

**USING THE REALISTIC MATHEMATICS EDUCATION APPROACH
TO ENHANCE THE TEACHING OF MATHEMATICAL LINEAR
PROGRAMMING AT TECHNICAL AND VOCATIONAL EDUCATION
TRAINING COLLEGES**

by

MOFOKENG KHEHLA PATRICK (2008088763)

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SUPERVISOR: Dr TJ Moloi

CO-SUPERVISOR: Mr TE Dlamini

DEDICATION

This study is dedicated to my late parents, Thabo Avril Mofokeng and Malwase Julia Mofokeng, who raised, taught, guided, and also provided me with love and support.

I further dedicate this study to my beautiful wife and daughter, Dimakatso Patricia Mofokeng and Boitumelo Dibuseng Mofokeng for their love, support, patience, and that they motivated me to pursue this project. More importantly, they sacrificed family time and a number of family resources.

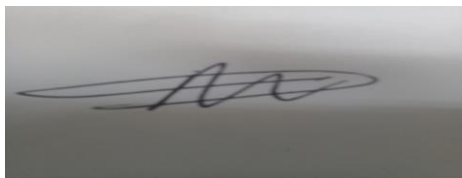
DECLARATION OF INDEPENDENT WORK

I KHEHLA PATRICK MOFOKENG solemnly declare that the dissertation,

**USING THE REALISTIC MATHEMATICS EDUCATION APPROACH TO
ENHANCE THE TEACHING OF MATHEMATICAL LINEAR PROGRAMMING AT
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hereby submitted for the qualification of Master of Art at the University of the Free State is my own work and that I have not previously submitted the same work for a qualification in/at another university.

I KHEHLA PATRICK MOFOKENG hereby declare that I am aware that the copyright is vested in the University of the Free State

A handwritten signature in dark ink, appearing to be 'KP Mofokeng', is centered on a light gray background.

KP Mofokeng

October 2020

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SUMMARY

This study aimed to enhance the teaching of Mathematical Linear Programming (MLP) at Technical Vocational Education and Training (TVET) colleges using the Realistic Mathematics Education (RME) approach. The various reports and literature revealed that MLP is one of the problematic mathematics topics. Moreover, most students do not perform well in MLP and this leads to poor performance in mathematics. Mathematical Linear Programming refers to mathematical theory that consists of a set of variables, limits, and numbers that are expressed in linear constraints, equations, and objective functions. Mathematical Linear Programming includes the following concepts, (i) MLP problem statement; (ii) linear inequalities problem solving; (iii) linear inequalities graphs; and (iv) feasible area. The RME approach refers to a teaching approach, and one vital characteristic of the RME approach is that realities and rich situations are given a prominent position in mathematics teaching and learning. As a result, lecturers are expected to incorporate real-life contexts to concretise the abstract MLP concepts. This study employed the RME principles as components solution to implement the RME approach successfully, namely with guidance, realities, level, intertwined, activity, and interactivity. In addition, TVET colleges are regarded as public colleges created to equip people with knowledge and skills highly needed in the corporate world and also essential for daily lives.

This study discovered that these challenges, abstract teaching, limited teaching aids, dominance of traditional pedagogical methods, the inability to search and actualise prior knowledge of students, lack of formative assessments, students' misconceptions and errors lead to poor performance in MLP. On other hand, this study uncovered that most students are not able to do the following: formulate linear inequalities and objective functions from MLP problem statements; sketch linear constraints and feasible region; and determine a maximum and minimum value through a search line and substitution methods. Thus, the study aimed to develop a strategy to respond to the above challenges using the RME approach at TVET colleges. This study adopted Bricolage as a theoretical lens to strengthen the teaching and learning of MLP at TVET colleges. In Bricolage, bricoleurs use available resources at hand to respond to a particular problem; they do not have time to look for the best resources to respond to unexpected problems. Bricolage enabled this study to uncover multiple challenges and

develop multi-layered components solutions to implement the RME approach successfully.

The generation of a multi-layered and multi-perspectival challenges and components solutions was made possible by this study's co-researchers, who shared their valuable Mathematical Linear Programming Content Knowledge (MLPCK). They also shared their mathematics teaching and learning experience, and best practices. The research team (myself as a principal investigator and my co-researchers) collaboration was made possible by the Participatory Action Research (PAR) methodology of this study, which created a suitable environment for us to share our knowledge in planning, organising, implementation, reflection, and re-planning phases. Therefore, this study used the PAR to generate data through our interactions in meetings, focus-group discussions, free interviews, lesson observations, presentations and reflection of this study. Students' mathematics scripts were analysed with the aim of identifying their misconceptions and errors in MLP. This study's co-researchers comprised a Council member, mathematics coordinator, Campus Manager, 2 Head of Departments, 3 Supervisors, 4 mathematics lecturers, an English lecturer, 3 parents and 25 mathematics students from 5 campuses of the examined TVET college. Various research tools, namely, audio devices, cameras, and notebooks were employed to record and capture generated data. Critical Discourse Analysis (CDA) was used to analyse and interpret generated data on three levels: textual, discursive, and social structure, in order to get a deeper meaning and understanding a text and talks in audio and other recordings. This study categorised generated data in five research objectives, namely challenges, solutions, conditions, threats, and success evidence, and also presented, interpreted and analysed. In conclusion, to solve the challenges mentioned on above, the following components solutions of the RME approach was recommended by this study; levels and intertwined doctrines, multiple teaching and learning aids; multiple components solutions; multiple activities and interactivity as a baseline and diagnostic assessments; and also continuous activities, feedback, and interactivity. As a result, the components solutions of the RME approach has improved both students and co-researchers MLP understanding and also students performance.

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LIST OF ABBREVIATIONS AND ACRONYMS

CAS	Computer Algebra System
CAT	College Academic Team
CBPR	Community-Based Participatory Research
CC	College Council
CCK	Conceptual Content Knowledge
CCWPR	Community Cultural Wealth Participatory Research
CDA	Critical Discourse Analysis
CDE	Centre for Development and Enterprise
CK	Content Knowledge
CM	Campus Manager
CMT	Campus Management Team
CRPK	Cultural Relevant Practice Knowledge
DHET	Department of Higher Education and Training
FET	Further Education and Training
FPAR	Feminist Participatory Action Research
FPR	Feminist Participatory Research
FRP	Feasible Region Point
HoD	Head of Department
MC	Mathematics Coordinator
MCK	Mathematics Content Knowledge
MLP	Mathematical Linear Programming
ICASS	Internal Common Assessment
ICT	Information Communication Technology
ILO	International Labour Organisation
ILP	Integer Linear Programming
MLPCK	Mathematical Linear Programming Content Knowledge
NATED	National Accredited Technical Education Diploma
NCS	National Curriculum Statement
NCV-L3 & L4	National Certificate Vocational Level 3 & 4
NPE	National Policy of Education
PAR	Participatory Action Research
PBL	Problem-Based Learning
PCK	Pedagogical Content Knowledge
PGCE	Post-Graduate Certificate in Education
POLC	Planning Organising Leading, Controlling

RME	Realistic Mathematics Education
RPL	Recognition of Prior Learning
SADC	Southern Development African Communities
SCK	Subject Content Knowledge
SITS	Socially Inclusive Teaching Strategy
SS/SL	Subject Specialist/Supervisor
SL	Senior Lecturer
TPACK	Technological Pedagogical Content Knowledge
TVET	Technical Vocational Education and Training
UFS	University of the Free State
UNESCO	United Nation Educational Scientific and Cultural Organisation
USA	United States of America

LIST OF PARTICIPANTS IN THE RESEARCH

PRINCIPAL INVESTIGATOR	
Mr Phoka [Myself]	
CO-RESEARCHERS/RESEARCH PARTICIPANTS	
Mr Mahapa	Council member and former FET phase mathematics teacher
Mrs Seeta	Mathematics Coordinator and academic board member
Mr Kapu	Campus Manager and academic board member
Mr Kia Ms Khaba	HODs
Mr Kanana Mrs Leeuw Ms Moleko	NCV supervisors
Mrs Lebone Ms Khodu Mr Koena Mr Takui Mr Kau	Mathematics lecturers
Mrs Mosomi	English lecturer
Mr Khetla	Business manager and parent
Mrs Matlou	Former mathematics teacher and parent
Mrs Mohlomi	Small farm co-owner and parent
Campus Management Team (CMT)	
Mathematics NCV L4 STUDENTS	
Dikotlolo; Dimpho; Gambu; Gladys; Khaya; Lebohang; Mampho; Mike; Morena; Neliswa; Poho; Sibongile; Teboho; Thabiso; Thabo; Tholoane; Tshepe; Wilson; Zinhle; Zulu	

CHAPTER 1:

THE ORIENTATION AND BACKGROUND TO THE STUDY

1.1 INTRODUCTION

This study aimed to use the Realistic Mathematics Education (RME) approach to enhance the teaching of Mathematical Linear Programming (MLP) at Technical Vocational Education and Training (TVET) colleges. This chapter gives an overview of the study.

1.2 BACKGROUND OF THIS STUDY

This study aimed to enhance the teaching and learning of MLP at TVET colleges. The diagnostic annual report of the Department of Basic Education (2013:127) reveals that students do not perform well in MLP. This is also supported by the Department of Higher Education Training (DHET) 2009, 2012/ 2013 reports and also Kgotsong TVET college markers and internal moderators reports for Mathematics National Certificate Vocational (NCV) Mathematics (cf. Annexure E), which show that students have serious challenges with MLP, poor performance in mathematics, and a high dropout level at TVET colleges (DHET, 2009:43-46; 2013b:85; Vale, 2016:5). The DHET uncovered that one of the causes of the high dropout rate is difficult subjects and topics, e.g. MLP, which further lead to poor results (DHET, 2013c:33-34). Moreover, researchers in the local, Southern Development African Communities (SADC), on the African Continent and also globally discovered that students struggling, perform poorly, and have a serious challenges with MLP (Olubukola, 2015:846; Chinamasa, Nhamburo & Sithole, 2014:57; Brown & Dell, 2007:153; Saleme & Etchells, 2016:158-159; White, 1995:91-95; Kar, 2016:652; Molnar, 2016:2-12).

Mathematical Linear Programming refers to a mathematical concept that consists of a set of variables and constraints that are expressed in linear constraints and objective functions (Daniel & Solomon, 2011:78). According to Nkambule (2009:7), MLP is used to find the best solution to the present problem of a particular project in various sectors, namely in manufacturing businesses, health, constructions, personnel planning and investment management, especially in maximising profit and lowering costs. Mathematical Linear Programming includes the following concepts: (i) problem

statement; (ii) linear inequalities problem solving; (iii) linear inequalities graphs; and (iv) feasible regions (Yoder & Kurz, 2015:18). Therefore, for the best solution to a particular problem presented in a statement, students need to formulate, solve and sketch linear inequalities on the Cartesian plane, and then find feasible area and calculate maximum and/or minimum value.

The Realistic Mathematics Education (RME) approach refers to a pedagogical method and theory developed by three mathematics didacticians, Edu Wiskobas, Fred Goffee and Adri Treffers during 1968 at the Wiskobas project in the Netherland (Van den Heuvel-Panhuizen & Drijvers, 2014:521). During 1973, the RME approach was established in secondary mathematics education by Hans Freudenthal, because mathematics education in the Netherlands had been dominated by the use of traditional pedagogical methods where mathematics was taught in a linear way or scientific disciplines, and students learned rules, principles and procedures to solve MLP. They were also confronted with ready-made mathematics. Thus, one vital RME characteristic is that realities and rich situations are given a prominent position in mathematics teaching and learning, whereby lecturers are expected to incorporate real-life contexts to concretise the abstract MLP concepts. The use of the RME approach includes the following principles: guidance, realities, level, intertwined, activity, and interactivity. In addition, Technical Vocational Education and Training (TVET) colleges create, design and develop ways to empower and equip students with the knowledge and skills demanded by the corporate world, but also essential for daily lives.

This study endeavoured to empower co-researchers to develop multiple strategies to use the RME approach to enhance the teaching and learning of MLP topic at TVET colleges: (i) investigating the MLP components that students do not understand; (ii) diagnose students' MLP misconceptions and errors; (iii) ascertain which challenges contribute to poor MLP performance; and (iv) collaboratively develop MLP lesson plans using the RME approach, and present MLP lessons in such a way that the students comprehend the abstract MLP content.

In South Africa, Indonesia, Australia, Kenya, India, the USA and other countries, various research and reports uncovered that students have difficulties with formulating linear inequalities and objective functions from problem statements; sketching linear

inequalities; showing feasible region (area); determining and drawing search lines on the Cartesian plane; and determining maximum and/or minimum values through search line method and substitution method (cf. Annexure E; Nkambule, 2013:190-240; Saleme & Etchells, 2016:158-159; White, 1995:91-95; Molnar 2016:2-12; Chinamasa *et al.*, 2014:57; Kar, 2016:652). Brown and Dell (2007:153) have found that the formulation of Integer Linear Programming (ILP) is hard to teach and even harder to learn. This means challenges contributing to poor MLP poor performance are not one-sided, while role-players such as lecturers, college management, communities and also students do contribute to this problem.

According to Moloi (2013a:138), the teaching of mathematics is too abstract. This is supported by White (1995:9), who observes that most lecturers in the USA teach all MLP components in one lesson. Sa'ad, Adamu and Sadiq (2014:37-39), Mapolisa and Tshabala (2014:164); and Mupa and Chinooneka (2015:129) are in agreement that inadequate pedagogical materials are one of the factors that impede mathematics teaching and learning and contribute to poor performance in mathematics in Nigeria, Zimbabwe and Edarabia. The traditional pedagogical approach is another challenge that contributes to poor performance in mathematics in South Africa, South Korea, Kenya, Tanzania and Greece (Molefe & Brodie 2010:10-11; Mji & Makgato, 2006:259-262; Nakhanu & Musasia, 2015:69; Gwambombo, 2013:99-100). Oyinloye and Popoola (2013:1) further exposes that most lecturers do not actively assess students prior mathematics Content Knowledge (MCK), which includes MLP.

This, according to Gafoor and Kurukkan (2015:238-239) thwarts students to grasp and learn new MCK. In addition, Moyosore (2015:484) indicates that one of the challenges in teaching and learning mathematics is lecturers' poor and insufficient assessment techniques. In South African, Saudi Arabia and Indonesia, reports and the literature reveal the following MLP errors and misconceptions: absent meaning of inequality; errors in basic arithmetic operations; errors in basic algebraic operations and deletion; confusion between inequality and equations; errors in inequality rules; errors in writing the solution set; comprehension; transformation; and encoding (Santoso, Farid & Ulum, 2017:3-7; Nkambule, 2013:190-240; El-khateeb, 2016:128-132). This contributes to poor performance in MLP. Blanco and Garrote (2007:223-226) uncovered that students have teething troubles dealing with MLP due to the lack of Procedural Content Knowledge (PCK) and Content Knowledge (CK) in mathematics

linear inequalities. Okello (2010:212-215) reveals these factors as a contributing to poor performance in mathematics: lack of students' interest to study mathematics; students' wrong perception about mathematics; students' MLP deficient vocabulary; students learning in a passive way; and students not asking questions during lessons and not practising at home. This is confirmed by Wanjiru and O'Connor (2015:143).

Therefore, various pedagogical methods have been developed and recommended to enhance the teaching and learning of mathematics, for instance, concretising the abstractness of MLP and mathematics concepts. Moloi (2013b:489-490), Chang, Lee and Koay (2017:1), and Lindsay (2011:605) recommend a combination of concrete and abstract pedagogical approaches. Karue and Amukowa (2013:106) and Sa'ad *et al.* (2014:32) recommend adequate teaching materials to improve the teaching and learning of mathematics in Nigeria and Zimbabwe. The significance of sufficient resources, according to Furner, Yahya and Duffy (2005:17), aids mathematics instructors to be creative and use tools and materials to enhance teaching and learning through manipulation. Moreover, numerous pedagogical approaches have been recommended locally, in SADC countries, on the African continent and globally, because Steven and Palocsay (2004: 38) indicate that there is no single method that is panacea to all teaching and learning challenges of MLP.

In South Africa, Kenya, Nigeria, the UK, Columbia, Croatia, Sweden, the USA, the Netherlands, and other countries, multiple pedagogical methods, such as Problem Based Learning (PBL), code switching, motivational tools, Information Communication Technology (ICT), computer software, procedural embodiment, traditionally oriented practices, student-centred practices, cooperative group work and the integration of games and cultural practices have been designed, developed and used to improve mathematics teaching and learning and performance (Nakhanu & Musasia, 2015:70-73; Schmidt, 1983:15-16; Stevens & Paclosay, 2004:42; Mosia, 2016:186-206; Granberg, 2016:33-35; McGlauglin, 2005:230; Mulaudzi *et al.*, 2016:50-52; Miletic & Lesaja, 2016:123-126; Ngu & Phan, 2015:9-18; Nogueira de Lima & Tall, 2008:1-8; Kolawole, 2008:35-37; Molefe & Brodie, 2010:10-11; Leone, Wilson & Mulcahy, 2010:1-3).

In these countries, the RME approach has been developed and employed to improve the teaching and learning of linear inequalities and other mathematics concepts (Van

den Heuvel-Panhuizen, 1996:2003; Ozkaya & Yetim, 2017:193-195; Ozdemir, 2017:425-426; Menanti, Sinaga & Hasratuddin, 2018:33-34; Van den Heuvel-Panhuizen & Drijvers, 2014: 521-524; Zaranis, 2017:47; Khuluq, 2015:30:58). However, the RME approach has not been used to teach and improve the MLP teaching and learning. Hence, Santoso *et al.* (2017:7) suggest future research focusing on developing and using the RME approach to enhance MLP teaching and learning and to improve students' MLP performance. This is what motivated me as the principal investigator to conduct this study, since MLP is confirmed as one of the problematic mathematics topics and other mathematics concepts in general at this TVET college. To search and activate students' prior content knowledge, Nurjaman and Sari (2017:75) suggest problem pose techniques, while Hailikari, Katajavuori, Lindblom-lanne (2008:4) and Leone *et al.* (2010:1-4) agree on the implementation of a diagnostic testing assessment. In addition, Ajogbeje, Ojo and Ojo (2013:8) suggest sufficient formative assessments to get instant feedback about students' progress in MLP teaching and learning.

For these developed and recommended pedagogical approaches to work, certain conducive conditions are essential that will enable these pedagogical methods to be executed well. Adequate lecturers' Mathematical Linear Programming Content Knowledge (MLPCK), adequate teaching time, dedicated lecturers and positive perceptions are fundamental to the combination of concrete and abstract pedagogical methods (Kim *et al.*, 2018:1; Moyers, 2001, cited in Kamina & Iyer, 2009:2).

Moreover, the availability of teaching proper resources heavily depends on adequate funds available for teaching and learning at TVET colleges. Therefore, the integration of such pedagogical aids in mathematics teaching and learning needs adequate lecturers' Content Knowledge (CK) in mathematics. On the other hand, it needs sufficient lesson planning, adequate infrastructure and continuous mathematics workshops and conferences. The successful use of the multiple teaching methods requires the following key conditions: appropriate lecturers' CK and Pedagogical Content Knowledge (PCK), sufficient teaching time, and a dedicated team of lecturers working collaboratively on lesson planning, preparations, presentations and reflections (Perkins & Wirth, 2008:12-14; Terry & Irving, 2010:110-111; Lenski, Caskey & Anfara, 2009:50; Khan, 2011:3958; Jita & Mokhele, 2014:1-15). In addition, better conditions for prior assessment techniques (diagnostic testing and problem posing) and formative

assessment to be executed successfully are lecturers' right thinking and mentality, enough time for teaching, realistic lecturers' workload, and an enthusiastic team of lecturers.

However, failure to consider these conditions leads to threats and risks to the successful implementation of the above pedagogical approaches developed and recommended. In South Africa and other countries threats identified as a hindrance to the successful use of the combination of concrete and abstract pedagogical approaches are limited time for lesson planning and preparations, wrong lecturers' perceptions, unmotivated lecturers, and inadequate lecturers' CK in mathematics (Moleje & Brodie, 2010:11; Uttal, Scudder & DeLoache, (1997 in Kamina & Iyer, 2009:2); Green, Piel & Flowers, (2008 in Kamina & Iyer, 2009:2); Turnuklu & Yesildere, 2007:12). Financial constraints, costly teaching aids and the absence of training and workshops lead to unavailable teaching aids and thus limiting the use of a multiple teaching aids (Thomas, 2006:267-270). Threats that hinder the successful use of multiple pedagogical methods, namely PBL, code switching, motivational tools, ICT with software, procedural embodiment, traditionally oriented practices, student-centred practices, cooperative groupwork, reform-oriented practices, RME, baseline and formative assessments, and the incorporation of games and cultural practices, according to Mulaudzi *et al.* (2016:50-52), Pelgrum (2001:171-177), Granberg (2016:46-47), Braza and Supapo (2014:20-21), Kim *et al.* (2018:2-3), Boikhutso (2010:207), Gafoor and Kurukkan (2015:238-241), Jita, Maree and Ndlalane (2008:475), Gwambombo (2013:99-100), and Zambo and Zambo (2008:159) at local, SADC, African continent and global levels are a lack of training, lack of computer skills, computer shortages, electricity cuts, limited time, overcrowding, limited interest, lecturers' CK and PCK deficiency in mathematics, lack of lesson planning and preparations, absence of teamwork, collaboration, a team that is not dedicated, and a heavy workload.

1.3 PROBLEM STATEMENT

The DHET's annual reports (cf. Annexure E) show poor performance in MLP at TVET colleges, including Thabong TVET college under investigation in this study. This has been confirmed by previous research conducted in South Africa and other countries at local, SADC, African continent and global levels. As a result, the DBE reports that

MLP is one of the problematic mathematics topics. The challenges contributing to the poor performance in MLP are the following: the teaching of MLP is too abstract; limited teaching tools; traditional pedagogical methods dominating MLP teaching and learning; lack of prior-knowledge and formative assessments; and students' MLP errors, mistakes and misconceptions. In addition, students are not able to formulate linear inequalities and objective functions from problem statements and word problems, or draw linear inequalities and show feasible region, as well as determine maximum and/or minimum values using a search line and substitution methods of MLP.

1.3.1 Research question

How can the RME approach be used to enhance the teaching of MLP at TVET colleges?

1.3.2 Research aim

To demonstrate how the RME approach can be used to enhance the teaching of MLP at TVET colleges.

1.3.3 Objectives of the study

The objectives of the study were:

- To investigate the challenges that hinder the MLP teaching and learning and contribute to poor MLP performance at TVET colleges;
- To demonstrate how to use the RME approach to enhance the teaching of MLP at TVET colleges;
- To examine critical conditions under which the RME approach enhances the teaching of MLP at TVET colleges;
- To investigate threats and risks involved in executing the RME approach to enhance the teaching of MLP at TVET colleges and make suggestions for preventing identified threats; and
- To identify evidence of success for using the RME approach to enhance the teaching of MLP at TVET colleges.

1.4 THEORETICAL FRAMEWORK

This section elucidates the adoption of Bricolage as a theoretical framework in developing a strategy to enhance the teaching and learning of MLP at TVET colleges using the RME approach. This is done through a discussion of the Bricolage origin and moments, objectives, formats, ontology and epistemology, the role of the researcher and the relationship between the researcher and the participants.

1.4.1 The origin and moments of Bricolage

According to Phillimore *et al.* (2016:08-09), Mahlomaholo (2012:4694), and Rogers (2012:1), Bricolage is an approach that is multidimensional, multi-perspectival and layered, because it wants to be as close to real life as possible, arguing that, to find a solution to an apparent problem, bricoleurs rely on a multiplicity of theories and ideas to find out what can work and under which conditions. Bricolage brings together and stitches diverse views into a coherent and logical story that can best explain a situation under consideration for its resolutions. Louvel (2013:672) indicates that Bricolage thrives on complexity and in this way mirrors the messiness of real-life situations. It is the science of making something out of nothing, as well as being action oriented and hands-on. Kincheloe (2004a:01) signposts that Bricolage is explained as a metaphor from an anthropological, managerial and sociological viewpoint. It is derived from the French term *bricoleur*, which means a handyman or woman who uses available resources and knowledge at his/her disposal from diverse environments to solve unexpected problems and to complete a task at hand.

This study adopted Bricolage as a theoretical framework guiding this study to achieve its research objectives. For instance, Bricolage assisted this study to view the poor performance in MLP and challenges in teaching and learning in a universal multi-perspectival manner. This was done through a literature review at local, regional, SADC, continental and global level to understand deeper-level common challenges in the teaching and learning of MLP, as well as looking into best practice. In this study, the available tools at hand were the co-researchers' MCK, teaching and learning experiences, and teaching methods; numerous teaching and learning aids; and eight moments of Bricolage, i.e. the traditional period; modernity phase; blurred genres; crisis of representation; postmodernity phase; post-experimental, methodologically contested representation; and the current fractured future.

This study used traditional-era ontology and epistemology to analyse traditional pedagogical methods' effectiveness, gaps, and limitations in the teaching of MLP. In the traditional period lecturers assume that students learn best when they are taught rules and procedures are demonstrated by their lecturers in a passive fashion. Lecturers are viewed as the only source of knowledge; a powerful individual who knows everything that students are supposed learn and master. This traditional-era learning epistemic has limitations in the 21st century; therefore, to respond to these limitations, this study makes its way into the modernity phase. The modernity phase acknowledges theories in qualitative research, because positivism was no longer the only method to discover the truth (Mahlomaholo, 2014:174). However, according to Mosia (2016:34), in the modernity phase, acceptable and good research indicated that its findings are produced by another academic scholar, in the sense administering research study using the same questionnaires and with different research sites, participants and time to prove validity and reliability. Thus, this procedure shows a limitation of the modernity phase because participants are not given space to air their emotions and beliefs regarding the investigation processes. As a result, this led to the third moment of Bricolage, blurred genres. The pedagogical methods in the modernity phase were analysed in this study, where after its limitations led to the next moment.

The Bricolage-blurred genres allow a multiplicity of theories, strategies, methods and paradigms (Kincheloe & Berry, 2004:23). This helped this study to generate data and achieve the study research objective, because the co-researchers were able to share their MCK, MLPCK, best practices, and teaching and learning experiences of MLP. However, blurred-genres epistemology and ontology stance allow fluid borrowings of ideas, leading to crises of representation, legitimation and praxis.

Therefore, this study used Participatory Action Research (PAR) as a research methodology to address the problem representation, because this allowed people who are directly faced with poor performance and the challenges in teaching and learning MLP on a daily and yearly basis in the post-modern, post-experimentalism period. The team worked together and shared their experiences, MCK, best practices and narratives regarding the teaching and learning of MLP. It enabled this study to use storytelling as a fragment of mixed representation to generate data. More importantly, as the principal investigator of this study, I ensured that the research processes were ethical and moral; the seventh moment ontological and epistemological stance.

The ethical principles underpinning the current study encouraged all team members to participate and share their narratives, experiences and knowledge. This helped this study to generate multiple methodologies and strategies how to teach MLP in a creative way, as Bricolage is seen as one of face of creativity to create something new out of available resources (Le Loarne, 2005:02). This evidenced what Mahlomaholo (2013b:4690) explains Bricolage is, namely the multi-layered and perspectival grounding of one research question, approached from diversity of theoretical stance.

1.4.2 Objectives of Bricolage

The objective of Bricolage is to use available resources to respond to an unexpected problem at hand. This means that bricoleurs, when faced with a problem, respond by using what is available, because they do not have the time to look for new resources to solve a problem. This agrees with what other scholars (Phillimore *et al.*, 2016:08-09; Mahlomaholo, 2012:4694; Rogers, 2012:1) articulate in defining Bricolage, namely that Bricolage refers to a process of creating something out nothing. This study sought to use the RME approach to enhance the teaching of MLP at TVET colleges, because at the specific college under investigation, mathematics is regarded as the worst-performing subject. Therefore, to achieve this study, research objectives, available resources, e.g. co-researchers' knowledge, skills and experiences are fundamental.

1.4.3 Formats of Bricolage

The numerous formats of Bricolage include methodological, theoretical, interpretive, political, narrative, social theory and the cognitive constructivism (Mosia, 2016:7-8), to strengthen and explain a need for Bricolage as theoretical lens. Bricolage enabled this study to use multiple methods to generate data in order to enhance the teaching of MLP. It helped us as a team to understand that there is no single teaching method that is panacea for all teaching and learning challenges; there is a need for multiple pedagogical methods. Moreover, theoretical Bricolage was used to view the research problem of this study through multiple co-researchers' lenses, because we all view the world through different angles. In this way multiple challenges and solutions could be uncovered to enhance the teaching of MLP. Interpretive Bricolage was applied in the analysis of students' scripts; lesson planning and preparations; presentations,

observations and reflections; and in focus-group discussions to allow for views and each team member's opinion from his/her analysis and interpretation.

The knowledge shared and strategy developed was aimed to benefit all people (participating or non-participating). More importantly, this study sought to empower and develop co-researchers, applying political Bricolage. Social theory was employed to bring the team together for us to share our personal experiences, knowledge and skills to enhance teaching of the MLP using the RME approach at Thabong TVET college (Nutchey, Grant & Cooper, 2016:373). This was possible, because individual team members used narrative Bricolage and cognitive constructivism to make meaning in order to contribute their knowledge. Thompson and Boro (2015:01) agree that knowledge constructed through social collaboration among co-researchers stems from diverse sociocultural practices, beliefs, and experiences from diverse contexts.

1.4.4 Ontology and epistemology of Bricolage

The ontology position of Bricolage is that within rural, urban and school communities as well as in the business world, there are valuable resources, such as tools, experiences, solutions, and knowledge of co-researchers that can be responsive to unexpected problems, leading to a complex ontology and epistemology of the Bricolage. According to Kincheloe (2004b:73), complex ontology further complements complex epistemology, which is created by a social web of reality that each individual person contributes unique knowledge to the rest of the team. Thus, poor performance in mathematics was a problem at this said TVET college. It has been regarded as the worst-performing and most problematic subject for years. The ontological and epistemological stance of Bricolage is fundamental to respond to this problem at this TVET college, because diverse and valuable resources are available.

This study agrees with the notion that within communities and around the world there are valuable resources, experiences and knowledge that can respond to many problems we have if enough space is created for the people involved to share their knowledge, experiences, back stories, emotional beliefs and values. This leads to multiple realities, complex ontology and epistemology, because people use their own lens to view and interpret the world and relate to the world.

1.4.5 The role of the researcher

The role of myself as a principal researcher in this study was to formally convene the team that can develop a strategy to implement and use the RME approach to teach and enhance the teaching of MLP at the TVET college. The aim of having the team in this study to develop a strategy to enhance the teaching and learning of MLP was because as a principal researcher I do not have enough knowledge, solutions, skills and experience to solve all problems lecturers, HoDs and this college face in MLP on my own, but together as a team we are powerful. McNiff and Whitehead (2006:36) indicate that people who are faced with problems are also the ones that are in a better position to solve their own problems if they are given space.

Hence, Kincheloe (2011:220) indicates that educational researchers offer little to help lecturers to respond to day-to-day problems. Thus, my role as a principal researcher in this study was to share my knowledge in a collaborative way with my co-researchers to solve and respond to MLP teaching and learning challenges; more importantly, to develop a strategy to use the RME approach successfully to enhance the teaching and learning of MLP at this TVET college. This can assist in addressing wrong perceptions that people have about the significance of educational research to daily problems in people's lives. Moreover, my role as a main investigator was to create a better space that enables co-researchers to share and participate freely in this study without any discrimination. Therefore, the co-researchers were allowed to share their knowledge in the language each are comfortable with. In addition, the study was mainly located in the third, fifth, sixth, seventh and eighth moments of Bricolage, which allow co-researchers to be active and improve their daily lives (Denzin, 2001:326), because this study believed in a complex epistemology that generate solutions.

1.4.6 The relationship between researcher and the participants

The relationship between myself as a principal investigator and the co-researchers was mainly working together to develop a strategy to implement the RME approach to enhance the teaching and learning of MLP at this TVET college successfully. In this study, during research meetings we had discussions, shared lesson observations, lesson presentations, and lesson reflections. Our roles were not fixed, as we used

Participatory Action Research (PAR) as a methodology, being guided by PAR doctrines and spiral phases.

1.5 SUMMARY OF RELATED LITERATURE

The literature from local, SADC, continental and international levels was reviewed and in line with this study research objectives, such as challenges, solutions, conditions, threats and also evidence of success.

1.5.1 Literature review

This section reviews related literature for the purpose of investigating challenges in the teaching and learning of MLP topic.

1.5.1.1 Difficult way of teaching MLP and other mathematics concepts

The literature revealed that lecturers teach all MLP components such as word problems, linear inequalities, drawing, analysing and interpreting MLP graphs in one lesson. As a result, this frustrates and confuses students and contributes to students' misunderstanding and poor performance in MLP (White, 1995:29). Mkhize (1999:36) uncovered that most school and college lecturers rely only on traditionally oriented practice, because they seldom employ cooperative group work, and then link this to the difficulty of teaching mathematics. Moloi (2013a:138) uncovered that teaching mathematics is too complex and difficult at school.

1.5.1.2 Inadequate pedagogical materials

Sa'ad *et al.* (2014:37-39), Mapolisa and Tshabala (2014:164), and Mupa and Chinooneka (2015:129) share the same sentiment, stating that inadequate teaching materials are one of the factors that hamper teaching and learning of MLP and other mathematics concepts, contributing to poor performance in mathematics, in amongst others, Nigeria, Zimbabwe and Edarabia. The above scholars, in their research study, further found that a lack of teaching facilities, and the poor quality of textbooks, libraries and laboratories contribute to mathematics poor performance. This leads to students' misunderstandings, misconceptions, errors and mistakes in mathematics concepts.

1.5.1.3 Traditional pedagogical methods dominate the teaching of mathematics

Sa'ad *et al.* (2014:37-39), Mapolisa and Tshabala (2014:164), and Mupa and Chinooneka (2015:129) found that the traditional pedagogical approach dominates the teaching and learning of MLP and other mathematics concepts. According to them, this approach also contributes to poor performance in mathematics in South Africa, South Korea, Kenya, Tanzania, and Greece (Molefe & Brodie 2010:10-11; Mji & Makgato, 2006:259-262; Nakhanu & Musasia, 2015:69; Gwambombo, 2013:99-100). Van der Wal (2015:49-50) observes the dominance of instructors in a mathematics class, which occurs when instructors provide students with ready-made material. Bature, Atweh and Treagust (2016:173) add that most mathematics class activities are teacher-centred, with students learning passively, being good listeners and recipients of knowledge, rather than active initiators of their knowledge. Centre for Development and Enterprise, (2004 in Mulaudzi *et al.*, 2016:44), share that most students find mathematics as a subject difficult to learn because of the outdated methods used in classrooms and because instructors mostly teach mathematics using the traditional paper and pencil approach. Waits, (1997 in Mulaudzi *et al.*, 2016:44-45) further explains the traditional paper and pencil approach as a manual procedure used to plot graphs in mathematics. They maintain this approach poses difficulties for students to learn.

1.5.1.4 Failure to search and actualise students prior knowledge in mathematics

Oyinloye (2013:1) discovered that most mathematics lecturers fail to search and actualise students' prior knowledge in the teaching and learning of MLP vocabulary. According to Gafoor and Kurukkan (2015:238-239), this hinders the MLP teaching and learning. Hailikari *et al.* (2008:04) signpost that a lack of this knowledge creates a lot of misconceptions and frustrations to students, and leads to poor results.

1.5.1.5 Lack of formative assessments

According to Moyosore (2015:484), one of the challenges in the teaching and learning of mathematics is lecturers' poor, insufficient assessments methods and lack of using formative assessments. The failure to implement formative assessments in mathematics teaching and learning impact students' performance negatively.

1.5.1.6 Students' misconceptions, mistakes and errors

Santoso *et al.* (2017:3-7), and El-khateeb (2016:128-132) have uncovered the following errors in solving linear inequalities, namely absence of meaning in inequality; errors in basic arithmetic and algebra operations and deletion; confusion between inequality and equations; errors in inequality rules; errors in writing solution set, comprehension, transformation and encoding. Blanco and Garrote (2007:223-226) discovered that students have difficulty dealing with MLP, because of a lack of Procedural Content Knowledge (PCK) and Conceptual Content Knowledge (CCK) in linear inequalities.

1.5.2 A review of literature for components solutions to enhance the teaching of MLP at TVET colleges

1.5.2.1 The use combination of concrete and abstract pedagogical methods

According to Moloi (2013b:489-490), Chang *et al.* (2017:1), and Lindsay (2011:605), to enhance the teaching and learning of mathematics, a combination of concrete and abstract teaching levels should be used to concretise abstractness of mathematics. Prangmsma *et al.* (2009:383-384) indicate that the teaching of mathematics should start from the concrete to abstract level to enhance students' mathematics understanding and performance. Kamina and Iyer (2009:2) explain manipulatives as a concrete object used in the teaching and learning of mathematics to help students understand the abstract concepts in the domain of mathematics. Furner *et al.* (2005:17) used manipulatives to provide mathematics teachers with the great potential to use their creativity to do further activities on mathematics concepts in the classroom. Students automatically learn mathematics concepts in a fascinating and hilarious way when teaching follows the sequence between the concrete and abstract levels.

Andrew (1990, in White, 1995:29-30) indicates that MLP should be divided into 5 to 6 lessons according its components:

- firstly recognising, describing variables and limits (understanding of problem situation);
- secondly, converting word problems into Mathematical linear inequalities;
- thirdly, drawing linear inequalities and find feasible region;

- fourthly, determining the maximum (or minimum) value using the feasible region (analysis and interpretation);
- fifthly, revising the whole process, checking students' solutions and prepare for test in order to improve the teaching and learning of MLP concepts.

Dantzig and Thapa (1997:8) recommend two ways to formulate a problem as a linear programme, namely column (recipe/activity) and row (material balance).

1.5.2.2 Sufficient teaching aids

Karue and Amukowa (2013:106) and Sa'ad *et al.* (2014:32) suggest that enough teaching materials be used to enhance the teaching and learning of mathematics. According to Furner *et al.* (2005:17), adequate resources help mathematics instructors to be creative and enhance the teaching and learning of MLP and other mathematics concepts through manipulation of materials. In 21st-century classroom, mathematics lecturers use various teaching and learning apparatus, including a technological tools and computer software packages (Thomas, 2006:268-269; Suleiman & Hammed, 2019:30).

1.5.2.3 Numerous teaching methods to teach MLP and other mathematics concepts

According to Steven and Palocsay (2004:38), a single teaching method does not respond to all challenges in the teaching and learning of mathematics. In South Africa, Kenya, Nigeria, the United Kingdom (UK), Columbia, Croatia, Sweden, the USA, the Netherlands, and other countries, numerous pedagogical methods have been designed, developed to enhance teaching and learning of MLP and other mathematics concepts, such as PBL, code switching, motivational tools, ICT with computer software, procedural embodiment, traditionally oriented practices, student-centred practices, cooperative groupwork and the integration of games, as well as cultural contexts, according to Nakhanu and Musasia (2015:70-73), Schmidt (1983:15-16), Mosia (2016:186-206), Granberg (2016:33-35), McGlauglin (2005:230), Miletic and Lesaja (2016:123-126), Ngu and Pha (2015:9-18), Nogueira de Lima and Tall (2008:1-8), Kolawole (2008:35-37), Molefe and Brodie (2010:10-11) and Leone *et al.* (2010:1-3). The RME approach was developed and employed in the Netherlands, Turkey, Greece, Indonesia, South Korea, the UK and the USA to enhance the teaching and learning of linear inequalities and other mathematics concepts (Van den Heuvel-

Panhuizen, 1996:2003; Ozkaya & Yetim, 2017:193-195; Ozdemir, 2017:425-426; Zaranis, 2017:47; Khuluq, 2015:30:58). As a result, the above pedagogical teaching methods have improved teaching and learning.

1.5.2.4 Activate and assess students' mathematics prior knowledge

Nurjaman and Sari (2017:75) recommend problem pose methods (probing questions), while Hailikari *et al.* (2008:4), and Leone *et al.* (2010:1-4) commend the use of diagnostic testing assessment. Diagnostic interviews activate and assess students' prior knowledge in mathematics. There are multiple ways to recognise, actualise and search students' prior knowledge, and many more not mentioned. The importance of conducting more prior-knowledge assessments is that they provide detailed information about students' prior knowledge. Hailikari *et al.* (2008:2) indicate that to develop prior knowledge assessment, the instructor should know what type of knowledge to be assessed, and distinguish between the different types of prior knowledge which are vital in a particular context, because not all types have similar relevance to students' achievement. Prior knowledge assessments are viewed as the most vital tool influencing learning and student achievements positively.

1.5.2.5 Sufficient formative assessments

Ajogbeje *et al.* (2013:8) recommend sufficient formative assessments to get immediate feedback about students' learning. Kolawole (2010, in Ajogbeje *et al.*, 2013:8) indicates that tests and other evaluation tools in the teaching and learning process serve to monitor students' learning progress to achieve lesson objectives and improve their performance results. The benefit of using formative assessments is that these remove the threatening effect of a single test given at the end of the subject course study. Gitaari *et al.* (2012:96) propose multiple types of assessments to identify students' abilities. Formative assessments play a key role in teaching and learning because it provides lecturers with instant feedback about presented lessons in order to re-plan next lessons (Tridane *et al.*, 2015:685).

1.5.2.6 Multiple languages and pedagogical methods

Blanco and Garrote (2007:227) share that, to teach linear inequalities of the MLP topic, lecturers should ensure that linear inequalities, jargon and symbols are differentiated and have semantic value for students. Furthermore, Blanco and Garrote (2007:228)

indicate that lecturers should use different strategies to approach questions related to inequalities; both enrich the learning process and allow more students to acquire the concept.

1.5.3 Critical conditions for solutions

1.5.3.1 Conditions enabling concrete and abstract approaches to working effectively

The conditions key to the successful implementing the combination of concrete and abstract teaching methods are the following: adequate lecturers' Mathematical Linear Programming Content Knowledge (MLPCK), enough teaching time, dedicated mathematics lecturers and also positive perceptions. Kim *et al.* (2018:1) indicate that the Content Knowledge (CK) has an influence on Pedagogical Content Knowledge (PCK) and students' results. Positive lecturer perception is needed to implement a combination of concrete and abstract teaching methods to teach mathematics concepts. Moyer (2001, in Kamina & Iyer, 2009:2) signposts that lecturers fail to incorporate realities, objects, stories, and background knowledge into the teaching of mathematics, because they think that it is a waste of time to do so.

1.5.3.2 Key conditions accessible and the use of teaching and learning resources

The availability of teaching sufficient resources requires enough funds or a budget allocated for teaching and learning at TVET colleges. The incorporation of teaching resources needs lecturers with adequate CK, adequate lesson preparations, adequate infrastructure and continuous mathematics workshops and conferences (Kim *et al.*, 2018:1; Khan, 2011:3958). Sufficient funds refer to money set aside for a particular obligation, for instance, acquiring teaching and learning resources. Adequate funds have a direct impact on teaching and learning resources. The literature revealed that adequate and available resources are fundamental to improving the teaching and learning of mathematics. More importantly, lecturers have to incorporate such teaching and learning aids successfully, requires a continuous mathematics in-house training, workshops, training and conferences for development and capacitation purposes. Lecturers can capacitate themselves by consulting different textbooks and pursue their studies to improve their CK and PCK, because adequate CK leads to adequate PCK

(DBE, 2018:16). Thomas (2006:265-266) shares that to operate technological tools and computer software, lecturers should have computer skills and knowledge.

1.5.3.3 Conducive conditions for various teaching approaches to work

The successful implementation of multiple teaching methods requires adequate lecturers' CK and PCK, enough teaching time, and a dedicated team of lecturers working as a team (Terry & Irving, 2010:110-111; Lenski *et al.*, 2009:50; Khan, 2011:3958; Jita & Mokhele, 2014:1-15). Furthermore, Malebese (2016:161) agrees with Perkins and Wirth (2008:12-14) on the fact that a dedicated team brings different knowledge to research and the teaching and learning, because it enables team members to learn from one another. Jita and Mokhele (2014:1-15), in their research study, have observed a dedicated team of instructors working in collaboration to plan and prepare lessons and jointly present the lessons. Furthermore, the team managed to develop a strategy to improve teachers Technological Pedagogical Content Knowledge (TPACK) through the integration of ICT and software in Mosia's work (2016:139). This is in line with the Japanese pedagogical approach, because Japanese instructors use a lesson study approach (Lenski *et al.*, 2009:50).

1.5.2.4 Conditions to enable prior knowledge assessments to work

For prior knowledge assessments to be successful, it needs lecturers' insight, enough time for teaching mathematics, fair lecturer workload and an enthusiastic team of lecturers and knowing what this type of assessment seeks to achieve (Ifeoma *et al.*, 2017:99; Ngema, 2016:123; Sa'ad *et al.*, 2014:35; Thanheiser *et al.*, 2010:12).

1.5.3.5 Conditions for formative assessments to be implemented successfully

Implementing formative assessments successfully necessitates lecturers' right thinking and attitude, sufficient time for mathematics teaching, manageable workloads and passionate team of lecturers. Formative assessments further help lecturers to become aware of students' abilities and deficiencies, and then to plan and re-plan next lessons. Therefore, this needs adequate time allocated for teaching and learning (Thanheiser *et al.*, 2010:12).

1.5.4 Threats to the successful use of solutions

1.5.4.1 Risks to hinder the successful use of concrete and abstract methods

The risks that pose a threat to the successful use of the combination of concrete and abstract teaching pedagogical approaches are the following: a lack of time for lesson planning and preparations; the negative attitude of lecturers; unmotivated lecturers; and a lack of lecturers CK in mathematics. Moyer (2001, in Kamina & Iyer, 2009:2) signpost that lecturers view manipulatives as a waste of time, while some lecturers simply use manipulatives for fun in their mathematics teaching, instead of using these to engage and interact with students to enhance mathematics teaching and learning. Turnuklu and Yesildere (2007:12) signpost that, if lecturers' CK in mathematics is deficient, it leads to a limited of PCK, putting concrete level of teaching at risk.

1.5.4.2 Threats to the successful use of multiple teaching resources

Threats to the successful use of various teaching aids to the teaching and learning of mathematics are a lack of financial resources, costly teaching aids and the absence of mathematics training and workshops (Becker, 2000b, in Thomas, 2006:266). Financial constraints leads to a lack of teaching and learning aids. This study agrees with Becker (2000b,in Thomas, 2006:266) that some teaching resources are expensive; hence, they are limited at schools and colleges.

1.5.4.3 Threats that hinder the use of multiple teaching methods

The following factors were identified and discovered as a threat to the successful use of multiple pedagogical methods: limited mathematics training; lack of computer skills; a shortage of computers; power failures; limited time for teaching; limited interest; lack of lecturers' CK and PCK; inadequate lesson planning and preparation; lecturers' working in silos; an non-dedicated team; and an unrealistic work load. This is confirmed by Turnuklu and Yesildere (2007:12), who state that inadequate lecturer CK in mathematics impedes the success of multiple pedagogical approaches, leading to inadequate lecturer PCK. According to Talis (2009:25), a lack of lesson planning and preparations also hinders the successful use of multiple methods to teach mathematics, because it disorganises instructors completely.

Moreover, Hu and Linden (2015:1104), and Jita *et al.* (2008:475) are in agreement that when instructors work in isolation, it denies them to learn from one another; share solutions; and team up in lesson planning, preparation, facilitation and evaluation, thus, this is also a risk. In addition, according to Pelgrum (2001:171-177), scarce

computer tools, mathematics lecturers' incorrect views, lack of infrastructure, limited internet connectivity, power cuts, lack of workshops and training, and limited time are threats/ risks to the successful use of technology to teach MLP and other mathematics concepts.

1.5.4.4 Risks to the successful implementation of prior knowledge assessments

Unrealistic lecturers' workload is one of the risks for the successful implementation of prior knowledge assessments, leading to poor performance in teaching and learning of mathematics and science (Ngema, 2016:106-107; Gwambambo, 2013:99-100; Mosia, 2016:21). In addition, a lack of motivation, poor attitude, and wrong perceptions among instructors and students are also threats and lead to poor mathematics performance (Gafoor & Kurukkan, 2015:234).

1.5.4.5 Threats to the successful to use of formative assessments

The factors in 1.5.4.4 that were identified as hindrances to the successful execution of prior knowledge assessment also threaten the successful execution of formative assessments.

1.5.5 Evidence of success for solutions

1.5.5.1 Concrete and abstract teaching methods

The combination of concrete and abstract teaching methods was used to enhance the teaching and learning of mathematics (Kamina & Iyer, 2009:2; Lindsay, 2011:605). The combination of these approaches should be used to mathematise abstract mathematics concepts. Kamina and Iyer (2009:3-9) demonstrate how to employ the combination of the concrete and abstract teaching approaches to teach abstract geometry concept. Moreover, concrete-pictorial-abstract sequence instructional strategy was employed to develop primary mathematics concepts in Singapore (Chang *et al.*, 2017:18-24).

Andrew (1990, in White, 1995:29) shows how to divide parts of MLP teaching into four to five lessons, namely recognising, describing variables and constraints; formulating linear constraints and objective functions from problem statement; sketching linear inequalities and feasible area; determining maximum and (or minimum) value using

feasible region; revision of whole process; check solutions and prepare for tests in order to enhance teaching of MLP concepts. This technique assists students to comprehend MLP topic well.

1.5.5.2 Multiple teaching apparatus

Sa'ad *et al.* (2014:32) uncovered that making provision for relevant teaching materials is one of the methods that enhance the teaching and learning of mathematics at schools and colleges. Majerek (2014:51) and Granberg (2016:33-35) discovered that the use of computer software has a positive impact on mathematics teaching and learning.

1.5.5.3 Multiple pedagogical methods

Sa'ad *et al.* (2014:32) suggest the use of proper methods to enhance the teaching and learning of mathematics and also results. Suleiman and Hammed (2019:30) demonstrate how to integrate games in the teaching of mathematics and have concretised the abstractness of mathematics concepts. On the other hand, Nakhanu *et al.* (2015:6) and Leone *et al.* (2010:6-7) uncovered that PBL and guided methods benefit and aid students to understand mathematics concepts and improve their performance.

Moreover, procedural embodiment and translation approaches have been used to teach MLP in a more meaningful way, leading to positive student understanding and results (Nogueira de Lima & Tall, 2008:1-8; Steven & Palocsay, 2004:42; Williams *et al.*, 2016:14-15). Stevens and Palocsay (2004:42) also use a combination of computer software and code switching methods to teach MLP in USA schools. In addition, the RME approach was employed to concretise the abstractness of mathematics concepts in the Netherlands, the USA, Indonesia, and other countries; thus, improving mathematics performance (Ozkaya & Yetim Karaca, 2017:194-195; Zaranis, 2017:58-59).

1.5.5.4 Prior knowledge assessments

According to Hailikari (2008:6-7), prior knowledge indeed aids students to pass a pharmaceutical chemical course, contributing positively to their learning. Most of the students in Hailikari's research study were in agreement that the use of prior

knowledge assessments benefited them because with this type of assessment they were more aware of their weak points.

1.5.5.5 Formative assessments

Ajogbeje *et al.* (2013:15) investigated the effect of formative testing with feedback on mathematics students' achievement. Their evaluation looked at three categories of students: students with formative test and regular feedback; students with formative tests only; and students with no formative tests and no regular feedback. As a result, they discovered that students who were provided with formative tests and regular feedback performed better than those provided with formative tests only and with no formative tests or feedback.

1.6 METHODOLOGY

This study used the Participatory Action Research (PAR) as a research methodology and guided by its principles to generate data which assisted this study to achieve its objectives (cf. Section 4.2- 4.5). According to Denzin and Lincoln (2007:277), PAR is a social process that allows a group of people to come together to share their problems, experiences and challenges to find solutions to their problems. This group of people is viewed as enabled people who can come together when they encounter problems to solve their problems, using their knowledge from diverse contexts, because PAR involves a number of stages, such as planning, actions, observations and reflections. McNiff and Whitehead (2006:36) advocate the philosophy that people who are affected and faced with problems should be involved in the decision-making process and developing solutions to their own problems through interaction with others. This is done through PAR as a research methodology in the iterative process, which circles from planning, to actions, observing and reflection. For this reason, this study has used PAR as methodology. In their study, people who face challenges with MLP teaching and learning and poor performance in mathematics were given ample space to share their daily lived experiences.

This study holds the view that these people, due to their mathematics Content Knowledge (MCK) and teaching experience, were in a good position to develop a strategy and find techniques to incorporate the RME approach into the teaching of MLP at TVET colleges. I realised from the start of the research that as a principal

investigator I did not have adequate experience and knowledge to accomplish the research objectives of this study on my own, or to provide solutions for all problems pertaining to the teaching and learning of MLP, especially to develop a strategy to enhance the teaching of MLP. In addition, the appropriateness of PAR to this study was that co-researchers' lived experiences, teaching experience and MCK were key to attaining the research objectives of this study. The rationale for using PAR as a methodology and research design is explained in Chapter 3.

1.7 ETHICAL CONSIDERATION

As the principal investigator, I requested a letter of permission and ethical clearance from the Education Ethics Committee at University of Free State (UFS) of the Faculty of Education to conduct this research study at TVET colleges. The committee approved the application and issued ethical clearance which allowed this study to be conducted. Thereafter, I further requested a letter of permission from the DHET to enter and conduct this research at TVET colleges. The DHET directed me to the colleges where this study would be conducted, because Clause 5.3 states that applicants (researchers) who wish to conduct research at fewer than 10 public colleges, for a specific study, must submit their application to the Head/s of the college/s concerned. According to Clause 5.4, researchers who wish to conduct research at 10 or more public colleges must submit their application forms to the DHET. The Department will then provide a letter of support, with which the applicant could gain access to the colleges concerned (DHET, 2016:7). Since the current study aimed to conduct research at fewer than 10 public colleges, I as the principal investigator requested a letter of permission from the college Principal, whereupon permission was granted and approved (cf. Annexure D). This letter enabled me to enter all campuses of this TVET college to generate data.

At the initial meeting I explained what this study was all about, what it sought to achieve, its procedures, aims, and objectives. Assent and consent forms were read and defined, then all people who were interested to participate in this study were provided with adequate time to read the document before signing and submitting it. Under-aged students' parents signed on behalf of them. More importantly, both assent and consent forms were written in multiple languages for all participants to understand. No individual person was forced to participate in this study. I as the principal

investigator informed all people that they may join, leave and re-join in this study at any time. I informed all participants that participation was voluntary in this study, and they were assured of anonymity regarding any information they supplied. Pseudonyms were used to maintain anonymity.

1.8 DESIGN, DATA GENERATION AND ANALYSIS

In Chapter 3 it will be discussed how PAR as a methodology was applied in this study. The research design firstly revealed the research site where this study was conducted, followed by the initial meetings for the purpose of inviting identified people by this study with common problems, challenges and experiences in teaching, learning and poor performance in MLP. Mr Kapu (pseudonym) as a Campus Manager understood what this study sought to achieve and he was excited and eager to participate, indicating that he would share this information with his Campus Management Team (CMT) members and mathematics lecturers as identified by this study. Mr Kapu suggested a Friday slot after 13:00, when everyone is free and off duty for research meetings. He further provided a class to convene the first meeting.

At the first meeting as a principal investigator I explained what this study was about, and what it aimed to achieve. This study paradigm, methodology, procedures, instruments, and methods (including concern forms) were elucidated. The invited people showed an interest to participate in this study. Thereafter, we as a team decided on the next meeting, which included the suitable venue, time, and weekly sessions for all team members. At the next meeting, the consent forms were handed in and we started to investigate challenges contributing to MLP poor performance in order to find solutions to respond to identified challenges. However, the team decided to prioritise the investigation of MLP challenges to determine problematic concepts, errors and misconceptions in MLP. This was done to develop a mathematics subject improvement plan that focuses on MLP. We drew up the action plan, consisting of three activities to generate data. In addition, to generate data, this study used students' scripts, DHET and DBE diagnostic reports, free interviews, focus group discussions, observations, presentations and reflections. Thereafter, the generated data were organised into five research objectives of this study.

The participants consisted of the college Council Member, mathematics Coordinator, Campus Manager, two Heads of Department (HoDs), 3 Supervisors, 4 mathematics lecturers, 1 English lecturer, 3 parents, and 25 mathematics students. However, in this study they were all regarded as co-researchers because as a main investigator, I worked together with them and shared duties with them. They were identified and chosen based on their valuable knowledge, expertise, and experiences from different contexts. More importantly, all team members were allowed to participate and share their knowledge and experience in the language they felt comfortable to use, which was then translated. This assisted this study develop a strategy to implement the RME approach to enhance the teaching of the MLP at this TVET college.

The following instruments were used to capture generated data, namely audio records, cameras, and notebooks, in agreement with the co-researchers. Moreover, the Critical Discourse Analysis (CDA) was employed on three levels, textual, discursive practices and social structure, to analyse and interpret data generated in order to gain a deeper understanding of the text and spoken words. Bloor and Bloor (2007:2) explain CDA as a cross-discipline that comprises the analysis of text and talk in almost all fields of humanities and social sciences. According to Malebese (2016:135, citing Fairclough, 1993), the main aim of CDA is to systematically explore often opaque relationships of causality and determination between discursive practices, events, and texts, as well as wider social and cultural structures, relations, and processes.

1.9 VALUE OF THIS STUDY

This study pursued to enhance the teaching of MLP at TVET colleges using the RME approach, because it is regarded as one of hard and difficult mathematics topics. This was proved by various researches and reports (diagnostic and markers) from DBE, DHET and Kgotsong TVET college. The RME approach enhanced students' understanding and their performance in MLP. In addition, this study envisaged contributing to efforts of the DHET to improve mathematics teaching and learning, skills, knowledge, and results in TVET colleges.

1.10 LAYOUT OF CHAPTERS

Chapter 2 reviews previous researches related to the teaching of MLP using the RME approach at TVET colleges. This chapter begins with a theoretical framework that guides this study. Thereafter, the related literature is unpacked with the purpose of developing component solutions of using the RME approach drawn from the best practices.

Chapter 3 elucidates PAR as a research methodology and its relevancy for this study. The second level in this chapter discusses the research design, which includes the research site, the initial meeting, research participants, action plan, and activities. In addition, data generation, which entails the research procedures, participants and instruments. Data analysis employing the Critical Discourse Analysis (CDA) is discussed.

Chapter 4 presents, interprets, analyses and discusses the data generated for each of this study's five research objectives. This chapter analyses the challenges of teaching MLP at TVET colleges. This was done to develop possible component solutions to use the RME approach to respond the challenges experienced and also enhance the teaching and learning during and beyond this study. Thus, conducive conditions will be examined to ensure that possible component solutions using the RME approach were sustainability. Moreover, the risks and threats that could impede the successful implementation of possible component solutions to using the RME approach to enhance the MLP teaching and learning at TVET colleges are investigated in this chapter. In addition, component solutions by using the RME approach that were tested and evaluated before recommendations could be made are indicated.

Chapter 5 once more elucidates the background and problem statement of the current study. Furthermore, this chapter presents the findings and recommends component solutions of the RME approach to enhance the MLP teaching and learning at TVET colleges developed in Section 4.3.

CHAPTER 2:

THEORETICAL FRAMEWORK AND LITERATURE STUDY

2.1 INTRODUCTION

This study aimed to enhance Mathematical Linear Programming (MLP) teaching and learning using the Realistic Mathematics Education (RME) approach in order to help students to perform well in MLP and in mathematics at TVET colleges. This chapter will discuss the historical background and objectives of Bricolage, and eight moments in depth as a lens guiding this study, discussing epistemology and ontology for each moment. This study will show how Bricolage was used to achieve this study's research objectives and explain its relevancy to this study. The role of the researcher, the relationship between the researcher and co-researchers, conceptual frameworks, formats, rhetoric and operational concepts have been outlined, explained and discussed. This study has reviewed literature studies of various scholars who share the same sentiments with the Technical Vocational Education and Training (TVET) colleges report annually, because both show that MLP is a problematic component of mathematics in South Africa and globally. The literature and reports showed that students perform poorly in mathematics.

2.2 THEORETICAL FRAMEWORK – BRICOLAGE

Next, the historical background, objectives, eight moments, epistemology, and ontology of Bricolage will be explained and discussed. This study will show how Bricolage has been employed to achieve research objectives, The relevance of Bricolage for this study will also be explained. The role of the researcher, the relationship between the researcher and co-researchers, formats and a rhetoric will be outlined and discussed.

2.2.1 Historical background of Bricolage

Bricolage is explained as a metaphor from the anthropologist, sociological, and also managerial point of view. It was derived from the French word *bricoleur*, which refers to a handyman or woman who uses the available tools or materials to execute an unexpected task; thus Bricolage means the artefacts necessary for transformative and

emancipatory agenda as participants are elevated to the position of co-researchers (Mosia, 2016:137; Mahlomaholo, 2014:172; Kincheloe, 2004a:01; Louvel, 2013:672; Levi-Strauss 1962:1-12; Mahlomaholo, 2013:4695). However, Lennerfors and Rehn (2014:378) explain Bricolage according to three attributes, namely making do with problems, rather than lingering over questions whether a workable outcome can be created from what is at hand; accumulation of resources at hand; and combining resources for new purposes. In light of the above definitions it is clear that Bricolage is relevant to different environments in the world, namely research, business, science, culture and education. However, a common assumption is that Bricolage is normally considered to be connected to an individual bricoleur – the person doing the Bricolage.

Lennerfors and Rehn (2014:378) indicate that Bricolage is a necessary component for small enterprises to grow and it should be conducted within the context of an economic downturn where business organisations continue to use existing resources to operate and even grow, developing new products or services in order to survive and prepare for an economic upturn, because Bricolage is primarily meant for businesses. Lennerfors and Rehn (2014:378) maintain that Bricolage processes represent an opportunity to understand innovation as a process of imbuing the previously worthless resources with value. Furthermore, they view Bricolage as a vital approach to solve business organisations' daily challenges, especially during difficult times, like during the Corona Virus Disease 2019 (COVID-19) pandemic where many businesses had constrained resources to achieve maximum profit and survive. This study sees a link between Bricolage and MLP, since both could be used to assist business organisations to overcome challenges of limited resources.

Mahlomaholo (2013b:4690) explains Bricolage as multi-layered and multi-perspectively grounded in one research question, as approached from a diversity of theoretical positions. Bricolage as a research approach is better posed, because paradoxically it thrives on making sense of what seems chaotic and contradictory. Le Loarne (2005:02) explains Bricolage as one face of creativity, which means to nourish the understanding of creativity. He explains Bricolage according to the following main aspects: the context within which Bricolage takes place; the process of Bricolage; status of bricoleurs; the relationship between Bricoleur and his/her environment; and the status of the final output.

2.2.1.1 The context in which Bricolage takes place

According to Le Loarne (2005:8), Bricolage happens when a person or business is faced with unexpected problems and is expected to solve them with available resources. Thus, this study understands that Bricolage occurs when it is time to improvise. This means that bricoleurs do not have time to look for resources that are firstly meant to address that particular problem, but have to utilise available resources to solve the unplanned problem. Lennerfors and Rehn (2014:378) indicate that within an entrepreneurial context, when a new project is established, unexpected problems might arise that need bricoleurs.

2.2.1.2 Process of Bricolage

Le Loarne (1962:9) signposts that bricoleurs gather resources to use them when they might be useful, e.g. solving unexpected problems. Bricoleurs do not gather specific resources for unpredicted problems because they work with heterogeneous resources, but a finite source of materials and tools.

2.2.1.3 Status of bricoleur

The bricoleur is a creative individual, not an individual who plans objectives. Secondly, bricoleurs choose among all objects gathered in a diverse environment and use them. In the process, they commit mistakes, but in the end they achieve their goals (Le Loarne, 1962:11). Mosia (2016:42) believes that bricoleurs are curious and always intersect with several worlds in order to understand diverse ideologies of these worlds.

2.2.1.4 Relationship of bricoleurs with their respective environment

Bricoleurs value their environment highly, because they gather resources from the world in order to use them in future. Le Loarne (1962:11) explains a bricoleur as a capable person who collects resources in a diverse environment; thereafter talks to them in order to solve unplanned problems. Therefore, this study's ontological stance is that there is a positive relationship between bricoleurs and the environment. For instance, individual cultures provide bricoleurs with basic frameworks to change nature and finality to use objects or resources to solve unexpected challenges.

2.2.1.5 Status of the final output

Bricolage aims to solve unplanned problems which in the first place were not anticipated to happen in the process of completing the project at hand. Therefore, its results could also be unexpected; so bricolage does not bring complete fulfilment, because it uses available resources to respond to unexpected problems and in the end solve those problems. Le Loarne (1962:12) implies that Bricolage results can be changed and replaced by a better solution in future, since it also relies on a multiplicity of methods to address the problems.

2.2.2 Moments of Bricolage as theoretical framework from its origin

2.2.2.1 Traditional period

The first historical moment of qualitative research of Bricolage started in 1900 and lasted until 1945 (Denzin & Lincoln, 2005:02). During this period and also prior to it, the research that was regarded as great was that done by natural scientists such as Isaac Newton, whose work was influential in the human, social, and economic sciences (Mosia 2016:46). The scholars were expected to conduct their research using proven and scientific approaches. Mahlomaholo (2014:174) adds that such investigations are seen to be meaningful only if its work can be tested through set scientific procedures such as reliability and validity, which is referred to as the positivist or traditional period among eight moments in the development of qualitative enquiry. This study agrees with Mosia (2016:29) who signposts that natural scientists' work used to dominate research because researchers were expected to follow certain principles in conducting research. Their work should be proven before accepted. Mahlomaholo (2014:174) adds that the traditional period aimed to investigate causes and effects of the particular problem, so that the researcher is able to predict future behaviour based on a previous pattern functioning, such as objectivity, reliability, validity, predictability and the formulation of general laws, among other things. In the traditional period, natural scientific methods overruled and dominated, to such an extent that science was key to liberating and bringing people together.

However, this study asks one vital question: How can we achieve social change if scientific research still holds on to scientific tools that exclude people's experience? The scientific process was characterised by risks, threats and weaknesses that were embedded in the processes that were supposed to generate social change, mainly

applicable to sociology and humanities researchers. A gap between a researcher and human beings affects people negatively, because it does not allow interaction among people. Denzin and Lincoln (2005:25) and Geertz (1988:81) agree that a positivist style needs to be changed because ethnographers were unable to give meaning to collected and generated data in the field using positivist rules, natural laws and generalisations, which led to the modernist phase of qualitative research. In the traditional era, lecturers played a big role in the classroom, where they were taken as the only people who knew everything. Secondly, lecturers relied only on Procedural Content Knowledge (PCK), rules and procedures in MLP teaching and learning, without integrating Conceptual Content Knowledge (CCK). As a result, this impacted teaching and learning, as well as MLP and mathematics performance negatively. The traditional period produced better results because some students performed well. However, it had some limitations, because not all students understood the PCK applied and recalled mathematical rules and procedures to solve MLP problems.

2.2.2.2 Modernity

The second moment of qualitative research, modernity, started around 1945 and ended in 1970 (Denzin & Lincoln, 2005:16). According to Mosia (2016:31), modernity acknowledges good work of traditional quality, as the aim was to make research difficult, where principles had to be followed internally and externally. Human background knowledge and experience were believed to be measured with the same rule as objects in natural sciences, which was believed to be the only research method to tell the truth. According to Mahlomaholo (2014:174), most of the researchers in this era started to question the value of taking this method as the only theoretical framework due to the difficulty of thinking all human behaviour can be measured with positivist rules and procedures. During the 1970s, some researchers came forward with multiple ways to do research, as positivism could no longer be the only method to uncover the truth. Mosia (2016:34) adds that good research in modernity should be one that is epitomised by reliability, which expects that discoveries of one research study must be reproducible by another researcher.

The research conducted in the past revealed that in the modernity phase, researchers were supposed to do research using the same questionnaires, but with different locations, times and samples to show validity and reliability. In addition, Turner (2001,

in Mosia, 2016:34) indicates that the relations and roles of researchers and the participants are the same as in the Bricolage period, except for the fact that they are regarded as respondents and not objects. According to this study, during the modern phase, mathematics lecturers were expected to teach MLP in an abstract way, without incorporating realities, because during this period, only traditional pedagogical methods were approved and recommended to teach MLP. Even to date at this TVET college, lecturers still rely and use only traditional methods to teach MLP, with its limitations. However, modern-phase limitations opened the way for blurred genres, the third moment of Bricolage.

2.2.2.3 Third moment: Blurred genres

The third moment emerged to make the modern phase more efficient, because in this era, research was no longer based only on positivism. Mosia (2016:35) signposts that the period of blurred genres (third moment of Bricolage) took place approximately between 1970 to 1986. He states that blurred genres admit and acknowledge an increase in different philosophies and theories, such as symbolic interactionism, constructivism, naturalistic inquiry, positivism, critical theory, ethnomethodology, post-positivism, phenomenology, neo-Marxist theory, semiotics, structuralism, feminism and various racial or ethnic paradigms. The blurred genres era accommodates different methods, strategies and paradigms, which show a maturity of qualitative research. The multiple blurred genres cover voluminous theoretical frameworks (Denzin & Lincoln, 2005:17; Kincheloe & Berry, 2004:23).

In the 21st century, mathematics lecturers should encourage students to use their background knowledge and experience in MLP, which is in line with the blurred genres, because it requires multiple readings, plurality, and inclusiveness. This study afforded co-researchers an opportunity to contribute knowledge and skills in the research study, allowing their voices to be heard. Kincheloe (2005, in Mahlomaholo 2014:175) confirms that in the blurred genres, participants from social and human sciences are allowed to interact and share their knowledge and experience. The blurred genres in teaching and learning of mathematics allow the use of multiple pedagogical approaches to enhance the teaching and learning. This study used Realistic Mathematics Education (RME) guided by its doctrines: activity, interactivity, levels, intertwined, realities and guidance.

2.2.2.4 Crisis of representation

The crises in representation started around during 1986 and lasted until 1990 (Mosia 2016:38). Mahlomaholo (2014:175) indicates that blurred genres directly led to a crisis of representation, because there were now multiple methods to uncover the truth and different people's views and experiences were considered to include findings. People could no longer depend on one researcher's expertise to interpret their own experiences. At the same time, new interpretive and qualitative views emerged, i.e. hermeneutics, cultural studies, semiotics, structuralism, and feminism. The ontological stance of this study is that there is no single method of telling the truth, because people use multiple methods to mine the truth. Different views of researchers challenged the idea of the objectivity, validity and reliability of positivism, because it became clear that all participants brought multiple methods to the research discussions from their own background knowledge.

In addition, the social sciences moment caused triple crises. Denzin and Lincoln (2005:19) and Mosia (2016:35) agree that it had to do with representation, praxis and legitimation. The literature explains crisis representation as researchers' difficult ways to report participants' experiences and this situation raised vital questions about whose participants' contexts should be represented in the findings. The representation led to a crisis of praxis, where people's experiences were represented within the context of researchers' opinions and there was no way to validate and verify its truthfulness. Mosia (2016:35), in his research, poses an important question, asking how findings that only exist in research reports can impact people's lives. Therefore this fourth moment (crisis of representation) made way for post-modernity, which was concerned with responses to the crises.

2.2.2.5 Postmodernity

Postmodernity emerged from the challenges of crises of representation (Mahlomaholo, 2014:175). This was seen in research where shared narratives of a small number of people's life stories were preferred to those of positivists and post-positivists. Mosia (2016:39) and Denzin and Lincoln (2005:18) add that postmodernity emerged in 1990 and lasted approximately until 1995, with the aim to solve a triple crisis representation. This moment, in turn, moved to an ontological stance, where the reality is not reflected within a particular context but rather in storytelling.

Postmodernity requires of the researcher to be closer to people participating in the research in order to get a better understanding of participants' narratives. This was achieved through their research, because the people participating in their research were promoted and regarded as co-researchers, rather than research objects. These co-researchers influenced the planning of the research, namely the conceptualisation, data collection, and interpretation phases.

Postmodernity was seen as vital a tool to challenge positivism principles, which trust that scholars are only the ones who can provide the truth about problems facing people in communities. Thus, post-modern researchers are against a scientific way of doing research and they probe a critical question: Who decides what the truth is? This movement rejects the narrative that educated, academic, intelligent, or powerful individuals are the only ones who can decide what the truth is. Mahlomaholo (2012:04; 2013:379) affirms that bricoleurs are not concerned with the truth, as the truth cannot be told by one researcher, because the truth is constructed from diverse environments.

This study also believes in the notion of a multifaceted epistemology, where people can construct their knowledge from diverse environments, namely politics, economics, education, and social and cultural environments to make sense of various issues. According to Mosia (2016:40), co-researchers should be encouraged to work together to find common solutions to the surrounding problems; thus advocated as appropriate theorisation. This study further agrees with Malebese (2016:161), and Terry and Irving (2010:110-111), who believe in teamwork and cooperation, because people bring multiple methods to solve problems and enable interaction among people. This study used a multi-layered and multi-methodological approach to generate data and solutions to teach MLP, because co-researchers and students participated actively in the research. This study also agreed that there is an epistemological shift to the idea that knowledge can be produced and reported via multiple genres, thus moving away from the post-modern phase to post-experimental research.

2.2.2.6 Post-experimentalism

According to Given (2008:311), post-experimental inquiry took place approximately between 1995 to 2000, which brought back a strong wave of blurring between the genres of the social sciences and the humanities. This moment placed research participants' experiences and knowledge at the centre of the research processes,

whereby people present themselves in way that they are comfortable (Mahlomaholo 2014:175). The literature explains post-experimental investigation as the concern for storytelling and composing ethnographies in new research methods, which gives participants in research a voice. Mosia (2016:40) affirms that post-experimental inquiry strongly anticipated to empower oppressed societies. Researchers in this era saw a need for a social-scientific type of research that promotes rigour and praxis, because the epistemic ideology of the traditional era was that a researcher had all the knowledge needed for knowledge production through research. However, in post-experimental inquiry, the dogmas of the tradition era are rejected (Van Dijk 2006:160-165). Post-experimental inquiry provides methodological tools to promote collaboration work between co-researchers.

This study used post-experimental inquiry to generate data through the collaboration among us as a team. During the research process, all co-researchers were treated fairly and afforded the time to share their teaching and learning experiences in MLP at this TVET college. Then, we worked together as a team to use the RME approach to teach MLP. Focus-group discussions, free interviews, and lesson presentations and observations took place. To capture the collected data, audio recordings, cameras and other tools were used, but the fluid borrowing of tools led to problems relating to evaluation, which paved the way for methodologically contested representation.

2.2.2.7 Methodologically contested representation

Given (2008:56) indicates that methodologically contested representation took place between 2000 and 2004. During the methodological contested representation period, criteria for evaluating important qualitative research were not only based on validity and reliability, but focused more on values, aesthetics, morals, epistemologies and ethics (Mosia, 2016:41; Denzin, 2001:326). Denzin (2001:326) maintains that in the traditional era, research participants' knowledge were powerless, because those who had power decided what was aesthetically pleasing and ethically acceptable.

In this study, we as a research team decided together on the agenda in the research meetings. This transformative agenda promoted the equality, diversity, inclusivity and transformation in the research process, because bricoleurs challenge positivism rules that ignore co-researchers' experiences and background stories.

2.2.2.8 *Current fractured futures*

The eighth moment of Bricolage is the current fractured future, beginning in 2004 (Mosia, 2016:42). During this phase, researchers were not regarded only as people who can use their intelligence to generate solutions to problems facing communities. The views of co-researchers were now recognised and pluralism of methodologies and theories were used to generate data. Clarke *et al.* (2015:182) argue that pluralism or diverse methods of conducting research discourage scholars to use a single method or principle in trying to make sense of a diverse social world. Thus, the significance of using a combination of different methods, multi-methodological research approaches, and multi-theoretical frameworks provides multiple solutions.

2.2.3 **Objectives of bricolage**

Bricoleurs use Bricolage by integrating available tools to solve unexpected problems. The context of Bricolage indicates that bricoleurs do not have the time to look for tools to solve problems; they use what they have at their disposal. Le Loarne (2010:02) views bricoleurs as creative people because they use whatever resources available to respond to unplanned problems. They do not collect tools that are meant for a particular problem. The DHET annual reports reveal that most students do not perform well in MLP and in mathematics in general. This study adopted Bricolage as a framework because Bricolage allows lecturers to use whatever they have available (teaching tools and people's diverse knowledge) to improve mathematics teaching and learning. This can also include cultural background, indigenous games, and different strategies to enhance the teaching of a problematic MLP topic. The annual diagnostic reports and various studies reveal that students have the following difficulties in solving MLP problems, namely formulating linear inequalities from a word problem; solving and sketching linear inequalities; showing feasible area; and determining a maximum and minimum value using a search line and substitution method. Therefore, the Bricolage in this study was aimed to diagnose apparent problems that hinder MLP teaching and learning in order to use available resources to respond to those challenges.

2.2.4 **Formats of bricolage**

The evolution of the Bricolage allowed multiple methods that created the density and multiple epistemologies and ontologies (cf. 2.2.3). Kincheloe (2005:335-336) indicates that there is no final or fixed description for bricoleurs, because they come with multiple methods to solve problems. Bricolage is structured in five dimensions: methodological, theoretical, interpretive, political, and narrative (Mosia, 2016:42).

According to Rogers (2012:05), methodological Bricolage is a process of using multiple research methods to generate data when conducting an investigation. Denzin and Lincoln (1999:3) and Berry (2004a:125) share that scholars use qualitative, quantitative, mixed, and Participatory Action Research (PAR) methods in the form of interviews, questionnaires, rhetorical, social, textual, discursive and semiotic analysis of symbols, phenomenological analysis of consciousness and inter-subjectivity, and psychoanalytical methods to gather and analyse data. Thus, adopting Bricolage allowed us as bricoleurs to use multiple methods and tools to generate data to solve MLP teaching and learning challenges. Consequently, the co-researchers were flexible and not bound to use one predetermined pedagogical method that is perceived to be panacea to all challenges in the teaching and learning of MLP. The multiple Bricolage moments provided adequate information about MLP teaching and learning challenges, which put us in a better position to understand the complexity of MLP teaching and learning. According to Denzin and Lincoln (1994:3), theoretical Bricolage began in blurred genres, because during that period, theoretical Bricolage acknowledged knowledge and experiences from social and cultural theories to understand challenges in the teaching and learning of MLP. It helped us as bricoleurs to develop a way to enhance the teaching and learning of MLP.

Interpretive Bricolage helps researchers to produce a variety of interpretive methods to interpret generated data from participants (Rogers, 2012:12). Rogers emphasizes that in this context, a bricoleur's duty is to locate his location in the web of reality in relation to intersecting axes of personal history, autobiography, socioeconomic class, sexual orientation, ethnicity, religion, race, gender, geographical place, and numerous other dynamics. These various views are used to determine the role of individuals in the interpretive process. The interpretive process allows people to bring knowledge from diverse environments to expand a particular topic; therefore it is viewed in relation to social, cultural, political, economic, psychological, and educational structures, as well as socio-theoretic stances. Denzin and Lincoln (2005:04) add that interpretive

bricoleurs believe that there is no single correct method to tell, because an individual person's telling is a reflection of his/her perspective. This study examined the MLP teaching and learning challenges through an interpretive lens. Secondly, it used the RME approach to enhance the teaching and learning of MLP. Thirdly, it identified conducive conditions for the RME approach to succeed. Fourthly, it also investigated factors that are threats to the successful implementation of the RME approach; and lastly, it identified the indicators of success of the RME approach.

Narrative Bricolage accepts and acknowledges that generated data and findings of conducted research are shaped by narratives shared by co-researchers about their background and experiences. Such stories are not innocently constructed, but rather reflect particular narratological traditions (Mosia, 2016:44). Knowledge constructed by bricoleurs from a narrative point of view allows a greater degree of insight into the forces that shape the nature of knowledge production. In the blurred genres era it was understood that complex and sophisticated research emerged from the historical background of Bricolage. The narrative bricoleur's perspective enabled the research team to allow a representation of multiple voices, perspectives and sources with respect to MLP teaching and learning challenges that contribute to poor performance.

Political Bricolage helps researchers to accept and learn that research processes hold political implications for manifestations of power. Bricoleurs study, analyse, and make sense of generated data; thus, the knowledge they analyse and interpret reveals unspoken forms of power. In light of such awareness, bricoleurs try to show the effect of different powers, such as ideological, hegemonic, discursive, disciplinary, regulatory and coercive power. Rogers (2012:6) indicates that the political bricoleur aims to produce knowledge that benefits all people that have been oppressed or marginalised in the past.

There are multiple theories that agree, support and close gaps that Bricolage could have, namely social theory, cognitive constructivism, critical theory, post-structuralism, phenomenology, ethnomethodology, critical hermeneutics, critical construction theory, and post-modernism.

The sociocultural theory grew from the work of seminal psychologist Lev Vygotsky, who believed that society contributed to individual development; learning takes place in social interaction that permits sharing of knowledge and skills inherited from

peoples' background knowledge and experiences (Nutchey *et al.*, 2016:373). Thompson and Boro (2015:01) add that knowledge constructed through social interaction among people and is suitable stems from sociocultural practices, beliefs and experiences from diverse communities. This study believes that all people are important and can bring valuable knowledge to the research study if they are given the opportunity to air their voices and share their individual knowledge. We as a research team had fruitful discussions during our meetings, and it is on that platform that we shared and learned about challenges in the teaching and learning of MLP. We shared best practices and planned together to implement the use the RME approach to enhance teaching and learning of MLP at this TVET college, because the socio-cultural theory aided us as bricoleurs to find multiple methods to solve teaching and learning challenges of MLP that contribute to poor performance in MLP.

The cognitive constructivist approach is based on research by the Swiss psychologist Jean Piaget, who fundamentally focused on the cognitive development. Arsalidou and Pascual-Leone (2016:1) signpost that the knowledge of human beings is constructed through their daily experiences; not from the information deposited into their mind. The literature shows that in the traditional era of Bricolage the research conducted in the past was mainly conducted in a positivist manner, whereby people's experiences were not considered in the research process, especially if they did not have formal education. However, the cognitive constructivist approach educates us as researchers to consider people's background knowledge and experiences when conducting research, because the ontological stance of this approach is the fact that people have valuable knowledge that can assist them to solve problems. The cognitive constructivism approach was accepted during the blurred genre era, allowing co-researchers to share their daily teaching and learning, and their personal experiences in generating data to achieve the research objectives and the purpose of this study with success. Theorists and scholars are of the view that the teaching and learning process is anchored through people actively constructing their knowledge, and who rely heavily on what they already know from their physical and social world. Actualising people's prior experience enables them to detect patterns and trends from their new experiences for the purpose of understanding the world around. Therefore, the constructivism method assisted this study to generate multiple solutions to respond to teaching and learning challenges of MLP topic.

2.2.5 Epistemology and ontology of bricolage

2.2.5.1 Epistemology of Bricolage

Schraw (2012:1-2) explains epistemology as a study of beliefs about the origin and acquisition of knowledge, while McNulty (2013:525) explains epistemology as being concerned about the origin, nature, limits, ways and explanation of human knowledge. Mertens and Yamashita (2010:48) signpost that the belief that all people construct their thinking by means of various approaches and background knowledge refers to the epistemic stance. Thus, to teach MLP, lecturers should be aware that for students to make meaning, they use multiple ways and background knowledge to solve MLP problems. Tennis (2008:102-103) and Audi (1999:5-7) justify epistemology as how people know. What justifies us to believe what we do? What standards of evidence should we use to pursue the truth about the world and human experience? Therefore, this confirms that in knowledge construction, we make implicit epistemic statements about knowledge of concepts, acts, entities, and systems. Tennis (2008:103) further states that an individual epistemic stance decides on a knowledge to be constructed, because each epistemic stance makes claims about what kind of knowledge to create, how it is gathered, presented and analysed in the research. According to Mertens and Wilson (2012:169-170), an epistemic stance assists us to construct knowledge. Thus, to enhance MLP teaching and learning at this TVET college, the co-researchers believed that students create knowledge (or meaning) by means of their background knowledge from diverse environments: cultural, political, economic, and social.

2.2.5.2 Ontology of Bricolage

According to Malebese (2016:36-37), reality is determined by diverse values and life experiences that participants bring to the research study. Mertens and Wilson (2012:172) add that people are shaped by diverse environments, namely social, political, cultural, economic, disability values and ethnicity, and these environments assist them in defining reality. Therefore this study relies heavily on the communication and interaction between co-researchers in order to uncover the truth and to find MLP teaching and learning challenges that students and lecturers faced on a daily basis. , this study views communication as an important tool to be utilised to bring different people from diverse contexts together. As a result, the communication tool enabled us as a research team to share our teaching and learning experiences regarding MLP.

The ontological stance of this study is that there is a wealth of knowledge inside and/or outside school communities that is not recognised and therefore not used in the teaching and learning of MLP. Denzin and Lincoln (2011:711) indicate that ontology refers to the study of what there is in the world and this has helped us as research team (bricoleurs) to use what is available in the world (tools and knowledge) to enhance MLP teaching and learning. Audi (1999:5-7) and Schraw (2012:2-3) agree that ontology refers to the nature of reality, also indicating that typical ontological questions include: What is reality? and What can be known about reality? So, we as a research team comprehended and made sense of those questions, which assisted us to become aware that students use their epistemic stance to make sense of MLP information in solving problems. Barry (2012:18) agrees with other scholars who are of the view that ontology refers to the nature of reality and belief that the nature of reality is socially constructed through social interaction and cultural conditions under which we live, and that learning shapes our knowledge and the way we view the world.

During the Bricolage traditional period, lecturers only relied on and used traditional methods to teach MLP and students were encouraged to follow certain rules, procedures and techniques to MLP problem solving. , during this period lecturers were believed to be very powerful people who knew everything (ontological stance) and students were expected to take note of everything their lecturers shared. Thus, the methods during this traditional era were influential and important in mathematics teaching and learning. However, this method was not successful, because apart from the success of the traditional era, there were some limitations which paved the way for other Bricolage moments. The epistemological position of this study is that knowledge and learning are socially constructed, because people (including students) employ multiple methods and background knowledge from cultural, historical, economic, social, political, educational dynamics and psychology contexts to make sense of knowledge (meaning). This study's ontological position and ideology are that if students' background knowledge is integrated into MLP lessons, students can easily understand and make sense of MLP teaching and learning. Therefore, lecturers must involve students in the teaching and learning process to link what students already know about MLP. According to Le Loarne (1962:2), bricoleurs implement and use available resources to respond to a crisis; therefore this study encouraged us as a research team to use funds of knowledge that are available from communities,

societies and culture to enhance MLP teaching and learning. In addition, the ontological stance of this study is that there is no one truth or reality, as evidenced in the blurred genres era (cf. 2.2.2.3) and it epistemises that the truth is interpreted by people from diverse environments. Thus, this ideology challenges and rejects the ontology in the traditional period, which indicates that there is only one method to find the truth and one pedagogic method to teach MLP.

2.2.6 THE ROLE OF THE PRINCIPAL RESEARCHER

The role of myself as principal investigator in this study was to select co-researchers, based on their mathematics teaching and learning experience, as well as valuable knowledge from diverse environments, namely educational, historical, social, economic, social etc. Their valuable knowledge assisted this study to achieve its research objectives (cf. 3.6.2 & 4.2-4.5). The reason for convening a team of co-researchers was the fact that as the principal researcher I did not have solutions for all challenges in the teaching and learning of MLP encountered by lecturers and students at this specific TVET college. Therefore, I encouraged people to participate in this research study, because this study aimed to enhance MLP teaching and learning and improve mathematics performance at this TVET college. This study changed people's minds when viewing education research. Kincheloe (2011:220) observes that people believe that education research has little to offer to people to solve their challenges. Moloi (2013b:480-483) adds that people do have wrong narratives about education research. For instance, some people believe that education research is just research and have nothing to offer to people's lives. Secondly, other people feel that scholars just come and steal their knowledge, because scholars do not come back with recommendations that can help them to respond to their problems. Lastly, my role as the principal investigator was to contribute to mathematics knowledge and work collectively with co-researchers to respond to the research question as well as achieve the research objectives of this study.

2.2.7 The relationship between the researcher and the co-researchers

This study relied heavily on the co-researchers' knowledge and experience shared in the research process. The team of co-researchers consisted of a council member, a mathematics coordinator, CM, HoDs, supervisors, lecturers and students, as well as

a former mathematics FET teacher, a farmworker, and a businessperson as parents. Co-researchers were chosen based on their valuable knowledge and teaching and learning experiences of mathematics. Their skills and knowledge assisted this study to achieve its research objectives. The ontological stance of Bricolage is that people are human beings, not an objects, because they have valuable knowledge, skills and teaching experience to be used to develop multiple pedagogical methods to enhance MLP teaching and learning. The Bricolage epistemological stance is that an individual person uses the knowledge, skills and teaching experiences available to make sense (meaning) of the Mathematical Linear Programming Content Knowledge (MLPCK) or any new knowledge. Therefore, Bricolage ontological and epistemology aided this study to achieve its objectives through the use of PAR as a methodology.

According to Mosia (2016:46), PAR has been developed with the purpose of promoting equality and empowering societies through the research process. I, together with my co-researchers in this study, decided on a suitable venue, suitable days, and suitable times for meetings, lesson observations, lesson presentations, and free interviews for this research study. , we as a team decided together on the agenda for meetings, lesson observations, lesson presentations, and free interviews. Thereafter, we shared roles and responsibilities, namely lesson presenters, lesson observers, chairpersons, secretaries, facilitators, photo capturers, and recorders to enhance, observe, chair and capture this study research proceedings (cf. 3.8 & 4.2-4.6). All team members were given an equal platform to voice their opinions, share their knowledge, skills and teaching and learning experiences to map a way to enhance MLP teaching and learning, as well as to solve challenges in mathematics teaching and learning.

Becker and Raveloson (2008:4) state that democracy addresses the imbalance of the past where some people were not treated fairly and equally on the basis of their skin colour, race and gender; in essence people were marginalised. During the traditional era of Bricolage, other groups of people were also marginalised in the research due to their lack of formal education; they were viewed as people who did not know (cf. 2.2.2.1). Therefore, this study endeavoured to address the limitations of the traditional period. We used democracy doctrines, PAR doctrines, Vygotsky's theorem and multiple theories and practices, which believe and acknowledge that knowledge is socially constructed and people have personal valuable knowledge, experiences and skills to solve their problems, if and only if they are provided with a space in the

research, regardless of their education level. The co-researchers of this study comprised different people, who were all allowed to share their knowledge and interact with one another in their preferred language (cf. 3.6.2 & 4.2-4.6). This study rejected the positivists' way to conduct research where the principal investigator works alone to find solutions to current issues concerning people.

Additionally, Moloi (2013b:480) agrees with other scholars on the fact that people must be included and consulted when data are generated in order for them to share the challenges they face on a daily basis, in this instance, MLP teaching and learning challenges. In addition, the ontological position of this study is that a research team brings unique skills, expertise, and knowledge that can help this study to find multiple pedagogies and strategies to enhance MLP teaching and learning.

2.2.8 RHETORIC

According to Mosia (2016:46), Bricolage refers to tools that are used to respond to specific problems. Tilley (1991:01) indicates that bricoleurs perform the process of Bricolage and are entrusted to use their creativity to respond to problems. This study aimed to involve people to generate strategies that can be used to solve MLP teaching and learning challenges. Section 1.1 and 2.4.2.1 of this study revealed that MLP is a problematic topic, which leads to poor performance in mathematics. Therefore, this study used Bricolage and PAR to generate data to enhance MLP teaching and learning through focus-group discussions, lesson presentations, lesson observations and free interviews during research meetings. More importantly, co-researchers' interactions were key to making this possible. They were expected to respect one another; acknowledge other people's differences; listen to one another; and share their knowledge, skills and experiences in their preferred language. Language played a very big role. Malebese (2016:107) signposts that research that involves human beings should take the issue of languages into consideration because the languages used in PAR have the power to empower marginalised people. This study's ontological stance is that all people can be empowered if, and only if, their indigenous languages are recognised. Freire (1970:75) indicates that researchers pose problems that come from students' lives, social issues and academic subjects in a mutually created dialogue using PAR as methodology. Thus, the Bricolage and PAR in this study do not

encourage discriminatory language under any circumstances, but accept all co-researchers with their diverse languages in the research process.

Among co-researchers we had people who understood diverse languages. They assisted us to understand discussions and generated data were shared in indigenous languages. This study employed the Critical Discourse Analysis (CDA) on three levels, namely, textual (written and spoken words in diverse languages), social and discursive in analysing generated data from co-researchers experiences, skills, and living conditions (Sisimwo, Rop & Ahmed, 2014:22). This analysis and reflection accommodate diversity of all co-researchers, irrespective of their background (Silva, Almeida & Martins, 2010:31). This study acknowledges the co-researchers' uniqueness and their different background knowledge in the research process that is meaningful in generating data and solutions for challenges in the MLP teaching and learning. Co-researchers as bricoleurs in this study interacted and shared their experiences, knowledge, practices and skills in their preferred language during research meetings. This happened in a polite way and, most importantly, respect and Ubuntu were promoted and practised.

2.2.9 RELEVANCE OF BRICOLAGE TO THIS STUDY

Bricolage refers to creating something new with available resources (cf. 2.2.1). Le Loarne (2010:02) regards bricoleurs as creative individuals who possess the ability to convert material to respond to the current problem in the process of Bricolage. Lennerfors and Rehn (2014:377-381) argue that Bricolage was created to help small businesses grow by utilising available resources to solve apparent problems that might rise. Nakhanu and Musasia (2015:68-72) signpost that there is a serious crisis in MLP teaching and learning, which results in poor mathematics performance. Urgent response is needed from co-researchers as bricoleurs, who uses resources available from diverse context, such as culture, theories, and real-life situations to enhance MLP teaching and learning using the RME approach at this TVET college. In this section, this study provided an overview of the appropriateness of Bricolage by referring to its research objectives. Bricolage was adopted as lens guiding this study for strengthening MLP teaching and learning using multiple methods to address the challenges of teaching and learning of MLP. This study encouraged co-researchers as bricoleurs to collect and analyse resources, knowledge, skills, practices and

experiences available on diverse contexts, and then use such resources to be creative and to strengthen MLP teaching and learning. In addition, the co-researchers (bricoleurs) decolonised education using the RME approach to teach MLP, in order for students to make meaning (sense) of their learning.

2.3 OPERATIONAL CONCEPTS

This section have discussed the operational concepts in depth to provide a reader with a good understanding with regard to key concepts of this study.

2.3.1 Mathematical Linear Programming

The literature and DHET annual diagnostic reports revealed that students do not perform well in mathematics as a critical subject (cf. 1.1 & 2.4.2.1). The South African government have changed the curriculum several times now with the aim to provide skills and knowledge that are in high demand in different sectors around the world. Therefore, the revised curriculum placed great emphasis on establishing proper connections between mathematics as a discipline and its application in real-world contexts. The objective of the mathematics curriculum is to address and assist people with generic knowledge of problem-solving skills. Nakhanu and Musasia (2015:68) share that mathematics content is key, because it include topics such as analytical geometry, algebra, financial, statistics, and MLP, which are relevant in our daily lives. However, in this section, MLP will be explained in depth, to give the reader a good understanding thereof. According to Nkambule (2013:46-47), MLP was introduced into the South African curriculum in the mid-eighties, meant for students in Grade 11 and 12 taking mathematics on the higher-grade level. However, in 2007 a new curriculum was implemented, the National Curriculum Statement (NCS) where students were no longer doing mathematics at a higher or standard grade level; MLP was henceforth offered to all students who did mathematics from Grade 11 level.

Mathematical Linear Programming refers to a mathematical model technique or tool to help us make decisions in diverse contexts and sectors, such as farming, manufacturing, industrial engineering, social, economic, technological and physical sciences. It also helps us to maximise profit and minimise costs with limited resources available (Daniel & Solomon, 2011:178). The MLP was developed by George Dantzig

who helped his colleagues to find a method that rapidly inserted time-staged operation, training and a logistical supply programme, according to Nkambule (2009:6-11). Dantzig was a mathematical advisor to the United State (US) Air Force Controller in the Pentagon and an expert in planning methods using a desk calculator. He was motivated and influenced by Leontief's work who initiated a large, but simple matrix structure named the inter-industry input-output for the American economy (Nkambule, 2009:6-11). In Leontief's model there was a one-on-one correspondence between the production process and products produced by these processes, which led to a situation where Dantzig used Leontiefs' ideas regarding the steadily economic growth and game theory to derive a dynamic model that can change over time.

Dantzig realised that the Air Force needed a tool with alternate activities; hence, he came with the simple method of MLP. The simple method is described as change linear programme with a staircase matrix structure. This method also generates a sequence of feasible iterates by repeatedly moving from one vertex of the feasible region set to an adjacent vertex with a lower value of objective function. When it is not possible to find an adjoining vertex with a lower value, the vertex must be optimal and termination occurs. Therefore, MLP is about using the simplex method to formulate real-world tasks in mathematical terms, nowadays solved by employing computers and software. The standard form of describing an MLP task consists of the following: linear function to be maximized or minimized, e.g. $2x + 3y \leq 30$; subject to linear constraints, e.g. $x + 6y \geq 24$ and $5x + 8y \leq 40$; and non-negative values, $x \geq 0$ and $y \geq 0$ (Nkambule, 2013:47). According to the South African National curriculum statement (DoE, 2003:31), MLP is introduced in Grade11 and students continue to study MLP in Grade 12. This is the same in the TVET college curriculum (assessment guideline mathematics NCV Level 3 & 4), because MLP is introduced in mathematics NCV L3, and then continued in mathematics NCV L4. Table 2.1 below indicates the curriculum statement indicating what needs to be covered when teaching MLP, as well as its assessment standards, while Table 2.2 shows the MLP content to be covered in mathematics NCV L4.

Proposed MLP content Grade 11	Assessment standards
The learner must be able to optimize a function in two variables subject to one or more linear constraints, by the numerical search along the boundary of the feasible region	We know this when the learner is able to: Solve linear programming tasks by optimizing a function in two variables, subject to one or more linear constraints, by numerical search along the boundary of the feasible region.
The learner solves a system of linear equations to find the coordinates of the vertices of the feasible region.	Solve a system of linear equations to find the coordinates of the vertices of the feasible region (DoE, 2003).

Table 2.1: MLP content under NCS curriculum

Subject content	
Assessment standards	Learning outcome
Linear constraints found and formulated from a given problem. Linear programming problems are solved by optimising a function in two variables, subject to one or more-linear constraints, using search line method. Method: Sketch the functions/constraints Determine and shade the feasible area Work out the gradient of the search line Use the search line to optimise the max or min from the objective function	Find and formulate linear constraints from a given problem Solve linear programming problems by optimising a function in two variables, subject to one or more linear constraints, using search line method

Table 2.2: MLP content under mathematics NCV L4 curriculum taken from assessment guideline mathematics NCV L4 (2013:15)

Both tables show that the mathematics curriculum needs students to learn mathematics within the context of real-life situations, because Mathematical Linear Programming Content Knowledge (MLPCK) is organised to develop students to become problem solvers. Dantzig (1963:20-21) and other scholars share the same sentiments by saying students should grasp and understand the following MLP terminologies, namely optimise, linear constraints and feasible region in order to solve MLP tasks and be in position to use MLP in their respective organisations and daily lives. Therefore, to solve MLP problem statement, individual students need to translate MLP word problems into symbolic mathematical language (linear inequalities), apply the necessary procedures to solve and sketch linear inequalities, and thereafter show and find coordinates for the vertices of feasible region. The following mathematics

concepts are a prerequisite before students move to learn how to solve MLP tasks, namely sketching of straight line graphs by using the table method and the intercept method; determining linear equations of straight line graphs; solving linear inequalities; $Z = (\dots; -3; -2; -1; 0; 1; 2; 3; \dots)$; R = Real numbers (Rational and Irrational numbers); number line concept; and mathematical word problems. Below is an example of a problem statement for MLP (Daniel & Solomon, 2011:171, 85):

A small factory makes two types of washing machines, a standard and a luxury model. The daily production cannot be more than 6 standard and 3 luxury washing machines. There are 8 factory workers and it takes one worker to make one standard model and 2 workers to make one luxury model per day. The profit on the standard model is R300 and on the luxury model R600.

Questions:

Write down the constraint inequalities **(06)**

How many of each type of machine must be manufactured per day to make a maximum profit? **(08)**

What is the maximum profit? **(06)**

Table 2.3: MLP problem statement example taken from mathematics NCV L3 textbook

Solution

Step 1

Let the standard model be x and the luxury model y :

Constraints are:

$$x \leq 6;$$

$$y \leq 3;$$

$$x + 2y \leq 8;$$

$$x \geq 0;$$

$$y \geq 0; x, y \in Z$$

Objective function is: $P = 300x + 600y$

Graph

First sketch on the Cartesian plane the graph: $x + 2y \leq 8$, firstly,

Let x be zero (0)

Substitute x with zero in $x + 2y = 8$

$$0 + 2y = 8$$

$$2y = 8$$

$$y = 4$$

Secondly, let $y = 0$

Substitute y with zero in $x + 2y = 8$

$$x + 2(0) = 8$$

$$x = 8$$

On the same Cartesian plane sketch line $x = 6$ and $y = 3$, indicate the constraints of the equations on the graph.

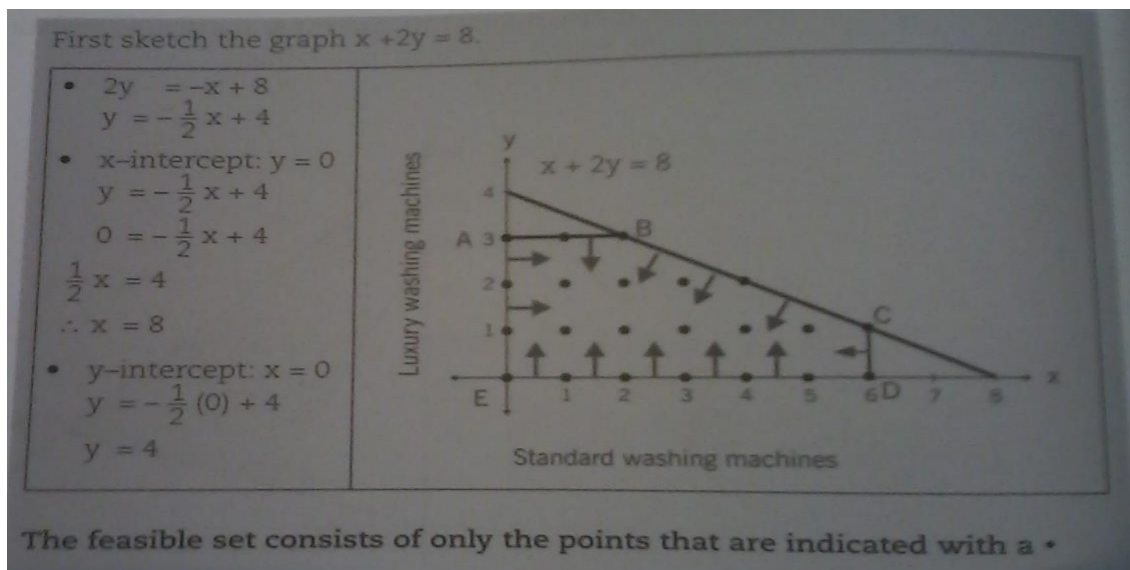


Figure 2.1: Mathematical linear programming graph (Adapted from mathematics for NCV level 3 textbook)

Step 2

Maximise $P = 300x + 600y$ subject to the constraints

$$x \leq 6$$

$$y \leq 3$$

$$x + 2y \leq 8$$

$$x \geq 0$$

$$y \geq 0$$

A numerical search is done as follows:

Determine the coordinates of the vertices of the feasible region by solving the linear equations simultaneously or read the coordinates from the graph.

Substitute each pair of coordinates into the objective function.

Find the pair that will give the optimal values.

This is called the vertex method.

Substitute A (0;3); X= 0; Y=3	Substitute B (2;3); X= 2; Y=3	Substitute C (6;1); X= 6; Y=1	Substitute D(6;0); X= 6; Y=0	Substitute E (0;0); X= 0; Y=0
$P = 300x + 600y$ $P = 300(0) + 600(3) = R1\ 800$	$P = 300x + 600y$ $P = 300(2) + 600(3) = R\ 2\ 400$	$P = 300x + 600y$ $P = 300(6) + 600(1) = R2\ 400$	$P = 300x + 600y$ $P = 300(6) + 600(0) = R\ 1\ 800$	$P = 300x + 600y$ $P = 300(0) + 600(0) = R\ 0$

Table 2.4: application of substitution method

Complete the table below:

Vertices	Coordinates	Profit (P) in rand
A	(0; 3)	R1 800
B	(2; 3)	R 2 400
C	(6; 1)	R 2 400
D	(6; 0)	R 1 800
E	(0; 0)	0

Table 2.5: minimum and maximum values

Step 3

Use the information on the table above and answer the questions:

It is clear from the table that the maximum profit is R2 400.

This is obtained by manufacturing 2 standard washing machines and 3 luxury washing machines per day or 6 standard washing machines and 1 luxury washing machine per day.

2.3.2 Realistic Mathematics Education

The Realistic Mathematics Education (RME) refers to a domain specific instruction or pedagogical theory for mathematics, which has been developed in the Netherlands (Van den Heuvel-Panhuizen & Drijvers, 2014:521). Vann den Heuvel-Panhuizen (2003:10) agree with other scholars when stating that one of the most vital characteristics of the RME is that the rich situations (realities) are given a prominent position in mathematics teaching and learning process. Therefore, real-world situations serve as a source for initiating development of mathematical concepts, tools, and procedures and as a context in which students can at later stage apply their mathematical knowledge and skills to solve problems, which then gradually has become more formal and general and less context specific. The co-researchers

(bricoleurs) in this study used multiple principles and resources; secondly, incorporated realities; and lastly, gave students MLP activities and problems that talk to real-life situations, either real or fictional. Ozkaya and Yetim Karaca (2017:185) indicate that the word *realistic* refers to something you can imagine in a Dutch expression; as a result, it helps students to make sense of abstract mathematics concepts. In light of the above information with regard to the RME approach, it is clear that it could be a solution to decolonise mathematics and abstract mathematics teaching education through the integration of realities, real-life stories, and indigenous knowledge in the teaching and learning of MLP, in order for students to make sense of their learning.

This study explains realities as events, practices and knowledge from diverse milieus that students can easily recognise, imagine, and see around the world. Van den Heuvel-Panhuizen and Drijvers (2014:521) add that the RME method was first introduced and developed by three mathematics didacticians, Wijdeveld, Goffree and Treffers at the Wiskobas project in 1968. In 1971, the Wiskobas project became part of the newly established IOWO institute, with Hans Freudenthal as first director and in 1973 when the IOWO was expanded with the Wiskivon project for secondary mathematics education. This basis received a decisive impulse to reform the prevailing method regarding mathematics education. In the 1960s, mathematics education in Netherlands was dominated by a mechanistic pedagogical approach, where mathematics was taught directly at a formal level, in an atomised manner. Mathematical content was derived from the structure of mathematics as a scientific disciplines, students learned procedures step by step showed by the mathematics lecturer how to solve mathematics problem solving. This led to an inflexible and reproduction-based generation (Zaranis, 2017:47). However, Hans Freudenthal supported the transformation in teaching mathematics subject.

Hans Freudenthal (1905-1990) was born in Germany. He became a professor of pure and applied and foundations of Mathematics at Utrecht University in the Netherlands in 1946. As a mathematician, he made substantial contributions to the domains of geometry and topology, and later in his career he became interested in mathematics education. Hans Freudenthal characterises a dominant approach to mathematics education in which scientific structured curricula were used to teach students while students were confronted with ready-made mathematics as an anti-didactic inversion.

The development of the RME approach was aimed to close limitations of mechanistic approach. One of the principles of RME requires of students to be active people in the teaching and learning process, whereby they develop mathematical tools and insights on their own, because Feudenthal regarded mathematics as a human activity and also argued that mathematics should not be learned as a closed system, but rather as an activity. Lastly, according to Van den Heuvel-Panhuizen and Drijvers (2003:12), Freudenthal (1991) adopted Treffers' (1987) perspective to explain the RME approach in two ways of mathematizing mathematics, which is horizontal and vertical mathematization. So, in horizontal mathematization, students would use mathematical tools to organise and solve problems situated in the real world by analysing MLP problem statements to formulate linear inequalities and objective functions, while vertical mathematization was taken as a process of reorganization (grouping like terms) within mathematical systems, resulting in shortcuts by using connections between concepts and strategies.

Treffers (1978 in Van den Heuvel-Panhuizen & Drijvers, 2014:522-523) reveal core principles of RME that guide instructors who view the RME approach as an essential pedagogical method in mathematics teaching and learning, such as activity, reality, levels, intertwined, interactivity, and guidance. The RME doctrines provided co-researchers with a good understanding about the Realistic Mathematics Education (RME) approach. This study regarded this instructional way as a significant tool to decolonise the teaching of mathematics in the world, because the RME method cannot be isolated from the worldwide reform movement in mathematics, which began decades ago. According to Freudenthal (1971:1973), activity is a principle of the RME approach and refers to work or activities prepared for students they are expected to complete in the teaching and learning process. This principle encourages students to be active participants in the teaching and learning. Therefore, instructors of the RME approach are expected to treat students as active participants in the teaching and learning of MLP by preparing a variety of activities and allow students to work on their own to measure themselves and discover a new knowledge.

The ontological position of this study is that MLP is best learned when students work on multiple activities themselves. Therefore, this principle helped this study to achieve its research objectives (cf. 4.3) and also helped students to understand that MLP is a human activity. The second principle of the RME approach is reality (reinvention rule)

and refers to real-life situations (realities) and background knowledge which is supposed to be incorporated in teaching of MLP and learning and also into students' class activities (Kwon, 2002:4). This principle aims to help students to make sense of abstract mathematics concepts. Thus, lecturers should be cognisant that mathematics do not end in the classroom, because at the end of each lesson students are expected to use what they have learned in mathematics classes (knowledge and skills) to solve their real-life problems, which is the call of people who advocate transformation, change and decolonisation of education (including abstract mathematics concepts).

The reality principle of RME approach assisted us as a research team in this study to concretise the abstractness of the MLP components and problematic MLP concepts (cf. 4.3), because we began MLP teaching from problem situations that students can make sense of. This offered students opportunities to interpret and gain MLP knowledge. Freudenthal (1973:77) explains the word *reality* as a framework to which mathematics attaches itself and contended that, if students learn mathematics in an isolated fashion, divorced from reality, it will easily be forgotten and they will not be able to apply it. Furthermore, he emphasises the teaching of mathematics using the RME approach. The teaching starts with problems in rich contexts that need mathematical organisation, rather than starting a lesson with definitions and procedures (rules) to be used and teaching mathematics in the abstract way. In other words, the lecturer actualises informal context-related solution strategies as a first step in the learning process. This context serves not only as a source for developing mathematics, but also as an area of application as well. Therefore to provide context-related strategies, the context can also elicit a shortcut; so, to teach mathematics in a realistic context, students should be given unsolvable problems and problems that can be solved in a variety of methods.

According to Gravemeijer and Doorman (1999, in Kwon, 2002:2), the third principle of the RME is instructional design or levels. This principle requires lecturers to prepare a lesson in such a way that students move from one level to the other, for instance, from informal to formal knowledge. The notion is that for students to understand and learn mathematics effectively, they have to graduate from various levels of understanding, which cycle from informal and context-related to formal mathematics. Levels can help students to understand different levels of shortcuts, schematization and models to acquire multiple strategies to solve mathematics problems. Most importantly, students

get an insight into how mathematics concepts and strategies are related. Therefore, the significance of levels is that mathematizing activity on the lower level at a later stage becomes a tool to solve the high level. In other words, the students can conduct all sorts of operations involving mathematics on an informal level and then later formalise them on next levels.

This principle helped us a team to enhance the MLP teaching and learning, because we divided the MLP topic in four lessons: teaching the MLP terminologies; formulation of linear inequalities and objective functions from MLP problem statements; sketching linear inequalities, determining a feasible area and calculating maximum and minimum values through search line way or substitution method; and lastly, teaching all MLP components in one lesson at the same time, repeat the whole process, do revision and provide a short test (cf. 4.3).

Treffers (1978, in Van den Heuvel-Panhuizen & Drijvers, 2014:522) signposts that the fourth RME principle is intertwinement, which highlights that mathematical content domains, namely geometry, measurement, data handling, and algebra are related and cannot be considered in the isolation from the mathematics curriculum. Therefore, students should be provided with rich problems that enable them to use mathematical tools, skills and knowledge they have learned in other chapters.

This study discovered that mathematical functions, linear graphs, simplification, and word problems relate to MLP, and therefore should be prerequisite for students to be able to understand MLP and solve its problems. The fifth principle of the RME approach is interactivity (didactical phenomenology), and it requires of lecturers to create a conducive learning setting where interaction takes place among students throughout the teaching and learning process (Freudenthal, 1993, in Kwon, 2002:4). Therefore, this study made interactivity possible, when students were given activities which encouraged them to interact (cf. 4.3) The RME approach in nature promotes social activity, rather than only individual activity in the teaching and learning process. To enhance the teaching and learning of MLP using the RME approach, co-researchers valued whole-class discussions, pairs and groupwork, in order to give students opportunities to share their knowledge, skills and techniques. Piaget (1990 in Pardjono, 2002:168) signposts that students learn best when they are doing activities or working on their own (active) rather than receiving ready-made lessons.

Interaction evokes reflection, which enables students to reach a higher level of understanding. In the past, lecturers believed that if teaching took place meritoriously, students are expected to cough up pre-digested material, because knowledge was deposited into the minds of students.

However, Freudenthal (1978, 1987) challenged that particular rationale by indicating that mathematics should make sense and students should be able apply its principles in their lives. In addition, according to Treffers (1978, in Van den Heuvel-Panhuizen & Drijvers, 2014:523), guidance is another doctrine of the RME approach. In this regard, lecturers are expected to guide students so that they can complete activities. This study viewed this principle as a scaffolding approach that takes place when students work on activities on their own; so, if students are lost, their lecturers provide them with hints and give them space to work on it again; in essence they allow them to make mistakes or even discover things on their own. Thus, lecturers now are facilitators because they are no longer the only source of knowledge. The study in Section 4.3 used baseline and formative assessments as well as multiple activities to let students discover MLP solutions and also make mistakes, in order for them to learn adequately.

2.3.3 Technical and Vocational Education and Training

Technical and Vocational Education and Training (TVET) colleges are colleges that are designed and developed to equip and provide students with skills and knowledge relevant for the workplace. Akhuemonkhan, Raimi and Dada (2014:04) explain TVET as a specialised education meant to empower students through the development of technical skills, human abilities, cognitive understanding, attitudes and work habits in order to prepare them for the world of work and/or position them for self-employment after graduations. The United Nation Educational Scientific and Cultural Organisation (UNESCO) and the International Labour Organisation (ILO) view TVET as a term referring to facets of the educational process involved. Dixon and Hutton (2016:102-104) share that to general education, the study of technologies and related sciences and acquisition of practical skills, attitudes, understanding and knowledge relating to the occupations in different sectors of the economic and social life. The Ministry of Education strongly expressed its commitment to TVET colleges in the National Policy of Education (NPE) due to the prospect of sustainable development, poverty eradication, job creation and actualisation of transformation agenda (Akhuemonkhan

et al., 2014:4). The former Ministry of Higher Education also encouraged people to consider the TVET sector, due to the fact that they serve as instruments for curbing social exclusion, where the cost of higher education is out of reach of the majority, and as an antidote for youth unemployment, where the labour market is saturated.

Agrawal (2013:20-23) upholds that if the TVET sector is well positioned, it could play the multidimensional roles of stimulating economic advance, social growth, improving conventional education, empowerment, wealth creation, poverty decrease, skills enhancement. In a country where recurring incidences of youth restiveness happen, the TVET sector is one of the solutions to assist youth and adults to become self-dependent and self-reliant. On the other hand, for those who work in the corporate world, it can be helpful for them in areas such as skills improvement, mitigation of high job turnover and risk of their skills becoming obsolete (Agrawal, 2013:16-24).

However, the literature shows that most people in communities view the TVET sector as a form of education designed for drop-outs, less intelligent while others view TVETs as low quality education meant for less-privileged and/or second-class citizens. In contrast, the main objective of TVET in the National Policy on Education (2004 in Akhuemonkhan *et al.*, 2014:02) is the inculcation of practical, applied skills, as well as basic scientific knowledge in students for useful living in their societies. Expected outcomes of TVET in National Policy on Education are: providing trained manpower in the applied science and business particularly at craft, advanced craft and technical levels; equipping students with technical and vocational skills essential for agricultural, commercial and economic development; and providing training in needed skills to individuals who will be self-reliant economically. TVET colleges are explained by Oketch (2007:220-227) as a range of learning experiences that are relevant to the world. It may take place in a variety of learning contexts, including educational institutions and the corporate world. Haolader, Foyso and Clement (2016:02) maintain that the TVET sector is designed to develop skills for practising, providing students with skills for practising particular jobs and preparing for entry and re-entry into the world of work. In addition, there are reasons behind the TVET initiative, namely: a cure for youth unemployment; giving youth pre-employment vocational education; taking the youth off the streets; giving hope to academically less able students who are not able to advance through the school system; and providing much-needed middle-level technicians.

In South Africa TVET colleges offer course programmes that are meant to equip students with knowledge and skills that are needed in the world (Terblanche, 2017: 2-3). TVET course programmes were categorised in the National Certificate Vocational (NCV) and National Technical Diploma (NATED) in South Africa. The Department of Higher Education and Training (DHET) named NATED programme as REPORT 191, which is known as an alternative post-school programme for engineering, business and utility studies offered on full-time basis. The examinations at TVET colleges are written in June and November (twice a year) for business and utility studies; however engineering studies is a trimester programme that enables the successful student to accumulate three National qualifications during the course of one year. The National Certificate N1-N3 engineering studies are viewed as a path for a student to become an artisan. , after students have completed N2 of engineering studies, an apprentice is required to complete the institutional modular training. Secondly, after successful completion and work-place experience the apprentice may apply for the Trade Test. Thirdly, after successfully completing N6 course and a 24-month in-service training period spent in environment relevant to the qualification, a student can apply for the National Diploma in Engineering. However, for business and utility studies, the N6 certificate can be converted into a National Diploma if, and only if students have acquired 18 months' relevant practice experience monitored at an approved place of employment. Therefore, for students to pass the required NATED subject, they should at least attain a minimum of 40% and also to sit for an examination students should have 80% attendance (DHET, 2013e:6; 2018a:6).

Table 2.6: TVET college courses for NATED programme (adapted by SA Webhosts, 2018:01)

Engineering studies (N1-N6)	Business studies (N4-N6)
Electrical Engineering (EE)	Business Management (BM)
Mechanical Engineering (ME)	Marketing Management (MM)
Civil Engineering (CE)	Financial Management (FM)
	Management Assistant (MA)
Utility studies (N4-N6)	Public Management (PA)
Hospitality (H)	Human Resources Management (HRM)
Clothing Production (CP)	Educare (E)
Art (A)	

The NCV programme is a new qualification introduced in 2007 offered at TVET colleges on Level 2, 3 and 4 (three levels) of the National Qualifications Framework (NQF) (SA Webhost, 2018:1). The NCV programme is designed to provide both the theory and practice to students. Thus, the practical component of these studies should be offered in a real or practical workplace environment, where students are provided with an opportunity to experience work situations during the period of study. An NCV qualification provides an opportunity to students to enter higher education studies at any university of their choice. However, students should meet the university minimum requirements, so that are accepted at a university of their own choice after they have completed and passed an NCV L4 Certificate. However, students must meet NCV L4 statutory requirements (Council for General and Further Education and Training) for a higher certificate, diploma and/or bachelor's degree. The government rallies behind students and encourages them to start their own businesses to enter into the world of work because these programmes were initiated to respond to scarce opportunities for employment and also the skills and knowledge that are needed in the corporate world. Apart from the benefits of above programmes, the NCV programme consists of seven subjects on L2, L3 and L4. Each level has three fundamental subjects, namely English, Life Orientation, Mathematics, Mathematical Literacy, and four vocational subjects offered in different course, according to the NQF. For example:

Table 2.7: TVET college courses for NCV programme (adapted by SA Webhost, 2018:01)

Engineering related field/course	Business related field/course
Information Technology (IT) L2-4	Marketing L2-4
Electrical Infrastructure Construction (EIC) L2-4	Finance Economic and Accounting (FEA) L2-4
Engineering and Related Design (ERD) L2-4	Office Administration (OA) L2-4
Civil Engineering & Building Construction (CEBC) L2-4	Primary Agriculture (PA) L2-4
Service related field/course	
Tourism	
Education and Development (ED) L2-4	
Hospitality L2-4	
Primary health L2-4	

For students to progress from one level to another in NC(V) NQF Level 2,3 and 4, they must pass all their seven subjects and will obtain NC(V) qualification. Therefore, for

students to pass fundamental subjects, they should obtain LO 40%, Eng. 1st Additional Language 40%, Mathematics 30% and Mathematical Literacy 30% in the final examination. Thereafter, they have to obtain 50% on four of their vocational subjects; only one fundamental subject may be condoned to a maximum of 5%; and no subjects may be condoned for a student who failed vocational subjects. Those failed must be repeated to get a valid Internal Common Assessment (ICASS) mark.

This study focused on the NCV programme in mathematics L4, because the DHET and this TVET college's reports showed that Mathematical Linear Programming (MLP) L4 is a problematic topic. Furthermore, the yearly reports at this examined TVET college show that mathematics is a problematic subject in which students do not perform well (cf. 1.1). Such reports are backed by the literature in Section 1.1. Next a literature review is provided that guided by this study to get a deeper understanding about the challenges contributing to poor performance in MLP topic.

2.4 LITERATURE REVIEW

This section will review literature for the purpose of developing a strategy to use the RME approach to enhance the teaching and learning of MLP at TVET colleges. This includes literature from local, continental and global in South Africa, Southern African Development Community (SADC), African continent, and international guided by its research objectives, namely challenges, best practices, conditions, threats and evidence of success of recommended best practices in the MLP teaching and other mathematics concepts.

2.4.1 Challenges in the teaching of Mathematical Linear Programming

The previous literature and DHET reports have revealed that students perform poorly in MLP. Students have difficulties in the following components of MLP, namely the formulation of linear inequalities from word problems; problem solving of linear inequalities; and drawing, analysing and interpreting MLP graphs (Stevens &

Palocsay, 2004:38; McGlaughlin *et al.*, 2005:223; Annexure E). According to this research, the following factors contribute to these difficulties of the students in MLP.

2.4.1.1 Abstract way of teaching MLP and other mathematics concepts

Moloi (2013a:138) asserts that teaching mathematical concepts in an abstract way only hampers the teaching and learning process, because it becomes very difficult for students to understand Mathematical Content Knowledge (MCK). This study explains an abstract way of teaching MCK, namely as teaching MCK in the linear and procedural way without integrating real-life stories or using hands-on activities with which the students are familiar from diverse environments. However, Prangma *et al.* (2009:383-384) indicate that, to teach MCK effectively, the teaching should start from the concrete level to abstract level; lecturers should use both procedural and conceptual methods. Previous investigations have shown that mathematics lecturers teach all MLP components in one lesson and at the same time, in contradiction with what Andrew (1990, in White, 1995:29) highlights, namely that it is not advisable to teach all components of MLP in one lesson at the same time because this might confuse students.

2.4.1.2 Limited teaching apparatus

Limited teaching apparatus is another challenge contributing to poor performance in MLP topic and or in other mathematics concepts. Limited teaching apparatus is mostly found where lecturers use a teacher-centred method to teach MCK in the classroom. Therefore, limited teaching apparatus is defined as insufficient or a lack of teaching materials, aids and resources that lecturers fail to organise in order to enhance students' knowledge and help students to comprehend abstract MCK well. Sa'ad *et al.* (2014:4), Mapolisa and Tshabalala, 2014:164), and Mupa and Chinooneka (2015:129) found that a lack of teaching facilities, poor quality of textbooks, inadequate mathematics teaching materials, libraries, or laboratories for teaching mathematics concepts, in this regard MLP topic, cause poor performance in mathematics and also create students' misunderstanding, misconceptions, errors and mistakes in MLP.

2.4.1.3 Traditional teaching methods dominate the teaching of mathematics

According to Bature *et al.* (2016:173), most mathematics class activities are teacher-centred with students being good listeners and recipients of knowledge, rather than

being an active initiators of their knowledge. CDE (2004 in Mulaudzi *et al.* 2016:44) shares that most of students find mathematics as a subject difficult to learn because of the use of an outdated methods in classrooms and also because instructors mostly teach mathematics through using the traditional paper and pencil approach. Mulaudzi *et al.* (2016:44-45) further explain the traditional paper and pencil approach as a manual procedure used to plot graphs in mathematics and maintain that the traditional paper and pencil approach poses difficulties for students to learn. Niklad (2004:207) found in his research that most of time the instructors demonstrated only one approach, which is the traditional teaching method to solve each mathematics problem. In the application of this method, students listen to their instructors' explanations and/or descriptions of the content and watch their instructors solve mathematical problems. However, the limitations of this method is that instructors keep on teaching algebra in a traditional way, which lacks real-life applications and actual problems.

On other hand, the participants in Niklad's research shared another limitation of the traditional teaching method, specifying that teaching and learning algebraic concepts in the traditional way, which stresses computational knowledge and skills, do not help students to in understand mathematics concepts. Mwenda *et al.* (2013 in Ngema, 2016:30) add that the teaching of mathematics is dominated by the use of traditional teaching methods, namely by demonstration and formal lecturing, making it challenging for students to learn and understand mathematics concepts (e.g. MLP). In the application of traditional teaching methods, lecturers are information providers, assessors and evaluators guiding students to get accurate answers, in order to prepare them for final examinations (summative assessment). The lecturers play big role in the teaching and learning process while students learn passively. Hawkar (2014:242) strongly argues that students prefer to learn differently. In essence, individual students has their own learning styles. For instance, auditory students learn well when they listen to their instructors, while visual students learn best when they watch their instructors showing them how to apply certain rules to solve mathematics problems to arrive to a correct solution. Therefore, auditory and visual students learn best by means of the traditional teaching method. However, kinaesthetic students learn best through interacting with others students or the lecturer, so if these students

are not catered for, they will not be able to learn adequately and as a result, not perform well in MLP.

In addition, Sa'ad *et al.* (2014:32) and Tshabalala and Ncube (2012:01) state that poor teaching methods contribute to poor mathematics performance. Thus, the use of a traditional method only, combined with poor teaching methods hampers the teaching and learning in mathematics, because these do not assist students to comprehend the abstractness of MLP components. Tshabalala and Ncube (2012:03) agree that most mathematics instructors do not make the teaching of mathematics practical and more exciting; as a result, creating negative attitudes among students towards mathematics as a subject.

This study viewed teacher-centred lectures from a textbook, telling, and demonstration as traditional teaching methods. Cottel and Millis (1993 in Dimitrios *et al.*, 2013:76) share the same sentiments, stating that traditional (or conventional) teaching methods are teacher-centred and include the use of lectures and discussions, with the problem-solving element only presented by instructors. The syllabus, teaching resources and student assessments are determined and transmitted to students in various lectures by the instructor. Therefore, by applying the teacher-centred method, an instructor plays an active role in the teaching and learning process while students listen passively when new information is presented. According to Johnson and Van Wyk (2016:109), the following approaches are typical of the teacher-centred method: direct instruction; whole-class discussion; teaching large classes; question and answer sessions; and the textbook method, because all share similar characteristics.

2.4.1.4 Inability to determine students' prior mathematics knowledge

The failure to determine students' prior mathematics knowledge takes place when instructors fail to use multiple techniques to find out what students know or not know before imparting new MCK, as discovered by the above researchers. Oyinloye (2013:1) found that most pre-service instructors fail to activate and assess prior knowledge; as a result students' are not able to not follow and comprehend new MCK presented. Contrary to what Oyinloye found, this study has discovered that, not only are pre-service instructors unable to determine students' prior MLP knowledge; senior and experienced lecturers also fail to search, activate and actualise students' prior knowledge when teaching MLP. This hinders the teaching and learning process,

resulting in students' poor performance in MLP as well as mathematics overall (cf. 1.1).

One of the repercussions of not determining students' prior mathematics knowledge is that instructors are unable to know what students already know, which can help them to unlock and deliver new knowledge. Hailikari *et al.* (2008:04) observe that most students lack prior MCK; hence they are unable to grasp new MCK. Hailikari *et al.* (2008:4) indicate that a lack of prior knowledge leads to failure of students to understand new lessons, while sufficient prior knowledge helps students to grasp new lessons well. Yuksel (2012:1197) indicates that prior knowledge plays a vital role in assisting students to comprehend a new lesson well. In addition, failure to determine students' prior mathematics knowledge leads to instructors failing to identify MLP gaps, misconceptions, and mistakes that students committed in the previous grades or levels, and/or lessons that students have struggled with for long time. As a result, students will not grasp new knowledge well (Gafoor & Kurukkan, 2015:238-239).

2.4.1.5 Lack of formative assessments

Another challenge associated with poor performance in mathematics is the lack of formative assessment while instructors impart new MCK. According to Moyosore (2015:484), a major problem identified as a challenge in teaching and learning is poor assessment techniques adopted by mathematics instructors in senior secondary schools. Poor assessment methods refer to a lack of proper implementation or integration of formative assessment in the teaching and learning process. This means that instructors do not assess students regularly, e.g. prior to, during and/or after the presentation of new lesson. Formative assessment is defined as process in which both instructors and students provide feedback during instruction to organise the learning and teaching process in order to increase students' achievement (Bloom, Hastings & Maddaus, 1971, in Guskey, 2014:03). The significance of this assessment technique is that it informs lecturers whether the students have learned and understood a lesson well, in order to plan a next lesson. In essence, formative assessment can be seen as an indicator, because it provides lecturers with a clear picture of a lesson presented, but if there is a lack of formative assessments in the teaching and learning process, students' achievements are always at risk.

2.4.6.1 Students misconceptions, mistakes and errors

Blanco and Garrote (2007:223-226) found that students experience two types of difficulties in dealing with linear inequalities, namely arithmetic and absence of meaning. This means students possess a lack of Procedural Content Knowledge (PCK) and Conceptual Content Knowledge (CCK) in linear inequalities (cf. 1.5.1.6). Blanco and Garrote (2007:227) further indicate that arithmetic is still a crucial referent for those students who make errors in algebraic procedures while the absence of meaning is an underlying cause of the failure to understand algebraic concepts and the algebraic process.

2.4.2 Solutions to respond to challenges in teaching mathematics

In this section, I review the literature as guided by the second research objective of this study. The literature revealed that to respond to the challenge mentioned in Section 2.4.1, mathematics lecturers should implement the following recommendations and/or use these solutions.

2.4.2.1 The use of concrete and abstract teaching methods

Kamina and Iyer (2009:2), Lindsay (2011:605), Chang *et al.* (2017:1), Moloi (2013b:489-490), and Prangma *et al.* (2009:383-384) agree that the teaching of MLP vocabulary, mathematics concepts or any other Subject Content Knowledge (SCK) should start from the concrete to abstract level to enhance students' understanding and performance. The concrete level of teaching takes place when a lecturer uses objects, background experience, realities and stories to mathematise mathematical content, as well as develop and help students to comprehend abstract mathematics concepts. The abstract level of teaching takes place when lecturers teach mathematical content in a formal way as presented in mathematics textbooks without incorporating realities, objects, background experience and stories for students to make sense of the teaching. Kamina and Iyer (2009:2) explain manipulatives as a concrete object used in the teaching and learning of mathematics to help students understand the abstract concepts in the domain of mathematics. Furner *et al.* (2005:17) indicate that the use of manipulatives provide mathematics teachers with a great potential to use their creativity to do further activities on mathematics concepts in the classroom. Thus, students automatically learn mathematics concepts in a sequence between the concrete and abstract levels. McNeil and Jarvin (2007, in Kamina & Iyer, 2009:2) point out that manipulatives provide additional resources in

learning mathematics, assist students to connect with real-world knowledge, and help to increase memory and understanding of students. Resnick and Ford (1981:38), Lehtinen and Hannula (2006, in Kamina & Iyer, 2009:2-3) agree that a powerful form of teaching is the use of the associationist theory of identical elements where simple concrete tasks help in the transfer of complex learning. This theory suggests that teachers should engage students in the learning process by mediating between the concrete objects and the characteristics of the problem situation.

Andrew (1990, in White, 1995:29-30) indicates that the MLP topic should be divided into five to six lessons according its components: firstly recognizing and describing variables and limits (understanding of problem situation); secondly, converting word problems into mathematical linear inequalities; thirdly, drawing linear inequalities and find feasible region; fourthly, determining the maximum (or minimum) value using the feasible region (analysis and interpretation); fifthly, revising the whole process; check students' solutions and prepare for tests to improve the teaching and learning of MLP concepts. MLP in National Certificate Vocational (NCV) L4 includes concepts such as word problems, linear inequalities and graphs, which White (1995:29) recommends should not be introduced and taught in one lesson. White (1995:29) further argues that no graphing of linear constraints (inequalities) to find feasible region must be done before the third of five lessons are allocated and understood by students. According to Dantzig and Thapa (1997:8), there are two ways in which one can formulate a problem as a linear programme, namely column (recipe/activity) and row (material balance). The column approach includes phases such as, defining the activity set; defining the item set; defining the input-output coefficients; specifying exogenous flows; and setting up the material balance equation, while the row approach defines the decision variables; defines the item set; and sets up constraints and objective function. Dantzig and Thapa (1997:12-16) further demonstrate how to apply column and row methods.

2.4.2.2 Sufficient teaching aids

Sa'ad *et al.* (2014:32), Mbugua *et al.* (2012:87), Gitaari *et al.* (2013:98), and Karue and Amukowa (2013:106) agree that adequate teaching materials are key to improve mathematics teaching and learning. They maintain that provision must be made for relevant teaching aids. Teaching aids in this study are defined as materials that

lecturers use to explain, demonstrate and emphasize components of MLP to students in the classroom. The teaching materials relevant for teaching MLP include a ruler, protractor, compass, real-life objects, real-life pictures, graph grid paper, and other teaching materials that enhance the teaching of MLP and graph representation. The literature confirms that there must be adequate teaching aids and resources to solve mathematics problems (cf. 2.4.1.2). In the 21st-century classroom, mathematics lecturers use multiple teaching and learning resources included technological tools such as computer software packages (spreadsheets; geogebra; statistics; graph paper, autograph, graphic calculator and other mathematics programs) (cf. 1.5.2.2).

2.4.2.3 Numerous teaching methods to teach mathematics

Stevens and Palocsay (2016:38) strongly point out that a single teaching method is not panacea to solving all challenges in the teaching and learning of mathematics. The literature confirms that students experience difficulties to solve mathematical word problems, linear equations and linear inequalities as well as drawing graphs, which are regarded as MLP components in this study. Thus, scholars recommended different teaching methods that allow numerous and multiple teaching pedagogies to respond to all MLP teaching and learning challenges identified in Section 2.4.1. Sa'ad *et al.* (2014:32) have found that using proper methods to teach mathematics has improved mathematics performance, while Gitaari *et al.* (2013:98) recommend the use of a variety of teaching methods to improve the mathematics performance of students. Suleiman and Hammed (2019:30) recommend four approaches to improve mathematics performance, firstly, arranging seating plan properly into groups and group students according to their abilities; secondly, use a constructivist strategy to teach mathematics for students to learn effectively and involve students in the teaching and learning process, allowing them to be in control of their learning; thirdly, using instructional resources; and lastly, integrate games and computer-aided instruction. This is also supported by Sa'ad *et al.* (2014:39) who suggest the use of a student-centred approach in teaching mathematics. NCTM (2000), in Nilklad's work (2004:207-208), points out that, in order to support multiple pedagogical methods, mathematics instructors need to cogitate the use of various methods to solve mathematics problems, alerting students that there is no single right method to solve mathematics problems. Therefore, mathematics instructors, teachers and lecturers should use and show different efficient techniques for solving problems.

However, Nilklad, in his mathematics lesson observations, found that most instructors use only traditional methods to teach mathematics concepts, where they interpret each problem situation and then show students how to arrive at the solution. Mathematics instructors do not emphasise this process and also do not focus on identifying more efficient ways. As a result, students do not experience more rigorous processes in solving mathematical problems and students are also not guided to use a variety of techniques to solve mathematical problems, being forced to memorise instructors' technique which is also difficult for other students. Hence, NCTM (2000, in Nilklad, 2004:207-208) proposes that mathematics instructors should inspire students to think critically, ask question, solve problems and discuss their ideas. The teaching methods of mathematics instructors should allow students to apply multiple techniques and adapt multiple appropriate techniques to solve mathematical problems. This can be achieved by encouraging students to practise their knowledge of techniques for identifying different methods. If the first method fails, students can consider second or third options. If those methods fail, they know how to reconsider the mathematical problem, break it down, and look at it from different angles. The constructivist approach links well with activity and interactivity principles of the RME method, because both encourage students to be active and cooperate among themselves while lecturers facilitate the teaching and learning process (cf. 2.3.2).

Leone *et al.* (2010:3), Suleiman and Hammed (2019:30), and Moloi (2013b:489-490) agree that games should be incorporated into mathematics teaching and learning. Games can be used by students to practise newly acquired skills, test their skills, and review their numeracy skills. Leone *et al.* (2010:3) further indicate that opening spaces where students can be competitive in constructing methods to solve mathematics problems can be through working or engaging motivational tools (sports games and mathematics bingo) to focus on numeracy. For instance, Michael Woods utilises a game of multiplication bingo at the end of many classes to assist students in practising multiplication, which is one of the important numeracy skills. This, according to Woods provides students with an opportunity to practise a skill that many of them struggle with and to compete in constructive way. Secondly, students can also work in pairs or small groups to present and solve algebraic equations using manipulatives such as algebra tiles. Then points are awarded to the group that accurately represent and solve the problem. Woods (in Leone *et al.*, 2010:3) articulates that battling, mathematics

bingo, and the representation and solution of examples can be helpful both in enhancing fluency and giving space for mathematics practice and productive student interaction, which in line with the student-centred approach. The use of motivational tools can also help teachers to bridge gaps of interest, social and generational culture that traditional methods may ignore. The awarding of point can enable students to earn incentives for to complete their academic tasks. The incentives can help to build individual and group responsibility and nurture a positive classroom climate. The behaviourist theory reveals that there are many incentives that can be used to motivate students to compete successfully and perform well. Leone *et al.* (2010:3) states that teachers, to encourage good behaviour and instil good performance, should allow students to listen to their own music during independent seatwork and allow them to have weekly parties. However, extrinsic motivational tools (incentives) are not intended to replace or undermine the intrinsic value of learning, but it recognises the limitations of the intrinsic motivational stance in practical application.

The other approach recommended by Leone *et al.* (2010:6-7) is the use of guided and independent methods to assess acquired mathematical competency and ability. The guided method allows students to show and evaluate their understanding, while teacher direction gradually decreases. Reducing teachers' dominance in the class allows students to take increasing responsibility for learning and practice. This allows independent practice to come in place, which is important for developing and maintaining mathematics knowledge. According to Leone *et al.* (2010:6-7), independent practice can takes place towards the end of the lesson where a teacher can use a short task with five questions as a ticket to pass out, or provide homework to complete outside the classroom. Homework needs strong collaboration between education staff, parents, residential and security staff. However, homework needs adequate time, appropriate space and tools to complete the work. More importantly, homework should be more practice, not an introduction to new MCK or skills. Therefore, it is the responsibility of teachers to make sure that students fully understand the lesson prior to giving students homework. Leone *et al.* (2010:3) believe that ensuring mathematical understanding can be achieved through Curriculum Based Assessment or other informal measure at the end of each lesson. They further suggest the incorporation of relevant cultural practices in the teaching of MCK. A culturally relevant practice is defined as a pedagogical method that integrates students' cultural

context that is relevant to their lives, so teachers are expected to connect learning to culture and encourage students to incorporate their own cultural stories, experiences, symbols, language into problem-solving opportunities and promote engagement and a sense of classroom community environment. The use of a guided and independent approach fits well into the activity and guidance doctrines of the RME approach, because the application of these methods require of students to be active participants in the teaching and learning process while lecturers facilitate and guide students in the process. The cultural practice fits into the reality principle of RME approach, in which expects lecturers incorporate real-life stories, background experiences, cultural stories and practices in the teaching and learning of mathematics (cf. 2.3.2).

Nakhanu, Toili and Nyongesa (2015:6) endorse the use of the Problem-Based Learning (PBL) approach to enhance the teaching of MLP, because they find this PBL method beneficial and helps students to understand and perform well in MLP. According to Nakhanu *et al.* (2015:72-73), the PBL approach is one of the student-centred pedagogies that allow students to learn effectively through problem solving and it gives students the opportunity to own or have a control over their studies. As a result, the PBL approach creates critical thinkers, because students learn both thinking strategies and domain knowledge. The intention of the PBL approach is to assist students in developing self-directed learning, effective collaboration skills, flexible knowledge, effective problem-solving skills and intrinsic motivation (Hmelo-Silver, 2004:2).

Nogueira de Lima and Tall (2008:1-8) recommend the use of the procedural embodiment approach to teach the MLP linear equations and constraints. Procedural embodiment is explained as procedures that have to do with embodied actions on the symbols that lead to picking the symbols up and moving them to the other side of an equation. This teaching method is regarded as an efficient tool to aid students to solve mathematics problems, since it is based on arithmetic operations or steps that lead to final answers. Procedural embodiment includes principles like changing operational signs or putting symbols in an equation to get the answer. I concur with Nogueira de Lima and Tall (2008:1-8), who are of the view that the procedural embodiment approach works adequately in aiding students to perform well in mathematics, but if students do not understand these procedures that lecturers use, then students are likely to commit many mistakes when sketching MLP graphs. The PBL approach

acquaintances well with activity, interactivity and guidance principles of the RME approach, because it promotes activeness among students by working through classroom and home activities individually and in groups to develop self-directed learning, effective collaboration, and effective problem-solving skills. Students also become critical thinkers, while lecturers guide and assist students where they are faced with difficulties while working on activities (cf. 2.3.2 & 4.3).

Kissane, McConney and Ho (2015:147) agree with Stevens and Palocsay (2004:38) that Information Communication Technology (ICT) and computer software should be integrated and employed to teach abstract mathematics. The literature evidences that ICT and software improve mathematics teaching and learning, students' mathematics understanding and students' mathematics performance. The technology used includes digital devices and software such as computers, tablets, graphical calculators, Computer Algebra System (CAS), geogebra and spreadsheets. More importantly, many scholars are of the view that technology in mathematics enables students to explore and investigate mathematics graphic examples to generate knowledge and test their understanding. For instance, graphical calculators provide students with multiple representations of mathematical functions and also assist them to solve mathematical activities with a large number of functions in multiple forms; graphical, tabular and symbolic.

According to the National Council of Teachers of Mathematics (2000, in Salani, 2013: 152-153), with calculators and computers students can investigate and explore more mathematics examples and representational forms than possible by hand. They can also estimate more easily. Computers and graphic calculators can also make precise values and graphs of functions. For example, students can show graphs of these functions, $f(x) = x$; $f(x) = x^2 - 4$; $f(x) = \frac{2}{x}$; and or $f(x) = 2^x$ without difficulty. This can also be relevant to the teaching and learning of MLP graphs, since MLP problems involve mathematical linear inequalities and equations. Burril *et al.* (2002, in Kissane *et al.*, 2015:13) recognise that students who use CAS calculators are better able to apply calculus than those without, because it enhances visualisation and aids students' understanding of mathematics concepts. CAS assists students to understand algebraic functions, sketching of graphs, evaluation of expressions (solutions) and how to transform one functional representation to another.

MacGlaughlin (2005:230) also proposes the use of computer software to address difficulties of constructing MLP graphs, as used in USA and Columbian schools. This computer software only displays numbers, not formulas and assist students to identify correct relationships between MLP components (word problems, linear inequalities, graphs, objective functions). Granberg (2016:33-35) agree with Mosia (2016:186-206) and Macglauglin (2005:230), who believe in the integration of computer software 'Geogebra' to instruct and teach MCK. According to Granberg (2016:33-35), geogebra is the best method that one can use to teach mathematical linear equations and inequalities to draw MLP graphs. Geogebra is explained as computer software that is used to teach MLP in Swedish schools. The benefit of using Geogebra is that it responds to or verify students' solutions instantly; it helps students to discover their mistakes in drawing MLP graphs by signifying their incorrectly constructed algebraic formulas; and allows students to examine their errors by giving them the opportunity to correct their algebraic formulas (cf. Fig. 2.3). Stevens and Palocsay (2004:38) strongly point out that computer software (spreadsheets) is not panacea to all mathematics teaching and learning teething troubles, because some students may inadvertently create non-linear programming or commit errors in identifying cells quantities that are outside their control. Direct spreadsheet formulation also does not really eliminate the need for traditional formulation skills and it recasts them as debugging skills (cf. Fig. 2.2). Hence, Steven and Palocsay (2004:42) believe in a multiple teaching approaches such as a combination of computer software and code switching in the teaching of MLP components in order to respond to the difficulties of students in MLP. Both researchers use the translation approach in improving the teaching of MLP in USA schools, because the translation approach is regarded as an efficient tool for converting mathematics word problems into mathematical expressions. The translation approach breaks down the process of constructing MLP topic into a series of small and well-defined steps. It also assists students to avoid many formulation errors and omissions (Williams *et al.*, 2016:14-15).

Prior the use of the translation approach, both lecturers and students should begin to understand MLP text to be converted and then generate meaningful linear inequalities and objective functions, because according to Williams *et al.*, (2016:14-15), translation method begins with a conceptual process of developing a measurable representation of problems, avoiding formulas and numbers, but rather encourages students to

construct their own thinking. Thus, I see this method as essential for use in the teaching and learning of MLP, because it assists both lecturers and students to convert one language into another, in order to understand MLP problem scenarios better. The translation approach is categorised into two stages: rephrasing word problems and constructing MLP graphs through semantic content and simple procedure (Mwihaki 2004:127-132). Semantic content is defined as a process of restating original problems in measurable quantities and should correspond to the steps of the translation approach, namely understanding MLP problems, describing objectives in words, describing each constraint in words, and defining decision variables. On the other hand, a simple procedure is taken as the second stage and it is used to transform each measurable quantity into proper mathematical form. It should also correspond to the remaining general steps (step 5-6 of 6), so that students are assisted with a rule that generalises mechanics for building mathematical expressions, linear equations and linear inequalities in mathematical graphs. Thus, I see the translation approach as having similar features that the RME approach and both approaches allow both lecturers and students to start teaching from informal to formal education. The use of semantic and simple stages provide both lecturers and students with a consistent step-by-step formulation method that emphasizes understandable concepts over mathematical hieroglyphics.

This study incorporated the RME approach to enhance the teaching and learning of MLP at TVET college. The RME approach is defined as a domain-specific teaching theory for mathematics, developed in the Netherlands and used by different scholars to improve the teaching and learning of mathematics (cf. 2.3.2). This approach has improved mathematics performance at different schools around the world. In applying the RME approach, the rich 'realistic' situations are given a prominent position in the teaching and learning process; these real-world situations serve as a source for initiating the development of mathematical concepts, tools and procedures within the context of students in order for them to use their mathematical knowledge to solve problems, which then gradually become more formal and general and less context specific. Therefore, I, as principal investigator, together with my co-researchers, aimed to show how to integrate the RME approach to improve the teaching of MLP topic at TVET colleges.

The following Figures are illustrations of MLP graphs using a spreadsheet and also geogebra as computer software.

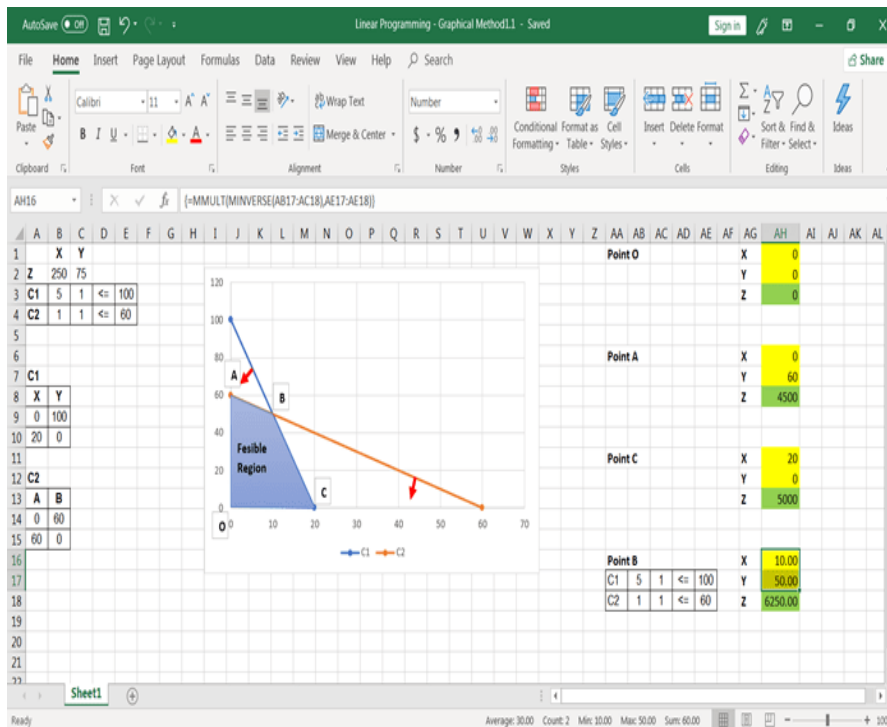


Figure 2.2: Spreadsheet taken from Google

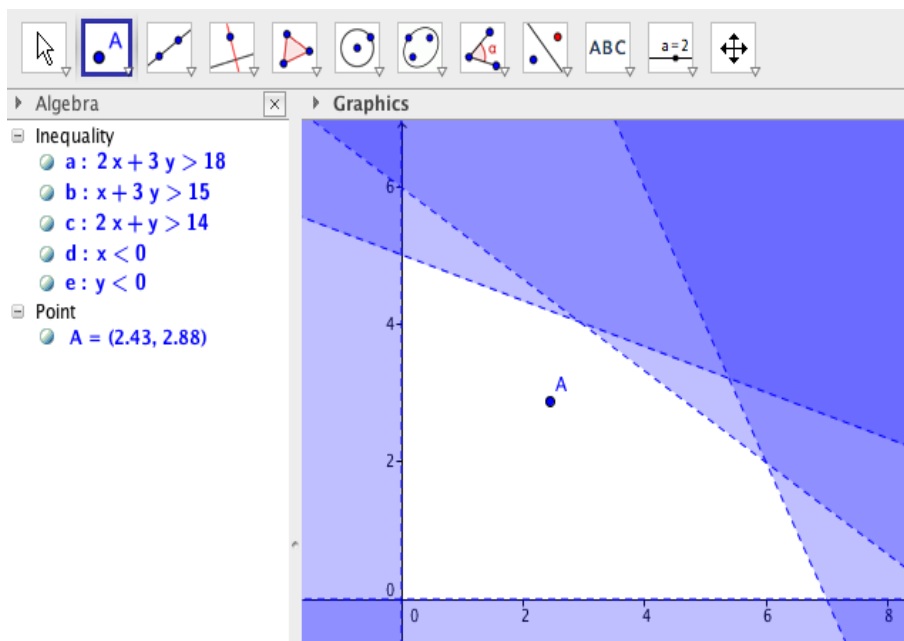


Figure 2.3: Geogebra taken from Google

2.4.2.4 Activate and assess students' mathematics prior knowledge

According to Hailikari *et al.* (2008:2), one efficient way to improve students' learning is through the use of prior knowledge assessment, i.e. diagnostic tests, to check what

students already know, identify gaps, misconceptions and evaluate the level of mathematics support students need. Leone, Wilson and Mulcahy (2010:3-4) recommend the use of diagnostic testing to recognise students' strengths and weaknesses. Also, diagnostic testing can provide valuable insights into foundational mathematics skills that will be key for a particular grade or NCV level. Leone *et al.* (2010:3-4) state that these skills should be outlined as a prerequisite, but teachers should be aware of test properties such as validity and reliability of tests for individual students. Another technique to search, recognise, assess and/or activate students' prior mathematics knowledge is through developing a diagnostic interview. In this interview, the instructor provided students with mathematics questions (activities), and then as students solved problems, the instructor asked them to explain the steps, process and thinking students used. Thus, diagnostic interviews are conducted to collect qualitative performance data about students in these areas: Content Knowledge (CK), cognitive processes, strategy use and disposition to mathematics. Leone *et al.* (2010:6) indicate that it is vital for an instructor to explicitly review and reteach previous knowledge.

There are multiple ways to recognise, actualise and determine students' mathematics prior knowledge in the mathematics classroom, namely prior knowledge assessments, probing questions, brainstorming and group discussions of mathematics concepts. However, this study sees prior knowledge assessments as the most significant tool to be sure of the pre-existing knowledge of students. Prior knowledge assessments provide detailed information about students' prior knowledge base, so this may be key and more beneficial as a diagnostic tool for students' support. Hailikari *et al.* (2008:2) state that when developing prior knowledge assessment, the instructor should be aware of what type of knowledge to be assessed, distinguish between the different types of prior knowledge, which is vital in a particular context, because not all these types have similar relevance to students' achievement. Students' prior knowledge is elucidated as a multidimensional and hierarchical entity that is dynamic by nature and involves what students already know, or learned knowledge as a result of previous learning in many settings. The students' prior knowledge consists of two types of knowledge; declarative and procedural prior knowledge. Declarative prior knowledge is knowing about or surface learning and it is described as rote learning. Students with this knowledge are able to answer simple reproduction problems which do not need

application of knowledge, while procedural prior knowledge the ability to integrate knowledge and understand relations between concepts at the highest level. Students with this knowledge are able to do problem-solving. Procedural knowledge is also referred to as knowing how; it is closely related to higher-order cognitive skills. Students with this knowledge perform better than students with only declarative prior knowledge. These results emphasise the importance of recognising students' prior knowledge at the beginning of the learning process. However, the focus should not only be on what students know, but also what on how well they know it (Hailikari *et al.*, 2008:2).

Prior knowledge has long been viewed as the key factor influencing learning and student achievements positively. The quality and amount of prior knowledge impact positively both on knowledge acquisition and the capacity to apply to higher-order, cognitive problem-solving skills. This creates a learning environment where students must actively construct their knowledge and skills on the basis of prior knowledge. The prior knowledge assessment provides detailed information about students' prior knowledge base. Hailikari *et al.* (2008:1-2) indicate that inadequate prior knowledge is a negative factor to consider, because if there is a mismatch between instructors' expectations of students' knowledge and the actual students' knowledge base, then learning may be hampered from the beginning of a lesson. As a result, trying to learn something new without adequate prior knowledge may lead to rote memorization and students will be unable to relate new knowledge to their existing knowledge.

2.4.2.5 Sufficient formative assessments

According to Alonge (2004 in Ajogbeje *et al.*, 2013:8), the crux of using tests and other evaluation tools during the teaching and learning process is to guide, direct and monitor students' learning progress to achieve lesson objectives and improve their performance. Ajogbeje *et al.* (2013:8-9) are in support of formative assessment (continuous tests) for effecting immediate student feedback results and to remove threatening effects of single tests (summative assessments) given at the end of the subject course.

Gitaari *et al.* (2012:96) propose different types of assessments to identify students' abilities. Sa'ad *et al.* (2014:35) agree with Gitaari *et al.* (2012:96) on the fact that one of the techniques to improve mathematics performance is more tests and quizzes

during the teaching and learning process. This can also instil competition among students and even schools. This study believes that sufficient formative assessment is one of the methods to increase students' mathematics performance at TVET colleges. The concept 'formative assessment' was first introduced by Bloom, Hastings and Maddaus (1971, in Guskey, 2014:3). They state that assessment does not need to be used solely for summative evaluations of students' performance at the end of the course; instructors should include a number of formative assessments, which at this examined TVET college are called additional supporting tasks during the phases of teaching. These episodes should provide student feedback and corrections as a method to remediate their work.

This study regards formative assessment as a class or home activity, quizzes and tests given on regular basis during or at the end of lessons, to check students' learning progress and, importantly, their level of understanding. The significance of implementing formative assessment is that it gives lecturers feedback about whether students have learned and comprehended a lesson presented or not, enabling lecturers to re-plan next lessons to increasing students' achievements and fill gaps and misconceptions (Tridane *et al.*, 2015:685). Fundamentally, formative assessment is an indicator, because it provides lecturers with a picture about the success of a lesson, while formative assessment is seen as a process through which lecturers and students gather evidence of learning and then use it to adapt the way they learn and teach. According to Guskey (2014:5-7), the following are elements of the formative assessment: the establishment of a classroom culture that encourages interaction; the use of assessment tools; the establishment of learning goals; tracking individual students' progress towards those goals; the use of varied instruction methods to meet diverse student needs; the use of multiple approaches to assess students understanding; feedback on student performance; adaptation of teaching to meet identified needs; and active involvement of students in the learning process.

2.4.2.6 Multiple languages and pedagogical methods

According to Blanco and Garrote (2007:227), when teaching linear inequalities, lecturers should ensure that linear inequality jargon and symbols (algebraic language) are clearly differentiated and have semantic value for students. mathematics lecturers should use different languages, like 'everyday' language, visual-geometric language,

and algebraic language to translate one geometric or algebraic concept to another for students' deeper understanding. Blanco and Garrote (2007:228) signpost that lecturers should use different strategies to approach questions related to inequalities both enriches the learning process and allows more students to acquire the concept.

2.4.3 Critical conditions for recommended solutions to work

This section outlines the conditions that enable solutions in Section 2.3 to work well and sufficiently.

2.4.3.1 Conditions for concrete and abstract approaches to work effectively

The solution recommended in Section 2.4.2.3 are the following: sufficient lecturer MLPCK, enough time and dedicated lecturers. The research conducted over the past years (Kim *et al.*, 2018:1) has found that CK has an influence on PCK and students' performance, which necessitates professional development of instructors CK, as an evidence-based practice for improving instructors PCK and in turn student results. Therefore, this study regards adequate lecturer MLPCK as a condition that enable lecturers to use and integrate real-life stories, objects, realities and other background experiences into the teaching of MLP topic. Positive lecturer perception is another condition to enable concrete and abstract methods to work sufficiently. Moyer (2001, in Kamina & Iyer, 2009:2) highlights that lecturers fail to incorporate realities, objects, and other background experiences into the teaching of mathematics, because they think that it is a waste of time. Lecturers need enough time to teach mathematics to be able to incorporate concrete objects, stories, realities and other background experiences into the teaching of MLP, because comprehension, application and analysis carry a weight 70% in the mathematics NCV L4 syllabus (cf. DHET assessment guidelines for mathematics NCV L4, 2013 26). In addition, a dedicated team is a key condition to allow the sequence of concrete and abstract instructional method to work adequately, because lecturers need to plan, organise and prepare the lesson in collaboration.

2.4.3.2 Key conditions for the use of teaching and learning resources

The literature revealed that one of the methods to improve the teaching and learning of mathematics is adequate and available teaching and learning resources (cf. 1.5.2.2 & 2.4.2.2). Adequate funds and a budget for each campus at this examined TVET

college is therefore. Adequate funds refer to money that is set aside for a particular purpose, namely the acquisition of teaching and learning resources. The availability of funds have a direct impact on teaching and learning resources, as insufficient funds for education lead to inadequate teaching and learning resources. For lecturers to incorporate teaching and learning resources into the teaching and learning of MLP well, they need continuous mathematics workshops, training and conferences to develop them. However, lecturers can also capacitate themselves by consulting more sources, such as different textbooks and by furthering their education to enhance their CK and PCK in mathematics, to teach mathematics in more meaningful and creative manner. Kim *et al.* (2018:1) emphasise that adequate CK leads to adequate PCK. Another condition for the successful incorporation of teaching and learning resources is adequate lesson planning, preparation, monitoring and evaluation which must be done on regular basis. According to Thomas (2006:265-266), to utilise technology tools and computer software, lecturers need to have adequate computer skills and knowledge and background experience, which need short courses, in-house training and regular workshops.

2.4.3.3 Conducive conditions for various teaching approaches to work

In Section 2.4.2.3 the multiple pedagogical approaches, such as PBL, code switching, translation, procedural embodiment, RME, guided, independent practice and the incorporation of games, cultural practices, computer software and other hi-tech tools were recommended to enhance the teaching of MLP. Therefore, to use these teaching approaches, lecturers need to possess adequate CK in mathematics. This study defines CK in mathematics as mathematics concepts, content, procedures, and illustrations. the DBE (2018:16) purports that the more lecturers possess a deep CK understanding, the more they possess sufficient PCK. PCK refers to various teaching methods that could be used to teach mathematics in more meaningful and understandable way (Shulman 1987, in Murray *et al.*, 2018:6-7). Another condition for the above teaching approaches and the incorporation of teaching aids and technological tools is sufficient time, since the mathematics NCV L4 syllabus contains 70% comprehension, application, and analysis (cf. 2.4.3.1).

A dedicated team is also regarded as one of the conditions for the above teaching methods to be executed and used well. Malebese (2016:161) agrees with Perkins and

Wirth (2008:12-14) and Terry and Irving (2010:110-111) that a dedicated team brings a diverse knowledge and share their background knowledge, teaching experience, and best practices, enabling them to learn from one another. Sufficient lesson planning and preparation are also necessary for the successful use of teaching methods and the integration of games, cultural background, real-life situations, and technological tools to teach MLP. Khan (2011:3958) signposts that sufficient lesson planning and preparation give lecturers a clear direction of what to teach, where, when and how to teach in order for students comprehend a lesson well and what teaching and learning aids are needed. The incorporation of cultural (indigenous knowledge, games and languages), social, political, technologic (tools and software), economic, and education contexts enhance MLP, but mathematics lecturers need to have knowledge of those contexts. For instance, they must know what is happening in a corporate world; know and accept diverse South African cultures; and know what and how to use technology.

2.4.3.4 Conditions for prior knowledge assessments to work

According to Ifeoma *et al.*, (2017:99), Ngema (2016:123), Sa'ad *et al.* (2014:35), and Thanheiser *et al.* (2010:12), lecturers need to possess the right thinking and mentality about what prior assessment seeks to achieve in order for them to implement prior knowledge assessments well. Another condition is adequate time for teaching and learning to administer diagnostic assessments successfully and to provide students with instant feedback, as well as to afford lecturers the time to plan and prepare lessons. The aim of using a diagnostic assessment is to detect students' mathematics background knowledge gaps of a particular topic, and to close these gaps before presenting a new lesson.

On other hand, convenient lecturers' workload is another condition to enable lecturers implement prior knowledge assessment, because they will be expected to administer diagnostic tests and provide students with instant feedback, thereafter, plan lessons to bridge students' gaps, deficiencies, misconceptions and difficulties, so that lecturers to pave way for new lesson to be delivered. An enthusiastic team is another key condition to implement prior knowledge assessment successfully, due to the fact that a team is expected to set, administer, mark diagnostic assessments and provide students with instant feedback in each topic to be presented.

2.4.3.5 Conditions for formative assessments to be implemented successfully

The success of formative assessments depends on lecturers' fair workload, because to implement formative assessments, lecturers are expected to administer this type of assessment during or at the end of a lesson. Another key condition for formative assessment to work well is adequate teaching time and a dedicated team of mathematics instructors to administer, assess, mark and provide students with feedback in time (Thanheiser *et al.*, 2010:12).

2.4.4 Threats to the successful use of solutions

2.4.4.1 Risks to hinder the successful use of concrete and abstract methods

The threats and risks that impede the success of using a combination of concrete and abstract instructional strategy are the following factors: lack of lecturers' CK, lecturers' incorrect perception about a combination of the use of concrete and abstract methods, limited time for teaching of mathematics and unmotivated mathematics lecturers. In Section 1.5.4.1, the literature revealed that most lecturers view manipulatives as play time or a waste of time, because they use manipulatives for fun in their teaching, instead of emphasising some abstractness of mathematics concepts. On the other hand, the inadequate CK in mathematics is a risk to the successful implementation of the combined use of concrete and abstract methods. As a result, the literature uncovered that, if lecturers lack CK, they will have limited PCK (Turnuklu and Yesildere, 2007:12).

Another threat to the successful use of a combination of concrete and abstract approaches is limited time for teaching, because it makes it hard for lecturers to start the teaching from a concrete to abstract level. Unmotivated lecturers are another threat to the successful use of the combination of concrete and abstract pedagogy methods, because unmotivated lecturers teach MLP topic in a linear fashion without incorporating manipulatives and do not see a need to put more effort, extra energy and time into their work.

2.4.4.2 Threats to the successful use of multiple teaching resources

The lack of financial resources leads to unavailable teaching and learning resources, such as different textbooks; lecturers' guide; graph pads, T-square rulers, autograph,

graphical calculators, graph paper, computers, proper infrastructure in classrooms, WiFi, and computer software, because according to Becker (2000b, in Thomas, 2006:266), some teaching and learning resources are expensive. The use of computer software has decreased in New Zealand due to its cost. Thus, unavailable teaching and learning resources impact the MLP teaching and learning negatively. The absence of mathematics workshops and conferences pose a threat to the successful integration of teaching and learning aids to improve MLP teaching and learning.

2.4.4.3 Threats that hinder the use of multiple teaching methods

An inadequate lecturer CK in mathematics impedes the success of multiple teaching methods to improve the MLP teaching and learning. As a result, it leads to inadequate lecturers' PCK. According to Murray *et al.* (2018:7-8) and Kim *et al.* (2018:2-3), CK has a direct influence to PCK. On other hand, lack of lesson planning and preparation impedes the successful use of the above teaching methods, because it disorganises instructors completely and hinders achievement (Boikhutso, 2010:207). Coppola *et al.* (2004, in Nasari & Heidari, 2014:26) strongly argue that if instructors do not prepare a lesson they should not find themselves in the classroom. Another threat is an undedicated team and the absence of collaboration between mathematics lecturers themselves as well as between lecturers and their management (Mosia, 2016:186).

Another threat that hinders the successful implementation of those teaching approaches is the absence of mathematics workshops and conferences. This denies lecturers the opportunity to share their frustrations and challenges in the teaching of mathematics and also learn how to use multiple pedagogical methods to solve them. In addition, insufficient number of computers, wrong mathematics lecturers' perception on the use technology to teach mathematics, improper infrastructure, absence of WiFi or internet, load shedding, and limited time allocated for teaching and learning are threats to the successful use of technology and software to teach mathematics (Pelgrum, 2001:171-177). Manoucherhri (1999, in Thomas, 2006:265-266), adds that most teachers are not convinced that the use of computers in their instruction is useful. The disadvantage of using technology and software is that students tend to rely on technology only without understanding the basic concepts and they are unable to calculate by hand (Thomas, 2006:271).

2.4.4.4 Risks to the successful implementation of prior knowledge assessments

According to Ngema (2016:106-107), Gwambambo (2013:99-100), and Mosia (2016:21), one of the factors that contribute to poor performance in the teaching and learning of mathematics and science is the heavy workload of instructors. A heavy workload is also a risk and threat to the successful implementation of prior knowledge assessment, e.g. baseline and diagnostic assessment, because to implement this type of assessments lecturers are expected to prepare such an assessment for individual topics, assess, mark and give feedback to students on regular basis. Therefore, this study agrees to the fact that if mathematics lecturers have a heavy workload, they will not be able to execute the prior knowledge assessment well. In addition, the research conducted in the past revealed that lack of motivation, poor attitude, and wrong perceptions among instructors and students contribute to poor performance in mathematics (Gafoor & Kurukkan, 2015:234).

2.4.4.5 Threats to the successful to use of formative assessments

The unrealistic workload of lecturers is a threat to the proper implementation of formative assessment. For instance, if lecturers have an overcrowded class, the implementation of formative assessments will not be possible, because mathematics lecturers will not be able administer, mark, and provide students with feedback effectively and in time. As a result, students' learning will be affected negatively, as they will not be become aware of their strengths and weaknesses.

Another threat that hampers the successful implementation of formative assessment is the limited time allocated to teach mathematics Thanheiser *et al.* (2010:12) indicate that formative assessment needs adequate time for teaching and learning. Thus, if there is limited time for teaching and learning, lecturers will not be able to teach, assess, mark and provide students with regular feedback. An unenthusiastic team of lecturers will also be another threat to the successful implementation of formative assessment.

2.4.5 Evidence of success

2.4.5.1 Concrete and abstract teaching methods

The combination of concrete and abstract methods is demonstrated by Prangma *et al.* (2009:383-384) and Moloi (2013c:129) on how to use it to teach mathematics concepts. Andrew (1990, in White, 1995:29) shows how to divide the MLP topic into four to five lessons according to its components in order to enhance the teaching of MLP concepts. This technique has assisted students to comprehend the topic of MLP well.

2.4.5.2 Multiple teaching apparatus

Sa'ad *et al.* (2014:32) have found that providing relevant teaching materials is one of the methods that improve the teaching of mathematics in schools. Thomas (2006:268-269) and Granberg (2016:33-35) have discovered that the use of technology and computer software, i.e. spreadsheets, geogebra, mathematics programme, statistics package, the internet, Graph Drawing, autograph, graphical package and graphical calculator also have a positive impact on the teaching and learning of mathematics. Thomas *et al.* (2006:268) further found that there was a significant increase in the use of technological tools, technology and computers software for the learning of statistics in New Zealand schools. There were used to do routine calculations as well as graphical and investigative work.

2.4.5.3 Multiple pedagogical methods

The integration of games into the teaching of mathematics is demonstrated by Moloi (2013b:485-489). The use of this approach enhances the teaching of abstract mathematics concepts. Leone *et al.* (2010:6-7) uncovered that the use of guided and independent methods aided students to understand mathematics concepts well, also to become more independent, take responsibility and own their education. Nakhanu *et al.* (2015:6) use the PBL approach to teach MLPCK and have found the PBL method beneficial to students; as a result, the students' performance has improved. The procedural embodiments, according to Nogueira de Lima and Tall (2008:1-8), aid mathematics lecturers in explaining the MLP steps in more meaningful way, with a positive impact on students' results. The translation approach was key in making the students' comprehend word problems in MLP (Williams *et al.*, 2016:14-15).

Steven and Palocsay (2004:42) also use the combination of computer software and code switching to teach MLP in USA schools, which helps lecturers to enhance the

teaching of MPL components. The use of technology and computer software has a significantly positive impact on the teaching and learning of mathematics positively, according to MacGlaughlin (2005:230), Granberg (2016:33-35), and Stevens and Palocsay (2004:42). The RME approach has also been used in the Netherlands, the United States of America (USA), Indonesia and other countries to improve mathematics performance (cf. 1.5.5.3 & 2.3.2).

2.4.5.4 Prior knowledge assessments

Hailikari (2008:6-7) indicates that prior knowledge from previous courses indeed aided students to pass a pharmaceutical chemical course and this contributed positively to their learning. Most students in Hailikari's work signposted that they benefited from the use of the prior knowledge assessments, because it assisted them be more aware of their weak points. Instructors were also mostly optimistic about the use of prior knowledge assessment where they could get a clear indication of students' prior knowledge, and which helped them to adapt their teaching to the needs of the student.

2.4.5.5 Formative assessments

Ajogbeje *et al.* (2013:15) investigated the effect of Formative Testing with Feedback on Students' Achievement in mathematics. They (Ajogbeje *et al.*, 2013:15-17) found that students who were given formative tests with regular feedback performed better than those who did formative tests only and with control (no formative and feedback). The weakest performance in mathematics came from the control group provided with expository teaching only, without any formative tests and feedback. Formative tests and feedback have improved students' results in internal assessments as well as in examinations.

CHAPTER3: THE PARTICIPATORY ACTION RESEARCH METHOD TO ENHANCE THE TEACHING AND LEARNING OF MATHEMATICAL LINEAR PROGRAMMING

3.1 INTRODUCTION

This study employed the Realistic Mathematics Education (RME) method to enhance the teaching and learning of Mathematical Linear Programming (MLP) at Technical Vocational Education and Training (TVET) colleges. As a principal researcher of this study, I realised that I did not have sufficient knowledge to use the RME approach to enhance the teaching and learning of MLP at TVET colleges. I believed that people who are faced with problems teaching and learning MLP are in a better position to use the RME approach to solve daily challenges. Moreover, this study has used Participatory Action Research (PAR) as a research methodology to generate data that assisted us to plan, organise, and implement the RME approach to enhance MLP teaching and learning at TVET colleges and evaluate the effectiveness of this RME approach. Hence, I saw PAR as appropriate methodology for this study, because I understood that to achieve the five objectives of this study, co-researchers should play a central role in generating data, as their lived experiences are regarded as the key factors that should be considered in the research process. This chapter substantiates the use of PAR as research methodology. The discussion is organised in the following manner: PAR as a research methodology, and as a research design.

3.2 PARTICIPATORY ACTION RESEARCH AS A RESEARCH METHODOLOGY

This study discusses PAR as methodology appropriate for conducting research. The importance of its relevancy and appropriateness is proven through a discussion of the origin of PAR, its objectives, principles, formats, ontology, and epistemology.

3.2.1 Origin of PAR

Mosia (2016:81-83) asserts that PAR is a subset of action research. Its origins can be traced back to 1944, as affirmed by MacDonald (2012:35). McNiff and Whitehead (2006:36) agree on the philosophy that people should be involved in the decision-making of their own teaching troubles. They further argue that these interactions

motivate them. According to Kindon, Pain and Kesby (2007:10), PAR is an iterative process that circles from action and reflection between co-researchers and me as the principal investigator. This process is known as an iterative cycle and/or spiral science, which is meant to address problems of the past such segregation and discrimination (MacDonald, 2012:35; Kindon *et al.*, 2007:10). This study afforded Campus Managers, HoDs, mathematics lecturers, former mathematics teachers, mathematics students and other stakeholders who are committed to enhancing the teaching and learning of MLP an opportunity to be at the forefront of decision-making processes (developing teaching strategies). They dictated this study research process through their involvement in planning, acting, observing and reflecting on the results of their actions.

According to Malebese (2016:91-92), PAR was observed in the work of Paulo Freire on a pedagogy of the oppressed in the late 1970s. The notion of his version of PAR, according to Malebese (2016:91-92), is that Freire (1970) advocated the empowerment of the poor and those who are pushed to the margin. Furthermore, PAR challenges positivistic paradigms that promote university-based research that makes no huge difference to peoples' lives, but instead, treats them as objects during the traditional era of qualitative research (Rocha-Schmid, 2010:344). The scholars who developed and used PAR are of the view that collaboration between two or more than two people are better than one person; thus, in their spiral approach to the proposed research actions and activities, they understood the impact of Freire's emancipatory agenda. Our research team decided to use the RME approach to respond to their daily challenges in MLP teaching and learning at TVET colleges.

The co-researchers in this study understood that they were the best people entrusted with incorporating the RME approach to enhance MLP teaching and learning at TVET colleges. This study also endeavoured to promote equality among citizens where the co-researchers and all people are treated fairly, equally and with respect, which is relevant to what South Africa and other countries in the rest of the world advocate. Maguire (1987:08, in Mosia, 2016:81-83) strongly affirms that PAR is also suitable for political or non-political organisations that share a vision of a society that seeks freedom from domination. MacDonald (2012:37) indicates that these kinds of movements are mostly found in arenas such as international development, social sciences communities, and adult education. He further links PAR with developments such as radical and reformist methodologies to international economic development

assistance; the view of adult education as an empowering alternative to traditional approaches to education; and an ongoing debate within the social sciences over the dominant social science paradigm. Reid, Tom and Frisby (2006:316) state that feminist researchers apply PAR to gather and analyse power differences based on gender. They also promote the significance of collaboration between the researcher and participants. Thus, this study allowed collaboration between Campus Managers, HoDs, Supervisors, mathematics lecturers, former teachers, mathematics students, stakeholders, and SRC members (including myself as principal investigator) at the research site. This team managed to identify MLP teaching and learning challenges and they mapped a way to solve those challenges identified (cf. 4.2 & 4.3). Malebese (2016:81-83) shares the same sentiment with other scholars who state that PAR is seen as a research methodology in the research process, since PAR encourages interaction between people and promotes cooperative efforts with an aim to empower a marginalised group of people. According to Reid *et al.* (2006:316), Feminist Participatory Action Research (FPAR) refers to conceptual and methodology framework that enables a critical understanding of women's multiple perspectives. FPAR works towards inclusion, participation and action, while challenging underlying assumptions that positivist researchers bring into the research process. Thus, PAR and FPAR allowed all co-researchers in this study to be treated equally, fairly and with respect. It also permitted all different genders to come together in a small cycle to investigate the challenges of teaching and learning of MLP concepts and find solutions to solve identified challenges at TVET colleges (cf. 3.4, 4.2 & 4.3). The methodology employed by Reid *et al.* (2016:316) was adopted and used in this study, since co-researchers' multiple perspectival views were recorded in the following: cycle planning, implementing, observing, reflecting, re-planning, implementing, observing, reflecting and evaluating processes of this study to achieve this study's research objectives. The co-researchers, and I planned, developed, presented, facilitated, observed, reflected, evaluated, re-planned, presented and facilitated MLP lessons to find what works or not during this research study processes (cf. 3.4). Rocha-Schmid (2010:344) states that the main goal of PAR is to allow people to be involved in the decision-making process and/or generate solutions to their social problems. PAR also includes identifying changes of immediate benefit to research participants, but the possibility of achieving goals of PAR is vested in a unique approach to research that is democratic by nature. Thus, the democratic nature principle in this study was

realised when all co-researchers and I as the principal investigator shared our experiences and challenges that contribute to poor performance in MLP. Together we would seek to find solutions to those identified problems (cf. 3.4, 4.2 & 4.3). The previous and current researchers employed PAR as an approach to solve different problems. However, the bottom line is to generate data and solutions for the problems experienced by lecturers and students in school communities, especially in education. Mhina (2009:159) argues that the power of PAR lies in its quest to work with the people who are the most affected by the problems that have been identified. This study agrees with this notion of using PAR in data generation, since the majority of the co-researchers are students and lecturers who are mostly affected by MLP (mathematics) teaching and learning.

3.2.2 Objectives of PAR

The methodology of Participatory Action Research (PAR) was relevant in this study. It helped this study to use the RME approach to enhance the teaching of MLP at TVET colleges, since it allowed interaction and sharing of knowledge among co-researchers and me as the principal investigator, offering us an opportunity to learn from one another (Malebese, 2016:90-93). Poland, Chen and Skinner (2007:127) signpost that in a process of implementing PAR methodology, people in or around school communities are involved in an investigation to address the issue of inequity and also to help people to resolve their own problems and initiate change while studying under the influence of those particular changes. Tsotetsi (2013:141) agrees with the other scholars on the fact that PAR methodology resolves complex issues related to social justice, inclusion and the employment of marginalised people while promoting interaction and collaboration amidst a principal investigator and co-researchers in a field of study. This study brings the PAR methodology in line with a Bricolage framework, because as a principal investigator, I have allowed and highly considered the knowledge that co-researchers brought into this study. Our main agenda was to advance equity, peace, freedom, hope and social justice at TVET colleges and communities. According to Moloi (2013b:480-482), PAR methodology contrasts with other approaches, which take information away from communities without giving back the recommendations on what has been found and learned. In PAR methodology, co-researchers actively participate in sharing information, experiences and ideas to guide

their future actions. The aim of PAR approach is to make sure that knowledge acquired on a daily basis is used to shape people's lives. Malebese (2016:92-93) argues that PAR practices generate data that are collaboratively analysed, knowledge rooted in social relations in social interactions, rather than the knowledge extracted for subsequent analysis by the academic researcher.

This study chose PAR as research method to show, expose and challenge inequality and the exploitation of co-researchers, but also to change co-researchers' social reality. PAR provides co-researchers with an opportunity to engage in acknowledging an existence of the problem; to study the problem; analyse the problem; and design methods to solve it. This participation can only be achieved by taking part in the act of change (Lybeck, 2010:91). The study also used PAR as a tool to empower co-researchers to become experts in solving mathematics problems through participation, according to the principles of PAR. MacDonald (2012:40-43) asserts that the principles of PAR, such as acknowledging and valuing that people have different values, there being no right or wrong suggestions; individuals learn by doing, strengthening their confidence or abilities and resources, as well as developing their skills in generation, analysis, interpreting and utilising information. For that reason I selected different people as research co-researchers; hence, generating a multiplicity of expertise to enhance the teaching of MLP at TVET colleges by means of the RME approach to help students to perform well in mathematics, rather than relying on a single teaching method.

3.2.3 Principles of participatory action research

According to MacDonald (2012:39), there are several principles of PAR methodology that guide researchers who rely on PAR as a research methodology to generate data; PAR is democratic by nature, and locates the voices of marginalised people at the centre of the research process. PAR works in a small cycle. It is a radical transformation of social reality and enhancement of the lives of people. According to MacDonald (2012:39), because PAR is democratic by nature, all people are given equal opportunities and space to participate fully in research processes. This study was guided by this principle, because all co-researchers were given equal time, opportunity and a platform to share their teaching and learning experiences freely. They were all seen as people with valuable knowledge and experiences, rather than

as an objects. The co-researchers were on the same level as the researcher in the modernist phase, unlike in the past, prior to or during the traditional era when only the researcher was entrusted to provide solutions to all problems facing people. The democratic system liberates, empowers and ensures that people are free from their oppressors or dictators; thus, I did not consider myself as an individual who knows everything, especially about using the RME approach to enhance the teaching of MLP.

The second principle of PAR is to locate the voice of marginalised people to construct knowledge in order to find solutions for their daily challenges (Mahlomaholo, 2012:2). Dupuis *et al.* (2014:95) affirm that the PAR involves a round, collaborative method whereby partnerships are built between those who have first-hand knowledge concerning the object of the study. This study relied on the co-researchers' teaching, learning, personal experience and their background knowledge to address challenges of MLP teaching and learning, integrating the RME strategy to teach MLP. MacDonald (2012:39) agrees that this principle is aligned with an active approach to improving the social practice through a change; congruency on authentic participation; collaboration; establishing self-critical communities; and involving people in theorising about their practices. Therefore, to achieve this objective, this study relied on collaborative processes, permitting different voices about MLP challenges in the research process as PAR and Bricolage require to find solutions.

According to McTaggart (1997:25), the third principle of PAR is about small cycles or groups. It expects participants to work as a team. A team together plans, organises, implements, evaluates, and re-plans to solve their challenges. This principle encourages participants to build records while allowing and expecting the participants to justify their views. MacDonald (2012:34) maintains that as people, we must accept and acknowledge that problems originate in communities. They should be defined, analysed, and solved by those affected in the communities. In this study I worked with people (co-researchers) who are affected by poor mathematics performance as well as those who are faced with the teething troubles of MLP. The co-researchers were given an opportunity to generate the multiplicity of teaching methods in responding to MLP problems.

The fourth principle of PAR is regarded as the radical transformation of a social reality and enhancement of the lives of people (Thompson & Boro, 2015:01-02); in this case,

the co-researchers in this study benefited because they gained knowledge from the interactions. The co-researchers shared their teaching experiences in terms of challenges and solutions of teaching and learning MLP (cf. 4.2 & 4.3). At the beginning of this study processes, this was exactly what they were assured of to benefit. Students were allowed to raise the learning problems they encountered in MLP to find solutions to overcome those challenges, whereas the lecturers, management and stakeholders were more aware of what frustrates or hampers the teaching of MLP, leading to poor students in MLP as well as in mathematics. All co-researchers (students, lecturers, management, other stakeholders) and I gathered around one table to found solutions for those identified challenges (cf. 4.2 & 4.3). This benefited all of us, since then we understood what were MLP problems and solutions, improving our understanding. In addition, lecturers shared what struggles they had, and their bosses (top or middle management) understood what struggles both students and lecturers experienced (cf. 4.2).

3.2.4 Formats of Participatory Action Research

This section of this study will discuss research methodologies such as Feminist Participatory Research (FPR), Community-Based Participatory Research (CBPR), and Community Cultural Wealth Participatory Research (CCWPR) as formats of the Participatory Action Research (PAR) that aim to close gap that PAR might have. In addition, these formats are in support of the use of PAR as research methodology and talks the same language with PAR.

3.2.4.1 Feminist Participatory Action Research

Maguire (1987, in Mosia, 2016:87) argues that the PAR can be explained in the feminist view, which combines different activities of social investigation, education and action in a collective process. According to Mosia (2016:87), a social investigation activity of PAR includes its research orientation, which anticipates investigating social challenges and finds its solutions through interaction with the subjugated ordinary people in the research processes. Moreover, Mahlomaholo (2009:224) asserts that, from a feminist point of view, PAR is used as a critical emancipatory approach to research that aims to empower oppressed and subjugated ordinary people. This study explained PAR from a feminist perspective and viewed PAR as an approach that seeks to open spaces for women, less educated and marginalised groups of people

in the research process in order to empower them with knowledge generated. This is possible when people who are faced with the problems are involved in the research processes in examining factors contributing to problems that they faced, finding its solutions in a collective processes. This study allowed us to:

- work together to investigate the challenges of teaching Mathematical Linear Programming (MLP);
- develop a way to use the Realistic Mathematics Education (RME) approach in enhancing the teaching of the Mathematical Linear Programming (MLP) at Technical Vocational Education (TVET) colleges;
- identify the conducive conditions to enable the RME approach to work well and successfully;
- examine threats that can hinder the success implementation of the RME approach; and
- find the evidence of success for using the RME approach (cf. 4.2-4.5).

MacDonald (2012:38) shares the same sentiment with scholars on the fact that action activity of PAR is designed and fit to create ways for researchers and marginalised co-researchers to come together and collectively take action, on the short and the long term, in order to pursuit a radical social change. In addition, from a feminist perspective, PAR involves three forms of change, namely development of critical awareness; advancement of lives; and transformation of societal structures and relationships of researchers and co-researchers (Van Dijk, 2008:86).

3.2.4.2 Community-Based Participatory Research

According to Minkler and Wallerstein (2008:1-2), the Community-Based Participatory Research (CBPR) is viewed as a collaborative approach in the research processes that take place within the community concerned where people decide on the kind of challenges to be researched. Therefore, CBPR rejects a traditional way of conducting a research where only a researcher decides what, when and how to ask questions and how to respond to those questions. During the traditional era, a powerful researcher is the only one who drives research processes by deciding what type of challenges to be investigated, whereas in CBPR a community decides together with a researcher what should to be researched, as is the case with PAR as methodology

(Minkler & Wallerstein 2008:2). Mahlomaholo and Netshandama (2012:36) argue that the research should focus on improving the lives of people by helping them to solve the challenges they face, furthering that one of the roles of co-researchers is ensuring that the research achieves this objective. According to Cornwall and Jewkes (1995:1667), CBPR and PAR generate a kind of action research that is more a multiplicity of methods; promoting and assuring mutual respect, co-learning, capacity building, and a balance between research and action. CBPR helped this study to select, invite and allow people who are affected the most by poor performance of mathematics, specifically MLP, to come together to root out the challenges in teaching and learning MLP at TVET colleges; develop ways to implement the RME approach in teaching MLP; investigate conducive conditions to enable the RME approach to work successfully; identify factors that are possible threats to the success of the RME approach; and to examine key success factors for the use of the RME approach. I as a principal investigator collaborated with my co-researchers to detect challenges contributing to poor performance in MLP at TVET colleges; find a way to use the RME method to enhance teaching of MLP at TVET colleges; identify the conducive conditions that enable the RME approach to work; detect risks that hinder the successful implementation of the RME approach; and examining evidence of the effectiveness and efficiency of the RME approach through the lesson presentations, lesson observations and discussions in the research meetings as one community.

Therefore, this study considered the CBPR as important approach, because it gave us clear direction; hence, we managed to achieve this study's research objectives. The CBPR as a collaborative approach allowed us to respond to complex challenges of teaching and learning MLP through the knowledge that team members brought in the research deliberations. The team's shared and learned knowledge enabled them to solve their teaching and learning challenges, since my co-researchers comprised people who faced these challenges directly. More vitally, during research meetings, we shared the duties and responsibilities (chairing meetings, noting and recording generated data, presenting lessons and observing lessons, organising venues and resources, keeping time). We planned the agenda for these meetings together as a team, guided by the study's research objectives. This is in accordance with one of the doctrines of the CBPR, which states that the community should play a central role in the research process. We practised mutual respect; hence all team members worked

freely in a relaxed environment. Co-learning and capacity building were possible in the research processes, since together we shared responsibilities and managed to achieve this study research objectives. In addition, balancing research and action was also achieved, as all benefited from this research; not only one person, as the aim of this study was to enhance the teaching of the MLP at TVET colleges.

3.2.4.3 Community Cultural Wealth Participatory Research (CCWPR)

This study explains Community Cultural Wealth Participatory Research (CCWPR) as a theory that informs researchers and other people in the public or private sector that there is valuable information in the communities from the social, economic, cultural and other backgrounds that needs to be considered in a research process. Thompson and Boro (2015:1) agree with Vygotsky that people are shaped by different environments – economic, social, cultural. Therefore, as researchers we should open space for different backstories. For that reason that this study allowed different people from different milieus be part of the research process; thus this study promoted social equality and recognition. According to Yosso (2005:69-71), Community Cultural Wealth (CCW) is meant to empower marginalised people and allow their voices in the research process. Yosso further argues that CCW is created to benefit oppressed people by means of aspirational, linguistic, familial, social, navigational and resistance, which should be adhered to when research is conducted. This study shared the sentiment with scholars who embrace and view CCW as a vital tool to empower people; hence, this study was aligned CCW with PAR because both were developed to benefit people. The six forms of CCW, according to Yosso (2005:69-71), are:

- Aspirational capital is viewed as hopes and dreams that people in communities, especially students have when they enter schools. For instance, communities, parents and other stakeholders have high educational aims for their children as they go to school despite persistent inequalities within our country. Aspirational capital alerted this study to this.
- Linguistic capital is viewed as a vital tool to understand people and the settings they come from. Linguistic capital refers to languages and communication skills of co-researchers which they use to participate in the research processes. This study employed co-researchers teaching and learning experience, background, backstories and cultural knowledge in this study research process. Moloi

(2013b:481) supports the accommodation of back stories as empirical data, because these stories are part of the co-researchers' lives, especially before arriving at any schools, colleges and universities. Yosso (2005:79), on the other hand, signposts that individual cultural experiences (storytelling) assist co-researchers, especially students with skills such as memorization, attention to detail, dramatic pauses, comedic timing, facial affect, vocal tone, volume, rhythm and rhyme. These are all skills that researchers, facilitators and students use as a solid foundation for academic success.

- Familial capital is explained as education that takes place through familial and community networks. This is affirmed by Moloi (2013b:482), arguing that before students come to any school, they already have background knowledge that should be searched and added onto in the teaching and learning process. In this regard, co-researchers in this study were treated as valuable people with knowledge rather than as an objects, assisting this study to generate data. As a result, co-researchers were also empowered to solve their challenges in the teaching of MLP at TVET colleges.
- Social capital occurs when people come together to solve the problems they face. This doctrine allowed co-researchers this study to implement, observe, reflect, re-plan, implement, observe, reflect and evaluate the research processes for this study achieve its objectives. Together we managed to investigate challenges in the MLP teaching and learning at TVET colleges; develop the way to use the RME approach to enhance the teaching of MLP; examine the conducive conditions to enable the RME approach to work; identify risk factors to hinder the successful implementation of the RME approach; and detect the key success factors using the RME approach.
- Navigational route: happens when individuals progress from one level to another. Yosso (2005:71) explains how students value this social contact. This allowed the research team to work together to plan, implement, observe, reflect, re-plan, and evaluate our research processes for this study achieve its own objectives. Together we decided the on agenda and shared responsibilities; thus, co-researchers learned new knowledge and skills from one another which helped them to navigate from one level to another.

- According to Yosso (2005:71), Resistance capital is rooted in the experiences of societies of colour to secure equal rights and collective freedom. The sources of resistance capital come from parents, community members and a historical legacy of engaging in social justice. This principle guided and directed this study to promote the equality, fairness and ensure the freedom in the research process.

3.2.5 Ontology PAR

The term *ontology* refers to what people believe to be reality or truth (Schraw, 2012:1). The ontology of the PAR in this study is that people should be involved in the decision-making, which involves their lives. Hence, the co-researchers in this study comprised people who experience challenges with the teaching and learning of MLP, leading to poor mathematics performance at TVET colleges. Another PAR ontology is that people have valuable knowledge that should be considered in research, especially when generating data, because this type of knowledge comes from their working and personal experiences, and community background. Lastly, the other PAR ontology in this study is that people working as a team or in collaboration achieve more, because the team has the ability to find multiple teaching and learning methodologies, pedagogies and strategies to solve apparent problems. Therefore, these ontological stances of PAR helped this study to be aware that people have valuable personal and working experiences, as well as back stories that can assist this study to achieve its own research objectives. For this reason, the co-researchers were given an opportunity to share their views with regard to MLP teaching and learning challenges, assisting them to understand individual problems (cf. 4.2.1-4.2.5). Thereafter, our team developed a way to employ the RME approach in enhancing the teaching of MLP at TVET colleges.

During the research processes, I realised that the co-researchers had valuable knowledge and the power of working in collaboration as researchers. I realised that I as a single researcher did not have the answers to all the frustrations and challenges of teaching MLP, but among my co-researchers the answers were right there. So, the collaboration led to multiple teaching and learning methods. This is affirmed by McTaggart (1997:8), who maintains that a socially inclusive approach of teaching and learning is grounded in several different ways of realities, rather than in other methods

of doing research. Social inclusivity involves the assistance of co-researchers to develop new abilities and be empowered. Malebese (2016:103) asserts that reality differs according to the social context, arguing that reality is constructed through human interaction and meaning of actions. Kincheloe (2009:109) is of the view that in a social environment there are a multiplicity of realities, because people use their own knowledge to make meaning. Therefore, this study relied more on the interactions between my co-researchers and I to generate data and to fulfil the principles of PAR principles. Additionally, the benefit of interacting and collaborating with other people is that each one has a different lens looking at the world, assisting this study to achieve its research objectives.

3.2.6 Epistemology of PAR

Epistemology refers to what people believe (consider) on: What is knowledge? How is knowledge acquired? What do people know? This enabled me as a principal investigator to understand that people have valuable knowledge (with or without a formal certificate), that was acquired in the formal and informal school, as well as in cultural, economic, political, technological, and other environments. This is supported by Community-Based Participatory Research (CBPR) and Community Cultural Wealth Participatory Research (CCWPR). The literature showed that knowledge is acquired through empiricism, rationalism, representationalism and constructivism (Moser 2010:3-6; Tennis 2008:109; Stables 2017:1-2; Tubbs:2016:1-3). In light of the above information, Tennis (2008:102) views constructivism as one of the theories that people use to acquire new knowledge. Hence, this study's epistemological stance is that students and lecturers make meaning in MLP concepts, using their background knowledge or experiences, especially regarding mathematical word problems, linear inequalities and equations, graphs of linear inequalities, and equations learned in previous grades or NCV levels. This has assisted this study to achieve its objectives, and this became possible when co-researchers were given equal opportunities to share their past experiences of teaching and learning MLP.

On the other hand, James (2006:9-10), Vygotsky (1978, in Topciu and Myftiu, 2015: 173) signpost that knowledge is social a thing and is socially constructed. This notion is supported by Kendall *et al.* (2011:4), who state that the PAR lens is that knowledge is socially constructed, embedded within the systems of values, beliefs and norm, and

understood through mental processes and promote interactions. Thompson (2000:14) furthers that PAR reflects an investigation into the nature of knowledge and also people conceptualise a knowledge in a unique way. However, Herr and Anderson (2005:2) highlight that knowledge must be shared in collaboration in order to improve the existing poor performance of literacy situation in schools, but in this regard the poor performance of MLP at TVET colleges. McNiff and Whitehead (2006:78) emphasise that conventional researchers believe that social reality is constructed; thus this notion enabled this study to negotiate knowledge in a social inclusive manner to develop a way to use the RME approach to enhance the teaching of MLP at TVET colleges.

This study's epistemological stance was that knowledge is owned equally by the co-researchers and is shared or learned through interactions in the culturally appropriate modes of communication; as a result co-researchers were empowered. This was also a key success in Molo'i's work (2013:126), because people or communities that were deprived in the past were empowered and encouraged by the work of PAR. In this regard, the knowledge was integral to me as a principal investigator's ability to interact in effective and respectable way with my co-researchers to avoid misunderstandings between us that could hamper the research process and lead to the failure of achieving this study's research objectives. The challenges contributing to MLP poor performance was examined to construct new knowledge to help this study to use the RME approach to enhance teaching of MLP at TVET colleges. This study has used PAR to allow co-researchers to share their knowledge, challenges, experiences, and best practices in the MLP at TVET colleges. This enabled us to integrate best practices in the principles of the RME approach to enhance the teaching and learning of MLP at TVET colleges. The co-researchers felt empowered through the use of PAR as a methodology.

3.3 ETHICAL CONSIDERATIONS AND DATA GENERATION PROCEDURES

As a principal investigator I applied for the ethical clearance all the institutions concerned, to enable me to enter this TVET college's premises on five Campuses to commence my research. I met with Campus Managers (CMs) and/or Head of Departments (HoDs) where CMs were not available. I introduced myself as the principal investigator and shared what this study was all about and what it aimed to achieve, providing the CMT with proof of my ethical clearance as a proof that I had

followed the correct channels to conduct this study. The CMT members was very pleased to hear about what this study aims to achieve and wanted to participate in this research study because they mentioned that mathematics was very challenging for their students at the TVET college.

CMT: *“Ntate mathematics mona ke mathata feela, jwale re dumela hore ka nnete something needs to done as urgent to assist students and college (Sir mathematics is very problematic here, so we believe that something needs to be done as urgent to assist students and college).”*

Furthermore, they wanted to know where, when, and how this research study would be conducted. I shared the invitation letter which revealed this study title, aim, objectives, value, venue, day, time and invited people (selected participants) (cf. Annexure F). The CMTs at Campuses of this TVET college were very helpful because they made sure that the people who would participate in this research got invitations; two CMs, two HoDs, two NCV Supervisors, NATED Supervisor, three mathematics lecturers, an English lecturer, a SRC member, a former mathematics teacher, and other stakeholders. However, I did not leave those responsibilities to the CMTs only, but I also invited the selected people to participate in this research study to the first research meeting. At the first research meeting of this research study, I explained what this study was all about.

More importantly, I informed the people who would participate in this study that they would be regarded as co-researchers, not participants, because I as the principal investigator would collaborate with them to plan, develop, organise, implement, reflect, re-plan and repeat this cycle in a spiral cycle of PAR, since this research study aimed to generate data through focus-group discussions, lesson observations, lesson presentations and free interviews (research techniques). I assured them that through this spiral cycle of PAR and research techniques, co-researchers would be empowered and gain knowledge on which challenges existed in the teaching and learning of MLP at TVET colleges; how to use the RME approach to enhance teaching and learning of MLP at TVET colleges; under what conditions the RME approach works or does not work; and whether the use of the RME approach was effective or not. This was achieved in this study (cf. 4.2-4.5) and proved to be more beneficial than what traditional paradigms offers.

Thereafter, I issued the consent forms drafted in multiple languages, English, Sesotho, isiZulu, isiXhosa, Sepedi, Tshivenda and Setswana, because I anticipated that the people invited spoke one of these languages, and each person was allowed to choose the consent form written in his/her language preference. The people who were present at the meeting were Sotho, Zulu and Xhosa people. I first requested a person who understood isiXhosa to read the isiXhosa consent form, followed by a person who understood isiZulu to read the isiZulu consent form, followed by a person who understood Sesotho to read the Sesotho consent form. Then I read the English consent form written and interpreted the content. I allowed them to ask questions where they did not understand to get clarity, but no-one asked questions. I informed them that people who were going to participate in this study were allowed to join, withdraw and also re-join at any time, when they wished so, but no-one would be forced or denied to participate. However, I alerted people who would be participating in this research study that they were all expected to follow and uphold ethical standards and principles. I also made them aware that this study aimed to use audios, videos, cameras and notebooks to capture generated data to do data presentations, analysis and interpretations. Audios, videos and cameras would be used only if the people participating were comfortable with those research instruments, although their faces would be hidden. If they were not comfortable with this, it would not be used. In this study co-researchers agreed to the use of all the instruments to capture data.

I notified the people invited that this study would need students' scripts and work in order to identify students' MLP errors, mistakes and misconceptions, which contribute to students' poor performance in mathematics. This would be used as a source and for data presentation, analysis and interpretation, but the students' personal details as well as the campus details would not be disclosed; instead, pseudonyms would be used. I also requested CMTs, HoDs, lecturers and parents to encourage mathematics students to attend the planned lesson presentations of this research study. Such lessons would disturb daily TVET college operations, work schedules, the calendar and assessment plans. Students aged 18 or more would be provided with consent forms to participate (cf. Annexure H), while the parents of those under the age of 18, would have to give permission for to their children to participate in the study (cf. Annexure I).

The CMTs arranged the venue, time, day and class for the lesson presentations, because they were of the view that it would benefit their students. I strongly maintained during the meeting that the research study would not endanger their lives and that there were no incentives. Everyone had to go home with the consent forms to read them in their own time, to ensure that they clearly knew what was expected. They could submit the consent forms at the next scheduled meeting. At the next research meeting, this study began, with planning, developing, organising, implementing, reflections, re-planning, etc., and forthwith they were regarded as co-researchers.

3.4 RESEARCH DESIGN

This section outlines how the current study was guided and executed by the principles of PAR. The following was done to pursue objectives of this study: establish a common goal; formulate research questions; create a profile of each co-researcher in order to know each co-researcher's strengths, weaknesses, opportunity, and threats.

3.4.1 The research site

This study was conducted at Kgotsong TVET college (pseudonym) located in the Free State Province. This TVET college consists of 300 lecturers and 10 000 students, of which 3 411 are males and 6 589 are females. There is good planning at this TVET college as it strives to achieve its vision and mission through involvement, dedication and commitment of its members. It is registered with the South African Bureau of Standards (SABS) and other bodies that promote and maintain quality education and administration. This TVET college is a public college; therefore it depends heavily on government support and most of its students come from needy communities. However, this TVET college was selected because it was not doing well in mathematics NCV (L4), and MLP was a problematic mathematics topic at this college.

The research meetings, where focus-group discussions, free interviews, lesson observations and lesson presentations took place, were held at one of its campuses (or satellites), which has 63 lecturers and 2 219 students, of which 818 students are male and 1 401 students are female. The majority of the research participants (co-researchers) came from this campus, because 25 students from the mathematics class, mathematics lecturer, SS, HoDs and CM came from this campus. However, the

other co-researchers (lecturers, SS and HoDs) came from the other three campuses that offer mathematics NCV (L4) in the same TVET college. However, the purpose was to find whether MLP was problematic only at Kgotsong TVET college or not. The other participants were included to share their personal teaching experience, challenges and strategies of teaching MLP.

3.4.2 Initial meeting

This study aimed to enhance the teaching of MLP at this TVET college to respond to the MLP teaching and learning challenges, which, as proven by annual reports (cf. Annexure E) and literature (cf. 2.4.1) were difficult for students. Apart from the interventions that the Department of Higher Education and Training (DHET), working together with colleges had done, students were still performing poorly in mathematics at TVET colleges. This became a regular task for lecturers and CMT at this TVET college to develop subject improvement plans for mathematics. The plan included the utilisation of mathematics tutors, a team teaching approach and the invitation of guest lecturers from within and/or outside this college. Furthermore, lecturers attended subject meetings of mathematics. The main purpose of these meetings was to enable lecturers from different campuses to come together to share mathematics teaching and learning challenges they encountered on a daily basis and to share best practices and find solutions to the identified problems. However, lecturers indicated that these efforts were not effective, because some were well coordinated and some did not happen. Therefore, mathematics was still one of the problematic subjects at this college, which were evident from students' poor performance in the 2019 November examination. Thus, this study was prompted by a need to respond to the mathematics teaching and learning challenges

After obtaining permission to enter this TVET college and to conduct research at this TVET college, I made an appointment with the Campus Manager who agreed to see me. Upon introducing myself as the principal investigator, I explained what this study aimed to achieve.

Campus Manager: *"You know, Ntate (Sir), mathematics is one of the problematic subject here at our campus, in fact at our college and for a long time. Then, I know this kind of research projects are very helpful, so I want my CMT members and mathematics lecturers to participate in this study. So, I will speak and ask them to participate, especially on Fridays because we are knocking off at 13:00. So, leave extra participants invitations. Again, I like the way this*

research aims to unfold like allowing us to come together to find problems and also share information on how to solve those problems.”

At the first meeting with the co-researchers we all introduced ourselves. I explained what this study was about, and what this study aimed to achieve. Co-researchers shared the sentiment of Mr Kapu (Campus Manager).

At the following meetings, we had fruitful discussions, understanding the nature of MLP teaching and learning challenges. Co-researchers were eager to share best practices and develop a strategy to employ the Realistic Mathematics Education (RME) approach that would be relevant to and/or realistic in relation to their daily experiences in mathematics teaching, specifically MLP, with the goal to improve students' performance. Interactive sessions assisted this study to find efficient ways of enhancing the MLP teaching utilising the RME approach. According to Chilisa (2012:250) and Dodson and Schmalzbauer (2005:953), initial meetings opens a room for a degree of trust and unity between co-researchers. During initial meetings, PAR was a core method to generate data in this study, because it promoted and empowered marginalised and oppressed people by giving them space and the freedom to express themselves and exchange ideas and opinions. This benefit helped this study to use Bricolage where co-researchers were free to raise their voices in producing multiplicity methods (Roger, 2012:1). This study's ontological stance is that research take place in an active process, where all participating people are equal, irrespective of their profession, and one only participates freely (Kincheloe, 2004a:1).

3.4.3 Formulating a research question

This study allowed interaction among co-researchers in finding ways to teach MLP, aligned to the RME approach. The team established a common goal, namely to mathematize the teaching of MLP. Together we identified teaching approaches that would make abstract MLP components more concrete and within the context of students, because annual reports and the literature revealed that the students struggled with MLP. The identified pedagogical methods were aligned with the RME approach, with the aim to improve the teaching and learning of MLP at TVET colleges, guided by doctrines of the RME approach. To achieve this study research objectives, teamwork between team members should be at the centre of the research process in order to work towards the same goal. Therefore, during research meetings, all co-

researchers (including myself as the principal investigator) shared teaching and learning experiences, best practices and MLP knowledge. We checked other countries that were doing well in mathematics to learn and understand what contributed to their successes. However, no country used the RME approach to teach MLP specifically, and all the co-researchers relied only on the traditional way of teaching mathematics. As a research team we formulated the question: How to use the RME approach to teach MLP?

3.4.4 Research participants (co-researchers)

This section will discuss the research participants (co-researchers) of this study and outline why the following co-researchers were selected.

3.4.4.1 Council Member

In the TVET sector, the highest body governing the institution is the College Council (CC). The College Council consists of people from each section of the college and members of the community. The CC's role is to establish and monitor processes to set strategic direction, establish the purpose, values, goals and objectives of the college, as well as to identify and manage results or performance expectations and monitor achievements. The CC is believed to have knowledge on what should be done to improve mathematics results, as mathematics is seen as a problematic subject. The Council member in the team was believed to have knowledge of mathematics, since she had worked as a mathematics teacher and facilitator at different schools (Grades 10-12) and university as well.

Table 3.1: Council members

Name (pseudonym)	Mrs Mahapa (former FET phase mathematics teacher)
Teaching experience	30 (mathematics)
Qualification	National Diploma in Education

3.4.4.2 Mathematics coordinator

The mathematics coordinator is the Campus Manager (CM) responsible for executing college plans, e.g. assessment plans, planning assessment schedules, in-house training, workshops and mathematics subject plans. The mathematics coordinator also identifies mathematics curriculum issues that concern mathematics lecturers and

students; and thereafter the mathematics coordinator works with the Deputy Academic Principal, Curriculum Manager and mathematics lecturers to improve mathematics teaching and learning. The mathematics coordinator is believed to have valuable information such as mathematics reports, intervention plans and annual minutes of mathematics meetings. These documents were very helpful to this study, because the mathematics coordinator brought very important information about the teaching and learning and intervention plans that aimed to strengthen MLP teaching and teaching to the meeting.

Table 3.2: Mathematics coordinators

Name (pseudonym)	Mrs Seeta
Mathematics coordinator experience	07
Managing experience (campus manager)	07
Committee serving	Academic board

3.4.4.3 Campus Managers (CMs)

Technical Vocational Education and Training (TVET) colleges today have grown and opened many campuses (satellites) with the purpose of empowering and enriching people's lives. TVET colleges appoint Campus Managers (CMs) to manage and ensure that teaching and learning take place at all campuses (satellites). This is why they were selected to participate in this research process. CMs are believed to have an interest in the teaching and learning process and improving mathematics results. CMs are also believed to know and understand the challenges in the teaching and learning of mathematics. Moreover, CMs were chosen because they have massive TVET knowledge and experience in the TVET sector. They were given an opportunity to share their past experience and we developed a way to implement the RME approach in teaching the MLP at this TVET college. CMs were also selected because we used the campus premises at the TVET college which they managed. Thus, CMs played an oversight role to check and monitor what this study promised in consent forms and request letters to conduct research, ensuring that this study was not harmful to co-researchers or the college premises.

Table 3.3: Campus managers

Names (pseudonyms name)	Mr Kapu
Managing experience as CMs	10 years plus

Mathematics teaching experience in Ls & Ns	20 years plus
Highest qualifications	Master's in mathematics
Professional Teaching Certificate	PGCE
Serving committee	Academic board

3.4.4.4 Heads of Departments (HoDs)

The focus of HoDs is mainly on teaching and learning at TVET college campuses. The HoDs' responsibilities are the following: develop class timetables; work with other CMTs members; allocate subjects to academic staff members; monitor the teaching and learning process; work with other CMTs to conduct and implement the Quality Management System (QMS); conduct in-house training or workshops to inform academic staff of DHET and TVET colleges policies; monitor files; conduct classes; and implement TVET college plans. The HoDs' other responsibility is to oversee and ensure that teaching and learning run smoothly. Moreover, if there are challenges in the teaching and learning process or in a particular subject, the HoDs work closely with lecturers to come with interventions for identified problems. Thereafter, HoDs will present the intervention plan to the College Academic Team (CAT) to be further helped by the college if strategies required financial assistance.

It was in light of the above duties and responsibilities of HoDs that this study selected HoDs to participate. The HoDs have an interest to see the enhancement of MLP teaching and learning at TVET colleges using the RME approach, as it is known that MLP is a problematic topic. HoDs would also bring vital MLP, mathematics and teaching and learning information to this study because they evaluated the teaching and learning process every year through class visits, spot checks and QMS.

Table 3.4: Heads of departments

Names (pseudonyms names)	Mr Kia	Ms Khaba
Managing experience as HoDs	06 years plus	12 years plus
Teaching experience in Ls & Ns	15 years plus (in Maths & Maths Lit.)	20 years plus (in Maths & Maths Lit)
Highest Qualifications	B Ed Degree	National Diploma & BTech
Professional Teaching Certificate	PGCE	PCGE

3.4.4.5 Supervisors

Supervisors at TVET colleges play a vital role in the teaching and learning environment and are regarded as Subject Specialists (SS) and Senior Lecturers (SL). Supervisors at TVET colleges are required to work with HoDs to develop class timetables; monitor the teaching and learning process; work with HoDs to conduct and implement the QMS; monitor files (checking necessary documents such as work schedules, lesson plans, syllabus and tasks); conduct classes; and implement TVET college plans. They also supervise and assist lecturers with problems they encounter in the teaching and learning process and work hand in hand with CMs and HoDs to ensure that teaching and learning take place. For these reasons, this study selected Supervisors to participate in this study, as they would bring valuable knowledge of MLP challenges to the meetings and also share their teaching methods.

Table 3.5: Supervisors

Names (pseudonyms)	Mr Kanana	Mrs Leeuw	Ms Moleko
Managing experience as Supervisors	13 years plus (former)	7 years plus	11 years plus
Teaching experience in Ls & Ns	20 years plus (Maths)	15 years plus	18 years (in Maths & Maths Lit)
Highest Qualifications	National Diploma	Science degree	BSc degree & BEd Honours
Professional Teaching Certificate	PGCE	PGCE	PGCE

3.4.4.6 Mathematics lecturers

Mathematics lecturers in the TVET sector's main responsibility is to do spadework, which is to teach students Mathematics, and they are expected to follow and implement DHET and TVET college plans and policies, namely the attendance policy, assessment policy, Internal Common Assessment (ICASS) policy and other policies. They are further expected to plan and prepare lessons; set internal tasks; teach and assess students; mark and give students feedback; and invigilate internal examinations. A lecturer or instructor is a person who teaches, educates, or train other persons or who provides professional educational services at any college (Kanyane, 2016:14; RSA, 2006:8). However, lecturers need to play many roles, according to different scholars. It is for this reason that mathematics lecturers were selected for this study, because they play a major role in the teaching and learning. Without them, the teaching process will not take place appropriately. Mathematics lecturers were

selected to participate in study because of their experience in the TVET sector and because they had taught mathematics NCV L3 and L4 for many years, so they also understood what challenges lecturers face in mathematics classrooms on a daily basis. Mathematics lecturers are believed to know the MLP challenges in teaching and learning. Therefore, they could share their teaching experiences, which assisted this study to achieve its objectives. More importantly, their involvement was very beneficial to other co-researchers and themselves, because we learned what the challenges of MLP teaching and learning were and what should be done to improve MLP results.

Table 3.6: Mathematics lecturers

Names (pseudonyms)	Mrs Lebone	Ms Khodu	Mr Koena	Mr Takui	Mrs Mosomi
Teaching experience in Ls & Ns (Maths & Maths Lit.)	10+ years	12 years plus	20 years	8 years	18 years English
Highest Qualifications	BEd degree	BSc & BEd Honours	BSc & BEd Honours	National Diploma	National Diploma
Professional Teaching Certificate	PGCE	PGCE	PGCE	None	PGCE

3.4.4.7 Mathematics NCV L4 students

Mathematics NCV L4 25 students were invited to participate in this study because they were doing mathematics NCV L4. The age group of these students ranged from 17-40 years, but only two students were 17; the rest were at 19 years and above. There were 15 female and 10 male students and the majority of these students came from disadvantaged communities and families, because of unemployment, poverty, crime and inequalities. During my first meeting with a CM at the campus, he raised concerns about mathematics results on all levels (L2-L4) and said,

“Students are not doing well in mathematics across levels (L2-L4) year in year out, especially in mathematics NCV 4. We are doing what we can do just to improve this results.”

Thus, the CM was very happy after I had shared with her what this study was all about and provided her with the UFS ethical clearance letter, college approval letter to conduct this research study and concern forms. At the next meetings, other CMT members shared the same sentiment with the CM. Mathematics lecturers went even

further and highlighted mathematics topics that are problematic and challenging to students:

“Students struggle with the following topic, linear programming, algebra, geometry, and trigonometry, especially with graphs.”

This was discovered during the lesson when one of student said:

“Linear programming e thata hle nna ya nhlola. Ke ya e tlola ha be batle ho l ferekanya (linear programming is difficult and I am unable to solve it. I skip it because I do not want to confuse myself).”

For this reason, they were included in this study.

3.4.4.8 Community members (parents and businessperson)

This study chose three community members as a parents and a businessperson due to the fact that schools, colleges and universities belong to the community. The community is believed to have an interest in the daily issues facing school activities and assists in solving those hindrances. Durisic and Bunijevac (2017:149) and Ntekane (2018:3), and the government agreed that a community and parents should be involved at schools. The parents are expected to play a central role in their children’s education, because instructors cannot succeed on their own. The Department of Higher Education and Training (DHET) and Technical Vocational Education and Training (TVET) college reports showed that MLP was a problematic topic because students had been performing badly in this topic for years. The community shared valuable knowledge, which helped this study to achieve its objectives. The community knew what was happening in the college environment; whether or not at this campus daily school activities were fulfilled; whether or not students attended school or classes; whether or not lecturers attended classes, knowing the challenges and other information. This community involvement is in line with PAR principles and Bricolage Postmodernity (cf. 3.2 & 2.2.2.5) because Mahlomaholo (2012:04; 2013:379) strongly maintained that amongst people within communities there was valuable knowledge that was key to improve MLP teaching and learning at TVET colleges using the RME approach.

Table 3.7: Community members

Name (pseudonyms)	Mr Khetla	Mrs Matlou	Mrs Mohlomi
Work experience	Business manager	Former Maths teacher	20 years (small farm co-owner)

3.4.5 Plan of action

The research team developed a plan of action that would respond to the five objectives of this study, namely

- investigating challenges in the teaching and learning of MLP;
- examining how to enhance MLP teaching and learning using the RME approach;
- exploring conducive conditions to enable the RME approach to work;
- examining risks and threats that can hinder success implementation of the RME approach; and lastly,
- examining the evidence of success of the RME approach.

The plan of action that was developed and drawn up comprised the following main constructs:

- a) activities that examined challenges in teaching MLP;
- b) activities that investigated students MLP misconceptions, errors, and mistakes;
- c) activities that investigated challenges to Mathematical Linear Programming Content Knowledge (MLPCK); and
- d) activities that examined challenges to the Mathematical Linear Programming Pedagogical Content Knowledge (MLPPCK).

The co-researchers agreed with one another that an investigation of MLP teaching and learning challenges should be prioritised. This study used PAR cycles that went through these stages: planning; acting; observing and reflecting; re-planning; acting; observing and reflecting again, in a spiral science fashion.

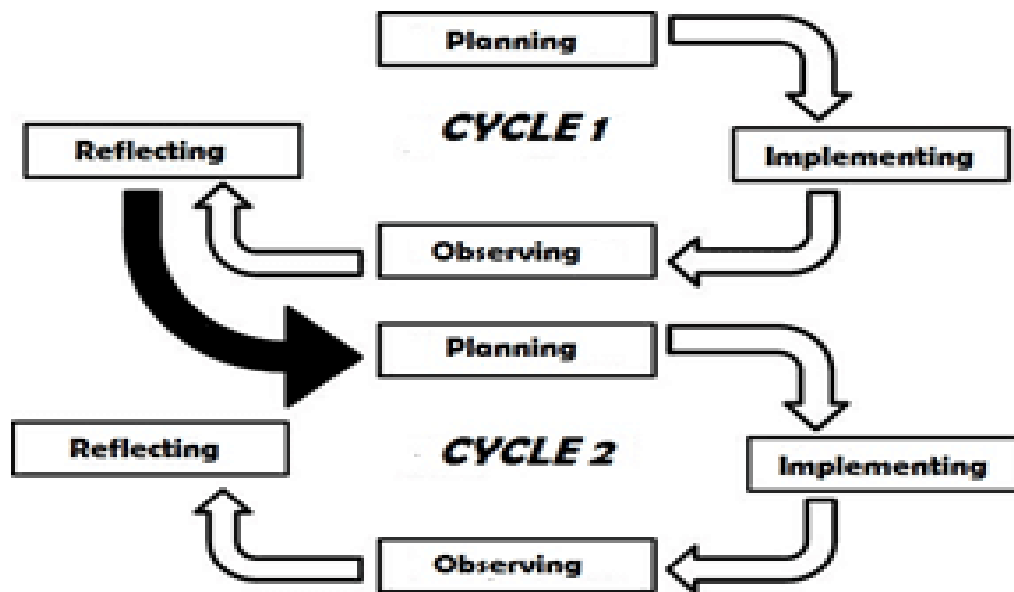


Figure 3.1: The spiral process of PAR (adapted from Reid et al., 2016:316)

3.4.6 Co-researchers' activities

At our first research meeting, all were present, except a council member, one HoD, two Supervisors and one parent (farmworker) due to other commitments on that particular day. The available team was dedicated to understanding the nature of the problem of MLP. They decided to come up with subject improvement plan for mathematics for this TVET college, as it was required for the worst-performing subjects.

3.4.6.1 Activity 1: Mathematics subject improvement plan

The team developed a mathematics subject improvement plan, which entailed the challenges (errors, mistakes, and misconceptions), possible causes and specific interventions and strategies. The team recommended the use of adequate student scripts of activities, tests and examinations; and markers and moderators reports from Department of Higher Education and Training (DHET), Department of Basic Education (DBE) and TVET college reports. The team was of the view that it was through understanding the nature of students' errors that one could understand MLP challenges and find the root causes. Copies were made of these, but their real names were hidden; pseudonyms were used for ethical purposes. This is in line with what Van Niekerk and Van Niekerk (2009:137) postulate because, according to these researchers, planning helps PAR research teams to allocate resources where they are

needed the most. The analysis of students' errors were done and every team member were given enough time to perform this activity and report back. We as a team had discussions about students' mistakes and uncovered the following errors: misconceptions and inabilities, namely formulating linear inequalities and objective functions from problem statements in MLP; problem solving and sketching of linear inequalities; showing feasible region; and determining maximum and minimum values using the search line method. This affirmed what the DHET markers and moderators signposted in their annual reports. The team members were guided by the spiral science of PAR (planning; acting; observing and reflecting), because the team planned ways to diagnose students' errors through an analysis of adequate DHET annual reports and students' scripts. All team members were afforded time to express his/her own view, in line with PAR principles and Bricolage post-modernity. The planning phase in PAR helped the research team to define objective of the activity and compare it with the objectives of this study. Priority was given to investigating and identifying the MLP concepts that were difficult for students. The discussions benefited all team members benefited and they learned from one another. Based on the discussions, the team developed a plan of action (Table 3.8).

Table 3.8: Mathematics subject improvement plan

Topic	Challenges	Possible causes	Specific interventions/ best practices
Q 3.2	Q3.2.1 Students write incorrect linear constraints. Some students skip this question	Students are unable to formulate linear inequalities Inability to understand MLP word problems Lack of MLP terms and language proficiency Inability to describe variables Abstract MLP teaching	Give students glossary of MLP terminologies with definitions and expression of MLP lexicons in mathematics symbols Give students more MLP word problem activities Use translation approach and code switching method to treat MLP word problems
	Q 3.2.2 Students draw linear inequalities incorrectly and also find incorrect feasible region	Students are not able to solve linear inequalities and graph them accurately which makes feasible region incorrect Cartesian plane is not accurate	Demonstrate to students how to apply procedures of x and y intercepts to find coordinates Write coordinates Plot and join coordinates Find feasible region

		Abstract MLP teaching	Do more and more activities
	Q 3.2.3 Students present search line graph is incorrect	Students are unable to find a search line function	Teach students how to make x and y subject of the formula Give more search line work or activities Show students how to draw a search line on the graph and move it to find maximum and/or minimum values
	Q 3.2.4 Students maximum profit (value) is incorrect	Incorrect feasible region and its co-ordinates	Teach students how to use a substitution method Do and give students a lot of activities for determination of maximum profit and minimum costs using the substitution methods

Elaboration and interpretation of Table 3.8

Table 3.8 shows a mathematics subject improvement plan developed by the team, which covers the topic of MLP as one of function and algebra concepts in Topic 2 of mathematics National Certificate Vocational (NCV) NQF Level 4 curriculum in Technical Vocational Education and Training (Assessment Guideline for mathematics NCV L4, 2013:15). This is in line with PAR, because it advocates that a research becomes a tool that improves people's lives. Therefore, using PAR, community interest drives the research process and in this regard, co-researchers' interest in the teaching and learning was to develop and complete a mathematics subject improvement plan, because mathematics was one of the problematic areas at this TVET college. This study only responded to activities related to MLP.

Focus

This study focused on the following MLP concepts:

- Formulation of linear inequalities and objective functions from the MLP problem statement;
- Problem solving and sketches of linear inequalities;
- Determination of the feasible region; and

- Determination of maximum and minimum values using the search line method and substitution method.

The analysis was done by categorising questions of the assessments activities in the MLP components on the above. Thus, this was done with the aim of finding common challenges and students' misconceptions.

Challenges

Table 3.8 revealed challenges, errors and misconceptions that students struggle with in the MLPCK. The mistakes that students made were investigated to gain knowledge about what they did not understand and what type of errors they committed. The findings of this analysis are further presented and discussed in Section 4.2.6. To response to these challenges, lecturers applied their SCK of MLP (Table 3.8 and Sections 4.2 & 4.3). In addition, this activity was used to enhance lecturers' MLPCK sharing of knowledge about analysis of students' work.

Possible causes

This section showed the possible causes of students' MLP challenges, misconceptions and errors that were investigated by the team. The team agreed that possible causes of students' misconceptions and challenges in MLPCK were key and needed to be examined to be in a better position to develop interventions and strategies to respond to those challenges. According to Mosia (2016:100), the investigation of possible causes enhances lecturers' MCK, because they have to apply Subject Content Knowledge (SCK) to move beyond correct and incorrect knowledge to find solutions at a deeper level. Therefore, in this study, this investigation of possible causes through sharing of knowledge and experiences enhanced lecturers' knowledge of MLP.

Interventions or strategies

This section explained strategies to be used to respond to the challenges, errors, and their possible causes. The team in this study recommended various teaching methods which assisted the incorporation of the RME approach to teach MLPCK and to respond to the errors identified on above, which will be addressed in Chapter 4.

3.4.6.2 Activity 2: lesson planning and preparation

Lesson planning and preparation were done collectively by the research team. The team had identified sub-topics on MLP to be presented over the next weeks (cf. 4.4.3). The team decided that lesson planning should be done collaboratively on a weekly basis to improve next lessons through re-planning after lesson reflection. The lesson planning and preparations sessions focused on MLPCK and MLPPCK; thus all the team members benefited from the preparation, even though mathematics lecturers were not teaching MLP during the following weeks. More importantly, they shared that these MLP lesson planning and preparations sessions were beneficial to them and would help them to prepare other mathematics concepts in the same fashion.

The lesson planning and preparations started with understanding the RME approach, but co-researchers agreed that the initial step was to have a deep understanding of the MLP (MLPCK), because strong MLPCK leads to more MLPPCK. During the lesson planning and preparation, we shared knowledge about MLP and its aim; then divided it into five components, namely MLP terms and terminologies; MLP word problems, linear inequalities, graphs with feasible region and maximum (or minimum values) using a search line and substitution method, together with lesson objectives. This was what the team identified as problematic areas in the mathematics subject improvement plan. They then shared their best practices to be used to teach the MLPCK identified. This evidenced that there was no single teaching method that was panacea to all MLP teaching and learning challenges, because multiple strategies were shared. This created a further opportunity for team members to share their knowledge and skills on how to integrate the RME approach to teach the MLP topic. As a result, the team PCK was improved.

3.4.6.3 Activity 3: Lesson presentation and facilitation

Collaboration was also used in the lesson facilitation activity, because in the lesson, Ms Moleko collectively worked with two assistant lecturers and me as the principal investigator. The other three team members observed the lessons to find out whether what we had prepared was effective or not, in order to re-plan and improve the next MLP lessons. What was noted in the lesson observation was shared with the rest of the team during the lesson reflection. Moreover, we as assistant lecturers teamed up with Ms Moleko who presented a lesson and shared our lesson reflection on every level of MLP lessons. This added to what lesson observers had found. As a result, a

multiplicity of views were generated by team approach, which informed the re-planning stage as spiral science of PAR to improve MLP teaching and learning.

As assistant lecturers we assisted Ms Moleko whenever she needed help, like facilitating students while working, or marking students' class/home activities.

3.5 DATA GENERATION

Next, I will outline how data were generated, explaining the procedures, co-researchers and instruments used.

3.5.1 Data generation procedures

The study aimed to enhance MLP teaching and learning using the RME approach through discussions during meetings. The co-researchers (including myself) interacted in order to generate data that assisted this study to achieve its research objective. This study used PAR as research methodology, because PAR advocates that people should collaborate and interact to find solutions to their problems, rather than one powerful person coming with solutions without involving the people affected. Shangase (2013:58) strongly criticises the traditional approach of conducting research where an individual person acts as an intelligent person who knows everything and where researchers obtain knowledge primarily through external observation and/or experimental manipulation of the human subject.

Shangase further signposts that conversations with human beings should be prioritised to understand their arguments and concerns. This implied that the co-researchers and I as the principal investigator were allowed to share our teaching experiences, fears problems and aspirations with regard to the teaching of MLP at a TVET college. Moloi (2013b:484) affirms that it is of paramount importance to generate data using PAR as method of research, because it allows all people to be free to make their voices heard in research processes focusing on MLP topic, in order to pinpoint challenges and develop solutions.

PAR helped this study to use a focus group discussions, lesson observations and presentations to generate data which enabled us as a team to achieve this study objectives. All mentioned co-researchers (included me as principal investigator) to

form a group that helped the teaching and learning of MLP to be enhanced and students' mathematics performance be improved. Therefore, this study had relied more on interaction through discussions among the research team so that we understood one another. Focus-group discussions, lesson observations and lesson presentations proceedings were recorded in audio recordings, together with notebook for the transcription purposes. It was also captured with cameras, but before the capturing and recordings began, I explained the reasons for using these instruments to all co-researchers, so that they are comfortable with this setup and procedures. At the first meeting, we together decided on the place, days and time for these study meetings at this TVET college that were conducive and comfortable to all co-researchers. This study allowed co-researchers to use language of their own choice for them to participate, speak freely and comfortably. This study informed co-researchers about the nature of the research, all ethical issues and the content of the consent forms. Co-researchers were requested to go through consent forms, sign and return them at the second meeting, but were made aware that participation was voluntary.

3.5.2 Participants in the study (co-researchers)

The research team in this study comprised a Council member, mathematics coordinator, the Campus Manager, 2 HoDs, 3 supervisors, 5 lecturers, 25 students, and 3 parents (including myself as the principal investigator). This team was selected because of their diverse knowledge and different experiences regarding the teaching and learning MLP. The roles of each team member were decided on and shared during the study meetings.

3.5.3 Instrumentation

This study used Bricolage and the PAR as a theoretical framework and research methodology to conduct research. Audio recordings, cameras, and notebooks were used to capture discussions, lesson observations and lesson presentations in the study meetings. Before using these instruments, I explained the importance of using these tools to capture the information from co-researchers. For instance, the audio recordings enabled us to compile transcripts, in particular when some of the information was not clear in a notebook. However, the co-researchers decided to

exclude video recording, because some were not comfortable with the use of this tool. This is in line with one of the PAR principles, namely promoting democracy; people being included in a decision that affects them. The co-researchers' rights were protected and respected, which made them feel free to participate in this study, since this study aimed to bring people affected by poor MLP performance, together to find solutions to enhance MLP teaching and learning using the RME approach. All co-researchers were allowed to employ the language of their own choice to facilitate lengthy participate. Ranganayakulu (2005:105, in Hlomuka, 2014:63) is of the view that if discussions are done in a way that all co-researchers feel free, they will be in the position to make a positive contribution to the study. They will freely ask questions, pose clarity-seeking questions and interact with one another. By means of this, the objectives of this study were achieved.

3.6 DATA ANALYSIS

This study aimed to enhance MLP teaching using the RME approach. According to Hlomuka (2014:65), there are six steps to generate, present and analyse data.

- This study used assessment guideline documents of mathematics NCV L4, which outlined MLP.
- To analyse data, this study used the words of co-researchers (including myself as principal investigator), to strengthen arguments and debate generated in this study.
- Extracted words were analysed in relation to Bricolage and PAR as theoretical framework and research methodology.
- Local, continental and global literature showing MLP is problematic for students and which reveal that students perform poorly in MLP was reviewed.
- Generated data were used to show what was currently happening at this TVET college to confirm what the literature revealed.
- In the sixth step we focused on the third level of CDA, social structure that helped this study to analyse recorded presentations and discussions

This study used Critical Discourse Analysis (CDA) on three levels: textual, discursive analysis and social. Van Dijk (2009:62-85) views CDA as a tool to analyse generated

data in depth, since the spoken words of the co-researchers are presented unchanged. This study adopted CDA as a method to analyse generated data, since the purpose was to analyse the language structure of meanings instead of presenting findings as they are in a mere description. According to Moloi (2013b:483), CDA allows researchers to construe meaning from day-to-day interaction, since the data generated relied more on team discussions and presentations during meetings as the primary data source.

To ensure that CDA was applied appropriately, interactions and discussions were recorded on a weekly basis to find a deeper meaning and to revisit the captured communication by means of audio recorders, cameras and notebooks. The recorded formal and informal discussions were transcribed verbatim, because this study focused on the analysis and interpreted written words and captured presentations, to determine the background environment and opinions of the co-researchers, namely the values, norms and beliefs about MLP teaching and learning. After the interpretive process the discursive phase followed. Hlomuka (2014:66) indicates that during the discursive phase, a set of statements are organised to give substance to the manner in which a particular subject, object, or process is discussed and taken into account. Mahlomaholo and Nkoane (2002:5) signpost that what people speak is a true, clear reflection of their practices. According to them, discourse is regarded as a socially accepted association on ways of using language, thinking, feeling, believing, valuing, and of acting that can be used to identify an individual as a member of a socially meaningful group and view that society does influence what we say.

3.7 CONCLUSION

This chapter outlined the research methodology of this study (PAR), explaining why it is relevant to this study. PAR helped this study to generate data to achieve the study research objectives. PAR also had empowered the co-researchers to work in collaboration to understand the problems, challenges and misconceptions observed in this study, and to find ways to implement the RME approach to enhance MLP teaching and learning. This chapter highlighted that we as team members together did the planning, observations, actions, reflections and re-planning on what are challenges in MLP teaching and learning and solve that challenges in an interactive fashion. At the end of the research meetings, we hoped to see progress towards the enhancement

of MLP teaching and learning. The chapter showed where the research was conducted, what happened at the initial meeting, how the research questions of this study were formulated, co-researchers' profiles, data-generation procedures, the participants, instruments used and what method was used to analyse data. In the next chapter, this the data will be presented, interpreted and analysed.

CHAPTER 4: DATA PRESENTATION, INTERPRETATION AND ANALYSIS OF THE RESULTS

4.1 INTRODUCTION

The aim of this study is to use the Realistic Mathematics Education (RME) strategy to enhance the teaching of Mathematical Linear Programming (MLP) at Technical and Vocational Education and Training (TVET) colleges. Diagnostic reports and literature show that MLP is one of the problematic mathematics topics. It also reveals that students have difficulty with MLP word problems, linear inequalities, drawing, analysing and interpreting MLP graphs on National Certificate Vocational Level 4 (NCV L4) at TVET colleges (cf. 1.1). This chapter will present, discuss, analyse and interpret data generated in five research objectives of this study.

Firstly, this chapter will present and analyse the challenges experienced by both lecturers and students in MLP teaching and learning. Secondly, solution components of the RME approach will be developed and formulated in such a way that they respond to the MLP teaching and learning challenges identified by the co-researchers. However, for the RME approach to work adequately, certain conditions should be considered, because if ignored, they may threaten to impede the successful implementation of the RME approach. For that reason, the RME approach is operationalised, assessed and evaluated before it can be considered as sufficiently conclusive to respond to the challenges of MLP teaching and learning challenges and to enhance the teaching of the Mathematical Linear Programming Content Knowledge (MLPCK).

The presentation, analysis, interpretation and discussion of the generated data in line with this study's research objectives were done in the following fashion. Firstly, this study used five research objectives to organise raw data generated in subtitles; secondly, an appropriate opening discussion followed with the idea to enlighten people what mathematics curriculum, syllabus, policy-related issues, theories and previous researches state on how to teach MLP. Thus, the generated data in this study have been presented in the form of written words (text), pictures, lessons and presentations.

Critical Discourse Analysis (CDA) was used to find the deeper meaning of the texts or spoken words and applied on three levels: textual, discursive practices and social structure. More importantly, the theoretical framework and research methodology of this study, namely the Bricolage and Participatory Action Research (PAR) have been infused in the analysis, discussion and interpretation.

4.2 NEED FOR AN RME APPROACH TO TEACH MLP

As the principal investigator, I, together with my co-researchers, observed MLP lesson presentations at the Kgotsoong Technical Vocational Education and Training (TVET) college in the Free State. Thereafter, we had vigorous discussions during the study meetings, analysing data generated from each presented, and then identified challenges contributing to students' poor performance in mathematics as well as in MLP (Figures 4.10–4.13). The challenges identified by the co-researchers during the lesson observations were the following: MLP is taught in abstract way; accompanied by limited teaching and learning resources; the method of teaching MLP is too teacher centred; the inability to recognise prior knowledge and learning; and the absence of formative assessments in the teaching and learning of MLP at the TVET college. In addition, during the research meetings, co-researchers agreed that the above challenges led to the inability of students to convert MLP word problems into linear inequalities; to solve linear inequalities; to sketch MLP graphs; and to analyse and interpret MLP graphs (Figures 4.10–4.13).

The following section will present, interpret and analyse the challenges identified in the teaching and learning of MLP.

4.2.1 A complex way of teaching MLP at TVET colleges

The literature in Section 2.4.2.1 revealed that to teach MLP well, lecturers should divide MLP into five to six lessons according to its components, such as recognising, describing variables and constraints (inequalities); formulating linear inequalities and objective functions from MLP word problems; drawing linear inequalities to find feasible region; determining the maximum (or minimum) values using feasible region [analysis and interpretation]; revising the whole process; checking solutions; and preparing short tests. White (1995:29) further argues that no drawing of linear

inequalities to find feasible region must be done until the third of five lessons has been completed and understood well. On other hand, other scholars maintain that the teaching of MLP should progress from a concrete to an abstract level so that students comprehend the abstract mathematics concepts.

However, this is not what happens at this TVET college, according to the co-researchers. Look at the way Mr Takui (mathematics lecturer) presented the MLP topic in the classroom (cf. Annexure J).

Mr Takui: “Good afternoon, students.”

Students: “Afternoon, Sir.”

Mr Takui: “Today we are going to do Mathematical Linear Programming.”

Mr Takui (taking Dikotlolo’s [student] mathematics L4 textbook): Ok, let’s go to page 80 of your textbook. Right this is MLP, so, to solve this kind of MLP problem in L4, we first need to convert MLP word problems into linear inequalities; secondly, we solve linear inequalities; and lastly we draw an MLP graph with that good sequence.

Mr Takui took the students through the example on page 84 of the mathematics L4 textbook, and thereafter showed and demonstrated **Problem statement 1** to show students how to solve the MLP problem.

Mr Takui: A small factory makes two types of washing machines, a standard and a luxury model. The daily production cannot be more than 6 standard and 3 luxury washing machines. There are 8 factory workers and it takes one worker to make one standard model and 2 workers to make one luxury model per day. The profit on the standard model is R300 and on the luxury model R600. **Questions:**

1. Write down linear constraint and inequalities.
2. How many of each type of machine must be manufactured per day to make a maximum profit?

Mr Takui (interpreting Problem Statement 1): Based on what I have read, firstly remember what I said earlier on, we need to write linear inequalities to answer question number 1. Secondly, we have to solve linear inequalities and then draw MLP graph so that we can find feasible region, in order to determine maximum profit in answering question number 2.

In answering question number 1, he first read the first sentence and let Type A (standard machine) be x , while Type B (luxury machine) is y . Then, he reminded students always to show these linear inequalities, $x \geq 0$ and $y \geq 0$. He continued:

Mr Takui: “The second sentence (**cf. above problem statement 1**), means company will not be able to produce more than 6 standard and 3 luxury machines, then to write it Mathematically we say...

$$x \leq 6 \text{ linear inequality 1}$$

$$y \leq 3 \text{ linear inequality 2}$$

and third sentence (**cf. above problem statement1**), means this company has employed only 8 workers to produce standard and luxury models, but per day it takes 1 worker to

produce standard washing machine and also it takes 2 workers to produce a luxury washing machine.

Then, we write...

$$1x + 2y \leq 8 \dots \text{linear inequality 3}$$

Lastly, the last sentence (**cf. above problem statement 1**), means for this company to make profit, it have to sell one standard washing machine at R300 and one luxury machine at R600 respectively. Then, we have ...

$$P = 300x + 600y \dots \text{objective function.}$$

But, this objective function will be used after we have drawn the MLP graph. So, now we have answered question number 1 (**cf. above problem statement 1**). Thus, these are linear inequalities." (cf. Figure 4.1).

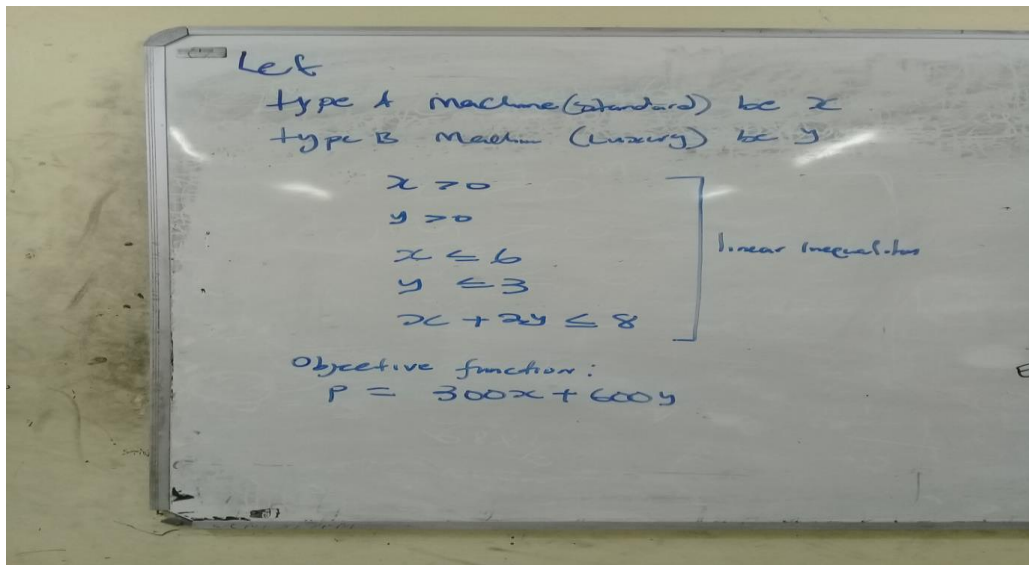


Figure 4.1: Variables, linear inequalities and objective function (extracted from classroom)

During Mr Takui's lesson, students were very silent, passive, listening and writing everything Mr Takui said. Then, he asked students:

Mr Takui: "Do you all understand?"

Students: "Yes, Sir."

Mr Takui: All right, now we move to the question number 2, which need a number to be manufactured for each type of a machine in order to maximise profit (**cf. the above problem statement 1**). This question needs us to determine a point where the profit will be maximised. So, to answer this question, we must draw an MLP graph. However, before we draw an MLP graph, we first need to solve all the above linear inequalities.

This is exactly what we did last year in NCV mathematics L3 and I am not going to repeat it again. In question number 1, we got these linear inequalities:

$$x \leq 6 \dots \dots \dots (1)$$

$$y \leq 3 \dots \dots \dots (2)$$

$$1x + 2y \leq 8 \dots \dots \dots (3)$$

Then, we change linear inequalities to linear equations so as to write coordinates.

$$x \leq 6 \text{ can be written as } x = 6$$

$y \leq 3$ can be written as $y = 3$

$1x + 2y \leq 8$ can be written as $1x + 2y = 8$

Therefore, coordinates will be.....

$x = 6$ A (6, 0)- when $x = 6$, then $y = 0$ (vertical line)

$y = 3$ B (0, 3)- when $y = 3$, then $x = 0$ (horizontal line)

$1x + 2y = 0$ C (8, 0) and D (0, 4)

With these coordinates we plot and join them on the Cartesian plane and this is MLP graph.
For each line on the graph, we write its linear inequalities." (cf. Figure 4.2).

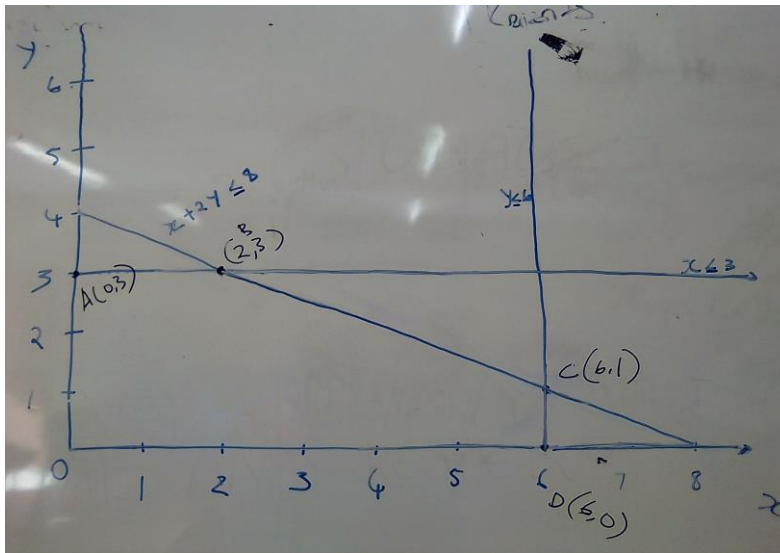


Figure 4.2: Plotting and joining coordinates drawing an MLP graph

Mr Takui drew a Cartesian plane and plot and joined the coordinates on the Cartesian plane. He then illustrated how to find the feasible region (cf. Figure4.3).

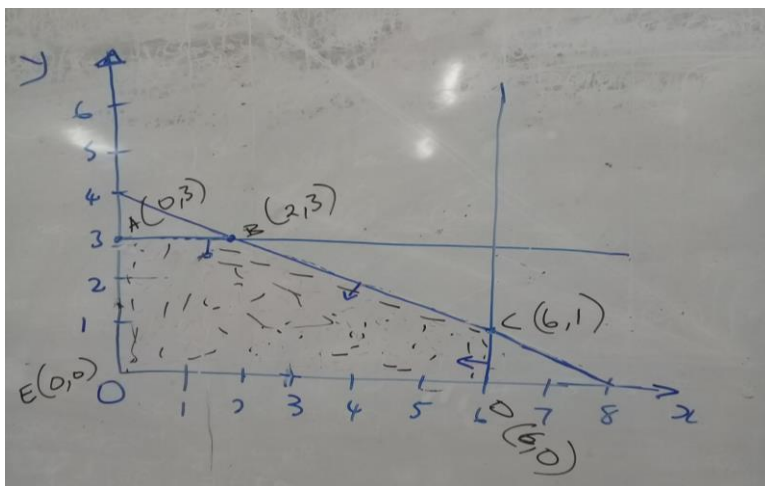


Figure 4.3: MLP graph for problem statement 1

Mr Takui continued: "Now, we draw MLP graph showing feasible area or region, which is made by following points (A (0, 3); B (2, 3); C (6, 1); D (6, 0); and E (0, 0)). So, like we did in NCV mathematics L3, with shaded area we can be able to determine number of machines to be manufactured in maximising profit. In answering question number 2, we substitute these points A (0, 3); B (2, 3); C (6, 1); D (6, 0); and E (0, 0) into this objective function $P = 300x +$

600y in order to determine maximum profit. So, let's practicalise or do calculations (he went to the board and did substitution)." (cf. Figure 4.4)

$$P = 300x + 600y$$
 Substitute points in above function:

$$P = 300(0) + 600(3) = 1800 \rightarrow A(0,3)$$

$$P = 300(2) + 600(3) = 2400 \rightarrow B(2,3)$$

$$P = 300(6) + 600(1) = 2400 \rightarrow C(6,1)$$

$$P = 300(6) + 600(0) = 1800 \rightarrow D(6,0)$$

$$P = 300(0) + 600(0) = 0 \rightarrow E(0,0)$$

Figure 4.4: calculation of maximum value (extracted from classroom presentation)

Mr Takui (in conclusion): "The calculation on the board proves that this company can maximise its profit when it produces 2 standard washing machines and also 3 luxury washing machines per day **OR** when it produces 6 standard washing machines and 1 luxury washing machines per day because both made profit of R 2 400. So, we have answered question 2, in conclusion question number 1 assisted us to do question number 2. This is what you should do when doing this kind of MLP problem."

The students looked surprised and some were tired. They started to make a noise, but some were talking to one another.

Tshepe (student): "Haah, what a long presentation; eish, I am very lost and confused. You know I have been struggling with this MLP topic, especially solving linear inequalities and sketching its graphs for years."

Poho (student): "Nna ke lahlehile qalong kwana, mme jwale wa tseba hore Ntate wa halefa, ha re botsa tse dipotso ngata, ha re sa utlwisisi teng (for me, I got lost at the beginning of this lesson and now you know Mr Takui get angry with us when we ask too much questions when we do not understand)."

Moreover, other students near to where we as lesson observers sat were talking.

Teboho (student): "Dipalo di thata Skeem, hai nna se mpa ke etsa dipalo hobane ke tlamehile course-ng ena ya ka. O hle, sheba MLP yona fela e batla o etse process e kae (Mathematics is difficult friend, hey I only just do mathematics because it is the must for me to do it in my course programme. Oh, to solve MLP problems only, it is a long process)."

Lebohang (student): "Skeem ke nako mang? Periot-o tse di fela neng, hah? (Friend, what time is it? (when these periods end?))"

Mr Takui: "All right, all right, students, since you are talking there I hope you have questions."

Students: "No we don't have questions,"

Mr Takui: "Yyah, I hope you understand. However, we will do another two examples hopefully tomorrow and on Thursday before moving to next topic, so that you get a picture and understand more."

The class ended and over the next two days Mr Takui did two more examples in the textbooks to help students to understand, but he was rushing, pushing the syllabus.

The above lesson presentation was analysed in identifying challenges hindering the teaching MLP graphs.

Mr Koena (chairperson): *“Colleagues what have you picked up from this lesson presentation as another challenges in teaching MLP?”*

Ms Moleko (NCV Supervisor) added: *“Ntate, mokgwa oo ho rutwang ka ona o thata ho bana ba rona ho re ba utlwise hobane ha ngata bo-teacher ba ruta dikarolo tsa MLP topic at the same time, like ho understand-er MLP scenarios, conversion of scenarios to linear inequalities, ho draw-ya, analys-a le ho interpret MLP graphs, jwale hona ho ferekanya bana. (Sir, the way students are taught MLP topic is difficult for our students to understand (he speaking looking on the lesson presentation on Annexure J) because teachers are teaching all components of MLP topic at the same time, like understanding of MLP word problems, conversion of word problems into Mathematical linear inequalities, solving Mathematical linear inequalities, drawing, analysing and interpreting MLP graphs, so, this confuse students.)”*

Ms Lebone (lecturer): *“Ke nnete, empa nako ha eyo, ya ho arolana le components tseo. Ha ke nahane hore re tla kgona ho cover syllabus ha re arola these components (It’s true, but we don’t have enough time to break these MLP topic. I don’t think we can cover the syllabus when we break MLP topic into these components.)”*

Mr Takui: *“When you teach all components of MLP topic at the same time is too challenging for students to understand and they find themselves confused. Most of time, you find them behind the lesson. Mohlala, when you solve linear inequalities, some tells you that they did not understand how we got linear inequalities from MLP word problem. When you have to sketch Mathematical linear inequalities, some tells you that they did not understand how we got the coordinates. Eish, is so confusing because we do not have enough time to cover even the syllabus. We find ourselves shouting them because you have to repeat repeat, repeat.”*

Ms Moleko: *“From this lesson, Mr Takui taught students MLP content the way it is in the book. I think also is other reason why the lesson is difficult for students to understand and it leads to the lacks interest to do mathematics.”*

The above lesson presentation exposed the abstract teaching of MLP. Mr Takui taught all MLP components, namely understanding of MLP word problems, conversion of word problems into linear inequalities, solving linear inequalities, drawing, analysing and interpreting MLP graphs in one lesson. At the same time, this contradicts what the literature postulates (cf. 2.4.2.1) with regard to best practices towards the teaching of MLP. In Section 2.4.1.1, the literature revealed that abstract teaching of MLP contributes to poor performance; thus this was also realised in the above lesson presentation. Mr Takui presented the MLP lesson in an abstract, complex and difficult way, according to the co-researchers:

Mr Takui: *“The way students are taught MLP topic is difficult for our students to understand ... teachers are teaching all components of MLP topic at the same time, like understanding of MLP scenarios, conversion of scenarios to linear inequalities, drawing, analysing and interpreting MLP graphs ... so, this confuses students.”*

This was drawn from the above MLP lesson presentation, which also showed the limitations, because students hardly learned. They misunderstood the entire lesson and it was evidenced by the following statement:

Poho: *"For me, I was lost from beginning of this lesson."*

Mr Takui had to repeat this lesson because students struggled to comprehend the lesson well. The other co-researcher supported Mr Lebone by saying:

"When you teach all MLP components at the same time is too challenging for students to understand ... Most of the time, you find them behind the lesson. E.g. when you solve linear inequalities, some tells you that they didn't understand how we got linear inequalities from MLP word problems."

I therefore concur with what has been put forward about this method used by Mr Takui and its consequences. Imagine you are in mathematics classroom where you are taught MLP word problems, converting MLP word problems into linear inequalities, solving the linear inequalities, drawing, analysing and interpreting MLP graphs in one lesson and at the same time for the first time in mathematics. It must be confusing and frustrating, because while you are struggling to understand one concept, the teacher moves to the next concept. This was also realised in the following statement

Teboho: *"Ooh, look! To solve MLP problems only, it is a long process."*

As a result, this long process usually discourages students. On the other hand, the above lesson lacks creativity; Mr Takui did not integrate real-life situations into his lesson and he also did not show students the significance of MLP in our daily lives. This was also articulated by Ms Moleko (HoD):

Ms Moleko: *"From this lesson, Mr Takui taught students MLP content the way it is in the book. I think also is other reason why the lesson is difficult for students to understand and it leads to the lacks interest to do mathematics."*

This is not in line with the Bricolage because in Section 2.2.1, Le Loarne (2005:02) explained Bricolage as one face of the creativity, so it nourishes the understanding of creativity. Therefore, the lack of lesson creativity will definitely have a negative impact on the students, because some will not be interested to study MLP, because MLP is taught in a difficult way. See the following conversation: between Teboho and Lebohang (students):

Teboho: *"Mathematics is difficult friend; hey I only just do Mathematics because it is the must for me to do it in my course programme. Oh, to solve MLP problems only, it is a long process)."*

Lebohang: *"Friend, what time is it? When these periods end, hah?"*

In addition, the above lesson presentation also shows discursive practices, which lead to social exclusion, Mr Takui taught the topic in an abstract and complex way, without regularly involving students during his lesson.

Co-researcher: *“Sir, the way students are taught MLP topic is difficult for our students to understand (he speaking looking on the lesson presentation on Annexure J) because lecturers are teaching all parts of MLP topic at the same time, like understanding of MLP word process, conversion of word problems into linear inequalities, solving linear inequalities, drawing, analysing and interpreting MLP graphs, so, this confuse students.”*

Other co-researchers agreed with her. This was also recognized when Mr Takui repeated the lesson in the same way he did in Annexure J. Thus, this seemed to be the norm of teaching MLP. The abstract way of teaching MLP was dominant during the Bricolage traditional period, where a lecturer was expected to teach students rules and procedures only to deal with mathematics problem solving. The other limitation of using only an abstract way is that students will not be able to work on their own, because they will always wait for their lecturers to lead and provide answers.

The above information in this Section 4.2.3 affirmed what the literature revealed (cf. 2.5.3) on the fact that an abstract, complex way of teaching MLP contributes to poor MLP performance. Moreover, both Sections (2.5.3 & 4.2.3) revealed that teaching all components of MLP: MLP word problems, conversion of MLP word problems to linear inequalities, solving Mathematical linear inequalities, drawing, analysing and interpreting MLP graphs at the same time in one lesson is difficult and challenging for students to comprehend properly. The empirical data further discovered that teaching all MLP components at the same time is regarded as a complex and abstract way. This method causes students to have limited interest in solving MLP problems, because it involves a long process to get the answer. In addition, most of the time students find themselves left behind, especially when the lecturer moves from one component to another.

In conclusion, the empirical data revealed that teaching all MLP components at the same time in one lesson makes it difficult for students to understand the lesson and this contributes to poor performance in MLP.

4.2.2 A need for multiplicity teaching and learning tools

According to Gitaari *et al.* (2013:98), one of the approaches to improve teaching and learning of mathematics is adequate and available teaching and learning resources

(cf. Section 2.4.3.2). The incorporation of various teaching and learning tools into mathematics teaching and learning is mainly employed to concretise the abstract MLP concepts, for students understand the lesson well. Moreover, the use of various teaching and learning resources improve both their Content Knowledge (CK) and Pedagogical Content Knowledge (PCK) in mathematics. This also impact students' mathematics performance positively. The examined TVET college expects all lecturers to gather and organise teaching and learning media or resources that will enable them to impart MCK in more meaningful and creative way in line with what the DHET expects. However, according to the co-researchers, this is currently not the case. In our mathematics lesson observations, we identified the following hindrances with regard to teaching and learning aids.

Mr Phoka (Myself, principal investigator): *"I observed that most of lecturers use only single mathematics textbook to teach mathematics content and according to me this disadvantage both lecturers and students because some of textbooks do not have enough information, clear examples and enough activities. Again, some of textbooks do not explain some concepts well and have errors and mistakes. So, using single textbook will disadvantage both lecturers and students and also will negative impact to the teaching and learning."*

Ms Moleko: *"No pictures, no teaching objects, no real life situations, and no indigenous knowledge integrated in the teaching and learning of MLP. Lecturers uses only one source which is textbook to teach mathematics."*

Mr Kia (HoD): *"Lesson plans of mathematics lecturers always indicates following teaching and learning aids or media, like textbook, whiteboard marker and calculator, for all mathematics topics."*

Mr Phoka: *"I found that most of time mathematics lecturers use a single textbook, whiteboard marker and calculator to teach the MLP topic and graphs as well. There were no available T-square rulers, and other teaching materials to help in teaching MLP graphs. So, what I realised is that MLP graphs drawn on the whiteboard were not accurate, straight and also graph scaling were not done properly. Then, this makes feasible region points not accurate."*

Ms Moleko: *"At the beginning of the lesson where lecturers introduced MLP topic. Lecturers do not inform students about resources that are going to be needed to treat MLP topic, such as graph paper grid or graph book, crayons, calculator, pencils, and different pens etc. to draw MLP graph and show feasible region accurately and clearly in order to determine the maximum (or minimum) values."*

Mr Phoka: *"Experienced lecturers sometimes do not even carry any resources even textbook to the classroom and just take students textbooks in the classroom to teach."*

This was also realised in the lesson presentation below extracted from the full lesson presentation in Section 4.2.1, where Mr Takui started the lesson in this fashion:

Mr Takui: *"Today we are going to do MLP. (He takes Dikotlolo's textbook.) Ok, let's go to page 80 of your textbook. Right this is MLP topic, so, to solve this kind of MLP problem in L4, we first need to convert MLP word problems into linear inequalities; secondly, we solve linear inequalities; and lastly we draw MLP graph with that good sequence."*

Mr Takui took students through example on page 84, and, thereafter demonstrated Problem statement 1 to show students how to solve MLP problem (cf. 4.2.1). Next, Mr Takui analysed, interpreted and explained the above MLP problem statement and then showed students how to formulate linear inequalities and objective function from that Activity 1 (cf. Figure. 4.1). During the lesson presentation, students were quiet, listening and writing everything that Mr Takui explained. Mr Takui further reminded students of what they did in NCV mathematics L3 and showed students how to solve the above linear inequalities, in order to answer question number 2 (cf. lesson presented in 4.2.1). Thereafter, Mr Takui approached the whiteboard, explained and showed how to plot and join the coordinates and how to identify feasible area (cf. Figure 4.5).

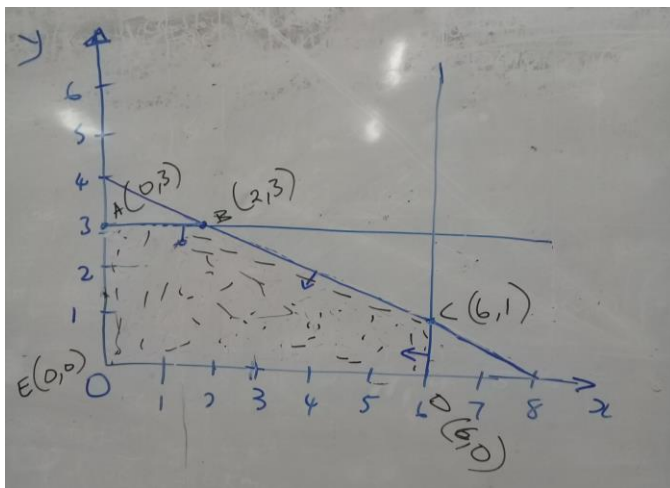


Figure 4.5: MLP graph (adapted from classroom)

Mr Takui: “Now, we draw an MLP graph showing feasible region, which is made by following points (A (0, 3); B (2, 3); C (6, 1); D (6, 0); and E (0, 0)). So, like we did in NCV mathematics L3, with shaded area we can be able to determine number of machines to be manufactured in maximising profit. In answering question number 2, we substitute these points A (0, 3); B (2, 3); C (6, 1); D (6, 0); and E (0, 0) into this objective function $P = 300x + 600y$ in order to determine maximum profit.”

Mr Takui ended the lesson presentation by demonstrating how to determine maximum value using a substitution method (cf. Figure 4.4), but some students did not agree with him regarding Point B.

Teboho: “Nna, I got point B as (1.4, 3) not (2, 3).”

Gladys: “No, nna, Sir, I got B (1.9, 3) not (2, 3).”

Mr Takui: “No you are not wrong; I just took approximate number. I will accept any value.”

The observations revealed that most mathematics lecturers do not plan and prepare MLP lessons properly. According to Khan (2011:3958), lesson planning and

preparation include which content to teach (topic), which lesson objectives to be achieved, which pedagogy methods to be used to unpack content in an easier way, which teaching and learning resources need to be integrated to unpack content more easily, and which activities (tools) are needed to measure students' understanding. The above extracts expose that there is a lack and absence of teaching and learning resources in teaching of MLP. This was proven by following statements,

"Most lecturers use only a single mathematics textbook to teach the MLP ... no pictures, no teaching objects, no real-life situations, and no indigenous knowledge integrated in the teaching ... of MLP. Lecturers use only one source to teach mathematics ... lesson plans of lecturers always indicate ... textbook, whiteboard marker and calculator (teaching media) for all mathematics topics."

These discursive practices were clear in the above MLP lesson, because Mr Takui used only one textbook to teach MLP, without any other resources or pictures, objects, real-life situations, indigenous knowledge, T-square rulers, and other relevant resources to teach the topic, particularly MLP graphs (cf. Figure 4.5 above).

Furthermore, the textbook used by Mr Takui belonged to Dikotlolo, because he took it from Dikotlolo without his consent, exposing power-related issues. Forcing students to share textbooks unnecessarily hinders students' learning. Another point observed with regard to teaching and learning resources was that most mathematics lecturers, when introducing MLP, fail to alert students about learning resources that are crucial in helping them to complete MLP problems, especially MLP graphs, feasible region and maximum (or minimum) values.

"At the beginning of the lesson where the lecturers introduced the topic of MLP, the lecturers do not inform students about resources that are going to be needed to treat the MLP topic, such as graph paper grid or graph book, crayons, calculator, pencils, and different pens etc. to draw MLP graph and show feasible region accurately and clearly in order to determine the maximum (or minimum) values."

This is not in line with Bricolage, because Bricolage allows and encourages lecturers as bricoleurs to use multiple teaching and learning aids to concretise the abstractness of MLPCK and also solve MLP challenges (cf. 2.2.1). In addition, this lack of teaching and learning resources forces mathematics lecturers to teach MLP in a more abstract way, which leads to MLPCK students' misconceptions, confusion and wrong perceptions, which impact the teaching and learning of MLP negatively. This was realised in Mr Takui's lesson when he identified the feasible area coordinates (points) from an inaccurate and not properly scaled MLP graph drawn on the whiteboard (cf. Figure 4.5 above).

Figure 4.5 shows the MLP graph presented by Mr Takui on the whiteboard. It was not accurately scaled and drawn, because the measurement on x and y axis is not equal or the same, inaccurate, and not straight, leading to confusion in identifying feasible region points and determining the maximum (or minimum) values. This was realised when students did not agree with Mr Takui with regard to point B coordinates, because students got 1.4, 3, 1.6, 3 & 1.9, 3, while Mr Takui got point B as (2, 3, and this confused students. Mr Takui further confused students by advising students to use an approximate numbers, not accurately determining values and coordinates, which is expected in tests, exams and also in future. This leaves students unsure as to what needs to be done when sketching and MLP graph, feasible region and determining maximum or (minimum) values, which emerged from the lack of teaching and learning resources.

The above section agrees with Section 2.4.1.2 on the fact that inadequate teaching aids is one of the challenges contributing to poor performance in mathematics, leading to misconceptions, errors and misunderstandings. This section showed that most lecturers rely on a single textbook as a source of information in teaching the MLP topic. They also do not plan and prepare MLP lessons in time to organise necessary and relevant teaching and learning resources to teach and concretise the abstractness of MLPCK. This section further revealed that most mathematics lecturers teach MLP graphs without using relevant teaching aids that assist in sketching accurate and clear graphs.

In conclusion, the empirical data showed that one of challenges leading to MLP poor performance, especially drawing MLP graphs, is a limited teaching and learning aids.

4.2.3 Method of teaching MLP at TVET colleges is too teacher centred

The literature in Section 2.4.2.3 showed that to teach MLP, mathematics lecturers should not rely on one teaching method, but rather use compound teaching methods such as computer software (spreadsheet & geogebra), technological resources, procedural embodiments, Problem-Based Learning (PBL) and/or any modern and traditional teaching methods to cater for different needs of students. On the other hand, all Critical Outcomes on mathematics NCV L4 Subject Guideline (TVET policy) expect all students to be active during the teaching and learning process (cf. subject

guideline NCV mathematics L4, 2007:2). For that reason, both the literature and TVET policy agree that students should at all times be centred during the learning process. However, the lesson presentations below contradict what TVET policy and literature prescribe, because from our lesson observations, we found that the method of teaching the MLP is too teacher centred (traditional method) and mathematics lecturers only rely on and use the traditional method to impart knowledge on MLP (cf. lesson presentations in Annexure J). In Section 2.4.1.3, the literature revealed that the use of a single teaching method is not enough. Failure to use multiple teaching methods impact the teaching process badly, because some students' learning styles are not accommodated, leading to poor performance in MLP.

The same lesson presentation below (taken from Section 4.2.1) evidences that the method of teaching MLP is too teacher centred.

Mr Takui: *"Today we are going to do MLP." (taking Dikotlolo's book), he said: "Ok, let's go to page 80 of your textbook. Right this is the topic of MLP topic, so, to solve this kind of MLP problem in L4, we first need to convert MLP word problems into linear inequalities; secondly, we solve linear inequalities; and lastly we draw MLP graph with that good sequence."*

The students were taken through the example on page 84 of the mathematics, and subsequently demonstrated Problem Statement 1 to show students how to solve MLP problems.

Mr Takui (interpreting the problem): *"Based on what I have read, firstly remember what I said earlier on, we need to write linear inequalities to answer question number 1. Secondly, we have to solve linear inequalities and then draw MLP graph so that we can find feasible region, in order to determine maximum profit in answering question number 2."*

*From the first sentence I have read (cf. **Problem Statement 1**), we find two types of machines manufactured by this company, so we first let ...*

Type A (standard machine) be x

Type B (luxury machine) be y

Remember, always to show these linear inequalities ...

$$x \geq 0$$

$$y \geq 0.$$

Tholoane (raising his hand): *"Sir, Sir."*

Mr Takui: *"Tholoane?"*

Tholoane: *"In which sentence does this $x \geq 0$ and $y \geq 0$ come from, from what you read, Sir?"*

Mr Takui: *"That's a good question, Tholoane. So, you must know that when we do MLP topic, we are always expected to show these linear inequalities. Thus, it is a rule, so you have to write them." (continuing The second sentence (cf. **problem statement 1**), means company will not be able to produce more than 6 standard and 3 luxury machines, then to write it Mathematically we say ...*

$$x \leq 6 \dots \text{linear inequality 1}$$

$$y \leq 3 \dots \text{linear inequality 2}$$

and the third sentence (**cf. problem statement 1**), means this company has employed only 8 workers to produce standard and luxury models, but per day it takes 1 worker to produce standard washing machine and also it takes 2 workers to produce a luxury washing machine.

Then, we write

$$1x + 2y \leq 8 \dots \text{linear inequality 3}$$

Lastly, the last sentence (**cf. problem statement 1**), means for this company to make profit, it have to sell one standard washing machine at R300 and one luxury machine at R600 respectively. Then, we have.....

$P = 300x + 600y \dots \text{objective function}$. But, this objective function will be used after we have drawn the MLP graph. So, now we have answered question 1 (**cf. Problem Statement 1**). Thus, these are linear inequalities.

Mr Takui explained and emphasised key points with regard to the formulation of linear inequalities from this MLP problem statement, thereafter wrote these linear inequalities and objective function, $x > 0$, $y > 0$, $x \leq 6$, $y \leq 3$, $x + 2y \leq 8$, and $P = 300x + 600y$ respectively (cf. Figure 4.1).

During the lesson, the students were very quiet, listening and writing down everything Mr Takui explained.

Mr Takui: All right, we move to the question numbers 2, which need a number of each type of machine to be manufactured to maximise profit (**cf. problem statement 1 on above**). This question requires us to determine a point where the profit will be maximised, so, to answer this question we must draw MLP graph. However, before we draw MLP graph, we first need to solve all above linear inequalities.

So, this is exactly what we did last year in NCV mathematics L3 and I am not going to repeat it again. In question number 1, we got these linear inequalities:

$$x \leq 6 \dots \dots \dots (1)$$

$$y \leq 3 \dots \dots \dots (2)$$

$$1x + 2y \leq 8 \dots \dots \dots (3)$$

Then, we change linear inequalities to linear equations so as to write coordinates.

$$x \leq 6 \text{ can be written as } x = 6$$

$$y \leq 3 \text{ can be written as } y = 3$$

$$1x + 2y \leq 8 \text{ can be written as } x + 2y = 8$$

Therefore, coordinates will be:

$$x = 6 \dots \dots \dots A (6, 0) \text{-when } x = 6, \text{ then } y = 0 \text{ (vertical line)}$$

$$y = 3 \dots \dots \dots B (0, 3) \text{-when } y = 3, \text{ then } x = 0 \text{ (horizontal line)}$$

$$x + 2y = 8 \dots \dots \dots C (8, 0) \text{ and } D (0, 4)$$

Now with these coordinates we plot and join them on the Cartesian plane and this is an MLP graph. For each line on the graph, we write its linear inequalities.

Figure 4.6 below shows the MLP graph that Mr Takui drew on the whiteboard, which clearly indicates the first quadrant of the Cartesian plane with linear inequalities and coordinates that can enable him to show feasible regions.

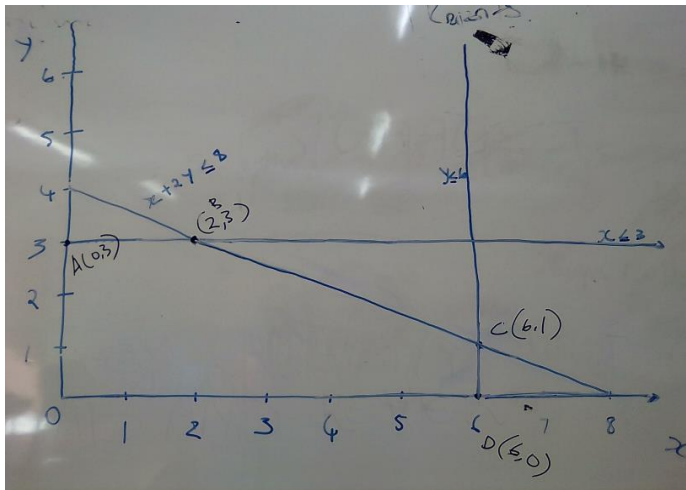


Figure 4.6: MLP graph without feasible area (taken from classroom)

Mr Takui: “The next step is to show feasible region on this graph (he pointed to Figure 4.6 on the board.) Firstly, we should put arrow directing where direction of inequality sign directs us. Example, if we have inequality sign is “ $\leq$ ”, the arrow of the line, will point down or to the left hand side and also if we have inequality sign “ $\geq$ ”, the arrow of the line, will point up or to the right hand side. Therefore, the area where these arrows face together is called feasible region and is what we did in NCV mathematics L3 last year.”

Mr Takui then explained and demonstrated all arrows of each line and shaded area where all arrows meet or point out (Figure 4.7).

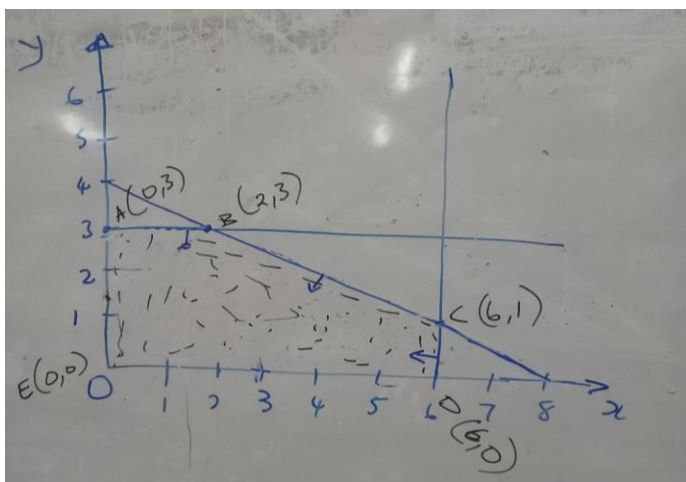


Figure 4.7: MLP graph (adapted from classroom)

Mr Takui: “Now, we draw an MLP graph showing feasible region, which is made by following points (A (0, 3); B (2, 3); C (6, 1); D (6, 0); and E (0, 0)). So, like we did in NCV Mathematics L3, with shaded area we can be able to determine number of machines to be manufactured in maximising profit. In answering question number 2, we substitute these points A (0, 3); B (2, 3); C (6, 1); D (6, 0); and E (0, 0) into this objective function $P = 300x + 600y$ in order to

determine maximum profit. So, let's practicalise or do calculations. (He went to the board and did substitution. See Figure 4.4)

Mr Takui: "The calculation on the board proves that this company can maximise its profit when it produces 2 standard washing machines and also 3 luxury washing machines per day **OR** when it produces 6 standard washing machines and 1 luxury washing machines per day because both made profit of R 2 400. So, we have answered question 2, in conclusion question number 1 assisted us to do question number 2. This is what you should do when doing this kind of MLP problem."

The students looked very surprised and tired. They started to make a noise; some were talking to one another.

Tshepe: "Haah, what a long presentation, eish I am very lost and confused. You know I have been struggling with this MLP topic, especially solving linear inequalities and sketching its graphs for years."

Poho added: "Nna ke lahlehile qalong kwana, mme jwale wa tseba hore Ntate wa halefa, ha re botsa tse dipotso ngata, ha re sa utlwisisi teng (for me, I got lost at the beginning of this lesson and now you know Mr Takui get angry with us when we ask too much questions when we do not understand.)"

Mr Takui: "I believe you understand. But, we will do another two examples hopefully tomorrow and on Thursday before moving to next topic, so that you get a picture and understand more."

The class ended and over the next two days, Mr Takui did two more examples in the textbooks to help students to understand. However, he was in a rush, pushing the syllabus.

The above lesson presentation reveals that the method of teaching of MLP is too teacher centred. From the beginning to the end of his lesson presentation, Mr Takui, only provided students with MLPCK. He also did not allow interaction among students, which discouraged the students to learn from their peers (cf. above presentation). This was affirmed by the following quote:

Mr Takui: "Ok, let's go to page 96 in your textbook. Right, to solve this kind of MLP problem, we first need to convert MLP word problems into linear inequalities; secondly, to solve linear inequalities; and lastly to draw, analyse and interpret MLP graph with this sequence."

He continued to read, analyse and interpret the MLP scenario. He also showed the students how to solve MLP problem questions:

Mr Takui: "Based on what I have read, firstly remember...we need to write linear inequalities ... secondly, we have to solve linear inequalities ... draw MLP graph so that we can find feasible region, so as to determine maximum profit.." and further said "from the first sentence I read (cf. **above problem statement 1**), we find two types of machines manufactured by this company, so we first let ... Type A (standard machine) be x and Type B (luxury machine) be y ... (he cautioned students by saying)... always remember to show these linear inequalities, $x \geq 0$ and $y \geq 0$..is a rule."

Subsequently, he continued to read, analysed, interpret and converted the MLP word problem into linear inequalities, then got the following linear inequalities, $x \leq 6$... (linear inequality 1), $y \leq 3$... (linear inequality 2), $x + 2y \leq 8$... (linear inequality 3) and also $P = 300x + 600y$, which is objective function (cf. above lesson). The lesson continued up to where Mr Takui drew an MLP graph and determined the maximum profit (cf. above lesson).

This is exactly the teaching process that mathematics lecturers applied at this TVET college. The teaching method Mr Takui used in the above lesson links well with the teacher-centred method. The teacher-centred method is one with traditional teaching methods and was dominant prior and during the first moment of bricolage. So, during and prior this era (1900-1945), traditional teaching methods were the only teaching ways and means approved and used to teach mathematics. As a result, the literature in Section 2.4.5.3 evidenced that traditional teaching methods improved MLPCK teaching and learning. In contrast, Section 2.4.1.3 of this study emphasised that there is no one single method that is panacea to all teaching and learning of mathematics challenges. For that reason, this notion encourages the use of multiple methodologies and strategies. The literature in Section 2.4.1.3 exposed the gaps and limitations of traditional teaching methods, because students have unique learning styles, which make it difficult for them to learn in a mathematics classroom where a lecturer uses traditional teaching methods to convey MLPCK, as confirmed by the following statements during Mr Takui's MLP lesson:

Poho: *"What a long presentation, eish I am very lost."*

Tshepe: *"I got lost at the beginning of this lesson."*

These words of students exposed the consequences of using only one single method in MLPCK teaching, which is teacher-centred method. The scholars in Section 2.4.1.3, agreed that students learn differently; for that reason an individual has his/her own learning styles, and ignoring these learning styles will lead to poor performance in mathematics (including the MLP). More importantly, limited traditional pedagogical methods (teacher-centred method) prior to or during the Bricolage traditional era opened space for Bricolage blurred genres, which endorsed multiple methodologies, pedagogies and strategies in teaching MLPCK to bridge the gaps (limitations) of the traditional era (cf. 2.2.2.3).

Section 2.4.1.3 of this study showed that the traditional teaching method includes the use of lectures and discussions, while MLPCK problem-solving elements (or any SCK) are presented or discussed by a lecturer. It means there is nothing that students can do or learn without involvement by their lecturers, because the traditional teaching method places lecturers in the highest position and their knowledge is valued more; hence, they are taken as powerful people. This further means that nothing is regarded as the truth, fact and/or correct without approval of their lecturer. Moreover, in the teacher-centred classroom milieu, lecturers use a specific textbook, which reveals the correct procedure to be applied in MLP problem solving. The lecturer becomes the dominant source of information where all students' questions are only answered by s/he without involving other students. All features of the teacher-centred method have been realised in Mr Takui's lesson, because

- use from the beginning to the end, he relied more on the textbook.
- Secondly, he dominated the MLPCK teaching and learning process (he played a central role).
- Thirdly, he gave students MLP information, which included the procedures that will enable them to solve MLP problems.
- Fourthly, he demonstrated processes or procedures on how to unpack MLP questions.
- Lastly, he failed to interact and involve students during his lesson presentation. He dominated the teaching and learning process and did everything alone without involving students (cf. the above MLP lesson presentation).

It is clear that the above characteristics of the teacher-centred method (traditional teaching method) are not in line with the PAR methodology of this study, because in Section 3.2.1, the PAR is defined as a methodology which nurtures, emboldens and promotes interactions or collaboration amongst lecturers and students and/or amongst students. In contrast, the above MLP lesson presentation at Khotsoeng TVET college exposed exactly what Mr Takui and Ms Lebone do when teaching MLPCK, which is mainly to deposit MLPCK into the students' minds and provide the answers to all the questions the students raised, without giving other students the opportunity to learn from their peers. They denied other students an opportunity to teach and link their prior knowledge to the new MLPCK. For example:

Nxumalo (student): “Sir, where $x \geq 0$ and $y \geq 0$ comes from? in the MLP problem statement you read?”

Mr Takui: “Good question, Nxumalo, you know its rule, when we do MLP we are always expected to show these two basic linear inequalities. Therefore, it’s a basic rule, so you have to write them.”

Ms Lebone: “Question 1.1 needs us to write linear inequalities from the above scenario on Activity 1 ... So, very first thing is to let one product/ item be x and the other be y ... the second step is to write linear inequalities. So, before we do that you have to understand and be able to write these words ... Mathematically ... “at most as ($\leq$), at least as ($\geq$), more than as ($>$), less than as ($<$), minimum as ($\geq$), maximum as ($\leq$), more than or equal to as ($\geq$), and less than or equal to as ($\leq$) ... these Mathematical signs ($>$) and ($<$) graphing it we use dotted lines, but for ($\leq$) and ($\geq$) we use solid line.”

These words show that even today students are regarded as the receiver of mathematics knowledge, so these MLP lesson presentations (cf. Annexure J) refute what Vygotsky signposted in Section 2.2.4, because Vygotsky indicates that effective teaching and learning take place when students interact with their peers, with the purpose of sharing their background knowledge gained in lower grades or NCV level 2 and 3 or in the community and other milieus, since learning is regarded as social activity. The above MLP lesson presentation revealed a discursive practice and social exclusion, because he used the teacher-centred method, where the lesson was mainly on depositing and transferring the MLPCK into students’ minds through the chalk-and-talk method. Look at the following statement:

Mr Takui: “Based on what I have read, firstly remember ... we need to write linear inequalities ... secondly, we have to solve linear inequalities ... draw MLP graph so that we can find feasible region, so as to determine maximum profit ... “from the first sentence I read (cf. **above problem statement 1**), we find two types of machines manufactured by this company, so we first let ... Type A (standard machine) be x and Type B (luxury machine) be y ... remember always to show these linear inequalities, $x \geq 0$ and $y \geq 0$.”

On the other hand, Ms Lebone in her lesson said:

Ms Lebone: “Question 1.1 needs us to write linear inequalities from the above scenario on Activity 1 ... So, very first thing is to let one product/ item be x and the other be y ... the second step is to write linear inequalities. So, before we do that you have to understand and be able to write these words Mathematically ... at most as ($\leq$), at least as ($\geq$) ... for ($>$) and ($<$) graphing them we use dotted lines, but for ($\leq$) and ($\geq$) we use solid line.”

These words show that at Khotsong TVET college it is the norm to teach MLPCK in this fashion where lecturers present MLP lessons without interacting with students or encouraging interactions among students (cf. Annexure J). This kind of discursive practice leads to social exclusion, because lecturers regard themselves as the only people with valuable knowledge; hence, they do not encourage participation among

students, so this thinking and belief were dominant in the Bricolage modernity phase and traditional era (cf. 2.2.2.1 & 2.2.2.2).

Section 2.4.1.3 of this study agrees with the above empirical data in Section 4.2.3 that the method of teaching MLPCK at TVET colleges is too teacher centred and contributes to poor MLP performance. Hence, it is regarded as a challenge. Both revealed that lecturers at TVET colleges rely solely on the teacher-centred method to impart MLPCK, but researchers (Section 2.4.1.3) argue that using solely the teacher-centred method is not adequate, because it has gaps and limitations. This opens the space for multiple teaching methods to teach MLPCK to cater for students different styles. This notion is supported by Palocsay and Steven (2004:38), who indicate that there is no single teaching method that is panacea to all teaching and learning challenges (cf. Section 2.4.2.3).

In conclusion, the literature and generated data revealed that using only the teacher-centred method to teach MLPCK makes it hard and difficult for kinaesthetic and other students with different or unique learning styles, which leads to poor performance of mathematics (including the MLP), as shown in Figures 4.10 to 4.13.

4.2.4 A need to recognise prior knowledge and learning

Recognising student prior knowledge is regarded as the teaching technique that can be used to improve the teaching and learning process and also students' achievement (cf. 2.4.2.4). There are multiple ways to recognise, actualise and determine students' prior knowledge, namely prior knowledge assessments (diagnostic tests), brainstorming a specific module, and random question posing. However, this study sees prior knowledge assessment as the most significant tool to determine pre-existing knowledge of students. This idea is supported by Leone *et al.* (2010:3-4), who maintain that one way to enhance student high-quality learning may be the use of prior knowledge assessment as a tool to evaluate the level of support a student will need (cf. 2.4.2.4).

Hailikari *et al.* (2008:2) indicate that in the prior knowledge valuation, as in assessment in general, the instructor (lecturer) should be aware of what type of knowledge is assessed and distinguish among different types of prior knowledge, which is vital in a particular context, because not all types of prior knowledge have similar relevance to

students' achievement. Prior knowledge is defined as a multidimensional and hierarchical entity that is dynamic by nature and which involves what students already know as a result of learning in the workplace and/or in other settings. They further indicate that prior knowledge consists of different types of knowledge and skills, namely the declarative and procedural prior knowledge (cf. 2.4.2.4). Yuksel (2012:1197) agrees with Hailikari *et al.* (2008:2) that prior knowledge has long been considered the most important factor influencing learning and student achievement.

The amount and quality of the prior knowledge positively impact knowledge acquisition and the capacity to apply higher-order cognitive problem solving skills. Adequate prior-knowledge assessments provide detailed information about students' prior knowledge base. It is also beneficial as a diagnostic tool for student support. The DHET policy on Recognition Prior Learning (RPL) policy, revealed that lecturers are required to search, recognise and actualise relevant prior learning. This must take place on individual and/or student-by-student basis through assessment and professional judgement of the prior learning (DHET, 2013d:73). However, learning outcomes and objectives must not be compromised as a result of RPL. Therefore, it is clear that all lecturers, before teaching new knowledge, are expected to give students prior knowledge assessments to get a picture about what students already know or where they are lacking.

In contradiction, Mr Takui started the MLP lesson by highlighting key procedures to follow in solving MLP problems. He presented MLP topic without recognising students' prior knowledge of MLP:

Mr Takui: *"Today we are going to do Mathematical linear programming. Let's go to page 80 of your textbook. Right this is the topic of MLP; so, to solve this kind of MLP problem in L4, we first need to convert MLP word problems into linear inequalities; secondly, we solve linear inequalities; and lastly we draw MLP graph with that good sequence."*

Mr Takui took the students through an example on page 84 in the textbook and thereafter demonstrated MLP Problem Statement 1 below to show students how to solve this problem (cf. 4.2.1). He interpreted Problem Statement 1 and highlighted key processes that one has to follow in order to answer the above questions. Mr Takui further interpreted the second, the third, and the fourth (last) sentence. He found the following linear inequalities and the objective function: $x \leq 6$; $y \leq 3$; $1x + 2y \leq 8$; and $P = 300x + 600y$, respectively.

Mr Takui: “This objective function will be used after we have drawn the MLP graph. So, now we have answered question number 1 ... all right, we move to the question numbers 2, which needed exactly number of each type of machine should be manufactured in order to maximise profit (**cf. problem statement 1**). This question requires us to determine a point where the profit will be maximised, so, to answer this question we must draw MLP graph. However, before we draw MLP graph, we first need to solve all above linear inequalities.”

Mr Takui solved and drew linear inequalities on the board (cf. Fig. 4.2), and determined feasible region and its points (cf. Fig. 4.3).

Mr Takui: “Now, we draw MLP graph showing feasible area or region. So, like we did in NCV mathematics L3, with shaded area we can be able to determine number of machines to be manufactured in maximising profit. In answering question number 2, we substitute these points into this objective function $P = 300x + 600y$ in order to determine maximum profit. So, let's practicalise or do calculations.” (He went to the board and applied substitution method (cf. Figure 4.4)

The calculation on the board proves that this company can maximise its profit when it produces 2 standard washing machines and also 3 luxury washing machines per day **OR** when it produces 6 standard washing machines and 1 luxury washing machines per day because both made profit of R 2 400. So, we have answered question 2, in conclusion question number 1 assisted us to do question number 2. This is what you should do when doing this kind of MLP problem.”

The students looked very tired and began to make a noise. Some were talking to one another. Tshepe said to Poho:

Tshepe: “Haah, what a long presentation, eish I am very lost and confused. You know I have been struggling with this MLP topic, especially solving linear inequalities and sketching its graphs for years.”

Poho: “Nna ke lahlehile qalong kwana, mme jwale wa tseba hore Ntate wa halefa, ha re botsa tse dipotso ngata, ha re sa utlwisisi teng (for me, I got lost at the beginning of this lesson and now you know Mr Takui get angry with us when we ask too much questions when we do not understand).”

Mr Takui: “All right, all right, yah, I hope you understand. However, we will do another two examples hopefully tomorrow and on Thursday before moving to next topic, so that you get a picture and understand more.”

The class ended and over the next two days Mr Takui did two more examples in the textbook to help students to understand. However, he was pushing the syllabus.

Therefore, from the above lesson presentation, we as a research team identified another challenge contributing to poor performance in mathematics.

Mr Koena: “Colleagues, what challenge have you also identified?”

Mr Kia: “Mr Takui did not search students MLP prior knowledge which made it difficult for students to understand the lesson presented.”

Ms Lebone: “I have different view on that point, but I am not disputing what has been said. I think lecturers should teach what they are supposed to teach. If students lack MLP prior knowledge, are you going to teach them that knowledge that was supposed to be taught at the previous level? Again, when are you going to complete the coverage of the syllabus?”

Ms Lebone: *“Exactly, you can’t teach it. So, I think we should teach what is in the book. Teach what you are supposed to teach and forget about prior knowledge assessments.”*

Ms Moleko: *“Prior knowledge is the key for new knowledge to be learned.”*

In the above lesson presentation Mr Takui did not search, recognise and actualise students’ MLP prior knowledge to know and be informed about what students already know. As a result students struggled to understand the MLP lesson presentation, while Mr Takui was unable to detect students’ MLP misconceptions and misunderstandings. This was also raised by one of co-researchers:

Co-researcher: *“Mr Takui did not search students MLP prior knowledge which made it difficult for students to understand the lesson presented.”*

This confirmed by Tshepe during this lesson:

“... you know I have been struggling with linear inequalities and its graphs for years.”

The quote:

“I have been struggling with linear inequalities and its graphs for years.”

proved that this student has inadequate prior knowledge in other components of MLP, namely linear inequalities and MLP graphs and this inadequate of prior knowledge contributed to or added confusion to Tshepe and other students in this lesson. In Section 2.4.2.4, the literature revealed that inadequate or fragmented prior knowledge is an important element to take into consideration, because if there is a mismatch between lecturers’ expectations and actual students’ base knowledge, learning may be hampered from the beginning of a lesson. Hailikari *et al.* (2008:4) add that, trying to learn new knowledge without having adequate prior knowledge, or having misconceptions, may result in rote memorisation (cf. Section 2.4.2.4).

This lesson presentation showed that Mr Takui hails from the Bricolage traditional era, because during this period, students were taken as people without prior or background knowledge actualised, while the lecturer mainly focused on depositing new knowledge in students’ minds (cf. 2.2.2.1). This is exactly what Mr Takui did in the above lesson. This is not in line with one of the principles of PAR that locate the voice of the voiceless at the centre of teaching and learning (cf. 3.2.3). In addition, MLP lesson presentations of Mr Takui in Annexure J also exposed a discursive practice, because Mr Takui did not recognise or actualise students’ MLP prior knowledge all lessons. This technique was strongly supported by his colleague, who argued that:

Ms Lebone: *“I have different view on that point ... I think lecturers should teach what they are supposed to teach. If students lack MLP prior knowledge, are you going to teach them that knowledge that was supposed to be taught at the previous level? Again, when are you going to complete the coverage of the syllabus?”*

These words reveal that it is the norm to teach MLP in this fashion at this TVET college, because Ms Lebone sees no problem with teaching new knowledge without recognising or actualising and searching students' prior knowledge. However, the literature shows the limitations of not valuing students' prior knowledge, which leads to low student achievement and poor performance.

Section 2.4.1.4 and 4.2.4 of this study agreed that failure to recognise prior knowledge of students in any subject content hampers the teaching and learning process because there will be a mismatch between the lecturers' expectations of students' knowledge and the students' knowledge base; thus this will result in students' low achievements.

In conclusion, the empirical data revealed that the baseline or diagnostic assessments are not implemented well.

4.2.5 A need for formative assessments in the teaching of MLP

Formative assessment is defined as a process in which both teachers and students provide feedback during instruction to organise the learning and teaching process in order to increase students' achievement (cf. 2.4.2.5). The vital importance of formative assessment approach is that it informs lecturers about whether the students have learned and understood the lesson well in order for the lecturer to plan the next lesson. In essence, the formative assessment can be seen as an indicator, because it gives a lecturer a clear picture about a lesson presented. The literature in Section 2.4.2.5 also revealed four main of formative assessments, namely explaining learning objectives and success criteria; increasing the quality of inquiry/dialogue; increasing the quality of marking/feedback/record keeping; and also using peer and self-assessment.

The assessment guideline for NCV mathematics developed by the DHET defines a formative assessment as assessment that takes place during the process of learning (DHET, 2013a:6). This assessment guideline emphasises that students need to be assessed through a process of continuous assessment, which will be conducted throughout the learning process. It should be noted that self-assessment, group assessment, and peer assessment cannot be recorded for the allocation of final

marks, but solely guide decisions about promotion. TVET colleges' curriculum instruction ICASS guideline for NCV implemented annually is in support of integration of formative assessments, which in this document is taken as additional supporting tasks, according to the DHET (2018b:07). These tasks include class tests or quizzes, observations, discussions and practical demonstrations that are used to monitor students' daily progress and are administered to support the teaching and learning process. Moreover, additional supporting tasks are used to give students feedback, to inform planning for teaching, and should serve as building blocks to assessment, but should not be seen as separate from learning activities taking place in the classroom. Lecturers can mark formative assessments or allow students to mark those of their peers, but these tasks do not contribute to students' ICASS mark; it must be recorded or filed separately.

In light of the above information about the formative assessment, it is clear that formative assessments are to be given during or at the end of a lesson. However, this is not what Mr Takui did in his lesson presentation (cf. 4.2.1). Look at the MLP lesson below presentation extracted from full MLP lesson presentation in Annexure J. Mr Takui took students through the example on page 84. Thereafter he demonstrated **Problem Statement 1** to show students how to solve an MLP problem (cf. 4.2.1).

He interpreted the **Problem Statement 1** and highlighted key processes that one has to follow in order to answer the above questions. Mr Takui next analysed and interpreted each sentence in the **Problem Statement 1**, then formulated the following linear inequalities and objective function $x > 0$, $y > 0$, $x \leq 6$, $y \leq 3$, $x + 2y \leq 8$ and $P = 300x + 600y$, respectively. Furthermore, Mr Takui solved and sketched the linear inequalities on the Cartesian plane (cf. Figure. 4.2) and also showed students how to find feasible region (c.f. Figure 4.8 below).

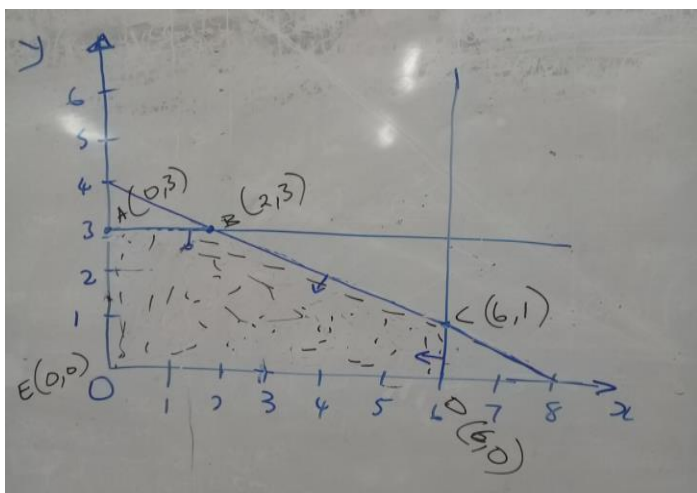


Figure 4.8: MLP graph (adapted from classroom).

Mr Takui: “Now, we draw MLP graph showing feasible area or region, which is made by following points (A (0, 3); B (2, 3); C (6, 1); D (6, 0); and E (0, 0)). So, like we did in NCV mathematics L3, with shaded area we can be able to determine number of machines to be manufactured in maximising profit. In answering question number 2, we substitute these points A (0, 3); B (2, 3); C (6, 1); D (6, 0); and E (0, 0) into this objective function $P = 300x + 600y$ in order to determine maximum profit. So, let's practicalise or do calculations. (He went to the board and applied substitution method (c.f. Figure 4.4 in Section 4.2.1))”

“This company can maximise its profit when it produces 2 standard washing machines and also 3 luxury washing machines per day **OR** when it produces 6 standard washing machines and 1 luxury washing machines per day because both made profit of R 2 400. So, we have answered question 2, in conclusion question number 1 assisted us to do question number 2. This is what you should do when doing this kind of MLP problem.”

Students seemed lost and they began to make a noise, some talking to one another.

Tshepe: “Haah, what a long presentation, eish I am very lost and confused. You know I have been struggling with this MLP topic, especially solving linear inequalities and sketching its graphs for years.”

Poho: “Nna ke lahlehile qalong kwana, mme jwale wa tseba hore Ntate wa halefa, ha re botsa tse dipotso ngata, ha re sa utlwisisi teng (for me, I got lost at the beginning of this lesson and now you know Mr Takui get angry with us when we ask too much questions when we do not understand).”

Mr Takui: “I believe and hope that you understand. However, we will do another two examples hopefully tomorrow and on Thursday before moving to next topic, so that you get a picture and understand more.”

The class ended and during the next two days Mr Takui did two more examples in the textbooks to help students to understand, but he was rushing to complete the syllabus.

From the above lesson presentation, the co-researchers analysed this MLP lesson presentation, trying to identify another challenge contributing to poor performance.

Mr Koena: “Colleagues, what have you also picked up?”

Ms Moleko: “Mr Takui from the entire MLP lesson presentation have not assessed students.”

Mr Kia: *"Lecturers do not assess students during the lesson. When we do class visit, IQMS and files monitoring we do not find any kind of formative assessment together with evidence. I think is the reason for them to perform bad in mathematics."*

Ms Lebone: *"Time is not enough to assess students during the lesson because we are trying to push the syllabus. Some of students take the whole period writing the activity, so to assess them during the lesson is the waste of time because we do not record the formative."*

Mr Takui: *"Formative assessments waste time, we have syllabus to cover so that they can be ready for tests and exam."*

The above MLP lesson presentation also showed the lack of formative assessment. In his entire MLP lesson, Mr Takui did not test students to check whether students understood the lesson well. This challenge was identified by Ms Moleko at one of this study research meetings.

Ms Moleko: *"Mr Takui in his entire MLP lesson presentation did not assess students."*

Mr Takui could not disagree with her, but maintained that assessing students during the lesson presentation was a waste of time because they had a syllabus to cover, to prepare students tests and examinations.

Mr Takui: *"Formative assessments waste time."*

This led to the lack of formative assessments (additional supporting tasks). The perception Mr Takui had about formative assessment, made him to teach all MLP components in one lesson without testing students during the lesson, in contradiction with the assessment policy of the TVET college. According to this policy, students need to be assessed through a process of continuous formative assessment conducted throughout the teaching and learning process. This was not done by Mr Takui, which is a deviation from TVET policy, In light of the importance of formative assessments revealed in Section 2.4.2.5, it is clear that poor assessment methods and a lack of additional supporting tasks (formative assessment) have a negative impact on mathematics performance because lecturers are unable to identify students' learning gaps in order to plan and prepare a next lesson. This was confirmed by this student, speaking to Tshepe during Mr Takui's lesson presentation.

Poho: *"I was lost from beginning of this lesson."*

The same students kept saying they followed and understood when asked. However, Poho's statement showed the opposite to what students kept saying. Mr Takui was not able to pick up some gaps in the students' progress, because he failed to test their knowledge during the teaching and learning process. The above lesson presentation of Mr Takui also revealed a discursive practice, as confirmed by his statement.

Mr Takui: *“Formative assessments waste time.”*

Mr Takui presented MLP lessons (cf. Lesson 1, 2, 3) without assessing students to check whether they grasped the lesson. This was his usual way of teaching MLP and other mathematics topics, because of his perception that they wasted time. Hence, he presented MLP lessons without testing students’ knowledge before moving on to another MLP concept. He presented lessons in this fashion because he wanted to complete the syllabus, so that students could be ready for summative assessments. The following statement showed that he was in a hurry to cover the syllabus.

Mr Takui: *“However, we will do another two examples hopefully tomorrow and on Thursday before moving to next topic, so that you get a picture and understand more.”*

This shows that Mr Takui functions in the Bricolage traditional period, where additional supporting tasks (formative assessments) are not implemented properly (cf. 2.2.2.1). During this period, summative assessments were dominant and at the centre of the teaching and learning process, which means students were only assessed towards the end of the course. The Centre for Education Research and Innovation (2008:7) signposts that the concept of formative assessment was first introduced and executed in 1971 (cf. 2.4.2.5). Tridane *et al.* (2015:685) and Ajogbeje (2013:8-9) suggest that teachers should include episodes of formative assessment (additional supporting tasks) in following phases of teaching, so that teachers should give students feedback as a way of remediating their work. The absence of formative assessment proves that mathematics lecturers rely only on summative assessments; hence students are not performing well in MLP topic (Figures 4.10–4.13).

The statement of Mr Takui above further exposes that he had power, because according to him, formative assessments wasted time; hence, he did not consider it. This exposes social structure, namely Mr Takui as a powerful person, deciding not to include formative assessment. Therefore, the manner in which Mr Takui operates in his class leads to social exclusion and discursive practices. This is not in line with the second principle of PAR and the sixth moment of Bricolage, namely locating the voice of side-lined students and post-experimental inquiry moment, with both supporting and promoting collaboration between students and lecturers or students among students (cf. 3.2.3 & 2.2.2.6). Mr Takui was supposed to give students an opportunity to provide their views by means of formative assessments, so that both can reach consensus if

time does not allow for him to cover the syllabus. Formative assessments are meant to assist and prepare students for summative assessments.

The literature in Section 2.4.2.5 confirmed that poor assessment methods contribute to poor performance of students. The above MLP presentation by Mr Takui exposed the lack or absence of formative assessment, which was identified by co-researchers when the team analysed the lesson. The literature in Section 2.4.2.5 further revealed that there is a positive relationship between formative and summative assessments. Thus, poor assessment methods and the lack of formative assessments contribute to poor performance in MLP and/or in mathematics. This study found that mathematics lecturers purposefully ignore formative assessment; hence students do not perform well in MLP or mathematics in general (cf. MLP lesson in Annexure J). In conclusion, the literature and the generate data revealed that a lack of formative assessments is another factor that contributes to poor performance in mathematics.

4.2.6 Students' difficulties, errors and misconceptions of Mathematical Linear Programming (MLP)

Mathematical Linear Programming is a part of mathematics taught in NCV L3 and L4 at TVET colleges. The MLP refers to a mathematical model method that helps people to make decisive decisions in different sectors (cf. 2.3.1). It comprises word problems, linear inequalities, straight line graphs, feasible regions and objective functions that assist in determining maximum and/or minimum values. According to the mathematics NCV assessment guideline, students are expected to use mathematical models to examine linear programming problems through the formulation of linear constraints from a given problem and solve problems by optimising a function in two variables, subject to one or more linear constraints, using a search line method (DHET, 2013:15). To solve MLP problems, one should sketch linear functions/constraints; secondly, determine and shade a feasible region; thirdly, work out the gradient of a search line; and lastly, use a search line to optimise maximum or minimum from the objective function. The co-researchers agreed that the topic of MLP is problematic and one of the most difficult mathematics topics for students.

***Mr Takui:** "Most students find it difficult to solve MLP questions, so they skip them by so saying they do not answer them at all."*

Ms Moleko: “Most students fail this question or topic, like out of 20 marks you find that the highest is student with 6/20, then the rest will be between 0-3 out of 20 marks. Is just a disaster.”

Mrs Mahapa (former FET phase mathematics teacher): “Most students are unable to formulate the MLP problem into linear constraints and objective functions. Again, determining feasible region is the hardest thing to them.”

Mr Koena: “Some students confuse equal sign with inequality signs, so this is the origin of most of problems. Hence, they find it hard to find the feasible region. Again, most of students confuse greater than, less than, maximum, minimum, greater or equal and also less or equal to, then graph automatically will be wrong.”

Mr Takui: “Most students are unable to convert MLP word problem into linear constraints, again they are unable to sketch linear constraints and also determine feasible area and this makes it hard to find max or min values.”

Mrs Matlou (former mathematics teacher): “Students get confused in sketching the Cartesian plane of bigger numbers and you find scale not accurate on both x and y axes, therefore, this make feasible region incorrect which also lead to wrong max or min value.”

Mrs Lebone: “A use of search line method is a problematic to most of students.”

Ms Khodu: “Students make so many mistakes in solving linear inequalities and sketching them. So that makes it difficult for students to find feasible region.”

The students’ responses from test question 3 also showed that students had difficulties with MLP (Figure 4.9 below).

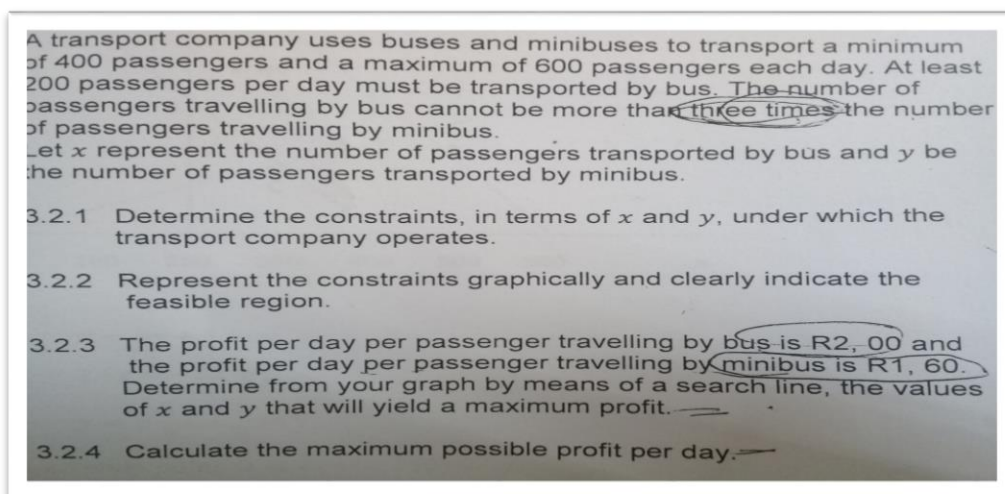


Figure 4.9: Question 3 extracted from Test 1

Students’ responses:

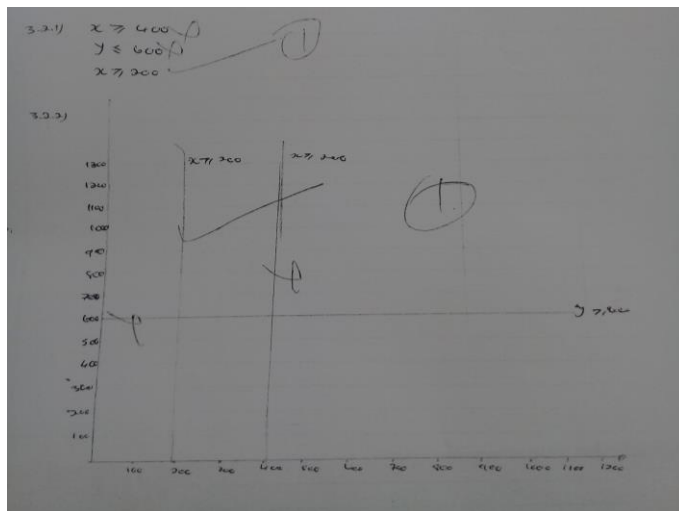


Figure 4.10: Naliswa's solution to question 3 (extracted from Test 1)

The above example reveals that Neliswa struggled to formulate linear inequalities from word problems appropriately and this affected the sketching of linear inequalities negatively, which made it hard for Neliswa to find feasible region and determine a maximum value, because if linear inequalities are incorrect then graph, feasible region and maximum value will also be incorrect. However, Neliswa was able to draw linear inequalities correctly (cf. Question 3.2.1) even though some were incorrect, but she was not able to show directions that satisfy inequality signs. Neliswa's Cartesian plane was also not drawn accurately (measurement) and did not indicate the x and y axes.

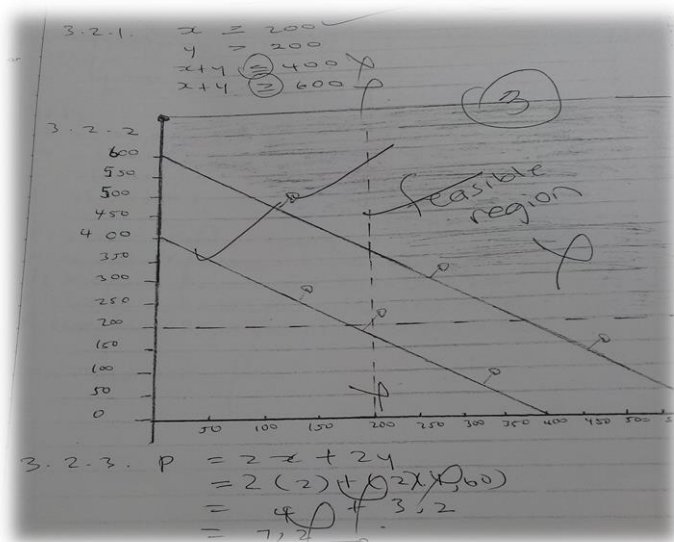


Figure 4.11: Mike's solution to question 3 (extracted from Test 1)

The above response revealed that Mike confused MLP jargon, namely minimum and maximum; hence the inequality signs were incorrect. Mike did not consider the third

statement (line) and this could mean that he did not understand this statement or skipped it by mistake; hence, he did not write this statement mathematically as $x < 3y$. This led to the inability to sketch the linear inequalities, determine feasible region and maximum value. Mike was also unable to formulate the correct objective function, evidencing that he lack an understanding of how to formulate objective function; hence he was not able to determine maximum value for Question 3.2.4. Mike's Cartesian plane was not drawn accurately (measurement), leading to incorrect feasible region and maximum value. He also did not label the x and y axes on the Cartesian plane.

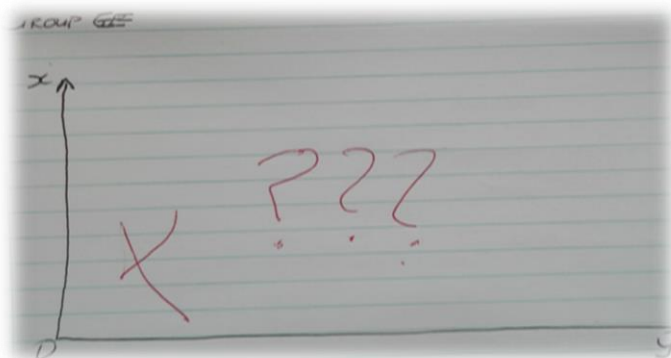


Figure 4.12: Poho's solution to question 3 (extracted from Test 1)

The above response revealed that Poho had serious difficulty with MLP, since he was not able to solve the questions (3.2.1-3.2.4). He was unable to formulate linear inequalities from the word problem, sketch linear inequalities, show feasible region and determine maximum value. His Cartesian plane was not measured, was inaccurate, and the mixed intercepts were incorrect.

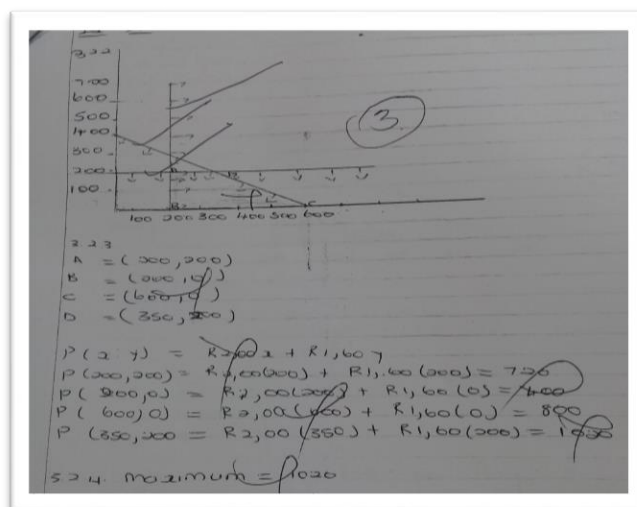


Figure 4.13: Wilson's solution to question 3 (extracted from Test 1)

Wilson's solution revealed that he struggled to formulate linear inequalities from MLP word problems appropriately; hence he got incorrect linear inequalities, which led to the incorrect graph and feasible region. However, he was able to sketch some linear inequalities and determine the feasible region, even though they were incorrect, but it showed that he has some idea about MLP graphs. He was further able to formulate objective function and determine a max value through the substitution method, even though they were incorrect. His measurement of the Cartesian plane on the x and y axes was not accurate, leading to inaccurate feasible region and maximum or minimum values.

The above statements of the co-researchers agree that MLP topic is problematic and challenging to students, because most students are unable to answer MLP questions (they skip them) and they do not perform well in this topic.

Mr Takui: "Most of the students find it difficult to solve MLP questions, so they skip them by so saying they do not answer them at all."

Ms Moleko: "Most of the students fail this question or topic, like out of 20 marks you find that the highest is student with 6/20, then the rest will be between 0-3 out of 20 marks. Is just a disaster."

This leads to students' poor mathematics performance overall. The co-researchers further highlighted factors that prevented students from performing well in MLP and to complete MLP questions. Look the following quotes of Mrs Mahapa and Mr Takui:

Mrs Mahapa & Mr Takui: "Most students are unable to formulate the MLP problem into linear constraints and objective functions. Again, determining feasible region is the hardest thing to them" and "most students are unable to convert MLP word problem into linear constraints, again they are unable to sketch linear constraints and also determine feasible area and this makes it hard to find max or min values" respectively".

The co-researchers also indicated that most students committed so many mistakes in formulating the linear inequalities and objective function from MLP problem statement; solving and sketching linear inequalities; and determining feasible area and max (or min) value, caused by misconceptions and a misunderstanding of MLP.

Mr Koena: "Some students confuse equal sign with inequality signs, so this is the origin of most of problems ... again, most of students confuse greater than, less than, maximum, minimum, greater or equal and also less or equal to, then graph automatically will be wrong."

Mrs Matlou: "Students get confused in sketching xy-Cartesian plane of bigger numbers and you find scale not accurate ... this makes feasible region incorrect ... also leads to wrong max or min value."

Ms Khodu: *"Students make so many mistakes in solving linear inequalities and sketches."*

This was evidenced by the above students' workings, as a result this leads to the MLP poor performance and mathematics results as well.

In conclusion, this Section agrees with the literature in Section 2.4.1.6, which revealed that MLP is one of the problematic mathematics topics. Students have difficulties with MLP word problems, linear inequalities, MLP graphs, and feasible region, and maximum (or minimum) values. In addition, this study found that the difficulties are caused by misconceptions.

4.3 THE USE OF THE RME APPROACH TO TEACH MLP

This Section considers the use of the RME approach to challenges in the teaching and learning of MLP. The RME approach principles are taken as components of the RME approach, such as levels and intertwined, multiple teaching and learning aids; reality, interactivity and guidance; activity and interactivity as baseline assessment; and the continuous activity and feedback to achieve the aim of using the RME approach to teach MLP.

4.3.1 Levels and intertwinedness

Kamina and Iyer (2009:2) and Chang *et al.* (2017:1) recommend the use of both concrete and abstract approaches. Therefore, according to them, the teaching of mathematics should start from the concrete level to the abstract level for students understand the concepts (cf. 2.4.2.1). As a result, students can understand that mathematics is a human activity, around us and within us. Andrew (1990, in White, 1995:30) suggests that to teach MLP, lecturers are expected to break the MLP topic into five components (lessons) that should be taught sequentially, namely recognising and describing variables with limits in problem statements of MLP; formulating linear constraints; sketching linear inequalities to find the feasible region (area); determining maximum (or minimum) value using feasible region [analysis and interpretation]; revising the whole process; checking solutions; and prepare for tests (cf. 2.4.2.1). This TVET college requires of all lecturers to break topics down into two or more lessons so that students comprehend a topic properly. This was also suggested by co-researchers in the planning and preparing phase of this study.

Co-researchers: *“Since MLP has many components, we need to break it down according to its components, but all components should link to one another. We must progress from the lower level to the higher level.”*

The team regarded level and intertwined principles of the RME approach as appropriate to teach this topic.

4.3.1.1 First level for teaching MLP topic: introducing MLP terminology

This study explains MLP terminology as the language that is used to unlock complex MLP concepts for students to understand, namely, less than, not more than, less or equal to, at most, greater than, not less than, more than, greater or equal to, minimum, at least, equal to, maximum, mathematics variables, half of variable, twice of variable, times, and, half, one third, etc. The literature revealed that teaching a subject content language is one technique that could be used to help students understand any Subject Content Knowledge (SCK) well. Blanco and Garrote (2007:227) signpost that lecturers teaching linear inequalities should ensure that the symbols used are clearly understood and differentiated and that they have semantic value for the students. They should use different languages translating one geometric or algebraic concept to another (cf. 2.4.2.6). This can be done by developing a table of linear inequalities terminologies or symbols, namely $=$, $<$, $>$, $\leq$, and $\geq$, which students are supposed to complete as they work; therefore, to complete a table, students will be required to explain the meaning of terms and concepts and provide examples using their own words that are not duplicates of the text in the textbook. The team agreed that one should start teaching MLP language and terminology before MLP word problems to unlock students' difficulties with MLP as a first step.

The first lesson was presented by Ms Moleko in the classroom at Thabong Campus.

Students: *“Good morning, Mme.”*

Ms Moleko: *“here with me, I am with my team, ‘mathematics experts’, who I am going to work with in enhancing the MLP teaching and learning using the RME approach. So, today we are going to do MLP, as I have indicated last week.”*

Students: *“Yes, Mme.”*

Ms Moleko: *“In your groups, write down the definition MLP and explain the relevance of MLP in our daily lives, in fact, its importance in our lives. Write these in your own words. You have only 5 minutes to do this work.”*

Students started to work in their groups. Ms Moleko together with her team facilitated this process.

Ms Moleko: “Ok, now lets’ give feedback, we start here with Poho’s group.”

Poho: “As the group, we think that MLP is just a mathematics topic and we do not see the importance of it in our lives.”

Ms Moleko: “All right, lets’ hear Mampho’s group.”

Mampho: “MLP refers to a Mathematical model or technique helping us to make decisions in business production, industrial, engineering, social and physical sciences. Moreover, they insist that MLP assist us in maximising profit and minimising costs with limited resources available. But, we do not know the importance of MLP in our lives.”

The groups went further to explain the definition of MLP, but all groups did not know the importance of MLP in our lives.

Ms Moleko: “Thanks to all groups for your presentations. You are correct; MLP is one of mathematics’ topics and is defined as the Mathematical model tool helping individuals to make informed decisions in public and private sectors, namely business services, business production, industrial engineering, social, physical sciences and different environments. MLP is used in these sectors to maximise profit or minimise costs with limited resources available. So, the importance of MLP topic is that it can help businesses and people to make informed decisions using constraints resources to generate profit or minimise costs. I know you do not understand what I am talking about, but practical examples will help you.”

Ms Moleko “So, before we go deeper into this topic, these are MLP components that we have to treat. (Ms Moleko wrote these components on the whiteboard.) These are also learning outcomes.”

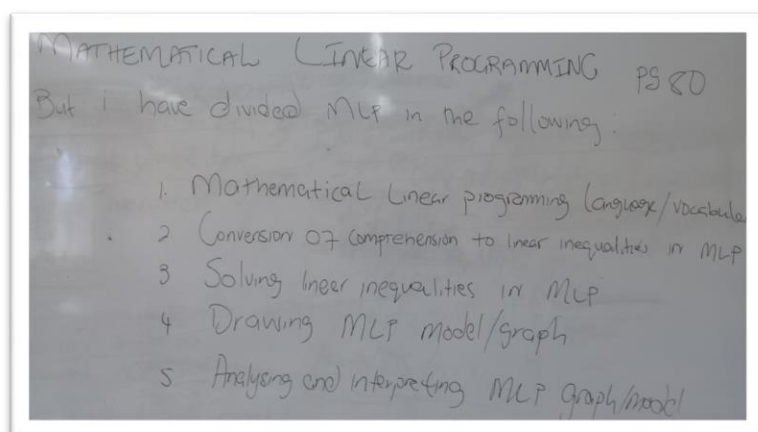


Figure 4.14: MLP components (taken from the classroom)

Ms Moleko “Introducing MLP, we first have to deal with MLP jargon; then move to the conversion of MLP word problems in linear inequalities; thereafter solving linear inequalities and drawing MLP models; and lastly analysing and interpreting MLP models. I, together with my team, have developed the following MLP terminologies which can help us unlock MLP word problems; however, these terminologies are not enough and please do not rely only on these them (Table 4.1)

Table 4.1: Activity A (developed by research team)

ACTIVITY A 2019 Complete below table:		
MLP terminologies	Definitions	Mathematical sign
1.1 Less than		

1.2 not more than		
1.3 Less or equal to		
1.4 At most		
1.5 Maximum		
1.6 Greater than		
1.7 Not less than		
1.8 Linear inequalities		
1.9 Greater or equal to		
1.10 Minimum		
1.11 At least		
1.12 Constraints		
1.13 Maths variables		
1.14 Feasible region		
1.15 Solid line		
1.16 Times		
1.17 Dotted line		
1.18 Half		
1.19 One third		

(27 marks)

Ms Moleko: “Let’s define these words together, apply real-life situations and write these words in mathematics symbols. So, in your own groups complete this table by defining these words, create practical examples and then, write mathematics signs representing those words. You have 22 minutes to complete this activity.”

Students worked in their respective groups as team, with Ms Moleko and her team facilitating the lesson. Ms Moleko asked the groups to exchange their work for marking and students did not perform well in the group activity provided (cf. Annexure L). This confirmed that students struggle and have limited knowledge of MLP language and its terminologies.

Ms Moleko “All right, let’s do it together. Let’s start with the word ‘less than’; you agreed that less than means lower than. Yes, I agree with you but this is a synonym of the word less than. So, let’s start here, what this word ‘less than’ mean in your mother tongue (language) like in Sesotho, isiZulu, isiXhosa and other indigenous languages. Ok, I see a hand, Mampho?”

Mampho: “Less than in Sesotho means e nyane ho.”

Khaya: “Less than in isiXhosa means incinane ku.”

Zinhle: “Less than in isiZulu means ngaphantsi kwa.”

Lebogang: “Less than in Sepedi means e nyane go.”

Ms Moleko: “Ok, thanks all of you. So, now I know that all of us understand well the word ‘less than’, since we used our indigenous languages to explain this word. Less than refer to smaller number or size especially when comparing two or more than two things. In mathematics context it occurs when we compare two or more things. So, the synonyms of

“less than” is lower than, below; under; smaller; shorter than; beneath; fewer than, etc. Any question?”

Students: “No questions, Mme.”

Ms Moleko: “Again, I agree with you to say mathematically we use this sign ‘<’ to explain the word ‘less than’ mathematically. So, let’s look at this example,

Mr Jacobs earns R2 000 per week while Ms Pretty receives R3 000 per week in their new jobs

From this statement I understand that Mr Jacobs’ wage is **less than** (<) what Ms Pretty receives on weekly basis as wage. Thus, to present this example mathematically, we can let:

Mr Jacobs’ wage be x (or we can use any alphabet)

Ms Pretty’s wage be y (or we can use any alphabet)

$$x < y$$

$$R\ 2000 < R\ 3000$$

(The number that is small comes before the ‘less than’ sign, then the bigger number will follow the sign)

Ms Moleko: “Let’s take another example,

In numbers, 1 is less than 10 (in values)

$$1 < 10$$

(This is true 1 is less than 10, so Mathematically we say $1 < 10$)

Ms Moleko: “So, the last example that I want to make to explain the word less than is by using these prices of cars and size of engines. Firstly, let’s look on this paper (she issued handouts to students).”

VQMAVARARA 1.6 Status+ auto (PICTURE A)	VQMAVARARA 1.3 Status (PICTURE B)
	
Price (including emission tax) R 334 700 Price excluding emission tax R 329 893 emission tax R 4 807	Price (including emission tax) R295 600 Price excluding emission tax R293 197 emission tax R 2 404

Figure 4.15: Cars features (adapted from classroom)

Ms Moleko: “The cars on this picture have different engine (features). The car on Picture A is 1.6, while car on Picture B is 1.3. So, the size and effectiveness of the engine for car in picture B is less than the size and effectiveness of engine for car in picture A. Therefore, writing this mathematically, we let:

Size and effectiveness of engine of a car in picture B be represented by m

Size and effectiveness of engine of a car in picture A be represented by n ,

$$m < n$$

$$1.3 < 1.6$$

(The size and effectiveness of engine for car that is small comes before the less than sign and followed by the bigger size and effectiveness of engine for car)

OR

Secondly, on this paper we can use the prices for cars in picture A and B.

The car in picture A cost R 334 700 (included emission tax) while the car in picture B is R 295 600 (included emission tax). So, it is clear that the price of the car in picture B is less than the price of a car in picture A. Mathematically we let:

Price of a car in picture B be represented by k

Price of a car in picture A be represented by l

$$k < l$$

$$R\ 295\ 600 < R\ 334\ 700$$

Ms Moleko continued to explain MLP terminologies developed by the team (Table 4.2).

Table 4.2: Ms Moleko working with her students to explain MLP terminologies and vocabulary

No.	Terminologies of MLP, synonyms, Maths signs and symbols	MLP terminologies in Sesotho, IsiXhosa, IsiZulu and Setswana; explanations and definitions; and also practical example in Maths)
1	Less than or equal to; at most; maximum; bottom; no more than ($\leq$)	In indigenous languages: - in Sesotho means ka tlase ho kapa ho lekana - in isiXhosa means incinane ku noma ilingana ne - in isiZulu means ngaphansi kwa noma ziyalingana - in Sepedi means e nyane go goba e lekona le Definition: - having a smaller amount or equal to. - it can be or any number greater than; greatest amount, extent, or intensity possible, permitted, or recorded Maths symbols: $a \leq b$, i.e alphabet a or b can be any alphabet (a-z), since they can represent words, number, and amount. So, $a \leq b$ means a is less than or equal to b. EXAMPLE: - Company A produces a maximum of 10 cars per week. Let: a number of cars produced be represented by a, so mathematically, $a \leq 10$
2	Greater than or equal to; minimum; at least; top; no less than ($\geq$)	In indigenous languages: - in Sesotho means ka hodimo ho kapa ho lekana - in isiXhosa means inkulu ku noma ilingana ne - in isiZulu means ngaphezulu kwa noma ziyalingana - Sepedi means e kgolo go goba e lekana le Definition: - it can be or any number greater than - the least quantity or amount possible, assignable, allowable, or the like Maths symbols: $a \geq b$, i.e alphabet a or b can be any alphabet (a-z), since they can represent words, number, and amount. So, $a \geq b$ means a is greater than or equal to b EXAMPLE:

		Company B produces a minimum of 11 cars per week. Let: a number of cars produced be represented by a, so mathematically, $a \geq 11$
3.	Less than ; lower than; below; under; smaller; shorter than; beneath; fewer than ($<$)	In <u>indigenous languages</u> - in Sesotho means ka tlase ho - in isiXhosa means incinane ku - in isiZulu means ngaphantsi kuka - in Sepedi means e nyane go <u>Definition:</u> - having a smaller number/amount or quantity <u>Maths symbols:</u> $a < b$, i.e alphabet a or b can be any alphabet (a-z), since they can represent words, number, and amount. So, $a < b$ means a is less than to b <u>EXAMPLE:</u> - Company A produces 10 cars weekly while Company B produces 11 cars per week. So, from this statement we see that a Company A produces less than what Company B produces since 10 is less than 11. mathematically, $10 < 11$
4	greater than; above; over; greater; larger than; exceeds; increased; longer than; higher than ($>$),	In <u>indigenous languages</u> - in Sesotho means ka hodimo ho - in isiXhosa means inkulu ku - in isiZulu means ngaphezulu kwa - in Sepedi means e kgolo go <u>Definition:</u> - Having a larger number/amount than <u>Maths symbols:</u> $a > b$, i.e alphabet a or b can be any alphabet (a-z), since they can represent words, number, and amount. So, $a > b$ means a is greater than <u>EXAMPLE:</u> - Company A produces 10 cars weekly while Company B produces 11 cars per week. So, from this statement we see that Company B produces more than what Company A produces since 11 is more than 10. Mathematically, $11 > 10$
5	Times; ($\times$)	In <u>indigenous languages</u> - in Sesotho means ho menahana - in isiXhosa means ukuphindaphinda - in isiZulu means ukuphindaphinda - in Sepedi means go oketsa <u>Definition:</u> - Means multiplication <u>Maths symbols:</u> $a \times b$, i.e alphabet a or b can be any alphabet (a-z), since they can represent words, number, and amount. So, $a \times b$ means a is greater than or equal to b <u>EXAMPLE:</u> - Thabo's age is three times the age of Lebo, Mathematically we can write this in this form: Thabo's age= $3 \times$ Lebo's age
6	And; add; plus ($+$)	In <u>indigenous languages</u> - in Sesotho means ho kopanya - in isiXhosa means ho dibanisa - in isiZulu means hlanganisa - in Sepedi means go hlakanya

		Definition: - used as a function word to indicate connection or addition especially of items within the same class or type —used to join sentence elements of the same grammatical rank or function Maths symbols: $a + b$, i.e alphabet a or b can be any alphabet (a-z), since they can represent words, number, and amount. So, $a + b$ means a plus b EXAMPLE: - The cost to produce a product A and product B is R2 000. This statement mean this company spent R2 000 to produce product A and product B. Cost of Pproduct A and product B = R 2 000 Cost of product A + cost of product B = R 2 000
7	half ; ($\frac{1}{2}$)	In indigenous languages - in Sesotho means halofo; - in isiZulu means uhhafu - in Sepedi means e tee e kemetsego e keketsa go tee godimo go pedi Definition: - When the whole of something is divided into two equal parts, we call each part as Half. It is numerically written as $\frac{1}{2}$. One half is the irreducible fraction resulting from dividing one by two (2) or the fraction resulting from dividing any number by its double. Maths symbols: $\frac{1}{2}$, i.e alphabet or b can be any alphabet (a-z), since they can represent words, number, and amount. So, $\frac{1}{2}$ means 1 over 2 EXAMPLE: - Thabo is tasked to share one apple equally to give his two sisters. When sharing one apply equally for two people, each will get half of apple, $\frac{1}{2} + \frac{1}{2} = \frac{1+1}{2} + \frac{2}{2} = 1$
8	one third ; ($\frac{1}{3}$)	In indigenous languages - in Sesotho means ke karolo e le nngwe hodima dikaralo tse tharo tse ntshitshweng nthong e le nngwe. Mohlala, ho arola lamunu ka karolo tse tharo tse lekanang, mme one third, re e fumana ka ho nka karolo e lenngwe hodima dikarolo tsohle tse tharo tse ntshitsweng lamunung. - in isiZulu means okukodwa ukuhlukasisa ngokuthathu - in Sepedi means e tee e kemetsego e keketsa go tee godimo go tharo Definition: - one of three equal parts of a divisible whole; it contains approximately a third of the minimum daily requirement Maths symbols: $\frac{1}{3}$, i.e alphabet a or b can be any alphabet (a-z), since they can represent words, number, and amount. So, $\frac{1}{3}$ means 1 over 3 EXAMPLE: - Thabo is now supposed to share his apple equally to three of his siblings. When sharing one apply equally for three people, each will get one third of apple,

		$\frac{1}{3} + \frac{1}{3} + \frac{1}{3} = \frac{1+1+1}{3} + \frac{3}{3} = 1$
9	Maths variables; (a-z)	In <u>indigenous languages</u> - in Sesotho means dihlaku tse sebediswang bakeng sa diratswana tsa dipalo ha o rarolla dipotso lefeleng e hodimo. <u>Definition:</u> ✓ A symbol for a number or words we don't know yet. It is usually a letter like a or b or c and /or d <u>Maths symbols:</u> ✓ a to z alphabet, representing unknown numbers, amounts, and EXAMPLE: ✓ Ms Koena has R1 500 in her savings account and she wants to surprise her sister's daughter to pay for school fees and transport, but she does not how much is the school fee and transport. So, since school fees and transport cost are unknown, then we let, School fee be p Transport cost be q Mathematically.... $p + q = 1\ 500$
10	Dotted line	In <u>indigenous languages</u> - in Sesotho ke mola o molelele o nang le digeogo. <u>Definition:</u> ✓ A line that is made up of a series of dots <u>Maths symbols:</u> _ _ _ _ _ EXAMPLE: ✓ We do dotted line when sketching linear inequalities with greater or less than sign ($<$ or $>$) on xy Cartesian plane
11	Solid line	In <u>indigenous languages</u> - in Sesotho ke mola o molelele o motenya <u>Definition:</u> ✓ printing with no space or leads between lines of type <u>Maths symbols:</u> EXAMPLE: ✓ We do a solid line when sketching linear inequalities and equation with less than or equal; greater than or equal to; and equal to sign ($\leq$ or $\geq$ or $=$) on the xy Cartesian plane.
12	Linear equation	In <u>indigenous languages</u> - in Sesotho ke Maths equation e nang le variable or variables (like a-z) the pedi mme re e fa straight line ha re e draw-ya on Cartesian plane empa e tlamehile hoba le equal sign. <u>Definition:</u> ✓ is an equation between one or two variables that gives a straight line when plotted on a graph. <u>Maths symbols:</u> ✓ $y = a$; $x = a$; and $y = mx + b$ NB: alphabet a, m and b in the above linear functions represents any integer while x and y represent axes EXAMPLE: ✓ $y = 2$; $x = 5$; and $2x + y = 10$ or $y = -2x + 10$
13	Linear inequality	In <u>indigenous languages</u>

		- in Sesotho ke Maths equation e nang le variables (like a-z) the pedi mme re e fa straight line ha re e draw-ya on Cartesian plane empa e tlamehile hoba le inequality signs such as less than, greater than, less than or equal to and greater than or equal to. Definition: ✓ involves a linear expression in two variables by using any of the relational symbols such as $<$, $>$, $\leq$, $\geq$ Maths symbols: ✓ $x > a$; $x < a$; $x \geq a$; and $x \leq a$ OR $y > a$; $y < a$; $y \geq a$; and $y \leq a$ ✓ $x + y > a$; $x + y < a$; $x + y \geq a$; and or $x + y \leq a$ NB: alphabet a, m and b in the above linear functions represent any number while x and y represent axes EXAMPLE: ✓ $x > 5$; $x < 15$; $x \geq 7$; and $x \leq -2$ OR $y > 5$; $y < 15$; $y \geq 7$; and $y \leq -2$ ✓ $x + 3y > 20$; $x + y < 8$; $2x + y \geq 2$; and $x + y \leq 13$

During the last 15 minutes of double class periods, after Ms Moleko presented and explained all MLP terminology developed by the team, and if anyone did not understand.

Students: “No, Ma’am”

Ms Moleko: “Thabiso, do you understand?”

Thabiso: “Yes, Ma’am, I understand and this lesson assisted me with questions I had before?”

Ms Moleko: “Ok, right, let’s do this activity and you have 12 minutes to complete it, Mme Leeuw, can you please assist me to distribute this work? Remember, this activity is individual work.”

Table 4.3: Activity B (developed by research team)

ACTIVITY B		
1. Complete below table:		
MLP terminologies	Explanation (include the practical examples)	Mathematical sign
1.1 Less than		
1.2 Greater than		
1.3 Linear equation		
1.4 Less or equal to		
1.5 Greater or equal to		
1.6 Linear inequalities		
1.7 Dotted line		
1.8 Solid line		
1.9 Constraints		
(18 marks)		

After the period ended, Ms Moleko collected the students' work, and she, so together with her team, immediately marked the work. They also reflected and evaluated Ms Moleko lesson presentation.

4.3.1.2 Second level for teaching the MLP topic: formulate linear inequalities and objective functions from MLP word problem

Adams (2003, in Sepeng & Sigola, 2013:325) defines word problems in mathematics as mathematical problems presented in a context of a real-life stories (situations) while Ahmad (2010) explains mathematical word problems as a story problem that requires of students to translate from the concrete to the abstract and from the abstract to the concrete; thus, mathematical exercises where the vital background information on the problem is given in the form of text, not in abstract (variables, operational signs, symbols, and graphs). This is also seen as a narrative or story problems. This study explains MLP word problems as mathematical problems presented in a text rather than in abstract mathematical notation (cf. examples in the scenarios below). Van Garderen (2007:550-551) and Lee, Yeo and Hong (2014:475) recommend a number of pedagogic approaches, i.e. metacognitive-based instruction, code switching and the use of diagrams.

The formulation of linear equations, linear inequalities, and objective functions from problem statement need the following steps, namely reading and understanding the Mathematical word problem; underlining key words and goal; developing a strategy to solve that specific problem; selecting variable(s) to represent the unknown quantity; reading every word in mathematical word problems (problem situation); translating the given information into the algebraic equations and inequalities; solving equations and inequalities; and writing answers in complete sentences. The team agreed that the second level to teach MLP topic was to teach mathematical word problems with one and two variables.

"The second level is to introduce the mathematical word problems with one and two variables separately so that students understand. So, to convert MLP word problems into linear inequalities, we first read MLP word problem twice with understanding and go through questions. Secondly, we read MLP word problem for the third time going line by line in order to understand, identify and underline the MLP terminologies. Thirdly, we identify products that a company is manufacturing or selling. Lastly, we identify resources constraints and identify MLP terminologies."

The lesson continued at Thabong Campus the next day. Ms Moleko went to the next lesson and introduced MLP word problems after dealing with the MLP terminologies. However, before that, Ms Moleko gave students feedback on Activity B from the previous lesson. The feedback was positive, because students performed well in that activity and seemed to understand MLP terminology (cf. Annexure N). Then Ms Moleko asked if there was anyone with questions on what they had done so far. As there was no question, the lesson continued:

Ms Moleko: “In the absence of questions, let’s move to the next level of MLP, which is conversions of MLP word problems into linear inequal components.”

Ms Moleko approached the whiteboard, wrote the topic to be presented together with the learning outcome to achieve on the whiteboard (cf. Fig. 4.16).

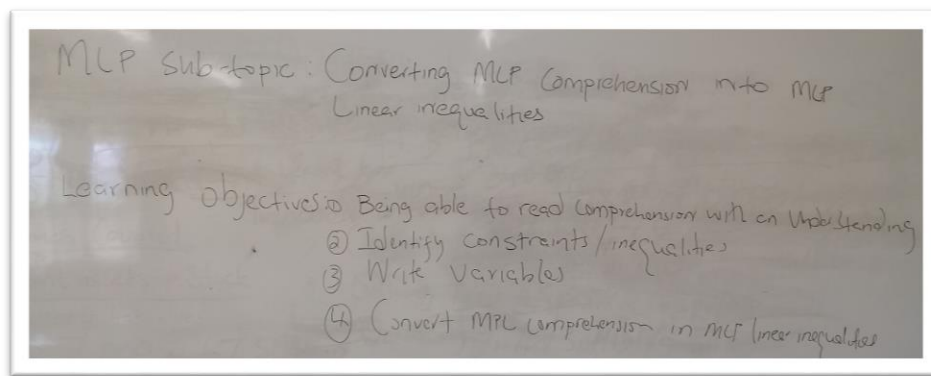


Figure 4.16: MLP problem statements lesson and objectives

Ms Moleko: “So, let’s solve this word problem together.”

Table 4.4: MLP scenario (taken from the classroom)

SCENARIO 1:

Mr Tshabalala produces at least 500 muffins per day with his limited resources.

Ms Moleko: “In your groups, solve this problem (write this statement mathematically).”

Students work in groups were provided given three minutes to do this. Then Ms Moleko and her team facilitated the lesson, checking whether students were working as expected. When marking, Ms Moleko found that most of the groups got it correct.

Ms Moleko: “It’s really really promising; most of groups got it right. So, is there anyone who can help us understand how you arrived to your solutions?”

Wilson stood up and came to show how they have reached their solution in his group (cf. Figure 4.17 below).

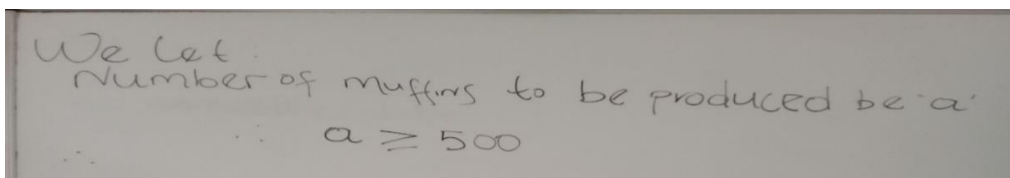


Figure 4.17: Wilson's scenario 1 solution

Ms Moleko: "Mr Tshabalala produces at least 500 muffins per day with his limited resources. So, to give proper background information, Mr Tshabalala bakes muffins at home with his resources and he can bake 500 and more muffins. The number of muffins to be produced is at least 500 and more.

Or

A number of muffins to be bake cannot be less than 500. So, mathematically this is represented by this sign $\geq$:

Firstly we let the number of muffins to be produced be represented by x (it can be any alphabet hence Wilson group used alphabet a)

$$x \geq 500$$

Ms Moleko continued with the lesson on MLP word problems, concentrating on **a single mathematics variable** and working together with students solving these following mathematics word problems.

1. **QUESTION A:** Ms Koba transport 80 school kids at most every day to different schools.
ANSWER: $x \leq 80$
2. **QUESTION B:** Touraus Firm manufactures not less than 1 000 golf shirts for NC TVET college per week. **ANSWER:** $x \geq 100$

Students understood because all were able to respond to the questions Ms Moleko asked based on mathematics word problems with one variable.

Ms Moleko: "OK, let's move to the next part of this lesson for mathematics word problems of **two variables**. So, in your groups you have 5 minutes to solve this mathematics problem."

Table 4.5: MLP scenario (taken from the classroom)

SCENARIO 2:

Mr Tshabalala makes chocolate and strawberry-flavoured muffins. He cannot make 750 of both chocolate and strawberry muffins per day.

Students worked together in groups and were given 5 minutes to do this work. Ms Moleko and his team facilitated the lesson by in checking whether students were working as expected. However, most of the students struggled to get it right (cf. Annexure N).

Ma Moleko: "this activity is involves two mathematics variables, look, Mr Tshabalala makes chocolate and strawberry flavours of muffins: He cannot make 750 of both chocolate and

strawberry muffins per day. Mr Tshabalala now bakes muffins with two flavours i.e. chocolate and strawberry. He baking two flavours of muffins per day is less than 750.

Let's let: number of chocolate muffins to be produced be represented x

number of strawberry muffins to be produced be represented y

$$x + y < 750$$

Ms Moleko continued with the lesson of **word problems for two mathematics variables** and worked together with students solving below problems, also illustrating them.

1. **QUESTION 1:** Ms Xulu uses a minibus and quantum taxi to transport school kids, workers and people to their destinies. A minibus cannot take twice the load of a quantum taxi to transport people to their destinies. **ANSWER:** $x < 2y$
2. **QUESTION 2:** A small factory makes two types of washing machines, a standard and a luxury model. The daily production cannot be more than 6 standard and 3 luxury washing machines. There are 8 factory workers and it takes one worker to make one standard model and 2 workers to make one luxury model per day. **ANSWER:** $x \leq 6; y \leq 3; x + 2y \leq 8$

Ms Moleko projected Figure 4.18 below on the wall next to the whiteboard and issued hand-outs of the same picture.

Ms Moleko: "In your groups, identify natural resources and man-made resources that you see in this picture (pointing and referring to the picture on the wall)."



Figure 4.18: A farm (adapted from www.google.co.za/search?q=hectares+of+land&source)

Students worked in their groups to identify natural and man-made resources in Fig. 4.18 and recognised land, plots, trees, grass, mountain, sky, plants, clouds, electric poles and wires.

Ms Moleko: "Very good, you are all correct. However, our focus today will be land, so is there anyone who can tell us why land is so important to all us?"

Mampho: "Ma'am, on the land we plant different types of food, we build houses, and buildings for its purposes and also we can find different natural resources."

Ms Moleko: "Very excellent, Mampho, and I believe all of us agree with her. However, to plant different food on a limited piece of land, we need to plan well in order to use it efficiently for maximising profit when we operate as a business. The MLP model is one of the techniques

we can use to plan well and use a limited piece of land well for reaching maximum profit. This technique has been used in different countries and sectors around the world, so it is also relevant to the farming sector. Let's do Activity1 to see how MLP can be used?"

Activity 1

A farmer has 80 hectares of his farm available for planting maize and cabbage. He must grow at least 10 hectares of maize and 20 hectares of cabbage to meet demands. He prefers to plant more cabbage than maize but his work force and equipment will only allow him to cultivate a maximum of three times the quantity of cabbages to that of maize.



Questions:

- 1.1 Represent the information above as a system of inequalities.
- 1.2 Sketch a graph of these inequalities.
- 1.3 If the profit on maize is R800 per ha and on cabbages R500 per ha, how should the farmer plant the two crops to make a maximum profit and what is this profit.

Figure 4.19: MLP activity1 developed by research team

Ms Moleko: *"We are going to integrate MLP components to find MLP model that determines a feasible area or region for profit maximization or cost maximisation."*

Ms Moleko: *"Question 1.1 indicates that we should represent this MLP word problem as a system of linear inequalities. So, converting MLP word problems into linear inequalities, we first read the MLP word problem twice with understanding and go through the questions. Secondly, we read the MLP word problem for the third time, going line by line to understand, identify and underline MLP terminologies. Thirdly, we identify products that a company manufactures or sells. Lastly, we identify resources constraints and identify MLP terminology."*

Ms Moleko: *"Let's go back to Activity 1; Teboho, read for us and interpret or say what you understand in that activity."*

Teboho read, interpreted the activity and other students were given an opportunity to share what they understood with regard to Activity 1. Ms Moleko, unpacked this activity further so that the students understood more.

Ms Moleko: “Activity 1 talks about a farmer who has only 80 hectares on his piece of land and he usually plant cabbage and maize for his business. So, we should first find and write variables which can be any alphabet (a-z) that represents these plants to be planted (i.e. maize and cabbage). We let:

number of maize to be planted be represented by variable “**a**”

number of cabbage to be planted be represented by variable “**b**”

NB: (**a** for x axes and **b** for y axes)

We should start to show these basic linear inequalities $a \geq 0$ and $b \geq 0$. For instance, $a \geq 0$ means if the farmer plant maize more than 0 hectares of maize will be harvested however, if the farmer do not plant maize, then 0 hectares of maize will be harvested. Therefore, this is the same with $b \geq 0$ because $b \geq 0$ means if the farmer plant cabbage more than 0 hectares of cabbage will be harvested but, if the farmer do not plant cabbage, then 0 hectares of cabbage will be harvested.

From this activity, it is clear that each sentence has the meaning, so we have go through this activity1 read line by line ... **the first sentence/line from Activity 1, which says...**

A farmer has 80 hectares of his farm available for planting maize and cabbages.

Ms Moleko: “What do you understand in this statement?”

Wilson: “I think it means that this farmer has maximum of 80 hectares (size of the piece of land) to plant maize and cabbages but planting both products cannot exceed 80 hectares because it is only land available him.”

So, number hectares of maize to be planted **plus** number hectares of cabbages to be planted **can be planted on 80 or less than hectares** because of limited piece of land.

NB, number hectares of maize to be planted is represented by variable “**a**” while number hectares of cabbage to be planted is represented by variable “**b**”, so, **mathematically** we can write...

$$a + b \leq 80$$

Ms Moleko: “Thanks, Wilson, I agree with you 100%. So, without any waste of time let’s move to **the second sentence/line from Activity 1 which says ...**

He must grow **at least** 10 hectares of maize and 20 hectares of cabbages to meet demands.

Therefore we have two sentences here, since it says he must grow at least 10 hectares of maize and he must also grow at least 20 hectares of cabbage to meet the demands. Moreover, the word **and** in this line was used to join two sentences. **The first part of this second sentence says ...**

He must grow at least 10 hectares of maize to meet the demands.

This means this farmer uses at least (minimum not less than) 10 hectares to plant maize.

...number hectares of maize to be planted on that available land is at least 10 hectares, thus mathematically we write...

$a \geq 10$, since number hectares of maize to be planted is represented by variable a ...

The second part of this statement, says...

He must grow at least 20 hectares of cabbages to meet demands.

This means this farmer uses **at least (minimum/not less than) 20 hectares** to plant cabbage.

number hectares of cabbage to be planted on that available land is at least 20 hectares, thus, mathematically we write...

$b \geq 20$, since number hectares of cabbage to be planted is represented by variable b ..

Third sentence/line from Activity 1 says

“He prefers to plant more cabbages than maize but his work force and equipment will only allow him to cultivate a maximum of three times the quantity of cabbages to that of maize.

This means the farmer usually plants more cabbage than maize because of limited resources, limited workforce and limited piece of land, so this farmer cultivate (encourage) a maximum of three times quantity of cabbages to that of maize. So, number hectares of cabbage are at most three number hectares of maize. Mathematically, we say...

$$b \leq 3a$$

Ms Moleko: *“Is there anyone who does not understand?”*

Students: *“No, Ma’am, we understand now because we did similar work in the previous lessons.”*

Ms Moleko: *“All right. Now we have the following linear inequalities:*

- 1.** $a \geq 0$
- 2.** $b \geq 0$
- 3.** $a + b \leq 80$
- 4.** $a \geq 10$
- 5.** $b \geq 20$
- 6.** $b \leq 3a$

Ms Moleko: *“So, tomorrow we are going to continue from this point to answer questions 1.2 & 1.3 of Activity 1. At home, please ensure that you do what we did in the previous lesson.”*

4.3.1.3 Third level of teaching the MLP topic: linear inequalities and MLP graphs

In Section 2.4.2.3, the literature showed different pedagogical methods to teach linear inequalities problem solving and mathematics graphs, namely RME, PBL, CAS, procedural embodiment, geogebra, spreadsheet. The arithmetical inequalities are mathematical sentences connecting expression to values, variables or another expression with an inequality sign, namely greater than, less than, greater or equal to, less than or equal to, not equal to, and equal to. According to El-khateeb (2016:124), linear inequality is like a linear equation e.g. $y = 2x + 1$, but will have inequality signs such as $>$, $<$, $\leq$, and $\geq$, instead of $=$. Such linear inequalities are presented on the Cartesian plane in this study.

The following are steps that one should apply to solve the linear inequalities, such as, simplifying each side of inequality; removing a parenthesis, if necessary; grouping like terms; adding like terms on each side of the inequality, so that all terms containing the variables are on one side and constant terms are on the other side; simplify each side

of the inequality by combining like terms; apply multiplication property of inequalities to isolate the variable; and lastly check whether solutions by substituting them into the original inequality. Moreover, to sketch linear inequalities one should firstly rearrange the equation making y subject of the formula on the left and everything else (variables, operational signs and constant numbers) on the right. Secondly, plot the $y =$ line, but make a solid line if the initial equation is $y \leq$ or $y \geq$, and/or dashed line if initial equation is $y >$, or $<$; and lastly, shade area above the solid or dash line if the initial line is greater than and/or greater than, or equal to ($y >$, $\geq$). Again, shade the region below the solid or dash line, if the initial line is less than or equal to ($<$, $\leq$). The co-researchers agreed that the third level should be solving the linear inequalities, sketching linear inequalities, and shading areas that satisfy inequality signs. The team agreed that the third level to teach MLP topic is to introduce and teach problem solving of linear inequalities and graphing of linear inequalities.

The third lesson were presented in the classroom at Thabong Campus.

Ms Moleko: *“Good morning, students, today we are going to continue where we ended yesterday. So, before we move further, is there anyone who still struggle with what we did so far?”*

There were no questions and students indicated that they were satisfied, but Ms Moleko asked them how they arrived at these linear inequalities

$$a \geq 0; b \geq 0; a + b \leq 80; a \geq 10, b \geq 20; \& b \leq 3a,$$

The students were able to explain.

Ms Moleko: *“Let’s move to the next level of MLP topic, which is linear inequalities and its graphs.”*

Ms Moleko wrote the topic to be presented together with learning outcome to achieve on the whiteboard (cf. Figure 4.20 below).

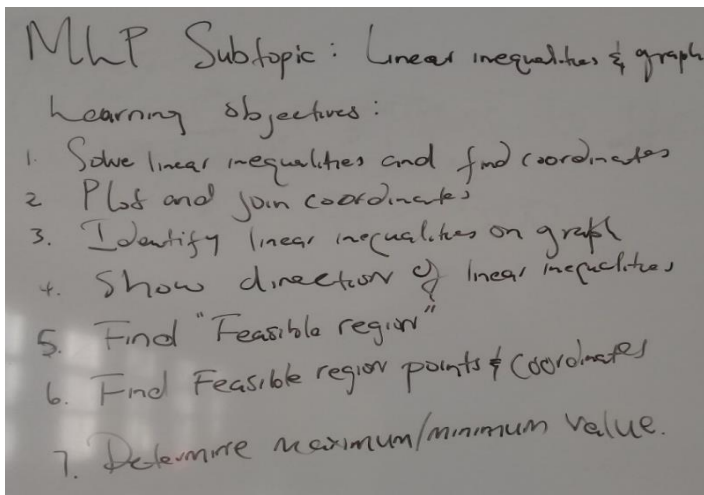


Figure 4.20: MLP linear inequalities lesson (taken from the classroom)

Ms Moleko: "On NCV level 2 we did linear inequalities, which paved the way for MLP on level 3, since to solve MLP we had to solve the linear inequalities, draw MLP graphs, show feasible region and lastly interpret MLP graphs to find a maximum profit or minimum costs. So, I think at this stage you will not have more problems to solve linear inequalities, to draw MLP graph, to show feasible region (area) and determine a maximise profit from limited resources. Again, as you have said, in the previous sessions earlier on, we have dealt with MLP terminologies, word problems with one variable, word problems two variables, linear inequalities and its' graphs."

Ms Moleko: "Let's now solve these linear inequalities to find, plot and join coordinates on Cartesian plane (draw MLP graph). Thereafter, determine feasible area and find maximum profit, so, as we have done earlier on, these are steps to solve linear inequalities:

Step 1: get deeper meaning of the linear inequalities,

Step 2: change linear inequalities to linear equations for the $<$ or $>$ OR for $\leq$ or $\geq$ focus on $=$ but keep on hold $<$ or $>$ for finding feasible region or area;

Step 3: solve each linear equation at one time

Step 4: apply intercepts, and do coordinates.

Ms Moleko: "Let's go back the linear inequalities on above, namely:

1. $a \geq 0$ **linear inequality 1**
2. $b \geq 0$ **linear inequality 2**
3. $a + b \leq 80$ **linear inequality 3**
4. $a \geq 10$ **linear inequality 4**
5. $b \geq 20$ **linear inequality 5**
6. $b \leq 3a$ **linear inequality 6**

Ms Moleko: "Let's apply above steps to these linear inequalities."

Step1: getting deeper understanding of these linear inequalities

Linear inequality 1: $a \geq 0$

= a represent number of maize to be planted but should be greater or equal to 0 hectares. If the farmer plants nothing then zero number of maize hectares will be harvested. Again, if the farmer plant then more than zero number of maize will be harvested (cf. Figure 4.22).

Linear inequality 2: $b \geq 0$

= **a** represent number of cabbages to be planted but should be greater or equal to 0 hectares. If farmer plants nothing then zero number of cabbages hectares will be planted. Again, if a farmer plant then more than zero number of cabbages will be harvested (cf. Figure 4.23).

Linear inequality number 3: **$a + b \leq 80$**

= **a** represents number of maize to be planted, **b** represents number of cabbages to be planted but both hectares of maize and cabbages should be less or equal to 80. Thus, it means this farmer has only 80 hectares of land to plant both maize and cabbage (cf. to Figure 4.24).

Linear inequality number 4: **$a \geq 10$**

= **a** represent number of maize to be planted, however should be greater or equal to 10 hectares of maize and it means this farmer is able to plant 10 or more hectares based on his ability (cf. Figure 4.25).

Linear inequality number 5: **$b \geq 20$**

= **b** represent number of cabbages to be planted but should be greater or equal to 20 hectares of cabbages and it means this farmer is able to plant 20 or more hectares based on his ability (cf. Figure 4.26).

Linear inequality number 6: **$b \leq 3a$**

= **b** represent number of cabbage to be planted, **a** represent number of maize. The linear inequality **$b \leq 3a$** means planting number of cabbages **b** less or equal to three times planting number of maize **a**. This also means this farmer plant maximum of cabbages that is three times planting maize (cf. Figure 4.27).

Step 2: focus on equal sign = for $\leq$ or $\geq$ but keep on hold $<$ or $>$ for determining the feasible region or area.

1. **$a \geq 0$ means $a = 0$ or $a > 0$**
2. **$b \geq 0$ means $b = 0$ or $b > 0$**
3. **$a + b \leq 80$ means $a + b = 80$ or $a + b < 80$**
4. **$a \geq 10$ means $a = 10$ or $a > 10$**
5. **$b \geq 20$ means $b = 10$ or $b > 20$**
6. **$b \leq 3a$ means $b = 3a$ or $b < 3a$**

Step 3: solve each equation at one time, apply intercepts, and find coordinates.

Linear equation 1: $a = 0$

If the straight line equation is **$a = r$ (r as any number)**, it will result in a vertical line on **ab Cartesian plane** but in this regard where **$a = 0$** , then **a** is zero on the **b axis**

Linear equation 2: $b = 0$

If the straight line equation is **$b = r$ (r as any number)**, it will result in a horizontal line on **ab Cartesian plane** but in this regard where **$b = 0$** , then **b** is zero on the **a axis**

Linear equation 3: $a + b = 80$

for **b intercept: $a = 0$**

sub **a** with 0 in the below equation

$$\begin{aligned} a + b &= 80 \\ (0) + b &= 80 \\ b &= 80 \end{aligned}$$

Therefore, **$a = 0$ and $b = 80$**

Coordinates will be **$(a, b) = A(0, 80)$**

(This means if this farm does not plant maize, it can plant 80 hectares of cabbage)

for a intercept: $b = 0$

sub b with 0 in the below equation

$$\begin{aligned} a + b &= 80 \\ a + (0) &= 80 \\ a &= 80 \end{aligned}$$

Therefore, $a = 80$ and $b = 0$

Coordinates will be $(a, b) = B(80, 0)$

(This means if this farm does not plant cabbage, then it can plant 80 hectares of maize because this farmer only has 80 hectares)

Therefore, this equation has two variables, and the line will cut both variables.

Linear equation 4: $a = 10$

If $a = 10$, $b = 0$, then $a = 10$ for every number of b on the **abCartesian plane** and this will be vertical line.

(This means this farm is able to plant 10 hectares of maize)

Coordinates will be $(a, b) = C(10, 0)$

Therefore, because this equation has one variable, this straight line will only cut one variable vertically (where $a = 10$).

Linear equation 5: $y = 20$

If $b = 20$, $a = 0$, then $b = 20$ for every number of a and this will be a horizontal line

(This means this farm is able to plant 20 hectares of cabbage)

Coordinates will be $(a, b) = D(0, 20)$

Therefore, because this equation has one variable, this straight line will only cut one variable horizontally (where $b = 20$)

Linear equation 6: $b = 3a$

(This symmetry line, derived from $b = a$ meaning all values of a and b are equal, for example if $a = 1$, then $b = 1$; if $a = 2$, then $b = 2$. However, for $b = 3a$, it means value of b is three times value of a . Then, for $b = 3a$ the straight line will be steeper. We can use table form to plot this line.

a	-60	-50	-40	-30	-20	-10	0	10	20	30	40	50	60	70	80
b	-180	-150	-120	-90	-60	-30	0	30	60	90	120	150	180	210	240

Earlier on we agreed that for Activity1 we use non-negative numbers because of $a \geq 0$ ($a = 0$ and $a > 0$) and $b \geq 0$ ($a = 0$ and $a > 0$). This means if we do not planted food there will be no food planted/ harvested, thus this will be represented by ($a = 0$ and $b = 0$). Again, if we planted food there will a number of planted/ harvested food and this will be represented by ($a > 0$ and $b > 0$). So, to present $y = 3a$ on the Cartesian plane under these restriction $a \geq 0$ and $b \geq 0$, we will only show this $b = 3a$ on the first quadrant (non-negative numbers) with these coordinates for **E, E1 (0,0), E2 (10,30), E3 (20,60), E4 (30,90), E5 (40,120), E6 (50,150), E7 (60,180), E8 (70,210), & E9 (80,240).**

Ms Moleko: "Is there anyone who does not understand or who need clarity?"

Students: "No, we understand, Mme."

Ms Moleko: "Please, feel free to ask questions if you do not understand or just indicate by your hand, then I will come to you for more clarity."

Khaya: "All right, Ma'am, but now we understand because we did this work for the past two days."

Ms Moleko: "Are you feeling the same?"

Students: "Yes, Ma'am."

Ms Moleko: "Good, now we move to the next MLP component, which is sketching the MLP graph. So, to draw MLP graph we need to find coordinates first, then plot and join them. Thereafter, use the inequality signs to find the feasible region. Let's continue the previous lesson and we already found the following coordinates for each linear inequality."

1. $a \geq 0$ coordinate A (0,0) and with greater or equal to sign
2. $b \geq 0$ coordinate B (0,0) and with greater or equal to sign
3. $a + b \leq 80$ coordinates C (80,0) and D (0,80) and with less or equal to sign
4. $a \geq 10$ coordinate E (10,0) and with greater or equal to sign
5. $b \geq 20$ coordinate F (0,20) and with greater or equal to sign
6. $b \leq 3a$ coordinates G1 (0, 0), G2 (10, 30), G3 (20, 60), G4 (30, 90), G5 (40, 120), G6 (50, 150), G7 (60, 180), G8 (70, 210), G9 (80, 240) and with less or equal to sign

Ms Moleko: "so, in your groups plot these coordinates, join coordinates and write linear inequalities on that joined line. You have only 10 minutes to do this work, but use these graph papers."

Ms Moleko issued graph paper to the students, and they began to work collectively in their groups. Ms Moleko and her team facilitated the process, ensuring that all students were working.

Ms Moleko: "All right, your time is up, so let's look whether you managed to get it correct. Exchange your work to other groups."

Ms Moleko: "Let's start, how much must be our scale in for this graph?"

Tholoane: "Ma'am?"

Ms Moleko: "Tholoane?"

Tholoane: "Our scale must be a factor of 10 for both **a-axis** and **b-axis** because the linear inequalities have the following numbers 0,3,10, 20 & 80. So, these coordinates fit well in this scale."

Ms Moleko: "All right, thanks, Tholoane), I agree with you, so please, ensure that if you choose 10 as a scale factor for your graph should be both on **a-** axes and **b-** axes and also ensure that the measurement between the spaces on a **ab Cartesian plane** are equal at all times. More importantly, as you have said, Tholoane, most of the numbers in these linear inequalities and its coordinates are covered with this scale factor (**ab Cartesian plane**)."

Ms Moleko showed the **ab Cartesian plane** on the first quadrant with the scale decided by students (cf. Figure 4.21).

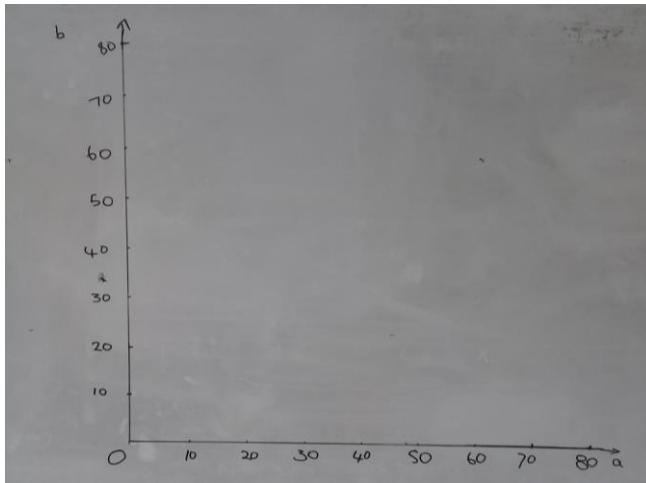


Figure 4.21: Image of **ab Cartesian plane** by Ms Moleko with 10 multiple factor

Ms Moleko: “Let’s move on, to show $a \geq 0$ ($a = 0$ or $a > 0$) on **ab Cartesian plane** at point A (0,0), then, we look where $a = 0$ and $b = 0$ which is point of origin. So, a is greater or equal to 0 on **a intercept** where $b = 0$ but starting from 0 to **positive a infinite**. We can use number line concept to show this $a \geq 0$ on **ab Cartesian plane**, so we put close point at the point of origin and put arrow direction to positive a infinite (right hand side **ab Cartesian plane**) on **a intercept** (horizontal line) to satisfy $a \geq 0$.”

Ms Moleko showed $a \geq 0$ on **ab Cartesian plane** and used a blue whiteboard marker to highlight the arrow direction and identify the line by $a \geq 0$ (Figure 4.22).

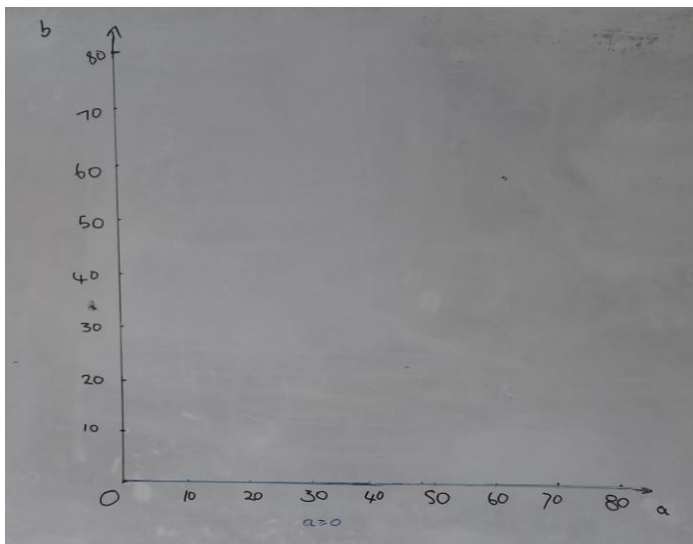


Figure 4.22: Image of $a \geq 0$ on **ab Cartesian plane** by Ms Moleko

Ms Moleko: “Is there any question? All right, let’s move on, to show $b \geq 0$ ”

Ms Moleko: “with the second linear inequality $b \geq 0$ ($b = 0$ or $b > 0$) on **ab Cartesian plane** at point B (0,0), then, we look where $a = 0$ and $b = 0$ which is point of origin. So, b is greater or equal to 0 on **b intercept** (vertical line) where $a = 0$ but starting from 0 to positive b infinite. We can again also use number line concept to show $b \geq 0$ on **ab Cartesian plane**, so we put close point at the point of the origin and put arrow direction on the vertical line to positive b infinite (upwards) on **b intercept** to satisfy $b \geq 0$.”

Ms Moleko indicated $b \geq 0$ on ab Cartesian plane and used a green whiteboard marker to highlight the arrow direction (Figure 4.23).

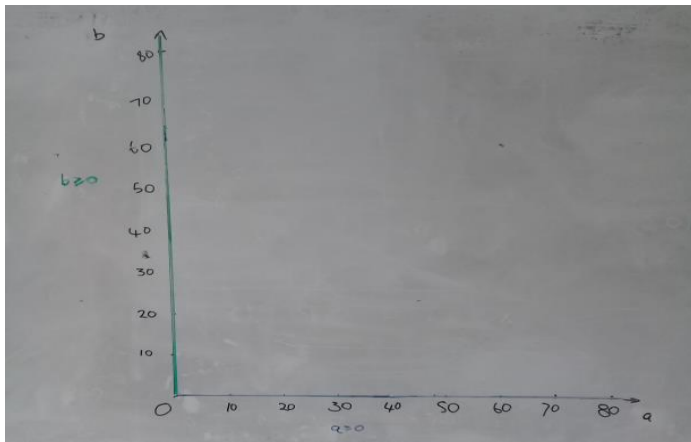


Figure 4.23: Image of $a \geq 0$ and $b \geq 0$ on ab Cartesian plane by Ms Moleko

Ms Moleko: “Do you understand?”

Students: “Yes, Mme.”

Ms Moleko: “Showing $a + b \leq 80$ on ab Cartesian plane with two coordinates C (80, 0) and D (0, 80), we first plot these coordinates on ab Cartesian plane and also join them. This will result to the straight line with negative slope ($b \leq -a + 80$). So, to satisfy the inequality sign of less or equal to ($\leq$) we make bold a straight line and shade the area under that bolded straight line, because this farmer can plant between 0 to 80 hectares of both cabbages and maize but not more than 80 hectares, so, in this regard we shade the area with vertical lines in the triangle shape.”

Ms Moleko interacted helped the students to understand more about how to draw the linear inequality where $a + b \leq 80$ and used the black whiteboard marker to draw $a + b \leq 80$ (Figure 4.24).

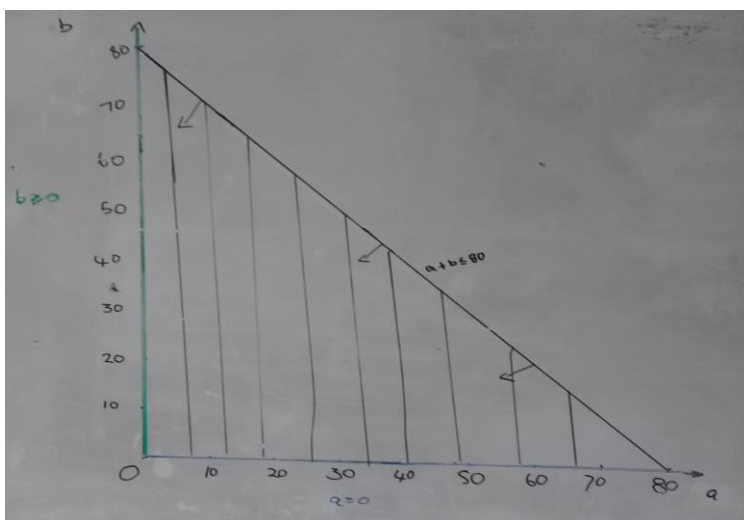


Figure 4.24: Image of $a \geq 0, b \geq 0$, and $a + b \leq 80$ on ab Cartesian plane by Ms Moleko

Ms Moleko: “Another linear inequality to show is $a \geq 10$, so, as we have said earlier on about linear equation with one mathematics variable. The linear equation with one mathematics variable cuts only one intercept vertically or horizontally. This linear inequality where $a \geq 10$ can be written as $a = 10$ and $a > 10$. So, $a = 10$ cuts a axis at number 10 and we are going to use red ink whiteboard marker to show this vertical line. Thus, to satisfy the inequality sign of greater or equal to ($\geq$) we make bold vertical line with red ink and shade the area where $a > 10$ with dotted red ink whiteboard marker to positive a infinite number, since the farmer is supposed to plant 10 and more hectares of maize to meet the customers’ demands, so, in this regard we shade the area with dots of red ink.”

Ms Moleko interacted with the students and helped them to understand how to draw the linear inequality where $a \geq 10$, using the red whiteboard marker to show $a \geq 10$. All students understood this, since Ms Moleko has used different colours of whiteboard markers to show the effect of inequality signs (Figure 4.25).

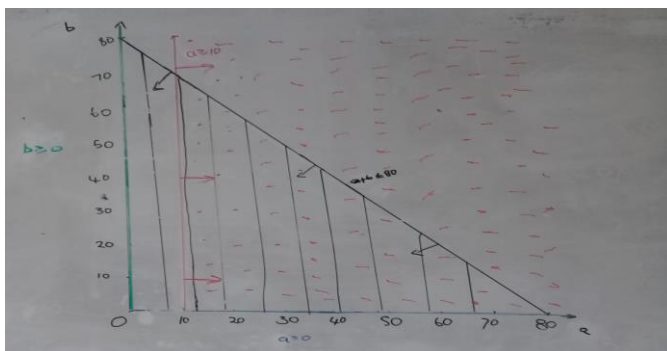


Figure 4.25: Image of $a \geq 0, b \geq 0, a + b \leq 80$, and $a \geq 10$ on ab Cartesian plane by Ms Moleko

Ms Moleko: “Another linear inequality to show is $b \geq 20$, so, as we have said earlier on about linear equation with one mathematics variable. The linear equation with one mathematics variable cuts only one intercept vertically or horizontally. This linear inequality where $b \geq 20$ can be written as $b = 20$ and $b > 20$. So, $b = 20$ cuts b axis at number 20 and we are going to use blue ink whiteboard marker to show this horizontal line. Thus, to satisfy the inequality sign of greater or equal to ($\geq$) we make bold horizontal line with blue ink and shade the area where $b > 20$ with blue ink arrow directions going up to positive b infinite number, since this farmer is supposed to plant 20 and more hectares of cabbages to meet customers’ demands, so, in this regard we shade the area with blue ink of arrow direction facing up.”

Ms Moleko further engaged students about graphing $b \geq 20$ on ab Cartesian plane, using a blue whiteboard maker to illustrate the effect of $b \geq 20$. The lines are identified by each straight line with their linear inequality and arrow direction (Figure 4.26).

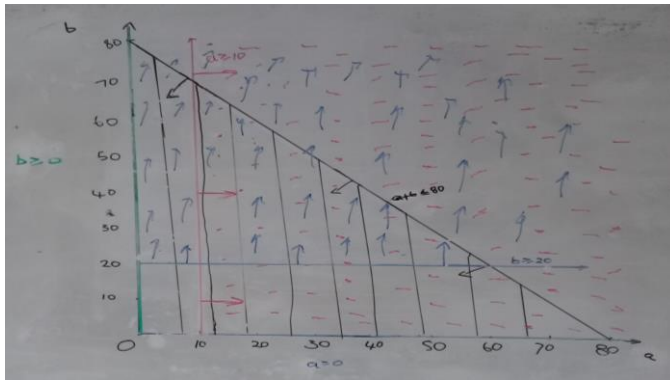


Figure 4.26: Image of $b \geq 20$ on the same Fig 4.24 demonstrated by Ms Moleko

Ms Moleko: “Lastly, $b \leq 3a$ can be written as $b < 3a$ or $b = 3a$, then drawing this linear equation or symmetric line where $b = 3a$. Now we plot its coordinates we got earlier on, E1 (0, 0), E2 (10, 30), E3 (20, 60), E4 (30, 90), E5 (40, 120), E6 (50, 150), E7 (60, 180), E8 (70, 210), & E9 (80, 240). Then, we join them and make bold line with black ink whiteboard marker to satisfy its inequality sign ($\leq$), then to satisfy and illustrate $b < 3a$ on **ab Cartesian plane**, we know that b will be less than $3a$ when this bold straight line $b \leq 3a$ shown on **ab Cartesian plane** move (shifts) to positive a infinite number of on the a axis so we have to move or shift this bold straight line ($b \leq 3a$) by doing black ink marks (horizontal straight small arrows) to our right hand side.”

Ms Moleko illustrated this linear inequality $b \leq 3a$ working together with her students, who understood this MLP component (Figure 4.27).

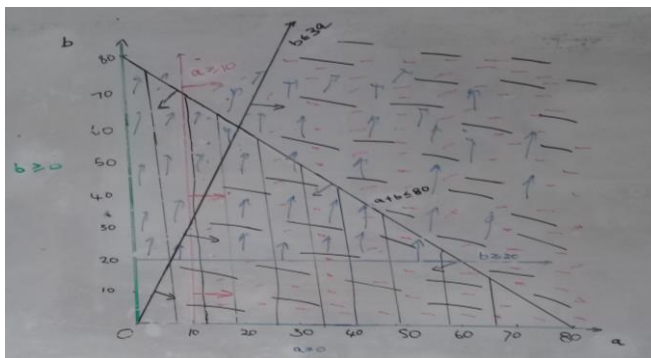


Figure 4.27: Image of $b \leq 3a$ on the same Fig 4.25 and was demonstrated by Ms Moleko

Ms Moleko: “All right, now we are done illustrating these linear inequalities $a \geq 0$, $b \geq 0$, $a + b \leq 80$, $a \geq 10$, $b \geq 20$ and $b \leq 3a$ on the **ab Cartesian plane**. Questions, please?”

Students: “No question, Ma’am.”

Ms Moleko: “Are you sure sure? Dimpho, are you sure?”

Dimpho (student): “Yes, Ma’am, we have done sketching of the linear inequalities several times now.”

Neliswa: “We are just waiting for the test, we are ready, Ma’am.”

Ms Moleko: “All right, let’s move forward, as we presented these linear inequalities $a \geq 0$, $b \geq 0$, $a + b \leq 80$, $a \geq 10$, $b \geq 20$ and $b \leq 3a$ on the **ab Cartesian plane** on above in Figure 4.27. We should find the feasible area (region). How can we find the feasible region or area?”

Ms Moleko: "Gambu?"

Gambu: "After we had presented and drawn the linear inequalities and shown the direction of inequality signs, we had to find the area where all arrow direction meet, then shade it so that shaded area is the feasible area."

Wilson: "For me, I think to find the feasible area if we used different colours to show inequalities signs (arrow directions), we have to check and find the area where all colours are found, then started to erase colours meets."

Ms Moleko: "All right, since you know how to find the feasible region and it shows that still remember what you did in NCV level 3 (previous schooling). So, in your respective groups find the area where all different colours indicating inequality signs of greater or less than and equal to meet together. You have 5 minutes to do that".

Students interacted in their groups in finding the feasible region or area. Ms Moleko facilitated and ensured that students were working in their groups.

Ms Moleko: "Let's talk now; from the graph we have drawn (she referred to above Figure 4.27) which area can give us feasible region?"

Morena: "Mem, I think we can show that area."

Ms Moleko: "All right, just come."

Morena came forward and showed feasible area on the whiteboard. He started to erase shaded areas from Figure 4.28 above that did not assist in finding feasible region.

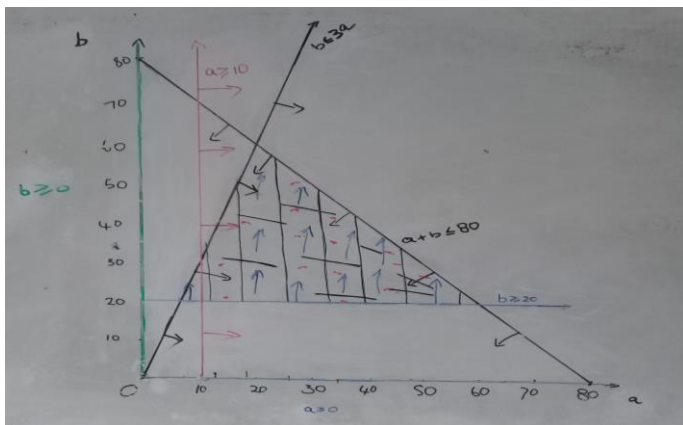


Figure 4.28: MLP graph with feasible region (area) shown by Morena in the classroom

Ms Moleko: "All right, thank you, Morena. Have you all found this area?"

Students: "Yes, Ma'am."

Ms Moleko: "So, what do we call this area and why is it important to find it?"

Gambu: "It is called a feasible area (region) and it is used to determine a maximum profit or a minimum costs in order to use limited resources efficiently. From this shaded area we have to find the points first that can assist us to find max profit or min costs"

Ms Moleko: "Come and show us, you can use FRP1, FRP2, because we already have coordinates A, B, C."

Gambu came forward and indicated a feasible region (area) coordinates. He also wrote feasible region inside the shaded area below (Figure 4.29).

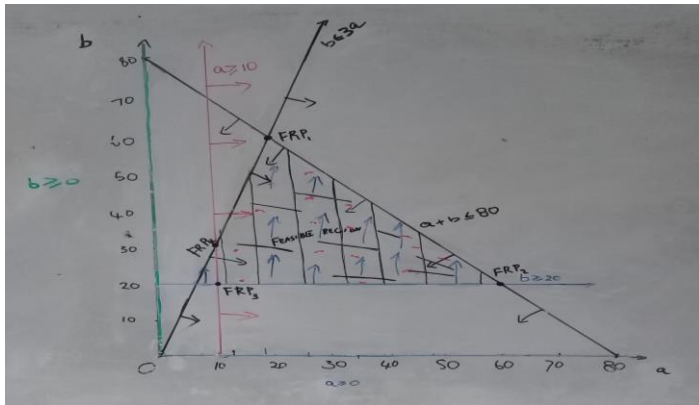


Figure 4.29: MLP graph with feasible region shown by Gambu

Ms Moleko: “All right, thanks Gambu. In your groups, find the coordinates for FRP1, FRP2, FRP3 and FRP4 on this graph. You only have 3 minutes to do this work.”

Students worked in their groups and Ms Moleko facilitated this process.

Ms Moleko: “Ok, your time is up. What are the coordinates for FRP1-FRP4? FRP1 is for Teboho; FRP2 is for Tshepe; FRP3 is for Zinhle; and FRP4 is for Khaya. Let’s hear.”

Teboho: “Point FRP1 is (20, 60).”

Tshepe: “Point FRP2 is (60, 20).”

Zinhle: “Point FRP3 is (10, 20).”

Khaya: “Point FRP4 is (10, 30).”

Ms Moleko: “All right, all right, thank you, did you all found these coordinates?”

Students: “Yes, Ma’am. We got the same number.”

Ms Moleko verified with students to read Figure 4.30 to find the coordinates for point FRP1-FRP4. They got the same (20, 60); (60, 20); (10, 20); & (10, 30).

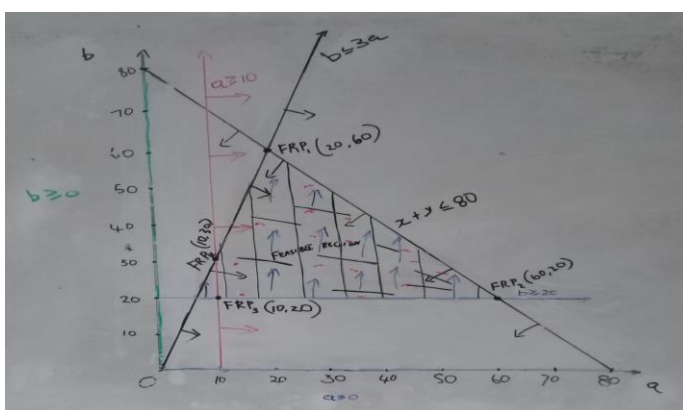


Figure 4.30: MLP graph with feasible region and its coordinates shown by Gambu

Ms Moleko: “Now we have answered question 1.1 and 1.2 so far which helped us to draw this graph (cf. Figure 4.30). Thus, with this graph (cf. Figure 4.30) we are at position to answer last question which is question 1.3 on above activity 1. Question 1.3 requires us to find objective function (profit formulae) or given objective function in order to find the business maximise profit or minimum costs.

(Reading question 1.3): "If the profit on maize is R800 per ha and on cabbages R500 per ha, how should the farmer plant the two crops to make maximum profit and what is this profit?" and interpreted "this question indicates that this business makes profit if the farmer charge R800 for each harvest of maize and R500 for each harvest of cabbage (a represent hectares of maize, b represent hectare of cabbage). Therefore, this can be written as **Profit = R800 per ha of maize + R500 per ha of cabbage** ($P = R\ 800\ a + R\ 500\ b$) then, we call this equation an objective function, how should the farmer plant two crops to maximise profit and what is the profit?

Since we have this objective function, do we end there?"

Zulu: "No, Mme, we use this objective function 'formula' to find maximum profit, same as we did in NCV L3 last year."

Ms Moleko: "Are you saying this is not the first time dealing with this objective function?"

Students: "Yes, Ma'am."

Ms Moleko: "All right, ok, in your groups find for me a maximum profit."

The students in their groups determined the maximum profit by substituting FRP1, FRP2, FRP3 and FRP4 into this objective function. All groups found the same answer and Ms Moleko asked Mike to show how they had arrived to their solution (Fig. 4.31).

Mike: "We did this table."

$$\begin{aligned} \text{FRP}_1 : P &= 800(30) + 800(60) = 46\,000 \\ \text{FRP}_2 : P &= 800(60) + 500(20) = 58\,000 \\ \text{FRP}_3 : P &= 800(10) + 500(20) = 18\,000 \\ \text{FRP}_4 : P &= 800(10) + 500(30) = 23\,000 \end{aligned}$$

Figure 4.31: Maximum profits determined by Mike

Mike: "As a result, the highest amount is R 58 000 here, so this means this farmer to make maximum profit, should plant 60 hectares of maize and 20 hectares of cabbage on available hectares because this farm has only 80 hectares of land."

Ms Moleko: "That's wonderful, students; tell you what you are so good. So, is there any other method we can use to determine the point that can help us to reach the maximum profit or minimum cost."

Students: "Yes, Mme, is through search line."

Lebohang: "We prefer the substitution method to find maximum profit than using the search line."

Mike: "I agree with Lebo, because the search line is difficult to apply for us."

Ms Moleko: Is there anyone who understand search line or who can try to apply search line? Using the search line we take the objective function and make b subject of the formula, so look how we do it."

$$\begin{aligned}
 P &= 800a + 500b \\
 -800a + P &= 500b \\
 \frac{-800a}{500} + \frac{P}{500} &= \frac{500b}{500} \\
 \frac{-800a}{500} + \frac{P}{500} &= b \\
 \therefore b &= \frac{-800a}{500} + \frac{P}{500}
 \end{aligned}$$

Search line

Figure 4.32 Calculations of search line presented by Ms Moleko

Ms Moleko: "Then, from above equation we use fraction $\frac{800}{500}$ in finding the search line, so we simplify this fraction $\frac{800}{500}$ to the simplest form which is $\frac{8}{5}$ however, since our graph scale which is multiple of 10. We can multiple $\frac{8}{5}$ both numerator and denominator by 10 and result to $\frac{80}{50}$ which best suit our scale. Moreover, in this fraction ($\frac{80}{50}$), 80 (numerator) will be **b** number on **b axis** while 50 will be **a** number on **a axis**, then we make points on the **ab Cartesian plane** and make it dotted line from where **b** is 80 and **a** is 50."

Ms Moleko explained the technique of finding the search line again and illustrated how to show the search line and move it up slowly by adding 10 (number) to each value. The last remaining point of the feasible region is the one which was used to find maximum profit. She further used a T-Square ruler to move the search line to the right (c.f. Figure 4.33).

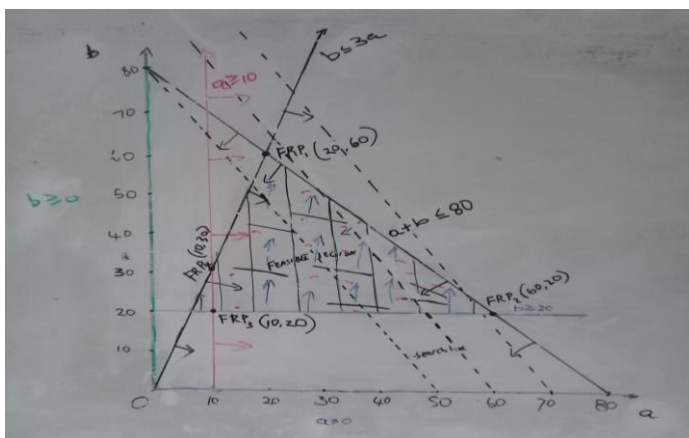


Figure 4.33: moving search line drawn (adapted in the classroom)

Ms Moleko "Then which point of this feasible region comes last when we move this search line?"

Students: "Point FRP2 with coordinates (60: 20)."

Ms Moleko: "All right, so what do we do with this point?"

Sibongile: "I think when it's here I can try?"

Ms Moleko: "Can you show us?"

Sibongile: "we start by substituting point FRP2 in this objective function where $P = 800a + 500b$ and determine maximum profit (showing it).

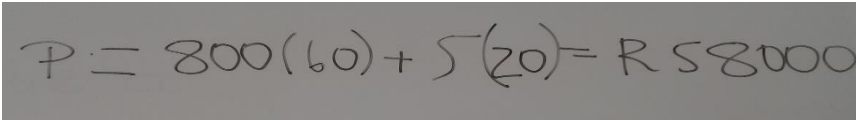

$$P = 800(60) + 5(20) = R 58000$$

Figure 4.34: Calculation of maximum profit presented by Sebongile

Ms Moleko: "Thank you very much, that's excellent. Questions please, I give this space for questions."

The students interacted with Ms Moleko in this part of the MLP lesson. Thereafter she worked with the students to sketch linear inequalities from previous lessons.

(i) $x \geq 500$; (ii) $x \leq 80$; (iii) $y \geq 100$; (iv) $x + y < 750$; (v) $x < 2y$
(vi) $15x - 12y \leq 60$ $x, y \in R$
 $x > 2$ and $1 \leq y < 5$
Objective $S = \frac{x}{2} + 2y$

4.3.1.4 The higher level for teaching of the MLP topic: teach all MLP components in one lesson

The higher level for teaching of the MLP topic takes place when all MLP components are taught in one lesson and at the same time. Therefore, we as a research team have agreed and decided to make this level the last one, because it integrates the previous levels (terminologies, word problems, linear inequalities and MLP graphs). Thereafter, all lessons should be repeated and link them, so that the students understand the interrelation between MLP components. This was evidenced by this statement of Mrs Seeta, who suggested that all components of MLP topic be taught in one lesson in the last level.

Ms Khodu: "Do we all agree, good people."

Co-researchers: "Yes, we agree."

Mr Takui: "This level should be the last one to be applied and should follow after all levels are taught. This will help students a lot."

Over the next two days the whole process was repeated and Ms Moleko presented the lessons in the same way in the first three levels. However, she taught the MLP components in one lesson (cf. example 1 & 2 on page 84-89 and Scenario 3).

Table 4.6: MLP scenario (developed by research team)

Scenario 3

A bottle of medicine contains vitamin C and vitamin E. The total number of units must not exceed 600. The medicine must contain at least 100 units, but not more than 400 units of vitamin C and at least 50 units of vitamin E. If vitamin C costs R5,50 per unit and vitamin E costs R2 per unit. Questions 1. Write down the equations of the inequalities. 2. Graph the feasible region 3. Find the maximum number of units of vitamin C and E 4. Find the minimum cost per bottle

Thereafter, Ms Moleko gave students a post-test based only on the MLP topic.

The lesson presentations above showed levels and intertwinement principles of the RME approach that Ms Moleko introduced to teach MLP. On the first level Ms Moleko taught MLP terminology, followed by a conversion of MLP word problems into linear inequalities. On the third level, Ms Moleko taught problem solving of linear inequalities, sketching MLP graphs, finding feasible area and determining maximum and minimum values. On the fourth level Ms Moleko taught all MLP components in one lesson, doing revision and lastly gave students a short test based on the MLP topic. Moreover, the above levels link and also are intertwined with one another, as planned and agreed upon by the research team. Look at the following statement:

"The MLP topic has many components we need to break according to its components but all components should link to one Who said this?"

This is in line with what the literature in Section 2.4.2.1. Andrew (1990, in White (1995:29-30) shared that for mathematics lecturers to teach MLP, they should break the topic into five sequential lessons, namely, recognising and describing variables with limits; formulating linear constraints; drawing linear inequalities to find the feasible region; determining maximum (or minimum) value through the analysis and interpretation of the MLP graphs; revising the whole process; checking solutions; and prepare for tests to improve the teaching of MLP concepts. He further maintained that no graphing of linear inequalities to find feasible region should be done until the third of five lessons are clear and comprehended by students. The Mathematical Linear Programming topic in mathematics NCV L4 includes MLP word problems, linear inequalities and straight line graphs; hence the above scholar suggested it should not be taught in one lesson and at the same time.

The same intertwined levels applied by Ms Moleko showed that this lesson began from the lower level to a more complex level, as articulated by the co-researchers. This is in line with concrete and abstract methods. Moloi (2013b:489-490), and Lindsay (2011:605) suggest the use of concrete and abstract methods to teach mathematics, starting from the concrete level to abstract level, so that students understand that mathematics is a human activity and is around us and within us (cf. 2.4.2.1). Multiple teaching approaches such as intertwined concrete and abstract levels were considered in the blurred genres era to enhance the teaching and learning of MLP. Ms Moleko integrated real-life situations, activities, pictures and indigenous languages.

The intertwined levels were also realised in the above four levels. In the first level, for instance, after Ms Moleko had brought her perspective on explaining the meaning and relevancy of the MLP topic to our lives, explained and wrote MLP learning outcomes on the whiteboard so that students were aware of what was expected from them at the end of the lesson.

“Understanding MLP terminologies and language; convert MLP word problems into linear inequalities; solve linear inequalities; draw MLP model/graph; analyse and interpret MLP models/graphs”

as guided by QMS as DHET policy. It is expected that lecturers should write, read and emphasise learning outcomes before unpacking and teaching SCK, in this regard MLP topic, in order for students to be aware of what they would be judged on.

Ms Moleko: *“Introducing the MLP topic, we first have to deal with MLP language (jargons) ... so we have developed the following terminologies (**less than, not more than, less than or equal to, at most, maximum, greater than, not less than, more than, greater or equal to, minimum, at least, equal to, mathematics variables, half, twice, times, and one third**) that can help us to unlock MLP word problems.”*

She alerted the students that the MLP lexicons were not limited only to those words; they should look for other words that would be of help to them. The students were given Activity A, based on MLP terminology to complete in their groups. This activity showed that students had difficulties with MLP terminology or language. Ms Moleko unpacked the MLP jargon, starting from the concrete to the abstract level of teaching to explain the jargons in their own indigenous languages. She brought her own views to explain MLP jargon and used practical examples that students could make sense of, to help students to understand MLP jargon well. Ms Moleko converted and solved practical MLP lexicons in a complex way (writing them in mathematical form). This technique assisted students to gain a deeper understanding of MLP terminology. This

allowed students to know and have a good understanding about where x and y variables come from and why they are used. More importantly, Ms Moleko made students aware that mathematics variables are not only about x and y to represent unknown numbers. As a result, Ms Moleko used other alphabets a and b as mathematics variable to represent unknown numbers and axes, instead of the usual x and y that are predominantly used in mathematics.

Ms Moleko: “Let’s start here, what the word less than mean in your own language.”

Students: “Less than in Sesotho means e nyane ho... In isiZulu it means ngaphansi kwa; in isiXhosa it means incicane ku”; and in Sepedi it means e nyane go.”

Ms Moleko: “Is there anyone with a different indigenous language to translate the word less than?”

The words ‘less than’ refer to a smaller number or size, especially when comparing two or more than two things in mathematics context. The following words are synonyms of the word less than, namely: lower than, below; under; smaller; shorter than; beneath; and fewer than.

Let’s look on this example:

Mr Jacobs earns R2 000 per week while Ms Pretty receives R3 000 per week.

From this statement, I understand that Mr Jacobs’ wage is less than ($<$) what Ms Pretty receives on a weekly basise. Thus, writing this statement mathematically, we can first let:

Mr Jacobs’ wage be x (or we can use any alphabet);

Ms Pretty’s wage be y (or we can use any alphabet); as a result $x < y$ ($R\ 2000 < R\ 3000$).

The number that is small comes before the less than sign, then follows the bigger number.”

Ms Moleko presented other MLP terminologies in this fashion to complete Table 4.2. The way in which Ms Moleko presented the first level of MLP topic also evidenced that Ms Moleko was guided by the principles of the RME approach, namely the activity, interactivity, realities, guidance, levels and intertwined principles. These principles assisted Ms Moleko to begin this lesson from the concrete level to the abstract level to concretise the MLP abstract concepts. This shows a discursive practice, because on the second level Ms Moleko presented the MLP word problems and linear inequalities lesson in this way. The lesson gradually went higher to a more complex level, because on the first level Ms Moleko taught the MLP terminology, followed by the MLP word problems with a single mathematics variable.

Table 4. 7: MLP word problems with a single mathematics variable

Scenario 1:

Mr Tshabalala produces at least 500 muffins per day with his limited resources

Ms Moleko: “To give proper background, **Mr Tshabalala bakes muffins** at home with his resources but he can **bake 500 and more muffins**. A number of muffins to be produced is **at least** 500 and more Or A number of muffins to be bake **cannot be less than** 500. So, Mathematically **at least is represented by this sign $\geq$**).

Then we let,

- a number of muffins to be produced be represented by x , then Mathematically

$$x \geq 500$$

This was followed by MLP word problems with two variables (cf. 4.3.1.1 & 4.3.1.2)

Table 4.8: MLP word problems with two variables

Scenario 2:

Mr Tshabalala now bakes muffins with two flavours, chocolate and strawberry. His baking of two flavours of muffins per day is less than 750.

Ms Moleko: “So, let’s let,

- a number of chocolate muffins to be produced be represented by x variable
- a number of strawberry muffins to be produced be represented by y variable,
 - therefore, mathematically,

$$x + y < 750$$

This sequence links well with the intertwined levels’ doctrines of the RME approach. According to Freudenthal (1973) and Gravemeijer and Doorman (1999, in Kwon, 2002:2), the level principle of RME approach requires a lecturer to break the MLP topic into two or more lessons based on its components, in order for s/he to start the teaching from an easier to a complex level, but ensure that those lessons are intertwined (cf. 2.3.2), with the aim to concretise the abstractness of mathematics concepts. This corresponds with what the literature revealed in Section 2.4.2.1, because Prangma *et al.* (2009: 383-384) recommend the use of the concrete and abstract method improve the teaching and learning of mathematics and improve results. On the third level, Ms Moleko introduced linear inequalities and MLP graphs and taught students how to solve and sketch linear inequalities. Subsequently, taught students how to find feasible area and determine a maximum (or minimum) values through the analysis and interpretation of MLP graphs (cf. Section 4.3.1.3). The above lessons in Sections 4.3.1.1, 4.3.1.2, 4.3.1.3 and 4.3.1.4 showed a chronological order that followed teaching of MLP applying the RME approach. Thus, this confirmed that the above levels cannot be treated in isolation, because one lesson led to another, which is exactly what Ms Moleko did. In light of the above information, levels and

intertwined principles of the RME approach are located in the Bricolage traditional era (cf. 2.2.2.1). However, in line with the RME approach, Ms Moleko involved her students throughout in the above lessons in line with PAR principles (cf. 3.2.2). This also shows a discursive practice and social inclusion levels, because Ms Moleko began her lessons from the concrete to the abstract level, to mathematise the MLP abstractness concepts. Students were allowed to interact with her and their peers, enabling them to learn from one another (cf. 3.5.5). This discursive practice evidenced that Ms Moleko did not act as a powerful person who is the only source of information, also in line with PAR, CDA social inclusion and postmodernity (cf. 3.2.3, 3.7 & 2.2.2.5), which locate the voice of the voiceless at the centre of the teaching and learning process.

This section agrees with Section 2.4.2.1 on the fact that teaching mathematics (including the MLP) should proceed from a lower level to the abstract level, breaking the MLP topics into five MLP components. This study further found that teaching MLP should start from the MLP lexicons in order to assist students to understand MLP concepts, which was not discovered by the research conducted in the past. In addition, in the teaching and learning process students were given class activities as assessments, prior to, during and after the lesson.

In conclusion, the empirical data showed that the concrete and abstract approach is key to concretise MLP abstract CK.

4.3.2 Multiple teaching and learning resources

Section 2.4.2.2 has explained teaching and learning media as teaching materials that assist lecturers to teach, elucidate, demonstrate, and emphasize key MLP concepts. Gitaari *et al.* (2013:98) and Karue and Amukowa (2013:106) agree that adequate teaching materials are key to improving mathematics teaching and learning. Sa'ad *et al.* (2014:32) maintain that the provision of a relevant teaching and learning materials needs to be made (cf. Section 2.4.2.2). The TVET college expects of mathematics lecturers, before lectures, to prepare and organise the teaching/learning resources that will be used to impart MCK and assist students in comprehending lessons. However, such teaching aids should be relevant to the individual mathematics topic.

The team agreed that multiple teaching and learning are crucial for MLP teaching and learning.

The following discussion took place during the meeting at Thabong Campus:

Mrs Matlou: “Colleagues MLP needs a lot of teaching and learning resources so that students are able to understand especially when we sketch linear inequalities, show the direction of inequalities and find feasible area. For instance, a T-square ruler is vital to draw an accurate MLP graph because a graph scale is still problematic.”

Mrs Lebone: “I agree with Mrs Matlou, because both lecturers and students do not scale the MLP graph properly on Cartesian plane.”

Ms Khodu: “What are other people saying?”

Mr Kia: “I agree, we have to look for resources and a lot of them. I like what Mrs Matlou said about a T-square ruler. Chair, can we suggest them now?”

Ms Khodu: “Colleagues, do you say?”

Co-researcher: “Let’s start now.”

During the lesson planning and preparation we identified teaching and learning aids such as a whiteboard; different ink markers, pictures, a T-square ruler, coins, paper money, scenarios, the internet, newspapers, erasers, colour crayons, pencils, different ink pens, rulers, graph pads, creative art books, exam pads and various textbooks.



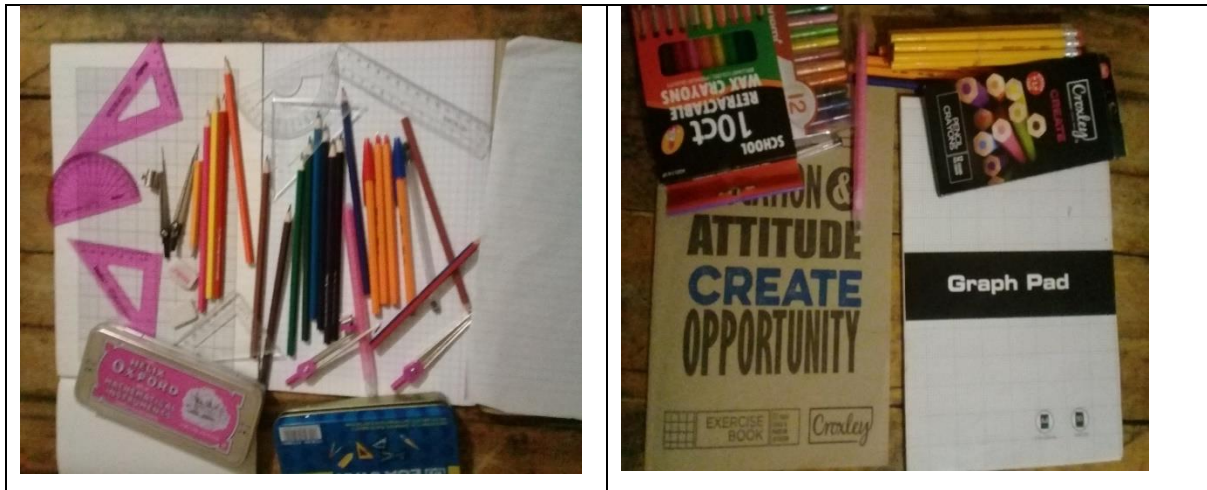


Figure 4.35: Teaching and learning materials for MLP topic (organised by a research team)

The above teaching and learning resources assisted Ms Moleko to impart the MLP components in the class (cf. the lesson presentation in Section 4.3.1), especially the sketching of the MLP, as proven by Figure 4.36 below.

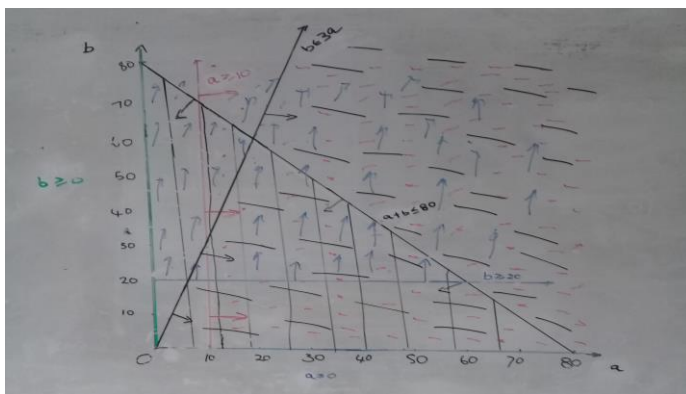


Figure 4.36: MLP graph (adapted from the classroom)

In the above Figure 4.36, we can identify the utilization of a T-square ruler, different markers (red, green, blue and black) to draw linear inequalities and demonstrate areas that satisfy the inequality signs on the whiteboard so that students understand how to draw linear inequalities. The use of these resources helped students to point out the feasible area. For instance, Morena was able to identify the feasible area and find the maximum value. On other hand, such resources aided Ms Moleko to show the search line on the MLP graph and used it to determine the maximum value (cf. Figure 4.37 below).

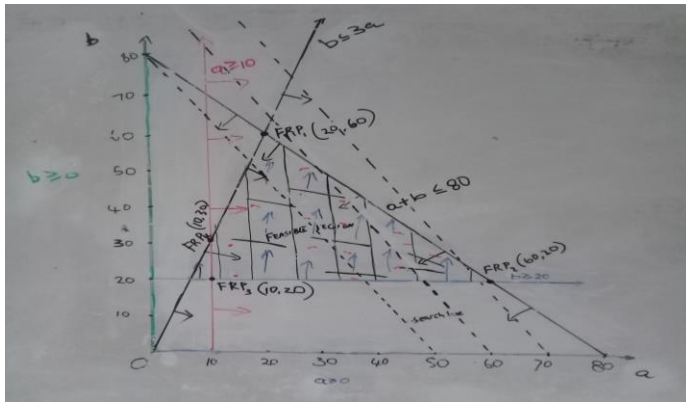


Figure 4.37: MLP graph with feasible area and search line (taken from classroom)

Thus, this is in line with the literature in Section 2.4.2.2 that supports the utilisation of relevant teaching and learning resources (Fig 4.35) to teach MLP. The organisation of teaching and learning resources showed that the research team are bricoleurs because they looked for resources at hand to enhance the teaching and learning of the MLP at TVET colleges. Moreover, the way Ms Moleko presented her lessons (cf. 4.3.1) evidenced that she was in the blurred genres and current fractured futures, moments of Bricolage, where multiple teaching and learning methods and the aids were considered important for teaching and learning (cf. 2.2.2.3 & 2.2.2.8). On the other hand, PAR was realised during the above discussions when the co-researchers decided on the necessary teaching and learning aids to teach the MLP at TVET colleges, because co-researchers collaborated with one another to reach consensus (cf. 3.2.1).

Mrs Matlou: "Colleagues, MLP needs a lot of teaching and learning resources."

Mr Kia: "I agree, we have to look for resources and a lot of them."

The team agreed on the benefit of teaching resources. Moreover, the co-researchers continuously used the word "we", so this showed a discursive practice and also social structure, since as a team we worked in partnership without any segregation.

This section revealed the teaching and learning resources that are relevant to teach the MLP components and how they can be employed (cf. the full lesson presentation in Annexure J). Therefore, this section agrees with what the literature recommended in Section 2.4.2.2 about the need of sufficient teaching and learning resources to teach mathematics. However, the literature did not reveal what kind of resources are key to teaching MLP components and how to use them. In addition, what was so exciting

was to see all the co-researchers coming under one roof and sharing their knowledge about resources relevant to teaching the MLP topic.

In conclusion, the generated data and the literature show that adequate teaching aids are essential component solution to enhance MLP teaching and learning.

4.3.3 Multiple principles of the RME approach to teach MLP topic

The literature in a Section 2.4.2.3 revealed that there is no single teaching method that is panacea to all mathematics teaching and learning challenges. Hence, many scholars have recommended various teaching approaches to teach the MLP, such as being student-centred; Problem Based Learning; Realistic Mathematics Education, using high-tech and computer software; code-switching translation, arranging students according to their capabilities; instructional resources; and awarding students (cf. 2.4.2.3). The DHET is in support of pedagogical methods that promote active learning among students, such as problem solving, collaboration, self-directed learning, project-based learning, challenge-based learning, and inquiry-based learning, because these methods foster more active learning experiences inside and outside the classroom, with students expected to be engaged in more critical thinking (*Government Gazette*, 2017:10). The examined TVET college expects of academic staff to use of a number of teaching methods, such as lectures, demonstrations and a student-centred method (cf. Annexure P).

In the planning phase of this study, the co-researchers were in agreement that to teach mathematics, especially MLP, lecturers should use multiple teaching methods because this topic includes other mathematics concepts such as a word problems, linear inequalities and graphs.

Mr Kapu: *“Colleagues, for me I think we should use realities, interactivity and guidance to apply the RME approach. We should use pictures and also what students see or what they can understand.”*

Mrs Matlou: *“The combination of traditional approach and guidance approach should be used because they fit into the traditional and student centred approaches because some students learn best through listening or working on activities.”*

Mrs Leeuw: *“We should also use students’ centred approach because most of time students should be active. So there must be an interactions between the lecturers and students and among students.”*

Mr Takui: *“For MLP word problems we should consider code switching while for linear inequalities and graphs we should consider procedure.”*

Thereafter, we as a team aligned suggested solutions into principles of the RME approach as these doctrines guided us to incorporate the RME approach to teach MLP. We then realised a link between solutions suggested by other co-researchers, with realities, interactivity and guidance as principles of the RME approach. We planned how to use the realities, interactivity, and guidance to teach the MLP components. Ms Moleko then presented this plan through lessons (cf. 4.3.1).

4.3.3.1 Reality

The reality principle of the RME approach requires of lecturers to incorporate realities into the teaching of mathematics so that students make sense of the lessons (cf. 2.3.2). Freudenthal (1973: 77) indicates that lecturers who use the RME approach to teach mathematics must be aware that the goal of mathematics education goal is that students can apply mathematics knowledge and skills gained in the class to solve real-life problems at the end of each lessons, as mathematics in this sense is attached to realities. Freudenthal (1973:77) strongly maintains that if students learn mathematics in isolation or divorced from reality, it is easy for them to forget what has been taught in the mathematics classroom. As a result, they will not be able to apply mathematics problem-solving skills to their real-life challenges (cf. 2.3.2).

The MLP component of mathematics National Vocational Certificate Level 4 (NCV L4) begins from problem situations because of resource constraints (cf. 2.3.1). It was suggested that teaching MLP using the RME approach should begin with problems in rich contexts that need mathematical organisation, rather than beginning the lesson with definitions to be applied or teaching MLP in an abstract way. In other words, the lecturer should actualise informal context-related solution strategies as a first step in the teaching and learning process. The context assists not only as a source for developing mathematics, but as area of application to stimulate problem-solving shortcuts. Thus, to provide context-related strategies and teach MLP within a realistic context, students should be given problems that can be solved in numerous ways (cf. 2.3.2). The above information suggests that to teach MLP should begin from problem situations students can make sense of, because those problem situations give students opportunities to interpret the MLP word problems and gain new problem-solving skills. During one of research meetings co-researchers agreed with Mr Kapu.

Mr Kapu: “To make mathematics interesting and enjoyable to all students we should link what to be taught with real life situations, we should show how the MLP is relevant in our lives or we should use pictures in order to apply reality as principle of the RME approach.”

As a research team, we then together planned lessons showing how to integrate realities in the teaching of MLP.

Ms Moleko delivered lessons exactly the way it was planned, prepared and developed integrated realities (cf. the lesson presentations in Section 4.3.1). The following extracts were taken from lesson presentations (cf. 4.3.1), showing how Ms Moleko integrated realities in teaching MLP topic.

Ms Moleko: “Write definition MLP and explain the relevance of MLP topic in our daily lives, in fact its importance in our lives.”

Poho’s group: “We think that MLP is just a mathematics topic and we do not see any importance of it in our lives.”

Mampho’s group: “MLP refers to a mathematical model or technique helping us to make decisions in business... But, we do not know the importance of MLP in our lives.”

Ms Moleko: “Correct, MLP is one mathematics topic and is defined as the mathematical model tool helping individuals to make an informed decisions in public and private sectors as well as different environments. MLP is used in these sectors to maximise profit or minimise costs with limited resources available. So, the importance of MLP topic is that it can help businesses and people to make informed decisions using constraints resources to generate profit.”

During the lesson Ms Moleko gave students Activity A below

MS Moleko: “Define these words, apply real-life situations and write these words in Mathematical symbol. So, in your own groups complete this table (**refer to Table 4.1 in Section 4.3.1.1**) by defining these words, create practical examples and then, write mathematical signs representing those words.”

The students did not perform well in this activity. Then, Ms Moleko taught the MLP terminologies using realities, pictures and indigenous languages (c.f. Table 4.2 in the Section 4.3.1.1). In the next session, Ms Moleko provided students with MLP activities word problems and linear inequalities (cf. Scenarios 1 & 2 in 4.3.1.2). Ms Moleko realised that students had difficulty formulating word problems into linear inequalities, and she then began to teach students how to formulate MLP word problems into linear inequalities. The lesson went well and students understood MLP word problems of a single and also two mathematical variables. Ms Moleko thereafter introduced a more complex MLP word problems and projected Figure 4.18 on the wall. She also issued hand-outs of Figure 4.18 and started this part of the lesson by saying:

Ms Moleko: “In your groups, identify natural resources and man-made resources that you see in this picture (**pointing and referring to Fig. 4.18 on the wall in Section 4.3.1.2**).”

The students managed to identify resources.

Students: “It is a farm with land, trees, polls, electric wires, clouds, mountain, sky and clouds.”

Ms Moleko: “Our focus today will be land, so is there anyone who can tell us why land is so important to all of us.”

Mampho: “On the land we plant different types of food, we build houses and buildings for its purposes.”

Ms Moleko: “Very, excellent, Mampho and I believe all of us agree with her. However, to plant different food on a limited piece of land, we need to plan well in order to use it efficiently for maximising profit when we operate as a business. So, MLP model is one of techniques we can use to plan well and use a limited piece of land well for reaching maximum profit. Therefore, this technique has been used in different countries and sectors around the world, so it is also relevant to the farming sector. Let’s do this work to see how the MLP can be used?”

Activity 1

A farmer has 80 hectares of his farm available for planting maize and cabbage. He must grow at least 10 hectares of maize and 20 hectares of cabbage to meet demands. He prefers to plant more cabbage than maize, but his workforce and equipment will only allow him to cultivate a maximum of three times the quantity of cabbages to that of maize.



Questions:

- 1.1 Represent the information above as a system of inequalities.
- 1.2 Sketch a graph of these inequalities.
- 1.3 If the profit on maize is R800 per ha and on cabbages R500 per ha, how should the farmer plant the two crops to make a maximum profit and what is this profit?

Figure 4.38: Activity 1 (taken from classroom)

Ms Moleko: “This activity talks about a farmer who has only 80 hectares on his piece of land and he usually plants cabbage and maize for his business. So, we should first find and write variables, which can be any alphabet letter (a-z) that represents these plants to be planted (i.e. maize and cabbage). We let:

number of maize to be planted be represented by variable “a”

number of cabbage to be planted be represented by variable “b”

NB: (a for x axes and b for y axes)

Ms Moleko: “We should start to show these basic linear inequalities $a \geq 0$ and $b \geq 0$. For instance, $a \geq 0$ means if the farmer plant maize more than 0 hectares of maize will be harvested however, if the farmer do not plant maize, then 0 hectares of maize will be harvested. Therefore, this is the same with $b \geq 0$ because $b \geq 0$ means if the farmer plant cabbage more than 0 hectares of cabbage will be harvested but, if the farmer do not plant cabbage, then 0 hectares of cabbage will be harvested.”

The lesson showed that Ms Moleko linked the MLP topic with real-life stories and its relevancy to our daily lives. This was realised in the following:

Ms Moleko: “Write definition MLP and explain the relevance of MLP topic in our daily lives, in fact its importance in our lives.”

It was a wise move to begin the MLP lesson in this fashion to alert students that MLP was fundamental in our daily lives and also a human activity. However, students did not know the relevancy of MLP in our daily lives, as was realised in the following statements of students.

Poho's group: “We think that MLP is just a mathematics topic and we don't see the importance of it in their lives.”

Mampho's group: “MLP refers to a mathematical model helping us to make decisions in business. But, we do not know the importance of MLP in our lives.”

Then, Ms Moleko made students aware that MLP is relevant in everyone's life.

Ms Moleko: “You are correct the MLP topic is one of mathematics topic and is defined as the mathematical model tool helping individuals to make an informed decisions in public and private sectors ... importance of MLP topic is that it can help businesses and people to make informed decisions using constraints resources to maximum profit or lower costs.”

She incorporated Fig. 4.18 of a farm where MLP can be used in agricultural planning, evidencing MLP is indeed a fundamental subject. This attracted students more and also their interest in the MLP lesson. Subsequently, Ms Moleko linked Activity 1 with Fig. 4.18 and showed students how an MLP model is used in the agricultural sector to plan and use available land well. This is in line with what the reality principle of the RME approach required from lecturers as it expects lecturers to integrate real-life situations into the MLP lesson for students to make sense of the lesson (cf. 2.3.2). The co-researchers agreed that this was another way that should be used to decolonise abstract mathematics concepts and contents. Fataar (2018:2) argues that education should be decolonised and make sense to students' daily lives. In essence, what is taught at schools should help students to solve their daily challenges. The integration of real-life situations into the teaching of mathematics was considered in the blurred genres, third moment of Bricolage, where multiple teaching methodologies, strategies,

pedagogies and techniques were allowed and used to close traditional era limitations (cf. 2.2.2.3). This is in line with Bricolage, because it allows bricoleurs to incorporate whatever is available at hand to enhance teaching and learning of MLP at TVET colleges, as Ms Moleko did in her lessons in Section 4.3.1. She integrated real-life pictures; real-life class activities; and indigenous languages to mathematize the abstract MLP concepts. This was evidenced by one of the activities used in the lesson presentations (cf. Activity 1). Thus, the Activity 1 assisted students to make sense of it, as it was associated with what they saw and heard about in a different context. For that reason they were able to read, analyse and interpret this activity.

Wilson: *“I think it means that this farmer has maximum of 80 hectares (size of the piece of land) to plant maize and cabbages but planting both products cannot exceed 80 hectares because it is only land available him.”*

Throughout the MLP lesson presentations (cf. 4.3.1), it was clear that Ms Moleko had prepared class activities to help students to make sense of mathematics concepts; especially understanding that MLP is key to solving real-life challenges and problems. This was evidenced by the following word problems

Mr Tshabalala produces at least 500 muffins per day with his limited resources” and “Ms Ncube uses a minibus and quantum taxi to transport school kids, workers and people to reach their destiny. A minibus cannot take twice of quantum taxi to transport people to their destiny

A farmer buys sheep at R600 each and goats at R800 each. He cannot spend more than R60 000. To conclude the transaction, he must buy at least 5 goats and 10 sheep. In future he can make a profit of R400 per sheep and R500 per goat.

This is in line with what Fataar (2018:02) maintains, namely that class activities should include real-life problems that make sense to students and assist them with their daily work and/or problems, one component of decolonising mathematics as a subject. Moreover, these types of class activities were encouraged and dominantly used in the Bricolage blurred genres (cf. 2.2.2.3). This notion was considered to close gaps of the Bricolage traditional era where class activities were prepared in an abstract fashion in isolation to people lives. These abstract class activities created wrong perceptions about mathematics among people, because some of the students stated:

“I am just doing mathematics because it is the must in our course, where are we going to apply x and y problem solving in our lives.”(cf. 4.2.1).

In essence this is is not a human activity. This perception contributes to poor mathematics performance and to a lack of students’ interest mathematics. According to Anigbo (2016:23), a lack of interest to in mathematics leads to poor performance in

mathematics. The reality principle was also realised when Ms Moleko explained MLP lexicons (cf. Table 4.2 & Fig. 4.15 in Section 4.3.1.1).

For instance, Ms Moleko employed the realities, rich information and real-life situations to explain and practicalise the MLP lexicons for students understand. This technique of integrating real-life situations is advocated by Van den Heuvel Panhuizen (2003:10), who maintains that lecturers should integrate rich information (real-life situations) into the teaching of Mathematical Content Knowledge (MCK) for students to comprehend and also make sense of their mathematics lesson (cf. 2.3.2). This also helps students to know that mathematics is within us and around us, quintessentially a human activity. For instance, Ms Moleko used hand-outs and practical examples which bring students closer and to understand the lesson. The technique that Ms Moleko applied also fits into concrete and abstract, because Ms Moleko accommodated and borrowed different indigenous languages to explain the MLP lexicons so that students first understood it their own mother tongue.

Subsequently, she explained MLP jargon in a mathematics context and gave practical examples to clarify MLP lexicons before presenting them in a complex form (cf. Table 4.2). Using a combination of concrete and abstract techniques helped students to understand the lesson well. Kamina and Iyer (2009:02) recommend the use of concrete and abstract techniques, proving its efficacy (cf. 2.4.2.1). The inclusion of realities in the teaching and learning of MLP lexicons and using the combination of the concrete and abstract method (multiplicity pedagogies) were first considered essential during blurred genres (cf. 2.2.2.3). In addition, Ms Moleko worked with students throughout her lessons (cf. 4.3.1) and that action showed PAR, discursive practices and social inclusion.

In this section, Ms Moleko integrated realities, pictures and indigenous languages to teach MLP topic, which agrees with the literature in Section 2.3.2 and 2.4.2.3. However, this section shows how to integrate realities principles of the RME approach, real-life situations, pictures and the use of indigenous languages to enhance the teaching of MLP at TVET colleges (cf. 4.3.1). Moreover, Ms Moleko worked with co-researchers to plan, prepare and develop lessons together on how to integrate real-life situations, pictures and indigenous languages. She interacted with the students throughout the lessons, showing collaboration.

In conclusion, the empirical data revealed that the incorporation of real life situations in the MLP teaching and learning makes it easier for students to comprehend the MLP components well.

4.3.3.2 Interactivity

The interactivity principle of the RME approach requires of lecturers to create a better mathematics classroom environment where the interaction among students and/or between the lecturers takes place. The literature suggests whole-class discussions and pair and group activities to promote social activity, rather than individual activity, during the learning process, but strongly maintains that both lecturers and students should consider and value whole-class activities, and pair and groupwork so that interaction becomes real and possible (cf. 2.3.2). This notion is recommended by Arsalidou and Pascual-Leone (2016:1) as they uphold that students learn best when they actively work on their own on activities developed by their lecturers in or outside the classroom rather than receiving every piece of information. The benefit of interaction is that it evokes reflection and enables students to reach a higher level of understanding. This study believes that interaction takes place when two or more people work together to solve their teething troubles. Freudenthal (1993, in Kwon, 2002:4), and Nakhanu *et al.* (2015:72-73) in Section 2.4.2.3 and 2.3.2, agree that interaction, collaboration and participation should be encouraged in the teaching and learning of MLP using *the interactivity principle* of the RME, PBL and student-centred methods. The DHET and TVET colleges are in support of student-centred methods (Government Gazette, 2017:10). We as a research team agreed with Mr Kapu.

Mr Kapu: *“That interactivity principle of the RME approach should be used to teach the MLP at TVET colleges because it has the same characteristics with the student-centred approach.”*

Then we planned together as a team how to use the interactivity principle. In the lessons presented by Ms Moleko we realized the use of interactivity principle of the RME approach (cf. 4.3.1). The following extracts were taken from the lesson presentation in Section 4.3.1.

Ms Moleko: *“In your groups, write the definition MLP and its importance in our lives. Write these in your own words.”*

Poho: *“As the group, we think that MLP is just a mathematics topic and we do not see any importance of it in our lives.”*

Ms Moleko: *“Thanks to all groups for your presentations, you are correct MLP is one of mathematics topic and is defined as the mathematical model tool used in public and private*

sectors ... MLP topic is that it can help businesses and people to make informed decisions using constraints resources to generate profit or minimise costs.”

Ms Moleko: “In your own groups complete this table by defining these words, create practical examples and then, write mathematical signs representing those words.”

Ms Moleko: “All right, let’s do it together, firstly, let’s start with the word ‘less than’.”

Then, Ms Moleko worked together with students to complete Table 4.2. In the next lesson, Ms Moleko presented the lesson in the same way as in the first lesson, which put students at the centre of the teaching.

Ms Moleko: “In your groups, solve this problem (write this statement mathematically).”

Table 4.9: MLP word problem (taken from classroom)

SCENARIO 1:

Mr Tshabalala produces at least 500 muffins per day with his limited resources.

The students worked together in groups and were granted three minutes to do this work. Ms Moleko and her team facilitated the lesson, checking whether students worked as expected. Ms Moleko marked and found that most groups got it correct.

Ms Moleko: “Is there anyone who can help us understand how you arrived to your solutions?”

Wilson came to show how they had reached their solution in his group (Figure 4.39).

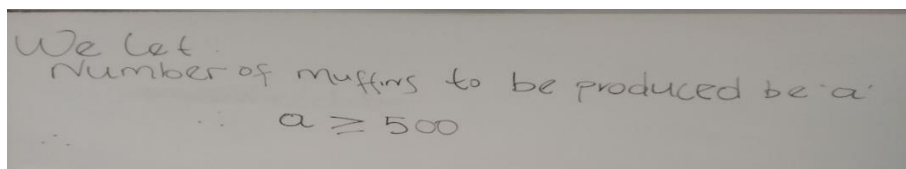


Figure 4.39: Wilson’s solution for scenario 1

Ms Moleko was very happy and satisfied with Wilson’s solution:

Ms Moleko: “OK, let’s move to the next part of this lesson for mathematics word problems of **two variables**. So, in your groups you have 5 minutes to solve this mathematics problem (cf. Table 4.5).”

Students next worked together and groups were given 5 minutes to do this work. Ms Moleko and assistant lecturers facilitated this process and checked whether students’ were working as expected. They marked students’ work and gave feedback. That feedback showed that students had difficulties with this part of the lesson (cf. Annexure N). She then guided them how to solve the MLP word problems with two variables. Ms Moleko further continued to teach MLP word problem (cf. Activity 1) and worked with students.

Ms Moleko: “Let’s go back to the above activity 1. Teboho, read and interpret or say what you understand in that activity.”

Teboho read, interpreted the activity and also other students were given an opportunity to share what they understood in Activity 1. Thereafter, Ms Moleko further unpacked this activity, and gave her perspective:

Ms Moleko: “This Activity 1 talks about a farmer who has only 80 hectares on his piece of land and he usually plant cabbage and maize for his business. So, we should first find and write variables which can be any alphabet (a-z) that represents these plants to be planted (i.e. maize and cabbage). We let:

number of maize to be planted be represented by variable “a”

number of cabbage to be planted be represented by variable “b”

NB: (a for x axes and b for y axes)

We should start to show these basic linear inequalities $a \geq 0$ and $b \geq 0$. For instance, $a \geq 0$ means if the farmer plant maize more than 0 hectares of maize will be harvested however, if the farmer do not plant maize, then 0 hectares of maize will be harvested. Therefore, this is the same with $b \geq 0$ because $b \geq 0$ means if the farmer plant cabbage more than 0 hectares of cabbage will be harvested but, if the farmer do not plant cabbage, then 0 hectares of cabbage will be harvested.

Ms Moleko completed Activity 1 converted the MLP word problem into the following linear inequalities: $a \geq 0$, $b \geq 0$, $a + b \leq 80$, $a \geq 10$, $b \geq 20$, $b \leq 3a$. Thereafter she showed the class how to solve those linear inequalities, resulting in these coordinates;

$a \geq 0$ coordinate A (0, 0) with greater or equal to sign;

$b \geq 0$ coordinate B (0, 0) and with greater or equal to sign;

$a + b \leq 80$ coordinates C (80, 0) and D (0, 80) and with less or equal to sign;

$a \geq 10$ coordinate E (10, 0) and with greater or equal to sign;

$b \geq 20$ coordinate F (0, 20) and with greater or equal to sign;

$b \leq 3a$ coordinates G1 (0, 0), G2 (10, 30), G3 (20, 60), G4 (30, 90), G5 (40, 120), G6 (50, 150), G7 (60, 180), G8 (70, 210), G9 (80, 240) and with less or equal to sign

Ms Moleko let the students plot and join the above coordinates on the Cartesian plane (cf. 4.3.1.3)

Ms Moleko: “In your groups plot these coordinates, join coordinates and write linear inequalities on that joined line, but use graph papers”.

Thereafter, Ms Moleko worked with the students to present those coordinates on a Cartesian plane, but the initial step was to ask students to decide on the scale factor for this Cartesian plane.

Tholoane: “Our scale must be a factor of 10 for both a-axis and b-axis because the linear inequalities have the following numbers 0,3,10, 20 and 80. So, these co- ordinates fit well in this scale.”

Ms Moleko: “All right, thanks Tholoane, I agree with you, so please ensure that if you choose 10 as a scale factor for your graph should be both on a - axes and b - axes and also ensure that the measurement between the spaces on a **ab Cartesian plane** are equal at all times. More importantly, as you have said, Tholoane, most of numbers in these linear inequalities and its coordinates are covered with this scale factor (**ab Cartesian plane**).

Ms Moleko continued to work together with students and drew Fig 4.40 below:

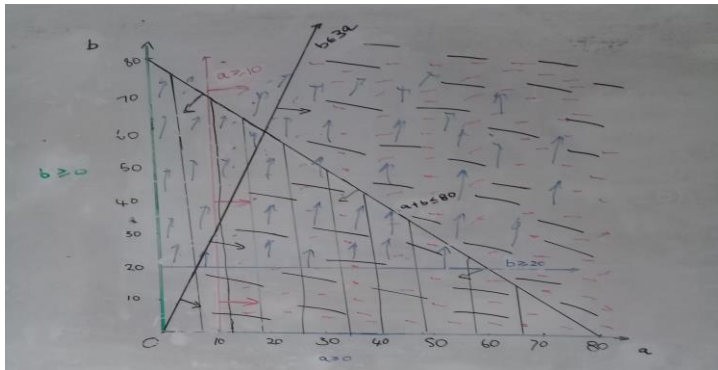


Figure 4. 40: Image of $b \leq 3a$ on the same Fig 4.25 and was demonstrated by Ms Moleko

Ms Moleko: “Let’s move forward, as we presented these linear inequalities $a \geq 0$, $b \geq 0$, $a + b \leq 80$, $a \geq 10$, $b \geq 20$ and $b \leq 3a$ on the **ab Cartesian plane** on above in Figure 4.40. We should find the feasible area (region). How can we find the feasible region or area?”

Gambu: “After we presented and drawn the linear inequalities and also shown the direction of inequality signs, we have to find the area where all arrow direction meet together, then shade it so that shaded area is feasible area.”

Wilson: “To find feasible area if we used different colours to show inequalities signs (arrow directions), we have to check and find the area where all colours are found, then start to erase colours.”

Ms Moleko: “Since you know how to find the feasible region and it shows that still remember what you did in NCV level 3 (previous schooling). So, in your respective groups find the area where all different colours indicating inequality signs of greater or less than and equal to meet together. You have 5 minutes to do that.”

Students interacted in their groups to find the feasible region. Ms Moleko ensured that the students worked in their groups. Thereafter, Ms Moleko allowed Morena to show a feasible region on the whiteboard (Fig. 4.40 on above), so all groups and Ms Moleko agreed on Morena’s answer. Moreover, Gambu identified feasible region points, then other students were asked to find feasible area coordinates. Students managed to find the feasible region coordinates.

Ms Moleko verified with students to read Fig. 4.41 to identify and find the coordinates for point FRP1, FRP2, FRP3 and FRP4, so they got the same coordinates as indicated on the graph (cf. Figure 4.41 below).

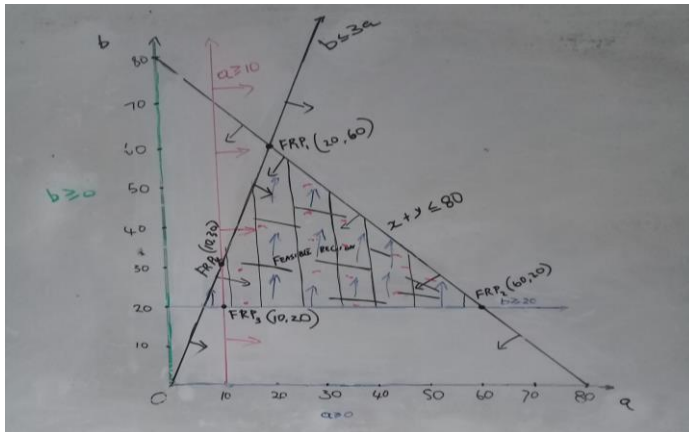


Figure 4.41: MLP graph with feasible region and its coordinates showed by Gambu

Ms Moleko: “Since we have this objective function; do we end there?”

Zulu: “No, Mme, we use this objective function to find max profit, same as we did in NCV L3”

Ms Moleko: “All right, ok, in your groups find for me a maximum profit.”

Students in their groups determined the maximum profit by substituting FRP1, FRP2, FRP3 and FRP4 into this objective function. All groups found the same answer and Ms Moleko asked Mike (student) to show how they arrived at their solutions.

Mike: “We did this table (substitution method) – as a result, the highest amount is R 8 000 here, so this means this farmer to make maximum profit, should plant 60 hectares of maize and 20 hectares of cabbage on available hectares because this farm has only 80 hectares of land (cf. Figure 4.31).”

The above lesson extracted from Section 4.3.1 revealed the interactivity principle of RME approach and student-centred approach. In Section 2.3.2, the literature showed that to apply the interactivity principle, lecturers are required to interact with students in the teaching and learning process, which is precisely what Ms Moleko did in the entire lesson. This was proven by the following statement

Ms Moleko: “In your groups plot these coordinates, join them (make straight lines) and also indicate each line with its linear inequality ... but use these graph papers.”

Then she distributed graph papers to all groups, one of the teaching aids that were used to assist students to draw the MLP graphs accurately (cf. 4.2.2) because co-researchers aired the same sentiment with Luenberger (2008:24) on the fact that most students have difficulties in drawing MLP graphs accurately, especially treating large numbers; thus, the scale factor is problematic. Ms Moleko and assistant lecturers facilitated this process in ensuring that all students are doing what were expected from them, in line with the team teaching approach. According to Davis and Winter (2019:33), the team teaching approach permits two or more lecturers to share duties,

roles and responsibilities in a particular subject and be available throughout the entire lesson in this regard. Moreover, this is also what TVET colleges advocate to all of its campuses in order to enhance the teaching and learning.

Ms Moleko: *“All right, your time is up, so let’s look whether you managed to get it correct. Exchange your work to other groups.”*

This permitted peer marking to take place effectively.

Ms Moleko: *“How much must be our scale in for this graph?”*

Tholoane: *“Our scale must be a factor (or multiple) of 10 for both a -axis and b -axis because these linear inequalities have these numbers 0, 3, 10, 20 and 80. So, these coordinates fit well in this scale of a factor 10.”*

Ms Moleko: *“I agree with you, so please ensure that if you choose 10 as a scale factor for your ab Cartesian plane, should be on both on a -axes and b -axes also ensure that measurement between the spaces (numbers) on ab Cartesian plane are equal”*

Students worked with Ms Moleko and drew Figure 4.36, and Ms Moleko further said:

“Let’s move forward, as we presented these linear inequalities $a \geq 0$, $b \geq 0$, $a + b \leq 80$, $a \geq 10$, $b \geq 20$ and $b \leq 3a$...We should find the feasible area (region). How can we find the feasible region or area?”

Gambu: *“After we presented and drawn the linear inequalities and also shown the direction of inequality signs, we have to find the area where all arrow direction meet together, then shade it and that shaded area is feasible area.”*

Wilson: *“I think to find feasible area if we used different colours to show inequalities signs (arrow directions), we have to check and find the area where all colours for arrow direction meet and erase parts where colours do not meet.”*

Ms Moleko realised that the students had adequate prior MLPCK in showing the feasible area. This enabled her to give students work to show the feasible area.

Ms Moleko: *“Since you know how to find the feasible region and it shows that still remember what you did in NCV level 3 (previous schooling). So, in your respective groups find the area where all different colours indicating inequality signs of greater or less than and equal to meet together. You have 5 min to do that.”*

Therefore, the students interacted in their groups finding feasible region, and all groups managed to show this area shown by Mampho and Gambu (Figure 4.42).

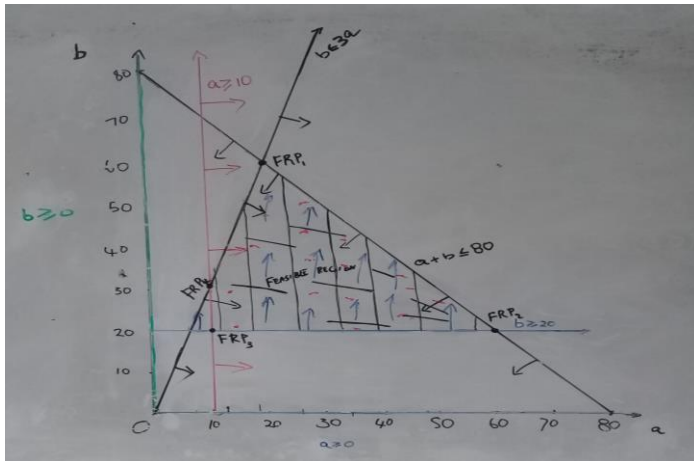


Figure 4.42: Shown by Mampho and Gambu (student)

Moreover, they also managed to find coordinates themselves without any help from Ms Moleko.

Teboho: "Point FRP1 is (20, 60)."

Tshepe: "Point FRP2 is (60, 20)."

Zinhle: "Point FRP3 is (10, 20)."

Khaya: "Point FRP4 is (10, 30)."

All groups agreed on the above coordinates and Ms Moleko made sure that they were accurate. Then Gambu was asked to write coordinates on the MLP graph (cf. Figure 4.41 above)

Ms Moleko: "Now we have answered question 1.1 and 1.2 so far, which helped us to draw this graph (cf. Figure 4.39). Thus, with this graph in Figure 4.39 we are in the position to answer last question, which is question 1.3 on Activity 1."

The students and Ms Moleko agreed that to answer the last question, 1.3, they needed to find the objective function. They managed to find the objective function: Profit = R800 per ha of maize + R500 per ha of cabbage ($P = R800a + R500b$). The students did not seem to struggle to find objective function as well as a maximum profit, because all agreed that they had done these concepts in NCV L3. Hence, they all managed to use an objective function to find the maximum profit. Mike illustrated how they got their solution in their group and explained how to choose the maximum profit (cf. Figure 4.31).

Mike: "As a result, the highest amount is R58 000 here, so this means this farmer to make maximum profit, should plant 60 hectares of maize and 20 hectares of cabbage on available hectares because this farm has only 80 hectares of land."

His teaching style is in line with the student-centred, interactivity principle of the RME and the constructivism approaches. These methods require of students to play a

central role in the teaching and learning, while lecturers are expected to guide students and facilitate the teaching and learning process (cf. 2.4.2.1). This study unfolded that to keep students as an active participants, lecturers should provide students with certain mathematics concepts to discuss and also report back in the form of presentations, in essence sharing their knowledge. Secondly, to keep students active, lecturers should provide students with practical work (activities) individually or in groups inside or outside the classroom in order for them to become hands-on, rather than being treated as receiver of information. This is what Ms Moleko did in the lesson presentation above. In light of above the expectation, using the student-centred, interactivity principle of the RME and constructivism methods, it is clear that these techniques are developed to bridge the Bricolage traditional era limitations, where lecturers were viewed as the only source of knowledge and/or powerful person who knows everything (cf. 2.2.2.1).

This lesson of Ms Moleko shows discursive practice and social inclusion, because students were continuously engaged in the lesson. They were also allowed to interact with their peers. This is in line with Bricolage blurred genres, postmodernity and post-experimentalism, because during these moments, students' prior knowledge are at the centre of the teaching and learning and lecturer-student collaboration is encouraged (cf. 2.2.2.3, 2.2.2.5 & 2.2.2.6).

In addition, this allows students to participate and engage with both their lecturers and their peers without any form of discrimination. Therefore, Ms Moleko's lessons are in line with the PAR principles (cf. 3.2.3), because throughout her lessons she continued to engage and involve students (cf. 4.3.1). Look the following statements by different students:

Wilson: *"This farmer has a maximum of 80 hectares to plant maize and cabbages but planting both products cannot exceed 80 hectares because it is only land available him. So, number hectares of maize to be planted plus number hectares of cabbages to be planted can be planted on 80 or less than hectares because of limited piece of land."*

Tholoane: *"Our scale must be a factor of 10 for both α -axis and b -axis."*

Gambu (after graphing the MLP mode): *"After we presented and drawn the linear inequalities and also shown the direction of inequality signs. We have to find the area where all arrow direction meet together, then shade it so that the shaded area is a feasible area."*

After presenting the feasible region, Tshepo, Zinhle and Khaya found feasible region coordinates:

“Point FRP2 is (60, 20), “Point FRP3 is (10, 20), and “Point FRP4 is (10, 30).”

Mike (using this to calculate a maximum profit): *“The highest amount is R 58 000 here, so this means this farmer to make maximum profit, should plant 60 hectares of maize and 20 hectares of cabbage on available hectares because this farm has only 80 hectares of land”.*

This liberty of social inclusion (collaboration) were denied or not permitted during the traditional era of Bricolage.

This Section showed how the interactivity principle can be applied into the teaching of the MLP topic using the RME approach. Ms Moleko used group discussions and group activities to foster interactions among students; thereafter, students provided feedback on their work. Moreover, Ms Moleko worked with her assistant lecturers and students in all the lesson presentations. This agrees with Sections 2.3.2 and 2.4.2.3 on the fact that the interactivity principle and a student-centred method should be used to encourage and allow interaction among students and also between lecturers and students. It was interesting was to see Ms Moleko, assistant lecturers and students working together in collaboration in the MLP teaching and learning.

In conclusion, the empirical data showed that the interaction among students and also between students and their lecturers make it easy for students to learn adequately.

4.3.3.3 Guidance

According to this study, guidance takes place when lecturers facilitate the teaching and learning process. In this regard, the students play a central role working on the class or home activities and engaging one another through discussions. In Section 2.3.2, Treffers (1978, in Van den Heuvel-Panuizen & Drijvers, 2014:523) maintains that lecturers should give support to students when teaching mathematics. However, students should play a central role while lecturers facilitate the teaching and learning process, so that the guidance principle of the RME approach is applied successfully. According to Travers (1949:211), the word *guidance* includes several social activities in which one or more persons learn new behaviours through the help of another. Guidance and, in a sense, all teaching, is a vital learning situation. In this regard, mathematics lecturers, by virtue of their position, are seen as persons with vast mathematics knowledge. Therefore they are expected to guide and help mathematics students in their MLPCK misconceptions and to acquire new knowledge. In all learning

situations, the obligation of the lecturer is one of guidance; they are expected to facilitate the teaching and learning process.

The lecturer is also viewed as a coach who teaches problem-solving skills and new techniques, and helps students to negotiate pro-social mathematics disciplines as they live and work with their peers. The guidance approach is categorised into two categories, namely direct instructional and minimal instructional guidance. Kirschner, Sweller & Clark, (2006:75) state that direct instructional guidance takes place when a lecturer provides the SCK that explains concepts and procedures in-depth to students, which students need to learn and understand. Direct instructional guidance is presented in a similar way with the teaching pedagogies lecture, presentation, demonstration and drills, and practice. Furthermore, minimal instructional guidance occurs when students discover crucial information on their own, rather than being fed with information by their lecturers. Therefore, the following techniques, discovery learning, problem-based learning, inquiry learning, experiential learning, and also constructivist learning are classified under the minimal instructional guidance (Kirschner *et al.*, 2006:75). This TVET college expects lecturers to use demonstration and other methods to teach SCK for students to understand the lesson well (cf. Annexure P). Lecturers also provide enough support for students. The co-researchers agreed with Ms Khodu in the research meetings.

Ms Khodu: *“Lecturers should be guide students in helping them to solve mathematics problems because some of students they do not have any clue about some mathematics topics, they do not know where to start. It can happen at beginning of the lesson before they solve mathematics problems.”*

Mr Koena *“I agree with my colleagues, guidance is so important, but lecturers should give students guidance while they are working on a problem in the form of scaffolding approach.”*

Thereafter, we as a team planned on how to use guidance approach to teach the MLP topic during this study research meetings. Ms Moleko executed the plan in the form of lesson presentations (cf. 4.3.1).

The guidance technique was also realised in the same lesson presentations of Ms Moleko. Look at the below extracts from full lesson presentations in Section 4.3.1:

Ms Moleko: *“Today we are going to do MLP topic, as I said last week.”*

Thereafter, Ms Moleko searched students MLP background knowledge, because it is a prerequisite in MLP NCV L4, realising that most of the students had a deficiency in MLP and its relevancy to our daily lives. Look at the following statements:

Poho: “As the group, we think that MLP is just a mathematics topic and we do not see any importance of it in our lives.”

Ms Moleko: “Thanks to all groups for your presentations, you are correct; MLP is one mathematics topic and is defined as the Mathematical model tool helping individuals to make informed decisions in public and private sectors, namely, the business services, business production, industrial engineering, social, physical sciences as well as different environments. MLP is used in these sectors to maximise profit or minimise costs with limited resources available. So, the importance of MLP is that it can help businesses and people to make informed decisions using constraints resources to generate profit or minimise costs. I know you do not understand what I am talking about but practical examples will help you.

So, before we go deeper into this topic, these are MLP components that we have to treat. (while writing) these are also learning outcomes (refer to Figure 4.14).

Introducing MLP, we first have to deal with MLP jargon; then move to the conversion of MLP word problems to linear inequalities; thereafter solve linear inequalities and draw an MLP model; lastly analyse and interpret the MLP model. I, together with my team, have developed the following MLP terminologies which can help us to unlock the MLP word problems . However, these terminologies are not enough and please do not rely only on these terminologies (refer to Table 4.1).

In your own groups complete this table by defining these words, create practical examples and then, write Mathematical signs representing those words.”

Students started to work in their respective groups as team, with Ms Moleko and her assistant lecturers facilitating. Thereafter, Ms Moleko asked students to exchange their groupwork to mark. Ms Moleko started to mark with students and realised that students did not perform well (cf. Annexure L). This confirmed that students struggle and have limited knowledge of MLP language and terminologies. Ms Moleko aided them look,

Ms Moleko: “All right, let’s do it together, firstly, let’s start with the word ‘less than’, you agreed that less than means lower than. Yes, I agree with you but this is a synonym of the word less than. So, let’s start here, what this word ‘less than’ mean in your mother tongue (language) like in Sesotho, isiZulu, isiXhosa and other indigenous languages. I see a hand, Mampho?”

Mampho: “Less than in Sesotho means e nyane ho.”

Khaya: “Less than in isiXhosa means incinane ku.”

Zinhle: “Less than in isiZulu means ngaphantsi kwa.”

Lebogang: “Less than in Sepedi means e nyane go.”

Ms Moleko: “Ok, thanks, all of you. So, now I know that all of us understand well the word ‘less than’, since we used our indigenous languages to explain this word. Less than refer to smaller number or size especially when comparing two or more than two things. In mathematics context it occurs when we compare two or more things. So, the synonyms of “less than” is lower than, below; under; smaller; shorter than; beneath; fewer than etc ... again, I agree with you guys, to say Mathematically we use this sign ‘ $<$ ’ to explain the word less than Mathematically. So, let’s look at this example,

Mr Jacobs earns R2 000 per week while Ms Pretty receives R3 000 per week in their new job.

So, from this statement I understand that Mr Jacobs wage is **less than** ($<$) what Ms Pretty receives on weekly basis as wage. Thus, to present this example mathematically, we can let:

Mr Jacob wage be x (or we can use any alphabet)

Ms Pretty wage be y (or we can use any alphabet)

$$\begin{array}{l} x < y \\ R2000 < R3000 \end{array}$$

(The number that is small comes before the less than sign, then the bigger number will follow the sign.)

Ms Moleko further unpacked MLP terminologies (lexicons) in the same fashion and completed Table 4.2 (cf. Table 4.2 in section 4.3.1.1).

In the next lesson, Ms Moleko introduced and taught MLP word problems and linear inequalities.

Table 4.10: MLP word problem (taken from classroom)

Scenario1

“Mr Tshabalala produces at least 500 muffins per day with his limited resources.

Ms Moleko: “...So, to give proper background information, Mr Tshabalala bakes muffins at home with his resources and he can bake 500 and more muffins.

A number of muffins to be produced is at least 500 and more OR a number of muffins to be bake cannot be less than 500. So, Mathematically at least is represented by this $\geq$ sign:

Firstly we let the number of muffins to be produced be represented by x (it can be any alphabet, hence Wilson’s group used alphabet a)

$$x \geq 500$$

Students were allowed to ask questions where they did not understand and Ms Moleko used another scenario with one mathematical variable (activities) to demonstrate how to solve them; therefore the students understood this part of the lesson well.

Ms Moleko: “OK, let’s move to the next part of this lesson for mathematical word problems of **two variables**. So, in your groups ... solve this mathematical problem (**refer to Table 4.5**).”

Ms Moleko, together with the assistant lecturers, marked and gave students feedback. The feedback was not good because most of students were unable to get it right (cf. Annexure N). Thereafter, Ms Moleko guided them how to solve it. Subsequently, Ms Moleko went further to introduced Activity 1 below,

Ms Moleko: “Let’s do this activity 1 together.”

Activity 1

A farmer has 80 hectares of his farm available for planting maize and cabbage. He must grow at least 10 hectares of maize and 20 hectares of cabbage to meet demands. He prefers to plant more cabbage than maize, but his workforce and equipment will only allow him to cultivate a maximum of three times the quantity of cabbages to that of maize.



Questions:

- 1.1 Represent the information above as a system of inequalities.
- 1.2 Sketch a graph of these inequalities.
- 1.3 If the profit on maize is R800 per ha and on cabbages R500 per ha, how should the farmer plant the two crops to make a maximum profit and what is this profit.

Figure 4.43: Activity 1 (taken from classroom)

Ms Moleko: “We are going to integrate MLP components to find MLP model that determine a feasible area or region for profit maximisation or cost minimisation. Question 1.1, indicates that we should represent this MLP word problem as a system of linear inequalities. So, converting MLP word problems into linear inequalities, we first read MLP word problems twice with understanding and go through questions. Secondly, we read MLP word problems for the third time going line by line in order to understand, identify and underline MLP terminologies. Thirdly, we identify products that a company is manufacturing or selling. Lastly, we identify resources constraints and identify MLP terminologies.”

Let's go back to the above activity 1, Teboho, read for us and interpret or say what you understand in that activity.”

Teboho read and interpreted the activity, and other students were also given an opportunity to share to us what they understood on the above Activity 1. Ms Moleko, further unpacked this activity so that students understood better.

Ms Moleko: “This activity 1 talks about a farmer who has only 80 hectares on his piece of land and he usually plant cabbage and maize for his business. So, we should first find and write variables which can be any alphabet (a-z) that represents these plants to be planted (i.e. maize and cabbage).

We let:

number of maize to be planted be represented by variable “a”

number of cabbage to be planted be represented by variable “b”

NB: (a for x axes and b for y axes)

We should start to show these basic linear inequalities $a \geq 0$ and $b \geq 0$. For instance, $a \geq 0$ means if the farmer plant maize more than 0 hectares of maize will be harvested however, if the farmer do not plant maize, then 0 hectares of maize will be harvested. Therefore, this is the same with $b \geq 0$ because $b \geq 0$ means if the farmer plant cabbage more than 0 hectares of cabbage will be harvested but, if the farmer do not plant cabbage, then 0 hectares of cabbage will be harvested.

From this above activity1, it is clear that each sentence has the meaning, so we have go through this activity1 read line by line ... **the first sentence/line from the above Activity 1, which says:**

A farmer has 80 hectares of his farm available for planting maize and cabbages.

What do you understand in this statement?”

Wilson: “I think it means that this farmer has maximum of 80 hectares (size of the piece of land) to plant maize and cabbages but planting both products cannot exceed 80 hectares because it is only land available him.”

So, number hectares of maize to be planted **plus** number hectares of cabbages to be planted **can be planted on 80 or less than hectares** because of limited piece of land.

NB, number hectares of maize to be planted is represented by variable “a” while number hectares of cabbage to be planted is represented by variable “b”, so, **mathematically** we can write...

$$a + b \leq 80$$

Ms Moleko: “Thanks, Wilson, I agree 100%with you . So, without any waste of time let’s move to **the second sentence/ line from the above activity 1 which says:**

He must grow **at least** 10 hectares of maize and 20 hectares of cabbages to meet demands.

Therefore we have two sentences here, since it says he must grow at least 10 hectares of maize and he must also grow at least 20 hectares of cabbage to meet the demands. Moreover, the word **and** in this line was used to join two sentences. **Then, the first part of this second sentence says...**

He must grow at least 10 hectares of maize to meet the demands.

This means this farmer uses at least (minimum/not less than) 10 hectares to plant maize.

...number hectares of maize to be planted on that available land is at least 10 hectares, thus Mathematically we write...

$a \geq 10$, since number hectares of maize to be planted is represented by variable a ...

The second part of this statement, says:

He must grow at least 20 hectares of cabbages to meet demands.

This means this farmer uses **at least (minimum/not less than) 20 hectares** to plant cabbage.

...number hectares of cabbage to be planted on that available land is at least 20 hectares, thus mathematically we write...

$b \geq 20$, since number hectares of cabbage to be planted is represented by variable b ...

Third sentence/ line from the above activity 1, says:

He **prefers to plant more cabbages than maize** but his work force and **equipment will only allow him** to cultivate a **maximum of three times** the quantity of **cabbages to that of maize**.

This means the farmer usually plants more cabbage than maize because of limited resources, limited workforce and limited piece of land, so this farmer cultivate (encourage) a maximum of three times quantity of cabbages to that of maize. So, number hectares of cabbage are at most three number hectares of maize. Mathematically, we say:

$$b \leq 3a$$

OK, all right. Now we have the following linear inequalities:

1. $a \geq 0$
2. $b \geq 0$
3. $a + b \leq 80$
4. $a \geq 10$
5. $b \geq 20$
6. $b \leq 3a$

So, tomorrow we are going to continue from this point to answer question 1.2 and 1.3 on this activity 1. Please at home ensure that you do what we did in the previous lesson."

In the third lesson Ms Moleko presented the lesson on linear inequalities and graphs.

Ms Moleko (starting the lesson): "On NCV level 2 we did linear inequalities which paved way for MLP topic on level 3, since to solve MLP topic we had to solve the linear inequalities, draw MLP graph, show feasible region and lastly interpreted MLP graph to find a maximum profit or minimum costs. So, I think at this level or stage you won't have more problems to solve linear inequalities, to draw MLP graph, to show feasible region (area) and determine a maximise profit from limited resources. Again, as you have said, in the previous sessions earlier on, we have dealt with MLP terminologies, word problems with one variable, word problems two variables, linear inequalities and its' graphs.

Let's now solve these linear inequalities to find, plot and join coordinates on Cartesian plane (draw MLP graph). Thereafter, determine feasible area and find maximum profit, so, as we have done earlier on, these are steps to solve linear inequalities:

Step 1: get deeper meaning of the linear inequalities,

Step 2: change linear inequalities to linear equations for the $< or >$ OR for $\leq or \geq$ focus on = but keep on hold $< or >$ for finding feasible region or area;

Step 3: solve each linear equation at one time

Step 4: apply intercepts, and do coordinates.

Let's go back the linear inequalities on above, namely:

1. $a \geq 0$ **linear inequality 1**
2. $b \geq 0$ **linear inequality 2**
3. $a + b \leq 80$ **linear inequality 3**
4. $a \geq 10$ **linear inequality 4**
5. $b \geq 20$ **linear inequality 5**
6. $b \leq 3a$ **linear inequality 6**

Let's apply above steps to these linear inequalities:

Step1: getting deeper understanding of these linear inequalities

Linear inequality 1: $a \geq 0$

= **a** represents number of maize to be planted but should be greater or equal to 0 hectares. If the farmer plants nothing then zero number of maize hectares will be harvested. Again, if the farmer plants more than zero units of maize, then he/she will harvest more than zero units of maize (cf. Figure 4.45).

Linear inequality 2: $b \geq 0$

= **a** represents number of cabbages to be planted but should be greater or equal to 0 hectares. If farmer plants nothing, then zero number of cabbages hectares will be planted. Again, if a farmer plants more than zero number of cabbages, then he/she will harvest more than zero units of cabbages (cf. Figure 4.45).

Linear inequality number 3: $a + b \leq 80$

= **a** represents number of maize to be planted, **b** represents number of cabbages to be planted but both hectares of maize and cabbages should be less or equal to 80. Thus, it means this farmer has only 80 hectares of land to plant both maize and cabbage (cf. Figure 4.46).

Linear inequality number 4: $a \geq 10$

= **a** represent number of maize to be planted, however should be greater or equal to 10 hectares of maize and it means this farmer is able to plant 10 or more hectares based on his ability (cf. Figure 4.47).

Linear inequality number 5: $b \geq 20$

= **b** represent number of cabbages to be planted but should be greater or equal to 20 hectares of cabbages and it means this farmer is able to plant 20 or more hectares based on his ability (cf. Figure 4.48).

Linear inequality number 6: $b \leq 3a$

= **b** represent number of cabbage to be planted, **a** represent number of maize. The linear inequality $b \leq 3a$ means planting number of cabbages **b** less or equal to three times planting number of maize **a**. This also means this farmer plants maximum of cabbages, that is three times planting maize (cf. Figure 4.49).

Step 2: focus on equal sign = for $\leq$ or $\geq$ but keep on hold $<$ or $>$ for determining the feasible region or area.

1. $a \geq 0$ means $a = 0$ or $a > 0$
2. $b \geq 0$ means $b = 0$ or $b > 0$
3. $a + b \leq 80$ means $a + b = 80$ or $a + b < 80$
4. $a \geq 10$ means $a = 10$ or $a > 10$
5. $b \geq 20$ means $b = 10$ or $b > 20$
6. $b \leq 3a$ means $b = 3a$ or $b < 3a$

Step 3: solve each equation at one time, apply intercepts, and find coordinates.

Linear equation 1: $a = 0$

If the straight line equation is $a = r$ (**r as any number**), it will result to vertical line on **ab Cartesian plane** but in this regard where $a = 0$, then **a** is zero on the **b axis**

Linear equation 2: $b = 0$

If the straight line equation is $b = r$ (**r as any number**), it will result in a horizontal line on **ab Cartesian plane**, but in this regard where $b = 0$, then **b** is zero on the **a axis**

Linear equation 3: $a + b = 80$

for **b** intercept: **a = 0**

sub **a** with 0 in the below equation

$$a + b = 80$$

$$(0) + b = 80$$

$$b = 80$$

Therefore, **a = 0** and **b = 80**

Coordinates will be **(a, b) = A(0, 80)**

(This means if this farm does not plant maize, then it can plant 80 hectares of cabbage)

for **a** intercept: **b = 0**

sub **b** with 0 in the below equation

$$a + b = 80$$

$$a + (0) = 80$$

$$a = 80$$

Therefore, **a = 80** and **b = 0**

Coordinates will be **(a, b) = B(80, 0)**

(This means if this farm does not plant cabbage, then it can plant 80 hectares of maize because this farmer has only 80 hectares)

Therefore, this equation has two variables, and then the line will cut both variables.

Linear equation 4: $a = 10$

If **a = 10, b = 0**, then **a = 10** for every number of **b** on the **ab** Cartesian plane and this will be vertical line.

(This means this firm is able to plant 10 hectares of maize)

Coordinates will be **(a, b) = C(10, 0)**

Therefore, because this equation has one variable, this straight line will only cut one variable vertically (where **a = 10**).

Linear equation 5: $y = 20$

If **b = 20, a = 0**, then **b = 20** for every number of **a** and this will be a horizontal line

(This means this firm is able to plant 20 hectares of cabbage)

Coordinates will be **(a, b) = D(0, 20)**

Therefore, because this equation has one variable, this straight line will only cut one variable horizontally (where **b = 20**)

Linear equation 6: $b = 3a$

(This symmetry line was derived from **b = a** meaning all values of **a** and **b** are equal, for example if **a = 1**, then **b = 1**; if **a = 2**, then **b = 2**. However, for **b = 3a**, it means value of **b** is three times value of **a**. Then, for **b = 3a** the straight line will be steeper. We can use table form to plot this line.

a	-50	-40	-30	-20	-10	0	10	20	30	40	50	60	70	80
b	-150	-120	-90	-60	-30	0	30	60	90	120	150	180	210	240

However, earlier on we agreed that for this activity¹ we use non-negative numbers because of $a \geq 0$ ($a = 0$ and $a > 0$) and $b \geq 0$ ($a = 0$ and $a > 0$). This means if we do not plant food, there will be no food planted/harvested, thus this will be represented by ($a = 0$ and $b = 0$). Again, if we planted food there will a number of planted/harvested food and this will be represented by ($a > 0$ and $b > 0$). So, to present $y = 3a$ on the Cartesian plane under these restriction $a \geq 0$ and $b \geq 0$, we will only show this $b = 3a$ on the first quadrant (non-negative numbers) with these coordinates for **E, E1 (0,0), E2 (10,30), E3 (20,60), E4 (30,90), E5 (40,120), E6 (50,150), E7 (60,180), E8 (70,210), and E9 (80,240)**.

Ms Moleko unpacked linear inequalities, continuously engaged students and said,

“Good, now we move to the next MLP component which is sketching the MLP graph. So, to draw MLP graph we need to find coordinates first, then plot and join them. Thereafter, use the inequality signs to find the feasible region. Let’s continue the previous lesson and we have already found the following coordinates for each linear inequality:

$a \geq 0$ coordinate A (0, 0) and with greater or equal to sign

$b \geq 0$ coordinate B (0, 0) and with greater or equal to sign

$a + b \leq 80$ coordinates C (80, 0) and D (0, 80) and with less or equal to sign

$a \geq 10$ coordinate E (10,0) and with greater or equal to sign

$b \geq 20$ coordinate F (0, 20) and with greater or equal to sign

$b \leq 3a$ coordinates G1 (0, 0), G2 (10, 30), G3 (20, 60), G4 (30, 90), G5 (40, 120), G6 (50, 150), G7 (60, 180), G8 (70, 210), G9 (80, 240) and with less or equal to signs

Ms Moleko: “In your groups, plot these coordinates, join coordinates and write linear inequalities on that joined line ... however, use these graph papers.”

Students drew the linear inequalities graphs and Ms Moleko allowed them to mark it themselves in order for them to realise their mistakes easier.

Ms Moleko: “Let’s start, how much must be our scale for this graph?”

Tholoane: “Our scale must be a factor of 10 for both **a-axis** and **b-axis** because the linear inequalities have the following numbers 0,3,10, 20 and 80. So, these coordinates fit well in this scale.”

Ms Moleko: “Is there anyone with different scale?”

Students: “No, even us we did chose 10 as a graph scale factor.”

Ms Moleko: “All right thanks Tholoane, I agree with you, so please ensure that if you choose 10 as a scale factor for your graph should be both on **a-** axes and **b-** axes and also ensure that the measurement between the spaces on a **ab Cartesian plane** are equal at all times. More importantly, as you have said, Tholoane, most numbers in these linear inequalities and its coordinates are covered with this scale factor (**ab Cartesian plane**).

Ms Moleko showed **ab Cartesian plane** on the first quadrant with the scale decided by students’ (cf. Figure 4.44).

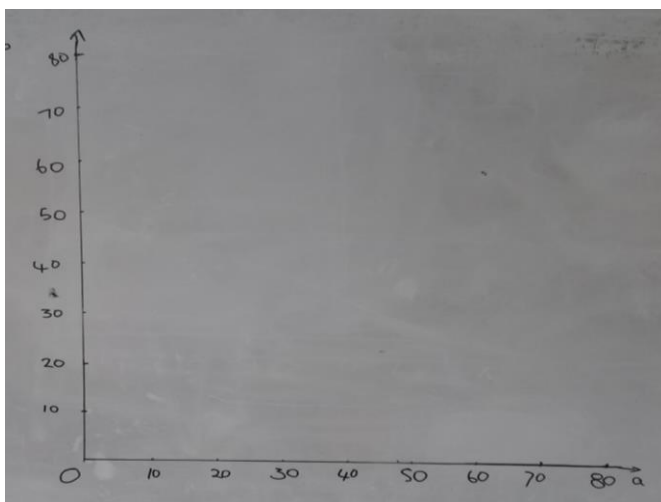


Figure 4.44: Image of **ab Cartesian plane** by Ms Moleko with 10 multiple factor

Ms Moleko “Let’s move on, to show $a \geq 0$ ($a = 0$ or $a > 0$) on **ab Cartesian plane** at point A (0, 0), then, we look where $a = 0$ and $b = 0$ which is point of origin. So, a is greater or equal to 0 on **a intercept** where $b = 0$ but starting from 0 to **positive a infinite**. We can use number line concept to show this $a \geq 0$ on **ab Cartesian plane**, so we put close point at the point of origin, put arrow direction to positive a infinite(right hand side **ab Cartesian plane**) on **a intercept** (horizontal line) to satisfy $a \geq 0$.

Ms Moleko showed $a \geq 0$ on **ab Cartesian plane** and used the blue whiteboard marker to highlight arrow direction and identify the line by $a \geq 0$ in Figure 4.45.

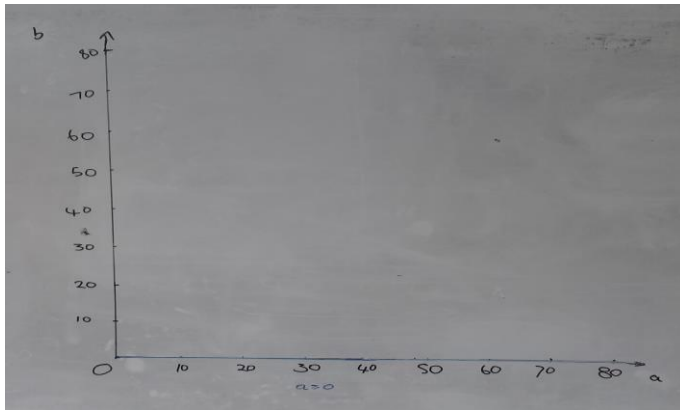


Figure 4.45: Image of $a \geq 0$ on **ab Cartesian plane** by Ms Moleko.

Ms Moleko: “Is there any question?”

Students: “No.”

Ms Moleko: “All right, let’s move on, to show $b \geq 0$ on the **ab-Cartesian plane**.”

Ms Moleko: “With the second linear inequality $b \geq 0$ ($b = 0$ or $b > 0$) on **ab Cartesian plane** at point B (0,0), then, we look where $a = 0$ and $b = 0$ which is point of origin. So, b is greater or equal to 0 on **b intercept** (vertical line) where $a = 0$, however starting from 0 to positive b infinite. We can again also use number line concept to show $b \geq 0$ on **ab Cartesian plane**, so we put close point at the point of the origin and put arrow direction on the vertical line to positive b infinite(upwards) on **b intercept** to satisfy $b \geq 0$.

Ms Moleko indicated $b \geq 0$ on **ab Cartesian plane** and used a green whiteboard marker to highlight the arrow direction (cf. Figure 4.45 below).

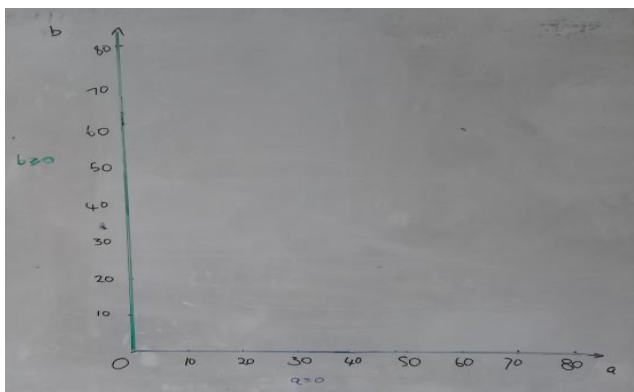


Figure 4.45: Image of $a \geq 0$ and $b \geq 0$ on **ab Cartesian plane** by Ms Moleko

Ms Moleko: “Showing $a + b \leq 80$ on ab Cartesian plane with two coordinates $C(80, 0)$ and $D(0, 80)$, we first plot these coordinates on ab Cartesian plane and also join them. This will result to the straight line with negative slope ($b \leq -a + 80$). So, to satisfy the inequality sign of less or equal to ($\leq$) we make bold a straight line and shade the area under that bolded straight line, because this farmer can plant between 0 to 80 hectares of both cabbages and maize but not more than 80 hectares; so, in this regard, we shade the area with vertical lines in the triangle shape.”

Ms Moleko interacted with students and helped them to understand more how to draw the linear inequality where $a + b \leq 80$. She used the black whiteboard marker to draw $a + b \leq 80$ (cf. Figure 4.46 below).

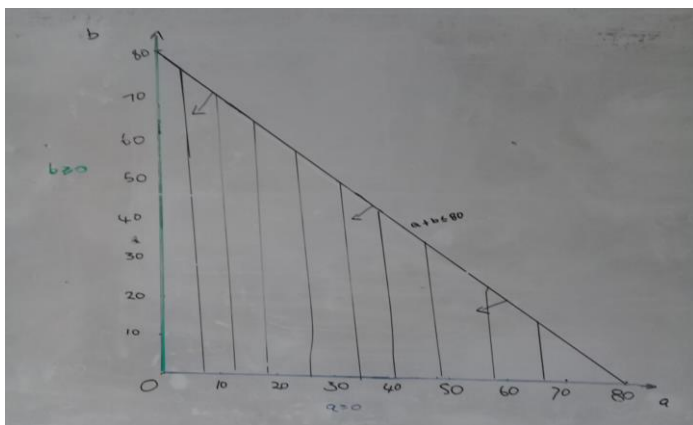


Figure 4.46: Image of $a \geq 0, b \geq 0$, and $a + b \leq 80$ on ab Cartesian plane by Ms Moleko

Ms Moleko: “Another linear inequality to show is $a \geq 10$, so, as we have said earlier on about linear equation with one mathematics variable. The linear equation with one mathematics variable cuts only one intercept vertically or horizontally. This linear inequality where $a \geq 10$ can be written as $a = 10$ and $a > 10$. So, $a = 10$ cuts a axis at number 10 and we are going to use red ink whiteboard marker to show this vertical line. Thus, to satisfy the inequality sign of greater or equal to ($\geq$) we make a bold vertical line with red ink and shade the area where $a > 10$ with dotted red ink whiteboard marker to positive a infinite number, since the farmer is supposed to plant 10 and more hectares of maize to meet the customers’ demands; so, in this regard we shade the area with dots of red ink.”

Ms Moleko interacted with students and helped them to understand how to draw the linear inequality where $a \geq 10$ and used the red whiteboard marker to show $a \geq 10$. All students understood this lesson presentation, since Ms Moleko used different whiteboard markers to show the effect of inequality signs (cf. Figure 4.47 below).

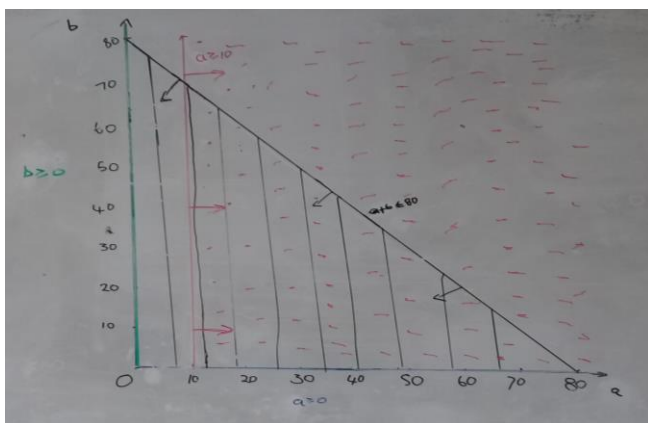


Figure 4.47: Image of $a \geq 0, b \geq 0, a + b \leq 80$, and $a \geq 10$ on ab Cartesian plane by Ms Moleko

Ms Moleko: “Another linear inequality to show is $b \geq 20$, and as we have said earlier on about linear equation with one mathematics variable. The linear equation with one mathematics variable cuts only one intercept vertically or horizontally. This linear inequality where $b \geq 20$ can be written as $b = 20$ and $b > 20$. So, $b = 20$ cuts b axis at number 20 and we are going to use a blue ink whiteboard marker to show this horizontal line. Thus, to satisfy the inequality sign of greater or equal to ($\geq$) we make a bold horizontal line with blue ink and shade the area where $b > 20$ with blue ink arrow directions going up to positive b infinite number, since this farmer is supposed to plant 20 and more hectares of cabbages to meet customers’ demands, so, in this regard we shade the area with blue ink of arrow direction facing up.”

Ms Moleko further engaged students about graphing $b \geq 20$ on ab Cartesian plane and used a blue ink whiteboard maker to illustrate the effect of $b \geq 20$. The lines are identified by each straight line with their linear inequality and arrow direction (cf. Figure 4.48).

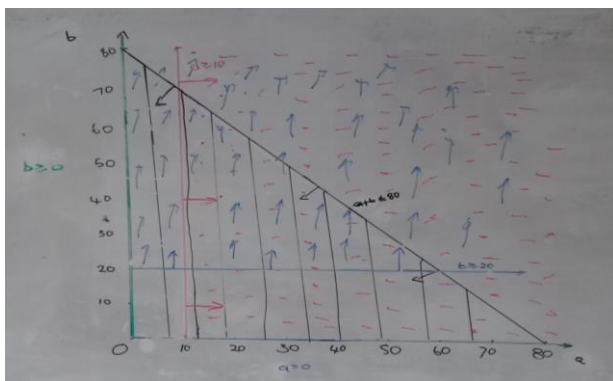


Figure 4.48: Image of $b \geq 20$ on the same Fig 4.47 and was demonstrated by Ms Moleko

Ms Moleko: “Lastly, $b \leq 3a$ can be written as $b < 3a$ or $b = 3a$, then drawing this linear equation or symmetric line where $b = 3a$. Now we plot its coordinates we got earlier on, E1 (0, 0), E2 (10, 30), E3 (20, 60), E4 (30, 90), E5 (40, 120), E6 (50, 150), E7 (60, 180), E8 (70, 210) and E9 (80, 240). Then, we join them and make a bold line with a black ink whiteboard marker to satisfy its inequality sign ($\leq$), then to satisfy and illustrate $b < 3a$ on ab Cartesian plane, we know that b will be less than $3a$ when this bold straight line $b \leq 3a$ is shown on the ab Cartesian plane move (shifts) to positive a infinite number of on the a axis, so we have to move or shift this bold straight line ($b \leq 3a$) by doing black ink marks (horizontal straight small arrows) to our right-hand side.”

Ms Moleko illustrated this linear inequality $b \leq 3a$, working together with her students, and they seemed understand this MLP component (cf. Figure 4.49).

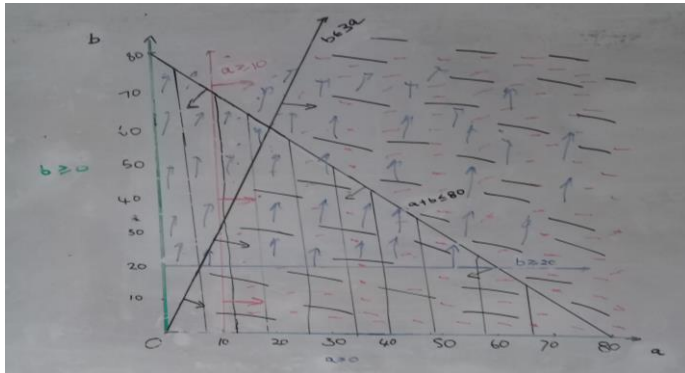


Figure 4.49: Image of $b \leq 3a$ on the same Fig 4.48 and was demonstrated by Ms Moleko

Ms Moleko: “All right, now we are done illustrating these linear inequalities $a \geq 0$, $b \geq 0$, $a + b \leq 80$, $a \geq 10$, $b \geq 20$ and $b \leq 3a$ on the **ab Cartesian plane**. Questions please?”

All right, let’s move forward, as we presented these linear inequalities $a \geq 0$, $b \geq 0$, $a + b \leq 80$, $a \geq 10$, $b \geq 20$ and $b \leq 3a$ on the **ab Cartesian plane** above in Figure 4.49. We should find the feasible area (region). How can we find the feasible region or area?”

Gambu: “After we have presented and drawn the linear inequalities and also shown the direction of inequality signs, we have to find the area where all arrow directions meet, then shade it so that shaded area is a feasible area.”

Wilson: “I think to find a feasible area from highlighted graph of linear inequalities as we have done in Figure 4.49. We have to check and find the area where all colours are found together, then start to erase colours where colours are not found together.”

Morena (student) were asked to come forward to show how to find a feasible region and he managed to do so (cf. Figure 4.50).

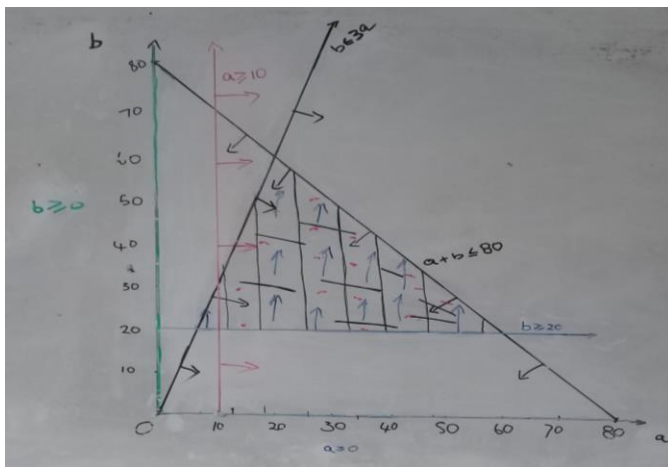


Figure 4.50: MLP graph with feasible region (area) showed by Morena

Ms Moleko: “All right, thank you, Morena. Have you all found this area?”

Students: “Yes, ma’am.”

Ms Moleko: “So, what do we call this area and why is it important to find it?”

Gambu: “It is called a feasible area (region) and it is used to determine a maximum profit or a minimum costs in order to use limited resources efficiently. From this shaded area we have to find the points first that can assist us to find max profit or min costs.”

Ms Moleko realised that the students understood this part, and then allowed them to find and identify feasible region points and coordinates. They found FRP1, FRP2, FRP3, and FRP4 as (20, 60); (60, 20); (10, 20) and (10, 30), respectively (cf. Figure 4.51). Ms Moleko, together with students, verified and then got the same coordinates.

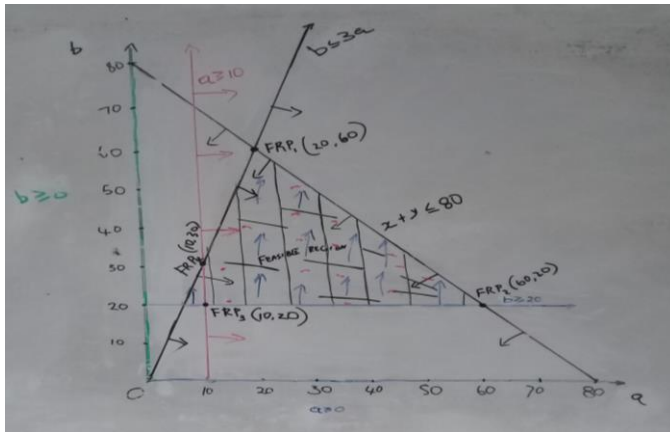


Figure 4.51: MLP graph with feasible area and its coordinates showed by Gambu

Ms Moleko: “Now we have answered question 1.1 and 1.2 so far, which helped us to draw this graph (cf. Fig. 4.51). Thus, with this graph (cf. Fig. 4.51) we are in a position to answer the last question, 1.3, on the above activity 1. Question 1.3 requires of us to find objective function (profit formulae) or given objective function in order to find the business maximise profit or minimum costs.”

(Question 1.3): “If the profit on maize is R800 per ha and on cabbages R500 per ha, how should the farmer plant the two crops to make maximum profit and what is this profit?” and interpreted “this question indicates that this business makes profit if the farmer charge R800 for each harvest of maize and R500 for each harvest of cabbage (a represent hectares of maize, b represent hectare of cabbage). Therefore, this can be written as **Profit = R800 per ha of maize + R500 per ha of cabbage** ($P = R\ 800\ a + R\ 500\ b$)

Then, we call this equation an objective function. How should the farmer plant two crops to maximise profit and what is the profit?

Since we have this objective function; do we end there?”

Zulu: “No Mme, we use this objective function ‘formula’ to find maximum profit, same as we did in NCV L3 last year.”

Ms Moleko: “All right, ok, in your groups find for me a maximum profit.”

Students in their groups determined the maximum profit by substituting FRP1, FRP2, FRP3 and FRP4 into this objective function. All groups found the same answer and Ms Moleko asked Mike to show how they arrived at their solutions (cf. Figure 4.31).

Mike: “As a result, the highest amount is R58 000 here, so this means this farmer to make maximum profit, should plant 60 hectares of maize and 20 hectares of cabbage on available hectares because this farm has only 80 hectares of land.”

Ms Moleko: "That's wonderful, students; tell you what you are so good. So, is there any other method we can use to determine the point that can help us to reach the maximum profit or minimum cost?"

Students: "Yes, Mme, through the search line."

Lebohlang: "We prefer the substitution method to find maximum profit than using the search line."

Mike: "I agree with Lebo (student), because search line is difficult to apply for us."

Ms Moleko: "Using search line we take the objective function and make **b** subject of the formula, so look how we do it."

$$\begin{aligned}
 P &= 800a + 500b \\
 -800a + P &= 500b \\
 \frac{-800a}{500} + \frac{P}{500} &= \frac{500b}{500} \\
 -\frac{800a}{500} + \frac{P}{500} &= b \\
 \therefore b &= \left(\frac{-800a}{500} \right) + \frac{P}{500}
 \end{aligned}$$

Search line

Figure 4.52: Calculation of search line function presented by Ms Moleko

Ms Moleko: "Then, from the above equation we use fraction $\frac{800}{500}$ in finding the search line, so we simplify this fraction $\frac{800}{500}$ to the simplest form which is $\frac{8}{5}$ however, since our graph scale, which is multiple of 10. We can multiply $\frac{8}{5}$ both numerator and denominator by 10 and result to $\frac{80}{50}$ which best suit our scale. Moreover, in this fraction ($\frac{80}{50}$), 80 (numerator) will be **b** number on **b axis** while 50 will be **a** number on **a axis**, then we make points on the **ab Cartesian plane** and make it dotted line from where **b** is 80 and **a** is 50."

Ms Moleko explained the technique of finding the search line again and illustrated to them how to show the search line and move it up slowly by adding 10 to each value. Thereafter, the last remaining point of the feasible region is the one that are used to find maximum profit. She further used a T-Square ruler to move the search line to the right (Figure 4.53).

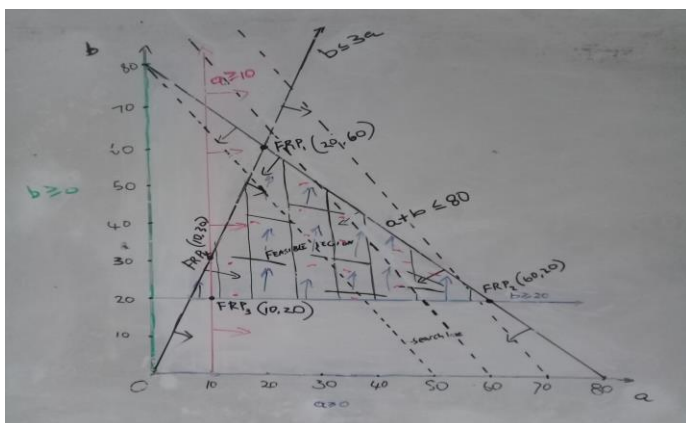


Figure 4.53: MLP graph with feasible region and search line drawn by Ms Moleko on the whiteboard

Ms Moleko: “Then, which point of this feasible region comes last when we move this search line?”

Students: “Point FRP2 with coordinates (60: 20).”

Ms Moleko: “All right, so, what do we do with this point?”

Sibongile (student): “I think when it’s here I can try. We start by substituting point FRP2 in this objective function where $P = 800a + 500b$ and determine maximum profit ... (showing it).”

$$P = 800(60) + 5(20) = R 58000$$

Figure 4. 54: Maximum profits determined by Sibongile

The above presentations showed that Ms Moleko used direct instructional guidance and minimal instructional guidance to enhance the teaching of MLP, which is in line with the guidance principle of the RME approach. Direct instructional guidance was realised.

Ms Moleko: “Today we are going to do MLP, as I have indicated last week. MLP is one of mathematics’ topics and is defined as the mathematical model tool helping individuals to make informed decisions in public and private sectors... make informed decisions using constraints resources to generate profit or minimise costs ... so, before we go deeper in this topic, these are the MLP components that we have to treat ... are also learning outcomes” (She wrote them on the whiteboard (cf. Figure. 4.14 in section 4.3.1.1).”

Ms Moleko: “We are going to integrate MLP components to find MLP model that determine a feasible area (region) for profit maximization or cost maximisation... question 1.1, indicates that we should represent this MLP word problem as system of linear inequalities. So, to convert MLP word problems into linear inequalities, we first read MLP word problem twice with understanding and go through questions. Secondly, we read MLP word problem for the third time going line by line in order to understand, identify and underline MLP terminologies. Thirdly, we identify products that a company is manufacturing or selling. Lastly, we identify resources constraints and identify MLP terminologies.”

She began the MLP lesson by probing students to define MLP and to explain its relevancy for our daily lives.

Ms Moleko: “in your groups, write the definition MLP and explain the relevance of MLP topic in our daily lives...in your own words ... (further introduced MPL terminology) ... in your own groups complete this table by defining these words, create practical examples and also write them Mathematically ... (then gradually introduced the word problems with one and two mathematics variables on one or two sentences) ... in your groups solve this problem.”

This required of students to convert these MLP word problems into linear inequalities. Look at these scenarios. The first and second scenarios, respectively say:

Mr Tshabalala produces at least 500 muffins per day with his limited resources. The second scenario,

Mr Tshabalala makes chocolate and strawberry flavours of muffins but he cannot make 750 of both chocolate and strawberry muffins per day.

Ms Moleko moved from one level to another, delighted with how the students responded. More importantly, throughout the above lesson presentation, she started each section of the MLP lesson by using questions and answers and/or activities to discuss a specific MLP concept, but she gave them 5-10 minutes to complete them (cf. Lesson presentations in 4.3.1). Therefore, throughout the lessons, Ms Moleko began each MLP part by asking questions. However, it did not end there, because the ontological stance of this study is that any assessment without feedback is worthless, which links well with what other scholars recommend in 2.4.2.5. As a result, in some instances, when students were done with their group discussion and/or solving a problem, Ms Moleko requested them to share their views and solutions.

Poho: “As the group, we think that MLP is just a mathematics topic and we do not see any importance of it in our lives.”

Wilson showed how the group arrived their solution (Figure 4.55).

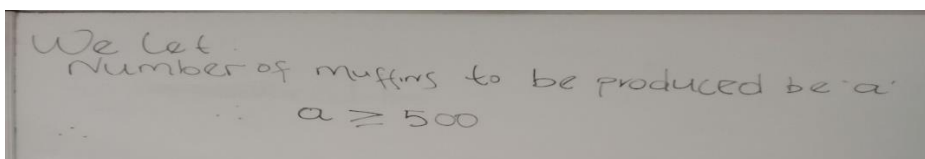


Figure 4.55: Wilson's solution for Scenario 1

This feedback aided Ms Moleko to detect students' MLP mistakes, misconceptions, errors and/or lack of MLP background. Thereafter, Ms Moleko was in a position to give guidance where necessary. This was evidenced by the following statement:

Ms Moleko: “Thanks to all groups for your presentations, you are correct MLP is one of mathematics topic and is defined as the Mathematical model tool helping individuals to make informed decisions in public and private sectors. So, the importance of MLP topic is that it can help businesses and people to make informed decisions using constraints resources to generate profit or minimise costs.”

Ms Moleko further added and brought a different view to solve this word problem:

Ms Moleko: “Mr Tshabalala produces at least 500 muffins per day with his limited resources. So, to give proper background information, Mr Tshabalala bakes muffins at home with his resources and he can bake 500 and more muffins. A number of muffins to be produced is at least 500 and more OR A number of muffins to be bake cannot be less than 500.

Mathematically it is represented by this sign $\geq$:

Firstly we let, number of muffins to be produced be represented by x (it can be any alphabet hence Wilson group used alphabet a)

$$x \geq 500$$

This shows that Ms Moleko used the guidance principle of the RME approach, because she allowed students to construct or discover MLP knowledge on their own, then started to give support where students have deficiencies.

Ms Moleko: “Our scale must be a factor of 10 for both **a -axis** and **b -axis** because the linear inequalities have these numbers 0,3,10, 20 and 80. So, these coordinates fit well in this scale.”

This is in line with the minimal instructional guidance, social constructivism, student-centred approach and also PBL, because these teaching pedagogies allow students to be active people in the teaching and learning in order to learn from their peers and discover knowledge on their own using their pre-existing knowledge (cf. 2.4.2.3 & 2.4.2.4). This is a discursive practice, because throughout the MLP lessons, Ms Moleko presented all lessons in this way. This promotes a social inclusion as well as PAR, because it provides students with the opportunity to work together as a team, learn from their peers and interact with their lecturers (cf. 3.2.1 & 3.5). This was also realised when Mampho (student) showed how to find a feasible area (Figure 4.56).

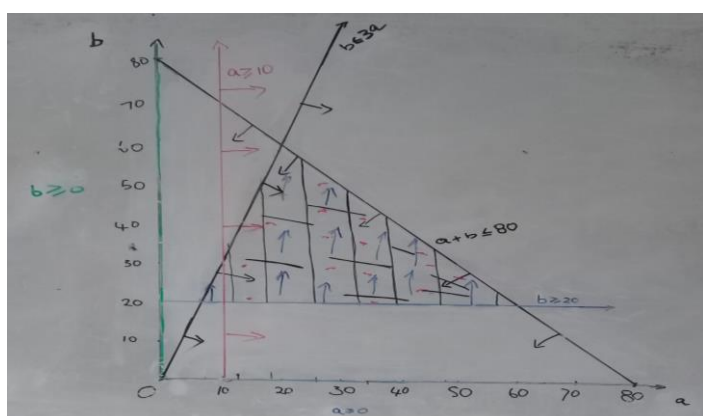


Figure 4.56: MLP graph with feasible region (area) showed by Mampho in the classroom

Discursive practice (pedagogies), which put students at the centre of the teaching and learning process was first considered effective in the blurred genres era (cf. 2.2.2.3).

Look at the following words and solutions of students, which prove that students are active in the classroom.

Gambu: *“It is called a feasible area (region) and it is used to determine a maximum profit or a minimum costs in order to use limited resources efficiently. From this shaded area we have to find the points first that can assist us to find max profit or min costs.”*

In addition, students were requested to find points of the feasible region which Gambu spoke about. Then Mike was asked to find the max or highest profit (cf. Figure 4.31).

Mike (student): *“... the highest amount is R58 000 here, so this means this farmer to make maximum profit, should plant 60 hectares of maize and 20 hectares of cabbage on available hectares because this farm has only 80 hectares of land.”*

However, Ms Moleko did not forget to fulfil her role as a lecturer because in the 21st century it is expected of a lecturer to show, demonstrate, facilitate, prompt, motivate, resource, inform, control, assess, organise, manage, and participate (Rindu & Ariyanti, 2017:94-99; Hinchliff, in Hayes and Marshall, 2016:52-56). For instance, after Ms Moleko had actualised students' background MLPCK at the beginning of each lesson of MLP components, she provided students with a deeper meaning of those MLP components. This shows that Ms Moleko fulfilled the roles: demonstrate, facilitate and resource. Look at the way she unpacked the MLP word problem in Activity 1,

Ms Moleko: *“This Activity 1 talks about a farmer who has only 80 hectares on his piece of land and he usually plant cabbage and maize for his business. So, we should first find and write variables which can be any **alphabet (a-z)** that represents these plants to be planted (i.e. maize and cabbage). We let:*

number of maize to be planted be represented by variable “**a**”

number of cabbage to be planted be represented by variable “**b**”

NB: (**a** for x axes and **b** for y axes)

We should start to show these basic linear inequalities $a \geq 0$ and $b \geq 0$. For instance, $a \geq 0$ means if the farmer plant maize, so more than 0 hectares of maize will be harvested. However, if the farmer does not plant maize, then 0 hectares of maize will be harvested. Therefore, this is the same with $b \geq 0$ because $b \geq 0$ means if the farmer plants cabbage, so more than 0 hectares of cabbage will be harvested. However, if the farmer does not plant cabbage, then 0 hectares of cabbage will be harvested...

*From this activity, it is clear that each sentence has the meaning, so we have go through this activity1 read line by line ... **the first sentence/ line from the above activity 1, which says:***

A farmer has 80 hectares of his farm available for planting maize and cabbages...

The second sentence/line from the above Activity 1 which says:

*He must grow **at least** 10 hectares of maize and 20 hectares of cabbages to meet demands.*

*Therefore we have two sentences here, since it says he must grow at least 10 hectares of maize and he must also grow at least 20 hectares of cabbage to meet the demands. Moreover, the word **and** in this line was used to join two sentences. **Then, the first part of this second sentence says...***

He must grow at least 10 hectares of maize to meet the demands.

➤ *This means this farmer uses **at least (minimum/ not less than) 10 hectares** to plant **maize**.*

... number hectares of maize to be planted on that available land is at least 10 hectares, thus Mathematically we write:

$a \geq 10$, since number hectares of maize to be planted is represented by variable a ...

Next, Ms Moleko analysed and interpreted Activity 1, and then showed students how to convert the MLP word problems into linear inequalities. From the above Activity1, Ms Moleko got these linear inequalities $a \geq 0$, $b \geq 0$, $a + b \leq 80$, $a \geq 10$, $b \geq 20$ and $b \leq 3a$. This extract shows that Ms Moleko believed in Conceptual Content Knowledge (CCK) pedagogy as the first level to teach MLP concepts before moving to Procedural Knowledge (PK) pedagogy in order for students get a deeper understanding of MLP word problems or concepts. This links well with the concrete and abstract teaching methods. Ms Moleko took this opportunity to show students that abstract linear inequalities are developed (constructed) from a particular situation (context), because some people have the perception that abstract linear inequalities do not have any meaning in our lives.

However, Ms Moleko helped the students to make sense of the MLP word problem in Activity 1 and further converted that particular word problem in Activity 1 into the following linear inequalities:

$a \geq 0$, $b \geq 0$, $a + b \leq 80$, $a \geq 10$, $b \geq 20$ and $b \leq 3a$

Ms Moleko told students that they could use any alphabet (**a to z**) as mathematics variables to present MLP word problems (or mathematics concepts) in the abstract level. This Ms Moleko did to help students understand that mathematics is not always about solving x & y as perceived prior to and during the traditional era and in the modernity phase (moments of Bricolage). This perception still exists today, because some people still think that mathematics is all about solving x & y , as we were taught, prior to and during the 20th century, when lecturers only used x & y as mathematics variables to represent some MLP word problem (or mathematics concepts)(cf. 2.2.2.1 & 2.2.2.2). The literature uncovered that lecturers teach mathematics exactly the way they have been taught at school. Ms Moleko enlightened students that there is a multiplicity of alphabets (**a to z**) to use as mathematics variables. This pedagogy of Ms

Moleko shows a discursive practice because throughout, Ms Moleko presented MLP lessons in this fashion, e.g.:

Ms Moleko: “Let’s move on, to show $a \geq 0$ ($a = 0$ or $a > 0$) on **ab Cartesian plane** at point A (0,0), then, we look where both $a = 0$ and $b = 0$ which is point of origin. Therefore, a is greater or equal to 0 on **a intercept** where $b = 0$ however starting from 0 to **positive a infinite**. We can use number line concept to show this $a \geq 0$ on **ab Cartesian plane**, so we put close point at the point of origin and put arrow direction to positive a infinite(right hand side **ab Cartesian plane**) on **a intercept** (horizontal line) to satisfy $a \geq 0$. (She showed $a \geq 0$ on **ab Cartesian plane** and used a blue whiteboard marker to highlight the arrow direction and identify the line by $a \geq 0$ on below)(Figure 4.57).

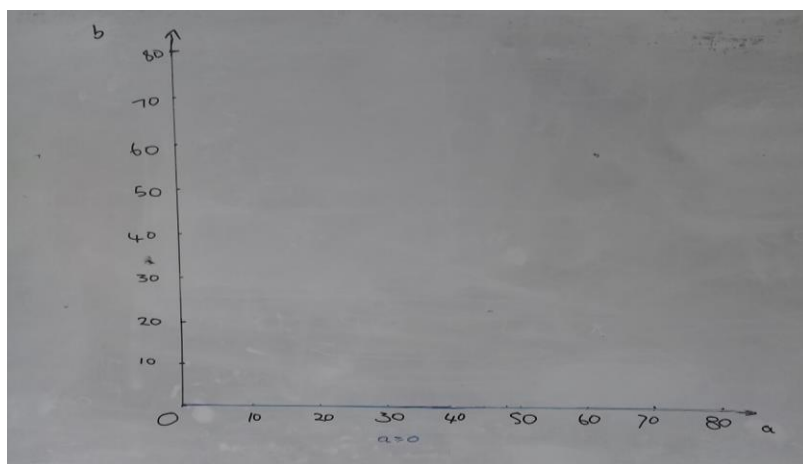


Figure 4.57: Image of Ms Moleko’s explanation: $a \geq 0$ on **ab Cartesian plane**

The statement,

“...to show $a \geq 0$ ($a = 0$ or $a > 0$) on **ab Cartesian plane** ... we look where $a = 0$ and $b = 0$ which is point of origin. So, a is greater or equal to 0 on **a intercept** where $b = 0$ but starting from 0 to **positive a infinite**. We can use number line concept to show this $a \geq 0$ on **ab Cartesian plane**,...”

confirms that there are many ways to draw linear inequalities on **ab Cartesian plane**. Ms Moleko to draw $a \geq 0$ ($a = 0$ and $a > 0$), told the students to look for a point on the Figure 4.57, where a & b is 0 (referring to the point of origin) in order to satisfy the linear equation $a = 0$. Then, to satisfy the linear inequality of $a > 0$ Ms Moleko indicated that a is greater or equal zero from the point of origin to **positive a infinite** and showed that effect on the whiteboard (cf. Figure 4.57). Furthermore, to further explain how to present $a \geq 0$, Ms Moleko used the number-line concept so that students could comprehend how to draw linear inequalities correctly. Ms Moleko did the same with the following linear inequalities:

$$b \geq 0, a + b \leq 80, a \geq 10, b \geq 20 \text{ and } b \leq 3a \text{ (cf. Figure 4.45–4.49).}$$

This shows that Ms Moleko function within the Bricolage blurred genres era where multiple teaching and learning strategies were encouraged to mathematise MLP (cf. 2.2.2.3). The literature in Section 2.4.2.3 also recommends numerous teaching methods to teach MLP, which saw the light during the Bricolage blurred genres.

This lesson also shows that Ms Moleko was guided by the principle of the RME approach (cf. 2.3.2) and direct instructional guidance as a component of guidance approach. This component assisted students to get a deeper meaning and understanding of MLP concepts. This proved that Ms Moleko fulfilled her role as a lecturer and demonstrator; however, this study ontology stance is that a lecturer cannot play these roles if s/he is not a mathematics expert. Ms Moleko, as a mathematics expert, provided students with steps to solve linear inequalities in the form of guidance.

Ms Moleko: “These are steps to solve linear inequalities. **Step1:** get deeper meaning of the linear inequalities; **Step 2:** change linear inequalities to linear equations for the $<$ or $>$ OR for $\leq$ or $\geq$ focus on = but keep on hold $<$ or $>$ for finding feasible region or area; **Step 3:** solve each linear equation at one time and also **Step 4:** apply intercepts, and do coordinates.”

She demonstrated how to apply those steps in solving linear inequalities ($a \geq 0$, $b \geq 0$, $a + b \leq 80$, $a \geq 10$, $b \geq 20$ and $b \leq 3a$). As a result she got the following coordinates from each linear equations:

A (0,0) for $a = 0$; B (0,0) for $b = 0$; C (80,0) & D (0,80) for $a + b = 80$; E (10,0) for $a = 10$; F (0,20) for $b = 20$; and G1 (0, 0), G2 (10,30), G3 (20,60), G4 (30,90), G5 (40,120), G6 (50,150), G7 (60,180), G8 (70,210), G9 (80,240) for $b = 3a$.

Next, Ms Moleko shared another procedure with her students:

Ms Moleko: “Now we move to the next MLP component which is sketching the MLP graph. So, to draw MLP graph we need to find coordinates first, then plot and join them. Let’s continue ... we have already found coordinates for each linear inequality.”

The students were then given the task to plot and join (draw) the above coordinates in their groups. They were provided with graph paper to do the graphs.

Ms Moleko: “In your groups, plot these coordinates, join coordinates and write linear inequalities on that joined line; use graph paper.”

Students were allocated 10 minutes to complete this; while Ms Moleko and the assistant lecturers facilitated. After the allocated time, Ms Moleko showed the students how to determine graph scale; plot and join coordinates; and illustrate directions that satisfy each linear inequality sign. She used multiple whiteboard markers to show the

linear inequalities (Figure 4.58). For example, for linear inequality ($b \geq 0$) she used a green whiteboard marker.

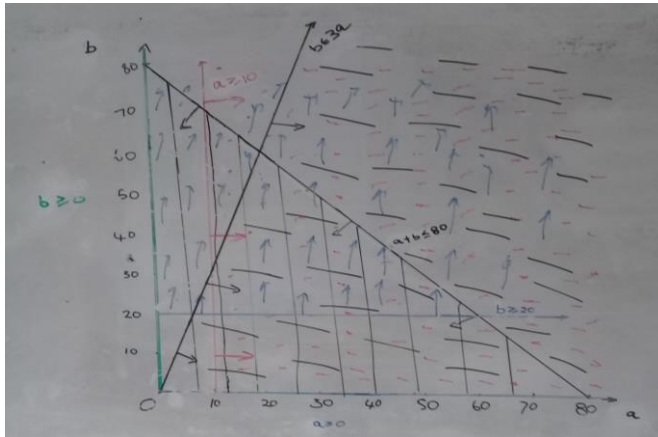


Figure 4.58: Image of all linear inequalities

This guidance assisted students to identify the feasible region, points, and coordinates (Figure 4.51) which helped Mike to find a maximum profit (Figure 4.59).

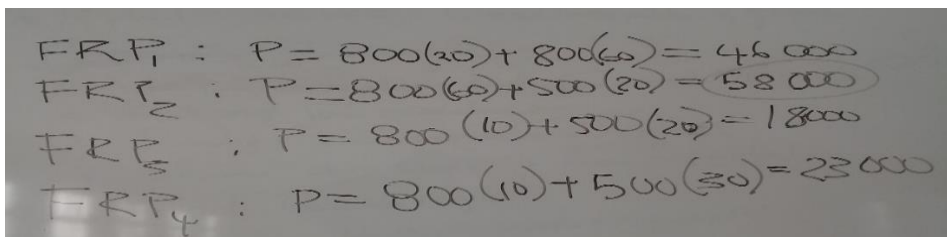


Figure 4.59: Maximum profits determined by Mike

Mike: “The highest amount is R58 000. This means this farmer to make maximum profit, should plant 60 hectares of maize and 20 hectares of cabbage on available hectares because this farm has only 80 hectares of land.”

Ms Moleko: “Wonderful, students; I tell you; you are so good.”

Ms Moleko understood that there is no single way to find a maximum or minimum point to determine maximum profit or minimum cost through MLP analysis and interpretation as a bricoleur. In Section 2.2 of this study it was revealed that bricoleurs understand that there are numerous ways, methodologies and strategies to solve any problem. This was realised when Ms Moleko said:

Ms Moleko: “Is there any other method we can use to determine the point that can help us to reach the maximum profit?”

Students: “Through search line.”

However, most of the students indicated that they had difficulty finding and using a search line to find maximum or minimum points.

Lebohang: "We prefer the substitution method to find maximum profit than using the search line."

Mike: "I agree with Lebohang, because search line is difficult to apply for us."

Ms Moleko did not immediately showed the students how to use a search line technique, but asked them to volunteer to show how to find and illustrate a search line on the MLP graph on the whiteboard. No-one came forward, which affirmed that students had a real challenge in using a search line technique, because it was the first time students did not volunteer. Ms Moleko took this opportunity to apply direct instructional approach.

Ms Moleko: Using search line we take the objective function ($P=800a+500b$) and make b subject of the formula, so look how we do it (showing it on the whiteboard – Figure 4.60)."

$$\begin{aligned}
 P &= 800a + 500b \\
 -800a + P &= 500b \\
 \frac{-800a}{500} + \frac{P}{500} &= \frac{500b}{500} \\
 \frac{-800a}{500} + \frac{P}{500} &= b \\
 \therefore b &= \left(\frac{-800a}{500} \right) + \frac{P}{500}
 \end{aligned}$$

Search line

Figure 4.60: Calculation of search line function presented by Ms Moleko

Ms Moleko: "From the above equation we use fraction $\frac{800}{500}$ in finding the search line, so we simplify this fraction $\frac{800}{500}$ to the simplest form which is $\frac{8}{5}$ however, since our graph scale which is multiple of 10. We can multiple $\frac{8}{5}$ both numerator and denominator by 10 and result to $\frac{80}{50}$ which best suit our scale. Moreover, in this fraction ($\frac{80}{50}$), 80 (numerator) will be b number on the b axis while 50 (denominator) will be a number on the a axis axis, then we make points on the ab Cartesian plane and make it dotted line from where b is 80 and a is 50."

She explained the technique of finding the search line again and illustrated to them how to show the search line and move it up slowly by adding 10 (number) to each value. The last remaining point of the feasible region is the one that will be used to find maximum profit. She used a T-Square ruler to move the search line to the right (Figure 4.61).

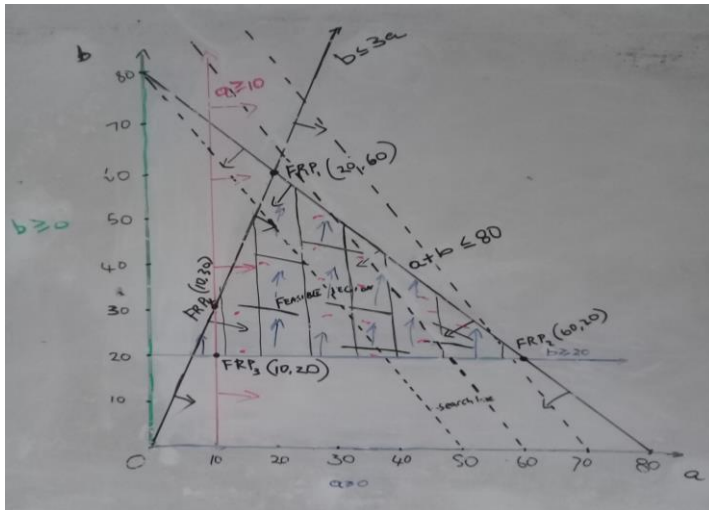


Figure 4.61: MLP graph with feasible region and search line

Ms Moleko: “Which point of this feasible region comes last when we move this search line?”

Then the students picked up the correct point, FRP2 (60:20), whereafter Sibongile was able to find a maximum profit (Figure 4.62).

Ms Moleko: “Thank you very much, that’s excellent.”

$$P = 800(60) + 5(20) = R 58000$$

Figure 4.62: Maximum profits

This part of the lesson showed that Ms Moleko applied direct instructional guidance: and this was realised when she said

Ms Moleko: “**Step1:** get deeper meaning of the linear inequalities; **Step2...**”

This statement is meant to assist students in solving Mathematical linear inequalities because in the earlier Sections (1.1, 2.1 & 4.2) it transpired that students had difficulties with solving linear inequalities. Ms Moleko guided students on how to apply those steps to solve the following linear inequalities, $b \geq 0$, $a + b \leq 80$, $a \geq 10$, $b \geq 20$ and $b \leq 3a$ and also to find a search line and use it to find a maximum point, which assisted students to determine a maximum profit. The manner in which Ms Moleko presented this part of the lesson is in line with the procedural embodiment, demonstration, and lecture method, which are classified under traditional teaching methods. This shows that Ms Moleko’s lesson is in the Bricolage traditional period, because in this era, traditional teaching methods were dominant (cf. 2.2.2.1). The traditional pedagogies approaches previously helped to improve mathematics

performance and in this study assisted students to understand a lesson well. However, other scholars (cf. 2.2.2.1) state that traditional teaching methods are vital important in mathematics teaching and learning and improve mathematics results. However there are limitations if they are only used and relied on to teach mathematics, because traditional methods make for passive students, which is not in line with PAR, as PAR encourages interaction between students and lecturers or among students (cf. 3.2.1). During the traditional period, the first moment of the Bricolage, it was the norm to teach MLP in this fashion, namely where students are the receivers of information, which helped them to pass summative assessments. This discursive practice was also realised in Ms Moleko's statement:

Ms Moleko: *"Step1: get deeper meaning of the linear inequalities."*

where students were given steps to follow to solve linear inequalities, while Ms Moleko led them in showing how to follow those steps. As a result, students comprehended a lesson well. This style of teaching promotes social exclusion, which takes place when a lecturer plays a central role in the teaching and learning. However, Ms Moleko closed the gaps and limitations of traditional teaching methods by using a combination of direct and minimal instructional guidance as multiple teaching methods, which were allowed in the Blurred genres era (cf. 2.2.2.3). Scholars mentioned in Section 2.4.2.3 suggest multiple teaching pedagogies to teach mathematics. Therefore, the multiple teaching techniques used by Ms Moleko promoted social interaction among students and also between her and students, which is in line with PAR ideology and CDA social inclusion (cf. 3.2.1 & 3.6). In addition, Ms Moleko did not act as very powerful person throughout the MLP lesson presentations (cf. 4.3.1).

Section 4.3.3.3 showed that the co-researchers were in agreement on the use of the guidance principle of the RME approach to execute the RME approach successfully. We as a research team planned how to execute the guidance method and Ms Moleko implemented the plan. In the above lesson by Ms Moleko we realised the use of the guidance approach in the form of the direct instructional and minimum instructional guidance. This agrees to what Section 2.3.2 and 2.4.2.3 signposted, namely that the guidance principle of the RME approach, direct instructional and minimal instructional guidance should be used in the teaching and learning of mathematics. This study revealed how to use a combination of direct and minimal instructional guidance to teach MLP, which the literature did not demonstrate.

In conclusion, the literature and generated data showed that the guidance approach is a significant component solution to enhance the teaching and learning of MLP.

4.3.4 Multiple activities and interactivity as a baseline assessment

Leone *et al.* (2010:3-4) and Hailikari *et al.* (2008:2) are in agreement that prior students' knowledge should be recognised and actualised through diagnostic testing (cf. Section 2.4.2.4). According to Leone *et al.* (2010: 3-4), diagnostic testing is used to recognise students' strengths and weaknesses; provide valuable insight into foundational mathematics skills that will be vital for a particular level; check what students already know; identify misconceptions and gaps, and evaluate the level of support needed by students in the teaching and learning of MLP in this regard. The DHET assessment policy for TVET colleges expects of lecturers to apply a baseline assessment approach to get a sense of what students already know (NCV assessment guidelines mathematics NQF L4 curriculum instruction, 2013:5-6). The co-researchers suggested that the students' prior knowledge of MLP should be tested through class activities prior to a new lesson.

Co-researchers: *"Determining students' prior MLP knowledge is the priority, so lecturers should begin every MLP lesson by giving students a short test in the form of a class activity as a baseline assessment, or use a question-and-answer technique to check what students already know to identify students' problem areas (gaps) or even strengths."*

This was exactly what Ms Moleko did in her lesson presentation (cf. 4.3.1):

Ms Moleko: *"In your groups, write the definition of MLP and explain the relevance of MLP in our daily lives; in fact, its importance in our lives. Write these in your own words. You have only 5 minutes to do this work."*

Poho: *"As a group we think that MLP is just a mathematics topic and we do not see the importance of it in our lives."*

Mampho: *"MLP refers to a Mathematical model or technique helping us to make decisions in business production, industrial, engineering, social and physical sciences. MLP assists us to maximise profit and minimise costs with limited resources available. However, we do not know the importance of MLP in our lives."*

Ms Moleko: *"Thanks to all groups for your presentations; you are correct, MLP is one mathematics topic and is defined as the Mathematical model tool helping individuals to make informed decisions in public and private sectors, namely business services, business production, industrial engineering, social and physical sciences as well as different environments. MLP is to maximise profit or minimise costs with limited resources available. So, the importance of MLP is that it can help businesses and people to make informed decisions using constraints resources to generate profit or minimise costs. I know you do not understand what I am talking about, but practical examples will help you"*

Let's define these words together, apply real-life situations and write these words in Mathematical symbols. In your own groups complete this table by defining these words, create

practical examples, and then write Mathematical signs representing those words. You have 22 minutes to complete this activity.”

Ms Moleko started to mark their work and found that the students did not perform well in the group activity (cf. Annexure L). This confirmed that students struggle and have limited knowledge of MLP language and its terminologies.

Ms Moleko: “All right, let’s do it together; firstly, let’s start with the word ‘less than’, you agreed that less than means lower than. Yes, I agree with you but this is a synonym of the word less than. So, let’s start here, what this word ‘less than’ mean in your mother tongue (language) like in Sesotho, isiZulu, isiXhosa and other indigenous languages. Ok, I see a hand, Mampho?”

Again, I agree with you, to say mathematically we use this sign ‘<’ to explain and represent the word ‘less than’. So, let’s look at this example,

Mr Jacobs earns R2 000 per week while Ms Pretty receives R3 000 per week in their new job.

So, from this statement I understand that Mr Jacobs’ wage is **less than (<)** what Ms Pretty receives on weekly basis as wage. Thus, to present this example mathematically, we can let:

Mr Jacobs’ wage be x (or we can use any alphabet)

Ms Pretty’s wage be y (or we can use any alphabet)

$$x < y$$

$$R\ 2000 < R\ 3000$$

So, let’s solve this word problem together, but first do it in your groups (**refer to Table 4.10**). Solve this problem by writing it mathematically.”

Ms Moleko marked and found that most of groups got it right.

Ms Moleko: “It’s really really promising; most groups got it right. So, is there anyone who can help us understand how you arrived at your solutions?”

OK, let’s move to the next part of this lesson for mathematics word problems of **two variables**. In your groups you have 5 minutes to solve this mathematics problem.”

Table 4.11: MLP word problem

SCENARIO 2:

Mr Tshabalala makes chocolate and strawberry-flavoured muffins. He cannot make 750 of both chocolate and strawberry muffins per day.

Ms Moleko found that most of the students struggled to get it right (cf. Annexure D)

Ms Moleko: “This activity involves two mathematics variables. Mr Tshabalala bakes two flavoured muffins, i.e. chocolate and strawberry. He bakes fewer than 750 of both flavours per day

We now let: a number of chocolate muffins to be produced be represented x
a number of strawberry muffins to be produced be represented y

$$x + y < 750$$

The third lesson were presented in the classroom at Thabong Campus and Ms Moleko began the lesson by saying:

“Good morning, students, today we are going to continue where we ended yesterday. So before we move further, is there anyone who still struggle with what we did so far.”

There were no questions and students indicated that they were satisfied, but Ms Moleko asked them how they had arrived at these linear inequalities $a \geq 0; b \geq 0; a + b \leq 80; a \geq 10, b \geq 20; \& b \leq 3a$. Then students were able to explain.

Ms Moleko: *“So, in your groups plot these coordinates, join coordinates and write linear inequalities on that joined line. You have only 10 minutes to do this work but use graph paper.*

In your respective groups, find the feasible area from this graph (refer to Fig. 4.58). You have 5 minutes to do that.”

Mampho came forward and showed feasible area on the whiteboard (Figure 4.63). He started to erase shaded areas that do not form part of feasible area from Figure 4.58.

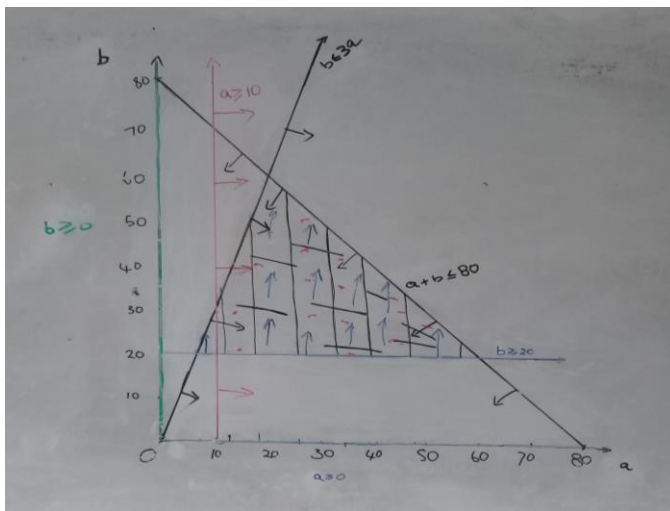


Figure 4.63: MLP graph with feasible region (area) shown by Mampho

Ms Moleko: *“All right, thanks, Gambu. In your groups, find the coordinates for FRP1, FRP2, FRP3 and FRP4 on this graph. You only have 3 minutes to do this work,”*

Students worked in their respective groups, engaging with one another and Ms Moleko facilitated this lesson. Students reported back and Teboho said:

“Point FRP1 is (20, 60).”

Tshepe: *“Point FRP2 is (60, 20).”*

Zinhle: *“Point FRP3 is (10, 20).”*

Khaya: *“Point FRP4 is (10, 30).”*

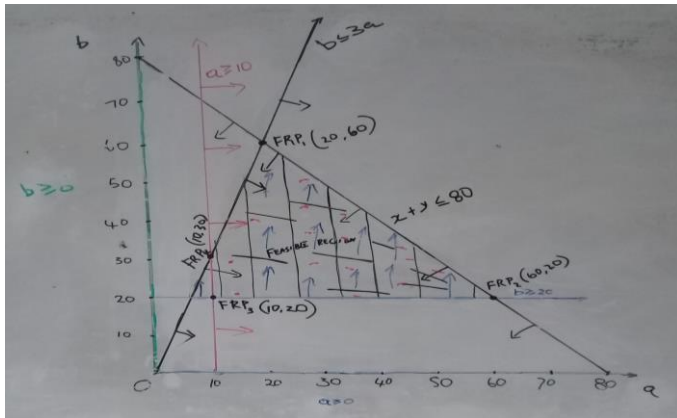


Figure 4.64: MLP graph with feasible region and its coordinates shown by Gambu

Ms Moleko: "Now we have answered question 1.1 and 1.2, which helped us to draw this graph (Fig. 4.64). So, with this graph (Fig. 4.64) we are in a position to answer the last question (1.3) of Activity 1. Question 1.3 requires us to find objective function (profit formulae) or given objective function in order to find the business maximise profit or minimum costs.

All right, ok, in your groups, find for me a maximum profit."

Students in their groups determined the maximum profit by substituting FRP1, FRP2, FRP3 and FRP4 into this objective function. All groups found the same answer and Ms Moleko asked Mike to show how they arrived at their solutions (cf. Figure 4.59 in Section 4.3.1.3).

Mike: "We did this table. As a result, the highest amount is R58 000 here, so this means this farmer to make maximum profit, should plant 60 hectares of maize and 20 hectares of cabbage on available hectares because this farm has only 80 hectares of land."

The above extracts from the lesson presentations in Section 4.3.1 evidence that Ms Moleko searched students' background knowledge on each level of teaching MLP. For instance, when Ms Moleko introduced MLP:

Ms Moleko (Before imparting the MLP lexicons): "In your groups, write the definition of MLP and explain its importance in our lives. You have only 5 minutes to do this work. In your own groups, complete this table by defining these words, create practical examples and then, write Mathematical signs representing those words. You have 22 minutes to complete this activity".

On the second level of teaching MLP before teaching the conversion of MLP word problems into linear inequalities:

Ms Moleko: "Write these word problems mathematically in scenarios 1 & 2."

In addition, prior to the teaching how to draw the MLP graphs, show the feasible region and determine the maximum (or minimum) value, she said:

Ms Moleko: "In your groups, plot these coordinates, join the coordinates and write linear inequalities on that joined line. You have only 10 minutes to do this work and use graph papers."

Next she demonstrated how to draw the MLP graph.

Ms Moleko: *“Since you know how to find the feasible region, it shows that you still remember what you did in NCV level 3 (previous schooling). So, in your respective groups find the area where all different colours indicating inequality signs of greater or less than and equal to meet together. You have 5 minutes to do that.”*

The students were able to get it right and Ms Moleko quickly went to the higher level, asking students to determine a maximum value.

Ms Moleko: *“All right, in your groups, find for me a maximum profit.”*

This approach has helped Ms Moleko to identify students’ comprehension and misconceptions using activities and interaction as a diagnostic assessments. The diagnostic tests revealed that students had difficulty in following concepts, MLP definitions and its relevancy on a daily basis; MLP lexicons; conversion of the MLP word problems with two variables into linear inequalities; and using the search line to determine the maximum (or minimum) values (cf. Annexures B & D). This is in line with the literature in Section 2.4.2.4, because scholars recommend actualisation and recognition of students’ prior mathematics CK. The advantage of determining students’ prior knowledge was highlighted in Section 2.4.2.4. The identification of students’ misconceptions aided Ms Moleko to focus on and tap into the above problematic areas, realised when Ms Moleko explained MLP topic and its relevancy, since students struggled with this concept.

Ms Moleko: *“You are correct, MLP is one mathematics topic and is defined as the Mathematical model tool helping individuals to make informed decisions in public and private sectors ... as well as different environments. It is used in these sectors to maximise profit or minimise costs with limited resources available. The importance of MLP topic is that it can help businesses and people to make informed decisions using constraints resources to generate profit or minimise costs.”*

Since Ms Moleko realised that students also had difficulty with the MLP lexicons, she provided students with a deeper meaning and unpacked MLP terminology, starting from the concrete to the abstract level of teaching, allowing students to explain MLP terminologies in their own indigenous languages. Thereafter, she explained the MLP lexicons and used practical examples that students could help students understand to MLP jargon. Ms Moleko converted and solved the MLP lexicons in a complex way (writing them in Mathematical form). This technique assisted students to gain a deeper understanding of MLP terminology. This allowed students to know and understand where x and y Mathematical variables come from and why they are used. More

importantly, Ms Moleko assisted students to know and understand that variables are only about x and y variables to represent unknown numbers and axes. Moreover, she used a and b alphabets as mathematics variables instead of x and y , which is dominantly used in the Bricolage traditional era (cf. the Section 2.2.2.1). This was affirmed by the following statements.

Ms Moleko: *“Let’s do this together ... Let’s start here, what the words ‘less than’ mean in your own languages, Sesotho, isiZulu, isiXhosa, Sepedi, since we have these tribes in this classroom.”*

Students: *“Less than in Sesotho means e nyane ho; in isiZulu, it means ngaphansi kwa; in isiXhosa it means incicane ku; and in Sepedi it means e nyane go.”*

Ms Moleko asked if there was anyone with a different indigenous language to translate the word, which the students denied.

Ms Moleko: *“So, now I know that all of us understand well the word ‘less than’, since we used our indigenous languages to explain this word. Less than refer to smaller number or size especially when comparing two or more than two things. In mathematics context it occurs when we compare two or more things. The synonym for “less than” is lower than, below; under; smaller; shorter than; beneath; fewer than. etc. Let’s look at this example,*

Mr Jacobs earns R 2 000 per week while Ms Pretty receives R 3 000 per week in her new job.

*From this statement I understand that Mr Jacobs’ wage is **less than** ($<$) what Ms Pretty receives on a weekly basis as wage. To present this example mathematically, we can let Mr Jacobs’ wage be x (or we can use any alphabet) Ms Pretty’s wage be y (or we can use any alphabet). As a result $x < y$ ($R2000 < R3000$). The number that is small comes before the less than sign, then the bigger number will follow the sign.”*

Ms Moleko followed the same technique to do revision of Activity A and also completed Table 4.2. Ms Moleko further showed students how to solve MLP word problems with two mathematical variables, because this concept was identified as a problem area.

Ms Moleko: *“This activity involves two mathematics variables; look, Mr Tshabalala makes chocolate and strawberry-flavoured muffins. He cannot make 750 muffins chocolate and strawberry muffins per day.”*

She then unpacked this MLP word problem with two mathematics variables.

Ms Moleko: *“Mr Tshabalala now bakes muffins with two flavours, i.e. chocolate and strawberry. His baking of flavours of muffins per day is fewer than 750. So, let’s let a number of chocolate muffins produced be represented by x variable, while the number of strawberry muffins to be produced is represented by y variable; therefore, **mathematically**, $x + y < 750$.”*

This explanation and demonstration given by Ms Moleko were in line with activity, interactivity, reality, levels, intertwined and guidance doctrines of the RME approach as a diagnostic assessments (cf. 2.3.2 & 2.4.2.4). The initial step that Ms Moleko took

was to give students a class activity to do in their groups to test their existing MLPCK. This addressed the activity and interactivity principles of the RME approach. Moreover, this class activity given to students entails realities that students are familiar with. These kinds of activities that integrate realities were first considered vital in mathematics teaching and learning in the Bricolage blurred genres (cf. 2.2.2.3). So, to complete these group activities students were required to work their groups as a team. This was recognised in the following statement,

Ms Moleko: *“In your groups you have 5 minutes to solve this mathematical word problem into linear inequalities.”*

This discursive practice is in line with PAR, because it promotes collaboration among students and allow students to work in their small cycle in their respective groups (cf. 3.2.3). Thereafter, Ms Moleko, together with students, marked the activity and she was able to identify and instantly address misconceptions and errors. Furthermore, this lesson of Ms Moleko showed that she was busy with the second level because she started introducing the MLP lexicon, followed by MLP word problems with one variable. Next, she dealt with MLP word problems with two mathematics variables, but all her lessons linked to one another and/or were intertwined. The above lesson proved that Ms Moleko’s teaching started from concrete, becoming more and more abstract (cf. 4.3.3.1 & 4.3.3.2), in line with what literature revealed (cf. 2.4.2.1). These lessons showed that they could not be treated in isolation, because one lesson led to the next, enabling students to understand well. Ms Moleko also applied the guidance principle by letting students to work on the class activities first to identify their errors and misconceptions.

Thereafter, Ms Moleko guided students on how to solve MLP terminologies and word problems with two variables. She did not act as a powerful individual who knew everything, in line with PAR, CDA social inclusion and postmodernity, the fifth moment (3.2.3, 3.7 & 2.2.2.5). Moreover, the way Ms Moleko delivered the lesson is similar to the student-centred method, because both techniques advocate that students should be encouraged to play a big role in the teaching and learning process, while lecturers guide and facilitate the lesson (cf. 2.4.2.3). On the other hand, after Ms Moleko had realised that the students understood the concepts of MLP, she allowed students to share their understanding, thereafter emphasising and highlighting the key points and

information. See Figure 4.65 below where students showed feasible region (area) and determined maximum value to make an informed decision.

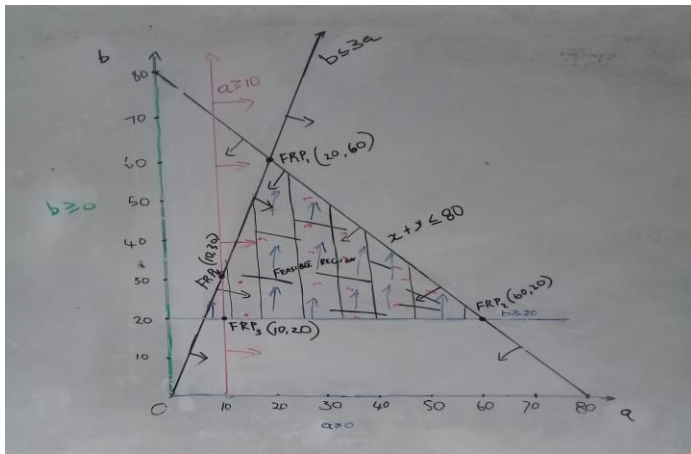


Figure 4.65: MLP graph with feasible region and its coordinates shown by Gambu

Students in their groups determined the maximum profit by substituting FRP1, FRP2, FRP3 and FRP4 into this objective function. All groups found the same answer and Ms Moleko asked Mike to show how they arrived at their solutions (Figure 4.66).

$$\begin{aligned} \text{FRP}_1 : P &= 800(20) + 800(60) = 46\,000 \\ \text{FRP}_2 : P &= 800(60) + 500(20) = 58\,000 \\ \text{FRP}_3 : P &= 800(10) + 500(20) = 18\,000 \\ \text{FRP}_4 : P &= 800(10) + 500(30) = 23\,000 \end{aligned}$$

Figure 4.66: Maximum profits determined by Mike

Mike: "As a result, the highest amount is R58 000 here, so this means for this farmer to make maximum profit, he should plant 60 hectares of maize and 20 hectares of cabbage on available hectares because this farm only has 80 hectares of land."

However, she did not spend much time on the areas students understood since one of the objectives of determining students' prior CK is that more time should be allotted to students' problematic areas in MLP rather than repeating what students already know and understand (cf. 4.3.1.1- 4.3.1.3), especially when students indicate that they have difficulties with showing and using a search line to determine a maximum (or minimum) value (cf. Figures 4.61 & 4.62).

This section revealed how Ms Moleko used the RME approach principles, activity and interactivity, as diagnostic assessments to check what students already knew and what they were lacking. At the beginning of each level of teaching MLP in Section

4.3.1.1 to 4.3.1.4, the students were tested on their prior MLPCK. This provided both students and Ms Moleko with urgent feedback needed before MLP teaching and learning. This agrees with what was postulated in Section 2.4.2.4 about actualising and searching students' prior knowledge CK. The most remarkable thing in this section of this study was to see Ms Moleko being consistent in applying the activity and interactivity principles of the RME approach as diagnostic assessments, because this baseline assessment is mostly not implemented well, or totally neglected, due to various reasons (cf. 4.2.1–4.2.5). Again, Ms Moleko, after recognising students' difficulties or misconceptions, used realities, guidance, levels and intertwinement to assist students to understand especially MLP definitions and its relevancy, MLP lexicons, MLP word problems and linear inequalities.

In conclusion, this section showed how to use the activity and interactivity principles of the RME approach as a baseline approach.

4.3.5 Continuous activities, feedback and interactivity

The literature in Section 2.4.2.5 have shown that formative assessments are vital in the teaching and learning. Tridane *et al.* (2015: 685) signposts that there is a need to assess students in the teaching and learning of mathematics because this guides, directs, and monitors students' progress to achieve course objectives and to improve students' results. According to Ajogbeje *et al.* (2013:8-9), formative assessments give instant feedback, removing the threatening effect of a single test (summative assessment) that is given at the end of the course study; also, this type of assessment instils competition among students. The DHET assessment policy requires of lecturers to administer and use formative assessment as a tool to improve teaching and learning of mathematics (NCV assessment guideline mathematics L4, 2013:5-6). The co-researchers highly recommended the administration of regular shorts test, quizzes, class activities and home activities on a regular basis together with regular feedback

Co-researchers: *"Short tests, like quizzes, class activities or home activities should be given to students on regular basis and in each and every mathematics lesson because these tests will give you as a lecturer problem areas or students ability."*

The research team (with myself as a principal investigator) planned how to integrate activities in MLP lessons.

Hence, Ms Moleko, in the lesson presentations in Section 4.3.1 above, continuously assessed students at the beginning, during and at the end of the lesson. This was proven by the following words from the lessons in Section 4.3.1. At the beginning of the lesson Ms Moleko said:

Ms Moleko: “in your groups, write the definition of MLP and explain the relevance of MLP in our daily lives; in fact, its importance in our lives. Write these in your own words. You have only 5 minutes to do this work.”

Poho: “As the group, we think that MLP is just a mathematics topic and we do not see any importance for it in our lives.”

Mampho: “MLP refers to a Mathematical model or technique helping us to make decisions in business production, industrial, engineering, social and physical sciences. They insist that MLP assist us in maximising profit and minimising costs with limited resources available. But, we do not know the importance of MLP in our lives.”

Ms Moleko: “You are correct that MLP is one mathematics topic and is defined as the mathematical model tool to help individuals to make informed decisions in the public and private sector and in different environments. MLP is used to maximise profit or minimise costs with limited resources available. Its importance is that it can help businesses and people to make informed decisions using constraints resources to generate profit or minimise costs.

(During the lesson) Define these words, apply real-life situations and write these words in mathematical symbols. In your own groups, complete this table by defining these words, create practical examples and then write mathematical signs representing those words. You have 22 minutes to complete this activity (refer to Activity A in Section 4.3.1.3).

All right, let's do it together; firstly, let's start with the word 'less than'. You agreed that 'less than' means 'lower than'. Yes, I agree with you but this is a synonym of the word 'less than'. So, let's start here, what does this word 'less than' mean in your mother tongue and other indigenous languages?”

Mampho: “Less than in Sesotho means e nyane ho.”

Khaya: “Less than in isiXhosa means incinane ku.”

Zinhle: “Less than in isiZulu means ngaphantsi kwa.”

Lebogang: “Less than in Sepedi means e nyane go.”

Ms Moleko: “Thanks, all of you. So now I know that all of us understand well the word 'less than', since we used our indigenous languages to explain this word. 'Less than' refers to a smaller number or size, especially when comparing two or more than two things. In a mathematics context it occurs when we compare two or more things. The synonyms of “less than” is lower than, below; under; smaller; shorter than; beneath; fewer than etc. Again, I agree with you guys, to say mathematically we use this sign ' $<$ ' to explain the word 'less than' mathematically.

Ms Moleko illustrated several practical examples to explain the word 'less than' and other MLP terminology (cf. Table 4.2). At the end of this lesson:

Ms Moleko: “Do this activity and you have 12 minutes to complete it, is an individual work (refer to Activity B in 4.3.1.1).”

Ms Moleko introduced students in the same way to another MLP levels (cf. 4.3.1).

Ms Moleko: “In your groups, solve this problem (refer to Scenario 1 in Table 4.10). Write this statement mathematically.

Thereafter, Ms Moleko gave students another work and said

*“OK, let’s move to the next part of this lesson for mathematical word problems of **two variables**. You have 5 minutes to solve this mathematical problem (cf. Table 4.11).*

In your groups, identify natural resources and man-made resources that you see in this picture (pointing and referring to the picture on the wall). What do you understand in this statement?”

Wilson: *“I think it means that this farmer has a maximum of 80 to plant maize and cabbage but planting both products cannot exceed 80 hectares because it is the only land available to him. So, the number hectares of maize to be planted **plus** the number hectares of cabbages to be planted **can be planted on 80 or fewer than 80 hectares** because of the limited piece of land.*

NB: The number hectares of maize to be planted is represented by variable “**a**” while number hectares of cabbage to be planted is represented by variable “**b**”, so, **mathematically** we can write $a + b \leq 80$

There were no questions and the students indicated that they were are satisfied, but Ms Moleko asked them how they arrived at these linear inequalities $a \geq 0; b \geq 0; a + b \leq 80; a \geq 10, b \geq 20; \& b \leq 3a$, then the students were able to explain.

Ms Moleko: *“In your groups, plot these coordinates, join coordinates and write linear inequalities on that joined line. You have only 10 minutes to do this work on graph paper.*

*All right, let’s move forward; as we presented these linear inequalities $a \geq 0, b \geq 0, a + b \leq 80, a \geq 10, b \geq 20$ and $b \leq 3a$ on the **ab Cartesian plane** above in Fig. 4.58. We should find the feasible area (region). How can we find the feasible region or area?”*

Gambu: *“After we have presented and drawn the linear inequalities and also shown the direction of inequality signs. We have to find the area where all arrow direction meet together, then shade it so that shaded area is feasible area.”*

Ms Moleko: *“All right, since you know how to find the feasible region and it shows that still remember what you did in NCV level 3 (previous schooling). In your respective groups, find the area where all different colours indicating inequality signs of greater or less than and equal to meet together. You have 5 minutes to do that.”*

Mampho came forward and showed feasible region on the whiteboard (Figure 4.63).

Ms Moleko: *“What do we call this area and why is it important to find it?”*

Gambu: *“It is called a feasible area (region) and it is used to determine a maximum profit or a minimum costs in order to use limited resources efficiently. From this shaded area we have to find the points first that can assist us to find max profit or min costs.”*

Ms Moleko: *“All right, thanks, Gambu. In your groups, find the coordinates for FRP1, FRP2, FRP3 and FRP4 on this graph. You only have 3 minutes to do this work.*

Your time is up. What are the coordinates for FRP1, FRP2, FRP3 and FRP4? FRP1 is for Teboho (student), FRP2 is for Tshepe;, FRP3 is for Zinhle; and FRP4 is for you, Khaya. Let’s start from Teboho.”

The students managed to find the feasible area coordinates and Fig. 4.65 was drawn. Thereafter, students were asked to determine a maximum value and managed to get that value. So, Mike was asked to illustrate it (cf. Fig. 4.66).

Mike: *“As a result, the highest amount is R58 000 here, so this means for this farmer to make a maximum profit, he should plant 60 hectares of maize and 20 hectares of cabbage on the available hectares because this farm has only 80 hectares of land.”*

The extracts above show sufficient continuous activities and continuous feedback, because Ms Moleko assessed the students on a regular basis in her lessons.

Ms Moleko: *“In your groups, write the definition MLP and explain the relevance of MLP in our daily lives. You have only 5 minutes to do this work. Define these words, apply real-life situations and write these words in mathematical symbols. In your own groups, complete this table by defining these words, create practical examples and, write mathematical signs representing those words. You have 22 minutes to complete this activity.”*

These activities identified problem areas which helped students to be aware of their strengths and difficulties with MLP through the feedback provided. This is in line with what the literature revealed earlier in Section 2.4.2.5, namely that assessments should go hand in hand with regular feedback to check students' base knowledge and track students' level of understanding. Tridane *et al.* (2015:685); Ajogbeje *et al.* (2013: 8-9), and Nakhanu, Toili and Nyongesa (2015:6) recommend formative assessments and a PBL approach to enhance students' mathematics understanding and performance (cf. 2.4.2.5 & 2.4.2.3). This type of assessment came in the teaching and learning process to close gaps by relying on summative assessments to determine the competency of students in MLP. This type of assessment gained more support in the Blurred genres where a multiple of the assessment methodologies were considered essential to improve mathematics results (cf. 2.2.2.2), since students were expected to be active in the teaching and learning process. Ms Moleko continued to teach the MLP concepts and at the end of the lesson gave students another class activity.

Ms Moleko: *“Do this activity, you have 12 minutes to complete it ... this activity is individual work.”*

These words show that Ms Moleko was guided by the following activity and interactivity principles of the RME approach to execute the RME approach well. The application of the activity principle occurs when a lecturer tests students' knowledge in a number of assessments methods. In this regard, Ms Moleko assessed students continuously on each level of teaching MLP, prior to, during and at the end of the lesson. The motive of assessing students continuously was to check students' base knowledge and comprehension; using instant feedback to remove all threats of summative tests; identify students' ability; improve mathematics results; and instil competition among students and colleges (cf. 2.4.2.5). On the other hand, the interactivity principle requires of a lecturer to give students class or home activities that encourage students

to work in pairs or in groups to enable them to interact to share knowledge and learn from one another (cf. 2.3.2). This is in line with the collaboration, participation and interaction method, which is encouraged by the Blurred genres, post-experimental and current fractured moments of Bricolage (cf. 2.2.2.3, 2.2.2.6 & 2.2.2.8), as well as the PAR principles such as small cycle and locating the voice of marginalised students (cf. 3.2.3).

In light of the above information, it is clear that activity and interactivity doctrines of the RME approach promote active students in teaching and learning. The epistemological and ontological stance of the RME approach in this regard is that students learn and comprehend well when they solve MLP activities in pairs, groups, and/or individually. This is exactly what Ms Moleko did in Section 4.3.1, where she presented MLP lessons in levels. She gave the students short quizzes to do in their groups and students were expected to discuss, find solutions and report back to the class.

Ms Moleko: “All right, since you know how to find the feasible region and it shows that still remember what you did in NCV level 3 (previous schooling). So, in your respective groups find the area where all different colours indicating inequality signs of greater or less than and equal to meet together. You have 5 minutes to do that.”

Thereafter, Morena was requested to come forward and show a feasible region on the whiteboard (Figure 4.67).

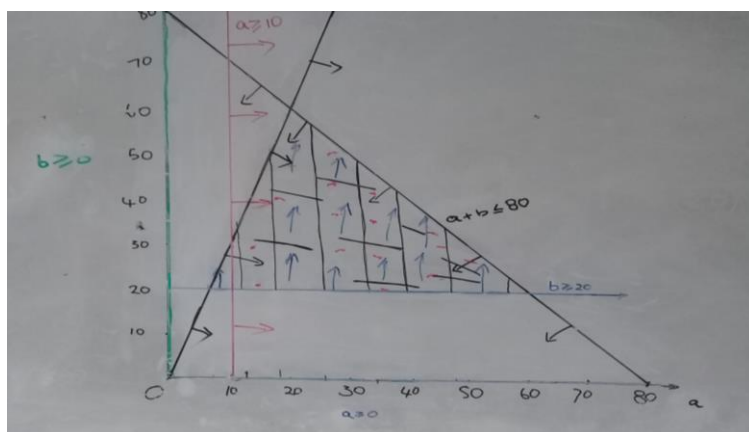


Figure 4.67: Morena showing the feasible area

The manner in which Ms Moleko presented the MLP lessons also shows a discursive practice and social inclusion, because in all the lessons, Ms Moleko worked together with students and understood that 21st-century students learn best when they are involved and play a central role in mathematics teaching and learning. She continuously gave students the MLP activities in pairs, groups and/or individually.

Ms Moleko: *“In your groups, find the coordinates for FRP1, FRP2, FRP3 and FRP4 on this graph. You only have 3 minutes to do this work.”*

This links well with Blurred genres because in this era, multiple assessments methods, symbolic interactionism, constructivism, numerous theories, paradigm, methods and strategies were considered to enhance teaching and learning of MLP topic rather than relying only on traditional pedagogies methods (cf. 2.2.2.3). This also links well with the principles of the PAR, such as democratic by nature, small cycles, and locating the voice of marginalised people (students) in the teaching and learning to encourage interaction and participation among students and their lecturers so that students are active participants in solving MLP activities (cf. 3.2.3). Ms Moleko and two members of our team (including me as principal investigator) monitored and facilitated lessons to ensure that all students did what they were supposed to do in their groups.

This Section showed adequate MLP activities with regular feedback that Ms Moleko gave to students prior to, during and at the end of the lesson with the aim to enhance MLP teaching and learning. These activities assisted the students to comprehend the MLP components well. This agrees with the literature in Section 2.4.2.5. They were all in agreement that mathematics lecturers should assess students on a regular basis in the teaching and learning of MLP so that both students and lecturers get feedback about the teaching and learning process. More importantly, what was so interesting was that most of the MLP activities were very rich, because those activities incorporated realities students could make sense of. Secondly, Ms Moleko and assistant lecturers monitored this part of the lesson to ensure that students did exactly what they were requested to do without cheating. Lastly, students instantly got feedback and were given a platform to share their solutions to learn from one another. In conclusion, the empirical data showed that formative assessments is one solutions component to enhance MLP teaching and learning.

4.4 CONDITIONS CONDUCIVE TO ENABLING THE IMPLEMENTATION OF THE RME APPROACH

This section of this study will discuss conducive conditions that enable the execution of the RME approach in Section 4.3 to teach MLP, sufficient RME knowledge,

adequate MLP knowledge, sufficient MLP lesson planning and preparation, and a dedicated team.

4.4.1 Adequate co-researchers' knowledge in the RME approach

Adequate co-researchers' knowledge of the RME approach is one of the conditions conducive to the successful implementation of the RME approach to teach MLP at a TVET college. According to this study, adequate knowledge of the RME approach refers to the proper understanding of the RME approach; firstly, knowing what the RME approach is about; secondly, knowing the principles and characteristics of the RME approach; and lastly what the RME aims to achieve. In one of the research meetings of this study, we as a research team agreed that the initial step to teach mathematics concepts using the RME approach is to have a proper understanding of the RME approach; one cannot execute what one does not understand.

Mrs Leeuw: *"Since we agreed to use the Realistic Mathematics Education (RME) approach to teach MLP, is there anyone who understands what the RME approach is all about, in order to for us to implement and use what we know and fully understand or have an idea with?"*

Ms Moleko: *"ke nahana nka leka Mme ... mmm ... RME approach ke tsela kapa mokgwa o bobebe oo re rutang thuto ya dipalo ka yona re sebedisa dintho tseo bana ba di tsebang, tseo ba ka dihopolang kapa tseo ba di bonang letsatsi le letsatsi lefatsheng ka bophara tseo thuto ya dipalo e sebedisitsweng ho tsona. Lebakabaka e le hore thuto ya dipalo ebe bonolo mme e natefele bana. Hona ho tla etsa bana ba sebetse hantle thutong ya dipalo (I think I can try Mem ... mmm ... the RME approach is a method or a way that makes mathematics more easier to be understood by students, using what students know, what they can remember or what they meet on daily basis around the world where mathematics concepts have been applied. The main reason is to make mathematics easier and enjoyable for students. This can make students to do well in mathematics.)"*

Mr Koena: *"I think using the RME approach, we should start the teaching with students linking prior knowledge from different environment before presenting the new mathematics content."*

Mrs Leeuw: *"Thank you, Mme le Ntate, for sharing your brighter knowledge. Is there any other views on the floor with regard to the RME approach?"*

The co-researchers agreed with the views of Ms Moleko and Mr Koena on what they shared with regard to what they thought about the RME approach.

Mrs Leeuw: *"Ok, let's Mr Phoka's view ... mmm ... Baba what is your intake?"*

Researcher: *"I agree with my colleagues because one of the RME approach principles requires us to use realities or real life situation. This means before we teach mathematics in procedurally or abstract way; we as lecturers need to concretise mathematics with what students can see, touch, and/or hear. In essence, we should prepare lessons that talk to real-life situations to lure students into lessons and also assist students to comprehend MLP easily. According to Freudenthal, the RME approach is guided by six principles, such as the activity, reality, levels, intertwined, interactivity and guidance which need to be followed by facilitators and lecturers to apply the RME successfully in the teaching and learning process. So, the*

RME approach refers to a Mathematical teaching method which requires of lecturers to incorporate real-life stories into mathematics teaching and learning with the intention to assist students to understand mathematics concepts. The RME approach was developed principally upon Freudenthal's view of mathematics as a human activity because, according to him, in order for mathematics to be of human value, it has to be taught in such a way that it makes sense to students, useful, close or relevant to the students and society as well (Van den Heuvel-Panhuizen, 2008). It has been developed and used in the Netherlands; thereafter the United Kingdom and other countries adopted the RME approach to teach mathematics as alternative to mechanistic approaches (or traditional approaches) which dominated the teaching and learning of mathematics, because the mechanistic approach (telling or textbook method) had limitations. Mostly this method made it difficult for other students to follow and understand mathematics lessons well. I will urge all people to go through this document; it will assist all of us to get a better understanding, but you can search and get more sources that explain the RME approach better."

The two-page document about the RME approach (cf. Annexure Q) was distributed among the co-researchers so that all co-researchers had a proper understanding. As the principal investigator, I drafted this document in different languages, some in English, Southern Sotho, isiZulu, isiXhosa and Sepedi, so that co-researchers were able to choose a paper with their language preference.

Mrs Leeuw (chairperson): *"I suggest that we called it a day, so that we get enough time to go through this document,"*

Mr Kia: *"I agree with you, chair."*

Ms Seeta: *"I second that idea."*

Co-researchers: *"Mmm ... we also agree."*

Mrs Leeuw *"Let's meet tomorrow at the same time at the place."*

(The next day):

Mrs Leeuw: *"Afternoon, colleagues, yesterday we agreed that we needed time to go through this doc and I hope all of us went through this doc which helped me to get a light and understanding of RME approach. Because, now I understand that Realistic means rich or real life situations or real life stories or imaginable or thinkable and should be integrated into the teaching of mathematics. What have you underlined or picked out from this document?"*

Mr Takui: *"I learned about principles that need to be followed in the teaching of mathematics, such as activity, interactivity, level, intertwined, realities and guidance. Then, each and every lesson of mathematics must have these principles, so that lesson observers can identify these principles from your lesson."*

Mrs Mohlomi: *"Ke ithutile hore ho tshwanetswe ho rutwa bana dipalo ka tsela e bobebe ho sebediswa ka dintho tse ba ka dibonang ha bonolo. Hape le ho bontsha bana hore se ba se etsang ka phapusing se molemo ha ka kang bophelong (I learned that mathematics should be taught in an easier way, using what students can see. Again, also showing them that what they are doing in class is important in real lives)."*

Mr Kapu: *"Use pictures, practical examples and also link what they already know from the previous lessons with new lesson to be presented. We should start teaching mathematics from the concrete level to the higher level."*

Mrs Leeuw: *"What should be done in the teaching of mathematics using the RME approach or I mean, where should we begin or what should be taken account using the RME approach."*

Ms Lebone: *"I think we should use these principles of the RME approach and follow them."*

Researcher: *"I agree with my colleagues, we should align ourselves with activity, interactivity, levels, intertwined, realities and guidance principles of the RME approach. Moreover, also use these principles as a check list when we plan, prepare, organise, present and reflect lessons."*

The co-researchers were in agreement that, before using the RME approach to teach mathematics, all that is needed is to have adequate RME approach knowledge to implement what we fully understand.

Co-researchers: *"Since we agreed to use the RME approach to teach MLP topic, is there anyone who understands what the RME approach is all about, in order for us to implement and use what we know ... or have an idea with?"*

The co-researchers shared their understanding of what they thought the RME method was all about. I as the principal investigator also shared my understanding of the RME approach understanding, guided by literature in Section 2.3.2. I shared the document I had prepared with the co-researchers. This included a definition, characteristics, principles and aim of the RME approach, including sources to be visited so that all co-researchers got a sense of what the RME approach is all about (cf. Annexure Q). The co-researchers were given ample time to study the content of the document. Thus, this document aided co-researchers to fully understand the definition, characteristics, principles, and aim of the RME approach, to link the RME approach with other teaching methods, as well as to suggest what needed to be integrated into the teaching and learning using the RME approach.

Mrs Leeuw: *"This document helped me to get a light and understanding of RME approach, now I understand that Realistic means rich or real life situations or real life stories or imaginable or thinkable and should be integrated into the teaching of mathematics ."*

Mr Takui: *"I learned about principles that needs to be followed in the teaching of mathematics , such as activity, interactivity, level, intertwined, realities and guidance."*

Mrs Mohlomi: *"I learned that mathematics should taught in an easier way, using what students can see ... showing them that what they are doing in class is important in real lives."*

Mr Kapu: *"Use pictures, practical examples and also link what they already know from the previous lessons with new lesson to be presented. We should start teaching mathematics from the concrete level to the higher level."*

This knowledge enabled the co-researchers and myself to plan, develop and prepare the MLP lesson in line with the RME approach (cf. 4.4.3). Moreover, adequate RME approach knowledge aided Ms Moleko to present the MLP using the RME approach through its principles (cf. 4.3.1).

4.4.2 Adequate Mathematical Linear Programming Content Knowledge (MLPCK) among co-researchers

Adequate MLPCK is another key condition to enable the RME approach to work well in improving teaching and learning of MLP. This study explains adequate MLPCK as a good understanding in MLP topic both conceptual and procedural knowledge. This is a condition that has been identified by co-researchers, because the argument brought forward was that a lecturer cannot effectively teach MLP topic if s/he has a deficiency in MLP knowledge or even struggles with MLP. The co-researchers indicated that if a lecturer had adequate MLPCK it would lead to adequate Mathematical Linear Programming Pedagogical Content Knowledge (MLPPCK). This was affirmed by the literature in Section 2.4.3.3, which showed that there is a direct impact between CK and PCK. The literature showed that if a lecturer understood a particular mathematics topic well, then s/he would come up with multiple ways to unpack that particular mathematics topic. Again, if a lecturer did not understand a mathematics topic well, s/he would have limited knowledge of instructional strategies to impart and/or teach that particular mathematics topic. This agrees with what the co-researchers shared in the research meeting.

Mrs Mahapa: *"We should first understand MLP topic before using the RME approach to improve the teaching of MLP topic as mathematics lecturers or lesson presenters and/ or facilitators."*

Ms Seeta: *"I understand mathematics content comes first before techniques, because if we understand content it will be easier for us, to develop many ways solve mathematics problems."*

Ms Khodu: *"I think the most important thing for us to have proper MLP topic understanding due to fact that we will have more ways or methods to teach the MLP topic using the RME approach. For instance, if we as lecturers or facilitators do not understand this topic how can we expected to effectively teach it or even improve students' MLP understanding?"*

Mrs Matlou: *"I agree with my colleagues, for me, I didn't teach this topic because I didn't understand this topic well or what I can say; I was not confident when I taught this topic. What I am saying is that content is very important and comes first."*

Mrs Leeuw: *"thank you, colleagues, I agree with you. What are other people saying?"*

Mrs Mahapa: *"We agree, before using the RME approach to improve the MLP topic we must first get more information on MLP."*

Ms Seeta: *"Since the lot of us here have done mathematics and also taught mathematics in different grades and NCV levels, lets summarise this theory of MLP and explain key information."*

Mrs Leeuw: *"I agree with you, Ma'am, but I think it will be fair if we give ourselves enough time to go through this module (MLP) and other material in order to remind ourselves about the MLP topic. It is just a suggestion, so what are other people think?"*

Ms Lebhone: *"I think we should go through this topic first individually alone, thereafter we can discuss about this topic and share key information."*

During the next session, the discussion about the MLP topic started in the research team and we shared what we understood about MLP.

Mr Kanana: *"In the book I read, MLP is explained as a Mathematical model technique or tool for helping people to make decisions in business production, industrial engineering, social and physical sciences. MLP helps people to maximise a profit and minimise costs with limited resources available. So, the importance of MLP is that it can help all people and businesses to make an informed decisions using constraints resources to generate profit or minimise costs."*

Mr Takui: *"MLP is a mathematics topic and it combines case studies, linear inequalities and also graphs. So, to solve MLP problem, we follow this process, firstly solve scenarios; secondly, solve linear inequalities and find coordinates; thirdly, sketch linear inequalities and identify feasible region; fourthly, determine maximum profit or minimum costs. Then, it will be then an individual person or business make decision in order to use resources at optimal level."*

Mr Kia: *"MLP consists of case studies, linear inequalities, feasible area, and maximum and/or minimum values. So, to teach this we should consider MLP terminologies and make sure that students understand language used in this topic."*

The discussion continued among the research team and all co-researchers shared their MLP content understanding and MLPPCK.

The above extracts show that the co-researchers were in agreement that there was a need for an adequate MLPCK before teaching of any MLP can take place. This was proven by the following statements.

Mrs Mahapa: *"We should first understand MLP before using the RME approach to improve the teaching of MLP topic."*

Ms Seeta: *"Understanding mathematics content comes first before techniques because if we understand content it will be easier for us, to develop many ways solve mathematics problems."*

Ms Khodu: *"The most important thing for us to have proper MLP topic understanding due to fact that we will have more ways or methods to teach the MLP topic using the RME approach."*

Mrs Mahapa: *"Before using the RME approach to improve the MLP topic we must first have more and more information of MLP topic."*

Moreover, the co-researchers strongly maintained that the first thing to do as mathematics lecturers or lesson facilitators is to have a full understanding in the MLP topic before one teaches the MLP topic using the RME approach, because an adequate MLPCK helps co-researchers improve their MLPPCK. This is in line with what the literature revealed that there is a positive (direct) relationship between CK and PCK. For instance an adequate CK leads to an adequate PCK. In contrast, a deficiency in CK leads to a lack of PCK. Therefore, this is what the co-researchers shared.

Ms Seeta: *“If we understand content it will be easier for us, to develop many ways solve mathematics problems.”*

Ms Khodu: *“The most important thing for us to have proper MLP topic understanding is due to fact that we will have more ways or methods to teach the MLP topic using the RME approach. If we as lecturers or facilitators don’t understand this topic. How can we expect to teach it effectively or even improve students’ MLP understanding?”*

Mrs Matlou: *“For me, I didn’t teach this topic because I didn’t understand this topic well or what I can say I was not confident enough when I taught this topic. What I am saying is that content is very important and comes first.”*

The adequate MLPCK helped us as a team to plan, develop, prepare and present MLP lessons using the RME approach successfully (cf. 4.3.1 & 4.4.3).

4.4.3 Sufficient lesson planning and preparation

Lesson planning is a blueprint that every effective lecturer should use, because this is the approach for teaching any SCK, in this regard MLPCK, in an attempt to save time, energy and give maximum output within the allocated time to teach (cf. 2.4.3.3). The importance of lesson planning is that it guides lecturers on the right path towards the achievement of pre-determined goals and also helps them to evaluate the efficacy of their teaching methods for further use of those techniques. Through lesson planning and preparation lecturers and facilitators are able to develop a lesson plan. According to Khan (2011:3958), lesson planning includes topic; lesson outcomes; relatedness to previous knowledge; organised subject matter; teaching resources; activities; suitable pedagogical approaches; reflection of teaching methods; and considering a future applicability of the lesson taught. The lesson planning and preparation are a daily activity for teachers, lecturers and facilitators, because it guides them on what to teach; what to achieve; how to teach and when to teach it; what resources to use; and how to reflect on the lesson plan, but all must be aligned to a syllabus, curriculum and work pace. The DHET through the Integrated Quality Management System (IQMS) policy requires of lecturers to do adequate lesson planning and preparation prior to teaching, because the DHET regards it as lesson observation tools (cf. Section C of IQMS for Public Further Education and Training [FET] college). A lesson plan and presentation format developed by the investigated TVET college in line with DHET guidelines includes the duration (date); module; specific outcomes; learning outcomes; lesson preparation; teaching/learning media; formative assessments; and reflection. The co-researchers were all in agreement that it was momentous for all lecturers to plan and

prepare MLP lessons adequately on a daily or weekly basis to impart the MLPCK well.

During the research meetings at TVET colleges co-researchers said:

Mrs Seeta: *"The department expects all lecturers to plan and prepare lesson daily or weekly so that lecturers deliver mathematics content. So, it is important also for us to plan and prepare the MLP lesson adequately in order for us to deliver MLP lesson well."*

Mr Kia: *"This is where we should start as we are expected by the department policy. If we do not plan and prepare lessons we are deviating from the department policy, which will not be accepted. So, I think we should together plan and prepare MLP lessons so that we present the MLP lesson in an easier way."*

Mr Phoka: *"We are now aware that it is imperative for us to plan and prepare lessons and develop a lesson plan. So, since we now had a discussion on the RME approach and MLP, I think we are in the position to plan and prepare on how to teach the MLPCK using the RME approach, because the successful implementation of the RME approach heavily relies on sufficient MLP lesson planning and preparation."*

Mr Kia: *"It is vital for all lecturers to plan and prepare MLP lessons sufficiently on a daily or weekly basis in order to teach MLP well."*

Mr Takui: *"I think planning and preparing lessons together will help me a lot, like to plan and prepare lessons, because usually I do teach this topic in my way."*

Ms Lebhone: *"Planning and preparing a lesson together are going to help us a lot and we are going to learn more from one another, because each one of us has a unique skill and knowledge to teach MLP."*

Mrs Leeuw: *"I agree with you, colleagues, we should plan and prepare MLP lesson before we teach this topic to students. So, where do we start, colleagues?"*

Mrs Matlou: *"We should have a lesson plan template, work schedule, textbook and syllabus in order for us to plan and prepare MLP lessons sufficiently."*

Mr Takui: *"Oho (looking surprised)."*

Mr Kapu: *"Yes, very true, Mme, that is the correct way to do it."*

Mrs Leeuw: *"Why do we have to such material, good colleagues?"*

Mr Kapu: *"Number one, a lesson plan template guides us on how to prepare a lesson; two, a work schedule or pace indicates how long in days/weeks we are expected to teach and complete this topic; three, textbooks provide a topic content to be delivered at an allocated time, including its learning outcomes; four, the syllabus shows what students will be judged on at the end of the lesson or what students are expected to have at the end of the lesson, the syllabus also provide content of the topic, specific and learning outcomes."*

Mrs Leeuw: *"Is there anyone who can indicate why we need such material to plan and prepare lesson?"*

Mr Koena: *"No, I think Mr Kapu explained it well and touched aspects that I would also bring to light when I respond to you."*

Mrs Leeuw: *"Ok, let's start planning and preparing the MLP lesson. Mr Takui, as our organiser today, can you organise and put lesson template, NCV L4 mathematics work schedules, NCV L4 mathematics syllabus and NCV L4 mathematics textbooks in place so that at the next session can start."*

Mr Takui organised teaching materials as per request and set enough materials and copies on each table all co-researchers. Co-researchers were also given extra notes

for the RME approach and the MLP topic discussed in the above Sections (cf. 4.4.1 & 4.4.2).

Mrs Leeuw: "Please go through these documents and textbooks, then in the next session we are going to plan, develop and prepare the MLP lesson."

And at the next session:

Mrs Leeuw: "Today our focus is on planning, developing, and preparing the MLP lesson and I know we all have an idea about planning and developing and preparing the MLP lesson. So where do we begin?"

Ms Moleko: "Let's start first to divide MLP topic into conversion of case studies into linear inequalities; secondly, solve linear inequalities and find coordinates; thirdly, sketch linear inequalities, find feasible region and also maximum (or minimum) values (make MLP graph) separately; fourthly, do a revision of the whole process and teach case studies, linear inequalities, graphs, feasible region and determining max (or min) values in one lesson because the MLP topic includes the mathematical word problems, linear inequalities and straight line graphs within which cannot be taught in one lesson at the first time because students will be confused and frustrated."

Mrs Matlou: "I agree with you, Ma'am, but don't you think that we should teach MLP terminologies first before teaching MLP case studies, linear inequalities, and graphs?"

Mr Kapu: "We should also continuously show students importance and relevancy of the MLP topic so that we attract students to the lesson and also increase their interest to mathematics."

We as a research team agreed with what Ms Moleko, Mrs Matlou and Mr Kapu suggested.

Mr Takui: "Colleagues, since now we have topic and subtopics to teach, how should we use the RME approach to teach MLP topic and subtopics?"

Mr Phoka: "Colleagues, remember in earlier meetings we agreed that we are going to employ activity, interactivity, level, intertwined, realities and guidance principles of the RME approach in order to implement the RME approach successfully."

Mrs Leeuw: "How should we apply these principles, Colleagues?"

Ms Lebone: "For activity principles, we should provide students with daily activities, like classwork or homework, maybe prior, during and/or at the end of the lesson."

Mrs Seeta: "Based on interactivity principle definition, we should encourage interaction among students and give them work that expect them to work together in pairs or in groups, so could be in the form of discussions and class activities. Again, we as lecturers and facilitators should interact and engage with students during the teaching and learning process."

Mr Phoka: "Applying the level and intertwined principles, we start teaching MLP from MLP terminologies; move to the next level, the conversion of MLP word problems and linear inequalities; move to the next level, the sketching of linear inequalities, identifying feasible area, finding a max (or min) values; move to the higher level for teaching MLP case study, linear inequalities, graphs, feasible region (area) and max (or min) values in one lesson at the same time; and lastly do revision of the whole process and test student's MLP knowledge by giving short test."

Mr Kapu: "Based on reality principle definition, we should use practical examples, realities, pictures, indigenous languages, use class or home activities talking to real-life stories and integrate real life situations in the teaching of the MLP topic."

Mr Koena: “To apply guidance principle, we can use a scaffolding technique and only assist students whenever they need help and leave them to complete tasks, but the idea is to allow students to make errors, mistakes and misconceptions so that they can learn from their mistakes. Again, we can guide and support students when they are unable to solve some MLP problems or unable to understand some of MLP concepts. Lastly, we can guide them and give students key information.”

Mrs Leeuw: “All right, thanks, Colleagues, so how can we see that the RME approach is working?”

Mr Phoka: “How about giving students a short MLP test before teaching the MLP topic using the RME approach and also give students a short MLP test after teaching the MLP using the RME approach.”

Mr Kia: “Results of students in tests or examinations will give us an indication whether the RME approach worked or not.”

Mr Phoka: “We can also use lesson observations to see whether through the RME approach work or not. To see whether students are following lesson presentations or not.”

We as a research team worked together to develop these lesson plans below,

The first part of the first lesson

Conduct pre-short test

Table 4.12: First lesson plan developed by the research team

The first lesson plan

Duration	50 min
Topic	MLP terminologies
Lesson objectives	List the MLP terminologies Define the MLP terminologies Create practical examples using MLP terminologies Write the MLP terminologies mathematically
Lesson preparation	
Teaching aids	Textbooks, whiteboards, markers, pictures, template of MLP terminologies developed by co-researchers, internet, coins, money papers, case studies, newspapers
Formative assessments	Class activity (Activity A) Home activity (Activity B)
Reflection	
Teaching method	The RME approach through the principles such as activity, interactivity, levels, intertwined, realities and guidance

Table 4.13: Second lesson plan developed by the research team

The second lesson plan

Duration	50 min
Topic	MLP word problems and linear inequalities
Lesson objectives	Understanding MLP word problems Identify variables and constraints Convert MLP word problems into linear inequalities
Lesson preparation	
Teaching aids	Textbooks, whiteboards, markers, pictures, case studies, newspapers, internet
Formative assessments	Class activity (question A and 1) Home activity (question B and 2)

Reflection	
Teaching method	The RME approach through the principles such as activity, interactivity, levels, intertwined, realities and guidance

Table 4.14: Third lesson plan developed by the research team

The third lesson plan

Duration	50 min
Topic	Linear inequalities and MLP graphs
Lesson objectives	Define linear inequalities and MLP graph Solve linear inequalities and find coordinates Plot and join coordinates and sketch linear inequalities Identify feasible region or area Determine maximum or minimum points Calculate maximum (or minimum) values
Lesson preparation	
Teaching aids	Textbooks, whiteboards, different ink markers blue-black-green-red, T-square rulers, internet, graph pads, crayons.
Formative assessments	Class activity Home activity
Reflection	
Teaching method	The RME approach through the principles such as activity, interactivity, levels, intertwined, realities and guidance

Table 4.15: Fourth lesson plan developed by the research team

Fourth lesson plan

Duration	100 min
Topic	MLP topic (word problems, linear inequalities and graphs)
Lesson objectives	List the MLP terminologies Define the MLP terminologies Create practical examples using MLP terminologies Write the MLP terminologies Mathematically Understanding MLP word problems Identify variables and constraints Convert MLP word problems into linear inequalities Define linear inequalities and MLP graph Solve linear inequalities and find coordinates Plot and join coordinates and sketch linear inequalities Identify feasible region or area Determine maximum or minimum points Calculate maximum (or minimum) values
Lesson preparation	
Teaching aids	Textbooks, whiteboards, pictures, template of MLP jargons and language developed by co-researchers, internet, coins, money papers, case studies, newspapers, T-square rulers, the different ink markers blue-black-green-red, graph pads, crayons.
Formative assessments	Class activity (activity C) Home activity
Reflection	
Teaching method	The RME approach through the principles such as activity, interactivity, levels, intertwined, realities and guidance

Fifth lesson plan: Do revision of the whole process and conduct post-short test

In the above conversations, co-researchers agreed that teaching MLP using the RME approach requires of mathematics lecturers and facilitators to plan, develop and

prepare the MLP lesson adequately before teaching of any kind can take place, because it is what the DHET expects of all lecturers at TVET colleges. The lesson planning and preparing, according to co-researchers, will make it easier for lecturers and facilitators to impart MLPCK well. This was proved by the following statements.

Mrs Seeta: *“The department expect all lecturers to plan and prepare lesson daily or weekly. So, it is important also for us to plan and prepare the MLP lesson adequately in order for us to deliver MLP lesson well.”*

Mr Kia: *“This is where we should start ... if we do not plan and prepare lessons we will be deviating from the department policy, which will not be accepted. So, I think we should together plan and prepare MLP lesson so that we present the MLP lesson in an easier way; it is vital for all lecturers to plan and prepare the MLP lesson sufficiently on daily or weekly basis in order to teach MLP topic well.”*

We then agreed together as a research team that the successful implementation of the RME approach relied on sufficient lesson planning and preparation. Subsequently, we started to plan, develop and prepare the MLP lesson together. To do this, we consulted and used the NCV lesson template for TVET colleges, mathematics work schedules, different mathematics textbooks and the mathematics syllabus. These were the reasons why such resources were consulted and used.

Mr Kapu: *“A lesson plan template guide us on how to prepare a lesson; ... work schedule indicate how long in number days/weeks we expected to cover and complete this topic; ... textbook provide a topic content to be delivered ... including its learning outcomes; ... syllabus shows what students will be judged on at the end of the lesson or what students are expected to know at the end of the lesson and also it provide content of the topic, specific and learning outcomes.”*

Moreover, adequate knowledge on the MLP topic and the RME approach assisted us to plan, develop and prepared the MLP lesson. As a result, we came up with lesson plans on the above Table 4.12-4.15, which helped us to impart the MLPCK easier, using the RME approach (cf. 4.3.1). This is in line what Coppola *et al.* (2004, in Nasari & Heidari, 2014:26) highlight in Section 2.4.3.3 and they are in agreement that educators should plan, develop and prepare lessons on a daily basis. However, they argue that educators should not be in class if they have not prepared and developed a lesson plan.

4.4.4 Dedicated and enthusiastic team

A dedicated and enthusiastic team in this study is explained as group of people working in collaboration, committed and work towards achieving a set common goal of a team. Thus, in this study the team was established to use and implement the RME

approach to teach MLP topic and everyone was eager and committed to participate in this study in order to look ways that improve mathematics performance. Jita and Mokhele (2014:1-15) and Lenski *et al.* (2009:50) recommend the use of team teaching, cluster, collaborative and lesson study approaches to improve the teaching and learning and also students' results (cf. Section 2.4.3.1). The successful implementation of these approaches rely on a dedicated and committed team that will plan, prepare and present lessons together. In Malebese's research study a dedicated team was formed to use the Socially Inclusive Teaching Strategy (SITS) to teach main language domain; secondly, the team of SITS was tasked to integrate diverse context and situations that learners come with to the class because learners' diverse contexts are at the centre of teaching English literacy in a socially inclusive way; and the team of SITS used a SWOT analysis to ensure that learners meet the expectations of the curriculum (Malebese, 2016:160).

In Mosia's (2016:138-139) work, a team was established an aim to create an enabling environment for the use of ICT software to teach Euclidean geometry. The TVET college encourages lecturers to work together and apply a team teaching approach, but this happens towards the final examination period during rallies.

During the research meeting, the co-researchers agreed that to use the RME approach to teach MLP topic at TVET colleges, co-researchers needed to work in collaboration.

Kapu: *"Colleagues, we are all aware that mathematics is a problematic subject at our college and also for our students. So, this need us to work together as a team and be committed to improve our teaching of mathematics. I believe in this research study we are going to benefit from one another because we are going to share our experiences, problems, challenges and more importantly, share our best practices. I urge you to bring all problems you are faced with daily so that we work together to find solutions."*

Mr Phoka: *"For me, I am happy to see everyone here, lecturers, Campus Manager, Campuses Head of Departments, NCV Supervisors, mathematics Coordinator, Council Member and parents. So, coming together under one roof will assist a lot, because we all understand what are MLP teaching and learning challenges or where are challenges and/or what contributes to the challenges (including mathematics in general) and together we will find a way to overcome those challenges and also see how to use the RME approach to teach MLP topic."*

Mrs Mohlomi: *"Le rona re thabile re le batswadi hoba karolo ya kopano tsena tsa mofuta ona hobane re tshwanetse ho bapala karolo ya rona thutong ya bana ba rona. Hape re tsebe hore bana ba rona ba hloleha kae kapa ha ba sebetse hantle kae, mme re tsebe ho fana ka tshihetso e tshwanetseng hobane matithere le management wa sekolo ba keke ba kgona ba le bang (Even us as parents are very happy to be part of these meetings, because we are supposed to play a role in our children's education, so that we know where our children*

struggle or where they do not perform well, in order for us to provide them with appropriate support, because lecturers and management of the school cannot succeed alone.)”

Mrs Seeta: *“I agree with everyone, because we need to work together and in collaboration as always. I am very happy to see our lecturers here with us, because I think they have enough information about what is happening in mathematics class level (on the ground), like they know what or where are problems in mathematics .”*

Ms Lebone: *“We are also happy to part of this kind of meetings that are looking deeply how to improve mathematics teaching and learning and also results. We believe that we are all going to learn from one another and share knowledge.”*

Mr Takui: *“I don’t know what to do now, I have tried a lot of strategies but these students are not serious. They do not honour their schoolwork; they are not committed at all. But I am looking forward to see what we can do together as a team to improve the work.”*

Mr Kapu: *“Thanks a lot, colleagues for appreciating this platform. Where do we go from here?”*

Ms Moleko: *“To use this approach what do we call ... I mean, the RME approach to teach the MLP topic, I think we need to work together as team because it will be the first time for most of us using this approach.”*

Mr Phoka: *“Again, we as research team need to understand our roles in this study. We need to work together to perform four functions that managers and or researchers namely, planning, organising, leading and controlling in this study. Firstly, we are going to plan meetings and lessons; secondly, organise research meetings and teaching and learning aids together; thirdly, share responsibilities in chairing research meetings, presenting and observing lessons; fourthly, evaluate lesson presentations and research processes in this study; and lastly re-plan, organise, lead and control and repeat the whole process.”*

Thereafter, we decided on a convenient research site to conduct research meetings, lesson observations and presentations. Together we developed weekly schedules for our research meetings, lesson observations and presentations in this study that were convenient for all of us (cf. 3.8). At next sessions, we together as a research team discussed how to enhance the MLP teaching using RME approach at TVET colleges.

Mrs Leeuw: *“Since we agreed to use the Realistic Mathematics Education (RME) approach to teach MLP topic, is there anyone who understands what the RME approach is all about, in order to for us to implement and use what we know and fully understand or have an idea with?”*

Ms Seeta: *“Understanding mathematics content comes first before techniques, because if we understand content it will be easier for us, to develop many ways solve mathematics problems. The department expect all lecturers to plan and prepare lesson daily or weekly so that lecturers deliver mathematics content. So, it is important also for us to plan and prepare the MLP lesson adequately in order for us to deliver MLP lesson well.”*

We then discussed the RME approach and planned how we would use it. Thereafter, we shared our knowledge about MLP and how to teach it (cf. 4.4.1 & 4.4.2). We concluded we had to integrate diverse contexts, realities, indigenous languages and real-life situations that students bring to the classroom. These contexts, realities, prior knowledge and situations are inevitable, and may include, but are not limited to the way students access and construct a new MLPCK, because a language facilitates

MLP word problems (Meirim *et al.*, 2010:43). Students' diverse experiences, prior knowledge and learning styles are at the centre of teaching and learning using an RME approach, because a variety of contexts and situations students exposed to explains why any SCK is difficult, complex and dynamic (Perkins & Wirth, 2008:12-14; Terry & Irving, 2010:110-111, in Malebese, 2016:161). Based on these complexities, multiple of teaching and learning approaches become inevitable (Grundy 1987:19; Bartlett, 2012:63). Hence, the RME approach becomes imperative, since it is multi-layered, multi-tiered and interdisciplinary by nature, linking well with Bricolage (cf. 2.2.2). This assisted us to plan, develop and prepare MLP lessons and also organised various teaching and learning aids that assisted in the implementation and use the RME approach to teach MLP (cf. 4.4.3).

Our plan as a team included one mathematics lecturer responsible for addressing the MLP topic; two assistant facilitators to assist in facilitations, marking and providing the students with feedback in time; dividing the MLP topic into MLP components; develop MLP formative assessments and grouping students according to their abilities (cf. 4.3). This was evidenced by the following statements:

Ms Moleko: *"Here with me, I am with my team 'mathematics experts' whom I am going to work with in enhancing the MLP teaching and learning using the RME approach.*

You have 12 minutes to complete it."

This collaboration was realised throughout lesson presentations in Section 4.3.1.

In the above extracts the co-researchers indicated that to enhance MLP teaching and learning using the RME approach, the co-researchers should work together.

Ms Moleko: *"To use this approach to teach the MLP topic we need to work together as a team."*

Mr Kapu: *"Colleagues, we are all aware that mathematics is a problematic subject for students at our college. We need us to work together as a team and be committed to improve our teaching of mathematics ."*

These quotes proved that co-researchers have a strong belief in one another and are of the view that working together as a committed team can improve MLP results. The collaboration and commitment aided the research team to fulfil and perform our functions in this study, such as Planning, Organising, Leading, Controlling (POLC). Together we planned research meetings, lesson observations and lesson presentations; organised research meetings and teaching and learning resources; directed meetings, presented lessons and observed lessons; and lastly, reflected on

lesson presentations, also evaluating the use of the RME approach to teach MLP (cf. 3.8 & 4.2–4.5). During research processes in this study, the co-researchers were treated fairly, equally, and with respect, enabling us to share our perspectives in our meetings without any form of discrimination.

Ms Moleko: *“Let’s start first to divide the MLP topic because it includes mathematical word problems, linear inequalities and straight line graphs, which cannot be taught in one lesson the first time because students will be confused and frustrated.”*

Mrs Matlou: *“I agree with you, Ma’am, but don’t you think that we should teach MLP terminologies first before teaching MLP case studies, linear inequalities, and graphs.”*

Mr Kapu: *“We should also continuously show students importance and relevancy of the MLP topic so that we attract students to the lesson and also increase their interest to mathematics.”*

These words show teamwork among co-researchers and are also in line with Bricolage postmodernity and post-experimentalism. During the Bricolage post-experimentalism and postmodernity era, the stories and experiences of a small number of people were preferred. Moreover, these Bricolage moments allow the principal investigator to work together with participants to find solutions to their problems; in essence, researchers are closer to participants. In addition, in these moments the research participants were elevated to researcher level because they could decide the agenda in the research meetings, which is different to other research paradigms (cf. 2.2.2.5 & 2.2.2.6). These actions link well with the PAR because people who were directly and indirectly faced with mathematics problems every day were involved in the research processes, performing POLC, decision-making process, and developing solutions to the problems.

Thus, the principles of PAR were also realised in this study, because we as a research team, we worked collaboratively. More importantly, we interacted, shared knowledge and allowed us to learn from one another, as a result, this proved discursive practice and social inclusion (cf. 3.2.1 & 3.8). The composition of the research team in this study gave us all an equal opportunity to share our views and experiences freely without any form of discrimination. This platform enhanced our MLPCK (including other mathematics concepts) and classroom management through our social interaction.

To succeed in implementing the RME approach, we conducted a SWOT analysis to evaluate our strengths, weaknesses, opportunities and threats to find the best ways to use the RME approach in responding to challenges in the teaching and learning of

MLPCK that had been identified in NCV mathematics L4 classes at TVET colleges (Rogoff, Correa-Chaves & Silva, 2011:155). This study also used a SWOT analysis as a tool and instrument of measurement, for generating data required for learning to happen. The team ensured that the RME approach was employed in a conducive environment, where interaction between students and their lecturers and also among students was possible. Moreover, the team incorporated real-life situations, indigenous languages, and teaching and learning aids, allowing students to play a major role in the teaching and learning process.

In addition, informing students prior to the commencement of lessons resulted in students who were more focused, involved, interested and motivated, because they took responsibility for their own learning, even prior to a learning activity, and offered opportunities for vital interactions between the facilitator and learners. The team also used the SWOT analysis to ensure that students meet the expectations of the NCV L4 Subject Guideline, which helped them to become critical thinkers, using reasoning skills to solve MLP problems. Thus, by so doing, the team prioritised the teaching and learning and showed dedication to learning. In addition, team members shared their knowledge and understanding of the RME approach, which included the principles of RME. The team was eager to improve the teaching of MLP at TVET colleges and mathematics performance, using the RME approach, since MLP and other mathematics concepts are problematic, thus impacting mathematics performance. We as a team managed to plan, develop, prepare and present MLP lessons together. This links well with what other scholars found effective (Lenski *et al.*, 2009:50; Jita & Mokhele, 2014:1-15; Mosia, 2016:139).

4.5 THREATS AND RISKS TO THE SUCCESSFUL IMPLEMENTATION OF THE RME APPROACH

This section of this study will discuss those factors that are threats to the proper use and successful implementation of the RME approach, and reveal how these threats can be addressed so that the RME approach can be implemented successfully.

4.5.1 Lack of teamwork

Lack of teamwork takes place when mathematics lecturers at the same institution work in isolation. In some instances, mathematics lecturers, NCV Supervisors, Head of Departments, Campus Managers, mathematics coordinators, parents and other stakeholders do not work in collaboration, impeding mathematics teaching and learning. According to Mosia (2016:11-12), there is a need for a team approach to improve TPACK for teaching and learning of Euclidean geometry using ICT and software because sharing of knowledge and skills not only contributes to enhance individual team members' knowledge, but also increases students' chances of learning. The TVET college involved encourage lecturers to work together and use a team teaching approach, but this usually happens towards the end of an examination period. The co-researchers in this study agreed that there was a lack of teamwork teaching MCK at this TVET college among mathematics lecturers, CMTs, mathematics coordinators, and other stakeholders. The co-researchers were of the view that this absence of teamwork was a threat to the success of the RME approach.

Mr Kapu: *"Most of my staff members prefer to work alone in their spaces, even if you try to show them the importance of teamwork or team teaching method, or inviting special guests from within or outside the college to assist one to teach a particular topic or a problematic topic. This situation will affect the implementation of the RME approach, because it needs of us to put minds together before using it to teach MLP topic. How is that possible if mathematics lecturers prefers to alone."*

Mr Kau: *"Nna, I prefer to teach my students, not someone who will confuse my students. Hee, Ntate, I once let someone to come to help my students, he confused my students. When I came trying to teach, students told me no, no, Ntate (referring to invited guest) said, not that, so they were no longer listening at all. So, now I prefer to teach my students alone."*

Mrs Leeuw: *"I think the lack of teamwork affect students' learning negatively, because If someone (including myself) do not understand some concepts (topics) well, we usually try to teach it as it is, straight, no examples, no pictures and we continue with other topics. So, if the lack of teamwork continues, we will not be able to use the RME approach to teach any topic because to use the RME approach to teach MLP topic we together plan and present lessons."*

Mr Kanana: *"The team teaching approach or teamwork does not happen at our college because time is not always enough especially on our side."*

Mrs Lebone: *"Always our Campus management talks about this approach ... mmm ... team teaching approach, but I have never seen it being implemented or tested properly. Again, there is no support or guidance from our management on how to use teamwork to teach mathematics."*

Mrs Lebone: *"Some of our colleagues are taking advantage of us or misuse this approach because they want us to teach their students, because they are always not there when we come to help. So, how can you help someone who is always not there but asks for help; tell you what, it is very painful because we take our precious time and add more load. If this continues we will not be able to use the RME approach because it needs us to work together if we want to implement it well."*

Mr Kanana: *"Teamwork is not happening at our college."*

Ms Khaba: *“Lecturers are not willing to work together at all. Some of our colleagues are anxious that other colleagues might laugh at them when they struggle with some of mathematics topics”*

Mr Kapu: *“I preached the team teaching long ago, but lecturers are not comfortable to work together at all. Some do not accept that they do not know.”*

The above quotations prove that there is a lack of collaboration and teamwork among mathematics lecturers, NCV Supervisor, HoDs and Campus managers. These quotations show that mathematics lecturers work in isolation. The absence of teamwork among mathematics lecturers transpired prior to and during the Bricolage traditional period, which encouraged people to work in segregation, because they are regarded as powerful individuals knowing everything (cf. 2.2.2.1). This is not in line with PAR due to the fact that mathematics lecturers at this TVET college work in isolation. According to Malebese (2016:81-83), the PAR encourages interaction between people and also promotes cooperation (cf. 3.2.1). The above statements show that it is the norm at this college that mathematics lecturers work individually and this shows a social exclusion. The limitations of the absence of a team teaching approach, a lack of teamwork and mathematics lecturers working in isolation denies them the opportunity to share their knowledge and skills, which could improve their PCK and form new knowledge (Cobb, 1994:17; Hu & Linden, 2015:1104). Moreover, according to Jita, Maree and Ndlalane (2008:475), this denies them the space and opportunity to collaboratively improve one another's lesson planning, preparation, assessment, and lesson facilitation.

The lack of collaboration between mathematics lecturers and also among lecturers and CMTs is the result of the following factors, according to co-researchers, namely, limited time, no support and guidance, manipulations, anxious and lack of acknowledgement about MCK deficiency.

Mr Kakana: *“Teamwork is not happening at our college because time is not always enough especially on our side.”*

Mrs Lebone: *“Our Campus management talks about this approach, but I have never seen it being implemented properly. There is no support or guidance from our management on how to use teamwork to teach mathematics. Some of our colleagues are taking advantage of us or misuse of this approach because they want us to teach their students because they are always not there when we come to help.”*

Ms Khaba: *“Some of our colleagues are anxious that other colleagues might laugh at them where they struggle with some of mathematics topics.”*

Mr Kapu: *“Some do not accept that they do not know.”*

These extracts demonstrate hindrances to the teamwork to work sufficiently. To overcome this threat and risk, the above factors had to be addressed to implement the RME approach successfully.

- One of the CMT members should be involved in the teaching and learning processes (lesson planning, development, preparations, presentations and reflection) so that team teaching is possible.
- CMTs should develop a programme for the implementation of team teaching approach on a monthly or quarterly basis.
- CMTs should support and guide mathematics lecturers on how to use a team teaching approach so that lecturers could have a picture of when a team teaching approach could be used, where and how to start.
- CMTs and lecturers teaching mathematics on the same level should plan, develop and prepare one rich MLP lesson together, and then share roles and responsibilities during lesson presentations. For instance, if one presents a lesson, others can assist to facilitate a lesson, mark activities, give feedback and lastly, reflect together on lessons together (cf. 4.3 & 4.4).

The successful execution of the RME approach in Section 4.3 heavily depends on better conditions in Section 4.4, specifically on a dedicated team. We as a team shared responsibilities, knowledge and skills about MLP and the RME approach, and planned, developed and prepared together. The teamwork approach was used to implement the RME approach to teach MLP (cf. 4.3).

4.5.2 Lack of lesson planning and preparation

This study defines lesson planning as lecturers' mental image on what SCK to teach; what pedagogy methods are suitable to teach identified SCK; what SCK objectives are to be achieved; what resources are needed to teach SCK; what tools should be used to measure SCK success; how much is time needed to teach SCK; and when to teach SCK. According Khan (2011:3958), lesson planning is a blueprint that every lecturer should prepare and use to teach any SCK, because it saves time allocated to teach, and energy and also it gives a maximum output (cf. 2.4.3.3). This blueprint refers to a lesson plan, which includes the module; lesson outcomes; link current with previous knowledge; prepared subject matter; teaching and learning aids; classroom

activities; relevant teaching methods and reflection. The lesson planning and preparation should be done on a daily basis, guiding lecturers and facilitators on what to teach, what to achieve, how teach and when to teach it, what resources to use, and how to reflect on the lesson plan. All of this must be aligned with the syllabus, curriculum and work pace (Khan, 2011:3958; Nasari & Heidari, 2014:25). If teachers, lecturers and facilitators do not plan and prepare lessons, they should not be in the classroom to deliver the lesson.

The DHET expect of all TVET college lecturers to plan and prepare lessons, because this is considered one of lecturers' tools of their trade. This blueprint is required by the DHET to be prepared and also be available all times. The examined TVET college designed and developed lesson plans and presentation frameworks in line with DHET guidelines. It includes date (duration), module, specific outcomes, learning outcomes, lesson preparation, teaching media, formative assessments, and reflection (cf. 4.4.3). However, at the research site, through observations, discussions and interviews, I found that most lecturers did not plan, prepare, and develop lesson plans before presentations but rather did lesson plans when files were about to be submitted for moderation. Others did not see the importance of lesson planning. Other lecturers printed lesson plans they developed in previous years, filled in the reflection section and file them long before going to class, keeping them in subject files for moderation.

Mr Kia: *"The problem that we have is that most lecturers prepare lesson plans towards files monitoring, because then they run all over the place trying to print all lesson plans for all needed files. Again, during internal files monitoring at campus level, we usually find that some files do not have lesson plans. So, if files don't have lesson plans, there is a likelihood that lecturers did not plan lesson. Again, other files have lesson plans, but we find those are not reflected."*

Mr Kanana: *"During the spot check I found that most of our lecturers attended classes without lesson plans and also without adequate resources, but during QMS, things are different; everything is in place, like lesson plans, resources and class activities. So, it might happen that they keep them in their offices or files."*

Mrs Lebone: *"You know, we do lesson plans prior and we keep them in our files so, I feel like we are throwing them away."*

During the registrations at the beginning of the year Mr Lehapu (Mathematical literacy lecturer) asked Mrs Lebone the following:

Mr Lehapu: *"What are you doing there, because we have not yet started with classes?"*

Mrs Lebone: *"I am changing the dates of the previous lesson plans because I want to print them and file them because when they are wanted during moderations, the printing paper usually is not there, electricity is a problem and also more people want to print. Ke semphete ke o fete (people are going up and down)."*

Mr Lehapu: “Hei, its very true. Hei, I remember well. On other day last year I left it late, I mean very, very late doing lesson plans and linking them to the work schedule, because we did not start at the same time with our sister campuses. I do not want to see that again. I think your strategy will work for me.”

Observation was done at the research site when lecturers engaged with one another regarding files, lesson planning and preparation and outcomes of the briefing session.

Ms Mosomi: “Hei, Meneer, have you heard that files are needed on Friday.”

Mr Koena: “No, eish.”

Ms Mosomi: “We heard that in the morning in the briefing session.”

Mr Koena: “Iyoh, I am going to start with lesson plans. Eish, these lesson plans, neh!”

Ms Mosomi: “Lesson plans just waste our time. We just want to teach and push the syllabus. We are behind with the syllabus and students are supposed to write test within the next two weeks. Nna, I just don’t remember when last I did plan a lesson? I just do them for compliance because they are needed in our files during files monitoring and must link with work schedules.”

Mr Koena: “Ienna, I just do that because what is important is to teach students.”

Mr Takui (mathematics lecturer) joined Mr Koena and Ms Mosomi.

Mr Koena: “Let’s hear Mr Takui about lesson plans. Mr Takui, do you prepare lesson plans?”

Mr Takui: “To tell you the truth I don’t (laughing). What I do I just take my book and whiteboard marker and go to class to teach. But, I do them during files monitoring and QMS.”

Ms Mosomi: “Hei, our Supervisor is very strict, so always we must ensure that they link with work schedule.”

Mr Koena: “Eish, eish, lesson plans, neh, but when are we going to get time to prepare lesson plans because we attend meetings that are planned with short period of time and also takes place in the teaching times? Again, look our workload. On top of that, today they want this, tomorrow that.”

The absence of lesson planning and preparation was also realised in the mathematics lesson presentation presented by Mr Takui (cf. 4.2.1). Look at the following extract.

Mr Takui: “Good afternoon, Students.”

Students: “Afternoon, Sir.”

Mr Takui: “Today we are going to do MLP.”

Mr Takui came into the classroom without a lesson plan, or even textbook. He took Dikotlolo’s (student) textbook.

Mr Takui: “Ok, let’s go to page 80 of your textbook. Right, this is MLP, so, to solve this kind of MLP problem in L4, we first need to convert MLP word problems into linear inequalities; secondly, we solve linear inequalities; and lastly we draw MLP graph with that good sequence.”

Mr Takui took students through an example on page 84, thereafter demonstrated Problem statement 1 to show students how to solve the MLP problem. Mr Takui read aloud:

A small factory makes two types of washing machines, a standard and a luxury model. The daily production cannot be more than 6 standard and 3 luxury washing machines. There are 8 factory workers and it takes one worker to make one standard model and 2 workers to make one luxury model per day. The profit on the standard model is R300 and on the luxury model R600. Questions:

1. Write down linear constraint and inequalities.
2. How many of each type of machine must be manufactured per day to make a maximum profit?

He then interpreted Problem statement 1.

Mr Takui: “Based on what I have read, firstly remember what I said earlier on, we need to write linear inequalities to answer question number 1. Secondly, we have to solve linear inequalities and then draw MLP graph so that we can find feasible region, in order to determine maximum profit in answering question number 2.”

From the first sentence I have read (cf. **above problem statement 1**), we find two types of machines manufactured by this company, so we first let...

Type A (standard machine) be x

Type B (luxury machine) be y

Remember, always to show these linear inequalities...

$$x \geq 0 \text{ \& } y \geq 0.$$

After Mr Takui had explained everything, he approached the whiteboard and wrote the linear inequalities and objective function (cf. Fig.4.1). Next, Mr Takui showed students how to draw linear inequalities, show feasible region and determine maximum value.

At the study research meetings, co-researchers indicated that lack of lesson planning and preparation were threats that hinder the successful implementation of the RME approach to mathematics and impacted negatively on the teaching and learning of mathematics, thus contributing to poor performance.

Mr Kia: “Lack of lesson planning and preparation also will impact the implementation of the RME approach negatively because to use the RME approach earlier on we had to plan and prepare MLP lessons thoroughly. Hence we succeed to implement the RME approach to teach the MLP topic.”

Mr Takui: “Doing lesson plans for only files moderation will not assist us at all to implement the RME approach.”

Mr Khodu: “I agree with my colleagues there. To say if we do not plan and prepare lesson we will not be able to use the RME approach well because it need us to cover and follow all six rules in order to organise enough resources.”

Mrs Seeta: “Absence of lesson plans prove that lecturers do not plan and prepare lessons and this affects the teaching and learning negatively and also contributes to poor performance

in mathematics . Again, if lesson plans are not done, what are lecturers teaching? How do they see whether they have achieved lesson objectives or not?"

Mr Kapu: *"Neglecting lesson planning and preparation affect the teaching and learning of mathematics badly, because a lecturer will not know what topic to treat first, what lesson objectives to achieve, what content to include or not to include, ... ehh ... mm ... what resources needed, what methods to use to teach that topic. So, this will waste allocated time to teach, the lecturer will be lost, ineffective and will even confuse students. So, students will not perform well. Again, if lecturers do not do lesson plans that is a serious deviation."*

Mr Matlou: *"An individual mathematics lecturer should plan, prepare and develop lesson plans well in order to use the RME approach to teach mathematics , or one can collaborate with other mathematics lecturers at the same campus or at other campuses or at other colleges, or even with his/her CMT members who understand mathematics , like we are doing now."*

The statements by Mr Kia and Mr Kanana exposed the absence of lesson plans, showing a lack of lesson planning and preparation; and secondly, showed that lesson plans were done, but were kept in lecturers' subject files or in their offices. This was confirmed by the following conversation

Mr Koena *(immediately after hearing that files were needed): "Iyoh, I am going to start with lesson plans."*

This statement proves the discursive practice mentioned by Mr Kia, which exposed that most of lecturers only did lesson plans when files were needed for moderation. This was also realised in the following statements:

Mrs Mosomi: *"Nna, I just don't remember when last I did lesson plan? I just do them for compliance because they are needed in our files during files monitoring."*

Mr Takui: *"I do them during files monitoring and QMS."*

The absence of lesson plans could be seen in Mr Takui lesson presentation extracted from a full lesson presentation in Annexure J. Lesson plans were only done with the aim of being included in files; not necessarily for guiding lecturers on what topic to be taught, what learning objectives to be achieved, what content to be covered, what teaching aids were needed, which teaching approaches were appropriate, and which tools were needed to measure success. According to Amininik *et al.* (2000, in Nasari & Heidari 2014:26) and Khan (2011:3958), lesson planning and preparation should be done adequately prior to lesson presentations, because these guide lecturers properly and also promote education quality (cf. Section 2.4.3.3). Coppola *et al.* (2004, in Nasari & Heidari, 2014:26) further argue that teachers, lecturers and facilitators should not be in class, delivering SCK without a lesson plan, as required for successful teaching.

The literature showed that the lesson planning and preparation are vital because it give lecturers clear direction about what topic to teach, lesson objectives, what content to teach or not to teach, what teaching and learning aids are needed, what suitable teaching methods are needed to teach a particular topic and what assessment tools are needed to measure students' success (cf. 2.4.3.3 & 4.4.3). In light of this, it is clear that the absence and lack of lesson planning and preparation impact teaching and learning negatively.

Mr Kapu: *"Neglecting lesson planning affects the teaching and learning of mathematics badly because a lecturer will not know what topic to treat first, what objectives lesson aims to achieve, what content to include or not to include, what resources needed, what methods to use to teach that topic."*

Mrs Seeta: *"Absence of lesson plans affect the teaching and learning negatively ... if lesson plans are not done, what are lecturers teaching? How do they see whether they have achieved lesson objectives or not?"*

The empirical data showed that the lack of lesson planning and preparation wasted teaching time, made lecturers very unproductive; compromised learning outcomes, and led to students' confusion and misconceptions; as a result contributing to students' poor performance (cf. 2.4.4.4 & 4.2).

Mr Kapu: *"This will waste allocated time to teach, lecturer will be lost, ineffective and will even confuse students. So, students will not perform well."*

Mrs Seeta: *"Absence of lesson plans contributes to poor performance in mathematics."*

The co-researchers maintained that a lack of lesson planning and preparation and/or developing lesson plans only for file moderation purposes was a threat and risk to the successful implementation of the RME approach to enhance and teach MLP (including other mathematics concepts). This was evidenced by the following statements

Mr Kia: *"Lack of lesson planning and preparation will impact the implementation of the RME approach negatively."*

Mr Takui: *"Doing lesson plans only for files moderation will not assist us at all to implement the RME approach."*

Mr Khodu: *"If we do not plan and prepare lessons, we will not be able to use the RME approach well, because it needs us to cover and follow all six rules in order to organise enough resources."*

Other co-researchers agreed with what had been shared about a lack of lesson planning and preparation as a threat and risk to the successful implementation of the RME approach to teach mathematics . According to the co-researchers, to overcome this threat and risk, an individual lecturer and/or as a mathematics team should appropriately plan and prepare lessons (develop a rich lesson plan) to implement the

RME approach successfully in teaching mathematics concepts, like we did earlier before teaching MLP using the RME approach (cf. 4.3 & 4.4).

Mr Kia: *“To use the RME approach earlier on we had to plan and prepare MLP lessons thoroughly.”*

Mrs Matlou: *“Individual mathematics lecturers should plan, prepare and develop lesson plan well in order to use the RME approach ... Like we are doing now in this study.”*

As a result, the co-researchers were in agreement that one of the conducive conditions towards the successful implementation of the RME approach was sufficient lesson planning and preparation.

Mr Kia: *“Hence we succeed to implement the RME approach to teach the MLP topic ... to use the RME approach earlier on **we** had to plan and prepare MLP lessons thoroughly”*

This shows collaboration among the research team because in this statement one sees the word ‘we’, representing more than one person, while Mr Kia acknowledged the team effort. The word ‘we’ fits well into PAR, because PAR promotes and encourages collaboration among people to solve problems (cf. 3.2.1). More importantly, people who team up are those who encountered a particular problem directly and indirectly, leading to social inclusion. The team managed to plan and prepare adequate lessons and also developed rich lesson plans that assisted in the successful implementation and use of the RME approach to teach MLP in this study at the TVET college. This was also realised in the statement:

“To use the RME approach earlier on we had to plan and prepare ... lessons thoroughly. Hence we succeed to implement the RME approach to teach MLP topics.”

The co-researchers realised the power of the team and learned what people could do together, because we as team members learned from one another and shared our individual MLPCK, MLPPCK, Realistic Mathematics Education (RME) knowledge and lesson planning, preparation and presentation. Therefore, from the beginning until the end of this study, we as a research team collaborated. Mrs Matlou (former mathematics teacher) strongly suggested mathematics lecturers to plan and prepare Math lessons adequately and develop a rich lesson plan so that they could execute the RME approach successfully. She encouraged the collaboration among mathematics lecturers at the campuses of the college or other colleges, as evidenced by the following statement:

“Individual mathematics lecturers should plan, prepare and develop lesson plan well in order to use the RME approach to teach mathematics or one can collaborate with another mathematics lecturers at the same campus or atn other campuses or other colleges or even

with his or her CMT members who understand mathematics . Like we are doing now in this study.”

So, this study promoted the collaborative approach, because this study consisted of different group of people not only lecturers, but the council member, mathematics coordinator, campus manager, HoDs, NCV supervisor, parents and students.

4.5.3 Lack of mathematics workshops and conferences

mathematics workshop/conferences refer to planned, prepared and organised meetings that are meant to train, empower and capacitate lecturers on CK, PCK, CCK, PK, mathematics curriculum, and DHET policies. Zambo and Zambo (2008:159) maintain that there is a need for ongoing professional development for mathematics teachers, because this will change teachers' beliefs about mathematics instruction and the instruction they provide, as well as about changing teachers' beliefs about their individual and collective efficacy. The literature revealed that mathematics workshops benefit mathematics teachers in a number of ways, namely assist them to learn new strategies; diminish teachers' urgency as source of knowledge that provides solutions to problems that produces mental blocks in students; learn from one another; inspire exchange of information among teachers of similar grades and classroom contexts; improve teachers' acuities of their Cultural Relevant Practice Knowledge (CRPK); form networks among mathematics teachers and team leaders to communicate after workshops and lastly, they may find new knowledge of CRPK to validate current practices (Cerdeña *et al.*, 2017: 22-24).

In light of the above information, a lack of mathematics workshops/conferences also denies mathematics lecturers to learn new CCK, PCK and improve areas where lecturers experience deficiencies. The White Paper for Post-School Education and Training revealed that the DHET requires of TVET colleges to build a relationship with universities for training and improving the quality of college lecturers' teaching skills (DHET, 2013:16). This type of training can be done in the form of workshops/conferences conducted within or outside TVET college premises.

The co-researchers revealed that there was a lack of mathematics workshops/conferences at this TVET college. They were of the view that this absence contributed to poor mathematics performance. According to co-researchers, the lack of

workshops/conferences were a threat to the successful implementation of the RME approach.

Mrs Lebone: *"Here at this college there are no mathematics workshops and conferences at all that we are taken to. I don't remember attending even one mathematics workshop and conference at our college."*

Mr Koena: *"I feel like our college is not developing us at all, because we are not taken into workshops and conferences that are specifically focusing on mathematics content and new strategies that are relevant to teach mathematics."*

Ms Khaba: *"There are subject meetings that college conduct to capacitate our staff."*

Mrs Leeuw: *"Our college again conduct in-house training and workshops for assessors, moderators, mentors, coaching and classroom management to develop our staff."*

Mr Kapu: *"We also give bursaries to our lecturers who further their studies."*

Ms Khodu: *"There are subject meetings that we attend, but they should not replace mathematics workshops and conferences, because these subject meetings mostly focus on who set tasks, who moderate tasks and also when we do post moderation for tasks and examinations. So, I think these subjects meetings are too general because they are not focusing on how we should improve teaching and learning of mathematics and also mathematics results."*

Mr Koena: *"What the college does well at is to organize in-house training where CMTs read DHET policies to us or made us aware of revised DHET policies. Secondly, we attend assessor, moderator, mentor, coaching and classroom management workshops which are also too general and once off thing. But, what is lacking is mathematics workshops and conferences. So, we need mathematics workshops so that we can improve our teaching strategies to reach students."*

Mr Takui: *"These subject meetings are not even effective. We are expected to share our problems we experience in the teaching and learning of mathematics and share best practices to improve mathematics results. So, usually most of time there is no Campus manager, HoD, Supervisor and even invited mathematics experts who guide us as mathematics lecturers on where and how we can improve our teaching, we are left alone there stranded ... I mean, to see to it how we can improve mathematics results, we are left there as mathematics lecturers alone and we are on the same level, So, some of our colleagues don't take part because they feel that there is nothing we can do alone because we expect guidance from our superiors. So, these subject meetings just waste our time, I rather work alone here."*

Mrs Lebone: *"The lack of mathematics workshops and conferences contribute to poor performance of mathematics because we do not get proper guidance on how we can improve mathematics. Most of us here at TVET college, like in Engineering, Mechanical, Electrical, Information Technology (IT), Finance Economic and Accounting (FEA) and agricultural campuses, we do not have a Post-Graduate Certificate in Education (PGCE), so we need these workshops."*

Ms Khodu: *"So, if we do not attend mathematics workshops, conferences and meetings, we will not know new teaching methods and how to use those methods to improve mathematics results. E.g. to use the RME approach to teach MLP topic, we had several meetings which focused on mathematics and also consisted with mathematics coordinator, CM, HoDs, NCV supervisors, mathematics lecturers, parents, council member and most who had knowledge in mathematics. If we do not have and attend mathematics workshops, conferences, meetings, we will not be able to implement the RME approach well."*

Co-researchers: *"True."*

Mrs Lebone: *"The assessor, moderator, mentor, coaching and class management workshop are not done regularly and are like short courses. When you have attended assessor workshop you are done with assessor. Next time you can go for moderator only if you are selected because we don't know the criteria used to select people who attend workshop."*

The conversation above between the mathematics lecturers and CMT shows that there is a lack of mathematics workshops, conferences and training.

Mrs Lebone *"There are no mathematics workshops and conferences at all that we can attend."*

Mr Koena: *"We are not taken to workshops and conferences that specifically focus on mathematics content and new strategies that are relevant to teach mathematics."*

These words affirm that there is indeed a lack of mathematics workshops and conferences. However, the CMT argued that the TVET college do provide lecturers with a number of workshops, such as assessors, moderators, mentors, coaching and the classroom management and also provide bursaries to develop them. The college conducted in-house training to inform academics about the DHET and the college policies and lastly, the college did schedule subject meetings that focused on curriculum issues. This was proven by the following statements:

Ms Khaba: *"There are subject meetings that college conduct to capacitate our staff."*

Mrs Leeuw: *"Our college conducts in-house training and workshops for assessors, moderators, mentors, coaching and classroom management to develop our staff."*

Mr Kapu: *"We also give bursaries to our lecturers who further their studies and this was also confirmed by mathematics lecturers."*

Mr Koena: *"What the college does well is to organize in-house training where CMTs read DHET and college policies to us or made us aware of revised DHET and College policies. Secondly, we also attend assessor, moderator workshops."*

Ms Khodu: *"There are subject meetings that we attend."*

The mathematics lecturers acknowledged that the college did conduct subject meetings, provided in-house training and provided bursaries and workshops for assessors, moderators, coaching, class management and mentors. However, they indicated that mathematics subject meetings mainly focused on the administrative part, not on problematic mathematics content and relevant pedagogies methods to teach mathematics. Other lecturers agreed that workshops provided by the college were too general, once-off, did not take place regularly and did not focus on the problematic mathematics CK and relevant PCK to improve mathematics performance. Hence, mathematics lecturers maintained that these efforts by the college should not replace mathematics workshops, conferences and training.

Ms Khodu: *"They (referring to subject meetings) should not replace mathematics workshops and conferences, because these subject meetings mostly focus on who set tasks, who moderate tasks and when we do post moderation for tasks and examinations...these subjects meetings are too general because they are not focusing on how we should improve teaching and learning of mathematics and also mathematics results."*

Mrs Lebone: *"Workshops are also too general and a once-off thing."*

Mr Koena: *"What is lacking is mathematics workshops and conferences. So, we need mathematics workshops so that we can improve our teaching strategies to reach students."*

The literature showed that a lack of mathematics workshops and conferences impacts mathematics lecturers negatively, because of the following:

- it denies them new teaching methods and learning from one another, as some lecturers come from diverse educational realities and had diverse sets of resources available;
- exchange of information among themselves of similar NCV levels and class context;
- improve their perception of CRP knowledge;
- create networks among them and team leaders, new post-conference mathematics lessons to implement in the classroom; and lastly,
- find new knowledge of CRP help to validate current practices (cf. 2.4.4.4).

Ms Khodu: *"... if we don't attend mathematics workshops and conferences, we will not know new teaching methods and how to use those methods to improve mathematics results ..."*

Mrs Lebone: *"The lack of mathematics workshops and conferences contribute to poor performance of mathematics because we do not get proper guidance on how we can improve mathematics. Most of us as mathematics lecturers here at the college, like in Engineering, Mechanical, Electrical, IT, FEA and agricultural campuses, we do not have PGCE, so we need these workshops."*

This extract also reveals that lecturers had inadequate pedagogical teaching methods due to a lack of professional development like PGCE and other professional education certificates.

Mr Takui: *"Subject meetings are not even effective. We are expected to share our problems we experience in the teaching and learning of mathematics and share best practices to improve mathematics results. So, usually most of time there is no Campus manager, HoD, Supervisor and even invited mathematics experts who guide us as mathematics lecturers on where and how we can improve our teaching, we are left alone there stranded. I mean, to see to it how we can improve mathematics results, we are left there alone as mathematics lecturers and we are on the same level, So, some of our colleagues don't take part because they feel that there is nothing we can do alone because we expect guidance from our superiors. So, these subject meetings just waste our time, I rather work alone here."*

This shows that it is a norm in this college that CMTs do not take part in the subject meetings aimed at improving mathematics results where mathematics lecturers are supposed to share their challenges in the teaching and learning of mathematics and best practices. This practice is not in line with the PAR, postmodernity and post-experimentalism, because CMTs and mathematics lecturers do not work together, mapping a way to enhance the teaching and learning and improve mathematics results. PAR, Bricolage postmodernity and post-experimentalism encourage all to come together and solve any mathematics teething troubles to improve mathematics results (cf. 3.2.1, 2.2.2.5 & 2.2.2.6).

The lack of mathematics workshops, conferences and training will also impact the execution of the RME approach according to co-researchers:

“If we do not have and attend mathematics workshops, conferences, meetings, we will not be able to implement the RME approach well.”

Due to the fact that co-researchers indicated that to use the RME approach to impart MLP in Section 4.3, we had several meetings as a team (cf. 3.8 & 4.4), where we shared our knowledge in planning and preparing lessons to teach the MLP topic using the RME approach.

“To use the RME approach to teach MLP topic, we had several meetings which focused on mathematics and also consisted with mathematics coordinator, CM, HoDs, NCV supervisors, mathematics lecturers, parents, council member and most who had knowledge in mathematics.”

To overcome this threat and risk to the successful implementation of the RME method, this college should conduct mathematics subject meetings that focus on improving teaching and learning of mathematics, addressing teaching and learning challenges facing lecturers and students, sharing the MLPCK and RME approach information, plan and prepare mathematics lessons together to teach mathematics CK using the RME approach, and also learn new pedagogic methods relevant to teaching mathematics. The CMT should fully participate in those subject meetings. The college should also take lecturers to mathematics workshops and conferences outside the college environment, accompanied by one of the CMT members, so that all workshop participants learn how to use the RME approach or other modern pedagogy methods to teach MLP topic and understand current challenges in the teaching and learning of mathematics. More importantly, they should share their teething troubles and learn how to solve those problems.

4.6 EVIDENCE OF SUCCESS OF USING RME APPROACH TO TEACH MLP TOPIC

One of the objectives of this study was to use the RME approach to enhance MLP teaching at TVET colleges. The success of this study was realised when the research team planned, developed and prepared lessons into four levels to teach MLP and organised teaching and learning resources significant to impart MLPCK (cf. 3.8). The NCV supervisor volunteered to present lesson presentations using the RME approach after planning and organising phases. This was agreed upon; hence, we had two assistant mathematics lecturers working with NCV supervisor in the implementation phase on lesson presentations (cf. 4.3.1). We also had another team who volunteered to observe lessons (including me a principal investigator).

4.6.1 The RME approach enhanced students' MLP understanding

Dickinson and Hough (2012:1) discovered that the RME approach helped students to make sense of their learning. As a result, the RME approach improved students' mathematics CK understanding and further improved mathematics students' results. The literature further reveals that the RME approach has been used to teach mathematics in the Netherlands, USA and other countries with the aim to close mechanistically approached gaps and imitations (cf. 2.3.2). The literature shows that one of the success indicators and effectiveness of developed, proposed and recommended pedagogy approaches is the enhancement of students' understandability and performance (cf. 2.4.5).

This study discovered that the RME approach enhances students' understanding of MLP concepts through its principles, namely activity, interactivity, level, intertwined, realities and guidance. The MLP lesson began from the concrete level and gradually progressed to the abstract level, as was suggested by this study research team. The teaching MLP was divided into five lessons, namely the teaching of MLP lexicons; the formulation of linear inequalities and objective functions from MLP word problems; problems solving of linear inequalities, sketching of linear inequalities; identification of the feasible regions; and determination of the maximum (or minimum) values; teaching all the MLP components in one lesson; and lastly, revision of the whole process and tests (cf. 4.3.1). The presentation of lessons in Section 4.3.1 incorporated the activity,

interactivity, reality, guidance, levels and guidance principles of the RME approach, helping co-researchers to mathematise the MLP abstractness in order for students comprehend lessons. As a result, the RME approach assisted students to make sense of the teaching and learning as well as comprehending the MLP concepts. This was affirmed by the extracts below.

Teboho: “Ma’am, on the internet I found what you talked about on the relevancy of MLP topic in our lives. In Ethiopia, smallholders of land have used MLP (model) to use land well and effectively.”

Neliswa: “Oho, now I see and understand these words clearly, a maximum, at most, less than or equal to ($\leq$) to can be used interchangeable as synonyms. Again, a minimum, at least, greater than or equal to ($\geq$) are also synonyms. I was confusing them. I learned a lot.”

Mike: “And remember skeem, less than and lower than ($<$) are synonyms while greater than, more than, higher than are also synonyms ($>$). Our mother tongues and practical examples used helped us a lot.”

Dikotlolo: “No, Xaba, Ma’am said we can use any alphabets, so let’s take a & b instead of x & y to represent MLP word problems or scenarios.”

Zinhle: “Mampho, remember what we did with Mem, we should first check or consider coordinates of linear inequalities we have, $A(0,0)$, $B(0,0)$, $C(5,0)$, $D(0,10)$, $E1(18,0)$, $E2(0,18)$ for these linear inequalities $x \geq 0$, $y \geq 0$, $x > 5$, $y \geq 10$ and $x + y \leq 18$ respectively, before deciding the graph scale factor on the Cartesian plane. If we don’t that we might experience a problem where some of coordinates are greater or less than the numbers of our graph scale on the Cartesian plane. Our graph is going to be accurate and we will not struggle to find feasible region points (coordinates) and max (or min) points (she referred to explanations and demonstrations Ms Moleko to draw Figure 4.44 -4.49).”

Sebongile: “Let’s take each linear inequality at the time, plot, join and shading area that satisfy linear inequality like we did with Mem because drawing all linear equations first and showing directions after confuses us (she referred to explanations and demonstrations Ms Moleko to draw Figure 4.44-4.49).”

Zinhle: “Ok mo-girl, so to show these $x \geq 0$, $y \geq 0$, $x > 5$, $y \geq 10$ and $x + y \leq 18$ on the Cartesian plane, let’s use number line concept. I found it interesting and easier for me to show linear inequalities because using our hands to show directions sometimes confuses us (she referred to explanations and demonstrations Ms Moleko to draw Figure 4.44-4.49).”

Students forwarded their comments and feedback to Ms Moleko with regard to lessons presented in this study.

Siyabonga: “Graph papers and crayons help a lot, especially to show make accurate graph, feasible region and also finding feasible region moo easier (He referred to Fig. 4.35).”

Teboho: “Thank you, Ma’am, now I understand that to how to sketch linear inequalities, you made it easier. I think now I will change my attitude towards MLP topic because I learned a lot from these lessons, now I understand terminologies and word problems, etc.”

Gambu: “Yes, Ma’am, I understand why do we have to include basic linear inequalities namely, $x \geq 0$ and $y \geq 0$. They help us to determine the quadrant and also feasible region, like we did in the third lesson (Figure 4.68).”

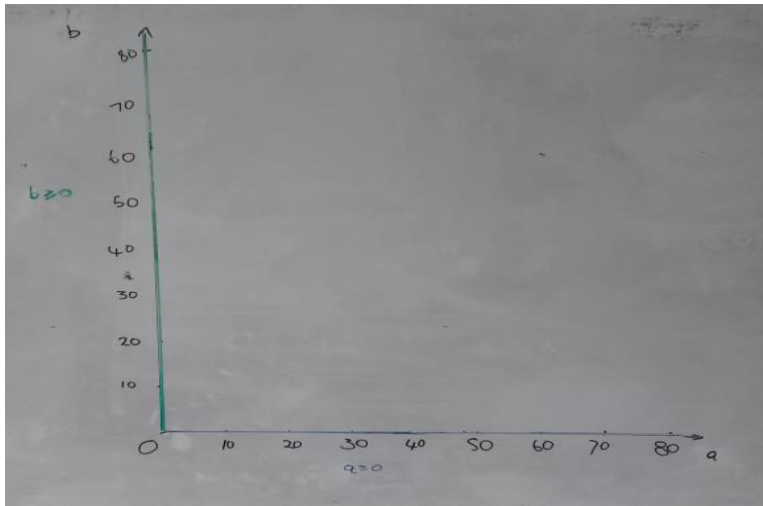


Figure 4.68: Showing $a \geq 0$ and $b \geq 0$ on ab Cartesian plane by Ms Moleko

Tholaone: “Again, Ma’am, like we agreed that x & y , can be used to represent types of products, items, and materials to be manufactured or purchased or sold. For example, let’s say we are in the business and selling muffins, the $x \geq 0$ could mean if we have not sold, 0 number of muffins will be sold OR if we have sold, a number of muffins will be sold.”

Wilson: “Can you remind me when do use solid line and broken lines?”

Poho: “Remember Wilson when we were doing MLP terminologies with Ms Moleko that we did saw that dotted line is only used on ($<$ or $>$) and solid line $\leq$ or $\geq$ only used on.”

Zulu: “Mme, o itse if we want search line, we take objective function and make y subject of formula, then on the left hand side we leave only y and all numbers and x should be on the left hand side. So, the coefficient of x is the one we make a search line with.”

The above extracts evidenced that the RME approach in this study played a significant role in the MLP teaching and learning. Students now understand that the MLP topic is essential to everyone, especially to businesses.

“Ma’am, on the internet I found what you talked about on the relevancy of MLP topic in our lives. In Ethiopia, smallholders of land have used the MLP (model) to use land well and effectively.”

This statement shows MLPCK is very vital to us and is also a human activity, because it can be used to address our daily problems. It is true that mathematics is within us and around us. These kinds of realities (practical examples) increased students’ interest in MLP, as realised in the following statements (cf. 4.3.1.2).

Ms Moleko: “To plant different food on a limited piece of land, we need to plan well in order to use it efficiently for maximising profit when we operate as a business. So, the MLP model is one of the techniques we can use to plan well and use a limited piece of land well for reaching maximum profit. Therefore, this technique has been used in different countries and sectors around the world, so it is also relevant to the farming sector. Let’s do Activity 1 to see how the MLP can be used in the farming sector ... so, here we are going to integrate the MLP components to find MLP model that determine a feasible area or region for profit maximisation or cost maximisation.”

Students paid attention and were interested to know how to solve Activity 1.

Bafana: *"How do we do that, Ma'am?"*

This was lacking in the lesson in Section 4.2.1. Mr Takui taught MLP in a complex way, without incorporating realities and demonstrating relevancy of MLP in our daily lives. This caused students not to be interested in the MLP lesson.

Teboho: *"mathematics is difficult, friend, to solve MLP problems only, is a long process."*

Lebohang: *"Friend, what time is it?"*

The literature revealed that a lack of interest in mathematics contributes to poor results in mathematics. The use of the RME approach increased students' MLP understanding and interest and changed their wrong perception about mathematics. This was affirmed by these words:

Teboho: *"Iyoo, now I realise that mathematics is so important."*

The teaching of MLP terminologies assisted students to understand the language used in MLP topic which further aided students to unlock difficult the MLP concepts such as word problems, linear inequalities, linear equations, objective functions, feasible area (region) and max (or min) value.

Neliswa: *"Oho, now I see and understand these words clearly, a maximum, at most, less than or equal to ($\leq$) to can be used interchangeable as synonyms ... a minimum, at least, greater than or equal to ($\geq$) are also synonyms ... I learned a lot."*

Mike: *"Our mother tongues, practical explanations and examples used in the first lesson helped us a lot."*

This affirmed the effectiveness of the RME approach in the teaching and learning of MLP lexicons and other concepts because most of the students had a deficiency of MLP lexicons to enable them to understand and formulate linear inequalities and objective functions from MLP problem statements (cf. Figures 4.10 to 4.13). This was confirmed by these statements of students:

Neliswa: *"I confused them."*

Poho: *"Language used in MLP case studies or word problems is difficult, so this enables me to understand well MLP case studies."*

Mike: *"These words, two times this or that, at least, maximum, at most, less than or equal and greater than or equal to confuses me."*

Smith (2004:10) states that misconceptions of students in mathematics concepts cause students' anxiety towards mathematics and lead to poor performance. This study believes that students' deficiency and misconception in MLP language and

terminologies are some of the factors contributing to poor performance in MLP. This study has succeeded to enhance MLP language and terminologies' understanding through the application of principles of the RME approach, such as activity, interactivity, level, intertwined, realities, guidance (cf. 4.3.1.1).

Students now understood that mathematics variables were not only all about x & y . but other variables (any alphabets), like a & b can be used instead of x & y to represent MLP word problems or scenarios.

"We can use any alphabets, so let's take a & b instead of x & y to represent this MLP word problems."

Ms Moleko: "Activity 1 talks about a farmer who has only 80 hectares on his piece of land and he usually plant cabbage and maize for his business. So, we should first find and write variables which can be any alphabet (a-z) that represents these plants to be planted (i.e. maize and cabbage). We let:

- number of maize to be planted be represented by variable "**a**"
 - number of cabbage to be planted be represented by variable "**b**"
- NB:** (**a** for x axes and **b** for y axes)

She also showed the students how to formulate linear inequalities and objective function from the MLP word problems in an easier way, using the RME approach (cf. Section 4.3.1.2), because most of the students found it difficult to formulate linear inequalities and objective functions (cf. Figures 4.10 - 4.13). It was detected that these students' inability is caused by finding it difficult to understand MLP word problems and its language. This was also confirmed by the literature, which revealed that students are unable to solve MLP word problems (cf. 1.1 & 2.4.2.1). This study have succeeded to tap into these students' difficulties through the use of the RME approach and Ms Moleko helped students to analyse the MLP word problems to formulate linear inequalities, equations, and the objective functions more easily.

Ms Moleko: "Let's move to **the second sentence** in Activity 1 which says He must grow **at least** 10 hectares of maize and 20 hectares of cabbages to meet demands."

There are two sentences here, which says he must grow at least 10 hectares of maize and at least 20 hectares of cabbage to meet the demands. The word **and** in this line was used to join two sentences.

The first part of this second sentence says ...

He must grow at least 10 hectares of maize to meet the demands.

*This means this farmer uses **at least (minimum/not less than) 10 hectares** to plant maize.*

- number of hectares of maize to be planted on that available land is at least 10 hectares, thus mathematically we write...

- $a \geq 10$, since number hectares of maize to be planted is represented by variable a ...

The second part of this statement, says ...

He must grow at least 20 hectares of cabbages to meet demands.

*This means this farmer uses **at least (minimum/ not less than) 20 hectares** to plant cabbage.*

- *number hectares of cabbage to be planted on that available land is at least 20 hectares, thus Mathematically we write...*
- $b \geq 20$, since number hectares of cabbage to be planted is represented by b ...

Students were now able to formulate linear inequalities, linear equations and objective functions (cf. Annexure R). This proved the enhancement of students' knowledge in MLP word problems.

The use of a & b mathematics variables shows a creativity which Bricolage promotes, because these variables (a & b) were the first to be used in this study as mathematics variables in MLP (cf. 2.2.1). It also shows that Ms Moleko is in Bricolage blurred genres and current fractured futures where various alphabets ($a - z$) were considered to be used as mathematics variables (cf. 2.2.2.3 & 2.2.2.8) with the aim of closing gaps of the traditional era and modernity phase, since in these eras, x & y were predominantly used as mathematics variables (cf. 2.2.2.1 & 2.2.2.2), which currently happens (cf. 4.2.1). This created phobia and a lack of interest in mathematics among people.

Mrs Matlou: *"The use of x & y as the only mathematics variables creates a phobia among students to study mathematics and as a result leads to a lack of interest in mathematics, because we were told that mathematics is all about solving x & y , mathematics problems, so if we cannot solve them we are not a type or material of mathematics."*

Ms Khodu: *"Colleagues, some of then students, when they see x & y , it automatically turns them off because of this phobia, so we should try to use other variables instead of always x & y , like a & b ."*

Hence, Ms Moleko presented MLP lessons in this fashion (cf. 4.3.1), because we as a research team agreed and decided that a & b should be used as mathematics variables to be creative in order to get the attention of students in the lesson.

During the lesson observations at the examined TVET college, we as a research team realised that it was the norm to teaching MLP topic without laying a deeper knowledge and understanding of MLP components, especially linear inequalities. For instance, we found that most lecturers taught linear inequalities in a complex manner without providing students with background knowledge:

Mr Takui: "Good people always remember to show these linear inequalities $x \geq 0$ & $y \geq 0$."

Tholoane: "In which sentence do these $x \geq 0$ and $y \geq 0$ come from, from what you read, Sir (referring to example 1 in 4.2.3)?"

Mr Takui: "Very good question, Tholoane. You should know that when we do MLP topics, we are always expected to show these linear inequalities. It is a rule, so you have to write them."

This confirms a lack of conceptual knowledge of $x \geq 0$ and $y \geq 0$; however, this study has achieved to enhance students' understanding of linear inequalities (cf. 4.3.1.3). This was proven by the following statements.

Gambu: "Yes, Ma'am, I understand now why do we have to include $x \geq 0$ and $y \geq 0$, basically they help us to determine a relevant quadrant (referred to the below Figure 4.20), secondly, to find feasible region and also max (or min) values, like we did in the third lesson."

Tholaone: "Again, Ma'am, like we agreed that x & y , can be used to represent types of products, items, and materials to be manufactured or purchased or sold. For example, let's say we are in the business and selling muffins, the $x \geq 0$ could mean if we have not sold, 0 number of muffins will be sold OR if we have sold, a number of muffins will be sold."

Therefore, the use of the RME approach in accordance with its principles has assisted students to give meaning to the abstract linear inequalities because students are now able to create practical examples to concretise and give meaning to the abstract linear inequalities. Moreover, they now understand the significance of $x \geq 0$ and $y \geq 0$, in MLP linear inequalities and graphs.

This study, through the use of the RME approach and multiple teaching and learning resources, enhanced students' understanding of how to draw MLP graphs accurately and identify feasible region easier.

Zinhle: "Mampho, remember what we did with Mem, we should first check or consider the coordinates and linear inequalities we have $(A(0,0), B(0,0), C(5,0), D(0,10), E(18,0), F(0,18))$ for these linear inequalities $x \geq 0, y \geq 0, x > 5, y \geq 10$ and $xy \leq 18$ respectively, before deciding the graph scale factor on the Cartesian plane. If we don't do that, we might experience a problem where some of coordinates are greater or less than the numbers of our graph scale on the Cartesian plane. Our graph is going to be accurate and we will not struggle to find feasible region points (coordinates) and max (or min) points (She referred to explanations and demonstrations Ms Moleko to draw Fig. 4.45-4.51.)"

Sibongile: "Let's take each linear inequality at the time, plot, join and shading area that satisfy linear inequality like we did with Ma'am, because drawing all linear equations first and showing directions after confuses us. (She referred to explanations and demonstrations of Ms Moleko to draw Figures 4.45-4.51.)"

Zinhle: "So, to show these $x \geq 0, y \geq 0, x > 5, y \geq 10$ and $x + y \leq 18$ on the Cartesian plane, let's use the number line concept. I found it interesting and easier for me to show linear inequalities because using our hands to show directions sometimes confuses us. (She referred to explanations and demonstrations by Ms Moleko to draw Figure 4.45-4.51.)"

Siyabonga: "Graph papers and crayons help a lot, especially in showing an accurate MLP graph and also finding feasible region easier."

Teboho: "Thank you, Ma'am, now I understand that to how to sketch linear inequalities, you made it easier. I think now I will change my attitude towards MLP."

Ms Moleko: "Are you sure sure, Dimpho?"

Dimpho: "Yes, Ma'am, we have done linear inequality sketches several times now."

Neliswa: "We are just waiting for test we are ready, Ma'am."

The above extracts proved that students understood the lesson on linear inequalities and MLP graphs because they managed to determine the graph scale factor, draw linear inequalities, find feasible region, and determine the max (or min) points of the following linear inequalities (Figure 4.69), namely $x > 5$; $y \geq 10$; $x + y \leq 18$; $x \geq 0$; $y \geq 0$; and $P = 3x + 2y$.

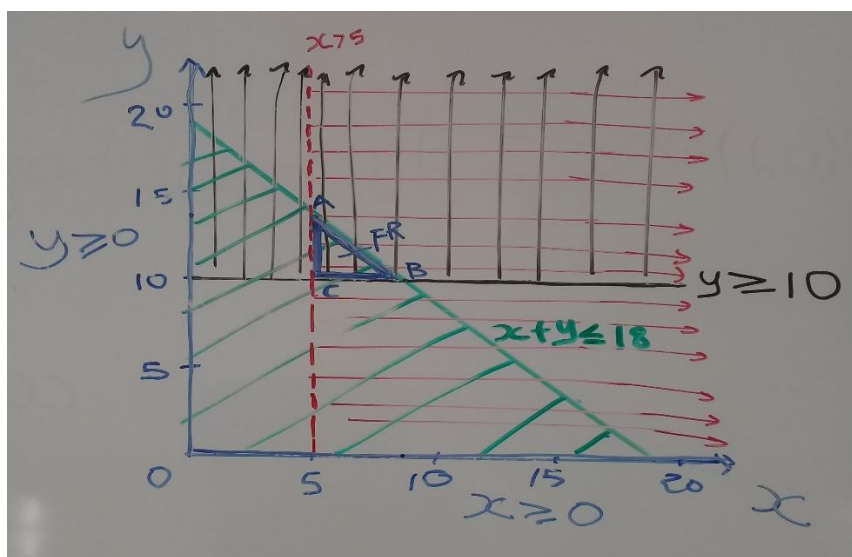


Figure 4.69: An MLP graph presented by students on whiteboard

$$\begin{aligned} FRP_A &= (5, 14); FRP_B = (9, 10); FRP_C = (5, 10) \\ FRP_A: P &= 3(5) + 2(14) = 43 \\ FRP_B: P &= 3(9) + 2(10) = 47 \\ FRP_C: P &= 3(5) + 2(10) = 35 \end{aligned}$$

Figure 4.70: Calculation of maximum profit presented by students

This proves the effectiveness of the RME approach through its principles (cf. 4.3.1.3). This study played a vital role in aiding to draw MLP graphs (cf. Fig. 4.69 & 4.70) that were identified as a difficult and problematic MLP concept for students (cf. 2.4.1, 4.2.6). Lastly, the effectiveness of this study was realised when Ms Moleko assisted

students to have an understanding of determining and using the search line method which was problematic for students.

Mike: "Search line is difficult to apply for us."

During the lesson teaching analysis and interpretation of MLP graphs, Ms Moleko gave students a platform to try before teaching these concepts, but the students were unable to find and use a search line to determine a max (or min) points and values. Ms Moleko helped students them with this.

Zulu: "Mme, o itse, if we want search line, we take objective function and make y subject of formula, then on the left hand side we leave only y and all numbers and x should be on the left hand side. So, the coefficient of x is the one we make a search line with, let's see how we can find search line in this objective function $p = 3x + 2y$."

Handwritten calculation showing the steps to find the search line for the objective function $p = 3x + 2y$:

$$\begin{aligned}
 p &= 3x + 2y \\
 -3x + p &= 2y \\
 -\frac{3}{2}x + \frac{p}{2} &= y \\
 \therefore y &= -\left(\frac{3}{2}\right)x + \frac{p}{2}
 \end{aligned}$$

The final expression is labeled "SEARCH LINE" with an arrow pointing to it.

Figure 4.71: Calculation of search line presented by Zulu

Zulu: "We use $\frac{3}{2}$ to draw search line, so 3 will be a point on y axis while 2 will be a point in x axis, then we point a pint and here then we join them with dotted line and move it to the right hand side."

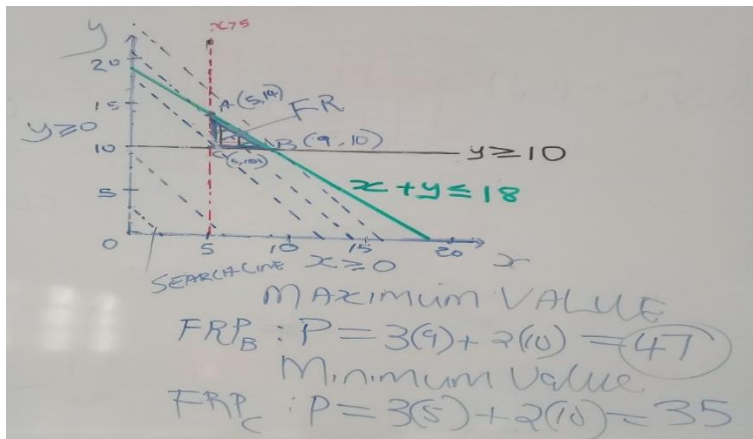


Figure 4.72: MLP graph, feasible area, max and min value presented by students

The above statement showed that students understood how to find a search line and used to it determine a max (or min) points and values, proving the effectiveness of the RME approach through its principles of activity, interactivity, level, intertwined, realities and guidance because these doctrines were integrated and used as components of the RME approach to teach MLP (cf. lesson presentations in 4.3.1).

The above section revealed the effectiveness of the RME approach in the teaching of MLP. This study enhanced students' understanding of MLP because students could finally do what they struggled with before (cf. 1.1, 2.4.1 & 4.2.1). This was achieved when

- students were provided with adequate activities as a baseline and formative assessments;
- they were allowed to interact among themselves in their groups to complete tasks (activities or discussions) at hand;
- the MLP topic was broken into five lessons (levels) according to its components but kept intertwined;
- Ms Moleko guided students on how to solve the MLP components;
- MLP components were taught in one lesson; and lastly,
- revision was done of the whole process and the test was based on an MLP topic (cf. Annexure R).

This agrees with the literature in Section 2.4.5.3, Ozkaya and Yetim Karaca (2017:194-195) and Zaranis (2017:58-59) found the RME approach effective, since it has improved students' mathematics understanding. However, this literature did not reveal which mathematics concepts was enhanced by the RME approach and how RME was used to enhance the mathematics concepts. Thus, this study has closed the gaps of previous researches by demonstrating how the RME approach can be used to teach MLP concepts (cf. lesson presentations in 4.3.1).

In conclusion, this Section revealed evidence of the effectiveness of the RME approach in the enhancement of students MLP understanding.

4.6.2 The RME approach improved MLP students' performance

The MLP students' performance is another indicator of success for the effectiveness of the RME approach used to teach the MLP topic at TVET colleges. Ozkaya *et al.* (2017:194-195) have uncovered that the use of the RME method improves students' performance (cf. 2.4.5.3), corroborating Zaranis' (2017: 58-59) view. Moreover, Van den Heuvel-Panhuizen (1996:9) signposts that the RME approach was developed with the purpose of mathematising mathematics concepts and also to close gaps of

mechanistic methods. As a result, the incorporation of the RME approach creates opportunities and space for learning. The aim of this study was to enhance the teaching of the MLP using the RME approach at TVET colleges, which automatically led to better performance, as confirmed by the co-researchers. This became clear from one of the research meetings where the impact of the use of the RME approach was evaluated.

Mr Takui: “These students’ understanding has improved a lot look the way they performed (looking at the students scripts), performance is better compared to what we have seen in the previous test.”

Mrs Lebone: “I am impressed, this strategy has indeed worked. Our students showed an understanding in MLP topic and their test performance is good.”

CMT: “We are so happy and indeed this approach is very helpful to students. It helped them to perform well and we are indeed impressed. So, we are going to propose this strategy to our lecturers across all levels in mathematics.”

Mrs Seeta: “The principles of the RME approach has contributed positively to students’ MLP results, performance, like activities, interactivity, level ...”

Mrs Matlou “I agree with my colleague on that point such principles improved students results and again has helped us to implement the RME approach successfully, just imagine if we didn’t have these principles as us our guide what should have happened? Where we should have started? We were going to struggle.”

In one of this study research meetings we analysed pre- and post-test results (Table 4.16).

Table 4.16: Analysis of pre-test and post-test results (taken from research meetings of this study)

Achievement codes	Competencies	%	Number of students (MLP pre-test)	Number of students (MLP Post-MLP test)
Level 1 (L1)	Not achieved	00-29	30	02
Level 2 (L2)	Elementary	30-39	03	04
Level 3 (L3)	Moderate	40-49	02	04
Level 4 (L4)	Adequate	50-69	00	09
Level 5 (L5)	Substantial	60-69	00	03
Level 6 (L6)	Meritorious	70-79	00	06
Level 7 (L7)	Outstanding	80-100	00	07

Then, we further analysed pre-test and post-test results by doing calculations on pass and fail percentages with the purpose of checking the impact of the RME approach (cf. Table 4.17).

Table 4.17: analysis of pre-test and post-test results (taken from research meeting of this study)

MLP pr-test	MLP post-test
Pass % = $\frac{\text{number passed}}{\text{total number wrote}} = \frac{05}{35} \times 100 = 14\%$	Pass % = $\frac{\text{number passed}}{\text{total number wrote}} = \frac{33}{35} \times 100 = 94\%$
Failed % = $\frac{\text{number failed}}{\text{total number wrote}} = \frac{30}{35} \times 100 = 86\%$	Failed % = $\frac{\text{number failed}}{\text{total number wrote}} = \frac{02}{35} \times 100 = 06\%$

In addition, we presented the information in Figure 4.73 on the pre-test and post-test results on the bar graph below.

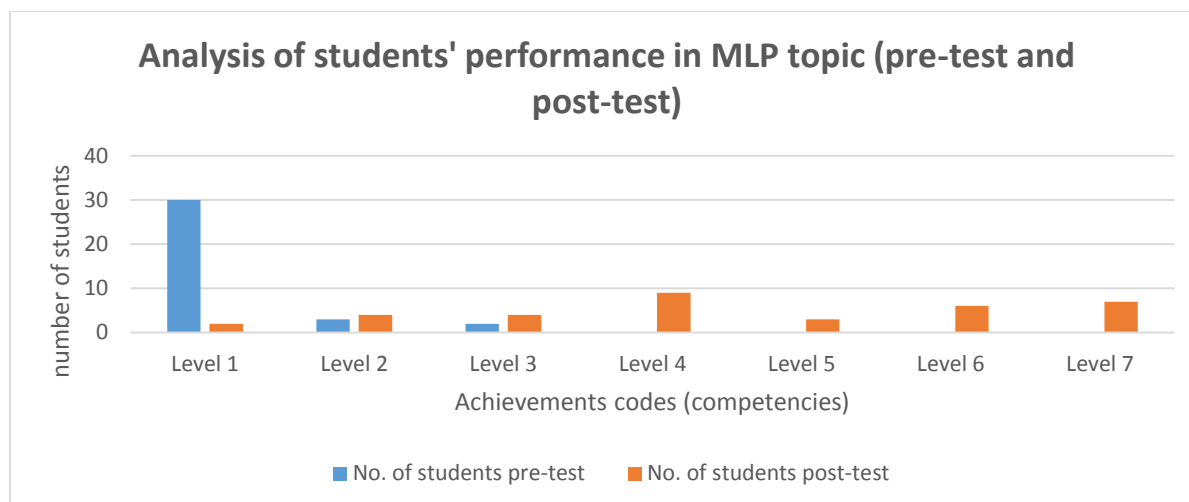


Figure 4.73: analysis of MLP pre- and post-test results in bar graph (taken from this study research meeting)

The above extracts proved the effectiveness of the RME approach in MLP teaching and learning because, according to co-researchers, this approach assisted students to perform better in the MLP test. This was realised in the following statements.

Mr Takui: “Performance is better compared to what we have seen in the previous test.”

Mrs Lebone: “This strategy has indeed worked ... test performance is good.”

CMT: “Indeed, this approach is very helpful ... It helped them to perform well.”

These statements were informed by the above Table 4.17 of the analysis of students’ performance in MLP test scores. This analysis confirms what the co-researchers articulated. From Mr Takui’s statement we realised that he had done a comparison between the pre-test and post-test to check the effectiveness of the RME approach. The information of the pre-test and post-test was projected on a bar graph (Figure 4.73).

The pre-test was conducted at the initial stage of this study before the RME approach was used to teach the MLP components with the following purposes; firstly, to show that MLP is a problematic mathematics concept for students at TVET colleges; secondly, to identify students’ MLP misconceptions, errors and mistakes. The pre-test results revealed that there was a need for the RME approach because out of 35 students who wrote the pre-test, 5 students passed (14%) and 30 students failed (86%). Thereafter, the RME was used to enhance the MLP concepts, and at the end of lessons the post-test was conducted to check the effectiveness of the RME

approach. The post-test results showed that out of 35 students who wrote, 33 students succeeded (96%) and 2 students failed (04%).

Figure 4.73 illustrates that out of 35 students who wrote the MLP pre-test, 30 of them failed, scored L1; 03 of them were in L2 and 02 of them are in L3. Therefore, 03 students in L2 on pre-test were at risk because they scored 30%, 31% and 31%, so they might fall into L1 if multiple pedagogies had not used to teach MLP. Hence, this study employed the RME approach to teach MLP. Thereafter, the post-test was conducted and the results improved, because out of 35 students who wrote the post-test, the score was as follows, L2 – 02 students; L2 – 04 students; L3 – 04 students; L4 – 09 students; L5 – 03 students; L6 – 06 students; L7 – 07 students. This means that after the incorporation of the RME approach in teaching MLP, most of the students scored higher and moved from L1 to L2 to L7, because only 2 students did not achieve, whereas in the pre-test 30 did not achieve. This confirms what co-researchers agreed to above, saying the RME approach improved the MLP results. Other co-researchers were of the view that the principles of the RME approach impacted positively on students' performance and results, and also assisted this study to use the RME approach effectively to teach MLP. This was proven by the following statements.

Mr Seeta: *"Principles of the RME approach have contributed positively to students' MLP results, performance, like activities, interactivity, level."*

Mrs Matlou: *"I agree with my colleague on that point such principles improved students results and again has helped us to implement the RME approach successfully. Just imagine if we didn't have these principles as us our guide what should have happened? Where we should have started? We were going to struggle."*

This Section agrees with Section 2.4.5.3 on the fact that the RME approach enhances students' mathematics results. This study further showed improved results in MLP, due to the incorporation of the RME approach through its principles. It was the first time the RME was used to teach MLP topic at TVET colleges, because the literature showed that the RME approach improved mathematics results, it but did not reveal which mathematics concepts results improved (cf. 2.4.5.3). Thus, this statement necessitated a need to conduct this study.

This Section showed extracts, presentation, analysis and the interpretation of pre-tests and post-tests, which confirms the effectiveness of the RME approach in the teaching of MLP.

4.6.3 The Mathematical Linear Programming Content Knowledge of lecturers, CMT and parents improved

This study also aimed to enhance mathematics lecturers, a CMT and other stakeholders' knowledge of MLP and the RME approach understanding through PAR; hence, action research paradigm was chosen. The co-researchers were included in this study's research routes (the spiral cycle), namely in the planning, designing, developing and preparing; organising; implementing, reflecting, re-planning, re-designing and so on (cf. 3.2.1). That is why people participating in this study were regarded as co-researchers, not participants, because they actively took part in the study research processes (cf. 3.5 & 4.2-4.6) with a notion to empower, capacitate and show them that they have valuable knowledge to solve their own problems in collaboration and as a team.

The TVET college encourages team teaching pedagogies because it has opened space at coordinating campuses for lecturers to meet and discuss subject matters. During these meetings, lecturers as mathematics experts were required to share teaching and learning challenges, hindrances and problems they faced every day. Thereafter, they shared solutions and strategies to solve those challenges in Math. This platform enabled lecturers to learn from one another in solving those teaching and learning challenges. mathematics lecturers, CMT and other stakeholders were in agreement that this study had empowered them with MLPCK, RME approach knowledge and processes that should be considered to conduct research studies of this nature.

At one of the research meetings, the co-researchers reflected on the processes of this study and the effect of this research study in their lives.

Mr Takui: *"I learned a lot from this study. I only use lecture method to teach mathematics but today I learned more about the RME approach and especially its principles that we used in the earlier lessons, so I think I am going to try this approach to teach MLP and other mathematics concepts, since students seemed to enjoy the lessons and also performed well."*

Ms Lebone: *"I usually do not consider teaching MLP terminologies when I teach MLP; however, today I have seen why I should consider teaching language and terminologies in MLP topic and also in all mathematics concepts."*

- maximum, at most, less than or equal to $\leq$ are synonyms ... solid/bold line
- minimum, at least, greater than or equal to $\geq$ are synonyms ... solid/ bold line
- Lower than, less than, not more than $<$ are synonyms ... dotted/ broken line ...
- greater than, not less than $>$ are synonyms ... dotted/ broken line."

Ms Lebone: “Again, this RME approach have assisted me to understand more MLP concepts.”

Mrs Mohlomi: “I am going to share this important information to my business partners, indeed the MLP model can help us to project future cash inflow. If they see a need and agree we will hire a temporary mathematics lecturer to advise and plan for us, because now we have old systems in place. We need to know how other countries use this model.”

Mr Khetla: “I see now that in business I can use MLP concepts and models to plan well to profit or lower costs. I will also share this information with my bosses; maybe they will partnership with the college.”

Mrs Seeta: “We have realised that the method that we usually use to teach mathematics is difficult for students to understand and today learned how to teach this topic in an easier way using the RME approach, like pictures, indigenous languages, practical examples relevant to the topic, show and indicate how mathematics concepts can be used in real life situations ... breaking the topic into subtopics and lessons and starting lessons from lower to higher level.”

CMT: “We are so happy and indeed this approach is very helpful to students. It helped them to perform well and we are indeed impressed. So, we are going to propose this strategy to our lecturers across all levels in mathematics.”

Mr Takui: “Some of MLP concepts were difficult for me to explain then, but now i can be able to explain and demonstrate those concepts. For instance, now I know that $x \geq 0$ & $y \geq 0$ have a meaning and are not just a rule but they have effect on the Cartesian plane especially in choosing quadrant and also they help us to find feasible regions and max (or min) values.”

Mrs Matlou: “In my time of teaching, I skipped this topic because it was difficult for me and my mathematics colleagues and they used to say do not stress this topic contributes between 10- 15 marks. Then I would ask my HoD to come and help me but he is always busy, sometimes he will come but most time he is unable. But today I think I can try because I learned a lot from this study.”

The above extracts evidenced that mathematics lecturers, CMT, mathematics coordinators, and parents gained much knowledge of the MLP concepts and the RME approach that were lacking in the past.

Mr Takui: “Some of MLP concepts were difficult for me to explain then.”

Mrs Matlou: “In my time of teaching, I skipped this topic because it was difficult for me and my mathematics colleagues and they used to say do not stress this topic contributes between 10- 15 marks. Then I would ask my HoD to come and help me but he is always busy, sometimes he will come but most time he is unable.”

These statements showed and evidenced the MLP deficiency among lecturers, which further impact negatively on students’ learning. However, after the use of the RME approach to enhance the teaching of the MLP topic, co-researchers seemed to have acquired more knowledge of the MLP concepts.

Mr Takui: “Some of the MLP concepts were difficult for me, but now I can explain and demonstrate those concepts. For instance, now I know that $x \geq 0$ & $y \geq 0$ have a meaning and they have an effect on the Cartesian plane ... in choosing quadrant; also they help us to find feasible regions and max (or min) values.”

Mrs Matlou: “In my time of teaching, I skipped this topic because it was difficult for me ... But today I think I can try because I learned a lot from this study.”

On the other hand, parents also learned and realised that MLP is important to the real world and to their space of work.

Mr Khetla: *“I see now that in my business I can use MLP model to plan well to profit or lower costs.”*

These statements confirm that co-researchers’ MLP conceptual understanding improved. Moreover, this study assisted co-researchers to reflect on the way the teach MLP topic and other mathematics concepts and also consider incorporating the RME method to teach MLP topic and other concepts.

Mr Lebone: *“I usually do not consider teaching MLP terminologies when I teach MLP, but today I have seen why I should consider teaching language and terminologies in MLP topic and also in all mathematics concepts”.*

Mrs Seeta: *We have realised that the method that we usually use to teach mathematics is difficult for students to understand and today learned how to teach this topic in an easier way using the RME approach, like pictures, indigenous languages, practical examples relevant to the topic, show and indicate how mathematics concepts can be used in real life situations ... breaking topic into subtopics and lessons and starting lessons from lower to higher level.”*

These quotes proved that this study has equipped co-researchers with valuable information about the RME approach and on how mathematics should be taught to concretise and Mathematise abstract mathematics concepts.

CMT: *“We are so happy and indeed this approach is very helpful to students. It helped them to perform well and we are indeed impressed. So, we are going to propose this strategy to our lecturers across all levels in mathematics.”*

This information on the above confirms the effectiveness of the RME approach in the teaching and learning of MLP at this TVET college.

This Section showed that from this study, lecturers, CMT and parents have gained much knowledge, which will assist them with daily problems, challenges, and activities. This agrees with Section 3.2.1 and 3.2.2, which revealed that the action research paradigm and PAR methodology empower, equip and capacitate co-researchers with information that assist them to solve problems and challenges they are faced with on a daily basis. In addition, this section revealed how lecturers, CMT and parents feel about the use of the RME approach to teach MLP and also how this study benefited them.

CHAPTER 5: FINDINGS, DISCOVERIES AND RECOMMENDATIONS

5.1 INTRODUCTION

The aim of this study was to use the Realistic Mathematics Education (RME) approach to enhance the teaching and learning of Mathematical Linear Programming (MLP) at Technical Vocational Education and Training (TVET) colleges, since the Department of Higher Education and Training (DHET) and the investigated TVET college reports revealed that mathematics is a problematic subject. The reports also showed that MLP is a difficult topic. This was confirmed by the literature in Sections 1.1 and 2.4.2.1. This led to poor performance in mathematics at this TVET college.

This chapter begins with a summary of the background of this study, which encapsulates the statement of the problem and research objectives of this study to remind the reader what this study aimed to achieve. Thereafter, this chapter will present the findings of this study as structured according to the research objectives of this study. Lastly, this chapter presents the tested and executed the RME approach to enhance the teaching of MLP at this TVET college using its multiple components solutions, which serve as a recommendation for all challenges identified as contributors to poor MLP performance during this research.

5.2 BACKGROUND AND STATEMENT OF THE PROBLEM

The RME is a mathematics education approach that was first developed and introduced by three mathematics didacticians, Wijdeveld, Goffree and Treffers, at Wiskobas in the Netherlands in 1968 (Van den Heuvel-Panhuizen & Drijvers, 2014:1-4). During 1971, the Wiskobas project became part of the newly established IOWO institute, with Hans Freudenthal as first director in 1973 when the IOWO was expanded with the Wiskivon project for secondary education. Altun (2006, in Ozkaya & Yetim (2017:186) indicate that Freudenthal is of the view that mathematics historically started with real-life problems, where real-life is mathematised, and the system is formalised; therefore it is anti-didactic to teach formal mathematics knowledge first and then apply later. He indicated that mathematics is a human activity; it was not discovered, but invented. The RME approach refers to a domain-pedagogical theory of mathematics and one of its important characteristics is that rich

situations (realities) are given a prominent position in the mathematics teaching and learning process (cf. 2.3.2). The realities serve as a source for the decolonisation and development of mathematical concepts, tools and procedures, and as a context within which students can use knowledge gained in the class at a later stage to solve problems, which then become more formal and general and less context specific. Van den Heuvel-Panuizen and Drijvers (2014:523-524) further point out that for instructors (lecturers) to use the RME approach successfully to enhance the teaching of mathematics it should be guided by the following principles, namely activity, realities, levels, intertwinedness, interactivity and guidance (cf. 2.3.2). According to Ozkaya and Yetim (2017:187), Freudenthal signposts that, for students to understand mathematics, they begin with sense-making and that sense-making should be key to all phases to do real mathematics. The students are enabled to reach mathematical concepts by contemplating daily life problems, discuss probable solutions and develop rational solutions to structure these concepts in their mind. The mathematics course, which is a source of concern, should thus become a mathematics activity, helping students to develop a positive approach towards the mathematics lesson and increase their success in mathematics. The RME problems presented to students can come from the real world, but also from the fantasy world of fairy tales or the formal world of mathematics, as long as the problems are experientially real in students' minds. The RME approach is categorised into two categories; horizontal and vertical mathematization (cf. Section 2.3.2). Horizontal mathematization is elucidated as the incorporation of mathematical tools to organise and solve problems based on real-life situations by analysing MLP word problems into mathematical symbols, while vertical mathematization is viewed as a process of reorganisation (grouping like terms) within a mathematical system, resulting in shortcuts by using connections between concepts and strategies.

Mathematical Linear Programming (MLP) is one of the mathematics topics for the National Certificate Vocational (NCV) Level 3 and 4 at TVET colleges. Mathematical Linear Programming refers to mathematical concepts that consist of a set of variables and constraints expressed in objective functions and linear inequalities (cf. 1.1). In Section 2.3.1, Nkambule (2009:6) defines the topic of MLP as a mathematical model technique that helps us to make decisions in diverse contexts and sectors of the economy, such as farming, industrial engineering, and social, economic, technological

and physical sciences. Many operational management decisions work towards making the most effective use of an organisation's resources, such as machinery, money, labour, raw materials and time. These resources may be used to produce products and provide services. Mathematical Linear Programming is a widely used mathematical technique designed to assist operations managers and people in different environmental planning and decision making necessary to allocate resources. The MLP content in mathematics NCV L4 includes the following: word problems, linear inequalities, linear graphs, feasible region, maximum and minimum values (cf. 2.3.1 & 4.4.2). In addition, Technical Vocational Education and Training (TVET) colleges are defined as colleges that are designed and developed to provide students with skills and knowledge relevant to the workplace and that help students to become entrepreneurs and future employers. According to Akhuemonkhan, Raimi and Dada (2014:04), TVET colleges offer specialised education meant to empower students in the development of technical skills, cognitive understanding, human abilities, and attitudes, as well as work habits to prepare them for the world of work and position them for self-employment after graduation.

The diagnostic reports from the Department of Higher Education and Training (cf. Annexure E) and the Department of Basic Education (2013:127) reveal that the topic of MLP is problematic and difficult for students to solve, because they perform poorly in LP. The annual reports of the TVET college that was investigated in this study also show that students have a serious challenge with MLP (cf. Annexure E). The reports show that most students perform poorly in mathematics, repeat mathematics and there is a high dropout rate (cf. 1.1). Scholars from South Africa, India, the USA, Kenya, Indonesia and Australia also affirm that most students experience serious challenges with MLP (Brown & Dell, 2007:153; Saleme & Etchells, 2016:158-159; White, 1995:91-95; Olubukola, 2015:846; Chinamasa *et al.*, 2014:57; Nkambule, 2013:190-240; Molnar, 2016:2-12; Kar, 2016:652). This research was motivated by the challenges of teaching and learning MLP observed in practice and those recorded in the literature. The literature from local, SADC, African and global countries shows the following factors and challenges contributing to poor performance in mathematics and in MLP topic: an abstract way of teaching of mathematics; limited teaching materials; mathematics teaching and learning being dominated by traditional teaching methods; inability to search and actualise MLP students' prior knowledge; absence of formative

assessments; errors and misconceptions; inability to formulate, solve and graph linear inequalities; inability to draw feasible areas; and to determine maximum and minimum values using a search line method as well as substitution method. Various challenges and factors impact negatively on the teaching and learning of MLP, which contribute to poor mathematics performance (cf. Section 1.1 & 2.4.2.1).

In response to the above teaching and learning challenges and factors contributing to poor mathematics, various studies conducted locally, continentally, and globally in the past recommend various pedagogical approaches, such as concrete and abstract methods, sufficient teaching aids, multiple teaching methods, the use of prior knowledge assessments, sufficient formative assessment and multiple languages (Chang, Lee & Koay, 2017:1; Moloi, 2013:489-490; Karue & Amukowa, 2013:106; Mbugua *et al.*, 2012:87; Palosay & Steven, 2016:38; Hailikari *et al.*, 2008:2; Leone *et al.*, 2010:3-4; Ajogbeje *et al.*, 2013:8-9; Sa'ad *et al.*, 2014:35; Blanco & Garrote, 2007:227). Multiple teaching methods were accepted in the early 1990s when Outcomes-Based Education (OBE) was introduced in South Africa, advocating students' involvement in the teaching and learning process. During this era, teaching and learning were no longer done in one way (linear way) only, where lecturers transmit knowledge to students' minds. More development and transformation have taken place to improve the teaching and learning environment in South Africa, which came with multiple pedagogical methods and tools, such as incorporating concrete and abstract ways; Information Communication Technology (ICT) with mathematics software; Code Switching (CS); Problem-Based Learning (PBL); calculators; student-centred, cooperative learning; and indigenous languages and indigenous games in the teaching and learning of mathematics, since mathematics is one of the problematic subjects in South Africa (Isaacs, 2007:7; Mosia, 2016:152-162; Moloi, 2013:489-490; Nkambule, 2009:116-189; Molefe & Brodie, 2010:10-11; Mkhize, 1999:34-36; Mulaudzi *et al.*, 2016:50-52). Despite this effort and the introduction of various pedagogical methods to enhance mathematics teaching and learning, mathematics results have not improved in the education sector in South Africa since the transformation. DHET and annual TVET college diagnostic reports still show that students perform poorly in MLP and mathematics. Studies in the USA (Dantzig & Thapa, 1997:8-12; Williams *et al.*, 2016:14-15; Macglauglin, 2005:230; Leone *et al.*, 2010:1-3; Ngu & Phan, 2015:23; Palocsay & Steven, 2016:38; Grandberg, 2016:33-

35; Murray *et al.*, 2018:19) report the use of the column-and-row method; the translation (code-switching) approach; ICT and software (spreadsheets, geogebra, tablets, computer algebra systems, graphical calculators); the RME approach; Content Knowledge (CK) and Pedagogical Content Knowledge (PCK); cultural practices; procedural embodiment; and Curriculum-Based Assessment (CBA). However, these practices are found mainly in basic education, as opposed to institutions of higher learning. Researchers in Kenya have observed the effectiveness of the PBL approach and mathematical vocabulary assessment in teaching MLP (Nakhanu *et al.*, 2015:6; Nakhanu & Musasia, 2015:70-73; Wanjiru & O'Connor, 2015:205-206). PBL was also observed by Schmidt (1983:15-16) in the Netherlands. The error analysis method was used in Indonesia to detect MLP teething troubles amongst students (Santoso *et al.*, 2017:3-7). They also signposted that further research should be done to check the effectiveness of the RME approach in teaching MLP. However, this was not done; thus there was a gap that this study aimed to bridge. In Indonesia, Khuluq (2015:30-58) employs balancing activities in line with the RME approach to teach linear equations with one variable. Van den Heuvel-Panhuizen (1996:9-33), Kwon (2002:6-9), Van den Heuvel-Panhuizen and Drijvers (2014:522-524), Ozdemir (2017:425:426), Zaranis (2017:54-59), and Ozkaya and Yetim (2017:193-195) report the use of the RME approach to teach mathematics in basic education in the Netherlands, South Korea, the USA, Turkey, and Greece, but not to teach MLP. Hence, this study saw a need to enhance the teaching and learning of MLP at TVET colleges by using the RME approach.

The success of the strategy that was developed to implement the RME approach was found to be evident when co-researchers were able to do the following:

- Identify MLP concepts that students find difficult to comprehend and lecturers find difficult to teach effectively;
- Collaboratively plan, design, and develop the RME approach lesson that would make abstract MLP concepts easy for students to comprehend;
- Teach and facilitate MLP lessons using the RME approach confidently;
- Provide students with regular feedback; and
- Reflect on MLP lesson presentations.

5.2.1 Research question

How can the RME approach be used to enhance the teaching of MLP at TVET colleges?

5.2.2 Aim and research objectives of this study

5.2.2.1 Research aim

To demonstrate how the RME approach can be used to enhance the teaching of MLP at TVET colleges.

5.2.2.2 The objectives of the study

The objectives of the study were to:

- To investigate the challenges that hamper MLP teaching and learning and contribute to poor MLP performance at TVET colleges;
- To demonstrate how to use the RME approach to enhance the teaching of MLP at TVET colleges;
- To examine critical conditions under which the RME approach enhances the teaching of MLP at TVET colleges;
- To investigate threats and risks involved in executing the RME approach to enhance the teaching of MLP at TVET colleges, making suggestions for preventing identified threats; and
- To identify evidence of success for using the RME approach to enhance the teaching of MLP at TVET colleges.

This chapter presents the findings and recommendations for the strategy that was developed to implement the RME approach in enhancing the teaching and learning of MLP topic.

5.3 FINDINGS AND RECOMMENDATIONS

This section presents the findings of the study as they emerged from the literature and during analysis of the data. The following are the findings of this study as it emerged in Chapter 4.

- A complex way of teaching MLP at TVET colleges;

- A need for multiple teaching and learning apparatus;
- The method of teaching MLP at TVET colleges is too teacher-centred;
- A need to recognise prior knowledge and learning;
- A need for formative assessment in the teaching of MLP; and
- Mathematical Linear Programming errors and misconceptions.

A discussion of each finding, followed by its respective recommendations, which were drawn from the tested pedagogical methods in Section 4.3, follows. To ensure that the recommended pedagogical methods work successfully, better conditions for their execution are recommended (cf. 4.4). However, it is understood that these recommended pedagogical methods could have embedded threats and risks; thus the risks and threats are discussed further and recommendations are made on the basis of the information in Section 4.5.

5.3.1 A complex way of teaching MLP at TVET colleges

This study revealed that mathematics lecturers teach the MLP in a complex way (cf. 4.2.1), which confirmed what the literature reported, showing that the abstract way of teaching mathematics concepts hampers teaching and learning process, because it becomes very difficult for students to understand the MLP parts (cf. 2.4.1.1). Moreover, prior to the study intervention, this study uncovered that mathematics lecturers at TVET colleges teach MLP in a complex way, by teaching all MLP components such as word problems, linear inequalities and graphs simultaneously in one lesson (cf. 4.2.1). Therefore, this study found that a limited time allocated to teach MLP and concerns with the completion the syllabus led to lecturers relying on the abstract method to teach MLP topic (cf. 4.2.1). So, the limitations of using the abstract method to teach MLP led to students' MLP misconceptions, mistakes and also to students' lack of interest in MLP, which also led to poor MLP performance and mathematics results (cf. 4.2.6). To respond to the challenges mentioned above, this study recommends the execution of the following stages of the strategy developed to implement the RME approach (cf. Section 4.3).

5.3.1.1 Recommended levels and intertwined principles of the RME approach

This study recommends the application of levels and intertwined doctrines of the RME approach as solution components to respond to the challenges mentioned in Section

5.3.1. We as a team used a combination of intertwined levels as solution components to enhance the MLP topic at the TVET college under investigation. We divided the MLP topic into four intertwined levels for teaching, starting from a lower level and proceeding to a higher level, such as introducing MLP terminology; formulating linear inequalities and objective functions from MLP word problems; problem solving of linear inequalities and MLP graphs; and lastly, teaching all MLP components (lexicon, word problems, linear inequalities, graphs and feasible region) simultaneously in one lesson (cf. 4.3.1–4.3.4). The aim of dividing MLP topic in four intertwined levels was to assist mathematics lecturers to concretise the abstractness of the MLP concepts, so that students comprehend them properly, as a result, this contributed positively to students learning (cf. Section 4.6).

In conclusion, recommended solutions (level and intertwined components solutions of the RME approach) to address the challenges identified in Section 5.3.1 assisted Ms Moleko to start teaching from the concrete to the abstract level, which helped students to understand the MLP content knowledge. This impacted positively on the results and performance of students in MLP when comparing the pre- and post-test results.

5.3.2 A need for multiple teaching and learning apparatus

Another challenge contributing to poor performance discovered by this study was that lecturers at specific TVET college used only one textbook to teach MLP. They also used a single-ink whiteboard marker to illustrate linear inequalities and find feasible region on the whiteboard. Students were not informed on what learning materials were significant for this topic (cf. 4.2.1). Therefore, limited teaching and learning materials had a negative impact on the teaching and learning of MLP, because limited teaching and learning materials created inaccurate MLP graphs and also confusion among students to determine feasible areas, coordinates, and maximum and minimum values (cf. Figure 4.3). This limited teaching media forced lecturers to teach MLP topic in a complex way where they were unable to integrate real life stories, objects and indigenous content knowledge to aid students to understand abstractness of MLP concepts (cf. 4.2.2). The lack of teaching and learning materials shows a lack of lesson planning, according to this study. To solve this challenge, this study recommends multiple teaching and learning resources.

5.3.2.1 Recommended multiple teaching and learning resources

This study recommends multiple teaching and learning resources to respond to the challenges identified in Section 5.3.2 and 4.2.6. During one of the research meetings of this study, co-researchers suggested the use of multiple teaching and learning resources (cf. Fig. 4.35). The teaching and learning materials identified and organised by us a research team helped Ms Moleko to concretise the abstractness of MLP concepts in all lesson presentations (cf. 4.3.1). For instance, to teach MLP graphs, Ms Moleko used the above teaching resources to sketch linear inequalities and determine feasible regions (cf. Figures 4.40 & 4.41); thus, such resources led to an accurate scale on the Cartesian plane; accurate sketch linear inequalities; accurate feasible area points and coordinates. This further helped them to determine the correct max value (cf. Figure 4.59).

The above illustrations evidenced the effectiveness of the use of multiple teaching aids and resources as bricoleurs and had a positive impact on students' efficacy in MLP graphs (cf. Annexure R).

5.3.3 Method of teaching MLP at TVET colleges is too teacher centred

This study discovered that the MLP teaching at the specific TVET college is too teacher centred and dominated by traditional teaching methods. From the beginning to the end of lesson presentations of MLP, Mr Takui provided students with MLP information and did not allow them to interact with one another (cf. 4.2.3). This confirms what the previous studies uncovered (cf. 2.4.1.3). Dominancy of traditional methods is caused by the following factors: lack of regular workshops and seminars of mathematics that concentrate on classroom management; managing 21st-century students and capacitation of various pedagogical approaches; perception and beliefs of lecturers that students learn best when they are told; lack of proper teaching qualifications; lack of participation by students; limited resources; resistance to change traditional methods; lack of lesson planning; limited teaching and learning media; the perception that 21st-century and modern pedagogical methods waste time allotted for teaching; and limited time to cover the syllabus. As a result, most of students became bored, lost attention and were confused in MLP lessons, because they were passive and inactive. This contributed to students' inability to formulate linear inequalities and objectives from MLP problem statements; solve and sketch linear inequalities; and to determine feasible area and max (or min) values, leading to poor performance in MLP

and mathematics in general (cf. 4.2.6). Thus, this study recommends multiple components solutions of the RME approach, such as realities, interactivity, and guidance to respond to the challenge.

5.3.3.1 Recommended multiple components solutions of the RME approach

The co-researchers recommended multiple components solutions of the RME approach, namely realities, interactivity and guidance as solutions to close gaps and limitations of the dominance traditional teaching approach at this TVET college (cf. 4.2.3). Ms Moleko used the reality method by incorporating real-life situations in the form of pictures, practical examples situations, and objects to unlock the abstractness of MLP terminologies that was problematic to students (cf. Table 4.2). Multilingual indigenous languages were used with the notion of assisting students to understand MLP jargon appropriately (cf. Table 4.2). Ms Moleko further integrated pictures into class and home MLP activities and constructed and looked for case studies (scenarios and activities) from which students could make sense (meaning) of and solve informally, gradually moving on to solve them in a formal mathematics fashion (cf. 4.3.1.2). She did this to show students that MLP is essential in our daily lives and all sectors of the world, e.g. farming, manufacturing, banking, industries, education and organisations in public and private sectors (cf. Fig. 4.18). This affirmed that mathematics is essential for all of us because it is a human activity. MLP is within us and around us, especially when we as people or businesses face limited resources, but aim to continue with minimum costs and maximum profits in operations. This technique assisted students to change their wrong perception of MLP and increased students' interest in MLP, because most of the students did not know the importance of these topics in our daily lives (cf. 4.3.1.1). As a result, this improved students' MLP knowledge and performance (cf. 4.6.1 & 4.6.2).

On other hand, interactivity was employed as one of multiple components solutions to enhance MLP teaching and learning at this TVET college. Ms Moleko firstly gave LP classroom activities to the students in their respective groups to allow them to interact with one another to share their MLP knowledge from previous learning. As a result, students became active participants (cf. 4.3.3.2). Students were able to report back to the rest of class. Throughout MLP lesson presentations, Ms Moleko kept students interacted through classroom activities and problem questions (cf. 4.3.1). This also

helped Ms Moleko to detect students' misconceptions, misunderstandings and mistakes of MLP more easily, but this did not mean that students did not have any background knowledge. Ms Moleko also used interactivity as the RME component solution by asking students questions, allowing students to interact and discuss the topic. The students' involvement enabled them to raise their ideas, considerations, knowledge, questions and frustrations with regard to MLP problematic concepts. This led to the situation where Ms Moleko had to provide proper guidance. In addition, the guidance component solution was applied in the scaffolding approach fashion. Ms Moleko provided students with MLP activities to complete, then started with facilitation and worked together with her team of class assistants (cf. 4.3.1). Therefore, where students encountered difficulties Ms Moleko was able to guide students (cf. 4.3.1).

In conclusion, the recommended multiple components solutions of the RME approach, such as realities, interactivity and guidance enhanced students understand-ability and performance in MLP (cf. 4.6.1 & 4.6.2). These multiple components solutions also increased students' interest in learning MLP topic and mathematics.

5.3.4 A need to recognise prior knowledge and learning

Another challenge this study was that mathematics lecturers at this TVET college fail to search and actualise Students' Prior Mathematical Linear Programming Content Knowledge (SPMLPCK). According to lecturers, searching SPMLPCK wasted allotted time for teaching and made it hard for them to complete the syllabus. Other co-researchers were of the view that the failure to search SPMLPCK was caused by lecturers' lack of MLP lesson planning. The limitations of not searching and actualising SPMLPCK make it hard for lecturers to detect what students understand or not in MLP, making it hard for students to comprehend new MLPCK. This contributed to poor MLP performance of students (cf. 4.2.4 & 4.2.6). Therefore, solve this challenge, this study recommends a multiplicity of activities and interactivity components solutions of the RME approach as baseline assessments.

5.3.4.1 Suggested multiple activities and interactive components solutions of the RME approach as baseline and diagnostic assessments

The multiple activities and interactive components solutions of the RME approach are recommended by this study to implement baseline and diagnostic assessments

identified as a challenge in Section 5.3.4 successfully. These types of assessments are planned to discover what MLPCK students bring to the classroom and what they lack, since the empirical data revealed that prior knowledge is key to new knowledge. It further revealed that there is a direct relationship between prior mathematics knowledge and students' performance. For instance, adequate prior mathematics knowledge leads to good mathematics performance, while inadequate mathematics prior knowledge leads to poor mathematics performance (cf. 4.3.4). This study used multiple activities in the form of prior knowledge assessment at each intertwined level of MLP lesson presentations in order to search and actualise the SPMLPCK. This assisted Ms Moleko to know what students understand or do not understand (cf. 4.3.1). In instances where students showed some lack of MLP understanding, Ms Moleko could not spend a lot of time on that; however, she emphasised key concepts and illustrations and then moved on. In essence, prior knowledge assessment assisted Ms Moleko in not spending much time on what students already understood, but rather to allocate more time to problematic MLP components (cf. 4.3.1). To determine students' prior MLP knowledge, Ms Moleko used probing questions, discussions and interactions throughout the MLP lesson presentations to get some sense about gaps, misconceptions and students' level of understanding of MLP concepts. In this way, multiple activities and interactivity component solutions of the RME approach as a baseline and diagnostic assessments enhanced students' MLP understanding and performance (cf. 4.6.1 & 4.6.2).

5.3.5 A need for formative assessment in the teaching of MLP

This study uncovered that mathematics lecturers at this specific TVET college do not test and assess students regularly during the MLP teaching and learning process to get a clear picture of whether students comprehend new knowledge or not (cf. 4.2.5). It was discovered that lecturers present MLP lesson presentations in a linear way, imparting knowledge to students without fitting short tests or activities into the teaching and learning process. According to lecturers, formative assessments wasted allotted time for MLP teaching and learning, created an unnecessary workload, and contributed to them not being able to finish the mathematics syllabus in time. The limitations of a lack of formative assessments added to students' MLP misconceptions and difficulties (cf. 4.2.6), which led to MLP poor performance. Lecturers were unable

to detect students' MLP misconceptions in time to enable them to tap into those problematic concepts (cf. 4.2). Thus, this study recommends the continuous activities, feedback and interactivity of the RME approach to respond to the challenge identified in Section 5.3.5.

5.3.5.1 Recommended continuous activities, feedback and interactivity of the RME approach

This study recommends continuous activities, feedback and interactivity of the RME approach to respond to the challenge identified in Section 5.3.5, because this study found that lecturers at the TVET college did not assess and test students during the teaching and learning process, one of the factors contributing to poor MLP performance. This affirmed what previous studies found (cf. 2.4.1.5). To solve this challenge, this study applied continuous activities with feedback through interactivity in every part of lesson presentations at the beginning, during and at the end of the lesson (cf. 4.3.1). These continuous activities were allocated adequate time for work that students were supposed to do, to enable Ms Moleko to give regular feedback to the students. Ms Moleko was not the only one who marks these activities, because she has classroom assistants to assist with facilitating, marking and providing students with regular and timely feedback. Peer marking was also allowed with the aim to reduce Ms Moleko's workload and allowing students to identify common errors their peers commit; thus learning from errors and avoiding them. This helped students to evaluate their MLP understanding and check whether they understood the new lesson presented, becoming aware of their strengths and weaknesses to improve on their weaknesses. Through continuous activities, Ms Moleko also got instant feedback about MLP lessons, namely whether it had achieved its objectives or not (cf. 4.3.5). This had a positive impact on students' understanding and performance in MLP (cf. 4.6.1 & 4.6.2).

5.3.6 MLP students' difficulties, misconceptions and errors

The study revealed that students' difficulties and misconceptions in MLP topic led to poor performance in MLP and in other mathematics concepts (cf. 4.2.6). The co-researchers, specifically mathematics lecturers, indicated that students were not able to formulate linear inequalities and objective functions from problem statements of

MLP; sketch linear inequalities; determine feasible areas; and calculate max or min values using a search line method. Moreover, students committed so many mistakes due to their misconceptions, as confirmed by students' work in Figure 4.10-4.13. This study's discoveries evidenced what the literature and reports revealed in Section 2.4.1.6. Therefore, this study recommends the use of multiple RME approach doctrines as components solutions, such as level, intertwined, realities, guidance, activity and interactivity (cf. 4.3.1–4.3.5).

In the section below, this study discusses better conditions to enable the implementation of the RME approach successfully, as well as risks and threats that hinder the successful implementation of the RME approach through its principles as component solutions in this study.

5.4 SUGGESTED CRITICAL CONDITIONS FOR COMPONENTS SOLUTIONS OF THE RME APPROACH TO WORK

The above components solutions of the RME approach, such as levels, intertwined, realities, guidance, activity and interactivity to enhance MLP teaching and learning at TVET college to work, this study recommends adequate co-researchers' (lecturers') knowledge of the RME approach. The successful execution of the RME approach in Section 4.3 was assisted by the knowledge shared by us as a research team (cf. 4.4.1). According to Mrs Leeuw (co-researcher), the initial step to enhance the MLP teaching and learning is a proper understanding of the RME approach and also know what this approach seeks to achieve (cf. 4.4.1). This enabled us as the research team to share our knowledge with respect to the RME approach (cf. 4.4.1). Therefore, from the knowledge shared, we agreed to use six principles of the RME approach, namely levels, intertwined, guidance, interactivity, reality, and activity as components solutions of the RME approach. It further helped us to concretise the abstractness of the MLPCK (cf. 4.3.1–4.3.5). As a result, these component solutions enhanced students' MLPCK knowledge and improved their MLP performance (cf. 4.6.1 & 4.6.2).

Another conducive condition to implement the RME approach successfully to enhance MLP, according to this study, is to have sufficient Mathematical Linear Programming Content Knowledge (MLPCK). The co-researchers agreed with one another that sufficient MLPCK is key and is also the first step that individual lecturers need. One of

the co-researchers signposted that an adequate MLPCK allows lecturers to develop many techniques to solve MLP problems and to concretise abstract MLP components using the RME approach. Thereafter we as a research team agreed to divide the MLP topic into five components, namely word problems, linear inequalities, graphs, feasible region and maximum (or minimum) values. In addition, it was suggested that the MLP terminology (language) be included as one of the MLP components to enable students to understand the MLP word problems.

This study recommends sufficient MLP lesson planning and preparation in order to implement the RME approach successfully, because lesson planning guides a lecturer on what to teach, what lesson outcomes he aims to achieve, which teaching methods are needed to teach content knowledge, and what teaching and learning aids and resources are needed. The critical conditions mentioned earlier helped us to plan and develop MLP lessons. This study divided the MLP theme into five lesson plans, identified teaching and learning aids, and planned and prepared formative assessments in line with the RME approach, which aimed to concretise the abstractness of MLP concepts and the sketching of MLP graphs (cf. 4.4.3). In addition, this study recommends an enthusiastic team to enhance MLP teaching and learning.

In conclusion, the recommended critical conditions, such as adequate co-researchers regarding the RME approach, adequate co-researchers' knowledge of MLP, sufficient lesson planning and a dedicated team would be able to enhance MLP teaching and learning at this TVET college, using the RME approach through its component solutions (cf. 3.8, 4.3 & 4.4).

5.5 FACTORS THREATENING THE USE OF RECOMMENDED COMPONENTS SOLUTIONS OF THE RME APPROACH

This study viewed these factors, namely lack of teamwork, lack of lesson planning and preparations, as well as a lack of mathematics workshops and conferences, as threats and risks to hinder the success implementation of the RME approach to teach and enhance MLP at TVET colleges. The components of the RME approach in Section 4.3, which dealt with difficulty of MLP components (cf. 4.2), also enhancing the MLP teaching and learning at this TVET college have inherent risks and threats, which included a lack of collaborations; limited time allocated for teaching; lack of MLPCK

and MLPPCK; and heavy workload. Thus, the aim this study was to circumvent identified threats and risks, recommending a team teaching, enthusiastic team, adequate RME approach knowledge in RME approach, sufficient MLPCK, and sufficient lesson planning. During this study, research meetings as a research team shared knowledge about the RME approach and MLP, then prepared lessons collectively (cf. 3.8 & 4.4).

5.6 VALUE OF THIS STUDY

This study sought to enhance the teaching of MLP at TVET colleges using the RME approach, because DHET yearly reports and various researches exposed that MLP is one of the problematic topics. The contribution of this study was that the use of the RME approach to respond to identified challenges in the teaching and learning of MLP topic at this TVET college was effectively implemented by co-researchers (cf. 4.3). This study enhanced students' MLP understanding and their performance in MLP. In addition, the use of PAR as a methodology in this study should improve the co-researchers of this study's MLP conceptual and procedural content knowledge.

5.7 LIMITATIONS OF THIS STUDY

The shortcomings of this study was to use ICT and software as one of multiple pedagogical approaches to enhance the teaching and learning of MLP at TVET colleges (as recommended by the literature in 1.5 & 2.4.2.3), because of limited technological resources and poor infrastructure in the TVET sector. Limited time was also a shortcoming of this study to integrate ICT and software. Therefore, further research is needed to enhance the MLP teaching and learning using ICT and software in multiple pedagogical approaches. In addition, this study realised the pivotal role played by PAR as a methodology in implementing the RME approach to enhance the teaching and learning of MLP at TVET colleges. As a result, further research is needed using PAR as a methodology in other settings to confirm the observations in this study and also to compare the effectiveness of using PAR in teaching and learning with resultant improvements.

5.8 CONCLUSION

This chapter reported the findings of this study, which revealed that MLP is taught in a complex way, with limited teaching and learning resources. This method of teaching MLP is too teacher centred, unable to search prior MLP knowledge because of the absence of formative assessments, causing misconceptions and errors among students. All this leads to ineffective MLP teaching and learning. Therefore, the findings of this study justified a need for the use of the RME approach to enhance MLP teaching and learning at TVET colleges. This chapter concludes by presenting a summary of the RME approach developed for this study.

5.9 SUMMARY OF THE STRATEGY

The Realistic Mathematics Education (RME) approach is a teaching approach for mathematics. It was developed and introduced during 1968 by Wijdeveld, Goffree and Treffers involved in a Wiskobas project as Mathematics didacticians in the Netherlands. The rich situations (realities) are given a prominent position in the classroom environment when using the RME approach to teach mathematics (cf. 2.3.2). The RME approach expects mathematics lecturers to incorporate realities into mathematics teaching and learning, which can help students to make sense of the abstractness of MLP topic. The realities, in the Dutch context, refer to something an individual can imagine, while this study explained realities as events, practices and knowledge from diverse milieus which students can easily recognise, imagine and see around the world. Real-world situations serve as source for initiating the development of mathematics concepts, tools, and procedures. It also serves as a context within which students can, at a later stage, use what they have learned to solve real-life problems and businesses operational problems, which then gradually become more formal, general and less context specific (cf. 2.3.2). The aim of developing and using the RME approach is to mathematize the abstract mathematics concepts and challenge the mechanistic approach (traditional method) according to which students learn best when they are told in a passive way (cf. 2.3.2). The RME approach regards students as active people. It further requires of lecturers to follow the RME principles, such as activity, interactivity, realities, guidance, levels, and intertwinedness. Therefore, this study used these RME principles as component solutions to implement the RME approach to enhance the MLP teaching and learning at this TVET college

(cf. 4.3). The following are phases that guide the planning, development and implementation of the RME approach.

PHASE 1: Set up an enthusiasm team

Identify people who are affected by poor mathematics results (or current situation), and determine an appropriate time and place to hold meetings. The identified people play a vital role in developing, planning and executing the RME approach and also share responsibilities to plan, organise, implement (lead) and evaluate. More importantly, these people could work collectively and share their teaching and learning experience, interests, opinions and best practices. This could be done by a lecturer, Campus Management Team (CMT), mathematics coordinator, local academic or any member of the community concerned. For instance, the CMT could be concerned about students' mathematics performance or about mathematics lecturers' development, because that is one of their roles to fulfil. In addition, they have to determine during the initial meeting what people already know and also what needs to be learned to enhance the MLP teaching and learning, thereby improving students' results (cf. 3.6.2).

PHASE 2: Shared vision and objectives

The identified and formulated team must share and reach a common vision. A shared vision should include the expectations, objectives and interests of all team members, which will guide all activities to be executed by the team (cf. 3.6.2).

PHASE 3: SWOT analysis

This stage requires of team members to do an analysis of their strengths, weaknesses, opportunities and threats in relation to common problems identified in the first phase. To identify and indicate the weaknesses and threats, the team must recruit new members with unique expertise from inside or outside the team. Opportunities and strengths, such as experience, knowledge and expertise should be used to help the team achieve its shared vision and goals. A SWOT analysis gives the team an opportunity to measure their effectiveness, limitations, opportunities and hindrances in order to re-plan, organise, implement and evaluate through the PAR cycle with the aim to achieve its objectives (cf. 3.7).

PHASE 4: Roles and responsibilities

The roles and responsibilities of team members should be flexible in order to develop all team members through the PAR cycle, namely planning, organising, implementing, evaluating and re-planning, and organising. This is possible when the team works in collaboration, respect and support one another, and is willing to change and participate. All team members should have the opportunity to experience different roles, which will allow all members to feel free and belonging. This will develop an individual member. In addition, multiple responsibilities and roles rotation enable all team members to participate freely and to look forward to their collaborative team interactions (cf. 4.2–4.5).

PHASE 5: Action plan

The action plan is a detailed plan showing all activities that a team aims to perform to achieve shared vision and goals. This plan shows the following parts:

- Activity to be completed
- Individual person responsible to perform activity (chairing meetings, capturing data, presentations and facilitations)
- Planned time to perform an activity
- Resources organised to complete an activity
- Indicators and success

The successful implementation of the RME approach requires of team members to be equipped or at least be informed about the subject matter and this is achieved through interaction among the team members (cf. 4.3 & 4.4).

PHASE 6: Lesson preparation as a developmental activity and presentation

Collaborative lesson planning and preparation as a developmental activity takes place when a team comes together to plan and prepare a lesson. The lesson planning and preparation should be divided into the following: topic, subject content knowledge, lesson objectives, teaching and learning resources, and multiple pedagogical methods. During the lesson planning and preparation, sharing subject content knowledge should be a priority. This is done with the understanding that students are expected to demonstrate the knowledge and skills at the end of the lesson as

embedded in the assessment guideline policy. Therefore it is vital for a lecturer to be knowledgeable and have mastered Mathematics Content Knowledge (MCK) to interpret the content of mathematics and its content aim. However, understanding the MLP content alone is not adequate. Therefore, it is essential that during collaborative lesson planning and preparation, lecturers share their MCK and best practices on how to teach the content being discussed, and this will create new knowledge (cf. 4.4). This shows that knowledge is a product of human activities, created through interaction among people as well as with the world around them. Social interaction made it easy for us as a team through a team-teaching approach to produce new knowledge. Moreover, the team could share how to use the RME approach to teach MLP or other mathematics concepts. As a result, due to team diversity, sharing PCK created room for all team members to learn from one another.

On the other hand, individual lecturers can do research to find more ways to integrate the RME approach to teaching MLP or other mathematics concepts, because a lecturer aims to apply his/her pedagogical methods when teaching. This step is necessary, as well as developmental and self-empowering, because individual lecturers do not wait for his/her superiors (CMT) to develop developmental training programmes, which most of the time are not always direct and responsive to a particular challenge.

The second step after lesson planning and preparation is lesson presentations using the RME approach to enhance the teaching of MLP (including other mathematics concepts) in the team teaching approach in order to achieve objectives set (cf. 4.3 & 4.6). Prior to the planning and teaching, lecturers need to ask themselves the following questions:

- How do the backgrounds and experiences of students impact their inspiration, engagement and learning?
- How can students' background knowledge and various materials be integrated into the teaching and learning?
- How can course exercises, assignments and/or exam papers be changed to enhance students' learning, as well as when to incorporate them in the teaching and learning?

- Who is going to present the lesson and what are other members are going to do?

The above questions lead to a team using multiple teaching strategies, activities and resources to cater for students' needs, irrespective of their diversity, which creates a conducive learning environment.

PHASE 7: Lesson facilitation as a further developmental activity

Lesson facilitation and evaluation take place when a lecturer reflects on the lesson presented to check what works or does not work. The main aim for taking this step is to improve the way a lesson is presented and/or facilitated. When lecturers use the team teaching approach, they are able to facilitate a lesson they have planned and prepared together in the presence of others, adding the richness of the knowledge produced in lesson reflection, because an individual lecturer shares his/her observations on how the teaching went and how it can be improved. Therefore this becomes a development activity.

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APPENDICES



Faculty of Education

16-Oct-2018

Dear Mr Khethla Mofokeng

Ethics Clearance: Using the Realistic Mathematics Education (RME) approach to enhance the teaching of mathematical linear programming at Technical and Vocational Education Training colleges

Principal Investigator: Mr Khethla Mofokeng

Department: School of Mathematics Natural Sciences and Technology Education Department
(Bloemfontein Campus)

APPLICATION APPROVED

With reference to your application for ethical clearance with the Faculty of Education, I am pleased to inform you on behalf of the Ethics Board of the faculty that you have been granted ethical clearance for your research.

Your ethical clearance number, to be used in all correspondence is: **UFS-HSD2018/0658**

This ethical clearance number is valid for research conducted for one year from issuance. Should you require more time to complete this research, please apply for an extension.

We request that any changes that may take place during the course of your research project be submitted to the ethics office to ensure we are kept up to date with your progress and any ethical implications that may arise.

Thank you for submitting this proposal for ethical clearance and we wish you every success with your research.

Yours faithfully

Prof. MM Mokhele Makgalwa
Chairperson: Ethics Committee

Education Ethics Committee
Office of the Dean: Education
T: +27 (0)51 401 3777 | F: +27 (0)86 546 1113 | E: MokheleMM@ufs.ac.za
Winkie Dinko Building | P.O. Box/Postbus 339 | Bloemfontein 9300 | South Africa
www.ufs.ac.za



CORRIE GELDENHUYS
POSBUS 28537
DANHOF 9310

☎ **083 2877088**
corrieg@mweb.co.za

26 September 2020

TO WHOM IT MAY CONCERN

Herewith I, **Cornelia Geldenhuys (ID 521114 0083 088)** declare that I am a qualified, accredited language practitioner and that I have edited the following dissertation:

**USING THE REALISTIC MATHEMATICS EDUCATION APPROACH TO ENHANCE
THE TEACHING OF MATHEMATICAL LINEAR PROGRAMMING AT TECHNICAL
AND VOCATIONAL EDUCATION TRAINING COLLEGES**

by

MOFOKENG KHEHLA PATRICK (2008088763)

All changes were indicated by track changes and comments **for the author/student to verify, clarify aspects that are unclear, make the necessary adjustments and finalise.**

The editor accepts no responsibility for any changes and corrections that are not made in the final text returned to the student.



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C GELDENHUYS
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Central Office
Mampoi Road
Private Bag X070
Witsieshoek
9870
Tel: 058 713 6100
Fax: 058 713 6975
officeman.cent@malutitvet.co.za



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Enquiries: Makole MIM

13 January 2019

TO WHOM IT MAY CONCERN

Re: Mr KP MOFOKENG-LETTER OF APPROVAL TO CONDUCT RESEARCH AT MALUTI TVET COLLEGE

This communique confirms that Maluti TVET College has granted approval for you to conduct research on Using the Realistic Mathematics Education (RME) approach to enhance the teaching of mathematics linear programme at Technical and Vocational Education Training Colleges. We are convinced that through the study the College will benefit hence the support.

We wish you all the best in your studies.

Regards.

ME Tsotetsi
Deputy Principal: Academic

Cell : 082 8066 181
Tel : 058 713 6100
Website : www.malutitvet.co.za
e-mail : academic.dep@malutitvet.co.za



Central Office
Mampoi Road
Private Bag X070
Witsieshoek
9870
Tel: 058 713 6100
Fax: 058 713 6975
officeman.cent@malutitvet.co.za



higher education
& training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

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PAPER 1 CONSOLIDATED MARKERS, INTERNAL MODERATORS, AND CHIEF MARKERS REPORT

2014 November NCV MATHEMATICS LEVEL 4 EXAM

PART 4: STUDENT PERFORMANCE

INSTRUCTION-

During the marking process each marker should complete the table below and submit it to the Chief Marker on conclusion of the marking process per subject. The Chief Marker must incorporate the inputs of all markers for the subject into the final Chief Marker's report.

Question Number	Content assessed in the question	Rate learner performance in question				Highlight areas of weakness in student performance in the content assessed in the question
		Poor	Average	Good	Excellent	
EXAMPLE	TRIAL BALANCE		X			Most candidates unable to calculate: - Cost of sales - Depreciation
1.	Question 1.1, 1.2, & 1.3 (8 marks)	X				- Simplification - Use DeMoivre's theorem
2.	Question 2.1, 2.2, 1.2, 2.2 & 2.3 (15 marks)	X				- Applying first principles - Using differentiation rule (derivatives)
3.	Question 3.1 & 3.2 (16 marks)	X				Doing Integrals
4.	Question 4.2 & 4.4 (14 marks)	X				- Simplification of cubic equation - Determining and drawing the inverse of graphs
5.	Question 5.1, 5.2, 5.3 & 5.4 (13 marks)	X				- Write down the constraints that represent the above scenario - Represent the system of constraints and indicate feasible region - Determine the minimum cost
OVERALL	XXXX	X				POOR

Annexure F

Researcher:	Supervisor:
KP Motokeng	Dr TJ Molei
159 Leiba Village	Faculty of Education UFS
Witsieshoek	Private bag X 12
9570	Phuthaditjaba
	6026
kgmotokeng95@gmail.com	moleiJT@ufs.ac.za
073 963 0360	058 718 5002/ 0822025670

15-01-2019

Dear Sir or Madam

Letter of invitation for research participants

I would like to invite you to take part in this research project: Using the Realistic Mathematics Education (RME) approach to enhance the teaching of Mathematical Linear Programming at Technical and Vocational Education Training Colleges. The aim of this project is to use Realistic Mathematics Education (RME) approach to enhance the teaching of mathematical linear programming at TVET Colleges. The previous researches and your reports showed that the students have difficulties to understand MLP in Mathematics which lead to poor performance in Mathematics at Technical and Vocational Education and Training Colleges. Thus, this study anticipate to strengthen the teaching and learning of mathematics especially MLP concepts which can improve students' performance and also college results.

This study planned to hold research meetings, lesson observations, lesson presentations, lesson reflection and free interviews in the way:

Venue: Kgotseng TVET College (Tlohang campus)

Days: Fridays

Time: 14:00-15:30

Research participants: Council member, Maths coordinator, Campus Manager, Head of Departments (HoDs), Supervisors, Maths lecturers, English lecturer, parents, and students.

NB: the information on above is subjective not fixed, because research participants (non-researchers) will be given opportunity to decide time and weekly days that will be fine with all of us.

For more information you may contact Dr Molei TJ (058 718 5002/ 0822025670 as my supervisor).

Thank you

Yours in sincerity

1.	2.
Student: Khentle P Motokeng (2009085795)	Supervisor: Dr TJ Molei
EMAIL: kgmotokeng95@gmail.com	EMAIL: moleiJT@ufs.ac.za
Cell No: 073 963 0360	Contact No: 058 718 5002/ 0822025670
Personnel No: 00001140	UFS (QWAGWA campus)
Male: TVET (DHET)	Faculty: Faculty of Education UFS (QwaQwa)
	Department: school of mathematics, science and technology

Researcher:	Supervisor:
Kir Mankeng	Dr T.J Maloi
101 Leran Village	Faculty of Education
Witwatersrand	UTS (Gwa Gwa)
2070	Private Bag 818
	Phuthaditaba
	9966

15/01/2019

Dear Sir/Madam

LETTER OF CONSENT FOR RESEARCH PARTICIPANTS (CO-RESEARCHERS)

(Council member, Maths coordinator, Campus manager, HoD, Supervisor, Maths lecturer, English lecturer, Parents, and students)

I would like to invite you to take part in this research project. Using the Realistic Mathematics Education (RME) approach to enhance the teaching of Mathematical Linear Programming at Technical and Vocational Education Training Colleges.

We would like you to participate with us in the research because this study believes that you have valuable knowledge, skills and information about the teaching and learning at TVET Colleges which can assist this study to achieve its objectives.

The aim of this project is to use Realistic Mathematics Education (RME) approach to enhance the teaching of mathematical linear programming at TVET Colleges. The previous researches and yearly reports showed that the students have difficulty to understand MLP in Mathematics which lead to poor performance in Mathematics at Technical and Vocational Education and Training Colleges. Therefore, this study anticipate to strengthen the teaching and learning of mathematics especially MLP concepts which can improve students' performance and also college results.

More importantly, this research project is not harmful to lives of research participants because it is guided by Participatory Action Research which seeks to promote social justice, equity, respect, hope, peace, and freedom between participants, which in this regard are called co-researchers. However, there will be no payments offered to you as research participants to participate in this research study. Therefore, we the co-researchers will work together in rooting out the challenges of teaching and learning mathematics and also share best practices, knowledge and skills on how a Realistic Mathematics Education (RME) approach can be used to teach Mathematical Linear Programming.

In addition, as principal investigator, I will uphold ethics considerations which protect concludes one moment in the research process. This research project will apply for ethics clearance from Research Ethics Committee at University of the Free State. You are informed that your participation is not compulsory and if you decide not to participate, that will not be held against you. So, you are free to join/withdraw/re-join this research project at any stage. Therefore, confidentiality, anonymity and legally issues of this project will be discussed with you, as it is important to all participants to fully understand the nature. Moreover, I will keep confidential information collected data through lesson presentations, discussions and observations and in the safe place where it is always locked as it will be used for this purpose only. However, if you want to be informed about the final research findings to verify whether your spoken words, written words, and captured information are well presented, you are free

ANNEXURE H

Researcher:	Supervisor:
KP Mokhele	Dr TJ Mole
159 Leribe Village	Faculty of Education UPS
Witsiesice	Private bag X 13
9870	Phuthaditjhaba
	5856
kpmokhele83@gmail.com	mole.tj@ufs.ac.za
073 903 0689	081 718 5002/ 0822025370

15-01-2019

Dumela Motswadi ya tlomphehang

Ka tlomphele le boikokobetso bo bohlo e mme wa ho nka karolo ya patliso tsa senleho sa: USING THE REALISTIC MATHEMATICS EDUCATION (RME) TO ENHANCE THE TEACHING OF MATHEMATICAL LINEAR PROGRAMMING AT TVET COLLEGE. Marara wa patliso ena, ke haka ka tsela e bobete wa ho matlafatsa tluto ya dipalo (Mathematics) nahale ho hana ba nna tlutong ya bona. Ka ho sebetsa ha nna o ho thusa saho la matijhere ho matlafatsa tluto ena ya dipalo nna banna ba tsela ho sebetsa hantle. Re ka thusa tluto ha o ka nka karo o dikozanang tse tlang ho tshwanwa projekeng ena ya ho thusa banna nna ho utlwisisa tluto ena ya dipalo. Ho nka karolo ha hao nna projekeng ena ho ka tlang phetolele e kgolo tlutong le sajjhabang sa nna, nna re ka tshela tshobotsana.

Ka tshenisa haka projeko ena e ka sa baha maphele a tlang tseleng ha o nka, karolo ha matlafatsang bokamoso bo bohe sehokang bona. Mosebetsi o monako projekeng ena ke hore tsotela ho ho sehokang re fapane mathata ao bona, matijhere, le sekole re kopanang le tanna, le ho sebetsa ka kopanako ho bala diharolo mathateng ao. Ha ho qotole, ka nna ha tsabisa balewadi hore projekeng ena ka hore ho kenya letsoho marang ena ke tshepo (ha bona potole), nna ha bala dikoto qobelo hobane ha ka se le sa tsela ho tsela pele, le ka nna ho le diphelelano tsa motuta oile kapa oile. Hape nna ke o matlafatsa hore ha tana le tlalele le dumelatswa ho thusa banna, bo le kopanang le bona ho matlafatsa wa dipatliso. Jwale ya ho qetela ya tsotela le hore matliso a nna a mme a keke a matlisoa tlutong ya dipatliso.

Ke ya leboha

Kheha P Mokhele

Dr TJ Mole- Promoter

ANNEXURE J.

Classroom MLP lesson presentation in Thabong campus at KHOTSONG TVET COLLEGE

On Tuesday the 3rd Mar 2016, we myself as principal investigator and Mr Kheena elected chairperson for morning meetings of this research study went to Thabong campus at the same KHOTSONG TVET COLLEGE to do observations on MLP lesson presentation. We met with Mr Kheena (Lecturer) who then introduced us to Mr Takai, the Mathematics lecturer, who was also about to go to his class. He took us to his Mathematics class, where he was going to present or start new mathematics topic which is MLP. Mr Takai (Maths lecturer) introduced us to his students and started his lesson by greeting students:

"Good afternoon students"

Students: "Afternoon Sir"

Mr Takai: "today we are going to do MLP"

He then took NCV Mathematics L4 textbook of D Kottler (student) and said

"Ok, let's go to page 56 of your textbook, right this is MLP topic, so, to solve this kind of MLP problem is 4, we first need to convert MLP word problems into linear inequalities; usually, we solve linear inequalities and lastly we draw MLP graph with this word sequence".

Mr Takai look students through Example of MLP on page 54. He read Example 1....

...A small factory makes two types of washing machines, a standard and a luxury model. The daily production cannot be more than 8 standard and 2 luxury washing machines. There are 8 factory workers and it takes one worker to make one standard model and 2 workers to make one luxury model per day. The profit on the standard model is R300 and on the luxury model R600. Questions:

1. Write down linear constraint and inequalities.
2. How many of each type of machine must be manufactured per day to make a maximum profit?

He interpreted the Example 1 and said

"based on what I have read, firstly remember what I said earlier on, we need to write linear inequalities to present constraint number 1. Secondly, we have to solve linear inequalities and then draw MLP graph so that we can find feasible region in order to determine maximum profit in questioning question number 2"

The lesson continued ...

...From the first sentence I have read prior to above Example 1, we first two types of machines manufactured by this company, so we first list ...

Type A (standard machine) be x

Type B (luxury machine) be y

Remember, always to state these linear inequalities...

$$x \geq 0$$

$$y \geq 0$$

Thuloane (student) said: "Sir, Sir... raised his hand"

Mr Takai: "Thuloane?"

The same (student): "in which sentence does the $x \geq 0$ and $y \geq 0$ come from, from what you read Sir?"

Mr Takai: "That's a good question Thuloane. So, you must know that when we do MLP topic, we are always expected to show these linear inequalities. Thus, it is a rule, so you have to write them"

Mr Takai continued with his lesson...

The second sentence (see above Example 1), means company will not be able to produce more than 8 standard and 2 luxury machines, then to write it mathematically we say...

$$x \leq 8 \text{ linear inequality 1}$$

$$y \leq 2 \text{ linear inequality 2}$$

PRE-TEST (50 marks)

Question 1 (17 marks)

1. Define and write the following words in mathe

NO.	MLP words	Explanation	Mathe symbol
1.1	less or equal to		
1.2	Maximum		
1.3	at most		
1.4	greater or equal to		
1.5	Minimum		
1.6	at least		
1.7	less than		
1.8	greater than		
1.9	times		
1.10	Half		
1.11	one third		
1.12	variables		

(17)

Question 2 (3 marks)

2.1 Ma Xulu use minibuses and minitaxis to transport school kids, workers and guests to reach their destiny. Wini has earned large sums of quantum tax to transport people in their destiny.

2.2 Ma Xulu has spent 30 so tool kids on most every day at different schools.

Question 3 (15 marks)

A factory makes two types of birds, type A and type B. Each month x of type A and y of type B are produced.

The following constraints control monthly production:

- (i) Not more than 50 birds of type A and 40 birds of type B can be made.
- (ii) To make a profit at least 30 birds in all must be made.
- (iii) The maximum number of birds that can be produced is 60.

PRE-TEST (50 marks)

Question 1 (17 marks)

1. Define and write the following words in maths

NO.	NLP words	Explanation	Maths symbol
1.1	less or equal to	Not above but not below or equal	$\leq$
1.2	Maximum	Higher than	$\leq$
1.3	at most	Cannot succeed	$\leq$
1.4	greater or equal to	Its either above or equal to	$\geq$
1.5	Minimum	It is the lowest	$\geq$
1.6	at least	minimum of your level	$\geq$
1.7	less than	It is for front	$<$
1.8	greater than	It is bigger than	$<$
1.9	times	It is multiplication	$\times$
1.10	Half	Part of the whole	$\frac{1}{2}$
1.11	one third	refers to divide by three	$\frac{1}{3}$
1.12	Maths variables	Refers to alphabets that are used in maths	x, y, z and any alphabets

Question 2 (3 marks)

2.1 Ms Xulu use mini bus and quantum taxi to transport school kids, workers and people to reach their destiny. Mini bus can not take twice of quantum taxi to transport people in their destiny.

Let mini bus be x
Let quantum taxi be y

2.2 Ms Xulu transport 80 school kids at most every day to different schools.

Let school kids be x
 $x \leq 80$

Question 3 (15 marks)

A factory make two types of beds, type A and type B. Each month x of type A and y of type B are produced.

The following constraints control monthly production:

- (i) Not more than 50 beds of type A and 40 beds of type B can be made.
- (ii) To show a profit at least 80 beds in all must be made.
- (iii) The maximum number of beds that can be produced is 80.

PRE-TEST (50 marks)

Question 1 (17 marks)

1. Define and write the following words in maths

NO.	MLP words	Explanation	Maths symbol
1.1	less or equals to	Smaller than/same as ✓	
1.2	Maximum	The higher ✓	
1.3	at most	Repeatedly ✓	
1.4	greater or equals to	Bigger / same as ✓	
1.5	Minimum	Less ✓	
1.6	at least	Worth a try ✓	
1.7	less than	Below ✓	
1.8	greater than	Bigger than ✓	
1.9	times	1/3 when we do multiplication ✓	
1.10	Half		
1.11	one third	Three equal parts ✓	
1.12	Maths variables	Stands for something ✓	

(17)

Question 2 (8 marks)

2.1 Ms Xulu use minibus and quantum taxi to transport school kids, workers and people to reach their destiny. Mini bus cannot take twice as quantum taxi to transport people to their destiny.

2.2 Ms Koba transport 80 school kids at most every day to different schools.

Question 3 (15 marks)

A factory makes two types of beds, type A and type B. Each month x of type A and y of type B are produced.

The following constraints on monthly production:

- (i) Not more than 50 beds of type A and 40 beds of type B can be made.
- (ii) To show a profit at least 60 beds in all must be made.
- (iii) The maximum number of beds that can be produced is 90.

PRE-TEST (50 marks)

Question 1 (17 marks)

1. Define and write the following words in maths

NO.	MLP words	Explanation	Maths symbol
1.1	less or equal to		
1.2	Maximum		
1.3	at most		
1.4	greater or equal to		
1.5	Minimum		
1.6	at least		
1.7	less than		
1.8	greater than		
1.9	times		
1.10	half		
1.11	one third		
1.12	Maths variables		

(17)

Question 2 (3 marks)

2.1 Ms Xulu use mini bus and quantum taxi to transport school kids, workers and people to reach their destiny. Mini bus cannot take twice of quantum taxi to transport people to their destiny.

2.2 Ms Koba transport 60 school kids at most every day to different schools.

Question 3 (15 marks)

A factory makes two types of beds, type A and type B. Each month x of type A and y of type B are produced.

The following constraints can't of monthly production:

- (i) Not more than 50 beds of type A and 40 beds of type B can be made.
- (ii) To show a profit at least 80 beds in all must be made.
- (iii) The maximum number of beds that can be produced is 80.

ACTIVITY A 2019

1. Complete below table:

NLP Competencies	Distribution	Marking
1.1 Less than		
1.2 Not more than		
1.3 Less or equal to		
1.4 At least		
1.5 More than		
1.6 greater than		
1.7 Greater than		
1.8 Less than or equal to		
1.9 Greater or equal to		
1.10 Minimum		
1.11 at least		
1.12 Greater than		
1.13 Value or variable		
1.14 Feasible region		
1.15 solution		
1.16 lines		
1.17 feasible line		
1.18 set		
1.19 optimal		

(27 marks)

ANALYSIS OF STUDENTS ACTIVITY A RESULTS

Achievement codes	Competencies	%	Number of students
Level 1 (L1)	Not achieved	20.00	25
Level 2 (L2)	Emerging	30.00	38
Level 3 (L3)	Intermediate	40.00	50
Level 4 (L4)	Adequate	50.00	63
Level 5 (L5)	Satisfactory	60.00	75
Level 6 (L6)	Meritorious	70.00	88
Level 7 (L7)	Outstanding	80.00	100

NLP ACTIVITY A

Pass % = $\frac{\text{Number of students who passed}}{\text{Total number of students}} \times 100 = 77\%$
Failed % = $\frac{\text{Number of students who failed}}{\text{Total number of students}} \times 100 = 23\%$

ACTIVITY A 2019

22%

1. Complete below table:

MLP terminologies	Definitions	Maths sign
1.1 Less than		$<$ ✓
1.2 not more than		
1.3 Less or equal to		$\leq$ ✓
1.4 At most		
1.5 Maximum		
1.6 greater than		$>$ ✓
1.7 not less than		
1.8 linear inequalities		
1.9 Greater or equal to		
1.10 Minimum		
1.11 at least		
1.12 Constraints		
1.13 Maths variables		
1.14 Feasible region		
1.15 solid line		
1.16 lines		$\times$ ✓
1.17 dotted line		
1.18 half		$\frac{1}{2}$ ✓
1.19 one third		$\frac{1}{3}$ ✓

(27 marks)

NB! Explain these terms in your own words

26%

ACTIVITY A 2019

1. Complete below table:

MLP terminologies	Definitions	Maths sign
1.1 less than	Below actual number ✓	1
1.2 not more than	The same as the number	1
1.3 Less or equal to	Below or same as ✓	1
1.4 At most	Nearby ✓	1
1.5 Maximum	Approximately ✓	1
1.6 greater than	Bigger ✓	1
1.7 not less than	Not below than ✓	1
1.8 linear inequalities	✓	1
1.9 Greater or equal to	Bigger or same as ✓	1
1.10 Minimum	The least ✓	1
1.11 at least	try ✓	1
1.12 Constraints	Difficulties Restrictions ✓	1
1.13 Maths variables	Stands for something ✓	1
1.14 Feasible region	Payable ✓	1
1.15 solid line	Stable ✓	1
1.16 times	Multiplication ✓	1
1.17 dotted line	Break us in between ✓	1
1.18 half	Not entirely ✓	1
1.19 one third	Three equal parts ✓	1

NB: Define these words, do not write synonyms only. Remember to write them mathematically (27 marks)

44%

ACTIVITY A 2019

1. Complete below table:

MLP terminologies	Definitions	Maths sign
1.1 Less than	minimum of something	$<$ ✓
1.2 not more than	maximum of something	$\leq$ ✓
1.3 Less or equal to	maximum of something or half of it.	$\leq$ ✓
1.4 At most	maximum	$\leq$ ✓
1.5 Maximum	as great	$\leq$ ✓
1.6 greater than	More than	$>$ ✓
1.7 not less than	not or minimum of something	$\geq$ ✓
1.8 linear inequalities		$\geq$ ✓
1.9 Greater or equal to	At most maximum or half of it.	$\geq$ ✓
1.10 Minimum	not less than or the least	$\geq$ ✓
1.11 at least	not less than	$\geq$ ✓
1.12 Constraints	or limitations	$\leq$ ✓
1.13 Maths variables	a symbol which functions as a placeholder	$\leq$ ✓
1.14 Feasible region	feasible set or solution space is the set of all possible points.	$\leq$ ✓
1.15 objective		$\leq$ ✓
1.16 times		$\leq$ ✓
1.17 divided by		$\div$ ✓
1.18 half	dividing something into two equal parts.	$\frac{1}{2}$ ✓
1.19 one third	one third is one part of three equal parts (27 marks)	$\frac{1}{3}$ ✓

1200
27.00

ACTIVITY B 2018

1. Complete below table

MLP Terminologies	Explanation (include the practical examples)	Mark's sign
1.1 Less than		
1.2 Greater than		
1.3 Inverse equation		
1.4 Less or equal to		
1.5 Greater or equal to		
1.6 Linear inequalities		
1.7 Dotted line		
1.8 Solid line		
1.9 Constraints		

(15 marks)

ANALYSIS OF STUDENTS ACTIVITY B RESULTS

Achievement codes	Competencies	%	Number of students
Level 1 (L1)	Not achieved	00-29	05
Level 2 (L2)	Elementary	30-39	09
Level 3 (L3)	Moderate	40-49	11
Level 4 (L4)	Adequate	50-59	09
Level 5 (L5)	Substantial	60-69	07
Level 6 (L6)	Meritorious	70-79	00
Level 7 (L7)	Outstanding	80-100	09

MLP ACTIVITY B			
Pass % =	$\frac{\text{number passed}}{\text{total number of students}} \times 100 = 86\%$		
Failed % =	$\frac{\text{number failed}}{\text{total number of students}} \times 100 = 14\%$		

ACTIVITY B 2019

1. Complete the table:

MLP terminologies	Explanation (include the practical examples)	Maths sign
1.1 Less than	Having a small number/amount or quantity. Mphah's car value is far from hereto's R234 000 R69 085	$<$
1.2 Greater than	Having a larger number/amount than Sipho's height is above mine	$>$
1.3 Linear equation	Is equation between one or two variables that gives a straight line when plotted on a graph.	$y = mx + b$
1.4 Less or equal to	Having a smaller amount/number or equal to. Mam Sadi during November at School makes maximum of 50 cup cakes per day	$\leq$
1.5 Greater or equal to	It can be or any number/values greater than. Mam Sadi during Jan makes minimum of 150 cup cakes per day	$\geq$
1.6 Linear inequalities	Involves a linear expression in two variables by using of the relational symbols such as $<$, $>$, $\leq$, and $\geq$	$y < mx + b$ $y \leq mx + b$ $y > mx + b$ $y \geq mx + b$
1.7 Dotted line	A line that is made up of a series of dots. In maths we sketch dotted line when given with less than/greater than sign	---
1.8 Solid line	A line that has no space between lines of type. In maths we sketch solid line when given linear equation and inequalities	—
1.9 Constraints	is restrictions and limitations of resources, or values to perform something. Mr Baco has only 20 hectares of land to plant Brown beans and Spinach	$\leq$

ACTIVITY B 2019

1. Complete below table:

NLP terminologies	Explanation (include the practical examples)	Maths sign
1.1 Less than	Is a maths symbols that denotes an inequality between two values. E.g. There are 4 stones in one bowl A and 7 stones in Bowl B. In comparison the two it is clear that Bowl A has fewer stones than Bowl B.	$<$ ✓
1.2 Greater than	Is a sign that denotes an inequality between two values. E.g. There are 4 stones in one bowl A and 7 stones in Bowl B. In comparison the two it is clear that Bowl B has greater stones than Bowl A.	$>$ ✓
1.3 Linear equation	An equation between two variables that gives a straight line when plotted on a graph.	$x=1$ $y=7$ $x+y=2$ ✓
1.4 Less or equal to	Statement that indicates value or expression is not equal to another. E.g. Her cell phone cost lower or equal to R2500.	$\leq$ ✓
1.5 Greater or equal to	It is either above or equal. E.g. Her shoe price is higher or equal to that of my cell phone.	$\geq$ ✓
1.6 Linear inequalities	A equation between two variables that gives a straight line when sketching it. It also shows area which satisfy inequality signs.	$x+y \geq 10$ ✓
1.7 Dotted line	A line made up of dots or dashes.	----- ✓
1.8 Solid line	A line that has no gaps that is continuous.	————— ✓
1.9 Constraints	Refers to restrictions.	A ✓

(18 marks)

15
18
83°

ACTIVITY B 2019

1. Complete below table.

MLP terminologies	Explanation (include the practical examples)	Maths sign
1.1 Less than	It is 2 nd from ✓	$<$ ✓
1.2 Greater than	Bigger than ✓	$>$ ✓
1.3 Linear equation	Is straight line equation made of variable, equal sign and values or numbers ✓	$x + 10 = 5$ ✓
1.4 Less or equal to	It is either below or equal to not above ✓	$\leq$ ✓
1.5 Greater or equal to	It is either above or equal ✓	$\geq$ ✓
1.6 Linear inequality	Is a straight line equation made of variable, unequal sign and value or numbers ✓	$x + 10 < 5$ ✓
1.7 Dotted line	Line that has spaces in between. ✓	----- ✓
1.8 Solid line	Line that has no spaces or gaps in the form of straight line. ✓	———— ✓
1.9 Constraints	Something that imposes a limit or restrictions that prevents something from happening ✓	X

NB: Define the above words and do practical examples. Do not write synonyms but explain them !!!

ACTIVITY C: MLP SCENARIO WITH TWO MATHS VARIABLES

SCENARIO 2:

Mr Tshabalala makes chocolate and strawberry flavors of muffins. He cannot make 750 of both chocolate and strawberry muffins per day.

(05)

ANALYSIS OF STUDENTS ACTIVITY C (SCENARIO 2) RESULTS

Achievement codes	Competencies	%	Number of students
Level 1 (L1)	Not achieved	14-29	31
Level 2 (L2)	Elementary	30-39	02
Level 3 (L3)	Modest	40-49	01
Level 4 (L4)	Adequate	50-59	01
Level 5 (L5)	Substantial	60-69	00
Level 6 (L6)	Meritorious	70-79	00
Level 7 (L7)	Outstanding	80-100	00

MLP ACTIVITY C

$$\text{Pass \%} = \frac{\text{number passed}}{\text{total number wrote}} \times 100 = 11\%$$

$$\text{Failed \%} = \frac{\text{number failed}}{\text{total number wrote}} \times 100 = 89\%$$

ACTIVITY C: MLP SCENARIO WITH TWO MATHS VARIABLES

SCENARIO 2:

Mr Tshabalala makes chocolate and strawberry flavors of muffins. He cannot make 750 of both chocolate and strawberry muffins per day.

750 chocolate and strawberry each
and day

NP: cannot make equation from the
above scenario

(05)

ACTIVITY C: MLP SCENARIO WITH TWO MATHS VARIABLES

SCENARIO 2:

Mr Tshabalala makes chocolate and strawberry flavors of muffins. He cannot make 750 of both chocolate and strawberry muffins per day.

chocolate + strawberry flavors muffins ≤ 750

let units of chocolate = x and strawberry be y

(05)

ACTIVITY C: MLP SCENARIO WITH TWO MATIS VARIABLES

SCENARIO 2:

Mr Tshabalala makes chocolate and strawberry flavors of muffins. He cannot make 750 of both chocolate and strawberry muffins per day.

0/5

0%



(05)

ANNEXURE Q

HOME ACTIVITY (20 marks)

Q1 (04 marks)

1. Find coordinates and sketch linear inequalities

$$(1.1) \quad x \leq 50 \quad (02)$$

$$(1.2) \quad y \leq 40 \quad (02)$$

Q2 (06 marks)

2. Find coordinates and sketch linear inequalities

$$(2.1) \quad \frac{20}{m}x - \frac{15}{n}y \leq 24 \quad (03)$$

$$(2.2) \quad 2x + y \leq 16 \quad (03)$$

Q3 (10 marks)

3. Find coordinates and draw MLP graph

$$(3.1) \quad 15x - 12y \leq 60, x, y \in \mathbb{R}$$

$$x \geq 2 \text{ and } 1 \leq y \leq 3$$

$$\text{Objective } G = \frac{x}{2} + 2y$$

(10)

ANALYSIS OF STUDENTS HOME ACTIVITY RESULTS

Achievement codes	Competencies	%	Number of students
Level 1 (L1)	Not achieved	00-20	12
Level 2 (L2)	Elementary	30-39	27
Level 3 (L3)	Moderate	40-49	18
Level 4 (L4)	Adequate	50-69	30
Level 5 (L5)	Substantial	60-69	20
Level 6 (L6)	Meritorious	70-79	00
Level 7 (L7)	Outstanding	80-100	00

MLP HOME ACTIVITY	
Pass % = $\frac{\text{number passed}}{\text{total number seats}} \times 100 = \frac{29}{40} \times 100 = 71\%$	
Failed % = $\frac{\text{number failed}}{\text{total number seats}} \times 100 = \frac{11}{40} \times 100 = 29\%$	



Lero La Tshaya Campus

Tseke Village

Purabe Road 4335

Pretoria

0075

Tel: 050 745 0031

Fax: 050 745 0011

office@ml.lero@matulivetha.za

ML LERO

2014-08-22



higher education
& training

Department
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

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10/06/14

JOB DESCRIPTION: LECTURER

JOB TITLE	JOB GRADE	SALARY RANGE	OCCUPATIONAL LEVEL
Lecturer	C2	To be confirmed	Technical Process Decision-making
UNIT/CAMPUS/BUSINESS STUDY UNIT	Campus (LERO LA TSHAYA)/Component: Academic Department		
JOB PURPOSE	To provide teaching and practical training that represents the highest level of professional excellence and to contribute to vocational and occupational education advancement in the community.		
KEY RESPONSIBILITIES	WEIGHT	SCHEDULE OF TASKS	STANDARDS AND PERFORMANCE OUTCOMES
1. Teaching and Learning Delivery	50%	- Teach students in classrooms or workshops providing theory and practical training through lectures, discussions, practical demonstrations and supervision. - Prepare lessons (lesson plans) and produce materials such as typed notes, diagrams, patterns and any consumables for use in teaching. - Maintain student attendance registers and produce student progress reports. - Monitor the provision of resources to libraries and/or access to IT facilities appropriate to the subject requirements and advise on these requirements. - Monitor and report on student engagement and responsiveness to teaching activities.	- Pass rates achieve agreed targets - Student and class representative indicate satisfaction with learning experience. - Lesson plans available - OHS principles and Health and Safety regulations exemplified with.
2. Assessment	20%	- Assess students by setting and marking exams and evaluating completed projects. - Ensure that work is set and assessed consistently, in line with OHS principles and the College Assessment policy and complete all related administrative records or reports. - Participate in subject related assessment and moderation committees. - Maintain all assessment records of student progress and performance.	- All assessment results and records submitted within required timeframes. - Compliance with external assessment related audits (e.g. UMAHSD). - All assessment documents (POB's) secure and available.
3. Teaching Administration	10%	- Maintain registers of all textbook/tool allocations and check returned textbooks/tools (ensuring charges for loss or damage). - Submit annual and term estimates of material needs (including consumables). - Report on or facilitate the maintenance of equipment and/or workshops. - Comply with all registration, assessment and other data entry requirements. - Participate in regular departmental meetings, informing colleagues of all matters pertaining to student performance.	- Student and textbook/tool registers accurately maintained and submitted within required timeframes. - Equipment maintenance practices ensure effective asset utilization and availability for teaching requirements.
4. Subject Development	10%	- Maintain up to date knowledge of industry trends in subject area. - Participate in INSET courses and contribute subject specific teaching enhancements. - Maintain and/or revise instructional material based on current industry trends. - Represent the subject area at 'open days' and other promotional events. - Provide guidance and work related insights to students, parents and the general community.	- Participation in a minimum of one (1) INSET OR INDUSTRY RELATED COURSE/VISIT PER ANNUM. - Employers satisfied with subject related skill levels. - Subject well represented in assigned promotional activities.
5. Student Guidance and Support	10%	- Provide advice on subject related career options and typical working scenarios. - Support students on an individual basis	Remedial and other academic support requirements reported timeously.

The Realistic Mathematics Education (RME) approach

Definition, characteristics, aim and history

- The Realistic Mathematics Education (RME) refers to a domain specific instruction or pedagogical theory for Maths, which has been developed in the Netherlands (Heuvel- Panghuizen & Drijvers, 2014, 147).
- According to Heuvel- Panghuizen (2008: 12) one of the most vital characteristics of the RME is that the real situations (realities) are given a prominent position in Maths teaching and learning processes.
- The real world situations serve as a source for initiating development of maths concepts, tools, and procedures as a context in which students at a later stage apply their mathematical knowledge and skills to solve problems, which then gradually become more formal and general and less context specific.
- Özgüye & Yetim-Kocman, (2017: 95) indicated that the word 'realistic' refers to something you can imagine in Dutch education, as a result, it helps students to make sense of abstract mathematical concepts.
- According to Heuvel- Panghuizen & Drijvers (2014: 147) the RME method was first introduced and developed by Ilse Kester, Joop van der Stoep, Fred Goffree and Adrie Treffers in 1984 at Wiskunde project. The Wiskunde project in 1971 became part of the newly established IOWO Institute, with Hans Freudenthal as first a director and in 1979 when the IOWO was expanded with the Wiskunde project for secondary Maths education; the basic research led to the purpose to reform prevailing methods of Mathematics education.
- In 1980s, Maths education in Netherlands was dominated by a mechanistic pedagogical approach, where Maths was taught directly at a formal level, by atomised 'teacher' and mathematical content was defined first in the structure of Maths as a scientific discipline, students learnt mathematics step by step showed by Maths lecturer or 'how to solve Maths' problem-solving, as this led to 'infinite' and 'reduction-based' (Zarembka, 2017: 7). However, Freudenthal was proponent of reconstruction of Maths education was not affected by formal approach of New Maths movement and this RME could be developed.
- Hans Freudenthal (1905-1990) was a mathematician born in Germany and became professor of pure and applied and foundations of Mathematics at Utrecht University in the Netherlands in 1948. As a mathematician, he made substantial contributions in domains of geometry and topology, where at later in his career became interested in Maths education. Moreover, he argued that seeing Maths is relevant for students, so carry out thought experiments to examine how students can be offered opportunities for guided reinvention of Maths. Therefore, Freudenthal characterised dominant approach to Maths education in which scientific structured curricula were used to teach students while students were confronted with ready-made Maths as an anti-didactic invention. However, the RME challenges mechanistic approach then because students were just receivers or 'copy paste Maths'.
- One of the principles of RME requires students to be active people in teaching and learning process, whereby they develop mathematical lens and insights on their own, because Freudenthal regard Maths as a human activity and can argue that Maths should not be learned as closed systems but rather as an activity of mathematizing reality and if possible even that mathematizing Maths.
- According to Heuvel- Panghuizen & Drijvers (2014: 521-522) Freudenthal (1981) took over Treffers (1982a) by characterising the RME approach in the horizontal and vertical mathematization. So, in horizontal mathematization, students use mathematical tools to organise and solve problems situated in the real world by analysing 'MD' problem statement, in order to formulate linear thought flow and also objective functions, while the vertical mathematization were focus as a process of organization (grouping like terms) within mathematical systems which results to one focus by using connectors between concepts and strategies.

Principles

Treffers (1982a) & Freudenthal, (1971, 1973); in Heuvel- Panghuizen & Drijvers, (2014: 523-524) revealed RME principles that guides instructors (lecturers) who view RME approach as research pedagogical model in Maths teaching and learning, such as activity, reality, levels, intertwined, interactivity, and guidance.

POST-TEST (50 marks)

Question 1 (20 marks)

Define and write the following words in maths

NO.	M.I.P words	Explanation	Maths symbol
1	less or equal to		
2	Maximum		
3	at most		
4	greater or equal to		
5	Minimum		
6	at least		
7	less than		
8	greater than		
9	times		
10	Half		
11	one third		
12	Maths variables		

(20)

Question 2 (8 marks)

- 2.1 A small factory makes two types of washing machines, a standard and a luxury model. The daily production cannot be more than 2 standard and 5 luxury washing machines. There are 3 luxury workers and it takes one worker to make one standard model and 2 workers to make one luxury model per day.

- 2.2 Tourist Firm manufactures not less than 1000 golf shirts for F.G TVET College per week.

Question 3 (10 marks)

- 3.1 solve and sketch the following linear inequalities and find feasible area

$$x > 5; y \geq 10; x + y < 16; x \geq 0; y > 0; \text{ and } y \leq 3x + 2y$$

POST-TEST: (50 marks)

Question 1 (20 marks)

Below you write the following words in maths

NO.	MLP words	Explanation	Maths symbol
1	less or equal to	Means it is smaller than or equal to	$\leq$
2	Maximum	Means it is the largest value that is less than or equal to	$\leq$
3	at least	Means it is the smallest value that is greater than or equal to	$\geq$
4	greater or equal to	Means it is the smallest value that is greater than or equal to	$\geq$
5	Minimum	Means it is the largest value that is less than or equal to	$\leq$
6	at least	Means it is the smallest value that is greater than or equal to	$\geq$
7	less than	Means it is smaller than	$<$
8	greater than	Means it is larger than	$>$
9	times	Means it is multiplication of	$\times$
10	Half	Means it is half of	$\frac{1}{2}$
11	one third	Means it is one third of	$\frac{1}{3}$
12	Maths variables	Means it is symbols representing unknown numbers	x, y, z

Question 2 (5 marks)

2.1 A small factory makes two types of washing machines, a standard and a luxury model. The daily production cannot be more than 8 standard and 3 luxury washing machines. There are 5 factory workers and 1 takes one worker to make one standard model and 2 workers to make one luxury model per day.

Let standard machines be represented by x
 luxury machines be represented by y
 $x \leq 8$
 $y \leq 3$
 $x + 2y \leq 10$

2.2 Tourists that visit facilities not less than 1000 dollars is for NO. 1741 Galapagos

Let golf shorts be z
 $z \geq 1000$

Question 3 (18 marks)

3.1 solve and sketch the following three inequalities and find feasible area

$x + 5y \geq 10$; $x + y \leq 18$; $x > 0$; $y > 0$; and $P = 3x + 2y$

$x \geq 0$
 $y \geq 0$
 $x + 5y \geq 10$
 $x + y \leq 18$
 $P = 3x + 2y$
 $y \geq \frac{10 - x}{5}$
 $y \leq 18 - x$
 if there are three coordinates above are your calculations.

POST-TEST (50 marks)

Question 1 (20 marks)

1. Define and write the following words in maths

NO.	Math words	Explanation	Maths symbol
1	use of equal to	It is either before or equal to	$=$
2	Maximum	It means the highest	$\leq$
3	at most	It means it cannot exceed	$\leq$
4	greater or equal to	It is either above or equal	$\geq$
5	Minimum	It means the lowest	$\geq$
6	at least	It means minimum of	$\geq$
7	less than	It is less than	$<$
8	greater than	It means bigger than	$>$
9	Since	refers to multiplication	$\times$
10	Part	is part of the whole	$\frac{1}{n}$
11	one third	refers to addition that whole number	$\frac{1}{3}$
12	Maths vs algebra	refers to algebra that we use x, y, z any alphabet in maths	x, y, z

Question 2 (5 marks)

2.1 A car factory makes two types of washing machines, a standard and a luxury model. The daily production cannot be more than 8 standard and 7 luxury washing machines. There are 6 factory workers and it takes one worker to make one standard model and 2 workers to make one luxury model per day.

Let standard machine be x
 and luxury machine be y
 $x \leq 8$
 $y \leq 7$
 $x + 2y \leq 12$

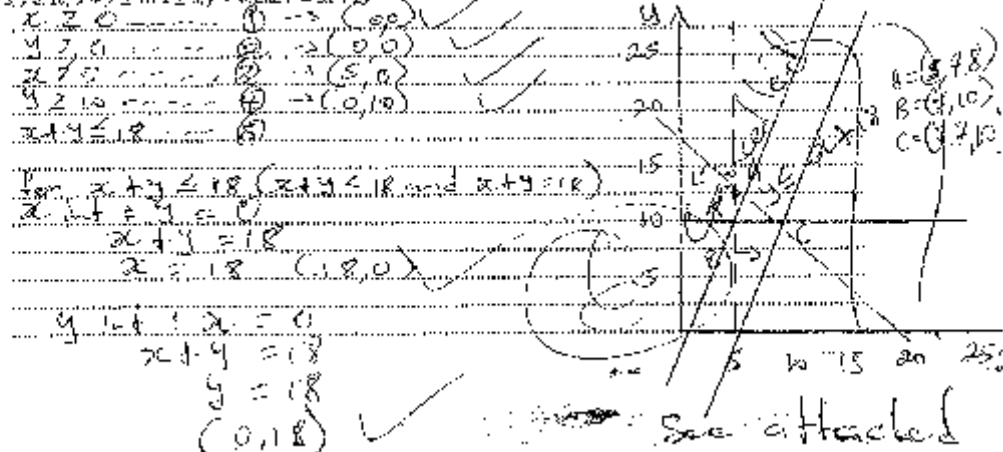
2.2 Touratec Firm manufactures not less than 1000 gold trinkets for GWET College per week.

Let number of gold trinkets be x
 $x \geq 1000$

Question 3 (10 marks)

3.1 solve and sketch the following lines in Cartesian and find feasible area

$x \geq 5, y \geq 10, x + y \leq 18, x \geq 0, y \geq 0$ and $P = 3x + 2y$



POST-TEST (50 marks)

Question 1 (20 marks)

i. Define and write the following words in matrix

NO.	Math words	Explanation	Math's symbol
1	Less or equal to	It is a symbol of equal to	$\leq$
2	Maximum	Means a small amount or equal to	$\leq$
3	at most	Means a smaller amount or equal to	$\leq$
4	greater or equal to	It can be of any number greater than	$\geq$
5	Minimum	It can be of any number greater than	$\geq$
6	at least	It can be of any number greater than	$\geq$
7	less than	Means smaller number than	$<$
8	greater than	Means larger number than	$>$
9	times	Means multiplication	$\times$
10	half	Dividing whole into two equal parts	$\frac{1}{2}$
11	one third	One of three parts of the whole	$\frac{1}{3}$
12	Maths variables	Letters or alphabets used to represent numbers	x, y

Question 2 (5 marks)

2.1 A small factory makes two types of washing machines, a standard and a luxury model. The daily production cannot be more than 8 standard and 3 luxury washing machines. There are 2 luxury workers and it takes one worker to make one standard model and 2 workers to make one luxury model per day.

Let number of standard machine be x
 Number of luxury machine be y
 (i) $x \geq 0, y \geq 0$ (ii) $x + 2y \leq 8$
 (iii) $x \leq 8, y \leq 3$

2.2 Fourus Hien manufactures not less than 1000 golf shirts for NO. 101 College per week.

Let number of golf shirts be x
 $x \geq 1000$

Question 3 (10 marks)

3.1 solve and sketch the following linear inequalities and find the feasible area

$x > 5; y > 10; x + y \leq 18; x \geq 0, y \geq 0$, and $P = 3x + 2y$

$x > 5 \rightarrow (5, 0)$
 $y > 10 \rightarrow (0, 10)$
 $x + y \leq 18$
 $1 \leq x + 18$
 $y \leq -x + 18$ or $y = -x + 18$

PRE & POST TEST RESULTS

ANALYSIS OF PRE-TEST AND POST-TEST RESULTS

Achievement codes	Competencies	%	Number of students (MLP pre-test)	Number of students (MLP Post-MLP test)
Level 1 (L1)	Not achieved	00-29	30	02
Level 2 (L2)	Elementary	30-39	03	04
Level 3 (L3)	Moderate	40-49	02	04
Level 4 (L4)	Adequate	50-59	00	09
Level 5 (L5)	Substantial	60-69	00	03
Level 6 (L6)	Meritorious	70-79	00	06
Level 7 (L7)	Outstanding	80-100	00	07

Table 4.3: developed by the research team in this study

Then, we further analysed pre-test and post-test results by doing calculations on pass and fail %s with the purpose of checking impact of the RME approach (see table 4.3),

MLP pre-test	MLP post-test
Pass % = $\frac{\text{number passed}}{\text{total number wrote}} = \frac{05}{35} \times 100 = 14\%$	Pass % = $\frac{\text{number passed}}{\text{total number wrote}} = \frac{33}{35} \times 100 = 94\%$
Failed % = $\frac{\text{number failed}}{\text{total number wrote}} = \frac{30}{35} \times 100 = 86\%$	Failed % = $\frac{\text{number failed}}{\text{total number wrote}} = \frac{02}{35} \times 100 = 06\%$

Table 4.4: developed by the research team

In addition, we presented information in Fig. 4.34 on the pre-test and post-test results on the bar graph below

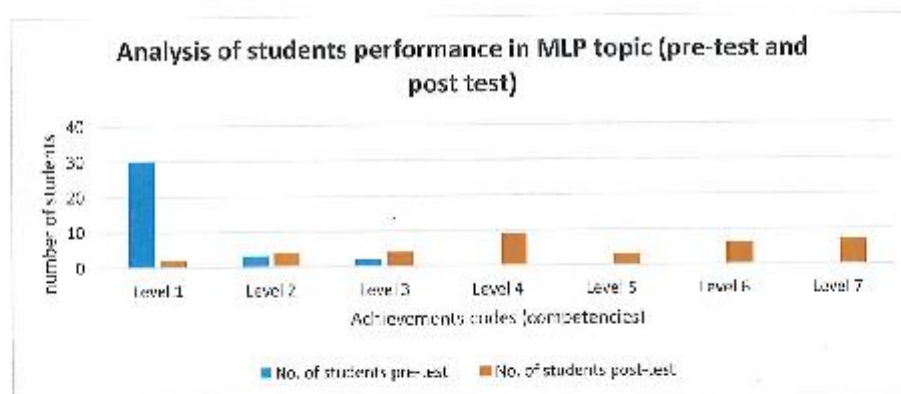


Fig. 4.34: (presented by the research team)