EXPLORING THE TEACHING AND LEARNING OF CIRCLE GEOMETRY IN RURAL SCHOOLS

By

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DEDICATION

I dedicate this dissertation work to my father, Mr. Mpheti Madishong Phineas; my mother, Mrs. Mpheti Phegisile Evah; my brothers Themba, Tshwarelo, and Godfrey Mpheti, and sisters, Ms. Nonhlanhla Mpheti, Ms. Vivian Mpheti, and the late Ms. Daphney Mpheti, may her soul rest in peace, for their support and encouragement.

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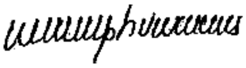
DECLARATION

I declare that **EXPLORING THE TEACHING AND LEARNING OF CIRCLE GEOMETRY IN RURAL SCHOOLS** is my work. All the sources I have used have been indicated and acknowledged utilising complete references. This work has not been submitted before for any other degree at any other institution.



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ABSTRACT

This report presents a qualitative case study exploring the teaching and learning of circle geometry in rural schools. The study involved six educators and was carried out in Bohlabela District, Mpumalanga. Data was collected through lesson observations of six consecutive lessons during the content workshop, lesson observations during class visits, learners' scripts, and semi-structured interviews with the educators. The Duval's cognitive theory guided the study. The findings indicate that educators that participated in this study were able to teach the correct content following the annual teaching plan. Learners were able to complete tasks set on circle geometry during teaching and learning in the classroom.

On the other hand, the educators could not emphasise the issue of composing and decomposing shapes, the application of visualising shapes before proving the theorems, the relationship of shapes within the orientation of the circle and encouraging learners in solving circle geometry problems. Learners could not solve geometric problems, provide statements and prove theorems. The findings of this study might be useful to all secondary school mathematics in improving the teaching and learning of circle geometry. This study recommends using the cognitive processes when teaching circle geometry and assessing learners in mathematics classrooms.

Keywords: Circle geometry, geometric spatial reasoning, geometric theorems, problem-solving and cognitive processes

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LIST OF ACRONYMS

ANA	Annual National Assessment
CAPS	Curriculum and Assessment Policy Statement
DBE	Department of Basic Education
GCP	Geometric Cognitive Processes
GRS	Geometric Spatial Reasoning
ISA	Information Sharing and Assessment
MKO	More Knowledgeable Other
NAEP	National Assessment of Educational Progress
NC	National Curriculum
NPA	National Protocol Assessment
PANDA	Performance and Data Analysis
TIMSS	Trends in International Mathematics and Science Study

CHAPTER 1 : INTRODUCTION AND BACKGROUND

1.1 INTRODUCTION

Circle geometry teaching nationally and internationally is situated in a cultural matrix. In South Africa, it is not the same as in the United States, Singapore, Hungary, Japan, or Germany, and the curricula are also different (Mesiti, Artigue, Hollingsworth, Cao & Clarke, 2021). South Africa participated in TIMSS 2019 (Trends in International Mathematics and Science Study), and the performance of rural schools was at a disappointingly low standard. South Africa performed below the TIMSS scale centrepoint in Grade 9, and as learners progress to Grade 11, they underperform in circle geometry. The findings from contextual data indicate that poor performance in rural schools resulted from an unsuitable environment for teaching and learning as well as the educators' teaching methods to assess skills of cognitive processes in circle geometry (DBE, 2019).

The current concerns in teaching and learning circle geometry in rural schools and the number of passes in Mathematics are insufficient. In the postmodern era, the learning of circle geometry is considered to be largely learner-centred and enhances learners' geometric thinking skills (Chew & Lim, 2013). However, circle geometry is trimmed in the South African Curriculum and Assessment Policy Statement (CAPS) to assess a few problem-solving skills and proving theorems (Shonhiwa, 2020). Educators use teaching methods to cover the Annual Teaching Plans (ATPs) and not assess the skills of the visualisation of geometric shapes, construction, and reasoning.

Geometry studies shapes' interconnectedness and properties (Luneta, 2015). In circle geometry, the relationship between shapes and their properties are within the orientation of the circle (Machisi & Feza, 2021). Logical arguments address geometric problems and establish interventions to teach circle geometry theorems (Karaman, 2017). A logical argument is a specialised connection of various properties of shapes (Ubah & Bansilal, 2019).

The effective teaching and learning of circle geometry involve skills to visualise geometric shapes and compose and decompose when constructing within the orientation of the circle (Naidoo & Kapofu, 2020). Firstly, visualisation is studying two-and three-dimensional objects and forming mental images of geometrical shapes and their properties in various orientations (Turer, 2020). Secondly, composing geometric shapes is about constructions within the orientation of shapes, and decomposing is how shapes are broken down into small parts (Hawes, Moss, Caswell, Naqvi & MacKinnon, 2017). The manipulation of shapes in terms of their properties plays an essential role in teaching circle geometry contexts (Sadiki, 2016). Lastly, reasoning is thinking, understanding, and logically forming judgements (Duval, 1998).

In the circle geometry curriculum, the processes of proving theorems have the following steps: 1.) Developing mental images of the shapes and their rotations into various orientations 2.) Tapping into previous knowledge of geometric concepts, proving and problem-solving procedures 3.) Manipulating geometric shapes and constructions 4.) Reproducing axioms to write the correct geometric statements and reasoning (Hanna, 2020).

This proposed study has been prompted by educators' inability to teach circle geometry as required by the curriculum and learners' inabilities to solve problems and proving theorems (Armah, Cofie & Okpoti, 2018). According to the circle geometry curriculum in South Africa, CAPS (2011) document and Alex (2019) outline teaching methods and cognitive levels to be used by educators to assess learners. The challenge of circle geometry curriculum coverage in rural schools has resulted from the lack of support, educator development, and the use of technological resources (Mahlaba, 2020; Mthethwa, Bayaga, Bossé & Williams, 2020).

Studies have indicated that educators in rural schools often encounter challenges using relevant and effective teaching strategies to involve all learners in the classroom to understand the content (Anggraini, Setyaningrum & Retnawati, 2020; Lee, 2016). Moreover, most educators in rural schools cannot teach the visualisation of geometric shapes and naming shapes in terms of their properties. Some educators have the challenge of teaching how to differentiate shapes if categorised as triangles, circles, or quadrilaterals. In circle geometry, triangles and quadrilaterals are within the orientation of the circle. These challenges resulted to learner inability to solve problems and (Sarkar, Kadam & Pillai, 2020; Ngirishi & Bansilal, 2019). As a result, educators cannot teach diagrams' spatial relations to prove theorems clearly. According to the Department of Basic Education (DBE) (2018), supported by

Mkhwanazi, Ndlovu, Ngema and Bansilal (2018), failure to prove theorems resulted in poor performance in Mathematics paper 2, grade 11 because of educators' inability to teach topic geometry. The diagnostic report indicates that most learners in rural schools skip circle geometry questions in the space provided for the answers. A blank space in the learners' scripts indicates that educators did not cover the circle geometry curriculum (Chiphambo, 2018).

1.2 BACKGROUND OF THE STUDY

Teaching circle geometry has been a challenge for some educators in rural schools. Studies conducted in geometry indicated that most educators use ineffective teaching methods to support learners' understanding of the content (Jones, Keith & Tzekaki, 2016). Cognitive processes in geometry are important for educators to understand the content they teach in proving theorems. In this exploratory study, educators were explored in terms of their teaching methods when proving circle geometry theorems and learners to solve problems. Proving theorems had challenges; some educators confirmed incorrect geometric reasoning in their lessons (Machisi, 2021). The incorrect geometric proving and reasoning resulted in learner poor performance in geometry because most of the marks in paper two were allocated to the topic of circle geometry (DBE, 2018).

Cognitive processes play an important role in understanding circle geometry (Hawes, Moss, Caswell, Naqvi & MacKinnon, 2017). Educators were expected to teach in class and use learning Tasks to prove circle geometry theorems, and to achieve this, educators were supposed to make sure that the diagrams were interpreted clearly in the Tasks to teach theorems correctly. A study by Mwadzaangati and Kazima (2019) indicated a need to develop spatial reasoning when proving circle geometry theorems. In support, educators should teach circle geometry theorems daily, use learning Tasks in their lesson plans, and solve geometric problems. Proving more geometric problems by learners involving all cognitive processes will improve performance in Mathematics.

A study conducted by Baah-Duodu, Osei-Buabeng, Cornelius, Hegan, and Nabie (2020) has shown the importance of involving educators to participate in workshops and train them on tasks involving cognitive processes to prove circle geometry. Therefore, the background of the study outlines how some educators perceived the teaching of circle geometry. As this study is classroom-based but specifically to educators on how they teach circle geometry, most educators perceive it as challenging content. In most rural schools, it has been reported how circle geometry has challenged educators to support learners in terms of scoring marks. In educator development content workshops, most educators and learners in rural schools were assessed with circle geometry Tasks to prove theorems and solve geometric problems.

Geometry studies indicated that teaching Tasks in geometry should involve geometric spatial reasoning. It is suggested that educators need to understand what is expected in geometry Tasks before assessing learners to prove the theorems. In doing so, educators need to visualise the geometric shapes, checking if there will be an application of construction and geometric reasoning. Visualising the shapes helps identify the figures and name shapes in terms of their properties. Educators understand and differentiate shapes if categorised as triangles, circles, or quadrilaterals. In circle geometry, triangles and quadrilaterals are within the orientation of the circle (Sarkar, Kadam & Pillai, 2020).

Construction within the circle is important for educators in the teaching process. After visualising geometric shapes, educators were expected to construct them within the circle. In most cases, educators and learners fail to understand when to decide to make constructions. Jayathirtha's (2018) study indicated that it is difficult to prove a theorem without constructions. Visualisation and construction are the levels of thinking educators should teach in geometry content. Constructing within the circle allows educators to apply reasoning and thinking of line segments, chords, arcs, perpendicular line, and angles. Educators are expected to teach learners how to manipulate geometric shapes in terms of their properties and proving the theorems with the correct geometric reasoning. Some studies in geometry showed that visualisation, construction, and reasoning are important in proving circle geometry. These processes allow educators to teach application of the correct geometric statement and reasoning when teaching circle geometry (Yaqin, 2020).

In conclusion, I support the application of visualisation in teaching and learning circle geometry because some educators have had challenges in proving something that has not been visualised before. Construction in circle geometry is important as educators understand how to teach the drawings of perpendicular lines, chords, arcs, tangent lines, and line segments. Moreover, circle geometry should be interesting to motivate educators in their lessons to achieve the expected outcomes at the end of the teaching and learning process. Educators must show how geometric reasoning must be provided in every geometric statement.

1.3 RESEARCH PROBLEM

Geometric spatial reasoning is one of the requirements for proving circle geometry theorems (Mwadzaangati, 2019). Circle geometry theorems require educators to elaborate on composing and decomposing shapes, visualising shapes, the relationships within the orientation of the circle, and their rotation in space. Furthermore, circle geometry is taught to understand how to solve geometric problems and prove theorems with reasons (Alex & Mammen, 2016). In addition, teaching circle geometry assesses the spatial visualisation of geometric shapes, making interpretations of the shapes' relationships and reasoning on the interpretations. Furthermore, educators can elaborate on making conjectures and develop arguments to prove theorems correctly (Stylianides, Bieda & Morselli, 2016).

The research conducted by Sinclair, Bussi, de Villiers, Jones, Kortenkamp, Leung, and Owens (2016) pointed out that teaching circle geometry remains somewhat limited in proving theorems. Educators in rural schools use different methods of teaching circle geometry. Still, they do not emphasise the issue of composing and decomposing shapes, the application of visualising shapes before proving the theorems, the relationship of shapes within the orientation of the circle, and rotating them in space (Anggraini, Setyaningrum & Retnawati, 2020; Novita, Putra, Rosayanti & Fitriati, 2018). In support, these studies highlighted that some teaching methods are ineffective in circle geometry. It results in learners' failure to prove the theorems correctly. Some learners in Tasks leave blank spaces with no geometric reasoning. Therefore, educators produce poor results in mathematics (Jones & Tzekaki, 2016).

This study aims to explore the teaching and learning of circle geometry. In Grade 11 content, lessons will be observed where educators teach circle geometry. Educators' presentations will be recorded for analysis at the end of every lesson. In addition, reflective interviews will be held with the participating educators in rural schools to clarify why they teach the way they do.

1.4 PURPOSE OF THE STUDY

This study explores the teaching and learning of circle geometry in rural schools. To address the shortcoming of educators failing to teach geometry problems involving proving theorems and reasoning. This will assist in terms of implementing the strategies to improve learner understanding of the content.

1.4.1 Research aims and objectives.

This study aims to explore the teaching and learning of circle geometry in rural schools. The study has the following objectives:

- To identify teaching methods utilized by Mathematics educators to teach circle geometry in Grade 11.
- To document instructional resources that Grade 11 Mathematics educators choose and use when teaching circle geometry.
- To list strategies educators in a rural setting use to teach geometrical proofs and theorems.
- To list strategies Grade 11 learners use to learn circle geometry theorems.

Also, this section addresses the potential value and findings of the circle geometry study. The findings of the study will have a positive impact on the teaching and learning of Circle Geometry. The knowledge of shapes should not focus on attaching names to the shapes only but go to the extent of thinking in geometry, proving theorems and reasoning. Geometric contexts should include the applied theorems, properties, and shapes within the circle's orientation (Mulligan, 2015).

Jones and Tzekaki (2016) stated that geometry develops educators' knowledge in the teaching process. It also encourages learners' understanding in the learning process. Regarding understanding geometric figures and concepts, educators' knowledge and understanding should be based on using the correct teaching methods to prove the theorems. Accurate geometric statements depend on the level of cognitive process in dealing with shapes within the circle's orientation.

In research aim 1, educators used different methods of teaching such as demonstration, small groups, learner-centred, question and answer to teach circle geometry. These teaching methods were utilized to teach and assess learners in the classroom. The lessons took place in two schools visited to observe the teaching and learning of circle geometry.

In research aim 2 the educators were provided with smart board and chalkboard to teach. The two educators choose to use a chalk board only. Learners had challenges to visualise the angles and sides of geometric shapes.

Strategies in research aim three were suggested to teach circle geometry proofs and theorems. The strategies included the use of short tests to assess learners weekly. Lastly, learners were motivated to complete all the tasks given in the classroom and seek clarity to challenging questions.

1.4.2 Research Questions

The study is guided by the following research question (RQ) and sub-questions:

How do Grade 11 mathematics educators in rural schools teach circle geometry?

1.4.3 Sub-questions

- What resources do Grade 11 mathematics educators use when teaching circle geometry?
- What teaching methods do Grade 11 mathematics educators use when teaching circle geometry?
- How do Grade 11 learners solve geometric problems and proving theorems?

1.5 LIMITATION AND DELIMITATIONS

Limitations affect the study's results and how the results are interpreted. Due to the nature of this study being qualitative with small number of participants, the results may not be generalizable. Some participants could not provide detailed explanations during lesson observations and semi-structured interviews, and that limited the results.

Delimitations affect the study in which the research generally has some degree of control. To ensure the management of the collected data in the study, the study's methodology focused on a case study design for exploration while teaching and learning circle geometry. Data required were collected and analysed to respond to the main research questions in the study.

1.6 ASSUMPTIONS OF THE STUDY

This section addresses the limitations that affected the study:

- In the study, the assumption is that gender of grade 11 educators will not affect their views.
- It is assumed that all the sampled participants will be observed when teaching circle geometry in their classrooms and semi-structured interviews for the educators in a rural school. The uses of audios are recorded as a data collection method, semi-structured interviews, and learners’ scripts to serve various purposes and questions honestly and to the best of their spatial abilities.

1.7 ORGANIZATION OF THE STUDY

This study is divided into five (5) chapters outlined in detail below:

Chapter one outlines the introduction and the background of the study. The introduction highlights the essential concepts and constructs and how they were used in the study. Secondly, this chapter addresses the literature that is concerning this study. Thirdly, this exploration aims to explore the teaching and learning of circle geometry in rural schools. The exploration findings contribute to geometry studies to clarify why grade 11 educators teach the way they do.

Chapter two reviews related literature by comparing and contrasting other researchers' findings in geometry studies. The findings by other researchers were also analysed to highlight the teaching and learning of circle geometry in rural schools. Therefore, the researchers have detailed that cognitive processes of geometric thinking are essential in teaching circle geometry.

Chapter three outlines the research methodology employed in this study, which was qualitative research to obtain quality results. Firstly, this chapter discusses the research approaches applied in the study. The method of data collected, and data analysed were detailed in the chapter. Data collection methods, data collection and data analysis are discussed in the chapter. This chapter also covers the quality criteria to ensure the quality results of the study.

Chapter four gives the interpretation of the findings and discussion in detail. Results were analysed and interpreted following the cognitive processes of Duval (1998). The findings from educators' responses during lesson observations, semi-structured interviews, and learners' scripts were captured in the tables.

Finally, **Chapter five** presents the summary of findings and the recommendations to explore the teaching and learning of circle geometry in rural schools. The contributions of this study to geometry studies and the limitations are clearly discussed in this chapter. The conclusion to the research questions pursued in the study and the problem statement is addressed.

CHAPTER 2 : LITERATURE REVIEW

2.1 INTRODUCTION

A literature review is conducted to compare previous studies related to this study. An outline of the results and findings of previous studies will be discussed and will show how this study will contribute to the teaching and learning of circle geometry.

2.1.1 Educator and learner-related variables

Educator's content knowledge in circle geometry is important in the performance of the learner. Most educators and learners do not enjoy teaching and learning of circle geometry, and they spend less time in this content (Mthethwa, Bayaga, Bossé & Williams, 2020). Ngirishi and Bansilal's (2019) study focused on exploring and understanding basic geometric concepts. The study's findings indicated the importance of teaching and learning how to compose and decompose shapes, visualising shapes, the relationship of shapes within the orientation of the circle and rotating them in space. It also outlined the challenges that resulted from educators failing to teach the topic properly (Novita et al., 2018).

Luneta's (2014) study focused on errors resulting from solving and proving geometric problems. The study's findings highlighted that errors were the cause of failing to apply correct geometric concepts. However, a study by Ngirishi and Bansilal (2019) presented contrasting findings. Their study did not focus on specific aspects, which is the reason when proving circle geometry theorems. The study by Luneta (2014) used shapes of circle geometry to understand geometry but did not show the procedures for proving the theorems. Therefore, educators continued using their teaching methods when proving theorems but did not produce good results.

a. Attitude of educators Towards Circle Geometry

In study, there has been an area focusing on educators' belief and attitude towards circle geometry. Luneta (2014) observed the importance of dealing with shapes and their properties to develop educators' content knowledge, the system of beliefs towards circle geometry and its teaching and learning. The teaching part provides educators' level of thought process of reflection. Educators are encouraged to associate themselves with its teaching and learning to develop positive attitude in this section.

b. Attitude of Educators towards Learners in Geometry

Studies related to the issue of attitude towards learners in circle geometry highlight factors in developing learners' geometry spatial ability in the classroom. This ability can be affected by the attitude of the educator towards the learners. It is the educators' belief about their learners' ability to learn circle geometry and how they teach it in the classroom (Brijlall & Abakah, 2022).

c. Educator Quality

A study by Darling-Hammond and Oakes (2021) indicated the importance of preparing educators for what life is like in the classroom and society. Anggraini, Setyaningrum, and Retnawati's (2020) study focused on improving geometric spatial reasoning. The findings of their study indicated that teaching and learning circle geometry should involve visualising geometric shapes and applying reasoning to support the geometric statements. Wongkia's (2018) study focused on the angle properties in circle geometry. The study's findings indicated that teaching tasks should involve the angle properties to understand circle geometry. In contrast to these studies, Anggraini et al. (2020) showed that geometric reasoning is underrepresented. As a result, some of the teaching Tasks did not involve spatial reasoning, and the consequence of lacking spatial reasoning resulted in educators' inability to produce good results in mathematics. Wongkia's (2018) study only presented an understanding of angle properties but not geometric reasoning.

2.1.2 Challenges related to learners' improvement when learning circle geometry

In my view, learners were not given high quality teaching at the lower grades. This has resulted because educators do not have the necessary content knowledge of circle geometry. The study conducted by Dhlamini, Chuene, Masha, and Kibirige (2019) on exploring geometry spatial mathematical reasoning outlined the results and findings related to this study. The study's results showed blended irrelevant prior knowledge in teaching geometry Tasks. Their study recommended that understanding geometry can be achieved using spatial visualisation, orientation, and relation.

a. Circle geometry Best Practices

Educators should know what learners need to learn based on prior knowledge to develop conceptual understanding in circle geometry. Learners are encouraged when the engagement is at a high level and providing scaffolding to support learners. The study conducted by Lowrie and Logan (2018), which focused on spatial reasoning, reported that spatial reasoning involves the correlation between geometric spatial reasoning and concepts relevant to the context of circle geometric theorems. Moreover, the reasoning is an essential cognitive aspect of understanding circle geometry. In support of the studies, educators could not teach spatial reasoning to prove circle geometric theorems.

b. Learners' Desire to Explore Geometric Figures

The focus of the learner should be on problem-solving, proving theorems, geometrical reasoning and making judgment. At some other point, Septia and Prahmana's (2018) study focused on improving spatial reasoning in circle geometry and highlighted that spatial reasoning consists of components to be applied in teaching and learning Tasks. Furthermore, the components of spatial tasks are spatial visualisation and spatial orientation. Spatial visualisation is the understanding and presenting geometric shapes in two and three-dimensional forms. Spatial orientation is the understanding of geometric shapes, relationships and properties. Their findings indicated the improvement effect, achievements, and reasoning in geometry.

In contrast, they only focused on visualisation, the orientation of shapes, and reasoning but were limited to proving circle geometric theorems. In this study, spatial reasoning and proving theorems were researched in detail. Research conducted by Fujita, Kondo, Kumakura, Kunimune, and Jones (2020) focused on identifying spatial reasoning and teaching how to solve geometrical problems. The results from the descriptive data showed a lack of spatial visualisation and application of spatial reasoning. These findings indicated that more research is needed about educators' skills to teach proving circle geometry theorems. However, the study did not address all steps to teach proving theorems.

Jones and Tzekaki's (2016) study focused on using geometric reasoning when teaching how to prove theorems. Their study showed that geometric spatial reasoning is vital for teaching and learning but was not often used when proving circle geometric theorems. A qualitative observation analysis indicated that most educators still have challenges when teaching proving theorems. In the findings, spatial reasoning was outlined but limited to addressing the specific interest in this capacity. Therefore, cognitive processes are important in evaluating geometrical competence. Miyazaki, Fujita, and Jones's (2017) study focused on construction and reasoning. The findings presented how to teach the construction of geometric shapes and apply geometric reasoning in circle geometry. In contrast to the studies, there was a lack of support and evidence on teaching geometric spatial reasoning when proving theorems.

A study by Seah and Horne (2019) focused on promoting the development of geometric reasoning progression. The study also outlined the importance of knowledge in teaching and learning geometry. Furthermore, the study supports the idea of teaching geometric reasoning when proving theorems. Moreover, Seah and Horne (2019) explained how the study aimed to improve geometric spatial relationships between knowledge and reasoning, which means that understanding geometry helps justify reasoning. The study's results showed the application of geometric shape properties but were limited to proving theorems, geometric knowledge, and reasoning. Data analysis indicated that most educators could not identify geometric shapes because of limited geometric knowledge. Therefore, findings in the study showed that educators should cover the geometric cognitive reasoning level. In contrast, geometric reasoning and knowledge development were addressed but were limited to proving theorems.

According to Luneta (2015), the focus on understanding misconceptions emerged when analysing performance in the final examination. In the study, understanding concepts in geometry is important because it enables educators to teach visualisation in geometric shapes, construct within the shapes, and prove theorems with reasons. The analysis from the study indicated that errors in the final examination were the results of educators limited geometric knowledge. Most of the educators were still following the old curriculum instead of CAPS. Research conducted by Mix and Battista (2018) focused on comparing geometric shapes and their connections.

Furthermore, the findings indicated that educators should use teaching Tasks that require comparing geometric shapes and their connections. Thus, further research should be conducted to address spatial reasoning and proving theorems. Spatial reasoning should correlate with spatial abilities and teaching outcomes to understand circle geometry (Jones & Tzekaki, 2016). Proving circle geometry theorems revolved around the ability to explore geometric thinking. Educators were expected to apply the knowledge of angles and shapes within the circle's orientation when teaching geometry (Zandieh, Roh & Knapp, 2014).

In conclusion, a literature review showed educator and learner-related variables, beliefs and attitudes towards the teaching and learning of circle geometry. It is indicated that most rural educators and learners perceive circle geometry as the challenging content. Educators in rural schools could not teach composing and decomposing geometric shapes, visualising shapes, the relationship of shapes within the orientation of the circle and rotating them in space. Learners had challenges to exhibit their skills to prove and solve circle geometry tasks (Novita et al., 2018). Teaching and learning tasks did not assess the spatial skills of geometry learning-rich concepts such as visualisation of shapes, construction of geometric shapes within the orientation of the circle, and spatial reasoning. However, in connection to proving circle geometric theorems, grade 11 educators in rural schools were expected to teach the content guided by the CAPS document.

2.2 THE ROLE OF THEORY

The cognitive theory of geometric reasoning (Duval, 1998) guided the study. The theory provided a detailed framework for analysing the results from geometric drawings, problem-solving and proving theorems. Furthermore, the theory illustrated how educators should teach circle geometry (Hershkowitz, Duval, Bussi, Boero, Lehrer, Romberg & Jones, 1998). Duval (1998) showed that geometric reasoning involved three (3) cognitive processes that fulfil specific epistemological functions and ways of teaching circle geometry.

2.2.1 Application of Cognitive processes

These cognitive processes were visualisation, construction, and reasoning. First, visualisation involves the extrapolation of space and illustrations of statements (Karaman, 2017). Secondly, construction refers to the tools and applications of properties constructed within the geometric shapes when proving theorems (Zandieh et al., 2014). Thirdly, reasoning relates to thinking, understanding, and forming judgments logically (Duval, 1998). In this study, geometric reasoning used logical argument and spatial reasoning to solve problems and find the relationships between concepts. Figure 2.1 below shows the connectedness of the cognitive processes, and the arrows indicate how the processes support one another when proving circle geometry theorems.

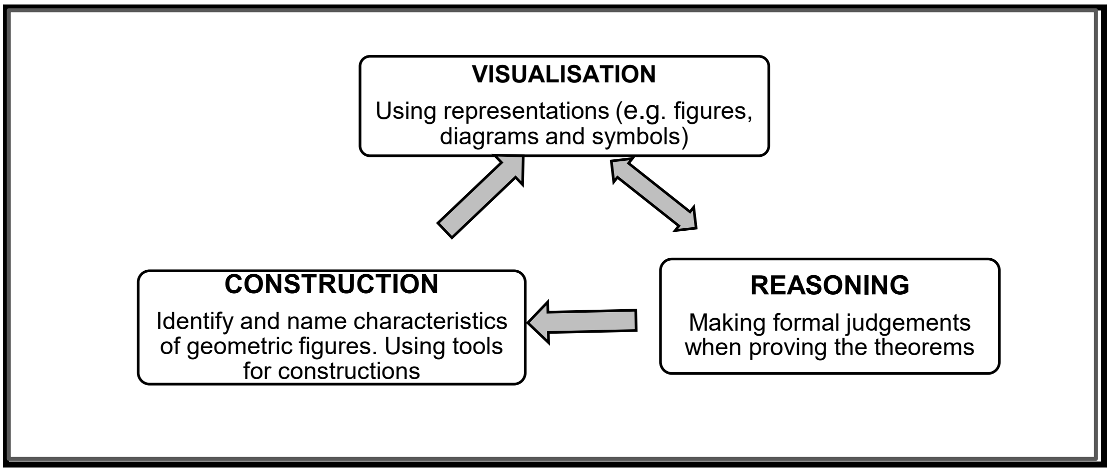


Figure 2.1: An adapted Duval cognitive theory of geometric reasoning by Kuzniak & Richard, 2014

In this study, Duval's cognitive processes were employed in the analysis of educators' lesson presentation as follows: 1.) Teaching visualisation in geometric shapes, recalling while noticing the properties of shapes, central angles, equal chords forming a triangle within the orientation of the circle and a tangent line. 2.) Construction process was through composing and decomposing geometric shapes. 3.) In reasoning, educators were expected to use geometric reasoning to support the statements when teaching how to prove the theorems and make connections between a circle's properties to reach a judgment **(see Appendices I (a) – I (n))** (Kuzniak et al., 2014). In the exploratory study, Duval's theory was employed following the five steps of the case study research design.

The cognitive theory of geometric reasoning stresses that educators should understand to connect geometric concepts and cognitive processes (Sinclair & Bruce, 2015). The underlying cognitive processes that educators should adopt in teaching circle geometry:

Table 2.1: Duval's cognitive processes of geometric reasoning

Cognitive processes	Description of processes
Visualisation	• Refers to the use of representations (e.g., figures, diagrams, and symbols) • Educators visualize geometric figures and how they are formed and notice the properties of shapes before teaching learners.
Construction	• Educators identify and name characteristics of geometric figures. • Educators make connections between properties and use tools for construction
Reasoning	• Educators make formal judgments during the lesson when proving the theorems. • Educators prove theorems correctly and provide geometric reasons to support geometric statements.

In Table 2.1, Duval's cognitive processes of geometric reasoning were outlined to explain the interactions between visualization, construction, and reasoning. Educators were expected to follow all these processes to prove the geometric theorems correctly (Conner, Singletary, Smith, Wagner & Francisco, 2014).

2.2.2 Spatial abilities in teaching visualizations

According to Münzer et al. (2018), educators had different spatial abilities in teaching visualization and manipulating mental visual-spatial representations of 2D shapes. In task-related to visualization, educators were expected to visualize a geometric figure before stating that lines FB is parallel to DC. As planned, the spatial visualisation in the task supported the statements to be true after proving the theorem (**see Appendix I**). The Grade 11 educators were expected to apply the knowledge of theorems and properties of shapes to prove Circle geometry theorems **(see Appendices I (a) – I (n))**. These allowed them to identify shapes such as circles, triangles, quadrilaterals, and other geometric parts like lines and angles based on their appearance. However, rural school educators were allowed to explore the teaching and learning of circle geometry (Sinclair et al., 2016). The description of the cognitive visualisation process identified on the content of geometry is highlighted in Table 2.2 below:

Table 2.2: Visualisation of geometric shapes and properties of shapes

Visualisation of geometric shapes and their properties	Content identified
1. Geometric Shapes	• Visualising different types of geometric shapes • Categories of shapes
2. Properties of shapes: Triangle, circles, and quadrilaterals	• If it is a triangle (Δ) within the orientation of the circle, the number of angles and sides (three) are calculated. • The number of angles and sides (four) are calculated if it is a quadrilateral. • The orientation of the circle is observed as its properties. • Angles are in terms of $\boldsymbol{x, y, \theta, \alpha, and\ \beta}$.

Table 2.2 above shows the content identified in visualising geometric shapes and their properties. It was also important to compare and contrast if the identified shapes and the properties were covered in teaching tasks set on circle geometry. Duval's cognitive processes outlined the importance of teaching and learning circle geometry in rural schools to assist educators with cognitive process application. Teaching tasks planned in their lessons should promote the proving of theorems to evaluate the final answer. Mathematical reasoning is usually described as inductive or deductive. But for this study, deductive reasoning was used to support geometric statements and reasoning.

This reasoning was applied to how geometric statements and reasons are written. Therefore, educators and learners were to explore the teaching and learning of circle geometry in rural schools to ensure that circle geometry theorems are proven correctly (Christou et al., 2014). As mentioned earlier, Duval (1998) tried to identify reasons why and how geometry should be taught in a mathematics classroom. The three cognitive processes should ultimately fulfil specific epistemological functions to circle geometry theorems. Educators should teach skills in constructing and describing geometrical figures and their properties.

2.2.3 Spatial abilities in teaching constructions

Berney, Bétrancourt, Molinari, and Hoyek (2015) argued that applying theorems and geometric spatial reasoning is developed after correctly visualising the shapes. Also, Grade 11 educators should teach geometric spatial reasoning in their teaching tasks (Marschark, Morrison, Lukomski, Borgna & Convertino, 2013). In tasks related to construction, the Grade 11 educators were expected to make constructions, for example, drawing a diameter of length NR and RM. The spatial relation allows the development of geometric spatial reasoning by stating that the radius is perpendicular to the tangent, the angle in a semi-circle, and the same segment. Table 2.3 below shows the construction process covered by the circle geometry tasks and how the shapes should be drawn:

Table 2.3: Duval's cognitive apprehensions in constructing geometric figures.

Cognitive apprehensions	Content identified
Perceptual	• visualizing shapes and linking with their properties
Sequential	• representations and constructions within the shapes
Discursive	• process whereby educators prove circle geometry theorems
Operative	• evaluation and judgments of educators' responses in circle geometry teaching tasks

Table 2.3 above shows the cognitive apprehensions and content covered when constructing geometric shapes within the orientation of the circle. In support, Duval (1995) used four (4) cognitive apprehensions: perceptual, sequential, discursive and operative as an analytical framework to explain how to teach construction in geometric figures. The following is a summary of cognitive apprehensions incorporated with the cognitive processes to analyze the tasks used to assess learners:

i.) *Perceptual Apprehension*, related to tasks that involved visualization and recognition of geometric figures (shape, representation, and size).

ii.) *Sequential Apprehension*, in geometric tasks, the construction of a figure and construction depends on knowing the names of figures and the relationship of their properties.

iii.) *Discursive Apprehension*, the ability to define and name shapes in terms of their properties, indicates what the figure shows and what it represents (see task 2); and iv.) *Operative Apprehension*, related to tasks, whereby judgments were made based on a given figure in various ways and understanding how to prove the theorems.

2.2.4 Spatial abilities in teaching geometric reasoning

In tasks related to geometric spatial reasoning, educators were expected to prove that MC is a tangent to the circle at C and if triangle ACB is similar to triangle CMD **(see Appendix I)**. Research done by Bergwall (2019) found that educators had difficulty understanding the role of theorems in circle geometry. Over the past years, research conducted in different mathematical studies indicated that some educators could not provide geometric statements when teaching tasks on circle geometry (Kuzniak et al., 2014). On the other hand, some researchers have disputed that what educators teach in the classroom cannot be separated from how they apply it in real-life situations. Therefore, to ensure that circle geometry is enjoyable, motivating, and appropriate as possible, the curriculum should be a combination of aspects that inform the importance of the topic (Siew, Chong & Abdullah, 2013). Table 2.4 shows the spatial reasoning in circle geometry when proving the theorems.

Table 2.4: The spatial reasoning in the geometric figure: circle geometry and the content identified.

Spatial reasoning in geometric figures	Geometric reasoning/Statement	Geometric statement Given (Good/acceptable/ Poor)
Orientation of the circle: - • Shape	Investigate and extend: - • The circle has the largest area for a given length of the perimeter	Good
• Properties	• The perpendicular line is drawn from the centre of the given circle.	Good

	• Dividing the chord into two the equal parts • The radius is drawn perpendicular to the tangent	
• Definitions	The diameter, chord and arc of a circle are as follows: • The diameter of a circle is twice as long as the radius. • A chord of a circle connects two points on a circle. • An arc is a part of that circle.	Good
The overall analysis of the agreement		**Good**

Table 2.4 above shows the spatial reasoning in the geometric figure: circle and geometric statements to be met. All content identified on the geometric statements and the research related to circle geometry indicated how educators should perceive teaching to prove circle geometry theorems and learners to understand the content. This suggests that educators and learners should follow the table above to understand the properties and definitions of the part of the circle.

Table 2.5: The spatial reasoning in the geometric figure: circle geometry theorems and the content identified.

Theorems	Geometric reasoning/Statement used in the proof of the theorem	Application of geometric reasoning (Good/acceptable/ Poor)
Theorem 1	• **A line from the centre of the circle to the chord**: *If a line is drawn from the centre of a circle perpendicular to a chord, it bisects the chord.*	Acceptable
Theorem 2	• **The perpendicular bisector of a chord**: *If the perpendicular bisector is drawn, it passes through the circle's centre*.	Acceptable
Theorem 3	• **The angle subtended by an arc or chord**: *The angle subtended by an arc at the centre of a circle is double the size of the angle subtended by the same arc at the circle.*	Acceptable
Theorem 4	• **Angles in a circle**: *If the same chord or arc subtends angles on a circle, the two angles are equal*.	Acceptable
Theorem 5	• **Cyclic quadrilaterals**: *If a quadrilateral is cyclic, then the opposite angles are supplementary.*	Acceptable
Theorem 6	• **Tangents to a circle**: *If two tangents are drawn from the same point outside the circle, they are equal in length.*	Acceptable
Theorem 7	• **The angle between a tangent and a chord**: *The angle between a tangent to a circle and a chord drawn from the point of contact is equal to an angle in the alternate segment.*	Acceptable
The overall analysis of the agreement		Acceptable

Table 2.5 above shows the spatial reasoning in the geometric figure: circle geometry theorems and statements to be met. All content identified on the geometric statements and the research related to circle geometry indicated that most educators performed poorly in the tasks completed during the content workshop. This suggests that

educators and leaners to follow the table above to understand how to prove the theorems and their application to circle geometry. Moreover, the overall analysis of agreement indicated that geometric reasoning used in proving theorems were Acceptable.

2.2.5 The existing research on interpreting geometric shapes representations

Luneta (2014) argued that educators would likely avoid teaching challenging content such as proving geometry theorems. The existing research on interpreting 2D shapes representations supports that geometric spatial reasoning should be through the cognitive processes of geometry tasks. This would allow educators to know how to present and teach all questions set on circle geometry.

Table 2.6: Sampled circle geometry tasks: Application of circle geometry theorems and the content identified.

Theorems and teaching tasks	Geometric reasoning/Statement used in the proof of the theorem	Application of geometric reasoning (Good/acceptable/ Poor)
Appendix: Teaching Task 1	• The angle at the centre = 2 x angle at the circumference	Acceptable
	• Equal chords; equal angles	Acceptable
Appendix: Teaching Task 2	• The angle at the centre = 2 x angle at the circumference	Acceptable
	• Opposite angles of cyclic quad	Acceptable
	• Sum of the angles in a triangle	Acceptable
Appendix: Teaching Task 3	• A line from the centre and perpendicular to the chord	Acceptable
	• Theorem of Pythagoras	Acceptable
Appendix: Teaching Task 4	• The angle at the centre = 2 x angle at the circumference	Acceptable
	• Angles around a point	Acceptable
The overall analysis of the agreement		**Acceptable**

The depth of knowledge consistency

The category, the depth of knowledge consistency, involves curriculum coverage, specific aims and teaching and learning tasks (Karuguti, Phillips, & Barr, 2017). Cognitive complexity assists in distinguishing the levels of knowledge and understanding and different levels of cognitive thinking. It is advisable to categorise the tasks into low, moderate and high degrees of difficulty. Morrison and Embretson (2014) explain that circle geometry tasks should emphasise specific geometric skills to show the differences in cognitive thinking. Geometric teaching and learning tasks should clearly highlight the various skills to be demonstrated and achieved at the lesson's end.

Therefore, analysing the cognitive processes in circle geometry tasks became an important theory to guide the study. Zambak and Tyminski (2020) emphasise that highlighting cognitive processes in teaching circle geometry can assist educators with approaches to teaching geometry and differentiating the cognitive processes of the assessment tasks. Musso, Boekaerts, Segers and Cascallar (2019) indicate that cognitive processes have been categorized into three levels: Level 1 (visualisation), level 2 (construction) and level 3 (reasoning). Firstly, level 1 deals with the ability to visualise geometric objects and apply geometric spatial reasoning. Educators should recall parts such as the properties of shapes and simple procedures to teach the content. Secondly, level 2 deals with important construction within shapes. In the case of a mental process, it allows educators to choose the appropriate teaching methods to find the unknown values of the angles and sides in shapes. Lastly, level 3 deals with spatial reasoning, requiring educators to explain and reason to learners using a particular method to reach the final answer. Educators come up with strategies to get to a geometric solution. With extended geometric spatial reasoning, the items that require a high level of reasoning and planning are well explained to the learners for some time before engaging in them to improve performance in grade 11 geometry.

As South Africa participated in TIMSS 2019, the thinking levels also link to Duval’s cognitive levels. This theory consists of four categories: knowledge, routine procedures, complex procedures and problem-solving. TIMSS’s cognitive levels have been described as introductory cognitive, and it deals with content-specific skills and separate parts of information like constructions and geometric reasoning. Vale, Pimentel and Barbosa (2018) describe ‘routine procedures as a level where educators are expected to teach learners to carry out simple calculations when solving or proving geometric problems. The level involves steps, such as solving problems in circle geometry. Shonhiwa (2020) describes the concept of ‘complex procedures’ as abstract problems that are an indirect route to the solution; here, educators are expected to use geometric concepts to solve problems and prove geometry theorems.

Lastly, DBE (2011) described problem-solving as non-routine problems requiring specific and complex cognitive skills and reasoning levels to solve geometric problems. The problem might require educators’ ability to compose and decompose geometric shapes into controllable parts.

The importance of bringing into line this geometric study through the recommended thinking levels by the DBE, the researcher employed Duval’s cognitive levels to analyse the way grade 11 educators are teaching circle geometry during Mathematics content workshop. The analysis of the abovementioned category was done qualitatively. The triangulation analysis focused on identifying and comparing cognitive levels covered in the Duval theory and circle geometry tasks that grade 11 educators taught during the content workshop. When the educators covered the circle, information displaying the geometry content measuring the same cognitive complexity was visible.

Here, the content analysts have drawn the content standards objectives to be achieved when teaching circle geometry problems and proving theorem. Ultimately, it was possible to compare various components in circle geometry to see if the same cognitive content processes were covered in teaching tasks.

Likewise, the comparison in the categories was the cognitive complexity of the content standards suggested by the DBE and the Duval cognitive processes. This ensured that the educators implemented the suggested strategies in circle geometry lessons. In the content workshop for grade 11 educators, the scale of the agreement were ‘full’, ‘acceptable’, and ‘insufficient’. The scale ‘full’ in educators’ presentations was applicable where the cognitive complexity of the content standards objectives was the same as the cognitive processes of Duval. Again, an ‘acceptable’ agreement was applicable where the cognitive complexity of the content standards objectives was partially the same as the cognitive processes of Duval. The ‘insufficient’ agreement was applicable where the content standards objectives' cognitive complexity did not correspond to the cognitive processes of many assessment tasks in circle geometry.

It was important to explore the depth of knowledge consistency in geometry, the teaching and learning of circle geometry in rural schools, and the alignment in terms of the cognitive complexity for both content standards objectives and Duval's cognitive processes in geometry. The following example illustrates the qualitative analysis of cognitive processes concerning circle geometry content standards objectives for Grade 11. Table 2.7 below shows the cognitive processes in Grade 11 content standards objectives.

Table 2.7: Grade 11 cognitive processes identified for content standards objectives.

<table>
<tr><th colspan="2">Content Standards objectives</th><th>Cognitive processes</th></tr>
<tr><td rowspan="5">1. Solve geometric problems determining the unknown angles and sides in shapes, properties of shapes between shapes, and geometric spatial reasoning:</td><td>• Has been represented in physical or diagram form</td><td>• Visualisation
• Construction</td></tr>
<tr><td>• not limited to proving other theorems</td><td>• Visualisation
• Reasoning</td></tr>
<tr><td>• of educators' own approach</td><td>• Visualisation
• Construction
• Reasoning</td></tr>
<tr><td>• represented in tables such as geometric statements and reasoning</td><td>• Construction
• Reasoning</td></tr>
<tr><td>• represented algebraically</td><td>• Reasoning</td></tr>
<tr><td colspan="2">2.Prove the theorem in a geometric shape</td><td>• Visualisation
• Construction
• Reasoning</td></tr>
</table>

Table 2.7 above shows the thinking levels identified by Duval, highlighting the content standards objectives. The cognitive levels identified for the content standards objectives were compared with those identified by the DBE for the assessment tasks in the CAPS document. Table 2.7 shows the cognitive levels identified for the Grade 11 circle geometry tasks.

Table 2.8: Grade 11 cognitive complexity identified in circle geometry tasks.

Assessment items	Cognitive complexity identified

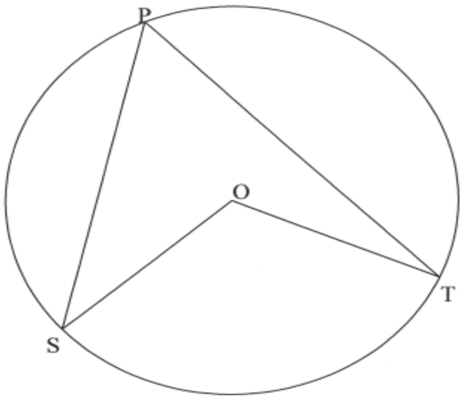
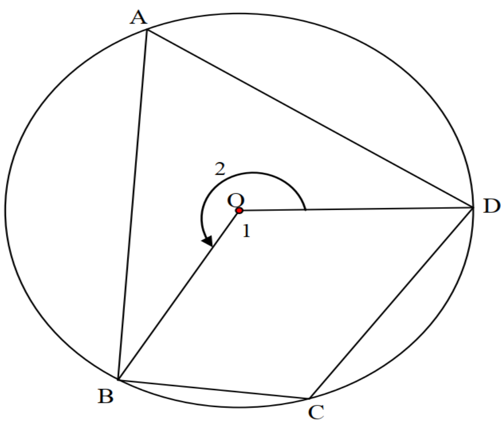
1. O as the centre of the circle, the length of AB is 24 cm, and OP is 13 cm • Determine the value of x. • Prove the theorem that AP = PB • Determine the values of unknown angles	• Knowledge • Routine procedures
2. In the diagram below, O is the circle's centre. P, T and S are points on the circumference of the circle. PS, PT and OT are drawn.  Figure 2.2: Circle geometry 1 Prove the theorem which states that $\text{SOT} = 2 \times \text{SPT}$.	• Knowledge
3. In the diagram below, a cyclic quadrilateral $ABCD$ is drawn within the circle with centre O. Figure 2.3: Circle geometry 2  Complete the given statement below: The same chord subtends the angle subtended by a chord at the centre of a circle at the circle's circumference.	• Knowledge

Table 2.8 above displays the cognitive levels identified for the assessment tasks taught by the educators during the content workshop. There was a comparison

between the cognitive levels identified for the content standards to see if they covered the same Duval cognitive processes. Table 2.9 below shows instructional time in Grades 11 circle geometry.

Table 2.9: Instructional time in Grades 11

Subject	**Time Allocated Per Week**
Mathematics	**4.5**
Content	**Number of weeks in 2022**
Circle Geometry	**Three (3)**
Assessment	Control

The table above shows the allocated time per week required for National Senior Certificate (NCS) Mathematics, and educators need to teach circle geometry to cover all geometric theorems. Educators are expected to use various teaching methods to accommodate all learners and to reach the planned assessment tasks in a topic.

Table 2.10: Specific Aim 3 in mathematics (Circle Geometry).

Subject	**Specific Aim**
Mathematics	**3**
Content	**Number of weeks in 2022**
Circle Geometry	**Three (3)**
Assessment	Control

The table above shows the requirements from the CAPS document on how to teach circle geometry effectively and ensure that the presented work is correct. One example of those specific aims in number three (3) aimed:

- To provide the opportunity to develop in learners the ability to apply a methodical, to generalize, make conjectures and try to justify or prove them.

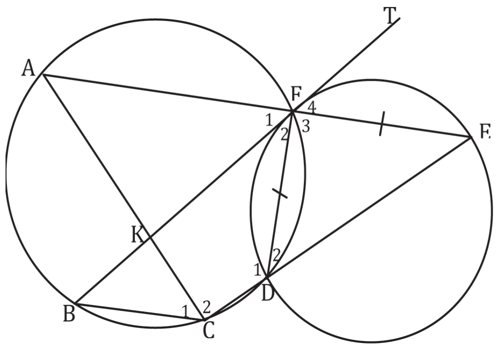
Table 2.11: Specific Skills

Subject	**Specific Aim**
Mathematics	• spatial skills and properties of shapes • objects to identify • pose and solve problems creatively and critically;
Content	
Circle Geometry	
Assessment	

Table 2.12: Weighting of the content (Circle geometry).

The weighting of the content	
Paper 2: Grades 11: theorems and/or trigonometric proofs: maximum 12 marks	
Description	**Grade 11**
Circle Geometry	40 ± 3

Table 2.13: he four cognitive levels used to guide educators in teaching circle geometry.

<table>
<tr><th>Cognitive levels</th><th>Description of skills to be demonstrated</th><th>Examples</th></tr>
<tr><td>Knowledge
20%</td><td>• a straight recall</td><td>1. The angle of $A\hat{O}B$ subtended by arc AB at the centre O of a circle</td></tr>
<tr><td>Routine Procedures
35%</td><td>• The proofs of prescribed theorems</td><td>2. Prove that the angle of $A\hat{O}B$ subtended by arc AB at the centre o of a circle is double the size of the angle $A\hat{C}B$ which the same arc subtends at the circle.</td></tr>
<tr><td>Complex Procedures
30%</td><td>• significant connections between different representations</td><td>3. Two circles intersect at F and D.


BFT is a tangent to the smaller circle at F. Straight line AFE is such drawn that FD = FE. CDE is a straight line, and chord AC and BF cut at K. Prove that:
3.1 BT // CE
(C)
3.2 BCEF is a parallelogram (P)
3.3 AC = BF (P)</td></tr>
</table>

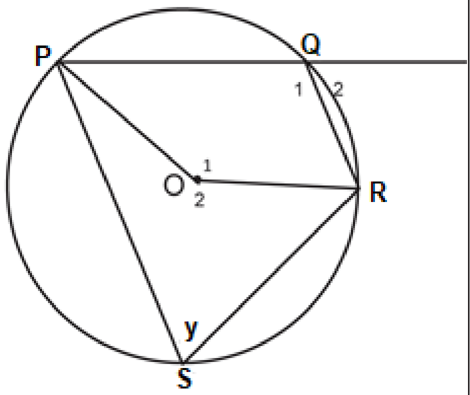
Problem-solving **15%**	• It might require the ability to break the problem down into its constituent parts	4. O is the centre of the circle below and $\hat{O}_2 = 2y$. Figure 2.4: Circle geometry 4 1  4.1 Determine $\hat{O}_2$ and $\hat{S}$ in terms of y.

Table 2.14: Cognitive levels identified on the content standards objectives and assessment for Grade 11.

Content standards objectives	Cognitive complexity of the content standards	Cognitive processes of Duval	The scale of the agreement for the presentation (Full/acceptable/ insufficient)
• Work example 1	• Knowledge	• Construction • Reasoning	• Acceptable
• Work example 2	• Routine procedures	• Construction • Reasoning	• Full
• Work example 3	• Complex procedures	• Visualisation • Construction • Reasoning	• Acceptable
• Work example 4	• Problem-solving	• Visualisation • Construction • Reasoning	• Acceptable
The overall scale of the agreement			**Acceptable**

Table 2.14 above shows the cognitive complexity levels and Duval’s processes covered by the identified content standards, objectives and assessment tasks. The analysis indicates that the assessment task that needed educators to teach circle geometry, specifically the theorem “The angle $A\hat{O}B$ subtended by arc ***AB*** at the centre **O**, " has the same cognitive complexity as the matching content standards objectives. They covered Duval’s cognitive processes, such as construction and reasoning. This indicates that the application of knowledge of geometric figures is important in circle geometry.

The assessment Task that needed educators to prove that the angle $A\hat{O}B$ is subtended by arc ***AB*** at the centre O of a circle is double the size of the angle of $A\hat{C}B$, which the same arc subtends at the circle. This fell under routine procedures. Duval's cognitive processes involving construction and reasoning covered the corresponding content standard objective indicating that the cognitive complexity of the content standard objective and Duval's processes are partly aligned.

The assessment Task that needed educators to prove theorems is if ***BFT*** is a tangent to the smaller circle at ***F*** and a straight line ***AFE*** is such drawn that ***FD = FE***. ***CDE*** is a straight line, chord AC, and BF is cut at **K**. Duval's cognitive processes, such as visualization, construction and reasoning, covering the corresponding content standard objective.

The assessment Task that needed educators to solve circle geometry theorems also required the ability to break the problem down into its basic parts to determine the angles if ***O*** is the centre of the circle and $\hat{O} = 2y$. Duval's cognitive processes covered the cognitive level of problem-solving with corresponding content standard objectives consisting of visualization, construction, and reasoning.

In conclusion, Duval's cognitive process theory emphasises that the content standards and assessment should be cognitively brought into one planned target. Thus, this study employed the theory of Duval's cognitive processes to explore the teaching and learning of circle geometry in rural schools. The following chapter aimed at focusing on the methodology employed in this study.

CHAPTER 3 : RESEARCH METHODOLOGY

3.1 INTRODUCTION

This chapter focuses on educational research approaches and explains why employing a qualitative study. It outlines the research designs under qualitative research and justifies employing the design in the exploration to focus on teaching and learning circle geometry in rural schools. The methods of data collection and data analysis and their related ethical issues are addressed in this chapter. Research methodology refers to the specific procedures used to identify, select, process, and analyse the information collected. Moreover, the chosen research methods were used to produce quality research results (Best & Kahn, 2016).

3.2 RESEARCH APPROACH

This study employed a qualitative research approach to provide insight into exploring the teaching and learning of circle geometry in rural schools (Flick, 2018). Qualitative research is an unfolding theory that enables researchers to improve their studies to a higher-level involvement in real experiences (Punch & Oancea, 2014).

Qualitative research as a significant educational research approach is applicable when one needs to understand the level of spatial reasoning in geometry values (Kalu & Bwalya, 2017). Anney (2014) further outlined that the trustworthiness of qualitative research is frequently questioned based on transparency and responsibility in all the research processes.

3.3 RESEARCH DESIGN

The study employed a case study research design. More specifically, the exploration focuses on teaching and learning circle geometry in rural schools (Harrison, Birks, Franklin & Mills, 2017). The case study design was appropriate to gain concrete, contextual, and in-depth knowledge about circle geometry. It allowed researchers to explore circle geometrics' meanings and implications (Hidayah, 2018). It focused on understanding the context and addressing different aspects of the research problem, whereby there was limited spatial reasoning in circle geometry. In addition, the design was based on knowledge and reasoning within a group of grade 11 educators when proving circle geometry theorems (Barut & Retnawati, 2020). Furthermore, the design assisted in exploring educators and learners' geometric spatial reasoning and how it influenced in proving of circle geometry theorems.

The data is first collected in the exploratory design. The results are explored in qualitative stages one to three to determine the participants' teaching experiences in circle geometry (Bishop & Holmes, 2013; Creswell & Plano Clark, 2011). The design is also useful in limited knowledge about the phenomenon under exploration and findings to change educators' experiences to prove theorems in teaching and learning tasks (Beyene, 2016). However, the researcher chose the exploratory design since there was limited knowledge of geometric concepts, geometric spatial reasoning application, and shapes' properties in the tasks under the literature. The study by Ponce and Pagan-Maldonaldo (2014) supports that design aims to explore more about teaching experiences and the research problem, especially when there is little knowledge about the phenomenon.

Nevertheless, this study focused on exploring the teaching and learning of circle geometry in rural schools. This was aimed at employing a case study design. This study examined a qualitative method to create a space for future research since there is limited knowledge about proving theorems in the tasks of circle geometry and spatial reasoning, a study conducted by Hoadley and Galant (2016). The case study design was appropriate for the study. Data were generated from a qualitative case study analysis focusing on educators teaching circle geometry theorems and problem-solving tasks during the workshop. Duval's (1998)'s cognitive processes were used to analyse whether educators use various methods in teaching visualisation, construction and reasoning in circle geometry content. Qualitative data was generated using the abovementioned theory to explore the teaching and learning of circle geometry in rural schools regarding their teaching experiences.

As guided by the case study design, the procedure of collecting and interpreting data was built from the educators teaching experiences into the semi-structured interviews qualitative phase. The methods were used to collect and analyse the qualitative data to explore teaching and learning of circle geometry content. After that, the results obtained from the teaching experiences of grade 11 mathematics educators were corroborated to produce comprehensive results to address the limited knowledge about circle geometry theorems. The case study design contributed to this study to complement the findings from the qualitative method and to capture different viewpoints and the teaching experiences of educators in circle geometry. The details of this research design are outlined in Table 3.1 below.

Table 3.1: Case study design for the research task

Case study	Observing teaching by educators	Interview conducted	Educators' presentation
Research Task 1	Teaching by the educator (A)	Not interviewed	Presented during the content workshop
Research Task 2	Teaching by the educator (B)	Using semi-structured interviews	Presented during the content workshop and class visit
Research Task 3	Teaching by the educator (C)	Not interviewed	Presented during the content workshop
Research Task 4	Teaching by the educator (D)	Using semi-structured interviews	Proving circle geometry theorems and supporting the statements with geometrical reasons
Research Task 5	Teaching by the educator (E)	Not interviewed	Not confident about presenting during the content workshop
Research Task 6	Teaching by the educator (F)	Not interviewed	Presented during the content workshop

Table 3.1 above indicates the structure and research Task generated from the qualitative data. It outlines the case, observations, interview process, and educators' presentation employed in the research design.

3.3.1 Sampling Strategy

There are various types of sampling in a qualitative study. For this study, purposive sampling has been employed, whereby a particular group of participants is targeted to participate in the study. The details of sampling strategies applied in the study have been discussed below:

3.3.1.1 Purposeful sampling strategy

In a case study, samples were selected purposively to the insight into a particular experience of grade 11 mathematics educators in rural schools. A purposeful sampling strategy was employed in this study to focus on the qualities and abilities the educators possess when teaching circle geometry theorems and problem-solving (Etikan, Musa & Alkassim, 2016). This study sought to explore the teaching and learning of circle geometry in rural schools; hence case study was employed.

3.3.1.2 Sampling of participants

Six (6) educators teaching mathematics were sampled in the circuit to participate in the study. Twelve (12) learners from the two schools were sampled to complete tasks set on circle geometry. The sampling of the participants was guided by the research purpose of the study (Bryman, 2016). Purposeful sampling is a non- probability category whereby the educators were selected according to their performance concerning the study (Etikan, Musa, & Alkassim, 2016). The strategy was employed to guide the sampling to achieve the study's aim and reach important information related to the phenomenon under exploration.

These six (6) educators were observed while teaching circle geometry during the Grade 11 mathematics content workshop and two (2) during class visits and learners in school A and B. The purpose was to find out why they teach the way they do. This assisted in conducting several content workshops to develop the educators on proving circle geometry theorems and problem-solving. They were tasked to teach one of the tasks to collect data, analyse and develop other educators.

The workshop assisted in terms of improving performance in the subject in the circuit. It was done after realising that there is limited knowledge about proving theorems in circle geometry and the limited number of tasks to assess learners in previous years. The sampled educators played their roles and responsibilities to present the relevant content. The only difference observed from the six (6) educators' presentations was an approach to teaching circle geometry questions. These educators employed various teaching methods in the content workshop. Most teaching methods were familiar to everybody who attended the content workshop.

3.3.1.3 Sampling documents

The study employed a purposeful sampling strategy to select the public documents from the Department of Education to reach the important information related to the phenomenon under exploration. The public documents from the Department of Education included: (1) Grade 11 geometry examination questions from Mathematics Paper 2 and the marking guidelines (DBE; 2018; 2019; 2020; 2022); (2) the National Senior Certificate Diagnostic Reports about geometry questions related in grade 11 content and intervention programmes; and (3) The number of answer sheets sampled on DBE from Mathematics Paper 2 for grade 11 question papers respectively.

3.3.2 Data Collection

There are various types of data collection methods that are employed in mathematics studies. Since the study uses qualitative research methods, data was collected to capture different viewpoints addressing the research problem. Initially, this was done to understand educators' differing perspectives when teaching circle geometry tasks, responses during the semi-structured interviews, and the use of learners' scripts. For quality criteria, the researcher involved the likes of More Knowledgeable Other (MKO), cluster leaders and Subject Advisors specialising in Mathematics to guide in conducting the content workshop and collecting data relevant to geometry studies. Data collection methods have been discussed in detail below:

3.3.2.1 Data collection approach and method

The appropriate data collection methods were employed when conducting this qualitative study of geometry (Phillips & Stawarski, 2016). Data were collected while observing educators teaching circle geometry and semi-structured structured interviews and the use of learners' scripts. The observation method of collecting data served various purposes (O'Keeffe, Buytaert, Mijic, Brozovic, & Sinha, 2016). Data collected from educators' interviews were after the educators taught their lessons and explained their viewpoints on proving circle geometry theorems.

Moreover, these qualitative methods identified the gaps addressed in the research problem and came up with ways to deal with limited knowledge about circle geometry.

The use of geometry studies assisted in collecting the data that was relevant to this study. In this study, the presentation and lesson observation served the following purposes: 1) providing data on the context of circle geometry; 2) providing additional research data related to circle geometry; 3) providing a means of tracking change and development in the learning process of circle geometry. The question papers were collected from the DBE website, and educators' worksheets were developed from grade 11 Mathematics Paper Two Answer Books of the same year. The six educators used the questions in the question papers to teach their lessons and assessed the others who attended the content workshop (O'Keeffe et al., 2016).

Data were collected in three phases. In the first phase, data were collected through educators' lesson presentations guided by Duval's (1998) cognitive processes in exploring teaching and learning circle geometry in rural schools. The use of this theory assisted in collecting data because it is so simple to follow and guide in its steps or processes. Lesson observation was done to see what educators are teaching as a true reflection of whether educators know the content, they are teaching to prove circle geometry theorems and problem-solving. In the second phase, data were collected using semi-structured interviews after checking how educators were teaching their lessons. In the same breath, some things did not emerge during the lesson presentation but appeared when responding to questions during the semi-structured interviews or vice versa. The last phase, data were collected from the learners' scripts to understand how they learn the content.

When categorising the qualitative data using the Duval (1998) theory of cognitive processes, themes were also used by breaking data into small parts to understand the insight of learning circle geometry. Data was collected with the assistance of some of the Subject Advisors and educators in the school by organising the content workshop venue for the participants. Managing the processes in the study, I ensured that the participants were comfortable sharing their experiences and belief in the teaching of circle geometry. These paved the way for the researcher to collect the appropriate data to address the study's problem. Therefore, data was in the form of codes that showed limited knowledge about proving circle geometry theorems.

The researcher used qualitative data methods, lesson observation, semi-structured interviews, and learners' scripts to answer the three research questions to clarify how educators should apply geometric spatial reasoning skills when teaching circle geometry theorems and problem-solving. These three research questions were categorised using both research methods to avoid data biases from the participants.

3.3.2.2 Development and testing of the data collection instrument

Qualitative data methods guided the study, and the instruments were developed on a word document, including the diagrams of circle geometry and space provided for statements and reasoning **(see Appendices I (a) – I (n))**. They were developed as a guide for educators to use when teaching the content and for exploring teaching and learning circle geometry in rural schools.

The instruments and diagrams mentioned above were used to track how educators teach circle geometry and assessing learners. In the tasks, learners were provided with answer sheets to complete the tasks.

3.3.2.3 Characteristics of the data collection instrument

More qualitative data collection tools were developed by following the three cognitive processes of Duval (1998)'s. From these data collection instruments, a case study design was followed. These instruments were developed to capture and categorise data and to ease data analysis. The instruments consist of columns and rows that guide the correct data analysis. Some instruments were developed to provide a space for participants to teach and prove circle geometry theorems with the reasons. The other instruments were used to capture data from semi-structured interviews. These instruments indicate the columns of Duval's (1998)'s cognitive processes and the space provided for participants' responses. Furthermore, these instruments played an essential role in saving data for future reference. This also eased in analysing of qualitative data for the study.

In addition, the qualitative data collection instruments were also developed following Duval's (1998) cognitive theory. These instruments ensured in capturing of relevant information concerning circle geometry theorems and problem-solving. Teaching and learning of circle geometry start in the lower grades, then the previous knowledge from those grades should have been applied in grade twelve within the topic of Euclidean geometry. According to the geometry studies, it was highlighted that most Grade 11 learners fail mathematics paper two (2) because of not understanding geometry, which indicates that some of the educators were not doing justice in terms of curriculum coverage. As guided by this case study design, educators should teach circle geometry to assist learners, improve performance in the circuit since it is situated in a rural area, and encourage learners to develop a positive attitude towards mathematics.

3.3.2.4 Data collection process

The three qualitative data methods were used in the study to collect data by coding the content for qualitative data. The researcher followed the coding process to group the participants' ideas, which assisted in developing instruments that captured the correct data relevant to the study.

In qualitative data collection, the case study analysis method is coded individually. In addition, coding focused on identifying the similar concepts of understanding circle geometry covered, as required, by the CAPS document, NPA and the DBE November past papers tasks in the question papers **(see Appendices I (a) – I (n))**. Each Appendix was developed to capture data from the participants. Some instruments had empty spaces indicating that some participants had no information to write in the provided spaces. The educators used these tasks during their lesson presentations. Some instruments capture enough data to be used and analysed. Related assessment tasks with similar objective aims were categorised together to avoid a repetition of questions and summary of answer sheet and proving of circle geometry theorems tasks, respectively. This was done to differentiate between an instrument used to capture data from lesson observation, learners' scripts and also the record audios from the semi-structured interviews.

Furthermore, the coding process gave in-depth knowledge of what should be captured during data collection. The circle geometry tasks had standard content objectives of finding limited knowledge about proving theorems and problem-solving. This strategy was implemented to address the study's three research questions to be answered. The coding on the concepts of circle geometry had to focus on circle geometry, geometric spatial reasoning, geometric theorems, geometric proofs, problem-solving, and cognitive processes. These concepts were important keys in the teaching and learning of circle geometry. Various textbooks give information on the teaching and learning of circle geometry within Euclidean geometry. Therefore, educators should be guided by textbooks to prepare learners to understand the content. In **Appendix E**, educators were expected to participate in the content workshop and teach some circle geometry tasks. In the space provided, educators should understand that geometric reasons are written to support the geometric statements in the tasks.

Over and above, qualitative data collection was done using lesson observations and semi-structured interviews to code the participants in the study. Here data collection focused on the lesson presentation and educators' responses to circle geometry tasks. Using Duval's (1998) theory, educators were grouped according to the matching cognitive processes. Their results from lesson observation were recorded in different categories. However, where educators' responses matched more than one cognitive process, data was generalised based on geometric spatial reasoning.

3.3.3 Data Analysis

Firstly, qualitative data was analysed following the three cognitive processes by Duval (1998) to explore the teaching and learning of circle geometry in rural schools. This was done in terms of how educators were teaching proving circle geometry theorems and problem-solving, namely 1.) educators being able to teach visualisation in geometric shapes, 2.) constructing within the shapes before proving the theorems, and 3.) checking whether there is an application of geometric spatial reasoning to support geometric statements. Educators' teaching methods in lesson presentations and semi-structured interviews were comprised of explaining geometric spatial reasoning of coding (**see Appendix D**).

Secondly, data were analysed from educators' audios captured during the semi-structured interviews. The two educators were visited in their respective classrooms in schools. An instrument to capture data was also included in table 4.3.

Lastly, learners completed the tasks set on circle geometry and the results were captured to understand how learners solve geometric problems as well as proving the theorems in grade 11. The responses of learners indicated how learners applied the geometric statement and reasoning in the tasks (**see Appendices K(a)-K(l)**).

The N-GCP was applicable when there was no application of geometric cognitive processes, blank space left by educators and no information related to proving circle geometry theorems and problem-solving. The V-GCP was applicable when there was a visualisation process without construction or reasoning. The procedure for proving the theorem and problem-solving was not correct. The VC-GCP was applied when there was visualisation and construction but no reasoning in the teaching tasks. The VCR-GCP was applicable when there was visualisation, construction, and reasoning. All the steps required when proving theorems and solving geometric problems were presented.

In analysing the geometric cognitive processes, the researcher outlined and indicated how the coding process would be used in the study. Geometry studies have shown that to understand the content, there is no need to skip the geometric cognitive processes. Educators were expected to go through all the cognitive processes in their lesson presentation to prove circle geometry theorems and problem-solving. The study design also highlighted in Table 3.1 the research Task involved in lesson observation, semi-structured interview, and learners' scripts to answer the study's research questions.

3.4 QUALITY CRITERIA

This study applied the following measures to ensure trustworthiness: 1. credibility, 2. transferability, 3. dependability (reliability), and 4. confirmability (Anney, 2014).

3.4.1 Credibility

A triangulation technique was employed to ensure credibility in the study when collecting data. The triangulation technique assisted as it was checked using lesson presentations, semi-structured interviews, and learners' scripts (Heale & Forbes, 2013). Theory triangulation was employed where Duval's (1998) cognitive theory was used as the qualitative study's theoretical framework. In addition, theory triangulation was done to capture data responding to three research questions. This measure justified the whole research process and the interaction among the participants. It ensured that there is a control of data to be collected based on truth value, consistency, and applicability (Noble & Smith, 2015).

3.4.2 Transferability

Transferability refers to the probability that the study findings have meaning to others in similar situations (Yilmaz, 2013). The results of this study will be compared to similar studies for consistency. This will be done to ensure truth value, consistency, and applicability.

3.4.3 Dependability

This measure ensured that there was consistency in terms of the data collection. Triangulation was considered to be a way of ensuring consistency in the study. It involves participants valuing the findings, the interpretation, and the recommendations of the study. The recordings of data from educators' lesson presentations ensured dependability. The semi-structured interviews were ensured by keeping records of all the participants and giving them enough time to respond to the questions based on circle geometry (Anney, 2014). Learners' scripts were analysed to find out how learners are learning the content in class. All the raw data from the three qualitative methods were captured and saved (Major & Savin-Baden, 2012). For this study, a manual in structural coding was used based on the first research question. This was done more than once, then compared their results (Meanwell, 2018). In this sense, cluster leaders and subject advisors were consulted after categorising educators to provide guidance and feedback based on the data.

3.4.4 Confirmability

Confirmability refers to the degree of supported viewpoints by other research studies. This measure ensured that data was collected and analysed correctly (Korstjens & Moser, 2018). This criterion was essential in not exercising bias (Bryman, 2016). The triangulation technique was used to ensure the confirmability of this study guided by Duval's (1998) cognitive processes.

3.5 CONCLUSION

The research approach employed in the study was qualitative to produce relevant and sufficient results. A case study design was used to guide the study in terms of sampling the participants, collecting data, and managing qualitative data. It was employed to provide evidence from the participants' results and to future researchers about the limited knowledge in geometric spatial reasoning, proving theorems and problem-solving. The gap has been identified in circle geometry teaching and learning as indicated that there were few tasks required planned by educators to apply spatial reason when proving the theorems and problem-solving. The exploratory case study addressed the challenges that made educators not teach all the questions as expected in the tasks during the content workshop. Various geometry studies were employed to support this study by indicating that in the past, geometry had been challenging to be understood by educators and learners.

Duval's (1998) cognitive theory was utilised to capture data from educators' lesson presentations and categorise them in reasoning capabilities. These assisted in knowing the level of understanding of how educators reason in supporting the geometric statements. Semi-structured interviews were used to capture data from educators' responses during the interview and recorded in the form of audio. Learners' scripts were analysed from the six tasks. The data from these participants' responses involved a group of six (6) educators teaching mathematics in rural schools and learners in school A and B. The study used tables, figures, and a geometric cognitive process theory to capture participants' data. The following chapter will focus on interpreting the results collected from mathematics educators.

CHAPTER 4 : DATA ANALYSIS AND DISCUSSION

4.1 INTRODUCTION

This section presents the findings of the exploration whereby Duval's cognitive processes were employed to guide on teaching and learning of circle geometry in rural schools. It presents procedures for dealing with geometry theorems and strategies for applying the correct geometry statements and reasoning. The qualitative data are presented in tables and interpreted rationally from Duval's theory.

The data was analysed from lesson observations and semi-structured interviews. In the lesson observations, data analysis was through the content analysis within the theory of Duval's cognitive processes guided by Merriam's (1998) five steps of the case study. The lessons presented by grade 11 educators were monitored to find out whether they possess relevant qualities and abilities to prove circle geometry theorems and analysis using applicable codes (**see Appendix D**). Coding is the process of analysing qualitative text and data, broken down into coding, reading data, breaking it into subcategories, and giving labels to the texts (Elliott, 2018).

There are various types of coding in qualitative research. For this study, a manual in structural coding was used based on the research question (Meanwell, 2018). The structural coding of geometric cognitive processes was well explained in the following columns: 1.) The first column presented no geometric cognitive processes. 2.) The second column captured the visualisation process; there is no construction or reasoning. 3.) The third column captured visualisation and construction but no reasoning, and 4.) The fourth column presented visualisation, construction, and reasoning. Thus, all the steps required when proving theorems and solving geometric problems were presented.

4.2 DATA MANAGEMENT AND ANALYSIS

Structural coding was used based on teaching techniques and interviews (Meanwell, 2018). The educators' responses to the interviews were analysed based on teaching methods, approaches, and understanding of circle geometry. The semi-structured interviews were about ten to fifteen minutes. The learning tasks were about ten to fifteen minutes and completed by twelve learners. These provided sufficient information about the participants from the past in teaching, present, and future knowledge (Saldana, 2013). Therefore, the sampled educators expressed themselves as they taught during lesson observations and responded to the interview questions that emerged from the lessons presented.

The findings present qualitative results from the three data collection methods. These analysed results were interpreted as guided by Duval's theory of cognitive processes. The theory assisted in categorising educators with and without qualities and abilities in teaching circle geometry theorems. The semi-structured interviews were conducted while guided by the following steps: 1. Gathering and recording important information about educators. 2. focusing on the exploration, being flexible based on the educators' responses, and 3. exploring the specific phenomenon in the study.

4.3 KEY PROCESSES OF ANALYSING DATA

Data was analysed from lesson presentations that took place during the content workshop. The two circuits consisting of fourteen educators were clustered as a common practice and directive from the District to conduct a circle geometry content workshop. Out of the fourteen (14) educators, the focus was on six (6) educators from one circuit to collect data. All fourteen educators in the workshop were willing to participate in the study. Still, the researcher explained to Subject Advisor with reason to focus on educators from one circuit identified to share how they teach circle geometry. Educators were named A to F to participate in the study. Educator A presented his lesson from teaching Task 1, Educator B's lesson was from Task 2, Educator C presented her lesson from Task 3, Educator D presented her lesson from Task 4, Educator E presented her lesson from teaching Task 5, and Educator F his lesson was taken from teaching Task 6. Learners were also involved to participate in the study. There were 12 (twelve) learners sampled from the two schools that were visited. The Grade 11 learners (LA-LL) completed tasks set on circle geometry.

4.3.1 Focus group and teaching experience.

The study involved both male and female educators within the circuit. Educators A to F were allowed to participate in the study based on the teaching strategies or methods used in teaching circle geometry. The use of resources to enhance the teaching of content was also considered.

Table 4.1: Focus group and experiences

Educators	Gender	Teaching experience	Teaching experience (Mathematics Grade 11)
A	Male	13 years	9 years
B	Male	10 years	10 years
C	Female	19 years	19 years
D	Female	23 years	23 years
E	Female	1 year	1 year
F	Male	14 years	8 years

These educators had their experiences in teaching mathematics in Grade 11. These experiences clearly indicated that the good part of sharing various teaching methods was to simplify the content teaching. Educators were in position and had the smart board in the board room, laptops, chalkboards, worksheets, calculators, and geo-gebra programs in the laptops to teach. Educators chose their own teaching resources and presented the lessons with the methods each preferred.

4.4 DATA TRIANGULATION ANALYSIS

Figure 4.1 below highlights the stages of analysing data from the participants. This process was followed and also guided by Duval's cognitive processes. The nature of collecting and analysing data was explained to all the participants.

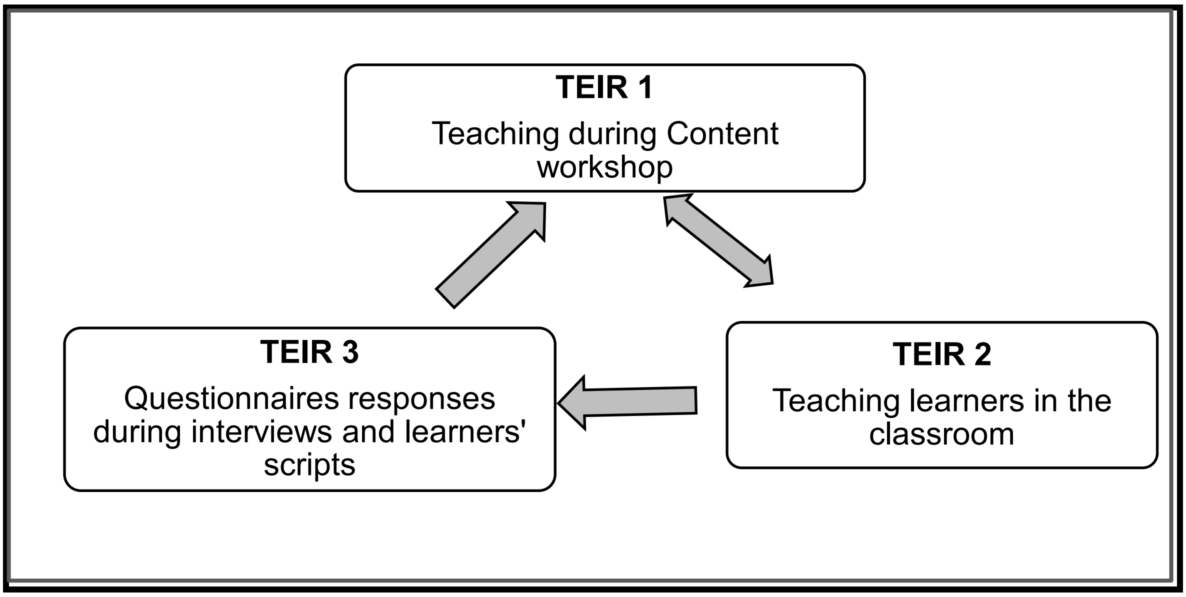


Figure 4.1: Data triangulation analysis

Figure 4.1 was used as a guide to collect data teaching during the content workshop, class visit, and questionnaires in the interviews. This figure was followed and linked with Figure 2.1, showing the cognitive processes of Duval (1998) in teaching and learning circle geometry in rural schools.

4.4.1 Tier 1: Teaching during the content workshop

This stage allowed educators to present their lessons in the content workshop to address the challenges they come across in circle geometry. These teachers had their own experiences in teaching the content, and they were sharing with other colleagues to teach circle geometry theorems. The data was analysed based on how the educators participated when teaching the content. Teaching resources were provided to the participants to use in their presentations.

4.4.2 Tier 2: Teaching learners in the classroom

At this stage, only two educators were visited in the classrooms in their respective schools. Educator (B) was visited at his school after realizing he has limited knowledge of geometric statements and reasoning in solving circle geometry problems. The educator showed steps to determine the value of x but did not provide the correct geometric reasoning. There was a need to visit him for developmental purposes. The reason for conducting the interviews was to find out whether this educator would implement the strategies shared during the content workshop and if there was an improvement in his teaching.

Educator D was visited based on her teaching experience in the subject since she managed to present her lesson during the content workshop and class visit. This was an intervention to support the educator and check progress and if she would ever implement the suggested strategies to teach circle geometry. There was an improvement in her lesson because, in the first Tier, she showed all the steps with reasoning [$\angle$ at centre $= 2 \times \angle$ at circ] but could not provide the reason that says [$\angle s$ around the points$= 360°$].

4.4.3 Tier 3: Semi-structured interviews and data from learners' scripts

This stage of analysing data was based on responding to the questionnaires that educators were asked during the interviews. In this research approach, data were collected from Educator B and Educator D and recorded in the form of audios and in the instrument. One of the reasons to interview these two educators was because they showed plenty of challenges from the other educators (A, C, E, and F). Also, learners completed the same tasks to track understanding when learning circle geometry.

4.5 DATA DISCUSSION

4.5.1 Task 1

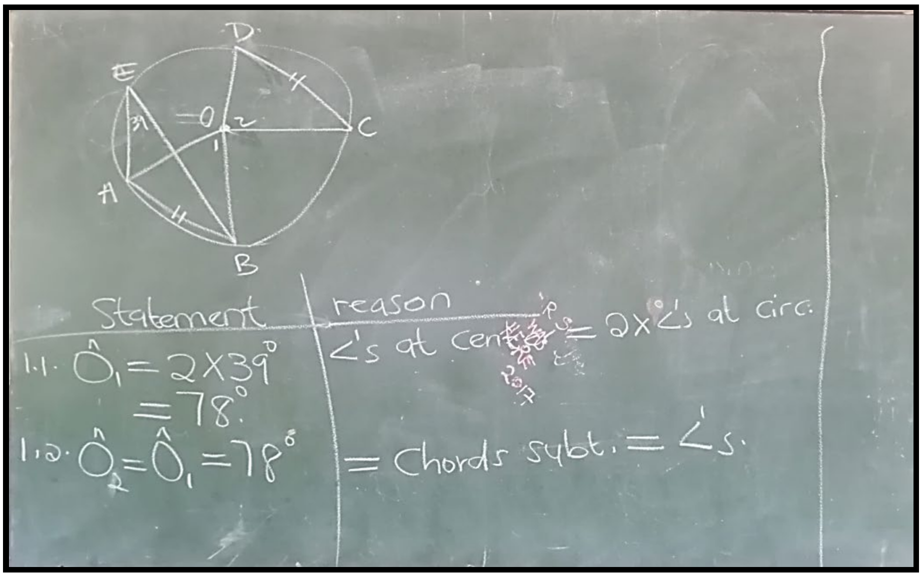


Figure 4.2: Lesson observation (Educator A)

Figure 4.2 shows a lesson presented by Educator A in question 1.1, indicating limited knowledge of geometric statements and reasoning in proving circle geometry theorems. The educator demonstrated steps to determine $\hat{O}_1$ and $\hat{O}_2$ but provided the incorrect geometric statement and reasoning. The educator was supposed to determine that $\hat{O}_1 = 2\text{AE}\hat{\text{B}} = 78°$ [$\angle$ at centre $= 2 \times \angle$ at circ]. The educator was able to provide the correct geometric statement and reasoning. The educator determined that $\hat{O}_2 = 78°$ $[\text{equal chords; equal angles}]$.

4.5.2 Task 2

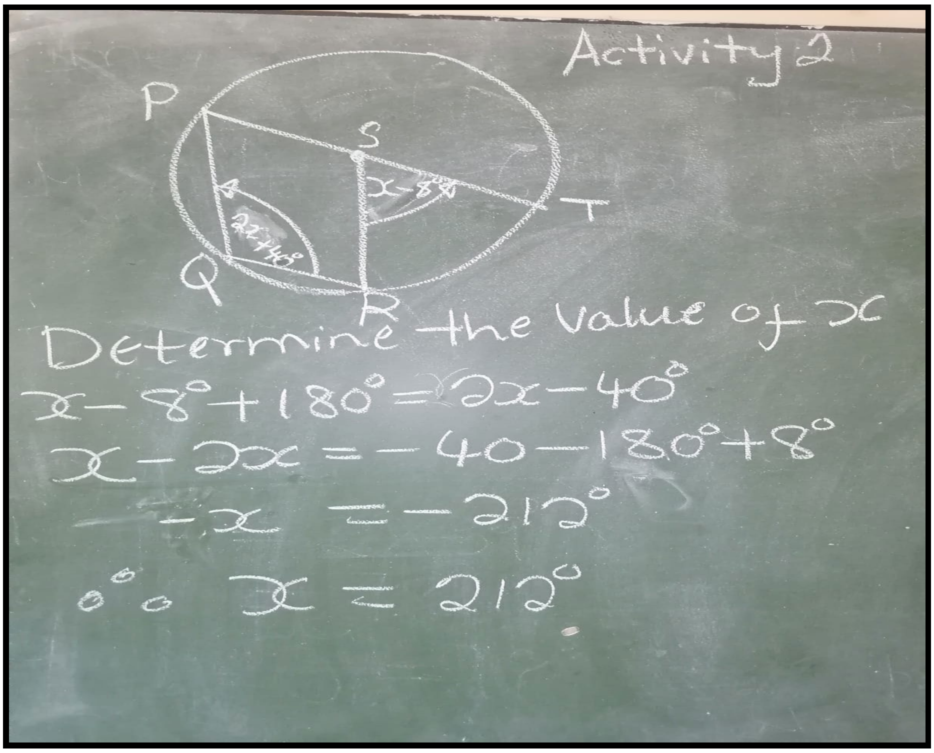


Figure 4.3: Lesson observation (Educator B)

Figure 4.3 indicate a lesson presented by Educator B and shows a few steps to determine the value of x but provides incorrect geometric statements and reasoning. The educator was supposed to indicate that [$\angle$ at centre $= 2 \times \angle$ at circ] or $[\text{opp } \angle\text{s of cyclic quad}]$ or$[\text{sum of } \angle\text{s of } \Delta]$.

4.5.3 Task 3

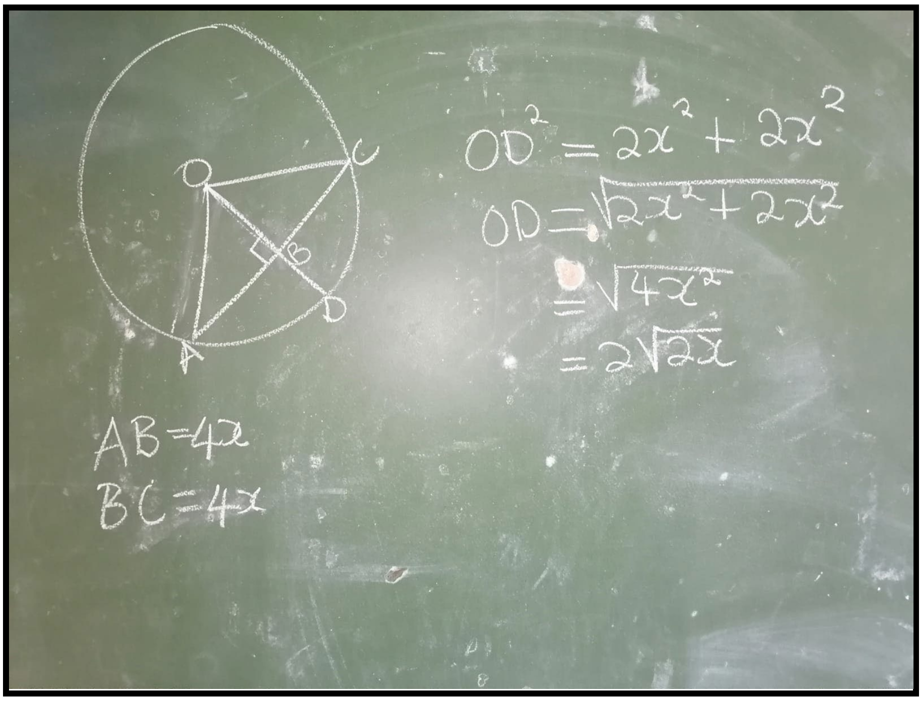


Figure 4.4: Lesson observation (Educator C)

Figure 4.4 shows a lesson presented by Educator C, who did not specify how to compose geometric figures. The educator provided incomplete steps in determining the value of OA and BD. The educator did not indicate that $[\text{line from centre} \perp \text{to chord}]$ when determining the length of OA. Also, she did not apply the correct theorem of Pythagoras. The educator could not identify the radii lengths to find the value of BD.

4.5.4 Task 4

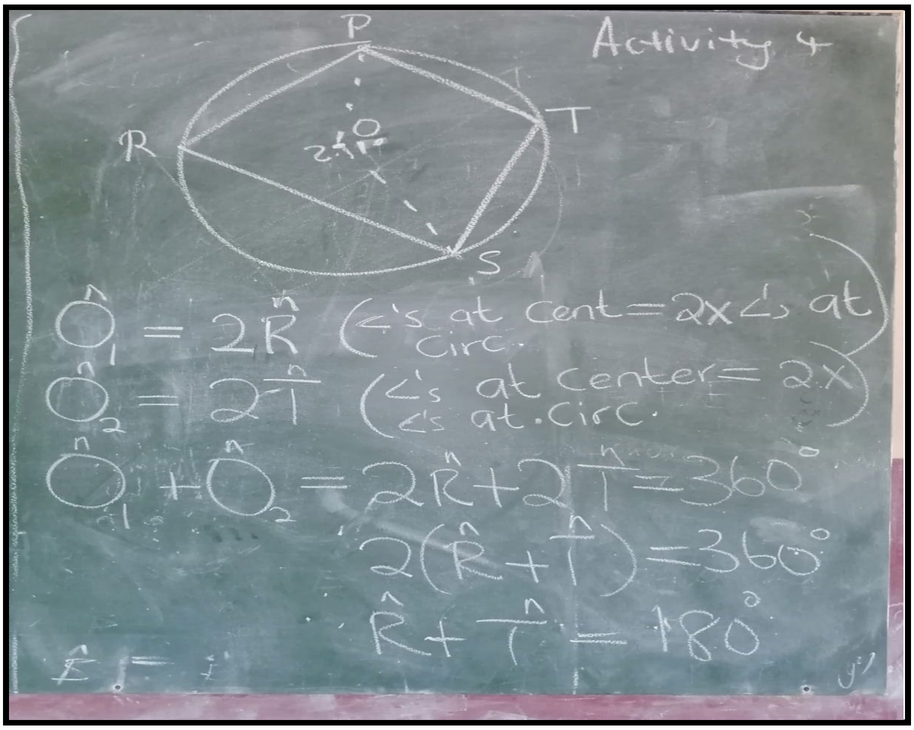


Figure 4.5: Lesson observation (Educator D)

Figure 4.5 above shows a lesson presented by Educator D was effective and clearly showed steps to determine $\hat{O}_1$ and $\hat{O}_2$ but provided incorrect geometric reasoning in one of the steps. The educator indicated that [∠ at centre= $2 \times$ ∠ at circumference]. Also, the educator indicated that [$\hat{O}_1 = 2 \times \hat{S} = 78°$] and $\hat{O}_2 = 2y$ as similarities. She proved that $2x + 2y = 360°$ such that $x + y = 180°$ but omitted a step and a reasoning that says angles around the point to prove that $\hat{S} + \hat{P} = 180°$.

4.5.5 Task 5

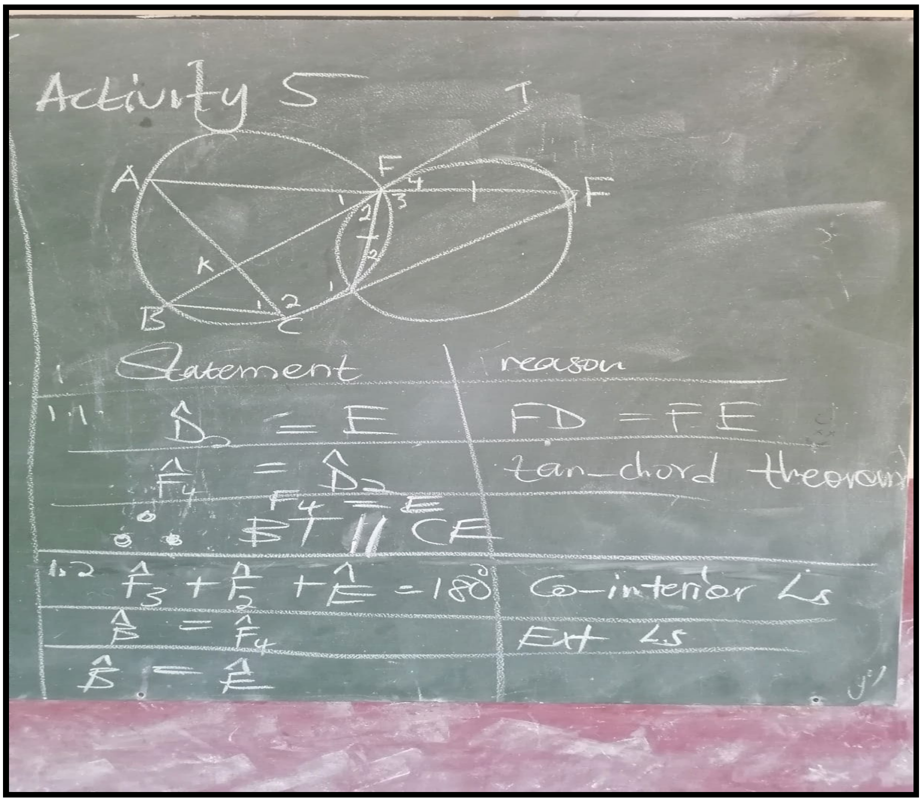


Figure 4.6: Lesson observation (Educator E)

Figure 4.6 above shows a lesson presented by Educator E, who had two questions and was ineffective. The educator did not demonstrate to prove *BT II CE* and indicate that BCEF is a parallelogram. She requested some other educators in the workshop to assist with the proof. The educator was not confident presenting her lesson in front of other educators. In this case, she needed assistance because she indicated that she did not specialize in mathematics.

4.5.6 Task 6

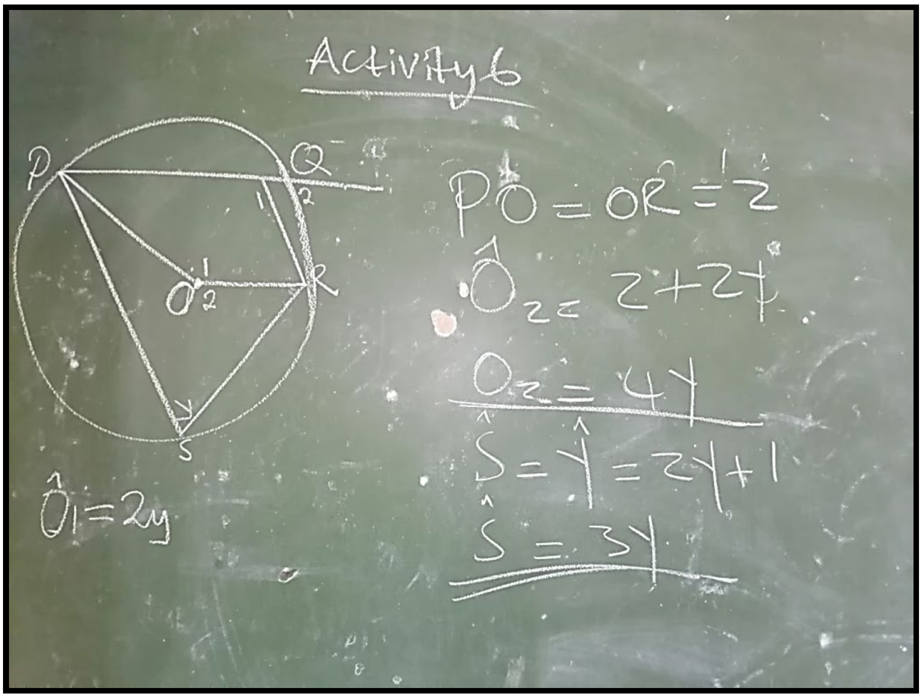


Figure 4.7: Lesson observation (Educator F)

Figure 4.7 above is a lesson presented by Educator F, and it had one question but required to determine $\hat{O}_2$ and $\hat{S}$ in terms of y. The educator provided incomplete geometric statements without reasoning, whereas the educator was supposed to say [∠ at centre =2×∠ at circumference].

Table 4.2: Analysis of lesson observations during the content workshop

PARTICIPANTS	**CHALLENGES IDENTIFIED IN TASK 1**
Educator (A)	No challenge was identified in questions 1.1 and 1.2
Educator (B)	Few steps to find the values Provided the incorrect geometric statements No geometric reasoning
Educator (C)	Provided the incorrect geometric reasoning without statement: [∠ at centre $= 2 \times \angle$ at circumference] in question No composition of geometric figures
Educator (D)	Unable to indicate that there were angles around the point. She omitted steps to prove that $\hat{S} + \hat{P} = 180°$.

Educator (E)	She tried to solve the problems She had no confidence in presenting her lesson
Educator (F)	Did not provide the correct geometric reasoning Some of the steps were omitted to determine $\hat{O}_2$ and $\hat{S}$ in terms of y.

The data captured addressed all the research questions of the study. In Tier 1, Educator B could not teach the formulation of geometric statements and reasoning in his teaching, whereas Educator D showed all the steps in her presentation.

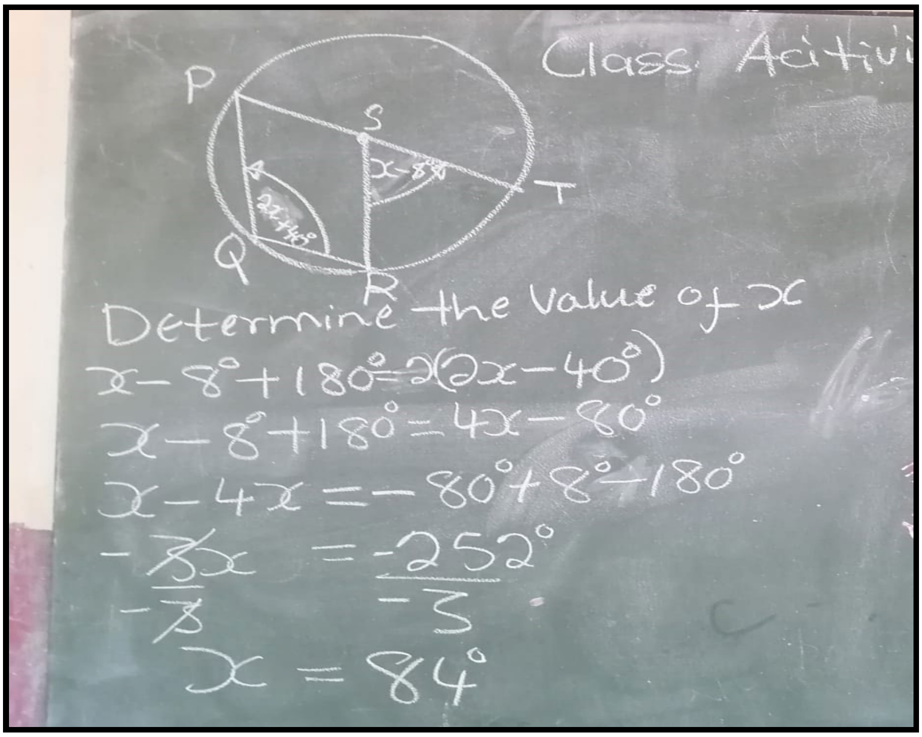


Figure 4.8: Class visit (Educator B)

In Tier 2, shown in Figure 4.8, Educator B solved the value of x and indicated all the required steps.

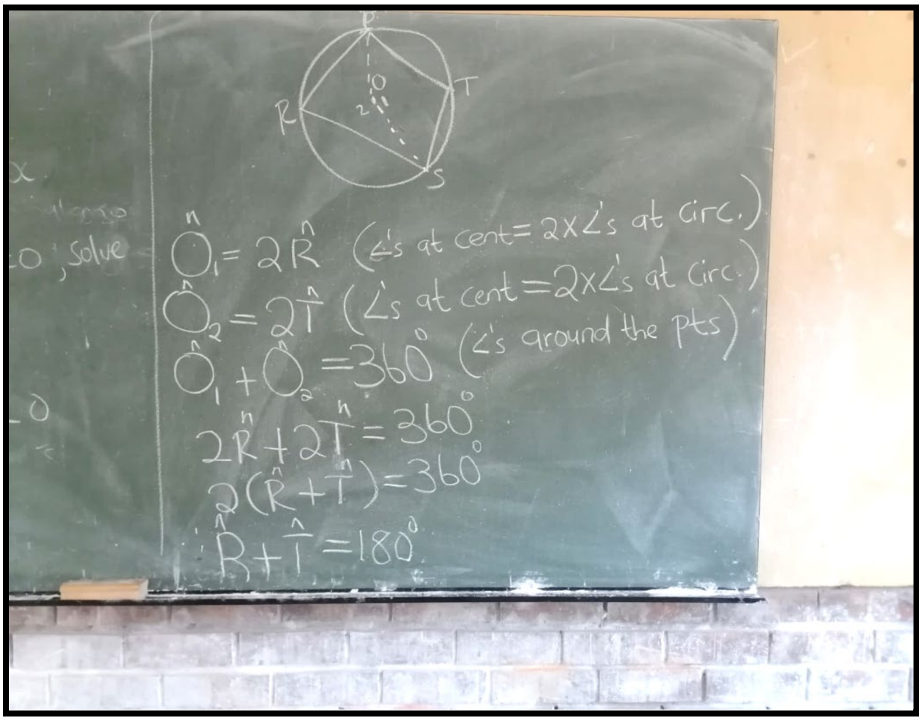


Figure 4.9: Class visit (Educator D)

In Tier 2, shown in Figure 4.9, Educator D proved the theorem by indicating the best ways to teach the correct geometric statements and providing the correct geometric reasoning. Therefore, the last stage (Tier 3) highlighted the two educators' responses during the interviews and data from learners' scripts.

Table 4.3: Semi-structured interviews (Educator B) and audio recordings

EDUCATOR B Subject: MATHEMATICS GRADE 11 Topic: GEOMETRIC CIRCLE	
Questions	**Participant's response**
1. How do you teach circle geometry?	Using each problem independently as a focus for a lesson. Selecting problems to tell a story. Thank you
2. What resources do you use when teaching circle geometry?	Using charts, textbooks and video lessons. Using environments as a stimulus for posing problems and supporting understanding of the circle. Thank you
3. What teaching methods do you use when teaching circle geometry?	Grouping of learners according to their skills. Thank you

EDUCATOR B Subject: MATHEMATICS GRADE 11 Topic: GEOMETRIC CIRCLE	
Questions	**Participant's response**
4. What are your experiences in teaching mathematics?	I have ten years of experience teaching mathematics and am a qualified educator in BSc and PGCE specializing in the subject. Thank you
5. In your own view, what are learners' dispositions toward learning circle geometry?	Since I usually taught learners who hated mathematics, the curriculum in mathematics is behind, and it is difficult to teach the subject. Thank you
6. Tell me more about the lesson on circle geometry that I observed	The learners work actively involved and participated in the lesson. Learners normally forget to provide reasons when solving or proving the solutions. Thank you.

Table 4.4: Semi-structured interviews (Educator D) and audio recordings

EDUCATOR D Subject: MATHEMATICS GRADE 11 Topic: CIRCLE GEOMETRY	
Questions	**Participant's response**
1. How do you teach circle geometry?	In class, when I teach circle geometry, I use a smart board
2. What resources do you use when teaching circle geometry?	Ok. Thank you. Use charts and smart board
3. What teaching methods do you use when teaching circle geometry?	To teach circle geometry, I use the display method
4. What are your experiences in teaching mathematics?	With this one, learners fail to give correct reasoning. Anxiety when learning circle geometry
5. In your own view, what are learners' dispositions toward learning circle geometry?	What I can say in this question when we begin with this topic, learners are not interested
6. Tell me more about the lesson on circle geometry that I observed	The time I was presenting the lesson in a workshop, some steps were omitted. This time around, there is an improvement

4.6 DATA PRESENTATION PERTAINING TO LEARNERS' SCRIPTS

4.6.1 Gender of the Learners

In the study, both males and female learners were involved in the study. The data also indicates that there was an equal distribution of male and female learners completing the assessment tasks (See **Appendix K**). This means there was a balance in the category participating in the study (Alghadari, Herman & Prabawanto, 2020).

4.6.2 Age Group

The learners were aged between 15 and 18 years. The age distribution revealed that most of the learners in schools A and B started primary school at the right age. This indicates that this is the accepted age group of learners in high school.

4.7 FACTORS THAT CONTRIBUTE TO THE LEARNING PROCESS OF LEARNERS IN CIRCLE GEOMETRY

4.7.1 Learning Environment

Geometry studies indicate that the learning environment should be conducive to learning of circle geometry. In schools A and B, the infrastructure and facilities were not conducive, and the schools are located in the rural area. These schools need to be developed because the school environment can assist in terms of determining the educator's effectiveness in the classroom. In these schools, there was overcrowding, in this case, the classroom environment size contributed to poor performance of learners in geometry. There was poor infrastructure in schools to the extent that some of the classrooms did not have electricity (Mensah & Nabie, 2021).

4.7.2 Resources Used in the Teaching and Learning of Circle Geometry

Educators in the schools were using chalkboards only to teach circle geometry. Teaching and learning resources such as projectors, smartboards, textbooks, and charts displaying geometric shapes were not available in schools. In geometry studies, it is indicated that the most significant contribution to improvement in circle geometry can be made by the development of charts to support the teaching and learning process (Adelabu, Marange & Alex, 2022).

4.7.3 Analysing Learners' Scripts in assessment tasks

Teaching using resources makes the learning of circle geometry more effective. Most of the learners have beliefs that learning by memorisation is important, whereas they could not apply geometric statements and reasoning when solving problems and proving theorems. In terms of their learning styles, memorisation made them perform poorly because they forgot whatever they learn in the classroom. Table 4.5 below presents an analysis of how learners associate themselves with the learning of circle geometry and when completing assessment tasks (Bayaga, Mthethwa, Bossé, & Williams, 2019).

Table 4.5: Analysing data from learners' scripts

LEARNER (A-L) Subject: MATHEMATICS GRADE 11 Topic: CIRCLE GEOMETRY		
PARTICIPANT	**GEOMETRIC STATEMENT**	**GEOMETRIC REASONING**
LEARNER A (LA)	X	✓
LEARNER B (LB)	X	X
LEARNER C (LC)	X	X
LEARNER D (LD)	X	✓
LEARNER E (LE)	X	X
LEARNER F (LF)	X	X
LEARNER G (LG)	X	X
LEARNER H (LH)	X	X
LEARNER I (LI)	X	X
LEARNER J (LJ)	X	X
LEARNER K (LK)	X	✓
LEARNER L (LL)	X	X

The results from the learners' scripts clearly showed how learners had challenges applying geometric statements in the tasks (Learner A-L). Learners A, D, and K were able to give geometric reasoning in the tasks and learners were not participating during the teaching and learning process in the classroom. Learners could not understand the "how "and "when" parts of providing the geometric statements and reasoning. It shows that during the class visit to the school A, Educator B failed to explain visualising, construction, and reasoning in geometric shapes (LA, LC, LE, LG, LI, and LK).

Learners in school B could not understand the “how “and “when” parts of providing the geometric statements and reasoning. It shows that during the class visit to school B, Educator D failed to explain visualising, construction, and reasoning in geometric shapes (LB, LD, LF, LH, LJ, and LL).

4.8 CONCLUSION

This chapter focused on the analysis of data collected from both the educators and the learners. The analysis of data gathered from educators’ interviews, lesson observations, and learners’ scripts responses. The analysis show that educators that participated in this study were able to teach the correct content of circle geometry by following the annual teaching plan. Learners were able to complete tasks set on circle geometry during teaching and learning in the classroom. The Duval’s cognitive theory of analysis also revealed educators’ understanding of geometric concepts when teaching learners in mathematics classrooms.

The interviews helped me to confidently classify Educators B and D as good and experienced mathematics educators during class visits. In this exploratory study, the written responses in the learners’ scripts and the educators’ interviews revealed that both the learners and the educators displayed a productive of key geometric content knowledge and problem-solving skills.

In the next chapter, I discuss the findings of the study as well as implications and suggested recommendations when exploring the teaching and learning of circle geometry.

CHAPTER 5 : SUMMARY, RECOMMENDATIONS, AND CONCLUSION

5.1 INTRODUCTION

This study sought to explore the teaching and learning of circle geometry in rural schools. This study was prompted by the fact that some educators use various teaching methods, while DBE encourages educators to use teaching methods that accommodate all the learners in the classroom. Teaching resources were developed primarily to supplement the teaching and learning of circle geometry during the content workshop and class visits in rural schools. Besides, the literature confirmed that the status of circle geometry in grade 11 was dropped. Hence, this study sought to explore the teaching of circle geometry and how educators utilize technological resources while teaching and learning the content. This study may enhance educators' development and teaching approaches in the future. Learner performance will improve in schools. This is anticipated since this study exposed the degree of teaching and learning circle geometry in rural schools.

5.2 RESEARCH DESIGN AND METHOD

The methodology adopted for this study is a qualitative method where data were triangulated to enhance the comprehensiveness of the results (Ngulube & Ngulube, 2015). Also, the research design employed was the case study design. The case study design had two parts and two phases. The design has been explored according to Duval's cognitive processes. This was done to explore the content structure in terms of visualising geometric figures and construction and applying reasoning to prove theorems. Content analysts coded the qualitative data individually by representing the content standards and the circle geometry teaching tasks with the cognitive levels.

Lesson observations, semi-structured interviews, and documents analysis were conducted to verify if the content taught by educators during the content workshop and class visits were the same in the teaching approach. Also, to check how learners are encouraged to understand circle geometry. The design also supported teaching and learning tasks according to their cognitive complexity. This was done to verify if the cognitive complexity of teaching geometry is consistent with the CAPS teaching tasks.

Lastly, the design was used to generate data where cognitive levels of tasks taught during the content workshop, class visits, and documents analysis were followed. These cognitive levels assisted in tracking performance and the levels that learners were operating at. In most cases, the levels of van Hiele are used to analyse data. Still, for this study, Duval's cognitive processes were employed to ensure that there is contribution and guidance to this study in the same course and producing comprehensive results.

5.3 SUMMARY AND THE INTERPRETATION OF THE RESEARCH FINDINGS

The teaching and learning of circle geometry can be examined using cognitive processes, as suggested by Duval (1998). This directed the consideration into one main research question and three sub-questions. The three sub-questions were established to support answering the main research question: How do Grade 11 mathematics educators in rural schools teach circle geometry?

5.3.1 Research Question One

The first sub-question was: What resources do Grade 11 mathematics educators use when teaching circle geometry?

In answering this research question, Duval (1998)'s theory consisting of three cognitive processes was employed into categories and the use of circle geometry teaching tasks during the content workshop and class visit in rural schools to unpack the content structure, proofs, and geometric reasoning. Data analysis was through the content analysis within the theory of Duval's cognitive processes guided by data triangulation analysis steps of the case study. In Tier 1, data was analysed from lesson observations during the content workshop, Tier 2 focused on data from lesson observations during the class visits in the rural schools, and lastly, data from the responses of two educators during semi-structured interviews and data from learners' scripts.

The content standards objectives and teaching and learning tasks had good alignment where the criteria were acceptably met to evaluate the educators. This indicates that the content standards objectives aligned well with circle geometry's teaching and learning tasks. The teaching and learning tasks in circle geometry were somewhat limited, and schools' educators identified and assessed learners with few tasks. This resulted in a lack of understanding of circle geometry, and most learners could not prove the theorems in the content covered in Grade 11. It is also important to plan more tasks that have been structured concerning the content standards. This may help improve the performance in circle geometry in the future.

5.3.2 Research Question Two

The second research sub-question focused on teaching methods used in teaching and learning circle geometry. The research question was: What teaching methods do Grade 11 mathematics educators use when teaching circle geometry? Duval (1998) emphasized the link between the three cognitive processes and the steps educators need to follow more, especially when teaching geometry in the classroom. The theory recommended by Duval also assists in terms of analysis to track whether learners are operating at level one (1) to level four (4) of van Hiele.

The study indicated that technological resources are necessary to assist educators in teaching circle geometry effectively and highlighted the importance of choosing teaching methods that accommodate the learners in the classroom. Teaching the content with geometric tools increases the chances of a high pass rate in schools. Moreover, it is indicated that the data collected by the triangulation analysis was reliable. According to the findings, educators teach the ways there were taught in their school time.

The Duval cognitive theory managed to answer the research questions effectively. The first cognitive process, visualisation, addresses the use of technological resources to teach geometry. Educators need to indicate the properties of geometric shapes and their relationships. The teaching and learning of circle geometry are recommended by teaching visualisation. The second cognitive process highlighted the importance of constructing within the shapes so learners can learn how to compose and decompose shapes. The third cognitive process also showed the importance of making informed judgments and bringing about correct geometric reasoning.

5.3.3 Research Question Three

The third research sub-question focused on how Grade 11 learners solve geometric problems and proving theorems. In this case, the sub-research question was answered with findings in the learners' scripts. Learners were able to write the assessment task but some left blank spaces in the answer sheet. The answer to the sub-question was that learners solve geometric problems without visualising the shapes. Learners do not interpret the questions before they answer. Learners are memorising the theorems and failing to apply in the tasks. Learners do not know how and when to write the geometric reasoning when proving the theorems.

5.4 RECOMMENDATIONS

The theory of Duval was adopted and employed in the study to understand how educators teach circle geometry and the use of teaching methods in the classroom. This includes the uniformity of the content covered and the cognitive complexity of the content in circle geometry. What the other theory mentioned earlier managed to expose was not highlighted by the other or instead highlighted in detail. Therefore, this study recommends the Duval theory to broadly explore the teaching and learning of circle geometry in rural schools.

Studies in circle geometry should be conducted to establish quality in the development of educators, teaching and learning resources, and teaching approaches in the future. The abovementioned strategies will enable learners to understand problem solving and proving theorems. Content standards should be specific to guide developers of learning materials, assessments, and instruction to minimise misalignment. The concepts and skills to be reached in the teaching and learning of circle geometry should be clearly outlined, not ambiguous and common statements to be strictly avoided at all costs because the correct statements in geometry are required. The match or similar findings were observed and noted when comparing the results from lesson observations, semi-structured interviews and learners' scripts. The two educators in semi-structured interviews talked about geometric statements only, whereas learners struggled with geometric statements and reasoning in the learning tasks.

Furthermore, circle geometry content coverage should be reflected in the content standards required by CAPS and is suggested as a guideline for developers of assessments, learning materials, and educators in the classroom. Above all, learning materials developers should be alert that it is imperative to teach circle geometry with content standards. Before assessment items can be developed, it is important to assess the cognitive level of the content standards; this will give direction on what cognitive complexity the assessment tasks must of standard. Duval (1998) recommended that geometry content be cognitively aligned with the assessment. Educators can teach circle geometry more effectively, and learners should be able to express themselves to respond to questions as expected.

Other areas to be recommended are the teaching resources such as Geogebra, smart boards, and projectors to enhance the teaching and learning of circle geometry. Educators are encouraged to attend the content workshop terms for development purposes. These will assist the educators in knowing the content coverage, especially circle geometry. It is also going to assist the learners in understanding circle geometry. Educators will be able to develop teaching and learning tasks that assess learners' understanding of circle geometry. The teaching methods and approaches can assist educators to deliver the content as expected, and learners can provide the correct geometric statements and reasoning. Therefore, learners should be assessed daily, and one of the keys to performing well in circle geometry is to practice every day.

Lastly, it is crucial to include all three cognitive processes when teaching circle geometry, while CAPS recommends four cognitive levels. The cognitive levels from CAPS document 'complex procedures' and 'problem-solving' should be covered in circle geometry tasks. Learning materials, assessments, and results could improve if all the challenges raised could be addressed.

5.5 CONTRIBUTIONS OF THE STUDY

This study may contribute knowledge to enable future researchers; meanwhile, several circle geometry studies were reported in South Africa. The developers of assessments and learning materials may benefit since guidelines on teaching, and learning circle geometry components with content standards have been highlighted.

Another contribution expected is skills development in the area of developing assessments and learning resources that are used in geometry. One more contribution is using both cognitive levels or processes theorys: Duval (1998) and van Hiele (2002) make up for one another and exposed the degree of links in delivering content knowledge and the teaching approach and learning of circle geometry. This study may benefit educators in comprehending the importance of curriculum coverage, assessing, and learning resources. Curriculum implementers and policymakers might be able to understand the connections and inconsistencies found in this study to support the core curriculum and the framework in the future. Educators might be able to spot qualitative assessments and learning resources in the future. Above altogether, this study has contributed to other geometry studies in terms of extending the knowledge of teaching and learning circle geometry, teaching approach, and resources, as most were recommended by Duval (1998) and van Hiele.

5.6 LIMITATIONS OF THE STUDY

This study has concentrated on exploring the teaching and learning of circle geometry in rural schools. This study concentrated on just one content area and one topic in circle geometry, which meets the sample requirements to be lesser. Additionally, the exploration outcomes emphasised the degree of teaching and learning of circle geometry, teaching approach, and teaching and learning resources. However, the area was extensively explored and highlighted the degree of circle geometry tasks and also brought to light issues of enhancing content standards, assessment, and learning resources. Though, the range of paramount research suit the level at which this study was conducted.

Regarding data triangulation analysis, the number of content analysts was also seen as a limitation since only two were engaged. Only twelve Grade 11 learners participated in writing the assessment tasks. Increasing the number of content analysts is believed to contribute to the study (Armes, 2016) positively. However, a study piloted by Hoadley and Galant (2016) on circle geometry confirmed that understanding is moderate to strongly positive to reach the study's aims. The discoveries of this study are related to the findings obtainable by Hoadley and Galant (2016), which add trustworthiness to this study.

My area of concern is the teaching and learning of circle geometry, the teaching approach, and the resources. Researching all the Tiers would paint a bigger picture of the status of circle geometry in rural schools. Lastly, another limitation is that this study focuses on circle geometry only, while other geometry sections also need to be explored. Therefore, the findings of this study assist as a base for future researchers on the comprehensive scope.

5.7 EXPERIENCE LEARNED FROM THIS STUDY

The researcher can agree that he learned more from this exploration study. What can be shown to future researchers is that work with Duval’s (1998) theory supported by van Hiele was worthy of knowledge and produced wide-ranging results. They give a clear indication of where issues of teaching and learning circle geometry exist. Duval’s cognitive processes theory pointed out that even the kind of content assessed is linked to the content standards. As a result, the study's aims in terms of content structure and expectation were highlighted comprehensively.

Another area learned from this study is that teaching and learning circle geometry content and assessment are critical and should be reflected in all other educational mechanisms. The researcher also learned what it means about problem-solving and proofs in geometry and the understanding thereof.

5.8 CONCLUDING REMARKS

This study explored the teaching and learning of circle geometry in rural schools. This study succeeded in exposing geometry in rural areas. The clear and simple methods used in this study and the findings may aid future geometry studies to cover the range of understanding circle geometry content, instruction, assessment, and learning resources. The results show visualisation, construction, and reasoning have positive links, which confirms that they have a good connection as they are intertwined.

Despite obtaining similar results from other geometry studies, replicating this study with an enormous scope is recommended. Therefore, I recommend similar studies to showcase the exploration in teaching and learning of circle geometry and other areas other than mathematics.

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APPENDICES

APPENDIX A: APPROVAL OF REQUEST TO CONDUCT RESEARCH AT CIRCUIT AND SCHOOL D

education
MPUMALANGA PROVINCE
REPUBLIC OF SOUTH AFRICA

Ikhamanga Building, Government Boulevard, Riverside Park, Mpumalanga Province
Private Bag X11341, Mbombela, 1200.
Tel: 013 766 5552/5115, Toll Free Line: 0800 203 116

Litiko le Temfundvo, Umnyango we Fundo | Departement van Onderwys | Ndzawulo ya Dyondzo

Mr Mpheti VM

PO BOX 5215
CASTEEL
1370

Dear Sir

APPROVAL OF REQUEST TO CONDUCT RESEARCH

Kindly be informed of the approval of your request to conduct research under Bohlabela District. The Research to be based on the "Exploring the teaching and learning of circle Geometry in rural schools, in Bohlabela District". The study must be carried out as per your request.

Furthermore, please be informed that the Mpumalanga Department of Education will require access to your research findings and recommendations. You are advised to communicate with our schools and ensure that no inconvenience is experienced at any given time. Teaching and learning must not be negatively affected as a result of this Research.

Your professionalism in this regard will be highly appreciated. Good luck on your research; your interest on matters of the District is applauded.

MS L.N GOBA
DISTRICT DIRECTOR: BOHLABELA

14/09/2022
DATE

APPENDIX B: CONSENT LETTER FOR PARTICIPANTS

UNIVERSITY OF THE FREE STATE
UNIVERSITEIT VAN DIE VRYSTAAT
YUNIVESITHI YA FREISTATA
UFS UV

RESEARCH STUDY INFORMATION LEAFLET AND CONSENT FORM

DATE

16 May 2022

TITLE OF THE RESEARCH PROJECT

Exploring the teaching and learning of circle geometry in rural schools

PRINCIPLE INVESTIGATOR / RESEARCHER(S) NAME(S) AND CONTACT NUMBER(S):

MPHETI VM	*2019516950*	*081 875 5570* *081 743 6649*

FACULTY AND DEPARTMENT:

Faculty of Education
Mathematics Education

STUDY LEADER(S) NAME AND CONTACT NUMBER:

Dr. N Mpalami)
078 678 1125/ 058 718 5340

WHAT IS THE AIM / PURPOSE OF THE STUDY?

This proposed study aims to explore the teaching and learning of circle geometry in rural schools. The study has the following objectives:

- *To identify teaching methods utilized by Mathematics educators to teach circle geometry in Grade 11;*
- *To document instructional resources that Grade 11 Mathematics educators choose and use when teaching the circle geometry;*
- *To list strategies that educators in a rural setting use to teach geometrical proofs and theorems.*
- To list strategies Grade 11 learners use to learn circle geometry theorems.

WHO IS DOING THE RESEARCH?

I MPHETI V.M (2019516950), a Masters student of the above-mentioned institution seek permission to conduct a research project in the Circuit. My study focuses on "Exploring the teaching and learning of circle geometry in rural schools".

HAS THE STUDY RECEIVED ETHICAL APPROVAL?

This study has received approval from the Research Ethics Committee of UFS. A copy of the approval letter can be obtained from the researcher.

Approval number: *N/A*

WHY ARE YOU INVITED TO TAKE PART IN THIS RESEARCH PROJECT?

Grade 11 Mathematics were identified to provide support and development during the content workshop. To address the gaps in teaching and learning of circle geometry in rural schools.

WHAT IS THE NATURE OF PARTICIPATION IN THIS STUDY?

Educators will be interviewed and observed during the content workshop to understand how they teach Circle Geometry. Grade 11 learners will complete tasks 1-6.

CAN THE PARTICIPANT WITHDRAW FROM THE STUDY?

Participants have the rights to withdraw from participating in the study if they wish to do so.

WHAT ARE THE POTENTIAL BENEFITS OF TAKING PART IN THIS STUDY?

Educators will be developed and gain experience in terms of teaching the content effectively. They will get support from the subject advisors and other educators.

WHAT IS THE ANTICIPATED INCONVENIENCE OF TAKING PART IN THIS STUDY?

Personal information will not be disclosed in public and responses will be protected with a password in a Google drive for five years.

WILL WHAT I SAY BE KEPT CONFIDENTIAL?

Confidentiality of information will be maintained participants will remain anonymous.

HOW WILL THE INFORMATION BE STORED AND ULTIMATELY DESTROYED?

Hard copies of responses will be stored by the researcher for a period of five years in a locked cupboard/filing cabinet (my home) for future research or academic purposes; electronic information will be stored on a password protected computer.

WILL I RECEIVE PAYMENT OR ANY INCENTIVES FOR PARTICPATING IN THIS STUDY?

Educators will benefit in terms of training during the content workshop and support from subject advisors in Bohlabela District, Mpumalanga.

CONSENT TO PARTICIPATE IN THIS STUDY

I, the undersigned,

____ (*participant's full names to be included*), (the "**Participant**")

confirm that I voluntarily agree to participate in the research study referred to as the

Exploring the teaching and learning of circle geometry in rural school (the "**Study**") in relation to

and which Study is being conducted by

(*Mpheti VM*), (the "**Researcher**").

I, the undersigned Participant, further confirm that–

1. the Researcher has explained the nature, procedure, potential benefits and anticipated inconvenience of my participation in the Study;
2. I have read (or had explained to me) and understood the Study as explained in the attached information sheet;
3. I have had sufficient opportunity to ask questions and am prepared to participate in the Study;
4. I understand that my participation in the Study is entirely voluntary and that I am free to withdraw at any time without penalty (if applicable);
5. I voluntarily provide the UFS and the Researcher with my personal information and consent to the UFS and the Researcher collecting, disclosing and processing my personal information in order to conduct the Study and any related activities in relation thereto;
6. I hereby acknowledge and confirm that I understand the purpose for which the UFS and the Researcher may collect, store, use, delete, destroy, outsource, transfer or otherwise process, as the context and circumstances may require and as contemplated in terms of POPIA, my personal information as set out herein;
7. I am aware that the findings of the Study will be anonymously processed into a research report, journal publications and/or conference proceedings and that my personal information will be aggregated and deidentified at such stage;
8. I also give the UFS permission to share, without notification, the collected data with other researchers at the UFS or other Higher Education Institutions. This permission is dependent on the same principles of ethical research practices, anonymity/confidentiality, safekeeping of information, and other issues listed above applying.

I, the Participant, agree to the recording of the *unstructured interviews/lesson observation/documentation*

Full Name of Participant: ______________________________

Signature of Participant: ______________________ Date: ______________

Full Name(s) of Researcher(s): MPHETI VINCENT MOLEBOGENG

Signature of Researcher: ____________ Date: 16.05.2022

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APPENDIX C: CONSENT LETTER FOR EDUCATORS (SEMI-STRUCTURED INTERVIEWS)

UNIVERSITY OF THE FREE STATE
UNIVERSITEIT VAN DIE VRYSTAAT
YUNIVESITHI YA FREISTATA

UFS UV

CONSENT TO PARTICIPATE IN THIS STUDY

I, the undersigned,

____ (*participant's full names to be included*), (the "**Participant**")

confirm that I voluntarily agree to participate in the research study referred to as the

____ (the "**Study**") in relation to

____

____

and which Study is being conducted by

____

(*Mpheti VM*), (the "**Researcher**").

I, the undersigned Participant, further confirm that–

1. the Researcher has explained the nature, procedure, potential benefits and anticipated inconvenience of my participation in the Study;
2. I have read (or had explained to me) and understood the Study as explained in the attached information sheet;
3. I have had sufficient opportunity to ask questions and am prepared to participate in the Study;
4. I understand that my participation in the Study is entirely voluntary and that I am free to withdraw at any time without penalty (if applicable);
5. I voluntarily provide the UFS and the Researcher with my personal information and consent to the UFS and the Researcher collecting, disclosing and processing my personal information in order to conduct the Study and any related activities in relation thereto;
6. I hereby acknowledge and confirm that I understand the purpose for which the UFS and the Researcher may collect, store, use, delete, destroy, outsource, transfer or otherwise process, as the context and circumstances may require and as contemplated in terms of POPIA, my personal information as set out herein;
7. I am aware that the findings of the Study will be anonymously processed into a research report, journal publications and/or conference proceedings and that my personal information will be aggregated and deidentified at such stage;
8. I also give the UFS permission to share, without notification, the collected data with other researchers at the UFS or other Higher Education Institutions. This permission is dependent on the same principles of ethical research practices, anonymity/confidentiality, safekeeping of information, and other issues listed above applying.

I, the Participant, agree to the recording of the *unstructured interviews and lesson observation*

Full Name of Participant: ____

Signature of Participant: ______________________________ Date: ________________

Full Name(s) of Researcher(s): MPHETI VINCENT MOLEBOGENG

205 Nelson Mandela Drive/Rylaan, Park West/Parkwes, Bloemfontein 9301, South Africa/Suid-Afrika
P.O. Box/Posbus 339, Bloemfontein 9300, South Africa/Suid-Afrika, T: +27(0)51 401 9111, www.ufs.ac.za

APPENDIX D: TABLE CODING OF LESSON OBSERVATIONS DURING CONTENT WORKSHOP

Teaching and learning through Duval's geometric cognitive processes (GCPs) codes

TICK WITH (**X**) WHERE APPLICABLE	• No application of geometric cognitive processes (**N-GCP**)	• visualisation process; no construction nor reasoning (**V-GCP**)	• visualisation and construction; but no reasoning (**VC-GCP**)	• visualisation, construction, and reasoning • All steps required when proving theorems and solving geometric problems are presented (**VCR-GCP**)
Educator A	•	•	•	• X
Educator B	•	• X	•	•
Educator C	•	• X	•	•
Educator D	•	•	• X	•
Educator E	•	• X	•	•
Educator F	•	•	• X	•

APPENDIX E: LESSON OBSERVATION FORM

Educator	A	B	C	D	E	F
Tick (X)	X					

:: MATHEMATICS GRADE 11

Topic: CIRCLE GEOMETRY

Peer/Observer: ________________________________

Date and Time __________________________

Questions	Recommendations
1. **SUBJECT MATTER CONTENT** (shows good command and knowledge of subject matter; demonstrates breadth and depth of mastery)	
2. **ORGANIZATION** (organizes subject matter; evidences preparation; is thorough; states clear objectives; emphasizes and summarizes main points in circle geometry)	
3. **RAPPORT** (holds interest of Peer/Observer; is fair, and impartial; provides feedback, encourages participation; interacts with colleagues)	
4. **TEACHING METHODS** (uses relevant teaching methods, aids, geometric shapes, materials, techniques, and technology; includes variety, balance, imagination, group involvement; uses examples that are simple, clear, precise, and appropriate; stays focused on and meets stated objectives)	

Questions	Recommendations
5. PRESENTATION (establishes classroom environment conducive to learning; maintains eye contact; uses a clear voice, strong projection)	
6. **MANAGEMENT** (uses time wisely)	
7. **PERSONAL** (evidences self-confidence; maintains professional comportment and appearance)	

Strengths observed:

Suggestions for improvement:

Overall impression of teaching effectiveness:

APPENDIX F: CLASS VISIT LESSON OBSERVATIONS IN SCHOOLS

Teaching and learning through Duval's geometric cognitive processes (GCPs) codes

TICK WITH (**X**) WHERE APPLICABLE	• No application of geometric cognitive processes (**N-GCP**)	• visualisation process; no construction nor reasoning (**V-GCP**)	• visualisation and construction; but no reasoning (**VC-GCP**)	• visualisation, construction, and reasoning • All steps required when proving theorems and solving geometric problems are presented (**VCR-GCP**)
Educator B	•	• X	•	•
Educator D	•	•	•	• X

APPENDIX G: INTERVIEW QUESTIONS

Educator	A	B	C	D	E	F
Tick (X)						

:: MATHEMATICS GRADE 11

Topic: CIRCLE GEOMETRY

Peer/Observer: ___________________________________

Date and Time _____________________________

Questions	Participant's response
1. How do you teach circle geometry?	
2. What resources do you use when teaching the circle geometry?	
3. What teaching methods do you use when teaching the circle geometry?	
4. What are your experiences in teaching mathematics?	
5. In your own views what are learners' dispositions toward the learning of circle geometry?	
6. Tell me more about the lesson on circle geometry that I observed	
7. What went well in the lesson presentation?	

Questions	Participant's response
8. What are the challenges identified in lesson you presented?	

Thank you for your time.

APPENDIX H: TABLE CODING OF SEMI-STRUCTURED INTERVIEWS

Teaching and learning through Duval's geometric cognitive processes (GCPs) codes

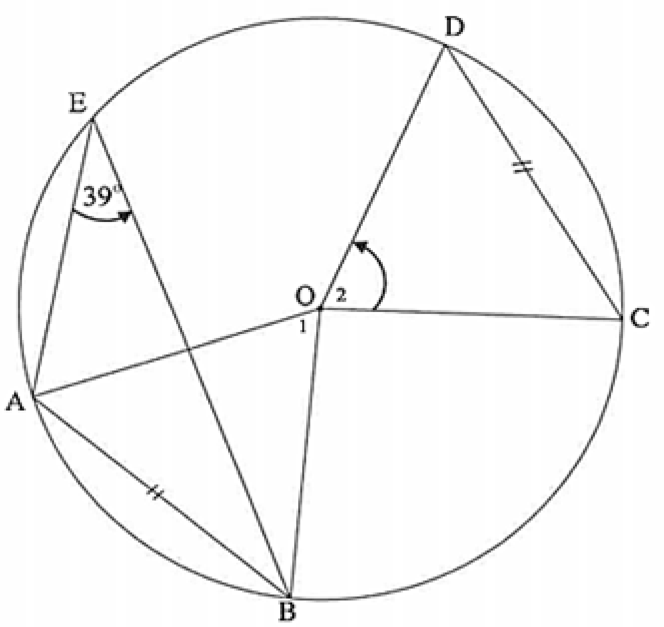
RESPONSES	• No application of geometric cognitive processes (**N-GCP**)	• visualisation process; no construction nor reasoning (**V-GCP**)	• visualisation and construction; but no reasoning (**VC-GCP**)	• visualisation, construction, and reasoning • All steps required when proving theorems and solving geometric problems are presented (**VCR-GCP**)
Educator B	•	• X	•	•
Educator D	•	•	• X	•

APPENDIX I: TEACHING TASKS

Appendix I (a): Teaching Task 1

(DBE: Mathematics Paper 2, Nov 2018, page12)

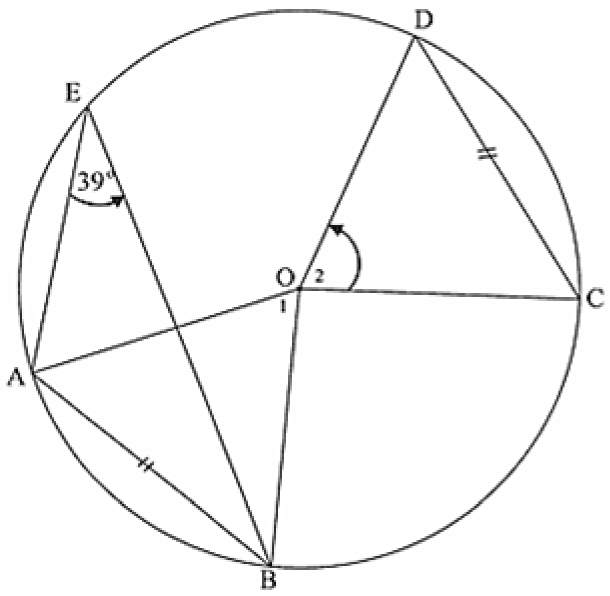
1. In the figure, O is the centre of the circle. A, B, C, D and E lie on the circle such that chord AB and chord DC are equal in length and $A\hat{E}B = 39°$.



1.1 Determine the size of $\hat{O}_1$. (2)

1.2 Determine the size of $\hat{O}_2$. (2)

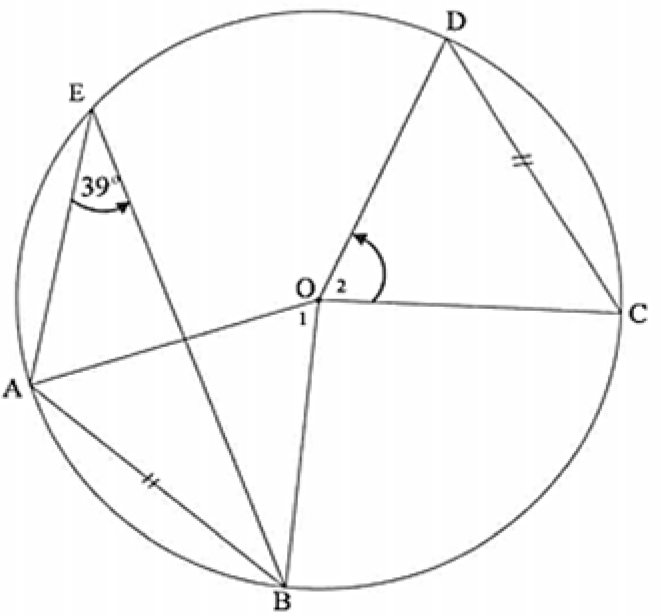
Appendix I (b): Answer Sheet (Task 1)



	Solution	Marks
1.1		
		(2)
1.2		
		(2)

Appendix I (c): Solution (Task 1)

(DBE: Mathematics Paper 2, Nov 2018)

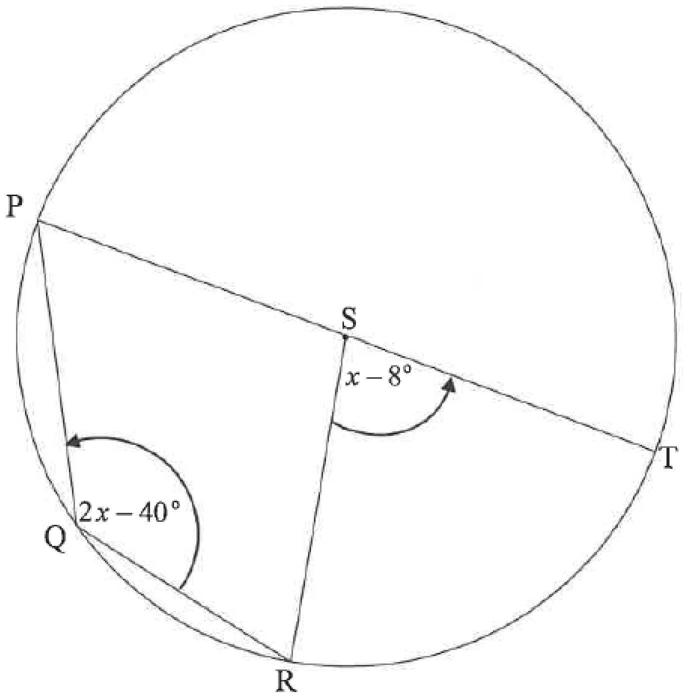


1.1	$\hat{O}_1 = 78°$ [angle at centre = $2 \times \angle$ at circumference]	✓S ✓R (2)
1.2	$\hat{O}_2 = 78°$ [equal chords; equal $\angle^s$	✓S ✓R (2)

Appendix I (d): Teaching Task 2

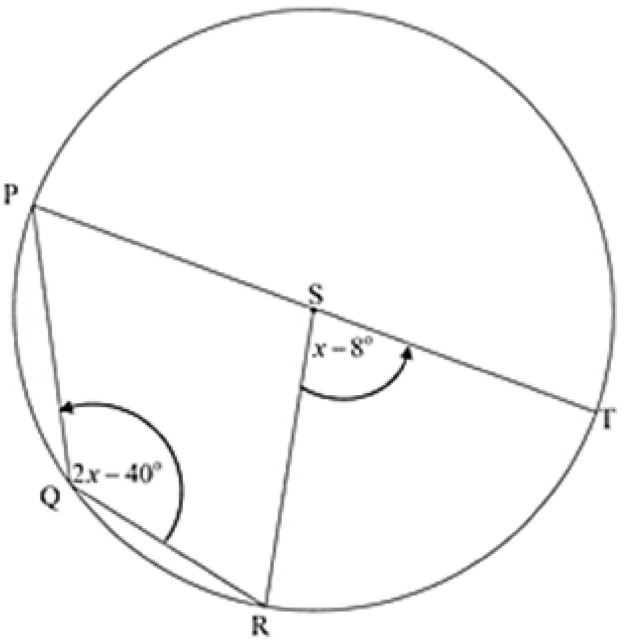
(DBE: Mathematics Paper 2, Nov 2018)

In the diagram, S is the centre of circle PQRT. PT is a diameter.
$R\hat{S}T = x - 8°$ and $P\hat{Q}R = 2x - 40°$.



Determine the value of x. (4)

Appendix I (e): Answer Sheet (Task 2)



2		
		(4)

Appendix I (f): Solution (Task 2)

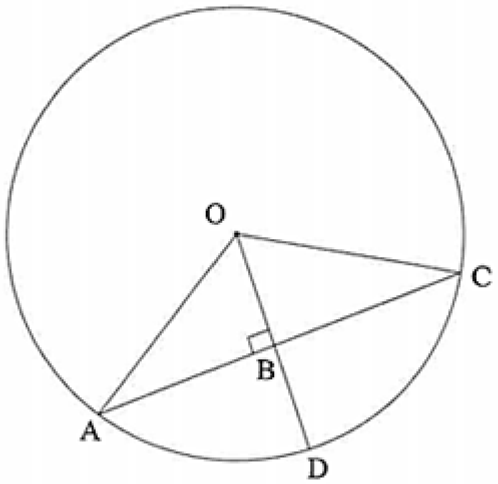
(DBE: Mathematics Paper 2, Nov 2018)

2	$x - 8° + 180° = 2(2x - 40°)$ [angle at centre $= 2 \times \angle$ at circumference $4x - 80° = 172° + x$ $3x = 252°$ $x = 84°$	✓ S ✓ R ✓ simplification ✓ answer (4)
	OR Join T and R $\hat{T} = 180° - (2x - 40°)$ [opp $\angle$'s of cyclic quad] $\hat{R} = \hat{T} = 220° - 2x$ [$\angle^s$ opp. = sides] $x - 8° + 220° - 2x + 220° - 2x = 180°$ [sum of int $\angle^s$ of Δ] $-3x = -252°$ $x = 84°$	✓ S ✓ R ✓ S ✓ answer/ (4)

Appendix I (g): Teaching Task 3

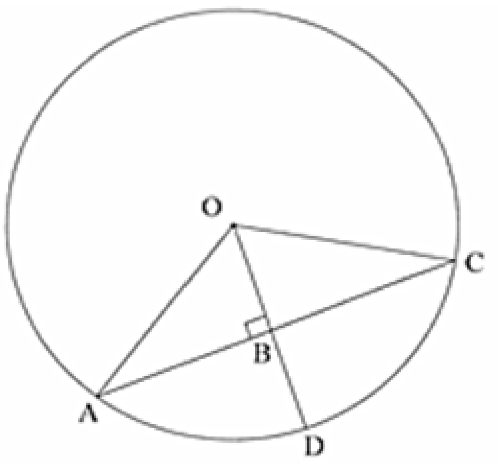
(DBE: Mathematics Paper 2, Nov 2018)

3. In the diagram, O is the centre of the circle. Chord AC is perpendicular to radius OD at B. OB = $2x$ units and AC = $8x$ units.



Show that the length of BD is $2x\left(\sqrt{5}-1\right)$ units. (5)

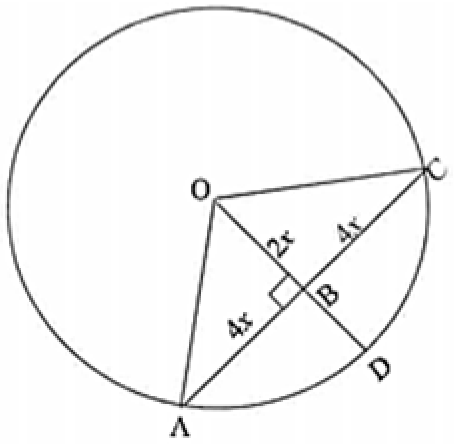
Appendix I (h): Answer Sheet (Task 3)



3		
		(5)

Appendix I (i): Solution (Task 3)

(DBE: Mathematics Paper 2, Nov 2018)

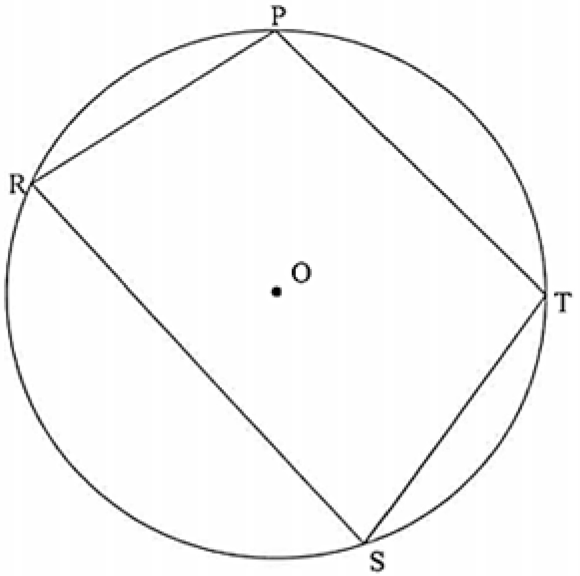


3	$AB = BC = 4x$ [line from centre ⊥ to chord] $OA^2 = (4x)^2 + (2x)^2$ [Pythagoras] $OA = \sqrt{16x^2 + 4x^2}$ $= \sqrt{20x^2}$ $= 2\sqrt{5}x$ $OD = OA = 2\sqrt{5}x$ [radii) $BD = 2\sqrt{5}x - 2x$ $= 2x(\sqrt{5} - 1)$	✓S ✓R ✓ Substitution ✓ length of OA ✓ $BD = 2\sqrt{5}x - 2x$ (5)

Appendix I (j): Teaching Task 4

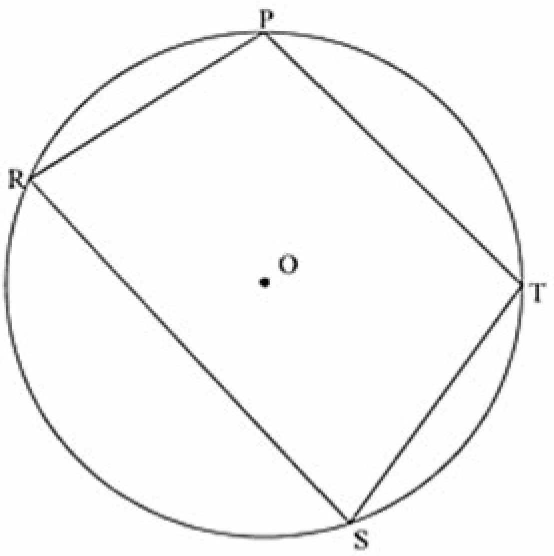
(DBE: Mathematics Paper 2, Nov 2018)

4. In the diagram below, O is the centre of the circle and PTSR is a cyclic quadrilateral.



Prove the theorem that states that $\hat{P} + \hat{S} = 180°$. (5)

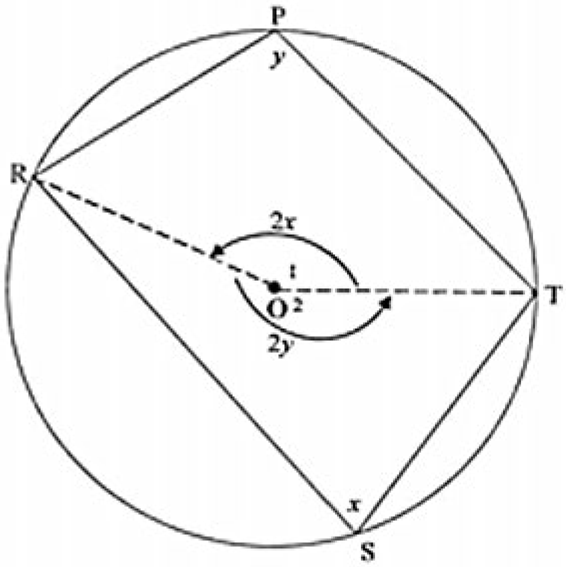
Appendix I (k): Task 4 Answer Sheet



4.	Solution	Marks
		(5)

Appendix I (I): Solution (Task 4)

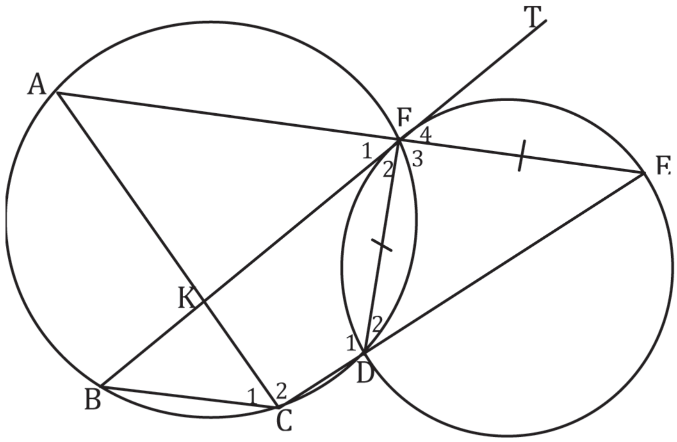
(DBE: Mathematics Paper 2, Nov 2018)



4.	Construction: Draw radii OR and OT Let : $\hat{S} = x$ and $\hat{P} = y$ $\hat{O}_1 = 2\hat{S}$ [angle at centre = 2 times angle at circumference] $\hat{O}_1 = 2x$ Similarly $\hat{O}_2 = 2y$ $2x + 2y = 360°$ [anlges around a pt] $x + y = 180°$ $\therefore \hat{S} + \hat{P} = 180°$	✓ construction ✓S ✓ R ✓S ✓ S/R (5)

Appendix I (m): Task 5 (CAPS Document)

Two circles intersect at F and D.



BFT is a tangent to the smaller circle at ***F***. Straight line ***AFE*** is such drawn that ***FD = FE***. ***CDE*** is a straight line and chord ***AC*** and ***BF*** cut at **K.** Prove that:

1.1 BT // CE (4)

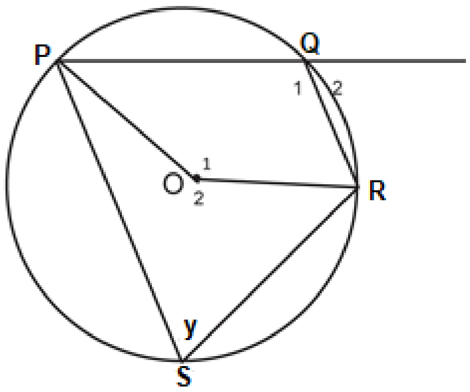
1.2 BCEF is a parallelogram (5)

1.1		(4)

1.2		(5)

Appendix I (n): Task 6 (CAPS Document)

O is the centre of the circle below and $\hat{O}_1 = 2y$



1. Determine $\hat{O}_2$ and $\hat{S}$ in terms of y.

1.		(6)

APPENDIX J: ETHICAL CLEARANCE APPROVAL

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YUNIVESITHI YA FREISTATA
UFS
UV

GENERAL/HUMAN RESEARCH ETHICS COMMITTEE (GHREC)

01-Nov-2022

Dear Mr Vincent Mpheti

Application Approved

Research Project Title:
Exploring the teaching and learning of circle geometry in rural schools

Ethical Clearance number:
UFS-HSD2022/0563/22

We are pleased to inform you that your application for ethical clearance has been approved. Your ethical clearance is valid for twelve (12) months from the date of issue. We request that any changes that may take place during the course of your study/research project be submitted to the ethics office to ensure ethical transparency. furthermore, you are requested to submit the final report of your study/research project to the ethics office. Should you require more time to complete this research, please apply for an extension. Thank you for submitting your proposal for ethical clearance; we wish you the best of luck and success with your research.

Yours sincerely

Dr Adri Du Plessis
Chairperson: General/Human Research Ethics Committee

Adri Du Plessis
Digitally signed by Adri Du Plessis
Date: 2022.11.03 08:34:20 +02'00'

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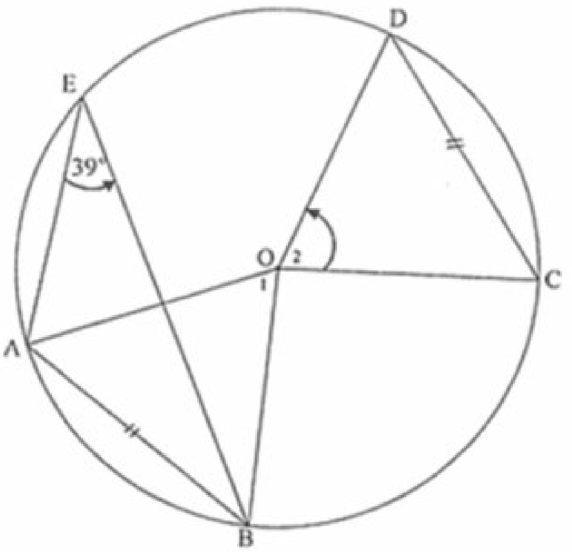
APPENDIX K (a): LEARNER SCRIPT

LEARNER A (LA)

Answer Sheet (Task 1)

1/4



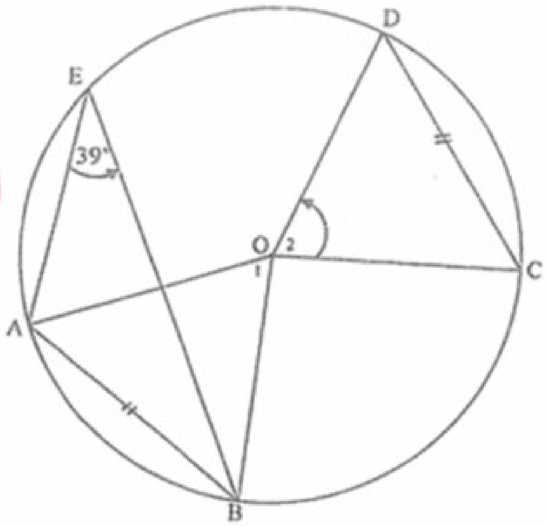
	Solution	Marks
1.1	$\hat{O}_1 = 2\hat{A}$ (∠ at cent = 2× ∠ at circumference)	
	$\hat{O}_1 = 2\hat{B}$ (∠ at cent = 2× ∠ at circumference)	
	∴ $2A = 2B$ $\hat{A} = B$	(2)
1.2	$\hat{O}_2 = 1D$ (∠ at cent = 2×∠ at circ)	
	$\hat{O}_2 = 1C$ (∠ at cent = 2×∠ at ∠s)	
	∴ $1D = 1C$ $\hat{D} = C$	(2)

APPENDIX K (b): LEARNER SCRIPT

LEARNER B (LB)

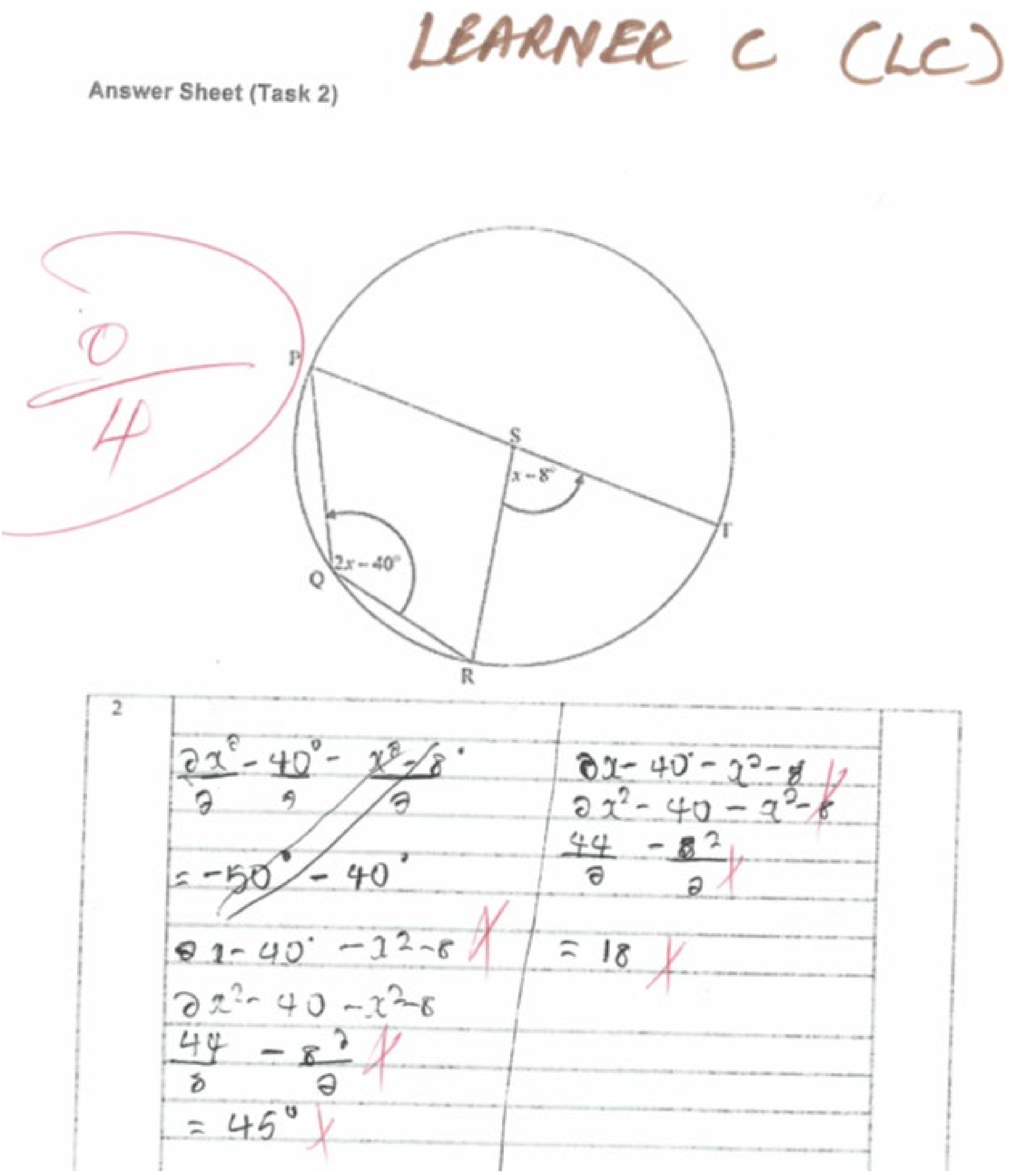
Answer Sheet (Task 1)

0/4



	Solution	Marks
1.1	$\hat{O}_1 = 30°$ ✗ $\hat{E} = \hat{O}_1$ (Alt ∠s) ✗	(2)
1.2	$\hat{O}_2 = 51 + 51$ ✗ (exterior ∠s of a Δ) ✗ $= 102$ ✗	(2)

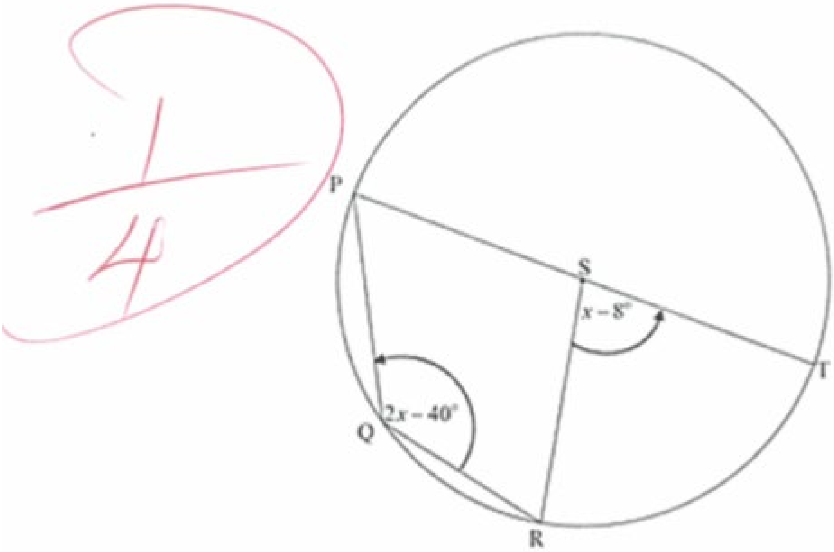
APPENDIX K (c): LEARNER SCRIPT



APPENDIX K (d): LEARNER SCRIPT

LEARNER D (CLD)

Answer Sheet (Task 2)



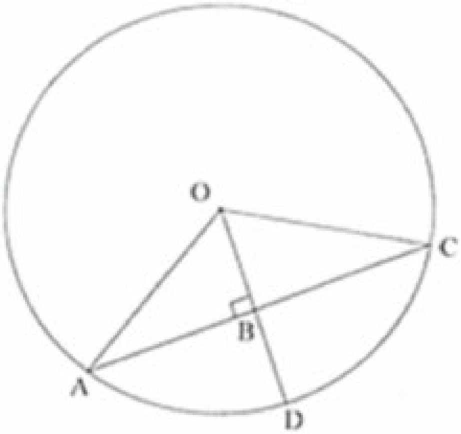
2	the value of x	reason	
	$\frac{2x}{2} - \frac{40^\circ}{2}$	exterior angles of cyclic quadrilateral	
	$x = 20^\circ$		
	$\frac{x}{1} - \frac{8^\circ}{1}$	∠'s on cyclic quadrilateral	
	$x = 8^\circ$		

APPENDIX K (e): LEARNER SCRIPT

LEARNER E (LE)

Answer Sheet (Task 3)

0/5



3

$BD = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ ✗

$= \sqrt{(8 - 2)^2 + (2 - 8)^2}$ ✗

$= \sqrt{(6)^2 + (-6)^2}$ ✗

$= \sqrt{36 + 36}$ ✗

$= \sqrt{72}$ ✗

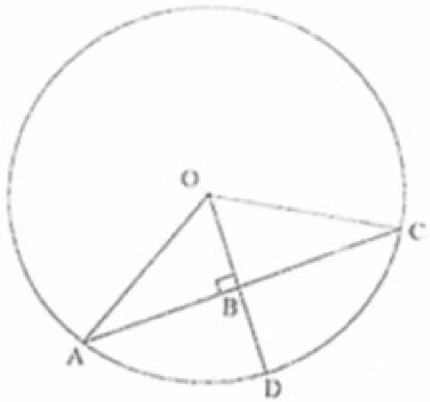
$= 8{,}48$ ✗

APPENDIX K (f): LEARNER SCRIPT

LEARNER F (LF)

Answer Sheet (Task 3)

0/5



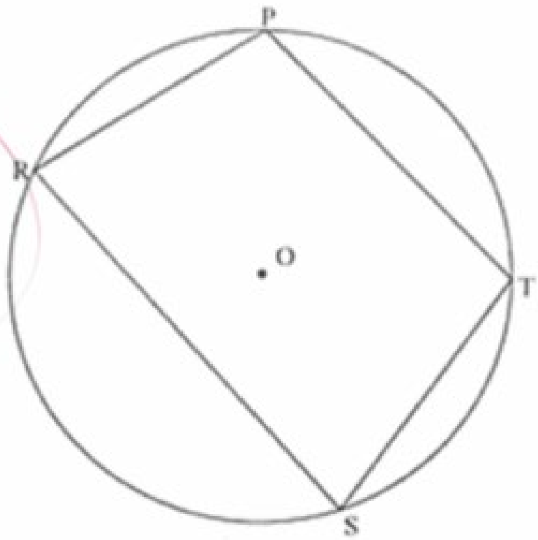
3	
	AC ⊥ OD
	mAC = mCD ✗
	mAC = $\frac{8x \text{ units}}{2}$ ✗
	= 4x units ✗
	mCD = $\frac{4x \text{ units}}{2}$
	= 2x units ✗
	BD = AC − OD ✗
	BD = 8x units − 2 units ✗
	BD = 4x units ✗

APPENDIX K (g): LEARNER SCRIPT

LEARNER G (LG)

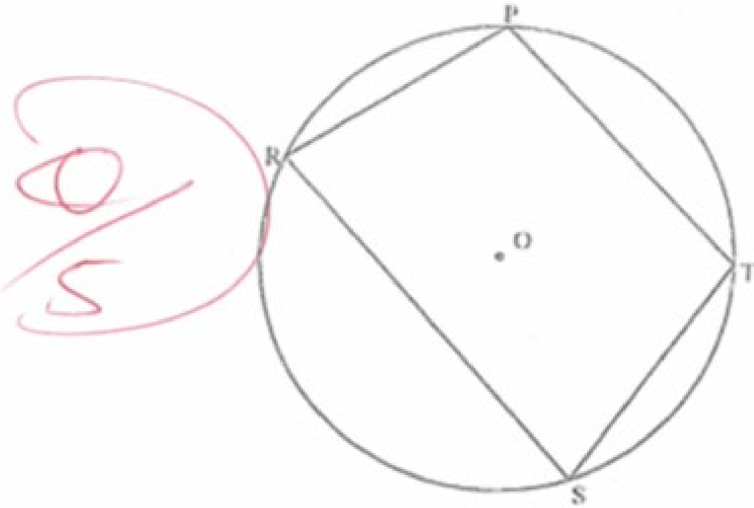
Task 4 Answer Sheet

0/5



4.	Solution	Marks
	$\hat{P} + \hat{S} = 180$ ✗	
	$\hat{R} + \hat{T} = 180$ ✗	
	Reason : opposite angles of cyclic quadrilateral ✗ ✗	

APPENDIX K (h): LEARNER SCRIPT



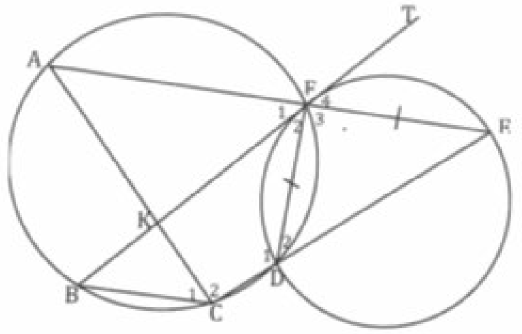
4.	Solution	Marks
	$\hat{P} + \hat{S} = 180^\circ$ ✗ $\hat{R} + \hat{T} = 180^\circ$ $\hat{P} = \hat{S} = 180^\circ$ ✗ $2 \times 90^\circ$ $= 180^\circ$ ✗ Reason : the opposite angle (cyclic quadrilateral) ✗ In the ~~mid~~ centre there ✗ is midpoint O. ✗	
		(5)

APPENDIX K (i): LEARNER SCRIPT

LEARNE I (LI)

Task 5 (CAPS Document)

Two circles intersect at F and D.



0/9

BFT is a tangent to the smaller circle at *F*. Straight line ***AFE*** is such drawn that ***FD = FE***. ***CDE*** is a straight line and chord ***AC*** and ***BF*** cut at **K**. Prove that:

1.1 BT // CE *(4)*

1.2 BCEF is a parallelogram (5)

1.1	$\frac{BT}{CE} = \frac{CE}{BT}$ ✗	(4)
	$90° + 180° = BT$ ✗	
	$= 180$ Δ ✗	

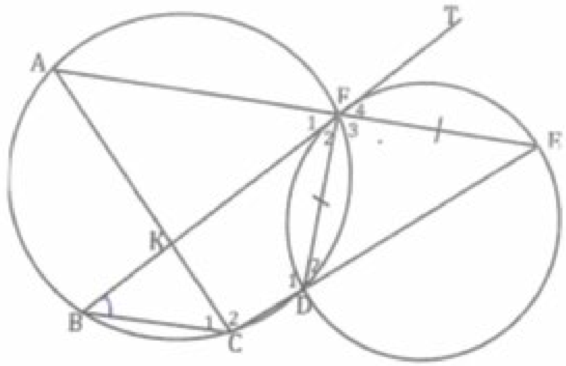
1.2	$\frac{BC}{FE} = \frac{EF}{1D} = \frac{F1}{C1}$ ✗	(5)
	$\frac{1}{4} = \frac{4}{1} = \frac{4}{3} = \frac{3}{4}$ ✗	
	DEF Tm Δ CDT (rop T) ✗	
	$E\hat{F}D = E\hat{C}D$ curespond ✗	

APPENDIX K (j): LEARNER SCRIPT

LEARNER J (LJ)

Task 5 (CAPS Document)

Two circles intersect at F and D.



BFT is a tangent to the smaller circle at *F*. Straight line *AFE* is such drawn that *FD* = *FE*. *CDE* is a straight line and chord *AC* and *BF* cut at K. Prove that:

1.1 *BT // CE* (4)

1.2 *BCEF* is a parallelogram (5)

1.1	Statement	reason	(4)
	BT = 90°	corresponding ∠'s	
	CE ∆ =	∠'s on a circumference	

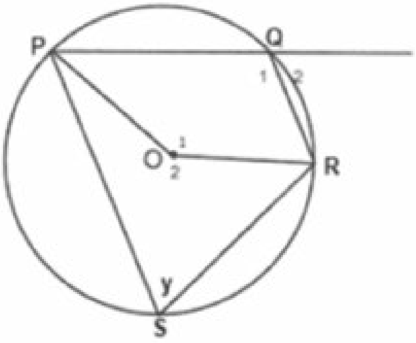
1.2		(5)
	B2E = ∠'s on a straight line = 18D	
	BCE? = corresponding ∠'s	
	∴ B2EF = sides are parallel	
	∴ AAA	

APPENDIX K (k): LEARNER SCRIPT

LEARNER K (LK)

Task 6 (CAPS Document)

O is the centre of the circle below and $\hat{O}_1 = 2y$



1/6

1. Determine $\hat{O}_2$ and $\hat{S}$ in terms of y.

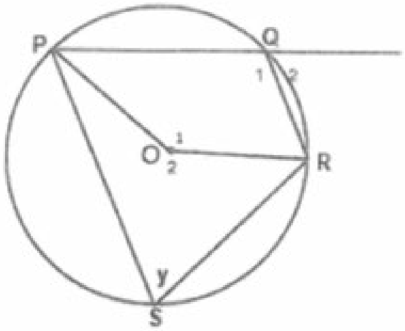
1.	$\hat{O}_1 = \hat{O}_2$ = ∠s around the point	(6)
	POQR = is a cyclic quadrilateral	
	$\hat{O}_2 = 2y$ Angles at centre = 2x angle at circ	
	$P\hat{O}$ = interior opposite ∠'s	

APPENDIX K (l): LEARNER SCRIPT

LEARNER L (LL)

Task 6 (CAPS Document)

O is the centre of the circle below and $\hat{O}_1 = 2y$



$\frac{0}{6}$

1. Determine $\hat{O}_2$ and $\hat{S}$ in terms of y.

1.	$\hat{O}_2 = 180°$ radies ⊥ tangent	(6)
	$O_1 = 90°$	
	$R = 51°$	
	$S = 157°$	
	$2 \times 180° + 90°$	
	$420°$	

APPENDIX L : LANGUAGE EDITING

Carmen
Nel
BA INF SCIENCE

EDITING SERVICES
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CERTIFICATE OF EDITING

This letter certifies that I have edited the work detailed below for language, as well as technically.

Title:

Exploring the teaching and learning of circle geometry in rural schools.

By

Mpheti Vincent Molebogeng
(Student no. 2019516950)

Regards

Carmen Nel
14 May 2023

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APPENDIX M: TURN IT IN REPORT

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