THE DEVELOPMENT OF A MATHEMATICS PROFICIENCY TEST FOR ENGLISH-, AFRIKAANS- AND SESOTHO-SPEAKING LEARNERS

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...because we know that suffering produces perseverance; perseverance, character, and character, hope. And hope does not disappoint us, because God has poured out His love into our hearts by the Holy Spirit, whom He has given us (Romans 5:3-5).

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1. INTRODUCTION

1.1 BACKGROUND

When the *ESSI* Reading and Spelling Test (Esterhuyse & Beukes, 1997) was compiled, the support teachers involved in the development of the test expressed an interest in a mathematics proficiency test that would serve a diagnostic purpose. The *VASSI* Mathematics Proficiency Test (Vassiliou, 2000) was then developed. The test was developed for grade one, two and three learners and standardised for English-speaking learners only. Not only did the Free State Department of Education express a need to standardise the test for Afrikaans- and Sesotho-speaking learners, but they also expressed a keen interest in the development of a mathematics proficiency test for learners in the intermediate phase.

These requests not only fuelled the researcher's decision to standardise the foundation phase mathematics proficiency test but also sparked the decision to develop and standardise a mathematics proficiency test for English-, Afrikaans- and Sesotho-speaking grade four, five and six learners. The request to standardise the grade one, two and three mathematics proficiency test for Afrikaans- and Sesotho-speaking learners was an extension of the development of the original *VASSI* Mathematics Proficiency Test and will therefore not be discussed in this study. The Free State Department of Education was then contacted to grant permission for the development of an intermediate phase mathematics proficiency test. According to the Free State Department of Education they viewed the process as a necessity and welcomed the research proposal with anticipation.

1.2 PROBLEM STATEMENT

Often in a young child's functioning, cognitive problems arise such as the inability to perform various mathematical tasks. The first task in helping a learner who is struggling with mathematics is to identify the problem. According to Skemp (1991) mathematics can be seen as a powerful example of the functioning of human intelligence. For learners to succeed at mathematics they need to go through various developmental phases. Various cognitive processes form part of these phases. Once mathematical thinking has developed in a learner, certain tasks are required of the learner. If a learner cannot carry out the various cognitive tasks on which mathematics builds, the learner may begin to dislike the subject (Dockrell & McShane, 1993). If this problem is not identified it could hinder the acquisition of more advanced mathematical skills. According to the Diagnostic and Statistical Manual of Mental Disorders, Fourth Edition (DSM IV, 1994) problems in mathematics could hinder linguistic skills, perceptual skills and attention skills. When learners fail to meet the expectations of the curriculum, mathematics becomes a major assessment concern.

Several changes are occurring in school mathematics in an effort to equip learners with mathematical skills necessary for living and working in the 21st century. New insights regarding how children learn are also influencing teaching. Too often, focus is placed on how mathematics is taught instead of exploring how children learn mathematics. Learners need to make sense of what is going on during a mathematics lesson. To help learners develop meaning, a teacher provides experiences that foster mental manipulations. Psychologists refer to these mental manipulations as cognitive processes. When a learner is unable to carry out the cognitive processes necessary for task completion, mathematics becomes a major assessment concern.

When a learner fails to meet the expectations of the curriculum or fails to carry out the cognitive processes necessary for task completion, the researcher aims to identify and address this. Just as in the case of the foundation phase mathematics proficiency test, the researcher wishes to develop an intermediate phase mathematics proficiency test with South African norms.

1.3 AIM OF THE STUDY

In view of the above, a need has arisen to construct a test that will enable a psychologist or educationist to evaluate learners at any stage of their intermediate phase. When a general scholastic evaluation is carried out, various psychometric tests are administered. The administration of a mathematics proficiency test can only serve to improve the quality of the evaluation and assist the educationist or psychologist to obtain results that are comprehensive and objective. The researcher proposes to develop a mathematics proficiency test with the following criteria in mind:

a) that the test will be applicable to grade four, five and six learners;
b) that the test will be standardised for English-, Afrikaans-, and Sesotho-speaking learners;
c) that the items selected for the final mathematics proficiency test will, as far as possible, be bias-free and culturally friendly - that is, the test will be cross-culturally adapted;
d) that the norms per term will be available, so that the test can be administered at any time of the year;
e) that the test can be administered to groups or individuals;
f) that the test can be used diagnostically to identify the learning strand in which the learner is experiencing problems, as well as the specific cognitive processes that could be hindering the learner's mathematical performance;
g) that the tests can be utilised by a psychologist or an educationist; and
h) that the test will be of value to future generations of learners.

1.4 CHAPTER EXPOSITION

There are four main focus areas in this study. In chapter two, the researcher wishes to explore the current educational system in South Africa. Specific reference will be made to the Mathematical Literacy, Mathematics and Mathematical Sciences (MLMMS) learning area (Mathematics Learning Area Statement, 2001).

Secondly, the focus will shift to the intermediate phase learner. Specific reference will be made to the intermediate phase learner's cognitive processes. Chapter three will therefore focus on the *learning-teaching process of the cognitive model for mathematics* proposed by Holmes (1985). The model consists of six key concepts. The first key concept is the three categories of representing experience, namely the enactive, iconic, and symbolic modes. The second key concept is motivation. This is of vital importance to promote learning amongst all children. The third key concept is the learner's individual differences. Each learner is unique and therefore this concept forms a vital part of the model. The next key concept is the categories of cognitive processes. Emphasis will be placed on the cognitive categories and cognitive processes necessary to conduct various mathematical tasks. Holmes (1995) states that there are six goals for teaching and learning, and these will be discussed in this section. The penultimate key concept is instructional procedures. The framework for teaching (Holmes, 1995) will form part of this concept. The final key concept is conceptual learning, which is the outcome of the learning process.

The third focus of the study is highlighted in chapter four. The standardisation of psychometric tests, as well as the measurement, item analysis, differential item functioning, reliability and validity of items that are selected for tests are discussed.

Chapter five summarises the phases of the research methodology. Phase one is the construction of the preliminary test. Phase two is the item selection and analysis, while phase three is the determination of the norms and phase four is the validity study. Finally, in chapter six, conclusions and recommendations for future research are considered. A schematic diagram representing the chapter exposition is presented in Figure 1.1.

Figure 1.1 Graphical representation of the chapter exposition

2. CURRENT EDUCATIONAL SYSTEM IN SOUTH AFRICA

The understanding of the current educational system in South Africa and the various learning areas, with specific reference to the mathematics learning area, is of vital importance prior to the development of the mathematics proficiency test.

According to Pretorius (1998) the greatest challenge in education since 1994 has been to create an educational system that would fulfil a vision to open the doors of learning to all. The challenge was also to integrate both prior learning and all forms of learning into an equitable system, which would produce quality education and training to learners, of all ages, throughout South Africa. To help achieve this challenge, the government started to put some developmental initiatives into place. Various educational White Papers were released and new policies and Acts of education were promulgated into legislation. One of the main developmental initiatives was the formation of the South African Qualifications Authority (SAQA), which was given the task of developing and implementing the National Qualifications Framework (NQF).

2.1 STRUCTURE OF THE SAQA

The SAQA consists of a chairperson and members nominated from diverse interest groups in education. The groups include labour, business, the teaching profession, the National Training Board, universities, technikons, the special education needs sector, the technical college sector and the basic adult education sector. The Minister of Education can also appoint no more than six members to serve on the SAQA (Pretorius, 1998).

The four main functions of the SAQA as outlined in SAQA Act No. 58 of 1995 (Pretorius, 1998) are to oversee the development of the NQF, to oversee the implementation of the NQF, to advise the Minister of Education on matters affecting the registration of standards and qualifications, and to consult with all affected parties. Various bodies and structures associated with the SAQA undertake and carry out the above four functions, namely the Standards Generating Bodies (SGB's), the National Standards Bodies (NSB's), the Qualifications Councils (QC's) and the Education and Training Quality Assurance Bodies (ETQA's) (Lemmer & Badenhorst, 1997).

2.2 STRUCTURE OF THE NQF

The NQF is a structure designed with the aim of improving the quality of education in South Africa. The main provision made by the NQF is the creation of opportunities for all, regardless of age, circumstances, gender and level of education, for lifelong learning in accordance with nationally agreed qualification levels. The NQF consists of eight levels. Upon completion of level one the learner will obtain a General Education and Training Certificate. Upon completion of levels two to four the learner will obtain a Further Education and Training Certificate. According to the Human Sciences Research Council (HSRC, 1995) levels five to eight comprise the Higher Education and Training Band. For purposes of this study emphasis will only be placed on level one. In Table 2.1, the General Education and Training Certificate is depicted in more detail.

Table 2.1: The National Qualifications Framework (HSRC, 1995)

NQF level	**Learning band**	**Types of qualifications**	**Locations of learning for units and qualifications**		
One	General Education and Training band	**Senior phase** grades seven to nine	Formal schools (urban/rural/ farm/special/ early childhood development centres)	Work-based training/ Occupational training/Re-construction and Development Programme/ Labour market schemes/ Upliftment programmes/ Community programmes/ Development schemes	Non-governmental Organisations/ Churches/ Adult Centres/ Private providers/ Industry/ Training boards/ Unions/ Workplace training
		Intermediate phase grades four to six			
		Foundation phase grades one to three			
		Preschool year five			

As indicated in the table above, the first band, namely the General Education and Training (GET) band, is made up of four phases, namely the preschool, foundation, intermediate and senior phases. It must be noted that English, Afrikaans and Sesotho are mediums of instruction in the foundation phase in the Free State. For purposes of this study, emphasis will be on the intermediate phase, and the learners in this phase are taught in English or Afrikaans only. The choice to be taught in Sesotho falls away in the intermediate phase.

2.3 THE INTERMEDIATE PHASE

This phase consists of grades four, five and six (previously known as standards two, three and four). According to Pretorius (1998) learning areas are the domains through which learners in the GET band experience a balanced curriculum. Learning takes place in the following learning areas (previously known as subjects):

a) Language, Literacy and Communication;
b) Human and Social Sciences;
c) Technology;
d) Mathematical Literacy, Mathematics and Mathematical Sciences;
e) Natural Sciences;
f) Arts and Culture;
g) Economic and Management Sciences; and
h) Life Orientation.

According to the Department of Education (1997) each of the above learning areas has been given notional learning time. Notional learning time refers to the time teachers spend on a specific learning area. This refers to the teacher's preparation time and actual contact time with learners, as well as the learners' efforts to master the outcome. Table 2.2 depicts a breakdown of the intermediate phase notional time. Emphasis is placed on the fact that the MLMMS learning area is given more notional time than most of the other learning areas and less notional time than the Language Literacy and Communication learning area.

Table 2.2: Notional time for each learning area (Department of Education, 1997)

INTERMEDIATE PHASE	
Learning program	**Notional time**
Language, Literacy and Communication	35%
Mathematical Literacy, Mathematics and Mathematical Sciences	15%
Natural Sciences and Technology	15%
Human, Social, Economic and Management Sciences	15%
Arts, Culture and Life Orientation	15%
Flexi Time	5%

All of the above learning areas are rooted in Outcomes Based Education (OBE). The focus OBE is discussed in greater detail below.

2.4 OUTCOMES BASED EDUCATION

According to the Draft Version of the Free State Mathematics Resources Development Project (2001) the introduction of Curriculum 2005 with its OBE approach to learning and teaching poses a challenge for all South African educators. According to the Mathematics Learning Area Statement (2001) the National Curriculum Statement (NCS) is guided by the developmental outcomes designed by the SAQA. These outcomes are critical because they apply to all learning areas, and every learning process should contribute to the achievement of the critical and developmental outcomes. The NCS is an OBE model with specific principles and goals. The foundation phase teachers started with the introduction of Curriculum 2005 in 1998, and this was followed by grades four and eight in 2001. The process was supposed to continue with the grade tens being exposed to Curriculum 2005 in 2003, but due to administrative shortfalls, the process had to be halted until further notice. OBE is therefore currently applied in grades one to nine.

Brodie (1997) states that some positive and negative feedback was given by the Association for Mathematics Education of South Africa (AMESA) National Curriculum Committee. The main positive comments were that the change in curriculum in mathematics was essential. Curriculum 2005 provided a vision and the potential for change. The potential for teacher professionalism to be developed was also greater. OBE also recognises the crucial role of the teacher in the classroom, making curriculum decisions in relation to the learners and their context. Some negative comments were that there is inadequate knowledge and experience of OBE in South Africa. Large classes and lack of resources are not conducive to effective implementation. The lack of confidence among teachers may disempower them from taking control of the new ideas.

The specific principles and goals of the OBE model, as well as the critical and developmental outcomes applicable to the learning areas, will be discussed below in more detail.

2.4.1 Principles and goals of OBE

Often one is faced with the question: What is OBE? According to the Mathematics Learning Area Statement (2001) the six principles and goals of OBE are: focus on *outcomes and process; design down; high expectations; expanded opportunity; participation; and learner- and activity-centred educational process.* It is important to consider each of the above in more detail.

OBE focuses on the *outcomes and process* of a specific learning area. Teachers and learners must focus on what the learners can do successfully. The results or outcomes expected at the end of each learning process are important, but the learning process is just as important as the desired outcomes.

Another principle is the *design down* goal where teachers and learners begin to teach and learn where the learners must 'end up'. Teaching and learning are planned from the end with the exit level and outcome in mind.

High expectations is the principle whereby all learners can learn to their full potential. Challenging standards of performance are set, where learners are challenged to do better in relation to the outcomes.

Expanded opportunity invites challenges and motivates educators and schools to take responsibility to do everything possible to afford learners opportunities to improve their performance.

The principle of *participation* means that OBE is best implemented in a participatory and democratic way. The community, teachers, learners and parents must share in the assessment thereof.

Finally, *learner- and activity-centred education* focuses on teaching according to the learner's needs. Teaching techniques and activities are designed to stimulate self-discovery. Learning is therefore interactive and the aim is to allow learners the opportunity to think critically and use their own experiences and those of others.

Another important aspect of OBE is the critical and developmental outcomes. What is meant by critical and developmental outcomes?

2.4.2 Critical and developmental outcomes of OBE

The critical and developmental outcomes are at the core of the NCS. All learning area outcomes contribute to the critical and developmental outcomes. The outcomes require an activity-based approach to learning and teaching. According to the Mathematics

Learning Area Statement (2001) the critical outcomes of the learning process are intended to enable learners to:

a) identify and solve problems in which responses display that responsible decisions using critical and creative thinking have been made;
b) work effectively with others as members of a team, group, organisation or community;
c) organise and manage themselves and their activities responsibly and effectively;
d) collect, analyse, organise and critically evaluate information;
e) communicate effectively using visual, mathematical and/or language skills in the modes of oral and/or written presentation;
f) use science and technology effectively and critically, showing responsibility towards the environment and the health of others;
g) demonstrate an understanding of the world as a set of related systems by recognising that problem-solving contexts do not exist in isolation;
h) reflect on and explore a variety of strategies to learn more effectively;
i) participate as responsible citizens in the life of local, national and global communities;
j) be culturally and aesthetically sensitive across a range of social contexts;
k) explore education and career opportunities; and
l) develop entrepreneurial opportunities.

Not only are there critical and developmental outcomes for OBE, but the NCS also has a vision for learners and teachers. Briefly this vision aims to assist the schooling system with political, social and economic challenges facing our country. The NCS wishes to achieve a balance between the need to develop high-level skills and knowledge and the requirements of a rights-based society. These rights refer to the principles of human rights education, which encompass democracy, participation, security, inclusivity, independence and freedom, privacy and anti-discrimination. In essence, it creates space for the growth of a skilled, critical, active, accountable and responsible citizen (Pretorius, 1998).

Each learning area also has specific outcomes. The specific outcomes represent knowledge, skills, attitudes and values within the particular context in which they are to be demonstrated. The critical and developmental outcomes referred to are applicable to every learning area. The specific outcomes of the mathematics learning area need to be explored in more detail.

2.5 LEARNING AREA: MLMMS

According to the Draft Statement on the National Curriculum for Grades 1 to 9 (1997) the above learning area is the construction of knowledge that deals with qualitative and quantitative relationships of space and time. This knowledge is expressed through language, symbols and social interaction. According to the Mathematics Learning Area Statement (2001) the teaching and learning of mathematics aims to instil in learners:

a) a critical awareness of how mathematical relationships are used in social, environmental, cultural and economic relations;
b) the necessary confidence to deal with any mathematical situation without being hindered by the fear of mathematics;
c) an appreciation for the beauty and elegance of mathematics;
d) curiosity about mathematics;
e) a passion for the learning area;
f) an ability to participate with confidence in the world of work and society by being mathematically literate;
g) an awareness of diverse historical, cultural and social practices in mathematics;
h) the recognition that mathematics is a creative part of human activity;
i) a deep conceptual understanding in order to make sense of mathematics;
j) the specific knowledge and skills necessary for the application of mathematics to physical, social and mathematical problems; and
k) the specific knowledge and skills necessary for the study of related subject matter and for further study in mathematics.

According to the Draft Statement on the National Curriculum for Grades 1 to 9 (1997) this learning area has nine specific critical outcomes. The learner must be able to:

a) manipulate number patterns in different ways;
b) demonstrate an understanding of the development of mathematics in various cultural contexts;
c) critically analyse how mathematical relationships are used in social and economic relations;
d) measure with competence and confidence in a variety of contexts;
e) use data from various contexts to make informed decisions;
f) describe and represent experiences with shape, space and motion;
g) analyse natural forms and processes as representations of shape, space and time;
h) use mathematical language to communicate mathematical ideas, concepts, generalisations and thought processes; and
i) use various logical processes to formulate, test and justify conjectures.

The teaching and learning of mathematics should be interactive and should provide opportunities for learners to engage in mathematical discussions and to develop communication, reasoning and problem-solving skills. It should not be limited to only the transmission and acquisition of mathematical knowledge. Therefore, according to the Mathematical Learning Area Statement (2001), certain mathematical skills and processes must be fostered to develop problem-solving abilities in learners. These skills (and various cognitive abilities that will be discussed in chapter three) must be taken into consideration when developing the mathematics proficiency test. These mathematical skills and processes include:

a) problem solving;
b) estimating and approximating;
c) looking for patterns;
d) reasoning;
e) predicting;

f) breaking down complex tasks into simpler steps;
g) proving and disproving;
h) using and testing hypotheses;
i) simplifying;
j) modelling;
k) calculating;
l) ordering;
m) analysing;
n) determining;
o) reflecting;
p) visualising;
q) measuring;
r) comparing and contrasting;
s) classifying;
t) interpreting;
u) making informed choices; and
v) justifying and validating.

Not only must the above skills be taken into consideration when developing the mathematics proficiency test, but the various learning outcomes for the MLMMS learning strands must also form the corner stone of the development process.

2.6 LEARNING OUTCOMES: MLMMS

The learning outcomes under the MLMMS learning area consisted of five learning strands. During the course of 2001 fractions were separated from numbers and operations, and this was described as a learning outcome on its own. The learning outcomes consist of six strands that focus on the attitudes and values required for the MLMMS learning area (as discussed on p. 15). The six strands consist of the following:

a) numbers and operations;
b) fractions;
c) patterns;
d) shapes and space;
e) measurement; and
f) data (Draft Version of the Free State Mathematics Resources Development Project, 2001).

The mathematics curriculum for each grade can be viewed in Annexure A. The researcher proposes to develop a mathematics proficiency test for each individual grade with the six learning strands in mind. The above strands need to be considered in more detail.

2.6.1 Numbers and operations

According to the Mathematics Learning Area Statement (2001) learners must be able to recognise, describe and represent numbers and their relationships. They must be able to count, estimate, calculate and check with competence and confidence when solving problems.

Koshy, Ernest and Casey (2000) state that amongst learners in their early years, the development of number deals with three aspects, namely the *cardinality of number*, the *ordinality of number* and the *use of number symbols*. The *cardinality of number* refers to the number of things in a group and the cardinal aspect of a number is used to describe the number in a set. The *ordinality of number* refers to the position or ranking of aspects and the ordinal aspect of a number refers to the number in relation to its position in a set. The *use of number symbols* refers to the symbol used to express the cardinality or ordinality of a number. The authors continue to state that learners need to be shown and taught a range of strategies for working with numbers and operations so that they can choose the most efficient strategy.

This strand leads to the development of number sense and a knowledge of basic number facts. Learners who have a good sense of number and operations have the confidence to make sense of problems in various contexts. This is also the foundation of further study in mathematics. Contexts in which learners have to count, estimate and calculate include social, economic, cultural, political, financial and environmental contexts. Learners have to expand their understanding of the concept of place value. Learners also need to be able to make accurate mental calculations and work confidently with a calculator. The outcome of this strand is that learners understand what different numbers mean, how they relate to one another, their relative size, and how they can be thought about and represented. Learners must be able to operate with numbers (Mathematics Learning Area Statement, 2001).

2.6.2 Fractions

This strand used to form part of the number and operations strand, but the need arose for it to be viewed as a separate outcome needing individual focus. According to the Mathematics Learning Area Statement (2001) learners must be able to recognise, describe and represent fractions and their relationships, and also to count, estimate, calculate and check, with competence and confidence, when working with fractions. Fraction concepts should be developed through the process of sharing problems involving physical quantities and/or drawings.

According to Koshy et al. (2000) research has highlighted the fact that learners find the concept of fractions and decimals difficult. Fraction concepts should be expanded through the use of number lines and diagrams. Learners must be able to order and compare fractions and they must develop a sense of decimal numbers. Learners must have sufficient confidence to practise calculations with decimal numbers.

2.6.3 Patterns

According to the Mathematics Learning Area Statement (2001) learners must be able to recognise, describe and represent patterns and relationships, and solve problems using algebraic language and skills.

According to Koshy et al. (2000) developing learners' number sense is of vital importance to enhance their accuracy, speed and confidence in numerical work. Reasoning abilities play an important role in predicting patterns, sequences and generalisations.

The Mathematics Learning Area Statement (2001) states that in the intermediate phase, numeric and geometric patterns are extended to the relationship between terms and between the number of the term and the term itself. These activities develop the understanding of concepts such as variable, relationship and function. The understanding of these relationships allows learners to describe the rules for generating patterns. The outcome of this strand focuses on the description of patterns and relationships through the use of symbolic expressions.

2.6.4 Shapes and space

According to the Mathematics Learning Area Statement (2001) learners must be able to describe and represent characteristics and relationships between 2-D and 3-D objects in a variety of orientations and positions. The outcome of this strand focuses on the properties, relationships, orientation, position and transformations of 2-D and 3-D objects. Learners should be encouraged to draw shapes and broaden their thinking towards an understanding of location, transformation and symmetry. Skills such as being able to visualise, interpret, calculate, reason, classify and justify are acquired from experiences with objects, drawings, construction and spatial relationships.

2.6.5 Measurement

According to the Mathematics Learning Area Statement (2001) learners must be able to use appropriate measuring units, instruments and formulae in a variety of contexts.

According to Koshy et al. (2000) length, area, volume, weight, time, angles and the use of scales all form a part of the concept of measurement. Anything which is measured is being compared to a unit of measurement. Most of the measuring work done by learners involves discrete data. Standard units of measure have to be taught to the learners, and the relationship between units must also be stressed. Learners must make conceptual links between units of measure and their uses in real life situations.

The last strand of the learning outcomes for the MLMMS learning area is data.

2.6.6 Data

According to the Mathematics Learning Area Statement (2001) learners must be able to collect, summarise, display and critically analyse data in order to draw conclusions and make predictions, as well as interpret and determine chance variation. Making sense of data involves collecting, organising, analysing, summarising, interpreting, drawing conclusions and making predictions. The focus of this learning outcome in the intermediate phase is on the acquisition of the skills needed to gather and summarise data. Through the study of data, learners develop a sense of how mathematics can be used to represent trends and patterns through the use of graphs, tables and charts.

Koshy et al. (2000) refer to data as the study of probability and statistics. Being able to enquire into any aspect of our surrounding, for the purpose of trying to increase our knowledge and improve our understanding, requires the collection and interpretation of data. The authors continue to state that learners must start handling data at a very early age. The terminology and methods of representation differ with age, but the purpose of handling data remains the same. Learners must be led to fresh inquiry.

The six learning outcomes mentioned above motivate the importance of the outcome for both the learner of mathematics and mathematics itself. These learning outcomes have associated assessment standards. Assessment standards are minimum standards for progression and should not be regarded as the limit for a learner's progress. The learning outcomes and assessment standards are interdependent and complementary. As stated previously, the curriculum (which represents the assessment standards) can be seen in more detail in Annexure A. Assessment in OBE is discussed below.

2.7 ASSESSMENT

According to Pretorius (1998) the outcomes based curriculum is strongly linked to assessment and therefore demands the implementation of reliable assessment procedures. The author continues to state that assessment has moved away from content assessment to developmental assessment. Content assessment occurs when emphasis is placed on a single evaluation event. Developmental assessment is the process of monitoring a learner's progress through an area of learning. Developmental assessment should be continuous, formative, diagnostic, criterion referenced, performance driven and authentic.

Developmental assessment implies the practice of continuous assessment (CASS). CASS is an ongoing formative assessment of the learner, which is associated with feedback, to monitor the strengths and weaknesses of a learner's performance (Pretorius, 1998). The intention in developmental assessment is to gain an estimate of a learner's current location with regard to his or her progress.

According to Pretorius (1998) the difference between content measurement and developmental assessment is the difference between a behavioural approach to learning and a cognitive approach to learning. The differences can be seen in Table 2.3 below.

Table 2.3: The shift from content measurement to performance assessment (Adapted from Pretorius, 1998, p. 84)

CONTENT MEASURMENT	**DEVELOPMENTAL ASSESSMENT**
Behavioural approach to learning and assessment: a) accumulation of isolated facts and skills; b) assessment activity separate from instruction; c) assessment of discrete, isolated knowledge and skills.	Cognitive approach to learning and assessment: a) application and use of knowledge; b) assessment integrated with teaching and learning; c) integrated and cross-disciplinary assessment.
Paper-pencil assessment: a) text-book based knowledge; b) academic exercises; c) implicit criteria.	Authentic assessment: a) use of knowledge in real life contexts; b) meaningful tasks; c) public criteria for assessment.
Single occasion assessment	Portfolios: samples over time
Single-attribute assessment: a) isolated knowledge or discrete skills.	Multidimensional assessment: a) knowledge, abilities, thinking processes, metacognition and affect.
Major emphasis on individual assessment: a) learners assessed individually with much secrecy surrounding tests.	Group assessment: a) collaborative learning and products.

It is clear that the new OBE and training approach is a shift away from the previous curriculum, where content was the main focus. This curriculum's driving force is the achievement of critical and specific outcomes with a cognitive approach to learning and assessment.

2.8 CONCLUSION

Chapter two focuses on the current educational system in South Africa, with special reference to OBE and the MLMMS learning area. The six strands, which are the learning outcomes in this learning area, are numbers and operations; fractions; patterns; shapes and space; measurement; and data. The researcher wishes to base the mathematics proficiency test upon these learning strands.

An integral part of OBE is the fact that assessment should include opportunities to assist teachers in fulfilling their task. The success of a teacher is dependent on sound assessment practices, which should motivate learners to achieve the outcomes. The proposed mathematics proficiency test must aim to be utilised as a tool for diagnostic assessment purposes.

The shift from content to developmental assessment implies a shift from behavioural to cognitive assessment. The focus on a child's means to achieve, retain and transform information has led to an increase in interest in the cognitive processes of children (Bruner, Goodnow & Austin, 1966). Chapter three discusses the learner in the intermediate phase with specific reference to the learning-teaching process of the cognitive model for mathematics (Holmes, 1985).

The reason this model was chosen is that the Mathematical Learning Area Statement (2001) states that the teaching and learning of mathematics should be interactive and should lead to the development of mathematical communication, reasoning and problem-solving skills. Emphasis here is on learning and teaching, which is the main focus of the model.

Another reason this model is suited to this study is that the core of the model stresses conceptual learning, and the Mathematics Learning Area Statement (2001) incorporates the progressive, conceptual development of mathematics, which reflects a high skill and high knowledge curriculum, in line with international standards.

In chapter three the focus shifts from the current educational system in South Africa to the intermediate phase learner in the system. Emphasis therefore shifts from the specific mathematics tasks that are expected from a learner to the cognitive processes that are necessary to master the performance of these tasks.

3. THE LEARNER IN THE INTERMEDIATE PHASE

As discussed in chapter two, learners in the intermediate phase at school are in grades four, five and six. A learner usually enters grade four at the age of 9 and turns 10 years of age in that year. The intermediate phase learner is therefore usually between 9 and 13 years of age.

In terms of developmental psychology, there are no boundaries in terms of age, as children of the same age can have mastered different processes. According to Mussen, Conger, Kagan and Huston (1990) development is defined as the orderly and relatively continuous changes over time in physical and neurological structures, thought processes, and behaviour. The authors go on to state that there are three goals when studying developmental psychology. The first goal is to understand *universal changes* that appear in all children, regardless of the culture in which they grow up or the experiences they have. The second goal is to explain the *individual differences* in children. The third goal is to understand how children's behaviour is influenced by their *environmental context* or situation. When discussing context, reference is made to the family, neighbourhood, cultural group, and socioeconomic group. This can also be referred to as the ecology of the child's behaviour.

According to Louw (1997) there are seven universal developmental processes to be mastered by the child in the intermediate phase:

a) the refining of motor skills;
b) the consolidation of gender-role identity;
c) the development of concrete operational thought;
d) the extension of knowledge and the development of scholastic skills;
e) an increase in social participation;
f) the acquisition of greater self-knowledge; and
g) the development of preconventional morality.

The mastering of the above seven tasks takes place within the boundaries of the child's physical, cognitive, moral, emotional, social and personality development. According to Mwamwanda (1995) the environmental context of rural African children differs significantly from that of urban children in that quite a significant proportion of rural African children further their non-formal education at home. While boys are tending grazing animals, they learn a lot about herding and social life in general. Girls help their mothers with housework, which includes cooking, washing dishes, drawing water, gathering and chopping firewood. Their development in this stage is significantly affected by their environmental context, as rural African children learn a great deal more from their environment.

In developmental psychology all the various areas of development, namely physical, cognitive, social, emotional and personality, are discussed in detail. No area should be seen in isolation, as each one contributes to the holistic development of the child. For purposes of this study the cognitive development of the intermediate phase learner will be explored as part of the learning-teaching process of the cognitive model for mathematics (Holmes, 1985). Holmes's (1985, 1995) research focuses on children learning mathematics, with specific emphasis on a cognitive approach to interactive teaching and learning. The mathematics proficiency test will be developed with this approach in mind.

Any model of cognitive development must include a descriptive and an explanatory component, and the learning-teaching model does just that. A descriptive component refers to a learner's conceptual resources at any point, noting how they change with age, and the explanatory component characterises the learning mechanisms causing them (Smith & Osherson, 1995).

As discussed in chapter one, the various key concepts in the learning-teaching model represent the different principles on which the model is based. According to Holmes (1985) four principles for teaching, based on how children learn, make up the cognitive model for mathematics. These principles include:

a) encouraging the use of cognitive processes;
b) stressing learning concepts and generalisations;
c) emphasising intrinsic motivation; and
d) providing for individual differences.

The learning-teaching process of the cognitive model for mathematics (Holmes, 1985) is based upon the theories of five cognitive psychologists, namely Ausubel (1968), Bruner (1966), Piaget (1969), Skemp (1965) and Wittrock (1980). It is essential to first draft an overview of various cognitive theorists, prior to the discussion of the learning-teaching process of the cognitive model for mathematics (Holmes).

3.1 OVERVIEW OF COGNITIVE THEORISTS

According to McShane (1991) there have always been many theories about cognitive development, yet the study of children's thought did not begin until the end of the 19th century. Hall (1887) was the first scientist to study children. He attempted to introduce a scientific approach to research with children, but his interpretation of data was very speculative. Binet (1907) was very interested in the assessment of children's cognitive abilities. Although these two scientists were influential in advocating the study of children, no significant contribution was made to the theoretical understanding of a child's mind.

Baldwin (1906) was the first modern developmental theorist. He contributed to developmental psychology in three areas, namely cognitive development, the social and cognitive foundation of personality development, and the relation between behavioural ontogeny and behavioural phylogeny. Baldwin postulated that development begins with reflexes and then progresses through a series of five stages, namely the sensorimotor stage, the ideational stage, the prelogical stage, the logical stage and the hyperlogical stage. He also investigated the mechanisms that moved a child from stage to stage. A very important aspect of his theory was the adaptation of the child to the environment through accommodation, oppositions and assimilation.

Baldwin (1906) invented the concept of ‘feedback mechanism’. He referred to this mechanism as a ‘circular action’ that gives rise to a mental representation of the environment, known as schemes. This feedback mechanism develops when a child learns to repeat movements that have had a pleasurable effect.

Watson (1924) was the founding father of behaviourism. He attempted to apply the conditioning methods of behaviourism to children’s learning. Theories of cognitive development began to lose their appeal towards the beginning of the 1960s. Two great forces that became the cornerstone of cognitive development in children are, Piaget’s (1966) theories and the information processing approach (Sternberg, 1985). Piaget’s theory is one of the foundations of the learning-teaching model (Holmes, 1985) and will be discussed in detail later in the chapter. The information processing approach, as well as the most recent research in cognitive development in children, are discussed below.

3.1.1 Information processing approach

The human brain receives and processes vast amounts of information. The functional organisation of the brain is referred to as the cognitive system. According to McShane (1991) the basic job of the cognitive system is to receive, process, store and retrieve information. The information processing approach focuses on the cognitive processes that operate to extract information from the environment. Within the information processing approach, there are two types of research on cognitive development.

The first is concerned with the development of the information processing system itself and the second focuses on the performance of particular tasks. The first entails studies of basic perceptual processes or memory processes and addresses the question of what develops in the basic information processing system. The second type of research is concerned with task performance in children that results from children applying computational rules to a task (McShane, 1991).

The information processing approach cannot be regarded as an alternative theory to Piaget's (1969) theory; instead theorists who derive their theoretical constructs from various frameworks use the approach. The information processing approach is therefore not a single theory but consists of various theories constructed within the information processing framework. The information processing approach (Sternberg, 1985 and Vygotsky, 1966) is important when considering cognitive development. Vygotsky arrived at the same conclusions about concept formation as Piaget, isolating stages of cognitive development. Vygotsky placed far greater emphasis on the role of communication, social interaction and instruction in determining the path of development (Wood, 1998).

3.1.2 Recent research

Flavell (1999) summarised the research carried out on the development of learners' knowledge over the past 15 years. Most of the theories stemmed directly or indirectly from Piaget's theory and research. Consistent with the Piagetian view, many studies since the 1950s have documented increases with age in various perspective-taking abilities. There has been an increase in metacognitive theories, which deals with children's memory, language, communication, perception, attention, comprehension and problem-solving abilities. According to the author there have been three main theories of development recently, namely the '*theory' theory; the modularity theory; and the simulation theory.*

The *theory theorists* argue that experience plays a formative role in learners' theory-of-mind development. They believe that experience provides learners with information that cannot be accounted for by their present theory of mind. *Modularity theorists* believe that experience may trigger modular mechanisms but that neurological maturation is of vital importance prior to mental representations. The *simulation theorists* state that learners are introspectively aware of their own mental states and can use this awareness to infer the mental states of others through a kind of role-taking or simulation process. This is of vital importance to acquire social cognitive knowledge and skills. Simulation theorists also assume that experience plays a formative role, but that it is through role taking that learners improve their simulation skills (Flavell, 1999).

It is important to remember that all the above resort to theories to predict and explain behaviour. It is also important to remember that even though the emphasis in research on learning has changed dramatically in the last 15 years, the connection between theories of instruction and theories of learning still remains an issue (Weaver, 1985).

The above is the very reason that the learning-teaching model was selected by the researcher as a cornerstone of this study. The learning-teaching process of the cognitive model for mathematics (Holmes, 1985) is based upon five cognitive psychologists' theories. The five theorists also made a significant contribution to the model.

3.2 FOUNDATIONS OF THE LEARNING-TEACHING PROCESS OF THE COGNITIVE MODEL FOR MATHEMATICS

As mentioned at the beginning of the chapter, the model has four principles, namely encouraging the use of cognitive processes; stressing learning concepts and generalisations; emphasising intrinsic motivation; and providing for individual differences. These principles are based upon the work of Piaget, Bruner, Ausubel, Skemp and Wittrock (Holmes, 1985). Each individual's contribution to the model is reviewed below.

3.2.1 J. Piaget

According to McShane (1991) Piaget has had the greatest influence on how cognitive development is viewed. Piaget's work, whether it is based on philosophy, biology or psychology, is aimed at elaborating a theory of knowledge of how the organism comes to know its world (Gruber & Voneche, 1982).

During the 1960s and 1970s the nature of learners' thinking and learning was dominated by the ideas of Piaget. The strengths and weaknesses of his ideas laid the foundation for the 'neo-Piagetian' theories, which emerged in the 1980s (Wood, 1998). Piaget's theory cannot be ignored though. According to Piaget, all learners pass through a series of stages before they construct the ability to perceive, reason and understand rational terms (Wood, 1998).

The stages of Piaget's theory emphasise that learners construct meaning and their thought processes change, by means of active involvement with objects and ideas. Learners' thinking develops and becomes more logical as they grow older and pass through the various stages. Piaget (1969) put three principles at the centre of this theory, namely a *biological*, a *maturational* and a *hierarchical* principle.

The *biological* principle states that higher processes such as mathematical processes evolve from biological mechanisms. The concept of adaptation is derived from biology (McShane, 1991). Adaptation involves assimilation and accommodation. Assimilation is the incorporation of new experiences into cognitive structures. Accommodation is the modification of ideas and the creation of new concepts. As adaptation occurs, cognitive structures become more organised, integrated and coordinated (Holmes, 1985). The *maturational* principle states that concept formation emerges during specific age ranges, while the *hierarchical* principle states that the stages must be experienced and passed through in a given order before any subsequent stages of development are possible (Piaget, 1969). It is necessary at this point to describe the various phases, as well known as they are, of Piaget's theory.

3.2.1.1 The stages of Piaget's theory

According to Piaget (1969) the first stage is called the *sensorimotor stage.* This stage lasts from birth until approximately the end of the second year. The child acquires object permanency in this stage, which includes tasks such as retrieving and retracing. The representational processes are developed through an internalisation of action schemes. This allows the child to represent spatial displacements in this way.

According to Keats, Collis and Halford (1978) the *preoperational stage* lasts from the age of two to about seven years. It is characterised by the rapid development of representational processes. There are six representational or semiotic functions, namely imitation, play, drawing, mental image, memory, and language. The main achievement during these years is the ability to use images and symbols. Language is an especially useful symbol system, which develops during this period (Wood, 1998). The child at this stage only recognises functional relations and the reasoning is precausal and prelogical. Much of the child's behaviour is defined by the absence of concrete operational achievements. According to McShane (1991) Piaget's earlier work describes this period as a period governed by principles of egocentrism and animism.

The *concrete operational period* extends from age 7 to age 12 or 13. This is the period in which the researcher is interested and which educational institutions refer to as the intermediate phase. Learners in this period can think logically about concrete experiences. Their ability to reason reflects the development of a principle of conservation, a principle that Piaget felt was fundamental to thought (McShane, 1991). They can classify and order objects and events with an ability to conserve or grasp the idea that certain properties of objects remain invariant, though changes have been made, with respect to other properties of the object (Holmes, 1985). During this period, learners become less egocentric. Decentered and reversible thought underlies the ability to conserve, classify, order and understand mathematical concepts. Decentering is being able to take various aspects into account and utilising them appropriately to solve a problem. Reversible thought is being able to carry out inverse operations, like knowing that addition and subtraction is reversible (Holmes).

According to the Rosskopf, Steffe and Sinclair (1971) ‘concrete’ in the Piagetian sense means that a learner can think in a logical and coherent manner about objects that do exist. Learners can perform mental operations when asked purely verbal questions and when manipulating objects. Two other processes common to the concrete operational phase are the concepts of classification and seriation. Classification is the grouping of objects according to their similarities, and seriation is the ordering of objects according to their differences.

The *formal operational stage* begins at the age of about 12 or 13 years. Learners in this phase become more scientific and abstract and are able to think about their thoughts (Holmes, 1985). There is a shift away from an ability to think and reason about concrete visible events towards an ability to think hypothetically, in other words, to entertain what-if possibilities about the world (McShane, 1991).

In summary, Piaget’s theoretical arguments about the nature of thinking and the relationships between what is seen, heard and understood has a direct implication for teaching and therefore the learning-teaching process of the cognitive model for mathematics (Holmes, 1985). Showing or explaining concepts to learners before they are cognitively ready will not foster development. The learner will only learn empty concepts. A teacher can provide materials and contexts for development so that learners are free to construct their own problem-solving mechanisms (Wood, 1998). Not all theorists agree with Piaget’s theory, and some of the criticism levelled at Piaget is discussed below.

3.2.1.2 Neo-Piagetian theorists

Although Piaget passed away in 1980 his theoretical legacy remains, but many neo-Piagetian theorists have emerged. Karmiloff-Smith (Wood, 1998) proposed a new theory of cognitive development. Her theory stems from Piaget's theory but she dispenses the concept of general stages of development. According to Lancy (1983) the theoretical position of Piaget has received qualified support. She states that Piaget's theory holds up reasonably well as long as the learners tested are undergoing formal schooling.

There are many people who do not accept Piaget's theory. Many theorists believe that there are many factors that need to be considered and evaluated when trying to decide how children conceptualise the world and reason about it at different ages. If learners do not understand the words and expressions involved in attempts to test their understanding, if they are so unfamiliar with the task they are asked to perform that they do not know what is relevant to the questions asked, and if they assume that there is more to the problem than meets the eye, they may appear logically incompetent. Piaget's theory did not provide explanations for these phenomena (Wood, 1998).

Piaget's theory is not a theory of intellectual development. Concerns have been expressed regarding the way in which Piaget's theory has been applied to different aspects of education. According to Weaver (1985) Piagetian tasks are not a useful readiness measure to determine learners' readiness for learning some or other aspect of school mathematics. The author goes on to state that many learners who fail Piagetian tasks are able to learn mathematical concepts and skills.

More recent studies have cast some doubt on Piaget's theory of homogeneous performance within a specific stage. It is now believed that performance varies greatly within each stage and depends more on the acquisition and development of language, perception and real world knowledge. We cannot reject the influence of language, communication and instruction, and Piaget paid no attention to these concepts (Wood, 1998).

The learning-teaching process of the cognitive model for mathematics (Holmes, 1985) attends to these aspects, and so even though the model has roots in Piaget's theory, it incorporates aspects that the neo-Piagetian theorists thought were essential for a holistic cognitive model. The theorists' influence on the development of the model is discussed below.

3.2.2 J.S. Bruner

Bruner (1966) pays much attention to the theory and practice of teaching. He believes that in learning, learners need experiences in the enactive, iconic and symbolic modes. This will be discussed in greater detail later. According to the author, concepts and principles are the keys to understanding. Conceptual learning provides the learner with a structure of knowledge, which makes subject matter easier to comprehend, remember and apply. Bruner also states that learners need examples to guide them to the discovery of major conceptual content. Bruner also stresses that learners only learn if there is a desire or the energy to learn. These energies or intrinsic motives are expressed in curiosity, a drive for competence, identification with models, and interaction with others in the effort to achieve common goals (Holmes, 1985). Intrinsic motivation also forms an important part of Bruner's theory.

Two other important concepts of Bruner's theory are the concepts of reinforcement and readiness. Bruner (1966) maintains that learning depends upon whether or not the knowledge of results is useful to the learner. Learners must also possess a readiness to learn subject matter that is appropriate to their intellectual development. According to Wood (1998) Bruner's theory stands between Piaget's and Vygotsky's. Like Piaget, Bruner emphasises the importance of biological constraints on human intelligence. Like Vygotsky, he stresses the way in which culture forms and transforms the child's development. Bruner (1966) also pays more attention to the role of social interaction, language and instruction in the formation of learning.

The similarities between Piaget's (1969) and Bruner's (1966) theories cannot be ignored. Both emphasise the importance of action and problem solving in learning. They adopted a similar position with regard to the different ways in which knowledge can be presented. Both theories agree that learners will only understand abstract mathematics if it is grounded in concrete, practical problem solving (Holmes, 1985). Another cognitive psychologist whose ideas contributed to the development of the cognitive model for mathematics (Holmes) is D.P. Ausubel.

3.2.3 D.P. Ausubel

Ausubel, Novak & Hansian (1978) state that learning is a process of gaining meaning from potentially meaningful material and that learners must have meaningful learning sets. It is important to understand each of the above.

Potentially meaningful material is material that is organised so that it can be logically related to learners' cognitive structures. A meaningful learning set is an ability to make sense out of an experience, in other words, to learn from experience and not only from rote learning. A learner must compare, contrast, relate, organise, question and reconcile ideas. Ausubel et al. (1978) state that the highest level of learning is when a learner integrates material into his or her cognitive structure. The teacher is responsible for the presentation of meaningful material, while the learner is responsible for engaging in meaningful learning. Ausubel et al. also believe that there are differences in the way learners learn, based on their developmental stage and age.

Ausubel et al. (1978) also place emphasis on *reception* versus *discovery* learning and *rote* versus *meaningful* learning. *Reception* learning is when the entire content of what is to be learned is presented to the learner. The learner's task is to internalise or incorporate the material so that it can be reproduced at a later stage. *Discovery* learning involves the learner having to discover the principle content on his or her own and incorporate it meaningfully into his or her cognitive structure. The two types of learning differ with respect to their roles in developing intellectual functioning. For large bodies of subject matter reception learning is mostly used, while for everyday problems of living, discovery learning is used. The ideal is obviously an overlap of the two (Ausubel et al.).

Meaningful and *rote* learning contribute an entirely independent dimension of learning and can take place during reception or discovery learning. *Meaningful* learning is when the learning task can be associated with something that the learner already knows. *Rote* learning occurs if the learner lacks the relevant prior knowledge necessary for making the learning task potentially meaningful and if the learner adopts the information in an arbitrary, verbatim fashion (Ausubel et al., 1978).

Ausubel's (1968) contribution to the learning-teaching process of the cognitive model for mathematics (Holmes, 1985) is that methods of teaching must be related to the nature of the learning processes in the classroom and to both the cognitive and affective social factors that influence it. R.R. Skemp's contribution to the model is discussed below.

3.2.4 R.R. Skemp

Skemp is a cognitive psychologist and a mathematician (Holmes, 1985). Skemp (1971) states that the development of mathematics involves the acquisition of knowledge structures and concepts. According to the author, there are primary and secondary concepts and higher and lower order concepts in the cognitive process. Primary concepts are abstracted from sensory experience and secondary concepts are abstracted from other concepts. Higher and lower order concepts refer to the greater or lesser degrees of abstraction.

In Skemp's (1991) more recent work he states that learners will not succeed at a cognitive task, specifically mathematics, if they are not taught to use their intelligence. The phases of mathematical processing involve the construction of mathematical knowledge. The continuity between mathematics and the everyday use of intelligence must be established. Mathematics, according to the author, does not need special cognitive abilities - the learners must just use their abilities in special ways. The author also agrees with Piaget in that the learners must have the mental capacity available for each new level of learning. The author goes on to state that sensory and motor experiences must precede paper and pencil work in the study of fundamental ideas.

Skemp (1991) states that learners need to observe, listen, reflect and discuss in order to increase their cognitive experiences. The experience will lead to knowledge, which in turn will lead to plans. A plan is necessary to achieve a goal and learners need to compare plans in order to select the most advantageous. Plans based on knowledge need skill. Skill, according to the author, is the ability to put plans easily and accurately into action. When a plan is executed often, the plan turns into a skill. Knowledge, plans and skills are therefore the cornerstone of Skemp's (1971) theory.

The teacher's role is to help the learners construct new concepts. These concepts need to become schemas. Schema construction entails three modes of building and testing. The first mode involves the building of knowledge from direct experience. The second mode is communication whereby the learner communicates knowledge of his schemas with others and compares others schemas with his own. The third mode is from within, where the learner forms higher order concepts by processes of imagination, intuition and creativity (Skemp, 1991). The last theorist to play a role in the foundation of the learning-teaching processes of the cognitive model for mathematics also, like Skemp, emphasises the important role of instructional procedures.

3.2.5 M.C. Wittrock

Wittrock (1980) emphasises two learning processes from a neuroscience and cognitive psychology perspective, which are attention and encoding. Attention involves the active construction of models of reality. Encoding is an information processing function, used to construct and store abstract and concrete representations and interpretations of experiences. Wittrock (1980) suggests that we do not try to discover and understand the information processing systems of learners or their individual differences but that we try to correlate environmental events with learning. Emphasis must be placed on how learners transform teaching and instruction into functional information. Research on cognitive processes emphasises the generative nature of learning and the interplay between the environment (teaching) and the learner's cognitive processes (Wittrock).

Wittrock (1979) developed a model for instruction, which he based on the premise that learners construct their own meaning during instruction. Wittrock's model has three levels, namely:

a) when teachers ascertain that learners are not meaningfully processing information, they make explicit the relations between previously learned material and new information. This is done by use of verbal or imaginal devices that highlight the organisation and specific details of the material;
b) when teachers ascertain that learners are meaningfully processing material when asked to do so, they use oral and written activities to ensure that learners generate relevant relations in a verbal or imaginal mode;
c) when teachers ascertain that learners spontaneously generate appropriate relations, they direct learners' attention to higher-level concepts, significant issues, and to complex interactions (Holmes, 1985, p.11).

Wittrock (1979) clearly indicates the interaction between the learner and the teacher. He states that teachers are responsible for planning activities and interactions that stimulate learning. Learners are responsible for the active processing of information and for attending to learning tasks. He also believes that learning is a discovery process. Learners are responsible for cognitive activity whereas teachers must stimulate learners to use their cognitive processes in the learning act.

All the above theorists emphasises the interaction between the teacher and the learner as being of vital importance. Let us explore how the above theories are molded to construct the learning-teaching processes of the cognitive model for mathematics (Holmes, 1985).

3.3 THE LEARNING-TEACHING PROCESS OF THE COGNITIVE MODEL FOR MATHEMATICS

The ideas of the above five cognitive psychologists, that is, Piaget, Bruner, Ausubel, Skemp and Wittrock, have been woven together to construct the cognitive model for mathematics (Holmes, 1985, 1995). Even though each theorist has a unique approach to the study of learning, each agrees that learning is a mental process involving the construction of meaning. The six key concepts of the model include modes of representing experience; motivation; individual differences; cognitive processes; teaching procedures; and conceptual learning. These six key concepts are graphically represented in Figure 3.1. Each of the six key concepts of the model (Holmes) is reviewed below.

Figure 3.1: The learning-teaching process of the cognitive model for mathematics (Holmes, 1985, p.18)

3.3.1 Modes of representing experience

The outer circle represents the modes of representing experience. This concept is based on Bruner's (1966) cognitive theories. He focused mainly on the theory and practice of teaching. The author believes that learners master subject matter using three modes of representing experience: *enactive, iconic and symbolic*. These modes parallel to a certain degree with Piaget's (1969) theory.

3.3.1.1 Enactive mode

The *enactive mode* refers to modelling behaviour, in other words, the fact that children learn from actions. The *iconic mode* is characterised by the use of images in understanding the world. The *symbolic mode* involves the representing of reality by the use of language (Bruner, 1966). Bruner believes that in learning, children need experiences in all three modes.

In terms of the *enactive mode*, children learn from actions and actions should be modelled on those of teachers. Long (1998) did a study to investigate mathematics teaching and learning at the grade four level. What happens in a mathematics classroom is complex, and there has been much debate on the best teaching strategies to use. Through her study, Long clearly deduced that there is a close link between learners' output and their classroom experience. Bear, Minke, Griffin and Deemer (1998) found that the teacher's feedback is the most common criterion used by learners to judge their academic performance. According to Hammill and Bartel (1990) ineffective instruction accounts for more problems in mathematics than any other factor.

With the implementation of Curriculum 2005 and OBE, there have been many attempts to improve mathematics teaching and learning. There has also been a shift away from focusing on the child and how the child learns and what conceptual levels must be reached, to focusing on the teacher. The focus is primarily on the style and attitude that the teacher employs in the classroom. What is also of importance is the teacher's understanding of the principles underlying the new curriculum.

In her research Long (1998) research she conducted a pre and posttest. She investigated the relationship between effective teaching practices and achievement. She first established the curriculum requirements at a grade four level and then set up the pre and posttest to cover number work, basic operations and problem solving. According to Long the test had the limitations of all paper and pencil tests. Language could have been a factor and even though the items were translated into Xhosa, the reading level might have been too difficult. However, the findings were as follows:

a) the learners in the sample group had little understanding of the knowledge and skills that are integral to the intended curriculum; and
b) learners had limited knowledge of how the number system works past two digits and had limited knowledge of place value. The application of addition, subtraction and multiplication was lacking in at least 75% of the learners tested.

Long (1998) agrees that what happens in a mathematics classroom is complex. The study clearly revealed that learners are given very few opportunities to practise difficult examples and that learners are not given opportunities to read, understand and use information in the mathematics texts.

Not all intermediate phase teachers are mathematics subject experts. Long (1998) states that Curriculum 2005 has a broad framework in place and therefore mathematical content and processes are not clear. What should be carried out in various grades is not clear. What is considered grade four work in one school could be grade one work in another. The author suggests that teachers should work collectively to interpret the curriculum and to produce the best lesson for each outcome.

3.3.1.2 Iconic mode

In terms of the *iconic mode*, the use of images in understanding the world should be represented in the books and notes that the learners receive. According to Webb (1998) mathematics education in previously disadvantaged schools at present is characterised by a 'cycle of mediocrity'. The infrastructure for teaching not only mathematics but science as well, is poor. Materials are in short supply. Teachers are under-qualified, which in turn produces weak and poorly prepared school learners. Webb also provides shocking statistics, which state that out of every 312 Black learners entering school, only 113 obtain a Senior Certificate and only 27 obtain matriculation exemption. Of these 27 only one offers mathematics and physical science as matriculation subjects. This one in 312 compares to one in five for White children, one in six for Indian learners and one in 46 for Coloured learners.

Some school textbooks in South African schools contain serious errors. They are not well written and not worth reading and therefore discourage the bright and motivated child (Webb, 1998). In addition to the problem of poorly qualified teachers and inferior textbooks, the curriculum is also in need of reform. Teachers, textbook writers and examiners were not prepared for the new curriculum. They do not understand the new material thoroughly and therefore broad consultation is necessary for each of the above.

3.3.1.3 Symbolic mode

In terms of the *symbolic mode*, according to Bruner (1966) language is highly significant in learning, as it aids communication and makes possible the acquisition of knowledge and efficient problem-solving skills. According to Smith (1991) language skills are highly related to mathematics success, because mathematics symbols are just another way of recording numerical language concepts. Despite the introduction of policies and legislation by South Africa's first democratically elected government to redress the imbalances with respect to language, the promotion of Black languages is still far from the ideal. The status and use of Black languages in schools has not improved at all (Mda & Mothata, 2000). According to Heugh (2000) international evidence is conclusive that education through English only for learners whose home language is not English, fails most of them.

As it was stated in chapter two, the intermediate phase learner is taught in Afrikaans or English. Most of the Sesotho schools' learners are taught in their mother-tongue up to the end of grade three. In grade four they have to start learning in a language other than their mother-tongue. According to Bruner's (1966) theory this could have an adverse effect on their acquisition of knowledge and on efficient problem solving.

According to Geary, Bow-Thomas, Liu and Siegler (1996) a complex mixture of cultural and maturational influences affect different aspects of a learner's development of mathematical competency. The learner's age, gender, language, experience, mathematical readiness, knowledge, understanding, imagination, creativity, mood, reading ability, confidence and motivation all contribute to the learning process.

The second key concept in the learning-teaching process of the cognitive model for mathematics (Holmes, 1985) is discussed below.

3.3.2 Motivation

'The beauty of mathematics is in the eye of the beholder' is perhaps a far cry from the ideal. Webb (1998) states that *enthusiasm* is an anagram of 'mathematics in use'. 'What is the use of mathematics?' is often a question posed not only by the learner but by the teacher as well. Even though there is no doubt that large parts of mathematics are very useful and that other portions are useless academic jargon, the most significant problem today is that learner and teacher alike don't understand mathematics, which in turn leads to a dislike of the subject.

The second key concept is based on Bruner's (1966) and Wittrock's (1980) cognitive theories. Bruner pays a lot of attention to motivation. He states that intrinsic motivation is expressed in a learner's curiosity about mathematics, in his or her drive for competence, and in his or her interaction with others to achieve common goals. Wittrock suggests in his theory that significant attention must be paid to the motivation that learners bring to their learning situations.

Webb (1998) states that mathematics is the most awesome construct of the human mind. He goes on to state that mathematicians and mathematics teachers need to promote mathematics as an enjoyable, stimulating, intellectually rewarding and useful activity. This can only happen if interesting and readable publications are written and lively, popular lectures are held with exciting problem-solving activities. Webb states that in 1994 he attended a talk given by a mathematics teacher that had studied in Russia and Africa. This teacher described how there are mathematics clubs in Russia being used to improve teaching and learning and promote the enjoyment of mathematics. One of the South African teachers was concerned that such 'mathematics clubs' would take extra time outside of school hours. The teacher stated that commitment and enthusiasm were required and that the creation of a positive attitude towards mathematics would take time, but that it was worth the consequence of reconstruction.

An attitude can either be positive or negative. A poor attitude or anxiety towards mathematics might inhibit the performance of some learners (Hammill & Bartel, 1990). Positive attitudes toward mathematics would include a liking for mathematics. Liking mathematics often leads to one being competent in the subject. When learners like a subject, they show interest and this in turn leads to greater participation, which inevitably leads to achievement (Grobler, 1994). A feeling of competence is related to intrinsic motivation (Holmes, 1995). According to Holmes, results of the 1986 National Assessment of Educational Progress in the United States of America indicated that 60% of third-grade children generally liked mathematics while 65% thought they were good at it, but only 40% wanted a job involving mathematics. The belief in the value of mathematics has to relate mathematics to life.

Anderman (1998) states that studies indicate that as learners pass from elementary to middle grade their achievement as well as motivation and attitude towards mathematics decline. Due to the abstract nature of intrinsic motivation, a child often lacks this type of motivation and therefore needs extrinsic motivation such as tangible motivators, activity oriented motivators and social motivators (Kapp, 1991).

Extrinsic motivation is directed towards attaining a goal that is set by parents, teachers or peers. Intrinsic motivation is directed towards attaining a goal that one sets for oneself. To improve a learner's intrinsic motivation, feelings of competence need to be fostered. The learner also needs to develop a positive attitude towards mathematics. Teachers play an important role in this regard and they must ensure that they provide challenges appropriate to the learner's developmental levels (Holmes, 1995).

Although extrinsic motivation often encourages learners to find answers to mathematical problems, intrinsic motivation leads to persistence and self-monitoring. Intrinsic motivation is also associated with parent/teacher expectations, the content of the mathematics lesson, the awareness of the relation between achievement and effort, and peer interaction. Cooperative learning, in terms of working with a supportive peer usually increases motivation (Holmes, 1985).

Feedback given to learners by teachers, parents or peers influences their motivation and levels of competency. According to Holmes (1995) informative feedback tells a learner the nature of his or her strengths and weaknesses, whereas controlling feedback tells a learner how his or her work meets certain standards of performance set by others. A child's level of motivation is unique. Individual differences amongst learners are explored below.

3.3.3 Individual differences

According to the learning-teaching process of the cognitive model for mathematics (Holmes, 1985) teachers must provide for differences with respect to each individual learner. This key concept is based upon Wittrock's (1980) cognitive theory. Wittrock clearly postulates in his theory that teachers must give extensive attention to the individual differences of learners, with specific reference to the manner in which they process information.

Individual differences in learning mathematics are apparent in each classroom. Some learners grasp mathematical concepts easier and are eager to work with challenging problems. Other learners need to study many concrete examples before they can construct an idea. Children at all levels of ability are expected to master content that has been selected for their specific grade. Teachers need to use different methods and learning experiences for different learners to help them attain the mathematical knowledge that is expected of them in a specific grade.

Wittrock (1980) places specific emphasis on the individual difference between males and females. He found that females surpass males in the retention of unstructured verbal and spatial information but that males surpass females in the abstraction and memorisation of logically organised verbal and spatial relationships.

Individual differences can be narrow or broad. Differences can vary from self-concept to emotional factors to socioenvironmental, socioeconomic or sociocultural factors. According to Kapp (1991) learners need successful social experiences during cooperative learning to build self-confidence and self-worth. A negative self-concept has an adverse effect on academic performance. According to Skemp (1991) cognitive factors cannot be separated from affective processes, as many learners experience strong emotions during their classroom experiences. Learners can experience pleasure or displeasure at school and this can affect their learning.

According to Smith (1991) many environmental factors create individual differences in learners. These factors can limit learners in reaching their full potential and these include insufficient stimulation, poor nutrition and a negative emotional climate. A child that is adequately stimulated before the age of five tends to adjust more easily to the schooling environment and performs better at school than a child that has limited learning opportunities (Myers & Hammill, 1990). According to Kapp (1991) the environment may limit the learner and hamper the child's cognitive development. Poor socioeconomic circumstances and an environment that lacks opportunities may also hinder the learner's cognitive development.

According to Lancy (1983), as development unfolds, differences between individuals arise. Some environments push cognitive growth better, earlier and longer than others. According to Bruner (1966) different cultures do not produce completely different and unrelated modes of thought, which could be due to biological influences. Lancy therefore postulates that there is a co-evolution between culture, cognition and schooling.

Myers and Hammill (1990) state that if learners attend a school that does not reflect the same values and attitudes as their cultural environment, the learners could experience academic problems. Language is also an important sociocultural factor that can affect individual differences. If the learner's mother-tongue differs from the school's medium of instruction, the learner could struggle with scholastic pressure due to language problems (Kapp, 1991).

Awareness of differences in learners is the first step in providing for their educational needs. Learners differ in their attitude towards mathematics and in the processes that they use to learn mathematics. Teachers must be sensitive to these individual differences and must try to help every learner learn mathematical concepts and generalisations to the best of that learner's unique ability and context. A teacher's success is the result of knowing how to guide the learning of learners according to their unique individual differences.

The four principles of the model as discussed earlier in the chapter were constructed from the views of cognitive learning theorists. The theorists' influence in terms of the next key concept of the model (Holmes, 1985), namely the cognitive processes, is discussed below. This is a very important aspect of a learner's development.

3.3.4 Cognitive processes

The next concept shows the categories of cognitive processes. The mathematics proficiency test is based upon the learning outcomes of the MLMMS learning area and the cognitive processes of this model. According to Molusi (2000) children's cognition or thinking is the responsibility not only of the teacher but of the child himself. The learner's role is to exchange ideas and to render spontaneous contributions rather than only scientific knowledge. This means that there is a special need for support and self-discovery. The teacher's role is to monitor the constructivist learning of the learner. Learners learn mathematics by thinking about their experiences and in so doing construct mathematical meanings. This is an abstract process, not a concrete one.

Norrell (2001) based her research on Piaget's (1969) theory and explored the various cognitive processes that a learner in the concrete operational phase must develop. These are:

a) development of reason – the ability to conserve, classify and decentre;
b) classification – the ability to group and order subgroups;
c) conservation – the ability to reverse and manipulate two dimensions;
d) combinational skills – the ability to add, subtract, multiply and divide;
e) seriation – the ability to arrange from big to small;
f) reversibility – the ability to mentally retrace actions and be aware of cause and effect;
g) decentration – the ability to consider more than one aspect at a time, for example to know that a father is a brother and a son and a spouse;

h) perspective thinking – a loss in egocentrism leads to the ability to understand someone else's perspective;
i) information processing – utilisation of memory;
j) other cognitive processes also include storage, placement, response, attention and memory;
k) metacognition – being able to think about your thinking, and knowing the difference between something that is easy and something that is difficult; and
l) strategic thinking – being able to be a reasonable problem solver.

Piaget (1969) and Ausubel (1968) had a significant influence with respect to the fourth key concept of this model, namely the cognitive processes. Holmes (1995) states that there are six goals for teaching and learning and Holmes (1985) gives six categories of cognitive processes for learning mathematics. Let us consider the latter first.

3.3.4.1 Categories of cognitive processes

Learners learn mathematics by thinking about their experiences. Teachers must provide the experiences so that learners can use their cognitive processes. The cognitive categories that Holmes (1985) refers to are: *receiving, interpreting, organising, applying, remembering, and problem solving*. Each category stresses a specific cognitive process. In the final mathematics proficiency test each item will reflect the cognitive category and cognitive process necessary to successfully complete the item. In Table 3.1 below there is a description and an example of each cognitive process.

Table 3.1: Cognitive processes for learning school mathematics (Adapted from Holmes, 1985, p. 3)

Category	**Cognitive Processes**	**Description**	**Example**
Receiving		**Being alert to stimuli.**	
	Attending	Maintaining awareness of; perceiving; observing.	Please watch what I am writing.
Interpreting		**Using past experience to understand present experience.**	
	Translating	Restating in another mode of expression (concrete, pictorial, or symbolic) or in another form of mode; labelling.	Show these fractions on the number line.
	Comparing	Noting likenesses and differences; discriminating.	How are 2 x 3 and 3 x 2 alike and different?
	Classifying	Grouping by critical or distinguishing attributes; categorising. Arranging elements or categories by levels of increasing (or decreasing) inclusiveness; hierarchical classification.	Which sums go together: 3+1; 2+3; 4+1; 2+2 Classify the following: circle, rectangle, square etc…
	Ordering	Placing in a series in terms of an increasing or decreasing attribute or characteristic; sequencing.	Put these numbers in order.

Organising		**Forming and structuring ideas.**	
	Relating	Connecting in terms of some qualitative or quantitative property; associating in terms of perceived or defined attributes or processes; transforming.	How are addition and multiplication alike?
	Questioning	Asking for clarification; noting inconsistencies; inquiring.	A pupil says: " I don't know why dividing by a decimal gives a larger number for a quotient than the dividend."
	Inferring	Using reason to abstract concepts, principles, patterns, or rules from particulars. Using reason to move from concepts or principles to examples or to conclusions; if/then reasoning.	What do you notice about: 80/40 = 2 40/20 = 2 60/30 = 2 What is the pattern? If 5+5 = 10, what is 5+6?
	Summarising	Condensing subject matter; noting main ideas; outlining.	Let's review what we learned today about rectangles.

Applying		**Using ideas in new situations.**	
	Predicting	Foretelling; stating consequences; estimating.	Estimate the nearest hundred.
	Evaluating	Concluding on the basis of evidence that criteria have been met; checking a solution; judging.	Use repeated subtraction to check that 12 ÷ 4=3.
	Hypothesising	Postulating a relationship.	How many addition facts have a sum of 8?
	Testing	Devising and carrying out a plan to verify a hypothesis.	Write the facts of 8 to check how many facts have a sum of 8.
Remembering		**Deliberate effort to retain.**	
	Rehearsing	Reviewing and organising actions and ideas with intent to recall later; practising.	Talk to yourself about the properties of triangles.
	Imaging	Using visual or auditory representations of objects and events; mental picturing; drawing.	Close your eyes and picture five boats. Think about what 'five' means.
	Retrieving	Bringing to mind; recovering ideas; focusing on past experience; using rules.	What do you know about regrouping of ones in addition?

Problem Solving		**Finding solutions to unresolved situations.**	
	Combinations of cognitive processes		What number between 25 and 50 is divisible by 4 and a multiple of the sum of 5+6?

According to Wood (1998) young learners can and will concentrate, remember and learn if given support by knowledgeable teachers. The author refers to the cognitive process of attending, as concentrating. He states that young learners' perception, attention and memory improve with age. According to Wood the above cognitive processes should be seen on a continuum for different age groups. He states further that teachers should not approach the above cognitive processes as activities, in other words, a teacher cannot command attention, concentration or learning. The teacher must provide external, observable and social learning situations that stimulate the cognitive processes and in return help the learner to internalise the mental activities.

If teachers taught mathematics with the cognitive category in mind, the work would be structured and the learners would use the appropriate cognitive process. Teachers should therefore plan activities designed for learning a particular cognitive process. Receiving, interpreting and remembering are important in the foundation phase. Learners in the intermediate phase should be assigned tasks that require more organising, applying and problem solving. The teacher's role is to encourage learners to use cognitive processes. Learners must take the responsibility of using their cognitive abilities. What are the goals then for teaching and learning mathematics?

3.3.4.2 Cognitive goals for teaching and learning mathematics

New insight into how learners learn is influencing teaching. Teaching should help learners construct knowledge, solve problems and think critically. Learners need skills to deal with the modern world. There are six goals for teaching and learning mathematics. As discussed above, the last category of the cognitive processes is problem solving. The first goal for teaching and learning, according to Holmes (1995), is to develop *problem-solving abilities*. It is therefore clear that the cognitive categories and the goals for teaching and learning distinctly overlap.

Intermediate phase learners must be able to find answers to questions. Problem-solving steps should be taught to learners. Problem solving should also include the ability to make connections between mathematics, the learner's own experience and other school subjects. The major outcome to problem solving is when learners are able to formulate their own problems and solve problems in different ways, being aware that some problems can have more than one solution (Holmes, 1995).

According to Holmes (1995) the second goal for learners and teachers is to develop *reasoning abilities*. To see relations and connections and make sense out of something is to be able to reason about it. When learners solve problems they are forced to reason and these reasoning abilities will develop as they engage in activities that are thought provoking. According to Wood (1998) reasoning is often described in terms of 'made a discovery'; 'got there'; 'thought it out'; 'proved'; 'explained'; or 'demonstrated'.

The third goal for learners and teachers is to develop *conceptual knowledge*, which is the knowledge of concepts, relations and patterns. This type of knowledge is attained when a learner integrates new information with what is already known. As learners have to reason, compare, relate and infer problems, conceptual knowledge is formed. The learners' conceptual knowledge is broadened when they are motivated to experience and expand their own understanding (Holmes, 1995). Conceptual knowledge will be discussed in more detail at a later stage.

The fourth goal for learners and teachers is to develop *procedural knowledge*. It is important to note that procedural knowledge is useless if it is attained without understanding. Meaningless routines are mindless. If learners do not understand what they are doing or why they are doing it, they find the study of mathematics unenjoyable. Procedural knowledge, when conveyed with understanding, leads to the recognition of symbols, rules, action sequences and mathematical skills (Holmes, 1995). According to Wood (1998) the intellectual divide between procedural and conceptual knowledge is that procedural knowledge is knowing how to 'solve' a problem and conceptual knowledge is the ability to 'grasp' the connection between the problem and the physical phenomena.

The fifth goal for learners and teachers is to develop *positive attitudes*. A positive attitude has a high correlation with intrinsic motivation (Holmes, 1995). Vrey (1987) concludes that a very definite relationship exists between attitude, self-concept and motivation. If a change occurs in any of the above, it will affect the other two. Del Popola and Shannon (1987) state that , "even though the causes are many, the dynamics of underachievement show a common core of cognitive and motivational factors which are interrelated and which are found in all underachievers" (p. 165).

According to Geldenhuys (2000) the ages of 9 to 11 seem to be critical for the development of attitudes and emotional reactions towards mathematics. A teacher can encourage learner participation by creating a classroom environment that increases learners' confidence. According to Geldenhuys teachers should also be sensitive to each learner's feelings; value everyone's contribution; recognise learners' need for success; involve learners in their own learning; and make mathematics exciting. Teachers are the key to helping learners develop positive attitudes. According to Koshy et al. (2000) most learners attribute their success or failure in mathematics to a particular teacher. The authors go on to state that a teacher's own enthusiasm is an important factor when developing a child's attitude towards mathematics.

The sixth and final goal for learners and teachers is to develop *the ability to work and communicate with others.* According to Koshy et al,. (2000) learners should grow personally as individuals from the study of mathematics. If learners develop the ability to work and communicate with others, they will develop an outer appreciation that will involve some awareness of the use of mathematics in everyday life. This will include an appreciation for the social use of mathematics for communication and persuasion, from advertisements to government statistics. It will also evoke an awareness of the role of mathematics across all cultures and in all school subjects.

To summarise, the six goals for teaching and learning school mathematics is to develop:

a) problem-solving abilities;
b) reasoning abilities;
c) conceptual knowledge;
d) procedural knowledge;
e) positive attitudes; and
f) abilities to work and communicate with others (Holmes, 1995, p.1).

Continuous emphasis has been placed on the role of the teacher. The fifth key concept of the cognitive model for mathematics (Holmes, 1985) is teaching procedures.

3.3.5 Teaching procedures

Skemp (1991) and Wittrock (1980) have played a major role with respect to the instructional procedures that are discussed under this heading. Wittrock postulates a model of instructional procedures, as discussed in 3.2.5, and Skemp feels that teachers play a vitally important role in terms of communicating concepts to learners through physical examples. To encourage learners to use cognitive processes, to provide for individual differences, to initiate intrinsic motivation, and to stress learning concepts and generalisations, Holmes (1985) emphasises two types of teaching procedures, namely *managerial procedures* and *instructional procedures*. *Managerial procedures* include planning, evaluating and organising and *instructional procedures* include telling, modelling, questioning and questioning methods.

Holmes (1995) also suggests a framework for teaching, including six components that can be combined in a unique manner to help learners gain mathematical power. These include *how children learn mathematics; knowledge of mathematics; instructional strategies; instructional methods; instructional planning;* and *assessing learning.*

3.3.5.1 A framework for teaching

Holmes (1995) considers six aspects that must be reviewed when considering a framework for teaching. The first and the sixth aspects are very important for the proposed development of the mathematics proficiency test. The other aspects will also briefly be discussed.

The first aspect that needs attention is *how children learn mathematics*. According to Holmes (1995) learners learn mathematics by thinking about their experiences. The three cognitive processes that the author emphasises in the construction of mathematical knowledge are representation, reflection and automatisation. These include doing, observing, abstracting, relating, inferring and developing skills. Representation refers to the oral and written language of a subject. Reflecting refers to the awareness of relations and patterns that enable learners to construct mathematical meaning. Automatisation involves the automatic or effortless performance of acting with minimal thought, which leads to procedural knowledge. Holmes unites the above three concepts under the umbrella term of motivation and concludes that motivation directs learning.

The second aspect of the framework for teaching that needs to be reviewed is the teacher's *knowledge of mathematics*. According to Cockburn (1999) a teacher with too little knowledge can affect a child's mathematical achievement. Such a teacher teaches the basic concepts and when the learner is ready to absorb more abstract information the teacher is unable to fill this void.

Holmes (1995) says that teachers use three strategies for guiding learners to develop mathematical knowledge. These include explanation, questioning, and cognitive modelling. Explanation is telling children what to do, observe and think. Questioning guides learners in their construction of mathematical knowledge, and cognitive modelling helps learners look at another person's thinking.

The fourth aspect of the framework for teaching is the *instructional methods*. According to Holmes (1995) teachers choose one of three instructional methods to foster learning. The first includes direction instruction, which is based on the idea that learners must be told how to think about mathematics. Interactive teaching is built upon the principle that understanding develops as we mentally process information and ideas. The last instructional method is cooperative learning, which involves learners working in groups to solve problems and complete tasks. Problem solving is the most important means for stimulating thought.

The fifth element of the framework for teaching is *instructional planning*. This includes the lesson structure, the formulating of objectives and the selecting of activities.

The sixth element of the framework for teaching is *assessing learning*. According to Holmes (1995) three types of assessment are prominent, namely assessment before instruction, assessment during instruction, and assessment after instruction. The assessment method is not as important as what is being assessed. Holmes continues to state that assessment includes gathering evidence about learners' understanding, reasoning abilities, competency in problem solving, skills and attitudes. Assessment should always equip learners with positive feelings about their ability to learn mathematics.

The modes of representing experience relate to the learners' motivation, individual differences and cognitive processes. The teacher's role in terms of instructional procedures has also been highlighted. The core of the learning-teaching process of the cognitive model for mathematics (Holmes, 1985) is conceptual learning. This concept is of utmost importance, as concepts and generalisations make up the content of mathematics.

3.3.6 Conceptual learning

Skemp (1991), being a mathematician and a cognitive psychologist, emphasises conceptual learning. He states that the core of understanding mathematics is to organise the system of concepts for each new level of learning. According to Koshy et al. (2000) a concept is a property. To learn what a property means and how it is defined is to learn a concept. A conceptual structure is a set of concepts with a linking relationship between them. It can be complex and continues to grow as a learner adds more concepts and links through learning. The authors go on to state that most mathematical knowledge learnt by children in school is organised into conceptual structures. The more connections learners make between concepts, facts and skills, the more applicable the knowledge they acquire.

According to Holmes (1985) a concept is an idea representing a class of objects that have certain characteristics in common and are represented by concrete objects or symbols. Concepts, when interrelated, form higher-level ideas called generalisations. These generalisations are mathematical principles. As learners process experience, they construct concepts and generalisations. Teachers cannot teach concepts, they can only provide experiences that enable learners to create their own concepts, which in time develop into generalisations. The author suggests that the learning of concepts be seen as a three-phase process:

a) Firstly, the development of prerequisite understanding;
b) Secondly, the constructing of a concept or generalisation; and

c) Thirdly, the extension and consolidation of conceptual knowledge.

These three phases are called the learning-teaching sequence and are discussed individually below.

Developing prerequisite understanding is the building of readiness to learn a concept or generalisation. It is the role of the teacher to provide the learning, but learners must be open to the process. This understanding can be immediate or it can develop over time. Prerequisite understanding can also entail previously learned knowledge that will contribute to the formation of other concepts or generalisations.

Constructing a concept or generalisation is the forming of an idea by relating past experience to present experience. The process involves prior learning. The learner will infer from previous examples and state a formal rule. When a learner is able to produce an example, then he or she has successfully formed an idea.

Extending and consolidating conceptual knowledge is the extension of understanding that is related to ideas recently constructed. By studying content that is related to ideas, learners consolidate knowledge and rehearse it to ensure retention. When learners extend their understanding they deepen the meaning they attach to the idea. Once learners have the prerequisite understanding to construct a concept or generalisation and they begin to extend and consolidate their conceptual knowledge, they start being able to solve problems relating to the idea. Understanding is extended to problem solving. When learners begin to extend and consolidate their conceptual knowledge, they begin to grasp the interrelatedness of mathematical ideas. This leads to the building of knowledge structures, which in return develops competency in performance.

Hammill and Bartel (1990) also refer to four stages of mathematical processing. The first stage is the acquisition stage, which correlates with the developing of prerequisite understanding where the teacher wants the learners to acquire a particular mathematical concept. The second stage is the proficiency stage, which correlates with the constructing of a concept or the generalisation phase where the learner's output increases and the teacher's input decreases. The third stage is the maintenance stage where the learner has to extend the concept by practising it. The fourth stage is the generalisation stage where the learners begin to apply their new concept and generalisation to a wide variety of situations. The third and fourth stages of mathematical processing correlate with the third stage of conceptual learning, namely the extending and consolidating of conceptual knowledge phase.

If knowledge acquired previously is a prerequisite to learning a concept or generalisation, assessment entails the evaluation of the learner's recall of prior learning. Learners demonstrate that they have constructed a concept or generalisation by identifying and producing examples of the idea. Extending and consolidating conceptual knowledge helps learners develop examples of the concept in an organised way. This enables the learner to develop study-related ideas, rehearse for retention and solve problems (Holmes, 1985).

The learning-teaching sequence was designed to ensure that learners internalise concepts and generalisations and develop skills.

3.4 CONCLUSION

Many interrelationships can be observed among the principles of the learning-teaching process of the cognitive model for mathematics (Holmes, 1985). The four key principles of the model are:

a) encouraging the use of cognitive processes;
b) stressing learning concepts and generalisations;
c) emphasising intrinsic motivation; and
d) providing for individual differences (Holmes).

It is important to briefly summarise each of the above.

Firstly, a cognitive view of learning and teaching focuses on the cognitive processes used to acquire knowledge. These cognitive processes include: receiving; interpreting; organising; applying; remembering; and problem solving. The mathematics proficiency test will be developed with a cognitive foundation. The encouragement of cognitive processes guides learners to use their thinking abilities in all grades.

Secondly, stressing learning concepts and generalisations offers a learning-teaching sequence that helps learners deepen and expand their understanding of what has been learnt. This sequence is a framework for planning instruction.

Thirdly, to emphasise intrinsic motivation means that teachers help learners set their own expectations for learning. Teachers help learners relate effort to achievement and discourage them from thinking that mainly outside factors are responsible for success and failure.

Lastly, learners' individual differences must be considered at all times. It is important that teachers use different methods and learning experiences for different learners, so as to ensure that all learners attain mathematical knowledge. This model is constructed by using three modes of representation, that is, enactive, iconic and symbolic. Enactive refers to learning from actions, iconic refers to the use of images in understanding the world, and symbolic involves the use of language.

To help learners develop meaning, teachers must provide experiences that foster mental manipulations. Psychologists call these mental manipulations cognitive processes. Teachers need to provide guidance and support to learners, but learners must control their own learning. The aforementioned cognitive model for mathematics (Holmes, 1985) provides an orientation of how teaching is helping learners grasp the meaning of mathematical ideas. The model is based on the cognitive theories of Piaget, Bruner, Ausubel, Skemp and Wittrock. Each of the theorists emphasise that learning is a mental process involving the construction of meaning. The new directions in school mathematics emphasise that learners assume responsibility for their own learning.

This chapter summarises some of the most important ideas about the teaching and learning of mathematics. The model must not be seen as six unrelated concepts. Each concept links with the others and should be seen as applicable to the real world situation. The model provides a framework within which we can examine what learners know and what they have to learn in order to master the foundations of mathematics. The analysis of the cognitive processes that are needed to learn helps us etch out the interplay between conceptual development and a learner's mathematical understanding.

With the MLMMS learning strand (chapter two) and the cognitive processes of the learning-teaching processes of the cognitive model for mathematics (chapter three) (Holmes, 1985) defined as the cornerstones of the proposed mathematics proficiency test, it is of vital importance to now review the various methodological processes that are necessary for test development. One of the main objectives of this study is to develop a culturally fair and bias-free mathematics proficiency test. Before this can take place the researcher must first explore the various statistical requirements that are essential for test development. In chapter four these requirements will be reviewed and specific emphasis will be placed on the standardisation of psychometric tests with respect to cross-cultural test adaptation.

4. STANDARDISATION OF PSYCHOMETRIC TESTS

4.1 INTRODUCTION

Psychological testing remains the flagship of applied psychology (Embretson & Reise, 2000). Testing should therefore be professional, accountable and free of bias. The classical measurement models and procedures for developing and standardising psychological and educational tests, have served testing specialists for a long time. The psychometric basis of tests has changed dramatically. Although the Classical Test Theory (CTT) has served the test developer for several decades, the Item Response Theory (IRT) has rapidly become the main theoretical basis for measurement. The IRT model can be applied in many ways, one of which is using IRT modelling to identify Differential Item Functioning (DIF). DIF is used to eradicate possible bias with regard to cross-cultural tests. Before we consider the above theories in detail, it is important to understand what is meant by cross-cultural test adaptation and the standardisation of psychological and educational tests.

4.2 CROSS-CULTURAL TEST ADAPTATION

According to Kanjee (Foxcroft & Roodt, 2001) the adaptation of assessment measures is a necessity in a multicultural and multilingual South Africa. Testees coming from different backgrounds, cultures or socioeconomic standards should be given equal opportunities to respond to a test item in a test. As in the *VASSI* Mathematics Proficiency Test (Vassiliou, 2003) it was essential to assess learners in the foundation phase with an assessment tool that was available in English, Afrikaans and Sesotho, since the medium of instruction in the foundation phase is still the learner’s mother-tongue. By adapting an assessment measure in this way, cross-cultural fairness is increased and test bias is decreased.

According to Kanjee (Foxcroft & Roodt, 2001) test adaptation is based on retaining the original meaning of an item but making it more applicable to a specific context or cultural group while using the same language. The reason for adaptation is to increase fairness and validity and decrease bias of assessment measures. Adaptation can be brought about by translating an assessment tool into a learner's mother-tongue or by eliminating an item that is biased towards a specific cultural group.

To be able to standardise tests for different cultures, assessment measures have to be equivalent. For assessment measures to be equivalent, individuals with the same position on a construct but from different groups should achieve a similar score on an item. If not, the item is said to be biased (Kanjee in Foxcroft & Roodt, 2001). To ensure that assessment measures are equivalent, they need to be standardised by using statistical designs such as DIF. This will be discussed later, but it is necessary to first consider what is meant by standardisation.

4.3 STANDARDISATION

Standardised tests provide methods of obtaining samples of information under uniform conditions. In essence, uniform conditions mean that fixed questions are administered with fixed directions, timing constraints and scoring procedures. Standardised tests are prepared instruments for which administrative and scoring procedures are carefully delineated, and typically, norms are provided as interpretive aids. Standardised tests serve as aids in instructional, guidance, administrative and research decisions. The results of a sample (the so-called standardised sample or norm group) are an estimate of the results which could have been obtained from the entire population. The results of the standardised sample are now known as the norms of the test (Huysamen, 1983).

According to Mehrens and Lehmann (1991) the word 'norm' is a synonym for 'average' and is the mean score for a group. This specified group is called the norm group or reference group. A table showing the performance of the norm group is called a norm table or norms. Norms show the correlation between raw scores and some type of derived score. An appropriate norm group must be recent, representative and relevant. The norm group in this research will be the grades four, five and six learners from schools throughout the Free State, whose home language is English, Afrikaans or Sesotho.

The scores achieved by the norm group are referred to as the norms of the test. Norms appear in various forms, namely percentile ranks, standard scores, linearly transformed standard scores, normalised standard scores, McCall's *T* scores, stanines and stens (Huysamen, 1983). For the sake of this study we shall consider stanines, percentile ranks and McCall's *T* scores in more detail. Stanines are integers one to nine that are assigned to normalised standard scores. Percentile ranks correspond to a raw score equal to the percentage of learners in the norm group who achieved a score lower than the raw score. McCall's *T* scores are normalised standard scores that are converted into McCall's *T* scores by means of the following equation:

$$\text{McCall's } T = z_n(10) + 50$$ (Huysamen, 1996).

An explanation of the relationship between stanines, percentile ranks and *T* scores is given in Table 4.1.

Table 4.1: Explanation of percentile ranks and stanines (Schepers, 1992)

Explanation	**Stanine**	**Percentile rank**	***T* score**
Very poor (below 11%)	1 2	0-4 5-11	≤ 32.49 32.50 – 37.49
Poor (next 12%)	3	12-23	37.50 – 42.49
Below average (next 17%)	4	24-40	42.50 – 47.49
Average (next 20%)	5	41-60	47.50 – 52.49
Above average (next 17%)	6	61-77	52.50 – 57.49
Good (next 12%)	7	78-89	57.50 – 62.49
Very good (upper 11%)	8 9	90-96 97-100	62.50 – 67.49 ≥ 67.50

Not only should a test be standardised according to an appropriate norm group but it should also be objective.

4.4 OBJECTIVITY

According to Anastasi and Urbina (1997) objectivity means that a learner should achieve the same score on a test, irrespective of the tester, the occasion or the situation. Objectivity is necessary in the administration, scoring and interpretation of psychometric tests. Before a test can be objective, correct item analysis and selection should take place.

4.5 ITEM ANALYSIS AND ITEM SELECTION

The most important step in item analysis and selection is to ensure that one determines exactly what type of information one expects from the items, why one expects that type of information, and how one intends to use that information once one has it. The first step in item selection is to determine the purpose for which testing is to be carried out. During item analysis, as much statistical information as possible must be gathered on the items, to determine whether certain metrical requirements are met. The goal of item analysis is:

a) to gather objective information on the items; and
b) to use the information to identify appropriate items for the test that will satisfy certain characteristics with regard to degree of difficulty, minimal item bias, reliability and validity (Esterhuyse, 1997).

With the aid of item analysis:

a) items that are too difficult or too easy can be identified and eliminated;
b) items that best discriminate between learners with good and poor ability can be identified;
c) items that must be improved and those that must be rejected can be identified; and
d) items that display bias towards a specific group of the sample can be identified and eliminated (Esterhuyse, 1997; Hambleton, Swaminathan & Rogers, 1991).

Item selection and item analysis are two separate processes. The objectives of item selection are to:

a) select the appropriate items for a test;
b) develop a test that satisfies the specific characteristics of degree of difficulty, reliability and validity (Esterhuyse, 1997); and
c) obtain items that are free of bias (as far as possible) (Hambleton et al., 1991).

The development of a test is dependent on the item analysis and item selection. A test is only as good or as poor as the items available to analyse. The item statistics are determined from certain quantities that are calculated from the learner's item scores. These quantities can be based on the CTT or on the more recent IRT (Esterhuyse, 1997). For purposes of this research, the researcher will utilise the CTT (Thorndike, Cunningham, Thorndike & Hagen, 1991) and the IRT (Barnard, 1991). Further investigation with regard to DIF will also be carried out (Embretson & Reise, 2000). According to Mehrens and Lehmann (1991) data that is gathered for item analysis should be interpreted with caution. The reasons for this are:

a) item analysis data are not analogous to item validity;
b) the discrimination index is not always a measure of item quality;
c) item analysis data are tentative; and
d) to avoid selecting test items purely on the basis of their statistical properties.

The CTT and the IRT are discussed in more detail below.

4.5.1 Classical Test Theory

4.5.1.1 Introduction

CTT (Thorndike et al., 1991) was originally used to develop a test that would determine a person's ability with respect to certain items included in the test that could satisfy certain predetermined characteristics and criteria. In CTT, item statistics are dependent on abilities, in other words, learners with a high ability will make items seem easy and learners with a low ability will make items seem difficult. The discrimination index is directly related to the homogeneity of the group of learners (Barnard, 1991). The statistics obtained from the CTT are dependent on the sample and the selection of items with respect to:

a) item difficulty;
b) item variance and test variance;
c) item-test correlation and the coefficient-alpha; and
d) item-criterion correlation and criterion-related validity (Esterhuyse, 1997; Huysamen, 1983).

Each of the above will now be discussed in more detail.

4.5.1.2 Item difficulty

According to Huysamen (1983) the proportion of learners who answered an item correctly is known as the *difficulty of the item*. The sum of the difficulty values of the items equals the test mean. The difficulty value (*p*) is defined as the mean item value of the item over the total number of learners in the group. During item selection, the items with a *p*-value of between 0.2 and 0.8 are selected. An item with a *p*-value of 0.2 or less is considered a difficult item and an item with a *p*-value of 0.8 or greater is considered an easy item. The items that will be used in this test are dichotomous items because they will be ascribed a value of 0 if answered incorrectly and 1 if answered correctly (Huysamen, 1983).

According to Huysamen (1996) and Anastasi and Urbina (1997) when the test items can only be correct (1) or incorrect (0), then the difficulty value is equal to the ratio of subjects that answered the item correctly to the total group of subjects. The difficulty level of the total test can be determined by selecting items of the appropriate difficulty values.

4.5.1.3 Item variance and test variance

According to Huysamen (1983) the variance of a dichotomous item is equal to the proportion of persons passing the item, times the proportion failing it:

item variance = item difficulty x (1,00 – item difficulty).

Item variance (s^2) is directly proportional to the degree of difficulty. If an item is too difficult or too easy, the item variance is very small. The ideal difficulty value of an item is 0.5 since this gives the largest possible item variance (Esterhuyse, 1997). The variance of a dichotomous item (as in this case) cannot exceed 0.25. The variance of the total scores on the test is always equal to or greater than the sum of the variances of the items comprising the test. This means that the researcher must select items with large variances, as this will result in a test with a large variance.

4.5.1.4 Item-test correlation and the coefficient-alpha

According to Huysamen (1983) the *item-test correlation* indicates the extent to which the learner's performance on an item correlates with the performance in other items in the test, in other words, the degree of consistency of a test. If an item has a high item-test correlation it means that persons who pass the item will probably pass other items too (the opposite also applies). It is therefore advisable to select items with high item-test correlation, as this will ensure a test with high internal consistency. The higher the item-test correlation of a collection of items with a fixed test variance, the higher the *coefficient-alpha* of the test (this will be referred to in more detail when considering the reliability of a test).

4.5.1.5 Item-criterion correlation and criterion-related validity

Item-criterion correlation is the correlation between item scores and criterion scores. The item-criterion correlation is usually referred to as the discrimination value of the item (Huysamen, 1983). According to the CTT the discrimination value (r_{it}) of an item is the product moment correlation between the item score and the total item scores on the test (also known as the point-biserial). This correlation is indicative of how the learner's performance correlates with the performance on other items in the test. The higher the discrimination value, the better the item discriminates among learners who differ in terms of their position on the criterion.

According to Barnard (1991) the discrimination values are represented as follows:

0.6 and higher	=	excellent
0.4 to 0.59	=	very good
0.25 to 0.39	=	good
0.2 to 0.24	=	fair
lower than 0.2	=	poor/unsatisfactory.

The higher an individual's item-criterion correlations of the items in a test with a fixed test variance, the higher the criterion-related validity of the test. It is therefore best to select items with high correlations with the criterion (Huysamen, 1983).

4.5.1.6 Advantages and disadvantages of CTT

According to Kanjee (2001) the CTT is a model that is based on weak assumptions that are easily met. The disadvantages of the CTT are that the item statistics are sample dependent; the ability estimates are item dependent; and the information is only available at the test level and not at the item level.

According to Hambleton et al. (1991) the CTT has a number of shortfalls. The most important shortfall is that testee characteristics and test characteristics cannot be separated. In the CTT the true score expresses the notion of ability. A testee's ability is defined only in terms of his or her performance on of a particular test. The difficulty of a test item is defined as the proportion of examinees in a group who answered the item correctly. Another source of dissatisfaction is that the CTT defines reliability as the correlation between test scores on parallel forms of a test. Satisfying this definition of a parallel test is almost impossible. As previously stated by Kanjee (2001) a final criticism of the CTT is that it is test oriented rather than item oriented. The classical true score provides no consideration of how a testee will respond to a given item.

For this reason psychometricians sought alternative theories and models. They desired a theory where:

a) the item characteristics are not group dependent;
b) scores describe the examinees' proficiency and are not test dependent;
c) the theory is expressed at the item level rather than at the test level;
d) no parallel test is necessary to assess reliability; and
e) the theory provides a measure of precision for each ability score (Hambleton et al., 1991).

An alternative framework with the above characteristics is the IRT.

4.5.2 Item Response Theory

4.5.2.1 Introduction

According to Hambleton et al. (1991) and Kanjee (2001) the IRT postulates the following six basic ideas:

a) the performance of a testee on a test item can be explained by a set of factors called traits, latent traits or abilities;
b) item parameters are independent of the population of testees, in other words, testee ability estimates are not test dependent or group dependent;
c) ability parameters are independent of the choice of items, in other words, item and ability parameters are invariant;
d) precision of ability estimates can be determined at each ability level;
e) items can be selected to yield a test with certain characteristics; and
f) the relationship between a testee's item performance and the set of traits underlying item performance can be described by a monotonically increasing function called an item characteristic function or item characteristic curve (ICC).

An ICC is a mathematical function expressing the testee's ability and the characteristics of the item. The possibility of a testee answering an item correctly is dependent on the ability of the testee and the characteristics of the item. In IRT an item can be characterised by its:

a) discrimination value (*a*);
b) level of difficulty (*b*); and
c) pseudo-chance level (*c*).

According to Thorndike et al. (1991) the discrimination value (*a*) has to do with the degree to which an item can differentiate between testees with high and low ability levels. According to Kanjee (2001) the greater the discrimination value, the better the item for separating testees into different ability levels. The higher the discrimination value, the steeper the ICC.

The difficulty value (*b*) represents the ability scale, where the probability of a correct response is 50%. Like the CTT, the greater the difficulty value, the more difficult the item and the smaller the difficulty value, the easier the item (Kanjee, 2001).

The pseudo-chance level (*c*) is the guessing factor. In other words, the (*c*) represents the testee's probability of guessing the item correctly. This factor is often brought into consideration in multiple-choice items, where a testee with a low ability might guess the item correctly (Esterhuyse, 1997). According to Barnard (1991) the numerical values of the IRT item parameters are:

***a*-parameter:**	smaller than 0.3	=	very low discrimination
	$0.35 < a < 0.65$	=	low discrimination
	$0.65 < a < 1.34$	=	average discrimination
	$1.34 < a < 1.70$	=	high discrimination
	greater than 1.70	=	very high discrimination
***b*-parameter:**	smaller than –1.96	=	very easy
	$-1.96 < b < -1.64$	=	easy
	$-1.64 < b < 1.64$	=	average
	$1.64 < b < 1.96$	=	difficult
	greater than 1.96	=	very difficult
***c*-parameter:**	smaller than 0.15	=	very small possibility of guessing
	$0.15 < c < 0.20$	=	small possibility of guessing
	$0.20 < c < 0.30$	=	acceptable possibility of guessing
	greater than 0.30	=	unacceptable possibility of guessing

Test items can be selected on the basis of the above values. The independence of the item parameters are ensured by expressing the parameters as a function of ability (Oakland & Hambleton, 1995). There are several popular models in IRT. It is important to investigate the three most popular IRT models, namely the one-, two, and three-parameter models, which will be discussed below. These models are appropriate for dichotomous items (Hambleton et al., 1991).

4.5.2.2 One-parameter model

The one-parameter model is often referred to as the Rasch model, in honour of its developer. The one-parameter logistic model is mathematically equivalent to Rasch's model (Hambleton et al., 1991). According to Lord (1990) Rasch's IRT assumes that all items are equally discriminating and that items cannot be answered correctly by guessing. If the sample size is small, Rasch's estimates are more accurate than the three-parameter-model estimates. This model is called the one-parameter model because it only incorporates one item parameter, namely the difficulty value (b). The discrimination value (a) remains constant. The b-parameter for an item is the point on the ability scale where the probability of a correct response is 0.50.

According to Barnard (1991), in Rasch's model the probability of a learner of a certain 'ability' achieving a certain score on an item of a certain difficulty is estimated from the data, instead of the difficulty of an item being calculated as the proportion of learners who answered the item correctly. Rasch's model proposes a relationship between a learner's ability and the item difficulty, and can be expressed as the probability of a certain response. The more able the learner, the better the learner's chance of answering an item correctly. If it is known how a learner has performed on other items, an estimate of his or her ability can be obtained, and if it is known how other learners performed on an item, an estimate of how difficult the item is can be obtained. The chance of an accurate response is therefore a function of the difference between the learner's ability and the difficulty of the item.

According to Barnard (1991) it is also important to note that Rasch's measures are traditionally expressed as 'person abilities' and 'item difficulties' on an interval scale, as opposed to raw scores on an ordinal scale. A *logit* is the unit of measurement that results when the Rasch model is used to transform raw scores to log odds ratios on the logit scale. The value of 0.00 *logits* is usually allocated to the mean of the item difficulty estimates, and typically, estimated values vary between –3 and 3 *logits*. Negative values indicate estimates below the mean and positive values indicate estimates above the mean.

An ability or difficulty measure is obtained by converting a raw score percentage into odds of success.

In the Rasch model, where items are scored dichotomously, ability estimates of learners and difficulty estimates of items are usually derived from the analysis. The point where the probability of an incorrect answer changes to a correct answer is also called a *threshold*. For dichotomous items, the difficulty of an item is the same as the *threshold* (Barnard, 1991).

According to Hambleton et al. (1991) the one-parameter model is represented by the following equation:

where

$P_i(\theta)$ = probability that a randomly chosen examinee with ability θ answers item i correctly

b_i = item i difficulty parameter

n = number of items in the test

e = transcendental number with a value of 2.718 (correct to three decimals).

The b_i parameter for an item is the point on the ability scale where the probability of answering an item correctly is 0.5. In other words, the greater the b_i parameter, the more difficult the item and the smaller the b_i parameter, the easier the item. Values of b_i near –2.0 indicate that the item is easy and values of b_i near 2.0 indicate that the item is difficult (Hambleton et al., 1991).

4.5.2.3 Two-parameter model

According to Hambleton et al. (1991) the two-parameter model is represented by the following equation:

where

$P_i(\theta)$ = probability that a randomly chosen examinee with ability θ answers item *i* correctly

a_i = item *i* discrimination parameter

b_i = item *i* difficulty parameter

n = number of items in the test

e = transcendental number with a value of 2.718 (correct to three decimals)

D = scaling factor with a value of 1.7.

The two-parameter model is also represented by the above equation except that the discrimination value a_i and the scaling factor D are added. The *a*-parameter is proportional to the slope of the ICC on the ability scale. High a_i values result in a steep ICC and low a_i values lead to an ICC that gradually increases as a function of ability. The two-parameter model is a generalisation of the one-parameter model in that it still considers the difficulty value but allows for differently discriminating items (Hambleton et al., 1991).

According to Kanjee (2001) a more compact way to write the above formula, where the meaning of the symbols remains the same, is:

4.5.2.4 Three-parameter model

According to Hambleton et al. (1991) the mathematical expression for the three-parameter model is:

where

$P_i(\theta)$ = probability that a randomly chosen examinee with ability θ answers item i correctly

a_i = item i discrimination parameter

b_i = item i difficulty parameter

c_i = pseudo-chance level parameter

n = number of items in the test

e = transcendental number with a value of 2.718 (correct to three decimals)

D = scaling factor with a value of 1.7.

The three-parameter model is the same as the two-parameter model, except that the parameter c_i is incorporated into the model to take into account performance at the low end of the ability continuum, where guessing is a factor in test performance on selected responses (Hambleton et al., 1991).

4.5.2.5 Model selection

According to Embretson and Reise (2000) several criteria can be applied to determine the best model. From the outset it can be said that the three-parameter model will not be applied in this study, as the pseudo-chance value will not be applicable in this test. The criteria that need to be considered are:

a) the weights of items for scores;
b) the desired scale properties for the measure;
c) fit to the data; and
d) the purpose of estimating the parameters.

If the items are to be equally weighted and if the strongest scale properties are required, then the Rasch model is favoured. If fit to the existing set of items or highly accurate parameter estimates are needed, then the two- and three-parameter models may be favoured (Embretson & Reise, 2000). The various criteria determine which model is best suited to a particular application. If items are equally important, the Rasch model may be selected. In this study, the one-parameter model (Rasch model) will be utilised.

4.5.2.6 Other computations

Fit statistics are also usually computed. A lack of fit for a learner indicates that the model is an inappropriate means of describing the behaviour of the individual on that set of items. When learners respond in accordance with the model's expectation, misfit of a learner can be attributed to anomalous test-taking behaviour of some kind. Whatever the underlying cause, a response vector which is inconsistent with an otherwise well-fitting model may indicate that the test, though possibly well functioning for the group as a whole, has failed to provide an appropriate measure of the relevant ability of that particular learner (Barnard, 1991).

According to Barnard (1991) four fit statistics are produced for each estimate, namely INFIT mean square, OUTFIT mean square, INFIT *t* and OUTFIT *t*. The INFIT mean square will be referred to in this study. A useful rule of thumb is that the INFIT mean square values should range approximately between 0.75 and 1.30. More accurately, for sample sizes of 30 or more, the range is approximately twice the standard deviation of the mean square statistic. Values greater than 1.30 show significant misfit and values below

0.75 show significant overfit. Misfitting items can be interpreted as indicating one of two things:

a) it may signal poorly written items; or
b) the item may be perfectly good in itself but not form part of a set of items, which together define a single measurement trait.

The INFIT mean square values in terms of the *t*-distribution will vary around a mean of zero. Values outside the range of –2 and 2 are said to indicate significant departure from the expectations of the model.

In other words, when an erratic response pattern is identified for a particular item across all the learners in the group, it may indicate that the item is flawed in some way. The item may also not tap into the same ability as the other items in the set, or certain systematic inconsistencies in the responses of subgroups may indicate that the item is biased. This bias is better known as DIF.

4.5.3 Differential Item Functioning

4.5.3.1 Introduction

According to Hambleton et al. (1991) the most highly charged issue surrounding testing is that of test fairness. Test bias against racial or ethnic groups is unacceptable to the public. The definition of DIF is that "…an item shows DIF if the majority and minority groups differ in their mean performances on the item" (p.109).

The definition accepted by psychometricians is that "…an item shows DIF if individuals having the same ability, but from different groups, do not have the same probability of getting the item right" (Hambleton et al., 1991, p. 110).

Working under the CTT framework, many researchers have investigated measurement invariance of the item bias by examining the comparability of classical item statistics such as the item-test correlation or the item proportions between groups. These statistics are not sample variant, because item means are influenced by valid group differences on the measured trait, and item-test correlations are affected by group variability on the measured trait. In terms of the CTT framework, traditional approaches to the investigating of item bias are mostly unsatisfactory (Embretson & Reise, 2000).

In the IRT literature, the term item bias has been replaced by DIF. DIF is said to occur when a test item does not have the same relationship to a latent variable across two or more sample groups. Much research has been carried out on the logic of IRT-based DIF detention approaches, with positive feedback. It should also be noted, though, that the most commonly used DIF detection procedures are not IRT based, for example, the Mantel-Haenszel statistic and the logistic regression approaches (Embretson & Reise, 2000).

According to the research done by Allalouf, Hambleton and Sireci (1999) on the causes of DIF in translated verbal items, there are four main causes of DIF, namely:

a) changes in difficulty of words or sentences after translation;
b) changes in content after translation;
c) changes in format after translation; and
d) differences in cultural relevance.

According to Katz, Bennett and Berger (2000) researchers have noted that some items are more difficult in the constructed response format than in the multiple-choice format. According to Kanjee (Foxcroft & Roodt, 2001) the bias analysis process implies an unfair disadvantage to one or more groups. When test development has been carried out, it is essential to eliminate any unfair advantage or disadvantage to any testee. There are two methods for detecting DIF, the one being the IRT approach and the other the Mantel-Haenszel approach. In terms of the IRT theory there are two types of approaches to detecting DIF, the one being the comparison of item parameters and the other the calculation of the area between ICC's. These approaches will now be discussed.

4.5.3.2 IRT: Comparison of item parameters

DIF can be investigated by comparing the item characteristic functions of two or more groups. The most direct approach is to compare the parameters that describe the ICC. The basis of IRT is the relationship between the ability level of the testee and the difficulty level of the test item (Kanjee in Foxcroft & Roodt, 2001).

According to Hambleton et al. (1991) if the parameters of two item characteristic functions are the same, then the function will be the same and the probability of a correct response will be the same. After the item parameter estimates are placed on a common scale, the variance-covariance matrix of the parameter estimates in each group is calculated. The test statistic is then asymptotically distributed as a chi-square (χ^2) with *p* degrees of freedom (where *p* is the number of parameters compared). In the one-parameter model, the test statistic for the χ^2 is:

Where $v(b_1)$ and $v(b_2)$ are the reciprocals of the information functions for the difficulty parameter estimates and $b_{\text{diff}} = b_2 - b_1$. In other words, items that do not exhibit a statistical difference in difficulty for the two groups are regarded as items with less DIF. This can be viewed on a map, where items that are within boundaries of –2 and 2 are accepted as exhibiting less DIF than those that fall outside the boundaries.

A valid criticism of this method is that the distribution of the test statistic is only known asymptotically. The asymptotic distribution is applicable only when item parameters are estimated in the presence of known ability parameters (Hambleton et al., 1991).

4.5.3.3 IRT: Area between ICC's

Another method of detecting DIF is to compare the ICC's by evaluating the area between them. If, after placing the parameter estimates on a common scale, the ICC's are identical, then the area between them should be zero. The smaller the area between the ICC's the less the DIF. The area is calculated by taking the y-values of the ICC's (that is, the probability of a correct response or theta) and obtaining the absolute value of the difference. This is done at 0.1 intervals, thus forming rectangles all the way across the theta scale from –3 to +3 and adding the area for each small rectangle together to get the overall total area (Osterlind, 1989). According to Hambleton et al. (1991), in determining the area, numerical procedures were used until Raju (1988) derived an expression for computing the area between ICC's. The numerical procedures involved:

a) dividing the ability range into k intervals;
b) constructing rectangles centred around the midpoint of each interval;
c) obtaining the values of the ICC's at the midpoint of each interval;
d) taking the absolute value of the differences between the probabilities; and
e) multiplying the difference by the interval width and summating (Hambleton et al., 1991).

Raju's (1988) expression for the three-parameter model is:

where

a_1	=	discrimination parameter of curve one
a_2	=	discrimination parameter of curve two
b_1	=	difficulty parameter of curve one
b_2	=	difficulty parameter of curve two
c	=	pseudo-chance level parameter
e	=	transcendental number with a value of 2.718 (correct to three decimals)
D	=	scaling factor with a value of 1.7.

For the two-parameter model, the term involving *c* falls away and for the one-parameter model, the expression reduces to the absolute difference between the *b*-values for the groups (Hambleton et al., 1991). The author goes on to state that the problem with this method is determining a cut-off value for the area statistic that can be used to decide whether DIF is present. In this study, the researcher will attempt to select items with the smallest area.

Another common problem with IRT approaches is that item parameters must first be estimated in both groups. A large number of testees with a wide range of ability is also needed (Hambleton et al., 1991). Swaminathan and Rogers (1990) provide a logistic regression procedure capable of detecting nonuniform as well as uniform DIF. This will not be utilised in this study and will therefore not be discussed in detail. The most popular non-IRT model is the Mantel-Haenszel method. This method is not sensitive to non-uniform DIF and is discussed below.

4.5.3.4 Mantel-Haenszel procedure

Oakland and Hambleton (1995) suggest that the Mantel-Haenszel procedure delivers the same results as the above DIF detection mechanisms. A researcher should therefore not obtain different results by utilising the different mechanisms. The only negative aspect of this technique is that the sample groups must be similar in size for matching purposes.

According to Roussos, Schnipke and Pashley (1999) DIF occurs in an item when testees of equal proficiency (on the construct of the test), but from different groups, differ in their probability of answering an item correctly. Many statistical procedures have been developed to detect DIF, but the Mantel-Haenszel procedure is recognised as the most widely used methodology in the testing industry.

According to Kanjee (Foxcroft & Roodt, 2001) this procedure is based on the assumption that an item does not exhibit DIF. In other words, the chances of answering an item correctly are the same for two groups of testees across all different ability levels. In this procedure, the testees are first matched into comparable groups based on ability levels. The pass and fail data are then computed for the two matched groups in a two-by-two table and then the matched groups are compared on every item on the measure, for each of the ability groups. The hypothesis that the odds of a correct response to the item is the same for both groups, on all ability levels, is tested using the χ^2 statistic with one degree of freedom.

Not only should a test be free of bias but it should also be reliable and valid.

4.6 RELIABILITY

"Reliability of a test refers to how consistently it measures whatever it measures…irrespective of when it was administered, which form of it was used, by whom it was scored" (Huysamen, 1983, p.24). According to Mehrens and Lehmann (1991) reliability is the degree of consistency between two measures of the same aspect. The variation in a learner's score is called error variance and the sources of this variation are known as the sources of error. The fewer the errors, the more reliable the measurement. With the sources of variation, reliability is estimated in different ways. There are five commonly used ways to estimate reliability, namely:

a) measures of stability (test-retest reliability);
b) measures of equivalence (parallel-forms reliability);
c) measures of equivalence and stability;
d) measures of internal consistency:
    i. split-half
    ii. Kuder-Richardson estimates and
    iii. coefficient-alpha;
e) test-scorer reliability (Mehrens & Lehmann, 1991).

Each is considered individually below.

4.6.1 Measures of stability

Measures of stability are also known as test-retest estimates of reliability and are obtained by administering a test to a group of persons and re-administering the same test to the same group at a later stage. The researcher then correlates the scores to determine the reliability (Mehrens & Lehmann, 1991). This coefficient is known as the stability coefficient and gives an indication of the consistency of the scores on different occasions (Huysamen, 1996). A person is more test ready at certain times than at others. This reliability explains the unsystematic source of variation. This type of reliability is immune to the test occasion. Therefore the scores obtained from one occasion may be generalised to the total population, which could have achieved similar scores on other comparable occasions (Huysamen, 1994). To determine test-retest reliability, a test must be administered on two occasions to a representative sample of the population for which the test is intended.

4.6.2 Measures of equivalence

The equivalent forms estimate of reliability is obtained by administering two forms of a test to the same group on the same day and then correlating the results. In constructing equivalent tests, the two measures must be the same with respect to means, variances and item intercorrelations. The equality of the content is also important. This type of reliability is also known as parallel-forms reliability (Mehrens & Lehmann, 1991). Parallel tests are composed on different items of the same difficulty, measuring the same construct equally well. Parallel-forms reliability is determined by correlating the two sets of scores obtained, and this score is known as the equivalent coefficient. If a tester questions the reliability of scores obtained, a parallel-form test can be administered to correlate the scores (Huysamen, 1994).

4.6.3 Measures of stability and equivalence

If the researcher questions whether a different but similar set of questions at a different point in time would give similar results, then the researcher is looking at measures of equivalence and stability. A coefficient of equivalence and stability is obtained by administering one form of a test, and then after some time another form of the test and correlating the results. This estimate of reliability is usually lower then either of the other two procedures but is common among traits such as intelligence, creativity, aggressiveness or even musical interest, where constructs are not dependent on a specific set of questions (Mehrens & Lehmann, 1991).

4.6.4 Measures of internal consistency

The previous three measures of reliability require two testing occasions, which often is not possible. It is possible to obtain reliability estimates from one set of test data by means of split-halt estimates, Kuder-Richardson (K-R) estimates and the coefficient-alpha. These estimates (with the exception of the split-half estimates) determine the degree to which the item score correlates with the total test score. If there is a high degree of internal consistency then the test is reliable. Each of the estimates are individually considered below.

4.6.4.1 Split-half estimates

Split-half reliability or internal consistency (as it is also known) is improved when a test is lengthened by means of items measuring the same construct (Huysamen, 1990). If someone performs well in a few items of the test, the chances of this person performing well in the rest of the test are high. To determine the reliability of the whole test, a correction factor is applied. The formula used is a special case of the Spearman-Brown formula, where:

$$\text{Estimated reliability of the whole test} = \frac{2 \times \text{reliability of one test half}}{1 + \text{reliability of one test half}}$$

(Mehrens & Lehmann, 1991).

The Spearman-Brown formula assumes that the variances of the two halves are equal.

4.6.4.2 Kuder-Richardson estimates

If items are scored dichotomously (right or wrong) the test can be split using the Kuder-Richardson (K-R) estimates (Mehrens & Lehmann, 1991).

Kuder-Richardson 20 (K-R 20) and Kuder-Richardson 21 (K-R 21) are two formulas given below:

$$\text{K-R 20} = \frac{n}{n-1}\left[1 - \frac{\Sigma pq}{s^2}\right]$$

$$\text{K-R 21} = \frac{n}{n-1}\left[1 - \frac{\bar{X}(n - \bar{X})}{ns^2}\right]$$

where:

n	=	number of items in the test
p	=	proportion of people who answered the item correctly
q	=	proportion of people who answered the item incorrectly
pq	=	variance of a single item scored dichotomously
Σ	=	summation sign indicating that *pq* is summed over all the items
s^2	=	variance of the total test
$\bar{X}$	=	mean of the total test (Mehrens & Lehmann, 1991).

The difference between K-R 20 and K-R 21 is that K-R 21 assumes that all items are of equal difficulty. To determine reliability using the K-R formula, given the number of items in the test, one only needs to compute the mean and the variance of the test and substitute the values into the formula. Due to the fact that dichotomous items will be used in this study, the K-R formula will be used to estimate the reliability of the test. The coefficient-alpha is often confused with the K-R formula, but there is a definite distinction.

4.6.4.3 Coefficient-alpha

The coefficient-alpha is an index that shows the degree to which all items in a test measure the same attribute. To compute the coefficient-alpha, both the variance of the total test and the variances of the individual items must be obtained. A high internal consistency implies that the test has a high degree of generalisability across items within the test and over other parallel tests (Huysamen, 1990). The coefficient-alpha is a generalisation of the K-R 20 formula when the items are scored dichotomously. The coefficient-alpha formula is the same as the K-R 20 formula except that the pq is replaced by S, where S is the variance of a single item (Mehrens and Lehmann, 1991).

The last estimate of reliability is the test-scorer reliability.

4.6.5 Test-scorer reliability

A test is usually only as good as the person administering it or scoring it. Standardised tests have standard instructions, so different individuals administering or scoring the test will not affect the reliability.

It is not sufficient for a test to merely be reliable. The validity of the test is equally important.

4.7 VALIDITY

Reliability of a test does not imply validity. Validity refers to the extent to which the test satisfies the intended purpose. For example, the researcher wishes to determine mathematics proficiency among learners in the intermediate phase. The extent to which the test measures this construct gives an indication of the validity of the test. In discussing validity, two inferences must be remembered.

The first is a statistical inference that refers to the performance that is not being measured, while the second inference is the measurement inference, which refers to the behavioural domain of the person that is being measured. When a score is used to infer performance, the researcher is actually predicting performance. The degree to which this prediction or inference is accurate depends on the content validity, criterion validity and construct validity (Mehrens & Lehmann, 1991). Each will be discussed in detail below.

4.7.1 Content validity

Content validity is the degree to which a test represents an inventory or universe or the extent to which the items in the test are representative of the total universe of items, which could have been compiled in terms of the appropriate curriculum and teaching objectives (Huysamen, 1990). This study involves the extent to which the items in the test are representative of the total universe of items, which could have been compiled in terms of the six learning strands, namely numbers and operations; fractions; patterns; shapes and space; measurement; and data. There is no single commonly used numerical expression for content validity. Each item used in the test is judged on whether or not it represents the total domain of items that it was meant to sample (Mehrens & Lehmann, 1991).

The only way a researcher can make sure the test is content valid is to ask specialists in the field. In this research, mathematics learning facilitators, support teachers, heads of departments, contact teachers, and intermediate phase mathematics teachers from schools throughout the Free State will be asked to comment on the content of the test with respect to the curriculum. In this manner, the content validity of the test will be questioned.

4.7.2 Criterion-related validity

According to Huysamen (1990) criterion-related validity refers to the degree to which diagnostic and selection tests correctly predict the relevant criterion. Comparing the obtained score with another score of the same construct tests criterion validity. Criterion validity in this research will be tested by correlating the obtained score on the mathematics proficiency test with a mark obtained in a school evaluation. Distinction is made between two types of criterion-related validity, namely predictive validity and concurrent validity.

Predictive validity refers to the accuracy with which a test predicts future behaviour. Concurrent validity refers to the accuracy with which the test identifies some current behaviour of the learner (Huysamen, 1983).

According to Steyn (1999) the significance of the predictive validity correlation results is also dependent on the practical interest of the result. The standardised difference in the means of two scores can be viewed as the point of departure of the effect size. Effect size is the relationship between nominal and interval scale variables. Cohen in Steyn (1999) states that the product moment correlation coefficient can be used as the effect size of a linear relationship between two variables that can be measured on an interval scale. From this the researcher can determine whether the correlation coefficients have a small, medium or large effect size. Steyn (1999) goes on to state that a correlation coefficient of 0.1 has a small effect size; a correlation coefficient of 0.3 has a medium effect size; and a correlation coefficient of 0.5 has a large effect size.

4.7.3 Construct validity

Construct validity is the degree to which the instruments used for measuring variables are measuring what they are supposed to measure. Any measuring instrument measures three components, that is, the actual construct, irrelevant constructs, and random measurement error. The last is an unsystematic source of variation because it refers to accidental factors, which may vary from one individual to the next. The actual construct and irrelevant constructs refer to systematic sources of variation because they remain constant for any given individual (Huysamen, 1990). If a test has acceptable construct validity, its content and criterion-related validity must also be satisfactory. The converse does not always apply.

4.8 CONCLUSION

In a multicultural society like South Africa, the adaptation of assessment measures and the elimination of bias from psychometric tools forms a vital part of the transformation process. According to Kanjee, "…assessment practitioners must accept responsibility for ensuring that the development and use of measures, and the interpretation and reporting of information are non-discriminatory, unbiased and fair towards all South Africans" (Foxcroft & Roodt, 2001, p.101).

During the development and standardisation of a psychometric test, it is important to make sure that each of the criteria discussed in this chapter is met. It is important that the desired construct is the construct that is being measured and that the measurement is objective and standardised. Item analysis and selection therefore plays a vital role to in ensuring that the test is objective. The reliability and validity of a test must also not be overlooked.

Before items are selected for a test they undergo extensive scrutiny. Despite this, some items function differently in various subgroups. According to Rudas and Zwick (1997) and Zwick, Thayer and Lewis (2000) the absence of DIF is regarded as an important aspect of test fairness. According to Muraki (1999) DIF refers to a test item that displays different statistical properties for different groups, but it is important to assume that the abilities of the groups are the same. DIF is strictly a statistical concept and free from any judgment of the fairness of the test items.

The researcher will use the CTT and the one-parameter model of the IRT to select items. Items will also be excluded on the basis of the fit statistics. Items will also be flagged according to the two IRT-DIF detection methods, namely the comparing of parameters and the calculating of the area between ICC's. The aim of the above procedures will be to obtain cross-cultural test adaptation. The results are discussed in the subsequent chapter.

5. METHOD, RESULTS AND DISCUSSION OF RESULTS

5.1 INTRODUCTION

The development of the mathematics proficiency test is based on the intermediate phase MLMMS learning strands (chapter two), the learner's cognitive processes (chapter three) and cross-cultural test adaptation methodology (chapter four). The researcher therefore developed a mathematics proficiency test with items that are representative of all the learning strands in the MLMMS learning area and linked these items to the various cognitive tasks expected from a learner in the intermediate phase. The method and results of the above processes will be discussed in this chapter, but it is important to first conceptualise the aim of this study.

5.2 AIM OF THE INVESTIGATION

The aim of the investigation is threefold:

a) to develop a grade four, a grade five and a grade six mathematics proficiency test and to standardise the test, with applicable norms, for English-, Afrikaans- and Sesotho-speaking learners;
b) to develop a culturally fair and bias-free (as far as possible) mathematics proficiency test through the use of DIF;
c) to link the cognitive processes that are necessary for successful task completion with each item in the mathematics proficiency test.

This will not only provide a means of identifying a learner with a mathematics problem, but will help to identify the cognitive processes that are lacking for the specific task. The test will have norms available per term so that the test can be administered throughout the year.

5.3 SAMPLE

The sample consisted of nine separate groups, namely English-speaking learners from grades four, five and six; Afrikaans-speaking learners from grades four, five and six; and Sesotho-speaking learners from grades four, five and six. The English-speaking learners' home language was English and they were chosen from schools where the medium of instruction was English. The Afrikaans-speaking learners' home language was Afrikaans and they were chosen from schools where the medium of instruction was Afrikaans. The Sesotho-speaking learners' home language was Sesotho and they were chosen from previously disadvantaged schools where the medium of instruction was English. An English test was therefore constructed for the English- and Sesotho-speaking learners and translated into Afrikaans for the Afrikaans-speaking learners. It is important to note that when this investigation was carried out, all intermediate phase schools in South Africa had either an English or an Afrikaans medium of instruction. The teachers at the previously disadvantaged schools requested that the learners not be tested in their mother-tongue but in their medium of instruction, namely English.

The learners were selected from schools throughout the Free State region. The sample consisted of boys and girls. The research process is discussed in more detail below.

5.4 RESEARCH PROCESS

The research was initially meant to consist of four phases to be carried out over a period of two years. Due to the fact that the effectiveness of the psychometric test is dependent on the type of items selected in compiling the test, the researcher felt that phase one had to be repeated since the original items selected were too difficult for the sample group. The researcher then decided to utilise these results in the form of a pilot study. The research therefore consisted of five phases carried out over a period of three years.

The test needed to distinguish between learners who were good at mathematics and learners who struggled with mathematics. For a tester to determine the level at which a child is functioning, the raw score needed to be converted into a standard score, and for this, norms were needed. The psychometric test must also be reliable and valid so that the test adheres to all psychometric requirements. To ensure that the above requirements were met, the following phases, as indicated in Table 5.1 below, were carried out while compiling the mathematics proficiency test for learners in the intermediate phase.

Table 5.1: Research process

PHASE	ACTIVITY	TIME PERIOD
One	Pilot study	Third and fourth terms, 2001
Two	Construction of preliminary test	Third term, 2002
Three	Item analysis and selection	Fourth term, 2002
Four	Determination of norms	First and fourth terms, 2003
Five	Validity study	Fourth term, 2003

The results obtained during each of the above stages are discussed in more detail below.

5.4.1 Phase one: Pilot study

5.4.1.1 Introduction

Before the research process could begin, the Free State Department of Education had to grant permission for the research to be conducted. Contact was established with the Head of Education on 15 February 2001 and he was requested to grant permission for research to be conducted in the Free State Province. A letter dated 12 March 2001 was then received from the department granting permission for research to be conducted in the Free State under certain conditions (see Annexure B).

The construction of the preliminary test was the most important phase of the research. Early in 2001, a Flemish representative who was working on the redrafting of the MLMMS curriculum requested the researcher to give a presentation on the development of the test. This presentation for all the Free State mathematics learning facilitators dealt with the work that was covered during the drafting of the grade one, two and three mathematics proficiency test and on the current development of the grade four, five and six mathematics proficiency test. Input and feedback were received from the relevant learning facilitators with regard to the research process. The learning facilitators advised that the preliminary test be set up according to the six MLMMS learning strands, namely numbers and operations; fractions; patterns; shapes and space; measurement; and data. These facilitators were very interested in the research and extremely satisfied with the grade one, two and three mathematics proficiency test.

During August 2001, contact was made with the schools in the Free State region, specifically in Bloemfontein and Welkom, to obtain permission from the headmasters to conduct research at their schools and to obtain items for the preliminary test. Six Afrikaans-medium schools and 12 English-medium schools (four of which were previously disadvantaged schools) agreed to participate in the research process. In other words, six schools were selected to represent the Afrikaans-speaking sample, eight schools were selected to represent the English-speaking sample and four schools were selected to represent the Sesotho-speaking sample. The reason why so many English-medium schools were asked to participate was that the home language of many of the learners at the English-medium schools was not English.

The entire research process was then explained to the headmaster at each school and the headmaster then gave the name of a contact person that could be corresponded with throughout the study. Some of the contact teachers were the same contact teachers that had participated in the development of the foundation phase test and were therefore aware of the process. The research phases were then explained to the contact persons who, in turn relayed the information to the other staff members (involved in intermediate phase mathematics teaching) at his or her school. The first task of the contact person (aided by all the intermediate phase mathematics teachers) was to set up the preliminary test.

The following guidelines concerning the identification of the preliminary items were recommended:

a) the contact teachers (aided by all the intermediate phase mathematics teachers) had to set up 20 mathematics items (per grade) based on what is expected of a learner in the applicable grade (grades four, five and six);
b) the items had to range in difficulty, in other words five items had to be relatively easy, five items had to be relatively difficult and ten items had to be of average difficulty for the relevant grade;
c) the 20 items had to cover the entire year's curriculum for each respective grade;
d) the items had to be representative of the MLMMS learning strands; and
e) the items could then be given to the respective grades as a class test.

5.4.1.2 Selection of questions for preliminary test

Once the preliminary test had been set up by the contact teachers (aided by the intermediate phase mathematics teachers) some of the schools then administered the test to the learners in the respective grades at the schools. This was to eliminate any uncertainty pertaining to the above instructions.

Phase one was completed by the end of the third term of 2001. In total 18 different mathematics proficiency tests per grade, consisting of approximately 20 items each, were set up by the contact teachers. There were approximately 360 items per grade from which to select, giving a total of 1080 items for all three grades. Some of the contact teachers set up two or three tests and some of the tests consisted of more than 20 items. Certain Free State mathematics learning facilitators were also approached and requested to set up items. The item bank therefore greatly exceeded what had been anticipated.

The items were worked through and possible items for the preliminary test were identified in the following ways:

a) items that were repeated were eliminated;
b) the items were then checked to ensure that the items corresponded with what was expected from a learner in the specific grade. The researcher utilised the current curriculum for this purpose (see Annexure A);
c) an even distribution of items according to the learning outcomes under the MLMMS learning area was also required. Items were then grouped according to each learning strand, namely numbers and operations; patterns; fractions; shapes and space; measurement; and data;
d) the items were then grouped even further to cover what is expected from a learner in that specific MLMMS learning strand for the specific grade. In other words, items that were different, but which tested the same aspect of the learning strand, were grouped together.

Using the above process, 10 items from each learning area were then identified as items that were representative of the learning strand. This gave a total of 60 items per grade. The 60-item test for grades four, five and six was then given to a mathematics learning facilitator to review, get comments from other learning facilitators, and give feedback. The 60-item test was also given to the contact teachers at the various schools for comments. The learning facilitator changed the way in which the items were asked but not the items themselves. Very few of the contact teachers gave feedback, but of those that responded appeared to be satisfied with the items. Only one school's contact teacher and intermediate phase mathematics teachers stated that the test was too difficult.

5.4.1.3 Application of preliminary test

The preliminary mathematics proficiency test (Annexure C), consisting of 60 items per grade, was then given to 18 schools during the last term of 2001. The instructions to the contact person during this phase was to administer the 60-item test to all the respective grade four, five and six learners at their school. The 60-item test was too long to administer to the learners at one time. The test was therefore divided into test one and test two. Test one consisted of questions pertaining to numbers and operations, fractions and patterns, and test two consisted of questions pertaining to shapes and space, measurement and data. Each test counted 30 marks. The teachers were not requested to mark the items. The researcher undertook the marking.

The researcher then randomly selected approximately 200 learners per grade, whose home language was Afrikaans, from the Afrikaans-medium schools and 200 learners per grade, whose home language was Sesotho, from the previously disadvantaged English-medium schools. Learners from several different schools were selected. Due to the fact that the Free State has a shortage of learners whose mother-tongue is English all such learners from the English-medium schools were selected. A control group of 100 learners per grade, whose home language was Sesotho but who attended an English-medium school (not a previously disadvantaged school) were also selected. The data was then analysed using the SAS (SAS Institute, 1985) and the SPSS computer programs (SPSS Incorporated, 1983).

The aim of the item analysis was to gather as much objective information on the items as possible. The information gathered was used to select the final 30 items for each grade. The items had to conform to certain characteristics, as previously discussed, in respect to the difficulty value, discrimination value, and item bias, as well as the reliability and validity of the items.

The contact teacher at each school was given instructions that had to be conveyed to all other teachers with grade four, five and six classes. The instructions were as follows:

a) no calculators may be used in the test;
b) no time limit is imposed (within reasonable limits);

c) you may read the word sums to the children, as their mathematical ability and not their English or Afrikaans reading ability is being tested;

d) important details such as the learner's name, home language, age and gender must be provided. Please make sure that the learner has completed his or her details correctly (especially the home language);

e) the test must be completed on the test paper;

f) extra paper may be given for rough work.

5.4.1.4 Sample and results

According to Huysamen (1996) it is important to obtain as large a sample as possible when analysing items. The recommendation is that the sample should be five times the number of items. In this phase of the research, with 60 items per grade, the sample per grade should not be less than 300. The composition of the sample for the pilot study is represented in Table 5.2.

Table 5.2: Sample distribution during the pilot study

Grade	**Home language**				
	English	**Afrikaans**	**Sesotho (previously disadvantaged schools)**	**Sesotho**	**Total**
4	164	202	200	100	666
5	154	201	199	100	654
6	180	201	201	100	682
Total	498	604	600	300	2002

The above sample distribution indicates that the sample size for each grade was ten times the number of items in the test. The sample size for each grade was relatively even. A total of 2002 learners' test results were analysed. The basic analysis variables of the learners in their respective grades are represented in Table 5.3.

Table 5.3: Basic statistical analysis results for the pilot study

GRADE	**Sample**	**N**	$\bar{X}$	**Percentile**	**Standard deviation**	**Minimum**	**Maximum**
4	English	164	37.4	**62**	9.26	14	56
	Sesotho (previously disadvantaged schools)	200	21.1	**35**	12.56	0	47
	Afrikaans	202	37.6	**63**	10.46	9	56
	Sesotho	100	27	**45**	9.46	8	50
	Total	666	30.8	***51***	---	0	56
5	English	154	23.2	**39**	8.98	3	48
	Sesotho (previously disadvantaged schools)	199	3.4	**6**	2.76	0	17
	Afrikaans	201	29.2	**49**	13.59	3	52
	Sesotho	100	14.4	**24**	7.19	3	35
	Total	654	17.6	***29***	---	0	52
6	English	180	26.1	**44**	9.43	2	52
	Sesotho (previously disadvantaged schools)	201	4.6	**8**	3.11	0	16
	Afrikaans	201	25.1	**42**	9.83	5	48
	Sesotho	100	17.5	**29**	7.76	2	41
	Total	682	18.3	***31***	---	0	52

The average percentage for each grade is indicated in the table above. The grade four learners achieved an average of 51%, the grade five learners achieved an average of 29% and the grade six learners achieved an average of 31%. These results were of great concern to the researcher. The preliminary test proved to be too difficult for the grade five and six learners. What was also of great concern was the poor performance of the Sesotho-speaking learners, with averages of 6% and 8% in the grade five and six tests. The CTT was utilised to obtain the difficulty values. At this stage, the researcher felt that it would not be advisable to continue with the development of the test. There was also concern as to whether a DIF process could not be performed if the averages differed so significantly between the various language groups.

5.4.1.5 Conclusion

At this stage there were various options with regard to the research process. Option one was to continue, while knowing that the original test was too difficult. Option two was to develop a test for English- and Afrikaans-speaking learners only, in other words, to discard the poor results of the Sesotho-speaking learners and continue with the research process. Option three was to restart the process and use the results of this phase as a pilot study, in other words, to utilise the difficulty values to discard the very difficult items in the test. Investigation as to why the Sesotho-speaking learners' results were substantially poorer than those of the other language groups was also necessary. The researcher began to hypothesise about the influence of mother-tongue education, the influence of poor teaching, and the selection of items that were biased towards a specific cultural group. Option three was decided upon. The researcher wished to see whether similar results would be obtained if the research process were repeated. The pilot study was completed at the end of 2001.

5.4.2 Phase two: Construction of preliminary test

5.4.2.1 Introduction

During 2002, the researcher was approached by the facilitator of support teaching in the Free State. She requested that the researcher attend various meetings throughout the Free State and give a presentation on the development of the mathematics proficiency test for the foundation phase to the support teachers. She also felt that the support teachers could give vital input into the development of the mathematics proficiency test for the intermediate phase. The researcher attended four meetings in Bloemfontein, as well as one meeting each in Kroonstad, Senekal, Bethlehem and Odendaalsrus. In so doing, input from support teachers throughout the Free State was obtained.

During the third term of 2002, the original 60-item test, used in the pilot study was utilised to identify items that had a good discrimination value and were not too easy or too difficult. Eight English-medium schools (three of which were previously disadvantaged schools) and four Afrikaans-medium schools were approached, for input with regard to the preliminary test. It was decided that despite the difficulty of the pilot study test, the preliminary test should not be of a substandard quality. It was important, according to the teachers, to maintain a good-quality test. The one recommendation that was made was to shorten the preliminary test from 60 items to 50 items. The teachers felt that despite the fact that the 60-item test was administered over two days, it would be a better option to shorten the test and administer it at one sitting. Once again, as with the pilot study, the items were reviewed and possible items for the preliminary test were identified in the following ways:

a) the original test was used as a baseline for the development of the preliminary test. Items that were too difficult were discarded;

b) if many items were discarded from an MLMMS learning strand, the researcher went back and selected items from the original tests that were supplied by the teachers in the third term of 2001;
c) each item was then revised to ensure that the item corresponded with what is expected from a learner in the specific grade. The current curriculum was utilised for this purpose (see Annexure A);
d) an even distribution of items, according to the learning outcomes under the MLMMS learning area, was also required. Items were then grouped according to each learning strand, namely numbers and operations; patterns; fractions; shapes and space; measurement; and data;
e) the items were then grouped even further to cover what is expected from a learner in that specific MLMMS learning strand for the specific grade. In other words, items that were different, but which tested the same aspect of the learning strand, were then grouped together; and
f) the researcher also attempted to obtain 15 easy items (at least 80% of the learners should answer correctly); 15 difficult items (only 20% of the learners should answer correctly); and 20 items with an average difficulty value.

Representative items from each learning strand were then identified. This gave a total of 50 items per grade. The 50-item test for grades four, five and six was then given to the contact teachers at the various schools for comments. The contact teachers were more satisfied with the preliminary test and gave positive feedback about the content of the test.

5.4.2.2 Application of preliminary test

The preliminary mathematics proficiency test (see Annexure D), consisting of 50 items per grade, was then given to 12 schools during the last term of 2002. The instruction to the contact person during this phase was to administer the 50-item test to all the respective grade four, five and six learners at their schools. The teachers were not requested to mark the items. The researcher undertook the marking.

Approximately 500 learners per grade consisting of learners whose home language was Afrikaans, from the Afrikaans-medium schools, English from English-medium schools and Sesotho, from the previously disadvantaged English-medium schools were randomly selected. Learners from several different schools were selected. Due to the fact that the Free State has a shortage of learners whose mother-tongue is English, all such learners from the English-medium schools were selected.

The aim of this phase was to gather as much objective information about the items as possible. The information gathered was used to select the final 20 items for each grade. The items had to conform to certain characteristics, as previously discussed, in respect to the difficulty value, discrimination value and item bias, as well as the reliability and validity of the items.

The contact teacher at each school was given instructions to be conveyed to all other teachers with grade four, five and six classes. The instructions were as follows:

a) no calculators may be used in the test;
b) no time limit is imposed (within reasonable limits);
c) you may read the word sums to the children, as their mathematical ability and not their English or Afrikaans reading ability is being tested;
d) important details such as the learner's name, home language, age and gender must be provided. Please make sure the learner has completed his or her details correctly (especially the home language);
e) the test must be completed on the test paper; and
f) extra paper may be given for rough work.

The data was then analysed in Bloemfontein using the SAS (SAS Institute, 1985) and the SPSS computer programs (SPSS Incorporated, 1983). The data was then sent to Pretoria where the MicroCAT program, with the aid of the Item and Test Analysis Program (ITEMAN) and the Rasch Model Item Calibration Program (Rascal) (Assessment System Corporation, 1989, 1995) was used to obtain the CTT and the IRT results. The DIF results were obtained by determining the area between the ICC's (Osterlind, 1989). The data was then forwarded to Australia where the Conquest–Generalised Item Response Modelling Software (Wu, Adams & Wilson, 1998) was used to check the IRT results and to obtain DIF results using the comparison of item parameters and the Mantel-Haenszel procedure. Goodness of fit results were also computed by using the above program.

5.4.2.3 Sample and results

As was previously stated, it is important to obtain as large a sample as possible when analysing items. The recommendation is that the sample should be five times the number of items. In this phase of the research, with 50 items per grade, the sample per grade should not be less than 250. The composition of the sample for phase two is represented in Table 5.4.

Table 5.4: Sample distribution for phase two

Grade	**Home language**			
	English	**Afrikaans**	**Sesotho**	**Total**
4	128	222	183	533
5	144	230	180	554
6	143	206	165	514
Total	415	658	528	1601

The above sample distribution indicates that the sample size for each grade was at least ten times the number of items in the test. The sample size for each grade was relatively erratic. It was relatively easy to obtain a sample of at least 200 Afrikaans-speaking learners per grade. The Sesotho-speaking sample was not that difficult to obtain, but many of the learners tested indicated that their home language was either Setswana or

IsiXhosa. There is only a small number of learners in the Free State whose home language is English. During phase two a total of 1601 learners' test results were analysed, due to the strict sample selection procedures.

The basic analysis variables of the learners in their respective grades are represented in Table 5.5.

Table 5.5: Basic statistical analysis results for phase two

GRADE	**Sample**	**N**	**$\bar{X}$**	**Percentile**	**Standard deviation**	**Minimum**	**Maximum**
4	English	128	36.2	**72**	7.5	10	49
	Sesotho	183	19.9	**40**	6.43	7	37
	Afrikaans	222	34	**68**	8.11	8	48
	Total	533	30	***60***	---	7	49
5	English	144	32.5	**65**	7.65	13	46
	Sesotho	180	11.4	**23**	5.23	0	37
	Afrikaans	230	28.6	**57**	9.16	6	46
	Total	554	24.2	***48***	---	0	47
6	English	143	27.6	**55**	7.54	8	44
	Sesotho	165	5.3	**11**	3.44	0	19
	Afrikaans	206	27	**54**	7.84	7	46
	Total	514	19.97	***40***	---	0	46

The average percentage for each grade is indicated in the table above. It is important to note that the learners that were tested in the pilot study were in different grades and therefore the results obtained in phase two are not for the same group of learners as in the pilot study. Despite this it is interesting to compare the results of phase one with phase two. The grade four learners achieved an average of 60%, compared to the 51% average achieved in the pilot study. The grade five learners achieved an average of 48%, compared to the 29% average that was achieved in the pilot study. The grade six learners achieved an average of 40%, compared to the 31% that was achieved in the pilot study. These results show that the means improved for each group, but that the Sesotho group's results were still of concern to the researcher. The grade four test proved to be on standard, but the grade five and six tests were still a little difficult, specifically for the Sesotho-speaking sample.

5.4.3 Phase three: Item analysis and selection

5.4.3.1 Introduction

Before the selection of the items for the grade four, five and six mathematics proficiency test could begin, the basis on which the items would be selected process had to be decided upon. The CTT results and IRT results would be utilised, but there was uncertainty as to the type of DIF results that would be used as the criteria to flag items. The DIF results of three respective groups, namely males versus females, English versus Afrikaans, and English and Afrikaans versus Sesotho, were computed using the comparison of item parameters, the area between ICC's, and, in some cases, the Mantel Haenszel procedure (due to the different cell sizes). The researcher had to decide on the criteria for flagging. The fit statistics were also computed.

Due to the significant differences between the means (see Table 5.5) there was concern that the DIF procedure would not be able to be carried out with the Sesotho sample group. For this reason, there was uncertainty as to whether the results of the Sesotho group should be discarded or whether the test should rather focus on being gender fair or translation fair. The researcher was adamant that the aim of the research was to set up a culturally fair mathematics proficiency test. The hypothesis at this point was that the English- and Afrikaans-speaking learners performed significantly better than the Sesotho-speaking learners in the mathematics proficiency test. Despite the significant differences with regard to the means of the English and Afrikaans groups versus the Sesotho group, the DIF results of this group would be investigated.

The abilities of the learners were assumed to have been the same, but due to the significant differences with regard to the means, the hypothesis was that most of the items would be easier for the English and Afrikaans group and would therefore show a major difference between the item parameters and a large area between the ICC's. Depite the fact that the means differed so significantly, there were items that showed minimal DIF in both the above DIF procedures. In other words, there were items in the test that were not biased towards a specific cultural group and therefore the items could be used to develop a culturally fair mathematics proficiency test. The Mantel-Haenszel procedure could not be carried out for this group, as the cell sizes differed and therefore individuals could not be matched. The researcher then contacted the statistician in Australia and questioned the probability of the above results. The statistician stated that a DIF procedure can be unpredictable but that what matters most with DIF is the interpretation thereof.

Twenty items were then selected for grades four, five and six, and the manner in which the items were selected is represented below:

a) for each grade, the DIF results were considered for the English and Afrikaans groups versus the Sesotho group. All items that showed minimal DIF (fell within the –2 and 2 parameter boundaries or just outside) were flagged;
b) the CTT results of the items flagged were then considered. The difficulty value (p) of each item was reviewed. Items with a difficulty value of less than 0.2 or more than 0.85 were discarded. The discrimination value (r_{it}) of each item was also taken into consideration. Items with a discrimination value of less than 0.2 were discarded. The item variance (s^2) was also noted;
c) the IRT results of the items flagged were then considered. The one-parameter Rasch model was utilised for this purpose. Items with a difficulty value (b) smaller than –1.96 and greater than 1.96 were discarded;

d) the ‘fit’ statistics were also taken into consideration. Items with an INFIT mean square value greater than or equal to 1.30 and smaller or equal to 0.75 were discarded;
e) these items were then double checked using the DIF results of the English and Afrikaans groups versus the Sesotho group with regard to the area between the ICC’s. Items that showed a large area were discarded. It must be stated at this point that the DIF results of the two different procedures correlated in most instances; and lastly
f) the items also had to be representative of all the learning strands, namely numbers and operations; fractions; patterns; shapes and space; measurement; and data.

DIF analysis with regard to the difference between item parameters and the area between the ICC’s was also computed for the male and female sample groups and the English and Afrikaans sample groups. Due to the item selection procedure, these results will not be discussed in detail in this study. The item selection procedure is reviewed in more detail below.

5.4.3.2 Results of item analysis

5.4.3.2.1 Introduction

According to Anastasi and Urbina (1997) there are certain statistical properties that need to be addressed when developing a test. These properties include the size of the sample, the number of items in each test, the means of test scores, the standard deviation of the test, the reliability coefficient, the skewness, and the kurtosis. These properties will be discussed in more detail in 5.4.4.5.1 for the final 20-item test for grades four, five and six. The above information for the 50-item test is represented in Table 5.6 below.

Table 5.6: Statistical properties of the 50-item mathematics proficiency test

GRADE	N	$\bar{X}$	Std. deviation	Skewness	Kurtosis	Co-efficient-alpha	Std. error of measurement	Mean difficulty value	Mean discrimination value	Mean point biserial value
4	533	29.7	10.3	-0.179	-0.920	**0.918**	2.948	0.594	0.445	0.587
5	554	24.0	11.7	0.041	-1.132	**0.942**	2.835	0.480	0.506	0.666
6	514	20.2	12.2	-0.096	-1.244	**0.947**	2.794	0.404	0.524	0.691

The only aspect that is highlighted above is the coefficient-alpha, which is greater than 0.9 in each of the above grades. This indicates that the test items are each measuring the same attribute and ensures that the test has a high internal consistency.

The first aspect that was considered when selecting items was the DIF results.

5.4.3.2.2 DIF results: Comparison of item parameters

The DIF results for each grade were computed separately. For grades four, five and six the researcher combined the English- and Afrikaans-speaking learners and compared them to the Sesotho-speaking learners. The reason for this is that the English- and Afrikaans-speaking learners obtained a similar mean on the test but the Sesotho-speaking learners did significantly poorer. It was assumed that the learners had a similar ability and therefore it had to be determined whether there were any items that were not culturally biased towards a specific group.

When comparing item parameters, the researcher wished to obtain items that were not significantly easier for the English- and Afrikaans-speaking group or for the Sesotho-speaking group. Items that ranged between –2 and 2 were flagged as showing less DIF. These items proved to be more linear, showing less homoscadasticity. These items were then investigated further. In other words, items with $p < 0.05$ were eliminated. The grade four and six test yielded a sufficient number of items (more than 20) for further inspection, but with respect to the grade five items, only 18 items were marked for further inspection and the researcher had to consider items that fell just outside the item parameters, (items equal to or between -2.8 and 2.8). In other words, items with a $p < 0.01$ were eliminated. As can be seen in Table 5.7.1, 5.7.2 and 5.7.3 items marked with a * show significant DIF at the 5% level for grades four and six and at the 1% level for grade five. These items were immediately eliminated. Items highlighted in yellow show minimal DIF and were inspected further.

It is important to note that the DIF of an item was not computed if one of the groups' minimum for that item was a zero. In grade five, item 17 was therefore excluded and in grade 6 items 3, 6, 9, 11, 12, 16, 17, 31, 38 and 42 were excluded. In all these items, none of the Sesotho-speaking learners answered the item correctly. Items with a positive parameter estimate were easier for the Sesotho-speaking learners and items with a negative parameter estimate were easier for the English- and Afrikaans-speaking learners. The DIF results were computed using the Conquest–Generalised Item Response Modelling Software (Wu et al., 1998). The DIF results for the comparison of item estimates for each grade are represented in Table 5.7.1, Table 5.7.2, and Table 5.7.3. The results are represented in Figure 5.1.1, Figure 5.1.2 and Figure 5.1.3 in the form of a scatter diagram.

Table 5.7.1: DIF results: Comparison of item parameters – grade four

Item	**Delta**		**Adjusted delta**		**Difference**		χ^2	**p**
	Afr/Eng	Sesotho	Afr/Eng	Sesotho	d_1-d_2	**d_1-d_2**		
1	-1.02	-3.93	-1.02	-3.93	2.92	**6.88**	47.30	0.00*
2	-1.80	1.24	-1.81	1.24	-3.00	**-10.02**	100.48	0.00*
3	-1.07	-0.15	-1.07	-0.15	-0.20	**-3.95**	15.59	0.00*
4	-1.16	0.00	-1.16	0.00	-1.16	**-4.85**	23.55	0.00*
5	-0.11	0.15	-0.11	0.15	-0.26	**-1.23**	1.51	0.22
6	0.63	0.93	0.63	0.93	-0.30	**-1.34**	1.78	0.18
7	1.34	1.63	1.34	1.63	-0.20	**-1.10**	1.20	0.27
8	1.04	0.66	1.04	0.66	0.37	**1.74**	3.01	0.08
9	0.74	-1.12	0.74	-1.12	1.87	**9.30**	86.44	0.00*
10	0.33	-0.27	0.33	-0.27	0.61	**3.03**	9.18	0.00*
11	-0.99	0.15	-0.99	0.15	-1.14	**-4.87**	23.77	0.00*
12	-0.04	-0.80	-0.04	-0.80	0.76	**3.71**	13.79	0.00*
13	1.31	0.34	1.31	0.34	0.97	**4.72**	22.29	0.00*
14	0.79	0.48	0.70	0.48	0.30	**1.45**	2.10	0.15
15	1.41	1.00	1.41	1.01	0.40	**1.77**	3.13	0.08
16	0.50	-1.30	0.50	-1.30	1.80	**8.77**	76.84	0.00*
17	1.92	0.18	1.92	0.18	1.75	**8.50**	72.20	0.00*
18	-1.47	-1.36	-1.47	-1.36	-0.11	**-0.42**	0.18	0.67
19	-2.13	-0.90	-2.13	-0.90	-1.23	**-4.15**	17.26	0.00*
20	-2.33	-1.00	-2.33	-1.00	-1.34	**-4.23**	17.86	0.00*
21	-1.71	-0.63	-1.71	-0.63	-1.08	**-4.07**	16.57	0.00*
22	-0.83	0.83	-0.83	0.83	-1.66	**-6.76**	45.72	0.00*
23	-0.96	-0.68	-0.96	-0.68	-0.28	**-1.23**	1.51	0.22
24	1.23	3.70	1.23	3.70	-2.47	**-4.10**	17.06	0.00*
25	1.47	2.46	1.46	2.46	-0.99	**-2.83**	7.98	0.00*
26	-1.16	-0.95	-1.16	-0.50	-0.22	**-0.90**	0.82	0.37
27	-0.94	-0.71	-0.94	-0.71	-0.23	**-1.01**	1.02	0.31

28	0.59	0.63	0.59	0.63	-0.05	**-0.21**	0.05	0.83
29	-1.39	-1.44	-1.39	-1.44	0.05	**0.18**	0.03	0.85
30	-1.32	-1.10	-1.32	-1.10	-0.23	**-0.92**	0.84	0.36
31	0.62	-1.25	0.62	-1.25	1.87	**9.17**	84.09	0.00*
32	2.02	-0.40	2.02	-0.39	2.41	**12.06**	145.46	0.00*
33	1.61	1.75	1.60	1.75	-0.14	**-0.52**	0.27	0.60
34	1.87	-0.22	1.87	-0.22	2.09	**10.44**	109.05	0.00*
35	-0.31	-0.35	-0.31	-0.35	0.04	**0.17**	0.03	0.86
36	-0.39	-0.22	-0.39	-0.22	-0.16	**-0.77**	0.60	0.44
37	0.01	1.24	0.01	1.24	-1.23	**-4.90**	24.91	0.00*
38	-0.07	0.63	-0.07	0.63	-0.71	**-3.18**	10.10	0.00*
39	-0.23	-1.44	-0.24	-1.44	1.21	**5.55**	30.77	0.00*
40	0.91	0.83	0.91	0.83	0.08	**0.38**	0.15	0.70
41	0.57	1.24	0.57	1.24	-0.67	**-2.78**	7.71	0.01*
42	0.93	0.97	0.93	0.97	-0.04	**-0.19**	0.03	0.85
43	-0.67	-0.51	-0.67	-0.51	-0.15	**-0.69**	0.48	0.49
44	-0.27	0.18	-0.27	0.18	-0.45	**-2.09**	4.38	0.04*
45	0.59	-0.08	0.59	-0.08	0.66	**3.32**	11.01	0.00*
46	0.80	0.31	0.80	0.31	0.49	**2.37**	5.63	0.02*
47	-1.80	-1.53	-1.81	-1.53	-0.28	**-1.00**	1.00	0.32
48	-0.94	-0.75	-0.94	-0.75	-0.18	**-0.79**	0.63	0.43
49	0.11	0.63	0.11	0.63	-0.52	**-2.37**	5.62	0.02*
50	1.81	0.93	1.81	0.93	0.87	**3.86**	14.90	0.00*

df = 49 $\chi^2 = 1087.85$ * $p < 0.05$

Table 5.7.2: DIF results: Comparison of item parameters – grade five

Item	Delta		Adjusted delta		Difference		χ^2	p
	Afr/Eng	Sesotho	Afr/Eng	Sesotho	d_1-d_2	**d_1-d_2**		
1	-0.33	0.11	-0.30	0.11	-0.40	**-1.50**	2.26	0.13
2	-0.21	0.35	-0.18	0.35	-0.53	**-1.85**	3.43	0.06
3	-1.06	-0.92	-1.02	-0.92	-0.11	**-0.46**	0.21	0.65
4	0.81	-1.53	0.85	-1.53	2.38	**11.76**	13.22	0.00*
5	0.77	-0.10	0.81	-0.10	0.91	**3.61**	13.00	0.00*
6	0.76	-1.64	0.80	-1.64	2.44	**12.08**	145.85	0.00*
7	-1.35	-0.10	-1.31	-0.10	-1.21	**-4.49**	20.13	0.00*
8	1.82	-1.11	1.86	-1.11	2.96	**13.61**	185.36	0.00*
9	-0.02	1.43	0.02	1.43	-1.41	**-3.39**	11.47	0.00*
10	-0.27	-1.25	-0.24	-1.25	1.02	**4.82**	23.25	0.00*
11	0.45	-0.15	0.48	-0.15	0.63	**2.55**	6.48	0.01
12	0.76	-0.41	0.80	-0.41	1.21	**5.16**	26.58	0.00*
13	1.09	2.04	1.13	2.04	-0.91	**-1.70**	2.88	0.09
14	0.72	2.34	0.76	2.34	-1.58	**-2.60**	6.75	0.01
15	1.62	0.82	1.65	0.82	0.83	**2.37**	6.08	0.01
16	0.58	3.48	0.61	3.48	-2.86	**-2.80**	7.83	0.01
17	---	---	---	---	---	**---**	---	---
18	-1.84	-3.46	-1.80	-3.46	1.66	**6.70**	41.87	0.00*
19	-0.93	-0.37	-0.89	-0.37	-0.52	**-2.10**	4.41	0.04
20	-2.17	-1.90	-2.14	-1.90	-0.23	**-0.92**	0.85	0.36
21	-1.00	-1.85	-0.97	-1.80	0.88	**4.13**	17.09	0.00*
22	0.06	-2.08	0.10	-2.08	2.18	**10.87**	118.19	0.00*
23	-2.33	-2.65	-2.30	-2.65	0.36	**1.34**	1.81	0.18
24	-2.06	-2.29	-2.03	-2.29	0.26	**1.05**	1.10	0.30
25	2.17	2.34	2.20	2.34	-0.14	**-0.23**	0.50	0.82
26	-1.56	-2.44	-1.53	-2.44	0.91	**4.00**	16.01	0.00*
27	-0.20	-2.00	-0.17	-2.00	1.84	**9.09**	82.71	0.00*

28	0.79	3.48	0.82	3.48	-2.65	**-2.59**	6.72	0.01
29	2.50	-0.89	2.54	-0.89	3.42	**14.55**	211.76	0.00*
30	-0.86	-3.17	-0.82	-3.17	2.35	**10.53**	110.86	0.00*
31	-1.35	-3.20	-1.31	-3.20	1.89	**8.05**	64.85	0.00*
32	0.41	0.05	0.44	0.05	0.39	**1.49**	2.21	0.14
33	-1.64	-1.85	-1.60	-1.85	0.25	**1.06**	1.13	0.29
34	-1.64	1.28	-1.60	1.28	-2.89	**-7.03**	49.47	0.00*
35	-0.70	-1.25	-0.43	-1.25	0.82	**3.87**	15.01	0.00*
36	1.97	3.48	2.10	3.48	-1.47	**-1.43**	2.05	0.15
37	-0.13	-0.05	-0.09	-0.05	-0.15	**-0.56**	0.31	0.56
38	-0.09	1.79	-0.05	1.79	-1.85	**-3.81**	14.53	0.00*
39	0.56	2.34	0.60	2.34	-1.74	**-2.85**	8.14	0.00*
40	-1.67	-1.51	-1.63	-1.51	-0.13	**-0.53**	0.28	0.59
41	1.15	3.48	1.18	3.48	-2.29	**-2.24**	5.02	0.03
42	1.02	1.60	1.06	1.60	-0.54	**-1.20**	1.45	0.23
43	-0.10	0.42	-0.07	0.42	-0.49	**-1.67**	2.78	0.10
44	1.12	3.48	1.15	3.48	-2.32	**-2.27**	5.14	0.02
45	0.72	2.77	0.76	2.77	-2.01	**-2.73**	7.43	0.01
46	-0.54	-1.40	-0.51	-1.40	0.89	**4.22**	17.78	0.00*
47	-1.26	-0.98	-1.23	-0.98	-0.24	**-1.05**	1.09	0.30
48	0.84	2.34	0.87	2.34	-1.47	**-2.40**	5.77	0.02
49	0.09	0.57	0.12	0.57	-0.44	**-1.45**	2.10	0.15
50	0.56	0.49	0.60	0.49	0.11	**0.36**	0.13	0.72

df = 48 $\chi^2 = 1419.89$ * $p < 0.01$

Table 5.7.3: DIF results: Comparison of item parameters – grade six

Item	**Delta**		**Adjusted delta**		**Difference**		χ^2	**p**
	Afr/Eng	Sesotho	Afr/Eng	Sesotho	d_1-d_2	**d_1-d_2**		
1	-0.77	-0.83	-0.56	-0.83	0.26	**1.05**	1.11	0.29
2	-0.40	0.33	-0.20	0.33	-0.52	**-1.53**	2.34	0.13
3	---	---	---	---	---	**---**	---	---
4	0.00	-2.33	0.21	-2.33	2.54	**12.32**	151.79	0.00*
5	-1.07	-0.92	-0.86	-0.92	0.06	**0.23**	0.05	0.82
6	---	---	---	---	---	**---**	---	---
7	-0.40	-1.16	-0.20	-1.16	0.96	**4.17**	17.36	0.00*
8	-1.38	-1.20	-1.18	-1.20	0.02	**0.08**	0.01	0.93
9	---	---	---	---	---	**---**	---	---
10	0.91	0.98	1.11	0.98	0.14	**0.31**	0.10	0.76
11	---	---	---	---	---	**---**	---	---
12	---	---	---	---	---	**---**	---	---
13	0.82	0.98	1.03	0.98	0.05	**0.12**	0.01	0.90
14	0.91	-1.00	1.11	-1.00	2.12	**8.87**	78.75	0.00*
15	1.28	1.69	1.49	1.69	-0.20	**-0.34**	0.11	0.74
16	---	---	---	---	---	**---**	---	---
17	---	---	---	---	---	**---**	---	---
18	-1.21	-0.30	-1.00	-0.30	-0.70	**-2.44**	5.94	0.01*
19	0.58	-1.20	0.79	-1.20	1.99	**8.67**	75.10	0.00*
20	-0.17	-0.78	0.04	-0.78	0.82	**3.30**	10.87	0.00*
21	1.14	-0.64	1.35	-0.64	1.98	**7.64**	58.41	0.00*
22	-1.38	-0.78	-1.18	-0.78	-0.40	**-1.51**	2.29	0.13
23	-1.35	-0.83	-1.14	-0.83	-031	**-1.20**	1.44	0.23
24	-0.24	0.55	-0.03	0.55	-0.58	**-1.56**	2.44	0.12
25	0.58	2.10	0.79	2.10	-1.31	**-1.81**	3.27	0.07
26	-1.37	-1.68	-1.16	-1.68	0.52	**2.26**	5.13	0.02*
27	-0.90	-1.00	-0.69	-1.00	0.31	**1.27**	1.62	0.20

28	-0.83	-1.58	-0.63	-1.58	0.95	**4.29**	18.36	0.00*
29	0.62	1.17	0.83	1.17	-0.34	**-0.72**	0.52	0.47
30	1.19	0.23	1.39	0.23	1.16	**3.49**	12.16	0.00*
31	---	---	---	---	---	**---**	---	---
32	1.31	1.69	1.52	1.69	-0.17	**-0.28**	0.08	0.78
33	0.69	0.98	0.89	0.98	-0.08	**-0.19**	0.04	0.85
34	-1.85	-1.65	-1.64	-1.65	0.01	**0.04**	0.00	0.97
35	0.47	0.82	0.68	0.82	-0.14	**-0.33**	0.11	0.74
36	-1.12	1.17	-0.91	1.17	-2.08	**-4.33**	18.76	0.00*
37	-0.80	1.40	-0.60	1.40	-1.99	**-3.78**	14.27	0.00*
38	---	---	---	---	---	**---**	---	---
39	-0.46	1.69	-0.25	1.69	-1.94	**-3.24**	10.49	0.00*
40	0.22	2.79	0.43	2.79	-2.35	**-2.33**	5.41	0.02*
41	2.77	2.79	2.97	2.79	0.19	**0.18**	0.03	0.86
42	---	---	---	---	---	**---**	---	---
43	-2.24	-1.52	-2.03	-1.52	-0.51	**-1.94**	3.77	0.05
44	-1.67	0.43	-1.46	0.43	-1.90	**-5.13**	26.31	0.00*
45	1.26	-0.02	1.47	-0.02	1.49	**4.85**	23.57	0.00*
46	-1.37	-1.24	-1.16	-1.24	0.08	**0.32**	0.10	0.75
47	-1.72	-1.68	-1.51	-1.68	0.17	**0.70**	0.49	0.48
48	-0.04	0.55	0.17	0.55	-0.38	**-1.02**	1.04	0.31
49	-0.77	-0.42	-0.56	-0.42	-0.14	**-0.53**	0.28	0.60
50	0.47	0.43	0.68	0.43	0.25	**0.70**	0.49	0.48

df = 39 $\chi^2 = 554.50$ * $p < 0.05$

5.4.3.2.3 CTT and IRT results

It is important to note that the CTT and IRT results were computed for the total group (English-, Afrikaans- and Sesotho-speaking learners) for each grade. According to the CTT, the difficulty value (*p*), the discrimination value (r_{it}) and the item variance (s^2) of each item must be considered when analysing an item. The CTT and IRT results were calculated by means of the MicroCAT program, ITEMAN and Rascal (Assessment Systems Corporation, 1989, 1995) and the Conquest–Generalised Item Response Modelling Software (Wu et al., 1998).

According to the IRT, the Rasch model focuses on one-parameter, namely the difficulty value (*b*). The discrimination value (*a*) remains constant. The above value can be centred on difficulty or ability. Both values for each item have been indicated below. In Rasch tradition, the probability value is equal to 0.50. All the above values are represented in Table 5.8.1, Table 5.8.2 and Table 5.8.3. The items that were marked for further investigation because of the small DIF that they exhibited have been printed in bold. For grade four 21 items were marked, for grade five 28 items were marked, and for grade six 24 items were marked to see whether they satisfied the CTT and IRT. Items that did not satisfy the CTT and the IRT have been highlighted. These items were discarded, along with the items that were discarded because the DIF was too large.

5.8.1: CTT and IRT results for English-, Afrikaans- and Sesotho-speaking grade four learners

Item	**CTT** **(*p*)**	**CTT** **(r_{it})**	**CTT** **(s^2)**	**IRT** ***b*- parameter centered on difficulty**	**IRT** ***b*-parameter centered on ability**
1	0.90	0.05	0.0900	-2.104	-2.238
2	0.67	0.76	0.2211	-0.334	-0.748
3	0.71	0.58	0.2059	-0.616	-0.985
4	0.71	0.50	0.2059	-0.582	-0.956
5	**0.61**	**0.56**	**0.2379**	**-0.029**	**-0.491**
6	**0.47**	**0.67**	**0.2491**	**0.676**	**0.103**
7	**0.35**	**0.63**	**0.2275**	**1.342**	**0.663**
8	**0.43**	**0.54**	**0.2451**	**0.879**	**0.274**
9	0.60	0.30	0.2400	0.001	-0.466
10	0.59	0.44	0.2419	0.069	-0.408
11	0.69	0.60	0.2139	-0.451	-0.846
12	0.68	0.41	0.2176	-0.387	-0.792
13	0.42	0.55	0.2436	0.967	0.348
14	**0.48**	**0.52**	**0.2496**	**0.647**	**0.079**
15	**0.36**	**0.61**	**0.2304**	**1.248**	**0.585**
16	0.65	0.28	0.2275	-0.241	-0.669
17	0.35	0.34	0.2275	1.342	0.663
18	**0.83**	**0.35**	**0.1411**	**-1.402**	**-1.647**
19	0.82	0.40	0.1476	-1.342	-1.597
20	0.83	0.45	0.1411	-1.449	-1.687
21	0.79	0.52	0.1659	-1.076	-1.373
22	0.63	0.75	0.2331	-0.149	-0.592
23	**0.75**	**0.49**	**0.1875**	**-0.825**	**-1.161**
24	0.33	0.69	0.2211	1.448	0.753
25	0.31	0.73	0.2139	1.556	0.844

26	**0.78**	**0.40**	**0.1716**	**-1.050**	**-1.350**
27	**0.75**	**0.51**	**0.1875**	**-0.825**	**-1.161**
28	**0.50**	**0.58**	**0.2500**	**0.561**	**0.006**
29	**0.83**	**0.23**	**0.1411**	**-1.418**	**-1.660**
30	**0.80**	**0.43**	**0.1600**	**-1.198**	**-1.475**
31	0.63	0.25	0.2331	-0.139	-0.583
32	0.38	0.03	0.2356	1.167	0.516
33	**0.31**	**0.45**	**0.2139**	**1.567**	**0.853**
34	0.38	0.33	0.2356	1.136	0.491
35	**0.67**	**0.52**	**0.2211**	**-0.345**	**-0.756**
36	**0.67**	**0.51**	**0.2211**	**-0.334**	**-0.748**
37	0.53	0.79	0.2491	0.369	-0.155
38	0.57	0.67	0.2451	0.157	-0.334
39	0.74	0.23	0.1924	-0.789	-1.130
40	**0.44**	**0.47**	**0.2464**	**0.840**	**0.241**
41	0.47	0.72	0.2491	0.714	0.135
42	**0.43**	**0.60**	**0.2451**	**0.889**	**0.282**
43	**0.71**	**0.56**	**0.2059**	**-0.604**	**-0.975**
44	0.62	0.63	0.2356	-0.109	-0.558
45	0.54	0.56	0.2484	0.312	-0.204
46	0.49	0.67	0.2499	0.599	0.038
47	**0.86**	**0.37**	**0.1204**	**-1.630**	**-1.839**
48	**0.75**	**0.55**	**0.1875**	**-0.849**	**-1.181**
49	0.55	0.68	0.2475	0.264	-0.244
50	0.32	0.46	0.2176	1.523	0.816

Twenty of the above 21 items that were marked for further inspection satisfied the requirements set out in 4.5.1 and 4.5.2.1. Only item 47 (highlighted in yellow) had a *p*-value that was greater than 0.85 and therefore had to be discarded. The CTT and IRT results of the grade five learners are represented in Table 5.8.2.

5.8.2 CTT and IRT results for English-, Afrikaans- and Sesotho-speaking grade five learners

Item	**CTT (*p*)**	**CTT (r_{it})**	**CTT (s^2)**	**IRT *b*- parameter centered on difficulty**	**IRT *b*-parameter centered on ability**
1	**0.50**	**0.76**	**0.2500**	**-0.124**	**0.000**
2	**0.48**	**0.68**	**0.2496**	**-0.002**	**0.086**
3	**0.63**	**0.74**	**0.2331**	**-0.839**	**-0.500**
4	0.43	0.40	0.2451	0.274	0.279
5	0.36	0.62	0.2304	0.683	0.565
6	0.45	0.28	0.2475	0.192	0.222
7	0.61	0.73	0.2379	-0.764	-0.448
8	0.28	0.11	0.2016	1.213	0.936
9	0.44	0.75	0.2464	0.253	0.265
10	0.56	0.59	0.2464	-0.451	-0.229
11	**0.41**	**0.71**	**0.2419**	**0.408**	**0.373**
12	0.38	0.66	0.2356	0.597	0.505
13	**0.28**	**0.73**	**0.2016**	**1.201**	**0.928**
14	**0.33**	**0.73**	**0.2211**	**0.890**	**0.711**
15	**0.23**	**0.51**	**0.1771**	**1.565**	**1.184**
16	**0.34**	**0.80**	**0.2244**	**0.791**	**0.641**
17	0.19	0.47	0.1539	1.816	1.359
18	0.85	0.28	0.1275	-2.448	-1.628
19	**0.59**	**0.80**	**0.2419**	**-0.606**	**-0.337**
20	**0.77**	**0.58**	**0.1771**	**-1.756**	**-1.143**
21	0.68	0.63	0.2176	-1.170	-0.732
22	0.57	0.46	0.2451	-0.523	-0.279
23	**0.83**	**0.42**	**0.1411**	**-2.232**	**-1.476**
24	**0.79**	**0.45**	**0.1659**	**-1.920**	**-1.257**
25	**0.15**	**0.43**	**0.1275**	**2.194**	**1.624**
26	0.77	0.41	0.1771	-1.769	-1.152

27	0.60	0.43	0.2400	-0.690	-0.396
28	**0.31**	**0.67**	**0.2139**	**0.969**	**0.765**
29	0.20	0.12	0.1600	1.787	1.339
30	0.75	0.19	0.1875	-1.664	-1.079
31	0.80	0.32	0.1600	-2.020	-1.328
32	**0.41**	**0.68**	**0.2419**	**0.419**	**0.380**
33	**0.73**	**0.49**	**0.1971**	**-1.513**	**-0.973**
34	0.60	0.86	0.2400	-0.701	-0.404
35	0.58	0.54	0.2436	-0.586	-0.323
36	**0.17**	**0.41**	**0.1411**	**2.044**	**1.519**
37	**0.48**	**0.74**	**0.2496**	**0.008**	**0.093**
38	0.44	0.79	0.2464	0.222	0.243
39	0.35	0.78	0.2275	0.758	0.618
40	**0.71**	**0.71**	**0.2059**	**-1.367**	**-0.870**
41	**0.27**	**0.57**	**0.1971**	**1.286**	**0.987**
42	**0.29**	**0.66**	**0.2059**	**1.118**	**0.870**
43	**0.46**	**0.77**	**0.2484**	**0.090**	**0.150**
44	**0.27**	**0.61**	**0.1971**	**1.261**	**0.970**
45	**0.32**	**0.73**	**0.2176**	**0.902**	**0.718**
46	0.60	0.56	0.2400	-0.690	-0.396
47	**0.65**	**0.67**	**0.2275**	**-0.969**	**-0.591**
48	**0.31**	**0.67**	**0.2139**	**0.992**	**0.781**
49	**0.44**	**0.79**	**0.2464**	**0.253**	**0.265**
50	**0.37**	**0.72**	**0.2331**	**0.618**	**0.520**

Twenty-five of the above 28 items that were marked for further inspection satisfied the requirements set out in 4.5.1 and 4.5.2.1. Only items 23, 25 and 36 (highlighted in yellow) either had a *p*-value that was smaller than 0.2 or a *b*-value that was greater than 1.96 or smaller than -1.96 and therefore had to be discarded. The CTT and IRT results of the grade six learners are represented in Table 5.8.3.

5.8.3 CTT and IRT results for English-, Afrikaans- and Sesotho-speaking grade six learners

Item	CTT (p)	CTT (r_{it})	CTT (s^2)	IRT *b*- parameter centered on difficulty	IRT *b*-parameter centered on ability
1	**0.54**	**0.68**	**0.2484**	**-0.833**	**-0.090**
2	**0.45**	**0.70**	**0.2475**	**-0.330**	**0.228**
3	0.30	0.61	0.2100	0.612	0.825
4	0.52	0.20	0.2496	-0.714	-0.015
5	**0.58**	**0.72**	**0.2436**	**-1.077**	**-0.245**
6	0.32	0.67	0.2176	-0.431	0.710
7	0.50	0.62	0.2500	-0.631	0.037
8	**0.62**	**0.69**	**0.2356**	**-1.383**	**-0.438**
9	0.47	0.75	0.2491	-0.422	0.170
10	**0.25**	**0.50**	**0.1875**	**0.891**	**1.000**
11	0.34	0.76	0.2244	0.337	0.650
12	0.31	0.69	0.2139	0.515	0.763
13	**0.26**	**0.52**	**0.1924**	**0.813**	**0.951**
14	0.31	0.43	0.2139	0.551	0.786
15	**0.20**	**0.50**	**0.1600**	**1.281**	**1.247**
16	0.05	0.14	0.0475	3.019	2.347
17	0.18	0.44	0.1476	1.419	1.335
18	0.57	0.78	0.2451	-1.027	-0.213
19	0.36	0.34	0.2304	0.209	0.569
20	0.45	0.64	0.2475	-0.330	0.228
21	0.26	0.19	0.1924	0.852	0.976
22	**0.60**	**0.82**	**0.2400**	**-1.253**	**-0.356**
23	**0.60**	**0.78**	**0.2400**	**-1.241**	**-0.348**
24	**0.43**	**0.73**	**0.2451**	**-0.170**	**0.329**
25	**0.29**	**0.65**	**0.2059**	**0.637**	**0.840**
26	0.65	0.67	0.2275	-1.555	-0.547

27	**0.56**	**0.73**	**0.2464**	**-0.978**	**-0.182**
28	0.58	0.59	0.2436	-1.127	-0.276
29	**0.29**	**0.51**	**0.2059**	**0.637**	**0.840**
30	0.23	0.40	0.1771	1.066	1.111
31	0.24	0.50	0.1824	0.970	1.051
32	**0.19**	**0.45**	**0.1539**	**1.311**	**1.266**
33	**0.28**	**0.53**	**0.2016**	**0.686**	**0.872**
34	**0.69**	**0.70**	**0.2139**	**-1.833**	**-0.723**
35	**0.32**	**0.57**	**0.2176**	**0.479**	**0.740**
36	0.53	0.87	0.2491	-0.797	-0.067
37	0.49	0.81	0.2499	-0.561	0.082
38	0.28	0.56	0.2016	0.686	0.872
39	0.45	0.76	0.2475	-0.284	0.257
40	0.34	0.69	0.2244	0.325	0.642
41	**0.06**	**0.16**	**0.0564**	**2.748**	**2.176**
42	0.16	0.35	0.1344	1.583	1.439
43	**0.70**	**0.73**	**0.2100**	**-1.963**	**-0.805**
44	0.59	0.90	0.2419	-1.190	-0.315
45	0.22	0.30	0.1716	1.093	1.129
46	**0.62**	**0.71**	**0.2356**	**-1.383**	**-0.438**
47	**0.68**	**0.68**	**0.2176**	**-1.776**	**-0.686**
48	**0.40**	**0.67**	**0.2400**	**0.001**	**0.438**
49	**0.52**	**0.75**	**0.2496**	**0.738**	**-0.030**
50	**0.32**	**0.49**	**0.2176**	**0.443**	**0.718**

Twenty-one of the above 24 items that were marked for further inspection satisfied the requirements set out in 4.5.1 and 4.5.2.1. Only items 32, 41, and 43 (highlighted in yellow) had a *p*-value smaller than 0.2 or a *b*-value greater than 1,96 or smaller than 1,96 and therefore had to be discarded. Item 41 did not satisfy any of the requirements.

The 20 items from grade four, the 24 items from grade five and the 21 items from grade six were then marked for further inspection with respect to the fit statistics.

5.4.3.2.4 Fit statistics results

The mean square INFIT values represent how well the items ‘hang together’, in other words, how well the items measure the same attribute, namely mathematics proficiency. The mean square INFIT values (as discussed in chapter four) are represented in Table 5.9. It is important to note that values between 0.75 and 1.30 are acceptable. Any item that did not fulfil this requirement was highlighted in yellow. The mean square INFIT values were calculated using the Conquest–Generalised Item Response Modelling Software (Wu et al., 1998). Only the mean square INFIT values of items that satisfied the DIF, CTT and IRT are represented in Table 5.9.

Table 5.9: Mean square INFIT values, for marked items

Grade four		Grade five		Grade six	
Item	Mean square INFIT	Item	Mean square INFIT	Item	Mean square INFIT
5	1.02	1	0.87	1	1.05
6	0.92	2	1.01	2	1.01
7	0.90	3	0.90	5	1.02
8	1.03	11	0.92	8	1.04
14	1.10	13	0.75	10	0.98
15	0.93	14	0.82	13	0.95
18	0.91	15	0.95	15	0.81
23	0.89	16	0.77	22	0.86
26	0.99	19	0.82	23	0.96
27	0.90	20	0.80	24	0.98
28	0.99	24	0.93	25	0.87
29	1.06	28	0.92	27	1.03
30	0.87	32	1.01	29	1.02
33	1.07	33	1.07	33	0.95
35	0.98	37	0.93	34	0.96
36	0.98	40	0.79	35	0.98
40	1.10	41	0.98	46	1.01
42	0.99	42	0.85	47	0.99
43	0.91	43	0.85	48	1.03
48	0.92	44	0.95	49	0.98
		45	0.84	50	1.08
		47	0.92		
		48	0.89		
		49	0.86		
		50	0.92		

As can be seen in the Table 5.9, only item 13 from grade five was discarded due to its overfit. The 20 items from grade four, the 24 items from grade five and the 21 items from grade six were then marked for further inspection using the DIF results with respect to the area between the ICC's.

5.4.3.2.5 DIF results: Area between ICC's

The DIF results with respect to the difference between item parameters and the area between ICC's were very similar. The calculated area between the ICC's of all the items is represented in Table 5.10.1 below. Each item's data was used to plot two graphs, one for the English- and Afrikaans-speaking learners and one for the Sesotho-speaking learners. The area between the two graphs was then calculated. A graphical representation of the DIF results of each grade, with respect to the area between ICC's, can be seen in Figure 5.2.1.1, Figure 5.2.1.2 and Figure 5.2.1.3. In Table 5.10.2, only the DIF results of the items that were marked for further inspection are represented and the graphical representation of each of the grades' DIF results of the marked items can be seen in Figure 5.2.2.1, Figure 5.2.2.2 and Figure 5.2.2.3. The DIF results were calculated by using the MicroCAT program, ITEMAN and Rascal (Assessment Systems Corporation, 1989, 1995).

Table 5.10.1: DIF results: Area between ICC's

Grade four		Grade five		Grade six	
Item	DIF results: Area between ICC's	Item	DIF results: Area between ICC's	Item	DIF results: Area between ICC's
1	11.0357	1	4.9604	1	0.8737
2	23.1239	2	5.1884	2	6.8096
3	6.9341	3	4.7682	3	16.2992
4	8.7471	4	16.2647	4	18.5820
5	2.0575	5	8.1239	5	2.0436
6	2.3677	6	16.3842	6	16.2409
7	2.1415	7	10.2330	7	6.2987
8	3.4953	8	22.6414	8	2.4817
9	14.8812	9	8.3848	9	16.8508
10	5.1055	10	5.7191	10	0.9800
11	8.8246	11	6.0733	11	16.2319
12	5.9563	12	9.6065	12	16.2605
13	8.6540	13	4.4031	13	0.9866
14	2.8326	14	7.1978	14	18.1511
15	3.8529	15	10.0008	15	2.4008
16	13.9323	16	14.4605	16	23.4786
17	15.3677	17	18.6582	17	17.2346
18	0.7567	18	4.0171	18	8.6654
19	7.2995	19	6.1695	19	16.4956
20	7.4633	20	5.8869	20	5.4741
21	7.1479	21	3.7901	21	17.3126
22	13.3997	22	12.5642	22	5.9273
23	2.0754	23	3.4720	23	5.2020
24	19.2200	24	3.8787	24	7.1995
25	7.9547	25	6.3972	25	12.5808
26	1.5410	26	2.9900	26	1.3528

27	1.7075	27	9.8629	27	0.8457
28	0.3401	28	12.6013	28	5.4060
29	0.2110	29	26.8409	29	4.2465
30	1.5602	30	10.0001	30	9.9320
31	14.6088	31	6.3208	31	16.5685
32	20.7584	32	5.2007	32	2.1018
33	0.8005	33	3.9273	33	1.8619
34	18.1319	34	20.1824	34	2.6756
35	0.3469	35	4.6682	35	2.4957
36	1.2684	36	4.0208	36	21.3062
37	10.3363	37	4.2980	37	20.4454
38	5.8032	38	10.8414	38	16.3374
39	8.5972	39	8.3764	39	19.6861
40	0.9954	40	5.2584	40	21.0708
41	5.5500	41	9.4285	41	1.1686
42	0.3120	42	4.2693	42	17.5784
43	1.1636	43	4.9213	43	6.2806
44	3.6107	44	9.6651	44	19.1626
45	5.7261	45	9.2629	45	12.9920
46	4.3801	46	4.6753	46	2.0382
47	1.6591	47	5.4386	47	1.5492
48	1.3565	48	6.3912	48	5.1858
49	4.2601	49	4.6270	49	3.5703
50	7.9694	50	4.7551	50	1.3024

Table 5.10.2: DIF results: Area between ICC's of marked items

Grade four		**Grade five**		**Grade six**	
Item	DIF results: Area between ICC's	Item	DIF results: Area between ICC's	Item	DIF results: Area between ICC's
5	2.0575	1	4.9604	1	0.8737
6	2.3677	2	5.1884	2	6.8096
7	2.1415	3	4.7682	5	2.0436
8	3.4953	11	6.0733	8	2.4817
14	2.8326	14	7.1978	10	0.9800
15	3.8529	15	10.0008	13	0.9866
18	0.7567	16	14.4605	15	2.4008
23	2.0754	19	6.1695	22	5.9273
26	1.5410	20	5.8869	23	5.2020
27	1.7075	24	3.8787	24	7.1995
28	0.3401	28	12.6013	25	12.5808
29	0.2110	32	5.2007	27	0.8457
30	1.5602	33	3.9273	29	4.2465
33	0.8005	37	4.2980	33	1.8619
35	0.3469	40	5.2584	34	2.6756
36	1.2684	41	9.4285	35	2.4957
40	0.9954	42	4.2693	46	2.0382
42	0.3120	43	4.9213	47	1.5492
43	1.1636	44	9.6651	48	5.1858
48	1.3565	45	9.2629	49	3.5703
		47	5.4386	50	1.3024
		48	6.3912		
		49	4.6270		
		50	4.7551		

As was stated in 4.5.3.3 the problem with this method is the determining of a cut-off value. The researcher wished to retain items with the smallest area between the ICC's. All the marked items in the grade four test exhibited a small area. Only item 25 in the grade six test was of concern to the researcher and was discarded. Six of the items in the grade five test, namely items 15, 16, 28, 41, 44 and 45, were reviewed. All six items could not be discarded as the test would then consist of only 18 items and not 20 items. The researcher reviewed the six items and found that three of the six items (items 41, 44 and 45) had a very similar area, namely 9.6651, 9.4285, and 9.2629. These items were retained and only the three items with the greatest area were discarded, namely items 15, 16 and 28. An arbitrary value of 10 was therefore decided upon as a cut-off value. After careful consideration with respect to the DIF results for both the difference between item parameters and the area between ICC's, the CTT and IRT results, and the fit statistics, the researcher obtained 20 items for grade four, 21 items for grade five and 20 items for grade six that satisfied all the above statistical properties. The 21 items of the grade five test were then re-examined and the item with the largest DIF result, with respect to the item parameters, was discarded. Item 45, with a DIF value of –2,73, was eliminated.

5.4.3.2.6 Representation of the learning strands

With the final 20-item test per grade, the ultimate prerequisite was to ensure that each learning strand, namely numbers and operations; fractions; patterns; shapes and space; measurement; and data, was adequately represented in each test. The number of items per learning area is represented in Table 5.11.

Table 5.11: Representation of the learning strands in the final 20-item test

Grade 4		Grade 5		Grade 6	
Learning strand	**No. of items represented**	**Learning strand**	**No. of items represented**	**Learning strand**	**No. of items represented**
Number and operations	4	Number and operations	3	Number and operations	4
Fractions	2	Fractions	2	Fractions	3
Patterns	2	Patterns	3	Patterns	3
Shapes and space	6	Shapes and space	2	Shapes and space	3
Measurement	4	Measurement	4	Measurement	2
Data	2	Data	6	Data	5

Not only has the 20 item grade four, five and six test satisfied the various statistical requirements, it is also representative of all the learning strands. The initial aim, that is, to develop a culturally fair and bias-free test, has been achieved. The item numbers of the mathematics proficiency test for grades four, five and six are represented in Table 5.12. The link between the cognitive processes (as discussed in chapter three), which is necessary for successful task completion, as well as each of the 20 items selected for grades four, five and six are discussed in 5.4.6.

As was stated earlier, the DIF results of the three respective groups, namely males versus females, English versus Afrikaans, and English and Afrikaans versus Sesotho, were computed using the comparison of item parameters, the area between ICC's, and in some cases the Mantel-Haenszel procedure. The English and Afrikaans versus Sesotho sample groups have been discussed in detail. The most important outcomes of the other two groups with respect to the 20 items selected for each test are reviewed below.

5.4.3.2.7 Other DIF computations

DIF was also computed for each grade, for the males versus females and the English versus Afrikaans sample groups. The difference between item parameters and the area between ICC's were calculated and the Mantel-Haenszel procedure was carried out for each of the above groups. Although the results were not taken into consideration when selecting the final test items, the results are still relevant. The Conquest–Generalised Item Analysis Software (Wu et al., 1998) and the MicroCAT software (Assessment Corporation, 1989, 1995) were used to compute the DIF. Only one of the DIF procedures, namely the comparison of item parameters, will be discussed in respect of the males versus females and the English versus Afrikaans sample groups.

With regard to the males versus females DIF results (comparison of item parameters) for the 20 items selected for the final grade four test, 14 of the items showed little or no DIF, while three of the items (6, 8 and 14) were easier for the males and three of the items (26, 30 and 33) were easier for the females. This represents a fairly balanced test with regard to the DIF results for the males versus females sample group. With regard to the English versus Afrikaans DIF results (comparison of item parameters) for the 20 items selected for the final grade four test, 17 of the items showed little or no DIF, while one item (14) was easier for the English-speaking learners and two items (7 and 29) were easier for the Afrikaans-speaking learners. This represents a fairly unbiased test with regard to the English versus Afrikaans sample group. The grade four tests are therefore not only culturally fair, but relatively translation and gender fair.

With regard to the males versus females DIF results (comparison of item parameters) for the 20 items selected for the final grade five test, 17 of the items showed little or no DIF, while two of the items (1 and 44) were easier for the males and one item (48) was easier for the females. This represents a fairly unbiased test with regard to the males versus females sample group. With regard to the English versus Afrikaans DIF results (comparison of item parameters) for the 20 items selected for the final grade five test, 14 of the items showed little or no DIF, while two of the items (1 and 37) were easier for the English-speaking learners and four of the items (2, 14, 32 and 44) were easier for the Afrikaans-speaking learners.

With regard to the males versus females DIF results (comparison of item parameters) for the 20 items selected for the final grade six test, 13 of the items showed little or no DIF, while three of the items (15, 22 and 24) were easier for the males and four of the items (5, 10, 27 and 34) were easier for the females. This represents a balanced test with regard to the DIF results for the males versus females sample group. With regard to the English versus Afrikaans DIF results (comparison of item parameters) for the 20 items selected for the final grade six test, 15 of the items showed little or no DIF, while two of the items (27 and 48) were easier for the English-speaking learners and three of the items (1, 2 and 33) were easier for the Afrikaans-speaking learners. This represents a fairly balanced test with regard to the English versus Afrikaans sample group.

The final grade four, five and six mathematics proficiency test is discussed below.

5.4.3.2.8 Final mathematics proficiency test items

The final draft of the mathematics proficiency test for grades four, five and six was then given to several of the contact teachers for advice before drafting the final test. The test was also given to the MLMMS learning facilitators at the Free State Department of Education for comments (see Annexure G). All involved were extremely satisfied with the final draft. The final test for grades four, five and six, in English and Afrikaans, is supplied as Annexure E. The final item numbers are indicated in Table 5.12 below.

Table 5.12 Final mathematics proficiency test items

Grade four		**Grade five**		**Grade six**	
Item	Original item no.	Item	Original item no.	Item	Original item no.
1	5	1	1	1	1
2	6	2	2	2	2
3	7	3	3	3	5
4	8	4	11	4	8
5	14	5	14	5	10
6	15	6	19	6	13
7	18	7	20	7	15
8	23	8	24	8	22
9	26	9	32	9	23
10	27	10	33	10	24
11	28	11	37	11	27
12	29	12	40	12	29
13	30	13	41	13	33
14	33	14	42	14	34
15	35	15	43	15	35
16	36	16	44	16	46
17	40	17	47	17	50
18	42	18	48	18	47
19	43	19	49	19	48
20	48	20	50	20	49

The final mathematics proficiency test is called the Intermediate Phase Mathematics Proficiency Test. A learner who is struggling with mathematics in grade four, five or six can be tested using an applicable test. Not only was the mathematics proficiency test developed but it was also standardised. Norms per term were calculated for each grade. The norm determination is discussed in the next section.

5.4.4 Phase four: Determination of norms

5.4.4.1 Introduction

The final mathematics proficiency test was sent to 12 schools during the first and fourth terms of 2003. The learners who were tested during the previous year were in the next grade and would therefore not be exposed to the same items. The norm determination consisted of testing the same grade four, five and six learners during the first and fourth terms respectively. In so doing, norms for the year were determined, with separate norms for each term. An estimation using inter-polarisation was used to determine the norms for the second and third terms.

The instructions to the contact teachers during phase four was to administer the 20-item test to all the respective grade four, five and six learners at their school during the first and fourth terms of 2003. They were not requested to mark the items. The researcher undertook the marking.

The contact teacher at each school was again given instructions to be conveyed to all other teachers with grade four, five and six mathematics classes. The instructions were as follows:

a) no calculators may be used;
b) no time limit is given (within limits);
c) you may read the questions to the learners;
d) important details such as the learner's name, home language, age and gender must be given in the space provided on the test;
e) the test must be completed on the test paper; and
f) extra paper may be given for rough work but need not be handed in.

5.4.4.2 Sample

The sample group for the norm determination phase was the same as that in all the previous phases. The researcher selected learners whose home language was English from the English-medium schools, Afrikaans from the Afrikaans-medium schools, and Sesotho from the previously disadvantaged English-medium schools. In this phase, the sample per grade should not be less than 100 (Huysamen, 1996). The composition of the sample according to grade and home language is represented in Tables 5.13.1 and 5.13.2.

Table 5.13.1: Sample distribution during phase four, first term

Grade	**Home language**			
	English	**Afrikaans**	**Sesotho**	**Total**
4	129	249	180	558
5	145	231	193	569
6	148	244	215	607
Total	422	724	588	1734

Table 5.13.1 indicates that the sample size for the English-speaking learners was smaller than for the Afrikaans- and Sesotho-speaking learners. A total of 1734 learners were tested during the first term of phase four.

Table 5.13.2: Sample distribution during phase four, fourth term

Grade	**Home language**			
	English	**Afrikaans**	**Sesotho**	**Total**
4	122	238	164	524
5	134	211	178	523
6	138	219	175	532
Total	394	668	517	1579

Once again Table 5.13.2 indicates that the sample size for the English-speaking learners was smaller than for the Afrikaans- and Sesotho-speaking learners. A total of 1579 learners were tested during the second administration of the test. A total of 28 English-speaking learners, 56 Afrikaans-speaking learners and 71 Sesotho-speaking learners could not be matched. This discrepancy between the number of learners being tested during the first and second administrations could be explained by school changes, absenteeism on the day of the test or other socioeconomic factors such as poverty (an inability to pay school fees which may have led to the termination of studies) or having to go and live with another family member or friend due to the death of one or both parents or the death of the guardian.

5.4.4.3 Calculation of norms

When a learner is tested by means of the mathematics proficiency test, a raw score out of 20 is obtained. This raw score does not indicate the learner's mathematical percentile. If this raw score is compared to the frequency of this raw score in the norm population, then relevant conclusions can be drawn from this information (Esterhuyse, 1997). A norm is sometimes a synonym for average and is the mean score for some specified group of people. A table showing the performance of the norm group is called a norm table or norms. This table shows the relationship between the raw score and some type of derived score (Mehrens & Lehman, 1991). To evaluate a learner's raw score, the score needs to be converted into a standard score that is obtained from the norm table. Two types of standard scores are stanines and percentile ranks. According to Mehrens and Lehmann (1991) stanines are normalised derived scores with a mean of five and a standard deviation of two. The integers one to nine occur. The authors go on to state that percentile ranks are the best scores to use. A percentile is defined as a point on the distribution below which a certain percentage of the scores occurs. A percentile rank gives a person's relative position or the percentage of a person's score. Percentile ranks have the advantage of being easy to compute and fairly easy to interpret. For a raw score to be normalised, it is necessary to use normal probability graph paper. On the left-hand side of the graph paper is a vertical axis where the cumulative proportion appears. At this point the percentile rank can be read. The percentile rank is a score from 0 to 100. On the right-hand side of the graph paper is the division that shows the standard scores or the stanines. The raw score is at the bottom on a horizontal axis. The raw scores vary from 0 to 20. The normalisation of the raw scores with respect to grade four, five and six English-, Afrikaans- and Sesotho-speaking learners was computed. The normalisation of the raw score for the total group of grade four, five and six learners is indicated in Figure 5.3.1, Figure 5.3.2 and Figure 5.3.3.

Figure 5.3.1: Normalisation of the raw scores of the mathematics proficiency test for the total group of grade four learners

Figure 5.3.2: Normalisation of the raw scores of the mathematics proficiency test for the total group of grade five learners

Figure 5.3.3: Normalisation of the raw scores of the mathematics proficiency test for the total group of grade six learners

By indicating the raw scores on the graph paper, the cumulative percentage of the mathematics proficiency test was calculated by means of the SAS computer program (SAS Institute, 1985). Once all the scores were indicated, a line was drawn through all the points in such a manner that there were an equal amount of points on both sides of the line. In this manner a graph that is continuous and smooth can be obtained. If the division of the points is normal, the line will be straight (Esterhuyse, 1997). Any raw score can, with the help of the graph, be converted into a standard score. In this study, stanines and percentile ranks were used as the normalised standard scores. This conversion was already discussed in Table 4.1 in chapter four.

5.4.4.4 Norm tables

Using the above method, the final norm tables for the grade four, five and six, English-, Afrikaans- and Sesotho-speaking learners and the total group of grade four, five and six learners were calculated. The norms were determined for each term, so that the test can be administered at any time of the year. Once a learner has completed the test, one mark is allocated for each correct answer, with no marks being awarded for an incorrect answer. The marks are then summated and the raw score obtained. This raw score can then be converted into a stanine or a percentile rank by looking up the raw score on the norm table. The norms for the appropriate grade must be used. If the learner writes the test in the second term, then the norms of the second term must be used. The norm tables for the grade four, five and six, English-, Afrikaans- and Sesotho-speaking learners and the norm tables for the total group of grade four, five and six learners can be found in Annexure F.

5.4.4.5 Statistical properties of the mathematics proficiency test

5.4.4.5.1 Introduction

Certain statistical properties have already been discussed, such as the norm determination and the item analysis. According to Esterhuyse (1997) there are certain statistical properties that need to be addressed when developing and standardising a psychometric test. These properties include the size of the sample, the number of items in each test, the means of test scores, the standard deviation of the tests, the reliability coefficient, the skewness, and the kurtosis. The above information for the 50-item test is represented in Table 5.6. The sample size, the means of the test scores, the standard deviation, the reliability coefficient, the skewness, and the kurtosis for the final 20-item test for grades four, five and six are represented in Table 5.14. This information was obtained from the first application of the final test during the first term of 2003.

Table 5.14: Statistical properties of the 20-item mathematics proficiency test

GRADE	Sample	N	NO. OF ITEMS	$\bar{X}$	S	RELIABILITY (K-R 20)	SKEWNESS	KURTOSIS
4	English	129	20	14.217	3.396	0.7440	-0.48370	-0.450020
4	Afrikaans	249	20	12.586	3.624	0.7326	-0.42070	-0.235420
4	Sesotho	180	20	8.861	3.899	0.7727	0.66147	-0.351250
4	**Total group**	**558**	**20**	**11.762**	**4.219**	**0.8032**	**-0.17953**	**-0.904600**
5	English	145	20	11.731	3.473	0.7452	-0.38489	0.177212
5	Afrikaans	231	20	10.329	4.003	0.7929	-0.16286	-0.623520
5	Sesotho	193	20	4.093	3.565	0.8133	1.273315	1.335488
5	**Total group**	**569**	**20**	**8.571**	**4.946**	**0.8734**	**-0.03061**	**-1.071040**
6	English	148	20	11.034	3.829	0.7850	-0.01443	0.061556
6	Afrikaans	244	20	10.455	3.924	0.7593	-0.06931	-0.612980
6	Sesotho	215	20	5.242	2.844	0.6465	0.467029	0.222320
6	**Total group**	**607**	**20**	**8.750**	**4.404**	**0.8175**	**0.204081**	**-0.646410**

According to the above statistical properties, each test has 20 items. The number of learners tested in grade four was 558, while 569 were tested in grade five and 607 were tested in grade six. The mean of the test for the total group of grade four learners was 58.81%, while it was 42.86% for the total group of grade five learners and 43.75% for the total group of grade six learners. This was the mean percentage achieved by the learners during the first term in their respective grades.

5.4.4.5.2 Means with respect to the first- and fourth-term test results

The norms for the year were determined by administering the test during the first- and fourth-terms of 2003. It would therefore be sensible to investigate the difference in the performance of the learners in each grade. Table 5.15 clearly shows the mean and standard deviation for the first- and fourth-term applications and the percentage by which the performance of the learners differed.

Table 5.15: Comparison of test means between the first- and fourth-terms of 2003

Grade	**Sample**	**First-term**			**Fourth-term**			**% increased**
		N	**$\bar{X}$**	**s**	**N**	**$\bar{X}$**	**s**	
4	English	129	14.217	3.396	122	15.795	2.718	7.890
4	Afrikaans	249	12.586	3.624	238	15.193	3.017	13.035
4	Sesotho	180	8.861	3.899	164	11.860	4.818	14.995
4	**Total group**	**558**	**11.762**	**4.219**	**524**	**14.290**	**3.977**	**12.640**
5	English	145	11.731	3.473	134	13.328	3.107	7.985
5	Afrikaans	231	10.329	4.003	211	12.806	3.966	12.385
5	Sesotho	193	4.093	3.565	178	3.500	2.256	-2.965
5	**Total group**	**569**	**8.571**	**4.946**	**523**	**9.772**	**5.558**	**6.005**
6	English	148	11.034	3.829	138	13.667	3.019	13.165
6	Afrikaans	244	10.455	3.923	219	12.283	3.823	9.140
6	Sesotho	215	5.242	2.844	175	4.491	2.602	-3.755
6	**Total group**	**607**	**8.750**	**4.404**	**532**	**10.079**	**5.119**	**6.645**

From the discussion in the literature review, the test is based upon the learning strands of the MLMMS learning area and the cognitive processes of the learning-teaching process of the cognitive model for mathematics (Holmes, 1985). The learners' performance should improve from the first- to the fourth-term due to the mastering of curriculum tasks and the cognitive development of the learners. Yet, with some of the groups, the learners' performance decreased or remained relatively the same. As was discussed in chapter three, it is important to remember that in terms of developmental psychology, there are no boundaries in terms of time and age, as children master different processes at different times. In other words, learners need not have mastered new cognitive processes by the fourth term, leading to their performance on the test remaining relatively the same or even decreasing. The mean percentage of the total group of grade four learners during the first term was 58.81%. This mean increased by 12.64% to an average of 71.45% during the fourth term. This increase was the largest amongst the three grades. According to Holmes (1985) this difference could also be explained by a phase change. The learners move from developing prerequisite understanding to constructing a concept or generalisation. According to Hammill and Bartel (1990) the difference in performance could also be due to a stage change where learners progress from the acquisition phase to the proficiency phase, in other words, where a teacher's input decreases and the learner's output increases. The mean percentage of the total group of grade five learners during the first-term was 42.86% and this increased by 6% to 48.86%. The mean percentage of the total group of grade six learners during the first-term was 43.75% and this increased by 6.65% to 50.40%. The constant performance of the grade five and six learners could be explained by the maintenance stage of Hammill and Bartel. The maintenance stage states that learners do not adopt new concepts - they simply extend the concepts they acquired in the proficiency phase by practising them.

5.4.4.5.3 Standard deviation

The standard deviation is the positive square root of the variance (Huysamen, 1990). According to Mehrens and Lehmann (1991) the standard deviation is the amount of variability in a distribution. The standard deviation is represented by an '*s*' in Table 5.14. According to Huysamen (1996) in a normal distribution, 99.74% of the testees' scores lie between –3 standard deviations and +3 standard deviations. In other words, 99.74% lie within six standard deviations of the calculated average of the distribution. On the basis of the above information, the ideal standard deviation would lie between +3.33 and –3.33 (20 divided by 6 = 3.33). Table 5.15 shows that the standard deviation of the total group of grade four, five and six learners in the first term was 4.219, 4.946 and 4.404 respectively. The standard deviation for grade four was 0.889 above the ideal; for grade five 1.616 above the ideal; and for grade six 1.074 above the ideal. The standard deviation of the total group of grade four, five and six learners in the fourth term was 3.977, 5.558 and 5.119 respectively. The standard deviation for grade four was 0.647 above the ideal; for grade five 2.228 above the ideal; and for grade six 1.789 above the ideal. The largest standard deviation occurred among the grade five learners.

5.4.4.5.4 Skewness

Data distributions can take on the shape of four types of distributions, namely a normal distribution, a positively skewed distribution, a negatively skewed distribution ,and a rectangular distribution (Mehrens & Lehmann, 1991). The ideal distribution is a normal distribution and the statistic measures the deviation from the norm. Once again the values vary from –3 to +3. A positive skewness value indicates that most of the learners achieved a score lower than the mean. This would give a positively skewed distribution and this might occur when a test is too difficult. A negative skewness value indicates that most of the learners achieved a score higher than the mean. This would give a negatively skewed distribution and could also indicate that the test was too easy (Esterhuyse, 1997). According to Table 5.14 the skewness values were negative for all the English- and Afrikaans-speaking learners and only slightly smaller than zero. This indicates a slightly negative skewness. The skewness values were also negative for the total group of grade four and five learners and only slightly smaller than zero, indicating a symmetrical distribution. The skewness values were positive for all the Sesotho-speaking learners and the total group of grade six learners. The results indicate that the test was difficult for the Sesotho-speaking learners, as the results were greater than zero. The test was also slightly difficult for the total group of grade six learners. It must be kept in mind that the raw scores were normalised before the norms were calculated.

5.4.4.5.5 Kurtosis

According to Huysamen (1990) kurtosis of a curve refers to the flatness or peakedness of the center of the curve. If a curve is more peaked it is said to be leptokurtic, and if it is more flat it is said to be platykurtic. The curve of a normal distribution is mesokurtic. A normal curve will have a kurtosis of zero, a peaked curve will have a positive kurtosis value, and a flat curve will have a negative kurtosis value. The kurtosis values for the total group of grade four, five and six learners were all negative, which indicates a more flat curve. The grade four English, Afrikaans and Sesotho groups and the grade five and six Afrikaans groups also all had a negative kurtosis value. The grade five and six English and Sesotho groups all had a positive kurtosis value, indicating a more peaked curve. The values are relatively small, so the curves all have relatively normal distributions.

5.4.4.5.6 Reliability

Two types of reliability will be considered in this paragraph, namely parallel-forms reliability and the test-retest reliability. Two processes were followed to determine the parallel-forms reliability, namely the coefficient-alpha and the split-half method. A discussion of each of the above is necessary to determine to what extent the mathematics proficiency test is reliable.

5.4.4.5.6.1 Parallel-forms reliability

a) Coefficient-alpha

According to Huysamen (1996) when the items are marked right and wrong, the coefficient-alpha is equal to the K-R 20 formula. The coefficient-alpha is given in Table 5.14. The preferred reliability coefficient for a standardised test is 0.80 or higher. The reliability coefficients for the total group of grade four, five and six were 0.8032, 0.8734, and 0.8175 respectively. The coefficient-alpha therefore indicates that the test is reliable and that it measures consistently.

b) Split-half method

The split-half method is discussed in paragraph 4.6.4.1. This method halves the length of the test by grouping all the even numbers as one half of the test and all the uneven numbers as the other half of the test. The two halves are then correlated. The results are given in Table 5.16.

Table 5.16: Split-half reliability results

Grade	Sample	N	Coefficient
4	English	129	0.628*
4	Afrikaans	249	0.575*
4	Sesotho	180	0.723*
4	**Total group**	**558**	**0.692***
5	English	145	0.633*
5	Afrikaans	231	0.693*
5	Sesotho	193	0.780*
5	**Total group**	**569**	**0.815***
6	English	148	0.688*
6	Afrikaans	244	0.654*
6	Sesotho	215	0.553*
6	**Total group**	**607**	**0.743***

* $p \leq 0.0001$

The results above indicate that the split-half reliability coefficients are smaller than the alpha reliability coefficients, but that the coefficients are still significant at the 0.01% level. The reliability results vary from satisfactory to good.

5.4.4.5.6.2 Test-retest reliability

According to paragraph 4.6.1, test-retest reliability is a measure of the stability of the test. To determine test-retest reliability, a test must be administered on two occasions, to a representative sample of the population for which the test is intended. In this research, the same mathematics proficiency test was administered on two occasions, during the first and fourth terms of 2003, to the same grade four, five and six learners. It could be assumed that the learners should, in general, achieve a higher score during the fourth term than the first term. A period of two terms elapsed between the testing, which is a relatively long period of time. Therefore the first administration should not have had an effect on the second administration. Test-retest reliability means that the same test can be administered at any time, to the same population, and that the same result should be achieved. Therefore if the test-retest reliability of the mathematics proficiency test is not significant, it does not necessarily mean that the test is not reliable; it could simply be an indication of the extent to which the variable being measured is unstable. The total group of grade five and six learners obtained a good correlation coefficient. The total group of grade four learners obtained a satisfactory correlation coefficient. The correlation coefficient of the grade four, five and six Sesotho groups was low. Despite this fact, the test-retest reliability was significant for each of the grade four, five and six tests at the 0.01% level. The mathematics proficiency test can therefore be said to be reliable and can, with confidence, be administered in practice, especially when using the norms for the total group of each grade. The results are represented in Table 5.17.

Table 5.17: Correlation coefficients between the first and second administrations

Grade	**Sample**	**N – first administration**	**N – second administration**	**Correlation coefficients**
4	English	129	122	0.668*
4	Afrikaans	249	238	0.691*
4	Sesotho	180	164	0.247*
4	**Total group**	**558**	**524**	**0.581***
5	English	145	134	0.669*
5	Afrikaans	231	211	0.749*
5	Sesotho	193	178	0.445*
5	**Total group**	**569**	**523**	**0.819***
6	English	148	138	0.671*
6	Afrikaans	244	219	0.719*
6	Sesotho	215	175	0.408*
6	**Total group**	**607**	**532**	**0.793***

$p \leq 0.0001$

5.4.5 Phase five – Validity

In this study emphasis is placed on the content validity and the criterion-related validity.

5.4.5.1 Content validity

In paragraph 4.7.1 content validity was defined as the degree to which the items in a test represent the total universe of items, which could have been compiled in terms of the curriculum and teaching objectives. Content validity of a measuring instrument cannot be given in terms of a quantitative analysis. As was stated in paragraph 5.4.3.2.8, the final draft of the mathematics proficiency test for grade four, five and six learners was submitted to several of the contact teachers for comment before drafting the final test. The teachers were asked to comment with respect to the type of items that were selected and the degree of difficulty of each. The teachers felt that the test covered the entire curriculum and would serve as a good measure of a learner's mathematical ability. The contact teachers were experienced in mathematics and could objectively evaluate the validity of the content of the items selected.

5.4.5.2 Predictive validity

Referring back to 4.7.2 distinction is made between predictive validity and concurrent validity. Concurrent validity was not investigated in this study, but the predictive validity of the test was explored. The way in which predictive validity was investigated was to correlate the score achieved in the mathematics proficiency test in the first term of 2003 with the score achieved in the most recent mathematics evaluation (third-term evaluation). The score given by the teachers as an evaluation mark was then converted to a percentage. The evaluation mark given by some schools was in terms of a percentage and that given by other schools was in terms of a symbol. The symbol was converted to a percentage as indicated in Table 5.18.

Table 5.18: Percentage intervals with respect to the intermediate phase symbols

SYMBOL	MEANING	% INTERVAL	% ASCRIBED
B	Beyond achieved	80 – 100	90
A	Achieved	60 – 79	70
P	Partially achieved	40 – 59	50
N	Not yet achieved	39 and below	20

The predictive validity of the test was then calculated by correlating the mark achieved in the mathematics proficiency test during the first term with the scholastic percentages achieved in the fourth term. The results are indicated in Table 5.19.

Table 5.19: Predictive validity of the mathematics proficiency test

Grade	Sample	N	Coefficient
4	English	122	0.563*
4	Afrikaans	238	0.576*
4	Sesotho	164	0.444*
4	**Total group**	**524**	**0.634***
5	English	134	0.625*
5	Afrikaans	211	0.700*
5	Sesotho	178	0.519*
5	**Total group**	**523**	**0.730***
6	English	138	0.470*
6	Afrikaans	219	0.699*
6	Sesotho	175	0.265*
6	**Total group**	**532**	**0.600***

* $p \leq 0.0001$

The above results indicate that the predictive validity of all the groups in all the grades is significant at the 0.01% level. At first glance some of the correlation scores appear small, especially for the Sesotho-speaking learners. According to paragraph 4.7.2 the predictive validity correlation coefficient of the total group of grade four, five and six learners has a large effect size. The grade four English and Afrikaans groups, all the grade five groups and the grade six Afrikaans group also have a large effect size. The grade four Sesotho group and the grade six English group have a medium effect size, despite the fact that the correlation coefficients appear to be small. The only correlation coefficient that is of concern is the grade six Sesotho group, as the effect size is small. The predictive validity correlation coefficient of the total group of each grade has a large effect size and this is of utmost importance.

5.4.6 Qualitative analysis

It is important that the test not only be used to determine whether a learner in grade four, five or six has a mathematics problem, but to determine in which learning strand the problem lies. The qualitative analysis is therefore important for diagnostic purposes.

The link between the cognitive processes (as discussed in chapter three), which is necessary for successful task completion, as well as each of the 20 items selected for grades four, five and six, are indicated in the tables below.

The various categories of the cognitive processes include *receiving*, *interpreting*, *organising*, *applying*, *remembering*, and *problem solving.* All 20 items of all the grades include the cognitive process of *attending.* This falls under the category of *receiving*. For successful task completion, the learner must be alert to stimuli, that is, maintaining awareness of, perceiving, and observing. It is important to state that cognitive processes are necessary for the successful task completion of each respective item.

Table 5.20.1: Qualitative analysis of the grade four mathematics proficiency test

Item	Learning strand	Category	Cognitive processes	Description
1	Numbers and operations	Problem solving Interpreting Applying	Combination of cognitive processes Comparing Evaluating	BODMAS (comparing and evaluating) Comparing two values: noting likenesses (similarities) or differences (< ; = ; >) Judging the two values obtained
2	Numbers and operations	Applying	Predicting	Estimating 347 to the nearest 100 (rounding off)
3	Numbers and operations	Interpreting Applying Remembering Problem solving	Ordering Evaluating Retrieving Combination of cognitive processes	Placing the series in terms of an increasing or decreasing attribute; sequencing Checking a solution by subtracting Remembering what the term difference means First sequencing and then subtracting (ordering and evaluating)
4	Numbers and operations	Applying	Evaluating	Evaluating the half of 525
5	Fractions	Applying	Testing	Devising and carrying out a plan to verify that 0,3 + ___ = 1
6	Fractions	Interpreting Applying Problem solving	Translating Evaluating Combination of cognitive processes	Restating kg in grams Checking a solution by adding Restating kg in grams and adding (translating and evaluating)

7	Patterns	Interpreting Organising Applying	Ordering Inferring Predicting	Sequencing the shapes Using reason to abstract the pattern Foretelling the sequence
8	Patterns	Interpreting Organising Applying	Ordering Inferring Predicting	Sequencing the arrows Using reason to abstract the pattern Foretelling the sequence
9	Shapes and space	Interpreting	Classifying	Classifying the shapes
10	Shapes and space	Interpreting Interpreting Organising Remembering	Comparing Classifying Relating Retrieving	Noting likenesses and differences between 2-D and 3-D shapes Classifying into 2-D and 3-D shapes Connecting in terms of a quantitative property (2-D/3-D) Bringing to mind what a 2-D and 3-D shape looks like
11	Shapes and space	Interpreting Interpreting	Comparing Classifying	Discriminating between all the shapes Arranging the shapes by hierarchical classification
12	Shapes and space	Interpreting Interpreting	Comparing Classifying	Discriminating between all the shapes Arranging the shapes by hierarchical classification
13	Shapes and space	Interpreting Interpreting Organising	Classifying Comparing Relating	Noting likenesses and differences Grouping to form a square Connecting the two shapes in

		Applying Remembering	Hypothesising Imaging	terms of a quantitative property Postulating a relationship between the two shapes Using visual representations; mental picturing
14	Shapes and space	Organising Remembering	Inferring Imaging	Using reason to abstract triangles from particulars Using visual representations of objects; mental picturing
15	Measurement	Interpreting Remembering	Comparing Retrieving	Comparing two values: noting likenesses or differences (< ; = ; >) Bringing to mind how many months in a year
16	Measurement	Interpreting Remembering	Comparing Retrieving	Comparing two values: noting likenesses or differences (< ; = ; >) Bringing to mind how many cents in a rand
17	Measurement	Remembering	Retrieving	Bringing to mind how many days in a leap year
18	Measurement	Applying	Predicting	Foretelling; stating consequences; estimating what the time would have been 10 minutes earlier
19	Data	Organising Applying	Summarising Evaluating	Condensing subject matter to determine which activities are not offered on Friday Concluding on the basis of evidence which activities are not offered on Friday

20	Data	Interpreting	Comparing	Comparing each shop, noting the likenesses and differences
		Applying	Evaluating	Concluding on the basis of evidence which shop spends the most money on salaries

Table 5.20.2: Qualitative analysis of the grade five mathematics proficiency test

Item	Learning strand	Category	Cognitive processes	Description
1	Numbers and operations	Interpreting	Ordering	Placing in a series in terms of decreasing attributes
2	Numbers and operations	Problem solving	Combination of cognitive processes	BODMAS
3	Numbers and operations	Interpreting	Translating	Restating 23504 in another mode
4	Fractions	Applying Remembering	Evaluating Retrieving	Concluding on the basis of evidence Using fraction rules; focusing on past experience with fractions
5	Fractions	Interpreting	Translating	Restating the fraction as a mixed number
6	Patterns	Applying Applying Problem solving	Evaluating Testing Combinations of cognitive processes	Judging, using the various numbers to find the correct solution Devising and carrying out a plan to verify a hypothesis Multiplying; adding; subtracting (evaluating and testing)
7	Patterns	Interpreting Organising Applying	Ordering Inferring Predicting	Sequencing the fractions Using reason to abstract the pattern Foretelling the sequence
8	Patterns	Interpreting Organising	Ordering Inferring	Sequencing the arrows Using reason to abstract the pattern

		Applying	Predicting	Foretelling the sequence
9	Shapes and space	Interpreting Interpreting Interpreting Organising Remembering	Translating Comparing Classifying Relating Imaging	Labelling the shape that is not a quadrilateral Discriminating between all the answers Grouping by distinguishing attributes (shapes with four sides) Connecting in terms of a quantitative property (four sides) Mental picturing of the shapes
10	Shapes and space	Interpreting Remembering	Classifying Retrieving	Discriminating between a 2-D and 3-D shape Bringing to mind what a 2-D and 3-D shape looks like
11	Measurement	Interpreting Organising Organising Applying Problem solving	Comparing Questioning Inferring Testing Combinations of cognitive processes	Discriminating between the two types of boxes of fishcakes Asking for clarification; inquiring about which is a better buy Using reason to move from concepts to examples and conclusions; reasoning about which is a better buy Devising and carrying out a plan to verify which is a better buy Calculating which is a better buy (comparing, questioning, inferring and testing)
12	Measurement	Organising	Inferring	Using reason to move from concepts to examples

		Applying Applying	Predicting Testing	Estimating how far he will travel Devising and carrying out a plan to verify how far he will travel in 4 hours
13	Measurement	Applying Remembering	Evaluating Retrieving	Calculating the perimeter of the figure Bringing to mind the meaning of perimeter
14	Measurement	Applying	Evaluating	Evaluating the solution on the basis of the information that has been given
15	Data	Interpreting Interpreting Organising	Translating Comparing Relating	Restating the number of learners represented on the pie chart in terms of a fraction Noting likenesses and differences between the number of learners represented on the pie chart Connecting in terms of a quantitative property (children who play netball)
16	Data	Interpreting Interpreting Organising	Translating Comparing Relating	Restating (quantifying) the number of learners represented on the pie chart who do not take part in sport Noting likenesses and differences between the number of learners represented on the pie chart Connecting in terms of a quantitative property (children

				who do not take part in sport)
17	Data	Interpreting	Comparing	Discriminating between all the scores to determine the lowest score in a single event
18	Data	Applying Remembering Problem solving	Evaluation Retrieving Combination of cognitive processes	Calculating the difference Remembering what the word difference means Finding the two total scores and remembering what difference is and calculating it (evaluating and remembering)
19	Data	Interpreting Organising	Translating Relating	Restating the number of learners with a mass of 42 kg from * into a quantity (where * = 5 children) Connecting in terms of a quantitative property
20	Data	Interpreting Organising	Translating Relating	Restating the number of learners with a mass of 42 kg from * into a quantity (where * = 5 children) Connecting in terms of a quantitative property

Table 5.20.3: Qualitative analysis of the grade six mathematics proficiency test

Item	**Learning strand**	**Category**	**Cognitive processes**	**Description**
1	Numbers and operations	Interpreting Applying Applying	Translating Predicting Evaluating	Restating 10^3 in another mode Estimating 10^3 Concluding what 10^3 equals
2	Numbers and operations	Applying	Evaluating	Determining the difference
3	Numbers and operations	Remembering	Retrieving	Bringing to mind what 'factor' means and using rules to state the factors of 12
4	Numbers and operations	Applying	Evaluating	Doubling 124,7
5	Fractions	Interpreting	Translating	Restating the fraction in the simplest form
6	Fractions	Applying Remembering Problem solving	Evaluating Retrieving Combination of cognitive processes	Concluding on the basis of evidence Using fraction rules; focusing on past experience with fractions Using fraction rules, addition and subtraction (retrieving and evaluating) BODMAS
7	Fractions	Interpreting Organising Applying	Translating Relating Testing	Restating $^1/_3$ of 24 as a $^1/_4$ of another number Connecting the two fractions of 24 by a quantitative property; transforming Devising and carrying out a plan to verify the hypothesis
8	Patterns	Interpreting Organising	Ordering Inferring	Sequencing the numbers Using reason to abstract the

		Applying	Predicting	pattern Foretelling the sequence
9	Patterns	Interpreting Organising Applying	Ordering Inferring Predicting	Sequencing the decimals Using reason to abstract the pattern Foretelling the sequence
10	Patterns	Interpreting Organising Applying	Ordering Inferring Predicting	Sequencing the numbers Using reason to abstract the pattern Foretelling the sequence
11	Shapes and space	Remembering	Retrieving	Bringing to mind what symmetry is; focusing on past experience of drawing lines of symmetry; using the rules of symmetry
12	Shapes and space	Applying Remembering	Evaluating Retrieving	Determining the diameter Bringing to mind what diameter is
13	Shapes and space	Applying	Testing	Devising and carrying out a plan to verify what *x* equals so that the angles add up to 90 degrees
14	Measurement	Interpreting Interpreting Remembering	Translating Comparing Retrieving	Restating a pictorial expression in words Discriminating between the various types of angles Bringing to mind the difference between acute, obtuse, and right angles
15	Measurement	Applying	Evaluating	Measuring the size of the angle
16	Data	Interpreting	Comparing	Noting the likenesses and differences between the two bean plants

<table>
<tr><td></td><td></td><td>Applying

Problem solving</td><td>Predicting

Combinations of cognitive processes</td><td>Predicting which bean plant grows the fastest between weeks 2 and 3
Reading the graphs first and then determining the results (comparing and predicting)</td></tr>
<tr><td>17</td><td>Data</td><td>Applying

Remembering</td><td>Evaluating

Retrieving</td><td>Determining the number of learners who took part in the survey
Bringing to mind what |||| stands for</td></tr>
<tr><td>18</td><td>Data</td><td>Interpreting

Organising

Organising
Applying</td><td>Comparing

Relating

Summarising
Predicting</td><td>Comparing all the various people and their birth dates
Connecting the people in terms of their birth dates
Condensing the subject matter
Estimating who was born first</td></tr>
<tr><td>19</td><td>Data</td><td>Applying

Applying</td><td>Predicting

Evaluating</td><td>Estimating at what age Estelle Swart died
Evaluating the age at which Estelle Swart died</td></tr>
<tr><td>20</td><td>Data</td><td>Interpreting

Organising

Organising
Applying

Applying</td><td>Comparing

Relating

Summarising
Predicting

Evaluating</td><td>Comparing all the various people with their birth and death dates
Connecting the people in terms of their birth and death dates
Condensing the subject matter
Estimating who died at the youngest age
Evaluating the various ages of the people and determining who died at the youngest age</td></tr>
</table>

5.5 CONCLUSION

In chapter five, the researcher focuses on the results of the development of the Intermediate Phase Mathematics Proficiency Test. Before the item analysis and item selection was discussed, the aim of the study in terms of developing a culturally fair test and standardising it with applicable norms for various subgroups had to be understood. Cross-cultural test adaptation and the process of standardisation were then reviewed. The five phases, namely the pilot study, the construction of the preliminary test, item analysis and selection, norm determination and the validity of the test, were discussed in detail. The sample group throughout this phase consisted of grade four, five and six learners whose home language was English and who were attending English-medium schools, as well as learners whose home language was Afrikaans and who were attending Afrikaans-medium schools and learners whose home language was Sesotho and who were attending previously disadvantaged English-medium schools.

In the pilot study the researcher, with the help of mathematics learning facilitators and teachers, selected 60 items for the mathematics proficiency test and gathered statistical information on the items. The initial mathematics proficiency test proved too difficult for the learners and the researcher decided to use the items and the results as a baseline study. This led to a more reliable construction of the preliminary test, which consisted of 50 items.

During the item analysis and selection phase 20 items were selected from the original 50-item test for the Intermediate Phase Mathematics Proficiency Test. The item selection was conducted in the following manner:

a) for each grade, the DIF results were considered for the English and Afrikaans groups versus the Sesotho group. All items that showed minimal DIF (fell within the –2 and 2 parameter boundaries or just outside) were flagged;
b) the CTT results of the items flagged for the total grade four, total grade five and total grade six sample were then considered. The difficulty value (*p*) of each item

was reviewed. Items with a difficulty value of less than 0.2 were discarded, as were items with a difficulty value of more than 0.85. The discrimination value (r_{it}) of each item was also taken into consideration. Items with a discrimination value of less than 0.2 were discarded. Note was also taken of the item variance (s^2);

c) the IRT results of the items flagged for the total grade four, total grade five and total grade six sample were then considered. The one-parameter Rasch model was utilised for this purpose. Items with a difficulty value (b) smaller than –1.96 and greater than 1.96 were discarded;

d) the 'fit' statistics were also taken into consideration. Items with an INFIT mean square value greater than or equal to 1.30 and smaller than or equal to 0.75 were discarded;

e) these items were then double checked using the DIF results of the English and Afrikaans groups versus the Sesotho group with regard to the area between the ICC's. Items that showed a large area were discarded; and lastly

f) the items also had to be representative of all the learning strands, namely numbers and operations, fractions, patterns, shapes and space, measurement, and data.

The fourth phase consisted of the norm determination where norms were determined for each of the 12 subgroups by administering the test in the first and fourth terms of 2003. The statistical properties of the test were also considered.

The final phase consisted of the reliability and validity study, where the researcher considered the parallel-forms and the test-retest reliability. The validity was examined by considering the predictive validity of the mathematics proficiency test. The Intermediate Phase Mathematics Proficiency Test proved to be reliable and valid, with specific reference to the total group of grade four, five and six learners. The test can therefore be administered in practice as a diagnostic tool that is cross-culturally adapted.

Not only can the tester statistically determine a learner's stanine and percentile with regard to his or her performance in the test, but the test can also give qualitative information. This information includes determining the learning strand where the learner is experiencing problems, as well as the various cognitive processes that could be lacking for the successful task completion of a specific item.

6. CONCLUSION AND RECOMMENDATIONS

6.1 INTRODUCTION

The development of a mathematics proficiency test for English-, Afrikaans-, and Sesotho-speaking learners enables the psychologist and the educationist to conduct a comprehensive evaluation of an intermediate phase learner's mathematical proficiency. The psychologist and educationist can immediately determine whether the learner has a mathematical problem, and if so, in which specific learning strand the problem lies, so that the applicable cognitive processes that are necessary to carry out the task can be addressed.

The test referred to above is called the Intermediate Phase Mathematics Proficiency Test and it complies with the following criteria:

a) the test is applicable to grade four, five and six learners;
b) the test is standardised for English-, Afrikaans-, and Sesotho-speaking learners;
c) the items selected for the final mathematics proficiency test are cross-culturally adapted;
d) the norms per term are available, so that the test can be administered at any time of the year;
e) the test can be administered to groups or individuals;
f) the test can be used diagnostically to identify the learning strand in which the learner is experiencing problems and the cognitive processes that could be hindering the learner's mathematical performance;
g) the tests can be utilised by a psychologist or an educationist; and
h) the test will be of value to future generations of learners.

If a learner fails to meet the expectations of the curriculum or fails to carry out the cognitive processes necessary for successful task completion, then, in accordance with the aim of this study, the Intermediate Phase Mathematics Proficiency Test can be used to identify and address this problem.

6.2 CONCLUSION OF THE RESEARCH

The learning-teaching process of the cognitive model for mathematics (Holmes, 1985, 1995) and the learning outcomes of the MLMMS learning area (Mathematics Learning Area Statement, 2001) are the foundation of the grade four, five and six mathematics proficiency test. Holmes (1985) found that the cognitive approach helps children become excited about mathematics. Holmes (1995) also reports that a cognitive orientation to mathematical learning gives new meaning and satisfaction to individuals working with children. The four key principles of the model are:

a) encouraging the use of cognitive processes;
b) stressing learning concepts and generalisations;
c) emphasising intrinsic motivation; and
d) providing for individual differences (Holmes).

In chapter two the current educational system in South Africa was discussed. The principles and goals of OBE were also highlighted. The principles seem very idealistic and very few schools visited during the research project were implementing these principles and goals. A combination of the 'old' and 'new' education style was far more prevalent. The aspect that many of the schools visited were attending to was the critical outcomes of each learning area. An integral part of OBE is that assessment should include opportunities to assist teachers in fulfilling their task. The success of a teacher is dependent on sound assessment practices, which should motivate learners to achieve the outcomes. The Intermediate Phase Mathematics Proficiency Test can be utilised as a tool for diagnostic assessment purposes.

New insights regarding how children learn are also influencing teaching. For this reason a cognitive model for mathematics (Holmes, 1985) was used to reflect upon six key concepts that influence learning and teaching in the Mathematical Literacy, Mathematics and Mathematical Sciences learning area. These concepts were discussed in chapter three and include the categories of representing experience; motivation; individual differences; cognitive categories and cognitive processes; instructional procedures; and conceptual learning.

Chapter four highlighted the standardisation of psychometric tests with specific emphasis on cross-cultural test adaptation. Differential Item Functioning was used to limit the possibility of cultural bias. All three DIF mechanisms, namely the comparison of item parameters the difference in area between the ICC's, and the Mantel-Haenszel procedure, were used to detect DIF with respect to gender differences and the English versus Afrikaans groups. The Mantel-Haenszel procedure could not be used with respect to the English and Afrikaans groups versus the Sesotho group, as the Sesotho sample was too small. Before items were selected for the test they underwent extensive scrutiny. Despite this, some items functioned differently in various subgroups.

The only DIF result that was of interest to the researcher was the English and Afrikaans groups versus the Sesotho group. During the item selection process, DIF was used to potentially eliminate items that could bias either one of these groups. An attempt was therefore made to select items that were culturally bias-free. As was stated in chapter four, according to Allalouf et al. (1999) one of the main causes of DIF is differences in cultural relevance. This was the possible cause of DIF in this study.

The Item Response Theory and the Classical Test Theory were also used for item analysis and selection. The test was standardised for English-, Afrikaans- and Sesotho-speaking grade four, five and six learners. During standardisation, separate norms for each term were calculated. These norms are available in both stanines and percentile ranks. In other words, if a learner in grade five achieves a raw score of 8 out of 20 in the second term, then the learner has a stanine of 5 and a percentile rank of 60. We may deduce that 60% of the norm group achieved a similar or lower score.

The test can also be used qualitatively to determine not only the learning strand in which the learner may be experiencing problems, such as numbers and operations, fractions, patterns, shapes and space, measurement and data, but also the specific cognitive process, such as receiving, interpreting, organising, applying, remembering and problem solving, which may be preventing the learner from reaching his or her full mathematical potential (see paragraph 5.4.6).

The Intermediate Phase Mathematics Proficiency Test is also a reliable and valid measuring instrument since the bias of the assessment measure has been decreased. This was done by eliminating any item that was biased towards a specific cultural group. The test can therefore be used in practice with confidence.

The reliability of the test for the total group of grade four, five and six learners was in all cases greater than 0.80. Despite this, the result of the Sesotho grade five and six test indicated that the test was still too difficult for the learners. If a bias-free (as far as possible) test was developed, the researcher can only hypothesise as to why this group of learners still performed poorly in the test. The lack of mother-tongue education or poor teaching mechanisms could be a reason for this poor performance. DIF refers to a test item that displays different statistical properties for different groups, but it is important to assume that the abilities of the groups are the same. This could be another reason for the difference in results. DIF is strictly a statistical concept and free from any judgment of the fairness of the test items. A valid criticism of DIF is that the distribution of the test statistic is only known asymptotically. The asymptotic distribution is applicable only when item parameters are estimated in the presence of known ability parameters (Hambleton et al., 1991). This could have had an effect on the results.

Irrespective of the above, care was taken during the construction of the test to ensure that the test was cross-culturally adapted. In a multicultural society like South Africa, the adaptation of assessment measures and the elimination of bias from psychometric tools forms a vital part of the transformation process.

6.3 RECOMMENDATIONS FOR FUTURE RESEARCH

In view of the above, the recommendations with regard to future research are as follows:

a) The items that were discarded due to their large DIF value with respect to the comparison between item parameters and the area between the ICC's for the English and Afrikaans groups versus the Sesotho group, should be considered in more detail to determine the type of items that favoured the Sesotho-speaking learners and the type of items that favoured the English- and Afrikaans-speaking learners. It would be interesting to determine whether there are certain cognitive processes or learning strands that are biased towards a specific cultural group.

b) The DIF results with regard to males and females and the English versus the Afrikaans group can also be explored in more detail. It would be interesting to determine whether there are certain cognitive processes or learning strands that are biased towards a specific gender or a specific language group.

c) The reason for the poor performance of the grade five and six Sesotho learners should also be investigated in more detail. The norms should also be revised for the grade five and six Sesotho-speaking learners.

d) The test can also be translated into Sesotho and the study can be repeated using the DIF analysis to see whether this would diminish the DIF of the test items.

e) By using the Intermediate Phase Mathematics Proficiency Test as a diagnostic tool, a mathematical programme can be developed to help learners overcome their problems in mathematics.

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ANNEXURE A

INTERMEDIATE PHASE MLMMS CURRICULUM

<u>Some attitudes and values for MLMMS</u>:

1. **Demonstrate an understanding of ways of working with numbers**
   Accuracy
   Sharing
   Empathy
   Sensitivity
   Ubuntu
   Problem solving
   Critical thinking
   Creativity
   Tenacity
   Neatness
   Confidence

2. **Manipulate number patterns in different ways**
   Creativity
   Perseverance
   Appreciation
   Curiosity

3. **Demonstrate an understanding of the historical development of mathematics in various social and cultural contexts**
   Respect
   Honesty
   Tolerance
   Sensitivity
   Free of bias
   Appreciation

4. **Critically analyse how mathematical relationships are used in social, political and cultural contexts**
   Critical thinking
   Confidence
   Accountability
   Honesty
   Discipline
   Self-management
   Justice

5. **Measure with competence and confidence in a variety of ways**
   Accuracy
   Confidence
   Honesty
   Neatness
   Discipline

6. **Use data from various contexts to make informed judgements**
   Accuracy
   Sensitivity
   Empathy
   Patience
   Free of bias
   Pro-activeness
   Ability to be methodical
   Responsibility
   Critical thinking
   Reliability
   Accountability
   Curiosity
   Creativity

7. **Describe and represent experiences with shape, space, time and motion, using all available senses**
   Accuracy
   Creativity
   Problem solving
   Ability to be analytical
   Abstract thinking
   Appreciation

8. **Analyse natural forms, cultural products and processes as representations of shape, space and time**
   Creativity
   Appreciation
   Innovation
   Respect
   Sensitivity
   Tolerance
   Neatness
   Responsibility
   Environmental awareness

9. Use mathematical language to communicate mathematical ideas, concepts, generalisations and thought

Sensitivity
Free of bias
Curiosity
Confidence
Accuracy
Self-esteem
Respect
Appreciation
Creative thinking

10. Use various logical processes to formulate, test and justify conjectures

Critical thinking
Problem solving
Creativity
Perseverance
Openness
Fairness
Confidence
Taking the initiative
Optimism

Your guide to intermediate phase Mathematical Literacy, Mathematics and Mathematical Sciences:

NUMBERS AND OPERATIONS		
Grade 4	**Grade 5**	**Grade 6**
Historical development - Numbers 1-10 in 3 SA languages - Counting: own community, Arabic, Roman - Counting styles - precolonial counting	Historical development - numbers 1-20 in 3 SA languages - counting: own community, Arabic, Roman - counting styles - precolonial counting	Historical development - numbers - multiples up to 100 in at least 3 SA languages - counting: own community, Arabic, Roman - counting styles - precolonial counting
Place value - number base based on counting in tens - simple expanded notation up to 4 digits. Use apparatus. Flash cards, abacus, etc… - place value of given digit	Place value - number base based on counting in tens - one more number base other than 10 and its application - expanded notation, up to 5 digits - place value of given digit	Place value - number base based on counting in tens - at least 2 more number bases other than 10 and its application - expanded notation, up to at least 6 digits - place value of given digit
Rounding off and estimation - compare sizes of numbers, e.g. number line to the nearest 10 and 100 - mathematical language, e.g. < ; = ; > - use to estimate and compare numbers up to 4 digits - answers and calculations - solve problems involving numbers up to 3 digits	Rounding off and estimation - compare sizes of numbers, e.g. number line to at least nearest 10, 100 and 1000 - mathematical language, e.g. < ; = ; > - use to estimate and compare numbers up to 5 digits, answers in calculations; square roots solving problems	Rounding off and estimation - compare sizes of numbers up to at least 10 000 - mathematical language, e.g. < ; = ; > up to six digits - use to estimate and compare answers in calculations; square roots solving problems (up to 100) - solve problems

	up to 30 - solve problems involving numbers up to 4 digits	
Basic operations - expanded notation for addition and subtraction up to at least 3 digits, subtract number from itself - develop an understanding of connection between repeated addition and multiplication - multiplication patterns and tables - halving and doubling - expanded notation to multiply and divide: multiply - 2 digits by 1 digit; by multiples of 10; doubling divide - number by itself; 2 digits by 1 to 10 with and without remainders develop an understanding of division by using word problems dealing with: equal sharing; equal grouping; sharing with remainders - relationship between multiplication and division - division rules for 2 and 10 - mathematical language, e.g. sum, difference, product,	Basic operations - build up and break down numbers for addition and subtraction - add and subtract in columns up to at least 4 digits, subtract number from itself - develop an understanding of connection between repeated addition and multiplication - multiplication patterns and tables - halving and doubling - expanded notation to multiply and divide up to: multiply - 3 digits by 2 digits; - by multiples of 10; - doubling divide - 3 digits by 2 digits, with and without remainders - develop an understanding of division by using word problems dealing with: - equal sharing - equal group	Basic operations - build up and break down for addition and subtraction - add and subtract in columns up to at least 5 digits, subtract numbers from themselves - develop an understanding of connection between repeated addition and multiplication - multiplication patterns and tables - halving and doubling - expanded notation to multiply and divide: multiply - 4 digits by 3 digits - by multiples of 1000 - doubling divide - 3 digits by 2 digits by 1000, with and without remainders - develop anunderstanding of division by using word problems dealing with: - equal sharing - equal grouping

quotient - strategies for mental calculation, doubling and halving - order of operation (division, multiplication, addition, subtraction)	- sharing with remainders - division rule for 2, 3, 5, 10 - develop an understanding of the connection between multiplication and division - order of mixed operations (mathematical convention BODMAS) - mathematical language, e.g. sum, difference, product, quotient strategies for mental calculation	- sharing with remainders - division rules for 2, 3, 5, 10, 100 - develop an understanding of the connection between multiplication and division - order of mixed operations (mathematical convention BODMAS) - mathematical language, e.g. sum, difference, product, quotient - strategies for mental calculation
Manipulating numbers - biases - different beliefs	Manipulating numbers - biases - different beliefs	Manipulating numbers - biases - different beliefs

FRACTIONS		
Grade 4	**Grade 5**	**Grade 6**
Historical development of commonly used fractions	Historical development of commonly used fractions	Historical development and application in other cultures
Number concept: fractions between 0 and 1 - develop an understanding of fractions by using word problems dealing with - equal sharing - equal grouping (1 to 10 objects shared into 2's, 3's, etc…) - Solve problems leading to simple fractions using - concrete representation in familiar situation - diagrammatic representation - name fractions (notation) - verbally and orally - writing in words - numbers and words, e.g. 4 fifths - introduce symbols for commonly used fractions - decimal notation as used with money	Number concept: fractions between 0 and 5 - develop an understanding of fractions using word problems dealing with - equal sharing - equal grouping leading to mixed numbers - solve problems using: - concrete representation in familiar and unfamiliar situations - diagrammic representation - name fractions (notation): numbers and words, e.g. 7 eights, 1 fifth, etc… - symbols (only if fractions concept is well understood) - for fractions with denominators from 1 to 10 - decimal notation up to 2 decimal places (e.g. in measurement)	Number concept: - develop an understanding in fractions using word problems dealing with: - equal sharing - equal grouping leading to proper and improper fractions and mixed numbers - solve problems using - diagrammatic representations - symbols - Name fractions (notation): - numbers and words - symbols (only if fraction concept is well understood) - Decimal notation up to three decimal places
Compare simple fractions (between 0 and 1) using: - concrete representation - diagram	Compare fractions between 0 and 5 using: - diagrams - number line - compare fractions	Compare fractions using: - diagrams - number line - symbols only - compare fractions

- compare fractions to one another by using words and symbols (< ; = ; >) - equivalent fractions	to one another - identify equivalent fractions - use mathematical language (< ; = ; >) language - compare tenths and hundredths	to one another using mathematical symbols (< ; = ; >) - equivalent fractions - compare tenths, hundredths and thousandths
Place value in decimal fractions - simple expanded notation - place value of given digit up to 2 decimal places	Place value in decimal fractions - simple expanded notation - place value of a given digit up to 3 decimal places	Place value in decimal fractions - place value of any given digit
Basic operations - addition and subtraction of fractions with the same denominator (1 to 10) and the sum < 1 in concrete situations - halving and doubling - operations on decimal fractions in contexts dealing with money - equivalency of common and decimal fractions - calculators: advantages and disadvantages - decimal fractions in shopping and budgeting - use of a calculator	Basic operations - addition and subtraction of fractions leading to mixed numbers - operations on decimal fractions up to 3 decimal places in familiar and unfamiliar contexts - addition and subtraction - halving and doubling - dividing and multiplying by ten and hundred - decimal fractions in shopping: price increases and decreases - Equivalency of common and decimal fractions between 0 and 1 - calculators: advantages and disadvantages	Basic operations - addition and subtraction of proper, improper and mixed fractions - word problems dealing with decimal fractions up to 3 digits - decimal fractions in shopping: price increases and decreases - dividing and multiplying by ten, hundred and thousand - equivalency of common and decimal fractions - halving and doubling in symbolic form - calculators: advantages and disadvantages - budgeting

PATTERNS		
Grade 4	**Grade 5**	**Grade 6**
Number patterns - recognise and discuss diagrammatic and number patterns in: - everyday experiences, e.g. calendar, watch, 100 block… - nature, e.g. pineapples, shells, reproduction of rabbits (fibonacci numbers)	Number patterns - identify number patterns in familiar and unfamiliar situations	Number patterns - identify and compare different number patterns
Complete patterns - identify missing terms in number patterns, e.g. even numbers, odd numbers, prime numbers, multiples of 3, 4, and 5 - skip counting in 2, 3, 4, 5, 10 leading to multiplication tables - arrange numbers in logical sequence, e.g. ascending and descending order	Complete patterns formed by: - adding or subtracting a particular number (a constant value) - adding or subtracting (not necessarily a constant number)	Complete patterns formed by: - adding or subtracting a particular number (a constant value) - adding or subtracting (not necessarily a constant value) - multiplication and division - other means e.g. squaring, cubing
Generalise patterns - find and describe the rule for the pattern verbally	Generalise patterns - find and apply the rule by completing a pattern	Generalise patterns - find and apply the rule for the pattern mathematical symbols
Create / generate own patterns - diagrammatic patterns number patterns using addition and subtraction	Create and generate own patterns - number patterns using basic operations	Create and generate own patterns - develop formulae for patterns created

Solve simple problems and test solutions using number patterns	Solve simulated problems and test solutions using number patterns	Model and solve real problems and test solutions
Geometric patterns - recognise and explore 2-D and 3-D patterns in concrete environments: - natural - cultural (at least 2 SA cultures) e.g. tessellations and transformations	Geometric patterns - identify and discuss 2-D and 3-D artistic patterns in SA context (at least 3 cultures) - recognise and explore 2-D and 3-D patterns in artistic environments: - natural - cultural (at least 3 SA cultures) e.g. tessellations and transformations	Geometric patterns - analyse, compare and explore abstract 2-D and 3-D patterns in SA and wider context, - e.g. tessellations and transformations
Generate and complete patterns using: - natural objects - cultural products from own environment - squares - triangles - available technologies, e.g. potato print, computer (logo)	Generate and complete patterns using: - a variety of objects - known geometric shapes - tessellations - available technologies, e.g. paper folding, computer (logo)	Generate and complete patterns using - a variety of objects - known geometric shapes using rigid shapes - congruent geometric shapes - tessellations - available technologies

SHAPES AND SPACE		
Grade 4	**Grade 5**	**Grade 6**
History of the development of geometry - local / provincial	History of the development of geometry - national	History of the development of geometry - international
Natural forms: - local nature, domestic plants and animals - one cultural product - misuse of nature and cultural products - recognise and classify shapes: triangle, rectangle, square, circle - terminology: circular, triangular, rectangular	Natural forms: - local nature, wild plants and animals - more than one cultural product - misuse of nature and cultural products - recognise shapes - combined shapes (overlapping) - classify shapes: circles and polygons - terminology: circular, triangular, rectangular, quadrilateral	Natural forms: - nature, wild plants and animals (other countries) - more than two cultural products - misuse of nature and cultural products - terminology: circular, compass center, radius, diameter, triangular, quadrilateral (rectangular) - congruency / similarity
Transformations: introduction to the following concepts: - shift / slide (translation) - turn (rotation) - flip (reflection symmetry)	Transformations: concepts applied to abstract figures: - shift / slide (translation) - turn (rotation) - flip (reflection) symmetry	Transformations: identify concepts and carry out: - shift / slide (translation) - turn (rotation) - flip (reflection) – symmetry - enlarge - contract (shrink) - stretch
Tessellate simple shapes - perspective (point of reference) - describe view of objects from different points of reference	Tessellate simple shapes - perspective (point of reference) - describe and sketch objects from different points of reference	Tessellate simple and complex shapes - perspective (point of reference) - draw and interpret objects from different points of reference

MEASUREMENT		
Grade 4	**Grade 5**	**Grade 6**
Length - history of measurement in own culture - measure length / distance using simple instruments, e.g. body parts, ruler, tape, meter wheels - use measurement in one context	Length - history of measurement in SA cultures - measure length / distance using simple instruments, e.g. ruler, tape, meter wheels - use measurement in various contexts	Length - history of measurement - measure length / distance: estimate, measure, verify
Units - use appropriate units - relationship: cm, m, km - convert: cm to m and m to km	Units - use appropriate units - relationship: mm, cm, m, km - convert: mm to cm, cm to m, m to km	Units - use appropriate units - relationship: mm, cm, m, km - convert: mm to cm, cm to m, cm to km, m to km
Rounding off to 1 decimal place and whole	Rounding off to 2 decimal places and whole	Rounding off to 3 decimal places and whole
Four basic operations in cm, m, km	Four basic operations in mm, cm, m km	Four basic operations in mm, cm, m, km
Mass - history of measurement in own culture - measure mass, compare mass (same size, different masses)	Mass - history of measurement in other SA cultures - measure mass, compare mass (same size, different masses)	Mass - history of measurement in other countries - measure mass: estimate, measure, verify
Units - understand, use - relationship: g, kg - convert: g to kg - round off: nearest kg - four basic operations	Units - understand, use - relationship g, kg - convert g to kg - round off: nearest kg - four basic operations	Units - understand, use - relationship: g, kg, ton (1000kg = 1 ton) - convert: g to kg to ton - round off: nearest kg - four basic operations

Location - read maps - determine shortest routes (immediate location) - plot routes - give directions - spatial orientation (terminology, e.g. N, S, W and E) - draw maps (school, home) - estimation: distance, maps	Location - read maps - determine shortest routes (in province) - give directions - directions: appropriate terminology (N, S, W and E) - estimation: distances, simple scale diagrams, maps - spatial orientation: draw maps (immediate location) - four main operations	Location - read maps (of country) - determine shortest routes - plot shortest routes - give directions - spatial orientation - directions (appropriate terminology; 8 cardinal points) - draw maps (own town) - estimation: distances, scale, diagrams, maps
	Angles - concept - recognise right angles in shapes	Angles - recognise and measure angles bigger than right angles and smaller than right angles
Temperature - history of development - thermometers: recognise different thermometers, use and read clinical thermometers - units: degrees Celsius - measure temperature - calculators	Temperature - history of development in different SA cultures - measure temperature: use different thermometers - calculations	Temperature - history of development - measure temperature - take accurate readings - calculations
Time - history of measurement - own culture - methods and tools (analogue, digital, time stick,	Time - history of measurement in other SA cultures - methods and tools (analogue, digital, time stick)	Time - history of measurement in other countries - methods and tools (analogue, digital) - time keeping: use

hourglass, etc…) - time keeping: use 24 - hour clock, calendar - units: understand, use - relationship: minutes, hours, days, weeks, months, years, leap years - estimate and verify time - calculations: duration (add and subtract)	- time keeping: use 24 - hour clock, stopwatch, calendar - units: understand, use - relationship: seconds, minutes, hours, days, weeks, months, years, leap years, centuries - estimate and verify time - calculations: duration (add and subtract)	24 - hour clock, stopwatch, calendar - units: understand, use - relationship: minutes, hours, days, weeks, months, years, leap years, decades, centuries - estimate and verify time - calculations: duration (basic operations)

DATA		
Grade 4	**Grade 5**	**Grade 6**
Identify situations for investigation	Identify situations for investigation in community	Identify situations for investigation in neighboring communities
Collect data - introduce learners to methods to collect data and the ability to choose the best one - use technology to collect data	Collect data - collect data using at least one method - use technology to collect data	Collect data - collect data using at least 2 different methods; the ability to choose the best one - use technology to collect data
Organise data - sort data according to one criteria - record data in simple tables	Organise data - various methods used to organise - sort and represent - construct, read and interpret tables, graphs and charts	Organise data - various methods used to organise - sort and represent - construct, read and interpret tables, graphs and charts - use tables to organise data on measurement of physical attributes of 2-D and 3-D objects - apply statistical tools - calculate mean / average and percentages - determine mode, frequency and range - decide on appropriate scale - use scale to interpret graph
Display data - draw and read simple graphs, e.g. picture and bar graphs, charts, e.g. tallying and tables - use technology to display data	Display data - draw and read simple graphs, e.g. picture and bar graphs, charts, e.g. tallying and tables - bar graphs: horizontal and	Display data - draw and read simple graphs, e.g. picture and bar graphs, charts, e.g. tallying and tables - histograms (frequency diagram

- display data with relevant level of accuracy	vertical - pie graphs: use technology to display data - display data with relevant level of accuracy	= bar chart) - use technology to display data - display data with relevant level of accuracy
Communicate findings - report findings	Communicate findings - interpret different types of graphs and tests	Communicate findings - interpret different types of graphs and test - apply statistical concepts and vocabulary
Evaluation of findings - make simple comparisons	Evaluation of findings: - make simple comparisons - make simple judgements and predictions	Evaluation of findings: - make simple comparisons - make informed judgements and predictions
Critical evaluation of data - make simple recommendations	Critical evaluation of data - biased use of data - read, analyse, interpret graphical representation - respect for different reasoning approaches	Critical evaluation of data - biased use of data - read, analyse, interpret graphical representation - respect for different reading approaches

SCORING OF THE TEST AND UTILISATION OF THE NORM TABLES

After completion of the mathematics proficiency test, one mark is allocated per question for the correct answer and zero for an incorrect answer. Write 1 or 0 at the end of the line on the space allocated. If a part of a question is wrong, then the entire question is marked as incorrect and a zero mark is allocated. All correct marks are then summated to determine the learner's raw score for mathematics. The raw score is then converted into a percentile rank by using the appropriate norm table and may be filled in at the end of the answer sheet for future reference.

The norms are represented in terms of stanines and percentile ranks. The norms were calculated per term so that the test can be administered at any time of the year. The norms for grade four appear in Table 1.1, Table 1.2, Table 1.3 and Table 1.4. The norms for grade five appear in Table 2.1, Table 2.2, Table 2.3 and Table 2.4. The norms for grade six appear in Table 3.1, Table 3.2, Table 3.3 and Table 3.4.

GRADE 4

Table 1.1: Conversion of mathematics raw score into stanines for English-, Afrikaans- and Sesotho-speaking grade four learners

Stanine	Percentile rank	First term	Second term	Third term	Four term
1	4	0-3	0-4	0-5	0-5
2	11	4-5	5-6	6-7	6-8
3	23	6-7	7-8	8-9	9-10
4	40	8-9	9-11	10-12	11-13
5	60	10-12	12-13	13-14	14-15
6	77	13-14	14-15	15-16	16-17
7	89	15-16	16-17	17	18
8	96	17	18	18	19
9	100	18-20	19-20	19-20	20
$\bar{X}$		**11.762**			**14.290**
s		**4.219**			**3.977**
N		**558**			**524**
KR-20		**0.8032**			

GRADE 4

Table 1.2: Conversion of mathematics raw score into stanines for English-speaking grade four learners

Stanine	Percentile rank	First term	Second term	Third term	Four term
1	4	0-7	0-8	0-9	0-10
2	11	8-9	9-10	10	11
3	23	10-11	11	11-12	12-13
4	40	12	12-13	13-14	14
5	60	13	14	15	15
6	77	14	15	16	16
7	89	15-16	16	17	17
8	96	17	17	18	18
9	100	18-20	18-20	19-20	19-20
$\bar{X}$		**14.217**			**15.795**
s		**3.396**			**2.718**
N		**129**			**122**
KR-20		**0.7440**			

GRADE 4

Table 1.3: Conversion of mathematics raw score into stanines for Afrikaans-speaking grade four learners

Stanine	Percentile rank	First term	Second term	Third term	Four term
1	4	0-5	0-6	0-7	0-8
2	11	6-7	7-8	8-9	9-10
3	23	8-9	9-10	10-11	11-12
4	40	10-11	11-12	12-13	13-14
5	60	12-13	13-14	14	15
6	77	14-15	15	15-16	16-17
7	89	16	16-17	17	18
8	96	17	18	18	19
9	100	18-20	19-20	19-20	20
$\bar{X}$		**12.586**			**15.193**
s		**3.624**			**3.017**
N		**249**			**238**
KR-20		**0.7326**			

GRADE 4

Table 1.4: Conversion of mathematics raw score into stanines for Sesotho-speaking grade four learners

Stanine	Percentile rank	First term	Second term	Third term	Four term
1	4	0-3	0-3	0-3	0-3
2	11	4	4	4-5	4-5
3	23	5	5-6	6	6-7
4	40	6	7	7-8	8-9
5	60	7-8	8-9	9-11	10-12
6	77	9-10	10-12	12-14	13-15
7	89	11-14	13-15	15-17	16-18
8	96	15-16	16-17	18	19
9	100	17-20	18-20	19-20	20
$\bar{X}$		**8.861**			**11.860**
s		**3.899**			**4.818**
N		**180**			**164**
KR-20		**0.7727**			

GRADE 5

Table 2.1: Conversion of mathematics raw score into stanines for English-, Afrikaans- and Sesotho-speaking grade five learners

Stanine	Percentile rank	First term	Second term	Third term	Four term
1	4	-	-	-	-
2	11	1	1	1	1
3	23	2-3	2-3	2-3	2-3
4	40	4-6	4-6	4-6	4-7
5	60	7-9	7-10	7-11	8-11
6	77	10-12	11-13	12-13	12-14
7	89	13-14	14	14-15	15-16
8	96	15	15-16	16	17
9	100	16-20	17-20	17-20	18-20
$\bar{X}$		**8.571**			**9.772**
s		**4.946**			**5.558**
N		**569**			**523**
KR-20		**0.8734**			

GRADE 5

Table 2.2: Conversion of mathematics raw score into stanines for English-speaking grade five learners

Stanine	Percentile rank	First term	Second term	Third term	Four term
1	4	0-4	0-5	0-6	0-6
2	11	5-6	6-7	7-8	7-8
3	23	7-8	8-9	9-10	9-10
4	40	9-10	10-11	11	11-12
5	60	11-12	12	12-13	13-14
6	77	13	13-14	14	15
7	89	14-15	15	15-16	16
8	96	16	16-17	17	17-18
9	100	17-20	18-20	18-20	19-20
$\bar{X}$		**11.731**			**13.328**
s		**3.473**			**3.107**
N		**145**			**134**
KR-20		**0.7452**			

GRADE 5

Table 2.3: Conversion of mathematics raw score into stanines for Afrikaans-speaking grade five learners

Stanine	Percentile rank	First term	Second term	Third term	Four term
1	4	0-2	0-3	0-4	0-4
2	11	3-4	4-5	5-6	5-7
3	23	5-6	6-7	7-8	8-9
4	40	7-8	8-9	9-11	10-12
5	60	9-10	10-11	12	13
6	77	11-13	12-13	13-14	14-15
7	89	14-15	14-15	15-16	16
8	96	16	16-17	17	17-18
9	100	17-20	18-20	18-20	19-20
$\bar{X}$		**10.329**			**12.806**
s		**4.003**			**3.966**
N		**231**			**12.385**
KR-20		**0.7929**			

GRADE 5

Table 2.4: Conversion of mathematics raw score into stanines for Sesotho-speaking grade five learners

Stanine	Percentile rank	First term	Second term	Third term	Four term
1	4	-	-	-	-
2	11	-	-	-	-
3	23	-	1	1	1
4	40	1	2	2	2
5	60	2-3	3	3	3-4
6	77	4-5	4-5	4-5	5
7	89	6-9	6-8	6-7	6
8	96	10-12	9-10	8-9	7
9	100	13-20	11-20	10-20	8-20
$\bar{X}$		**4.093**			**3.500**
s		**3.565**			**2.256**
N		**193**			**178**
KR-20		**0.8133**			

GRADE 6

Table 3.1: Conversion of mathematics raw score into stanines for English-, Afrikaans- and Sesotho-speaking grade six learners

Stanine	Percentile rank	First term	Second term	Third term	Four term
1	4	1	1	1	1
2	11	2	2	2	2-3
3	23	3-4	3-4	3-4	4
4	40	5-6	5-7	5-7	5-8
5	60	7-9	8-10	8-10	9-11
6	77	10-11	11-12	11-13	12-14
7	89	12-14	13-14	14-15	15-16
8	96	15-16	15-16	16-17	17
9	100	17-20	17-20	18-20	18-20
$\bar{X}$		**8.750**			**10.079**
s		**4.404**			**5.119**
N		**607**			**532**
KR-20		**0.8175**			

GRADE 6

Table 3.2: Conversion of mathematics raw score into stanines for English-speaking grade six learners

Stanine	**Percentile rank**	**First term**	**Second term**	**Third term**	**Four term**
1	4	0-3	0-5	0-6	0-7
2	11	4-5	6	7-8	8-9
3	23	6-7	7-8	9	10
4	40	8-9	9-10	10-11	11-12
5	60	10-11	11-12	12-13	13
6	77	12-13	13-14	14	14-15
7	89	14-15	15-16	15-16	16-17
8	96	16-17	17-18	17-18	18
9	100	18-20	19-20	19-20	19-20
$\bar{X}$		**11.034**			**13.667**
s		**3.829**			**3.019**
N		**148**			**138**
KR-20		**0.7850**			

GRADE 6

Table 3.3: Conversion of mathematics raw score into stanines for Afrikaans-speaking grade six learners

Stanine	Percentile rank	First term	Second term	Third term	Four term
1	4	0-2	0-3	0-4	0-4
2	11	3-4	4-5	5-6	5-6
3	23	5-6	6-7	7-8	7-8
4	40	7-9	8-9	9-10	9-10
5	60	10-11	10-11	11-12	11-12
6	77	12-13	12-13	13-14	13-14
7	89	14-15	14-15	15-16	15-16
8	96	16	16-17	17	17-18
9	100	17-20	18-20	18-20	19-20
$\bar{X}$		**10.455**			**12.283**
s		**3.923**			**3.823**
N		**244**			**219**
KR-20		**0.7593**			

GRADE 6

Table 3.4: Conversion of mathematics raw score into stanines for Sesotho-speaking grade six learners

Stanine	Percentile rank	First term	Second term	Third term	Four term
1	4	-	-	-	-
2	11	1	1	1	1
3	23	2	2	2	2
4	40	3	3	3	3
5	60	4-5	4-5	4-5	4
6	77	6-7	6	6	5-6
7	89	8	7-8	7	7
8	96	9-10	9	8-9	8
9	100	11-20	10-20	10-20	9-20
$\bar{X}$		**5.242**			**4.491**
s		**2.844**			**2.602**
N		**215**			**175**
KR-20		**0.6465**			

SUMMARY

Mathematical Literacy, Mathematics and Mathematical Sciences is a learning area in the intermediate phase, which forms part of the General Education and Training band. This learning band is level one of the National Qualifications Framework and is overseen by the South African Qualifications Authority. The curriculum of this learning area consists of various learning strands, namely numbers and operations, fractions, patterns, shapes and space, measurement, and data. When learners fail to meet the expectations of the curriculum, mathematics becomes a major assessment concern. If this problem is not identified it could hinder the acquisition of more advanced mathematical skills.

The first task in helping a learner who is struggling with mathematics is to identify the problem. For learners to succeed at mathematics they need to go through various developmental phases. Various cognitive processes form part of these phases. Often in a young child's functioning, cognitive problems arise such as the inability to perform various mathematical tasks. For this reason a cognitive model for mathematics was used to reflect upon six key concepts that influence learning and teaching in the Mathematical Literacy, Mathematics and Mathematical Sciences learning area. These concepts include the categories of representing experience; motivation; individual differences; cognitive categories and cognitive processes; instructional procedures; and conceptual learning. Learners need to make sense out of what is going on during a mathematics lesson. To help learners develop meaning, a teacher provides experiences that foster mental manipulations. Psychologists refer to these mental manipulations as cognitive processes. When a learner is unable to carry out the cognitive processes necessary for task completion, mathematics becomes a major assessment concern.

If a learner fails to meet the expectations of the curriculum or fails to carry out the cognitive processes necessary for successful task completion, then, in accordance with the aim of this study, the Intermediate Phase Mathematics Proficiency Test can be used to

identify and address this problem. During the construction of this test, care was taken to ensure that the test was cross-culturally adapted. Differential Item Functioning was used to limit the possibility of cultural bias. The Item Response Theory and the Classical Test Theory were also used for item analysis and selection. The test was standardised for English-, Afrikaans- and Sesotho-speaking grade four, five and six learners. During standardisation, separate norms for each term were calculated. These norms are available in both stanines and percentile ranks.

The test can also be used qualitatively to determine not only the learning strand in which the learner may be experiencing problems, but also the specific cognitive process, such as receiving, interpreting, organising, applying, remembering and problem solving, which might be preventing the learner from reaching his or her full mathematical potential.

The Intermediate Phase Mathematics Proficiency Test is also a reliable and valid measuring instrument since the bias of the assessment measure has been decreased. This was done by eliminating any item that was biased towards a specific cultural group. The test can therefore be used in practice with confidence.

In a multicultural society like South Africa, the adaptation of assessment measures and the elimination of bias from psychometric tools forms a vital part of the transformation process. The Intermediate Phase Mathematics Proficiency Test is a multicultural test with South African norms.

OPSOMMING

Wiskundige Geletterdheid, Wiskunde en Wiskundige Wetenskappe is ’n leerarea in die intermediêre fase, wat deel vorm van die afdeling Algemene Onderwys en Opleiding. Hierdie leerafdeling is vlak een van die Nasionale Kwalifikasiesraamwerk en toesig word gehou deur die Suid-Afrikaanse Kwalifikasiesoutoriteit. Die kurrikulum van hierdie leerarea bestaan uit verskeie leerafdelings, naamlik getalle en bewerkings, breuke, patrone, vorms en ruimte, meting en data. Wanneer leerders nie daarin slaag om aan die verwagtinge van die kurrikulum te voldoen nie, word wiskunde ’n groot assesseringsvraagstuk. Indien hierdie probleem nie geïdentifiseer word nie, kan dit die aanleer van meer gevorderde wiskundige vaardighede belemmer.

Die eerste taak tydens hulpverlening aan die leerder wat sukkel met wiskunde, is om die probleem te identifiseer. Vir leerders om sukses te behaal in wiskunde, moet hulle deur verskillende ontwikkelingsfases beweeg. Verskeie kognitiewe prosesse vorm deel van hierdie fases. Dit gebeur dikwels dat daar in die jong kind se funksionering kognitiewe probleme ontstaan, soos byvoorbeeld die onvermoë om verskeie wiskundige take uit te voer. Om dié rede is ’n kognitiewe model vir wiskunde gebruik om te besin oor die ses sleutelkonsepte wat die leer en onderrig van Wiskundige Geletterdheid, Wiskunde en Wiskundige Wetenskappe beïnvloed. Hierdie konsepte sluit in: die kategorieë van die weergee van ervaring; motivering; individuele verskille; kognitiewe kategorieë en kognitiewe prosesse; onderrigprosedures en konseptuele leer. Leerders moet sin maak uit dit wat aangaan gedurende ’n wiskunde-les. Om leerders te help om betekenis te ontwikkel, verskaf die onderwyser ervaringgeleenthede wat verstandelike bewerkings aanmoedig. Sielkundiges noem hierdie verstandelike bewerkings, kognitiewe prosesse. Wanneer ’n leerder nie in staat is om die nodige kognitiewe prosesse uit te voer wat nodig is vir taakvoltooiing nie, word wiskunde ’n groot assesseringsvraagstuk.

Indien ’n leerder nie daarin slaag om aan die verwagtinge van die kurrikulum te voldoen nie of misluk in die uitvoering van kognitiewe prosesse wat nodig is vir suksesvolle taakvoltooiing, kan die Intermediêre Fase Wiskundige Vaardigheidstoets, in

ooreenstemming met die doel van hierdie studie, gebruik word om hierdie probleem te identifiseer en aan te spreek. Tydens die opstelling van hierdie toets is sorg geneem om te verseker dat die toets kruis-kultureel aangepas is. Differensiële Itemfunksionering is gebruik om die moontlikheid van kulturele vooroordeel te beperk. Die Itemresponsteorie en die Klassieke Toetsteorie is ook gebruik vir itemanalise en -seleksie. Die toets is gestandaardiseer vir Engels-, Afrikaans- en Suid-Sotho-sprekende leerders van graad vier, vyf en ses. Gedurende standaardisasie is aparte norms vir elke term bereken. Hierdie norms is beskikbaar in beide staneges en persentielrange.

Die toets kan ook kwalitatief gebruik word om nie net die leeraspek waarmee die leerder probleme ondervind te bepaal nie, maar die spesifieke kognitiewe proses, soos byvoorbeeld ontvangs, interpretering, organisering, toepassing, onthou en probleemoplossing, wat die leerder daarvan kan weerhou om sy of haar volle wiskundige potensiaal te bereik.

Die Intermediêre Fase Wiskundige Vaardigheidstoets is ook ’n betroubare en geldige meetinstrument omdat die vooroordeel van die assesseringsmeting verminder is. Dit is gedoen deur enige item wat bevooroordeeld teenoor ’n spesifieke kulturele groep is, uit te skakel. Om die rede kan die toets met vertroue in die praktyk gebruik word.

In ’n multikulturele samelewing soos Suid-Afrika, vorm die aanpassing van assesseringsmaatstawwe en die uitskakeling van vooroordeel in psigometriese toetse, ’n baie belangrike deel van die transformasieproses. Die Intermediêre Fase Wiskundige Vaardigheidstoets is ’n multilkulturele toets met Suid-Afrikaanse norms.

KEY TERMS

Current educational system in South Africa

Intermediate phase

Outcomes Based Education

Mathematical Literacy, Mathematics, Mathematical Sciences learning area

Learning strands

Assessment

Learning-teaching process of the cognitive model for mathematics

Cognitive theorists

Cognitive processes

Development of a test

Standardisation of a test

Item analysis and selection

Cross-cultural test adaptation

Differential Item Functioning

Classical Test Theory

Item Response Theory

Norm determination

Mathematics proficiency test

ANNEXURE B

LETTER OF PERMISSION
FROM THE
FREE STATE DEPARTMENT OF EDUCATION

FREE STATE PROVINCE

Enquiries : Mrs M V Wessels/LB
Reference no. : 16/4/1/6 - 2001

Tel :(051) 404 8075
Fax :(051) 4048074

2001-03-12

Mrs C P Vassiliou
Department of Psychology
Faculty of the Humanities
UOFS
P O Box 339
BLOEMFONTEIN
9300

Dear Mrs Vassiliou

REQUEST TO CONDUCT RESEARCH IN THE FREE STATE DEPARTMENT OF EDUCATION

1. This letter is in reply to your letter dated 15 February 2001.

2. Research topic: **The Development and Standardardisation of a Mathematics Proficiency Test for learners in the Intermediate Phase.**

3. Permission is granted for you to conduct research in the Free State Department of Education under the following conditions:

   3.1 The names of learners and educators must be provided by the principals.
   3.2 Educators and learners will participate voluntarily in the project.
   3.3 The names of the schools, educators and learners involved must remain confidential.
   3.4 This letter must be shown to all participating persons.
   3.5 You are requested to donate a report on this study to the Free State Department of Education after completion of the project. It will be placed in the Education Library, Bloemfontein.

4. You are requested to address a letter to the Head: Education, for attention
   CES: Educational Planning
   Room 1213
   C R Swart Building
   Private Bag X20565
   BLOEMFONTEIN
   9301
   accepting the above conditions.

5. We wish you every success with your research.

Yours sincerely

pp **HEAD: EDUCATION**
MEESRES

ANNEXURE C

60-ITEM MATHEMATICS PROFICIENCY TEST

GRADE 4 **(Test 1)**

Name: ______________________ **Home language:** ______________

Gender: ______________________ **Age:** ______________________

NUMBERS AND OPERATIONS

1. Complete the following table. Start with the number 335 each time.

	+ 49	- 99	x 20	÷ 5	x 0
335					

2. Double 357 = ______________________________

3. Half 525 = ______________________________

4. Find the product of 4 and 17 and then add the quotient of 60 and 2 to your original answer______________________________

5. Round off 347 to the nearest 100 ______________________________

6. 321 can be written as 300 + 20 + 1. Write 2468 in the same way _________

____

7. What is the place value of the underlined number: <u>2</u>987 ______________

8. What is the difference between the biggest and smallest numbers in this group of numbers: 74 ; 47 ; 88 ; 57 ; 38 ; 83 ______________

9. Fill in < or = or > in the place of the *

   37 – 26 * (15 + 9) ÷ 6

10. Mrs Naidoo is catering for a party. She makes 764 snacks altogether. There are 153 samoosas, twice as many chilli bites as samoosas but only a third as many pies as samoosas. The rest of the eats are meatballs. How many meatballs did Mrs Naidoo make?

____

FRACTIONS

11. What is ⅓ of 90 kg? ______________________________

12. 250 g + $\frac{1}{4}$ kg + ____________ g = 1 kg

Fill in < or = or > in place of the *

13. $\frac{1}{4}$ * $\frac{1}{8}$

14. $\frac{1}{2}$ * 0,5

15. Mom gives you R1,00 to buy sweets. You buy toffees for 74 c. What fraction of the R1,00 did you spend? ____________________

16. How many quarters in 4 apples? ____________________

17. $\frac{1}{8} + \frac{3}{8} =$ ____________________

18. Colour in $^{3}/_{5}$ of the rod:

19. 0,3 + ____________ = 1

20. Max cuts a pie into 8 pieces. He eats 3 pieces. What fraction of the pie is left over? ____________________

PATTERNS

Complete the pattern:

21. 50c ; R1,00 ; R1,50 ; R2,00 ; ________ ; ________.

22. 10:00 ; 10:30 ; 11:00 ; ________ ; ________ ; ________.

23. □ ; ▲ ; □ ; Δ ; ________ ; ▲ ; ________ ; ________.

24. 4 ; 8 ; 12 ; 16 ; ________ ; ________ ; ________ .

25. 48 ; 45 ; 42 ; 39 ; ________ ; ________ ; ________ .

26. 7 $\frac{1}{2}$; 9 ; 10 $\frac{1}{2}$; 12 ; ________ ; ________ ; ________ .

27. 45 ; 90 ; 180 ; ________ ; ________ ; ________ .

28. 1 ; 2 ; 4 ; 7 ; ________ ; 16 ; ________ .

29. $\frac{1}{8}$; ________ ; $\frac{3}{8}$; ________ .

30. ← ; ↑ ; → ; ↓ ; ← ; ________ ; ________ ; ________ .

GRADE 4 **(Test 2)**

Name: ____________________ **Home language:** ______________

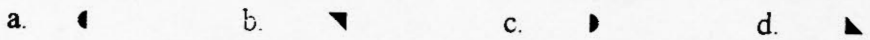
Gender: ____________________ **Age:** ______________

SHAPES AND SPACE

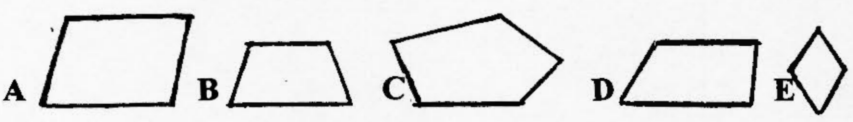
31. Identify each of the following shapes and write the name in the space provided:

__________ __________ __________ __________

32. Underline two of the shapes below, that when put together, will form a square?



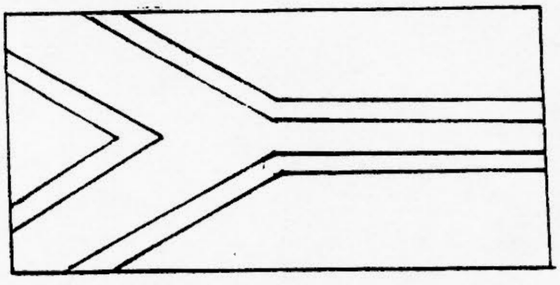
33. Underline the shape which does not fit with the others?

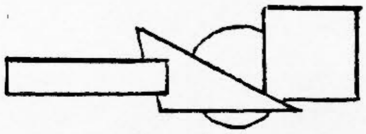


34. Which of the following shapes are two-dimensional and which are three-dimensional? Fill in 2-D for two dimensional and 3-D for three dimensional in the space provided.

__________ __________

35. Below is a picture of the South African Flag. Draw a line of symmetry through the flag.



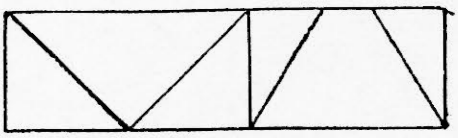


In the figure above there are two-dimensional shapes packed onto each other.

36. Which shape lies on top? ______________________________

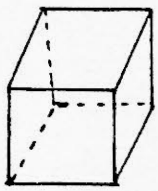
37. Which shape lies at the bottom? ______________________________

38. Colour in the triangles in the figure.



39. The total perimeter of a triangle is 45 mm. If all the sides are equal in length what will the length of one side be? ______________________________

40. What is this figure called? ________________



MEASUREMENT

41. How many days in a leap year? ____________

42. Double 135 c and write the answer in rand ______________________

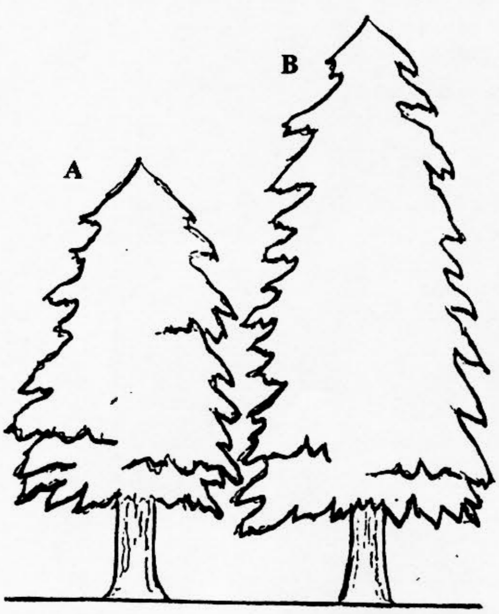
43. 6000 m = ____________ km

44. Mrs Brown buys 4 kg of mince for R74,00. How much will 1 kg cost?

____

45. A plane departs from Johannesburg airport at 07:30 and arrives in Cape Town at 10:30. How long did the flight take? ____________________

46. Measure the height of tree A and tree B. If 1 cm is equal to 2 meters, how much taller is tree B than tree A? Write your answer in meters.



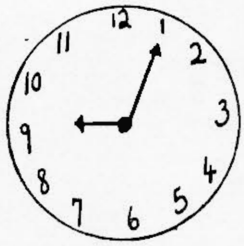
____

____

____

____

47. What was the time 10 minutes earlier? ______________________



48. My brother had to be at school at 14:15 for a cricket match which should have ended at 17:45. I had to be at ballet class from 17:30 until 18:30. His match ended 15 minutes late. My dad collected my brother first and then drove 20 minutes to get to the ballet studio. What time did dad arrive at the ballet studio?______________________________________

____

____

____

Fill in < or = or >:

49. 60 minutes __________ 1 hour

50. 13 months __________ 1 year

DATA

51. The following table shows the sale of CD's from a specific shop (* represents 1 CD). 1 CD sells for R80,50. How much money did the shop owner make?

____

____

____

South African music	* * * *
Pop music	* * * * * *
Rave music	* *
Classical music	*

Study the table and then answer the questions that follow. * means that the sport is offered on that day.

	Monday	Tuesday	Wednesday	Thursday	Friday
Netball		*	*		
Swimming	*		*		*
Gymnastics		*		*	
Athletics		*		*	*
Choir practice	*				*

52. On which day(s) of the week are there swimming lessons? ____________

____

53. On which day(s) of the week are there swimming lessons **and** athletics?

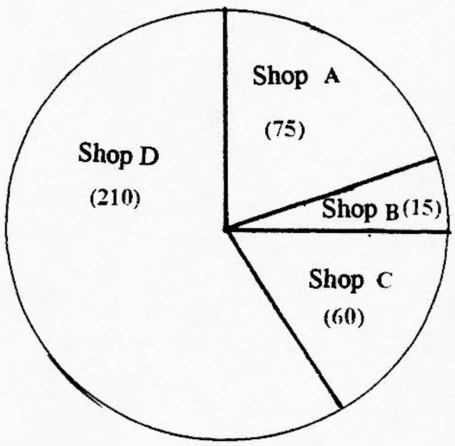
____

54. On which day(s) is neither netball nor choir offered? ________________

55. What activities are not offered on Friday? ____________________

____

Study the Pie Chart below: The numbers indicate the number of workers per shop.



56. If each worker is paid the same salary. Which shop would spend the most money on salaries? ____________________

57. How many workers are there in the smallest shop? ____________

58. What fraction of the pie chart is shop B and C together? ____________

59. Write the fraction in question 58 in decimal form ____________

60. How many workers are there altogether? ____________________

GRADE 5 **(Test 1)**

Name: ______________________ **Home language:** ______________

Gender: ______________________ **Age:** ________________________

NUMBERS AND OPERATIONS

1. Write 23 564 in expanded notation ______________________________

   ____

2. Write from biggest to smallest: 0,007 ; 7 ; 0,07 ; 70 ; 0,7

   _________ ; _________ ; _________ ; _________ ; _________.

3. 2 and 11 are both prime numbers. Name two other prime numbers______&_____
4. The dividend is 826 and the divisor is 36. Find the quotient:

   ____

   ____

5. 226 x 37 = ____

   ____

6. (6348 + 3481) - (1368 + 843) = ______________________________

   ____

7. 7 + ½ of 10 x 2 = ____
8. A farmer prepares 32 rows for fruit trees and each row has 54 holes waiting for the trees to be planted. The farmer receives 1700 trees from the nursery. How many holes still need trees? ________________________________

   ____

   ____

9. Subtract (2 x 7) from 7^2? ________________________________
10. The December holidays are 6 weeks and 3 days long. The July holidays are 3 weeks and 6 days long. How much longer is the December holiday than the July holiday? ________________________________

    ____

    ____

FRACTIONS

11. Write $^{29}/_7$ as a mixed number ______________________________

12. Write $4\,^{4}/_7$ as an improper fraction ______________________________

13. If $^{2}/_5 = {}^{y}/_{15}$, then y = ____________________

14. If $^{45}/_{60} = {}^{9}/_{y}$, then y = ____________________

15. If 78 ÷ 1000 = y , then y = ____________________

16. Arrange from biggest to smallest: $^{1}/_5$; $^{1}/_9$; ½ ; ¼ ; ¾

_______ ; _______ ; _______ ; _______ ; _______.

17. $13\,^{9}/_{12} - 7\,^{5}/_{12} =$ ______________________________

18. What fraction of an hour is 20 minutes? ______________________________

19. Susan has saved R1386,00. She spends $^{4}/_9$ of her savings. What amount does she have left? ______________________________

20. $2\,⅓ + 3\,^{2}/_5 =$ ______________________________

PATTERNS

Complete the pattern in questions 21 to 27:

21. 1212 ; 1218 ; 1224 ; _______ ; _______ ; _______.

22. 3 ; 9 ; 27 ; _______ ; 243.

23. 1 ; 4 ; 9 ; 16 ; _______ ; _______ ; _______.

24. 6 ; 6 ⅓ ; 6 ⅔ ; _______ ; _______.

25. $^{5}/_{12}$; $^{5}/_6$; 1 ¼ ; 1 ⅔ ; _______ ; _______.

26. 0,1 ; 0,15 ; 0,2 ; _______ ; _______ ; _______.

27. $^{35}/_{100}$; $^{30}/_{100}$; _______ ; _______ ; _______.

28. Put the number 2 ; 3 ; 5 and 7 in the boxes below to make the equation true.

☐ x ☐ + ☐ – ☐ = 24

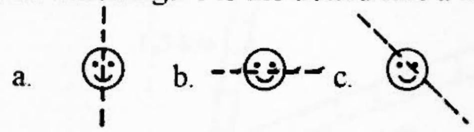
29. Fill in the missing blocks to complete the pattern:

□	Δ	Ọ
Ọ	□	Δ
Δ		

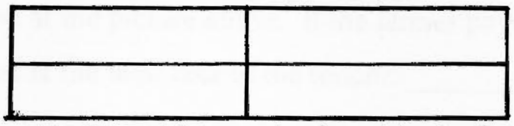
Fill in the missing numbers in the circles that will make the sum true:

30. ◯ $\xrightarrow{+\ 15}$ ◯ $\xrightarrow{\div\ 8}$ ◯ $\xrightarrow{\times\ 20}$ = 80

39. In which figure is the dotted line a line of symmetry? ______________



40.



How many rectangles can be found in the figure above? ____________

MEASUREMENT

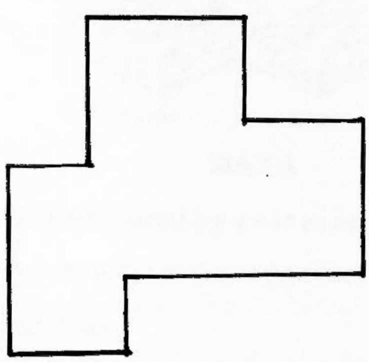
41. Find 0,367 x 23 = ____

____

42. 1,436 kg + 3,447 kg + 32 kg + 1136 g = ________________kg

____

43. With your ruler, measure the perimeter of the figure below: _________ cm



44. (2,5 km + 100 m) - (236 cm + 10 mm) = ____________ m

____

45. Round 4 km 499 m off to the nearest km ________________

46. A motorist travels at an average speed of 110km per hour. How far will he travel if he travels for 4 hours and 30 minutes? ________________km

____

____

____

GRADE 6 **(Test 1)**

Name: ______________________ **Home language:** ______________

Gender: ______________________ **Age:** ______________________

NUMBERS AND OPERATIONS

1. Write 105,681 in expanded notation ______________________

   ____

2. (Double 124,7) - (Half 244,1) = ______________________

   ____

3. (3 x 100 000) + (4 x 10^4) + (8 x 10^3) + (2 x 1) = ______________

   ____

4. Determine the difference between the sum of 127,3 and 19 and the product of the same two numbers ______________________

   ____

5. Estimate whether 8 538 x 380 is closer to: ______________________

   a. 10 000 x 400 b. 5 000 x 400 (Just write the letter)

6. If 5y = 30 - 5 then y = ______________________

7. Circle all the prime numbers

   1 ; 2 ; 3 ; 4 ; 5 ; 6 ; 7 ; 8 ; 9 ; 10 ; 11 ; 12

8. Circle all the factors of 12

   1 ; 2 ; 3 ; 4 ; 5 ; 6 ; 7 ; 8 ; 9 ; 10 ; 11 ; 12

9. Calculate the quotient of 5^2 and 5. Then add the answer to 2^5. ____________

   ____

10. Each child in a certain school gets 250 ml of milk every morning. If the children together drink 90,25 liters of milk every day, how many children are there in the school? ______________________

    ____

    ____

FRACTIONS

11. Simplify: $3\frac{1}{2} + 4\frac{1}{3} - 5\frac{5}{6}$ = ______________________________

____

12. Simplify: $3\frac{1}{3} \div 2\frac{1}{2} \times 2{,}25$ = ______________________________

____

13. Simplify: $\frac{25}{30} \times \frac{10}{15}$ = ______________________________

14. If $\frac{1}{3}$ of 24 = $\frac{1}{4}$ of y, then y = ______________________________

15. If y = 10 and n = 2 ; calculate y^n ______________________________

16. A man earns R360 per month. He saves $\frac{3}{9}$ of his salary and spends the rest. How much money does he save in one and a half years? ______________

____

____

17. The quotient is $16\frac{3}{8}$. The denominator is 8. What is the numerator? ______

____

____

18. What number is exactly halfway between $2\frac{3}{4}$ and $5\frac{1}{6}$? ______________

____

19. What fraction is 42 of 105? ______________________________

____

20. Alexis divides a number by 12 and gets an answer of $8\frac{1}{12}$. What is the original number? ______________________________

____

PATTERNS

Complete the pattern in questions 21 to 25:

21. 16 ; 8 ; 4 ; 2 ; ______ ; ______ ; ______.

22. 2 ; 3 ; 5 ; 8 ; ______ ; ______ ; ______.

23. 0,6 ; 1,2 ; 1,8 ; ______ ; ______.

24. 1 ; 2 ; 6 ; 24 ; ______ ; ______.

25. 1 ; 8 ; 27 ; ______ ; ______.

26. Nina packed her collection of matchboxes into a suitcase. She had three layers altogether. In each layer there were 5 rows with 6 boxes. How many boxes did she have in her suitcase? ______________________________

____

____

27. Fill in the missing numbers using number patterns.

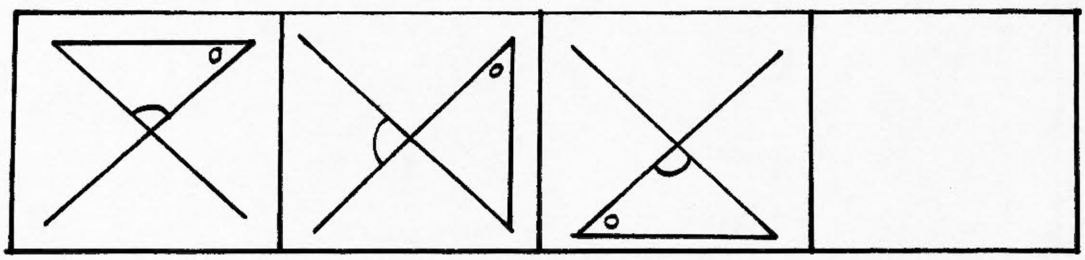
2	**4**	*3*	*6*		
8	**6**	*12*	*9*		

28. Arrange in decreasing order:

76,507 ; 76,570 ; 76,057 ; 76,705

__________ ; __________ ; __________ ; __________

29. Complete the pattern:



30. Complete the pattern by filling in the open spaces in the table.

8	4	4	28
2	5	7	
5	1	3	2
11	19		

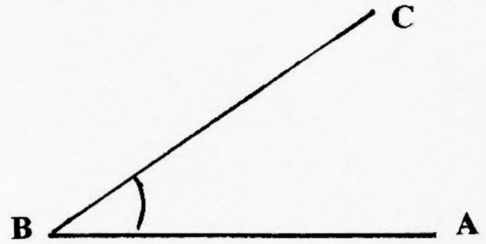
GRADE 6 (Test 2)

Name: ______________________ **Home language:** ______________

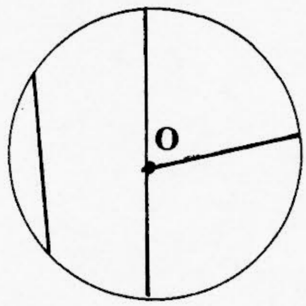
Gender: ______________________ **Age:** ______________________

SHAPES AND SPACE

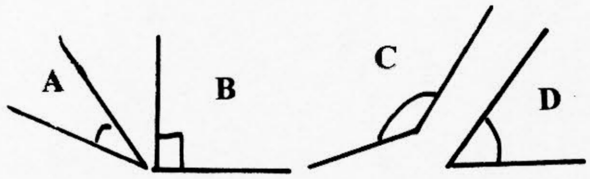
31. Measure ∠ABC ______________________________ °



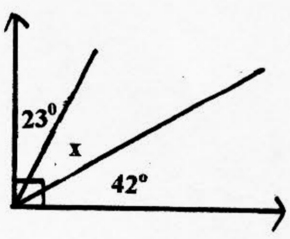
32. O is the center of the circle. Measure the diameter ______________ cm



33. Estimate which is the largest angle and write only the letter __________



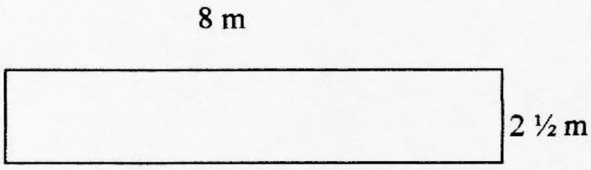
34.



x = ______________________________ °

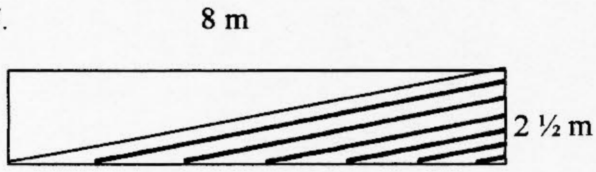
35. Draw 1 line of symmetry through the shape

36.



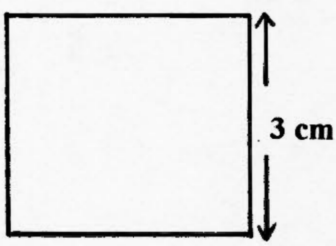
Area of the shape = ______________ m^2

37.



Area of the shaded part of the shape = ______ m^2

38.

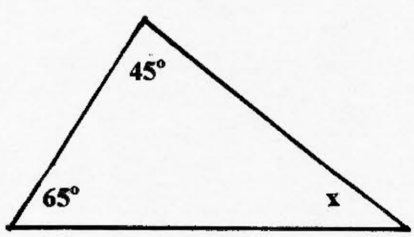


Area of the square = ______________ cm^2

39. If the radius of a circle is 8 cm. What is the diameter of the circle?

__________ cm

40.

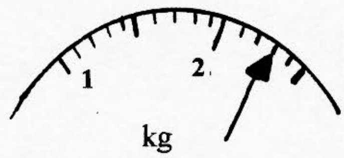


x = ____________________ °

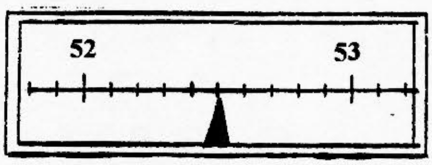
MEASUREMENT

41. Calculate 15 % of 400 = ______________________

42. The mass on this scale = ______________ kg

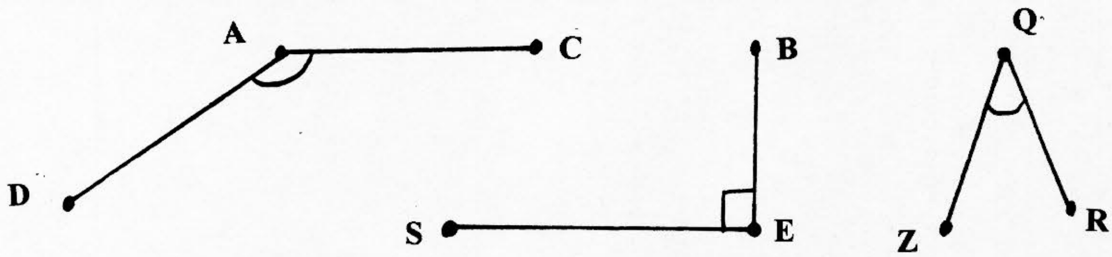


43. The speed in km per hour = ______________________ km/h



speed in km per hour

Look at the angles shown in the figure. Complete the table given below:



Question	Angle	Kind of angle (acute, obtuse, right angle)	Degrees
44.	∠ DAC	Obtuse	
45.	∠ BES		90 °
46.	∠ ZQR	Acute	

47. Calculate: 524 kg + 8 tons + 4 tons 33 kg + 26,462 tons + 0,003 tons

____

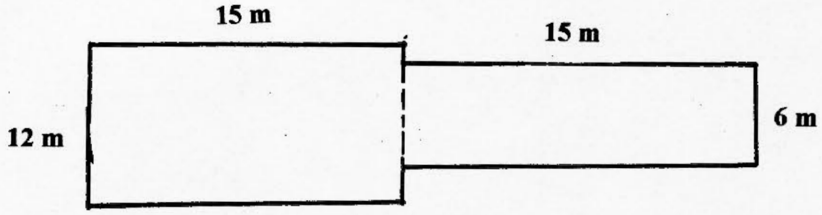
____

48. How long was it from 27 February 1996 at 18:45 to 3 March 1996 at 07:55 in hours and minutes? ______________________

____

49. A tap leaks 0,445 ml of water per hour. How much water will be wasted in one and a half days? ______________________

50. Calculate the perimeter of the figure below: ________________m



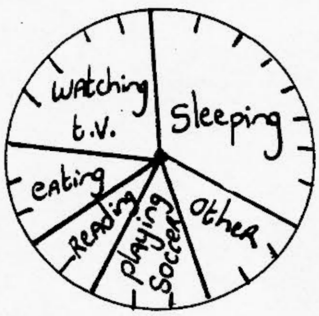
DATA

51. What is the arithmetic mean of 25 ; 46 ; 37 ; 16 ; 36. ____________

52. Elize gets the following results for her mathematics tests. The marks are out of 20: 17 ; 15 ; 13 ; 17 ; 18 ; 19 ; 20.

The average % for her mathematics tests is? ________________

53. The pie chart shows how John spends his Saturday. How many more hours does John spend sleeping than watching television? ____________

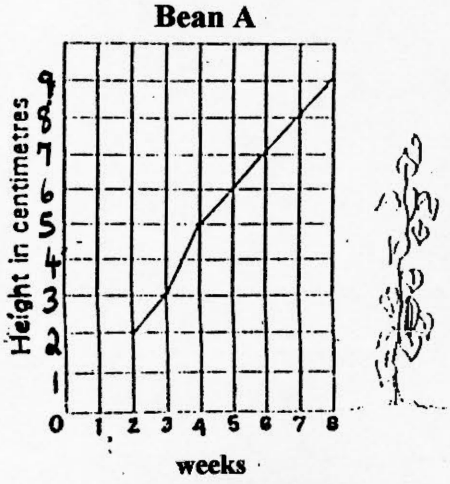


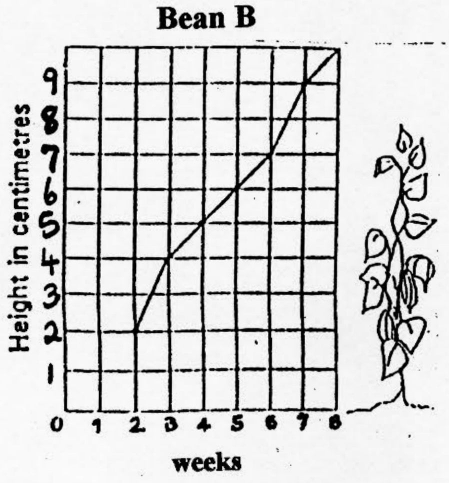
Mrs Gardener compares two kinds of climbing beans (refer to the graphs below):

54. Which bean grows the highest in the first 3 weeks? ____________

55. How many weeks does it take for bean A to grow 7 cm? ____________

56. Which bean reaches a height of 4 m first? ____________





Khani is researching her family history. She has collected the following information:

NAME	GENDER	DATE BORN	DATE DIED
Dreyer, Noel	Male	**1867-04-13**	**1914-12-27**
Solomon, Gladys	Female	**1845-07-10**	**1901-06-01**
Koen, Maria	Female	**1899-05-11**	**1899-05-11**
Swart, Estelle	Female	**1903-01-01**	**1959-01-17**
Samuels, Johanna	Female	**1899-05-12**	**1956-08-19**
Koen, Petrus	Male	**1904-09-13**	**1983-03-31**
May, Constance	Female	**1936-08-27**	**1958-05-30**
Cilliers, Morris	Male	**1938-10-26**	**1999-07-29**
Koen, Gerty	Female	**1903-04-17**	**1990-07-31**
Els, Willem	Male	**1891-03-17**	**1963-09-12**

57. Who was born first? ______________________________

58. At what age did Estelle Swart die? ______________________

59. Who was the youngest to have died? ____________________

60. A survey was done to determine the type of transport the learners use to go to school. The table shows the number of learners that use the various transport systems. How many learners took part in the survey? _______________

Type of Transport	Number of learners
Motor car	𝍸 𝍸 𝍸 ||
Bicycle	𝍸 |
Taxi	𝍸 𝍸 𝍸 𝍸 𝍸
Bus	𝍸 𝍸 |||
Walk	||

ANNEXURE D

50-ITEM MATHEMATICS PROFICIENCY TEST

GRADE 4

Name: ______________________ **Home language:** ______________

Gender: ______________________ **Age:** ______________________

NUMBERS AND OPERATIONS

1. 335 x 0 = ____________________
2. 321 can be written as 300 + 20 + 1. Write 2468 in the same way ________

   ____
3. Double 357 = ____
4. What is the place value of the underlined number: 2987 ______________
5. Fill in < or = or > :

   37 – 26 ________ (15 + 9) ÷ 6
6. Round off 347 to the nearest 100 ______________________________
7. What is the difference between the biggest and smallest numbers in this group of numbers: 74 ; 47 ; 88 ; 57 ; 38 ; 83 ______________________
8. Half of 525 = ____
9. Find the product of 4 and 17__________________________________

FRACTIONS

10. Fill in < or = or > :

    $^1/_4$ __________ $^1/_8$
11. $^2/_8$ + $^3/_8$ = ____
12. How many quarters in 4 apples? ____________________________
13. What is $^1/_3$ of 90 kg? ____
14. 0,3 + _________ = 1
15. 250 g + $^1/_4$ kg = __________ g
16. Max cuts a pie into 8 equal pieces. He eats 3 pieces. What *fraction* of the pie is left over? ____

17. Colour in $^{3}/_{5}$ of the rod:

PATTERNS

Complete the pattern:

18. □ ; ♦ ; □ ; ◊ ; ________ ; ♦ ; ________ .

19. 4 ; 8 ; 12 ; ________ ; ________ .

20. 10:00 ; 10:30 ; 11:00 ; ________ ; ________ .

21. $^{1}/_{8}$; ________ ; $^{3}/_{8}$; ________ .

22. 48 ; 45 ; 42 ; ________ ; ________ .

23. ← ; ↑ ; → ; ↓ ; ← ; ________ ; ________ .

24. $7\,^{1}/_{2}$; 9 ; $10\,^{1}/_{2}$; ________ ; ________ .

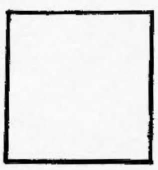
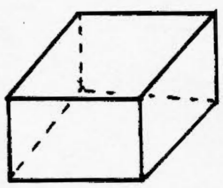
25. 45c ; 90c ; R1.35 ; ________ ; ________ .

SHAPES AND SPACE

26. Identify each of the following shapes and write the name in the space provided:

____________ ____________ ____________ ____________

27. Which shape below is two-dimensional and which is three-dimensional?
Fill in 2-D for two-dimensional and 3-D for three-dimensional in the space provided.

 ________  ________

In the figure below there are two-dimensional shapes packed onto each other.

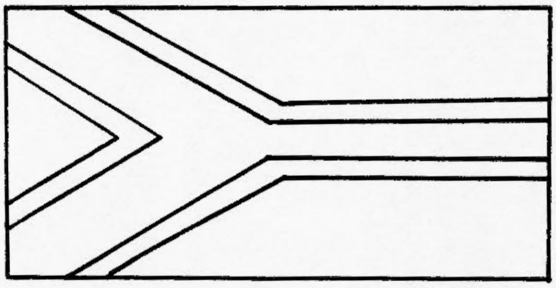
28. Which shape lies on top? ________________________________

29. Which shape lies at the bottom? ________________________________

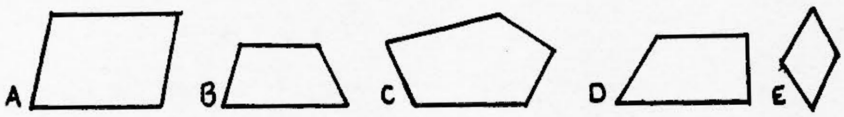
30. Underline two of the shapes below, that when put together, will form a square?

a.    b.   c.    d.

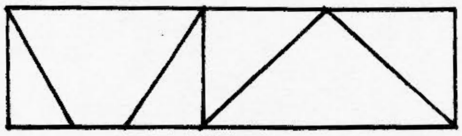
31. Below is a picture of the South African Flag. Draw a line of symmetry through the flag.



32. Underline the shape that does not belong with the others.



33. How many triangles are there in the figure below? ________________



34. The total perimeter of a triangle is 45 mm. If all the sides are equal in length what will the length of one side be? ________________________

MEASUREMENT

Fill in < or = or > :

35. 13 months __________ 1 year

36. 225c __________ R1.60

37. Double 135 c and write the answer in Rand ________________________

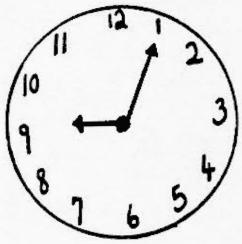
38. 6000 m = ___________ km

39. Measure the height of tree A ___________ cm

40. How many days in a leap year? ____________

41. A plane departs from Johannesburg airport at 07:30 and arrives in Cape Town at 10:30. How long did the flight take? ____________________

42. What was the time 10 minutes earlier? ______________________



DATA

Study the table and then answer the questions that follow. * means that the sport is offered on that day.

	Monday	**Tuesday**	**Wednesday**	**Thursday**	**Friday**
Netball		*	*		
Swimming	*		*		*
Gymnastics		*		*	
Athletics		*		*	*
Choir practice	*				*

43. Which activities are not offered on Friday? ______________________

____

44. On which day(s) of the week are there swimming lessons? ___________

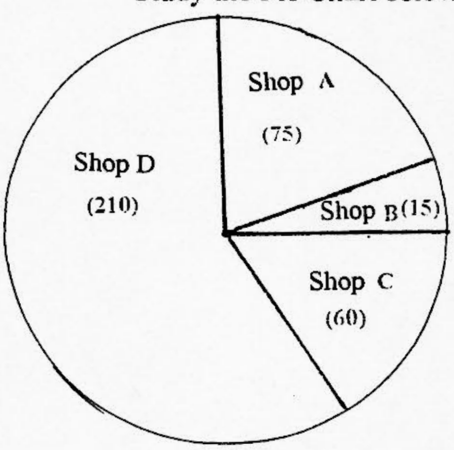
____

45. On which day(s) is neither netball nor choir offered? ________________

46. Which day(s) of the week offer both swimming lessons **and** athletics lessons?

____

Study the Pie Chart below: The numbers indicate the number of workers per shop.



47. How many workers are there in the smallest shop? ____

48. If each worker is paid the same salary. Which shop would spend the most money on salaries? _________

49. How many workers are there altogether? _________

50. Which *fraction* of the pie chart is shop A and B together? ________________________

GRADE 5

Name: ______________________ **Home language:** ______________

Gender: ______________________ **Age:** ______________________

NUMBERS AND OPERATIONS

1. Write from biggest to smallest: 0,007 ; 7 ; 0,07 ; 70 ; 0,7

   ________ ; ________ ; ________ ; ________ ; ________ .

2. (6348 + 3481) - (1368 + 843) = ______________________________

   ____

3. 321 can be written as 300 + 20 + 1. Write 23504 in the same way________

   ____

4. 226 x 37 = ______________________________________

5. The December holidays are 6 weeks and 3 days long. The July holidays are 3 weeks and 6 days long. How much longer are the December holidays than the July holidays? ______________________________

6. 5 ; 7 and 11 are all examples of: (underline the correct answer)

   a) fractions b) composite numbers c) prime numbers

7. If 18 is divided by 6 the answer is called the (underline the correct answer)

   a) sum b) difference c) quotient d) product

8. Fill in < or = or > :

   (2 x 7) __________ 7^2

9. Mrs Naidoo is catering for a party. She makes 153 samoosas and twice as many chilli bites as samoosas. How many eats did Mrs Naidoo make all together?

   ____

FRACTIONS

10. $13\ ^9/_{12} - 7\ ^5/_{12}$ = ______________________________

11. $^1/_2 + 1\ ^1/_4$ = ____________________

12. If $^2/_5 = {}^y/_{15}$ then y = ______________

13. How many minutes in $^1/_3$ of an hour? ______________________

14. Write $^{29}/_7$ as a mixed number ______________________________

15. Write $4\,^{1}/_{2}$ as an improper fraction ________________________________

16. Arrange from biggest to smallest: $^{1}/_{5}$; $^{1}/_{2}$; $^{1}/_{4}$; $^{3}/_{4}$

______ ; ______ ; ______ ; ______ ; ______ .

17. $^{1}/_{3} + {}^{2}/_{5} =$ _________________________

PATTERNS

18. Fill in the missing spaces to complete the pattern:

□ Δ O
O □ Δ
Δ __ __

19. Put the number 2 ; 3 ; 5 and 7 in the boxes below to make the equation true.

[ ] x [ ] + [ ] − [ ] = 24

Complete the pattern in questions 20 to 25:

20. $^{35}/_{100}$; $^{30}/_{100}$; ______ ; ______ .

21. 1212 ; 1218 ; 1224 ; ______ ; ______ .

22. 6 ; $6\,^{1}/_{3}$; $6\,^{2}/_{3}$; ______ ; ______.

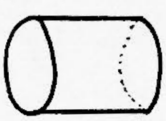
23. 0,10 ; 0,15 ; 0,20 ; ______ ; ______ .

24. ↑ ; ↓ ; → ; ↑ ; ______ ; ______ ; ______ .

25. 1 ; 3 ; 9 ; ______ ; 81 .

SHAPES AND SPACE

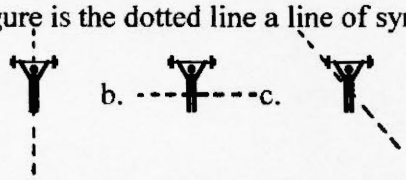
26. Underline the name of the shape given below:



a. triangular prism b. cylinder c. hexagon d. octagon

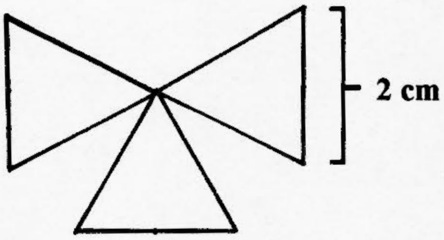
27. In which figure is the dotted line a line of symmetry? ________________

a.



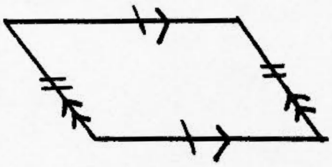
28. Determine the perimeter of the figure below: ____________________

The triangles are all equilateral.



29. A 90° angle is called a ________________ angle.

30.



What is the above shape called? ______________

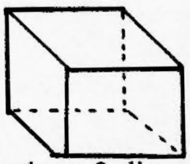
a) rectangle b) square c) triangle d)parallelogram

31. Draw a rectangle in the space below:

32. Which of the following is not an example of a quadrilateral? ____________

a. square b. rectangle c. kite d. triangle

33.



Is the above shape 2-dimensional or 3-dimensional? ______________

MEASUREMENT

34. 1136 g = __________ kg

35. 10 mm = ___________ cm

36. 236 cm + 12 m = _______________

37. You need to 20 fishcakes. Will you buy a large box of 20 fishcakes for R15,40 or two small boxes of 10 fishcakes for R7,50 each?

____

38. Round off 4,45 km to the nearest km ________________________

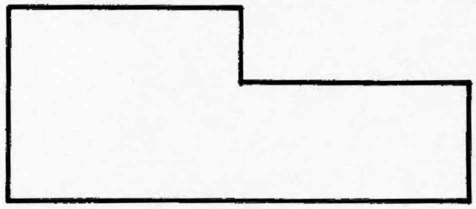
39. A kitchen sink holds 5,55 litres of water. A large bucket holds 20 litres. How much more water is there in the bucket? ____________________

____

40. A motorist travels at an average speed of 110km per hour. How far will he travel if he travels for 4 hours? ________________km

____

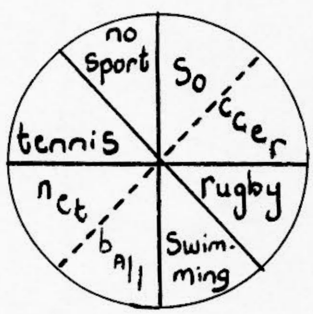
41. With your ruler, measure the perimeter of the following figure below:______cm



42. 1,436 kg + 3,447 kg + 32 kg = ________________ kg

DATA

In the grade 5 class there are 40 children (The pie chart is divided into 8 equal pieces).



43. What *fraction* of the class plays netball?

____

44. How many children do *not* take part in sport? ____________________

45. What *fraction* of the class does take part in sport? ________________

NAMES	EVENT 1	EVENT 2	EVENT 3	TOTAL SCORE
WARREN	8,20	7,15	7,55	22,9
FEZILE	8,20	8,50	9,55	26,25
RORY	7,55	8,70	4,50	20,75

Warren, Fezile and Rory all take part in the Southern Gymnastics Competition. The scoreboard looks like this after the first three events.

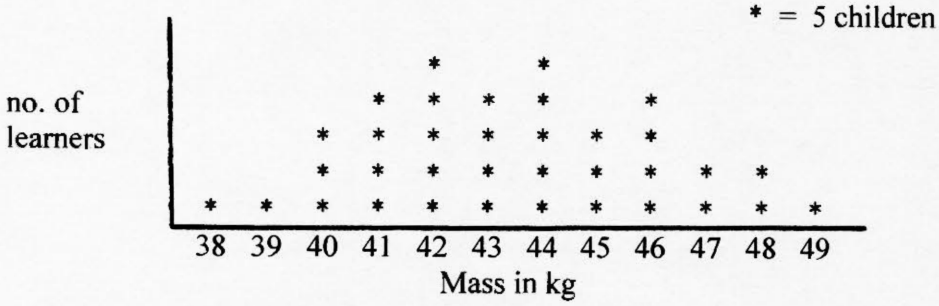
46. Who had the highest score in a single event? ______________________

47. Who had the lowest score in a single event? ________________________

48. What is the difference between Fezile's total score and Rory's total score?

____

The number of learners with a specific mass is given in the graph below:



49. How many learners have a mass of 42 kg?____________________

50. How many learners are there altogether? ____________________________

GRADE 6

Name: ______________________ **Home language:** ______________

Gender: ______________________ **Age:** ______________________

NUMBERS AND OPERATIONS

1. 10^3 = (underline the correct answer)

   a) 30 b) 1000 c) 300 c) 100

2. 9,2 - 5,786 = ______________________

3. The quotient of 826 and 36 = ______________________

4. Estimate whether 8 538 x 380 is closer to: (underline the correct answer)

   a) 10 000 x 400 b) 5 000 x 400

5. Circle all the factors of 12:

   1 ; 2 ; 3 ; 4 ; 5 ; 6 ; 7 ; 8 ; 9 ; 10 ; 11 ; 12

6. $(4 \times 10^4) + (8 \times 10^3) + (2 \times 1)$ = ______________________

7. If y – 28 = 7, then y = ______________________

8. Double 124,7 = ______________________

9. 321 can be written as 300 + 20 + 1. Write 105,6 in the same way.

   ____

   ____

FRACTIONS

10. Write $^{120}/_{25}$ in simplest form ______________________

11. Reduce the following fraction to a decimal fraction: $^1/_5$ ______________

12. What *fraction* of an hour is 40 minutes? ______________________

13. Simplify: $3\,^1/_2 + 4\,^1/_3 - 5\,^5/_6$ = ______________________

    ____

14. If y = 10 and n = 2 ; calculate y^n ______________________

15. If $^1/_3$ of 24 = $^1/_4$ of y, then y = ______________________

16. What fraction is 42 of 105? ______________________

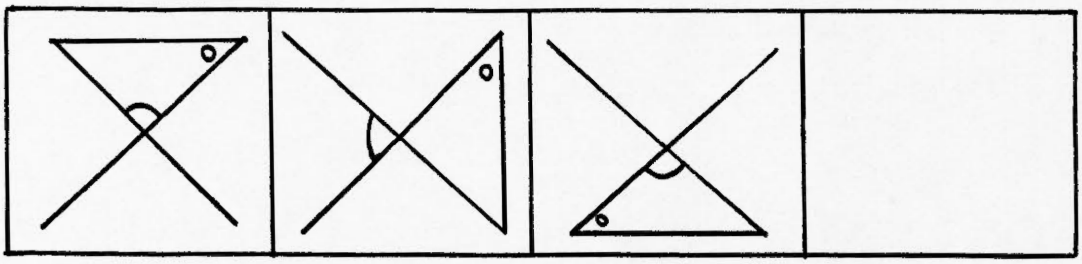
17. A man earns R4900 per month. He saves $^2/_7$ of his salary every month and spends the rest. How much does he save in a year? ______________________

PATTERNS

18. Arrange in descending order: 76,507 ; 76,570 ; 76,057 ; 76,705

__________ ; __________ ; __________ ; __________

19. Complete the pattern:



20. Nina packed her collection of matchboxes into a suitcase. She had three layers altogether. In each layer there were 5 rows with 6 boxes. How many boxes did she have in her suitcase? ______________________________

____

Complete the pattern in questions 21 to 25:

21. O ; Δ ; □ ; □ ; Δ ; O ; ___ ; ___ .

22. 3 ; 6 ; 12 ; 24 ; ___ ; ___ .

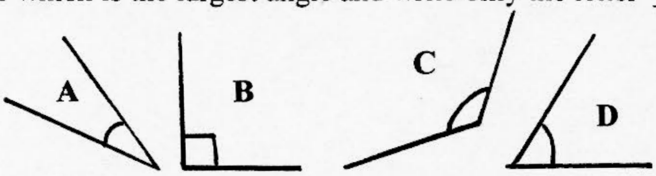
23. 0,6 ; 1,2 ; 1,8 ; _____ ; _____.

24. 2 ; 3 ; 5 ; 8 ; _____ ; _____ .

25. 16 ; 8 ; 4 ; 2 ; _____ ; _____ .

SHAPES AND SPACE

26. Estimate which is the largest angle and write only the letter __________



27. Draw 1 line of symmetry through the shape

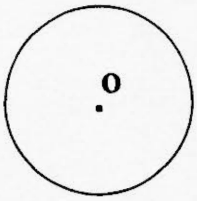
♣

28. If the radius of a circle is 8 cm. What is the diameter of the circle?

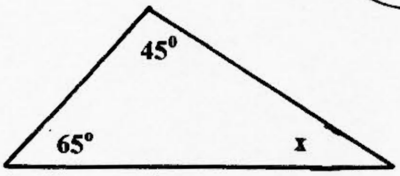
__________cm

29. O is the center of the circle. Measure the diameter

______________cm

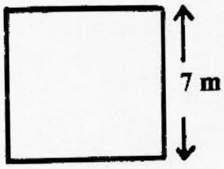


30.



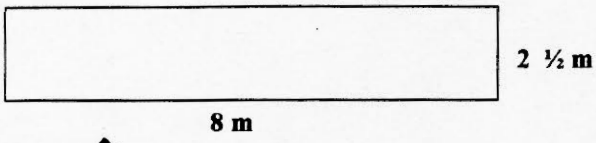
x = ____________________°

31.



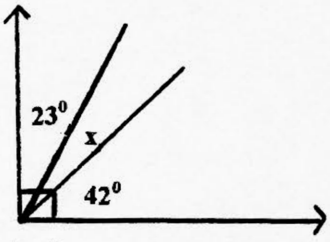
Area of the square = ______m^2

32.



Area of the rectangle = ______________m^2

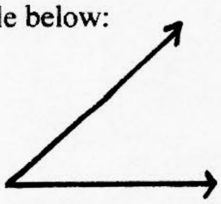
33.



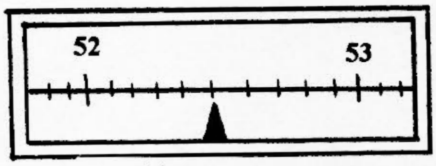
x = _________________________°

MEASUREMENT

Complete the table below:



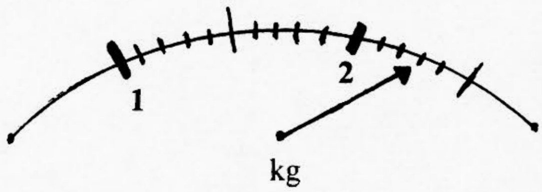
Angle	Kind of angle (acute, obtuse, right angle)	Degrees
Angle ZQR	34.	35.



speed in km per hour

36. The speed in km per hour = ______________________ km/h

37. The mass on this scale = ______________ kg



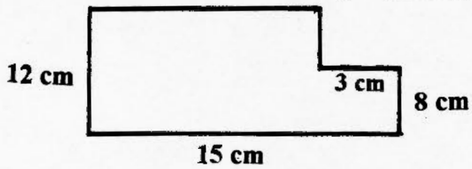
38. A tap leaks 0,445 ml of water per hour. How much water will be wasted in one day? ____

39. 17 minutes 43 seconds + 18 minutes 34 seconds = _____min______sec

____

40. 0,59 kg + 300 g + 2,36 kg + 48,003 kg = ______________________kg

41. Calculate the perimeter of the shape ________________________ m



42. If you go to sleep at 18:45 and you awaken at 07:25. How long did you sleep?

____________ h ____________ min

DATA

Mrs Gardener compares two kinds of climbing beans (refer to the graphs below):

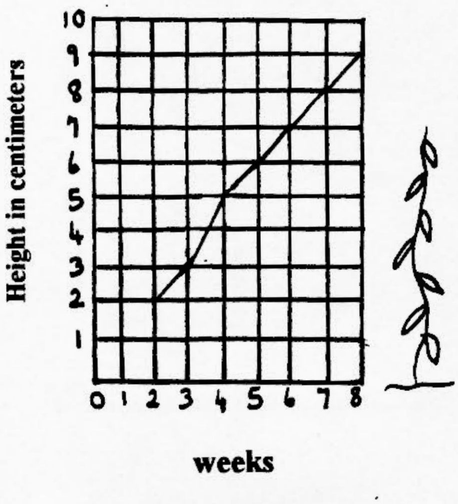
43. Which bean plant grows the highest in the first 3 weeks? ______________

44. How many weeks does it take for bean plant A to grow 7 cm? ____________

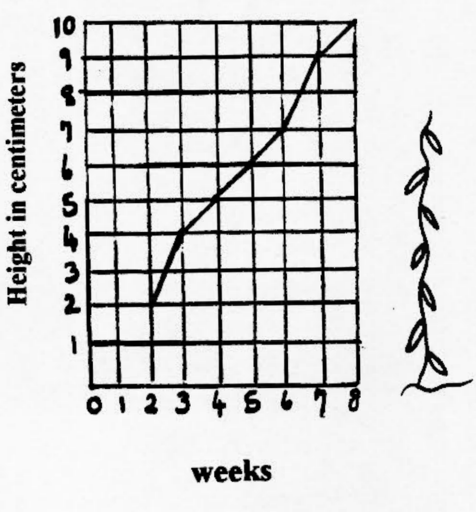
45. For how many weeks are the bean plants the same height? ______________

46. Which bean plant grows the fastest between weeks 2 and 3? ____________

Bean Plant A



Bean Plant B



Khani is researching her family history. She has collected the following information:

NAME	GENDER	DATE BORN	DATE DIED
Dreyer, Noel	Male	**1867-04-13**	**1914-12-27**
Solomon, Gladys	Female	**1845-07-10**	**1901-06-01**
Koen, Maria	Female	**1899-05-11**	**1899-05-11**
Swart, Estelle	Female	**1903-01-01**	**1959-01-17**
Samuels, Johanna	Female	**1899-05-12**	**1956-08-19**
Koen, Petrus	Male	**1904-09-13**	**1983-03-31**
May, Constance	Female	**1936-08-27**	**1958-05-30**
Cilliers, Morris	Male	**1938-10-26**	**1999-07-29**
Koen, Gerty	Female	**1903-04-17**	**1990-07-31**
Els, Willem	Male	**1891-03-17**	**1963-09-12**

47. Who was born first? ________________________________

48. At what age did Estelle Swart die? ______________________

49. Who died at the youngest age? _____________________

A survey was done to determine the type of transport the learners use to go to school. The table shows the number of learners that use the various transport systems.

50. How many learners took part in the survey? ______________________

Type of Transport	Number of learners
Motor car	𝍸 𝍸 𝍸 II
Bicycle	𝍸 I
Taxi	𝍸 𝍸 𝍸 𝍸 𝍸
Bus	𝍸 𝍸 III
Walk	II

ANNEXURE E

INTERMEDIATE PHASE
MATHEMATICS PROFICIENCY TEST

INTERMEDIATE PHASE
MATHEMATICS
PROFICIENCY TEST

ANSWER SHEET: GRADE 4

Name of Learner: .... **Gender (m/f):** ....

Date: .... **Age:** ....

1.) **Fill in < or = or > :**

37 - 26 ___________ (15 + 9) ÷ 6 ___

2.) **Round off 347 to the nearest 100** _______________ ___

3.) **What is the difference between the biggest and smallest numbers in this group of numbers:**

74 ; 47 ; 88 ; 57 ; 38 ; 83 _________________ ___

4.) **Half of 525 =** ___________ ___

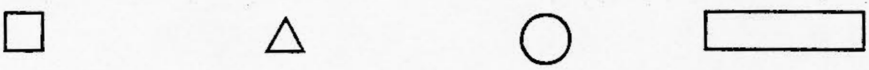
5.) **0,3 +** ___________ **= 1** ___

6.) **250 g + ¼ kg =** ______ **g** ___

7.) **Complete the pattern:** □ ; ◆ ; □ ; ◇ ; ___ ; ◆ ; ___ . ___

8.) **Complete the pattern:** ← ; ↑ ; → ; ↓ ; ← ; ___ ; ___ . ___

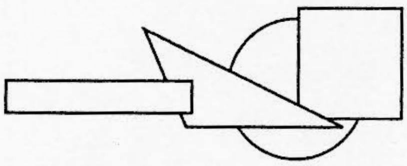
9.) **Identify each of the following shapes and write the name in the space provided:**



__________ __________ __________ __________ ___

10.) **Which shape below is two-dimensional and which is three-dimensional? Fill in 2-D for two-dimensional and 3-D for three-dimensional in the space provided.**

In the figure below there are two-dimensional shapes packed onto each other.



11.) **Which shape lies on top?** ____________________ ___

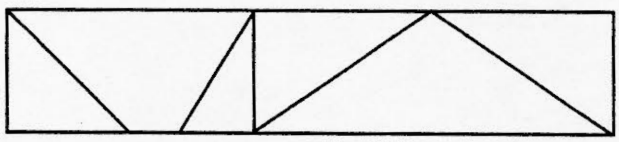
12.) **Which shape lies at the bottom?** ________________ ___

13.) Underline two of the shapes below, that when put together, will form a square.

a.  b.  c.  d. 

14.) How many triangles are there in the figure below?

____



Fill in < or = or > :

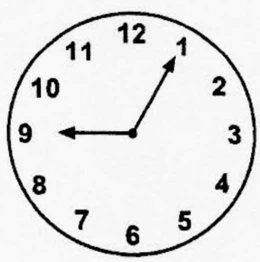
15.) 13 months ______ 1 year

Fill in < or = or > :

16.) 225c ______ R1.60

17.) How many days in a leap year? ________________

18.) What was the time 10 minutes earlier? ____________



Study the table and then answer the questions that follow.

*** means that the sport is offered on that day.**

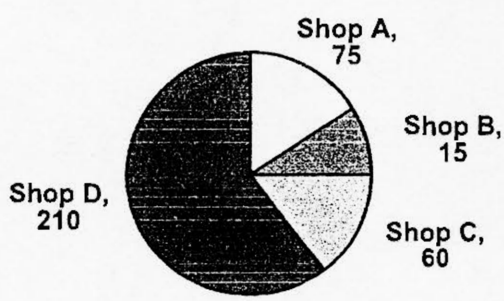
	Monday	Tuesday	Wednesday	Thursday	Friday
Netball		*	*		
Swimming	*		*		*
Gymnastics		*		*	
Athletics		*		*	*
Choir practice	*				*

19.) Which activities are not offered on Friday?

______________________________________ ___

Study the Pie Chart below: The numbers indicate the number of workers per shop.

20.) If each worker is paid the same salary. Which shop would spend the most money on salaries? __________ ___



RAW SCORE	STANINE	PERCENTILE RANK
/ 20		

INTERMEDIATE PHASE
MATHEMATICS
PROFICIENCY TEST

ANSWER SHEET: GRADE 5

Name of Learner: .... **Gender (m/f):** ....

Date: .... **Age:** ....

1.) **Write from biggest to smallest:**

0,007 ; 7 ; 0,07 ; 70 ; 0,7

_______ ; _______ ; _______ ; _______ ; _______ . __

2.) **(6348 + 3481) - (1368 + 843) =** _______________

____ __

3.) **321 can be witten as 300 + 20 + 1. Write 23504 in the same way** _______________

____ __

4.) $^1/_2 + 1^1/_4 =$ _______________ __

5.) **Write** $^{29}/_7$ **as a mixed number** _______________ __

6.) **Put the numbers 2 ; 3 ; 5 and 7 in the boxes below to make the equation true.**

☐ x ☐ + ☐ - ☐ = 24 __

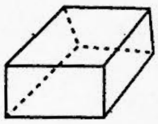
7.) **Complete the pattern:** $^{35}/_{100}$; $^{30}/_{100}$; ______ ; ______. | __

8.) **Complete the pattern:** ↑ ; ↓ ; → ; ↑ ; __ ; __. | __

9.) **Underline the answer below that is not an example of a quadrilataral?**

a. square b. rectangle c. kite d. triangle | __

10.)



Is the above shape 2-dimensional or 3-dimemsional?

____________________ | __

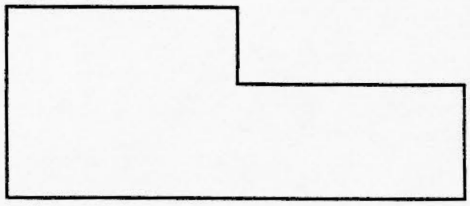
11.) **You need 20 fishcakes. Will you buy a large box of 20 fishcakes for R15,40 or two small boxes of 10 fishcakes for R7,50 each?**

____________________________________ | __

12.) **A motorist travels at an average speed of 110 km per hour. How far will he travel if he travels for 4 hours?**

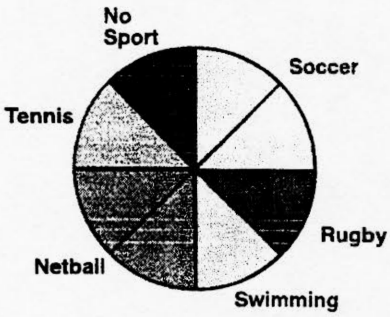
____________________ km | __

13.) **With your ruler, measure the perimeter of the following figure below: _________ cm**



14.) **1,436 kg + 3,447 kg + 32 kg = _______________ kg**

In the grade 5 class there are 40 children (The pie chart is divided into 8 equal pieces).



15.) **What *fraction* of the class plays netball?**

____

16.) **How many children do *not* take part in sport?**

____

Names	Event 1	Event 2	Event 3	Total Score
Warren	8,20	7,15	7,55	22,9
Fezile	8,20	8,50	9,55	26,25
Rory	7,55	8,70	4,50	20,75

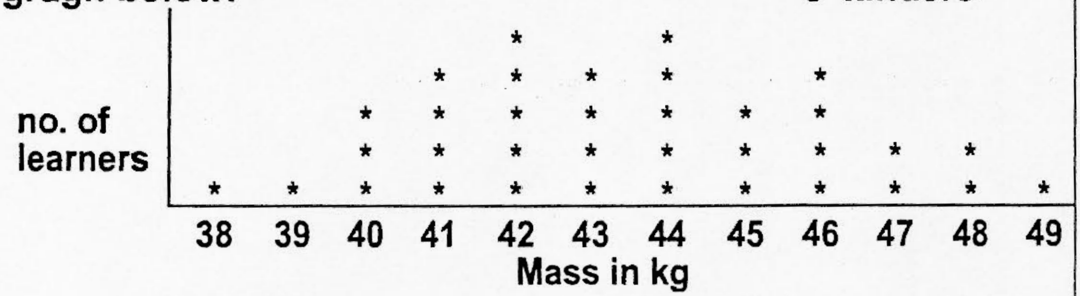
Warren, Fezile and Rory all take part in the Southern Gymnastics Competition. The scoreboard looks like this after the first three events.

17.) Who had the lowest score in a single event?

______________________ __

18.) What is the difference between Fezile's total score and Rory's total score? ______________________ __

The number of learners with a specific mass is given in the gragh below: *** = 5 kinders.**



19.) How many learners have a mass of 42 kg?

______________________ __

20.) How many learners are there altogether? __________ __

RAW SCORE	STANINE	PERCENTILE RANK
/ 20		

INTERMEDIATE PHASE
MATHEMATICS
PROFICIENCY TEST

ANSWER SHEET: GRADE 6

Name of Learner: .... **Gender (m/f):** ....

Date: .... **Age:** ....

1.) 10^3 = (underline the correct answer)

a) 30 b) 1000 c) 300 d) 100 ___

2.) 9,2 - 5,786 = ____________________ ___

3.) Circle all the factors of 12:

1 ; 2 ; 3 ; 4 ; 5 ; 6 ; 7 ; 8 ; 9 ; 10 ; 11 ; 12 ___

4.) Double 124,7 = __________ ___

5.) Write $^{120}/_{25}$ in simplest form ____________________ ___

6.) Simplify: $3\,^{1}/_{2} + 4\,^{1}/_{3} - 5\,^{5}/_{6}$ = __________ ___

7.) If $^{1}/_{3}$ of 24 = $^{1}/_{4}$ of y, then y = __________ ___

8.) Complete the pattern: 3 ; 6 ; 12 ; 24 ; ____ ; ____

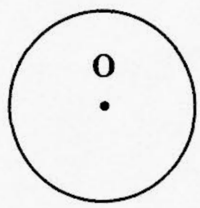
9.) Complete the pattern: 0,6 ; 1,2 ; 1,8 ; ____ ; ____

10.) Complete the pattern: 2 ; 3 ; 5 ; 8 ; ____ ; ____

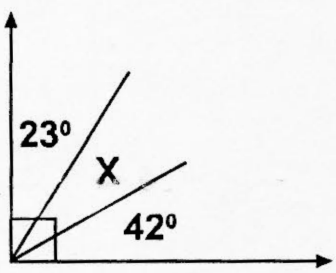
11.) Draw 1 line of symmetry through the shape

12.) O is the centre of the circle. Measure the diameter

____________________ cm

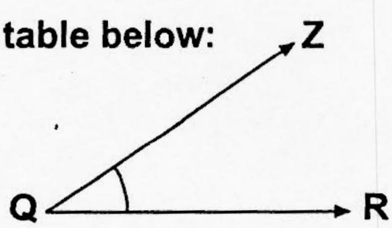


13.)



X = ______________ °

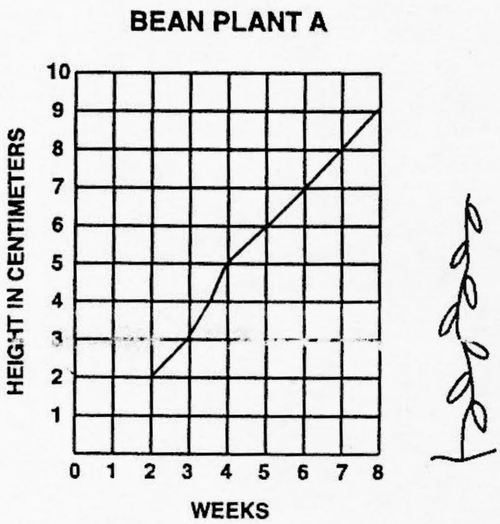
Complete the table below:

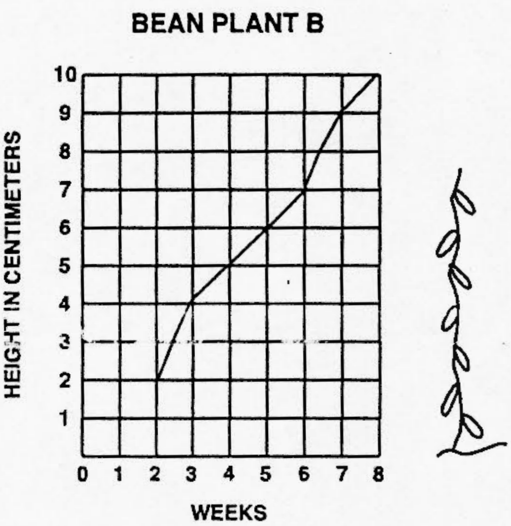


Angle	Kind of angle (acute, obtuse, right angle)	Degrees
ZQR	14.	15.

Mrs Gardener compares two kinds of climbing beans (refer to the graphs below):

16.) Which bean plant grows the fastest between weeks 2 and 3? ____________________





A survey was done to determine the type of transport the learners use to go to school. The table shows the number of learners that use the various transport systems.

17.) How many learners took part in the survey?

____

Type of Transport	Number of learners
Motor Car	卌 卌 卌 II
Bicycle	卌 I
Taxi	卌 卌 卌 卌 卌
Bus	卌 卌 III
Walk	II

Khani is researching her family history. She has collected the following information:

NAME	GENDER	DATE BORN	DATE DIED
Dreyer, Noel	Male	1867-04-13	1914-12-27
Solomon, Gladys	Female	1845-07-10	1901-06-01
Koen, Maria	Female	1899-05-11	1899-05-11
Swart, Estelle	Female	1903-01-01	1959-01-17
Samuels, Johanna	Female	1899-05-12	1956-08-19
Koen, Petrus	Male	1904-09-13	1983-03-31
May, Constance	Female	1936-08-27	1958-05-30
Cilliers, Morris	Male	1938-10-26	1999-07-29
Koen, Gerty	Female	1903-04-17	1990-07-31
Els, Willem	Male	1891-03-17	1963-09-12

18.) Who was born first?

____ __

19.) At what age did Estelle Swart die?

____ __

20.) Who died at the youngest age?

____ __

RAW SCORE	STANINE	PERCENTILE RANK
/ 20		

ANNEXURE F

NORM TABLES FOR THE INTERMEDIATE PHASE MATHEMATICS PROFICIENCY TEST

ANNEXURE G

REFERENCE LETTER
FROM THE
FREE STATE DEPARTMENT OF EDUCATION

FREE STATE PROVINCE

Enquiries: WF RETIEF
Reference no. :

Tel: 051 – 4048230
Fax: 051 – 4048233

TO Mrs Colleen Vassilion

Thank you for the information on the utilization of the VASSI Mathematics Proficiency Test. The diagnostic value of the test is highly recommended. The latter will assist educators to develop strategies to support those learners who experience barriers to mathematics.

We wish you all the best

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