USING EYE-TRACKING TO ASSESS THE APPLICATION OF DIVISIBILITY RULES WHEN DIVIDING A MULTI-DIGIT DIVIDEND BY A SINGLE DIGIT DIVISOR

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DECLARATION

I hereby declare that the work that is submitted here is the result of my own independent investigation and that all sources I have used or quoted have been indicated and acknowledged by means of complete references. Furthermore, I declare that the work is being submitted for the first time at this university and faculty towards the Philosophiae Doctor degree and that it has never been submitted to any other university or faculty for the purpose of obtaining a degree.

2017/09/01

Pieter Henri Potgieter **Date**

ABSTRACT

The Department of Basic Education in South Africa has identified factorisation as a problem area in Mathematics for Grade 9 learners. Establishing the foundation for factorisation begins at earlier grades. If learners know the divisibility rules, they can help them to determine the factors of numbers. The divisibility rules are presented to learners in Grade 5 for the first time. When a true/false question is used to assess learners' ability to determine whether a dividend is divisible by a certain divisor, the teacher has no insight in the learners' reasoning because he or she is only in possession of the final answer, which could be correct or incorrect. If the answer is correct, the teacher does not know if the learner (i) guessed the answer, (ii) correctly applied the divisibility rule, or (iii) incorrectly applied the divisibility rule. To improve the credibility of the assessment, learners can be requested to provide a reason for their answer. However, if the reason is correct, the teacher still does not know whether the learners correctly applied the divisibility rule – regardless of whether the answer is correct or not.

A pre-post experiment design was used to investigate the effect of revision on the performance of learners and also the difference in gaze behaviour of learners before and after revision of divisibility rules. About 1000 learners from Grade 4 to Grade 7 of two schools were assessed by means of a paper-based assessment on their knowledge of the divisibility rules before and after revision. The gaze behaviour of 155 learners was also recorded before and after revision.

It was found that revision had an impact on learner performance per divisor for nearly all grades that participated in the test for both schools. The gaze behaviour was measured as the percentage of fixation time on the digits of the dividend. It was found that revision had an effect on the gaze behaviour for learners who indicated the reason incorrectly before revision and the answer and reason correctly after revision. However, revision did not have an impact on the gaze behaviour of learners who indicated the answer and reason correctly before and after revision.

It was found that the correctness of the answer did not have an impact on the gaze behaviour (except for divisor 6) for learners who indicated the reason correctly. However, revision had an impact on the gaze behaviour for learners who indicated the answer incorrectly and reason

correctly before revision, as well as for learners who had both the answer and reason correctly after revision for divisor 6. This infers that eye-tracking can be used to determine whether the divisibility rule was applied correctly or incorrectly. Eye-tracking also revealed that learners who did not know the divisibility rules, only inspected the last two digits of the dividend before indicating their answer.

The study suggests that when a teacher has access to the learner's answer, reason and gaze behaviour, he or she will be in a position to identify if the learner (i) guessed the answer, (ii) applied the divisibility rule correctly, (iii) applied the divisibility rule correctly but made mental calculation errors, or (iv) applied the divisibility rule incorrectly.

An instrument is proposed that can be used by teachers to assess learners on divisibility rules where learners only have to indicate whether a dividend is divisible by a divisor. Eye-tracking will predict whether the learner knows the divisibility rule. For 85% of learners who provided the correct answer, their gaze behaviour corresponded with the reason provided.

The study concluded, therefore, that eye-tracking can, to a large extent, correctly identify whether learners, who indicated correctly if a dividend is divisible by a certain single digit divisor, applied the divisibility rules correctly.

OPSOMMING

Die Departement van Basiese Onderwys in Suid-Afrika het faktorisering as ’n probleem area vir Graad 9-leerders geïdentifiseer. Die boustene van faktorisering begin alreeds in vorige grade. Indien leerders die deelbaarheidsreëls ken, kan dit hulle help om die faktore van getalle te bepaal. Die deelbaarheidsreëls word vir die eerste keer behandel in Graad 5. Wanneer ’n waar/vals vraag gebruik word om leerders se vermoë om te bepaal of ’n deeltal deelbaar is deur ’n sekere deler, te assesseer, het die onderwyser geen idee watter benadering die leerder gebruik het nie omdat die onderwyser slegs die finale antwoord beoordeel. Indien die antwoord reg is, weet die onderwyser nie of die leerder (i) die antwoord geraai het, (ii) die deelbaarheidreëls reg toegepas het, of (iii) die deelbaarheidsreël verkeerd toegepas het nie. Om die geloofwaardigheid van die toets te verhoog, kan van die leerders verwag word om ’n rede vir hul antwoord te verskaf. Indien die rede korrek is, weet die onderwyser egter steeds nie of die leerder die deelbaarheidsreël reg toegepas het nie – ongeag of die antwoord korrek is of nie.

’n Pre-post eksperiment ontwerp is gebruik om die effek van hersiening op die prestasie van die leerders asook die verskil in blik-gedrag (“gaze behaviour”) voor en na hersiening van die deelbaarheidsreëls, te bepaal. Ongeveer 1000 leerders van twee skole vanaf Graad 4 tot Graad 7 het ’n papier-gebaseerde toets omtrent hulle kennis van deelbaarheidsreëls voor en na hersiening geskryf. Die blik-gedrag van 155 leerders is ook opgeneem voor en na hersiening.

Dit is gevind dat hersiening ’n impak het op die leerder se prestasie per deler vir omtrent alle grade van die twee skole. Die blik-gedrag, as die persentasie fiksasie-tyd op die syfers van die deeltal, is ook gemeet. Dit is gevind dat hersiening ook ’n effek het op die blik-gedrag van leerders wat die rede vir hul antwoord voor hersiening verkeerd gehad het en na hersiening die antwoord en rede korrek aangedui het. Hersiening het egter nie ’n impak gehad op die blik-gedrag van leerders wat die antwoord en rede reg gehad het voor en na hersiening nie.

Dit is gevind dat die korrektheid van die antwoord nie ’n impak gehad het op die blik-gedrag (behalwe vir deler 6) vir leerders wat die rede korrek aangedui het nie. Hersiening het egter wel ’n impak gehad op die blik-gedrag van leerders wat die antwoord verkeerd en die rede reg gehad het voor hersiening, sowel as dié van leerders wat die antwoord en rede reg gehad het na hersiening vir deler 6. Dit volg dus dat oog-volging (“eye-tracking”) gebruik kan word om

te bepaal of die deelbaarheidsreël reg of verkeerd toegepas is. Oog-volging het ook aan die lig gebring dat indien leerders nie die deelbaarheidsreëls ken nie, hulle slegs na die laaste twee syfers van die deeltal kyk voordat hulle hul antwoord verskaf.

Hierdie studie dui daarop dat indien ’n onderwyser toegang het tot die leerder se antwoord, rede asook blik-gedrag, hy of sy in ’n posisie is om te identifiseer of die leerder (i) die antwoord geraai het, (ii) die deelbaarheidsreël reg toegepas het, (iii) die deelbaarheidsreël reg toegepas het maar berekeningsfoute gemaak het, of (iv) die deelbaarheidsreël verkeerd toegepas het.

’n Instrument wat deur onderwysers gebruik kan word om leerders te toets oor die deelbaarheidsreëls, word voorgestel. Leerders hoef slegs aan te dui of die deeltal deelbaar is deur die deler waarna oog-volging kan voorspel of die leerder die deelbaarheidsreël ken. Vir 85% van die leerders wie se antwoord reg was, het die rede wat aangevoer was ooreengestem met hul blik gedrag.

Die finale gevolgtrekking van die studie is dus dat oog-volging tot ’n groot mate kan identifiseer of ’n leerder die deelbaarheidsreël reg toegepas het indien hy of sy die antwoord reg het.

Dedicated to my three children:

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GLOSSARY

ANA	Annual National Assessments
ANOVA	Analysis of variance
AOI	Area of interest
Dividend	A number to be divided by another number
LCD	Least common denominator
LCM	Lowest common multiple
LSM	Living Standard Measure
NCTM	National Council of Teachers of Mathematics
PIRLS	Progress in Reading and Literacy Study
STATISTICA	Data analysis software system
TIMSS	Trends in Mathematics and Science Studies

CHAPTER 1
PROJECT OVERVIEW

1.1 INTRODUCTION

The Centre for Development and Enterprise investigated the education system of South Africa, and came to the conclusion that “there is an on-going crisis in South African education, and that the current system is failing the majority of South Africa’s youth” (Spaull, 2013a:3). South Africa participates in international assessments in order to benchmark learner performance in various subjects, such as Literacy, Science and Mathematics (Department of Basic Education, 2013a). South Africa’s poor performance in international assessments, such as the *Trends in Mathematics and Science Studies* (TIMSS) and the *Progress in Reading and Literacy Study* (PIRLS), highlights the concern about the quality of education in the country (Archer & Howie, 2013).

Learners in South Africa also participate in the Annual National Assessments (ANA), and these results were not at all promising (Department of Basic Education, 2013b). The Department of Basic Education (2013c, 2014a) has, through the ANAs, identified certain problem areas of which the factorisation of numbers was identified as a problem area for Grade 9 learners. This study will focus on the divisibility rules that can assist learners with factorisation of numbers, and will do so by making use of eye-tracking technology as a tool to observe learners’ gaze behaviour while they are applying the divisibility rules.

The following aspects will be discussed in this Chapter:

- South Africa’s participation in international assessments (Section 1.2)
- Annual National Assessments (ANA) in South Africa (Section 1.3)
- Areas of concern identified after analyses of the ANA results (Section 1.4)
- Problem statement (Section 1.5)
- Research question (Section 1.6)
- Thesis statement (Section 1.7)
- Secondary hypotheses (Section 1.8)
- Research methodology (Section 1.9)
- Significance of the study (Section 1.10)

- Limitations of the study (Section 1.11)
- Outline of the dissertation (Section 1.12)

1.2 SOUTH AFRICA'S PARTICIPATION IN INTERNATIONAL ASSESSMENTS

The Department of Basic Education has been committed since 1994 to participate in international assessment programs such as the *Trends in Mathematics and Science Studies* (TIMSS) and the *Progress in Reading and Literacy Study* (PIRLS) (Department of Basic Education, 2013a). The performance of Grade 9 learners in Mathematics and Science are reported in TIMSS, and the performance of Grade 4 and Grade 5 learners in Literacy are reported in PIRLS.

PIRLS showed that Grade 4 learners performed fairly poor, and worse than their counterparts in other countries. The performance patterns of TIMSS were similar to the results of PIRLS and the Annual National Assessments (ANA) in South Africa (Department of Basic Education, 2013a).

In 2015, 57 countries participated in the international TIMSS benchmarking assessments for Grade 4 and Grade 8 learners. Of these, 49 and 39 countries participated in the division for Mathematics for Grade 4 and Grade 8 learners respectively (Mullis, Martin, Foy & Hooper, 2016). South African Grade 5 and Grade 9 learners took part in the assessments for Grade 4 and Grade 8 respectively since these curricula were better matched (Mullis et al., 2016). Table 1.1 indicates the distribution of Mathematics achievements for participating countries. At each grade, the scale has a range of zero to 1000.

Table 1.1 Distribution of Mathematics achievements for participating countries during 2015

Grade 4			Grade 8		
Rank	**Country**	**Average scale score**	**Rank**	**Country**	**Average scale score**
1	Singapore	618	1	Singapore	621
2	Hong Kong	615	2	Korea	606
3	Korea	608	3	Chinese Tapei	599
:	:	:		:	:
47	Morocco	377	37	Morocco	384
48	South Africa	376	38	South Africa	372
49	Kuwait	353	39	Saudi Arabia	368

1.3 ANNUAL NATIONAL ASSESSMENTS (ANA) IN SOUTH AFRICA

The school system in South Africa is divided into four phases, namely the Foundation Phase (Grade R to Grade 3), the Intermediate Phase (Grade 4 to Grade 6), the Senior Phase (Grade 7 to Grade 9) and the Further Education and Training (FET) Phase (Grade 10 to Grade 12) (Department of Basic Education,2014b). In 2013 and 2014, around 7 million learners from Grade 1 to Grade 6 and Grade 9 took part in the Annual National Assessments (ANA) (Department of Basic Education, 2013b, 2014c). The ANA results for Mathematics from 2012 to 2014 were reported as percentages (Table 1.2). The acceptable achievement is expressed as the percentage of learners who achieved more than 50% in Mathematics for Grade 3, Grade 6 and Grade 9. In 2015, all the learners from Grade 1 to Grade 9 were supposed to participate in the ANA. However, the ANA was disrupted at a critical stage due to some teacher unions who announced that their members would boycott the ANA (Fredericks, 2016a). Some schools already had test papers in their custody and other schools did not. It could not be guaranteed that all the schools administered the ANA under standardised conditions to produce reliable and credible results and therefore the results for 2015 were not released in public (Fredericks, 2016a). The current ANA will be replaced by a new systemic assessment and will be written every three years starting in 2018 (Fredericks, 2016b).

Table 1.2 Annual National Assessments (ANA) results for Mathematics from 2012 to 2014

Grade	Mathematics average percentage mark			Percentage of learners achieving 50% or more		
	2012	2013	2014	2012	2013	2014
1	68	60	68			
2	57	59	62			
3	41	53	56	36	59	65
4	37	37	37			
5	30	33	37			
6	27	39	43	11	27	35
9	13	14	11	2	2	3

1.4 AREAS OF CONCERN IDENTIFIED AFTER ANALYSES OF THE ANA RESULTS

The Department of Basic Education highlighted several problem areas in the Foundation, Intermediate and Senior Phases. Some of the problem areas that are applicable to this study were that learners in the Intermediate Phase used the wrong strategies when they do calculations with fractions and applied the wrong mathematical rules when working with the numerator and the denominator (Department of Basic Education, 2013c).

A grave concern regarding Senior Phase learners was their inability to do factorisation. Learners in this phase also demonstrated an inability to simplify algebraic fractions (Department of Basic Education, 2013c; 2014a).

Multiple choice questions related to factorisation were included in the ANA papers of 2013 to 2015 for Grade 4 to Grade 6. The following examples appeared in the ANA papers for Grade 4 (Department of Basic Education, 2013d, 2014d):

- "*Which number is a factor of 12?*"
- "*Which number is not a factor of 15?*"

The following examples appeared in the ANA papers for Grade 5 (Department of Basic Education, 2013e, 2014e, 2015a):

- "*Which factor of 18 is missing in the list 1, 2, 3, 6, 18?*"
- "*Which one of the following numbers is not a factor of 54?*"
- "*Which one of the following shows all the factors of 18?*"

The following examples appeared in the ANA papers for Grade 6 (Department of Basic Education, 2013f, 2014f, 2015b):

- "*Which one of the following numbers is a factor of 81?*"
- "*Which number is not a factor of 96?*"
- "*Which of the following numbers has a factor of 9?*"

It is therefore clear that the building blocks for factorisation should already have been established in Grades 4, 5 and 6.

1.5 PROBLEM STATEMENT

If one looks at the results that were published in the TIMSS and ANA reports, the performance of South African learners in national and international Mathematics assessments can be labelled as unsatisfactory. Evidence of formal instructions to apply divisibility rules for divisors 2 to 12 was found in the workbook for Grade 5 to Grade 7 learners (Department of Basic Education, 2015c; 2015d; 2015e), but despite this early exposure, the fact that Grade 9 learners lack the ability to do factorisation (Department of Basic Education, 2013c), is concerning.

During the pilot study (*cf.* Section 4.3.1) it was found that learners from Grade 4 to Grade 7 knew the divisibility rules for divisors 2, 5 and 10, but it seemed as though most learners did not know the other divisibility rules to determine if a dividend is divisible by single digit divisor. Although some learners could indicate if a dividend is divisible by a 3, 4, 6, 8 or 9, they failed to provide the corresponding divisibility rule. Knowing the divisibility rules would enable learners to quickly determine if a number, referred to as the dividend, is divisible by a specific single digit divisor without having to do long calculations.

When a learner has to indicate, by only giving the answer, if a dividend is divisible by a certain single digit divisor, the teacher has no insight in the learner's reasoning. If the answer is correct, the teacher does not know if the learner guessed the answer or applied the divisibility rule correctly or incorrectly.

1.6 RESEARCH QUESTION

Knowing the divisibility rules will assist learners to simplify mathematical calculations such as factorisation of numbers, manipulating fractions and determining if a given number is a prime number. The research objective of this study is to inspect learners' gaze behaviour before they respond to a true/false question on divisibility (*cf.* Section 2.9). If it can be determined through gaze analysis that learners do not apply the divisibility rules correctly, the teacher can intervene and explain the learning material again. See Appendix A for a typical scenario.

The research question to be addressed in this study is:

> *Is it possible that learners' gaze behaviour can indicate whether they applied the divisibility rules correctly when they correctly indicated if a dividend is divisible by a specific single digit divisor?*

In an effort to answer the above-mentioned research question, Grade 4 to Grade 7 learners will, as part of the assessment, have to indicate with a reason whether a dividend is divisible by a single digit divisor. An eye-tracker will record the learner's gaze behaviour to determine whether the specific learner's gaze behaviour corresponds with the reason that the learner provided. For example, if learners inspect only the last digit of the dividend to identify if the dividend is divisible by 2, 5 or 10, it is reasonable to infer that they apply the tests for divisibility.

1.7 THESIS STATEMENT

The primary thesis statement for the study is:

H_0: *Eye gaze cannot be used to determine if learners who indicate that a dividend is divisible by a certain divisor correctly, have applied a divisibility rule correctly.*

In order to investigate the above null hypothesis, the following factors (independent variables) were considered:

1. School (implicitly referring to socio-economic conditions and mother tongue instruction)
2. Grade (Grades 4 to 7)
3. Revision (Before and after revision)
4. Divisibility (5-digit dividend is divisible by a given single digit divisor or not)
5. Divisor (2, 3, 4, 5, 6, 8, and 9)
6. Digit (One to five, numbered from right to left in a five-digit dividend)
7. Correctness of answer (Correct or incorrect)
8. Correctness of reason (Correct or incorrect).

Three dependent variables were used to determine the effect of a specific factor:

1. Performance was measured as the mean percentage achieved by learners for Assessment 1 (paper-based, before revision) and Assessment 3 (paper-based, after revision). The assessments consisted of 14 questions (7 divisors × 2 questions per divisor) and a learner scored one (1) mark for each question if both the answer and reason were correct (A✓R✓), otherwise a zero (0).
2. The effect of a specific factor could also be expressed in terms of the number of responses in a specific combination of answer/reason correct/incorrect.
3. Gaze behaviour was expressed as the percentage of fixation time per digit during Assessment 2 (before revision) and Assessment 4 (after revision).

1.8 SECONDARY HYPOTHESES

In addition to the primary thesis statement above, a set of secondary hypotheses can be formulated for the effect of the above-mentioned factors on the dependent variables. A summary of the secondary hypotheses follows in Table 1.3.

$H_{0,1}$: *There is no difference in the overall performance of learners before and after revision.*

$H_{0,2}$: *There is no difference in the performance of learners per divisor before and after revision.*

There were two questions per divisor for each assessment. The two dividends were selected such that one of them was divisible by the specific divisor and the other not. Therefore, it can be hypothesised that:

$H_{0,3}$: *There is no difference in the percentage of responses which indicated the answer and reason correctly between the dividends that were divisible by the divisor and those that were not divisible.*

$H_{0,4}$: *There is no difference in gaze behaviour between learners of different grades for learners who provided the correct answer and reason.*

When learners provided the incorrect reason before revision, it means that they did not know the divisibility rule, irrespective of their answer. The question is whether there was a difference in the percentage of fixation time on the digits per divisor if they provided the correct answer and reason after revision.

$H_{0,5}$: *There is no difference in gaze behaviour before and after revision for learners who provided an incorrect* ***reason before revision*** *and the correct* ***answer and reason after revision*** *(**All learners, not pairwise**).*

$H_{0,6}$: *There is no difference in gaze behaviour before and after revision for learners who provided the correct* ***answer and reason before and after revision*** *(**Same learners, pairwise**).*

It is possible, for example, that before revision only 10% of learners know the divisibility rules while after revision the number could rise to 80%. The question is whether there will be a difference in the percentage of fixation time on the digits between the 10% of the learners before revision and the 80% of the learners after revision.

$H_{0,7}$: *There is no difference in gaze behaviour before or after revision between learners who know the divisibility rules (**All learners, not pairwise**) .*

$H_{0,8}$: *There is no difference in gaze behaviour between learners from schools in different socio-economic environments who provided the correct answer and reason.*

$H_{0,9}$: *There is no difference in gaze behaviour between the different divisors for learners who provided the correct answer and reason.*

Learners who provided the correct reason but the wrong answer could have made a calculation error. If gaze behaviour is indicative of whether or not the divisibility rules are applied correctly, it should not be different between learners who provided the correct answer and reason and those who provided the correct reason along with an incorrect answer.

$H_{0,10}$: *There is no difference in gaze behaviour between learners who provided the correct answer and reason and learners who provided the correct reason along with an incorrect answer (**All learners, not pairwise**).*

The question arises whether gaze behaviour can reveal if learners applied the divisibility rule incorrectly before revision if **the same learners** provided an incorrect answer along with a correct reason before revision and the correct answer and reason after revision:

$H_{0,11}$: *There is no difference in gaze behaviour between learners who provided an incorrect answer with a correct reason before revision and learners who provided the correct answer and reason after revision (**Same learners, pairwise**).*

$H_{0,12}$: *There is no difference in gaze behaviour between the different digits of the dividend for learners who provided the correct **answer and reason**.*

The question arises whether there is a trend with regard to gaze behaviour when learners provide an incorrect answer and divisibility rule.

$H_{0,13}$: *There is no difference in gaze behaviour between the different digits of the dividend when learners provide an **incorrect answer and divisibility rule**.*

The above-mentioned secondary hypotheses are summarised in Table 1.3. The variables that were controlled for were determined as part of the analysis (Chapter 5), but they are included in the table for easier reference further on.

Table 1.3 Summary of secondary hypotheses

No	Factor	Controlled variables	Uncontrolled variables	Limitations to sample	Dependent variable
$H_{0,1}$	Revision	School, Grade ***(Same learners, pairwise)***			Mean performance of learners
$H_{0,2}$	Revision	School, Grade, Divisor ***(Same learners, pairwise)***			Mean performance of learners
$H_{0,3}$	Divisibility	School, Grade, Divisor, Revision ***(All learners, not pairwise)***		Answer ✓, Reason ✓	Number of responses
$H_{0,4}$	Grade	Grade, Divisor, Digit, Revision ***(All learners, not pairwise)***	Divisibility	School A Answer ✓, Reason ✓	Gaze behaviour
$H_{0,5}$	Revision	Divisor, Digit ***(Same learners, pairwise)***	Divisibility, Grade	School A Before: Reason × After: Answer ✓, Reason ✓	Gaze behaviour
$H_{0,6}$	Revision	Divisor, Digit ***(Same learners, pairwise)***	Divisibility, Grade	School A Answer ✓, Reason ✓	Gaze behaviour
$H_{0,7}$	Revision	Divisor, Digit ***(All learners, not pairwise)***	Divisibility, Grade	School A Answer ✓, Reason ✓	Gaze behaviour
$H_{0,8}$	School	Divisor, Digit ***(All learners, factorial pairwise)***	Divisibility, Grade	After revision only Answer ✓, Reason ✓	Gaze behaviour
$H_{0,9}$	Divisor	School, Digit ***(All learners, not pairwise)***	Divisibility, Grade, Revision	Answer ✓, Reason ✓	Gaze behaviour
$H_{0,10}$	Answer	Divisor, Digit ***(All learners, not pairwise)***	Divisibility, Grade, Revision	School A Reason ✓	Gaze behaviour
$H_{0,11}$	Revision	Divisor, Digit ***(Same learners, pairwise)***	Divisibility, Grade, Revision	School A Before: Answer ✓, Reason × After: Answer ✓, Reason ✓	Gaze behaviour
$H_{0,12}$	Digit	School, Divisor ***(All learners, not pairwise)***	Divisibility, Grade, Revision	Answer ✓, Reason ✓	Gaze behaviour
$H_{0,13}$	Digit	School, Divisor ***(All learners, not pairwise)***	Divisibility, Grade, Revision	Answer ×, Reason ×	Gaze behaviour

1.9 RESEARCH METHODOLOGY

The research methodology can be succinctly stated as follows. Refer to Chapter 4 for a complete discussion.

Learners from Grade 4 to Grade 7 from two different schools will participate in the study. All the learners in these grades will write a paper-based assessment (Assessment 1) where after selected learners will participate in an eye-tracking assessment (Assessment 2) on the divisibility rules. After the initial assessments, revision will be done on the divisibility rules and then a follow-up paper-based assessment (Assessment 3) and another eye-tracking assessment (Assessment 4) will be done to determine if revision had an effect on these learners' performance and gaze behaviour.

1.10 SIGNIFICANCE OF THE STUDY

When teachers use true/false questions to assess learners' knowledge of the divisibility rules, it is possible that learners could guess the correct answer (Section 2.8.1). Although teachers can improve the credibility of an assessment by requiring learners to provide reasons to support their answers (Section 2.8.1), learners could provide a correct answer with a correct reason but use the wrong strategy (Section 2.8.3). It may be possible that the gaze behaviour of learners can indicate whether they applied the divisibility rules correctly when they answered a question correctly. If this is true, software can be developed that will enable the teacher to inspect the gaze behaviour of learners in real-time, as illustrated in Appendix A. Development of this software is beyond the scope of this study.

1.11 LIMITATIONS OF THE STUDY

At the time of the study, the researcher did not have access to a computer laboratory where all the computers were equipped with eye-trackers and connected to a network (Appendix A). Furthermore, no software was available to calculate the percentage of fixation time for each digit from the recordings that were captured during Assessment 2 and Assessment 4. Therefore, real-time testing where learners could do the assessment online and the recordings analysed automatically, was not possible.

The fixation time per digit as a percentage of the total time that the stimulus was displayed, was calculated manually. For each divisibility rule, a set of minimum requirements was established for each digit. The percentages of fixation time could not be sent to a central computer so that the teacher could immediately identify whether learners applied the divisibility rules correctly. Instead, the percentages of fixation times per digit were manually compared with the minimum requirements for each divisor to determine to what extent gaze behaviour could identify whether learners applied the divisibility rules correctly (Section 6.5).

Although divisibility rules exist for divisors greater than 9, only the single digit divisors 2, 3, 4, 5, 6, 8 and 9 will be used in this study.

1.12 OUTLINE OF THE DISSERTATION

This chapter (Chapter 1) provides the introduction to the study. A motivation for the study was provided and the research question and problem statement were defined. The primary and secondary hypotheses were also formulated. A brief overview of the research methodology was given and the significance of the study was highlighted.

Chapter 2 will provide the background and rationale of the study. A literature study will be conducted to discuss the role of divisibility rules in Mathematics and the application thereof in factorisation and Algebra. The different memory structures and knowledge that participants will be using while dealing with the divisibility rules will also be discussed. The applicable areas where the divisibility rules can be used and the proof of the divisibility rules will be provided. The question format that will be used in this study, will be discussed. The role of teachers and the methods that they use in class will also be addressed. The value of eye-tracking as a tool to observe learners' gaze behaviour will be stipulated. The limitations of eye-tracking for the study, specifically regarding peripheral vision and the fact that eye-tracking cannot be used in isolation, will also be discussed.

Chapter 3 will discuss previous attempts where eye-tracking was used to analyse gaze behaviour while participants solve mathematical problems. The potential of eye-tracking in classrooms will also be investigated.

The method that will be used in the study will be discussed in Chapter 4. This chapter will also discuss the pilot study and elaborate on the research design and the methodology that will be used, which includes (i) the design, purpose, reliability and limitations of the research instruments; (ii) the way that the dividends will be compiled for the study; (iii) how the data will be collected and analysed; and (iv) the ethical procedures that will be followed.

The data will be analysed and the results will be reported in Chapter 5. This chapter will elaborate on the results for the secondary hypotheses as stated in Section 1.8.

Chapter 6 sets the parameters for an instrument to evaluate learners' knowledge of divisibility rules. The parameters will include the effective compilation of dividends for each divisor, as well as the establishment of minimum gaze behaviour per divisor to determine whether learners applied the divisibility rules correctly. The chapter will also report on special software that will be required for the optimal use of eye-tracking to assess learners in real-time on the divisibility rules. The functionality of the instrument will be determined by testing the proposed instrument against existing recordings.

The conclusions and recommendations will be discussed in Chapter 7. Recommendations for future research will also be made.

1.13 SUMMARY

In this chapter, the poor Mathematics results of South African learners in international and national assessments were discussed. Based on the Annual National Assessments (ANA), factorisation was highlighted as an area of concern for Grade 9 learners – even though factorisation has already been introduced in the earlier grades.

Some of the questions on factorisation in the ANA papers for Grade 4 to Grade 6 were multiple choice questions. When teachers make use of true/false or multiple choice questions, they do not know if the learners guessed an answer, or used the correct method to get to an answer.

A motivation for the study was provided and the research question and problem statement were defined. The primary and secondary hypotheses were also formulated. A brief overview of the research methodology was given and the significance of the study was highlighted.

The structure and sequence of presentation of this dissertation was also provided.

In the next chapter, a literature study on the background and rationale of the study, based on the application of the divisibility rules in Grade 4 to Grade 7, will be conducted. The value of eye-tracking for this study will also be discussed.

CHAPTER 2
BACKGROUND AND RATIONALE OF THE STUDY

2.1 INTRODUCTION

The poor performance of South African learners in national and international assessments was mentioned in the previous chapter. Factorisation of numbers was highlighted by the Department of Basic Education as a main concern for Grade 9 learners (Department of Basic Education, 2013c; 2014a) – despite being introduced to factorisation of numbers since Grade 4 (Section 1.4). It was also discussed that learners could benefit from knowing the divisibility rules as it would enable them to quickly determine if a number, referred to as the dividend, is divisible by a specific single digit divisor without having to do long calculations (Section 1.5).

This chapter will provide the background and rationale for the study. The following aspects will be discussed:

- Factorisation and divisibility (Section 2.2)
  - Basic principles
  - What is expected of learners
- Divisibility rules and their value in primary school Mathematics (Section 2.3)
  - Motivation for using divisibility rules
  - Divisibility rules
  - Alternative implementations for divisors 4 and 8
  - Presentation of divisibility rules
  - Save time during assessments
  - Relationship between the divisibility rules and Algebra
- Application areas of divisibility rules (Section 2.4)
- Proof for the divisibility rules (Section 2.5)
- Teaching (Section 2.6)
  - The teachers’ role in developing learners’ strategic skills
  - The effect of a second language as the medium of instruction on learners’ understanding of mathematical concepts
  - The improvement in learners’ performance with *drill and practice* activities

- Problem solving (Section 2.7)
  - Memory structures
  - Conceptual and procedural knowledge
- Assessment of knowledge of divisibility rules (Section 2.8)
  - Format of questions
  - Distractors
  - Possible answers and reasons for responses
- Using eye-tracking to observe gaze behaviour (Section 2.9)
- Eye-tracking as a tool to observe learners' gaze behaviour during problem solving (Section 2.10)

2.2 FACTORISATION AND DIVISIBILITY

2.2.1 BASIC PRINCIPLES

The concepts of divisors, multiples, prime and composite numbers had already been known and studied since 350 B.C. (Niven, Zuckerman & Montgomery, 1991). Learners in the early grades learn that ***a*** is a factor of ***b*** if ***a*** can divide into ***b*** without a remainder (Department of Basic Education, 2015c).

"***a*** divides ***b*** if and only if ***a*** is a factor of ***b***. When ***a*** divides ***b***, we can also say that ***a*** is a divisor of ***b***, ***a*** is a factor of ***b***, ***b*** is a multiple of ***a***, and ***b*** is divisible by ***a***" (Musser, Burger & Peterson, 2011:186). As mentioned in Section 1.5, divisibility rules present a short way of determining whether ***a*** divides into ***b*** without actually performing the division.

"Encapsulation of divisibility as an object could result in an understanding of the concept of divisibility as an essential property of whole numbers (*sic*) independent of the procedural aspects of division" (Zazkis & Campbell, 1996:545). Zazkis and Campbell (1996) further stated that divisibility can be seen in terms of a "*yes*" or a "*no*" property of integers. A natural number is an integer greater than zero (Musser et al., 2011). Therefore, for any two natural numbers ***a*** and ***b***, ***a*** is either divisible by ***b*** or ***a*** is not divisible by ***b***.

2.2.2 WHAT IS EXPECTED OF LEARNERS

The Department of Basic Education issues a grade-specific Mathematics workbook for all learners in South Africa on an annual basis. The examples in the Grade 5 workbook illustrate that calculations should be performed to determine if an integer is a factor of another number. When learners have to determine whether the number 3 is a factor of 87, they have to perform division or use a calculator to determine if 3 divides into 87 with or without a remainder. However, if learners are familiar with the divisibility rules, they only need to inspect the relevant digits of the number.

2.3 DIVISIBILITY RULES AND THEIR VALUE IN PRIMARY SCHOOL MATHEMATICS

As mentioned above, divisibility rules are used to determine if a dividend is divisible by a divisor, by only examining the relevant digits of the dividend. In other words, the division is not actually done to determine the answer.

2.3.1 MOTIVATION FOR USING DIVISIBILITY RULES

Performing division can be time consuming and complex. If, however, the quotients or remainders are not required, divisibility rules can be used to determine if one integer is divisible by another (Zazkis & Campbell, 1996). For example, if learners want to determine if a number is a prime number, they are only interested in whether a certain divisor divides the number (dividend), irrespective of the values of the quotient and the remainder. Divisibility rules are exemplary of mathematical rules that can be learned easily and allow learners to get to an answer without understanding a concept in full (Skrandies & Klein, 2015). Skrandies and Klein (2015) also stated that when these rules are known, the tasks that involve the rules become easy.

2.3.2 DIVISIBILITY RULES

Learners should encounter the divisibility rules during the first two terms of the Grade 5 to Grade 7 school year (Department of Basic Education, 2015c; 2015d; 2015e). The workbook of Grade 5 and Grade 6 contain the divisibility rules whereas the workbook of Grade 7 only does revision thereof. The workbook of Grade 5 during 2015 contained the divisibility rules as indicated in Table 2.1. The divisibility rule for divisor 8 is, however, stated incorrectly in the 2015 workbook for Grade 5 but correctly in the 2015 workbook for Grade 6. It reads that "if the sum of the last three digits is divisible by 8, the whole number is also divisible by 8". It should read that "if the number formed by the last three digits is divisible by 8, the whole number is divisible by 8". All the divisibility rules, as stated in Table 2.1, will be proven in Section 2.5.2 to Section 2.5.12. Although the divisibility rules for divisors 2 to 12 appear in the workbook, only divisors 2 to 9 (excluding 7) will be used in the study.

Table 2.1 Divisibility rules as published in the workbooks of the Department of Basic Education (Department of Basic Education, 2015c:172)

Divisor	Rule
2	A number is divisible by 2 if the last digit is an even number.
3	If the sum of the digits is divisible by 3, the whole number is also divisible by 3.
4	If the number formed by the last two digits is divisible by 4, the whole number is also divisible by 4.
5	If the last digit is 5 or 0, the number is divisible by 5.
6	If the number is divisible by both 2 and 3, it is also divisible by 6.
7	Multiply the last digit by 2 and subtract it from the number formed by the first four digits. If the answer is divisible by 7 (including 0), then the whole number is divisible by 7.
8	If the sum of the last three digits is divisible by 8, the whole number is also divisible by 8. (Note that this is incorrect, as explained in the paragraph above.)
9	If the sum of all the digits is divisible by 9, the number is also divisible by 9.
10	If the number ends in 0, it is divisible by 10.
11	Subtract the sum of the even digits from the sum of the odd digits; if the difference, including 0, is divisible by 11, the number is also divisible by 11.
12	If the number is divisible by both 3 and 4, it is also divisible by 12.

2.3.3 ALTERNATIVE IMPLEMENTATIONS FOR DIVISORS 4 AND 8

Chakraborty (2007) suggested the following easy reasoning to apply the divisibility rule for divisor 4: If the second-to-last digit of the dividend is an even number, the last digit of the dividend should be 0, 4 or 8. If the second-to-last digit of the dividend is an odd number, the last digit of the dividend should be 2 or 6.

Chakraborty (2007) also simplified the divisibility rule for divisor 8. If the third last digit of the dividend is an odd number, the number formed by the last two digits should be divisible by 4, but not by 8. If the third last digit of the dividend is an even number, the number formed by the last two digits should be divisible by 8.

2.3.4 PRESENTATION OF DIVISIBILITY RULES

The structure of Mathematics is hierarchical as all related topics fit into an interconnected dependency pattern, where these dependencies form the structure of Mathematics (Wilson, 2009). Wilson (2009) further emphasised that specific topics have to be taught prior to other topics, and learners must understand one topic properly before the teacher moves to the next topic.

Rules can be used for problem solving, but it must be applied with understanding (Ploger & Rooney, 2005). In other words, divisibility rules should not be discussed until learners have properly grasped the concepts of division (Zazkis & Campbell, 1996).

The understanding of the concepts of division is beyond the scope of the study. Therefore, learners' ability on the concepts of division was not assessed. The study focused on the assessment of the precise execution of divisibility rules and investigated learners' reasoning while they are solving mathematical problems.

2.3.5 USING THE DIVISIBILITY RULES TO SAVE TIME DURING ASSESSMENTS

Divisibility rules make it easy to determine if a number (called the dividend) is divisible by a divisor, since one only has to examine the relevant digits of the dividend, without having to perform the division (Zazkis & Campbell, 1996).

The possible advantage of knowing the divisibility rules is that learners could spend less time on calculations to determine if a specific divisor is a factor of a specific dividend. According to the Mathematics syllabi for Grade 4 to Grade 7 (Department of Basic Education, 2011a; 2011b), learners are exposed to factors of integers, fractions, equivalent fractions and prime numbers - mathematical concepts where knowledge of the divisibility rules could help learners (*cf*. Section 2.4). In cases where the divisibility rules are not part of the curriculum, not many teachers know, without actually doing the calculation, if a simple division of two integers will give an integer quotient (Nahir, 2008). "Probably the main reasons for studying rules of divisibility are to help students experience the thrill of creating mathematics, and to give one a sense of intellectual accomplishment" (Nahir, 2008:17).

This study focused on learners' ability to determine, through the application of the divisibility rules, if an integer is divisible by another integer (discussed in Section 2.3.2). The assessments that will be conducted in the study will expect learners to indicate "*yes*" or "*no*" along with a motivation to the questions on divisibility.

2.3.6 RELATIONSHIP BETWEEN DIVISIBILITY RULES AND ALGEBRA

In a study by Zazkis and Campbell (1996), it was argued that insufficient pedagogical emphasis is placed on the understanding of elementary concepts of arithmetic. They further believed that the development of conceptual understanding of the structures of numbers in general, depends on the conceptual understanding of divisibility and factorisation.

Cimen and Campbell (2013) agreed that the concepts of division and divisibility are important factors that help learners move from Arithmetic to Algebra. In short, Algebra is the part of Mathematics where letters and symbols are used to represent numbers in equations and formulae. Further study in Mathematics depends on the mastering of Algebra (Maher & Weber, 2009). The proficiency of students' knowledge of Algebra is debated worldwide, although it is generally accepted that students' procedural knowledge and understanding of Algebra contribute to their proficiency in Algebra (Van Stiphout, Drijvers & Gravemeijer, 2013).

It follows, therefore, that knowledge of divisibility rules will enhance learners' performance in Algebra because it addresses the concept of division and divisibility directly.

2.4 APPLICATION AREAS OF THE DIVISIBILITY RULES

Knowledge of the divisibility rules can make life easier for learners in various application areas. Some of these possibilities are discussed below.

2.4.1 DETERMINE IF A DIVISOR IS A FACTOR OF A DIVIDEND

The Annual National Assessment (ANA) papers of 2013 to 2015 contained questions that required learners to demonstrate the ability to indicate if a divisor is a factor of a dividend (*cf*. Section 1.4). Learners who knew the divisibility rules would find it easy to determine if there was a remainder, without having to do actual division.

When learners were confronted with a question such as *"Is 1731 divisible by 3?"*, they would easily determine the answer if they knew the divisibility rules. If they did not know the rules, they would have to do the actual calculation in order to determine if there is a remainder.

Grade 6 learners must be able to find the factor pairs of a number (Department of Basic Education, 2015b). For example, the factor pairs for 12 are (1, 12), (2, 6) and (3, 4). If a learner can determine if a specific number is a factor, calculating the other number of the factor pair becomes easier.

2.4.2 PRIME NUMBERS AND PRIME FACTORS

A prime number is an integer greater than 1 that has no positive divisors other than 1 and itself (Niven et al., 1991). According to the South African Mathematics syllabus, Grade 6 learners are expected to recognize and list prime numbers to at least 100 (Department of Basic Education, 2011a). Grade 7 learners are expected to list all prime factors of three-digit numbers (Department of Basic Education, 2011b). Questions where learners had to write down the next prime number in the sequence 19, 23, 29, ___, appeared in the 2013 ANA papers for Grade 6 learners (Department of Basic Education, 2013f).

In order to determine if a number is a prime number, learners could apply the divisibility rules using prime divisors from 2 to at least the square root of the number. For example, to determine if 91 is a prime number, learners could apply the divisibility rules for divisors 2, 3, 5 and 7 in succession and find that 7 is a factor of 91. Therefore, 91 is not a prime number.

In order to express a number in terms of its prime factors, learners should do repetitive division by the smallest prime factor. For example, for the number 450:

$$\begin{array}{r|r} 2 & 450 \\ 3 & 225 \\ 3 & 75 \\ 5 & 25 \\ 5 & 5 \\ & 1 \end{array}$$

Therefore, 450 could be written as the product of its prime factors: $2 \times 3 \times 3 \times 5 \times 5$.

2.4.3 FRACTIONS

A common fraction can be described as the ratio or quotient of two integers ***a*** and ***b***, expressed in symbolic form, $\frac{a}{b}$, where ***b*** is not zero (Wong & Evans, 2007). Two common fractions are considered equivalent if they have the same value (Wong & Evans, 2007). Grade 4 to Grade 6 learners should be able to recognise and use equivalent fractions in which the denominator of one is a multiple of the other (Department of Basic Education, 2011a).

Where $\frac{a}{b}$ represents any fraction and ***n*** is a non-zero integer, then $\frac{a}{b} = \frac{a \cdot n}{b \cdot n}$ (Musser et al., 2011). When $\frac{a \cdot n}{b \cdot n}$ is replaced by $\frac{a}{b}$ where $n \neq 1$, it means that $\frac{a \cdot n}{b \cdot n}$ has been simplified. By applying the divisibility rules, learners will be able to tell if both the numerator and denominator are divisible by ***n*** – thereby determining if a fraction can be simplified (Baily, Hoard, Nugent & Geary, 2012).

When learners have limited ability to work with fractions and do division, they have to rely mainly on memorisation for mathematical learning (Siegler et al., 2012). Poor ability to work with fractions normally leads to low performance in Mathematics in high school. Therefore, the ability to work with fractions is important for Mathematics learning and success in advanced Mathematics studies (Jordan et al., 2013; Siegler, Fazio, Baily & Zhou, 2013).

2.4.4 LOWEST COMMON MULTIPLE AND LEAST COMMON DENOMINATOR

In arithmetic and number theory, the lowest common multiple of two integers ***a*** and ***b***, usually denoted by LCM(***a***, ***b***), is the smallest positive integer that is divisible by both ***a*** and ***b*** (Musser et al., 2011). In Mathematics, the least common denominator (LCD) of two or more non-zero integers is the largest integer that divides into the dividends without a remainder (Musser et al., 2011). Grade 7 learners should be able to find the lowest common multiple (LCM) and the least common denominator (LCD) for three-digit numbers by inspection or factorisation (Department of Basic Education, 2011b). Learners who know the divisibility rules do not have to rely on inspection but will be able to use a valid strategy to find the LCD or LCM of two integers.

To calculate the least common denominator (LCD) of two numbers, learners could use the method described in Section 2.4.2 to present the numbers as the product of their prime factors. The LCD is the product of the prime factors that exist in both numbers. For example, for the two numbers 12 and 18, 12 = $\underline{\mathbf{2}}\times2\times\underline{\mathbf{3}}$ and 18 = $\underline{\mathbf{2}}\times\underline{\mathbf{3}}\times3$. Therefore, the product of $2\times3=6$ represents the LCD of 12 and 18.

Learners who can calculate the LCD as mentioned above, can easily determine the lowest common multiple (LCM) by using the following method (Burkman, 2013): For any two integers ($\boldsymbol{a}$ and $\boldsymbol{b}$) where $\boldsymbol{g}$ = LCD($\boldsymbol{a}$,$\boldsymbol{b}$), there are prime numbers, $\boldsymbol{c}$ and $\boldsymbol{d}$, for which $\boldsymbol{a}=\boldsymbol{cg}$ and $\boldsymbol{b}=\boldsymbol{dg}$. Then

$$\begin{aligned} \mathrm{LCM}(\boldsymbol{a},\boldsymbol{b}) &= \boldsymbol{cdg} \\ &= \frac{\boldsymbol{cgdg}}{\boldsymbol{g}} \\ &= \frac{\boldsymbol{ab}}{LCD(\boldsymbol{a},\boldsymbol{b})} \end{aligned}$$

For example, the lowest common multiple of 12 and 18 is $\frac{12\times18}{LCD(12,18)} = \frac{12\times18}{6}$ = 36.

2.5 PROOFS OF DIVISIBILITY RULES

Musser et al. (2011), as well as Nahir (2008), explained the divisibility rules by proving the following statement: if $\frac{a}{b}=\boldsymbol{c}$, where $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$ are integers and $\boldsymbol{b}\neq 0$, then $\boldsymbol{a}$ is divisible by $\boldsymbol{b}$. The divisibility rules in the following sections were adapted for a five-digit dividend, ***abcde***, where each one of ***a, b, c, d*** and ***e*** represents a single digit from 0 to 9. Note that ***abcde*** does not imply $\boldsymbol{a}\times\boldsymbol{b}\times\boldsymbol{c}\times\boldsymbol{d}\times\boldsymbol{e}$, but a natural number, i.e. an integer greater than zero (Section 2.2.1).

2.5.1 LEMMA

The following lemma is central to all the subsequent proofs: If ***a*** divides ***m*** and ***a*** divides ***n***, then ***a*** divides ***m***+***n*** (Musser et al., 2011:187).

Proof:

If $\boldsymbol{a} \mid \boldsymbol{m}$, then $\boldsymbol{ax} = \boldsymbol{m}$ for an integer $\boldsymbol{x}$.

If $\boldsymbol{a} \mid \boldsymbol{n}$, then $\boldsymbol{ay} = \boldsymbol{n}$ for an integer $\boldsymbol{y}$.

Therefore, adding the respective sides of the two equations:

$\boldsymbol{ax} + \boldsymbol{ay} = \boldsymbol{m} + \boldsymbol{n}$, or $\boldsymbol{a}(\boldsymbol{x}+\boldsymbol{y}) = \boldsymbol{m} + \boldsymbol{n}$.

Since $\boldsymbol{x}+\boldsymbol{y}$ is an integer, the last mentioned equation implies that $\boldsymbol{a} \mid (\boldsymbol{m}+\boldsymbol{n})$ (Musser et al., 2011:187).

2.5.2 DIVISIBILITY BY 2

A natural number $\boldsymbol{N} = abcde$ is divisible by 2 if the last digit of $\boldsymbol{N}$ is even.

Proof:

$$\begin{aligned}\boldsymbol{N} &= abcde \\ &= a(10^4) + b(10^3) + c(10^2) + d(10^1) + e \\ &= \underbrace{10(a\cdot10^3 + b\cdot10^2 + c\cdot10^1 + d)}_{\boldsymbol{m}} + \underbrace{e}_{\boldsymbol{n}}\end{aligned}$$

Two divides 10, therefore 2 divides $\boldsymbol{m}$ $[10(a\cdot10^3 + b\cdot10^2 + c\cdot10^1 + d)]$. If e is even, then 2 divides e, i.e. 2 divides $\boldsymbol{n}$. Consequently, 2 divides $\boldsymbol{m}+\boldsymbol{n}=\boldsymbol{N}$, as stated in the lemma above.

2.5.3 DIVISIBILITY BY 3

A natural number $\boldsymbol{N} = abcde$ is divisible by 3 if the sum of its digits is divisible by 3. In other words, *abcde* is divisible by 3 if $(a + b + c + d + e)$ is divisible by 3.

Proof:
$$\begin{aligned}\boldsymbol{N} &= abcde\\ &= a(10^4) + b(10^3) + c(10^2) + d(10^1) + e\\ &= a(9999+1) + b(999+1) + c(99+1) + d(9+1) + e\\ &= a\cdot 9999 + b\cdot 999 + c\cdot 99 + d\cdot 9 + a + b + c + d + e\\ &= \underbrace{9(a\cdot 1111 + b\cdot 111 + c\cdot 11 + d\cdot 1)}_{\boldsymbol{m}} + \underbrace{a + b + c + d + e}_{\boldsymbol{n}}\end{aligned}$$

Three divides 9, therefore 3 divides $\boldsymbol{m}$ $[9(a\cdot 1111 + b\cdot 111 + c\cdot 11 + d\cdot 1)]$. If 3 divides $\boldsymbol{n}$, i.e. $a + b + c + d + e$, then 3 divides $\boldsymbol{m+n=N}$**,** as stated in the lemma above.

2.5.4 DIVISIBILITY BY 4

A natural number $\boldsymbol{N} = abcde$ is divisible by 4 if the number formed by the last two digits of $\boldsymbol{N}$ is divisible by 4.

Proof:
$$\begin{aligned}\boldsymbol{N} &= abcde\\ &= a(10^4) + b(10^3) + c(10^2) + d(10^1) + e\\ & \underbrace{10^2\,(a\cdot 10^2 + b\cdot 10^1 + c)}_{\boldsymbol{m}} + \underbrace{d(10^1) + e}_{\boldsymbol{n}}\end{aligned}$$

The number 4 divides $\boldsymbol{m}$ because 4 is a factor of 10^2. If 4 divides $\boldsymbol{n}$, i.e. the two-digit number formed by *de*, then 4 divides $\boldsymbol{m+n=N}$**,** as stated in the lemma above.

2.5.5 DIVISIBILITY BY 5

A natural number ***N*** =*abcde* is divisible by 5 if the last digit of ***N*** is either 5 or 0.

Proof: $\boldsymbol{N} = abcde$

$$= a(10^4) + b(10^3) + c(10^2) + d(10^1) + e$$

$$= \underbrace{10(a\cdot 10^3 + b\cdot 10^2 + c\cdot 10^1 + d)}_{\boldsymbol{m}} + \underbrace{e}_{\boldsymbol{n}}$$

Five divides 10, therefore, 5 divides ***m***, i.e. $10(a\cdot 10^3 + b\cdot 10^2 + c\cdot 10^1 + d)$. If *e* is either 5 or 0, then *e* (and hence ***n***) is divisible by 5. Consequently, as stated in the lemma above, 5 divides ***m***+***n***=***N***.

2.5.6 DIVISIBILITY BY 6

A natural number ***N*** = *abcde* is divisible by 6 if ***N*** is divisible by 2 and 3. According to a theorem stated by Musser et al. (2011:190), "a number is divisible by the product, ***a·b***, of two nonzero whole numbers (integers) ***a*** and ***b*** if it is divisible by both ***a*** and ***b***, and ***a*** and ***b*** have only the number 1 as common factor". Therefore, 6 = 2×3 with one (1) as the least common denominator (LCD) of two (2) and three (3). The proof for divisibility by 2 and 3 was discussed above.

2.5.7 DIVISIBILITY BY 7

If the difference between the integer part of a natural number (*abcde*) divided by 10 (*abcd*) and double the last digit (2·*e*) is divisible by 7, then the entire number is divisible by 7. For example, 52794 is divisible by 7 since 5279 – 8 is divisible by 7.

Proof : $\boldsymbol{N} = abcde$

$$= 10(abcd) + e$$

$$= 10(abcd) + 21\cdot e - 20\cdot e$$

$$= \underbrace{10(abcd - 2\cdot e)}_{\boldsymbol{m}} + \underbrace{21\cdot e}_{\boldsymbol{n}}$$

Seven divides $\boldsymbol{n} = (21\cdot\boldsymbol{e})$. Seven also divides $\boldsymbol{m} = 10(abcd - 2\cdot e)$ if 7 divides $\boldsymbol{abcd} - 2\cdot\boldsymbol{e}$. Hence, it also divides $\boldsymbol{m+n=N}$, as stated in the lemma above.

The procedure can be repeated if ($\boldsymbol{abcd} - 2\cdot\boldsymbol{e}$) is still too big.

Example: $\boldsymbol{N} = \overset{\boldsymbol{abcde}}{52801}$

	5280	
–	2	(Subtract 2×1 from 5280)
	5278	(Repeat procedure)
	527	
–	16	(Subtract 2×8 from 527)
	511	(Repeat procedure)
	51	
–	2	(Subtract 2×1 from 51)
	49	(7 divides 49, therefore 7 divides 52801. Repeat procedure if necessary)
	4	
–	18	(Subtract 2×9 from 4)
	- 14	(7 divides -14, therefore 7 divides 52801)

2.5.8 DIVISIBILITY BY 8

A natural number $\boldsymbol{N} = abcde$ is divisible by 8 if the number formed by the last three digits of $\boldsymbol{N}$ is divisible by 8.

Proof : $\boldsymbol{N} = abcde$

$$= \underbrace{a(10^4) + b(10^3)}_{\boldsymbol{m}} + \underbrace{c(10^2) + d(10^1) + e}_{\boldsymbol{n}}$$

Eight divides 10^4 and 10^3, therefore 8 divides $\boldsymbol{a}.10^4 + \boldsymbol{b}.10^3$ [$\boldsymbol{m}$]. If 8 divides the number formed by the last three digits of $\boldsymbol{N}$ [$\boldsymbol{n}$], then 8 divides $\boldsymbol{m+n=N}$, as stated in the lemma above.

2.5.9 DIVISIBILITY BY 9

A natural number $N = abcde$ is divisible by 9 if the number formed by the sum of its digits is divisible by 9.

Proof: The proof for divisibility by 9 is the same as the proof for divisibility by 3 that was discussed above.

2.5.10 DIVISIBILITY BY 10

A natural number $N = abcde$ is divisible by 10 if the last digit of N is 0.

Proof :

$$\begin{aligned} N &= abcde \\ &= a(10^4) + b(10^3) + c(10^2) + d(10^1) + e \\ &= \underbrace{10(a\cdot 10^3 + b\cdot 10^2 + c\cdot 10^1 + d)}_{m} + \underbrace{e}_{n} \end{aligned}$$

Ten divides 10, therefore 10 divides m. If 10 divides n [e], then 10 divides $m+n=N$, as stated in the lemma above.

2.5.11 DIVISIBILITY BY 11

A natural number $N = abcde$ is divisible by 11 if 11 divides the difference of the sum of the digits with place values that are odd powers of 10 and the sum of the digits with place values that are even powers of 10. In other words, N is divisible by 11 if 11 divides (b+d) – (a+c+e).

Proof:

$$\begin{aligned} N &= abcde \\ &= a(10^4) + b(10^3) + c(10^2) + d(10^1) + e \\ &= a(9999+1) + b(1001-1) + c(99+1) + d(11-1) + e \\ &= a\cdot 9999 + a + b\cdot 1001 - b + c\cdot 99 + c + d\cdot 11 - d + e \\ &= a\cdot 9999 + b\cdot 1001 + c\cdot 99 + d\cdot 11 + a - b + c - d + e \\ &= a\cdot 9999 + b\cdot 1001 + c\cdot 99 + d\cdot 11 + a + c + e - b - d \\ &= \underbrace{a\cdot 9999 + b\cdot 1001 + c\cdot 99 + \cdot d\cdot 11}_{m} + \underbrace{(a + c + e) - (b + d)}_{n} \end{aligned}$$

Eleven divides $a \cdot 9999$, $b \cdot 1001$, $c \cdot 99$ and $d \cdot 11$, therefore 11 divides $\boldsymbol{m}$ [$a \cdot 9999 + b \cdot 1001 + c \cdot 99 + d \cdot 11$]. If 11 divides $\boldsymbol{n}$ [$(a + c + e) - (b + d)$], then 11 divides $\boldsymbol{m+n=N}$.

However, the divisibility rule for 11 states that the difference of the sum of the digits with place values that are odd powers of 10 and the sum of the digits with place values that are even powers of 10 should be divisible by 11. With above-mentioned proof, $\boldsymbol{n}$ [$(a + c + e) - (b + d)$] represents the difference of the sum of the digits with place values that are even powers of 10 and the sum of the digits with place values that are odd powers of 10. The following proof illustrates that if a number $\boldsymbol{p}$ divides $\boldsymbol{n}$ [$(a + c + e) - (b + d)$], then the number $\boldsymbol{p}$ divides $(b+d) - (a + c + e)$.

Suppose $\boldsymbol{p}$ divides $(a+c+e) - (b+d)$. Then $\frac{(\boldsymbol{a+c+e})-(\boldsymbol{b+d})}{\boldsymbol{p}} = \boldsymbol{q},$ where $\boldsymbol{p}$ and $\boldsymbol{q}$ are integers. However, $\frac{(\boldsymbol{b+d})-(\boldsymbol{a+c+e})}{\boldsymbol{p}} = -\boldsymbol{q} = \boldsymbol{r}$, where $\boldsymbol{r}$ is an integer. Hence $\boldsymbol{p}$ also divides $(b+d) - (a+c+e)$.

For example, if $(a+c+e) = 3$ and $(b+d) = 14$, then $(a+c+e) - (b+d) = 3 - 14 = -11$, and $(b+d) - (a+c+e) = 14 - 3 = 11$. Eleven divides -11 [$(a+c+e) - (b+d)$] and 11 [$(b+d) - (a+c+e)$].

For example: $\boldsymbol{N} = \overset{\boldsymbol{abcde}}{42394}$ $\quad \overset{\boldsymbol{a}}{4 \times 10^4} + \overset{\boldsymbol{b}}{2 \times 10^3} + \overset{\boldsymbol{c}}{3 \times 10^2} + \overset{\boldsymbol{d}}{9 \times 10^1} + \overset{\boldsymbol{e}}{4 \times 10^0}$

$\boldsymbol{b} + \boldsymbol{d} = 2 + 9 = 11$

$\boldsymbol{a} + \boldsymbol{c} + \boldsymbol{e} = 4 + 3 + 4 = 11$

$\boldsymbol{a} + \boldsymbol{c} + \boldsymbol{e} - (\boldsymbol{b} + \boldsymbol{d}) = 11 - 11 = 0$

Eleven divides zero (0) and therefore 11 divides 42394.

2.5.12 DIVISIBILITY BY 12

A natural number $\boldsymbol{N} = abcde$ is divisible by 12 if $\boldsymbol{N}$ is divisible by 3 and 4. According to the theorem stated in Section 2.5.6, divisibility by 12 can be determined by applying the tests for 3 and 4 at the same time ($12 = 3 \times 4$, where 1 is the only common factor of 3 and 4).

2.6 TEACHING

To be successful in reasoning, learners need (i) teachers who are capable of explaining their thinking, as well as (ii) effective engagement with learning material (Baig & Halai, 2006). When learners are confronted with well-defined problems, such as logical reasoning or mathematical calculations, they should approach the problem step-by-step and follow the applicable procedure to reach a conclusion (Ellis, 2012).

2.6.1 THE TEACHER'S ROLE IN DEVELOPING LEARNERS' STRATEGIC SKILLS

Teachers' knowledge of the contents of Mathematics play an important role in the teaching of mathematical content to learners (Hill, Rowan & Ball, 2005). The teachers' role is to transfer knowledge and mathematical rules to learners, while the learners are expected to process the knowledge and reproduce it (Nisbet & Warren, 2000). Nisbet and Warren (2000) also suggested that teachers must only *facilitate* the learning process while learners must *create* their own mathematical knowledge.

Learners with lower than expected mathematical knowledge at the time they enter school, will continue to perform poorly in Mathematics unless there is active intervention by the teacher (Wright, 2013). Teachers can determine if learners grasped certain concepts and are in a position to decide when to intervene if learners struggle with specific concepts (Van den Bogert, Van Bruggen, Kostons & Jochems, 2014). Learners' understanding of mathematical concepts improve when they recognised their misunderstandings or mistakes, or when the teacher point out their mistakes or misunderstandings (Tan & Lim, 2010). Learners' incorrect reasoning that accidentally lead to the correct answer should be used as the opening remarks of a lesson during which the reasons why a correct answer was achieved, are investigated (Covillion, 1995).

Teachers should, by providing guidance, encourage learners to express themselves and improve their reasoning skills (Baig & Halai, 2006). Learners should also be prompted to verbalise their reasoning, because if the reasoning is incorrect, the teacher can address the misconception directly (Maher & Weber, 2009). Teachers should guide learners through effective lessons and material and should also monitor the learners' understanding of concepts

(Witzel & Riccomini, 2007). Furthermore, the teacher should accommodate learners who need extra help through revision or additional explanations during lessons.

Learners could also be assessed by evaluating their verbal responses. The teacher can easily evaluate the correctness of a response when learners explain their thinking aloud (Baig & Halai, 2006). Unfortunately, only confident learners may respond to a teacher's prompt to verbalise their reasoning, while others then miss out on the opportunity to confirm that they also understand the specific mathematical concept (Wiliam, 2014). The reason why learners do not participate in discussions in class can be a language constraint and it would be wrong to assume that these learners struggle with Mathematics *per se* (Machaba & Mokhele, 2014). Eye-tracking can be used to determine if these learners have the correct understanding of a specific concept in Mathematics.

2.6.2 THE EFFECT OF A SECOND LANGUAGE AS THE MEDIUM OF INSTRUCTION ON LEARNERS' UNDERSTANDING OF MATHEMATICAL CONCEPTS

South Africa has 11 official languages, namely Afrikaans, English, Ndebele, Northern Sotho, Sotho, Swazi, Tsonga, Tswana, Venda, Xhosa and Zulu. However, the Grade 4 to Grade 7 Mathematics workbooks are available in Afrikaans and English only.

Teachers at rural schools in South Africa where English is not the learners' home language, mentioned that the language of learning poses a problem because learners are from different ethnic groups and speak different languages (Machaba & Mokhele, 2014). They further stated that it is quite challenging to explain mathematical concepts in African languages. Learners could have misconceptions or be confused about the meaning of mathematical terminology in English if it was not their home language (Mji & Makgato, 2006).

The brain systems that are involved with executive function, language and sensory-motor processes give rise to different arithmetic strategies that learners follow to solve problems (Tschentscher & Hauk, 2014). The two schools that participated in this study used different languages of instruction. One of the schools used Afrikaans, the learners' home language, as medium of instruction whereas the other school used English, the learners' second language. This study investigated learners' progress after the divisibility rules had been revised, and also reported whether the language of instruction (home language or second language) of the revision sessions had an influence on the performance of the learners from the two schools (Section 5.4.3).

2.6.3 IMPROVEMENT IN LEARNERS' PERFORMANCE WITH DRILL AND PRACTICE ACTIVITIES

Drill and practice is an activity whereby exercises are given to learners continuously to practise a specific concept until they can perform it almost without making any mistakes (Watson, 2015). Therefore, *drill and practice* can be used to sharpen learners' mathematical skills – especially when they have to recall facts (Suppes & Jerman, 1970). The drill and practice approach can be used with great success in various subjects in primary and secondary schools (Suppes & Jerman, 1970).

Mathematics is hard work and takes discipline to master and therefore it is recommended that drill and practice activities be used to practise learners' skills in an effort to build a good foundation (Slosky, 2005). Some teachers also use this type of teaching to ensure that their learners perform well (Van der Merwe, 2011). However, if it is done without conceptual knowledge, it might result in ineffective mathematical learning (Lim, 2015).

It was proved that learners show an improvement in performance after practice time in the effective execution of the correct strategy and/or a change in the strategy that they used (Ganor-Stern & Weiss, 2016; Susac, Bubic, Kaponja, Planinic & Palmovic, 2014). Clarification of mathematical rules should be done carefully and a step-by-step approach should be followed to repeat mathematical rules, especially with weaker learners (Steinbring, 1989).

2.7 PROBLEM SOLVING

2.7.1 MEMORY STRUCTURES

There are different types of memory structures that must be considered when learners are introduced to new concepts. The focus in this section is on the types of memory structures that are applicable when dealing with divisibility rules.

The *Adaptive Character of Thought* theory distinguishes between three types of memory structures, namely declarative, procedural and working memory (Torio, 2015). The theory states that all knowledge begins with declarative information where the declarative memory underlies the learning and storage about facts and events (Ullman et al., 1997). The second memory structure is procedural knowledge, which is learned by making suggestions from factual knowledge that already exists (Section 2.7.3). The procedural memory is for the learning and processing of motor, perceptual and cognitive skills (Ullman et al., 1997). The third memory structure is working memory, which is part of the long term memory and is activated by the first two structures. Therefore, working memory refers to the temporary storage of information in connection with the performance of other cognitive tasks such as problem-solving or learning (Baddeley, 1983).

When learners are introduced to divisibility rules, the explanation of these rules makes an appeal on the declarative memory structure. Procedural knowledge is involved when learners learn the divisibility rules and perform exercises for each divisor. During assessments and examinations, the working memory is tested on the recall of knowledge that was captured in the declarative and the procedural memory.

The first assessment in this study made an appeal on learners' working memory as it was an unprepared assessment (Section 4.3.8.4). Thereafter, the divisibility rules were revised to refresh the learners' declarative and procedural memory (Section 4.2). Finally, the working memory was assessed again to determine if revision had an effect on it.

2.7.2 CONCEPTUAL KNOWLEDGE

There are different types of knowledge that learners could use to solve a mathematical problem. The most common types of knowledge are conceptual and procedural knowledge.

Conceptual knowledge can be defined as "the ability to construct situated conceptualizations of the category that serve agents in particular situations" (Barsalou, Simmons, Barbey & Wilson, 2003:89) and also as "knowledge of concepts, which are abstract and general principles" and "conceptual knowledge can be implicit or explicit and thus does not have to be verbalizable (*sic*)" (Rittle-Johnson, Schneider & Star, 2015:2). Conceptual knowledge is also called conceptual understanding or principled knowledge (Rittle-Johnson & Schneider, 2014).

Learners in Grade 4 should already have knowledge of integer operations such as addition (of at least four-digit numbers) and division (three-digit dividends by a single digit divisor) (Department of Basic Education, 2011a). For the purposes of this study, learners were expected to add five single digit numbers (divisors 3, 6, 9) and do division of a one, two or three-digit dividend by a single digit divisor (divisors 2, 4, 5, 8 and divisors 3, 6, 9 after adding the digits). Learners did not need to know the proofs of the divisibility rules (Department of Basic Education, 2015c), as discussed in Section 2.5.

In summary, for the purpose of this study it was expected that learners had conceptual knowledge of numbers and divisibility as mentioned above.

2.7.3 PROCEDURAL KNOWLEDGE

Procedural knowledge can be defined as the "knowledge of procedures" while "a procedure is a series of steps, or actions, done to accomplish a goal" (Rittle-Johnson et al., 2015:2). It further proposes that learners know how to solve a problem or know the steps to solve a problem (Rittle-Johnson & Schneider, 2014). Procedural knowledge also requires that procedures should be performed accurately and appropriately and, to promote the fluency of the procedural knowledge, the procedures should also be performed frequently (Stott, 2013).

Certain skills and strategies can be classified as procedures (Rittle-Johnson & Schneider, 2014). An essential characteristic of improved procedural knowledge is the increased use of the correct procedures and, in turn, the decreased use of incorrect procedures (Godau, 2015).

When learners use rules to solve mathematical problems, it can be classified as procedural knowledge (Star & Stylianides, 2013). The action to determine divisibility without actually doing it, is a type of procedural activity and procedural understanding (Zazkis & Campbell, 1996).

For learners to perform the divisibility rules correctly (Table 2.1), they should know the steps to apply the appropriate divisibility test for each divisor. The knowledge to apply the correct steps could be classified as procedural knowledge. For example, if (i) the divisor is 4, learners should only inspect the number formed by the last two digits of the dividend; or (ii) if the divisor is 9, learners should add all the digits of the dividend and determine if the sum is divisible by 9.

2.7.4 INTEGRATION OF CONCEPTUAL AND PROCEDURAL KNOWLEDGE

Procedural and conceptual understanding complement each other (Wong & Evans, 2007; Zazkis & Campbell, 1996). Both conceptual and procedural knowledge should be developed to achieve competency in Mathematics (Rittle-Johnson & Schneider, 2014; Stott 2013). The integration of conceptual and procedural knowledge should be developed in primary school or even before primary school (Godau et al., 2014). There is little value in skills if they are not properly understood and the same goes for understanding without skills – both are important (Wilson, 2009). It is possible, however, that procedural knowledge could be achieved without understanding, and procedures could be performed that are independent of conceptual knowledge (Engelbrecht, Bergsten & Kågesten, 2009; Rittle-Johnson & Schneider, 2014).

In the current study, learners should have the necessary conceptual knowledge as discussed in Section 2.7.2. Furthermore, they should have procedural knowledge of how the divisibility rules should be applied, but they do not need any conceptual knowledge as the proofs of the divisibility rules are not yet part of the curriculum.

2.8 ASSESSMENT OF KNOWLEDGE OF DIVISIBILITY RULES

2.8.1 FORMAT OF QUESTIONS

The most basic formats of questions for assessments are multiple choice and true/false questions, which are used widely (Singh, 2014). As mentioned in Section 1.5 (problem statement), a teacher has no insight in a learner's reasoning when multiple choice questions or true/false questions are asked, because the teacher has access to the learner's final answer only.

The probability of guessed answers for true/false questions is high (Burton, 2001; Haffejee & Sommerville, 2014). Learners can obtain an average of 50% by only guessing the answer, thereby passing an assessment regardless of their ability to solve the problem (Singh, 2014).

When a teacher assesses the learners on a mathematical question where they have to show all the steps they followed to reach the answer, the teacher is in a position to observe which method of reasoning the learners used. The teacher can also identify whether the learners followed the correct procedure and applied the correct mathematical rules. If learners have to answer true/false or multiple choice questions, where they only have to provide the answer, the teacher does not know which method the learners used and there is a possibility that the learners could have guessed the answer.

The credibility of true/false questions can be improved by asking learners to provide reasons for their answers (Singh, 2014). In order to prevent guessing, an "*I don't know*" option can also be included (Zhu, 2006).

These suggestions were applied in the current study. Learners had to indicate "*yes*" if the dividend was divisible by the divisor, or "*no*" if not and they had to provide reasons to support their answers. An "*I don't know*" option was also available.

2.8.2 DISTRACTORS

The probability that learners can correctly guess answers in assessments should be limited as far as possible. Dickinson (2014) defined distractors, also called foils or misleads, as incorrect options and stated that distractors are used to identify learners who do not know how to arrive at the correct answer. Dickinson (2014) also mentioned that a distractor is only effective if it attracts some response. Distractors should, furthermore, be logical or reasonable (Zhu, 2006).

When participants have to choose an answer, their minds can sometimes suggest a way to provide a sensible answer quickly and they could, therefore, base their answer on the given distractor (Rożek et al., 2014). The dividends used in the study had to be chosen in such a way that learners would find it difficult to guess the divisibility, but easy to perform division without using a calculator (Zazkis & Campbell, 1996). For example, if the dividend is 75133 and the divisor is 3, it is difficult to guess the correct answer, but easy to calculate. 7+5+1+3+3=19 and therefore 3 does not divide 19.

The dividends for the assessments in this study were compiled in such a way that learners needed to know the divisibility rules to choose the correct answer without having to use a calculator (Section 4.3.7).

2.8.3 POSSIBLE ANSWERS AND REASONS FOR RESPONSES

In the current study (*cf.* Section 1.9), learners were exposed to paper-based assessments as well as assessments where eye-tracking technology was used. Learners had to provide an answer with a reason for each question in all the assessments. Learners who do not know any strategies to solve a problem, may apply correct or incorrect strategies, whereas learners who know applicable strategies and/or shortcuts, may apply these (Godau et al., 2014).

When learners are expected to provide a reason for their answers to true/false questions, their responses can fall into one of five possible categories: (i) answer correct and reason incorrect, (ii) answer incorrect and reason correct, (iii) answer incorrect and reason incorrect, (iv) answer correct and reason correct, or (v) no answer. These categories of answers will be discussed next.

Reasons incorrect

With regard to the assessments that formed part of this study, learners were not credited for correct answers that were accompanied by invalid explanations.

Each class has its "Lucky Larry", who manages to get the correct answer from incorrect reasoning (Covillion, 1995). Hill et al. (2005) conducted a study among Mathematics teachers and found cases where even teachers applied incorrect reasoning, but ended up with correct answers. Thanheiser, Whitacre and Roy (2014) also did a study on the subject knowledge of prospective Mathematics teachers of the elementary phases. They found that some of these teachers often forgot about the divisibility rules or applied them incorrectly. This could also be applicable to learners when they are dealing with the application of divisibility rules.

Learners want to provide satisfactory answers to questions and they will often act as if they know the correct strategy – although there is no evidence that they are aware of it (Andrà et al., 2015). In other words, learners may seem confident about their reasoning even though the reasoning is incorrect (Storm & Zullo, 2015; Wall, Thompson & Morris, 2015). Such trends, where learners created their own incorrect reasoning, were indeed observed in the current study as well (Section 5.4.1).

When learners are introduced to new mathematical topics, they often create their own, incorrect assumptions that influence their reasoning (Maher & Weber, 2009). In the current study, for example, learners could add all the digits as expected for divisor 3, but could do so where divisor 8 was assessed and not divisor 3.

For the purpose of this study, learners had to provide a reason in their own words that relates to the applicable divisibility rule. It could happen that learners provided an answer but did not provide a reason or provided a reason that did not make any sense. For example, reasons such as "Because it is so", "It is a prime number" or "It is difficult" do not relate to the divisibility rule.

When learners are expected to provide a reason for an answer, they may have difficulty in expressing how they reached a specific answer (Schneider et al., 2008). During a study where learners had to use the divisibility rules to determine if a number is a prime number, some learners used the divisibility rules incorrectly (Zazkis & Liljedahl, 2004). Zazkis and Liljedahl (2004) stated that some learners based their answers on (i) whether the sum of the digits was a prime number, or (ii) whether the last digit was a prime number. They also mentioned that earlier research revealed several similar cases of incorrect applications of the divisibility rules – something that could occur when learners do not have a thorough understanding of rules and procedures (Star & Stylianides, 2013). The value that eye-tracking could add in this regard will be discussed in Section 2.10.1.

Reasons correct

The use of certain strategies indicates the understanding of mathematical concepts and learners should have the ability to recognise where and when specific strategies could be used to simplify processing of a problem (Godau et al., 2014). Therefore, learners should know which divisibility rule to follow for each divisor.

Although learners may indicate the correct reason for the divisibility rule, it does not necessarily imply that the answer is correct. It is possible that learners know the divisibility rule, but make calculation errors or apply the divisibility rule incorrectly, and therefore provide an incorrect answer (Hill et al., 2005).

Teachers cannot identify the incorrect application of a divisibility rule if the correct answer and motivation were provided. However, analysis of gaze behaviour can be used to determine if learners applied the divisibility rule correctly. For example, if the dividend was 27612 and the divisor was 6, learners' gaze behaviour could indicate that they inspected only the last two digits of the dividend, determined that 12 was divisible by 2 and 3, and therefore, concluded that 27612 is divisible by 3. This aspect will be elaborated upon in Section 2.10 as well as in Section 5.3.9, when the results of the eye-tracking tests will be discussed.

In this study, learners' gaze behaviour were analysed to determine if there is a difference between the gaze behaviour of learners who knew the divisibility rules and those who did not know the divisibility rules.

2.9 USING EYE-TRACKING TO OBSERVE GAZE BEHAVIOUR

Two main fields of application of gaze tracking are diagnostic and interactive (Duchowski, 2007; Hansen & Ji, 2010). Therefore, the eye-tracker can be used as a diagnostic tool to provide objective and quantitative information about the user's intended gaze-patterns over a specific stimulus. Duchowski (2007) highlight eye-tracking applications in the following areas:

- Reading (Example of cognitive processing)
- Visual search (Visually scan a scene to form a conceptual image or notion of the seen as assembled by the brain.)
- Illness (Patients with scotomas and visual neglect)
- Dynamic situations (Such as driving under different conditions)
- Aviation (Testing procedural training to evaluate gaze behaviour like a cockpit with predefined AOIs)
- Advertising (Gaze behaviour with different stimuli)
- Computer Science (Domain of computer science and human-computer interaction).

In the domain of illness, eye-tracking can also be used with schizophrenia and bipolar disorder patients to investigate the visual scan path to reveal differences within the psychoses that were not necessarily detectable by temporal measures alone (Bestelmeyer et al., 2006).

Preoccupation can have a negative influence on the execution of the task at hand (Savage, Potter & Tatler, 2013) and tiredness can be measured by the number and durations of blinks (Stern, Boyer & Schroeder, 1994) where slower eye movements are an indication thereof (Di Stasi et al., 2012). In the current study it will be the first time that selected learners are introduced to eye-tracking procedures, and therefore preoccupation about the assessment and also tiredness could have an influence on their gaze behaviour.

Tatler & Tatler (2013) stated that the task at hand influence strategic allocation of fixations and objects that should be visited inside an AOI received a higher number of fixations. This implies that the percentage of fixation time on relevant digits in the current study should be more than the fixations on irrelevant digits.

Above mentioned applications investigate the fixations of participants. The current study will make use of diagnostic approach to investigate the gaze behaviour of learners. Task specific problems lead to more and longer fixations (Duchowski, 2007). The next section will elaborate on the gaze behaviour of participants during problem solving in an educational setting.

2.10 EYE-TRACKING – A TOOL TO OBSERVE LEARNERS' GAZE BEHAVIOUR DURING PROBLEM SOLVING

The use of eye-tracking during the past 50 years has been quite promising and the rapid development of the technology has already made many eye movement applications possible (Jacob & Karn, 2003).

Low-cost devices, like webcam devices, can be used as more affordable eye-tracking devices (Ferhat, Vilariño & Sánchez, 2014; San Agustin, Skovsgaard, Hansen & Hansen, 2009). A tracking method that is based on particle filtering and the EM Contour algorithm is well suited for both high quality and low cost eye-tracking (Hansen & Pece, 2005). There is a possibility that, in the near future, all laptops and smart devices such as tablets, will contain a built-in eye-tracker. There is an increasing trend in schools to use tablets for interactive activities, such as surveys and quizzes (Ali, 2013). Therefore, if learners use laptops or tablets equipped with eye-trackers to complete a quiz on the divisibility rules, the teacher would be able to not only investigate the learners' gaze behaviour while they are doing the quiz, but also identify, in real-time, which learners are not applying the divisibility rules when answering the questions. The teacher can then explain the divisibility rules to this specific group of learners again.

2.10.1 THE VALUE OF EYE-TRACKING FOR THIS STUDY

Eye movements reveal detailed information on the procedure of how a learner approaches a problem and how the learner reaches the solution to the problem (Nyström & Ögren, 2012). Visual search is the process of visual scanning a scene where a perception is formed by the brain and eye movements provide visualisation of a participant's process, and therefore provide an instance of the process measures (Duchowski, 2007). By inspecting learners' gaze behaviour when they are confronted with a particular mathematical problem, one could attempt to analyse their reasoning. This could also enable a teacher to detect whether a learner grasps the specific concepts or if intervention is necessary.

Since most eye-tracking systems make use of infrared illumination that is invisible to the eye and does not distract or annoy the user (Rozado, Agustin, Rodriguez & Varona, 2012), eye-tracking can be used as a non-intrusive technology to investigate learners' gaze behaviour (Anderson, Carter & Koedinger, 2000) while they are doing Mathematics.

In the current study, eye-tracking was used to determine if learners applied a divisibility rule correctly if they provided the correct answer and reason to a divisibility question.

2.10.2 FIXATIONS

Two of the terms used in eye-tracking research that are applicable to this study, are fixations and saccades. A fixation is defined as the maintaining of the visual gaze on a single location, and a saccade is a quick motion of the eye from one fixation to another (Holmqvist et al., 2011a; Salvucci & Goldberg, 2000). The eyes are blind during saccades in the sense that no visual information is cognitively processed (Breidegard et al., 2008; Ekanayake, Karunarathna & Hewagamage, 2009). The duration of a fixation is at least 100 to 150 milliseconds and a fixation represents the gaze on an attention area (Rozado et al., 2012).

In the current study, learners' fixations were captured to indicate which parts of the dividend they inspected. The total duration of a fixation on a digit of the dividend was captured and used as a percentage of fixation time to analyse the captured data, as illustrated in Figure 2.1.

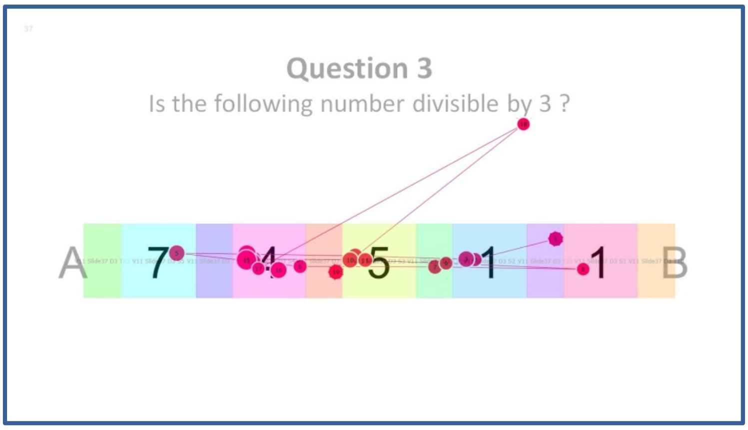


Figure 2.1 Fixations on dividend

2.10.3 EYE-TRACKING REVEALS AREAS OF INTERESTS THAT WERE INSPECTED

People normally divert their attention to the area where they fixate (Duchowski, 2007). As such, eye-tracking can be used to inspect participants' distribution of visual attention while they are solving problems (Nyström & Ögren, 2012). Eye-tracking can thus also reveal which areas participants are not inspecting (Paruchuri, 2012). The correct patterns of gaze behaviour could be associated with learners who possess solid factual knowledge of a problem (Mason, Pluchino, Tornatora & Ariasi, 2013b).

Bolden, Barmby, Raine and Gardner (2015) performed an eye-tracking study to investigate how young learners (aged 9 - 10) viewed mathematical representations. Presentations in the form of groups, arrays and number lines were presented to the learners. The duration of learners' fixations on the picture representations was measured. The percentage of time was measured when learners fixated on the picture representation. It was found that when learners inspected the number line, they mostly focused on the last number only.

In the current study, eye-tracking was also used to reveal where learners fixated and whether the correct digits of the dividend were inspected to reach the correct answer. The percentages of fixation time per digit of learners who had a solid factual knowledge of the divisibility rules and those who did not know how to solve a divisibility problem, were used to identify trends in gaze behaviour.

2.10.4 COGNITIVE PROCESSES IDENTIFIED BY EYE-TRACKING

Eye-tracking is not only used to report where participants look. It can also be used to identify processes such as cognitive thinking and problem solving. Therefore, analysis of gaze behaviour can be used to identify the academic potential and cognitive activities of learners (Anderson et al., 2000; Gluck, Anderson & Douglas, 2000; Lai et al., 2013; Mavrikis, 2016).

The full potential of eye-tracking with regard to numerical cognition has not yet been reached. There is also huge potential in investigating how complex numerical tasks are performed and how the strategies of well-performing learners differ from those of poor-performing learners (Hartmann & Fisher, 2016). As eye-tracking technology improves, more accurate feedback on the cognitive processes that take place, will be provided (Wang, Chignell & Ishizuka, 2006).

In this study, eye-tracking was used to investigate learners' gaze behaviour and whether they performed the cognitive and problem solving processes correctly to get to an answer.

2.10.5 EYE-TRACKING COMPLEMENTS VERBAL RESPONSES

Gaze behaviour cannot be obtained from verbal responses (Van Gog, Paas & Van Merriënboer, 2005) and the verbal explanation of reasoning can on its own not be used to derive which strategies were used (Paruchuri, 2012). When learners do not clearly verbalise their reasons, as discussed in Section 2.8.3, eye-tracking technology can be used to determine whether the correct cognitive activities took place. Verbal responses, together with eye movement data, can be used to examine cognitive processes (Van Gog et al., 2005).

Verbalisations can be obtain from participants during the execution of the task (concurrent think-aloud method) or after execution of the task (retrospective think-aloud method) (Hyrskykari, Ovaska, Majaranta, Räihä, & Lehtinen, 2008). The retrospective think-aloud method is a usability method that suggests that participants' verbalisations of the task at hand provide valid information about their task performance (Guan, Lee, Cuddihy & Ramey 2006). The retrospective think-aloud method produces more and quality data than the concurrent think-aloud method (Hyrskykari et al., 2008). Although participants could find it difficult to perform a task and want to provide a verbal response to the experimenter (Donker & Markopoulos, 2002), gaze behaviour provides additional information (Hyrskykari et al., 2008).

Learners' eye movements can potentially provide more information than traditional written assessments and also reveal the strategies that learners use when dealing with mathematical problem solving (Susac et al., 2014). In addition, learners' gaze behaviour might also be used to investigate if there were implicit forms of understanding that accompany incorrect verbal responses (Heine et al., 2010).

During the capturing of eye movements, participants could be asked additional questions in order to supplement the data (Bolden et al., 2015). In the current study, for example, learners could be asked to clarify their intentions although participant performance should not be disturbed (Bolden et al., 2015). For example, if a learner provided a reason such as "Add the digits and determine if the sum is divisible by 3", the researcher might respond with the following prompt: "Which digits?".

In this study, eye-tracking technology and verbal responses were used to determine whether gaze behaviour complements learners' verbal responses. This information may enable teachers to identify whether learners applied the divisibility rules correctly.

2.10.6 PERCENTAGE OF FIXATION TIME

The absolute duration of a fixation on a digit should not be used to indicate if a learner applied the divisibility rule correctly as some learners could be faster to produce an answer than others. The fixation duration on a digit as a percentage of the time spent on all digits could, however, reveal where learners spent most of their time and it could be used to determine whether learners applied the divisibility rule correctly.

Participants who know how to perform a task spend more time on relevant areas than on irrelevant areas (Susac et al. 2014). In a study on the identification of safety hazards in a work environment, Dzeng, Lin and Fang (2016) found that it required adequate knowledge and experience to establish correct search patterns.

In the current study, learners had to inspect different digits of the dividend. The percentage of fixation time on each digit of the dividend will be used to identify whether or not the learners applied the correct divisibility rule.

2.10.7 PERIPHERAL VISION

A specific limitation of eye-tracking, known as the dissociation problem, is that participants may fixate on a certain area while they are paying attention to another area (Bolden et al., 2015). Participants may use peripheral vision to inspect areas of interests that they do not directly fixate on (Schneider et al., 2008). The size of the area where participants can absorb meaningful information with a single fixation, is called the functional visual field or the visual span (Holmqvist et al., 2011a). Legge, Ahn, Klitz & Luebker (1997) found that the visual span decreases from ten letters for 1° characters to five letters for 6° characters.

When eye movements are used as indication of attention shifts, visual attention may be overt or covert (Mock, Huber, Klein & Moeller, 2016). Mock et al. (2016) also stated that attention shift during overt attention is accompanied with overt eye movements, whereas no eye movements are perceived during covert attention shifts.

Forward and backward eye movements, the so-called regressions, take place during arithmetic calculations where the target is fixated on or skipped (Mock et al., 2016). Information within an AOI, which were observed by peripheral vision or direct fixations, can be absorbed by the short-term memory if it stays there long enough to be recalled (Itti & Koch, 2000).

In the current study, it could happen that learners fixate on one digit but use peripheral vision to observe the neighbouring digits. It is also possible to fixate between the digits of the dividend and then perceive two digits with a single fixation. In order to limit the uncertainties that go with peripheral vision, elements of a stimulus can be larger and more widely spaced than usual (Sottilare, Graesser, Hu & Holden, 2013). This strategy was followed in the current study as it was important to determine the fixation time on each digit. Additional AOIs were inserted between digits of the dividend and the fixations on these AOIs were divided between the neighbouring digits (Section 4.3.5), as illustrated in Figure 2.1.

2.10.8 EYE MOVEMENTS DO NOT REVEAL ENOUGH EVIDENCE TO BE USED IN ISOLATION

There are limitations when eye-tracking is used to investigate learners' reasoning while they are completing mathematical assessments, because one can only determine where they are looking and not what they are thinking at any specific moment (Bolden et al., 2015). Although gaze behaviour is indicative of cognitive processes, it will not be fully known what the brain absorbs while the participant is fixating on an object (Bolden et al., 2015). Eye-tracking is not an alternative for any other method of assessment, but it could be used in combination with other methods, such as verbal responses, to analyse thought processes (Bolden et al., 2015; Mason, Tornatora & Pluchino, 2013a; Van den Bogert et al., 2014).

In the current study, learners had to verbalise their reasoning with regard to the divisibility of dividends and their responses were used to confirm if their eye movements agree with their thought processes.

2.10.9 THE NEED FOR CALIBRATION OF EYE TRACKING EQUIPMENT

In order to determine where a participant is looking, calibration is usually needed (Hansen & Pece, 2005). Gaze estimation methods have to determine a set of parameters through calibration where a participant inspects predefined points on a monitor (Hansen & Ji, 2010).

In the current study, a nine point calibration was used for each learner before each assessment.

2.11 SUMMARY

Learners in Grade 5 to Grade 7 should encounter the divisibility rules during the first two terms of the school year but they do not need to know the proofs. The ability to determine the divisibility of numbers (dividends) by specific divisors is necessary for mathematical development in Algebra. When learners know the divisibility rules, their work, when dealing with factorisation, is simplified. The application of the divisibility rules can assist learners in various application areas in Mathematics, such as (i) finding factors, (ii) determining whether a number is prime, (iii) calculating the prime factors of a number, (iv) simplifying fractions, and (v) calculating the LCM and LCD of two numbers.

Learners are exposed to three types of memory structures when dealing with divisibility rules, namely declarative, procedural and working memory. Although conceptual knowledge of numbers and divisibility are essential for dealing with divisibility rules, divisibility rules can be seen as procedural knowledge where the application thereof is independent of the conceptual knowledge.

Teachers play an important role in the development of learners' strategic skills and learners' performance can be improved with *drill and practice* activities. The language of instruction also plays an important role as there are challenges when Mathematics is offered in the learners' second language.

Multiple choice and true/false questions are popular formats in mathematical assessments and the credibility of these forms of assessment can be increased if learners are required to provide motivations for their answers. Appropriate distractors should also be used with these types of questions in order to prevent learners from correctly guessing the answer.

Eye-tracking is a useful tool to determine where participants look but there are also limitations with respect to peripheral vision and the fact that eye-tracking cannot be used in isolation.

Gaze behaviour will be analysed in the current study to determine if learners followed the correct strategy to solve questions on divisibility. The study will also investigate to what extent gaze behaviour can be used without a verbal response to identify if the divisibility rule was applied correctly.

Previous attempts to identify strategies in mathematical problem solving through eye-tracking will be discussed in the next chapter.

CHAPTER 3

PREVIOUS ATTEMPTS TO IDENTIFY GAZE BEHAVIOUR IN MATHEMATICAL PROBLEM SOLVING

3.1 INTRODUCTION

The background and the rationale of the current study were discussed in the previous chapter. The different types of memory structures and the knowledge that learners should have to apply the divisibility rules correctly, were also discussed. Application areas where divisibility rules can assist learners and the influence of teachers on the development of strategies used by learners, were highlighted. The value of eye-tracking for this study was also mentioned.

Eye-tracking can be used for disciplines other than Mathematics, as mentioned in Section 2.10.6. Although many eye-tracking studies have already been performed on graphs, formulas, geometry, etc. (Andrà et.al, 2009; Andrà et al., 2015; Holmqvist et al., 2011a; Holmqvist et al., 2011b; Krichevets, Shvarts, & Chumachenko, 2014; Lin & Lin, 2014; Nugrahaningsih, Porta & Ricotti, 2013; Tai, Loehr & Brigham, 2006), the discussions in this chapter will focus on the use of eye-tracking in applicable mathematical areas relating to the divisibility rules (Section 2.3.2).

The following aspects will be discussed:

- Using gaze patterns to identify problem solving strategies (Section 3.2)
- Gaze behaviour of novice and expert participants (Section 3.2.1)
- Gaze behaviour when solving problems that involve fractions (Section 3.2.2)
- Gaze behaviour while solving problems that involve associativity and commutativity principles (Section 3.2.3)
- The effect of illustrations on participants’ responses to true/false questions (Section 3.2.4)
- The effect of the order of fixations on problem solving when the order is irrelevant (Section 3.2.5)
- The effect of fixation time per relevant AOI on problem solving (Section 3.2.6)
- Gaze behaviour when solving problems that involve long division (Section 3.2.7)
- Gaze behaviour when studying and answering questions on division theory (Section 3.2.8)
- Number processing based on eye-tracking data in numerical cognition (Section 3.3)
- The use of eye-tracking during teaching (Section 3.4)

3.2 USING GAZE PATTERNS TO IDENTIFY PROBLEM SOLVING STRATEGIES

Eye-tracking can be used to identify the strategies that participants use to solve mathematical problems, especially where other methods, such as verbal responses, are less reliable (Obersteiner et al., 2014). For example, if learners cannot verbally express themselves clearly (*cf.* Section 2.8.3), eye-tracking can still reveal if they have inspected the correct digits of the dividend.

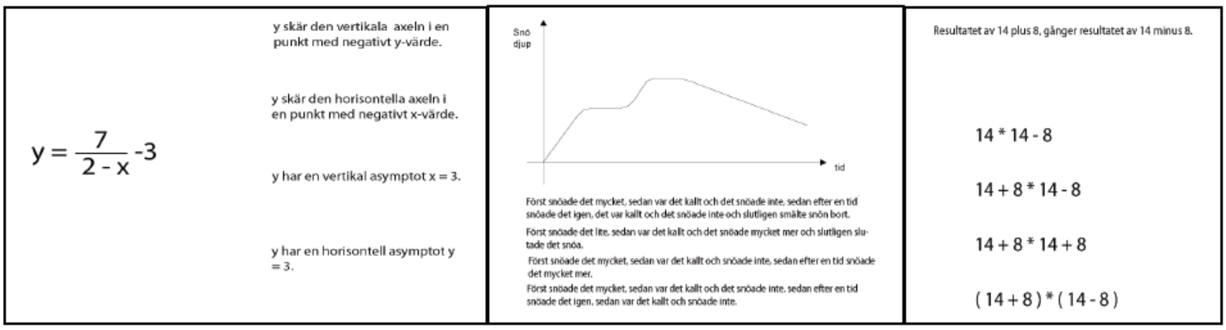
Many eye-tracking studies, some of which will be discussed in this chapter, were conducted to analyse participants' gaze while they are solving problems as the application of different problem solving strategies reveals different eye movement patterns (Godau et al., 2014). Ganor-Stern and Weiss (2016) found that participants shift their gaze and attention after practice to more appropriate areas of interest to perform a relevant task.

When doing mental calculations, learners should learn to integrate conceptual knowledge of mathematical principles with procedural knowledge of shortcuts (Godau et al., 2014). The conceptual knowledge and procedural knowledge that learners need to have for the purpose of this study were discussed in Section 2.7.4.

3.2.1 GAZE BEHAVIOUR OF NOVICE AND EXPERT PARTICIPANTS

It is known that, when analysing stimuli to do problem solving, experts inspect more relevant aspects of the stimuli than novices and experts use knowledge-based shortcuts (Jarodzka, Scheiter, Gerjets, & Van Gog, 2010a). In the context of this study, expert learners can be classified as learners who know the divisibility rules and also know how to apply them. Novice learners, on the other hand, can be classified as learners who are introduced to the divisibility rules for the first time or learners who have been introduced to the divisibility rules previously but still lack good understanding thereof.

Andrà et al. (2009) conducted an eye-tracking study to investigate the gaze behaviour of 46 novice and expert students while answering questions on mathematical equations, expressions and graphs. The stimuli were presented as multiple choice questions with an input and four alternatives, as illustrated in Figure 3.1. Three types of stimuli were used, namely (i) input as a formula and the alternatives as text (Figure 3.1a); (ii) input as a graph and the alternatives as text (Fig. 3.1b); and (iii) input as text and the alternatives as formulas (Figure 3.1c). Eye movements revealed that experts tend to read the question carefully before they pay attention to the possible answers.



a b c

Figure 3.1 Multiple choice questions used by Andrà et al. (2009:51)

Krichevets et al. (2014) analysed the gaze behaviour of participants while they performed a visual search for coordinates on a Cartesian plane. Ten learners from Grade 9 to Grade 11, 23 first year university students and 11 graduate students in Mathematics took part in the study. It was found that the expert participants performed the tasks faster and with fewer fixations than the novice participants. These findings also complement the findings of a study by Okamoto and Kuroda (2014), which will be discussed in Section 3.2.7.

In the current study, the gaze behaviour of novice and expert learners was investigated to determine if there are differences in gaze behaviour between the two groups when they apply divisibility rules.

3.2.2 GAZE BEHAVIOUR WHEN SOLVING PROBLEMS THAT INVOLVE FRACTIONS

Many learners experience challenges with fractions (Mazzocco, Myers, Lewis, Hanich & Murphy, 2013). Learners often treat the numerator and denominator as separate integers and struggle to understand the relationship between them (Fazio & Siegler, 2011; Stafylidou & Vosniadou, 2004).

Since it is a complex cognitive process to deal with fractions, learners may adapt to a strategy that simplifies the mental workload (Huber, Moeller & Nuerk, 2014). Huber et al. (2014) did a study that involved 36 university students and found that participants fixated more on the denominators than the numerators. They also found that the initial fixations were mainly on the denominator because of the assumption that it is more difficult to process denominators.

Obersteiner and Tumpek (2015) used gaze behaviour to determine whether 25 adult participants used integrated numerical magnitudes or componential processing when comparing fractions. It was found that participants preferred to use integrated numerical magnitudes when the fractions had common numerators or denominators, but used componential processing with fractions without common numerators or denominators.

Unlike the study by Huber et al. (2014), Obersteiner and Tumpek (2015) found that participants fixated more on the numerators than on the denominators. They suggested that the difference in findings could be because Huber et al. (2014) used single digit numbers while they used two-digit numbers.

Wall, Thompson & Morris (2015) reported that when participants (14 undergraduate students) compared two unequal fractions, there were longer fixations on the larger fraction – even when there was a common numerator or denominator. Wall et al. (2015) agreed with Huber et al. (2014) that if the fractions are equal, there are more fixations on the denominators than on the numerators.

The above-mentioned studies determined where participants spend most of their time while dealing with fractions. In the current study, the percentage of fixation time on each digit of the dividend will be used to determine if learners apply the relevant divisibility rule correctly.

3.2.3 GAZE BEHAVIOUR WHILE SOLVING PROBLEMS THAT INVOLVE ASSOCIATIVITY AND COMMUTATIVITY PRINCIPLES

Godau et al. (2014) conducted a study to determine the ability of Grade 1 to Grade 3 learners to spontaneously notice commutativity ($\boldsymbol{a}+\boldsymbol{b}+\boldsymbol{c} = \boldsymbol{a}+\boldsymbol{c}+\boldsymbol{b}$). The questions were compiled in such a way that the learners could not use the associativity principle [$(\boldsymbol{a}+\boldsymbol{b}+\boldsymbol{c}) = (\boldsymbol{a}+(\boldsymbol{b}+\boldsymbol{c})$]. For example, 4+7+6, it might be easier to calculate (4+6) + 7, i.e. “Ten-strategy”. Learners were presented with six questions on a stimulus, and they could return to previous questions on the same stimulus to identify a strategy to use. The eye-tracking data suggest that the ten-strategy and the addend-strategy could be identified by specific fixation patterns. By using the ten-strategy where the first and last addends add up to ten (**4** + 3 + **6**), little fixation time was needed to conclude to ten and more fixation time was spend on the middle addend. Godau et al. (2014) concluded that learners used search processes to identify a suitable strategy to use.

Godau (2015) did a study on instruction, association and estimation factors that support or hamper spontaneous strategies where learners are using shortcuts based on commutativity (***a***+***b***=***b***+***a***). For example, sets of questions such as “3+5+4”, “5+3+4”, “4+3+5” etc. were used to identify if participants used the commutativity principle. Godau (2015) used paper-based tests where participants had to answer as many mathematical questions as possible in a limited time. The participants were also tested individually in an eye-tracking laboratory where problems were presented on a computer screen. The percentage of fixations on specific areas of interest were measured along with the distribution of saccade distances. Godau (2015) found that better understanding of concepts led to an improvement in the strategy used and vice versa.

Schneider, Maruyama, Dehaene and Sigman (2012) did an eye-tracking study with 35 adult participants on the sequence of fixations on an arithmetic expression, such as [4*(3*2+5)], and found that the gaze behaviour was organised according to precedence of operations (e.g. multiplication before addition) rather than the parentheses that were present.

In the current study, learners were supposed to add all the digits of the dividend when dividing by 3, 6 or 9. Learners could also use associativity or commutativity to add the digits, and therefore, the sequence of fixations was not indicative of the correct application of the divisibility rule. Unlike the study by Godau et al. (2014), only a single question at a time was visible to learners and learners were not allowed to return to previous questions.

3.2.4 THE EFFECT OF ILLUSTRATIONS ON PARTICIPANTS’ RESPONSES TO TRUE/FALSE QUESTIONS

Nyström and Ögren (2012) did an eye-tracking study on vector calculus where 36 university students were confronted with true/false questions. The participants were divided into two groups. One group was offered true/false questions with illustrations, as indicated in

Figure 3.2, and the other group was offered true/false questions without illustrations. It was found that the questions with illustrations did not benefit participants' overall performance, but it influenced the participants to indicate that the answer was "*true*" even when the correct answer was "*false*". Cimen (2013) also made use of true/false questions in a study on divisibility, which will be discussed in Section 3.2.8.

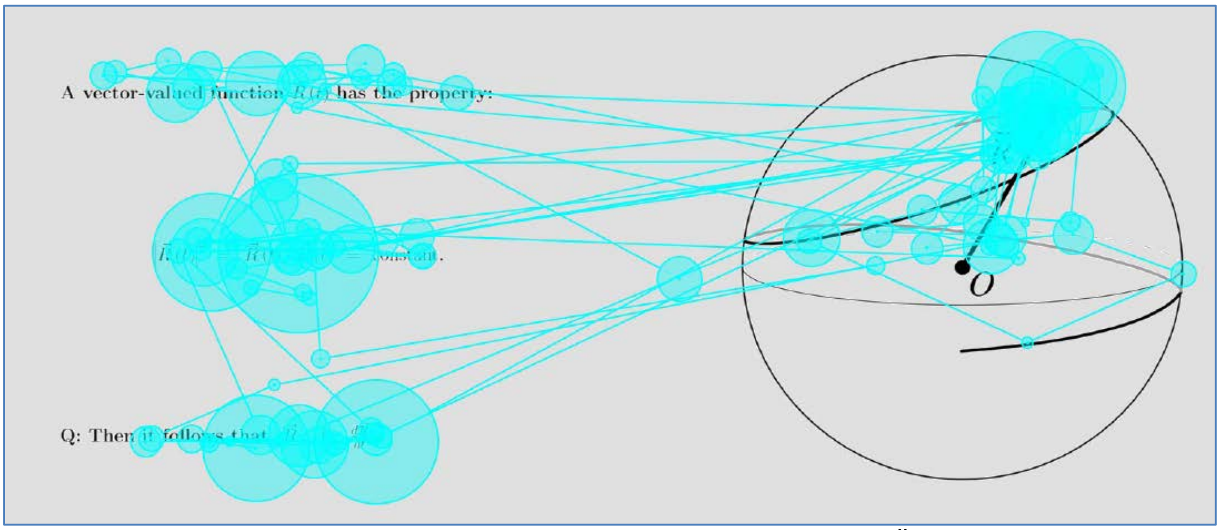


Figure 3.2 Multiple choice questions used by Nyström and Ögren (2012:2)

In this study, as mentioned in Section 2.8.1, true/false questions without any illustrations were used to determine whether learners could apply the divisibility rules correctly. There were an equal number of "*true*" and "*false*" statements in all the assessments. Although no illustrations were used, the dividends were compiled (Section 4.3.7) such that learners would probably provide an incorrect answer if they did not apply the divisibility rules correctly.

3.2.5 THE EFFECT OF THE ORDER OF FIXATIONS ON PROBLEM SOLVING WHEN THE ORDER IS IRRELEVANT

When learners have to determine whether a five-digit number is divisible by a certain single digit divisor, they should inspect only the relevant digits of the dividend according to the applicable divisibility rule. It does, however, not prevent them from inspecting other digits and neither does the order in which the digits are visited, matter.

Chesney, McNeil, Brockmole and Kelley (2013) did a study to determine if participants used correct relational strategies rather than typical arithmetic strategies to solve a mathematical equation in the form $\boldsymbol{a}+\boldsymbol{b}+\boldsymbol{c} = \boldsymbol{d}+$___. Relational strategies refer to the equal sign as a relational symbol where both sides of the equation have the same value (Chesney et al., 2013), whereas arithmetic strategies refer to addition and subtraction only to get to an answer. Participants were expected to verbalise their answers and the order in which they inspected the numbers in the equation was determined through eye-tracking. As in the current study, the order and number of fixations should not matter and Chesney et al. (2013) did indeed not find any evidence that the number of times that participants moved their gaze across the equal sign had an effect on whether or not the correct strategy was used.

For the purpose of the current study, the order in which the learners inspected the digits was not measured because of the associativity and commutativity principle that learners could apply (*cf.* Section 3.2.3). The number of visits on a digit was also not measured, but instead the percentage of total fixation time on each digit was calculated to reveal where the learners fixated most of their time.

3.2.6 THE EFFECT OF FIXATION TIME PER RELEVANT AOI ON PROBLEM SOLVING

In this study, individual digits in the dividend were regarded as areas of interest and the fixation time on each digit was expressed as a percentage of the total fixation time on the dividend. Susac, Bubic, Kaponja, Planinic and Palmovic (2014) followed a similar approach to determine the amount of attention on each element in an equation. They did a study with 40 university students on equation rearrangements that involved division. Three types of equations were used to solve $\boldsymbol{x}$ ($\boldsymbol{a} \cdot \boldsymbol{x} = \boldsymbol{b}$; $\frac{\boldsymbol{x}}{\boldsymbol{a}} = \boldsymbol{b}$; $\frac{\boldsymbol{a}}{\boldsymbol{x}} = \boldsymbol{b}$), as illustrated in Figure 3.3. Only the gaze behaviour of students who had the answer correct were analysed. Susac et al. (2014) found that (i) the accuracy improved and the reaction times of students decreased after practice and concluded that practice improve performance even when students knew how to solve the equations; (ii) there were lesser fixations for students who solve the equations correctly after practice that suggest that students developed more efficient strategies by knowing where to look for each of the equation problems and (iii) there was a correlation between the number of fixations and the ability of the participants to solve the equation.

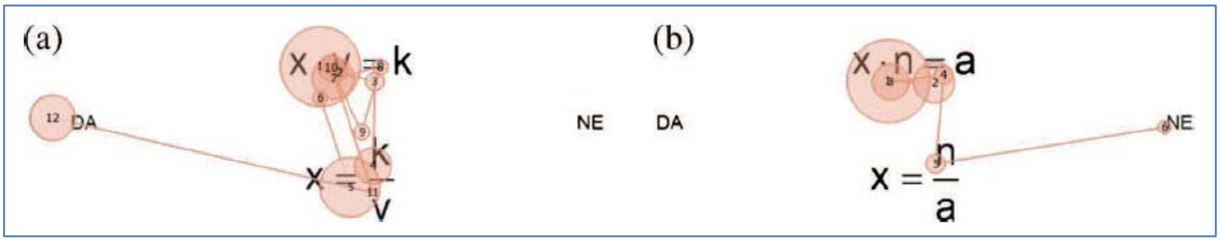


Figure 3.3 Equations used by Susac et al. (2014:567)

3.2.7 GAZE BEHAVIOUR WHEN SOLVING PROBLEMS THAT INVOLVE LONG DIVISION

In an eye-tracking study by Okamoto and Kuroda (2014), 12 university students had to solve a division puzzle, as indicated in Figure 3.4a. An EMR-9 head-mounted eye-tracker (NAC Image Technology) was used (Figure 3.4b). The purpose of the study was to determine if participants used the correct bottom-up strategy.

It was found that when participants were aware of the bottom-up approach, there was little eye movement, whereas participants who did not know the specific strategy displayed more eye movements. Although all the participants solved the division puzzle correctly, eye-tracking revealed that participants who knew the bottom-up strategy did not fixate on the previously visited AOIs again, whereas the other participants repeatedly fixated on previously visited AOIs. Such trends, where participants repeatedly fixated on previously visited AOIs, were observed in the current study (Section 4.3.1). The revisit of AOIs were not measured in this study because learners had to do mental calculations while inspecting the appropriate digits and it did not matter how many times a learner visited an AOI to conclude his/her answer.

Previous research studies found that the magnitude of numbers determine fixation duration (Mock, Huber, Klein & Moeller, 2016). When learners apply the divisibility rules for divisors 3, 6 and 9, they have to fixate on the digits and add them mentally. The duration of the fixation could be influenced by the magnitude of the digits as that could affect the time that learners need to do the mental calculations. The dividends used in the current study will be discussed in Section 4.3.7.4.

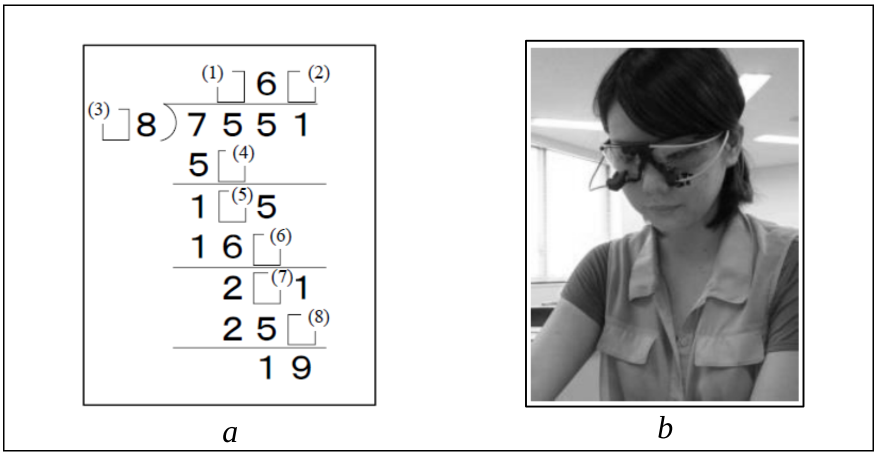


Figure 3.4 Illustration of division puzzle and eye-tracker (Okamoto and Kuroda, 2014:95)

3.2.8 GAZE BEHAVIOUR WHEN STUDYING AND ANSWERING QUESTIONS ON DIVISION THEORY

Cimen (2013) did a case study on division theory where (i) mathematical instruction was offered to elucidate the relationship between a divisor, dividend, quotient and remainder; and (ii) where participants were assessed on the learning material. The study included (i) surveys to measure epistemological beliefs, metacognitive strategies and levels of mathematical anxiety; (ii) behaviour data captured from audio-visual materials and eye-tracking; and (iii) physiological data that included heart, respiration and eye blink rates.

Participants studied the learning material on division theory online in their own time and without interference from the researcher. Their gaze was captured while they were engaging with the learning material, as illustrated in Figure 3.5. Thereafter, participants were assessed on their understanding of the concepts of division theory. The assessment contained 10 multiple choice or true/false questions (Figure 3.6). True/false questions, such as "*If 43 = 2(18)+6, then 18 is a divisor of 42*" and "*7 is a divisor of 42*", appeared in the assessment. Multiple choice questions, such as "*In the equation 42 = 2(18)+6, the dividend is*" followed by four options, also appeared in the assessment. With the use of eye-tracking, Cimen (2013) found that participants built connections between related concepts of divisibility as indicated in Figure 3.7.

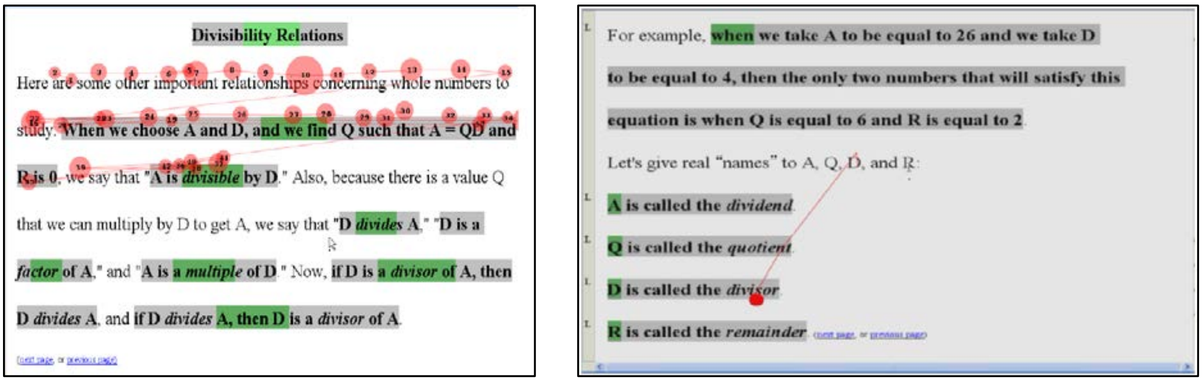


Figure 3.5 Illustration of mathematical instruction (Cimen, 2013:51-52)

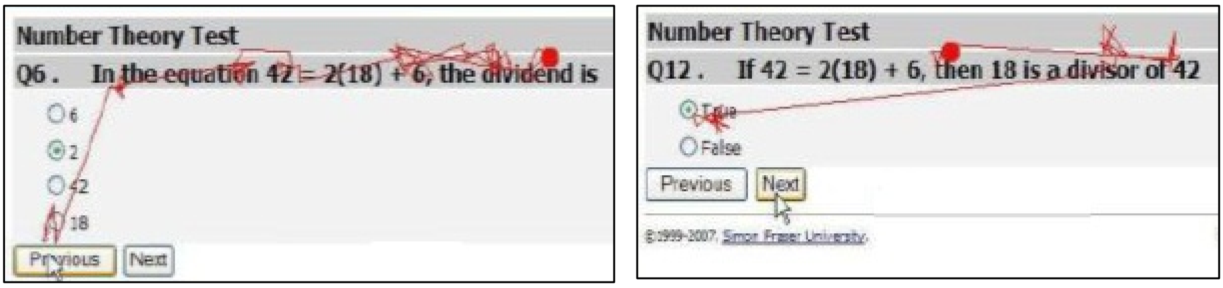


Figure 3.6 Examples of assessment questions (Cimen, 2013:122)

The current study and the study by Cimen (2013) share a common platform, namely divisibility. There are, however, differences between the two studies as indicated in Table 3.1.

Although Cimen (2013) did not set a time limit per question, learners should be able to answer three true/false questions or three short-answer questions per minute (Gronlund, 1993). In the current study, the stimuli with the questions were visible for only 20 seconds before the learner was prompted for a response. If learners provided an answer before 20 seconds had expired, the researcher continued to the next stimulus to avoid unnecessary fixations on the stimulus. The purpose of this study was to determine if learners knew the divisibility rules and therefore a time restriction was enforced to prevent learners from doing long division mentally. When learners provided an answer, it was assumed that they have decided on a strategy and are done with the mental arithmetic.

Figure 3.7 Connections between related concepts of divisibility (Cimen, 2013:157)

Table 3.1 Differences between the current study and the study performed by Cimen (2013)

This study	Study by Cimen (2013)
Investigated divisibility rules	Investigated division theory
Unprepared assessment on divisibility rules	Self-evaluation on mental calculations
Used five digit dividends	Used a maximum of two-digit dividends
Determined the percentage of fixation time on digits of dividends	Did not determine the percentage of fixation time on digits of dividends
Researcher revised the divisibility rules with the learners	Participants studied the learning material on their own
Performed two assessments	Performed one assessment
Learners could not return to a previous question during an assessment	Participants could return to a previous question during the assessment
Time limit of 20 seconds per question	No time limit per question
Learners motivated their answers	Participants did not motivate their answers
Participants expressed their answers and motivations verbally	Participants indicated answers by selecting an option with a mouse

3.3 NUMBER PROCESSING BASED ON EYE-TRACKING DATA IN NUMERICAL COGNITION

Mock et al. (2016) have identified more than 40 eye-tracking studies on numerical cognition that were performed over the past 40 years and suggested a temporal dynamics model of number processing (Figure 3.8).

The rectangular shapes in Figure 3.8 represent the different processing stages. The top rectangle with the white background (Rectangular 1) represents the early processing stage, which is largely an automatic process and where the numeric representations are conceptualised. The light grey rectangle (Rectangular 2) represents later stages of processing where the context of the task at hand is integrated with previous numerical representations. The dark grey rectangle (Rectangular 3), which is assumed to be under cognitive control, represents the plausibility that the task at hand was performed successfully. If a specific task is not performed successfully, it is repeated until a satisfactory answer is formulated.

The current study can relate to the above-mentioned model. For example, if the dividend is 75133 and the divisor is 3, learners who know the divisibility rule will calculate 7+5+1+3+3=19. Therefore, they will fixate on a digit of the dividend (digit 1 or digit 5, depending from which side the learner wants to add), as illustrated in Figure 3.8a. They will recognise the digit (Figure 3.8b) and recall the steps of the divisibility rule for 3 (Figure 3.8c). They will add the value of the current digit to the sum of the digits that they have inspected already (Figure 3.8d). If they have not inspected all the digits yet (Figure 3.8e), they will fixate on the next digit (Figure 3.8a) and repeat the process. After adding all the digits, learners have to decide whether they are confident about their proposed answer (Figure 3.8f). If they are satisfied with their answer (Figure 3.8g), they respond, otherwise they repeat the process.

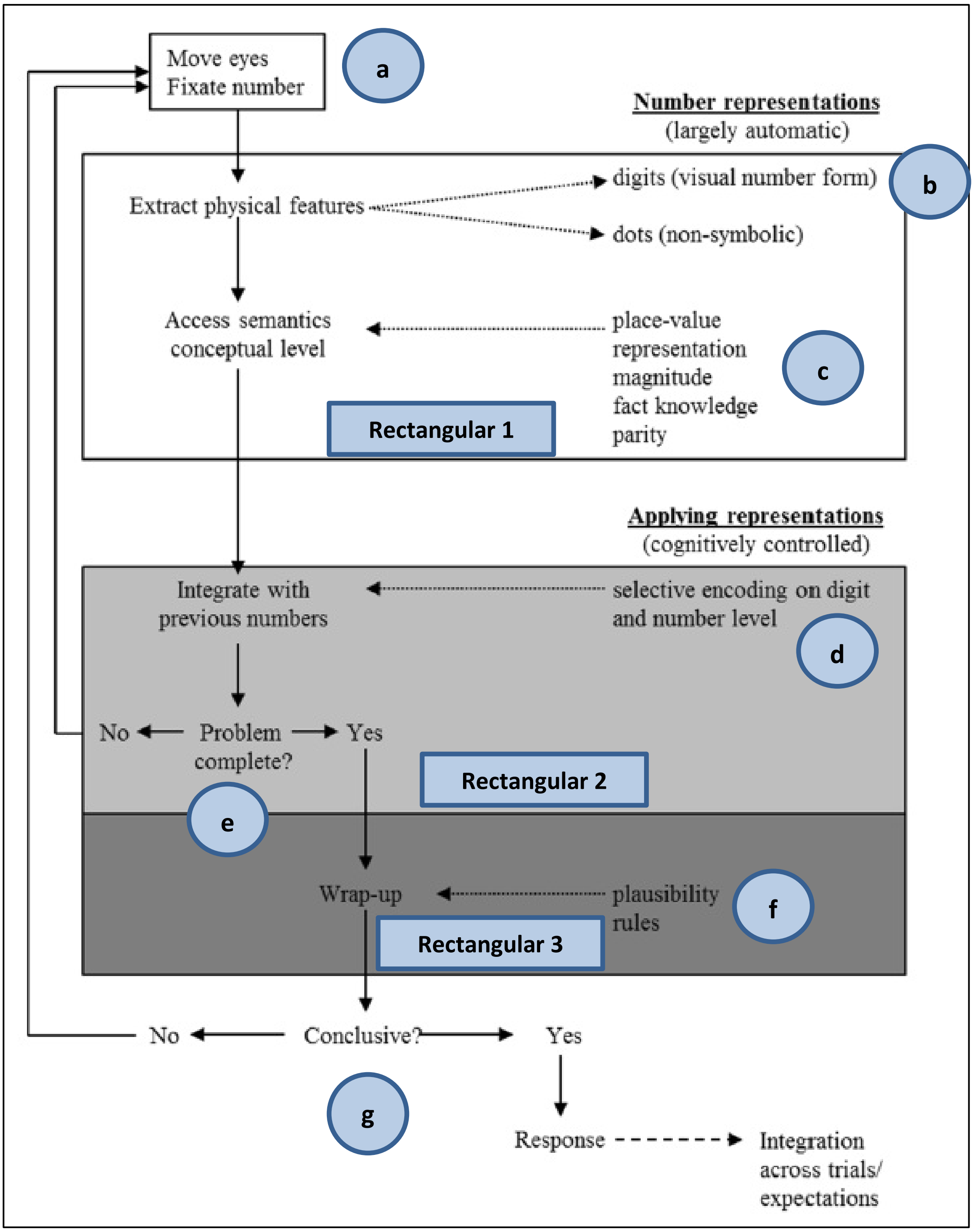


Figure 3.8 Model of the temporal dynamics of number processing based on eye-tracking data in numerical cognition, as suggested by Mock et al. (2016)

3.4 THE USE OF EYE-TRACKING DURING TEACHING

It is challenging for a teacher to manage the learning that takes place in many learners' minds at the same time (Wiliam, 2014). Teachers are not in a position to provide immediate feedback to individual learners in a large group, whereas a computer-based system could provide immediate feedback and also keep record of learners' responses (Suppes & Jerman, 1970). Furthermore, with an electronic system, learners could take part in classroom activities without the pressure of raising hands and having to answer questions in class (Zhu, 2006; Wiliam, 2014).

According to the National Council of Teachers of Mathematics (NCTM) in the Unites States of America, technology could be used for the marking of answers (Burkman, 2013; Singh, 2014) and for immediate feedback (Doukas & Andreatos, 2007). Immediate feedback on the incorrect application of a divisibility rule, based on gaze behaviour, would prevent learners from adopting an incorrect strategy.

3.4.1 CLICKER TECHNOLOGY AS A FRAMEWORK FOR EYE-TRACKING

Although clicker technology is beyond the scope of the current study, it is briefly discussed here to provide a framework for the use of eye-tracker technology in a classroom environment.

Clickers are interactive remote devices that learners use to respond to questions, and these responses are available to the teacher immediately (Lantz & Stawiski, 2014). Responses can be anonymous or linked to learners, which would provide the teacher with immediate knowledge of who responded with a correct or incorrect answer (Blasco-Arcas, Buil, Hernandez-Ortega & Sese, 2013).

Clicker technology creates an environment that enables teachers to quickly inspect learners' responses in real-time and decides what to do next (Wiliam, 2014) and can be used to assess learners' current knowledge and to identify any misunderstandings that learners may have (Zhu, 2006).

On the downside, when clickers are used, it means that all responses from learners are saved - which may conflict with the idea that a classroom is supposed to be a safe place, where mistakes can be made without the evidence being stored (Wiliam, 2014). Furthermore, the cost to establish a classroom with the necessary equipment to assess 20 to 30 learners with clickers, as well as the time to set it up, limit the implementation thereof (Zhu, 2006; Wiliam, 2014).

3.4.2 ELECTRONIC MATHEMATICAL EDUCATIONAL GAMES

Although educational computer games can be entertaining, there is no certainty that learning takes place (Conati & Zhao, 2004). *Prime Climb* is an electronic educational game, illustrated in Figure 3.9. The aim of the game is to practice the factorisation of numbers and learners are guided by an agent or a tool that displays the prime factors of a dividend (Conati & Manske, 2009), as discussed in Section 2.4.2. Players must move to numbers that do not share common factors with their opponent's number, otherwise they lose the game (Muir & Conati, 2012).

Muir and Conati (2012) further mentioned that *Prime Climb* helps learners when they made incorrect moves caused by lack of factorisation skills, or when they correctly guessed the next move. Muir and Conati (2012) did an eye-tracking study to investigate the intention patterns of learners. Their study was based on a student model (Conati & Zhao, 2004) that uses artificial intelligence to predict the behaviour and intentions of the learner. They found that the attention to given hints decreased as the game proceeded and learners with limited knowledge of factorisation seldom paid attention to the given hints.

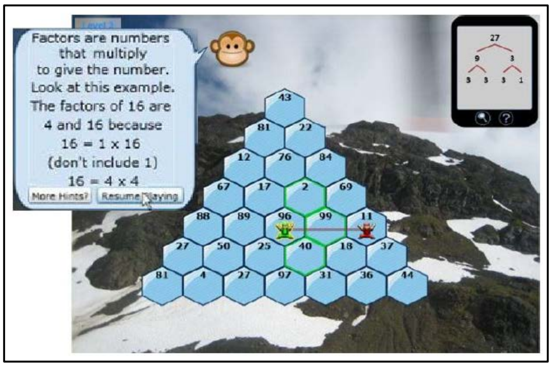


Figure 3.9 Illustration of Prime Climb (Muir & Conati, 2012:114)

The suggested use of eye-tracking in a classroom environment (Appendix A) neither included hints as stated above, nor did it make use of mathematical education (Section 3.2.8). The intention with the use of eye-tracking in a classroom environment in this study is purely for assessment purposes, which will be discussed in the next section.

3.4.3 EYE-TRACKING TECHNOLOGY IN CLASSROOMS

The correct application of Mathematics rules is crucial to the accuracy of a solution (Conley, 2011). Clickers can assist the teacher to determine whether learners understand the learning material, but the teacher would have no evidence that learners applied the correct mathematical rules to get to their answer as far as true/false and multiple choice questions are concerned. If, however, the learners' gaze behaviour were available, the teacher could possibly identify which method the learners followed to get to a specific answer. Appendix A outlines possible classroom scenarios if eye-tracking technology is available in classrooms.

Eye-trackers are valuable interactive systems (Duchowski, 2002) that respond to gaze behaviour in real-time and can improve participants' learning (Sottilare, Graesser, Hu & Holden, 2013). Although only one eye-tracker was used in the current study, it is worthwhile to explore the use of eye-tracking in a classroom environment, as suggested in Appendix A.

The reduced cost of eye-trackers, combined with the general improvement of computer technologies, make it feasible to do experiments with more than one eye-tracker simultaneously in a classroom environment (Nyström, Niehorster, Cornelissen & Garde, 2016). In such a setting, eye-tracking could be used as an additional tool to assist teachers in the early identification of misconceptions regarding the divisibility rules. The teacher could also display an expert's gaze behaviour while explaining the application of divisibility rules because it may have an influence on their gaze behaviour and also enhance their performance (Jarodzka, Scheiter, Gerjets, Van Gog & Dorr, 2010b).

Lai et al. (2013) reviewed the use of eye-tracking in the learning process from 2000 to 2012. All the studies that they reviewed, used one eye-tracker in a laboratory environment. Although current online assessment systems do not include a function to track the gaze behaviour of the learners (Tsai, Hou, Lai, Liu & Yang, 2012), Lai et al. (2013) recommended that two or more eye-trackers could be used to investigate learners' moment-to-moment eye fixations in a classroom environment.

In this study, parameters were developed for an instrument (Chapter 6) to determine the required percentage of fixation time per digit per divisor to identify if a learner applied a divisibility rule correctly or not. The instrument also provides guidelines for teachers to compile effective dividends when the divisibility rules are assessed. Application of this instrument in a real-time classroom setting would enable a teacher to detect misconceptions of the divisibility rules at an early stage and intervene before learners adapt to an incorrect strategy. Development of software for the instrument was, however, beyond the scope of this dissertation.

3.4.4 ADVANTAGES AND DISADVANTAGES OF EYE-TRACKING IN A CLASSROOM ENVIRONMENT

Assessments that take place in classrooms where learners have to write their responses on paper, have advantages over assessments in laboratories because the learners are in a familiar environment. There are, however, fewer distractions in laboratories and eye-tracking could be used to determine the strategies that learners use to solve given problems (Godau, 2015).

Gluck, Anderson and Douglass (2000) found that it is possible that an eye-tracker in an eye-tracking tutoring system can predict if a learner is going to make a mistake. The tutoring system could then intervene and assist the learner before the mistake is made. Teachers can also make use of heat maps to identify difficulties that learners may have with mathematical reasoning (Bolden, Barmby, Raine & Gardner, 2015).

3.5 SUMMARY

Previous eye-tracking research proved that gaze can be used to identify the strategies that participants use to solve Mathematical problems. Eye-tracking can also reveal on which areas participants fixate while dealing with division and divisibility and therefore eye-tracking can be a useful tool in identifying whether participants followed the correct strategy.

For the purpose of this study, gaze behaviour will be analysed to determine if learners follow the correct strategy to solve questions where they have to apply the divisibility rules. The study will also investigate to what extent gaze behaviour can be used on its own, i.e. without a verbal response, to determine if a divisibility rule is applied correctly.

The next chapter will focus on the methodology followed in the current study.

CHAPTER 4
THE METHOD

4.1 INTRODUCTION

In the previous chapter, earlier attempts to analyse gaze behaviour when mathematical problems are solved, were discussed. Specifically, participants' gaze behaviour was analysed while they attempted problems that involved long division and divisibility.

In this study, learners participated in four different assessments. In each of these assessments, the learners were requested to indicate if a certain number, referred to as the dividend, is divisible by a specified single digit divisor. The first and third assessments (before and after revision respectively) were done in a classroom with all learners participating. The questions were displayed by means of a data projector and the learners wrote their responses (answer and a reason/motivation for their answer) on an answer sheet. The second and the fourth assessments (before and after revision respectively) were individual sessions led by the researcher. The questions were displayed on a laptop equipped with an eye-tracker and learners had to verbalise their responses.

The following aspects will be discussed in this chapter:

- Research design (Section 4.2)
- Methodology (Section 4.3)
- Pilot study (Section 4.3.1)
- Research tools (Section 4.3.2)
- Presentation of questions for Assessments 1 and 3 (Section 4.3.3)
- Answer sheets (Section 4.3.4)
- Presentation of questions for Assessments 2 and 4 (Section 4.3.5)
- Calibration and data quality (Section 4.3.6)
- Compilation of questions (Section 4.3.7)
- Data collection (Section 4.3.8)
- Data analysis (Section 4.3.9)
- Ethical procedures (Section 4.4)

4.2 RESEARCH DESIGN

Common approaches used by researchers in Information Technology include literature reviews, experiments, models, surveys, case studies, prototypes, arguments, mathematical proofs and various combinations of these (Olivier, 2009). Experimental design, that includes experiments and observational studies, are commonly used for eye-tracking studies (Duchowski, 2007).

All the learners in Grade 4 to Grade 7 from two schools in Bloemfontein (which will be discussed in Section 4.3.8.1), who were present on the day participated in the first paper-based assessment on divisibility rules (Assessment 1, as illustrated in Figure 4.1). Selected learners from those who were present during Assessment 1, participated in the second assessment on divisibility rules (Assessment 2) where an eye-tracker was used. Thereafter, the divisibility rules were explained or revised and all learners present at the time completed a third assessment (Assessment 3) with the same structure as Assessment 1. Assessment 4 was done after the revision session to collect gaze data from the same learners who took part in Assessment 2. Figure 4.1 shows a diagrammatic representation of the sequence of events.

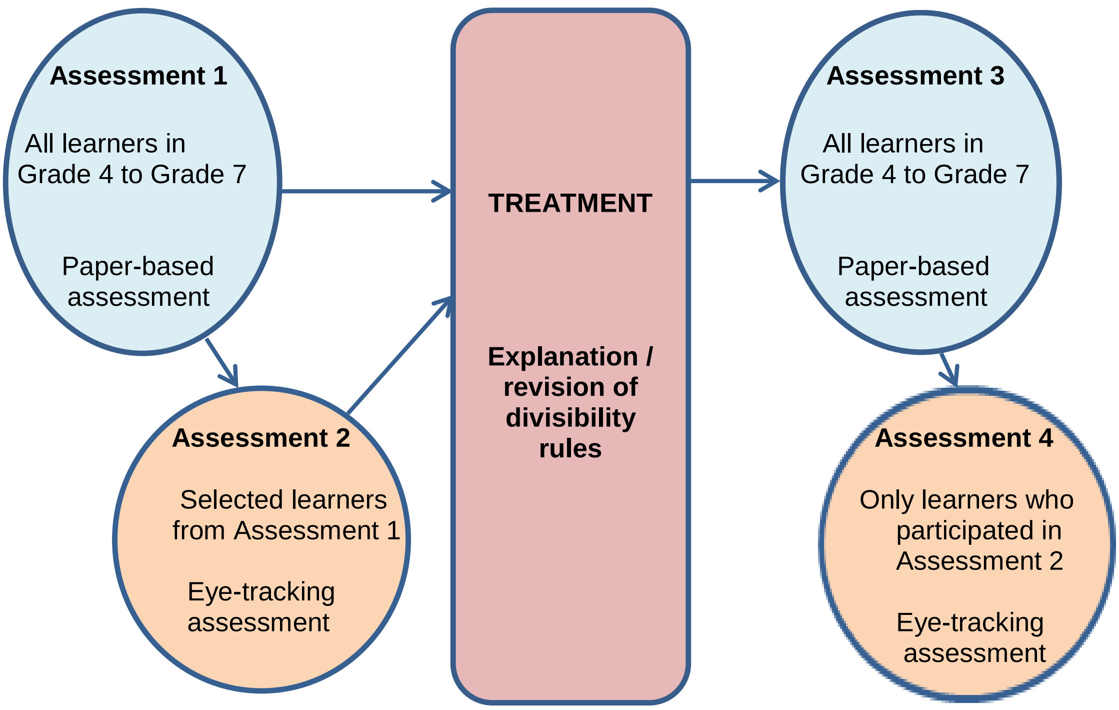


Figure 4.1 Sequence and details of assessments

The following design methods were considered for the current study.

4.2.1 SURVEYS

Four assessments consisting of questions where the respondents had to indicate whether a dividend was divisible by a divisor, were completed. Although Hofstee (2006) discouraged the use of open questions, the current study needed responses from learners in their own words to determine if they knew the divisibility rules.

4.2.2 GENERAL PRE-POST EXPERIMENTAL DESIGN

For the purpose of the current study, the general pre-post experimental design was chosen because there was no control group and the same learners participated in the study before and after the divisibility rules were explained or revised. This design is also a common approach in eye-tracking research (Duchowski, 2007).

The general pre-post experimental design requires the same group of individuals to be measured prior to the treatment and after the treatment (Duchowski, 2007). Therefore, data was analysed only for learners who participated in both Assessment 1 and Assessment 3. Only learners who participated in both Assessment 2 and Assessment 4 were used for the analysis of the eye-tracking recordings.

4.3 METHODOLOGY

4.3.1 PILOT STUDY

Seven learners from Grade 4 to Grade 7, and not attending any of the two schools that participated in the actual study, took part in the first pilot assessment. Only one of these learners knew the divisibility rules at the time of the assessment. The questions were displayed on a laptop, and learners had to indicate "*yes*" if the dividend was divisible by the divisor, or "*no*" if that was not the case. The learners were not expected to give reasons to motivate their answers. However, the dividends were not carefully selected (*cf.* Section 4.3.7) and learners could guess the correct answer. The result was that nearly no difference in the performance of the learner who knew the divisibility rules and the other learners was found.

For the second pilot assessment, the dividends were compiled as discussed in Section 4.3.7 below and provision was made for learners to give reasons for their answers. The researcher was, therefore, in a position to identify if the learners knew the divisibility rules or guessed the answers.

Staff members from a local institution of higher education took part in the pilot eye-tracking assessment. Gaze behaviour that supports the findings of Okamuto and Kuroda (2014), namely that there are more eye movements when participants are unaware of an efficient strategy (Section 3.2.7), were observed and are illustrated in Figure 4.2. The participant in Figure 4.2a did not know the divisibility rule for divisor 8 and probably did mental long division, whereas the participant in Figure 4.2b knew the divisibility rule. Fixations were observed between the digits of the dividend, and this observation will be addressed in Section 4.3.5.

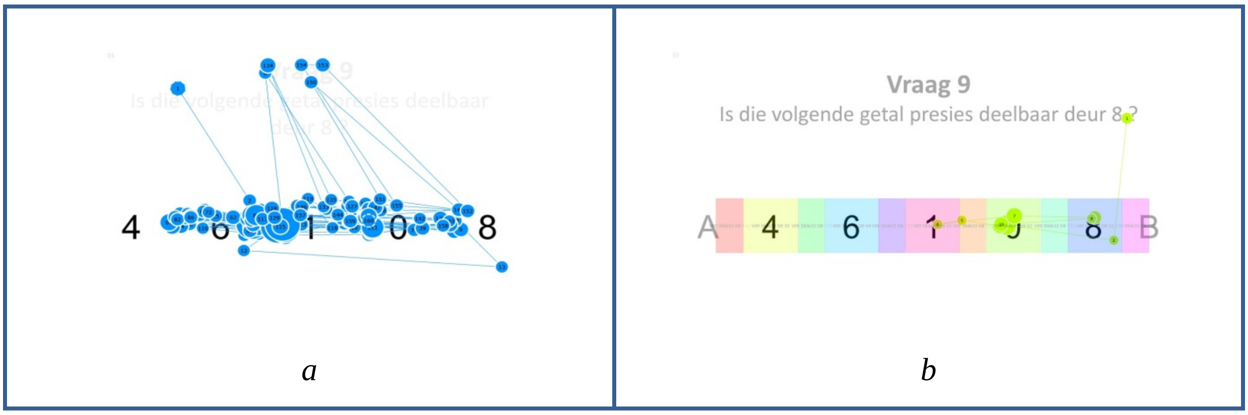


Figure 4.2 Gaze behaviour during preliminary investigation for (a) a participant who did not know the divisibility rule and (b) a participant who knew the divisibility rule

The data that were captured during the eye-tracking recordings of the pilot study were to the researcher's satisfaction. Only small adjustments to the stimuli were made before the research study continued.

4.3.2 RESEARCH TOOLS

Presentation of questionnaires and stimuli

Microsoft PowerPoint was used to present the questionnaires for Assessment 1 and Assessment 3 by means of a data projector. PowerPoint was also used for the revision lesson and to present the stimuli during Assessment 2 and Assessment 4 (*cf*. Section 4.3.3).

Answer sheets were issued to all the learners for Assessment 1 and Assessment 3 (Appendix B) (*cf*. Section 4.3.3). The researcher recorded learners' responses for Assessment 2 and Assessment 4 on answer sheets as well (Appendix C).

Hardware

A 60 Hz Tobii X2-60 Wide eye-tracker was used to capture the gaze behaviour of learners of School A (*cf*. Section 4.3.8.1) for Assessment 2 and Assessment 4 (*cf*. Figure 4.1). It was also used for Assessment 2 for learners of School B (*cf*. Section 4.3.8.1). Due to a lack of accuracy that was observed during this assessment (possibly due to the fact that all learners had dark brown irises) (Section 5.3.1), a 300 Hz Tobii TX-300 eye-tracker was used for Assessment 4 for learners of School B.

Also due to the lack of accuracy that was observed during Assessment 2 for learners of School B, the AOIs were enlarged to ensure that all fixations on the digits were recorded. The recordings of Assessment 2 for School B were analysed separately and were not compared with Assessment 4 for School B or any recordings for School A. These recordings were in any case not considered for the final analysis because of the insufficient number of correct answers and reasons that learners of School B provided (Table 5.11). The recordings obtained from learners of School A were adequate to continue and conclude the study.

An Acer 2630QM laptop with Intel Core i7 CPU (2.9 GHz) and 4 GB RAM with Windows 7 as operating system was used for Assessment 2 and Assessment 4.

Software

Tobii Studio (version 12) (Tobii AB, 2014) was used to obtain the percentage of total fixation time on each AOI for all the eye-tracking recordings. These percentages, together with learners' responses (answer and reason), were entered into an **MS Excel** spreadsheet to prepare the data for statistical analysis in **STATISTICA** version 12 (Dell Inc., 2015). MS Excel and STATISTICA were used to produce the graphs that are presented later in this dissertation.

4.3.3 PRESENTATION OF QUESTIONS FOR ASSESSMENTS 1 AND 3

Presentation of questions for Assessment 1 and Assessment 3 by means of a data projector would not be unfamiliar to the learners as they were used to information being presented in this way. In order to avoid distractions, a white background with black text was used for presenting the questions (Figure 4.3). All slides were presented in landscape format. The numbers were displayed in an Arial typeface with font size 138 and with one space between digits. The numbering of each question, for example "*Question 1*", was presented with font size 48 while the instructions were presented with font size 40 (Appendix D).

The first slide, which was displayed before the assessment commenced, was a colourful picture to attract attention. The second to tenth slides displayed the instructions and a number of examples which the researcher used to familiarise the learners with what they could expect.

The actual questions that learners had to complete started on the eleventh slide. The instructions for each question were at first displayed without the dividend (Figure 4.3). Once the learners had time to inspect the instruction, the dividend was displayed (Figure 4.4). The instruction was still visible in case the learner forgot which divisor to use, but it was displayed in grey to minimise distraction from the dividend.

As mentioned in Section 3.2.8, a learner should be able to answer three true/false questions or three short-answer questions per minute (Gronlund, 1993). The nature of the questions in the current study resembled that of true/false questions and therefore the dividend was displayed for 20 seconds only before it disappeared from the screen.

Finally, a slide that instructed the learners to write down their responses was displayed (Figure 4.5).

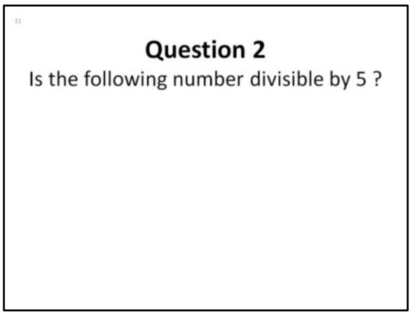


Figure 4.3 Instruction with divisor only

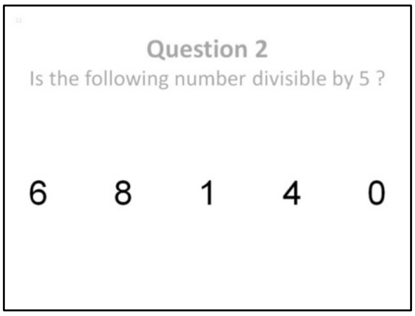


Figure 4.4 Instruction with divisor and dividend

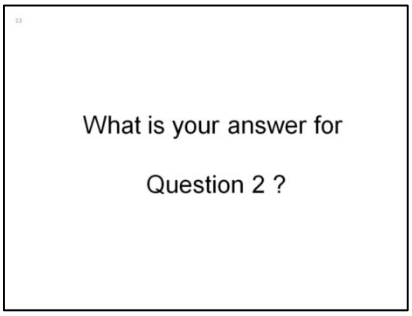


Figure 4.5 Prompt to respond

4.3.4 ANSWER SHEETS

The answer sheets were designed to capture the learners' ability to apply the divisibility rules correctly (Appendix B). The surname, name, grade, section, age and gender for each learner were acquired. The name and the surname were required to provide confidential feedback to the teachers and learners. The gender was needed to keep a balance between the genders when learners were identified for participation, but was not used as a factor during the analysis.

Learners had to inspect the dividend and divisor displayed on the screen and write down their responses on the answer sheet (Figure 4.6). For each question, learners had to tick either "*Yes*", "*No*" or "*Do not know*" and write down a motivation in the space provided.

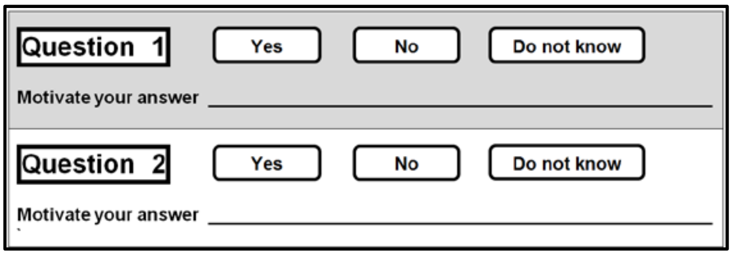


Figure 4.6 Answer sheet for Assessment 1 and Assessment 3

The motivations were required to indicate if the learner provided the correct answer because he/she knew the divisibility rule or guessed. When learners provided an acceptable motivation along with an incorrect answer, they probably knew the divisibility rule but made calculation errors or applied the rule incorrectly.

If learners did not answer a question and/or did not provide a motivation, the answer and/or motivation was recorded as incorrect.

4.3.5 PRESENTATION OF QUESTIONS FOR ASSESSMENTS 2 AND 4

As for the PowerPoint presentations of the questions in Assessments 1 and 3, the background of the stimuli for Assessments 2 and 4 was white with black text. The stimuli were presented as on-screen slides in landscape format and optimised for the laptop display with a resolution of 1600×900 pixels (36.09°×20.77°). An "*A*" was placed in front of and a "*B*" at the end of the dividend to minimise the positional preference that the first and last digits would enjoy (Figure 4.7).

The numbering of each question, for example "*Question 1*", was presented in Calibri with font size 61 and the instruction for the question was displayed in Calibri with font size 50. The numbers, with spaces between the digits, were displayed in an Arial typeface with font size 72 (0.88°×1.32°). The dividend and all characters were spread out evenly across the display. The distance between the digits of the dividend was 3.97°. The distance between digits was such that a neighbouring digit would not be perceived within the acute or parafoveal regions (Duchowski, 2007). The spaces between the digits were a trade-off between the confidence in eye-tracking data and ecological validity. In this study reliable eye-tracking data is more important than ecological validity – especially in the light of the fact that learners got an opportunity to practice before the actual assessment commence. The distance between digits was such that a neighbouring digit would not be perceived within the acute or parafoveal regions (Duchowski, 2007).

Each digit was included in an AOI of 160×160 pixels (3.26°×3.26°) (Figure 4.7). Between each of these AOIs was another AOI of 80×160 pixels (1.63°×3.26°). Fixations on the AOIs between the digits were divided 50-50 between the two digits on either side. AOIs of 80×160 pixels were also placed at the ends of the dividend. The fixation time on the AOIs were used to calculate the percentage of total fixation time on the entire dividend for each one of the five digits.

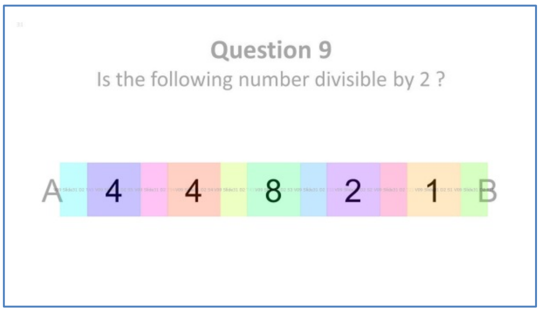


Figure 4.7 Areas of interest

The first stimulus displayed the same picture as the opening slide of Assessment 1. The second to seventh stimuli displayed the instructions and an example that the researcher used in his instruction to the learners. The learners were prompted to verbalise a dividend that were spaced out (Section 4.3.5) and also to exercise divisibility on two dividends (Appendix D). The questions started at slide eight. The same order as with Assessment 1 and Assessment 3 was followed, except that the researcher initiated the move to the next stimulus as soon as the learner provided an answer or after 20 seconds have expired (see the discussion in Section 4.3.8.4 below). The researcher also recorded the learners' verbal responses.

4.3.6 *CALIBRATION AND DATA QUALITY*

A nine point calibration was done prior to the assessment to ensure good quality of the eye gaze recordings. Learners could make sudden head movements that could influence the recordings. Although they were instructed to keep their eyes fixated on the display, it could happen that they looked elsewhere during the recording. Such behaviour would have an effect on the quality of the recordings. The accuracy and the precision for the Tobii X2-60 were 0.4° and 0.34° respectively and for the Tobii TX-300 it were 0.4° and 0.08° respectively (Tobii, 2014) (*cf*. Section 3.9.1). The events in the eye-tracking data were detected by manufacturer software with default settings for dispersion and minimum fixation duration as illustrated in Figure 4.8. It was assumed that for the short duration of a recording session (less than 10 minutes), it was not necessary to check for drift and therefore no intermediate calibrations were done.

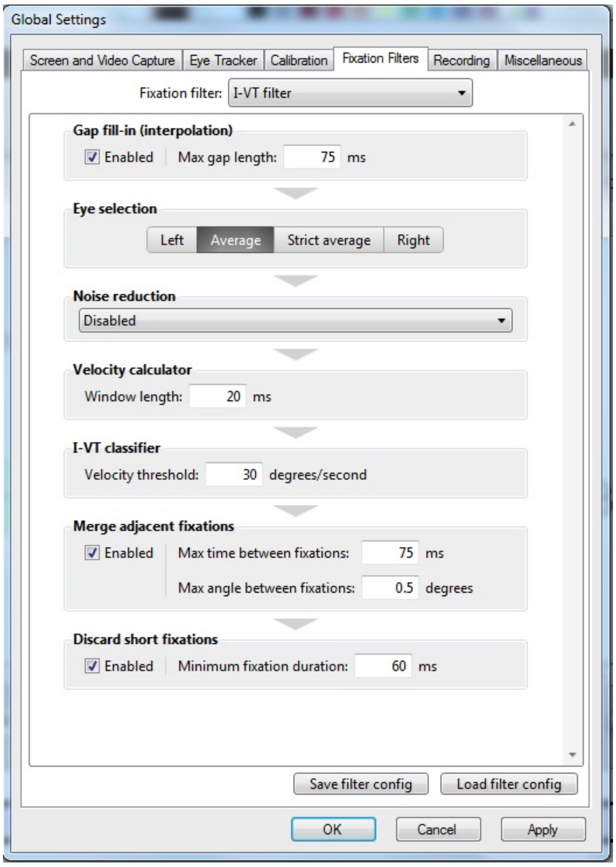


Figure 4.8 Default settings of manufacturer software

4.3.7 *COMPILATION OF QUESTIONS*

4.3.7.1 RANGE OF DIVIDENDS

All the dividends that were presented to learners were five-digit numbers as Grade 4 learners are supposed to identify a five-digit number (Department of Basic Education, 2014g). The numeric value of the dividends was irrelevant for the testing of divisibility, because if the learners knew the divisibility rules, their focus would be on the applicable digits only.

A calculator was not allowed during the assessments and the choice of digits in the dividends was such that learners could do mental calculations. For example, for 10199 / 9, the learners should add all the digits and calculate 1+0+1+9+9=20. For 63172 / 4, learners should inspect only the number formed by the last two digits of the dividend (63172) and, since they were supposed to know the mathematical tables, they should know that 72 is divisible by 4. If they did not know the mathematical tables, they could do long division (4 divides into 7 with a remainder of 3; 4 divides into 32 without a remainder.) All dividends were chosen such that the sum of all the digits varied from 18 to 20 (for example, 6+3+1+7+2=19).

4.3.7.2 RANGE OF DIVISORS

Only divisors two (2) to nine (9), excluding seven (7), were used in the assessments. Divisor seven (7) was excluded because the rule for divisibility by seven is more complicated than the rules for the other divisors in the range.

4.3.7.3 TOTAL NUMBER OF QUESTIONS

Fourteen questions, two per divisor, were asked in each of the four assessments. For each divisor there was a question where the dividend was divisible by the divisor and a question where the dividend was *not* divisible by the divisor.

4.3.7.4 COMPILATION OF DIVIDENDS FOR EACH QUESTION

During the pilot phase of the study (Section 4.3.1), it was observed that learners inspected only the last digit or the number formed by the last two digits if they did not know the divisibility rule. Therefore, most of the dividends were compiled such that when learners inspected the last digit or the number formed by the last two digits, the answer would probably be incorrect. For example, for 10199 / 9, learners would inspect the “9” or “99” and decide that the dividend was divisible by 9, which is incorrect.

The reasoning behind each of the questions in Assessment 1 is discussed below (Appendix E):

<u>QUESTION 1 - 72154 / 5</u>

The learner should inspect only the last digit of the dividend. The last digit is 4 and therefore a "*no*" response would be correct. The digit 5 in the dividend is used as a distractor, as discussed in Section 2.8.2.

<u>QUESTION 2 - 73134 / 2</u>

The learner should inspect only the last digit of the dividend. The last digit is an even number and therefore a "*yes*" response would be correct. The fact that all digits but the last one are odd, serve as distractor.

<u>QUESTION 3 - 75133 / 3</u>

The learner should add all digits of the dividend. The sum of the digits is not divisible by 3 and therefore a "*no*" response would be correct. The last and second-to-last digits are used as distractors, because if learners inspect only the last digit or the number formed by the last two digits, they would probably indicate that the dividend is divisible by 3.

<u>QUESTION 4 - 10199 / 9</u>

The learner should add all digits of the dividend. The sum of the digits is not divisible by 9 and therefore a "*no*" response would be correct. The last and second-to-last digits are used as distractors, because if learners inspect only the last digit or the number formed by the last two digits, they would probably indicate that the dividend is divisible by 9.

<u>QUESTION 5 - 63172 / 4</u>

The learner should inspect only the number formed by the last two digits of the dividend. The number formed by the last two digits is 72, and 72 is divisible by 4. Therefore, a "*yes*" response would be correct. Some of the digits are odd and the last digit is not a multiple of four, which is used as the distractor.

<u>QUESTION 6 - 46108 / 8</u>

The learner should inspect only the number formed by last three digits which is 108 and not divisible by 8. Therefore, a "*no*" response would be correct. The last digit and the number formed by the last two digits are used as distractors because 8 is divisible by 8.

QUESTION 7 - 52336 / 6

The learner should apply the divisibility rule for divisor 2 and divisor 3. Both conditions do not apply and therefore a "*no*" response would be correct. The last digit and the number formed by the last two digits are used as distractors because 6 and 36 are divisible by 6.

QUESTION 8 - 61254 / 4

The learner should inspect only the last two digits of the dividend. The number formed by the last two digits is 54 which is not divisible by 4. Therefore, a "*no*" response would be correct. The fact that the last digit is a 4, is used as the distractor.

QUESTION 9 - 44821 / 2

The learner should inspect only the last digit of the dividend. It is not an even number and therefore a "*no*" response would be correct. The fact that all digits, but for the last one, is even, was used as the distractor.

QUESTION 10 - 68140 / 5

The learner should inspect only the last digit of the dividend. The last digit is zero (0) and therefore a "*yes*" response would be correct. The last digit, a zero, is used as a distractor because the learners might reason that the number should end in a five to be divisible by 5.

QUESTION 11 - 45432 / 6

As for question 7, the learner should inspect if the dividend is divisible by both 2 and 3. Both conditions apply and therefore a "*yes*" response would be correct. The last digit and the number formed by the last two digits are used as distractors because neither 2, nor 32, is a multiple of 6.

QUESTION 12 - 74511 / 3

The learner should add all digits of the dividend. The sum of the digits is divisible by 3 and therefore a "*yes*" response would be correct. The last digit and the number formed by the last two digits are used as distractors because neither 1, nor 11, is a multiple of 3.

QUESTION 13 - 41346 / 9

The learner should add all digits of the dividend. The sum of the digits is divisible by 9 and therefore a "*yes*" response would be correct. The last and the second-to-last digits are used as distractors, because if learners inspect only the last digit or the number formed by the last two digits, they would probably indicate that the number is not divisible by 9.

QUESTION 14 - 23960 / 8

The learner should inspect only the number formed by the last three digits which is 960 and divisible by 8. Therefore, a "*yes*" response would be correct. The number formed by the last two digits is used as distractor because 60 is not a multiple of 8.

4.3.8 DATA COLLECTION

4.3.8.1 RESEARCH SITE

There are seven grades in primary schools in South Africa, namely Grade 1 to Grade 7. A learner in a South African school must attend Grade 1 in the calendar year that the learner turns seven (Department of Education, 1998). Even though it is already more than 20 years after the end of the Apartheid era, concerns are being raised that the majority of the learners in South Africa are still part of the historically disadvantaged system. In other words, most learners are black and coloured and the schools that they attend often reflect low proficiency in reading, writing and numeracy (Van der Berg, Taylor, Gustafsson, Spaull, & Armstrong, 2011). Spaull (2013b) further emphasised that schools that serve mainly white learners, stayed functional. Beets (2012) stated that policies were formulated after 1994 to re-structure the education system in South Africa in an effort to reduce the inequalities of the past. Therefore, two schools were selected who had done the divisibility rules and who were willing to take part in the project. One of the schools (School A) was located in an upper class urban area, whereas the other school (School B) was located in a township area with the average LSM[1] of the population probably being below 6 (Van Biljon and Jansen van Rensburg, 2011).

[1] Living Standard Measure (http://www.saarf.co.za/LSM/lsms.asp)

4.3.8.2 RESEARCH POPULATION

In order to participate in the current study, learners were required to be of a certain age, possess specific mathematical skills and display stable gaze. Specific requirements regarding (i) knowledge of numbers, (ii) the ages at which learners are supposed to be able to do the required mathematical calculations, and (iii) the academic profile of the schools participating in the current study, are discussed below:

Knowledge of numbers

Elementary Mathematics consists of five building blocks, namely (i) numbers, (ii) place values, (iii) integer operations, (iv) fractions and decimals, and (v) problem solving (Wilson, 2009). It is important that children from an early age have knowledge of numbers in order to develop their arithmetic ability (Östergren & Träff, 2013). Mathematical content in primary schools are focused and straightforward, but it has to be learned properly for learners to progress (Wilson, 2009).

In order to participate in this study, learners had to possess knowledge of numbers and know how to perform mathematical operations, such as addition and division, on numbers. Specifically, learners had to identify a five-digit dividend. Learners should be able to add single digit numbers and do division on three-digit dividends to apply the divisibility rules.

Minimum age to perform required mathematical calculations

Learners perform better as they grow older, and a significant improvement with respect to which strategies are best to solve proportional reasoning problems can be detected between Grade 3 and Grade 4 (Boyer & Levine, 2012). Nine and ten year olds already have a good idea about integers as they can count and do simple additions (Martin & Schwartz, 2005).

At the time of the first assessment, learners were requested to indicate their age on the answer sheet (Appendix B). The average age for Grade 4 learners from School A was 9.99 years ($\sigma = 0.19$) and 12.96 years ($\sigma = 0.22$) for Grade 7 learners. The average age for Grade 4 learners from School B was 10.32 years ($\sigma = 0.64$) and 13.75 years ($\sigma = 0.80$) for Grade 7 learners. The learners were, therefore, deemed old enough to participate in the study.

The academic profile of the schools participating in the current study

Grade 4 learners are exposed to the divisibility rules of divisor 2 and divisor 5. According to the Mathematics workbooks that are used by South African learners, the other divisibility rules are explained in Grade 5 (*cf*. Section 1.5). Grade 4 learners were, however, also included in the study to investigate their performance after they were introduced to the other divisors. Grade 6 and Grade 7 learners were included because the divisibility rules are revised with them during the year.

Table 4.1 indicates the results of the Annual National Assessments for Mathematics for the years 2012 to 2014 for the two schools participating in the current study in comparison with the national average. But for Grade 5 in 2012, School A performed better than School B. For every grade, the averages for School A were also much better than the national average. Grade 4 and Grade 5 learners from School B also performed better than the national average for Mathematics. It should be noted that the integrity of the results in Table 4.1 could not be verified due to the fact that the teachers of the respective schools marked the scripts themselves.

Table 4.1 Average percentage on the Annual National Assessments (ANA) for Mathematics for School A and School B

Grade	Average National percentage			School A			School B		
	2012	2013	2014	2012	2013	2014	2012	2013	2014
4	37	37	37	73	71	66	51	46	50
5	30	33	37	51	74	56	69	52	34
6	27	39	43	49	69	63	23	39	38

4.3.8.3 BACKGROUND INFORMATION ON THE TEACHING OF DIVISIBILITY RULES

A questionnaire was issued to all the Mathematics teachers of Grade 4 to Grade 7 of both schools to collect background information about the teaching of the divisibility rules (Appendix F). A follow-up questionnaire was also issued to the teachers of School B to investigate their opinion as to why learners did not perform as expected during the assessments (Appendix F).

4.3.8.4 ASSESSMENTS

Assessment 1

The first assessment (Assessment 1) was conducted among all the learners in Grade 4 to Grade 7 to test their knowledge of divisibility rules (Appendix D). Assessment 1 and Assessment 3 took place in the classrooms per grade and per section. Assessment 1 was an unprepared test as learners were not informed about the test before the time. Refer to Section 4.3.3 for a discussion on the presentation of the questions.

Each assessment consisted of fourteen questions, two questions per divisor. One mark was allocated if learners provided the correct answer as well as the correct reason (the reason was considered as correct if it related to the appropriate divisibility rule). Therefore, the maximum total score for each assessment was 14.

Assessment 2

A sample of 20 learners per grade per school were identified to take part in the first eye-tracking test (Appendix G). The learners were chosen according to their performance in Assessment 1 according to the following selection criteria: All learners in Grade 4 to Grade 7 were supposed to know the divisibility rules for divisors 2 and 5. Therefore, they could score at least four out of 14 (29%). If they succeeded in only three questions, their final result would be 21%. To evaluate the hypothesis ($H_{0,5}$) stated in Section 1.8, there should be room for improvement for learners after revision. Therefore, only learners who achieved between 20% and 50% were selected to take part in Assessment 2.

The learners were seated approximately 65 cm from the screen of the laptop and instructed to put their hands in their lap. They were also instructed not to make movements and to look at the screen of the laptop only. The instructions offered to the learners were part of the recordings. The learners called out their responses aloud and the researcher wrote them down. Assessment 2 used the same set of questions as Assessment 1 but in random order (Appendix E).

The researcher moved to the next stimulus immediately after a learner responded to the current question to prevent unnecessary fixations on the areas of interests (AOIs). The researcher moved to the next stimulus also if a learner did not respond within 20 seconds.

Assessment 3

Learners in Grade 5 to Grade 7 were already exposed to the divisibility rules (divisors 2 to 9) prior to Assessment 1 and Assessment 2. Therefore, the divisibility rules were taught to the Grade 4 learners prior to the third assessment (Assessment 3) while revision was done with Grade 5 to Grade 7 learners (Appendix H). Assessment 3 was then conducted in the same way as Assessment 1 to determine if there was any improvement in the learners' ability to apply the divisibility rules. The same set of questions used in Assessment 1 was used for Assessment 3, but in a different order.

Assessment 4

Assessment 4 was done soon after Assessment 3 and in the same way as Assessment 2. It involved all the learners who took part in Assessment 2, excluding those who did not attend the revision session and learners who moved to other schools in the meantime or who were absent on the day of the test. As it was believed that learners would not remember the questions from the previous assessment and for the sake of unbiased comparison, the same set of questions was used as in the previous assessments and presented in random again.

4.3.8.5 COMMUNICATION MEDIUM OF ASSESSMENTS

The instruction medium for School A was Afrikaans - the learners' mother tongue. Therefore, all PowerPoint presentations and answer sheets were compiled in Afrikaans. The mother tongue of learners from School B was Sesotho or IsiXhosa or one of the other indigenous South African languages. For this school, however, the instruction medium was English and therefore all PowerPoint presentations and answer sheets were compiled in English.

4.3.9 ANALYSIS OF DATA

4.3.9.1 ANALYSIS OF ASSESSMENT 1 AND ASSESSMENT 3

As mentioned in Section 4.3.8.4, the performance of learners was measured as such that learners only scored one mark per question if the answer and the reason were correct. In order to populate the responses of learners into MS Excel, a 1 (one) was entered if the learner indicated the answer correctly, otherwise a 0 (zero). The same procedure was followed with the reason that the learner provided as a 1 (one) was entered if the reason corresponded with the required divisibility rule, otherwise a 0 (zero) (Figure 4.9). The correctness of the answers and the reasons were recorded separately in MS Excel for further analyses, as indicated in Table 5.2 to Table 5.5.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	AA	AB	AC	AD	AE	AF	AG
1				**Dividend**		72154		73134		75133		10199		63172		46108		52336		61254		44821		68140		45432		74511		41346		23960	
2				**Divisor :**		5		2		3		9		4		8		6		4		2		5		6		3		9		8	
3						**Q1**		**Q2**		**Q3**		**Q4**		**Q1**		**Q2**		**Q3**		**Q4**		**Q1**		**Q2**		**Q3**		**Q4**		**Q1**		**Q2**	
4	**Name**	**Surname**	**Grade**	**Section**	**Age**	**A**	**R**	**A**	**R**	**A**	**R**	**A**	**R**	**A**	**R**	**A**	**R**	**A**	**R**	**A**	**R**	**A**	**R**	**A**	**R**	**A**	**R**	**A**	**R**	**A**	**R**	**A**	**R**
5	Name 1	Surname 1	5	A	11	1	1	1	1	1	1	1	1	0	0	1	0	1	1	0	1	1	1	1	1	0	0	1	1	1	1	1	0
6	Name 2	Surname 2	5	A	11	1	1	1	1	0	0	1	0	0	0	1	0	0	1	1	0	1	1	1	1	0	0	0	0	0	0	1	0
7	Name 3	Surname 3	5	A	11	1	1	1	1	1	1	1	0	0	0	0	0	1	1	1	0	1	1	1	1	1	0	1	1	1	0	1	0
8	Name 4	Surname 4	5	A	11	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	0	1	1	1
9	Name 5	Surname 5	5	A	10	1	1	1	1	1	1	1	0	1	0	1	0	1	1	1	0	1	1	1	1	1	1	0	0	0	0	1	0
10	Name 6	Surname 6	5	A	11	1	1	1	1	1	1	0	1	1	1	0	0	0	0	0	0	1	1	1	1	0	0	0	0	0	0	1	0
11	Name 7	Surname 7	5	A	11	1	1	1	1	1	1	1	1	0	1	1	0	0	1	1	1	1	1	1	1	0	1	1	1	1	1	1	0
12	Name 8	Surname 8	5	A	11	1	1	1	1	1	0	1	0	0	0	0	0	0	0	1	0	1	1	1	1	0	0	1	0	0	0	0	0
13	Name 9	Surname 9	5	A	12	1	1	1	1	0	0	1	0	1	0	1	0	0	0	1	0	0	0	1	1	0	0	0	0	0	0	1	0
14	Name 10	Surname 10	5	A	11	1	1	1	1	1	1	1	1	1	0	0	0	1	1	1	1	1	1	1	1	0	1	1	1	0	1	0	0
15	Name 11	Surname 11	5	A	10	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	1	0
16	Name 12	Surname 12	5	A	11	1	1	1	1	1	0	0	0	0	0	0	0	0	1	1	0	1	1	1	1	1	1	1	0	1	0	0	0

Figure 4.9 Example of captured marks in MS Excel
(Key: A: Answer R: Reason 1: Correct 0: Incorrect)

<u>Calculating learners' final scores for Assessment 1 and Assessment 3</u>

As mentioned earlier, a learner obtained a mark if the answer of a question was correct and the reason correlated with the specific divisibility rule, otherwise, the learner did not score a mark for the question. The maximum score that a learner could obtain for the assessment was 14. Therefore, all the marks per learner were added to calculate the final score as a percentage.

<u>Calculating the distribution of marks for Assessment 1 and Assessment 3</u>

The *frequency()* formula in MS Excel was used to calculate how many learners scored between 0%-9%, 10%-19%, etc. by using the final score of the learners as a percentage.

Preparations of data for statistical calculations: Assessment 1 and Assessment 3

The data were prepared in MS Excel for use in STATISTICA (version 12) (Dell Inc., 2015). The data were then imported into STATISTICA to perform statistical calculations for Assessment 1 and Assessment 3, such as a within-subjects, repeated measures ANOVA, to test different effects. Significance results will be reported based on the minimum alpha level from .05, .01 and .001 that can be attained. For p values > .05, no significance will be reported.

Figure 4.10 illustrates the layout of the imported data in STATISTICA.

	A	B	C	D	E	F	G	H	I	J
1	Nr	Grade	Divisor	Number	Aswer1	Reason1	Answer2	Reason2	Mark1	Mark2
2	1	4	2	1	1	1	1	1	1	1
3	1	4	2	2	1	1	1	1	1	1
4	1	4	3	1	0	0	1	1	0	1
5	1	4	3	2	0	0	1	0	0	0
6	1	4	4	1	0	0	0	0	0	0
7	1	4	4	2	0	0	1	1	0	1
8	1	4	5	1	1	1	1	1	1	1
9	1	4	5	2	0	0	1	1	0	1
10	1	4	6	1	0	0	1	1	0	1
11	1	4	6	2	0	0	0	1	0	0
12	1	4	8	1	0	0	1	1	0	1
13	1	4	8	2	0	0	0	1	0	0
14	1	4	9	1	0	0	1	0	0	0
15	1	4	9	2	0	0	1	1	0	1

Figure 4.10 Example of prepared data for use in STATISTICA
(Key: Mark1: Answer and reason correct for Assessment 1
Mark2: Answer and reason correct for Assessment 3)

Analysis

The effect of revision was investigated to determine whether it had an influence on the performance of the learners who indicated the correct answer and reason. Statistical analyses were performed for Assessment 1 and Assessment 3, such as repeated measures analysis of variance (ANOVA), to challenge some of the hypotheses stated in Table 1.3.

4.3.9.2 ANALYSIS OF ASSESSMENT 2 AND ASSESSMENT 4

The answer sheets for Assessment 2 and Assessment 4, which were recorded by the facilitator during the individual sessions with the selected learners, were marked in the same manner as Assessment 1 and Assessment 3. Learners' responses to every question were also entered into MS Excel.

The duration of the fixation time per AOI and the total duration fixation time on all the AOIs were captured by Tobii Studio. These times were imported into MS Excel to match the entries for the learners whose answers and reasons were entered, as illustrated in Figure 4.11. If there was a fixation on an AOI between the digits, half of the fixation time was allocated to the AOI on the digit to the left and the other half to the AOI on the digit to the right.

The percentage of fixation time was then calculated for each digit of the dividend by using the fixation time on the digit and the total fixation time on the AOIs.

<u>Preparations of data for statistical calculations: Assessment 2 and Assessment 4</u>

The data were also prepared in MS Excel for use in STATISTICA. The data were imported into STATISTICA in order to perform statistical calculations for Assessment 2 and Assessment 4, such as a one-way, factorial or repeated measures analysis of variance (ANOVA), applicable to the relevant hypothesis to test the different effects listed in Table 1.3. Figure 4.12 illustrates the layout of the imported data in STATISTICA for a one-way ANOVA, and Figure 4.13 illustrates the layout for a repeated measure ANOVA.

Digit / Divisor / Question / Slide

A	D	G	H	I	J	K	L	M	DK	DL	DM	DN	DO	DP	DQ	DR	DS	DT	DU
		Q 1		Q 2		Q 3													
Nr	Grade	A	R	A	R	A	R	Continue	Total Fixation Duration_Slide9.JPG_V01 Slide09 D2 S1_Sum	Total Fixation Duration_Slide9.JPG_V01 Slide09 D2 S2_Sum	Total Fixation Duration_Slide9.JPG_V01 Slide09 D2 S3_Sum	Total Fixation Duration_Slide9.JPG_V01 Slide09 D2 S4_Sum	Total Fixation Duration_Slide9.JPG_V01 Slide09 D2 S5_Sum	Total Fixation Duration_Slide9.JPG_V01 Slide09 D2 T1B_Sum	Total Fixation Duration_Slide9.JPG_V01 Slide09 D2 T21_Sum	Total Fixation Duration_Slide9.JPG_V01 Slide09 D2 T32_Sum	Total Fixation Duration_Slide9.JPG_V01 Slide09 D2 T43_Sum	Total Fixation Duration_Slide9.JPG_V01 Slide09 D2 T54_Sum	Total Fixation Duration_Slide9.JPG_V01 Slide09 D2 TA5_Sum
1	7	1	1	1	1	0	0		0.367	0.15		0.15	0.233	0.117	0.283	0.055	0.083		0.117
2	7	1	1	1	1	0	0		0.65	0.317					0.117				
3	7	1	1	1	1	1	1		0.189		0.117		0.083		0.1				
4	7	1	1	1	1	1	1		1.113	0.367									
5	7	1	1	1	1	0	0		0.45				0.117		0.117			0.083	
6	7	1	1	1	1	0	0		0.233	0.083		0.084				0.2			
7	7	1	1	1	1	0	0		0.3	0.283			0.117		0.35				
8	7	1	1	1	1	0	0			0.2					0.133		0.324		
9	7	1	1	1	1	0	0		0.4	0.233	0.067	0.083			0.167				
10	7	1	1	1	1	1	1		0.947	0.133		0.15			0.1				
11	7	1	1	1	1	0	0		0.1	0.417					0.637				
12	7	1	1	1	1	1	1		1.405	0.55									
13	7	1	1	1	1	0	0		0.383	0.317	0.244	0.1							
14	7	1	1	1	1	0	0		0.1										
15	7	1	1	1	1	0	0		0.2	0.567			0.3		0.117				
16	7	1	1	1	1	1	1		1.167	0.083				0.083	0.133	0.133			

Figure 4.11 Example of prepared data with fixation durations per AOI

A	B	C	D	E	F	G	H	I	J
LearnerNo	Grade	Divisor	Test	Question	Answer	Reason	Digit	Percentage	Mark
12	7	2	2	2	1	1	5	0	1
12	7	2	2	2	1	1	AB	0	1
13	7	2	1	1	1	1	1	36.68582375	1
13	7	2	1	1	1	1	2	30.36398467	1
13	7	2	1	1	1	1	3	23.37164751	1
13	7	2	1	1	1	1	4	9.578544061	1
13	7	2	1	1	1	1	5	0	1
13	7	2	1	1	1	1	AB	0	1
13	7	2	1	2	1	0	1	18.19757366	0
13	7	2	1	2	1	0	2	29.16811092	0
13	7	2	1	2	1	0	3	13.49220104	0

Figure 4.12 Example of prepared data for use in STATISTICA for a one-way ANOVA during Assessment 2 and Assessment 4
(Key: Mark: Answer and reason correct)

A	B	C	D	E	F	G	H	I	J	K	L	M
LearnerNo	Grade	Divisor	Number	Answer1	Reason1	Answer2	Reason2	Digit	Percentage Ass2	Percentage Ass4	Mark1	Mark2
23	4	3	2	0	0	0	0	5	0	28.95362663	0	0
23	4	3	2	0	0	0	0	AB	0	0.713436385	0	0
24	4	3	1	0	0	1	1	1	60.77170418	14.28571429	0	1
24	4	3	1	0	0	1	1	2	27.70632369	21.68253968	0	1
24	4	3	1	0	0	1	1	3	11.62915327	30.6984127	0	1
24	4	3	1	0	0	1	1	4	0	13.23809524	0	1
24	4	3	1	0	0	1	1	5	0	20.0952381	0	1
24	4	3	1	0	0	1	1	AB	0	0	0	1
24	4	3	2	0	0	1	1	1	68.75	3.921568627	0	1
24	4	3	2	0	0	1	1	2	31.25	15.70588235	0	1
24	4	3	2	0	0	1	1	3	0	17	0	1
24	4	3	2	0	0	1	1	4	0	57.52941176	0	1
24	4	3	2	0	0	1	1	5	0	5.882352941	0	1
24	4	3	2	0	0	1	1	AB	0	0	0	1

Figure 4.13 Example of prepared data for use in STATISTICA for a repeated ANOVA during Assessment 2 and Assessment 4
(Key: Number: Appearance of number per divisor during assessment
Mark1: Answer and reason correct for Assessment 2
Mark2: Answer and reason correct for Assessment 4)

4.4 ETHICAL PROCEDURES

The divisibility rules were taught to learners in Grade 4 to Grade 7 as part of the normal curriculum during the academic year. The nature of this research study overlapped with this teaching and learning experience in a classroom environment and the study was done as an extension to the normal teaching and learning activities. The principals of both schools gave permission that the study may be conducted at their respective schools (Appendix I). The procedure of the study was discussed with the heads of the Mathematics departments of the schools, as well as the teachers responsible for teaching Mathematics to the participating grades. They were all enthusiastic to be part of the project and everyone offered their unconditional support.

4.5 SUMMARY

The method that was discussed in this chapter was used to structure the body of the following chapter. The general pre-post experimental design was chosen to be used with this study. A pilot study was done to simulate the assessments that were used in the study. The research tools were discussed and the presentations of questions and the design of the answer sheets were discussed in detail.

The compilation of the questions was discussed and the justifications of the compiled dividend were also mentioned. The data collection procedures were stipulated and the ethical procedures that were followed were highlighted.

The results of Assessment 1 and Assessment 3 will be analysed in the next chapter to determine the effect of revision on the performance of the learners. Thereafter, the results of Assessment 2 and Assessment 4 will be analysed to determine which factors had an effect on learners' gaze behaviour as indicated by the percentage of fixation time on the digits. The common reasons that learners provided when they do not know the divisibility rules, and the similarities and differences between the two schools will be discussed as well.

CHAPTER 5
DATA ANALYSIS

5.1 INTRODUCTION

In the previous chapter, the research methodology and research design were discussed in detail. Learners from Grade 4 to Grade 7 took part in four assessments, two with eye-tracking and two without. Each assessment focused on the ability of the learners to apply the divisibility rules on divisors 2 to 9, excluding 7. Assessment 1 was an unprepared test where learners wrote their responses on paper (Appendix B). Assessment 2 was an eye-tracking assessment where selected learners verbalised their responses while looking at the relevant five-digit dividends on a computer screen. Assessments 3 and 4 were done after revision of the divisibility rules. For Assessment 3, the learners wrote their responses on paper as in Assessment 1. The same learners who took part in Assessment 2, also participated in Assessment 4, which was another eye-tracking assessment.

The secondary hypotheses were summarised in Table 1.3. This chapter is going to analyse the data to conclude whether each stated null hypothesis in Table 1.3 could be rejected or not.

The following aspects will be discussed in this chapter:

- The effect of revision on the performance of learners (Section 5.2)
- Analysis of gaze behaviour before and after revision (Section 5.3)
- Tracking percentage during Assessment 2 and Assessment 4 (Section 5.3.1)
- The effect of divisibility on performance of learners (Section 5.3.2)
- The effect of grade on percentage of fixation time on the digits (Section 5.3.3)
- The effect of revision on the percentage of fixation time per digit for learners who benefited from the revision (Section 5.3.4)
- The effect of revision on the percentage of fixation time per digit for learners who provided the correct answer and reason before and after revision (Section 5.3.5)
- The effect of revision on the percentage of fixation time per digit for learners who provided the correct answer and reason before or after revision (Section 5.3.6)
- A comparison between School A and School B with regard to the percentage of fixation time on the digits (Section 5.3.7)
- The effect of divisor on the percentage of fixation time per digit (Section 5.3.8)

- The effect of the correctness of answer on the percentage of fixation time per digit for learners in A×R✓ or A✓R✓ (Section 5.3.9)
- The effect of digit position on the percentage of total fixation time per digit (Section 5.3.10)
- Comparison between School A and School B with regard to knowledge of the divisibility rules (Section 5.4)

5.2 EFFECT OF REVISION ON THE PERFORMANCE OF LEARNERS

In this section, Assessment 1 and Assessment 3 will be compared to determine the effect of revision on the learners' performance. All analyses are done separately for the two schools. Table 5.1 indicates the number of learners who took part in Assessment 1 and Assessment 3, as well as the number of learners who took part in both assessments.

Table 5.1 Number of learners who participated in Assessment 1 and Assessment 3

Grade	School A			School B		
	Ass 1	**Ass 3**	**Did both assessments**	**Ass 1**	**Ass 3**	**Did both assessments**
4	130	129	123	51	48	48
5	129	127	120	183	177	172
6	116	117	106	163	160	153
7	125	129	120	187	140	132
Total	**500**	**502**	**469**	**584**	**525**	**505**

5.2.1 PERFORMANCE OF LEARNERS BEFORE REVISION

Figure 5.1a and Figure 5.1b indicate the distribution of learners' performance for Assessment 1 for School A and School B respectively. The majority of learners in School A achieved between 20% and 29% for Assessment 1. The majority of learners in School B achieved between 0% and 9% for Assessment 1.

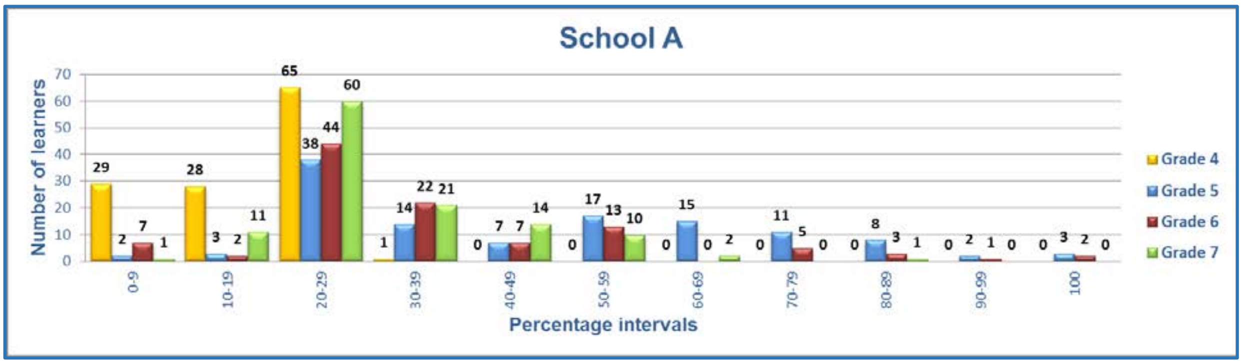


Figure 5.1a Assessment 1: Distribution of marks for Grade 4 to Grade 7 for School A

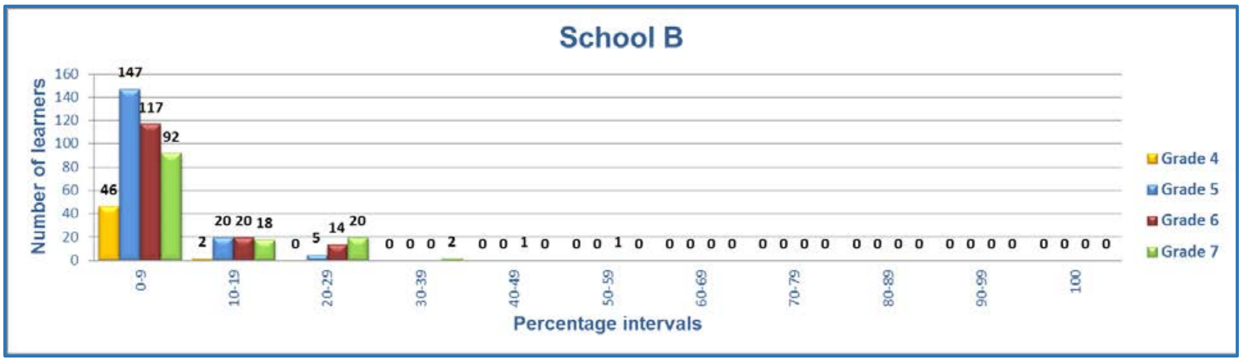


Figure 5.1b Assessment 1: Distribution of marks for Grade 4 to Grade 7 for School B

Table 5.2 shows the percentage of responses in each of the possible answer/reason combinations per school and grade. The "DNK" column represents the percentage of responses where learners indicated that they did not know the answer – a value that was included in the A×R× category. Figure 5.3 provides a visualisation of the data before and after revision.

Table 5.2 Percentage of responses over the possible answer/reason combinations per school and grade for Assessment 1. (Key: A=Answer; R=Reason; ×=Incorrect; ✓=Correct)

Grade	Percentage: School A					Percentage: School B				
	A×R×	**A×R✓**	**A✓R×**	**A✓R✓**	DNK (Included in **A×R×**)	**A×R×**	**A×R✓**	**A✓R×**	**A✓R✓**	DNK (Included in **A×R×**)
4	67.1	0.2	14.5	18.2	5.6	58.9	0	39.9	1.2	8.0
5	30.4	6.9	15.1	47.6	5.2	61.0	0	35.7	3.3	4.2
6	46.3	4.8	11.5	37.5	11.9	66.3	0.6	26.9	6.2	1.5
7	53.0	1.4	13.9	31.7	4.8	72.0	0.1	20.5	7.5	1.6

Grade 5 learners in School A performed the best of all the grades as nearly 50% of the responses were correct with a correct motivation (Table 5.2). Grade 6 learners in School A performed second best, followed by the Grade 7 and Grade 4 learners. Grade 5 learners in School A covered the divisibility rules about 3 months prior to Assessment 1. Grade 6 learners in School A revised the divisibility rules about 7 months prior to Assessment 1. Grade 7 learners in School A did not revise the divisibility rules at all. At the time of Assessment 1, Grade 4 learners were only exposed to the divisibility rules for divisors 2 and 5.

Learners in School B showed limited knowledge of the divisibility rules before revision, although the teachers indicated that the divisibility rules were presented to Grade 5 learners about 5 months and to Grade 6 learners about 1 month prior to Assessment 1. Grade 7 learners only revised the divisibility rules for divisors 2, 5 and 10 about 7 months prior to Assessment 1. Teachers in School B pointed out that the learners are not good listeners, they easily forget what they had been taught, and they are too lazy to apply the rules. Some teachers also mentioned the language of instruction, namely English, as a possible reason for their poor performance since it was the learners' second language. The learners spoke one or more of the indigenous languages, such as Sesotho and Zulu, and the teachers could not offer instruction in the mother tongue of all learners.

For both schools and all grades, the percentages of responses for which learners indicated that they did not know the answer are much lower than those with an incorrect answer and reason. Therefore, it seems as though most learners guessed the answer rather than admitting that they did not know it (*cf.* Section 2.8.3).

The results in Table 5.2 are further broken down in Table 5.3 with the percentage of responses in each of the possible answer/reason combinations per school, grade and divisor.

Grade 5 to Grade 7 learners in School A showed decent knowledge of divisors 2 and 5. Although Grade 4 learners in School A scored fairly well with divisors 2 and 5, they had limited knowledge of the other divisors. They will be exposed to the other divisors in Grade 5. Grade 5 to Grade 7 learners in School B showed limited knowledge of divisors 2 and 5, and almost no knowledge of the other divisors. Grade 4 learners in School B showed limited knowledge of divisor 2 and no knowledge of the other divisors.

The percentage of responses of Grade 5 and Grade 6 learners in School A with the wrong answer and a correct reason (the dividend was divisible by 2 and 3) for divisor 6 in Assessment 1, is extremely high in comparison with the other divisors (Table 5.3). It is reasonable to infer that learners did not apply the rule correctly. In Section 5.3.9 eye-tracking analysis revealed that some learners, who knew the rule for divisor 6, only inspected the last and the second-to-last digits of the dividend instead of all the digits to determine if the dividend is divisible by 3.

Table 5.3 Percentage of responses over the possible answer/reason combinations per school, grade and divisor for Assessment 1. (Key: A=Answer; R=Reason; ×=Incorrect; ✓=Correct)

Grade	Divisor	Percentage of responses: School A				Percentage of responses: School B			
		A×R×	A×R✓	A✓R×	A✓R✓	A×R×	A×R✓	A✓R×	A✓R✓
4	2	17.9	0.8	8.5	72.8	40.6	0	51.0	8.3
	3	88.2	0.0	11.4	0.4	61.5	0	38.5	0
	4	83.3	0.4	15.5	0.8	55.2	0	44.8	0
	5	26.4	0.4	19.9	53.3	58.3	0	41.7	0
	6	81.7	0.0	18.3	0.0	63.5	0	36.5	0
	8	82.9	0.0	17.1	0.0	74.0	0	26.0	0
	9	89.0	0.0	11.0	0.0	59.4	0	40.6	0
5	2	2.1	0.4	3.3	94.2	42.2	0	41.0	16.9
	3	37.9	6.7	14.2	41.3	64.0	0	36.0	0
	4	30.8	10.4	25.8	32.9	64.0	0	35.8	0.3
	5	2.1	0.4	4.6	92.9	55.5	0	38.4	6.1
	6	36.7	23.9	8.3	31.3	64.2	0	35.8	0
	8	61.7	2.5	28.3	7.5	65.7	0	34.3	0
	9	41.7	3.8	21.3	33.3	71.5	0	28.5	0
6	2	3.8	0.9	5.2	90.1	29.7	0.3	42.8	27.1
	3	72.6	1.9	9.9	15.6	73.2	0.7	25.8	0.3
	4	49.1	3.8	25.5	21.7	73.9	1.3	24.5	0.3
	5	6.1	0.5	3.8	89.6	49.7	0	35.6	14.7
	6	49.1	22.6	11.3	17.0	75.8	0.7	23.5	0
	8	74.5	3.3	9.9	12.3	81.4	0	18.6	0
	9	68.9	0.5	14.6	16.0	80.4	1.0	17.6	1.0
7	2	3.8	0.0	2.5	93.8	27.7	0	39.0	33.3
	3	87.1	1.7	6.7	4.6	92.4	0	7.6	0
	4	42.1	4.6	25.8	27.5	84.1	0	15.2	0.8
	5	4.6	0.4	13.8	81.3	45.5	0.4	36.0	18.2
	6	75.4	2.1	19.6	2.9	75.4	0	24.6	0
	8	73.8	0.8	14.6	10.8	90.9	0	9.1	0
	9	84.6	0.0	14.2	1.3	87.9	0	12.1	0

5.2.2 PERFORMANCE OF LEARNERS AFTER REVISION

Revision of the divisibility rules were done by the researcher prior to Assessment 3 (Appendix H). Figure 5.2a and Figure 5.2b show the distribution of learners' performance for Assessment 3 for School A and School B respectively. For Assessment 1, the majority of the learners in all the grades in School A achieved between 20% and 29%, but for Assessment 3 it was only Grade 4 learners who still achieved between 20% and 29%. The majority of the learners in School B still achieved between 0% and 9% after revision.

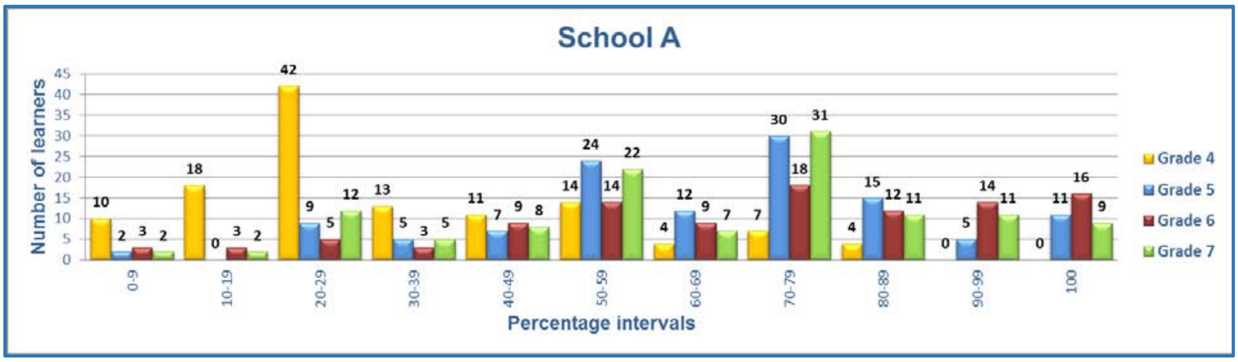


Figure 5.2a Assessment 3: Distribution of marks for Grade 4 to Grade 7 for School A

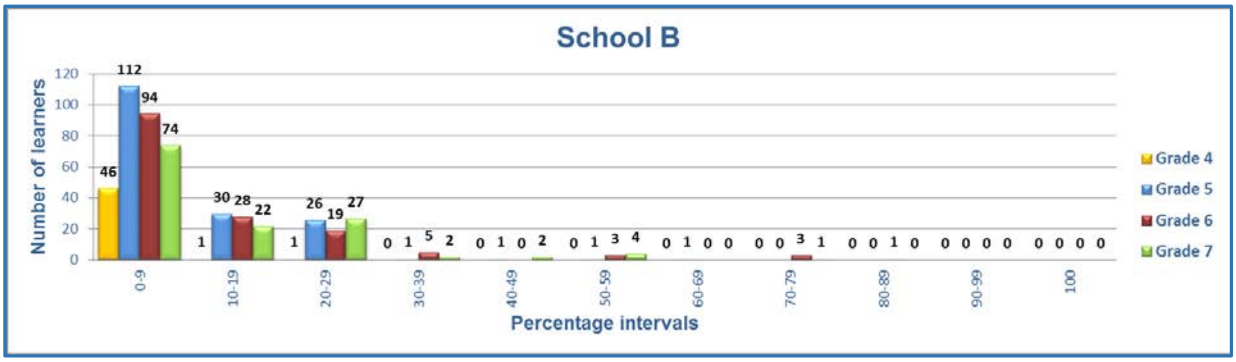


Figure 5.2b Assessment 3: Distribution of marks for Grade 4 to Grade 7 for School B

Table 5.4 shows the percentage of responses in each of the possible answer/reason combinations per school and grade. Once again, the "DNK" column represents the percentage of responses where learners indicated that they did not know the answer – a value that was included in the A×R× category. The majority of learners' responses in Grades 5 to Grade 7 in School A were correct with a correct reason. In School B, however, the majority of the responses were still wrong. Figure 5.3 provides a visualisation of the data before and after revision.

Table 5.4 Percentage of responses over the possible answer/reason combinations per school and grade for Assessment 3. (Key: A=Answer; R=Reason; ×=Incorrect; ✓=Correct)

Grade	Percentage: School A					Percentage: School B				
	A×R×	**A×R✓**	**A✓R×**	**A✓R✓**	DNK (Included in **A×R×**)	**A×R×**	**A×R✓**	**A✓R×**	**A✓R✓**	DNK (Included in **A×R×**)
4	44.3	3.0	18.4	34.4	10.2	61.9	0.1	35.9	2.1	4.5
5	15.1	6.4	12.9	65.6	4.8	56.3	0.4	34.7	8.7	3.6
6	13.8	6.2	11.0	69.1	3.1	54.4	1.4	32.8	11.3	2.0
7	14.0	5.4	16.5	64.1	2.0	53.7	0.9	33.2	12.2	0.9

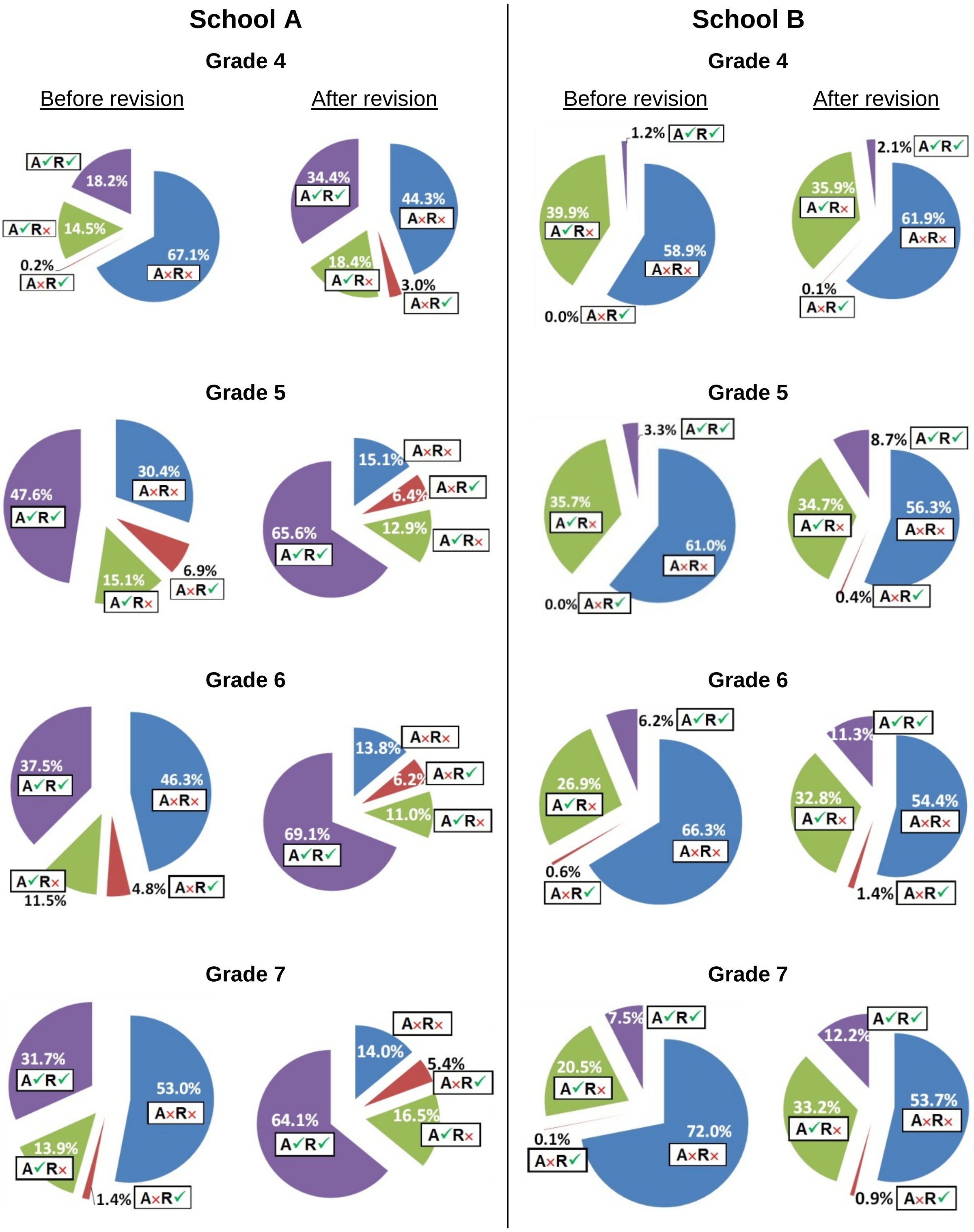


Figure 5.3 Percentage of responses over the possible answer/reason combinations per school and grade before and after revision. (Key: A=Answer; R=Reason; ×=Incorrect; ✓=Correct)

The results in Table 5.4 are further broken down in Table 5.5 where the percentage of responses in each of the possible answer/reason combinations are given per school, grade and divisor. The information in Table 5.5 will be used in Section 5.2.3 to illustrate the effect of revision (Assessment 1 to Assessment 3) on the learners' performance.

Table 5.5 Percentage of responses over the possible answer/reason combinations per school, grade and divisor for Assessment 3. (Key: A=Answer; R=Reason; ×=Incorrect; ✓=Correct)

Grade	Divisor	Percentage of responses: School A				Percentage of responses: School B			
		A×R×	A×R✓	A✓R×	A✓R✓	A×R×	A×R✓	A✓R×	A✓R✓
4	2	2.4	0.4	11.0	86.2	43.8	0	46.9	9.4
	3	54.1	2.9	17.1	26.0	63.5	0	36.5	0
	4	69.9	5.7	16.3	8.1	69.8	0	29.2	1.0
	5	9.8	2.0	17.9	70.3	50.0	1.0	45.8	3.1
	6	51.6	3.3	33.3	11.8	61.5	0	38.5	0
	8	70.7	5.3	15.9	8.1	78.1	0	20.8	1.0
	9	51.2	1.2	17.1	30.5	66.7	0	33.3	0
5	2	1.3	0.0	1.3	97.5	33.7	0.6	34.9	30.8
	3	8.8	5.0	12.9	73.3	67.7	0	30.2	2.0
	4	29.2	14.6	16.7	39.6	59.3	0.3	39.5	0.9
	5	0.8	0.8	2.9	95.4	33.1	1.2	40.4	25.3
	6	8.3	11.3	22.9	57.5	56.7	0	43.0	0.3
	8	45.8	9.6	16.3	28.3	76.5	0.6	22.7	0.3
	9	11.7	3.8	17.1	67.5	66.9	0	32.0	1.2
6	2	1.9	0.0	1.9	96.2	25.2	2.0	40.2	32.7
	3	13.7	3.3	6.1	76.9	71.9	0.7	21.2	6.2
	4	24.1	10.9	15.6	49.5	62.1	1.0	34.0	2.9
	5	2.4	0.50	3.3	93.9	21.6	3.6	45.8	29.1
	6	11.3	8.5	22.2	58.0	57.5	0	41.2	1.3
	8	32.1	12.5	16.0	39.2	75.8	2.0	20.9	1.3
	9	10.9	7.6	11.8	69.8	67.0	1.0	26.5	5.6
7	2	0.4	0.0	2.5	97.1	20.1	1.5	43.9	34.5
	3	15.0	4.2	15.8	65.0	80.7	0.8	13.6	4.9
	4	22.9	10.4	19.2	47.5	61.4	1.1	36.7	0.8
	5	0.8	0.4	7.5	91.3	18.2	2.3	41.3	38.3
	6	16.3	4.6	37.5	41.7	60.6	0	39.4	0
	8	32.9	13.8	15.8	37.5	75.0	0.8	23.5	0.8
	9	9.6	4.6	17.1	68.8	59.8	0	33.7	6.4

5.2.3 THE EFFECT OF REVISION ON LEARNER PERFORMANCE

Learner performance per school and grade

Figure 5.4 indicates the distribution of learner performance per school and grade for all the learners who participated in both Assessment 1 and Assessment 3.

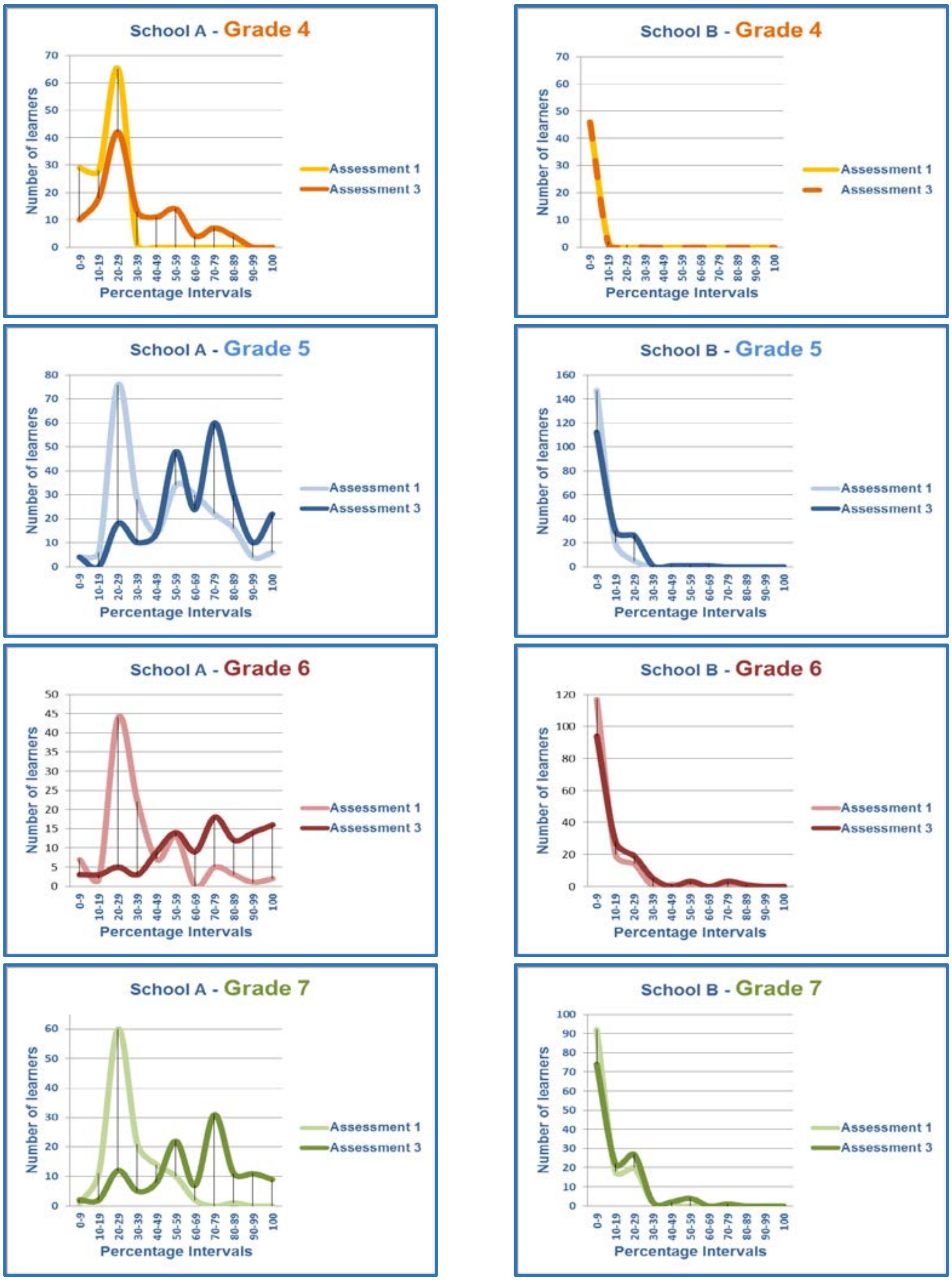


Figure 5.4 Distribution of marks for Assessment 1 and Assessment 3 per school and grade where the answer and reason were correct

According to Figure 5.4, there was a substantial difference in the distribution of marks of learners in School A after revision. In School A, only learners in Grade 4 still achieved between 20% and 29% while learners in the other grades performed much better after revision. Learners in all grades in School B still achieved between 0% and 9% after revision and the distribution of marks did not change much for any of the grades.

Percentage of responses with the correct answer and reason (overall per school and grade)

Figure 5.5 illustrates the overall effect of revision on the percentage of responses with the correct answer and reason (A✓R✓) in Assessment 1 and Assessment 3. The results of a within-subjects repeated-measures analysis of variance (ANOVA) for the effect of revision on the percentage of responses in A✓R✓ are presented in Table 5.6 per school and grade. Significant p values (α = .001) were found for all the grades for both schools except for Grade 4 in School B.

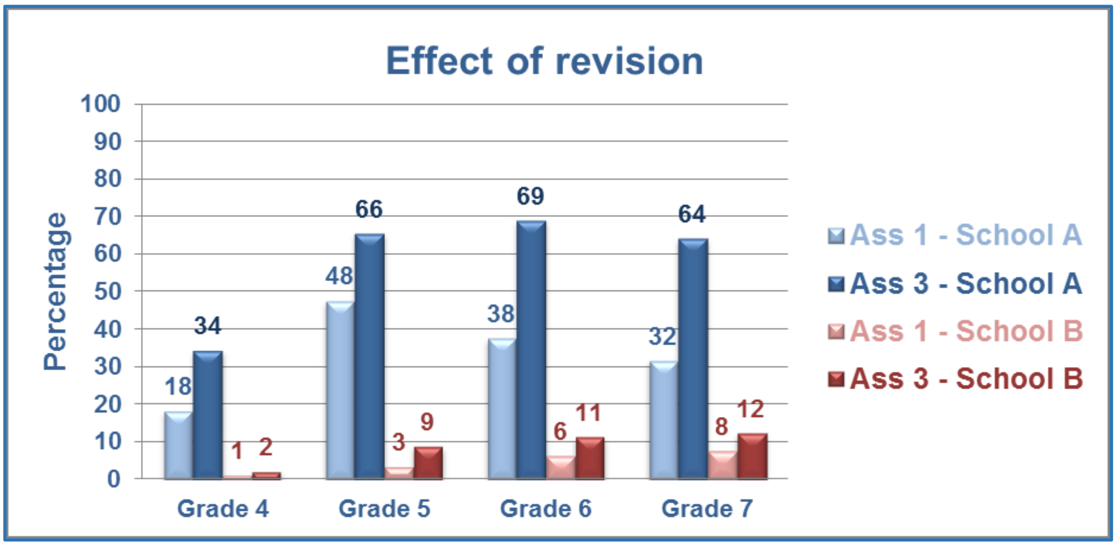


Figure 5.5 The effect of revision on the percentage of responses in A✓R✓

Table 5.6 Results of a within-subjects repeated-measures analysis of variance for the effect of revision per grade on the percentage of responses in A✓R✓

Grade	School A		School B	
	Current effect	p	Current effect	p
4	F(1, 1721)=235.67	< .001	F(1, 671) =2.5775	.109
5	F(1, 1679)=223.56	< .001	F(1, 2407)=88.437	< .001
6	F(1, 1483)=551.22	< .001	F(1, 2141)=58.949	< .001
7	F(1, 1679)=643.17	< .001	F(1, 1847)=38.352	< .001

Significantly more responses of Grade 5 to Grade 7 learners in School A were correct with a correct reason after revision. Although the effect is also significant for Grade 4, the performance was still poor. This could be due to the short time (30 minutes per class group) that was available to explain the divisibility rules. According to the Grade 5 teacher in

School A, about a week of contact time is necessary to explain the divisibility rules to Grade 5 learners.

Although a significant improvement (α = .001) was found for Grade 5 to Grade 7 learners in School B, the results for School B were still poor.

Percentage of correct responses with a correct motivation per school, grade and divisor

In Figure 5.6, the results per grade are further broken down per divisor. The figure shows the percentage of responses with the correct answer and reason (*cf.* Table 5.3 and Table 5.5). Table 5.7 shows the effect of revision on this percentage per grade and divisor.

The general trend was that more learners could provide the correct answer and reason for questions on divisors 3, 6 and 9 (as indicated in Figure 5.6) than for questions on divisors 4 and 8 – especially in School A. For divisibility by divisors 3, 6 or 9, the learners only had to add the digits and determine divisibility of the sum (which were never more than 20 (*cf.* Section 4.3.7.1) by 3 or 9. For divisors 4 and 8, learners had to determine divisibility by 4 or 8 of the bigger numbers (54 and 72, 108 and 960 respectively) that was formed by the last 2 or 3 digits of the dividends. Looking at Table 5.5, for divisors 4 and 8, more responses fell in the category A×R✓ than in any one of the other categories. This supports the impression that the learners knew the divisibility rules but probably made mental calculation errors.

Table 5.7 shows the results of a within-subjects repeated-measures ANOVA for the effect of revision on the percentage of responses in A✓R✓ per school, grade and divisor. Significant *p* values (α = .05) were found for School A for all the divisors except for divisor 2 (Grade 7) and divisor 5 (Grade 5). This is due to the fact that the divisibility rules for divisors 2 and 5 were already covered before Grade 5. Significant *p* values (α = .05) were found for School B for all the divisors except divisor 2 (Grade 4 and Grade 7) and divisor 4 (Grade 5). There are, however, many divisors where there were no variances (Table 5.7), because nobody provided the correct answer and reason for these divisors in Assessment 1.

In summary, the results show that revision had an influence on the percentage of responses with the correct answer and reason for both School A and School B for most grades and divisors, although School B did not perform as well as School A (Table 1.3: $H_{0,1}$, $H_{0,2}$).

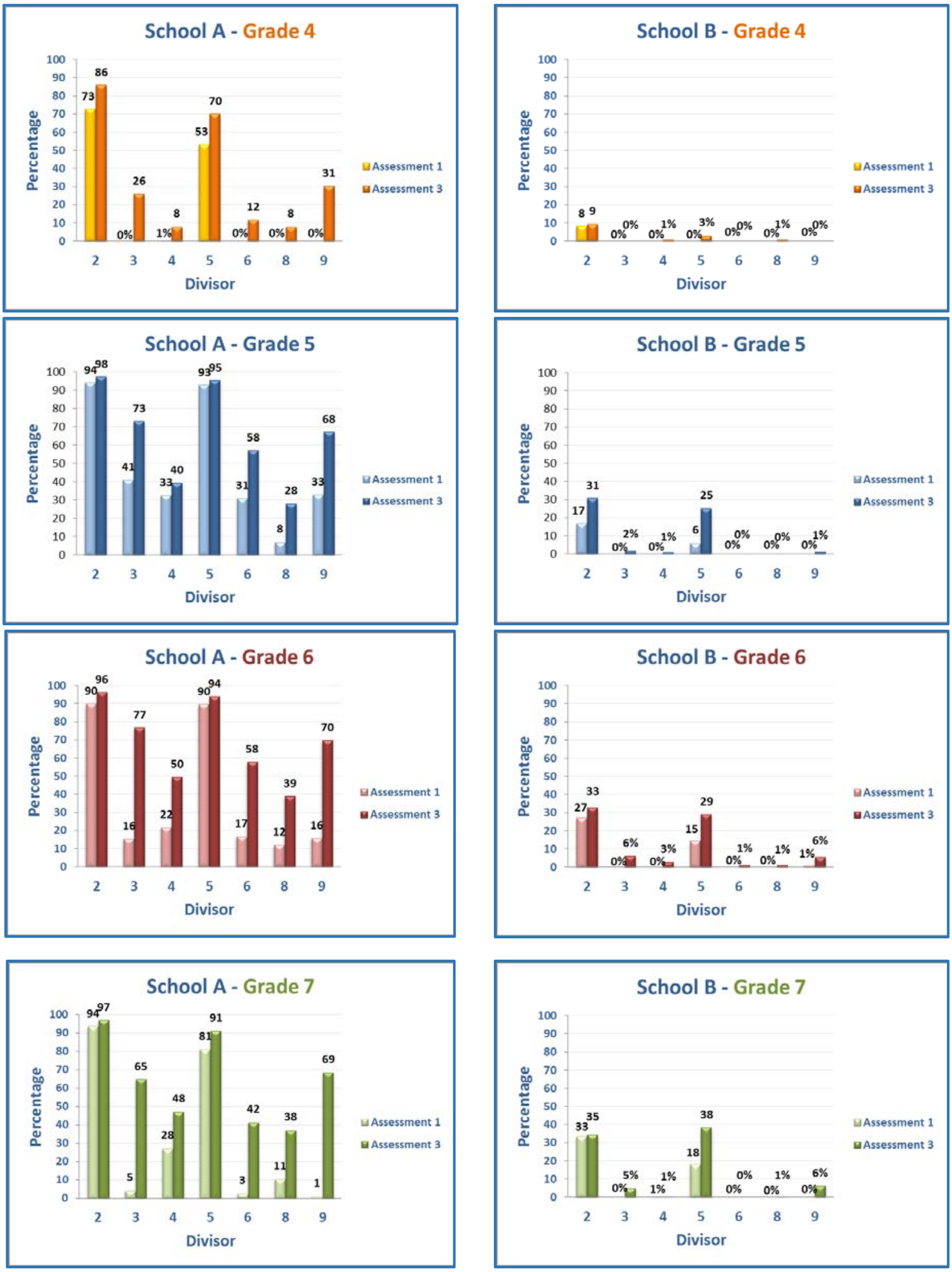


Figure 5.6 Average percentage of responses in A✓R✓ per divisor for Assessment 1 and Assessment 3 per school and grade

Table 5.7 Effect of revision per grade and divisor

Grade	Divisor	School A		School B	
		Current effect	p	Current effect	p
4	2	F(1, 245)=18.517	< .001	F(1, 95)=.11	.741
	3	F(1, 245)=80.892	< .001	-	
	4	F(1, 245)=17.272	< .001	-	
	5	F(1, 245)=17.776	< .001	-	
	6	-		-	
	8	-		-	
	9	-		-	
5	2	F(1, 239)=5.4318	.021	F(1, 343)=28.26	< .001
	3	F(1, 239)=81.669	< .001	No variance	
	4	F(1, 239)=4.1838	.042	F(1, 343)=1.00	.318
	5	F(1, 239)=1.6407	.201	F(1, 343)=56.16	< .001
	6	F(1, 239)=42.747	< .001	-	
	8	F(1, 239)=50.210	< .001	-	
	9	F(1, 239)=93.021	< .001	-	
6	2	F(1, 211)=9.2405	.003	F(1, 305)=3.89	.049
	3	F(1, 211)=309.86	< .001	F(1, 305)=19.06	< .001
	4	F(1, 211)=53.648	< .001	F(1, 305)=8.19	.005
	5	F(1, 211)=4.3301	.039	F(1, 305)=25.50	< .001
	6	F(1, 211)=108.71	< .001	-	
	8	F(1, 211)=56.068	< .001	-	
	9	F(1, 211)=236.47	< .001	F(1, 305)=12.72	< .001
7	2	F(1, 239)=3.2297	.074	F(1, 263)=.12	.733
	3	F(1, 239)=352.51	< .001	-	
	4	F(1, 239)=30.031	< .001	F(1, 263)=--	-
	5	F(1, 239)=11.565	.001	F(1, 263)=34.00	< .001
	6	F(1, 239)=136.80	< .001	-	
	8	F(1, 239)=77.058	< .001	-	
	9	F(1, 239)=496.38	< .001	-	

5.3 ANALYSIS OF GAZE BEHAVIOUR BEFORE AND AFTER REVISION

This section will focus on the eye-tracking assessments (Assessment 2 and 4, Appendix G). In order to establish minimum gaze behaviour per divisor to verify that learners inspect the correct digits of a dividend (Section 6.5), only learners' responses in A✓R✓ were used for the analyses (*cf*. 5.3.3). Factors (school, grade, revision, divisibility, divisor, digit, correctness of answer and correctness of reason) , as stated in Section 1.7, that may have an influence on learners' responses in A✓R✓ were firstly investigated to identify whether it could be discarded or not for further analyses. Thereafter, learners' responses in other combinations (A×R✓, A✓R×, A×R×) will be investigated. Therefore, a multivariate ANOVA with so many variables are complex and it is preferred to rather do separate ANOVAs for each variable while controlling for the others. Specifically, the following aspects will be analysed:

- The quality of recordings in terms of the percentage of expected gaze data that was captured (Section 5.3.1).
- The effect of divisibility on the performance of learners (Section 5.3.2)
- The fixation time per digit as a function of grade (Section 5.3.3), revision (Section 5.3.4, 5.3.5 and 5.3.6), school (Section 5.3.7), divisor (Section 5.3.8), correctness of answer (Section 5.3.9) and digit position (Section 5.3.10).

Table 5.8 indicates the number of learners who took part in Assessment 2 and Assessment 4 as well as the number of learners who took part in both assessments.

Table 5.8 Number of learners who participated in Assessment 2 and Assessment 4

Grade	School A			School B		
	Ass 2	Ass 4	Did both assessments	Ass 2	Ass 4	Did both assessments
4	20	20	20	9	8	8
5	20	18	18	25	23	23
6	20	20	20	27	26	26
7	20	20	20	29	20	20
Total	**80**	**78**	**78**	**90**	**77**	**77**

5.3.1 TRACKING PERCENTAGE DURING ASSESSMENT 2 AND ASSESSMENT 4

Table 5.9 indicates the average tracking percentages per school and grade as reported by the eye-tracking software. Although the learners were instructed to look at the screen only, it was observed that some of them turned their heads towards the researcher while he was explaining the instructions and therefore tracking percentages less than 100% do not necessarily imply bad data.

Table 5.10 indicates the percentage of recordings per question where there was at least one fixation on any of the AOIs. The complete data set, from which Table 5.10 was derived, is presented in Appendix J. As explained in Section 4.3.2, another eye-tracker was used for Assessment 4 for School B and therefore analyses for Assessment 2 and Assessment 4 of this school will be done separately.

Table 5.9 Tracking percentage for Assessment 2 and Assessment 4

Grade	School A						School B					
	Assessment 2			Assessment 4			Assessment 2			Assessment 4		
	n	Avg	SD	n	Avg	SD	n	Avg	SD	n	Avg	SD
4	20	74.1	7.5	20	79.5	4.6	9	81.0	5.5	8	85.4	7.0
5	20	72.7	10.7	18	79.6	7.1	25	83.2	4.5	23	89.6	5.0
6	20	82.3	6.2	20	83.8	6.4	27	79.3	8.0	26	86.8	5.8
7	20	82.3	6.6	20	86.2	5.2	29	81.4	7.4	20	86.5	5.4

Table 5.10 Percentage of recordings for which there were at least one fixation on any of the AOIs per school, assessment, grade, divisor and question

Grade	Divisor	School A				School B			
		Assessment 2		Assessment 4		Assessment 2		Assessment 4	
		Q1	Q2	Q1	Q2	Q1	Q2	Q1	Q2
4	2	95	100	95	100	88	63	100	100
	3	95	100	95	100	100	88	100	100
	4	100	100	100	100	100	75	100	100
	5	95	95	95	95	100	100	100	100
	6	100	95	100	95	88	88	88	100
	8	95	95	95	95	88	75	100	100
	9	95	100	95	100	75	75	100	100
5	2	100	100	100	100	96	78	100	100
	3	94	94	94	94	91	78	100	100
	4	94	94	94	94	83	61	100	100
	5	100	100	100	100	83	65	100	100
	6	94	94	94	94	78	78	100	100
	8	94	94	94	94	91	61	100	100
	9	94	94	94	94	74	70	100	100
6	2	100	100	100	100	100	85	100	100
	3	100	100	100	100	88	88	100	96
	4	100	100	100	100	92	88	100	100
	5	100	100	100	100	85	85	100	100
	6	100	100	100	100	81	88	100	100
	8	100	100	100	100	85	73	100	100
	9	100	100	100	100	88	77	100	100
7	2	100	100	100	100	90	55	100	100
	3	100	100	100	100	90	85	100	100
	4	100	100	100	100	85	55	100	100
	5	100	100	100	100	90	75	100	100
	6	100	100	100	100	80	65	100	100
	8	100	100	100	100	80	85	95	95
	9	100	100	100	100	75	70	100	100

Summary of percentages per Assessment

Grades	School A		School B	
	Assessment 2	Assessment 4	Assessment 2	Assessment 4
4	97.1	97.1	85.9	99.1
5	95.7	95.7	77.6	100.0
6	100.0	100.0	85.9	99.7
7	100.0	100.0	77.1	99.3

5.3.2 THE EFFECT OF DIVISIBILITY ON PERFORMANCE OF LEARNERS

Each assessment contained two questions per divisor – thus 14 questions in total. For one of the questions, the dividend was divisible by the divisor and for the other one it was not divisible. Table 5.11 shows the percentage of responses in A✓R✓ for each question as well as the results of a within-subjects one-way analysis of variance (ANOVA) for the effect of divisibility on the percentage of responses in A✓R✓ per assessment, school, grade and divisor. Recordings where the quality of the eye-tracking data was such that there were no fixations on the areas of interest (Figure 5.7), were discarded.

Table 5.11 The effect of divisibility on the percentage of responses in A✓R✓ *per assessment, school, grade and divisor. (Key: A=Answer; R=Reason; ✓=Correct)*

		Assessment 2 (before revision)								Assessment 4 (after revision)							
		School A				School B				School A				School B			
Grade	Divisor	% Q1 in A✓R✓	% Q2 in A✓R✓	F	p	% Q1 in A✓R✓	% Q2 in A✓R✓	F	p	% Q1 in A✓R✓	% Q2 in A✓R✓	F	p	% Q1 in A✓R✓	% Q2 in A✓R✓	F	p
	2	95	85	0.22	.643	29	20	0.37	.554	95	90	0.00	1.000	25	13	0.37	.554
	3	0	0	-		0	0	-		26	20	0.14	.714	0	0	-	
	4	11	11	0.00	1.000	0	0	-		25	25	0.00	1.000	0	0	-	
4	5	90	95	0.35	.560	0	0	-		89	89	0.00	1.000	0	0	-	
	6	0	0	-		0	0	-		20	26	0.14	.714	0	0	-	
	8	0	0	-		0	0	-		26	11	1.54	.222	0	0	-	
	9	0	0	-		0	0	-		37	30	0.11	.744	0	0	-	
	2	89	89	0.00	1.000	32	28	0.44	.513	89	94	0.35	.560	70	43	3.27	.077
	3	11	17	0.22	.641	0	0	-		59	76	1.06	.312	4	4	0.00	1.000
	4	17	17	0.00	1.000	5	0	1.00	.323	35	29	0.12	.727	4	0	1.00	.323
5	5	83	89	0.22	.641	21	13	0.75	.393	94	94	0.00	1.000	26	52	3.38	.073
	6	6	22	2.10	.157	0	0	-		47	59	0.43	.519	4	4	0.00	1.000
	8	0	0	-		0	0	-		12	12	0.00	1.000	4	4	0.00	1.000
	9	11	11	0.00	1.000	0	0	-		71	47	1.79	.190	4	9	0.34	.561
	2	100	95	1.00	.324	50	50	0.68	.415	100	100	-	.324	65	46	1.95	.169
	3	0	5	1.00	.324	4	4	0.00		65	55	0.40	.531	19	28	0.42	.520
	4	25	20	0.14	.714	8	4	0.34	.561	50	60	0.39	.537	8	15	0.74	.395
6	5	100	100	-		32	27	0.10	.755	100	95	1.00	.324	35	54	1.95	.169
	6	0	0	-		0	0	-		65	45	1.60	.214	15	8	0.74	.395
	8	0	0	-		0	5	1.00	.322	35	30	0.11	.744	12	12	0.00	1.000
	9	15	0	3.35	.075	4	0	0.00	1.000	75	60	1.00	.324	27	23	0.099	.755
	2	100	95	1.00	.324	56	64	0.90	.350	100	100	-		95	70	4.61	.038
	3	25	35	0.458	.503	0	0	-		75	80	0.14	.714	0	10	2.11	.154
	4	45	60	0.88	.355	6	0	1.00	.324	70	70	0.00	1.000	0	5	1.00	.324
7	5	100	85	3.35	.075	33	47	0.11	.744	100	100	-		50	55	0.10	.759
	6	15	10	0.22	.643	0	0	-		65	60	0.10	.752	0	5	1.00	.324
	8	25	45	1.75	.194	0	0	-		65	50	0.90	.350	0	5	1.00	.324
	9	20	10	0.76	.389	0	0	-		95	65	6.22	.017	5	10	0.35	.560

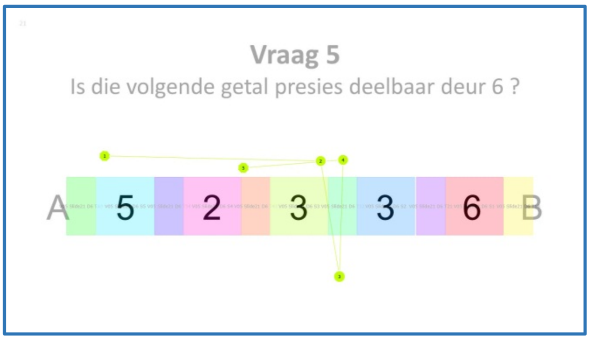


Figure 5.7 Example of a recording where fixations were not on AOIs

With the exception of two cases (underlined in Table 5.11), the results show (α = .05) that it does not matter whether a dividend is divisible by a divisor and therefore the influence of divisibility was discarded for further analysis (Table 1.3: $H_{0,3}$).

5.3.3 THE EFFECT OF <u>GRADE</u> ON PERCENTAGE FIXATION TIME ON DIGITS

The percentage of fixation time on specific digits would indicate where learners spend their time when determining if a dividend is divisible by a specific divisor. This could indicate if a learner applied the correct divisibility rule and/or whether it was applied correctly.

Only responses where learners provided the correct answer and reason (A✓R✓) were considered for the analysis because:

(i) These learners probably applied the divisibility rules correctly, and therefore, fixated on the required digits to conclude their answers. Learners who provided an incorrect reason (A✓R×) probably did not know the relevant divisibility rule and their fixations on the digits would not be reliable. Learners who provided a correct motivation but an incorrect answer (A×R✓) probably made mental calculation errors or applied the divisibility rule incorrectly and therefore their fixations on the digits could be misleading.

(ii) The percentage of fixation time on the digits for these learners will be used as a benchmark for the proposed instrument (Chapter 6).

Only learners in School A were used for this analysis because of the limited number of learners in School B who provided the correct answer and reason (A✓R✓).

Learners were presented with five-digit dividends. In Table 5.12, "*Digit 5*" refers to the leftmost digit, while "*Digit 1*" refers to the rightmost digit. The A and B "digits" were placed at the sides of the dividend to minimise the positional preference that the first and last digits would enjoy (Figure 5.7). As can be seen in Table 5.12, these two "digits" attracted less than 1% of attention.

Table 5.12 shows the results of a within-subjects one-way analysis of variance (ANOVA) for the effect of grade on the percentage of fixation time (%) while controlling for assessment (implicitly the effect of revision), divisor and digit. The number of responses in A✓R✓ is presented by **N**.

With the exception of four of the 70 cases (7 divisors, 5 digits, before and after revision), all during Assessment 2, it was found that the effect of grade on the percentage of time that learners fixate on the respective digits, was not significant (α = .05). Hence, the influence of grade was discarded for further analyses (Table 1.3: $H_{0,4}$).

5.3.4 THE EFFECT OF REVISION ON THE PERCENTAGE OF FIXATION TIME PER DIGIT FOR LEARNERS WHO BENEFITED FROM THE REVISION

It was found in Section 5.2.3 that revision had an influence on the performance of learners. This section will investigate if revision had an influence on the percentage of fixation time on the digits if learners did not know the divisibility rules before revision. This result will inform the development of an instrument to use eye-tracking to evaluate learners' knowledge of divisibility rules. Only learners in School A will be used for the analysis as very few learners in School B provided the correct answer and reason after revision.

Learners who provided an incorrect reason for a question in Assessment 2, irrespective of what their answers were (A✓R× or A×R×), did not know the relevant divisibility rule. If the same learners provided the correct answer and reason for the same question after revision (A✓R✓), they probably benefited from the revision session and would now be able to apply the divisibility rule.

Table 5.13 shows the results of a within-subjects repeated-measures ANOVA for the effect of revision on the percentage of fixation time while controlling for divisor and digit for learners who benefited from the revision. **N** indicates the number of responses in the respective groups (A✓R× or A×R× before revision and A✓R✓ after revision).

Table 5.12 The effect of <u>grade</u> of learners in A✓R✓ on the percentage of fixation time (%) on a digit while controlling for the effects of revision (assessment), divisor and digit for School A. (Significant p values are underlined (α=.05))
(Key: N=number of responses)

Divisor	Digit	Assessment 2 (before revision)									Assessment 4 (after revision)								
		Grade 4		Grade 5		Grade 6		Grade 7		p	Grade 4		Grade 5		Grade 6		Grade 7		p
		N	%	N	%	N	%	N	%		N	%	N	%	N	%	N	%	
2	A&B	35	0.27	32	0.20	39	0.48	39	0.45	.765	36	0.57	33	0.31	40	0.07	40	0.21	.403
	5		2.32		4.51		5.82		3.38	.379		0.89		1.77		1.79		0.81	.712
	4		1.77		6.01		6.39		3.76	.076		1.03		2.52		1.66		0.75	.454
	3		5.74		7.84		8.68		5.58	.413		3.25		6.65		6.79		3.05	.144
	2		33.99		30.14		35.62		34.86	.727		36.90		38.04		32.60		29.73	.521
	1		55.92		51.30		43.04		51.97	.163		57.35		50.73		57.09		65.46	.205
3	A&B	0	-	5	0	1	0	12	0.38	.518	9	0	23	0.04	24	0.11	31	0.04	.568
	5		-		4.43		0		12.59	.186		10.88		10.60		14.96		11.07	.583
	4		-		38.25		0		36.44	.297		27.85		29.16		23.82		31.33	.347
	3		-		22.91		18.94		19.88	.880		22.56		22.05		23.44		22.44	.985
	2		-		22.89		48.52		21.91	.150		24.53		25.15		24.54		24.15	.995
	1		-		11.53		32.56		8.78	.148		14.20		13.00		13.12		10.98	.775
4	A&B	4	0	6	0.39	9	0.63	21	0.32	.900	10	0	11	0.14	22	0	28	0	.140
	5		0		1.53		1.50		2.16	.687		0.89		3.12		1.87		2.09	.904
	4		7.32		6.37		1.15		2.45	.272		0.41		2.02		2.38		2.27	.731
	3		4.87		9.25		18.26		10.35	.559		4.65		3.40		5.40		6.54	.812
	2		37.93		51.14		53.86		60.22	.159		58.82		56.97		57.77		58.10	.998
	1		49.88		31.34		24.60		24.51	**.047**		35.19		34.43		32.57		30.99	.944
5	A&B	37	0.19	31	1.11	40	0.06	37	0.27	.038	34	0.06	34	0	39	0.09	40	0.54	.235
	5		0.68		6.25		4.13		2.27	**.040**		2.33		2.27		2.01		1.79	.988
	4		0.94		5.23		4.50		2.91	.089		2.14		2.19		3.22		0.56	.321
	3		4.14		5.57		8.26		5.13	.349		2.48		1.60		7.00		2.89	.153
	2		47.08		32.99		37.13		47.12	**.024**		38.91		36.52		38.25		35.77	.960
	1		46.98		48.85		45.90		42.31	.749		54.09		57.41		49.44		58.46	.586
6	A&B	0	-	5	0.95	0	-	5	0	.347	9	0	18	0.61	22	0.19	25	0.08	.283
	5		-		9.24		-		15.91	.407		9.70		9.21		9.65		11.52	.751
	4		-		7.22		-		18.78	**.049**		16.41		19.27		17.75		15.18	.733
	3		-		6.18		-		20.82	.065		18.99		16.73		19.75		20.71	.583
	2		-		40.45		-		35.60	.544		39.82		32.86		27.13		28.24	.102
	1		-		35.77		-		8.89	.088		15.10		21.32		25.52		24.29	.399
8		No statistics available, because there were correct responses for Grade 7 only.									7	0	4	0.44	13	0	23	0.20	.582
												0		1.68		0.85		0.53	.401
												0		9.65		2.25		3.06	.178
												28.48		38.31		27.93		32.06	.712
												60.24		37.61		54.37		52.75	.266
												11.27		12.31		14.60		11.42	.934
9	A&B	0	-	4	0	3	0	6	0.14	.600	13	0.24	20	0.19	27	0.32	32	0.03	.459
	5		-		4.47		4.28		9.88	.185		7.39		10.52		12.73		10.62	.500
	4		-		13.68		14.34		14.53	.987		19.11		16.62		14.47		15.81	.583
	3		-		35.65		18.39		23.00	.239		17.71		27.22		21.15		24.10	.121
	2		-		26.96		42.53		25.48	.182		34.88		30.81		30.21		27.74	.435
	1		-		19.23		20.47		26.99	.495		20.66		14.63		21.12		21.71	.315

No significant difference ($\alpha = .05$) between the percentage fixation time on the A and B "digits" before and after revision were found for anyone of the divisors. There were also no significant differences ($\alpha = .05$) for divisor 2. This could be due to the trend (*cf.* Section 5.3.10) that if learners did not know the reason, they fixated mainly on the last two digits of the dividend which correlates with the divisibility rule of divisor 2 (Section 2.3.2). Although the same argument applies to divisor 5, a significant *p* value (α =

.05) was found on digit 1 for divisor 5. Significant *p* values (α = .05) were found for all the digits for divisors 3, 6 and 9 except for divisor 3, digit 3. Significant *p* values (α = .05) were found for some of the digits for divisors 4 and 8. Therefore, it can be surmised that revision has a significant impact on the percentage of fixation time on at least one of the digits for all the divisors but 2 (Table 1.3: $H_{0,5}$).

Table 5.13 The effect of revision on the percentage of fixation time on a digit for learners who benefited from revision. ***N*** *indicates the number of responses in the respective groups (A✓R× or A×R× before revision and A✓R✓ after revision)*

			Percentage of total fixation time			
Divisor	**Digit**	**N**	**Before revision**	**After revision**	**Current effect**	**p**
2	A&B	7	0.25		-	
	5		19.21		-	
	4		10.18	0.79	F(1, 6)=3.10	.129
	3		7.88	1.35	F(1, 6)=3.14	.127
	2		23.16	35.32	F(1, 6)=1.08	.339
	1		39.33	62.54	F(1, 6)=2.56	.161
3	A&B	70	0.23	0.07	F(1, 69)=2.59	.112
	5		1.99	13.28	F(1, 69)=48.01	**< .001**
	4		6.49	27.51	F(1, 69)=119.24	**< .001**
	3		18.35	22.18	F(1, 69)=3.71	.058
	2		44.17	24.17	F(1, 69)=68.25	**< .001**
	1		28.78	12.73	F(1, 69)=30.68	**< .001**
4	A&B	32	0.34	0.05	F(1, 31)=1.15	.292
	5		2.39	3.37	F(1, 31)=.21	.653
	4		3.28	2.33	F(1, 31)=.42	.520
	3		7.98	3.65	F(1, 31)=4.34	**.046**
	2		49.50	52.83	F(1, 31)=.33	.569
	1		36.52	37.76	F(1, 31)=.05	.819
5	A&B	3	0.04		-	
	5		6.97		-	
	4		14.68		-	
	3		14.80		-	
	2		40.63	8.88	F(1, 2)=2.41	.261
	1		22.89	91.12	F(1, 2)=163.08	**.006**
6	A&B	54	0.19	0.13	F(1, 53)=.37	.548
	5		2.77	10.59	F(1, 53)=38.93	**< .001**
	4		3.33	16.08	F(1, 53)=42.83	**< .001**
	3		13.87	20.13	F(1, 53)=5.74	**.020**
	2		48.11	31.66	F(1, 53)=26.10	**< .001**
	1		31.74	21.42	F(1, 53)=7.79	**.007**
8	A&B	32	0.15	0.06	F(1, 31)=.56	.459
	5		6.32	0.78	F(1, 31)=6.45	**.016**
	4		8.04	2.81	F(1, 31)=5.29	**.028**
	3		19.29	29.91	F(1, 31)=7.10	**.012**
	2		45.40	51.30	F(1, 31)=1.46	.236
	1		20.79	15.14	F(1, 31)=2.79	.105
9	A&B	78	0.13	0.20	F(1, 77)=.38	.538
	5		3.44	10.95	F(1, 77)=37.28	**< .001**
	4		6.78	15.98	F(1, 77)=33.21	**< .001**
	3		16.51	22.79	F(1, 77)=10.11	**.002**
	2		44.06	29.92	F(1, 77)=34.81	**< .001**
	1		29.07	20.17	F(1, 77)=10.16	**.002**

Figure 5.8(a-g) provides a visualisation of the results in Table 5.13.

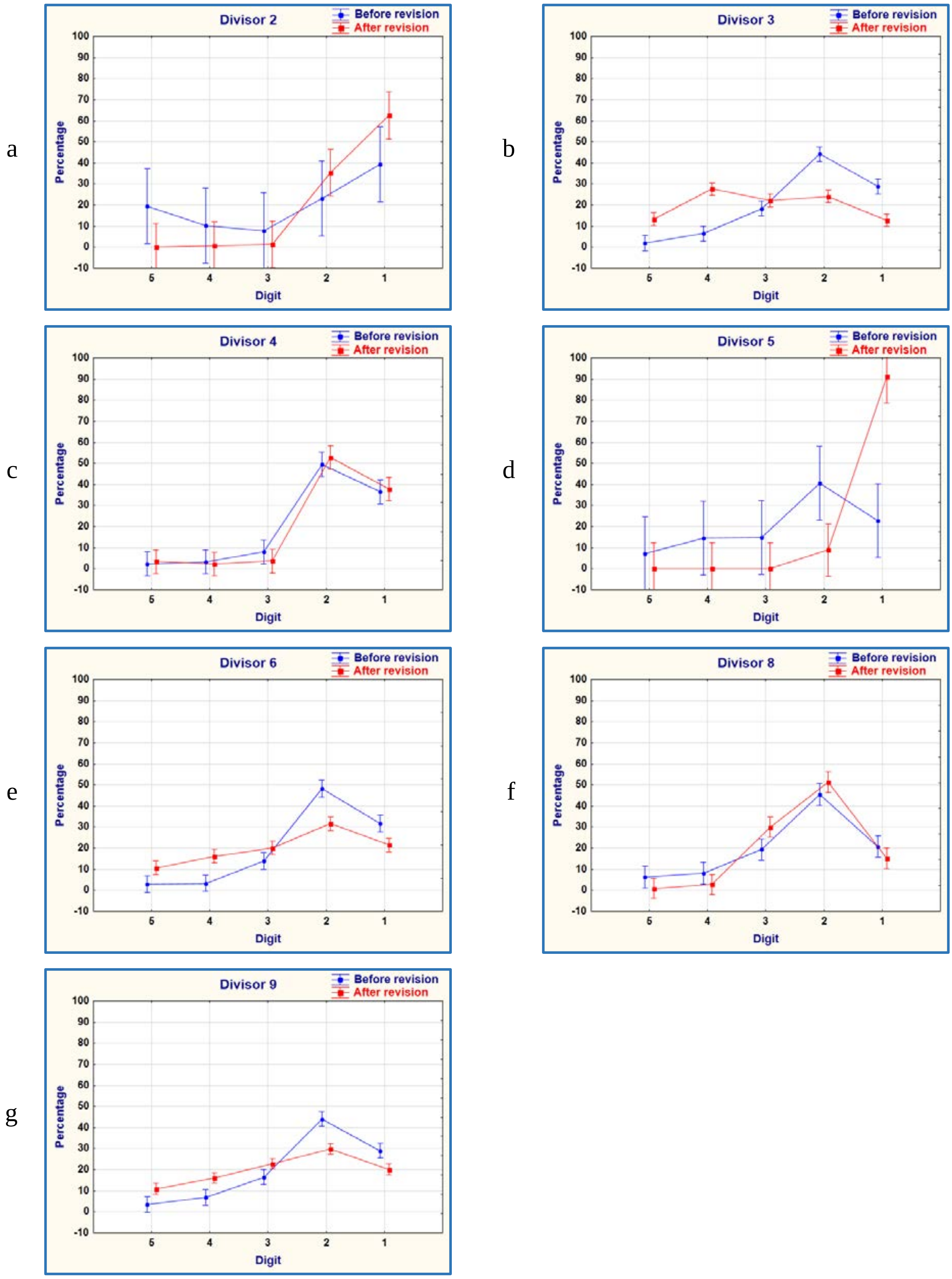


Figure 5.8 The effect of revision on the percentage of fixation time on a digit for learners who benefited from revision

For divisor 2, learners do not need to fixate on digit 2 to digit 5. Figure 5.8a shows, however, that before revision, learners looked at the other digits also. After revision, the percentage of fixation time on the last two digits (digit 1 and digit 2) increased and there was nearly no attention on digits 3 to 5.

Figure 5.8b and Figure 5.8e illustrates that, after revision, learners are aware that they should examine all digits for divisor 3 and 6.

The divisibility rule for divisor 4 states that learners only have to inspect the number formed by the last two digits (Section 2.3.2). It will be demonstrated in Section 5.3.10 that when learners did not know the rule, their fixations were mainly based on the last two digits. Therefore, the percentage of fixation time on the digits as seen in Figure 5.8c, seems to be the same before and after revision, although the percentage of fixation time on the last two digits increased slightly after revision and the percentage of fixation time on the rest of the digits was nearly zero. Figure 5.8f shows the same trend for divisibility by 8 where the percentage of fixation time on the last three digits was marginally higher after revision.

Figure 5.8d shows clearly how the percentage of fixation time on the last digit increased after revision of the divisibility rule for divisor 5 while the other digits enjoyed considerably less attention.

Figure 5.8g illustrates that, before revision of the divisibility rule for 9, the percentage of fixation time on the last two digits (digit 1 and digit 2) were high – a confirmation of the trend mentioned above. After revision, the attention on all digits was at least 10% which agrees with the trends for divisors 3 and 6.

In conclusion, Figure 5.8(a-g) shows how learners' gaze behaviour change after revision. After revision, learners mainly inspected the appropriate digits according to the divisibility rule per divisor (Section 2.3.2).

5.3.5 THE EFFECT OF REVISION ON THE PERCENTAGE OF FIXATION TIME PER DIGIT FOR LEARNERS WHO PROVIDED THE CORRECT ANSWER AND REASON BEFORE AND AFTER REVISION

If the same learners provided the correct answer and reason in Assessment 2 and again in Assessment 4, it probably means that they knew the divisibility rules before revision and that revision was actually unnecessary. Table 5.14 shows the results of a within-subjects repeated-measures ANOVA for the effect of assessment (before or after revision) on the percentage of fixation time on the respective digits for learners in A✓R✓ at both occasions. As before, only learners in School A were used for the analysis. Figure 5.9(a-g) provides a visualisation of the results in Table 5.14.

Table 5.14 The effect of revision on the percentage of fixation time on a digit for learners who had the answer and reason correct before and after revision. (Significant p values are underlined (α=.01)) (Key: N=number of responses)

			Percentage of total fixation time		
Divisor	**Digit**	**N**	**Before revision**	**After revision**	**p**
2	A&B	140	0.35	0.30	.742
	5		3.9	1.39	**<u>.003</u>**
	4		4.52	1.51	**<u>< .001</u>**
	3		6.86	5.15	.096
	2		33.80	33.79	.997
	1		50.54	57.88	**<u>.008</u>**
3	A&B	15	0	0	-
	5		10.66	7.12	.291
	4		37.52	34.17	.680
	3		21.24	22.81	.666
	2		19.64	23.97	.389
	1		10.63	11.94	.680
4	A&B	30	0	0	-
	5		1.88	1.08	.344
	4		2.32	2.05	.828
	3		13.36	8.71	.087
	2		56.94	62.45	.129
	1		25.29	25.71	.884
5	A&B	139	0.39	0.20	.308
	5		2.20	3.13	.253
	4		3.43	2.12	.073
	3		5.96	3.79	.110
	2		40.63	38.77	.488
	1		46.47	52.92	.013
6	A&B	6	0	0	-
	5		9.78	10.03	.950
	4		13.89	18.76	.555
	3		14.58	11.72	.509
	2		42.39	31.18	.085
	1		18.57	28.30	.203
8	A&B	11	0	0	-
	5		0.13	0.47	.517
	4		4.14	2.24	.310
	3		40.85	32.54	.247
	2		44.25	58.03	.023
	1		10.64	6.39	.121
9	A&B	10	0	0	-
	5		7.01	11.07	.506
	4		13.95	14.20	.924
	3		27.04	22.53	.498
	2		27.55	31.51	.485
	1		24.38	20.69	.442

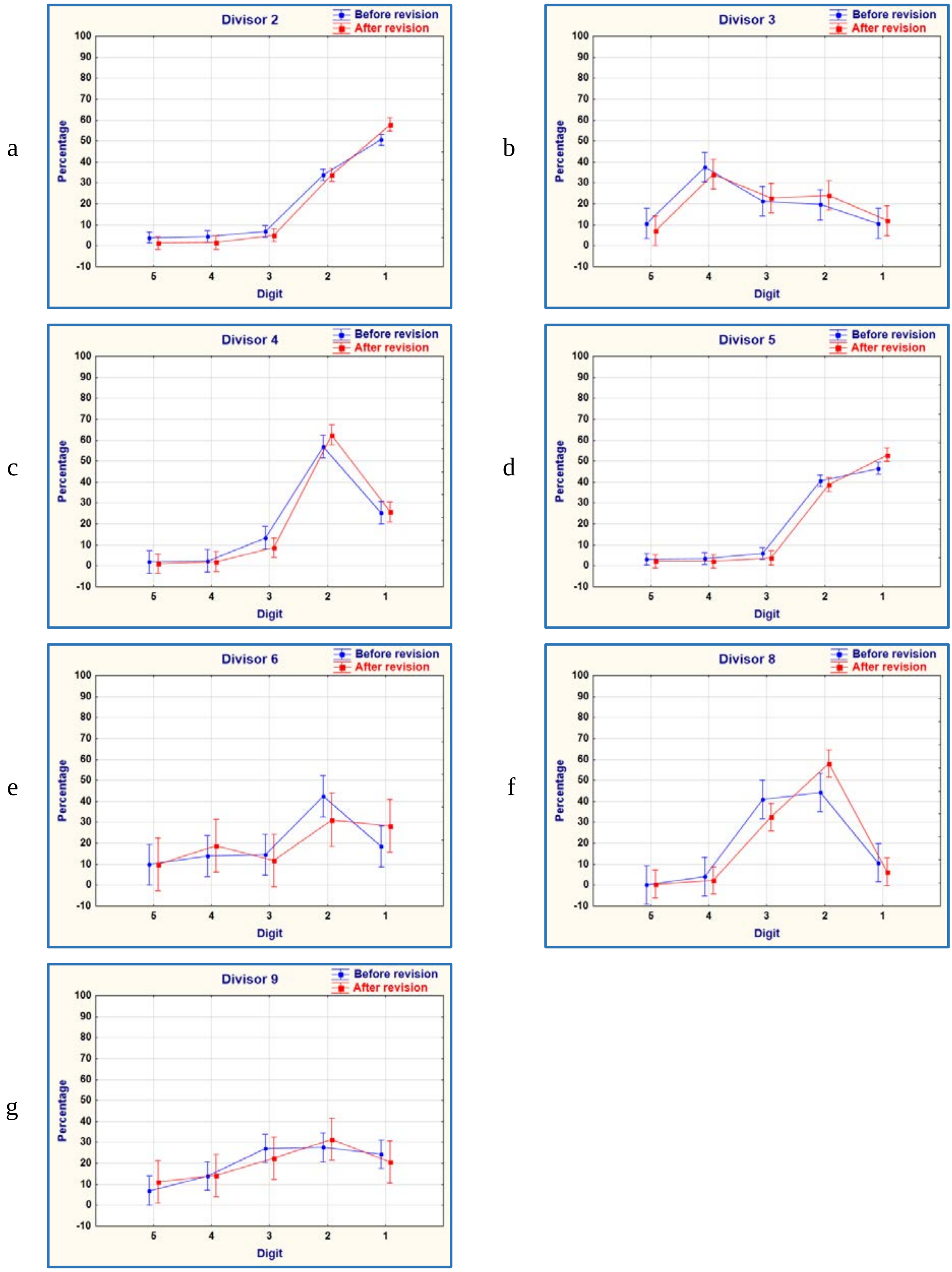


Figure 5.9 The effect of revision on the percentage of fixation time on a digit for learners who knew the divisibility rules before and after revision

As expected, the results show that learners who knew the divisibility rules before revision spent nearly the same percentage of fixation time on the different digits before and after revision. Significant *p* values (α = .01) were only found for digits 1, 4 and 5 for divisor 2. Therefore, it seems that revision does not have an influence on the percentage of time spent on a digit if learners knew the divisibility rule before revision (Table 1.3: $H_{0,6}$). For these learners, the two assessments for School A were combined in subsequent analysis for the percentage of fixation time. The two assessments for School B were not combined because a different eye-tracker was used for Assessment 4 (*cf.* Section 4.3.2).

5.3.6 THE EFFECT OF REVISION ON THE PERCENTAGE OF FIXATION TIME PER DIGIT FOR LEARNERS WHO PROVIDED THE CORRECT ANSWER AND REASON BEFORE OR AFTER REVISION

It was shown above that the number of responses offered by learners who provided the correct answer and reason before and after revision, was quite low for most of the divisors (Table 5.14). Table 5.13 indicated that many responses by learners who did not know the divisibility rule before revision, provided the correct answer and reason after revision. The question can be raised whether the percentage of fixation time on the digits will be different after learners gained new or refreshed knowledge about the divisibility rules? Table 5.15 indicates the effect of revision on the percentage of fixation time on a digit where learners provided the answer and reason correctly before or after revision. **N** indicates the number of responses. A one-way analysis of variance (ANOVA) (as opposed to a within-subjects analysis above) was used to do the analysis.

The same trend is observed in Table 5.14 and Table 5.15. Significant *p* values (α = .01) were only found for divisor 2 (digit 4 and digit 5) and divisor 5 (digit 1). Although more learners provided correct answers and reasons after revision, the percentage of fixation time on the digits did not change significantly. Based on the trends that were observed in Table 5.14 to Table 5.15 and Figure 5.9(a-g), it can be concluded that if learners know the rules, they spend about the same percentage of fixation time on the digits per divisor. Therefore, it seems that there is no difference in gaze behaviour before or after revision between learners who know the divisibility rules (Table 1.3: $H_{0,7}$). Therefore, the two assessments for School A were combined in subsequent analysis for the percentage of fixation time.

Table 5.15 The effect of revision on percentage of fixation time on a digit for learners who had the answer and reason correct either before or after revision – School A. (Key: N=number of responses)

		Percentage of fixation duration				
		Before revision		After revision		
Divisor	**Digit**	**N**	**%**	**N**	**%**	**p**
2	A&B	145	0.36	149	0.28	.622
	5		4.03		1.30	**.001**
	4		4.48		1.45	**< .001**
	3		6.95		4.90	.066
	2		33.81		34.07	.926
	1		50.37		57.99	.019
3	A&B	18	0.25	87	0.05	.023
	5		9.63		12.00	.440
	4		34.92		28.32	.132
	3		20.67		22.63	.547
	2		23.66		24.56	.797
	1		10.87		12.44	.556
4	A&B	40	0.37	71	0.02	.042
	5		1.70		2.01	.784
	4		3.23		2.01	.267
	3		11.42		5.43	.020
	2		55.20		57.92	.491
	1		28.09		32.61	.255
5	A&B	145	0.37	147	0.18	.290
	5		3.23		2.08	.219
	4		3.34		2.01	.108
	3		5.84		3.59	.076
	2		41.33		37.33	.190
	1		45.89		54.82	**.008**
6	A&B	10	0.47	74	0.23	.496
	5		12.57		10.18	.384
	4		13.00		17.09	.307
	3		13.50		19.25	.084
	2		38.13		30.44	.109
	1		22.33		22.82	.934
8	A&B	14	0	47	0.13	.440
	5		0.10		0.64	.233
	4		3.30		2.94	.863
	3		36.61		30.91	.328
	2		48.69		53.03	.448
	1		11.31		12.35	.801
9	A&B	13	0.06	92	0.18	.544
	5		6.92		10.76	.191
	4		14.22		16.06	.525
	3		25.83		23.01	.444
	2		29.87		30.14	.945
	1		23.10		19.85	.425

5.3.7 A COMPARISON BETWEEN SCHOOL A AND SCHOOL B WITH REGARD TO THE PERCENTAGE OF FIXATION TIME ON THE DIGITS

Although learners in School B were not included in Table 5.14 and Figure 5.9(a–g) due to their poor performance in Assessment 2, they showed improvement after revision (*cf.* Section 5.3.8, Table 5.17). Figure 5.10 illustrates similarities between School A and School B for the percentage of fixation time on the digits per divisor for learners in A✓R✓ for Assessment 4.

Table 5.16 shows the results of a within-subjects factorial-measures analysis of variance (ANOVA) for the effect of School on percentage of fixation time while controlling for the divisor and digit for learners in A✓R✓. Although there were significant *p* values (α = .01), as underlined in Table 5.16, the trend of the percentage of fixation time on the digits (Figure 5.10) was nearly the same for both schools. Therefore, it seems that there is no difference in gaze behaviour between learners from schools in different socio-economic environments who provided the correct answer and reason (Table 1.3: $H_{0,8}$).

5.3.8 THE EFFECT OF <u>DIVISOR</u> ON THE PERCENTAGE OF FIXATION TIME PER DIGIT

<u>Answer and reason correct (A✓R✓)</u>

When learners provided a correct answer and reason, one can infer that the learners knew the divisibility rule and also how to apply it. Table 5.17 shows the average percentage of fixation time of learners in A✓R✓ per school, digit and divisor. As discussed above, Assessment 2 and Assessment 4 were combined for School A, but the analysis for School B were done separately for Assessment 2 and Assessment 4 because of the lack of accuracy that was observed during Assessment 2 (*cf.* Section 5.3.1). **N** represents the total number of responses in A✓R✓. The maximum possible value of **N** for School A is 312 (78 learners took part in both assessments × 2 assessments × 2 questions per divisor). The maximum possible value of **N** for School B is 154 (77 learners per assessment × 2 questions per divisor).

Table 5.17 also shows the results of a series of one-way analyses of variance (ANOVA) for the effect of divisor on the percentage of fixation time for each of the five digits. It was proven above that there is no need to control for revision (Section 5.3.6) or grade (Section 5.3.3) for learners who provided the correct answer and reason.

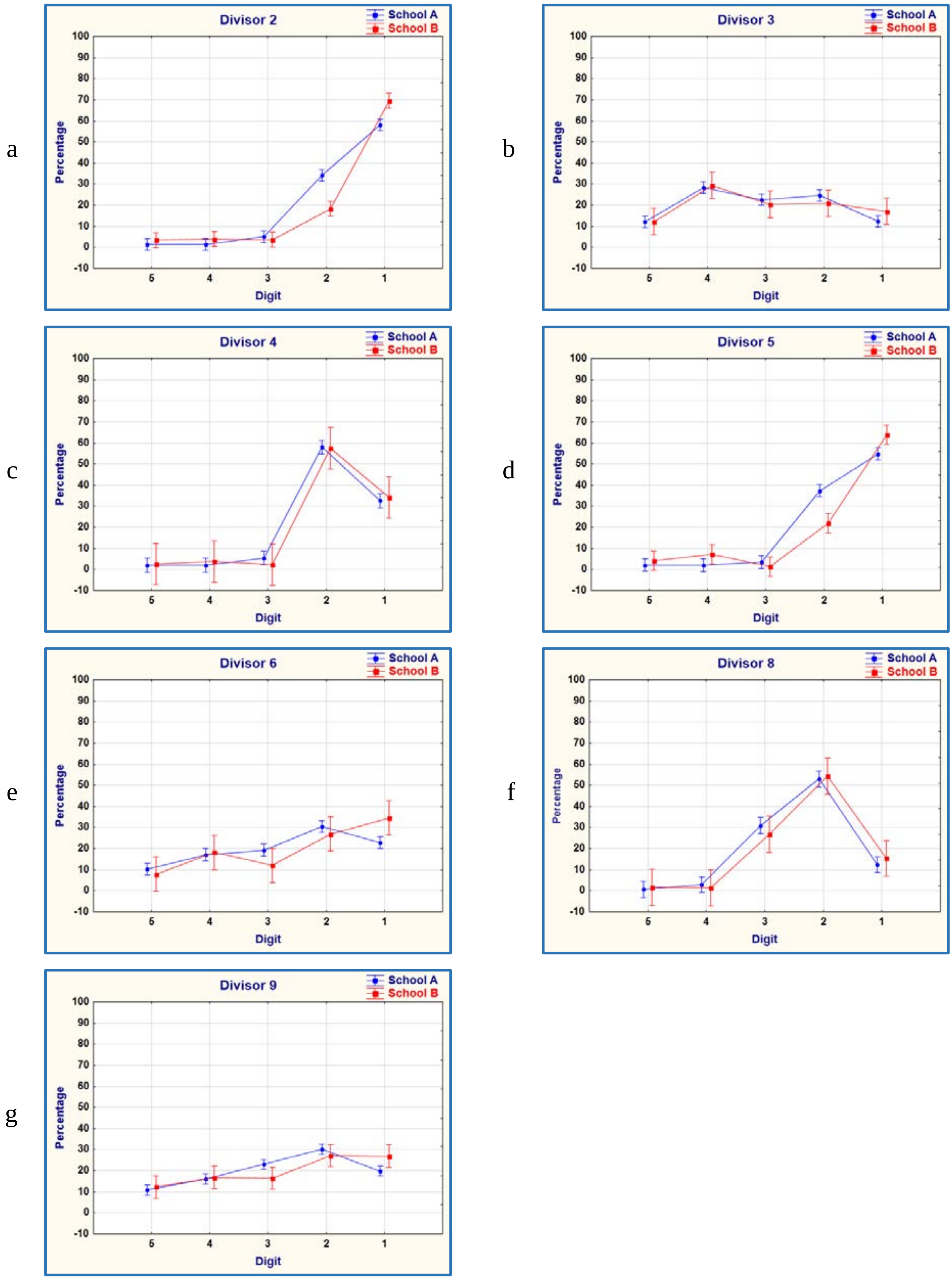


Figure 5.10 Comparison between School A and School B with regard to the percentage of fixation time on a digit for learners who knew the divisibility rules after revision (Assessment 4)

Table 5.16 The effect of school on the percentage of fixation time per divisor for Assessment 4 (after revision) and learners in A✓R✓

		Percentage of total fixation time			
Divisor	**Digit**	**School A**	**School B**	**F**	**p**
2	A&B	0.28	1.53	F(1, 238)=8.54	**.004**
	5	1.30	3.42	F(1, 238)=6.19	.014
	4	1.45	3.82	F(1, 238)=7.11	**.008**
	3	4.90	3.57	F(1, 238)=1.13	.289
	2	34.07	18.26	F(1, 238)=25.24	**< .001**
	1	57.99	69.41	F(1, 238)=9.99	**.002**
3	A&B	0.05	0.22	F(1, 101)=2.32	.131
	5	12.00	12.18	F(1, 101)=.00	.955
	4	28.32	29.34	F(1, 101)=.06	.808
	3	22.63	20.32	F(1, 101)=.42	.516
	2	24.56	20.91	F(1, 101)=1.00	.319
	1	12.44	17.04	F(1, 101)=2.80	.098
4	A&B	0.02	0.00	F(1, 77)=.11	.739
	5	2.01	2.56	F(1, 77)=.05	.824
	4	2.01	3.75	F(1, 77)=.98	.325
	3	5.43	2.22	F(1, 77)=.93	.338
	2	57.92	57.46	F(1, 77)=.00	.951
	1	32.61	34.01	F(1, 77)=.03	*.858*
5	A&B	0.19	1.61	F(1, 207)=7.76	**.006**
	5	2.08	4.11	F(1, 207)=2.58	.110
	4	2.01	7.28	F(1, 207)=16.75	**< .001**
	3	3.59	1.32	F(1, 207)=2.42	.121
	2	37.33	21.90	F(1, 207)=15.80	**< .001**
	1	54.82	63.79	F(1, 207)=4.19	.042
6	A&B	0.23	0.62	F(1, 81)=1.14	.289
	5	10.18	7.93	F(1, 81)=.72	.398
	4	17.09	18.08	F(1, 81)=.05	.820
	3	19.25	11.96	F(1, 81)=4.75	.032
	2	30.44	26.88	F(1, 81)=.53	.469
	1	22.82	34.53	F(1, 81)=3.80	.055
8	A&B	0.133	0.601	F(1, 54)=2.00	.163
	5	0.64	1.67	F(1, 54)=2.04	.159
	4	2.94	1.32	F(1, 54)=.46	.501
	3	30.91	26.79	F(1, 54)=.45	.505
	2	52.74	54.33	F(1, 54)=.04	.841
	1	12.35	15.30	F(1, 54)=.33	.567
9	A&B	0.18	0.63	F(1, 109)=4.64	.033
	5	10.76	12.23	F(1, 109)=.36	.551
	4	16.06	16.82	F(1, 109)=.10	.755
	3	23.01	16.33	F(1, 109)=5.30	.023
	2	30.14	27.07	F(1, 109)=.91	.342
	1	19.85	26.93	F(1, 109)=4.08	.046

Only one significant *p* value (α = .05) was found for digit 1 to digit 5 for School B in Assessment 2. The low values of **N** could have contributed to this lack of effect. Significant *p* values (α = .001) were found for digit 1 to digit 5 for School A, and in Assessment 4 for School B. The "digits" A and B did not have any effect on the percentage of fixation time on the digits. Based on these results, one can conclude that the divisor had an effect on the percentage of fixation time on the respective digits for learners who provided the correct answer and reason (Table 1.3: $H_{0,9}$).

Table 5.17 Effect of divisor on the percentage of fixation time (%) on a digit for learners who provided the correct answer and reason. (Key: N=number of responses)

		Divisor														
		2		3		4		5		6		8		9		
School	Digit	N	%	N	%	N	%	N	%	N	%	N	%	N	%	p
School A (Assessment 2 and 4 combined)	A & B	294	0.32	105	0.09	111	0.15	292	0.28	84	0.26	61	0.10	105	0.17	.475
	5		2.65		11.59		1.90		2.65		10.47		0.51		10.28	< .001
	4		2.95		29.45		2.45		2.67		16.60		3.02		15.83	< .001
	3		5.91		22.29		7.59		4.70		18.56		32.22		23.36	< .001
	2		33.95		24.41		56.94		39.32		31.35		52.03		30.11	< .001
	1		54.23		12.17		30.98		50.38		22.76		12.11		20.25	< .001
School B (Assessment 2)	A & B	56	21.08	2	21.20	5	7.40	32	23.59	0	-	1	12.76	2	26.57	.582
	5		3.70		13.24		1.92		7.03		-		0		8.34	.539
	4		5.61		26.54		25.94		4.31		-		0		15.62	.008
	3		8.00		5.86		18.26		9.19		-		0		0	.809
	2		19.74		24.17		15.68		17.81		-		66.71		26.49	.426
	1		41.87		9.01		30.81		38.09		-		20.65		22.98	.381
School B Assessment 4)	A & B	91	1.53	16	0.22	8	0	62	1.61	9	0.62	9	0.60	19	0.63	.833
	5		3.42		12.18		2.56		4.11		7.93		1.67		12.23	< .001
	4		3.82		29.34		3.75		7.28		18.08		1.32		16.82	< .001
	3		3.57		20.32		2.22		1.32		11.96		26.79		16.33	< .001
	2		18.26		20.91		57.46		21.90		26.88		54.33		27.07	< .001
	1		69.41		17.04		34.01		63.79		34.53		15.30		26.93	< .001

More answer/reason combinations

The previous analysis was performed for learners in A✓R✓. When learners provided an incorrect reason (R×), one can infer that the learners did not know the divisibility rule and guessed the answer – either correct or incorrect. When learners provided an incorrect answer but a correct reason (A×R✓), it could mean that the learners knew the divisibility rule but probably made mental calculation errors or applied the rule incorrectly. This was confirmed when learners verbalised their calculations aloud. Examples of such verbalisations were: "3 is a factor of 19"; and "7+5+1+3+3 = 18".

Table 5.18 shows that if a learner provided an incorrect answer and reason (A×R×), the most attention was on digit 1 and digit 2 – irrespective of the divisor. Learners who provided the correct answer but with a wrong explanation of how they arrived at their answers (A✓R×) also focused mainly on digit 1 and digit 2.

Table 5.18 Average percentage of fixation time on a digit for answer/reason combinations. (Key: A=Answer; R=Reason; ×=Incorrect; ✓=Correct; N=number of responses)

			Divisor													
			2		3		4		5		6		8		9	
	Combinations	Digit	N	%	N	%	N	%	N	%	N	%	N	%	N	%
School A	A×R×	A & B	11	0	166	0.28	137	0.59	9	0	142	0.14	172	0.16	164	0.37
		5		18.57		2.25		3.72		0		2.62		3.96		2.94
		4		2.94		5.32		5.08		1.20		5.11		6.66		5.05
		3		4.87		11.66		8.19		21.08		11.47		15.97		13.08
		2		20.59		46.73		43.13		44.68		46.84		43.92		40.28
		1		53.02		33.78		39.29		33.04		33.83		29.33		38.29
	A✓R×	A & B	5	0	21	0.49	39	0.28	6	0	47	0.60	45	0.10	27	0.09
		5		12.66		3.76		4.32		3.53		5.77		4.20		3.87
		4		10.72		10.00		2.75		6.78		8.44		6.89		8.95
		3		5.19		24.97		9.42		8.93		11.20		13.80		16.70
		2		37.17		37.06		42.57		31.30		41.60		45.85		50.22
		1		34.27		23.72		40.66		49.49		32.41		29.16		20.16
	A×R✓	A & B	0	-	14	0	21	0.25	1	0	33	0.33	26	0.15	10	0
		5		-		9.73		2.74		0		5.45		0.42		7.35
		4		-		38.98		4.46		0		7.17		3.25		13.16
		3		-		22.49		7.04		2.18		11.69		25.87		20.10
		2		-		20.85		51.82		34.15		34.87		50.05		39.79
		1		-		7.94		33.69		63.62		40.49		20.25		19.60
School B – Assessment 2	A×R×	A & B	31	22.68	104	23.19	68	24.64	45	17.99	81	24.29	99	24.49	91	22.14
		5		15.03		6.28		6.79		9.81		8.47		5.78		5.85
		4		9.88		7.12		4.75		7.94		6.51		5.74		5.71
		3		9.96		6.72		5.01		11.93		7.93		6.07		5.87
		2		12.03		17.41		16.98		18.59		19.03		17.31		19.03
		1		30.43		39.30		41.84		33.72		33.79		40.62		41.40
	A✓R×	A & B	41	22.17	28	15.03	48	20.31	50	26.15	42	20.95	21	10.20	22	23.61
		5		9.96		6.56		6.82		5.00		7.74		10.59		4.37
		4		11.90		8.94		7.50		3.04		3.85		15.47		5.61
		3		8.35		11.08		11.90		4.20		5.80		18.10		5.91
		2		12.48		17.93		16.43		19.38		16.91		25.19		14.97
		1		35.17		40.47		37.05		42.23		44.72		20.43		45.51
	A×R✓	A & B	0	-	1	22.34	1	41.24	0	-	0	-	1	39.65	2	39.16
		5		-		19.81		0		-		-		0		1.21
		4		-		35.40		0		-		-		0		1.30
		3		-		6.74		0		-		-		0		5.18
		2		-		4.21		13.58		-		-		12.93		3.40
		1		-		11.53		45.18		-		-		47.41		49.74
School B – Assessment 4	A×R×	A & B	29	2.31	100	0.58	85	0.62	33	0.47	87	1.49	113	0.43	108	0.18
		5		4.17		2.62		4.26		4.22		4.42		2.58		3.14
		4		6.76		5.42		5.72		8.64		5.53		5.27		6.38
		3		8.00		6.15		7.59		5.10		4.44		4.95		6.04
		2		29.44		25.63		21.27		14.07		25.47		26.61		20.14
		1		49.31		59.60		60.55		67.52		58.65		60.17		64.13
	A✓R×	A & B	33	1.00	29	0.60	53	0.91	58	1.74	54	0.40	23	0.77	25	0.74
		5		5.67		3.22		7.48		5.40		4.86		1.97		6.35
		4		8.46		5.20		7.44		5.06		4.73		4.78		8.88
		3		7.02		5.79		3.41		4.10		5.91		13.03		7.52
		2		21.84		16.07		13.23		20.81		19.03		19.84		13.99
		1		56.02		69.13		67.53		62.89		65.09		59.62		62.53
	A×R✓	A & B	1	0	8	0.14	8	0	1	0	3	0.16	7	0	2	0
		5		44.51		17.19		0.31		31.19		21.70		0.37		11.10
		4		6.12		28.73		2.77		34.93		20.00		1.92		12.35
		3		15.80		16.28		7.27		0		10.14		27.82		25.73
		2		4.78		18.05		60.20		22.74		33.46		53.42		25.54
		1		28.88		19.61		29.45		11.14		14.57		16.48		25.26

5.3.9 THE EFFECT OF THE CORRECTNESS OF ANSWER ON THE PERCENTAGE OF FIXATION TIME PER DIGIT FOR LEARNERS IN A×R✓ OR A✓R✓

Table 5.19 indicates the average percentage of fixation time on a digit for learners in School A (Assessment 2 and 4 combined) in A×R✓ and A✓R✓. Divisor 2 is not included because no participants applied the rule correctly along with a wrong answer. The results of a one-way ANOVA for the effect of correctness of the answer on the percentage of fixation time are shown per divisor and digit.

Table 5.19 The effect of correctness of answer on percentage of fixation time per digit for learners in A×R✓ or A✓R✓. (Key: N=number of responses)

		Reason correct				
		Answer incorrect		Answer correct		
Divisor	Digit	N	%	N	%	p
3	A&B	14	0	105	0.09	.341
	5		9.73		11.59	.566
	4		38.98		29.45	.044
	3		22.49		22.29	.955
	2		20.85		24.41	.335
	1		7.94		12.17	.135
4	A&B	21	0.25	111	0.15	.608
	5		2.74		1.90	.545
	4		4.46		2.45	.154
	3		7.04		7.59	.853
	2		51.82		56.94	.278
	1		33.69		30.98	.575
5	A&B	1	0	292	0.28	.853
	5		0		2.65	.740
	4		0		2.67	.707
	3		2.18		4.70	.816
	2		34.15		39.32	.843
	1		63.62		50.38	.646
6	A&B	33	0.33	84	0.26	.748
	5		5.45		10.47	<u>.003</u>
	4		7.17		16.60	<u>< .001</u>
	3		11.69		18.56	<u>.001</u>
	2		34.87		31.35	.256
	1		40.49		22.76	<u>< .001</u>
8	A&B	26	0.15	61	0.10	.687
	5		0.42		0.51	.775
	4		3.25		3.02	.879
	3		25.87		32.22	.128
	2		50.05		52.03	.656
	1		20.25		12.11	.019
9	A&B	10	0	105	0.17	.424
	5		7.35		10.28	.364
	4		13.16		15.83	.416
	3		20.10		23.36	.421
	2		39.79		30.11	.036
	1		19.60		20.25	.883

Non-significant *p* values (α = .01) were found for all the divisors, except for divisor 6, which will be discussed below. Since learners applied the rule correctly and showed no difference in gaze behaviour, it confirms earlier speculation (*cf.* Section 5.2.3) that learners made mental calculation errors for divisors 3, 4, 5, 8 and 9. Therefore, it seems that there is no difference in gaze behaviour between learners in A×R✓ before revision and A✓R✓ after revision except for divisor 6 (Table 1.3: $H_{0,10}$).

Special case for divisor 6

In Assessment 2, some learners stated the divisibility rule for divisor 6 correctly, but when they were prompted for more details, it became evident that they only inspected the last one or two digits of the dividend. The dividends for divisor 6 were compiled such that inspection of only the last or last two digits would result in an incorrect answer (Section 4.3.7.4).

Table 5.20 indicates the percentage of fixation time on the digits for divisor 6 where learners provided the correct divisibility rule before and after revision, but indicated an incorrect answer before revision (A×R✓) and a correct answer after revision (A✓R✓). The results of a within-subjects repeated-measures ANOVA for the effect of revision on percentage of fixation time are also shown.

Table 5.20 The effect of revision on the percentage of fixation time per digit for divisor 6 for learners in A×R✓ before revision and A✓R✓ after revision. (Key: N=number of responses)

			Percentage of fixation time		
Divisor	**Digit**	**N**	**Assessment 2 A×R✓**	**Assessment 4 A✓R✓**	**p**
6	A & B	11	0.43	0.15	.569
	5		2.33	9.81	**.033**
	4		3.68	18.00	**.018**
	3		6.04	15.71	**.004**
	2		32.51	26.39	.371
	1		55.01	29.93	**.004**

Significant *p* values (α = .05) were found for all the digits except for digit 2. Therefore, it can be inferred that some learners who knew the divisibility rule for divisor 6, applied the rule incorrectly (did not actually focus on all digits) before revision.

In conclusion, the results of Table 5.19 and Table 5.20 show that the percentage of fixation time on digits can indicate if a learner applied the rule correctly (Table 1.3: $H_{0,11}$).

5.3.10 THE EFFECT OF <u>DIGIT POSITION</u> ON THE PERCENTAGE OF TOTAL FIXATION TIME PER DIGIT

<u>Answer and reason correct (A✓R✓)</u>

The average percentage of fixation time per digit for learners who provided the correct answer and also the correct divisibility rule (A✓R✓) is indicated in Table 5.21. As explained before, the assessments were combined for School A but not for School B. The table also shows the results of a series of one-way analyses of variance (ANOVA) for the effect of digit position on the percentage of fixation time for each divisor.

Significant p values (α = .001) were found for all the divisors for School A and Assessment 4 for School B, which means that learners were decisive on the time spent on each of the digits for all divisors (Table 1.3: $H_{0,12}$).

Table 5.21 The effect of digit position on the percentage of fixation time per digit for learners in A✓R✓. (Key: N=number of responses)

	Divisor	N	Percentage of fixation time spent per digit						p
			5	4	3	2	1	A & B	
School A Assessment 2 & 4	2	294	2.65	2.95	5.91	33.95	54.23	0.32	< .001
	3	105	11.59	29.45	22.29	24.41	12.17	0.09	< .001
	4	111	1.9	2.45	7.59	56.94	30.98	0.15	< .001
	5	292	2.65	2.67	4.70	39.32	50.38	0.28	< .001
	6	84	10.47	16.60	18.56	31.35	22.76	0.26	< .001
	8	61	0.51	3.02	32.22	52.03	12.11	0.10	< .001
	9	105	10.28	15.83	23.36	30.11	20.25	0.17	< .001
School B Assessment 2	2	56	3.70	5.61	8.00	19.74	41.87	21.08	< .001
	3	2	13.24	26.54	5.86	24.17	9.01	21.20	.505
	4	5	1.92	25.94	18.26	15.68	30.81	7.40	.400
	5	32	7.03	4.31	9.19	17.81	38.09	23.59	< .001
	6	0	-	-	-	-	-	-	-
	8	1	0	0	0	66.71	20.65	12.76	-
	9	2	8.34	15.62	0	26.49	22.98	26.57	.074
School B Assessment 4	2	91	3.42	3.82	3.57	18.26	69.41	1.53	< .001
	3	16	12.18	29.34	20.32	20.91	17.04	0.22	< .001
	4	8	2.56	3.75	2.22	57.46	34.01	0	< .001
	5	62	4.11	7.28	1.32	21.90	63.79	1.61	< .001
	6	9	7.93	18.08	11.96	26.88	34.53	0.62	< .001
	8	9	1.67	1.32	26.79	54.33	15.30	0.60	< .001
	9	19	12.23	16.82	16.33	27.07	26.93	0.63	< .001

Answer and reason incorrect (A×R×)

As motivated previously, a trend was observed that if a learner provided an incorrect answer and did not know the divisibility rule, the highest percentage of fixation time was mainly on the last two digits (digit 1 and digit 2).

The average percentage of fixation time per digit for learners who provided the incorrect answer and also the incorrect divisibility rule (A×R×) is shown in Table 5.22. The results of a series of one-way analyses of variance (ANOVA) for the effect of the digit on the percentage of fixation time for each divisor are also shown. Figure 5.11 provides a visualisation of the results in Table 5.22.

Significant *p* values ($\alpha = .001$) were found for all the divisors for School A and School B, which means that learners were decisive on the time spent on each of the digits for all divisors (Table 1.3: $H_{0,13}$).

Table 5.22 The effect of the digit position on the percentage of fixation time per digit for learners in A×R×. (Key: N=number of responses)

	Divisor	N	Percentage of time spent per digit						p
			5	4	3	2	1	A & B	
School A Assessment 2 & 4	2	11	18.57	2.94	4.87	20.59	53.02	0	.014
	3	166	2.25	5.32	11.66	46.73	33.78	0.28	< .001
	4	137	3.72	5.08	8.19	43.13	39.28	0.59	< .001
	5	9	0	1.20	21.08	44.68	33.04	0	< .001
	6	142	2.62	5.11	11.47	46.84	33.83	0.14	< .001
	8	172	3.96	6.66	15.97	43.92	29.33	0.16	< .001
	9	164	2.94	5.05	13.08	40.28	38.29	0.37	< .001
School B Assessment 2	2	31	15.03	9.88	9.96	12.03	30.43	22.68	< .001
	3	104	6.28	7.12	6.72	17.41	39.30	23.19	< .001
	4	68	6.79	4.75	5.01	16.98	41.84	24.64	< .001
	5	45	9.81	7.94	11.93	18.59	33.72	17.99	< .001
	6	81	8.47	6.51	7.93	19.03	33.79	24.29	< .001
	8	99	5.78	5.74	6.07	17.31	40.62	24.49	< .001
	9	91	5.85	5.71	5.87	19.03	41.40	22.14	< .001
School B Assessment 4	2	29	4.17	6.76	8.00	29.44	49.31	2.31	< .001
	3	100	2.62	5.42	6.15	25.63	59.60	0.58	< .001
	4	85	4.26	5.72	7.59	21.27	60.55	0.62	< .001
	5	33	4.22	8.64	5.10	14.07	67.52	0.47	< .001
	6	87	4.42	5.53	4.44	25.47	58.65	1.49	< .001
	8	113	2.58	5.27	4.95	26.61	60.17	0.43	< .001
	9	108	3.14	6.38	6.04	20.14	64.13	0.18	< .001

Figure 5.11a and Figure 5.11b illustrate the percentage of time fixation on digits per divisor where the answer and reason were incorrect, for learners from School A (both assessments) and School B (Assessment 4) respectively.

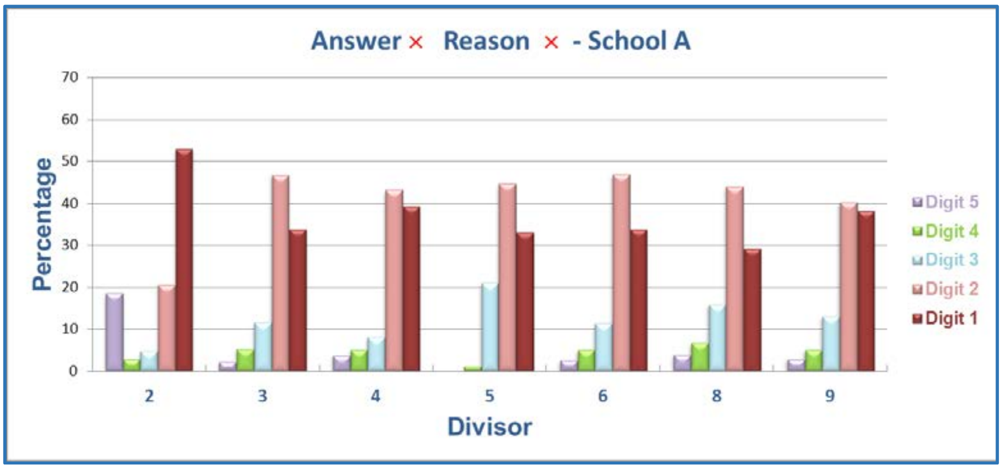


Figure 5.11a Percentage of total fixation time per divisor and digit for learners in School A in A×R× (Both assessments)

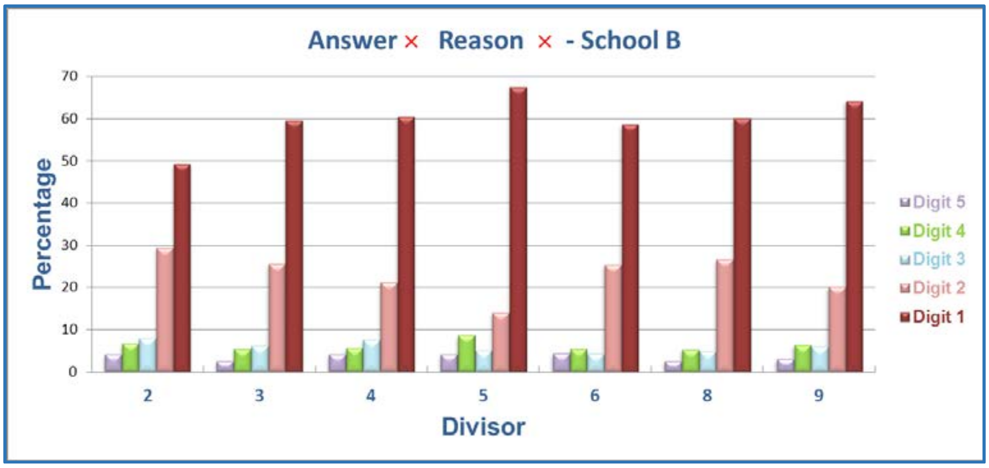


Figure 5.11b Percentage of total fixation time per divisor and digit for learners in School B in A×R× (Assessment 4)

The instrument for predicting whether learners know the divisibility rules (Chapter 6) should consider learners' gaze behaviour when they do not know a divisibility rule. According to Figure 5.11, the fixations of learners could be correct for divisors 2, 4, 5 and 8 even if they did not know the divisibility rules for these divisors. Therefore, careful consideration is needed when deciding on dividends in an assessment as it is important to ensure the reliability of an assessment. Teachers must strive to restrict learners from guessing the answer correctly by constructing the dividend in such a way that if a learner does not know the divisibility rule, the learner would probably guess wrongly.

5.4 COMPARISON BETWEEN SCHOOL A AND SCHOOL B WITH REGARD TO KNOWLEDGE OF THE DIVISIBILITY RULES

It is clear that there is a significant difference between the two schools with regard to the knowledge of divisibility rules – even after revision (Figure 5.5). The following sections will compare various aspects between the two schools.

5.4.1 REASONS OFFERED WHEN LEARNERS DID NOT KNOW THE DIVISIBILITY RULES

The majority of learners in School A who did not know the divisibility rules, based their reasons for an answer mainly on inspection of the last digit of the dividend. Table 5.18 (A×R× and A✓R×) confirms this statement because the fixations of learners who did not know the divisibility rule were mainly on the last digits of the dividend - irrespective of the divisor. These learners created their own incorrect strategy about the divisibility rule and acted confidently when stating it.

Learners in School B also based their reasons for an answer mainly on inspection of the last digit of the dividend. Furthermore, many learners, especially Grade 4 and Grade 5 learners, referred to the fact that the dividend was an even or an odd number. Grade 6 and Grade 7 learners used the same approach as learners in School A if they did not know the divisibility rule. The dividends that were used for divisors 4, 6 and 8 were all even numbers because these divisors cannot divide into an odd dividend. If an odd dividend was used with divisors 4, 6 and 8, it would have been correct if learners observed that it was an odd number.

Learners of both schools had difficulty when a dividend ended in zero (divisor 5 and divisor 8). Some indicated that zero was not divisible by 5 or 8 and others based their answer on the number formed by the last two digits (681**40** for divisor 5 and 239**60** for divisor 8).

Table 5.23 provides a summary of the reasons that learners offered when they did not know the divisibility rules. It was obvious that some learners of both schools did not know the mathematical tables as they still used their fingers to count. They also made unnecessary mental calculation errors when doing simple calculations such as “19 is divisible by 3”; “18 is not divisible by 3”, etc. Some learners in School A, when dealing with divisors 4 and 8, tried to count in multiples of the divisor to inspect if they reach the number formed by the last two or three digits. Most of the time these learners did not have enough time to complete their calculations because there was a time limit of 20 seconds per question.

Table 5.23 General incorrect reasons provided by learners per divisor

Divisor	Reason	School A	School B
2	It is not an even number (The number was indeed an even number)		x
3	Sum of the last 2 or the last 3 digits is not divisible by 3	x	
4	Sum of the last two digits is not divisible by 4	x	
	Last digit must be odd		x
	Number does not end on 4 or multiple of 4	x	x
5	Number must end on a 5	x	x
	Last digit is odd		x
6	Last digit is divisible by 2 and 3	x	
	Last 2 digits is divisible by 2 and 3	x	
	2 can divide into 2 and 3 can divide into 3 (Number was 45432)	x	
	Last digit is an even number		x
	Last digit must be odd		x
	Sum of the digits is divisible by 6	x	x
8	Sum of the last 3 digits is divisible by 8	x	
	Last digit must be odd		x
	It ends on a zero		x
9	Sum of all the digits is divisible by 3	x	
	Sum of the last 2 or last 3 digits is divisible by 9	x	
	3 times 3 is equal 9 (Number was 10199)		x
	6 is a multiple of 9 (Number was 41346)		x

Besides the incorrect reasons for specific divisors in Table 5.23, learners in School B also exhibited general uncertainty or incorrect reasoning about other aspects:

(i) Grade 4 and Grade 5 learners had difficulty to distinguish between odd and even numbers;

(ii) Learners based their reasons on whether the last digit or last two digits forms a prime number;

(iii) Some learners indicated that the last digit should be the same as the divisor for the dividend to be divisible by the specific divisor;

(iv) Learners indicated that the divisor had to be a multiple of the last digit or the last digit had to be a multiple of the divisor;

(v) Many of the learners, especially those in Grade 4 and Grade 5, based their answers on whether the divisor was bigger or smaller than the last digit;

(vi) Some Grade 4 learners inspected the first digit of the dividend to arrive at their answers, for example: “the number **4**4821 is divisible by 2 because 4 (digit 5) was divisible by 2”;

(vii) Some learners indicated that the last digit is not divisible by itself, for example: “the digit 3 is not divisible by 3”; and “the number 61254 is not divisible by 4 because the last digit is a 4”;

(viii) A few learners based their answers on whether the divisor was one of the digits of the dividend, for example, “the number 73134 is not divisible by 2 because there is no 2 in the number”.

5.4.2 CREATIVE REASONS OFFERED WHEN LEARNERS KNOW THE DIVISIBILITY RULES

Some learners in School A used creative methods to produce correct answers as listed below. Learners in School B did not use any of these problem solving strategies if they did not know the divisibility rules.

- Divisor 4, dividend 63172: Count in 4’s to a known value near 72, for example 72, 76, and 80. The learner indicated that the number 80 is divisible by 4, and therefore 72 also had to be divisible by 4.
- Divisor 4, dividend 63172: 72 = 8 × 9. Since 4 could divide into 8, therefore, 4 could divide into 72.
- Divisor 8, dividend 46108: 100 is not divisible by 8; therefore, the number 108 would also not be divisible by 8.
- Divisor 8, dividend 23960: 1000 is divisible by 8. 1000 – 960 = 40. Since 8 can divide into 40, therefore, 8 can divide into 960.
- Divisor 9, dividend 10199: The sum of the digits was not divisible by 3, therefore, the sum of the digits could not be divisible by 9.

5.4.3 EFFECT OF REVISION ON THE PERFORMANCE OF LEARNERS

When comparing School A with School B in Figure 5.6, one notices that revision had a greater impact on the performance of the learners in School A than for the learners in School B. Three reasons or combinations thereof can be presented for this observation:

- Grade 5, 6 and 7 learners in School A formed a solid foundation of the divisibility rules during Grade 5, and if they forgot the divisibility rules, it only took a brief session to refresh their memory. Learners in School B did not master the divisibility rules as well as the learners in School A did – probably because learners in School B did not have a solid foundation of these divisibility rules.

- Revision at School A was done in the learners' home language, while at School B the revision was done in the learners' second language. It is possible that learners in School B could not grasp the new or revised knowledge in their second language.

- The revision time was limited to 30 minutes per group. Because of the larger backlog of learners in School B, the 30 minutes were too short to explain also basic principles or determine at what stage of the explanation the learners got lost.

5.4.4 PARTICIPATION OF LEARNERS DURING REVISION SESSION

Learners in School A participated enthusiastically in the discussions during the revision session. In contrast, the participation of learners in School B during the revision sessions was disappointing. Maybe the poor performance in School B illustrates the concerns of Letseka (2014), who stated that the education system in South-Africa can be labelled as a crisis and a national disaster.

5.5 SUMMARY

In this chapter, the data that were collected on the knowledge of divisibility rules were analysed. Learners from two schools took part in the project and had to complete four assessments. Assessment 1 was an unprepared test where learners from Grade 4 to Grade 7 wrote down their answers and reasons on answer sheets. Selected learners from each school took part in Assessment 2, where learners provided their answers and reasons verbally and an eye-tracker captured their fixations on the individual digits of five-digit dividends. Assessment 3, which was the same format as Assessment 1, and Assessment 4, which was the same format as Assessment 2, were done after revision of the divisibility rules.

Table 5.24 shows a summary of the results that were found in this chapter along with a reference to the relevant hypothesis as stated in Chapter 1 (Table 1.3).

Table 5.24 Summary of results

Hypothesis	Factor	DV	Indicator	Significant?	Reference
$H_{0,1}$	Revision	Performance	Percentage of responses over Answer/Reason combinations (Overall effect)	Yes	§5.2.3
$H_{0,2}$	Revision	Performance	Percentage of responses over Answer/Reason combinations per divisor	Yes	§5.2.3
$H_{0,3}$	Divisibility	Performance	Percentage of responses over Answer/Reason combinations	No	§5.3.2
$H_{0,4}$	Grade	Gaze behaviour	Percentage fixation time per digit	No	§5.3.3
$H_{0,5}$	Revision	Gaze behaviour	Percentage fixation time per digit (R× before revision and A✓R✓ after revision). Same learners, pairwise	Yes	§5.3.4
$H_{0,6}$	Revision	Gaze behaviour	Percentage fixation time per digit (A✓R✓ before revision and A✓R✓ after revision). Same learners, pairwise	No	§5.3.5
$H_{0,7}$	Revision	Gaze behaviour	Percentage fixation time per digit (A✓R✓ before or after revision). All learners, not pairwise	No	§5.3.6
$H_{0,8}$	School	Gaze behaviour	Percentage fixation time per digit (A✓R✓) for both schools. before or after revision) All learners, not pairwise A✓R✓ A✓R×	No	§5.3.7
$H_{0,9}$	Divisor	Gaze behaviour	Percentage fixation time per digit	No	§5.3.8
$H_{0,10}$	Answer	Gaze behaviour	Percentage fixation time per digit (A×R✓ before revision and A✓R✓ after revision). All learners, not pairwise	No, except divisor 6	§5.3.9
$H_{0,11}$	Revision	Gaze behaviour	Percentage fixation time per digit (A×R✓ before revision and A✓R✓ after revision for divisor 6). Same learners, pairwise	Yes	§5.3.9
$H_{0,12}$	Digit	Gaze behaviour	Percentage fixation time per digit (A✓R✓)	Yes	§5.3.10
$H_{0,13}$	Digit	Gaze behaviour	Percentage fixation time per digit (A×R×)	Yes	§5.3.10

It can be concluded that revision had a significant impact on percentage of responses with the correct answer and reason if the divisibility rules were covered properly at an earlier stage. If, however, learners did not master the divisibility rules at any time in the past, revision did not contribute to learners' understanding thereof.

Each assessment contained two questions per divisor. It was found that there was no difference in the performance of learners between the two questions per divisor. Eye-tracking revealed furthermore that grade does not have an effect on the percentage of fixation time on the digits when learners knew the divisibility rules.

Revision had a major effect on the percentage of fixation time on the digits if learners did not know the divisibility rules before division. Revision had, however, no effect on the percentage of fixation time on the digits when learners knew the divisibility rules and applied them correctly before and after revision. The only two factors that had an effect on the percentage of fixation time for the latter group of learners, were the divisors and the digits of the dividends.

Using eye-tracking to determine if learners apply the divisibility rules correctly, proved to be extremely valuable for divisor 6. Although some learners provided the correct answer and reason, eye-tracking revealed that these learners applied the rule incorrectly as they mainly fixated on the last 2 digits.

In the following chapter, parameters will be set for a proposed instrument that will be based on the percentage of fixation time on the digits to determine if learners know the divisibility rules and if they apply the rules correctly.

CHAPTER 6

ESTABLISHMENT OF MINIMUM REQUIRED ATTENTION LEVELS TO EVALUATE LEARNERS' ABILITY TO APPLY DIVISIBILITY RULES

6.1 INTRODUCTION

In the previous chapter, the collected data was analysed and discussed. It was found that only the divisor and the digit position had an influence on the percentage of fixation time on the digits. The value of eye-tracking was also highlighted, especially for divisor 6, where the gaze behaviour revealed that even though learners might know a divisibility rule, they do not necessarily apply it correctly. A trend was found for each divisor for responses in A✓R✓ with regard to the percentage of fixation time on the digits.

Using the trend that was found for each divisor (Chapter 5), the minimum required attention levels per digit will be determined for each divisor to evaluate learners' ability to apply the divisibility rules. Using these levels, an instrument can be devised that will expect learners to only indicate if a dividend is divisible by a divisor. The percentage of fixation time on the digits can then be used to determine if the divisibility rule was applied correctly.

The following aspects will be discussed in this chapter:

- The intentions with an instrument to assess learners' ability to apply the divisibility rules correctly (Section 6.2)
- Software required (Section 6.3)
- Compilation of dividends for each divisor (Section 6.4)
- Minimum attention required per divisor (Section 6.5)
- Validation of the minimum required attention levels (Section 6.6)

6.2 INTENTIONS WITH AN INSTRUMENT TO ASSESS LEARNERS' ABILITY TO APPLY THE DIVISIBILITY RULES CORRECTLY

The intentions with the instrument are to:

(i) assist the teacher to compile dividends per divisor when assessing learners on their ability to apply divisibility rules;
(ii) determine if learners applied the relevant divisibility rule correctly when they provided a correct answer on a divisibility question;
(iii) identify the reason why the answer was incorrect. The reason could be either that the learner guessed the answer, made mental calculation errors or applied the divisibility rule incorrectly.

Should a teacher not have access to an eye-tracker, he or she can still use the proposed strategies to compile the dividends (Section 6.4) for assessments. Teachers can also make use of the procedures followed in Assessment 1 (Section 4.3.8.4) to assess the learners.

If a teacher has access to one eye-tracker only, he or she can use the strategies to compile dividends and then follow the same procedure as for Assessment 2 where learners were assessed individually. The eye-tracking recordings can then be used diagnostically to give feedback to the learners individually. Special software (Section 6.3) can also be used to determine the aggregated percentage of fixation time of all the learners' recordings after they had been assessed individually. The teacher can then analyse the reports to determine if learners applied the divisibility rules correctly.

The ultimate aim with the instrument is to assist teachers who have access to a computer laboratory where all the computers are equipped with eye-trackers. All learners can then be assessed simultaneously in real-time (Appendix A). Although there are many challenges to implement customised software (Section 6.3), as discussed in the limitations of the study (Section 1.11), questions on divisibility can be displayed on a computer screen and the learners will have to answer by clicking on a "*Yes*" or "*No*" button. The data will be analysed automatically to determine if the learner has applied the divisibility rule correctly. As with clicker technology[2], the teacher can interact with the learners to provide feedback on the application of the divisibility rules (Appendix A). Teachers can also immediately identify learners who do not apply the divisibility rules correctly. Real-time feedback can be given

[2] http://er.educause.edu/articles/2007/1/clickers-in-the-classroom-an-active-learning-approach

(i) during introduction of the divisibility rules to identify the problem long before learners acquire wrong habits; (ii) while revision is being done to control learners' application of the divisibility rules; and (iii) during an assessment to create reports that the teacher can use in a discussion with learners afterwards.

6.3 SOFTWARE REQUIRED

When a teacher uses software such as Tobii Studio to analyse learners' gaze behaviour, it is easy to view the recordings on the same computer and draw statistics on the fixation time on each AOI afterwards. These recordings are usually done at a high framerate and they could easily overload a network if they are transferred to another computer. Angelone (2012), who did research in translator training, stated that eye-tracking still has a long way to go before it is ready to be used in classrooms. It is, however, worthwhile to speculate about the benefits that eye-tracking can bring into a classroom.

The software that is needed to assess learners on the divisibility rules only need the percentage of fixation time on each digit and an image (jpeg file) of the fixations as illustrated in Figure 6.1. Therefore, if every computer in the laboratory has the software to calculate the percentage of fixation time on the digits, only one value per digit and a *jpeg* file have to be transferred to the main computer for every question and less constraints will be placed on the network.

To fulfil the requirements of the software, all the computers in a laboratory have to be connected to a network and equipped with an eye-tracker. The teacher's computer (the main computer) should control all the computers in the laboratory. For example, when the teacher gives an instruction on the main computer that the assessment must commence, the assessment should automatically be activated on the learners' computers. All the computers in the laboratory should run the software that calculates the percentage of fixation time for each digit and generates a *jpeg* file for each question (Figure 6.1).

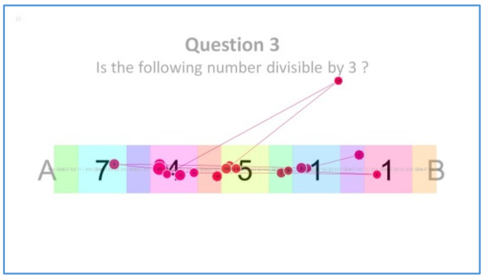


Figure 6.1 Example of an image to transfer over a network

The teacher can also use the software to compile an assessment. The software can be developed in such a way that it automatically generates the dividends to be used during the assessment (Appendix A) according to the strategies to minimise the possibility of correct guesses (Section 6.4). The teacher can decide if the learners can do the assessment in their own time or if they have to do it question by question to enable the teacher to provide feedback between questions. The teacher can inspect which learners provided (i) an incorrect answer with an incorrect gaze pattern; (ii) an incorrect answer with a correct gaze; (iii) a correct answer with an incorrect gaze; and (iv) a correct answer with a correct gaze pattern. This information can be displayed graphically and the teacher can also inspect the learners' final results. A scenario of using the proposed software is illustrated in Appendix A. Please note that development of the software is beyond the scope of this thesis.

6.4 COMPILATION OF DIVIDENDS FOR EACH DIVISOR

If a learner does not know a divisibility rule, the percentage of fixation time on the digits could correspond with the expected gaze behaviour for divisors 2, 4, 5 and 8 because of the trend that learners look at the last one or two digits in any case (Section 5.3.10). If only eye-tracking is used to determine if a learner applied the divisibility rule correctly, it could happen that the fixations are accepted as correct even if the learner did not know the divisibility rule (labelled as **false accept responses**). If a learner knew the divisibility rule, applied it correctly but the eye-tracking rejects the fixations as being incorrect, they can be labelled as **false reject responses**. It is very important that the dividends per divisor should be compiled in such a way that a false accept response is avoided. When a learner provides an incorrect response on a question of divisibility, irrespective of the percentage of fixation time on the digits, the teacher has to investigate whether the learner did not know the divisibility rule, applied the divisibility rule incorrectly, or made mental calculation errors.

Section 5.4.1 reported on the reasons that learners provided when they did not know the divisibility rules. These reasons will be used to suggest strategies for teachers regarding the "*Do's and don'ts*" when compiling the dividend to be used when it has to be divisible by the divisor (Table 6.1). Table 6.2 indicates the strategies that can be followed to compile the dividend when it must <u>not</u> be divisible by the divisor. Although some of these strategies are the opposite of the strategies stated in Table 6.1, they are listed again.

Table 6.1 Strategies to follow if the dividend must be divisible by the divisor

Divisor	Suggested strategies to adopt	Motivation (Section 5.4.1)	Examples
2	Let the dividend end in zero (0).	Some learners struggled to identify if zero is divisible by 2. If the dividend ended in zero, learners inspected the last two digits (such as divisor 5 in Figure 6.2j), and the percentage of fixation time will probably be less than the minimum criteria for percentage of fixation time on the last digit as discussed in Section 6.5.2.	31530 39670 13590
	Let the dividend start with an odd digit and end in an even digit.	Some learners inspected the first digit of the dividend to determine if it was even. These learners had difficulties to identify an odd or even dividend during the assessments.	13138 93176 77998
	Compile the dividend in such a way that the dividend is even, but there is no digit with value 2.	Some learners inspected the digits of the dividend for the appearance of 2.	
	Dividend should not end in 2.	Learners inspect if the last digit is the same as the divisor.	
3	Dividend should not end in 3 or a multiple of 3.	Some learners inspected the last digit of the dividend to identify if the last digit was divisible by 3.	38625 28317 43647
	The number formed by the last two digits should not be divisible by 3.	Some learners inspected the number formed by the last two digits to identify if it is divisible by 3.	
	The sum of the last three digits should not be divisible by 3.	Some learners added up the last three digits of the dividend to determine if the sum is divisible by 3.	
4	Dividend should not end in 4 or 8.	Some learners inspected the last digit of the dividend to identify if it was divisible by 4.	52796 37976 91292
	Use a relative high value for the second-to-last digit.	Some learners counted in fours to determine if they reached the number formed by the last two digits. The higher the second-to-last digit, the longer it will take the learner to reach that number. The high value of the second-to-last digit will ensure that learners use mathematical tables or creative ways (Section 5.4.2) to get to an answer.	
	The sum of the last two digits should not be divisible by 4.	Some learners added the last two digits to determine if the sum was divisible by 4.	
5	Let the dividend end in zero.	Some learners reasoned that the dividend had to end with 5.	21690 74880 79890
	Use a high value for the second-to-last digit if the dividend ends in zero.	Some learners inspected the number formed by the last two digits. If the value of the second-to-last digit is high, learners will spend more time fixating on the second-to-last digit and probably not meet the minimum criteria for percentage of fixation time on the last digit (Section 6.5.2).	
6	Dividend should not end in 6.	Some learners inspected the last digit to determine if it was divisible by 6. Some learners, who knew the divisibility rule, inspected the last digit to determine if it is divisible by 2 and 3.	73182 29574 10404
	The number formed by the last two digits should not be divisible by 6.	Some learners inspected the number formed by the last two digits to determine if it was divisible by 6.	
	The sum of the last three digits should not be divisible by 2 and 3.	Some learners calculated the sum of the last three digits and determined if the sum was divisible by 2 and 3.	
	Dividend should not end in 23 or 32.	Some learners only inspected the last two digits and reasoned that the digit “3” was divisible by 3 and the digit “2” was divisible by 2.	
	The sum of the digits should not be divisible by 6.	Some learners calculated the sum of the digits and determined if the sum was divisible by 6.	
8	Dividend should not end in 8.	Some learners inspected the last digit to determine if it was divisible by 8.	39160 79792 32536
	The number formed by the last two digits should not be divisible by 8.	Some learners inspected the number formed by the last two digits to determine if it was divisible by 8.	
	The sum of the last three digits should not be divisible by 8.	Some learners added the last three digits to determine if it was divisible by 8.	
9	Dividend should not end on 9.	Some learners inspected the last digit to determine if it was divisible by 9.	75285 65817 34164
	The number formed by the last two digits should not be divisible by 9.	Some learners inspected the number formed by the last two digits to determine if it was divisible by 9.	
	The sum of the last three digits should not be divisible by 9.	Some learners added the last three digits to determine if it was divisible by 9.	

Table 6.2 Strategies to follow if the dividend must not be divisible by the divisor

Divisor	Suggested strategies to adopt	Motivation (Section 5.4.1)	Examples
2	Let the dividend end in 1.	If the last digit is less than the divisor, learners found it difficult to determine if the last digit is divisible by the divisor.	79591 88421 28261
3	Let the dividend end in 3, 6 or 9.	Learners, who did not know the divisibility rule, inspected the last digit of the dividend to determine if it was divisible by 3.	35933 17696 55369
	The number formed by the last two digits should be divisible by 3.	Some learners inspected the number formed by the last two digits to determine if it was divisible by 3.	
	The sum of the last two digits should be divisible by 3.	Some learners added the last two digits to determine if it was divisible by 3.	
	The sum of the last three digits should be divisible by 3.	Some learners added the last three digits to determine if it was divisible by 3.	
4	Let the dividend end in 4 or 8.	Learners, who did not know the divisibility rule, inspected the last digit of the dividend to determine if it was divisible by 4.	37574 48538 44554
	The sum of the last two digits is divisible by 4.	Some learners added the last two digits to determine if the sum was divisible by 4.	41722 95262 26426
5	Dividend ends in odd number except 5.	Some learners reasoned that the dividend was divisible by 5 because it ended in an odd number.	31753 61479 52417
6	Dividend ends with 6.	Some learners inspected the last digit of the dividend to determine if it was divisible by the divisor.	42766 13006 14536
	The number formed by the last two digits should be divisible by 6.	Some learners inspected the number formed by the last two digits to determine if it was divisible by 6.	
	The sum of the last two digits should be divisible by 3 or 6.	Some learners added the last two digits to determine if it was divisible by 3 or 6.	32260 35612 65348
8	Dividend ends in 8.	Some learners inspected the last digit of the dividend to determine if the last digit was divisible by the divisor.	82388 32108 84348
	The number formed by the last two digits should be divisible by 8.	Some learners inspected the number formed by the last two digits to determine if it was divisible by 8.	48316 94124 16164
	The sum of the last three digits should be divisible by 8.	Some learners added the last three digits to determine if it was divisible by 8. The textbook indicated the divisibility rule for divisor 8 incorrectly by stating that the sum of the last three digits should be divisible by 8. (Section 2.3.2)	88332 24602 22116
9	Dividend ends in 9.	Some learners inspected the last digit of the dividend to determine if it was divisible by 9.	34509 33999 60609
	The number formed by the last two digits should be divisible by 9.	Some learners inspected the number formed by the last two digits to determine if it was divisible by 9.	
	The sum of the digits should be divisible by 3.	Some learners added all the digits and then determined if the sum was divisible by 3.	

6.5 MINIMUM ATTENTION REQUIRED PER DIVISOR

Two schools participated in the project. Learners of School A revealed some knowledge about the divisibility rules before revision was done and showed remarkable improvement after revision (Section 5.2.3). The learners of School B revealed limited knowledge about the divisibility rules before revision and their improvement after revision was limited as well

(Section 5.2.3). Therefore, only the fixations of learners of School A who provided the correct answer and reason in Assessment 2 or Assessment 4, will be used to establish the minimum required percentage time of fixation on the digits per divisor.

6.5.1 THE EFFECT OF DIGIT POSITION ON PERCENTAGE OF FIXATION TIME

As Table 5.21, Table 6.3 shows the effect of the digit position on the percentage of fixation time per divisor as well as the lower limit of the 95% confidence interval. This extra column was added to determine a trend for the minimum requirements for testing learners' ability to apply divisibility rules. Only the percentage of fixation time of learners who provided the correct answer and reason, were used. **N** represents the total number of responses with the correct answer and motivation (A✓R✓) per divisor. The digits that learners had to inspect for the relevant divisibility rule (Section 2.3.2) are underlined in the table. The *p* values (Table 5.21) are obtained from a within-subjects repeated-measures ANOVA on the effect of digit position on the percentage of fixation time for the respective divisors. The last column of figures in Figure 6.2 provides a visualisation of the data in Table 6.3.

6.5.2 MINIMUM ATTENTION REQUIRED FOR DIVISORS 2 AND 5

A dividend is divisible by 2 if the last digit is even (Section 2.3.2), and divisible by 5 if the last digit is zero (0) or five (5) (Section 2.3.2) and therefore only the last digit of a dividend should be inspected. Table 6.3 indicates that the average percentage of fixation time that learners spent on the last digit for divisor 2 and 5 was 54.23% and 50.38% respectively. The lower limits of the confidence intervals indicate that we can be 95% sure that the percentage of fixation time will be higher than 51.04% and 47.08% respectively. These values were used as the cut-off values for the minimum attention criteria.

Figure 6.2j indicates that when the dividend ends in a zero, learners base their answers on whether five can divide into the number formed by the <u>last two</u> digits. The average percentage of fixation time was approximately 42% on digit 1 and digit 2 where the dividend ends in a zero (6814**0**). A learner, who spent too much time on the <u>second-to-last</u> digit of the dividend (Figure 6.2j), would probably spend less than 47% on the last digit. Therefore, one can assume that the learner did not know the divisibility rule for divisor 5. Since both the rules for divisors 2 and 5 expects the learners to focus on the last digit only, it could be argued that the same criteria should be applied for these two divisors. According to Figure 6.2(a-c, and j-l), the minimum percentage of fixation time of 47% will be acceptable for divisors 2 and 5. Therefore, one can surmise that for divisibility by divisors 2 and 5 learners had to spend 47% of their time on the last digit to be 95% sure that the the divisibility rule was applied correctly.

Table 6.3 Percentage of fixation time per divisor and digit for learners in A✓R✓ of School A. (Learners should inspect the underlined digits according to the divisibility rule)

Divisor	N	Digit	Percentage fixation time: %	Percentage fixation time: Lower limit of 95% confidence interval
2	294	A&B	0.32	0.17
		5	2.65	1.81
		4	2.95	2.13
		3	5.91	4.82
		2	33.95	31.16
		1	54.23	51.04
3	105	A&B	0.09	0.02
		5	11.59	9.31
		4	29.45	26.19
		3	22.29	19.88
		2	24.41	21.82
		1	12.17	10.19
4	111	A&B	0.15	0.02
		5	1.90	0.82
		4	2.45	1.40
		3	7.59	5.13
		2	56.94	53.20
		1	30.98	27.22
5	292	A&B	0.28	0.11
		5	2.65	1.74
		4	2.67	1.86
		3	4.70	3.46
		2	39.32	36.31
		1	50.38	47.08
6	84	A&B	0.26	0.03
		5	10.47	8.71
		4	16.60	14.04
		3	18.56	16.42
		2	31.35	28.27
		1	22.76	19.00
8	61	A&B	0.10	-0.04
		5	0.51	0.14
		4	3.02	1.31
		3	32.22	27.37
		2	52.03	47.27
		1	12.11	8.70
9	105	A&B	0.17	0.04
		5	10.28	8.38
		4	15.83	13.96
		3	23.36	20.97
		2	30.11	27.55
		1	20.25	17.61

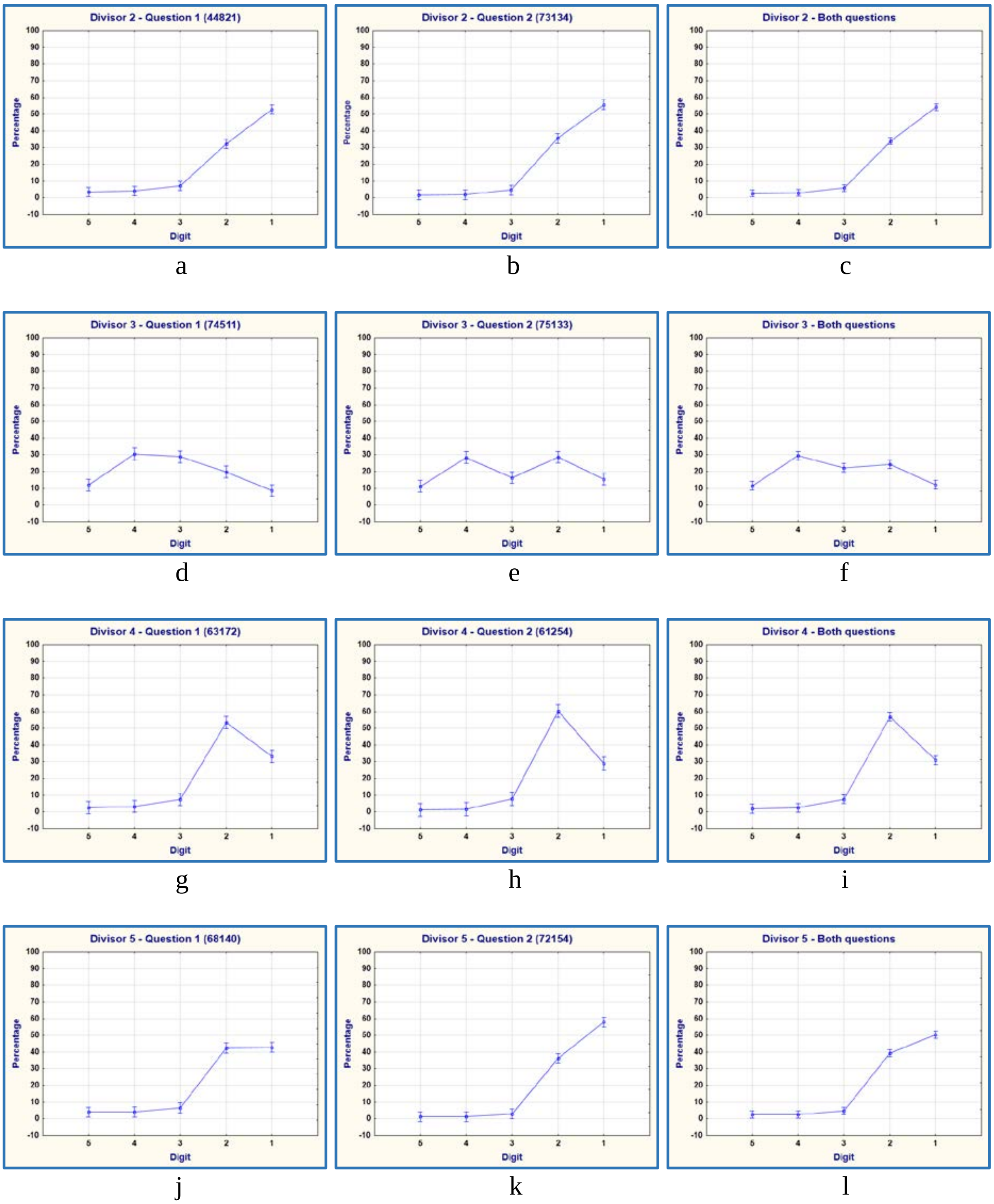
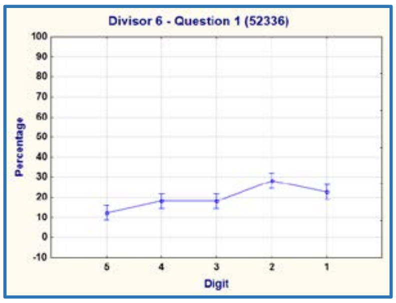
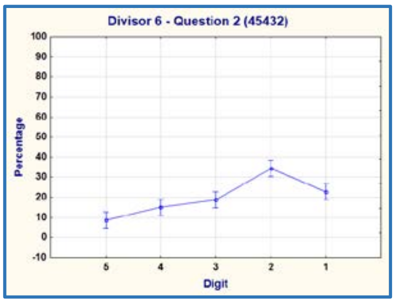
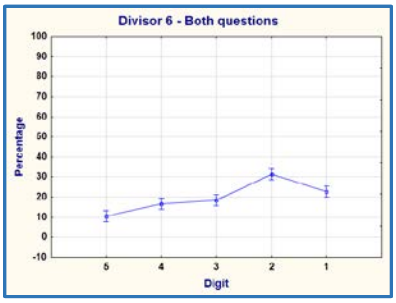


Figure 6.2 Percentage of fixation time on the digits per question and divisor (continued on next page)







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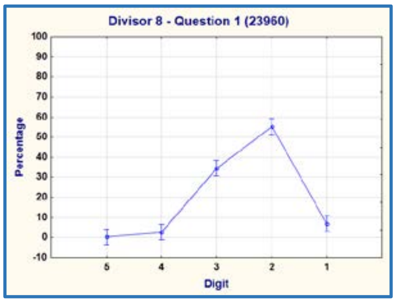

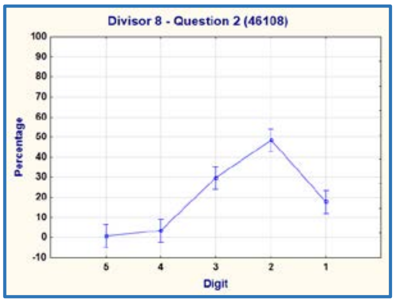

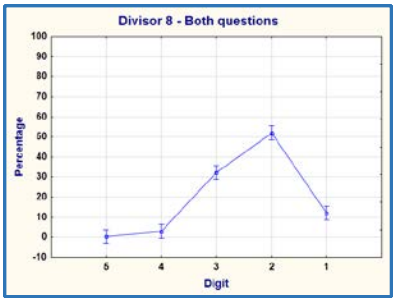

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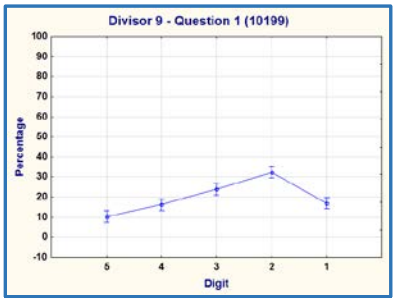

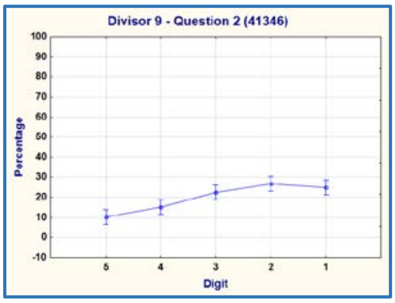

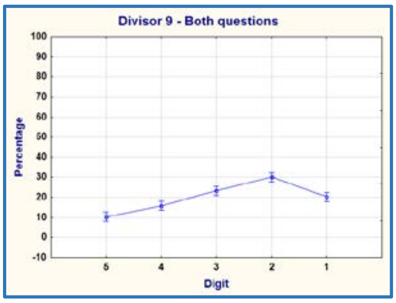

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Figure 6.2 (continued) Percentage of fixation time on the digits per question and divisor

6.5.3 MINIMUM ATTENTION REQUIRED FOR DIVISOR 4

A dividend is divisible by 4 if the number formed by the last two digits is divisible by 4 (Section 2.3.2). Table 6.3 indicates that the average percentage of fixation time that learners in A✓R✓ spent on the last two digits for divisor 4 was 87.92% and that we can be 95% sure that the total fixation time on the last two digits will be higher than 80.42%. Figure 6.2(g-i) shows that the same trend was observed for both questions. Therefore, one can surmise that for divisibility by 4, learners had to fixate on both digit 1 and digit 2, and they had to spend 80% of the time on the last two digits to be reasonably sure that the learner applied the divisibility rule correct.

6.5.4 MINIMUM ATTENTION REQUIRED FOR DIVISOR 8

A dividend is divisible by 8 if the number formed by the last three digits is divisible by 8 (Section 2.3.2). Table 6.3 indicates that the average percentage of fixation time that learners in A✓R✓ spent on the last three digits for divisor 8 was 96.36% and that we can be 95% sure that the total fixation time on the last three digits will be higher than 83.34%. Figure 6.2(p-r) shows that the same trend was observed for both questions. All the learners in A✓R✓ fixated on digit 2 and digit 3, but 9% of these learners did not fixate on digit 1. This agrees with Figure 6.2(p and q) where it can be seen that the percentage of fixations on the last digit was much lower than on digits 2 and 3 for both questions. Therefore, one can surmise that for divisibility by divisor 8, learners had to spend 80% of the time on the combined last three digits for the researcher to be 95% sure that the learner applied the divisibility rule correctly, and the learners also had to fixate on digit 2 and digit 3.

6.5.5 MINIMUM ATTENTION REQUIRED PER DIGIT WHEN LEARNERS HAVE TO INSPECT ALL THE DIGITS

Divisor 3

A dividend is divisible by 3 if the sum of the digits is divisible by 3 (Section 2.3.2) which means that a learner should inspect all the digits. According to Figure 6.2(d-f) the percentage of fixation time on the digits vary between the two questions that were asked, but all digits enjoyed at least 10% of the attention.

Figure 6.2(d-f) shows that the smallest percentage of fixation time were on digits 1 and 5. It is understandable that there will be a low percentage of fixation time on the first digit that learners inspect, regardless of whether they start from the left or right (While analysing the recordings, it was observed that some learners started adding the digits from the left and others started from the right (*cf.* Section 3.2.3).), because they only have to remember the first digit. Thereafter, learners have to accumulate the values of the digits – thus adding to the mental effort with consequential longer fixations.

Although every digit enjoyed at least 10% attention, it will not be justified to require a minimum percentage of fixation time per digit. If a digit is zero (0) or one (1), learners will quickly fixate on it and move on. It is, however, important that learners fixate on all the digits.

Divisor 6

A dividend is divisible by 6 if the dividend is an even number and the sum of the digits is divisible by 3 (Section 2.3.2), which means that the learner should inspect all the digits as for divisibility by 3. Because of the general trend that learners who do not know the divisibility rule fixate mainly on the last two digits (Section 5.3.10), it is important that the dividend is even when testing divisibility by 6.

Divisor 9

A dividend is divisible by 9 if the sum of the digits is divisible by 9 (Section 2.3.2). This means that learners had to inspect all digits. The same arguments that hold for divisor 3 also apply to divisor 9.

Number of digits to be visited

Although learners should inspect all the digits for divisors 3, 6 (if it is even) and 9, it was found that some learners in A✓R✓ did not do so. Table 6.4 indicates the number of responses in A✓R✓ where learners fixated on four of the five digits only. For example, for Grade 4, 4 of the 9 learners in A✓R✓ skipped one of the digits. It could be argued that the fixations were too short to be registered by the eye-tracker (as in cases where easy digits, such as 0 or 1, were not fixated on), or it could be that digits are perceived in learners' peripheral vision without explicit fixations. It could, therefore, be argued that in order to determine divisibility by 3, 6 (if the dividend is even) or 9, learners have to fixate on at least four of the five digits.

*Table 6.4 Number of responses in A✓R✓ who did not fixate on specific digits for divisors 3, 6 and 9. (**N** = number of responses with no focus on a specific digit)*

				Number of responses with no focus on digits				
Divisor	**Grade**	**A✓R✓**	**Total N**	**Digit 5**	**Digit 4**	**Digit 3**	**Digit 2**	**Digit 1**
3	4	9	4	1		1	1	1
	5	28	6	4				2
	6	25	5	1	1			3
	7	43	11	5	2	2		2
6	4	9	4	2	2			
	5	23	13	4	5	4		
	6	22	2					2
	7	30	4	1	2	1		
9	4	13	3	2	1			
	5	24	4	2	1		1	
	6	30	2		4			
	7	38	10	4	1	1	1	3

6.5.6 SUMMARY

Table 6.5 shows a summary of the gaze requirements for each divisor as discussed above.

Table 6.5 Summary of gaze requirements per divisor

Divisor	Requirements	Reference
2	The percentage of fixation time on the last digit (digit 1) must be greater or equal than 47%.	Section 6.5.2
3	There must be fixations on at least four of the five digits	Section 6.5.5
4	The total fixation time on the last two digits must be greater or equal than 80%. Learners must fixate on both the last two digits.	Section 6.5.3
5	The percentage of fixation time on the last digit (digit 1) must be greater or equal than 47%.	Section 6.5.2
6	There must be fixations on at least four of the five digits if the dividend is even.	Section 6.5.5
8	The total fixation time on the last three digits (digits 1, 2 and 3) must be higher or equal than 80%. Learners must fixate on digit 2 and digit 3 at least.	Section 6.5.4
9	There must be fixations on at least four of the five digits.	Section 6.5.5

6.6 VALIDATION OF THE MINIMUM REQUIRED ATTENTION LEVELS

The percentage of fixation time on the digits were analysed in the previous chapter and minimum requirements were set in Section 6.5 to determine if learners applied the divisibility rule correctly. To validate the required minimum attention levels that were set, all the eye-tracking recordings with the learners' answers and reasons were used to compare these levels with the reasons that learners provided. Although recordings were discarded in the analysis of data in the previous chapter where there were no fixations on the areas of interest (Section 5.3.2), it was used during the validation because it could happen in practice that there are no fixations on the AOIs as illustrated in Figure 5.7. Only the recordings of learners of School A who participated in both assessments were used for the validation (Table 5.8). Table 6.6 shows the percentage of responses in each of the possible answer/reason/gaze combinations combined for all divisors per grade for Assessment 2 and Assessment 4. Figure 6.3 provides a visualisation of the results in Table 6.6. The results in Table 6.6 are further broken down in Table 6.7 with the percentage of responses in each of the possible answer/reason/gaze combinations per divisor per grade for Assessment 2 and Assessment 4. Appendix K provides a visualisation of the results in Table 6.7.

Table 6.6 Percentage of responses in each of the possible answer/reason/gaze combinations combined for all divisors per grade for Assessment 2 and Assessment 4. (Key: A=Answer; R=Reason; G=Gaze behaviour; ×=Incorrect/Inadequate; ✓=Correct/Adequate)

	Percentage															
	Assessment 2								Assessment 4							
Grade	××× ARG	××✓ ARG	×✓× ARG	×✓✓ ARG	✓×× ARG	✓×✓ ARG	✓✓× ARG	✓✓✓ ARG	××× ARG	××✓ ARG	×✓× ARG	×✓✓ ARG	✓×× ARG	✓×✓ ARG	✓✓× ARG	✓✓✓ ARG
4	41.8	20.0	0.0	0.4	5.7	5.0	1.4	25.7	26.4	14.3	0.4	3.9	5.0	7.1	2.5	40.4
5	27.0	25.4	2.8	2.8	4.4	4.8	3.6	29.4	12.7	17.9	0.8	2.8	2.0	6.3	3.2	54.4
6	30.7	23.9	2.5	2.5	2.5	5.0	3.6	29.3	8.9	12.9	0.4	6.8	2.9	0.7	3.2	64.3
7	18.6	17.9	0.7	2.9	5.7	6.4	4.3	43.6	3.9	5.4	1.1	5.4	2.1	3.9	4.3	73.9

Table 6.7 Percentage of responses in each of the possible answer/reason/gaze combinations per divisor and grade for Assessment 2 and Assessment 4. (Key: A=Answer; R=Reason; G=Gaze behaviour; ×=Incorrect/Inadequate; ✓=Correct/Adequate)

		Percentage															
		Assessment 2								Assessment 4							
Divisor	Grade	××× ARG	××✓ ARG	×✓× ARG	×✓✓ ARG	✓×× ARG	✓×✓ ARG	✓✓× ARG	✓✓✓ ARG	××× ARG	××✓ ARG	×✓× ARG	×✓✓ ARG	✓×× ARG	✓×✓ ARG	✓✓× ARG	✓✓✓ ARG
2	4	2.5	7.5	0.0	0.0	0.0	2.5	5.0	82.5	2.5	2.5	0.0	0.0	0.0	5.0	0.0	90.0
	5	5.6	2.8	0.0	0.0	0.0	2.8	8.3	80.6	8.3	0.0	0.0	0.0	0.0	0.0	0.0	91.7
	6	0.0	2.5	0.0	0.0	0.0	0.0	10.0	87.5	0.0	0.0	0.0	0.0	0.0	0.0	2.5	97.5
	7	0.0	0.0	0.0	0.0	0.0	2.5	0.0	97.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	100.0
3	4	75.0	17.5	0.0	0.0	2.5	5.0	0.0	0.0	57.5	12.5	0.0	0.0	0.0	2.5	5.0	22.5
	5	30.6	44.4	0.0	0.0	5.6	5.6	0.0	13.9	2.8	22.2	0.0	0.0	0.0	5.6	5.6	63.9
	6	60.0	25.0	0.0	0.0	5.0	7.5	2.5	0.0	10.0	17.5	0.0	5.0	2.5	0.0	0.0	65.0
	7	37.5	25.0	0.0	0.0	2.5	5.0	0.0	30.0	2.5	2.5	2.5	10.0	2.5	2.5	5.0	72.5
4	4	20.0	57.5	0.0	0.0	0.0	12.5	5.0	5.0	15.0	37.5	2.5	7.5	0.0	12.5	2.5	22.5
	5	38.9	16.7	2.8	8.3	8.3	8.3	5.6	11.1	22.2	30.6	5.6	0.0	5.6	5.6	2.8	27.8
	6	20.0	37.5	0.0	5.0	2.5	12.5	5.0	17.5	5.0	25.0	0.0	7.5	5.0	2.5	10.0	45.0
	7	12.5	10.0	5.0	2.5	10.0	7.5	22.5	30.0	7.5	7.5	2.5	5.0	0.0	7.5	10.0	60.0
5	4	0.0	5.0	0.0	0.0	0.0	2.5	0.0	92.5	5.0	2.5	0.0	0.0	0.0	7.5	2.5	82.5
	5	11.1	2.8	0.0	0.0	0.0	0.0	5.6	80.6	5.6	0.0	0.0	0.0	0.0	0.0	2.8	91.7
	6	0.0	0.0	0.0	0.0	0.0	0.0	7.5	92.5	0.0	2.5	0.0	0.0	0.0	0.0	7.5	90.0
	7	0.0	0.0	0.0	2.5	2.5	2.5	5.0	87.5	0.0	0.0	0.0	0.0	0.0	0.0	2.5	97.5
6	4	67.5	10.0	0.0	2.5	17.5	2.5	0.0	0.0	32.5	12.5	0.0	5.0	20.0	7.5	5.0	17.5
	5	27.8	22.2	16.7	11.1	5.6	2.8	5.6	8.3	8.3	13.9	0.0	11.1	2.8	13.9	2.8	47.2
	6	55.0	15.0	17.5	7.5	5.0	0.0	0.0	0.0	17.5	7.5	2.5	10.0	7.5	0.0	0.0	55.0
	7	35.0	32.5	0.0	0.0	7.5	12.5	0.0	12.5	7.5	12.5	0.0	2.5	2.5	12.5	2.5	60.0
8	4	47.5	30.0	0.0	0.0	12.5	10.0	0.0	0.0	22.5	22.5	0.0	12.5	12.5	12.5	0.0	17.5
	5	38.9	47.2	0.0	0.0	8.3	5.6	0.0	0.0	30.6	41.7	0.0	2.8	2.8	11.1	2.8	8.3
	6	22.5	65.0	0.0	2.5	2.5	7.5	0.0	0.0	17.5	22.5	0.0	22.5	2.5	2.5	2.5	30.0
	7	15.0	25.0	0.0	10.0	10.0	5.0	2.5	32.5	5.0	12.5	0.0	15.0	5.0	5.0	2.5	55.0
9	4	80.0	12.5	0.0	0.0	7.5	0.0	0.0	0.0	50.0	10.0	0.0	2.5	2.5	2.5	2.5	30.0
	5	36.1	41.7	0.0	0.0	2.8	8.3	0.0	11.1	11.1	16.7	0.0	5.6	2.8	8.3	5.6	50.0
	6	57.5	22.5	0.0	2.5	2.5	7.5	0.0	7.5	12.5	15.0	0.0	2.5	2.5	0.0	0.0	67.5
	7	30.0	32.5	0.0	5.0	7.5	10.0	0.0	15.0	5.0	2.5	2.5	5.0	5.0	0.0	7.5	72.5

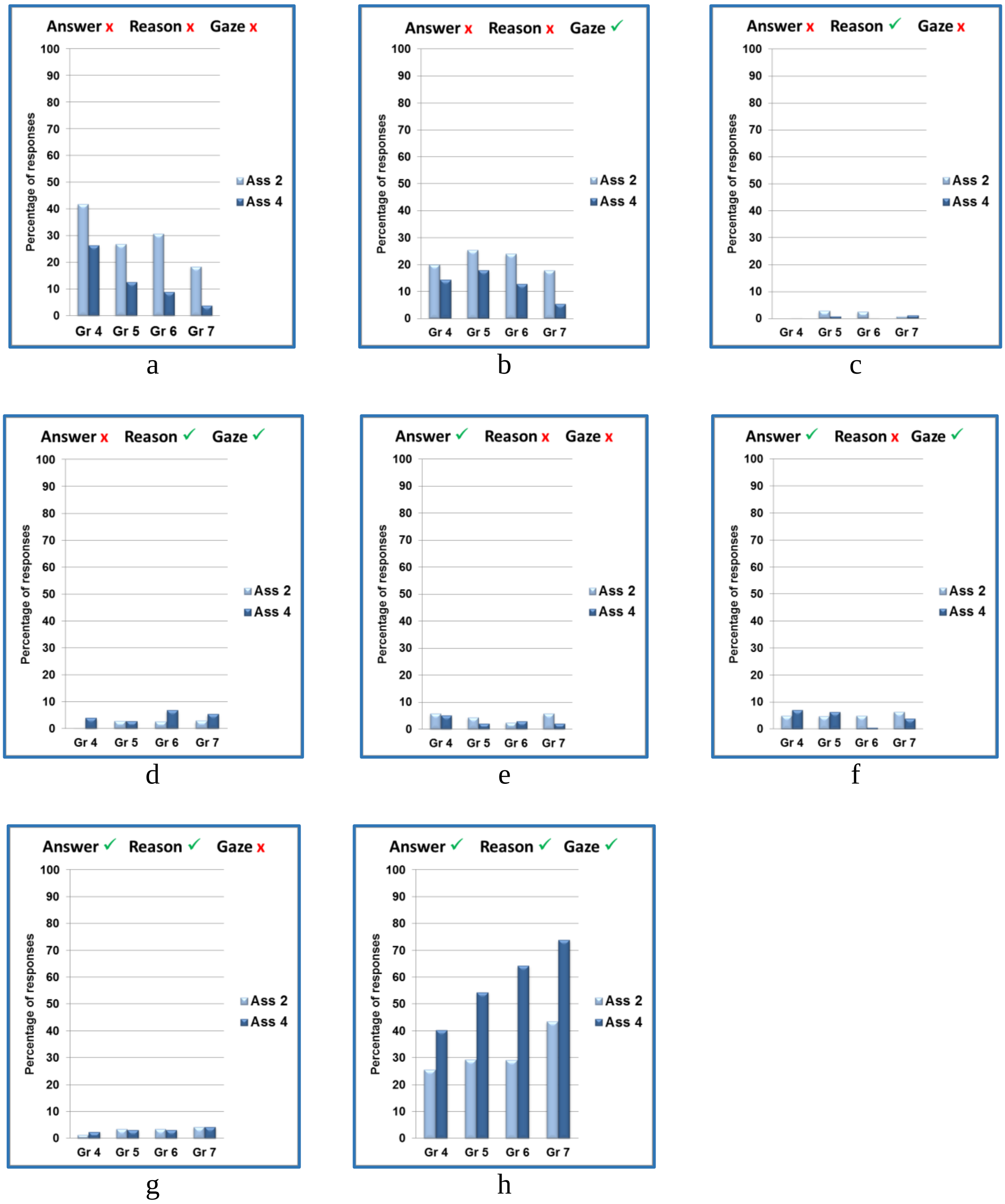

Figure 6.3 Percentage of responses per grade, answer, reason and gaze for all the divisors with minimum gaze requirements

If a learner provides an incorrect answer, a teacher must determine the reason, because it is possible that the learner (i) does not know the divisibility rule (Figure 6.3a), (ii) knows the divisibility rule but applied it incorrectly (Figure 6.3c), or (iii) knows the divisibility rule but made calculation errors (Figure 6.3d). For divisors 3, 6 and 9, the percentage of fixations could readily indicate which one of these categories applies. For the other divisors, it is less obvious since the trend that learners who do not know the divisibility rule focus on the last two digits (Section 5.3.10), overlaps with the correct gaze behaviour. Figure 6.3b illustrates this trend, because there were a percentage of responses in A×R× with acceptable gaze behaviour. It is therefore important that the teacher prompts the learners for a motivation if their answers are incorrect.

In cases where learners does not know the divisibility rule but provided the correct answer (Figure 6.3e and f) they probably guessed the correct answer.

False rejects would occur when the learners' gaze (G) show too little attention on the respective digits but both the answer (A) and reason (R) is correct (example Figure 6.3g). It could happen when (i) a learner knew the divisibility rule but spent too little attention on the required digits or (ii) fixations were above or below the AOIs around the digits (*cf.* Figure 5.7). However, it could also happen that the false reject is actually a correct rejection in cases where learners knew the definition of the divisibility rule, but applied it incorrectly. For example, 45432 is divisible by 6 and there was a learner that indicated the divisibility correctly and stated that the dividend was divisible by 2 and 3, which was correct. However, the response time of this learner was very quick and the learner was prompted to elaborate. The learner reasoned that the digit "3" was divisible by 3 and the digit "2" was divisible by 2 (Table 6.1).

False accepts can occur when the relevant digits enjoy an acceptable amount of attention but the answer and/or the motivation is incorrect (example Figure 6.3f). It could happen that a learner did not know the divisibility rule but guessed the answer correctly and the gaze behaviour fulfil the minimum gaze requirements. However, it could also happen that a learner could not verbalise the divisibility rule correctly although he/she knew the divisibility rule and applied it correctly.

Table 6.8 indicates the percentage of false accept (R×G✓) and false reject (R✓G×) responses where the reasons that learners provided are compared with the minimum gaze requirements for Assessment 2 and Assessment 4, for learners who answered correctly. The overall success rate where learners provided the correct answer and where the attention corresponded with the minimum required attention levels (or the lack thereof in A✓R×), was 85.74%. The false accepts and false rejects for incorrect answers were not calculated, because the learners were not going to receive any marks for incorrect answers.

One of the purposes of this study is to set the parameters for an instrument that can be used by teachers to identify learners who struggle to apply the divisibility rules correctly. The relatively low false accept and false reject responses indicate that the instrument is reasonably reliable to assist teachers to assess learners on their ability to apply divisibility rules.

Table 6.8 Percentage of false accepts and false rejects for Assessment 2 and Assessment 4 for learners who provided the correct answer. (Key: N=answer ✓; R=Reason; G=Minimum gaze requirements; ×=Incorrect/Inadequate; ✓=Correct/Adequate)

	Assessment 2					Assessment 4				
Grade	**N**	**R×G✓**	**R✓G×**	False accept %	False reject %	**N**	**R×G✓**	**R✓G×**	False accept %	False reject %
4	106	14	4	13.21	3.77	154	20	7	12.99	4.55
5	106	12	9	11.32	8.49	166	16	8	9.64	4.82
6	113	14	10	12.39	8.49	199	2	9	1.01	4.52
7	168	18	12	10.71	7.14	236	11	12	4.66	5.08
Total:	**493**	**58**	**35**			**755**	**49**	**36**		
			Average :	11.76	7.10			**Average :**	6.49	4.77
								Total average for both assessments :	**8.57**	**5.69**

6.7 SUMMARY

In this chapter, parameters were proposed for an instrument to assist teachers in assessing learners on their ability to apply divisibility rules. The instrument consisted of two parts. The first part provided strategies for compiling dividends such that learners who do not know a divisibility rule, will not be able to guess the answer correctly. The second part established minimum attention levels per divisor for the correct application of divisibility rules.

The minimum required attention levels were compared with the existing recordings and 85.74% of responses in A✓R✓ and A✓R× were identified successfully. The false accept rate, where the instrument would be in error if it identifies a learner who have applied a divisibility rule correctly, was 8.57%. The false reject rate, where the instrument would mistakenly indicate that a learner does not apply the divisibility rule correctly, was 5.69%.

The relatively high success rates indicate that eye-tracking can assist teachers when they assess learners on their ability to apply divisibility rules.

The following chapter provides a summary of the study, a discussion of the findings, final conclusions and recommendations for future research.

CHAPTER 7
CONCLUSION

7.1. INTRODUCTION

In the previous chapter, guidelines were proposed for (i) strategies to compile dividends in an assessment on the application of divisibility rules, and (ii) establish minimum attention levels per divisor which would indicate if a learner applied a divisibility rule correctly.

The following aspects will be discussed in this chapter:

- Summary of the findings (Section 7.2)
- Concluding remarks (Section 7.3)
- Recommendations for implementation (Section 7.4)
- Summary of the contributions (Section 7.5)
- Suggestions for future research (Section 7.6)

When teachers assess learners by using true/false ("*yes*" or "*no*") questions, they do not have access to the learners' reasoning. This study investigated whether eye-tracking can assist teachers in discovering learners' reasoning while they are thinking about their answers.

7.2 SUMMARY OF THE FINDINGS

7.2.1 REFLECTION ON THE PRIMARY HYPOTHESIS

The study found that eye-tracking can, to a large extent, identify if learners who correctly indicated if a dividend is divisible by a certain single digit divisor, applied the divisibility rule correctly. When the eye-tracking results of learners who provided the correct answer (A✓R✓ and A✓R×) were compared with the reasons that they provided, the success rate was 85.74% (Section 6.6). Therefore, the primary hypothesis that eye-tracking cannot be used to determine if learners apply a divisibility rule correctly when they answer correctly, (H_0), is rejected.

7.2.2 REFLECTION ON THE SECONDARY HYPOTHESES

The secondary hypotheses were summarised in Table 1.3. In the study it was found that:

$H_{0,1}$ Revision had a significant (α = .001) impact on the percentage of responses with the correct answer and reason (A✓R✓) for all grades of the two schools, except for Grade 4 learners of School B (Section 5.2.3). Therefore, the hypothesis that there is no difference in the performance of learners before and after revision, is rejected.

$H_{0,2}$ Revision had a significant (α = .05) impact on the percentage of responses with the correct answer and reason (A✓R✓) per divisor for all grades of the two schools, except for divisor 2 (Grade 7) and divisor 5 (Grade 5) for learners of School A, and divisor 2 (Grade 4 and Grade 7) and divisor 4 (Grade 5) for learners of School B (Section 5.2.3). Therefore, the hypothesis that there is no difference in the performance of learners per divisor before and after revision, is rejected.

$H_{0,3}$ The divisibility per divisor did not have a significant (α = .05) impact on the number of responses with the correct answer and reason for any grade/divisor combination (Section 5.3.2). There was no conclusive evidence to reject the null hypothesis.

$H_{0,4}$ The grade of the learners of School A did not have a significant (α = .01) impact on the percentage of fixation time per digit for learners in A✓R✓ (Section 5.3.3). The analysis failed to reject the null hypothesis. Therefore, there was no conclusive evidence that grade has an effect on gaze behaviour.

$H_{0,5}$ Revision had a significant (α = .05) impact on the percentage of fixation time on at least one of the digits for nearly all the divisors for learners of School A who provide the wrong reason before revision (A✓R× or A×R×) and the correct answer and reason after revision (A✓R✓) (Section 5.3.4). Therefore, the hypothesis that there is no significant difference in the percentage of fixation time between these two groups, is rejected.

$H_{0,6}$ Revision did not have a significant (α = .01) impact on the percentage of fixation time per digit when the same learners in A✓R✓ were compared pairwise before and after revision (Section 5.3.5). Significant values were only found for divisor 2 with digits 1, 4 and 5, but one must remember that digits 4 and 5 do not contribute to the divisibility rule of divisor 2. Therefore, the analysis failed to reject the null hypothesis. There was no conclusive evidence that revision has an effect on gaze behaviour where learners provided the correct answers and reasons before **and** after revision.

$H_{0,7}$ Revision did not have a significant ($\alpha = .01$) impact on the percentage of fixation time per digits for learners (all learners, not pairwise) of School A who provided the correct answer and reason before **or** after revision (Section 5.3.6). Significant values were only found for divisor 2 with digits 4 and 5 and divisor 5 with digit 1. Therefore, the analysis failed to reject the null hypothesis.

$H_{0,8}$ Different socio-economic environments, as defined by school, did not have a significant ($\alpha = .01$) impact on the percentage of fixation time per digit for learners who provided the correct answer and reason after revision (Section 5.3.7). There were only significant values for divisors 2 and 5, and therefore the analysis failed to reject the null hypothesis that the different socio-economic environments of the two schools have an effect on the gaze behaviour of learners in A✓R✓.

$H_{0,9}$ The divisor had a significant ($\alpha = .001$) impact on the percentage of fixation time on the digits for learners in A✓R✓ of School A before and after revision and School B after revision (Section 5.3.8). Therefore, the hypothesis that there is no difference in gaze behaviour (as measured in terms of percentage of fixation time per digit) between the different divisors, is rejected.

$H_{0,10}$ The correctness of the answer did not have a significant ($\alpha = .01$) impact on the percentage of fixation time per digit (except for divisor 6) for learners who provided the correct reason (Section 5.3.9). Therefore, there was no conclusive evidence that, except for divisor 6, the correctness of answer has an effect on gaze behaviour of learners.

$H_{0,11}$ For divisor 6, revision had a significant ($\alpha = .05$) impact on the percentage of fixation time per digit (except for digit 2) when the learners in A×R✓ before revision and A✓R✓ after revision were compared pairwise (Section 5.3.9). This indicates that some learners applied the divisibility rule for divisor 6 incorrectly before revision. Therefore, the hypothesis that there is no difference in gaze behaviour between learners who provided the correct reason but with an incorrect answer before revision and the correct answer and reason after revision, is rejected.

$H_{0,12}$ The position of a digit had a significant ($\alpha = .001$) impact on the percentage of fixation time on the digits for responses in A✓R✓ (Section 5.3.10). A significant *p* value

(α = .001) was found for all the divisors for learners of School A before and after revision and for learners of School B after revision. Therefore, the hypothesis that there is no difference in gaze behaviour (as measured in terms of percentage of fixation time per digit) for responses A✓R✓ between the different digits of the dividend, is rejected.

$H_{0,13}$ The position of a digit had a significant (α = .001) impact on the percentage of fixation time on the digits for responses in A×R× (Section 5.3.10). The majority of the learners who provided the wrong answer and reason, inspected only the last digit of the dividend and they fixated mostly on the last two digits of a dividend – irrespective of the divisor (Section 5.3.10, Figure 5.11). Therefore, the hypothesis that there is no difference in gaze behaviour (as measured in terms of percentage of fixation time per digit) between the different digits of the dividend for responses in A×R×, is rejected.

7.2.3 MINIMUM REQUIRED ATTENTION LEVELS

As a result of the above-mentioned secondary hypotheses, the following minimum required attention levels were set as a benchmark to investigate whether the reason that the learners provided, correlated with attention level for learners who provided the correct answer. It is postulated that:

1. If only the last digit of the dividend has to be inspected, the learner should spend at least 47% of fixation time on the last digit for the teacher to be reasonably sure that the learner applied the divisibility rule correctly (Section 6.5.2);

2. If only the last two digits of the dividend have to be inspected, the learner should fixate on both the last two digits and spend at least 80% of fixation time on them for the teacher to be reasonably sure that the learner applied the divisibility rule correctly (Section 6.5.3);

3. If only the last three digits of the dividend have to be inspected, the learner should fixate on at least the third and second-to-last last digits and spend at least 80% of fixation time on the combined last three digits for the teacher to be reasonably sure that the learner applied the divisibility rule correctly (Section 6.5.4);

4. If all five digits have to be inspected, there should be clear fixations on at least four of the five digits for the teacher to be sure that the learner applied the divisibility rule correctly (Section 6.5.5).

The proposed minimum required attention levels proved to predict the correct or incorrect application of divisibility rules accurately for 85.7% of the cases where learners provided the correct answer (Section 6.6, Figure 6.3(e-h)). In 8.6% of the cases, incorrect applications of the divisibility rules were predicted to be correct and in 5.7% of the cases, correct applications were predicted to be incorrect (Table 6.8).

7.3 CONCLUDING REMARKS

The study suggests that when a teacher is in possession of the learner's answer, reason and fixation data, the teacher is in a position to identify if the learner (i) guessed the answer, (ii) correctly applied the divisibility rule, (iii) correctly applied the divisibility rule but made mental calculation errors, or (iv) incorrectly applied the divisibility rule (Figure 6.3).

The study found that eye-tracking cannot predict with 100% confidence that learners apply the divisibility rules correctly if they indicate correctly if the dividend is divisible by a certain divisor, and can therefore not be used with formal assessments where the results of the learners reflect on their progress reports. However, eye-tracking can still assist teachers to identify whether the gaze behaviour of learners who had the answer correct, agree with the minimum required attention levels as stated in Section 7.2.3 and therefore provide the teacher with useful information regarding application of the divisibility rule.

If learners do not provide verbal or written motivations for their answers, gaze behaviour cannot distinguish between learners who guessed the answers and learners who knew the divisibility rules but applied them incorrectly (as illustrated with divisor 6 in the study). The gaze behaviour will in both cases be incorrect as suggested by the postulated minimum required attention levels above.

7.4 RECOMMENDATIONS FOR IMPLEMENTATION

7.4.1 COMPILATION OF DIVIDENDS TO USE WHEN ASSESSING THE APPLICATION OF DIVISIBILITY RULES

The study has revealed some of the strategies that learners follow when they have to decide if a dividend is divisible by a single digit divisor (Section 5.4). A trend was discovered that learners inspected only the last two digits of the dividend when they do not know the divisibility rules (Section 5.3.10). The strategies to compile a dividend per divisor (Section 6.4) limit the possibilities of learners guessing the correct answers and can be used by teachers to assess learners' knowledge of the divisibility rules.

7.4.2 USING EYE-TRACKING TO IDENTIFY IF LEARNERS APPLIED DIVISIBILITY RULES CORRECTLY

It was recommended that a software system should be developed for an instrument to assess learners' abilities to apply the divisibility rules correctly (Appendix A). This system will capture the percentage of fixation time on each digit of the dividend and transfer the data to a central computer where it will be processed and aggregated. The teacher can use the results to identify shortcomings and provide feedback to learners. It is beyond the scope of this study to implement the system due to the limitations that were discussed in Section 1.11.

7.5 SUMMARY OF CONTRIBUTIONS

Learners who did not know the divisibility rules, mainly inspected the last two digits of the dividend. Although they indicated verbally that they only inspected the last digit of the dividend, it was revealed through eye-tracking that there was also a high percentage of fixation time on the second-to-last digit. Susac, Bubic, Kaponja, Planinic and Palmovic (2014) also concluded that information gained from participants is not very objective and stated that eye-tracking can be a useful complementary source of information to identify the strategies that participants use when solving a problem. Therefore, when teachers compile dividends to be used in the assessment of divisibility rules, they have to keep in mind that learners who do not know the divisibility rules, mainly inspect the last two digits of the dividend.

Eye-tracking can assist teachers in identifying whether learners applied the divisibility rules correctly when their answers were correct. Eye-tracking can also reveal if learners applied the divisibility rule incorrectly, or applied the divisibility rule correctly but made mental calculation errors if they provide correct reasons for their answers. However, due to the relatively high probability of error, eye-tracking cannot be used exclusively to determine if learners applied the divisibility rules correctly or not.

7.6 SUGGESTIONS FOR FUTURE RESEARCH

It may be worthwhile to develop a system as illustrated in Appendix A. With such a system, similar procedures as were followed in this study could be performed and the results compared with the results of this study to report on the similarities or disagreements when a group of learners perform the assessment simultaneously.

The proposed instrument to predict whether learners applied the divisibility rules correctly has a relatively high probability of error (Section 6.6). Research can be conducted to establish benchmarks for an acceptable error rate.

With regard to the concerns about the mathematical performance of learners in South-Africa, eye-tracking can also be used in other problem areas in Mathematics as highlighted by the Department of Basic Education (Department of Basic Education, 2013c; 2014a) to reveal if learners follow the correct strategies when thinking about their answers.

7.7 SUMMARY

When true/false questions have to be answered, teachers have no insight with regard to learners' reasoning. Learners could answer correctly because they knew how to solve the question, but they could also have guessed the answer. It is also possible that learners knew how to solve a question but made calculation errors and therefore provided an incorrect answer. Learners could also have misunderstood a teacher's explanation of how to solve a problem and adapt a wrong way on how to perform a procedure.

It does not matter which of the above-mentioned cases apply - the teacher judges the correctness of the answer and assumes that learners who provided a correct answer, know how to apply the procedure and learners who answered incorrectly, do not. If learners motivate their answers, it can help teachers to evaluate whether the correct procedure was followed. However, as was indicated in this study, some of the learners motivated their answers for divisibility by 6 correctly, but applied the rule incorrectly.

This study proved that eye-tracking can assist teachers in assessing learners on their ability to apply divisibility rules. If teachers have access to learners' answers, reasons and fixation data, they are able to judge if learners applied the divisibility rules correctly or made calculation errors during the assessment. When teachers have access to this information, they are in a better position to assist learners. If learners applied the divisibility rules incorrectly, as was illustrated for divisor 6 in the current study, teachers will know where the learners made mistakes in their reasoning and they can explain the divisibility rules again. If learners made calculation errors, the teachers will know what actions to take to enhance learners' calculation abilities. Teachers will also be in a position to identify which learners guessed the answers, and revise the divisibility rules with them.

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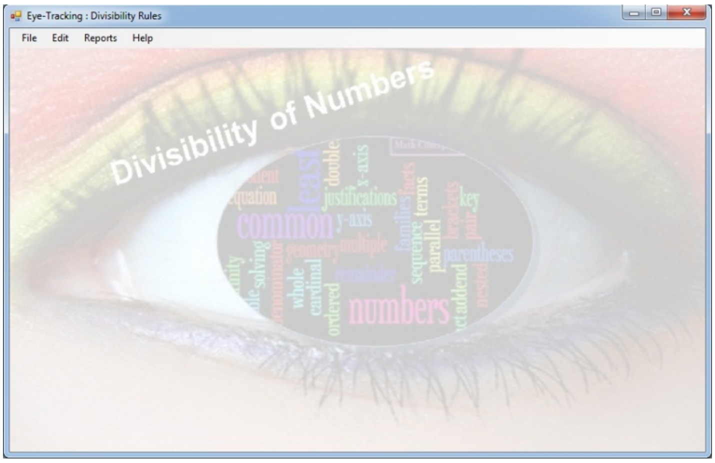
APPENDIX A

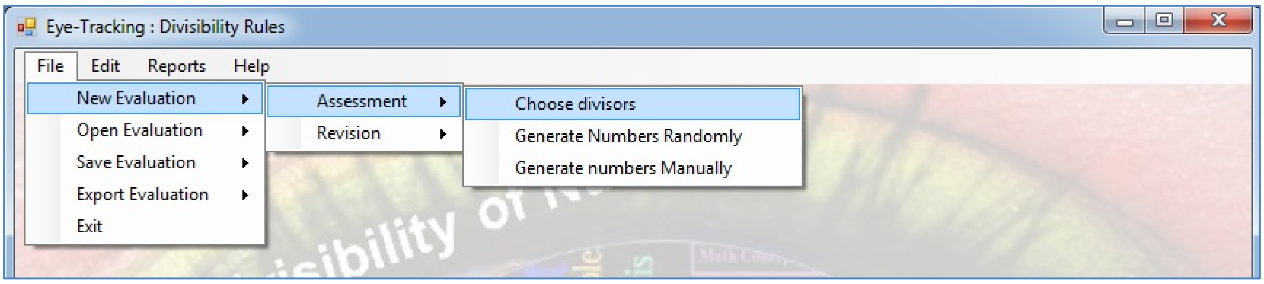
CLASSROOM SCENARIO WITH REAL-TIME EYE-TRACKING

Setting the scene

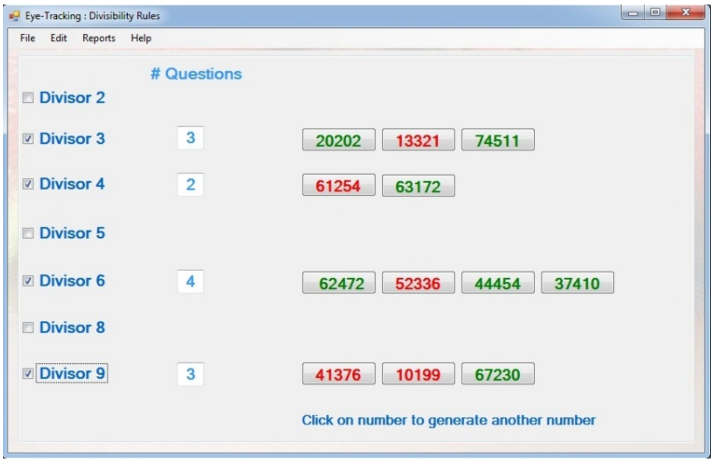
Ms Robertson, the Mathematics teacher, walks into the empty computer lab. "I have to set an assessment for the learners before they arrive in 30 minutes" she thinks by herself. She switches on the main computer in front of the class and opens the software application that Dave installed for her to assess the learners on the divisibility rules.



She clicks on the menu to create the assessment. She already compiled the revision questions beforehand.

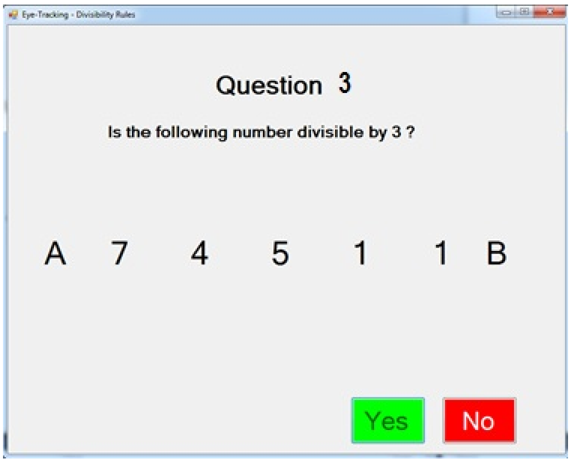


She chooses the "*Choose divisors*" option, and decides which divisors she wants to use, as well as how many questions per divisor to ask. She does not have the time to compile the dividends herself and therefore she allows the computer to generate the dividends randomly. Dave told her that the program uses certain strategies to compile the dividends so that learners, who do not know the divisibility rules, would probably guess the answers incorrectly.



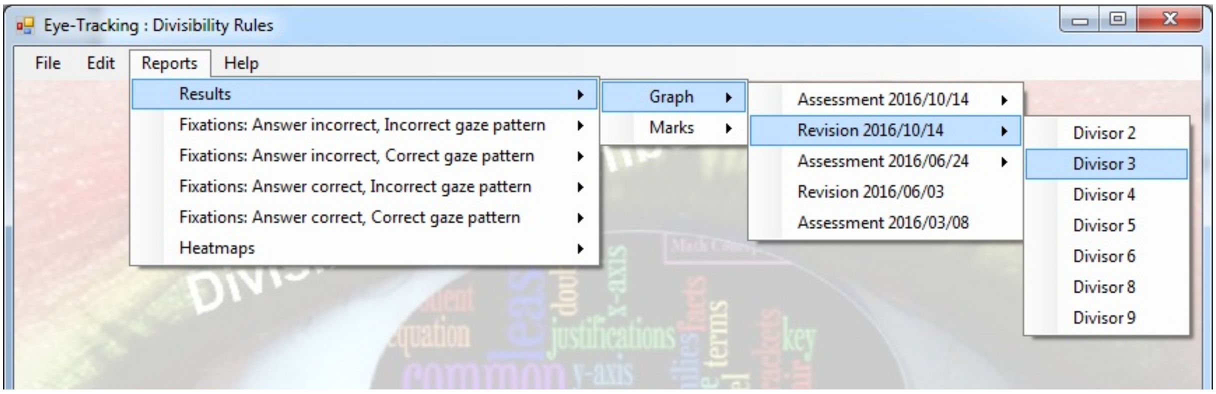
She inspects the generated dividends. The three dividends, which were generated for divisor 3, are fine. The "*green*" dividends are divisible by the divisors and the "*red*" dividends are not divisible by the divisors. Last time when she used the software, she clicked on one of the dividends, because she was not satisfied with the generated dividend, and the program then generated another. Today, she is satisfied with the generated dividends. She opens the revision questions that she is going to use before the learners arrive.

The bell rings and the learners arrive. Twenty learners listen in silence while Ms Robertson revises the divisibility rules. Each learner sits in front of a computer that is equipped with an eye-tracker. All the computers are connected to a network. After the explanations, Ms Robertson instructs the learners to do the revision questions on the computer and she helps them with the first two questions. Then the following question appears on the screen.

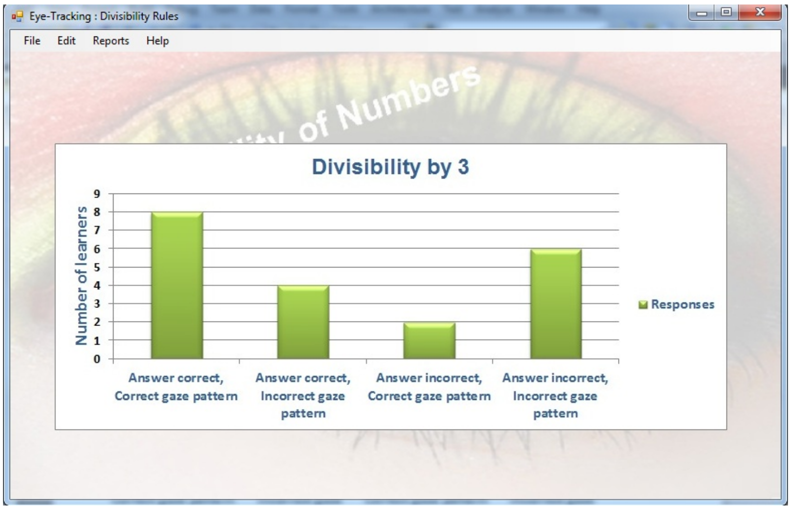


Each learner inspects the question and click “*yes*” or “*no*” to respond. Each computer analyses the eye-tracking recordings and transfers the calculated percentage of fixation time on each digit of the dividend, together with a jpeg file of the fixations, to the teacher’s computer.

Ms Robertson chooses the following option on her screen.

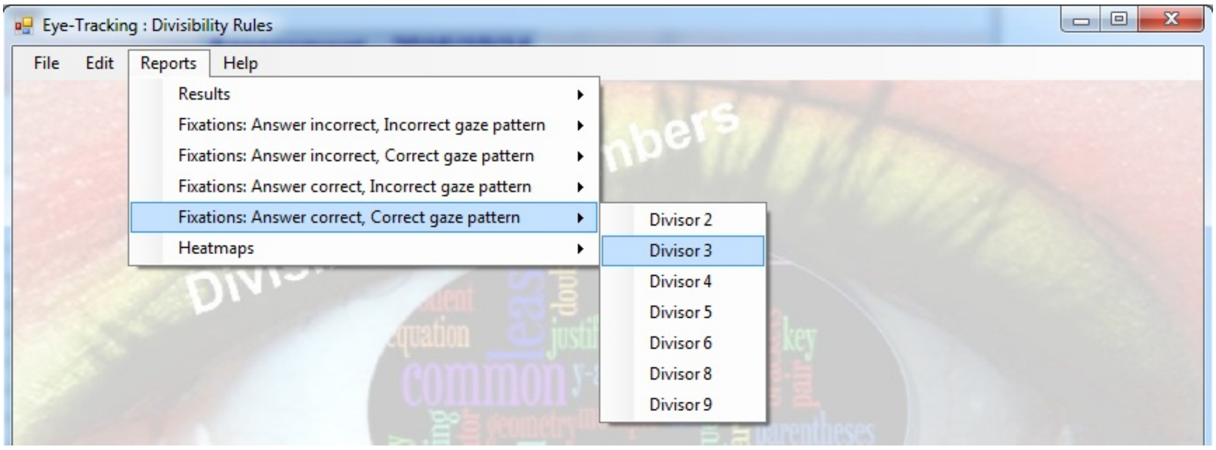


Similar to the graphs generated by clicker technologies, the following graph appears on her screen.

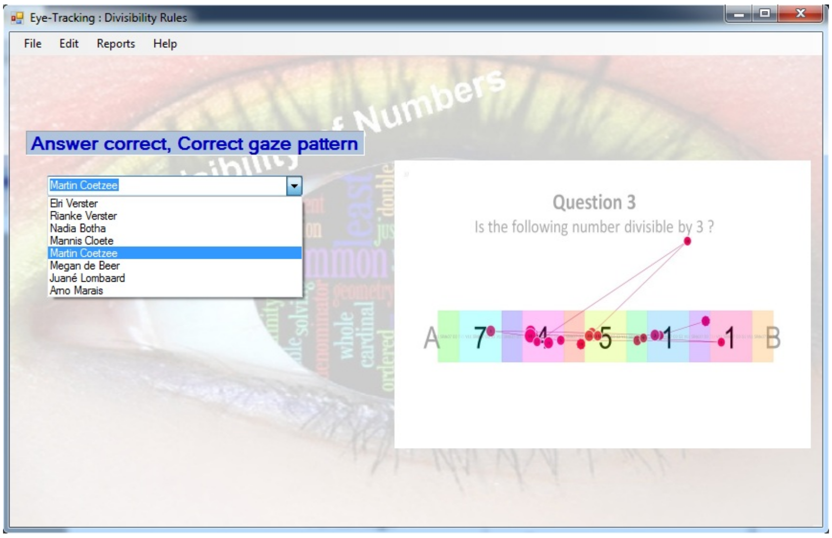


Conversations in class

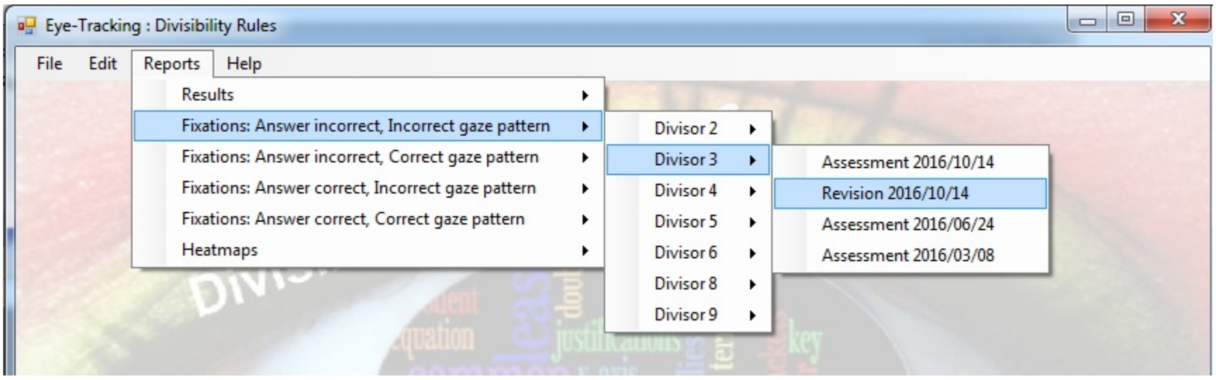
"There are many learners who provided an incorrect answer and/or used an incorrect strategy to solve the problem. I have to inspect their fixations on the digits", Ms Robertson thinks by herself while inspecting the graph. She chooses the following option on the menu:



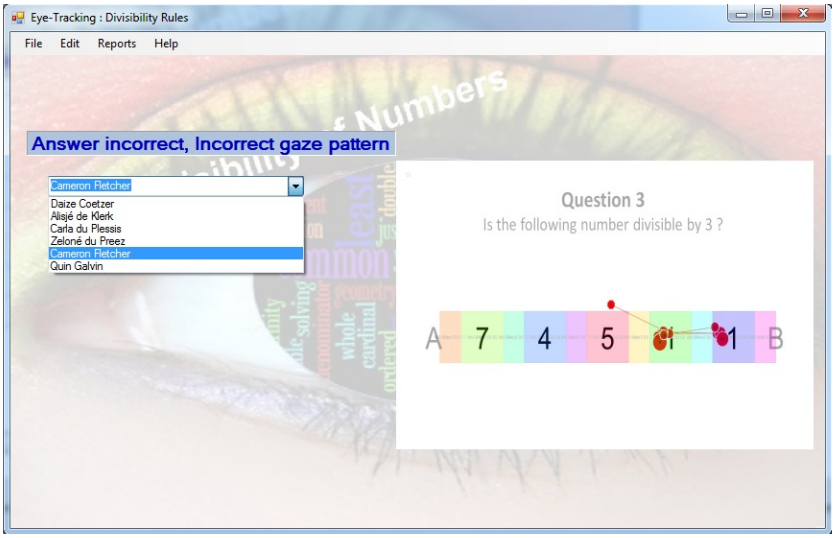
She checks the gaze patterns of some of the learners who provided a correct answer. She chooses Martin's name from the dropdown list and sees that he looked at all the digits.



"It looks fine to me", she thinks. She inspects more fixations of learners who had the answer correct and applied the correct problem solving strategy, and feels satisfied. She then chooses the following option on the menu:

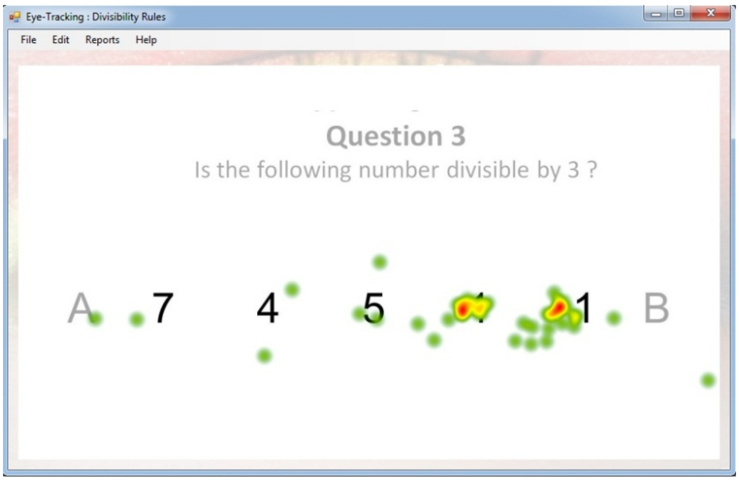


The names of learners who provided an incorrect answer and incorrect gaze pattern appear on the screen:



She clicks on Cameron's name, and the information associated with him is shown on her computer screen.

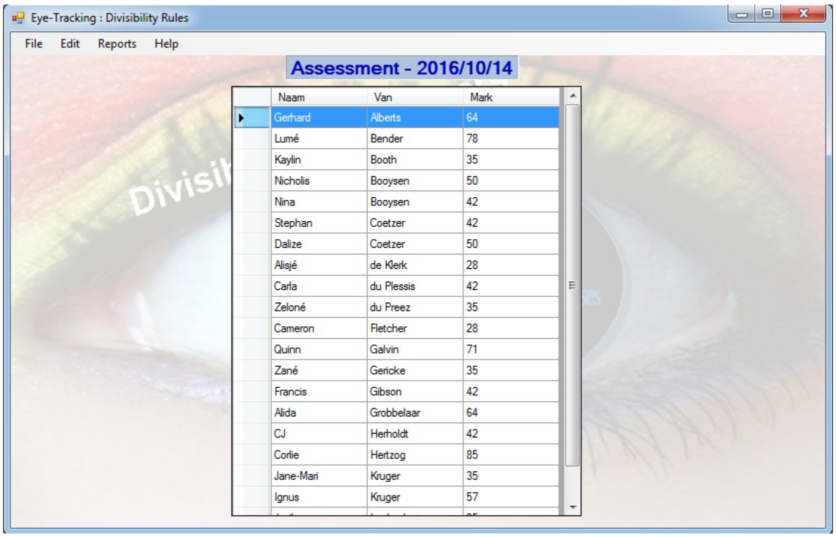
"Cameron, how did you get to your answer?" the teacher asks. "Ma'am, I saw that 11 is not divisible by three, therefore I reasoned that 74511 is not divisible by three", Cameron replies softly, because he was not paying attention when the teacher explained the divisibility rule of divisor 3. Some of the learners giggle behind their hands. "The rest of you do not have anything to giggle about. Look at this heatmap. Many of you inspected only the last two digits!" the teacher said. The heatmap appears on the big screen behind her.



“Let me repeat the rule again. To determine if a number is divisible by three, you add all the digits and decide if the sum is divisible by three." Rodney raises his hand and asks, “Ma’am, can you please show my fixations, because I added the digits but my answer is still wrong?” The teacher calls him to her desk and shows his fixations to him on her computer. She asks what the sum of the digits is. “The sum is 19 and it is not divisible by three”, Rodney says confident. “Yes, look at your fixations, it is correct, you fixated on all the digits, but you added incorrectly because the sum is 18 and not 19” the teacher explains.

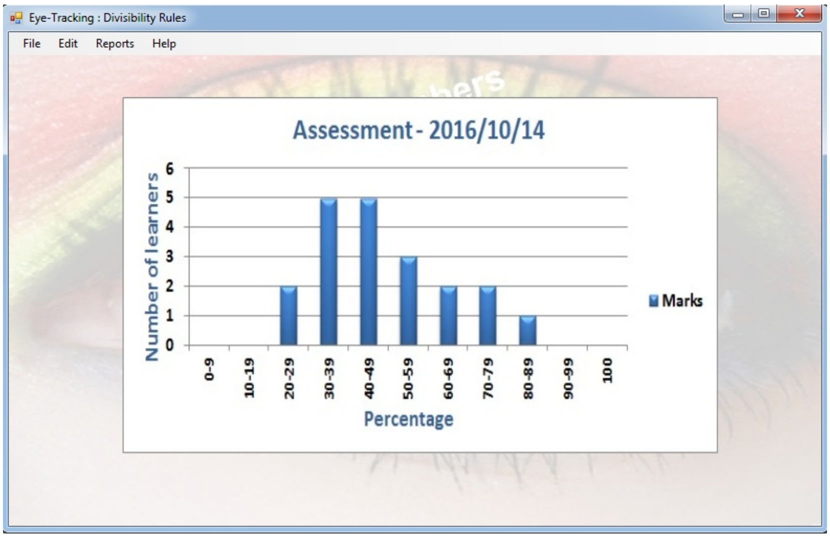
“Okay, now we are going to write a test. You can work at your own pace. As soon as you click on “*yes*” or “*no*”, the next question will appear. There are 12 questions. You will have 20 seconds per question to answer. Ready?”

The class do the assessment in silence - each learner at his/her own pace. After the test, the teacher inspects the responses again as she did during the revision session. A final progress report appears on her computer screen.



She looks at the results to determine which learners performed poorly. The teacher then analyses their fixations to determine if they guessed the answer or applied the divisibility rule incorrectly. She again explains to the learners the rules that they struggled with. When learners try to disagree with her about their fixations, she calls each one to her desk and displays the fixations on her computer to them and takes them through the process step-by-step to determine if a dividend is divisible by a certain divisor.

After the learners left the class, she inspects the distribution of the marks.



“Not acceptable at all. I have to explain the divisibility rules again tomorrow” she thinks by herself. She exits the computer program and shuts down the computer.

APPENDIX B

ANSWER SHEET FOR ASSESSMENT 1 AND ASSESSMENT 3

Surname :			**Name :**	
Grade :	**Section :**	**Age:**	**Gender**	**Male** / **Female**

Make a cross in the block of your choice.

Question 1	**Yes**	**No**	**Do not know**
Motivate your answer ____________________			
Question 2	**Yes**	**No**	**Do not know**
Motivate your answer ____________________			
Question 3	**Yes**	**No**	**Do not know**
Motivate your answer ____________________			
Question 4	**Yes**	**No**	**Do not know**
Motivate your answer ____________________			
Question 5	**Yes**	**No**	**Do not know**
Motivate your answer ____________________			
Question 6	**Yes**	**No**	**Do not know**
Motivate your answer ____________________			

Question 7	Yes	No	Do not know
Motivate your answer ____			
Question 8	Yes	No	Do not know
Motivate your answer ____			
Question 9	Yes	No	Do not know
Motivate your answer ____			
Question 10	Yes	No	Do not know
Motivate your answer ____			
Question 11	Yes	No	Do not know
Motivate your answer ____			
Question 12	Yes	No	Do not know
Motivate your answer ____			
Question 13	Yes	No	Do not know
Motivate your answer ____			
Question 14	Yes	No	Do not know
Motivate your answer ____			

APPENDIX C

ANSWER SHEET FOR ASSESSMENT 2 AND ASSESSMENT 4

Answer sheet used by researcher to record the learners' responses for Assessment 2. A similar answer sheet was used for Assessment 4.

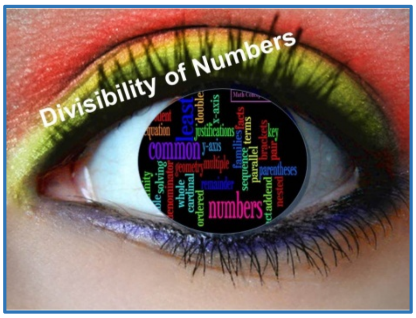
Eye-Tracking Assessment 2 – September 2015						
Name:					Grade	Section
No	Divisor	Number	Answer			Reason
1	2	44821	Yes	No	Don't know	
2	5	68140	Yes	No	Don't know	
3	3	74511	Yes	No	Don't know	
4	4	63172	Yes	No	Don't know	
5	6	52336	Yes	No	Don't know	
6	8	23960	Yes	No	Don't know	
7	9	10199	Yes	No	Don't know	
8	5	72154	Yes	No	Don't know	
9	8	46108	Yes	No	Don't know	
10	4	61254	Yes	No	Don't know	
11	2	73134	Yes	No	Don't know	
12	3	75133	Yes	No	Don't know	
13	6	45432	Yes	No	Don't know	
14	9	41346	Yes	No	Don't know	

APPENDIX D

ASSESSMENT 1

The PowerPoint presentation that was used for School B during Assessment 1. The same layout was used for Assessment 3. The same PowerPoint presentation was used for School A but translated into Afrikaans.

1



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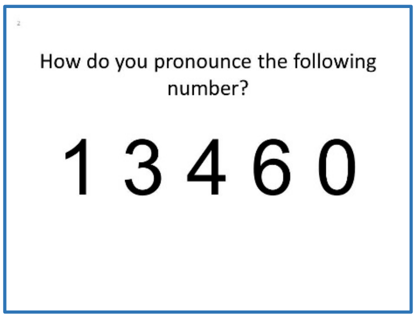

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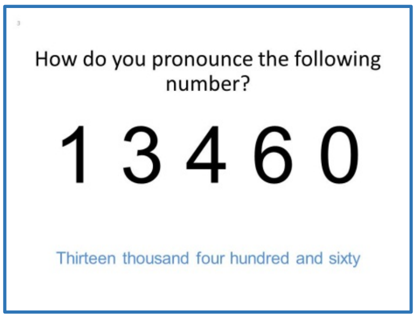

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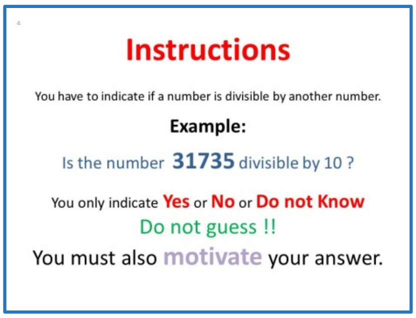

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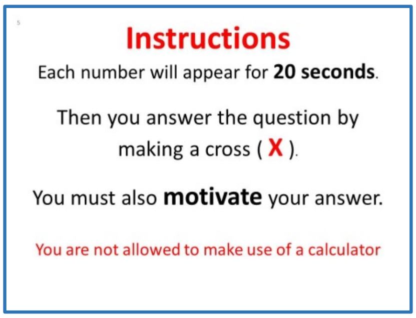

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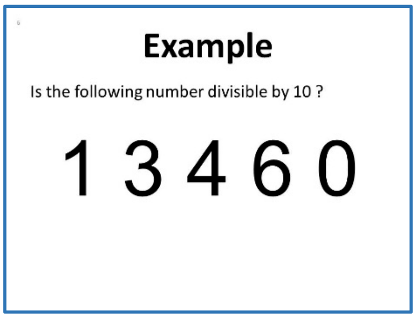

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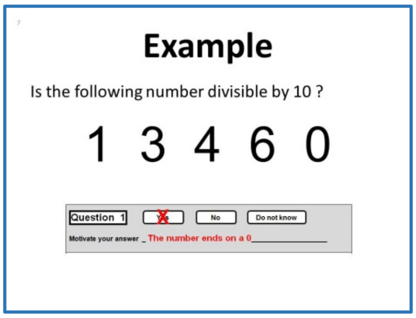

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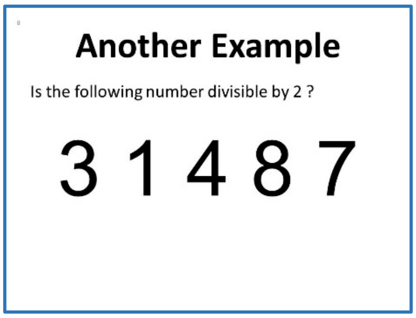

9

Another Example

Is the following number divisible by 2 ?

3 1 4 8 7

Question 2	Yes	X	Do not know
Motivate your answer	The last digit is an odd number *or* *The last digit is not divisible by 2*		

10

Are you ready?

11

Question 1

Is the following number divisible by 5 ?

12

Question 1

Is the following number divisible by 5 ?

7 2 1 5 4

13

Question 1

Is the following number divisible by 5 ?

Provide your answer for **Question 1**.

Also **motivate** your answer

14

Question 2

Is the following number divisible by 2 ?

15

Question 2

Is the following number divisible by 2 ?

7 3 1 3 4

16

Provide your answer for question 2

17

Question 3

Is the following number divisible by 3 ?

18

Question 3

Is the following number divisible by 3 ?

7 5 1 3 3

19

Provide your answer for question 3

20

Question 4
Is the following number divisible by 9 ?

21

Question 4
Is the following number divisible by 9 ?

1 0 1 9 9

22

Provide your answer for question 4

23

Question 5
Is the following number divisible by 4 ?

24

Question 5
Is the following number divisible by 4 ?

6 3 1 7 2

25

Provide your answer for question 5

26

Question 6
Is the following number divisible by 8 ?

27

Question 6
Is the following number divisible by 8 ?

4 6 1 0 8

28

Provide your answer for question 6

29

Question 7
Is the following number divisible by 6 ?

30

Question 7
Is the following number divisible by 6 ?

5 2 3 3 6

31

Provide your answer for question 7

32

Question 8
Is the following number divisible by 4 ?

33

Question 8
Is the following number divisible by 4 ?

6 1 2 5 4

34

Provide your answer for question 8

35

Question 9
Is the following number divisible by 2 ?

36

Question 9
Is the following number divisible by 2 ?

4 4 8 2 1

37

Provide your answer for question 9

38

Question 10
Is the following number divisible by 5 ?

39

Question 10
Is the following number divisible by 5 ?

6 8 1 4 0

40

Provide your answer for question 10

41

Question 11
Is the following number divisible by 6 ?

42

Question 11
Is the following number divisible by 6 ?

4 5 4 3 2

43

Provide your answer for question 11

44

Question 12
Is the following number divisible by 3 ?

45

Question 12
Is the following number divisible by 3 ?

7 4 5 1 1

46

Provide your answer for question 12

47

Question 13
Is the following number divisible by 9 ?

48

Question 13
Is the following number divisible by 9 ?

4 1 3 4 6

49

Provide your answer for question 13

50

Question 14

Is the following number divisible by 8 ?

51

Question 14

Is the following number divisible by 8 ?

2 3 9 6 0

52

Provide your answer for question 14

53

APPENDIX E

DIVISORS AND DIVIDENDS FOR ASSESSMENTS

Question Number	Assessment 1			Assessment 2			Assessment 3			Assessment 4		
	Divisor	Dividend	Answer	Divisor	Dividend	Answer	Divisor	Dividend	Answer	Divisor	Dividend	Answer
1	5	72154	×	2	44821	×	5	68140	✓	2	73134	✓
2	2	73134	✓	5	68140	✓	2	44821	×	5	72154	×
3	3	75133	×	3	74511	✓	9	41346	✓	4	61254	×
4	9	10199	×	4	63172	✓	4	61254	×	6	45432	✓
5	4	63172	✓	6	52336	×	3	75133	×	8	46108	×
6	8	46108	×	8	23960	✓	8	23960	✓	3	75133	×
7	6	52336	×	9	10199	×	6	45432	✓	9	41346	✓
8	4	61254	×	5	72154	×	2	73134	✓	5	68140	✓
9	2	44821	×	8	46108	×	5	72154	×	2	44821	×
10	5	68140	✓	4	61254	×	4	63172	✓	9	10199	×
11	6	45432	✓	2	73134	✓	9	10199	×	3	74511	✓
12	3	74511	✓	3	75133	×	8	46108	×	4	63172	✓
13	9	41346	✓	6	45432	✓	6	52336	×	8	23960	✓
14	8	23960	✓	9	41346	✓	3	74511	✓	6	52336	×

Key: ✓ **:** Dividend divisible by divisor. × **:** Dividend not divisible by divisor

APPENDIX F

QUESTIONNAIRE FOR MATHEMATICS TEACHERS

Grade 7

Teacher name: ______________________________

Cell phone number: ______________________________

e-mail: ______________________________

Number of learners in Grade 7: ____________________

Learners in Grade 7 have already encountered the following divisibility rules in previous years.

2 ☐ 3 ☐ 4 ☐ 5 ☐ 6 ☐ 7 ☐ 8 ☐ 9 ☐ 10 ☐

The following divisibility rules are discussed in Grade 7 for the first time.

2 ☐ 3 ☐ 4 ☐ 5 ☐ 6 ☐ 7 ☐ 8 ☐ 9 ☐ 10 ☐

Please list the advantages of knowledge of divisibility rules for learners:

____

____

____

____

Thank you for your time.

Pieter

Follow-up questionnaire for teachers of School B.

Grade ________

The following divisibility rules were discussed during 2015 (please tick the numbers):

2 ☐ 3 ☐ 4 ☐ 5 ☐ 6 ☐ 7 ☐ 8 ☐ 9 ☐ 10 ☐

During which month did you discuss the divisibility rules? _______________

Please provide examples of how you tested the learners' knowledge of divisibility rules?

____

____

____

____

____

____

Why do you think learners did not perform well with the test(s) on divisibility rules?

____

____

____

Thank you for your time.

Pieter

APPENDIX G

ASSESSMENT 2

Stimuli used for School B during Assessment 2. The same layout was used for Assessment 4. The same stimuli were used for School A, but translated into Afrikaans.

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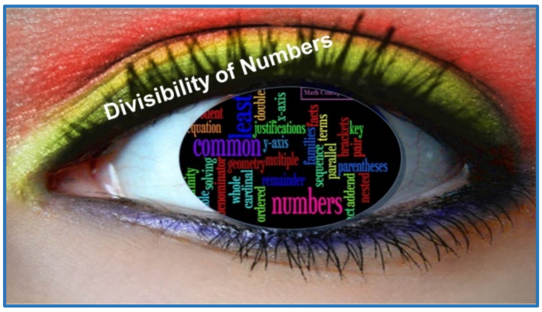

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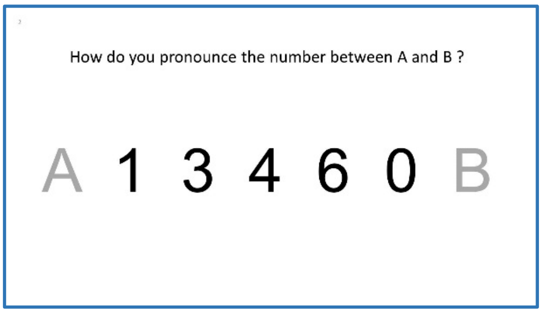

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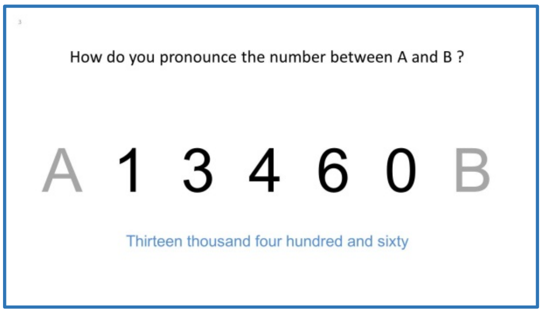

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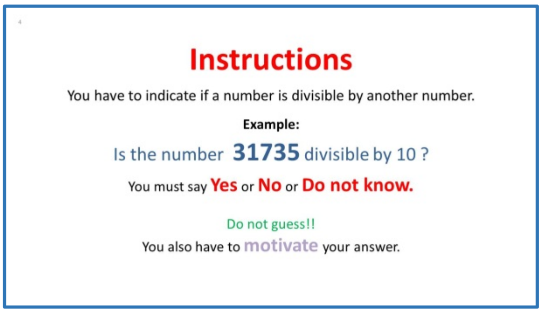

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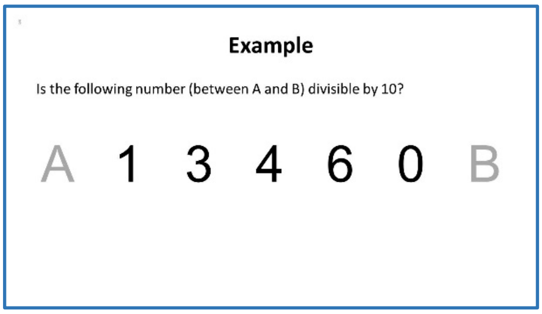

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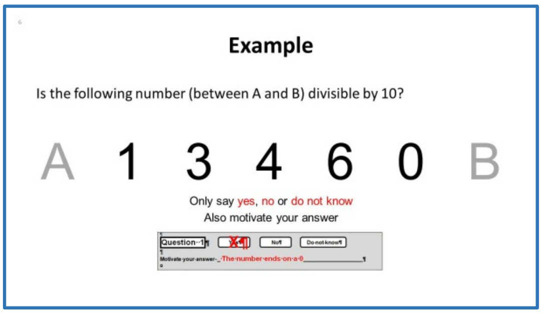

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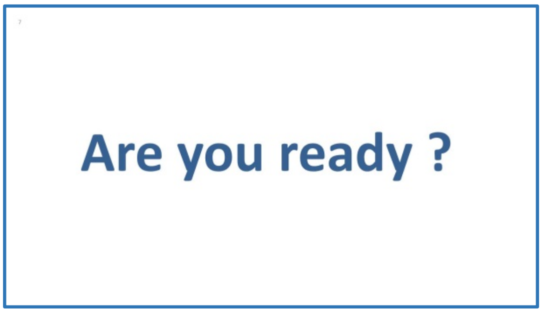

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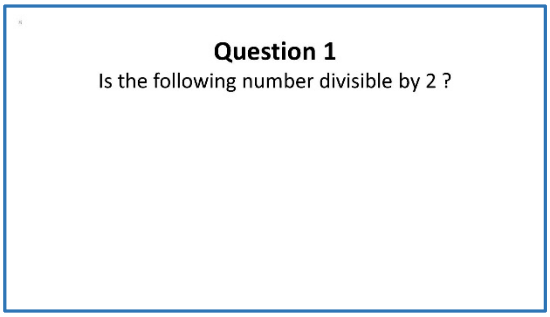

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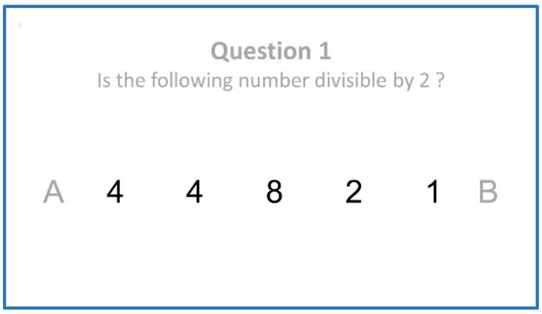

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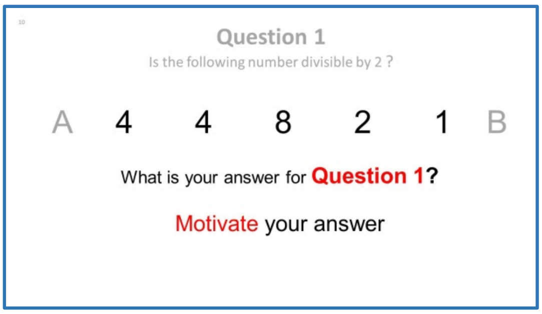

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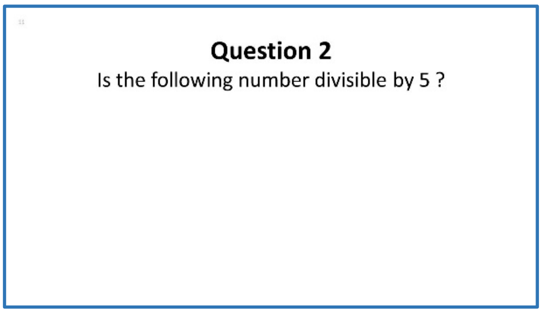

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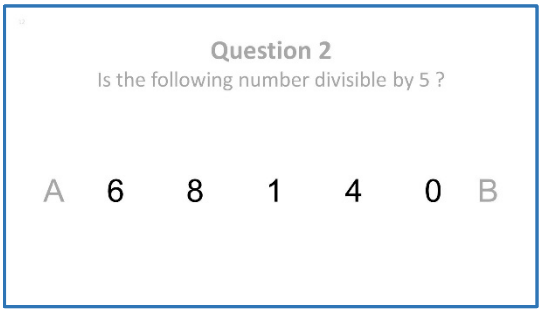

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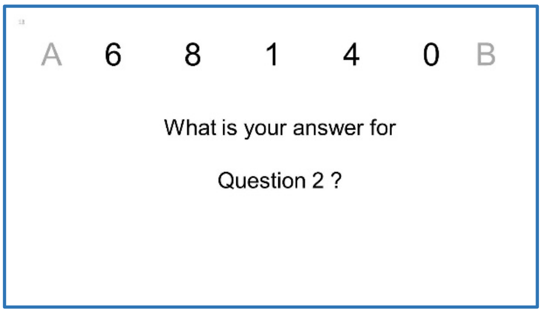

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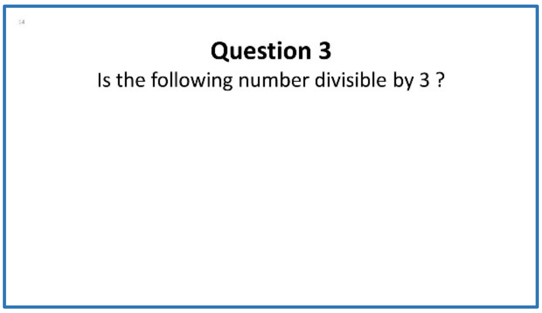

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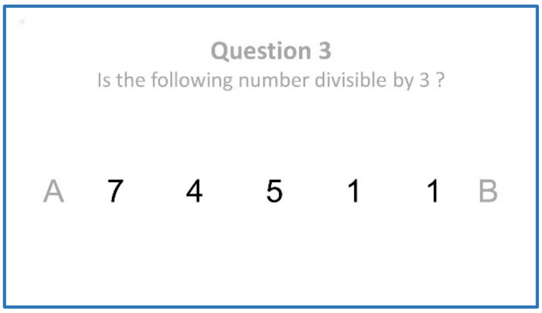

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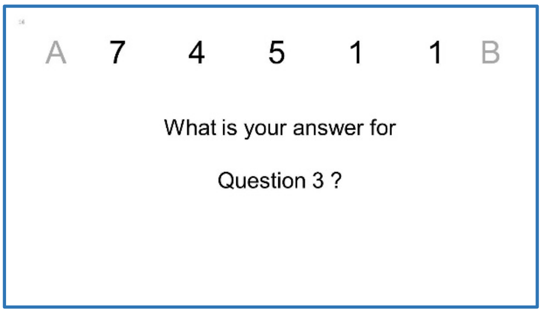

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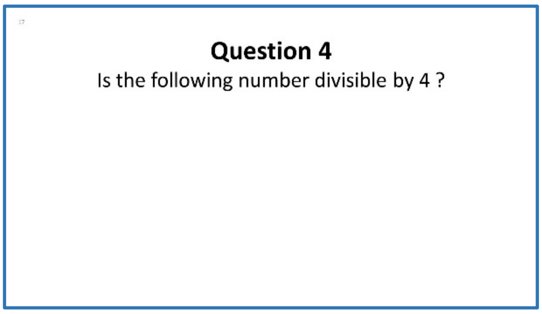

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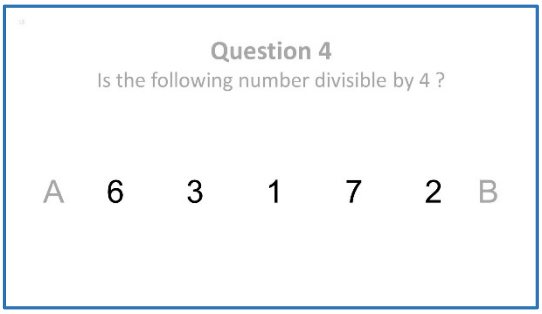

19

A 6 3 1 7 2 B

What is your answer for

Question 4 ?

20

Question 5

Is the following number divisible by 6 ?

21

Question 5

Is the following number divisible by 6 ?

A 5 2 3 3 6 B

22

A 5 2 3 3 6 B

What is your answer for

Question 5?

23

Question 6

Is the following number divisible by 8 ?

24

Question 6

Is the following number divisible by 8 ?

A 2 3 9 6 0 B

25

A 2 3 9 6 0 B

What is your answer for

Question 6?

26

Question 7

Is the following number divisible by 9 ?

27

Question 7

Is the following number divisible by 9 ?

A 1 0 1 9 9 B

28

A 1 0 1 9 9 B

What is your answer for

Question 7 ?

29

Question 8

Is the following number divisible by 5 ?

30

Question 8

Is the following number divisible by 5 ?

A 7 2 1 5 4 B

31

A 7 2 1 5 4 B

What is your answer for

Question 8 ?

32

Question 9

Is the following number divisible by 8 ?

33

Question 9

Is the following number divisible by 8 ?

A 4 6 1 0 8 B

34

A 4 6 1 0 8 B

What is your answer for

Question 9 ?

35

Question 10

Is the following number divisible by 4 ?

36

Question 10

Is the following number divisible by 4 ?

A 6 1 2 5 4 B

37

A 6 1 2 5 4 B

What is your answer for

Question 10 ?

38

Question 11

Is the following number divisible by 2 ?

39

Question 11

Is the following number divisible by 2 ?

A 7 3 1 3 4 B

40

A 7 3 1 3 4 B

What is your answer for

Question 11 ?

41

Question 12

Is the following number divisible by 3 ?

42

Question 12

Is the following number divisible by 3 ?

A 7 5 1 3 3 B

43

A 7 5 1 3 3 B

What is your answer for

Question 12 ?

44

Question 13

Is the following number divisible by 6 ?

45

Question 13

Is the following number divisible by 6 ?

A 4 5 4 3 2 B

46

A 4 5 4 3 2 B

What is your answer for

Question 13 ?

47

Question 14

Is the following number divisible by 9 ?

48

Question 14

Is the following number divisible by 9 ?

A 4 1 3 4 6 B

49

A 4 1 3 4 6 B

What is your answer for

Question 14 ?

50

51

Thank you for your participation

(F10 to end)

APPENDIX H

REVISION LESSON

The PowerPoint presentation that was used for School B during the revision session. The same PowerPoint presentation was translated into Afrikaans for School A.

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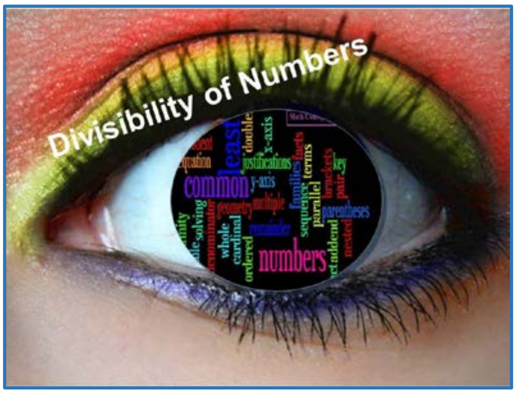

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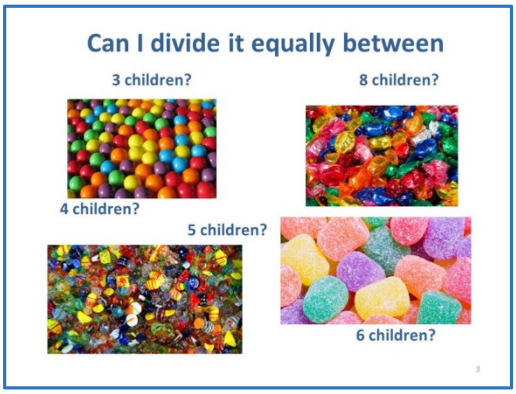

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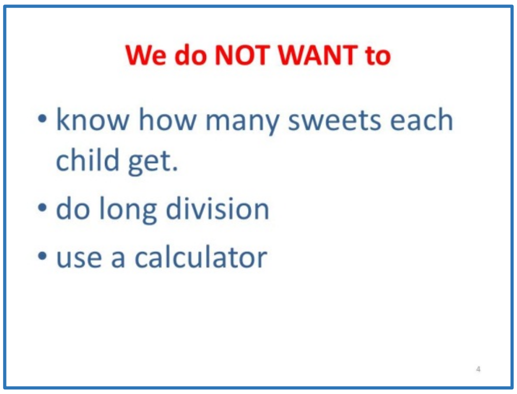

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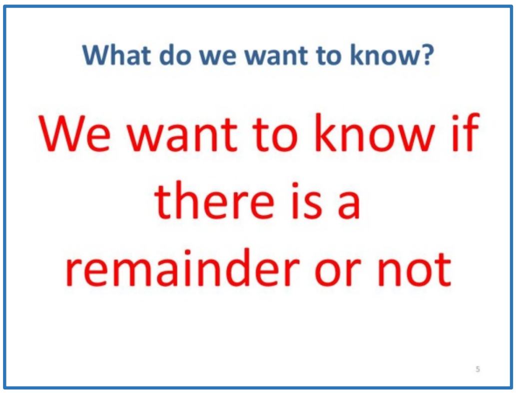

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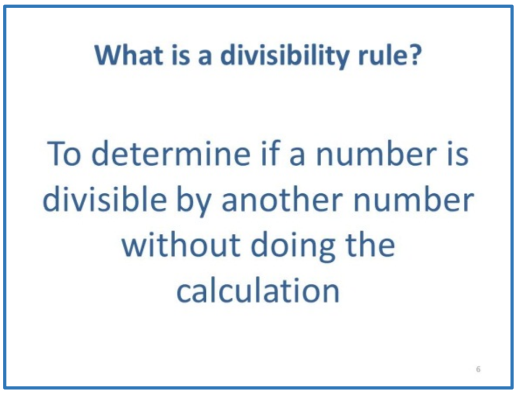


7

Where do we use divisibility rules?

- Is 3 a factor of 38241 ?
- Simplify : $\frac{2637}{5214}$
- Is 42853 a prime number?

8

Divisibility rule for 2

2		

9

Divisibility rule for 2

2	164	
4		
6		
8		
10		
12		
14		
16		
18		
20		
22		

10

Divisibility rule for 2

2	164	3250
4	166	3252
6	168	3254
8	170	3256
10	172	3258
12	174	3260
14	176	3262
16	178	3264
18	180	3266
20	182	3268
22	184	3270

11

Divisibility rule for 2

2	164	3250
4	166	3252
6	168	3254
8	170	3256
10	172	3258
12	174	3260
14	176	3262
16	178	3264
18	180	3266
20	182	3268
22	184	3270

12

Divisibility rule for 2

The last digit has to be an even number.

(0,2,4,6,8)

13

Divisibility rule for 5

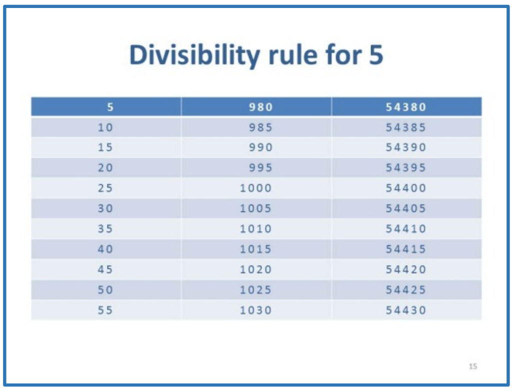
5		

14

Divisibility rule for 5

5	980	
10		
15		
20		
25		
30		
35		
40		
45		
50		
55		

15



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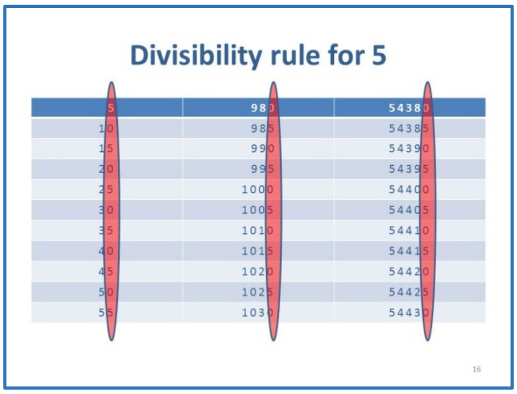

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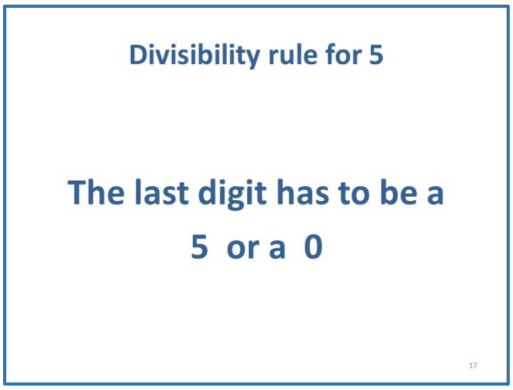

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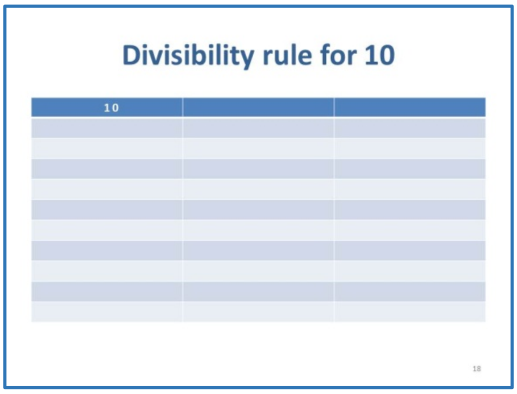

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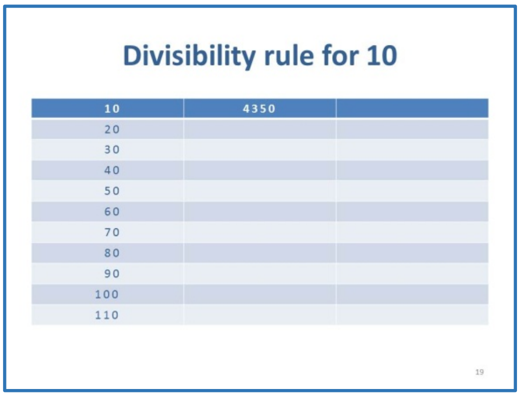

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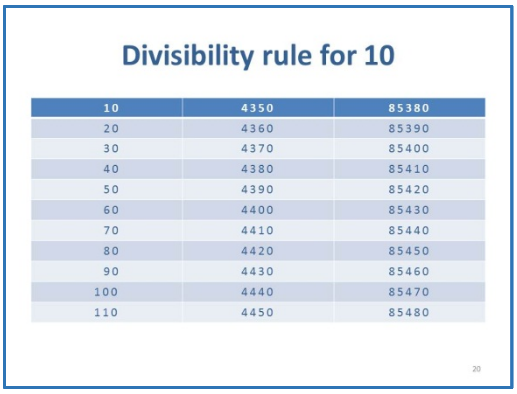

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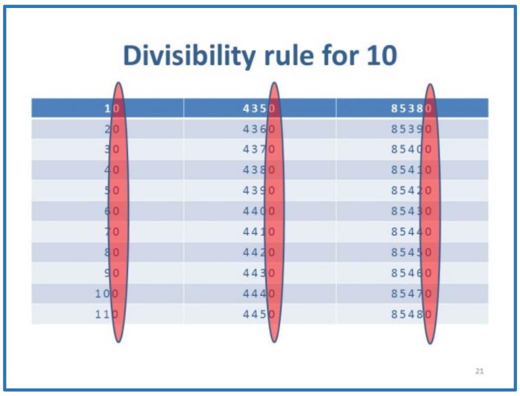

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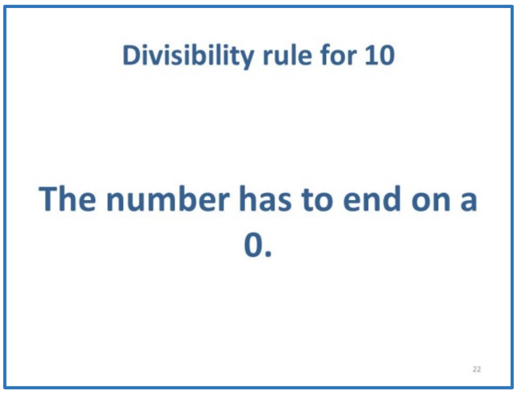

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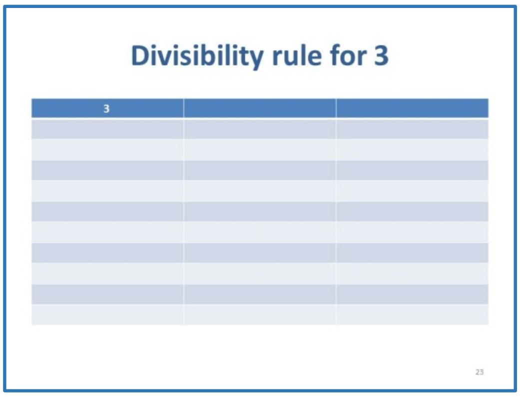
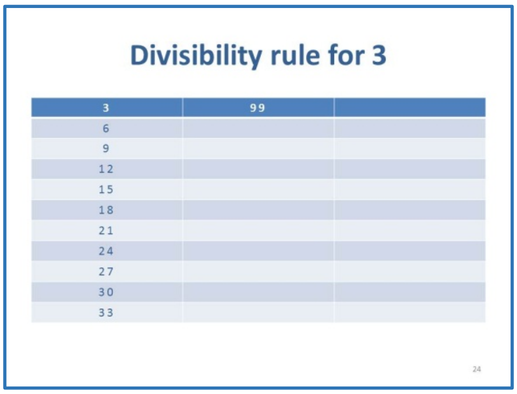


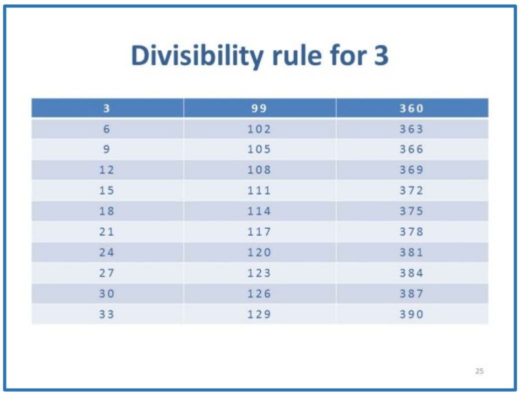
3		

24



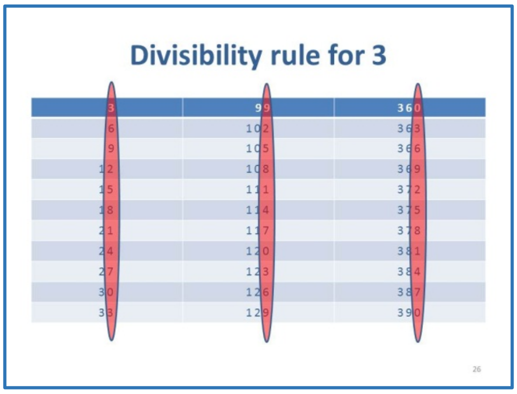
3	99	
6		
9		
12		
15		
18		
21		
24		
27		
30		
33		

25



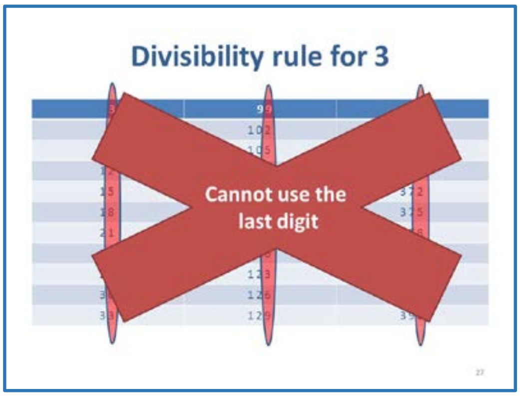
3	99	360
6	102	363
9	105	366
12	108	369
15	111	372
18	114	375
21	117	378
24	120	381
27	123	384
30	126	387
33	129	390

26



3	99	360
6	102	363
9	105	366
12	108	369
15	111	372
18	114	375
21	117	378
24	120	381
27	123	384
30	126	387
33	129	390

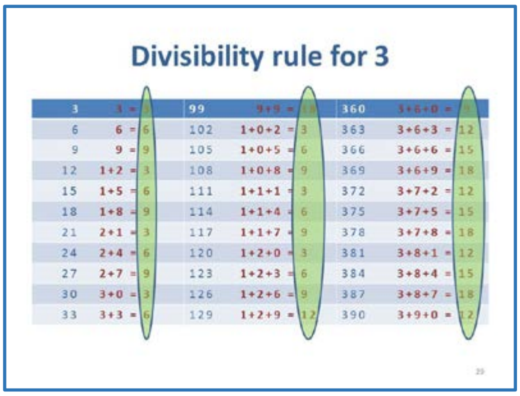
27



28

3	3 = 3	99	9+9 = 18	360	3+6+0 = 9
6	6 = 6	102	1+0+2 = 3	363	3+6+3 = 12
9	9 = 9	105	1+0+5 = 6	366	3+6+6 = 15
12	1+2 = 3	108	1+0+8 = 9	369	3+6+9 = 18
15	1+5 = 6	111	1+1+1 = 3	372	3+7+2 = 12
18	1+8 = 9	114	1+1+4 = 6	375	3+7+5 = 15
21	2+1 = 3	117	1+1+7 = 9	378	3+7+8 = 18
24	2+4 = 6	120	1+2+0 = 3	381	3+8+1 = 12
27	2+7 = 9	123	1+2+3 = 6	384	3+8+4 = 15
30	3+0 = 3	126	1+2+6 = 9	387	3+8+7 = 18
33	3+3 = 6	129	1+2+9 = 12	390	3+9+0 = 12

29



3	3 = 3	99	9+9 = 18	360	3+6+0 = 9
6	6 = 6	102	1+0+2 = 3	363	3+6+3 = 12
9	9 = 9	105	1+0+5 = 6	366	3+6+6 = 15
12	1+2 = 3	108	1+0+8 = 9	369	3+6+9 = 18
15	1+5 = 6	111	1+1+1 = 3	372	3+7+2 = 12
18	1+8 = 9	114	1+1+4 = 6	375	3+7+5 = 15
21	2+1 = 3	117	1+1+7 = 9	378	3+7+8 = 18
24	2+4 = 6	120	1+2+0 = 3	381	3+8+1 = 12
27	2+7 = 9	123	1+2+3 = 6	384	3+8+4 = 15
30	3+0 = 3	126	1+2+6 = 9	387	3+8+7 = 18
33	3+3 = 6	129	1+2+9 = 12	390	3+9+0 = 12

30

3003	3+0+0+3 = 6	60699	6+0+6+9+9 = 18
3006	3+0+0+6 = 9	60702	6+0+7+0+2 = 15
3009	3+0+0+9 = 12	60705	6+0+7+0+5 = 18
3012	3+0+1+2 = 6	60708	6+0+7+0+8 = 21
3015	3+0+1+5 = 9	60711	6+0+7+1+1 = 15
3018	3+0+1+8 = 12	69333	6+9+3+3+3 = 24
3021	3+0+2+1 = 6	69993	6+9+9+9+3 = 36
3024	3+0+2+4 = 9	69999	6+9+9+9+9 = 42
3027	3+0+2+7 = 12	70002	7+0+0+0+2 = 9
3030	3+0+3+0 = 6	86211	8+6+2+1+1 = 18
3033	3+0+3+3 = 9	92220	9+2+2+2+0 = 15

31

Divisibility rule for 3

The sum of the digits has to be divisible by 3

32

Revision of 2,3,5 & 10

- 285
- 46112
- 87531

33

Divisibility rule for 9

What do you think is the divisibility rule for 9?

34

Divisibility rule for 9

9	9 = 9	999	9+9+9 = 27
18	1+8 = 9	1008	1+0+0+8 = 9
27	2+7 = 9	1017	1+0+1+7 = 9
36	3+6 = 9	1026	1+0+2+6 = 9
45	4+5 = 9	81099	8+1+0+9+9 = 27
54	5+4 = 9	99216	9+9+2+1+6 = 27
63	6+3 = 9	42417	4+2+4+1+7 = 18
72	7+2 = 9	65124	6+5+1+2+4 = 18
81	8+1 = 9	31041	3+1+0+4+1 = 9
90	9+0 = 9	19989	1+9+9+8+9 = 36
99	9+9 = 18	10008	1+0+0+0+8 = 9

35

Divisibility rule for 9

The sum of the digits has to be divisible by 9

36

Divisibility rule for 6

6 = 2 x 3

6 has 2 rules

The number has to be divisible by 2 and 3.

37

APPLY BOTH RULES

38

Divisibility rule for 6

The last digit has to be an even number (0,2,4,6,8)

The sum of the digits has to be divisible by 3

39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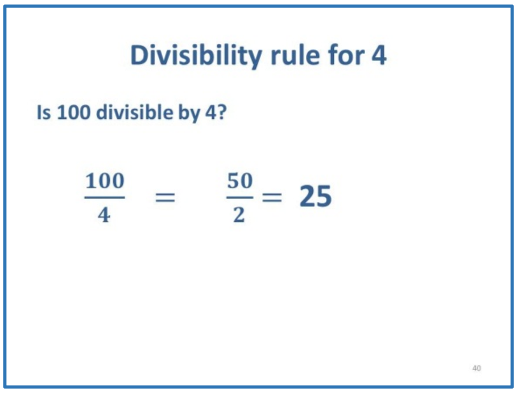

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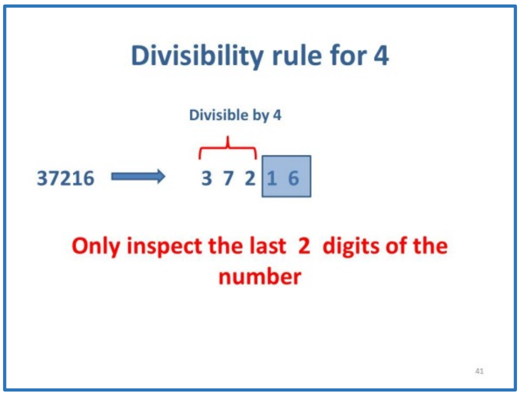

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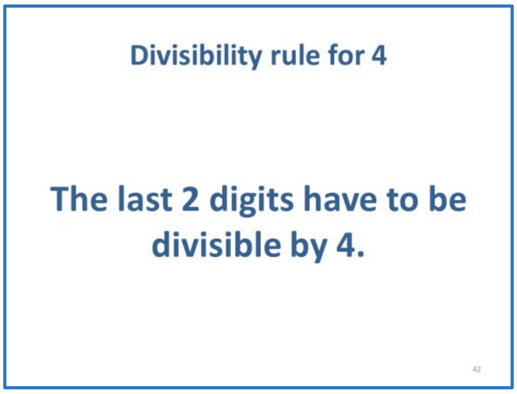

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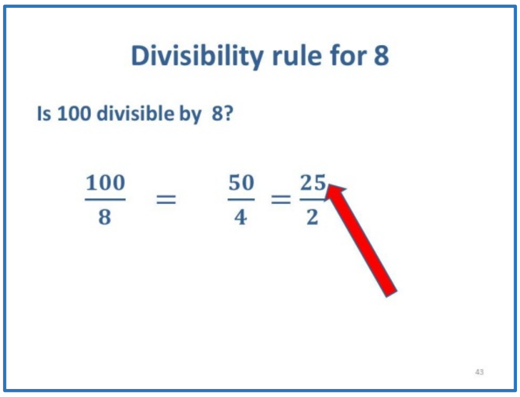

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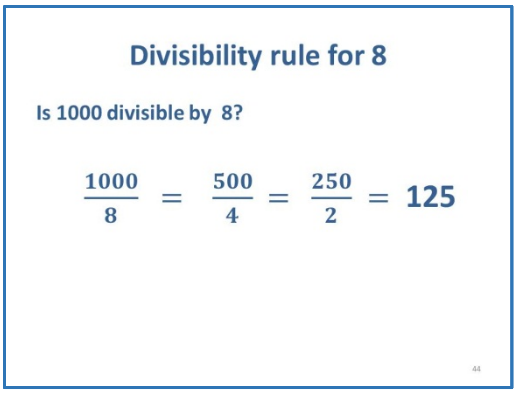

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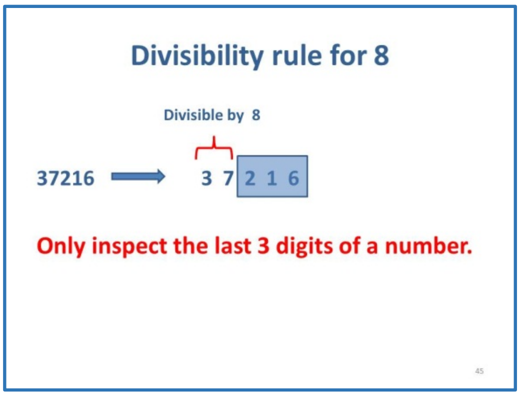

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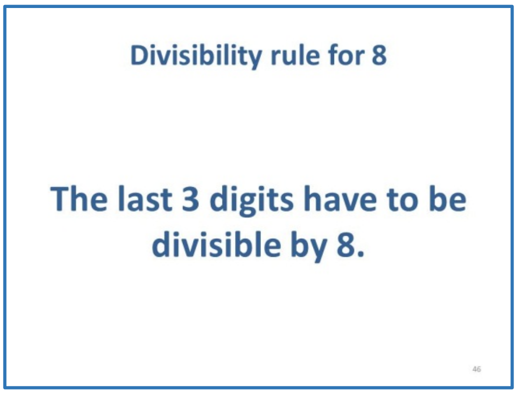

47

How to remember rules for 2,4 and 8

	2	4	8
	2	2x2	2x2x2
How many 2's	1	2	3
	Inspect last digit	Inspect last 2 digits	Inspect last 3 digits

47

48

Revision

- 2, 5 and 10 – Inspect only the LAST digit
- 3, 6 and 9 – Calculate the sum of ALL digits
  6 also have to be an EVEN number
  CALCULATE SUM ONLY WITH
  3, 6 and 9
- 4 – Inspect LAST 2 digits
- 8 – Inspect LAST 3 digits

48

49

Provide a value for the last digit

4 2 1 7 ?

49

50

Summary

2	If the last digit is an even number. (0,2,4,6,8)
3	If the sum of the digits is divisible by 3, the whole number is also divisible by 3
4	If the number made by the last two digits is divisible by 4, the whole number is also divisible by 4
5	If the last digit is a 5 or a 0, the number is divisible by 5
6	If the number is divisible by both 3 and 2, it is also divisible by 6
8	If the number made by the last three digits is divisible by 8, the whole number is also divisible by 8
9	If the sum of all the digits is divisible by 9, the number is also divisible by 9
10	If the number ends in 0, it is divisible by 10

50

51

APPENDIX I

REQUEST FOR PERMISSION TO CONDUCT A RESEARCH STUDY AT A SCHOOL

UNIVERSITY OF THE FREE STATE
UNIVERSITEIT VAN DIE VRYSTAAT
YUNIVESITHI YA FREISTATA

UFS·UV
NATURAL AND AGRICULTURAL SCIENCES
NATUUR- EN LANDBOUWETENSKAPPE

27 Augustus 2014

Die Skoolhoof: Laerskool Fichardtpark

Geagte Meneer du Preez

Ek het 'n PhD student, mnr Pieter Potgieter, wat besig is met 'n navorsingsprojek wat handel oor Wiskunde en "eye-tracking". "Eye-tracking" word gebruik om te bepaal waarna 'n persoon kyk op die rekenaar. Volgens die Departement van Basiese Onderwys is faktorisering as een van die probleemareas by Graad 9 geïdentifiseer. Een manier om te help met faktorisering is om die delingreëls te ken. Delingreëls word gebruik om te bepaal of 'n getal deelbaar is deur 'n ander getal sonder om dit fisies uit te werk. Byvoorbeeld, indien 'n getal op 'n ewe getal eindig is dit deelbaar deur 2. Hy wil graag die delingreëls by die leerlinge toets. Leerlinge ken die delingreëls van 2, 5 en 10, maar die syfers 3, 4, 6, 8 en 9 is nie deel van die leerplan nie (7 word uitgelaat omdat dit meer kompleks is). Hy sal graag leerders (Graad 4 tot Graad 7) wil gebruik om aan die studie deel te neem.

Die beplande prosedure is soos volg:

'n Vergadering word gehou met al die Wiskunde onderwysers vanaf Graad 4 tot 7. Met 'n informele vraelys word vasgestel watter van die delingreëls in die klas behandel word. Inligting met betrekking tot die leerders se deelname sal ook met hulle bespreek word.

Al die leerders in Graad 4 tot Graad 7 skryf op 'n spesifieke dag, soos gereël met die onderwysers, 'n assesseringstoets op papier oor die delingreëls. Hierdie toets word gedurende 'n periode deur die onderwysers gedoen. Die vrae word met 'n dataprojektor vertoon en hulle skryf dan die antwoorde op papier. Daar is 14 vrae wat aan die leerders gestel word. Byvoorbeeld: "Is 27564 deelbaar deur 3?". Die leerders moet dan net 'n korrekte antwoord (ja, nee of weet nie) merk. Onderaan hierdie dokument is 'n voorbeeld van die antwoordblad. By elke vraag kry die leerders slegs 20 sekondes om 'n opsie te kies en dan moet hulle aandui hoekom hulle so sê. Die hele toets behoort nie langer as 15 minute in beslag te neem nie.

Die antwoordstelle word dan deur hom gemerk en dan word 20 leerders per graad (totaal 80 leerders) gekies om aan die "eye-tracking" deel te neem. Dit sal leerders wees wat minimum 30% vir die toets behaal het, maar nie meer as 50% nie. Tye sal dan bepaal word wanneer

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hierdie leerders met die "eye-tracking" getoets gaan word. Daar is reeds 'n proeflopie gehou en dit neem ongeveer 10 minute per leerder. Dus sal leerders slegs vir omtrent 10 minute uit die klas onttrek word. Die leerders word een-een getoets.

Op 'n latere stadium word die delingreëls dan aan al die leerders verduidelik. Hier sal hy dan graag self die klas wil behartig vir al die leerlinge. Die klas behoort so 20 minute te wees, waarna die leerders weer 'n papiergebaseerde toets sal aflê. Al die leerders wat betrokke was by die "eye-tracking" (80 leerders), sal dan weer die "eye-tracking" toets aflê.

Hy het die ANA uitslae van 2012 en 2013 vanaf die internet gekry, maar dit is per provinsie asook nasionaal. Dit sal waardeer word indien 2012 asook 2013 se ANA uitslae van Fichardtpark gekry kan word indien beskikbaar.

Voorgestelde datums:

- Mnr Potgieter het 'n afspraak met u gemaak vir eerskomende Vrydag, 29 Augustus 2014 om 11:30.
- 'n Vergadering word gehou met al die Wiskunde onderwysers op Dinsdag, 2 September 2014 direk na skool. Al die benodigde materiaal, soos antwoordskrifte en PowerPoint, sal aan die onderwysers voorsien word.
- Die voorgestelde datum om die skriftelike toets te doen is 4 of 5 September 2014. Dit word gefasiliteer deur onderwysers. Die toets is 'n PowerPoint aanbieding met die vrae waar leerders slegs die antwoorde op papier skryf (± 15 minute).
- 20 Leerders per graad word geïdentifiseer. Hierdie leerders word dan getoets met "eye-tracking" op 8 en 9 September 2014. Die leerders word een vir een uit klasse onttrek vir die toets. Die toets is ongeveer 10 minute per leerder.
- In die eerste week na die Oktober vakansie word die delingreëls aan die leerlinge verduidelik. Die klas word deur mnr Potgieter behartig. Dit behoort 20 minute te neem. Direk hierna skryf hierdie leerders weer 'n toets op papier, om te bepaal of hulle die delingreëls kan toepas ('n dubbel-periode sal voldoende wees om die reëls te verduidelik asook om toets af te lê.)
- In die daaropvolgende week word dieselfde leerders wat aan die "eye-tracking" deelgeneem het, weer getoets met die "eye-tracking". Dieselfde prosedure word gevolg soos voorheen.

Ek sal dit waardeer indien u hierdie projek kan ondersteun, aangesien die leerders ook gaan baat vind by die projek. Die delingreëls is veral bruikbaar waar leerders te doen kry met die vereenvoudiging van breuke, ekwivalente breuke, bewerkings met breuke asook bewerkings met priemgetalle.

Baie dankie

____

PROF PJ BLIGNAUT

AKADEMIESE DEPARTEMENTSHOOF

UNIVERSITY OF THE FREE STATE
UNIVERSITEIT VAN DIE VRYSTAAT
YUNIVESITHI YA FREISTATA
UFS·UV
NATURAL AND AGRICULTURAL SCIENCES
NATUUR- EN LANDBOUWETENSKAPPE

16 March 2015

The Principal: Botlehadi Primary School

Dear Mr Mlamleli

I have a PhD student, Mr Pieter Potgieter, who is busy with a research project that involves Mathematics and eye-tracking. Eye-tracking is used to determine where a person is looking on a computer screen.

The Department of Basic Education has identified factorization as a problem area for Grade 9 learners. Knowing the divisibility rules are one way to help with factorization. Divisibility rules are used to determine if a number is divisible by another number without doing the actual calculation. For example, if a number is an even number, it is divisible by 2. The divisibility rules are done in Grade 5 and revision is done in Grade 6 and Grade 7. According to the text book, the divisibility rules are covered during the second term. The research project involves only the divisibility rules of digits 2, 3, 4, 5, 6, 8 and 9 (7 is omitted because it is more complex).

Mr Potgieter wants to test the divisibility rules among the learners. He wants to involve learners from Grade 4 to Grade 7 to take part in the study. He has talked to Mrs Soato on Wednesday, 11 March about the project. He also wants to meet with you in person to discuss the project.

The planned procedure is as follows:

A meeting will be held with all the Mathematics teachers from Grade 4 to Grade 7. The purpose of this meeting is to identify when the divisibility rules are explained to the learners, and also to gather information about the class sizes, time tables, number of classes etc. Information regarding the participation of the learners will also be discussed with them.

All the learners in Grade 4 to Grade 7 will write on a specific date, as arranged with the teachers, an assessment on paper about the divisibility rules. This assessment will take place during the Mathematics periods and will be facilitated by the teachers. The assessment contains 14 questions, for example "Is 27564 divisible by 3?". The learners will have to indicate their responses (yes, no or don't know) together with a reason on an answer sheet. The learners will have only 20 seconds to answer each question. The test should not take longer than 15 minutes.

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Mr Potgieter will then mark the scripts and 20 learners per grade (total of 80 learners) will be identified to take part in the eye-tracking phase of the study. These will be learners who scored between 20% and 50% for the test and they will be tested individually. They will sit in front of a laptop computer (the eye-tracker is connected to the laptop) and the questions will be displayed on the screen. This assessment should not take longer than 10 minutes per learner. Therefore, each learner will be absent from class for 10 minutes only.

On a later stage during the year, Mr Potgieter will revise the divisibility rules with the learners. This will take about 30 minutes, and then the learners will write a paper-based assessment again. All the learners involved in the initial eye-tracking study (80 learners) will do the eye-tracking test again.

Mr Potgieter has the national and provincial Mathematical ANA results for 2012, 2013 and 2014 (Grade 4 to Grade 6). It will be appreciated if he can get the Mathematical ANA results for your school for 2012 to 2014 (Only the average per grade, for example: 2013 Grade 5 = 60%).

Proposed dates:

- Mr Potgieter has scheduled a meeting with you on Wednesday, 18 March 2015 at 09:00 to get your support for the project. His e-mail address is pieter@cut.ac.za and his cell number is 083 409 7328.
- A meeting with all the Mathematics teachers from Grade 4 to Grade 7 on Wednesday, 25 March 2015 during tea time (Please confirm the tea time). The meeting will not take longer than 15 minutes.
- The proposed date to write the paper-based assessment is Tuesday, 25 August 2015 during the Mathematics periods. All the necessary stationery will be delivered to the school on Monday, 24 August 2015. This assessment will take approximately 15 minutes.
- 20 Learners per grade will be identified. These learners will take the eye-tracking test from 31 August 2015 until 2 September 2015. Learners will be assessed separately. This assessment will take approximately 10 minutes per learner.
- During the week of 12 to 15 October, the first week after the holidays, the divisibility rules will be explained to the learners. All the learners will write a paper-based assessment again. A double period is necessary for the teaching and assessment.
- The learners involved in the initial eye-tracking assessment will repeat the assessment.

I will appreciate it if you support this project, because the learners will also gain from it. The divisibility rules are useful when learners simplify fractions, determine equivalent fractions, calculations with fractions and also calculations with prime numbers.

Thank you

____

PROF PJ BLIGNAUT

ACADEMIC HEAD OF DEPARTMENT

APPENDIX J

PERCENTAGE OF RECORDINGS PER QUESTION

Number of learners where there were at least one fixation on the AOIs per question, grade and divisor, for Assessment 2 and Assessment 4 (*cf.* Section 5.3.1).

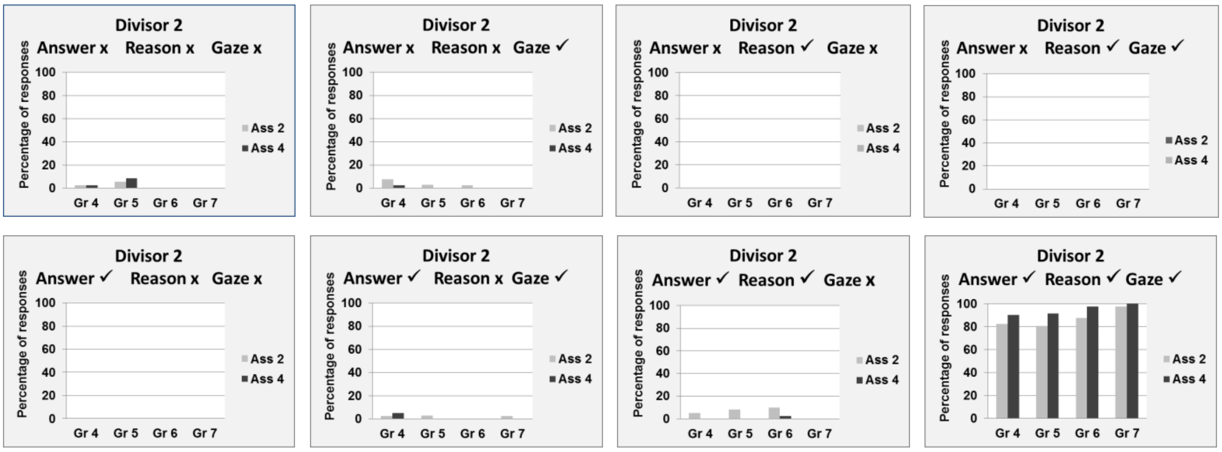
(Key: N = Number of learners; # Rec = Number of recordings; Q = Question; % = Percentage)

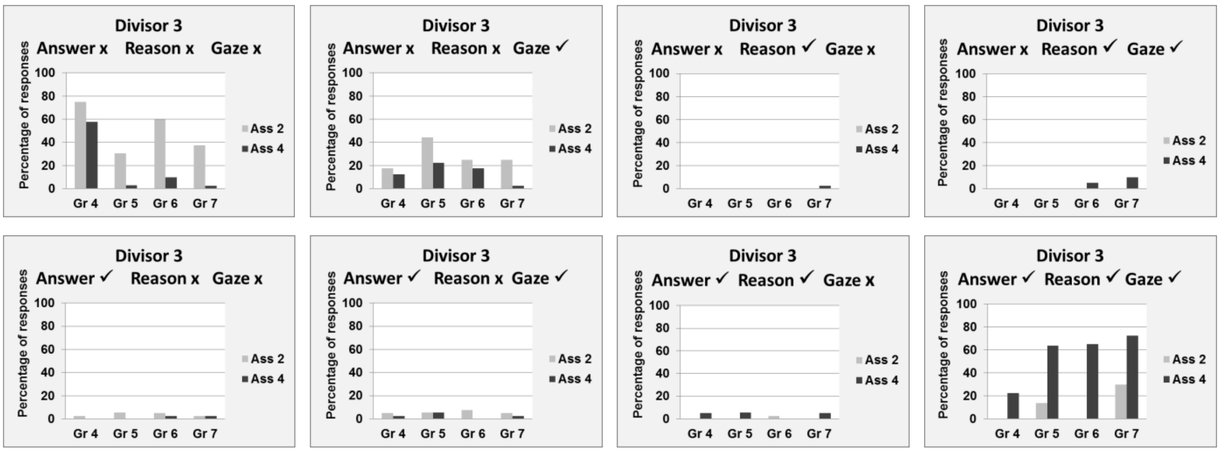
		School A										School B									
			Assessment 2				Assessment 4						Assessment 2				Assessment 4				
Grade	**Divisor**	**N**	**# Rec Q1**	**%**	**# Rec Q2**	**%**	**# Rec Q1**	**%**	**# Rec Q2**	**%**		N	**# Rec Q1**	**%**	**# Rec Q2**	**%**	**# Rec Q1**	**%**	**# Rec Q2**	**%**	
	2	20	19	95	20	100	19	95	20	100		8	7	88	5	63	8	100	8	100	
	3	20	19	95	20	100	19	95	20	100		8	8	100	7	88	8	100	8	100	
	4	20	20	100	20	100	20	100	20	100		8	8	100	6	75	8	100	8	100	
4	5	20	19	95	19	95	19	95	19	95		8	8	100	8	100	8	100	8	100	
	6	20	20	100	19	95	20	100	19	95		8	7	88	7	88	7	88	8	100	
	8	20	19	95	19	95	19	95	19	95		8	7	88	6	75	8	100	8	100	
	9	20	19	95	20	100	19	95	20	100		8	6	75	6	75	8	100	8	100	
	2	18	18	100	18	100	18	100	18	100		23	22	96	18	78	23	100	23	100	
	3	18	17	94	17	94	17	94	17	94		23	21	91	18	78	23	100	23	100	
	4	18	17	94	17	94	17	94	17	94		23	19	83	14	61	23	100	23	100	
5	5	18	18	100	18	100	18	100	18	100		23	19	83	15	65	23	100	23	100	
	6	18	17	94	17	94	17	94	17	94		23	18	78	18	78	23	100	23	100	
	8	18	17	94	17	94	17	94	17	94		23	21	91	14	61	23	100	23	100	
	9	18	17	94	17	94	17	94	17	94		23	17	74	16	70	23	100	23	100	
	2	20	20	100	20	100	20	100	20	100		26	26	100	22	85	26	100	26	100	
	3	20	20	100	20	100	20	100	20	100		26	23	88	23	88	26	100	25	96	
	4	20	20	100	20	100	20	100	20	100		26	24	92	23	88	26	100	26	100	
6	5	20	20	100	20	100	20	100	20	100		26	22	85	22	85	26	100	26	100	
	6	20	20	100	20	100	20	100	20	100		26	21	81	23	88	26	100	26	100	
	8	20	20	100	20	100	20	100	20	100		26	22	85	19	73	26	100	26	100	
	9	20	20	100	20	100	20	100	20	100		26	23	88	20	77	26	100	26	100	
	2	20	20	100	20	100	20	100	20	100		20	18	90	11	55	20	100	20	100	
	3	20	20	100	20	100	20	100	20	100		20	18	90	17	85	20	100	20	100	
	4	20	20	100	20	100	20	100	20	100		20	17	85	11	55	20	100	20	100	
7	5	20	20	100	20	100	20	100	20	100		20	18	90	15	75	20	100	20	100	
	6	20	20	100	20	100	20	100	20	100		20	16	80	13	65	20	100	20	100	
	8	20	20	100	20	100	20	100	20	100		20	16	80	17	85	19	95	19	95	
	9	20	20	100	20	100	20	100	20	100		20	15	75	14	70	20	100	20	100	

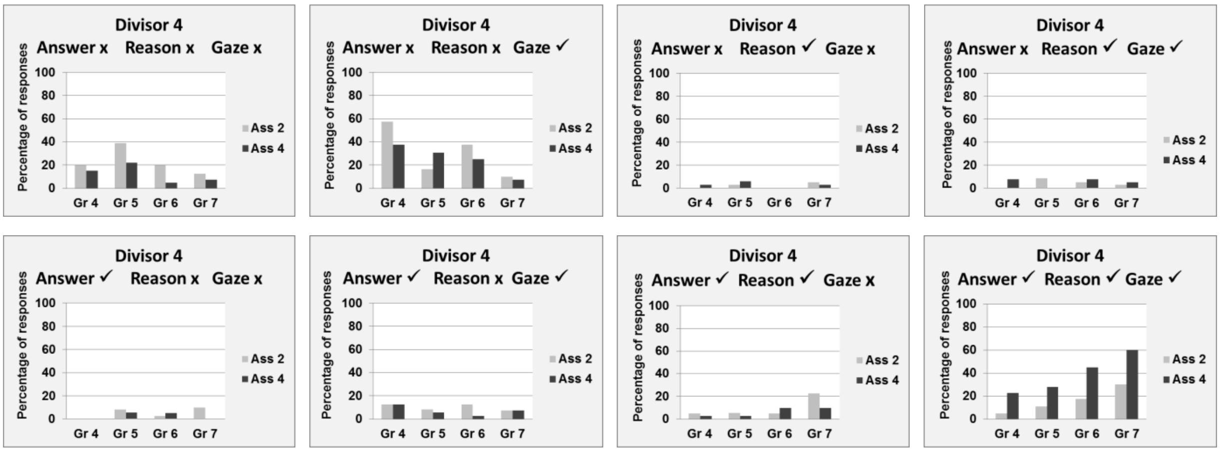
APPENDIX K

VALIDATION OF THE MINIMUM REQUIRED ATTENTION LEVELS PER DIVISOR

The percentage of fixation time on the digits was analysed and minimum requirements were set to determine if learners apply the respective divisibility rules correctly (Section 6.5). The learners' responses were compared with their gaze behaviour. Figure 6.3 indicates the percentage of responses in each of the possible answer/reason/gaze combinations combined for all divisors per grade for Assessment 2 and Assessment 4 (Section 6.6). This appendix shows the same output as Figure 6.3, but the outcomes are broken down per divisor (Table 6.7).






Divisor 4
Answer x Reason x Gaze x
Divisor 4
Answer x Reason x Gaze ✓
Divisor 4
Answer x Reason ✓ Gaze x
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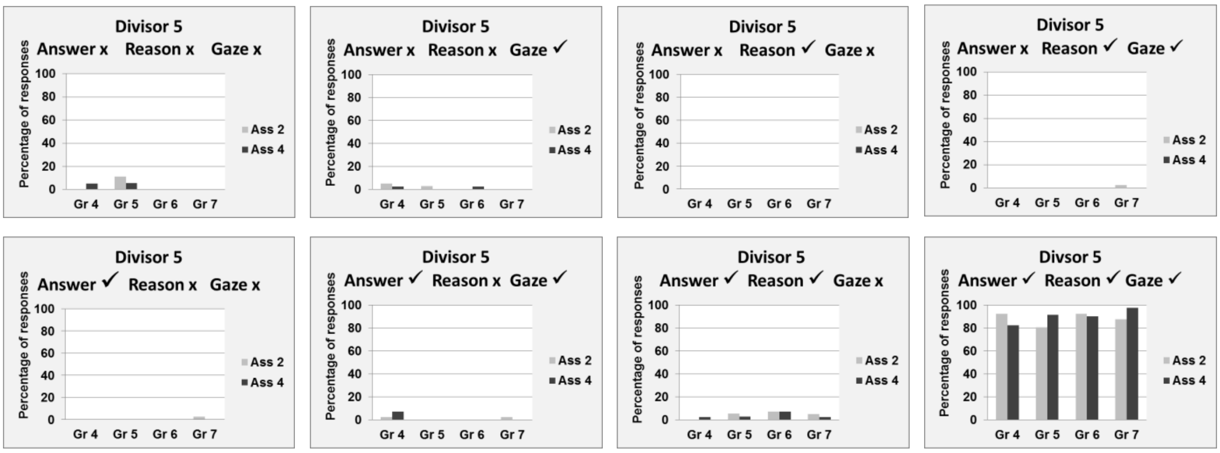
Divisor 4
Answer ✓ Reason x Gaze x
Divisor 4
Answer ✓ Reason x Gaze ✓
Divisor 4
Answer ✓ Reason ✓ Gaze x
Divisor 4
Answer ✓ Reason ✓ Gaze ✓
Percentage of responses
Ass 2
Ass 4
Gr 4
Gr 5
Gr 6
Gr 7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Divisor 5
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Divisor 5
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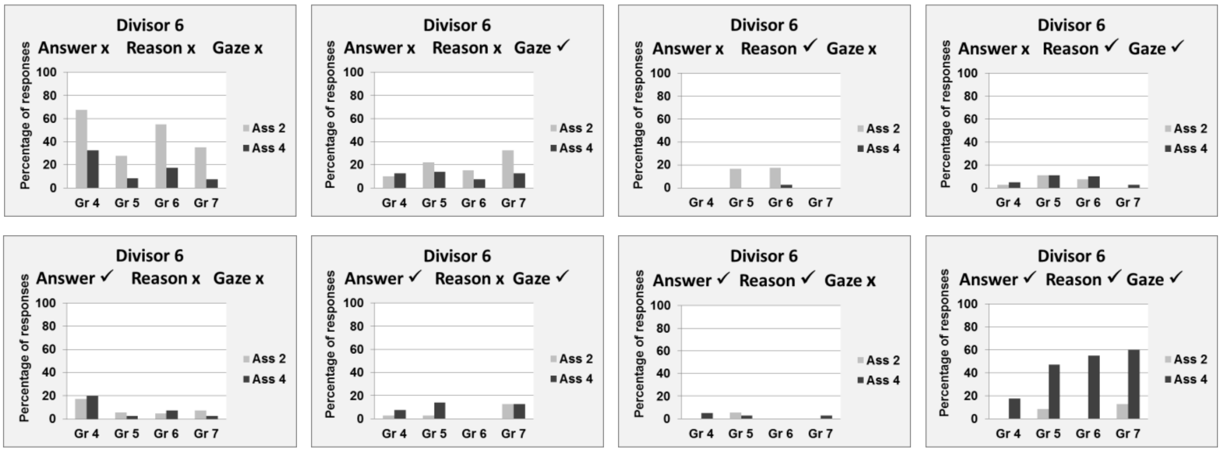
Answer x Reason ✓ Gaze ✓
Divisor 5
Answer ✓ Reason x Gaze x
Divisor 5
Answer ✓ Reason x Gaze ✓
Divisor 5
Answer ✓ Reason ✓ Gaze x
Divsor 5
Answer ✓ Reason ✓ Gaze ✓
Percentage of responses
Ass 2
Ass 4
Gr 4
Gr 5
Gr 6
Gr 7


Divisor 6
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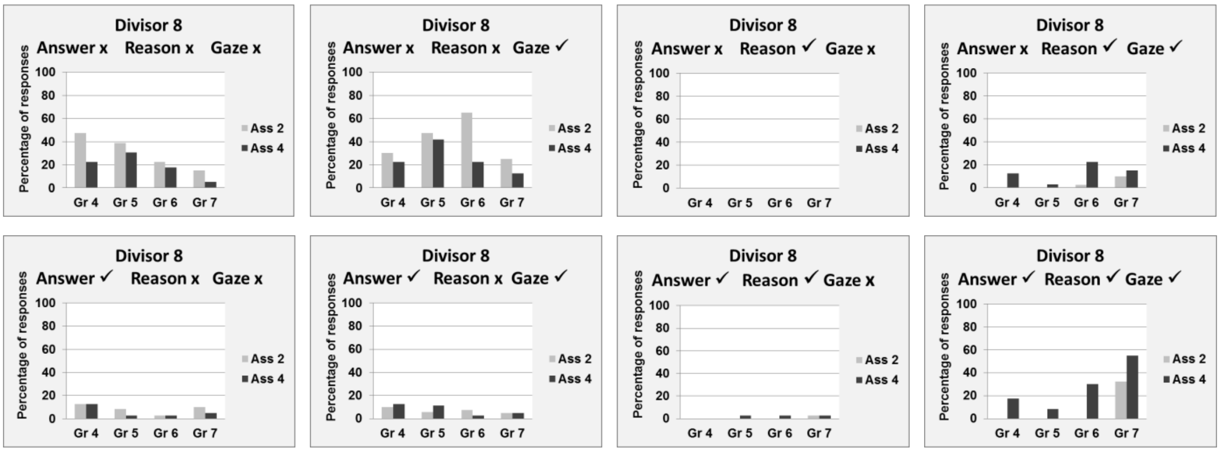
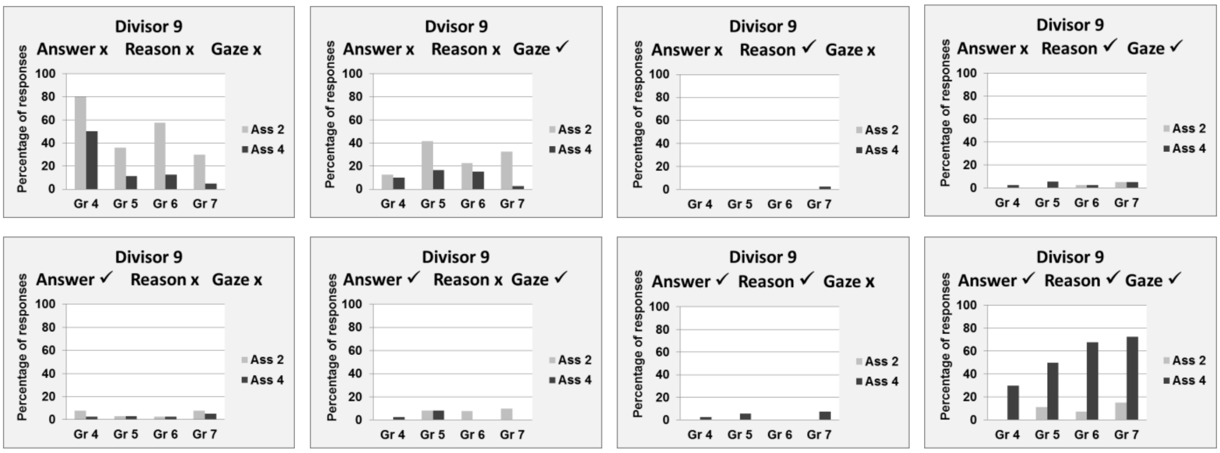
Divisor 6
Answer x Reason x Gaze ✓
Divisor 6
Answer x Reason ✓ Gaze x
Divisor 6
Answer x Reason ✓ Gaze ✓
Divisor 6
Answer ✓ Reason x Gaze x
Divisor 6
Answer ✓ Reason x Gaze ✓
Divisor 6
Answer ✓ Reason ✓ Gaze x
Divisor 6
Answer ✓ Reason ✓ Gaze ✓
Percentage of responses
Ass 2
Ass 4
Gr 4
Gr 5
Gr 6
Gr 7


Divisor 8
Answer x Reason x Gaze x
Divisor 8
Answer x Reason x Gaze ✓
Divisor 8
Answer x Reason ✓ Gaze x
Divisor 8
Answer x Reason ✓ Gaze ✓
Divisor 8
Answer ✓ Reason x Gaze x
Divisor 8
Answer ✓ Reason x Gaze ✓
Divisor 8
Answer ✓ Reason ✓ Gaze x
Divisor 8
Answer ✓ Reason ✓ Gaze ✓
Percentage of responses
0
20
40
60
80
100
Gr 4
Gr 5
Gr 6
Gr 7
Ass 2
Ass 4


Divisor 9
Answer x Reason x Gaze x
Dvisor 9
Answer x Reason x Gaze ✓
Divisor 9
Answer x Reason ✓ Gaze x
Divisor 9
Answer x Reason ✓ Gaze ✓
Divisor 9
Answer ✓ Reason x Gaze x
Divisor 9
Answer ✓ Reason x Gaze ✓
Divisor 9
Answer ✓ Reason ✓ Gaze x
Divisor 9
Answer ✓ Reason ✓ Gaze ✓
Percentage of responses
0
20
40
60
80
100
Gr 4
Gr 5
Gr 6
Gr 7
Ass 2
Ass 4