THE EFFECT OF METACOGNITIVE INTERVENTION ON LEARNER METACOGNITION AND ACHIEVEMENT IN MATHEMATICS

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DECLARATION

I hereby declare that the work which is submitted here is the result of my own independent investigation and that all sources I have used or quoted have been indicated and acknowledged by means of complete references. I further declare that the work was submitted for the first time at this university/faculty towards the Philosophiae Doctor degree and that it has never been submitted to any other university/faculty for the purpose of obtaining a degree.

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D.S. DU TOIT **DATE**

Teaching is not a science; it is an art. If teaching were a science there would be a best way of teaching and everyone would have to teach like that. Since teaching is not a science, there is great latitude and much possibility for personal differences. ... the main point in mathematics teaching is to develop the tactics of problem solving.

- George Polya

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ABSTRACT

International and national measures point to the poor mathematics achievement of South African learners. The enhancement of the quality of mathematics education is a key priority of the Department of Basic Education in South Africa.

Several studies have found a correlation between learner metacognition and mathematics achievement. Metacognition entails knowledge and regulation of one's cognitive processes. Previous studies point to the positive effect of metacognitive interventions on learner metacognition and mathematics achievement.

The purpose of this study was to investigate the effect of a metacognitive intervention (MI) on learner metacognition and the mathematics achievement of Grade 11 learners in the Free State from a predominantly pragmatic perspective. The MI was developed by combining aspects of a mathematical perspective on De Corte's (1996) educational learning theory with aspects of previous metacognitive intervention studies in mathematics.

A mixed methods research design was employed where qualitative data were embedded within a quasi-experiment. Data were collected from an experimental group (N=25) and a control group (N=24). Quantitative data on learner metacognition were obtained from the Metacognitive Awareness Inventory (MAI), while quantitative data on mathematics achievement were obtained from the learners' Terms 1 and 4 report marks. Qualitative data were acquired by means of teacher interviews, problem-solving sessions, and learner and teacher perspectives on the MI process. The mixed methods research question investigated the extent to which the findings from the qualitative phase of the study support the findings from the quantitative phase regarding the effect of MI on learner metacognition and mathematics achievement.

The quantitative findings indicated that MI had a statistically significant impact on learner metacognition in respect of the MAI total score, the *Knowledge of cognition (KC)* factor, the *Regulation of cognition (RC)* factor, and the subscales *Declarative knowledge, Planning*, and *Monitoring.*

The impact of MI on mathematics achievement was less pronounced, as inferences had to be drawn from the correlation between learner metacognition and mathematics achievement. The quantitative findings showed a statistically significant correlation between *KC* and mathematics achievement, as well as between *Declarative knowledge* and mathematics achievement. Since MI had a statistically significant impact on *KC* and *Declarative knowledge*, it is concluded that MI also had a positive impact on mathematics achievement.

The qualitative findings strongly support the quantitative findings regarding the positive impact of MI on learner metacognition. The quantitative findings in respect of the correlation between learner metacognition and mathematics achievement were only partially supported by the qualitative data.

Main recommendations emerging from this study relate to the improvement of learners' mathematics achievement by enhancing their *Declarative knowledge*, the enhancement of learners' problem-solving skills, and the need to implement metacognitive interventions in mathematics particularly in schools where the teachers are inexperienced or underqualified.

KEY TERMS

Metacognition; metacognitive intervention, mathematics achievement; metacognitive awareness inventory; problem-solving; educational learning theory; mixed methods; knowledge of cognition; regulation of cognition; self-regulated learning.

OPSOMMING

Die swak wiskunde-prestasie van Suid-Afrikaanse leerders word deur internasionale en nasionale maatstawwe aangetoon. Die verbetering van die kwaliteit van wiskunde-onderwys is ‘n kernprioriteit van die Departement van Basiese Onderwys in Suid-Afrika.

Verskeie studies het bevind dat daar ‘n korrelasie tussen leerdermetakognisie en wiskunde-prestasie is. Metakognisie behels die kennis en regulering van ‘n persoon se kognitiewe prosesse. Vorige studies dui op die positiewe effek van metakognitiewe intervensies op leerdermetakognisie en wiskunde-prestasie.

Die doel van hierdie studie was om die effek van ‘n metakognisie intervensie (MI) op leerdermetakognisie en die wiskunde-prestasie van Graad 11 leerders in die Vrystaat vanuit ‘n grotendeels pragmatiese wêreldsiening te ondersoek. Die MI is ontwikkel deur aspekte van ‘n wiskundige perspektief op De Corte (1996) se opvoedkundige leerteorie met aspekte van vorige metakognisie-intervensiestudies in wiskunde te kombineer.

‘n Gemengde-metodes navorsingsontwerp is gebruik waar kwalitatiewe data in ‘n kwasi-eksperiment ingebed was. Data is van ‘n eksperimentele groep (N=25) en ‘n kontrole groep (N=24) verkry. Kwantitatiewe data van leerdermetakognisie is verkry uit die “Metacognitive Awareness Inventory” (MAI) terwyl kwantitatiewe data van wiskunde-prestasie uit die Kwartaal 1- en Kwartaal 4 rapportpunte verkry is. Kwalitatiewe data is uit onderhoude met onderwysers, probleemoplossingsessies, en leerder- en onderwyserperspektiewe oor die MI-proses verkry. Die gemengde-metodes navorsingsvraag het die mate ondersoek waartoe die bevindinge van die kwalitatiewe fase van die studie die bevindinge ondersteun van die kwantitatiewe fase met betrekking tot die effek van MI op leerdermetakognisie en wiskunde-prestasie.

Die kwantitatiewe bevindinge het aangedui dat MI ‘n statisties-beduidende impak op leerdermetakognisie gehad het met betrekking tot die totale MAI-telling, die *Kennis van kognisie*-faktor, die *Regulering van kognisie*-faktor, en die subskale *Verklarende kennis*, *Beplanning,* en *Monitering.*

Die impak van MI op wiskunde-prestasie was minder prominent omdat afleidings gemaak moes word uit die korrelasie tussen leerdermetakognisie en wiskunde-prestasie. Die kwantitatiewe bevindinge het aangetoon dat daar ‘n statisties-beduidende korrelasie tussen *Kennis van kognisie* en wiskunde-prestasie was, en ook tussen *Verklarende kennis* en wiskunde-prestasie. Omdat MI ‘n statisties-beduidende impak op *Kennis van kognisie* en *Verklarende kennis* gehad het, word die gevolgtrekking gemaak dat MI ook ‘n positiewe impak op wiskunde-prestasie gehad het.

Die kwalitatiewe bevindinge ondersteun tot ‘n groot mate die kwantitatiewe bevindinge wat verband hou met die positiewe impak van MI op leerdermetakognisie. Die kwantitatiewe bevindinge ten opsigte van die korrelasie tussen leerdermetakognisie en wiskundeprestasie was slegs gedeeltelik deur die kwalitatiewe data ondersteun.

Hoofaanbevelings voortspruitend uit hierdie studie het betrekking op die verbetering van leerders se wiskunde-prestasie deur die verbetering van hulle *Verklarende kennis*, die verbetering van die leerders se probleemoplossingsvaardighede, en die noodsaaklikheid om metakognitiewe intervensies in wiskunde te implementeer veral in skole waar die onderwysers onervare is of onvoldoende gekwalifiseer is.

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CAPS	Curriculum and Assessment Policy Statement
DBE	Department of Basic Education
DSMK	Domain-specific metacognitive knowledge
GCR	Global Competitiveness Report
GSE	Graduate School of Education
KC	Knowledge of Cognition
MAI	Metacognitive Awareness Inventory
MI	Metacognitive Intervention
MPSAT	Mathematical Problem-Solving Achievement Test
MSRLS	Mathematics Self-Regulated Learning Scale
MSLQ	Motivated Strategies for Learning Questionnaire
NCS	National Curriculum Statement
NCTM	National Council of Teachers of Mathematics
NDE	National Department of Education
NRC	National Research Council
NSC	National Senior Certificate
PGCE	Postgraduate Certificate in Education
RC	Regulation of Cognition
SRL	Self-regulated Learning
TIMSS	Trends in International Mathematics and Science Study

CHAPTER 1

ORIENTATION

1.1 INTRODUCTION

The complexity of the physical universe continues to inspire awe. The difference in scale from the infinitesimally small subatomic particles to galaxies millions of light years apart suggests its intricacy. For the past thousands of years, humankind has been unravelling nature's mechanisms one by one. In fact, we have made such progress that some top scientists are attempting to formulate a "theory of everything" that would unify existing scientific theories

Will a "theory of everything" also include philosophical, spiritual and ethical issues? Despite some scientists' claims to the opposite, can we really disregard philosophical, spiritual and ethical issues in our quest for a "theory of everything"? We immediately realise how the introduction of these issues increases our world's complexity, since we could still fathom the possibility of uniting theories of physical phenomena, but the fusion of different philosophical, spiritual and ethical perspectives seems highly unlikely.

Both the physical world and the different philosophical, spiritual and ethical issues pose serious challenges to humankind's survival. Despite many setbacks, our history bears witness to our ability to adapt to a hostile environment and overcome oppressive systems of authority. How did we manage to not only survive, but also flourish to such an extent that the earth's ability to sustain humankind is in jeopardy?

A major reason for our survival is our ability to learn from our experiences and to convey those lessons to our children. Initially, these lessons were conveyed informally, but more formalised systems of education gradually developed. Arguably, not all formal education systems succeed equally well in preparing learners to survive in this multifaceted environment. In fact, many past debates centred on the characteristics of quality education and no finality on this issue has yet been reached – even if there is such a possibility. Nonetheless, it remains a crucial and continual quest to explore the characteristics of quality education that will enable learners to survive and flourish, as learners and as adults.

One characteristic of quality education should entail enabling learners to adapt to a rapidly changing environment in terms of knowledge acquisition and technological advances. In order to enable learners to cope with life's complexities and explore unique problems related to physical and ethical issues, it is crucial to facilitate thinking skills instead of conveying knowledge only. The successful facilitation of these cognitive skills should include first, the enhancement of learners' awareness of what cognitive skills entail and, secondly, the enhancement of learners' ability to monitor and regulate their cognitive processes. These two aspects entail learners' metacognitive awareness, which is the focus of this investigation.

1.2 BACKGROUND

The governments of most countries regard quality education as a top priority (Barber & Mourshed, 2007: 3). In South Africa, the importance of education is acknowledged, as the largest share of the national budget (21%) was allocated to education in 2011 (Gordhan, 2011). The value of quality education is reflected in a speech by South Africa's Minister of Basic Education and Training, Angie Motshekga, in which she states that education plays a fundamental role in human development, poverty eradication, economic growth and social transformation (Motshekga, 2011). Despite the funding allocated to education and the government's acknowledgement of the importance of education, there are national and international concerns about the quality of the South African education system as 60% to 80% of schools are considered to be dysfunctional (Bloch, 2009: 17).

In an effort to improve the quality of education, the Department of Basic Education (DBE) introduced a draft education sector plan in 2010, entitled "Action Plan to 2014: Towards the Realisation of Schooling 2025" (DBE, 2010a). This Action Plan sets out 27 goals to address deficiencies in the following areas: teachers; learner resources; whole school improvement; school funding; school infrastructure, and support services.

Eight of the first nine goals address specific subjects in different grades. Three of these eight goals pertain to the mastering of minimum competencies in specific subjects such as language and numeracy for Grade 3 (Goal 1); language and mathematics for Grade 6 (Goal 2), and language and mathematics for Grade 9 (Goal 3). A further two goals address increasing the number of learners who pass mathematics in Grade 12 (Goal 5) and physical science in Grade 12 (Goal 6).

The last three of these eight goals relate to increasing the average performance in Grade 6 languages (Goal 7), Grade 6 mathematics (Goal 8), and Grade 8 mathematics (Goal 9) (DBE, 2010a: 5-6). It is evident that improving mathematics achievement is regarded as a major concern, as five of these eight goals deal with mathematics. However, this concern stems not only from assessment results in the South African context, but also from results obtained from an international study on the Trends in International Mathematics and Science Study (TIMSS) and other international reports.

The TIMSS compares mathematics and science achievement between different countries. In 2003, the international average score for Grade 8 learners on the mathematics scale was 466. Singapore scored the highest, with a score of 605, and South Africa's Grade 8 learners scored the lowest of 46 countries, with a score of 264 (TIMSS, 2003: 5, 7). A further indication of the poor performance of South Africa's learners is evident from the fact that South Africa was placed third from last out of 134 countries with respect to the quality of mathematics and science education in 2009 (Dutta & Mia, 2009: 326) and 138th out of 142 countries in 2011 in the Global Competitiveness Report (GCR) (GCR, 2012: 343).

The small number of learners who obtain good results in Grade 12 mathematics and physical science is a cause for serious concern, as this leads to a shortage of professionals in the fields of medicine and financial management (DBE, 2010a: 17). Table 1.1 represents the 2010 National Senior Certificate (NSC) examination results for mathematics and physical science, and shows the total number and the percentage of candidates who obtained above 30% and 40%, respectively (DBE, 2011: 55-56).

Table 1.1: 2010 NSC examination results in mathematics and physical science

Subject	Total number of candidates	Achieved at 30% and above	Achieved at 40% and above
Mathematics	263 034	47.4%	30.9%
Physical science	205 364	47.8%	29.7%

Table 1.1 reflects similar achievement levels for mathematics and physical science. The achievement levels are indicated as 30% and 40%, respectively, because a learner can pass mathematics by either achieving 30% or 40%, depending on the learner's overall achievement. A learner passes mathematics by obtaining 30%, on the condition that 30% is obtained for two other subjects and 40% for home language and two more subjects (DBE, 2010a: 3-4). It is obvious that a 30.9% pass rate in mathematics (on the 40% level) raises serious concerns about learners' performance in mathematics, and thus the standard of mathematics education in South Africa.

An earlier discussion referred to the Action Plan's 27 goals for the improvement of the education system (see 1.2). Goal 5 of the Action Plan is to increase the number of Grade 12 learners who pass mathematics. The Action Plan does not explicitly mention whether Goal 5 refers to a pass percentage on the 30% or the 40% level, but it does state that presently "around one in seven youths leave school with a Grade 12 pass in mathematics" (DBE, 2010a: 17). It is argued that Goal 5 refers to a pass percentage on the 40% level in mathematics, because 552 073 learners wrote the 2009 Grade 12 NSC examination and one in seven (14.29%) learners obtained at least 40% for mathematics (DBE, 2010a: 3-4). The objectives of Goal 5 are to raise the number of learners who pass mathematics from 14.29% to 20% of the total number of candidates who obtain the NSC in 2014, and then to 33.3% in 2025 (DBE, 2010a: 17). Obviously, these targets can only be achieved first, if more learners take mathematics as a subject and, secondly, if learner achievement in mathematics improves.

This study focuses on the second aspect, namely the improvement of learner achievement in mathematics. From the previous discussion, it appears that there is a dire need to improve learner achievement in mathematics in South Africa. Concerns about mathematics achievement cannot be viewed from a mathematical point of view only, since many indirect factors emanating from the broader educational and community context of South Africa – for example, socio-economic and political factors – could influence mathematics achievement. However, it is important to identify potential factors that play a more direct role in mathematics achievement.

1.3 FACTORS CONTRIBUTING TO HIGH ACHIEVEMENT

Campione (1987: 136) suggests that, for high achievement in a specific domain, learners need knowledge about that domain, specific procedures for operating in that domain, and general task-independent regulatory processes. De Corte (1996: 34-36) affirms the importance of these aspects, adding affective components as another prerequisite by stating that studies in cognitive science have led to a broad agreement that expert performance in a given domain necessitates the integrated acquirement of four categories of aptitude, namely a structured, accessible domain-specific knowledge basis; heuristic methods; affective components, and metacognition.

These four categories of aptitude provide a more focused perspective on factors that influence achievement, in general, and mathematics, in particular. The role played by learner metacognition in mathematics achievement is of particular interest in this study. The reason for this is that learner metacognition involves first, knowledge of factors that could influence learners' mathematics achievement and, secondly, the ability to control and regulate their own process of learning in order to achieve well.

Flavell (1987: 27) sheds more light on the meaning and applicability of metacognition as follows:

> Metacognition is especially useful for a particular kind of organism, one that has the following properties. First, the organism should obviously tend to think a lot; by definition an abundance of metacognition presupposes an abundance of cognition. Second, the organism should be fallible and error-prone, and thus in need of careful monitoring and regulation. Third, the organism should want to communicate, explain and justify its thinking to other organisms as well as to itself; these activities clearly require metacognition. Fourth, in order to survive

> and prosper, the organism should need to plan ahead and critically evaluate alternative plans. Fifth, if it has to make weighty, carefully considered decisions, the organism will require metacognitive skills. Finally, it should have a need or proclivity for inferring and explaining psychological events in itself and others, a penchant for engaging in those metacognitive acts termed social cognition. Needless to say, human beings are organisms with just these properties.

Flavell's exposition of the relevance of metacognition for learners underscores the main elements of metacognition, namely knowledge of cognition; the monitoring and regulation of cognition, and being conscious of one's affective state. It is evident from these ideas that learners, in general, exhibit all the characteristics that warrant the application of metacognition in the process of learning. In the next section, a more detailed exposition of the role of metacognition in high achievement will be briefly discussed.

1.4 METACOGNITION AND ACHIEVEMENT

When solving problems, learners perform better when they become aware of their own thinking (Paris & Winograd, 1990: 15). Butler and Winne (1995: 245) affirm that learner awareness of thinking processes enhances effective learning and improves learner achievement. Shraw (1998: 114) supports this claim, stating that learner performance is enhanced by metacognitive regulation, because learners utilise resources and existing strategies more effectively. A study conducted by Camahalan (2006: 194) also affirms that students' academic achievement is more likely to improve when they are given the opportunity to monitor and regulate their learning strategies. Larkin (2010: 16) agrees that the enhancement of metacognition improves academic attainment and states that it also leads to the holistic development of learners. Further studies on learner metacognition and achievement in mathematics have also established a correlation between learner metacognition and mathematics achievement (see 2.3).

The ideas expressed in this and the previous sections point to the importance of metacognitive processes for better academic performance. Therefore, the enhancement of learner metacognition could enable them to perform better in mathematics. The following question arises: “How can learner metacognition be enhanced?” In the next section, some of the studies that explored this question by implementing metacognitive interventions are briefly discussed.

1.5 METACOGNITIVE INTERVENTION STUDIES

In an international context, some studies focused mainly on metacognitive intervention programmes aimed at enhancing mathematics achievement (see 2.4). The majority of these studies reported a significant, positive effect on the post-test measures of mathematics achievement (see 2.4). Some of these studies also investigated the effect of metacognitive intervention on learner metacognition (see 2.4.1). The majority of these studies reported a significant, positive effect on post-test measurements of learner metacognition (see 2.4.6).

In South Africa, relatively little has been published about metacognition in mathematics (Van der Walt, Maree & Ellis, 2006). One of these studies (Van der Walt, Maree & Ellis, 2008: 205-235) investigated the use of metacognitive strategies in mathematics (senior phase) and recommended that the implementation of metacognitive strategies be facilitated in schools and at universities. A different study by Van der Walt and Maree (2007: 223-241) investigated the value of metacognitive strategies in the learning of mathematics. These studies focused mainly on the use of metacognitive strategies, and not on the enhancement of metacognition. As no extensive metacognitive intervention study in mathematics has been done in the South African context, it seems imperative to conduct an investigation into the effectiveness of a metacognitive intervention to enhance learner metacognition and achievement in mathematics.

1.6 THE RESEARCHER’S PERSPECTIVE ON MATHEMATICS EDUCATION IN SOUTH AFRICA

As discussed earlier, the poor mathematics results reflect negatively on the quality of mathematics education in South Africa. However, a balanced perspective on the South African education system is needed. There are schools with good discipline, well-qualified teachers and well-equipped classrooms that deliver good results in general, and in mathematics, in particular (see Table 1.2). The researcher has first-hand experience of the South African education system that shaped his view on mathematics education and stimulated an interest in research.

The researcher taught secondary mathematics for twelve years, namely ten years in South Africa and two years in England. In terms of different education systems, this teaching experience ranged from different cultural contexts, different mediums of instruction (Afrikaans and English), and different ability groups (Mathematics Higher Grade and Mathematics Standard Grade) to different quintile schools, namely very well-resourced schools and very poor schools. A further five years' experience in the Higher Education Sector as a mathematics education lecturer consolidated many of the researcher's perspectives, but also challenged many of his beliefs about the effective teaching-and-learning of mathematics. A brief discussion of the researcher's views on mathematics education is necessary to reveal his interest in the research problem.

Despite continuing variations and growth in one's perspectives, the researcher always believed, as a teacher, in the value of challenging learners to think. Instead of delivering mathematics lectures, he endeavoured to enhance active learner involvement and understanding by challenging their responses, asking them to motivate their answers, and establishing a safe and friendly classroom context. The researcher also developed his appreciation for the opportunity authentic problems pose for deep engagement with mathematical content. His actual teaching method often conflicted with these perspectives due primarily to time constraints.

In his years of teaching, the researcher reflected on the difficulties learners experience in the learning of mathematics. These difficulties involve mainly the following four aspects. First, many learners found it difficult to understand how to link mathematical topics with one another. A second aspect concerns the ability of learners to study mathematics effectively. Often, the researcher had to answer questions related to an effective way of studying mathematics. This aspect includes some learners' lack of ability to take control of their own studies. Thirdly, the way in which colleagues presented mathematics often reflected the lecturing method as their preferred teaching method. This aspect can be related to poor understanding, as active learner involvement is not encouraged. Fourthly, many learners would enquire about the application of mathematics in everyday life. This aspect can also be related to poor understanding and to the way in which mathematics is presented.

As mathematics education lecturer, the researcher had more opportunity to reflect on these aspects for five years, during which he developed a special interest in the role of learner metacognition in the learning of mathematics. In 2007, the researcher completed a master's study on the implementation of metacognitive strategies by Grade 11 mathematics learners. Since then, the researcher has taken an interest in the same issue investigated in this study, namely the enhancement of learner metacognition and achievement in mathematics.

1.7 PROBLEM STATEMENT

In Section 1.2, some indications were given of the poor mathematics results in the 2010 NSC examination.

Although the DBE does not explicitly state the enhancement of learner metacognition as a key objective, the enhancement of mathematics results is a key priority. The DBE initiated a strong drive, in terms of policy, to improve mathematics results (see 1.2). Metacognition, on the other hand, was identified as one of the categories of aptitude required for high achievement in any domain (see 1.3 and 1.4). In fact, several studies reported that the enhancement of learner metacognition leads to better academic performance (see 1.4 and 1.5).

The research problem, therefore, entails the exploration of how learner metacognition can be enhanced in an attempt to improve the mathematics performance of South African learners in the NSC examination.

1.8 PURPOSE STATEMENT

The purpose of this study was to investigate the effect of a metacognitive intervention on learner metacognition and learner achievement in mathematics. The researcher used an embedded mixed methods design, where qualitative data obtained from a case study were embedded within a quasi-experiment. The Metacognitive Awareness Inventory (MAI) was used to measure the effect of metacognitive intervention (MI) on

Grade 11 learner metacognition at a secondary school in the Free State. The impact of MI on learner achievement was measured indirectly by determining the correlation between learner metacognition and mathematics achievement. Concurrently, the learners' metacognitive awareness in a problem-solving context was investigated by analysing written statements of their thinking processes and mathematical calculations. Open-ended questionnaires were used to explore learner and teacher perspectives on the MI process, while teacher interviews were conducted to explore their views on issues related to the subject mathematics and the teaching-and-learning of mathematics.

1.9 RESEARCH QUESTIONS

In a mixed methods study, an ideal approach may be to write quantitative research questions and qualitative research questions separately, followed by a mixed methods question (Creswell, 2009: 139). In the next sections, this approach is followed as qualitative research questions and a mixed methods research question ensue from the quantitative primary research question.

1.9.1 Primary research questions

The following quantitative primary research question arises from the foregoing discussion:

- Primary research question 1: Does MI have a statistically significant positive effect on learner metacognition and achievement in mathematics?

The following three qualitative primary research questions are explored:

- Primary research question 2: What is the effect of MI on learner metacognition and mathematics achievement in a problem-solving context?
- Primary research question 3: What are the teachers' views on the nature of mathematics and aspects related to the teaching-and-learning of mathematics?
- Primary research question 4: What are the perspectives of the experimental group's learners and their teacher on the MI process?

Some important aspects are evident in the first primary, quantitative research question. The following aspects need to be investigated: the conceptualisation of metacognition; the relationship between metacognition and achievement in mathematics; features of successful metacognitive interventions; features of an educational learning theory in mathematics, and the statistical significance of the impact of MI on learner metacognition and achievement in mathematics.

The second primary, qualitative research question requires an investigation into two aspects, namely learner metacognition in a problem-solving context prior to and after MI, and learners' problem-solving skills prior to and after MI.

The third primary, qualitative research question entails an exploration of the nature of mathematics and aspects related to the teaching-and-learning of mathematics from the perspective of both the experimental group's teacher and the control group's teacher.

The fourth primary, qualitative research question establishes the need to investigate two aspects, namely the perspectives of the experimental group's learners on the process of MI, and the process of MI from the perspective of the experimental group's teacher.

1.9.2 Secondary research questions

In this section, the secondary questions, ensuing from the four primary research questions, are stated.

1.9.2.1 Secondary research questions arising from the first primary research question

Perspectives gained from literature will enable the researcher to explore the following four secondary questions ensuing from the first primary research question:

- Secondary research question 1: How is metacognition conceptualised?
- Secondary research question 2: What is the relationship between metacognition and achievement in mathematics?
- Secondary research question 3: What are the features of some previous metacognitive interventions in mathematics?

- Secondary research question 4: What are the features of a proposed framework for a metacognitive intervention in mathematics?

The following two secondary research questions, ensuing from the first primary research question, are investigated by the representation, analysis, and interpretation of the empirical data collected in this study:

- Secondary research question 5: Does MI have a statistically significant positive effect on the metacognitive awareness of the experimental group's learners?
- Secondary research question 6: Is there a statistically significant positive relationship between learner metacognition and mathematics achievement?

The null and alternative hypotheses that result from secondary research questions 5 and 6 are stated in Chapter 4 (see 4.4.1.3c).

1.9.2.2 Secondary research questions arising from the second primary research question

The following secondary research questions, arising from the second primary, qualitative research question, are investigated with the focus on the experimental group:

- Secondary research question 7: What is the impact of MI on the level of learner metacognition in a problem-solving context?
- Secondary research question 8: What is the impact of MI on the level of mathematics achievement in a problem-solving context?

1.9.2.3 Secondary research questions arising from the third primary research question

The third primary, qualitative research question necessitates an exploration of the following two secondary research questions:

- Secondary research question 9: What are the perspectives of the experimental group's teacher on the nature of mathematics and aspects related to the teaching-and-learning of mathematics?

- Secondary research question 10: What are the perspectives of the control group’s teacher on the nature of mathematics and aspects related to the teaching-and-learning of mathematics?

1.9.2.4 Secondary research questions arising from the fourth primary research question

The following secondary questions ensue from the fourth primary, qualitative research question:

- Secondary research question 11: What are the perspectives of the experimental group’s learners on the MI process?
- Secondary research question 12: What are the perspectives of the experimental group’s teacher on the MI process?

1.9.3 Mixed methods research question

The purpose of this study is to investigate the effect of a metacognitive intervention on learner metacognition and learner achievement in mathematics (see 1.8). To accomplish this purpose from a mixed methods perspective, a mixed methods question is stated that combines some aspects from the quantitative and qualitative research questions, namely:

- Mixed methods research question: To what extent do the results from the qualitative phase of the study support the findings obtained from the quantitative phase of the study regarding the effect of MI on learner metacognition and mathematics achievement?

1.10 PHILOSOPHICAL WORLD VIEW

A researcher’s philosophical world view is a general way of viewing reality and the nature of research (Creswell, 2009: 6). In this study, reality is viewed as multifaceted. Therefore, hypotheses are tested, but findings are also explained from multiple perspectives, offering both unbiased and biased points of view. Hence, the pragmatic world view serves as the point of departure in the undertaking of this study.

Many authors regard pragmatism as the world view that corresponds best with mixed methods research. It enables researchers to employ practices that work well, to use varied approaches, and to regard objective and subjective knowledge as important (Creswell & Plano Clark, 2007: 26).

1.11 RESEARCH DESIGN

Research data were collected by using both quantitative and qualitative research methodologies. Babbie (1998: 38) confirms the legitimacy and usefulness of both types of research. In this study, quantitative and qualitative research methodologies are integrated to form a mixed methods approach. The rationale for combining quantitative and qualitative research is to give a more comprehensive description of the extent to which the main purpose of the study was achieved (Bryman, 2006: 106). The purpose of social research may be exploratory, descriptive, or explanatory. It may also serve more than one of these purposes (Babbie & Mouton, 2001: 79-81). In this study, the quantitative section serves an explanatory purpose as causality between variables is indicated. The purpose of the qualitative section is evident in the exploration of perspectives on the MI process.

The main assertion of mixed methods research is that a combination of a qualitative and a quantitative approach leads to a better understanding of the research problem (Creswell & Plano Clark, 2007: 5). Quantitative and qualitative data are combined, because the qualitative data provide a supportive role to the quantitative, primary data, and different questions require different types of data in order to address these questions (Creswell & Plano Clark, 2007: 67-69). In addition, the research problem could be better understood by triangulating the broad quantitative tendencies with rich, qualitative detail.

1.11.1 Quantitative methodology

In education, it is not always possible to randomly assign participants to experimental or control groups. Therefore, a pre-test–post-test non-equivalent group design was employed as it is one of the most common quasi-experimental designs in educational research (Cohen, Manion & Morrison, 2007: 283).

1.11.1.1 Sampling

Quantitative data were obtained from two intact Grade 11 classes from different schools. These classes were as similar as possible regarding characteristics such as race, gender, achievement in mathematics, socio-economic background, and aspects of the teaching-and-learning situation such as time allocated to teaching, teacher qualifications and experience, as well as the school environment.

1.11.1.2 Data collection

The questionnaire used to determine the learners' level of metacognition in both the pre-test and the post-test was the MAI, developed by Schraw and Dennison (1994). The MAI assesses metacognitive awareness in adolescents and adults (Schraw & Dennison, 1994: 461). Learners' report marks were used as a measure of their achievement in mathematics.

1.11.1.3 Reliability and validity

Reliability in quantitative research refers to the consistency and dependability of the instrument. A high degree of internal consistency was reported for the MAI with a Cronbach's *alpha* value of 0.95, and the two-factor model of metacognition, namely knowledge of cognition and regulation of cognition, was strongly supported = 0.90) (Schraw & Dennison, 1994: 460, 464). In other studies, the MAI was used to assess metacognitive awareness in mathematics (Mevarech & Fridkin, 2006; Mevarech & Amrany, 2008; Yunus & Ali, 2008) as well as metacognitive awareness in strategic learning and learning skills (Turan, Demirel & Sayek, 2009).

Validity is the main aspect to be considered in the development and evaluation of measuring instruments (Ary, Jacobs & Sorenson, 2010: 225). Validity is the extent to which an instrument measures what it claims to measure and the degree to which the interpretations of the instrument's scores are supported by evidence and theory. The validity of the interpretations of an instrument's scores is regarded as the salient feature of the concept validity. An instrument may, therefore, be valid in one situation for a specific purpose, but not in a different situation for a different purpose (Ary *et al.*, 2010:

225, 235). In this study, the purpose of the MAI was to measure learner metacognition for an experimental group and a control group.

Chapter 4 provides a more detailed discussion relating to the reliability and validity of the quantitative measurement employed in this study, namely the MAI (see 4.4.1.2a).

1.11.1.4 Data analysis

Data collected from two Grade 11 classes from different schools were used in this study. However, due to the small number of participants (25 learners in the experimental group, and 24 learners in the control group), non-parametric tests were used to test whether the hypotheses are supported, as suggested by Pietersen and Maree (2007a: 231).

The statistical significance of the possible differences in medians of the MAI total scores on both the pre-test and the post-test between the experimental group and the control group was determined by using the Mann-Whitney test, which is the non-parametric equivalent of the *t*-test for independent samples (Pietersen & Maree, 2007a: 231-233).

The Wilcoxon signed-rank test was used to determine the statistical significance of the possible differences within each group. The Wilcoxon test is the non-parametric test equivalent of the *t*-test for two related (dependent) samples (Pietersen & Maree, 2007a: 231-232). The relationship between learner metacognition and academic achievement was determined by calculating the Spearman rho correlation coefficient, which is a non-parametric measurement (Pietersen & Maree, 2007a: 237).

1.11.1.5 Role of the researcher

The researcher's role in the quantitative section of a study should be informed by the characteristic attitudes required of quantitative researchers (Ary *et al.*, 2010: 13-14). In this study, the researcher endeavoured to display a sceptical attitude towards the obtained data; to be objective and impartial; to focus on facts instead of values, and to integrate and organise the findings.

1.11.2 Qualitative methodology

In this study, a case study research methodology was employed to explore the qualitative research questions. The aim of the case study was to gain a deep understanding of the metacognitive awareness of the learners of the experimental group and to explore learner and teacher perspectives on the MI process.

1.11.2.1 Participants

Qualitative data were obtained from those participants who formed part of the experimental group in the quantitative part of this study.

1.11.2.2 Data-collection procedures

Multiple sources of evidence were used – for example, learners' written statements of their thinking processes in a problem-solving context; open-ended interviews on the teachers' perspectives on aspects related to the teaching-and-learning of mathematics; open-ended learner questionnaires on the experimental group's perspectives on the MI process, and reflections of the experimental group's teacher on the MI process.

1.11.2.3 Reliability and validity

Reliability (trustworthiness) in qualitative research refers to the consistency of the researcher's interactive approach and the recording, analysis and interpretation of data. These issues were addressed by maintaining documentary evidence of the raw data that were collected and by means of stepwise replication (see 4.4.2.3c).

The internal validity (credibility) and external validity (transferability) of the study were enhanced by using multiple methods of data collection (interviews, document analysis, open-ended questionnaires) and by involving a peer researcher during data interpretation (Nieuwenhuis, 2007: 80; see 4.4.2.3a, b).

1.11.2.4 Data analysis and interpretation

In this case study, a detailed description of the study's setting was followed by an analysis of the data for themes (Creswell, 2009: 184). Data were coded to generate themes for analysis. These themes were presented in tabular form for the purposes of interpretation, and in a chronological format to reflect the time schedule of the MI's different phases (Creswell, 2009: 189).

The interpretation of the data was related to the researcher's personal point of view, as influenced by his own history and experiences. Applicable references to related theory were made when the data are interpreted (Creswell, 2009: 189-190).

1.11.2.5 Role of the researcher

Case study reports are typically written from an emic (insider) and an etic (outsider) perspective (Ary *et al.,* 2010: 456). The researcher and the teacher of the experimental class, who acted as co-researcher, first strived to provide an emic perspective by focusing on the experiences of the case study's participants. They also provided an etic perspective by describing their interpretation of the data obtained.

1.12 DEMARCATING THE FIELD OF STUDY

Secondary school education serves as the contextual background of this study. The field of mathematics education and, more specifically, the achievement of learners in the NSC examination and the role metacognition plays in mathematics achievement, demarcate the field of this study.

The specific focus was on an experimental group, which was one Grade 11 class from School A and a control group, consisting of one Grade 11 class from School B. These schools for girls are located in the Motheo district of the Free State province and are multicultural, with English as the medium of instruction. Both schools are ranked as Quintile 5 schools. All public schools in South Africa are grouped in Quintiles where the level of poverty of the school's surrounding community determines the school's Quintile ranking, with Quintile 5 being the least poor (Giese, Zide, Koch & Hall, 2009: 30). Both schools obtained a 100% pass rate in the 2010 NSC examination (DBE, 2011: 111-112).

Table 1.2 illustrates the 2010 NSC results for both schools in mathematics, mathematical literacy and physical science (2011 Free State NCS results).

Table 1.2: Mathematics, mathematical literacy, and physical science results (2010 NSC)

	School A	School B
Mathematics		
Number of learners	34	98
Average	52.2%	79.3%
Achieved at 30% and above	97%	100%
Achieved at 40% and above	82%	100%
Mathematical literacy		
Number of learners	19	66
Average	69.4	78.4
Achieved at 30% and above	100%	100%
Achieved at 40% and above	100%	100%
Physical science		
Number of learners	21	80
Average	55.4	74.5
Achieved at 30% and above	100%	100%
Achieved at 40% and above	91%	100%

Table 1.2 clearly indicates the excellent results obtained by School B in mathematics, mathematical literacy, and physical science. It is evident that the performance of learners from School A, particularly in mathematics, was not as good as that of learners from School B. However, the performance of the mathematics learners of School A was still relatively good, as 97% of them passed mathematics on the 30% level, whereas only 47.4% of the total number of learners in South Africa passed mathematics on the 30% level in the 2010 NSC examination (see Table 1.1).

1.13 THESIS STRUCTURE

The next six chapters of this study are structured as follows:

- Chapter 2 comprises a literature review on the origin and definition of metacognition. The relationship between metacognition and achievement in mathematics, with specific reference to metacognitive interventions in mathematics and the features of successful metacognitive interventions in mathematics are discussed. Secondary questions 1, 2 and 3 are addressed in this chapter.

- The nature of mathematics, mathematical proficiency and the relation between general educational learning theory and effective learning in mathematics are discussed in Chapter 3. These aspects and the features of successful metacognitive interventions in mathematics, as discussed in Chapter 2, are synthesised to propose a framework for metacognitive interventions in mathematics. In this chapter, secondary research question 4 is addressed.
- The research design is described in Chapter 4. The following aspects of the research design are discussed: the researcher's philosophical world view; research methodologies, and the specific research methods.
- The representation, analysis and interpretation of the quantitative research data are given in Chapter 5. The results pertaining to secondary research questions 5 and 6, as well as to the five hypotheses are addressed in this chapter.
- In Chapter 6, the representation, analysis and interpretation of the qualitative research data are discussed. Secondary questions 7 to 12 and the mixed methods research question are addressed in this chapter.
- Chapter 7 contains a summary of the results for each primary research question in the form of findings, conclusions and recommendations. The significance and limitations of the study are discussed, while recommendations for further research are proposed.

Table 1.3 indicates in which chapter the research questions and hypotheses will be addressed.

Table 1.3: Research questions and/or hypotheses per chapter

Chapter	Research question and/or hypothesis
2	First primary research question; secondary research questions 1, 2 and 3.
3	First primary research question; secondary research question 4.
5	First primary research question; secondary research questions 5 and 6; hypotheses 1 to 5.
6	Primary research questions 2 to 4; secondary research questions 7 to 12 and mixed methods research question.

1.14 CONCLUSION

In this chapter, the need to improve the quality of mathematics education in South Africa was established. Metacognition was identified as one of the four categories of aptitude that plays a role in high achievement in mathematics. The need to implement MI in the South African context became evident when the effectiveness of international studies on metacognitive interventions was considered. The purpose of the study, namely to investigate the effect of MI on learner metacognition and mathematics achievement, was outlined and specific research questions were stated. Brief references to the researcher's philosophical world view and aspects of the research design were made. The final part of this chapter focused on the demarcation of the research area and the thesis structure.

In the next chapter, three main themes are explored and discussed as part of the literature review on metacognition, namely the origin and definitions of metacognition; the relationship between metacognition and mathematics achievement by focusing on metacognitive intervention studies, and the features of metacognitive interventions in mathematics. These themes are explored with the intention of identifying aspects that should be included in the development of this study's MI.

CHAPTER 2

METACOGNITION: CONCEPTUAL BASIS, RELATION TO MATHEMATICS ACHIEVEMENT, AND INTERVENTIONS

2.1 INTRODUCTION

In Chapter 1, an orientation to this study was given. Four primary research questions and twelve secondary research questions were stated. The aim of this chapter is to gain perspectives from literature in order to explore the first three secondary research questions. From these perspectives, aspects of previous metacognitive interventions were identified in order to include them in the development of this study's MI.

Secondary research question 1 seeks to explore the conceptualisation of metacognition. First, the origin of the term *metacognition* and different definitions thereof are discussed (see 2.2.1 and 2.2.3). The relationship between metacognition and cognition is examined as part of the discussion on the definition of metacognition (see 2.2.2). Flavell's (1979: 906) definition of metacognition serves as the basis for the exploration of the four categories of metacognition (see 2.2.4). The relationship between the four metacognitive categories in the monitoring and regulation of cognitive processes is investigated (see 2.2.5). As metacognition, self-regulation and self-regulated learning (SRL) are very similar concepts, the relationship between these three concepts is discussed (see 2.2.6). A summary of the aspects related to the conceptualisation of metacognition concludes the exploration of secondary research question 1 (see 2.2.7).

Secondary research question 2 deals with the relationship between learner metacognition and achievement in mathematics (see 2.3). This relationship is discussed with specific reference to eight studies that investigated this relationship and one study that examined the link between self-regulation and mathematics achievement. Six of these nine studies implemented metacognitive interventions.

In order to address secondary research question 3, the features of these metacognitive interventions are investigated. These studies are discussed, with specific reference to the following aspects: aims; age and gender of the participants; intervention period; theoretical basis; method of intervention, and assessment of metacognition (see 2.4).

Table 2.1 provides an overview of the different sections of this chapter and their relation to the applicable research questions.

Table 2.1: The relation between the different sections of Chapter 2 and the first three secondary research questions

Research question	Section
Secondary research question 1: How is metacognition conceptualised?	2.2 (2.2.1-2.2.7)
Secondary research question 2: What is the relationship between metacognition and achievement in mathematics?	2.3 (2.3.1-2.3.10)
Secondary research question 3: What are the features of some previous metacognitive interventions in mathematics?	2.4 (2.4.1-2.4.6.4)

2.2 CONCEPTUALISING METACOGNITION

In the following sections (see 2.2.1-2.2.7), the first secondary research question is explored. The first use of the term *metacognition* in literature is examined next.

2.2.1 Origin of the term *metacognition*

Research activity in metacognition was initiated by John Flavell who is regarded as the "father of the field" (Papaleontiou-Louca, 2003: 9). He began his research in metacognition when he realised that children aged between six and nine did not apply their knowledge on memory-enhancing strategies (Boekaerts & Simons, 1995: 89). Metacognition was first used as a term in the 1970s, and sporadic references were made to metacognition in the literature of the early 1980s. Although metacognition was not well understood, it became a frequently used term in the latter part of the 1980s and a foremost topic in the field of cognitive developmental research (Schoenfeld, 1992: 9; Papaleontiou-Louca, 2003: 9).

An analysis of the literature on metacognition reveals that it became a more precisely defined concept in the 1990s (Hacker, 1998: 11). Larkin (2010: 12) notes that there are two strands evident in metacognition research. First, research focusing on information-processing and cognition has been dominant since the 1970s. The second strand entails social psychology, which addresses the social and cultural context of metacognitive awareness, building on the work of Vygotsky (Larkin, 2010: 12-13).

The next section provides a more detailed description of various definitions of metacognition.

2.2.2 Cognition

Flavell (1976: 232) views metacognition as referring "… among other things, to the active monitoring and consequent regulation and orchestration of those processes in relation to the cognitive objects or data on which they bear, usually in the service of some concrete goal or objective". This initial definition underscores individuals' active involvement in performing a task and awareness of their thinking processes during the performing of the task. Flavell's definition reveals the nature of metacognition, namely the awareness, monitoring and regulation of cognition. Therefore, a brief discussion of the concept *cognition* is important in order to enhance understanding of the concept *metacognition.*

Cognition can be defined as "... knowing in its broadest sense. It includes the reorganization of objects and attributing meaning to them. It also distinguishes between the self and others. It includes the more specific aspects such as perception, concept formation, reasoning, thinking, fantasy and imagination" (Van den Aardweg & Van den Aardweg, 1993: 41).

McMillan (2011: 148) views reasoning as the overarching higher level thinking skill. According to McMillan (2011: 148), the three elements of reasoning are mental skills, knowledge, and task. In reasoning, mental skills are used in the manipulation of declarative or procedural knowledge needed to perform a task. The mental skills such as classify, compare, analyse, and synthesise are differentiated from the task such as problem-solving, creative thinking, critical thinking, hypothesising, and generalising (McMillan, 2011: 148).

Another definition states that cognition "... refers to a variety of higher level mental processes such as comprehending, analyzing, reasoning, problem-solving, and evaluating" (Lock, 2003: 61). In contrast to McMillan's view, cognition, rather than reasoning, is viewed as an encompassing term for higher level thinking skills in this definition. In this study, cognition will be viewed as the overarching higher order thinking skill for the following reasons.

In the first definition by Van den Aardweg and Van den Aardweg (1993: 41), cognition is regarded as knowing in its broadest sense. They include reasoning as a subcomponent of cognition. The second definition of cognition (Lock, 2003: 61) also refers to the variety of higher level mental processes associated with cognition, once again referring to reasoning as an aspect of cognition. Therefore, as reasoning is only regarded as an aspect of higher level thinking, according to these definitions of cognition, the remaining discussion will treat the concept cognition as the overarching term for the different levels of thinking skills.

Theorists are not clear on the specific nature of the interaction between metacognition and cognition (Larkin, 2010: 16). Cognitive strategies are used when a mathematics problem is solved, but metacognitive processes are employed when learners are aware of their thinking about the problem, or when they begin to evaluate their progress in solving the problem (Larkin, 2010: 16). Although a theoretical distinction can be made between metacognition and cognition, in practice learners constantly alternate between metacognitive and cognitive processes (Larkin, 2010: 16). This interchange can be brief – for example, when learners perceive that they do not understand some aspects of the problem. Longer periods of metacognitive awareness are evident when learners consciously draw on past experience to devise problem-solving strategies (Larkin, 2010: 16). It remains difficult to distinguish between metacognitive and cognitive processes, unless learners discuss their thinking processes (Larkin, 2010: 16). Veenman (2011: 205) also stresses the difficulty of distinguishing between these concepts, because strategic processes have cognitive and metacognitive features.

Although the distinction between cognition and metacognition is not precise, a better understanding of metacognition could be achieved when different definitions thereof are explored.

2.2.3 Definitions of metacognition

In literature, the first definition of metacognition is by Flavell (1979: 906) who distinguishes between the following four categories of metacognition, namely metacognitive experience; metacognitive knowledge; metacognitive goals, and metacognitive strategies. Larkin (2010: 8) regards Flavell's original conceptualisation of metacognition as possibly the prime theory of metacognition to date.

Prior to discussing these four categories (see 2.2.4.1-2.2.4.4), some later definitions of metacognition are examined in order to identify possible relationships between different conceptualisations of metacognition.

As definitions of metacognition differ, Schoenfeld (1992: 2, 38-39) asserts that "metacognition has multiple and almost disjoint meanings (for example, knowledge about one's thought processes, self-regulation during problem-solving) which make it difficult to use as a concept". He summarises various definitions of metacognition into different categories, namely the declarative knowledge learners have about their cognitive processes; self-regulation; beliefs, and affect.

In a later definition, De Corte (1996: 35-36) states that metacognition is the knowledge and beliefs about cognition, in addition to the skills and strategies enabling the self-regulation of cognitive processes. It is apparent that De Corte's definition reflects the ideas expressed in Schoenfeld's definition, but it does not include declarative knowledge about beliefs and affect. Hacker's definition (1998: 11) includes all the aspects referred to in Schoenfeld's earlier definition by asserting that the definition of metacognition should at least include the following aspects: knowledge of one's knowledge; the conscious monitoring and regulating of one's knowledge, and cognitive and affective states.

In a more recent definition, Papaleontiou-Louca (2003: 12) states that metacognition refers to "… all processes about cognition, such as sensing something about one's own thinking, thinking about one's thinking and responding to one's own thinking by monitoring and regulating it". This definition also stresses the awareness, monitoring and regulating of cognition without explicitly referring to beliefs and affect. One could argue that the use of the phrase "sensing something about one's own thinking" includes awareness of one's beliefs and attitudes during the performance of a cognitive task. This definition does not explicitly mention the knowledge aspect of cognition.

Larkin (2010: 3) refers specifically to the knowledge aspect of cognition when she states that "meta" indicates a change of position, or a second order or higher level, and "cognition" refers to a person's faculty of knowing or thinking. Therefore, metacognition refers to one's ability to be aware of and reflect on one's thoughts. As in the above-mentioned definition by Papaleontiou-Louca's (2003: 12), no direct references are made to beliefs and affect, although awareness of and reflection on one's thoughts would also imply awareness of and reflection on one's emotional state.

These various definitions of metacognition have in common the subcomponents *knowledge of cognition* and *regulation of cognition*. The summaries of the different facets of metacognition by Hacker (1998: 11) and Schoenfeld (1992: 38-39) contain an additional subcomponent that refers to the *knowledge and regulating of one's affective state*. In respect of this study, metacognition is viewed as the *knowledge of cognition* and the *regulation of cognition* as these subcomponents are common to all definitions of metacognition that were discussed.

Reference was made earlier in this section to Flavell's (1979: 906) distinction between the four categories of metacognition. The next sections (see 2.2.4.1-2.2.4.4) present a more comprehensive analysis of these four categories of metacognition.

2.2.4 The four categories of metacognition

The following categories of metacognition are discussed: metacognitive experiences; metacognitive knowledge; metacognitive goals, and metacognitive strategies.

2.2.4.1 Metacognitive experiences

These are conscious experiences about one's feelings, thoughts, and attitudes, and concern any aspect of the cognitive processes – for example, when one suddenly perceives that one does not understand what the teacher said. It can also be related to learners' awareness of their progress towards a goal (Flavell, 1979: 906; Flavell, 1981: 286; Larkin, 2010: 12). Metacognitive experiences may also occur when a person experiences a feeling that a certain task or question is difficult to understand, remember, or solve. Tasks that require careful and conscious thinking and concentration increase metacognitive experiences (Papaleontiou-Louca, 2003: 15). As metacognitive experiences involve emotions and feelings, they can influence a person's mood which, in turn, affects a person's motivation during the problem-solving process (Larkin, 2010: 12). Metacognitive experiences are only beneficial if they are worked through and not disregarded as being too time-consuming or psychologically demanding (Larkin, 2010: 9).

2.2.4.2 Metacognitive knowledge

Metacognitive knowledge stems from metacognitive experiences (Larkin, 2010: 9). Flavell (1979: 906) states that metacognitive knowledge may be declarative ("knowing that") or procedural ("knowing how"). Declarative knowledge is a person's conscious knowledge about him-/herself, others and reality. From a metacognitive point of view, declarative knowledge equates with knowledge of persons as cognitive beings and knowledge of mental processes (Larkin, 2010: 10). Procedural knowledge entails a person's knowledge of how to do something by applying different strategies or skills. In terms of metacognition, this involves reflecting on a specific task and on the use of strategies (Larkin, 2010: 10). Both types of metacognitive knowledge can be employed – for example, one might "know that" it is helpful to identify the main concept in a mathematics question and "know how" to do that (Flavell, 1979: 906; Papaleontiou-Louca, 2003:14; Larkin, 2010: 8). A third type of metacognitive knowledge is conditional knowledge of when, why and how to use one's knowledge (Schraw & Dennison, 1994; Larkin, 2010: 11). In metacognitive terms, conditional knowledge is associated with a person's knowledge of the thinking processes of monitoring and control (Larkin, 2010: 11).

An individual's metacognitive knowledge of the factors that act and interact on cognitive tasks comprises three variables, namely person, task and strategy. The person variable includes beliefs about intra-individual differences – for example, a belief that one remembers facts better than another; inter-individual – for example, a belief that one studies better in total silence, and cognitive aspects in general – for example, the awareness that one forgets many things that have been learned with time and an understanding of how performance is affected by attention, concentration and remembering (Papaleontiou-Louca, 2003: 14, 15; Larkin, 2010: 8). The person variable is interactively linked with the task variable (Larkin, 2010: 8).

Task variables necessitate different ways to deal with tasks that are different in nature. Learners may ask questions about the similarities and differences between tasks. For example, it is better to really understand a geometry proof rather than to study the proof by heart, whereas another task could require the learner to study in a different way (Papaleontiou-Louca, 2003: 15; Larkin, 2010: 8).

The strategy variable of metacognitive knowledge deals with knowledge about the use of the most effective strategy in different tasks (Papaleontiou-Louca, 2003:14; Larkin, 2010: 9). A learner, for example, realises that identifying the main points of a new concept and rephrasing it leads to effective learning.

2.2.4.3 Metacognitive goals

Metacognitive goals refer to the purposes of a cognitive activity – for example, when a learner sets him-/herself specific goals when studying (Flavell, 1979: 906). Flavell (1981: 286) posits that metacognitive knowledge and metacognitive experiences significantly influence the development, pursuit and achievement of goals. These goals range from short-term to long-term goals and involve "expectations about the intellectual, social and emotional outcomes for students as a consequence of their classroom experiences" (Artzt & Armour-Thomas, 1998: 9).

2.2.4.4 Metacognitive strategies

Metacognitive strategies refer to the conscious monitoring and regulation of one's cognitive strategies to achieve specific goals – for example, when learners ask themselves questions about the work and then observe how well they answer these questions (Flavell, 1981: 273). Boekaerts and Simons (1995: 91) view metacognitive strategies as the decisions learners make prior to, during and after the process of learning. For example, a learner may know of the metacognitive strategy *Thinking aloud* (metacognitive knowledge) and then decide to implement that strategy during the learning process.

There are various metacognitive strategies, applicable to any grade or subject, that can enhance learners' metacognition, namely planning strategy; generating questions; choosing consciously; setting and pursuing goals; evaluating the way of thinking and acting; identifying the difficulty; paraphrasing, elaborating and reflecting learners' ideas; clarifying terminology; thinking aloud; journal-keeping; cooperative learning; modelling by teachers, and problem-solving activities (Costa, 1984: 59-61; Blakey & Spence, 1990: 2-4; Brown, in Boekaerts & Simons, 1995: 91; Flavell, Koutselini & Trilianos, in Papaleontiou-Louca, 2003: 18).

2.2.4.4a Planning strategy

Planning plays a role prior to, during and after the completion of a learning activity. When learners plan their approach to a learning activity well, it can enhance their prospects of successfully completing the learning activity. The role of the teacher is to assist learners in planning by clearly communicating time restrictions, goals and ground rules pertaining to a learning activity. If learners internalise this information and instructions, they will bear them in mind during the learning activity and monitor their performance against the criteria of the learning activity. After a learning activity, learners should assess their performance and the effectiveness of their planning (Costa, 1984: 59; Blakey & Spence, 1990: 3; Koutselini & Trilianos, in Papaleontiou-Louca, 2003: 18-19).

2.2.4.4b Generating questions

At the start of a learning activity, learners should ask themselves what they know and what they do not know about the task. Further questions include whether they understand the question, and whether they can link the main concept in the question to other concepts and to prior knowledge (Ratner, 1991: 32). The connections between prior knowledge and new concepts should also be sought during the learning activity, since the integration of prior knowledge and new concepts enables learners to understand the unified and interconnected nature of knowledge, while facilitating profound understanding of subject matter (Ornstein & Hunkins, 1998: 240; Muijs & Reynolds, 2005: 63).

2.2.4.4c Choosing consciously

Learners make many decisions during the completion of a learning activity, and the results of their decisions and choices should be fully explored. As learners become more aware of the consequences of their choices, they will recognise the importance of contemplating the consequences of their decisions before the decision is made (Costa, 1984: 60; Koutselini & Trilianos, in Papaleontiou-Louca, 2003: 19).

2.2.4.4d Setting and pursuing goals

The setting and pursuing of goals can improve learners' regulation of their cognitive strategies if they remain aware of their goals during the learning process. Teachers should assist learners in setting goals and help them reflect on their progress during the learning process (Trilianos, in Papaleontiou-Louca, 2003: 19). This metacognitive strategy differs from the third metacognitive category *metacognitive goals* (see 2.2.4.3) by focusing more on the monitoring and regulation of one's progress towards a goal, whereas *metacognitive goals* place more emphasis on learners becoming aware of the necessity of setting goals.

2.2.4.4e Evaluating one's way of thinking and acting

Learners' awareness of their own thinking processes and actions can be enhanced if they complete evaluative criteria about the learning activity and their performance during a learning activity. Teachers could initially develop these criteria in cooperation with the learners to support them in evaluating their own thinking until their self-evaluation skills become more independent. For example, learners could be asked to evaluate the learning activity by stating easy and difficult aspects and what they liked and did not like in the learning activity. Consequently, learners bear the criteria in mind when classifying their opinions about the learning activity and motivate the reasons for those opinions. Guided self-evaluation can be introduced by means of checklists with a focus on thinking processes, and hence self-evaluation will increasingly be applied more independently (Costa, 1984: 60; Blakey & Spence, 1990: 3; Koutselini & Trilianos, in Papaleontiou-Louca, 2003: 20).

2.2.4.4f Identifying the problem

When learners find it difficult to complete a learning activity successfully, using phrases such as “I can’t”; “I am too slow to …”, or “I don’t know how to …”, teachers should support learners in identifying the specific problems they experience. Specific problems could be the resources, skills and information they need in order to attain the learning outcome. As a result, learners grow in their ability to distinguish between their current knowledge and the knowledge they still require (Costa, 1984: 60; Presseisen, in Papaleontiou-Louca, 2003: 20).

2.2.4.4g Paraphrasing, elaborating and reflecting learners’ ideas

When learners are encouraged to restate, translate, compare and paraphrase problem statements, the teacher’s statements, and other learners’ ideas, they will become more conscious of their own thinking and their communication skills will improve. The term “articulation” could be used to describe learners’ expression of their thoughts and ideas (Costa, 1984: 61; Koutselini & Trilianos, in Papaleontiou-Louca, 2003: 21; Muijs & Reynolds, 2005: 64).

During this articulation process of problem statements and others’ ideas, learners will demonstrate whether they have a profound understanding of the concepts involved (Carpenter & Lehrer, 1999: 22; Muijs & Reynolds, 2005: 64). Articulation of one’s thoughts and ideas can be enhanced by paired problem-solving: learners discuss their thinking processes in pairs and help one another clarify their thinking by listening and asking questions (Blakey & Spence, 1990: 2).

2.2.4.4h Clarifying learners' terminology

Learners may make vague statements when referring to aspects of a learning activity – for example, "The question is not fair", or "The question is too difficult". These statements should be clarified by encouraging learners to explain why the question is too difficult or unfair (Costa, 1984: 61). Learners are, therefore, challenged to refer to specific parts of a question or activity with which they experience problems.

2.2.4.4i Thinking aloud

Learners should be encouraged to express their thoughts verbally, that is, to "think aloud" (Costa, 1984: 61). Talking about their thinking will help learners become more aware of their thinking processes (Blakey & Spence, 1990: 2). As stated earlier, Muijs and Reynolds (2005:64) use the term "articulation" to describe learners' expression of their thoughts and ideas. They recommend that learners should discuss complex tasks and present their ideas to fellow learners.

Thinking aloud may also enable learners to act more reflectively and reduce impulsive behaviour (Diaz, Neal & Amaya-Williams, 1990: 135-136). Camp, Blom, Hebert and van Doornick, (1977: 160) developed a program called *Think Aloud* to improve learners' self-control. Learners were taught to use the following four questions when solving problems: "What is my problem?"; "How can I do it?"; Am I using my plan?"; and "How did I do?" These four questions resemble the four stages of Polya's problem-solving model (see 2.2.4.4m).

2.2.4.4j Journal-keeping

Throughout the learning process, learners will encounter new insights, common mistakes, misconceptions and knowledge on how to deal with these mistakes and misconceptions. This information could be recorded in a personal diary that reflects learners' growth when their preliminary insights are compared with subsequent insights noted in their personal journals (Costa, 1984: 61; Blakey & Spence, 1990: 3).

2.2.4.4k Cooperative learning

Cooperative learning may promote awareness of learners' personal thinking and of that of others. When learners act as "tutors", the process of planning what they are going to teach leads to independent learning and clarifies the learning material (Blakey & Spence, 1990: 2). Although cooperative learning is regarded as a teaching strategy, it may also be viewed as a metacognitive strategy, as it stimulates learners' awareness of their thinking processes. As a metacognitive strategy, *cooperative learning* has a great deal in common with the metacognitive strategies *thinking aloud* and *paraphrasing, elaborating and reflecting learners' ideas*, because it also involves the continuous articulation of one's ideas and reflection on the contributions of other group members.

2.2.4.4l Modelling by teachers

Larkin (2010: 7) states that metacognitive behaviour should be explicitly modelled to learners with little skill in metacognitive processing. Modelling occurs when teachers *think aloud* when they teach, thereby demonstrating their thinking and motivation for selecting certain strategies when solving problems. This enhances learners' awareness of their own thinking (Blakey & Spence, 1990; 2; Muijs & Reynolds, 2005: 63).

Schoenfeld (1987: 200) refers to the importance for educators of not always displaying the finished, neat presentation of the answers on the board, but to sometimes model and work through the problems step-by-step. Consequently, the processes yielding the correct answer – for example, false starts, recoveries from false starts, and interesting insights – are exposed and the chief purpose of the modelling approach is achieved, namely the centring of learners' awareness on metacognitive behaviours**.** Larkin (2010: 7) supports the notion of teachers modelling each step of the problem-solving process by arguing that metacognition will develop if learners become aware of their thinking during the process of thinking, rather than reflecting on the task afterwards. Post-task reflection on thinking involves a possibly wrong interpretation of what was thought during the completion of the task; therefore, learners' thinking about their thinking during the execution of a task is preferable.

Other aspects that denote educators' metacognitive behaviour include explaining their planning, goals and objectives to the learners, and motivating their actions; acknowledging their temporary inability to answer a question, but developing pathways for finding the answer; making human mistakes, but demonstrating how to correct those mistakes; requesting comments and assessment of their actions; acting in accordance with an explicitly stated value system; explaining what their strengths and weaknesses are, and expressing an understanding and valuing of the ideas and feelings of other people (Costa, 1984: 61; Costa & Trilianos, in Papaleontiou-Louca, 2003: 21).

From the previous discussion, it is evident that modelling comprises many aspects. Costa (1984: 61) suggests that modelling could be the most effective strategy to enhance metacognition among learners, because they learn best by imitating teachers. Teachers, therefore, have a great responsibility because "... a fair proportion of the learning problems in mathematics are actually taught to the children …" (Moodley, 1992: 8).

2.2.4.4m Problem-solving activities

Problem-solving is discussed as one of the metacognitive strategies, but it may also be regarded as the overarching metacognitive strategy, since the application of all other metacognitive strategies is required for successfully solving a problem.

In problem-solving, existing knowledge is applied to an unfamiliar situation in order to gain new knowledge (Killen, 2000: 129). Problem-solving activities present ideal opportunities for the enhancement of learner metacognition, since learners who are good at problem-solving are usually also self-aware thinkers. These learners demonstrate their advanced metacognitive skills by their ability to analyse their problem-solving strategies and reflect on their thinking processes (Blakey & Spence, 1990: 2; Panaoura, Philippou & Christou, 2001: 3).

The mathematician George Polya is best known for his conceptualisation of problem-solving in mathematics (Schoenfeld, 1992: 339). Polya (1945: v) emphasises the importance of the problem-solving process by stating that learners' interest, intellectual development and motivation can be impeded if mathematics teachers focus solely on routine, drill-and-practise exercises. Schoenfeld (1992: 334-335) concurs, stating that it is generally agreed that the most important objective of mathematics instruction should be to assist learners to become competent problem-solvers. Polya (1945: 6-19) suggests that a teacher should stimulate learners' curiosity by posing interesting problems, proportionate to their knowledge, in order to enhance their independent thinking abilities. Sufficient time should be allowed to present the problem in an interesting way.

Polya distinguishes between four phases of the problem-solving process: understanding the problem; devising a plan; carrying out the plan, and looking back (Polya, 1945: 6-19). These four phases are discussed next.

- ***Understanding the problem***

Learners must understand the verbal statement of the problem to such a degree that they can easily repeat the problem statement. The main feature of the problem should be identified and that aspect, for which a solution must be obtained, the unknown, should be clearly pointed out. All given information, or data, and any conditions must also be indicated. This process of understanding the problem should be repeated, and learners should attempt to understand the problem from different perspectives (Polya, 1945: 6-7). During this process of understanding the problem, the teacher should ask questions such as, among others, "Are you all convinced that you understand the problem?" (Schoenfeld, 1987: 202).

- ***Devising a plan***

A plan consists of an outline of the calculations and constructions that will lead to the solution and is regarded as the core aspect of finding the solution. Polya (1945: 8, 12) views the making of a plan as difficult, because it requires the learners' concentration, good luck and tenacity. Learners could receive unobtrusive help in this phase.

The importance of factual mathematical knowledge, prior knowledge and past experience are prerequisites for devising a plan. Consequently, learners should attempt to think of a related problem with a similar unknown that has been solved previously. If no link with a related problem can be established, the problem should be modified, that is, restated differently. Learners could first try to solve a related problem, but they risk losing sight of the original problem, if attention is only focused on the modified problem. As a result, learners should always bear in mind that they should use all given information and that their plan involves all given conditions (Polya, 1945: 8-10).

- ***Carrying out the plan***

Polya (1945: 12) regards this phase as easier than devising a plan, because learners only need to exercise patience in order to successfully carry out the plan. During this phase, each element of the plan needs to be scrutinised. Each step should be checked and, if possible, proved correct. In this regard, Costa (1984: 61) notes that teachers, instead of merely correcting the learners, should encourage them to clarify their course of action. Polya (1945: 13) perceives a potential danger in this instance, namely that learners would not be able to clarify their course of action if they had forgotten their original plan. For that reason, it is imperative that learners carry out their own instead of their teacher's plans. Schoenfeld (1987: 202) suggests that these plans could be changed. He states that, after the learners have worked on the problem for approximately five minutes, the teacher could ask them whether the process is going well, and if not, to reassess their strategies. If the learners decide to reject their strategies, the teacher could ask whether anything helpful could be recovered from their efforts. In contrast to Polya's statement that learners only need to exercise patience in order to successfully carry out the plan, Schoenfeld anticipates the possibility that learners would have to change their problem-solving strategies.

- ***Looking back***

Polya (1945: 14) regards this phase as very instructive, because learners' knowledge is consolidated and their problem-solving skills are enhanced if they re-examine the problem-solving process. When a solution is re-examined by checking the argument and the result, three important aspects of the problem-solving process emerge.

First, no problem is ever fully explored; further involvement will yield alternative and often better solutions. The initial understanding of the problem can also be enhanced

(Polya, 1945: 15). These alternative solutions serve to more strongly convince learners of the correctness of the answer, just as verification by sight and touch is stronger evidence than verification by sight or touch only (Polya, 1945: 15).

Secondly, as learners try to find alternative solutions to a problem, they recognise the connections between different mathematical problems and topics and between mathematics and other topics. Polya (1945: 15) stresses the duty of a teacher to help learners discover these connections. The process of looking back can also stimulate learners' levels of interest if they are encouraged to find alternative solutions.

Thirdly, common aspects of seemingly unrelated problems should be recognised and, therefore, learners should ask themselves how the problem-solving strategy or result could be useful in solving other problems (Polya, 1945: 15-16).

Schoenfeld (1987: 202) also recommends that a teacher should review the problem-solving process with the learners and guide them in finding alternative solutions to the problem. This verification of the effectiveness of problem-solving strategies and the comparing of different solutions are also viewed as important elements of the reflective thinking process (Muijs & Reynolds, 2005: 64).

Current research indicates that the problem-solving process in mathematics is not a four-phase linear process, as put forward by Polya, but rather a complex recursive process (DuBois, Clinton, Trowell & Fincher, 2011: 369). However, when the four phases of problem-solving are considered, the following three aspects indicate that Polya's problem-solving model is a complex process with strong recursive elements.

First, learners are encouraged in the first phase to understand the problem from different perspectives, implying that learners re-visit the problem statement in a recursive manner. Secondly, learners should attempt to modify the problem statement (first phase) when they do not make progress in devising a plan (second phase). A clear interaction between the first two phases is evident. Thirdly, each element of the plan is scrutinised in the third phase, thereby indicating a direct interaction with the second phase. Hence, an indirect interaction with the first phase also occurs due to the recursive link between the first two phases. A fourth aspect pointing very convincingly to the recursive nature of Polya's problem-solving model involves the fourth phase. The re-examining of the problem-solving process by re-visiting the previous three phases indicates the recursive nature of Polya's problem-solving model. In addition, learners with experience in applying this model (the fourth phase, in particular) will tend to increase the degree to which they work in a recursive manner, as they have the requirements of the fourth phase in their minds when they start with the first phase.

In conclusion, successful problem-solving may be viewed as evidence of high competency in mathematics. The successful integration of all metacognitive strategies could enhance the prospects of solving the problem. In addition, the integration of all four components of metacognition may enhance holistic learner engagement during the problem-solving process.

In the next section, the interaction between the four categories of metacognition is examined.

2.2.5 Interaction between the four categories of metacognition

The four categories of metacognition were discussed in the previous section (see 2.2.4). It was not the intention of Flavell to view the four categories of metacognition as discrete subsets, but as active and interactive categories through which cognitive monitoring occurs (Flavell, 1981: 286; Brown, 1984: 214; Papaleontiou-Louca, 2003: 13). The significant effect of metacognitive experiences on metacognitive goals, metacognitive knowledge, and metacognitive strategies can serve as an example of this interaction. A sense of failure (metacognitive experience) can cause the redefining of metacognitive goals. Metacognitive knowledge can be adapted – for example, when learners sense (metacognitive experience) a lack of knowledge of a certain topic and therefore study that topic again to improve their knowledge. Finally, the use of metacognitive strategies may be triggered when learners wonder (metacognitive experience) whether they understand a topic correctly; consequently, they employ the metacognitive strategy of asking themselves questions about the topic.

It is clear that the "monitoring" role played by metacognitive experiences leads to the "regulation" of cognitive aspects. In discussing the definition of metacognition, Flavell distinguishes between "monitoring" and "regulation" (see 2.2.2). However, Flavell (1981: 272-273) asserts that "monitoring" incorporates "regulation", because monitoring entails both a regulatory and a feedback function. His argument seems plausible, especially in the light of the role metacognitive experiences play in cognitive monitoring.

Even so, a clearer picture of metacognition emerges when both "monitoring" and "regulation" are used in a discussion of metacognition. The processes of monitoring and regulation require conscious engagement by a learner, but it is still debatable whether metacognition is only a conscious act, or whether automatic processing that requires little control is also viewed as metacognition (Larkin, 2010: 6). In this regard, Veenman (2011: 205-211) states that monitoring processes may be consciously applied, but they may also be of an involuntary nature and thus only emerge when errors or anomalies are detected.

However, a number of definitions of metacognition refer to its self-regulatory aspect (see 2.2.3). Reference was made earlier to the similarity between the concepts *metacognition*, *self-regulation* and *SRL* (see 2.1). It is, therefore, important to examine the relationship between these three concepts.

2.2.6 Metacognition, self-regulation and SRL

The specific nature of the interrelationships between metacognition and other self-regulated constructs is not clear (Sperling, Howard, Staley & DuBois, 2004: 120). However, educational literature reveals that metacognition, self-regulation and SRL are three interrelated concepts that display many similarities (Alexander, 2008: 369). There had been a growing interest in metacognition, self-regulation and SRL in the decade leading up to 2008, but these concepts are not precisely defined. In fact, other concepts such as knowledge, motivational constructs, and learning also lack conceptual clarity (Alexander, 2008: 369). Until 2008, the conceptual boundaries between metacognition, self-regulation and SRL had also not been investigated (Alexander, 2008: 370).

The different conceptualisations of metacognition, self-regulation and SRL necessitate a review and synthesis of educational literature with the aim to identify further areas of investigation (Alexander, 2008: 369). Alexander (2008: 370) states that it is not crucial, or even advisable, to have one definition for a multifaceted educational construct, because cross-fertilisations between different constructs occur frequently. However, it is imperative to investigate the original conceptualisations of metacognition, self-regulation and SRL and to compare these with recent conceptualisations (Alexander, 2008: 370).

Although the terms metacognition, self-regulation and SRL only started to appear in literature since the 1970s, the combined work of three foundational theorists – William James, Jean Piaget and Lev Vygotsky – provides an integrated view of these concepts (Fox & Riconscente, 2008: 373).

James's work on introspection was foundational to many aspects of "modern" psychology. His view on introspection relates to metacognition, whereas his extensive writings on Habit and Will relate to self-regulation and SRL (Fox & Riconscente, 2008: 376). The process of introspection that James investigated involved aspects such as depths of consciousness, attention, the Self, and Will. For James, one's view of metacognition and self-regulation is determined by one's relation to Self, because control of attention, self-knowledge and self-awareness are prerequisites for introspective observation and the purposeful control of one's behaviour (Fox & Riconscente, 2008: 374-375). To him, it was imperative that teachers encourage habits – practical, emotional, and intellectual – in their learners, as habits are signs of automated mental activity. James's view of the Will can be linked to SRL, because he described Will as the voluntary enacting of strategies and effort (Fox & Riconscente, 2008: 376-377).

Piaget developed an extensive and systematic body of work on human development and learning (Fox & Riconscente, 2008: 378). The progress of children through the different developmental stages, described by Piaget, involves the awareness of, interaction with, and efforts to control objects in one's environment. These processes relate closely to the processes involved in metacognition and self-regulation (Fox & Riconscente, 2008: 378). Links with metacognition are apparent in Piaget's notion of formal operations, as the ability to think about one's own thoughts is a characteristic of formal operations (Fox & Riconscente, 2008: 379). As Fox and Riconscente (2008: 380) view self-regulation as the deliberate control of one's thoughts and actions, they recognise links with self-regulation in two components of reason in Piaget's work, namely intellect and affect. Piaget regards intellect as the purposeful regulation of one's thoughts and problem-solving strategies, and affect as one's will, that is, the control of one's emotions and needs (Fox & Riconscente, 2008: 380).

To Vygotsky, a learner's psychological development results from internalised social interactions, mainly through the use of language. Internalisation enhances conscious abstraction, reflection and deliberate control. In Vygotsky's work, metacognition corresponds with the concept of consciousness. In addition, self-regulation is strongly associated with metacognition, because the intentionality of self-regulation requires consciousness (Fox & Riconscente, 2008: 383).

The literature of the 1970s reveals that self-regulation initially indicated behavioural, emotional, and motivational regulation in interaction with the environment. In contrast to the behavioural roots of self-regulation, metacognition has its roots in a cognitive orientation (Dinsmore, Alexander & Loughlin, 2008: 393-394). A current distinction between self-regulation and metacognition relates to the trigger of self-awareness in a learner. In self-regulation, the environment stimulates awareness, but in metacognition the mind of the individual is regarded as the factor that initiates self-awareness (Dinsmore *et al.,* 2008: 405).

Later literature on self-regulation focused more on self-regulatory aspects in academic settings, which led to the emergence of the term SRL. A key aspect of SRL is its roots in an academic setting, whereas metacognition and self-regulation were not initially focused on academic settings only (Dinsmore *et al.*, 2008: 405). Despite their different developmental paths, there is a common conceptual bond between metacognition, self-regulation and SRL (Dinsmore *et al.*, 2008: 404). However, Dinsmore *et al.* (2008: 405) state that metacognition, self-regulation and SRL should not be regarded as synonymous constructs, although they have a common conceptual bond.

Kaplan (2008: 479) confirms this conceptual bond, by stating that the boundaries between metacognition, self-regulation and SRL are very vague and porous. He does not view them as three clearly distinguished concepts, but argues that metacognition, self-regulation and SRL are different subtypes of the phenomenon *self-regulated action.* Instead of using the term *boundaries* to distinguish between the three concepts, Kaplan (2008: 480) proposes the term *dimensions.* He argues that the term *boundaries* positions these concepts against one another, whereas the term *dimensions* indicates the possibility for one concept to gradually merge into the other (Kaplan, 2008: 480, 483).

Hence, metacognition, self-regulation and SRL could be placed at different points on, first, a dimension with *cognition* and *external behaviour* as the two poles and, secondly, a dimension with *individual* and *environment* as the two poles (Kaplan, 2008: 480). Metacognition is closer to the *cognition* and *individual* poles; SRL is nearer to the *external behaviour* and *environment* poles, and self-regulation is positioned near the centre of the two poles.

It is evident from the discussion in this section that metacognition cannot be separated conceptually from self-regulation and SRL, although it has more pronounced *individual* and *cognition* dimensions than self-regulation and SRL. It is likely that there is no hierarchical differentiation between these concepts, as they represent different subtypes of *self-regulated action.*

The aim of Section 2.2 was to address secondary research question 1 by exploring the conceptualisation of metacognition from a literature perspective. In the next summary, important aspects emerging from this discussion are highlighted.

2.2.7 Summary

A summary of the literature perspectives on the concept *metacognition* reveals the following aspects. First, cognition may be regarded as the overarching term for thinking skills. Initially, metacognition was viewed as the awareness, monitoring and regulation of cognition. Although this may imply that metacognition and cognition are separate concepts, it is difficult to achieve a clear distinction between metacognitive and cognitive processes (see 2.2.1-2.2.2).

Secondly, metacognition has been defined in many different ways, but Flavell's original definition of metacognition may yet be the key definition. It entails the following four categories: metacognitive experiences; metacognitive knowledge; metacognitive goals, and metacognitive strategies (see 2.2.3-2.2.4). Table 2.2 presents Flavell's definition of metacognition compared to that of other authors (see 2.2.3).

Table 2.2: The definition of metacognition

Flavell (1976, 1979)	**Schoenfeld (1992), De Corte (1996), Hacker (1998), Papaleontiou-Louca (2003), Larkin (2010)**
Metacognitive knowledge Declarative; procedural. Person variable; task variable; strategy variable.	*Knowledge of cognition* Declarative; procedural; conditional.
Metacognitive experiences Awareness of one's affective state.	*Knowledge and regulating of one's affective state*
Metacognitive goals Awareness of the importance of setting goals.	
Metacognitive strategies Monitoring and regulation of one's cognitive strategies.	*Regulation of cognition*

Table 2.2 shows that Flavell's definition of metacognition encompasses other conceptualisations, with the exception that Flavell did not explicitly mention conditional knowledge. His definition includes two aspects not explicitly referred to by the other authors, namely the different variables under *metacognitive knowledge* and *metacognitive goals.*

Thirdly, these four categories interact closely during the process of cognitive monitoring and regulation (see 2.2.5).

Fourthly, metacognition, self-regulation and SRL are interrelated concepts on the same hierarchical level, but metacognition has more prominent *individual* and *cognition* dimensions than self-regulation and SRL.

These four aspects shed more light on the concept *metacognition*, but an exploration of perspectives from literature indicates that the facets of metacognition and metacognitive processes are not precisely defined and understood.

In the next section, secondary research question 2 is explored.

2.3 METACOGNITION AND ACHIEVEMENT IN MATHEMATICS

In this section, the relationship between learner metacognition and achievement in mathematics is discussed. Specific references will be made to eight studies (Studies 1-3; 5-9) which investigated aspects related to metacognition and achievement in mathematics, and to a study (Study 4) by Camahalan (2006: 194-205) that examined the role of self-regulation learning strategies. Study 4 is included, first, because metacognition and self-regulation have a common conceptual bond (see 2.2.6, 2.2.7) and, secondly, because it investigates the use of some foundational self-regulated learning strategies (see 2.4.5.3).

Table 2.3 presents a brief summary of the author(s), year of publication, and title of these nine studies.

Table 2.3: Studies that investigated the relationship between metacognition and achievement in mathematics

Study	Author(s)	Year	Title
1	Mevarech, Z.R. & Kramarski, B.	1997	IMPROVE: A multidimensional method for teaching mathematics in heterogeneous classrooms.
2	Kapa, E.	2001	A metacognitive support during the process of problem-solving in a computerised environment.
3	Cetinkaya, P. & Erktin, E.	2002	Assessment of metacognition and its relationship with reading comprehension, achievement, and aptitude.
4	Camahalan, F.M.G.	2006	Effects of self-regulated learning on mathematics achievement on selected Southeast Asian children.
5	Mevarech, Z.R. & Fridken, S.	2006	The effects of IMPROVE on mathematical knowledge, mathematical reasoning, and metacognition.

Study	Author(s)	Year	Title
6	Desoete, A.	2007	Evaluating and improving the mathematics teaching-learning process through metacognition.
7	Van der Walt, M.S., Maree, J.G. & Ellis, S.M.	2008	Metacognition in the learning of mathematics in the senior phase: Some implications for the curriculum.
8	Mevarech, Z.R. & Amrany, C.	2008	Immediate and delayed effects of metacognitive instruction on regulation of cognition and mathematics achievement.
9	Özsoy, G. & Ataman, A.	2009	The effect of metacognitive strategy training on mathematical problem-solving achievement.

2.3.1 Study 1 (Mevarech & Kramarski, 1997)

In Study 1, Mevarech and Kramarski (1997: 365-394) investigated the effect of a metacognitive intervention programme (IMPROVE) on the mathematics achievement of Grade 7 learners. This acronym stands for Introduction of new concepts; Metacognitive questioning; Practising; Reviewing and reducing difficulties; Obtaining mastery; Verification, and Enrichment (Mevarech & Kramarski, 1997: 369).

The study consisted of two parts. In the first part, a 36-item algebra test was used to measure achievement. The first section of the test consisted of 25 factual knowledge items which were objective in nature, while the second section comprised 11 open-ended items which required of the learners to explain their reasoning. The purpose of including the open-ended items was to better understand the learners' reasoning processes (Mevarech & Kramarski, 1997: 372-373). Scoring criteria were developed for the qualitative part of the study in order to perform statistical tests of significance on the qualitative section.

The results for the quantitative section show that the experimental group performed significantly better than the control group in the mathematics post-test, but only in the middle and higher achieving learner groups. Significantly better mathematics performances by the lower-, middle-, and higher achieving learners of the experimental group were only obtained on the qualitative section of the post-test (Mevarech & Kramarski, 1997: 380-381).

Mevarech and Kramarski (1997: 382) offer some possible reasons why the lower achieving learners only performed significantly better on the qualitative section of the test. It was possible, first, that the lower achieving learners did not attempt to solve the higher level questions of the quantitative section that required several steps. Secondly, the quantitative section of the test did not have sufficient items that measured precisely enough the variations in the achievement of the lower achieving learners. Lastly, the metacognitive intervention was strongly focused on improving learners' articulation of reasoning processes. Therefore, although lower achieving learners in the experimental group showed improved articulation of their reasoning processes, they possibly found basic factual knowledge items on the quantitative section of the test difficult (Mevarech & Kramarski, 1997: 382).

2.3.2 Study 2 (Kapa, 2001)

Results of Study 2 (Kapa, 2001: 317-336) on Grade 8 learners' mathematics problem-solving achievement indicated a significant improvement in the performance of learners with low prior knowledge. The instrument that measured their problem-solving ability during a pre-test and a post-test consisted of 10 word problems with authentic contexts such as selling and buying, velocity, time and rate (Kapa, 2001: 326).

The treatment groups were structured as follows: the first group was given metacognitive support during the problem-solving process and at the end of the process; the second group received metacognitive support only during the process, and the third group was given metacognitive support only at the end of the problem-solving process. A fourth group was designated as the control group.

Learners with low prior knowledge in the treatment groups showed a significant improvement in their performance in comparison to learners with high prior knowledge in the treatment and control groups. Learners in the first two treatment groups with low prior knowledge also performed significantly better in the post-test than learners with low prior knowledge in the third treatment group (Kapa, 2001: 328-329).

Kapa (2001: 329-330) explains this finding by stating that metacognitive directives given during the problem-solving process stimulate the learners' prior knowledge which activates relevant thought processes. By contrast, metacognitive directives given only at the end of the problem-solving process do not stimulate prior knowledge, but require of the learner to think creatively. A second possible reason may be related to the problem-solving habits of the learners. As learners are normally more product-oriented in mathematics, metacognitive prompts given during the problem-solving process may help enhance the learners' organisation and comprehension during the phases of problem-solving that are more process-oriented, thereby guiding them to the solution (product). This is more effective than metacognitive prompts that are only given at the end of the problem-solving process (Kapa, 2001: 329-330).

It was also found that learners with high prior knowledge in the treatment groups did not perform significantly better than learners with high prior knowledge in the control group. Moreover, no significant differences were found when the performances of the learners with high prior knowledge in the three treatment groups were compared (Kapa, 2001: 329, 331). Kapa (2001: 332) explains these findings by claiming that learners with high prior knowledge already possess strong problem-solving skills and, therefore, do not need metacognitive directives to the same extent as learners with low prior knowledge.

2.3.3 Study 3 (Cetinkaya & Erktin, 2002)

Findings that failed to relate mathematics achievement to learner metacognition were reported in Study 3 (Cetinkaya & Erktin, 2002: 1-10). As part of their study, they examined, among other things, the relationship between Grade 6 learners' metacognition and achievement in mathematics. Their score on a 32-item metacognition inventory was correlated with their average mathematics score at the end of the academic year (Cetinkaya & Erktin, 2002: 5).

A possible explanation for the absence of a significant relationship between metacognition and mathematics achievement could be the result of the measure used for mathematics achievement, namely their course grades, which the researchers view as possibly invalid measures of true achievement (Cetinkaya & Erktin, 2002: 9).

2.3.4 Study 4 (Camahalan, 2006)

In Study 4, Camahalan (2006: 194-205) investigated, as part of a bigger study, whether Grade 4 and Grade 6 mathematics learners who received instruction in self-regulation learning strategies achieved significantly better in a mathematics achievement post-test than the control group. The structure of the mathematics achievement pre-test and post-test is not discussed in the study. Results indicate that the treatment groups performed significantly better than the control groups in the mathematics achievement post-test (Camahalan, 2006: 199).

In a second part of the study, the effect of instruction in self-regulation on mathematics school grade was examined. A learner's mathematics school grade at the end of the third term comprised the pre-test score, whereas the post-test score was the mathematics school grade at the end of the fourth term. The findings of the second part of the study, however, do not indicate that mathematics school grades of the treatment groups were significantly better (Camahalan, 2006: 201).

Camahalan (2006: 201) explains this finding by stating that the mathematics school grade is the weakest measure employed in her study, because it is related to the way in which mathematics is taught in the traditional classroom where the focus is on learner skills such as copying and memorising. In these classrooms, the teacher is the most important source of information, and the emphasis on speed and accuracy may influence learners to view mathematics as answer-centred and not process-centred (Camahalan, 2006: 201). Camahalan's views correspond with Cetinkaya and Erktin's (2002: 9) opinion that course grades could be a less accurate measure of achievement in mathematics.

2.3.5 Study 5 (Mevarech & Fridken, 2006)

In Section 2.3.1, the effects of IMPROVE on mathematics achievement was discussed. In Study 5, Mevarech and Fridken (2006: 85-97) investigated the effects of IMPROVE on mathematical knowledge, mathematical reasoning, and metacognition. Unlike Study 1 that involved Grade 7 learners, Study 5 involved pre-college students in mathematics. Study 1 involved conventional tests to measure algebra achievement. Study 5 investigated the effect of metacognitive instructional methods on non-conventional, authentic mathematical tasks (Mevarech & Fridken, 2006: 88).

In Study 5, the students had three hours to solve five open-ended problems (mathematical knowledge) and write justifications for five correct mathematical propositions (mathematical reasoning). Results indicate that the experimental group performed significantly better than the control group on mathematical knowledge and mathematical reasoning in the post-test (Mevarech & Fridken, 2006: 93). The fact that the participants in Study 5 elected to take the course and were highly motivated to achieve well lends more weight to the results of Study 5, when compared to Study 1. Therefore, the control group did not perform worse due to lack of motivation (Mevarech & Fridken, 2006: 96).

2.3.6 Study 6 (Desoete, 2007)

In a longitudinal study (Study 6) by Desoete (2007: 705-730), the relationship between metacognitive skills and arithmetic reasoning skills of Grade 3 mathematics learners was investigated. The mathematics measures consisted of two parts. The first part involved mental arithmetic and number knowledge tasks. The second part measured the number of questions on basic arithmetic which the learners could solve in five minutes (Desoete, 2007: 710). Similar measures were applied one year later in Grade 4. The learners' metacognitive skills, as measured by teacher ratings, showed a significant positive relationship with arithmetic reasoning skills (Desoete, 2007: 715).

2.3.7 Study 7 (Van der Walt, Maree & Ellis, 2008)

In a study (Study 7) by Van der Walt *et al.* (2008: 205-235), a further indication of the relationship between metacognition and achievement in mathematics was reported. The metacognitive skills of Grade 9 learners were measured and correlated with their examination marks in mathematics. Positive significant relationships between examination marks and the metacognitive skills of prediction, monitoring, evaluation, and reflection were obtained (Van der Walt *et al.*, 2008: 221). However, mixed results were obtained when the *Motivated Strategies for Learning Questionnaire* (MSLQ) scores were correlated with mathematics achievement.

The MSLQ measured two main metacognitive skills, namely cognitive strategies and self-regulation (Van der Walt *et al.*, 2008: 216). A significant positive relationship was found between mathematics achievement and self-regulation, but not between mathematics achievement and cognitive strategies (Van der Walt *et al.,* 2008: 221). A possible explanation for the absence of a significant positive relationship between mathematics achievement and the use of cognitive strategies could be that the poor-performing learners assessed their cognitive strategies as too high due to a lack of metacognitive knowledge (Van der Walt *et al.,* 2008: 227).

2.3.8 Study 8 (Mevarech & Amrany, 2008)

In Studies 1 and 5, the effect of IMPROVE on mathematics achievement was discussed. A third study (Study 8), conducted by Mevarech and Amrany (2008: 147-157), examined the effects of IMPROVE on mathematics achievement. The participants in this study were Grade 12 learners, as opposed to Grade 7 learners and pre-college students in Studies 1 and 5, respectively.

The mathematics achievement pre-test contained 13 items about the completed unit of work that was regarded as foundational knowledge for the new unit of work they would do. The post-test consisted of 16 items on the completed new unit of work. In both tests, all items, with the exception of the first one, were open-ended and the learners had to explain in writing how they obtained each answer. The findings indicate that the experimental group achieved significantly better than the control group in the post-test (Mevarech & Amrany, 2008: 152).

2.3.9 Study 9 (Özsoy & Ataman, 2009)

In a study (Study 9) conducted by Özsoy and Ataman (2009: 67-82), the effect of metacognitive strategy training on mathematical problem-solving achievement was investigated. The *Mathematical Problem-Solving Achievement Test* (MPSAT) was used to measure the mathematical problem-solving achievement of Grade 5 learners. The MPSAT consists of 20 items that test behaviours in line with Polya’s four stages of problem-solving (Özsoy & Ataman, 2009: 73). The results show that the experimental group performed significantly better than the control group in the mathematics post-test (Özsoy & Ataman, 2009: 78).

2.3.10 Summary

A summary of important aspects relating to these nine studies is provided in this section. Table 2.4 provides a summary of the mathematics achievement measuring instruments and the results of these nine studies. The information contained in Table 2.4 arises from the discussion in sections 2.3.1-2.3.9.

Table 2.4: Measurement instruments and results of studies that investigated the relationship between metacognition and mathematics achievement

Study	Mathematics achievement measuring instrument(s)	Results
1	Quantitative: 36-item algebra test (25 factual knowledge items) (pre-test and post-test). Qualitative: (11 open-ended questions). (pre-test and post-test).	The experimental group (middle- and higher achieving groups) performed significantly better than the control group on the quantitative section in the post-test. The experimental group (lower-, middle- and higher achieving groups) performed significantly better than the control group in the post-test.
2	Qualitative: Ten word problems in an authentic context (pre-test and post-test).	Learners with low prior knowledge showed a significant improvement in the post-test when compared to learners with high prior knowledge. Learners with low prior knowledge in the first two treatment groups also performed significantly better in the post-test when compared to learners with low prior knowledge in the third treatment group. No significant differences were found for learners with high prior knowledge. No control groups were used.
3	Not stated whether quantitative or qualitative. Average of mathematics course grades.	No significant relationship was found between metacognition and mathematics achievement.

Study	Mathematics achievement measuring instrument(s)	Results
4	[Not stated whether quantitative or qualitative. Mathematics achievement test (pre-test and post-test).] [Not stated whether quantitative or qualitative. Mathematics grade at the end of Term 3 (pre-test).] [Not stated whether quantitative or qualitative. Mathematics grade at the end of Term 4 (post-test).]	[Experimental group performed significantly better than the control group in the post-test.] [Experimental group did not perform significantly better than the control group in the pre-test.] [Experimental group did not perform significantly better than the control group in the post-test.]
5	i) Qualitative: Five open-ended problems (pre-test and post-test). ii) Qualitative: Writing justifications for correct mathematical propositions (pre-test and post-test).	The experimental group performed significantly better than the control group in the post-test for both measures.
6	i) Qualitative: Mental arithmetic and number knowledge tasks (pre-test and post-test). i) Qualitative: Number of questions on basic arithmetic solved in five minutes (pre-test and post-test).	Learners' metacognitive skills showed a significant positive relationship with both measures of arithmetic reasoning skills.
7	Not stated whether quantitative or qualitative. Examination marks in mathematics.	Positive, significant relationships were found between examination marks and the following metacognitive skills: prediction, monitoring, evaluation, and reflection. A significant positive relationship was found between mathematics achievement and self-regulation, but not between mathematics achievement and cognitive strategies.
8	Qualitative: Thirteen items regarded as foundational for new unit (pre-test). All items, with the exception of the first one, were open-ended. Qualitative: Sixteen items on a completed unit (post-test). All items, with the exception of the first one, were open-ended.	The experimental group performed significantly better than the control group on the post-test.
9	Qualitative: MPSAT (20 items that test behaviours in line with Polya's problem-solving model) (pre-test and post-test).	The experimental group performed significantly better than the control group in the mathematics post-test.

Table 2.4 indicates that all these studies, with the exception of Study 3, point to a significant positive correlation between learner metacognition and mathematics achievement. Possible explanations for the lack of a positive significant correlation between learner metacognition and mathematics achievement, offered by the authors of Study 3, were discussed earlier (see 2.3.3). In Study 7, a significant positive relationship between mathematics achievement and one category on the MSLQ scores (cognitive strategies) could not be established. Possible reasons for this finding were discussed earlier (see 2.3.7).

In interpreting the results of these nine studies, one should bear in mind that mathematics achievement is a broad concept which was measured quantitatively (Study 1) and qualitatively (Studies 2, 5, 6, 8 and 9). All studies, with the exception of Studies 3 and 7, employed pre-test and post-test measurements in the quantitative and qualitative measurements.

In two studies (Studies 3 and 4), it is explicitly stated that course grades are not very accurate measures of true mathematics achievement. It is likely that these two studies, as well as Study 7, used a combination of quantitative and qualitative measurements of mathematics achievement.

It was shown that these results must be interpreted with caution, due to the many different measures of mathematics achievement that were used, although they definitely point to a positive, significant correlation between learner metacognition and mathematics achievement, especially mathematics achievement measured by means of open-ended questions and involving problem-solving contexts. Larkin (2010: 17) also warns against a rigid interpretation of the relationship between metacognition and mathematics achievement, due to the difficulty of accurately measuring a construct such as metacognition that is not precisely defined. She further states that other variables such as emotions and motivation also affect learner performance (Larkin, 2010: 17).

In summary, the purpose of this section was to address secondary research question 2 that seeks to explore perspectives from literature on the relationship between learner metacognition and achievement in mathematics. Next, secondary research question 3 is addressed by exploring features of previous metacognition intervention studies in mathematics. Six of the nine studies discussed in this section also implemented a metacognitive intervention. In the next section, the features of these metacognition interventions are examined.

2.4 FEATURES OF METACOGNITIVE INTERVENTIONS IN MATHEMATICS

This section focuses on the six studies that examined the effects of metacognitive intervention on mathematics achievement. It is important to note that these six studies (Studies 1, 2, 4, 5, 8 and 9) all reported learner improvement in mathematics-related aspects that include mathematical reasoning, mathematical problem-solving skills, mathematics achievement, and mathematics knowledge (see Table 2.4). The aim of four of these studies was to enhance self-regulation or metacognition (see 2.4.6). Where necessary, a distinction will be made between the two studies that only measured mathematics-related aspects and the four studies that measured self-regulation or metacognition.

First, these six studies are discussed with reference to the following aspects: aims; age and gender of participants; intervention period; theoretical basis; methods of intervention, and the assessment of metacognition (see 2.4.1-2.4.6). A summary of the aspects of metacognitive interventions in mathematics concludes this section (see 2.4.7).

2.4.1 Aims

A brief overview of the aims of these studies shows that they endeavoured to improve mathematics achievement in the following areas: mathematical reasoning (Studies 1 and 5); mathematical knowledge (Study 5); mathematical problem-solving skills (Studies 2 and 9), and scores in a mathematics achievement test (Studies 4 and 8). Four of these studies also stated the following aims: the enhancement of self-regulation (Study 4) and the enhancement of metacognition (Studies 5, 8 and 9).

2.4.2 Age and gender of participants

The participants' ages represent a broad range. These studies included learners from Grade 4 (Study 4); Grade 5 (Study 9); Grade 6 (Study 4); Grade 7 (Study 1); learners aged 13-14 (Study 2); Grade 12 (Study 8), and pre-college students (Study 5). The participants in all these studies were both males and females.

2.4.3 Intervention period

The metacognitive intervention periods also display a wide range, namely four weeks (Studies 5 and 8); six weeks (Study 4); eight weeks (Study 2); nine weeks (Study 9), and the entire academic year (Study 1).

2.4.4 Theoretical basis

The theoretical basis of each intervention study, as it relates to the aim(s) of the study (see 2.4.1), is examined next.

2.4.4.1 Study 1 (Mevarech & Kramarski, 1997)

In Study 1, Mevarech and Kramarski (1997: 365-394) implemented a metacognitive intervention programme called IMPROVE.

The theoretical basis of IMPROVE relates to the role of the following three aspects that may enhance mathematical reasoning: strategy acquisition and metacognitive questioning should take place in a problem-solving context; learning should also incorporate cooperative settings, and corrective feedback and enrichment opportunities should be provided (Mevarech & Kramarski, 1997: 369).

2.4.4.2 Study 2 (Kapa, 2001)

Study 2 reflects a similar theoretical grounding as evident in Study 1. Kapa (2001: 318) elaborates the well-established link between metacognition and problem-solving in order to enhance problem-solving skills. In addition, Kapa (2001: 319) used a theoretical model that suggests distinct metacognitive skills for each phase of the problem-solving process.

The six phases of the problem-solving process, as evident in Study 2, differ from Polya's four-phase problem-solving model in the following ways. In Study 2, Polya's first phase (understanding the problem) is divided into the following two parts, namely *identifying and defining the problem,* and *mental representation of the problem. Identifying and defining the problem* refers to the coding of the main elements of the problem, whereas the *mental representation of the problem* denotes the synthesis of the main elements of the problem.

The third and fourth phases in Study 2 are similar to Polya's second and third phases, but Polya's last phase (looking back) is divided into two parts, namely *evaluation of one's performance*, and *reaction to feedback*. *Evaluation of one's performance* relates closely to Polya's fourth phase, as it also signifies the checking of one's solution and finding alternative approaches and solutions to the problem. However, *reaction to feedback* emphasises, in a more direct way than Polya's fourth phase, the value of corrective feedback in enhancing metacognition and problem-solving skills.

Another obvious aspect in Study 2 is the role of feedback in enhancing problem-solving skills. Unlike Study 1, cooperative learning is not an explicit feature of the theoretical grounding of Study 2.

2.4.4.3 Study 4 (Camahalan, 2006)

The first aim of Study 4 relates to the enhancement of mathematics achievement. The theoretical grounding of this study is rooted in the positive effect of enhanced learner self-regulation on learning (Camahalan, 2006: 194).

The theoretical basis that relates to the enhancement of self-regulation in Study 4 involves the following aspects. First, instead of external control imposed by the teachers, the learners' internal control of their learning process informed the intervention that focused on the training of self-regulated strategies (Camahalan, 2006: 194-195). Camahalan (2006: 196) distinguishes between the following four aspects that are important to the explicit training of self-regulated strategies. The first aspect refers to learner knowledge and beliefs. Learner knowledge entails the knowledge of why, how and when to use self-regulating learning strategies, whereas learner beliefs refers to the setting of goals, motivation, and self-efficacy (Camahalan, 2006: 196). The emphasis on learner beliefs denotes a distinct difference between this study and Studies 1 and 2.

The second aspect relates to the notion that self-regulated strategies can be explicitly taught, similar to the third aspect in Study 1.

The third aspect of Study 4 highlights the importance of teaching self-regulating strategies in an environment that is appropriately structured for the practising of self-regulated skills. Since a learning environment, in which problem-solving is prominent, enhances self-regulation, the third aspect correlates with Study 2 which is set in a problem-solving environment.

The fourth aspect of the theoretical basis of Study 4 stresses the importance of monitoring learner performance and providing feedback, thus showing another link with Study 2, in which corrective feedback was regarded as an important feature of the problem-solving process.

2.4.4.4 Study 5 (Mevarech & Fridken, 2006)

Reference was made earlier to the theoretical basis of a study (Study 1) that dealt with the design of the instructional method IMPROVE. In Study 5, the first aim of Mevarech and Fridken (2006: 85-97) was to investigate the effects of IMPROVE on mathematical knowledge and mathematical reasoning. Therefore, the theoretical basis of Study 5 includes the same aspects as those in Study 1.

In Study 5, Mevarech and Fridken not only focused on the effects of metacognitive intervention on mathematics achievement, as in Study 1, but also investigated the effects of metacognitive intervention on learner metacognition. It builds on theory pertaining to general and domain-specific metacognitive knowledge (Mevarech & Fridken, 2006: 86).

General metacognitive knowledge refers to the control and regulation of problem-solving processes in any domain, whereas domain-specific metacognitive knowledge is applicable to a specific domain. General metacognitive knowledge is rooted in the two main components of the MAI, namely *knowledge of cognition* and *regulation of* *cognition.* Domain-specific metacognitive knowledge in mathematics, as evident in this study, relates to specific metacognitive processes activated prior to, during, and after solving mathematical problems (Mevarech & Fridken, 2006: 86).

The distinction between metacognitive processes applicable to specific different phases of the problem-solving process in Study 5 clearly shows parallels with the six phases of problem-solving in Study 2 (see 2.4.4.2). However, in Study 5, Mevarech and Fridken (2006: 86) only differentiate between three phases of the mathematics problem-solving process (*prior to the problem, during the problem,* and *after the problem*), whereas in Study 2, Kapa (2001: 319-320) distinguishes between six phases.

2.4.4.5 Study 8 (Mevarech & Amrany, 2008)

In a third study (Study 8) that investigated the effects of the metacognitive intervention programme IMPROVE on mathematics achievement, Mevarech and Amrany (2008: 147-157) drew on the same theoretical basis as the previously discussed studies (Studies 1 and 5) that also implemented IMPROVE.

The theoretical basis for the extent to which metacognitive knowledge can be applied in contexts different to contexts in which the metacognitive intervention took place could not be established, due to a lack of literature on that topic (Mevarech & Amrany, 2008: 148).

2.4.4.6 Study 9 (Özsoy & Ataman, 2009)

This study focused first on the enhancement of mathematical problem-solving skills. The theoretical grounding of this study in respect of the enhancement of mathematical problem-solving skills involves the positive correlation between metacognitive skills and problem-solving (Özsoy & Ataman, 2009: 70).

Study 9 also focused on the enhancement of metacognition. Analogous to previously discussed studies (Studies 2 and 5) that grounded the metacognitive interventions in problem-solving, in Study 9, Özsoy and Ataman (2009: 67-82) also based their metacognitive intervention on the established relationship between metacognition and mathematical problem-solving skills such as understanding the problem by asking self-directed questions, and seeking links between prior knowledge and new information (Özsoy & Ataman, 2009: 68-69). Partly in line with previously discussed studies in which metacognition is regarded as the *knowledge of cognition* and the *regulation of cognition* (Studies 5 and 8), Özsoy and Ataman (2009: 68) view metacognitive knowledge and metacognitive control as the two main components of metacognition.

An earlier discussion pointed out the similarities between metacognitive knowledge and knowledge of cognition (see Table 2.2). Metacognitive control, also viewed as metacognitive strategies in Study 9, entails the ability to use metacognitive knowledge effectively by employing four skills, namely prediction, planning, monitoring, and evaluation (Özsoy & Ataman, 2009: 68).

Özsoy and Ataman (2009: 69) view active learner participation, learner autonomy, and a supporting social environment as crucial elements in the design of a metacognitive intervention. They stress the importance of teaching metacognitive skills in an integrated manner by using activities structured around content, as this promotes learner understanding of where, when and how to apply metacognitive skills (Özsoy & Ataman, 2009: 69-70).

The incorporation of these aspects in the methods of metacognitive intervention is discussed in the next section.

2.4.5 Methods of intervention

2.4.5.1 Study 1 (Mevarech & Kramarski, 1997)

In Section 2.4.4.1, the theoretical basis of the metacognitive intervention programme IMPROVE, designed by Mevarech and Kramarksi (1997: 365-394), was discussed. The method of intervention of this study is structured around three interdependent design features.

The first design feature involves three kinds of metacognitive questions, namely comprehension questions – for example, learners stating the main ideas in the problems in their own words; strategic questions – for example, stating the strategies that could be used to solve the problem, and connection questions – for example, stating what the similarities and differences are between the problem learners are currently solving and the problems they have solved in the past. The metacognitive questions are arranged according to the four stages of Polya's problem-solving model (Mevarech & Kramarski, 1997: 369-370). The first design feature links with the first aspect of the theoretical basis of Study 1 which involves metacognitive questioning in a problem-solving context (see 2.4.4.1).

The second design feature, cooperative learning, involves metacognitive questioning by Grade 7 learners in cooperative groups after the teacher has introduced a new concept by modelling metacognitive questioning applicable to the new concept. The teacher spends 10-15 minutes with a different group every day, also modelling metacognitive questioning. Each cooperative group consists of four learners with different levels of prior knowledge and achievement, namely one high-achieving learner, one low-achieving learner, and two learners with average achievement (Mevarech & Kramarski, 1997: 369, 377-378). The second design feature clearly corresponds with the second aspect of the theoretical basis of Study 1, namely cooperative learning (see 2.4.4.1). This design feature, namely cooperative learning, also overarches the first design feature, since metacognitive questioning takes place in a cooperative setting.

The third design feature, feedback-corrective-enrichment, involves the administration of a formative test at the end of a unit, approximately every 10 lessons. Learners obtaining less than 80% in the test engage in corrective activities, whereas the other learners receive enrichment activities applicable to the unit (Mevarech & Kramarski, 1997: 378). A clear correlation between the third design feature and the third aspect of the theoretical basis of Study 1, namely corrective feedback and enrichment, is observed. This feature could act as a support to the theoretical basis and design features of Study 1, because it could focus and motivate the learners when they work through a unit.

2.4.5.2 Study 2 (Kapa, 2001)

In the study by Kapa (2001: 317-336), a computerised metacognitive intervention programme in a word-problem-solving context was implemented. The intervention was structured according to the theoretical basis of Study 2, namely the six phases of problem-solving (see 2.4.4.2). The teacher modelled metacognitive questioning at the first problem-solving session; thereafter the Grade 8 learners worked through different problems on the computers (Kapa, 2001: 325-326).

Throughout the problem-solving process, learners are asked the following questions for five of the six phases: "What are you asked to find?"; What is given in the problem?" (Problem identification); "In what way is this problem similar to the example?" (Problem representation); "What is the strategy?" (Planning the solution); "Is the solution suitable for the problem's conditions?" (Problem evaluation), and "Is there any other way to solve the problem you have already solved? If so, what is it?" (Directing feedback). If a learner's solution is incorrect, the following questions are asked: "Why is your answer wrong? Where is the mistake?" (Correcting feedback).

The third phase, namely executing the solution, differs from the other five phases, because the learners are not prompted with a single question, but rather with an instruction to write the full solution on a draft paper. They may click buttons labelled *problem mapping* and *solved example* to assist them in finding a solution (Kapa, 2001: 322).

Several similarities and differences between the method of intervention of Study 2 and the previously discussed method of intervention in Study 1 (see 2.4.5.1) are noted. First, both designs incorporate metacognitive questioning according to the different phases of problem-solving, although the phases are more detailed in the second study. Secondly, in the first study, the metacognitive questions are asked by the learners in their cooperative groups, whereas the learners work individually in the second study by answering questions on the computer. Thirdly, both studies incorporate feedback as an essential aspect of metacognitive intervention, but only the first study offers enrichment opportunities. Fourthly, the feedback in the first study is only given after a unit of approximately 10 lessons, whereas feedback is given after every problem in the second study. Fifthly, the first study incorporated continuous teacher involvement, whereas the second study involved the teacher only at the first problem-solving session. Arguably, the computer fulfils the role of the teacher to a certain extent. This implies that there is some "teacher" involvement.

2.4.5.3 Study 4 (Camahalan, 2006)

The six-week intervention programme used in Study 4 is rooted in the four aspects of its theoretical basis: learners' knowledge and beliefs; explicit teaching of self-regulated strategies; structuring the environment optimally for the enhancement of self-regulation, and monitoring of learner performance and giving feedback (see 2.4.4.3).

These four aspects were addressed as follows in the 30-session (six weeks) intervention programme. The first five sessions were used to teach the Grade 4 learners the importance of personal responsibility, self-efficacy, the setting of goals, and motivation (Camahalan, 2006: 198). These aspects address learners' awareness of their affective states, in contrast to Studies 1 and 2.

In the next six sessions, self-regulated learning strategies were introduced and the learners practised each strategy individually. These strategies include self-evaluation; organising and transforming; goal-setting and planning; seeking information; keeping records and monitoring; environmental structuring; self-consequences; rehearsing and memorising; seeking social assistance, and reviewing records (Zimmerman, 1989: 337). It is obvious that these strategies are applicable to the learning of mathematics in a broader context than the two studies discussed earlier, which mainly address self-regulation in a problem-solving context.

Learners had to apply the strategies in the remaining 19 sessions. They had to monitor their performance by completing a self-evaluation report at the end of each week, and they were observed daily to record their use of the strategies (Camahalan, 2006: 198).

This intervention programme correlates with the theoretical basis of the study, although it is not made clear how feedback is incorporated, except for the reference to the daily observation of the learners. When compared to the intervention programmes of the two previously discussed studies, the following additional observations are made. First, similar to Study 1, the teacher plays a very active role, especially in the first 11 sessions of this intervention programme. Secondly, the learners work more individually, as in Study 2, but they are encouraged to seek social assistance from peers, teachers and other adults, similar to Study 1. Thirdly, learner self-evaluation in this study takes place once a week, whereas the correcting feedback is given after each problem in Study 2.

2.4.5.4 Study 5 (Mevarech & Fridken, 2006)

In Study 5, Mevarech and Fridken (2006: 85-97) also employed IMPROVE, the metacognitive intervention programme used in Study 1. Although the one month's intervention also took place in a problem-solving context, there were some differences in the way in which IMPROVE was implemented in Study 5.

In Study 1, only three types of metacognitive questioning are used, namely comprehension questions, strategic questions, and connecting questions (see 2.4.5.1). In Study 5, reflection questions follow connecting questions to guide the learners to reflect on the mistakes they made or on alternative ways to solve the problem (Mevarech & Fridken, 2006: 87). This could be an improvement on the first study's intervention programme, because trying to solve a problem in alternative ways is likely to enhance mathematical understanding.

Another observed difference is that the learners worked individually or in cooperative settings in Study 5, whereas they only worked in cooperative settings in Study 1. A possible explanation could be that the participants in Study 5 were better able to work individually as they were older (pre-college) students, as opposed to Grade 7 learners (Study 1) who probably needed the support provided by cooperative settings.

2.4.5.5 Study 8 (Mevarech & Amrany, 2008)

In Study 8, Mevarech and Armany (2008: 147-157) also implemented the metacognitive intervention IMPROVE in a problem-solving context. The intervention programme corresponds with the adapted IMPROVE in Study 5 by first including reflection questions and, secondly, choosing problems such as optimisation problems and investments that are of interest to learners (Mevarech & Fridken, 2006: 89-90; Mevarech & Amrany, 2008: 151). In contrast to Study 5, the Grade 12 learners only worked individually during the two months' intervention (Mevarech & Amrany, 2008: 151).

2.4.5.6 Study 9 (Özsoy & Ataman, 2009)

In Study 9, Özsoy and Ataman (2009: 67-82) also implemented a metacognitive intervention programme in a problem-solving context. The participants were Grade 5 learners. However, in contrast to Study 1 in which the Grade 7 participants worked in cooperative groups, the participants in Study 9 worked individually.

Learners were first informed about the general aspects of metacognition in two lessons of 40 minutes each. As part of these introductory lessons, each learner received a list of metacognitive skills applicable to problem-solving. The intervention period totalled nine weeks during which the learners worked through 23 word problems by using the metacognitive skills list. The teacher played an active role by guiding the learners to ask self-directed questions about the problem and their thinking processes, to share their thinking processes with the class, and to evaluate themselves at the end of each problem-solving activity. In addition, the teacher and researchers gave feedback on the worksheets which the learners completed individually during each problem-solving session. The feedback focused on assisting learners to monitor their own development (Özsoy & Ataman, 2009: 75-76).

The intervention design corresponds with the theoretical basis of this study in the following ways. First, the structuring of the intervention around the solving of word problems clearly links with the importance of enhancing metacognition in an environment integrated with content. Secondly, learner autonomy and active participation are enhanced when learners attempt to solve the problems individually by completing their worksheets. Thirdly, the continuous teacher involvement and the sharing of their thinking processes by the entire class provide a supporting social environment.

In this section, some important features of metacognitive interventions were identified. The effect of these metacognitive interventions on learner metacognition was determined by quantitative and qualitative measurements. In the next section, the measurement of metacognition in intervention studies is discussed.

2.4.6 Measurement of learner metacognition

Six studies that investigated the effects of a metacognitive intervention on mathematics achievement were discussed in Section 2.3. In two of these studies (Studies 1 and 2), the level of learner metacognition was not measured. In the other four studies (Studies 4, 5, 8 and 9), the effects of the metacognitive intervention on learner metacognition were also examined. The pre-test and post-test measurements of metacognition in these four studies will be discussed next.

2.4.6.1 Study 4 (Camahalan, 2006)

In Study 4, Camahalan (2006: 194-205) reported a significant difference in the post-test scores of the experimental and the control groups on the Mathematics Self-Regulated Learning Scale (MSRLS). The study did not report a detailed analysis of the differences on specific self-regulated learning strategies between pre-test and post-test scores of the experimental and control groups on specific self-regulated learning strategies.

2.4.6.2 Study 5 (Mevarech & Fridken, 2006)

In Study 5, Mevarech and Fridken (2006: 85-97) provided a more detailed analysis of the effect of a metacognitive intervention on learner metacognition. They used two instruments to measure learner metacognition. As a first measure, they used the MAI to assess general learner metacognition. They found a significant difference in post-test scores of the experimental and control groups for the two main components of the MAI, namely *knowledge of cognition* and *regulation of cognition*. In addition, there was a significant difference on all subscales of the MAI, namely declarative knowledge, conditional knowledge, procedural knowledge, planning, information management, monitoring, debugging, and evaluation (Mevarech & Fridken, 2006: 93-94). A second instrument measured domain-specific metacognitive knowledge (DSMK). They found significant differences on the use of metacognitive strategies prior to, during, and at the end of the problem-solving process (Mevarech & Fridken, 2006: 95).

2.4.6.3 Study 8 (Mevarech & Amrany, 2008)

In Study 8, Mevarech and Amrany (2008: 150) used an adapted MAI, consisting of only 24 items, to measure learner metacognition prior to and after an intervention. The adapted MAI contained all the subscales of the original MAI. In contrast to the study referred to in the previous paragraph, no significant difference was found on the *knowledge of cognition* component of the MAI, but a significant improvement in post-test scores on the *regulation of cognition* component was reported (Mevarech & Amrany, 2008: 152). Mevarech & Amrany (2008: 155) offer two possible explanations for the failure of the intervention programme to establish a significant difference on the *knowledge of cognition* component. They argue, first, that a certain level of *knowledge of cognition* does not guarantee a corresponding level of *regulation of cognition.* Secondly, the intervention programme mainly emphasised the *regulation of cognition* component (Mevarech & Amrany, 2008: 155).

A qualitative measure of metacognition was also employed by interviewing learners from the experimental and the control groups on their use of metacognitive strategies in the mathematics matriculation examination (Mevarech & Amrany, 2008: 150). These interviews measured metacognition in a delayed situation; that is, learner metacognition was measured in a different context to that of the intervention. Learner responses were classified into four categories, namely understanding the problem, making connections, using problem-solving strategies, and evaluating the solution.

It was found that the control group applied the first category (understanding the problem) to a greater degree than the experimental group. However, the experimental group applied the other three categories to a greater extent than the control group. Mevarech and Amrany (2008: 155) suggest that the experimental group's learners reported a lesser degree of conscious engagement with the first category (understanding the problem), because they automatically applied the first category as a result of solving many problems during the intervention programme.

The following learner comment supports the researchers' explanation: "I sure did it, how can one solve a problem without understanding what it is all about?" However, the researchers caution against a too rigid interpretation of the qualitative measure of learner metacognition used in their study, as the number of interviewees was relatively small (Mevarech & Amrany, 2008: 151, 154).

2.4.6.4 Study 9 (Özsoy & Ataman, 2009)

In Study 9, Özsoy and Ataman (2009: 67-81) used an instrument that contains the same main components and subscales as the MAI to measure learner metacognition. It differs from the MAI in that it also assesses the learners' application of each subscale qualitatively by requesting the learners to explain their reasoning processes (Özsoy & Ataman, 2009: 73). A significant difference was found between the post-test scores of the experimental and the control groups for the quantitative and qualitative aspects (Özsoy & Ataman, 2009: 77).

2.4.7 Summary

In this section, a brief summary of the different features of metacognitive interventions in mathematics (see 2.4.1-2.4.6) is provided. Some of these aspects will be grouped together to facilitate the discussion.

2.4.7.1 Aim(s) of the study, grade/age of participants, and the intervention period

Table 2.5 represents the aim(s) of the study, the grade/age of the participants, and the intervention period.

When the relationship between the aims of the study and the grade/age of the participants is considered (see Table 2.5), the following observations are made. First, the enhancement of problem-solving skills and mathematical reasoning could already start in the primary school (see Studies 1 and 9). Secondly, self-regulation and metacognition intervention studies can also involve primary school learners (see Studies 4 and 9).

Table 2.5: Aim(s) of the study, grade/age of participants, and the intervention period

Study	Aim(s): Improvement of ...	Grade/age of participants	Intervention period
1	Mathematical reasoning.	Grade 7.	Entire academic year.
2	Mathematical problem-solving skills.	Aged 13-14.	Eight weeks.
4	Mathematics achievement; self-regulation.	Grades 4, 6.	Six weeks.
5	Mathematical reasoning; mathematical knowledge; metacognition.	Pre-college.	Four weeks.
8	Mathematics achievement; metacognition.	Grade 12.	Four weeks.
9	Mathematical problem-solving skills; metacognition.	Grade 5.	Nine weeks.

The link between the aim(s) of the study and the intervention period shows that mathematics metacognition may be enhanced within a period of four weeks (see Studies 5 and 8). However, these two studies involved older participants, namely pre-college students (Study 5) and Grade 12 learners (Study 8). The other two studies relating to the enhancement of self-regulation (Study 4) and metacognition (Study 9) involved Grade 4 learners (Study 4), Grade 5 learners (Study 9), and Grade 6 learners (Study 4). Their intervention periods were longer, namely six weeks (Study 4) and nine weeks (Study 9), than the interventions where older learners were involved.

2.4.7.2 Theoretical basis

All six studies stated aims relating to mathematics, and four of these studies also stated aims relating to self-regulation or metacognition. The theoretical basis of the six studies with aims relating to mathematics is discussed first, followed by a discussion of the theoretical basis of the four studies that also stated aims relating to self-regulation and metacognition.

Table 2.6 represents a summarised version of the aspects of the theoretical basis of the six studies which state mathematics-related aims.

Table 2.6: Aspects related to the theoretical basis of the studies that state mathematics-related aims

Study	Aim of the study: Improvement of …	Problem-solving context	Cooperative settings	Corrective feedback	Enrichment activities
1	Mathematical reasoning.	x	x	x	x
2	Mathematical problem-solving skills.	x		x	
4	Mathematics achievement.	x			
5	Mathematical reasoning; mathematical knowledge.	x	x	x	x
8	Mathematics achievement.	x	x	x	x
9	Mathematical problem-solving skills.	x			

Some important aspects emerge when considering the theoretical basis of these studies. First, framing mathematics within problem-solving contexts is regarded as important for the enhancement of mathematical reasoning (Study 1), mathematical problem-solving skills (Studies 2 and 9), mathematics achievement (Studies 4 and 8), and mathematical knowledge (Study 5). Secondly, cooperative settings improve mathematical reasoning processes (Studies 1 and 5) and mathematical knowledge (Study 5). However, cooperative settings are not explicitly mentioned in the two studies related to problem-solving skills (Studies 2 and 9) and in one of the studies related to mathematical achievement (Study 4). Thirdly, in all the studies, with the exception of Study 9, corrective feedback is regarded as an important aspect of the theoretical basis of those studies. It enhances mathematical reasoning (Studies 1 and 5), mathematical problem-solving skills (Study 2), and mathematics achievement (Studies 4 and 8). Enrichment activities only feature in three studies (Studies 1, 5 and 8).

Four studies also stated aims involving self-regulation or metacognition. Table 2.7 presents aspects related to the theoretical basis of these studies.

Table 2.7: Aspects of the theoretical basis of the studies that state aims related to self-regulation or metacognition

Study	Aim(s): Improvement of ...	Learner autonomy	Knowledge of strategy use	Beliefs (goals, motivation)	Regulatory strategies	Problem-solving context	Feedback
4	Self-regulation.	x	x	x	x	x	x
5	Metacognition.		x		x	x	
8	Metacognition.		x		x	x	
9	Metacognition.	x	x		x	x	

Based on Table 2.7, the following observations are made. First, the learning of mathematics within problem-solving contexts is regarded as vital for the enhancement of learner metacognition. In all four studies, the metacognition intervention took place within a problem-solving context in which the classroom environment was optimally structured for engaging learners in the problem-solving process. Secondly, *knowledge of strategy use* and *regulation of strategy use* were the two main components of all four studies. A third aspect relates to learner affect such as *beliefs and motivation*. Although only Camahalan (2006: 196) explicitly refers to learner beliefs and motivation, learner affect is an important element in the problem-solving process (see 2.2.4.4m). The first aspect refers to problem-solving contexts as a theoretical basis for metacognitive interventions; therefore, learner affect may be viewed as a crucial aspect in the theoretical basis of the metacognitive interventions of all four studies. Fourthly, learner autonomy only explicitly features in Studies 4 and 9. However, learner autonomy is enhanced in a problem-solving context, as in the case of these four metacognitive interventions. Therefore, it could be posited that learner autonomy is a feature of the theoretical bases of all four studies.

2.4.7.3 Methods of intervention

Table 2.8 presents the features of the metacognition intervention methods that were implemented in the six studies.

Table 2.8: Features of the metacognition intervention methods

	Study 1	Study 2	Study 4	Study 5	Study 8	Study 9
Problem-solving context.	x	x	x	x	x	x
Broader mathematics context.			x			
Cooperative settings.	x			x		
Corrective feedback.	x	x		x	x	x
Enrichment.	x			x	x	
Individual settings.	x	x	x	x	x	x
Learner affect.			x			
Active teacher involvement.	x	x	x	x	x	x

The following features emerge as important aspects in the implementation methods of metacognitive interventions in all six studies. The implementation methods of the six studies are discussed jointly, because Table 2.8 indicates that all aspects present in the two studies (Studies 1 and 2), which stated only mathematics-related aims, are also present in the other four studies.

First, the establishing of problem-solving contexts creates a very suitable environment for the application of metacognitive strategies. All studies' interventions took place in problem-solving contexts, whereas in Study 4, the intervention incorporated a problem-solving context into a broader mathematics context.

Secondly, corrective feedback plays an important role in guiding the learners to develop their metacognitive and mathematical skills. All studies, with the exception of Study 4, implemented corrective feedback. However, the self-evaluation aspect of Study 4 could be viewed as corrective feedback provided by the learner and not by the teacher.

Thirdly, teacher involvement during the process of metacognitive intervention may enhance learner metacognition if teachers create a safe environment in which learners can freely share their thoughts and observe the metacognitive skills modelled by the teacher. The important role of the teacher is acknowledged in all the studies, except in Study 2 where learners interacted actively with the computer that provided corrective feedback; therefore, there arguably was “teacher” involvement to some extent.

A fourth feature emerging from Table 2.8 is the implementation of cooperative and/or individual settings for the metacognitive intervention. Two studies (Studies 1 and 5) implemented cooperative and individual settings, whereas the other four studies used individual settings only. No study implemented only cooperative settings. It appears that individual settings are viewed as an important element in the enhancement of learner metacognition.

Fifthly, enrichment opportunities may enable learners to develop their metacognitive skills. Only three studies (Studies 1, 5 and 8) implemented enrichment opportunities. Since all studies are grounded in a problem-solving context, enrichment opportunities implicitly feature in all studies, due to the scope for enrichment activities during problem-solving. In the fourth phase of Polya’s problem-solving model, learners are encouraged to note alternative solutions or the application of the solution in different contexts, thereby allowing for enrichment activities (see 2.2.4.4m).

A sixth feature involves learner affect. Only Study 4 implemented measures of making learners explicitly aware of their beliefs and emotions. However, this feature may be implicitly present in all interventions, because active teacher involvement and corrective feedback processes will ensure that learners are aware of their affective states.

Larkin (2010: 5-6) highlights two concerns related to metacognitive interventions. First, she states that the effective use of time permeates Western culture and, therefore, schools mirror this emphasis on getting as many things done as quickly as possible. Unfortunately, the enhancement of learner metacognition is a slow process with no tangible outcome to measure learner progress. Secondly, teachers often view metacognition as the reflective part of a teaching session in which learners are asked to reflect on what they have learned and to verbalise their problem-solving strategies. However, learners may be reluctant to partake in this reflective process.

On considering these two concerns in respect of the six intervention studies, it is apparent that some of the metacognitive interventions did require explicit metacognitive strategy training. This could be problematic in contexts in which teachers have very limited time for work not directly related to the mathematics curriculum. Regarding the second concern, it is argued that these six studies established an environment that encouraged learner involvement *during* the process of learning, instead of only asking learners to reflect on what they *have* learned.

2.4.7.4 Measurement of learner metacognition

Table 2.9 presents metacognition measuring instruments and results of the four studies that measured metacognition, namely Studies 4, 5, 8 and 9.

Table 2.9: Metacognition measuring instruments and results of studies that measured metacognition

Study	Metacognition measuring instrument(s)	Results
4	Self-regulated learning scale (quantitatively). Pre-test and post-test.	Significant differences were found between post-test scores of the experimental and control groups.
5	General learner metacognition measured by the MAI (quantitatively). Pre-test and post-test. Domain-specific metacognitive knowledge measured by the DSMK questionnaire (quantitatively). Pre-test and post-test.	Significant differences were found in post-test scores of both subscales of the MAI for the experimental group, but not for the control group. Significant differences were found in post-test scores on the use of metacognitive strategies prior to, during, and at the end of the problem-solving process for the experimental group, but not for the control group.
8	Adapted MAI (quantitatively). Pre-test and post-test. Interviews (qualitatively) on their use of the four categories: understanding the problem; making connections; using problem-solving strategies, and evaluating the solution. Post-test only.	No significant difference was found on *knowledge of cognition;* significant improvement in post-test scores on the *regulation of cognition* component for the experimental group. Experimental group had less engagement than the control group with the first category, but greater application of the other three categories than the control group.

Study	Metacognition measuring instrument(s)	Results
9	MAI (quantitatively). Pre-test and post-test. Application of MAI (qualitatively).	Significant differences were found between post-test scores of the experimental and the control groups for the quantitative and qualitative aspects.

The metacognition intervention programmes of these four studies had significant, positive effects on learner metacognition, although in Study 8, Mevarech and Amrany (2008: 147-157) only reported significant results for the *regulation of cognition* component. In two studies (Studies 4 and 5), learner metacognition was only measured quantitatively. Learner metacognition was measured both quantitatively and qualitatively in Studies 8 and 9. Quantitative measurements were done by means of standardised instruments – for example, the MAI was used in two studies (Studies 5 and 9) and an adapted MAI in one study (Study 8). Qualitative measures of metacognition included interviews (Study 8) and the analysis of learners' responses in problem-solving situations, according to the MAI's subscales (Study 9). In all four studies, the quantitative measurements employed pre-test and post-test measurements.

The following factors influence measurements of metacognition: numbers of measurements; the time of the measurement, and the unit of analysis (Azevedo, 2009: 88-89). Contradictory results in published journals imply that different measures of metacognition could be unrelated, as different facets of metacognition are measured (Azevedo, 2009: 89). Since the four studies employed a different number of measurements and different measurement instruments over different time periods, some caution needs to be exercised in the interpretation of these studies' metacognition measurements.

2.4.7.5 Secondary research question 3

The aim of Section 2.4 was to address secondary research question 3, which seeks to explore features of some previous metacognition interventions in mathematics. The following features were examined: aims; age and gender of participants; intervention period; theoretical basis; method of intervention, and measurement of learner metacognition. Similarities and differences between the studies in respect of each feature were discussed (see 2.4.7.1-2.4.7.4).

In order to conclude the exploration of secondary research question 3, the following observations about each feature of the metacognitive interventions in mathematics are made. First, all studies formulated aims relating to the improvement of learner performance in different aspects of mathematics, namely mathematics achievement, mathematical reasoning, and mathematical problem-solving skills. Some studies also formulated aims relating to the enhancement of learner self-regulation or learner metacognition. Secondly, participants ranging in age from Grade 4 to pre-college students may benefit from metacognition interventions.

Thirdly, the intervention period in the studies with mathematics-related aims only ranged from eight weeks to the entire academic year, whereas the studies that also stated aims related to self-regulation or metacognition had intervention periods ranging from four weeks to six weeks. Fourthly, the theoretical basis of studies that only expressed mathematics-related aims includes the following key aspects: problem-solving contexts; cooperative settings; corrective feedback, and enrichment.

Fifthly, the theoretical basis of the studies that stated mathematics-related aims and aims involving self-regulation or metacognition comprises the following aspects: problem-solving contexts; the conceptualisation of metacognition as *knowledge of cognition* and *regulation of cognition*; learner affect, and learner autonomy. Sixthly, the methods of intervention include the following aspects: problem-solving contexts; corrective feedback; active teacher involvement; cooperative settings; individual settings; enrichment opportunities, and learner affect. Seventhly, the measurement of self-regulation or metacognition involved quantitative and qualitative pre-test and post-test measures.

Table 2.10 provides a summary of the features of metacognitive interventions in mathematics. A distinction is made between studies that only stated mathematics-related aims (Studies 1 and 2); studies that stated mathematics-related aims and/or aims relating to self-regulation or metacognition (Studies 1, 2, 4, 5, 8 and 9), and studies that stated aims relating to mathematics and self-regulation or metacognition (Studies 4, 5, 6, and 9). Table 2.10 also includes information on the mathematics measuring instruments (see 2.3.10) in order to provide a more comprehensive picture of metacognition intervention studies in mathematics.

Table 2.10: Features of some previous metacognitive intervention studies in mathematics

Feature	**Studies that only stated mathematics-related aims(Studies 1 and 2)**	**Studies that stated mathematics-related aims and/or aims relating to self-regulation or metacognition (Studies 1, 2, 4, 5, 8, and 9)**	**Studies that stated aims relating to mathematics and self-regulation or metacognition (Studies 4,5, 8, and 9)**
Age of participants	Grade 7-14 years.	Grade 4 to pre-college.	Grade 4 to pre-college.
Intervention period	Eight weeks to one year.	Four weeks to one year.	Four weeks to nine weeks.
Theoretical basis	*Related to mathematics:* Problem-solving contexts; corrective feedback; enrichment.	*Related to mathematics:* Problem-solving contexts; cooperative settings; corrective feedback; enrichment. *Related to self-regulation or metacognition:* Problem-solving contexts; knowledge of cognition; regulation of cognition; learner beliefs and motivation; learner autonomy.	*Related to mathematics:* Problem-solving contexts; cooperative settings; corrective feedback; enrichment. *Related to self-regulation or metacognition:* Problem-solving contexts; knowledge of cognition; regulation of cognition; learner beliefs and motivation; learner autonomy.
Method of intervention	Problem-solving contexts; corrective feedback; active teacher involvement;	Problem-solving contexts; corrective feedback; active teacher involvement;	Problem-solving contexts; corrective feedback; active teacher involvement;

Feature	Studies that only stated mathematics-related aims(Studies 1 and 2)	Studies that stated mathematics-related aims and/or aims relating to self-regulation or metacognition (Studies 1, 2, 4, 5, 8, and 9)	Studies that stated aims relating to mathematics and self-regulation or metacognition (Studies 4,5, 8, and 9)
	cooperative settings; individual settings; enrichment opportunities.	cooperative settings; individual settings; enrichment opportunities; learner affect.	cooperative settings; individual settings; enrichment opportunities; learner affect.
Measurement of mathematics-related aspects	Quantitative and qualitative measurements. Pre-test and post-test.	Quantitative and qualitative measurements. Pre-test and post-test.	Quantitative and qualitative measurements. Pre-test and post-test.
Measurement of self-regulation or metacognition	Not applicable.	*Quantitative measures*: MSRLS; MAI; DSMK. Pre-test and post-test. *Qualitative measures:* Interviews; applied MAI.	*Quantitative measures*: SRLS; MAI; DSMK. Pre-test and post-test. *Qualitative measures:* Interviews; applied MAI.

2.5 CONCLUSION

In this chapter, three secondary research questions were explored.

2.5.1 Secondary research question 1

The first theme explored in this chapter dealt with metacognition as a concept. Different definitions of metacognition were discussed, with Flavell's definition serving as a focus for this discussion. The relationship between metacognition, self-regulation and SRL was examined by exploring their original and current conceptualisations.

2.5.2 Secondary research question 2

The second theme focused on the role of learner metacognition in mathematics achievement. Metacognitive intervention studies were analysed according to different aspects. Similarities and differences between these aspects in the different studies were examined. One very important feature emerging from these intervention studies is the role of problem-solving contexts in enhancing learner metacognition. Many studies reported a significant, positive relationship between learner metacognition and mathematics achievement, although some studies did not report similar results. A significant enhancement of learner metacognition was indicated in most studies that assessed learner metacognition by using pre-test and post-test measures.

2.5.3 Secondary research question 3

The third theme explored features of some previous metacognitive interventions in mathematics. The following features were examined: aims; age and gender of participants; intervention period; theoretical basis; method of intervention, and measurement of learner metacognition. Problem-solving contexts also emerged as vital to the implementation of metacognition interventions.

The purpose of this study is to investigate the effect of metacognition intervention on learner metacognition and achievement in mathematics. In this chapter, the investigation of the conceptual basis of metacognition, the relation between metacognition and mathematics achievement, and features of previous metacognitive interventions provided guidelines that were used in the development of this study's MI.

Although the exploration of the three themes in this chapter also revealed some aspects related to the teaching-and-learning of mathematics, a further investigation into the nature of mathematics and effective learning in mathematics needs to be conducted that could be used further in the development of this study's MI. These aspects will serve as the focus for the next chapter in order to address secondary research question 4 which seeks to propose a framework for metacognitive intervention in mathematics.

CHAPTER 3

A PROPOSED FRAMEWORK FOR METACOGNITIVE INTERVENTIONS IN MATHEMATICS

3.1 INTRODUCTION

In Chapter 2, the concept *metacognition* was explored and the relationship between learner metacognition and achievement in mathematics was investigated. Features of previous metacognitive interventions in mathematics were identified, and the first three secondary research questions were addressed.

The purpose of this study is to investigate the effect of MI on learner metacognition and achievement in mathematics (see 1.8). The aim of Chapter 3 is to address secondary research question 4 which seeks to propose a framework for a metacognitive intervention in mathematics that incorporates features of previous metacognitive interventions in mathematics and aspects related to the nature of mathematics and effective learning in mathematics.

Three themes are addressed in this chapter in order to propose a framework for a metacognitive intervention in mathematics. First, the nature of mathematics, the aims of mathematics education, and aspects of mathematical proficiency are explored by examining international and national perspectives from literature (see 3.2). Secondly, De Corte's (1996) educational learning theory and its relationship with effective learning and expert performance in mathematics is examined (see 3.3). Thirdly, a synthesis of the first two themes is provided in order to establish a mathematical perspective on De Corte's (1996) educational learning theory (see 3.4).

The features of a mathematical perspective on De Corte's (1996) educational learning theory are combined with those of previous metacognitive interventions in mathematics in order to address secondary research question 4 (see 3.5).

Next, international and national perspectives on the nature of mathematics, the aims of mathematics education, and aspects of mathematical proficiency are discussed.

3.2 THE NATURE OF MATHEMATICS

3.2.1 Introduction

If the question “Did you like school mathematics?” is posed to people in general, the prospects are good that many responses will be accompanied by groans of dislike. It appears that the views of mathematics as a school subject are not expressed in moderate terms, but rather with responses conveying either utter loathing or genuine interest and enjoyment. Therefore, it is important to investigate the nature of mathematics as expressed by mathematicians and writers of policy documents in order to establish some official viewpoints on what school mathematics is supposed to be like. The nature of mathematics forms the basis on which the aims of mathematics education are established. Therefore, the discussion also includes views on the aims of mathematics education and on mathematical proficiency, although *the nature of mathematics* is viewed as the overarching concept in the next discussion.

The nature of mathematics is discussed from both an international and a national perspective (see 3.2.2 and 3.2.3). Then, differences and similarities between these views are highlighted in order to present a synthesised view on the nature of mathematics in the summary (see 3.2.4).

3.2.2 International perspectives

3.2.2.1 Hans Freudenthal

Earlier perspectives on the nature of mathematics are provided by Hans Freudenthal, probably one of the most influential mathematics educators of his time (Gravemeijer & Terwel, 2000: 777). Freudenthal refers indirectly to the problem-solving nature of mathematics by pointing to the applications of mathematics. He also highlights the search for thinking patterns which would not necessarily have immediate application value (Freudenthal, 1973: 8):

> Mathematics has always been ahead of its applications; it is the way of mathematics – to look for patterns of thought from which the appliers make their choice.

Freudenthal (1973: 29) also views human characteristics, in this instance language, as crucial to making sense of mathematics:

> The conscious occupation with language as a tool of exact expression is called formalizing […] modern mathematics shows a strong tendency to organization, and formalizing is one of its means.

As human involvement implies change, a strong reference is also made to the continual development and flexibility of mathematics (Freudenthal, 1973: 47, 75):

> Mathematics is never finished – anyone who worships a certain system of mathematics should take heed of this advice […] The greatest virtue of mathematics is its flexibility.

Freudenthal (1973: 38) offers an interesting perspective on the historical development of mathematics. His discussion of the renewed role of geometry relates to his previous statement about the greatest virtue of mathematics being its flexibility:

> Viewing the present structure of mathematics, it looks as though geometry had had its day. For centuries, even after the successes of algebra and analysis, geometry was esteemed as the only true mathematics, as the paragon of mathematical rigour. Not until the second half of the 19^{th} century, after algebra and analysis had been put on rigorous foundations independent of geometry, did it come out that traditional geometry was not as rigorous as people had believed, and this of course eroded the firm position of geometry. In fact, since antiquity up to the end of the 18^{th} century, geometry had hardly progressed and had hardly contributed anything to the growth of mathematics. Then, at the beginning of the 19^{th} century, geometry awoke to a new life and by its flourishing it contributed greatly to the development of group theory and to many chapters of algebra, and helped prepare the shape of modern axiomatic.

Freudenthal (1973: 44) warns against regarding the conceptual aspect of mathematics as more important than its algorithms. His view on the cyclic nature of the links between conceptual thinking and algorithmisation over the ages is expressed as follows:

> It is often asserted that modern mathematics is distinguished from the old by the stress on the conceptual component as opposed to algorithms. I agree that this is true and several times I have mentioned that the most striking innovations that start the process of modernizing – set theory, abstract algebra and analysis, topology – were eruptions of conceptual thinking, which burst through the petrified crust of algorithmic tradition. But all lava petrifies eventually. Each conceptual innovation encloses in itself the germ of algorithmization – this is the way of mathematics […] Without the algorithm of calculus, analysis would never have flourished. Algorithmizing means consolidating, starting from a platform to jump even higher. Algorithms provide the technical means of fathoming greater conceptual depth. It is not fair to confront algorithmic and conceptual mathematics with one another as though one is a lofty tower from which you may look down on the other, and we certainly cannot identify this opposition with that between new and old.

Freudenthal underscores the emphasis on understanding reality and solving real-life problems. However, he warns against the teaching of an applied mathematics as opposed to learning how to apply mathematics (Freudenthal, 1973: 44, 75, 77):

> Organizing the reality with mathematical means is today called mathematizing. The mathematician, however, is inclined to disregard reality as soon as the *logical* connection promises faster progress […] Reality is the framework to which mathematics attaches itself […] A mathematics tailored to some applications is beside the mark, it fossilizes. While I do not urge that the pupil learns applied mathematics, I do wish that he learns how to apply mathematics. This does not mean utilitarism. Therefore, instead of applied mathematics, I would prefer to speak of *multi-related* mathematics.

In summary, Freudenthal emphasises the following five aspects related to the nature of mathematics. First, mathematics enables one to establish modes of thinking (thinking patterns) whose application value may not be of immediate interest. Secondly, language is a crucial aspect of mathematical sense-making. In this regard, Polya advises that learners should be able to repeat the problem statement easily (see 2.2.4.4m).

Thirdly, mathematics continually evolves due to its flexible nature, and it should not be viewed in absolute terms. A philosophical perspective on the nature of mathematics reveals a degree of disagreement on what the nature of mathematics entails. Ernest (1991: 7, 18) states that the absolutist view of mathematical knowledge as comprising certain and unchallengeable truths has been refuted. Each of the three schools of thought on absolutism (logicism, formalism and constructivism) depends on a set of assumptions, namely fallible beliefs (Ernest, 1991: 13-14). He, therefore, asserts that the fallibilist view of mathematical knowledge as fallible, and never beyond revision, should be adopted. Vergnaud (1997: 7), however, reports an ongoing controversy about the nature of mathematics, namely whether mathematical activity discovers timeless truths regardless of context, or whether mathematical knowledge is relative. The fourth phase of Polya's problem-solving model also encourages the flexible use of mathematics as alternative solutions and applications in different contexts are sought (see 2.2.4.4m). A square, for example, has a bigger area than a rectangle when their perimeters are equal. In a different context, however, a rectangle may have a bigger area than a square. This occurs when one has the option to construct either a square or a rectangle by combining only three sides (with a total length of 12m for the three sides). The fourth side is given as part of a line of infinite length. A rectangle with a length of 6m and a breadth of 3m will yield an area of $18m^2$ which is larger than the area of the square ($16m^2$).

Fourthly, the conceptual and algorithmic aspects of mathematics are equally important aspects of the dynamic process of mathematical sense-making. Fifthly, it is imperative to use real-life problems in the teaching of mathematics, although it should not be the primary aim of mathematics education.

3.2.2.2 Alan Schoenfeld

Alan Schoenfeld is a leading mathematics researcher on mathematical problem-solving. The *Senior Scholar Award, Special Interest Group for Research in Mathematics Education* is among some of the numerous awards he has received (GSE, n.d., 1, 6 of 6). Schoenfeld points out that conceptualisations of mathematics range, at the one end, from viewing mathematics as facts and procedures dealing with quantities, magnitudes and forms, while at the other end, mathematics is regarded as the science of seeking patterns based on experiential findings (Schoenfeld, 1992: 334-335).

Schoenfeld (1992: 3) also states that, although the tools of mathematics are abstraction, symbolic representation, and symbolic manipulation, the use of these tools alone would not enable one to be mathematically proficient. To him, mathematical proficiency is first about developing a mathematical viewpoint, that is, to value and apply the processes of mathematisation and abstraction and, secondly, to understand how to use those tools to understand reality. Schoenfeld stresses the importance of translating events of everyday life into mathematical language in order to understand real-life events. One notes clear links with Freudenthal's view of organising mathematics by using language through the process of formalising, and the organising of reality (see 3.2.2.1). It is evident that Schoenfeld's view of mathematical proficiency also relates to Freudenthal's problem-solving in an authentic context.

In a later work, Schoenfeld (2007: 59) refers to the cognitive revolution that led to a greater emphasis on the way in which knowledge is applied, instead of focusing on knowledge alone. An important aspect of mathematical proficiency, therefore, is the ability to use mathematical knowledge in appropriate contexts (Schoenfeld, 2007: 59). Schoenfeld (2007: 60) also states that views of mathematical proficiency progressively developed from merely having a good understanding of mathematics concepts, skills and procedures to the current view of mathematical proficiency as learners possessing good problem-solving skills. However, it appears that this point of view was expressed earlier, because Schoenfeld (1992: 334-335) previously referred to the general agreement among mathematics educators that the most important objective of mathematics instruction should be to assist learners to become competent problem-solvers (see 2.2.4.4m).

Problem-solving skills involve cognitive aspects such as flexibility, imaginativeness, thinking in different ways about a problem, using alternative strategies if difficulties are encountered, extending a solution to a broader or new context, and finding new solutions (Schoenfeld, 2007: 60). This view shows strong links with Polya's problem-solving model, especially the fourth phase where alternative solutions are sought and where the solution is applied in a different context, thus requiring imaginativeness from a learner (see 2.2.4.4m). Good problem-solvers also have a positive attitude towards problem-solving, that is, a willingness to continue trying, even if it requires days or weeks to solve the problem (Schoenfeld, 2007: 60). Schoenfeld's emphasis on positive learner attitudes corresponds with Polya's statement that learners may become uninterested and unmotivated if mathematics is not applied in problem-solving contexts (see 2.2.4.4.m).

3.2.2.3 The National Research Council (USA)

Another perspective on mathematical proficiency comes from a prominent report by the National Research Council (NRC) in the USA on how learners learn mathematics. In this report, Kilpatrick, Swafford and Findell (2001: 5) describe mathematical proficiency as those skills and procedures a learner needs in order to learn mathematics effectively. They divide mathematical proficiency into five interwoven strands: *conceptual understanding* – the understanding of the concepts, operations and relations in mathematics; *procedural fluency* – the completion of mathematical procedures in precise, flexible, effective and appropriate ways; *strategic competence* – learners' ability to pose and solve mathematics problems; *adaptive reasoning* – learners' ability to think logically, to reflect, explain and justify, and *productive disposition* – learners' consistent inclination to regard mathematics as useful and applicable to their lives, coupled with a belief in their own effectiveness and diligence.

In considering these five strands, the following four observations are made. First, higher order thinking skills in mathematics require a solid knowledge and skills foundation, as described by the first two strands, *conceptual understanding* and *procedural fluency*. These two strands correspond with Schoenfeld's view of the importance of understanding concepts, skills and procedures. They also relate to the first phase of Polya's problem-solving model where the understanding of a problem requires knowledge of the mathematical concepts pertaining to the problem (see 2.2.4.4m). Polya also regards factual mathematical knowledge as one of the prerequisites in devising a plan (see 2.2.4.4.m). The fourth phase of Polya's problem-solving model encourages the identification of the relationship between different mathematical topics (see 2.2.4.4m).

Secondly, *strategic competence* clearly links with Schoenfeld's view of the importance of problem-solving in developing mathematical proficiency. It is interesting to note the reference to the learners' ability to pose problems, because the posing of problems requires an excellent understanding of mathematical concepts and procedures.

Thirdly, *adaptive reasoning* clearly links with the third phase of Polya's problem-solving model where learners have to clarify their thinking processes when carrying out their plan (see 2.2.4.4m).

Fourthly, *productive disposition* epitomises mathematical proficiency, as it refers to learners taking control of mathematics by using it with confidence and a positive attitude to solve problems related to their life experiences. This last strand also confirms the two most important aspects of mathematical proficiency highlighted by Schoenfeld and Polya, namely effective problem-solving in an affective nurturing environment (see 2.2.4.4m and 3.2.2.1). It is important to note the interwoven aspect of these five strands. For example, a learner could find aspects related to the last strand (*productive disposition*) difficult due to deficiencies in the first two strands (*conceptual understanding* and *procedural fluency*).

In this section, some international perspectives on the nature of mathematics were discussed. Common elements of the perspectives by mainly Freudenthal, Schoenfeld, Polya, and Kilpatrick *et al.* were highlighted. In the next section, perspectives on the nature of mathematics, as stated in South African policy documents, are examined.

3.2.3 National perspectives

The National Curriculum Statement (NCS) (NDE, 2003: 9) states the following about the nature of mathematics:

> Mathematics enables creative and logical reasoning about problems in the physical and social world and in the context of mathematics itself. It is a distinctively human activity practiced by all cultures. Knowledge in the

> mathematical sciences is constructed through the establishment of descriptive, numerical and symbolical relationships. Mathematics is based on observing patterns, which, with rigorous logical thinking, leads to theories of abstract relations. Mathematical problem-solving enables us to understand the world and make use of that understanding in our daily lives. Mathematics is developed and contested over time through both language and symbols by social interaction and is thus open to change.

An analysis of the NCS on the nature of mathematics reveals the following aspects. First, mathematics enables learners to enhance their thinking skills. Secondly, these thinking skills focus on real-life problems which improve mathematical understanding. A third aspect relates to the universal human practising of mathematics. Fourthly, the examination of relationships between phenomena enables the construction of knowledge. The recognising of patterns is a fifth aspect that enables the establishment of novel theories and abstract thought, a sixth aspect. The seventh aspect recognises the social element of knowledge construction through language and symbols through the ages. A last aspect refers to the changing nature and flexibility of mathematics due to the influence of human activity.

The Curriculum and Assessment Policy Statement (CAPS) views mathematics as follows (DBE, 2010b: 6):

> Mathematics is the study of quantity, structure, space and change. Mathematicians seek out patterns, formulate new conjectures, and establish axiomatic systems by rigorous deduction from appropriately chosen axioms and definitions. Mathematics is a distinctly human activity practiced by all cultures, for thousands of years. Mathematical problem solving enables us to understand the world (physical, social and economical) around us, and, most of all, to teach us to think creatively.

A comparison of the descriptions of mathematics by the NCS and the CAPS shows a common view regarding the importance of viewing mathematics as a human activity, although the CAPS does not explicitly refer to the changing nature of mathematics. The

CAPS also views problem-solving as enabling one to understand the world better. However, unlike the NCS, it emphasises the development of creative thinking during the process of problem-solving. Both documents also refer to basic mathematical knowledge of space and shape; patterns; the formulation of new theories, and the construction of mathematical knowledge through social interaction throughout history. The CAPS does not explicitly refer to the use of language as a tool for knowledge construction in mathematics nor to the flexible nature of mathematics, although it is probably implied in viewing mathematics as a "distinctly human activity".

3.2.4 Summary

These international and national perspectives on the nature of mathematics reveal many common viewpoints. Table 3.1 presents a synthesis of the international and national perspectives on the nature of mathematics. This synthesis includes views relating to mathematical proficiency and the aims of mathematics education. Table 3.1 also includes Polya's (1945) views relating to the nature of mathematics and mathematical proficiency, as expressed in the discussion of his problem-solving model (see 2.2.4.4m).

Table 3.1: A synthesis of international and national perspectives on the nature of mathematics

Thinking and reasoning skills
"… patterns of thought …" (Freudenthal, 1973); … developing a mathematical viewpoint. … imaginativeness, thinking in different ways about a problem (Schoenfeld, 2007); "adaptive reasoning – learners' ability to think logically, to reflect, explain and justify …" (*Kilpatrick et al.*, 2001); "… creative and logical reasoning …" … "rigorous logical thinking …" (NDE, 2003); "think creatively …" (DBE, 2010b); … clarify their thinking processes …(Polya, 1945).
Problem-solving in authentic contexts
"… how to apply mathematics …" (Freudenthal, 1973); … to understand real-life events … use mathematical knowledge in appropriate contexts (Schoenfeld, 2007); … strategic competence – the ability of learners to pose and solve mathematics problems …(Kilpatrick *et al.* , 2001); "… enables us to understand the world …" (NDE, 2003); "Mathematical problem solving enables us to understand the world (physical, social, economical) …" (DBE, 2010b).

Mathematics as a human activity
"… distinctively human activity practiced by all cultures …"(NDE, 2003); "… distinctly human activity practiced by all cultures …"(DBE, 2010b).
Conceptual and procedural knowledge
"… set theory, abstract algebra and analysis, topology – were eruptions of conceptual thinking …"; "Each conceptual innovation encloses in itself the germ of algorithmisation …"; "Algorithms provide the technical means of fathoming greater conceptual depth …" (Freudenthal, 1973); "… viewing mathematics as facts and procedures … good understanding of mathematics concepts, skills and procedures (Schoenfeld, 1992, 2007); "…*conceptual understanding* – the understanding of the concepts, operations and relations in mathematics; *procedural fluency* – the completion of mathematical procedures in precise, flexible, effective and appropriate ways …" (Kilpatrick *et al.*, 2001).
Relationships
"… set theory, abstract algebra and analysis, topology – were eruptions of conceptual thinking …" (Freudenthal, 1973); "… the understanding of the … relations in mathematics" (Kilpatrick *et al.*); "… descriptive, numerical and symbolical relationships …" (NDE, 2003); "… the connections between different mathematical topics (Polya, 1945).
Patterns
"… the science of seeking patterns …" (Schoenfeld, 1992); "… based on observing patterns …" (NDE, 2003); "… seek out patterns …" (DBE, 2010b).
New conjectures and abstract thought
"… ahead of its applications … abstract algebra …" (Freudenthal, 1973); "… tools of mathematics are abstraction … extending a solution to a broader or new context, finding new solutions …" (Schoenfeld); "…to pose and solve mathematics problems …" (Kilpatrick *et al.*, 2001); "… theories of abstract relations …" (NDE, 2003); "… formulate new conjectures …" (DBE, 2010b); "… alternative solutions and applications in different contexts …" (Polya, 1945).
Knowledge construction by means of language
"…conscious occupation with language …" (Freudenthal, 1973); "…translating events of everyday life into mathematical language …" (Schoenfeld, 1992; 2007); "… to reflect, explain and justify …" (Kilpatrick *et al.*, 2001); "… through … language …" (NDE, 2003); "…learners should be able to repeat the problem statement easily …" (Polya,

1945).
Changing nature and flexibility
"… traditional geometry was not as rigorous as people had believed …"; "Mathematics is never finished … "; "… its flexibility …" (Freudenthal, 1973); "… different conceptualizations of the nature of mathematics … cognitive aspects like flexibility …" (Schoenfeld, 1992, 2007); "… the completion of mathematical procedures in … flexible … ways" (Kilpatrick *et al.*, 2001); "… by social interaction and is thus open to change …" (NDE, 2003); "… alternative solutions and applications in different contexts …" (Polya, 1945).
Historical development
"… since antiquity …" (Freudenthal, 1973); "… developed and contested over time …" (NDE, 2003); "... for thousands of years …" (DBE, 2010b).
Attitudes
"… positive attitude towards problem-solving …" (Schoenfeld, 2007); "...learners' consistent inclination to regard mathematics as useful and applicable to their lives, tied with a belief in their own effectiveness and diligence (Kilpatrick *et al.*, 2001); "… learners may become uninterested and unmotivated if mathematics is not applied in problem-solving contexts ... it requires ... tenacity of the learners ..." (Polya, 1945).

In Table 3.1, aspects related to the nature of mathematics, the aims of mathematics education and mathematical proficiency are identified. Next, these aspects are discussed briefly by referring to the following sources: Freudenthal (1973); Schoenfeld (1992, 2007); Kilpatrick *et al.* (2001), the NCS (NDE, 2003), and the CAPS (DBE, 2010b).

- ***Thinking and reasoning skills***

All sources (see Table 3.1) refer to thinking and reasoning skills. Two main categories of thinking are mentioned, namely logical thinking and creative thinking.

- ***Problem-solving in authentic contexts***

This aspect also features in all sources. A clear link between problem-solving in authentic contexts and mathematical understanding is established.

- ***Mathematics as a human activity***

Only the South African policy documents explicitly refer to this aspect. It is probably implicitly assumed by the other sources in their references to problem-solving in authentic contexts, knowledge construction by means of language, and the historical development of mathematics.

- ***Conceptual and procedural knowledge***

The NCS (NDE, 2003) and the CAPS (DBE, 2010b) do not directly refer to conceptual and procedural knowledge, but these aspects are basic to an aspect mentioned by both policy documents, namely problem-solving in authentic contexts. In problem-solving, conceptual knowledge is needed in the understanding of the problem (Polya's first phase) when the mathematics concepts and topics related to the problem are identified. Procedural knowledge is applied when the plan is carried out (Polya's third phase) by applying applicable algorithmic procedures.

- ***Relationships***

Only Schoenfeld (1992, 2007) and the CAPS (DBE, 2010b) do not refer directly to mathematical relationships. However, the identification of relationships between mathematics concepts and topics is a crucial part of problem-solving which is an important aspect to which Schoenfeld (1992, 2007) and the CAPS (DBE, 2010b) refer. Specifically, the making of a plan (Polya's second phase) requires an understanding of these relationships.

- ***Patterns***

Only Schoenfeld (1992), the NCS (NDE, 2003) and the CAPS (DBE, 2010b) refer to the examining of patterns. One could argue that an investigation into patterns relates to the establishing of relationships between phenomena. As relationships feature directly or indirectly (see the previous paragraph) in all sources, patterns are most likely an aspect included in all sources.

- ***New conjectures and abstract thought***

All sources cite this aspect. The emphasis in most sources is on abstract thought, but only Kilpatrick *et al.* (2001), the NCS (NDE, 2003) and the CAPS (DBE, 2010b) mention the stating of conjectures. The stating of conjectures could possibly be viewed as a consequence of abstract thought, which implies a higher level of mathematical engagement.

- ***Knowledge construction by means of language***

The CAPS (DBE, 2010b) is the only document that does not directly indicate language as a tool in the construction of mathematical knowledge, but its reference to the formulation of new conjectures implies the appropriate use of language.

- ***Changing nature and flexibility***

This aspect features strongly in all sources, with the exception of the CAPS (DBE, 2010b). In a previous discussion, the implicit reference in the CAPS (DBE, 2010b) to the flexibility of mathematics was discussed (see 3.2.3).

- ***Historic development***

Despite the lack of references by Schoenfeld (1992, 2007) and Kilpatrick *et al.* (2001) to the historical development of mathematics, it is commonly accepted that the roots of mathematics lie in antiquity.

- ***Attitudes***

Freudenthal (1973) and the South African policy documents do not mention attitudes directly. However, Freudenthal's (1973) reference to "the greatest virtue of mathematics" reveals the possibility of having a positive attitude towards mathematics. It seems reasonable to argue that a possible consequence of problem-solving in authentic contexts is an appreciation for the utility value of mathematics. Therefore, the references to problem-solving in authentic contexts in the NCS (NDE, 2003) and the CAPS (DBE, 2010b) may indirectly reflect the possibility of positive attitudes, due to the sense of achievement when problems are solved successfully.

In summary, these aspects point to the nature of mathematics, the aims of mathematics education and aspects of mathematical proficiency from an international and a national perspective. Thus, the first of the three themes explored in Chapter 3 was addressed in this section (3.2). A discussion of the second theme follows in the next section.

3.3 THE RELATIONSHIP BETWEEN DE CORTE'S (1996) EDUCATIONAL LEARNING THEORY AND LEARNING IN MATHEMATICS

In order to establish a broader basis for mathematical proficiency, Schoenfeld (1992: 345-348) examined studies undertaken in the 20th century by educational researchers, psychologists, social scientists, philosophers, and cognitive scientists, but he also acknowledges the influence of Plato and Aristotle on thinking and learning. He concluded his investigation by stating that there is apparently an emerging consensus that the following aspects are important in order to become proficient in a domain: a knowledge basis; problem-solving strategies; monitoring and control, as well as beliefs and affect.

These aspects relate to aspects of De Corte's (1996: 33-43) educational learning theory for any domain. Although learning psychologists find it difficult to agree on a list of the aspects that describe effective learning, De Corte's (1996) educational learning theory represented a temporary consensus (Tella, 1996: 5 of 7). It entails two subthemes, namely effective learning and expert performance.

The characteristics of effective learning in mathematics are discussed next, using De Corte's (1996) educational learning theory (applicable to any domain) as a theoretical basis (see 3.3.1). The four categories of aptitude that relate to expert performance in mathematics will conclude the discussion on De Corte's (1996) educational learning theory (see 3.3.2).

3.3.1 Effective learning

De Corte (1996: 33-34) defines effective and meaningful learning as follows:

> Learning is a constructive, cumulative, self-regulated, goal-directed, situated, collaborative, and individually different process of meaning construction and knowledge building.

This definition lists the characteristics of effective learning and states the effect of learning as "meaning construction and knowledge building" that occurs in individuals. From a more recent perspective, the effect of learning is viewed as long-term changes in an individual's knowledge, skills, attitude, understanding of the world, or behaviour (Geren & Leahey, 2011: 264-265). These two perspectives are similar, as the process of meaning construction could also include attitudinal changes.

In the next sections, frequent reference will be made to the National Council of Teachers of Mathematics (NCTM) due to the high international regard for their work (Van de Walle, 2004: 1). The NCTM is a professional, non-profit organisation from the USA that is regarded as a world leader of the reform movement in mathematics education. They published the following documents that have guided the reform movement: *Curriculum and Evaluation Standards for School Mathematics (1989); Professional Standards for Teaching Mathematics (1991),* and *Assessment Standards for School Mathematics (1995)* (Van de Walle, 2004: 1-2). After an extensive revision process, they published *Principles and Standards for School Mathematics* in 2000. This document describes features of high-quality mathematics education (NCTM, 2000: ix and 10). In the next sections, references to this document will form an integral part of the discussion. In addition, the definitions of the aspects of De Corte's (1996) educational learning theory discussed in the next sections are provided by De Corte, Verschaffel and Masui (2004: 365-384).

3.3.1.1 Constructive

De Corte *et al*. (2004: 369) define the constructive aspect of effective learning as follows:

> Learning is an effortful and mindful process in which students actively construct their knowledge and skills through reorganization of their already acquired mental structures in interaction with the environment.

This definition underscores active learner participation in the learning process. The use of the word *mindful* indicates the importance of learners knowing how to learn effectively, otherwise their effort could be misdirected. The reference to *reorganization of their already acquired mental structures* points to learners' awareness of their current cognitive levels when they encounter new knowledge. It also indicates learners' ability to successfully integrate new knowledge. This definition recognises the vital role the environment plays in enhancing active knowledge building.

In the past, mathematics educators have recognised the importance of the constructive aspects of learning. The construction theorem, formulated by Bruner (in Bell, 1978: 143), states that learners would understand mathematical concepts, rules and principles better if they first construct their own representation of the concepts, rules and principles. The implication of this theorem for effective learning is twofold. First, it shows the relationship between effective teaching and effective learning. Mathematics teachers should construct learning activities that allow learners to discover rules and principles. Secondly, learners should take more responsibility in the process of understanding mathematics by not relying solely on the teacher to explain mathematical concepts.

Other authors also point to the importance of learners actively constructing meaning and understanding in the learning process and being able to interpret the vast amount of data they receive daily (Schoenfeld, 1992: 335; NCTM, 2000: 2, 10; Van de Walle, 2004: 31). In order to interpret data effectively, learners need to be mindful of effective learning practices, as pointed out by the De Corte *et al*.'s (2004) definition.

3.3.1.2 Cumulative

De Corte *et al*. (2004: 369) define the cumulative aspect of effective learning as follows:

> This characteristic stresses the important impact of students' prior formal as well as informal knowledge on subsequent learning.

This definition clearly points to the need to connect new knowledge to already existing knowledge. Prior knowledge refers to aspects of mathematics such as basic facts, principles and procedures that learners are being taught explicitly. Prior knowledge also includes knowledge obtained by way of informal means.

In his connectivity theorem, Bruner (in Bell, 1978: 145) states that all principles, concepts and skills in mathematics are related to other principles, concepts and skills. He argues that analytical and synthetic reasoning in mathematics is made possible by these structured and interrelated connections in mathematics. Prior knowledge can be used to construct relationships of difference, that is, different forms of representation of the same mathematical concept. In addition, relationships of similarity can be constructed, that is, the same form of representation for different mathematical concepts. Therefore, these two forms of representation address the following questions: "In which way are these mathematical concepts different?" and "In which way are these mathematical concepts similar?" (Bruner, in Bell, 1978: 145).

The cumulative aspect of effective learning can also be beneficial in problem-solving contexts. Prior knowledge enables learners to apply knowledge from a variety of mathematical topics in a problem-solving situation (NCTM, 2000: 2). Better learning takes place as new knowledge is constructed by linking it with experience and prior knowledge (NCTM, 2000: 10).

The above first two aspects of effective learning, namely the constructive aspect and the cumulative aspect, are similar in that they stress the importance of linking new knowledge with prior knowledge. These two aspects differ in what they require a learner to focus on. In bearing the constructive aspect in mind, learners may ask themselves how they can represent new information in their own way by building on prior knowledge. In bearing the cumulative aspect in mind, learners may focus less on how to present new information in their own way, but rather on which specific mathematics concepts and topics are linked with the new information.

These two aspects are vital elements of mathematical understanding, because mathematical understanding is a "measure of the quality and quantity of connections that an idea has with existing ideas" (Van de Walle, Karp, & Bay-Williams 2010: 23). This definition of understanding clearly relates to the constructive and cumulative aspects of effective learning with its reference to the connection between existing and new ideas. One's mind operates on existing knowledge networks that are represented internally in a structured way. As mental operations are not directly observable, a discussion of how ideas are represented is based on assumptions. Cognitive science makes two assumptions about the mental representation of knowledge. The first assumption is that there is a relationship between the external and the internal representations of a concept. Secondly, different internal representations of concepts can be associated in useful ways (Hiebert & Carpenter, 1992: 66). The first assumption permits the internal representation of any new information which a learner encounters, while the second assumption allows for different concepts to be linked together in order to produce networks of knowledge. The notion of "networks of knowledge" provides a helpful framework in the discussion of mathematical understanding (Hiebert & Carpenter, 1992: 66).

The need for learners to understand mathematics is one of the most widely acknowledged ideas among mathematics teachers (Hiebert & Carpenter, 1992: 65). In support, the NCTM (2000: 19) regards understanding as one of the most important elements of effective learning in mathematics, as stated in the Learning Principle:

> Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge.

In clarifying the Learning Principle, the NCTM states that learning without conceptual understanding has been a problematic feature of mathematics education since at least the 1930s (NCTM, 2000: 19). Conceptual understanding is "… knowledge about the relationships or foundational ideas of a topic" (Van de Walle *et al*., 2010: 24). This definition of conceptual understanding clearly links with the constructive and cumulative aspects of effective learning. The NCTM (2000: 19-20) mentions some advantages of learning with conceptual understanding.

First, learning with conceptual understanding makes subsequent learning easier. Secondly, in a novel situation such as a problem-solving context, it is a crucial aspect of the required knowledge to successfully solve the problem. Thirdly, as change is a common feature of everyday life, learning with conceptual understanding enables learners to deal with new kinds of problems. Fourthly, learning with conceptual understanding enables learners to become more autonomous.

3.3.1.3 Self-regulated

De Corte *et al*. (2004: 369) define the self-regulated aspect of effective learning as follows:

> This feature refers to the metacognitive nature of productive learning; indeed, self-regulation of learning means that students manage and monitor their own processes of knowledge building and skill acquisition. The more students become self-regulated, the more they assume control and agency over their own learning; consequently they become less dependent on external instructional support for performing those regulatory activities.

An earlier discussion focused on the similarities and differences between self-regulation and metacognition (see 2.2.6). Metacognition as a concept was extensively discussed (see 2.2.3 and 2.2.4). Therefore, the definition of self-regulation by De Corte *et al*. (2004: 39) is discussed only briefly.

De Corte *et al*. (2004) use the terms “metacognitive” and “self-regulation” interchangeably, thereby indicating the similarities between the two concepts. They also stress the importance of learners taking responsibility for their own learning processes. The definition also refers to students “becoming” self-regulated. This indicates that a learner’s self-regulation can be enhanced over time. In fact, the NCTM (2000: 20) regards enhancing learner self-regulation as one of the major goals of mathematics education.

The NCTM (2000: 20) lists a number of positive effects of SRL. First, self-regulated learners learn more effectively, because they define their own goals and monitor their progress. Secondly, they have more confidence in their attempt to solve problems. Thirdly, self-regulatory learners are more flexible in their use of problem-solving strategies and are more willing to try alternative ways of solving problems. Fourthly, they show more perseverance when confronted with a complex mathematical task.

3.3.1.4 Goal-directed

This aspect is explained by De Corte *et al*. (2004: 369) in the following definition:

> Effective and meaningful learning is facilitated by an explicit awareness of, and orientation toward a goal. Because of its constructive and self-regulated nature, it is plausible that learning will be most productive when students choose and determine their own objectives. Therefore, it is desirable to stimulate and support goal-setting activities in students.

This definition highlights learner awareness of goals which they have chosen themselves, and emphasises the role of the teacher to assist learners with goal-setting activities. The necessity of teacher involvement in goal-setting activities may be due to two reasons. First, learners may not be aware of the importance of being goal-oriented in the process of learning. Secondly, some learners may need guidance to set themselves appropriate goals; otherwise, their goals may either be unchallenging or unrealistically high.

Goal-setting activities are addressed in two of the six principles stated for mathematics education by the NCTM (NCTM, 2000: 10). In the first of these two principles, the Equity Principle (NCTM, 2000: 10), it is stated that:

> Excellence in mathematics education requires equity – high expectations and strong support for all students.

The NCTM (2000: 11) discusses this principle by stating the following:

> Making the vision of the Principles and Standards for School Mathematics a reality for all students […] is both an essential goal and a significant challenge. Achieving this goal requires raising expectations for students' learning, developing effective methods of supporting the learning of mathematics by all students, and providing students and teachers with the resources they need.

> Educational equity is a core element of this vision. All students, regardless of their personal characteristics, backgrounds, or physical challenges, must have opportunities to study – and support to learn – mathematics. Equity does not mean that every student should receive identical instruction; instead, it demands that reasonable and appropriate accommodations be made as needed to promote access and attainment for all students.

The NCTM's discussion emphasises the following two aspects. First, learner goals need to be really challenging. Although De Corte *et al*. (2004: 369) do not explicitly stress the "high expectations" aspect of goal-setting, this aspect relates to the first part of De Corte *et al*.'s definition (2004: 369) which refers to "… an explicit awareness of, and orientation toward a goal". This aspect underscores the teacher's responsibility to assist the learners in setting goals that really challenge them. The NCTM (2000: 12) recommends that high expectations be communicated to learners by means of teacher-learner interactions; comments on learner answer sheets; assigning learners to instructional groups, and the level of teacher support learners receive.

A second aspect relates to learners' need to be supported in achieving these goals by taking personal differences into account. This aspect is discussed in more detail in a subsequent section that deals with the "individually different" aspect of effective learning (see 3.3.1.6).

3.3.1.5 Situated and collaborative

De Corte *et al*. (2004: 370) define the "situated and collaborative" aspects of effective learning as follows:

> Learning is conceived as an interactive activity between the individual and the physical, social and cultural context and artefacts, and especially through participation in cultural activities and contexts. In other words, learning is mostly not a purely 'solo' activity, but distributed one: the learning effort is distributed over the individual student, his partners in the living environment, and the resources and (technological) tools that are available.

De Corte *et al*.'s (2004) definition highlights three main features. First, learners should partake in activities within an authentic and cultural context, using available resources. Schoenfeld (1992: 35) refers to the first feature by stating that mathematically strong learners are able to apply mathematics practically in both simple contexts such as scale models, and complex contexts such as statistical analysis. Learners should view these contexts as important (NCTM, 2000: 10). In fact, as De Corte (2000: 254) and Kilpatrick *et al.* (2001: 5) point out, the authentic contexts in which learning takes place should have personal meaning for learners, that is, learners should view mathematics as useful and applicable to their lives.

Secondly, learning is viewed as an interaction between an individual learner and fellow learners. Learners' cognitive activities are enhanced in a social context (see 2.2.4.4 k), because mathematics learning takes place within a structure of specific social practices. The social context of learning is so important that the question is raised as to whether it is possible for an individual learner to learn abstract mathematics autonomously (Schubauer-Leoni & Perret-Clermont, 1997: 269). The NCTM (2000: 2) and De Corte (2000: 254) also support the importance of collaborative learning (see 2.2.4.4. k).

Thirdly, reference is made to available resources and technological tools that may improve effective learning. In Section 3.3.2.1, a more detailed discussion of the impact of resources and technological tools is presented.

3.3.1.6 Individually different

For learning to be effective, learners should be aware of their differences regarding some aspects of learning. De Corte *et al*. (2004: 370) view individual differences in learning as follows:

> The process and outcomes of learning vary among students due to individual differences in a diversity of aptitudes that affect learning, such as prior knowledge, conceptions of learning, learning styles and strategies, interest, motivation, self-efficacy, beliefs, and emotions. To induce productive learning in students, instruction should take into account these differences in aptitudes.

This definition refers to many aspects regarding individual differences. The key part of this definition states that "… instruction should take into account these differences in aptitudes". Therefore, this discussion will mainly focus on these individual differences.

The essential role that *prior knowledge* plays in the constructive and cumulative aspects of effective learning was discussed in Sections 3.3.1.1 and 3.3.1.2. A *conception of learning* refers to a consistent system of knowledge and beliefs about oneself as a learner, learning objectives, learning activities and strategies, general aspects of learning and studying (Vermunt & Vermetten, 2004: 362). Learners use diverse ways to acquire knowledge and understanding (Van de Walle, 2004: 32). Therefore, strong support should be given to learners, and differences in learning should be accommodated (NCTM, 2000: 2, 10).

Learning styles are learners' preferred modes of learning, that is, the cognitive and affective behaviours that indicate how learners interact with the learning environment (Baden & Rightmeyer, 2011: 266). Several factors may influence learners' preferred style of learning. First, intellectual preferences refer to multiple intelligences such as spatial, linguistic, and bodily-kinaesthetic intelligence. Secondly, family culture may determine the degree to which learners display interdependent or independent learning styles. A third factor is psychological attributes – for example, sensitivity to different types of sensory stimuli. A fourth factor is the sociological histories, that is, the thought and behavioural patterns that learners display as a result of socialisation (Baden & Rightmeyer, 2011: 266).

Learning strategies can be divided into cognitive strategies, metacognitive strategies, and management and organisation of learning resources (Radovan, 2011: 217). Cognitive strategies include repetition, organisation, elaboration and elements of critical thinking. Metacognitive strategies were discussed in a previous section (see 2.2.4.4). The organisation of learning resources points to learners' organisation of the learning environment and time management (Radovan, 2011: 218).

Learners differ not only in their cognitive aptitudes, but also in their affective characteristics (De Corte, 2000: 254). These affective characteristics such as *interest,* *motivation, self-efficacy, beliefs and emotions* are discussed as affective components of effective learning in a later section (see 3.3.2.3).

In this section, effective learning as the first subtheme of De Corte's (1996) educational learning theory was discussed. The second subtheme of De Corte's (1996) educational learning theory, namely *expert performance,* is discussed in Section 3.3.2.

3.3.2 Expert performance

For expert performance in a given domain, a learner needs to integrate the four categories of aptitude, namely a structured, accessible domain-specific knowledge basis; heuristic methods; affective components, and metacognition (De Corte, 1996: 34-36; De Corte *et al*., 2004: 368-369).

These aspects of expert performance correspond in three ways to the aspects necessary for proficiency in any domain, as identified by Schoenfeld (see 3.3). First, two aspects (knowledge basis and affective components) feature in both Schoenfeld's and De Corte *et al*.'s definitions. Secondly, Schoenfeld refers to problem-solving strategies which relate closely to De Corte *et al*.'s (2004) view of heuristics (see 3.3.2.2). Thirdly, the monitoring and control aspect in Schoenfeld's definition corresponds with De Corte *et al*.'s (2004) view of metacognition (see 3.3.2.4).

3.3.2.1 Knowledge basis

According to De Corte *et al*. (2004: 368), this category of aptitude is defined as follows:

> A well-organized and flexibly accessible domain-specific knowledge basis involving the facts, symbols, concepts, and rules that constitute the contents of a subject-matter field.

The obvious important aspect in this category of aptitude is the necessity to make quality learning resources available to learners. These resources, whether textbooks, worksheets, physical models or software programmes, should sufficiently cover the domain-specific knowledge.

Resources are a theme common to two of the six principles for school mathematics stated by the NCTM (2000: 10-11). In the Equity principle (NCTM, 2000: 11), reference is made to the necessity of providing students and teachers with the resources they need in order to perform well in mathematics. In the Technology principle (NCTM, 2000: 24), the important role that technology plays in enhancing learner performance is underscored:

> Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning.

This principle points to the positive impact that technology can have on the teaching-and-learning of mathematics. The NCTM (2000: 24-26) lists several aspects of effective learning that could be enhanced by the use of technology. First, technology may enable learners to focus to a greater extent on problem-solving skills, because the organising and analysis of data are done more efficiently and accurately. Secondly, learners' understanding of mathematics may improve, as conjectures are explored with more ease. Learners' understanding may be further enhanced when technological tools enable learners to view a problem from multiple perspectives. Thirdly, the use of technology could greatly extend the range of problems that can be explored. Fourthly, feedback on the learning process can be provided more rapidly, as learners immediately notice the effect when variable values are changed in, for example, a spreadsheet programme. Fifthly, the use of technology may cater better for special learner needs.

3.3.2.2 Heuristics

De Corte *et al*. (2004: 368) define heuristics as follows:

> Heuristic methods are systematic search strategies for problem analysis and transformation.

De Corte *et al*. (2004: 368) further view search strategies as follows:

> Search strategies are used for problem analysis and transformations (e.g., decomposing a problem into sub goals, making a graphic representation of a

> problem) which do not guarantee, but significantly increase the probability of finding the correct solution of a problem because they induce a systematic approach to the task.

The goal of applying systematic search strategies is to find the solution to a problem. In problem-solving, previously acquired knowledge and experience are applied to an unfamiliar situation that contains major obstacles in order to find a solution to the problem (Killen, 2010: 246; DuBois, Clinton, Trowell & Fincher, 2011: 369). Problem-solving in mathematics enhances learners' understanding and effective learning in mathematics (see 2.2.4.4m).

3.3.2.3 Affective components

De Corte *et al*. (2004: 369) define affective components as follows:

> Positive beliefs about the self in relation to learning and problem solving in a domain, about the social context in which learning activities take place, and about the content domain and learning and problem solving in that domain.

In this definition, a distinction is made between three types of learner beliefs. First, beliefs about themselves as learners and problem-solvers are identified. Beliefs about self are strongly associated with metacognition, self-regulation and self-awareness (Gorno & Rohrkemper, in McLeod, 1992: 580). Gender differences are one aspect of beliefs about self that has been researched in detail (McLeod, 1992: 580). There are definite gender differences regarding self-concept and confidence in learning mathematics (Reyes, Meyer & Fennema, in McLeod, 1992: 580).

A second type is learners' beliefs about their social surroundings – for example, learners may believe that learning is mostly competitive. In a study by Grouws and Cramer (in McLeod, 1992: 581), it was found that secondary school mathematics teachers who enable their learners to be good problem-solvers establish a supportive classroom environment in which learners enjoy problem-solving. Broader social influences that affect learner beliefs include parental and cultural influences.

Thirdly, learners hold beliefs about mathematics and the nature of effective learning and problem-solving in that subject. Research on learners' beliefs about mathematics reveals that learners regard mathematics as complicated, important, and based on rules (Brown *et al*., in McLeod, 1992: 579). Although these beliefs are mainly cognitive, they are significant in the development of learner attitudes and emotional reactions to mathematics (McLeod, 1992: 579). Learners' beliefs about mathematics may have a negative effect on their problem-solving abilities. If learners believe that problems must be solved in less than five minutes, they may find it difficult to persevere when they attempt to solve problems that take considerably longer to solve (Schoenfeld, in McLeod, 1992: 579).

Expressing a broader point of view, McLeod (1992: 575, 578) suggests that beliefs, attitudes, and emotions constitute the three subdomains of affective components. As a rule, beliefs are stable and cognitive depictions of that which the person considers applicable or valid. Attitudes are reasonably stable predispositions, such as enjoyment of geometric proof, and emotions are rapidly changing feelings, such as joy in solving a problem or an aesthetic response to mathematics. These three categories (beliefs, attitudes, and emotions) vary in terms of affective involvement (increasing); cognitive involvement (decreasing); intensity of response (increasing), and response stability (decreasing) (McLeod, 1992: 575-578). Goldin (in Leder & Forgasz, 2006: 404) also refers to these three subdomains, but adds values, ethics and morals as a fourth subdomain. These are viewed as strongly held preferences that may be highly affective and cognitive.

A different viewpoint is held by Aiken (in Leder & Forgasz, 2006: 404) who views affect as a subdomain of attitude. He categorises attitude as cognition (knowledge of intellect), affect (emotion and motivation), and performance (behaviour). This view is supported by Triandis (in Leder & Forgasz, 2006: 404) who states that attitudes "… involve what people *think* about, *feel* about, and how they would like to *behave* toward an attitude object".

It could be argued that the three subdomains of affective components (McLeod, 1992: 578) are similar to the three categories of attitude (Aikin, in Leder & Forgasz, 2006:

404), because of the obvious similarities between beliefs and cognition, and between emotions and affect. A link between the two remaining aspects, attitudes and performance (behaviour), could be established if one were to argue that one's attitude has a strong effect on performance. Therefore, a synthesis of views on affective components should include the three aspects of thinking, feeling and acting in addition to the fourth subdomain of values, ethics and morals (Goldin, in Leder & Forgasz, 2006: 404).

The importance of affective components for expert performance is affirmed not only by Schoenfeld (1992) and De Corte *et al*. (2004) (see 3.3 and 3.3.2.3), but also by McLeod (1992: 575, 578) who suggests that affective matters need to form part of studies on cognition and instruction in mathematics, because informal observation confirms that affective issues play an important role in the teaching-and-learning of mathematics. Goos and Galbraith (1996: 231) also state that beliefs about the nature of mathematics and about themselves could have a positive or negative effect on learners' cognitive and metacognitive processes involved in problem-solving.

3.3.2.4 Metacognition

De Corte *et al*. (2004: 368-369) use two categories in their definition of metacognition, namely metaknowledge and self-regulatory skills. They view metaknowledge as follows:

> Metaknowledge, involving knowledge about one's cognitive functioning […] on the one hand, and knowledge about one's motivation and emotions that can be used to deliberately improve volitional efficiency […] on the other hand.

It is evident that metaknowledge involves knowledge of two aspects, namely cognitive aspects and affective aspects. This view of metaknowledge corresponds with the first two categories of Flavell's original conceptualisation of metacognition, namely metacognitive experiences and metacognitive knowledge (see 2.2.4.1 and 2.2.4.2).

De Corte *et al*. (2004: 369) define self-regulatory skills as follows:

> Self-regulatory skills, involving skills relating to regulating one's cognitive processes/activities […] on the one hand, and skills for regulating one's volitional processes/activities […] on the other hand.

In this view, regulation bears on two aspects: cognitive aspects and affective aspects. Therefore, this corresponds with the fourth category of Flavell's definition of metacognition, namely metacognitive strategies (see 2.2.4.4). As metacognitive strategies are also used to monitor and regulate goals, they also relate to the third category of Flavell's definition, namely metacognitive goals (see 2.2.4.3). The term *self-regulatory*, used in this definition, does not feature explicitly in Flavell's definition. However, as there are strong links between self-regulation and metacognition (see 2.2.6), it can be concluded that De Corte *et al*.'s (2004: 369) view of metacognition is similar to Flavell's original conceptualisation of metacognition (see 2.2).

In this section, the second theme was addressed by discussing De Corte's (1996) educational learning theory and its link with learning in mathematics. Next, in the discussion of the third theme, a synthesis of the first two themes is provided in order to establish a mathematical perspective on De Corte's (1996) educational learning theory.

3.4 A MATHEMATICAL PERSPECTIVE ON DE CORTE'S (1996) EDUCATIONAL LEARNING THEORY

Sections 3.2 and 3.3 addressed many aspects related to effective learning in mathematics. Table 3.2 presents the relationships between these aspects.

Table 3.2: A synthesis of aspects related to De Corte's (1996) educational learning theory and aspects related to the nature of mathematics

De Corte's (1996) educational learning theory (as it relates to learning in mathematics)	Aspects related to the nature of mathematics
Constructive	Thinking and reasoning skills; mathematics as a human activity; language; conceptual and procedural knowledge; changing nature and flexibility.
Cumulative	Thinking and reasoning skills; relationships; patterns; historic development.

De Corte's (1996) educational learning theory (as it relates to learning in mathematics)	Aspects related to the nature of mathematics
Self-regulated	Mathematics as a human activity; changing nature and flexibility; attitudes.
Goal-directed	Mathematics as a human activity; attitudes; problem-solving in authentic contexts.
Situated and collaborative	Mathematics as a human activity; language; changing nature and flexibility; problem-solving in authentic contexts.
Individually different	Mathematics as a human activity; language; changing nature and flexibility; attitudes.
Knowledge basis	Conceptual and procedural knowledge.
Heuristics	Problem-solving in authentic contexts; new conjectures and abstract thought; changing nature and flexibility.
Affective components	Mathematics as a human activity; attitudes.
Metacognition	Mathematics as a human activity; conceptual and procedural knowledge; attitudes; problem-solving in authentic contexts.

De Corte's (1996) educational learning theory, as it relates to learning in mathematics, and aspects related to the nature of mathematics (see Table 3.2) are linked in many ways. Although a complex network of direct and indirect interactions between all the mentioned aspects is likely, only some of the most obvious relationships are highlighted. In addition, these relationships are not fixed, and may be interpreted in different ways. A discussion of these relationships follows next.

- **Constructive**

The *constructive* element emphasises active learner involvement which relates to human activities such as thinking skills, reasoning skills and language. Active learner involvement also implies the application of conceptual and procedural knowledge and the likelihood that learners will demonstrate a flexible use of mathematics during the process of knowledge-building and meaning-construction. The important role that language plays in the construction of knowledge should also be stressed. The *constructive* aspect shows the most relationships with the individual aspects related to the nature of mathematics.

- **Cumulative**

The *cumulative* aspect focuses on relationships between prior and new knowledge. Therefore, relationships and patterns are key aspects relating to the *cumulative* aspect. It could be argued that prior knowledge also entails cumulative knowledge acquired through the ages, thereby linking the *cumulative* aspect to the historic development of mathematics. Of course, thinking and reasoning skills are an important aspect in linking topics in mathematics.

- **Self-regulated**

This aspect relates strongly with mathematics as a human activity which implies the flexible use of mathematics, due to individual differences. In addition, the central role that attitudes play in learner regulative behaviour must be stressed.

- **Goal-directed**

When learners are challenged to set their own goals, individual differences relating to ability and attitudes will result in different goals. Problem-solving in authentic contexts could stimulate learner curiosity and enhance learner attitudes (see 3.2.2.2). Therefore, a clear relationship between learner goals and problem-solving activities is likely.

- **Situated and collaborative**

The *situated* aspect links with problem-solving in authentic contexts, whereas the *collaborative* aspect relates strongly to the human aspect in the mathematics learning process. Possible changes in learner attitudes, due to the nature of the mathematics being taught and the social context of learning, indicate the link with attitudes.

- **Individually different**

Learners' individual differences in connection with learning in mathematics (see 3.3.1.6) clearly relate to the aspect that views mathematics as a human activity; hence, a flexible use of mathematics is a possibility. In addition, aspects such as language proficiency and attitudes are probably also related to individual differences in learners.

- **Knowledge basis**

As a knowledge basis involves the facts, symbols, concepts, and rules of a domain (see 3.3.2.1), a clear relation with conceptual and procedural knowledge is evident.

- **Heuristics**

Heuristics and problem-solving in authentic contexts are closely linked. Since problem-solving may also involve the finding of alternative solutions and solutions in different contexts, *heuristics* also relates to the aspect of new conjectures and abstract thought.

- **Affective components**

It is obvious that this aspect of De Corte's (1996) educational learning theory relates to attitudes. It also plays a role in how mathematics is practised as a human activity.

- **Metacognition**

The concept *metacognition* has been discussed extensively (see 2.2). De Corte *et al*.'s (2004: 368-369) definition of metacognition (see 3.3.2.4) focuses, first, on the knowledge of cognition and affect and, secondly, on the regulation of cognition and affect. The knowledge and regulation of cognition clearly relate to conceptual and procedural knowledge, whereas the knowledge and regulation of affect correspond with attitudes. In addition, a link between metacognition and problem-solving in authentic contexts is obvious as metacognition enhances achievement and problem-solving ability in mathematics (see 2.3).

- **Aspects related to the nature of mathematics**

Table 3.2 indicates the following relationships between aspects representing the nature of mathematics and De Corte's (1996) educational learning theory:

- thinking and reasoning skills (*constructive*; *cumulative*);
- problem-solving in authentic contexts (*goal-directed*, *situated and collaborative*, *heuristics*, *metacognition*);

- mathematics as a human activity (*constructive*, *self-regulated*, *goal-directed*, *individually different*, *affective components*, *metacognition*);
- conceptual and procedural knowledge (*constructive*, *knowledge basis*, *metacognition*);
- relationships (*cumulative*);
- patterns (*cumulative*);
- new conjectures and abstract thought (*heuristics*);
- knowledge construction by means of language (*constructive*, *situated and collaborative*, *individually different*);
- changing nature and flexibility (*constructive*, *self-regulated*, *situated and collaborative*, *individually different*, *heuristics*);
- historic development (*cumulative*), and
- attitudes (*self-regulated*, *goal-directed*, *individually different*, *affective component*s, *metacognition*).

A few observations result from this analysis. First, *mathematics as a human activity*, *changing nature and flexibility,* and *attitudes* feature most frequently. Secondly, *problem-solving in authentic contexts* also relates to many aspects of De Corte's (1996) educational learning theory. *Relationships, patterns, new conjectures and abstract thought,* and *historic development* are only linked to one aspect of De Corte's (1996) educational learning theory. All aspects related to the nature of mathematics illustrate a relation with at least one aspect of De Corte's (1996) educational learning theory.

In conclusion, it is apparent that De Corte's (1996) educational learning theory indicates a strong relationship with, first, aspects related to the nature of mathematics, which include the *aims of mathematics education* and *mathematical proficiency* (see 3.2)*,* and, secondly, learning in mathematics (see 3.3). In this section, a mathematical perspective on De Corte's (1996) educational learning theory was provided. In the next section, a framework for metacognitive interventions in mathematics is proposed.

3.5 A PROPOSED FRAMEWORK FOR METACOGNITIVE INTERVENTIONS IN MATHEMATICS

The aim of this chapter is to address secondary research question 4. In this section, the third theme explored in this chapter (see 3.4) and the secondary research question 3 explored in Chapter 2 (see 2.4.7.5) are combined in order to propose a framework for metacognitive interventions in mathematics.

In Section 2.4.7.5, the features of previous metacognitive interventions in mathematics were summarised. In Table 3.3, the information contained in Table 2.10 (see 2.4.7.5) is combined with a mathematical perspective on De Corte's (1996) educational learning theory, as explored in the third theme of Chapter 3 (see 3.4).

Table 3.3: Features of metacognitive intervention studies in mathematics and aspects of De Corte's (1996) educational learning theory

Features	Studies that stated mathematics-related aims and/or aims relating to self-regulation or metacognition (Studies 1, 2, 4, 5, 8, and 9)	A mathematical perspective on De Corte's (1996) educational learning theory
Age of participants	Grade 4 to pre-college.	Not applicable.
Intervention period	Four weeks to a year.	Not applicable.
Theoretical basis	*Related to mathematics:* Problem-solving contexts. Cooperative settings. Corrective feedback. Enrichment. *Related to self-regulation or metacognition:* Problem-solving contexts. Knowledge of cognition. Regulation of cognition. Learner beliefs and motivation.	 Heuristics. Situated and collaborative. Goal-directed; self-regulated; metacognition. Heuristics. Heuristics. Self-regulated; metacognition. Self-regulated; metacognition. Affective components.

Features	Studies that stated mathematics-related aims and/or aims relating to self-regulation or metacognition (Studies 1, 2, 4, 5, 8, and 9)	A mathematical perspective on De Corte's (1996) educational learning theory
	Learner autonomy.	Constructive; cumulative; knowledge basis; individually different; goal-directed; self-regulated; metacognition.
Method of intervention	Problem-solving contexts. Corrective feedback. Active teacher involvement. Cooperative settings. Individual settings. Enrichment opportunities. Learner affect.	Heuristics. Goal-directed; self-regulated; metacognition. Situated and collaborative. Individually different; cumulative. Heuristics. Affective components.
Measurement of mathematics-related aspects	Quantitative and qualitative measurements. Pre-test and post-test.	Not applicable.
Measurement of self-regulation or metacognition	*Quantitative measures*: SRLS; MAI; DSMK. Pre-test and post-test. *Qualitative measures:* Interviews; applied MAI.	Not applicable.

In Table 3.3, parallels are drawn between the theoretical basis, the method of intervention of previous metacognitive interventions, and De Corte's (1996) educational learning theory. In the next discussion, each aspect of De Corte's (1996) educational learning theory is associated with features of the theoretical basis and/or features of the method of intervention of previous metacognitive interventions in mathematics. The following observations are made.

- **Constructive**

The *constructive* characteristic, which points to learning as an effortful and mindful process, links with *learner autonomy*. It is also an implicit characteristic of problem-solving contexts.

- **Cumulative**

This characteristic is also associated with *learner autonomy*, because learners' level of prior knowledge could influence how autonomously they progress in their learning. It, therefore, also relates to *individual settings*, as individual differences in prior knowledge may influence the structuring of individual settings.

- **Self-regulated and metacognition**

These two aspects correlate with *corrective feedback*, because learners must demonstrate an awareness of their mistakes when they receive feedback. They should also be able to incorporate this feedback by implementing regulatory strategies. *Self-regulated* and *metacognition* strongly relate to *learner autonomy*, because knowledge of one's cognitive functioning and the ability to regulate cognitive behaviours could enhance autonomous learning.

- **Goal-directed**

The explicit awareness of, and orientation towards a goal correlate with *corrective feedback*, since learners become conscious of their progress in relation to their goals when they receive feedback. *Goal-directed* is also associated with *learner autonomy*, since the setting of goals could stimulate autonomous learning.

- **Situated and collaborative**

There is a clear relationship between these aspects and *cooperative settings*. The *situated and collaborative* aspects point to learning as a distributed activity that involves the individual, the environment, resources, and other learners.

- **Individually different**

Individually different entails the variation in aptitudes that affect learning. These aptitudes may influence *learner autonomy*. It also relates to *individual settings* adapted to accommodate differences in individual aptitudes.

- **Knowledge basis**

A well-organised *knowledge basis,* which entails quality learning resources that sufficiently cover the domain-specific knowledge, could enhance learners' ability to learn more independently, thereby linking with *learner autonomy.*

- **Heuristics**

Heuristics is strongly associated with *problem-solving contexts*, since the systematic search strategies of *heuristics* are problem-solving strategies that enhance the prospects of success when problems are solved.

- **Affective components**

Affective components link with *learner beliefs and motivation* and *learner affect*, since affective components entail positive beliefs both about the self in relation to problem-solving and about the social context of learning.

In summary, secondary research question 4 explores the proposed features of a metacognitive intervention in mathematics. In Section 3.5, a broader perspective on De Corte's (1996) educational learning theory and its links with the theoretical basis and methods of intervention of metacognitive intervention studies in mathematics were discussed. It was shown that the different aspects of a mathematical perspective on De Corte's (1996) educational learning theory incorporate all aspects related to the theoretical basis and methods of intervention of previous metacognitive intervention studies in mathematics.

Therefore, in addressing secondary research question 4 it is proposed that a framework for metacognitive interventions in mathematics is structured according to the aspects contained in Table 3.3.

3.6 CONCLUSION

In Chapter 3, three themes were explored and secondary research question 4 was addressed. In Theme 1, aspects relating to international and national viewpoints on the nature of mathematics were identified (see 3.2).

In Theme 2, the relationships between De Corte's (1996) educational learning theory and learning in mathematics were established (see 3.3).

In Theme 3, a mathematical perspective on De Corte's (1996) educational learning theory was provided by combining aspects of the discussion in Sections 3.2 and 3.3 (see 3.4).

Finally, secondary research question 4 was addressed by highlighting the relationship between a mathematical perspective on De Corte's (1996) educational learning theory and features of metacognitive interventions (see 3.5). The theoretical framework for a proposed metacognitive intervention in mathematics, established in this chapter, was used to develop this study's MI and to inform key aspects of the research methodology and research methods of this study.

In the next chapter, the research design of this study is discussed. Specific references are made to the researcher's philosophical world view, the mixed methods research methodology, and the quantitative and qualitative research methods.

CHAPTER 4

RESEARCH DESIGN

4.1 INTRODUCTION

The research design of a study involves an interaction between three components, namely the researcher's philosophical world view, the research methodology, and the specific research methods that translate the research methodology into practice (Creswell, 2009: 5). In the first part of this chapter, the researcher's philosophical world view and its relation to the philosophical world view that informs this study, namely pragmatism, are discussed. It is indicated how pragmatism links with the research methodology and research methods employed in this study. This is followed by a discussion of the research methodology evident in this study, namely mixed methods. This chapter concludes with a discussion of the third component of the research plan, namely specific research methods (see Table 4.1). Table 4.1 represents the structure of Chapter 4 according to the elements of research design and the corresponding section numbers.

Table 4.1: Structure of Chapter 4 according to the elements of research design

Introduction (4.1)	
Philosophical world view (4.2)	
Research methodology (4.3)	
Mixed methods research methodology (4.3.1)	
Mixed methods research methodology: Quantitative aspect (4.3.1.1)	Mixed methods research methodology: Qualitative aspect (4.3.1.2)
Research methods (4.4)	
Quantitative research methods (4.4.1)	Qualitative research methods (4.4.2)
Conclusion (4.5)	

4.2 PHILOSOPHICAL WORLD VIEW

Philosophical worldviews, or research paradigms, are general beliefs about reality and the nature of research that researchers hold. These beliefs will determine whether the researcher follows a quantitative, qualitative, or mixed methods approach (Johnson & Christensen, 2004: 29-30; Creswell, 2009: 6).

4.2.1 The researcher's philosophical world view

In Sections 1.1 and 1.6, two aspects relating to the philosophical world view of the researcher were discussed. First, Section 1.1 mainly focused on the complexity of reality and the implications of a multifaceted view of reality for education. These implications primarily involve developing learner thinking skills in order to deal with complexities. Secondly, in Section 1.6, it was emphasised that the researcher's philosophy regarding the teaching of mathematics involves the challenge to stimulate and enhance learner thinking skills. A more detailed description of the researcher's general philosophical world view as it relates to these two themes is given next.

The first theme entails reality's complexity. The researcher believes that reality entails more than what can be perceived through one's five senses and he, therefore, acknowledges that one's philosophical world view is not absolute, but open to change, because reality's complexities are never fully explored. The researcher's views on the intricacy of reality have shaped his approach to this study – in the context of learner metacognition and achievement in mathematics – in that he acknowledges that there are many factors, apart from learner metacognition, that influence the learning process in mathematics and that mathematics achievement is a complex concept that reflects the multifaceted nature of reality (see 3.3). One may ask, in the South African context, whether learners who perform well in the mathematics NSC examination are really proficient in mathematics (see 3.2.2.3).

A second theme relates to the role of teachers in enhancing learner thinking skills. The researcher believes that teachers should engage with reality from different perspectives. Many instances can be cited where "the truth" may involve a compromise of two opposite points of view. In the evolution versus creation debate, for example, there are two opposing views. Life either evolved due to the influence of environmental factors, or life was created by a higher dimensional power. A third point of view involves a synthesis of these opposing perspectives, namely that life was created by a higher dimensional power through the process of evolution.

It is obvious that teachers may not enforce their modes of thinking, but learners should be made aware of different perspectives and of the importance of gathering as much evidence as possible from various sources. Consequently, more informed choices are made while still allowing for more evidence to emerge that could influence those choices. From a mathematics perspective, this could entail the establishing of the validity of a problem's solution and the further exploration of a problem by seeking alternative solutions and applying the solutions in different contexts. In addition, a varied mathematical learning process should be encouraged according to individual differences in learner aptitudes.

In summary, two main themes of the researcher's general philosophical world view and some implications thereof for education, in general, and mathematics education, in particular, were discussed in this section. There is, however, a specific philosophical world view associated with the research methodology followed in this study, namely mixed methods. According to Cresswell and Plano-Clark (2007: 5, 23) and Creswell (2009: 10), pragmatism is a world view that relates strongly with a mixed methods methodology, and it directs the data-collection methods and the analysis of data. In the next section, some basic premises of pragmatism and its ethos are discussed. Subsequently, the researcher's general philosophical world view is compared with the pragmatic world view in order to identify possible common aspects.

4.2.2 Pragmatism

Pragmatism has its roots in the work of Peirce, James and Dewey (Bernstein, 1988: 6). Pragmatism is a form of philosophy that "... takes the continuity of experience and nature as revealed through the outcome of directed action as the starting point for reflection" (Thayer-Bacon, 2011: 363). In this definition, it is evident that a person's observable action is not regarded as a random event, but as a result of factors stemming from that person's experience or nature. However, these factors are only viewed as a point of departure for further reflection, and not as ultimate causes, thereby allowing for different perspectives to emerge. Although diverse perspectives are accommodated in pragmatism, some basic premises can be identified.

4.2.2.1 Basic premises of pragmatism

Bernstein (1988: 6) regards the diversity of perspectives in pragmatism – evident in the late nineteenth century – as a reflection of the fluidity and lack of clearly defined boundaries of academic disciplines at the time. Early pragmatic thinkers displayed openness in their philosophical views, but they were also critical of the metaphysical and epistemological dichotomies in traditional and modern philosophy such as, for example, mind/body, reason/will, thought/purpose, reason/emotions, self/others, belief/action, theory/practice (Bernstein, 1988: 7; Thayer-Bacon, 2011: 363). Although they opposed scientism, they supported philosophical reflection on scientific developments (Bernstein, 1988: 7). Pragmatism can be distinguished from other world views in respect of the following aspects: the nature of reality (ontology); the way in which knowledge is obtained (epistemology); the influence of values on research (axiology); the research process (methodology), and linguistic aspects related to the research (rhetoric) (Cresswell & Plano-Clark, 2007: 23). These aspects are discussed in Section 4.2.2.4.

A more detailed view of pragmatism emerges when considering the ethos of pragmatism, as discussed by Bernstein (1988: 7-11).

4.2.2.2 The ethos of pragmatism

Bernstein (1988: 7-11) discusses five interrelated themes characterising the ethos of pragmatism. First, the anti-foundational theme which opposes the notion that knowledge has fixed foundations, and that a person can know these foundations. Absolute certainty is, therefore, unattainable (Bernstein, 1988: 7- 8). Creswell (2009: 11) supports this view by stating that pragmatists view truth as related to context and to practices that work best in that specific context.

Secondly, anti-foundationalism does not imply that pragmatism is similar to scepticism or relativism. Instead, fallibilism is regarded as an alternative to foundationalism. This means that all inquiries start with preconceptions and are open to ongoing interpretation. Philosophy is regarded as tentative and intrinsically fallibilistic (Bernstein, 1988: 8-9; see 3.2.2.1). In fact, pragmatism does not support a specific system of philosophy, but draws from quantitative and qualitative world views (Creswell, 2009: 10).

The third theme is regarded as an essential aspect of the pragmatic ethos (Bernstein, 1988: 9). It states that an individual's limited perspective necessitates the scrutiny of one's ideas by a critical community of inquirers (Bernstein, 1988: 9). Therefore, the social, historical, political, and other contextual backgrounds of the research play an important role in the interpretation of the data, because this broadens the inquirer's perspective (Creswell, 2009: 11).

The fourth theme evident in the pragmatic tradition is its stance on a fundamental problem in philosophy, namely contingency and change (Bernstein, 1988: 9). The philosophical quest to master and restrict contingency is countered by the pragmatist's position. Pragmatists do not view contingency and chance as evidence of human ignorance, but as integral, permanent features of the universe (Bernstein, 1988: 9-10). The universe is viewed as open and as a source of failure and success. Accordingly, pragmatists advocate a state of readiness to successfully deal with contingencies (Bernstein, 1988: 10). Creswell (2009: 11) also refers to the pragmatic notion of an external, uncontrollable world that operates independently from the human mind.

The final, encompassing theme deals with the plurality of perspectives and philosophical viewpoints in pragmatism (Bernstein, 1988: 10). Plurality poses some challenges, because it encourages an environment in which radically different opinions and ideas flourish. These challenges are the following (Bernstein, 1988: 15). First, a "fragmenting pluralism" could jeopardise constructive communication between philosophers that have different perspectives. Secondly, a "flabby pluralism" reflects a superficial synthesis of different philosophical orientations. A third challenge is posed by a "polemical pluralism" where the advancement of personal ideologies takes preference above a willingness to learn from others. Lastly, in “defensive pluralism”, other viewpoints are considered without the willingness to be influenced by them.

To counter these challenges, Bernstein (1988: 15) suggests that “engaged fallibilistic pluralism” represents the ethos of pragmatism best. This means that one is willing to listen to others and realise that other ideas cannot always be translated completely due to one’s own entrenched vocabulary. There are no undisputed rules for determining the validity of philosophical statements; hence, it requires an ongoing process of seeking the shared viewpoints and differences with rival philosophies. Pragmatists engage in dialogues that enhance mutual understanding which does not exclude differences (Bernstein, 1988: 15-16). This inclusiveness corresponds with Creswell’s view (2009: 11) that pragmatism accepts multiple and varied world views, methods, data-collection techniques, and data-analysis strategies.

4.2.2.3 The researcher’s philosophical world view and pragmatism

Three aspects of pragmatism resonate with the researcher’s general philosophical world view.

First, and most important, are the complementary notions expressed in the third and fifth themes of the pragmatic ethos (see 4.2.2.2). The third theme emphasises one’s willingness to acknowledge a limited perspective and, consequently, to allow external examination of one’s ideas. This theme relates to the fifth theme which expresses the inability of language to accurately portray one’s ideas. Hence, even if one allows external scrutiny of one’s ideas, the appropriate impact of that critique is jeopardised by the inadequacy of the instrument that conveys the message. Conversely, one’s critique of others’ ideas suffers the same fate. Therefore, as the fifth theme states, seeking the validity of philosophical statements “requires an ongoing process of seeking the shared viewpoints and differences with rival philosophies”.

The main theme of the researcher’s general philosophical world view entails the acknowledgement of the complexity of reality and the continual quest to grow in one’s understanding thereof. Clear parallels with the combined third and fifth themes of the pragmatic ethos emerge. The researcher concurs that one’s limited perspective necessitates the continual engagement with one's own and other viewpoints. As these views correspond with "engaged fallibilistic pluralism", viewed as central to the pragmatic ethos (see 4.2.2.2), the researcher's general philosophical world view corresponds strongly with the core of pragmatism.

Secondly, philosophical inquiry is viewed as evolving and intrinsically flawed, as is evident in the second ethos of pragmatism. The researcher acknowledges that a better understanding of reality is only obtained by an ongoing examination of one's preconceptions and ideas. However, this search for better understanding is flawed, as it seems unlikely that a total awareness of one's preconceptions will be obtained.

A third aspect of pragmatism that relates strongly to the researcher's world view entails the contingent and uncontrollable nature of the universe, as described in the fourth theme of the pragmatic ethos. The researcher concedes that a perfect execution of one's plans is not possible due to the intricate network of interrelationships in any situation. However, this aspect does not encourage pessimism, as the consideration of the contingent nature of reality may motivate one to plan even better to limit errors.

These three aspects correspond sufficiently with the researcher's general philosophical world view to serve as the philosophical world view informing this study. Yet, the very nature of pragmatism allows one to continually examine one's understanding and application thereof. Therefore, pragmatism is not an absolute point of departure in this study, but a useful guide in the execution of this research. Some implications of the pragmatic world view for this study are discussed next.

4.2.2.4 The implications of pragmatism

Pragmatism differs from other world views in respect of five aspects, namely ontology; epistemology; axiology; methodology, and rhetoric (see 4.2.2.1). These five aspects influenced this study as follows.

4.2.2.4a Ontology

Reality comprises singular and multiple aspects; researchers may test hypotheses, but also explain findings from multiple perspectives (Cresswell & Plano-Clark, 2007: 24). In this study, hypotheses are tested, but qualitative data play a supporting role in providing a holistic picture of the impact of the intervention.

4.2.2.4b Epistemology

A researcher views data collection from a practical perspective by asking “what works” in order to address the research question (Cresswell & Plano-Clark, 2007: 24). The way in which knowledge was obtained in this study reflects the notion that the use of a standardised questionnaire such as the MAI only is not the best way to determine the learners’ level of metacognition. What “worked best” was to also obtain knowledge from other sources, for example, the learners’ responses during two problem-solving sessions.

4.2.2.4c Axiology

Biased and unbiased perspectives on the data-collection process and results are offered (Cresswell & Plano-Clark, 2007: 24). In this study, the researcher’s report on his teaching experience and philosophy of teaching gives an indication of his views of aspects related to values and “truth” in the teaching of mathematics. In addition, the interview with the co-researcher and his personal reflections provide evidence of his orientation towards specific values and the ideal way of teaching mathematics.

4.2.2.4d Methodology

In research informed by a pragmatic world view, quantitative and qualitative data are mixed (Cresswell & Plano-Clark, 2007: 24). In this study, the testing of hypotheses is supported by the following qualitative data: teacher interviews; the learners’ written responses during two problem-solving sessions; the learners’ perspectives on the process of MI, and the experimental group’s teacher perspectives on the process of MI.

4.2.2.4e Rhetoric

Formal and informal styles of writing are used (Cresswell & Plano-Clark, 2007: 24). In this study, the interpretation of the quantitative data is conducted in a more formal style of writing, but the interpretation of the qualitative data reflects a more informal writing style.

In this section, the philosophical world view applicable to this study was discussed. The next section focuses on the second element of research design, namely research methodology.

4.3 RESEARCH METHODOLOGY

In this section, the mixed methods research methodology employed in this study is discussed. The quantitative and qualitative aspects of the mixed methods research methodology are also addressed. Lastly, ethical concerns relating to this study are discussed.

4.3.1 Mixed methods research methodology

In earlier years both quantitative and qualitative data were collected in the same study before the term “mixed methods research” was used. The novel aspect of mixed methods research lies in the fact that it is regarded as a distinct research design or methodology with its own notation system, terminology, diagrams of procedures, and aspects of different mixed methods designs (Creswell & Plano Clark, 2007: 1).

Creswell and Plano Clark (2007: 5) offer the following broad definition of mixed methods research that incorporates different definitions:

> *Mixed methods research* is a research design with philosophical assumptions as well as methods of inquiry. As a methodology, it involves philosophical assumptions that guide the direction of the collection and analysis of data and the mixture of qualitative and quantitative approaches in many phases in the research process. As a method, it focuses on collecting, analyzing, and mixing both quantitative and qualitative data in a single study or series of studies. Its central premise is that the use of quantitative and qualitative approaches in combination provides a better understanding of research problems than either approach alone.

The fundamental principle of mixed methods research is that research methods should be combined in such a way as to complement the strengths of the different research methods without overlapping of their weaknesses (Johnson & Christensen, 2004: 162). There are several reasons why a better understanding of the research problem emerges with a combination of quantitative and qualitative approaches. The following reasons are offered.

First, the deficiencies of both quantitative and qualitative research are addressed. Weaknesses in quantitative research include the lack of contextual understanding, the viewpoints of the participants that are not explicitly heard, and the fact that the researcher's bias and interpretations are not discussed. Limitations of qualitative research mainly refer to personal interpretations by the researcher that could lead to bias, and the lack of generalisation of the results due to the limited number of participants (Creswell & Plano Clark, 2007: 9).

Secondly, more complete substantiation for the results of the research study is provided, because the researcher is not restricted to only certain types of data collection (Creswell & Plano Clark, 2007: 9).

Thirdly, answers can be obtained to research questions that could not be answered by using a single research approach (Creswell & Plano Clark, 2007: 9).

Fourthly, the use of multiple paradigms is encouraged, or a researcher can use a single paradigm such as pragmatism that includes paradigms associated with quantitative or qualitative research only (Creswell & Plano Clark, 2007:10).

Fifthly, the practicality of mixed methods research enables the researcher to use all methods available in order to address a research problem (Creswell & Plano Clark, 2007: 10).

In this study, an embedded mixed methods design was used. In an embedded mixed methods design, one set of data supports the primary data set on which the study is based. The basis for this design is threefold: one set of data is insufficient; different questions need to be answered, and different data sets are needed in order to answer the different questions (Creswell & Plano Clark, 2007: 67). These three aspects are evident in the design of this study, since the quantitative data are regarded as insufficient and the qualitative data support the quantitative, primary data; different questions relating to the quantitative and qualitative aspects of the study are stated, and qualitative and quantitative data sets are collected.

In an embedded mixed methods design, two data sets are mixed during the design process in two ways. Quantitative data can be embedded within a qualitative methodology, or, as in this study, qualitative data are embedded within the quantitative methodology (Creswell & Plano Clark, 2007: 67). The embedded design has two variants, namely the experimental model and the correlational model.

The embedded experimental model (used in this study) is probably the most frequently used variant of the embedded design (Creswell & Plano Clark, 2007: 69). In this model, qualitative data are embedded within an experimental design based on a true experiment, or (as is the case in this study) on a quasi-experiment. In this design, further differentiation is made between a one-phase approach and a two-phase approach. The one-phase approach implies that qualitative data are embedded within the quantitative methodology during the intervention phase in order to examine the process of intervention, whereas in the two-phased approach, the qualitative data are embedded within the quantitative methodology prior to and after the intervention phase (Creswell & Plano Clark, 2007: 69). In this study, both the one-phase approach and the two-phase approach are applied, because the qualitative data are used to examine the process of intervention (one-phase approach), and the qualitative data (two problem-solving sessions) are also embedded within the quantitative methodology prior to and after the intervention phase (two-phase approach).

4.3.1.1 Mixed methods research methodology: Quantitative aspect

In education, random (equivalent) assignment of participants to experimental or control groups is not always possible. In this study, a pre-test – post-test non-equivalent group design is employed which is one of the most common quasi-experimental designs in educational research (Cohen *et al.,* 2007: 283). At least two groups are needed, because the purpose of experimental research is to compare the effect that one condition (independent variable) has on the first group with the effect that a different condition has on a second group (McMillan & Schumacher, 2001: 322). In this study, the effect of a metacognition intervention (independent variable) on learner metacognition (dependent variable) and mathematics achievement (dependent variable) is investigated for the experimental group. The metacognitive intervention did not require any extra time allocated to teaching, but it can rather be viewed as a different way of teaching. Therefore, the control group was not disadvantaged with respect to contact time.

4.3.1.2 Mixed methods research methodology: Qualitative aspect

In this study, a case study research methodology was employed to study the qualitative research questions. A case study focuses on a bounded system, for example one individual, one group or one programme, with the aim of understanding and describing the "case" in detail. Generalisations to theory may result from case study research. Multiple methods, for example interviews, observations, and document analysis, may be used to gather data (Babbie & Mouton, 2001: 280-281; Cohen *et al.,* 2007: 253; Ary *et al.,* 2010: 29). In case studies, cause and effect can be verified, because effects are studied in real contexts. Context plays a major role in establishing cause and effect (Cohen *et al.,* 2007: 253).

There are four general design principles in conducting a case study, namely conceptualisation, contextual detail, multiple sources of data, and analytical strategies (Babbie & Mouton, 2001: 282-283).

4.3.1.2a Conceptualisation

Broad conjectures or theoretical expectations may be stated at the start of the study. These theoretical expectations are based on a review of literature and the researcher's experience, and it gives more structure in the collection of data (Babbie & Mouton, 2001: 282). In this study, the proposed framework for a metacognitive intervention is considered as the theoretical basis for the MI (see 3.5). This proposed framework provides the structure of the data-collection process in this study.

4.3.1.2b Contextual detail

Environmental factors have an influence on the participants, the researcher and the data-collection process; therefore, the context of the study must be described in detail. (Babbie & Mouton, 2001: 282). In this study, general environmental factors that may impact on the learners of both School A and School B are discussed (see 1.12; 4.4.1.1d and 4.4.2.1a).

4.3.1.2c Multiple sources of data

When multiple sources of data are used, a thick description of the participants' and inquirer's experiences can emerge (Babbie & Mouton, 2001: 282). In this study, qualitative data were collected as follows: interviews with the teacher of the experimental group and with the teacher of the control group; learners' written responses during two problem-solving sessions, and open-ended questionnaires on the perspectives of the experimental group's teacher and learners on the process of MI.

4.3.1.2d Analytical strategies

When the case study is analysed, at least three aspects should be addressed, namely organising the findings; establishing whether generalisation is possible, and addressing the issue of theory development (Babbie & Mouton, 2001: 283). In this study, these aspects are addressed when the qualitative data are analysed (see Chapter 6).

4.3.2 Ethical concerns

Ethical issues arise when researchers seek to collect reliable and valid data. Informed consent forms the basis of these ethical procedures (Cohen *et al.*, 2007: 51-52).

Informed consent stems from the participants' right to be free and it is especially important if they are going to experience stress, pain, or invasion of privacy (Cohen *et al.,* 2007: 52). In this study, the participants' right to freedom was not put at risk by the quantitative and qualitative data-collection procedures. In fact, the MI was structured in such a way as to encourage learner participation and feedback on the method of MI (see 4.4.2.1b; 4.4.2.1c).

The learners of the experimental group were also not exposed to any danger or stressful situations during the course of the MI. The MAI pre-test and post-test (experimental group and control group) and the qualitative pre-test and post-test (experimental group) were conducted in a familiar context, namely the learners' mathematics classrooms.

Official permission to conduct the research was granted by the Free State Department of Education on condition that the learners participate on a voluntary basis and without mentioning the names of the schools (see Appendix A4). Oral consent to conduct research at School A and School B was given by the respective headmasters in response to the researcher's written request (see Appendix A2). The parents (or custodians) of the experimental group's learners gave written consent for them to participate in the research. The consent form stipulated that the research report would not mention the names of the learners, the teacher and the school (see Appendix A3).

The consent given by the Free State Department of Education, the headmasters and the parents (or custodians) allowed the researcher to conduct data-collection procedures. However, procedural ethics do not suffice; the research purposes, results and reporting need to adhere to ethical principles (Cohen *et al.,* 2007: 51). The purpose of this study is not to reflect negatively on any teacher or learner, but to study the impact of MI on learner metacognition and mathematics achievement. Some results of this study could point to poor learner achievement in mathematics or to some learners' poor problem-solving skills. These data are not presented in a judgmental, but rather in an exploratory manner that seeks to interpret the results from different perspectives.

Specific research methods are associated with the research methodology discussed in this section. These research methods are discussed next.

4.4 RESEARCH METHODS

The third element of research design entails the specific research methods employed for the collection, analysis and interpretation of the data (Creswell, 2009: 15). This section focuses first on the quantitative research methods and, subsequently, on the qualitative research methods used in this study.

4.4.1 Quantitative research methods

In this section, the following main aspects regarding the quantitative research methods relating to this study are discussed: sampling, data collection, and data analysis. Before these aspects are addressed, an explanation of the time frame that relates to sampling and data collection is illustrated in Table 4.2.

Table 4.2: Time frame of the quantitative data-collection procedures

Date	Event
October 2009	Initial discussions with headmaster, deputy headmaster and Mark about the possibility of doing research at their school in 2010.
November 2009	Initial discussion with the teacher of the control group about the possibility of doing research with her Grade 11 class in 2010.
November 2009	Informal permission to do research obtained from the headmaster.
26 January 2010	Requested permission from the Free State Department of Education to do a research project.
15 February 2010	Requested permission from the headmaster of School A to do research that involves one Grade 11 class.
15 February 2010	Requested permission from the headmaster of School B to collect data from the Grade 11 learners.
3 March 2010	Quantitative pilot questionnaire: School B Grade 11 learners.
5 March 2010	Quantitative pilot questionnaire: School A Grade 11 learners.
5 March 2010	Letter to the parents of the Grade 11 experimental group's learners requesting permission to conduct research that involves the learners.
16 March 2010	Quantitative pre-test: control group.
1 April 2010	Obtained Term 1 report marks: experimental group and control group.
1 April 2010	Quantitative pre-test: experimental group.
25 October 2010	Quantitative post-test: control group.
7 November 2010	Quantitative post-test: experimental group.
November 2010	Obtained Term 4 report marks: experimental group and control group.

4.4.1.1 Sampling

In this study, the most important factor in choosing a sample was to find a teacher who would be willing and keen to implement the MI. The teacher had to commit to a six-month period that would demand of him/her to steer the MI under guidance of the researcher and to reflect on the process of MI.

In addition, four key factors had to be taken into account in choosing a sample: the sample size; representativeness of the sample; access to the sample, and the sampling strategy to be used (Cohen *et al.,* 2007: 100).

4.4.1.1a Sample size

The sample size is related, first, to the purpose of the study and, secondly, to the characteristics of the population (Cohen *et al.,* 2007: 101). The purpose of this study is to investigate the effect of MI on learner metacognition and achievement in mathematics. Quantitative and qualitative data were collected in order to achieve the purpose of this study (see 1.8). The extent of the qualitative data-collection process implied that the sample had to be small enough, because all the learners in the Grade 11 class were included in order to obtain a rich understanding of the MI process.

A sample size of 30 is regarded as the minimum number of cases to perform parametric statistical analyses, although a significantly greater number of cases are recommended (Cohen *et al.,* 2007: 101). Therefore, non-parametric statistical procedures were performed on the data obtained from the sample of less than 30 learners (see 4.4.1.3b).

A second aspect that determines the size of the sample is the characteristics of the population. In this study, the population is defined as Grade 11 female mathematics learners in multicultural Quintile 5 schools who receive instruction through the medium of English.

4.4.1.1b Representativeness of the sample

The researcher needs to determine the extent to which the sample represents the different subgroups of the population (Cohen *et al.,* 2007: 108). The subgroups of the population of this study include gender, home language, socio-economic background, and language of instruction. Although the sample size limits the generalisation value of the results (see 4.4.1.1a), the different subgroups of the sample relate to a population of multicultural female learners from good socio-economic backgrounds who have English as a medium of instruction.

4.4.1.1c Access to the sample

Access is a crucial element in the sampling process. It relates to permission to do research as well as to practicality (Cohen *et al.,* 2007: 109). The researcher only considered easily accessible schools, as a substantial amount of time would be spent visiting the teacher of the experimental group. Once the teacher of the experimental group agreed to be involved, permission from the relevant stakeholders was obtained in order to conduct the study (see Table 4.2).

4.4.1.1d Sampling strategy

In non-probability sampling, a particular group is selected (Cohen *et al.,* 2007: 113). Two intact Grade 11 classes from different schools were used in this study. There were 25 learners in the experimental group (School A) and 24 learners in the control group (School B).

As a non-equivalent group design is used, the experimental group and the control group may display different characteristics that could influence the independent variables, namely learner metacognition and achievement in mathematics. Therefore, the researcher used both an experimental and a control group that were as similar as possible regarding extraneous variables that could influence the independent variables and, therefore, reduce the internal validity of this design. These variables include *home language*, *age*, *gender*, *achievement in mathematics*, and aspects of the teaching-and-learning situation such as time allocated to teaching, teacher qualifications and teaching experience. A more specific discussion of these characteristics follows in Chapter 5 (see 5.2).

A further two aspects that may influence the independent variables are the general school environment and the learners' broad socio-economic background.

- **School environment**

First, School A (experimental group) and School B (control group) are similar in various respects. They are situated within close proximity of a city in the Free State province. Both schools offer excellent opportunities in respect of academic aspects, as well as cultural and sport activities. Both schools pride themselves on quality education offered to girls for over 100 years. Although English is the medium of instruction in both schools, there is a vibrant interaction of girls from different home languages, cultures and races.

These two schools differ in respect of their academic results and the number of learners in each school. School B has received numerous awards for its mathematics and physical science NSC results. Many of these awards relate to achievements for the period 2000 to 2009. By contrast, School A has not received specific recognition for its mathematics and physical science results, although the 2010 NSC results indicate that it performed very well in the South African context (see Table 1.2). School A may radiate a more personal atmosphere, as the total number of learners is approximately a third of the number of learners in School B. In 2010, School A had 53 Grade 12 learners grouped into two classes, whereas School B had 164 Grade 12 learners grouped into four classes (DBE, 2011: 111-112; see Table 1.2).

- **Socio-economic background**

A second aspect entails the broad socio-economic background of the learners. Both schools are classified as Quintile 5 schools. Quintile ranking determines the amount of funding that a school receives. Quintile 5 schools receive the smallest allocation per learner, because it is argued that schools in less poor communities are able to raise their own funds (Giese *et al*., 2009: 30). In 2010, there were 5 915 schools in South Africa of which 832 (14%) were ranked as Quintile 5 schools. In the Free State, 51 (17%) of the 299 schools were ranked as Quintile 5 schools in 2010 (DBE, 2011: 13). Table 4.3 reflects the percentage of schools, according to their Quintile rank, that achieved a pass rate of 80% and above and a pass rate of 100% in the 2010 NSC examination (DBE, 2011: 52-54).

Table 4.3: Pass rates in the 2010 NSC examination according to the quintile ranking of schools

Pass rate in the 2010 NSC examination	Quintile 1	Quintile 2	Quintile 3	Quintile 4	Quintile 5
Pass rate of 80% and above	20.4%	23.8%	26.5%	35.7%	65.5%
Pass rate of 100%	2.6%	3.3%	3.2%	6.9%	21.3%

It is evident that there is not a big difference in results between schools in the Quintile 1 to Quintile 4 grouping, but Quintile 5 schools had markedly better results. School A and School B both obtained a 100% pass rate in the 2010 NSC examination (DBE, 2011: 111-112).

4.4.1.2 Data collection

Data collection involves executing the research plan by using a variety of instruments such as tests and attitude scales (Ary *et al.,* 2010: 32). In this study, quantitative data were collected in respect of two aspects, namely learner metacognition and mathematics achievement.

4.4.1.2a Learner metacognition

An adapted MAI was used as a pre-test and post-test measure of learner metacognition for both the experimental and the control group. The original MAI was developed to determine the metacognitive awareness of adolescents and adults, because other measures such as on-line experimental testing were very time-consuming (Schraw & Dennison, 1994: 461).

The MAI assesses the two main factors of metacognition, namely *knowledge of cognition (KC)* and *regulation of cognition (RC). KC* comprises learners' awareness of their strengths and weaknesses; knowledge about strategies, and knowledge about the use of strategies. The three subscales of *KC* are *Declarative knowledge*, *Procedural knowledge*, and *Conditional knowledge*.

Declarative knowledge is knowledge about self and strategies; *Procedural knowledge* refers to knowledge about the use of strategies, and *Conditional knowledge* refers to knowledge about when and why to use strategies (Schraw & Dennison, 1994: 460). These three subscales and their corresponding items on the MAI are indicated in Appendix B2.

RC refers to the control that learners exercise over their learning processes. It has five subscales that facilitate the regulation aspect of learning. According to Schraw and Dennison (1994: 460), these aspects are:

- *Planning* ("planning, goal setting, and allocating resources *prior* to learning");
- *Information management* ("skills and strategy sequences used *during* learning to process information more efficiently, for example organising, elaborating, summarising, selective focusing");
- *Monitoring* ("assessment of one's learning or strategy use");
- *Debugging* ("strategies used to correct comprehension and performance errors"), and
- *Evaluation* ("analysis of performance and strategy effectiveness *after* a learning experience").

During its original developmental process, the MAI still consisted of 120 items that assessed the two main factors of metacognition, namely *KC* and *RC* (Schraw & Dennison, 1994: 462). The items were piloted by Schraw and Dennison (1994) on 70 undergraduate students and all items with extreme mean scores or high variability were dropped. A total of 52 items remained. Each of the eight subcomponents of metacognition is represented by at least four items.

The pilot phase in the development process of the original MAI consisted of two experiments that investigated three issues. First, it was determined whether *KC* and *RC* are valid factors of metacognition. Secondly, it also examined the possible relationship between *KC* and *RC*. Thirdly, the possible relationship between either of *KC* and *RC* and achievement was explored (Schraw & Dennison, 1994: 470).

The following findings were reported for the three issues that were investigated (Schraw & Dennison, 1994: 470-472). First, the validity of viewing *KC* and *RC* as the two factors of metacognition was confirmed. Secondly, a statistically significant positive correlation was found between these two factors (r = 0.54). Thirdly, a significant correlation was found between *KC* and achievement only, but not between the MAI total score and achievement, or between *RC* and achievement. Schraw and Dennison (1994: 470-472) explain their findings by stating that *KC* and *RC* influence performance in different ways, although *KC* and *RC* share a statistically significant relationship. In addition, they suggest that the correlation between the MAI and higher order thinking skills would be higher than the correlation between the MAI and lower order thinking skills for two reasons. First, the variation between individual scores is greater on complex tasks. Secondly, the completion of difficult tasks requires a higher level of metacognitive awareness (Schraw & Dennison 1994: 470-472).

Other important findings that emerged during the pilot phase of the original MAI were the following. First, the internal consistency of the eight subscales was only marginally acceptable; this implies that the eight subscales are not very reliable measures of metacognition. Secondly, the MAI is a reliable instrument for measuring metacognitive awareness among adolescents (Schraw & Dennison, 1994: 471-472).

In this study, the MAI was selected to assess learner metacognition, as it reliably measures adolescents' metacognitive awareness. In addition, the MAI has been used to assess the metacognitive awareness of mathematics learners in several previous studies (see Table 2.9).

The original MAI questionnaire by Schraw and Dennison was adapted by changing some words to more familiar words in the South African education context. In addition, as the original MAI measures general learner metacognitive awareness, it was adapted to reflect a mathematical context. A detailed description of the way in which the original MAI was adapted to construct the MAI pilot questionnaire is given in Chapter 5 (see 5.3.1).

Various stakeholders in mathematics education completed the pilot MAI questionnaire, namely two university lecturers; the experimental group's teacher and the control group's teacher; eight Grade 11 learners from School A who were not part of the experimental group, and 35 Grade 11 learners from School B who were not part of the control group. The stakeholders' feedback was used to finalise the MAI pre-test, which was identical to the MAI post-test. This feedback is discussed in Chapter 5 (see 5.3.1).

4.4.1.2b Learners' mathematics achievement

The mathematics achievement of the learners from both groups consisted of their first term and fourth term report marks. Term 1's report marks were composed of assessment activities completed prior to the intervention. Term 4's report marks consisted of assessment activities completed throughout the intervention and of two 150-mark examinations written after the intervention ended (see Table 4.2; 5.3.13).

In this section, aspects of the data-collection process were discussed. In the next section, relevant aspects of data analysis are addressed.

4.4.1.3 Data analysis

As quantitative data are generally in numerical format, various statistical procedures are used to analyse these data (Ary *et al.,* 2010: 32).

There are a number of related themes in the analysis of quantitative research data. First, quantitative measurement is regarded as the best way of measuring constructs. Secondly, variable analysis is central in describing human behaviour. Thirdly, experimental and statistical controls for sources of error play a crucial role in quantitative research studies (Babbie & Mouton, 2001: 49).

The first two themes are not entirely consistent with the philosophical world view that informs this study, because both quantitative and qualitative data-collection and data-analysis methods are regarded as informative. The notion of the third theme, which states the importance of control for sources of error in quantitative measurement, is recognised in this study.

4.4.1.3a Statistical significance

Scientific conclusions do not reflect absolute truth, but are statements that have a high probability of being the truth. Statistical significance refers to the use of statistical tests to determine whether the findings for the sample have a high probability of being correct and not due to chance (Cohen *et al.,* 2007: 515). Researchers need to determine how strong the evidence should be to not support the null hypothesis. This pre-established level of probability, which will be used to decide whether the null hypothesis is not supported, is called the level of significance (p) (Ary *et al.,* 2010: 165).

The most common levels of significance used in educational research are 0.01 and 0.05. If the level of significance is set at 0.05, it means that the researcher limits the probability of making a Type I error to 5%, or stated otherwise, that the researcher is 95% certain that the difference in medians between the pre-test scores and the post-test scores is not due to chance (Ary *et al.,* 2010: 166-167). In this study, the level of significance is set as 0.05.

4.4.1.3b Non-parametric tests

Due to the small number of participants, non-parametric tests were used to test whether the hypotheses are supported or not (Pietersen & Maree, 2007a: 231). When non-parametric data are used, no assumptions about population characteristics and the distribution of data are made, whereas parametric data relate to data that are normally distributed. In educational research, nominal and ordinal data are classified as non-parametric. Non-parametric data are usually obtained from questionnaires and surveys. Interval and ratio data are viewed as parametric data and are usually obtained from experiments and tests. This distinction between parametric and non-parametric data is important, as some statistical tests are applicable to parametric data only (Cohen *et al.,* 2007: 503).

The statistical significance of the possible differences between, first, the experimental group's median of the pre-test MAI and the control group's median of the pre-test MAI and, secondly, between the experimental group's median of the post-test MAI and the control group's median of the post-test MAI was determined by using the Mann-Whitney test. The Mann-Whitney test is the non-parametric equivalent of the *t*-test for independent samples and is used to compare two independent groups based on the median of a single ranked variable (Cohen *et al.*, 2007: 552; Pietersen & Maree, 2007a: 233; Ary *et al.,* 2010: 175). This test should be used when each sample is small (less than 30) and when the study variable is not normally distributed. The study variable's ranks, and not its actual values, are used. This, and the fact that medians are used instead of means, minimise the effect of extreme values (Pietersen & Maree, 2007a: 233).

The Wilcoxon signed-rank test was used to compare variables within each group in order to determine the statistical significance of the possible differences between the medians of the pre-test MAI and the medians of the post-test MAI. The Wilcoxon test is the non-parametric test equivalent of the *t*-test for two related (dependent) samples. The Wilcoxon test is applied when the same group's score on the study variable is measured at two different times such as in a pre-test and a post-test. Similar to the Mann-Whitney test, the Wilcoxon test is applied when sample sizes are smaller than 30 and a normal distribution of the study variable cannot be assumed. The influence of extreme values is reduced, because ranks and medians (instead of means) are used (Cohen *et al.,* 2007: 552-554; Pietersen & Maree, 2007a: 231-232, 237; Ary *et al.,* 2010: 177).

4.4.1.3c Hypotheses

A hypothesis is a formal, tentative statement that emerges from the research question. It states the researcher's prediction of the relationship between the dependent and independent variables investigated in the study; a hypothesis can either be supported or not supported by empirical evidence (Johnson & Christensen, 2004: 80-81).

- **Null hypothesis**

When pre-test and post-test scores are compared, the observed differences may be attributed to a relationship between the independent and the dependent variables, or they may be due to chance (sampling error). A null hypothesis states that there is no relationship between variables and that the observed relationship is due to chance. This implies that there is no statistically *significant* difference in the means of the pre-test and the post-test scores (Cohen *et al.,* 2007: 83; Ary *et al.*, 2010: 162).

Cohen *et al*. (2007: 515) state that the use of a null hypothesis compels the researcher to prove that the null hypothesis is not supported. They also caution against the terminology used in respect of the null hypothesis: for example, stating that the null hypothesis is 'accepted or not accepted, confirmed or rejected'. According to Cohen *et al*. (2007: 515), these terms imply absolute statements of truth about the results of the research. They thus suggest the use of the terminology 'the null hypothesis is supported' or 'the null hypothesis is not supported'.

- **Alternative hypothesis**

The alternative hypothesis, as opposed to the null hypothesis, states a relationship between variables or a difference between the pre-test and the post-test results. It is considered to be a weaker hypothesis than the null hypothesis, because of the rigour that is required to support the null hypothesis (Cohen *et al.*, 2007: 515-516).

Cohen *et al.* (2007: 82) state that a good hypothesis displays the following features: it is either directional or non-directional; it is stated in such a way that it can be tested by an experiment or a survey, and its results are clearly measurable, because the concepts used in the hypothesis are clearly defined. These three aspects are discussed next.

- **Directionality of the hypothesis**

If two different treatments are compared, the researcher is interested in differences in either direction; therefore, the alternative hypothesis will be non-directional. A non-directional hypothesis implies that there are two alternative hypotheses of interest, and a two-tailed test is used to determine whether the null hypothesis is supported. A directional hypothesis indicates the direction of the differences between the dependent variable and the independent variable. A one-tailed test is used to establish whether the null hypothesis is supported or not (Cohen *et al.,* 2007: 82, 504; Ary *et al.,* 2010: 166-167).

In this study, two null hypotheses (Hypotheses 1a and 2a) and two non-directional alternative hypotheses (Hypotheses 1b and 2b) are stated for the Mann-Whitney test. Two null hypotheses (Hypotheses 3a and 4a) and two directional hypotheses (Hypotheses 3b and 4b) are stated for the Wilcoxon signed rank test, respectively. One null hypothesis (Hypothesis 5a) and one directional hypothesis (Hypothesis 5b) are stated in respect of the correlation between learner metacognition and mathematics achievement.

- **Testability of the hypothesis**

The procedures used to test the hypothesis should be clearly explained (Cohen *et al.,* 2007: 82). The procedures used to test the hypotheses in this study are the Wilcoxon test, the Mann-Whitney test, and the interpretation of the Spearman rho correlation coefficient.

- **Measurability of the results**

The concepts in the hypothesis should be clearly defined in order to measure the results accurately (Cohen *et al.*, 2007: 82). The first concept in the hypotheses, namely metacognition is clearly defined in terms of the two factors *KC* and *RC* of the MAI (see 4.4.1.2a). The second concept, mathematics achievement, is defined as Term 1 and Term 4 report marks (see 4.4.1.2b and 5.5.13).

- **Hypotheses tested in this study**

The statistical significance of the possible differences in medians of the MAI total scores (dependent variable) on the pre-test and the post-test between the experimental group and the control group is determined by the Mann-Whitney test. It tests the following null hypotheses which states that the median of the MAI total scores of the experimental group and the control group is equal for, first, the pre-test and, secondly, the post-test:

- *Hypothesis 1a*

  H_0: $Me_{\text{(experimental group pre-test MAI total score)}} = Me_{\text{(control group's pre-test MAI total score)}}$

- *Hypothesis 2a*

  H_0: Me $_{\text{(experimental group's post-test MAI total score)}}$ = Me $_{\text{(control group's post-test MAI total score)}}$

The alternative hypotheses state that the median of the MAI total scores of the experimental group is not equal to the median of the MAI total scores of the control group for, first, the pre-test and, secondly, the post-test. The alternative hypotheses are:

- *Hypothesis 1b*

  H_1: Me $_{\text{(experimental group pre-test MAI total score)}}$ ≠ Me $_{\text{(control group's pre-test MAI total score)}}$

- *Hypothesis 2b*

  H_1: Me $_{\text{(experimental group post-test MAI total score)}}$ ≠ Me $_{\text{(control group's post-test MAI total score)}}$

The Wilcoxon signed-rank test is used to compare variables within each group to test the null hypotheses that the medians of the MAI total scores (dependent variable) are equal in respect of the following two aspects. First, the MAI pre-test and post-test scores of the experimental group and, secondly, the MAI pre-test and post-test scores of the control group. The null hypotheses are:

- *Hypothesis 3a*

  H_0: Me $_{\text{(experimental group's pre-test MAI total score)}}$ = Me $_{\text{(experimental group's post-test MAI total score)}}$
- *Hypothesis 4a*

  H_0: Me $_{\text{(control group's pre-test MAI total score)}}$ = Me $_{\text{(control group's post-test MAI total score)}}$

The alternative hypotheses state that the medians of the MAI post-test scores are greater than the medians of the pre-test scores in respect of the following two aspects. First, the MAI pre-test and post-test scores of the experimental group and, secondly, the MAI pre-test and post-test scores of the control group. The alternative hypotheses are:

- *Hypothesis 3b*

  H_1: Me $_{\text{(experimental group's pre-test MAI total score)}}$ < Me $_{\text{(experimental group's post-test MAI total score)}}$
- *Hypothesis 4b*

  H_1: Me $_{\text{(control group's pre-test MAI total score)}}$ < Me $_{\text{(control group's post-test MAI total score)}}$

The statistical significance of the observed relationship between learner metacognition and achievement in mathematics (dependent variable) is determined by interpreting the Spearman rho correlation coefficient. The null hypothesis states that there is no statistically significant positive relationship between learner metacognition and achievement in mathematics:

- *Hypothesis 5a*

  H_0: There is not a statistically significant positive relationship between learner metacognition and achievement in mathematics.

The alternative hypothesis states that there is a statistically significant positive relationship between learner metacognition and achievement in mathematics:

- *Hypothesis 5b*

  H_1: There is a statistically significant positive relationship between learner metacognition and achievement in mathematics.

4.4.1.3d Correlation between learner metacognition and mathematics achievement

Correlations point to the direction and magnitude of the relationship between paired scores (Ary *et al.,* 2010: 128-129). In this study, the correlation between learner metacognition and academic achievement was determined by calculating the Spearman rho correlation coefficient, which is a non-parametric measure, because it does not assume that the two variables are normally distributed. In addition, the Spearman rho correlation coefficient is determined when the scale is at least ordinal and the data are ranked (Cohen *et al.,* 2007: 588-559; Pietersen & Maree, 2007a: 237; Ary *et al.,* 2010: 354).

The interpretation of the Spearman rho is similar to the Pearson product moment coefficient of correlation (Pearson's *r*). The maximum and minimum values of the correlation coefficient are +1.00 and -1.00, respectively. The maximum value indicates a perfect positive relationship and a perfect negative relationship is indicated by -1.00. A value of 0.00 indicates no relationship (Ary *et al.*, 2010: 129).

It is important to consider the following three aspects in the interpretation of the correlation coefficient. First, correlation does not necessarily imply causation. Variables may be associated, but it does not indicate that the one variable causes changes in the other variable (Cohen *et al.,* 2007: 535; Ary *et al.*, 2010: 135). Secondly, a larger variability in the two distributions that are going to be correlated will lead to a higher correlation coefficient value (Ary *et al.*, 2010: 135, 355). When the number of participants increases, thereby increasing the variability, a smaller correlation coefficient value will be statistically significant (Cohen *et al.,* 2007: 535).

A third aspect to bear in mind is not to interpret the correlation coefficient value in terms of percentages. A correlation coefficient value of 0.80 does not indicate a correlation that is twice as strong as a correlation coefficient value of 0.40. The degree to which one variable can be used to predict the value of the other variable is related to the coefficient of determination which is the square of the correlation coefficient ($\mathbf{r^2}$). It shows the percentage of variance in the one variable that is directly linked to the variance in the other variable. Therefore, a correlation coefficient value of 0.80 indicates a 64% related variance between two variables, but a correlation coefficient value of 0.40 only indicates a 16% related variance (Cohen *et al.,* 2007: 535-536; Ary *et al.,* 2010: 135-136).

4.4.1.3e Reliability

Reliability is a prerequisite for validity, but it does not guarantee validity. Reliability refers to the degree of consistency of an instrument’s measurement, that is, whether consistent and dependable scores are obtained within a particular time frame (Cohen *et al.,* 2007: 133, 146; Ary *et al.*, 2010: 236). It thus refers to the effect of error on the scores. These errors are random errors of measurement that may influence scores in unpredictable ways (Ary *et al.*, 2010: 237).

Random errors of measurement result from inconsistencies in three domains: a participant’s behaviour, the administration of the instrument, and the instrument itself. First, inconsistent scores may result from participants’ fluctuating levels of motivation, interest, health, and other mental and emotional factors (Ary *et al.,* 2010: 237). In this study, the teachers of the control group and experimental group and the researcher were present when the MAI pre-test and post-test were administered. They did not observe any signs of distress, but it is obvious that they could not accurately determine the learners' emotional and mental states.

Secondly, the instrument may be administered by an inexperienced person who does not follow the correct procedures. Environmental factors such as light, heat and ventilation in the venue where the test is administered may influence the test results (Ary *et al.,* 2010: 237). In this study, the researcher administered both the pre-test and the post-test for the control group and the experimental group, and the post-tests were administered under similar environmental conditions and in the same settings as for the pre-tests.

Thirdly, the duration of the test influences reliability, because an instrument with very few items improves the probability that the correct answers are obtained by guessing (Ary *et al.,* 2010: 237). The MAI instrument used in this study does not measure the correctness of answers, but the degree to which learners display metacognitive behaviours. Therefore, the possibility that the reliability of this instrument was influenced by the guessing of answers is probably negligible.

A further indication of the reliability of an instrument is given by the value of the Cronbach's *alpha* coefficient. It is the only internal-consistency coefficient applicable to Likert scales and it measures the degree of similarity between an item and the sum of all other items that measure the same construct (Cohen *et al.*, 2007: 148; Pietersen & Maree, 2007b: 215). If the inter-item correlation is high, the internal consistency is high. When items that measure the same construct are poorly formulated, they will not correlate strongly and the *alpha* coefficient will be close to zero.

The following scale may be used to interpret Cronbach's *alpha*'s coefficient: 0.90 to 1 (high reliability); between 0.80 and 0.89 (moderate reliability); between 0.70 and 0.79 (low reliability), and from 0.60 to 0.69 (marginal reliability). Values below 0.60 are not sufficient to indicate that the instrument displays internal reliability (Pietersen & Maree, 2007b: 216).

Cohen *et al.* (2007: 506) provide a slightly different interpretation of the coefficient's value. They suggest the following: values greater than 0.90 (very highly reliable); 0.80-0.90 (highly reliable); 0.70-0.79 (reliable); 0.60-0.69 (marginally reliable), and values smaller than 0.60 (unacceptably low reliability). The two interpretations are in agreement that a coefficient value of smaller than 0.60 indicates that the instrument is not reliable. In this study, the interpretation suggested by Cohen *et al.* (2007: 506) is used, as it more closely corresponds with the interpretation by Schraw and Dennison (1994: 471).

A high degree of internal consistency was reported for the original MAI with a Cronbach's *alpha* value of 0.95. The two-factor model of metacognition, namely knowledge of cognition and regulation of cognition, was strongly supported = 0.90) (Schraw & Dennison, 1994: 460, 464). The Cronbach's *alpha* values of the pilot MAI, the pre-test MAI and the post-test MAI used in this study are discussed in Chapter 5 (see 5.3.2 and 5.3.3).

4.4.1.3f Validity

Validity is the main aspect to be considered in developing and evaluating measuring instruments (Ary *et al.*, 2010: 225). Validity is the extent to which an instrument measures what it claims to measure and also the degree to which the interpretations of the instrument's scores are supported by evidence and theory. The validity of the interpretations of an instrument's scores is regarded as the salient feature of the concept validity. An instrument may, therefore, be valid in one situation for a specific purpose, but not in a different situation for a different purpose (Ary *et al.*, 2010: 225, 235).

A construct such as learner metacognition is an abstract variable in contrast to constructs such as length and volume that can be measured directly. To measure an abstract construct, an operational definition describing observable behaviours that serve as indicators of the theoretical construct needs to be developed. The validity of an instrument measuring critical thinking would depend on the level of correspondence between the operational definition and the theoretical definition (Ary *et al.*, 2010: 225).

The validation of an instrument entails the process of gathering and evaluating evidence about the interpretation of the instrument's scores. Construct-related validity relates to evidence that the instrument measures an abstract construct in a valid way. This implies that the definition of the construct used in the development of the instrument is based on theory and previous research. It also means that the instrument's items reflect the aspects that define the construct (Cohen *et al.,* 2007: 138; Ary *et al.,* 2010: 231). The validity of the MAI in assessing the learner metacognition according to the two factors *KC* and *RC* was established by Schraw and Dennison (1994: 470-471) (see 4.4.1.2a).

More specific issues that relate to the internal and external validity of the MAI instrument are discussed next.

4.4.1.3f(i) Internal validity

Internal validity refers to the validity of the conclusions that are drawn from an experiment. Conclusions are validated if changes in the dependent variable are due to the influence of the independent variable and not due to extraneous factors (Cohen *et al.,* 2007: 135; Ary *et al.,* 2010: 271-272). The following threats to internal validity need to be controlled in order to draw valid inferences from an experimental study.

- **History**

Other events may occur at the same time as the intervention period and influence the dependent variables, namely learner metacognition and mathematics achievement. The effect of these events becomes more pronounced when the intervention period becomes longer. One way of reducing this threat is to use a control group, but then the control group and experimental group must be influenced similarly by these events (Creswell, 2009: 163; Ary *et al.,* 2010: 273-274). In this study, a control group and an experimental group are used, but as the two groups are from different schools, they would not have been affected equally by certain events. To the best knowledge of the researcher, no events occurred during the intervention period that could have significantly influenced the independent variables.

- **Maturation**

Participants may undergo biological or psychological maturation during the intervention period. These changes can influence the independent variable, especially in children, as they undergo faster biological and psychological changes than adults (Creswell, 2009: 163; Ary *et al.*, 2010: 274). Since the participants in the control group and those in the experimental group were of similar age (see 5.2.3), the rate of biological and psychological maturation was probably very similar.

- **Testing**

Post-test performance may be affected by the writing of a pre-test. Several factors that may influence post-test performance are identified. For example, participants could have remembered some of the items on the pre-test or they could have become acquainted with the test format; participants could also have developed strategies for performing better in the post-test, and participants could be less nervous the second time they write the test. Pre-testing effects are less pronounced during a lengthy intervention period (Creswell, 2009: 164; Ary *et al.,* 2010: 274-275). The use of a control group and an experimental group in this study implies that these aspects had most likely very similar effects on both groups. In addition, a lengthy intervention period of six months ensured minimum pre-testing effects.

- **Instrumentation**

When different instruments are used for the pre-test and the post-test, the observed changes in the dependent variables may be due to a change in instruments. Therefore, the instruments used for the pre-test and the post-test must be similar in terms of the type of instrument, the level of difficulty and the way the test was administered (Creswell, 2009: 164; Ary *et al.*, 2010: 275-276). In this study, the MAI measured learner metacognition in both the pre-test and the post-test. The post-test was administered in the same way as the pre-test.

- **Statistical regression**

Participants who score very high or very low on a pre-test tend to obtain scores closer to the mean on a post-test. Therefore, the sample needs to be selected from different subgroups, or the participants must be randomly assigned to an experimental group and a control group (Creswell, 2009: 163; Ary *et al.,* 2010: 277-278).

In this study, random assignment of the participants was not possible as intact classes were used as the control group and the experimental group. The MAI pre-test standard deviation scores indicated that the variation within the groups was very similar. This implies that the distribution of low and high scores, relative to the median MAI total score of each group, included all subgroups in respect of their MAI total scores. In addition, the standard deviation scores of the experimental and control groups' Term 1 report marks also pointed to a similar variation in pre-test achievement scores.

- **Selection bias**

Existing differences between the experimental group and the control group may pose a threat to the validity of any observed post-test differences between the two groups. Quasi-experimental studies are especially prone to this threat, because intact groups are used and the participants are not randomly assigned (Creswell, 2009: 163; Ary *et al.*, 2010: 278). The MAI pre-test and post-test differences between the groups are discussed in Chapter 5 (see 5.4.1-5.4.4).

- **Experimental mortality (attrition)**

Internal validity is threatened when participants who wrote the pre-test are no longer part of the study when the post-test is written. This effect is particularly pronounced if a significant number of low performers or high performers drop out (Creswell, 2009: 163; Ary *et al.*, 2010: 279). In this study, seven learners from the control group and one learner from the experimental group changed from Mathematics to Mathematical Literacy during the course of the intervention. These learners' pre-test scores did not threaten the internal validity of the results, because their pre-test MAI scores were not used in the Mann-Whitney test and the Wilcoxon test.

- **Selection-maturation interaction**

Participants in the experimental group can mature at a faster rate than the participants in the control group, although they may be equivalent on pre-test measures. This higher rate of maturation can occur in the experimental group, because participants may be more motivated than participants in the control group to improve on their pre-test scores (Ary *et al.,* 2010: 279). In this study, the experimental group did not know that they would be writing a quantitative and qualitative post-test. This could have reduced the effects of selection-maturation interaction. In addition, neither group was informed of another group taking part in the study.

- **Experimenter**

The researcher's characteristics such as age, race and gender may unintentionally influence the participants (Ary *et al.,* 2010: 280). Since the researcher's direct involvement with the experimental group only involved the administering of the pre-test and post-test MAI, and two problem-solving sessions, it seems unlikely that the experimenter had a significant influence on the participants.

- **Participant effects**

Participants in the experimental group may perform better in the post-test, because of the attention they receive during the intervention period due to the Hawthorne effect. Participants in the control group may also perform better, because they experience the research study as a competition (Ary *et al.,* 2010: 281-282).

In this study, the researcher and co-researcher did not tell the learners that they would be writing a post-test MAI, or that their Term 1 and Term 4 report marks would be used. The aim of the intervention was to gradually enhance learner awareness of effective learning in mathematics without drastically changing the structure of the mathematics lessons to which the learners were accustomed. The intention of the researchers was to counter the Hawthorne effect by establishing a natural setting without constantly reminding the learners that they were part of a research project. The fact that the learners also had the opportunity to critique the intervention instrument and method could have countered the Hawthorne effect to a certain degree, as they were given the freedom to display their normal behaviour.

The researcher did not inform the control group that they were part of an experiment. This reduced the possibility of them viewing the research study as a competition.

- **Diffusion**

Participants in the experimental group could influence the performance of participants in the control group by informing them about the treatment (Creswell, 2009: 163; Ary *et al.,* 2010: 282). In this study, diffusion effects were unlikely, as the control group and the experimental group were in two different schools.

4.4.1.3f(ii) External validity

External validity refers to the extent to which the findings of the study can be generalised to other participants and contexts. Many of the factors that threaten internal validity also influence external validity. The factors that threaten both internal and external validity are selection bias, testing, participant effects, history, and experimenter effects (Cohen *et al.,* 2007: 137; Ary *et al.,* 2010: 292-293). These factors were discussed in the previous section (see 4.4.1.3f(i)).

A factor that threatens external validity only is the setting in which the study was conducted. An artificial setting may limit the application value of the study to general contexts (Cohen *et al.,* 2007: 137; Creswell, 2009: 165; Ary *et al.,* 2010: 292-294). This research was conducted in the natural mathematics classroom setting of both the control group and the experimental group.

In this section, the quantitative research methods were discussed in respect of sampling, data collection, and data analysis. In the next section, the qualitative research methods employed in this study are discussed with reference to data-collection procedures, data analysis, and the interpretation of the qualitative data.

4.4.2 Qualitative research methods

This section describes the qualitative research methods employed in this study, namely data-collection procedures; data analysis, and the interpretation of the qualitative data.

4.4.2.1 Data-collection procedures

Data-collection procedures entail the setting of boundaries for the study, collecting data, and recording data (Creswell, 2009: 178).

Apart from an interview conducted with the teacher of the control group, the qualitative data that were collected in this study only involved the learners of the experimental group and their teacher, Mark. Data were collected in respect of the following aspects:

- an initial discussion with Mark on the structuring of the MI process;
- interviews with the teachers of both groups about their views on the nature of mathematics and the teaching-and-learning of mathematics;
- a qualitative pre-test problem-solving session;
- the learners' and the teacher's perspectives on both cycles of the MI process, and
- a qualitative post-test problem-solving session.

Table 4.4 indicates the time frame of these qualitative data-collection procedures.

Table 4.4: Time frame of the qualitative data-collection procedures

Date	Event
27 February 2010	Interview with Mark.
5 March 2010	Letter to parents requesting their permission to conduct research that involves the Grade 11 learners of the experimental group.
26 March 2010	Initial discussion with Mark about the MI process.
4 April 2010	First problem-solving session (qualitative pre-test).
26 May 2010	Learners' perspectives on the first cycle of the MI process.
29 May 2010	Interview with the teacher of the control group.
18 June 2010	Mark's perspectives on the first cycle of the MI process.

Date	Event
2 September 2010	Mark's perspectives on the second cycle of the MI process.
8 September 2010	Learners' perspectives on the second cycle of the MI process.
15 September 2010	Second problem-solving session (qualitative post-test).

Table 4.4 indicates that the qualitative data-collection period stretched over the major part of 2010. Most of the data were collected from the experimental group's learners. A brief discussion of their learning environment could enhance understanding of contextual factors that relate to the data-collection process.

4.4.2.1a The learners' environment

Some general aspects relating to the school environment of School A and School B were discussed earlier (see 1.12 and 4.4.1.1). Additional aspects relating to the experimental group's learning environment are discussed next.

Their school day consists of 10 academic periods, each lasting half an hour, and a break of half an hour. Nine periods a week are allocated to mathematics. Each day starts with a chapel period of 20 minutes. The chapel period is probably one factor that contributes to the good relationships among learners and between learners and teachers. During the researcher's visits to the school, he observed a friendly and disciplined atmosphere, not only in the mathematics classroom, but also in the behaviour of learners during change of classes, during breaks, and during sport events such as athletics and the swimming gala. The majority of the secondary learners are hostel boarders, which could also contribute to a general sense of team spirit. Each learner is part of one of three houses that serve as the teams when the learners compete in athletics and swimming events. The learners are proud of their school's rich traditions and they are given appropriate responsibilities in the governing of the school. A fully functioning learner representative council, with representation on the school's governing body, further serves as a measure which ensures healthy relationships among all stakeholders. The learners are also involved in the organisation of school events, thus further promoting their sense of ownership.

In any organisation, matters do not always run smoothly. A code of conduct encourages positive behaviours, but it also indicates punitive measures. Positive conduct is further encouraged by holistic learner support. Learners have daily access to the computer room with internet access. A well-equipped library, with a full-time library assistant, provides learning support. Professional emotional and spiritual support services are accessible via the school's minister. Sports coaching are done by qualified coaches, and medical support is provided by a hospital and physiotherapy practices in close proximity. These aspects are evidence of a nurturing school environment that provides a solid basis for effective learning.

A climate for effective learning was definitely established in the mathematics classroom. The mathematics classroom of the experimental group was neat, well-lit, fully resourced with textbooks, additional learning material, and mathematics posters. Each learner had her own textbooks and pocket calculator. Although the learners sat in rows, there was ample space to move around and form groups, if necessary. The classroom was situated in a fairly quiet part of the school, with views of the school garden and the sports fields, respectively. From observing the teacher's behaviour towards the learners, his views on teaching as expressed in the interview, and the learners' behaviour during their interaction with the researcher and the teacher, the researcher concluded that there was a positive learning atmosphere in the mathematics classroom. One point of critique relates to the length of the periods. In practice, the learners take approximately five minutes to settle, which leaves only 25 minutes for teaching. That is probably not enough time to ensure full learner engagement in, for example, the conducting of a problem-solving session.

In conclusion, the experimental group's learners learned mathematics in an environment in which optimal learning could take place.

4.4.2.1b Structuring of the MI process

The metacognitive intervention in this study was structured according to the proposed framework for metacognitive interventions (see 3.5). Table 3.3 indicates features of metacognitive intervention studies and how those features relate to a mathematical perspective on De Corte's (1996) educational learning theory (see 3.5). The MI used in this study relates as follows to the features of the proposed framework.

- **Age of the participants**

The age of the experimental group's learners is well within the age range of the proposed framework, namely Grade 4 to pre-college learners.

- **Intervention period**

The first cycle of the intervention extended from the first problem-solving session on the 4th of April to the 26th of May 2010, whereas the second round extended from the 13th of July to the 15th of September 2010 (see Table 4.4). From the 27th of May to the 12th of July, the learners wrote examinations and had a holiday during which no explicit metacognitive intervention took place. Therefore, 14 weeks were used explicitly for MI. This implies that the intervention period of this study corresponds well with the intervention period proposed in the framework.

- **Theoretical basis**

The MI of this study was constructed according to the aspects of the mathematical perspective in De Corte's (1996) educational learning theory, as indicated in the proposed framework for metacognitive interventions (see 3.5). Each aspect of De Corte's (1996) educational learning theory was interpreted from a mathematical perspective and presented to the learners in a booklet called the *codes booklet* or what the teacher of the experimental group initially called the *reflection sheet* and later *the tool* (see Appendices B5 and B6). The aspects of De Corte's (1996) educational learning theory were stated in more learner-friendly terms. Table 4.5 indicates these alternative terms.

Table 4.5: Alternative terms for the aspects of De Corte's (1996) educational learning theory

Aspects of De Corte's (1996) educational learning theory	**Alternative terms used in the codes booklet**	**Codes**
Constructive	**S**tarting Up	S1-S9
A structured knowledge base	Solid **F**oundations	F1-F2
Cumulative	**B**uilding Blocks	B1-B2
Goal-oriented	My **G**oals	G1-G3
Collaborative	**T**alk Time	T1-T3
Situated	**L**iving maths	L1-L3
Individually different	My **W**ay	W1-W4
Heuristics	**P**roblems can be solved	P1-P12
Affective components	Matters of the **H**eart	H1-H7

Two aspects of De Corte's (1996) educational learning theory, namely self-regulation and metacognition, were not included in the codes booklet as the purpose of the codes booklet was to enhance learner metacognition in mathematics. In other words, by applying the codes during the learning of mathematics, learners were also implicitly *forced* to apply the *knowledge of cognition* and *regulation of cognition* aspects of metacognition. In a previous discussion, the similarities and differences between metacognition and self-regulation were discussed (see 2.2.6). The use of the codes booklet is described in the next section.

4.4.2.1c Method of intervention

In Table 3.3, the following aspects relating to the method of intervention were mentioned: problem-solving contexts; corrective feedback; active teacher involvement; cooperative settings; individual settings; enrichment opportunities, and learner affect. Table 3.3 also indicated how these aspects relate to the aspects of De Corte's (1996) educational learning theory and are incorporated in the codes booklet. Therefore, when learners applied the codes during the learning of mathematics, these aspects were already addressed. The first cycle of MI entailed the use of the codes booklet. The process was structured as follows.

The researcher and Mark had initial discussions about the implementation of the codes (see Appendix E1). Mark handed out the codes booklets during the first week of the second term and he explained their use to the learners. The idea was for learners to indicate in their workbooks when they apply a certain code; for example, if a learner could identify the main topic of a mathematics question, she would write the main topic down and then write the code B1 next to it. Learners were guided by Mark during this process that lasted for the entire second term. Mark would, for example, tell the learners to write the code W2 in their workbooks where they made a common mistake or had a misconception about a certain section of the work. For most of the second term, the researcher and Mark communicated about the process of MI via e-mail, as the researcher was overseas for a three-week period. At the end of the second term, Mark's perspectives and the learners' perspectives on the first cycle of the MI process were used to adapt the MI process (see 6.4).

At the beginning of the third term, a letter was handed to each learner thanking them for being part of the research and for their willingness to continue with the research project. An appendix, containing the learners' feedback on the first cycle of the MI, was attached to the letter. The learners could, therefore, determine to what extent the researchers incorporated their suggestions (see 6.4).

The second cycle of the MI process extended from the 13th of July to the 2nd of September 2010. At the end of the second cycle, Mark and the learners gave their perspectives on the second cycle of the MI process (see 6.4).

4.4.2.1d First problem-solving session (qualitative pre-test) and second problem-solving session (qualitative post-test)

The first problem-solving session served as the qualitative pre-test. A word problem was given to the learners which they had to solve individually. The word problem related to the area and perimeter/circumference of two-dimensional shapes. These topics were addressed in the years prior to Grade 11. An analysis of the mathematics topics and concepts of this world problem, and the solution, are discussed in Chapter 6 (see 6.2).

The learners had to record, first, their thoughts relating to the problem and, secondly, the calculations that corresponded to their thoughts. After the individual activity, a whole-class discussion followed in which the solution to the problem was established and recorded by the learners. Their completed worksheets were taken in and they were not told that there would be a second problem-solving session on the same problem.

4.4.2.1e Interviews with the teachers

Interviews afford participants the opportunity to share their perspectives and interpretations of the world in which they live. Interviews enable researchers to further explore the participant's answers about complex issues (Cohen *et al.,* 2007: 349). In structured interviews, the content, wording and sequence of questions are established before the interview takes place; in unstructured interviews, the researcher has more freedom and flexibility to adapt the content, wording and sequence of questions. However, an unstructured interview still requires careful planning (Cohen *et al.,* 2007: 355).

Unstructured interviews were conducted with the teachers of both groups on their views of mathematics and the teaching-and-learning of mathematics. The purpose of the interviews was to gain insight into their experiences as mathematics teachers and their reflections on their daily practices in the teaching of mathematics. The interview with Mark was conducted before the MI started in order to determine his perspectives, without him possibly being influenced by knowledge gained from the MI process. Lisa's interview took place during the second term of 2010. The proposed interview questions were given to both teachers a few days prior to the interviews, but the researcher explained that those questions would only serve as a guide to the interview process. The interviews were audio-recorded, transcribed, coded, and subthemes and themes were identified (see Appendices D1-D6).

The experimental group's teacher checked the transcription of his interview and changes were incorporated. The control group's teacher was not asked to check the transcription of her interview, because her South African accent enabled the researcher to transcribe more accurately as compared with the British accent of the experimental group's teacher.

4.4.2.2 Data analysis and interpretation

The most complicated and mystifying part of qualitative research is data analysis (Ary *et al.,* 2010: 481). The organisation of the findings is a complex task due to the amount of data collected (Babbie & Mouton, 2001: 283). It is an untidy and non-linear process that may seem overwhelming at first. However, qualitative data analysis could be less threatening if the following three stages are followed: organising and familiarising; coding and reducing, and interpreting and representing (Ary *et al.,* 2010: 481).

Organising and familiarising enables the researcher to easily retrieve the data. The researcher becomes immersed in the data by repeatedly listening to audio-recorded interviews and by reading and re-reading through transcripts and field notes (Creswell, 2009: 185; Ary *et al.,* 2010: 481). In this study, the researcher became immersed in the data by transcribing the interviews with the teachers. In addition, the researcher converted all learner responses to both problem-solving sessions and the learners' perspectives on both cycles of the MI process into electronic form. This afforded him the opportunity to deeply engage with the learners' responses.

The coding and reducing process is the main aspect of qualitative analysis. During coding, concepts emerge from the raw data through data categorisation and the identification of themes. Units of meaning – whether words, phrases, sentences, behaviour patterns or events – that appear regularly and are considered important are sorted and categorised (Creswell, 2009: 186-189; Ary *et al.,* 2010: 483). In this study, the concepts that emerged from the problem-solving sessions were coded in respect of the item numbers on the MAI (see Appendix C2). In both problem-solving sessions, the learner errors were categorised as conceptual errors or calculation errors (see Appendices C5-C7). The teacher interviews were analysed by first identifying subthemes, and then by grouping the subthemes into main themes (see Appendices D2-D3 and D5-D6). Learners' perspectives on the first cycle of the MI process were categorised into themes (see Appendix E4). In the analysis of learners' perspectives on the second cycle of the MI process, subthemes were identified which were then grouped into main themes (see Appendix E14).

Interpreting is an inductive process that entails reflecting on the data and abstracting significant understanding from the data. Connections between categories are made which may lead to generalisations and hypotheses. There are no set rules for data interpretation, although the researcher's background, perspective, knowledge, and intellectual skills may enhance the quality of the interpretation (Creswell, 2009: 189-190; Ary *et al.,* 2010: 490). In this study, the interpretation of the qualitative data was mainly done by relating the teachers' and learners' responses to the theoretical underpinnings of the MI process, as evident in the proposed framework for a metacognitive intervention in mathematics (see 3.5).

4.4.2.3 Rigour in qualitative research

Rigour in research refers to the validity of the inferences and the consistency of the collected data. In their discussion of rigour in qualitative research, Ary *et al.* (2010: 497-504) compare the standards for rigour in quantitative and qualitative research. They state that internal validity in quantitative research relates to credibility in qualitative research. Similarly, external validity relates to transferability, and reliability relates to dependability or trustworthiness. However, the terms 'internal validity', 'external validity', and 'reliability' are used in this discussion on rigour in the qualitative section of this study in order to be consistent with the terms used in the section dealing with the quantitative data-collection procedures and also because these terms are pertinent in qualitative inquiry (Morse, Barrett, Mayan, Olson & Spiers; 2002).

4.4.2.3a Internal validity

Internal validity refers to the truthfulness of the findings based on the research design, participants and context. Internal validity in a qualitative study can be improved by five types of evidence: structural corroboration; consensus; interpretive adequacy; theoretical adequacy, and control of bias (Ary *et al.,* 2010: 498).

- **Structural corroboration**

Structural corroboration relates to the use of different sources of data (data triangulation) and different methods (methods triangulation) (Creswell, 2009: 191; Ary *et al.,* 2010: 498-499). In this study, different sources of qualitative data were obtained from the teachers and the learners. Apart from the interview conducted with Mark, in which he stated his views on the teaching-and-learning of mathematics, his perspectives on both cycles of the MI process also included further references to the teaching-and-learning of mathematics (see 6.4). Different sources of qualitative learner data on their level of metacognition in a problem-solving context were obtained from two problem-solving sessions (see 6.2). In addition, data on learner metacognition were obtained by using two different qualitative methods, namely two problem-solving sessions and their perspectives on both cycles of the MI process. The learners' perspectives on both cycles of the MI process entailed broader aspects than learner metacognition, but there were also references to learner metacognition (see 6.4).

- **Consensus**

Consensus is demonstrated through peer review (peer debriefing) and investigator triangulation. The process of peer review requires that colleagues of the inquirer reach consensus about the interpretation of the data. Investigator triangulation refers to other researchers who also collect data independent from the main researcher and then comparing the collected data (Creswell, 2009: 192; Ary *et al.,* 2010: 499). Peer review featured in this study in respect of two aspects. First, Mark acted as a colleague by also analysing and interpreting the learners' level of metacognition and achievement in a problem-solving context (see 6.2.4). He also analysed and interpreted the learners' feedback on the first cycle of the MI process (see 6.4.1). Investigator triangulation only featured indirectly in this study. Only the researcher collected data on learner metacognition during both problem-solving sessions, but Mark observed learner metacognitive behaviour which occurred as part of his normal teaching activities during both cycles of the MI process. These observations were reflected in his perspectives on the first and second cycles of the MI process (see 6.4).

- **Interpretive adequacy**

Evidence based on interpretive adequacy concerns the accurate portrayal of the participants' experiences by using member checks and low-inference descriptors. Member checks (participant feedback) refer to the feedback given by the participants about the study's findings. Low-inference descriptors are direct quotations by the participants that shed light on the participants' world (Creswell, 2009: 191; Ary *et al.*, 2010: 499-500). In this study, member checks were done by the learners after the second cycle of the MI process. They stated their perspectives on the new MI codes booklet which was adapted by incorporating the findings of the first cycle of the MI process (see 6.4). Low-inference descriptors were recorded in both problem-solving sessions and in the learners' perspectives on both cycles of the MI process (see Appendices C2, E3 and E13).

- **Theoretical adequacy**

Theoretical adequacy (plausibility) refers to the sufficiency of the theoretical explanation emerging from the study, that is, the correspondence between the theoretical explanation and the data. Extended fieldwork, theory triangulation, and pattern matching promote theoretical adequacy.

Extended fieldwork allows the researcher to gain the trust of the participants in order to obtain truthful answers. It also allows for multiple activities that could help the researcher identify relationships and patterns in the data. Theory triangulation concerns the interpretation of the findings by using multiple theories. Pattern matching involves the prediction of patterns emerging from the study. These predictions are based on theory (Ary *et al.,* 2010: 500). In this study, extended fieldwork was done, in a sense, by Mark as he taught the learners during the two cycles of the MI process which extended over the second and third terms of 2010. He had already established a relationship of trust with the learners prior to the start of the research study, as he had taught them since the first term of 2010.

Theory triangulation was employed to a minor degree in this study. The qualitative interpretation of the level of learner metacognition was based on the subscales of the

MAI which incorporates common elements of different theories about metacognition (see 4.4.1.2a). The qualitative data interpretation in respect of mathematics achievement was mainly based on De Corte's (1996) educational learning theory, which is a synthesis of many aspects of various learning theories (see 3.3).

The last aspect of the theoretical adequacy of this study relates to the prediction of patterns emerging from the study. This aspect did not feature prominently, but predictions can be made about the effectiveness of future metacognitive interventions based on the adapted MI codes booklet.

- **Researcher bias**

Researcher bias, the last aspect that relates to internal validity in qualitative studies, occurs when researchers are not objective in their interpretation of data. Bias can be reduced when researchers record self-reflections in a journal and then refer to the journal when the data is interpreted. Bias can also be controlled by negative case sampling which involves the deliberate search for contradictory data (Creswell, 2009: 192; Ary *et al.,* 2010: 500-501). In this study, the researcher did not record self-reflections, but Mark continually reflected on the MI implementation process during the second term of 2010 (see 6.4.2). The researcher endeavoured to be objective in the interpretation of the data by adhering to the ethos of pragmatism, namely engaged fallibilistic pluralism which requires an acknowledgement of one's limited understanding and a continued willingness to seek more insight (see 4.2.2.2). Although the researcher did not deliberately search for contradictory data, learners were given the opportunity to freely state their thoughts during the problem-solving sessions. They also completed open-ended questionnaires when they gave their perspectives on both cycles of the MI process. This provided them with the opportunity to express their ideas without restrictions.

4.4.2.3b External validity

External validity refers to the generalisability of the findings. In qualitative research, generalisability is not normally a goal, but the inquirer must still provide rich, thick descriptions of the different aspects of the study in order to enhance comparisons between data sources and allow judgments about the similarity between the context of the study and other contexts (Creswell, 2009: 191-193; Ary *et al.,* 2010: 501). Potential users of the study determine the similarity between the study's context and other contexts in contrast to quantitative research where the researcher makes the generalisations. External validity is enhanced when methods are described in detail and the researcher's biases are stated (Creswell, 2009: 192-193; Ary *et al.,* 2010: 502). In this study, the qualitative methods in respect of the learners' environment, the teacher's background, the MI process, and the problem-solving sessions are described in detail to enable potential users of the study to determine to what extent the findings could be generalised to their own settings (see 4.4.2).

4.4.2.3c Reliability

Reliability is viewed as the extent to which inconsistencies and variations in results can be explained when studies are replicated. Documentation and consistent findings played a role in establishing the reliability of a study (Creswell, 2009: 190-191; Ary *et al.,* 2010: 502). These two aspects are discussed next.

An audit trail provides documentary evidence of the raw data collected in interviews, observations, documentary analysis and a record of the researcher's activities describing the reasons for certain decisions that were made (Ary *et al.*, 2010: 502-503). In this study, the raw data in respect of the teachers' interviews, the problem-solving sessions, and the learners' perspectives on the MI process have been maintained. Raw data were also converted into electronic form (see Appendices C-E).

Reliability is enhanced if consistent findings are demonstrated by means of stepwise replication in which two inquirers analyse the data separately and then compare their findings (Creswell, 2009: 190; Ary *et al.,* 2010: 503). In this study, the researcher and Mark analysed both problem-solving sessions separately and then the researcher compared the findings (see 6.2.4-6.2.6). Mark also analysed the learners' perspectives on the first cycle of the MI process (see 6.4.1). The researcher and Mark adapted the MI process by taking his analysis and the researcher's analysis of the learners' perspectives into account (see 6.4.2).

This section concludes the discussion of the qualitative research methods employed in this study. Next, an overview of the concepts discussed in Chapter 4 is provided and subsequently the objectives of Chapter 5 are stated.

4.5 CONCLUSION

In this chapter, the relation between the researcher's philosophical world view and pragmatism was established. Pragmatism informs this study's mixed methods research methodology as multiple and varied methods, data-collection techniques, and data-analysis strategies are employed. In this embedded experimental design, a quasi-experiment was conducted and qualitative data were embedded within the quantitative methodology prior to, during, and after the intervention phase.

The quantitative research methods were discussed regarding the comparison between the experimental group and the control group in respect of their scores on the MAI pre-test and post-test, and their mathematics achievement scores. The development and reliability of the original MAI were examined, and the structuring of this study's MI was explained.

The qualitative research methods were discussed in respect of a pre-test and post-test problem-solving session; the teachers' perspectives on the nature of mathematics and the teaching-and-learning of mathematics, and the perspectives of both the experimental group's learners and their teacher on the MI process. Furthermore, aspects related to the reliability, internal validity and external validity of the quantitative and qualitative data-collection procedures of this study were discussed.

In the next chapter, the data emerging from the quantitative data-collection procedures employed in this study are presented, analysed, and interpreted in order to address secondary research question 5, secondary research question 6, and Hypotheses 1-5.

CHAPTER 5

PRESENTATION, ANALYSIS, AND INTERPRETATION OF THE QUANTITATIVE RESEARCH DATA

5.1 INTRODUCTION

In Chapter 4, the research design of this study was discussed. This chapter addresses the first primary research question in respect of secondary research questions 5 and 6:

- Secondary research question 5: Does MI have a statistically significant positive effect on the metacognitive awareness of the experimental group's learners?
- Secondary research question 6: Is there a statistically significant positive relationship between metacognitive awareness and mathematics achievement for learners in both the experimental group and the control group?

These questions are addressed by means of the presentation, analysis, and interpretation of the data collected from both groups in respect of the MAI pre-test and post-test, and the Term 1 and Term 4 report marks in mathematics.

This chapter is structured as follows:

- First, the extraneous variables of this study and the reliability of the pilot, pre-test, and post-test MAI questionnaires are presented and analysed (see 5.2).
- Secondly, results from the pre-test and the post-test MAI, and from Term 1 and Term 4 report marks are presented and described by means of descriptive statistical procedures (see 5.3).
- Thirdly, inferential statistical procedures are used to determine whether the observed changes in the data collected from the MAI pre-test and post-test, and from the correlation between learner metacognition and Term 1 and Term 4 report marks are likely to be observed in the larger population (see 5.4).
- Fourthly, the data that are presented and analysed by means of descriptive and inferential statistical procedures are interpreted in respect of the first two primary research questions (see 5.5).

5.2 EXTRANEOUS VARIABLES

This study investigates the effect of MI (the independent variable) on two dependent variables, namely learner metacognition and mathematics achievement. In this section, some extraneous variables that could influence the internal validity of this study are discussed (see 4.4.1.1d).

5.2.1 Teachers' qualifications

Mark (a pseudonym), the experimental group's teacher, holds a M.Sc. in Mathematics Education and a Bachelor of Arts (Honours) degree in Philosophy from a British University. He successfully completed Mathematics at second-year level. Lisa (a pseudonym), the control group's teacher, holds a B.Sc. degree with Physics and Chemistry as her two majors, and Mathematics at second-year level. She also completed a Postgraduate Certificate of Education (PGCE).

5.2.2 Teaching experience

At the start of the intervention, Mark had two years and two months' post-qualification experience, teaching mathematics in a UK secondary school, and one year and 11 months' teaching experience in South African secondary schools. In the UK, he taught learners aged 11 to 17 (AS level) from June 2005 to August 2007. He also coordinated the school's provision programme for talented and gifted students in mathematics. In South Africa, he taught at two different schools for girls. At the first school, he taught Grades 8, 10, 11 and 12 mathematics from January 2008 to December 2008. At that school, he also organised a Maths Evening which involved fun mathematics activities. Since May 2009, he taught at the second school, School A, where he has since been responsible for the Grade 11 and Grade 12 mathematics classes. This implies that he only started teaching the experimental group in the year of the intervention (2010) and not when they were in Grade 10 in 2009.

At the start of the intervention in 2010, Lisa had 15 years' teaching experience in the FET phase. In 1995, she started teaching a Grade 11 class at School B. In 1996, she taught this class again in Grade 12. This process of teaching learners in a Grade 11 class and teaching the same group again in Grade 12 repeated itself until the start of the intervention in 2010. This implies that, at the start of the intervention, she had eight years' teaching experience of Grade 11 mathematics and seven years' teaching experience of Grade 12 mathematics. She also taught Grades 8, 9 and 10 classes during those 15 years.

5.2.3 Learners' age

The mean age of the experimental group's learners (School A) was 16.40 when they completed the pre-test. Fifteen (60%) of the 25 learners were aged 16, and 10 learners (40%) were aged 17 when the pre-test data were collected.

The data obtained from the control group's learners (School B) pre-test show that 18 (75%) of the 24 learners were aged 16, whereas six learners (25%) were aged 17. The mean age of the control group was 16.25 when the pre-test data were collected.

5.2.4 Learners' home language

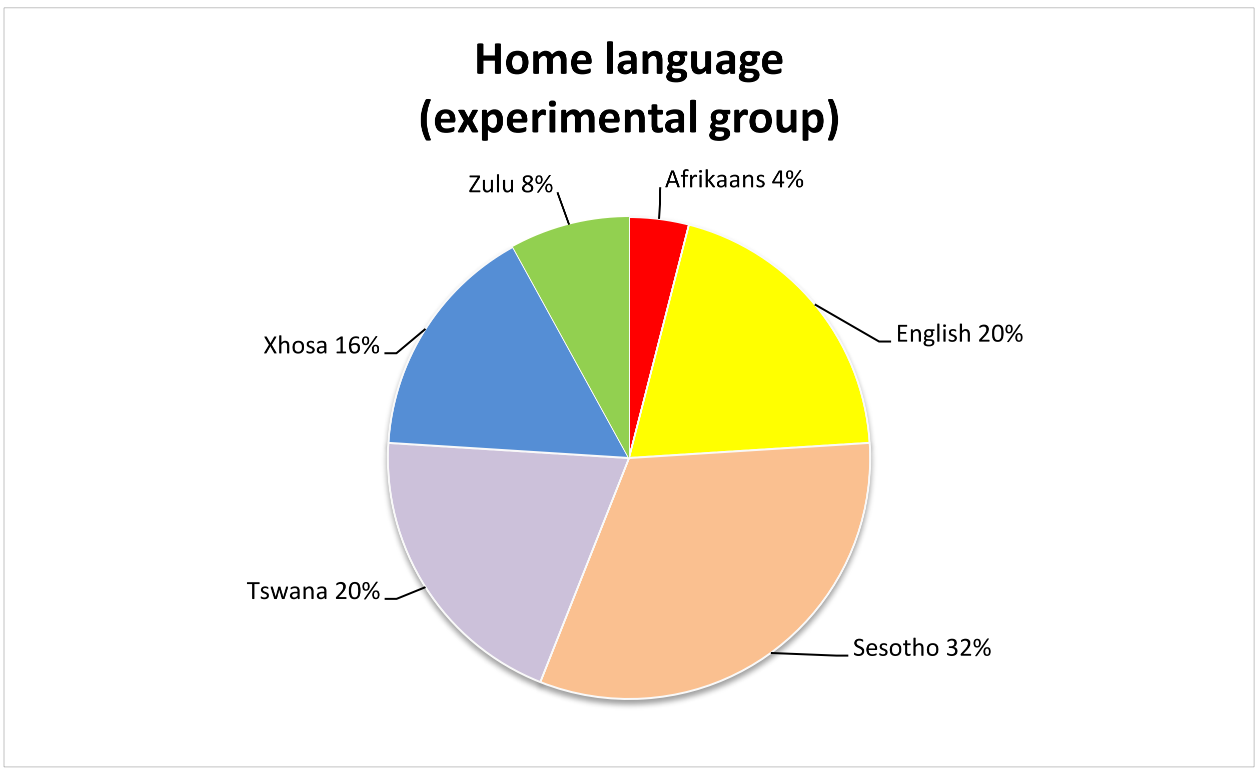

Figure 5.1: Home language distribution of the experimental group

Figure 5.1 indicates that the highest percentage (32%) of learners had Sesotho as a home language. A significant percentage of the learners had English (20%) and Tswana (20%) as a home language. Xhosa (16%), Zulu (8%), and Afrikaans (4%) were the home languages of the lowest percentage of learners.

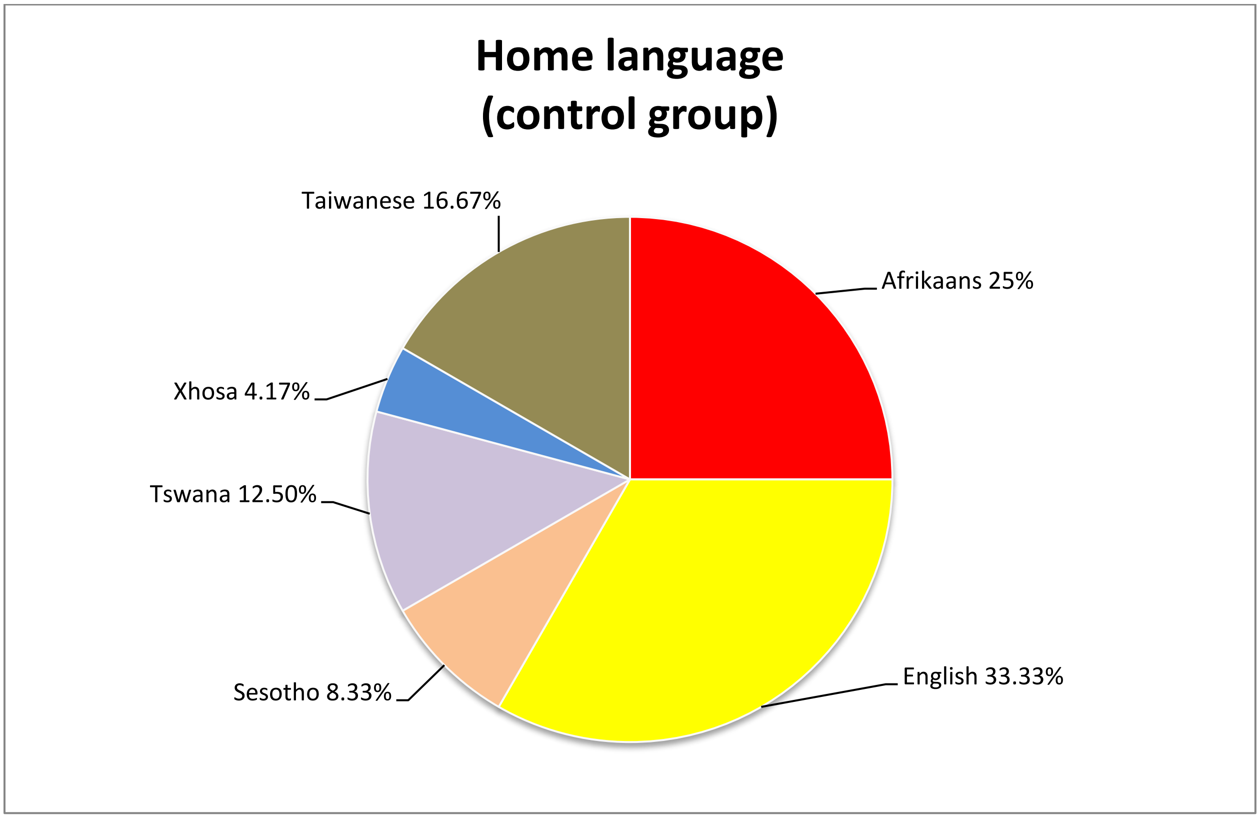

Figure 5.2: Home language distribution of the control group

The data displayed in Figure 5.2 show that English was the home language of 33.33% of the learners, which was the highest percentage. Afrikaans was the home language of the second highest percentage (25%) of the learners. The other home languages were Taiwanese (16.67%), Tswana (12.50%), Sesotho (8.33%), and Xhosa (4.17%).

5.2.5 Time allocated to teaching

The experimental group received a total of four hours and 30 minutes of formal mathematics teaching per week. This consisted of nine 30-minute periods. The control group's mathematics periods lasted 40 minutes and were spread over a 10-day cycle with 13 periods allocated to mathematics. They thus received four hours and 20 minutes of formal mathematics teaching per week.

The experimental group received two hours of extra classes per term. The extra classes were structured around problem areas identified by the teacher. The control group received two hours of extra classes per week per term. These extra classes were compulsory and were used to explain new work, address problem areas and do revision. The control group had eight extra classes during Term 2 and seven extra classes during Term 3. As the intervention extended over Terms 2 and 3, this implies that the control group had a total of 30 hours of extra classes, whereas the experimental group had four hours of extra classes.

The interpretation of these aspects addresses the potential influence of these extraneous variables on learner metacognition and mathematics achievement (see 5.5.1-5.5.5).

Next, the quantitative data collected in this study are presented, using descriptive statistical procedures.

5.3 DESCRIPTIVE STATISTICS

Descriptive statistics are statistical procedures that are used to organise, summarise, and describe observations (Ary *et al.,* 2010: 101). In this section, data obtained from the pilot MAI, the MAI pre-test and post-test, and the Term 1 and Term 4 report marks of both groups are discussed.

5.3.1 Pilot MAI

The pilot questionnaire was constructed by adapting the original MAI. Some words were changed to more familiar words that reflect the South African education context. In addition, as the original MAI measures general learner metacognitive awareness, the pilot questionnaire was adapted to reflect a mathematical context. The original MAI was also adapted in respect of the following aspects.

First, Item 41 was excluded, because the researcher found it problematic to rephrase the wording of Item 41, namely “I use the organisational structure of the text to help me learn”, in order to reflect a mathematical context. Schraw and Dennison (1994: 474) give no indication of under which of the eight subscales Item 41 is classified.

Secondly, Items 51 and 52 were combined, because the wording of the two items is similar, namely “I stop and go back over new information that is not clear” (Item 51), and “I stop and reread when I get confused” (Item 52). As both items are classified under the subscale *Debugging*, it means that *Debugging* only comprises four items in the pre-test MAI, instead of five items as in the original MAI.

Thirdly, the rating scale of the original MAI was adapted to reflect a more learner-friendly format. In the original MAI, ratings for each item were made on a 100mm, bipolar scale where the right-hand of the scale indicated that the statement was false, and the left-hand indicated that the statement was true. Respondents had to draw a slash across the rating scale at a point that indicated how true or false a statement was about their metacognitive behaviours (Schraw & Dennison, 1994: 463). In the adapted MAI used in this study, learner responses are indicated on a Likert scale with the categories Strongly Disagree, Disagree, Neutral, Agree, and Strongly Agree. The researcher believed that learners would give more accurate responses when choosing a specific Likert scale category, instead of drawing a slash across a rating scale of which only the end-points were labelled.

The pilot questionnaire was completed by two university lecturers in mathematics education. The first lecturer commented on the format of the questionnaire by advising that numbers 1-5 be used instead of the codes SD, D, U, A, and SA for each question. He also indicated some grammatical errors. The main concerns voiced by this lecturer involved Items 12 and 20 and the questions that used the term “problem-solving strategies”. As far as Item 12 was concerned, he wondered whether the phrase “information I receive” referred to textbooks or information conveyed orally. He also queried the clarity of Item 20, which asks learners whether they have control over how well they learn in mathematics.

The second lecturer remarked that the phrase “I think about what I really need to learn” in Item 6 was unclear, because it did not indicate whether it refers to mathematics principles and topics. She queried the clarity of Item 14 “I have a specific purpose for each problem solving strategy I use when I solve a problem in mathematics”. As indicated by the first lecturer, the second lecturer also queried the clarity of Item 20. The clarity of Item 39 and the phrase “… overall meaning ...” in Item 47 were also queried.

The pilot questionnaire was given to 35 Grade 11 mathematics learners of School B to complete. These learners were in a different class to those in the control group. They were also asked to state any unclear aspects by writing down the item number and the reason(s) why they find those items unclear. The researcher found that all learners completed the questionnaire within 10 minutes. Their written responses of those items they found unclear are indicated in Appendix B3.

Furthermore, the pilot questionnaire was completed by eight Grade 11 mathematics learners of School A who were in a different class to those in the experimental group, due to differences in subject choices. They also completed the pilot MAI questionnaire and made notes of all vague aspects in the pilot questionnaire. They expressed their understanding of the terms “problem-solving strategies” and “learning strategies”. Four of the eight learners indicated that Item 29 was unclear. One learner stated that Item 31 was unclear, while two learners did not understand Item 39.

In addition, the experimental group’s teacher and the control group’s teacher perused the questionnaire. Their main query was whether the word “problem” referred to a problem or merely a question. They suggested that the phrase “problem-solving strategy” be replaced with “problem-solving method”, as the learners would understand that better.

Appendix B4 indicates how the items of the original MAI compare with those on both the pilot MAI and the adapted MAI used in this study. The pre-test MAI was constructed by taking in to account the feedback from the two university lecturers, the two pilot groups from School A and School B, and the teachers. These changes involved items 3, 11, 14, 20, 29, 31, 35, 39, 40, 46, and 47, as indicated in Appendix B4.

5.3.2 Reliability of the pilot MAI

The reliability of the MAI pilot questionnaire and the MAI questionnaire used for both the pre-test and the post-test was determined, as shown in Tables 5.1 and 5.2. The Cronbach's *alpha* value was computed for the instrument as a whole, the two larger factors (*KC* and *RC*), and the various subscales.

Table 5.1: Cronbach's *alpha* values for the pilot MAI

Metacognitive scale	Number of items	Cronbach's *alpha* (N=35)
MAI total score	50	0.91
Knowledge of cognition	17	0.81
Declarative knowledge	8	0.66
Procedural knowledge	4	0.60
Conditional knowledge	5	0.36
Regulation of cognition	33	0.89
Planning	7	0.76
Information management	9	0.67
Monitoring	7	0.64
Debugging	4	0.69
Evaluation	6	0.40

Table 5.1 indicates that the highest Cronbach's *alpha* value of 0.91 was obtained for the MAI total score. *RC* (0.89) and *KC* (0.81) had the second highest and third highest Cronbach's *alpha* values, respectively. The Cronbach's *alpha* values of the eight subscales ranged from 0.36 (*Conditional knowledge*) to 0.76 (*Planning*).

5.3.3 Reliability of the MAI pre-test and post-test

Table 5.2 presents the Cronbach's *alpha* values for the MAI pre-test and post-test.

Table 5.2: Cronbach's *alpha* values for the MAI pre-test and post-test

Metacognitive scale	Number of items	Cronbach's *alpha*	
		Pre-test (N=49)	Post-test (N=49)
MAI total score	50	0.89	0.93
Knowledge of cognition	17	0.82	0.82
Declarative knowledge	8	0.66	0.65
Procedural knowledge	4	0.60	0.62
Conditional knowledge	5	0.50	0.59
Regulation of cognition	33	0.83	0.91
Planning	7	0.66	0.75
Information management	9	0.69	0.78
Monitoring	7	0.56	0.73
Debugging	4	0.34	0.54
Evaluation	6	0.23	0.56

Table 5.2 shows that the highest pre-test Cronbach's *alpha* values were obtained for the MAI total score (0.89), *RC* (0.83), and *KC* (0.82), respectively. The Cronbach's *alpha* values of the eight subscales ranged from 0.23 (*Evaluation*) to 0.69 (*Information management*).

The MAI total score (0.93), *RC* (0.91), and *KC* (0.82) had the highest post-test Cronbach's *alpha* values, whereas the Cronbach's *alpha* values for the eight subscales were between 0.54 (*Debugging*) and 0.78 (*Information management*).

5.3.4 Comparison between the pre-test MAI scores of both the experimental group and the control group

Table 5.3 presents the means, medians, the difference between the mean and the median, and the difference between the two groups' medians. The median values are considered a more valid measurement of both groups' metacognitive score, due to the effect of outliers on the mean values in small samples.

Table 5.3: Mean, median, difference between mean and median, and the difference between the medians (pre-test: experimental group and control group)

Metacognitive scale	Group	Mean	Median	Difference between mean and median	Control group's median minus experimental group's median
MAI total score: pre-test	Experimental	3.24	3.26	-0.02	0.25
MAI total score: pre-test	Control	3.49	3.51	-0.02	
Knowledge of cognition: pre-test	Experimental	3.34	3.35	-0.01	0.47
Knowledge of cognition: pre-test	Control	3.75	3.82	-0.07	
Declarative knowledge: pre-test	Experimental	3.30	3.25	0.05	0.69
Declarative knowledge: pre-test	Control	3.90	3.94	-0.04	
Procedural knowledge: pre-test	Experimental	3.18	3.25	-0.07	0.50
Procedural knowledge: pre-test	Control	3.69	3.75	-0.06	
Conditional knowledge: pre-test	Experimental	3.53	3.40	0.13	0.20
Conditional knowledge: pre-test	Control	3.68	3.60	0.08	
Regulation of cognition: pre-test	Experimental	3.18	3.18	0	0.18
Regulation of cognition: pre-test	Control	3.35	3.36	-0.01	
Planning: pre-test	Experimental	2.95	2.86	0.09	0.50
Planning: pre-test	Control	3.46	3.36	0.1	
Information management: pre-test	Experimental	3.28	3.25	0.03	0.19
Information management: pre-test	Control	3.43	3.44	-0.01	
Monitoring: pre-test	Experimental	2.95	2.86	0.09	0.35
Monitoring: pre-test	Control	3.20	3.21	-0.01	

Metacognitive scale	Group	Mean	Median	Difference between mean and median	Control group's median minus experimental group's median
Debugging: pre-test	Experimental	4.00	4.25	-0.25	-0.25
Debugging: pre-test	Control	3.89	4.00	-0.11	
Evaluation: pre-test	Experimental	3.04	3.00	0.04	-0.08
Evaluation: pre-test	Control	2.94	2.92	0.02	

Table 5.3 indicates that the mean values corresponded closely with the median values for both groups. The absolute values of the difference between mean values and median values for each group respectively were equal to or less than 0.10 for all metacognitive scales, except for *Conditional knowledge* of the experimental group (0.13); *Debugging* of the experimental group (0.25), and *Debugging* of the control group (0.11).

The control group had higher median values on the MAI total score (0.25 higher than the experimental group) and for both main factors of the MAI, namely *KC* and *RC*. For *KC*, the control group's median value was 0.47 higher than that of the experimental group. For *RC*, the median value was 0.18 higher than that of the experimental group.

The control group had a higher median value on all three subscales of *KC*. The biggest difference was in respect of *Declarative knowledge*, namely 0.69. However, the control group's median values were higher for only three of the five subscales under *RC*, namely *Planning*, *Information management*, and *Monitoring*. For these three subscales, the biggest difference was found for *Planning* (0.50). The experimental group performed better on two subscales, namely *Debugging* and *Evaluation*.

5.3.5 The five items with the highest and lowest means in the pre-test (experimental group and control group)

This study determines the level of learners' metacognitive awareness, as indicated primarily by the MAI total score. The purpose of this discussion is to identify items that indicate the highest level of metacognitive awareness in both groups, and items that point to the lowest level of metacognitive awareness in both groups. Although these items are not an indication of the learners' broader level of metacognitive awareness (MAI total score), they provide valuable insight into the level of metacognitive awareness in respect of more detailed aspects of the MAI.

Tables 5.4 to 5.7 present, for each group, the five items with the highest means and the five items with the lowest means in the pre-test. These items are presented in rank-order.

Table 5.4: The five items with the highest means in the pre-test (experimental group)

Subscale	Mean of item	SD	D	N	A	SA	SD + D	A + SA
45. I learn better when I am interested in a specific mathematics topic.								
Declarative knowledge	4.52	0%	0%	12%	24%	64%	0%	88%
50. When I read a mathematics question, I stop and reread any section of the question that is not clear.								
Debugging	4.36	0%	0%	16%	32%	52%	0%	84%
15. I learn best when I already know something about the mathematics topic I am studying.								
Conditional knowledge	4.28	0%	4%	20%	20%	56%	4%	76%
25. I ask other learners for help when I do not understand something in mathematics.								
Debugging	4.08	8%	0%	16%	28%	48%	8%	76%
3. When I solve a mathematics problem, I try to use methods of solving a problem that have worked in the past.								
Procedural knowledge	4.00	0%	8%	16%	40%	32%	8%	72%

Table 5.4 indicates that Item 45 (*Declarative knowledge*) had the highest mean (4.52). Of the responses, 88% were in the combined Agree and Strongly Agree categories. The item with the second highest mean (4.36) was Item 50 (*Debugging*). Of the responses, 84% were in the combined Agree and Strongly Agree categories. The next three items with the highest means were Item 15 (*Conditional knowledge*); Item 25 (*Debugging*), and Item 3 (*Procedural knowledge*), with means of 4.28, 4.08, and 4.00, respectively.

Next, the five items with the lowest means in the pre-test of the experimental group are presented.

Table 5.5: The five items with the lowest means in the pre-test (experimental group)

Subscale	Mean of item	SD	D	N	A	SA	SD + D	A + SA
4. I pace myself when I study for a mathematics test or examination in order to finish studying in time.								
Planning	2.04	32%	48%	4%	16%	0%	80%	16%
31. I create my own examples to make new information I receive in mathematics more meaningful and understandable.								
Information management	2.40	36%	28%	12%	8%	16%	64%	24%
19. After I have solved a mathematics problem, I ask myself if there was an easier way to solve the problem.								
Evaluation	2.48	28%	28%	12%	32%	0%	56%	32%
38. After I have solved a mathematics problem, I ask myself whether I have considered different ways to solve the problem.								
Evaluation	2.52	8%	44%	36%	12%	0%	52%	12%
21. I periodically do revision in order to understand important relationships in mathematics.								
Monitoring	2.52	8%	44%	36%	12%	0%	52%	12%

Table 5.5 presents the distribution of the responses on the Likert scale of the five items that had the lowest means in the pre-test of the experimental group. Item 4 (*Planning*) had the lowest mean (2.04). Of the responses, 80% were in the combined Disagree and Strongly Disagree categories. Item 31 (*Information management*) had the second lowest mean (2.40) and 64% of the responses in the combined Strongly Disagree and Disagree categories. Two items (Items 19 and 38) on the *Evaluation* subscale had the third lowest (2.48) and combined fourth lowest (2.52) means. The mean of Item 21 (*Monitoring*) was also 2.52. Of the responses, 56% for Item 19 were in the combined Strongly Disagree and Disagree categories, while 52% of the responses for Items 38 and 21 were in those two categories.

Table 5.6 presents the five items with the highest means in the pre-test of the control group.

Table 5.6: The five items with the highest means in the pre-test (control group)

Subscale	Mean of item	SD	D	N	A	SA	SD + D	A + SA
45. I learn better when I am interested in a specific mathematics topic.								
Declarative knowledge	4.58	4.17%	0%	4.17%	16.67%	75%	4.17%	91.67%
15. I learn best when I already know something about the mathematics topic I am studying.								
Conditional knowledge	4.50	0%	0%	8.33%	33.33%	58.33%	0%	91.67%
5. I understand my intellectual strengths and weaknesses in mathematics.								
Declarative knowledge	4.46	0%	8.33%	0%	29.17%	62.50%	8.33%	91.67%
25. I ask other learners for help when I do not understand something in mathematics.								
Debugging	4.25	0%	4.17%	8.33%	45.80%	41.70%	4.17%	87.5%
50. When I read a mathematics question, I stop and reread any section of the question that is not clear.								
Debugging	4.21	0%	4.17%	16.67%	33.33%	45.83%	4.17%	79.16%

Item 45 (*Declarative knowledge*) had the highest mean (4.58) in the pre-test of the control group. The percentage of responses in the combined Agree and Strongly Agree categories was 91.67%. An almost similar percentage of responses (91.67%) were obtained in the combined Agree and Strongly Agree categories for the item with the second highest mean, namely Item 15 (*Conditional knowledge*). Item 5 (*Declarative knowledge*) had the third highest mean (4.46) and the percentage of responses in the combined Agree and Strongly Agree categories equalled that of Item 45, namely 91.67%. Two items of the subscale *Debugging* (Items 25 and 50) had the fourth highest and fifth highest means, namely 4.25 and 4.21, respectively.

In the next table, Table 5.7, the five items with the lowest means in the pre-test of the control group are presented.

Table 5.7: The five items with the lowest means in the pre-test (control group)

Subscale	Mean of item	SD	D	N	A	SA	SD + D	A + SA
19. After I have solved a mathematics problem, I ask myself if there was an easier way to solve the problem.								
Evaluation	2.08	25%	45.83%	25%	4.17%	0%	70.83%	4.17%
31. I create my own examples to make new information I receive in mathematics more meaningful and understandable.								
Information management	2.17	29.17%	41.67%	20.83%	0%	8.33%	70.84%	8.33%
38. After I have solved a mathematics problem, I ask myself whether I have considered different ways to solve the problem.								
Evaluation	2.17	16.67%	54.17%	25%	4.17%	0%	70.84%	4.17%
11. I ask myself if I have considered different methods of solving a problem when solving a mathematics problem.								
Monitoring	2.67	8.33%	45.83%	16.67%	29.17%	0%	54.16%	29.17%
35. I know in which situation each problem- solving method I use will be most effective.								
Conditional knowledge	2.79	4.17%	37.50%	41.67%	8.33%	8.33%	41.67%	16.66%

Table 5.7 indicates that Item 19 (*Evaluation*) had the lowest mean (2.08) in the pre-test. Of the responses, 70.83% were in the combined Strongly Disagree and Disagree categories. Item 31 (*Information management*) had the combined second lowest mean (2.17), but there was a higher percentage of responses (70.84%) in the combined Strongly Disagree and Disagree categories than the percentage of responses in Item 19. It is likely that Item 31 had a better mean than Item 19, due to the higher percentage of responses (8.33%) in the Strongly Agree category as opposed to 0% responses in that category for Item 19.

Items 31 and 38 (*Evaluation*) both had a mean of 2.17 and 70.84% of the responses in the combined Strongly Disagree and Disagree categories. However, Item 31 had a higher percentage of responses (29.17%) in the Strongly Disagree category. Item 11 (*Monitoring*) had the fourth lowest mean (2.67) and 54.16% of the responses in the combined Strongly Disagree and Disagree categories. An item of the subscale

Conditional knowledge (Item 35) had the fifth lowest mean (2.79) and 41.67% of the responses were in the combined Strongly Disagree and Disagree categories.

5.3.6 Comparison between the pre-test and the post-test MAI scores (experimental group)

Table 5.8 presents the medians and the difference in medians between the pre-test and post-test medians of the experimental group.

Table 5.8: Median and difference in medians (pre-test and post-test: experimental group)

Metacognitive scale	Median	Difference in medians (post-test median minus pre-test median)
MAI total score: pre-test	3.26	0.30
MAI total score: post-test	3.56	
Knowledge of cognition: pre-test	3.35	0.36
Knowledge of cognition: post-test	3.71	
Declarative knowledge: pre-test	3.25	0.50
Declarative knowledge: post-test	3.75	
Procedural knowledge: pre-test	3.25	0.00
Procedural knowledge: post-test	3.25	
Conditional knowledge: pre-test	3.40	0.40
Conditional knowledge: post-test	3.80	
Regulation of cognition: pre-test	3.18	0.28
Regulation of cognition: post-test	3.46	
Planning: pre-test	2.86	0.57
Planning: post-test	3.43	
Information management: pre-test	3.25	0.19
Information management: post-test	3.44	
Monitoring: pre-test	2.86	0.43
Monitoring: post-test	3.29	

Metacognitive scale	Median	Difference in medians (post-test median minus pre-test median)
Debugging: pre-test	4.25	0.00
Debugging: post-test	4.25	
Evaluation: pre-test	3.00	0.33
Evaluation: post-test	3.33	

Table 5.8 shows that the median values of the MAI total score, factors *KC* and *RC*, and six of the eight subscales improved from the pre-test to the post-test. The improvement in median values of the MAI total score, *KC* and *RC*, was 0.30, 0.36, and 0.28, respectively. The improvement in the median values of six of the eight subscales ranged from 0.19 (*Information management*) to 0.57 (*Planning*). The two subscales that did not experience any changes in their median values were *Procedural knowledge* and *Debugging*.

5.3.7 Comparison between the pre-test and the post-test MAI scores (control group)

In Table 5.9, the medians and the difference in pre-test and post-test medians of the control group are presented.

Table 5.9: Median and difference in medians (pre-test and post-test: control group)

Metacognitive scale	Median	Difference in medians (post-test median minus pre-test median)
Metacognition total score: pre-test	3.51	0.18
Metacognition total score: post-test	3.69	
Knowledge of cognition: pre-test	3.82	0.00
Knowledge of cognition: post-test	3.82	
Declarative knowledge: pre-test	3.94	-0.06
Declarative knowledge: post-test	3.88	

Metacognitive scale	Median	Difference in medians (post-test median minus pre-test median)
Procedural knowledge: pre-test	3.75	0.00
Procedural knowledge: post-test	3.75	
Conditional knowledge: pre-test	3.6	0.20
Conditional knowledge: post-test	3.8	
Regulation of cognition: pre-test	3.36	0.22
Regulation of cognition: post-test	3.58	
Planning: pre-test	3.36	0.28
Planning: post-test	3.64	
Information management: pre-test	3.44	0.12
Information management: post-test	3.56	
Monitoring: pre-test	3.21	0.15
Monitoring: post-test	3.36	
Debugging: pre-test	4	0.00
Debugging: post-test	4	
Evaluation: pre-test	2.92	0.50
Evaluation: post-test	3.42	

Table 5.9 indicates the results of the post-test for the control group. The median of the MAI total score and *RC* improved by 0.18 and 0.22, respectively. However, the median value of *KC* did not change. The medians of the eight subscales show that there was an improvement in five subscales. The improvement in medians of these five subscales ranged from 0.12 (*Information management*) to 0.50 (*Evaluation*). The median values for the subscales *Procedural knowledge* and *Debugging* remained constant, whereas the median value of *Declarative knowledge* was 0.06 lower than in the pre-test.

5.3.8 Comparison between the post-test MAI scores of the experimental group and the control group

Table 5.10 presents the means, medians, and the difference between the post-test medians of the experimental group and the control group.

Table 5.10: Mean, median, and the difference between the medians (post-test: experimental group and control group)

Metacognitive scale	Group	Mean	Median	Difference in medians (control group's median minus experimental group's median
MAI total score: post-test	Experimental	3.49	3.56	0.13
MAI total score: post-test	Control	3.63	3.69	
Knowledge of cognition: post-test	Experimental	3.65	3.71	0.11
Knowledge of cognition: post-test	Control	3.85	3.82	
Declarative knowledge: post-test	Experimental	3.77	3.75	0.13
Declarative knowledge: post-test	Control	3.92	3.88	
Procedural knowledge: post-test	Experimental	3.43	3.25	0.50
Procedural knowledge: post-test	Control	3.69	3.75	
Conditional knowledge: post-test	Experimental	3.64	3.80	0.00
Conditional knowledge: post-test	Control	3.86	3.8	
Regulation of cognition: post-test	Experimental	3.41	3.46	0.12
Regulation of cognition: post-test	Control	3.51	3.58	
Planning: post-test	Experimental	3.37	3.43	0.21
Planning: post-test	Control	3.54	3.64	
Information management: post-test	Experimental	3.49	3.44	0.12
Information management: post-test	Control	3.53	3.56	
Monitoring: post-test	Experimental	3.21	3.29	0.07
Monitoring: post-test	Control	3.35	3.36	
Debugging: post-test	Experimental	4.07	4.25	-0.25
Debugging: post-test	Control	4.05	4	
Evaluation: post-test	Experimental	3.13	3.33	0.09
Evaluation: post-test	Control	3.26	3.42	

Table 5.10 indicates that the control group had higher post-test median values than the experimental group in the following metacognitive scales: the MAI total score, *KC*, *RC*, and six of the eight subscales. The difference in median values ranged from 0.07

(*Monitoring*) to 0.50 (*Procedural knowledge*). There was no difference in the median values of *Conditional knowledge*, whereas the median value of *Debugging* was 0.25 lower in the control group.

Table 5.11 shows the difference in pre-test and post-test medians of the experimental and control group.

Table 5.11: Difference in medians (pre-test and post-test: experimental group and control group)

Metacognitive scale	Difference in pre-test medians (control group's median minus experimental group's median)	Difference in post-test medians (control group's median minus experimental group's median)
MAI total score	0.25	0.13
Knowledge of cognition	0.47	0.11
Declarative knowledge	0.69	0.13
Procedural knowledge	0.50	0.50
Conditional knowledge	0.20	0.00
Regulation of cognition	0.18	0.12
Planning	0.50	0.21
Information management	0.19	0.12
Monitoring	0.35	0.07
Debugging	-0.25	-0.25
Evaluation	-0.08	0.09

Table 5.11 shows that the control group had higher medians than the experimental group for all metacognitive scales in the pre-test, with the exception of *Debugging* and *Evaluation*. In the post-test, the control group also had higher medians than the experimental group for all metacognitive scales, with the exception of *Debugging*.

The differences between the two groups in respect of the medians of the metacognitive scales were smaller in the post-test, with the exception of *Procedural knowledge*, *Debugging*, and *Evaluation*. For *Procedural knowledge* and *Debugging*, the differences in medians between the control group and the experimental group were equal in respect of the pre-test and the post-test, and for *Evaluation* this difference was bigger in the post-test.

5.3.9 Comparison of the rank-order of the experimental group's pre-test MAI and post-test MAI medians

The rank-order of the pre-test and post-test MAI medians of the experimental group is indicated in Table 5.12.

Table 5.12: Rank-order of the pre-test and post-test medians (experimental group)

Experimental group (pre-test)			Experimental group (post-test)		
Rank	**Subscale**	**Median**	**Rank**	**Subscale**	**Median**
1	Debugging	4.25	1	Debugging	4.25
2	Conditional knowledge	3.40	2	Conditional knowledge	3.80
3	Declarative knowledge	3.25	3	Declarative knowledge	3.75
3	Procedural knowledge	3.25	4	Information management	3.44
3	Information management	3.25	5	Planning	3.43
6	Evaluation	3.00	6	Evaluation	3.33
8	Planning	2.86	7	Monitoring	3.29
8	Monitoring	2.86	8	Procedural knowledge	3.25

Table 5.12 indicates that *Debugging* and *Conditional knowledge* had the highest and second highest medians, respectively, in both the pre-test and the post-test. *Declarative knowledge* had the shared third highest median in the pre-test and the third highest median in the post-test. *Procedural knowledge* had the shared third highest median in the pre-test, but the lowest median in the post-test. *Information management* also dropped from the shared third highest median in the pre-test to the fourth highest median in the post-test. *Evaluation*'s median was ranked sixth in both the pre-test and the post-test, whereas *Planning*'s median improved its rank position from the shared seventh position to the fifth position. *Monitoring*'s position remained fairly constant; it had the shared eighth highest median in the pre-test and the seventh highest median in the post-test.

5.3.10 Comparison of the rank-order of the control group's pre-test MAI and post-test MAI medians

Table 5.13 presents the rank-order of the pre-test and post-test MAI medians of the control group.

Table 5.13: Rank-order of the pre-test and post-test medians (control group)

Control group (pre-test)			Control group (post-test)		
Rank	Subscale	Median	Rank	Subscale	Median
1	Debugging	4.00	1	Debugging	4.00
2	Declarative knowledge	3.94	2	Declarative knowledge	3.88
3	Procedural knowledge	3.75	3	Conditional knowledge	3.80
4	Conditional knowledge	3.60	4	Procedural knowledge	3.75
5	Information management	3.44	5	Planning	3.64
6	Planning	3.36	6	Information management	3.56
7	Monitoring	3.21	7	Evaluation	3.42
8	Evaluation	2.92	8	Monitoring	3.36

Table 5.13 shows that *Debugging* had the highest median in both the pre-test and the post-test of the control group. The three subscales of *KC* had the second, third and fourth highest medians in the pre-test and the post-test. However, *Procedural knowledge* and *Conditional knowledge* switched places. *Information management* had the fifth highest median in the pre-test and the sixth highest median in the post-test, whereas *Planning* had the sixth highest median in the pre-test and the fifth highest median in the post-test. *Monitoring* and *Evaluation* had the lowest medians in both the pre-test and the post-test but *Evaluation* showed an improvement from the lowest median to the second lowest median, whereas *Monitoring* dropped from the second lowest median to the lowest median.

The five items with the highest and lowest means in the post-test of the experimental group and the control group are discussed in the next section.

5.3.11 The five items with the highest and lowest means in the post-test (experimental group and control group)

Tables 5.14 to 5.17 present, for each group, the five items with the highest means and the five items with the lowest means in the post-test MAI. These items are presented in rank-order.

Table 5.14 indicates the five items with the highest means in the post-test of the experimental group.

Table 5.14: The five items with the highest means in the post-test (experimental group)

Subscale	Mean of item	SD	D	N	A	SA	SD + D	A + SA
45. I learn better when I am interested in a specific mathematics topic.								
Declarative knowledge	4.68	0%	0%	8%	16%	76%	0%	92%
50. When I read a mathematics question, I stop and reread any section of the question that is not clear.								
Debugging	4.52	0%	0%	8%	32%	60%	0%	92%
15. I learn best when I already know something about the mathematics topic I am studying.								
Conditional knowledge	4.44	0%	0%	4%	48%	48%	0%	96%
25. I ask other learners for help when I do not understand something in mathematics.								
Debugging	4.32	0%	4%	4%	48%	44%	4%	92%
9. I read slower when I encounter important information in a mathematics question.								
Information management	4.28	0%	0%	12%	48%	40%	0%	88%

Table 5.14 shows that the item with the highest mean (4.68) in the post-test of the experimental group (Item 45) is part of the subscale *Declarative knowledge*. Of the responses, 92% were obtained in the combined Agree and Strongly Agree categories. The same percentage of responses in these categories was obtained for Items 50 and 25 (*Debugging*), although they had the second highest mean (4.52) and fourth highest mean (4.32), respectively. The highest percentage of responses (96%) in the combined

Agree and Strongly Agree categories was obtained in the item with the third highest mean, Item 15 (*Conditional knowledge*). Item 9 (*Information management*) had the fifth highest mean (4.28) and 88% of the responses were in the combined Agree and Strongly Agree categories.

Table 5.15 shows the five items with the lowest means in the post-test of the experimental group.

Table 5.15: The five items with the lowest means in the post-test (experimental group)

Subscale	Mean of item	SD	D	N	A	SA	SD + D	A + SA
21. I periodically do revision in order to understand important relationships in mathematics.								
Monitoring	2.52	16%	32%	40%	8%	4%	48%	12%
38. After I have solved a mathematics problem, I ask myself whether I have considered different ways to solve the problem.								
Evaluation	2.52	12%	36%	40%	12%	0%	48%	12%
19. After I have solved a mathematics problem, I ask myself if there was an easier way to solve the problem.								
Evaluation	2.56	24%	16%	44%	12%	4%	40%	16%
4. I pace myself when I study for a mathematics test or examination in order to finish studying in time.								
Planning	2.72	16%	40%	8%	28%	8%	56%	36%
31. I create my own examples to make new information I receive in mathematics more meaningful and understandable.								
Information management	2.72	12%	32%	36%	12%	8%	44%	20%

Table 5.15 indicates that Item 21 (*Monitoring*) and Item 38 (*Evaluation*) had the lowest means (2.52) in the post-test of the experimental group. They also had the same percentage of responses in the Strongly Disagree and Disagree categories. Item 19 (*Evaluation*) had the third lowest mean (2.56). Item 4 (*Planning*) and Item 31 (*Information management*) had the fifth lowest means (2.72). However, Item 4's

percentage of responses (56%) in the combined Strongly Disagree and Disagree categories was higher than that of Item 31 (44%).

Table 5.16 presents the five items with the highest means in the post-test of the control group.

Table 5.16: The five items with the highest means in the post-test (control group)

Subscale	Mean of item	SD	D	N	A	SA	SD + D	A + SA
45. I learn better when I am interested in a specific mathematics topic.								
Declarative knowledge	4.58	4.17%	0%	0%	25%	70.83%	4.17%	95.83%
15. I learn best when I already know something about the mathematics topic I am studying.								
Conditional knowledge	4.50	0%	0%	4.17%	41.67%	54.17%	0%	95.84%
50. When I read a mathematics question, I stop and reread any section of the question that is not clear.								
Debugging	4.33	0%	4.17%	12.50%	29.17%	54.17%	4.17%	83.34%
3. When I solve a mathematics problem, I try to use methods of solving a problem that have worked in the past.								
Procedural knowledge	4.29	0%	0%	8.33%	54.17%	37.50%	0%	91.67%
29. I use my strengths in mathematics to compensate for my weaknesses in mathematics.								
Conditional knowledge	4.29	0%	0%	8.33%	54.17%	37.50%	0%	91.67%

Table 5.16 shows that an item of the subscale *Declarative knowledge* (Item 45) had the highest mean (4.58) in the post-test of the control group. For this item, 95.83% of the responses were in the combined Agree and Strongly Agree categories. Although Item 15 (*Conditional knowledge*) had the second highest mean (4.50), it had a higher percentage of responses (95.84%) in the combined Agree and Strongly Agree categories. Item 50 had the third highest mean (4.33) and 83.34% of responses in the combined Agree and Strongly Agree categories. Item 3 (*Procedural knowledge*) and Item 29 (*Conditional knowledge*) both had a mean of 4.29

Table 5.17 presents the five items with the lowest means in the post-test of the control group.

Table 5.17: The five items with the lowest means in the post-test (control group)

Subscale	Mean of item	SD	D	N	A	SA	SD + D	A + SA
31. I create my own examples to make new information I receive in mathematics more meaningful and understandable.								
Information management	2.71	20.83%	37.50%	8.33%	16.67%	16.67%	58.33%	33.34%
19. After I have solved a mathematics problem, I ask myself if there was an easier way to solve the problem.								
Evaluation	2.71	12.50%	29.17%	33.33%	25%	0%	41.67%	25%
38. After I have solved a mathematics problem, I ask myself whether I have considered different ways to solve the problem.								
Evaluation	2.92	4.17%	29.17%	41.67%	20.83%	4.17%	33.34%	25%
7. I know how well I did once I finish a mathematics test or examination.								
Evaluation	3.00	8.33%	20.83%	37.50%	29.17%	4.17%	29.16%	33.34%
37. I draw pictures or diagrams in order to understand while I am learning mathematics.								
Information management	3.04	16.67%	12.50%	29.17%	33.33%	8.33%	29.17%	41.66%

Table 5.17 indicates that the items with the lowest means (2.71) in the post-test of the control group were Item 31 (*Information management*) and Item 19 (*Evaluation*). Item 31, however, had a higher percentage of responses (58.33%) in the combined Strongly Disagree and Disagree categories compared to Item 19's percentage of 41.67%. Items 38 and 7, part of the *Evaluation* subscale, had the third lowest and fourth lowest means, namely 2.92 and 3.00, respectively. Item 37 (*Information management*) had the fifth lowest mean (3.04).

A summary of the five items with the highest and lowest means in the pre-test and the post-test is provided next.

5.3.12 Summary of the five items with the highest and lowest means in the pre-test MAI and the post-test MAI (experimental group and control group)

Table 5.18 provides, for both groups, a summary of the five items with the highest means and the five items with the lowest means in the pre-test and the post-test.

Table 5.18: The five items with the highest and lowest means in the pre-test and the post-test (experimental group and control group)

Group	Item number and mean of the five items with the highest means	Item number and mean of the five items with the lowest means
Experimental (pre-test)	**45 (4.52); 50 (4.36)**; **15 (4.28)**; 25 (4.08); 3 (4.00)	4 (2.04); **31 (2.40)**; **19 (2.48); 38 (2.52)**; 21 (2.52)
Experimental (post-test)	**45 (4.68); 50 (4.52)**; **15 (4.44)**; 25 (4.32); 9 (4.28)	21 (2.52); **38 (2.52); 19 (2.56)**; 4 (2.72); **31 (2.72)**
Control (pre-test)	**45 (4.58)**; **15 (4.50)**; 5 (4.46); 25 (4.25); **50 (4.21)**	**19 (2.08); 31 (2.17); 38 (2.17)**; 11 (2.67); 35 (2.79)
Control (post-test)	**45 (4.58)**; **15 (4.50)**; **50 (4.33)**; 3 (4.29); 29 (4.29)	**31 (2.71); 19 (2.71); 38 (2.92)**; 7 (3.00); 37 (3.04).

Table 5.18 indicates that Items 15, 45, and 50 featured in both groups for both the pre-test and the post-test in the highest mean category. Item 45 had the highest mean for both groups in both tests. Item 50 had the second highest mean in both tests for the experimental group, but the fifth highest and third highest means for the control group in the pre-test and the post-test, respectively. The means of these items improved from the pre-test to the post-test, with the exception of Items 45 and 15 in the control group whose means remained the same.

Except for the control group (pre-test), Item 25 featured three times in the highest means category. The mean of Item 25 improved from 4.08 to 4.25. Item 3 featured twice, namely for the experimental group (pre-test) and for the control group (post-test). Item 5 was among the five items with the highest means for the control group (pre-test), and Item 9 for the experimental group (post-test).

In the lowest means category, Items 19, 31, and 38 featured in both groups for both tests. With the exception of Item 38 (experimental group), the means of these items improved from the pre-test to the post-test. Items 4 and 21 were part of the five items with the lowest means in the experimental group for both tests. Item 4's means was higher in the post-test, whereas Item 21's pre-test and post-test means were equal. Items 11 and 35 (control group, pre-test) and Items 7 and 37 (control group, post-test) only featured once.

In summary, the first part of this section (see 5.3.1-5.3.12) addressed aspects related to the MAI pre-test and post-test. In the next part of Section 5.3, aspects that relate to the relationship between learner metacognition and mathematics achievement are presented and analysed (see 5.3.13-5.3.14).

5.3.13 Mathematics achievement (experimental group and control group)

Table 5.19 presents the mathematics achievement of the experimental group for Terms 1 and 4.

Table 5.19: Mathematics report marks (experimental group)

Learner number (N=25)	Term 1 report mark (100)	Term 4 report mark (400)	Term 4 report mark (%)
1	65	200	50%
2	84	340	85%
3	48	136	34%
4	59	212	53%
5	48	188	47%
6	72	308	77%
8	56	168	42%
9	74	272	68%
10	87	308	77%
11	67	172	43%
12	70	252	63%
13	59	148	37%
14	72	172	43%
15	67	220	55%
16	66	292	73%
17	68	292	73%

Learner number (N=25)	Term 1 report mark (100)	Term 4 report mark (400)	Term 4 report mark (%)
18	74	220	55%
19	67	192	48%
20	70	224	56%
21	71	256	64%
22	56	224	56%
23	46	152	38%
24	63	212	53%
25	71	288	72%
26	70	248	62%
Mean	**66%**		**56.96%**

Table 5.19 indicates that the report marks of Term 1 represent the learners' academic results prior to the start of the intervention, since the intervention started at the beginning of Term 2. The intervention ended at the end of Term 3; thus, the Term 4 report marks represent the learners' academic achievement one term following the intervention.

The Term 1 report mark consisted of the following assessment activities:

- one formal test of 50 marks (weight 66.67%);
- four class tests of 10 marks each (weight 16.67%), and
- one assignment of 100 marks (weight 16.67%).

Term 4's report mark consisted of a continuous assessment (CASS) mark of 100 (weight 25%) and two formal examinations of 150 marks each (weight 75%). The CASS mark was compiled by the combined marks of all formal tests and examinations written during the year (weight 75%). All class tests written during the year, an assignment and an investigation all contributed to 25% of the CASS mark. Therefore, the formal component of the Term 4 report mark had a weight of 75% (examinations) plus 18.75% (75% of the CASS mark) which equals 93.75%. The mean of the Term 1 report mark was 66% and the mean of the Term 4 report mark was 56.96%.

Table 5.20 presents the report marks of the control group.

Table 5.20: Mathematics report marks (control group)

Learner number (N=24)	Term 1 report mark (100)	Term 4 report mark (400)	Term 4 report mark (%)
1	70	304	76%
2	47	248	62%
3	73	308	77%
4	67	204	51%
5	52	240	60%
6	69	324	81%
7	66	324	81%
8	73	328	82%
9	75	324	81%
10	52	220	55%
12	75	292	73%
13	85	344	86%
14	54	256	64%
15	72	328	82%
17	68	284	71%
18	43	160	40%
19	57	188	47%
22	49	272	68%
24	64	212	53%
25	63	216	54%
26	55	252	63%
27	90	356	89%
28	69	328	82%
29	83	324	81%
Mean	**65.46%**		**69.13%**

Term 1's report mark consisted of:

- three formal tests of 40 marks, 45 marks, and 40 marks, respectively (weight 70%);

- three class tests of 10 marks each and one class test of 20 marks (weight 15%); an assignment of 50 marks (weight 10%), and
- four homework evaluations of 20 marks each (weight 5%).

The Term 4 report mark consisted of CASS of 100 marks (weight 25%) and the examination (75%). The CASS mark consisted of formal tests and examinations written during the year (weight 75%); class tests, an assignment, an investigation and homework evaluations made up the remaining weight of 25%. Similar to the experimental group, the formal component of the Term 4 report mark had a weight of 93.75%. The mean of Term 4's report mark (69.13%) was 3.67% higher than that of Term 1's report mark (65.46%).

5.3.14 Correlation between learner metacognition and mathematics achievement (experimental group and control group)

In this section, Tables 5.21 to 5.24 present the MAI rank scores and the report mark rank scores for the experimental group pre-test (Table 5.21); the experimental group's post-test (Table 5.22); the control group's pre-test (Table 5.23), and the control group's post-test (Table 5.24).

These tables contain the following information: the learner number according to the MAI pre-test; the ranks of the learners' total MAI rank score; their *KC* rank score; their *RC* rank score, and their report mark rank score. In each of the second, third and fourth columns, the metacognitive rank score that corresponds most closely with the rank of the Term 1 report mark is highlighted. The last column represents the difference score which is the absolute value of the difference between the highlighted metacognitive rank score and the report mark rank score. The data are tabulated in ascending order according to the learners' rank on the total MAI scores.

Table 5.21: Correlation between learner metacognition and the mathematics Term 1 report mark (pre-test: experimental group)

Learner number (N=25)	Metacognitive scale			Term 1 report mark (rank score)	Difference score
	MAI total (rank score)	Knowledge of cognition (rank score)	Regulation of cognition (rank score)		
25	1	1	2	**7**	5
13	2	6	1	**19**	13
2	3	2	9	**2**	0
16	3	3	4	**16**	12
23	5	5	8	**25**	17
19	6	7	4	**13**	6
24	6	16	3	**18**	2
20	8	4	13	**9**	1
18	9	10	9	**3**	6
21	9	7	16	**7**	0
26	9	10	9	**9**	0
6	12	15	4	**5**	1
14	13	12	13	**5**	7
11	14	13	13	**13**	0
12	14	16	4	**9**	5
8	16	19	9	**21**	2
17	16	9	20	**12**	3
10	18	14	19	**1**	13
15	19	16	17	**13**	3
1	20	20	21	**17**	3
5	21	21	22	**23**	1
3	22	25	18	**23**	1
9	23	22	23	**3**	19
4	25	23	25	**19**	4
22	25	24	24	**21**	3

Table 5.21 indicates that the MAI total rank score only corresponded perfectly with the *KC* and *RC* rank scores in respect of Learner 22 who had a MAI total rank score of 25. There was a perfect correlation between one of the metacognitive scales rank scores and the Term 1 report mark in respect of *KC* (Learner 21); *RC* (Learner 26), and *KC* and *RC* (Learner 11). The largest difference score was 19 (Learner 9).

Table 5.22: Correlation between learner metacognition and the mathematics Term 4 report mark (post-test: experimental group)

Learner number according to pre-test (N=25)	Metacognitive scale			Term 4 report mark (rank score)	Difference score
	Total MAI (rank score)	Knowledge of cognition (rank score)	Regulation of cognition (rank score)		
6	1	1	1	2	1
23	2	1	2	23	21
18	3	8	3	13	5
21	4	4	6	8	2
13	5	12	4	24	12
16	5	3	8	4	1
11	7	4	10	20	10
24	7	12	5	15	3
10	9	9	11	2	7
8	10	4	12	22	10
2	11	4	14	1	3
12	12	18	8	9	1
19	12	11	12	18	6
3	14	21	6	25	4
20	15	14	14	11	3
1	16	15	14	17	1
25	17	10	18	6	4
15	18	15	17	13	2
14	19	17	19	20	1
22	20	21	21	11	9

Learner number according to pre-test (N=25)	Metacognitive scale: Total MAI (rank score)	Knowledge of cognition (rank score)	Regulation of cognition (rank score)	Term 4 report mark (rank score)	Difference score
5	21	24	20	19	1
26	22	20	22	10	10
17	23	19	24	4	15
9	24	25	22	7	15
4	25	23	25	15	8

Table 5.22 shows that there was only one instance where the MAI total, *KC* and *RC* rank scores corresponded perfectly (Learner 6). There was also no difference score of 0. The lowest difference score was 1 (Learners 1, 5, 6, 12, 14 and 16) and the highest difference score was 21 (Learner 23).

Table 5.23: Correlation between learner metacognition and the mathematics Term 1 report mark (pre-test: control group)

Learner number (N=24)	Metacognitive scale: Total MAI (rank score)	Knowledge of cognition (rank score)	Regulation of cognition (rank score)	Term 1 report mark (rank score)	Difference score
3	1	1	1	**6**	5
5	2	3	3	**20**	17
13	2	6	2	**2**	0
17	4	1	6	**12**	6
4	5	4	4	**13**	8
1	6	6	5	**9**	3
14	7	6	6	**19**	12
12	8	5	11	**4**	1
24	9	6	10	**15**	5
26	10	16	8	**18**	2
8	11	18	9	**6**	3

Learner number (N=24)	Metacognitive scale			Term 1 report mark (rank score)	Difference score
	Total MAI (rank score)	Knowledge of cognition (rank score)	Regulation of cognition (rank score)		
19	12	13	12	**17**	5
9	13	6	14	**4**	2
25	14	12	14	**16**	2
15	15	16	13	**8**	5
29	16	11	18	**3**	8
27	17	14	17	**1**	13
6	18	14	19	**10**	4
7	19	21	14	**14**	0
28	20	19	22	**10**	9
22	21	21	21	**22**	1
2	22	23	20	**23**	0
18	23	20	23	**24**	1
10	24	24	23	**20**	3

Table 5.23 indicates that the MAI total, *KC* and *RC* rank scores correlated perfectly in respect of Learners 3 and 22. Difference scores of 0 occurred in respect of *KC* (Learner 2); RC (Learner 7), and for both the MAI total and *RC* rank scores (Learner 13). The highest difference score was 17 (Learner 5).

Table 5.24 presents the correlation between learner metacognition (post-test) and the Term 4 report mark for the control group.

Table 5.24: Correlation between learner metacognition and the mathematics Term 4 report mark (post-test: control group)

Learner number (N=24)	Metacognitive scale			Term 4 report mark (rank score)	Difference score
	MAI total (rank score)	Knowledge of cognition (rank score)	Regulation of cognition (rank score)		
3	1	1	1	10	9
17	2	2	2	13	11
13	3	4	4	2	1
4	4	10	3	22	12
15	5	5	5	3	2
5	6	8	7	18	10
12	7	3	13	12	1
1	8	6	11	11	0
26	9	9	10	16	6
8	10	16	8	3	5
14	11	19	6	15	4
7	12	17	9	6	3
19	13	12	12	23	10
22	14	10	14	14	0
6	15	6	18	6	0
27	16	12	20	1	11
24	17	20	16	21	1
28	17	21	15	3	12
25	19	12	22	20	1
2	20	22	19	17	2
9	21	12	23	6	6
29	21	17	21	6	13
10	23	24	16	19	3
18	24	23	24	24	0

Table 5.24 indicates that the MAI total, *KC* and *RC* rank scores corresponded exactly in the case of Learners 3, 15, and 17. In addition, there were exact correlations between one of the metacognitive scales' rank-order and the Term 4 report marks in respect of Learners 1, 6, 18, and 22. The highest difference score was 13 (Learner 29).

Based on Tables 5.21 to 5.24, the following tables (Tables 5.25 to 5.28) present four aspects. First, the rank scores of the applicable metacognitive scales. Secondly, the difference scores of each metacognitive scale are indicated and difference scores that have values of less than three are highlighted, because they represent a high correlation between the metacognitive scale rank and the report mark rank. Thirdly, the frequency of difference scores that have values less than three is indicated. Lastly, the total frequency of all difference scores is indicated.

The difference scores of the experimental group's pre-test are represented in Table 5.25.

Table 5.25: Difference scores (pre-test: experimental group)

Metacognitive scale rank score	Difference scores (pre-test and Term 1 report mark)	Frequency of difference scores with values less than 3	Total frequency of difference scores per metacognitive scale (N=25)
MAI total rank score	**1; 1**	2	2
KC rank score	**0; 0; 2; 2;** 3; 3; 4; 6; 7; 13; 13; 19.	4	12
RC rank score	**0; 1; 1**; 5; 12; 17	3	6
MAI total rank score and KC rank score	3	0	1
MAI total rank score and RC rank score	5; 6	0	2
KC rank score and RC rank score	**0**	1	1
MAI total rank score, KC rank score and RC rank score	3	0	1

A consideration of the difference scores shows that *KC* rank score had a difference score with values smaller than three in most cases, namely in four instances. The *RC* rank score had the second highest number of difference scores less than three, namely, in three instances.

The total frequency of difference scores indicates that, in 12 instances, the *KC* rank score was most closely associated with the Term 1 report mark. The *RC* rank score was most closely associated with the Term 1 report mark in six instances. The number of instances where each of the other metacognitive rank scores corresponded most closely with the Term 1 report mark occurred at the most only twice.

Table 5.26 presents the difference scores of the experimental group's post-test.

Table 5.26: Difference scores (post-test: experimental group)

Metacognitive scale rank score	Difference scores (post-test and Term 4 report mark)	Frequency of difference scores with values less than 3	Total frequency of difference scores (N=25)
MAI total rank score	**1;** 9.	1	2
KC rank score	**1; 2;** 3; 3; 4; 4; 5; 8; 10; 12; 15.	2	11
RC rank score	**1; 1; 2**; 10; 10; 15.	3	6
MAI total rank score and KC rank score	7	0	1
MAI total rank score and RC rank score	**1**; 6; 21.	1	3
KC rank score and RC rank score	3	0	1
MAI total rank score, KC rank score and RC rank score	**1**	1	1

Table 5.26 indicates that the highest frequency of difference scores with values less than three was obtained in the *RC* rank score category, namely in three instances. The second highest frequency of difference scores was evident in the *KC* rank score category, namely in two instances. The *KC* rank scores and *RC* rank scores, respectively, also had the highest and second highest total frequency of difference scores, namely 11 (*KC*) and six (*RC*). The total frequency of difference scores for the other metacognitive rank scores varied between one and three.

The difference scores of the control group's pre-test is presented in Table 5.27.

Table 5.27: Difference scores (pre-test: control group)

Metacognitive rank score category	Difference scores (pre-test and Term 1 report mark)	Frequency of difference scores with values less than 3	Total frequency of difference scores (N=24)
MAI total rank score	8; 12	0	2
KC rank score	**0; 1; 1; 2; 2**; 4; 8; 9; 13	5	9
RC rank score	**0;** 3; 3; 5; 5; 6	1	6
MAI total rank score and KC rank score	3	0	1
MAI total rank score and RC rank score	**0; 1; 2**; 5	3	4
KC rank score and RC rank score	17	0	1
MAI total rank score, KC rank score and RC rank score	5	0	1

Table 5.27 shows that the *KC* rank score category had the highest frequency of difference scores with values less than three. In five instances, the difference score was less than three, whereas the *RC* rank score category and the combined MAI total rank score and *RC* rank score category had the second highest frequency in three instances. The *KC* rank score category also had the highest total frequency of difference scores, namely in nine instances. The *RC* rank score category, with six instances, had the second highest total frequency of difference scores.

Table 5.28 presents the difference scores for the control group's post-test.

Table 5.28: Difference scores (post-test: control group)

Metacognitive scale rank score	Difference scores(post-test and Term 4 report mark)	Frequency of difference scores with values less than 3	Total frequency of difference scores(N=24)
MAI total rank score	**1; 1;** 10	2	3
KC rank score	**0; 1**; 6; 10; 11; 12; 13	2	7
RC rank score	**0; 1; 2;** 3; 3; 5; 6; 12	3	8
MAI total rank score and KC rank score	4	0	1
MAI total rank score and RC rank score	**0; 0**	2	2
KC rank score and RC rank score	0	0	0
MAI total rank score, KC rank score and RC rank score	**2**; 9; 11	1	3

Table 5.28 indicates that the *RC* rank score category had the highest frequency of difference scores with values less than three, namely in three instances. It also had the highest frequency of difference scores, namely in eight instances. Three categories had the second highest frequency of difference scores with values less than three: MAI total rank score; *KC* rank score, and the combined MAI total rank score and the *RC* rank score. The *KC* rank score category, however, also had the second highest frequency of difference scores, namely in seven instances.

Table 5.29 summarises the results contained in Tables 5.21 to 5.28. It indicates two aspects related to the difference scores. First, the metacognitive scale with the highest frequency of difference scores with values less than three and frequency indicated in columns three and four, respectively. Secondly, the metacognitive scale with the highest total frequency of difference scores is indicated in column 5, and the corresponding highest total frequency of difference scores is indicated in column 6. These two aspects are presented in respect of the correlation between academic achievement and learner metacognition for each group in Term 1 and Term 4, respectively.

Table 5.29: Summarised difference scores

Group	Pre-test or post-test x report mark	Metacognitive scale rank score category with the highest frequency of difference scores with values less than 3	Highest frequency of difference scores with values less than three	Metacognitve scale rank score category with the highest frequency of difference scores	Highest total frequency of difference scores
Experimental	Pre-test x Term 1 report mark	KC	4	KC	12
Experimental	Post-test x Term 4 report mark	RC	3	KC	11
Control	Pre-test x Term 1 report mark	KC	5	KC	9
Control	Post-test x Term 4 report mark	RC	3	RC	8

Table 5.29 indicates that the *KC* rank score category had the highest frequency of difference scores with values less than three for both the experimental group and the control group in Term 1. In Term 4, the *RC* rank score category had the highest frequency of difference scores with values less than three for the experimental group and the control group.

The *KC* rank score category had the highest total frequency of difference scores for the experimental group in both Term 1 (12) and Term 4 (11), and for the control group in Term 1 (9). In Term 4, the *RC* rank score category had the highest total frequency of difference scores for the control group (8).

In summary, this section (see 5.3) presented and analysed the quantitative data collected in this study by means of descriptive statistical procedures. In the next section, inferential statistical procedures are used to determine the statistical significance of the improvement in learner metacognition and the correlation between learner metacognition and mathematics achievement.

5.4 INFERENTIAL STATISTICS

Inferential statistics are statistical procedures that determine the probability that phenomena observed in a sample are likely to be observed in the larger population from which the sample was drawn (Ary *et al.,* 2010: 101). In the first part of this section (see 5.4.1-5.4.4), the statistical significance of the differences in medians of the MAI total scores are presented and analysed in respect of:

- the pre-test MAI total scores of the experimental group and the control group;
- the post-test MAI total scores of the experimental group and the control group;
- the pre-test and post-test MAI total scores of the experimental group, and
- the pre-test and post-test MAI total scores of the control group.

In the second part of this section (see 5.4.5), the correlation between learner metacognition and mathematics achievement is presented and analysed.

5.4.1 Differences between the pre-test MAI scores of the experimental group and the control group

Table 5.30 presents the Mann-Whitney comparison between the pre-test scores of the experimental group and the control group.

Table 5.30: Mann-Whitney comparison between the experimental group and the control group on pre-test scores

Metacognitive scale	Pre-test median (experimental group)	Pre-test median (control group)	p-value
MAI total score: pre-test	3.26	3.51	0.0198*
Knowledge of cognition: pre-test	3.35	3.82	0.0043*
Declarative knowledge: pre-test	3.25	3.94	0.0006*
Procedural knowledge: pre-test	3.25	3.75	0.0303*
Conditional knowledge: pre-test	3.40	3.60	0.4146

Metacognitive scale	Pre-test median (experimental group)	Pre-test median (control group)	p-value
Regulation of cognition: pre-test	3.18	3.36	0.0942
Planning: pre-test	2.86	3.36	0.0041*
Information management: pre-test	3.25	3.44	0.3720
Monitoring: pre-test	2.86	3.21	0.1239
Debugging: pre-test	4.25	4.00	0.4242
Evaluation: pre-test	3.00	2.92	0.6077

* $p < 0.05$

Table 5.30 indicates that there were statistically significant differences between the pre-test medians of the experimental group and the control group in respect of the following metacognitive scales:

- MAI total score ($p = 0.0198$);
- *KC* ($p = 0.0043$);
- *Declarative knowledge* ($p = 0.0006$);
- *Procedural knowledge* ($p = 0.0303$), and
- *Planning* ($p = 0.0041$).

For all these metacognitive scales the control group performed significantly better than the experimental group on the MAI pre-test.

5.4.2 Differences between the post-test MAI scores of the experimental group and the control group

The Mann-Whitney comparison between the post-test scores of the experimental group and the control group is shown in Table 5.31.

Table 5.31: Mann-Whitney comparison between the experimental group and the control group on post-test scores

Metacognitive scale	Post-test median (experimental group)	Post-test median (control group)	p-value
MAI total score: post-test	3.56	3.69	0.2981
Knowledge of cognition: post-test	3.71	3.82	0.1763
Declarative knowledge: post-test	3.75	3.88	0.3242
Procedural knowledge: post-test	3.25	3.75	0.1056
Conditional knowledge: post-test	3.80	3.80	0.2331
Regulation of cognition: post-test	3.46	3.58	0.4530
Planning: post-test	3.43	3.64	0.3203
Information management: post-test	3.44	3.56	0.9201
Monitoring: post-test	3.29	3.36	0.3502
Debugging: post-test	4.25	4.00	0.9517
Evaluation: post-test	3.33	3.42	0.3690

Table 5.31 shows that none of the p-values were less than 0.05 and, therefore, there were no statistically significant differences between the experimental group and the control group in respect of the post-test medians of the metacognitive scales.

5.4.3 Differences between the pre-test and the post-test MAI scores of the experimental group

Table 5.32 presents the Wilcoxon comparison between the pre-test and post-test scores of the experimental group.

Table 5.32: Wilcoxon comparison between the experimental group's pre-test and post-test scores

Metacognitive scale	Median	p-value
Metacognition total score: pre-test	3.26	0.002*
Metacognition total score: post-test	3.56	
Knowledge of cognition: pre-test	3.35	0.001*
Knowledge of cognition: post-test	3.71	
Declarative knowledge: pre-test	3.25	<0.001*
Declarative knowledge: post-test	3.75	
Procedural knowledge: pre-test	3.25	0.13
Procedural knowledge: post-test	3.25	
Conditional knowledge: pre-test	3.40	0.397
Conditional knowledge: post-test	3.80	
Regulation of cognition: pre-test	3.18	0.005*
Regulation of cognition: post-test	3.46	
Planning: pre-test	2.86	0.014*
Planning: post-test	3.43	
IMS: pre-test	3.25	0.101
IMS: post-test	3.44	
Monitoring: pre-test	2.86	0.014*
Monitoring: post-test	3.29	
Debugging strategies: pre-test	4.25	0.533
Debugging strategies: post-test	4.25	
Evaluation: pre-test	3.00	0.379
Evaluation: post-test	3.33	

*p < 0.05

Table 5.32 shows that there were statistically significant differences between the pre-test and the post-test medians of the following metacognitive scales:

- MAI total score (p = 0.002);

- *KC* (0.001);
- *RC* (0.005);
- *Declarative knowledge* (p < 0.001);
- *Planning* (0.014), and
- *Monitoring* (0.014).

5.4.4 Differences between the pre-test and the post-test MAI scores of the control group

The Wilcoxon comparison between the control group's pre-test and post-test scores is indicated in Table 5.33.

Table 5.33: Wilcoxon comparison between the control group's pre-test and post-test scores

Metacognitive scale	Median	p-value
MAI total score: pre-test	3.51	0.076
MAI total score: post-test	3.69	
Knowledge of cognition: pre-test	3.82	0.189
Knowledge of cognition: post-test	3.82	
Declarative knowledge: pre-test	3.94	0.576
Declarative knowledge: post-test	3.88	
Procedural knowledge: pre-test	3.75	0.288
Procedural knowledge: post-test	3.75	
Conditional knowledge: pre-test	3.6	0.038*
Conditional knowledge: post-test	3.8	
Regulation of cognition: pre-test	3.36	0.057
Regulation of cognition: post-test	3.58	
Planning: pre-test	3.36	0.219
Planning: post-test	3.64	
Information management: pre-test	3.44	0.348
Information management: post-test	3.56	

Metacognitive scale	Median	p-value
Monitoring: pre-test	3.21	0.116
Monitoring: post-test	3.36	
Debugging: pre-test	4	0.196
Debugging: post-test	4	
Evaluation: pre-test	2.92	0.004*
Evaluation: post-test	3.42	

*p < 0.05

Table 5.33 indicates that two subscales experienced a statistically significant improvement in pre-test medians, namely *Conditional knowledge* (p = 0.038) and *Evaluation* (p = 0.004). *RC* had a p-value of 0.057 which indicates that its improvement was almost statistically significant.

5.4.5 Correlation between learner metacognition and mathematics achievement (experimental group and control group)

Table 5.34 presents the Spearman *rho* correlations between learner metacognition and mathematics achievement of the experimental group and the control group respectively.

Table 5.34: Spearman *rho* correlations between learner metacognition and mathematics achievement (experimental group and control group)

Metacognitive scale	Experimental group (N=25		Control group (N=24)	
	Spearman *rho* value (Term 1 report mark x pre-test)	**Spearman *rho* value** (Term 4 report mark x post-test)	**Spearman *rho* value** (Term 1 report mark x pre-test)	**Spearman *rho* value** (Term 4 report mark x post-test)
MAI total score	0.20	-0.07	0.33	0.21
Knowledge of cognition	0.35	0.11	0.39	0.29
Declarative knowledge	0.42*	0.13	0.38	0.29

Metacognitive scale	Experimental group (N=25		Control group (N=24)	
	Spearman *rho* value (Term 1 report mark x pre-test)	**Spearman *rho* value** (Term 4 report mark x post-test)	**Spearman *rho* value** (Term 1 report mark x pre-test)	**Spearman *rho* value** (Term 4 report mark x post-test)
Procedural knowledge	0.16	-0.01	0.29	0.41*
Conditional knowledge	0.21	-0.02	0.34	0.17
Regulation of cognition	0.08	-0.20	0.28	0.13
Planning	0.04	-0.23	0.20	0.21
Information management	0.26	0.07	0.18	0.01
Monitoring	-0.30	-0.43	0.23	-0.01
Debugging	0.08	0.22	0.13	0.09
Evaluation	0.09	-0.20	0.49*	0.38

* $p < 0.05$

The Spearman *rho* coefficient values, indicated in Table 5.34, were positive in the majority of instances, except in the following: *Monitoring* (Term 1, experimental group); MAI total score, *Procedural knowledge*, *Conditional knowledge*, *RC*, *Planning*, *Monitoring*, *Evaluation* (Term 4, experimental group), and *Monitoring* (Term 4, control group).

The strongest positive correlations were obtained for the following subscales:

- *Evaluation* (0.49, Term 1, control group);
- *Declarative knowledge* (0.42, Term 1, experimental group);
- *Procedural knowledge* (0.41, Term 4, control group);
- *KC* (0.39, Term 1, control group);
- *Declarative knowledge* (0.38, Term 1, control group), and
- *Evaluation* (0.38, Term 4, control group).

The correlation between the first three subscales (*Evaluation*, *Declarative knowledge* and *Procedural* knowledge) and mathematics achievement was statistically significant ($p < 0.05$).

The Spearman *rho* correlations between learner metacognition and mathematics achievement of the combined experimental and control groups are presented in Table 5.35.

Table 5.35: Spearman *rho* correlations between learner metacognition and mathematics achievement (experimental group and control group combined)

Metacognitive scale	Experimental and control group **Spearman *rho* value** Term 1 report mark x pre-test (N=49)	Experimental and control group **Spearman *rho* value** Term 4 report mark post-test (N=49)
MAI total score	0.27	0.14
Knowledge of cognition	0.33*	0.32*
Declarative knowledge	0.35*	0.29*
Procedural knowledge	0.24	0.34*
Conditional knowledge	0.28	0.18
Regulation of cognition	0.19	0.05
Planning	0.10	0.08
Information management strategies	0.19	0.07
Monitoring	-0.01	-0.11
Debugging strategies	0.11	0.14
Evaluation	0.30*	0.18

* $p < 0.05$

Table 5.35 indicates that there were positive relationships between all metacognitive scales and mathematics achievement, with the exception of *Monitoring* (-0.01, Term 1; -0.11, Term 4). The five strongest correlations were also statistically significant ($p< 0.05$). The subscales that had statistically significant correlations with academic achievement were:

- *Declarative knowledge* (0.35, Term 1);
- *Procedural knowledge* (0.34, Term 4);
- *KC* (0.33, Term 1);
- *KC* (0.32, Term 4), and
- *Evaluation* (0.30, Term 1).

In summary, in this section inferential statistical procedures were used to present and analyse the quantitative data collected in this study. In the next section, the data presented and analysed in Sections 5.2 to 5.4 are interpreted.

5.5 INTERPRETATION OF THE RESULTS

In the typical stages of the research process, data analysis is followed by the interpretation of the findings in terms of the research problem (Ary *et al.*, 2010: 31-32). This section interprets the presentation and analysis of the extraneous variables, the descriptive statistics, and the inferential statistics (see 5.2-5.4).

5.5.1 Teachers' qualifications

Mark and Lisa both studied Mathematics up to second-year level at university, although Mark studied in the UK and Lisa in South Africa. Therefore, it is likely that the standard and content of the mathematics they studied were not identical. Lisa's B.Sc. degree was more mathematically oriented than Mark's B.A. degree, as she had Chemistry and Physics as majors, whereas Mark majored in Philosophy. On the other hand, Mark's M.Sc. in Mathematics Education probably gave him a more thorough theoretical grounding in principles related to the teaching of mathematics than the theoretical grounding which Lisa had acquired in the PGCE. It is, therefore, possible that Mark's teaching methods could have enhanced the metacognitive awareness of the experimental group to a greater extent than what the impact of Lisa's teaching methods was on the enhancement of the control group's metacognitive awareness.

5.5.2 Teaching experience

There was a pronounced difference in Mark's and Lisa's teaching experience. At the start of the intervention, Mark had just over four years' teaching experience, whereas Lisa had 15 years' teaching experience. Their teaching experience was similar in respect of the grades they had taught. Compared to Mark, Lisa had 11 years more experience in teaching mathematics; However, as Mark taught for two years in the UK, Lisa had, in fact, 13 years more experience in the South African context.

Hence, Lisa's vast teaching experience could have impacted more positively on the mathematics achievement of the control group when compared to the impact of Mark's teaching experience on the mathematics achievement of the experimental group.

5.5.3 Learners' age

The ages of the experimental and control group's learners were very similar at the start of the intervention (see 5.2.3). The experimental group had a greater percentage of learners aged 17, namely, 40%, in comparison with 25% of the control group's learners. The age difference between the two groups is considered not to have had an effect on the pre-test MAI scores, as there was a slight difference in the mean age of the experimental group's learners (16.40) and the control group's learners (16.25). In addition, as all learners were in Grade 11, they were exposed to similar cognitive challenges at school and it is, therefore, likely that age-related differences in cognitive and metacognitive development were minimised.

5.5.4 Learners' home language

The home language distribution shows that five of the six home languages of the experimental group's learners were also present among the learners in the control group. Zulu was the home language of 8% of the experimental group's learners, but none of the control group's learners had Zulu as a home language. Instead, Taiwanese was the home language of 16.67% of the control group's learners. It is argued that the differences (between the two groups) in the percentage of learners having a certain home language probably did not have a significant impact on the pre-test and the post-test results, because all learners received instruction through the medium of English. Although 13% more learners in the control group had English as a home language, it is not considered a major difference that could have had a significant impact on the pre-test and the post-test results.

5.5.5 Time allocated to teaching

Although the control group received 10 minutes less of formal mathematics teaching than the experimental group per week, the difference was probably negligible, as the control group had less mathematics periods per week and they, therefore, spent less time (per week) on entering the classroom and settling down.

There was a marked difference of 26 hours between the two groups in respect of the time allocated for extra classes during Terms 2 and 3. As the experimental group received four hours and 30 minutes of formal mathematics teaching per week, this difference of 26 hours means that the control group received more than five weeks extra formal teaching in mathematics during the course of the intervention, as compared to the experimental group. These extra teaching hours could have impacted significantly on the mathematics achievement of the control group.

5.5.6 Reliability of the pilot MAI

In the interpretation of Cronbach's *alpha*, the findings of the developers of the MAI, namely Schraw and Dennison (1994: 460-475), need to be taken into account. They found strong statistical support for the postulation that metacognition consists of two main factors, namely *KC* and *RC*. In addition, the statistical analysis revealed that the three subscales of *KC* combined, and the five subscales of *RC* combined, are reliable indicators of the *KC* and *RC* factors of metacognition, respectively. However, the statistical evidence for the division of metacognition into eight subscales was not as convincing, as indicated by the Cronbach's *alpha* values of the eight subscales that had a lower reliability (less than 0.80) than the two factors *KC* and *RC* (Schraw & Dennison, 1994: 461-466).

The Cronbach's *alpha* value of 0.91 for the MAI total score indicates that the pilot questionnaire was very highly reliable (see Table 5.1). The *KC* and *RC* Cronbach's *alpha* values of 0.81 and 0.89, respectively, indicate that these two factors were highly reliable. One subscale, namely *Planning*, had a Cronbach's *alpha* value of between 0.70 and 0.79, which means that it was reliable. The five subscales that had Cronbach's *alpha* values of between 0.60 and 0.69, indicating that they were marginally reliable, were *Declarative knowledge*, *Procedural knowledge*, *Information management*, *Monitoring*, and *Debugging*. Two subscales, namely *Conditional knowledge* and *Evaluation*, displayed low reliability with Cronbach's *alpha* values of less than 0.60.

5.5.7 Reliability of the pre-test and the post-test MAI

Table 5.2 indicates that the MAI pre-test questionnaire was highly reliable ($\alpha = 0.89$), whereas the post-test MAI questionnaire was very highly reliable ($\alpha = 0.93$). *KC* was highly reliable in the pre-test and the post-test ($\alpha = 0.82$), whereas *RC* was highly reliable in the pre-test ($\alpha = 0.83$) and very highly reliable in the post-test ($\alpha = 0.91$).

In the pre-test, the following subscales displayed low reliability: *Conditional knowledge* ($\alpha = 0.50$); *Monitoring* ($\alpha = 0.56$); *Debugging* ($\alpha = 0.34$), and *Evaluation* ($\alpha = 0.23$). The other four subscales were all marginally reliable with Cronbach's *alpha* values ranging from 0.60 (*Procedural knowledge*) to 0.69 (*Information management*).

In the post-test, three subscales displayed low reliability, namely *Conditional knowledge* ($\alpha = 0.59$); *Debugging* ($\alpha = 0.54$), and *Evaluation* ($\alpha = 0.56$). Two subscales were marginally reliable, namely *Declarative knowledge* ($\alpha = 0.65$) and *Procedural knowledge* ($\alpha = 0.62$), whereas three subscales were reliable, namely *Monitoring* ($\alpha = 0.73$), *Planning* ($\alpha = 0.75$), and *Information management* ($\alpha = 0.78$).

The findings for the pilot MAI, the pre-test MAI and the post-test MAI corroborate two main findings of Schraw and Dennison (1994: 461-466). First, the MAI is very highly reliable in measuring metacognitive awareness, in general, and highly reliable in measuring *KC* and *RC*, in particular. Secondly, the MAI is less reliable ($\alpha < 0.80$) in assessing the eight subscales.

5.5.8 Comparison between the pre-test MAI scores of the experimental group and the control group

5.5.8.1 Descriptive statistics

It is clear from Table 5.3 that, at the start of the intervention, the control group functioned on a higher level of metacognitive awareness than the experimental group, as the latter only performed better than the control group on *Debugging* and *Evaluation*. *Debugging* involves "strategies used to correct comprehension and performance errors" and *Evaluation* involves the "analysis of performance and strategy effectiveness after a learning episode" (see 4.4.1.2a). Both these metacognitive scales involve the performance of learners. It may be argued that learners who perform worse in mathematics would apply *Debugging* and *Evaluation* to a greater degree than learners who perform better, because they are more aware of their need to correct their mistakes (*Debugging*) and they, therefore, analyse their performance to a greater degree (*Evaluation*).

The question is whether, at the start of the intervention, the experimental group performed worse in mathematics than the control group. Although the average of the experimental group's Term 1 report marks was higher (0.54%) than the control group's average, they neither wrote the same tests nor were their report marks composed similarly (see Tables 5.19 and 5.20). However, if past results of the two schools are considered, it is evident that School B had a 27.10% better average in the 2010 NSC mathematics examination (see Table 1.2). School B received recognition for their outstanding results in mathematics and physical science in previous years (see 4.4.1.1). Therefore, there is the possibility that the control group consisted of learners who generally perform better in mathematics than the learners of the experimental group.

The biggest difference in median values between the two groups (0.69 for *Declarative knowledge*) points to the control group's superior knowledge of their skills, intellectual resources, and abilities as learners (see Table 5.3). It speaks of learners who were confident about what kinds of information in mathematics was important to learn and what the teacher expected them to learn. In addition, it points to learners who were confident in, first, their intellectual strengths and weaknesses and, secondly, their judgment on how well prepared they were for assessment.

5.5.8.2 Inferential statistics

Table 5.30 indicates that, at the start of the intervention, the control group had a significantly higher level of metacognitive awareness than the experimental group. The median of the *KC* factor and the medians of two of its subscales (*Declarative knowledge* and *Procedural knowledge*) were also significantly higher than the experimental group's corresponding medians. Therefore, the control group had a significantly higher level of knowledge, first, about their skills, intellectual resources, and abilities as learners (*Declarative knowledge*) and, secondly, about how to implement problem-solving methods and learning strategies (*Procedural knowledge*).

There was no significant difference in respect of the *RC* factor. This implies that, at the time of the intervention, both groups controlled their learning processes to a similar extent.

However, the significant difference ($p = 0.0198$) between the two groups in respect of the median of the pre-test MAI total score implies that Hypothesis 1a, the null-hypothesis, is not supported:

- *Hypothesis 1a*

  H_0: $Me_{\text{(experimental group pre-test MAI total score)}} = Me_{\text{(control group's pre-test MAI total score)}}$

Consequently, Hypothesis 1b, the alternative hypothesis, is supported which states that there was a significant difference between the two groups' median pre-test MAI score:

- *Hypothesis 1b*

  H_1: $Me_{\text{(experimental group pre-test MAI total score)}} \neq Me_{\text{(control group's pre-test MAI total score)}}$

5.5.9 Comparison between the post-test MAI scores of the experimental group and the control group

5.5.9.1 Descriptive statistics

The data presented in Table 5.10 strongly suggest that, at the end of the intervention, the control group had a higher level of metacognitive awareness than the experimental group. The biggest difference in post-test medians was in respect of *Procedural knowledge* for which the control group's median was 0.50 higher than that of the experimental group. Therefore, the biggest difference between the two groups was in respect of their knowledge of *how* to implement problem-solving methods and learning strategies. *Conditional knowledge* was the only subscale of the control group whose median was not higher than that of the experimental group. This indicates that both groups had a similar level of knowledge about *when* and *why* to use learning strategies.

The differences between the control group and the experimental group's post-test medians of the metacognitive scales were, in the majority of instances, less pronounced than in the pre-test (see Table 5.11). There was an improvement in both groups' medians of the MAI total scores (see Tables 5.8 and 5.9), and a smaller difference between the two groups in respect of the post-test medians of the MAI total score. This indicates that, during the course of the intervention, the metacognitive awareness of the experimental group was enhanced to a greater extent than the metacognitive awareness of the control group.

5.5.9.2 Inferential statistics

Although there were significant differences between the control group and the experimental group's pre-test medians in respect of five metacognitive scales (see Table 5.30), there were no significant differences for any of the metacognitive scales in the post-test (see Table 5.31).

Therefore, Hypothesis 2a, the null hypothesis, is supported. It states that there is no significant difference between the experimental group and the control group in respect of the median of the post-test MAI score:

- *Hypothesis 2a*

  H_0: Me $_{\text{(experimental group's post-test MAI total score)}}$ = Me $_{\text{(control group's post-test MAI total score)}}$

Hence, Hypothesis 2b, the alternative hypothesis, is not supported:

- *Hypothesis 2b*

  H_1: Me $_{\text{(experimental group post-test MAI total score)}}$ ≠ Me $_{\text{(control group's post-test MAI total score)}}$

5.5.10 The five items with the highest and lowest means in the pre-test (experimental group and control group)

5.5.10.1 The five items with the highest means in the pre-test (experimental group)

Table 5.4 points to the strong influence of affective components on the learning process, as indicated by the fact that the experimental group's learners rated interest in a mathematics topic of foremost importance for learning. The item with the second-highest mean, Item 50, involves the rereading of unclear sections in a mathematics question. This is probably an indication of the level of difficulty of most mathematics questions and the learners' past experiences of misinterpreting a question. Item 15 points to the learners' realisation that effective learning is cumulative, that is, their learning experiences were enhanced when new knowledge was connected to prior knowledge.

The important role of the collaborative aspect of effective learning is evident from Item 25, which had the fourth highest mean. It also points to a safe classroom atmosphere in which learner interaction is encouraged. Item 3, the item with the fifth highest mean, illustrates the heuristic aspect of effective learning where knowledge of problem-solving methods was activated when the experimental group encountered problems.

Next, the five items with the lowest means in the pre-test of the experimental group are presented.

5.5.10.2 The five items with the lowest means in the pre-test (experimental group)

Table 5.5 indicates that Item 4 had the lowest mean, which involves the goal-orientation aspect of effective learning, as the experimental group found it difficult to reach their goal of being well-prepared for a test or examination. Item 31 involves the constructive aspect of effective learning and points to the learners' perceived difficulties in actively constructing their knowledge. Items 19 and 38, which had the third lowest and fourth lowest means, respectively, relate to the heuristic aspect of effective learning where reflection on the problem-solving process is required after the problem-solving process. Item 21 relates to the cumulative aspect of effective learning, as the experimental group had a perceived inability to revise their work with the specific emphasis on understanding the relationships between different topics.

5.5.10.3 The five items with the highest means in the pre-test (control group)

Table 5.6 shows that the control group also regarded interest in a mathematics topic as crucial in the learning process, as Item 45 had the highest mean for both the control group and the experimental group. In fact, with respect to the pre-test items with the highest means, the control group and the experimental group had four of the five items in common, although the rank-order sequence differed. Yet, these common items (Items 15, 25, 45, and 50) indicate that the two groups were very similar in respect of those aspects they regarded as most important in the mathematics learning process.

The control group differed from the experimental group in respect of Item 5, which only featured in the control group's pre-test with the third highest mean among the control group. This item demonstrates the control group's perceived knowledge about their strengths and weaknesses in mathematics.

5.5.10.4 The five items with the lowest means in the pre-test (control group)

It is obvious that the control group perceived aspects related to problem-solving as most problematic in the learning of mathematics, since four of the five items with the lowest means (Items 11, 19, 35, and 38) relate to problem-solving aspects, that is, the heuristic aspect of effective learning (see Table 5.7). Two of these items (Items 19 and 38) were also among the five items with the lowest means regarding the experimental group. Item 31, which relates to the constructive aspect of effective learning, was also common to both the control and the experimental group. Therefore, Items 11 and 35 featured only in the control group. Both these items relate to problem-solving aspects, whereas the two items with the lowest means, that only related to the experimental group, entail the learners' preparation for tests and examinations (Item 4), and the periodical revision of the relationships in mathematics (Item 21).

5.5.11 Comparison between the pre-test and the post-test MAI scores (experimental group)

5.5.11.1 Descriptive statistics

Table 5.8 indicates that the experimental group experienced an improvement in their level of metacognitive awareness as the median MAI total score improved by 0.30. In addition, the post-test medians of the *KC*, *RC* and six subscales were higher than the pre-test medians. The biggest improvement (0.57) in median values was for the subscale *Planning*, namely from 2.86 in the pre-test to 3.43 in the post-test. This indicates an improvement in their planning, goal-setting, and the assigning of resources *before* learning. It is interesting to note that one of the items of *Planning*, namely Item 4, had the lowest mean of all the items in the pre-test.

The other five subscales all improved from the pre-test to the post-test with 0.40 or more, except for *Information management* which only improved by 0.19. As *Information management* involves skills and strategies applied *during* the learning process, it indicates that the experimental group gained more by the intervention in terms of learning activities before the learning process starts (*Planning*) as opposed to learner strategy use during the learning process (*Information management*).

The median values of *Procedural knowledge* did not improve from the pre-test to the post-test. Therefore, the experimental group's knowledge of *how* to implement learning strategies was not enhanced. *Procedural knowledge* had the joint third highest median in the pre-test, but the lowest median in the post-test (see Table 5.12). This suggests that the experimental group's learners found it most difficult, in respect of all the MAI subscales, to improve their knowledge of how to implement problem-solving methods and learning strategies.

Debugging was the second subscale that did not improve, but as *Debugging* had the highest median of all metacognitive subscales in the pre-test (see Table 5.12), the lack of improvement may be due to the fact that the learners already used strategies to correct comprehension and performance errors to a great degree at the start of the intervention and that it was therefore more difficult to improve.

5.5.11.2 Inferential statistics

Table 5.32 indicates that the medians of the MAI score, *KC*, *RC*, and only three of the eight subscales experienced a statistically significant improvement. These three subscales were *Declarative knowledge*, *Planning* and *Monitoring.* Although only one subscale of *KC* (*Declarative knowledge*) and two subscales of *RC* (*Planning* and *Monitoring*) improved statistically significantly, the overall improvement for *KC* and *RC* was still statistically significant. These results may be interpreted by considering the findings of Schraw and Dennison (1994: 464-465) when they developed the MAI. They found that the factors *KC* and *RC* were very reliable in measuring knowledge of cognition and regulation of cognition, respectively, whereas the eight subscales were less reliable in measuring the respective subscales.

Therefore, the intervention was successful in enhancing the learners' metacognitive awareness (MAI total score) in general, and their knowledge aspect of learning (*KC*) and control aspect of learning (*RC*) specifically.

The implication is that Hypothesis 3a (the null hypothesis) is not supported. It states that the median of the experimental group's pre-test MAI total score is equal to the median of the experimental group's post-test MAI total score:

- H_0: $Me_{\text{(experimental group's pre-test MAI total score)}} = Me_{\text{(experimental group's post-test MAI total score)}}$

Hence, Hypothesis 3b (the alternative hypothesis) is supported. It states that the median of the experimental group's post-test MAI score was significantly higher than the median of the experimental group's pre-test MAI total score.

- H_1: $\text{Me}_{\text{(experimental group's pre-test MAI total score)}} < \text{Me}_{\text{(experimental group's post-test MAI total score)}}$

5.5.12 Comparison between pre-test and post-test MAI scores (control group)

5.5.12.1 Descriptive statistics

Table 5.9 indicates that the control group also experienced an improvement in the median of the MAI total score, *RC*, and five subscales.

Evaluation had the biggest improvement in medians (0.50). This improvement could be partially explained by the fact that *Evaluation* had the lowest pre-test median (see Table 5.13) and, therefore, it had a greater possibility for improvement. In addition, in the seven-month period between the pre-test and the post-test, the control group was probably becoming more aware of their final school year lying ahead and the great importance of the NSC examination at the end of Grade 12. Therefore, they could have started focusing, to a greater degree, on the analysis of their performance and strategy use in mathematics during the intervention period. Another factor that could have caused an improvement in *Evaluation* or any other metacognitive scale is the biological and psychological maturation that the learners experienced during the intervention period (see 4.4.1.3f(i)).

Three subscales did not experience an improvement, namely *Debugging*, *Declarative knowledge* and *Procedural knowledge*. The reason for this may be that they had the highest, second highest and third highest medians in the pre-test, respectively, and that they, therefore, had a smaller scope for improvement than the other subscales.

5.5.12.2 Inferential statistics

Table 5.33 shows that the control group did not experience a statistically significant improvement in their MAI total score, *KC*, or *RC*. However, two subscales improved statistically significantly, namely *Conditional knowledge* and *Evaluation*. Since the MAI does not assess the eight subscales as reliably as the two factors *KC* and *RC*, the findings for *Conditional knowledge* and *Evaluation* are not as convincing as the findings for the MAI total score, *KC*, and *RC* which did not indicate a statistically significant improvement for these metacognitive scales.

The implication is that Hypothesis 4a (the null hypothesis) is supported. It states that there is no statistically significant difference between the medians of the control group's pre-test and post-test MAI total score:

- H_0: Me $_{\text{(control group's pre-test MAI total score)}}$ = Me $_{\text{(control group's post-test MAI total score)}}$

Hence, Hypothesis 4b (the alternative hypothesis) is not supported. It states that the control group's median of the post-test MAI total score is significantly greater than the median of the pre-test MAI total score:

- H_1: Me $_{\text{(control group's pre-test MAI total score)}}$ < Me $_{\text{(control group's post-test MAI total score)}}$

5.5.13 Comparison of the subscale rank-order of the experimental group's pre-test and post-test MAI scores

The rank-order comparison of the subscales indicates that the subscales were fairly constant in their rank position, with the exception of *Procedural knowledge* which was ranked jointly third in the pre-test, but eighth in the post-test (see Table 5.12). *Procedural knowledge* was one of only two subscales that did not improve from the pre-test to the post-test; this partly explains its drop in rank. As *Procedural knowledge* entails knowledge about how to implement learning strategies and problem-solving methods, it indicates that the experimental group's learners did not apply these skills to a greater degree after the intervention.

In the pre-test, it is evident that *KC*'s subscales were ranked higher than *RC*'s subscales, with the exception of the *RC* subscale *Debugging*. In the post-test, the same order is evident except for *Procedural knowledge*'s drop in rank. The higher rank of *KC*'s subscales – especially in the pre-test – and the higher median of *KC* (see Table 5.8) indicate that the learners were more confident in their knowledge of the learning process in mathematics than in their ability to control these learning processes before and after the intervention.

5.5.14 Comparison of the subscale rank-order of the control group's pre-test and post-test MAI scores

The rank-order of the subscales also remained very similar in the pre-test and the post-test (see Table 5.13). No subscale improved or dropped by more than one position and the three subscales of *KC* were also ranked higher than all the subscales of *RC*, with the exception of *Debugging* which was the *RC* subscale in the top position in both the pre-test and the post-test. Hence, the control group's learners also had a better knowledge of the mathematical learning process than their ability to regulate their learning processes.

5.5.15 Comparison of the five items with the highest and lowest means in the pre-test and the post-test MAI (experimental group and control group)

5.5.15.1 Highest means

Table 5.18 points to the strong influence of affective components on the learning process, as indicated by the fact that both groups' learners rated interest in a mathematics topic (Item 45) as of foremost importance for learning.

The item with the second-highest mean in both the pre-test and the post-test of the experimental group, Item 50, involves the rereading of unclear sections in a mathematics question. This may be an indication of the level of difficulty of most mathematics questions and the learners' past experiences of misinterpreting a question. The rereading of unclear sections of a question in order to improve understanding relates to the heuristic aspect of effective learning, as understanding of a problem is the first step in problem-solving. For the control group, interest in a mathematics topic was also regarded as crucial in the learning process, as Item 50 had the fifth highest mean in the pre-test and the third highest mean in the post-test.

Item 15 had the third highest mean in both the pre-test and the post-test of the experimental group, and the second highest mean in both the pre-test and the post-test of the control group. Item 15 relates to learning being more effective when learners already know something about a certain topic. Therefore, it points to the learners' realisation that effective learning is cumulative, that is, their learning experiences were enhanced when new knowledge was connected to prior knowledge.

The important role of the collaborative aspect of effective learning is evident from Item 25, which refers to learners asking other learners for help when they do not understand something in mathematics. It had the fourth highest mean for the experimental group in both the pre-test and the post-test, and the fourth highest mean in the pre-test of the control group. This also points to a safe classroom atmosphere in which learner interaction is encouraged. The fact that Item 25 did not feature under the five items with the highest means in the post-test of the control group could indicate that those learners have become more independent and confident in their ability to identify and correct their own mistakes.

Item 3 had the fifth highest mean in the pre-test of the experimental group and the fourth highest mean in the post-test of the control group. This item entails the application of problem-solving methods that have worked in the past when solving a problem; it, therefore, relates to the heuristic aspect of effective learning. It was not under the five items with the highest means in the post-test of the experimental group. This could indicate that the learners encountered progressively more difficult problems during the course of the intervention which diminished their chances of applying problem-solving methods that have worked in the past. However, as Item 3 also featured under the five items with the highest means in the post-test of the control group, the explanation offered does not seem valid, because both groups followed the same mathematics curriculum. The control group had markedly more extra classes than the experimental group during the intervention period; this could have enhanced their application of past problem-solving methods, because more time was available to practise problem-solving skills.

Item 9 only featured once; it had the fifth highest mean in the post-test of the experimental group, and states “I read slower when I encounter important information in a mathematics question”. It is likely that, during the course of the intervention, the experimental group’s learners realised that some of their mistakes were caused by not reading a question properly and, therefore, their regulation of this aspect was relatively better than in the pre-test.

Item 5, which entails the understanding of one’s intellectual strengths and weaknesses in mathematics, only featured in the pre-test of the control group. This indicates that the control group’s learners had been aware of what mathematical skills and procedures they could perform well (or find difficulty with) in March, but that they were (relative to the pre-test) not as confident about their understanding of their strengths and weaknesses in October. It is challenging to explain this finding, because the control group achieved better in Term 4. This would most likely indicate an even better understanding of what they can do well or find difficulty with in mathematics. More light is shed on this finding by the fact that Item 29 (“I use my strengths in mathematics to compensate for my weaknesses in mathematics”) only features in the control group’s post-test. It could be that, as the control group improved their ability to compensate for their weaknesses, they also became less aware of their weaknesses.

5.5.15.2 Lowest means

The three items common to both groups in the pre-test and the post-test (see Table 5.18) indicate that both groups’ learners found the following aspects difficult: creating their own examples in mathematics in order to understand new information better (Item 31); finding easier ways to solve a problem (Item 19), and finding different ways to solve a problem (Item 38).

Item 31 involves the constructive and cumulative aspects of effective learning, and it may indicate that both groups’ teachers played a dominant role in helping the learners to make sense of new information.

The low means of Items 19 and 38 may point to two aspects. First, the learners were not made aware of the importance of finding different or easier solutions to problems after they had solved a problem. Secondly, time constraints could have prevented the teachers from emphasising those aspects of problem-solving. It is understandable that mathematics teachers in general do not emphasise finding easier or different solutions (after a solution has been obtained), because it is perceived that the goal is reached when a problem is solved, but mathematical proficiency entails, among others, the search for alternative methods (see 2.2.4.4m and 3.2.2.2).

Item 4 only featured in the pre-test and the post-test of the experimental group. This indicates that the learners' struggled with pacing themselves when studying for a test or examination in order to finish studying in time was still pertinent after the intervention. This finding is related to Item 21 ("I periodically do revision to help me understand important relationships in mathematics") which also featured in the pre-test and the post-test of the experimental group. It seems reasonable to propose that the scant revision impacted negatively on their knowledge of the relationships in mathematics which, in turn, could impede effective studying.

Item 11 ("I ask myself if I have considered different methods of solving a problem when solving a mathematics problem") and Item 35 ("I know in which situation each problem solving method I use will be most effective") only featured in the pre-test of the control group. Therefore, in relation to other items, the control group's learners considered these aspects of problem-solving less problematic after the intervention. This may indicate that the learners became more aware of different and more effective methods of problem-solving, due to the fact that their extra classes enabled them to solve many problems during the course of the intervention.

Two items only featured in the post-test of the control group, namely Item 7 ("I know how well I did once I finish a mathematics test or examination") and Item 37. The pre-test mean and post-test mean of Item 7 was equal (3.00), which implies that the control group's learners had the same level of awareness (in the pre-test and the post-test) of their achievement after they had written a test or examination.

Item 37 ("I draw pictures or diagrams to help me understand while I am learning mathematics") had a pre-test mean of 3.42, but a post-test mean of 3.04. This could indicate that the teacher took more control of the mathematics learning process during the course of the intervention. The more prominent role of the teacher in facilitating learner understanding could have been due to the following factors. The final Grade 11 examination covered the entire year's work and, in order for learners to be successful in that examination, they had to have a good understanding of the relationship between many different concepts and topics in mathematics. Therefore, more demands were made on their ability to understand; the teacher could have played a bigger role in the process of understanding these relationships.

5.5.16 Mathematics achievement (experimental group and control group)

In this study, the effect of MI on learner metacognition and academic achievement in mathematics is investigated. The learners' levels of metacognition were measured by the pre-test and the post-test MAI. The pre-test and the post-test MAI were identical instruments and were completed by both the experimental group and the control group. Their academic achievement in mathematics, however, was measured in ways that make a comparison within and between the groups more difficult.

The following problematic aspects need to be considered when the academic achievement of the two groups is interpreted. The Term 1 and Term 4 report marks that were used to determine the relationship between learner metacognition and academic achievement were not compiled identically for the experimental group and the control group (see 5.3.13). Within each group, the Term 1 and Term 4 report marks were obviously not identical. Therefore, a comparison within or between the groups in respect of their academic achievement is problematic.

Nevertheless, a favourable effect of MI on learner achievement in mathematics may be indirectly observed if a statistically significant positive relationship ($p < 0.05$) between learner metacognition and academic achievement can be established. This implies that an improvement in the learners' level of metacognition will probably lead to an improvement in their academic achievement as measured in Term 1 and Term 4. However, if the Term 4 report mark is composed of assessment activities that assess learners on a higher mathematical level than in Term 1, it could cause a drop in learners' marks from Term 1 to Term 4, despite an improvement in the learners' levels of metacognition. Conversely, learners' marks could improve from Term 1 to Term 4, due to the fact that Term 4 assessment activities are easier than those of Term 1. The increasing volume of work from Term 1 to Term 4 and the increasing role that formal assessment activities such as tests and examinations play make it improbable that learners would find it easier to perform better in Term 4.

5.5.16.1 Term 1 report marks (experimental group and control group)

Although the experimental and control groups' Term 1 report marks were composed differently, they were nearly similar in respect of the weights of the formal (experimental group: 66.67%; control group: 70%) and informal assessment activities (see 5.3.13).

It can be argued that a formal assessment activity (such as a formal test) is a more valid and reliable indication of a learner's mathematics achievement, because it tests individual work as opposed to assignments which are not necessarily done individually. Informal assessment activities (such as class tests of 10 marks each and homework evaluations) are less valid and reliable, due to the little scope of work that is tested as opposed to a formal test which covers more work.

The Term 1 report mark averages were very similar for the two groups, namely 66% (experimental group) and 65.46% (control group). The control group wrote three formal tests, whereas the experimental group only wrote one formal test (see 5.3.1.3). Therefore, the control group's Term 1 report mark is probably a more valid measure of the learners' achievement than the experimental group's Term 1 report mark.

5.5.16.2 Term 4 report marks (experimental group and control group)

The drop of 9.04% in the mean of the experimental group's report marks could, to a certain extent, be ascribed to the following factors. First, Term 4's assessment was of a more formal nature than that of Term 1. The formal assessment activities of the Term 4 report mark had a weight of 93.75% as opposed to 66.67% of the Term 1 report mark that comprised formal assessment activities. Secondly, the nature of the formal assessment activities was more demanding, as the learners wrote two examination papers of 150 marks each as opposed to one test of 50 marks in Term 1. Thirdly, the examination covered all the work done during the year and thus posed a bigger challenge than Term 1's assessment activities.

Despite the bigger weight in the composition of the report mark and the more demanding nature of Term 4's formal assessment activities – as opposed to Term 1's report marks' composition and nature of the assessment activities – the control group's achievement was better in Term 4. It is suggested that the extra classes which the control group received had a significant impact on their achievement, considering that they received extra classes totalling 26 hours more than the total hours of extra classes which the experimental group received during the course of the intervention (see 5.5.5). In addition, the significantly greater teaching experience of the control group's teacher could have contributed to the control group's better achievement relative to the experimental group's achievement (see 5.5.2).

5.5.17 Correlation between learner metacognition and achievement in mathematics (experimental group and control group)

In the interpretation of the correlation between learner metacognition and academic achievement, it is emphasised that the Term 1 and Term 4 report marks show a great deal of variation in their compilation, both within groups and across groups, regarding the following aspects.

First, the two groups' Term 1 report marks were composed of different types of assessment activities with different weightings (see 5.3.13). Secondly, the CASS component of the Term 4 report marks of both groups also consisted of different types of assessment activities with different weightings (see 5.3.13). Thirdly, it is likely that the content of the Term 1 assessment activities was not similar for both groups. For example, the formal tests which the experimental group and the control group wrote in Term 1 probably differed in respect of both the mathematics topics they covered and the cognitive level of the questions, as the tests were set by different teachers for different groups. Fourthly, although the groups did not write identical examination papers in Term 4, it is likely that the papers were very similar in respect of the mathematics topics they covered, as the examination papers were based on the prescribed Grade 11 syllabus. However, the cognitive levels of questions in the experimental group's examination paper were most likely not identical with the cognitive levels of the questions on the same topic in the control group's examination paper.

Consequently, it is suggested that, due to these variations in the manner in which mathematics achievement was measured, a more balanced perspective on the correlation between learner metacognition and mathematics achievement is obtained when the Term 1 and Term 4 report marks of both groups are considered collectively.

5.5.17.1 Descriptive statistics

In this section, the focus is mainly on Table 5.29, as it is a summary of Tables 5.21 to 5.28.

Table 5.29 shows that, in general, the knowledge aspect of learning (*KC*) and the regulation aspect of learning (*RC*) individually showed a stronger correlation with academic achievement than the two aspects combined (MAI total score), because the MAI total score does not feature in Table 5.29. In addition, *KC* shows a stronger correlation with academic achievement than *RC* in two ways. First, although *KC* and *RC* featured twice as the metacognitive rank score category with the highest frequency of difference scores with values less than three (column 3), *KC*'s frequency of difference scores with values less than three was higher in both instances (column 4). Secondly, *KC* featured three of the four times as the metacognitive scale rank-score category with the highest frequency of difference scores (column 5). In all three instances, *KC* also had a higher total of frequency of difference scores than *RC* (column 6).

5.5.17.2 Inferential statistics

Table 5.34 shows that only three metacognitive scales had a significant correlation with mathematics achievement. However, when the Term 1 and Term 4 report marks of the experimental and control groups were combined, six metacognitive scales showed a significant correlation ($p < 0.05$) with mathematics achievement (see Table 5.35). These six metacognitive scales had lower Spearman *rho* values than the three metacognitive scales which had a significant correlation with mathematics achievement (see Table 5.34). This confirms that a smaller correlation coefficient value will be statistically significant when the number of participants increases (see 4.4.1.3d).

This discussion focuses on the results displayed in Table 5.35, as the combined experimental and control groups' report marks provide a more balanced perspective on the mathematics achievement marks (see 5.5.16). The significant correlation between *KC* and mathematics achievement for both the Term 1 and Term 4 report marks supports the findings by Schraw and Dennison (1994: 470-472) who also found a significant correlation between *KC* and mathematics achievement, but not between *RC* and achievement, or between the MAI total score and achievement (see 4.4.1.2a).

In this study, the absence of a significant relationship between the MAI total score and mathematics achievement could partly be explained by considering Schraw and Dennison's (1994: 472) suggestion that a greater level of metacognitive awareness is required to complete complex tasks. The correlation between learner metacognition and mathematics achievement would, therefore, be less observable in less complex tasks. One may ask whether the assessment activities that comprised the mathematics achievement marks in this study were sufficiently complex to reveal this correlation.

A weight of 33.33% of the experimental group's Term 1 report mark was allocated to assessment tasks such as class tests of 10 marks each and an assignment, whereas 25% of the control group's Term 1 report mark consisted of assessment activities such as class tests, an assignment, and homework evaluations (see 5.3.13). It is likely that these assessment activities did not assess higher order thinking skills to a great extent (see 5.5.16.1).

By contrast, 93.75% of the Term 4 report mark of both groups was composed of formal assessment activities (see 5.3.13). The Term 4 report marks, therefore, were more formal in nature than the Term 1 report marks. However, one could not assume that the Term 4 report marks were composed of more complex assessment activities than the Term 1 report marks, as formal assessment activities such as tests and examinations assess both higher order and lower order thinking skills.

It is suggested that the Term 1 and Term 4 report marks were not the best indicators of learner performance of the higher cognitive levels in mathematics. Cetinkya and Erktin (2002: 9) also state that course grades are not good measures of true mathematics achievement (see 2.3.3). In addition, previous studies that investigated the relationship between learner metacognition and mathematics achievement used a range of mathematics achievement measures, for example, open-ended word problems and writing justifications for mathematical propositions (see Table 2.4).

In summary, there was a significant correlation between only one of the two factors of metacognition and mathematics achievement. This correlation was not sufficiently strong to ensure a significant correlation between the total MAI total score and mathematics achievement. Therefore, Hypothesis 5a, the null hypothesis, is supported:

> H_0: There is not a statistically significant positive relationship between learner metacognition and achievement in mathematics.

Consequently, Hypothesis 5b, the alternative hypothesis, is not supported:

> H_1: There is a statistically significant positive relationship between learner metacognition and achievement in mathematics.

5.6 CONCLUSION

Chapter 5 addressed secondary research questions 5 and 6. Secondary research question 5 investigated whether MI had a statistically significant positive effect on the metacognitive awareness of the experimental group's learners. In respect of secondary research question 5, it was found that MI had a statistically significant positive effect on the metacognitive awareness of the experimental group's learners. This result is based on the interpretation of two aspects. First, the experimental group's MAI total score had a significantly lower median than the control group in the MAI pre-test, but no differences in the medians of the MAI total scores were observed in the post-test (see 5.5.8 and 5.5.9). Secondly, the experimental group had a significantly higher median MAI total score in the post-test, whereas the control group's median MAI total score did not change significantly (see 5.5.11 and 5.5.12).

Secondary research question 6 investigated whether there was a statistically significant positive relationship between metacognitive awareness and mathematics achievement for the learners of both the experimental group and the control group. It was found that there was a statistically significant positive relationship between one of the two factors of the MAI (*KC*) and mathematics achievement. However, when the two factors *KC* and *RC* were combined, a statistically significant positive relationship between metacognitive awareness and mathematics achievement was not observed.

In this chapter, quantitative data were presented, analysed and interpreted in order to address secondary research questions 5 and 6. In Chapter 6, secondary research questions 7 to 12 and the mixed methods research question are explored.

CHAPTER 6

PRESENTATION, ANALYSIS AND INTERPRETATION OF THE QUALITATIVE RESEARCH DATA

6.1 INTRODUCTION

The purpose of this study is to investigate the effect of MI on learner metacognition and mathematics achievement. In the previous chapter, quantitative data were presented, analysed, and interpreted in order to address secondary research question 5, secondary research question 6, and five hypotheses. In this chapter, qualitative data are presented, analysed and interpreted in order to address the following secondary research questions and mixed methods research question:

- Secondary research question 7: What is the impact of MI on the level of learner metacognition in a problem-solving context? (see 6.2).
- Secondary research question 8: What is the impact of MI on the level of mathematics achievement in a problem-solving context? (see 6.2).
- Secondary research question 9: What are the perspectives of the experimental group's teacher on the nature of mathematics and aspects related to the teaching-and-learning of mathematics? (see 6.3).
- Secondary research question 10: What are the perspectives of the control group's teacher on the nature of mathematics and aspects related to the teaching-and-learning of mathematics? (see 6.3).
- Secondary research question 11: What are the perspectives of the experimental group's learners on the MI process? (see 6.4).
- Secondary research question 12: What are the perspectives of the experimental group's teacher on the MI process? (see 6.4).

The following mixed methods research question is addressed:

- To what extent do the results from the qualitative phase of the study support the results obtained from the quantitative phase of the study regarding the effect of MI on learner metacognition and mathematics achievement? (see 6.5).

The main purpose of this study is to investigate the impact of MI on learner metacognition and mathematics achievement. As the MI implemented in this study is structured according to the components of the two subthemes of De Corte's (1996) educational learning theory (see Appendix B5), specific references to these components are made in the interpretation of the qualitative data.

6.2 FIRST AND SECOND PROBLEM-SOLVING SESSIONS

In this section, the following two secondary research questions are addressed:

- Secondary research question 7: What is the impact of MI on the level of learner metacognition in a problem-solving context?
- Secondary research question 8: What is the impact of MI on the level of mathematics achievement in a problem-solving context?

First, the researcher's analysis of the learner responses in respect of both secondary research questions is presented. This is followed by Mark's perspectives on both problem-solving sessions. Subsequently, both secondary research questions are addressed in the interpretation of Mark's perspectives and the learners' responses.

6.2.1 Problem analysis

The experimental group's learners participated in two problem-solving sessions to determine their level of metacognitive awareness during a problem-solving activity. The first problem-solving session took place at the start of the intervention, and the second problem-solving session took place nearly six months later at the end of the intervention. The same problem statement was given at both problem-solving sessions (see Appendix C1).

The problem is phrased such that as few direct instructions as possible are given so that no direct route to the answer can be identified. The first clue of what the problem is about is the fact that the farmer is not certain as to which shape to use for the enclosure. The learners are explicitly asked to advise the farmer as to which shape to use on condition that the advice must be based on calculations.

The mathematical concepts hidden in the problem statement involve the enclosure and the sheep contained within the enclosure. Learners should realise that the enclosure relates to the perimeter of the shape and that the sheep contained within it relate to the area of the shape.

Learners are expected to identify the core of this problem, namely that farming is a business that aims to maximise its profits. Therefore, if different shapes have different areas when the perimeter stays constant, the shape with the largest area would be most cost-effective. No direct references are made to maximum area or optimum use of the shape in terms of its area. The length of the fence is given as 100m, which is an arbitrary number that somewhat simplifies the calculations for the dimensions of squares and rectangles. It is, however, not an easy number to work with when calculating the diameter of the circle. The number of sheep (650) is, in fact, a distracter, as one does not know what area each sheep occupies. Yet, if learners would estimate the 'area' one sheep occupies, it could help them visualise how the sheep are arranged in the enclosure.

One would expect the learners to consider the following shapes: square, rectangle, triangle, and circle. The learners have dealt with the area and perimeter formulas for these shapes – circumference formula in the circle's case – since Grade 8. Therefore, the learners were expected to cope with these formulas. As there are various rectangles with integral side lengths and a perimeter of 100m, only one rectangle's area – with dimensions that closely resemble a square – is calculated. The areas of these shapes are, in increasing order, 481.13m^2 (triangle); 624m^2 (rectangle); 625m^2 (square), and 795.77m^2 (circle) (see Appendix C3). Therefore, the circle is the most optimum shape in respect of area, as there is a pronounced difference of 27.33% between the area of the circle and the area of the square.

Another aspect that should be considered in the analysis of this problem relates to the practicality of the shapes. Would it, for example, be easier to construct a square enclosure than to construct a circular enclosure? What limitations would a triangular enclosure impose on the movement of the sheep? Learner 2, for example, stated that:

> The triangle has sharp corners it wouldn't be an ideal shape to use we can work out the area for the other three shapes.

These practical aspects, however, should not distract from the core of the problem, namely that it would be most cost-effective to construct a circular enclosure, as it yields the largest area for the given perimeter.

6.2.2 Analysis of the level of learner metacognition during the first and second problem-solving sessions

In both problem-solving sessions, I was surprised by the extent to which the learners recorded their thoughts (see Appendix C2). The majority of the learners expressed their thoughts freely in written form. Learner 10, for example, reminded herself about an important piece of information in the problem-statement:

> Need to use most of the fence to fit all his sheep. Remember that he only has 100m.

I coded the learners' responses in terms of items on the MAI (see Appendices C2 and C4). During the first and the second problem-solving sessions, the learners' metacognitive awareness primarily related to the following subscales of the MAI: *Declarative knowledge, Planning, Information management*, and *Monitoring*. These four subscales featured as follows.

First, *Declarative knowledge* refers to knowledge about one's skills, intellectual resources, and abilities as a learner. Item 17 ("I am good at remembering mathematics facts and principles") was very prominent as it relates to knowledge of the correct formulas for the areas of the different shapes. Generally, the formulas for the perimeter/circumference and area of the triangle, rectangle, square, and circle were applied correctly, although there were common misconceptions about the length of the radius (see 6.2.3). An example of the application of Item 17 is evident in the following response by Learner 6:

> Square = l x b (sides same)
> Rectangle = l x b (x 2 sides same)

Secondly, *Planning* entails the following aspects: planning, goal-setting, and the allocation of resources prior to a learning activity. Two items were applied frequently, namely, Item 22 ("I ask myself questions about the problem before I begin to solve a mathematics problem") and Item 23 ("When I start to solve a mathematics problem, I think of several ways to solve the problem and choose the best one"). An example of an application of Item 22 is evident from the next question by Learner 26:

> What is the formula for the area of a square?

Learner 22 applied Item 23 as follows:

> I first thought of the word shape and decided to write down all my formulas of shapes and by maybe substituting the information given in my formulae.

As Item 23 is very similar to Item 2 ("I first consider different ways of solving the problem before I start solving a problem in mathematics") of the subscale *Monitoring*, I coded the majority of learner references to different shapes as falling under Items 22 and 2.

Thirdly, *Information management* entails skills and strategy sequences during learning in order to process information more efficiently. Two items featured very prominently during both problem-solving sessions, namely Item 13 ("I consciously focus my attention on important information in a mathematics question") and Item 30 ("When I receive new information about a familiar topic or a new topic in mathematics, I focus on the meaning and significance of the new information"). As these two items are very similar, I also coded learner references to the important information in the problem statement as belonging to Items 13 and 30. Learner 16, for example, realised that the core of the problem was about the concept 'maximum area', as she stated the following:

> A circular kraal would be the best option because there should be more space.

Fourthly, *Monitoring* comprises assessment of one's learning or of one's use of strategy. Item 2 was applied frequently in both problem-solving sessions. As stated earlier in this section, I grouped Item 2 with Item 22, as both items refer to the consideration of different problem-solving methods at the start of a problem-solving session. In the context of these problem-solving sessions, different problem-solving methods refer to the area calculations of different shapes.

Therefore, the learners' metacognitive awareness during both problem-solving sessions related mainly to the metacognitive behaviours described in the following six items of the MAI: Item 17 (*Declarative knowledge*); Items 22 and 23 (*Planning*); Items 13 and 30 (*Information management*), and Item 2 (*Monitoring*).

6.2.3 Analysis of the level of mathematics achievement during the first and second problem-solving sessions

Only one learner solved the problem successfully in the first problem-solving session, whereas five learners were successful in the second problem-solving session. The learner who solved the problem during the first problem-solving session used very similar calculations during the second problem-solving session. Although the other four learners correctly calculated the circle's area, they did not compare the circle's area, in each case, to the area of a triangle, rectangle, and square (see Appendix C8).

An analysis of learner errors in both problem-solving sessions provides some insight into some reasons for their poor performance (see Appendices C5-C7). Conceptual errors were very common, in particular those relating to the length of the diameter and the radius. A diameter of 50m was used in a number of calculations, probably because those learners were of the opinion that the diameter equals half the circumference, since the diameter splits the circle into two 'halves'.

A substantial number of conceptual errors related to the length of the radius. The radii lengths varied between 25m and 100m. Arguably, the most difficult calculation in this problem was to determine the radius' length. Yet, these mistakes point to a lack of understanding as to how the radius and diameter relate to the circumference.

As expected, another conceptual error concerned the confusion between the area and circumference formulas for a circle. In addition, some learners equated the circumference length or the 650 sheep to the area of the enclosure. Some learners put the 650 sheep equal to 100m. These errors indicate serious learner misunderstanding of the concepts ‘area’ and ‘circumference’. The only conceptual errors that did not involve the circle were noticed in the answers of a few learners who used a square or a rectangle with perimeters that did not equal 100m.

In contrast to these various conceptual errors, there were very few calculation errors (see Appendix C7). These errors were in respect of the wrong calculations of the quotient of 100 and 4, the product of 100 and 100, and the height of a triangle.

6.2.4 Mark’s perspectives on both problem-solving sessions

I asked Mark to share his perspectives on the following aspects of the learners’ problem-solving behaviour during both problem-solving sessions: common mistakes and reasons for making those mistakes; possible learner improvement in their attempts to solve the problem, and the learners’ level of thinking awareness and thought processes (see Appendix C9). Mark’s perspectives were as follows:

> One idea that came up a few times was that a particular number of sheep could fit in each metre as opposed to square metre. It is not clear exactly what causes this mistake, but it indicates a lack of understanding of the jump from one to two dimensions. One learner in particular referred to having ‘only 100m space’. Some other learners took it that the area was to be $100m^2$.
>
> Another issue that came up repeatedly was a tendency to use a specific rectangle rather than a generalised one. The learners seem uncomfortable with converting between a word problem and algebra. This is a common theme in many classrooms as far as I can tell; the learners will rather pick an arbitrary rectangle and work with that than attempt to produce a formula based on the perimeter.

Another issue was with circles. Though some learners seemed to have a good grasp of how to get the radius of the circle from the perimeter, some simply took the 100m as the diameter so concluded that the radius must be 50m or even took the radius as 100m. Again, I am not sure exactly what mechanism is behind this, but it does seem that it relates to a gap between mathematics on paper and real-life situations and I suspect they are used to the type of situation they would have encountered when first learning about circles, where the radius or diameter was always given first and the other dimensions were to be extrapolated.

The main difference that I noted with learners who were doing the activity for the second time was a greater readiness to dive into the algebra. Perhaps they were now somewhat primed that this was the approach that was expected. It led, in some cases, to greater mathematical success, but there was little discussion. The first time around the learners were liable to make suggestions based on practical considerations (e.g. corners are a bit wasted, particularly sharp ones like in a triangle as sheep will not fit there, hence a circle is better).

Generally speaking the approach the second time was more technically mathematical. It is probable that a lot of learners took the algebraic approach the second time around because they had seen the solution and were now exhibiting a behaviourist approach and responding to training; i.e. ‘we have seen a problem like this and this is the way to approach it’ as opposed to engaging with the problem and abstracting to the algebra because they worked out that it was the best approach.

However, for many of the learners, their second approach contained more diagrams and more detailed on-going reflection throughout the task. Factors affecting this include an improved familiarity with the task as well as a general level of comfort with reflection and communicating mathematical ideas. [...] I would suggest that the approach the second time around indicates that they had developed in this regard.

6.2.5 Interpretation of the level of learner metacognition during the first and second problem-solving sessions

When the analysis of the learners' level of metacognition during both problem-solving sessions is considered, it is evident that there was hardly any difference between the levels of metacognition during the two problem-solving sessions. The same four subscales and individual items featured to a very comparable extent (see Appendix C4).

Before the first problem-solving session, I expected that these metacognitive behaviours would link to a varying degree with the four phases of Polya's problem-solving model. Next, I compare the learners' level of metacognition to the four steps of Polya's problem-solving model as this model provides the background to metacognitive behaviour in a problem-solving context.

To me, the secret to unlocking this problem lies in identifying the hidden mathematical concepts. In writing down their thoughts, I would have expected the learners to use phrases that relate to the relationship between the area and perimeter of different shapes. In addition, I expected them to think of basic shapes such as the triangle, rectangle, square, and circle (*Understanding the problem*). Furthermore, I expected them to state that they were going to compare the areas of these different shapes (*Devising a plan*). Moreover, I anticipated that they would identify the correct formulas for the area and perimeter of these shapes and apply these formulas correctly (*Carrying out the plan*). Finally, I did not expect that many of the learners would evaluate their answers and reflect on the practicality of their solution (*Looking back*).

Of the six items that featured strongly in both problem-solving sessions, Item 22 ("I ask myself questions about the problem before I begin to solve a mathematics problem"), Item 13 ("I consciously focus my attention on important information in a mathematics question"), and Item 30 ("When I receive new information about a familiar topic or a new topic in mathematics, I focus on the meaning and significance of the new information") relate to Polya's first phase, namely *Understanding the problem*.

Item 23 ("When I start to solve a mathematics problem, I think of several ways to solve the problem and choose the best one") and Item 2 ("I first consider different ways of solving the problem before I start solving a problem in mathematics") point to metacognitive behaviours when *Devising a plan,* Polya's second phase.

The last of these six items is Item 17 ("I am good at remembering mathematics facts and principles"). This item was very prominent, as it relates to the application of the correct formulas for the areas of the different shapes. Therefore, this item links with Polya's third phase, *Carrying out the plan.*

Thus, there is strong evidence that the learners' metacognitive behaviours corresponded to the first three phases of Polya's problem-solving model. The question arises: Did Polya's fourth phase feature in these problem-solving sessions? Items that relate to Polya's fourth phase are Item 19 ("After I have solved a mathematics problem, I ask myself if there was an easier way to solve the problem") and Item 38 ("After I have solved a mathematics problem, I ask myself whether I have considered different ways to solve the problem"). These two items are part of the subscale *Evaluation.* They were not applied once during both problem-solving sessions. Time constraints could have prevented the learners from reflecting on the problem-solving process and, in particular, from finding alternative solutions for the problem.

Apart from *Evaluation,* it is worth considering which other subscales of the MAI did not feature prominently during the two problem-solving sessions. These were *Procedural knowledge* (knowledge about how to implement problem-solving strategies); *Conditional knowledge* (knowledge about when and why to use learning procedures), and *Debugging* (strategies used to correct comprehension and performance errors). In all fairness, it must be stated that most of the items of these four subscales relate to the broader mathematical learning process and are not applicable to a problem-solving situation. *Conditional knowledge*, for example, relates more to learners' study habits than to their problem-solving behaviours. However, I would have expected the two items of *Debugging* to feature to a greater extent. These were Item 40 ("I change my problem-solving method when I fail to make progress when I try to solve a mathematics problem") and Item 43 ("If I do not make progress when I solve a mathematics problem, I ask myself whether my first understanding of the problem was correct").

Mark made only one reference to learner metacognition in his analysis of the learner responses in both problem-solving sessions (see 6.2.4):

> ... their second approach contained more diagrams and more detailed on-going reflection throughout the task. Factors affecting this include an improved familiarity with the task as well as a general level of comfort with reflection and communicating mathematical ideas. [...] I would suggest that the approach the second time around indicates that they had developed in this regard.

According to Mark, the learners exhibited a greater level of metacognitive awareness during the second problem-solving session, as there was more "detailed on-going reflection throughout the task". His perspective was that the learners had developed in respect of their level of metacognition, because they displayed "a general level of comfort with reflection and communicating mathematical ideas".

The purpose of the interpretation of the learners' level of metacognition during both problem-solving sessions was to explore secondary research question 7: What is the impact of MI on the level of learner metacognition in a problem-solving context?

My thoughts on this question are as follows. First, although the learners recorded their thoughts surprisingly well during both problem-solving sessions, a clear improvement in their level of metacognition was not evident, because the same subscales and items featured to a similar extent.

Secondly, in both sessions, the learners' responses mainly related to the first three phases of Polya's problem-solving model. In neither problem-solving session did the learners reflect on their solutions to a noticeable extent, in other words, metacognitive awareness corresponding to the *Looking back* phase only occurred to a minor degree.

Mark stated that the learners had displayed greater on-going reflection and communication of mathematical ideas during the second problem-solving session (see 6.2.4). Based on my analysis of their level of metacognition, I am of the opinion that there was not a prominent improvement in the learners' on-going reflection.

In conclusion, it is my opinion that the level of learner metacognition in a problem-solving context was very similar prior to and after MI and that MI, therefore, did not have a prominent effect on the learners' level of metacognition as displayed in a problem-solving context.

6.2.6 Interpretation of the level of mathematics achievement during the first and second problem-solving sessions

I wonder what the prediction of mathematics teachers will be in respect of the number of learners in the experimental group who would have solved the problem successfully. I am of the opinion that there would not have been a great deal of consensus, as I presume that mathematics teachers, in general, do not have many opportunities to facilitate this type of problem-solving sessions. However, I estimate that most mathematics teachers would have expected – as I did – that, say, at least 10 of the 24 learners (Learner 16 did not take part in the first problem-solving session) would show that the circle has the biggest area. However, only one learner calculated that the circle would be the best option after comparing the areas of a rectangle, a square, and a circle with one another. One should consider a few factors in order to understand why all learners, except one, failed to solve the problem in the first problem-solving session.

Time constraints could have played a role. The learners were given 20 minutes to solve the problem and an analysis of the learners' answers indicates that the majority of them made a substantial number of calculations (see Appendix C2). Yet, one must take into account that the format of the problem-solving session was novel in respect of at least one, and probably two aspects. First, the learners did not previously give a written account of their thoughts during a problem-solving session. Secondly, the problem statements of previous problem-solving sessions, in which the learners took part during normal school periods, were probably not stated in such vague terms in respect of what they were expected to do. Normally, textbook problems follow after certain topics have been addressed. This implies that learners generally have a good idea of where to start, as opposed to the problem where the mathematical topics are concealed to a considerable degree.

Only one learner solved the problem successfully in the first problem-solving session, whereas five learners were successful in their second attempt. I expected more learners to be successful in the second problem-solving session, because I facilitated a whole-class discussion after the first problem-solving session in which the solution was provided. However, the second problem-solving session took place more than five months after the first problem-solving session, and some learners probably forgot the solution. In addition, the learners were not aware that there would be a second problem-solving session that entails exactly the same problem. Therefore, some of the learners probably did not concentrate well enough when I facilitated the process of finding the solution. Yet, I also expected more learners to be successful with their second attempt, because they would have experienced growth in their mathematical ability over the intervention period.

In Mark's analysis of the learner responses in both problem-solving sessions, he referred to common mistakes; for example, not calculating the length of the radius and using an arbitrary rectangle in their calculations (see 6.2.4). His statement that "learners seem uncomfortable with converting between a word problem and algebra" is indicative of the complexity of translating everyday language into the language of mathematics; in fact, Mark stated that this difficulty "is a common theme in many classrooms as far as I can tell".

He definitely observed an improvement in their level of achievement as "learners who were doing the activity for the second time [displayed] a greater readiness to dive into the algebra" and "the approach the second time was more technically mathematical". Although Mark used the term "algebra", there was no evidence of algebraic procedures in which variables were manipulated.

Nevertheless, according to Mark, this improvement was not necessarily due to the metacognitive intervention. He attributed the learners' improvement in the use of algebraic procedures to, possibly, the fact that they were "primed that this was the approach that was expected" or "because they had seen the solution and were now exhibiting a behaviourist approach and responding to training".

The second secondary research question which this section seeks to explore is secondary research question 8: What is the impact of MI on the level of mathematics achievement in a problem-solving session? Although the learners were more successful the second time round, I do not believe that even with the improvement accounted for did the learners perform adequately. My statement is debatable, as it would be difficult to reach consensus on what an acceptable level of achievement for Grade 11 learners is in respect of this problem. My verdict is based mainly on the numerous conceptual errors in both problem-solving sessions.

Yet, more learners were successful in their second attempt, but the improvement cannot be readily ascribed to the impact of MI. Mark ascribed this improvement either to a greater inclination to address the problem algebraically – which, I believe, could partly be due to the impact of MI – or because they remembered what the solution was. I believe that, although there was an improvement in the number of learners who solved the problem successfully, the many conceptual errors that were still present in the second problem-solving session indicate that the majority of the learners did not improve. The level of mathematics achievement in a problem-solving context prior to and after MI, therefore, was very similar, with some evidence of an improvement which could be attributed partly to MI.

6.3 TEACHER INTERVIEWS

I conducted semi-structured interviews with Mark and Lisa in which they were requested to share their perspectives on the nature of mathematics and aspects related to the teaching-and-learning of mathematics.

Both teachers' interviews were transcribed and analysed to identify themes and sub-themes (see Appendices D1-D6). A comparison of their views is presented in Appendix D7. An interpretation of the interview with Mark follows in order to explore secondary research question 9:

- What are the perspectives of the experimental group's teacher on the nature of mathematics and aspects related to the teaching-and-learning of mathematics?

Subsequently, an interpretation of the interview with Lisa follows in order to address secondary research question 10:

- What are the perspectives of the control group's teacher on the nature of mathematics and aspects related to the teaching-and-learning of mathematics?

6.3.1 Interpretation of the interview with Mark

The exploration of secondary research question 9 and secondary research question 10 involves the interpretation of the aspects that emerged in the analysis of the interviews with Mark and Lisa (see Appendices D1-D7).

Mark's perspective on the nature of mathematics entailed the following. He viewed mathematics as an important subject, but not as the only subject that could develop analytical reasoning skills. His statement about the higher regard that universities have for mathematics, as compared to mathematical literacy, is very relevant, as it is commonly perceived that mathematical literacy is much easier than mathematics. In addition, he stated that the applications of mathematics in everyday life were not always directly related to learners' lives. It seems, therefore, that Mark had a high regard for mathematics, but that he did not lend it more importance than other subjects.

My strongest impression after the interview with Mark relates to his statements on the importance of problem-solving, the difficulty that learners experience with this aspect, and the negative attitude towards mathematics that some learners have due to their lack of problem-solving ability.

He also pointed out the inadequacy of the lower grades in preparing mathematics learners to successfully address the problems in the FET-phase. Mark linked effective teaching with the development of learner understanding in a problem-solving context. In fact, he regarded a learner-centred lesson, where learners completed a group activity and in which he only played a facilitative role, as one of the best classes that he had ever presented. Although he stressed the value of facilitating mathematics by using group work, he mentioned time constraints as an obstacle to the effective teaching of mathematics.

In exploring secondary research question 9, therefore, it is evident that Mark viewed mathematics as an important subject that could be beneficial to learners' cognitive development. In addition, his perspectives on the teaching-and-learning of mathematics centred on the importance of problem-solving, learners' general lack of problem-solving skills, and the negative impact that learners' lack of problem-solving skills has on their attitude towards mathematics.

The two strongest aspects that emerged from Mark's interview were problem-solving in a group context, and learner affect. These two aspects relate to De Corte's (1996) educational learning theory as follows.

First, it was shown that the development of learners' problem-solving ability could be regarded as one of the main objectives of mathematics teaching (see 2.2.4.4m and 3.2.2.2). The strong relationship between problem-solving skills and mathematical proficiency has been established in a previous section (see 3.2.2.3). Hence, the *heuristics* aspect of De Corte's (1996) learning theory features prominently in Mark's interview. Mark's reference to the value of facilitating a problem-solving session by using group work relates to the *situated and collaborative* aspect of De Corte's (1996) educational learning theory which entails, among others, the enhancement of learners' cognitive activities in a social context (see 3.3.1.5).

Secondly, Mark's views on learner affect confirm the importance of *affective components* in the effective learning of mathematics.

6.3.2 Interpretation of the interview with Lisa

Lisa shared the following two most prominent perspectives on the nature of mathematics. The first perspective relates to her view of mathematics as a tough subject that demands commitment, a fighting spirit and hard work in order to survive. A second perspective entails the excessive importance given to the subject mathematics at the cost of learners' positive attitudes and quality of life.

Lisa's first perspective relates to the severe demands that mathematics makes on learners. In Lisa's view, learners will cope better with these demands if they study in such a way as to re-create the pressure situation of examinations. Lisa's reference to the hard work required to be successful in mathematics relates strongly to the definition of the *constructive* element of effective learning which states that learning "is an effortful and mindful process" (see 3.3.1.1). Her statement that the effort required in order to succeed in mathematics especially applied to problem-solving due to the necessity of continually practising the solutions. Therefore, a clear link with the importance of the *heuristic* aspect of expert performance is evident.

In respect of the second perspective, Lisa stated that effective learning was also jeopardised by negative learner attitudes resulting not only from the undue importance given to mathematics, but also by learners' lack of aptitude to successfully solve problems. In fact, a poignant statement which, in a sense, overarches Lisa's perspectives on the teaching-and-learning of mathematics is her description of learner behaviour when facing a problem-solving situation. She stated that learners "get frightened and they get scared because they feel insecure and they just close up". As *affective components* are "positive beliefs about the self in relation to learning and problem-solving in a domain ...", the important role of affective components in the learning process is highlighted by Lisa's second perspective.

Therefore, in addressing secondary research question 10, it is clear that Lisa highlighted the cognitive and emotional demands that mathematics makes on learners. Her perspectives on the nature of mathematics impact on her perspectives on the teaching-and-learning of mathematics, as the demands of mathematics influence effective learning by causing negativity among some learners. Lisa also stressed that the high demands made on teacher by time constraints diminish the opportunity to do a variety of problems which will benefit those learners who do not possess a natural flair for problem-solving.

6.4 LEARNER AND TEACHER PERSPECTIVES ON THE FIRST AND THE SECOND CYCLES OF THE MI PROCESS

At the end of Term 2, after the first cycle of MI, the learners were asked to give feedback on the use of the tool. They had to respond to two open-ended questions. The first question enquired about their experiences relating to the use of the codes. The second question allowed the learners to make suggestions about possible better or easier ways to use the tool (see Appendix E2).

After the first cycle of the MI process, Mark and I discussed his and the learners' feedback on the MI process during Term 2. Our discussion resulted in the adaptation of the codes booklet which was used during Term 3 (second cycle of the MI process).

The adapted MI codes booklet contained all the aspects (codes) of the original MI codes booklet, but the format was changed due to the incorporation of the teacher's and learners' feedback (see Appendix E9). The adapted codes booklet was a more visual representation of the codes, but the original MI codes booklet still had to be consulted, as it contained the explanation of each code. At the start of Term 3, Mark and I wrote a letter to the learners explaining to them how we have incorporated their feedback into the adapted MI codes booklet (see Appendix E11).

The learner and the teacher perspectives on both cycles of the MI process were analysed (see Appendices E4, E5, E7, E14, E15, and E17). This analysis forms the basis for the interpretation of these perspectives in order to address the last two secondary research questions:

- Secondary research question 11: What are the perspectives of the experimental group's learners on the MI process?
- Secondary research question 12: What are the perspectives of the experimental group's teacher on the MI process?

6.4.1 Interpretation of the learners' perspectives on the MI process

There was a vast difference between the learners' perspectives on the first cycle and the second cycle of the MI process. After the first cycle, their responses contained many negative references, especially in respect of the time it took to complete the MI codes booklet. Their perspective is understandable if one considers that the codes booklet was comprehensive in its attempt to include all the aspects related to the effective learning of mathematics according to De Corte's (1996) educational learning theory. In the case of learners who have not given their learning experiences much conscious thought in the past, it seems obvious that it could have been a daunting task to be confronted with so many aspects to consider when engaging with mathematics.

Mark also identified time and effort issues as the learners' most prominent complaint after the first cycle. By contrast, there were very few negative remarks about the use of the adapted MI codes booklet during the second cycle of the MI process.

Therefore, in addressing secondary research question 11, the following aspects emerge as indications of the learners' perspectives on the MI process. First, although many learners were negative after the first cycle of the MI process, the fact that their feedback was incorporated into the adapted MI codes booklet enhanced their experience to such an extent that the majority of the learners were very positive about the use of the adapted MI codes booklet. Once again, the importance of affective components in the learning process was affirmed by the learners' positivity about the use of the adapted tool. Although there was no difference in the theoretical foundations of the first tool and the adapted tool, the fact that the second tool was presented in a more learner-friendly format was the major cause for the improved learner attitudes.

Secondly, Learner 15's statement "All in all it satisfies the components of smart learning of mathematics" perhaps summarises many learners' perspectives best. Learner 15 probably did not have knowledge of what literature states about effective learning in mathematics, but she experienced the tool as addressing the components that enable effective learning in mathematics. In fact, many learners commented on improved mathematical understanding that could be related to De Corte's (1996) educational learning theory. These aspects are discussed next.

The learner references to improved mathematical understanding mainly relate to the *constructive* component of effective learning, but they also mentioned an improvement in problem-solving skills which links with the *heuristic* aspect of expert performance. Specific references to understanding a question and alternative solutions relate to two phases of Polya's problem-solving model, namely *Understanding the problem* and *Looking back* (see 2.2.4.4m). The learner references to thinking outside the box and an improved awareness of matters that were not related to mathematics only demonstrate a feature of *Looking back,* namely to seek applications of a solution in other contexts.

Learner comments about the following aspects relate to the *cumulative* aspect of effective learning: an improved understanding about the relationships between mathematics topics; the value of mind-maps, and better understanding of the subtopics of trigonometry. Of particular interest is one learner's comment about the benefit of taking the *cumulative* aspect of learning into account, namely that it put "some sense" into what she was doing.

The majority of the learners did not really experience an enhancement in their *goal-setting* ability after the first cycle of the MI process. Learner 2 was the only learner who made an explicit reference to improved goal-setting ability after the second cycle of the MI process. As the teacher plays a prominent role in assisting learners with goal-setting activities (see 3.3.1.4), it is possible that Mark did not emphasise this aspect to a great degree.

Learner references to the applicability of mathematics relate to the *situated* aspect of effective learning. The following learner responses relating to an improved mathematical understanding also point to an improved understanding of the applicability of mathematics: thinking outside the box; having a broader view of mathematics; discovering alternative solutions, and the application of mathematics in the "outside world".

Particular references were made to the *collaborative* aspect of effective learning in the learners' feedback after the second cycle of the MI process. Learners mentioned that the tool enabled them to learn from peers.

References to the *individually different* aspect of effective learning were also evident from the learners' perspectives. The strongest references to this aspect after the first cycle of the MI process were evident from the learners' suggestions for the improvement of the MI process; in particular, suggestions about the format of the tool. Individual differences in respect of the learning process were apparent from the variety of suggestions offered about the format of the tool. After the second cycle of the MI process, some learners referred explicitly to their learning style preferences. Specific learners referred to a better response to graphics, and a number of learners referred to the benefits they derived from the visual representation of the mathematical concepts.

A *knowledge basis* needs to be “well-organised and flexibly accessibly [..] involving the facts, symbols, concepts, and rules that constitute the contents of a subject-matter field” (see 3.3.2.1). Some learner comments after the second cycle of the MI process related to the *knowledge basis* aspect of expert performance, because they mentioned the effective organisation of the mathematical concepts by using a visual representation. Other learners reported an improved understanding of the links between mathematics topics. To some learners, the adapted tool enabled them to summarise the relevant facts and skills of the topic in an organised manner. The organisation of the mathematics content through the identification of common questions also contributed to learners’ preparation for the examination. Although the tool was initially developed to accompany a learner’s daily learning activities, the flexibility of the tool is illustrated by a learner’s comment on the value of the adapted tool as a revision aid.

Although learners were much more positive about the second cycle of the MI process, a significant number of learners also expressed positivity about the use of the codes after the first cycle of the MI process. References to *affective components* were very prominent in the learners’ perspectives on both cycles of the MI process. After the second cycle, a number of learners mentioned an improved attitude due to the more visually oriented adapted tool. In addition, the identification of common mistakes and common questions enhanced some learners’ attitude.

Links with the *self-regulation* and *metacognition* components of effective learning and expert performance were evident from the learners’ comments about enhanced awareness of their thinking processes. The comments of Learner 16 and Learner 21 were very interesting, as they referred to an improved awareness of matters which not only related to the mathematics context. To me, their comments imply that these two learners internalised the idea behind the codes to such an extent that their general metacognitive awareness was enhanced. Further links with the *self-regulation* and *metacognition* aspects are evident from their comments on an improved awareness of the following aspects: the mathematics topic; their learning styles; common mistakes; common questions; level of understanding, and personal feelings. Mark also referred to some learners' experiences of an enhancement in their self-assessment ability which relates to their "understanding of their understanding".

6.4.2 Interpretation of the teacher's perspectives on the MI process

In our initial discussion on the implementation of the MI process, Mark played an active role in adapting my initial idea into something more workable. His active involvement continued right through the first cycle of the MI process by continually reflecting on the process. Mark played a prominent role, in conjunction with the researcher, in adapting the MI process after the learners' feedback on the first cycle of the MI process, and his reflections were analysed. Therefore, I suggest that Mark really took ownership of the MI process and that his perspectives reflect his authentic experiences.

In our first discussion, Mark realised the value of the reflection sheet (as the MI codes booklet was called initially), but he also anticipated the extra demands it will make on learners. At the end of the first cycle of the MI process, Mark referred to the tool (the MI codes booklet) as "an excellent tool with all the potential to promote self-reflection and improved understanding". Although his fear concerning the expected constraints imposed by the MI process on the time available for teaching-and-learning was proven valid, he was still very positive about the potential of the tool to "aid reflection and comprehension of mathematics".

Some important aspects relating to Mark's perspectives become evident from the analysis of his responses after the second cycle of the MI process. The following aspects give a good indication of Mark's perspectives on the MI process. First, Mark was still very positive about the value of the MI codes booklet in enhancing mathematical reasoning processes. Secondly, Mark was willing to change his teaching habits in order to integrate the use of the MI codes booklet in his teaching and for revision purposes.

The last secondary research question, secondary research question 12, explores Mark's perspectives on the MI process. Upon considering the interpretation of Mark's perspectives on the first and second cycles of the MI process, I conclude that Mark viewed the MI process as very valuable in enhancing learners' self-reflection, mathematical understanding, and mathematical reasoning processes.

6.5 MIXED METHODS RESEARCH QUESTION

In this section, the mixed methods research question is explored:

- To what extent do the results from the qualitative phase of the study support the results obtained from the quantitative phase of the study regarding the effect on MI on learner metacognition and mathematics achievement?

First, the mixed methods research question is addressed by focusing on learner metacognition (see 6.5.1) and the effect of MI on mathematics achievement is subsequently explored from a mixed methods perspective (see 6.5.2). Relevant aspects related to learner metacognition and mathematics achievement of the control group are also discussed when they shed more light on the aspects that relate to the experimental group which received MI.

6.5.1 The effect of MI on learner metacognition from a mixed methods perspective

In the quantitative phase of this study, it was found that the experimental group's post-test median of the MAI total score was significantly higher than the median of their pre-test MAI total score (see 5.5.11.2). There was an improvement in respect of the MAI total score, and the two main factors, namely *KC* and *RC*. The score of only one of the three subscales of *KC*, namely *Declarative knowledge*, improved significantly. The score of two subscales of *RC* improved significantly, namely *Planning* and *Monitoring.* The scores of all other subscales of *KC* and *RC* also improved, with the exception of *Procedural knowledge* and *Debugging* (see Tables 5.8 and 5.32).

The control group experienced an improvement in the median score of five subscales, of which only *Conditional knowledge* and *Evaluation* improved significantly. The MAI total score and *RC* also improved but not significantly. *KC* and the following three subscales did not experience an improvement: *Declarative knowledge*, *Procedural knowledge*, and *Debugging* (see Tables 5.9 and 5.33).

In this section, the problem-solving sessions, the teacher interviews, and the teacher's and learners' perspectives on the MI process are explored in order to establish a broader perspective on the results obtained from the quantitative phase of this study.

6.5.1.1 First and second problem-solving sessions

In the qualitative section of this study, the experimental group's level of learner metacognition in two problem-solving sessions was explored.

Three of the four subscales that featured to the greatest extent in the problem-solving sessions – *Declarative knowledge*, *Planning*, and *Monitoring* – also showed a significant improvement in their scores on the pre-test and the post-test MAI. The post-test MAI score of the fourth subscale that featured prominently in the problem-solving sessions, *Information management*, also improved. These four subscales featured to a similar extent in both problem-solving sessions. I concluded that the MI did not have a prominent effect on the learners' level of metacognition, as observed in a problem-solving context (see 6.2.5). Mark, on the other hand, was of the opinion that the learners displayed a greater level of ongoing reflection during the second problem-solving session.

When comparing the quantitative and the qualitative data on learner metacognition, it must be borne in mind that the level of learner metacognition, as measured by the MAI, relates to many aspects in the learning of mathematics and is, therefore, much broader than the level of metacognition which learners can display in a problem-solving context. In addition, as problem-solving skills can be regarded as the most difficult mathematical skills to attain, it is plausible that the learners' level of metacognition pertaining to problem-solving would also be the most difficult to improve on.

The results from the quantitative data showed a significant improvement in some aspects of learner metacognition. The qualitative section of this study only revealed a partial improvement in learner metacognition. From a problem-solving perspective, I conclude that the results from the qualitative data only support the quantitative results (in respect of a significant improvement in learner metacognition) to some extent.

6.5.1.2 Teacher interviews

The purpose of the discussion in this section is to establish how the teachers' normal teaching methods could have impacted on the enhancement of learner metacognition.

6.5.1.2a Interview with Mark

In Mark's interview, the aspects relating to learner metacognition portrayed his points of view prior to the intervention. This gives an idea of which aspects of learner metacognition Mark could have enhanced as part of his daily teaching activities, prior to and during the intervention. His statement about the need for learners to identify their mistakes and to recognise why they have made a particular mistake relates to the *Conditional knowledge* subscale of the MAI, as it relates to knowledge of *when to* and *why* use a specific learning strategy. He also explicitly referred to the necessity for learners to be aware of their thinking processes which relate to the *KC* factor of the MAI.

His comments on problem-solving requiring true understanding, and the application of solutions in different contexts link with aspects of *Planning* (Item 22) and *Evaluation*, respectively. As Mark referred to the negative impact of time constraints on developing learner problem-solving skills, the learners' metacognitive awareness in respect of problem-solving skills was probably not enhanced to a great extent. This implies that the MAI subscales that show strong links with problem-solving most likely did not improve, namely *Procedural knowledge, Planning, Monitoring, Debugging,* and *Evaluation.*

Mark was aware of the impact of attitudes on the learning of mathematics, and he probably enhanced learners' awareness of their attitudes towards the learning of mathematics. Thus, learners' *Declarative knowledge* (Items 20 and 45) and *Conditional knowledge* (Item 26) could have been enhanced to some extent.

These perspectives that emerge from Mark's interview support the results from the quantitative data in respect of, first, a statistically significant improvement in the learners' *KC* and *Declarative knowledge* and secondly, a lack of improvement in the learners' *Procedural knowledge* and the *Debugging* subscale of the MAI. Hence, these results obtained from the qualitative data indicate that Mark's general teaching activities could have partially been responsible for the positive impact on learner metacognition that was ascribed to MI in the quantitative section.

The quantitative results that indicated a significant improvement in *Planning* and *Monitoring* are not supported by the perspectives obtained from Mark's interview. Therefore, MI was probably most effective in respect of the enhancement of these subscales, as it appears that Mark's normal teaching activities did not really support learner improvement in these areas.

6.5.1.2b Interview with Lisa

The two strongest aspects emerging from Lisa's interview were her emphasis on the demanding nature of mathematics, and the negative effect that the pressure associated with the learning of mathematics has on learners' attitudes. Lisa's perspectives on the demanding nature of mathematics probably enhanced the metacognitive awareness of the control group in respect of some aspects of *Declarative knowledge* (Item 5), *Planning* (Items 4 and 8), and *Monitoring* (Item 1).

The control group's metacognitive awareness in respect of their attitudes towards the learning of mathematics was most likely enhanced due to the academic pressure they experienced. Thus, the control group learners' *Declarative knowledge* (Items 20 and 45) and *Conditional knowledge* (Item 26) were probably also enhanced. The quantitative results pertaining to the significant improvement in the learners' *Conditional knowledge* is therefore supported. The quantitative section showed that the learners' *Declarative knowledge* was not enhanced significantly, although the qualitative data suggest an improvement in this aspect of learner metacognition. Thus, the perspectives obtained from Lisa's interview in respect of learner metacognition partly explain the improvement – although statistically insignificant – in *Declarative knowledge.*

Learner metacognition in respect of *Evaluation* also improved significantly in the quantitative measurement. Lisa referred to the lack of time to properly develop and practise learner problem-solving skills. Consequently, items relating to problem-solving of the subscales *Procedural knowledge, Planning, Monitoring, Debugging,* and *Evaluation* were most likely not enhanced to a great extent. It appears that the qualitative data obtained from the interview with Lisa does not support the significant improvement of the *Evaluation* subscale in the quantitative measurement.

6.5.1.3 Learner and teacher perspectives on both cycles of the MI process

The following learner viewpoints of the experimental group on both cycles of the MI process provide a broader perspective on the impact of MI on learner metacognition.

The fact that many learners complained about the time demands made on them during the first cycle of the MI process indicates that they put in a serious attempt to apply the instructions in the MI codes booklet. Consequently, their general metacognitive awareness was most probably enhanced. The learners' perspectives on both cycles of the MI process included positive references to all aspects of De Corte's (1996) educational learning theory. Since there is a strong relationship between aspects of the MAI and De Corte's (1996) educational learning theory (see Appendices B5-B7), these positive references implicitly relate to an enhanced metacognitive awareness. The learners also made explicit comments related to an enhanced metacognitive awareness.

Mark's perspectives on both cycles of the MI process, particularly his references to learners' improved self-reflection, also point to the positive impact of MI on learner metacognition.

Thus, the learner and teacher perspectives on both cycles of the MI process support the results obtained from the quantitative data of this study which indicated that MI had a significant effect on the MAI total score of the experimental group's learners.

6.5.2 The effect of MI on mathematics achievement from a mixed methods perspective

In the quantitative section of this study, the impact of MI on mathematics achievement was explored by examining the correlation between learner metacognition and mathematics achievement.

From a descriptive statistics point of view, it was found that *KC* had the strongest correlation with mathematics achievement. The correlation between *RC* and mathematics achievement was stronger than that between the MAI total score and mathematics achievement.

Inferential statistics indicated that only *KC* and two subscales (*Declarative knowledge* and *Evaluation*) had statistically significant correlations with the Term 1 report marks. There were statistically significant correlations between the following factors and subscales of metacognition and the Term 4 report marks: *KC*, *Declarative knowledge*, and *Procedural knowledge* (see Table 5.35). It is important to note that Hypothesis 5a was supported, namely that there is not a statistically significant positive relationship between learner metacognition (as measured by the MAI total score) and mathematics achievement.

6.5.2.1 First and second problem-solving sessions

In the quantitative section of this study, mathematics achievement was measured by using the Term 1 and Term 4 report marks. However, due to the many differences in how the report marks were composed, it was not realistic to compare the Term 1 and Term 4 report marks in terms of their scores only (see 5.3.13 and 5.5.16).

The qualitative section of this study afforded a better opportunity to study the impact of MI, as the same problem had to be solved in both problem-solving sessions. There was an improvement in the number of learners who solved the problem successfully. Only one learner was successful in the qualitative pre-test and five learners were successful in the qualitative post-test. The learners also improved in respect of the following aspects: the use of diagrams; the use of algebraic manipulations, and the communication of mathematical ideas. The improvements observed in the second problem-solving session can, to some extent, be ascribed to MI.

Learners' *Declarative knowledge* featured very prominently in both problem-solving sessions. Although the learners' level of metacognitive awareness – and more specifically *Declarative knowledge* – was very similar in both problem-solving sessions, their level of mathematics achievement was better in the second problem-solving session. Thus, the learners' better achievement cannot be ascribed to a higher level of *Declarative knowledge,* but to other factors. Yet, it is possible that some learners could have performed better if they had a higher level of *Declarative knowledge* in the second problem-solving session.

Therefore, the results from the quantitative section, in which it was found that *Declarative knowledge* had a significant correlation with mathematics achievement, are not fully supported, although the prominent role that *Declarative knowledge* played in both problem-solving sessions points to its relationship with mathematics achievement.

6.5.2.2 Teacher interviews

In this section, aspects emerging from the teacher interviews related to mathematics achievement are discussed.

6.5.2.2a Interview with Mark

Mark mentioned that one of the best lessons he had ever presented was learner-centred, where hardly any formal teaching took place. Learners' problem-solving skills were possibly enhanced, because the learners had to work more independently than usual in that lesson. The main perspectives emerging from Mark's interview were his references to the importance of developing learner problem-solving skills in a group context and the crucial role of learner attitudes in the learning of mathematics. Mark was already aware of the importance of these aspects prior to the start of the intervention. It is possible that Mark – out of his own volition and not due to the MI that was implemented – continually emphasised the importance of learner problem-solving skills during the intervention period.

It was shown in the quantitative section that MI enhanced the *KC* factor and *Declarative knowledge* subscale of learner metacognition significantly, and that there was a statistically significant correlation between, first, *KC* and mathematics achievement and, secondly, between *Declarative knowledge* and mathematics achievement (see Tables 5.32 and 5.35). *KC* and *Declarative knowledge*, as measured by the MAI, contain a number of items related to problem-solving. Consequently, Mark's general way of teaching could partly have been responsible for the improvement in *KC* and *Declarative knowledge*, with a subsequent improvement in mathematics achievement.

The perspectives gained from Mark's interview indicate that the significant improvement in *KC* and *Declarative knowledge*, as measured quantitatively, was possibly not only due to MI but also to Mark's general teaching activities.

6.5.2.2b Interview with Lisa

During the intervention period, Lisa most probably continued her efforts to stress the importance of hard work in the learning of mathematics. The number of extra classes she taught was evidence of her commitment to instil this principle. Thus, aspects of *Declarative knowledge, Planning, Monitoring,* and *Conditional knowledge* were probably enhanced during the intervention period. Of these subscales, only *Declarative knowledge* had a statistically significant correlation with mathematics achievement. Therefore, the control group's mathematics achievement, as measured by the Term 4 report marks, could partly be attributed to elements of Lisa's teaching that enhanced the learners' *Declarative knowledge.*

The qualitative data obtained from Lisa's interview support the findings from the quantitative section in respect of the significant correlation between *Declarative knowledge* and mathematics achievement to some extent.

6.5.2.3 Learner and teacher perspectives on both cycles of the MI process

A broader perspective on the impact of MI on mathematics achievement is provided by considering the experimental group's views and Mark's views on the MI process. In this discussion, the focus is on learner perspectives relating to the MAI factor and subscale that had a statistically significant correlation with the Term 1 and Term 4 report marks, namely *KC* and *Declarative knowledge* (see 5.5.17.2 and Table 5.35).

Declarative knowledge entails the following aspects. First, it involves learner knowledge about their own skills and abilities as mathematics learners. Secondly, it involves learner knowledge and understanding of what the important information in mathematics is and how to organise that information (see Appendix B2).

After the first cycle of the MI process, the learners' references to improved mathematical understanding relate to knowledge about important information in mathematics and knowledge about their mathematical skills. The learners' feedback after the second cycle of the MI process contained many references relating to De Corte's (1996) educational learning theory. These references relate to *Declarative knowledge* as they point to the learners' knowledge about which important aspects to concentrate on when they study mathematics. Thus, the learner perspectives on both cycles of the MI process support the finding in the quantitative section that the MI process improved the aspect of metacognition that had a significant relationship with mathematics achievement, namely *Declarative knowledge.*

In his feedback on both cycles of the MI process, Mark also highlighted an aspect of *Declarative knowledge*, namely the impact of MI on the learners' mathematical understanding. His statement on enhanced self-reflection by the learners due to the MI process relates to another aspect of *Declarative knowledge*, namely learners' knowledge about their skills and abilities as mathematics learners. Hence, Mark's perspectives on both cycles of the MI process also support the finding in the quantitative section that the MI process enhanced *Declarative knowledge* which had a significant correlation with mathematics achievement.

6.5.3 Summary

In Sections 6.1-6.4, perspectives on the MI process emerging from the qualitative data in respect of learner metacognition and mathematics achievement were explored in order to answer the mixed methods research question. In respect of learner metacognition, qualitative perspectives on the results obtained from the quantitative phase of the study are the following:

- The first and second problem-solving sessions only revealed a partial enhancement of learner metacognition in contrast to the statistically significant improvement in learner metacognition as measured by the MAI (see 6.5.1.1).
- The interview with Mark indicated that his general teaching methods could have been partly responsible for the statistically significant improvement in the learners' *KC* and *Declarative knowledge.* The statistically significant improvement in *Planning* and *Monitoring* was probably mainly due to the MI process and not to Mark's general teaching activities (see 6.5.1.2a).
- Perspectives from the interview with Lisa indicate that the statistically significant improvement in *Conditional knowledge* and the improvement in *Declarative knowledge* of the control group's learners could to a great extent be ascribed to her general teaching activities. However, the statistically significant improvement of *Evaluation* is not supported by the way in which she generally teaches (see 6.5.1.2b).
- The statistically significant improvement in learner metacognition due to MI, as measured by the MAI total score, is corroborated by the learners' and Mark's feedback on both cycles of the MI process (see 6.5.1.3).

The following perspectives emerged from the qualitative data in respect of mathematics achievement:

- The first and second problem-solving sessions partially support the findings from the quantitative section that failed to establish a significant correlation between the MAI total score and mathematics achievement. The quantitative findings that established a significant correlation with *Declarative knowledge* and mathematics achievement are also supported to some extent (see 6.5.2.1).
- The interview with Mark revealed that his general teaching activities – and not the MI process only – could also have contributed to the quantitative results that established a statistically significant improvement in the two aspects that had a

statistically significant correlation with mathematics achievement, namely *KC* and *Declarative knowledge* (see 6.5.2.2a).

- The qualitative data obtained from the interview with Lisa revealed how her daily teaching activities could have contributed to the improvement in the aspect of metacognition that had a significant correlation with mathematics achievement, namely *Declarative knowledge* (see 6.5.2.2b).
- The learners' and teacher's perspectives on the MI process support the finding in the quantitative section which stated that the MI process enhanced the aspect of metacognition that had a statistically significant correlation with mathematics achievement, namely *Declarative knowledge* (see 6.5.2.3).

The qualitative data provide broader perspectives on the findings of the quantitative section in respect of learner metacognition and mathematics achievement. These perspectives mainly support the quantitative findings. Although the quantitative findings are not fully supported in all instances, a richer understanding of the impact of MI on learner metacognition and mathematics achievement emerged from the analysis and interpretation of the qualitative data.

Table 6.1 provides an integrated summary of the findings related to the quantitative and qualitative phases of this study that were discussed in this section. The main focus of Table 6.1 is on the impact of MI on learner metacognition (first column) and mathematics achievement (second column) from a quantitative perspective. Where applicable, it is indicated to what extent the qualitative data support the quantitative data.

Table 6.1: Integrated summary of the quantitative and qualitative results

<table>
<tr><th colspan="2">Quantitative phase
(impact of MI on learner metacognition)</th><th colspan="2">Quantitative phase
(impact of MI on mathematics achievement)</th></tr>
<tr><td>Experimental group:
Statistically significant improvement:
(Yes / No)</td><td>Control group:
Statistically significant improvement:
(Yes / No)</td><td colspan="2">Both groups
Statistically significant correlation
(Yes / No)</td></tr>
<tr><td></td><td></td><td>Term 1</td><td>Term 4</td></tr>
<tr><td colspan="4">MAI total score</td></tr>
<tr><td>Yes</td><td>No</td><td>No</td><td>No</td></tr>
<tr><td colspan="2">Qualitative phase (Experimental group): The improvement due to MI is partially supported (problem-solving sessions; learner and teacher perspectives on the MI process).</td><td colspan="2">Qualitative phase (Experimental group): The failure to establish a significant correlation is partially supported (problem-solving sessions).</td></tr>
<tr><td colspan="4">KC</td></tr>
<tr><td>Yes</td><td>No</td><td>Yes</td><td>Yes</td></tr>
<tr><td colspan="2">Qualitative phase (Experimental group): The improvement due to MI is not supported as Mark's general teaching activities could have caused the improvement (interview with Mark).</td><td colspan="2">Qualitative phase (Experimental group): The correlation is partially supported (interview with Mark).</td></tr>
<tr><td colspan="4">Declarative knowledge</td></tr>
<tr><td>Yes</td><td>No</td><td>Yes</td><td>Yes</td></tr>
<tr><td colspan="2">Qualitative phase (Experimental group): The improvement due to MI is not supported as Mark's general teaching activities could have caused the improvement (interview with Mark).</td><td colspan="2">Qualitative phase (Experimental group): The correlation is partially supported (problem-solving sessions, interview with Mark, and learner and the teacher perspectives on the MI process).
Qualitative phase (Control group): The correlation is partially supported (interview with Lisa).</td></tr>
<tr><td colspan="4">Procedural knowledge</td></tr>
<tr><td>No</td><td>No</td><td>No</td><td>No</td></tr>
</table>

Quantitative phase (impact of MI on learner metacognition)		Quantitative phase (impact of MI on mathematics achievement)	
Experimental group: Statistically significant improvement: (Yes / No)	Control group: Statistically significant improvement: (Yes / No)	Both groups Statistically significant correlation (Yes / No) Term 1	Term 4
Conditional knowledge			
No	**Yes**	No	No
Qualitative phase (Control group): Lisa's general teaching activities could have caused the improvement (interview with Lisa).			
RC			
Yes	No	No	No
Planning			
Yes	No	No	No
Qualitative phase (Experimental group): The improvement due to MI is supported (interview with Mark).			
Information management			
No	No	No	No
Monitoring			
Yes	No	No	No
Qualitative phase (Experimental group): The improvement due to MI is supported (interview with Mark).			
Debugging			
No	No	No	No
Evaluation			
No	**Yes**	No	No
Qualitative phase (Control group): Lisa's general teaching activities probably did not cause the improvement (interview with Lisa).			

6.6 CONCLUSION

In this chapter, qualitative data were presented, analysed and interpreted in order to explore secondary research questions 7 to 12. The secondary research questions support the primary research purpose, namely to investigate the impact of MI on learner metacognition and mathematics achievement.

The qualitative data were obtained from problem-solving sessions, teacher interviews, as well as learner and teacher perspectives on the MI process. This data revealed perspectives related to learner metacognition and mathematics achievement which were then used to explore the mixed methods research question.

In exploring the mixed methods research question, it was found that the qualitative data enriched the understanding of the quantitative results by providing broader perspectives on the impact of MI on learner metacognition and mathematics achievement.

The findings, conclusions and recommendations emerging from this study are presented in Chapter 7.

CHAPTER 7

FINDINGS, CONCLUSIONS AND RECOMMENDATIONS

7.1 INTRODUCTION

The purpose of this study, namely to investigate the effect of MI on learner metacognition and mathematics achievement, originated from the concern about the poor performance of South African mathematics learners in the NSC examination (see Chapter 1). Four primary and 12 secondary research questions were stated in order to achieve the purpose of this study.

The first primary research question and secondary research questions 1-6 were explored by means of both a literature review (see Chapters 2 and 3) and a quantitative empirical investigation (see Chapter 5). The research design of this study was discussed in Chapter 4. In Chapter 6, the qualitative data obtained from the empirical investigation were presented, analysed, and interpreted in order to explore primary research questions 2-4, secondary research questions 7-12, and the mixed methods research question.

What follows is a summary of this study’s findings, conclusions, and recommendations that relate to the literature and the empirical investigation in respect of the research questions. The limitations of the study and suggestions for further research are discussed. The significance of the study is then framed within an international and a national context. This is followed by concluding remarks relating to this study.

7.2 FINDINGS, CONCLUSIONS AND RECOMMENDATIONS RELATED TO THE RESEARCH QUESTIONS

7.2.1 First primary research question

- Does MI have a statistically significant positive effect on learner metacognition and achievement in mathematics?

In order to explore this question, six secondary research questions were formulated. Perspectives gained from literature enabled the researcher to investigate the first four secondary research questions, namely:

- How is metacognition conceptualised?
- What is the relationship between metacognition and achievement in mathematics?
- What are the features of some previous metacognitive interventions in mathematics?
- What are the features of a proposed framework for a metacognition intervention in mathematics?

The next two secondary research questions ensuing from the first primary research question were explored by means of an empirical investigation.

- Does MI have a statistically significant positive effect on the metacognitive awareness of the experimental group's learners?
- Is there a statistically significant positive relationship between learner metacognition and mathematics achievement?

7.2.1.1 Findings

Although metacognition and metacognitive processes are not precisely defined and understood, the following aspects shed light on the conceptualisation of metacognition.

Metacognition involves both the knowledge and the regulation of cognition. Knowledge of cognition may be further categorised as declarative knowledge, procedural knowledge, and conditional knowledge. Knowledge of cognition also includes metacognitive experiences, which entail awareness of one's affective state, and metacognitive goals, which involve awareness of the importance of setting goals (see 2.2.7).

Regulation of cognition relates to the monitoring and regulation of one's metacognitive strategies. Metacognitive strategies include planning; generating questions; choosing consciously; setting and pursuing goals; evaluating one's way of thinking and acting;

identifying the difficulty; paraphrasing, elaborating and reflecting learners' ideas; clarifying learners' terminology; thinking aloud; journal-keeping; cooperative learning; modelling, and problem-solving activities. During cognitive monitoring and regulation there is a close interaction between the aspects of knowledge of cognition and regulation of cognition (see 2.2.5; 2.2.7).

These aspects related to the conceptualisation of metacognition aid in the distinction between metacognition, self-regulation and SRL as they point to metacognition's more prominent *individual* and *cognition* dimensions as compared to self-regulation and SRL (see 2.2.6; 2.2.7).

In respect of secondary research question 2, caution needs to be exercised when the relationship between metacognition and mathematics achievement is interpreted, because metacognition is not precisely defined and, therefore, difficult to measure accurately. In addition to learner metacognition, other factors such as learner motivation can also impact on mathematics achievement. Mathematics achievement is a broad concept that can be measured either quantitatively or qualitatively (see 2.3.10).

Six of the nine previous studies explored in this study employed a pre-test and a post-test in the quantitative and qualitative measurements. In two of these studies, it was stated that course grades are not very reliable measures of mathematics achievement. Except for one study, these studies all reported a significant positive correlation between learner metacognition and mathematics achievement, especially mathematics achievement measured by open-ended questions and involving problem-solving contexts (see 2.3.10).

In order to address secondary research question 3, previous metacognitive intervention studies were examined in respect of the following features: aims; age of participants; intervention period; theoretical base; method of intervention, and measurement of learner metacognition.

The aims of all these studies related to, first, the improvement of mathematics achievement, mathematics reasoning skills, and mathematical problem-solving skills.

Secondly, some studies were aimed at enhancing learner metacognition or learner self-regulation (see 2.4.7.5).

In previous metacognitive intervention studies, the participants' ages ranged from Grade 4 to pre-college students. The intervention period in the studies, which stated only mathematics-related aims, ranged from eight weeks to the entire academic year, whereas the intervention periods of the studies, which also stated aims related to self-regulation or metacognition, ranged from four to six weeks (see 2.4.7.5).

The studies that only stated mathematics-related aims based their intervention on theory relating to problem-solving contexts, cooperative settings, corrective feedback, and enrichment. In the studies where aims related to the enhancement of metacognition or self-regulation were also stated, the intervention was based on theory relating to the conceptualisation of metacognition as knowledge of cognition and regulation of cognition; learner affect, and learner autonomy (see 2.4.7.5).

The following elements were addressed in the intervention methods of these studies: problem-solving contexts; corrective feedback; active teacher involvement; cooperative settings; individual settings; enrichment opportunities, and learner affect. In respect of the last feature of previous metacognitive intervention studies, it was found that the measurement of self-regulation or metacognition involved quantitative and qualitative pre-test and post-test measures (see 2.4.7.5).

Findings relating to the fourth secondary research question suggest that a proposed framework for a metacognitive intervention in mathematics should integrate the elements of a mathematical perspective on De Corte's (1996) educational learning theory with the features of previous metacognition interventions in mathematics (see 7.5).

The empirical investigation relating to the fifth secondary research question revealed three aspects which indicated that MI had a statistically significant positive effect on learner metacognition. First, Hypothesis 1b was supported which states that the median of the experimental group's pre-test MAI total scores was significantly lower than that of the control group. Secondly, Hypothesis 2a was supported which states that there is not a statistically significant difference in the medians of the post-test MAI total scores of both the experimental group and the control group (see 5.5.8.2; 5.5.9.2). Thirdly, the median of the experimental group's post-test MAI total score was significantly higher than the median of the pre-test MAI total score, but there was not a significant improvement in the control group's median of the post-test MAI total score. Therefore, Hypotheses 3b and 4a were supported (see 5.5.11.2; 5.5.12.2).

Apart from the statistically significant improvement in the median of the experimental group's post-test MAI total score, the following factors and subscales of the MAI also improved significantly: *KC, RC*, *Declarative knowledge*, *Planning* and *Monitoring* (see 5.5.11.2). All other subscales, except *Procedural knowledge* and *Debugging* also improved but not significantly.

The empirical investigation relating to the sixth secondary research question failed to establish a statistically significant correlation between the MAI total score and mathematics achievement. Therefore, Hypothesis 5a was supported which states that there is no statistically significant positive relationship between learner metacognition and achievement in mathematics (see 5.5.17.2).

Although a statistically significant correlation between the MAI total score and mathematics achievement could not be established, there was a significant correlation between *KC* and mathematics achievement, and between *Declarative knowledge* and mathematics achievement (see 5.5.17.2). These statistically significant correlations were obtained when a more balanced perspective on the achievement of the learners was provided by combining the report marks of the experimental group and the control group for Terms 1 and 4, respectively (see 5.5.17.2).

7.2.1.2 Conclusions

The first primary research question focuses on the impact of MI on learner metacognition and mathematics achievement. Despite the different conceptualisations of metacognition, in literature the MAI is regarded as a valid instrument for measuring learner metacognition according to the factors *KC* and *RC.* The researcher regards the MI implemented in this study as a valid metacognitive intervention, as it is based on past metacognitive interventions and a mathematical perspective on De Corte's (1996) educational learning theory. Therefore, based on the findings, it is concluded that MI had a statistically significant impact on learner metacognition.

The impact of MI on mathematics achievement was less pronounced, as inferences had to be drawn from the correlation between learner metacognition and mathematics achievement. The findings indicated a statistically significant correlation between *KC* and mathematics achievement, as well as between *Declarative knowledge* and mathematics achievement. Since MI had a statistically significant impact on *KC* and *Declarative knowledge*, it is concluded that MI had a positive – although not necessarily a statistically significant – impact on mathematics achievement.

7.2.1.3 Recommendations

The different definitions of the concept *metacognition* should be borne in mind in the conceptualisation of metacognition, but knowledge of cognition and regulation of cognition are the two main factors that should be present in the conceptualisation of metacognition. In addition, mathematics achievement should be measured in a variety of ways in order to accommodate different viewpoints on what true mathematics achievement entails, and metacognitive interventions should include the elements of the proposed framework for metacognitive interventions.

To improve mathematics achievement in the South African context, it is recommended that aspects relating to *Declarative knowledge* are addressed in the mathematics teaching-and-learning situation, as it was the only subscale of *KC* that had a significant correlation with mathematics achievement. Therefore, teachers should endeavour to enhance the following learner-related aspects: understanding of their strengths and weaknesses in mathematics; knowledge of what kind of information is the most important to learn in mathematics; ability to organise the information they receive in mathematics; knowledge of what the teacher expects them to study for a mathematics test or examination; ability to remember mathematics facts and principles; ability to control how well they learn in mathematics; ability to judge how well they understand different aspects of mathematics, and their attitudes towards mathematics.

7.2.2 Second primary research question

- What is the effect of MI on learner metacognition and mathematics achievement in a problem-solving context?

Qualitative secondary research questions 7 and 8 were formulated to address the second primary research question. These qualitative secondary research questions, which were explored by means of an empirical investigation, are:

- What is the impact of MI on the level of learner metacognition in a problem-solving context?
- What is the impact of MI on the level of mathematics achievement in a problem-solving context?

7.2.2.1 Findings

In both problem-solving sessions, an application of individual items of the following four MAI subscales featured to the greatest extent in the learners' responses: *Declarative knowledge, Planning, Information management*, and *Monitoring.* The extent to which the applicable items of each of these four subscales were applied in the first problem-solving session was very similar in the second problem-solving session (see 6.2.2; 6.2.5). The learners' responses mainly related to the first three phases of Polya's problem-solving model. They did not really reflect on their solutions, indicating a lack of metacognitive awareness corresponding to Polya's fourth phase (*Looking back*) (see 6.2.5).

The learners possibly displayed a higher level of metacognitive awareness in respect of reflection and the communication of mathematical ideas in the second problem-solving session, but an overall analysis indicates that MI did not have a prominent effect on the level of learner metacognition in a problem-solving context in this study (see 6.2.5).

In respect of secondary research question 8, it was found that the level of mathematics achievement possibly improved, as more learners solved the problem successfully in the second problem-solving session, and algebraic procedures were employed to a greater extent (see 6.2.3; 6.2.6). However, many of the conceptual errors that had been made in the first problem-solving session were repeated in the second problem-solving session. When the few learners who were successful in both problem-solving sessions and the numerous conceptual errors are considered, it becomes evident that the learners' level of mathematics achievement in both problem-solving sessions was not satisfactory (see 6.2.6).

7.2.2.2 Conclusions

MI did not have a prominent effect on learner metacognition in a problem-solving context, as the subscales *Declarative knowledge, Planning, Information management,* and *Monitoring* were applied to the same extent during both problem-solving sessions. The fact that *Looking back*, the fourth phase of Polya's problem-solving model, featured to a very limited extent in both problem-solving sessions points to a lack of learner reflection on the validity of their answers.

The findings point to a positive impact of MI on mathematics achievement, but the numerous conceptual errors in both problem-solving sessions, however, indicate that the positive impact of MI on the experimental group's achievement was not very prominent.

7.2.2.3 Recommendations

Learners' problem-solving skills should be enhanced by paying specific attention to those subscales that did not feature to a great extent during both problem-solving sessions, namely *Procedural knowledge, Conditional knowledge, Debugging,* and *Evaluation.* Learners' basic conceptual errors should be identified and addressed, as these have a negative impact on the level of achievement in a problem-solving context.

7.2.3 Third primary research question

- What are the teachers' views on the nature of mathematics and aspects related to the teaching-and-learning of mathematics?

Secondary research questions 9 and 10 result from this question. These qualitative questions are:

- What are the perspectives of the experimental group's teacher on the nature of mathematics and aspects related to the teaching-and-learning of mathematics?
- What are the perspectives of the control group's teacher on the nature of mathematics and aspects related to the teaching-and-learning of mathematics?

7.2.3.1 Findings

The experimental group's teacher viewed mathematics as an important subject that may enhance learners' analytical reasoning skills. He stressed five aspects related to the teaching-and-learning of mathematics.

First, the development of learner understanding in a problem-solving context was the most important aspect in the teaching-and-learning of mathematics. Secondly, learners generally have a lack of problem-solving skills in mathematics. A third aspect relates to the negative impact that learners' poor problem-solving skills have on their attitudes towards mathematics. Fourthly, effective learning may be promoted in learner-centred lessons within a group context. Fifthly, time constraints have a negative impact on the effective teaching of mathematics (see 6.3.1).

The control group's teacher emphasised the difficulty of mathematics as a subject and the cognitive and emotional demands it places on learners (see 6.3.2). She highlighted three aspects in respect of the teaching-and-learning of mathematics.

First, the problem-solving aspect of mathematics is very important, but it requires natural ability and the continual practising of problems and their solutions in order to experience success. Secondly, the learners' attitudes towards mathematics are negatively influenced both by the undue importance given to mathematics and by the learners' poor problem-solving skills. Thirdly, time constraints hamper the proper development of learner problem-solving skills (see 6.3.2).

7.2.3.2 Conclusions

Both teachers highlighted the cognitive demand that mathematics places on learners, especially in a problem-solving context. Both teachers affirm the importance of enhancing learners' problem-solving skills, and it was evident that time constraints had a major impact on the development of this aspect.

7.2.3.3 Recommendations

The enhancement of learners' problem-solving skills should be a key priority in the mathematics teaching-and-learning situation. In addition, the development of support mechanisms that will enable learners to better cope with the high demands of mathematics should be explored. Furthermore, teachers should strive to use time optimally so that problem-solving sessions can be conducted.

7.2.4 Fourth primary research question

- What are the perspectives of the experimental group's learners and their teacher on the MI process?

The next qualitative questions, secondary research questions 11 and 12, follow from the fourth primary research question:

- What are the perspectives of the experimental group's learners on the MI process?
- What are the perspectives of the experimental group's teacher on the MI process?

7.2.4.1 Findings

The experimental group's learners referred to negative and positive aspects concerning the first cycle of the MI process. Many learners experienced the application of the MI codes booklet as very time-consuming and laborious. A number of learners commented on an improvement in respect of the following aspects of the learning process: mathematical understanding, awareness of one's thinking processes; goal-setting ability, and attitude. There were also some positive statements about the way in which the MI codes booklet was implemented (see 6.4.1).

Since the learners' feedback on the first cycle of the MI process was used to adapt the MI codes booklet, the majority of the learners were positive about the MI process after the second cycle. Their perspectives on the second cycle of the MI process attest to an enhancement of their learning processes in respect of all aspects of De Corte's (1996) educational learning theory (see 6.4.1).

The teacher's perspectives reflect the prominent role he played in the implementation and the adaptation of the MI process. Before the start of the first cycle, he anticipated that the MI process would place extra demands on the learners. Despite some negative learner comments on the first cycle of the MI process, he viewed it as an excellent way to improve learner self-reflection and mathematical understanding (see 6.4.2).

After the second cycle of the MI process, the teacher's positivity about the MI process became evident from the three aspects he mentioned. First, he was willing to integrate the use of the MI codes booklet with his daily teaching activities. Secondly, he also valued the MI codes booklet as a revision aid. The third aspect relates to his experiences about learner improvement in the following areas: self-reflection, mathematical understanding, and mathematical reasoning processes (see 6.4.2).

7.2.4.2 Conclusions

The learners' views on the MI process (especially the second cycle) as being valuable in enhancing effective learning in mathematics point to the applicability of this study's MI in the South African context. The teacher's perspectives on the effectiveness of the MI process in enhancing learners' self-reflection, mathematical understanding and mathematical reasoning processes lend further support to this statement.

7.2.4.3 Recommendations

In the implementation of metacognitive interventions, relevant learner and teacher perspectives on the improvement of the intervention process should be incorporated judiciously in order to facilitate effective learning and to improve learner attitudes.

7.2.5 Mixed methods research question

- To what extent do the results from the qualitative phase of the study support the results obtained from the quantitative phase of the study regarding the effect of MI on learner metacognition and mathematics achievement?

The mixed methods research question was addressed by exploring the perspectives obtained from the first and the second problem-solving sessions, the teacher interviews, and the learners' and teacher's perspectives on both cycles of the MI process.

7.2.5.1 Findings

In the quantitative section of the study, *Declarative knowledge, Planning,* and *Monitoring* had a statistically significant improvement in respect of their pre-test and post-test MAI total scores. *Information management* also improved, but not on a statistically significant level. These four subscales featured to the greatest extent in both problem-solving sessions. In contrast to the findings in the quantitative section of the study, these subscales did not feature more prominently in the second problem-solving session. Although the experimental group's learners displayed a greater level of on-going reflection during the second problem-solving session, MI did not have a prominent impact on learner metacognition in a problem-solving context. In respect of the impact of MI on learner metacognition, the qualitative data only support the quantitative results to a limited extent (see 6.5.1.1).

The interview with Mark revealed that his general way of teaching could have enhanced the learners' *KC* and *Declarative knowledge.* Therefore, the significant improvement in the median scores of these two aspects of metacognition – as reported in the quantitative section – is supported by the qualitative data. The quantitative results that indicated a significant improvement in *Planning* and *Monitoring* were not supported by the perspectives obtained from Mark's interview in respect of the negative impact of time constraints on the development of learner problem-solving skills (see 6.5.1.2a).

The perspectives that emerged from Lisa's interview support the statistically significant improvement in the learners' *Conditional knowledge* and the improvement in the learners' *Declarative knowledge*. The significant improvement of *Evaluation* due to MI was not supported by the qualitative data (see 6.5.1.2b).

The feedback of the experimental group's learners on both cycles of the MI process contained direct references to improved awareness of their thinking processes. The teacher also referred to the enhancement of the learners' self-reflection due to MI. Thus, the quantitative results that showed a significant improvement in the level of learner metacognition due to MI were supported by the learners' and teacher's perspectives on both cycles of the MI process (see 6.5.1.3).

A significant correlation between learner metacognition and mathematics achievement, as measured by the MAI total score, could not be established in the quantitative section. The perspectives obtained from both problem-solving sessions partially supported these findings. The quantitative results pertaining to the significant correlation between *Declarative knowledge* and mathematics achievement were also supported to some extent in both problem-solving sessions (see 6.5.2.1).

The interview with Mark revealed that his general way of teaching could have contributed to the significant improvement in *KC* and *Declarative knowledge*, the two aspects of metacognition that had a significant correlation with mathematics achievement in the quantitative section (see 6.5.2.2a).

The perspectives that emerged from the interview with Lisa indicated how her general way of teaching could have enhanced the subscale *Declarative knowledge* that had a significant correlation with mathematics achievement in the quantitative section (see 6.5.2.2b).

The learners' and teacher's perspectives on the MI process revealed an improvement in the learners' *Declarative knowledge.* This supports the finding in the quantitative section which stated that the MI process enhanced the one subscale, namely *Declarative knowledge,* of the MAI that had a statistically significant correlation with mathematics achievement in both terms (see 6.5.2.3).

7.2.5.2 Conclusions

In respect of the impact of MI on learner metacognition, the first and second problem-solving sessions only support the quantitative results to a limited extent. The interviews with the teachers support the quantitative results with regard to *KC*, *Declarative knowledge,* and *Conditional knowledge*, but the quantitative results were not supported in respect of *Planning, Monitoring,* and *Evaluation.* The qualitative data obtained from the learner and teacher interviews support the quantitative results regarding the positive impact of MI on learner metacognition.

The qualitative data obtained from both problem-solving sessions, the teacher interviews, and the learners' and teacher's perspectives on the MI process support the quantitative results in respect of the failure to establish a correlation between learner metacognition and mathematics achievement. However, the quantitative results pertaining to the correlation between *Declarative knowledge* and mathematics achievement were supported by the qualitative data to a certain extent.

7.2.5.3 Recommendations

The impact of metacognitive interventions on learner metacognition should be determined by means of qualitative and quantitative measures as a more holistic picture of the impact is obtained.

Qualitative and quantitative measures of the impact of metacognition interventions on mathematics achievement provide broader and more balanced perspectives on mathematics achievement.

7.3 LIMITATIONS OF THE STUDY

Some factors need to be considered when the findings relating to the impact of MI on learner metacognition and mathematics achievement are generalised.

First, the quasi-experimental design restricts the extent of the inferences made, as the learners were not randomly allocated to the control group and the experimental group. Secondly, both groups consisted of girls only, and this further confines the findings as possible gender differences may exist in respect of metacognition and mathematics achievement. Thirdly, only learners from high-achieving schools (Quintile 5 schools) participated in this study. A fourth factor relates to the high qualification level of the experimental group's teacher and his interest in developing learner problem-solving skills. Teachers with lower qualification levels, or teachers who do not strive to promote learners' problem-solving skills during their daily teaching activities, may implement the MI in a less effective manner. Fifthly, the impact of MI on mathematics achievement was only investigated indirectly by determining the correlation between learner metacognition and mathematics achievement. Inferences about the statistical significance of the impact of MI on mathematics achievement were made on the basis of that correlation.

The qualitative measurement of learner metacognition in a problem-solving context was based on the researcher's interpretation of the metacognitive behaviours displayed by the learners. It is possible that some metacognitive behaviours could be interpreted differently. Due to the open-ended nature of the problem statement, the learners' responses in the qualitative measurement of mathematics achievement were very varied, thus making it difficult to compare their pre-test and post-test responses**.**

The learners' perspectives were only obtained at the end of each cycle of the MI process; this hindered the swift implementation of refinements to the MI process. Hardly any provision was made for the teacher to adapt the MI process on a more regular basis, as it was only adapted after the feedback of the learners and the teacher once the first cycle had been incorporated.

The qualitative data involved qualitative measurements of learner metacognition and mathematics achievement in a problem-solving context. As such, learners' metacognitive behaviours and achievement in a problem-solving context only represent a small section of the quantitative measurements of learner metacognition and achievement, namely the MAI and the Terms 1 and 4 report marks. Therefore, it limits the extent to which comparisons between the qualitative and the quantitative measurements in respect of learner metacognition and mathematics achievement could be made.

7.4 FURTHER RESEARCH

Further research in respect of the following areas of interest can be conducted:

- The impact of MI on learner metacognition and mathematics achievement, by randomly allocating boys and girls to both an experimental and a control group, thus following a true experimental design.

- The impact of MI on learner metacognition and mathematics achievement in Quintile 1-4 schools.
- The effect of MI on learner metacognition and mathematics achievement of learners whose mathematics teachers are underqualified.
- The impact of MI on mathematics achievement, by using equivalent pre-test and post-test measures of mathematics achievement.
- The use – or development of – first, an assessment of learner metacognition scale in a problem-solving context and, secondly, an assessment scale that measures mathematics achievement in a problem-solving context.
- Teachers' perspectives on the nature of mathematics and aspects related to the teaching-and-learning of mathematics, by focusing on aspects related to De Corte's (1996) educational learning theory.
- The adaptation of the MI process that incorporates more regular learner feedback and teacher feedback on the implementation process.
- The feasibility of increasing the official time allocated to the teaching of mathematics in the South African school system.
- Factors that impact on the correspondence or non-correspondence between the quantitative and the qualitative findings in respect of the impact of MI on learner metacognition and mathematics achievement.
- The development of qualitative measurements of learner metacognition and mathematics achievement that align closely with quantitative measurements.
- The enhancement of the problem-solving facilitation skills of pre-service and in-service teachers.
- The training of pre-service and in-service teachers to successfully implement metacognitive interventions in mathematics.

7.5 SIGNIFICANCE OF THE STUDY

It is proposed that this study contributes to the field of metacognitive intervention in mathematics, both internationally and nationally.

Previous metacognitive interventions in mathematics, discussed in this study, that were implemented in an international context had different design features, but none was based on a comprehensive learning theory as evident in the MI of this study. This study, therefore, contributes to the theoretical underpinnings of metacognitive interventions in mathematics. In addition, a contribution to the practice of metacognitive interventions in mathematics is made, as these studies did not implement learner and teacher perspectives during the process of metacognitive intervention.

In South Africa, no evidence could be found of an extensive metacognitive intervention in mathematics. The significance of the MI implemented in this study does not only lie in its novelty within the South African context, but also in the incorporation of the learners' and teachers' perspectives in the adaptation of the MI process to produce a more flexible and learner-friendly MI tool. It is suggested that this study's MI could establish the basis of further mathematics metacognitive interventions within the South African context.

The MI tool implemented in this study could have a positive impact on the mathematics results, as it facilitates effective learning and learners' ability to regulate their learning processes in mathematics. Inexperienced or underqualified teachers could benefit from the structured yet flexible approach to the teaching-and-learning of mathematics. Therefore, the most significant aspect of this study relates to its possible impact on the teaching-and-learning of mathematics in dysfunctional schools. However, the MI tool could have a broader impact as, even in well-performing schools, many learners may lack true understanding and problem-solving skills.

This study also impacted on the researcher's views on the teaching-and-learning of mathematics. The emphasis in literature on the importance of problem-solving, and the poor performance of the experimental group in a problem-solving context, strengthened his belief in the value of problem-solving sessions facilitated by teachers. This study also confirmed the researcher's belief that the mathematics teaching-and-learning environment should be conducive to the development of positive learner attitudes towards mathematics as a subject. The most significant impact on the researcher's views relates to a new understanding of the importance of involving learners in the learning process by developing a flexible MI tool that incorporates learner perspectives.

7.6 CONCLUDING REMARKS

The quality of mathematics education could be improved, first, if mathematics teachers and learners have knowledge of what effective learning in mathematics entails and, secondly, if learners can regulate their learning processes. The MI implemented in this study provided the teacher with an opportunity to guide the learners in respect of these two aspects. It also enabled the learners to enhance their knowledge of effective learning in mathematics and their ability to regulate their learning processes.

Time constraints, however, impact negatively on the effective teaching-and-learning of mathematics. Above all, these time constraints affect the enhancement of learner problem-solving skills – which is regarded as the true aim of mathematics education, as it represents the highest level of achievement. Consequently, learners may develop negative attitudes towards mathematics due to their poor problem-solving skills and the generally demanding nature of mathematics.

It is imperative to address factors that play a role in mathematics achievement, for example teachers' knowledge of what effective learning entails; teachers' ability to vary their teaching methods to suit different learner needs; learners' knowledge of the elements of effective learning and the ability to regulate their learning processes; time constraints; learners' problem-solving skills, and learners' attitudes.

Naturally, there are many other factors that impact on mathematics achievement. To explore problems in mathematics education holistically, one should consider the implications of, among others, the following factors: the multifaceted and contingent nature of reality; peoples' limited and biased perspectives of reality; the inability of language to convey ideas accurately, and peoples' different ethical, spiritual, and philosophical paradigms.

Regardless of peoples' different perspectives on what the problems in mathematics education are and how to address these problems, there are, in my opinion, two elements that are of utmost importance in mathematics education: the enhancement of learner thinking skills in order to explore problems from different perspectives and to find alternative solutions where applicable, and the enhancement of learner attitudes towards mathematics, in particular, and towards life, in general. Thus, their thinking skills could be productively applied to the obstacles they face as learners and in the future.

A metacognitive intervention in mathematics could, in addition to its positive effect on learner metacognition and mathematics achievement, also enhance those thinking skills necessary to cope better with life's challenges and the complex problems that confront humankind.

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APPENDICES A1-A4

REQUESTS FOR PERMISSION TO CONDUCT RESEARCH

APPENDIX A1

REQUEST FOR PERMISSION TO CONDUCT RESEARCH: THE FREE STATE DEPARTMENT OF EDUCATION

University of the Free State

Faculty of Education

Department of Curriculum Studies

26 January 2010

The Director: Quality Assurance

Room 401

Syfrets Building

Free State Department of Education

Dear Mr …

PERMISSION TO CONDUCT RESEARCH

I am a lecturer at the Department of Curriculum Studies, University of the Free State. I am currently studying for a Ph. D degree in Mathematics Education.

I plan to undertake research on the enhancement of learner metacognition in mathematics. Metacognition entails firstly the knowledge one has about the elements of effective learning in mathematics, and secondly the ability to control and monitor one's learning. Research literature indicates a strong relation between learner metacognition and achievement in mathematics. The title of the research is:

The effect of metacognitive intervention on learner metacognition and achievement in mathematics.

Attached, please find the application form to register my research in the Free State Department of Education. The following documents are also attached: a letter from my supervisor confirming my registration; a draft of the letter that will be sent to the principals; a draft of the letter that will be sent to the parents; a copy of the questionnaire; and a list of questions that will be used in the interviews.

Thank you for the opportunity to apply for permission to conduct this research.

Yours sincerely,

Stephan du Toit

Lecturer

Department of Curriculum Studies

Tel: ...

e-mail: ...

APPENDIX A2

REQUEST FOR PERMISSION TO CONDUCT RESEARCH: THE HEADMASTERS OF SCHOOL A AND SCHOOL B

University of the Free State

Faculty of Education

Department of Curriculum Studies

15 February 2010

The Principal

... Secondary School

Dear Mr ...

PERMISSION TO CONDUCT RESEARCH

I am a lecturer at the Department of Curriculum Studies, University of the Free State, and I am currently studying for a Ph. D degree in Mathematics Education.

I plan to undertake research on the enhancement of learner metacognition in mathematics. Metacognition entails firstly the knowledge one has about the elements of effective learning in mathematics, and secondly the ability to control and monitor one's learning. Research literature indicates a strong relation between learner metacognition and achievement in mathematics. The title of the research is: **The effect of metacognitive intervention on learner metacognition and achievement in mathematics.**

I request permission to do research in your school by means of questionnaires that will be administered during a non-academic period. The questionnaires will be administered to grade 11 mathematics learners and their mathematics educator and will take a maximum of 45 minutes to complete. I also plan to gather data by means of the following methods: observation of grade 11 mathematics classes; interviewing grade 11 mathematics learners during non-academic periods or after hours; and analysing the reflective journals of the learners (after hours, learners are not directly involved). As a post-test, the same questionnaire will be administered after a period of roughly four months.

I pledge that the names of the learners, the educator and the school will not be mentioned in the research study, and that the results will be made available if required. Attached to this letter, please find a letter of permission from the Free State Department of Education to conduct this research.

I believe that this study could be of value to the learners regarding the process of learning in mathematics, and your assistance in this regard will be highly appreciated.

Yours sincerely,

Stephan du Toit

Lecturer

Department of Curriculum Studies

Tel: ...

e-mail: ...

APPENDIX A3

REQUEST FOR PERMISSION TO CONDUCT RESEARCH: THE GRADE 11 PARENTS OF SCHOOL A

University of the Free State

Faculty of Education

Department of Curriculum Studies

28 February 2010

Dear Grade 11 parents / custodians

PERMISSION TO CONDUCT RESEARCH

I am a lecturer at the Department of Curriculum Studies, University of the Free State. I am currently studying for a Ph. D degree in Mathematics Education.

I plan to undertake research on the enhancement of learner metacognition in mathematics. Metacognition entails firstly the knowledge one has about the elements of effective learning in mathematics, and secondly the ability to control and monitor one's learning. Reseach literature indicates a strong relation between learner metacognition and achievement in mathematics. The title of the research is: **The effect of metacognitive intervention on learner metacognition and achievement in mathematics.**

I have obtained permission to do research at ... by the Free State Department of Education and the headmaster, Mr ... The research will be conducted by means of questionnaires that will be administered during a non-academic period. The questionnaires will be administered to grade 11 mathematics learners and their mathematics educator and will take a maximum of 45 minutes to complete. I also plan to gather data by means of the following methods: observation of grade 11 mathematics classes; interviewing grade 11 mathematics learners during non-academic periods or after hours; and analysing the reflective journals of the learners (after hours, learners are not directly involved). I pledge that the names of the learners, the educator and the school will not be mentioned in the research study, and that the results will be made available if required.

I believe that this study could be of value to the learners regarding the process of learning in mathematics, and your assistance in this regard will be highly appreciated.

Yours faithfully,

Stephan du Toit

Lecturer

Department of Curriculum Studies

Tel: ...

e-mail: ...

………………………………………………………………………………………………………

It would be much appreciated if you could complete the following form and return it to Mr …:

I, ………………………, the parent / guardian of …………………………….. give permission that Mr Stephan du Toit conducts research that involves the learner mentioned above. I understand that no names will be used in the research report.

Signature: ……………………… Date:……………………..

APPENDIX A4

PERMISSION TO CONDUCT RESEARCH: LETTER FROM THE FREE STATE DEPARTMENT OF EDUCATION

education
Department of Education
FREE STATE PROVINCE

Enquiries : IM Malimane
Reference no. : 16/4/1/01-2010

Tel: 0514048662
Fax: 051 4477318

2010-02-01

Director: Motheo Education District
Room 413
Jubilee Building
Bloemfontein

Dear Mr Motsetse

NOTIFICATION OF A RESEARCH PROJECT IN YOUR DISTRICT

Please find attached copy of the letter giving **Mr. DS DU TOIT** permission to conduct research in two sampled school in the Motheo Education District. Mr. Du Toit is a Ph.D. in Mathematics education student at the University of the Free State. He is also a lecturer at the same university

Yours sincerely

FR SELLO
DIRECTOR: QUALITY ASSURANCE

Directorate: Quality Assurance
Private Bag X20565, Bloemfontein, 9300
Syfrets Center, 65 Maitland Street, Bloemfontein
Tel: 051 404 8750 / Fax: 051 447 7318
E-mail: quality@edu.fs.gov.za

APPENDICES B1-B7

MAI QUESTIONNAIRE AND THE MI CODES BOOKLET

APPENDIX B1

MAI LEARNER QUESTIONNAIRE (PRE-TEST AND POST-TEST)

Official use

This questionnaire consists of two sections:

Section A: Biographic particulars.

Section B: Metacognitive strategies.

General information

1. This questionnaire will roughly take 20 minutes to complete.
2. Your response will be valuable for research purposes.
3. These questionnaires will only be handled by the researcher.
4. Your name will not be used in the reporting of the research findings, but is only used to correlate your responses with the post-test.

SECTION A: BIOGRAPHIC PARTICULARS

Please complete the following and then answer the questions:

Name and surname**:**…………………………………………………….

INSTRUCTIONS:

Circle (O) the number of your answer:

1. Home language (please choose only one)

Afrikaans	1
English	2
Sesotho	3
Tswana	4
Xhosa	5
Zulu	6
Other (Please write it down)	7

2. Language of instruction

Afrikaans	1
English	2

3. What is your current age?

15 years	1
16 years	2
17 years	3
18 years	4
19 years	5

4. What was your report mark for mathematics at the end of grade 10?

0 – 9%	01
10 – 19%	02
20 – 29%	03
30 – 39%	04
40 – 49%	05
50 – 59%	06
60 – 69%	07
70 – 79%	08
80 – 89%	09
90 – 100%	10

SECTION B

The purpose of the following questions is to investigate various aspects of learning in mathematics.

INSTRUCTIONS:

Choose **one** of the following five possible answers by circling the number that

corresponds with the following options:

Strongly disagree	**Disagree**	**Neutral (*Neither agree nor disagree*)**	**Agree**	**Strongly agree**
1	**2**	**3**	**4**	**5**

	PLEASE READ EACH QUESTION CAREFULLY	*Strongly disagree*	*Disagree*	*Neutral*	*Agree*	*Strongly agree*
1.	I ask myself periodically if I am meeting my goals in mathematics.	1	2	3	4	5
2.	I first consider different ways of solving the problem **before** I start solving a problem in mathematics.	1	2	3	4	5
3.	When I solve a mathematics problem, I try to use methods of solving a problem that have worked in the past.	1	2	3	4	5
4.	I pace myself when I study for a mathematics test or examination in order to finish studying in time.	1	2	3	4	5
5.	I understand my intellectual strengths and weaknesses in mathematics.	1	2	3	4	5
6.	I think about what I **really need to learn** before I begin studying for a mathematics test or examination.	1	2	3	4	5

7.	I know how well I did once I finish a mathematics test or examination.	1	2	3	4	5
8.	I set specific goals before I begin to study for a mathematics test or examination.	1	2	3	4	5
9.	I read slower when I encounter important information in a mathematics question.	1	2	3	4	5
10.	I know what kind of information is most important to learn in mathematics.	1	2	3	4	5
11.	I ask myself if I have considered **different methods** of solving a problem when solving a mathematics problem.	1	2	3	4	5
12.	I am good at organizing the information I receive in mathematics.	1	2	3	4	5
13.	I consciously focus my attention on important information in a mathematics question.	1	2	3	4	5
14.	I have a specific purpose for each problem solving method I use when I solve a problem in mathematics.	1	2	3	4	5
15.	I learn best when I already know something about the mathematics topic I am studying.	1	2	3	4	5
16.	I know what the teacher expects me to learn when I study for a mathematics test or examination.	1	2	3	4	5
17.	I am good at remembering mathematics facts and principles.	1	2	3	4	5
18.	I use different learning strategies, depending on the situation, when I study mathematics.	1	2	3	4	5
19.	After I have solved a mathematics problem, I ask myself if there was an easier way to solve the problem.	1	2	3	4	5
20.	I can control how well I learn in mathematics.	1	2	3	4	5
21.	I periodically do revision to help me understand important relationships in mathematics.	1	2	3	4	5
22.	I ask myself questions about the problem before I begin to solve a mathematics problem.	1	2	3	4	5
23.	When I start to solve a mathematics problem, I think of	1	2	3	4	5

	several ways to solve the problem and choose the best one.					
24.	I summarize what I learn when I study.	1	2	3	4	5
25.	I ask other learners for help when I do not understand something in mathematics.	1	2	3	4	5
26.	I can **motivate myself** to study for a mathematics test or examination.	1	2	3	4	5
27.	I am aware of what learning strategies I use when I study mathematics.	1	2	3	4	5
28.	I ask myself how useful my learning strategies are while I study for a mathematics test or examination.	1	2	3	4	5
29.	I use my strengths in mathematics to compensate for my weaknesses in mathematics.	1	2	3	4	5
30.	When I receive new information about a familiar topic or a new topic in mathematics, I focus on the meaning and significance of the new information.	1	2	3	4	5
31.	I create my own examples to make new information I receive in mathematics more meaningful and understandable.	1	2	3	4	5
32.	I am a good judge of how well I understand something in mathematics.	1	2	3	4	5
33.	I find myself using helpful learning strategies in mathematics automatically (without consciously thinking about it).	1	2	3	4	5
34.	When I solve a mathematics problem, or when I study for a mathematics test or examination, I find myself pausing regularly to check my comprehension.	1	2	3	4	5
35.	I know in which situation each problem solving method I use will be most effective.	1	2	3	4	5
36.	I ask myself how well I accomplished my goals once I am finished studying for a mathematics test or an examination.	1	2	3	4	5
37.	I draw pictures or diagrams to help me understand while I	1	2	3	4	5

	am learning mathematics.					
38.	**After** I have solved a mathematics problem, I ask myself whether I have considered different ways to solve the problem.	1	2	3	4	5
39.	I try to put mathematics questions into my own words.	1	2	3	4	5
40.	I change my problem solving method when I fail to make progress when I try to solve a mathematics problem.	1	2	3	4	5
41.	I read the question carefully before I answer a mathematics question.	1	2	3	4	5
42.	When I read a mathematics question, I ask myself if what I am reading is related to what I already know.	1	2	3	4	5
43.	If I do not make progress when I solve a mathematics problem, I ask myself whether my first understanding of the problem was correct.	1	2	3	4	5
44.	I organize my time to best accomplish the goals I set in mathematics.	1	2	3	4	5
45.	I learn better when I am interested in a specific mathematics topic.	1	2	3	4	5
46.	When I study mathematics, I try to break down the work into smaller sections.	1	2	3	4	5
47.	When I study mathematics, I focus on how the specific topic I study fits in with the other topics in mathematics.	1	2	3	4	5
48.	I ask myself questions about how well I am doing **while** I am solving a mathematics problem.	1	2	3	4	5
49.	I ask myself if I have learned as much as I could have once I finish studying.	1	2	3	4	5
50.	When I read a mathematics question, I stop and reread any section of the question that is not clear.	1	2	3	4	5

Please make sure that you have answered all questions, and that you have written down your name and surname.

Thank you very much for your co-operation!

APPENDIX B2

MAI SUBSCALES

KNOWLEDGE OF COGNITION

Declarative knowledge (Items 5; 10; 12; 16; 17; 20; 32; 45)

(Knowledge about *what* the important information is and knowledge of one's skills, intellectual resources, and abilities as a learner).

Procedural knowledge (Items 3; 14; 27; 33)

(Knowledge about *how* to implement problem-solving methods and learning strategies).

Conditional knowledge (Items 15; 18; 26; 29)

(Knowledge about *when* and *why* to use learning strategies).

REGULATION OF COGNITION

Planning (Items 4; 6; 8; 22)

(Planning, goal setting, and allocating resources *prior* to learning).

Information management (Items 9; 13; 30; 31; 37; 39; 42; 46; 47)

(Skills and strategy sequences used *during* learning to process information more efficiently, for example, organising, elaborating, summarising, selective focusing).

Monitoring (Items 1; 2; 11; 21; 28; 34; 48)

(Assessment of one's learning or strategy use).

Debugging (Items 25; 40; 43; 50)

(Strategies used to correct comprehension and performance errors).

Evaluation (Items 7; 19; 24; 36; 38; 49)

(Analysis of performance and strategy effectiveness *after* a learning experience).

APPENDIX B3

LEARNERS' FEEDBACK ON PILOT QUESTIONNAIRE (SCHOOL B)

Questions were understandable
I understood questions.
No confusion – I understand all the questions.
Understood all questions.
All questions were clear.
All the questions were clear and understandable.
…(B)ut overall all the questions are understandable.
The first question I didn't really understood it at first but I got it after a while.
Questionnaire format
Please just staple questionnaires on other corner.
The unsure in one of the options to choose is not made clear, does this option say that I don't know what to answer to the question or is it in the middle of disagree and agree?
The options you gave us (SD; D; U; A; SA) you could rather use options such as Never, Sometimes, Usually and Always.
Q 29: Whole question on 1 page.
Questions were not understandable
Instead of using the term periodically, I suggest you use the term regularly.
Try to put questions in a more simple way.
A lot of questions were repeated. Some questions were ambiguous and therefore I would have different answers for different sections in mathematics.
Question 29: I wasn't sure what 'those things' were so specify what it is.
Question 21 I periodically is a bit difficult to understand it would be easier to use regularly.
Repeating of some question. Q 29: Use 'those things' more specific.
Q 47. This question makes me kind of insecure. Because it's kind of confusing…

Q 17: I think can be asked in a better way. I’m not sure but using good at doing something is not really correct English although people use it a lot these days.
General
The reason I don’t always consider what different methods I can use to solve a maths problem is because there is limited time when writing a test, so I just use the method I can relate to the question first.

APPENDIX B4

THE LINK BETWEEN THE ORIGINAL MAI, THE PILOT MAI, AND THE PRE-TEST MAI

Original MAI	**Pilot MAI** **(pre-test MAI items that differ from pilot MAI are indicated in italics)**
1. I ask myself periodically if I am meeting my goals.	1. I ask myself periodically if I am meeting my goals in mathematics.
2. I consider several alternatives to a problem before I answer.	2. I first consider different ways of solving the problem before I start solving a problem in mathematics.
3. I try to use strategies that have worked in the past.	3. When I solve a mathematics problem, I try to use problem solving strategies that have worked in the past.
	3. Pre-test: When I solve a mathematics problem, I try to use methods of solving a problem that have worked in the past.
4. I pace myself while learning in order to have enough time.	4. I pace myself when I study for a mathematics test or examination in order to finish studying in time.
5. I understand my intellectual strengths and weaknesses.	5. I understand my intellectual strengths and weaknesses in mathematics.
6. I think about what I really need to learn before I begin a task.	6. I think about what I **really need to learn** before I begin studying for mathematics test or examination.
7. I know how well I did once I finish a test.	7. I know how well I did once I finish a mathematics test or examination.

8. I set specific goals before I begin a task.	8. I set specific goals before I begin to study for a mathematics test or examination.
9. I slow down when I encounter important information.	9. I read slower when I encounter important information in a mathematics question.
10. I know what kind of information is most important to learn.	10. I know what kind of information is most important to learn in mathematics.
11. I ask myself if I have considered all options when solving a problem.	11. I ask myself if I have considered different problem solving strategies when solving a mathematics problem.
	11. *Pre-test: I ask myself if I have considered* ***different methods*** *of solving a problem when solving a mathematics problem.*
12. I am good at organizing information.	12. I am good at organizing the information I receive in mathematics.
13. I consciously focus my attention on important information.	13. I consciously focus my attention on important information in a mathematics question.
14. I have a specific purpose for each strategy I use.	14. I have a specific purpose for each problem solving strategy I use when I solve a problem in mathematics.
	14: Pre-test: I have a specific purpose for each problem solving method I use when I solve a problem in mathematics.
15. I learn best when I know something about the topic.	15. I learn best when I already know something about the mathematics topic I am studying.
16. I know what the teacher expects me to learn.	16. I know what the teacher expects me to learn when I study for a mathematics

	test or examination.
17. I am good at remembering information.	17. I am good at remembering mathematics facts and principles.
18. I use different learning strategies depending on the situation.	18. I use different learning strategies, depending on the situation, when I study mathematics.
19. I ask myself if there was an easier way to do things after I finish a task.	19. After I have solved a mathematics problem, I ask myself if there was an easier way to solve the problem.
20. I have control over how well I learn.	20. I have control over how well I learn in mathematics.
	20. *Pre-test: I can control how well I learn in mathematics.*
21. I periodically review to help me understand important relationships.	21. I periodically do revision to help me understand important relationships in mathematics.
22. I ask myself questions about the material before I begin.	22. I ask myself questions about the problem before I begin to solve a mathematics problem.
23. I think of several ways to solve a problem and choose the best one.	23. When I start to solve a mathematics problem, I think of several ways to solve the problem and choose the best one.
24. I summarize what I've learned after I finish.	24. I summarize what I learn when I study.
25. I ask others for help when I don't understand something.	25. I ask other learners for help when I do not understand something in mathematics.
26. I can motivate myself to learn when I need to.	26. I can **motivate myself** to study for a mathematics test or examination.
27. I am aware of what strategies I use when	27. I am aware of what learning

I study.	strategies I use when I study mathematics.
28. I find myself analyzing the usefulness of strategies when I study.	28. I ask myself how useful my learning strategies are while I study for a mathematics test or examination.
29. I use my intellectual strengths to compensate for my weaknesses.	29. I use those things in mathematics that I can do well to compensate for those things that I cannot do well.
	29. Pre-test: I use my strengths in mathematics to compensate for my weaknesses in mathematics.
30. I focus on the meaning and significance of new information.	30. When I receive new information about a familiar topic or a new topic in mathematics, I focus on the meaning and significance of the new information.
31. I create my own examples to make information more meaningful.	31. I create my own examples to make new information I receive in mathematics more meaningful.
	31. *Pre-test: I create my own examples to make new information I receive in mathematics more meaningful and understandable.*
32. I am a good judge of how well I understand something.	32. I am a good judge of how well I understand something in mathematics.
33. I find myself using helpful learning strategies automatically.	33. I find myself using helpful learning strategies in mathematics automatically (without consciously thinking about it).
34. I find myself pausing regularly to check my comprehension.	34. When I solve a mathematics problem, or when I study for a mathematics test or examination, I find myself pausing regularly to check my

	comprehension.
35. I know when each strategy I use will be most effective.	35. I know in which situation each problem solving strategy I use will be most effective.
	35. *Pre-test: I know in which situation each problem solving method I use will be most effective.*
36. I ask myself how well I accomplished my goals once I'm finished.	36. I ask myself how well I accomplished my goals once I am finished studying for a mathematics test or an examination.
37. I draw pictures or diagrams to help me understand while learning.	37. I draw pictures or diagrams to help me understand while I am learning mathematics.
38. I ask myself if I have considered all options after I solve a problem.	38. After I have solved a mathematics problem, I ask myself whether I have considered different ways to solve the problem.
39. I try to translate new information into my own words.	39. I try to translate mathematics questions into my own words.
	39. *Pre-test: I try to put mathematics questions into my own words.*
40. I change strategies when I fail to understand.	40. I change my problem solving strategy when I fail to make progress when I solve a mathematics problem.
	40. *I change my problem solving method when I fail to make progress when I try to solve a mathematics problem.*
41. I use the organizational structure of the text to help me learn.	

42. I read instructions carefully before I begin a task.	41. I read the question carefully before I answer a mathematics question.
43. I ask myself if what I'm reading is related to what I already know.	42. When I read a mathematics question, I ask myself if what I am reading is related to what I already know.
44. I re-evaluate my assumptions when I get confused.	43. If I do not make progress when I solve a mathematics problem, I ask myself whether my first understanding of the problem was correct.
45. I organize my time to best accomplish my goals.	44. I organize my time to best accomplish the goals I set in mathematics.
46. I learn more when I am interested in the topic.	45. I learn better when I am interested in a specific mathematics topic.
47. I try to break studying down into smaller steps.	46. When I study mathematics, I try to break down the studying into smaller steps.
	46. *Pre-test: When I study mathematics, I try to break down the work into smaller sections.*
48. I focus on the overall meaning rather than specifics.	47. When I study mathematics, I focus on the overall meaning of the specific topic I study.
	47. *Pre-test: When I study mathematics, I focus on how the specific topic I study fits in with the other topics in mathematics.*
49. I ask myself questions about how well I am doing while I am learning something new.	48. I ask myself questions about how well I am doing **while** I am solving a mathematics problem.

50. I ask myself if I have learned as much as I could have once I finish a task.	49. I ask myself if I have learned as much as I could have once I finish studying.
51. I stop and go back over new information that is not clear.	50. When I read a mathematics question, I stop and reread any section of the question that is not clear.
52. I stop and reread when I get confused.	

APPENDIX B5

THE LINK BETWEEN THE ASPECTS OF DE CORTE'S (1996) EDUCATIONAL LEARNING THEORY AND THE MI CODES BOOKLET

Code	Aspect of De Corte's (1996) educational learning theory	Metacognitive strategy
	Starting up **[Constructive]**	
S1	When I start with a mathematics activity, I make sure that I know what the instructions are *(for example, whether it is an individual activity or a group work activity.)*	Planning strategy
S2	I make sure that I know how much time I have available to complete a mathematics activity (*for example, the project must be finished in two weeks' time.)*	Planning strategy
S3	When the teacher explains a new mathematics topic, I can identify those parts of the explanation that I understand well *(for example, if the new topic is "Trigonometry", I understand that in a right-angled triangle* *sin* = ——————*)*	Evaluating one's way of thinking and acting
S4	When the teacher explains a new mathematics topic, I can identify those parts of the explanation that I **do not** understand well *(for example, if the new topic is "Trigonometry", I* ***do not*** *understand what the difference between the adjacent side, the hypotenuse and the opposite side is.)*	Evaluating one's way of thinking and acting

S5	When I answer a mathematics question, I can identify that part of the mathematics question that I understand well *(for example, when the question states: "Solve the quadratic equation by completing the square", the part of the question that I understand well is the part that requires me to solve the quadratic equation.)*	Evaluating one's way of thinking and acting
S6	When I answer a mathematics question, I can identify those aspects of the mathematics question that I **do not** understand well *(for example, when the question states: "Solve the quadratic equation by completing the square", the part of the question that I do not understand well is the part that requires me to complete the square.)*	Evaluating one's way of thinking and acting
S7	When the teacher gives the solution to a mathematics question, I know **which** specific part of my answer is wrong (*for example, if the question states "Simplify ", I did it as follows: = . I know that I had the following part of my answer wrong: .*	Identifying the difficulty
S8	When the teacher gives the solution/answer to a mathematics activity, I can identify **why** I had some part(s) of the solution/answer incorrect (for example, if the question states: *"Simplify ", then is wrong because I only added the exponents but I was suppose to multiply the exponents.)*	Identifying the difficulty
S9	If I think that a mathematics question is too difficult, I can give a **reason** why I think that the question is too difficult *(for example, if I regard the following question as too difficult: "Prove that the following pairs of numbers have a linear relationship: (1; – 7); (2; – 5); (3;– 3), then I could*	Identifying the difficulty

	say that the question is too difficult because it is unclear what is meant by "a linear relationship.")	
	Solid **F**oundations **[A structured knowledge base]**	
F1	I make a summary of the **basic facts** that I must have on each mathematics topic *(for example, if the topic is "Sketching of parabolas", my summary refers to the following ways of sketching the parabola:* *a) Using the table method;* b) *Calculating the x-intercepts, y-intercept and the turning point.)*	Generating questions.
F2	I make a summary of the **basic principles/procedures** of each mathematics topic *(for example, if the topic is "Sketching of parabolas", and my summary of basic facts refers to the following ways of sketching the parabola:* *a) Using the table method;* *b) Calculating the x-intercepts, y-intercept and the turning point,* *then my summary of basic principles/procedures of how to calculate the turning point is to use the formula: — — .*	Generating questions.
	Building blocks **[Cumulative]**	
B1	When I answer a mathematics question, I can identify the main topic in mathematics that the question is about *(for example, if the question states: "Solve for x: $x^2 - 4x = 0$, then the main topic is "Solving of quadratic equations.")*	Generating questions.
B2	When I answer a mathematics question and I can identify the main topic in mathematics that the question is about, I can also identify a supporting topic/supporting topics that	Generating questions.

	the question is about *(for example, if the question states: Solve for x: $x^2 - 4x = 0$, then the main topic is "Solving of quadratic equations.", and supporting topics are "Factorisation of quadratic equations" and "Using the formula to solve a quadratic equation.")*	
	My **G**oals **[Goal-oriented]**	
G1	During the completion of a mathematics activity, I identify what goal(s) the teacher wants me to achieve *(for example, the teacher wants me to achieve 70% in the examination; or to understand and apply the sine rule.)*	Setting and pursuing goals
G2	Before I start with a mathematics activity, I set myself specific goals that I would like to achieve *(for example, I want to have all my home work questions correct; or I want to score 80% in the test.)*	Setting and pursuing goals
G3	During a mathematics activity, I am aware of my progress towards the goals I want to achieve *(for example, I have had all my homework correct for the past two weeks so I think that I am on track to obtain 80% for the test).*	Setting and pursuing goals
	Talk Time **[Collaborative]**	
T1	When it is allowed to talk in class, I explain mathematics to other learners.	Cooperative learning
T2	When it is allowed to talk in class, I ask other learners to help me with mathematics.	Cooperative learning
T3	I do mathematics in a group with other learners whenever there is an opportunity.	Cooperative learning
	Living Maths **[Situated]**	

L1	Whenever a new topic/concept in mathematics is explained, I determine the application/relevancy of that topic/concept to my life specifically *(for example, I can use the surface area formula of a rectangle to determine how much paint I need to paint my room.)*	Problem-solving activities
L2	Whenever a new topic/concept in mathematics is explained, I determine the application/relevancy of that activity to real life in general *(for example, the height that a rocket reaches after a certain time can be determined by using quadratic equations.)*	Problem-solving activities
L3	Whenever a new topic/concept in mathematics is explained, I determine the application/relevancy of that activity to my other subjects *(for example, calculating the surface area of organisms, as part of the subject Life Sciences, can help me determine how those organisms have adapted to the amount of sunlight their skins absorb.)*	Problem-solving activities
	My **W**ay **[Individually different]**	
W1	When I study mathematics or do homework in mathematics, I think aloud *(express my thoughts verbally.)*	Thinking aloud
W2	I make notes of mistakes I make or misconceptions I have during the completion of a mathematics activity *(for example, I make a note of the fact that I always forget to check the solution when I solve an equation where I square both sides if there is a square root sign in the equation.)*	Journal keeping
W3	I make notes of ways to correct the mistakes I make or correct the misconceptions I have during the completion of a mathematics activity *(for example, I have written a note*	Journal keeping

	that reminds me to always substitute the answer into the first line of an equation to check whether the answer satisfies the equation.)	
W4	I make a summary of new mathematical knowledge I acquire during the completion of a mathematics activity *(for example, I use a table to represent the similarities and differences between simple interest and compound interest.)*	Journal keeping
	Problems can be solved **[Heuristics]**	
P1	When I am busy with a mathematics activity, I can identify a specific computer software program that can help me to complete the mathematics activity successfully *(for example, I know that the software program "Microsoft Office Excel" can be used to draw straight line graphs.)*	Use of interactive technology
P2	When I have access to a computer, I use a specific computer software program to help me complete a mathematics activity successfully *(for example, I use the software program "Microsoft Office Word" to draw different objects like cylinders and cubes.)*	Use of interactive technology
P3	I can state a mathematics question in my own words or in a different way that could help me to understand the question better *(for example, the question "Solve for x" can also be stated as "Determine the x-intercept(s)" or "Determine the root(s) of the equation.")*	Paraphrasing, reflecting and elaborating questions .
P4	Before I answer a mathematics question, I first make sure that I understand the question very well by answering the following questions: What information is given? What is	Problem-solving activities

	the unknown? What are the conditions?	
P5	Before I start with a mathematics activity, I remind myself of different problem solving strategies that I could possibly use (*for example, pattern recognition; try-and-improve; imagine that the problem has been solved and then working back to the question; drawing a diagram or sketch; first solve a simpler problem (modification of the problem); represent the data in table form; generalisation.)*	Problem-solving activities
P6	If I do not make progress when I answer a mathematics question, I change the strategy/method that I am using *(for example, if I have to solve a quadratic equation, my strategy is to first factorise the equation. If I cannot factorise the equation, I decide to change my strategy by using the quadratic formula to solve the equation.)*	Problem-solving activities
P7	During the completion of a mathematics activity, I can give a reason (or reasons) why I have decided to use a particular method/strategy *(for example, I draw a diagram of the information in the question because the information is about different shapes a farmer can use to make an enclosure, and a diagram with different shapes helps me to order my thoughts.)*	Problem-solving activities
P8	When I answer a mathematics question, I can explain why I do each step of the answer in that particular way *(for example, if the question states "Solve for x if = 25", I can explain the next step as follows: = I take square roots both sides, but the right-hand side must have the sign because my answer must have two values as I solve a quadratic equation.)*	Problem-solving activities

P9	I use different problem solving strategies to solve a mathematics problem *(for example, when I have to solve a mathematics problem, I make use of the following strategies: pattern recognition; try-and-improve; imagine that the problem has been solved and then working back to the question; drawing a diagram or sketch; first solve a simpler problem (modification of the problem); represent the data in table form; generalisation.)*	Problem-solving activities
P10	After I have found the solution to a mathematics problem, I check/test whether my solution is correct *(for example, I substitute my answer into the original equation to check whether my answer makes the original equation true.)*	Problem-solving activities
P11	After I have found the solution to a mathematics problem, and there is enough time available, I try to find the solution by using a different method *(for example, if the question states "Simplify — , and find the answer in the following way: — = =8 . A different method to find the answer is: — = — = 8)*	Problem solving
P12	After I have solved a mathematics problem, I determine whether any aspect(s) of the mathematics problem solving activity can be applied to other mathematics problems *(for example, if I find that a circle has a bigger area than a square if the circumference of the circle equals the perimeter of the square, then I apply the same principles when I compare differently shaped 3D objects.)*	Problem solving

	Matters of the **H**eart **[Affective components]**	
H1	When I start to solve a mathematics problem, I know whether I will be able to successfully solve the problem or not *(for example, I recognise this problem as similar to a problem I have done before, so I will be able to do it.)*	Problem solving
H2	I know whether I like a specific mathematics topic or not *(for example, we are busy with financial mathematics, I do not like it at all.)*	Problem-solving activities
H3	I am aware of my feelings when I successfully answer a mathematics question *(for example, I feel great because I could never solve this type of problem before.)*	Evaluating one's way of thinking and acting
H4	I am aware of my feelings when I **am not** successful in answering a mathematics question *(for example, I feel bad because I have really studied hard for the test, and I had most of the questions wrong.)*	Evaluating one's way of thinking and acting
H5	I know what my level of motivation is when I am busy with a mathematics activity *(for example, I am not very motivated because I do not really see where I am going to use this section of mathematics when I am an adult.)*	Evaluating one's way of thinking and acting
H6	I know which topics/concepts/procedures in mathematics I find easy to do *(for example, I find it easy to work with linear equations and to represent the linear function graphically.)*	Evaluating one's way of thinking and acting
H7	I know which topics/concepts/procedures in mathematics I find difficult to do *(for example, I struggle to solve quadratic equations by completing the square.)*	Evaluating one's way of thinking and acting

APPENDIX B6

THE MI CODES BOOKLET

SECTION A: FOR EACH MATHEMATICS ACTIVITY **(doing class work, homework, listening to the teacher, projects, etc.)**		
Code	Description of the code	When to apply the code
My **G**oals		
G1	During the completion of a mathematics activity, I identify what goal(s) the teacher wants me to achieve *(for example, the teacher wants me to achieve 70% in the examination; or to understand and apply the sine rule.)*	Indicate short term goals (can be for a specific lesson) and longer term goals (can be stated only once a term.)
G2	Before I start with a mathematics activity, I set myself specific goals that I would like to achieve *(for example, I want to have all my home work questions correct; or I want to score 80% in the test.)*	Indicate short term goals (can be for a specific lesson) and longer term goals (can be stated only once a term.)
Starting up		
S1	When I start with a mathematics activity, I make sure that I know what the instructions are *(for example, whether it is an individual activity or a group work activity.)*	Only at beginning of activity.
S2	I make sure that I know how much time I have available to complete a mathematics activity (*for example, the project must be finished in two weeks' time.)*	Only at beginning of activity.

SECTION B: ANSWERING QUESTIONS IN CLASS AND HOME WORK QUESTIONS (understanding the problem)		
Matters of the **H**eart		
H1	When I start to solve a mathematics problem, I know whether I will be able to successfully solve the problem or not *(for example, I recognise this problem as similar to a problem I have done before, so I will be able to do it.)*	For each question.
Starting up		
S5	When I answer a mathematics question, I can identify that part of the mathematics question that I understand well *(for example, when the question states: "Solve the quadratic equation by completing the square", the part of the question that I understand well is the part that requires me to solve the quadratic equation.)*	For each question, unless the question is stated in one word only like "solve" or "factorise".
S6	When I answer a mathematics question, I can identify those aspects of the mathematics question that I **do not** understand well *(for example, when the question states: "Solve the quadratic equation by completing the square", the part of the question that I do not understand well is the part that requires me to complete the square.)*	For each question, if applicable.
Problems can be solved		
P3	I can state a mathematics question in my own words or in a different way that could help me to understand the question better *(for example, the question "Solve for x" can also be stated as "Determine the x-intercept(s)" or "Determine the root(s) of the equation".)*	For each question.
P4	Before I answer a mathematics question, I first make sure that I understand the question very well by answering the following questions: What information is	For each question, especially problem solving questions.

	given? What is the unknown? What are the conditions?	
P5	Before I start with a mathematics activity, I remind myself of different problem solving strategies that I could possibly use (*for example, pattern recognition; try-and-improve; imagine that the problem has been solved and then working back to the question; drawing a diagram or sketch; first solve a simpler problem (modification of the problem); represent the data in table form; generalisation.)*	Here you do not have to use full sentences, just write a tick next to the code if you have reminded yourself, e.g. P5 ✓
	Building blocks	
B1	When I answer a mathematics question, I can identify the main topic in mathematics that the question is about *(for example, if the question states: Solve for x: $x^2 - 4x = 0$, then the main topic is "Solving of quadratic equations.")*	For all questions.
B2	When I answer a mathematics question and I can identify the main topic in mathematics that the question is about, I can also identify a supporting topic/supporting topics that the question is about *(for example, if the question states: Solve for x: $x^2 - 4x = 0$, then the main topic is "Solving of quadratic equations.", and supporting topics are "Factorisation of quadratic equations" and "Using the formula to solve a quadratic equation.")*	For all questions.
	Starting up	
S9	If I think that a mathematics question is too difficult, I can give a **reason** why I think that the question is too difficult *(for example, if I regard the following question as too difficult: "Prove that the following pairs of*	For each question, if applicable.

	numbers have a linear relationship: (1; – 7); (2; – 5); (3;– 3), then I could say that the question is too difficult because it is unclear what is meant by "a linear relationship.")	
SECTION C: ANSWERING QUESTIONS IN CLASS AND HOME WORK QUESTIONS (making a plan, carry out the plan, look back)		
Problems can be solved		
P1	When I am busy with a mathematics activity, I can identify a specific computer software program that can help me to complete the mathematics activity successfully *(for example, I know that the software program "Microsoft Office Excel" can be used to draw straight line graphs.)*	For each topic.
P2	When I have access to a computer, I use a specific computer software program to help me complete a mathematics activity successfully *(for example, I use the software program "Microsoft Office Word" to draw different objects like cylinders and cubes.)*	For each topic.
P6	If I do not make progress when I answer a mathematics question, I change the strategy/method that I am using *(for example, if I have to solve a quadratic equation, my strategy is to first factorise the equation. If I cannot factorise the equation, I decide to change my strategy by using the quadratic formula to solve the equation.)*	Refer to the problem solving strategies in P5 whenever you solve a problem, or too other methods/strategies like the one mentioned in P6.
P7	During the completion of a mathematics activity, I can give a reason (or reasons) why I have decided to use a particular method/strategy *(for example, I draw a diagram of the information in the question because the*	For each question, especially problem solving questions.

	information is about different shapes a farmer can use to make an enclosure, and a diagram with different shapes helps me to order my thoughts.)	
P8	When I answer a mathematics question, I can explain why I do each step of the answer in that particular way *(for example, if the question states "Solve for x if = 25", I can explain the next step as follows: — = — I take square roots both sides, but the right-hand side must have the sign because my answer must have two values as I solve a quadratic equation.)*	It would be a bit too much to do it for every single step, just indicate with a tick that you have considered every step and that you can explain it, e.g. P8 ✓
P9	I use different problem solving strategies to solve a mathematics problem *(for example, when I have to solve a mathematics problem, I make use of the following strategies: pattern recognition; try-and-improve; imagine that the problem has been solved and then working back to the question; drawing a diagram or sketch; first solve a simpler problem (modification of the problem); represent the data in table form; generalisation.)*	For each problem, indicate your strategy.
P10	After I have found the solution to a mathematics problem, I check/test whether my solution is correct *(for example, I substitute my answer into the original equation to check whether my answer makes the original equation true).*	For each question where applicable e.g. just write P10 x = 3 makes the first line of the question true.
P11	After I have found the solution to a mathematics problem, and there is enough time available, I try to find the solution by using a different method *(for example, if the question states "Simplify — , and*	Whenever possible, if the time allows.

	find the answer in the following way: — = =8 . *A different method to find the answer is:* — = — = 8)	
P12	After I have solved a mathematics problem, I determine whether any aspect(s) of the mathematics problem solving activity can be applied to other mathematics problems *(for example, if I find that a circle has a bigger area than a square if the circumference of the circle equals the perimeter of the square, then I apply the same principles when I compare differently shaped 3D objects.)*	Whenever possible, if the time allows.
SECTION D: UPON COMPLETION OF A LEARNER ACTIVITY		
My **W**ay		
W1	When I study mathematics or do homework in mathematics, I think aloud *(express my thoughts verbally).*	Indicate at the end of your daily study to which degree you have thought aloud.
Talk time		
T1	When it is allowed to talk in class, I explain mathematics to other learners.	Each time you do.
T2	When it is allowed to talk in class, I ask other learners to help me with mathematics.	Each time you do.
T3	I do mathematics in a group with other learners whenever there is an opportunity.	Each time you do.
My **G**oals		
G3	During a mathematics activity, I am aware of my progress towards the goals I want to achieve *(for example, I have had all my homework correct for the*	Indicate progress towards goals.

	past two weeks so I think that I am on track to obtain 80% for the test.)	
Matters of the **H**eart		
H5	I know what my level of motivation is when I am busy with a mathematics activity *(for example, I am not very motivated because I do not really see where I am going to use this section of mathematics when I am an adult).*	At least once a week.
SECTION E: WHEN THE TEACHER GIVES FEEDBACK ON ANY ACTIVITY		
Starting up		
S7	When the teacher gives the solution to a mathematics question, I know **which** specific part of my answer is wrong (*for example, if the question states "Simplify ", I did it as follows: = . I know that I had the following part of my answer wrong: .*	For each answer, if applicable.
S8	When the teacher gives the solution/answer to a mathematics activity, I can identify **why** I had some part(s) of the solution/answer incorrect (for example, if the question states: *"Simplify ", then is wrong because I only added the exponents but I was suppose to multiply the exponents.)*	For each answer, if applicable.
S9	If I think that a mathematics question is too difficult, I can give a **reason** why I think that the question is too difficult *(for example, if I regard the following question as too difficult: "Prove that the following pairs of numbers have a linear relationship: (1; – 7); (2; – 5); (3;– 3), then I could say that the question is too difficult because it is unclear what is meant by "a linear relationship.")*	For each question, if applicable.
My **W**ay		
W2	I make notes of mistakes I make or misconceptions I	For all mistakes.

	have during the completion of a mathematics activity *(for example, I make a note of the fact that I always forget to check the solution when I solve an equation where I square both sides if there is a square root sign in the equation.)*	
W3	I make notes of ways to correct the mistakes I make or correct the misconceptions I have during the completion of a mathematics activity *(for example, I have written a note that reminds me to always substitute the answer into the first line of an equation to check whether the answer satisfies the equation.)*	For all mistakes.
	Problems can be solved	
P11	After I have found the solution to a mathematics problem, and there is enough time available, I try to find the solution by using a different method *(for example, if the question states "Simplify —— , and find the answer in the following way: —— = =8 .* *A different method to find the answer is:* *—— = —— = 8)*	Whenever possible, if time allows.
	Matters of the **H**eart	
H3	I am aware of my feelings when I successfully answer a mathematics question *(for example, I feel great because I could never solve this type of problem before.)*	At least once a week.
H4	I am aware of my feelings when I **am not** successful in answering a mathematics question *(for example, I feel*	At least once a week.

	bad because I have really studied hard for the test, and I had most of the questions wrong.)	
SECTION F: WHEN THE TEACHER EXPLAINS NEW WORK		
Starting up		
S3	When the teacher explains a new mathematics topic, I can identify those parts of the explanation that I understand well *(for example, if the new topic is "Trigonometry", I understand that in a right-angled triangle* *sin = ——————)*	Each time the teacher explains.
S4	When the teacher explains a new mathematics topic, I can identify those parts of the explanation that I **do not** understand well *(for example, if the new topic is "Trigonometry", I* ***do not*** *understand what the difference between the adjacent side, the hypotenuse and the opposite side is.)*	Each time the teacher explains.
Living maths		
L1	Whenever a new topic/concept in mathematics is explained, I determine the application/relevancy of that topic/concept to my life specifically *(for example, I can use the surface area formula of a rectangle to determine how much paint I need to paint my room.)*	For each topic.
L2	Whenever a new topic/concept in mathematics is explained, I determine the application/relevancy of that activity to real life in general *(for example, the height that a rocket reaches after a certain time can be determined by using quadratic equations.)*	For each topic.
L3	Whenever a new topic/concept in mathematics is explained, I determine the application/relevancy of that activity to my other subjects *(for example, calculating*	For each topic.

	the surface area of organisms, as part of the subject Life Sciences, can help me determine how those organisms have adapted to the amount of sunlight their skins absorb.)	
SECTION G: TOPIC SUMMARY		
F1	I make a summary of the **basic facts** that I must have on each mathematics topic *(for example, if the topic is "Sketching of parabolas", my summary refers to the following ways of sketching the parabola:* *c) Using the table method;* d) *Calculating the x-intercepts, y-intercept and the turning point.)*	For all topics, continue as the topic progresses and complete summary when the topic has been dealt with.
F2	I make a summary of the **basic principles/procedures** of each mathematics topic *(for example, if the topic is "Sketching of parabolas", and my summary of basic facts refers to the following ways of sketching the parabola:* *c) Using the table method;* *d) Calculating the x-intercepts, y-intercept and the turning point,* *then my summary of basic principles/procedures of how to calculate the turning point is to use the formula:* — — .	Similar to the summary in F1, but concentrate on principles/procedures.
My **W**ay		
W4	I make a summary of new mathematical knowledge I acquire during the completion of a mathematics activity *(for example, I use a table to represent the similarities and differences between simple interest and compound interest).*	At least once a week, combine this with the summaries of F1 and F2.

Matters of the **H**eart		
H2	I know whether I like a specific mathematics topic or not *(for example, we are busy with financial mathematics, I do not like it at all.)*	For each topic.
H6	I know which topics/concepts/procedures in mathematics I find easy to do *(for example, I find it easy to work with linear equations and to represent the linear function graphically.)*	Make a summary of all topics / concepts / procedures you find easy.
H7	I know which topics/concepts/procedures in mathematics I find difficult to do *(for example, I struggle to solve quadratic equations by completing the square.)*	Make a summary of all topics / concepts / procedures you find difficult.

APPENDIX B7

THE LINK BETWEEN THE MAI AND THE MI CODES BOOKLET

1	I ask myself periodically if I am meeting my goals in mathematics.
G3	During a mathematics activity, I am aware of my progress towards the goals I want to achieve *(for example, I have had all my homework correct for the past two weeks so I think that I am on track to obtain 80% for the test.)*
2	I first consider different ways of solving the problem **before** I start solving a problem in mathematics.
P5	Before I start with a mathematics activity, I remind myself of different problem solving strategies that I could possibly use (*for example, pattern recognition; try-and-improve; imagine that the problem has been solved and then working back to the question; drawing a diagram or sketch; first solve a simpler problem (modification of the problem); represent the data in table form; generalisation.)*
3	When I solve a mathematics problem, I try to use methods of solving a problem that have worked in the past.
P7	During the completion of a mathematics activity, I can give a reason (or reasons) why I have decided to use a particular method/strategy *(for example, I draw a diagram of the information in the question because the information is about different shapes a farmer can use to make an enclosure, and a diagram with different shapes helps me to order my thoughts.)*
4	I pace myself when I study for a mathematics test or examination in order to finish studying in time.
S2	I make sure that I know how much time I have available to complete a mathematics activity (*for example, the project must be finished in two weeks’*

	time.)
5	I understand my intellectual strengths and weaknesses in mathematics.
H6	I know which topics/concepts/procedures in mathematics I find easy to do *(for example, I find it easy to work with linear equations and to represent the linear function graphically.)*
H7	I know which topics/concepts/procedures in mathematics I find difficult to do *(for example, I struggle to solve quadratic equations by completing the square.)*
6	I think about what I **really need to learn** before I begin studying for a mathematics test or examination.
S1	When I start with a mathematics activity, I make sure that I know what the instructions are *(for example, whether it is an individual activity or a group work activity.)*
7	I know how well I did once I finish a mathematics test or examination.
G3	During a mathematics activity, I am aware of my progress towards the goals I want to achieve *(for example, I have had all my homework correct for the past two weeks so I think that I am on track to obtain 80% for the test.)*
8	I set specific goals before I begin to study for a mathematics test or examination.
G2	Before I start with a mathematics activity, I set myself specific goals that I would like to achieve *(for example, I want to have all my home work questions correct; or I want to score 80% in the test.)*
9	I read slower when I encounter important information in a mathematics question.
P4	Before I answer a mathematics question, I first make sure that I understand the question very well by answering the following questions: What information is given? What is the unknown? What are the conditions?

10	I know what kind of information is most important to learn in mathematics.
F1	I make a summary of the **basic facts** that I must have on each mathematics topic *(for example, if the topic is "Sketching of parabolas", my summary refers to the following ways of sketching the parabola:* *e) Using the table method;* f) *Calculating the x-intercepts, y-intercept and the turning point.)*
F2	I make a summary of the **basic principles/procedures** of each mathematics topic *(for example, if the topic is "Sketching of parabolas", and my summary of basic facts refers to the following ways of sketching the parabola:* *e) Using the table method;* *f) Calculating the x-intercepts, y-intercept and the turning point,* *then my summary of basic principles/procedures of how to calculate the turning point is to use the formula: — — .*
11	I ask myself if I have considered **different methods** of solving a problem when solving a mathematics problem.
P5	Before I start with a mathematics activity, I remind myself of different problem solving strategies that I could possibly use (*for example, pattern recognition; try-and-improve; imagine that the problem has been solved and then working back to the question; drawing a diagram or sketch; first solve a simpler problem (modification of the problem); represent the data in table form; generalisation.)*
12	I am good at organizing the information I receive in mathematics.
F1	I make a summary of the **basic facts** that I must have on each mathematics topic *(for example, if the topic is "Sketching of parabolas", my summary refers to the following ways of sketching the parabola:* *g) Using the table method;*

	h) *Calculating the x-intercepts, y-intercept and the turning point.)*
F2	I make a summary of the **basic principles/procedures** of each mathematics topic *(for example, if the topic is "Sketching of parabolas", and my summary of basic facts refers to the following ways of sketching the parabola:* *g) Using the table method;* *h) Calculating the x-intercepts, y-intercept and the turning point,* *then my summary of basic principles/procedures of how to calculate the turning point is to use the formula: — — .*
13	I consciously focus my attention on important information in a mathematics question.
B1	When I answer a mathematics question, I can identify the main topic in mathematics that the question is about *(for example, if the question states: Solve for x: $x^2 - 4x = 0$, then the main topic is "Solving of quadratic equations.")*
B2	When I answer a mathematics question and I can identify the main topic in mathematics that the question is about, I can also identify a supporting topic/supporting topics that the question is about *(for example, if the question states: Solve for x: $x^2 - 4x = 0$, then the main topic is "Solving of quadratic equations.", and supporting topics are "Factorisation of quadratic equations" and "Using the formula to solve a quadratic equation".)*
14	I have a specific purpose for each problem solving method I use when I solve a problem in mathematics.
P7	During the completion of a mathematics activity, I can give a reason (or reasons) why I have decided to use a particular method/strategy *(for example, I draw a diagram of the information in the question because the information is about different shapes a farmer can use to make an enclosure, and a diagram with different shapes helps me to order my thoughts.)*

15	I learn best when I already know something about the mathematics topic I am studying.
F1	I make a summary of the **basic facts** that I must have on each mathematics topic *(for example, if the topic is "Sketching of parabolas", my summary refers to the following ways of sketching the parabola:* *i) Using the table method;* j) *Calculating the x-intercepts, y-intercept and the turning point.*
16	I know what the teacher expects me to learn when I study for a mathematics test or examination.
S1	When I start with a mathematics activity, I make sure that I know what the instructions are *(for example, whether it is an individual activity or a group work activity.)*
G1	During the completion of a mathematics activity, I identify what goal(s) the teacher wants me to achieve *(for example, the teacher wants me to achieve 70% in the examination; or to understand and apply the sine rule.)*
17	I am good at remembering mathematics facts and principles.
F1	I make a summary of the **basic facts** that I must have on each mathematics topic *(for example, if the topic is "Sketching of parabolas", my summary refers to the following ways of sketching the parabola:* *k) Using the table method;* l) *Calculating the x-intercepts, y-intercept and the turning point.)*
F2	I make a summary of the **basic principles/procedures** of each mathematics topic *(for example, if the topic is "Sketching of parabolas", and my summary of basic facts refers to the following ways of sketching the parabola:* *i) Using the table method;* *j) Calculating the x-intercepts, y-intercept and the turning point,*

	then my summary of basic principles/procedures of how to calculate the turning point is to use the formula: — — .
18	I use different learning strategies, depending on the situation, when I study mathematics.
T1	When it is allowed to talk in class, I explain mathematics to other learners.
T2	When it is allowed to talk in class, I ask other learners to help me with mathematics.
T3	I do mathematics in a group with other learners whenever there is an opportunity.
W1	When I study mathematics or do homework in mathematics, I think aloud *(express my thoughts verbally.)*
W2	I make notes of mistakes I make or misconceptions I have during the completion of a mathematics activity *(for example, I make a note of the fact that I always forget to check the solution when I solve an equation where I square both sides if there is a square root sign in the equation.)*
W3	I make notes of ways to correct the mistakes I make or correct the misconceptions I have during the completion of a mathematics activity *(for example, I have written a note that reminds me to always substitute the answer into the first line of an equation to check whether the answer satisfies the equation.)*
W4	I make a summary of new mathematical knowledge I acquire during the completion of a mathematics activity *(for example, I use a table to represent the similarities and differences between simple interest and compound interest.)*
19	After I have solved a mathematics problem, I ask myself if there was an easier way to solve the problem.

P11	After I have found the solution to a mathematics problem, and there is enough time available, I try to find the solution by using a different method *(for example, if the question states "Simplify —— , and find the answer in the following way:* *—— = =8 .* *A different method to find the answer is: —— = —— = 8)*
20	I can control how well I learn in mathematics.
S1	When I start with a mathematics activity, I make sure that I know what the instructions are *(for example, whether it is an individual activity or a group work activity.)*
S2	I make sure that I know how much time I have available to complete a mathematics activity (*for example, the project must be finished in two weeks' time.)*
G2	Before I start with a mathematics activity, I set myself specific goals that I would like to achieve *(for example, I want to have all my home work questions correct; or I want to score 80% in the test.)*
G3	During a mathematics activity, I am aware of my progress towards the goals I want to achieve *(for example, I have had all my homework correct for the past two weeks so I think that I am on track to obtain 80% for the test.)*
21	I periodically do revision to help me understand important relationships in mathematics.
G3	During a mathematics activity, I am aware of my progress towards the goals I want to achieve *(for example, I have had all my homework correct for the past two weeks so I think that I am on track to obtain 80% for the test.)*
22	I ask myself questions about the problem before I begin to solve a mathematics

	problem.
P4	Before I answer a mathematics question, I first make sure that I understand the question very well by answering the following questions: What information is given? What is the unknown? What are the conditions?
23	When I start to solve a mathematics problem, I think of several ways to solve the problem and choose the best one.
P5	Before I start with a mathematics activity, I remind myself of different problem solving strategies that I could possibly use (*for example, pattern recognition; try-and-improve; imagine that the problem has been solved and then working back to the question; drawing a diagram or sketch; first solve a simpler problem (modification of the problem); represent the data in table form; generalisation.)*
24	I summarize what I learn when I study.
F1	I make a summary of the **basic facts** that I must have on each mathematics topic *(for example, if the topic is "Sketching of parabolas", my summary refers to the following ways of sketching the parabola:* *m) Using the table method;* n) *Calculating the x-intercepts, y-intercept and the turning point.)*
F2	I make a summary of the **basic principles/procedures** of each mathematics topic *(for example, if the topic is "Sketching of parabolas", and my summary of basic facts refers to the following ways of sketching the parabola:* *k) Using the table method;* *l) Calculating the x-intercepts, y-intercept and the turning point,* *then my summary of basic principles/procedures of how to calculate the turning point is to use the formula: — — .*
25	I ask other learners for help when I do not understand something in mathematics.

T2	When it is allowed to talk in class, I ask other learners to help me with mathematics.
26	I can **motivate myself** to study for a mathematics test or examination.
G2	Before I start with a mathematics activity, I set myself specific goals that I would like to achieve *(for example, I want to have all my home work questions correct; or I want to score 80% in the test.)*
G3	During a mathematics activity, I am aware of my progress towards the goals I want to achieve *(for example, I have had all my homework correct for the past two weeks so I think that I am on track to obtain 80% for the test.)*
27	I am aware of what learning strategies I use when I study mathematics.
28.	I ask myself how useful my learning strategies are while I study for a mathematics test or examination.
29	I use my strengths in mathematics to compensate for my weaknesses in mathematics.
H6	I know which topics/concepts/procedures in mathematics I find easy to do *(for example, I find it easy to work with linear equations and to represent the linear function graphically.)*
H7	I know which topics/concepts/procedures in mathematics I find difficult to do *(for example, I struggle to solve quadratic equations by completing the square.)*
30	When I receive new information about a familiar topic or a new topic in mathematics, I focus on the meaning and significance of the new information.
L1	Whenever a new topic/concept in mathematics is explained, I determine the application/relevancy of that topic/concept to my life specifically *(for example, I can use the surface area formula of a rectangle to determine how much paint I need to paint my room.)*

F2	I make a summary of the **basic principles/procedures** of each mathematics topic *(for example, if the topic is "Sketching of parabolas", and my summary of basic facts refers to the following ways of sketching the parabola:* *m) Using the table method;* *n) Calculating the x-intercepts, y-intercept and the turning point,* *then my summary of basic principles/procedures of how to calculate the turning point is to use the formula: — — .*
L2	Whenever a new topic/concept in mathematics is explained, I determine the application/relevancy of that activity to real life in general *(for example, the height that a rocket reaches after a certain time can be determined by using quadratic equations.)*
L3	Whenever a new topic/concept in mathematics is explained, I determine the application/relevancy of that activity to my other subjects *(for example, calculating the surface area of organisms, as part of the subject Life Sciences, can help me determine how those organisms have adapted to the amount of sunlight their skins absorb.)*
31	I create my own examples to make new information I receive in mathematics more meaningful and understandable.
L1	Whenever a new topic/concept in mathematics is explained, I determine the application/relevancy of that topic/concept to my life specifically *(for example, I can use the surface area formula of a rectangle to determine how much paint I need to paint my room).*
L2	Whenever a new topic/concept in mathematics is explained, I determine the application/relevancy of that activity to real life in general *(for example, the height that a rocket reaches after a certain time can be determined by using quadratic equations.)*

L3	Whenever a new topic/concept in mathematics is explained, I determine the application/relevancy of that activity to my other subjects *(for example, calculating the surface area of organisms, as part of the subject Life Sciences, can help me determine how those organisms have adapted to the amount of sunlight their skins absorb.)*
32	I am a good judge of how well I understand something in mathematics.
S5	When I answer a mathematics question, I can identify that part of the mathematics question that I understand well *(for example, when the question states: "Solve the quadratic equation by completing the square", the part of the question that I understand well is the part that requires me to solve the quadratic equation.)*
S6	When I answer a mathematics question, I can identify those aspects of the mathematics question that I **do not** understand well *(for example, when the question states: "Solve the quadratic equation by completing the square", the part of the question that I do not understand well is the part that requires me to complete the square.)*
33	I find myself using helpful learning strategies in mathematics automatically (without consciously thinking about it).
34	When I solve a mathematics problem, or when I study for a mathematics test or examination, I find myself pausing regularly to check my comprehension.
P4	Before I answer a mathematics question, I first make sure that I understand the question very well by answering the following questions: What information is given? What is the unknown? What are the conditions?
P5	Before I start with a mathematics activity, I remind myself of different problem solving strategies that I could possibly use (*for example, pattern recognition; try-and-improve; imagine that the problem has been solved and then working back to the question; drawing a diagram or sketch; first solve a simpler problem*

	(modification of the problem); represent the data in table form; generalisation.)
P6	If I do not make progress when I answer a mathematics question, I change the strategy/method that I am using *(for example, if I have to solve a quadratic equation, my strategy is to first factorise the equation. If I cannot factorise the equation, I decide to change my strategy by using the quadratic formula to solve the equation.)*
P7	During the completion of a mathematics activity, I can give a reason (or reasons) why I have decided to use a particular method/strategy *(for example, I draw a diagram of the information in the question because the information is about different shapes a farmer can use to make an enclosure, and a diagram with different shapes helps me to order my thoughts.)*
P8	When I answer a mathematics question, I can explain why I do each step of the answer in that particular way *(for example, if the question states "Solve for x if = 25", I can explain the next step as follows: = I take square roots both sides, but the right-hand side must have the ± sign because my answer must have two values as I solve a quadratic equation.)*
35	I know in which situation each problem solving method I use will be most effective.
P7	During the completion of a mathematics activity, I can give a reason (or reasons) why I have decided to use a particular method/strategy *(for example, I draw a diagram of the information in the question because the information is about different shapes a farmer can use to make an enclosure, and a diagram with different shapes helps me to order my thoughts.)*
36	I ask myself how well I accomplished my goals once I am finished studying for a mathematics test or an examination.
G3	During a mathematics activity, I am aware of my progress towards the goals I want to achieve *(for example, I have had all my homework correct for the past*

	two weeks so I think that I am on track to obtain 80% for the test.)
37	I draw pictures or diagrams to help me understand while I am learning mathematics.
F1	I make a summary of the **basic facts** that I must have on each mathematics topic *(for example, if the topic is "Sketching of parabolas", my summary refers to the following ways of sketching the parabola:* *o) Using the table method;* p) *Calculating the x-intercepts, y-intercept and the turning point.)*
F2	I make a summary of the **basic principles/procedures** of each mathematics topic *(for example, if the topic is "Sketching of parabolas", and my summary of basic facts refers to the following ways of sketching the parabola:* *o) Using the table method;* *p) Calculating the x-intercepts, y-intercept and the turning point,* *then my summary of basic principles/procedures of how to calculate the turning point is to use the formula: — — .*
W4	I make a summary of new mathematical knowledge I acquire during the completion of a mathematics activity *(for example, I use a table to represent the similarities and differences between simple interest and compound interest.)*
P5	Before I start with a mathematics activity, I remind myself of different problem solving strategies that I could possibly use (*for example, pattern recognition; try-and-improve; imagine that the problem has been solved and then working back to the question; drawing a diagram or sketch; first solve a simpler problem (modification of the problem); represent the data in table form; generalisation.)*
38	**After** I have solved a mathematics problem, I ask myself whether I have considered different ways to solve the problem.
P11	After I have found the solution to a mathematics problem, and there is enough

	time available, I try to find the solution by using a different method *(for example, if the question states "Simplify —— , and find the answer in the following way:* *—— = =8 .* *A different method to find the answer is: —— = —— = 8)*
39	I try to put mathematics questions into my own words.
P3	I can state a mathematics question in my own words or in a different way that could help me to understand the question better *(for example, the question "Solve for x" can also be stated as "Determine the x-intercept(s)" or "Determine the root(s) of the equation.")*
40	I change my problem solving method when I fail to make progress when I try to solve a mathematics problem.
P6	If I do not make progress when I answer a mathematics question, I change the strategy/method that I am using *(for example, if I have to solve a quadratic equation, my strategy is to first factorise the equation. If I cannot factorise the equation, I decide to change my strategy by using the quadratic formula to solve the equation.)*
41	I read the question carefully before I answer a mathematics question.
P4	Before I answer a mathematics question, I first make sure that I understand the question very well by answering the following questions: What information is given? What is the unknown? What are the conditions?
42	When I read a mathematics question, I ask myself if what I am reading is related to what I already know.
B1	When I answer a mathematics question, I can identify the main topic in mathematics that the question is about *(for example, if the question states: Solve*

	for x: $x^2 - 4x = 0$, then the main topic is "Solving of quadratic equations.")
B2	When I answer a mathematics question and I can identify the main topic in mathematics that the question is about, I can also identify a supporting topic/supporting topics that the question is about *(for example, if the question states: Solve for x: $x^2 - 4x = 0$, then the main topic is "Solving of quadratic equations.", and supporting topics are "Factorisation of quadratic equations" and "Using the formula to solve a quadratic equation.")*
43	If I do not make progress when I solve a mathematics problem, I ask myself whether my first understanding of the problem was correct.
S2	I make sure that I know how much time I have available to complete a mathematics activity (*for example, the project must be finished in two weeks' time.)*
44	I organize my time to best accomplish the goals I set in mathematics.
S2	I make sure that I know how much time I have available to complete a mathematics activity (*for example, the project must be finished in two weeks' time.)*
45	I learn better when I am interested in a specific mathematics topic.
H2	I know whether I like a specific mathematics topic or not *(for example, we are busy with financial mathematics, I do not like it at all.)*
46	When I study mathematics, I try to break down the work into smaller sections.
F1	I make a summary of the **basic facts** that I must have on each mathematics topic *(for example, if the topic is "Sketching of parabolas", my summary refers to the following ways of sketching the parabola:* *q) Using the table method;* r) *Calculating the x-intercepts, y-intercept and the turning point.)*

F2	I make a summary of the **basic principles/procedures** of each mathematics topic *(for example, if the topic is "Sketching of parabolas", and my summary of basic facts refers to the following ways of sketching the parabola:* *q) Using the table method;* *r) Calculating the x-intercepts, y-intercept and the turning point,* *then my summary of basic principles/procedures of how to calculate the turning point is to use the formula: — — .*
47	When I study mathematics, I focus on how the specific topic I study fits in with the other topics in mathematics.
B1	When I answer a mathematics question, I can identify the main topic in mathematics that the question is about *(for example, if the question states: Solve for x: $x^2 - 4x = 0$, then the main topic is "Solving of quadratic equations.")*
B2	When I answer a mathematics question and I can identify the main topic in mathematics that the question is about, I can also identify a supporting topic/supporting topics that the question is about *(for example, if the question states: Solve for x: $x^2 - 4x = 0$, then the main topic is "Solving of quadratic equations.", and supporting topics are "Factorisation of quadratic equations" and "Using the formula to solve a quadratic equation.")*
48	I ask myself questions about how well I am doing **while** I am solving a mathematics problem.
P6	If I do not make progress when I answer a mathematics question, I change the strategy/method that I am using *(for example, if I have to solve a quadratic equation, my strategy is to first factorise the equation. If I cannot factorise the equation, I decide to change my strategy by using the quadratic formula to solve the equation.)*
G3	During a mathematics activity, I am aware of my progress towards the goals I want to achieve *(for example, I have had all my homework correct for the past*

	two weeks so I think that I am on track to obtain 80% for the test.)
49	I ask myself if I have learned as much as I could have once I finish studying.
P12	After I have solved a mathematics problem, I determine whether any aspect(s) of the mathematics problem solving activity can be applied to other mathematics problems *(for example, if I find that a circle has a bigger area than a square if the circumference of the circle equals the perimeter of the square, then I apply the same principles when I compare differently shaped 3D objects.)*
50	When I read a mathematics question, I stop and reread any section of the question that is not clear.
P4	Before I answer a mathematics question, I first make sure that I understand the question very well by answering the following questions: What information is given? What is the unknown? What are the conditions?

APPENDICES C1-C9

FIRST AND SECOND PROBLEM-SOLVING SESSIONS

APPENDIX C1

PROBLEM STATEMENT FOR BOTH PROBLEM-SOLVING SESSIONS

Instructions

1. Please write down the date, your name and surname.
2. Read the problem statement and complete the table.

Name and surname:...
Date:...

Problem statement
A farmer has 100m of fence available to build an enclosure (kraal) for his 650 sheep. The farmer is not yet sure which shape he is going to use for the kraal. What advice can you give to the farmer? Base your advice on your calculations.

Write down all your thoughts and questions relating to the problem. Also write down all sketches and calculations that correspond with your thoughts.

Written account of thoughts	**All sketches and calculations that correspond with your thoughts**

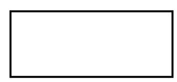
APPENDIX C2

ANALYSIS OF BOTH PROBLEM-SOLVING SESSIONS

(Examples of three learners' work: Learner 1; Learner 8; Learner 15)

Learner 1 (Qualitative pre-test)

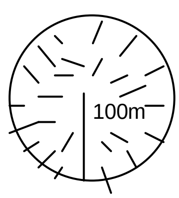
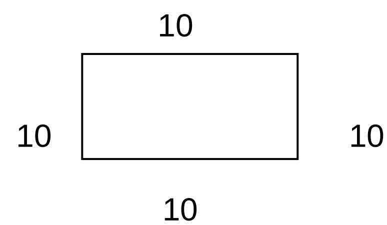
100m and 650 sheep underlined. "Base your advice on your calculations" underlined. **(IMS 13).**

Written account of thoughts:	**All sketches and calculations that correspond with your thoughts**
Check the shape of the sheep first. **Discussion: She focused on the meaning and the significance of the given information. (IMS 30).** Then it is a square shape the side length. **Discussion: The learner made a conceptual mistake by assuming that the shape of the kraal and the sheep's shape must be similar. She focused on the meaning of new information but did not do it correctly.**	10m 10  10m 10m x 10m = $100m^2$ 10 **Discussion: She drew a diagram. (IMS 37).** **Discussion: The learner made a conceptual mistake by working with a perimeter of 40m, instead of 100m. She drew a diagram with an area of $100m^2$, instead of the diagram having a perimeter of 100m.**
Okay so then how many sheep would fit in one side.	5 x 10 = 50 6 by 6 **Discussion: She assumed that 5 sheep would occupy 1m of the side fence. It is unclear what she means by the statement** 6 by 6".

Learner 1 (Qualitative post-test)

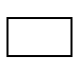
Nothing in the problem statement was underlined.

Written account of thoughts:	All sketches and calculations that correspond with your thoughts
Make a square kraal. **Discussion: She considered different ways of solving the problem. (M 2)**	Area = l x b = 10 x 10 = 100m so 650 – 100 = 550 sheep that can fit in. So this is not the best option. **Discussion: She asked herself question about the problem and checked her comprehension. (M 34, M 48).** **Discussion: The learner made a conceptual mistake by working with a perimeter of 40m instead of a perimeter of 100m, and calculated the area as 100m. She equated 100m to 100 sheep. It is unclear why she subtracted 100 from 650. She evaluated her solution and realized that it is not correct according to her calculations.**
Circle kraal. **Discussion: She considered different ways of solving the problem. (M 2).** **Discussion: The learner considered two different ways of solving the**	at the coast line of the circle there could be 43 sheep placed. Plus another circle of row of 31 then we can see that 650 sheep would fit in perfectly because the area of a circle is bigger than of the square. **Discussion: She remembered that the circle has a bigger area than the**

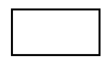
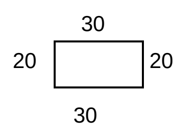
problem by working with a square and a circle. (P 23).	**square. (DK 17).** Area = = 31 415,93m^2 31 415,93 – 650 sheep = 30 765,93 area that the sheep would use, or take up. **Discussion: The learner made many conceptual errors here. She assumed that 43 sheep would touch the fence. She also worked with a radius of 100m (note the corresponding sketch in the next row's left column). It is unclear why 650 was subtracted from the area.**
 **Discussion: She drew a diagram. (IMS 37).**	 **Discussion: She drew a diagram. (IMS 37).**

Learner 8 (Qualitative pre-test)

She underlined "shape he is going to use for the kraal." She also wrote down "100" and drew a rectangle.

100 

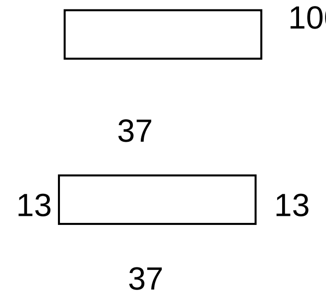
Problem statement: A farmer has 100m of fence available to build an enclosure (kraal) for his 650 sheep. The farmer is not yet sure which <u>shape he is going to use for the kraal.</u> What advice can you give to the farmer? Base your advice on your calculations.

Written account of thoughts:	**All sketches and calculations that correspond with your thoughts**
How many sheep.	650 sheep
Size of the fence. **Discussion: She used the word "size" but probably meant "length". She put the problem statement into her own words. IMS 39.**	100m
Shape of the sheep. **Discussion: This is not really relevant to the problem. Incorrect application of IMS 13.**	
Shape for kraal. **Discussion: She assumed that a rectangle is the appropriate shape. She did not apply P 23.**	Rectangle
Dividing 100 into 4 (2 equal sides and 2 larger equal sides in a rectangular form.	 **Discussion: She used a diagram. (IMS 37). She chose dimensions without motivating why.**

Learner 8 (Qualitative post-test)

She underlined "shape".

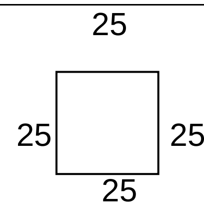
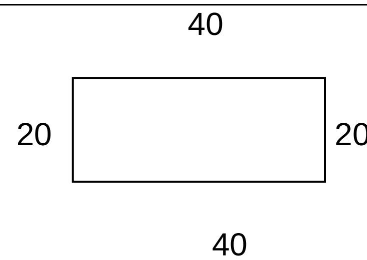
Written account of thoughts:	**All sketches and calculations that correspond with your thoughts**
A shape that wouldn't have any spaces between if the 650 are placed in the enclosure (kraal) e.g. a circular shape which will have a greater volume.	

Discussion: She made a conceptual error by referring to the volume of a circle.	
Rectangle shape. **Discussion: She considered different shapes. (P 23).**	100 4 = 25 25 2 = 12,5  **Discussion: She drew diagrams of different rectangles. (IMS 37).**

Learner 15 (Qualitative pre-test)

Nothing in the problem statement was underlined.

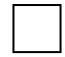
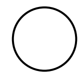
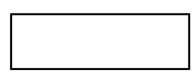
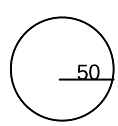
Written account of thoughts:	All sketches and calculations that correspond with your thoughts
A square or a circle? **Discussion: She took different problem solving methods into account. (P 23; M 2).**	
Area of circle. **Discussion: She understood that the main part of the problem is about the area concept. (IMS 13; IMS 30).**	**Discussion: She used the correct formula. (DK 17).**
Area of square.	$s^2 = 10^2 = 100$m **Discussion: She made a conceptual error by using side lengths of 10m. Another conceptual error was made when she put the area calculation equal to the perimeter.**

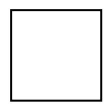
	 **Discussion: She calculated the square's dimensions correctly. (DK 17).**
Area of rectangle.	 **Discussion: She calculated a rectangle's dimensions correctly. (DK 17).**
Size of average sheep? **Discussion: She asked herself questions about the problem. (P 22).**	unknown…?
Does that matter? **Discussion: She tried to figure out what the main part of the problem is. (IMS 13; IMS 30).**	650 = **Discussion: She made a conceptual error by assuming that the area of the circle is 650m^2.** 206,9 = = 14,38

Learner 15 (Qualitative post-test)

Nothing in the problem statement was underlined.

Written account of thoughts:	All sketches and calculations that

	correspond with your thoughts
Square, circle or triangle? **Discussion: She took different problem solving methods into account. (P 23; M 2).**	Area of square = s x s  **Discussion: She used the correct formula. (DK 17). By working with the concept area, she identified the main part of the problem. (IMS 13; IMS 30).**
	Area of circle =  **Discussion: She stated the formula for the area of the circle correctly. (DK 17).**
Not work with triangle because of corners. **Discussion: She realized that the triangle is not the best option. (P 23).**	Area of triangle = – b.h **Discussion: She used the correct formula. (DK 17).** Area of rectangle = l x b **Discussion: She used the correct formula. (DK 17).**
Perimeters (as I have 100m). Investigate the vast difference between area and perimeter. **Discussion: She understood the difference between the concepts perimeter and area. (IMS 30; IMS 31)**	30 20  20 Area = 30 x 20 = $600m^2$ 30 **Discussion: Discussion: She used the correct formula. (DK 17).**  Area = P = = $.50^2$ 100 = = 7854 — **Discussion: She stated the correct**

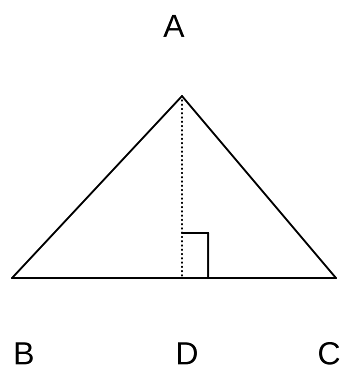
	formulas for the area and circumference of the circle. (DK 17). She worked out the length of the radius correctly, but she made a conceptual error by using a radius length of 50m when she calculated the area.  Area = s x s = 25 x 25 = 625 m^2 25 **Discussion: Her calculations were correct. (DK 17).**
The circle will be the best option. **Discussion: She based her answer on the correct calculations.**	Area = = = **Discussion: Her calculations were correct assuming that she rounded the radius to 16m. (DK 17).**

APPENDIX C3

SOLUTION FOR BOTH PROBLEM-SOLVING SESSIONS

EQUILATERAL TRIANGLE

Solution: Length of one side = —— = 33, m



Altitude: $AD^2 = AB^2 - BD^2$ (Theorem of Pythagoras)

$AD^2 = (33,\quad ^2 - (16.66667)^2$

$AD^2 = 1111.111 - 277.7778$

$AD^2 = 833.3333$

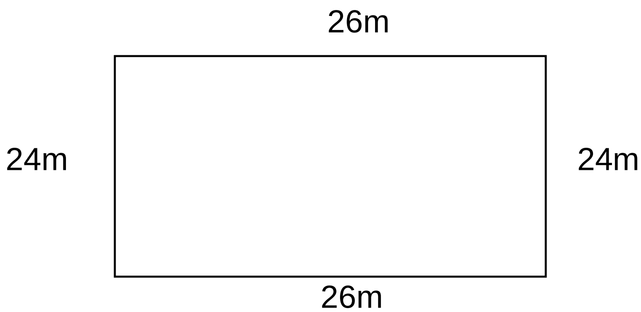
$AD = 28.86751$ m

Area of triangle = – x BC x AD

= – x 33, x 28.86751

= 481.13 m^2 (Rounded off to two decimal digits)

RECTANGLE



Area of rectangle =

SQUARE

Area of square =

CIRCLE

Circumference of circle =

Area of circle

APPENDIX C4

THE LEVEL OF LEARNER METACOGNITION DURING BOTH PROBLEM-SOLVING SESSIONS

MAI item number applied by learners in the first problem-solving session.	Frequency of MAI items applied by all learners in the first problem-solving session.	MAI item number applied by learners in the second problem-solving session	Frequency of MAI items applied by all learners in the second problem-solving session.
Declarative knowledge			
Item 5	1	Item 5	1
Item 17	20	Item 17	33
	Total: 21		**Total:34**
Procedural knowledge			
Item 14	1	Item 14	3
	Total:1		**Total:3**
Conditional knowledge			
Item 35	1	Item 35	1
	Total:1		**Total:1**
Planning			
Item 22	17	Item 22	8
Item 23	17	Item 23	32
	Total:34		**Total:40**
Information Management			
Item 13	21	Item 13	11
Item 30	18	Item 30	11
Item 31	1	Item 31	3
Item 37	6	Item 37	7
Item 39	1	Item 39	1
	Total:47		**Total:33**
Monitoring			

Item 2	13	Item 2	23
Item 11	2		
Item 34	2	Item 34	1
Item 48	3	Item 48	1
	Total:20		**Total:25**
	Debugging		
Item 25	1		
		Item 40	1
	Total:1		**Total:1**
	Evaluation		
Item 7	2	Item 7	1
Item 36	1		
	Total:3		**Total:1**

APPENDIX C5

ANALYSIS OF LEARNERS' CONCEPTUAL ERRORS AND ACHIEVEMENT LEVELS IN BOTH PROBLEM-SOLVING SESSIONS

(Examples of three learners' work: Learner 10; Learner 17; Learner 24)

Learner number	i) Conceptual errors in the pre-test and the post-test problem-solving activities. ii) Comparison of pre-test and post-test learner activities.
10	**i) Conceptual errors:** **Pre-test:** She made a conceptual error because the problem is about whether the area is big enough, not whether there is enough material. The material represents the perimeter. She made a conceptual error by taking the dimensions of a square while she referred to a rectangle. She made a conceptual error by confusing the perimeter of 100m with the area. **ii) Comparison of pre-test and post-test learner activities:** In the pre-test she only considered the square and the rectangle, but in the post-test she considered the square and the circle. **She solved the problem in the post-test.**
17	**i) Conceptual errors:** **Pre-test:** She assumed that the length of a sheep is 2m. She made a conceptual error by not accounting for the breadth of the sheep. She made a conceptual error by dividing the 1300 by 4 to work out the area that the sheep

	will cover. **ii) Comparison of pre-test and post-test learner activities:** In the pre-test, she only worked with the square. In the post-test, she only worked with the circle and she solved the problem, but she did not compare the circle with other shapes. **She did not solve the problem in either problem-solving activity.**
24	**i) Conceptual errors:** **Pre-test:** She made a conceptual error by taking the diameter as 50m. **Post-test:** She made a conceptual error by expressing the radius as half of the circumference. She made a conceptual error by dropping She made a conceptual error by using the perimeter's length as the side length of the square. **ii) Comparison of pre-test and post-test learner activities:** In the pre-test, she only considered a circle, but in the post-test she considered a circle and a square. **She did not solve the problem in either problem-solving activity.**

APPENDIX C6

COMMON CONCEPTUAL ERRORS IN BOTH PROBLEM-SOLVING SESSIONS

Common conceptual errors	Learner number	Frequency
Using a diameter length of 50m.	18; 23; 24	3
Using a radius length of 50m, instead of 15,92m.	4, 9; 15; 24	4
Using a radius length of 25m.	16; 26	2
Using a radius length of 100m.	1;20; 23	3
Using the wrong formulas for the circumference and the area of a circle.	6; 21; 26	3
Using the 100m as the area instead of the perimeter.	1; 4; 5; 6; 10	4
Using 650 sheep as the area.	5, 9; 15	3
Using a square with incorrect side lengths.	1;5;13;15; 24	5
Putting 650 sheep equal to 100m.	6; 13; 15	3
Using a rectangle with incorrect side lengths.	10; 13; 22	2

APPENDIX C7

CALCULATION ERRORS IN BOTH PROBLEM-SOLVING SESSIONS

Learner number	Calculation errors
14	**Pretest:** She made a calculation error by getting 20 as the answer when she divided 100 by 4.
24	**Posttest:** She made a calculation error by obtaining a product of 200 when she multiplied 100 by 100.
25	**Posttest**: She made a calculation error here by using a height of 25m. If one assumes that she used a triangle with a base of 40m and side lengths of 30m each, then the height must be 22m.

APPENDIX C8

CORRECT LEARNER SOLUTIONS IN EITHER PROBLEM-SOLVING SESSION

Learner number **[First problem-solving session (Qualitative pre-test)]**	**Learner number** **Second problem-solving session (Qualitative post-test)**
2 (She compared the circle's area to the area of a rectangle and a square).	2 (She compared the circle's area to the area of a rectangle and a square).
	10 (She compared the circle's area to the area of a square, and she made a reference to a rectangle without calculating its area).
	12 (She calculated that the circle has a bigger area than a rectangle and a triangle, but she did not explicitly state that the circle was the better option).
	15 (She did not compare the circle's area with the areas of other shapes).
	19 (She planned to compare the circle, square and rectangle, but eventually only compared the square and the circle).

APPENDIX C9

LETTER TO MARK: REQUEST FOR ASSISTANCE WITH ANALYSIS OF LEARNER RESPONSES TO BOTH PROBLEM-SOLVING SESSIONS

...

Thank you for your continued willingness to explore mathematical thinking processes. I attach three files. The one named "Problem solving session 1" shows the worksheet that the learners worked from in the first problem solving session. I also attach a file called "2010 Farmer and sheep problem complete answer" that has a lot of extra information but shows the correct square and circle calculations. As there are many possible calculations concerning triangles and rectangles, I have not included them.

The last file is "Problem solving session 1 and 2 learner answers". It contains the learners answers to the first problem-solving session [...] and also the answers to the the second problem solving session that asked exactly the same question as Activity 1 of the first session. You will notice that each learner's answer to the second session follows their answer to the first session. [...]

From my side, what I ask of you is to look at the learners answers and to note common mistakes and try to identify reasons why they made those mistakes. Also, to compare learners answers of the first session to the second session (6 months later) and comment on similar mistakes being made and growth (in terms of whether they have learned from the first session). Remember that the complete answer to the first session has been done with the learners, but they were not made aware that they were going to get the same problem again. Then, you can perhaps comment on the learners' level of thinking awareness and thought processes, as evident in their left columns [...]

These are just suggestions for analysis; anything from your side that you feel may be important will be appreciated. [...]

All the best, please let me know if anything is unclear.

APPENDICES D1-D7

TEACHER INTERVIEWS

APPENDIX D1

INTERVIEW WITH MARK

(All the questions and the answers pertaining to some selected questions are given as examples of Mark's responses)

Question 1

R: Mark, thank you very much for the opportunity, I appreciate it, your willingness, and the time, you are very busy with . um . other work, so thank you very much for that.

Question 2

R: As I have told you .. um .. previously, I gave you a quick scan of all the questions,.. um ..I am always interested to know from students also why did they become a teacher, perhaps also a mathematics teacher?

M: Okay, .. um .. I am afraid it's going to be quite a long answer .. um ... I ... didn't ... plan from the outset to become a teacher, when I studied at the university, in fact I .. I studied for two years, I studied mathematics then .. um ..after two years I ..um.. I fall out fallen out of the subject completely .. um dropped it totally and did a three year philosophy degree instead .. um ... and then towards the end of that career options were were coming into my head and I wanted, I wanted something that that fitted in with my own personal ethical framework, that was one thing I wanted ..um I wanted to actually give something ..um and and not just going to a career where I am doing doing it for myself or ..um.. just earning money ..um..and so my first thought was I wanted to work for ..um.. some sort of NGO, but I didn't really know how ..um..I didn't really have a big plan as how it would work and then an opportunity came up of doing some ..um.. teaching practice ..um.. as a ..um.. final year university student, actually as a a religious education teacher because that's what that's what they offered philosophy students ..um.. and I took it up, and before that I'd I'd been interested in teaching because, and it's funny because I was in the UK ..um.. at the university at this stage but I'd been to South Africa and I'd seen ..um.. the Liberty Learning Channel ..um.. and I'd seen this guy this this William Smith xxx and he was, he was ..um.. doing this ..these things where he explained something and ..um.. ..um..learners phoned in and and I saw what a difference this this this individual was making by doing this ..um.. and the power of that and it just started a ... it it sowed a seed in my head the value of teaching and what a teacher can do ..um.. and I and I thought that I had probably some of the qualities that would that would be suitable for that ..um.. I've got patience ..um.. ..um.. I was at least at one stage capable in mathematics ..um.. so I ... what I did I started doing some .. I did some mathematics courses to get myself back up to speed which didn't take long ..um.. and I did this teaching course as part of my ..ah.. towards the end of my degree and and then from that point I was I was sold and and I was definitely going to be a teacher ..um.. specifically mathematics? ..um... I could have chosen a few different subjects to teach but I chose mathematics because ..um.. partly because I knew that I was going to... not going to want to stay in the same country so I knew I was going to need something which was transferable out of the UK system and ..um.. secondly something that I genuinely enjoyed enjoyed at school and at school even though at university I fell out with it at school I loved mathematics it was it was just my in the last couple of years of school it was my favourite subject by a long way .. um .. so .. that's yeah .. so .. teaching becau .. a .. because I felt it's something I would enjoy and fit in with my ethical framework and mathematics because it was at school my favourite subject and it had that international transferability.

Question 3

R: Thank you .. um .. if you say it was your favourite subject at school why, why do you say that?

Question 4

R: Um .. if you now think of ..a.. learners and the way they learn mathematics, what do you regard as effective, or quality learning in mathematics?

Question 5

R: Um .. if you think of teaching, in general, perhaps your own practices, what would you regard as quality teaching?

M: The best teaching, I think I have done so far this year has been the bits where I speak the least so .. um .. I .. I think .. and it is is always difficult because I feel .. I feel the most effective teaching is .. um .. when you when you facilitate an activity where where they're busy doing an activity .. um .. and learning through the activity whilst you .. are able to .. sort of go around and and and give go around in the class and ... um ... point in the right direction .. tweak them a little bit .. but allow them to .. um .. allow them to work out where they are going .. I'll give you this specific example this .. this one particular thing that I have done this year that was quite pleased with I made .. um .. a card-sort activity for .. um .. factori .. well quadratic equations, it was for grade 11. It was used to introduce them to completing the square, so I had I had three sets of... well it was all on paper I had to cut it out but they had .. they had the expanded form of the quadratic .. um .. a number of them, the factorised form and the completing the square form, and they hadn't seen completing the square before, so it was an opportunity .. an opportunity to realise that these three expressions were the same, they they mean the same thing, there there's the standard form, the factorised form, the completed squared form .. um .. and so the the the first stage was to match them all up and then I wanted them to see .. um .. they could see how you got the completed square to the standard form but I wanted them to try and see how you got from .. did it the other way round, but now the really good thing about that activity is instead of me standing there .. um .. and talking at them which requires them all to be awake and pay attention which is which is even in a small class like we have in this school, it's quite difficult because they .. they might be a bit tired and it's .. it's hard to listen to somebody the whole time .. whereas this activity they had to engage with it and moving things around and they're thinking and they're, they're they're applying their understanding and that .. that was, I think the best activity I have done all term so far .. um .. and I think that was my most effective piece of teaching .. um .. but ... it's not the bread and butter of what I do unfortunately.

Question 6

R: Thank you. Um .. If you think of the learning process also what what are some of the most common problems you think learners experience in mathematics?

Question 7

R: I .. I would say, if you look at the general activities happening in the mathematics class, you know, studying for exams, doing homework, concentrating in class, perhaps solving problems in group .. um .. what are some of the complaints, perhaps even after you have taught a new concept?

Question 8

R: Ok. Um ... how would you address some of those common problems, how could one, like you've mentioned now, they apply procedures but they can't really transfer to other situations, what can one perhaps do to assist them?

Question 9

R: So talking about effective learning in mathematics .. um .. independent learning skills, study skills in mathematics, taking control of your learning process, what are your views on that?

Question 10

R: If you talk about learning methods, could you perhaps explain more?

Question 11

R: Ok. You have referred earlier to .. um .. the transfer of skills .. um .. in a more open-ended situation .. problem solving situation.

Question 12

R: What are your views on the importance of problem solving .. um .. methods in mathematics, problem solving situations?

M: Ah yeah it's it's interesting, because .. in .. in the context of the of the .. well I'm now, you know fully inside the FET system, the South African FET system .. in the context of that, problem solving accounts for only fifteen percent of the marks .. um .. in the exam .. um ,, but the ability to problem solve counts probably for about sixty percent of the marks overall because the ability to, the ability to transfer the skills .. that is the problem solving ability .. um and if they've got that then the rest should fall into place .. um .. I think it's .. for .. for half the learners who are taking mathematics at the moment in my school it's very important to have problem solving skills .. um .. for the other half .. I, I feel that .. well .. for, for a lot of them it's changing now because more are doing maths literacy .. um .. but .. the, the .. being able to deal with routine questions .. um .. is for them more important, not because of the skills it gives them because the brute reality that is the, the matric certificate that's coming up .. um .. and they, to .. to be able to pass the .. um .. exam as best they can .. um .. unfortunately the, the importance .. in immediate practical terms of the problem solving falls away a little bit but I, I mean I think in a .. in a .. in a pure way I think problem solving is what mathematics is, should be mostly about .. um which is why I like it more than .. um .. the, the .. they didn't have the .. the desire or interest in doing that .. um .. or the .. or even perhaps the capability .. um should rather be in focusing their attentions on something like mathematical literacy .. um .. because … to be honest I think there's, there's limited .. um .. use in mathematics as a, as a pure subject without the problem solving aspect.

Question 13

R: Ok. I just want to figure out one thing, you referred to about half the learners that .. um.. um.. I don't remember your exact words .. words, but they are not really interested, I think in problem solving. Do you mean half of the learners that takes mathematics, or is that the combined mathematics and math lit groups?

Question 14

R: Ok. Thanks. That really cleared it up. May I ask you, say you present a word problem, or a problem solving situation on the board, and .. um .. you are going to facilitate a whole class discussion on that problem .. how would you go about that?

Question 15

R: Thank you. Um [long pause] if you look at the importance of mathematics, of the sub .. as as a subject, do you do you view it as a very important subject, and perhaps connected to that question .. um .. the applicability of mathematics to everyday life .. ok .. so it's kind of a double question, this, but you can answer it separately perhaps .. "What is, is mathematics important at all?" and perhaps then also "Do, do you think it's important .. um .. for learners' everyday lives' .. a .. the application in everyday life?

Question 16

R: You, you referred to the transferability of skills that you pick up in mathematics that .. um .. that other subjects could also teach those skills. Could you perhaps just explain what do you mean by the, what type of skills?

Question 17

R: Ok. You referred to the fact that you encountered some people that say they have a block to mathematics.

Question 18

R: What do you think are the .. are the reasons for, for .. um .. people developing that block?

M: It’s so difficult without .. without being able to sort of see inside their heads, I think .. um [long pause] part of me believes that, really, the the the the .. almost everybody .. um .. is is capable of, of doing .. a .. understanding mathematical concepts, but I, I think these people that have a block, I think it’s partly to do with early experiences of mathematics, I think, if somebody experiences a failure early on in mathematics they lose the, the confidence and I think the emotional impact is enormous .. um .. now, obviously the, the FET teacher, by the time they get to me, it’s kind of .. for a lot of them it’s too late, they’ve had this block for a long time .. um .. and they they they’ve missed concepts from early on and so they just completely, they're switched off to it .. um .. but .. I think that one other thing it’s an emo…, it’s an emotional response .. um .. a .. which is funny because I always think it's funny that mathematics people think of as a cold subject, for such a cold subject people get very emotional about it .. um .. and from conversations with, with learners, they .. they will .. it’s funny, they often .. they often talk about a .. a teacher they had .. um .. who made them feel bad about the subject and I don’t .. the teachers don’t consciously do it but they, it’s probably because they felt .. they felt that they were a failure in the eyes of the teacher because they weren’t getting the right answer and so, and and if their response is just sort of close up on the whole thing, and I think that’s part of it, but I do think there are some people who .. um .. they have .. a .. they have .. less .. I mean, we have to accept that there are those people who are mathematical geniuses you have, you've got, the other side of the coin which is people who have a sort of a mathematical disability if you like, for them it’s just much harder to deve… to develop the concepts. So, I think it is, it’s a combination of early experiences and there are people who don’t have the same innate capacity as others. [long pause] Yeah.

Question 19

R: Mark, thank you very much! Is there anything else you would like to add perhaps, or anything .. related to, to this whole concept of quality learning, teaching?

Question 20

R: Ok, you, you referred now in your response to .. um .. the fact that you concentrate too much content and that you regard the .. um .. teaching of skills as, as better teaching than the teaching of content. Could you perhaps just .. um .. explain that?

Question 21

R: You referred to activity spread in the class, could you explain that?

Question 22

R: Do you .. do you distinguish between the level of difficulty xx so that you assign some groups a different activity perhaps?

Question 23

R: Ok. Um .. Mark, I thought the interview was over, but, but you just used the word "maths club".

Question 24

R: Um .. if you just perhaps tell us something about that, a .. what, what type of learners do you draw, and if you say "No holds barred", what happens in that class?

Question 25

R: Did I understand you correctly that the learners will bring problems, that you also encounter problems that you haven't prepared?

R: Mark, thank you very much, I really enjoyed talking to you, and I really xxxx, thank you very much!

M: Don't hold me to it!

APPENDIX D2

ANALYSIS OF THE INTERVIEW WITH MARK: LEVEL 1

(Examples of how the selected responses of Appendix D1 were analysed on the first level)

Question 2 …I am always interested to know from students also why did they become a teacher, perhaps also a mathematics teacher?	
Mark's responses	**Sub-theme**
I … didn't … plan from the outset to become a teacher,	Aspiration as learner
I wanted something that that fitted in with my own personal ethical framework, that was one thing I wanted ..um I wanted to actually give something ..um and and not just going to a career where I am doing doing it for myself or ..um.. just earning money … it sowed a seed in my head the value of teaching and what a teacher can do…	Service rendered
…I thought that I had probably some of the qualities that would that would be suitable for that ..um.. I've got patience…	Personal characteristics
I was at least at one stage capable in mathematics ..um.. so I … what I did I started doing some .. I did some mathematics courses to get myself back up to speed which didn't take long…	Capability in mathematics
…not going to want to stay in the same country so I knew I was going to need something which was transferable out of the UK system… it had that international transferability…	Employment opportunities
…something that I genuinely enjoyed … at school I loved mathematics it was it was just my in the last couple of years of school it was my favourite subject by a long way … it was at school my favourite subject…	Love of mathematics

teaching … because I felt it's something I would enjoy and fit in with my ethical framework…	Love of teaching
Question 5 Um .. if you think of teaching, in general, perhaps your own practices, what would you regard as quality teaching?	
…where I speak the least… when you when you facilitate an activity where … they're busy doing an activity .. and learning through the activity whilst you are able to sort of go around and … and give go around in the class and point in the right direction , tweak them a little bit but allow them to work out where they are going … it's hard to listen to somebody the whole time .. whereas this activity they had to engage with it and moving things around and they're thinking and … they're applying their understanding and that … was, I think the best activity I have done all term…	Active learners
…they had the expanded form of the quadratic, a number of them, the factorised form and the completing the square form, and they hadn't seen completing the square before, so it was … an opportunity to realise that these three expressions were the same, they … mean the same thing, … there's the standard form, the factorised form, the completed squared form…	Different representations of the same concept
…they could produce some sort of diagram to help them or notes or or whatever .. anything, so long as they're processing information…	Processing of information
Question 12 What are your views on the importance of problem solving .. um .. methods in mathematics, problem solving situations?	
…problem solving accounts for only fifteen percent of the marks .. um .. in the exam .. um ,, but the ability to problem solve counts probably for about sixty percent of the marks …because the ability to, the ability to transfer the skills .. that is the problem solving ability .. um and if they've got that then the rest should fall into place…	Percentage of exam marks allocated to problem solving
…for half the learners who are taking mathematics at the moment in	Relative

my school it's very important to have problem solving skills .. um .. for the other half .. I, I feel that .. well .. for, for a lot of them it's changing now because more are doing maths literacy .. um .. but .. the, the .. being able to deal with routine questions .. um .. is for them more important … they didn't have … the desire or interest in doing that … or even perhaps the capability … should rather be in focusing their attentions on something like mathematical literacy…	importance of problem solving skills
…problem solving is what mathematics is, should be mostly about…	Nature of mathematics
Question 18 What do you think are the .. are the reasons for, for .. um .. people developing that block?	
…I think it's partly to do with early experiences of mathematics, I think, if somebody experiences a failure early on in mathematics they lose the, the confidence and I think the emotional impact is enormous…	Early experiences of mathematics
…it's an emotional response .. um .. a .. which is funny because I always think it's funny that mathematics people think of as a cold subject, for such a cold subject people get very emotional about it … a teacher they had .. um .. who made them feel bad about the subject and I don't .. the teachers don't consciously do it but they, it's probably because they felt .. they felt that they were a failure in the eyes of the teacher…	Emotional responses
…almost everybody .. um .. is is capable of, of doing .. a .. understanding mathematical concepts … there are those people who are mathematical geniuses you have, you've got, the other side of the coin which is people who have a sort of a mathematical disability … there are people who don't have the same innate capacity as others…	Mathematical ability

APPENDIX D3

ANALYSIS OF THE INTERVIEW WITH MARK: LEVEL 2

(Mark's responses were coded and sub-themes were identified. These sub-themes were grouped together and the following themes emerged: Mark's background and experience; the nature of mathematics; effective learning in mathematics; effective teaching in mathematics; learner affect; problem-solving, and the importance of mathematics as a subject. An example of the analysis of one of these themes, namely ***Effective learning in mathematics*** is *shown)*

Mark's responses	**Sub-theme**
Main theme: Effective learning in mathematics	
…the most obvious part is that they have to do mathematics .. um .. and that's that's unfortunately the last thing that happens in the class sometimes … effective learning is is engaging, engaging with it...	Engaging with mathematics
…making mistakes and then see where the mistakes come from and sort of backing up… and reconceptualising their their … image of .. of the concept...	Learning from mistakes
…it's encountering problems…	Problem solving context
…realising that the inconsistency in their thought processes were .. where it was ..	Awareness of one's thinking processes
...we haven't seen this before… have we done this before?	Learners do not recognise the question
...taking the concepts you have learnt in the lesson and applying them in a slightly different context...	Questions are asked in a different context

...the consequence of this sort of rote style learning is that .. they will .. they will understand something completely when you do the topic but later on .. um .. they mix things up and they confuse things …broadly speaking, lack of understanding but it's it's .. inappropriate use of routine methods...	Rote learning
...they mix things up and they confuse things they bring in ideas that you .. taught them in in one topic and and procedures that in fact is not ideas, it's procedures they have learnt in one topic and they apply it to a different topic...	Confusion between different topics
...understands how we learn best...	Learner knowledge of effective learning
...with the independent learning there has to be sort of self .. self-discipline...	Self-discipline required
...they have to really fully understand the benefits of it .. and the, and the motivation...	Learner understanding of benefits of independent learning skills
and the, and the motivation...	Learner motivation
...people find mind mapping to be a really useful way of learning… who learns in a very visual way … a learning aid, if you like, or a revision aid for later or something that actually helps now in the form of a mind map .. um .. to help them understand the concept...	Mind mapping
...they could produce some sort of diagram to help them or notes or or whatever .. anything, so long as they're processing information...	Processing of information

APPENDIX D4

INTERVIEW WITH LISA

(All the questions and the answers pertaining to some selected questions are given as examples of Lisa's responses)

Question 1

R: Perhaps I can just ask you in general why you became a teacher, a mathematics teacher, .. and your reasons?

L: I think teaching is a calling .. it's definitely not something that you do for the money .. you can't be … from an early age I .. I can remember that teaching was, was a .. was an option .. for me, coming from a … a family, where teaching is quite prominent .. I saw what it entailed and I, I saw what a difference you can make and a .. yeah, I would say it was a calling, I think that is the .. the best way to summarise it. I think why a maths teacher .. why a maths teacher is .. um … because I knew it would ensure me a job, I knew that I would never be without a job, and I knew that I would always be in demand .. and that's the reason.

Question 2

R: Okay, thank you. Um .. what do you regard as effective learning in mathematics, quality learning?

Question 3

R: Yeah .. um .. a learner that achieves success in mathematics, what, what type of …

Question 4

R: Yeah, yeah their secret basically, what do you think?

Question 5

R: All right, great. Um .. what are some of the most common problems you think that learners experience in mathematics?

L: I think … they misjudge their own … knowledge, I think they misjudge the knowledge that they have, they always think they know more than what they really know .. um .. you, you find it when they write test and when you give the test back "Miss, I did learn" and you believe them, they did learn .. but, "Did they learn enough? "Did they start learning early enough?" And when they practised while they were learning, "Did they do it under rele .. relevant circumstances?", meaning that, "Was there a time limit given?" .. ah .. "Did you re-create the pressure situation?" I, I find that is the single most biggest factor .. problem factor is that girls know a lot, but as soon as you put them in a pressure environment, many, many of them are not able to put down on paper what they know. They become their own worst enemies.

Question 6

R: Okay. Thank you. Um .. do you regard it as important that learners must be able to regulate their own learning, take control of their own learning .. um .. could they perhaps be too overly dependent giving guidance or when, when to learn .. perhaps it links with your point about the pressure situation, do they take control of their own learning enough, do you think?

Question 7

R: Um .. the concept problem-solving in mathematics .. um .. what are your, your views on problem solving, and by that I mean .. um .. word problems that you don't immediately have a specific formula or rule to apply to get to the answer, where the learners first have to place the .. the mathematical topic, they, they have to basically make sense of the problem .. um ..what are your views on that, is that an important part of mathematics?

Question 8

R: Okay. Um .. have you perhaps picked up some particular methods or strategies that learners know of that they use when they .. um .. solve word problems, for example, always drawing a sketch or a diagram, do you think they approach word problems in a structured method?

Question 9

R: Um .. then perhaps just .. ah .. if we talk about mathematics in general .. um .. the importance of mathematics. What are you views about mathematics as a school subject and its importance?

L: I'm going to answer this from my heart. I think xxx they place a lot of emphasis on mathematics, which is not a bad thing, but I think a lot of people end up attempting mathematics … that, in the first place, don't have the passion for it, and in the second place, don't have the aptitude for it .. and that puts them in a very awkward situation, because maths is a jealous subject without having to have a jealous teacher. It demands a lot of your time, a lot of mental energy. You've got to fight many ghosts in trying to survive, very often, in senior mathematics .. and I, I find it sad that parents put so much pressure .. on learners to take mathematics .. when they don't have the passion or the aptitude for it, because .. we can lead a rich and a full life without having mathematics. God created us in such a way that He put in us the talents and the passion that we need to accomplish His plan for our lives .. and if we want to go and do other things .. outside of that plan that He has, it's not really successful and rewarding xxx so I find it sad that the government emphasizes mathematics so much.

Question 10

R: Um .. I think, linked to that question .. um I've heard learners asking me many times "Sir, where are we going to use this in everyday life?" Is that a question you get often .. um "How do we apply mathematics in everyday life?"

Question 11

R: Lisa, and then perhaps the um .. last question at this stage, I look at the class situation, learners trying to make sense of mathematics. Do you think that group work can contribute perhaps to an effective mathematics lesson and in which ways perhaps, if it can?

Question 12

R: Perhaps we can finish off with a more general question .. um .. if you think of mathematics teaching in general .. um .. what are perhaps some concerns or suggestions you have that could .. um .. perhaps improve your teaching .. ah .. improve the quality of learners in general .. um .. if you think of some of the obstacles you face perhaps in terms of time for example. What would you chance if you had the ability to change mathematics education?

L: I think time is a big, big factor. I think if sometimes a weaker girl can here it one or two or three more times, or just do five or six or seven more sums .. um .. it would help them .. and the thing is, is that you can create the opportunity for them to do it, but remember that your subject is competing with other subjects for time .. and for attention and there is also only twenty four hours in a day. Yeah, I think um .. I, I think time, if I had more time that would really, that would really help me to be able to practice more. I also think .. um .. somehow to, to have a marking assistant, I think would help me .. um .. to be able to have more time available to help the people that are battling .. but then on the other hand you, you also learn something of what the pupils absorbed by marking their papers .. but I think if I had to choose I would say I, I .. an assistant xxx one could use in whichever way um .. you can, marking maybe ah .. tasks of lesser importance, and time, those are the two things.

R: Lisa, thank you very much.

APPENDIX D5

ANALYSIS OF THE INTERVIEW WITH LISA: LEVEL 1

(Examples of how the selected responses of Appendix D4 were analysed on the first level)

Question 1 Perhaps I can just ask you in general why you became a teacher, a mathematics teacher, .. and your reasons?	
Lisa's responses	**Sub-theme**
I think teaching is a calling .. it's definitely not something that you do for the money … what a difference you can make…	Teaching as a calling
...because I knew it would ensure me a job, I knew that I would never be without a job, and I knew that I would always be in demand...	Job security
Question 5 Researcher: All right, great. Um .. what are some of the most common problems you think that learners experience in mathematics?	
...they misjudge their own … knowledge, I think they misjudge the knowledge that they have, they always think they know more than what they really know...	Misjudging of the level of one's own knowledge
Did they learn enough? "Did they start learning early enough?"	Not learning enough
...Did they do it under rele .. relevant circumstances?", meaning that, "Was there a time limit given?" .. ah .. "Did you re-create the pressure situation?" I, I find that is the single most biggest factor .. problem factor is that girls know a lot, but as soon as you put them in a pressure environment, many, many of them are not able to put down on paper what they know...	Simulating the pressure situation of tests and examinations
Question 9 Um .. then perhaps just .. ah .. if we talk about mathematics in general .. um .. the	

importance of mathematics. What are you views about mathematics as a school subject and its importance?	
...they place a lot of emphasis on mathematics, which is not a bad thing … so I find it sad that the government emphasizes mathematics so much...	Importance of mathematics
...I think a lot of people end up attempting mathematics … that, in the first place, don't have the passion for it...	Lack of passion for mathematics
...don't have the aptitude for it...	Lack of aptitude for mathematics
...because maths is a jealous subject without having to have a jealous teacher. It demands a lot of your time, a lot of mental energy. You've got to fight many ghosts in trying to survive...	Demands of mathematics on learners
...I find it sad that parents put so much pressure .. on learners to take mathematics...	Parental pressure on learners
...we can lead a rich and a full life without having mathematics...	Fulfilling lives without mathematics
Question 12 Perhaps we can finish off with a more general question .. um .. if you think of mathematics teaching in general .. um .. what are perhaps some concerns or suggestions you have that could .. um .. perhaps improve your teaching .. ah .. improve the quality of learners in general .. um .. if you think of some of the obstacles you face perhaps in terms of time for example. What would you chance if you had the ability to change mathematics education?	
...I think time is a big, big factor. I think if sometimes a weaker girl can here it one or two or three more times, or just do five or six or seven more sums .. um .. it would help them .. and the thing is, is that you can create the opportunity for them to do it, but remember that your	Time available for teaching

subject is competing with other subjects for time .. and for attention and there is also only twenty four hours in a day...	
...I also think .. um .. somehow to, to have a marking assistant, I think would help me .. um .. to be able to have more time available to help the people that are battling .. but then on the other hand you, you also learn something of what the pupils absorbed by marking their papers...	Teacher assistant

APPENDIX D6

ANALYSIS OF THE INTERVIEW WITH LISA: LEVEL 2

(Lisa's responses were coded and sub-themes were identified. Then, the sub-themes were grouped together and the following themes emerged: Lisa's motivation for becoming a mathematics teacher; effective learning in mathematics; affect; problem-solving; the importance of mathematics as a subject; the nature of mathematics; the applicability of mathematics; group work, and the effective teaching of mathematics. An example of the analysis of one of these themes, namely ***Problem-solving*** *is shown.)*

Lisa's responses	**Sub-theme**
Main theme: Problem-solving	
...It is an important part, it is also a difficult part...	Importance of problem solving
...difficult to teach someone that does not have the natural ability and flair to be able to approach something like that … I think natural flair gets you far...	Natural ability
Methods that you can employ would be, to just expose them to .. ah .. as great a variety of those difficult things that you can, and after exposing them to the different kinds or types that they can encounter to try and practice as many of them.	Exposing learners to a variety of problems
...repeating that kind of problem they can try and repeat the structure … repeating that kind of problem they can try and repeat the structure and if you expose them to many kind of structures and plans, I think some of them, not all of them, some of them will um .. employ those plans...	Doing the same kinds of problems in a structured way
...I think pupils prefer to be spoonfed and to know exactly what to do...	Spoon feeding
...when you are faced with an unfamiliar situation, I think many of them just .. they, they get frightened and they get scared because they feel insecure and they just close up...	Learner fears

APPENDIX D7

A COMPARISON OF MARK'S AND LISA'S INTERVIEWS

- **Motivation for becoming a mathematics teacher**

Both Mark and Lisa became teachers in order to have a positive impact on learners' lives. They also anticipated that they would have job security as mathematics teachers. Additionally, Mark's love for and capability in mathematics further contributed to his decision to become a mathematics teacher.

- **The importance of mathematics**

The importance of the subject mathematics was affirmed by Mark and Lisa. Mark emphasized the positive impact of mathematics on the development of learners' reasoning skills and study habits. Both teachers affirmed the application of mathematics to everyday life, but they also stated that it was not always easy to demonstrate mathematical applications. Mark, for example, explained that few people would encounter applications of quadratic equations unless they are in mathematical or scientific careers. Lisa felt that apart from obvious applications of geometry and trigonometry, the study of mathematics itself is also worth pursuing.

- **Problem-solving**

For both teachers, a further aspect that demonstrates the importance of mathematics is the opportunity it creates for the fostering of learner problem-solving skills. Mark views problem-solving as the core of mathematics and also as an aspect of mathematics which requires true learner understanding. Lisa viewed problem-solving as a higher-level activity that is difficult to teach if a learner does not possess a natural ability in problem-solving. Mark actually stated that only half of his mathematics learners have the desire or interest to have good problem-solving skills. Mark's statement reflects the role of affect in mathematics achievement, but when Lisa's comment about learners' lack of natural ability in mathematics is considered one wonders whether the lack of interest in problem-solving that Mark experienced could stem from learner inability rather than from a lack of interest in problem-solving.

- **Learner affect**

The significant role of learner affect in mathematics achievement was acknowledged by both teachers. Mark attributed learners' negative attitudes towards mathematics to the way teachers treat learners and also to early experiences of failure in mathematics. To him, intelligent learners could struggle with mathematics because they have "a block on mathematics". Lisa differed in respect of this viewpoint; she felt that learners are negative because they do not have the intellectual skills to handle the pressures of the mathematics teaching-and-learning situation.

- **Effective teaching of mathematics**

Both teachers identified time constraints as the most prominent factor that impacts negatively on their teaching. To Mark, a lack of time implied that less time are spent on facilitating problem-solving sessions in order to complete the prescribed syllabus while Lisa felt that weaker learners are especially affected if less time is available for the practicing of extra questions.

- **Effective learning in mathematics**

Mark and Lisa agreed that true engagement with mathematics is necessary to achieve success. Lisa emphasized the "fighting spirit" that a learner should display. This fighting spirit, she asserted, stems from a learner's motivation of choosing mathematics as a subject.

- **Group work**

Mark and Lisa placed much value on the advantages of group work. Mark highlighted the independent work by group members and a shared responsibility of the final answer as the main benefits of group work while Lisa viewed group work as an opportunity to teach more effectively as some group members could act as teachers in the group.

APPENDICES E1-E17

LEARNER AND TEACHER PERSPECTIVES ON THE MI PROCESS

APPENDIX E1

INITIAL DISCUSSION ON THE IMPLEMENTATION OF THE MI PROCESS

Before the intervention started, Mark and I had an initial discussion about the practical issues related to the MI process. Mark's perspectives on that discussion are indicated next:

> *When I first saw the reflection sheet it seemed like there was a lot of information and as useful as it could be it was going to be difficult to get the learners to understand the potential value as well as to get around the issue of how to use the reflection sheet. However, when we went over approaching a question we realised that one can distinguish different activities in class and indicate which reflections should be used for each activity (that is, learner work, activity feedback, new material teaching and topic review) and the order in which they should be used. By the end of the session we had a system for using the reflection sheet and a plan for a more user-friendly version. I am aware that the learners will perceive it as more work, but shall try to balance this out by reducing mundane homework tasks as well as making the benefits clear. I think this will present a challenge, but the learners are generally quite willing and motivated, so provided they trust my judgment and see that this is a solution to an existing problem they will get on board.*

APPENDIX E2

LEARNER QUESTIONNAIRE: LEARNER PERSPECTIVES ON THE FIRST CYCLE OF THE MI PROCESS

Name:..

Surname:.......................................

1. How did you experience the whole process of using the codes while you learn mathematics? Please describe your feelings and give reasons why you experienced the use of the codes in the way that you have described.

2. What suggestions do you have that could enable us to use the codes in a better and/or easier way, if necessary?

APPENDIX E3

LEARNER FEEDBACK ON THE FIRST CYCLE OF THE MI PROCESS

(Examples of three learners' responses to the questionnaire in Appendix E2: Learner 11; Learner 14; Learner 20)

Learner 11

1. It made me very aware of my approach to maths. I do my activities because I like them not because I have to. I also realised that I enjoy doing maths my way and not by always following specific rules and formulae. The codes brought back the "uniformity" in my work, because my work has no order but when I work with the codes I actively implement a few of the things I think of but don't put down on paper.

2. It's not easier as such but if forces you to think and assess at the same time. But here is an example: Instead of asking if I can also solve my sums in different ways, give a restriction to certain formulae, for example "Complete the activity without using quadratic equation formulae". In that way I think outside the box and if I can't then I can assess, correct myself and improve.

Learner 14

1. Helpful at times but sometimes I was too lazy cause I wanted to get my maths over and done with. And sometimes too lazy to do my homework so I felt guilty for not doing them.

2. Less questions but either than that I think this was a great experiences and I think I will work well in the future thank you.

Learner 20

1. Sometimes very helpful, to get my mind more 'maths orientated' but there were times when I really thought they were unnecessary because the work was sometimes a lot and the codes to add to that would just be a drag.

2. Not make them so many and more yes/no questions to make it too much to write.

APPENDIX E4

ANALYSIS OF LEARNER PERSPECTIVES ON THE FIRST CYCLE OF THE MI PROCESS: LEVEL 1

NEGATIVE ASPECTS

Themes emerging from learners' comments	Learner number	Frequency
Time considerations		
…time taking…	1	17
…took time to do the codes…	2	
…take up time.	3	
…and the amount of time that I have on my hands decreases so does my patience for this type of activity.	6	
… I only do have the codes when required because it is a lot. …matter of time we have to do them.	9	
…it really took me a while to get through the codes.	10	
…it just added on to my work load in math… It was like have to do the same home work three times. It took up too much time.	12	
Felt it took too long.	15	
…time consuming...	13	
There are too many codes, it takes long to take the booklet out and use the codes.	16	
The process is slowly…	17	

…it took a lot of time doing…	19	
…there were times when I really thought they were unnecessary because the work was sometimes a lot…	20	
I felt the codes very long and to do the codes and the homework was exhausting.	21	
Took most of my time …	22	
…I felt as though they were time consuming.	25	
…they took up a lot of time…	26	

POSITIVE ASPECTS

Themes emerging from learners' comments	**Learner number**	**Frequency**
Mathematical understanding		
…understand the homework…	1	12
In a way it did help… …because I read the codes I knew what I had to do.	4	
In the beginning it was very useful and relevant to improving my maths skills… …I know it is meant to help improve my math skills.	6	
…the codes are relevant. …one can also see her mistakes in the mathematical equations. …I can see little improvement in my work.	8	

But re-thinking the codes help because I might feel like I know the work.	9	
The codes brought back the uniform [sic] in my work, because my work has no order but when I work with the codes I actually implement a few of the things I think of but don't put down on paper... In that way I think outside the box...	11	
…helped me understand what I was doing and understand my work … it made my homework more easier and understandable	13	
Helpful at times…	14	
It made me see a wider spectrum of work I was doing ...	15	
It broadens my way of thinking, allowing me to see the other ways of achieving the final answers – breaking the questions down … it helps get to the bottom of the answer more comprehensively and effectively.	17	
When I'm using the codes it becomes gradually easy to deal with the topic. It makes the process of understanding and solving easier than normal.	18	
It helped in many ways because you would have to go through every question step by step. It helped to understand the question and how to solve it … in the end it made it easier for us.	19	

SUGGESTIONS

Themes emerging from learners' comments	Learner number	Frequency
Number and complexity of the codes		
…make the codes short…	1	12

…repeat one question too many times.		
Lessen the amount of codes.	4	
…not be too long sentences to explain the codes…	8	
Not necessary for every sum or maybe less codes.	9	
There should be less codes. We should only do them a few before we start and a few when we are done.	12	
Less questions…	14	
Colour coding would be fun and having a few codes. If you group them as well.	16	
Reducing the number of codes and making the entire process shorter.	17	
…it would be easier if we understood all the codes but we haven't been through all of them.	19	
Not make them so many…	20	
Keep working with the codes but lessen the amount of codes.	24	
Not having so many codes, you can group similar codes under one heading that would save time.	26	

APPENDIX E5

ANALYSIS OF LEARNER PERSPECTIVES ON THE FIRST CYCLE OF THE MI PROCESS: LEVEL 2

Negative aspects relating to the MI process

- **Time considerations**

A very common complaint related to the extra demand the use of the codes placed on the available time to complete their homework. It seems as if the majority of the learners only saw the use of the codes as adding to their work load instead of helping them to learn mathematics more effectively.

- **Attitude**

It is quite logical that the time aspect should have influenced the learners' attitudes relating to the use of the codes. Some learners expressed the negative effect the use of the codes had on their attitudes and they were quite frank in their responses, using phrases like "...groan when we were told to use the codes...", "...a bit irritated..." and "I generally despise the codes..."

- **Clarity of the codes**

Most learners experienced the codes wording as clear and understandable. The only negative feedback came from two learners. One stated that the codes looked the same to her, but she did not indicate which codes specifically she referred to. Another learner stated that she had not been given enough information, presumably about how to apply the codes.

- **Relevancy of the codes**

How relevant were the codes to the process of learning mathematics? Although most learners experienced the codes as relevant, there were some learners that apparently saw no value in the use of the codes. Learner 10, for example, stated that the codes focus too much on problem-solving, while Learner 17 complained about some codes being repeated. Although Learner 26 saw some purpose in the use of the codes, the amount of writing she had to do made her doubt their value.

- **Implementation of the codes**

There was little criticism about the way that the codes were implemented. Learner 10 again referred to the difficulty of applying the codes when there was no problem-solving involved. Some complaints related to the excessive number of codes and the mental demands it posed.

Positive aspects relating to the MI process

The learners' experiences of the positive aspects during the first round of MI involve the following facets: mathematical understanding; awareness of own thinking; goal-setting; attitude; and, implementation of the codes.

- **Mathematical understanding**

Almost half of the learners reported improved mathematical understanding. Improvement in the following areas were mentioned: doing homework; applying mathematics skills; identifying mistakes; knowledge of the work; and, a better structuring of one's work. Some of these learners referred to an improved mathematical understanding in respect of the following problem-solving skills: thinking outside the box; having a broader view of mathematics; broader thinking skills; discovering alternative solutions; analysing a question; and, understanding the question.

- **Awareness of own thinking**

A fair number of learners stated an enhanced awareness relating to aspects of their thinking processes that relate to mathematics procedures or to more general thinking procedures. One learner reported an improved awareness of her progress towards the goals she has set. Another learner also became more aware of her progress as she assessed herself while doing mathematics. Although one learner mentioned an enhanced mental orientation towards mathematics in general, another learner was more specific by referring to a better awareness of how prior knowledge impact on her current practices. Learner 16 experienced an improved awareness of matters that were not related to mathematics only. Learner 21 stated that she had an awareness of the codes even when it was not necessary to apply them.

- **Goal-setting**

Very few learners mentioned that the use of the codes assisted them in setting specific goals they wanted to achieve. Their goals ranged from specific goals relating to homework and finishing an exercise to broader goals that relate to their overall achievement.

- **Attitude**

A number of learners explicitly reported a positive attitude about the use of the codes during the first cycle of MI. Some learners only stated their feelings without motivating their responses, while other learners mentioned the following aspects as motivation for their positive attitudes towards the process of MI: improvement in one's work quality; improvement in one's mathematical knowledge; future benefits in respect of how one will deal with mathematics; and, a greater awareness of one's thinking processes.

- **Implementation of the codes**

A few learners stated that they experienced the implementation of the codes as practical. These learners mentioned that the codes were well-structured and self-explanatory, and that the codes would become more familiar as time goes on.

Suggestions

- **The number of codes**

The learners' main suggestion relates to the excessive number of the codes and they specifically suggested that the number of codes should be reduced. The benefits, according to some of them, would be improved productivity and attitudes.

- **Different format**

The learners also voiced strong criticism against the format of the codes booklet. There were suggestions that more codes should only require yes or no responses instead of requiring an explanation of one's reasoning processes. Others proposed that the codes should not be used for each question during class activities, but rather after a section of work has been done or during homework activities. Also, it was suggested that a learner indicates the code she uses in a different book than the mathematics workbook.

- **Relevancy of the codes**

Very few learners suggested that the codes should be made more relevant to the work they do. Some of these learners also suggested that it should be made easier to understand how the codes link with the work.

- **Familiarising**

Not all learners, however, felt a need to change the MI tool as three learners suggested that their understanding of the codes would improve as time goes on.

APPENDIX E6

LETTER TO MARK: REQUEST FOR ASSISTANCE WITH ANALYSIS OF LEARNERS' FEEDBACK ON THE FIRST CYCLE OF THE MI PROCESS

...

Please find attached the learner feedback after the second term's implementation of the codes. Could you please analyse it in terms of strengths, weaknesses and suggestions for improvement.

I suggest the following:

1. Mainly in Question 1, but also in Question 2, they have identified those aspects of the codes that caused their feelings towards the use of the codes. I suggest that we classify it as strengths and weaknesses, and order it in terms of the frequencies. For example, Weakness: Took too much time (12 learners).
   Strength: Ordered my thoughts (5 learners).

2. In Question 2, we can use a similar approach where we identify common suggestions and record the frequencies, for example:
   Use True/False questions (7 learners).

...

APPENDIX E7

MARK'S ANALYSIS OF THE LEARNERS' FEEDBACK ON THE FIRST CYCLE OF THE MI PROCESS

I requested Mark to share his perspectives in respect of the learners' feedback on the first cycle of MI (see Appendix E4). His response was as follows:

From my analysis of what the learners have written I have broken the positives down into the following table:

Improvement of approach	Improvement of understanding	Self-assessment
6	6	7

From this analysis, all three factors are significant; the learners feel that the support programme has the potential to develop these three things. By self-assessment I mean that they specifically state something that suggests that the codes develop their understanding of their understanding.

The negatives break down as follows:

Time/effort issues	Purpose issues	Overlap
17	2	1

Clearly the length of time and effort required was too high for the learners, and it was only a minority that mentioned the lack of purpose in the codes that they perceived.

In terms of the specific suggestions for improvements, the ones that interested me most were as follows:

- *Prepared sheets*

- *Use of yes/no answers*
- *A system not requiring more than five answers*
- *Regular homework that is code-based and handed in for evaluation*

It may not work out to incorporate all of these issues, but it would certainly be worth considering some. The regular homework issue would require week-by-week planning, or perhaps two designed activities per topic, but it would help set expectations and reduce the 'groan effect' every time the codes are announced in class. It would also help me to monitor how they are being used. Following a poor set of mid-year exam results I am already intending to implement a weekly test system on Fridays to help make the learners aware of their progress with the routine questions and this will complement that.

APPENDIX E8

MARK'S PERSPECTIVES ON THE FIRST CYCLE OF THE MI PROCESS

In response to my request, Mark also wrote a synthesis of his reflections on the first cycle of the MI process.

> *The following summarises my thoughts over the term:*
>
> *Initially I felt that we had produced an excellent tool with all the potential to promote self-reflection and improved understanding. However, as soon as I began to implement it, I realised that the amount of time required for the activity would become a problem and the learners also made this known to me. Despite this, I explicitly asked the learners to use the codes several times and used the principles behind them in my own planning and explanations. I now feel positive about the potential for reworking the tool whilst retaining the purpose behind it and producing something that will effectively aid reflection and comprehension of Mathematics.*

APPENDIX E9

THE SECOND CYCLE OF THE MI PROCESS: ADAPTED TOOL

Name:……………………………………………………………………………………..

Level 1 (Complete as topic progresses)

Solid Foundations: What are the basic facts/subtopics/procedures/skills of this topic?

Codes F1, F2, W4:

Main topic:..

:

Name:………………………………………………………………………………………..

Level 2 (Complete as topic progresses)

Building blocks and **Living Maths**: With which other topics and/or subtopics of other topics does this topic link (also in real-life and/or other subjects)?

Codes B1, B2, L1 to L3:

Main topic:..

Name:……………………………………………………………………………………..

Level 3 (Complete as topic progresses)

What are common mistakes, reasons for this mistake and ways to correct them for this topic?

Codes S7, S8, W2, W3:

Main topic:...

Common mistakes	**Reasons for mistakes and ways to correct them**

What are common mathematics questions in this topic and ways to state the question in your own words or differently?

Code P3:

Common questions	**Questions stated in your own words or stated differently**

Name:………………………………………………………………………………………..

Level 4 (Complete as topic progresses)

Main topic:..

Problems can be solved: Give at least one example of a worked-out problem (higher level question) where the codes P1 to P12 can be applied:

Question and answer	**Corresponding codes**

Name:…………………………………………………………………………………..

Personal reflection (As topic progresses and at the end of the topic)

SECTION A

Main topic:...

Starting up (Codes S1 to S6, S9): When you reflect on this topic, to which degree did you:

a) Follow instructions? *Never Sometimes Mostly Always*

b) Adhere to the time limits of activities? *Never Sometimes Mostly Always*

c) Identify those parts of the teacher's explanation that you understood well? *Never Sometimes Mostly Always*

d) Identify those parts of the teacher's explanation that you **did not** understand well? *Never Sometimes Mostly Always*

e) Identify those parts of a mathematics question that you understood well? *Never Sometimes Mostly Always*

f) Identify those parts of a mathematics question that you **did not** understand well? *Never Sometimes Mostly Always*

g) Give a reason why you thought that a

mathematics question was too difficult, if

applicable? *Never Sometimes Mostly Always*

My Goals (Codes G1 to G3): Complete the following table by writing down the respective goals, and then by answering **Yes** or **No**. Please supply a reason if you answer **No**.

Goals	**I achieved the goals (Yes)**	**I did not achieve the goals (No) Please supply a reason**
Were you aware of your progress toward your goals during the completion of this topic?	**Yes**	**No (Please supply a reason)**

Name:………………………………………………………………………………………..

Personal reflection (As topic progresses and at the end of the topic)

SECTION B

Main topic:..

Talk time (Codes T1 to T3, W1): When you reflect on this topic, to which degree did you:

a) Explain mathematics to other learners when allowed to? *Never Sometimes Mostly Always*

b) Ask other learners to help you with mathematics when allowed to? *Never Sometimes Mostly Always*

c) Do mathematics in a group with other learners when allowed to? *Never Sometimes Mostly Always*

d) Think aloud when allowed to? *Never Sometimes Mostly Always*

Matters of the Heart (Codes H1 to H7)

a) How confident are you that you will be able to answer test and exam questions on this topic?

Not confident *Somewhat confident* *Mostly confident* *Very confident*

b) Did you like this topic? *Not at all* *Somewhat* *Mostly* *Very much*

c) How did you feel when you answered questions correctly? ……………………………..

d) How did you feel when you answered questions **incorrectly**? ……………………………...

e) How motivated were you whilst doing this topic? ……………………………………...

f) Which part(s) of this topic did you find easy to do?

……
……
……
……
………………………………………………………………………………………………

g) Which part(s) of this topic did you find difficult to do?

……
……
……
……
………………………………………………………………………………………………

APPENDIX E10

THE LINKS BETWEEN THE ADAPTED MI CODES BOOKLET (THE ADAPTED TOOL) AND THE ORIGINAL MI CODES BOOKLET

De Corte's (1996) educational learning theory	Codes	First cycle of MI (Original MI codes booklet; see Appendix B6)	Second cycle of MI (Adapted MI codes booklet; see Appendix E5)
Starting up [Constructive]	S1 – S9	S1 – S2: Section A S3 – S4: Section F. S5 – S6, S9: Section B S7 – S9: Section E.	S1 – S6, S9: Personal reflection (Section A). S7 – S8: Level 3.
Solid Foundations [A structured knowledge base]	F1 – F2	F1 – F2: Section G.	F1 – F2: Level 1
Building blocks [Cumulative]	B1 – B2	B1 – B2: Section B.	B1 – B2: Level 2.
My Goals [Goal-oriented]	G1 – G3	G1 – G2: Section A G3: Section D.	G1 – G3: Personal reflection (Section A).
Talk Time [Collaborative]	T1 – T3	T1 – T3: Section D.	T1 – T3: Personal reflection (Section B.)
Living Maths [Situated]	L1 – L3	L1 – L3: Section F.	L1 – L3: Level 2.
My Way [Individually different]	W1 – W4	W1: Section D. W2 – W3: Section E. W4: Section G.	W1: Personal reflection (Section B.) W2 – W3: Level 3. W4: Level 1
Problems can be solved [Heuristics]	P1 – P12	P3 – P5: Section B P1 – P12: Section C. P11: Section E.	P1 – P12: Level 4. P3: Level 3.

Matters of the Heart [Affective components]	H1 – H7	H1: Section B. H2: Section G. H3 – H4: Section E. H5: Section D. H6, H7: Section G.	H1 – H7: Personal reflection (Section B).

APPENDIX E11

LETTER TO THE LEARNERS AT THE START OF THE THIRD TERM

Dear Grade 11 learners,

I want to thank you at this stage for your co-operation and willingness to continue participating in this research project. As stated at the start of the project, we as researchers would like to enhance your ability to be aware of your thinking processes in mathematics. From the feedback that we have received from you (on how you experienced the use of the codes during the second term), Mr H... and I compiled a list of positive aspects on the use of the codes, negative aspects on the use of the codes, and your suggestions for using the codes more effectively.

Regarding the positive aspects, many of you felt that there was real value in using the codes in terms of mathematical understanding and awareness of your thinking processes. Negative aspects that featured strongly were time considerations and how your attitudes were affected due to the way the codes were implemented. A main theme that featured in your suggestions was the need to simplify and lessen the codes ***[Please see the attached sheet (Appendix A) which shows the feedback that all learners have given].***

We appreciate the thoughtfulness that went into your feedback and reflection on the use of the codes. During the third term we plan to use the codes in an adapted way. The adapted format incorporate many of the suggestions you made to make the use of the codes easier. We hope that you will experience it as easier to use and less time consuming ***[Please see the attached sheet (Appendix B) that gives some guidelines on the use of the codes for the third term)]***

All the best!

Mr ... and Mr Du Toit

APPENDIX E12

LEARNER QUESTIONNAIRE: FEEDBACK ON THE SECOND CYCLE OF THE MI PROCESS

Name and surname: ……………………………………………………………………………

You have completed a new tool for the topic trigonometry during the third term. The tool was adapted after the second term by taking your feedback and the teacher's feedback into account. Please answer the following questions about the **format** of the tool.

1. If you find the new tool easier to use than the previous tool, which aspects of the format of the new tool make the tool easier to use than the tool used during the second term? Please explain your answer fully.

……………………………………………………………………………………………………
……………………………………………………………………………………………………
……………………………………………………………………………………………………
……………………………………………………………………………………………………
…………………………………………………………………………………………………
……………………………………………………………………………………………………

2. If you find the new tool more difficult to use than the previous tool, which aspects of the format of the new tool make the tool more difficult to use than the tool used during the second term? Please explain your answer fully.

……………………………………………………………………………………………………
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APPENDIX E13

LEARNER FEEDBACK ON THE SECOND CYCLE OF THE MI PROCESS

(Examples of three learners' responses: Learner 4; Learner 18; Learner 25)

Learner 4

1. The different levels made it easier to understand. The codes were brought in but was [sic] easy to do. By using level 1 I understood what sub-topics we dealt with in a topic. By seeing the sub-topics I could see which topics I knew well or understood and which I didn't. I also saw/knew what I had to study or go over at home every day.
2. We ran out of space in certain levels so we couldn't write out fully.

Learner 18

1. This format is easier, because we wrote what we needed, the common questions and mistakes we make. I felt it was more personal and made me realise why we do trigonometry, the mistakes I'm likely to make as well as my peers and questions that are more frequently asked. And it taught me to look at the questions differently.

2. Did not find it difficult at all.

Learner 25

1. The new tool is that it's easier because we didn't have to do it every time we had an activity to do. It was also easier because for Level one and two we put topic in the centre of the page then spider diagrammed it according to thing associated with it. Which made it easier in the sense that our minds think in terms of mind-maps. Level 3 was effective because it snow what you did wrong and why.

2. (left out)

APPENDIX E14

ANALYSIS OF LEARNER PERSPECTIVES ON THE SECOND CYCLE OF THE MI PROCESS: LEVEL 1

Examples of learner comments relating to Level 1 and Level 2 are given.

1. If you find the new tool easier to use than the previous tool, which aspects of the format of the new tool make the tool easier to use than the tool used during the second term? Please explain your answer fully.		
Learners' comments	**Learner number**	**Sub-theme**
Main theme: Level 1 and Level 2 (mind-map; spider diagram)		
I found the spider diagram easier.	1	Visual representation.
...I respond better to graphics and not to writing...	3	Visual representation.
By using Level 1 I understood what sub-topics we dealt with in a topic. By seeing the sub-topics I could see which topics I knew well or understood and which I didn't. I also saw/knew what I had to study or go over at home every day.	4	Visual representation.
I personally loved the blocks that were provided in Levels 1 – 2 as I believe that I am more of a visual person.	10	Visual representation.
…it has nice easy blocks to use and it explains clearly…	14	Visual representation.
It is visually more pleasing…The 'mind-map' on the facts / sub-topics / topic / procedures and skills are more able to stay in our memory.	15	Visual representation.
…giving us information visually and helps understand how things interlink with each other	19	Visual representation.

… Visual aid.		
This tool also outlined things easier and the more 'pictured format' it was the easier it is to see how trigonometry consists of.	20	Visual representation.
Because now you can visualize it (blocks) it's easier to read and understand the flow of the subject e.g trig, financial maths. The blocks and headings with sub-headings makes the new tool so much easier because it also tells us exactly what you should know under trig, financial maths etc.	22	Visual representation.
The variety of ways this theme would be used was visualized and gave me an idea of what to look forward too.	24	Visual representation.
It was also easier because for Level one and two we put topic in the centre of the page then spider diagrammed it according to thing associated with it. Which made it easier in the sense that our minds think in terms of mind-maps.	25	Visual representation.
Having it grouped on one page makes it easier to understand what exactly is being ask. Having them grouped together on one page made it less frustrating to do. This applies to the other levels too.	26	Visual representation.
...you get more ideas for Level 1 and 2...	9	Applicability of mathematics.
I like the way we have a Level 2 which basically proves to us that maths is used in the outside world.	10	Applicability of mathematics.
…the format of the tool looked at the broadness of the topic and made me realise	13	Applicability of mathematics.

how broad the topic was…		
…Level 2 is very interesting because we learn where we apply the topic and we know why we have to do it.	16	Applicability of mathematics.
Understanding the application if certain topics in real-life gives one more knowledge and comprehension.	17	Applicability of mathematics.
…made me realise why we do trigonometry…	18	Applicability of mathematics.
...the blocks make it more fun because we, as a class, brainstorm to fill in the…	2	Attitude
...I respond better to graphics and not to writing...	3	Attitude
I personally loved the blocks that...	10	Attitude
Level 2 is very interesting...	16	Attitude
...made it less frustrating to do.	26	Attitude
The fact that when you write down certain topics you get to link the ones that relate to each other and see how they relate and that also makes things easier for us...	5	Relationship between topics.
It also allows us to make links which puts some sense into what I was doing.	10	Relationship between topics.
…the format of the tool looked at the broadness of the topic and made me realise how broad the topic was…	13	Relationship between topics.
…helps understand how things interlink with each other…	19	Relationship between topics.
I find it better because it allows you to summarise your work...	6	Summary
It was easier because it was sort of a summary of the things we dealt with within the topic of	11	Summary

trigonometry.		
… we, as a class, brainstorm to fill in the blocks which make us more aware of the topic...	2	Awareness

2. If you find the new tool more difficult to use than the previous tool, which aspects of the format of the new tool make the tool more difficult to use than the tool used during the second term? Please explain your answer fully.	
Learners' comments	**Learner number**
Main theme: No aspects were more difficult	
Nothing was difficult to use for me.	1
N/A	2
I did not find it more difficult.	3
(left open).	5
I find it easier in fact so there are no difficulties that I experienced as such; the focus was specifically on my capability and my lack of mathematical ability.	6
(left open).	11
The format was not difficult at all but easier and took less time, it was also more beneficial to us.	12
(left open).	13
I find the new tool easier.	14
N/A	15
(left open).	16
None. The new tool is way USER FRIENDLY.	17
Did not find it difficult at all.	18
(left open).	20
(left open).	21

(left open)	22
I find this tool less difficult and easier to deal with unlike the second term tool.	23
(left open).	24
(left out).	25
N/A	26

APPENDIX E15

ANALYSIS OF LEARNER PERSPECTIVES ON THE SECOND CYCLE OF THE MI PROCESS: LEVEL 2

In the analysis of the learners' feedback, learner responses to Question 1 were grouped in respect of specific references made to the format of Level 1 – 4, the format of the personal reflection section, the format of the complete tool, and the use of the tool by the whole class.

Positive learner responses about the adapted MI codes booklet

- **Level 1 and Level 2**

Most learners were very positive about the new tool's format in respect of Level 1 and Level 2 as they experienced it as easier to use than the first cycle's tool. They experienced an improvement in the areas of visual effects, the applicability of mathematics, the relationship between mathematics topics, summary of one's work; the fun aspect; and awareness of the mathematics topic.

The majority of the responses involved the visual representation of the aspects related to Level 1 and Level 2. Some learners were aware that it suited their style of learning better, for example, Learner 3 stated that she "respond(s) better to graphics" and Learner 10 explained that she is "more of a visual person". Other learners indicated their knowledge about the learning process by stating that memory is enhanced when mind-maps are used and that our minds "think in terms of mind-maps" (Learner 25).

Some learners referred to another advantage of the visual representation in terms of an enhanced mathematical understanding. One learner's mathematical understanding improved in respect of the identification of topics she understood or did not understand. They also reported better understanding of the sub-topics of trigonometry and financial mathematics and better understanding of the knowledge that was required to deal with these topics. It is interesting to note that the visual representation enabled Learner 24 to "look forward" to future work.

To some learners, Level 1 and Level 2 were easier to use because the applicability of mathematics became evident. Learner 10 stated that Level 2 proved to her that mathematics could be applied in the “outside world”. Other learners' understanding of the broad application value of the topic and the reasons for doing that topic was enhanced.

The visual representation of mathematical topics had a positive effect on some learners' attitudes. One learner had more fun, while another learner stated that she responds better to visual representations. Learner 10 stated that she “personally loves” the visual representation. Further comments related to a raised level of interest and less frustration as compared to the first cycle of MI.

Several learners stated that the relationship between mathematics topics was easier to understand. One learner said it put “some sense” into what she was doing. Learner 13 realized “how broad the topic was”, while other learners stated an improved understanding of the links between the topics in mathematics.

Level 1 and Level 2 gave some learners the opportunity to summarize their work in an organized way, and one learner's awareness of the required facts and skills of the topic was enhanced.

- **Level 3A**

Various learners referred specifically to Level 3A as an easier aspect of the new tool. Themes that were identified from their responses are: Easier identification of mistakes; improve attitudes; helpfulness in respect of the learning process; an enhanced awareness of common mistakes in mathematics; and, the opportunity to learn from peers.

Some mentioned that they were better able to identify their mistakes when they used the new format. Learner 18 was aware of the fact that one is “likely to make” mistakes. Learners also commented on their improved attitudes, one learner actually stated that Level 3A was the “most helpful” of all the levels. Some reported that their awareness of common mistakes was enhanced while two learners experienced Level 3A as an opportunity for the whole class to “learn from each other”.

- **Level 3B**

A few learners experienced Level 3B as useful to identify common questions. The benefits from identifying common questions entailed better preparation for the exam; identifying differences between questions; and, identifying questions that are frequently asked. The format of Level 3B also had a positive effect on two learners’ attitudes while Learner 18 also expressed a changed attitude as Level 3B taught her “to look at the questions differently”.

- **Level 4**

Some learners pointed out that they found Level 4 easier to use than the previous format because Level 4 assisted them in identifying higher-level questions. One learner expressed a better attitude because Level 4 helped her “to see where and how the codes are used” while another learner highlighted alternative solutions as an aspect that she found easier to apply in the new format.

- **Personal reflection**

Various learners found the personal reflection section easier to use in the new format. Some stated that it made them aware about their levels of understanding. Learner 2 mentioned two more advantages that she experienced, namely, an improved awareness of what the topic involves, and goal-setting. Another learner stated that the personal reflection section enabled her to do proper revision and also compelled her to state her feelings about the topic. The new format improved the attitude of Learner 18 as she experienced the personal engagement with the topic as positive.

- **Applying the complete tool**

A fair number of learners commented on the application of the tool in general. To some of them, the fact that they did not have to refer to specific codes anymore made the tool easier to use and more time effective. Further comments about the tool in general involved shows that some learners found it easier to understand (Learner 2, Learner

22); easier to use (Learner 4); less intrusive (Learner 11); a more convenient format (Learner 17); and, that it assists with revision (Learner 23). Of particular importance is the statement by Learner 15 in which she said:

> *All in all it satisfies the components of smart learning of mathematics.*

- **Whole class involvement**

Several learners referred to the collaborative involvement of the whole class as an easier aspect of the new tool because they learn about their peers' views on certain topics.

Negative learner responses about the adapted MI codes booklet

In response to Question 2, many learners explicitly stated that they did not find the adapted tool more difficult to use; in fact, some of them stated that it was "easier and took less time", "user friendly", "less difficult and easier to deal with unlike the second term tool". Several learners did not respond to Question 2, thereby indicating that they did not find any aspects of the adapted tool more difficult to use than the second term tool.

A few learners found some aspects of the new tool more difficult to use than the second term tool. Some considered the lack of space provided in some levels problematic, because, as Learner 19 stated, the new tool "required more writing than the previous tool". One learner referred to the importance of considering the second term tool before the adapted tool is used because the second term tool explained the meaning of the codes in detail. Her remark was important as the new tool was not supposed to replace the second term tool, but rather to represent it in a more user-friendly format. The second term tool, therefore, still served as the basis of the new tool.

Learner 9 found it harder to use the adapted tool and to rely on Mark for assistance, but she also saw that as beneficial to the learning process as the new tool was used in a more constructive manner instead of "just trying to finish quickly like previously".

APPENDIX E16

MARK'S PERSPECTIVES ON THE SECOND CYCLE OF THE MI PROCESS

Hi Mark,

Thanks for our conversation this morning. If I understood you correctly, you are still positive about the use of the tool, although it is not that easy to integrate it during every lesson.

Could you please give me your thoughts on the following?

1. How did you use the tool during the third term for the topic trigonometry in terms of:

1.1 Frequency (every lesson, once a week etc.)?

More frequently at the start of the topic – for the first three lessons or so it was used. After that it was once a week. This was probably an issue of habit more than anything else; it is a different pattern than normal, so it takes a deliberate conscious decision to take it out.

1.2 Method, did they write it down etc. (you explained that you gave them an initial outline for level one and two, and that you later added to that with their contributions?)

I explained the process, then I wrote on the board and they contributed ideas. They copied from the board onto their sheets.

1.3 Use of the tool with respect to the different levels (which levels were the easiest / most difficult to use; did the learners set themselves goals at the start of the topic when they were made aware of the personal section etc.?)

The easiest level to complete was Level one as we could discuss it nicely. Level two was similarly easy, but the discussion of Level one naturally flowed into this, so we had to separate the issues carefully. We did not set goals at the start of the topic, though in retrospect this was a mistake – I should have made the goals clear to them.

1.4 Ease of use (what could some stumbling blocks be that prevent the tool being used as an integral part of every lesson).

The main stumbling block is habit. You get into certain lesson routines, and to break them requires conscious effort. The reason given for this is often time constraints and this is, to some extent, valid. However, the value of the tool is, in my view, great enough to warrant time spent on it and changing my habits so that it becomes integrated is my personal obstacle.

2. What was your explanation to the learners at the beginning of the topic trigonometry in terms of how it is going to work (what your role is going to be, and what their roles are going to be?).

I explained to them that for this topic I would be showing them how to use the tool and the aim would be for them to develop an understanding until they can use it themselves without teacher input. I told them that the tool can also be used for revision purposes once it is completed. I also said that once we get onto revision in class we can go over old topics using the tool as a guideline.

3. Any further thoughts / reflections / suggestions about the future use of the tool and adaptations?

The tool will, I believe, continue to be useful in lessons. It seems to me that one can either use it on an ongoing basis as an integral part of the lesson or retrospectively for revision purposes. In the latter case the different levels will serve to provide a reminder of the main issues in the topic and the connections to other topics. However, on balance my personal belief is that the tool is best used as an integral part of each topic, with the learners developing it according to their experiences (with teacher input on key issues) and then using it retrospectively for revision. The act of completing the tool seems to model the processes involved in good mathematical learning.

APPENDIX E17

ANALYSIS OF MARK'S PERSPECTIVES ON THE SECOND CYCLE OF THE MI PROCESS

During the third term, Mark only used the tool about once a week. He played a more active role than during the second term by writing on the board and asking the learners for contributions. The learners would then use information on the board to complete the tool. Mark found Level 1 and Level 2 easy to complete, but he stated that he neglected to guide the learners in the setting of goals.

I find it very interesting how the efficient use of the tool reflected a continually evolving nature over the course of the second term and the third term. Apart from the changes in the format of the tool, it was originally conceptualised as something that would be used during every lesson. However, it gradually became more practical to use it only once a week, and then not only "as an integral part of each topic" but also "retrospectively for revision purposes."

I belief the key perspectives that Mark shared is highlighted by the following comment:

> ...my personal belief is that the tool is best used as an integral part of each topic, with the learners developing it according to their experiences (with teacher input on key issues) and then using it retrospectively for revision. The act of completing the tool seems to model the processes involved in good mathematical reasoning.

To me, these perspectives represent some core elements of what mathematics teaching should be about because it involves, first, the opportunity for learners to be actively involved in the teaching and learning process by completing the tool "according to their experiences". Also, the vital role of the teacher in facilitating the learner process by giving "input on key issues" is acknowledged. Mark also affirmed his belief that the use of the tool could enhance the learners' mathematical reasoning processes. Additionally, Mark's positive view of the tool is underscored by his assertion that "the value of the tool is "great enough to warrant time spent on it and changing my habits so that it becomes integrated”. His willingness to change his habits to accommodate the use of the tool further indicates his belief in the value of the tool.