THE DEVELOPMENT AND STANDARDISATION OF A MATHEMATICS PROFICIENCY TEST FOR LEARNERS IN THE FOUNDATION PHASE

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1. INTRODUCTION

1.1 BACKGROUND

When the *ESSI* Reading and Spelling test (Esterhuyse & Beukes, 1997) was compiled, the remedial teachers involved expressed an interest in a mathematics test that would serve the same diagnostic purpose. The need has therefore arisen to set up a mathematics test that can help to isolate mathematics problems at a young age. The only two subjects, that are evaluated during the foundation school phase, are language and mathematics. There is a high positive correlation (correlation coefficients vary between 0,45 and 0,59; N=250) between the ability to read and spell (language proficiency) and mathematics performance (Esterhuyse & Beukes). The sample consisted of English speaking grade one to grade seven learners.

1.2 PROBLEM STATEMENT

Often in a young child's functioning, cognitive problems arise such as the inability to perform mathematical calculations. Psychologists and educationists go to great lengths to determine the problem, so that a plan of action can be put into place to help a child function at his/her optimal level. Intelligence tests, visual-motor perceptual tests and even reading and spelling tests are performed. A need has arisen for a new mathematics test, which has South African norms, that will enable a psychologist or an educationist to identify a mathematics problem. From a young age children are often told that, of all the subjects they will encounter at school, mathematics will be the most difficult.

Cited from De Wet (1994):

> At present, substantial parts of mathematics that is taught – especially in work on number, and especially junior years – are based on a conceptual model that children are 'empty vessels', and that it is the teacher's duty to fill those vessels

> with knowledge about how calculations are performed by standard methods, and to provide practice until the children can perform these methods accurately...Recent work would suggest that another model of mathematics learning is in fact a better one; learners are conceptualized as active mathematical thinkers, who try to construct meaning and make sense for themselves of what they are doing, on the basis of their personal experience...and who are developing their ways of thinking as their experience broadens, always building on the knowledge which they have already constructed. (p.145)

If children are struggling to be active mathematical thinkers and are unable to construct and make sense for themselves of what they are doing in mathematics, then the researcher wishes to identify this and address it. This test could enable teachers to identify and assist a child with a mathematics problem at an early age. This test can prevent a learner from experiencing future mathematics problems, if the problem is identified and dealt with timeously.

1.3 AIM OF THE STUDY

In view of the above, the researcher proposes to set up an English mathematics proficiency test for the Free State Education Department and to standardise it with the following in mind:

a) that the test will be applicable to grades one, two and three;
b) the norms per term will be available, so that the test can be administered at any time of the year;
c) that the test will consist of universal mathematics concepts, so that the usage of the test will not be limited;
d) that the test can be administered to groups or individuals;
e) that the test can be used diagnostically (i.e. to identify the area in which the learner is experiencing problems); and
f) that the test will be of value to future generations of learners.

1.4 CHAPTER EXPOSITION

There are five main focus areas in this study. In chapter two the researcher wishes to explore learning problems, with specific reference to the manifestation and classification thereof. Secondly, the focus is shifted to the three main factors that influence learning problems, namely; cognitive factors, non-cognitive factors and socio-environmental factors. In chapter three, the researcher emphasizes one specific learning area, namely mathematics. The definition, components and processing of mathematics are then considered. Focus is also placed on the tasks that need to be performed once mathematical thinking has developed in the child. Fourthly, in chapter four, with mathematics defined and mathematical processing clearly considered, the researcher highlights problems that could hinder mathematics achievement in children. Focus is placed on the types of problems that manifest, the causes thereof and the assessment, classification and intervention of learning problems in mathematics. Lastly, the empirical side of the study is discussed in detail. In chapter five, the standardisation of psychometric tests, the measurement, reliability and validity of items that are selected for the tests are discussed. In chapter six the researcher reviews the five phases of the research method. Phase one is the compilation of the preliminary questionnaire. Phase two is the item selection and analysis, phase three is the determination of the norms. Phase four is the validity of the study and in phase five the results are considered and discussed. Finally conclusions and recommendations for future research are considered. A schematic diagram representing the chapter exposition is presented in Figure 1.1.

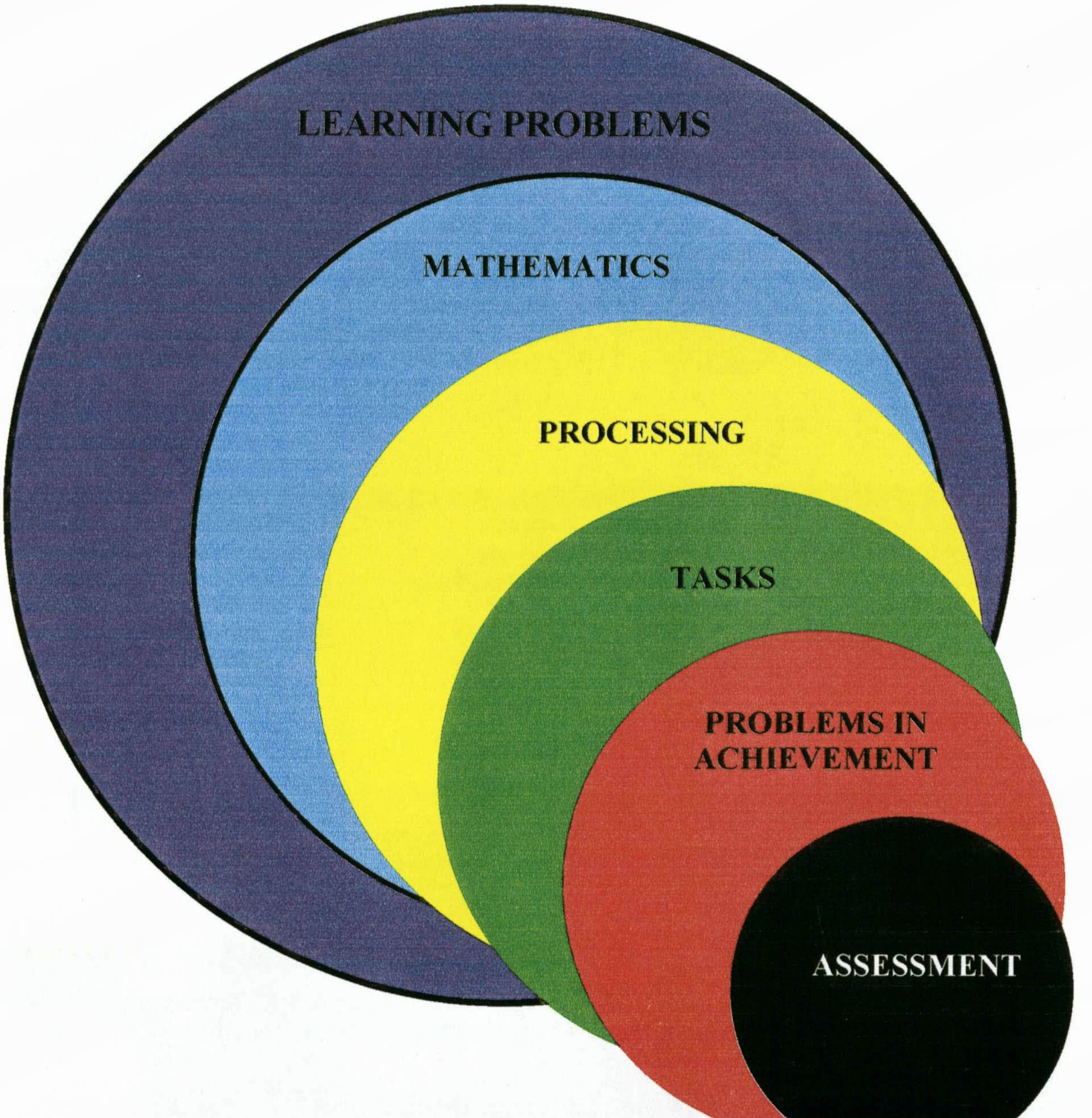
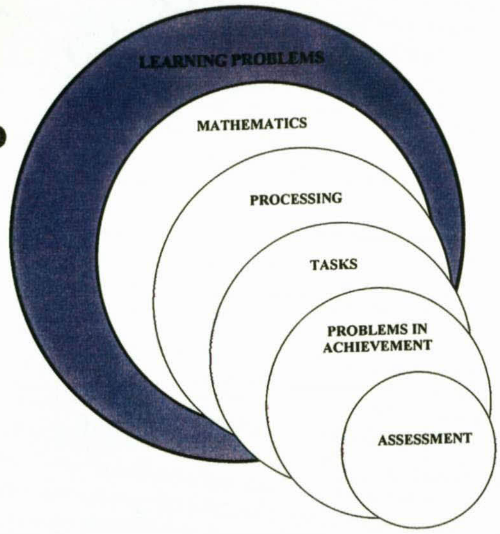


Figure 1.1: Graphical representation of the chapter exposition

2. LEARNING PROBLEMS DURING CHILDHOOD



We are still waiting for the revolution in educational techniques that will make children willing callaborators instead of grudging accomplices in the marshalling and structuring of their intellectual powers. How can something so natural as learning be made to seem so hostile? (Harris, 1983)

2.1 INTRODUCTION

While surveying the literature about learning problems, a dilemma arose between the various terms used to describe learning problems. According to Kapp (1991) it is necessary to make a distinction between children with learning restraints and children with learning disabilities. A learning restraint develops when certain factors cause a child not to actualise his/her potential. Their level of achievement, development and behaviour does not correspond with their intellectual potential. A gap or discrepancy then occurs between the level the child should be on and the actual level the child is functioning on. A child with a learning disability on the other hand, has an identifiable deficiency in his/her given potential, such as a sensory, neural, intellectual or physical deficiency. This is usually a permanent condition, which hinders a child's education. The distinction between learning restraints and learning disabilities is more complex than the above definition. Kapp continues to state that learning restraints and learning disabilities often overlap. Some children can become restrained to the extent that their deficiencies are greater. They are therefore both disabled and restrained. The effect of a restraint could be so comprehensive that it permanently affects the child's development and learning. Such a child may then, by definition, also be disabled. The two terms overlap to the extent that for several years, clinicians have struggled to define what a learning disability is. According to the American Psychiatric Association's Diagnostic and Statistical Manual of Mental Disorders, Fourth Edition (DSM IV, 1994), reference is made to the term Learning Disorders (formerly Academic Skills Disorders). According to this reference:

> Learning Disorders are diagnosed when the individual's achievement on individually administered, standardised tests in reading, mathematics, or written expression is substantially below that expected for age, schooling, and level of intelligence. The learning problems significantly interfere with academic achievement or activities of daily living that require reading, mathematical, or writing skills. (p.46)

A learning disorder can therefore be diagnosed when a learning restraint or learning disability is so severe that it substantially affects a child's achievement. The above three terms overlap to the extent that it is often difficult to distinguish between them. Therefore, for the purpose of this study, learning restraints, learning disabilities and learning disorders will be considered under the umbrella term of learning problems. When discussing the classification and manifestation of learning problems, the researcher will consider learning restraints, learning disabilities and learning disorders individually. On the other hand, the causes of the above three terms overlap to the extent that the causes of learning problems as a whole will be explored. In chapters three and four, reference will be made to learning problems. It is essential to consider how learning problems are classified and how they manifest.

2.2 CLASSIFICATION AND MANIFESTATION OF LEARNING PROBLEMS IN CHILDREN

As noted above, Kapp (1991) postulates that it is important to classify learning problems in children according to learning restraints and learning disabilities. Let us first consider the broad categories of the above two definitions. The following are classified as restraints:

- special forms of learning difficulties;
- emotional and behavioural disturbance;
- environmental deprivation;
- didactical neglect;

- gifted underachieving: and
- school-readiness problems.

The following are classified as disabilities:

- epilepsy;
- cerebral palsy;
- mental handicap;
- aural handicap;
- visual handicap;
- learning disabilities;
- physical handicaps;
- autism and other childhood psychoses; and
- multiple handicaps.

According to the DSM IV (1994), Learning Disorders fall into the umbrella category of Disorders usually First Diagnosed in Infancy, Childhood or Adolescence. All the disorders in this category include:

- Mental Retardation;
- Learning Disorders;
- Motor Skills Disorder;
- Communication Disorders;
- Pervasive Developmental Disorders;
- Attention-Deficit and Disruptive Behaviour Disorders;
- Feeding and Eating Disorders of Infancy or Early Childhood;
- Tic Disorders;
- Elimination Disorders; and
- Other Disorders of Infancy, Childhood or Adolescence.

If we focus for a moment on Learning Disorders, the DSM IV (1994) states that Learning Disorders include:

- Reading Disorder;
- Mathematics Disorder;
- Disorder of Written Expression; and
- Learning Disorder Not Otherwise Specified.

For the purpose of this study, focus will be placed on one specific learning area, namely mathematics. The exposition of a mathematics learning restraint, a mathematics learning disability and a mathematics disorder will be discussed in chapter four.

According to Dockrell and McShane (1993) a learning problem can either be specific or general. A specific learning problem occurs when a child experiences a problem with a specific task, such as reading. A general learning problem occurs when learning is slower across a range of tasks. Children who initially experience a specific learning problem sometimes experience other difficulties as a result. For example, language difficulties can lead to reading difficulties and reading difficulties can lead to difficulties in mathematics. According to the DSM IV (1994) prevalence of learning disorders range from two percent to ten percent, depending on the nature of the disorder.

With learning problems clearly classified, knowledge of the manner in which they manifest is of vital importance. Learning problems manifest according to their classification. According to Myers and Hammill (1990) there are certain characteristics that manifest in children with learning problems. The observable characteristics are as follows:

- poor speech and communication;
- academic problems;
- delayed thinking processes;
- impairments of concept formation;
- test performance that is erratic or unpredictable;
- impairments in perception;
- specific neurological indicators;
- poor motor function;

- various physical characteristics: drooling, enuresis or slow toilet training;
- emotional characteristics: impulsiveness, maladjustment, explosiveness or low tolerance frustration;
- sleep characteristics: irregular sleep patterns, abnormally light or deep sleep;
- irregular relationship capacities;
- variation in physical development;
- irregularities in social behaviour;
- variations and irregularities in personality; and
- inability to pay attention and concentrate.

According to the DSM IV (1994) learning disorders must be differentiated from normal variations in academic performance and from scholastic difficulties due to lack of opportunity or poor teaching or cultural factors. Now that the manifestations of learning problems have been clarified, emphasis must be shifted to the causes of these learning problems.

2.3 GENERAL CAUSES OF LEARNING PROBLEMS IN CHILDREN

There are three main causes of learning problems in children, cognitive factors, non-cognitive factors and socio-environmental factors.

2.3.1 Cognitive factors

Cognitive factors can be the cause of many learning problems in children. A number of cognitive factors are necessary to learn. Cognitive factors not only cause mathematical problems but problems in many learning areas. The four main cognitive causes of learning problems are aptitude, ability, psycho-neurological factors and concentration.

2.3.1.1 Aptitude

According to Mussen, Conger, Kagan and Huston (1990) aptitude refers to the inherent potential to learn a new skill or to do well in some future learning situation. Aptitude is genetic. According to Kapp (1991) many learning problems can be related to genetic components such as aptitude. According to Smith (1991) studies on heritability of aptitude are important because of the role of intelligence. Aptitude correlations increase in direct proportion to the increase in genetic relationships. Wigle, White and Parish in Anderman (1998) compared the reading, mathematics and writing achievement of both low and high IQ students with learning disabilities during childhood. They found that the IQ score and the mathematics score declined over time, while the reading and writing scores remained constant. According to Nigg, Quamma, Greenberg and Kusche (1999) IQ and delinquency have been hypothesised for decades and have received empirical support. A low aptitude can therefore be a cause of a learning problem.

2.3.1.2 Ability

To successfully cope with general scholastic requirements, the learner must possess a general intellectual ability. These abilities can be vastly altered through parenting and teaching. Heredity establishes an upper limit to ability and therefore learning, but whether one reaches the limit depends on the environment. Ability is therefore dependent on the environment, but aptitude is inherent and cannot be changed (Smith, 1991).

2.3.1.3 Psycho-neurological factors

According to Westman (1990) learning disabilities originated from studies of brain-damaged adults and children. Cited from Kapp (1991):

> ...all learning is neurological...No learning can take place without the nervous system being involved. Emotions are neurological. Sensation is neurological. Perception is neurological...each filters down to...efferent and afferent nerves and to the extraordinarily significant structures called synapses, to actions within the cortex, thalamus, the cerebellum, or involving among other structures, the association fibres. (p.384)

Smith (1991) does not only mention brain injury or structural brain differences as a physiological learner based contributor. Emphasis is also placed on hereditary and biochemical differences. Different brain structures, patterns of brain maturation and biochemical irregularities can impair brain functioning. This may be genetically transmitted. Emphasis is also placed on biochemical irregularities which may lead to learning problems. Biochemical irregularities may lead to brain injury, hyperactive or hypoactive behavioural states and could also result in learning problems. According to Kapp (1991) a learning problem does not only arise from structural abnormality in the central nervous system, but from a dysfunction related to biochemical, electrochemical, and molecular chemical systems within the neurons of the brain. Certain children with learning problems show evidence of abnormal brain impulses that are measured by an Electro-encephalogram (EEG). Some children show such a slight neurological dysfunction that it is difficult to diagnose. Sometimes a child exhibits soft neurological signs, such as motor clumsiness, hyperactivity, perceptual disturbances, emotional disturbances, disturbances in memory and uncommon behaviour. A neurological examination does not always reveal positive neurological signs. A neurological dysfunction can still be deduced from manifestations in the behaviour of the child. Neuropsychological or cognitive function is often seen as a causal mediator in childhood for the development of psychopathology. Mild early neuropsychological risk may exert a small but significant causal effect on later behavioural adjustment. (Nigg et al., 1999). Teachers comment that children with learning problems are immature in far more than just academic achievement (Smith, 1991). The author continues to state that there is a high degree of asymmetry in the brain in children with learning problems. This refers to the learner's left and right hemisphere usage. Children with learning problems usually have a natural preference for one hemisphere. The one hemisphere is therefore slightly more active than the other hemisphere during certain activities. The nervous systems of children with learning problems also respond more slowly than normal. They therefore require more time to process information and complete tasks. Motor in-coordination is also common because of the brain's inability to integrate all incoming and stored information. The brain is vitally important in learning and most theories on learning problems emphasize that brain dysfunction is often the point of departure. This will be discussed in greater detail in Chapter 3.

2.3.1.4 Concentration

Adequate attention and concentration is critical to learning efficiency. Teachers often comment that children with learning problems cannot concentrate. One needs to determine whether concentration is a cognitive or non-cognitive factor. If concentration is regarded as a cognitive factor, the neurological component of an inability to concentrate needs to be explored. As children pay greater attention, their brain wave amplitudes increase and the time needed for the brain to respond decreases (Westman, 1990). This author continues to state that children with learning problems have brain cells that inhibit responses and these cells develop a lot slower. These children cannot help but respond to distractions. If we consider concentration as a non-cognitive term we view concentration as something that is a voluntary action. Focused attention is to devote attention to all relevant information and withhold attention from irrelevant information. The young child struggles to withhold relevant information from irrelevant information and at this point is accused of not concentrating. Concentration is the extension of focused attention into the sustained processing of stimuli and can be measured by its intensity and span (Westman). The argument between whether concentration is a voluntary or involuntary action parallels with whether it is a cognitive or non-cognitive factor. Concentration is a useful concept because it implies active processing of stimuli in a task. Whether this is cognitive or non-cognitive can be debated.

2.3.2 Non-cognitive factors

Students with learning problems have unique strengths and weaknesses with respect to ability and learning styles. Whether these unique ability patterns and learning styles will influence their achievement is partly dependent on non-cognitive factors. Learning success is facilitated when the school task matches the child's ability level and learning style. Efficient learning is enhanced by environmental factors. These include emotional factors, motivation and self-concept. Each of these factors requires further exploration.

2.3.2.1 Emotional factors

Skemp (1991) states that we cannot separate cognitive from affective processes, as many students experience strong emotions during their classroom experience. A child can either experience pleasure at school or displeasure. These two states can influence or cause learning problems. Pleasure is experienced when emotions signal changes towards a goal state. Displeasure is experienced when signals change away from the goal state. A goal state is usually achieved when one is working towards something one enjoys and understands. An anti-goal state is experienced when one is working towards something one does not fully enjoy or understand. The author continues to comment that there are three emotional states common in learning problems, the state of fear, frustration and relief. Fear signals change towards an anti-goal state. Frustration occurs in the state of displeasure. Relief is experienced when changes occur away from the anti-goal state. Emotions experienced when working towards one's goal are in relation to a feeling of competence. These emotions include confidence and security. Confidence is usually associated with pleasure. Security is an emotion experienced in the state of relief. When a learner experiences frustration and anxiety in the state of displeasure or fear, learning problems usually develop. Skemp states:

> This confidence in one's ability to learn is a crucial factor in any learning situation. How long a person goes on trying, and how much frustration he can tolerate, will depend on the degree of confidence he brings to the learning skill

initially. His likelihood of success will also depend partly on how long he goes on trying. So a good level of initial confidence tends to be a self-fulfilling prophecy: the learner succeeds because he thinks he can. Lack of confidence will have the opposite effect. (p.201)

2.3.2.2 Motivation

Anderman (1998) states that a number of studies indicate that as the learners move from elementary to middle grades, achievement, motivation and attitude decline. To add to this decline, the learning disabled/restrained child often lacks motivation from the beginning. This is due in part to the numerous past failures. The learning disabled/restrained child has little or no intrinsic motivation and therefore needs to be motivated extrinsically with tangible motivators, activity orientated motivators and social motivators (Kapp, 1991). Given their repeated academic and social failure, the children with learning problems believe that there is little relationship between effort and success. Perceiving that their efforts are in vain, the children believe that success is out of their control, this leads to what is known as a state of learned helplessness. This has a profound effect on the developing self-concept.

2.3.2.3 Self-concept

According to Kapp (1991) a child with a learning problem often has a poor self-concept. This is due to the child having unrealistic goals and constantly underachieving according to his/her expectations. The child therefore feels inferior which leads to a poor self-image and poor self-identity. Often the learning disabled/restrained child is also unable to meet the expectations of the parents. The deficiencies are often over-emphasized and any positive attributes are not even mentioned. The children also compare themselves to their peer group. Bear, Minke, Griffin and Deemer (1998) argue that children with learning disabilities tend to hold realistic self-perceptions of their academic difficulties.

They perceive themselves negatively in a domain of self-concept compared to most school-aged children. Kapp (1991) continues to state that children with learning problems often experience rejection by their social group, which leads to negative self-identification. Learners need successful social experiences to build self-confidence and self-worth. Children with poor self-concepts sometimes exhibit aggressive behaviour or seem to withdraw from their peer group. This may attribute to their poor self-image. A negative self-concept has an effect on academic performance. Not only does the learning problem hinder academic performance but a poor self-concept may also contribute to academic failure. According to the DSM IV (1994), demoralisation, low self-esteem and deficits in social skills are often associated with learning disorders. Many cognitive and non-cognitive factors underlie learning problems but another cause could be ascribed to socio-environmental factors.

2.3.3 Socio-environmental factors

According to Smith (1991) many environmental factors create learning problems in normal, healthy children. Various environmental factors limit a child to reach his/her full potential, such as insufficient stimulation, poor nutrition and a negative emotional climate. A child that is adequately stimulated before the age of five adjusts quicker and achieves better at school than a child that has limited learning opportunities (Myers & Hammill, 1990). According to these authors malnutrition is also an environmental factor that could have an adverse effect on a child's development. If the child has suffered from malnutrition during infancy or early childhood, due to economic, disease related or psychogenic reasons, this condition could cause learning problems. A hungry child is not motivated to put effort into schoolwork. This is the same for children who are sick (allergies or chronic colds). Nutritional deficiencies negatively affect the maturation of the brain; this could cause learning problems.

An adverse emotional climate may also be the cause for learning problems. Atypical emotional climates which are associated with learning problems are family disorganisation, divorce, emotional instability, critical caregiving, parental job/income loss, difficult parental temperaments and behaviours contradictory to school success. These children put so much of their energy into coping with this negative emotional environment, that they struggle to pay attention and compute the mental activities. This environmental stress can trigger emotional states that could contribute to learning problems (Smith, 1991). Cited from Westman (1990):

> A report card on public education is a report card on the nation. Schools can rise no higher than the communities that surround them. It is in the public school that this nation has chosen to pursue enlightened ends for all its people. (p.51)

Socio-environmental factors will be looked at in more detail in three broad categories, socio-economic factors, socio-cultural factors and educational factors.

2.3.3.1 Socio-economic factors

Often in the third world countries, educational problems arise due to poor socio-economic circumstances (Kapp, 1991). The author continues to state that the environment may limit the child and hamper the child's development and learning to an extent that his/her potential cannot fully develop. The child's problems are often associated with the environment or the circumstances in which they grow up. Poor socio-economic circumstances and an environment which is culturally poor and lacks opportunities may hinder the child's development. Children who grow up in poor socio-economic circumstances may not be prepared for school and the consequence thereof is poor school achievement (Kapp). However, some children grow up in good homes, under sound economic circumstances and receive adequate stimulation and still develop learning problems. The exposition may be a simplistic division of possible causes, as it is often impossible to determine the precise cause of a learning problem. From the literature explored it is clear that socio-economic factors may cause or contribute to the development of learning problems. Other factors that fall into the same exposition are socio-cultural factors and educational factors.

2.3.3.2 Socio-cultural factors

Children from cultures that live in a deprived economic environment usually experience language problems, socialising problems or have poor self-concepts (Kapp, 1991). The author continues to state that this could affect the child's potential and may lead to underachievement. According to Myers and Hammill (1990) if a child comes from a cultural environment that does not reflect the same values and attitudes as the school, concerning the importance of education, the child could experience academic problems. If the child's family speaks a language other than the medium of instruction, the child could struggle with scholastic pressure due to language problems. If children are more proficient in their home language than the medium of instruction, they could struggle to adapt in the foundation phase.

2.3.3.3 Educational factors

According to Kapp (1991) the inadequate actualisation of educational structures may be the cause of a child's behavioural and learning problems. The fault does not only lie with the teachers but with the school system as a whole. The curriculum and teaching methods need to be questioned and measured against a comprehensive education system. Teachers often associate with the child in an educationally purposeless manner, concentrating on the child's mental development and neglecting the affective and normative aspects. If a child's educational career has been interrupted for whatever reason and the child was absent from school, this could cause learning problems. Other educational factors that could negatively influence a child's ability to learn is, exposure to instruction by unqualified teachers, physical or psychological abuse by a previous teacher or a history of repeated failures with no educational intervention (Kapp). Cited from Westman (1990):

> An educational system and the society in which it flourishes are reciprocal. You cannot improve a society without changing its education; but you cannot lift the educational system above the level of the society in which it exists. (p.610)

The above cognitive, non-cognitive and socio-environmental factors could be the cause of most learning problems in children. It is essential that the various causes be known, as each child deals with a learning problem differently. Knowledge of the cause of a learning problem could aid the identification and intervention process.

2.4 CONCLUSION

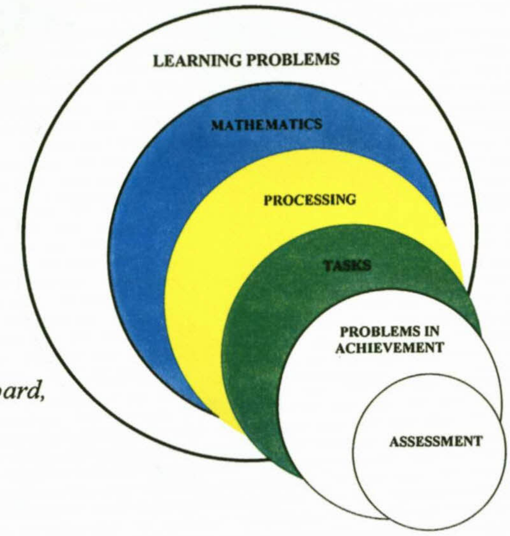
A learning restraint applies to a child who has difficulty acquiring academic skills. A learning disability exists when a child is unable to perform academic tasks in the educational mainstream. According to the DSM IV (1994) a learning disorder exists when a child is functioning significantly below what is expected of him/her according to age, measured intelligence and age appropriate education, with regard to reading, mathematics or written expression. The above three terms are viewed under the umbrella term of learning problems. There is a readiness to blame children, parents, teachers, clinicians and the brain itself as the cause of learning problems. The issue of organic versus psychological is crucial. There is a great need for research to empower the multisystem with information that can help identify and prevent learning problems. There is no place for single factor causation and blame. The barriers to learning at school exist at many levels within and outside the child. We need to adopt a child-centered team involving family, school personnel and professionals in overcoming these learning problems. Training in the multisystem approach with a clinical process could benefit the children immensely. The clinical process must be developmentally based with diagnostic understanding of the individual child and the child's family. The identification and treatment plan must be monitored over time. Cited from Westman (1990):

> We are moving into a new age in which the learning potentials of even the most handicapped individuals are being recognized. We know that children vary in reading and ciphering talents, as they do in musical and athletic aptitude. We can help children who encounter difficulty working in school by identifying their functional and educational disabilities before they become educational handicaps. We have paid too much attention to what children cannot do and not enough to

> what they can do. We must convert their presently insurmountable barriers into surmountable hurdles. (p.729)

In this chapter learning problems have been clearly defined, classified and the manifestations and causes have all been considered. Emphasis needs to shift from general learning problems to one specific learning area, namely, mathematics. What now needs to be explored is the processes through which a child must go to avoid the clutches of a learning problem.

3. DEVELOPMENT OF MATHEMATICAL THINKING IN THE FOUNDATION PHASE



The human mind is a stream running after some half-sensed goal, yet capable of attention, forming objects like an artist and concepts like a geometrician, while the whole organism, acting like a sounding-board, generates the emotions that reason is meant to serve. (Barzun, 1985)

3.1 INTRODUCTION

Mathematics may be seen as a powerful example of the functioning of human intelligence. Mathematics is an adaptable tool that amplifies human intelligence. Learners of any age will not succeed at mathematics unless they are taught to use their intelligence in learning mathematics (Skemp, 1991). For learners to succeed at mathematics, they need to go through various developmental processes. Once mathematical thinking has developed in the child, certain tasks are required of the child in the foundation phase. These tasks are universal, but it is also essential that focus be placed on the tasks that are expected of a child in the Free State. The above themes will be the focus of this chapter. For the reader to understand mathematics, a clear definition of the term is required.

3.2 DEFINITION OF MATHEMATICS

In order to understand the definition of mathematics, it is necessary to understand that mathematics is: an ordered field of knowledge with many branches such as algebra, geometry, trigonometry, statistics and arithmetic; it is a particular way of thinking using inductive and deductive reasoning; it has its own language using mathematical terms and symbols; it is the study of patterns and relationships; and it requires the search for solutions to various approaches (Kapp, 1991).

According to Meuller (1980) mathematics is a logical study of shape, quantity, arrangement and concepts that are all related. This general definition of mathematics can be narrowed to a specific definition provided by the Free State Department of Education (1998):

> Mathematics is the construction of knowledge that deals with qualitative and quantitative relationships in space and time. It is a human activity that deals with patterns, problem-solving and logical thinking in an attempt to understand the world and make use of that understanding. This understanding is expressed, developed and contested through language, symbols and social interaction. Mathematical literacy, mathematics and mathematical sciences as domains of knowledge are significant cultural achievements of humanity. They have both utilitarian and intrinsic value. All people have a right of access to these domains and their benefits. These domains provide powerful numeric, spatial, temporal, symbolic, communicative and other conceptual tools, skills, knowledge, attitudes and values to enable us to analyse, make and justify critical decisions and take transformative action. (p.1)

With mathematics defined, the researcher wishes to investigate the developmental processes necessary for mathematical processing.

3.3 MATHEMATICAL PROCESSING IN THE FOUNDATION PHASE

According to Skemp (1991) mathematical processing involves knowledge, plans and skills. The knowledge necessary for mathematical processing is structured knowledge. Structured knowledge is not just a mere collection of isolated facts but the ability to combine this knowledge with a plan for dealing with the requirements of the situation. Although structured knowledge is the first requirement, the learner needs a plan of action. This plan is what is necessary to reach a goal from a particular starting point. Various plans will be used to get to the same goal and learners need to compare plans in order to select the most advantageous. This is the way we use our intelligence in everyday life.

Plans based on knowledge need skill. Skill is the ability to put plans easily and accurately into action. Children need a firm foundation of structured knowledge, with an adequate repertoire of plans. Children need to practise these plans in frequently encountered tasks, until they become a skill. The author continues to state that the best preparation for a future in mathematics is a combination of knowledge, plans and skills. This together with the enjoyment of the subject and confidence to continue learning it, allows the student to apply what they already know to new situations. The development and phases of mathematical processing all overlap with the theories thereof. Yet, it is important that the reader has the theories as a background before the development and phases of mathematical processing are placed in context.

3.3.1 Theories on mathematical processing

To become competent in mathematics, the child must go through various developmental processes, namely, cognitive, personal and social development. Various theories have been constructed about the above processes and each needs to be explored separately.

3.3.1.1 Theories on cognitive development

The three main theorists who explored concept formation and cognitive development were J. Piaget, L. Vygotsky and J.S. Bruner (Child, 1993). Bruner focused on cognitive development in adults. Focus will therefore be placed on the theories of Piaget and Vygotsky. Let us consider each individually.

Piaget (1969) had three principles at the center of his theory, a genetic one, a maturational one and a hierarchical one. These three principles each have an effect on the manner in which mathematical processing is viewed. The *genetic* principle states that higher processes (like mathematical processes) are seen to evolve from biological mechanisms. These biological mechanisms are rooted in the development of an individual's nervous system. The *maturational* principle states that the process of concept formation follows an invariant pattern through several stages which emerge during specific age ranges (this is discussed in more detail in Chapter 4). Lastly, the *hierarchical* principle of Piaget states that the stages must be experienced and passed through in a given order before any subsequent stages of development are possible (Piaget). What implications does the above have on teaching? According to Child (1993) the first implication is the existence of maturational unfolding of conceptual skills which are linked with certain periods in the lives of learners. Neurological development and a progression of concept forming skills must appear before full intellectual maturation is possible. Teaching in lower grades should begin with concrete considerations. Building up schemata requires practical experience of concrete situations. Explanation should therefore accompany experience. According to this theory, cognitive development is a cumulative process. The hierarchical nature requires the formation of lower-order schemata on which more advanced work is based. Therefore, mathematical acquisition is highly individual and the teacher must use the pattern of development in each child as a means of assessing attainment in respect to the child's progress and mental age group. Piaget's theory therefore postulates that conceptual growth (mathematical acquisition) occurs because the child actively attempts to adapt to the environment and in so doing, organises actions into schemata through the process of assimilation and accommodation (Child).

Vygotsky (1966) arrived at the same conclusions about concept formation as Piaget, isolating only three stages of cognitive development. The first stage is *vague syncretic*, this is when a child at an early stage randomly piles and heaps objects without any recognisable order. This grouping results from trial and error, random arrangement or from the nearness of the objects. The second stage of thinking is called *complexes*. This is where a child groups attributes by criteria, which are not the recognised properties, which could be used for the classification of the concepts. Five sub-stages of this stage are: associative complexes, this is when the child classifies according to one common characteristic; collections, this is when the child classifies according to a group characteristic, for example, a knife, a fork and a spoon are all eating utensils; chain complexes, this is when the child classifies according to more than one common characteristic, for example, where a child picks up all the shapes that look like triangles and then notices that some of them are green and proceeds to pick up only the green triangles; diffuse complexes, where the child is able to classify objects according to characteristics that are not the same, for example the child separates the different shapes and colours and; pseudo-concepts arise when the child perceives superficial similarities based on the physical properties without having grasped the full significance of the concept. The third phase identified by Vygotsky is called the *potential concept stage.* This is when a child can cope with one attribute at a time but is not yet able to manipulate all the attributes at once (add, subtract, divide and multiply). According to Child (1993) when a child can manipulate all the above attributes, maturity in concept attainment is reached.

3.3.1.2 Theories on personal-social development

There are biological, social learning and psychoanalytic theories on personality and affective development. *The Biological Theory* holds that differences in how we feel about ourselves, others and circumstances are due to temperaments that we inherit from our parents. The *Social Learning Theory* holds that personality differences are acquired through a process of modeling and the *Psychoanalytic Theory* holds that personality differences are the result of the complex interplay between maturational forces, cognitive development and experience (Borich & Tombari, 1997). Let us consider each of the above in more detail.

The *Biological Theory* (Borich & Tombari, 1997) refers to three types of traits or temperaments that are inherited from parents. The first temperament is the child's activity level. How energetic or lethargic a child is can depend on the parent's activity level. The second temperament is adaptability. This refers to an individual's ability to adjust to new people and places, or the inability to do so. The last temperament is emotionality. This describes the degree to which individuals become upset, fearful or angry. Temperaments affect not only the way individuals react to their environment but also how the environment reacts to the individuals with these traits.

The *Social Learning Theory* (Bandura, 1977) states that children learn social skills through a fundamental developmental process called modeling. Bandura also emphasized the important developmental tasks that a child must master from infancy to adolescence, which must be acquired through the social learning process. The tasks that a child must master are the ability to establish relationships, to acquire appropriate sex roles, to behave morally and ethically, to learn important expectations and develop a self-concept through perceived self-efficacy.

The *Psychoanalytic Theory* (Erikson, 1965) shares some characteristics with the biological and social learning approaches. Like the biological theory, emphasis is placed on instinctual tendencies. Like the social learning theory, emphasis is also placed on the role of the environment but where the psychoanalytic theory is different is the emphasis that is placed on stages of identity. According to Erikson (1983) the psychoanalytic approach mentions discrete periods of personality development during which the individual confronts an identity crisis which the child must overcome to pass successfully into the next stage. According to the stages of Erikson, children in the foundation phase are going through the phase of industry versus inferiority. School places three important demands on children: the mastering of academic tasks, to get along with others and to follow the rules of the classroom. Children who succeed at these developmental tasks develop a sense of industry. For a child to succeed at mathematics, a sense of industry needs to develop. If children acquire a basic sense of inferiority, then they believe and expect that they can't do anything right. For mathematical acquisition to take place in a child in the foundation phase, competence must be the synthesis of this stage.

With a clear knowledge of concept formation, cognitive and personal-social development, how do the above theories influence the development and phases of mathematical processing?

3.3.2 Development of mathematical processing

Skemp (1991) states that the development of mathematics involves the acquisition of knowledge structures and concepts. The author continues to state that there are primary and secondary concepts and higher and lower order concepts. Primary concepts are those abstracted from sensory experience. Secondary concepts are abstracted from other concepts. Higher and lower order concepts refer to the greater or lesser degrees of abstraction. Each learner has to construct new concepts for themselves. In the foundation phase, the teacher can greatly help learners to construct new concepts by communicating a concept or making a concept available to the learner's mind. These concepts need to become schemas. Schema construction entails three modes of building and testing. The *first mode* involves the building of knowledge from direct experience. The learner tests this knowledge by comparing it to events in the physical world, this is called prediction. The *second mode* is communication. The learner communicates knowledge from the schemas of others and compares his/her schemas with others. This in turn leads to discussion. The *third mode* is from within. The learner forms higher order concepts by the process of imagination, intuition and creativity. This comparison of one's own knowledge and beliefs leads to internal consistency. Learning situations need to be favorable so that learners can construct their own schemas. These schemas need to include methods and materials that will bring into use all three of the above modes (Skemp). Schematically this may be represented as follows:

	Building	Testing
Mode 1	experience	prediction
Mode 2	communication	discussion
Mode 3	creativity	internal consistency

The author continues to state that healthy learning situations provide opportunities where all of the above can be used. Children need to observe, listen, reflect and discuss to increase their experience of mathematics. This experience will lead to knowledge, which in turn will lead to plans. These plans need to be executed often to turn it into a skill.

According to Meuller (1980) there are three main processes in mathematics, *basic processes*, *operation processes* and *transition processes*. *Basic processes* consist of describing, classifying, comparing and ordering. *Operation processes* consist of equalising, joining, separating, combining equivalent disjoint sets and grouping and partitioning into equivalent disjoint sets. *Transition processes* consist of representing and validating. The four *basic operation processes* form the basis from which complex mathematical operations and complex relations evolve. Describing is the process of characterising an object, set or event in terms of attributes. Classifying requires the child to compare how things are alike and from this generalisations can be made. Comparing allows the child to focus on the attribute and decide what is common or different about that attribute. Ordering allows the child to order natural numbers. The five *operation processes* involve equalising, which is a process of making two objects or sets the same on an attribute. Equalising teaches the concept of whole numbers. Joining is the putting together of two objects or sets so that they have a common attribute. Joining introduces addition. Separating takes place when one takes an object, set or a relation apart. Separation introduces subtraction. Combing equivalent disjoint sets in the process of putting together two or more sets that are equal in number to form another set. Joining leads to addition, whereas combining leads to multiplication. Partitioning and grouping into equivalent disjoint sets is the process of arranging a set of objects into equal groups with the possibility of remainders. This is the process of division. The *transition processes* consist of two operations, representing and validating. Representing enables the child to progress from solving problems directly to solving them abstractly. A learner gradually learns to use physical representations, then pictorial representations and then symbolic representations to solve problems. Validating is the process of determining whether a proposed solution is acceptable. A child learns to validate solutions to problems about comparing, ordering, equalising, joining, separating, grouping and partitioning. The above processes each emphasize the same attributes, which are introduced in the foundation phase. These attributes are length, mass, capacity, shape, colour and direction. Children learn to make mathematical processes their own using attributes exhibited by objects, sets and representation. Therefore, attributes become the vehicle though which various mathematical processes are learned.

Mathematics is a symbol system. The power of a symbol is great and must not be taken for granted. It must not be overlooked that a child can learn to speak their mother tongue before the age of five but so many children have difficulty in learning to understand mathematical symbols. Children need to assimilate mathematical symbols into appropriate schemas. Symbols do not exist alone; they form a symbol system that consists of a set of symbols corresponding to a set of concepts. This takes place together with a set of relations between systems corresponding to a set of relations between concepts. Mathematics is the communication of conceptual structures through writing, reading and speaking. Mathematics depends on ideas but access to these ideas and the ability to communicate them depends on mathematical symbols (Skemp, 1991). Symbol systems are surface structures in our minds, conceptual structures are deeper embedded. Mathematics is the process of manipulating these deep mathematical concepts using symbols. In children these concepts do not exist. So they learn to manipulate empty symbols without content. It is therefore important for teachers to use methods that help learners to build up their conceptual structures. This includes sequencing of new material and using structures practical activities with a do and say approach. This should be followed by written work only when the connection between thoughts and symbols are established (Skemp).

3.3.3 Phases of mathematical processing

According to Hammill and Bartel (1990) the five phases of writing, stating, identifying, displaying and manipulating develop during four stages of learning in mathematics. The first stage is the *acquisition stage*, where the teacher wants students to acquire a particular mathematical skill. During this stage the learner writes, states, identifies and displays. In stage two the learner manipulates objects so as to achieve mastery and a high level of accuracy in the various skills acquired. This is the stage of *proficiency*, where the teacher's input decreases and the learner's output increases. Stage three is the *maintenance stage*, where the learner must practice the skills acquired to be able to manipulate them with ease. The fourth stage is the *generalisation phase*. The teacher must provide enough opportunities throughout the school day for the learner to apply his/her newly learnt skill to a wide variety of situations. The learner must apply the knowledge to everyday life by applying the skill to new situations in new contexts

According to Skemp (1991) the phases of mathematical processing involve the construction of mathematical knowledge. The continuity between mathematics and the everyday use of intelligence must be established. Mathematics does not need special mental abilities. Mathematics just requires that a person use their abilities in special ways.

According to Hammill and Bartel (1990) the phases of mathematical processing are an interactive unit between the teacher's input and the child's output. There are five phases in the process and the children must be able to complete each of the five phases on their own. During the first phase the teacher shows the child how to write mathematical symbols. The child then mimics the teacher. In the second phase the teacher states the mathematical concepts and the child must understand and remember how to state them on their own. The third phase comprises the identifying of symbolic options and understanding the meaning of each of them. In the fourth phase the teacher identifies the fixed representations and displays them. The final phase is a combination of writing, stating, identifying and displaying the symbols so that the learner is able to eventually manipulate the symbols.

It is now clear how mathematical thinking develops in the child in the foundation phase and through which processes a child must go to comprehend. How will we know that these processes have developed? Certain tasks give an indication. If a child is able to complete the task at hand then it should be clear that the child is able to comprehend mathematics. The tasks are universal, yet it is important in this study to focus on the tasks that need to be carried out in the foundation phase in our specific context, namely the Free State.

3.4 MATHEMATICAL TASKS DURING CHILDHOOD

Before a mathematics proficiency test can be developed and standardised, a thorough knowledge of what is expected from a child in the foundation phase is needed. The researcher wishes to first consider the mathematical tasks that need to be carried out in the foundation phase from a universal point of view.

3.4.1 Universal mathematical tasks during childhood

According to Westman (1990) the basic mathematical tasks that must be executed with ease after the foundation phase is: the saying, reading and writing of words for numbers; writing and reading the figures for numbers; counting; understanding the relative value of a number compared with other numbers; reading, writing, and understanding the mathematical signs; recognising the arrangement of numbers to do addition, subtraction, multiplication and division; understanding the calculation and placement significance of 0; understanding the placement value of all numbers; doing arithmetic mentally without the use of concrete objects or written material and developing the necessary conditioned responses so that basic mathematical acts become automatic.

According to Wallace, DeWolfe and Herman (1992) eight specific tasks are universal in mathematics in childhood. The tasks include numeration, operations, money, time, measurement, geometry, fractions and word problems. Let us consider each of the tasks individually.

Numeration means the ability to match objects to objects for 1:1 correspondence and use the 1:1 correspondence. A child must also be able to name and use number symbols 0-10 and identify and use the terms "more than" and "less than". Numeration includes counting, naming and writing number symbols to 100. Counting by 2's, 5's and 10's are also included in this task (Wallace et al., 1992).

Operations is the process of joining or separating using addition, multiplication, subtraction or division. The adding and taking away of whole numbers, with the relevant use of language and symbols involved in the addition and subtraction process, also forms part of operations. The use of language and symbols of multiplication and division are also operations. The addition and subtraction of four or more numbers is also a task the child must complete in the foundation phase. The multiplication of a two digit number by a three digit number and the division of a two digit number into a three digit number with a remainder, is the biggest division and multiplication task for a child in the foundation phase (Wallace et al., 1992).

Money includes the labeling of coins and the identification of the value of a coin in the currency best known to the child. A child must be able to count money, determine which coin is worth more or less, change the notes of the currency into coins and label written symbols for money (Wallace et al., 1992).

Time is a task that children seem to struggle with. A child in the foundation phase must be able to identify the hands of a clock. They must also be able to tell time by the hour, half-hour and quarter-hour, tell time in five-minute intervals and tell time using minutes. The learner must be able to use a clock to tell time and be aware of the days of the week, the seasons and the months (Wallace et al., 1992).

Measurement involves several processes. A learner in the foundation phase should be able to measure length using objects and measure lengths using a centimeter ruler or meter ruler. The learner must be aware that liquid can be measured and has a unit known as liters. They must be aware that weight can be measured and has a unit known as kilograms or grams and lastly must be aware that temperature can be measured and represented by a unit know as Celsius (Wallace et al., 1992).

Geometry involves the identifying of shapes, namely, the circle, square, triangle and rectangle. Calculating the perimeter of shapes, the area and the volume can also be expected from the learner (Wallace et al., 1992).

Fractions is another task that learners struggle with throughout their primary school phase. The learner must be able to identify one-half, one-quarter and one-third and identify and use the vocabulary of fractions and symbols of fractions. The learner must be able to write a fraction for a shaded part of a region and write a fraction for a designated number of several items. Adding and subtracting of fractions with like denominators is also a task that must be carried out and finding a fractional part of a number can also be asked (Wallace et al., 1992).

Lastly, *word problems* is another difficult task the child has to become proficient in. The combination of language and mathematics causes endless problems throughout the learner's schooling career. Word problems include the solving of addition and subtraction problems with or without pictures. The learner must be able to specify the appropriate addition or subtraction process to solve the problem and solve multiplication and division word problems. They must also be able to solve two or three step word problems (Wallace et al., 1992).

The above represents eight universal tasks that learners in the foundation phase need to become proficient in. Focus now needs to shift to specific tasks in a specific setting.

3.4.2 Specific mathematical tasks during childhood

The Free State Department of Education (1998) set out expected levels of performance for Mathematical Literacy, Mathematics and Mathematical Sciences in the foundation phase. The expected levels form the components of mathematics in the foundation phase. The ten components are as follows:

- demonstrate understanding about ways of working with numbers;
- manipulate numbers and number patterns in different ways;
- demonstrate an understanding of the historical development of mathematics in various social and cultural contexts;

- critically analyse how numerical relationships are used in social, political and economic relations;
- measure with competence and confidence in a variety of contexts;
- use data from various contexts to make informed judgements;
- describe and represent experiences with shape, space, time and motion using all available senses;
- analyse natural forms, cultural products and processes as representations of shape, space and time;
- use mathematical language to communicate mathematical ideas, concepts and generalisations and thought processes; and
- use of various logical processes to formulate, test and justify conjectures (Free State Department of Education, 1998).

Each of the above ten components will be considered in a bit more detail but a comprehensive curriculum will be attached (Annexure B).

3.4.2.1 Number operations

The development of number concept is an important part of mathematics. All learners have an intuitive understanding of the number concept. This outcome wishes to teach learners to know the history of the development of number, number systems and how to use numbers as part of their tool kit when working with other outcomes. Solving problems, handling information, attitudes and awareness may depend crucially on a confident understanding of the use of number. The assessment criteria under this characteristic are:

- evidence of use of heuristics to understand number concept;
- evidence of knowledge of number history;
- estimation as a skill;
- performance of basic operations;
- knowledge of fractions; and

- solving of real life and simulated problems.

3.4.2.2 Manipulation of numbers and number patterns

Mathematics involves observing, representing and exploring patterns in social and physical environments and within mathematical relationships. Learners have a natural interest in investigating relationships and making connections. Mathematics offers ways of thinking, structuring, organising and making sense of the world. The assessment criteria under this component are:

- identification of the use of numbers for various purposes;
- evidence that number patterns and geometric patterns are recognised and identified using a variety of media;
- completion and generation of patterns; and
- exploration of patterns in abstract and natural contexts using mathematical processes.

3.4.2.3 Historical development of mathematics

Mathematics is a human activity. Many individuals throughout the world have contributed to the development of mathematics. Learners must be able to understand the historical background of the use of mathematics. The assessment criteria under this component is:

- evidence that mathematics is understood as a human activity.

3.4.2.4 Critical analysis of numerical relationships

Mathematics is used as a means of expressing ideas from a wide range of fields. The use of mathematics in these fields often creates problems. This outcome aims to give a critical outlook to enable learners with issues that concern their lives individually, in their communities and beyond, to develop critical thinking about these issues. The assessment criteria under this component are:

- evidence of knowledge of the use of mathematics in the economy;
- evidence of the understanding of budget; and
- demonstrate knowledge of the use of mathematics in determining location.

3.4.2.5 Measurement

Measurement in mathematics is a skill for universal communication. The aim of this outcome is to familiarise learners with appropriate skills of measurement, relevant units used and issues of accuracy. The assessment criteria under this component are:

- evidence of knowledge of the importance of measurement;
- evidence of knowledge of standards;
- evidence of knowledge of concepts used in measurement;
- evidence of knowledge of the concept of time; and
- evidence of knowledge of the concept of temperature.

3.4.2.6 Numerical judgement

In this technological age of rapid information expansion an ever-increasing need exists to understand how information is processed and translated into usable knowledge. Learners should acquire these skills that enable them to handle information and make informed decisions. The assessment criteria under this component are:

- identification of situations for investigation;
- collection of data;
- organisation of data;
- application of statistical tools;
- display of data;
- communication of findings;

- critical evaluation of findings: and
- understanding of the concept of probability.

3.4.2.7 Shape, space, time and motion

Mathematics helps to formalise the ability to grasp, visualise and represent the space in which we live. Space and shape do not exist in isolation from motion and time. Learners should be able to display an understanding of spatial sense and motion in time. The assessment criteria under this component are:

- description of the position of an object in space;
- descriptions of changes in shape of an object;
- descriptions of orientation of an object; and
- demonstrate an understanding of the interconnectedness between shape, space and time.

3.4.2.8 Analysis of natural forms

Mathematical relationships and processes embedded in the natural world and cultural representation is often unrecognised. Learners should be able to understand, critically analyse and make sense of these forms, relationships and processes. The assessment criteria under this component are:

- recognition of natural forms, cultural products and processes and their value;
- representation of natural forms, cultural products and processes in mathematical form;
- generation of ideas through natural forms, cultural products and processes; and
- extensions of natural forms, cultural products and processes in the economy.

3.4.2.9 Mathematical language

Mathematics uses notation symbols, terminology, conventions, models and expression to process and communicate information. The section of mathematics where this language is mostly used is algebra. Learners will be expected to develop the use of this language. The assessment areas of this component are:

- use of language to express mathematical observations;
- use of mathematical notation and symbols;
- use of mathematical conventions and terminology;
- interpretation and analysis of models; and
- representation of real life and simulated situations.

3.4.2.10 ***Formulation of logical processes***

Reasoning is very important in mathematical activity. Active learners question, examine, and experiment. Mathematics should provide opportunities for learners to develop their reasoning skills. Learners need varied experiences to construct arguments in problem settings and to evaluate the arguments of others. The assessment criteria for the above component are:

- evidence of logical reasoning in addressing problems;
- ability to justify familiar and unfamiliar hypotheses; and
- evidence of use of empirical or theoretical rationale in justifying conjectures.

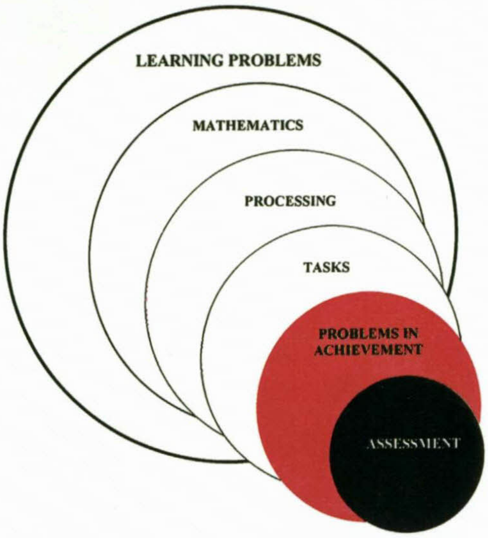
3.5 CONCLUSION

In this chapter, emphasis was placed upon the definition, processing and tasks of mathematics. Mathematics is a term that has a perceived negative connotation. This needs to be broken and in the process reviewed. We need to consider which goals in mathematics can be affected by learning problems. The following is a set of five broad goals for the acquisition of mathematics skills:

- The learner must develop a problem solving ability.
- The understanding of basic mathematical concepts is important.
- The learner must develop the ability to understand and perform measurements of distance, weight, temperature, quantity, area, speed, volume and money.
- The development of the ability to perform mathematical computations using calculators must be taught.
- The recognition of the fact that mathematics is a tool that can serve in daily living must be harbored (Hammill & Bartel, 1990).

Some children understand mathematics quicker and better, others enjoy it more. Some only do mathematics when they have to. Ultimately, if children believe that they can do mathematics, the chances of them doing it with success are far greater. Yet, this is a simplistic opinion of the problem. Problems in mathematical achievement do occur. The classification, manifestation and specific causes of the problems in mathematical achievement will be discussed in the next chapter.

4. PROBLEMS OCCURRING IN MATHEMATICAL ACHIEVEMENT



The true spirit of delight, the exaltation, the sense of being more than Man, which is the touchstone of the highest excellence, is to be found in mathematics as surely as poetry. (Russel, 1961)

4.1 INTRODUCTION

Children learn to count and solve problems by counting. Some children have difficulty in learning to count. This is where mathematical problems begin. If this is not overcome, it can lead to a cycle which could hinder the acquisition of more advanced mathematical skills. It could also hinder linguistic skills, perceptual skills and attention skills (DSM IV, 1994). Some children have problems with the basic operations of counting, adding and subtracting. If the child can't grasp the basic skills on which mathematics builds, the child may start to dislike mathematics because of the lack of success (Dockrell & McShane, 1993). Some children cannot learn the rules for manipulating numbers. Some children cannot relate mathematics to the real world. The first task in helping a child who is struggling with mathematics is to identify the problem area. The mathematics proficiency questionnaire wishes to fulfil this goal. Next would be to discover which set of principles the child is using. Thirdly an intervention plan must be set out to help the child deal with his/her backlog.

Similar to the look and say method of teaching, Westman (1990) states that the New Maths method of teaching mathematics can have an adverse impact on many children. A teacher can neglect the basic principles of learning mathematics because the method does not teach practical computational skills. A child's ability to learn and perform mathematics is complicated by the process of memorisation. Children who solve problems in this way experience anxiety when confronted with new levels of mathematical operations. There is also evidence of a spatial type of development dyscalculia exhibited by vertical and horizontal spatial confusion in addition and subtraction. Developmental anarithmetria refers to mixing procedures involving addition, subtraction, multiplication and confusion in carrying out the operations of written mathematics. Children with attentional sequential dyscalculia add and subtract inaccurately by omitting figures in adding, forgetting to carry figures, and omitting decimal points and signs (Westman).

Before we can discuss how mathematical problems manifest, we need a clear classification of what a learning problem in mathematics entails.

4.2 CLASSIFICATION OF PROBLEMS OCCURRING IN MATHEMATICS

Just as general learning problems can be classified according to learning restraints, learning disabilities and learning disorders, so mathematics problems can also be subdivided into each of the above. A *learning restraint* in mathematics occurs when certain factors cause a child not to actualise his/her mathematical potential. A mathematical *learning disability* occurs when a child has an identifiably deficiency in his/her given mathematical potential. When a mathematical learning restraint or disability becomes significant, a diagnosis of *Mathematics Disorder* can be made. The prevalence of Mathematics Disorder is estimated at 1 % of school-age children (DSM IV, 1994). According to the DSM IV a number of different skills can be impaired in Mathematics Disorder, these include: linguistic skills, which refer to the understanding or naming of mathematical terms, operations, concepts and the decoding of written problems into mathematical symbols; perceptual skills refer to the recognition or reading of numerical symbols and clustering objects into groups; attention skills refer to the copying of numbers and figures correctly, remembering to add and observing operational signs; and mathematical skills refer to the following of steps in mathematical sequences, counting of objects and learning multiplication tables. The diagnostic criteria for Mathematics Disorder are as follows:

A. Mathematical ability, as measured by individually administered standardized tests, is substantially below that expected given the person's chronological age, measured intelligence, and age-appropriate education.

B. The disturbance in Criterion A significantly interferes with academic achievement or activities of daily living that require mathematical ability.

C. If a sensory deficit is present, the difficulties in mathematical ability are in excess of those usually associated with it (DMS IV, 1994, p.51).

With a mathematics restraint, disability and disorder clearly defined and classified, we can now consider how mathematical problems manifest in children in the foundation phase.

4.3 MANIFESTATION OF PROBLEMS OCCURRING IN MATHEMATICS

Children experience various problems in mathematics. Mathematical problems manifest in the inability to recognise or read numerical symbols or signs and the inability to cluster objects into groups, here perceptual skills are not yet developed. The copying of numbers or figures incorrectly, the observing of operational signs incorrectly and the remembering to add in "carried" numbers are all manifestations of the problem where attention skills are lacking. Furthermore, the following of mathematical steps, counting objects and learning multiplication tables are an indication of mathematical skills that are not yet developed (DSM IV, 1994). The problem may manifest in the understanding of number, quantity, counting, arithmetic, addition, subtraction, word problems, division, multiplication or fractions. According to Kapp (1991) the most common errors in the foundation phase are errors in addition, subtraction, multiplication, division and common fractions. These errors will be discussed below.

Errors in *addition* include a lack of knowledge of basic combinations of adding, errors in carrying over, errors relating to sums containing zero and adding of tens, hundreds and thousands.

Errors in *subtraction* include a lack of knowledge of basic combinations of subtracting, decomposition of numbers, errors relating to sums containing zero, incorrect methods of subtracting and a confusion of when to subtract and when to add.

Errors in *multiplication* include a lack of knowledge of tables for multiplication, errors in 'carrying over', errors relating to multiplication with zero, incorrect methods of multiplication, confusion of multiplication with other operations (such as addition) and a lack of arrangement of products for long multiplication.

The most common errors in *division* include a lack of knowledge of tables, decomposition errors, errors relating to zero, incorrect methods of division and the inability to divide the divisor into the number.

Children in the foundation phase struggle with *fractions* because the division and multiplication rule is often incorrectly applied. Children find it difficult to simplify fractions or to find a common denominator, so that the fractions can be added or subtracted. Errors are made with mixed fractions, improper fractions and proper fractions where the numerator and the denominator are not multiplied or divided by the same number.

Kapp continues to state that *other mathematical errors* occur when a child finds it difficult to determine whether his/her answer is correct. Rounding off of numbers to the nearest ten, hundred or thousand is often too abstract for some learners. The selection of relevant information from word sums and the formulation of a number sentence are also major problems for most learners. The place value principle in the naming of numbers is often not completely understood and therefore the transfer and decomposition of numbers becomes a problem.

4.4 GENERAL CAUSES OF PROBLEMS OCCURRING IN MATHEMATICS

Besides the general cognitive, non-cognitive and socio-environmental causes of learning problems in general, there are also specific factors that contribute to learners experiencing problems in mathematics. These specific factors are indicated in figure 4.1 and are the three most common sources of mathematical errors. According to Cockburn (1999) these three sources of mathematical errors include the child, the teacher and the task. It is important to note that the sources are not mutually exclusive and we cannot predict how the various factors interact.

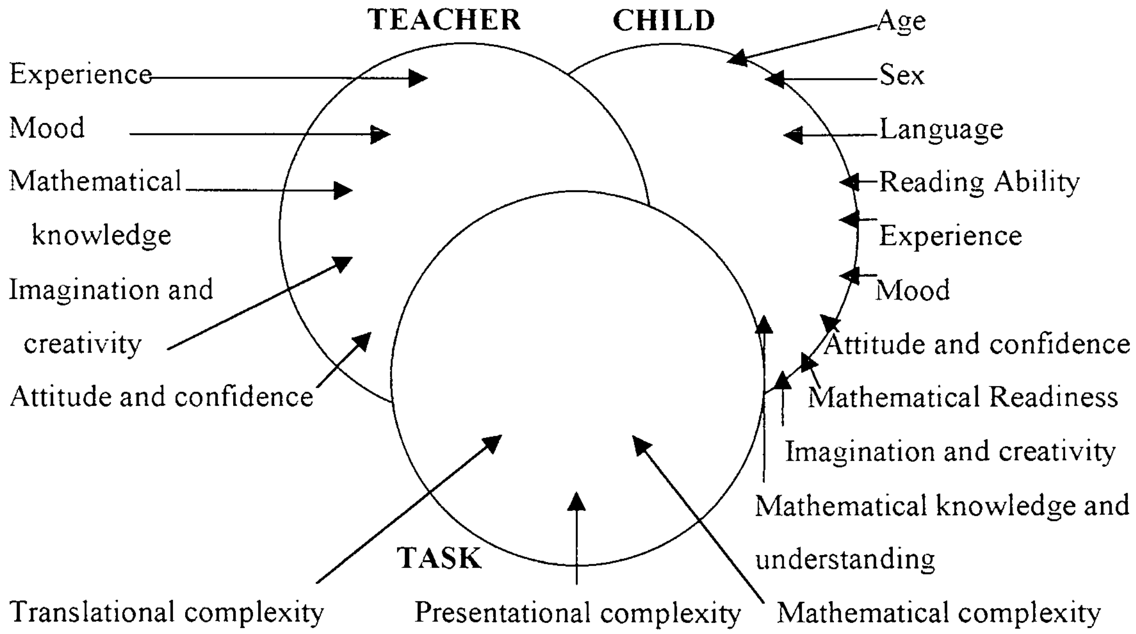


Figure 4.1 Sources of problems in mathematics (Adapted from Cockburn, 1999, p.8)

4.4.1 Child

According to Geary, Bow-Thomas, Liu and Siegler (1996) a complex mixture of cultural and biological factors influence children's mathematical development. Different cultural and maturational influences affect different aspects of children's developing mathematical competencies. The child's age, sex, language, experience, mathematical readiness, knowledge, understanding, imagination, creativity, mood, reading ability, attitude and confidence all contribute to his/her mathematical achievement. We will investigate how each of the above can cause problems in mathematics.

4.4.1.1 Age

Problems in mathematics can be influenced by age. According to Schminke, Maertens and Arnold (1978) it is important that the teacher understands how a child grows in ability to encounter and understand mathematics and how the educational and environmental climate affects mathematical growth. Although symptoms of problems in mathematics may occur as early as the foundation phase or grade one, Mathematics Disorder is seldom diagnosed before the end of the first grade. Cone, Wilson, McDonald, Bradley & Reese in Anderman (1998) found that the discrepancy between IQ and achievement for students with learning disabilities increased with age. Mathematical problems can be dependent on the cognitive quantitative growth of the child from a developmental point of view. We have to revert to the Swiss psychologist Jean Piaget's developmental theory. Children learn mathematical concepts according to Piaget's levels of intellectual development. Accordingly, children experience problems according to these levels. Children cannot absorb information that they are unable to understand. Piaget (1969) states that there are four stages of development. The *sensorimotor stage*, takes place from birth to two years of age. The *preoperational stage*, takes place from two years to seven years. The *concrete operational stage*, takes place from seven years to eleven years and the *formal operational stage*, takes place from adolescence through to adulthood. In the foundation phase we only have to do with two of the phases of development, the preoperational stage and the concrete operational stage. In Chapter 3, certain mathematical tasks were mentioned that must be achieved in the above phases of development.

According to Souviney (1989) the preoperational stage is characterised by the development of symbolic thinking. Assimilation and accommodation are two capabilities that the child must develop. For example, a child must be able to understand that five is a particular quantity, a quantity gained from experience and which exists independently of the operations that may be performed with it. Reversibility is the ability to reverse actions mentally. For example, 4 + 3 = 7 can be reversed, 7 – 3 = 4. Juxtaposition is where the child prefers factual descriptions to cause and effect explanations. These limitations render the child incapable of making a coherent whole out of the explanation. At the age of seven or eight, the child begins to reflect, unify and contradict. At this time the child becomes conscious of the logic of actions. The teacher must observe when a child appears to appreciate temporal, cause-effect and logical relationships. To push ahead prior to such understanding only invites associative (rote) learning (Schmike et al., 1978).

In the concrete operational stage there is a significant increase in a child's ability to carry out mental actions related to physical objects and actual events. The child is not able to conserve quantity, length, mass and volume. The ability to group objects according to attributes, or classification, and order objects according to size, or seriation, improves drastically during this period (Souviney, 1989). Geometric and measurement topics are introduced, as are the concepts of number, place value, regrouping, and operations. Most elementary mathematics depends on the mental operations that develop during the concrete operational stage. If these mental operations do not develop the child will not be able to proceed from elementary mathematics to more advanced levels of the subject.

Third grade appears to be the transitional phase in which children make the transition from counting mathematics facts to recalling number facts automatically (Smith, 1991). The author continues to state that half of the third grade class will still count, while the other half will retrieve knowledge from their memory. By grade four, children automatically retrieve mathematical knowledge.

4.4.1.2 Sex

According to Mussen et al. (1990) males and females, on average, have different patterns of performance in different intellectual areas. According to the DSM IV (1994) 60-80 % of children diagnosed with Reading Disorder are males. Referral may be biased towards males because they display disruptive behaviour which is in association with learning disorders. When careful diagnosis and stringent criteria is used, the prevalence is equal in males and females. In the foundation phase, girls perform better than boys in verbal ability, reading, verbal fluency and verbal comprehension (Mussen et al., 1990). The authors continue to state that girls and boys perform equally well in mathematics, but males begin to do better, on average, in high school. Boys are better, on average, in tasks that require visual-spatial reasoning, girls tend to be more proficient than the boys in computation and recall aspects of mathematics. Boys are superior with regard to complex mathematics and analytical reasoning, such as word problems that involve multiple steps. According to Smith (1991) male superiority in mathematical reasoning is due to society's gender-stereotyped beliefs. These beliefs are on the decline but many tertiary mathematics courses have a greater male enrollment. According to Visser (1985) there are no sex differences with regard to learner's general mathematical ability during early adolescence. Sex differences do begin to occur at a later stage in favour of the male learners.

4.4.1.3 Language

According to Geary et al. (1996) there appears to be at least two ways in which language might influence mathematical development. The first is the influence of the structure of number words on emerging numerical competencies and the second way is the speed with which basic number names (e.g., one, two, three) can be pronounced. The speed of the number pronunciation influences the number of digits that can be retained in working memory. This may influence the strategies that children can use to solve simple mathematical problems. According to Smith (1991) language skills are highly related to mathematics success because mathematics symbols are just another way of recording numerical language concepts. Word problems need to be read and understood. Mathematics operations use high-level reasoning that depends on a good deal of language flexibility. The greater the language complexities of the problem, the more difficult mathematics solutions become for learners with learning problems. Children enter school with a quantitative reasoning ability. Often transition to working with numerical symbols is believed to suppress the reasoning or disrupt the mathematics acquisition process. As was stated in paragraph 1.1, according to Esterhuyse and Beukes (1997) there is a high positive correlation between the ability to read and spell (language proficiency) and mathematics performance. Geary et al. (1996) suggest that age, schooling and language can differentially affect the emergence of different components of children's early numerical competencies.

4.4.1.4 Experience

Most children in South Africa are exposed to diverse environments. A mistake a teacher can make is to assume that all her learners have common experiences. To demonstrate fractions using a cake, for example, can cause a problem to the child who has never seen or eaten a cake before. Teachers too often make the assumption that all learners use money in the same manner. In more recent times, due to safety reasons, children are no longer allowed to venture alone into shopping centers. Therefore experience of money and basic mathematical calculations are no longer experienced at a young age (Cockburn, 1999).

4.4.1.5 Mathematical readiness

According to Kapp (1991) children need to acquire certain mathematical skills by progressing through certain consecutive developmental stages. If they do not master a certain stage, mathematical problems will arise in more advanced stages. Classification of objects is an important skill that will help a child classify visually on the basis of differences and similarities. Determining a one-to-one relationship between elements of two sets is a prerequisite for the classification of sets and the establishment of number concepts. Seriation is of importance for number comprehension and number operations. Flexibility and reversibility of thought are preconditions for mathematical readiness. Reversibility of thought plays an important role in mathematics, with regard to inverse operations such as subtraction and division. Piaget (1969) showed that this principle is discovered by the child and mastered on different levels, varying from total lack of insight to total insight on the abstract level. Inadequate mathematical readiness can lead to serious problems in mathematics.

4.4.1.6 Mathematical knowledge and understanding

In the mathematics syllabus the same operations appear in successive years. The mastery of this knowledge and the understanding of the operations are essential prerequisites for the mastery of these operations on a higher level. If there is no continuity in the teaching, be it due to the absence from school or changing of schools, the child may experience problems due to inadequate prior knowledge of the work (Kapp, 1991). A child may also struggle to understand the work due to the language used in mathematics. It is essential that the teacher and child discuss the meaning of various interpretations of words and phrases so that the child can express him/herself more coherently (Cockburn, 1999). Difficulties in abstract or symbolic thinking will also interfere with the learner's ability to conceptualise the relationship between numbers and objects. This difficulty can limit the child's knowledge and understanding of mathematics (Hammill & Bartel, 1990).

There are three kinds of cognitive factors that influence mathematical achievement: the ability to remember arbitrary associations; the ability to understand basic relationships; and the ability to make lower-level generalizations (Hammill & Bartel, 1990). According to these authors there are six levels where cognitive factors have a relationship with mathematics understanding. The first level is the *arbitrary association* between mathematical symbols, digits and words. The second level is the *basic relationship* between one-on-one correspondence, sequencing, succession and topological relationships. The third level is the *lower-level generalisations*, where the child has to sort, correspond, estimate, understand the concept of greater and less than, create equivalent sets and understand seriation. The fourth level of cognitive processes is the *understanding of basic concepts* such as number, shape, colour, weight, time and age. The fifth level is the *high-level relationships* where mathematical language, equivalence relationships, number operations, set operations, transformations, cause-effect, inequalities and conservation need to be understood. The highest level of cognitive ability is the *higher-level generalisations*, where the child must be able to understand place value, axioms, basic facts, probability, statistics, graphs and geometrics. The child must also compute word problems and be able to apply the mathematics daily.

4.4.1.7 Imagination and creativity

In the new mathematics syllabus, emphasis is placed on allowing the child to use his/her initiative in solving a problem. Children use their imagination and creativity to firstly understand and secondly solve the problem at hand. This changes the way we view children in totality. Cockburn (1999) states:

> ...although mathematical processes are to be valued, mathematical products are too often what it is all about: finding the right answer, by whatever means, is considered important. As busy teachers we may not always be tuned in to thinking about how a pupil's imagination or creativity might have contributed to a wrong answer mathematically but a perfectly logical response in everyday terms. (p.11)

Children are no longer just sponges, soaking up information. Children must become excited about information and want to soak up as much of it as possible. The information they soak up must be information they have discovered for themselves using their imagination and creativity.

4.4.1.8 Mood

Mood can influence a child's performance. Motivation is influenced by a child's mood. If children are not in the mood to learn, their performance will not be a true reflection of their ability. Tiredness also affects a child's mood, which can result in careless mistakes while completing a mathematics task (Cockburn, 1999). Teachers must never forget that children also have emotions and feelings that need to be dealt with. They also have mood changes, which need to be taken as a factor that could influence a child's mathematical achievement.

4.4.1.9 Attitude and confidence

Poor attitude or anxiety about mathematics may inhibit the performance of some learners (Hammill & Bartel, 1990). Many children are told from a young age that mathematics is the most difficult subject they will encounter at school. They may therefore find it difficult to experiment or participate due to fear of failure. The authors continue to state that when a child does not comprehend a section of mathematics they often become fearful of asking a question. Instead of asking they remain silent and the teacher misjudges this silence as confidence. This misjudged level of confidence can influence performance. The authors continue to state that children need to believe that they can do mathematics before it can become a reality. Once a child believes that they are capable, they try harder. This extra effort usually leads to better results, which boosts the learner's self-confidence. A single boost of this nature, results in the learners believing in themselves. The cycle of inability is replaced with a cycle of mathematical achievement. According to Visser (1985) there is a strong relationship between self-concept and self-confidence regarding mathematical ability. Visser continues to state that self-confidence in mathematical ability and the enjoyment of the subject is a predictor of mathematical achievement.

4.4.1.10 Reading ability

Language comprehension and vocabulary can affect a child's mathematical performance. Extensive research shows a significant overlap between reading, achievement and behavioural problems in children (Nigg et al., 1999). A child with a reading problem does well in a mathematics test that does not require a high degree of reading skill, but performs poorly when it comes to problems that are formulated in words (Cockburn, 1999). As was stated in paragraph 1.1, Esterhuyse (1997) found a high positive correlation between the English speaking learners reading and spelling marks and their mathematical achievement.

4.4.2 The teacher

According to Skemp (1991) mathematics is a powerful and concentrated example of the functioning of human intelligence. Learners of any age will not succeed at mathematics unless they are taught in ways which enable them to bring their intelligence, rather than rote learning, into use for learning mathematics. Bear et al. (1998) assessed the self-perceptions of teacher feedback, social comparison of reading competence, reading satisfaction and general self-worth of third and sixth grade children with learning disabilities. As predicted, the teacher's feedback was the most common criterion children used to judge their academic performance. The teacher's experience, mathematical knowledge, imagination, creativity, mood, attitude and confidence can all have an influence on the child's mathematical achievement. According to Hammill & Bartel (1990) ineffective instruction accounts for more problems in mathematics than any other factor. We will discuss how each of the above can cause problems in mathematics performance.

4.4.2.1 Experience

Teachers not only acquire information about children's errors over the years, they also become aware of the behaviour patterns of children who struggle with mathematics. This information can help teachers predict possible mathematical errors and this can help them minimise future problems (Cockburn, 1999).

4.4.2.2 Mathematical knowledge

Kapp (1991) states that deficiencies in the teaching of mathematics should be considered as a cause of learning problems. Unlike languages and the social sciences, where subject knowledge can be obtained in informal ways, mathematics is a subject where the child is directly dependent on the teacher's mathematical knowledge. Before the child can use his/her insight to solve a problem, the teacher must have mastered the concepts, methods and manner of reasoning. According to Kapp:

> If the teacher himself does not possess the required subject knowledge and subject didactical expertise and proficiency, he will not be able to present the mathematics lessons with the necessary interest, dedication and enthusiasm, and will also not be able to anticipate learning problems that may develop. (p.101)

According to Cockburn (1999) a teacher with too much or too little knowledge can affect a child's mathematical achievement. The author continues to state that teachers with too much knowledge often take it for granted that children will grasp a simple concept before they actually do. They quickly explain something in great detail and bombard the child with intellectual detail. Teachers with too little knowledge often only teach the basic concepts and when the children are ready to absorb more abstract information the teacher is unable to fill this void. This inability is usually due to a lack of knowledge (Cockburn).

4.4.2.3 Imagination and creativity

A creative and imaginative teacher is able to fit into a learner's environment. In our diverse society it is of crucial importance that the teacher explains the work in terms and with examples that the children in the class will understand. The children should not have to fit into the teacher's frame of reference, the teacher must fit into theirs. A creative and imaginative teacher can also easily identify when something has gone wrong or when children are unable to understand. The teacher then reflects on the situation and changes their manner of teaching to do something about the problem (Cockburn, 1999). An imaginative or creative teacher will explain subtraction in terms of apples that are eaten. For example, I have six apples, if I eat two, how many apples will I have left? From this the problem will be presented as six minus two. An imaginative and creative teacher will also bring six apples to class and ask two of her children to eat an apple. This visual stimulation also makes the mathematics more concrete.

4.4.2.4 Mood

Teachers are also human but a teacher's mood can adversely affect a child's achievement (Cockburn, 1999). For example, if a teacher is in a bad mood and on the day that the concept of division is being explained, the child will be hesitant to ask questions for fear that the teacher will react negatively. This division concept might never be understood and the consequences of this lack of knowledge will affect the learner until the concept is re-explained in terms that he/she understands. With curriculums that are very complex and classes that are growing in number, the teacher is under enormous pressure. This could affect the speed and depth of explanations. These external demands on teachers can increase problems in mathematics.

4.4.2.5 Attitude and confidence

Just as a child's attitude and confidence can affect mathematical performance, so too can a teacher's. Bear et al. (1998) suggest that teachers can have a significant impact on students' attitude. The authors continue to state that in their study teacher feedback was the most powerful predictor of satisfaction among learners with respect to their academic progress. Cockburn (1999) states that a teacher that dislikes a subject will, without consciously being aware of it, influence a child's attitude towards a subject. The 'self-fulfilling prophecy' should not be overlooked in teaching. If a bright child is labeled as incompetent, this label will affect the child's performance, confidence and attitude. Teachers sometimes view certain children in a more favourable light. This creates a situation where erroneous expectations can lead to behaviour that causes expectations to come true.

4.4.3 The task

The manner in which the child views the mathematical task can lead to learning errors. The child's perspective on the task is dependent on its mathematical complexity, presentational complexity and its translational complexity (Cockburn, 1999). We will investigate how each of the above can influence a child's mathematical achievement.

4.4.3.1 Mathematical complexity

According to Cockburn (1999) if a task is too difficult for a child, the child will experience problems. Cockburn states:

> If I had to reduce all educational psychology to just one principle I would say this: the most important single factor influencing learning is what the child already knows. Ascertain this and teach him [her] accordingly. (p.13)

In mathematics this simple principle must be applied. If a child struggles with mathematics, check the mathematical complexity of the task in relation to the child's knowledge.

4.4.3.2 Presentational complexity

Mathematics is a complex subject that needs to be presented in simple terms. The basic concepts need to be taught and once the student has grasped these concepts, then the beauty of the application of mathematics can be discussed. Errors will occur in mathematics if the presentation of a task is too complex or inappropriate. This can interfere with the child's ability to overcome problems in mathematics (Cockburn, 1999).

4.4.3.3 Translational complexity

Translational complexity involves the children's ability to translate a task in the manner intended, in other words, to make the subject matter their own. According to Cockburn (1999) there are five main errors that hinder a child's translational complexity. The five aspects are; *translating the mathematics*, *reading errors*, *comprehension errors*, *encoding errors* and *implication errors*. Let us consider each of the above individually.

Translating the mathematics is usually required when a child is presented with a word problem. The child needs to know what is required of him/her before executing the task. The child's mathematical ability is not necessarily tested but rather the child's ability to translate the mathematics. Translation problems also arise when a child has been computing addition sums and suddenly he/she is faced with a subtraction sum. The child struggles to translate what is expected at that moment.

Reading errors often occur in the foundation phase. A child is not only in the process of deciphering new mathematics symbols but he/she is also learning to read for the first time. A child's reading ability will adversely affect his/her mathematics performance as a lot of the work is questioned with word sums. In South Africa it is also common that a learner will be placed into a school with a medium of instruction other than the mother tongue. The inability to read is also influenced by the child's language vocabulary and comprehension.

Comprehension errors occur when the child has difficulty understanding the written aspects of the problem. Translational errors often overlap with comprehension errors but not all translational errors are comprehension errors. In translational errors the child understands the word but does not know how to apply it. In comprehension errors the child does not understand the meaning of the word.

Encoding errors occur when the child understands the mathematics at hand and can compute the sum, but writes down the incorrect answer. For example, a child will write 19 for 91 or 6 rather than R6. This is a common error in the foundation phase, as the children are still learning to grasp and write all the new symbols that they are taught.

Implication errors are when the child does not 'play the game' in mathematics. For example, if you have sixteen sweets and you share them with three of your friends, how many sweets will each child get? The child could answer that he will keep thirteen sweets and only give one sweet to each friend. This is an example of an implication error. An implication error can arise due to one of the above difficulties or the child just does not appreciate the reality of the problem at hand.

4.5 ASSESSMENT OF PROBLEMS OCCURRING IN MATHEMATICS

According to Myers and Hammill (1990) individuals who would assess mathematical abilities should consider four skills, namely, *prerequisite abilities*, *computation*, *reasoning and geometry*. *Prerequisite skills* are the Piagetian concepts of conservation, constancy and permanence. *Computation skills* refer to the ability to add, subtract, multiply and divide. *Reasoning* is mostly associated with word problems and *geometry* deals with lines, angels, surfaces and their relationships to each other. According to McLoughlin and Lewis (1994) there are two types of assessment, informal assessment and test-based assessment. Informal assessment refers to the measurement of cognitive abilities that underlie mathematics proficiency. The authors continue to state that the manner in which these abilities are assessed is through a series of activities. Activities include classification by means of function, colour, size, shape and units. The assessment is very practical and active. Informal assessments can also be done with teacher checklists, informal inventories, error analysis (writing a class test, checking homework) and conducting interviews. The test-based assessment measures the child's level of mathematical ability. Various American tests, with American norms, have been developed to measure arithmetic abilities, namely, Diagnostic Test of Arithmetic, Fountain Valley Teacher Support System in Mathematics, Hudson Education Skills Inventory-Math, KeyMath-Revised, Stanford Diagnostic Mathematics Test, Test of Early Mathematics Ability and the Test of Mathematical Abilities (Myers & Hammill, 1990). Each of these tests measures one or more of the four mathematical skills mentioned above. The age range for these tests is four to eighteen years.

According to Myers and Hammill (1990) six basic levels need to be assessed. These levels provide general guidelines for educational planning. The levels are *attention, response, order, exploratory, social* and *mastery*. The *attention level* is concerned with children making contact with the environment; the *response level* is concerned with them becoming motorically and verbally active; the *order level* teaches the children routine; the *exploratory level* allows them to explore their environment; the *social level* allows them to handle approval and disapproval from others; and the *mastery level* helps them learn skills related to self-care, academics and vocational pursuits.

In this research a test-based assessment was developed that should help identify a learner, in the foundation phase, with a mathematics problem. The assessment method is a norm-referenced test that will be useful in helping to confirm suspicion that a child is currently performing below the norm. Various questions had to be asked while conducting the research and reference had to be made to 'The assessment question model' (McLoughlin & Lewis, 1994). These statements were continual yardsticks by which the test-based assessment had to be measured:

- The researcher wished to determine whether there was a school performance problem and if it was related to a disorder.
- The status of classroom behaviour, social-emotional development and the student's educational needs was also considered.
- The relationship between a learning problem and general classroom success was taken into account.
- All learners are influenced by social and cultural factors, therefore the way in which these factors influence educational goals were looked at.
- The learner is the priority and therefore the least restrictive and most appropriate educational assessment had to be considered.
- The most important issue the researcher had to constantly consider was the effectiveness of the intervention.

4.6 INTERVENTION

The most important question in intervention is, 'what needs to be addressed?' According to Dockrell and McShane (1993) the test-teach-test cycle is very useful when intervention into a mathematical problem is required. Assessment forms the first step in the test-teach-test process. Learning problems will come to light in learning situations. The teacher needs to identify the intensity and degree of the problem. If the problems are not too serious and necessary stimulation will decrease the backlog, then the teacher is able to compile an intervention program on his/her own. If the problem is serious, the teacher needs to refer the child to a psychologist so that a formal assessment can be conducted. The cycle will once again begin with the identification of the problem. A diagnosis then needs to be made and an intervention program laid out and discussed. The program must be monitored and an evaluation of the child's skill development and improvement must be conducted. The intervention can be targeted at the child, the teacher or the environment. The level of intervention with the child will first be to determine the child's level of mathematical proficiency and then to concentrate on the knowledge the child lacks so that new mathematical strategies can be adopted. The level of intervention with the teacher will concentrate on the task. The level of intervention concerning the environment will focus on the classroom organisation and parental involvement. The ideal would be to focus on all three facets.

According to Myers and Hammill (1990) intervention should focus on the following 21 facts:

1. encourage self-appraisal by the child;
2. gear instruction to underlying concepts and procedures the child is accustomed to;
3. make sure the child has the goals of instruction clearly in mind;
4. protect and strengthen the child's self-image;
5. personalise corrective instruction;
6. base corrective instruction on your diagnosis;
7. structure instruction in a sequence of small steps;
8. choose instructional procedures that differ from the way the child was previously taught;
9. use a variety of instructional procedures and activities;
10. encourage the child to use aids as long as they are of value;
11. let the child choose from materials available;
12. have the child explain his/her use of materials;
13. let the child state his/her understanding of a concept in his/her own language;
14. move toward symbols gradually;
15. emphasize ideas that help the child organise what he/she learns;
16. stress the ability to estimate;
17. emphasize proper alignment of symbols and digits;
18. make sure the child understands the process before assigning practice;
19. select practice activities which provide immediate confirmation;
20. spread practice time over several short periods; and
21. provide the child with a means to observe any progress.

In Piaget's (1969) view, the goal of education is to encourage intellectual autonomy. Children's arguments and exchanges of views are important learning experiences. The teacher must encourage the children to think in their own way rather than to recite correct answers. Children acquire logical thinking in their everyday activities. Mathematical knowledge is constructed in a manner that each child invents addition, subtraction, multiplication and division. Social interaction is important to children's thinking and construction of knowledge. Everyday life experiences and group games should replace the old worksheet. Mathematics should not just be work for children but they should experience it as being fun (Mussen et al., 1990). Cited from Westman (1990):

> The whole aim of good teaching is to turn the young learner, by nature a little copycat, into an independent, self-propelling creature, who cannot merely learn but study... This is to turn pupils into students... (p.297)

4.7 CONCLUSION

In the above chapter the researcher focused on problems occurring in mathematical achievement. The manifestations, classification and general causes of learning problems in mathematics were considered. Much attention was centered on the child, the teacher and the task. Mathematics is a complex body of knowledge, but its hierarchy is still developmental. Mathematical thinking begins in the preschool years. In the foundation phase the written language of mathematics is taught and built on a conceptual framework. Elementary skills taught, in the foundation phase, are prerequisite to the higher mathematics of the secondary grades. One of the primary aims of the foundation phase is the development of proficiency in mathematical thinking and computation. When learners fail to meet the expectations of the curriculum, mathematics becomes a major assessment concern. Teachers must continually monitor the learner's progress in mathematics. According to McLoughlin and Lewis (1994) there are general cognitive and non-cognitive mathematics goals for all learners in the foundation phase:

Non-cognitive goals include:

- they must learn to value mathematics;
- they must become confident in their ability to do mathematics.

Cognitive goals include:

- they must become mathematical problem solvers;
- learn to communicate mathematically; and
- learn to reason mathematically.

When an educationist identifies a learner who is unable to achieve the above goals, the learner must be referred to a psychologist for an assessment. The aim of this research is to develop and standardise a mathematics proficiency test that could serve as an assessment tool, by helping to identify learners in the foundation phase with problems in mathematical achievement. For a test to be developed and standardised, certain empirical procedures need to be carried out to make the test reliable and valid. The one aspect that needs to be considered is the standardisation of a psychometric test. This will be discussed in more detail in Chapter 5.

5. STANDARDISATION OF PSYCHOMETRIC TESTS

For the researcher to be able to standardise a test, psychological testing first has to take place. Psychological testing has been an issue of contention for several years. Does the outcome of a psychological test give the true reflection of what is being tested? Cited from Mehrens and Lehmann (1991) are two contradictory quotes about psychological testing. The positive quote about psychological testing is reflected below:

> Educational and psychological testing represents one of the most important contributors of behavioural science to our society. It has provided fundamental and significant improvements over previous practices in industry, government, and education. It has provided a tool for broader and more equitable access to education and employment.... The proper use of well-constructed and validated tests provides a better basis for making some important decisions about individuals and programs than would otherwise be available. (p.3)

The opposite opinion is reflected below:

> I feel emotionally toward the testing industry as I would toward any other merchant of death. I feel that way because of what they do to the kids. I'm not saying they murder every child – only 20 percent of them. Testing has distorted their ambitions, distorted their careers. (p.11)

Due to the fact that the researcher wishes to develop and standardise a psychometric test, the positive and negative influences of psychological testing should never be overlooked.

5.1 INTRODUCTION

In the previous chapter, the topic of assessment and intervention came to the fore. The question now is how can assessment be done in the foundation phase? This is when it is necessary to utilise psychometric tests to quantify a learner's ability with regard to a specific construct. The standardisation of psychometric tests is essential. If different individuals administer a test on various occasions, the test norms must enable these individuals to arrive at the same interpretation given the same raw score. A test is only as good as the person administering it, yet the fact that a test is standardised allows the error of interpretation to be that much narrower (Huysamen, 1983). It is important that a test only be used if it is going to aid your decision and add information to your conclusions. According to Mehrens and Lehmann (1991):

> If knowledge of a test result does not enable one to make a better decision than the best decision that could be made without the use of the test, then the test serves no useful purpose and might just as well not be given. (p.291)

In this chapter, standardisation, reliability and validity of a test is going to be discussed. Before these aspects can be looked at in detail it is important to understand the concept of measurement.

5.2 MEASUREMENT

The word measurement, testing, assessment and evaluation are often used interchangeably. Testing refers to a set of questions to be answered. Measurement can refer to both obtaining a score and the method used. Evaluation is the process of obtaining and providing information for decision alternatives. Assessment is the clinical diagnosis of an individual's problems (Mehrens & Lehmann, 1991). We can never measure or evaluate people, we measure and evaluate characteristics of people, their school potential and mathematical potential. According to van der Westhuizen (1980) measurement is only a part of evaluation, although it is a substantial part.

Measurement aids the teacher and the learner. It aids the teacher by helping to provide knowledge about the learner's entry behaviour. It helps to set and clarify realistic goals for each learner. It helps determine the degree of objectives achieved and helps to determine and evaluate teaching techniques. Measurement aids the student by communicating the goals of the teacher, increasing motivation, encouraging good study habits and providing feedback on strengths and weaknesses. Measurement is also good for administrative decisions, to select, classify and place learners (Mehrens & Lehmann, 1991). Measurement also aids the psychologist as a tool in the decision making process.

In measurement, we must distinguish between four levels of measurement. On the basis of these four levels of measurement, numbers are assigned. The four levels of measurement are: *distinguishability* (the number two is different from the number one); *order of rank* (two is a higher rank than one); *equal intervals between successively higher numbers*; and *absolute size*. Corresponding to these four levels are four types of measurement: nominal measurement; ordinal measurement; interval measurement; and ratio measurement (Huysamen, 1994). After measurement has taken place, the raw score must be converted into a standard score.

5.2.1 Standardisation

Standardised tests provide methods of obtaining samples of information under uniform conditions. In essence uniform conditions mean fixed questions are administered, with fixed directions, timing constraints and scoring procedure. Standardised tests are prepared instruments for which administrative and scoring procedures are carefully delineated and typically, norms are provided as interpretive aids. Standardised tests serve as aids in instructional, guidance, administrative and research decisions. The results of a sample, the so-called standardised sample, or norm group, is an estimate of the results which could have been obtained from the entire population. The results of the standardised sample are now known as the norms of the test (Huysamen, 1983). According to Mehrens and Lehmann (1991) the word norm is a synonym for average and is the mean score for a group. This specified group is called the norm group or reference group. A table showing the performance of the norm group is called a norm table or norms. Norms show the correlation between raw scores and some type of derived score. An appropriate norm group must be recent, representative and relevant. The norm group in this research is the grade one, two and three learners from schools throughout the Free State, whose home language is English.

The scores obtained by the norm group are referred to as the norms of the test. Norms appear in various forms, namely, percentile ranks, standard scores, linearly transformed standard scores, normalised standard scores, McCall's *T* scores, stanines, stens and deviation IQ's (Huysamen, 1983). For the sake of this study we shall consider stanines, percentile ranks and McCall's *T* scores in more detail. Stanines are integers one to nine that are assigned to normalised standard scores. Percentile ranks correspond to a raw score equal to the percentage of learners in the norm group who obtained a score lower than that raw score. McCall's *T* scores are normalised standard scores that are converted into McCall's *T* scores by means of the following equation:

$$\text{McCall's } T = z_n(10) + 50 \text{ (Huysamen, 1996).}$$

The explanation of the relationship between stanines, percentile ranks and *T*-scores is represented in the Table 5.1.

Table 5.1: Explanation of percentile ranks and stanines (Esterhuyse, 1997)

Explanation	**Stanine**	**Percentile rank**	***T*-score**
Very poor (below 11 %)	1 2	0-4 5-11	< 32,49 32,50 - 37,49
Poor (next 12 %)	3	12-23	37,50 - 42,49
Below average (next 17 %)	4	24-40	42,50 - 47,49
Average (next 20 %)	5	41-60	47,50 - 52,49
Above average (next 17 %)	6	61-77	52,50 - 57,49
Good (next 12 %)	7	78-89	57,50 - 62,49
Very good (upper 11 %)	8 9	90-96 97-100	62,50 - 67,49 > 67,50

Not only should a test be standardised according to an appropriate norm group but it should also be objective.

5.2.2 Objectivity

According to Anastasi and Urbina (1997) objectivity means that a learner should obtain the same score on a test, irrespective of the tester, occasion or the situation. Objectivity is necessary in the administration, scoring and interpretation of psychometric tests. Before a test can be objective, correct item analysis and selection should take place.

5.2.3 Item analysis and item selection

The most important steps in item analysis and selection is to ensure that you determine exactly what type of information you expect from the items, why you expect that type of information and how you intend to use that information once you have it. The first step in item selection is to determine the purpose for which testing is to be carried out. During item analysis, as much statistical information must be obtained about the items, to determine whether certain metrical requirements are met. The goal of item analysis is:

a) to obtain objective information about the items; and
b) to use the information to identify appropriate items for the test that will satisfy certain characteristics with regard to degree of difficulty, reliability and validity (Esterhuyse, 1997).

With the help of item analysis the:

a) too difficult or too easy items can be identified and eliminated;
b) items that best discriminate between learners with good and poor ability can be identified; and
c) items that must be improved and those that must be rejected can be identified (Esterhuyse, 1997).

Item selection and item analysis are two separate processes. The goal of item selection is to:

a) select the appropriate items for a test; and
b) develop a test that satisfies the specific characteristics of degree of difficulty, reliability and validity (Esterhuyse, 1997).

The development of a test is dependent on the item analysis and item selection. A test is only as good or as poor as the items available to analyse. The item statistics are determined from certain quantities that are calculated from the learner's item scores. These quantities can be based on the classical test theory, or on the more recent item response theory (Esterhuyse, 1997). For the purpose of this research, the researcher used the Classical Test Theory (Thorndike, Cunningham, Thorndike & Hagen, 1991) instead of the Item Response Theory (Barnard, 1991) as the sample size of the grade one learners was too small (N < 220). According to Mehrens and Lehmann (1991) item analysis data should be interpreted with caution. The reason for this is:

- item-analysis data are not analogous to item validity;
- the discrimination index is not always a measure of item quality;
- item-analysis data are tentative; and
- avoid selecting test items purely on the basis of their statistical properties.

Item selection and item analysis are based on the classical test theory.

5.2.4 Classical test theory

The Classical Test Theory (CTT) (Thorndike et al., 1991) was originally used to develop a test that would determine a person's ability with respect to certain items included in the test that could satisfy certain predetermined characteristics and criteria. The statistics obtained from the CTT is dependent on the sample and the selection of items with respect too:

a) item difficulty;

b) item variance and test variance;

c) item-test correlation and the coefficient-alpha; and

d) item-criterion correlation and criterion-related validity (Esterhuyse, 1997; Huysamen, 1983).

Each of the above will now be discussed in more detail.

According to Huysamen (1983) the proportion of learners who answered an item correctly is known as the *difficulty of the item*. The sum of the difficulty values of the items equals the test mean. The difficulty value (p) is defined as the mean item value of the item over the total number of learners in the group. During item selection, the items with a p-value of between 0,2 and 0,8 are selected. The items used in this test are better known as dichotomous items because they are ascribed a value of 0 if answered incorrectly and 1 if answered correctly (Huysamen, 1983). According to Huysamen (1996) and Anastasi and Urbina (1997) when the test items can only be correct (1) or incorrect (0), then the difficulty value is equal to the ratio of subjects that had the item correct with respect to the total group of subjects. The difficulty level of the total test can be determined by selecting items of the appropriate difficulty values.

According to Huysamen (1983) the variance of a dichotomous item is equal to the proportion of persons passing the item times the proportion failing it:

item variance = item difficulty x (1,00 – item difficulty).

Item variance (s^2) is directly proportional to the degree of difficulty. If an item is too difficult or too easy, the item variance is very small. The ideal difficulty value of an item is 0,5 because this gives the largest possible item variance, namely, 0,25 (Esterhuyse, 1997). The variance of a test is always equal to or greater than the sum of the variances of the items comprising the test. This means that the researcher must select items with large variances as this will result in a test with a large variance.

According to Huysamen (1983) the *item-test correlation* indicates the extent to which the learner's performance on an item correlates with the performance in other items in the test. If an item has a high item-test correlation it means that persons who pass the item will probably pass other items too, the opposite also applies. It is therefore advisable to select items with high item-test correlation as this will ensure a test with high internal consistency. The higher the item-test correlation of a collection of items with a fixed test variance, the higher the *coefficient-alpha* of the test (this will be referred to in more detail when considering the reliability of a test).

Item-criterion correlation is the correlation between item scores and criterion scores. The item-criterion correlation is usually referred to as the discrimination value of the item ((Huysamen, 1983). According to the CTT the discrimination value (r_{it}) of an item is the product moment correlation between the item score and the total item scores on the test. This correlation is indicative of how the learner's performance correlates with the performance on other items in the test. The higher the discrimination value, the better the item discriminates among learners who differ in terms of their position on the criterion.

According to Barnard (1991) the discrimination values are represented as follows:

0,6 and higher	=	excellent
0,4 to 0,59	=	very good
0,25 to 0,39	=	good
0,2 to 0,24	=	fair
lower than 0,2	=	poor/unsatisfactory.

The higher the item-criterion correlations of the items for an individual, of a test with a fixed test variance, the higher the criterion-related validity of the test. Therefore it is best to select items with high correlations with the criterion (Huysamen, 1983).

Now that the ground rules of measurement and the explanation of the CTT have been clarified, it is important to know that standardised tests should be reliable and valid. What is reliability?

5.3 RELIABILITY

'Reliability of a test refers to how consistently it measures whatever it measures... irrespective of when it was administered, which form of it was used, by who it was scored' (Huysamen, 1983, p.24). According to Mehrens and Lehmann (1991) reliability is the degree of consistency between two measures of the same thing. The variation in a learner's score is called error variance and the sources of this variation are known as the sources of error. The fewer the errors, the more reliable the measurement. With the sources of variation, reliability is estimated in different ways. Five common ways are used to estimate reliability, namely:

1. measures of stability (test-retest reliability);
2. measures of equivalence (parallel-forms reliability);
3. measures of equivalence and stability;
4. measures of internal consistency;

4.1 split-half;

4.2 Kuder-Richardson estimates;

4.3 coefficient-alpha; and

5. test-scorer reliability (Mehrens & Lehmann).

Let us consider each individually.

5.3.1 Measures of stability

Measures of stability are also known as test-retest estimates of reliability. This is obtained by administering a test to a group of persons and re-administering the same test to the same group at a later stage. The researcher then correlates the scores to determine the reliability (Mehrens & Lehmann, 1991). This coefficient is known as the stability coefficient and gives an indication of the consistency of the scores at different occasions (Huysamen, 1996). Sometimes a person is more test ready than other times. This reliability explains the unsystematic source of variation. This type of reliability is immune to the test occasion. Therefore the scores obtained from one occasion may be generalised to the total population, which could have been obtained on other comparable occasions (Huysamen, 1994). To determine test-retest reliability, a test must be administered on two occasions, on a representative sample of the population for which the test is intended.

5.3.2 Measures of equivalence

The equivalent-forms estimate of reliability is obtained by giving two forms of a test to the same group, on the same day and then correlating the results. In constructing equivalent tests, the two measures must be the same with respect to means, variances and item intercorrelations. The equality of the content is also important. This type of reliability is also known as parallel-forms reliability (Mehrens & Lehmann, 1991). Parallel tests are composed on different items of the same difficulty, measuring the same construct equally well. Parallel-forms reliability is determined by correlating the two sets of scores obtained, this score is know as the equivalent coefficient. If a tester questions the reliability of scores obtained, a parallel-form test can be given to correlate the scores (Huysamen, 1994).

5.3.3 Measures of stability and equivalence

If the researcher questions whether a different but similar set of questions at a different point in time would give similar results, then the researcher is looking at measures of equivalence and stability. A coefficient of equivalence and stability is obtained by giving one form of a test and after some time another form of the test and correlating the results. This estimate of reliability is usually lower then either of the other two procedures but is common among traits such as intelligence, creativity, aggressiveness or even musical interest, where constructs are not dependent on a specific set of questions (Mehrens & Lehmann, 1991).

5.3.4 Measures of internal consistency

The previous three measures of reliability require two testing occasions, which often is not possible. It is possible to obtain reliability estimates from one set of test data by means of split-halt estimates, Kuder-Richardson (K-R) estimates and the coefficient-alpha. These estimates (besides the split-half estimates) determine the degree to which the item correlates with the total test score. If there is a high degree of internal consistency than the test is reliable. Let us consider each of the estimates individually.

5.3.4.1 Split-half estimates

Split-half reliability or internal consistency (as it is also known) is improved when a test is lengthened by means of items measuring the same construct (Huysamen, 1991). If someone performs well in a few items of the test, the chances of them performing well in the rest of the test is great. To determine the reliability of the whole test, a correction factor is applied. The formula used is a special case of the Spearman-Brown formula, where:

$$\text{estimated reliability of the whole test} = \frac{2 \times \text{reliability of one test half}}{1 + \text{reliability of one test half}}$$

(Mehrens & Lehmann, 1991). The Spearman-Brown formula assumes that the variances of the two halves are equal.

5.3.4.2 Kuder-Richardson estimates

If items are scored dichotomously (right or wrong) the test can be split using the K-R (Mehrens & Lehmann, 1991). K-R 20 and K-R 21 are two formulas given below:

$$\text{K-R 20} = \frac{n}{n-1}\left[1 - \frac{\Sigma pq}{s^2}\right]$$

$$\text{K-R 21} = \frac{n}{n-1}\left[1 - \frac{\bar{X}(n - \bar{X})}{ns^2}\right]$$

where:

n	=	number of items in the test
p	=	proportion of people who answered the item correctly
q	=	proportion of people who answered the item incorrectly
pq	=	variance of a single item scored dichotomously
Σ	=	summation sign indicating that pq is summed over all the items
s^2	=	variance of the total test
$\bar{X}$	=	mean of the total test (Mehrens & Lehmann, 1991).

The difference between K-R 20 and K-R 21 is that K-R 21 assumes that all items are of equal difficulty (the items have the same p value). To determine the reliability using the K-R formula, given the number of items in the test, one only needs to compute the mean and the variance of the test and substitute the values into the formula. Due to the fact that dichotomous items are used in this research study, the K-R formula will be used to estimate the reliability of the test. The coefficient-alpha is often confused with the K-R formula, there is a definite distinction.

5.3.4.3 Coefficient-alpha

The alpha-coefficient is an index that shows the degree to which all items in a test measure the same attribute. To compute the alpha-coefficient, both the variance of the total test and the variances of the individual items must be obtained. A high internal consistency implies that the test has a high degree of generalisability across items within the test and over other parallel tests (Huysamen, 1991). The coefficient-alpha is a generalisation of the K-R 20 formula when the items are scored dichotomously. The coefficient-alpha formula is the same as the K-R 20 formula except that the pq is replaced by S, where S is the variance of a single item (Mehrens and Lehmann, 1991). The last estimate of reliability is the test-scorer reliability.

5.3.5 Test-scorer reliability

A test is usually only as good as the person administering it or scoring it. Standardised tests have standard instructions so different individuals administering or scoring the test won't affect the reliability.

It is not sufficient if a test is only reliable. The validity of the test is just as important.

5.4 VALIDITY

Reliability of a test does not imply validity. Validity refers to the extent to which the test satisfies the intended purpose. For example, the researcher wishes to determine mathematics proficiency among learners in the foundation phase. The extent to which the test measures this construct, gives an indication of the validity of the test. In discussing validity two inferences must be remembered. The first is a statistical inference which refers to the performance that is not being measured and the second inference is the measurement inference which refers to the behavioural domain of the person that is being measured. When a score is used to infer performance, the researcher is actually predicting performance. The degree to which this prediction or inference is accurate depends on the content validity, criterion validity and construct validity (Mehrens & Lehmann, 1991). Each will be discussed in detail below:

5.4.1 Content validity

Content validity is the degree to which a test represents an inventory or universe or the extent to which the items in the test are representative of the total universe of items which could have been compiled in terms of the appropriate curriculum and teaching objectives (Huysamen, 1991). There is no single commonly used numerical expression for content validity. Each item used in the test is judged on whether or not it represents the total domain of items that it was meant to sample (Mehrens & Lehmann, 1991). The mathematics proficiency test is an achievement test and its primary goal is to determine to what degree the learner has achieved the teaching goals in mathematics (Huysamen, 1996). The only way a researcher can make sure the test is content valid is to ask specialists in the field. In this research, teachers and head of departments from many schools were asked to ask to comment on the content of the test with respect to the curriculum. In this manner, the content validity of the test was questioned.

5.4.2 Criterion-related validity

According to Huysamen (1989) 'criterion-related validity refers to the degree to which diagnostic and selection tests correctly predict the relevant criterion'. Comparing the obtained score with another score of the same construct, tests criterion validity. Criterion validity in this research was tested by correlating the obtained score on the mathematics proficiency test with a mark obtained in a school evaluation. Distinction is made between two types of criterion-related validity, namely, predictive validity and concurrent validity. Predictive validity refers to the accuracy with which a test predicts future behaviour. Concurrent validity refers to the accuracy with which the test identifies some current behaviour of the learner (Huysamen, 1983). According to Steyn (1999) the significance of the predictive validity correlation results is also dependent on the practical interest of the result. The standardised difference in means of two scores can be viewed as the point of departure of the effect size. Effect size is the relationship between nominal and interval scale variables. Cohen in Steyn states that the product moment correlation coefficient can be used as the effect size of a linear relationship between two variables that can be measured on an interval scale. From this the researcher can determine whether the correlation coefficients have a small, medium or large effect size. Steyn continues to state that a correlation coefficient of 0,1 has a small effect size; a correlation coefficient of 0,3 has a medium effect size; and a correlation coefficient of 0,5 has a large effect size.

5.4.3 Construct validity

Construct validity is the degree to which the instruments used for measuring variables are measuring what they are supposed to measure. Any measuring instrument measures three components, the actual construct, irrelevant constructs and random measurement error. The last is an unsystematic source of variation because it refers to accidental factors, which may vary from one individual to the next. The actual construct and irrelevant constructs refer to systematic sources of variation because they remain constant for any given individual (Huysamen, 1991). If a test has acceptable construct validity, its content and criterion-related validity has to be satisfactory as well. The converse does not always apply.

5.5 CONCLUSION

During the development and standardisation of a psychometric test, it is important to make sure that each of the above criteria is met. It is important that the desired construct is the construct that is being measured and that the measurement is objective and standardised. Item analysis and selection therefore plays a vital role to ensure that the test is objective. The model used to carry out the item analysis is the CTT. The CTT was used to develop and standardise the mathematics proficiency test and the results are discussed in the following chapter.

6. METHOD, RESULTS AND DISCUSSION OF RESULTS

6.1 INTRODUCTION

According to Swanson (1991) there are two reasons for assessment in learning problems, the one reason is to enhance communication and the other is to improve instruction. Assessing children with possible learning problems should facilitate communication between psychologists, educationists and researchers concerning the needs of those suspected of having problems in mathematics. All assessment should provide outcome predictions. The assessment can only be carried out with a standardised mathematics test.

6.2 GOAL OF THE INVESTIGATION

The aim of assessment should be to provide information about prevention and treatment. The aim of the mathematics proficiency test for learners in the foundation phase is to provide a means of identifying a learner with a mathematics problem so that intervention can take place as early as possible. This identification will be done with a standardised mathematics test. This test will be applicable for learners in grade one, two and three and will have norms available per term so that the test will be applicable throughout the year. The test can be administered on groups or individuals and can be used diagnostically. The norms of the test were determined according to the sample.

6.3 SAMPLE

The sample consisted of learners in grades one, two and three whose home language was English. The norms were determined for learners whose home language was English but the test consisted of universal mathematics concepts that would not prevent another language group from using the test. The learners were selected from thirteen schools throughout the Free State region, namely Bloemfontein, Welkom, Kroonstad, Virginia, Ladybrand and Bethlehem. The sample consisted of boys and girls. The various groups that were used during different phases of the research will be discussed in more detail in the next paragraph.

6.4 RESEARCH METHOD

The research consisted of four phases that was carried out over a period of two years. The effectiveness of the psychometric test is dependent on the type of items selected to compile the test. The test needs to distinguish between learners who are good at mathematics and learners who struggle with mathematics. The raw score on a test does not give an indication of how well or how poorly a learner is performing. For a tester to determine the level a child is functioning on, norms are needed. The psychometric test must also be reliable and valid so that the test adheres to all psychometric requirements. To make sure that the above requirements are met, the following phases were carried out while compiling the mathematics proficiency test for learners in the foundation phase:

Phase one: Construction of preliminary questionnaire
(Third term, 1999)

Phase two: Item analysis and selection
(Fourth term, 1999)

Phase three: Determination of norms
(First and Fourth term, 2000)

Phase four: Validity study
(Fourth term, 2000)

The results obtained during each of the above stages will be discussed in more detail.

6.4.1 Phase one: Construction of preliminary questionnaire

6.4.1.1 Introduction

Before any contact could be made with schools selected to participate in the research study, permission had to be granted by the Free State Education Department to conduct the research. Contact was made with the Head of Education on 29 April 1999, requesting permission to conduct research in the Free State Province. A letter dated 4 May 1999 was then received from the department granting permission to conduct research in the Free State under certain conditions (see Annexure A).

The construction of the preliminary questionnaire is one of the most important phases of the research. During June/July 1999 the researcher contacted seven schools in the Free State region, from Bloemfontein and Welkom, to obtain permission from the headmasters to conduct research at their schools and to obtain items for the preliminary questionnaire. The entire process of the research was then explained personally to the headmaster and the head of department of the foundation phase at each respective school. The head of department at each school then became the contact person. The rest of the research phases were then explained to the contact person who gave the information through to the other staff members at their school. The first task of the contact person was to set up the preliminary questionnaire.

The following guidelines concerning the identification of the preliminary items were recommended:

a) During phase one, the contact teachers had to set up 20 mathematics items (per grade) based on what is expected from a child in the applicable grade (grade one, two or three).
b) The contact teachers could receive as much input as required from the other foundation phase teachers when selecting the most applicable items.
c) The items had to range in difficulty.

d) Five items had to be relatively easy for the applicable grade. five items had to be relatively difficult and ten items of average difficulty.
e) The 20 items had to cover the curriculum for the whole year for each respective grade.
f) The items then had to be given to the respective grades as a class test.
g) The test could be completed on a loose piece of paper with the child's grade, home language and age written on the top of the page.
h) The test need not be marked, the researcher would mark the items and in so doing the researcher would confirm the preliminary effectiveness of the items selected.

6.4.1.2 Application of preliminary questionnaire

Once the items were selected, the preliminary questionnaire was given to approximately two to three classes of 30 learners, in the respective grades, at each of the seven schools. This was to (a) eliminate any uncertainty pertaining to the above instructions, (b) to determine the relevance of the items selected and (c) to determine the order of the items. An attempt was made to ensure that learners, both boys and girls, with above average mathematical ability, average mathematical ability and below average mathematical complete the test.

6.4.1.3 Selection of questions for experimental test

Phase one was completed by the end of the third term of 1999. In total 21 different mathematics tests, consisting of approximately 20 items each, were set up by the contact teachers. There were approximately 140 items per grade to select from, giving a total 420 items. All the tests were written in English. Several learners from each grade wrote their respective tests but only the results of learners whose home language was English, were taken into account. These results were then computerised and preliminary calculations were carried out. For every grade (one, two and three) 40 items were selected in the following manner:

a) 10 easy items (at least 80 % of the learners obtained the correct answer);
b) 10 difficult items (only 20 % of the learners obtained the correct answer); and
c) 20 items with an average difficulty value.

The discrimination value of each item was also taken into account. Only items with a discrimination value of 0,6 or higher were considered at first. When not enough items could be obtained with a discrimination value of 0,6 or higher then items with a discrimination value of 0,4 or higher were also selected. During selection, note was also taken of the amount of times a specific item came to the fore.

The selected items were then arranged from easiest to most difficult and the selected items were then given to a few of the contact teachers for comment before finalising the experimental mathematics proficiency test. This then completed phase one. During phase two the selected items had to be analysed and the final mathematics proficiency test drafted.

6.4.2 Phase two – Item analysis and selection

6.4.2.1 Introduction

The experimental mathematics proficiency test, consisting of 40 items per grade, was given to 13 schools during the last term of 1999. The seven original schools who helped determine the preliminary questionnaire were once again involved. During this phase it was necessary to extend the input to schools from other Free State towns or cities. The 13 schools were all English medium schools selected from Bloemfontein, Welkom, Kroonstad, Bethlehem, Virginia, and Ladybrand. Due to a written contract between the researcher and the Free State Department of Education, the names of the various schools involved must be kept confidential. The process of the research was explained to the headmasters and heads of departments of the foundation phase of all the respective schools and a contact person was established. The contact person's role was to serve as the liaison between the researcher and the school. The instruction to the contact person during this phase was to give the 40-mark questionnaire to all the respective grade one, two and three learners at their school. They did not need to mark the items, the researcher would do the marking. If they wished to mark the items it was important to note that each question counted only one mark. The researcher then selected the learners whose home language was English and analysed the data. The goal of the item analysis was to obtain as much objective information about the items as possible. The obtained information was used to select the final 20 items for each grade. The items had to conform to certain characteristics previously discussed with reference to the difficulty value, discrimination value, as well as the reliability and validity of the items.

Extra instructions were given to the contact teacher at each school, this had to be conveyed to all other teachers with grade one, two and three classes. The instructions were as follows:

1. No calculators may be used in the test.
2. No time limit is imposed (within reasonable limits).
3. You may read the word sums to the children as their mathematical ability and not their English reading ability is being tested.
4. In the grade two test, the word 'decompose' (break down) has been used. If the children do not know what the word is, please explain it to them.
5. Important details such as the name, home language, age and gender must be completed. Please make sure the learner has completed his/her details correctly (especially the home language).
6. The test must be completed on the questionnaire.
7. Extra paper may be given for rough work.

6.4.2.2 Sample during phase two

According to Huysamen (1996) it is important to obtain as large a sample as possible when analysing items. The recommendation is that the sample should be five times the number of items. In this phase of the research, with 40 items per grade, the sample per grade should not be less than 200. The composition of the sample during phase two is represented in Table 6.1:

Table 6.1: Sample distribution during phase two

Grade	**Boys**		**Girls**		**Total**	
	N	**%**	**N**	**%**	**N**	**%**
1	112	53,3	98	46,7	210	33,6
2	101	47,6	111	52,4	212	33,9
3	110	54,2	93	45,8	203	32,5
Total	323	51,7	302	48,3	625	100,0

The above sample distribution indicates that the sample size for each grade was relatively even and the number of boys and girls relatively proportional. A total of 625 learners were tested during phase two. The minimum age of a child tested in the above sample was 72 months and the maximum age was 129 months. The mean age of the learners in their respective grades is represented in Table 6.2:

Table 6.2: Mean age distribution (in months) of learners in their respective grades during phase two

Grade	**Boys**			**Girls**			**Total**		
	N	$\bar{X}$	**s**	**N**	$\bar{X}$	**s**	**N**	$\bar{X}$	**s**
1	112	88,94	5,55	98	86,36	6,33	210	87,73	6,05
2	101	101,19	5,21	111	101,05	5,32	212	101,11	5,26
3	110	113,33	5,93	93	112,99	7,44	203	113,17	6,65

6.4.2.3 Results of item analysis

According to the CTT, the difficulty value (p), discrimination value (r_{it}) and the item variance (s^2) of each item must be considered when analysing an item. The explanation of each of the above variables was discussed in chapter five. The CTT results were calculated by means of the SAS-(SAS Institute, 1985) and the SPSS-computer programs (SPSS Incorporated, 1983). The 40 items selected for grade one follow.

GRADE ONE

NAME: ____
HOME LANGUAGE: ________________________________
AGE: ________________YEARS ________________MONTHS
GENDER: MALE / FEMALE

1.) **WHICH NUMBER COMES BEFORE 8 ?** ________

2.) **WHICH NUMBER COMES AFTER 10 ?** ________

3.) **HALF OF 4 =** ________

4.) **HALF OF 8 =** ________

5.) **HALF OF 12 =** ________

6.) **HALF OF 7 =** ________

7.) **DOUBLE 9 =** ________

8.) **DOUBLE 5½ =** ____

9.) **10 + 5 =** ________

10.) **21 + 5 =** ________

11.) **9 + 2 + 3 =** ________

12.) **R2 + R1 + R5 + R5 =** ________

13.) **5 + ____ = 9**

14.) 4 = ________

15.) 8 $\boxed{-2}$ = ________

16.) 7 = ________

17.) 3 x 2 = ____

18.) 5 x 2 = ____

19.) 3 x 4 = ____

20.) 16 ÷ 2 = ____

21.) 7 ÷ 2 = ____

22.) 18 ÷ 3 = ____

23.) 14 ; 15 ; ____ ; 17 ; 18 ; 19

24.) 16 ; 18 ; ____ ; 22 ; 24 ; 26

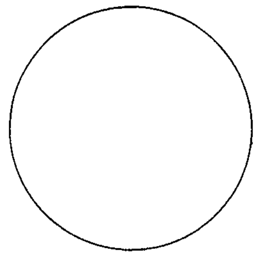
25.) 18 ; 21 ; 24 ; ____ ; 30 ; 33

26.) 5 ; 10 ; ____ ; 20 ; 25

27.) 28.) 29.) 30.)

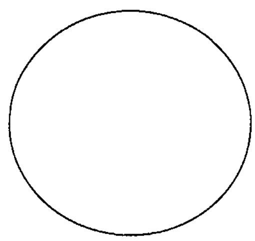
18 ; ____ ; 30 ; 36 ; ____ ; 48 ; ____ ; ____ ; 66 ; 72

31.) DRAW THE HANDS TO SHOW THE TIME ON THE CLOCK



5 O' CLOCK

32.)



HALF PAST 8

33.) I SHARE 12 SWEETS BETWEEN 2 CHILDREN. HOW MANY SWEETS EACH ? ____________________________________

34.) 4 CATS HAVE ____ LEGS ALTOGETHER ?

35.) KABELO HAD 11 MARBLES. HE LOST 5 AND LATER FOUND 2. HOW MANY DOES HE HAVE THEN ?

____

36.) MEG HAD 13 EGGS. SHE LOST 4 EGGS AND THE FOX TOOK 3 EGGS. HOW MANY EGGS DOES MEG HAVE NOW ? ____

37.) THERE ARE 4 LEMONS, 8 ORANGES AND 2 APPLES IN THE BOWL. HOW MANY FRUIT ARE IN THE BOWL ALTOGETHER ? ________________________________

38.) ANN PICKS 16 ROSES. SHE SHARES THEM AMONG HER FOUR FRIENDS. HOW MANY ROSES DO THEY EACH GET?

____

39.) PAT HAS 9 SWEETS. SHE EATS 6 SWEETS. HOW MANY SWEETS ARE LEFT ? ______________________________

40.) JOHN HAS 6 BLOCKS. PAT HAS 5 BLOCKS. BEN HAS 4 BLOCKS. HOW MANY BLOCKS DO THEY HAVE ?

The item analysis results for the above 40 items are represented in Table 6.3.1.

Table 6.3.1: Item analysis results for grade one

ITEMS	p	r_{it}	s^2
1	0,919	0,205	0,075
2	0,867	0,122	0,116
3	0,943	0,431	0,054
4	0,929	0,445	0,067
5	0,871	0,353	0,113
6	0,624	0,522	0,236
7	0,819	0,394	0,149
8	0,471	0,528	0,250
9	0,933	0,429	0,063
10	0,752	0,308	0,187
11	0,886	0,254	0,102
12	0,852	0,468	0,127
13	0,838	0,360	0,136
14	0,938	0,212	0,059
15	0,924	0,333	0,071
16	0,948	0,320	0,050
17	0,867	0,445	0,116
18	0,876	0,534	0,109
19	0,724	0,527	0,201
20	0,771	0,469	0,177
21	0,548	0,474	0,249
22	0,671	0,494	0,222
23	0,929	0,409	0,067
24	0,848	0,437	0,123
25	0,719	0,611	0,203
26	0,905	0,388	0,086
27	0,629	0,566	0,234
28	0,586	0,604	0,244
29	0,529	0,546	0,250
30	0,538	0,521	0,250
31	0,900	0,314	0,091
32	0,681	0,505	0,218
33	0,838	0,451	0,136
34	0,762	0,403	0,182
35	0,743	0,407	0,192
36	0,724	0,425	0,201
37	0,829	0,367	0,143
38	0,638	0,524	0,232
39	0,819	0,425	0,149
40	0,848	0,476	0,130

$\bar{X} = 31{,}433$ $s = 6{,}687$ $N = 210$

The final test consisting of 20 mathematics items for grade one was selected from the above 40 items. The statistical analysis of the 20 selected items is represented in Table 6.3.2:

Table 6.3.2: Item analysis results of the mathematics proficiency test for grade one

ITEM	ORIGINAL NO.	p	r_{it}	s^2
1	9	0,933	0,429	0,063
2	23	0,929	0,409	0,067
3	4	0,929	0,445	0,067
4	18	0,876	0,534	0,109
5	12	0,852	0,468	0,127
6	24	0,848	0,437	0,123
7	40	0,848	0,476	0,130
8	33	0,838	0,451	0,136
9	39	0,819	0,425	0,149
10	20	0,771	0,469	0,177
11	34	0,762	0,403	0,182
12	35	0,743	0,407	0,192
13	19	0,724	0,527	0,201
14	36	0,724	0,425	0,201
15	25	0,719	0,611	0,203
16	32	0,681	0,505	0,218
17	22	0,671	0,494	0,222
18	38	0,638	0,524	0,232
19	6	0,624	0,522	0,236
20	8	0,471	0,528	0,250

$\overline{X} = 15{,}400$ $s = 4{,}071$

The 40 items for grade two follow.

GRADE TWO

NAME: ____

HOME LANGUAGE: ___________________________________

AGE: _______________YEARS _______________MONTHS

GENDER: MALE / FEMALE

1.) DECOMPOSE: 26 _______________

2.) DECOMPOSE: 96 _______________

3.) DECOMPOSE: 136 _______________

4.) WRITE FROM SMALLEST TO BIGGEST
15 ; 7 ; 24 ; 31 ; 9
____ ; ____ ; ____ ; ____ ; ____

5.) HALF OF 16 = ____

6.) HALF OF 23 = ____

7.) HALF OF 47 = ____

8.) HALF OF 49 = ____

9.) HALF OF 99 = ____

10.) DOUBLE 26 = ____

11.) DOUBLE 75 = ____

12.) DOUBLE 9¼ = ____

13.) 37 + 10 = ____

14.) 40 + 32 = ____

15.) 46 + 63 = ____

16.) 72 - 60 = ____

17.) 54 × 3 = ____

18.) 12 ÷ 2 = ____

19.) 74 ÷ 8 = ____

20.) 2½ + 4¼ = ____

21.) FILL IN < ; > ; =
18 - 3 ____ ½ OF 15

22.) (2 x 4) + 20 = ________

23.) (6 x 2) + 1 = ________

24.) 30 ; 45 ; 60 ; ____ ; ____ ; ____.

25.) 350 ; 300 ; 250 ; ____ ; ____ ; ____.

26.) ____ ; ____ ; 105 ; 110 ; 115.

27.) HOW MANY MINUTES ARE THERE IN AN HOUR ?________

28.) HOW MANY HOURS ARE THERE IN A DAY? _____________

29.) THE TWO NUMBERS BEFORE 48 ARE ____ AND ____

30.) HOW MANY CENTS ARE THERE IN R1 ? ____

31.) 9 CHILDREN HAVE HOW MANY LEGS ? ____

32.) ONE FLY HAS 6 LEGS. HOW MANY LEGS WOULD 8 FLIES HAVE ? ____

33.) GRANNY BAKES 36 COOKIES EVERY DAY. SHE SHARES THEM BETWEEN 3 CHILDREN. HOW MANY COOKIES DOES EACH CHILD GET ? ____________________________

34.) VUSI HAS 15 MARBLES. HE LOSES 8. HOW MANY MARBLES ARE LEFT ? ______________________________

35.) HOW MANY 20c PIECES ARE THERE IN R2 ? ___________

36.) IN A SHOP THERE ARE 25 RED BALLOONS AND 28 BLUE BALLOONS. IF 19 OF THE BALLOONS POP, HOW MANY BALLOONS ARE LEFT ? ______________________________

37.) SIPHO, LINDY, SARA, MARCUS, ESTER, JANE, JOHN AND MANDLA EACH HAVE R5. HOW MUCH MONEY DO THEY HAVE TOGETHER ? __________________________________

38.) ANNE HAS 79 BEADS. 33 WERE BLUE. HALF OF THE REST WERE RED. HOW MANY RED BEADS DID SHE HAVE ? ____

39.) ARRANGE THE FOLLOWING NUMBERS FROM BIGGEST TO SMALLEST.
98 ; 67 ; 101 ; 24 ; 50 ; 19 ; 91 ; 15
____ ; ____ ; ____ ; ____ ; ____ ; ____ ; ____ ; ____ .

40.) ARRANGE THE FOLLOWING NUMBERS FROM SMALLEST TO BIGGEST.
11 ; 56 ; 29 ; 9 ; 44 ; 14 ; 87 ; 78
____ ; ____ ; ____ ; ____ ; ____ ; ____ ; ____ ; ____ .

The item analysis results for the above 40 items are represented in Table 6.4.1.

Table 6.4.1: Item analysis results for grade two

ITEMS	p	r_{it}	s^2
1	0,901	0,272	0,089
2	0,906	0,256	0,086
3	0,825	0,286	0,144
4	0,948	0,249	0,049
5	0,934	0,293	0,062
6	0,769	0,485	0,179
7	0,712	0,527	0,206
8	0,708	0,513	0,208
9	0,349	0,503	0,228
10	0,726	0,416	0,200
11	0,533	0,558	0,250
12	0,439	0,477	0,247
13	0,926	0,341	0,036
14	0,792	0,434	0,166
15	0,741	0,536	0,193
16	0,708	0,412	0,208
17	0,377	0,485	0,236
18	0,825	0,449	0,144
19	0,231	0,285	0,179
20	0,160	0,264	0,135
21	0,340	0,329	0,226
22	0,825	0,382	0,144
23	0,811	0,303	0,154
24	0,476	0,483	0,250
25	0,670	0,600	0,222
26	0,698	0,562	0,212
27	0,759	0,542	0,183
28	0,726	0,576	0,200
29	0,693	0,439	0,213
30	0,759	0,435	0,183
31	0,830	0,347	0,141
32	0,542	0,412	0,249
33	0,604	0,485	0,240
34	0,858	0,496	0,122
35	0,637	0,565	0,232
36	0,467	0,459	0,250
37	0,769	0,452	0,179
38	0,212	0,332	0,168
39	0,830	0,615	0,141
40	0,825	0,625	0,144

$\bar{X} = 26{,}882$ $s = 7{,}430$ $N = 212$

The final test consisting of 20 mathematics items for grade two was selected from the above 40 items. The statistical analysis of the 20 selected items is represented in Table 6.4.2:

Table 6.4.2: Item analysis results of the mathematics proficiency test for grade two

ITEM	ORIGINAL NO.	p	r_{it}	s^2
1	34	0,858	0,496	0,122
2	39	0,830	0,615	0,141
3	40	0,825	0,625	0,144
4	37	0,769	0,452	0,179
5	27	0,759	0,542	0,183
6	15	0,741	0,536	0,193
7	28	0,726	0,576	0,200
8	16	0,708	0,412	0,208
9	8	0,708	0,513	0,208
10	26	0,698	0,562	0,212
11	29	0,693	0,439	0,213
12	25	0,670	0,600	0,222
13	35	0,637	0,565	0,232
14	33	0,604	0,485	0,240
15	32	0,542	0,412	0,249
16	11	0,533	0,558	0,250
17	24	0,476	0,483	0,250
18	36	0,467	0,459	0,250
19	12	0,439	0,477	0,247
20	17	0,377	0,485	0,236

$\bar{X} = 13{,}061$ $s = 4{,}831$

The 40 items for grade three follow.

GRADE THREE

NAME: ______________________________

HOME LANGUAGE: ______________________

AGE: ____________ **YEARS** ____________ **MONTHS**

GENDER: MALE / FEMALE

1.) COUNT THE NUMBER

96 +  **= ____**

2.) 24 + 31 + 56 = ____

3.) 26 - ____ = 13

4.) 571 - 348 = ____

5.) 17 x 6 = ____

6.) 27 x 5 = ____

7.) 1500 x 5 = ____

8.) 96 ÷ 4 = ____

9.) HALF OF 241 = ____

10.) UNDERLINE THE NUMBER WHICH CAN BE MADE BY MULTIPLYING A NUMBER BY ITSELF ?
(48 ; 36 ; 34 ; 18)

11.) (9 x 5) - 6 = ____

12.) (27 ÷ 3) + 5 = ____

13.) (110 ÷ 10) - 7 = ____

14.) COUNT 101 ; 106 ; ____ ; ____ ; ____ ; ____ .

15.) ____ ; 80 ; 130 ; 180 ; 230 ; ____ .

16.) 2 ; 4 ; 8 ; ____ ; 32 ; ____ .

17.) -7 : 110 ; ____ ; ____ ; ____ ; ____.

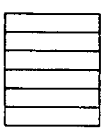
18.) + 13 : 111 ; ____ ; ____ ; ____ ; ____.

19.) ____ IS 10 MORE THAN 65.

20.) 8 IS ____ LESS THAN 23.

21.) ¼ HOUR + ½ HOUR + 15 MINUTES = ____

22.) WHAT FRACTION OF THIS SHAPE IS SHADED AND WHAT FRACTION IS UNSHADED ?



________ SHADED
________ UNSHADED

23.) ½ OF 100 = ____

24.) ¾ OF 200 = ___

25.) 1/5 OF 350 = ____

26.) 4/4 OF R10 = ____

27.) THERE ARE 32 CHILDREN IN THE CLASS. IF 18 ARE BOYS, HOW MANY GIRLS ARE THERE ? ________________________

28.) SUSAN HAS 47 DRINKING STRAWS. SHE NEEDS 72 STRAWS. HOW MANY STRAWS DOES SHE STILL NEED ?

____

29.) A FARMER HAD 160 LITRES OF MILK. HE SOLD 75 LITRES. HOW MANY LITRES OF MILK HAS HE LEFT ? ___________

30.) MY WATCH IS 10 MINUTES SLOW. IF IT SHOWS TEN PAST TEN, WHAT IS THE RIGHT TIME ? ___________

31.) I HAVE 3 FIFTY CENT COINS, 2 TWENTY CENT COINS, 2 TEN CENT COINS, 1 FIVE CENT COIN AND 4 ONE CENT COINS IN MY PURSE. HOW MUCH MONEY DO I HAVE ALTOGETHER ? ___________

32.) I HAVE 76 KG OF SWEETS TO SELL. I WANT TO SHARE THEM INTO PACKETS EACH WEIGHING 4KG. HOW MANY PACKETS OF SWEETS CAN I MAKE ? ___________

33.) IF YOU SUBTRACT 1 FROM A NUMBER AND THEN ADD 2, THERE IS 8 LEFT. WHAT WAS THE NUMBER ? ___________

34.) MY DOG EATS 4 BISCUITS A DAY. HOW LONG WILL A PACKET OF 64 BISCUITS LAST HIM ? ___________

35.) IF I WALK 12KM EACH DAY FOR A WEEK, HOW FAR DO I WALK ? ___________

36.) I TAKE 5ml OF MEDICINE 5 TIMES A DAY. HOW MANY ml DO I TAKE IN ONE DAY ? ___________

37.) JANE TOOK 58 MINUTES TO DO HER HOMEWORK. JACK DID HIS IN 89 MINUTES. IF THOMAS TOOK TWICE AS LONG AS JANE TO DO HIS HOMEWORK. HOW LONG DID IT TAKE THE THREE CHILDREN TO DO THEIR HOME WORK ? ___________

38.) THE 49 GRADE 3 PUPILS EACH HAD TO THROW 4 BALLS INTO A CONTAINER. ONLY 6 BALLS DID NOT LAND IN THE CONTAINER. HOW MANY BALLS WERE IN THE CONTAINER ? ___________

39.) A FAMILY USES 21 KG OF SUGAR EVERY TWO MONTHS. HOW MANY KILOGRAMS OF SUGAR WILL THE FAMILY USE IN 8 MONTHS ? _________________________________

40.) IF THE PERIMETER OF A SQUARE IS 24cm WHAT IS THE MEASUREMENT OF EACH SIDE ? ______________________

The item analysis results for the above 40 items are represented in Table 6.5.1.

Table 6.5.1: Item analysis results for grade three

ITEMS	p	r_{it}	s^2
1	0,921	0,302	0,073
2	0,862	0,263	0,120
3	0,906	0,276	0,085
4	0,709	0,368	0,207
5	0,709	0,449	0,207
6	0,778	0,470	0,173
7	0,660	0,513	0,226
8	0,670	0,512	0,222
9	0,724	0,563	0,201
10	0,606	0,320	0,240
11	0,877	0,315	0,108
12	0,695	0,547	0,213
13	0,724	0,600	0,201
14	0,872	0,376	0,112
15	0,833	0,327	0,140
16	0,665	0,511	0,224
17	0,675	0,444	0,221
18	0,690	0,436	0,215
19	0,788	0,296	0,168
20	0,788	0,380	0,168
21	0,680	0,661	0,219
22	0,571	0,550	0,246
23	0,956	0,250	0,042
24	0,478	0,435	0,251
25	0,374	0,534	0,235
26	0,291	0,484	0,207
27	0,773	0,393	0,176
28	0,527	0,518	0,251
29	0,724	0,435	0,201
30	0,468	0,526	0,250
31	0,562	0,542	0,247
32	0,640	0,566	0,231
33	0,788	0,383	0,168
34	0,616	0,601	0,238
35	0,724	0,513	0,201
36	0,724	0,360	0,201
37	0,271	0,479	0,199
38	0,261	0,401	0,194
39	0,325	0,493	0,221
40	0,500	0,598	0,251

$\bar{X} = 26{,}485$ $s = 8{,}020$ $N = 202$

The final test consisting of 20 mathematics items for grade three was selected from the above 40 items. The statistical analysis of the 20 selected items is represented in Table 6.5.2:

Table 6.5.2: Item analysis results of the mathematics proficiency test for grade three

ITEM	ORIGINAL NO.	p	r_{it}	s^2
1	23	0,956	0,250	0,042
2	14	0,872	0,376	0,112
3	6	0,778	0,470	0,173
4	9	0,724	0,563	0,201
5	13	0,724	0,600	0,201
6	35	0,724	0,513	0,201
7	12	0,695	0,547	0,213
8	21	0,680	0,661	0,219
9	8	0,670	0,512	0,222
10	16	0,665	0,511	0,224
11	7	0,660	0,513	0,226
12	4	0,709	0,368	0,207
13	32	0,640	0,566	0,231
14	22	0,571	0,550	0,246
15	31	0,562	0,542	0,247
16	28	0,527	0,518	0,251
17	40	0,500	0,598	0,251
18	30	0,468	0,526	0,250
19	25	0,374	0,534	0,235
20	39	0,325	0,493	0,221

$\bar{X} = 12{,}824$ $s = 4{,}857$

The final draft of the mathematics proficiency test for grade one, two and three learners was then given to several of the contact teachers for advice before drafting the final test. The test was also given to the Mathematics, Mathematics Literacy and Mathematical Science Learning Facilitator of the Free State Education Department for comment. The only change that was brought about to the test was that the teachers recommended that the font of the final mathematics proficiency test be in Arial with a font size of 16.

A final mathematics test, namely the *VASSI* Mathematics Proficiency Test, was then developed for grade one, two and three learners whose home language is English (see Annexure C). This implies that a learner with a mathematics problem in grade one, two and three can be tested using one of the applicable tests. Not only was the mathematics proficiency test developed but it was also standardised and in so doing, norms were calculated for each grade. The norm determination is discussed in the next paragraph.

6.4.3 Phase three – Determination of norms

6.4.3.1 Introduction

The final mathematics proficiency test was sent to the same thirteen schools during the first and fourth terms of 2000. The learners who were tested during the previous year were in the next grade and would therefore not be exposed to the same items. The norm determination consisted of testing the grade one, two and three learners during the first and fourth terms respectively. During the fourth term the same sample of learners that were tested during the first term, were tested. In so doing, norms for the year could be determined, with separate norms for each term. An estimation using inter polarisation was used to determine the norms for the second and third terms.

The instructions to the contact persons during phase three were to give the 20 mark questionnaire to all the respective grade one, two and three learners at their school during the first and fourth terms of 2000. They did not need to mark the items, the researcher would do the marking. If they wished to mark the items it was important to note that each question counted only one mark. The researcher would then select the learners whose home language was English and analyse the data.

Extra instructions were given to the contact teacher at each school, this had to be passed on to all other teachers with grade one, two and three classes. The instructions were as follows:

1. No calculators may be used in the test for grade one, two, and three.
2. No time limit is imposed (within limits).
3. You may read the word sums to the children (we wish to test their mathematics ability and not their English reading ability).
4. You may explain the meaning of words to the children.
5. Important details such as the name, home language, age and gender must be completed. Please make sure the learner has completed his/her details correctly (especially the home language).
6. The test must be completed on the questionnaire.
7. Extra paper may be given for rough work.

6.4.3.2 Sample during phase three

In this phase of the research, with 20 items per grade, the sample per grade should not be less than 100 (Huysamen, 1996). The composition of the sample according to grade and gender is represented in Tables 6.6.1 and 6.6.2:

Table 6.6.1: Sample distribution during phase three. first term

Grade	Boys		Girls		Total	
	N	%	N	%	N	%
1	92	48,9	96	51,1	188	29,6
2	115	52,0	106	48,0	221	34,8
3	113	50,0	113	50,0	226	35,6
Total	320	50,4	315	49,6	635	100,0

Table 6.6.1 indicates that the sample size for grade two and three was relatively even but grade one's sample was slightly smaller. The number of boys and girls were relatively proportional per grade and for the sample. The number of grade three boys and girls was exactly even. A total of 635 learners were tested during the first term of phase three.

Table 6.6.2: Sample distribution during phase three, fourth term

Grade	Boys		Girls		Total	
	N	%	N	%	N	%
1	83	48,8	87	51,2	170	29,7
2	106	53,0	94	47,0	200	35,0
3	104	51,5	98	48,5	202	35,3
Total	293	51,2	279	48,8	572	100,0

Once again Table 6.6.2 indicates that the sample size for the grade two and three learners was relatively even but that sample size for the grade ones was slightly smaller. The number of boys and girls per grade and for the sample was relatively proportional. A total of 572 learners were tested during the second administration of the test. This discrepancy between the number of learners being tested during the first and second administrations could be explained by school changes and absenteeism on the day of administration. A total of 63 learners could not be accounted for in this phase.

6.4.3.3 Calculation of norms

When a child is tested with the mathematics proficiency test, a raw score is obtained. This raw score does not indicate the child's mathematical percentile. If this raw score is compared to the frequency of this raw score in the norm population, then relevant conclusions can be drawn from this information (Esterhuyse, 1997). A norm is sometimes a synonym for average and is the mean score for some specified group of people. A table showing the performance of the norm group is called a norm table or norms. This table shows the relationship between the raw score and some type of derived score (Mehrens & Lehman, 1991). To evaluate a child's raw score, the score needs to be converted to a standard score that is obtained from the norm table. Two types of standard scores are stanines and percentile ranks. According to Mehrens and Lehmann, stanines are normalised derived scores with a mean of five and a standard deviation of two. The integers one to nine occur. The authors continue to state that percentile ranks are the best scores to use. A percentile is defined as a point on the distribution below which a certain percentage of the scores occurs. A percentile rank gives a person's relative position or the percentage of students' scores occurring below this obtained score. Percentile ranks have the advantage of being easy to compute and fairly easy to interpret. For a raw score to be normalised, it is necessary to use normal probability graph paper. On the left-hand side of the graph paper is a vertical line where the cumulative proportion appears. At this point the percentile rank can be read. The percentile rank is a score from 0 to 100. On the right-hand side of the graph paper is the division that shows the standard scores or the stanines. The raw score is at the bottom on a horizontal line. The raw scores vary from 0 to 20. The normalisation of the raw scores with respect to grade one, two and three are indicated in figures 6.1; 6.2; and 6.3.

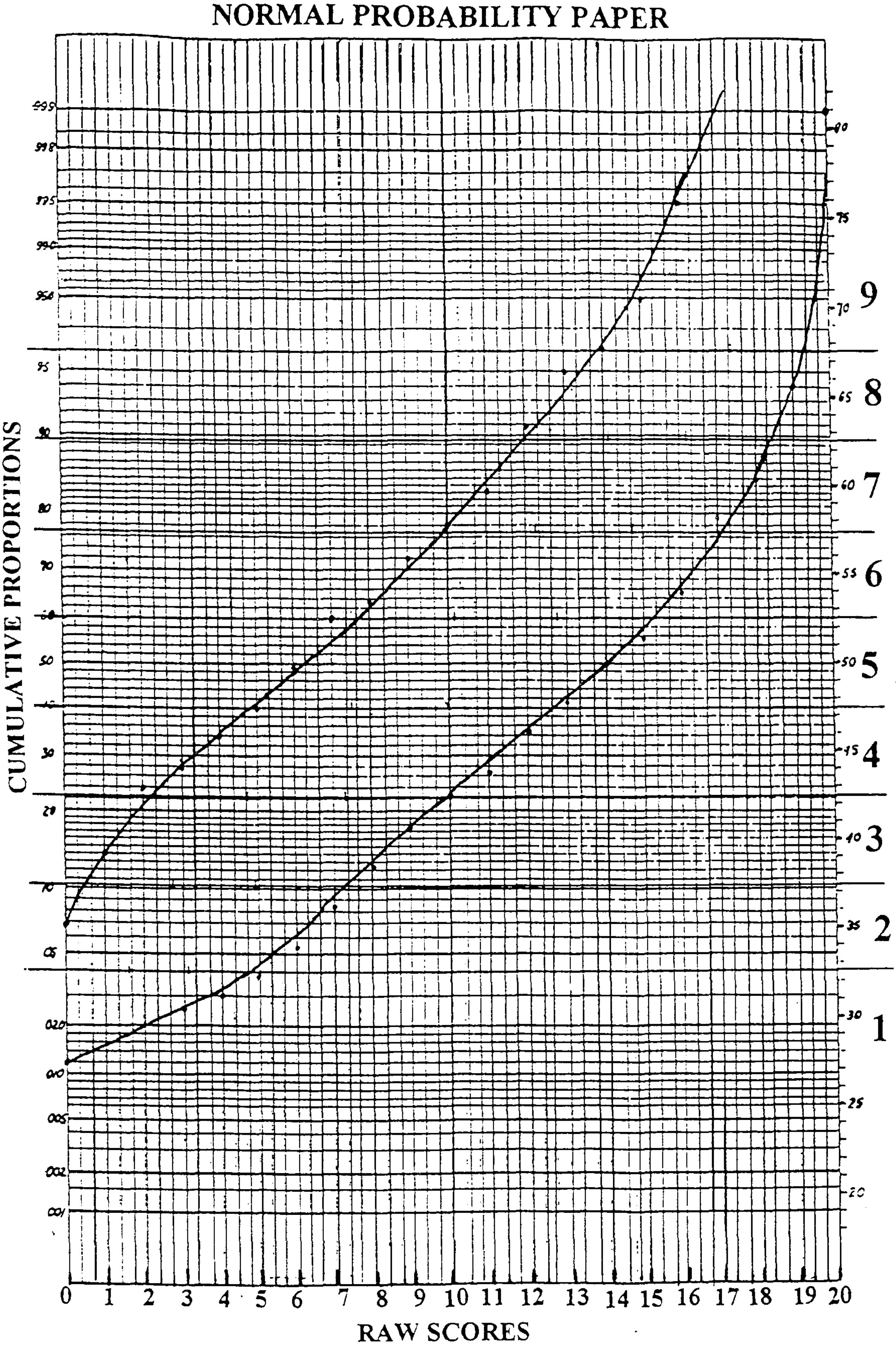


Figure 6.1: Normalisation of the raw scores of the grade one mathematics proficiency test

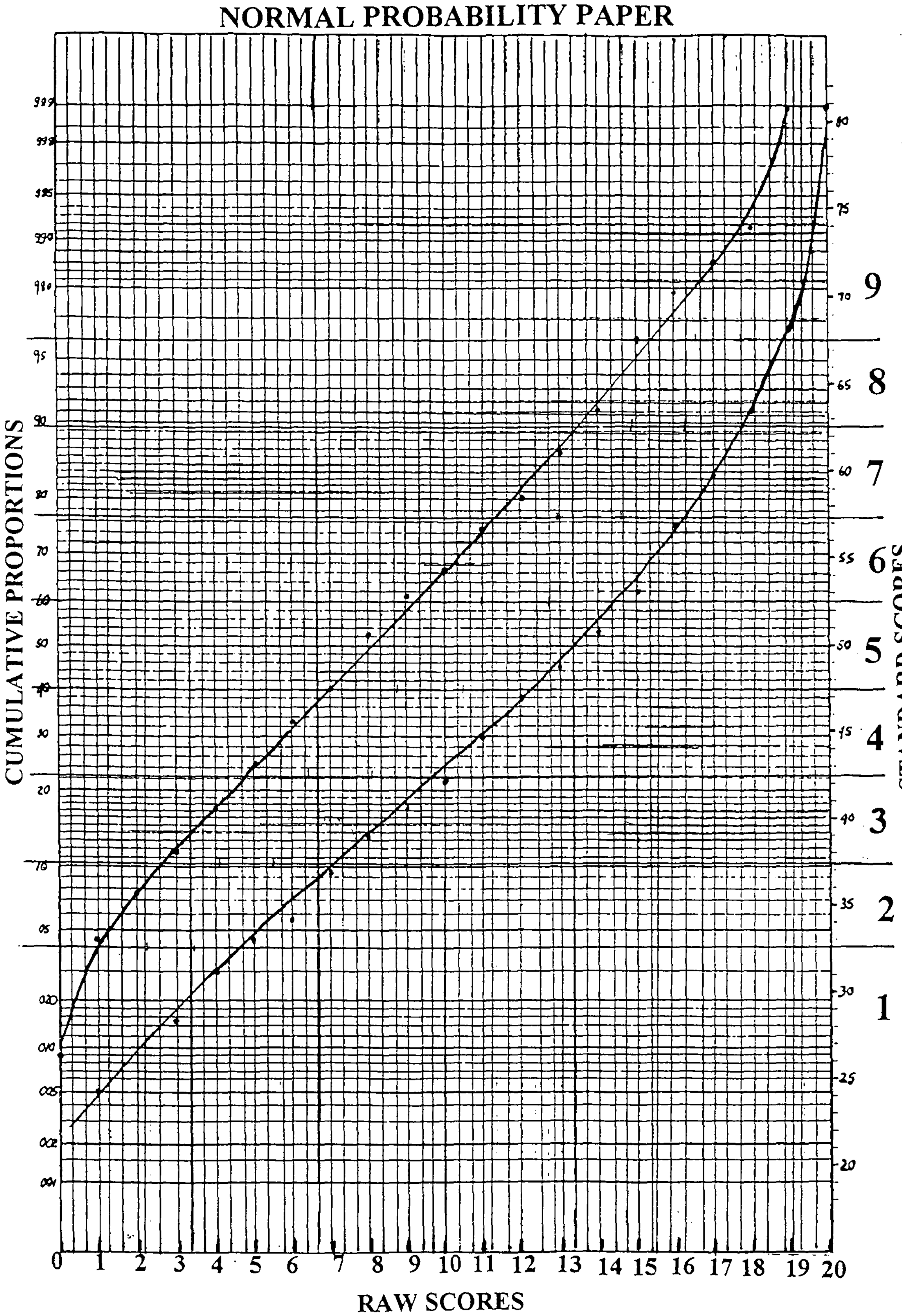


Figure 6.2: Normalisation of the raw scores of the grade two mathematics proficiency test

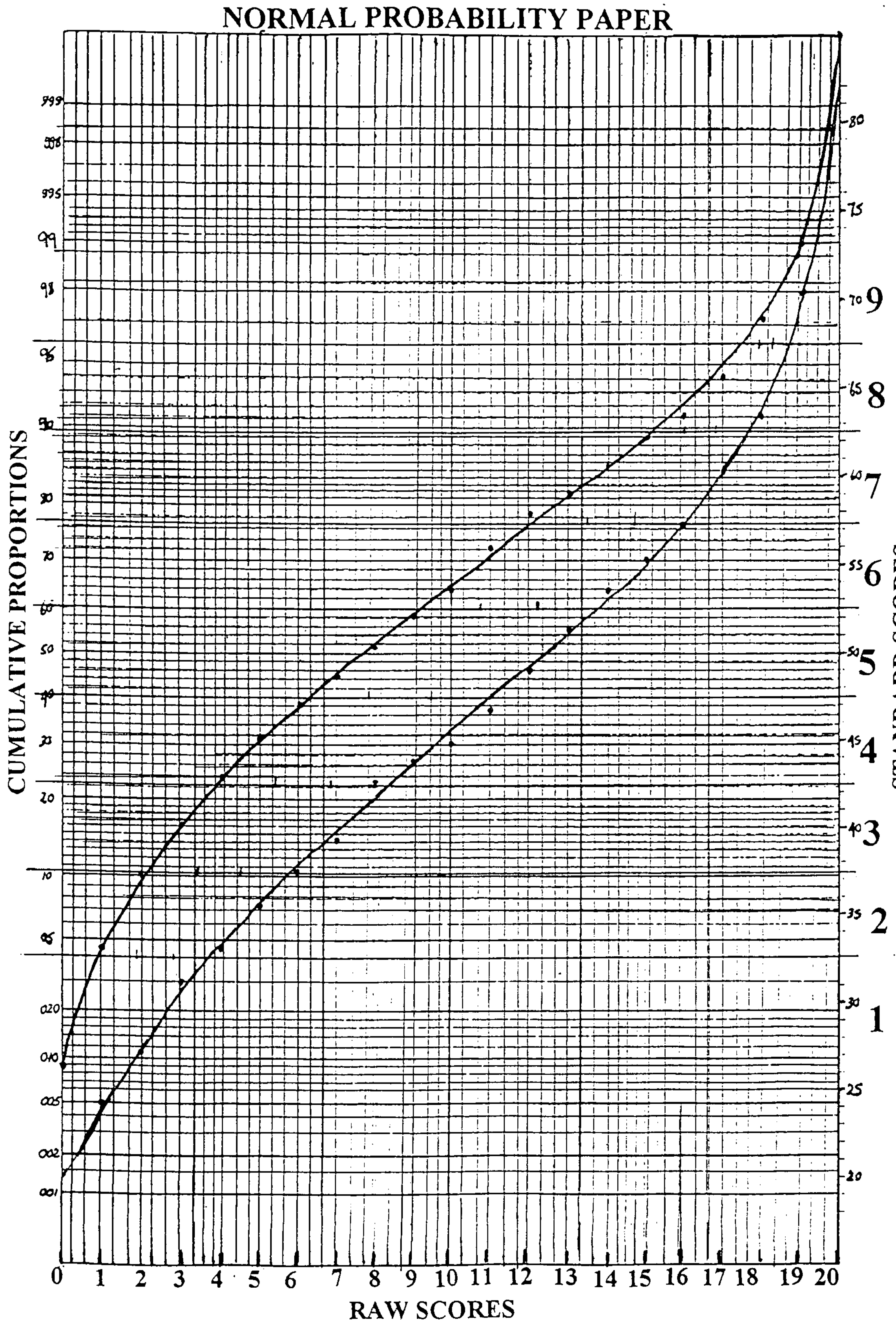


Figure 6.3: Normalisation of the raw scores of the grade three mathematics proficiency test

By indicating the raw scores on the graph paper, the cumulative percentage of the mathematics proficiency test was calculated by means of the SAS-computer program. Once all the scores had been indicated, a line was drawn through all the points in such a manner that there were an equal amount of points on both sides of the line. In this manner a graph that is continuous and smooth can be obtained. If the division of the points is normal, the line will be straight (Esterhuyse, 1997). Any raw score can, with the help of the graph, be converted into a standard score. In this study, stanines and percentile ranks were used as the normalised standard scores. This conversion was already discussed in Table 5.1 in chapter five.

6.4.3.4 Norm tables

Using the above method, the final norm tables for the grade one, two and three mathematics proficiency test were calculated. The norms were determined for each term, so that the test could be administered at any time of the year. Once a learner has completed the test, one mark is allocated for each correct answer and no marks for each incorrect answer. The marks are then tallied and the raw score obtained. This raw score can then be converted into a stanine or a percentile rank by looking up the raw score on the norm table. The norms for the appropriate grade must be used. If the learner wrote the test in the second term, then the norms of the second term must be used. The norms for the grade one, two and three mathematics test can be obtained in the manual (see Annexure D). The test is completed on the question sheet, but the answers to the questions can be obtained in the manual.

6.4.3.5 Statistical properties of the mathematical test

6.4.3.5.1 Introduction

Certain statistical properties have already been discussed, such as the norm determination and the item analysis. According to Esterhuyse (1997) there are certain statistical properties that needs to be addressed when developing and standardising a psychometric test. These properties include: the size of the sample; the number of items in each test; the means of test scores; the standard deviation of the tests; the reliability coefficient; the skewness and the kurtosis. The above information is represented in Table 6.7. This information was obtained from the first application in the final test during the first term of 2000.

Table 6.7: Statistical properties of the mathematics proficiency test

GRADE	N	NO. OF ITEMS	$\bar{X}$	s	RELIABILITY (K-R 20)	SKEWNESS	KURTOSIS
1	188	20	6,670	4,300	0,849	0,193	-0,882
2	221	20	8,548	4,143	0,808	0,119	-0,589
3	226	20	8,690	4,878	0,867	0,357	-0,681

According to the above statistical properties, each test has 20 items. The number of learners tested in grade one was 188, in grade two, 221 and in grade three, 226. The mean of the grade one test differed significantly from the mean of the grade two and three tests. The reason for this could be that during the first term of grade one the learners have not as yet been exposed to a vast number of concepts and basic mathematical operations. The mean of the grade one test was 33,35 %, the mean of the grade two test was 42,74 % and the mean of the grade three test was 43,45 %. This is the mean percentage obtained by the learners during the first term of their respective grades. The other statistical properties represented in the above table will be discussed later, but it is necessary to continue the discussion of means with the focus shifting to means with respect to the first and second applications of the test, and with respect to sex and age differences.

6.4.3.5.2 Means with respect to the first and fourth term test results

The norms for the year were determined by administering the test during the first and fourth terms of 2000. It would therefore be sensible to investigate the increase in the performance of the learners in each grade. Table 6.8 clearly shows the mean and standard deviation for the first and fourth term applications and the percentage that the performance of the learners increased.

Table 6.8: Comparison of test means between the first and fourth terms of 2000.

GRADE	FIRST TERM 2000			FOURTH TERM 2000			% INCREASED
	N	$\bar{X}$	s	N	$\bar{X}$	s	
1	188	6,670	4,300	170	13,900	4,341	36,15
2	221	8,548	4,143	200	13,470	4,072	24,61
3	226	8,690	4,878	202	12,515	4,463	19,13

From the discussion in the literature review (see paragraph 3.3.3) the basic mathematical acts that must be executed with ease improve from the first term to the fourth term. The mean percentage of the grade one learners during the first term was 33,35 %. This mean increased by 36,15 % to an average of 69,5 % during the fourth term. This increase was the greatest of the three grades. The reason for this could be that the grade one learners are not exposed to many mathematical concepts during the first term. As their exposure to the subject increases, so their mathematical knowledge improves dramatically. The improvement of the grade two and three learners is less because they have already been exposed to number concepts and basic mathematical operations, and their knowledge acquisition is more continuous. The grade two's mean was 42,74 % during the first term and this increased by 24,61 % to 67,35 %. The grade three's mean was 43,45 % during the first term and this increased by 19,13 % to 62,58 %.

It is also necessary to investigate the difference between means with respect to sex differences.

6.4.3.5.3 Means with respect to sex differences

According to the discussion in the literature review (see paragraph 4.4.1.2) the prevalence of Mathematics Disorder is equal in males and females. According to Mussen et al. (1990) girls and boys in the foundation phase perform equally well in mathematics. Males begin to do better in high school. Visser (1985) also found that there are no sex differences in mathematics achievement during early adolescence but differences do begin to occur in favour of the male learner at an older age. A *t*-test for independent groups (Huysamen, 1996) was used to determine whether there was a significant difference between the means with respect to sex differences. The results were obtained from the first administration of the mathematics proficiency test. The results are indicated in Table 6.9.

Table 6.9: Means with respect to sex differences in the first term of 2000

	FIRST TERM OF 2000													
GRADE	**t-value**	**p-value**	**MALES**					**FEMALES**				
			N	$\bar{X}$	**s**	**Min**	**Max**	**N**	$\bar{X}$	**S**	**Min**	**Max**
1	2,347	0,0200 *	92	7,413	4,490	0	17	96	5,958	4,005	0	16
2	-1,235	0,2181	115	8,217	4,487	0	19	106	8,906	3,723	1	17
3	0,900	0,3693	113	8,982	4,730	0	19	113	8,398	5,026	1	20

$p \leq 0,05$

The results indicate that boys in the first term of grade one perform significantly better in mathematics than girls in the first term of grade one. The mean percentage in the grade one test in the first term was 37,07 % for the boys and 29,79 % for the girls. The boys therefore performed on average 7,28 % better than the girls in the grade one mathematics proficiency test. The t-test also indicates a significant difference between the boys and girls averages at the 5 % level. Both girls and boys had the same minimum value of zero out of 20. The maximum value out of 20 was higher for the boys than the girls. In the grade two test, the girls performed better than the boys did in the first term. The mean percentage in the grade two test in the first term was 44,53 % for the girls and 41,09 % for the boys. The girls performed 3,44 % better on average than the boys in the mathematics proficiency test. According to the t-test, the means with respect to sex were not significant among the grade two's. In the grade two test, the minimum value out of 20 was lower for the boys than for the girls and the maximum value out of 20 was higher for the boys than for the girls. In the grade three test, the boys performed on average 2,92 % better than the girls did. Once again the means with respect to sex, according to the t-test, were not significant. The mean percentage in the grade three test in the first term was 44,91 % for the boys and 41,99 % for the girls. The maximum value obtained in the grade three test was lower for the boys than for the girls and the boys also had a lower minimum score than the girls did. There was therefore no significant difference between the means of the boys and girls in grade two and three. The boys did significantly better than the girls, in the grade one test and this could be due to various nuisance variables. The fact that the girls perform poorer in the first term of grade one could be due to variables such as adjustment to school, separation anxiety or less exposure to mathematics than boys. It would also be interesting to see if the same differences are prevalent among other language groups.

6.4.3.5.4 Means with respect to age differences

According to Geary et al. (1996) age, schooling and language can differentially affect the emergence of different components of children's early numerical competencies (see paragraph 4.4.1.3). The discussion from the literature (see paragraph 4.4.1.1) indicates that teachers much understand how children grow in their ability to encounter and understand mathematics. Piaget (1969) emphasizes the different stages of development. The majority of this sample falls into the concrete operational stage. The tests for the grade one, two and three learners differ significantly from each other. Therefore means with respect to age cannot be compared as different mathematical competencies are expected from the different age groups. Despite this, the researcher investigated the means with respect to age differences during the first administration of the test. Table 6.10 indicates the mean score for a specific age group, per grade. The researcher tried to keep the sample size the same for each respective age group.

Table 6.10: Means with respect to age differences

GRADE	**1**				**2**				**3**			
AGE IN MONTHS	**72-75**	**76-80**	**81-83**	**84-108**	**82-90**	**91-93**	**94-97**	**98-111**	**88-100**	**101-104**	**105-108**	**109-123**
N	47	44	44	53	56	44	54	67	58	59	62	47
$\bar{X}$ OUT OF 20	5,2	5,7	7,9	7,8	8,1	9,5	8,8	8,0	9,2	9,1	8,7	7,6
TOTAL $\bar{X}$	6,670				8,548				8,690			

From the above results it is evident that the total grade means increase with age. This supports the literature review in paragraph 4.4.1.1. There is a difference between the grade one, two and three means. The reason for this could be that the grade one mathematics curriculum focuses on concrete operational tasks, whereas more than three-quarters of the grade one learners fall into the pre-operational phase during the first term of grade one. The grade two and three means are approximately the same and this could be due to that fact that all these learners fall into the same phase, namely the concrete operational phase.

From the grade one results, it is evident that the older grade ones (81 – 108 months) had a better mean than the younger grade ones (72 – 80 months). The mean of the older grade ones mathematics score was above the total grade one mean. The younger grade ones achieved below the grade one mean. The Free State Department pertinently states that a learner must turn seven years in his/her grade one year. The above result supports this fact. In grade two, the younger (82 – 90 months) and older (98 – 111) groups achieve below the grade two mean. The middle age groups (91 – 97 months) obtained an average that was above the grade two mean. In grade three the mean per age group decreased with age. The younger grade three learners (88 – 100 months) obtained the highest mean, whereas the older grade three learners (109 – 123 months) had the lowest mean. The first three age groups in grade three all obtained an average that was higher than the grade mean. The older grade three learners obtained an average that was below the grade mean.

The older grade one learners obtained the highest mean in grade one and this result is supported by the literature review. In grade three, it appears that the younger learners start catching up to the older learners with regard to achievement. No literature could be found to support this occurrence.

6.4.3.5.5 Standard deviation

The standard deviation is the positive square root of the variance (Huysamen, 1990). According to Mehrens and Lehmann (1991) the standard deviation is the amount of variability in a distribution. According to Huysamen (1996) in a normal distribution 99,74 % of the testees scores lie between –3 standard deviations and +3 standard deviations. In other words 99,74 % lie within six standard deviations of the calculated average of the distribution. From the above information, the ideal standard deviation would lie between +3,33 and –3,33 (20 divided by six = 3,33). From Table 6.7 the standard deviation of the grade one, two and three learners in the first term was 4,300; 4,143; and 4,878 respectively. The standard deviation for grade one was 0,97 above the ideal; for grade two 0,813 above the ideal; and for grade three 1,548 above the ideal. The standard deviation of the grade one, two and three learners in the fourth term was 4,341; 4,072; and 4,463 respectively. The standard deviation for grade one was 1,011 above the ideal; for grade two 0,742 above the ideal; and for grade three 1,133 above the ideal. The greatest standard deviation occurred among the grade three learners. According to Esterhuyse (1997) the greatest standard deviation in the *ESSI Reading and Spelling Test* was in the English reading and spelling test. The author continues to state that this could be attributed to the heterogeneous school populations in the English medium schools.

6.4.3.5.6 Skewness

Data distributions can take on the shape of four types of distributions: a normal distribution; a positively skewed distribution; a negatively skewed distribution; and a rectangular distribution (Mehrens & Lehmann, 1991). The ideal distribution is a normal distribution and the statistic measures the deviation from the norm. Once again the values vary from -3 to $+3$. A positive skewness value indicates that most of the learners obtained a score lower than the mean, this would give a positively skewed distribution and this might occur when a test is too difficult. A negative skewness value indicates that most of the learners obtained a score higher than the mean, this would give a negatively skewed distribution and could also indicate that the test was too easy (Esterhuyse, 1997). According to Table 6.7 the skewness values are all positive and only slightly greater than zero. This indicates a slightly positive skewness. The reason for this could be that the test was administered in the first term and some of the items could be too difficult for that time of year. The values are only slightly greater than zero, indicating a symmetrical distribution. It must be kept in mind that the raw scores are normalised before the norms are calculated (see paragraph 6.4.3.3).

6.4.3.5.7 Kurtosis

According to Huysamen (1990) kurtosis of a curve refers to the flatness or peakedness of the centre of the curve. If a curve is more peaked it is said to be leptokurtic, if it is more flat it is said to be platykurtic. The curve of a normal distribution is mesokurtic. A normal curve will have a kurtosis of zero, a peaked curve will have a positive kurtosis value and a flat curve will have a negative kurtosis value. The kurtosis values for the grade one, two and three test are all negative which indicates a more flat curve, but the values are so small that the curves all have relatively normal distributions.

6.4.3.5.8 Reliability

Two types of reliability will be considered in this paragraph, namely; parallel-forms reliability and the test-retest reliability. Two processes were followed to determine the parallel-forms reliability, namely; the alpha-coefficient and the split-half method. A discussion of each of the above is necessary to determine to what extent the mathematics proficiency test is reliable.

6.4.3.5.8.1 Parallel-forms reliability

a) Alpha-coefficient

According to Huysamen (1996) when the items are marked right and wrong, the alpha-coefficient is equal to the K-R 20 formula. In Table 6.7 the alpha-coefficient is given. The preferred reliability coefficient for a standardised test is 0,85 or higher. The reliability coefficients for grade one, two and three were: 0,85; 0,83; and 0,87 respectively. The grade one and grade three reliability coefficients were above 0,85 and even though the grade 2 reliability coefficient was below 0,85 it is still greater than 0,8. The alpha-coefficients therefore indicate that the tests are reliable and measure consistently.

b) Split-half method

The split-half method is discussed in paragraph 5.3.4.1. This method halves the length of the test by grouping all the even numbers as one half of the test and all the uneven numbers as the other half of the test. The two halves are then correlated. The results are given in Table 6.11.

Table 6.11: Split-half reliability results

Grade	N	Coefficient
1	188	0,796 *
2	221	0,680 *
3	226	0,770 *

* $p \leq 0{,}0001$

The above results indicate that the split-half reliability coefficients are smaller than the alpha reliability coefficients but the coefficients are still significant on the 0,01 % level. The reliability results vary from satisfactory to good.

6.4.3.5.8.2 Test-retest reliability

According to paragraph 5.3.1, test-retest reliability is a measure of a test's stability. To determine test-retest reliability, a test must be administered on two occasions, on a representative sample of the population for which the test is intended. In this research, the same mathematics proficiency test was administered on two occasions, during the first and fourth terms of 2000, to the same grade one, two and three learners. It could be assumed that the learners should, in general, achieve a higher score during the fourth term than the first term. A period of two terms elapsed between the testing, which is a relatively long period of time. Therefore the first administration should not have had an effect on the second administration.

Test-retest reliability means that the same test can be administered at any time, to the same population and the same result should be obtained. Therefore if the test-retest reliability of the mathematics proficiency test is not significant, it does not necessarily mean the test is not reliable, it could just be an indication of the extent to which the variable being measured is unstable. Despite this fact, the test-retest reliability was significant for each of the grade one, two and three tests on the 0,01 % level. The mathematics proficiency tests can therefore be said to be reliable and can, with confidence, be administered in practice. The results are represented in Table 6.12.

Table 6.12: Correlation coefficients between the first and second administrations

Grade	N – First Administration	N – Second Administration	Correlation coefficients
1	188	170	0,530 *
2	221	200	0,593 *
3	226	202	0,690 *

* $p \leq 0{,}0001$

6.4.4 Phase four – Validity

In this study emphasis is placed on the content validity and the criterion-related validity.

6.4.4.1 Content validity

In 5.4.4 content validity was defined as the degree to which the items in a test represent the total universe of items which could have been compiled in terms of the curriculum and teaching objectives. Content validity of a measuring instrument cannot be given in terms of a quantitative analysis. As was referred to in 6.4.1.3 the final draft of the mathematics proficiency test for grade one, two and three learners was submitted to several of the contact teachers for comment before drafting the final test. The teachers were asked to comment with respect to the type of items that were selected and the degree of difficulty of each. The teachers felt that the test covered the entire syllabus and would serve as a good measure of a child's mathematical ability. The test was also given to the Mathematics, Mathematics Literacy and Mathematical Science Learning Facilitator of the Free State Education Department for comment. The learning facilitator gave the test to various experienced foundation phase teachers and the test was returned with no changes. The contact teachers and the learning facilitator are experienced in their field and could objectively evaluate the validity of the content of the items selected.

6.4.4.2 Predictive validity

Referring back to 5.4.5 distinction is made between predictive validity and concurrent validity. Concurrent validity was not investigated in this study but the predictive validity of the test was explored. The way in which predictive validity was investigated was to correlate the score obtained on the mathematics proficiency test in the first term of 2000 with a score obtained from the latest mathematics evaluation (third or fourth term evaluation). The score given by the teachers for an evaluation mark was then converted to a percentage. The evaluation mark given by some schools were in terms of a percentage and some in terms of a symbol. The symbol was converted to a percentage as indicated in Table 6.13:

Table 6.13: Percentage intervals with respect to the foundation phase symbols

SYMBOL	MEANING	% - INTERVAL	PERCENTAGE ASCRIBED
B	Beyond achieved	80 – 100	90
A	Achieved	60 – 79	70
P	Partially achieved	40 – 59	50
N	Not yet achieved	39 and below	30

The predictive validity of the test was then calculated by correlating the mark obtained on the mathematics proficiency test during the first term with the scholastic percentages obtained in the fourth term. The results are indicated in Table 6.14.

Table 6.14: Predictive validity of the mathematics proficiency test

Grade	N	Correlation coefficients
1	170	0,380 *
2	200	0,573 *
3	202	0,540 *

* $p \leq 0{,}0001$

The above results indicate that the predictive validity of the grade one, two and three tests are all significant on the 0,01 % level. At first glance the correlation scores appear small, especially for the grade one learners. According to paragraph 5.4.5 the significance of the results is also dependent on the practical interest of the result. Therefore the predictive validity correlation coefficient of the grade one test has a medium effect size, despite the fact that the correlation coefficient appears to be small. The grade two and three correlation coefficients have a large effect size.

6.4.5 Qualitative analysis

The main aim of this research was to develop and standardise a mathematics proficiency test, that would enable a psychologist or an educationist to quantify a learner's mathematical level of functioning. While quantifying the learner's mathematical level of functioning, certain qualitative properties can be observed. In paragraph 4.2, a clear definition of a learning restraint, learning disability and learning disorder was given. The psychologist or educationist can use this test to diagnose any one of the above conditions. A *learning restraint* in mathematics occurs when certain factors cause children not to actualise their mathematical potential. A mathematical *learning disability* occurs when a child has an identifiably deficiency in his/her given mathematical potential. When a mathematical learning restraint or disability becomes significant, a diagnosis of *Mathematics Disorder* can be made.

Just as this test can serve as a diagnostic tool, manifestations of problems occurring in mathematical achievement can be isolated. Errors in addition, subtraction, multiplication, division and fractions can be identified if the learner continually gets a question wrong that has to do with one specific mathematical task, for example, fractions. In so doing the psychologist or educationist can identify an area in which a learner is experiencing problems. The context of universal and specific mathematics tasks must always be kept in mind to determine whether the learner is struggling with a specific aspect of the Free State Department of Education curriculum or whether it is a universal task the learner is unable to grasp.

The psychologist and educationist should also take into consideration the developmental aspects that can have an effect on mathematical achievement. The tester must be able to understand capabilities such as assimilation and accommodation which lead to the development of symbolic thinking. The tester must also be aware of the fact that if a child is unable to do subtraction, it could be due to an inability to reverse actions mentally. Elementary mathematics depends on the mental operations that develop during the concrete operational stage. If these mental operations do not develop the child will not be able to proceed from elementary mathematics to more advanced levels of the subject.

The instructions of the test clearly state that the word sums can be read to the testees. If the tester becomes aware that a child is unable to do the word sums when reading the sums on their own, but experiences no problems when the word sums are read to him/her, then certain hypotheses about the child's language ability can be formulated.

The mathematics proficiency test can be administered with a battery of tests, so that a holistic view of the child's scholastic level of functioning can be determined. From this, an extensive qualitative analysis can be made.

6.5 CONCLUSION

In the above chapter the researcher focused on the results of the development and standardisation of the *VASSI* Mathematics Proficiency Test for learners in the foundation phase. Before the statistical results were investigated and discussed, emphasis was placed on the goal of the investigation and the research method. The four phases namely: the construction of the preliminary questionnaire; item analysis and selection; norm determination; and the validity of the test were discussed in detail. The sample in each phase was represented. During the item analysis and selection phase, which was made possible by using the CTT, the original 40 item test for each grade also was given. The final mathematics proficiency test, as well as the norms per term, were determined and are represented in Annexure C and D. The discrimination values of most of the items selected are excellent, very good or good. All the statistical properties were investigated and emphasis was placed on the differences of means with respect to the first and second administration, sex differences and age differences. From this certain hypotheses could be made and future research considered. The *VASSI* Mathematics Proficiency Test also proved to be reliable and valid and can therefore be administered in practice as a diagnostic tool.

7. CONCLUSION AND RECOMMENDATIONS

7.1 INTRODUCTION

Mathematics proficiency is a competency area assessed by many schools for grade advancement. The barriers to learning at school exist at many levels within and outside the child. We need to adopt a child-centered team involving family, school personnel and professionals in overcoming these learning problems. When mathematics is identified as an area of need for a particular learner, assessment is necessary to determine whether mathematics is actually the problem. Other factors such as ability, psycho-neurological causes, reading and spelling ability also need to be assessed. The development and standardisation of a mathematics proficiency test enables the psychologist and educationist to conduct a comprehensive evaluation, with a specific proficiency test, which could identify mathematics as the learner's problem area.

In view of the above, the researcher developed and standardised an English mathematics proficiency test for the Free State Department of Education. The test is called the *VASSI* Mathematics Proficiency Test. The test complies to the following criteria:

a) the test is applicable to grade one, two and three learners;
b) the norms per term are available, so that the test can be administered at any time of the year;
c) the test consists of universal mathematics concepts, so that the usage of the test will not be limited;
d) the test can be administered to groups or individuals;
e) the test can be used diagnostically; and
f) the test should be of value to future generations of learners.

As was stated in paragraph 3.4.1 and in point (c) above, there are universal mathematical tasks during childhood. With the development of the *VASSI* Mathematics Proficiency

Test it is necessary to investigate whether these universal tasks are included in the developed test. According to Westman (1990) there are basic mathematical tasks that must be executed with ease after the foundation phase. In the *VASSI* Mathematics Proficiency Test the following universal tasks are included: the saying, reading and writing of words for numbers; writing and reading the figures for numbers; counting; understanding the relative value of a number compared with other numbers; reading, writing, and understanding the mathematical signs; recognising the arrangement of numbers to do addition, subtraction, multiplication and division; understanding the calculation and placement significance of 0; and understanding the placement value of all numbers. Of the eight mathematical tasks that Wallace et al. (1992) mention, the mathematics proficiency tests contain seven of the tasks, namely: numeration; operations; money; time; fractions; geometry and word problems. Measurement is the only task that is not assessed in the tests.

The rationale of this test is based on the fact that the first task in helping a child who is struggling with mathematics is to identify the problem area. Knowledge breeds wisdom as the identification and diagnosis will isolate the problem and enable teachers to assist a child in mathematics from an early age. The perceived negative connotation attached to mathematics performance at school could be addressed if a child is equipped with the basic skills necessary to do mathematics. The learner will continue his/her school career with a positive perception of the subject. This positive perception is half the battle won.

The development and standardisation of the *VASSI* Mathematics Proficiency Test consisted of four phases that were carried out over a period of two years. The conclusion of the results, per phase, are given below.

7.2 CONCLUSION OF RESULTS

The following phases were carried out while developing the *VASSI* Mathematics Proficiency Test for learners in the foundation phase:

Phase one: Construction of preliminary questionnaire
(Third term, 1999)

Phase two: Item analysis and selection
(Fourth term, 1999)

Phase three: Determination of norms
(First and Fourth term, 2000)

Phase four: Validity study
(Fourth term, 2000).

A psychometric test has to be reliable and valid so that it adheres to all psychometric requirements. The effectiveness of the psychometric test was dependent on the type of items selected to compile the test. For a tester to determine the level a child is functioning on, norms were needed. If we consider the results of phase one, the construction of the preliminary questionnaire, it is important to note that the sample consisted of learners in grade one, two and three, whose home language was English. During phase one, the seven contact teachers set up 20 mathematics items (per grade) that ranged in difficulty and covered the whole years curriculum. The items were given to the respective grades as a class test. The results were then computerised and 40 items, per grade, were selected in the following manner:

a) 10 items that 80 % of the learners answered correctly;
b) 10 difficult items that only 20 % of the learners answered correctly; and
c) 20 items that were of average difficulty.

The discrimination value of each of the items selected was higher than 0,4. The 40 item test was then given to the contact teachers for comment before the final experimental mathematics proficiency test was compiled.

In phase two, the 40 item test was given to the grade one, two and three learners at 13 schools throughout the Free State. The goal of this phase was to obtain as much objective information about the items as possible. It is important to note that the teachers were encouraged to read the word problems to the children, as an accurate estimation of the child's mathematical ability, without the hindrance of language, was investigated. A total of 625 learners were tested during this phase. To determine the final 20 items the Classical Test Theory (CTT) was used to determine the items difficulty value and item variance. The CTT results were calculated with the help of the SAS-(SAS Institute, 1985) and SPSS-computer programs (SPSS Incorporated, 1983). From these results the *VASSI* Mathematics Proficiency Test was compiled (see Annexure C). The discrimination values of the items selected are from average to excellent and the items range in difficulty from easiest to most difficult.

A child's mathematical achievement cannot be determined by obtaining a raw score. Therefore in phase three, norms had to be determined for the grade one, two and three tests. To determine the norms for each term, the test was given to the same learners in the first and fourth terms. Inter polarisation was used to determine the norms for the second and third terms. The instructions to the teachers during this phase were as follows:

1. No calculators may be used in the test for grade one, two, and three.
2. No time limit is imposed (within limits).
3. You may read the word sums to the children (we wish to test their mathematics ability and not their English reading ability).
4. You may explain the meaning of words to the children.
5. Important details such as the name, home language, age and gender must be completed. Please make sure the learner has completed his/her details correctly (especially the home language).
6. The test must be completed on the questionnaire.
7. Extra paper may be given for rough work.

A total of 635 learners were tested during the first term of 2000 and 572 learners were tested during the fourth term of 2000. The number of boys and girls were relatively proportional per grade and in total. The cumulative frequency percentage of the mathematics proficiency test was calculated with the help of the SAS-computer program

by indicating the raw scores on graph paper. Any raw score can, with the help of the graph, be converted into a standard score. In this study, stanines and percentile ranks were used as the normalised standard scores. Using this method, the final norm tables for the grade one, two and three tests were calculated (see Annexure D). Certain statistical properties of the tests were also determined. The Kuder-Richardson reliability coefficients of the grade one and three test were greater than 0,85 and of the reliability coefficient of the grade two test was greater than 0,80. This indicates that the test is reliable and consistently measures mathematical ability. Other interesting statistical properties were also investigated. The means with respect to the first and fourth term results indicated that the grade one, two and three means increased by 39,15 %,
24,61 % and 19,13 % respectively. The basic mathematical acts that must be executed, with ease, improved from the first to the fourth term. The grade one mean increased the most and this could be due to various factors. In the first term of grade one, the learners have not been exposed to a lot of mathematical concepts. As their exposure increases their knowledge increases. The grade two and three's difference in means was smaller, this could be due to a more consistent acquisition of mathematical knowledge.

Another aspect that was investigated was the means with respect to sex differences. According to Mussen et al. (1990) males and females, on average, have different patterns of performance in different intellectual areas but the authors continue to state that girls and boys do equally well in mathematics. Visser (1985) obtained similar results. Visser found that no sex differences, in general mathematical achievement, occurred in early adolescence. While determining norms for the *VASSI* Mathematics Proficiency Test, the researcher found that according to the *t*-test, the means with respect to sex differences were not significant among the grade two and three learners. These results therefore support the above findings. Yet, according to the *t*–test for independent groups, the grade one learners obtained a significant difference with respect to means between the boys and girls at the 5 % level. This is contrary to the findings of Esterhuyse (1997). Esterhuyse found that the girls do significantly better than the boys in reading and spelling and these results correlate with their mathematical achievement. Therefore according to Esterhuyse the girls should perform better at mathematics than the boys. The fact that the girls perform poorer than the boys in the first term of grade one, in this study, could be due to various factors. Certain hypotheses, such as the fact that girls could be exposed to less mathematics than boys, need to be investigated. Other variables such as adjustment to school, emotional factors, and social factors, as well as whether this is prevalent among other language groups need to be investigated.

The means with respect to age differences were also investigated. The grade one's mean is less than the grade two and three means, which are relatively the same. This could be due to the fact that the grade one learners fall into two of Piaget's (1969) phases, namely the preoperational phase and the concrete operational phase, whereas the grade two and three learners are both in the concrete operational phase. This study also shows that the younger grade one learners obtain a lower mean than the older grade one learners. According to Piaget (1969), the preoperational phase is from two to six years and the concrete operational phase if from seven to eleven years. Many of the younger learners in grade one are still in the pre-operational phase. This limits them from making a coherent whole out of an explanation. According to Souviney (1989) at the age of seven, the child begins to reflect, unify and contradict. In this concrete operational phase there is a significant increase in the child's ability to carry out mental actions. The above research of Souviney supports the results of this study. The grade two's means with respect to age differences are rather erratic as the younger and older grade two learners have a lower mean than the two middle age groups. The grade three results need careful consideration, as the younger grade three learners perform better than the older grade three learners. The mean of the four age groups of the grade three learners decrease consistently from the youngest grade three learners to the oldest grade three learners. No literature could be found that supported this result. According to Smith (1991) third grade appears to be the transitional phase in which children make the transition from counting mathematics facts to recalling number facts automatically. The author also states that half the third grade class will still count, while the other half will retrieve knowledge from their memory. Whether this is the case among these grade three learners is a possible hypothesis. Other variables, such as the child, the teacher or the task, could have had an adverse effect on the above result.

Lastly, according to the statistical results, the criterion-related validity was significant on the 0,01 % level. The content validity was investigated by the Mathematics, Mathematics Literacy and Mathematical Science Learning Facilitator of the Free State Department o Education and various competent contact teachers. Content and criterion-related validity proves that this test satisfies its intended purpose.

The *VASSI* Mathematics Proficiency Test is not only a diagnostic tool. According to paragraph 6.4.5, certain qualitative conclusions can be made that can aid an identification and intervention process.

As the above results were obtained and certain conclusions were made, several recommendations for future research came to the fore.

7.3 RECOMMENDATIONS FOR FUTURE RESEARCH

In view of the above, the recommendations with regard to future research are as follows:

1. A mathematics proficiency test for grade four to grade seven learners can be developed and standardised in the same manner as the *VASSI* Mathematics Proficiency Test.

2. Norms for other language groups can be determined for the *VASSI* Mathematics Proficiency Test.

3. Due to the difference of the grade one means with respect to sex, an investigation can be carried out to determine whether these results are prevalent among other language groups.

4. Due to the fact that the there was no significant difference between the grade two and three means with respect to sex, investigation can be carried out to determine whether this is true for all primary school grades.

5. The results of the grade one means, with respect to age differences, can be investigated in other competency areas, so that an optimal school going age could be determined.

6. The grade three means with respect to age differences can be investigated, among other language groups, to see if this occurrence was only a once-off result or whether grade three is a significant developmental period for certain learners.

7. The prevalence of Mathematics Disorder among South African children can be investigated.

8. The influence of the child, the teacher and the task on South African learners in the foundation phase can be investigated, so that specific causes of problems in mathematics can be determined.

9. The prevalence of the various basic operation errors could be quantified.

10. Using the *VASSI* Mathematics Proficiency Test as an assessment tool, a remedial program can be developed that will help learners, in the foundation phase, to overcome their problems in mathematics.

When a general scholastic evaluation is carried out, various psychometric tests are administered such as aptitude tests, reading tests, spelling tests and developmental tests. The administration of a mathematics psychometric test can only improve the quality of the evaluation and help the psychologist to obtain results that are comprehensive and objective. These results can help to identify a problem area or aid the psychologist in making a diagnosis, so that an intervention program can be recommended as early as possible.

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ANNEXURE A

LETTER OF PERMISSION

FREE STATE PROVINCIAL GOVERNMENT

Education

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55 Elizabeth Street • CR Swart Building • Bloemfontein
Tel. : +27 (0) 51 - 4074911 • Fax : +27 (0) 51 - 4074036

Enquiries : Mr W.B. van Rooyen/LB — Tel. : 051-405 5504
Reference no. : 0.1/11/3/3 — Fax. : 051-403 3421

04 May 1999

Mrs C P Vassiliou
38 Lorraine Street
Bayswater
BLOEMFONTEIN
9300

Dear Mrs Vassiliou

REQUEST TO CONDUCT RESEARCH IN THE FREE STATE DEPARTMENT OF EDUCATION

1. Your request dated 29 April 1999 refers.
2. Research title applied for: **The development and standardisation of a mathematics proficiency test for Junior Primary learners.**
3. Permission is granted for your request to conduct research in the Free State Department of Education under the following conditions:
   - 3.1 The names of learners/teachers must be provided by the principals.
   - 3.2 Principals/Teachers/Learners participate voluntarily in the project.
   - 3.3 The names of the schools and teachers/learners involved remain confidential in all respects.
   - 3.4 Taking down tests with learners must take place outside normal tuition time of the school.
   - 3.5 This letter must be shown to all participating persons.
   - 3.6 A report on this study must be donated to the Free State Department of Education after completion of the project where it will be accessed in the Education Library, Bloemfontein.
   - 3.7 You must address a letter to the Head: Education, for attention
     W.B. van Rooyen
     Room 1211
     C.R. Swart Building
     BLOEMFONTEIN
     9301
     accepting the conditions as laid down.
4. We wish you every success with your research.

Yours sincerely

HEAD: EDUCATION

ANNEXURE B

FREE STATE DEPARTMENT OF EDUCATION EXPECTED LEVELS OF PERFORMANCE FOR MATHEMATICAL LITERACY, MATHEMATICS AND MATHEMATICAL SCIENCES IN THE FOUNDATION PHASE

MATHEMATICAL LITERACY, MATHEMATICS & MATHEMATICAL SCIENCES

Definition:

Mathematics is the construction of knowledge that deals with qualitative and quantitative relationships of space and time. It is a human activity that deals with patterns, problem solving, logical thinking, etc, in an attempt to understand the world and make use of that understanding. This understanding is expressed developed and contested through language, symbols and social interaction.

Rationale:

Mathematical literacy, mathematics and the mathematical sciences as domains of knowledge are significant cultural achievements of humanity. They have both utilitarian and intrinsic value. All people have a right of access to these domains and their benefits. These domains provide powerful numeric, spatial, temporal, symbolic, communicative and other conceptual tools, skills, knowledge, attitudes and values to:

- analyse;
- make and justify critical decisions; and
- take transformative action.

Thereby empowering people to:

- work towards the reconstruction and development of South African society;
- develop equal opportunities and choices
- contribute towards the widest development of society's cultures
- participate in their communities and in the South African society as a whole in a democratic, non-racist and non-sexist manner;
- act responsibly in protecting the total environment
- interact in a rapidly-changing technological global context
- derive pleasure and satisfaction through the pursuit of rigour, elegance and the analysis of patterns and relationship;
- understand the contested nature of mathematical knowledge; and
- engage with political organisational systems and socio-economic relations.

1. DEMONSTRATE UNDERSTANDING ABOUT WAYS OF WORKING WITH NUMBERS

The development of number concept is an integral part of mathematics. All learners have an intuitive understanding of the number concept. This outcome intends to extend that understanding. Its aim is to enable learners to know the history of the development of numbers, number systems and use of numbers as part of their tool kits when working with other outcomes. Solving problems, handling information, attitudes and awareness may depend crucially on a confident understanding and use of number.

ASSESSMENT CRITERIA	RANGE STATEMENT	PERFORMANCE INDICATORS	LEVEL 1	LEVEL 2	LEVEL 3	LEVEL 4
1. Evidence of the use of heuristics to understand number concept	1.1 Use of personal experiences to show the significance of number	Learner: Identifies situations in their environment where numbers are used	Learner: Identifies uses of numbers in personal environment (myself, home)	Learner: Identifies uses of numbers in immediate environment (school, village)	Learner: Identifies uses of numbers in large environments (town, shops)	Learner: Identifies uses of numbers in larger environment (farms, factories)
		Counts a collection of objects, maintaining order in numbers	Counts real objects	Counts up to 100	Counts up to 500	Counts up to 5000
	1.2 Express number in words and symbols	Writes number symbols and number names	Does prewriting exercises	Writes and reads ordinal and cardinal numbers (0 to 24)	Writes and reads ordinal and cardinal number (0 to 104)	Writes and reads ordinal and cardinal numbers (0 to 1004)
		Skip-counts forwards and backwards from a given number	Skip-counts using body experiences	Counts in 2's and 3's	Counts in 2's, 3's, 5's and 10's	Counts in 2's, 3's 5's, 10's and 100's)
		Uses number knowledge to develop strategies to solve problems	Solves incidental problem situations (one operation with two numbers < 12	Solves problems (one operation with two numbers < 24)	Solves problems (one operations with two numbers < 104)	Solves problems (one operation with two or three numbers < 1004)
2. Evidence of knowledge of number history	2.1 Understand counting as an historical activity	Represents numbers in different cultures	Counts on own fingers	Counts by matching with pebbles, tallies, etc	Counts by matching (larger numbers)	Counts by matching (larger numbers
	2.2 Show knowledge of the history of counting in their own communities, history of Roman numerals and Arabic numerals	Tells stories about the development of counting practices in their own communities	Listens to stories about counting	Describes different ways of counting	Compares different counting styles in different cultures and communities in South Africa	Counts in different languages
		Recognises, writes and reads Roman numerals	Recognises I, II and III (on clocks)	Recognises I, XII (on clocks)	Uses I to XII	Uses I to XX

		2.3	Understand importance of place value	Performs operations where place value is used	Performs operations with one digit numbers	Performs operations with two digit numbers up to 24	Performs operations with two and three digit numbers up to 104	Performs operations with three digit numbers up to 1004
3.	Estimation as a skill	3.1	Estimate lengths, heights, masses, volumes and time	Uses body parts to estimate and measure length and height	Uses smaller, bigger, the same, etc	Uses language of estimation, e.g. nearly, about, as much as , the same as, etc	Uses language of estimation	Uses language of estimation
				Estimates length (cm, m, km)	Uses right vocabulary to compare lengths of objects	Estimates and measures lengths (cm, up to 1 m)	Estimates and measures lengths (cm, m)	Estimates and measures lengths (cm, m, 1km)
				Estimates mass (g, kg)	Uses right vocabulary to compare mass of objects	Estimates and compares mass of objects	Estimates and measures mass of objects (kg)	Estimates and measures mass of objects (g, kg)
				Estimates volume (liter)	Uses right vocabulary to compare volumes	Compares and estimates volumes	Estimates and measures volumes (up to 1 liter)	Estimates and measures using liter and ml
				Estimates time (minutes, hours, days, weeks, months, seasons, years) Chooses and uses an appropriate measuring instrument and an appropriate measuring unit to check	Tells the day of the week, the month and parts of the day	Knows the four seasons Reads the time (hours)	Knows days, weeks, months, seasons, years Reads the time (hours, quarters)	Reads the time (hours, minutes
4.	Performance of basic operations	4.1	Add and subtract positive whole numbers	Solves a variety of computational problems (basic operations) posed in various contexts	Solves different types of concrete simple problems	Solves different kinds of word problems (uses one-step operations)	Solves different kinds of word problems (uses two-step operations)	Solves different kinds of word problems (uses multi-step operations)
		4.2	Multiply and divide positive whole numbers	Uses and explains preferred methods in a coherent way				
		4.3	Do simple mental calculations	Uses multi-step operations	Adds and takes away groups and shares	Adds, subtracts, multiplies and divides positive whole numbers up to 24 Uses mental calculations	Adds, subtracts, multiplies and divides positive whole numbers up to 104 Uses mental calculations	Adds, subtracts, multiplies and divides positive whole numbers up to 1004 Uses mental calculations
		4.4	Use calculators to check					

5.	Knowledge of fractions	5.1	Share and divide as an introduction to fractions	Solves practical problems where real objects are shared equally, leading to fractions	Share/groups real objects equally (Up to 4 parts)	Solves concrete problems with fractions as answers	Represents halves and quarters	Uses graphic representations of $\frac{3}{4}$, $\frac{1}{2}$, $\frac{1}{4}$, 1/5, 2/5, 1/10
		5.2	Use decimal fractions and place value	Demonstrates that fractional parts can be put together again to form a whole Recognises, reads and writes the names and notation for common fractions	Puts parts of one object together Recognises 'a half'	Knows two halves equal a whole Reads 'a half'	Knows four quarters equal a whole Reads names of $\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{4}$	Puts together to form a whole Reads and writes names of fractions
		5.3	Do operations on money	Is able to do addition and subtractions using Rands and cents	Recognises R and cents	Adds and subtracts R and cents	Adds and subtracts R and cents apart	Knows one R = 100 cents Calculates change
6.	Solving of real life simulated problems	6.1	Solve real life or simulated problems	Uses available knowledge of numbers and their environment to solve problems from familiar contexts	Identifies and solves concrete problems	Solves simple word problems	Solves a variety of word problems	Solves a large variety of word problems

2. MANIPULATE NUMBERS AND NUMBER PATTERNS IN DIFFERENT WAYS

Mathematics involves observing, representing and investigating patterns in social physical phenomena and within mathematical relationships. Learners have a natural interest in investigating relationships and making connections between phenomena. Mathematics offers ways of thinking, structuring, organising and making sense of the world.

ASSESSMENT CRITERIA	RANGE STATEMENT	PERFORMANCE INDICATORS	LEVEL 1	LEVEL 2	LEVEL 3	LEVEL 4
1. Identification of the use of numbers for various purposes	1.1 Give own understanding of number manipulation from personal experiences	Learner: Gives examples of the use of numbers in everyday activities Gives purposes for the use of numbers in given examples Gives examples of the use of numbers in media and information	Learner: Gives examples of the use of numbers from own environment	Learner: Gives examples of the use of numbers from direct environment	Learner: Gives examples and purposes of the use of numbers from home, at school in village, in newspaper, etc	Learner: Gives examples and purposes of the use of numbers in different media
	1.2 Show link between patterning and repetition	Gives examples of repetitions in life Identifies common patterns from immediate environment	Observes given patterns	Recognises repetitions in poems and songs	Gives examples of repetitions in music and stories	Identifies patterns in art and nature
	1.3 Identify, repeat and continue patterns of sounds, body movement, body position, art, music and stories	Repeats given patterns	Repeats body movements and positions	Identifies and repeats patterns in art and music	Identifies and continues patterns in art	Identifies and continues patterns in arc and nature
2. Evidence that number patterns and geometric patterns are recognised and identified using a variety of media	2.1 Identify and/or copy simple number patterns in rows, columns and diagonals	Copies patterns in order given Identifies simple number patterns in row, columns and diagonals	Copies patterns with concrete objects Copies patterns in given quantities	Copies and continues number patterns in rows or columns	Identifies simple number patterns in grids	Identifies number patterns in grids
	2.2 Show a knowledge of skip counting starting at any number	Shows a knowledge of skip counting starting at any number	Skip counts in songs and poems	Skip-counts in 2's and 3's	Skip counts in 2's, 3's, 5's and 10's	Skip-counts in 2's, 3's, 4's, 5's, 10's and 100's

		2.3 Identify and/or copy linear patterns using two and three dimensional shapes	Identifies linear patterns using 2D and 3 D shapes	Identifies patterns with 3D shapes	Identifies and copies patterns with 3D and 2D shapes	Identifies and copies patterns with 3D and 2D shapes	Identifies and copies patterns with 3D and 2D shapes
		2.4 Identify artistic patterns in South African cultures	Identifies artistic patterns in artifacts produces within South Africa	Identifies artistic patterns in own environment	Identifies artistic patterns in direct environments	Arrange numbers up to 104	
3.	Completion and generation of patterns	3.1 Arrange numbers in a logical sequence	Arranges numbers in a logical sequence	Uses number sequences in songs and games	Arranges numbers up to 24	Fills in two to five missing terms in number and geometric patterns	
		3.2 Identify missing terms of number and geometric patterns	Identifies missing terms of numbers and geometric patterns	Identifies missing terms in songs and games	Fills in one to two missing terms in numbers and geometric patterns	Creates patterns using 2D or 3D shapes	
		3.3 Extend or create linear patterns using 2D and/or 3/D	Extends or creates linear patterns using 2D and/or 3D	Extends patterns using 3D objects	Extends patterns using 2D or 3D shapes	Draws border patterns	
		3.4 Use concrete objects to extend, create and depict tiling or grid patterns	Constructs (build and draw) patterns Constructs patterns and objects in their environment	Extends border patterns	Builds border patterns	Extends number pyramids	
		3.5 Generate step patterns		Extends pyramids using 3D objects	Constructs pyramids using number blocks		
4.	Exploration of patterns in abstract and natural contexts using mathematical processes	4.1 Explore tessellation	Identifies 2D shapes used to cover surfaces in the world around them	Uses similarly shaped blocks to cover surfaces	Identifies 2D shapes used to cover surfaces without gaps		
		4.2 Use plane shapes and solid objects to investigate step patterns and symmetrically growing or shrinking patterns	Identifies and describes informally 2D shapes that can cover a surface with gaps	Identifies growing or shrinking patterns using 3D objects	Identifies growing or shrinking patterns using 3D or 2 D objects		
		4.3 Use available technology to generate patterns		Identifies symmetrical patterns using concrete objects	Identifies and completes symmetrical patterns		

3. DEMONSTRATE AN UNDERSTANDING OF THE HISTORICAL DEVELOPMENT OF MATHEMATICS IN VARIOUS SOCIAL AND CULTURAL CONTEXTS

Mathematics is a human activity. All people of the world have contributed to the development of mathematics. The view that mathematics is a European product must be challenged. Learners must be able to understand the historical background of their communities' use of mathematics.

ASSESSMENT CRITERIA	RANGE STATEMENT	PERFORMANCE INDICATORS	LEVEL 1	LEVEL 2	LEVEL 3	LEVEL 4
1. Evidence that mathematics is understood as a human activity	1.1 Demonstrate counting and measurement in everyday life	Learner: Gives examples about counting and measuring at home	Learner: Gives examples of familiar tools and ways for counting and measuring in own environment	Learner: Gives examples of familiar tools and ways for counting and measuring in direct environment	Learner: Gives examples of standardised tools and ways for counting and measuring in larger environments	Learner: Uses standardised tools and ways for counting and measuring
	1.2 Illustrate at least two mathematical activities at home	Gives examples about counting and measuring from other environments	Gives examples of tools used in the past to count and measure	Compares tools and ways for measuring and counting from different environments	Compares standardised tools and units from different environments for counting and measuring	Compares standardised tools and units from different environments for counting and measuring
	1.3 Show the link between mathematics and technology	Gives examples that show the use of mathematics in technology Gives examples that show the use of technology in mathematics	Gives examples of use of counters in own environment	Gives examples of use of counters in different environments	Gives examples of use of abacus in different environments	Gives examples of use of calculators in different environments

4. CRITICALLY ANALYSE HOW NUMERICAL RELATIONSHIPS ARE USED IN SOCIAL, POLITICAL AND ECONOMIC RELATIONS

Mathematics is used as an instrument to express ideas from a wide range of other fields. The use of mathematics in these fields often creates problems. This outcome aims to foster a critical outlook to enable learners with issues that concern their lives individually, in their communities and beyond. A critical mathematics curriculum should develop critical thinking about how social inequalities, particularly concerning race; gender and class are created and perpetuated.

ASSESSMENT CRITERIA	RANGE STATEMENT	PERFORMANCE INDICATORS	LEVEL 1	LEVEL 2	LEVEL 3	LEVEL 4
1. Evidence of knowledge of the use of mathematics in economy	1.1 Demonstrate understanding of the use of mathematics in shopping	Learner: Compares prices	Learner: Compares prices using games and role play	Learner: Compares prices by visiting shops and playing shop keeper	Learner: Compares prices using advertisements	Learner: Compares prices using different media
		Calculates the total cost of three to four items	Shows awareness of cost of items	Calculates cost of two items	Calculates cost of two to five items	Calculates cost of three to ten items
		Calculates change	Shows awareness of 'change' when shopping	Calculates change in R or cents	Calculates change in R and cents	Calculates change in R and cents
	1.2 Show understanding of price increases	Calculates a price increase over a period of time	Show awareness of price increase	Shows awareness of price increase	Compares prices over a certain period	Calculates price increase over period of time
2. Evidence of the understanding of budget	2.1 Show understanding of family budgeting	Demonstrates an awareness of how money is spent on food, clothes, fuel, etc	Plans how to spend part of pocket money	Plans how to spend part of pocket money	Draws shopping list (R and cents; two to five items)	Draws shopping list (R and cents; five to ten items)
	2.2 Show understanding of saving	Demonstrates an understanding of saving	Plans how to save part of pocket money	Shows awareness of need to save	Shows awareness of advantages of saving	Shows awareness of advantages to save and calculates time and amount
3. Demonstrate knowledge of the use of mathematics in determining location	3.1 Mapping of immediate locality	Maps immediate locality informally	Knows street and village where the school is located	Describes how to get to school using relevant vocabulary	Describes location of shops, farms, etc	Draws maps of immediate locality
	3.2 Word descriptions of directions and local transport	Draw rough sketches of paths	Shows awareness of directions		Shows awareness of North, South, East and West	Uses North, South, East and West
		Describes directions and local transport	Knows different forms of transport	Compares different forms of transport (fastest)	Compares different transport (most expensive)	Shows awareness of transport (sea, air, road)

5. MEASURE WITH COMPETENCE AND CONFIDENCE IN A VARIETY OF CONTEXTS

Measurement in mathematics is a skill for universal communication. People measure physical attributes, estimate and develop familiarity with time. The aim is to familiarise learners with appropriate skills of measurement, relevant units used, and issues of accuracy.

ASSESSMENT CIRTERIA	RANGE STATEMENT	PERFORMANCE INDICATORS	LEVEL 1	LEVEL 2	LEVEL 3	LEVEL 4
1. Evidence of knowledge of the importance of measurement	1.1 Show knowledge of measurement from experience	Learner: Describes situations in which measurement is used at home Gives examples of measured goods from shops	Learner: Describes how measurement is taken (own environment)	Learner: Gives examples of measured goods from shops	Learner: Gives examples of measured goods from shops and notes units in which goods are sold	Learner: Compares units in which goods are sold
	1.2 Comparison of masses of objects	Measures different objects by comparison				
2 Evidence of knowledge of standards	2.1 Show some knowledge of non-standard forms of measurement	Shows knowledge of the use of non-standard units Gives and explains examples where different non-standard units of measurement give different results according to the size of the unit of measurement	Measures masses, lengths and volumes with non-standard units	Measures and compares masses, lengths and volumes with non-standards units	Measures and compares masses, lengths and volumes with non-standards units Explains different results	Measures and compares masses, lengths and volumes with non-standard units Explains different results
	2.2 Demonstrate understanding of reasons for standardisation	Shows knowledge of the approximate sizes of cm, m, km, ml, l, g, kg Measures with SI units Selects a suitable unit of measurement for a given situation	Recognises one liter and half a liter Measures personal mass	Measures lengths in cm Measures mass in kg Measures volumes in liter and half a liter	Measures lengths in cm and m Measures mass in kg or g Measures 1l, $\frac{1}{4}$ l, $\frac{1}{2}$ l, $\frac{3}{4}$ l	Measures lengths in cm, m and km Measures mass in g and kg Expresses 1m in cm, 1kg in g and 1l in ml
	2.3 Demonstrate knowledge of SI units	Knows different forms of units and their application				
3 Evidence of knowledge of concepts used in measurement	3.1 Understand concepts used in the measurement of space in 2D/3D	Compares areas of different sizes Compares volumes of different sizes	Covers areas with objects Fills different containers	Covers areas with shapes Compares volumes of different sizes	Compares sizes of areas Compares volumes of different sizes	Compares sizes of areas Compares volumes of different sizes

		3.2	Understand money as a unit of measurement	Shows understanding of how and when money is used in everyday life	Gives examples of uses of money	Recognises different coins and bank notes	Compares values of different coins and bank notes	Compares values of different coins and bank notes
4	Evidence of knowledge of the concept of time	5.1	Use language to express times of the day	Shows knowledge of the periods of the day	Recognises periods of days related to activities	Shows knowledge of different periods of the day Knows the four seasons	Shows knowledge of different periods of the day Knows days of the week and months of the year	Shows knowledge of different periods of the day Knows days of the week and months of the year
		5.2	Show knowledge of how to read time	Reads time	Recognises time related to activities	Reads time in hours, half hours and five minute intervals	Reads time in hours and minutes	Reads time using different types of watches
5	Evidence of knowledge of the concept of temperature	5.1	Explain difference between hot and cold	Explains the difference between hot and cold	Distinguishes hot and cold by touching and feeling	Shows awareness of the difference between hot and cold	Shows awareness of the day and night temperatures	Compares day and night temperatures
		5.2	Explain dangers of very high or very low temperature	Gives examples of dangerous situations involving very high or low temperatures	Shows awareness of danger of fire, boiling water, sunburn, stoves, electrical equipment, etc	Gives examples of danger of fire, boiling water, sunburn, stoves, electrical equipment, etc	Gives examples of danger of fire, boiling water, sunburn, stoves, electrical equipment, etc	Gives examples of danger of fire, boiling water, sunburn, stoves, electrical equipment, etc

6. USE DATA FROM VARIOUS CONTEXTS TO MAKE INFORMED JUDGEMENT

In this technological age of rapid information expansion, the ability to manage data and information is an indispensable skill for every citizen. An every increasing need exists to understand how information is processed and translated into usable knowledge. Learners should acquire these skills for critical encounters with information and make informed decisions.

ASSESSMENT CRITERIA	RANGE STATEMENT	PERFORMANCE INDICATORS	LEVEL 1	LEVEL 2	LEVEL 3	LEVEL 4
1. Identification of situations for investigation	1.1 Identify situations for data collection	Learner: Identifies situations and their characteristics for data collection	Learner: Gives oral information about own family	Learner: Gives data of own environment and interests	Learner: Gives data of large environment	Learner: Gives data of large environment
2. Collection of data	2.1 Choose methods of data collection	Asks appropriate questions to gather data/formulates questionnaires to collect data	Formulates simples questions to collect data (data from own environment)	Formulates appropriate questions to collect data (data from direct environment)	Formulates questionnaire to collect data Organises interviews (data from larger environment)	Formulates questionnaire and identifies sources of information Organises interviews using a tape recorder
	2.2 Use interviews and sampling	Shows logical sequence in questions asked				
	2.3 Use technology	Names sources and media where information can be kept				
3. Organisation of data	3.1 List and arrange data in a logical order	Records data correctly on paper Can read relevant information from variety of sources	Records simple information using pictures and drawings	Records simple information in a table	Records collected information in tables	Arranges data logically and records in tables
	3.2 Sort relevant data	Interpret and extract relevant information from simple tables	Answers questions using simple tables	Reads simple tables	Reads and interprets tables	Extracts and interprets relevant information from tables
4. Application of statistical tools	4.1 Choose relevant methods	Arranges data in a particular order Can identify the most common element (mode)	Identifies most common element. Comments on 'how often...'	Arranges data in a given order	Calculates average of a small number of measurements	Calculates average and deviation from the mean
	4.2 Show understanding of average, variance, frequency	Calculates averages of a small number of measurements Calculates deviation from the mean for at least one of the elements		Identifies most common element Comments on 'how often...'	Identifies the mean Comments on frequency	Comments on frequency

		Comments on the frequency of an occurrence				
5. Display of data	5.1 Draw summary 5.2 Represent data using graphs, charts, tables, text	Presents the data in a simple table and in simple graphical form Gives clear and labeled charts about data Gives verbal/written description of data Uses different instruments and constructions to display data	Presents simple information on pictographs Comments on pictographs Uses posters	Presents information from tables on pictographs and simple graphs (Classroom posters) Gives verbal description of pictographs and graphs Uses posters	Presents information from tables on graphs (group work) Describes graphs and gives the basis of classification Uses posters	Presents information from tables on graphs (group work and individual work) Gives verbal and written descriptions of graphs Uses poster or transparencies
6. Communication of findings	6.1 Show understanding of use of simple and statistical language	Identifies trends or most/least popular groupings in the data Identifies misconceptions from the use of phrases like: 'statistics show that …' to mean 'everything shows that…'	Identifies ' the most' and 'the least' in pictographs	Identifies trends in graphs	Identifies and comments on trends in graphs	Identifies and comments on trends in graphs Shows awareness of misuse of information
7. Critical evaluation of findings	7.1 Explain meaning of information 7.2 Analyse validity 7.3 Analyse the impact of results 7.4 Make projections over time	Discuss the possible reasons for the trends Discuss whether these trends may change in time	Gives examples of how information from pictographs can be used	Gives reasons for trends noticed in graphs	Analyses validity of findings Justifies how information can be used Makes projections over time	Analyses validity of findings Justifies how information can be used Makes and justifies projections over time
8. Understanding of the concept of probability	8.1 Make predictions 8.2 Use to address real or simulated problems	Solves simple problems involving chance, probability and predictions	Makes predictions using games	Makes predictions	Makes and justifies predictions	Makes and tests predictions

7. DESCRIBE AND REPRESENT EXPERIENCES WITH SHAPE, SPACE, TIME AND MOTION USING ALL AVAILABLE SENSE

Mathematics enhances and helps to formalise the ability to grasp visualise and represent the space in which we live. In the real world, space and shape do not exist in isolation from motion and time. Learners should be able to display an understanding of spatial sense and motion in time.

ASSESSMENT CRITERIA	RANGE STATEMENT	PERFORMANCE INDICATORS	LEVEL 1	LEVEL 2	LEVEL 3	LEVEL 4
1. Description of the position of an object in space	1.1 Represent object in various forms of Geometry 1.2 Show links between Algebra and Geometry	Learner: Explains position of objects in space Uses own understanding of position to state in words the 'co-ordinate of a point'	Learner: Sorts shapes, finds differences and similarities Describes position of objects with simple words	Learner: Names and describes geometric shapes (3D and 2D) Describes positions of objects in space	Learner: Names and describes geometric shapes (3D and 2D) Describes position of one object in relation to another	Learner: Names and describes geometric shapes (3D and 2D) Uses mathematical language to describe position of objects
2. Descriptions of changes in shape of an object	2.1 Demonstrate movement of points with time an irrelevant variable	Describes geometrical positions formed by movement of shapes Identifies different types of movements of shapes	Identifies shapes and their positions in picture, art, etc	Classifies objects according to shapes and sizes	Draws 2 D shapes of different sizes and different positions	Draws 2D and 3D shapes of different sizes and different positions
	2.2 Transform and tessellate shapes	Represents these transformations in various ways Recognises changes that occur when object move	Uses shapes (2D and 3D) to make patterns	Recognise and copy patterns with shapes (3D and 2D)	Tessellates 2D shapes	Tessellates 2D shapes
3. Descriptions of orientation of an object	3.1 Show understanding of the concept of point of reference in 2D and 3D	Describes a set of objects from different points of view and different distances Identifies different projections in their environment	Describes position of objects using self as reference Investigates projected images (e.g. shadows)	Describes position of objects using self as reference Identifies projected images (e.g. shadows)	Identifies symmetrical pictures, shapes and figures Investigates projections when light source changes position	Identifies symmetrical pictures, shapes and figures Investigates and compares images of 2D and 3D objects
	3.2 Show understanding of perceptions by an observer from different reference points	Describe the effect of distance from the light source on the size of the projection				

	3.3 Work with projections 3.4 Use available technologies in simulations	Describe the effects of changing points of reference on the 'co-ordinates' Describe distortion of projections taken from different reference points	Describes position of shapes	Describes position and movements of shapes	Identifies font and side views	Identifies and draws front and side views Shows awareness of interpretation of pictures
4. Demonstrate an understanding of the interconnectedness between shape, space and time	4.1 Show the effect of movement and shape 4.2 Demonstrate an understanding of changes of perceptions of space and shape through different media	Show some understanding of the effect of movement on shapes Shows some understanding of the effect of the viewing medium on shape Shows some understanding of the	Draws similar shapes using translation	Draws and creates similar shapes using translation and rotation	Combines two or more shapes to create new geometric shapes	Combines shapes to create figures using translation, rotation and reflection
	4.3 Visualise and represent objects from various spatial orientations	effects of growth in nature (people) on shapes	Constructs shapes from 3D and 2D shapes	Constructs objects from 2D and 3D shapes	Solves geometrical puzzles	Solves geometrical puzzles

8. ANALYSE NATURAL FORMS, CULTURAL PRODUCTS AND PROCESSES AS REPRESENTATIONS OF SHAPE, SPACE AND TIME

Mathematical forms relationships and processes embedded in the natural world and in cultural representations are often unrecognised or suppressed. Learners should be able to unravel, critically analyse and make sense of these forms, relationships and processes.

ASSESSMENT CRITERIA	RANGE STATEMENT	PERFORMANCE INDICATORS	LEVEL 1	LEVEL 2	LEVEL 3	LEVEL 4
1. Recognition of natural forms, cultural products and processes and their value	1.1 Observe nature, cultural products and processes 1.2 Explain use and value of cultural products and processes 1.3 Analyse different cultural products and processes at different epochs	Learner: Identifies 2D and 3D shapes and their patterns in nature Identifies cultural products and cultural processes Explains the use or need of at least one of the products or processes Identifies at least one artifact that has changed over time	Learner: Gives examples of natural forms and cultural products Recognises changes of cultural products	Learner: Describes natural forms and cultural products Describes changes of cultural products (music instruments, beadwork, etc)	Learner: Describes and explains uses of cultural products and processes Describes changes of cultural products (drawings, painting, pottery, etc)	Learner: Describes and designs cultural products and processes Describes and explains changes of cultural products (housing, clothing, transport, etc)
2. Representation of natural forms, cultural products and processes in a mathematical form	2.1 Represent cultural products and processes in various mathematical forms – 2D and 3D 2.2 Represent nature in mathematical form	Describes cultural products informally Draws, where suitable, 2D and 3D representations of these artifact Identifies shapes in nature Represents shapes identified in nature	Draws shapes identified in nature and cultural products	Draws and describes shapes identified in nature and cultural products	Explains why certain shapes are used in cultural products	Generates new ideas to use shapes and patterns in cultural products
3. Generation of ideas through natural forms, cultural products and processes	3.1 Use representations to generate new idea	Shows some knowledge of why certain shapes and patterns are used in particular situations	Recognises natural forms used in cultural products	Recognises natural forms used in cultural products	Makes examples of artifacts (bead work, etc)	Makes own examples of artifacts using shapes and patterns
4. Extensions of natural forms, cultural products and processes in the economy	4.1 Critically analyse the misuse of nature and cultural products and processes	Shows some knowledge of minimum space required for natural life processes	Describes the role of nature in some cultural processes	Describes and analyses the role of nature in some cultural processes	Describes the effects that some cultural processes have on nature	Demonstrates respect for nature, cultural products and processes

9. USE MATHEMATICAL LANGUAGE TO COMMUNICATE MATHEMATICAL IDEAS, CONCEPTS, AND GENERALISATIONS AND THOUGHT PROCESSES

Mathematics is a language that uses notations, symbols, terminology, conventions, models and expressions to process and communicate information. The branch of mathematics where this language is mostly used is Algebra. The use of this language will be developed.

ASSESSMENT CRITERIA	RANGE STATEMENT	PERFORMANCE INDICATORS	LEVEL 1	LEVEL 2	LEVEL 3	LEVEL 4
1. Use of language to express mathematical observations	1.1 Share observations using all available forms of expression, verbal and non-verbal	Learner: Explains personal solution methods for problems, relationship and patterns verbally and/or through drawings	Learner: Presents simple problems using real objects	Learner: Presents and explains solution of simple problems	Learner: Presents and explains strategies to solve problems	Learner: Presents and explains strategies to solve problems
		Comments on other people's work in a coherent way	Shows awareness of peer and self-assessment by drawings	Shows awareness of peer and self-assessment by drawing	Uses peer and self-assessment to improve learning	Uses peer and self-assessment to improve learning
2. Use of mathematical notation, symbols	2.1 Represent ideas using mathematical symbols	Represents relationships between numerical quantities using mathematical symbols	Demonstrates awareness of one-to-one correspondence	Represents numerical equalities	Represents two different relations between numerical quantities	Represents two to five different relations between numerical quantities
	2.2 Use mathematical symbols	Uses mathematical notation appropriate for the level	Uses mathematical notation in games	Presents simple word problems I one numerical sentence	Presents word problems in one or two numerical sentences	Presents word problems in numerical sentences
		Reads and interprets numerical statements	Uses mathematical notation in games	Puts numerical sentence into words	Puts a numerical sentence into a word problem	Puts two numerical sentences into a word problem
3. Use of mathematical conventions and terminology	3.1 Formulate expressions, relationship and sentences	Formulates own stories that involve the use of numbers	Presents simple word problems in role play	Writes simple similar word problems	Writes new word problems	Generates creative word problems
				Represents a problem situation with real objects	Presents problem situations through drawings or tables	Represents problem situations through tables or graphs
4. Interpretation and analysis of models	4.1 Read and explain models 4.2 Analyse models and give meaning 4.3 Use models to solve problems	Represents problem situations in mathematical notation, tables or drawings Explains verbally a numerical expression, relationship, tables, or drawing which represents a problem situation	Solves concrete problem situations	Estimates answer and solves problems using own strategies	Estimates answer and solves problems using own strategies	Estimates answer and solves problems using own strategies

5.	Representation of real life and simulated situations	5.1 Use abstraction to simulate word problems 5.2 Represent real life or simulated situations in a mathematical format 5.3 Use technology to represent and process observations	Represents problem situations and patterns where suitable physically, through drawings or constructions to show understanding of the situation or to aid in the solving of the problem		Explains strategies used to solve problems Reflects on solutions Checks answers Checks answers with calculator	Explains strategies used to solve problems Reflects on solution Checks answers Checks answers with calculator	Compares and justifies used strategies to solve problems Reflects on solutions Checks answers Checks answers with calculator

10. USE VARIOUS LOGICAL PROCESSES TO FORMULATE, TEST AND JUSTIFY CONJECTURES

Reasoning is fundamental to mathematical activity. Active learners question, examine, conjecture and experiment. Mathematics Programmes should provide opportunities for learners to develop their reasoning skills. Learners need varied experiences to construct convincing arguments in problem setting and to evaluation the arguments of others.

ASSESSMENT CRITERIA	RANGE STATEMENT	PERFORMANCE INDICATORS	LEVEL 1	LEVEL 2	LEVEL 3	LEVEL 4
1. Evidence of logical reasoning in addressing problems	1.1 Demonstrate reasoning processes of association, comparison, classification and categorization	Learner: Demonstrates reasoning processes of association, classification and categorization used to solve a problem	Learner: Compares and classifies according to given criteria Finds differences and similarities	Learner: Demonstrates reasoning processes of associations and classification to solve problems	Learner: Demonstrates reasoning processes of associations and classification to solve problems	Learner: Demonstrates reasoning processes of associations and classification to solve problems
	1.2 Report mathematical reasoning processes verbally and visually	Reports mathematical reasoning process used to solve a problem verbally and visually	Reports mathematical processes verbally using concrete objects	Reports mathematical processes verbally and visually by drawing	Reports mathematical processes verbally and visually by drawing	Reports mathematical processes verbally and visually using numbers
2. Ability to justify familiar and unfamiliar hypotheses	2.1 Recognise familiar or unfamiliar situations	Explains how a problem was solved	Explains how problem was solved	Explains how problem was solved	Explains and justifies how problem was solved	Explains and justifies how problem was solved
	2.2 Infer from know experiences 2.3 Demonstrate respect for different reasoning approaches	Demonstrates willingness to listen to other learner's reasoning approaches Asks question or makes statements that show that he/she listened attentively	Listens attentively to different reasoning approaches Asks questions	Listens attentively to different reasoning approaches Formulates questions	Listens attentively and evaluates different reasoning approaches	Listens attentively to different reasoning approaches and presents comments based on own conjectures
3. Evidence of use of empirical or theoretical rationale in justifying conjectures	3.1 Choose relevant data as a basis for prediction 3.2 Construct logical steps in an understandable order 3.3 Test validity of judgement	Makes a conjecture based on logically relevant existing knowledge Tests the conjecture in a suitable way, justifies the conjecture Alters the conjecture on the basis of conflicting evidence	Makes conjectures based on existing knowledge	Makes and test conjectures	Tests and justifies different approaches or/and conjectures	Test, justifies or alters conjecture Presents comments based on own conjectures

ANNEXURE C

ANSWER SHEET FOR THE *VASSI* MATHEMATICS PROFICIENCY TEST

VASSI MATHEMATICS
PROFICIENCY TEST

ANSWER SHEET: GRADE 1

Name of Learner: .... **Gender (m/f):** ....

Name of School: .... **Age:** ....

1.) 10 + 5 = __________ ___

2.) 14 ; 15 ; ___ ; 17 ; 18 ; 19 ___

3.) Half of 8 = __________ ___

4.) 5 x 2 = __________ ___

5.) R2 + R1 + R5 + R5 = R__________ ___

6.) 16 ; 18 ; ___ ; 22 ; 24 ; 26 ___

7.) John has 6 blocks. Pat has 5 blocks. Ben has 4 blocks. How many blocks do they have altogether? __________ ___

8.) I share 12 sweets between 2 children. How many sweets will each child have? __________ ___

9.) Pat has 9 sweets. She eats 6 sweets. How many sweets are left? __________ ___

10.) $16 \div 2 =$ __________ ___

11.) 4 cats have __________ legs. ___

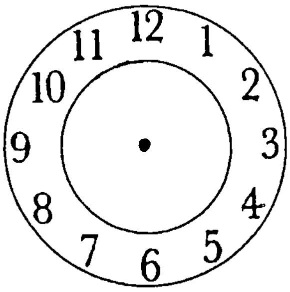
12.) Kabelo had 11 marbles. He lost 5 and later found 2. How many did he have then? __________ ___

13.) $3 \times 4 =$ __________ ___

14.) Meg had 13 eggs. She lost 4 eggs and the fox took 3 eggs. How many eggs does Meg have now? __________ ___

15.) 18 ; 21 ; ___ ; 27 ; 30 ; 33

16.) Draw the hands to show the time on the clock.



Half past 8

17.) 18 ÷ 3 = __________

18.) Anne picks 16 roses. She shares them among her 4 friends. How many roses do they each get? __________

19.) Half of 7 = __________

20.) Double 5 ½ = _________

RAW SCORE	STANINE	PERCENTILE RANK
/20		

VASSI MATHEMATICS
PROFICIENCY TEST

ANSWER SHEET: GRADE 2

Name of Learner: .... **Gender (m/f):** ....

Name of School: .... **Age:** ....

1.) **Vusi has 15 marbles. He loses 8. How many marbles are left? __________** —

2.) **Arrange the following numbers from biggest to smallest**

98 ; 67; 101 ; 24 ; 50 ; 19 ; 91 ; 15

___ ; ___ ; ___ ; ___ ; ___ ; ___ ; ___ ; ___ . —

3.) **Arrange the following numbers from smallest to biggest**

11 ; 56 ; 29 ; 9 ; 44 ; 14 ; 87 ; 78

___ ; ___ ; ___ ; ___ ; ___ ; ___ ; ___ ; ___ . —

4.) **Sipho, Lindy, Sara, Marcus, Ester, Jane, John and Mandla each have R5. How much money do they have altogether? __________** —

5.) How many minutes are there in an hour?

____

6.) 46 + 63 = __________

7.) How many hours are there in a day? __________

8.) 72 – 60 = __________

9.) Half of 49 = __________

10.) ___ ; ___ ; 105 ; 110 ; 115.

11.) The two numbers before 48 are ___ and ___.

12.) 350 ; 300 ; 250 ; ___ ; ___ ; ___ .

13.) How many 20c pieces are there in R2?

____

14.) Granny bakes 36 cookies. She shares them among 3 children. How many cookies does each child get? __________

15.) One fly has 6 legs. How many legs would 8 flies have? __________

16.) Double 75 = __________

17.) 30 ; 45 ; 60 ; ___ ; ___ ; ___ .

18.) In a shop there are 25 red balloons and 28 blue balloons. If 19 of the balloons pop, how many balloons are left? __________

19.) Double 9 ¼ = __________

20.) 54 x 3 = __________

RAW SCORE	STANINE	PERCENTILE RANK
/20		

VASSI MATHEMATICS
PROFICIENCY TEST

ANSWER SHEET: GRADE 3

Name of Learner: .... **Gender (m/f):** ....

Name of School: .... **Age:** ....

1.) Half of 100 = __________ | ___

2.) Count 101 ; 106 ; ___ ; ___ ; ___ ; ___ . | ___

3.) 27 x 5 = __________ | ___

4.) Half of 241 = __________ | ___

5.) (110 ÷ 10) – 7 = __________ | ___

6.) If I walk 12 km each day for a week, how far do I walk? __________ | ___

7.) (27 ÷ 3) + 5 = __________ | ___

8.) ¼ hour + ½ hour + 15 minutes = __________

9.) 96 ÷ 4 = __________

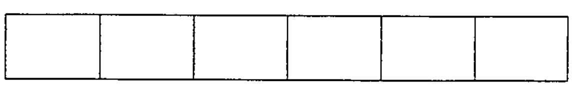
10.) 2 ; 4 ; 8 ; ___ ; 32 ; ___ .

11.) 1500 x 5 = __________

12.) 571 – 348 = __________

13.) I have 76 kg of sweets to sell. I want to pack them into packets each weighing 4 kg. How many packets of sweets can I make? __________

14.) What fraction of the shape is shaded and what fraction unshaded?



__________ Shaded

__________ Unshaded

15.) **I have 3 fifty cent coins, 2 twenty cent coins, 2 ten cent coins, 1 five cent coin and 4 one cent coins in my purse. How much money do I have altogether? __________**

16.) **Susan needs 72 drinking straws. She already has 47 straws. How many straws does she still need?**

____

17.) **If the perimeter of a square is 24 cm, what is the measurement of each side? __________**

18.) **My watch is 10 minutes slow. If it shows ten past ten, what is the correct time? __________**

19.) **$\frac{1}{5}$ of 350 = __________**

20.) **A family uses 21 kg of sugar every two months. How many kilograms of sugar will the family use in 8 months? __________**

RAW SCORE	STANINE	PERCENTILE RANK
/20		

ANNEXURE D

MANUAL FOR THE *VASSI* MATHEMATICS PROFICIENCY TEST

MANUAL FOR THE

VASSI MATHEMATICS PROFICIENCY TEST

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A SPECIAL WORD OF THANKS TO THE FOLLOWING PEOPLE ASSOCIATED WITH THE DEPARTMENT OF EDUCATION, FREE STATE PROVINCIAL GOVERNMENT:

1. **Mr W.B. van Rooyen**

2. **Mr P. de Villiers**

3. **Principles and staff of the participating schools – especially the contact teachers**

4. **All the learners who were tested**

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1. INTRODUCTION

1.1 BACKGROUND

When the ESSI Reading and Spelling test (Esterhuyse & Beukes, 1997) was compiled, the remedial teachers involved all expressed an interest in a mathematics test that would serve the same diagnostic purpose. The need has therefore arisen to set up a mathematics test that can help to isolate a mathematics problem at a young age. The only two subjects, which are evaluated during the foundation school phase, are language and mathematics. There is a high positive correlation (correlation co-efficients vary between 0,45 and 0,59; N=250) between the ability to read and spell (language proficiency) and mathematics performance (Esterhuyse & Beukes). The sample consisted of English speaking grade one to grade seven learners.

Often in a young child's functioning, cognitive problems arise such as the inability to perform mathematical calculations. Psychologists and educationists go to great lengths to determine the problem, so that a plan of action can be put into place to help a child function at his/her optimal level. Intelligence tests, visual-motor perceptual tests and even reading and spelling tests are performed. A need has arisen for a new mathematics test, which has South African norms, that will enable a psychologist or an educationist to identify a mathematics problem. From a young age children are often told that, of all the subjects they will encounter at school, mathematics will be the most difficult.

1.2 RATIONALE

The rationale for these tests is based on the assumption that if children are struggling to be active mathematical thinkers and are unable to construct and make sense for themselves of what they are doing in mathematics, then this test can help to identify and address the problem. This test can enable teachers to identify and assist a child with a mathematics problem at an early age. This test can prevent a learner from experiencing future mathematics problems, if the problem is identified and dealt with timeously.

In view of the above, the VASSI is an English mathematics proficiency test developed for the Free State Education Department, that fulfills the following needs:

a) the test is applicable to grades one, two and three learners;
b) the norms per term are available, so the test can be administered at any time of the year;
c) the test can be administered to groups or individuals;
d) the test can be used diagnostically (i.e. to identify the area in which the learner is experiencing problems); and
e) the test can be of value to future generations of learners.

1.3 TEST MATERIAL

The test material consists of:

(a) Manual

(b) Answer sheets: Each grade has its own mathematics answer sheet.

2. TEST INSTRUCTIONS

Allow the learner to complete the information at the top of the answer sheet. Show the learner the space allocated where the answer to each question must be written. Say to the learner: "I want you to complete this worksheet. Some questions may be easy but others are more difficult. Don't worry if you can't do them all. Just do your best."

Instructions to the tester:

(a) No calculators may be used in the test;

(b) No time limit is imposed (within reasonable limits);

(c) You may read the word sums to the learner, as mathematical ability and not English reading ability is being tested;

(d) You may explain the meaning of the words to the learner;

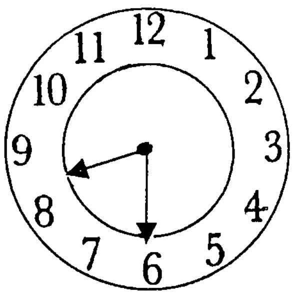
(e) The test must be completed on the answer sheet; and

(f) Extra paper may be given for rough work.

3. TEST ANSWERS

GRADE 1

1.)	15
2.)	16
3.)	4
4.)	10
5.)	13
6.)	20
7.)	15
8.)	6
9.)	3
10.)	8
11.)	16
12.)	8
13.)	12
14.)	6
15.)	24
16.)	



17.)	6
18.)	4
19.)	3 ½ *or* 3,5
20.)	11

GRADE 2

1.)	7
2.)	101 ; 98 ; 91 ; 67 ; 50 ; 24 ; 19 ; 15
3.)	9 ; 11 ; 14 ; 29 ; 44 ; 56 ; 78 ; 87
4.)	R40 *or* 40 *or* 40 rand *or* forty rand
5.)	60
6.)	109
7.)	24
8.)	12
9.)	24 ½ *or* 24,5
10.)	95 ; 100
11.)	46 and 47 *or* 47 and 46
12.)	200 ; 150 ; 100
13.)	10
14.)	12
15.)	48
16.)	150
17.)	75 ; 90 ; 105
18.)	34
19.)	18 ½ *or* 18,5
20.)	162

GRADE 3

1.) 50

2.) 111 ; 116 ; 121 ; 126

3.) 135

4.) 120 ½ *or* 120,5

5.) 4

6.) 84 km *or* 84

7.) 14

8.) 1 hour *or* 60 minutes (acceptable alternatives are 1 *or* 60)

9.) 24

10.) 16 ; 64

11.) 7500

12.) 223

13.) 19 (if the learner wrote 19 kg allocate one mark)

14.) ½ Shaded, ½ Unshaded *or* 3/6 Shaded, 3/6 Unshaded

15.) R2,19 *or* 2,19 Rand *or* 219 cents *or* 219c (acceptable alternatives are 2,19 *or* 219)

16.) 25

17.) 6 cm *or* 6

18.) 10:20 *or* 10h20 *or* 20 past ten *or* 20 past 10 *or* twenty past ten *or* twenty past 10

19.) 70

20.) 84 kg *or* 84

4. SCORING THE TESTS

After completion of the mathematics test, one mark is allocated per question for the correct answer and zero for an incorrect answer. Write 1 or 0 at the end of the line on the space allocated. If a part of a question is wrong, then the entire question is marked as incorrect and a zero mark is allocated. On specific items the learner can indicate the unit. If the learner just writes the number and the number is correct, without indicating the unit, the tester can allocate one mark. All correct marks are then summated to determine the individual's raw score for mathematics. The raw score is then converted into a stanine and percentile rank by using the appropriate norm table and may be filled in at the bottom of the answer sheet for future reference.

5. NORMS

Norms for the mathematics test have been calculated in the form of stanines and percentile ranks. Due to the fact that the norms had to be available for each term, the same testees were tested during the first and fourth term of 2000.

5.1 STANINES

The stanine scale is a normalised nine point standard scale. It produces standard scores which range from one to nine, with a mean of five and a standard deviation of 1,96. Each stanine value represents a specific percentage of cases as indicated in Table 5.1.

Table 5.1: Percentile range and description of stanine scale

Percentage testees	Stanine	Cumulative percentage	Description	Estimated % of testees
Lowest 4,01 %	1	4,01 %	Very poor	4 %
Next 6,55 %	2	10,56 %	Poor	19 %
Next 12,1 %	3	22,66 %	Poor	
Next 17,47 %	4	40,13 %	Average	54 %
Middle 19,74 %	5	59,87 %	Average	
Next 17,47 %	6	77,34 %	Average	
Next 12,1 %	7	89,44 %	Good	19 %
Next 6,55 %	8	95,99 %	Good	
Highest 4,01 %	9	100 %	Very Good	4 %

5.2 PERCENTILE RANKS

The percentile of a test score is equal to the percentage of testees in the norm group who obtained a score equal to or lower than that specific score. With reference to Table 5.1, we may deduce that if a learner's raw score is converted into a stanine of six, that 59,87 % of the norm group obtained a lower score and 40,13 % obtained a higher score than that specific learner. We may further deduce that 77,37 % of the norm group obtained a similar or lower score.

6. STANDARDISATION OF THE TESTS

6.1 IDENTIFICATION OF PRELIMINARY ITEMS

During June/July 1999 the researcher contacted seven schools in the Free State region, from Bloemfontein and Welkom, to obtain items for the preliminary questionnaire. Once each school had selected preliminary items, the preliminary questionnaires were given to approximately two to three classes of 30 learners, in the respective grades, at each of the seven schools. Only the result of learners whose home language was English was taken into account. The results were then computerised and 40 items were selected per grade in the following manner:

(a) 10 easy items (at least 80 % of the learners obtained the correct answer);
(b) 10 difficult items (only 20 % of the learners obtained the correct answer); and
(c) 20 items with an average difficulty value.

6.2 ADMINISTRATION FOR ITEM ANALYSIS

In this phase the experimental mathematics proficiency test, consisting of 40 items per grade, was given to 13 schools during the last term of 1999. The seven original schools who helped determine the preliminary questionnaire were once again involved. The 13 schools were all English medium schools selected from Bloemfontein, Welkom, Kroonstad, Bethlehem, Virginia, and Ladybrand. During the selection of the items an attempt was made to obtain similar numbers of below average, average and above average achievers as well as boys and girls. The composition of the sample is represented in Table 6.1.

Table 6.1: Sample to which the mathematics tests were administered for item analysis

Grade	Boys		Girls		Total	
	N	%	N	%	N	%
1	112	53,3	98	46,7	210	33,6
2	101	47,6	111	52,4	212	33,9
3	110	54,2	93	45,8	203	32,5
Total	323	51,7	302	48,3	625	100,0

A total of 625 pupils were tested and approximately the same number of boys and girls were tested. The results of the item analysis are contained in a comprehensive technical report (Vassiliou, 2000) which is available from the test compiler. The results of the Classical Test Theory (Thorndike, Cunningham, Thorndike & Hagen, 1991) are reported.

6.3 ADMINISTRATION FOR ESTABLISHING NORMS

The final mathematics proficiency test was sent to the same thirteen schools during the first (first administration) and fourth term (second administration) of 2000. Table 6.2 and Table 6.3 reflect the composition of the sample according to gender during the first and second application.

Table 6.2: Composition of the samples to which the mathematics tests were administered for establishing norms, first administration

Grade	Boys		Girls		Total	
	N	%	N	%	N	%
1	92	48,9	96	51,1	188	29,6
2	115	52,0	106	48,0	221	34,8
3	113	50,0	113	50,0	226	35,6
Total	320	50,4	315	49,6	635	100,0

Table 6.3: Composition of the samples to which the mathematics tests were administered for establishing norms, second administration

Grade	**Boys**		**Girls**		**Total**	
	N	**%**	**N**	**%**	**N**	**%**
1	83	48,8	87	51,2	170	29,7
2	106	53,0	94	47,0	200	35,0
3	104	51,5	98	48,5	202	35,3
Total	293	51,2	279	48,8	572	100,0

A high degree of success was achieved in obtaining an equal number of boys and girls for the norm group during the first administration. Far less learners were involved in the second administration than the first. The most common reasons were that the learners had changed schools or were sick on the day of testing.

7. TECHNICAL DETAIL

7.1 INTRODUCTION

Certain statistical properties are represented in Table 7.1. According to Esterhuyse (1997) there are certain statistical properties that needs to be addressed when developing and standardising a psychometric test. These properties include: the size of the sample; the number of items in each test; the means of test scores; the standard deviation of the tests; the reliability coefficient; the skewness and the kurtosis. This information was obtained from the first application in the final test during the first term of 2000.

Table 7.1: Statistical properties of the mathematics proficiency test

GRADE	N	NO. OF ITEMS	$\overline{X}$	s	RELIABILITY (K-R)	SKEWNESS	KURTOSIS
1	188	20	6,670	4,300	0,849	0,193	-0,882
2	221	20	8,548	4,143	0,808	0,119	-0,589
3	226	20	8,690	4,878	0,867	0,357	-0,681

7.2 MEANS AND STANDARD DEVIATIONS

The above means indicate that the grade one learners had a lower mean than the grade two and three learners. This could be due to the fact that the learners are expected to reason concretely and many of the grade one learners, according to Piaget (1969), are still in the preopertional phase. The standard deviation of the grade one, two and three learners in the first term was 4,300; 4,143; and 4,878 respectively. The standard deviation for grade one was 0,97 above the ideal; for grade two, 0,813 above the ideal; and for grade three, 1,548 above the ideal.

7.3 SKEWNESS AND KURTOSIS

The ideal distribution is a normal distribution and the skewness statistic measures the deviation from the norm. The ideal variation is from –3 to +3. A positive skewness value indicates that most of the learners obtained a score lower than the mean, this would give a positively skewed distribution and this might occur when a test is too difficult. A negative skewness value indicates that most of the learners obtained a score higher than the mean, this would give a negatively skewed distribution and could also indicate that the test was too easy (Esterhuyse, 1997). According to Table 7.1 the skewness values are all positive and only slightly greater than zero. This indicates a slightly positive skewness. The values are only slightly greater than zero, indicating a symmetrical distribution.

According to Huysamen (1990) kurtosis of a curve refers to the flatness or peakedness of the centre of the curve. A normal curve will have a kurtosis of zero, a peaked curve will have a positive kurtosis value and a flat curve will have a negative kurtosis value. The kurtosis values for the grade one, two and three test are all negative which indicates a more flat curve, but the values are so small that the curves have relatively normal distributions.

7.4 RELIABILITY

7.4.1 Kuder-Richardson-formula-20

According to Table 7.1 the Kuder-Richardson reliability coefficients for grade one, two and three are: 0,85; 0,83; and 0,87 respectively. The grade one and three reliability coefficients are both greater than 0,85; which indicates an excellent reliability score. Even though the grade 2 reliability coefficient is below 0,85 it is still greater than 0,8.

The reliability co-efficients therefore indicate that the tests are reliable and measure consistently.

7.4.2 Test-retest reliability

Test-retest reliability is a measure of a test's stability. To determine test-retest reliability, a test must be administered on two occasions, on a representative sample of the population for which the test is intended. In this research, the same mathematics proficiency test was administered on two occasions, during the first term and fourth term of 2000, to the same grade one, two and three learners. It could be assumed that the learners should, in general, achieve a higher score during the fourth term than the first term. A period of two terms elapsed between the testing, which is a relatively long period of time and therefore the first administration should not have an effect on the second administration.

Test-retest reliability means that the same test can be administered at any time, to the same population and the same result should be obtained. Therefore if the test-retest reliability of the mathematics proficiency test is not significant, it does not necessarily mean the test is not reliable, it could just be an indication of the extent to which the variable being measured is unstable. Despite this fact, the test-retest reliability was significant for each of the grade one, two and three tests on the 0,01 % level. The results are represented in Table 7.2.

Table 7.2: Correlation coefficients between the first and second administrations

Grade	N – First Administration	N – Second Administration	Correlation coefficients
1	188	170	0,530 *
2	221	200	0,593 *
3	226	202	0,690 *

* $p \leq 0{,}0001$

7.5 VALIDITY

7.5.1 Prediction validity

The prediction validity of the test was calculated by correlating the mark obtained on the mathematics proficiency test during the first term, with the latest school mathematics mark. The results are indicated in Table 7.3.

Table 7.3: Predictive validity of the mathematics proficiency test

Grade	N	Correlation coefficients
1	170	0,380 *
2	200	0,573 *
3	202	0,540 *

* $p \leq 0{,}0001$

The above results indicate that the predictive validity of the grade one, two and three tests are all significant on the 0,01 % level. According to Steyn (1999) the significance of the results is also dependent on the practical interest of the result. The standardised difference in means of two scores can be viewed as the point of departure of the effect size. Effect size is the relationship between nominal and interval scale variables. Cohen in Steyn states that the product moment correlation coefficient can be used as the effect size of a linear relationship between two variables that can be measured on an interval scale. Steyn continues to state that a correlation coefficient of 0,1 has a small effect size, a correlation coefficient of 0,3 has a medium effect size and a correlation coefficient of 0,5 has a large effect size. Therefore the predictive validity of the grade one test has a medium effect size, despite the fact that the correlation coefficient appears to be small. The grade two and three correlation coefficients have a large effect size.

7.5.2 Content validity

Content validity is the degree to which the items in a test represents the total universe of items which could have been compiled in terms of the curriculum and teaching objectives. Content validity of a measuring instrument cannot be given in terms of a quantitative analysis. The final draft of the mathematics proficiency test for grade one, two and three learners, was then given to several of the contact teachers for comment before drafting the final test. The teachers were asked to give comment with respect to the type of items that were selected and the degree of difficulty of each, with the easier items in the beginning and the more difficult items at the end. The teachers felt that the test covered the entire syllabus and would serve a good measure of a child's mathematical ability. The test was also given to the Mathematics, Mathematics Literacy and Mathematical Science Learning Facilitator of the Free State Education Department for comment. The learning facilitator gave the tests to various experienced foundation phase teachers and the tests were returned with no changes. The contact teachers and the learning facilitator are experienced in their field and could objectively evaluate the validity of the content of the items selected.

8. NORM TABLES

The norms are represented in terms of stanines and percentile ranks. The norms were calculated per term so that the test can be administered at any time of the year. The norms appear in Table 8.1, Table 8.2 and Table 8.3.

GRADE 1

Table 8.1: Conversion of mathematics raw score into stanines for grade one learners

Stanine	Percentile rank	First Term	Second Term	Third Term	Fourth Term
1	4	-	1	1-3	1-5
2	11	-	2	4-5	6-7
3	23	1-2	3-4	6-7	8-10
4	40	3-4	5-7	8-10	11-12
5	60	5-7	8-10	11-12	13-15
6	77	8-9	11-12	13-14	16-17
7	89	10-11	13-14	15-16	18
8	96	12-14	15	17	19
9	100	15-20	16-20	18-20	20
$\bar{X}$		**6,670**			**13,900**
s		**4,300**			**4,341**
N		**188**			**170**
KR-20		**0,849**			

GRADE 2

Table 8.2: Conversion of mathematics raw score into stanines for grade two learners

Stanine	Percentile rank	First Term	Second Term	Third Term	Fourth Term
1	4	1	1-2	1-3	1-4
2	11	2	3-4	4-5	5-7
3	23	3-4	5-6	6-8	8-9
4	40	5-6	7-8	9-10	10-12
5	60	7-9	9-11	11-12	13-14
6	77	10-11	12-13	13-14	15-16
7	89	12-13	14	15-16	17
8	96	14-15	15-16	17	18
9	100	16-20	17-20	18-20	19-20
$\bar{X}$		**8,548**			**13,470**
s		**4,143**			**4,072**
N		**221**			**200**
KR-20		**0,808**			

GRADE 3

Table 8.3: Conversion of mathematics raw score into stanines for grade three learners

Stanine	Percentile rank	First Term	Second Term	Third Term	Fourth Term
1	4	1	1	1-2	1-3
2	11	2	2-3	3-4	4-6
3	23	3	4-5	5-6	7-8
4	40	4-6	6-7	7-9	9-11
5	60	7-9	8-10	10-12	12-13
6	77	10-11	11-13	13-14	14-16
7	89	12-14	14-16	15-16	17
8	96	15-17	17-18	17-18	18
9	100	18-20	19-20	19-20	19-20
$\bar{X}$		**8,690**			**12,515**
s		**4,878**			**4,463**
N		**226**			**202**
KR-20		**0,867**			

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SUMMARY

Learning problems, during childhood, manifest in the form of learning restraints, learning disabilities and learning disorders. The concepts can be classified separately but in this research, the three concepts are viewed under the umbrella term of learning problems. There are three main causes of learning problems in children namely: cognitive factors; non-cognitive factors; and socio-environmental factors. In this study emphasis shifted from general learning problems to one specific learning area, namely; mathematics. Mathematics is a construction of knowledge that deals with qualitative and quantitative relationships in space and time. For a child to become competent in mathematics, the child must go through various processes, namely, cognitive, personal and social developmental processes. The child in the foundation phase must go through mathematical processes to comprehend certain mathematical tasks. The comprehension of specific mathematical concepts is operationalised by the successful execution of related mathematical tasks. These tasks are universal, yet in this study emphasis was placed on the tasks required of a child in the foundation phase in the Free State. If the child is not able to complete a task, problems in mathematical achievement can occur. Mathematical learning problems can be classified in terms of mathematical learning restraints, disabilities or Mathematics Disorder. The problems manifest in the form of various mathematical errors and the most common sources of these problems include the child, the teacher and the task. An assessment must be done to identify a child with a mathematics problem and provide information about prevention and treatment. In this research the *VASSI* Mathematics Proficiency Test was developed and standardised, for

the Free State Department of Education, to serve as an assessment tool, to identify learners in the foundation phase with problems in mathematical achievement. The sample consisted of learners in grade one, two and three whose home language was English. Four phases were carried out while compiling the test. The first phase consisted of the construction of the preliminary questionnaire. In this phase seven schools compiled preliminary questionnaires for grade one, two and three learners and from these items a preliminary questionnaire consisting of 40 items was compiled. In phase two, the 40 item tests of grade one, two and three were given to 13 schools to administer. These results were analysed and the final 20 items were selected. The items were selected according to the Classical Test Theory with respect to their difficulty value, discrimination value, item variance, test variance, item-test correlation, item-criterion correlation, criterion-related validity and the coefficient-alpha. In phase three norms were determined by administering the test during the first and fourth terms of 2000. Inter polarisation was carried out to determine the second and third term norms. Various statistical properties were investigated, these included the size of the sample; the number of items in each test; means with respect to sex, age, and the first and fourth administrations; standard deviations of the tests; reliability of the tests and skewness and kurtosis. The validity of the test was also investigated with regard to the content and prediction validity. The reliability and validity correlation coefficients varied from good to excellent. The obtained results suggest that the test can be administered in practice with confidence. The *VASSI* Mathematics Proficiency Test complies to the following criteria:

- the test is applicable to grade one, two and three learners;

- the norms are available per term so that the test can be carried out at any time of the year;
- the test consists of universal mathematics concepts;
- the test can be administered to groups or individuals;
- the test can be used diagnostically; and
- the test should be of value to future generations of learners.

OPSOMMING

Leerprobleme gedurende die kinderjare manifesteer in die vorm van leergeremdhede, leergestremdhede en leerversteurings. Hierdie konsepte kan afsonderlik geklassifiseer word, maar in hierdie navorsing word die terme onder 'n sambreelterm, naamlik, leerprobleme, bespreek. Die hoof oorsake van leerprobleme by kinders is kognitiewe-, nie-kognitiewe- en omgewingsfaktore. In hierdie studie verskuif die klem van algemene leerprobleme na een spesifieke leerarea naamlik, wiskunde. Wiskunde is 'n konstruksie van kennis wat handel oor kwalitatiewe en kwantitatiewe verhoudings in tyd en ruimte. Alvorens 'n kind bevoegdheid in wiskunde verwerf, moet hy/sy deur verskeie wiskundige prosesse vorder. Hierdie prosesse omvat verskillende kognitiewe, persoonlike en sosiale ontwikkelingsfases. Die kind in die grondslagfase moet deur hierdie wiskundige prosesse vorder ten einde sekere wiskundige take te bemeester. As 'n kind nie 'n bepaalde wiskundige taak kan voltooi nie, dui dit daarop dat die kind nog nie 'n sekere deel van die wiskunde verstaan nie. Die wiskundige take wat van 'n kind, in die grondslagfase verwag word, is universeel. In hierdie navorsing word daar op die take wat van 'n kind in die Vrystaat tydens die grondslagfase verwag word, gefokus. Indien 'n kind nie 'n bepaalde wiskundige taak kan voltooi nie, kan probleme in wiskundeprestasie voorkom. Wiskunde leerprobleme kan geklassifiseer word in terme van wiskundige geremdhede, gestremdhede of 'n Wiskundeversteuring. Die probleme manifesteer normaalweg in die vorm van verskeie wiskundige foute en die mees algemene bronne van hierdie probleme is gewoonlik die kind, die onderwyser en die wiskundetaak self. Die vroegtydige evaluering van 'n wiskundeprobleem kan inligting oor die voorkoming en behandeling

van die probleem verskaf. Vir hierde doel het die navorser die *VASSI* Wiskunde Bekwaamheidstoets vir die Vrystaatse Departement van Onderwys opgestel en gestandaardiseer. Die ondersoekgroep het bestaan uit graad een, -twee, en –drie leerders wie se huistaal Engels was. Die navorsing het uit vier fases bestaan wat oor twee jaar gestrek het. Die eerste fase het die identifisering van die voorlopige wiskunde items behels. In hierdie fase het die navorser sewe skole betrek om voorlopige wiskunde items op te stel vir graad een, -twee en -drie. Uit hierdie voorlopige items het die navorser 40 items geselekteer vir insluiting in die voorlopige wiskundetoets. In fase twee, het die leerders van 13 skole hierdie voorlopige toetse voltooi. Die navorser het die resultate van hierdie toetse ontleed en die finale 20 items geselekteer. Die items is geselekteer op grond van die Klassieke Toetsteorie met betrekking tot die items se moeilikheidsgraad, diskriminasiewaarde, itemvariansie, toetsvariansie, item-toetskorrelasie, item-kriteriumkorrelasie, kriteriumverwante geldigheid en die alpha-koëffisiënt. Fase drie het die normbepaling van die toets behels. Die finale toets is tydens die eerste en vierde kwartaal van 2000 op die graad een, -twee en –drie leerders van 13 skole toegepas. Die navorser het van die interpolêre normaliseringsmetode gebruik gemaak om die tweede en derde kwartaal se norms te bereken. Verskeie statistiese eienskappe van die toets wat ondersoek is, het die volgende ingesluit: die grootte van die steekproef; hoeveelheid items in elke toets; gemiddeldes ten opsigte van geslag, ouderdom, eerste en vierde kwartaal se toepassings; standaardafwykings van die toetse; betroubaarheid; skeefheid en kurtose. Die inhoudsgeldigheid en voorspellingsgeldigheid van die toets is ook ondersoek. Die betroubaarheids- en geldigheidskoëffisiënte wissel van goed tot uitstekend, derhalwe kan die *VASSI* Wiskunde Bekwaamheidstoets met vertroue in die praktyk afgeneem word. Die *VASSI* Wiskunde Bekwaamheidstoets voldoen ook aan die volgende kriteria:

- die toets is geskik vir graad een, -twee en –drie leerders;
- die norms per kwartaal is beskikbaar, sodat die toets enige tyd van die jaar afgeneem kan word;
- die toets bestaan uit universele wiskunde konsepte;
- die toets kan op groepe en op individuele leerders toegepas word;
- die toets kan gebruik word as 'n diagnostiese instrument; en
- die toets behoort van waarde te wees vir toekomstige generasies leerders.

KEY TERMS

Learning problems

Cognitive factors

Non-cognitive factors

Socio-environmental factors

Mathematics

Mathematical processing

Mathematical tasks

Problems in mathematical achievement

Assessment

Foundation phase

***VASSI* Mathematics Proficiency Test**

Item analysis and selection

Norm determination