TEACHING AND LEARNING OF FRACTIONS IN PRIMARY SCHOOLS IN MASERU

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DEDICATION

This is dedicated to my mother, ’MaliemisoKuleile, and my late father, Sentebale Cletus Kuleile who both instilled in all their children the love of education and perseverance in everything they do.

DECLARATION

I, the undersigned, declare that the thesis hereby submitted by me for the MAGISTER EDUCATIONIS (M.Ed.) degree at the University of the Free State is my own independent work and that I have not previously submitted the same work for a qualification at another university. I further cede copyright of this thesis in favour of the University of the Free State.

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ABSTRACT

Throughout the world governments and other education stakeholders advocate quality education and education for all. Among others, mathematics education is seen by governments as essential in the advancement of the development of countries. Lesotho is no exception in this regard hence mathematics is one of the core subjects in Lesotho's education system. Though Mathematics education is seen as pivotal to the development of countries, analysis of mathematics Junior Certificate (JC) examination results in Lesotho indicates that performance in mathematics is not good. This study therefore aspired to investigate teaching strategies predominantly employed by primary mathematics teachers and assess their effect on learners' meaningful learning of fractions. In order to meet this aim the study attempted to determine what literature said about effective learning and teaching of fractions, the level of training given to mathematics teachers and determine whether effective learning and teaching materialised in the three classrooms that were studied.

The existing literature proposed different teaching strategies that resulted in significant learning of fractions. To investigate dominant teaching strategies that teachers used in the teaching of fractions, class observations of three teachers were conducted. Teachers were observed in their classrooms over a period of time and follow-up interviews were conducted. Samples of the teachers' documents and the learners' work were analysed to evaluate the extent to which effective learning and teaching of fractions were taking place in these respective classes. Literature indicates that effective learning, of fractions, entails meaningful construction of the concept through handling of concrete materials and formation of relationship between concepts. Effective teaching on the other hand entails the ability to create situations in which learning is facilitated. Teachers are said to possess both mathematical knowledge for teaching (MKT) and Pedagogical content knowledge (PCK) in order to be able to teach effectively.

In order to fully understand the level of training that the teachers received teacher trainers were interviewed. It was found that teachers did not engage learners in high order reasoning and problem solving, instead they gave close-ended questions which

learners answered by practising rules and procedures that teachers taught. Learners therefore did not use their own strategies when writing solutions to questions. It was recommended that teachers should use readily available materials like paper and papers and when planning lessons they should think of possible errors, misconceptions and difficulties that learners were likely to have.

KEY WORDS

- **Effective Teaching**
- **Effective Learning**
- **Fractions**
- **Mathematics**
- **Learning Theories**
- **Subconstructs**
- **Problem-Solving**

ACRONYMS

ECoL	**Examinations Council of Lesotho**
JC	**Junior Certificate**
ZPD	**Zone of Proximal Development**
PCK	**Pedagogical Content Knowledge**
MKT	**Mathematical Knowledge for Teaching**
CKM	**Common Knowledge of Mathematics**
SKM	**Specialised Knowledge of Mathematics**

TABLE OF CONTENTS

CHAPTER 3

TEACHING MATHEMATICS

CHAPTER 6

FINDINGS, CONCLUSIONS AND RECOMMENDATIONS

APPENDICES

LIST OF FIGURES

LIST OF TABLES

CHAPTER 1

BACKGROUND TO THE STUDY

1.1 INTRODUCTION

Throughout the world governments and other education stakeholders advocate quality education and education for all. Mathematics teachers are no exception in this regard, hence so much research is going on regarding contextualising mathematics for better understanding and making it realistic (Romberg 2006). Despite several research findings of mathematics teaching, learners still perform poorly in mathematics.

Lesotho is no exception in this regard. This is evidenced by the Examinations Council of Lesotho's (ECoL) (2007) comment in the Junior Certificate (JC) pass list, "it has observed with great concern the deterioration of candidates' performance in Mathematics over the years." The situation poses questions such as, "are mathematics teachers in Lesotho still employing the drill and practice methods of teaching?"

Goldman (2007:75-76) ascribes poor performance of mathematics to teachers' incompetence and lack of deeper understanding of mathematics concepts. Consequently they tell learners what to do and thereafter give them practice questions to help them memorize the procedures.

Alibali (2005) articulates that decimal fraction knowledge is central to mathematics understanding, and it is believed that if effectively taught it can lay solid foundations to mathematics learning. On the other hand, there are difficulties associated with fractions knowledge, some of which are: its multifaceted nature and the language used (Charalambos, Charalambous & Pitta-Pantazi 2006:293-316). If this is the case, then perhaps the question one can ask is how do teachers teach fractions? Are learners given chance to construct their own meaning of fraction?

As a high school mathematics teacher, the researcher made observations regarding students' misconceptions pertaining to fractions and strategies teachers use to teach fractions. When studying the current primary school mathematics syllabus, it depicts that fractions are taught from standard 2, known as grade 4 in other countries. If fractions are taught so early in primary schools, what is it that makes learners demonstrate so much incompetence in fractions when they get to secondary schools? Could it perhaps be that primary school teachers contribute towards learners' shallow understanding of fractions?

1.2 RESEARCH PROBLEMS, RESEARCH QUESTIONS AND AIMS

Evidence as highlighted by primary literature in the foregoing section demonstrates that effective teaching and learning of mathematics is essential. In order to attain this goal Yetkiner and Caprano (2009) highlight the importance of effectively teaching different mathematical concepts by citing fractions in particular. Yetkiner and Caprano (2009) emphasise the significance of effectively teaching fractions by stating that, "fractional concepts are important building blocks of elementary and middle school mathematics curricula." They further state that "conceptually based instruction of fractions requires teachers to have a complete understanding of the subject matter." This opinion is supported by Shulman, Ball and Bass (as cited by Hristovitcn & Mitcheltree 2004) as they state that high quality teacher preparation both in content and pedagogy is critical for the improvement of student outcomes in mathematics. On this basis teachers should have the skills and competences to teach fractions.

Torrence (2002) testifies that realistic mathematics makes sense for learners. She further asserts that if learners are given the opportunity to construct their own understanding they learn to think and they learn that they could think. They can also learn to see mathematics as creative and pleasurable. Torrence (2002) is also of the opinion that if learners are helped to acquire this attitude they could be competent to solve problems in any situation.

The value of realistic mathematics cannot be underestimated as it appears to be a promise to the improvement of learners' mathematical proficiency. It is in this regard that Troutman and Lichtenberg (2003:365) underline learners' difficulties to learn mathematics, fractions in particular, to insufficient exposure of manipulating concrete materials.

Research into classroom-based teaching is necessary, as there is literature that enunciates the importance and value of realistic mathematics (Romberg 2006).

There are also suggestions in literature on how to improve mathematics teaching. Regardless of these research findings mathematics performance in Lesotho schools is persistently low. Learners' construction of mathematical knowledge and creativity in solving novel problems does not seem to improve.

The study will therefore attempt to answer the following questions:

- What does literature say on effective learning of fractions?
- What does literature say on mathematics teaching and effective teaching of fractions?
- What level of training did the teachers get regarding mathematics teaching?
- Which teaching strategies are predominantly used by mathematics teachers?
- What do teachers do to verify that students have effectively learned fractions?
- How can fractions be taught effectively?

The purpose of the proposed study is to explore and assess how primary school mathematics teachers teach fractions and how learners learn fractions. In this research study, the general aim is thus to investigate teaching strategies predominantly employed by primary mathematics teachers and assess their effect on learners' meaningful learning of fractions. The research will therefore seek to address the following objectives:

- Determine what literature is saying on effective teaching of fractions
- Determine what literature is saying on effective learning of fractions by learners
- Determine the level of training given to mathematics teachers
- Determine whether effective teaching is taking place
- Determine whether effective learning of fractions materialises in selected classes
- Recommend guidelines on how to teach fractions effectively

1.3 THEORETICAL FRAMEWORK

Mathematics skills required for people to function at the work place today are different from those which were required yesterday, hence the emergence of mathematics education reform (Romberg 2006). Romberg (2006) further articulates that mathematics learning is no more regarded as a drill and practice process but students actively construct their own mathematics by making connections, building mental schemata and developing mathematics based on prior knowledge through interactions with others.

This research intends to construct meaningful reality of the phenomenon under study and the way the members understand it. This research will therefore be based on both interpretivism and constructivism paradigms. Allan, Carmona, Calvin and Rowe (n.d.) state that "... constructivism is synonymous with interpretivism ... and that they both share the goal of understanding the complex world of lived experiences from the point of view of those who live it." Higgs (1995:97-98) further explains that people understand social reality by constantly interpreting what they see. Constructivism is therefore a way of thinking that advocates personal construction of reality by interacting with the environment. During interactions people observe, relate and connect to what they already know (Troutman & Lichtenberg 2003:18). Constructivism argues that learning is meaningful and effective when learners generate and construct knowledge for themselves, either individually or within social contexts during learning (Muijs & Reynolds 2005:62). Torrence (2002) reveals that in realistic mathematics education, students develop mathematics through their social interactions in their endeavours to solve problems hence this reform supports constructivism.

The researcher believes that people's actions are influenced mainly, by their cultural beliefs, experiences, values and attitudes. So in order for the researcher to fully understand the situation there must be direct interactions between the researcher and the respondents. She is also, of the opinion that actions do not speak for themselves, instead they need to be interpreted in order to capture the actual meaning (Pring 2000: 96).

1.4 RESEARCH DESIGN AND RESEARCH METHODOLOGY

In order to achieve the stated objectives, a qualitative approach was used in this study. The researcher intended to get an in-depth understanding of the participants' world by giving rich descriptions on how they construct their knowledge (Maree 2007:87). Maree further contends that the truth can be gained through observations and interviews.

The researcher observed lessons where 11 to 12 year old (standard six) children were taught fractions. The emphasis of the observation of lessons was primarily to determine dominant teaching strategies used when teaching fractions and to find out how 11 to 12 year old children conceptualize fraction knowledge.

In order not to influence the dynamics of the lesson, the observer was part of the lesson, but remained uninvolved (de Vos 2002:279; Nieuwenhuis 2009:85). The researcher did the study with the aim of understanding the assumptions, values and beliefs of the participants. In order to improve the efficiency of the observations, a self-designed observation sheet was used.

At the end of the observation period learners' mathematics exercise books were collected and analysed to determine how they performed calculations of problems on fractions.

At the end of the observation period, that is when the whole topic as planned by the teacher is taught, interviews with the observed teachers were conducted using a self-designed, semi-structured interview. Interviews were done after school when the learners and other teachers had gone home, so that they would not be disturbed.

The advantages of using semi-structured interviews are that, if questions are prepared in advance, the researcher will be able to plan the logic in which questions should be asked, identify ambiguous and complex questions and then change them before the interview session. In semi-structured interviews the interviewer probes the participants so that they elaborate on their responses (de Vos 2002:302; Maree 2007:87).

The use of both observations and interviews aimed at ensuring trustworthiness of the research findings. This approach, according to de Vos *et al.* (2002:341) and Shenton

(2003:65), is called triangulation of measures. According to these authors, a concept or phenomenon measured with multiple instruments has greater a chance of being valid.

The sampling of schools was both convenient and purposeful due to restrictions with regard to resources such as time and money. Three Government schools formed part of the sample but in the end only two participated. In these schools grade 8 learners and three teachers were observed and the teachers were interviewed.

Lecturers from the Lesotho College of Education (L.C.E.) were also interviewed. Information pertaining to the type of training given to the teachers was sought. Two lecturers were included in the study.

According to de Vos *et al.* (2002:337), pilot studies are done prior to the actual study and few a respondents having the same characteristics as those in the main population are used. Pilot study helped the researcher to test the questions, gain confidence and improve on their own interview skills. It also helped in building relationships and estimating time and costs that would be involved in the main study. In this regard the pilot study was done with one school and one teacher.

1.5 VALUE OF THE RESEARCH

This research will help to build understanding and enhance both current and further literature on the teaching and learning of fractions within the Lesotho context. Fractional concepts are some of the practical topics in the mathematics curriculum, so this study is intended to uncover the classroom practices of some teachers in Lesotho, in order that they can reflect on them and improve on their teaching.

1.6 ETHICAL CONSIDERATIONS

Consent from the Ministry of Education was asked for to undertake the study. When permission was granted, the schools were asked for consent and the teachers who agreed to participate in the study filled in the consent form.

Teachers voluntarily participated in the research after the researcher had explained the purpose of the study to them. To ensure confidentiality, the names of the schools, teachers and learners are not used. The schools are referred to as school A and school B. Teachers are coded as teacher A, teacher B and teacher C. Fictitious names are used to refer to the learners. The researcher ensured that the participants will not be harmed emotionally, physically or psychologically.

1.7 RESEARCH LAYOUT

In chapter 1, the background to this study, theoretical framework, problem statement, research problem, aims and objectives of the research, research design and methodology, value of the research and ethical considerations were discussed.

To really understand the context a literature study on learning Mathematics with emphasis on fractions was discussed in chapter 2.

In Chapter 3, a literature study on teaching Mathematics was discussed.

In chapter 4, the research methodology was discussed with reference to the sampling procedure engaged, the research design and the analysis on the research findings.

In chapter 5, data was analysed and interpreted.

In chapter 6, research findings, conclusions and recommendations were outlined.

1.8 CONCLUSION

The statement of the problem, research questions and aims were established in this chapter. Interpretivism which is viewed to be synonymous to constructivsm was discussed as the theoretical framework of this study. This study attempts to construct the meaning of the phenomenon, teaching and learning of fractions, as interpreted by the participants. The research design and methodology were discussed and a qualitative approach was used. Class observations, interviews and document analysis were utilised as data collecting instruments.

The value and contribution of this study, the ethical considerations together with the chapter layout were also discussed.

Chapter two reviews literature on effective learning of mathematics with the emphasis on fractions.

CHAPTER 2

LEARNING OF MATHEMATICS

2.1 INTRODUCTION

“Teaching and learning of fractions has traditionally been one of the most problematic areas in primary school mathematics (Charalambos, Charalambous & Pitta-Pantazi 2006)”. This has therefore led to poor performance in Mathematics. This study focuses on how children learn fractions and how teachers teach them. Memory formation will be discussed then learning will be conceptualized from a psychological perspective especially with the focus on the work of Piaget, Vygotsky and Bruner. The theoretical framework that informs this study is constructivism; hence these theorists’ perspectives on learning will be of interest. Their theories on learning inform the teaching process especially teaching that enhances construction of knowledge.

After examining in broad terms how children learn, more focus will be on how children learn Mathematics. The importance of a learning environment, how it promotes or hampers effective learning will all be looked into together with the barriers to effective learning. From the constructivist perspective, Mathematics must be conceptualized so that it is learned effectively. Learners’ prior knowledge helps in making Mathematics meaningful and worth learning as it becomes realistic when related to learners’ experiences. Prior knowledge; how it can be used to build mathematical concepts and enhance experiential learning will therefore be discussed with more emphasis on fractions. The relationship and dependence of both procedural and conceptual learning will be clarified.

Learning can be affected and/or damaged, if barriers to effective learning such as learners’ learning styles, language and fragmentation of knowledge are not addressed. In this chapter barriers to effective learning will be discussed.

In the teaching and learning situation teaching and learning are interdependent, that is teaching should result in learning and for learning to take place there has to be teaching. Since effective teaching should be based on how children learn, the approach in this study is to first discuss how children learn mathematics followed by discussing effective teaching of mathematics.

2.2 MEMORY FORMATION

In our everyday lives we rely on memory for our everyday chores but most importantly we need memory to learn. For learning to occur, there has to be the processing of information. Information processing involves gathering and representing information meaningfully so that it can be stored and retrieved when needed (Hamachek 1990:190). This definition brings in an important characteristic of learning, which is memory.

The degree to which one remembers a piece of information depends on where the information is stored. Where a piece of information is stored also depends on what is done to that information the moment it is received (Entwistle 1981:121-122).

At times when people receive information they do not pay attention to it, it simply gets into the system without being processed. This means if no meaningful representations are made, the information will be stored in the short term memory where it will be lost within twenty seconds depending on individuals. But if the information is dealt with by perhaps associating it with something already known then it is likely to be stored in the long term memory (Entwistle 1981:223-224; Hamachek 1990:195-199).

One of the concerns of a teacher should be how to help learners develop long term memory. To help teachers address this concern Hamachek (1990:201-202) illustrates some of the things learners may do to ensure that what they learn will be stored in long term memory.

Rehearsal strategies: This involves going through the idea again and again. In the case of learning fractions, the partitioning exercise could be done again and again for different fractions so that the concept and the equality of parts can be stored in long term memory. Repetition not only enhances rote learning but it also fosters deep rooted understanding. This is because each time an exercise is repeated learners are likely to view what they are doing from different perspectives, hence getting more insight of the concept they are building.

Elaboration strategies: New ideas can be elaborated upon by associating them with what is already known. In the case of fractions again, learners may associate sharing with the actual sharing of items in real life situations.

Organizational strategies: New information may also be organized in a manner that makes sense to the learner. Fraction $\frac{1}{2}$ may be represented as part of a whole,

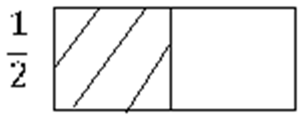


Figure 2.1 Fraction as a part-whole

and as a measure

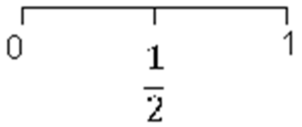


Figure. 2.2 Fraction as a measure

Comprehension and monitoring strategies: Include things we do to keep up with our learning. This may include note taking and self-questioning to check for understanding. Learners for example should question themselves why $\frac{50}{100}$equals $\frac{1}{2}$?

Affective strategies: The way we feel can have a powerful impact on memory. If learners feel bored, their motivation level drops and then learning is impeded. Affective strategies include time management, establishing and maintaining motivation and focusing attention on what is being learned.

Memory, as one important aspect of learning has been discussed and what teachers can do to enhance long term memory. The next section discusses learning from the cognitive psychologists' point of view.

2.3 PSYCHOLOGICAL PERSPECTIVES ON LEARNING

Learning is a complex concept defined from different points of view by different people. In this section, learning is going to be examined from the psychological point of view of three theorists: Piaget, Bruner and Vygotsky.

2.3.1 Piaget

Piaget's interest in the nature of knowledge led him speculate about the development of thinking by children (Biehler, Snowman, D'Amico & Schmid 1999: 54). He postulates that human beings try to understand their environment by organising information into coherent general systems and adapting it to their knowledge base (Biehler *et.al.* 1999:54-56). According to Piaget, organisation occurs before adaptation and through it organised patterns of behaviour or thought called schemes result. But if new knowledge does not fit into the existing scheme learners engage in the adaptation process. In this process learners try to make sense of the experience so that new knowledge can be incorporated into their existing scheme, or they restructure and modify their schemes which may be faulty, so that they accommodate the new knowledge (Biehler *et.al.* 1999:54-56; Mavugara-Shava 2005: 41).

It is mentioned above that two things are likely to happen when one encounters new knowledge. That is new knowledge may either fit properly into the existing scheme or a misfit may occur. Biehler *et al.* (1999:56) call these situations equilibration and disequilibration. The authors accentuate that equilibration occurs when new knowledge fits perfectly with the existing scheme, while disequilibration results when there is a discrepancy between the existing scheme and the new knowledge.

When learning the concept, fractions, learners need to compare them with whole numbers which are an existing scheme. For example if they use paper folding to investigate fractions, they should first compare it with whole numbers. That is one fold gives two pieces each of which is less than one whole (Troutman & Litchtenberg 2003:342). Ability to form this logic may result in accommodation of the new knowledge. The processes of accommodation and assimilation result in mental growth because new knowledge gained may result in the change of one's reasoning, thinking and perceptions which is referred to as maturation (Bell 1978:100).

Through interactions with their environment children gain experiences that Piaget calls physical, logico-mathematical and social knowledge (Bell 1978:100; Kamii 1990:23). Bell (1978:100) defines physical knowledge as the experience gained through experiments and observations, such as sharing concrete objects; children gain different sets of knowledge such as addition of fractions. Logico-mathematical knowledge is the mental actions children perform during accommodation. An example of logico-mathematical knowledge is for example where learners give meaning to the addition of fractions.

Social knowledge consists of conventions worked out by society. An example of social knowledge is telling learners that the ‘1’ is known as the numerator and the ‘2’ is known as the denominator in the fraction ‘$\frac{1}{2}$’. When interacting with their environment, children engage in the actual manipulation of concrete materials and co-operation with other people. By so doing, children gain knowledge of concepts like fractions, their meaning and operation, just to mention a few, hence new knowledge is adapted. A typical example of an experience that leads to physical knowledge is to engage learners in an activity to cut a pizza into parts, thus working with concrete material to assist in conceptualizing fractions.

As mentioned above Piaget points out that when there is a discrepancy between existing schemes and new knowledge, confusion occurs. This confusion if not cleared will make it difficult to adapt new knowledge, which will in turn force one to find out more about new information so that it can fit the present cognitive structure, or for it to be discarded (Fisher 1995:12). This results in cognitive growth. If the confusion is left unattended it becomes difficult to learn because of the gap that results from the “disequilibration”. Fisher (1995:12) further posits that teachers should look for signs of cognitive conflict and challenge the learners ideas so that they attain higher levels of thinking.

Piaget insists that schemes undergo systematic changes at particular points in time from birth to adulthood; hence he theorizes that there are distinct stages of cognitive development through which children transcend.

2.3.1.1 Sensorimotor stage

This is the first stage at which infants to two year old develop schemes by exploring their own bodies, senses and later explore external objects and situations.

2.3.1.2 Preoperational stage

The two to seven or eight year old thinking centres on mastery of symbols. Children cannot think of conservation problems as those needing logical thinking. Piaget (as cited by Biehler *et al.* 1999:58), clarifies this by citing this example; a teacher pours fruit juice into two identical short glasses until the child agrees that each contains an equal amount, and may be half full. Then the juice is poured from one of the glasses into a taller thin glass. The child is asked; if there is more juice in the tall thin glass or the short one?

Children at this stage will think there is more juice in the tall thin glass than in short squat glass (Biehler *et al.* 1999:58).

They cannot think of more than one quantity at a time, mentally reverse the situation and take another person's point of view.

2.3.1.3 Concrete operational

This is the third stage at which the eight to 11 or 12 year old begin to develop schemes that allow understanding of logic based tasks. But their thinking is limited to concrete objects. Piaget postulates that they are not able to solve abstract problems by engaging in mental explorations; usually they need to manipulate concrete objects physically. For instance, when teaching fractions learners should be physically engaged like; cutting of real objects so that they can develop schemes such as; ordering, fair share and equivalence. Piaget in Biehler *et al.* (1999:59-60) encourages manipulation of concrete objects because children at this stage cannot generalise from one situation to a similar one with any degree of consistency. Repeated manipulation of real objects helps to develop schemes which will later enable children to generalize.

2.3.1.4 Formal operational stage

Children at this stage are able to generalize and engage in mental processes by thinking up hypotheses and testing them in their head (Biehler *et. al.*1999:60-61). Knowledge gained through the formation of relationships created by an individual is called logico-Mathematical knowledge (cf. 2.3.1; Kamii 1990:23). They can list equivalent fractions without necessarily sharing concrete objects. So children who have reached this stage are able to create relationships, hypothesize and test them.

This study focuses on how the 11 to 12 year old learn fractions, so the last two stages are of importance as far as this study is concerned.

On social interactions, Piaget believed that peer interactions do more to spur cognitive development than do interactions with adults. His argument is that children are able to discuss, analyze and debate merits of another child's point of view. On social interaction Piaget comments that, no amount of teaching could accelerate the rate at which children could progress through stages. By this Piaget perceives that human development precedes learning (Bobis *et al.* 2004:7).

2.3.2 Bruner

In his study on the cognitive development of children, Bruner (n.d.) came up with three stages, which are not necessarily hierarchical in nature. The first one is an enactive (action-based) representation, in which learners handle concrete objects in order to understand their environment (Biehler *et al.* 1999:140). They do so without the use of words. The second stage is the iconic (image-based) representation in which learning depends on visual or sensory organization. In this stage learners use diagrammatic sharing to represent concepts like fractions. The third stage is symbolic (language-based) representation. In order to understand the environment, learners use language, logic and mathematics. The symbolic representation enables learners to arrange ideas and store them so that they can be retrieved when needed. Bruner maintains that these stages are evident in both young and adult learners when they are faced with new situations (Biehler *et al.* 1999:140).

Based on the stages of cognitive development discussed above, Bruner views learning as an active process in which learners construct and discover things for themselves. When solving problems, learners bring in their informal, current and/or prior knowledge to solve the problems. In doing so, they discover new truths about what they are learning and they also develop their own strategies to solve problems (Bruner n.d.; Orten 2004:194-195).

During knowledge construction, learners may use one or all of the three stages of learning, select and transform information, formulate hypotheses and make decisions (Biehler *et al.* 1999:140). As learners categorize their knowledge to try and understand whatever they are learning they may gain more understanding, find other ways of explaining and in so doing they discover new knowledge hence Bruner views learning as the process of discovery and enquiry. In order to enhance discovery learners are given a variety of examples, facts and information and they are encouraged to find answers or underlying rules or principles (Hamachek 1990: 222).

Another critical feature of discovery learning is that learners make errors in drawing conclusions. Instead of discarding them they should serve as points for instruction in which the teacher may ask some learners to explain why the answer is wrong. This can be afforded in the classroom climate that tolerates wrong hypotheses and incorrect answers (Hamachek 1990:224).

For effective knowledge construction, activities should be of the difficulty that is not too complex for learners to handle; hence Bruner (n.d.) advocates a spiral approach to curriculum design at classroom level. It is the teacher's task to design curriculum at classroom level because they know the learners' context and needs better hence they can make informed decisions regarding what to teach and what not to teach at different stages. He posits that spiralling benefits learners in that they continually build on what they have already learned. For example learners first learn addition with whole numbers and they learn addition with fractions. Hence spiralling promotes the use of one's prior knowledge.

Learners could be allowed to work in small groups so that they can share their experiences (Orton 2004:198-199). In these cooperative learning groups as they are referred to by Bruner (n.d.) teachers and learners engage in active dialogue. They analyse and reflect one another's experiences and hence construct their own understanding. During dialogue, teachers or learners may explain and ask questions. Teachers do so to direct the learners' focus to key concepts of what they are learning whereas learners may do so while sharing their experiences with their peers or trying to make their point heard (Fisher 1995:12). This process Bruner refers to it as scaffolding. During scaffolding the mediator provides help and suggestions, but gradually withdraws as learners reach a level of constructing their own internalized understanding (Donald, Lazarus & Lolwana 2002:04-105).

Unlike Piaget, Brunner theorizes that teachers should engage in spiral curriculum organization because learners can learn anything as long as it is simple enough for them to understand (Bruner n.d.; Fisher 1995:12). In relation to fractions, learners should first be helped to build the concept of fractions before ordering fractions because having conceptualized fractions it is only then that learners will be able to determine their sizes.

2.3.3 Vygotsky

Like Bruner, Vygotsky believes that cognitive development is largely due to social processes particularly interactions with others who are more knowledgeable and competent (Biehler *et al.* 1999:69-71). He assures that children are first introduced to a culture's major psychological tool (speech, writing and numbers) through social interactions with parents and later with teachers.

He argues that children can learn anything not very far from what they already know, as long as they are exposed to it through collaborations with experts such as teachers, parents and/or experienced learners. While Piaget's theory encourages teachers to plan their teaching basing themselves on their learners' cognitive developmental stage, Vygotsky advises teachers to expose and guide their learners through challenging situations. He believes that well designed instruction aimed slightly higher than what children know and can do at the present time will pull them along, helping them master things they cannot learn on their own, hence the zone of proximal development (ZPD) (Biehler *et al.* 1999:69-71).

Vygotsky in Biehler *et al.* (1999:69-71) defines the zone of proximal development (ZPD) as "the difference between problem solving ability that a child has learned and the potential that the child can achieve from collaboration with a more advanced peer or expert, such as a teacher." Teachers are cautioned though that the tasks they give to learners should not be at a level where learners can work without guidance because they will practise previously learned material and result in no new learning (Biehler *et al.* 1999:69-71). They should also not be too far above learners' current level of competence because the result is that the learners will be off task and no learning takes place. So tasks should be pitched between these two extremes as it is the learning zone (ZPD) in which learners can operate only with some form of help (Mavugara-Shava 2005: 45).

The three theorists agree on the issue that learning is an active process in which knowledge is constructed. Piaget focuses on individual construction of knowledge through the use of one's' cognitive structures while Bruner and Vygotsky focus on knowledge construction in social groups. The next section focuses on constructivism and how it relates to teaching and learning.

2.4 CONSTRUCTIVISM

The way people understand the world around them is determined by their epistemological underpinnings. It is mentioned in chapter one that the epistemological basis of this study is constructivism. Troutman and Lichtenberg (2003:14-18) give account of the origin of constructivism from the pioneering work of Piaget and Vygotsky.

Constructivism focuses on how knowledge is acquired and emphasizes knowledge construction not knowledge transmission (Mudork-Steward 2005:14). Constructivism also views human beings as agents of their own development because they actively shape their development by interacting with their environment which enhances the development of skills essential in their every day functioning. Through interactions with the environment learners make hypotheses, test them and draw conclusions. In so doing they create knowledge, adapt and assimilate it to create new internal representations (Troutman & Lichtenberg 2003:18). Constructivism therefore views learning as an active process.

As learners explore their environment, they sometimes find things they do not understand and hence fail to be accommodated in their existing schemes. In an attempt to understand this new information learners engage in the process of eliminating fallible information and adjusting their schemes. This process is referred to by Piagetian psychologists and educators as constructivism (Biehler *et al.* 1999:54-56). Hence Piaget is also seen as a constructivist.

This in turn creates what Piaget refers to, as cognitive conflict (cf. 2.3.1). As Bruner mentioned, teachers should now play the role of the facilitator and mediator by either asking questions or explaining to learners so that they help them construct meaningful knowledge. This process is referred to by Bruner as scaffolding (cf. 2.3.2; Donald *et al.* 2002:111). Scaffolding as one of the teaching strategies of constructivism should be done intensively during the beginning of instruction. Then withdrawn gradually so that learners become independent (Donald *et al.* 2002:111). Constructivism does not stipulate what should be done in the teaching and learning situation, it guides teachers as to how they can maximise learning by giving suggestions of what could be done such as pitching activities to be at the learners' level (cf. 2.3.3; Mudork-Steward 2005:15).

Since learning is viewed from a constructivist perspective as an active construction of knowledge, before going to class, teachers should prepare activities that will enable learners to construct knowledge. In order to do that teachers may structure the teaching and learning situation that will enable the formation of co-operative groups in which learners will share information as they engage in class discussions. As Bruner indicated, (cf. 2.3.2) co-operative learning is characteristic to constructivism, because in these groups learners share ideas and then alter their schemes as they use their peers' ideas to build on new knowledge. Through discussions, learners organize their experiences and prior knowledge and then adapt it in their schemes as indicated by Piaget (Orton 2004:197). It is also through co-operative groups that learners develop communication and interpersonal skills, which is a basic skill needed for effective functioning in social groups (Donald *et al* 2002:108).

The role of the teachers is to create opportunities for action in which they give learners tasks which will engage them in action (Biehler *et al.* 1999:396; Donald *et al.* 2002:108). Tasks should be grounded in learners' contexts so that learning becomes realistic for them to make connections easily. Learners may be asked to work in groups and share a pizza among six people and write the fraction of the pizza each person will get. Through this co-operative exercise, learners share their experiences and strategies, they engage in arguments and give each other support and assistance (cf. 2.3.3). Hence, learning is a social activity and knowledge is socially constructed.

Learners have different experiences and background knowledge, so teachers in constructivist learning should vary activities and provide many experiences that require manipulation of materials, so that all learners are offered equal chances to construct meaningful knowledge (Bobis *et al.* 2004:7-8). Learners should also be a given chance to make decisions regarding how and what they want to learn and teachers should create and structure learning environments that promote effective learning. It is true that it may not be possible to let every learner decide on what and how to learn, but giving everybody a chance to participate and come up with own their own strategies and views during learning may in some way give learners a sense of ownership and control, hence, effective learning may result. For retention and long term acquisition of the new knowledge, Bruner and Vygotsky suggest that some form of apprenticeship should be given (cf. 2.3.2 & 2.3.3). The authors further suggest that learners must take greater responsibility for their own learning; hence learning must be learner centred and personal.

From the above discussion about constructivism one can deduce that Piaget, Bruner and Vygotsky are constructivists. Piaget advocates individual knowledge construction. He indicates that human beings always try to make sense of their environment by organising their knowledge, comparing what they know to new information therefore adjusting their schemes. This is an individual activity hence Piaget talks of individual knowledge construction.

Bruner and Vygotsky add that to effectively construct new knowledge learners need other people. When working in groups, as Bruner indicated, learners may exchange ideas and understandings of what they are learning (cf. 2.3.2). Vygotsky on the other hand adds that to construct knowledge, learners need somebody who knows more than them to guide them. This more knowledgeable other could be a teacher or other learners (cf. 2.3.3).

2.5 LEARNING MATHEMATICS

Learning, as defined from the cognitive psychology point of view is the acquisition of knowledge. It comprises all activities that increase the individual's knowledge, skills and understanding of the world so that they can interact effectively and successfully with their environment (Steffens 2001). From the works of Piaget, Bruner and Vygotsky, learning is an individual construction and accumulation of knowledge through collaborations with other people like peers and/or the more knowledgeable other who could be a teacher, parent or other children (cf. 2.3.3; Bruner n.d.; Biehler *et al.* 1999).

Learning is also perceived as a personal activity which is determined by a variety of factors such as; learning environment, individual preferences, context in which the content is presented and the quality of instruction (Hierbert *et al.* 1997:5-6;Fennema

& Romberg 1999: 5). Knowledge is constructed as learners make sense of the new information and connecting it to their experiences so that it makes more sense, hence, it is cumulative. Some people learn best when they collaborate with others and share ideas, argue their strategies or findings and explain what they learned to others, hence, learning is a social process (Mueller, Yankelewitz & Maher 2010).

Russell (1999:1) as well as Haylock and Cockburn (2008:226) view Mathematics as a discipline which deals with abstract entities, as a collection of ways of thinking and reasoning and ways of organizing and internalizing information received from the external world. This conceptualization of mathematics accords with Piaget's theory of accommodation and assimilation (cf. 2.3.1), in which learners need to think about the new knowledge, compare it with what they already know and organize it in such a way that it makes more sense to them. Mudork-Steward (2005:13) adds that mathematics is a language that is used to describe the physical and non-physical aspects of the world we are living in. For example learners may be asked what the fraction of girls in their classroom is. Therefore learners need to have a good command of this language so that they can communicate effectively.

It is mentioned earlier in the works of Bruner and Vygotsky that learning is socially constructed. Through social learning learners become mature in terms of knowledge possession and reasoning capability. This indicates that social learning is vital when learning Mathematics as it is a language which needs to be communicated. So through learning in a social setting, learners have the chance to share their understanding, difficulties and observe one another. Through these sharing sessions, learners have the chance to modify their understandings, rethink their conceptions and do things they were not able to do before engaging in group work.

One of the research questions is to determine what literature says on effective learning of fractions, so this section will reveal what a learning environment is, how it enhances the learning of mathematics; what is effective learning, what factors contribute to effective learning and what are its indicators? Lastly the barriers to effective learning will be discussed.

2.5.1 Learning environment

A learning environment is viewed in relation to the factors in the teaching and learning situation which affect learning positively. Bobis *et al.* (2004:302) refer to the learning environment conducive to the development of mathematical power for all learners as those that:

- encourage students to explore
- help students verbalize their mathematical ideas
- show students that many mathematical questions have more than one right answer
- teach students the importance of careful reasoning and disciplined understanding through experience
- build confidence in all students so they can learn mathematics

Troutman and Lichtenberg (2003:561-562) suggest that teachers must create an environment that promotes respect and empathy. When learners know that they are respected, they are free to explore their environment and participate in class activities knowing that their contributions will be appreciated. Teachers may allow learners to express what they like, value and believe. Troutman and Lichtenberg (2003:561-562) presume that, these interactions will provide richer content for problem solving tasks as teachers will formulate tasks around the learners' context.

A learning environment is more than a designated room or physical space in which instruction occurs; it is created by the hidden messages conveyed about what is important in learning and doing mathematics (Bobis *et al.* 2004:304). It is therefore socially, emotionally and physically created. If teachers hold the belief that mathematics is a set of rules and procedures to be transferred from one person to the other, this will be depicted in the classroom setting in which there will be silence and the teacher will be the only one responsible and talking (Troutman & Lichtenberg 2003:304). Learners will not be given chance to verbalize their mathematical ideas.

In the learning process which supports knowledge construction by learners, mistakes are inevitable (Ding 2007). In an attempt to construct knowledge learners are likely to make wrong connections and generalizations. So the situation should afford learners to ask questions to seek for clarification and guidance. Teachers should also ask learners questions which will direct their focus to important issues hence result in a healthy classroom discourse (Huerta 2009). In rich learning environments learners' mistakes are used as opportunities for learning. The whole class analyzes mistakes in order to find why they are mistakes instead of rejecting them without giving justification. Teachers and learners in healthy and rich learning environments are free to take risks and deal with difficulties (Ding 2007; Hiebert *et al*. 1997). This helps learners to develop confidence and hence face real life problems and challenges and solve them confidently (Abramovich & Brouwer 2007).

2.5.2 Effective learning

Effective learning is an ultimate goal of knowledge formation and does not occur in a vacuum but in a learning environment that supports it. Some learners are naturally motivated and have high self-confidence but if this is not nurtured they end up de-motivated. In learning environments that support effective learning, most learners become free to experiment and hence discover things for themselves (Brown & Quinn 2007). The knowledge base learners get from experimentation (physical knowledge) enables the process of accommodation since what is learned makes more sense (Kamii 1990:22).

So this new knowledge is meaningful and effective if it benefits the learner (Beihler, *et al.*1999). It is essential that learners are helped to learn effectively by exposing them to situations that will facilitate knowledge construction through cooperative groups, stipulating the goals, employing spiral approach to curriculum and encouraging independence and autonomy (Bruner n.d.). Beihler, *et al.* (1999:387) emphasize that effective learning results when:

- Learners understand the fundamental ideas of a subject and how they are related. That is, understanding $\frac{1}{2}$as a number which can be marked on a number line
- Learners construct their own knowledge

- When constructed knowledge can be applied to different situations. That is, understanding $\frac{1}{2}$ as the ratio between two numbers like 1 and 2 and dividing sweets amongst children in the ratio 1 to 2

Employing a spiral approach to mathematics teaching makes use of learners' prior knowledge. That is when learning fraction addition learners make use of the knowledge of addition of whole numbers. Hence spiralling results in the accumulation of knowledge. Learning is cumulative and effective because relationships are established between concepts.

Brown and Quinn (2007) suggest that learners should be given freedom to experiment and come up with their own strategies when answering questions and solving mathematical problems. Mathematical problems should be rooted in learners' real life so they can make use of their prior knowledge (Troutman & Lichtenberg 2003: 561-562). In this way learners will have effective learning because they gained skills and insight into the problem. Thus learning should be contextualised and situated. So when faced with the same problem they will use the acquired skills. This suggestion is supported by Bruner (in Beihler *et al.* 1999:389) as he emphasized that learning should be "discovery because conceptions that children arrive at on their own are usually more meaningful than those imposed by others." Therefore learning is personal and discovery learning results in effective learning.

Learners who have effectively learned a concept are able to communicate it effectively to others as it makes more sense. Communication is regarded as another tool for effective learning (Mueller *et al.* 2010). Through communication learners share ideas and help each other view the same situation or problem from different perspectives. This could be achieved through collaborative groups.

During collaborative encounters, learners ask questions to seek for clarifications and they explain their thinking. As they communicate they rethink their strategies and hence use their peers' reasoning to improve on their own (Vygotsky in Dahl, n.d.). Through these social encounters learners can learn and acquire skills which are thought to be beyond their level of comprehension (cf. 2.3.3).

Mueller *et al.* (2010) underline that understanding, which is an indication of effective learning, is meaningless without a serious emphasis on reasoning. One who understands has the capacity to explain, justify and think critically. As a result learning is facilitated because they are now able to learn quickly and are able to use their knowledge flexibly in novel situations (Newton 2000:7-8). When learning about sharing, learners should be able to give reasons behind their solutions such as why the shaded part is $\frac{2}{5}$ not $\frac{2}{3}$ (Charalambos *et al.* 2007).

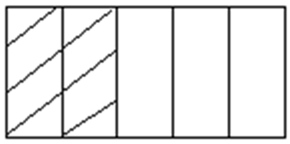


Figure 2.3 Representation of $\frac{2}{5}$

If they are able to argue that; the fraction is $\frac{2}{5}$ because there are two shaded parts out of five parts that the whole is divided into. This will indicate that learners use reasoning to understand relationships and make connections to new ideas.

The definition of effective learning of De Corte (1996:34-37) summarises the previous discussions on effective learning and is further more based on international research on learning. In this definition effective learning is considered a constructive, cumulative, goal-directed, situated, collaborative, self-regulated and individually different process of knowledge-building through meaningful construction.

Having discussed what effective learning is, it is worth explicating the character traits needed to enhance effective learning. In the next section, the three levels of maturity one needs to have to participate effectively in the teaching and learning situation, will be discussed.

2.5.2.1 Dependent, independent and interdependent

From the preceding discussions it is mentioned that learning is acquired through interactions with one's environment including the people around. People interact effectively when they have attained higher levels of maturity, being independent and interdependent. For clarity the discussion will commence by discussing the lowest level of maturity which is dependence.

- Dependent

Children at Piaget's first stage of development (sensorimotor stage) (cf. 2.3.1.1) are entirely dependent on other people for everything. But as they develop to the second stage they begin to have some physical independence.

- Independent

As children advance to Piaget's second stage of cognitive development they begin to have some independence. That is, they decide for themselves when they want to walk or play. But they still lack emotional and intellectual independence. In the teaching and learning situation, learners should be helped to be intellectually and emotionally independent. Covey (1989: 50) explains that people are said to be emotionally and intellectually independent when they do not need other people's opinion to feel good and are able to make their own decisions, think and reach their own conclusions.

Since learning is defined among others as collaborative, then for effective learning to take place learners should be emotionally and intellectually independent. If not, they will be unable to participate in collaborative groups because of the fear of threatening their self-worth.

There is a higher level of development, one in which learners are aware that they need other people's thinking to complement their own. This is interdependence.

- Interdependent

This is the highest level of maturity in which one accepts that they are capable to produce, but they are more capable and can accomplish more when working with others (Covey 1989:50-51). These people participate in co-operative groups effectively as they take criticism and disagreements positively. They acknowledge that they need other people's thinking to complement their own and they are ready to change their point of view after analysing other peoples thinking.

2.5.2.2 Prior knowledge

It is mentioned in 2.3.1 that people learn effectively when they are able to assimilate and accommodate new knowledge to what is already known, otherwise confusion results and learning becomes minimal. Bruner on the other hand in 2.3.2 advocates spiralling of curriculum design. With regard to prior knowledge, spiralling of curriculum design makes it possible for learners to revisit the basics of a concept-fraction in this case-at different levels of schools. When learning fraction addition spiralling will call for reviewing the meaning of the concept, fraction. In this way learners' prior knowledge is tapped so that it assists in the effective learning of fraction addition.

Prior knowledge comprises all the formal and informal knowledge together with experiences learners bring into the learning environment which needs to be confirmed or refuted (Mueller *et al.* 2010). If no match exists between the learners' prior knowledge and the new knowledge, effective learning does not occur (cf. 2.3.1; Bobis *et al.* 2004:13). As Piaget indicated earlier, that a misfit of old and new knowledge leads to learners being confused (Fisher 1995:12; Murdock-Steward 2005:14-17).

The use of learners' prior knowledge as reference for learning increases their curiosity and they get deeply involved in learning. This enables learners to create their own ways of understanding and build representations and understanding based on their previous knowledge and experiences (Theunessen 2003; Mueller *et al.* 2010).

Mueller *et al.* (2010) contend that learners make maximum use of their prior knowledge when the learning materials used in the lesson, provide them with opportunities to have full experience of the content. When adding mixed fractions learners may be asked to write how much chocolate do the girls have if Neo has one bar and a half ($1\frac{1}{2}$) and Lipalesa has one and a quarter ($1\frac{1}{4}$). With real chocolate or slabs made from clay, learners may use them to do addition.

The use of learning materials provides, what Piaget calls, physical experience. Hence, it is imperative for teachers, to have a thorough knowledge of the learners' socio-cultural background from which they would formulate problems and activities that make sense to all learners, hence, situated learning is enhanced (Bobis *et al.* 2004:302). If the concept fraction is introduced through sharing, learners should then engage in the actual sharing of concrete materials they share in their everyday life, or what Dewey (1969) calls experiential learning. In that way learners will be able to compare old and new knowledge, contrast what is and what is not, separate out patterns with respect to fixed conditions, make generalizations after seeing the same phenomenon under different tool usages, and simultaneously bring together different and varying aspects to construct new knowledge (Mueller *et al.* 2010).

The use of learners' prior knowledge seems to be crucial as it appears to bridge the gap between informal and formal knowledge. The fact that it encompasses experiential learning and enhances critical thinking also seems to be an answer to the educational needs of all nations (Freire 1973:36)**.**

2.5.2.3 Experiential learning

Dewey (1969:118) argues that experience is an active-passive process in which "We do something to the thing and it does something to us in return", that is, the experience should result in the change. The change might be the growth of knowledge or an insight into whatever was experienced.

With regard to learning, it is indicated above that incorporating learners' prior knowledge enhances effective learning and that learning should begin with the actual manipulation of concrete objects. During manipulations of real objects learners use in their real-life, learners should move backward and forward connecting what they do, what they know and the outcomes of their actions (Dewey 1969:118). In the case of sharing as they learn fractions, learners should experience the feelings and emotions of unfair share. By this it means the pain or the joy of the sharing experiences. In this way Brookfield (1996:750), Beard and Wilson (2006:2) and Dewey (1969:119) say that it is learning from experience. Having experienced sharing, learners may be in a position to form relationships between fractions and equal sharing.

Unlike traditional learning which starts with abstractions which may not bear any relationship to learners' experiences, experiential learning emphasises starting with concrete experiences. Teachers should therefore design activities which will help learners have meaningful experience of what they learn. When learners have learned from experience and they have established the connections and relationships between activity and the consequence, it then becomes possible to make accurate and comprehensive foresight (Dewey 1969:123). Establishment of connections and formation of relationships result in the cumulative growth of knowledge hence experiential learning is continuous.

In order to maximize learning, activities should demand the use of multiple senses. It is indicated in previous paragraphs that, in the sharing exercise to build the fraction concept, learners do not only share but they use their hands to touch as they share, their eyes to compare the size of the pieces and their brains to give reasons to their observations and discuss their observations amongst themselves. This is supported by Dewey (1969:120), Beard and Wilson (2006:6-7) when they contend that experiential learning results in deeper learning when the experience requires the use of most of the senses, mind and body and when the activities provide challenge to engage and test mental and physical endurance.

Dewey (1969) referred to traditional classrooms as those in which, learners are expected to sit passively in class as the teacher talks, moves and demonstrates. Dewey (1969:120) contends that this is divorcing the bodily activity from the perception and meaning. When this is done, mechanical use of bodily actions, or rote learning, is encouraged. In this manner learners are denied the opportunity to learn using their senses, namely seeing, hearing, feeling and tasting. Even today that arrangement is still referred to as a traditional classroom.

Caution should be taken though, that experiential learning could result in gaps that may hinder future learning (Beard & Wilson 2006: 21). The authors explain that during experiences two situations may arise. The first one may be where learners have established connections and relationships. That is, they have developed methods and strategies to act in a certain situation. They are also able to discern why they work and why they do not work.

The second situation according to Beard and Wilson (2006:21) is where learners do not develop connections and relationships but act in a trial and error fashion. In this situation they have not developed strategies and cannot discern why this works and that does not work. As a result they are not in a situation to decide with confidence how to act in the future.

The discussion indicates therefore that teachers should be careful they facilitate learning so that learners make sense of the experiments they engage with. This is to ensure that experiential learning results in effective learning. Conceptual understanding of the concept learned is contributing towards effective learning.

2.5.3 Conceptual and procedural knowledge

2.5.3.1 Differentiating between conceptual and procedural knowledge

In every day endeavours when people encounter problems they try different approaches to solve such problems. When doing thus they often search for relationships and connections to what they already know and then determine the strategy they think could work (cf. 2.3.1 & 2.5.2.2). At other times relationships are not established and no connections are made hence disequilibration occurs. So when one is faced with this situation and cannot confidently establish connections and relationships one tends to search more into the problem and use a trial and improvement approach to solve the problem (cf. 2.5.2.2).

In the first situation where relationships and connections are established it is said that conceptual knowledge is gained. Conceptual knowledge is therefore defined as knowledge of facts and properties of mathematics that are recognised as being related in some way (Owen & Super 1995:138; Effandi 2009). The authors further explain that the interconnections between ideas explain and give meaning to mathematical procedures and algorithms hence conceptual knowledge is gained. Owen and Super (1995:138) maintain that a piece of information becomes conceptual knowledge only when it is integrated into a larger network that is already in place. This once again links to Piaget’s theory of accommodation.

Building conceptual knowledge also makes use of one’s prior knowledge. With regard to fractions, learners are said to have conceptual knowledge if, for example, they learn fraction position values by relating them to whole numbers and/or fractions (Owen & Super 1995:138). Since learners already know about the whole number system and their position on the number line, then using this knowledge will facilitate fraction knowledge construction.

In order to enhance conceptual understanding learners should be encouraged to handle and experiment with physical concrete objects, as it is through manipulation of concrete objects that they gain physical and logico-mathematical knowledge (cf. 2.3.1). These are believed to facilitate construction, understanding and retrieval of mathematical concepts (Mudorck-Steward 2005: 35).

In most cases people are satisfied with the knowledge of facts and procedures to follow when doing certain tasks. Most people do not ask the why of what they are doing. This type of knowledge is called procedural knowledge. It is defined by (Barker & Czarnocha 1995; Effandi 2009: 202) as the mastery of computational skills and familiarity with procedures for identifying mathematical components, algorithms and definitions. It is characterised by lack of relationships and of connections to prior knowledge (Owen & Super 1995:138). A learner who can only read 14 as one quarter without realising that it is a number greater than zero but less than one has procedural not conceptual knowledge (Owen & Super 1995:138).

Learners who have conceptual knowledge are able to tackle a variety of problems in different contexts whereas those with procedural knowledge are able to compute procedures but fail to solve problems in different contexts from those they were drilled on.

2.5.3.2 Connecting conceptual and procedural knowledge

Barker and Czarnocha (1995) point out that learning begins with actions on existing conceptual knowledge and with increasing knowledge, learners acquire procedural knowledge. When learning fractions learners have some informal knowledge regarding sharing but in most cases they are not aware of the importance of equal sharing. So when teachers emphasize equal sharing their knowledge becomes modified and increases. Learners then gain procedures regarding how to share objects of different shapes in order to come up with different fractions.

Fennema and Romberg (1999:4) and Meagher (2002) caution that conceptual knowledge should precede procedural knowledge so that effective learning can occur. They concur that learners who possess procedural knowledge only, simply practise procedures and algorithms without understanding what they are doing. When faced with a novel problem such learners do not have alternative methods should they forget the algorithm or the learned procedure (Carruthers & Worthington 2006:75; Meagher 2002).

On the other hand learners could be helped to develop conceptual knowledge by means of procedures. As they execute procedures learners could be encouraged to embark on the cognitive process of reflecting on the procedure by looking for patterns and relationships (Hiebert 1990:34). When adding fractions by embarking on the algorithm of finding the lowest common multiple, learners could be asked to find how and why the algorithm works. If, through procedures learners could develop conceptual understanding, they will have a cushion to fall back on should they encounter difficulties when confronted with novel problems.

Hiebert as (cited by Owen and Super 1995:139-140) describes three types of connections between conceptual and procedural knowledge. The first being attaching meaning to mathematical symbols, that is, when computing $2\frac{1}{2}$ x $6\frac{2}{5}$ =, a learner must have an understanding of $2\frac{1}{2}$; x; $6\frac{2}{5}$; =; and ___. Without conceptual knowledge, gained through social, physical and/or logico-mathematical knowledge of these symbols, learners will not be able to make sense of the multiplication of fractions.

The second connection is with regard to the internal steps of the procedure and their conceptual underpinnings. Learners are expected to understand that the product would have the denominator as a multiple of 5 (Hiebert in Owen and Super 1995:139-140).

The last connection would be to realize that $2\frac{1}{2}$x $6\frac{2}{5}$ = ___ must be somewhere between 12 and 21. Twelve is 2 x 6 while 21 is obtained from 3 x 7 since $2\frac{1}{2}$ is between two and three, while $6\frac{2}{5}$ is between six and seven. Owen and Super (1995:139-140) cite Hiebert as explaining that interpreting a computed answer with meaning is highly dependent on conceptual underpinning of the number and operation symbols.

Possession of conceptual understanding of mathematical concepts enables learners to reason from different perspectives because conceptual learning is believed to enhance more mathematical content (Mudork-Steward 2005:23).

2.6 BARRIERS TO EFFECTIVE LEARNING

As discussed, learning is very complex and depends on a number of factors: there are some that facilitate learning and some that impede learning. This section discusses the factors that impede learning.

2.6.1 Lack of motivation

Some people are naturally motivated while others are naturally demotivated. Motivation can be viewed as “the engine that powers and directs behaviour” (Entwistle 1981:193; Hamachek 1990:262). Some people have the inner natural powers that direct their behaviours while others need the external drive to behave. Such people are said to have intrinsic and extrinsic motivation respectively.

Learners may enter the learning situation with intrinsic motivation but this may diminish due to circumstances beyond the individual, such as persistent failure no matter how hard the learner tries to succeed; lack of opportunities for enhancing personal adequacy; and lack of self-worth (Hamachek 1990:263). To help them feel fulfilled they should be afforded the opportunity to participate in cooperative learning groups. In such groups learners may have the platform to demonstrate their abilities which may boost their motivation level.

Learners with high self-esteem perceive themselves as achievers and in most cases strive to maintain high standard, whereas those with low self-esteem view themselves as failures and also maintain that standard (Hamacheck 1990:282-283). Learning environments should therefore provide extrinsic motivation to those learners with low self-esteem so that their perception of themselves may change for the better and hence boost their motivation level.

Learning situations should be structured with learners’ characteristics in mind, that is, learners’ intelligence, motivation, personality and prior knowledge (Entwistle 1981:83). For low achievers, they may be given tasks of low cognitive order so that their chances to succeed are increased. On the other hand, high achievers may be given demanding task which will challenge them so they stay motivated (Hamachek 1990:283).

For any activity that learners do they should be made aware of its relevance and be helped to see how it relates with their prior knowledge. If no relationship is established then some learners are likely to be demotivated. Learners whose level of motivation is low do not put in any effort to understand or learn. They are, in most cases pushed from behind in order to learn something.

2.6.2 Learners learning styles

Learning has been defined, among other things as a personal process (cf. 2.5). It was mentioned earlier that learners receive information, assimilate, accommodate and construct knowledge differently due to their different characteristics. Hence, people adopt different strategies to learning.

When confronted with a situation, in order to understand it, others prefer to take bits and pieces of information and link them logically to build a global understanding or generalisation. Others prefer to begin with a big idea and then break it into smaller pieces of information, hence they develop deep understanding. The former group is referred to as inductive or serialist learners while the latter is the deductive or holistic learners (Entwistle 1981:90-92; Marwaha 2011). The general tendency that one adopts when learning is referred to as learning style (Entwistle 1981: 93).

Within the two broad learning styles discussed above, there are those who understand best when they write down the steps. That is, talk about what they do and demonstrate it in different forms, like drawing concepts maps, diagrammatically or otherwise. There are those who understand when they start with a bigger picture like a concept map or a diagram and then discern why and how, concepts relate.

The implication to education is that learning situations in classrooms should promote the use of multiple senses (cf. 2.5.2.3). If the classroom situation favours one type of learning style, say inductive learners, those who are deductive are disadvantaged hence, minimal learning results. In the classroom learners should be encouraged to use all their senses; otherwise they will experience learning difficulties (cf. 2.5.2.3).

2.6.3 Fragmented learning

Street or informal mathematics is the knowledge base acquired out of school and it is mainly oral arithmetic. School or formal mathematics on the other hand is the knowledge base acquired in schools (Nunes & Bryant 1996:104).

Informal mathematics is mostly physical knowledge gained through experimentation (cf.2.3.1). The authors explicate that representation of numbers in oral arithmetic allow learners to calculate and think of values they are working with at the same time. Whereas at school, the meaning of numbers or logico-mathematical knowledge is not thought of during calculations, instead numbers are operated with and spoken about outside their context (Nunes & Bryant 1996:104-108). The question can be asked whether this is still the situation now in 2012. Kamii (1990:22-23) states that this happens especially when learners are not helped to identify relationships. This approach to learning results in fragmented learning.

Fragmented learning is manifested by the lack of solving problems similar to those that learners encounter in real life. With reference to fractions, fragmented learning is explicated by incidences such as: two pizzas of the same size cut into different pieces. One is cut into six pieces while the other one is cut into eight pieces, if one person gets a piece from each pizza, from which pizza does this person get more? The authors indicate that if learning is fragmented, learners will say the person gets more pizza from one that is cut into eight pieces. The reason being that, learners know that eight is greater than six. The question can be asked whether the fraction concept was developed from only one representation?

2.6.4 Language

Learning mathematics requires understanding of carefully worded expositions as well as appreciation of carefully drawn distinctions. Mathematics is a language; therefore to understand it teachers should create appropriate experiences in which symbols, pictures, context and language are meaningfully connected (Association of Teachers of Mathematics 2010:1). This accords with Bruner's cooperative groups in which learners work together during which they explain and ask questions (cf. 2.3.2). During these collaborative encounters learners use language and hence may become conversant with the appropriate language.

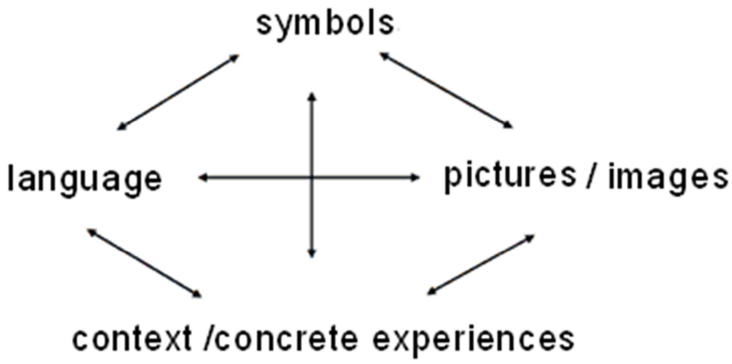


Figure 2.4 The connective model of learning mathematics (taken from Association of Teachers of Mathematics 2010)

Regarding fractions, learners are not immersed in fractions as they are in numbers. They are not exposed to situations in which they actively engage with them. Therefore learners do not become familiar with the language and as a result they do not work and move confidently from context to symbols and images (Association of Teachers of Mathematics 2010:1).

Learners could be immersed in a situation where they share a chocolate bar or any concrete material among four people (*concrete experience*). To represent the chocolate bar pictorially, a rectangle cut into four equal parts (*pictures*) may be drawn. When giving each of the four people their share, that is, one quarter or one part out of the four parts then the appropriate *language is used*. The (*symbol*) representing one part out of four $\frac{1}{4}$ can now be written (Association of Teachers of Mathematics 2010:1).

With this type of representation, learners would have got a complete picture of the concept and would have gained a complete meaning of fractions. In this regard if this holistic approach to fractions is not presented, learners will encounter difficulties in the use of language of fraction and fragmented learning will result.

Other difficulties that language impose on the learning of mathematics bear reference to the restricted and imprecise use of language in describing fractions such as representing fractions by shapes. Karslake (1991:86-87) exemplifies this by giving a situation in class where a circle is divided into two parts to give two semicircles. Each semicircle is labelled one-half or $\frac{1}{2}$ but a semicircle is a whole in its own right. Another example is taking an example of $\frac{1}{2}$ an apple and $\frac{3}{4}$ of a bun.

Karslake (1991:86-87) explain that if learners are exposed to fractions as parts of a whole and they are to operate them, then problems of meaning arise as there is no possible meaning that can be attached to adding one-half of an apple to three-quarters of a bun. This is because it is numbers that are added not pieces of food. It is even more inexplicable to interpret multiplication of 12 of an apple and 34 of a bun. This implies therefore that teachers should first help learners conceptualise fractions as numbers and represent them as different constructs (part-whole, measure, ratio, quotient and operator) as presented by Charalambos, Charalambous and Pitta-Pantazi (2007). When learners have fully conceptualised fractions as numbers, now they can be taught how to operate them. Based on Karslake's (1991:86-87) example it is also advisable that learners, especially in primary schools should operate fractions with meaning. One example could be to find the amount of fruit juice John would have drank if Sana gives him one quarter ($\frac{1}{4}$) of a litre and Lebo gives him two fifths ($\frac{2}{5}$).

Classroom teaching is full of jargon Karslake (1991:88). Some of the phrases and words that teachers use, intentionally or unintentionally have some impact on the learning process. Some of this jargon, just to mention a few examples, are 'to share by' and 'cancelling'. If there are sweets to be shared among a certain number of children, teachers or learners use the phrase 'share by'. To share sweets by children is bizarre. The author attributes this confusion to stem from the comparison with 'dividing by', which applies strictly to numbers only.

The word 'cancelling' also has evidenced to cause difficulties in the learning process. In general speech 'cancel' has the idea of: to remove, to undo and to annul and this may explain the way many children appeal to the notation of cancelling to get them out of the difficulty (Karslake 1991:90). It is true that some learners do not understand that cancelling in mathematics is used when dividing. As a result they use it as a convenient technique mostly when simplifying fractions but they do not relate it clearly to the concept involved. This incorrect use of cancelling Barmby, Bilsborough, Harries and Higgins (2009:81- 82) attribute it to often teaching through memorization. That is if teachers do not use symbols, pictures, context and language learners will tend to misuse cancelling as they often lack understanding of the concept.

When learning new abstract concepts, learners are often helped to make sense of this new knowledge by relating it to their prior knowledge (cf.2.5.2.2). In order to do this, teachers often use metaphors. Nolder (1991:105) mentions that "metaphors enable people to deal with novel experiences whether describing something which they have never previously encountered, or seeking to comprehend a new idea."

Nolder (1991:106) explains that teachers use metaphors with the purpose of

- offering something concrete and familiar to help learners understand an unfamiliar abstract idea
- linking a new concept to the learners' experiences

The author points out that, teachers do this with the intention of making the concept more secure and hope that the metaphor by its novelty may enhance memorability. He points out that research has indicated that metaphors can create difficulties to learn mathematics.

When learning number concept and thus multiplication, learners are often told that multiplication makes bigger, but when applying multiplication to fractions, the generalization no more holds because $\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$ and $\frac{1}{6}$ is less than both $\frac{1}{2}$ and $\frac{1}{3}$so learners become confused and encounter difficulties when multiplying fractions.

This possible difficulty implies that teachers should use metaphors, but with great care, so that the learning should not be polluted. That is teachers should first examine the metaphors for their danger of causing confusion and misunderstanding.

In class communication teachers often ask questions and learners give responses. Depending on the learners' response, teacher gives feedback. Cangelosi (2003:63) distinguishes between two types of language teachers often use in classroom communication. These are descriptive and judgmental languages.

In classrooms teachers often label learners as good or bad depending on their responses. Cangelosi (2003:64) explains that the use of judgmental language may have negative impact on learning. This is because learners may refrain from responding to questions with the fear of being labelled as either good or bad.

Instead of using labels the author advocates the use of language that provides rich information which provides learners with specific feedback. When learners explain correctly why $\frac{1}{2}$ is greater than $\frac{1}{4}$, descriptive feedback could be; ok I understand that half ($\frac{1}{2}$) a loaf is bigger than a quarter ($\frac{1}{4}$) loaf. Judgmental language such as ‘correct, not quite, wrong, no’ is often used and that does not provide learners with important information.

Cangelosi (2003:63-64) therefore encourages teachers to use descriptive language as learners are more likely to listen and pay attention to what their teachers say if the teachers’ comments are full of helpful information.

2.7 CONCLUSION

This chapter has elucidated that reform in mathematics teaching requires an understanding of how memory is formed and how children learn. Therefore learning was discussed from the psychological point of view of Piaget, Bruner and Vygotsky.

The aspects of effective learning of mathematics such as ensuring a favourable learning environment that support cognitive growth, using learners’ prior knowledge and engrossing learners in manipulations of concrete materials were discussed. It was also indicated that helping learners develop conceptual knowledge before procedural knowledge facilitates effective learning.

Since learning could be facilitated, it can also be impeded. It is in this regard that barriers to effective learning were also discussed. These are lack of motivation, learning styles, fragmented learning and language. De-motivated learners find it difficult to learn so do learners whose learning styles are not incorporated in the learning encounter. It was also indicated that learners who are not exposed to different representations of a concept may not develop a holistic understanding of the concept. Inappropriate language use may also be a barrier to learning.

CHAPTER 3

TEACHING MATHEMATICS

3.1 INTRODUCTION

Teaching is a very complex activity which goes beyond the management of classroom discipline, motivation and imparting of knowledge from the teacher to the learners. It is dependent on a number of things such as the learners' characteristics; context and teachers' beliefs about their role in the teaching situation and their beliefs about the subject they are teaching (Kroll & Miller:1993).

3.2 TEACHING MATHEMATICS

Mathematics has been defined in section 2.5 as a discipline that deals with abstract entities, a collection of ways of thinking and reasoning and of organising and internalising information received from the external world. Because of the abstract nature of Mathematics, one of the tasks of teachers is to try to make it concrete. This is because learners of concern in this study understand best when they work with concrete materials, since they are at the beginning of the concrete operational stage as discussed in section 2.3.1.3.

It will be discussed later in this chapter that one way of teaching learners to think, reason and organise mathematical information is through problem solving. It is in this regard that Strutchens (2007) as well as Fajemidagba and Olawoye (n.d.) state that, teachers' beliefs determine how they plan their teaching activities. Teachers who hold the belief that mathematics is a way of thinking and reasoning will use teaching strategies which align with this belief. But those who believe that it is a set of isolated facts and skills or, as rules to be memorised and practised, then they will not teach it for understanding (Hiebert *et al.* 1997 xvi). Such teachers employ traditional methods of teaching in which the main emphasis of the lesson is to practise procedures rather than develop conceptual knowledge (Fennema & Romberg 1999:4). This implies that teachers should work on their beliefs so that their perception of mathematics may also change as will their teaching strategies.

Teaching is about assisting others develop intellectual growth. Shulman (1987:7) states that;

> A teacher knows something not understood by others, presumably the students. The teacher can transform understanding, performance skills, or desired attitudes or values into pedagogical representations and actions.

Teachers therefore need to understand what is to be learned and how it is to be taught as this is the basis for teaching (Ball n.d.:1). Shulman (1987) adds that it is also imperative for teachers to have multifaceted understanding of the concept to be taught so that they can guide learners, to a better understanding. By the multifaceted nature of the concept, the author means, its representation-either pictorial or otherwise-then how the concept relates with other concepts, within and outside the subject.

In addition to the knowledge of the subject matter, Shulman (1987:14) cite the following pedagogical reasoning and actions teachers need to undergo.

- Comprehension of subject matter within and outside the discipline
- Transformation of comprehended ideas by critically preparing the given text. Then representing the ideas in the form of analogies and metaphors by selecting from a variety of teaching methods. Since learners are different and have different needs, teachers may adapt the representations to learners' characteristics
- Instruction, which includes, *inter alia,* organizing and managing the classroom, presenting clear explanations and descriptions; assigning and checking work and interacting effectively with students through questioning and probes; answers and reactions and praise and criticism
- Assessment which involves checking for learners understanding during interactive teaching and at the end of a lesson and evaluating the teacher's own performance and making adjustments
- Reflection involves reviewing and critically analyzing teacher's own performance and that of the learners

In order to achieve this goal of teaching, teachers should have a good command of the subject matter because if they are largely ignorant or uniformed they can do much harm by passing on inaccurate bodies of knowledge to their students. Such teachers may even fail to realize and correct students' misconceptions (Ball & McDiarmid n.d.; Haylock & Cockburn 2008:6-7; Zakaria 2009). Because children understand structures of mathematical ideas differently from adults, teachers should find what understanding learners have regarding a concept because Haylock and Cockburn (2008:7) believe that this will enhance and strengthen the teachers' own understanding of mathematics and help teachers with identifying learners' misconceptions.

Murdock-Steward (2005:21) communicates that mental activities which contribute to the development of mathematical understanding are; (a) constructing relationships, (b) extending and applying mathematical knowledge, (c) reflecting about experiences, (d) articulating what one knows and (e) making mathematical knowledge one's own. Teachers should go through these activities so that they are able to help learners do so (Haylock & Cockburn 2008).

In the above discussion there has been mention of teachers' knowledge of the subject matter and the reasoning and actions teachers need to undergo for them to teach. The next section will discuss the knowledge base teachers need to have that will help them be effective teachers.

3.3 PEDAGOGICAL CONTENT KNOWLEDGE (PCK) AND MATHEMATICAL KNOWLEDGE FOR TEACHING (MKT)

The knowledge base that teachers should possess must be different from the one possessed by any educated adult. This section unfolds the knowledge base that is essential for teachers, especially mathematics teachers.

Ball (n.d.:3) suggests that teachers should have subject matter content knowledge which goes beyond knowing topics on the curriculum. The author defines subject content knowledge as the knowledge of the topic, concepts and procedure. Teachers must be able to explain why concepts are correct, why it is worth knowing and how they relate to other concepts.

Shulman (1987:7) and Ball (n.d.:3) corroborate that knowledge of content alone is not enough; in addition, teachers must also possess knowledge of pedagogy. Apart from the general and broad principles and strategies of classroom management and organization, teachers should know how to help learners access the subject matter easily. Shulman (1987) therefore conceptualises pedagogical content knowledge (PCK) as the knowledge a teacher has regarding how to represent and formulate the subject matter so that it becomes accessible to learners. PCK is essential for planning and executing lessons that facilitate effective learning. This is because PCK enables teachers to predetermine possible misconceptions and difficulties learners are likely to have.

Mathematical knowledge needed by a mathematics teacher is different from the one needed by a mathematics specialist. Hence Hill, Schilling and Ball as cited by Research Summary (2009:1 of 8) explain that a mathematics teacher needs to possess both common knowledge of mathematics (CKM) and specialized knowledge of mathematics (SKM). They explain that common knowledge is mathematics possessed by any mathematically educated adult while specialized knowledge of mathematics is the possession of mathematical knowledge and skills such as explaining why an algorithm works or being able to provide learners with multiple representations to address their diverse learning styles. SKM in relation to fractions include being able to explain fractions in different ways *inter alia*: as a number on the number line or a relation between two quantities.

In addition to CKM and SKM, Hill, Schilling, and Ball (Research summary 2009:2 of 8) further explain that teachers need to possess knowledge of learners and their ways of thinking about the content. Like SKM knowledge of learners and their ways of thinking about the content are unique to teachers. This body of knowledge specifically relevant to mathematics teachers is an extension of PCK and it is theorized as the Mathematical Knowledge for Teaching (MKT).

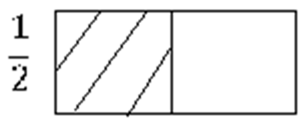
Both PCK and MKT enable teachers to know and understand what makes learning of certain topics easy or difficult and hence represent and formulate them in a way that they become easily conceptualized by the learners (Izs´ak 2008). PCK also enables teachers to predetermine learners' misconceptions and difficulties. MKT on the other hand goes further by equipping teachers with skills to analyse and evaluate errors and determine their source (Ding 2007:20). Therefore Ding concurs that MKT is an extension of PCK and it enables effective teaching of mathematics.

It is mentioned in section 2.5.1 that when learning fractions learners make mistakes. Apart from mistakes they ask questions, make representations and rethink their strategies when solving problems. When teachers attempt to respond to learners' difficulties and mistakes, they make representations. What remains crucial is deciding on the representation that effectively explains the concept; connects alternative representations; explains the mathematics underlying each representation and deciding on the representation which will be understood easily by most learners. So teachers need to possess MKT in order to make effective pedagogical decisions (Meixia 2007).

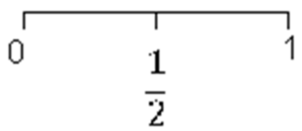
What teachers do in mathematics classrooms should be determined by their MKT. Regarding the teaching of fractions teachers should undergo the process of pedagogical reasoning, as discussed in the second paragraph above. That is, they should comprehend the concept fraction, transform the content by sequencing the activities in a way that will make the fraction concept easir to understand, for example to build conceptual understanding of equivalence before adding fractions. This is because equivalence serves as the basis for addition, so when the concept is clearly developed fraction addition becomes easy to understand (Charalambos *et al.* 2006).

Apart from the reasoning, teachers should engage in actions that help learners learn that a fraction can be presented differently, that is, half should be presented, among others as

a part-whole



a measure.



Teachers who are able to employ multiple representations of a concept are said to posses MKT.

Teachers should also anticipate difficulties learners are likely to encounter so that they plan strategies to help learners overcome them. Difficulties in ordering fractions are associated with inadequate exposure of learners to manipulate concrete materials (Troutman & Lichtenberg 2003:365). In this way teachers will plan in advance how to help learners to avoid such difficulties (Meixia 2007). MKT also enables teachers to interpret and evaluate learners' diverse solutions to a problem and respond to them in a way that will challenge them. Prospective teachers should also make connections by moving flexibly among subtopics so that learners can see relationships in them (Izs'ak 2008; Meixia 2007).

It was mentioned in section 2.4 that mathematics is viewed as a way of thinking and reasoning, so the next section discusses types of mathematical thinking and how it enhance the teaching of mathematics.

3.4 INDUCTIVE AND DEDUCTIVE REASONING

The importance of the manipulation of concrete objects in developing mathematical conceptions has already been emphasized. This experience results in the development of physical knowledge as indicated by Piaget (cf. 2.3.1).

Kamii and Warrington (1999:83) emphasize that, physical, social and logico-mathematical knowledge are important to learners, therefore teachers should avoid teaching only the physical and social knowledge at the expense of logico-mathematical knowledge.

Cooney, Davis and Henderson (1975:152) suggest a teaching strategy in which teachers give mathematical instances to learners and guide them with questions, to the objective of the teaching. An example could be to ask learners to share equally a whole, like a pizza, between two, four and six people. Then ask them to compare the size of the piece each person gets. From this they will see that when the number of people sharing one whole increases, the resulting pieces become small. They would have learned that $\frac{1}{2}>\frac{1}{4}>\frac{1}{6}$ and so on.

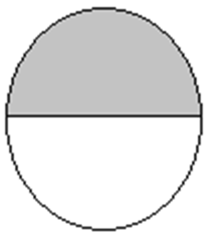
This type of reasoning is called inductive reasoning (Marwaha 2011) whereas Cooney *et al*. (1975:152) call it an inductive discovery teaching strategy. Marwaha

(2011) and Cooney *et al*. (1975:152) agree that the strategy moves from the specific idea to the general idea.

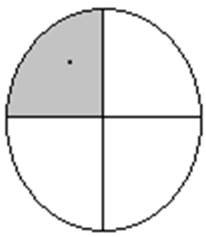
In the traditional methods of teaching, teachers give learners algorithms to use when solving mathematics problems. Instead of letting learners use these algorithms without understanding them, Cooney *et al*. (1975:153-158) suggest teachers to employ deductive discovery strategy. If learners work out $\frac{1}{2}+\frac{1}{4}$ as $\frac{2}{4}+\frac{1}{4}=\frac{3}{4}$, then teachers can design the learning experience in which learners will discover how the answer comes about.

The situation could be; two sisters are each given a piece of pizza, one is given one half ($\frac{1}{2}$) and the other one quarter ($\frac{1}{4}$), so what fraction of the whole pizza do the girls get?

The teacher may ask learners to draw diagrams to represent this situation or the teacher may do it on the board. But eventually they should see one half given to one girl.



and one quarter given to the second girl.



Then to find the fraction of the pizza they got, the pizza divided into two parts will now be divided into four parts

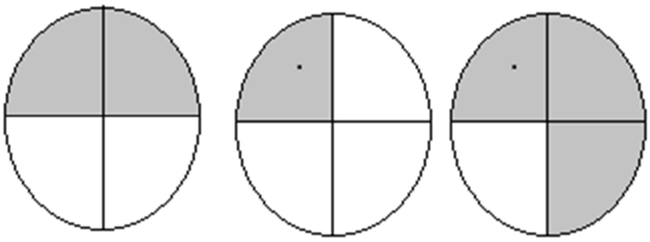


Figure 3.1Representing addition of fraction

and the shaded quarters now added equal three quarters.

From this and similar situations, learners are expected to deduce the generalization that when adding fractions they should have the same denominator or they should find the lowest common multiple.

Cooney *et al*. (1975:157) highlight that “the key success in a deductive discovery strategy is the teacher’s ability to ask a sequence of leading questions that will guide the students to deduce the generalization the teacher wishes to teach.”

Inductive and deductive reasoning can therefore be regarded as the stepping stone to problem solving as a mathematical teaching strategy (Marwaha 2011). Cooney *et al.* (1975:157) also link with Piaget’s theory of learning, discussed earlier in this chapter, in that in the processes of accommodation and assimilation one uses both inductive and deductive reasoning (cf. 2.3.1). Take the example of sharing a pizza, learners assimilate the new knowledge and then accommodate it into their schemes.

Both inductive and deductive reasoning are therefore pivotal in the learning of Mathematics, but at different stages as indicated by Piaget who points out that children begin to think inductively and deductively when they are at the formal operational stage (cf. 2.3.1.4). Both deductive and inductive reasoning provide one with conceptual knowledge as decisions they make may not be based on guess work but on Mathematical facts. Da Ponte (2001:53) adds that inductive reasoning is essential in creating new knowledge while deductive reasoning is necessary for organizing new knowledge and deciding what is valid and what is not.

3.5 PROBLEM SOLVING

As mentioned earlier, learning is a social activity, so is mathematics (Fajemidagba & Olawoye n.d.; Hiebert 1997:5-6). Mathematicians engage in mathematical activities using systematic steps to observe, study and experiment to come up with generalizations and new rules. Quinones (2005: 8-10) as well as Brown and Quinn (2007) suggest that one way of helping learners think and behave like real mathematicians is to use problem solving as a teaching strategy.

Through problem solving learners learn to think critically about what to do, how to do it and then monitor their progress. The National Council of Teachers of Mathematics (NCTM) in Ding (2007) state that “effective mathematics teaching requires understanding what students know and need to learn and then challenging and supporting it well.” Zakaria (2009) supports this as he states that students learn Mathematics through classroom experiences and problems given by teachers. The actions teachers take in class are determined by both their PCK and MKT.

Kroll and Miller (1993:59-60) as well as Bobis *et al.* (2004:40-41) define a problem as a situation in which an individual sees no apparent means to obtaining a solution. They further point out that a mathematical task is a problem only if the problem solver reaches a point where he or she does not know how to proceed. Problem solving as a teaching strategy, engages learners in the thinking process as they attempt to come up with appropriate solutions.

A problem solving program should include instruction to help learners know how to approach unfamiliar problems (Bobis *et al.* 2004:46). One way would be to familiarize learners with a variety of problem solving strategies or heuristics like drawing a diagram, completing a table or making a list (Kroll & Miller 1993:64-68; Quinones 2005:9). These are methods and rules of mathematical inventions and discoveries used by mathematicians to represent problems in different forms that make more sense to them. Kroll and Miller (1993:64-68) as well as Quinones (2005:9) also indicate that experience with using different heuristics gives learners a better understanding of the problem solving process and of possible strategy choices.

Being able to interpret a problem and present it in a different form is motivating to learners and it aids as an encouragement to continue with the attempt to solve the problem.

Polya (in Kroll & Miller 1993:61) suggests a problem solving approach in which learners are expected to read and understand the problem, devise a plan, carry out the plan on how to solve the problem and look back to verify the effectiveness of their plan of action.

3.5.1 Polya's four phases of problem solving

Polya indicates that the purpose of a teacher is to help students gain much experience and independent work. He warns teachers though, that the help they give should not be too much or too little because if it is too much it deprives learners of the chance to develop cognitively. So learners should be left with a reasonable amount of work that will result in some learning when the task is completed. This notion is in accordance with Vygotsky's ZPD and Bruner's process of scaffolding as discussed in sections 2.3.2 and 2.3.3 respectively. Bruner indicated that teachers should provide assistance to a certain degree and then withdraw gradually for learners to gain independence and hence grow cognitively. Vygotsky on the other hand warned that tasks given to learners should be just above their zone of proximal development so that they remain engaged and challenged.

In order to help learners, teachers should ask learners questions and make recommendations that will make learners think and engage in mental operations. Learners should be made aware that mathematics problems require the use of common sense and ask questions they would ask themselves in real life situations (Polya 1985:2). Teachers can facilitate and enhance learners' problem solving skills by creating situations in which learners can imitate and practise how teachers solve problems.

Polya (1985) maintains that these stages can be followed to solve any problem from any discipline because, as mentioned, they are more of common sense than academic. These stages are:

- understanding the problem

In order to understand the problem, (Polya 1985:6; Troutman & Lichtenberg 2003:57-58) suggest that problem solvers should ask questions such as; Can I identify needed information? What are the given data? Can I generate all possibilities? These questions are essential to ask because, according to Troutman and Lichtenberg (2003:43) as well as Kroll and Miller (1993:66), problems may sometimes contain extraneous information or may not contain all the necessary information. So it is the problem solvers' task to identify these. This indicates that learners should possess deductive reasoning skills as discussed in section 3.4, in order to be able to solve mathematical problems.

Care should be taken when using problem solving to teach mathematical generalization that, problems do not contain information that may result in no learning. When building the concept fraction, from the problems given learners should learn that a whole is shared equally in order to be able to say this is a certain fraction.

Teachers should ask learners questions that will focus their attention to the unknown.

Example: on the number line what fraction is represented by each division?

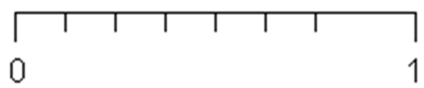


Figure 3.2 Representing fractions on the number line

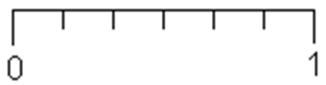
The teacher and learners can now engage in a dialogue that will elicit understanding. Learners should indicate that the unknown is the fraction that represents each division on the scale. The given data are zero and one, which is one whole. They should also be able to tell whether the problem is reasonable or not.

- Devising a plan

With the necessary knowledge of mathematics to solve the problem, learners should be given the chance to come up with their own plan (Polya 1985:9). Polya suggests that this can be achieved by engaging in the dialogue which will provoke the idea of how to solve the problem. Learners' attention should once again be focused to the unknown. They may also be asked to come up with a familiar problem having the same or similar unknown.

It is also essential to devise more than one plan so that one can compare the effectiveness of each plan (Troutman & Lichtenberg 2003:57-58). They further suggest that when carrying out the plan and it seems not to be giving positive results, problem solvers should not be afraid to start all over again as one of the qualities of problem solvers is perseverance.

In the example explicated above, a familiar problem would be the one in which learners are to find the fraction represented by each division when all the divisions are of equal width.



If they are able to come up with a similar problem this may suggest that they have a plan.

- Carrying out a plan

Coming up with a plan is not very easy since it requires one to engage in deep mathematical thinking (Polya 1985:12). Once the plan is conceived, to implement it, is not as challenging as coming up with it. Now that learners have realized that the divisions in the problem are not even then they should be urged to carry out the plan of making divisions of equal width and then find the solution, which is one eighths in this problem.

- Looking back

This is the last of Polya's stages of problem solving. This calls learners to check the effectiveness and accuracy of their plans and the reasonableness of their solution. Also to reconstruct and re-examine the result and the path that led to the solution (Polya1985:14).

If problem solvers feel that the solution is not reasonable enough then they can go through the steps to find where they might have gone wrong. When looking back to how the problem is solved, learners are likely to identify their mistakes and then see why they are mistakes. As another strategy to verify one's solution, learners should be given the chance to discuss each other's plan so that they could criticize them and hence build on their own and attain cognitive growth (Leung, 2010; Mueller *et al.* 2010).

Polya's model gives learners the freedom to solve problems the way they make sense to them. If problems are rooted in learners' real life situations they will understand and interpret the problem the way they exist in their context and hence employ strategies they find suitable (Fennema & Romberg, 1999:5; Hiebert *et al.* 1997:4). The model also aligns with the constructivist philosophy in which learners construct knowledge through social interactions and by making connections to their own experiences (cf. 2.3).

3.5.2 Problem solving strategies

Ding (2007) indicates that trial and improvement is a problem solving strategy that can be used to solve mathematical problems. Through it, learners struggle, make mistakes and then see why they are mistakes. Trial and improvement as a problem solving strategy could be done in small groups or through whole class discussions in which learners criticize each other's problem solving strategies and hence build on their own strategies (cf. 2.3.2; Leung 2010; Mueller *et al.* 2010).

The above discussion indicates that emphasis should not be put on correct answers but on the process of attaining that answer (Kroll & Miller 1993: 61).The authors posit that one of the hindrances to effectively learning of mathematics is emphasizing the attainment of correct answer. Problem solving, as a teaching strategy, therefore can contribute to effective learning of mathematics.

Problem solving can be used in mathematics as a teaching strategy, that is, learners may be given a problem to work on individually or in groups, and later the whole class engage in the whole-class discussion of their solutions. Teachers may structure the discussion such that learners come up with their own conclusions. When learners have thorough understanding of what they did, the teacher may now introduce the new mathematical concept learned through that problem. Carpenter (1989:191) calls this teaching through problem solving.

The author defines teaching through problem solving as giving learners a problem to solve using their own strategies and through the problem solving process they learn new mathematical concepts.

As discussed teachers at times find it worth teaching learners the strategies they could use when solving mathematical problems, like Polya's model of problem solving. This is called teaching problem solving by Carpenter (1989:199). In this way learners are taught the problem solving strategies to follow when faced with a problem. In this type the prime focus is not to learn new mathematics but to acquire the problem solving process. Like in the teaching through problem solving, learners need to use all the mathematics they know to come up with the solution to the problem.

Meaningful learning is the outcome if teaching is effective. As argued above teaching through problem solving enhances deep and meaningful understanding of mathematics.

3.5.3 Factors Influencing problem solving

Kroll and Miller (1993:61-65) suggest that teachers should consider the following factors which may affect problem solving; knowledge factors; beliefs and affective factors.

- Knowledge factors

In order that learners can solve problems Schoenfeld in Thompson (1989) cite that they should have mathematical resources, which he describes as the knowledge of concepts, facts and procedures. Learners often encounter difficulties when solving problems if they lack the necessary resources which Kroll and Miller (1993:61-65) call algorithmic or computational skills.

It was discussed that procedural knowledge should follow conceptual knowledge (cf. 2.5.3.1). In this regard teachers should help learners develop procedural knowledge, it should not be compromised. For learners who did not develop procedural knowledge it sometimes becomes difficult for them to decide on the operation to use. This difficulty Kroll and Miller (1993) attribute to the development of procedural knowledge, that is, the ability to calculate without paying attention to developing understanding. Teachers are encouraged not to spend time teaching learners to rely on key words to decide what operation to use as this strategy has been observed to break down quickly as problems become more complex.

- Beliefs and Affective factors

At times learners may hold believe that mathematical problems should be able to be solved quickly. They may also belief that there is just one right way to solving a mathematical problem. These beliefs may have a negative impact on the learners' motivation and hence imply that when a mathematical problem does not have a straight forward solution it is not solvable and hence be poor problem solvers.

When using problem solving as a teaching strategy, teachers should help learners develop positive beliefs about problem solving. One way could be by introducing problem solving by giving learners easy tasks which have low cognitive demand as this will help them improve their motivation level. Again teachers should allow learners see them as they struggle with a problem (Owen 1993: 66).

Teachers often undermine the learners' capability and hence give them problems with low cognitive demand. Teachers who do this are said to have low expectations for their learners and such learners often do not perform well. For learners to do well in mathematics, teachers should give problems, within the learners' cognitive capability, but which are more demanding. This is what Spady (1994:180) calls high expectations for the learners. Spady (1994:16) accentuates that most learners, whose teachers have high expectations of them, will raise their performance in more challenging levels of learning.

Kroll and Miller (1993) point out that some researchers have linked work on meta-cognition with work on problem solving. In the words of Schoenfeld in (Lesh & Zawojewski 2007:771) metacognition is described as "self regulation, or monitoring and control … [of] … resource allocation during cognitive activity and problem solving." Problem solvers or learners should be able to decide; on what they know, what they do not know and what they need to know; on the mathematical knowledge needed to solve the problem; what to do to solve the problem.

Since learning can also be socially constructed (cf. 2.3.2 & 2.3.3), metacognition can be enhanced through group work. In those groups, learners can provide to each other, explanations to each other regarding their understanding of the problem, the method they use and others may ask questions seeking for clarification or indicating a mistake.

That is, learners should be helped to be aware of their own knowledge and thinking process and monitor their thinking and progress when involved in a problem situation. This ability, of learners, to regulate and monitor their own problem solving is what is called metacognition (Thompson 1989:234).

3.5.4 Nature of problems

The nature of problems given to learners has direct impact on the learners' motivation and perception as problem solvers. Teachers should try their best to compose meaningful and solvable problems (Kroll & Miller 1993:66). For a start teachers should give problems which are solvable and require low order cognitive reasoning (cf. 3.5.3) so that learners' confidence is increased. When they have gained confidence, then learners could be exposed to problems that require high order cognitive reasoning which are within their cognitive development stage.

Real-life application problems should be used as they often seem more meaningful and interesting to students. This could be done by adapting problem details from learners' interests such as using a popular context (Kroll & Miller 1993:66). Learners could be asked to find how many apples a friend will have after he is given half ($\frac{1}{2}$) a box of apples by one friend and a quarter ($\frac{1}{4}$) box of apples by the other. Giving learners this type of problems may indicate to them that mathematics is real and hence it may become meaningful to them.

While introducing learners to problem solving, care should be taken when formulating problems. Kroll and Miller (1993: 66) warn teachers that problems should not contain extraneous information or incomplete or contradictory information. This is likely to confuse learners and hence they perceive that problem solving is difficult and not possible to do. It is true that in Polya's first stage of understanding the problem, learners should be able to identify what belongs and what does not. But this could well be done by learners who are at the advanced stages of problem solving. For young learners teachers should avoid confusing them with a lot of unnecessary or wrong information. Instead they should begin with simple problems with no extra unnecessary information, then gradually introduce them to extra information.

In order to monitor learners' progress and/or understanding of what they are doing in the classroom when solving problems, teachers should continually assess the learners' understanding. The next section discusses assessment and how it enhances teaching.

3.6 ASSESSMENT

The major goal of mathematics teaching is to develop an understanding of mathematical concepts so that learners will have the capacity to integrate, apply and communicate their mathematical understanding in and out of school. To achieve this learners must be confident, interested, curious and inventive (Lambdin 1993:10). One of the tasks of teachers is to design instructional activities that will enhance the achievement of mathematical teaching. Teachers give instructional activities that do not only facilitate learning but give them information regarding the learners' competence. Webb (1993: 1) indicates that it is the teachers' tasks to engage in an on-going assessment that will help learners attain the goals of curriculum.

"Assessment is a comprehensive accounting of a student or a group of student's knowledge (Webb 1993:1)." Engelbrecht, Green, Naicker & Engelbrecht (1999:117) on the other hand define assessment as:

> The process of identifying, gathering and interpreting information about learner's learning. The central purpose of assessment is to provide information on learner achievement and progress and set the direction for ongoing teaching and learning.

This definition brings us to the discussion of two types of assessment; formative and summative assessment.

3.6.1 Types of assessment

3.6.1.1 Formative assessment

It has been discussed in section 3.3 above that during the teaching and learning, teachers should identify learners' difficulties so that they can assist them. This could be done through observing and questioning learners as they engage in classroom activities. The information that teachers get from such encounters should guide their decision-making and actions to facilitate learning.

This continuous monitoring of learners' strengths and needs facilitates and informs teachers of the actions that will help them pace learners accordingly. This form of information gathering is called formative assessment and it is referred to as the assessment for learning (Cangelosi 2003:147; Engelbrecht *et. al.* 1999:110).

Cangelosi (2003:147) and Chambers (1993:17) warn that assessment must be an interaction between teachers and learners with the teacher continually seeking to understand what a learner can do and how a learner is able to do it and then using this information to guide instruction. Cangelosi (2003:147) call this formative assessment. So assessment must be part of every mathematics lesson so that effective teaching can occur.

Since the purpose of assessment for learning is to facilitate learning, learners should be given specific and detailed feedback immediately (Earl & Kartz 2006:33). Descriptive feedback enhances learning as is the case with descriptive language (cf. 2.6.4). Descriptive feedback gives learners information regarding what they know, what they do not know and what they should do in order to learn effectively. Coupling assessment with descriptive feedback therefore facilitates effective learning.

3.6.1.2 Summative assessment

Other than gathering information that will help teachers make informed teaching actions, information may also be gathered at the end of a learning period. That is at the end of a learning unit, a term, semester or year. The intention being to provide other stake holders about the teaching and learning that has taken place (Engelbrecht *et al*. 1999:111).

Effective assessments require teachers' to have thorough knowledge of the mathematical domain under investigation and the nature of the learners' thinking within that domain or MKT (cf. 2.5.3; Lambdin 1993:10).

Both formative and summative assessments are important in teaching and learning and more specifically in the teaching and learning of fractions. Since this study is concerned about effective teaching, this study will focus mainly on formative assessment as one of the aims of the study is to determine whether effective teaching is taking place (cf. 1.2). As discussed in 2.6.1, through formative assessment teachers should be able to identify learners' errors, misconceptions and difficulties and hence effective learning would take place.

3.6.2 Assessment methods

Different assessment methods that teachers could use include; interviewing, observations, portfolio, performance, essay writing and objective tests.

3.6.2.1 Interviewing

This is not necessarily a separate technique; it can be part of the observation and can be done during the normal teaching. As learners participate in classroom activities, teachers may notice certain behaviour. In order to make sure that they understand what that behaviour means, teachers may have to interview or question the concerned learner (Engelbrecht *et al.* 1999:117).

From interviews, teachers may get a lot of information pertaining to the learners' feelings and beliefs about themselves and about mathematics (Engelbrecht *et al*. 1999:117). They may also gather information about the thinking behind their actions as they perform mathematics tasks. Information obtained can inform teachers about the learners' level of understanding of the subject, expectations and/or beliefs. Hence the information obtained may be of use to the teacher when designing further instruction.

3.6.2.2 Observation

Through observations of learners as they participate in class, teachers can also judge learners' level of motivation and confidence when they engage in a mathematics task and when they interact with other learners. However as said in the above paragraph teachers should make sure that they do not misjudge learners. They could do this by verifying an observation with an interview (Engelbrecht *et al*. 1999:117; McMillan 1997:52).

Engelbrecht *et al.* (1999:117) warn teachers that observations should not be done by chance but should be a planned assessment strategy. For example if a teacher wants to determine whether learners can competently demonstrate addition of fractions, then they should plan activities from which they can competently draw a conclusion.

3.6.2.3 Portfolio

A collection of learners' work over an extended period may also serve as an assessment strategy. From this collection of the learner's work, teachers may make informed decisions about the learner's progress. It can be deduced whether there is improvement or not. Based on this evidence, teachers may discuss the learners' work with the concerned learner or other parties, like parents and/or the principal (Engelbrecht *et al*. 1999: 120).

The next section discusses features of assessment as identified by Webb (1993:3-4).

3.6.3 Features of assessment

3.6.3.1 Assessment situation, task or question

This could be a problem presented to learners, a classroom discussion or activity or any other action that elicits learners' responses.

3.6.3.2 Response

In order for teachers to be able to have some information regarding learners' knowledge, teachers should ask learners to display this knowledge either by

- Writing numerical answers
- Writing a paragraph explaining the thinking process behind the solution
- Performing oral presentation
- Performing journal entry
- Keeping a portfolio of their work over an extended period of time
- Lastly teachers should Interpret student's responses

This can be attained by comparing responses to defined goals or expectations. This could also involve understanding what the response reveals about the learners' structure of mathematical knowledge. It requires making deductions on the basis of an understanding of mathematics, learning mathematics and knowledge of the students.

3.6.3.3 Assigning some meaning to the interpretation of the student's responses

This could be demonstrated by locating learners' responses on a scale that represents the range of all possible responses.

3.6.3.4 Recording and reporting

Recording can be done by writing 'good' on the margin of the book or by writing the marks in the teachers record book. Reporting can be by verbally talking to the learner to explain how they have performed, what problems are identified and what improvements need to be done and then recording the findings in a book for reference when designing future instruction. This form of communication gives learners feedback (Earl & Kartz 2006:33) on their work and performance as a whole and therefore facilitates learning.

Learners who have effectively learned a concept should have the capacity for integrating, applying and communicating their mathematical understanding (cf. 2.5.2), and this ability is referred to by Lambdin (1993:10) as mathematical power. The author further explicates that learners should show confidence, interest, curiosity and inventiveness in working with mathematics ideas. This he calls mathematical disposition. If learners demonstrate mathematical power through multiple behaviours, then teachers must also use a variety of methods for assessment. That is, teachers should assess whether learners have all the aspects that culminate to the ultimate goal of mathematical learning (Lambdin 1993:11).

3.7 EFFECTIVE TEACHING OF FRACTIONS

It has already been discussed that effective learning requires assimilation and accommodation of one's experiences into the already existing schemes. Based on what effective learning in mathematics is, effective teaching in mathematics, more particularly fractions, will be discussed from that angle. The discussion will begin by focusing on what effective teachers do and what effective teaching is.

Children learn by manipulating concrete materials, making sense of what they are learning by relating it to what they already know and then articulating it within collaborative groups (cf. 2.5.2).

Ball and McDiamid (n.d.:4) conceptualise effective teaching as a result of what teachers do, how they understand their work and decide what to do. The author explicates that effective teaching depends on what the teachers' believe about their teaching and about the subject matter.

Teachers who perceive mathematics as the set of rules to be learned through drill and practice teach learners rules and then give them exercises to practise them. Those who believe mathematics is learned by constructing their own understanding and meaning, either individually or in groups, create situations which will enable them to construct meaning and understanding. Ball and McDiamid (n.d.:36) elucidates that what teachers decide to do depends on their subject matter knowledge. For teachers who give learners opportunities to construct meaning and understanding, it is because they have the knowledge that enables them to reason and relate concepts. So they are able to explain why learners' reasoning is mathematically wrong or correct. That is, prospective teachers have MKT (cf. 3.3).

So effective teaching should include giving learners activities which are rooted in their contexts with the attempt to extend, answer their questions and give meaning to what they already know (Donald *et al.* 2002:109).This enables learners to construct their own meaning and understanding (cf. 2.5.3.1).

3.7.1 Fractions

Fractions can be understood from different angles of their usage. They can be understood as numbers, positions on the number line or ruler and as the ratio between a part and a whole (Haylock & Cockburn 2008:49-50). They are elements of the set of rational numbers and it is defined as an integer divided by an integer not equal to zero.

3.7.2 Difficulties of teaching fractions

As stated earlier, Charalambos *et al.* (2006) and Leung (2009:3) emphasize that teaching and learning of fractions is one of the most problematic areas in primary school mathematics. Haylock and Cockburn (2008:49) and Barmby *et al.* (2009:61) corroborate with them and then posit that the problems relate to the way fractions are represented as numbers and the standard ways that we use to represent fractions in diagrams and pictures. They suggest that the number concept should first be established in learners before they can be introduced to fractions. This is because the concept of a fraction as a number can be overwhelming to learners because initially fractions were rejected as numbers.

Some of the causes of these difficulties arise from the misconceptions associated with fractions that learners bring along and the multifaceted nature of fractions (Bezuk & Bieck 1993:119; Charalambos *et al.* 2006).

3.7.2.1 Handling misconceptions

The resistance to accept fractions as numbers can lead to the misconception that a fraction is two different whole numbers, that is, to find the fraction of the shaded part in the diagram below

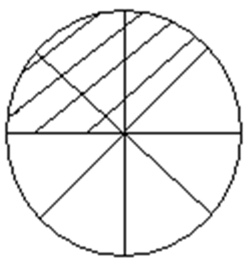


Figure 3.3 Fraction as a part-whole

They count the number of shaded sectors as 3 and write the fraction as $\frac{3}{8}$. To get the fraction of the unshaded part, instead of subtracting fraction of the shaded part from one whole the learners would count again (Barmby *et al.* 2009:62). That is, they count the unshaded sectors as 5 and then write the fraction as $\frac{5}{8}$. The authors explain that this could be that learners do not see the connection between the fraction of shaded and unshaded or complimenting addition, that is, $\frac{3}{8} + \frac{5}{8} = \frac{8}{8} = 1$.

Troutman and Lichtenberg (2003:290) advise teachers that if they realise that their learners have this difficulty they should engage them in the hands on manipulation of concrete materials. They contend that this will help learners realise that they do not have to count every time they encounter this situation.

In order to handle this difficulty, teachers could engage learners in the counting activities when finding fractions. When they are satisfied that every learner can do this exercise, then they can ask learners to deduce how to find one fraction when they know the other (cf. 3.4).

Another misconception as cited by Nunes, Bryant, Hurry, and Pretzlik (2006) together with Murdock-Steward (2005:6) is equating the number of shaded parts to the numerator and the number of un-shaded parts to the denominator, which is $\frac{3}{4}$, instead of $\frac{3}{7}$.

Figure 3.4 (a) Fraction misconception

and hence result in computational errors when operating fractions like $\frac{3}{4} + \frac{2}{5} = \frac{5}{9}$. As discussed earlier, teachers are advised to engage learners in manipulating concrete materials. They should also help learners conceptualise fraction as a number (Charalambos *et. al.* 2006; Haylock & Cockburn 2008: 45-52).

So effective teaching and learning will take place if teachers can assist learners not to develop misconceptions or to clear them if they already exist. This could be achieved by creating learning situations in which learners interdependently construct understanding of all the constructs (cf. 2.5.2). Misconceptions such as equating the shaded part as $\frac{1}{5}$

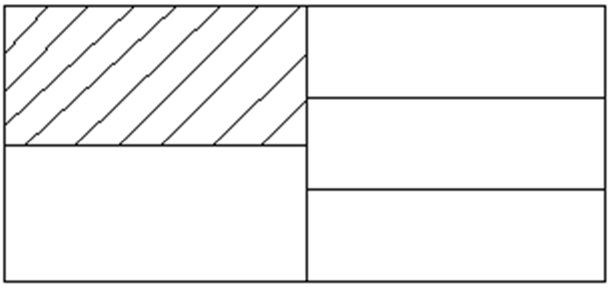


Figure 3.4 (b) Fraction misconception

and calculating $\frac{1}{2} + \frac{3}{8} = \frac{4}{10}$ are attributed to inappropriate exposure of learners to manipulate real life concrete materials and hence acquire physical knowledge (cf. 2.3.1; Murdock-Steward 2005: 6; Troutman & Lichtenberg 2003: 365).

3.7.2.2 Difficulties associated with contextual teaching

Difficulties associated with learning fractions among others are that fractions are not related to the learners' real-world and fraction learning experiences are not sequenced in a cohesive way(Murdock-Steward 2005:4-5). If this is the case, teachers should, as mentioned, use examples from the learners' context and approach to the topic should be spiral (cf. 2.3.2). Charalambos *et al.* (2006); Leung (2009) as well as Pearn (2007) do not agree with Murdock-Steward that fractions are not related to the learners' real-world. They argue that when teaching fractions, teachers use language that is used in learners' real-life. The example being when talking about sharing, teachers refer to concrete objects that are familiar to learners, like sharing a pizza, sweets and many more (Charalambos *et al.* 2006; Leung 2009; Pearn 2007).

On the other hand Kerslake (1991:86-87) argue that fractions are mostly represented as parts of things learners use in their everyday life, such as an apple or a bun (cf. 2.5.2). So according to Kerslake, the language learners use in their every-day life with an attempt to contextualise their teaching imposes difficulties especially when operating fractions as the author indicated where no possible meaning could be attached to adding one half of an apple to three quarters of a bun (cf. 2.5.2). So contextualising could also result in learners experiencing difficulties.

In order for teachers to help learners not to experience difficulties, especially when using their context for teaching, they should invest more time in helping learners develop conceptual understanding by engaging them in hands-on activities using concrete objects. This is because in this study learners are at both Piaget's concrete operational and formal operational stages; the entry point of cognitive development (cf. 2.3.1.3 & 2.3.1.4). They should also help learners build the concept of a fraction as a number and they should build it from different perspectives (Caswell 2007; Haylock & Cockburn 2008: 6-7; Kerslake 1991: 88).

3.7.2.3 Diagnosing and remedying learners' difficulties

It is inevitable that, in the teaching and learning environment, learners will experience difficulties. So, it is indicated in section 2.6.3 that teachers should posses the skills to identify and remedy learners' difficulties. Cooney, Davis and Henderson (1975:202-239) suggest that teachers should approach diagnosis and remedying of learners' difficulties as problem solving. They should go through the four stages of problem solving as discussed by Polya (cf. 2.5.2.1).

Firstly teachers should be aware of the problem. For example a learner cannot add fractions. Then they should define it. By this he means that teachers should state the kind of errors learners make, what misunderstanding they manifest and what it is that they cannot do (Troutman & Lichtenburg 2003:287; Cooney *et al.* 1975:203).

Having defined the problem, teachers may now collect data that will enable them to solve the problem. If the problem is: learners make mistakes when adding fractions, that is, they write $\frac{1}{4} + \frac{1}{2} = \frac{2}{6}$. Teachers then have to decide on the cause of this error. They may conjecture that learners do not have conceptual knowledge regarding sharing. To test the conjecture teachers may now engage in formative assessment by asking learners questions that need them to have the possession of conceptual knowledge of sharing. Data will now be the answers that learners. Once the correct conjecture has been determined, teachers may now decide on what kind of remedial teaching to do (Cooney *et al.* 1975:202-214).

Point of clarification is given to teachers though that, they should remember that learning difficulties are not attributed to learners' intelligence only. There are other factors such as; physiological, social, emotional and pedagogical factors. As it is easier to correct one's own problems than others, teachers should start by doing introspection before looking at other factors (Cooney 1975: 214). That is, teachers should assess their teaching. Teachers should ask themselves the following questions; "Have I used the learners' prior knowledge to assist in the development of conceptual knowledge? What is it that I have done wrong? What should I do to improve my teaching" (Cooney *et al.* 1975:214)?

The next section discusses the five subconstructs teachers are advised to consider when teaching fractions.

3.7.3 Subconstructs

Haylock and Cockburn (2008:6-7) indicated that teachers should engage with structures of mathematical ideas in terms of how children come to understand them so that they can teach them effectively. So in this section the five subconstructs pertaining to fractions as identified by Behr *et al.* (1983) and Charalambos *et al.* (2006) will be discussed. These are part-whole, measure, operator, quotient and ratio subconstructs.

3.7.3.1 Part-whole subconstruct

This is defined as the situation in which a whole or a set of discrete objects is divided into equal parts (Charalambos *et al.* 2006; Leung 2009).

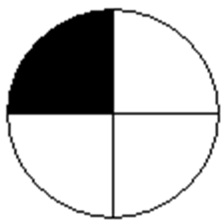


Figure 3.5 A whole pizza divided into 4 equal parts.

The shaded part represents one quarter ($\frac{1}{4}$) of a pizza

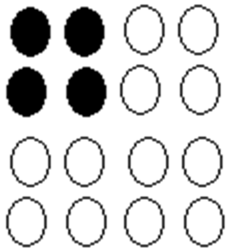
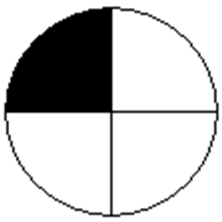


Figure 3.6 A set of 16 balls

The shaded balls represent one quarter ($\frac{1}{4}$) of the whole set of balls. In both cases the fraction, $\frac{1}{4}$ represents the comparison between the number of parts of the partitioned unit to the total number of parts in which the unit is partitioned (Charalambos *et al.* 2006; Leung 2009:4). The authors indicate that to understand the part-whole subconstruct, learners should develop the concepts that; a) the parts into which the whole is divided into must be equal; b) the more parts the whole is divided into, the smaller the produced parts become; c) the parts taken together must be equal to the whole; d) the relationship between the parts and the whole is conserved regardless of the size, shape and orientation of the equivalent parts.

Learners should also be made aware that they should count the parts twice, once for the numerator and then for the denominator (Charalambos *et al.* 2006; Nunes & Bryant 1996:203). Learners who have not developed this concept will write the shaded part as $\frac{1}{3}$, instead of $\frac{1}{4}$ as discussed above.



Learners should further develop the concept of repartitioning already partitioned units. For example, from the whole partitioned into fourths, learners should construct $\frac{2}{8}$ or any fraction equivalent to $\frac{1}{4}$.

3.7.3.2 Ratio subconstruct

Fraction as a ratio conveys the notion of comparison between two quantities. Carraher (in Charalambos *et al.* 2006) posits that a fraction is therefore a comparative index not a number. Teachers should assist learners to understand that for a fraction to be a comparative index, similar quantities should be compared. Two girls share a pizza and three boys share a pizza, which one gets more pizza, a boy or a girl? To answer this question so that the comparative notion is employed, learners should compare the same thing that is the amount of pizza each girl gets to the amount of pizza each boy gets.

Since this study is based on age group 11 to 12, it may not be important to let them know the difference between rate and ratio. However problems and activities that are given should develop what they are intended to develop, either rate or ratio (Charalambos *et al.* 2006).

The authors further make teachers aware that, for learners to fully develop the subconstruct, they need to construct the idea of relative amounts. Learners need also to understand what it means to say that there is a relationship between two quantities and that when quantities in the ratio relationship change together the relationship between them remains invariant (Charalambos *et al.* 2006).

3.7.3.3 Operator subconstruct

A fraction as an operator is viewed as a function applied to some number, an object or a set. Lamon (in Charalambos *et al.* 2006) says that a fraction as an operator could be viewed as a composite function which results from two multiplicative operations or as two discrete but related functions applied consecutively, that is, $\frac{3}{4}$ of 8 could be viewed as $\frac{3\ x\ 8}{4}$ or as $3\ x\ 2$. An operator is also defined as a transformer that lengthens or shortens the length of a line segment, increases or decreases the number of tasks in a set of discrete objects and takes a figure in the geometric plane and maps it onto a larger or smaller figure of the same shape.

From the above analysis, Lamon (in Charalambos *et al.* 2006) further decomposes the operator subconstruct into two subconstructs: stretcher/shrinker and as duplicator/partition reducer. But Behr (in Isz'ak 2008) have the third subconstruct, namely multiplier/divisor. This then brings in the other subconstruct, the quotient.

3.7.3.4 Quotient subconstruct

A fraction can be viewed as a result of a division situation (Charalambos *et al.* 2006; Iza'ks 2008; Leung 2009). That is $\frac{x}{y}$ is a numerical value obtained when a whole number x is divided by another whole number y (y ≠ 0). Situations usually used to help learners construct the quotient subconstruct involve finding fair shares of continuous quantities. Because of its numerical value, a fraction as a quotient can be of any size; less than, equal to, or greater than a unit. A problem situation in which three pizzas are shared by four children depicts two different measure spaces; *pizzas* and *children*. The pizzas are shared into four equal parts, the dividends, which is the number of parts in each share, and each child gets three quarters of a pizza, the divisor, which is the fraction name of each share.

3.7.3.5 Measure subconstruct

Fraction as a measure is connected to two inter-related notions. The first one is how big the fraction is, and the second one is associated with the measure assigned to some interval (Charalambos *et al.* 2006; Leung 2009:5). A unit fraction is defined as $\frac{1}{a}$ so when used repeatedly from the starting point then the distance can be determined. For example, $\frac{3}{4}$ corresponds to the distance 3 ($\frac{1}{4}$ unit) from a given point. Hence to build the measure subconstruct, number lines and measuring devices like scales and rules are usually used.

3.8 TEACHER DOCUMENTS

Curriculum design on a micro level can enhance effective teaching of mathematics, and more specifically effective teaching of fractions. Lesson plans, work schedules, assessment activities, grading documents are all examples of teacher documents.

In this study focus will be on lesson plans and learners' mathematical exercise books or assessment products.

3.8.1 Lesson plan

The effective ways of teaching mathematics, fractions in particular, were reflected on in the previous paragraphs. Before the actual teaching of their classes, teachers have other responsibilities (Cangelosi 2003:17). Cangelosi (2003:20-22) name these, among others, as planning and organising courses by writing syllabi; arranging and organising the classroom.

A lesson plan, according to the Department of Education (2003:2) "describes concretely and in detail teaching, learning and assessment activities that are to be implemented...'. It could be a daily, weekly or quarterly lesson plan.

A lesson plan prepared by an individual teacher and its contents are *inter alia*: topic, date, objectives, list of materials needed, teaching approach, method, procedures, potential problems and pitfalls and methods of evaluation (Cangelosi 2003:54; Department of Education 2003:4; Wagner 2004).

- Objectives

Objectives or learning objectives are short and measurable statements that describe what learners will be able to do as a result of learning. Objectives should begin with action an action verb and when possible they should include performance standards on how the learners will be evaluated

- List of materials

List of materials needed. When thought of well before the lesson they save the teacher much frustration and time.

- Procedures

Procedure is a list of all the steps the teacher intends to follow in the lesson. This may be how to introduce the lesson, how to activate prior knowledge and so on.

- Potential problems and pit falls

Potential problems and pit falls that may occur are that learners do not have the requisite knowledge necessary for the lesson, so teachers should have a plan if this be the case (Wagner 2004).

- Assessment tasks

Assessment task is a question, exercise or problem that a teacher intends to give to the learners during or at the end of the lesson. The purpose of which is to determine whether learners have learned what was taught. Learners' performance on the assessment task provides teachers with information pertaining to the achievement of the lesson objective/s.

Wagner (2004) further indicates that there are different types of lesson plans. The type of lesson plan each teacher uses depends on individual circumstances, lesson being taught, cognitive readiness and social stance of learners involved.

Cangelosi (2003:194) agrees with Wagner (2004) about different types of lesson plans. But Cangelosi (2003:194) describes a four stage lesson plan that can be used by mathematics teachers regardless of the intention of the lesson. The four stages as described by Cangelosi (2003:182-194) are:

- Experimenting

It is discussed in section 2.5.2.3 that mathematics can be learned effectively through experiments. Hence teachers should plan in advance experiments they intend to give to learners.

- Reflecting and explaining

In this section of the lesson, teachers should plan how learners will be helped to reflect on the experiment. Learners may be asked to explain the problems and difficulties they encountered and how they overcame the difficulties. As learners reflect on their experiences, they get used to using the appropriate mathematical language and also improve on reasoning skills (cf. 2.3.2 & 2.5).

- Hypothesising and articulating

At the end of an experiment, learners should be encouraged to discuss the findings and come up with their own generalizations. This makes learning personal and hence learners gain conceptual knowledge (cf. 2.5.3.2).

- Verifying and refining

Teachers may now conclude the lesson by helping learners to verify their findings and make corrections where needed.

3.8.2 Learners' mathematics exercise books

The researcher is of the opinion that exercise books of learners can provide valuable information to the researcher regarding the performance of learners. This information may include, *inter alia*, the type of mathematics questions given to learners, the strategies learners use to perform calculations and the feedback that teachers give to learners.

The researcher believes that by studying learners' mathematics exercise books, information can be gained regarding performance, errors and misconceptions that learners may have.

3.9 CONCLUSION

Chapter 2 elucidated what effective learning is and how learners could be helped to achieve effective learning. This is because the teaching of mathematics is informed by the knowledge of how children learn and what effective learning is.

This chapter discussed effective teaching strategies that teachers can use to assist learners to learn effectively. These teaching strategies are the use of problem solving as Quinones (2005:8-10) suggests. It is one way of helping learners think and behave like real mathematicians. Teachers could use this strategy to teach mathematical concepts. Using problem solving as a teaching strategy could help learners develop both inductive and deductive reasoning, which are high order reasoning skills essential in solving every-day real-life problems.

In order to be effective in this regard mathematics teachers should have both PCK and MKT. This is because these bodies of knowledge help teachers make informed decision such as: sequencing of the subject matter and teaching activities such that learning becomes accessible; the teachability of the topic; identifying and rectifying learners' difficulties and misconceptions; and selecting appropriate assessment strategies that will facilitate learning.

Teachers are also advised on how to teach fractions, that is the five subconstructs pertaining to fractions were discussed. It is said that teachers who teach all these constructs and help learners identify the relationships are embarking on effective teaching.

Lesson plan and learners' mathematical exercise books are discussed as important documents in this study. Lesson plan books indicate the type of preparation that teachers do. That is, type of activities and questions they intend to give their learners. Learners' mathematics exercise books on the other hand indicate the layout and strategies that learners use when solving problems.

In the next chapter describes the research methodology that will be used in this study.

CHAPTER 4

RESEARCH METHODOLOGY

4.1 INTRODUCTION

The purpose of this study as was stated in chapter 1 is to explore and assess how primary school mathematics teachers teach fractions and how students learn fractions. In Chapters 2 and 3 literature on the effective teaching and learning of mathematics with emphasis on fractions was reviewed. This chapter, on methodology, will explicate how empirical data was collected that helped fulfil the purpose of this study.

4.2 RESEARCH METHODOLOGY

Methodology is defined by Silverman (2000:13) as “a general approach to studying research topics.” Wagner and Okeke (2009:69) indicate that research methodology is determined by the epistemological underpinning of the researcher. In chapter 1 it was communicated that constructivism, a branch of the interpretive approach (Nieuwenhuis 2009:58), is the epistemological underpinning of this study in which the phenomenological approach will be employed. The phenomenon of this study is teaching and learning fractions. The research intends to give a descriptive report regarding these phenomena. Since phenomenologists believe that phenomenology is the study that, “wants us to see clearly and to describe accurately what we see, before we start explaining in a scientific manner what we have seen” (Higgs 1995:9). This study is therefore qualitative in nature.

4.3 QUALITATIVE RESEARCH

The theoretical perspective of interpretivism sees the world as too complex to reduce it to observable laws (Gray 2004:31). This is because the behaviour of human beings consists of the physical actions that we see and the reasons or the meanings attached to such actions. Hence to understand these behaviours fully the observer should understand the reasoning that human beings attach to their actions (Carr & Kemmis 1986:88). The observer can understand by doing some form of interpretation.

It was mentioned in chapter 1 that this research study is intended to get insight and understanding of the strategies teachers use to teach fractions and how learners learn fractions. In order to achieve this, the researcher will observe and interpret both the teachers and learners actions. Nieuwenhuis (2007:50) together with Wagner and Okeke (2009:63) describe qualitative studies as those that attempt to get rich descriptive data regarding a phenomenon under study. So this study intends to do that hence it is qualitative. What teachers and learners do in their classrooms is sometimes determined by other factors, so the researcher entered the situation without predetermined knowledge but tried to understand what teachers and learners do from their point of view.

Invankova, Creswell and Clark (2007:257) state the following characteristics of qualitative studies:

- the research questions are broad and general
- seek to understand participants' experiences with the central phenomenon

The selected sample is small and purposeful.

With regard to the above characteristics of qualitative research, Gray (2004:320) adds that qualitative research:

- be conducted through intense contact within a "field" or real life setting
- main focus is to understand the ways in which participants (teachers) act (teach) and account for these actions
- supports the researcher's role of getting a "holistic" or integrated overview of the study, including the perceptions of the teachers
- provides data open to multiple interpretations
- allows participants (teachers) to review the themes emerging from the data
- allows for the simultaneous data collection and analysis but not necessarily analysis occurring sequentially after data collection

With regard to the above characteristics, this is a phenomenological study because the phenomenon under study is studied intensely in its real setting.

4.4 A PHENOMENOLOGICAL STUDY

The phenomenon in this study is teaching and learning fractions. Phenomenological research is a complicated process since the researcher has her own conceptions which she brings to the study (Smith & Eatough 2007:38). This is true because in this case the researcher is a Mathematics teacher and she has her own understanding and experiences regarding the phenomenon under study. The authors purport this as they state that the researcher’s conceptions help her to make sense of the participants’ worlds’ view through a process of interpretative activity. The purpose of phenomenology is not to theorise but to describe what all participants have in common as they experience the phenomenon. In this research therefore the researcher works from the participants’ specific statements and experiences rather than abstracting from their statements to construct a model (Lichtman 2011:243). In the words of Berglund (2007:76) “the goal of phenomenological methods is to study the meaning of phenomena, and human experiences in specific situations…”

In this study the researcher, aspires to determine a view into the teachers and learners’ life worlds, which is their teaching and learning of fractions respectively, and how learners learn fractions (cf.1.3; Johnson & Christensen 2012:383). Therefore this is a phenomenological study.

In order to obtain this view the researcher laid aside her understanding of the phenomena and opened up for new meanings by focusing on what she is experiencing at that moment. This she did so that she could get the subjective experiences of the teachers and learners (Gray 2004:21-22; Johnson & Christensen 2012:383-384).

4.5 DATA COLLECTING STRATEGIES

4.5.1 Observations

Observation as a qualitative data gathering technique is defined as “a systematic process of recording the behavioural patterns of participants, objects and occurrences without necessarily questioning or communicating with them” (Nieuwenhuis 2007:83). Observations can be done by watching and/or listening (Thomas 1998:136). There are three types of observation namely, casual, participant and systematic (Dyer 2006:80-81). Dyer (2006:80-81) indicates that in systematic observations the researcher is detached from the subjects and data collection is systematically organized, whereas in casual and participant observation, the researcher is fully involved in the social activities and data collection is not organized, the observer acts like one of the subjects. This study therefore followed the systematic observation as the researcher took the role of the detached observer (Nieuwenhuis 2007:83-87).

The researcher sat in front of the class, facing in such a way that most of the learners' faces, together with the teacher, could be seen throughout the lesson.

A voice recorder was used to record the classroom discourse. Together with the voice recorder an observation guide was used to supplement the recorded information by noting non-verbal cues such as facial expressions of both teachers and learners, or any other behaviour that may be of value during data analysis and interpretation (Nieuwenhuis 2007:86). The observation sheet used is as shown in appendix B.

The observation guide defines the behaviours of interest. Dyer (2006:89) warned that pre-determined behaviours should be defined in such a way that they can be easily identified and not confused with other behaviours. Since in a normal classroom even those behaviours which are of no concern of the study will occur, the observation form was as exhaustive as possible. Maree (2007:86) warns that excluding other behaviours which might be of value may create problems during data interpretations. He maintains that it is better to have excess observation records than to have inadequate. In addition to the predetermined observations, other behaviours that occurred during the observations, like late coming, were also recorded as they may add value during interpretations. Some of the behaviours observed were:

- how the lesson was introduced, (do teachers remind the learners about the previous lesson, if they do how: do they give questions, ask learners to explain what they learned etc)
- teaching strategies used during the lesson, (questioning, demonstration, problem solving, discussions etc)
- types of problems given to learners (real life, abstract non-relevant)

- how teachers respond to learners difficulties; and
- the time given to learners to execute the given task

One of the disadvantages of observations is that, if people who understand what is happening are observed, like the teachers in this study, they may change their behaviours and behave in a way they perceive desirable (Dyer 2006:94-95). As mentioned a voice recorder was used to help minimize the cues that may signal that observations are in progress (Dyer 2006:4-95).

In school A, observations were done over a period of eight days while in school B they were done for five days in one class and for 11 days in the other class.

The main focus of this study is to get a deep understanding of how teachers teach fractions and how learners learn fractions, the researcher felt it is not enough to just observe classrooms. She went further to interview teachers as an attempt to get teachers' perspectives regarding the actions that transpired in their respective classrooms. She also collected the learners' mathematics exercise books in order to determine the types of questions given to them, how they answered them and the type of feedback that teachers gave to learners.

The next section explains how interviews were conducted.

4.5.2 Interviews

Interview as a qualitative data collecting technique is defined as "attempts to understand the world from the participant's point of view, to unfold the meaning of people's experiences and to uncover their lived world prior to scientific explanations" (Greeff 2002:292). It is discussed in 4.3 that the distinct feature of human beings' actions is the motive and intention in performing the action. Such actions become intelligible to others when they understand the meaning attached. The purpose of supplementing observations with interviews is therefore to find out what is in someone else's mind in order to minimize subjectivity in the study (Best & Kahn 2003: 255). Greeff (2002:292) and Lankshear and Knobel (2004:198) warn that if observations are not followed by interviews there is a danger of making assumptions regarding people's behaviours.

Each of the three teachers was interviewed and all of their lessons, on fractions, were observed. Each teacher was given chance to determine the time that would be convenient to have an interview. They all chose to be interviewed after school when everybody had gone home so that we would not be disturbed. The interviews were semi-structured in that, there were predetermined themes the interviewer intended to cover (Dyer 2006:32; Wellington 2000:71-80). The topics mapped the interview but as the interview proceeded the questioning were guided by the content of the respondent's answers (Dyer 2006:32). The researcher was therefore able to probe and hence the interviewees were able to clarify themselves and make it possible for the researcher to get a deeper understanding of their view on teaching fractions. This is because one of the characteristics of qualitative research is that the researcher should neither be objective nor detached, but should be engaged (Greeff 2002:299). Interview questions for class teachers and teacher trainers are attached as appendix C and D respectively.

Interview sessions were voice recorded and supplemented with notes on non-verbal cues. The use of a recorder helped the researcher to maintain eye contact and show interest in the person being interviewed (Dyer 2006:40; Lankshear & Knobel 2004:199; Nieuwenhuis 2007:89). The authors indicate that if eye contact is maintained the researcher can get the respondent's feelings and emotions even without them being said.

Some of the precautions the researcher took were to open up and show interest in what the interviewees say and asked more questions to elicit more information. This helped in determining if the interviewees were just trying to be smart or they are being authentic (Lankshear & Knobel 2004:198).

Teachers' choice of pedagogical actions is influenced by their beliefs, attitudes and the type of training they got while being trained as teachers. Some of them model themselves on their lecturers at teacher training institutions while others teach the way they were taught while they were in primary and high school. So to get more insight into their choice of pedagogical practices, Mathematics lecturers from the Lesotho College of Education (LEC) were interviewed.

In this regard lecturers were asked to explain to the researcher the level of training given to these teachers that would enable them to teach Mathematics. That is, were they ever given a chance to demonstrate their understanding of mathematical concepts they would teach at the end of training or to demonstrate how they would use different teaching strategies, such as problem solving and/or group work to teach mathematics and fractions in particular.

The interviews were also voice recorded and supplemented with notes to capture those cues which could not be recorded.

4.5.3 Document analysis

Observations and interviews such as data gathering techniques are classified as interactive or obtrusive measures (Gray 2004:263). Document analysis on the other hand is a non-obtrusive strategy for obtaining qualitative data with little or no reciprocity between the researcher and the participant (McMillan & Schumacher 2001:451). Documents, among others, are organizational records, personal records and school records. The documents of concern in this study were teachers' lesson plans and learners' mathematics exercise books. These documents were viewed as important in this study because, as discussed in section 3.8, lesson plans and learners exercise books enhance effective learning.

Punch (2009:158) maintains that documents are rich sources of data and may be collected in conjunction with observations and interviews.

4.5.3.1 Teachers' lesson plans

From these documents the researcher determined how teachers prepared for their mathematics lessons. Among other things the researcher wanted to find out whether teachers wrote lesson plans before teaching. If they did, then what is it that they write in their lesson plans? The contents of the teachers' lesson plans will be studied in relation to the contents stipulated in section 3.8 as:

- topic
- date
- objectives
- methods

- procedure
- list of materials needed
- potential problems and pitfalls
- methods of evaluation

Apart from the contents of a lesson plan effective ways of teaching mathematics were discussed in chapter 3, so from lesson plans the following information will be sought:

- teachers' anticipation of learners' mistakes and difficulties regarding the concept to be taught
- pre-determination of problems to be solved to elicit learners understanding
- whether the types of problems used are derived from learners' contexts or not

4.5.3.2 Learners mathematics exercise books

From these documents, information regarding effective teaching of mathematics will be depicted. That include:

- how learners layout their work, that is, whether their work is neat and easy to read
- the use of learners' own strategies when solving problems enhances conceptual knowledge hence effective learning (cf. 2.5.3.1)
- whether learners use algorithms all the time when they solve problems. When this is done learners acquire procedural knowledge which promotes procedural knowledge (cf. 2.5.3.1)
- whether they use their own methods. It is discussed in section 2.5.2.3 that through experiential learning, learners come up with their own methods of solving mathematics problems

4.6 POPULATION, SAMPLE AND SAMPLING TECHNIQUES

4.6.1 The population

Population is defined as the total set of individuals or target group from which the individuals of the study are chosen and on which to generalize and apply the research findings (Strydom & Venter 2002:198). In this study the population refers to all 11 to 12 year old pupils in primary schools in Morija (in Maseru district), primary Mathematics teachers in the district of Maseru Lesotho and Mathematics teacher trainers at the Lesotho College of Education (LCE).

Maseru is a large district of 4279 square kilometres, so it is not feasible to study the entire population. The researcher therefore selected a small proportion of the population especially as the study is qualitative in nature and the aim of this study is not to generalize but to get an understanding of how fractions are taught and how learners construct understanding.

The next section describes the sampling techniques used in this study.

4.6.2 The sample and sampling techniques

Sampling is defined by Nieuwenhuis (2007:79) as a small proportion of the population of study which has all the characteristics of the population. The researcher did this study and at the same time she was a fulltime teacher, so studying the entire population is not feasible due to the constraints of time, accessibility and finances (Nieuwenhuis 2007:80).

Purposive sampling of the participants was employed. The majority of Primary school teachers in Lesotho are trained at the Lesotho College of Education and those who further their studies often do so with the National University of Lesotho (NUL). In the recent years some cross the borders of Lesotho and train with the University of Free State (UFS).

Initially the researcher intended to study three teachers who trained with the above institutions so that comparison could be made regarding how they taught fractions.

However this was not possible because standard six teachers in the schools which the researcher was able to observe trained at the LCE.

In this regard the participants comprise three primary Mathematics teachers teaching standard six (known to be grade eight in other countries such as South Africa). The researcher's place of residence is Morija, therefore primary schools around Morija participated in this study. All the learners in these three classrooms and two mathematics teacher trainers both from LCE formed part of this study. I did not find it relevant anymore to interview lecturers from NUL because none of the observed teachers studied at NUL.

Silverman (2000:105) purports that purposive and theoretical sampling are often treated as synonyms. The author cites three features of theoretical sampling as:

- choosing cases in terms of our theory
- choosing deviant cases
- changing the size of our sample during the research

The researcher's decision to study learners in standard six was based on the theory that they were at the end of the concrete operational stage and were moving towards the beginning of the formal operational stage of cognitive development (cf. 2.3.1.3 and 2.3.1.4). Children at this stage are expected to begin to reason inductively and deductively.

The investigations, in the form of fieldwork, took place in three classrooms in two schools. Two of the classrooms are in the same school but they were observed at different times. All three were standard six classes. These three cases provide an opportunity to examine and understand in depth the phenomenon in a classroom setting and determine how feasible information about teaching and learning of fractions from the literature translates into existing classroom practice (McMillan & Schumagher 2001:398).

In school A the researcher took eight days without interruptions. In the first class in school B she also took five days without interruptions. In the second classroom she took overall 11 days. After seven days we were interrupted for one day by the debating activity which was held one Friday. The last four days went without interruptions. Details of the context in which the teaching took place were recorded, including information about the physical environment and social factors that were relevant to the situation (Punch 2009:119).

Interviewing mathematics teacher trainers from LCE provided information on whether teachers practiced the training they got or whether their teaching was mainly influenced by their beliefs, environment, and culture of the schools or other factors. From these interviews the researcher gained a deeper understanding of the type of content matter, pedagogical content knowledge and the attitudes of teachers which helped them develop into becoming effective teachers.

4.7 PILOT STUDY

Pilot studies are referred to as either small scale feasibility studies done to prepare for the major study or a study done to test a particular research instrument (Gall, Borg & Gall 1996:65; Van Teijlingen & Hundley 2001). In this study piloting was done with both aims of testing the feasibility of the study and testing the research instruments.

The pilot study was done in a school with the same environment and socio cultural background as those two included in this research study. This helped the researcher improve on the skills she needed to conduct the major study; these include observation and interviewing skills together with document analysis. In so doing the researcher gained confidence and hence embarked on the major study with a positive attitude and higher level of competence. These views are supported by Van Teijlingen and Hundley (2001) as they state that one advantage of pilot studies is to train researchers about different elements of the research process.

After conducting the preliminary study and having identified problems, adjustments were done accordingly. Some of them included the focus of observations and structuring of the interview questions.

4.8 VALIDITY AND RELIABILITY

Creswell (2008:266) as well as Whittemore, Chase and Mandle (2001: 528-529) warn qualitative researchers that throughout the entire research process they should make sure that the findings and interpretations are accurate. Whittemore *et al.* (2001:528-529) state the criteria that should be used by qualitative researchers to maximise quality of their research studies. These are credibility, authenticity, sensitivity and creativity, just to mention a few.

Care was therefore taken in this study to be accurate while reporting on the findings and interpretations. The researcher supplemented the findings and interpretations with verbatim statements of the participants. The researcher was also conscious of the conceptions she brought to the study and that her involvement in the research process could hinder the participants to speak authentically for themselves. The precautions that the researcher took were according to Whittemore *et al.* (2001:529) are important to increase credibility and authenticity of qualitative research.

Criticality and integrity of qualitative research as discussed by Whittemore *et al.* (2001:531) refers to whether the researcher worked repeatedly with data and every time tried to find different and/or multiple understanding. The researcher therefore followed this procedure because she engaged with data continuously. She engaged with it individually and collaboratively. At the end of the observation she asked observed teachers to each check on the transcriptions to verify their accuracy. She also asked her study leader to reflect with her on the raw and coded data. This is referred to by (Creswell 2008:267) as member checking and it is another strategy used to ensure validity of qualitative studies.

Gray (2004:90) on the other hand states that to ensure validity, research instruments must measure what they are intended to measure. In this regard Hopkins (1980:45) defines validity as the degree to which a research instrument accurately describes the attribute being observed. It also ensures that collected data is relevant to research (Birly & Moreland 1998:41).

Observations pose the danger of producing invalid results (Gray 2004:256). This is because when observing one event, two people may come up with different interpretations. So this indicates that observations, as data gathering instruments,

are subjective. To reduce this error and hence increase the credibility and authenticity of data and findings, triangulation of data collecting instruments was used (Creswell 2008:266; Golafshani 2003:603; Nieuwenhuys 2007:113). That is, observed data was supplemented with interviews with the teachers who presented the authenticity of their behaviour and actions in their respective classrooms.

Triangulation of data collecting instruments can be upheld as a claim for validity. The reason being that data and research finding becomes credible if data collected by one instrument is confirmed by data collected from another instrument.

When designing interview schedules, the researcher ensured that the content of the interview questions directly concentrates on the research objectives (Gray 2004:218-219). Gray further suggests to researchers to do the following in order to increase validity of interviews:

- using interview techniques that build rapport and trust, thus giving informants the scope to express themselves
- prompting interviewees to illustrate and expand on their initial responses
- ensuring that the interview process is sufficiently long for subjects to be explored in depth
- constructing interview schedules that contain questions drawn from the literature and from pilot work with respondents

All the points suggested by Gray were taken care of because all teachers observed are known by the researcher as they work in the neighbouring schools and one of them is the researchers' former student. So a good rapport was established as all seemed to be at ease during observations and interviews. Interview questions were also drawn from the literature.

It is mentioned above that the researcher was in direct contact with data throughout the process. So during data analysis the researcher constantly inspected and compared all data fragments that arose in a single case. The data from one case was further compared with data from the other two cases. This validating method is referred to as constant comparative method by Silverman (2000:179). Apart from this the number of instances a category emerged from data was recorded as this was believed to provide grounds for generating conclusions. This is also as depicted by Silverman (2000:184-185)

The steps taken to ensure validity of the research study according to Lincoln and Guba in Golafshani (2003:601-602) also maintains reliability as they state that "Since there can be no validity without reliability, a demonstration of the former [validity] is sufficient to establish the latter [reliability]".

4.9 DATA ANALYSIS AND INTERPRETATIONS

Notes on teachers' planning and learners' workbooks; transcriptions of observations and interviews form data to be analysed. The number of learners in each class, their age distribution and number of repeaters; age of the teacher, highest qualifications, years of teaching experience, their qualifications and the set-up of the class formed the basis for the description (Gray 2004). In qualitative research data collection and analysis are done simultaneously. At the research site, the researcher took notes of the events that happened as she thought some might be helpful during data analysis.

One teacher was observed at a time, so this gave the researcher enough time to work with one set of data. Data from each teacher was filed and put aside while the next teacher was observed. The data that is referred to here are; field notes, lesson and interview transcripts, teachers' and learners' documents from the same school.

The next phase was to use a word processor to transcribe the audio recorded lessons (de Vos 2002:347; Gray 2004:331; Nieuwenhuis 2007:108). When typing even the non-verbal cues like silence, laughter and words such as "*er"… "well"…* were included as they portray, for instance, uncertainties and will add value during data interpretation (Nieuwenhuis 2007:108-110).

After transcribing the observations and interviews, the researcher then read the field notes, listened to the tape and read the transcripts over and over so that she became familiar with the data. She further wrote impressions she had as she went through the data which Nieuwenhuis (2007:104-105) referred to as a reflective journal.

The process of analysis and interpretation was done based on the six steps as depicted by Radnor (2002:71). These wre:

- topic ordering; which is listing topics that appear on reading the whole text
- constructing categories; some of which were derived from literature and some as explicated by data

- reading for content; highlighting and attaching codes in the text
- completing the coding sheet; under each category, the codes from all teachers were written in order to give the researcher an idea of how much data there was for each category
- generating coded transcripts; copied and pasted chunks of data to appropriate categories in the coding sheet
- analysis and interpretation; written statements that supported and summarised the findings within each category

To address steps one and two the researcher drew a table, as shown in table 4.1, in which she filled in the topics, codes and categories (Nieuwenhuis 2004:108; Radnor 2002:85) as derived from literature discussed in chapters two and three.

Table 4.1 Coding table

Topics	Code	Category
Effective learning	EL	Manipulation of concrete materials Using learners' prior knowledge Conceptual knowledge before procedural knowledge Learning environment Group work
Effective teaching	ET	Problem solving Inductive reasoning Deductive reasoning
Teachers'' PCK and MKT	TK	Analysing learners' mistakes Sequencing of subtopics Multiple representations
Traditional teaching strategies	TTS	Rules before concept Drill and practice of rules Lecture

The researcher was flexible enough to remain open and include other themes as they emerged in data during data analysis (Mertens 2010:428). She did not only focus on identifying themes from literature.

She once again read the transcripts for content, by highlighting and attaching codes to chunks of data. The convention used here was such that EL 2a represents the topic effective learning (EL), category number two (using prior knowledge) appeared for the first time (a) (Radnor 2002:71-90). These were then filled in the coding sheet so as to give the researcher an idea of the amount of data there was for each category.

Then coded data, for each topic, was copied and pasted on a master assessment file (Radnor 2002:80). This enabled the researcher to have access of data from all three teachers at the same time so that analysis could be done with ease.

After arranging data in this manner, the researcher wrote statements that summarise the findings within each category as interpreted by the researcher. It is worth noting that phenomenologist do not assume that individuals are completely unique, in this regard the researcher not only focused on the differences between the teachers' teaching approaches but also looked for the commonality of experience (Johnson & Christensen 2012:385).

4.10 ETHICAL ISSUES

The researcher had the responsibility to adhere to the ethical issues. Before doing the study, first the researcher sought permission to do the study in the schools from the Ministry of Education and Training (MoET). When she had permission she approached the principals of the schools and informed them of the purpose of the study. The concerned teachers were approached, told the purpose of the study and they willingly agreed to participate in this study. Teachers who willingly accepted to participate in this study also signed a consent form which stated clearly the purpose of the investigation (Kvale 1996:112).

Confidentiality of the identity of teachers, learners and schools and results was observed. This was done by labelling schools by letters A, B and C and identifying learners as learner 1, learner 2, teachers as 'Teacher A, Teacher B and Teacher C'.

Where names were mentioned the researcher used fictitious names (Kvale 1996:112; Radnor 2002:34-35).

While doing class observations, the researcher did not interfere with the class timetables, the normal running of the schools or the personal activities of the teachers. She did class observations at the timetabled time and did not ask for favours that would suit her schedule.

During class observations the researcher sat at the front of the class where she could not interfere with the smooth running of the class but where she had a good view of both the teacher and learners. When learners were given class work, the researcher went around with the purpose of looking at how the learners' did their work.

4.11 CONCLUSION

This chapter dealt with the research design followed in this study which helped to answer the research questions as stated in chapter one. In this chapter the research methodology that was followed while conducting the study was discussed together with the rationale for following the phenomenology as the research approach.

The study followed a qualitative approach in which class observations, teachers and teacher trainers' were interviewed. Interviews were conducted in order to supplement information gathered through class observation teachers' documents and learners', mathematics books.

The population and the sampling techniques employed were discussed. The data collecting strategies used in this study, which were observations, interviews and document analysis together with their validity, reliability and rationale for using them, were discussed. Data analysis and interpretations together with ethical issues were dealt with.

In the next chapter collected data will be analysed, interpreted and discussed.

CHAPTER 5

ANALYSIS AND INTERPRETATIONS

5.1 INTRODUCTION

In chapter four the empirical research was described as a phenomenological study. It was discussed that teachers were observed in their natural setting as they teach fractions to standard six learners. The purpose of this study was stated in chapter one to assess and explore how mathematics teachers teach fractions and how learners learn fractions.

In chapters three and four of this study, effective methods of learning and teaching mathematics, with emphasis on teaching and learning fractions, were discussed. Expectations regarding what teachers should do to help learners learn effectively were elucidated together with the knowledge base teachers need to possess in order to be effective in this regard.

This chapter therefore, interprets and analyses data collected through class observations, teachers' lesson plans, learners' exercise books, interviews of class teachers and also teacher trainers.

5.2 SCHOOL A

5.2.1 Schools' environment

School A was situated at about 8 kilometres South of where the researcher resided. The school was in a rural village in which the community mainly relied on subsistence farming and the rearing of cattle.

The school was a three block building where each block comprised of three rooms. There was no fence around the school hence the villagers pass freely next to the classes. The first block comprised of standard five, the principal's office, standards six and seven.

The inside walls of the classroom were not clean. They were painted with white and blue paint, but most of the paint on the lower parts of the walls was removed possibly by the rubbing of desks or hard objects. There were neither posters nor charts on the walls except one hand written chart of multiplication tables on the wall.

The area of the classroom was five by four metres and there were 50 students in all. There were two chalk boards, one on the front wall and the other one on the back wall. There were two teachers responsible for standard six, but they taught different subjects, and their tables were placed on either side of the class. The teacher who participated in this study taught mathematics, science and social studies while the other one taught the remaining subjects.

The floor map below represents the sitting arrangement of both teachers and the learners

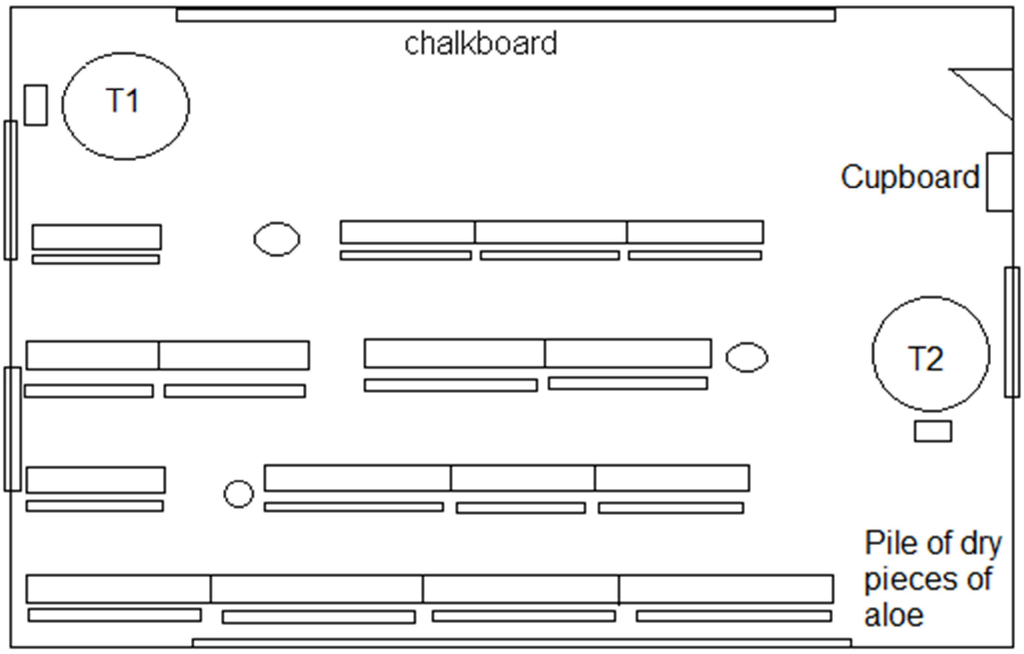


Figure 5.1 Floor map of teacher A’s classroom

T1 and T2 refer to the teachers’ tables while the small rectangles refer to the learners’ desks with chairs. The small circles represent stools made of aloe that were used by some of the learners who did not have desks.

The observer sat at T1’s table. This was the table of the mathematics teacher who participated in this study.

5.2.2 Teacher's profile

Teacher A is a male teacher in the late twenties. He has been teaching for five years. His highest qualification is a three year Diploma in Education Primary.

5.2.3 Class observation

In this section the researcher describes the observations as she conducted them. There were 50 learners in teacher A's class. There were pieces of dry aloe at the back of the class and some of the pieces were sculptured such that they served as stools on which some learners sat. The teacher was involved in a project which involved integrating technology in teaching, and he used aloe in this project.

As mentioned in section 5.2.1 there were two teachers assigned in this class but they taught different subjects. During mathematics lesson the other teacher sat at her table doing other business such as marking or preparing for her next lesson. There was one incidence in which she helped the mathematics teacher to mark class work but most of the time she would be busy with her work.

At the beginning of every lesson of teacher A one learner took the mathematics text books from the cupboard situated behind the door and gave each learner a copy of this book.

Primary school education is free and compulsory in Lesotho and learners do not buy their own stationery and text books. Instead the government supplies schools with stationery and text books. In this school text books are kept at school and they are given to the learners only when they are given work from the books as homework. Schools are not supplied with text books every year, so it is their responsibility to take care of them. Hence, they keep them at school.

As the learner distributed the books the teacher would be writing the date, subject and class on the chalk board.

All the mathematics lessons were in the morning hours, that is, from eight to eleven. A single lesson took forty minutes and a double lesson eighty minutes. There were two single lessons and three double lessons. During each of these morning lessons the teacher would be interrupted by a learner who was late. He never seemed bothered as he often allowed them in without a comment.

The series of his lessons were such that he taught fraction concepts, comparing fractions, equivalent fractions, and addition of both proper and mixed fractions, multiplication and division by a whole number and making fractions. These were taught in eight consecutive lessons.

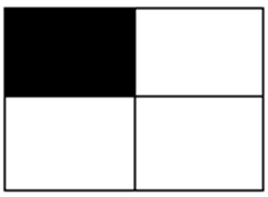
When the teacher reviewed fraction concepts, he introduced the lesson by telling a story about a mother who shared an orange among her four children. He had an orange and was holding it up for all the learners to see. He did not share the actual orange instead he had a chart on which an orange was divided into four parts. Holding this chart he asked learners to name the fraction of an orange that each child would get.

All learners answered one quarter. He also asked how many parts would be left after the mother has given one part to one of the children. The learners all said three quarters.

The teacher wrote both fractions on the chalkboard and then told learners that one of the quarter is the numerator and the four is the denominator. He then wrote the fraction $\frac{1}{2}$ and asked learners to name the numerator and denominator.

The teacher then drew rectangles on the board and shaded part of each rectangle. He asked the learners to name the fraction of the shaded parts. Learners were able to name the fractions and name numerators and denominators.

Towards the end of the lesson the teacher drew this shape



He asked one learner to name the fraction of the shaded part.

The learner said

One third

The teacher

Is it one third?

He called another learner who gave the correct answer which was one quarter. He ended the lesson by giving a written exercise in which the learners repeated the same exercise of writing fractions of shaded parts.

The given diagrams were a partitioned circle, a rectangle and a group of twelve eggs, three of which were coloured black.

The second lesson was about comparing fractions, the teacher introduced the lesson by saying:

> *Today we will compare fractions and before we can do that I want us to look back and remind ourselves about what we did yesterday, we were using the fraction wall or the fraction tables to identify the fractions, so I want someone to remind me these fractions that I'm putting on the board here.*

After doing the oral exercise with four fractions the teacher drew a fraction wall like the one below.

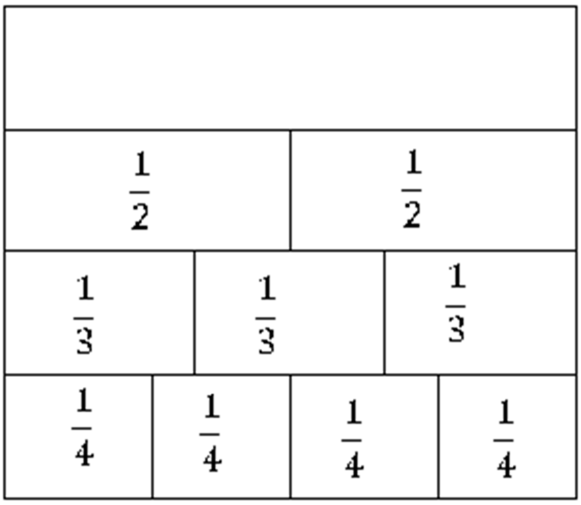


Figure 5.2 (a) Fraction board

After drawing it he informed learners about what they were going to use the fraction wall for.

> *We are going to use the fraction wall and the number line to compare fractions. Do you think these fractions* $\frac{1}{2}$ *and* $\frac{1}{5}$ *are the same?*

Learners: ***(****Together) No sir*

Teacher A: *You are correct. That means the other fraction is bigger than the other, isn't it?*

Learner: *(Together). It is so.*

The teacher asked learners to name the fractions of different sections of the fraction wall, may be to make sure that they understood it.

Teacher A: *What do we have on the first bar of the fraction wall? (Students shout sir, sir!)*

Tumi*: One whole*

Teacher A*: We have one whole right? One whole, that's correct. On the second bar we have?*

Hloni*: Half*

Teacher A*: We have half; I'm not going to ask further questions, I want to ask you this, how many halves make up a whole? That's the question I want to ask. How many halves make up one whole?*

The teacher straight away asked learners to name the number of halves in one whole. The learner who was chosen was able to say

Two halves in one whole.

This is what the teacher said after the response

Teacher A: *That means we must have 2 halves and you remember that when we add fractions we only have to add them when the denominators are?*

Learners*: (Together) Same*

Teacher A*: When denominators are the same. Are the denominators the same?*

Learners*: (Together) Yes*

Teacher A*: Are they the same?*

***Learners**: (Together) Yes*

***Teacher A**: Alright let's get right away, how do we do it? That is… yes Thabisang!*

***Thabisang**: 1+1 is 2*

***Teacher A**: One plus one is two next, yes 'Malebo*

***'Malebo**: Over 2*

***Teacher A**: Over the denominator 2. Do not add the denominators. So we are saying that is 2, let us try to simplify this fraction that is 2 over 2 simplified, yes Katji*

***Katji**: 1 whole*

***Teacher A**: One whole correct that's correct. So this is how we can compare fractions. So you will realize that um… 2 out of 2 is equivalent to 1 that is 1.*

Learners were then told to look at the fraction wall in their text books and find the number of quarters that make up one whole. In order to find the number of quarters in one whole the teacher encouraged them to add the fractions as above.

The teacher gave four sets of fraction slabs to four desks and asked those who did not have the slabs to move to the desks where the fraction slabs were placed. He asked the groups to compare the halves, thirds, quarters, fifths sixths and eighths to one whole. The teacher went from group to group and looked at what they were doing.

After this exercise, the teacher wrote $<, >\text{and}=$ and asked learners to name them. One learner said; *It means greatest.*

The teacher said greater than and then showed half and one fifth slab and asked;

Which is bigger than the other?

***'Malebo**: One fifth.*

***Teacher A**: I would advise you to answer the question looking at the fraction wall but not on this. Between half and one fifth which is bigger? Yes 'Mamo.*

***'Mamo**: One fifth.*

***Teacher A**: One fifth is bigger! Why? Look at the fraction wall not this. Between half and one fifth which one is bigger?*

***Bongi**: One fifth.*

***Teacher A**: One fifth is bigger? He moves the chalk around* $\frac{1}{2}$ *and* $\frac{1}{5}$*and then asks which is bigger? I think if I can rub that people can now get to where I want them to, to drive them to. Half and one fifth which one is bigger?*

***Katji**: Half*

***Teacher A**: Half is bigger. We say* $\frac{1}{2}$ *is greater than*$\frac{1}{5}$. (The teacher moved the chalk around the two fractions) Please ensure that there are spaces before and after fractions (and formulas).

After comparing these set of fractions, $\frac{1}{3}$ *and* $\frac{1}{9}$, $\frac{1}{5}$ *and* $\frac{1}{12}$, the teacher asked learners to come up with the generalisation regarding comparing fractions. He asked:

So what does this tell you? What does this tell you as you compare fractions? Say something about the denominator.

The teacher accepted the answer

The fraction that comes after half is smaller than half.

After this answer he asked:

If you are to choose which orange to take between one eighth and half, which one would you choose? And why would you choose that one?

The learner who responded to the question was able to say half because it is bigger than one eighth.

The teacher assessed the lesson by giving learners an exercise from the text book. The question was to compare two pairs of fractions by writing the appropriate sign between$<$, $>$and$=$.

When teaching equivalent fractions, which was the third lesson, the teacher introduced the lesson by drawing a fraction wall on the chalk board and pointed at the whole slab, half, one third up to the eighths and asked. Each time he pointed, the learners named the fraction in unison.

He then shaded the fraction board as shown below and pointed to the line that divided a whole into two halves.

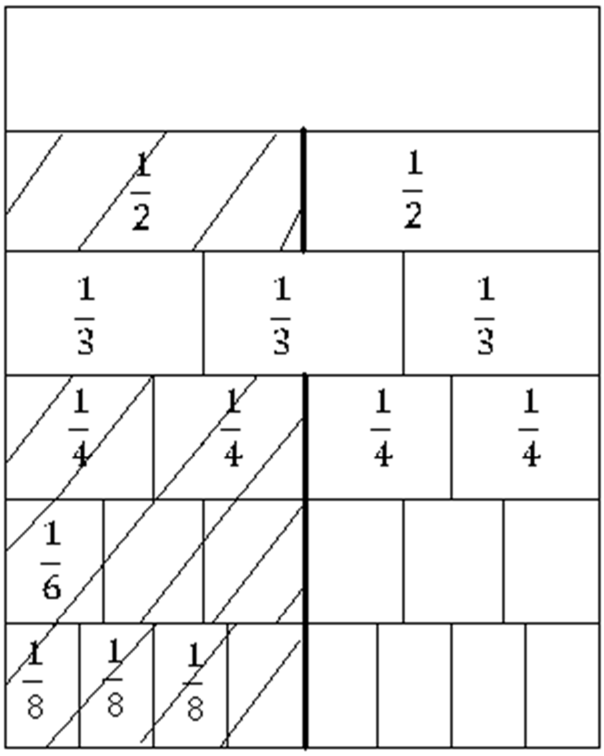


Figure 5.2 (b) Fraction board

He then told the class that

> *This line divides the fraction bar into two halves.*

The learners then named the fractions whose bars shared the same half line (drawn in bold in the diagram above). The learners stated that they are $\frac{1}{4}$, $\frac{1}{6}$, and $\frac{1}{8}$

Having listed the above fractions the teacher then asked learners the following.

> ***Teacher A**: I would like us to compare these 2 fractions, writes* $\frac{1}{2} = \frac{}{6}$ *on the board. But this fraction is incomplete. We have half here and I want us to have*

a fraction that we can say it is equal to this one, and for us to come and find the missing numerator here. I would like you to tell us what we need to do alright? Suppose we want to find the fraction that is equal er… equivalent to half, what do we do? We only have this part here. Ntate (sir)

Makaja: *We take the half over 6*

Teacher A: *Let me get this clear what did we do to get 6 from 2, the denominator 6 from 2 We only have denominators right? We only have one denominator I mean numerator rather and we have another one which is missing so we only have this two which we know so what did we do to 2 so that we get 6?*

'Malebo*: We multiply by 3*

Teacher A*: We multiply 2 by 3 and that will give us 6. Is that true?*

Learners together*: That is true*

From this point the teacher helped learners practise the rule by calculating equivalent fractions and every time that they did it the teacher would be saying:

What we do to the numerator we also do it to the denominator

Having drilled learners on how to calculate equivalent fractions, he asked learners to find the smallest fraction which was equivalent to $\frac{12}{24}$ and $\frac{15}{30}$. These are the responses:

Lala*: 2*

Teacher A: *The fraction, 2 is the whole number*

Lala*: Half*

To check whether it is half the teacher divided the numerator 15 with 15 and the denominator 30 with 15. He ended the lesson by asking learners to complete the statements. He then reminded learners that they should remember the rule of equivalent fractions.

$$\frac{3}{9} = \frac{}{3}$$

$$\frac{5}{20} = \frac{}{3}$$

$$\frac{1}{4} = \frac{}{12}$$

Lesson four was on addition of fractions. This time the teacher did not review the work done in the previous lesson. Instead he asked learners to give him two fractions.

> *Can you, can you just give me any, any fraction, yes*

The learners shouted *half, One quarter*

The teacher told the class that they are going to add fractions and reminded them to remember the rule that they talked about yesterday.

> ***Teacher A**: I would like us to come and add the two fractions, now. (Pauses). Before we add there is one rule of fractions that we talked of yesterday. What is that rule?* A few hands were up and he repeats, *what rule of fractions that we talked of? Yes girl.*

> ***'Mato**: … (*Talks with a very low voice such that the teacher did not hear and he said*)*

> ***Teacher A**: Don't talk to the window but to me*

The student did not seem confident so the teacher called on another to give the answer.

> ***TK**: What you do to the numerator you also do to the denominator*

> ***Teacher A**: That is the rule for the fractions. What we are going to do here which is addition we are going to use that rule, especially when we add fractions that have different denominators, alright?*

***Learners**: (Together) Yes sir*

From this point the teacher asked open questions, explained and demonstrated to the class how they to add fractions and emphasized rules they should use.

***Teacher A**: You cannot add fractions if they have different denominators and that means we have to make those denominators. …That means you have to make the denominators the same. What are the denominators of the fractions that we are adding now? What are the denominators of the two fractions that we are adding now? (Pause as he waits for the hands and call! Yes boy).*

***Tboss**: 2 and 4*

***Teacher A**: Repeats; 2 and 4. You cannot add fractions that have different denominators. …So like when we are, when we are dealing with the equivalent fractions we had to make the two denominators the same, right?*

***Learners**: (In chorus) Yes sir*

***Teacher A**: We had to make them the same. Which is the bigger denominator? Which is the bigger denominator? (Yes girl, Likeleli)*

***Likeleli**: The bigger denominator is 4*

***Teacher A**: The bigger denominator is 4. Now if the bigger denominator is 4 we have to look for means of making the smaller denominator the same as the bigger denominator, alright?*

***Learners**: (Together) Yes sir*

***Teacher A**: And in this case I must explain that for making the smaller denominator being equal to the bigger one we are to multiply the smaller denominator by a certain number or…r see if the smaller denominator can divide the bigger one without having a remainder, ok?*

There are 2 methods; multiply the smaller denominator by a certain number to get the bigger denominator or see if the smaller denominator can divide the bigger one without getting a remainder. Those are the two methods. But I *would like us to go for this one of multiplying. Which number can we multiply 2 so that it becomes 4?*

Learners: *(Shout) sir, sir*

Teacher A*: Calls a student to give the answer*

TK*: We multiply by 2*

Teacher A*: 2 multiplied by 2 equal 4. Correct 2 by 2 correct. Tell me the rule now*

Sir, sir, sir

'Malebo*: 4*

Teacher A: *The rule sir, not the answer. The rule that we must apply,* (Calls another student ntate! *(sir))*

Tboss*: We add. (Others shout sir, indicating that they disagree with him)*

Teacher A*: Do we add?*

Lala*: Whatever we do on the denominator we also do on the numerator*

Teacher A*: Repeats and then asks, what do we do on the denominator? So what do we do to the denominator or what should we do to the denominator and what should we do to the numerator?*

TK: *Multiply 2 by 1*

Teacher A*: We multiply 2 by 1, he writes it*

TK*: And we multiply 2 by 2*

Teacher A*: This is the rule that we have to apply. What ever that we do on the numerators we do to the denominators. As simple as that, and now it goes something like that, so let us try to simplify this, what is 1 by 2?*

Learner 10*: 1 plus 2*

***Teacher A**: It is not plus but it's… (*somebody shout times). *Teacher then says the answer is?*

***Learners**: (Together) 2 times 1*

***Teacher A**: The answer is 2, right*

***Learners**: (Together) Two times two equals four (2 x 2 is 4)*

***Teacher A**: Two times 2 is 4, now here we go, here we go, (*as he writes on the chalkboard) *do you see what we mean?*

***Learners**: (Together) Yes sir*

***Teacher A**: Now we have this now we have this as he writes*

$\frac{1\,x2}{2\,x2}+\frac{1}{4}=\frac{2}{4}+\frac{1}{4}$.*Now the denominators are*

***Learners**: (Together) The same*

***Teacher A**: Ke eona ntho e re e batlang (that is what we were looking for). That's a rule that is a rule, you cannot add the fractions that have different denominators why? So now we can go on adding. What do we say?*

***Teacher A**: Another rule, if the denominators are not the same, you don't add them. Some make the mistake of saying $\frac{2}{4}+\frac{1}{4}=\frac{2+1}{4+4}$. We don't do that ok? We only add the numerators, the top numbers, that is, what we only do. So this is how fractions that have different denominators are being added. Look at the picture, look at the pictures*

***Learners**: (Together) Yes sir*

***Teacher A**: Ah… ah, I would like to put the question here on the board and someone should come and solve it. (He writes $\frac{3}{5}+\frac{1}{10}$ and $\frac{1}{2}+\frac{5}{8}$)*

After demonstrating, he called upon two learners to come to the board and add fractions. They worked out the sums at the same time. They worked quietly and applied the rule they had just learned. They worked them like this;

$\frac{3\text{ x }2}{5\text{ x }2}+\frac{1}{10}=\frac{6}{10}+\frac{1}{10}=\frac{7}{10}$

$$\frac{1 \text{ x } 3}{2 \text{ x } 3} + \frac{3}{6} = \frac{3}{6} + \frac{3}{6} = \frac{6}{6}$$

When they finished working out these two questions he showed the learners how to add fractions with denominators that are not multiples. Like in the first case, he taught the rule.

***Teacher A**: Here is another instance I want you to look at. Let us look at this one now, writes* $\frac{1}{3}+\frac{1}{4}$*. The denominators are 3 and 4 is there any number you can multiply 3 by in order to make it 4*

***Learners**: (Together) No*

***Teacher A:** The second option; can 3 get into 4 without leaving a remainder?*

***Learners**: (Together) No sir*

***Teacher A**: So what is 3 into 4? It goes?*

***Learners**: (In chorus) It goes once and leaves remainder 1*

***Teacher A**: You are right, and the other one is the? That's the remainder I'm talking about. In this case, look at this carefully please; in this case if we try to balance the 2 fractions, since we don't have the number that can multiply 3 to be 4. We try to balance the fractions. Why? By exchanging, exchanging the denominators, you see? (multiplies one third by 4 and one quarter by three.)*

***Learners**: (In chorus) Yes sir.*

***Teacher A:** Through multiplying. That means what we do is this. Here we have the denominator 4. To balance it we bring that denominator to this side.*

He demonstrates on the board

$$\frac{1}{3} + \frac{1}{4}$$

$$= \frac{1 \, x \, 4}{3 \, x \, 4} + \frac{1 \, x \, 3}{4 \, x \, 3}$$

$$= \frac{4}{12} + \frac{3}{12}$$

$=\frac{7}{12}$.

So we apply the rule of fractions, whatever that is done on the numerator we do on the denominator. The bottom line is to get the denominator the same.

I know some will have problems especially (ha se li le peli) when they are two.

You see here we are working with one side, mona re sebetsa (here we work) with two sides. Do you all see? (re a bona). So I want you to help each other in realising how we should treat the sides. That is, under which circumstances do we work with one side? (Na ha re treata side le le leng ke ha denominator li le joang) and under which circumstances do we work with two side? (le hore na ha re treata two sides ke ha re reng). So I want us to work in groups so that we help each other and every one, everybody is going to participate ok?

After teaching addition of simple fractions, the fifth lesson was on addition of mixed fractions.

This was how the lesson transpired:

Teacher A: *(raises up 3 pieces of chalk and says to the class**)** I have this in my hand, now ah… how many pieces are here? Yes boy!*

Sako: *There are 3 pieces*

***Teacher A**: Of what?*

Sako: *There are 3 pieces of chalk.*

Teacher A: *… of chalks. (Breaks the other piece and asks) how many pieces are here? Yes girl!*

Lebo: *There are 2 and half pieces*

Teacher A: *Can somebody come and write 2 and half on the board. Boy*

Sups: *Writes* $2\frac{1}{2}$

***Teacher A**: She has done this, pointing to the fraction written on the board, is he right?*

***Learners in chorus**: Yes sir*

***Teacher A:** Now how many wholes are there in this number that Sups has written? Yees (Did he stretch out the word as he spoke? If so – keep it as such)*

***Learners in chorus**: 2 wholes*

***Teacher A:** 2 wholes. So which is the whole number there? Ntate TK (sir)*

***TK-** 2 is a whole number*

***Teacher A**: 2 is a whole number, 2 is aaa… whole number. Now here (pointing to the fraction on the board) 2 doesn't go alone, what else goes with 2? What else goes together with 2?*

***TK**: Fraction*

***Teacher A**: What is that fraction? Yes man!*

***Katso**: Half*

***Teacher A**: Half is that fraction. So the whole number goes together with a fraction. So this whole number together with the fraction, what do we call them … together? What do you call a fraction and a whole number together? Lebo (girl)*

***Lebo**: Improper fraction*

***Teacher A:** I don't think so*

***Lebo**: With a low voice, a mixed number*

***Teacher A**: Did you hear that over the corner?*

***Learners at the back of class together**: No sir*

***Teacher A**: Speak aloud*

***Lebo**: A mixed number*

Teacher A: *That is a mixed number. That means the numbers are being mixed, a whole number and a…*

Learners: *(Together) fraction*

Teacher A: *As you have seen there were 2 whole pieces of chalks right? And they went together with*

Learners together: *Half*

Teacher A: *(Repeats) half. That means 2 chalks are whole that means they are complete. Which is why we have the 2 here right?*

Learners together: *Yes sir*

Teacher: *And we also have the half part of a piece of chalk and we have here next to the whole number 2 so we say that it is a mixed number. The numbers have been brought together, so we say they are mixed numbers, so now we are going to change this mixed number into another type of fraction which is going to… which is not going to be a mixed number any more. So we have 212 as a mixed number, as a mixed number. Remember, remember here in fractions we work with rules. How many rules do you remember now? Yes man!*

Thisa: *One*

Teacher A: *One! Thisa only remembers one rule. Which is that rule Thisa?*

Thisa: *What you do (teacher interrupts)*

Teacher A: *I can't hear you*

Thisa: *Whatever you do on the denominator you must also do on the numerator*

Teacher A: *(Clears his voice) we are saying that we are now changing the, the mixed number to something else, when we change um… the mixed number to a fraction we multiply the denominator with the whole number and add the whole number, oh… the product with the numerator like this; as he writes what he's saying on the board. A ke re?*

***Learners together**: Yes sir*

***Teacher A**: And the total that you'll get you put it over the denominator (writes $\frac{5}{2}$) the very same denominator. Somebody come to the board and do just that*

Learners shout- sir, sir sir

***Teacher A**: TK*

***TK**: (*As he writes teacher asked him to talk to the class*) talk to us*

***TK:** 2 times 2 is 4 plus 1is 5 (writes 5)*

***Teacher A**: There is TK is he right?*

Some learners said yes some said no

***Teacher A**: Yes?*

Learners together- no, yes

***Teacher A**: Those who say TK is not correct, he is not correct, raise up your hands*

(Learners who said he is not correct raised their hands)

***Teacher A**: It is obvious that those who did not raise their hands say he is correct. (Learners raise their hands shouting sir, sir.)*

***Teacher A:** Yes Mamo (girl)*

***Mamo**: 2 times 2 plus 1 is 5. (She writes $\frac{5}{2}$ and says 2 into 5 goes 2 and leaves remainder 1 (writes $2\frac{1}{2}$)*

***Teacher A**: Repeats, 2 by 2 is*

***Learners in chorus**: Four*

***Teacher A**: And 4 plus 1 that is*

***Learners together**: Five*

Teacher A: *And she puts 5 over here (writes 5) and she said 5 out of 2 (writes $\frac{5}{2}$). So she puts it over 2, and now Mamo has further simplified this er… fraction. She said 2 into 5, she said it goes 2 times (writes it), as a whole number and it leaves 1 out, out of*

Learners in chorus: *Two (says it with the learners. which is the same as the first one. You see? (*Directs them to the first one with an arrow?) *We are saying… what is the name of this type of fraction?*

(No response and the teacher called) *every body… over here… what is the name of this type of fraction (taps on $\frac{5}{2}$ on the chalkboard) (girl)*

Lebo: *Improper fraction*

Teacher A: *We call it improper fraction (*learners said it with them*). You will see that improper fraction has denominator… a…a… it has its numerator being bigger than the denominator. A ke re?(isn't it?)*

Learners together: *Yes*

Teacher A: *Yes. An improper fraction has its numerator being bigger than its denominator. That' an imp…*

Learners finish off the sentence: *...roper fraction*

Teacher A: *Now I would like us to go further. But before we go further, err… we are going to do a very quick piece of work from our text books.*

Very quickly lets do this in your exercise books and I'm going to mark it right now. You change this thing into improper fraction. Change (pauses as he looks around to check whether they are getting started).

The fractions are

a) $2\frac{1}{3}$
b) $1\frac{1}{2}$
c) $3\frac{1}{4}$

> ***Teacher A:*** *After a while he asks one learner, ha eka ha uso qale tje u etsa joang? U etsa joang ha eka hau so qale tje (you have not yet started, why have you not started?)*
>
> *When the denominators are the same, a ke re.*

While other learners worked on the given exercise, others moved up and down the class making a noise. The teacher did not try to stop them, he marked a few learners then stopped for learners to collect the books and put them on his table he would mark them later.

At the beginning of the class, the teacher told the learners that today they were going to add fractions. But they only changed mixed fractions to improper fractions and they did not add them.

Lesson six was on multiplication of fractions. The teacher used the same approach that he used for addition. He began the lesson by asking learners to give a fraction they would multiply by a whole number.

> ***Teacher A****: I would like you to give me just any fraction that, er… you will have to multiply with a whole number, a fraction multiplied with a whole number. Can you just give me that? (Girl)*
>
> ***Lebo:*** *Two times one half.*
>
> ***Teacher A****: Writes* $2 \times \frac{1}{2}$*. (boy)*
>
> ***Sako****: One times one quarter.*
>
> ***Teacher A****: Something like this (writes)*$\frac{1}{4} \times 1$

He then asked one learner to work out $2\times \frac{1}{2}$ *on the chalkboard"*

> ***Mamo-*** *She writes* $\frac{2}{1} \times \frac{1}{2} = \frac{2 \times 1}{1 \times 2} = \frac{2}{2} = 1$ *repeating everything she writes (2 times 1 is 2, and 1 times 2 is 2, 2 into 2 goes once).*
>
> ***Teacher A****: Asks the class why did she write 2 whole as 2 over 1? (that is* $2 = \frac{2}{1}$*)*

Learners: (In unison) *No sir.*

***Teacher A**: Do you want to know.*

Learners: (In unison) *yes sir.*

***Teacher A**: The thing is, let's get this clear; the thing is a whole number can be changed to a fraction, alright? If you have a whole number the thing that you must do is that you must put it over 1 because when you divide that whole number by 1, you are going to get the same whole number, right?*

***Learners**: Together they say, Yes*

***Teacher A:** For instance we have 2 out of 1($\frac{2}{1}$). What is 1 into 2?*

***Learners**: Some say one, some say two.*

***Teacher A**: It goes two times, right. So it takes you back to that whole number. That is why we are putting it over 1. Now somebody come and do number. (*As a student goes to the board he said) *I think you must look at the differences when you are adding and when you are multiplying, alright? You are going to tell the differences. So be very observant as you are watching here. (yes boy)*

Phisa: $\frac{1}{4} \times 1 = \frac{1}{4} \times \frac{1}{1} = \frac{1\times1}{4\times1} = \frac{1}{4}$

The product to the fractions they were multiplying were all proper fractions in their simplified form. He told learners to simplify the answers if they found that they were not in their simplified form.

He performed fraction multiplications whose answer was an improper fraction. Here again he told learners what to do in order to change improper fractions to mixed fractions. When he finished demonstrating, he asked one learner to work out $3 \times \frac{3}{5}$ on the chalk board and this is how he worked it $\frac{3}{1}$x $\frac{3}{5}$= $\frac{3\ x\ 3}{1\ x\ 5}$ = $\frac{9}{5}$ = $2\frac{1}{5}$. Some of the learners were able to see that he made a mistake when changing it to a mixed fraction and raised their hands with an attempt to be given the chance to correct it. So this was what they did

***Lebo**: $\frac{3 \times 3}{1 \times 5}$ = $\frac{9}{5}$ = $1\frac{3}{5}$ (*repeats every thing she does*)*

She quickly realised the mistake and wrote the correct answer *$1\frac{4}{5}$*

Towards the end of the lesson the teacher used the diagram taken from the text book and asked learners the following question

***Teacher A:** How many shapes are there which are whole numbers? How many shapes are there that are whole numbers that are coloured? In the first example, look in the text books you'll see what I'm talking about. How many coloured shapes are there?*

***Lebo**: Two coloured*

***Teacher A**: Where are they? Show me this is one this is two.* (Still Lebo did not know what to say).

***Learner:** There are 5 shapes that are coloured.*

***Teacher A:** There are 5 shapes that are coloured is that so?*

***Learners:** (Together) It is so*

***Teacher A:** So the whole number will be? The whole number will be…?*

***Learners:** (Together) The whole number will be five.*

***Teacher A:** The whole number will be five. So we are going to, we are going to multiply that by half, right?*

***Learners:** (Together) Yes sir.*

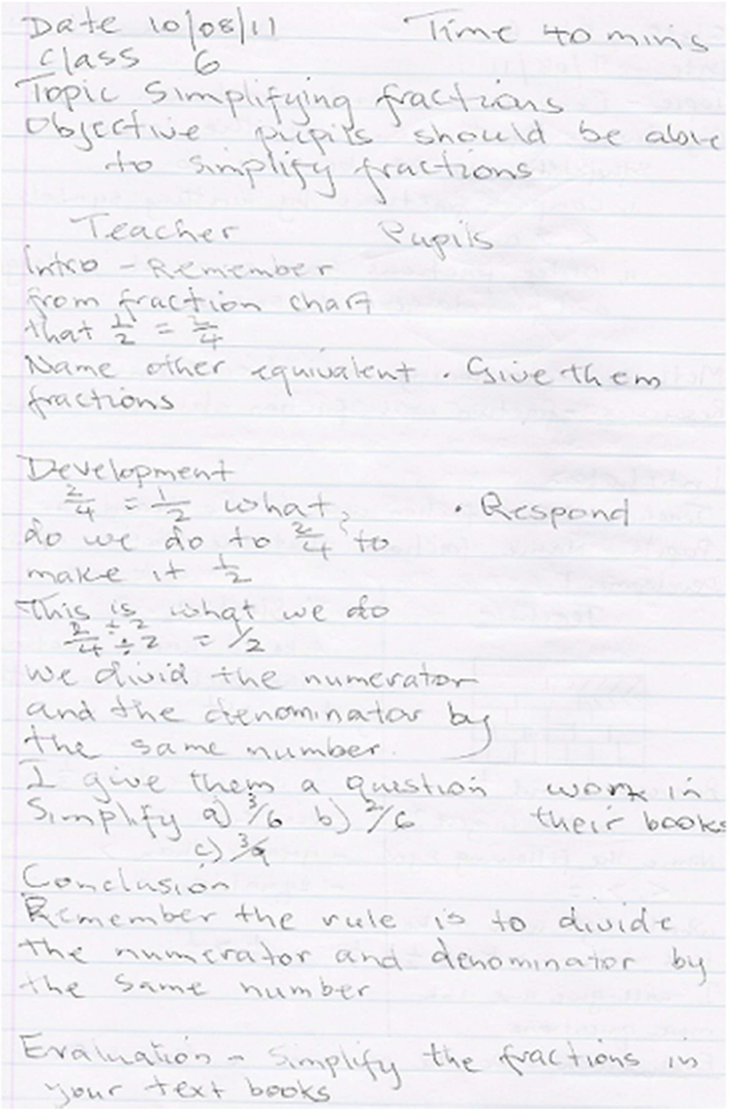
At the end of the lesson learners were given class work to do which required the application of the rule that the teacher had given them. In most cases learners did the work individually except in one instance in which the teacher encouraged learners to help each other understand the rule used to add fractions. This was what he said

> *So I want you to help each other in realising how we should treat the sides ... I want us to work in groups so that we help each other and every one, everybody is going to participate ok?*

When he gave class work the teacher used questions in the text book and he went round, from desk to desk checking whether all were doing the work. After making that round he would stand in front of the class and learners one by one would go to him so that he can mark their work. He was not able to mark the work of all learners, so he collected their books and marked them as the other teacher taught.

5.2.4 Teacher A's lesson plan

It was discussed in section 3.8 that there were documents that teachers kept that helped them organise their teaching. Teacher A's lesson plans had the same format as indicated below.


Date 10/08/11 Time 40 mins
Class 6
Topic Simplifying fractions
Objective pupils should be able to simplify fractions

Teacher	Pupils
Intro – Remember from fraction chart that $\frac{1}{2} = \frac{2}{4}$ Name other equivalent fractions	· Give them
Development $\frac{2}{4} = \frac{1}{2}$ what do we do to $\frac{2}{4}$ to make it $\frac{1}{2}$	· Respond
This is what we do $\frac{2 \div 2}{4 \div 2} = \frac{1}{2}$ we divid the numerator and the denominator by the same number.	
I give them a question Simplify a) $\frac{3}{6}$ b) $\frac{2}{6}$ c) $\frac{3}{9}$	work in their books

Conclusion
Remember the rule is to divide the numerator and denominator by the same number

Evaluation – Simplify the fractions in your text books

Figure 5.3 Teacher A's lesson plan

The structure of his lesson plans was the same for all lessons. The format was such that he wrote the class, date, duration and topic. He then listed the objectives, wrote the materials that would be used and the teaching methods. Under the procedure, he wrote the steps he would follow when teaching, together with the content he would be teaching. In the assessment he wrote a list of questions he would give to the learners at the end of the lesson.

As discussed in section 3.8.1 teacher A wrote short and measurable objectives that could be attained within the duration of the lesson. In the example lesson plan above there was one objective and the duration of the lesson was 40 minutes. He also wrote a list of materials he would use where applicable. Regarding the procedure, he wrote what he would do such as asking questions and/or demonstrating on the chalkboard, but did not indicate potential problems and/or misconceptions that learners were likely to have. He always concluded his lessons by giving learners an assessment task, in the form of closed ended questions which were done individually.

The teacher did not keep the lesson plan book next to him when teaching. As a result he did not use the examples that he had written in the lesson plan but he used similar ones. On the plan he indicated that he would ask learners to identify the numerator and denominator in the fraction $\frac{2}{5}$, in the actual teaching he wrote $\frac{2}{3}$.

5.2.5 Learners' exercise books

Learners divided each page into two columns, in this way the layout of their work was neat and easy to read.

It was indicated in the discussion above that learners used procedures provided by the teacher when answering questions. They were not given a chance to come up with their own strategies. The teacher always provided them with procedures that they should follow when answering questions. The example below indicates what procedure they were given when subtracting fractions and all learners used it.

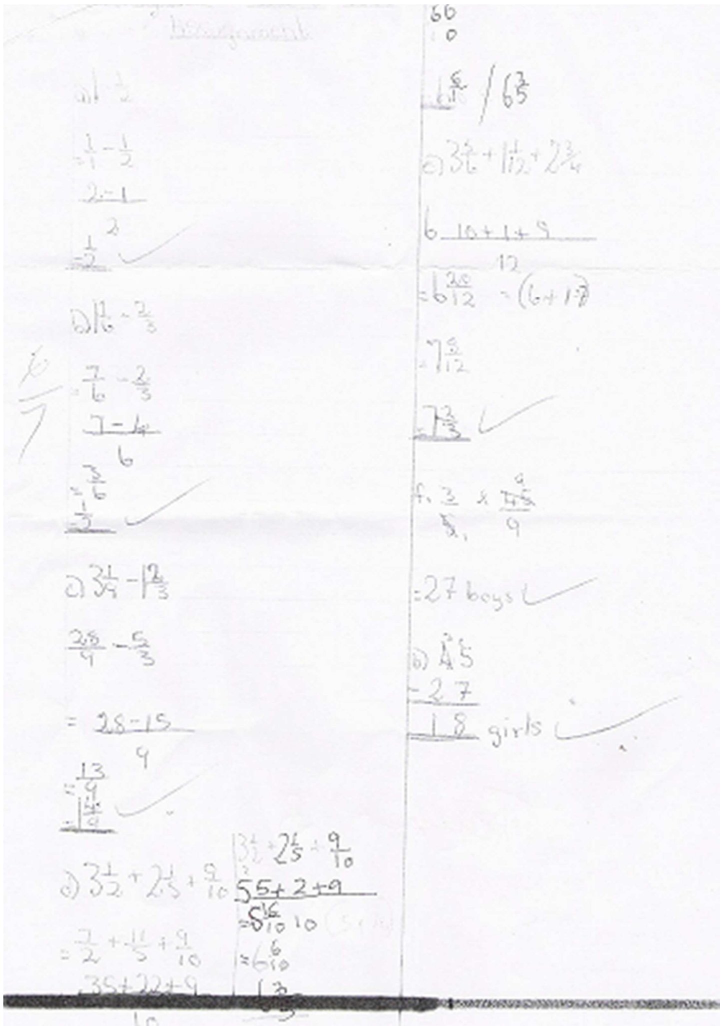


Figure 5.4 Learner's working in teacher A's class

When learners had forgotten the procedure they did not know what to do. There was one incidence in which the learner multiplied like this

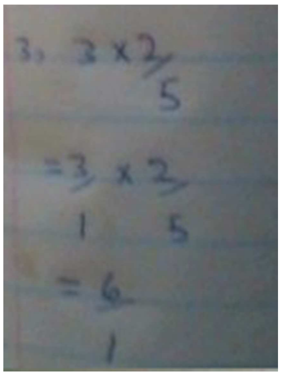


Figure 5.5(a) Errors made by teacher A's learners

When the teacher asked the learner to explain where one, the denominator of six, comes from, he said;

It is this one (pointing to the denominator of three)

Another learner had multiplied like this

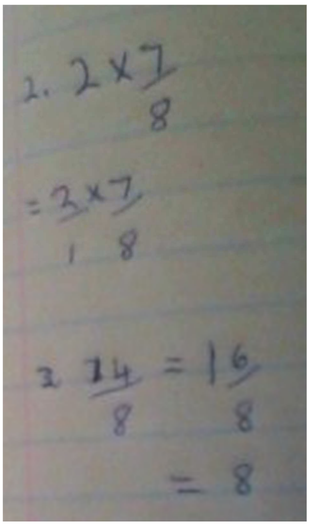


Figure 5.5 (b) Errors made by teacher A's learners

When the teacher asked where the eight comes from, she said:

6 into 6 goes 1 and that 1 times 8 is 8.

From the learner's explanation one deduced that this learner had not fully conceptualised the concept of simplifying fractions and fraction multiplication. The learner was practising the procedure which she did not conceptualize. This could also be that the learner had fragmented learning because the teacher did not help learners understand and relate multiplication of fractions to multiplication of whole numbers. May be this difficulty arose from the fact that throughout the whole observation period, the teacher used was the part-whole representation only. In no single case did he engage learners in the other four subconstructs, namely; ratio, operator, quotient and measure (cf. 3.7.3).

Other than these spotted difficulties those who were said to have understood were those who were able to repeat the procedures.

During the lesson not all learners' work were marked, so the teacher would normally collect the books he had not marked and then mark them as the other teacher taught her subjects. In general the amount of work given to learners was little. During the lessons they were given not more than four questions, and for assignments, which were not always given, learners were given again not more than four questions.

Learners were not given constructive feedback. The teacher assigned a tick and said good to learners who got the questions correct. To those who experienced difficulties, he would say things like, not quite or no we do not do it like that go and correct it.

5.3 SCHOOL B

5.3.1 School's environment

School B is situated about two kilometres from the researcher's place of residence. The school is surrounded by a number of institutions, hence the socio-economic status of the learners' families is diverse as most parents are employed in these institutions.

Unlike in school A, school B has a fence around and villagers do not pass through the school grounds. There is a senior and junior block. The senior block is comprised of standard six and seven. There are three standard six and three standard seven classrooms.

The observed two standard six teachers in this school, but did it in two successive years. The classes were both painted white though the paint was worn out and the walls appeared to have been exposed to rubbing off by hard objects like desks and/or chairs. There were no posters or pictures of any form on the walls.

The first teacher the researcher observed in school B is referred to as teacher B and the second one as teacher C. There were 40 learners in teacher B's class and 39 learners in teacher C's class.

Both classrooms are of the area 5 by 3 meters. The floor map of each of these classrooms, the arrangement of both the teachers' and learners' desks and the position where the observer sat are shown below.

Teacher B’s classroom

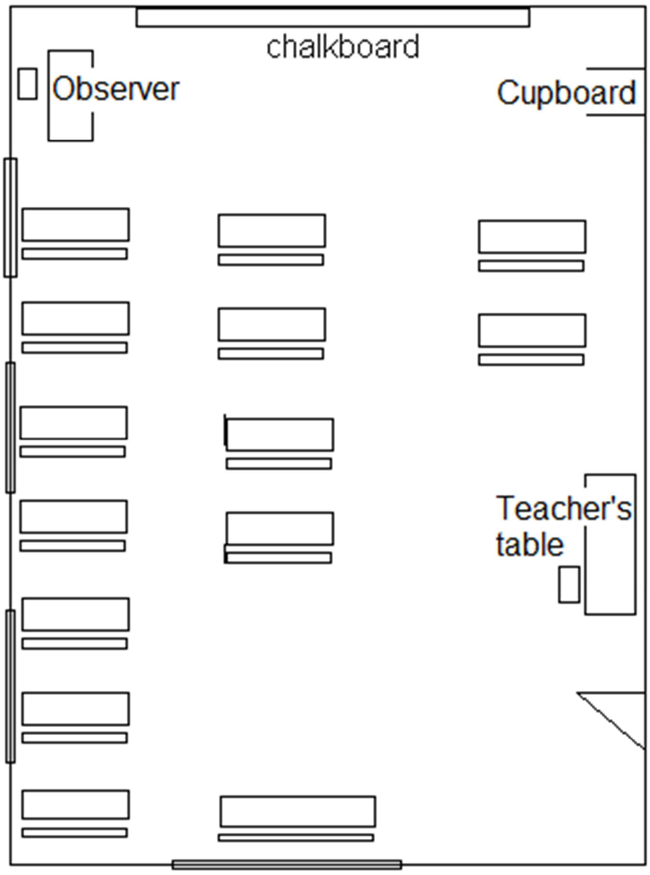


Figure 5.6 Floor map of teacher B’s classroom

The large rectangles are the learners’ desks while the small rectangles represent the chairs. There are two teacher’s desks so the observer used the one which was in front of the class so that she could have a clear view of both the teacher and learners during the lesson.

Teacher C's classroom

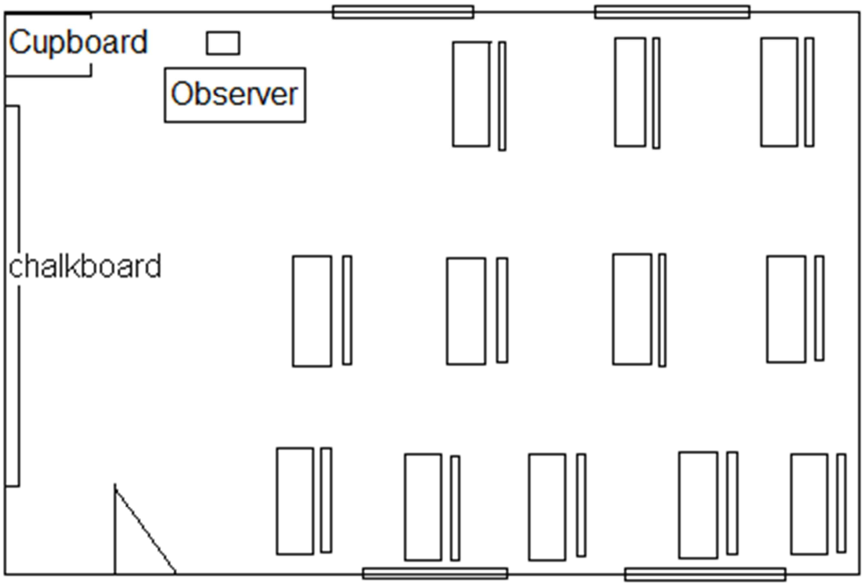


Figure 5.7 Floor map of teacher C's classroom

Large rectangles represent the learners' desks and small rectangles represent the learners' chairs. The observer used the teacher's table during observations.

5.3.2 Teachers' profiles

Teacher B is a female teacher in her early forties and has been teaching for 21 years. Her highest teaching qualification is a three year Diploma in Education Primary.

Teacher C is a female teacher in her early fifties who has been teaching for 17 years. Her highest qualification is also a three year Diploma in Education Primary.

5.3.3 Teacher B

5.3.3.1 Class observations

On the first day of the observation, the mathematics lesson was at eight o'clock in the morning. The researcher arrived just after the morning assembly when learners were getting into their respective classrooms. The teacher greeted the researcher and they entered the classroom before all learners could enter. The researcher had already been to the school before the observation in which she and the class teacher made arrangements as to how they were going to work. The researcher took her place as the learners entered the class.

When they were all seated the teacher introduced the researcher and they greeted her enthusiastically.

There were three class six classrooms and this one that the researcher observed was standard 6B. There were no repeaters this class. They were all doing standard five the previous year.

Unlike teacher A, who taught fractions in eight lessons, teacher B taught fractions in five lessons. The duration was 80 minutes for two lessons and 40 minutes for the other three lessons. The sequence in which she taught fractions was sharing, denominators and numerators, comparing, ordering and equivalent fractions.

Like teacher A all lessons were in the morning hours. The class activities were not very different from those of teacher A. Introductions to the lessons were the same for all lessons. She asked learners to state what was done in the previous lesson and then reviewed the content by asking questions like:

> ***Teacher B:*** *Who can remind us what we have already done yesterday in fractions? Who can tell us what we have already done yesterday in fractions, (Calls a learner by his name)*
>
> ***Lele:*** *We have done numerators and denominators*
>
> ***Teacher B****: We have already done the numerators and the denominators. What did we say the numerators are and what did we say the denominators are? Who can tell us what numerators stand for and what denominators stand for?*

The teacher then linked the previous lesson to the present lesson. The first lesson that I observed was about sharing and its duration was 40 minutes. The teacher used fraction slices (ready-made fraction pieces) to demonstrate sharing. Before that she told a short story about a woman who stayed with her children.

***Teacher B**: Ok, there was an old woman who was just staying in her hut with her children and you know what! The lady was alone in the house so everything she had she shared. She used to share everything she had with her… children.*

She used two learners to show how they would share a fraction slab between two people. This was what transpired in this class.

***Teacher B:** So in life, in real life,* we *are just going to see how we share things and in maths we also learn to… we are going to see how we… share… (Pause then takes out fraction slabs and asks one student to come to the front of the class)*

The teacher gives one student a square fraction slab and then asks

What do we have in here?

***Learners:** (Together) One whole.*

***Teacher B:** We take this as one whole. Should any one come and hold this (fraction slabs representing 1 whole). Can I have two learners to come in front, so the other learner should try to share this (whole slab) into two people but we consider this, to share this whole number into 2 people.*

(The student took two $\frac{1}{2}$ slabs and covered the whole slab with them, then he gave each of the half slabs to the students he is sharing to).

***Teacher B:** So are you aware from this whole number how many parts do we have so far?*

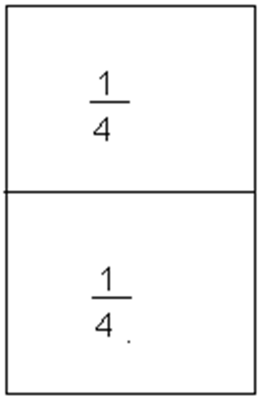
***Learners:** (Together) 2 parts*

***Teacher B**: Let us see when we want to divide this one, suppose we want to share this half to two people. Two people to come and share this one and the other one to come and share that half (two more students came in front took $\frac{1}{4}$ slabs covers $\frac{1}{2}$ slab with 2 of the $\frac{1}{4}$ slabs.*

What can you see and what can you say about this exactly?

***Seth**: In half there are 2 quarters.*

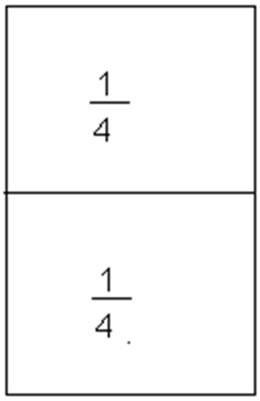
***Teacher B:** In half we have got these two quarters meaning that in a half we have*



***Learners**: (Together) 2 quarters*

***Teacher B**: 2 quarters. May I have another 2 learners to come (they volunteered). So let us divide the second half [the second half] that is held by (calls her by name).*

***Seth**: Takes 2 quarters and place them over the* $\frac{1}{2}$ *slab*



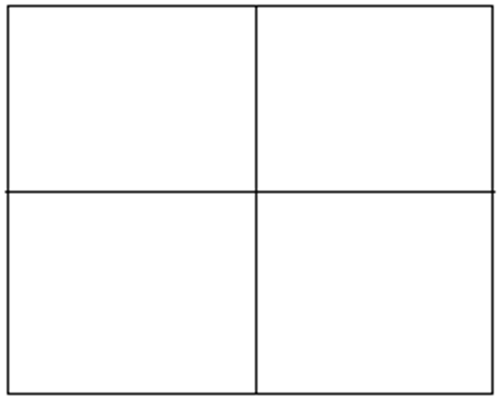
***Teacher B**: How many parts are there? Before you go*

***Seth**: Two parts*

***Teacher B:** Two parts. So let us just take this one half and put them here so that you could see how many parts are there in here*

How many parts do we have (calls him by name)?

***Lele**: 4 parts*



***Teacher B**: Why do we say it's a quarter?*

***Tebo**: (Shouts) we say it is a quarter because there are 4 parts in a whole. This whole has been divided into how many parts?*

***Teacher B:** We say it is a quarter because there are 4 parts in a whole. This whole has been divided into how many parts?*

***Learners**: (Together shout) four*

***Teacher B**: It has been shared to 4 …*

***Learners**: (Together shout) 4 parts*

***Teacher B**: 4 parts. This is why we are saying that it's one quarter, it's one over 4, and this one over four it's a quarter. It has been divided into 4, ok?*

***Learners**: (Together) yes*

***Teacher B**: Thanks go back*

(Learners who were sharing went back to their seats) pauses

***Teacher B**: Let us just go to our exercises and in your exercise books, just draw a whole number and then divide the whole number into 2 equal parts* *then from that one you also divide the same whole number into 4 parts. You do it quickly.*

At the end of the lesson learners were asked to draw a rectangular bar three times. The first one was to be labelled one whole, and then the second bar was to be divided into two equal parts and the third into four equal parts.

Teacher B: *(Calls a learner by his name) Seth go to the chalkboard and show us what the teacher is saying.*

Seth drew the fraction wall from one whole to quarters.

Teacher B*: Let us look at what he is showing and you will see that you can do it within a very short time.*

When he finished Seth turned back trying to go back to his seat but the teacher stops him.

Teacher B*: Go and explain what you have done.*

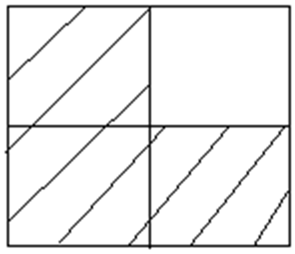
Seth*: These two parts give the whole of this fraction. When we divide this fraction into 2 parts is going to give a half and when you divide the half to 2 equal parts again it's going to be quarters and a half, a whole is equal to 2 halves and 2 halves are equal to 4 quarters.*

Teacher B*: So what do we mean by a fraction? We mean what? We mean parts. In order to make this whole we must have at least 2 parts and we call these 2 equal parts half ok?*

Learners together*: Yes*

In the second lesson the teacher asked what was done in the previous lesson, having stated what they did the teacher then drew a rectangle, divided it into four equal parts and shade three of those. She then asked learners to name the fraction of the shaded part. Learners did not give the fraction straight away, they struggled a little. The teacher then became irritated and raised her voice as she repeats and directs learners to what the question wants. She straight away provided learners with the meaning of the denominator and numerator.

Teacher: *Yesterday we have seen a whole then we divided a whole into two equal parts and then we also divided it into four equal parts. Therefore today let us try to… let us try to look at this diagram on the board*



By looking at this diagram on the chalkboard, who can tell us what fraction err… what part is shaded. What fraction is shaded on this diagram on the chalkboard?

Nka: *3 is shaded*

Teacher B*: Ah…*

Nka*: 3 is shaded*

Teacher B*: Who can correct him, who can add something on what he has already said?*

Tebo*: Three quarters*

Teacher B*: Three quarters, listen to the question, the question says what (raises voice) fraction is shaded, what fraction is shaded? Tebo.*

Teacher B*: This numerator tells us about the parts which are shaded. What about the denominator? Who can say something about the denominator? If the numerator tells us about the number which are shaded what about the denominator?*

Tebo*: The denominator tells us how many of them are there. The total parts*

Fractions were mainly represented by partitioning shapes and mainly rectangles. There was one instance though where she drew a circle and partitioned it unequally. This is the discourse that transpired

Teacher B*: The denominator is dealing with the total parts of the*

***Learners**: (Together) Fraction*

***Teacher B**: Fraction. So from now you should know that the top numbers are called the numerators and the bottom numbers are called the denominators only in fractions.*

Let us have another diagram.

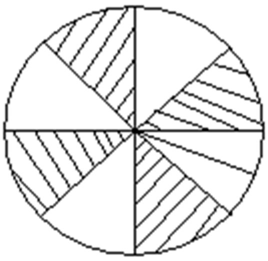


Figure 5.8 (a) Faulty representations that Teacher B allowed

***Teacher B**: How many parts are shaded in this diagram? How many parts are shaded, what fraction is shaded in the second diagram?*

You may even come closer and count, but don't tell others.

***Lele**: Four parts are shaded out of nine*

***Teacher B**: Is he correct?*

***Learners**: (Together) Yes madam*

***Teacher B**: 4 parts are shaded out of nine* $\frac{4}{9}$*. When looking at these two numbers 4 and 9 which one is the numerator and which one is the denominator?*

***Luke**: 4 is the numerator and 9 is the denominator*

***Teacher B**: Ok good. Four is the numerator whereas nine is the denominator (writes the words denominator and numerator). So let me just quickly write these numbers for you so that you show me the numerators and the denominators.*

The teacher concluded the lesson by asking learners to write a list of numerators and denominators from a set of fractions she wrote on the chalk board.

Writes on the board $\frac{1}{2}, \frac{3}{4}, \frac{7}{8}$

i. *write down all the numerators*

ii. *Write down all the denominators*

The teacher marked this work in class. She went around from desk to desk. When she came across one girl, Thato, she found that she had exchanged the numerators with denominators and this is what transpired.

> ***Teacher B****: (Shouting) You must have copied; you wrote them right here and because you copy, you erased your correct answers and wrote the wrong one. Stop copying from others.*

The third lesson was comparison of fractions. In this lesson she asked questions that reviewed the previous day's lesson:

> ***Teacher B:*** *Who can remind us what we have already done yesterday in fraction? Who can tell us what we have already done yesterday in fractions? Tumi!*
>
> ***Tumi:*** *We have done numerators and denominators.*
>
> ***Teacher B:*** *We have already done the numerators and denominators. What did we say the numerators are and what did we say the denominators are? Who can tell us what numerators stand for and what denominators stand for? Famo!*
>
> ***Famo:*** *Denominators mean the bottom number.*
>
> ***Teacher B:*** *But what do they mean? (The tone of her voice increases) Fine numerators are the top numbers. The top number we call it the numerator and the bottom number we call it the denominator but what does numerator tells us? Luke, what does numerator tells us?*
>
> ***Luke:*** *Numerator stands for the shaded parts.*

After reviewing denominators and numerators the teacher stated what they were going to do today.

> ***Teacher B:*** *For now we are just going to count or compare, who can just go to the fraction board and just show us where half should be, where is half?*

The teacher drew the big rectangle and then divided it into four rectangles. She then asked learners to indicate where one whole was, another one was to divide the second rectangle into halves, quarter and eighths.

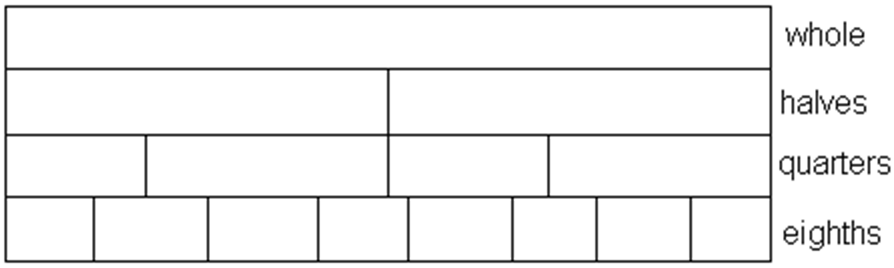
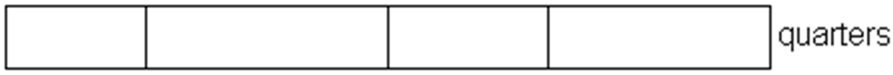


Figure 5.8 (b) Faulty representations that Teacher B allowed

The teacher accepted the partitioning of quarters and eighths as they are demonstrated above. This is how the discussions went.

> ***Teacher B****: Pointing to the first bar of the fraction board, what do we have here?*
>
> ***Lele****: Whole*
>
> ***Teacher B****: This one is a whole, she writes whole next to the bar*
>
> *What about here? Pointing to the second bar*
>
> ***Famo****: Half*
>
> ***Teacher B****: How many halves are there?*
>
> ***Seth****: Two*
>
> ***Teacher B****: Who can come up here and show us quarters on the fraction board*
>
> *One learner goes to the board and divides the third bar into 4 parts*



***Teacher B**: What can you say about these?*

***Lele**: Madamm… they are not equal*

***Teacher B**: They are not equal?*

***Lele**: Yes madam.*

***Teacher B**: But let us talk about the number**.** Even though they are not equal are they not the quarters?*

***Lele**: Yes*

***Teacher B**: Well even though they are not exactly equal are they not the quarters?*

***Learners**: (Together) they are quarters.*

***Teacher B**: Uh… so in this part this part (pointing at the bar of quarter) has been divided into?*

***Learners**: (Together) 4 parts.*

The teacher asked learners to compare fractions using the fraction board which was not partitioned equally. She asked them to count the number of quarters that make up one half and the number of eighths that made three quarters. This was an oral exercise and those learners who answered the questions answered them correctly.

After this oral exercise, the teacher gave an individual exercise in which learners were to complete the following statements to make them true using the fraction wall on the chalk board. The statements were

$\frac{\ }{4} = \frac{1}{2}$

$\frac{\ }{8} = \frac{1}{2}$

$\frac{\ }{4} = 1$

These are solutions from some of the learners

$\frac{2}{8} = \frac{1}{2}$

$\frac{5}{8} = \frac{1}{2}$

$\frac{1}{4} = 1$

$\frac{8}{4} = 1$

This is what the teacher said to the learner who have written ($\frac{5}{8} = \frac{1}{2}$)

> ***Teacher B**: ‘Mantho Let us just come to the fraction board just count five eighths.*
>
> ***‘Mantho**: Counts 1, 2, 3, 4, 5*
>
> ***Teacher B**: Are they equal to eight? Where is half? Show me half, are these equal to half?*
>
> ***‘Mantho**: No madam [with a shaky voice as if she is afraid]*
>
> ***Teacher B:** So you said five eighths are equal to half? Mm… use the fraction board [throwing the students books on the floor.*

The teacher became frustrated and lost her temper. She shouted at learners and threw their books down. While this happened learners laughed at those whom the teacher was shouting at. This is how the teacher wrapped up the activity and the lesson.

> *There are people who did not use the fraction board therefore they come up with the wrong answers. You may find that if ever you use this you could have got the answers correct. Someone said 3 eighths are equal to half, yet the fraction board is already there, people you must always think and use the teaching material which are already on the chalkboard. So we shall see each other tomorrow.*

The fourth lesson was ordering fractions. In this lesson again the teacher drew a fraction board and this time it was partitioned equally.

She directed the learners' attention to half, one third and one quarter parts on the fraction board. She then asked which part they would prefer to have if those were the slice of bread.

> ***Teacher B****: So if they are the slice of bread, which one would you like to take and why? Seli!*
>
> ***Seli****: Half, one half because it is bigger than one third.*

Some of the learners struggled to decide which slice they would prefer. One of the learners said he would take half because it is bigger than one whole.

The teacher ran the chalk around one whole and around half and asked

> *Is this half bigger than this one whole?*

The fraction board was extended to the eighths and the teacher asked learners to compare fractions orally.

A written exercise was to order two sets of fractions starting with the smallest. The fractions were.

1. $\frac{5}{8}, \frac{1}{8}, \frac{8}{8}, \frac{3}{8}$
2. $\frac{4}{5}, \frac{1}{5}, \frac{3}{5}, \frac{2}{5}$

When they finished they formed a queue in front of their teacher in order for her to mark their work.

After marking most of the learners work she gave them an extra exercise. This was

$$\frac{1}{4}, \frac{1}{5}, \frac{1}{6}, 1$$

While learners tried to order the fractions, she took out fraction pieces and showed them to the whole class, each time she raised a piece she asked: *What is this?* And learners shouted the fraction.

With these fraction pieces arranged on the teacher's table she asked

> ***Teacher B:*** *So which is the smallest?*

Learners together: ***(****Shouted) One sixth.*

Teacher B: *Followed by?*

Learners: ***(****All shouted) One fifth,*

The teacher listed the fractions as

$\frac{1}{6}, \frac{1}{5}, \frac{1}{4}, 1$

After this exercise the teacher concluded the lesson by asking learners to say something about the denominator. She then directed their attention to each of the slabs and named its denominator.

Teacher B: *Say something about the denominator. Is there anyone who can say something about the denominator? And tell us something about the denominator and the size of the fraction. Do you want to say something?*

Lele: *When the denominator is big the fraction is small.*

The lesson that followed ordering fractions was equivalent fractions. As usual the teacher reviewed the previous lesson and then informed learners of what they are going to do today.

Teacher B: *When you just want to make the order be simple, you have to change and make the denominators the same, if the denominators are the same is now that you will be able to arrange the fractions in order of their sizes and the size will be easy for you. Do you get me?*

Learners*: (Together) Yes madam*

The teacher straight away tells learners what to do

Teacher B*: If you can change the denominators and make the denominators to be the same ordering the fractions will be easier, ok?*

Learners*: (Together) Yes madam*

Teacher B*: Now who can tell me what is this? Lifts the fraction board*

Learners*: (Together) Fraction board*

Teacher B: (*Raising fraction slices and asks) What is this?*

Learners*: (Together) Fraction slice*

Teacher B*: So for today we are just going to divide ourselves into 3 groups because we are just going to use these 2 fraction boards and fraction slices, therefore we are just going to divide ourselves into 3 groups. I think it should be groups of 8; we should be 8 in each group. Let us just be 8 in each group and one member to come and get the fraction board and fraction slices*

When you are 8 come and take the fraction boards and fraction slices.

Learners formed groups but they were far more than eight in each group. In two groups they were 13 and in the other group they were 12. Some moved in front and took the fraction board and fraction slices. The learners made a lot of noise as they moved into groups.

Teacher B: *(Talking to one of the groups that have already taken the material): you are going to use this fraction board and you are just going to use this desk to place them, ok?*

Learners*: (Together) Yes madam*

Teacher B*: Just place them on a desk so that they end up making what you really see how these things are done. Now that you have fraction board don't just take out the parts before, don't just take out the parts before.*

Gave instructions and moved around to check whether they were following

Try to show me the whole number, where is the whole number?

Learners show a whole on the fraction board and fraction slices

Teacher B*: Are you sure this is a whole number?*

Learners*: (Together) Yes madam*

Teacher B*: Ok fine if this one is a whole number, where is the whole number in this?*

The teacher continued going from one group to the next asking them to show fractions that were equal to one whole. She got answers like $\frac{4}{4} = 1$, $\frac{6}{6} = 1$

After finding fractions that were equivalent to one whole the teacher used the fraction pieces from the fraction stacks and asked:

> ***Teacher B:*** *What does this red (pieces are coloured differently) stand for?*
>
> ***Learners****: (Together) One quarter.*
>
> ***Teacher B****: One quarter, so we say that these half and two quarters what do they mean?*
>
> ***Learners****: (Together) Half*
>
> ***Teacher B****: They are equal, you know what even though they are not exactly but the value is the same, ok?*
>
> ***Learners****: (Together) Yes madam*
>
> ***Teacher B:*** *The value of two quarters and a half is equal, even if when we write them they are not the same, we may say that half is equal to two quarters, ok?*
>
> ***Learners****: (Together) Yes madam*
>
> ***Teacher B****: What we are trying to say is that this half is equal to two quarters, so we are saying that half is equivalent to two quarters, ok? The value of half and this two quarters is the same, ok? Mm… so I want you to take out your math books so that you see what I mean by saying, talking about equivalent fractions. Let us quickly go back to our seats [students take the board and slices in front, on the teacher's table as they move to their seats]*
>
> ***Teacher B:*** *The equivalent fractions are equal in what?*
>
> ***Learners****: (Together) Value*

***Teacher B**: In value, so to make fractions to be equivalent, suppose that we have may be this $\frac{3}{4}$ so we want another equivalent fraction so we can multiply this by two, so we are going to have $\frac{3 \, x \, 2}{4 \, x \, 2} = \frac{6}{8}$therefore*

$\frac{3}{4}$is equivalent to$\frac{6}{8}$. Normally when we are just doing equivalent we do fractions in pairs, ok?

***Learners**: (Together) Yes*

***Teacher B**: We do them in pairs three quarters is equivalent to six eighths, ok?*

***Learners**: (Together) Yes*

***Teacher B**: This is the same as when we are saying err… we are just doing this, we are trying to multiply this to help you to work out the fractions when the fraction boards are not available, you should also be able to … work out the equivalent fractions. Do you get me? Mm… so this is how we can get the equivalent fractions by multiplying both the numerators and denominators so when you say $\frac{3}{4}$ is equivalent to $\frac{6}{8}$, ok?*

***Learners**: (Together) yes*

***Teacher B:** Mm … so you write all the fractions in twelve fraction board at the moment I'm going to work out this one with you, the other ones you shall work it alone.*

Let us have; $\frac{1}{4} = \frac{}{}$ who can just tell us how we work out this one?

***Lele**: Madam it's two eighths*

***Teacher B**: How did you get that? Come and show us on the chalkboard*

***Lele:** Madam I multiplied by 2; one times 2 is equal to two and four times two is equal to eight $\frac{1 \, x \, 2}{4 \, x \, 2} = \frac{2}{8}$*

***Teacher B**: So let us go back to the fraction board and try to prove what he is saying. Look at one quarter; do you all see one quarter?*

***Learners**: (Together) (some say) Yes (some say) No*

***Teacher B**: So let's go back to one eighth and see*

The teacher used the fraction slices to verify the answer. She lifted the $\frac{1}{4}$ slice and the $\frac{2}{8}$ slice**.** She then put $\frac{1}{4}$ slice over $\frac{2}{8}$ slice and they exactly covered each other

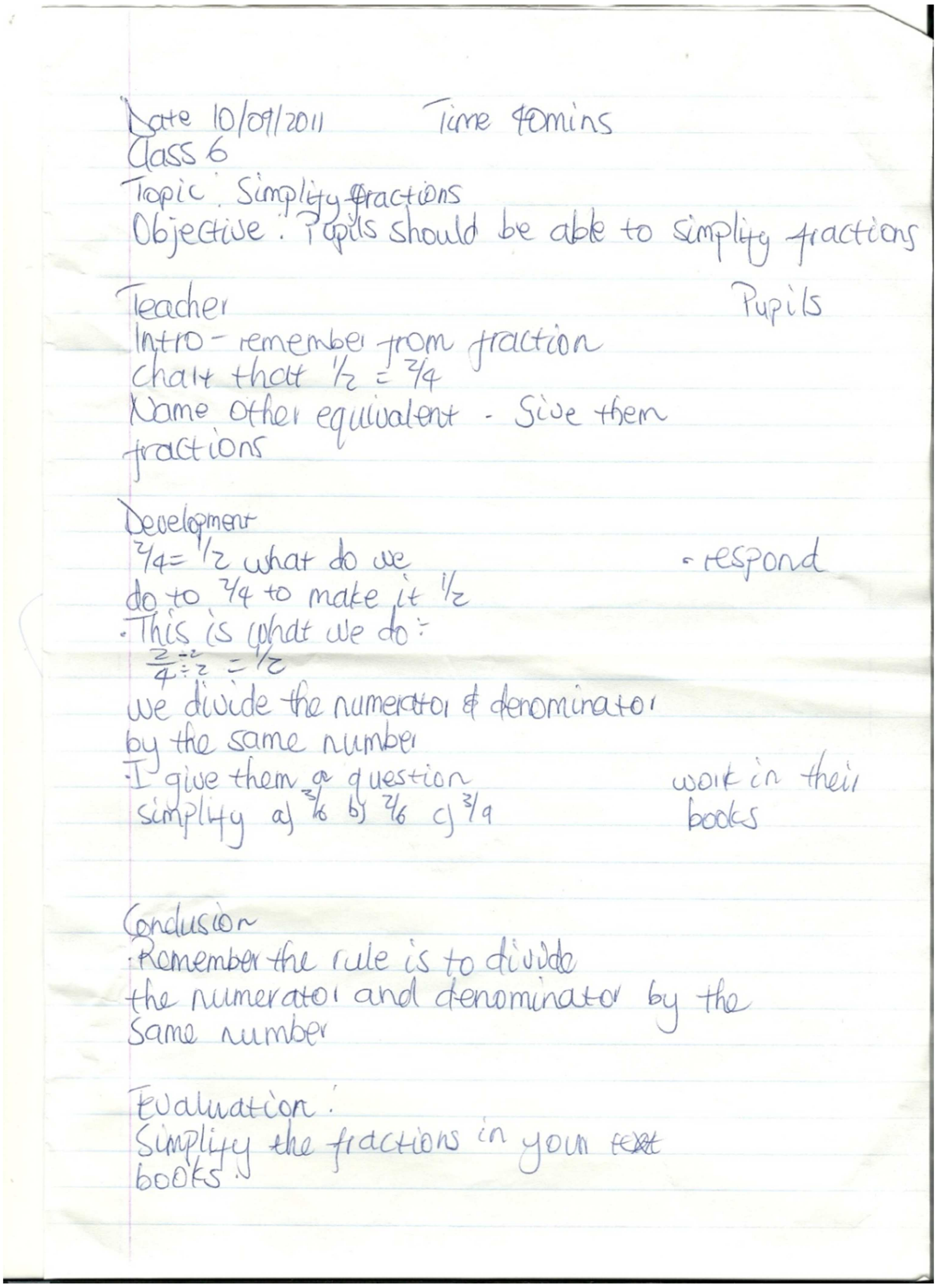
***Teacher B**: Is this one quarter equal to two eighths.*

***Learners**: (Together) Yes madam*

The lesson was ended with an exercise in which learners changed fractions to their equivalence.

5.3.3.2 Teacher B's lesson plan

The lesson plans of teacher B that the researcher looked at all had the same format. As indicated in the interview which will be discussed in section 5.4. She stated the date, duration and the number of learners, topic, objectives, teaching materials, introduction, development and the assessment (3.8.1). The format was as indicated below.



Date 10/09/2011 Time 40mins
Class 6
Topic: Simplify fractions
Objective: Pupils should be able to simplify fractions

Teacher Pupils
Intro – remember from fraction
chart that $\frac{1}{2} = \frac{2}{4}$
Name other equivalent - Give them
fractions

Development
$\frac{2}{4} = \frac{1}{2}$ what do we - respond
do to $\frac{2}{4}$ to make it $\frac{1}{2}$
· This is what we do:
$\frac{2 \div 2}{4 \div 2} = \frac{1}{2}$
we divide the numerator & denominator
by the same number
· I give them a question work in their
simplify a) $\frac{3}{6}$ b) $\frac{2}{6}$ c) $\frac{3}{9}$ books

Conclusion
· Remember the rule is to divide
the numerator and denominator by the
same number

Evaluation:
Simplify the fractions in your text
books.

Figure 5.9 Teacher B's lesson plan

Objectives were short, clear and measurable. She made a list of the teaching methods and teaching materials she intended to use. The teacher also wrote the procedure she intended to follow and the possible responses she expected to get from the learners. Regarding potential problems and misconceptions learners were likely to have, the teacher did not indicate them. She ended her lessons by giving learners closed-ended questions which required learners to remember and use procedures.

It was discussed in section 3.8.1 that teachers could follow a four stage lesson in which learners experimented, reflected and/or explained, hypothesised and verified their findings. There was no lesson in which teacher B employed this procedure. She also did not engage learners in high order thinking and/or problem solving.

5.3.3.3 Learner’s exercise books

Like teacher A’s learners, the learners drew a line in the middle of the page to divide the pages into two. In this way the layout of their work was neat and easy to read. As they answered questions, they wrote the steps going down the page.

There was not much information that the researcher got from the learners’ exercise books regarding the conceptual learning. When learners answered the questions they used procedures that their teacher told them in class. None of the learners had done anything different from what the teacher did, except those who forgot the procedure and/or rule and then got everything wrong.

This is how the learners worked out questions given as class work.

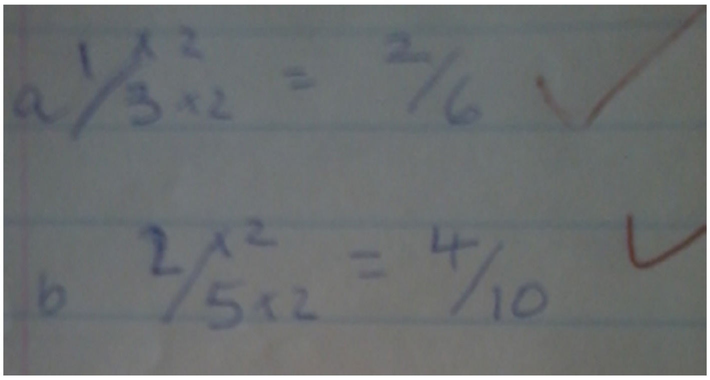


Figure 5.10 Workings of learners i teacher B’s class

In this case learners were given this statement, $\frac{-}{8} = \frac{1}{2}$, to complete. This was what one of the learners did

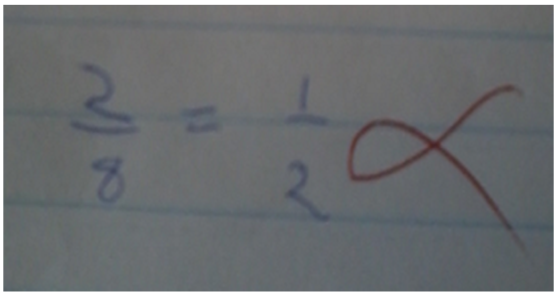


Figure 5.11 Errors in teacher B's class

As indicated in section 5.3.3.1, teacher B was not patient and hence did not assess formatively. She lost her temper and threw down learners' books when she was supposed to give them feedback that would enhance understanding and learning. In this particular case, the teacher did not write any comment on the learner's exercise book. This meant that this particular learner did not know why she got the question wrong.

The teacher gave two or three close ended questions as an assignment. She often did not mark all the learners' class work, but marked all of their assignments.

5.3.4 Teacher C

5.3.4.1 Class observations

The first day of observation of teacher C the researcher arrived while the learners were still outside for a short break. The teacher let me in and we looked for a place to sit which was where the researcher would be able to see most of the learners. It was agreed that her table was the best option.

There were 39 learners altogether in this class. They were all doing standard six for the first time. There were three standard six classes; these are 6A, 6B and 6C. All repeaters are in class 6C. The class that the researcher observed was standard 6B.

When the learners arrived the researcher was already seated, so they looked at her and looked around as if they were wondering who she was. The teacher introduced me to the class and their faces loosened and they greeted me enthusiastically.

All lessons were in the morning hours. On Mondays, Tuesdays and Fridays the lessons were at eight o'clock in the morning and they were 80 minutes long. On

Wednesdays and Thursdays they were 40 minutes long. On Wednesday the lesson was at eleven o'clock and Thursday it was at 0950 hours.

Like the other two teachers, the teacher wrote the date and subject at the top of the chalk board while one of the learners was distributing textbooks to the rest of the learners.

The teacher taught fractions in nine consecutive lessons. The first lesson was a review of fractions, such as, what is a fraction, distinguishing between denominators and numerators and so on.

The teacher introduced the lesson by relating a short story about sharing.

> *'M'e 'Malipuo has one loaf of bread. She wants to share the loaf equally among her 4 children. How much will each child get?*

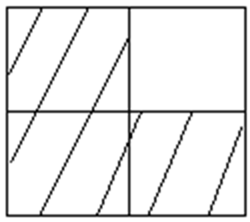
Learners wrote the answers and the teacher marked ten learners who were the first to finish writing. After marking she drew a rectangle and divided it into four equal parts and provided this explanation

> *This is 'm'e Malipuo's one loaf of bread. (She drew a rectangle on the board). This loaf has to be given to 4 children, so all you have to do is to see to it that the pieces that you give to each child are the same size. There shouldn't be a child who gets a bigger share, shares or parts should be the same size. So this is one, one, one (as she writes on each part). She then writes each piece as* $\frac{1}{4}$*… it has been cut into 4 parts. One child will get one out of 4, the other 1 over 4 and so on and so forth.*

$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$

> *What do we call this type of numbers? What do we call this type of numbers where we have a number on top of another, what do we call this type of numbers?*

The teacher drew a square, divided it into four equal parts and shaded three of them. She asked learners to name the fraction of the shaded part. Learners gave the fraction as three quarters.



The teacher wrote the fraction on the chalkboard and told learners which were denominators and numerators

Teacher C*: 3 parts are shaded (writes it on the board). What i would like you to learn now is that the number of divisions in which a whole has been divided or shared is called a denominator. Denominator is the equal part of a whole and they are the ones at the bottom. And the ones that are the shaded ones, we said there are 3 shaded parts. The three shaded parts are called the*

Learners: *(Together with the teacher said) Numerators*

The teacher gave other examples for learners to name the fractions they represented and to state numerators and denominators.

Teacher C*: We can also have the whole group, like you are a group of standard 6B, let's say there are 40 of you and there are 19 boys, so the fraction of boys will be* $\frac{19}{40}$*. From our book there are some oranges, the basket has some oranges. How many oranges are in the basket? Yes girl, Keny*

Keny*: There are 12 oranges*

Teacher C: *There are 12 oranges then how many oranges are shaded*

Lefa*: 5 orange are shaded*

Teacher C*: Which is the denominator when we have* $\frac{5}{12}$*? Yes Mary*

Mary*: 12*

Teacher C*: 12 is the denominator and what is the numerator? Yes boy!*

Bobo*: 5 is the numerator*

This oral exercise was followed with a short written exercise in which learners were to identify numerators and denominators of given fractions. These were

$\frac{1}{3}$ and $\frac{2}{5}$

In the same lesson fractions were compared. Like the other two teachers, she drew the fraction board and partitioned it up to the eighths. She asked learners to name the fraction of each strip as she pointed it. Answers were given in chorus.

She asked how many quarters would give one whole when brought together. Learners answered in chorus. She directed the learners' attention to the sizes of different fractions on the fraction chart and said:

> *Looking at this it is smaller than this (pointing at the halves, quarter, thirds and so on) now what we have to do is try to compare them. We use this sign* $<$*, what sign is it?*

Learners responded together. She gave an oral exercise in which she asked learners to use the fraction board and put the correct sign between these fractions to make the statement true.

$$\frac{1}{2} * \frac{1}{3}$$

She took out half and put one third over it and showed them to the learners and asked; what do you see? Learners together said half is greater than one third. She then called one learner to insert the symbol on the chalk board.

Individual written work that she gave towards the end of the lesson was to insert either $<, >\mathrm{or} =$ in the place of * in order to make the following true: $\frac{5}{8} * \frac{3}{8}$ and $\frac{3}{4} * \frac{1}{2}$.

The second lesson was on equivalent fractions. The teacher began the lesson by reviewing denominators and comparing fractions by using inequality signs.

After marking the first ten learners who had finished the review questions she asked learners to complete this statement by writing the correct sign. $\frac{8}{8} * \frac{4}{4}$, $\frac{6}{8} * \frac{3}{4}$ and $\frac{1}{4} * \frac{3}{8}$. These were answered orally.

Teacher C: *Now we are going to fill in any of the three signs to the different statements that we have. She writes*

a) $\frac{8}{8} * \frac{4}{4}$

Nt'sala: *8 over 8 equals 4 over 4*

Teacher C: *Ee! Do you agree with him?*

Learners: *(Together) Yes madam.*

Teacher C: *Writes $\frac{8}{8} = \frac{4}{4}$. Number b) (writes $\frac{6}{8} * \frac{3}{4}$). Yes boy, Lebo*

Lebo: *6 eighths is greater than 3 quarters*

Teacher C: *Ee! Is he correct?*

Learners: *(Together) No*

Teacher C: *Sefiso is trying, yes*

Sefiso: *6 eighths equals 3 quarters*

Teacher C: *(Repeats and writes) $\frac{6}{8} * \frac{3}{4}$, $\frac{6}{8} = \frac{3}{4}$. We are coming to verify this so that e! People who might have missed the point here can you understand. I hope you still have the fraction chart and you are using them.*

Then number c) $\frac{1}{4} * \frac{3}{8}$,

Lillo: *1 quarter is less than 3 eighths*

Teacher C: *Is it less than 3 eighths? Mm! is that so?*

Learners: *(Together) Yes.*

Teacher C: *I want everybody to say yes or no. I want to be given answers by all of you because I have allowed you to use... what do you say class?*

Learners: *(Together) 1 quarter is less than 3 eighths*

Teacher C: *Repeats as she writes $\frac{1}{4} < \frac{3}{8}$*

The teacher then demonstrated on the fraction wall that 8 eighths $\frac{8}{8} = \frac{4}{4}$, $\frac{6}{8} = \frac{3}{4}$ and that $\frac{1}{4} < \frac{3}{8}$.

Then she drew a fraction wall on the chalkboard, she made sure that the partitions were equal because she used the fraction pieces from the fraction chart. That is, when drawing a whole she used the piece which was a whole from the fraction pieces, for half she used the piece which has a half and so forth.

Teacher c*: How many halves are equal to 1 whole? Let's observe the fraction wall. Lebo*

Lebo*: 2 halves*

Teacher C: *Aha! We have this 1 half and this 1 half. (Pointing at the halves on the fraction wall). These 2 halves are equal to*

Learners: *(Together) 1 whole*

Teacher C: *How many quarters are equal to half? Writes* $\frac{}{4} = \frac{1}{2}$ *as she repeats 3 times. Yes girl!*

Puseletso: *2 quarters.*

Teacher C: *2 quarters. I don't think we need more exercise, all we can do is to try on our own.*

Learners*: (Together) Yes madam.*

Teacher C: *Can we? Or are there some people who still have some difficulty?*

Learners*: (Together) No madam.*

Teacher C: *Do you see? You people. This halves are equal (writes = on the chalkboard) to 1 whole, pointing to 2 halves and then 1 whole, akere?*

Learners: *(Together) Yes madam.*

Teacher C: *Counts the halves 1, 2 is equal (writes = on the board) to 1 whole. Alright now let's try to do this in our exercise books.*

Having done this oral exercise she gave class work, with this she ended the lesson, in which they were to fill in the missing numbers in the given fractions

1. $\frac{\ }{4} = 1$

2. $\frac{\ }{3} = 1$

3. $\frac{\ }{8} = \frac{1}{2}$

In the third lesson they ordered fractions. She did this by engaging learners in the paper cutting exercise.

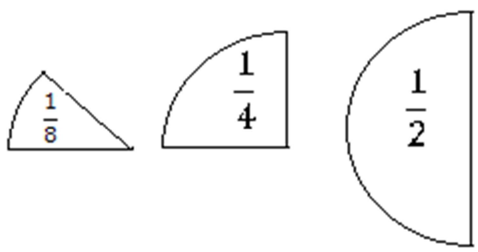
On each desk the teacher put three pieces of paper cut in circular form and told them to imagine those papers as cakes. She instructed each group to cut one of the cakes into quarters, halves and eighths and gave them a hint on how to share the cakes

> *You share the cakes in different fractions, the first one you make it in quarters, the second one in eighths and the third one in halves.*
>
> *It is easy if you fold your cake (demonstrates by folding her paper such that the 2 parts are equal).*

Having finished sharing all the three cakes, the teacher asked learners to count the number of divisions they have on each cake. When counting on the cake cut in to eighths this is how they did it.

> ***Learners**: (Together) Count together 1, 2, 3, 4, 5, 6, 7, 8*

The teacher then asked learners to take out one piece from each cake and then arrange them in order so that they can compare their sizes.



After this exercise learners were given a set of fractions to arrange in order of size starting with the smallest. The fractions were $\frac{8}{8}$, $\frac{3}{8}$, $\frac{5}{8}$, $\frac{1}{8}$

When learners were trying to settle and arrange the fractions the teacher directed their attention to the fraction slabs

***Teacher C**: Looking at the column of eighths where is 8 eighths. Let's have somebody to come and count for us. Yes girl! Puseletso*

***Puseletso**: (Counts the eighths) 1, 2, 3, 4, 5, 6, 7, 8*

***Teacher C**: These are 8 eighths a ke re? (Isn't it?)*

***Puseletso**: Yes madam.*

***Teacher C**: What's the other number? 5 eighths, let's have another one to come and show us 5 eighths.*

***Bobo:** (Went to the board and counts the eighths) 1, 2, 3, 4, 5. Learners counted the 5 sixths*

***Teacher C**: Are they the 5 eighths?*

***Learners**: (Together) Yes*

***Teacher C**: Ee?*

***Learners**: (In chorus) No madam*

***Teacher C**: 5 eighths*

***Sefiso**: (Counted the eighths) 1, 2, 3, 4, 5*

***Teacher C**: Now from here to here (indicating on the fraction wall where 5 eighths are)*

Then 1 eighth

***Lebo**: Points where 1 eighth is*

***Teacher C**: Ok then 3 eighths*

Learners shouted "Madam" so that they can be given chance to go to the board and show where 3 eighths are.

***Nka**: Counts 1, 2, 3*

Teacher used different colours to indicate where each of the four fractions were.

She then directed the learners' attention to the marked fractions by saying as she points

This is 3 eighths, 1 eighth, 5 eighths and 8 eighths

***Teacher C**: Where do we start, from the biggest or from the smallest?*

***Lefa**: Smallest*

***Teacher C**: From the smallest*

***Mary**: Yes*

***Teacher C**: Ok somebody to come and write the one that is the smallest*

One boy volunteered and the teachers asked

***Teacher C**: Are you not coming for the second time?*

***Nt'sala**: No. Went to the board and struggled to write. The teacher reminds him.*

The smallest, you write the smallest

***Learner**: Writes 1 eighth*

***Teacher C**: What comes next?*

***Molemo**: Writes 3 eighths.*

He did not write it next to 1 eighth

***Teacher C**: Do you see that you disturb the arrangement write it next to the first one. Next, yes boy! Thabiso.*

***Thabiso**: (Writes 5 eighths and 8 eighths)

The final arrangement was;

$\frac{1}{8}, \frac{3}{8}, \frac{5}{8}, \frac{8}{8}$

***Teacher C**: Do you all agree that 1 eighth is the smallest?*

***Learners**: (The whole class) Yes madam.*

***Teacher C**: Ok. With the help of what we have been doing here try to locate this err on our chart we do see that 1 eighth is smaller than other fractions and then comes one err 3 eighths, 5 eighths and lastly 8 eighths which is a whole a ke re?*

***Learners**: (Together) Yes madam.*

The teacher then gave class work.

a) Arrange the fractions from the smallest to the greatest $\frac{4}{5}, \frac{1}{5}, \frac{3}{5}, \frac{2}{5}$

b) Arrange from greatest to smallest $\frac{1}{10}, \frac{1}{2}, \frac{1}{5}, \frac{1}{3}$

Most learners were able to order fractions. The teacher wrapped up the lesson by demonstrating with fractions stacks that one fifth is the smallest and four fifths is the largest

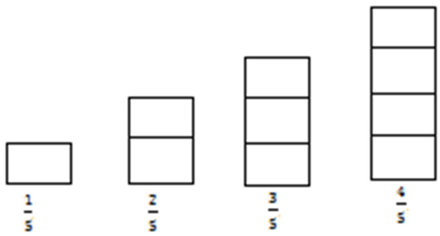


Figure 5.12 (a) Teacher C's representations of ordering fractions

In the fourth lesson the teacher wrote two sets of fractions for the learners to arrange and she marked the first 10 learners as usual. As she marked she made comments

***Teacher C:** Yesterday you were able to order fractions, but now there are only two people who got these ones correct. Now if we have these eight, ten and five parts, (showing eighths, tenths and fifths on the fraction wall), really which is smaller*

***Thabiso:** 10*

***Teacher C:** Then?*

***Bobo:** Eight*

Teacher C: *And then 5. So do you see that this shows that you did not work with understanding yesterday, you were just excited. She wrote another set of fractions with different denominators and they also order it orally. The fractions were;* $\frac{1}{10}, \frac{5}{8}, \frac{3}{5}$

After ordering the above fractions correctly, some learners shout *no, no,* while others complained, *Teacher you cheat me.* When the teacher checks the fractions again she found that she had ordered them incorrectly and hence marked learners wrong yet they were correct. This is what she said; *Okay thank you. So you see a teacher can also go wrong. Ok that is good of you. I nearly get you confused yet you understood what you were doing.*

The teacher then proceeded with the lesson of the day which was equivalent fractions. She drew a fraction wall on the chalkboard. She covered one half of the fraction wall and gave the following instruction

Teacher C: *You look through this half and wherever there is a line cutting through you should say the fractions. This is half, where do we find another line that goes straight from half, which is that other line?*

The lesson proceeded as follows

Learners: (*Together) 2 quarters*

Teacher C*: 2 quarters, which one else? Yes boy, Sello*

Sello*: 4 eighths*

Teacher C*: Counts 1, 2, 3, 4 eighths. Now try to do that in your text books. Cover 1 half of your fraction table then say the fractions we have which are equal to half. Ok Mary!*

Mary*: 1 sixth*

Teacher C*: 1 sixth why are you skipping others? Yes another one.*

Mary*: 2 quarters*

Teacher C*: 2 quarters*

***Learners:** (Together) 3 sixths, 4 eighths, 5 tenth*

***Teacher C**: Alright there is something that is a bit new today. You have half equals*

Learners did not wait for the teacher to finish they answered*: 2 quarters*

***Teacher C**: Another half is equal to...*

***Learners**: (Together) 4 eighths*

***Teacher C**: Do you have another one in your head?*

***Learners**: (Together) Half equal to 6 twelfths*

***Teacher C**: Ok thanks, you can go on and on to find fractions that are equal to half. So in mathematics when we have this kind of err fractions we say they are equivalent, that is, they are equal in value. Equi means equal. Let's try to find a fraction that is equivalent to 1 quarter. Let's try that from our err... fraction chart. Try to find the fraction that is equivalent to a quarter. That is you have to cover the fraction chart.*

Learners shout madam

***Teacher C:** Yes*

***Keny**: 2 sixth*

Others shout no madam, madam

***Teacher C**: Yes calling him by the name. Lefa!*

***Lefa**: 2 eighths*

***Teacher C**: Two?*

***Lefa**: Eighths*

***Teacher C**: Another one yes*

***Learner**: 4 twelfths*

***Teacher C**: Alright we need to know how we can find this without the help of the fraction chart. We have to learn this. Let us suppose you are in the exam room and you are to write an equivalent fraction to a quarter or a half. You won't have a fraction chart in front of you; all you have to do is to find a way out to find this.*

Instead of waiting for the learners to come up with their way of finding equivalent fractions without using the fraction wall, the teacher told them what to do. This is what she said;

***Teacher C:** So you will have to find one number that you are going to multiply both the numerator and the denominator with.*

For example looking at this number we will say half times something (writes $\frac{1\ \times}{2\times}$–). This number should be the same. A ke re? (Isn't it?)

If it is 2 it is going to be 2, if you decide to make it 3, we will have 3 on both sides. So 1 times 2

***Learners: (**Together) 2*

***Teacher C**: 2 times 2*

***Learners:** (Together) 4*

***Teacher C**: So you see $\frac{1\ \times 2}{2\times\ 2}$= $\frac{2}{4}$.. Half is equals to 2 quarters*

The lesson that followed this one was about simplifying fractions. This was the fifth lesson. After reviewing equivalent fractions, she asked, what is this number $\frac{2}{4}$? (Writes $\frac{2}{4}$ on the chalkboard)

***Learners**: (Together) 2 quarter*

***Teacher C**: What?*

***Learners**: (Together) 2 quarters*

***Teacher C**: Just think and think, how can you make this fraction the smallest or smaller than it is now yet it doesn't change the value (pause) how can you make this (points at the fraction)*

***Learners**: Divide*

***Teacher C**: Divide, do you agree with him?*

What followed after was that the teacher demonstrated how to divide the fraction by the same number to simplify it. The learners then simplified on the chalkboard and later into their exercise books.

As the teacher marked she came across the situation where one learner did not fully simplify. Instead of having $\frac{50}{100} = \frac{1}{2}$, one learner had written $\frac{50}{100} = \frac{5}{10}$. Another one had written $\frac{15}{20} = \frac{7}{10}$.

The teacher helped the first learner to identify that he had not fully simplified and asked him to think of a number that could divide both five and 10. The boy was able to find that number and further divided by five to get half.

The second learner was not marked, and hence his misconception was left unattended.

The sixth lesson was comparing fractions. She compared fractions in the first lesson when reviewing fractions, but this time she engaged learners in activity. She demonstrated with fraction slabs and drawings.

The teacher asked, orally, learners to write each of these signs $<, =$ or $>$ to make each of the statement true

1. $\frac{5}{10} * \frac{3}{10}$
2. $\frac{3}{4} * \frac{7}{8}$

Learners gave the correct answers

1. $\frac{5}{10} > \frac{3}{10}$
2. $\frac{3}{4} < \frac{7}{8}$

The teacher then asked learners to explain how they got $\frac{3}{4} < \frac{7}{8}$

The responses were as follows;

***Moshe**: Madam 7 eighths is nearer to a whole*

***Teacher C**: What about if I add 1 eighths here (to 7 eighths) and I add a quarter here (to 3 quarters), wont they make a whole? So how do you get that $\frac{3}{4} < \frac{7}{8}$? Err? Talk to each other, I agree with you 3 quarters are less than 7 eighths. Yes boy, Malinga*

***Malinga**: Err... madam it's because the eighths are smaller than the quarters and the eighths are close to being a whole there*

***Teacher C:** Laughs and repeats, the eighths are close to being a whole and add the 3 quarters are close to being a whole*

The teacher then drew this on the chalkboard in order to clarify what Malinga said.

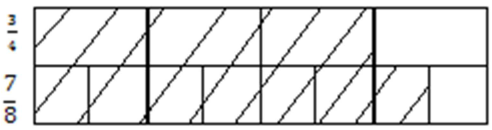


Figure 5.12 (b) Teacher C's representations of ordering fractions

After the teacher finished drawing this rectangle Malinga raised his hand and said:

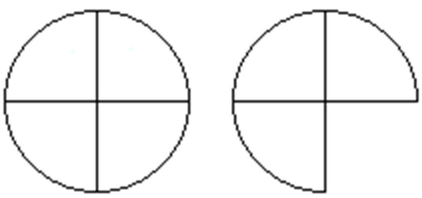
Madam when you times 3 quarters by 2 you get 6 eighths

The teacher was very happy and asked everybody to clap for him and then said:

Yes all we do is to make denominators the same, common denominators, that is the fact. Ke eona taba a ke re? (That is the fact, isn't it?)

She then demonstrated how to make the denominators the same $\frac{3\,x\,2}{4\,x\,2} = \frac{6}{8}$. She gave learners the chance to practise the procedure on the chalkboard and later in their exercise books as the evaluation of the lesson.

The seventh lesson was on mixed numbers, the teacher drew this on the chalkboard



She asked learners whether they remember this. They struggled for some time but quickly they said

Mixed number

The teacher together with the learners together counted the parts and got seven. She then asked seven over what?

Learners: *(Together) four*

Teacher C: *Writes* $\frac{7}{4}$ *and said: Somebody might not know why we say over 4 yet here we have only 3. That person who does not understand, should know that there was a part here, it is only that it has not been included. It might happen that we have all these parts and only these are shaded, so don't think it is not there, right?*

From this point the teacher asked learners what type of fraction $\frac{7}{4}$ is. The teacher then showed learners how to change mixed fractions to improper fractions and improper fractions to mixed fractions. This was how she proceeded

$$1\frac{3}{4} = \frac{1\times4+3}{4}$$

As usual she gave learners two mixed fractions and asked them to change to improper fractions. These were

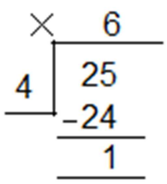
3. $1\frac{3}{6}$

4. $\frac{25}{4}$

Learners used the procedure their teacher has shown them. That is;

1. $1\frac{3}{6} = \frac{1\times6+3}{6} = \frac{9}{6}$

2. $\frac{25}{4} =$



$\frac{25}{4} = 6\frac{1}{4}$

Teacher C taught addition and subtraction of mixed numbers in the last two lessons, lesson eight and nine. She drew two rectangles and partitioned them as shown below.

$\frac{1}{5} + \frac{1}{5} + \frac{1}{5} = \frac{3}{5}$ shaded $\qquad$ $\frac{1}{5} + \frac{1}{5} + \frac{1}{5} = \frac{3}{5}$ shaded

She then asked learners what fraction of each rectangle is shaded and the teacher wrote the fractions below each rectangle. She also asked

Teacher C: *Now here how many parts are shaded in the 2 wholes? How many are shaded altogether? Yes boy! Mosa.*

The learner struggled but she probed him until he gave the correct answer.

Mosa: *6 tenths.*

Teacher C*: 6 tenths? E? How much is this whole?*

Malinga*: 1 whole*

Teacher C*: Ee?*

Puseletso*: 1 whole*

Teacher C*: Ee! How much is it?*

Lefa*: 5 fifths*

Teacher C*: and this one?*

Lefa*: 5 fifths*

Teacher C*: And how many fifths are shaded altogether?*

Nt'sala*: 6*

***Teacher C**: Do you agree with him? (She writes $\frac{6}{5}$).*

From this the teacher demonstrated how to add mixed fractions. This was what she did:

$1\frac{1}{4} + 2\frac{2}{4} = (1+2)\ \frac{1}{4} + \frac{2}{4} = 3\frac{3}{4}$

The learners practiced the same procedure and towards the end of the lesson they were given two sums to perform in their exercise books.

5.3.4.2 Teacher C’s lesson plan

Below is an example of a lesson plan of teacher C.

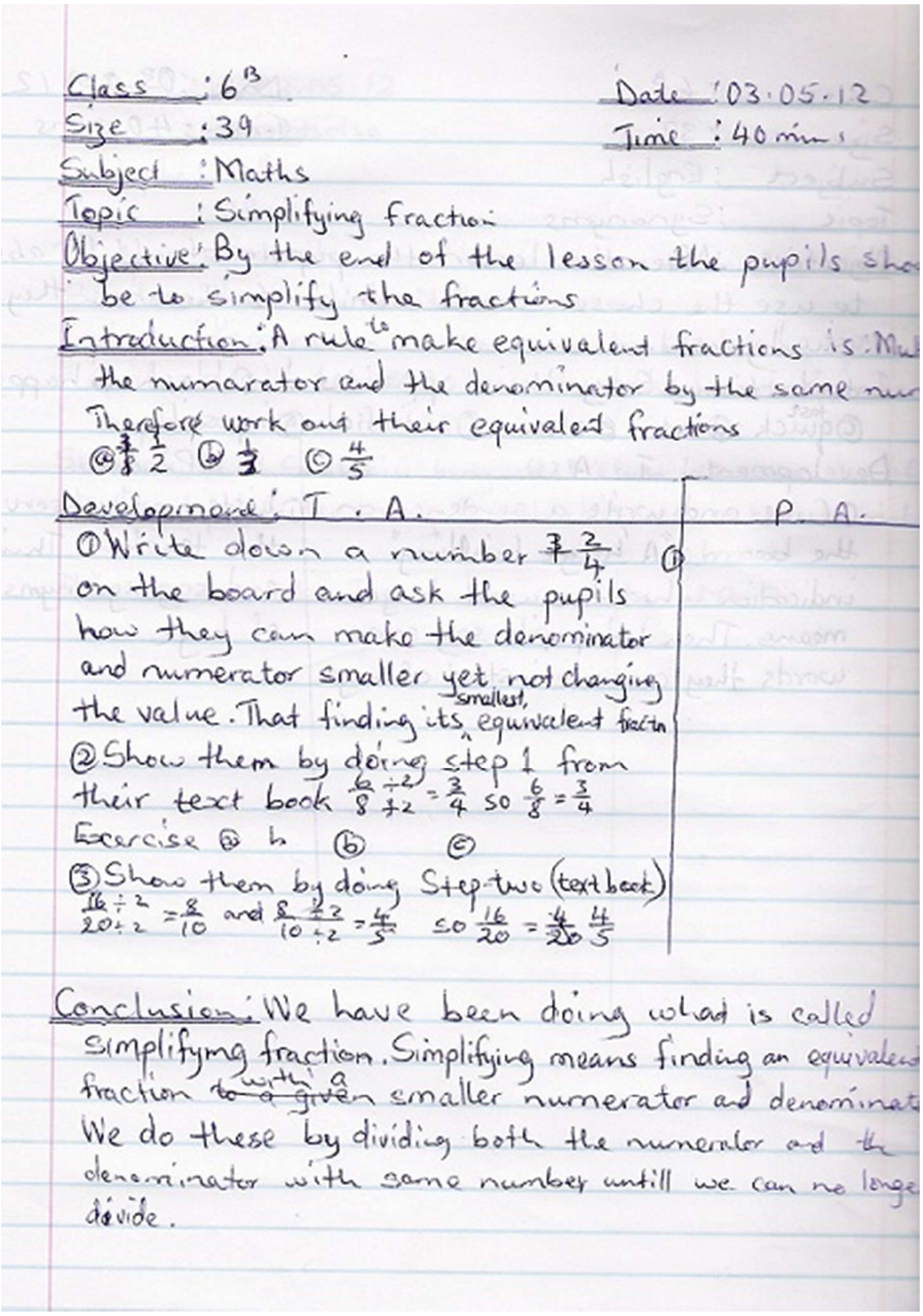

Class : 6B
Size : 39
Date : 03.05.12
Time : 40 mins
Subject : Maths
Topic : Simplifying fraction
Objective: By the end of the lesson the pupils sho
be to simplify the fractions
Introduction: A rule to make equivalent fractions is: Mu
the numerator and the denominator by the same nu
Therefore work out their equivalent fractions
(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{4}{5}$
Development: T. A. | P. A.
(1) Write down a number $\frac{2}{4}$ on the board and ask the pupils how they can make the denominator and numerator smaller yet not changing the value. That finding its smallest, equivalent fraction
(2) Show them by doing step 1 from their text book $\frac{6 \div 2}{8 \div 2} = \frac{3}{4}$ so $\frac{6}{8} = \frac{3}{4}$
Exercise (a) (b) (c)
(3) Show them by doing Step two (text book) $\frac{16 \div 2}{20 \div 2} = \frac{8}{10}$ and $\frac{8 \div 2}{10 \div 2} = \frac{4}{5}$ so $\frac{16}{20} = \frac{4}{5}$
Conclusion: We have been doing what is called simplifying fraction. Simplifying means finding an equivalent fraction with a smaller numerator and denominat
We do these by dividing both the numerator and the denominator with same number untill we can no longe
divide.

Figure 5.13 Teacher C’s lesson plan

Like the other two teachers, teacher C wrote short clear objectives, listed materials that would be used and wrote the procedure she intended to follow in the lesson. She did not use teaching materials in all lessons, so in that case she did not write them in her lesson plan.

There is no evidence, in her lesson plan, of predetermined problems, mistakes and/or difficulties learners were likely to make. In the lesson on simplifying fractions one learner simplified like this $\frac{15}{20} = \frac{7}{10}$. The teacher did not address this error, may be because she did not anticipate this error so she did not know how to address it. She also did not use a four stage lesson plan in any of her lessons. Learners were not given chance to work with concrete materials and then come to their own conclusions or discoveries.

During the lesson, the teacher always had her lesson plan book around and referred to it whenever need arouse. The examples she used were those that are in her plan book even the class work questions she copied them from her book.

It was true that she sometimes thought of fractions on the spot, but in most of her lessons she used what she had planned to use.

The questions that she gave to learners were abstract and closed. She did not give questions that were extracted from the learners' context. Learners were therefore not challenged and always used the rules and algorithms that the teacher gave in class.

5.3.4.3 Learners' exercise books

Learners' exercise books were also divided into two sections by drawing a vertical line in the middle of each page. In this regard learners' work was neat and easy to read.

The teacher even emphasised that learners should write the steps to a question vertically, such that the answer should be written at the end with an arrow pointing to the answer as this particular learner did in his working.

The teacher gave closed ended questions similar to the ones she used in class. All learners were therefore obliged to use the procedures and algorithms that the teacher gave in class. No single student used a different method, may be because the questions did not call for high order thinking.

An example of their working is shown below.

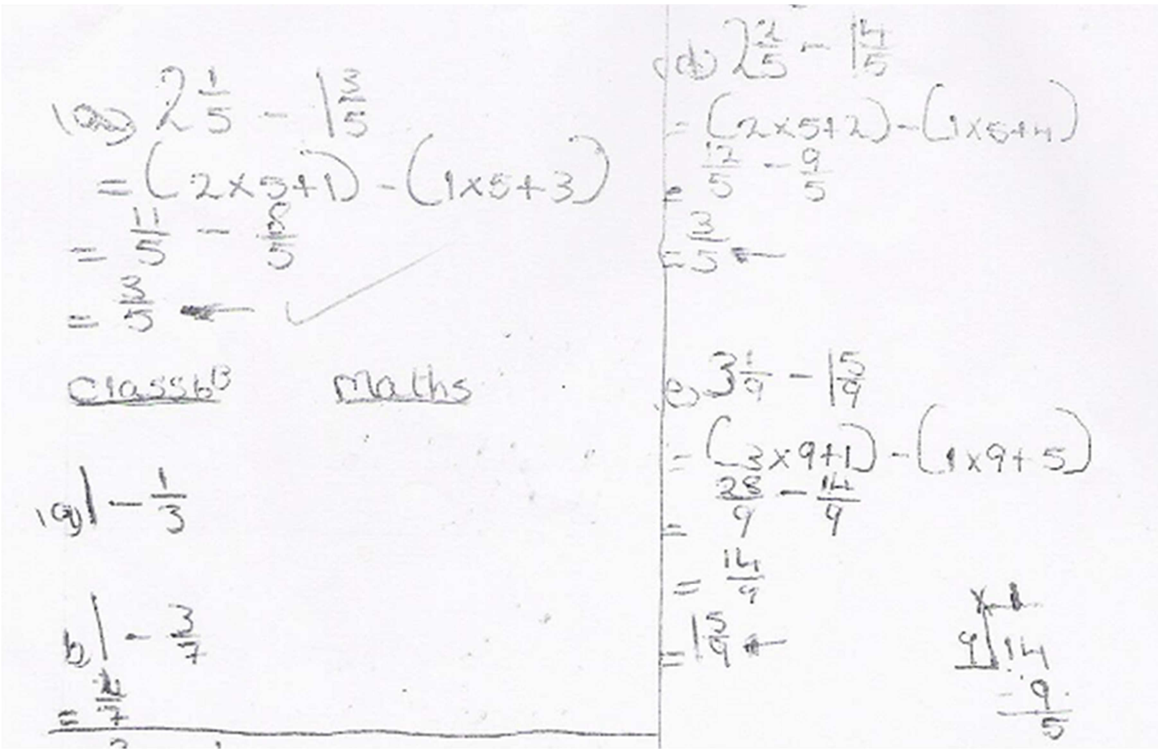


Figure 5.14 Workings of teacher C's learners

When marking the learners' work, the teacher either wrote a tick or a cross. Sometimes for wrong answers she drew a circle around the mistake. She did not give feedback on what learners should have done. She often marked the first ten learners to finish the class work. The rest of the class would know whether they got them correct or not when the questions were done on the chalkboard.

During each lesson the teacher gave introductory review questions and assessment tasks. In both of these exercises, the teacher marked the first ten learners to finish the work. For both exercises the teacher gave learners chance to work the questions on the chalkboard. This is where learners whose work was not marked would see whether they got the questions correct or wrong. The last exercise that she gave was at the end of the lesson. In this exercise the teacher marked most of the learners' work but often questions were not solved on the chalkboard.

5.3.5 Reflection on the observation and analysis of the documents

5.3.5.1 Reflection on the observation of teachers A, B and C

When teaching fractions all three teachers did not use concrete teaching materials that are found from the learners' context. They used ready-made fraction boards and fraction charts. Even when using these ready-made materials, learners did not manipulate them; instead teachers demonstrated with them and drew representations on the chalkboard. Learners then answered questions referring to the fraction board drawn on the chalkboard.

In the earlier lessons, teacher C made an attempt to elicit reasoning from the learners before giving them answers. She also tried to engage learners by giving them paper cutting exercise through which learners were able to order fractions in order of size. This experience provided learners with an opportunity to handle concrete materials visualise fraction parts and hence gain conceptual knowledge (cf. 2.5.3.2).

Teachers asked closed-ended questions in which learners did not need to think but recall what was done earlier. These were either oral or written. In the case of written work learners worked individually. These types of questions did not give learners the opportunity to explain or reason, hence no high order thinking skills such as inductive and deductive reasoning (cf. 3.4) were incorporated in these teachers' classrooms. On few occasions teachers B and C asked learners to give reasons for their answers.

It was discussed in chapter two that concepts should be related since learning is cumulative (cf. 2.5.2). Regarding learners' prior knowledge teachers did not use it to its greatest advantage. The prior knowledge that teachers used was with regard to what was done in previous lessons. They would ask learners what they did in the previous lesson but did not use all the knowledge of mathematics that they knew. For example, when adding fractions, teachers A and C did not bring in addition of whole numbers and compare it with addition of fractions. Literature recommends that teachers use learners' out of school mathematics so that learners could understand that mathematics is not abstract and meaningless (cf. 2.5).

It was mentioned, above that learners did not handle and manipulate materials, and that the questions given did not allow them to explore and experiment. Teachers therefore provided rules and procedures to use when working with fractions. The class room environment did not give learners the opportunity to construct their “own” rules and hence answer questions using their own strategies (cf. 2.3.1). When teaching about ordering of fractions, teacher C tried to help learners find the pattern by themselves. In as far as other concepts were concerned she also provided learners with rules and procedures to use when working with fractions.

Teacher C taught equivalent fractions in two separate lessons, the second and the fourth lesson. The first time she used fraction board and diagrams to help learners identify that different fraction pieces were not equal. The second time, equivalent fractions were calculated. When asked why she did not teach in two consecutive lessons she indicated that she remembered that she did not teach how to find equivalent fractions by calculation.

Learners experience different barriers to learning (cf. 2.6). One of the teachers’ tasks is to be aware of these barriers to effective learning so that they could help learners overcome them. The teachers did not seem to consider learners’ barriers because they did not engage different teaching methods in one lesson so that most of the learners’ learning styles could be catered for (2.6.2). Learners who learn best by doing were not catered for because teachers did not design their lessons in such a way that learners could handle and experiment with real objects. This approach could lead to learners experiencing very limited level of motivation and lack of experiential leaning (cf. 2.5.2.3 & 2.6.1).

It is indicated above that learners’ prior knowledge was not used to its maximum, hence learners were not helped to identify relationships. This may result in fragmented learning as discussed in section 2.6.3. Fragmentation result when teachers do not relate fraction addition and subtraction to equivalent fractions.

Inappropriate language may result in a learning difficulty; hence teachers are encouraged to use appropriate language. When building fraction concepts, all three teachers moved from context, pictures, symbols and language.

Teacher B talked of sharing a number. This might result in learners encountering difficulties as it was discussed that inappropriate language use could result in learning difficulties (cf. 2.6.4). When calculating equivalent fractions, teacher C referred to numerators and denominators as sides so this is a language issue that could result in learners experiencing difficulties.

Incorporating assessment, when teaching, has been found to maximise teaching and learning (cf. 3.6). Through formative assessment, teachers engage in the diagnosis of learning, such as asking questions to determine understanding and/or difficulty and hence providing learners with feedback to assist learning. Teachers did not make an attempt to find causes of difficulties and errors that learners made. They either endorsed a cross next to the learners' answer or just made a verbal comment that they were wrong.

All three teachers did not give learners novel mathematics problems to challenge them. Learners were given questions through which they executed rules and procedures they learned in class. Learners were neither taught problem solving or were taught through problem solving as discussed in literature (cf. 3.5.2).

Section 3.7.3 discussed the recommendations that teachers should follow in order to teach fractions effectively. It was suggested that the five subconstructs of fractions should be taught because this provides a holistic approach to fractions. All three teachers only taught the part-whole subconstruct. Teachers did not even incorporate the two aspects of part-whole subconstruct, which are, partitioning the whole object and/or a set of similar objects. Teachers mainly partitioned the whole object and engaged learners in finding fractions of each piece. As discussed this approach could result in fragmented knowledge of fractions and hence learners incur learning difficulties in higher mathematics (cf. 2.6.3).

The three teachers are compared in table 5.1 below.

Table 5.1 Summary of class observations

Effective learning EL	**Teacher A**	**Teacher B**	**Teacher C**
Manipulation of concrete materials	None	None	Paper cutting once
Prior knowledge	No out of school mathematics	No out of school mathematics used	Prior mathematics learned was used. No out of school mathematics
Conceptual knowledge before procedural knowledge	Taught procedures. No relationships formed	Taught procedures. No relationships formed	Taught procedures. No relationships formed
Learning environment	Did not enhance discovery through experimentation	Did not enhance discovery through experimentation	Did not enhance discovery through experimentation
Group work	None	None	None
Formative assessment	Very little	Very little	Very little
Handling of learning barriers	Not well done	Not well done	Not well done
Inductive and deductive reasoning	Not done	Very limited	Very limited

5.3.5.2 Reflection on the lesson plans used by teachers A, B and C

All three teachers wrote the topic, date and objectives in all their lesson plans but did not write teaching methods and teaching materials in all lesson plans. Regarding procedures to follow during the lesson, teachers wrote them clearly. They indicated the teacher's and learners' activities. For all three teachers under the teacher's activities, they gave sample questions that they would ask learners. In some cases teachers wrote the answers they expected learners to give.

None of the teachers wrote problems, errors and/or difficulties they expected learners to have. When such errors occurred, teachers were unable to help learners, hence did not give learners constructive feedback.

Lessons were assessed by giving closed questions extracted from the prescribed text book. Such questions were not rooted in the learners' context.

In most of the lessons, teachers asked oral questions, demonstrated with ready-made materials and/or one learner would write answers on the chalkboard. Learners were not engaged in the manipulation of concrete materials hence teachers did not use experimentation in their lessons.

Table 5.2 Summary of teachers' lesson plans

Contents	**Teacher A**	**Teacher B**	**Teacher C**
Topic, date, objectives	Done	Done	Done
Teaching methods and teaching materials	Not always	Not always	Not always
Procedure	Clearly stated	Clearly stated	Clearly stated
Potential problems and pitfalls	Not indicated	Not indicated	Not indicated
Methods of assessment	Written answers to closed questions	Written answers to closed questions	Written answers to closed questions
Experimentation	Not done	Not done	Done once in the series of lessons

5.3.5.3 Reflection on the exercise books of learners in schools A, B and C

Learners in all three classes partitioned the pages of their mathematics books in two and this made the presentation of their work neat and easy to read. When answering questions, they wrote the steps going down the page such that the final answer appeared at the bottom. Teacher C even asked learners to draw an arrow pointing to the final answer. She emphasised this because she did not mark any learners' work that had not drawn an arrow.

All three teachers did not employ formative assessment because on the learners' exercise books, teachers did not write comments that would guide learners. They simply wrote a tick for correct answers or a cross for wrong answers.

The amount of work given to learners was also little because teacher A only gave assessment tasks and sometimes did not give assignments (cf. 5.2.5) while teacher B gave two or three assignment questions (cf. 5.3.3.3). Teacher C gave more work than the other two teachers because it was indicated that she gave learners three different sets of tasks in one lesson (cf. 5.3.4.3).

All teachers gave learners rules and algorithms, so all learners used rules and algorithms given by the teachers when answering questions. Table 5.3 summarises the findings from learners' exercise books.

Table 5.3 Summary of learners' exercise books

Contents	**Teacher A**	**Teacher B**	**Teacher C**
Layout and neatness	• Page divided into two • Writing steps going downwards • Neat	• Page divided into two • Writing steps going downwards • Neat	• Page divided into two • Writing steps going downwards • Neat
Use of own strategies	Not	Not	Not
Use of algorithms	Every time	Every time	Every time
Marking	Marked all learners	Did not mark all learners	Did not mark all learners

5.3.6 Interviews with teachers

It was indicated in chapter one and four that after conducting class observations the researcher would interview each teacher in order to understand each teacher's view regarding the teaching of fractions. From the three interviews five themes were identified and they were: planning, effective teaching, effective learning, teaching materials and choice of teaching strategies.

Tables 5.4 (a), (b) (c) (d) and (e) below summarise the analyses of the interviews with the three teachers.

5.3.6.1 Planning

Planning is an essential element of teaching. It was discussed in chapter two that children learn differently at different stages of development, hence Piaget's stages of cognitive development (cf. 2.3.1). It was discussed that teachers should know the learners context before teaching. It is in this regard that planning is an essential aspect of teaching.

From interviews, teachers indicated that they planned before teaching. Teacher A said that while planning he decided on the teaching materials that would help him deliver the content he intended to teach.

Teacher B on the other hand said that when planning she bore in mind the age of learners, their prior knowledge and then she wrote a lesson plan.

Teacher C also considered learners' age, prior knowledge, teaching materials and methods.

Table 5.4(a) Planning

Themes	**Teacher A**	**Teacher B**	**Teacher C**
Planning	*I firstly look at the topic that I want to treat and I would go for those teaching aids which I think and believe that they would err… reinforce the content*	*I consider the age of the group I am planning for, the content they have. ...I write a lesson plan*	*The things that I consider are the; class, topic, prerequisite knowledge that the learners in that class are supposed to have, then the materials, and the methods*

5.3.6.2 Effective learning

Chapter two discussed features of effective learning which teachers should be aware of when teaching. The table below displays teachers' views regarding effective learning.

From interviews teacher A indicated that when teaching he considered learners' prior knowledge as it helps build new knowledge on it and hence effective learning results.

Teacher B on the other hand indicates that when learners are able to answer questions it indicated that learners have acquired effective learning.

Teacher C considered using concrete materials to enhance effective learning.

Table 5.4(b) Effective learning

Theme	Teacher A	Teacher B	Teacher C
Effective learning	*I first started with the topic which I think they were familiar to. Their prior knowledge that would help me build up on that knowledge for this new one that I want to teach*	*If they are able to answer questions and when I give them an exercise they get them right I move on to something else because that shows that they have understood*	*When you take an orange and share it among say 3 children equally they see that the pieces are equal*

5.3.6.3 Effective teaching

Teacher A said he ensured effective teaching by connecting learners' prior knowledge to the new content that was being taught.

Teacher B on the other hand said that she emphasised rules because that was what mathematics is.

Teacher C said that she engaged her learners in the partitioning of whole objects when teaching fractions. When the partitioning was done she then asked learners questions to ensure that they have understood what they were doing.

Table 5.4(c) Effective teaching

Theme	Teacher A	Teacher B	Teacher C
Effective teaching	*Normally emm… take them from what they already know to this thing that I want to introduce*	*Mathematics is full of rules so I make sure that they are able to use those rules because they need them when they write examinations*	*We only partition wholes in class. I do not use other methods I only give them questions when I come across them in the text book*

5.3.6.4 Teaching materials

Teacher A said he used materials that are easy to get. Teachers B and C said that they were aware that they should use concrete materials to help learners understand what is taught. But teacher B added that concrete materials like food items were not easy to get as they were expensive

Table 5.4(d) Teaching materials

Theme	Teacher A	Teacher B	Teacher C
Teaching materials	*I choose materials firstly by looking at how available they might be, I don't look for those abstract and are… hard to get*	*I know that they should use real things like paper, oranges, sweets and you name them. ...And if you buy them yourself it becomes expensive*	*So I have learned that working with tangible things help them to understand*

5.3.6.5 Teaching strategies

Teachers A and C did not say anything about how they decided on teaching strategies. Teacher B specified that she chose teaching strategies depending on the number of learners in the class, the topic and the availability of teaching materials that learners could use.

Table 5.4(e) Teaching strategies

Theme	Teacher A	Teacher B	Teacher C
Choice of teaching strategies		*I use teaching strategies depending on the topic, availability of teaching materials and the number of learners in the class.*	

5.4 INTERVIEWS WITH TEACHER TRAINERS

5.4.1 Profiles of teacher trainers

Lecturer 1

Lecturer 1 is a male in the early thirties. He has been training mathematics teachers for six years. His highest qualification is Master of Education in mathematics.

Lecturer 2

Lecturer 2 is a female in the late forties. She has been training mathematics teachers for 15 years. Her highest academic qualification is Master of Education in curriculum studies.

5.4.2 Interviews

For both lecturers interview sessions were conducted in the morning in their offices. The interview session with lecturer 1 was scheduled at eight o'clock in the morning. With lecturer 2 it was on the same day at ten o'clock in the morning.

The interview with lecturer 1 began at quarter past eight and it took about one and a half hours. With lecturer 2 we began at ten as we planned.

The lecturers used different offices. During the interview sessions they were the only one present. I did not ask about the whereabouts of the other office occupants.

I asked the lecturer to explain in detail the type of training they offered mathematics teachers especially those who would be teaching in primary schools.

The lecturers indicated that the college offers a three year teachers' diploma. Student teachers spend two years, first and third year in the college while they spend the second year in schools doing teaching practice. During the first year they are offered two courses, namely methodology and content.

The analysis of the interviews of teacher trainers was done according to the themes identified from literature discussed in chapters two and three.

5.4.2.1 Methodology

The lecturers indicated from the interviews that, in the methodology course, mostly done in first year, student teachers were taught the essentials of teaching such as: the psychology of child development and learning; designing and selection of appropriate teaching materials; different teaching methods; and designing lesson plans.

5.4.2.1.1 Child development

Effective teaching encompasses, among others, teachers' knowledge of the children's level of development. This knowledge helps teachers to decide on appropriate classroom discourse (cf. 2.3 & 2.5.2.2).

Table 5.5 Child development

Theme	Lecturer 1	Lecturer 2
Child development	We train student teachers about the child development. You know children learn differently at different stages of development. So since primary teachers are going to work with children at different stages we train them in that regard. When they teach fractions in class one and class five or six, they should know what activities they should give them depending on what they are capable to learn.	First of all I think the most important thing that we expect our student teachers to have is to have the knowledge of a child, as child as in the um… psychology that surrounds the child as a person, that is, how the child develops so that they can approach the children knowing exactly what they expect out of them, secondly when they have learned about the child then we expect them to know how the child learns for example using concrete materials, using pictures using all those things, we expect them to know those things and how to use them at different stages

Unlike Lecturer 1, Lecturer 2 explained that in standard 1 learners are supposed to manipulate concrete materials whereas in higher classes they begin to learn by using pictures and symbols.

> ***Lecturer 2:*** *In standard one we expect our students to be using concrete materials most of the time, but when you go further you find that when you go to upper classes they learn by using pictures and then from there we can look at abstract symbols that we use in mathematics, we can go to symbolism now.*

5.4.2.1.2 Teaching materials

Regarding teaching materials, both lecturers explained that they helped student teachers to develop teaching materials for different topics. This is what they said regarding teaching aids:

Table 5.6 Teaching materials

Theme	Lecturer 1	Lecturer 2
Teaching materials	ok one of the things that we help them do here is to construct teaching materials	So when we have addressed the teaching methods which we think are more effective in the teaching of mathematics then we help them to because we understand that if the student can be able to understand the learning aids, the teaching aids then they are going to own the concept and it also makes mathematics easier to teach

5.4.2.1.3 Lesson planning

The lecturers further indicated that they teach student teachers how to write a lesson plan and the importance of having one before one enters the classroom.

Table 5.7 Lesson planning

Theme	Lecturer 1	Lecturer 2
Lesson planning	We train them on how to design lesson plans, to do this they should know how to construct objectives, decide on teaching materials and teaching methods that will help them achieve the objectives We give them chance to do presentations in class.	So if they have the teaching methods, they understand the child, they have teaching aids, and then we plan our lesson, because it's through planning the lesson that you can deliver. That is what we have discussed. So we help them to make the lesson plan and to make sure that they really understand how a lesson has to be taught we have what we call peer teaching, where they present in the class.

5.4.2.1.4 Teaching methods

Regarding teaching methods, both lecturers explained that they help student teachers to use teaching methods that help learners discover rules for themselves. The choice of appropriate teaching methods seems to correlate with the teachers' PCK and MKT. This is because in section 3.3 it is mentioned that both PCK and MKT enable teachers to know and understand what makes learning of certain topics easy or difficult. This is what they said regarding teaching methods:

Table 5.8 Teaching methods

Theme	Lecturer 1	Lecturer 2
Teaching methods	We actually discourage them from giving rules because mathematics is not about rules; rules only help them when they have understood the concept then they can use the algorithm just for efficiency	We also have those teaching aids that we say if you use this one then you ask students different questions based on this teaching aid then they will be able to say, at the end of the day, oh… this is the formula for finding this and if they have gone through that process of knowing exactly how we got the formula then they will not have the problem of forgetting and mixing things up.

Lecturer 1 explained in detail how they train student teachers to teach fractions constructively.

> ***Lecturer 1****: In the methodology part we have a section where we teach fractions. First we unpack the knowledge of what is a fraction, we start with a whole and then we start dissecting some pieces. Recently we have introduced something very interesting, the concept of slicers where we show them how to add fractions practically or by means of drawing, they can draw some sketches to add, subtract multiply and divide fractions.*

Lecturer 1 raised his concern that, regardless of what they were taught, student teachers teach rules when they get to schools.

Lecturer 2 explained that they teach teaching strategies appropriate for teaching mathematics. She said these are: discussions, discoveries and investigations.

5.4.2.2 Content

Section 3.3 indicated that teachers needed to possess different bodies of mathematical knowledge, CKM, SKM and MKT. So the content that student teachers were taught was both CKM and SKM because they needed both knowledge base as mathematics professionals and mathematics teachers. The teachers needed CKM not only for their day-to-day teaching but also when they furthered their studies.

During peer teaching the lecturers have indicated that student teachers use the mathematics they used from high school. But they selected the content from the primary mathematic syllabus.

> ***Lecturer 2:*** *We give them content that will help them teach effectively but which will also help them further their studies. In other words we don't say, since you are going to teach primary students we are going to give you primary content ah… ah… (laughs) we go further and say even if you want to go further with your studies we have to give you enough content that will enable you to further your studies*.

Lecturer 1 did not say anything about the type of content they taught. He paid more attention on the methodology.

5.4.3 Analysis

Both lecturers have indicated that during the first year of training, student teachers are taught both content and methodology. By methodology they meant the child development in relation to learning, selection and design of teaching materials, types of teaching strategies, lesson preparation and actual teaching. They indicated that under the methodology they also unpacked the knowledge base of different topics such as fractions.

The lecturers have indicated that student teachers selected content from the primary school mathematics syllabus. Since most of the student teachers have not taught before, they worked in groups when preparing for lessons. They brought their lesson plans to the lecturers and they discussed it together.

Then one of the group members would teach the lesson to the whole class after which the whole class would analyse the lesson.

During year two, student teachers go to schools to practise teaching for the whole year. During this period, lecturers visit student teachers at their respective schools to observe them teach.

Regarding the content, the lecturers indicated that the type of content they teach is not what the teachers will teach when they get to schools. This is the mathematics content that will help when they further their studies. This is CKM that will help teachers gain higher reasoning skills and participation in higher institutes of learning (cf. 3.3).

Teachers A, B and C teaching strategies seem to accord with lecturer 1's concern that teachers taught rules when they got to schools (cf. 5.2.7) regardless of how they were trained to teach. Again the teachers presented fractions by partitioning and by symbols as said by lecturer 2 that in lower classes learners must handle concrete materials but in higher classes teachers should teach by partitioning and symbols.

5.5 CONCLUSION

In this chapter, data collected from all the three schools was analysed and interpreted. The biographical data of each teacher, the context of each school and each class room were discussed. Class observations were discussed and compared in order to identify the similarities and differences in the teachers' classroom activities. Class observations were then followed by interviewing teachers. As discussed in chapter four, this was done to get deep understanding of each teacher's view on the teaching of fractions.

The teachers' lesson plans were evaluated in order to determine how teachers plan for their lessons and to see the type of questions they gave to the learners. Learners' mathematics exercise books were also analysed with the purpose of determining whether learners used their own strategies or rules when doing mathematics.

Teacher trainers were also interviewed with the intention of determining the level of training that teachers received. These interviews were discussed and analysed.

In the analysis the teachers’ actual teaching were compared with their responses to the interview questions, the learners’ workings to given questions, and the teacher trainers’ responses to interview questions.

CHAPTER 6

FINDINGS, CONCLUSIONS AND RECOMMENDATIONS

6.1 INTRODUCTION

Chapter one introduced the study by presenting the problem, the purpose and the significance of the study. The research methodology and the procedure that were applied in the study were also outlined. Chapters two and three reported on the literature in which the aspects that determine effective learning and teaching of mathematics with emphasis on fractions were discussed. Chapter four then discussed in detail the empirical research. In chapter five interpretations of class observations and interviews, teachers' lesson plans' and learners' mathematics exercise books were done. From this analysis and interpretation, the objectives of the research were met. In this chapter therefore the research findings regarding the research objectives will be discussed. These objectives are:

- Determine what literature is saying on effective learning of fractions by learners
- Determine what literature is saying on effective teaching of fractions
- Determine the level of training given to mathematics teachers
- Determine whether effective teaching is taking place
- Determine whether effective learning of fractions materializes in selected classes
- Recommended guidelines on how to teach fractions effectively

6.2 RESEARCH FINDINGS

6.2.1 Effective learning of fractions

The first and fifth objectives of this study are addressed in this section.

6.2.1.1 Findings from literature

From literature it is found that effective learning is a constructive, cumulative, goal oriented, collaborative, situated, self-regulated and individually different process of knowledge building through meaningful construction (cf. 2.5.2). Literature therefore indicates that, in order for teachers to enhance effective learning, especially of fractions, in this instance they should help learners manipulate concrete materials which learners use in their real life.

Section 2.3.2 suggests that learners could be allowed to work in small groups as they experiment with concrete materials. In those cooperative groups learners should communicate among themselves. Through communication and experimentation learners may connect new knowledge to their prior knowledge and come up with their own strategies to solve problems. If learners manage to accommodate and assimilate new knowledge to the existing schema, they will develop conceptual knowledge and hence develop effective learning results (cf. 2.3.1 & 2.5.3.1).

Literature also depicts that learners who have conceptual knowledge are able to solve unfamiliar problems as they do not have fragmented knowledge (2.6.3). Fragmented knowledge is said to result when learners are not helped to establish relationships among different concepts such as, equivalent fractions and addition of fractions.

It is also said that for effective learning to take place, the learning environments should permit learners to explore and work cooperatively with others (cf. 2.5.1). When learners explore they use most of their senses and in this regard effective learning is said to take place. If learners are not allowed to employ multiple senses when learning, this may result in a learning barrier as learners are individuals and learn differently.

Motivation is also found to be an important aspect of learning. Lack of or very low motivation may serve as a learning barrier (cf. 2.6.1). Learners who are motivated are able to learn even under difficult conditions but those with very little or no motivation give up when they face challenges. It is in this regard that learners' motivation level should be kept high so that learning can occur.

6.2.1.2 Findings from empirical research

From the empirical research it was found that learners' context and hence their out of school prior knowledge was not used when learning mathematics, fractions. Even when they did, they used the knowledge of mathematics that learners learned in school not the informal knowledge gained out of school.

Learners memorised procedures and reproduced them when answering questions. They did not handle concrete materials, instead, teachers demonstrate with ready-made teaching materials. The most dominant ready-made teaching materials in all three classrooms were fraction boards and fraction stacks.

6.2.1.3 Conclusion

The findings regarding effective learning indicated that learning environments in the three classrooms did not facilitate effective learning. Learners memorised procedures and rules and reproduced them when answering questions.

The teaching and learning discourse that transpired in the classrooms may result in fragmented knowledge as learners were not exposed to experimentation and discovery of relationships and rules.

6.2.2 Effective teaching of fractions

This section presents findings regarding the second and fourth objectives:

6.2.2.1 Findings from literature

It was found from literature that effective teaching begins with thorough planning. Teachers should know, *inter alia*, the age group of the learners in class and their socio-economic status. This information helps teachers select appropriate content, teaching materials and activities that will facilitate teaching and learning.

It was found from literature that from age 11 years, learners are at the beginning of the concrete operational stage (cf. 2.3.1.3). Learners at this age and class have learned basic concepts of fractions. It was explained that for effective teaching of fractions, teachers should help learners gain conceptual knowledge before procedural knowledge. In order to do this, teachers are advised to create opportunities for learners to experiment by handling concrete materials. During experimentation, learners may come up with their own methods and hence gain conceptual knowledge. As learners experiment they may work in groups in which they share ideas and help each other. Through collaborative groups learners are said to gain essential skills like communications.

Teachers are also encouraged to teach fractions through problem solving. It is said in literature that through problem solving learners can gain skills like critical thinking, communication and many more essential skills for a mathematician (cf.3.5).

Teachers are again advised that during problem solving, teachers should ask learners a series of questions that will help them engage intensively in analysis and critically think of what they do (cf. 3.5.1). If teachers are able to create situations in which learners are able to solve and deduce solutions then effective teaching would have occur.

Literature also depicts that for effective teaching to take place, teachers should engage in a continuous assessment of learning and provide learners with constructive feedback. That is teachers should engage in formative assessment (cf. 3.6.1.1). Assessment for learning is said to enhance learning because learners are provided with useful information that helps them know what they should do. Embarking continuously on assessing learners, teachers may even identify learners' difficulties and/or errors and hence help learners overcome them.

Teachers are also encouraged to engage in multiple representations of fractions. That is they should help learners conceptualise fractions as a part-whole, quotient, ratio, measure and operator (cf. 3.7.3). Literature indicates that learners, who have a holistic understanding of the concept fraction, will not have problems handling fractions in different situations. Then learners who are able to work with fractions in different contexts are said to have effectively learned the concept (cf. 2.5.3.1).

It is worth noting that literature explains that in order for teachers to be able to achieve all of the above, they should have two bodies of knowledge, PCK and MKT (cf. 3.3). Both PCK and MKT help teachers to plan their teaching and assess both teaching and learning (cf. 3.3).

6.2.2.2 Findings from empirical research

It seemed that the teachers had a grasp of the psychology of learning hence they all used concrete materials but not really relevant to the context of the learners. They did not though create learning environments in which learners handled and experimented with concrete materials. Instead teachers demonstrated with these materials and provided learners with rules.

Teachers did not help learners develop conceptual knowledge. The cumulative and collaborative aspects of learning were not considered as teachers did not consider learners' prior knowledge and they did not employ group work in their classes. Learners' out of school knowledge of mathematics and real life concrete materials were not used. In this regard teachers did not give learners opportunities to solve mathematical problems. Learners did not engage in inductive and deductive reasoning. They were taught rules and procedures which they memorised and retrieved when needed, and were not given time to do mathematics their own way.

Continuous formative assessment and more specifically constructive feedback was not used. This was evident because in their lesson plans teachers did not identify and/or specify difficulties that learners may experience so that they could prepare strategies to address them should they occur during their lessons. The importance of predetermining possible errors and difficulties was evidenced in teacher B's lesson when she was not able to help a learner who made an error when simplifying $\frac{2}{8}$ to $\frac{1}{4}$ (cf. 5.3.3.3). So teachers did not handle learners' difficulties and provide them with feedback that facilitated learning.

All three teachers taught the part-whole sub-construct only. They did not represent fractions as measure, ratio, quotient and operator. This may result in fragmented learning and hence impede further learning of fractions or mathematics in general.

6.2.2.3 Conclusion

Teachers who participated in this study seemed not to indulge in effective teaching. The type of learning environments they created in their respective classes did not support knowledge construction. They did not engross learners in tasks which demanded high order thinking, formation and discovery of relationships among mathematical concepts. Learners learned and practised rules and procedures used to operate fractions.

Formative assessment, and specifically feedback, was used to the minimum because teachers did not try to identify difficulties that learners experienced. Even when they identified that certain learners experience difficulties they did not help such learners overcome them.

The part-whole sub-construct was the dominant sub-construct in these three classrooms. The other four were not used hence effective teaching was very minimal in these classrooms.

6.2.3 Level of training given to mathematics teachers

In this section findings regarding the third objective are presented.

6.2.3.1 Findings from literature

It is found from literature that teachers should possess different bodies of knowledge. These are: the psychology of child development and learning (cf. 2.3) factors that determine effective learning and how to enhance effective learning (cf. Chapter 2) common and specialised knowledge of mathematics (cf. 3.3) MKT and PCK (cf. 3.3). All aspects of assessment, ranging from assessing learners' socio-economic status of the learners to assessing learning (cf. 3.6). Designing and planning of the curriculum through scheming and lesson planning.

6.2.3.2 Findings from empirical research

From interviews with teacher trainers it was found that mathematics teachers are taught the psychology of child development and learning, designing teaching materials and lesson plans. Teacher trainers indicated that designing teaching materials help teachers to understand the structures of mathematics that are to be delivered through those materials. This is what is called specialised knowledge of mathematics (cf. 3.3). Teachers are also taught common knowledge of mathematics.

Teacher trainers also indicated that teachers are trained on how to plan for teaching. By this they referred to designing a lesson.

6.2.3.3 Conclusion

What teachers did in class seems to correlate with what teacher trainers indicated that they have taught. All three teachers wrote a lesson plan before going to class. The lesson plan of all three teachers contained the same contents. This is supportive to the information obtained from the teachers, namely that that they were trained at the same institution.

When teaching fractions all three teachers used concrete materials, although not contextualised, indicating that they knew how learners learn at this age.

Based on the conclusions of the first two objectives the effectiveness of the teacher training can be questioned.

Teacher trainers did not say anything about teaching teachers to use assessment and how to incorporate assessment when teaching. It was evident in the classrooms that teachers did not have appropriate training on how to continuously assess learning through implementing formative assessment.

6.3 RECOMMENDATIONS ON HOW TO TEACH FRACTIONS EFFECTIVELY

The following are some of the recommendations that teachers may follow when teaching fractions in order to enhance effective learning:

- Teachers should use readily available materials such as; paper, small stones and the learners themselves. They are readily available and teachers do not need money to have them
- Learners must also work collaboratively in groups where they share ideas
- Teachers should facilitate learning by helping learners identify relationships between and among different concepts

- When planning for a topic, teachers should identify misconceptions that learners may have, errors they may make and difficulties they may encounter and plan accordingly to address these barriers to learning

- Teachers should do thorough lesson planning and address all aspects of teaching that will facilitate effective learning

- Teachers should read research on mathematics learning and teaching so that they stay up to date with recent developments in the field

- Teachers should have forums in which they discuss their teaching and help each other improve their teaching

- The number of learners in class should be reduced to between 20 and 30. A smaller teacher learner ratio helps teachers identify possible barriers to learning such as different learning styles, cultural background and other aspects. Teachers can then adapt their teaching to ensure effective learning for all

6.4 RESEARCHER'S CHALLENGES

The researcher is a full time teacher so she faced a lot of challenges when conducting this study. These challenges were with regard to

- Time: The researcher struggled to keep a balance between work, family and her studies.

- Participation of teachers: Teachers did not like to be observed when teaching. The researcher wanted to include teachers from other schools as well but those teachers refused to participate in this study because they did not want to be observed.

6.5 SUGGESTIONS FOR FUTURE RESEARCH

The following areas on researching classroom teaching and learning need to be investigated:

- The implementation of formative assessment to improve teaching and learning of mathematics

- How to engage learners in the manipulation of concrete materials that are contextualised so that they can construct their own knowledge
- How to create powerful learning environments that will enhance critical thinking and discovery of procedures
- How to identify barriers to learning and how to address them so that effective learning could be enhanced

6.6 OVERALL CONCLUSION

This study was aimed at understanding how mathematics teachers in Lesotho primary schools teach fractions. In order to execute this study, objectives were formulated as stated in chapter one.

The research process as described in chapter one, was such that class observations and analysis of teachers' lesson plans and learners' exercise books were conducted. This was done to determine the type of contents the teachers included in their preparation and also to find out the type of questions they posed to learners. At the end of the whole class observations, the respective teachers were interviewed together with teacher trainers.

The aim of the research was met as the researcher determined that teachers predominantly use lecture and demonstration methods when teaching fractions. It is also found that learners in these classes did not meaningfully learn fractions. This was because of insufficient manipulation of concrete objects by learners and also that learners are not helped to connect concepts and formulate relationships and generalisations.

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APPENDICES

Appendix A: OBSERVATION SHEET

Date ______________________________

Class_____________________________

Topic _____________________________

Duration __________________________

Behaviours	How behaviours are demonstrated	Other behaviours
1. Teaching strategies:		
2. Nature of problems: • recall questions • Application questions		
3. Individual, Group work or whole class discussions		
4. Are rules given and practiced?		
5. Do learners manipulate concrete materials to develop conceptual knowledge before they are given rules		

6. Are students given opportunities to communicate their understanding with their classmates		
7. Identifying learners' misconceptions and difficulties		
8. Addressing and rectifying learners' difficulties and misconceptions		
9. Alerting learners of mistakes they are likely to make		
10. Use of concrete materials		

Appendix B: Interview questions for class teachers

1. When planning to teach a topic, which things do you consider, that will help enhance effective learning and teaching of that particular topic?
2. Can you also explain what influences you when you decide on the teaching strategy?
3. Which teaching strategy do you often use and why?
4. In your own opinion does it work for you?
5. What makes you think so?
6. When teaching the first lesson on fraction what did the learners know about fraction?
7. How do you perceive the importance of prior knowledge in your teaching?
8. What is your understanding of a mathematics problem?
9. How do you use problem solving as a teaching strategy?
10. How do you think problem solving as a teaching strategy enhances effective learning of mathematics?

Appendix C: Interview question for teacher trainers

Can you please tell me what knowledge base you expect teachers to have at the end of training that will help them teach mathematics effectively?

Appendix D: Application letter to the Ministry of Education

P.O. Box368

Morija 190

16 May 2011

The Principal Secretary

Ministry of Education and Training

Maseru 100

Dear Sir

RE: Application for conducting research in Morija Primary Schools in the district of Maseru

I wish to apply for permission to conduct research in primary schools in Morija.

The title of my research dissertation is:

Teaching and learning of Fractions in schools in Maseru

The research is part of my studies I am doing with the University of Free State. The research is intended to inform myself and other teachers and empower us in our teaching career.

The study will not interfere with the normal running of the school and all the ethical considerations, such as confidentiality of the results and findings of the study and the protection of participants' identities will be observed.

I am thanking you in advance for your attention.

Yours sincerely

'Maphole Marake

Appendix E: Permission letter from the Ministry

THE KINGDOM OF LESOTHO
MINISTRY OF EDUCATION AND TRAINING

ED/X/1 | 08th July, 2011

'Maphole Marake
University of Free State
Department of Curriculum Studies
Faculty of Education
Bloemfontein

Dear Madam,

Re: APPROVAL TO CONDUCT RESEARCH WITHIN THE MINISTRY AMONG THE PRIMARY TEACHERS IN MORIJA

The captioned matter bears reference.

The Ministry of Education and Training appreciates your interest in conducting this study and hence grants its approval.

It is hoped that you shall share your research findings with the Ministry. The information thus obtained will help us to improve this programme and others similar to it. Looking forward to your report.

Yours faithfully,

THUTO NTŠEKHE- MOKHEHLE (MS)

Cc: SEO - Maseru

Appendix F: Application letter to the schools

P.O. Box 368

Morija 190

01 August 2011

The Principal

Dear Madam

<u>RE: Permission to do research at your school</u>

I am a registered student for the Masters' degree at the University of the Free State. As part of my degree, I need permission to conduct a research project at your school.

The title of my research is

Teaching and learning of Fractions in primary schools in Maseru

Three of my research aims are to:

Reflect on the teaching strategies predominantly used by teachers.

Reflect on what teachers do to elucidate that learners have meaningfully learned fractions.

Reflect on the students' explanations to the strategies they use to solve problems regarding fraction.

I am therefore asking for permission to observe all the lessons on fractions in standard six.

The study will not interfere with the normal running of the school and all the ethical considerations, such as confidentiality of the results and findings of the study and the protection of participants' identities will be observed.

Attached is a copy of the letter of approval from the Ministry of Education and Training.

I am thanking you for your attention.

Yours faithfully,

‘Maphole Marake (Mrs)

Appendix G: Letter to teachers

P.O. Box 368

Morija 190

01 August 2011

Dear teacher

I am a registered student for the Masters' degree at the University of the Free State. As part of my degree, I am cordially asking for your permission to observe your lessons on fractions.

The title of my research is:

Teaching and learning of Fractions in primary schools in Maseru

I am thanking you in advance for your participation.

Yours faithfully

'Maphole Marake (Mrs) Teacher's signature

…………………………………………………

Appendix H: Application letter to interview lecturers at LCE

P.O. Box 368

Morija 190

12 August 2011

The Director

Lesotho college of Education

Maseru 100

Dear Sir

RE: Application to interview mathematics teacher trainers

I am a registered Masters student at the University of Free State and I wish to apply for permission to conduct research in your institution. The title of my dissertation is

Teaching and learning of Fractions in primary schools in Maseru

One of my research aims is to determine the level of training given to teachers to enable them to teach mathematics effectively. I am therefore asking for permission to interview two lecturers from your institution.

The ethical considerations, such as confidentiality of the results and findings of the study and the protection of participants' identities will be observed.

Attached is a copy of the letter from my Supervisor.

I am thanking you for your attention.

Yours faithfully,

'Maphole Marake (Mrs)