DBE, SMT, Mathematics facilitator and Instructional Strategy: PBL. The study shows that teachers' MPCK has an impact on learners' teaching, and that DBE, SMT, the Mathematics facilitator and, the use of PBL as a preferred instructional teaching strategy has a positive effect on the classroom environment. Accordingly, many scholars and researchers, as well as schools that have a long-standing reputation for producing high-performance levels in the NSC have embraced PBL as the favoured instructional strategy while placing prominence on teacher education. Although several studies have focused on teachers' MPCK and the PBL approach, this study aimed to specifically investigate how these current bodies of knowledge can be used together to address a relatively new

phenomenon that was identified in the years 2018, 2019 and 2020, and as recent as 2022. After I concluded the data analyses in Chapter 5, the general feeling I had was to create a framework for teaching and learning Grade 12 Mathematics using PBL. To generate the framework, I analysed current literature and arrived at the important aspects that were applicable, investigated and used. Research question 7 from the focus groups and research question 5 from the teachers' interviews, the theoretical framework in chapter 2 and the literature review in chapter 3, as well as the theoretical framework, have guided the development of the proposed framework. The next section presents a framework for teachers' MPCK and how it affects learners learning Grade 12 Mathematics using PBL.

Figure 7.1: A framework for teachers MPCK and how it affects learners learning Grade 12 Mathematics using PBL.



Source: Prepared by the researcher, with insights from Sunday and Nkopodi (2013).

The framework in Figure 7.1 is based on MPCK, DBE, SMT, Mathematics Facilitator and Instructional Strategy: PBL. The framework attempts to demonstrate teachers' MPCK and how it affects learners learning Grade 12 Mathematics using PBL. The important components that form part of the framework in Figure 7.1 are explained immediately hereunder and follow this particular sequence.

7.5.5.1 MPCK

Teachers practising good Grade 12 Mathematics education should know how to teach the content to a classroom of learners who have mixed learning abilities. MPCK is the knowledge that enables teachers to use various teaching styles like PBL. Teachers with higher MPCK use learner-centred teaching strategies (PBL) and successfully facilitate learners' learning of new knowledge and this can influence the quality of their outcomes in the NSC (Makofane & Maile, 2019).

7.5.5.2 DBE

DBE is a major role player in the development and support of learners and teachers concerned with Grade 12 Mathematics at the NSC level. It is incumbent upon DBE to generate more precise policies on the use of PBL as an instructional learning strategy in Grade 12 Mathematics.

7.5.5.3 SMT

Mathematics teachers teaching Grade 12 Mathematics require “on the spot” support and this highlights the importance of the DH. They should be able to provide immediate support and development on the use of PBL in the teaching and learning of Mathematics and lead teachers in designing and implementing learner-centred education activities.

7.5.5.4 Mathematics facilitator

Mathematics teachers need to transform from their old ways of teaching, as the only knowledge source to the modern form of facilitating learning in learner-centred groups who have a mathematics knowledge base. Therefore, teachers need to be upskilled. Again, DBE can provide more bursaries and attractive incentives to young Mathematics teachers so schools can have teachers rich in MPCK employing PBL

and facilitating group acquisition of knowledge and improving learners' performances in Grade 12 Mathematics, which this study sought to investigate.

7.5.5.5 Instructional strategy: PBL

PBL is a learner-focused learning strategy where learners critically present creative ideas and communicate mathematically with peers. The literature discussed has shown that teachers' MPCK positively affects the teaching and learning of Grade 12 Mathematics using PBL. This translates into higher achievement levels in learners' assessments. Teachers need to implement PBL as the main instructional tool to improve Grade 12 learners' performance levels at the NSC level.

7.6 Recommendations

This unit displays the layout of the proposals to DBE and school managing committees that may firmly affect teachers' pedagogical content knowledge in learning Grade 12 Mathematics using PBL. Through reasonable processes, the study's findings propose actions to be undertaken to enhance the learning of Grade 12 Mathematics, which are as follows:

- Grade 12 Mathematics teachers should be trained in using PBL as an instructional approach to teaching Mathematics.
- Workshops and seminars should be presented to Mathematics teachers in primary and secondary schools to employ PBL in their lessons.
- Professional learning committees (PLCs) should be supported by the Mathematics subject advisors to implement PBL to improve the academic achievements of Grade 12 learners.
- Bursaries and incentives should be made available to allow Mathematics teachers to study further at tertiary institutions to improve their knowledge and skills in teaching Grade 12 Mathematics.
- Mathematics teachers require more effective support from their DHs to render quality Mathematics teaching and learning using PBL.
- More stringent policies need to be developed by DBE on the use of PBL in the teaching and learning of Grade 12 Mathematics.
- DBE needs to review the scope of the Grade 12 CAPS document so that PBL,

as the preferred instructional strategy, can be adopted in teaching and learning Mathematics.

- The curriculum planners of Grade 12 Mathematics may decide to incorporate the PBL approach in the design of new textbooks and also suggest activities in the Mathematics curriculum that are representative of real-life situations.

7.7 Recommendations for future research

The study recommends the following for future research.

An empirical review could be conducted into the effects of school effectiveness on the learning and teaching of Grade 12 Mathematics at the NSC and how DH's MPCK affects teachers teaching Mathematics in the classroom. This is important because professional development and support can immensely influence the quality of teaching and learning of Grade 12 Mathematics and its learners' performance levels at NSC.

Research could be carried out on the effects of progressing Mathematics learners in Grade 10 and Grade 11 into Grade 12 and what constitutes an effective support programme. This study is deemed important because progressed learners have not met the minimum pass requirements in Grade 10 and Grade 11 but are advanced to Grade 12 and eventually fail Grade 12 Mathematics and therefore do not achieve their NSC certificate.

Further studies should be done to investigate the influence of Artificial Intelligence (AI) on the effective teaching and learning of Grade 12 Mathematics. This study is vital because, with the immense development of AI, it warrants an investigation.

7.8 Limitations

Limitations can refer to the weaknesses or limits within a research study that could affect the findings of the study and are not within the investigator's reach (Creswell & Creswell, 2018). The limitations of the study seek to discuss the investigator's awareness of the constraints to the research design and methodological approach thereby, improving the investigator's credibility (Dimitrios & Antigoni, 2019).

These are the limitations I identified in the research study that were as follows.

- The findings from the qualitative research were on a single case study, which meant, the findings that emanated from the methodology I used for this empirical study could not be used to generalise the entire Phumelela circuit.
- The site selected for interviews was only in town and it excluded rural schools.
- I was responsible for conducting the classroom observations and interviews for the teachers and focus groups, my presence may have had an undue influence on the data obtained since I did not disclose the reason for my attendance and therefore the learners assumed I was a senior “inspector” from DBE.
- I employed thematic analysis to critique the data into categories, then codes and themes and there may have been a personal bias. However, all themes were concluded from data gained and were founded on inferences reached.

7.9 Conclusion

This study investigated the effects of pedagogical content knowledge in learning Grade 12 Mathematics using PBL in two selected secondary schools in the Amajuba District in Newcastle, KZN. The study's outcome relates explicitly to the aims and objectives of the investigation undertaken. The study was to identify the following themes; namely, teachers' MPCK positively affects Grade 12 learners' Mathematics learning; the PBL facilitator uses inclusive educational strategies for teaching and learning to improve Grade 12 Mathematics learners' performance, and lastly, the teachers' role as mediators of Grade 12 Mathematics enables content discovery through practical scenarios.

The study is additional to the existing body of subject knowledge on the effects of pedagogical content knowledge in learning Grade 12 Mathematics using PBL. I recommend that the investigation parameters should be reviewed to include other circuits in the Newcastle region, increase the time spent observing the lesson presentations, and scholars could explore this phenomenon in other subjects. Based on these provisos, the new study's results could be generalised for the Amajuba district

in Newcastle. The findings may guide Grade 12 Mathematics teachers on the effects of their MPCK on the learning and teaching of Grade 12 learners through the employ of PBL. This empirical study will enrich all vested stakeholders' knowledge and comprehension of the elements affecting learners' understanding of Grade 12 Mathematics.

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Appendices

Appendix 1: Ethical clearance certificate



GENERAL/HUMAN RESEARCH ETHICS COMMITTEE (GHREC)

31-Oct-2022

Dear Mr Yudvir Bhagwonparsadh

Application Approved

Research Project Title:

The effects of pedagogical content knowledge in learning Grade 12 mathematics using Problem-based learning

Ethical Clearance number:

UFS-HSD2022/1659/22

We are pleased to inform you that your application for ethical clearance has been approved. Your ethical clearance is valid for twelve (12) months from the date of issue. We request that any changes that may take place during the course of your study/research project be submitted to the ethics office to ensure ethical transparency. Furthermore, you are requested to submit the final report of your study/research project to the ethics office. Should you require more time to complete this research, please apply for an extension. Thank you for submitting your proposal for ethical clearance; we wish you the best of luck and success with your research.

Yours sincerely

Dr Adri Du Plessis

Chairperson: General/Human Research Ethics Committee

Appendix 2: Permission from the Department of Education



KWAZULU-NATAL PROVINCE
EDUCATION
REPUBLIC OF SOUTH AFRICA

OFFICE OF THE HEAD OF DEPARTMENT

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Ref.:2/4/8/41143

Mr Y Bhagwonparsadh
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2940

Dear Mr Bhagwonparsadh

PERMISSION TO CONDUCT RESEARCH IN THE KZN DoE INSTITUTIONS

Your application to conduct research entitled: "THE EFFECTS OF PEDAGOGICAL CONTENT KNOWLEDGE IN LEARNING GRADE 12 MATHEMATICS USING PROBLEM-BASED LEARNING", in the KwaZulu-Natal Department of Education Institutions has been approved. The conditions of the approval are as follows:

1. The researcher will make all the arrangements concerning the research and interviews.
2. The researcher must ensure that Educator and learning programmes are not interrupted.
3. Interviews are not conducted during the time of writing examinations in schools.
4. Learners, Educators, Schools and Institutions are not identifiable in any way from the results of the research.
5. A copy of this letter is submitted to District Managers, Principals and Heads of Institutions where the intended research and interviews are to be conducted.
6. The period of investigation is limited to the period from 26 September 2022 to 31 August 2025.
7. Your research and interviews will be limited to the schools you have proposed and approved by the Head of Department. Please note that Principals, Educators, Departmental Officials and Learners are under no obligation to participate or assist you in your investigation.
8. Should you wish to extend the period of your survey at the school(s), please contact Miss Phindile Duma at the contact numbers above.
9. Upon completion of the research, a brief summary of the findings, recommendations or a full report/dissertation/thesis must be submitted to the research office of the Department. Please address it to The Office of the HOD, Private Bag X9137, Pietermaritzburg, 3200.
10. Please note that your research and interviews will be limited to schools and institutions in KwaZulu-Natal Department of Education.

AMAJUBA DISTRICT

Mr GN Ngcobo
Head of Department: Education
Date: 29 September 2022

GROWING KWAZULU-NATAL TOGETHER

Appendix 3: Permission from the Principals of Schools



Dear Mr _____ the Principal of _____ School

I am doing research in the district of Amajuba in Newcastle and would like to request permission to conduct my research in your school.

DATE

2022/09/26

TITLE OF THE RESEARCH PROJECT

The Effects Of Pedagogical Content Knowledge In Learning Grade 12 Mathematics Using Problem-Based Learning (PBL)

PRINCIPLE INVESTIGATOR / RESEARCHER(S) NAME(S) AND CONTACT NUMBER(S):

Yudvir Bhagwonparsadh Student number: 2017560745 Contact number: 083226824924

FACULTY AND DEPARTMENT:

Name of Faculty: Education: Curriculum Studies

Name of Department: Education

STUDY LEADER(S) NAME AND CONTACT NUMBER:

Name of Study Leader (UFS staff member): Dr G Pule

Contact number: 063 910 0127

WHAT IS THE AIM / PURPOSE OF THE STUDY?

The purpose of the study is to investigate the effects Of Pedagogical Content Knowledge In Learning Grade 12 Mathematics Using Problem-Based Learning (PBL)

WHO IS DOING THE RESEARCH?

I will be personally doing the research. I am based in SE Vawda Primary School and I am the Deputy Principal. The reason for me doing the research is, in my experience, learners are indiscriminately condoned in mathematics and then promoted into the Further Education and Training (FET) Phase. In 2018 during a Promotions Committee meeting, I raised the concern at the district level that many learners lack the competency to take mathematics in the FET Phase. I have observed that there is a high failure rate of Grade 12 mathematics learners and those learners prefer to take mathematics literacy in the FET Phase. This has a negative impact on learners when they apply to study science subjects at higher education institutions; hence the need for this study.

HAS THE STUDY RECEIVED ETHICAL APPROVAL?

This study has not received approval from the Research Ethics Committee of UFS. Seeking consent from you is part of the ethical application. Once Ethical Approval is granted, I will present to you a copy as proof of approval.

Approval number: UFS-HSD2022/1659/22

WHY IS YOUR INSTITUTION/ORGANISATION/COMPANY INVITED TO TAKE PART IN THIS RESEARCH PROJECT?

I chose the Amajuba district because it is where I have identified the learners who are indiscriminately condoned in mathematics and then promoted into the Further Education and Training (FET) Phase.

WHO IS DOING THE RESEARCH?

I will be personally doing the research. I am based in SE Vawda Primary School and I am the Deputy Principal. The reason for me doing the research is, in my experience, Grade 12 mathematics learners struggle to achieve good levels of performance in the National Senior Certificate (NSC). Furthermore,

Grade 12 learners' performance in mathematics for 2019, 2020, and 2021 is poor because they cannot analyse and evaluate problem-based questions (DBE, 2021). This is attributed to Grade 12 learners learning mathematics under the supervision of unqualified teachers with insufficient MPCK (Lee et al., 2018; Anney & Hume, 2014). Against this backdrop, the proposed study will investigate the effects of MPCK in learning Grade 12 mathematics using Problem-based learning.

HAS THE STUDY RECEIVED ETHICAL APPROVAL?

This study has not received approval from the Research Ethics Committee of UFS. Seeking consent from you is part of the ethical application. Once Ethical Approval is granted, I will present to you a copy as proof of approval.

Approval number: UFS-HSD2022/1659/22

WHY ARE YOU INVITED TO TAKE PART IN THIS RESEARCH PROJECT?

The research population of this study comprises the Grade 12 mathematics learners as well as the mathematics teachers from two selected schools in the Amajuba District in Newcastle. The focus group will consist of 8 learners. The reason for using a focus group is to collect rich data from the learners regarding their thoughts and experiences. The focus group will be able to provide rich data concerning the effects of pedagogical content knowledge in learning Grade 12 mathematics using Problem-based learning.

WHAT IS THE NATURE OF PARTICIPATION IN THIS STUDY?

The participant's actual role in the study will be to answer questions. The study involves focus groups and structured interviews that will be audio recorded; The main research question was formulated as follows: How do teachers' MPCK affect the learning of Grade 12 mathematics using PBL? Sub-Research questions: What are the effects of teachers' MPCK on Grade 12 learners learning mathematics? How does the PBL approach in teaching and learning mathematics impact the Grade 12 mathematics learners' performance? What is the teachers' role in teaching Grade 12 mathematics using PBL? The expected duration of participation and the time needed to complete specific research activities (focus groups, interviews and observation) will be 30 mins each. The possible risk to the children may experience emotions of a negative self-image due to being teased by other learners. Working with children is a given that they are a vulnerable group therefore all ethical considerations and protocols will be followed.

CAN THE PARTICIPANT WITHDRAW FROM THE STUDY?

Participation is voluntary and there is no penalty or loss of benefit for non-participation. Being in this study is voluntary and you are under no obligation to consent to participate. If you do decide to take part, you will be given this information sheet to keep and be asked to sign a written consent form. You are free to withdraw at any time and without giving a reason.

WHAT ARE THE POTENTIAL BENEFITS OF TAKING PART IN THIS STUDY?

The findings are intended to add value to current and further research on the effects of condonation on the promotion of senior phase mathematics learners into the FET phase. The findings will help mathematics advisors to make possible changes to teaching practices; District support structures may also introduce ongoing workshops on MPCK and the results will fill in gaps in the literature for future studies.

WHAT IS THE ANTICIPATED INCONVENIENCE OF TAKING PART IN THIS STUDY?

Participating learners will be removed from normal teaching in order to participate fully in the research. At the learners' soonest, it is expected that learners will cover up with lost teaching time done in the class by the class teacher.

WILL WHAT I SAY BE KEPT CONFIDENTIAL?

The names of participants will not be recorded anywhere therefore no one will be able to link the participants to their answers. The use of pseudonyms and a coding system for the questionnaires will ensure participants' confidentiality and the participants will be referred to in this way in the data, any publications, or other research reporting methods such as conference proceedings. A transcriber and I will only have access to the data. The transcriber will sign a confidentiality agreement. The data may be made available to other people who have a vested interest in the study. I shall, based on my discretion make the data available only after a confidentiality agreement is signed. Participants will be informed that their anonymous data may be used for research reports, journal articles and conference presentations. Participant's privacy will be protected in any publication of the information. Individual participants will not be identifiable in a report. While every effort will be made to ensure that participants will not be connected to the information that you share during the focus group, I cannot guarantee that other participants in the focus group will treat the information confidentially. I shall, however, encourage all participants to do so. For this reason, I will advise participants not to disclose personal and sensitive information in the focus group. Participants will be explained that they can refuse to take part even if their parents/gatekeepers have agreed to their participation. Participants may also leave the study at any time without any repercussions forthcoming.

HOW WILL THE INFORMATION BE STORED AND ULTIMATELY DESTROYED?

Hard copies of your answers will be stored by the researcher for a period of five years and locked in the researcher's safe located in the researcher's residential address for future research or academic purposes. Electronic information will be stored on my password-protected computer. Future use of the stored data will be subject to further Research Ethics Review and approval if applicable. After a period of five years, all hard copies of the data will be shredded. All electronic data will be deleted. There is no possible inconvenience and/or discomfort to the participants because all precautions will be taken not to disclose their identities.

WILL I RECEIVE PAYMENT OR ANY INCENTIVES FOR PARTICIPATING IN THIS STUDY?

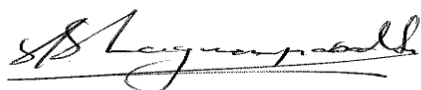
There is no payment or reward offered, financial or otherwise to the school.

HOW WILL THE PARTICIPANT BE INFORMED OF THE FINDINGS / RESULTS OF THE STUDY?

If you would like to be informed of the final research findings, please contact Y. Bhagwonparsadh on 083 226 8924 or yudvirbhagwon@gmail.com. The findings are accessible for the year 2023 only. Should you have concerns about the way in which the research has been conducted, you may contact my supervisor Dr G Pule of the UFS on 063 910 0127 or PuleKG@ufs.ac.za

Thank you for taking the time to read this information sheet and for participating in this study.

Yours sincerely



Researcher: Y.Bhagwonparsadh

Date: 2022/09/25



I, _____ the Principal of

_____ School in Newcastle,

- I hereby give/do not give free and informed consent to your study to be undertaken in the Newcastle district.
- I give/do not give the researcher permission to make use of the data gathered, subject to the stipulations indicated in the above letter.
- I understand what the study is about and what the risks and benefits are to the district and participants.

Signature of the Principal

Date _____

SCHOOL STAMP

Signature of the Researcher

Date: _____

Appendix 4: Consent indemnity forms from parents



RESEARCH STUDY INFORMATION LEAFLET AND CONSENT FORM

DATE

2022 09 26

TITLE OF THE RESEARCH PROJECT

The effects of pedagogical content knowledge in learning Grade 12 mathematics using Problem-based learning.

PRINCIPLE INVESTIGATOR / RESEARCHER(S) NAME(S) AND CONTACT NUMBER(S):

Yudvir Bhagwonparsadh Student number: 2017560745 Contact number: 083226824 924

FACULTY AND DEPARTMENT:

Name of Faculty: Education: Curriculum Studies

Name of Department: Education

STUDY LEADER(S) NAME AND CONTACT NUMBER:

*Name of Study Leader (UFS staff member): Dr G Pule
Contact number: 063 9100 127*

TO THE PARENT/ GUARDIAN OF _____

OF _____ GRADE _____

WHAT ARE THE AIM / PURPOSE OF THE STUDY?

The purpose of the study is to investigate the effects of pedagogical content knowledge in learning Grade 12 mathematics using Problem-based learning.

WHO IS DOING THE RESEARCH?

I will be personally doing the research. I am based in SE Vawda Primary School and I am the Deputy Principal. The reason for me doing the research is, in my experience, Grade 12 mathematics learners struggle to achieve good levels of performance in the National Senior Certificate (NSC). Furthermore, Grade 12 learners' performance in mathematics for 2019, 2020, and 2021 is poor because they cannot analyse and evaluate problem-based questions (DBE, 2021). This is attributed to Grade 12 learners learning mathematics under the supervision of unqualified teachers with insufficient MPCK (Lee et al., 2018; Anney & Hume, 2014). Against this backdrop, the proposed study will investigate the effects of MPCK in learning Grade 12 mathematics using Problem-based learning.

HAS THE STUDY RECEIVED ETHICAL APPROVAL?

This study has not received approval from the Research Ethics Committee of UFS. Seeking consent from you is part of the ethical application. Once Ethical Approval is granted, I will present to you a copy as proof of approval.

Approval number: UFS-HSD2022/1659/22

WHY ARE YOU INVITED TO TAKE PART IN THIS RESEARCH PROJECT?

The research population of this study comprises the Grade 12 mathematics learners as well as the mathematics teachers from two selected schools in the Amajuba District in Newcastle. The focus group will consist of 8 learners. The reason for using a focus group is to collect rich data from the learners regarding their thoughts and experiences. The focus group will be able to provide rich data concerning the effects of pedagogical content knowledge in learning Grade 12 mathematics using Problem-based learning.

WHAT IS THE NATURE OF PARTICIPATION IN THIS STUDY?

The participant's actual role in the study will be to answer questions. The study involves focus groups and structured interviews that will be audio recorded; The main research question was formulated as follows: How do teachers' MPCK affect the learning of Grade 12 mathematics using PBL? Sub-Research questions: What are the effects of teachers' MPCK on Grade 12 learners learning mathematics? How does the PBL approach in teaching and learning mathematics impact the Grade 12

mathematics learners' performance? What is the teachers' role in teaching Grade 12 mathematics using PBL? The expected duration of participation and the time needed to complete specific research activities (focus groups, interviews and observation) will be 30 mins each. The possible risk to the children may experience emotions of a negative self-image due to being teased by other learners. Working with children is a given that they are a vulnerable group therefore all ethical considerations and protocols will be followed.

CAN THE PARTICIPANT WITHDRAW FROM THE STUDY?

Participation is voluntary and there is no penalty or loss of benefit for non-participation. Being in this study is voluntary and you are under no obligation to consent to participate. If you do decide to take part, you will be given this information sheet to keep and be asked to sign a written consent form. You are free to withdraw at any time and without giving a reason.

WHAT ARE THE POTENTIAL BENEFITS OF TAKING PART IN THIS STUDY?

The findings are intended to add value to current and further research on the effects of condonation on the promotion of senior-phase mathematics learners into the FET phase. The findings will help mathematics advisors to make possible changes to teaching practices; District support structures may also introduce ongoing workshops on MPCK and the results will fill in gaps in the literature for future studies.

WHAT IS THE ANTICIPATED INCONVENIENCE OF TAKING PART IN THIS STUDY?

Participating learners will be removed from normal teaching in order to participate fully in the research. At the learners' soonest, it is expected that learners will cover up with lost teaching time done in the class by the class teacher.

WILL WHAT I SAY BE KEPT CONFIDENTIAL?

The names of participants will not be recorded anywhere therefore no one will be able to link the participants to their answers. The use of pseudonyms and a coding system for the questionnaires will ensure participants' confidentiality and the participants will be referred to in this way in the data, any publications, or other research reporting methods such as conference proceedings. A transcriber and I will only have access to the data. The transcriber will sign a confidentiality agreement. The data may be made available to other people who have a vested interest in the study. I shall, based on my discretion make the data available only after a confidentiality agreement is signed. Participants will be informed that their anonymous data may be used for research reports, journal articles and conference presentations. Participant's privacy will be protected in any publication of the information. Individual participants will not be identifiable in a report. While every effort will be made to ensure that participants will not be connected to the information that you share during the focus group, I cannot

guarantee that other participants in the focus group will treat the information confidentially. I shall, however, encourage all participants to do so. For this reason, I will advise participants not to disclose personal and sensitive information in the focus group. Participants will be explained that they can refuse to take part even if their parents/gatekeepers have agreed to their participation. Participants may also leave the study at any time without any repercussions forthcoming.

HOW WILL THE INFORMATION BE STORED AND ULTIMATELY DESTROYED?

Hard copies of your answers will be stored by the researcher for a period of five years and locked in the researcher's safe located in the researcher's residential address for future research or academic purposes. Electronic information will be stored on my password-protected computer. Future use of the stored data will be subject to further Research Ethics Review and approval if applicable. After a period of five years, all hard copies of the data will be shredded. All electronic data will be deleted. There is no possible inconvenience and/or discomfort to the participants because all precautions will be taken not to disclose their identities.

WILL I RECEIVE PAYMENT OR ANY INCENTIVES FOR PARTICIPATING IN THIS STUDY?

There is no payment or reward offered, financial or otherwise to the participants.

HOW WILL THE PARTICIPANT BE INFORMED OF THE FINDINGS / RESULTS OF THE STUDY?

If you would like to be informed of the final research findings, please contact Y. Bhagwonparsadh on 083 226 8924 or yudvirbhagwon@gmail.com. The findings are accessible for the year 2023 only. Should you have concerns about the way in which the research has been conducted, you may contact my supervisor Dr G Pule of the UFS on 063 910 0127 or PuleKG@ufs.ac.za

Thank you for taking the time to read this information sheet and for participating in this study.

Yours sincerely



Researcher: Y.Bhagwonparsadh

Date: 2022/09/25

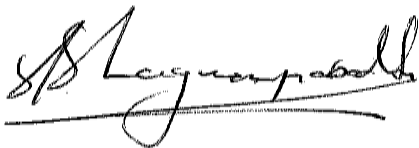
I, _____,

the parent / guardian of _____ in grade _____,

- I hereby give free and informed consent to my child/ward to participate in the abovementioned research study.
- I understand what the study is about why my child/ward is participating and what the risks and benefits are.
- I give the researcher permission to make use of the data gathered from my child/ward's participation, subject to the stipulations indicated in the above letter.

Signature of the parent/guardian

Date: _____



Researcher: Y. Bhagwonparsadh

Date: 2022/09/25

Appendix 5: Consent indemnity forms from teachers



RESEARCH STUDY INFORMATION LEAFLET AND CONSENT FORM

DATE

2022 09 26

TITLE OF THE RESEARCH PROJECT

The effects of pedagogical content knowledge in learning Grade 12 mathematics using Problem-based learning.

PRINCIPLE INVESTIGATOR / RESEARCHER(S) NAME(S) AND CONTACT NUMBER(S):

Yudvir Bhagwonparsadh Student number: 2017560745 Contactnumber: 0832268924

FACULTY AND DEPARTMENT:

Name of Faculty: Education: Curriculum Studies

Name of Department: Education

STUDY LEADER(S) NAME AND CONTACT NUMBER:

*Name of Study Leader (UFS staff member): Dr G Pule
Contact number: 063 9100 127*

To The Mathematics and Mathematic Literacy Teachers of _____
_____ School.

Name of Teacher: _____

Current Subject teaching: _____

WHAT ARE THE AIM / PURPOSE OF THE STUDY?

The purpose of the study is to investigate the effects of pedagogical content knowledge in learning Grade 12 mathematics using Problem-based learning.

WHO IS DOING THE RESEARCH?

I will be personally doing the research. I am based in SE Vawda Primary School and I am the Deputy Principal. The reason for me doing the research is, in my experience, Grade 12 mathematics learners struggle to achieve good levels of performance in the National Senior Certificate (NSC). Furthermore, Grade 12 learners' performance in mathematics for 2019, 2020, and 2021 is poor because they cannot analyse and evaluate problem-based questions (DBE, 2021). This is attributed to Grade 12 learners learning mathematics under the supervision of unqualified teachers with insufficient MPCK (Lee et al., 2018; Anney & Hume, 2014). Against this backdrop, the proposed study will investigate the effects of MPCK in learning Grade 12 mathematics using Problem-based learning.

HAS THE STUDY RECEIVED ETHICAL APPROVAL?

This study has not received approval from the Research Ethics Committee of UFS. Seeking consent from you is part of the ethical application. Once Ethical Approval is granted, I will present to you a copy as proof of approval.

Approval number: *To follow*

WHY ARE YOU INVITED TO TAKE PART IN THIS RESEARCH PROJECT?

The research population of this study comprises the senior phase and FET phase learners as well as the mathematics and mathematics literacy teachers from a selected school in the Amajuba District in Newcastle. The focus group will consist of 6 learners. The reason for using a focus group is to collect rich data from the learners regarding their thoughts and experiences. The focus group will be able to provide rich data concerning the effects of condonation on their promotion into the FET phase.

WHAT IS THE NATURE OF PARTICIPATION IN THIS STUDY?

The research population of this study comprises the Grade 12 mathematics learners as well as the mathematics teachers from two selected schools in the Amajuba District in Newcastle. The focus group will consist of 8 learners. The reason for using a focus group is to collect rich data from the learners regarding their thoughts and experiences. The focus group will be able to provide rich data concerning the effects of pedagogical content knowledge in learning Grade 12 mathematics using Problem-based learning.

CAN THE PARTICIPANT WITHDRAW FROM THE STUDY?

Participation is voluntary and there is no penalty or loss of benefit for non-participation. Being in this study is voluntary and you are under no obligation to consent to participation. If you do decide to take

part, you will be given this information sheet to keep and be asked to sign a written consent form. You are free to withdraw at any time and without giving a reason.

WHAT ARE THE POTENTIAL BENEFITS OF TAKING PART IN THIS STUDY?

The findings are intended to add value to current and further research on the effects of condonation on the promotion of senior-phase mathematics learners into the FET phase. The findings will help mathematics advisors to make possible changes to teaching practices; District support structures may also introduce ongoing workshops on MPCK and the results will fill in gaps in the literature for future studies.

WHAT IS THE ANTICIPATED INCONVENIENCE OF TAKING PART IN THIS STUDY?

Participating learners and teachers will be removed from normal teaching in order to participate fully in the research. At the learners' soonest, it is expected that participants will cover up with lost teaching time done in the class.

WILL WHAT I SAY BE KEPT CONFIDENTIAL?

The names of participants will not be recorded anywhere therefore no one will be able to link the participants to their answers. The use of pseudonyms and a coding system for the questionnaires will ensure participants' confidentiality and the participants will be referred to in this way in the data, any publications, or other research reporting methods such as conference proceedings. A transcriber and I will only have access to the data. The transcriber will sign a confidentiality agreement. The data may be made available to other people who have a vested interest in the study. I shall, based on my discretion make the data available only after a confidentiality agreement is signed. Participants will be informed that their anonymous data may be used for research reports, journal articles and conference presentations. Participant's privacy will be protected in any publication of the information. Individual participants will not be identifiable in a report. While every effort will be made to ensure that participants will not be connected to the information that you share during the focus group, I cannot guarantee that other participants in the focus group will treat the information confidentially. I shall, however, encourage all participants to do so. For this reason, I will advise participants not to disclose personal and sensitive information in the focus group. Participants will be explained that they can refuse to take part even if their parents/gatekeepers have agreed to their participation. Participants may also leave the study at any time without any repercussions forthcoming.

HOW WILL THE INFORMATION BE STORED AND ULTIMATELY DESTROYED?

Hard copies of your answers will be stored by the researcher for a period of five years and locked in the researcher's safe located in the researcher's residential address for future research or academic

purposes. Electronic information will be stored on my password-protected computer. Future use of the stored data will be subject to further Research Ethics Review and approval if applicable. After a period of five years, all hard copies of the data will be shredded. All electronic data will be deleted.

WILL I RECEIVE PAYMENT OR ANY INCENTIVES FOR PARTICIPATING IN THIS STUDY?

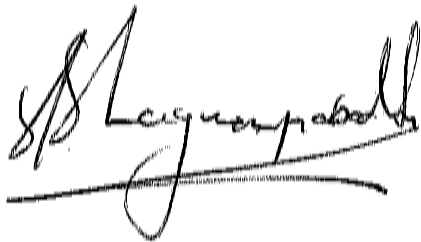
There is no payment or reward offered, financial or otherwise to the participants.

HOW WILL THE PARTICIPANT BE INFORMED OF THE FINDINGS / RESULTS OF THE STUDY?

If you would like to be informed of the final research findings, please contact Y. Bhagwonparsadh on 083 226 8924 or yudvirbhagwon@gmail.com. The findings are accessible for the year 2023 only. Should you have concerns about the way in which the research has been conducted, you may contact my supervisor Dr G Pule of the UFS on 063 910 0127 or PuleKG@ufs.ac.za

Thank you for taking the time to read this information sheet and for participating in this study.

Yours sincerely

A handwritten signature in black ink, appearing to read 'Y. Bhagwonparsadh', with a horizontal line underneath it.

Researcher: Y.Bhagwonparsadh

Date: 2019/03/15


I, _____,

the mathematics literacy teacher of _____ School;

- Hereby give free and informed consent to my participation in the abovementioned research study
- Understand what the study is about why I participate and what the risks and benefits are.
- Give the researcher permission to make use of the data gathered from my participation, subject to the stipulations indicated in the above letter.

Signature of Participant:

Date: _____



Researcher: Y. Bhagwonparsadh

Date: 2022/09/25

Appendix 6: Assent forms from learners



TITLE OF THE RESEARCH PROJECT:

The effects of pedagogical content knowledge in learning Grade 12 mathematics using Problem-based learning.

RESEARCHERS NAME(S): Bhagwonparsadh Yudvir

ADDRESS: 1 a Toucan Place, Newcastle, 2940

CONTACT NUMBER: 083 2268 924

What is Research?

Research is a process of finding new knowledge about things. We conduct research or a study to help find out more information about phenomena that affect us. Research also helps us to find better ways of solving our problems.

What is this research project all about?

The purpose of the study is to investigate the effects of pedagogical content knowledge in learning Grade 12 mathematics using Problem-based learning.

Research interest OR problem

Grade 12 learners' performance in mathematics for 2019, 2020, and 2021 is poor because they cannot analyse and evaluate problem-based questions (DBE, 2021). This is attributed to Grade 12 learners learning mathematics under the supervision of unqualified teachers with insufficient MPCCK (Lee et al., 2018; Anney & Hume, 2014). Against this backdrop, the

proposed study will investigate the effects of MPCK in learning Grade 12 mathematics using Problem-based learning.

The findings will help mathematics advisors to make possible changes to teaching practices and support District structures to introduce ongoing workshops on MPCK. Furthermore, the results will fill in gaps in the literature for future studies.

Why have I been invited to take part in this research project?

You are currently taking Grade 12 mathematics. You have “rich information” to share. You are the subject of the study.

Who is doing the research?

The researcher is Yudvir Bhagwonparsadh. I am a deputy principal of SE Vawda Primary School. My employer is the Department of Basic Education. I am doing this study so that I can provide scientific data on the effects of pedagogical content knowledge in learning Grade 12 mathematics using Problem-based learning.

What will happen to me in this study?

You will be in a group of Grade 12 mathematics learners. You will be asked a series of questions. You will be required to freely engage in the group and contribute honestly to the discussion.

I will be observing the discussions and I will make written observations. There will be audio recordings of the discussions. You will remain anonymous. You may withdraw from the group at any time without any consequence. The duration of the discussion should be no longer than 30 minutes on a school day which I will negotiate with the principal.

Is my Participation voluntary?

Your participation in this study is voluntary. If you decide not to participate in this study, your decision will not affect your future relations with the University of the Free State, its personnel, and associated institutions. If you decide to participate in the study, you are free to withdraw your consent from participating at the time you choose to do so.

What if I do not want to do this?

You may not participate in the study although your parent may have given consent. You may choose to stop participating in the study at any time without any repercussions.

Can anything bad happen to me?

There are possible risks that your disclosures may be exposed to other learners first; they may come to know your identities from other participants in the group. They may tease you and this may affect your self-confidence. Should this occur, you may immediately contact your parents and me to resolve the matter.

Can anything good happen to me?

There are plenty of benefits to your participation in the research. Your contribution to the study may add more value to the learning of mathematics using problem-based learning. You will be part of the study that is sanctioned by the University of the Free State (UFS) and therefore you will be exposed to the university which you may consider going to study in the near future.

You will not be financially compensated for your participation in the study.

Will anyone know I am in the study?

If you agree to participate in this study by signing this document, I may disclose the findings to the University of the Free State or the General and Health Research Ethics Council (GHREC) for auditing purposes. Any data that is obtained in connection with this study which can identify you will remain confidential and will be disclosed only with your permission.

What does my authorisation mean?

You are making a decision as to whether or not you are willing to participate in this study. Your signature below indicates that you have read and understood the information provided, you have had all your questions answered in relation to the study, and have decided to participate in it of your own free will.

Who can I talk to about the study?

You may contact my supervisor on the following details:

Title	Dr
Initials	G
Surname	Pule
*Institutional Affiliation	University of the Free State
Physical Address	
Academic Unit	School of Mathematics, Natural Sciences and Technology Education
Office Number	058 718 5214
Building	KGORONG
Institution	University of the Free State
Street Name	KESTELL ROAD
City	PHUTHADITJHABA
Post Code	9870

Telephone no.	063 910 0127
Email address	PuleKG@ufs.ac.za

Do you understand this research study and are you willing to take part in it?

YES

NO

Has the researcher answered all your questions?

YES

NO

Do you understand that you can pull out of the study at any time?

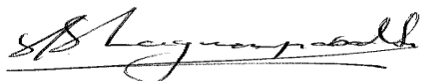
YES

NO

Name of child

Signature of Child

Date: _____



Researcher: Y. Bhagwonparsadh :

Date: 2022/09/26

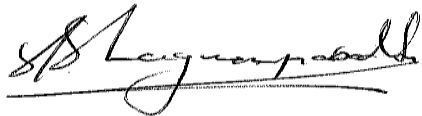
Appendix 7: Focus group interviews guide



1. You are currently doing Grade 12 mathematics. Describe a regular teaching and learning experience in your mathematics classroom.
2. Describe a lesson your mathematics teacher conducted in the classroom that you enjoyed the most.
3. Describe the personality and relationship you have with your mathematics teacher.
4. Please describe your current performance levels in Grade 12 mathematics and your feelings about it.
5. Problem-based learning or PBL is a learner-focused learning strategy where learners communicate mathematically with peers in smaller groups while the teacher facilitates the lesson. How many of you favour being taught mathematics in smaller groups rather than the teacher instructing? Explain why?
6. Do you think a mathematics teacher who knows the content very well can produce good results? Why do you think so?
7. What are some of the learning difficulties you experience in Grade 12 mathematics?
 - 7.1 Why do you think you experience such learning difficulties in learning mathematics?
 - 7.2 Is there anything that your teachers do in the classroom that makes learning difficult?
8. How have your mathematics learning difficulties/challenges/fears/attitudes affected your performance levels in Grade 12?

9. Suggest some ways you would like your mathematics teacher to use to help you improve your performance levels into Grade 12.

10. Is there any other experience you would like to share regarding your teacher's content knowledge of mathematical topics and how it affects your learning and your performance levels?

A handwritten signature in black ink, appearing to read 'Y. Bhagwonparsadh', with a horizontal line underneath.

Researcher: Y.Bhagwonparsadh Date: 2022/10/11

Appendix 8: Semi-structured interviews guide

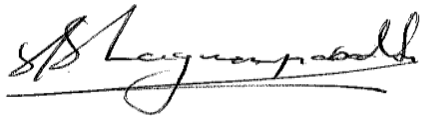
1. Do you have any information on Problem-based learning (PBL) and teaching strategy?
2. Where did you receive your information?
3. Explain what you understand about PBL within the Grade 12 mathematics curriculum.
4. What are some of the skills and knowledge you think a Grade 12 mathematics teacher must have to implement PBL in the mathematics classroom successfully?
5. How do teachers' Mathematics Pedagogical Content Knowledge (MPCK) affect the learning of Grade 12 mathematics using PBL?
6. What are some of the examples of PBL that you used in the teaching of Grade 12 mathematics?
7. Explain the PBL strategies you have used in teaching Grade 12 mathematics and how these strategies have impacted learning.
8. Which of these learning strategies did you use in the teaching and learning of Grade 12 mathematics positively affected learners' performance in Grade 12?
9. Would you consider PBL as an effective learning strategy for improving learners' problem-solving skills and performance in Grade 12 mathematics? Explain.

10. How does your mastery of the content influence the choice of teaching styles in teaching Grade 12 mathematics?

11. In your experience teaching Grade 12 learners, how have your teaching styles affected the learners' performance in mathematics?

12. How do you think knowing Grade 12 learners' preconceptions, misconceptions, and difficulties they may experience when taught a specific topic will ensure effective learning of Grade 12 mathematics?

13. Discuss an effective teaching strategy you used in teaching Grade 12 mathematics after discovering that learners did not grasp the content initially teaching the concept.



Researcher: Y. Bhagwonparsadh

Date: 2022/10/11

Appendix 9: Observation tool



A qualitative observation is when the researcher takes field notes on the behaviour and activities of individuals at the research site and records his observations. The researcher will record the classroom's teaching and learning activities in these field notes. Where necessary, the researcher may pose questions to the participants, allowing them to provide their views freely. The participants in this study include mathematics teachers and Grade 12 mathematics learners.

The role of the researcher is to identify personal values, assumptions and biases at the outset of the study.

Situation	Action observed	Researcher's Reflections
1. Teaching and learning support materials used and displayed in the classroom		
2. Managers the teaching and learning activities effectively		
3. Has adequate subject knowledge to teach effectively.		
4. Uses appropriate tasks suitable to the learners' level		
5. Tasks were representative of real-life problems		
6. Uses various resources and LTSM (Technology,		

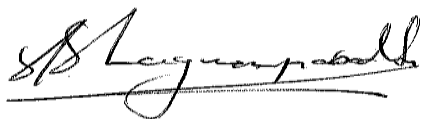
textbooks, different sources of information) to teach mathematics.		
7. The lesson is logical, coherent and meaningful to learners		
8. Learners are intrinsically motivated to learn		

9. The lesson is built on past knowledge and experience of learners		
10. Encourages interactive learning, including class discussions, learner questions and demonstrations		
11. Responds appropriately to learner questions and inputs		
12. Feedback is meaningful and regular		
13. Uses different forms of assessment in line with CAPS to test learner performance		
14. Intervention strategies accommodate learners with various learning abilities		
15. Learner results of various forms of the assessment show that they are attaining the set outcomes		
16. Remedial / Enrichment work supports learner progress		

17. PBL Facilitation The teacher clearly stated goals and tasks.		
18. The teacher facilitated the students to remain on-task		
19. The teacher asked effective open-ended questions.		
20. The teacher worked with members of all small groups.		
21. The teacher achieved the objectives he/she identified.		
Student Participation		
22. The students were actively engaged.		
23. Learners were given opportunities to work independently		
24. Learners were involved in problem-solving activities and displayed critical thinking skills		
25. The students could explain tasks and solution strategies		
26. The students could explain the goal(s).		
Assessment		
27. The assessment(s) was/were continuous and varied		
28. The evidence of holistic assessments existed (e.g. rubrics for		

participation/engagement, early stages of the PBL, or group work		
29. The students understood how the rubric would be used as an assessment.		
Classroom Learning Environment. 30. The teacher identified and engaged students around their prior knowledge.		
31. The teacher identified and engaged the students around their diverse cultural contexts.		
32. Learners displayed social skills while interacting with each other		
33. The teacher mainly controlled classroom discipline		
34. There is a fair amount of noise levels due to group work engagements		

It is adapted from the ELRC, Collective Agreement 2 of 2020 and Stearn, Morgan, Capraro, and Capraro, 2012.



Researcher: Y. Bhagwonparsadh Date: 2022/09/26

Appendix 10: Proof of title registration



16 September 2022

APPLICATION FOR TITLE REGISTRATION

Applicant: Bhagwonparsadh, Y
Student Number: 2017560745
Discipline: Curriculum Studies
Study Code: Doctoral (EDC19100)

Dear Mr Bhagwonparsadh

Your registered title is as follows: "The effects of pedagogical content knowledge in learning Grade 12 mathematics using Problem-based learning".

All of the best with your studies.

Yours sincerely,

Prof Patrick Mafora
Chair: CTR committee

Ms CS Duvenhage
Secretary: CTR committee

Appendix 11: Editing certificate



EDITING CERTIFICATE

TO WHOM IT MAY CONCERN

This certifies that the research document whose title appears below has been edited for proper English language, grammar, punctuation, spelling and overall style.

Level of document: PhD thesis

Title: The effects of pedagogical content knowledge in learning Grade 12 Mathematics using problem-based learning (PBL)

Author: Yudvir Bhagwonparsadh

Date: June 2022

Signed

A handwritten signature in blue ink, appearing to read "Tony Kambi Masha".

Dr Tony Kambi Masha (PhD in Education; D. Admin: Public Administration)

PhD in Education; Doctor of Administration: Public Administration – Leadership Development, Mentoring and Training (Fort Hare); Master's in Administration: Public Administration (Fort Hare); Honours B.A. (Group Dynamics - NMMU); Honours B.A. (Social Sciences - WSU); B.A. (Education - WSU); Postgraduate Diploma in Education (UWC)

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Appendix 12: Turnitin certificate

THE EFFECTS OF PEDAGOGICAL
CONTENT KNOWLEDGE IN
LEARNING GRADE 12
MATHEMATICS USING
PROBLEM-BASED LEARNING
(PBL)

by Yudvir Bhagwonparsadh

Submission date: 18-Jul-2023 01:14PM (UTC+0200)

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