Perspectives in Education 2013: 31(3) *http://www.perspectives-in-education.com*
ISSN 0258-2236

Selecting and sequencing mathematics tasks: Seeking mathematical knowledge for teaching

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In this article, I present an initial analysis of an empirical study that was undertaken in an attempt to elicit what subject-matter knowledge, pedagogic content knowledge and curriculum knowledge teachers bring to bear on decisions for teaching. The analysis is based on interview data with 46 Grade 3 teachers, who were presented with two mathematical tasks taken from the 2010 NDBE Grade 2 and Grade 3 Numeracy Workbooks. Teachers were required to justify the selection and sequencing of the two mathematical tasks for teaching multiplication. In so doing, they provide some indication of what they know or do not know about the mathematical concepts in the tasks; about the connections between mathematical concepts; about the representations of those concepts, and about how learners learn those concepts.

Teachers' responses varied from an articulation of the pedagogic and mathematical intentions of the tasks, to the use and consequences of pictorial representations in the tasks and how learners would respond to the tasks. The variation in responses reflected different criteria that teachers used to justify the selection and sequencing of the tasks. The analysis raises critical questions regarding the interplay between teachers' subject-matter knowledge, their pedagogic content knowledge and curricular knowledge, which they bring to bear on pedagogic decisions. The analysis raises further critical questions concerning the pedagogic and mathematical explicitness of tasks in the NDBE Numeracy Workbooks. The analysis suggests that careful consideration must be given to the construction of mathematical tasks in Grade 3, and probably the Foundation Phase, to ensure that the mathematical purpose of tasks is explicit, and that 'contextual noise' is not introduced that distracts from the pedagogic and mathematical intent of the tasks.

Keywords: subject-matter knowledge, pedagogic content knowledge, curricular knowledge, mathematical tasks, numeracy workbooks

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Introduction

In South Africa, the long and winding road of curriculum development has seen policy initiatives first veer away from, and then more recently back to, prescription and specification of contents and coverage. The 'milestones' curriculum of the Foundations For Learning (2008) started the path back to specification of contents, and most recently, the Curriculum and Assessment Policy Statements (CAPS) (2011) extended this specification with prescriptions for weekly planning. These prescriptions specify the content to be covered, the sequence in which they are to be covered, and the time to be spent on each content area (pace) per week. In addition, CAPS provides some 'teaching guidelines' as to how to cover some of the content areas. To support these policy initiatives, the National Department of Basic Education (NDBE) went even further by producing prescribed workbooks for Foundation Phase learners, providing teachers with ready-made materials and exemplars of the contents to be covered in each grade.

The collection of studies in Taylor and Vinjevold (1999), which reviewed the implementation of Curriculum 2005 through a range of empirical contexts, showed the disastrous consequences of having unspecified curriculum statements that leave the interpretation of contents, sequencing and pacing, entirely over to teachers. Hence, all of this recent curricular specification and prescription can be regarded as a means to circumvent the teacher from having to make these decisions of content, sequencing and pacing, based on their interpretation of more general curriculum statements.

Given the greater specification of curricular contents that makes progression in mathematics more visible, are teachers in a better position to select and sequence mathematics tasks for use in classrooms? Has the shift in curriculum policy regimes changed the discourses with which teachers engage in the construction of mathematics and mathematics tasks? Have the teaching guidelines and exemplars provided teachers with more insight into the mathematics they need to know for teaching? This article reflects on these and presents a brief empirical analysis of how teachers talk about the selection and sequencing of mathematics tasks for use in classrooms. The analysis is based on interview data with 46 Grade 3 teachers in 2012, in which they were asked to comment on the selection and sequencing of two mathematics tasks related to multiplication. The tasks were taken from the 2010 NDBE Grade 2 and Grade 3 Numeracy Workbooks.

Research contexts

The analysis presented in this article may be located within a mathematics education research context in which teachers' 'pedagogic content knowledge' and 'mathematical knowledge for/in teaching' have been soundly interrogated (Adler & Davis, 2006; Askew, Venkat & Mathews, 2012; Ball, Hill & Bass, 2005; Ball, Thames & Phelps, 2008; Brodie, 2004; Goulding, Rowland & Barber, 2002; Ma, 1999; Marks, 1990;

Petrou & Goulding, 2011; Rowland & Ruthven (eds), 2011; Shulman, 1987). Using various theoretical models and empirical settings, these studies collectively identify different 'categories' of knowledge that teachers need for effective mathematics teaching. These include specialised subject-matter knowledge, knowledge of the student, curricular knowledge and knowledge of teaching. They demonstrate that a combination of these 'knowledge categories' enables teachers, for example, to make decisions about the selection and sequencing of mathematics tasks; to respond appropriately to students' misconceptions; to notice the advantages or disadvantages of using different representations of mathematics in their teaching, and to be aware of the progression and development of mathematical topics across Grades.

A further research context in which the empirical analysis in this article may be located are those studies that consider the teaching and learning of mathematics in terms of making interconnections within mathematics and using and interpreting different modes of representation in mathematics (Barmby, Harries, Higgins & Suggate, 2009; Businskas, 2008; Hodgson, 1995; Mhlolo, Venkat & Shafer, 2012; Stylianou, 2010). These studies suggest that the ability to recognise interconnections between mathematical topics or different representations of the same mathematical concepts, demonstrates a deeper level of understanding of mathematics than those who cannot make these connections.

Most of the studies referred to earlier have investigated these issues through observation of teachers' classroom practices. However, the empirical analysis in this article is based on interviews with teachers. The analysis thus also contrasts with Ball et al.'s approach (2008). They developed a bank of structured multiple choice assessment items to measure what they call teachers' 'common and specialized mathematical knowledge for teaching'. The assessment items were used in large-scale surveys to test teachers' subject-matter knowledge. The aim of Ball et al.'s (2008) study was to identify the content knowledge needed for effective practice and to develop measures for that knowledge. The empirical study in this article is much less ambitious, but it is also an attempt to elicit what subject-matter knowledge and pedagogic content knowledge teachers bring to bear on decisions for teaching. We draw on both Ball et al.'s (2008) and Petrou & Goulding's (2011) models of mathematical knowledge for teaching, which recognises an essential interplay between teachers' subject-matter knowledge, pedagogic content knowledge and curriculum knowledge. This interplay between these domains of knowledge enables teachers to make decisions about what topics in mathematics to teach, in what order and how to teach them. In this model of mathematical knowledge for teaching, pedagogic content knowledge includes teachers' knowledge of students.

Teachers were required to justify the selection and sequencing of two mathematical tasks. In so doing, they provide some indication of what they know or do not know about the mathematical concepts in the tasks; about the connections between mathematical concepts; about the representations of those concepts, and about how learners learn those concepts. In particular, this small-scale empirical study sought to explore first, on what basis do teachers sequence mathematical tasks in the classroom (for example, choose to do a particular task before another) and, secondly, what connections between mathematical topics do teachers make when selecting tasks for teaching multiplication?

Description of empirical study

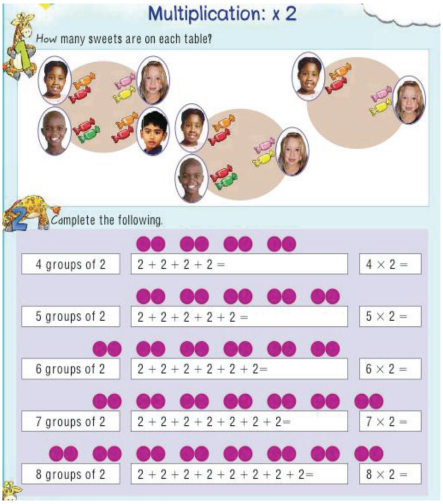
The analysis is based on data collected within the context of the broader SPADE research project concerned with schools in poor neighbourhoods performing above demographic expectations. Six schools, four that performed above average in systemic tests and two comparator schools that performed just below average form the sample.

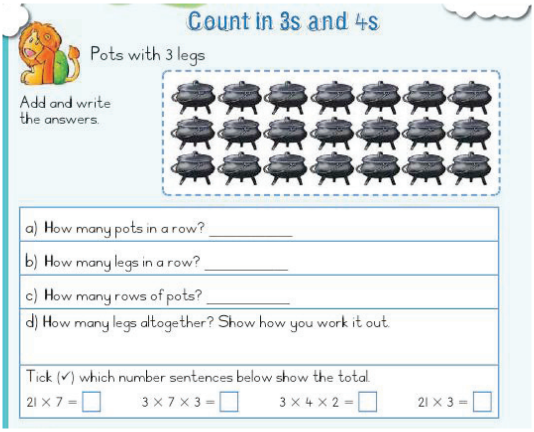
The data was collected as part of the SPADE project, which focuses on schools in poor communities in the Western Cape. Schools were selected on the basis of performance on systemic Grade 3 numeracy and literacy tests. Test scores were measured over four cycles of the Western Cape Education Department (WCED) numeracy and literacy systemic tests between 2004 and 2008. There are 14 schools in the study sample, nine of which are above average performing and five below average performing. Schools were matched according to socio-economic and demographic profiles. Data collection comprised classroom observations as well as interviews with school leaders and teachers. The analysis in this article is based on individual interviews of one hour, with a total of 46 Grade 3 teachers at all the schools.

The interviews were conducted during the second term of 2012. All the schools in the sample had NDBE numeracy workbooks in use in the Foundation Phase. Teachers were not asked explicitly about whether they had received training from the WCED on CAPS in 2011, but they all reported that their teaching and planning for 2012 was based on CAPS curriculum policy documents.

The two tasks below were shown to 46 Grade 3 teachers during individual interviews. Teachers were only told that these tasks were taken from the NDBE workbooks, but not from which Grades. They were given time to peruse both tasks before the interview questions were posed.

The first task was taken from the 2010 Grade 2 Numeracy Workbook, Book 1, on page 33, which was labelled ‘Term 2, Week 1’. The second task was taken from the 2010 Grade 3 Numeracy Workbook, Book 1, on page 57, which was labelled, ‘Term 2, Week 7’. There was thus a clear sequencing of the two tasks in the workbooks, in that one appears in the Grade 2 workbook and the other in the Grade 3 workbook. However, this sequencing was not made explicit to teachers.





Task 1: Multiplication: x2 ***Task 2: Counting in 3s and 4s***

Only two interview questions relating to the tasks were posed: (1) How would you explain to a new teacher what the difference between these two tasks is for teaching multiplication in Grade 3? (2) In which order would you suggest they be taught and why? These were open response questions, and interviewers were instructed to probe for sequencing of tasks and the reasons for sequencing. Before presenting the analysis of teachers' responses, I briefly examine how multiplication is mentioned in the FP CAPS document and consider features of the workbook tasks that were given to teachers.

Multiplication in FP CAPS

In the FP CAPS documents, under the topic Number, Operations and Relationships, multiplication is referred to explicitly under the sub-topic Repeated addition leading to multiplication. The Overview for Grade 3 Number, Operations and Relationships also includes Problem Types for Grade 3 under two topics: (i) Repeated addition and (ii) Grids, with some examples of each. In the Grade 3 Clarification Notes for Number, Operations and Relationships, there are additional references to arrays, groups, and order of multiplication as well some illustrations. It is interesting to note that the problem types for arrays are given in words only, without illustrations. The term 'arrays' is first used in the Clarification Notes for Grade 2, 1.14, where the notes are more explicit about what is meant by arrays and counting equal rows, and includes illustrations. The notes also specify that by Grade 2 learners must understand repeated addition, and must 'relate skip counting and repeated addition to an understanding of multiplication'.

The Clarification Notes for Grade 2, 1.14 Term 2, includes pictures of 'four fingers' and tricycles to illustrate 'groups of 3' and 'groups of 4' as well as circles with 3 balloons in them to illustrate 'repeated images of repeated addition'. The Grade 2 notes thus give more clarification than the Grade 3 notes. However, these notes appear in one document: Foundation Phase Grades 1-3. The CAPS Foundation Phase document thus assumes that a Grade 3 teacher will read the Clarification Notes for every Grade per topic, i.e. Grade 1 Terms 1-4; Grade 2 Terms 1-4, and Grade 3 Terms 1-4.

Features of the workbook tasks

Rather than provide an in-depth analysis of these two tasks at this point, I merely wish to highlight features of the tasks that are clearly noticeable at first glance, as teachers might have done before answering the interview questions. Both tasks clearly signify multiplication, with the use of the multiplication signs in the number sentences which learners have to complete. However, the titles are very different – Task 1 is explicit about it being multiplication, i.e. 'Multiplication: x2', while Task 2 is titled 'Count in 3s and 4s'. From the illustration and questions asked, it is unclear why Task 2 is labelled 'Count in 3s and 4s', as there is no obvious grouping of 4s in the task. Task 1 requires only completion of number sentences, whereas Task 2 also requires answering written questions. The illustrations in the two tasks are also very different. The first illustration in Task 1 is ambiguous; it appears that it could be about 'equal sharing', with 18 sweets shared among 9 children, where each child gets 2 sweets. The illustrated circles in the second part of Task 1 suggest repeated addition of groups of 2. The illustration of pots in Task 2 suggests an array, in that there are 3 rows, with 7 pots in each row. In both tasks, the illustrations may be viewed as a pedagogic resource to help learners complete the task. Both tasks may be considered to be 'structured' tasks: it appears that questions have been posed and sequenced in a purposeful manner.

Analysis

Teachers' responses to the tasks varied from an articulation of the pedagogic and mathematical intentions of the tasks, to the use and significance of pictorial representations in the tasks. Teachers suggested a sequencing of the tasks based on their assessments of mathematical and cognitive demands of the tasks and how Grade 3 learners would respond to the tasks. The actual criteria for selection and sequencing varied across the teachers. In the analysis that follows, I grouped teachers' responses in terms of differences and similarities, i.e. noting what some teachers were saying that was similar to the others, and what some teachers were saying that was different to the others. In this way, the analysis begins to tease out the responses in terms of different aspects of teachers' pedagogic content knowledge that emerge from the interviews.

Justifying the sequencing

Eight out of the 46 teachers chose Task 2 as the first task to be done.

Teacher	*Reasons for first choice*
ARE	I would use the one with the pots first, because it is more concrete, and especially if it is a new concept that you're starting with, it's to take them back to the concrete; if I look at this one, then the learner sees the pot and he sees the three legs.
HAR	I will definitely start with the one with the pots, because it has the context and they can physically see the legs of pots in there. They're using the different forms of multiplication problems in a sense, because they're using the grid and grouping which is the legs of the pots.
MBE	I would recommend this [pots] as the first technique to use, because it's word sums and like problem-solving; it has pictures so they can count, so learners can count the first one.
GOR	I would actually do the grouping pots first before I go to the multiplication; counting in 3s and 4s, this is grouping, this is good.

In these responses, we notice a kind of fetishizing of the 'physical representation' within the task – a representation that allows learners 'to look, see and count'. For these teachers, the selection of this task as the first task seems to be based on a 'cognitive sequencing' where:

- 'concrete' and 'contextual' comes before 'abstract' or 'non-contextual';
- 'grouping' and 'counting' comes before 'multiplication'

This 'cognitive sequencing' is, in fact, consistent with the way in which the curriculum statements are organised in the Foundation Phase CAPS document in which 'Solve problems in context' comes before 'Context-Free Calculations', and 'Counting' and 'Grouping' comes before 'Multiplication'.

In this sense, the title of Task 2 as 'Counting in 3s and 4s' may have contributed to this interpretation of what the task is about, and hence it's sequencing before Task 1 for these teachers. Only one teacher, HAR, identifies this task as involving 'different forms of multiplication problems', which she identifies as 'grids and grouping', although the teacher does not elaborate on this.

The reasons given by these teachers for selecting Task 2 as the first task suggest that they have used the title and illustrations in the task to make their choice, rather than consider what the learning objects of the tasks might be following the way they have been structured or designed.

Expressing ambivalence about the sequencing

Two out of the 46 teachers, were ambivalent about which task should be done first.

Teacher	*Reasons for ambivalence*
STE (1) [Task 1]	Multiplication of two, I will do this first, because I teach them multiplication of two before I teach them multiplication of three and four; it's easier, they count in 2s; [...] and there are counters, he can count, he can count everything, he can see the groups, he doesn't even need to know it's a multiplication sum.
STE (2) [Task 2]	Child can count how many pots in three rows, how many rows are there and how many threes in a row, so this also a multiplication sum, and both are multiplication and addition; there he will not see the pots in a row, here he sees it's in a row, I will do this one first, then I will do that one, but I don't know about these sentences here at the bottom, now it gets difficult.
MBE	This one [pots], because it has a lot of counting on it, because they need to have a sense of numbers, even this one [sweets], because it has equal sharing you know; and also [sweets] has repeated addition so it makes it easier for them to do the multiplication, because repeated addition is in, so I can also give them this one [sweets] and then come to this one [pots]

The ambivalence about sequencing for these two teachers seems to stem from their recognition of counting and their interpretation of what can be counted in each activity, i.e. they view counting as the primary activity in both tasks.

However, teacher STE also recognises that both tasks involve 'multiplication and addition'. She first selects Task 1 on the basis of the smaller number range, then changes her mind about the sequencing and selects Task 2 as the first task on the basis of the physical representation of pots in rows: 'there he will not see the pots in a row, here he sees the pots in a row'. This again suggests an apparent fetishizing of the 'physical representation' in the task.

Teacher MBE describes Task 2 as having 'lots of counting', but recognises 'repeated addition' only in Task 1, and uses this to change her mind about the sequencing, suggesting that 'repeated addition makes it easier for them to do multiplication'. Teacher MBE also responds to the ambiguity of the illustration in Task 1, by describing it as having 'equal sharing'.

Responding to ambiguity in illustrations

In addition to teacher MBE, at least 3 other teachers also responded to the ambiguity of the illustration in Task 1.

Teacher	*Equal Sharing/Division/Grouping/ Addition/Multiplication?*
MVA	The first one has got the learners, you can see each learner how many he gets, he gets two sweets so that is division, division is involved here, and then grouping them altogether, and addition, and a short method for addition is four times two, then you will be able to get the answer;
NQO	Here [sweets] you are having two groups of four which make eight, here they have to divide first, they have to divide the balls like make the circles
NGA	You are adding there sweets to each learner and multiplying; you can make some story around this so that learners can picture more in numbers, even there's sharing and also adding, addition is also in part here and multiplication is also in there; I would prefer to go with this one so that they can group it, group, group, group.

While these teachers all recognise 'grouping' of some form in the task, the illustration elicits an ambiguity with respect to whether this 'grouping' is about 'equal sharing' or 'repeated addition'.

Choosing the smaller number range

Some teachers chose Task 1 as the first task primarily on the basis of the smaller number used for multiplication in Task 1. In both tasks, of course, they are single digit numbers.

Teacher	*Counting in 2s comes before counting in 3s or 4s*
ZIM	I start at the small number first, then the numbers become bigger; you will start at multiplying two and then later on you will multiply three and then later on by four, because you start at the familiar first.
EIM	Will use the first one first, because it is easier for the child to see the picture together with the sum; child can count much easier in 2s. Child has been counting much longer in 2s.
GAR	I will obviously start with multiplication of 2, then three and multiplication of four comes later.
STE	Multiplication of two, I will do this first, because I teach them multiplication of two before I teach them multiplication of three and four.

In basing their choice simply on the smaller number used in the task, these teachers do not really engage with the structure or cognitive demands of either task. So, for example, if the number range for Task 1 had been 8, these teachers might have selected Task 2 as the first task instead, because it has a smaller number range; yet the structure and cognitive demands of Task 1 would have been simpler than those of Task 2.

Making Task 1 concrete

Earlier I showed that some of those teachers, who chose Task 2 first, motivated their choice by describing Task 2 as more 'concrete' and 'contextual' than Task 1. Yet several teachers, who chose Task 1 as the first task, similarly argued that Task 1 is more 'concrete' and 'contextual' or rather, can be made more concrete and contextual.

Teacher	*How to make Task 1 concrete?*
SKE	Here is sweets it is also addition, I think for some of them it is easier to do addition, instead if I don't have sweets I can use beans to show them or demonstrate for them.
DAM	I will do the first task first, children can use body parts, it is easier to count; it can be done practically and can be integrated with doubling.
THO	First do repeated addition, must be packed out concretely in groups, children must learn to count in twos.
SMA	I would teach this one first, because they use counters, they're familiar with counters in school; they must pack it out; you can't just pack it out, you maybe say there's four girls, how many eyes, ok? Now pack out the four girls and then you do the eyes, so it's two eyes for the girls, two, two, two, so let's count.

Even though this is, in fact, a written task given in a workbook, these teachers conceive of Task 1 only as something to be done 'with the help of counters' or 'beans' or 'body parts', thereby making it a more 'concrete' activity for learners than Task 2. The use of pots in Task 2 appears to mitigate against teachers conceiving how 'counters' or 'beans' may be used for Task 2. These teachers' responses again suggest a 'fetishizing' of the concrete.

Making connections between 'equal groups', 'repeated addition' and 'multiplication'

Surprisingly, in their descriptions of Task 1, only seven teachers out of the 46 make explicit reference to 'repeated addition that leads to multiplication' or 'multiplication as a short way of writing repeated addition', or using 'multiplication for finding how many equal groups there are'.

Teacher	Repeated Addition Leading to Multiplication
FRE	They must know multiplication is the short method for adding or addition; they must be able to count before they can get to multiplication and then see it's grouping; here they must still know it is adding but multiplication is the short way to get the answer;
PAU	If you have taught a child to count in 2s then he will tell you immediately four 2s is eight; he will be able to see immediately groups of 2, 4, 6; then you will learn to double, then I will say two times table;
NTI	This one is simpler because you can add two plus two plus two plus two, it's like four times two; she can easily see four times two is the same like two plus two plus two plus two.
KAL	So multiplication is a short cut for addition; for instance, you say two plus two plus two plus two is eight where you can just say two times four is eight;

In these descriptions, the teachers display some recognition of the 'structure' and learning objective of Task 1, i.e. that 'repeated addition leads to multiplication' or 'multiplication as a short way of writing repeated addition'. The remaining 39 teachers merely mention that the task involves 'addition and multiplication' or 'grouping, addition and multiplication', without being explicit about why they are being done together in one task.

Making connections between 'arrays' and 'multiplication'

Some of the teachers show some recognition of the arrangement of the pots in an array, i.e. equal rows of objects arranged in rows and columns, and make some connection between this arrangement and multiplication.

Teacher	Multiply equal rows?
PIE	This multiplication is actually very confusing; there are three rows, there are seven groups in each row, isn't it something like that? I plant three rows of onions, in each row there are seven onions, how many are there? It's a multiplication sum, there are three rows, there are seven in each row; this is multiplication and act 1 is grouping.
REM	Count how many pots in this row, repeated addition, how many times do you see three, then you do the addition sum first that which you see every time, then you multiply the number let's say the three with the number of pots there are in a row.
STE	Child can count how many pots in the rows, how many rows are there and how many threes in a row, so this also a multiplication sum, and both are multiplication and addition.
DUP	In the second activity, the child would have to count the number of pots and then times by three, they would have to know that they have to do a times sum in order to know how many legs are in the room

Even these few teachers weakly state the connection between the array and multiplication, and only teacher PIE explicitly mentions the numerical relationship in the array, i.e. 3 rows and 7 pots in a row. The majority of these teachers recognise the arrangement of rows, but few pick up on the idea of 'equal rows', and hence repeated addition of equal groups, or multiplying by the number of rows. In other words, there is very little recognition of the representation of 'arrays' as a pedagogic resource for teaching multiplication.

(Mis)recognising arrays

Teachers expressed at least three forms of misrecognition of arrays. The first form of '(mis) recognition' of the array, shown in Task 2, is regarding Task 2 as simply a counting activity.

Teacher	*More counting?*
SMA	Here they can use the picture to come to the answer. How many pots in a row? So they count 1-7. Then how many legs in a row 3, 6, 9, 12, 15, 18, 21. How many rows of pots? Three rows. How many legs altogether? I am thinking now, ok so it's just a counting thing.
KLE	This one is almost like a graph; they ask questions about it, but it comes down to the same thing ... how many? Then you count how many legs in a row, then you count in 3s; this is what we call data handling.
DAM	Counting in 3s, it's more difficult to count uneven numbers.
EIM	If the child sees the picture with pots, he will get scared and not want to do it; we don't count in sevens.
SKE	They have to count the legs, count the legs how many pots, how many pots altogether.

Some of these teachers recognise that the counting includes 'counting in 3s' for the legs, but few recognise the arrangement as 'three equal groups of 7'.

The second form of '(mis)recognition' of the array is perceiving Task 2 as a problem that requires learners to make their own decisions about how to group and count the objects.

Teacher	*Making your own groups?*
GAB	When doing this activity, the child must discover on his own how much and what he must add; it is like a comprehension, it is more difficult to get; with this activity you can integrate mathematics and literacy.
HEN	The second one you must use more your imagination; you must demarcate the pots on your own or group them to do your multiplication.
MBET	This one, the bottom one, you choose yourself how to do it [grouping].
NQO	Here [pots] she has to make the groups herself; she has to make the groups then count them.

In other words, these teachers do not recognise the array as already an arrangement of equal groups.

The third form of (mis)recognition of arrays is similar to what teacher GAB describes above, namely perceiving Task 2 as primarily a 'comprehension' exercise that requires reading and interpretation from learners.

Teacher	*A reading exercise?*
JON	It is very complicated for Grade 3; it's about reasoning and thinking. The second task must be guessed, because I am not sure how they will work it out.
MBE	This one it's a wording, because if you say you ask how many pots in a row, maybe a child can't even read, so how is she or he going to answer the question.
NTI	This one will take a long time to explain, you have to have enough time, it's a long method.
MGW	You see it's word sums, because you have to read first, there's lots of reading in here.

In these responses, there is again very little recognition of what the learning objective might be of the 'reading and interpretation' within the task. It is interesting to note that these teachers view only Task 2 as a task that requires reasoning and thinking, and Task 1 as merely an exercise in grouping and multiplication.

Reflections and conclusion

The responses from teachers to how they would select and sequence two tasks in these interviews highlights weaknesses in teachers' subject-matter knowledge, their pedagogic content knowledge and curricular knowledge that are brought to bear upon them when making decisions about what to teach and how. In deciding on the sequencing of the two tasks, several teachers demonstrate an obsession with the 'concrete' in mathematics and 'physical representations' that 'show' the mathematics to be done. This basis for selection would appear to be based on teachers' knowledge of how children learn mathematics, i.e. that 'concrete' learning comes before 'abstract' learning. The teachers' obsession with the 'concrete' leads to little recognition or engagement with the structure or learning objects of the tasks, i.e. teachers' recognition of tasks as a series of questions that are sequenced in a way that provides opportunities for learners to see patterns and reason logically. The analysis suggests that of the two components of pedagogic content knowledge identified in Ball et al.'s (2008) model, namely Knowledge of Content and Students (KCS) and Knowledge of Content and Teaching (KCT), KCS predominates at the expense of KCT.

Several teachers also show a lack of engagement with, or understanding of progression and development of mathematical concepts and processes, in this instance, multiplication. The analysis suggests, again following Ball et al.'s

(2008) model, that teachers display weak 'specialized content knowledge' and weak 'mathematical knowledge at the horizons'. Only some of the teachers make mathematical connections across different 'numeracy topics', for example in this instance, between counting, repeated addition, and multiplication, and show an awareness of how one develops the other. In addition, the teachers appear to lack exposure to a range of cognitive and pedagogic resources and strategies to teach the same mathematical concept or 'topic', in this instance, multiplication. This suggests both weak pedagogic content knowledge and weak curricular knowledge. While the analysis in this article examined all 46 Grade 3 teachers across the sample schools, the design of the SPADE project opens up the possibility to investigate in more detail the relationship between teachers' subject-matter knowledge, pedagogic content knowledge, curricular knowledge and learner outcomes.

This initial analysis also raises critical questions concerning the pedagogic and mathematical explicitness of tasks in the NDBE Numeracy Workbooks. It raises questions about how teachers select and use the tasks from the Workbooks in classrooms without a 'User Guide' that makes explicit the learning objects of the task, i.e. the intended pedagogic and mathematical outcomes of the tasks. It also raises questions about how teachers are engaging with curriculum documents, in particular, the 512-page-document FP CAPS, which assumes, or requires that teachers read clarification notes and assessment exemplars for each Grade per term, in order to get a sense of progression from Grade to Grade and for explanatory notes on concepts and terms that are introduced in earlier Grades and extended in later Grades. There are connections between topics in CAPS, but they are buried in the Clarification Notes. CAPS has the correct language, but how explicit this is made to teachers is questionable.

The analysis further suggests that careful consideration must be given to the construction of mathematical tasks in Grade 3, and probably the Foundation Phase, to ensure that the mathematical purpose and learning objectives of the tasks are explicit, and that 'contextual noise' is not introduced that distracts from the pedagogic and mathematical intent of the tasks. Furthermore, in engaging with both curriculum policy documents and teaching materials, teachers must be made aware of honing both their subject-matter knowledge and pedagogic content knowledge. This includes their 'specialized mathematical content knowledge', 'knowledge at the mathematical horizon', 'knowledge of content and students', 'knowledge of content and teaching', and 'knowledge of curriculum'.

References

Adler J & Davis Z 2006. Opening another black box: Researching mathematics for teaching in mathematics teacher education. *Journal for Research in Mathematics Education,* 37(4): 270-296.

Askew M, Venkat H & Mathews C 2012. Coherence and consistency in South African primary mathematics lessons. *Philosophy of Mathematics Education* (PME) 36:1-8.

Ball D, Bass H & Hill H 2005. Knowing mathematics for teaching: Who knows

mathematics well enough to teach third grade, and how can we decide? *American Educator,* 29(1): 14-46.
Ball D, Thames M & Phelps G 2008. Content knowledge for teaching: What makes it special? *Journal of Teacher Education*, 59(5): 389-407.
Barmby P, Harries T, Higgins S & Suggate J 2009. The array representation and primary children's understanding and reasoning in multiplication. *Educational Studies in Mathematics*, 70(3): 217-241.
Brodie K 2004. Re-thinking teachers' mathematical knowledge: A focus on thinking practices. *Perspectives in Education*, 22(1): 65-80.
Businskas A 2008. Conversations about connections: How secondary mathematics teachers conceptualise and contend with mathematical connections. Unpublished doctoral dissertation. Simon Fraser University, Burnaby, Canada. Available fromhttp://ir.lib.sfu.ca/handle/1892/10579.
Goulding M, Rowland T & Barbar P 2002. Does it matter? Primary teacher trainers' subject knowledge in mathematics. *British Educational Research Journal*, 28(5): 689-704.
Hodgson T 1995. Connections as problem-solving tools. In PA House & AF Coxford (eds), *Connecting mathematics across the curriculum* (pp. 13-21). Reston, VI: National Council of Teachers of Mathematics.
Ma L 1999. *Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in China and the United States*. NJ Lawrence Erlbaum.
Marks R 1992. Pedagogic content knowledge: From a mathematics case to a modified conception. *Journal of Teacher Education*, 4(3): 3-11.
Mhlolo M, Venkat H & Schäfer M 2012. The nature and quality of the mathematical connections teachers make. *Pythagoras*, 33(1), Art.#22, pages. http://dx.doi.org/10.4102/ pythagoras.v33i1.22
Petrou M & Goulding M 2011. Conceptualising teachers' mathematical knowledge in teaching. In T Rowland & K Ruthven (eds), *Mathematical knowledge for teaching* (pp. 9-25). London: Springer.
Rowland T & Ruthven K 2011. *Mathematical knowledge for teaching*. London: Springer.
Taylor N & Vinjevold P 1999. *Getting learning right. Report of the President's education initiative research project*. Johannesburg: Joint Education Trust.
Shulman L 1987. Knowledge and teaching: Foundations of new reforms. *Harvard Educational Review,* 57: 1-22.
Stylianou D 2010. Teachers' conceptions of representation in middle school mathematics. *Journal of Mathematics Teacher Education*, 13: 325-343.