STOCHASTIC EFFICIENCY OPTIMISATION ANALYSIS OF ALTERNATIVE AGRICULTURAL WATER USE STRATEGIES IN VAALHARTS OVER THE LONG- AND SHORT-RUN

BY BENNIE GROVÉ

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DECLARATION

I, Bennie Grové, hereby declare that this thesis work submitted for the degree of Philosophiae Doctor in the Faculty of Natural and Agricultural Sciences, Department of Agricultural Economics at the University of the Free State, is my own independent work, conducted under the supervision of Prof. L.K. Oosthuizen.

________________________ ____________________

Bennie Grové Date

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"Work is not primarily a thing one does to live, but the thing one lives to do. It is, or should be, the full expression of the worker's faculties, the thing in which he finds spiritual, mental and bodily satisfaction, and the medium in which he offers himself to God."

Dorothy Sayers

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ABSTRACT

The main objective of this research was to develop models and procedures that would allow water managers to evaluate the impact of alternative water conservation and demand management principles in irrigated agriculture over the long-run and the short-run while taking risk into account.

One specific objective was to develop a generalised whole-farm stochastic dynamic linear programming (DLP) model to evaluate the impact of price incentives to conserve water when irrigators have the option to adopt more efficient irrigation technology or cultivate high-value crops over the long-run. The DLP model could be characterised as a disequilibrium known life type of model where terminal values were calculated with a normative approach. MOTAD (Minimising Of Total Absolute Deviations) was used to model risk. Another specific objective was to develop an expected utility optimisation model to economically evaluate deficit irrigation within a multi-crop setting while taking into account the increasing production risk of deficit irrigation in the short-run.

The dynamic problem of optimising water use between multiple crops within a whole-farm setting when intraseasonal water supply may be limited was approximated by the inclusion of multiple irrigation schedules into the short-run model. The SAPWAT model (South African Plant WATer) was further developed to quantify crop yield variability of deficit irrigation while taking the non-uniformity of irrigation applications into account. Stochastic budgeting procedures were used to generate appropriately correlated inter- and intra-temporal matrixes of gross margins necessary to incorporate risk into the long-run and short-run water use optimisation models. A new procedure (standard risk aversion) was developed to standardise values of absolute risk aversion with the objective of establishing a plausible range of risk aversion levels for use with stochastic efficiency analysis techniques. A procedure was developed to conduct stochastic efficiency with respect to a negative exponential utility function using standard risk aversion. The standardised risk aversion measure produced consistent answers when the risk premium was expressed as a percentage of the range of the data.

Long-run results showed that the elasticity of irrigation water demand was low. Overall risk aversion and the individual farming situation will have an important impact on the effectiveness of water tariff increases when it comes to water conservation. Although the more efficient irrigation technology scenario had a higher net present value when compared to flood irrigation, the ability to pay for water with the first mentioned scenario was lower because the lumpy irrigation technology needs to be financed. Failure to take risk into account would cause an over- or underestimation of the shadow value of water, depending on whether water was valued as relatively abundant or scarce. The conclusion was that care should be taken when interpreting the derived demand for irrigation water (elasticity) without knowing the conditions under which they were derived. Cognisance should also be taken of the fact that higher gross margins per unit of applied water would not necessarily result in greater willingness to pay for water when the alternatives were evaluated on a whole-farm level.

The main conclusion from the short-run analyses was that although deficit irrigation was stochastically more efficient than full irrigation under limited water supply conditions, irrigation farmers would not willingly choose to conserve water through deficit irrigation and would be expected to be compensated to do so. Deficit irrigation would not save water if the water that was saved through deficit irrigation were used to plant larger areas to increase the overall profitability of the strategy. Standard risk aversion was used to explain the simultaneous increasing and decreasing relationship between the utility-weighted premiums and increasing levels of absolute risk aversion and was shown to be more consistent than when constant absolute risk aversion was assumed.

The modelling framework and the models that were developed in this research provide powerful tools to evaluate water allocation problems that are identified while busy implementing the National Water Act. Only through the application of these type of models linked to hydrological models will a better understanding of the mutual interaction amongst water legislation, water policy administration, technology, hydrology, human value systems and the environment be gained to enhance water policy formulation and implementation.

CHAPTER 1

INTRODUCTION

1.1 BACKGROUND AND MOTIVATION

The South African water sector has experienced significant changes during the past decade with respect to the way in which water is allocated between competing uses and the manner in which water resources are managed. After an extensive consulting process the fundamental principles and objectives for a new South African water law were develop and published as the Water Law Principles (DWAF, 1996) followed by the White Paper on a National Water Policy (DWAF,1997). The broad objectives of the National Water Policy are to achieve equitable access to water and to ensure sustainable and efficient use of water for optimum social and economic development. The legal framework for achieving these policy goals is provided for by the National Water Act (Act 36 of 1998) (NWA), which provides comprehensive provisions for the protection, use, development, conservation, management and control of water resources. A legal requirement of the NWA is the development of a National Water Resource Strategy (NWRS), which was published during 2004 (DWAF, 2004a). The NWRS provides a framework for implementing the NWA. An integral part of the strategy is the development of a National Water Conservation and Demand Management Strategy. The importance of water conservation and demand management is usually motivated by increasing scarcity of water resources and the South African case is no exception.

World Bank predictions are that water scarcity in South Africa will increase drastically in the nearby future moving its status from a water scarce to a water stressed country between the years 2005 to 2040 (Seckeler, Baker and Amarasinghe, 1999). The NWRS indicated that more than half of the water management areas are in deficit while the country as a whole is still in surplus (DWAF, 2004a). The problem is that in many instances it is not practical or economically viable to transfer water from surplus to deficit areas. Furthermore, the potential options for supply augmentation are limited and attention will have to be given to managing the increasing demand for water as an alternative to reconcile imbalances between water requirement and availability through the use of water conservation and demand management (WC&DM) principles (Backeberg, 2006). WC&DM relate to measures to increase the efficiency of water use and the reallocation of water from lower to higher benefit uses within or between water use sectors. Important to note is that the NWA gives priority of use over all other uses to the Reserve, which includes the quantity and quality of water to meet basic human needs and to protect aquatic ecosystems. Implementation of WC&DM will have some serious implications for irrigated agriculture since it accounts for 62% of all the water used in South Africa and in many instances, the use is highly inefficient (DWAF, 2004b). A WC&DM strategy for the agricultural sector was finalised during 2004 with the overall objective of ensuring that WC&DM principles are applied by the agricultural sector in order to release some water for use within the sector, to open up irrigation opportunities for emerging farmers, to release more water to cater for the needs of competing water users and to protect the environment (DWAF, 2004b). The strategy will provide the regulatory support and incentive framework to improve irrigation efficiency in the sector by influencing water users to use water optimally. Central to the strategy is the use of a pricing strategy as a powerful tool to reduce water demand and increase water use efficiency (DWAF, 2004b). Each water user association is also required to develop and submit a water management plan in which current practices are stated and how they will proceed to achieve WC&DM. From the above it is clear that irrigated agriculture is targeted as a potential source of water and that the sector will experience increasing pressure to improve irrigation efficiency with the aim of conserving water.

According to Weinberg, Kling and Willen (1993), irrigated agriculture may conserve water in at least three ways: a) improved efficiency of water applications, b) alternative crops, and c) deficit irrigation. Water application efficiency may be improved through the adoption of more efficient irrigation technology and the use of information to ensure that irrigation water is being applied in accordance with the requirements of the crops that are grown. Within a South African context decision support systems to estimate water requirements of crops (Crosby and Crosby, 1999) and simulation models to enhance real time irrigation scheduling whereby water applications are minimised to achieve maximum crop yields (Annandale, Benadé, Javanovic, and Sautoy, 1999) have been developed and the technology transferred to the end users (Van Heerden, Crosby and Crosby, 2001; Annandale, Steyn, Benadé, Javanovic, and Soundy, 2005). English, Solomon and Hoffman (2002) argue in favour of a new paradigm whereby irrigation applications will be based on economic efficiency principles rather than applying irrigation water to achieve maximum crop yield. Optimising water use based on economic principles implies taking into consideration the costs, revenues and the opportunity cost of water (scarcity value) while allowing the crop to sustain some level of water stress resulting in yield reductions due to deficit irrigation. A complicating factor with the adoption of such a strategy is that not only will crop yields decrease but the variability thereof will increase (English *et al*., 2002). Currently government is emphasising irrigation modernisation through the adoption of more efficient irrigation technology, irrigation scheduling and the cultivation of high valued crops (DWAF, 2004b).

The question is, however, not whether irrigators should adopt water conserving irrigation technology, apply irrigation water efficiently or cultivate higher valued crops. Rather, the problem is how to proceed. Many farm-level variables will determine farmers' use of water conserving farming practices and generally, the interaction among these variables is not well understood. Optimising water use at farm level to achieve maximum profit is especially challenging since the farmer needs to integrate information regarding irrigation technology, crop water requirements, crop yield response to water deficits, infrastructural constraints that limit water supply, credit availability and input and output prices of multiple crops simultaneously. Furthermore, farmers are operating within a deregulated marketing environment with increased price volatility (Jordaan, Grové, Jooste and Alemu, 2006). Backeberg (2004) states that the need for tools to give timely management and/or policy advice has increased due to the deregulated market environment and the devolvement of water management to the local level. The WC&DM strategy for the agricultural sector furthermore underlines the importance of research and the use of different tools to generate information that will enhance the ability of the sector to achieve WC&DM (DWAF, 2004b). The importance of developing procedures that will enable better decision support also increases if one considers that many irrigation schemes in South Africa are operated at low levels of assurance of water supply, which makes quota reductions common (Breedt, Louw, Liebenberg, Reinders, Nell and Henning, 2003; Scott, Louw, Liebenberg, Breedt, Nell and Henning, 2004). A clear need exists for decision support that is able to integrate relevant information from different sources to achieve optimal water use at farm-level.

1.2 PROBLEM STATEMENT AND OBJECTIVES

Water managers are currently unsure about the effectiveness of alternative WC&DM instruments such as increasing water charges and the promotion of alternative water conserving management practices that hamper WC&DM in the agricultural sector. The uncertainty stems from a lack of understanding of the interaction of farm-level variables that influence optimal water use and profitability of alternative water management options within the dynamic and stochastic environment in which farmers have to make decisions. A lack of models that are able to model these interactions satisfactorily while taking cognisance of the dynamics within irrigated agriculture, the development of the farm firm and the risks of agriculture further hamper the identification of feasible and profitable alternatives that will conserve water in the irrigated agricultural sector.

Various researchers have optimised agricultural water use over the short-run by means of linear programming (LP) (Hancke and Groenewald, 1972; Van Rooyen, 1979; Brotherton and Groenewald, 1982). Typically, these researchers did not include deficit irrigation or risk in their analyses. Deficit irrigation has been researched in South Africa by means of simulation and optimisation methods. The simulation studies mainly concentrated on the impact of production risk of predefined irrigation schedules (Grové, Nel and Maluleke, 2006; Botes, 1990). These simulation studies ignore the opportunity cost of water, which may increase the benefits of deficit irrigation if water that is saved through deficit irrigation is used to irrigate larger areas (English and Raja, 1996). Optimisation studies, on the other hand, failed to appropriately represent the non-linear relationship between water consumed by the crop and applied water (Mottram, De Jager, Jackson and Gordijn, 1995) or the dynamic relationship of soil water availability between different crop growth stages (Mottram *et al.*, 1995; Grové and Oosthuizen, 2002). Furthermore, these optimisation studies ignored risk. An exception is the research by Botes (1994) who linked a sophisticated optimisation search algorithm to a crop growth simulation model to optimise water use for different levels of irrigation information strategies while taking risk into account. A drawback of the procedure is that it is highly specialised and difficult to apply within a whole-farm set up where decisions need to be made regarding water use between multiple crops within multiple seasons. Grové (2006a) proposed a more robust procedure to optimise water use within a whole farm set up. The procedure is based on the optimisation of water use by choosing amongst multiple irrigation strategies simulated with a simulation model. Other South African researchers acknowledge the importance of a longer time frame to model irrigation technology adoption and the cultivation of long-term crops more satisfactorily. As a result, deterministic dynamic linear programming (DLP) is applied frequently as a method of assisting water managers with optimal water usage over the long-run (Backeberg, 1984; Oosthuizen, 1995; Maré, 1995; Louw and Van Schalkwyk, 1997; Haile, Grové and Oosthuizen, 2003). Typically, these researchers do not include risk in their analysis. Incorporating risk into DLP models is difficult and requires quantification of price risk, crop yield risk and making assumptions about intra- and inter-temporal correlation structures between these variables. Furthermore, these applications are very problem specific, which makes it difficult to transfer the models from one situation to another.

Since agricultural prices and production are inherently variable, most researchers and decision-makers acknowledge the importance of taking risk into account when conducting profitability and feasibility analyses. However, most researchers choose to assume risk away due to a lack of data to quantify risk, increased modelling time and expertise necessary to conduct risk analyses and the difficulty in choosing realistic absolute risk aversion levels. Choice of absolute risk aversion levels is especially difficult since the invariance property of arbitrary linear transformations of the utility function does not apply to arbitrary rescaling of the outcome variable (Raskin and Cochran, 1986). By implication, some form of rescaling of the absolute risk aversion coefficient is necessary to represent risk aversion consistently. The problem is that more than one procedure exists in literature to scale absolute risk aversion levels. Furthermore, some of these methods will provide consistent scaling under restrictive conditions.

The main objective of this research is to develop models and procedures that will allow water managers to evaluate the impact of alternative WC&DM principles in irrigated agriculture over the long-run and the short-run while taking risk into account.

Specific objectives are to develop:

- A generalised whole-farm stochastic DLP model to evaluate the impact of price incentives to conserve water when irrigators have the possibility to adopt more efficient irrigation technology or cultivate high-valued crops.

In order to achieve the objective this research built on the research by Grové (2006b) who developed GAMS (General Algebraic Modelling System) (Brooke, Kendrick, Meeraus and Raman, 1998) code to construct a DLP matrix based on the inputs that are provided. The structure is general in that the model structure is easily transferred between different applications. GAMS code is also developed to generate the necessary risk matrixes from irrigation technology specific subjectively elicited crop yield distributions and historical price information for the DLP model.

- An expected utility optimisation model to economically evaluate deficit irrigation within a multi-crop setting as a strategy to conserve water while taking into account the increasing production risk of deficit irrigation.

  To achieve the above objective the capability of SAPWAT (South African Plant WATer) (Crosby and Crosby, 1999) was extended to generate crop yield indices regarding different irrigation schedules. The crop yield indices were then combined with subjectively elicited crop yields under conditions of no water stress to quantify production risk of alternative deficit irrigation schedules. Direct expected utility maximisation was then used to determine optimal water use and cropping combinations, which were further evaluated with stochastic efficiency with respect to a function (SERF) procedures.

- A procedure to standardise choice of Arrow-Pratt absolute risk aversion coefficients for application with stochastic efficiency analysis techniques.

  Central to the application of the two programming models developed as part of this research is the choice of the level of risk aversion. Constant absolute risk aversion (CARA) utility functions have the property that adding or subtracting a constant to all payoffs does not alter risk aversion. The last mentioned property is explored in this research to derive a standardised risk aversion measure. The standardised risk aversion measure will give consistent answers when the risk premium is expressed as a percentage of the range of the data.

A description of the research data area is provided in the following section.

1.3 RESEARCH AREA

The research is conducted at the Vaalharts irrigation scheme, which is located east of the Ghaap plateau, on the Northern Cape and North West Province border. The border is currently running through this scheme. The area covers about 36 950 ha, and is one of the largest irrigation areas in the world. Water is provided to some 680 farmers. The scheme is supplied with water abstracted from the Vaal River at the Vaalharts Weir about 8 km upstream of Warrenton. A canal is used to convey the water to the scheme. When the canal reaches the scheme it divides into two main canals, the north canal and the west canal. The north canal feeds the greater Vaalharts area with water; this includes places like Jan Kempdorp, Tadcaster, Hartswater and Magogong. The west canal provides water to Ganspan, Hartsvallei and Bull Hills. These canals provide water to a network of feeder and community canals. Additionally there are drainage canals, draining water out of the scheme to the Harts River, west of the scheme.

A Water User Association (WUA) was recently formed to help the community carry out their water-related activities more effectively.

1.3.1 CLIMATE

Vaalharts irrigation scheme has an average rainfall of 442 mm per annum. The rainfall is mostly in the form of heavy thunder, although soft frontal rainfall also occurs, and hailstorms are a common phenomenon (De Jager, 1994). Not only is the rainfall low, but also seasonal and irregular. The irregularity of rainfall makes rainfall more important than would otherwise have been the case.

To gain a better idea of the distribution of rainfall within the year, the average monthly rainfall for years with normal weather conditions as well as years with favourable and severely unfavourable weather conditions are shown in Figure 1.1. It is clear that Vaalharts is in a summer rainfall area receiving the highest rainfall from November to March. The rainfall is the lowest from April to October. In some years (severe years), it did not rain at all in the months May to October.

Temperatures play an important role in determining evaporation. January seems to be the warmest month with maximum and minimum temperatures of 32.7 °C and 17.4 °C. July is the coldest month with a day temperature that can fall to 2.4 °C (Viljoen, Symington and Botha, 1992). Common to this area is the significant difference between the maximum and minimum temperatures as the seasons change. The evaporation for the three different weather scenarios given in Figure 1.1 is shown in Figure 1.2. The highest evaporation values are observed in the summer and the lowest during the winter, which corresponds to the rainfall distribution. However, there is a negative correlation between the rainfall and the evapotranspiration for the different years. The severe year has the highest evaporation and the lowest rainfall. The favourable year has the lowest evaporation values and the highest rainfall.

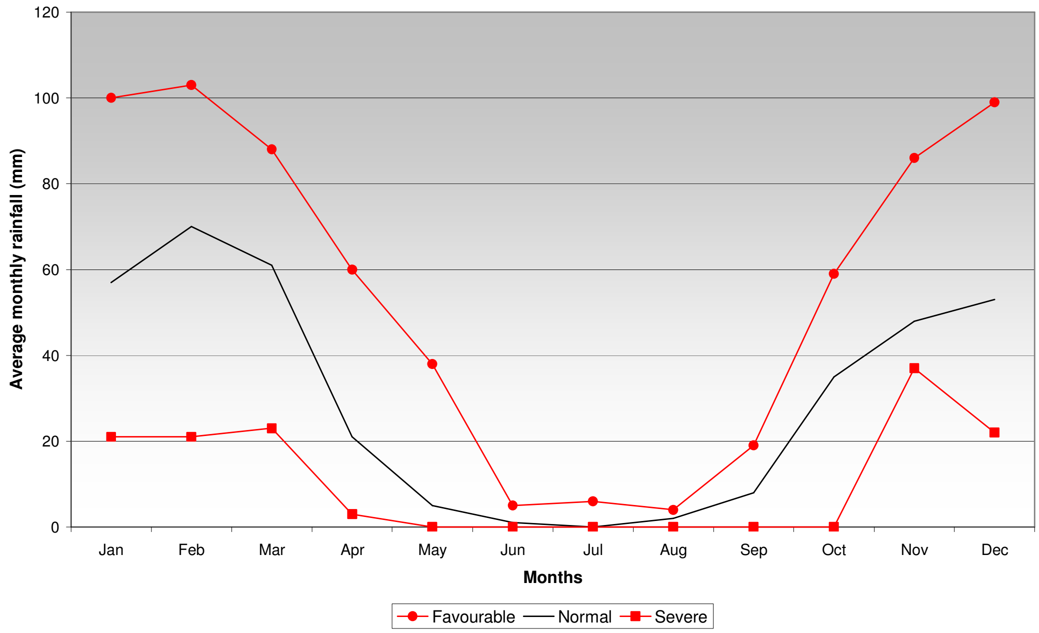


Figure 1.1: Monthly rainfall data from SAPWAT for the rainfall at the Jan Kempdorp weather station for years with normal, favourable and severe weather conditions.

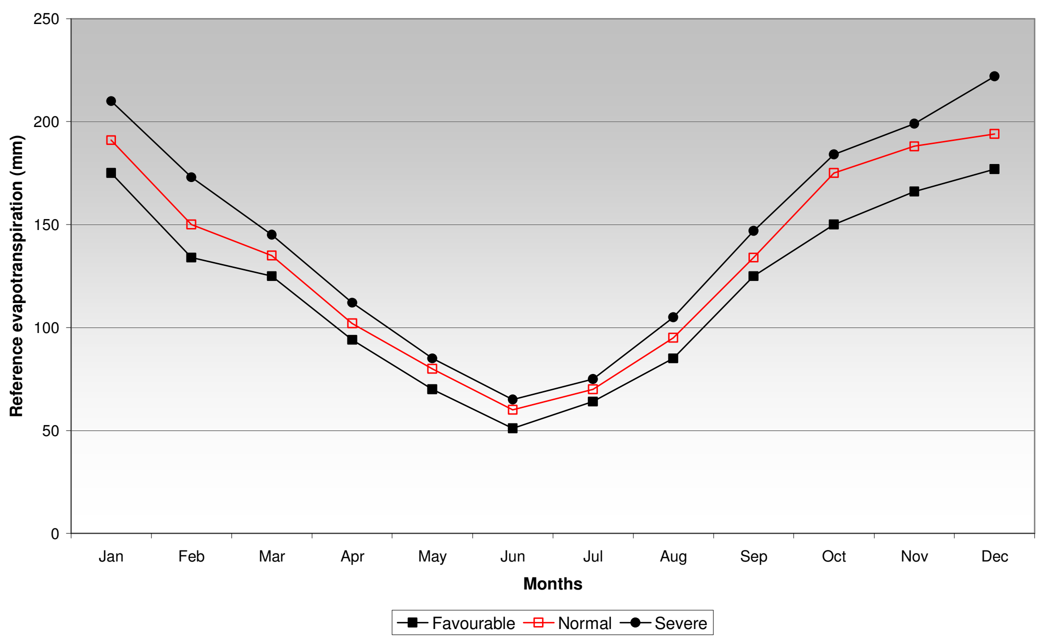


Figure 1.2: The monthly evaporation for three different weather years (normal, favourable and severe) at the Jan Kempdorp weather station.

From the above it is clear that evaporation is greater than rainfall, which necessitates irrigation. A problem is that the capacity of the canals is limiting in terms of supplying water to the farmers (Viljoen *et al.*, 1992).

1.3.2 SOILS

The two main types of soil found in Vaalharts are Hutton/Mangano and Clovelly/Sunbury (Herold and Bailey, 1996). The soils have a high sand context, which leads to compactation and puts a constraint on potential root depth. The soil also has a low water holding capacity, low fertility, high bulk density and limited depth (Herold and Bailey, 1996; Streutker, 1977). According to Viljoen *et al.* (1992), the largest proportion (±70%) of soil is the Mangano type, which is a sandy loam with silt and clay contents that fluctuate between 10 and 16 per cent.

About 12.9 per cent of the area's soil depth is less than 0.9 m. More or less 10.9 per cent of the area's soil depth is between 0.9 m and 1.2 m, while 15.4 per cent of the soil depth is between 1.2 m and 1.8 m. The greater part of the scheme, 60.9%, has a soil depth of more than 1.8 m (Herold and Bailey, 1996).

1.3.3 WATER DEMAND, DISTRIBUTION AND ALLOCATION

Canals supply the water to the irrigation plots. The two main canals, the northern canal and the western canal, feed a network of feeder and community canals. The water quota for the north and west canal is 9 140 m^3 per ha, resulting in an annual water use right of 209 744 720 m^3 for the north canal and 57 143 280 m^3 for the west canal (Van Heerden, 2001). Crop water requirements for the north and west canal are similar, the reason why so much more water is allocated to the north canal is that it provides water to a larger area.

The feeder canals are supplied directly by the two main canals. Each feeder provides water for the community canals. Typically the community canals, which receive water via feeders out of the northern canal, provide water for six plots. Most of these community canals can supply water for two plots at a time due to limitations on community canal capacities. Therefore, farmers need to take turns to water their plots. Each turn is 24 hours long. When it is a particular plot's turn, it receives about 150 m^3 water per hour. Each week the farmers of a community canal fill in the water requested for the coming week. These forms are handed in, on or before the Thursday before the water is needed.

Traditionally water is supplied for five and a half days, from Monday mornings to Saturday afternoons. Centre pivots enable farmers to irrigate any day of the week because the need for labour is minimal. The increase in the number of centre pivots in the area will result in an increase in pressure from farmers on the water authorities to be supplied with water for seven days a week.

The total water use charge in Vaalharts is 8.77 cents per cubic meter of water, which consists of a charge of 8.24 cents for irrigation water use, a catchment management charge of 0.5 cents per cubic meter and a water research charge of 0.03 cents per cubic meter of water. The farmer pays this tariff to the Vaalharts WUA.

1.3.4 REPRESENTATIVE FARMS

Only a short overview of the representative farms is given in this section. Detail on the data and procedures used to compile representative farms are contained in Louw (2002) and Grové (2006b).

1.3.4.1 Farm size

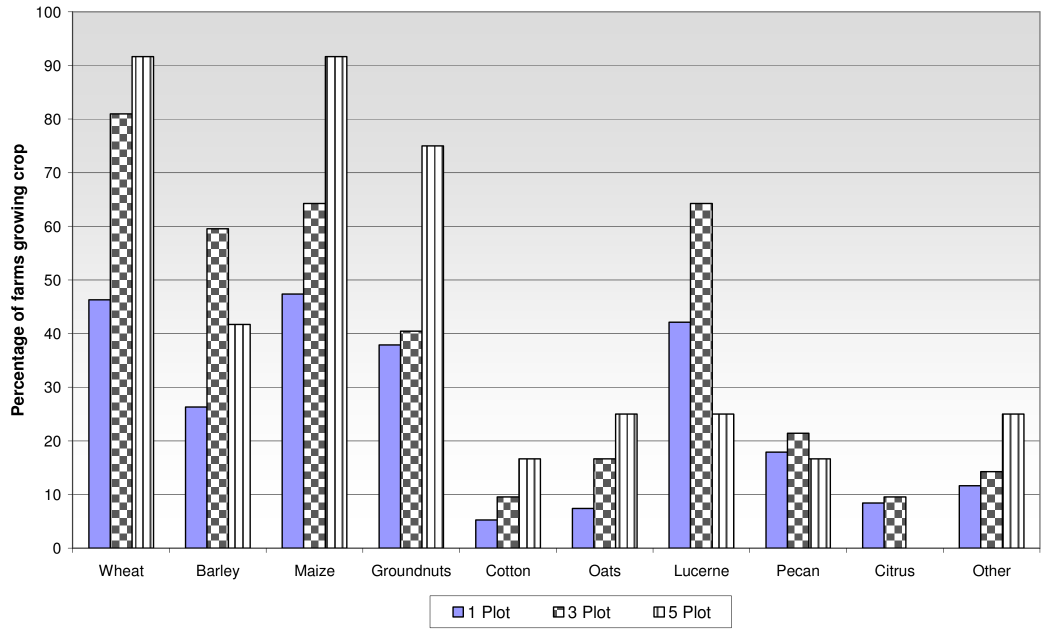
Information obtained from WAS (Water Administration System) that is used by Vaalharts Water to administrate water allocation was used to determine the distribution of farm sizes in the Vaalharts irrigation scheme.

Six hundred and eighty five farming units were counted for the total irrigation scheme. Two hundred and twenty two (32%) were one-plot farms and a hundred and fifty (22%) were two-plot farms. The rest were 63 (9%) three-plot farms, 70 (10%) four-plot farms, 36 (5%) five-plot farms and 37 (5%) six-plot farms. The frequencies are available to well into the thirty-plot farms, but the six-plot farms are the last group of farms that is significant. These six groups of farms represent 84% of the total number of farm-units in the Vaalharts irrigation scheme.

Louw (2002) compiled small, medium, large and extra large representative farms for Vaalharts. Given a standard plot size of 25.7 ha, small farms correspond to one plot, medium farms to three plots, large farms to five plots and extra large farms to nine plots.

1.3.4.2 Crop production

Figure 1.3 shows the percentage of one-plot, three-plot and five-plot farms producing a specific crop. Cash crops are by far the most important crops cultivated in the Vaalharts irrigation scheme area. The most commonly found cash crops are wheat/barley, maize, groundnuts and cotton. Wheat is a winter crop and is produced in rotation with maize and/or groundnuts. Maize and groundnuts grow in the summer and compete for resources. The specific area allocated to a specific crop is determined by product price expectations at the time of planting. The low cotton prices have generally resulted in only a few farmers producing cotton recently.



Source: Badenhorst (2003)

Figure 1.3: Percentage of one-plot, three-plot and five-plot farms growing a specific crop.

Permanent crops that are produced in Vaalharts include lucerne, pecan nuts, grapes, olives and some other fruits. Of these permanent crops, lucerne and pecan nuts are the most important. Olives do well in the irrigation scheme, but are not as popular as lucerne and pecan nuts. Unfortunately, severe frost in 2003 damaged much of the citrus and other fruit orchards, which resulted in a decline in the acreage under fruit.

Vaalharts was originally designed for flood irrigation. In the past few years centre pivot irrigation has increased tremendously. Table 1.1 gives the distribution of irrigation system by farm type. From Table 1.1 it is clear that on average about 67% of all the farms use flood irrigation while more or less 30% of all the farms use pivot irrigation. Thus, flood irrigation and centre pivot irrigation are the dominant methods of irrigation. Other irrigation systems such as micro- and drip irrigation are predominantly used to irrigate tree crops.

Table 1.1: Utilisation of irrigation system by farm size.

	Percentage of farm type utilising irrigation system (%)		
Irrigation system	1 Plot	3 Plot	5 Plot
Flood	76	60	66
Pivot	24	37	26
Other	0	3	9

Source: Badenhorst (2003)

1.3.4.3 *Irrigation requirements*

Irrigation water demand is defined as the amount of water that should be applied to a specific crop irrigation system combination. Each farmer is allocated 914 mm per ha water per annum, which the user may distribute between crops. Seasonal crop water requirements for the most important crops are shown in Table 1.2

Table 1.2: Monthly SAPWAT estimated gross irrigation water requirements (mm.ha) for selected crops under flood and pivot irrigation in Vaalharts.

	Gross irrigation water requirement (mm.ha)												
	Nov	**Dec**	**Jan**	**Feb**	**Mar**	**Apr**	**May**	**Jun**	**Jul**	**Aug**	**Sep**	**Oct**	**Total**
Flood													
Maize		120	60	240	240	60							720
Groundnut		60	120	120	240	240	60						840
Wheat								60	120	120	240	300	840
Lucerne	240	300	240	240	120	60	120	120	60	60	120	120	1800
Pecan nuts	240	240	300	120	120	120	120	60	60	120	120	300	1920
Pivot													
Maize		60	75	120	150	15							420
Groundnut		30	75	105	120	105							435
Wheat	30							15	45	90	195	210	585
Lucerne	105	120	120	105	60	60	60	30	15	105	120	120	1020

Source: Van Heerden (2001)

Table 1.2 shows that the gross water requirements vary from a low of 420 mm with centre pivot to a high of 1920 mm with flood irrigation. Annual crops require a gross of between 420 to 585 mm water with centre pivot and between 720 and 840 mm with flood irrigation. The gross crop water requirement for late maize (both flood and centre pivot) is also less than that required for groundnut (flood and centre pivot). The difference between perennial crops irrigated by centre pivot and flood is substantial, e.g., the gross water requirements for lucerne flood is 1 800 mm, while for lucerne centre pivot it is only 1 020 mm. What is obvious from the table is that the water requirements for centre pivot are less than that of flood irrigation, because of efficiency differences in the irrigation systems.

1.3.4.4 *Cash expenses and income*

The crops grown are the most important generators of income. The overhead costs per annum of the one-plot, three-plot and five-plot farms are R47 000, R67 000 and R107 000 respectively. However, these costs do not include electricity, land rent, income tax and the water tariffs. Household expenses per annum are R27 000, R62 000 and R76 000 respectively for the small, medium and large farms while fixed liabilities per annum are on average R31 000, R54 000 and R163 000 respectively.

1.4 THESIS LAYOUT

The thesis consists of an introduction, five additional chapters and an abstract.

A review of the literature pertaining to crop water use optimisation is conducted in Chapter 2 and provides the basis for developing the two optimisation models. The theoretical part of the review relies heavily on the work done by Bernardo (1985). The theoretical principles are then used to evaluate local and international research regarding crop water use optimisation after which some implications for this research are discussed.

Chapter 3 provides an overview of some of the methods to scale Arrow-Pratt absolute risk aversion coefficients to consistently represent risk aversion. A new method is proposed whereby absolute risk aversion is scaled based on the dispersion of the risky prospect. The method is then applied to determine plausible ranges of risk aversion that can be used with stochastic efficiency analysis methods.

The main objective of Chapter 4 is to provide a description of the procedures used to quantify the risk matrixes of the long-run and short-run water use optimisation models and the specification of the programming models. The procedure developed in Chapter 3 to standardise risk aversion relies on a measure of the dispersion of the risky prospect. Since the dispersion of the optimised water use plan is determined endogenously, the relationship only holds *ex post*. A procedure is therefore developed to conduct a SERF analysis of the optimised water use plans while using the standardised risk aversion levels. The procedure is presented in the last part of the chapter.

The results and conclusions made by applying the models and procedures developed in this research are given in Chapter 5. A summary and recommendations for water conservation policy and further research are provided in Chapter 6.

CHAPTER 2

LITERATURE REVIEW ON CROP WATER USE OPTMISATION

The chapter is structured into two parts. The first part motivates a paradigm shift from applying water to achieve maximum crop yield to one that optimises economic efficiency and gives an overview of the theory of crop water use optimisation. The theoretical principles are then used to evaluate research efforts pertaining to water use optimisation in South Africa and internationally.

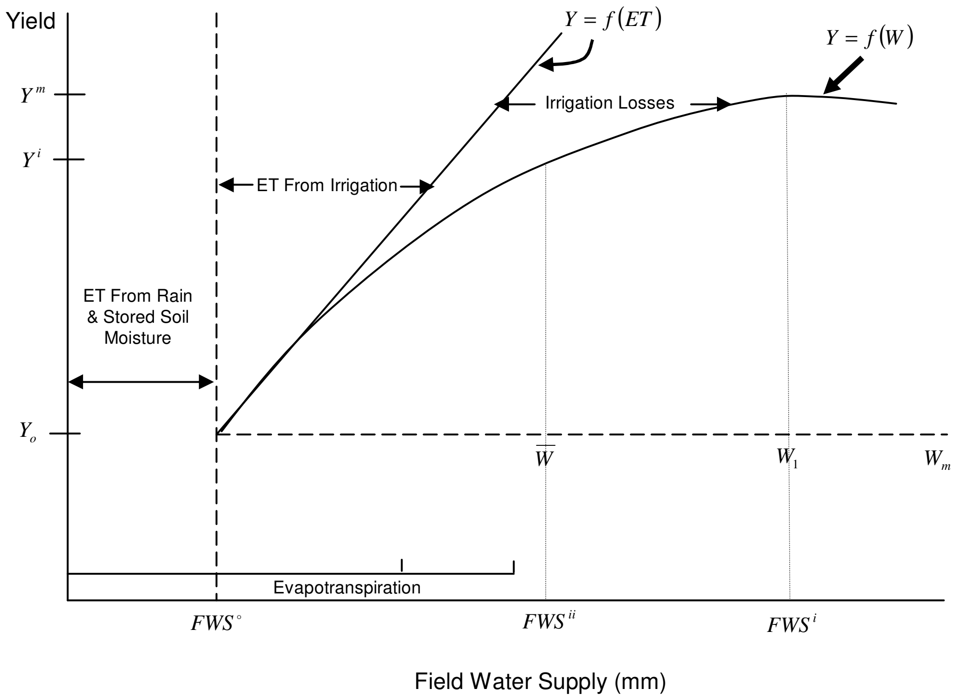
2.1 PARADIGM SHIFT IN IRRIGATION MANAGEMENT

English *et al.* (2002) argue that irrigation based on economic efficiency principles will be the new paradigm that will govern irrigation management in the future. The old paradigm where water was managed to achieve maximum yields will be replaced with one where water use between multiple alternatives is optimised to achieve economic efficiency. The change in the paradigm is motivated by the increasing scarcity of water and a more intense competition for water.

Irrigation optimisation should not be confused with scientific irrigation scheduling which relies on the systematic tracking of soil moisture or crop water status to determine when and how much to irrigate (English *et al.*, 2002). Scientific irrigation scheduling is typically done to minimise water applications with the aim of achieving maximum yield. Thus, no explicit consideration is given to costs, revenues and the opportunity cost of water. Optimisation of water use with the aim of maximising economic efficiency implies some form of deficit irrigation. Deficit irrigation is defined as an optimising strategy under which the crops are deliberately allowed to sustain some degree of water deficit resulting in yield reduction in order to achieve maximum profit (English and Raja, 1996). Benefits from deficit irrigation stem from reduced operating cost, increased water use efficiency and the opportunity cost of water. However, adoption of deficit irrigation is difficult and implies appropriate knowledge about crop evapotranspiration, yield response to water deficits, gross irrigation applications and the economic impacts of deficit irrigation (Pereira, Oweis and Zairi, 2002).

In order to optimise agricultural water use one needs to relate applied water to some measure of crop water consumption since consumptively used water is directly related to crop yield. Evapotranspiration (ET) is preferred by many researchers as a measure of crop consumptive water use although it does include evaporation from the soil. The reason is that a considerable number of researchers found a linear relationship between ET and crop yield (Vaux and Pruitt, 1983; Stewart and Hagan, 1973). However, the relationship between applied water and crop yield is non-linear. Figure 2.1 is used to explain the relationship between field water supply

(FWS) and ET where FWS consists of irrigation water stored in the root zone, effective rainfall and soil water carry-over.



Source: Vaux and Pruitt (1983)

Figure 2.1: Relationship between crop yield evapotranspiration and applied water.

Figure 2.1 shows that some crop yield (Y_o) is possible without applying any water. Y_o corresponds to dryland crop yield and Y_m to maximum crop yield under irrigation. A linear relationship between crop yield and ET is shown. However, the relationship between applied water and crop yield is non-linear. The horizontal difference between ET and applied water constitutes irrigation losses such as deep percolation and runoff after wind drift is taken into account. One should note that crop yield increases linearly with applied water up to about 50% of full irrigation whereafter the relationship starts to become non-linear (Doorenbos and Kassam, 1979). As more water is applied, the relationship between applied water and crop yield becomes curvilinear due to increasing losses resulting from increased surface evaporation, runoff, and deep percolation (English *et al.*, 2002). Thus, the relationship between crop yield and ET is more or less independent of soils, irrigation system, management and other factors that may influence the shape of the relationship between applied water and crop yield. Some important implications for this research are discussed below based on the relationships discussed above.

Irrigation managers do not have direct control over ET but have control over the amount of water applied to satisfy ET. Various researchers (Ascough, 2001; Li, 1998; De Juan, Tarjuelo, Valiente and Garcia, 1996; Mantovani, Villalobos, Orgaz and Fereres, 1995) have demonstrated that the uniformity with which water is applied influences the water application efficiency (curvature of applied water yield relationship). Choice of irrigation technology and the amount of water applied (irrigation management) will therefore determine irrigation application efficiencies. The conclusion is that proper optimisation of water use needs to take into account the non-linear relationship between applied water and crop yield.

Figure 2.1 shows a seasonal relationship between ET and crop yield and therefore a constant rate by which ET is transformed into crop yield. However, it is a fact that crop water stress in different crop growth stages impacts differently on crop yields (Doorenbos and Kassam, 1979). Deficit irrigation may further increase yield variability (Botes, 1990, Grové *et al.,* 2006). English *et al.* (2002) argue that when the opportunity cost of water is taken into account and it is optimal to reduce water application and at the same time increase the area irrigated, any losses that may incur will be amplified by the increased area under irrigation. A complete evaluation of deficit irrigation therefore requires that risk be taken into account.

Several operations research techniques, each with its own strengths and weaknesses, are available that can be used to optimise water use. However, application of these techniques within a multicrop intraseasonal setting requires a thorough understanding of the economic theory of water use allocation. The theory is reviewed next.

2.2 ECONOMIC THEORY OF WATER USE OPTIMISATION

The review presented in this section follows the work done by Bernardo (1985:71-91). First, the principle of allocating a given amount of water over a season is reviewed. Secondly, the impact of sequential irrigation decisions in different time periods on the optimality condition is presented. The last part of this section is concerned with allocating water between multiple crops taking intraseasonal water supply capacity constraints into account.

2.2.1 SINGLE PERIOD

Assuming energy and labour requirements may be specified as a function of water use *(W)*, the profit function may be specified as:

$$\Pi = P_y \cdot f\left(x_1, \ldots, x_n, w\right) - A - \sum_{i=l}^{n} r_i x_i - r_w w - r_e \cdot E(w) - r_\ell \cdot L(w) \qquad (2.1)$$

In this specification, E and L are expressions relating energy and labour use to the seasonal irrigation depth and r_e, r_ℓ, and r_w are the prices of energy, labour, and water, respectively. A

represents fixed cost and x_i represents other inputs. Under this scenario, the first-order conditions for profit maximisation are:

$$P_y \cdot f_i - r_i = 0 \qquad (i = 1, \ldots, n) \tag{2.2}$$

$$P_y \cdot f_w - r_w - r_e \cdot \partial L / \partial w = 0$$

When land is the limiting input, the objective is to maximise profit per unit land area. The optimal seasonal irrigation depth is the water application required to equate the marginal value product (MVP) of water with the marginal factor cost of applying a unit of water (including the energy and labour requirements). Mathematically this condition is given by:

$$P_y \cdot f_w = r_w + r_e \cdot \partial E / \partial w + r_\ell \cdot \partial L / \partial w \tag{2.3}$$

The optimisation problem when annual water availability is limited to the quantity $(\overline{W})$ becomes:

Max $\quad \Pi = P_y \cdot f(x_1, \ldots, x_n, w) - A - \sum_{i=l}^{n} r_i x_i - r_w w - r_e \cdot E(w) - r_\ell \cdot L(w) \qquad (2.4)$

s.t. $\quad w \leq \overline{W}$

The Lagrangian function defined by the constrained optimisation problem is:

$$T = P_y \cdot f(x_1, \ldots, x_n, w) - A - \sum_{i=l}^{n} r_i x_i - r_w w - r_e \cdot E(w) - r_\ell \cdot L(w) + \lambda(\overline{W} - w) \tag{2.5}$$

The resulting first-order conditions, assuming the available water supply is totally exhausted are:

$$P_y \cdot f_i - r_i = 0 \qquad (i = 1, \ldots, n) \tag{2.6}$$

$$P_y \cdot f_w - r_w - \partial E / \partial w \cdot r_e - \partial L / \partial w \cdot r_\ell - \lambda = 0$$

$$\overline{W} - w = 0$$

In this case, the optimal irrigation depth is the quantity which results in the equality:

$$P_y \cdot f_w = r_w + \partial E / \partial w \cdot r_e - \partial L / \partial w \cdot r_\ell + \lambda \tag{2.7}$$

The Lagrangian multiplier *(λ)* represents the scarcity value of water in the production of the output y. Instituting a water supply restriction results in a further decrease in the optimal annual irrigation depth. The MVP of water is equated to the sum of the marginal factor cost of applying a unit of water (the market price of water + energy + labour) and its scarcity value.

The example above implicitly assumes that $\overline{W}$ will be distributed optimally over the growing season of the crop. Thus, in terms of decision support to irrigation farmers little information is gained in terms of water allocation if the farmer does not know how to distribute the water optimally.

2.2.2 MULTIPERIOD

Dealing with the optimal allocation of water is difficult because water applications in different crop growth stages will impact differently on final crop yield. Bernardo (1985) uses a relatively simple example of time dependent response to illustrate the interdependency of the sequential decisions defining an optimal intraseasonal water allocation.

To evaluate the effect of time on irrigator decision-making, consider the case of allocating a finite water supply to a single crop. For simplicity, it is assumed the irrigation season comprises n discrete subperiods. The management objective may be defined mathematically using the following separable objective function:

$$Max\sum_{i=1}^{n} NR_i(WA_i, I_i) \tag{2.8}$$

where: NR_i = net returns from stage i

WA_i = the state vector describing the soil moisture status in period i

I_i = the quantity of water applied in period i

In the usual reverse order of dynamic programming, i is used to denote that period after which $i-1$ further runs of the response process are made.

The irrigator seeks to maximise returns over the n periods by choosing irrigation quantities in each of the n periods $(I_1, I_2, ..., I_n)$. If an irrigation is to be applied, it is assumed to occur at the beginning of each subperiod. Thus, the soil-moisture status in period i (WA_i) is defined by the soil moisture carried over into period i (R_i) and the depth of irrigation in the period (I_i). Therefore, a response function relating yield to soil-moisture status in period i may be defined as:

$$f_i[WA_i(R_i, I_i)] \tag{2.9}$$

Note that yield is a function of the state of the soil-plant system rather than the total physical quantity of input applied in the season. The function f_i is assumed to exhibit diminishing returns so that the required second-order conditions for optimality hold. In addition, the specification of f_i differs among periods, accounting for the changing marginal productivity of water use over time.

The optimisation problem includes a transformation function (a recursion relation) that describes the transition of soil-moisture status from the initial stage to the final stage. This expression may be written as:

$$R_i = V_i\left(I_{i+1}, R_{i+1}\right) \tag{2.10}$$

As in the timeless case presented in Section 2.2.1, the problem is subject to the proviso that the total amount of water applied must be less than or equal to the total available water *(* $\overline{W}$ *)*. That is:

$$\sum_{i=1}^{n} I_i \leq \overline{W} \tag{2.11}$$

Abstracting from any uncertainties in price or yield, recurrence equations of the usual dynamic programming form may be formulated. Net returns are determined by subtracting variable costs from total revenue. For the case with only one period remaining (i.e., $n = 1$), the objective function may be defined as:

$$\text{Max } T_1\left(WA_1\right) = P_y \cdot f_1\left[WA_1\left(R_1, I_1\right)\right] - r_w I_1 - r_e \cdot E_1\left(I_1\right) - r_\ell \cdot L_1\left(I_1\right) + \lambda_1\left(\sum_{i=1}^{n} I_i - \overline{W}\right) \tag{2.12}$$

Differentiating the expression with respect to the decision variable I_1, yields the final-period condition for profit maximisation:

$$P_y \cdot \partial f_1 / \partial WA_1 \cdot \partial WA_1 / \partial WA_1 = r_w + r_e \cdot \partial E_1 / \partial I_1 \cdot r_\ell \cdot \partial L_1 / \partial I_1 + \lambda_1 \tag{2.13}$$

Water is applied to the level required to equate the MVP of water applied in period one to its marginal factor cost. Continuing for the case with two periods remaining, the objective function becomes:

$$T_2\left(WA_2\right) = P_y \cdot f_2\left[WA_2\left(R_2 I_2\right)\right] - r_w I_2 \cdot r_e \cdot E_2\left(I_2\right) - r_L \cdot L_2\left(I_2\right) - \lambda_2\left(\sum_{i-2}^{n} I_n - \left(\overline{W} - I_1\right)\right) \tag{2.14}$$

$$+\begin{Bmatrix} P_y \cdot f_1 - \{WA_1[R_1(R_2, I_2), I_1^*]\} - r_w I_1^* \\ - r_e \cdot E_1(I_1) - r_\ell \cdot L_1(I_1) \end{Bmatrix}$$

The resulting second-period condition for optimality is:

$$\begin{aligned} & P_y(\partial f_2/\partial WA_2 \cdot \partial WA_2/\partial I_2 + \partial f_1/\partial WA_1 \cdot \partial WA_1/\partial R_1 \cdot \partial R_1/\partial I_2) \\ & = r_w + r_e \cdot \partial E_2/\partial I_2 + r_\ell \cdot \partial L_2/\partial I_2 + \lambda_2 \end{aligned} \tag{2.15}$$

Equation (2.15) illustrates the interdependence of the sequential irrigation decisions. This expression states that the sum of MVP of a unit of water applied in period two and the impact on period one revenues resulting from applying a unit of water in period two, must equal the marginal factor cost of applying a unit of water in the second period. The interaction between the two periods is a consequence of the value of the additional soil moisture from I_2 carried over to period one.

2.2.3 *MULTIPLE CROPS*

To illustrate the influence of time on the optimality conditions for intercrop water allocation, the irrigation season is divided into three discrete subperiods. Two independent response processes are assumed and expressed as:

$$Y_h = f_h(w_{1h}, w_{2h}, w_{3h}, x_h) \qquad (h = 1, 2) \tag{2.16}$$

where: w_{ih} = water applied to the h^{th} crop in period i

x_h = the quantity of a composite input x applied to crop h

The problem faced by a profit-maximising producer seeking to allocate a finite seasonal water allotment $(\overline{W})$ over the three periods may be represented as:

$$\begin{aligned} \text{Max} \quad & P_1 \cdot f_1(w_{11}, w_{21}, w_{31}, x_1) + P_2 \cdot f_2(w_{12}, w_{22}, w_{32}, x_2) \\ & - r_w\left(\sum_{h=1}^{2} \sum_{i=1}^{3} w_{ih}\right) - r_x(x_1 + x_2) \end{aligned} \tag{2.17}$$

$$\text{s.t.} \quad \sum_{h=1}^{2} \sum_{i=1}^{3} w_{ih} \leq \overline{W}$$

where: P_h = the price of product h

r_x = the price of composite input x

Assuming the water supply is totally exhausted, optimal irrigation sequencing and intercrop water allocation must satisfy the following conditions:[1]

$$P_h \cdot \partial f_h / \partial w_{ih} = r_w + \lambda \qquad (h = 1, 2;\ i = 1, 2, 3) \qquad (2.18)$$

$$P_h \cdot \partial f_h / \partial w_h = r_x \qquad (h = 1, 2)$$

$$\sum_{h=1}^{2} \sum_{i=1}^{3} w_{ih} = \overline{W}$$

The optimal intraseasonal water allocation requires that the marginal value product of water between crops and among the three subperiods be equivalent. In this case, the necessary conditions for optimality require that the marginal value products of water equal the sum of the price of water and its shadow price.

When a limitation is placed on the seasonal water allotment, the optimal irrigation schedules will result in the maximum yields attainable from the seasonal irrigation depths allocated to each crop. That is, the assumption of technical efficiency is met, and the optimal schedule may be represented as a point on the upper envelope curve shown in Figure 2.2. This curve represents the locus of points of maximum water-use efficiency for each level of seasonal irrigation depth. Technically efficient points must be characterised by the condition that the marginal product of water for a particular crop must be equivalent across subperiods. That is, no reallocation of water between subperiods may increase yield. The condition is met in the above first-order conditions, where the marginal product of water used in each subperiod is equivalent to the constant $[(r_w+\lambda)/P_h]$.

Typically, seasonal water allocation is constrained by irrigations system capacities, capacity of infrastructure to supply irrigation water and seasonal patterns in the distribution of irrigation water availability. When restrictions are imposed on the intraseasonal distribution of water, optimal irrigation activities need not be technically efficient. Distributional considerations may dictate the selection of activities represented by points below the envelope of technically efficient irrigation activities. To analyse the effect of restrictions on the temporal distribution of water use, a constraint on the quantity of water available in period three (W_3) is added to the formulation presented in Equation (2.17).

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[1] λ is the Lagrangian multiplier on the annual water constraint and represents the shadow price of water.

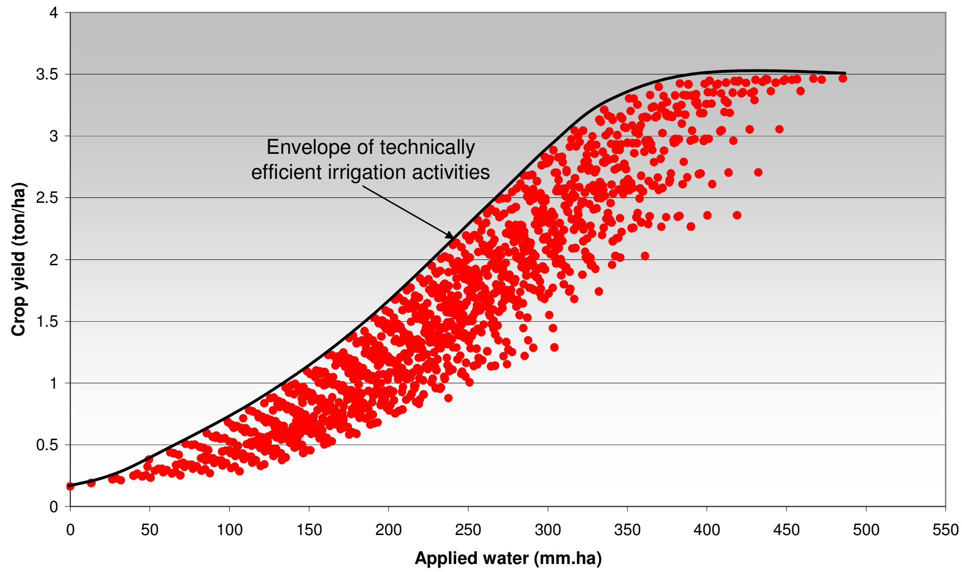

Figure 2.2: Envelope of technically efficient irrigation activities.

The optimisation problem becomes:

$$\text{Max} \quad P_1 \cdot f_1(w_{11}, w_{21}, w_{31}, x_1) + P_2 \cdot f_2(w_{12}, w_{22}, w_{32}, x_2) - r_w\left(\sum_{h=1}^{2} \sum_{i=1}^{3} w_{ih}\right) - r_x(x_1 + x_2) \tag{2.19}$$

$$\text{s.t.} \quad \sum_{h=1}^{2} \sum_{i=1}^{3} w_{ih} \leq \overline{W}$$

$$w_{31} + w_{32} \leq \overline{W_3}$$

The resulting first-order conditions are:

$$P_h \cdot \partial f_i / \partial w_{ih} = r_w + \lambda_1 \qquad (h = 1, 2;\ i = 1, 2) \tag{2.20}$$

$$P_h \cdot \partial f_h / \partial w_{3h} = r_x + \lambda_1 + \lambda_2 \qquad (h = 1, 2)$$

$$P_h \cdot \partial f_h / \partial x_h = r_x \qquad (h = 1, 2)$$

$$\sum_{h=1}^{2} \sum_{i=1}^{3} w_{ih} = \overline{W}$$

$$w_{31} + w_{32} \leq \overline{W_3}$$

λ_1 and λ_2 represent the Lagrangian multipliers on the annual and third-period water supply constraints. For water applied in periods one and two these conditions are equivalent to those in Equation (2.18). Optimality conditions for period three water also require that the MVP of water equals the price of water plus its shadow price. In this case however, the shadow price has an added scarcity value dictated by the constraint on period-three water use.

These conditions can be used to illustrate that economically efficient irrigation schedules need not be technically efficient. Algebraic manipulation of the first-order conditions gives:

$$\begin{aligned} \partial f_1/\partial w_{11} &= [(r_w + \lambda_1)/P_1] \\ \partial f_1/\partial w_{21} &= [(r_w + \lambda_1)/P_1] \\ \partial f_1/\partial w_{31} &= [(r_w + \lambda_1 + \lambda_2)/P_1] \end{aligned} \tag{2.21}$$

Therefore, the conditions for technical efficiency hold only if the constraint on third-period water availability is non-limiting. This result illustrates the limitations of applying neoclassical production functions, which presuppose technical efficiency, to the intraseasonal water allocation problem.

When water allocation between multiple crops is of concern and intraseasonal water supply is constraining, economic theory suggests that water allocation does not need to be technically efficient. To optimise intraseasonal water use a multiple period model is necessary where the impact of decisions in previous periods is linked to current period decisions.

2.2.4 CONCLUSIONS

Optimising water use is complicated due to the fact that the marginal productivity of water between different crop growth stages is different. Furthermore, when seasonal water production functions are used to optimise water use the assumption is implicitly made that the distribution of water over the growing season is optimal. To provide farmers with more relevant information the interdependencies between irrigation decisions in different crop growth stages need to be acknowledged when optimising water use. Probably the most serious implication for water use optimisation is that the optimal irrigation schedules need not be technically efficient when restrictions are imposed on the intraseasonal distribution of water when multiple crops compete for water. Thus, the employed methodology used to optimise agricultural water use needs to allow for these technical inefficiencies.

In the next section, alternative methods of optimising water use in South Africa and internationally are reviewed.

2.3 AGRICULTURAL WATER USE OPTIMISATION

Numerous research studies have been done in the area of water use optimisation and the review in this section is by no means exhaustive. Rather, the review concentrates on selected studies which influenced the methodology used in this research to optimise water use taking deficit irrigation into account. The review of the South African literature is, however, thoroughly done.

English *et al.* (2002) argue that modelling deficit irrigation realistically is critical in efforts to optimise agricultural water use. Some of the issues in agricultural water use optimisation that need to be considered are the following: (i) non-linear relationship between applied water and crop yield, (ii) interdependencies between water use in different crop growth stages and (iii) production risk.

2.3.1 NON-LINEAR RELATIONSHIP BETWEEN APPLIED WATER AND CROP YIELD

Modelling the non-linear relationship between applied water and crop yield is very important since the non-linear relationship gives rise to declining marginal productivity of applied water, which is a necessary condition to maximise profits. The existence of a non-linear relationship between applied water and crop yield and a linear relationship between ET and crop yield are discussed at the beginning of this chapter. Therefore, the specific objective of this section is to evaluate alternative procedures to quantify the relationship between applied water and crop yield and to evaluate how researchers have incorporated the relationship in their analyses.

2.3.1.1 International research

Recent international research emphasised the importance of non-uniform water applications on crop yields. Two alternatives exist to model the non-linear relationship as a result of non-uniform water applications. The first approach simulates spatial variability in soil depths, water holding capacities, infiltration characteristics, and distribution of applied water by dividing irrigated fields into sectors and using Monte Carlo simulation to assign variable values randomly to each sector (Hamilton, Green and Holland, 1999). As a result some portion of the irrigated field will be over-irrigated and some portion under-irrigated, which gives rise to a non-linear relationship between applied water and crop yield. Hamilton *et al.* (1999) used the stochastic simulation approach with CropSyst (Cropping Systems Simulation model) to estimate crop water production functions for different crops under various irrigation technologies. These crop water production functions were then utilised in a mathematical programming model to evaluate water reallocation possibilities in the Snake River.

The second approach assumes a statistical distribution for the non-uniform applications to calculate the average area that is respectively under-irrigated and over-irrigated. The second approach is used extensively in recent agricultural water use optimisation literature to characterise the relationship between applied water and crop yield (Mantovani *et al.*, 1995; De Juan *et al.*, 1996; Reca, Roldán, Alcaide, López and Camacho, 2001; Ortega, de Juan, Tarjuelo and López, 2004; Sepaskhah and Ghahraman, 2004; Ortega, de Juan and Tarjuelo, 2005). The overall procedure is based on the integration of an estimate of the average water deficit due to non-uniform applications and a relative ET formula to calculate crop yield. Relative ET formulae calculate crop yield by relating relative yield percentage (Y_a/Y_m) to relative evapotranspiration percentage (ET_a/ET_m) by means of a crop yield response factor which indicates the sensitivity of the crop to water deficits (Doorenbas and Kassam, 1979). Most of the researchers that have adopted the procedure use the Stewart multiplicative relative ET formula to calculate crop yield because it takes into account the impact of water deficits in different crop growth stages on crop yield. The most frequently used distributional assumptions for water applications are the normal and uniform distributions. Information regarding the non-linear relationship between applied water due to non-uniform water applications is then used in some kind of an optimisation procedure to optimise water use.

2.3.1.2 South African research

Various South African researchers optimised agricultural water use by means of linear programming (LP) (Hancke and Groenewald, 1972; Van Rooyen, 1979; Brotherton and Groenewald, 1982) or dynamic linear programming (DLP) (Backeberg, 1984; Oosthuizen, 1995; Maré, 1995; Louw and Van Schalkwyk, 1997; Haile, *et al.*, 2003). Typically, these researchers use one point estimate on a crop water production function to represent the relationship between applied water and crop yield. Although the crop yield estimates correspond to actual crop yields, the water use is typically derived for conditions of no water deficits. These research efforts are not reviewed in this section since the main objective of this section is to review the South African literature that considers the economics of allowing the crop to sustain some level of water stress commonly referred to as deficit irrigation.

Viljoen, Symmington, Botha and Du Plessis (1993) used a crop growth simulation model to simulate the impact of alternative deficit irrigation scheduling strategies on water use and crop yield. The outputs of the model were used to estimate polynomial crop water production functions to represent the non-linear relationship between applied water and crop yield. Point estimates on these functions were then included in a DLP model to evaluate the impact of alternative canal capacities on agricultural water use in Vaalharts. By implication, these researchers are implicitly assuming that water applications are distributed optimally over the growing season. However, theory suggests that the assumption will be violated if intraseasonal water allocations are limited by canal capacities when multiple crops compete for water.

Mottram *et al.* (1995) adopted a procedure that will correctly optimise water use between multiple crops when intraseasonal water allocations are limiting but assumed a linear relationship between applied water and crop yield. The procedure relies on the inclusion of different activities consisting of different combinations of 10 mm deficits in each of the growth stages in their programming model. Crop yield was estimated for each combination using an additive law of calculating crop yield as a function of ET deficits. Two critical assumptions were made by these researchers. Firstly, they assumed that water use in any of the crop growth stages is independent of the other. Thus, the influence of irrigation decisions early in the season have no influence on decisions made later in the season. Secondly, they assumed that reductions in ET are proportional to reductions in applied water. Thus, these researchers did not account for the non-linear relationship between applied water and crop yield and therefore the increasing water use efficiencies as the crop is deficit irrigated. Results from their analyses indicated that deficit irrigation is not viable and that the areas planted should be reduced and fully irrigated. These results may be the direct result of the inability of these researchers' procedures to account for increasing irrigation efficiencies when the crop is deficit irrigated.

Grové and Oosthuizen (2002) optimised agricultural water use while quantifying economic environmental tradeoffs of maintaining instream flow requirements. Rather than generating discrete activities of alternative deficit irrigation schedules these researchers optimised a continuous function that relates ET to crop yield. The Stewart multiplicative function has the property of modelling more than proportional yield reductions if the crop is stressed in more than one crop growth stage. Increasing water use efficiencies as the crop is deficit irrigated were modelled using procedures developed by Willis (1993) whereby efficiencies are assumed to increase linearly between maximum water application and a given maximum allowed deficit. The results of the analyses indicated that it is profitable to practise deficit irrigation while spreading available water over larger irrigation areas. Although these researchers were able to model increasing irrigation efficiencies as the crop was deficit irrigated no link exists between the water budgets in different crop growth stages. Furthermore, these researchers did not account for any changes in yield variability as the crop is increasingly deficit irrigated.

The work done by Lecler (2004) is not specifically aimed at optimising water use but provides an important simulation application that acknowledges the importance of the uniformity with which irrigation technology applies water to the relationship between applied water and crop yield. The water use efficiency of alternative irrigation schedules and irrigation technologies was evaluated by simulating multiple water budgets with ZIMsched (Zimbabwe Irrigation Scheduling model) to incorporate the impact of non-uniform water applications of alternative irrigation technologies on sugarcane yields. Recently Grové (2006a) used a simulation model that incorporates the impact of non-uniform water applications on crop yield to generate activities for a linear programming model to optimise water use.

2.3.1.3 Conclusions

At the international level researchers are increasingly focussing on modelling the non-linear relationship between applied water and crop yield using the non-uniformity with which irrigation systems apply water linked to the Stewart multiplicative relative ET formula. Modelling procedures to simulate the impact of non-uniform applications on crop yields have only recently being adopted by South African researchers.

The review of the South African research indicated that a large number of optimisation studies have followed the old paradigm of allocating water to achieve maximum yield. The difference in the results of the research by Mottram *et al.* (1995), which assumed constant irrigation efficiencies, and the research by Grové and Oosthuizen (2002), who modelled increasing efficiencies as the crop is deficit irrigated, emphasises the importance of modelling the non-linear relationship between crop yield and applied water. The conclusion is that failure to model the non-linear relationship between applied water and crop yield will result in an under estimation of the potential benefits of deficit irrigation if it is profitable to deficit irrigate the crop.

2.3.2 INTERDEPENDENCY BETWEEN WATER USE IN DIFFERENT CROP GROWTH STAGES

Optimising agricultural water use is difficult because irrigation water differs from other production inputs since it can be dynamically adjusted as the growing season progresses (Peterson and Ding, 2005). A further complicating factor is that water deficits in different crop growth stages will impact differently on final crop yield (Doorenbos and Kassam, 1979). In order to model deficit irrigation satisfactorily the modelling procedure should be able to model the interdependency of sequential irrigation decisions on crop yield. Modelling these interdependencies is especially important in systems where multiple crops compete for limited water supplies.

2.3.2.1 International research

Dynamic programming (DP) is frequently used by researchers to optimise water use within a growing season. One of the problems with DP is that many simplifying assumptions are necessary to cope with the problem of dimensionality (Shütze, de Paly, Wöhling and Schmitz, 2005). Typically, water use optimisation between multiple crops is achieved by a multi-tier approach.

Reca *et al.* (2001) use DP to derive optimal seasonal production functions. The relationship between applied water and crop yield is based on normally distributed water applications and the Stewart multiplicative relative ET formula to account for the impact of ET deficit in different crop growth stages. DP is used to allocate a limited amount of irrigation water optimally over the growing season. Repeating the optimisation for different levels of water availability yields the necessary information to estimate a crop water production function based on optimally distributed irrigation quantities over the growing season. The optimal crop water production functions of different crops are used in a second optimisation model to optimise water use between multiple crops. Since the production functions of the individual crops are non-linear Reca *et al.* (2001) transformed it into a linear problem by approximating the benefit function to a discrete function. Shangguan, Shao, Horton, Lei, Qin and Ma (2002) adopted a similar procedure to optimise water use between multiple crops. In the first stage, DP is used to distribute alternative limited amounts of water optimally over the growing season. Regression analysis is used to estimate m-order polynomial crop water production functions. These functions are used in a second DP optimisation model to optimise water use between competing crops given a limited amount of water is available. A problem with using optimal production functions to optimise water use between multiple crops is that the solutions may not be optimal if intraseasonal water allocations are limiting.

Ortega *et al.* (2004) developed a comprehensive water use optimisation model, which forms the basis of the irrigation advisory service provided to farmers in Castilla-La Mancha (Ortega *et al.*, 2005). Rather than developing optimal production functions to generate the necessary information for a second optimisation model, the model utilises a genetic algorithm (GA) to optimise the whole system. Crop yields are estimated using the Stewart multiplicative relative ET formula while the non-linear relationship between applied water and crop yield was modelled assuming normally distributed water applications over the entire field. Historical weather data is used to drive the system where ET is calculated using Penman-FAO and Penman-Monteith procedures. The cropping pattern and corresponding irrigation schedule are optimised for each year with the GA. The recommended strategy is chosen based on the lowest accumulative measure of risk. The cumulative risk associated with a specific alternative corresponds to the sum of deviations from a reference gross margin, determined for each year, as a consequence of the application of this crop rotation throughout the climatic series (Ortega *et al.*, 2004:67).

Bernardo, Whittlesey, Saxton and Bassett (1987) developed a procedure to approximate the dynamic problem of optimising water use between multiple crops with LP. The approximation is based on the inclusion of a large number of discrete activities representing alternative ways of distributing water over the growing season. Information for the activities is simulated with a crop growth simulation model. The methodology is appealing since it uses procedures that are easily understandable by a large community and does not require highly specialised software or modelling expertise. The procedure has recently been applied by Scheierling, Young and Cardon (2004) to determine the price responsiveness of demands for irrigation water deliveries and consumptive use.

2.3.2.2 South African research

The most sophisticated example of crop water use optimisation is the work done by Botes, Bosch and Oosthuizen (1996). These researchers linked a crop growth simulation model to an optimisation procedure to optimise different irrigation scheduling strategies for maize under dynamic plant growth conditions in order to estimate the value of information for irrigation scheduling for different soils. Results indicated that the value of irrigation information is sensitive to the plant extractable soil water of the soils and water availability.

As an alternative to the highly specialised applications of water use optimisation above Grové (2006a) used a more robust procedure to optimise water between competing crops that can be applied within a whole farm setup. The procedure is based on simulating the effect of multiple irrigation quantity combinations on crop yield. Information on water applications in different time periods and crop yields are then used in a mathematical programming model to optimise water use (Bernardo *et al.,* 1987).

Although not specifically aimed at deficit irrigation[2] the research by Viljoen, Dudley, Gakpo and Mahlaha (2004) needs mentioning because theirs is one of the few South Africa African studies that employed LP and stochastic dynamic programming (SDP) to optimise water use. A rather simple LP model in terms of water use optimisation was used to derive gross margin as a function of the total amount of water allocated to the farm. The first derivatives of these functions provide estimates of the MVP of water allocated to a specific farm under consideration. These values were used in the SDP model to optimise the water allocation for different capacity shares in the Vanderkloof dam. Linking the results of the LP with the SDP model clearly demonstrates the inability of DP approaches to handle more complex problems due to the curse of dimensionality.

2.3.2.3 Conclusions

DP procedures are typically preferred to optimise crop water use within a growing season to derive optimal crop water yield production functions. Simplifying assumptions are, however, necessary to keep the model tractable because adding more detail quickly results in too large a model. Incorporating information regarding optimal production functions in a second tier optimisation model to allocate water optimally between competing crops will violate optimality conditions if intraseasonal water availability is limiting. Use of GA to optimise complex systems seems to be a practical alternative to DP and should be further investigated.

[2] These researchers pre-assumed irrigation requirements consistent with maximum yield.

South African research that focused on optimising the interdependency between water usage in different crop growth stages is scant. Botes *et al.* (1996) treated the problem comprehensively. However, application of the methodology requires computer-programming skills and is time consuming to implement. Furthermore, application of such a methodology will be highly complicated if water use needs to be optimised between competing uses where the decision-makers have to decide upon areas planted and irrigation quantities. Grové (2006a) adopted the procedures developed by Bernardo *et al.* (1987) to optimise water use with standard mathematical programming procedures while adhering to the theory of water use optimisation. The same procedure was recently applied by Scheierling *et al.* (2004). The simplicity of the approach is appealing because incorporating the non-linear relationship between applied water and crop yield, while taking cognisance of the impact of water deficits in different crop growth stages, production risk and other farm level constraints, is straightforward. The conclusion is that less complicated procedures that conform to economic theory may provide a framework for optimising water use between multiple crops within a whole-farm setup while taking cognisance of production risk.

2.3.3 PRODUCTION RISK

To evaluate deficit irrigation thoroughly production risk needs to be taken into account because adjusting irrigation amounts during the growing season is viewed as the producer's primary tool for managing production risk (Peterson and Ding, 2005). English *et al.* (2002:272) furthermore argue that when the opportunity cost of water is taken into account and it is optimal to reduce water applications and at the same time increase the area irrigated, any losses that may incur will be amplified by the increased area under irrigation. The need to take production risk into account is accentuated by the fact that irrigation farmers in South Africa are found to be risk averse (Botes, 1994; Meiring, 1993). The main objective of this section is to review the impact of deficit irrigation on production risk.

2.3.3.1 International research

Reca *et al.* (2001) used optimal production functions derived from DP models to demonstrate the impact of climate variability on income. Analyses were conducted for both winter and summer crops. Results indicated that higher climatic variability causes the overall income variability between crops to increase while increased levels of deficit irrigation cause increased levels of income variability.

Peterson and Ding (2005) developed a risk programming model to quantify the effect of irrigation efficiency on water use in the High Plains of America taking account of the impact of irrigation timing on production risk. Data simulated with a crop growth simulation model is used to estimate a Just-Pope production function to determine the impact of irrigation timing on expected crop yields and the variability thereof. Results indicated that irrigation water applications are risk reducing in some crop growth stages and in others it is risk increasing. These results were explained by differences in crop growth development resulting from different irrigation scheduling practices during the season.

2.3.3.2 South African research

Botes (1990) evaluated the risk efficiency of alternative wheat irrigation strategies taking plant extractable soil water-holding capacities of different soils into account. Only one deficit irrigation strategy was simulated by allowing the crop to sustain 20% crop water stress before triggering the next irrigation. Simulated crop yields for the deficit irrigation strategy showed increased variability in crop yields over the other irrigation strategies. Stochastic dominance with respect to a function (SDRF) (Meyer, 1977) was used to show that risk averse irrigators will not choose to deficit irrigate their crop. Unfortunately, Botes (1990) did not include alternative levels of deficit irrigation in his analysis.

Grové *et al.* (2006) extended the research by Botes (1990) by including increasing levels of deficit irrigation for wheat and maize in their risk efficiency analyses. A more robust alternative to SDRF, called stochastic efficiency with respect to a function (SERF) (Hardaker, Richardson, Lien and Schumann, 2004) was used to rank alternative water use strategies for decision-makers with varying degrees of risk aversion. Results of the analyses indicated that gross margins of both crops are more variable under deficit irrigation. In contrast with the findings of Botes (1990) results also indicated that there might be some level of maize deficit irrigation that will be preferred by risk averse irrigators under limited water supply conditions whereas full irrigation is preferred for wheat. These results highlight the importance of weather on the risk efficiency of deficit irrigation since maize is produced during periods of relatively higher expected rainfall while wheat is produced during periods of lower expected rainfall conditions.

The research efforts discussed above used simulation procedures to determine the risk efficiency of alternative deficit irrigation schedules. A shortcoming of simulation is that it shows the impact of predefined alternatives which ignore the opportunity cost of water. Botes *et al.* (1996) enhanced their previous efforts (Botes, Bosch and Oosthuizen, 1995) to quantify the value of irrigation information for risk averse decision-makers. However, these researchers did not allow changes in the area planted while optimising limited water availabilities. Grové (2006a) incorporated risk into his analysis to evaluate the potential of deficit irrigation to conserve irrigation water. Results indicated that it is profitable to use the water that is saved through deficit irrigation to irrigate larger areas.

2.3.3.3 Conclusions

The international studies show some important aspects that need to be taken into account when evaluating deficit irrigation. Firstly, deficit irrigation will decrease expected crop yield and most likely increase yield variability as the crop is deficit irrigated. Secondly, the importance of using appropriate crop growth simulation models to quantify the impact of deficit irrigation on crop yield is highlighted by the fact that water applications might be risk reducing or risk increasing in some crop growth stage.

The South African studies emphasise the importance of weather conditions on the deficit irrigation profitability. The conclusion is made that the potential to use rainfall more efficiently has a significant impact on the adoption of deficit irrigation strategies by risk-averse decision makers. Any information that will increase the potential to use rainfall more efficiently, such as improved localised weather forecasts, will improve the adoption of deficit irrigation strategies. However, use of deficit irrigation in areas where rainfall is minimal may cause risk averse farmers to adopt full irrigation. The overall conclusion is that the risk aversion will impact significantly on the adoption of deficit irrigation in different regions because the impact of deficit irrigation is highly dependent on prevailing weather conditions.

In the next chapter, choice of risk aversion coefficients for use with stochastic efficiency analysis techniques is reviewed.

CHAPTER 3

CHOICE OF RISK AVERSION LEVELS FOR STOCHASTIC EFFICIENCY ANALYSIS

The principal theory that is used to guide decision-making under risk is subjective expected utility theory[3] (SEU). SEU requires the decision-maker to integrate his/her subjective views of the variability of a specific outcome variable (risk quantification) and his/her preferences for those outcomes (utility). Since the preferences of decision-makers are not always known, stochastic efficiency criteria were developed. Efficiency criteria allow some ranking of risky alternatives when the preferences for alternative outcomes of decision-makers are not exactly known. Usually some assumptions are made with respect to preferences, which translate into evaluating risky alternatives over a range of risk aversion levels to establish an efficient set of alternatives. The efficient set contains all the alternatives that a decision-maker to whom the assumptions apply will prefer. The main objective of this chapter is to determine plausible ranges of risk aversion for use with stochastic efficiency analysis methods. To achieve this objective alternative methods to scale Arrow-Pratt absolute risk aversion between different situations are reviewed. A new method of scaling is then developed and applied to determine plausible ranges of risk aversion for further use in this research.

3.1 STOCHASTIC EFFICIENCY WITH RESPECT TO A FUNCTION (SERF)

The most recent advance in ranking risky alternatives is Stochastic Efficiency with Respect to a Function (SERF) (Hardaker *et al.*, 2004). SERF finds its theoretical foundation in expected utility theory where expected utility theory is based on the existence of an ordinal utility function by which alternatives can be ranked. The axioms that guarantee the existence of such an ordinal utility function that allows alternatives to be ranked are ordering, transitivity, continuity and independence (Hardaker, Huirne, Anderson and Lien, 2004:35-36).

Ordering implies that when faced with two risky prospects, a_1 and a_2, a decision-maker either prefers one to the other or is indifferent to both. *Transitivity* implies that if a_1 is preferred to a_2, and a_2 is preferred to a_3 then a_1 is preferred to a_3. *Continuity* implies that if a_1 is preferred to a_2 and a_2 to a_3, then a subjective probability $P(a_1)$ exists, not zero or one, that makes the decision-

[3] Although expected utility theory has come under criticism and most recently by Rabin and Thaler (2001) it is still considered to be the most appropriate theory for decision–making under risk (Hardaker *et al.*, 2004; Hardaker, *et al.*, 2004; Meyer, 2001).

maker indifferent to a_2 and a lottery yielding a_1 with probability $P(a_1)$ and a_3 with probability $1-P(a_1)$. *Independence* requires that if a_1 is preferred to a_2 and a_3 is any other risky prospect then the decision-maker will prefer a lottery yielding a_1 and a_3 as outcomes to a lottery yielding a_2 and a_3 when $P(a_1)=P(a_2)$.

Given the axioms are not violated, the ordinal utility function will allow one to rank alternatives based on utility because if a_1 is preferred to a_2 then $U(a_1)>U(a_2)$ where $U(\cdot)$ presents the utility function. Thus, the best alternative is chosen by maximising expected utility (EU) where the utility of a risky prospect is the probability weighted average of all the discrete outcomes.

Hardaker *et al.* (2004) developed a technique called stochastic efficiency with respect to a function that is based on the notion that ranking risky alternatives in terms of utility is the same as ranking alternatives with certainty equivalents (CE). CE is defined as the sure sum with the same utility as the expected utility of the risky prospect (Hardaker *et al.*, 2004). Thus, the decision-maker will be indifferent to both the CE and the risky prospect. CE is calculated as the inverse of the utility function and is therefore dependent on the form of the utility function. Assuming an exponential utility function and a discrete distribution of x, CE is calculated as (Hardaker *et al.*, 2004:257):

$$CE\left(x, r_a(x)\right) = \ln\left\{\left(\frac{1}{n}\sum_j^n e^{-r_a(x)x_j}\right)^{\frac{-1}{r_a(x)}}\right\} \tag{3.1}$$

where $r_a(x)$ is the level of absolute risk aversion and n defines the size of the random sample of risky alternative x. The relationship between risk aversion and CE is determined by evaluating Equation (3.1) over a range of $r_a(x)$ values. Repeating for different risky alternatives yields the relationship for several alternatives, which are best compared by means of graphing the results (Hardaker *et al.*, 2004).

The alternatives are ranked based on CE whereby the alternative with the highest CE is preferred given the specific level of risk aversion. Figure 3.1 shows that Alternative 1 dominates the other alternatives for risk aversion levels $r_a(x)_L$ to $r_a(x)_2$ whereas Alternative 2 dominates the other two alternatives between $r_a(x)_2$ and $r_a(x)_U$. The utility efficient set over $r_a(x)_L$ to $r_a(x)_U$ therefore consists only of Alternative 1 and 2. The vertical distance between two alternatives at a specified $r_a(x)$ level yields a utility weighted risk premium[4], which is defined as the minimum sure

[4] Note that this concept is different from the risk premium defined by Pratt (1964).

amount that has to be paid to a decision-maker to justify a switch between a preferred and a less preferred alternative (Hardaker *et al.*, 2004).

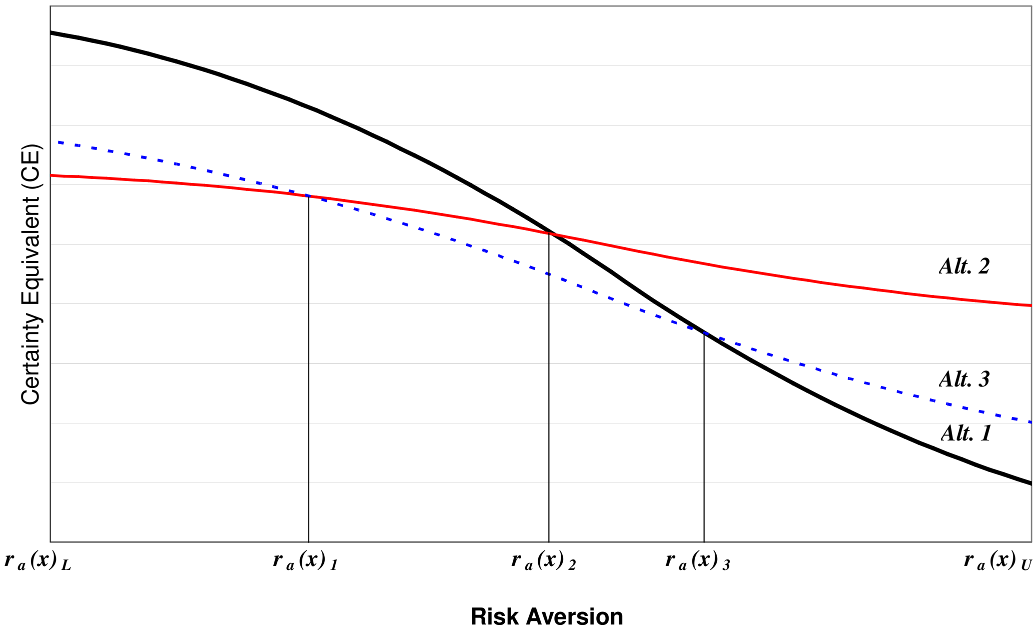

Figure 3.1: Illustration of stochastic efficiency with respect to a function comparing three alternatives over risk aversion levels $r_a(x)_L$ to $r_a(x)_U$.

Key to the application of SERF is the choice of utility function and the representation of risk attitudes in accordance with the chosen utility function. Two commonly used functions are the negative exponential and the power utility functions. The negative exponential utility function utilises absolute risk aversion whereas the power function uses relative risk aversion. Important to note is that the same risk aversion applies to all risky alternatives even though the expected outcomes and the variability between alternatives may differ significantly.

3.2 RISK ATTITUDES AND MEASURES OF RISK AVERSION

The shape of the utility function reflects the preferences of the decision-maker. Generally it is accepted that decision-makers will always prefer more wealth to less, which is true for utility functions with a positive slope $(U'(x) \geq 0)$ over the whole range of payoffs. The second derivative of the utility function gives an indication of the decision-maker's attitude towards risk where $U''(x) < 0$ implies risk aversion (concave function); $U''(x) = 0$ implies risk indifference (linear function); and $U''(x) > 0$ implies risk preference (convex function). Risk attitudes can therefore be inferred from the sign of the second derivative of the utility function (Pratt, 1964). However, it is important to note that the second derivative of the utility function is not in itself a meaningful measure of concavity and therefore the magnitude of a decision-maker's risk attitude. The reason is that the scale used to measure utility is arbitrary and U is defined only for any positive linear transformation. Multiplying U by any positive number, for example, does not alter behaviour but does alter $U''(\cdot)$ (Pratt, 1964).

To obtain a measure of the magnitude of risk attitude Pratt (1964) and Arrow (1971) have taken the rate of change in the slope (second derivative) of the utility function and normalised this function by the slope (first derivative) of the utility function. The result is the Arrow-Pratt measure of absolute risk aversion, denoted by $r_a(x)$:

$$r_a(x) = -U''(x)/U'(x) \tag{3.2}$$

where x is the appropriate performance indicator (outcome variable).

According to Hey (1979) the Arrow-Pratt measure of absolute risk aversion, $r_a(x)$ has the following properties:

1. if $r_a(x)<0,\ = 0\ or > 0$ then the individual displays risk-seeking, risk-neutral or risk-averse preferences, respectively.
2. $r_a(x)$ is larger for a more risk-averse individual than for a less risk-averse individual.
3. $r_a(x)$ is unaffected by an arbitrary linear transformation of the utility function.

Raskin and Cochran (1986) demonstrated that the invariance property of arbitrary linear transformation of the utility function does not apply to arbitrary rescaling of the outcome variable x. Due to the before mentioned; $r_a(x)$ cannot be transferred from one study to another without applying some sort of rescaling. McCarl and Bessler (1989) also questioned the validity of employing $r_a(x)$ values elicited in one study in another, because wealth levels and the dispersion of risky prospects would change between studies. Ferrer (1999) used the relationship between the Pratt risk premium, the variance of the risky prospect and $r_a(x)$ to demonstrate the impact of both scale and range on $r_a(x)$.

Following McCarl and Bessler (1989), Pratt (1964:125) defined the risk premium as follows:

$$\prod(x,Y) = 0.5\sigma_y^2\ r_a(x) + o(\sigma_y^2) \tag{3.3}$$

where $\prod(x,Y)$ is the risk premium given a level of wealth and a risky prospect Y, σ_y^2 is the variance of the risky prospect, $r_a(x)$ is the Arrow-Pratt risk aversion at level of wealth x, and

$o(\sigma_y^2)$ are the higher order terms in the Taylor series expansion of the expected utility function around a mean of x. By rearranging the above expression to make $r_a(x)$ the subject yields:

$$r_a(x) = 2[\prod(x,Y) - o(\sigma_y^2)] / \sigma_y^2. \tag{3.4}$$

Following Tsaing (1972), if the dispersion of the risk prospect is assumed small relative to wealth, then the term $o(\sigma_y^2)/\sigma_y^2$ may be neglected. Thus, $r_a(x)$ is approximately given by:

$$r_a(x) \cong 2\prod(x,Y) / \sigma_y^2. \tag{3.5}$$

Equations (3.4) and (3.5) show that both the level of x and σ_y^2 affect the magnitude of $r_a(x)$. Because the risk premium is divided by σ_y^2 and not $E[Y]$ it is concluded that the magnitude of $r_a(x)$ is not affected by the use of incremental rather than absolute returns, or *vice versa* (Ferrer, 1999).

From the discussion above it is clear that $r_a(x)$ is affected by arbitrary rescaling of x or a change in the dispersion of the risky prospect (σ_y^2). Thus, assuming same $r_a(x)$ values to discriminate between risky prospects may imply different levels of risk aversion if σ_y^2 between the alternatives differ. Intuitively Hardaker *et al.* (2004) do some scaling in their SERF analyses when they do not base their choice of $r_a(x)$ on the overall mean of all the alternatives, but on ones with similar mean values, thereby recognising that the distributions need to be similar to be ranked. However, no formal scaling of $r_a(x)$ is done to ensure consistent representation of risk aversion between alternatives.

The need to rescale $r_a(x)$ for consistent representation of risk attitude is not new. In the next section alternative procedures to adjust $r_a(x)$ for consistent representation of risk attitude is reviewed.

3.3 CONSISTENT PRESENTATION OF RISK AVERSION

The objective of this section is to evaluate alternative methods to scale $r_a(x)$ for consistent representation of risk attitude and to establish plausible ranges of risk aversion in the absence of specific utility functions for decision-makers. Two alternative methods frequently used in literature to scale $r_a(x)$ are considered as well as other methods whereby $r_a(x)$ are scaled based on the dispersion of the risky prospect. The first method found in literature that is frequently used, is referred to as mean scaling after the work of Raskin and Cochran (1986) and the second method is based on the work done by Babcock, Choi and Feinerman (1993) on risk and probability premiums. Nieuwoudt and Hoag (1993) proposed that the range (maximum-minimum) of the risky prospects be used to standardise risk aversion. The caveats of such a procedure are highlighted and a new method that uses the standard deviation in the standardisation procedure is formalised. The description of all four these methods will be followed by a numerical example to demonstrate the implications of these methods on risk aversion. The last part of this section will be used to establish plausible risk aversion bounds for use in stochastic efficiency analysis techniques.

3.3.1 MEAN SCALING

Raskin and Cochran (1986:206-207) introduced two theorems that can be used to guide the rescaling of Arrow-Pratt absolute risk aversion levels.

THEOREM 1: Let $r_a(x)=-U''(x)/U'(x)$. Define a transformation of scale on x such that $w=x/c$, where c is a constant. Then $r_a(w)=cr_a(x)$.

Given the transformation of scale, relative risk aversion for x $(r_r(x))$ and w $(r_r(w))$ is respectively defined as:

$$r_r(x) = r_a(x)x \tag{3.6}$$

$$r_r(w) = r_a(w)\frac{x}{c} \tag{3.7}$$

Now assuming $r_r(\cdot)$ is constant yields:

$$r_a(x)x = r_a(w)\frac{x}{c} \tag{3.8}$$

$$\therefore\ r_a(w) = cr_a(x) \tag{3.9}$$

which demonstrates what Raskin and Cochran's (1986) first theorem implies constant relative risk aversion.

THEOREM 2: If $v=x+c$, where c is a constant, then $r_a(v)= r_a(x)$. Therefore the magnitude of risk aversion is unaffected by the use of incremental rather than absolute outcome levels (or *vice versa*).

McCarl (1987:228) demonstrated that the $r_a(v)$ at income level x is not always equal to the $r_a(v)$ at wealth level $v=x+c$. However, Cochran and Raskin (1987:231) argued that McCarl (1987) failed to recognise the distinction between wealth and incremental income given a base level of wealth. Later Cochran and Raskin (1987) altered their theorem to especially recognise the wealth/incremental distinction. They presented the following equation:

$$r_w(x+c) = r_{i/c}(x) \tag{3.10}$$

where r_w is risk aversion to wealth and $r_{i/c}$ is the risk aversion to incremental returns given previous wealth level c. According to this theorem, the willingness to deviate from, for example, a \$110 000 wealth level will be equivalent to the willingness to deviate from a \$10 000 incremental return (annual income) level given wealth is already \$100 000, if the decision-maker can mentally account whether a wealth dollar or an annual income dollar is at stake.

It should be recalled that it was demonstrated in the previous section that the variability of a risky prospect affects $r_a(x)$. Ferrer (1999) proposed an amendment to Raskin and Cochran's (1986) first theorem such that $c=\sigma_x/\sigma_w$ thereby recognising that the standard deviation of the risky prospects alters the presentation of risk aversion and not the changes in the expected values of the risky prospects. However, this procedure was never applied in literature. Rather, Ferrer (1999) used a standardisation procedure developed by Nieuwoudt and Hoag (1993) whereby $r_a(x)$ is scaled based on the difference between the minimum and maximum of the risky prospect.

3.3.2 *RISK PREMIUMS AS A FRACTION OF THE GAMBLE SIZE*

Babcock *et al.* (1993) demonstrated in their work on probability and risk premiums that the size of the gamble greatly influences reasonable interpretations of a given $r_a(x)$. These researchers argue that $r_a(x)$ should be selected such that the risk premium expressed as a fraction (θ) of the gamble size is equal between alternatives.

Following Babcock *et al.* (1993:18) let us consider an individual with certain income w and random income z. Let $z=[h, -h; .5, .5]$ be a bet to gain or lose a fixed amount $h\in(0, w)$ with equal chances. The risk premium, expressed as a fraction θ of the gamble h, is implicitly defined by the equation:

$$0.5U(w+h)+0.5U(w-h) = U(w-\theta h) \tag{3.11}$$

where $U(\cdot)$ is an increasing von Neumann-Morgenstern utility function and where $0 \leq \theta \leq 1$.

If constant absolute risk aversion (CARA) is assumed and the negative exponential utility function is used, θ was derived as:

$$\theta = \frac{\ln\left(0.5\left(e^{-r_a(x)h} + e^{r_a(x)h}\right)\right)}{r_a(x)h} \tag{3.12}$$

What is interesting to note about the relationship is that θ is calculated independently of wealth. Furthermore, the only parameters that effect θ are the size of the gamble and the level of absolute risk aversion. Given the relationship, information about θ and the gamble size is sufficient to calculate $r_a(x)$.

Babcock *et al.* (1993) convincingly argued that θ portrays more of the degree of risk aversion than $r_a(x)$ itself. For example, if the gamble size is \$10 000, a person with a risk-aversion coefficient of 0.0001 would have a θ of 43% of the gamble. A θ of this amount suggests a relatively high level of risk aversion. The same coefficient with a gamble of \$1 000 implies a θ of only 5%, suggesting a relatively low level of risk aversion. Therefore alternatives should be compared such that θ is equal between alternatives. In cases where the outcomes are more than one, Babcock *et al.* (1993) suggests that the standard deviation be used as a gamble size indicator.

3.3.3 STANDARD DEVIATION SCALING

The motivation of the use of σ to standardise $r_a(x)$ between alternatives is based on the assumption of CARA. CARA implies that risk preferences are not changed by adding/subtracting a constant to all payoffs. By implication CARA imposes the requirement that alternative distributions may be compared with the same $r_a(x)$ if the σ of these distributions are equal. Any distribution may be transformed such that the resulting σ after the transformation is equal to one. This result is achieved if each payoff is divided by the σ of the distribution of payoffs. Once the distributions are standardised such that σ of all the distributions are equal, the same risk aversion coefficient may be used with a utility function that exhibits CARA to represent risk aversion for the transformed data series. The relationship between $r_a(x)$ and a standardised measure of risk aversion $(r_s(x^s))$ is derived below.

Given a transformation of x such that:

$$x^s = x / \sigma_x \tag{3.13}$$

$$\therefore\ x = x^s \sigma_x \tag{3.14}$$

where x is the original data with standard deviation of σ_x and x^s is the standardised data with a standard deviation of $\sigma_x^s = 1$.

Assuming a negative exponential function $U(x)$ and $U(x^s)$ are given by:

$$U(x) = -e^{-r_a(x)x} \text{ and } U(x^s) = -e^{-r_s(x^s)x^s} \tag{3.15}$$

If utility is assumed constant whether the outcome variable is x or x^s gives:

$$r_a(x)x = r_s(x^s)x^s \tag{3.16}$$

since e is a constant. Substituting (3.13) into (3.16) gives the relationship between $r_s(x^s)$ and $r_a(x)$:

$$r_a(x)x = r_s(x^s)x / \sigma_x \tag{3.17}$$

$$\therefore\ r_s(x^s) = r_a(x)\sigma_x \quad \text{and} \tag{3.18}$$

$$\therefore\ r_a(x) = r_s(x^s)/\sigma_x \tag{3.19}$$

Equation (3.18) indicates that $r_s(x^s)$ is a function of $r_a(x)$ and the size of the gamble given by σ_x. Thus, the value of $r_s(x^s)$ can be calculated exactly for any $r_a(x)$ value with Equation (3.18) without altering utility. One should observe that if σ is used to approximate the gamble size in Equation (3.12), $r_s(x^s)$ can be used as a substitute for the term $r_a(x)h$ to calculate θ. Thus, there is a direct relationship between θ and $r_s(x^s)$.

3.3.4 RANGE SCALING

Nieuwoudt and Hoag (1993) suggested that the $r_a(x)$ be standardised by expressing the data as a percentage of the range. After Ferrer (1999) applied the approach it became the standard procedure at the University of KwaZulu Natal to represent risk aversion. Recent applications of the procedure include Gillet, Nieuwoudt and Backeberg (2005) and Nieuwoudt, Gillet and Backeberg (2005). The relationship between the range based risk aversion coefficient $(r_\lambda(x^r))$ and $r_a(x)$ is derived below.

Given a transformation of x such that:

$$x^r = (x - x_{\min})/(x_{\max} - x_{\min}) \tag{3.20}$$

$$\therefore \quad x = x_{\min} + x^r (x_{\max} - x_{\min}) \tag{3.21}$$

where x is the original data, x^r the transformed data, x_{min} and x_{max} respectively the minimum and the maximum values of x. In essence, the transformation produces a distribution of values between zero and one.

Assuming a negative exponential function $U(x)$ and $U(x^r)$ are given by:

$$U(x) = -e^{-r_a(x)x} \quad \text{and} \quad U(x^r) = -e^{-r_\lambda(x^r)x^r} \tag{3.22}$$

If the utility is assumed constant whether the outcome variable is x or x^r gives:

$$r_a(x)x = r_\lambda(x^r)x^r \tag{3.23}$$

since e is a constant. Substituting Equation (3.20) into Equation (3.23) and realising that $r_\lambda(x^r)x_{min}$ is constant yields the following relationship between $r_a(x)$ and $r_\lambda(x^r)$:

$$r_\lambda(x^r) = r_a(x)(x_{\max} - x_{\min})$$
$$\therefore r_a(x) = r_\lambda(x^r)/(x_{\max} - x_{\min})$$

The standardisation procedure above has been used on numerous occasions to represent risk aversion estimated with certainty equivalents where the risky prospect is represented by a coin toss (Ferrer, 1999; Gillet *et al.*, 2005; Nieuwoudt, *et al.*, 2005). Important to note is that for a risky prospect with only two equal likely outcomes the standard deviation is equal to half the range $(x_{max}\text{-}x_{min})$ of the risky prospect. Thus, the values of $r_\lambda(x^r)$ will be twice that of $r_s(x^s)$ and therefore a relationship exists with θ. However, this relationship is incidental since the standard deviation is altered when more outcomes are included in the risky prospect or when the probabilities are altered.

Next, it will be shown by means of a numerical example that restrictive assumption needs to be applied in order for Raskin and Cochran's (1986) first theorem to result in consistent representation of risk attitude between alternatives. Further, it is shown that scaling of $r_a(x)$ based on the dispersion of the outcomes as measured by the standard deviation results in consistent risk aversion. The results, after appropriate scaling is done, are also shown to be consistent with the argument of Babcock *et al.* (1993) that the risk premium as a fraction of the gamble size should be equal between alternatives. Range based scaling is not considered in the numerical example since the relationship between $r_\lambda(x^r)$, $r_s(x^s)$ and θ is circumstantial.

3.3.5 NUMERICAL EXAMPLE

The properties of the distributions will be discussed first; whereafter the consistent presentation of risk aversion between the alternatives will be explained. Table 3.1 shows selected statistical moments of the distributions that are compared while Figure 3.2 shows the cumulative distributions of the alternatives.

Table 3.1: Hypothetical linearly related distributions of outcome variable x

	Values of outcome variable x for alternative i			
		Linear transformations of A: $x_i=a+bx_A$		
	A	A+200 a=200 b=1	A/3 a=0 b=1/3	A/3+200 a=200 b=1/3
	100	300	33	233
	200	400	67	267
	300	500	100	300
	400	600	133	333
	500	700	167	367
Mean $(E[x_i])$	300	500	100	300
Standard deviation (σ_i)	158	158	53	53
Coefficient of variation $(CV_i=\sigma_i /E[x_i])$	53	32	53	18

Alternative A is the base case. The other alternatives were derived using linear transformations of A i.e. A/3+200 was formed by dividing alternative A by 3 and adding a constant of 200. Table 3.1 shows that adding a constant to a base scenario (A+200) does not change the dispersion of the outcomes around the mean (i.e. $\sigma_A= \sigma_{A+200}$), but does change the position on the scale. Since adding a constant does not change σ, but increases the mean, the relative variability is reduced (i.e. $CV_A > CV_{A+200}$). Dividing the base by 3 (A/3), contracts the mean and σ by the same factor. Therefore dividing or multiplying a base scenario with a factor will contract or expand these statistical moments by the same factor resulting in a constant relative dispersion around the mean (constant CV). Inspection of Figure 3.4 clearly demonstrates that a parallel shift (adding a constant) does not alter the shape and therefore the associated variability of the distribution.

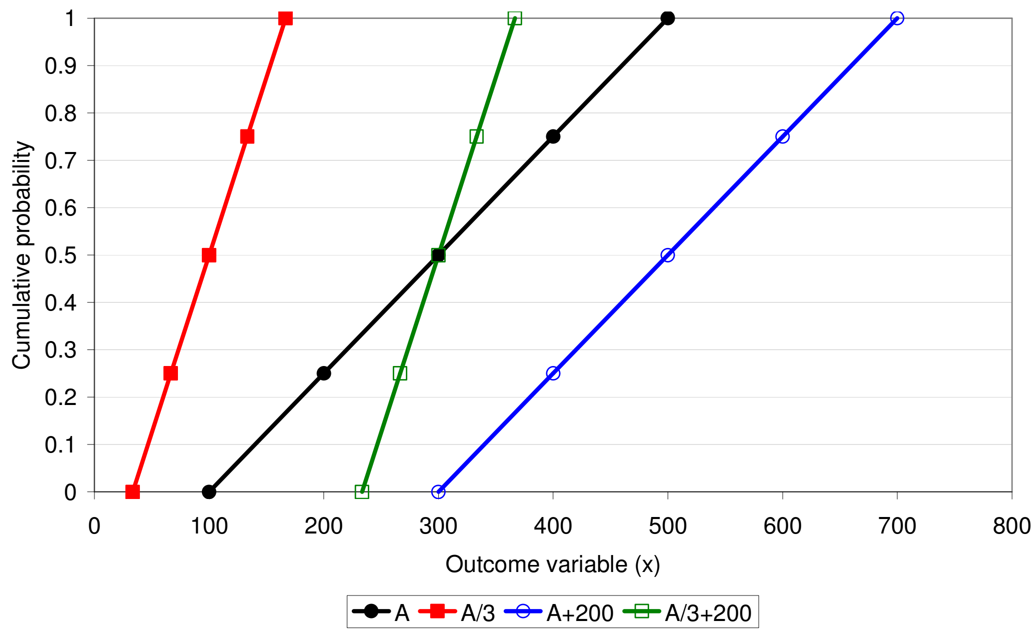

Figure 3.2: Cumulative probability distributions of hypothetical linearly related distributions

Table 3.2 is used to demonstrate the effects of Raskin and Cochran's (1986) first theorem and the standard deviation scaling method presented in the previous section on relative risk aversion $(r_r(x))$, absolute risk aversion $(r_a(x))$, standard risk aversion $(r_s(x^s))$, the Pratt risk premium and θ as defined by Babcock *et al.* (1993). The different terms in Table 3.2 are defined as follows:

Arrow-Pratt absolute risk aversion: $$r_a(x) = -\frac{U''(x)}{U'(x)}$$

Relative risk aversion: $$r_r(x) = x r_a(x)$$

$$\therefore \quad r_a(x) = r_r(x)/x$$

Standard risk aversion: $$r_s(x^s) = r_a(x)\sigma$$

$$\therefore \quad r_a(x) = r_s(x^s)/\sigma$$

Certainty equivalent: $$CE(x, r_a(x)) = \ln\left\{\left(\frac{1}{n}\sum_j^n e^{-r_a(x)x_j}\right)^{\frac{-1}{r_a(x)}}\right\}$$

Pratt risk premium $$\Pi(x, r_a(x)) = E[x] - CE(x, r_a(x))$$

Risk premium as fraction of gamble: $$\theta(r_a(x), \sigma) = \frac{\ln\left(0.5\left(e^{-r_a(x)\sigma} + e^{r_a(x)\sigma}\right)\right)}{r_a(x)\sigma}$$

where x defines the outcome variable, σ the standard deviation and $E[x]$ the expected value of x. Important to note is that θ is calculated using σ and not the gamble size[5].

Table 3.2: Numerical example of the impact of alternative scaling procedures on implied risk aversion

	Alternative distributions (i)			
		Linear transformations of A: $x_i=a+bx_A$		
	A	A+200 a=200 b=1	A/3 a=0 b=1/3	A/3+200 a=200 b=1/3
	Constant $r_a(x)$			
Scaling factor = 1	1.00	1.00	1.00	1.00
$r_a(x_i) = r_a(x_A)$	0.01	0.01	0.01	0.01
$r_r(x_i) = r_a(x_i)E[x_i]$	3.00	5.00	1.00	3.00
$r_s(x^s_i) = r_a(x_i)\sigma_i$	1.58	1.58	0.53	0.53
$CE_i = U_i^{-1}$ *	215.75	415.75	89.15	289.15
$\Pi_i = E[x_i]-CE_i$	84.25	84.25	10.85	10.85
θ_i	0.59	0.59	0.25	0.25
	Mean scaling of $r_a(x)$: $E[x_A]/E[x_i]$			
Scaling factor = $E[x_A]/E[x_i]$	1.00	0.6	3	1.00
$r_a(x_i) = r_a(x_A)\cdot E[x_A]/E[x_i]$	0.01	0.006	0.03	0.01
$r_r(x_i) = r_a(x_i)E[x_i]$	3.00	3.00	3.00	3.00
$r_s(x^s_i) = r_a(x_i)\sigma_i$	1.58	0.95	1.58	0.53
$CE_i = U_i^{-1}$ *	215.75	444.11	71.92	289.15
$\Pi_i = E[x_i]-CE_i$	84.25	55.89	28.08	10.85
θ_i	0.59	0.42	0.59	0.25
	Standard deviation scaling of $r_a(x) : \sigma_A / \sigma_i$			
Scaling factor = : σ_A / σ_i	1.00	1.00	3	3
$r_a(x_i) = r_a(x_A)\cdot \sigma_A / \sigma_i$	0.01	0.01	0.03	0.03
$r_r(x_i) = r_a(x_i)E[x_i]$	3.00	5.00	3.00	9.00
$r_s(x^s_i) = r_a(x_i)\sigma_i$	1.58	1.58	1.58	1.58
$CE_i = U_i^{-1}$ *	215.75	415.75	71.92	271.92
$\Pi_i = E[x_i]-CE_i$	84.25	84.25	28.08	28.08
θ_i	0.59	0.59	0.59	0.59
Mean $(E[x_i])$	300	500	100	300
Standard deviation (σ_i)	158	158	53	53
Coefficient of variation $(CV_i=\sigma_i/E[x_i])$	53	32	53	18

** Negative exponential utility function*

5 Babcock *et al.* (1993) indicated that the gamble size can be approximated with the standard deviation.

In Table 3.2, A presents the base case for which the $r_a(x_A)$=0.01 is known and appropriate values of $r_r(x)$, $r_s(x^s)$, Π and θ are derived. No scaling was done in the first part to demonstrate the impact of assuming a value of 0.01 for all the alternatives. Raskin and Cochran's (1986) first theorem is used in the second part to scale the assumed value of $r_a(x_A)$=0.01 for the other alternatives. The last part demonstrates the use of σ rather than expected values to scale risk aversion appropriately to be consistent.

Table 3.2 shows that assuming the same $r_a(x)$=0.01 for all the alternatives implies different levels of $r_r(x)$ and $r_s(x^s)$. Raskin and Cochran's (1986) second theorem predicts that alternatives A and A+200 as well as A/3 and A/3+200 may be compared with the same $r_a(x)$ respectively. The standard risk aversion measure, $r_s(x^s)$, and θ also indicated that these pairs of distributions may be compared. These predictions make sense logically since adding a constant to all payoffs does not alter the dispersion (σ) around the mean. Rather it acts as a scale shifter and therefore the difference between the CE is equal to 200. Furthermore, because the dispersion was not changed Π between these pairs are constant (i.e. $\Pi_A = \Pi_{A+200}$). Thus, the higher CE of A+200 was not a result of risk aversion as such, but the expected value of A+200 was higher. However, $r_s(x^s)$ further indicates that $r_a(x)$=0.01 implies a less risk averse attitude for A/3 and A/3+200, because their $r_s(x^s)$ values are 0.53 compared to 1.58 for A and A+200. A property of the negative exponential function is that of CARA. CARA implies that adding/subtracting a constant to all payoffs does not alter risk preferences (Hardaker *et al.*, 2004) because the variability around the mean (σ) is not changed. Thus, the fact that there are two pairs of distributions that might be compared is in line with the assumption of CARA. Lower risk averseness towards A/3 and A/3+200 makes perfect sense since the dispersion of these alternatives is three times less compared to the other distributions. Thus, in order to compare A/3 and A/3+200 relatively to the other distributions scaling of these two distributions $r_a(x)$ is necessary.

When Raskin and Cochran's (1986) Theorem 1 is applied $r_a(x^s_{A/3})$ is scaled upwards by a factor of 3 to 0.03, which resulted in a $r_s(x^s_{A/3})$ of 1.58, which is the same as A. Furthermore, $CE_{A/3}$ and $\Pi_{A/3}$ is 3 times smaller than that of A. Although the $\Pi_{A/3}$ is 3 times smaller than that of A the value of θ is equal to 0.59, which is an indication that these two distributions are consistent in terms of their implied risk aversion. Given the assumption of CARA, this result is anticipated since A/3 was constructed through division of A by 3. Although Theorem 1 was able to scale A/3 to be comparable with A it failed to consistently scale all alternatives to be compared. The reason why Theorem 1 resulted in the correct scaling of A/3 is because the $CV_A = CV_{A/3}$. It should be recalled that division by 3 contracted both the mean and σ by a factor of 3; therefore the correct scaling will be done whether it is done by mean or σ scaling. Thus, when researchers apply Theorem 1, they implicitly assume that relative risk aversion between alternatives is the same.

The last section of the table indicates that appropriate scaling factors are obtained when σ scaling is done. In order to compare the distributions consistently at the same levels of standard risk aversion, CE should be calculated using $r_a(x)$ of 0.01 for alternatives A and A+200 whereas a value of 0.03 should apply to A/3 and A/3+200. In absolute terms Π seems to be a good measure of risk aversion if $r_a(x)$ is appropriately scaled, since these values for A/3 and A/3+200 are 3 times smaller than that of A and A+200, which resulted in the same θ across the alternatives.

The numerical example confirms the conclusion made in Section 3.2 that $r_a(x)$ is affected by the dispersion of the risky prospect. The property of the negative exponential utility function that risk aversion is unchanged when a constant is added to all payoffs requires that all the distributions that are compared have to have the same dispersion of outcomes (i.e. σ) in order to use the same $r_a(x)$ to represent the level of risk aversion. When the distributions differ in terms of their dispersion, quite different risk aversion behaviour may be implied if a constant absolute risk aversion coefficient is used to compare alternatives. The conclusion is that assuming a constant absolute risk aversion coefficient does not necessarily imply constant risk aversion. Scaling of $r_a(x)$ values using the ratio of standard deviations is shown to produce a more consistent presentation of risk aversion.

3.4 PLAUSIBLE ABSOLUTE RISK AVERSION RANGES

In the previous section, it is argued that $r_a(x)$ needs to be standardised to present risk aversion consistently between alternatives. However, no indication is given with respect to plausible risk aversion ranges that can be used to discriminate between alternatives. In this section, possible options are reviewed.

3.4.1 APPLICATIONS OF CONSTANT RISK PREMIUMS AS A FRACTION OF THE GAMBLE SIZE

In Section 3.3.2 it is argued that the risk premium as a fraction of the gamble size (θ) be equal between alternatives to compare the alternatives with the same level of risk aversion. Furthermore, it was also shown that such a decision rule will yield the same $r_s(x^s)$. Knowledge of possible ranges of θ will therefore also provide possible ranges of $r_s(x^s)$.

Babcock *et al.* (1993) reason on theoretical grounds that for an approximate two state gamble where σ is used as size indicator, values of θ should range between 2%-98%. However, values of θ greater than 85% imply severe risk aversion since such a decision-maker will require a probability of winning in excess of 99%. A θ = 85% translates into a $r_s(x^s)$=4.65. Next the values of θ which are used by some researchers who have implemented the procedure of Babcock *et al.* (1993) are reviewed.

Hart and Babcock (2001) used θ values of 10%, 25% and 50% to rank risk management strategies combining crop insurance products and marketing strategies in Iowa. Vedenov and Barnett (2004) derived values of $r_a(x)$ using values of θ equal to 5% and 10% to determine the risk efficiency of alternative weather derivatives as primary crop insurance instruments. The impact of risk on the adoption of BT corn is demonstrated by Hurley, Mitchell and Rice (2004) using a value of θ = 20%. Mitchell, Gray and Steffey (2004) used a negative exponential utility function to demonstrate the impact of two alternative models to estimate pest damage functions for the western corn rootworm variant in Illinois on the implied certainty equivalents of risk averse farmers. The $r_a(x)$ values for the exponential utility function were derived using θ equal to 20% and 40%. An interesting application of the procedure is that of Fuasti and Gillespie (2006) who used the procedure to establish appropriate ranges of risk aversion for use with the interval approach to elicit risk preferences of agricultural producers. An upper value was derived based on an θ = 66%, which translates to a $r_s(x^s)$=2.

All the studies that were reviewed, with the exception of Fuasti and Gillespie (2006), used values of θ below 50% with an associated value for $r_s(x^s)$=1.25.

3.4.2 ELICITED

Given the procedure to standardise $r_a(x)$ described above, one possible option is to use risk aversion ranges elicited in other studies. Previously risk preferences of South African agricultural producers were elicited by various researchers in the past (Ferrer, 1999; Botes, 1994; Meiring and Oosthuizen, 1993; and Lombard and Kassier, 1990). Botes (1994) used the interval approach to elicit the risk preferences for income and wealth in the Winterton irrigation area. Although a scale adjustment was made between income and wealth, no scaling was done between different levels of wealth and different levels of income and the elicited values may be biased[6]. Only coefficients elicited at the R60 000 income level and the R600 000 wealth level are evaluated here since these values were appropriately scaled since the coefficients of variation of the two alternatives were the same. The elicited absolute risk aversion ranges were converted to $r_s(x^s)$ to enable comparison between studies. All the respondents' $r_s(x^s)$ values with respect to

[6] A scale factor of 10 was used to adjust $r_a(x)$ appropriately between income of R60 000 (σ=9000) and wealth of R600 000 (σ =90 000). However, the same scaled $r_a(x)$ values were used to elicit for instance $r_a(x)$ at a wealth level of R950 000 (σ =142 500). An appropriate scale factor is 15.83 for wealth level R950 000 and therefore the elicited $r_a(x)$ values will be biased.

income and wealth fell within the range -2.7 to 15 with corresponding θ value of 95% on the upper level of risk aversion. A $r_s(x^s)$ of 15 may seem extreme. It should be noted, however, that the interval ranged between 0.9 and 15.3, which is very wide. The next largest upper bound on a risk aversion interval was 2.7 with a corresponding value for θ of 74%. Ferrer (1999) used a certainty equivalent approach to elicit risk preferences of sugarcane farmers in KwaZulu Natal. The results of the analyses indicated that $r_s(x^s)$ varied from a minimum of –1.5 to a maximum of 3.5 over all the gambles[7] with a corresponding θ = 80% for the upper bound on risk aversion. More than 80% of the estimated $r_s(x^s)$ values ranged between -0.9 to 1.65 with most outliers being extremely risk averse (Ferrer, 1999). A $r_s(x^s)$=1.65 translates into a θ = 60%.

3.4.3 APPLIED MOTAD STUDIES

Before evaluating the ranges of risk aversion parameters in applied MOTAD studies, it is important to understand the link between MOTAD and mean-variance (EV) quadratic programming model specifications. Freund (1956) assumed a negative-exponential utility function of the form $U(Y)=1-e^{-ay}$ and showed that maximisation of:

$$\mu - a/2\left(\sigma^2\right) \tag{3.24}$$

is equivalent to maximising $U(Y)$ if Y is normally distributed with mean μ and variance σ^2. In essence, the model maximises the certainty equivalent. The decision-maker's attitude towards risk is given by a, which has to be greater than zero if the decision-maker is averse to risk. The negative exponential utility function exhibits CARA and a is the Arrow-Pratt absolute risk aversion coefficient.

Hazell (1971) developed a linear alternative (MOTAD) to the quadratic programming problem presented above. An estimate of the standard error of the optimal solution can be obtained if the mean absolute deviation of the MOTAD model is multiplied by Fisher's constant. Boisvert and McCarl (1990) present a specification of the MOTAD model as:

$$\max \quad \sum_{j=1}^{n} \overline{C}_j X_j \ - \alpha\sigma \tag{3.25}$$

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7 Ferrer (1999) used the difference between the minimum and maximum to standardise $r_a(x)$, which resulted in values that ranged between -3 and +7. These values were converted to be consistent with the proposed standardisation procedure by using the standard deviations of the gambles to scale $r_a(x)$.

where the summation term calculates the expected value, α represents a risk aversion parameter and σ the approximation of the standard error. Given the link between mean absolute deviations of the MOTAD specification and the standard error holds McCarl and Bessler (1989) derived a link between the MOTAD and the mean-variance quadratic programming problem.

Following Boisvert and McCarl (1990) the link may be developed as follows:

MOTAD

$$\begin{aligned} &Max \quad CX - \alpha\,\sigma \\ &St \\ &\qquad AX \le b \\ &\qquad\; X \ge 0 \end{aligned}$$

EV

$$\begin{aligned} &CX - 0.5 r_a(x)\,\sigma^2 \\ &St \\ &\qquad AX \le b \\ &\qquad\; X \ge 0 \end{aligned}$$

The Kuhn-Tucker conditions with respect to X of these two models are:

MOTAD

$$\begin{aligned} &C - \alpha\frac{\partial \sigma}{\partial X} - uA \le 0 \\ &\left(C - \alpha\frac{\partial \sigma}{\partial X} - uA\right)X = 0 \\ &X \ge 0 \end{aligned}$$

EV

$$\begin{aligned} &C - 2\cdot 0.5 r_a(x)\sigma\frac{\partial \sigma}{\partial X} - uA \le 0 \\ &\left(C - 2\cdot 0.5 r_a(x)\sigma\frac{\partial \sigma}{\partial X} - uA\right)X = 0 \\ &X \ge 0 \end{aligned}$$

For these two models solutions to be identical in terms of X and u, then

$$\alpha = 2\cdot 0.5\, r_a(x)\sigma \tag{3.26}$$

$$\alpha = r_a(x)\sigma \tag{3.27}$$

Equation (3.27) shows that the risk aversion parameter of the MOTAD model is equivalent to the $r_s(x^s)$ risk aversion parameter derived from the EV programming model if the risk aversion parameters are scaled to produce model solutions to be identical in terms of X and u[8].

[8] The relationship between the risk aversion parameters of the MOTAD and EV models presented in Equation (3.27) is different from the relationship presented by Boisvert and McCarl (1990) because their specification treats 0.5$r_a(x)$ as the E-V risk aversion parameter.

McCarl and Bessler (1989) derived an upper bound on $r_a(x)$ using estimates of α in applied MOTAD studies. More specifically the bound was established by relating a positive Pratt risk premium to the MOTAD risk premium as:

$$\alpha\sigma = 0.5\, r_a(x)\sigma^2 \tag{3.28}$$

$$\alpha\ = 0.5\, r_a(x)\sigma \tag{3.29}$$

$$2\alpha = r_a(x)\sigma \tag{3.30}$$

Given a maximum of 2.5 usually reported as the upper bound on α in applied MOTAD studies, McCarl and Bessler (1989) derived an upper bound on $r_a(x)$ of $5/\sigma$ or equivalently $r_s(x^s)$=5. Important to note is that the relationship only holds *ex post* since it is derived by equating the risk premiums of the two models and is not derived from the Kuhn-Tucker conditions necessary to achieve optimality. Thus, it is argued that Equation (3.27) specifies the correct relationship between the risk aversion parameters of the two models since the solutions will be identical in terms of X and u. The objective functions will however, be different. Substituting Equation (3.27) into Equation (3.28) it is easy to show that the estimated risk premium of the MOTAD model will be twice that of the EV model specification even though the solutions will be identical in terms of X and u.

McCarl and Bessler (1989) state that α=2.5 are typically reported as the maximum value in applied MOTAD studies. Therefore $r_s(x^s)$=2.5 according to the reasoning above. Recently Conradie (2002) compared the observed crop mixes of 16 different farm types to those simulated with MOTAD in the Fish-Sundays irrigation scheme in South Africa. Reported α values varied from 0.25 to 5 with only two farms having values greater than α=2.5 or equivalently θ = 73%.

3.4.4 DISCUSSION

The review of the range of risk aversion levels used in literature revealed widely varying values of $r_s(x^s)$. Researchers whose choice of $r_a(x)$ was based on constant θ tend to use $r_a(x)$ values that corresponds to relatively low values of $r_s(x^s)$ (<1.25) when compared to values of $r_s(x^s)$ (typically < 2.5) used in applied MOTAD studies. Cognisance should be taken of the fact that the values of $r_s(x^s)$ derived from applied MOTAD studies may be biased if the model specification is poor or data errors occur. The $r_s(x^s)$ values elicited by Ferrer (1999) also suggest relatively low levels of $r_s(x^s)$ if one considers that the right tails of the cumulative distributions of the elicited risk aversion levels become fairly flat for $r_s(x^s)$>1.5. The conclusion is that an upper value of $r_s(x^s)$=2.5 is more than sufficient even to correspond to severely risk averse decision-makers.

3.5 CONCLUSIONS

Recent trends in stochastic efficiency analyses use certainty equivalents to discriminate between risky alternatives while assuming a specific utility function. Typically, the same absolute risk aversion coefficient is used to discriminate between multiple alternatives even though the variability of the distributions may differ substantially. Various researchers have recognised that the dispersion of the risky prospect will impact on the level of absolute risk aversion. Applying the same coefficient, researchers are failing to recognise that the Arrow-Pratt absolute risk aversion measure is a function, not a constant. Therefore, some form of scaling of Arrow-Pratt absolute risk aversion coefficients is necessary before one may apply them. The notion of scaling risk aversion coefficients is not new.

Evaluation of Raskin and Cochran's (1986) research indicated that a prerequisite for using their first theorem to scale $r_a(x)$ is that all the alternatives must exhibit constant relative risk (same coefficient of variation). Cognisance should be taken that $r_a(x)$ is dependent on both the scale of the alternatives and the variability of the risky prospect. Therefore assuming same levels of $r_a(x)$ may imply vastly different levels of risk aversion when alternatives with varying levels of variability are compared relatively to each other. Previous efforts to scale $r_a(x)$ based on the dispersion of the risky prospect is lacking and a new procedure is formalised whereby $r_a(x)$ is scaled between alternatives based on the standard deviation of the risky prospects, hence it is referred to as standard risk aversion $(r_s(x^s))$. The newly formalised method is shown to be consistent with the standardisation procedure of Babcock *et al.* (1993), which scale $r_a(x)$ such that the risk premium as percentage of the size of the gamble (θ) is constant. Furthermore, $r_s(x^s)$ is also shown to be equivalent to the risk aversion parameter used in applied MOTAD studies. A plausible range for $r_s(x^s)$ to characterise risk aversion is between zero and 2.5. Thus, the magnitude of the coefficient by itself allows for easier comparison between studies.

Lastly, it is important to note that CARA was assumed in this section. Thus, the impact of wealth on risk aversion was ignored. Recent research by Meyer and Meyer (2005) argues in favour of using relative risk aversion as a measure for comparison.

CHAPTER 4

RISK QUANTIFICATION AND CROP WATER USE OPTIMISATION MODEL DEVELOPMENT

The main objective of this chapter is to describe the procedures used to quantify gross margin variability of alternative irrigation strategies and how the information is used to optimise water use over the long-run and short-run. The chapter commence with a description of the procedures that are used to augment SAPWAT to simulate crop yield risk of alternative irrigation schedules under limited water supply conditions, taking the non-uniformity with which irrigation water is applied into account. An explanation of the stochastic budgeting procedure that is used to combine crop yields and water use data from SAPWAT with crop price variability to simulate intra- and inter-temporally correlated gross margin risk matrixes for the programming models then follows. The last part of the chapter is devoted to a description of the programming models that uses the intra- and inter-temporal risk matrixes to optimise water use over both the long-run and short-run.

4.1 DEVELOPMENT OF A PLANNING MODEL FOR SIMULATING IRRIGATION STRATEGIES UNDER LIMITED WATER SUPPLY CONDITIONS

SWB (Annandale *et al*., 1999) and SAPWAT (Crosby and Crosby, 1999) are two models that are locally available to estimate crop water requirements of a variety of crops in South Africa. SWB is a very sophisticated crop growth simulation model that was specifically developed to improve real time irrigation scheduling. The development of the model was in line with the old paradigm of determining when to irrigate and how much to irrigate to sustain maximum crop yield. The adoption of the model was relatively slow and the user base is still very small. Two of the reasons why irrigation managers do not adopt the model are that until recently it was cumbersome to setup the model and to simulate predefined irrigation schedules (Jordaan, Grové, Steyn, Benadé, Annandale and Pott, 2006). Thus, the usefulness of the model to evaluate the relative profitability of alternative irrigation schedules was hampered.[9]

Although SWB is a state-of-the-art crop growth model SAPWAT was chosen for this research. Several factors motivated the choice of SAPWAT in favour of SWB. Firstly, SAPWAT is founded on internationally accepted principles of estimating crop water requirements (FAO-56, Allan, Pereira, Raes and Smith, 1998) and is regarded as the standard in estimating crop water

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[9] The model was recently improved to simulate predefined irrigation strategies. (Pott , Benadé, Van Heerden, Grové, Annandale and Steyn, 2007)

requirements in South Africa. As a result, the user base is large. Furthermore, government also see it as a tool to benchmark lawful water use under the National Water Act (Act 56 of 1998). Thus, any improvement in SAPWAT will benefit a large community. Secondly, it is easy to use the model because it is supplied with weather data and a unique set of crop parameters that is specific to South Africa. Thirdly, the model has a function to write the water budget equations to Excel©. From a research perspective, this is of benefit since the researcher does not need to have any experience in computer programming to evaluate the code in order to improve the model.

For the above reasons SAPWAT was chosen for this research. Next, the calculation of crop water requirements with SAPWAT will be discussed. A description of the improvements that were made to simulate the impact of different irrigation schedules on crop yield under water limiting conditions will then follow.

4.1.1 SAPWAT WATER BUDGET CALCULATIONS[10]

SAPWAT uses the basic methodology proposed in FAO-56 (Allan *et al*., 1998) to calculate crop water requirements based on a reference evapotranspiration rate, ETo. Given a good estimate of ETo is obtained through the use of appropriate weather data, the maximum evapotranspiration rate (ETm) under conditions of no water stress is determined by:

$$ETm = kcETo \tag{4.1}$$

where kc is the crop coefficient, which relates the water use of a specific crop to ETo. Scientists are continuously updating the values of kc incorporated in SAPWAT to ensure that calculated crop water requirements are relevant for the cultivars that are grown in a specific region.

The soil water balance in any time period determines the actual level of evapotranspiration (ETa). SAPWAT utilises a simple cascading water budget routine that distinguishes between water in the root zone and below the root zone. The total available moisture (TAM) in the soil that potentially can be used by the crop is a function of soil water holding capacity of a specific soil (AWC) and the rooting depth (RD) of the crop. However, only a portion of TAM is readily available for crop consumption (RAW). RAW is determined by the soil characteristics and the type of crop that is planted. The rate at which a crop consumes water is reduced from its potential level when soil moisture deficits (SMD) are greater than RAM and therefore ETa fall below ETm. Given these conditions ETa is determined by:

[10] The calculations in this section conform to the formulas written to Excel ©. The calculation procedures in SAPWAT may thus be different. No literature is available describing the actual procedures used in SAPWAT.

$$ETa_t = \begin{vmatrix} ETm_t & \text{if } SMD_t \leq RAM_t \\ ETm_t\left(\dfrac{RWC_t}{TAM_t - RAM_t}\right) & \text{if } SMD_t > RAM_t \end{vmatrix} \tag{4.2}$$

where SMD defines the difference between the water holding capacity in the root zone $(RWCAP)$ and the actual water content in the root zone (RWC). RWC is a function of ETa, net irrigation (IR), rainfall (R) and water that drain below the root zone (BR) and any additions to RWC due to root growth (TR). The following equation indicates that BR is not explicitly accounted for in the calculation of RWC but indirectly because it is capped to a maximum of TAM:

$$RWC_t = \min\begin{vmatrix} RWC_{t-1} - ETa_{t-1} + R_{t-1} + IR_{t-1} + TR_t \\ RWCAP_t \end{vmatrix} \tag{4.3}$$

The water content of water below the root zone $(BRWC)$ is determined by:

$$BRWC_t = \min\begin{vmatrix} BRWC_{t-1} + BR_t - TR_t \\ (RD_{\max} - RD_t)AWC \end{vmatrix} \tag{4.4}$$

where BR and TR are calculated as:

$$BR_t = \max\begin{vmatrix} RWC_{t-1} - ETa_{t-1} + IR_{t-1} + R_{t-1} + TR_t - TAM_t \\ 0 \end{vmatrix} \tag{4.5}$$

$$TR_t = \begin{vmatrix} (RD_{t-1} - RD_t)/(RD_{\max} - RD_{t-1})BRWC_{t-1} & \text{if } RD_{t-1} \neq RD_{\max} \\ 0 \quad \text{if } RD_{t-1} = RD_{\max} \end{vmatrix} \tag{4.6}$$

The last equation indicates that TR is directly attributed to root growth and the availability of water below the root zone $(BRWC)$.

To initialise the whole water budget the user has to specify the soil water holding capacity (mm/m) and the water content in percentage terms. RWC and $BRWC$ are then adjusted accordingly to give the same water content in percentage terms.

Gross irrigation requirements are calculated in SAPWAT by adjusting IR by an efficiency factor. The efficiency factor is determined by your yield expectation and the efficiency of the irrigation system. Typically, centre pivot irrigation systems have an efficiency of 85%, which means that

15% of the water is lost due to wind drift and evaporation losses. Due to the non-uniformity of water applications SAPWAT further adjusts the irrigation system efficiency based on a specific crop yield expectation. The user may choose between maximum or normal crop yield expectations. In essence, these two estimates are point estimates of the function used in literature to convert net irrigation to gross irrigation requirements based on the uniformity with which water is applied. Once a specific yield expectation and the irrigation system application efficiency are chosen, the efficiency factor is fixed. For example, an irrigation system efficiency of 85% combined with a normal yield expectation results in an overall efficiency factor of 72.25%. Thus, SAPWAT assumes 27.75% of all the applied water does not enter the water budget. Thus, SAPWAT is unable to quantify increasing application efficiencies associated with deficit irrigation. Efficiency, however, is a function of the irrigation timing, amount of the application and the status of the soil water content (Lecler, 2004).

In the next section the procedures used to incorporate a dynamic relationship between applied water, efficiency and crop yield based on the uniformity with which irrigation water is applied are discussed.

4.1.2 SIMULATING THE IMPACT OF IRRIGATION STRATEGY ON CROP YIELD

The literature review indicated the importance of modelling the non-linear relationship between applied water and crop yield. Furthermore, irrigators have control over the amount of water that is applied to sustain crop water requirements. The approach used is therefore to model the impact of water applications on crop yield while endogenously taking inefficiencies resulting from non-uniform water applications into account. As a result, the irrigation efficiency is a function of the timing of water applications, amount of water applied and the soil water status at the time of the irrigation.

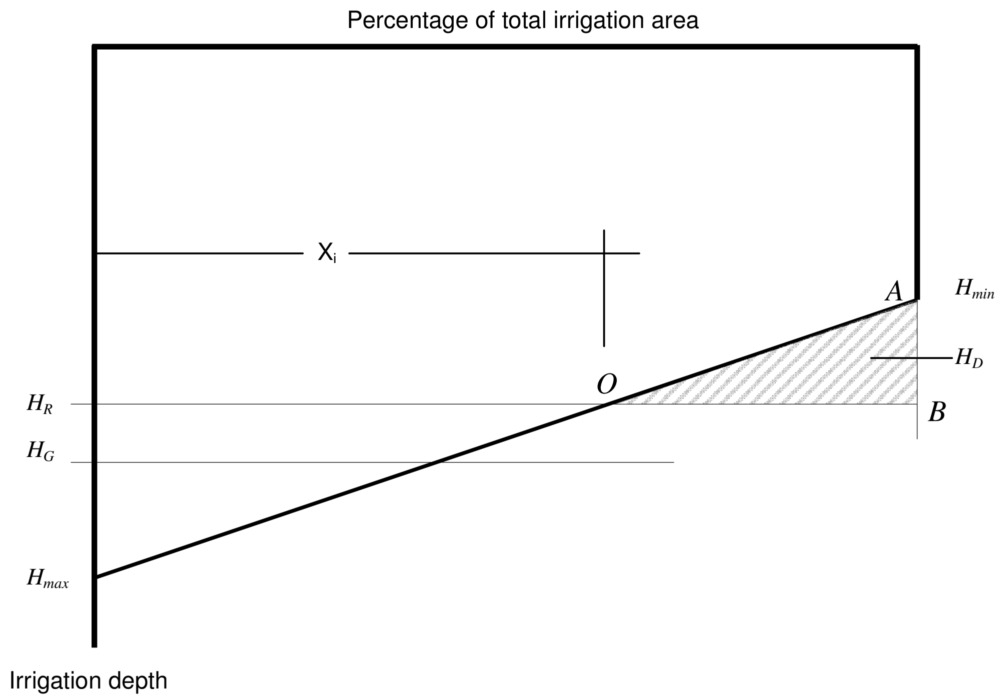
Next, the procedure used to determine the efficiency of water applications endogenously is discussed whereafter the procedure used to calculate crop yield is explained.

4.1.2.1 Incorporating coefficient of uniformity

The relationship between the uniformity with which water is applied and water deficits in the soil is discussed by Li (1998). Figure 4.1 will be used to describe the procedures in more detail.

Let us assume the irrigator needs to compensate for a soil water depletion or required depth (H_R). In normal practice, one will apply gross irrigation depth (H_G). Due to non-uniform applications some portion of the field will receive more water and some less with an average deficit of H_D. Assuming a uniform distribution, an irrigation system will apply water uniformly between a minimum (H_{min}) and maximum (H_{max}) level. The result is that triangle H_ROH_{max}

represents areas where too much water is applied and triangle BOH_{min} or H_D areas where too little water is applied. A deficit coefficient (C_D), which gives the percentage deficit is defined as: $C_D = H_D / H_R$.



Source: Li (1998)

Figure 4.1: Probability distribution of irrigation depths assuming a uniform distribution.

The relationship between the coefficient of uniformity (CU), H_{max} and H_G is given by:

$$CU = 1 - \frac{H_{\max} - H_G}{2H_G} \tag{4.7}$$

While the average amount of water applied is calculated as:

$$H_G = \frac{1}{2}\left(H_{\max} + H_{\min}\right) \tag{4.8}$$

For known values of CU and H_G it is possible to calculate H_{max} and H_{min} through manipulation of Equations (4.7) and (4.8). The relationship between C_D and CU, H_r and H_D is given by:

$$C_D = \begin{cases} \dfrac{(1-2CU + H_R / H_G)[1 - H_G / H_R (2CU-1)]}{8-8CU} & if \quad H_{\max} \geq H_R \\ 1 - \dfrac{H_G}{H_R} \quad if \quad H_{\max} < H_R \end{cases} \tag{4.9}$$

Equation (4.9) indicates that the relationship between H_G and C_D is linear if H_{max} is less than H_R and nonlinear if H_{max} is greater than H_R.

The relationship described above was explicitly incorporated in the water budget calculations by modelling three different water budgets simultaneously in Excel©. The main water budget represents areas that received the required amount of water whereas the lower and upper water budgets keep track of the areas that are respectively under and over irrigated. Lecler (2004) used a similar approach to incorporate non-uniformity into the ZIMsched 2.0 cane growth simulation model. The amount of water added or subtracted from the average water application to represent the water application of the upper and lower water budget respectively is calculated as: $(1/3*(H_{max} - H_{min}))/2$. Important to note is that the average water budget (main) is used to determine the crop water requirements and that the other two budgets are simulated to capture the impacts of non-uniform water applications on crop yields.

4.1.2.2 *Crop yield estimation*

Crop yield was calculated with the use of crop yield response factors (ky), which relate relative yield decrease $(1\text{-}Ya/Ym)$ to relative evapotranspiration deficit $(1\text{-}ETa/ETm)$. More specifically the Stewart multiplicative (De Jager, 1994) relative evapotranspiration formula was used to calculate crop yield taking the effect of water deficits in different crop growth stages into account:

$$Ya = Ym \times \prod_{g=1}^{4} \left(1 - ky_g \left(1 - \left(\frac{\sum_{t=1}^{m} ETa_{tg}}{\sum_{t=1}^{m} ETm_{tg}} \right) \right) \right) \tag{4.10}$$

with:

Y_a	actual crop yield (ton/ha)
Y_m	maximum potential crop yield (ton/ha)
ET_a	actual evapotranspiration (mm.ha)
ET_m	maximum potential evapotranspiration (mm.ha)
ky	crop yield response factor

g growth stages

t length of crop growth stage in days

The latter part of Equation (4.10) calculates a yield indice that is used to adjust the expected yield under conditions of no water stress. Crop yield indices were estimated for each of the three district water budgets and the average were used for further analyses. The ky values used to determine the responsiveness of a crop to water deficits were taken from Doorenbos and Kassam (1979).

4.1.3 MODEL APPLICATION

The original version of SAPWAT (Crosby and Crosby, 1999) was used in collaboration with PICWAT consultancy (Van Heerden, 2001) to determine crop water requirements of the predominant crops grown under flood irrigation. During the same time, the necessary parameters for crops grown under centre pivot irrigation were determined. Appropriate values for ET_o, kc, RAM and weather variables were exported to Excel©. These values were then used to simulate the impact of alternative irrigation strategies on crop yield and irrigation water requirements of centre pivot irrigated crops within Excel. All water budget calculations were done for AWC of 140mm/m.

Economic theory requires that technically inefficient irrigation schedules should be included in the LP framework to optimise water use between multiple crops when intraseasonal water availability may be limiting. SAPWAT was used to simulate water use and crop yield indices of 1296 different irrigation schedules for three different states of nature included in the SAPWAT weather database. Full irrigation schedules were developed first for each of the states of nature by simulating an irrigation strategy whereby the irrigator is allowed to irrigate once a week to refill the soil to field capacity. The irrigation amounts were recorded. Alternative strategies where then simulated by reducing the amount applied in each of the four growth stages by different combinations of 15% deficits from the full irrigation strategy. Since a full irrigation strategy is developed for each state of nature, the implicit assumption is that the farmer was able to keep track of the soil water status in each year. Simulation of the alternatives was carried out by writing a macro in Excel© to loop over the alternatives.

4.2 QUANTIFICATION OF RISK MATRIXES FOR THE MATHEMATICAL PROGRAMMING MODELS

An intra-temporal risk matrix is needed for the short-run model that is used to evaluate deficit irrigation for maize, wheat and groundnut production under pivot irrigation when water supply is limited. An inter-temporal matrix is needed to model risk in the DLP model that is used to estimate the derived demand for irrigation water.

The risk matrixes for the short-run and long-run programming models consist of randomly generated gross margins that were generated with the use of stochastic budgeting procedures (Hardaker *et al.,* 2004). Risk enters the budgets as price and yield variability. Price variability was quantified using historical price information while the yield indices simulated with SAPWAT were combined with subjectively elicited yield distributions to quantify crop yield variability. Since a mixture of distributions are used in the stochastic budgeting procedure a general procedure for simulating multivariate probability distributions are discussed next whereafter the specific procedure used to generate the risk matrices for the short-run and long-run analyses will be discussed.

4.2.1 GENERAL PROCEDURE FOR SIMULATING MULTIVARIATE PROBABILITY DISTRIBUTIONS

The general procedure to simulate multivariate probability distributions follows the procedure developed by Richardson, Klose and Gray (2000). Risk simulation is concerned with random draws from a specified distribution that is used to characterise risk. In this research the empirical and triangle distributions are used to characterise risk.

The empirical distribution has no fixed function for the cumulative probability distribution, $F(x)$, and is characterised as discrete points on a cumulative probability function. A continuous function of $F(x)$ can be found through interpolation with the following formula:

$$F(x) = \frac{(x - x_i)}{(x_{i+1} - x_i)}(p_{i+1} - p_i) + p_i, \qquad x_i \leq x < x_{i+1} \tag{4.12}$$

where x represents the empirical values and p the cumulative probability of occurrence. The values of x and the corresponding calculated values of p should be arranged from small to large as inputs. In Equation (4.12), i and $(i+1)$ are the lower and upper bounds for which the value of x should be interpolated. Given 10 observations are used the minimum value has a 10% chance to be realise. A pseudo-minimum is used to interpolate between 10% and 0% on the cumulative probability distribution. Pseudo-minimum and maximum values are defined to be very close to the observed minimum and maximum and cause the simulated distribution to return the extreme values with approximately the same frequency they were observed in the past (Richardson *et al.*, 2000).

The inverse transformed continuous empirical function, which was used to draw stochastic variables from an empirical function, can be written as:

$$x = \frac{(u - p_i)(x_{i+1} - x_i)}{(p_{i+1} - p_i)} + x_i \quad with \quad p_i \le u < p_{i+1} \tag{4.13}$$

When risk is characterised by the triangular distribution the following equation specifies *F(x)*, which is defined completely in terms of the minimum (a), maximum (b) and the most probable value (mode) *(m)* (Hardaker, Huirne and Anderson, 1997):

$$F(x) = \begin{vmatrix} (x-a)^2 / (b-a)(m-a) & if \quad x \le m \\ 1-(b-x)^2 / (b-a)(b-m) & if \quad x > m \end{vmatrix} \tag{4.14}$$

To facilitate simulation of risk through inverse transformation, the following equations are used for the triangular distribution:

$$x = \begin{vmatrix} a + (u(b-a)(m-a))^{0.5} & if \quad 0 \le u \le (m-a)/(b-a) \\ a - ((1-u)(b-a)(b-m))^{0.5} & if \quad (m-a)/(b-a) < u \le 1 \end{vmatrix} \tag{4.15}$$

By substituting appropriately correlated uniform random values for u into Equations (4.13) and (4.15) it is possible to simulate correlated random entities from the empirical and triangle probability distributions used to characterise risk.

To generate correlated random uniform values independent standard normal deviates and the Cholesky matrix of the correlation matrix are needed. More specifically the following procedure is used to generate appropriately correlated uniformly distributed random values. First independent standard normal deviates *(ISND)* equal to the number of random iterations are generated for each of the risk parameters. In the next step, the *ISNDs* are correlated through the multiplication of the deviates with the Cholesky matrix of the correlation matrix. The following procedure is used to calculate the Cholesky matrix (Dagpunar, 1988:157):

$$\begin{aligned} c_{ii} &= \sqrt{\left(V_{ii} - \sum_{m=1}^{i-1} c_{im}^2\right)} \\ c_{ij} &= \left(V_{ij} - \sum_{m=1}^{i-1} c_{im} c_{jm}\right) / c_{ii} \quad for \quad j > i \end{aligned} \tag{4.16}$$

Through integration the correlated standard normal deviates *(CSND)* are transformed to correlated uniformly distributed values *(CUD)* that are then used in the inverse transform functions of the empirical and triangular distribution to simulate risk.

With respect to long-run analyses, the simulated values should also be correlated inter-temporally. The intra-temporally $CSND$ are adjusted in a second step by multiplying it with the Cholesky matrix of the inter-temporal correlation matrix. The inter-temporal correlation matrix is constructed by calculating a one-year lagged correlation coefficient and then assuming no higher order autocorrelation. More specifically the inter-temporal correlation matrix for variable X_{it}'s correlation to X_{it-1} is given by[11]:

$$P_{i(t,t-1)} = \begin{bmatrix} 1 & P_{(X_{it},X_{it-1})} & 0 \\ & 1 & P_{(X_{it},X_{it-1})} \\ & & 1 \end{bmatrix} \tag{4.17}$$

The general procedure described above was coded in GAMS to generate risk matrixes for the long-run and short-run programming models. The code for generating the short-run risk matrix is presented in Appendix A. Section (A) of the code generates appropriately correlated uniformly distributed random numbers. Section (B) of the code uses the generated random numbers to simulate correlated yields, prices, gross irrigation amounts and yield indices from triangle and empirical distributions and lastly Section (C) of the code generates GDX[12] files of the outputs.

4.2.2 CHARACTERISING PRICE RISK

The importance of price risk increased after the deregulation of the South African marketing boards due to increased price volatility (Jordaan *et al.,* 2006) Price data published by the National Department of Agriculture (NDA, 2005) was used to characterise price risk due to a lack of availability of farm-level data. Empirical distributions were used to characterise price variability and only data after the deregulation of the markets was considered. The data was deflated to 2005 values and the deflated time series did not exhibit any trends in the data at a p=0.05 level of significance. The historical data shows that groundnut has the highest expected value but also the highest coefficient of variation (0.34). The crop with the second highest price variability is maize (0.29) followed by lucerne (0.17) and wheat (0.09).

No published data on pecan nut prices were available and price variability of pecan nuts was approximated by means of a triangle distribution. Expert opinion was used to determine the arguments of the triangle distribution.

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[11] Richardson *et al.* (2000) uses the unsorted error terms from a time trend regression. In this case X_{it} represents the detrended values of X and therefore produces the same results.

[12] GDX (GAMS Data Exchange) file is a file that saves the values of one or more GAMS symbols.

4.2.3 CROP YIELD VARIABILITY AND APPLIED WATER

A survey was conducted by Jordaan (2006) to determine factors affecting maize producers' use of forward pricing methods. A part of the questionnaire elicited irrigation farmers subjective yield expectations for flood and centre pivot irrigation systems with the triangle distribution. On average about 15 farmers were willing to express their yield expectations for flood-irrigated maize, groundnuts, wheat and lucerne. More or less the same number of farmers provided data for the same crops under centre pivot irrigation with the exception of lucerne for which only five farmers provided data. Five farmers provided data for pecan nuts that are flood irrigated.

Visual inspection of the crop yield distributions showed that they are similar and it was decided to aggregate the information in order to characterise the farmers' subjective views by means of empirical distributions. The following procedure was used to aggregate the distributions. First 100 crop yields for a specific crop irrigation technology were simulated from each farmer's triangle distribution. The entire randomly generated crop yields were then used to present an empirical distribution of crop yields with n x 100 observations where n is the number of triangle distributions (farmers). To reduce the number of values used to characterise yield variability 100 values were randomly drawn from the distributions. The resulting empirical distributions with 100 observations were taken to represent the average yield variability of crops grown in Vaalharts under normal production conditions. These distributions were further adjusted with the simulated crop yield indices to give the crop yield variability of a specific irrigation strategy.

The modified version of SAPWAT developed for this research was used to simulate the impact of alternative irrigation strategies on water use and a crop yield indice (Equation (4.10)). Given the three states of nature included in SAPWAT a triangle distribution is used to represent the variability of relative yield changes under limited water supply conditions. The information on the crop yield indice was combined with the aggregated empirical distributions of farmers' subjective views of crop yield variability to present the crop yield variability associated with alternative irrigation strategies. Under unlimited water supply the crop yield indice approximates one under all the states of nature resulting in the same distribution of crop yields as the aggregated subjectively elicited crop yield distribution. Associated with each crop yield indice is a gross irrigation amount for each state of nature. A triangle distribution was therefore used to characterise the variability of gross amounts of water applied.

4.2.4 SIMULATING GROSS MARGIN RISK

Stochastic gross income, operating cost and resulting gross margins are simulated using the general procedure described to simulate multivariate probability distributions as well as the distributions of prices, crop yield, yield indices and gross water applications. The specific combination of the beforementioned variables are determined by the correlation structure between the variables.

More specifically the following equation is simulated to generate gross margin variability for a specific crop irrigated with a specific irrigation system:

$$GM_{yi} = \left(\tilde{P}_y - YC_y\right) \times \tilde{Y}_{yi} \times \tilde{I}_{yi} - \tilde{W}_{yi} \times \left(T + IC_i\right) - AC_{yi} \tag{4.18}$$

where

GM_{yi}	gross margin of crop y irrigated with irrigation system i (R/ha)
$\tilde{P}_y$	Empirically distributed deflated prices of crop y (R/ha)
$\tilde{Y}_{yi}$	Empirically distributed aggregate subjective yields of crop y irrigated with irrigation system i (ton/ha)
$\tilde{I}_{yi}$	Triangle distributed yield indice for crop y irrigated with irrigation system i
$\tilde{W}_{yi}$	Triangle distributed amount of applied water to crop y with irrigation system i
YC_y	Yield dependent cost for crop y (R/ha)
T	water tariff (R/m^3)
IC_i	Variable irrigation cost for irrigation system i (R/m^3)
AC_{yi}	Area dependent cost of crop y irrigated with irrigation system i (R/ha)

The correlations that were assumed between crop yield ($\tilde{Y}_{yi}$) and prices ($\tilde{P}_y$) are shown in Table 4.1.

Table 4.1: Correlations between prices and crop yields

	Price			Yield		
	Maize	Wheat	Groundnuts	Maize	Wheat	Groundnuts
Price						
Maize	1.000	0.723	-0.050	-0.239	-0.424	-0.365
Wheat		1.000	0.426	-0.203	-0.263	-0.716
Groundnuts			1.000	-0.101	-0.166	-0.174
Yield						
Maize				1.000	0.003	0.619
Wheat					1.000	-0.058
Groundnuts						1.000

Pecan nut and lucerne prices and crop yields were assumed to be uncorrelated with the other entities due to a lack of data to satisfactorily calculate the correlation matrix. A correlation of 1.0 is also assumed between $\tilde{Y}_{yi}$ and $\tilde{I}_{yi}$. The assumption is justified by the fact that random weather occurrences are the major factor affecting expected crop yields. Under favourable weather conditions high crop yields as well as a high-simulated yield indice is expected for the same irrigation strategy. A correlation of -1.0 is assumed between the simulated $\tilde{I}_{yi}$ and $\tilde{W}_{yi}$. The major reason for justifying this assumption is that a correlation of greater than -0.9 was estimated between the yield indice and applied water simulated with SAPWAT. Thus, in a favourable year when rainfall is high less water is applied with a relatively higher crop yield indice.

All the relevant costs were obtained from a local agricultural cooperative while the irrigation costs associated with centre pivot irrigation were estimated with IRRICOST (Meiring, Oosthuizen, Botha, Crous, 2002). SAPWAT was only modified to simulate the impact of alternative irrigation schedules for centre pivot irrigation. No deficit irrigation strategies were allowed for flood irrigation and information provided by Van Heerden (2001) was used to quantify irrigation requirements for flood. A yield indice of one was assumed for all flood-irrigated crops when Equation (4.18) is simulated.

Minor changes to the GAMS code presented in Appendix A were necessary to simulate the stochastic variables for the long-run optimisation model.

4.3 LONG-RUN WATER USE OPTIMISATION

The DLP model described in this section closely follows the generalised whole-farm DLP model developed by Grové (2006b). The objective of this section is, however, not to provide the reader with the code that is used to develop the correct data structure for the DLP model, rather the objective is to give a description the equations used to construct the programming matrix.

The model specification follows a disequilibrium known life type of DLP model specification (McCarl and Spreen, 2003), which is used to optimise water usage over a period of 15 years. Known life means that resources and fund flows are committed for a fixed period of time, whereas disequilibrium implies that the same activity does not need to follow the previous activity and can be replaced with another activity. The model includes five alternative crops (pecan nuts, lucerne, maize, groundnuts and wheat) and three alternative irrigation technologies (flood, small pivot, and large pivot). Risk is incorporated in the model with MOTAD. Important to note is that only full irrigation strategies were included in the model due to the increased size of the programming model over multiple years. A more detailed description of the model follows, with capital letters representing variables. All the input parameters were discounted to present values before entering the optimisation model, and therefore no discounting is shown when the model is specified.

4.3.1 OBJECTIVE FUNCTION

The objective of the model is to maximise the present value of after-tax cash surpluses at the end of the planning horizon, plus terminal values for any activity beyond the planning horizon, minus a risk aversion parameter *(α)*, multiplied by the approximate standard error *(SE)* of the solution.

$$CF_{15} + \sum_{i}^{3}\sum_{c}^{5}\sum_{it}^{15} Q_{i,c,it}\, qt_{i,c,it} + \sum_{i}^{3}\sum_{t}^{15} IR_{i,it}\, irt_{i,it} - \sum_{c}^{5}\sum_{it}^{15} P_{c,it}\, pt_{c,it} - \sum_{i}^{3}\sum_{it}^{15} IL_{i,it}\, ilt_{i,it} - \alpha SE \tag{4.25}$$

CF_{15}	cash flow in year 15
$Q_{i,c,it}$	quantity of crop c established in year it utilising irrigation system i
$qt_{i,c,it}$	terminal value associated with cropping activities established in year it
$IR_{i,it}$	investment in irrigation system i in year it
$irt_{i,it}$	terminal value associated with irrigation investment i in year it
$P_{c,it}$	production loan for financing production cost of crop c in year it
pt_{it}	terminal value associated with production loan in year it
$IL_{i,it}$	borrowed capital to finance irrigation system i in year it
$ilt_{i,it}$	terminal value associated with borrowed capital in year it
α	standardised risk aversion parameter
SE	approximate standard error

Equation (4.25) shows three distinct parts. The first part is concerned with the generation of cash flows at the end of the planning period, the second part with terminal values and the last part with risk. These parts are discussed in a little more detail below.

4.3.1.1 *Calculation and utilisation of cash surpluses*

The main purpose of this section is to describe how cash surpluses are calculated in each year as well as the factors that will determine its level.

The following two equations are used to calculate the cash surpluses in each year of the planning horizon:

$$\sum_{i}^{2}\sum_{c}^{5}\sum_{it}^{15}Q_{i,c,it}\,pi_{i,c,t,it}+B_t(1+ri)-\sum_{c}^{5}\sum_{it}^{15}P_{c,it}\,pay_{c,t,it}+\sum_{i}^{3}\sum_{it}^{15}IR_{i,it}sal_{i,t,it}-fix_t$$

$$-\sum_{i}^{3}\sum_{it}^{15}IL_{i,it}ipay_{i,t,it}-lc_t-TI_t tax=CS_t \qquad (4.26)$$

$$\sum_{i}^{3}\sum_{c}^{5}\sum_{it}^{15}Q_{i,c,it}(pi_{i,c,t,it}-pc_{i,c,t,it})-\sum_{i}^{3}\sum_{it}^{15}IR_{i,it}(dep_{i,t,it}-sal_{i,t,it})+B_t ri$$

$$-\sum_{c}^{5}\sum_{it}^{15}P_{c,it}\,payi_{c,t,it}-\sum_{i}^{3}\sum_{it}^{15}IL_{i,it}ipayi_{i,t,it}-fix_t-TT_{t-1}+TT_t=TI_t \qquad (4.27)$$

$pi_{i,c,t,it}$	production income in year t of crop c established in year it utilising irrigation system i
$pc_{i,c,t,it}$	production cost in year t of crop c established in year it utilising irrigation system i
B_t	bank balance in year t
ri	interest rate on a positive bank balance
$pay_{c,t,it}$	instalment in year t to finance production cost of crop c established in year it
$ipay_{c,t,it}$	instalment in year t to finance irrigation system i established in year it
$payi_{c,t,it}$	interest portion of instalment in year t to finance production cost of crop c established in year it
$ipayi_{c,t,it}$	interest portion of instalment in year t to finance irrigation system i established in year it
fix_t	overheads in year t
lc_t	living expenses in year t
tax	marginal tax rate
$dep_{i,t,it}$	parameter specifying the tax deductions in year t associated with irrigation system i established in year it
$sal_{i,t,it}$	salvage value in year t of irrigation system i purchased in year it
TI_t	taxable income in year t
TT_t	taxable income transferred in year t due to a negative taxable income
CS_t	cash surplus in year t

A cash surplus in any given year exists if the sum of production income, money in the bank account (including interest earnings) and any salvage income is more than the sum of all overhead expenses, loan repayments, living expenses and tax liabilities. Equation (4.26) does not account for operating capital, as the bank balance is net of operating capital. Taxable income is a function of production income, operating expenses, salvage income, overheads, interest and depreciation deductions, as well as any losses transferred from the previous year.

The DLP model has the unique ability to defer tax payments until a positive taxable income is calculated.

A link is established between different years through the bank account. Equations (4.28) to (4.32) are used to determine how the generated cash surplus of the previous year will be utilised in the current production year.

$$B_t + \sum_c CP_{c,t} + CI_t - CS_{t-1} \leq 0 \tag{4.28}$$

$$\sum_i^3 \sum_{it}^{15} Q_{i,c,it} \, pc_{i,c,t,it} - CP_{c,t} - P_{c,t} \leq 0 \tag{4.29}$$

$$\sum_i^3 \sum_{it}^{15} IR_{i,it} inv_{i,t,it} - \sum_i^3 IL_{i,t} - CI_t \leq 0 \tag{4.30}$$

$$\sum_i^3 IL_{i,t} + \sum_{\substack{it \\ it<t}} IL_{i,it} ipayo_{i,t,it} \leq icf_t \tag{4.31}$$

$$\sum_c P_{c,t} + \sum_{\substack{it \\ it<t}} P_{c,it} \, payo_{c,t,it} \leq cf_t \tag{4.32}$$

$CP_{c,t}$	money used to finance production cost of crop c in year t
CI_t	money used to finance irrigation systems investments in year t
$inv_{i,t,it}$	investment cost in year t of irrigation system i established in year it
$payo_{c,t,it}$	outstanding capital year t of production loan used to finance production cost of crop c established in year it
$ipayo_{i,t,it}$	outstanding capital in year t of borrowed capital used to finance irrigation system i established in year it
cf_t	credit facility for financing production costs in year t
icf_t	credit facility for financing irrigation investment cost in year t

Cash surpluses from the previous year can be used to purchase new irrigation technology and/or to finance operating expenses with any surplus deposited in a bank account. The model furthermore allows for the use of production loans as a means to finance production cost, and borrowed capital to finance irrigation investments. The amount of money that might be borrowed in any given year is limited by the credit facilities and the amount outstanding.

4.3.1.2 Terminal values

The normative approach proposed by Rae (1970) is used to account for any cash flow streams beyond the planning horizon. With the normative approach a terminal value is calculated for each activity as the present value of future net revenue discounted from infinity for an assumed replacement cycle, given the planning horizon, is exceeded. Terminal values ensure that capital investments with cash flow streams beyond the planning horizon are not penalised.

Adding an annuity assumes that the activity composition in the terminal year of the model will be followed to infinity. Terminal values were calculated for the cropping activities, borrowing activities, irrigation investments and production loans since all these activities may extent past the planning horizon. For details of the calculation procedures and a description of the GAMS code used to generate appropriate terminal values the reader is referred to Grové (2006b).

4.3.1.3 Risk

Risk is incorporated into the DLP model by means of MOTAD and the objective function follows the specification given by Boisvert and McCarl (1990). In their formulation, risk is accounted for by subtracting a risk aversion parameter multiplied by an approximation of the standard error of the activity combinations. The following equations were used to calculate the approximate standard error:

$$\sum_{n}^{100} D_n^- \Big/ \left(2\Pi / (n(n-1))\right)^{-0.5} = SE \tag{4.33}$$

$$\sum_{i}^{3}\sum_{c}^{5}\sum_{it}^{15} Q_{i,c,it}\left(C_{n,i,c,it} - \overline{C}_{i,c,it}\right) + D_n^- \geq 0 \tag{4.34}$$

$C_{n,i,c,it}$	random *(n)* gross margin of crop i established in year it with irrigation system i
$\overline{C}_{i,c,it}$	average of $C_{n,i,c,it}$
D_n^-	negative deviations below the mean
Π	constant equal to 22/7

Equation (4.34) calculates the negative deviations from mean gross margins. The sum of these negative deviations is divided by a constant to give the approximate standard error of the solution (Equation 4.33). The deviations of long-term crops were based on the present value of gross margins. Procedures described in Section 4.2 are used to generate the risk matrix for the programming model. Deficit irrigation activities are not included in the DLP-model due to the increase in the size of the programming model over multiple years.

4.3.2 RESOURCE CONSTRAINTS

Resource constraints are discussed in two groups. The first group is concerned with land availability and general resource use and the other group with irrigation water supply limitations.

4.3.2.1 Land availability and general resource use

The following constraints are used to restrict resource use to be less than or equal to its availability and to determine land occupation by irrigation system and crop:

$$\sum_{c}^{3}\sum_{it}^{15} Q_{i,c,it}\, lo_{c,t,it} - \sum_{it}^{15} IR_{i,it}\, io_{i,t,it} \leq 0 \tag{4.35}$$

$$\sum_{it}^{15} IR_{i,it}\, io_{i,t,it} \leq land_{i} \tag{4.36}$$

$$\sum_{c}^{3}\sum_{it}^{15} Q_{i,c,it}\, ru_{r,i,c,t} \leq ra_{r,t} \tag{4.37}$$

$io_{i,t,it}$	land occupation in year t of irrigation system i established in year it
$lo_{c,t,it}$	land occupation in year t of crop c established in year it
$land_{i}$	land availability for irrigation system i
$ru_{r,i,c,t}$	use of resource r by crop c planted with irrigation technology i in year t
$ra_{r,t}$	availability of resource r in year t

Equation (4.35) is used to ensure that an investment in an irrigation system is made first, before any cropping activities can take place. Thus, the cultivation of a specific crop is linked to the availability of a specific irrigation technology. The total irrigation development is restricted to available land resources with Equation (4.36) while Equation (4.37) ensures that resource use is equal to or less than resource availability in any time period.

4.3.2.2 Irrigation water supply

Several factors including irrigation system capacity, conveyance capacity, the capacity of the canal off takes and the total water allocation can limit the amount of water that can be applied to a crop within a specific time period. Equations (4.38) to (4.42) are used to model these restrictions on irrigation water supply.

$$\sum_{it}^{15} Q_{i,c,it}\, gir_{i,c,t,it,wt} - \sum_{ws} AP_{ws,i,c,t,wt} \leq 0 \tag{4.38}$$

$$\sum_{c}^{5} AP_{ws,i,c,t,wt} \leq \sum_{it}^{15} IR_{i,it}\, io_{i,t,it}\ wsi\_cap_{ws,i} \tag{4.39}$$

$$\sum_{i}^{3}\sum_{c}^{5} AP_{ws,i,c,t,wt} \leq ws\_cap_{ws} \tag{4.40}$$

$$\sum_{i}^{3}\sum_{c}^{5}\sum_{ws}^{6} AP_{ws,i,c,t,wt} \leq con\_cap_{wt} \tag{4.41}$$

$$\sum_{i}^{3}\sum_{c}^{5}\sum_{ws}^{6}\sum_{wt}^{52} AP_{ws,i,c,t,wt} \leq alloc_{t} \tag{4.42}$$

$AP_{ws,i,c,t,wt}$	water application to crop c in year t week wt from water source ws with irrigation system i
$gir_{i,c,t,it,wt}$	gross irrigation requirement in year t week wt of crop c established in year it irrigated with irrigation system i
$wsi\_cap_{ws,i}$	irrigation application rate of irrigation system i from water source ws
ws_cap_{ws}	capacity off take *(ws)* on the tertiary canal
con_cap	tertiary canal conveyance capacity
$alloc$	annual water allocation in year t

The amount of water that is applied to a specific crop is related to the source of water application by Equation (4.38). A water source can be tertiary conveyance canal or on-farm storage. The specific amount of water applied from a specific water source is restricted by the application rates of the irrigation systems from a specific source, which is a function of irrigation system investments. The system application rates are 72m^3/day, 124m^3/day and 130m^3/day respectively for flood irrigation operated 12 hours a day, large and small pivots. A tertiary canal has on average six off takes with a capacity of 150m^3/hour, which limits the amount of water that can be abstracted from the canal (Equation (4.40)). Abstraction of all the off takes from the canal is limited by the conveyance capacity (10 800m^3/day) with Equation (4.41). Parameter ws_cap is specified on a weekly basis to account for the number of days in a week the WUA is supplying water to the farmers. Typically, the canals are operated on a five and a half day week. Equation (4.42) is used to ensure that no more water than the total annual water allocation of 9 140m^3/ha is abstracted.

4.4 SHORT-RUN WATER USE OPTIMISATION MODEL

The main objective of the short-run analysis is to evaluate the impact of limited water supply conditions on optimal water allocation decision between multiple crops while explicitly taking the production risk of deficit irrigation into account. To achieve the objective a non-linear mathematical programming model was developed. The basic structure of DLP model in terms of land availability, resource use and irrigation water supply was retained for the short-run model specification. Cognisance should be taken that the model only includes pivot irrigation activities for maize, groundnuts and wheat and that the model only includes two production seasons. The objective function of the model follows the Direct Expected Maximisation Non-linear Programming (DEMP) specification as presented by Boisvert and McCarl (1990) with the exception that CE is maximised. Formally the objective function of the model is:

$$Max\ \ CE = \frac{\ln\left(-\sum_{r}^{n} \frac{1}{n}\left(-e^{-r_a(x)}\left(gm_{cjr} X_{cj} \right)\right)\right)}{-r_a(x)} \tag{4.43}$$

gm_{cjr}	random (r) gross margin of crop c irrigated with irrigation schedule j (mm.ha)
X_{cj}	variable indicating number of hectares planted with crop c using irrigation schedule j (ha)
$r_a(x)$	Arrow-Pratt absolute risk aversion coefficient
r	random realisations

The random gross margins for each irrigation alternative were generated using procedures described in Sections 4.1 and 4.2

Choice of $r_a(x)$ is of critical importance to ensure that the level of risk aversion presented in the model is in accordance with the actual risk averseness of decision-makers. In Chapter 3, it is argued that $r_a(x)$ should be chosen such that $r_s(x^s)$ is less than 2.5. Unfortunately the relationship between $r_a(x)$ and $r_s(x^s)$ can only hold *ex post* because $r_s(x^s)$ is dependant on the standard deviation of the optimised farm plan. The model was therefore run for alternative levels of $r_a(x)$ until $r_s(x^s)$ exceeded 2.5. Parameterisation of $r_a(x)$ yields the set of crop irrigation combinations that is stochastically efficient over all the other combinations for a specific utility function and risk aversion level. Figure 4.2 is used to illustrate the procedure graphically.

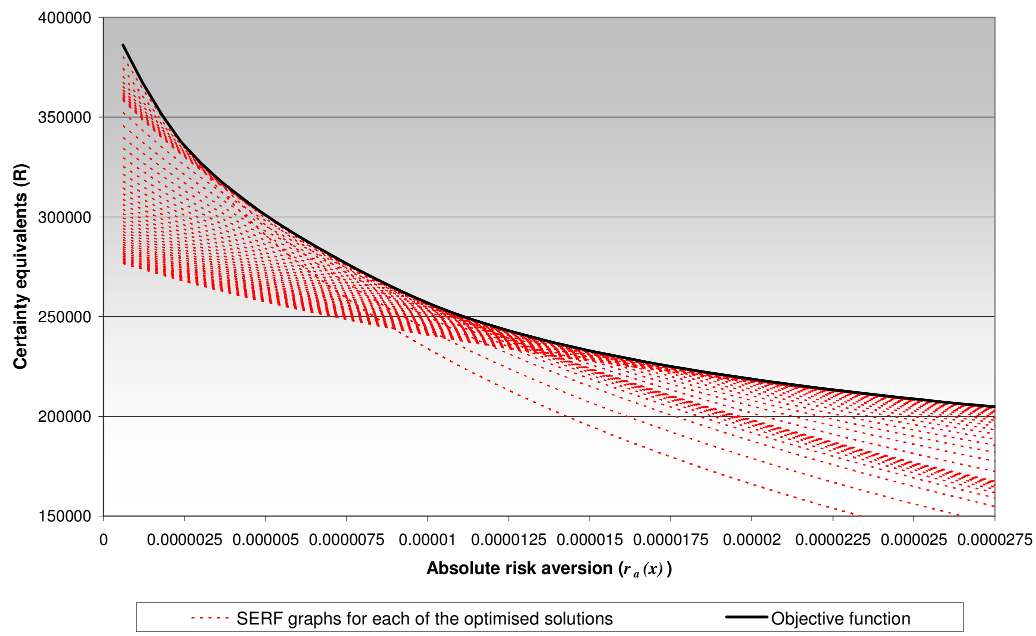

Figure 4.2: Illustrating stochastic efficiency with respect to a function for optimised solutions.

Figure 4.2 is constructed by conducting a SERF analysis for each of the optimised solutions. It should be recalled that the alternative with the largest CE at a specified level of risk aversion represents the preferred alternative. Thus, the upper bound of the graph represents the stochastic efficiency frontier. Figure 4.2 clearly shows that the objective function values of the model solution produce the stochastic efficiency frontier when graphed against the risk aversion parameters used during optimisation.

As already mentioned the relationship between $r_a(x)$ and $r_s(x^s)$ only holds *ex post* to the optimisation and it is anticipated that the *ex post* calculations of $r_s(x^s)$ may differ substantially between different scenarios. The optimise farm plans for each of the $r_a(x)$ values considered during optimisation are also evaluated by conducting a SERF analysis with constant $r_s(x^s)$ values.

4.5 STOCHASTIC EFFICIENCY WITH RESPECT TO A FUNCTION (SERF) ANALYSIS WITH CONSTANT STANDARD RISK AVERSION

To conduct a SERF analysis without specialised software is tedious because expected utility is calculated by evaluating a distribution of values where each of the values needs to be weighted separately. SIMETAR© (Simulation for Excel To Analyze Risk) (Richardson, Schumann and Feldman, 2004) provides a means to conduct a SERF analysis of multiple alternative easily in Excel©. The user is required to specify the distribution of outcomes for each of the alternatives for which the SERF analysis is needed and the range of risk aversion levels for which the *CE's* are calculated. The software automatically constructs all the necessary formulas to calculate the *CEs* of each alternative over the range of risk aversion levels specified and graphically shows the results. In order to calculate *CE's* with constant $r_s(x^s)$ requires scaling of $r_a(x)$ according to each alternative's dispersion of outcomes, which renders the use of the SERF analysis tool in SIMETAR© inappropriate. In this section a procedure is developed that will allow one to specify $r_s(x^s)$ while conducting a SERF analysis by means of SERF analysis tool in SIMETAR©.

In Chapter 3, Section 3.3.3 it is shown that for a transformed data set, x^s, such that $x^s = x/\sigma_x$, $r_s(x^s) = r_a(x)\sigma_x$ under the condition that utility calculated with the negative exponential utility function stays the same. If one assumes a negative exponential utility function the *CE's* of x and x^s are calculated as:

$$CE(x) = -\ln(E[U(x)]) / r_a(x) \tag{2.44}$$

$$CE(x^s) = -\ln(E[U(x^s)]) / r_s(x^s) \tag{2.45}$$

Given the assumption that utility remains constant:

$$CE(x) r_a(x) = CE(x^s)\, r_s(x^s) \tag{2.46}$$

Substituting $r_a(x)\sigma_x$ for $r_s(x^s)$ gives

$$CE(x) r_a(x) = CE(x^s)\, r_a(x)\, \sigma_x \tag{2.47}$$

$$\therefore\ CE(x) = CE(x^s)\, \sigma_x \tag{2.48}$$

Equation (2.48) shows that $CE\ (x^s)$ gives the number of σ, which makes the decision-maker indifferent between the *CE* and the gamble. To obtain $CE\ (x)$ $CE\ (x^s)$ is multiplied with σ given the assumption that $r_a(x)$ values are appropriately scaled to keep utility constant. Thus, the data transformation allows for the calculation of *CE* with the same $r_s(x^s)$. However, it should be remembered that the calculated *CE's* needs to be multiplied by the standard deviation of the alternative to equal the *CE's* calculated with $r_a(x)$. To conduct a SERF analysis with SIMETAR© therefore requires a standardised dataset of the alternatives that needs to be compared. After conducting the SERF analysis with $r_s(x^s)$ the calculated *CE's* of a specific alternative is multiplied with its respective standard deviation to complete the analysis. A numerical example of the procedure is given in Appendix B.

The next chapter provides the results of the long-run and short-run analyses.

CHAPTER 5

LONG-RUN AND SHORT-RUN MODELLING RESULTS

The first part of this chapter is used to report the results of the stochastic DLP model which is used to evaluate the impact of price incentives on water conservation when irrigators have the possibility of adopting more efficient irrigation technology or cultivating high-value crops (long-run analysis). In the second part of this chapter, the results of the stochastic efficiency analysis of deficit irrigation (short-run analysis) are reported.

5.1 LONG-RUN

The results presented in this section were generated with the stochastic DLP model described in the previous chapter. More specifically the model was used to study the impact of risk aversion, and starting capital availability on the derived demand for irrigation water and the associated expected net present value (NPV) for three alternative farm developing scenarios. Scenario FLOOD presents the base case where the decision-maker is only allowed to produce maize, groundnuts, wheat and lucerne with flood irrigation. Scenario PIVOT is the same as scenario FLOOD but allows for centre pivot irrigation adoption possibilities. The last scenario, PECAN, is also the same as the base scenario but allows for the production of pecan nuts under flood irrigation. Starting capital of R150 000 (C150) and R300 000 (C300) were used. The risk aversion levels correspond to a risk neutral (N) and a risk averse (A) decision-maker. The $r_s(x)$ level of N was set at zero which corresponds to profit maximisation without considering risk and an upper bound on $r_s(x)$ of 2.5 for A. The tradeoffs with respect to the expected NPV will be discussed next, followed by the derived demand for irrigation water. The last section of the long-run analyses is used to determine whether a price increase will result in significant reductions in the quantity irrigation water demand.

5.1.1 WATER AVAILABILITY NET PRESENT VALUE TRADEOFFS

Figure 5.1 shows the impact of risk aversion on the water availability NPV tradeoffs for the three alternative farm developing strategies for C150, while Figure 5.2 shows the impact of risk aversion for C300. The two graphs are presented on the same scale to allow easy comparison of the impact of starting capital availability on the tradeoffs. Please note that the tradeoff curves are not graphed from zero water allocation since no alternatives to irrigation are included in the model. Therefore, a threshold level of water availability is needed to generate a feasible solution. A graph that combines the tradeoffs presented in Figure 5.1 and 5.2 is shown in Appendix C.

Significant differences are observed between the different farm development strategies. Let's consider NC150 first. FLOOD generates feasible solutions when water allocations are at least 85% of the current full water allocation of 9 140m^3/ha. Once a feasible solution is generated, the NPV increases from R1.85 mil to a maximum of R4.0 mil at a water allocation of 60% more than the current allocation. A plateau is reached at water allocation levels greater than 1.25 times the current water allocation. With water allocations greater than 1.25 times the current water allocation, only marginal increases in the NPV are observed. PECAN also generates feasible solutions when water allocations are at least 85% of the current full water allocation. However, the rate at which the expected NPV increased as more water was allocated is much greater than that of FLOOD. Furthermore, PECAN takes longer to reach a plateau and as a result, the NPV of PECAN is about R4.0 mil more than that of FLOOD at its maximum. PIVOT dominates FLOOD over the whole range of water allocations. Pivot irrigation is a more efficient use of irrigation water when compared to flood irrigation and as a result feasible solutions were generated when only 60% of the current water allocation is allocated when scenario PIVOT is considered. A steady increase in the NPV of PIVOT is observed as water allocations are increased. The maximum NPV of R5.3 mil is reached at a water allocation of 1.4 times the current allocation whereafter it stays constant as water allocations are increased. Thus, when PIVOT is considered the maximum is reached more rapidly. The reason is that PIVOT is a more efficient use of irrigation water resulting in larger areas being irrigated which again causes land and canal capacities in critical time periods to become limited. At current water allocations, the NPV of PIVOT is only R0.31 mil less than PECAN and at water allocation reductions of more than 5%, PIVOT starts to dominate PECAN.

Increasing starting capital availability does not alter the general shape of the tradeoff curves for the alternative scenarios presented in Figure 5.2. However, it does increase the level of the NPV. The NPV of PECAN is increased the most and that of FLOOD the least. More significant is the fact that feasible solutions were generated at water allocations in excess of 75% of the current water allocation. The shifts in the tradeoff curves result in a situation where PIVOT dominates PECAN only if water allocations are less than 85% of the current allocation.

When risk aversion is considered a lower NPV is generated irrespective of the level of starting capital or the specific farm development scenario. Interesting to note is that the impact of risk aversion on the NPV decreases as water becomes scarcer. At very low water allocations, the impact of risk aversion is insignificant.

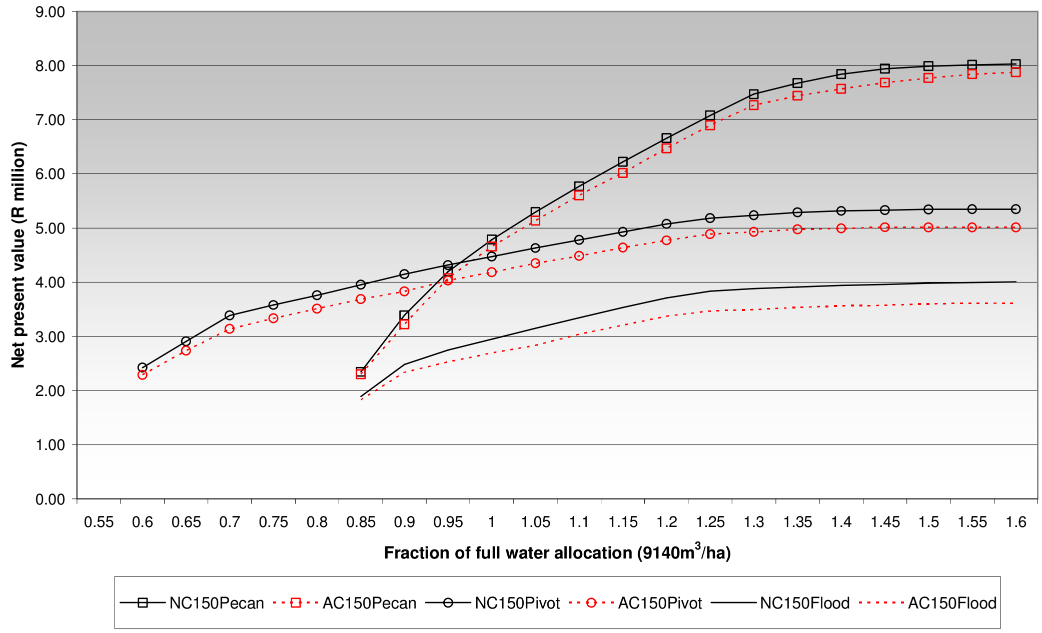

Figure 5.1: Net present value water availability tradeoffs for alternative farm development scenarios (PECAN, PIVOT and FLOOD), two levels of risk aversion (A and N) and starting capital of R150 000 (C150).

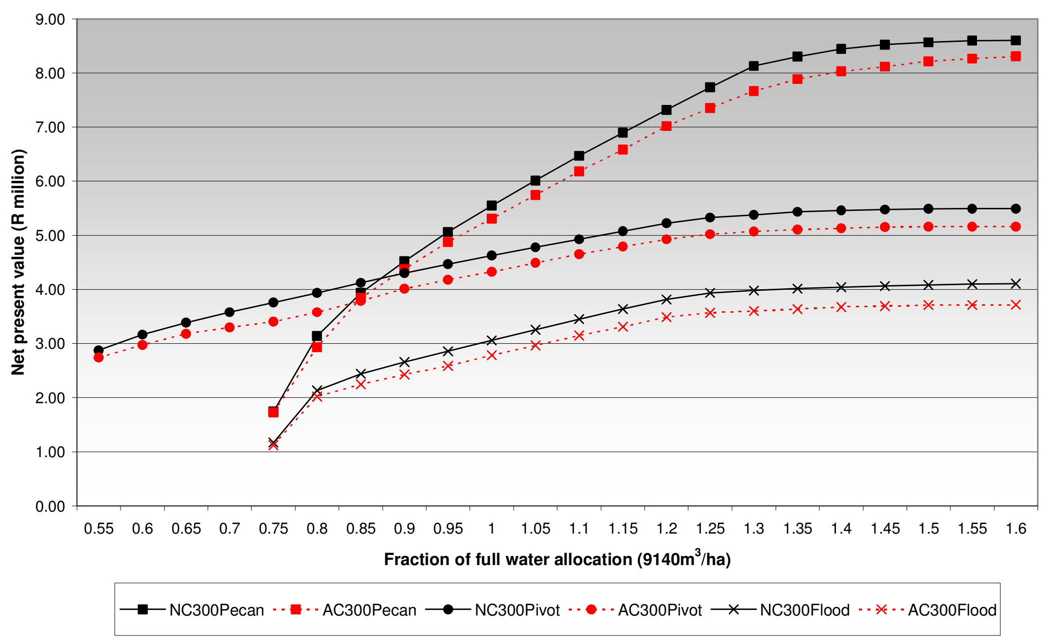

Figure 5.2: Net present value water availability tradeoffs for alternative farm development scenarios (PECAN, PIVOT and FLOOD), two levels of risk aversion (A and N) and starting capital of R300 000 (C300).

From the discussion above it is clear that the type of farm developing scenario most significantly influenced the water availability NPV tradeoff curves. More specifically, it is important to realise that the production of high-value crops such as pecan nuts may not be as feasible and profitable as the adoption of more efficient irrigation technology such as centre pivots when water is curtailed. The importance of producing crops that will generate cash flows increases when water is limiting. Thus, when water is limited farmers will reduce the establishment of pecan nuts. As a result, water allocation reductions will impact most severely on farmers with high-value crops. Risk seems to be of less importance in terms of expected NPV differences when water is severely limiting the financial feasibility of the farming operation. Furthermore, the financial position (starting capital) of the farmer will significantly affect the ability of the farmer to sustain higher levels of irrigation water allocation reductions as indicated by shifts in the tradeoff curves to the left when starting capital was increased. The conclusion is that the relative profitability of alternative water use strategies and cash flows, which determine the ability to adopt modern irrigation technology and to establish high-value crops, play a significant role in the farmer's ability to sustain water curtailments.

In the next section the maximum willingness to pay for irrigation water for the alternative scenarios is evaluated.

5.1.2 DERIVED DEMAND FOR IRRIGATION WATER

The three irrigation development scenarios are discussed separately using two graphs each to present the derived demand for water. The first graph gives an indication of the total derived demand curve over all feasible price ranges. The second graph highlights the lower price ranges that are more relevant for decision-making purposes. The scales of the two different sets of graphs are the same to enable a quick comparison between alternative farm developing scenarios. Graphs, which combine the results of the different scenarios, are presented in Appendix C.

5.1.2.1 FLOOD scenario

Figure 5.3 shows the derived demand for irrigation water over all the feasible price ranges for scenario FLOOD. Graphically the graph shows two distinct regions. The first is a relatively inelastic region that covers the low to very high price range and a relatively elastic region at a very low price range. For both the levels of starting capital availability and the two levels of risk aversion considered, the maximum marginal values calculated were in a range of R3.5/m^3 – R3.9/m^3. The total water charge payable to the WUA is R0.0877/m^3. The total water charge is in the very lower end of the overall price range and Figure 5.4 is used to highlight the differences between risk aversion levels and starting capital availability at the lower price ranges.

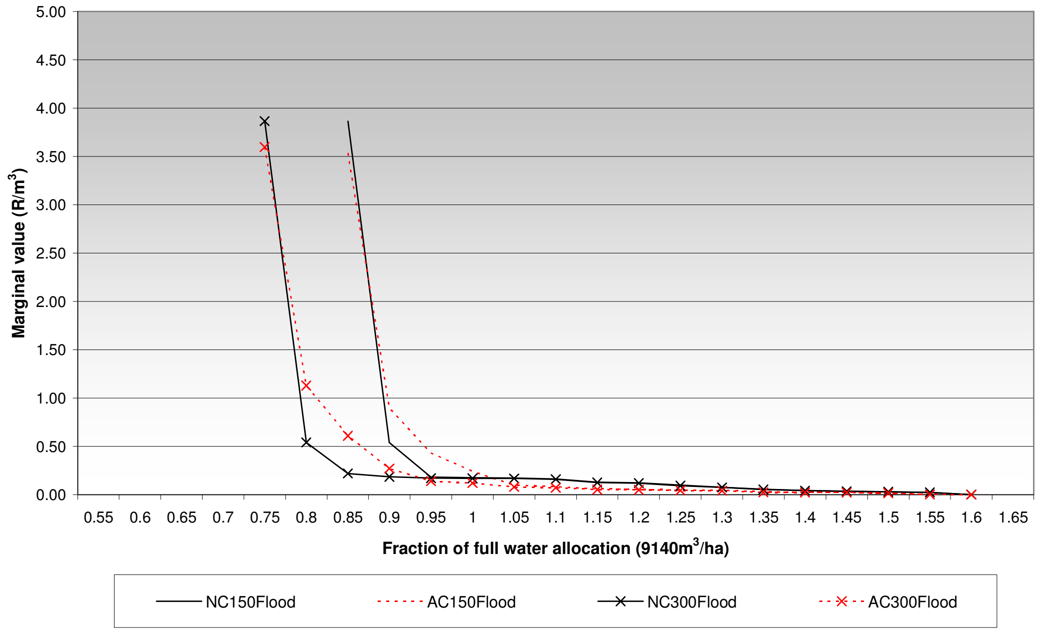

Figure 5.3: Irrigation water derived demand for the FLOOD farm development scenario with two levels of starting capital (C150 and C300) and two levels of risk aversion (N and A).

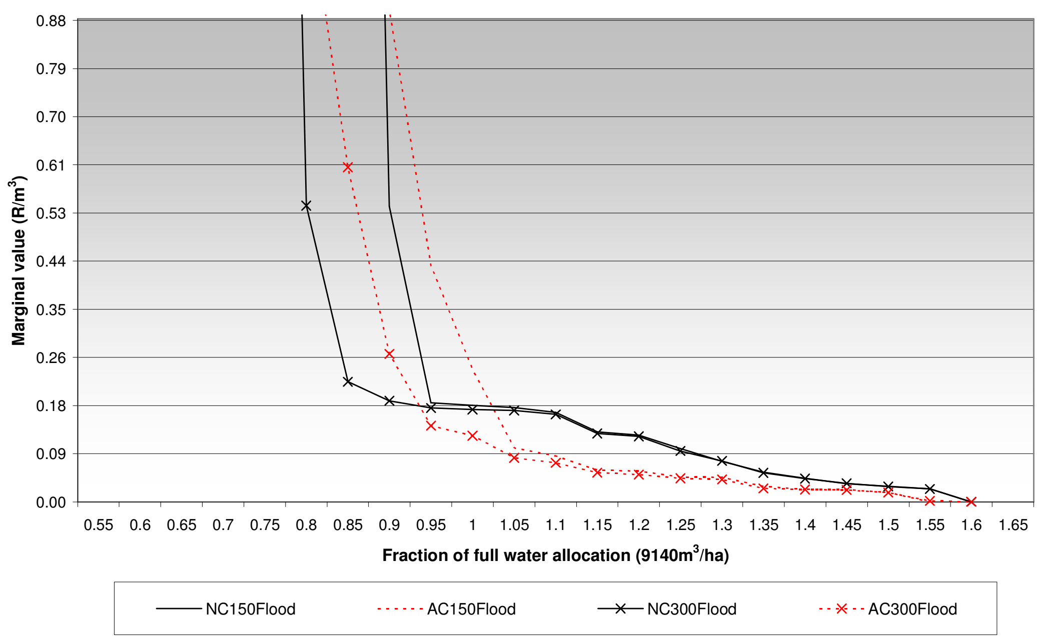

Figure 5.4: Lower price range irrigation water derived demand for the FLOOD farm development scenario with two levels of starting capital (C150 and C300) and two levels of risk aversion (N and A).

Figure 5.4 shows that no difference in the willingness to pay for irrigation water exists for different levels of starting capital availability if quantity irrigation water demand exceeds about 1.1 times the current water allocation for the risk neutral irrigator. When less water is allocated, irrigators with less starting capital are willing to pay slightly more for their irrigation water. More significant is that C150 reaches the relatively inelastic phase of the derived demand curve at a fraction of 0.95 whereas C300 reaches this stage at a fraction of 0.85 of the current allocation. After these switching points, the willingness to pay for water increases very significantly over relatively small changes in the amount of water allotted. The general impact of starting capital availability on irrigator's willingness to pay for water is not altered by risk aversion and for water allotments greater than 1.15 little difference in willingness to pay is modelled. However, the water allocation fraction at which a risk averse irrigator reaches the switching point between the relatively elastic and inelastic regions is 10 percentage points higher when compared to the risk neutral case. Also evident from Figure 5.4 is that there is a range of prices (water allotments) where the willingness of a risk averter to pay for irrigation water will be lower than that of a risk neutral irrigator. Such a situation is present for water allotments greater than 1 and 0.9 respectively for C150 and C300. The NPV water availability tradeoffs presented in the previous section clearly demonstrated that risk aversion implies lower expected NPV. Due to these lower expected NPV a risk averse irrigator's cash flow situation will come under pressure more rapidly resulting in significant increase in the willingness to pay for water. The increase in willingness to pay is associated with increased levels of specialisation in crops with relatively higher gross margins per unit water use such as groundnuts.

5.1.2.2 PIVOT scenario

Significant differences exist between the derived demand for irrigation water for FLOOD and PIVOT (Figure 5.5) scenarios. Firstly, the value of the last unit of water before infeasible solutions were generated is much lower when compared to FLOOD. Furthermore, feasible solutions were generated at lower water availabilities (55% for C300 and 65% for C150) compared to FLOOD (75% for C300 and 85% for C150). Secondly, although Figure 5.5 shows a relatively elastic and a relatively more inelastic region the inelastic region is not nearly as inelastic as the FLOOD scenario. Thirdly, the willingness to pay for irrigation water with PIVOT is, generally speaking, less than FLOOD. These changes highlight the fact that pivot irrigation is more efficient than flood irrigation and therefore add to the manoeuvrability of the irrigation farmer under water limiting conditions. The result that the willingness to pay for water is less with PIVOT than FLOOD is interesting. Especially in view of the fact that PIVOT has higher less variable crop yields that are obtainable with less irrigation water when compared to FLOOD. The decrease in willingness to pay is because the pivot irrigation systems are financed with borrowed capital. Figure 5.6 is used to evaluate the impact of starting capital availability and risk aversion in more detail at the lower price ranges.

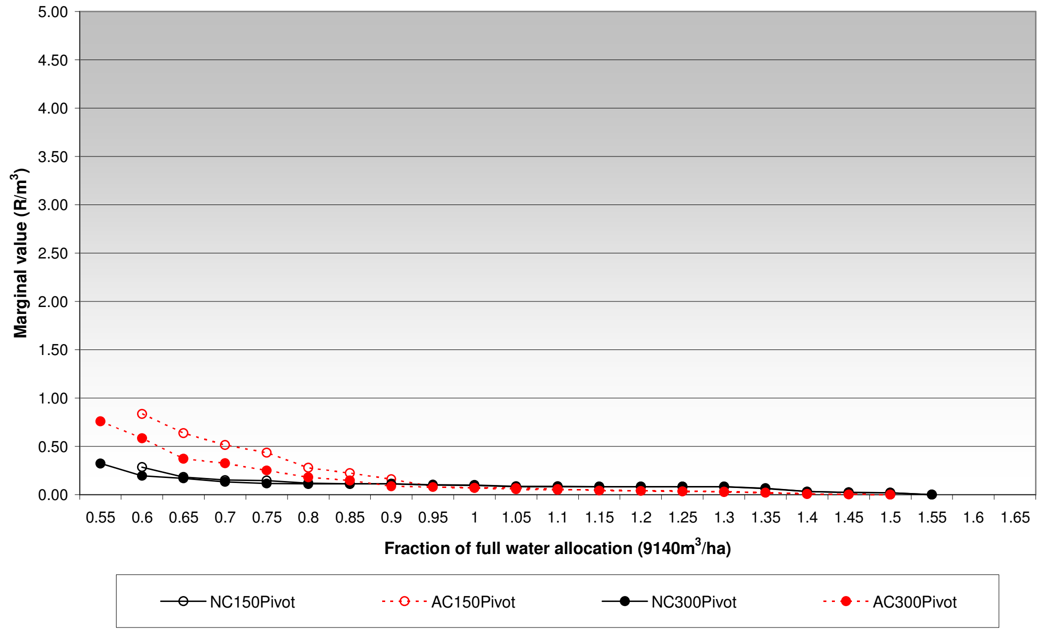

Figure 5.5: Irrigation water derived demand for the PIVOT farm development scenario with two levels of starting capital (C150 and C300) and two levels of risk aversion (N and A).

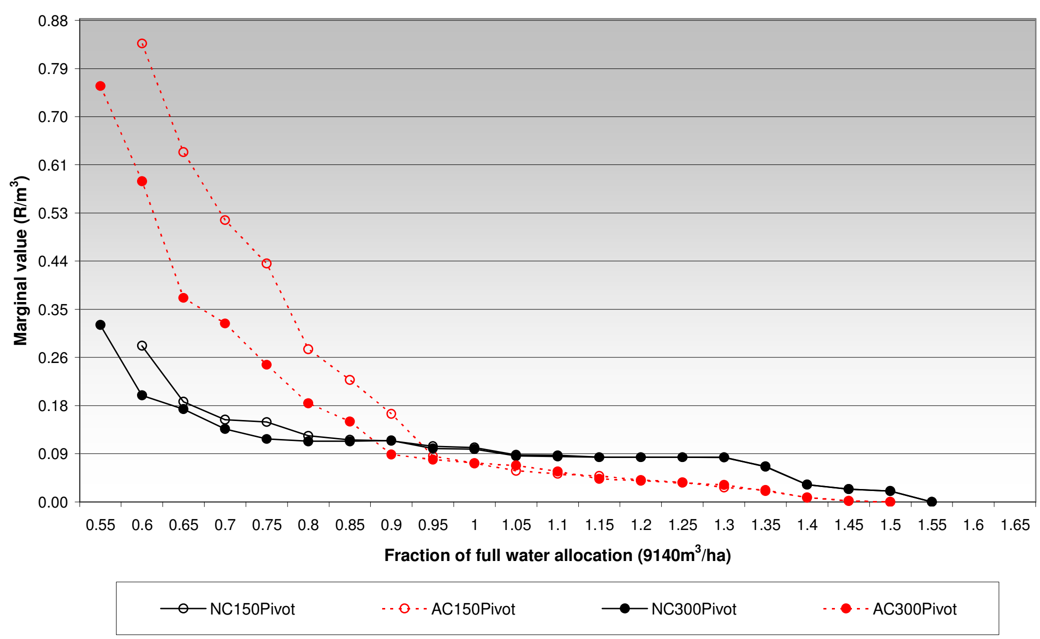

Figure 5.6: Lower price range irrigation water derived demand for the PIVOT farm development scenario with two levels of starting capital (C150 and C300) and two levels of risk aversion (N and A).

Figure 5.6 again shows that under very low price ranges no difference in the willingness to pay for water exists between C150 and C300. When water allotments are less than 0.95 and 0.85 for A and N respectively lower starting capital availabilities increase the irrigator's willingness to pay for water. The same positive and negative relationship between risk aversion and willingness to pay for irrigation water is shown for PIVOT when compared to FLOOD. However, a positive relationship is modelled for PIVOT water allotments less than 0.95 compared to 1.05 for FLOOD when C150 is considered. When C300 is considered a positive relationship is modelled if water allotments are less than 0.9 compared to 0.95 for FLOOD. Again, when cash flows are constraining the financial feasibility of the farming operation, risk averse farmers are willing to pay more for irrigation water as water allocations are reduced. Under these conditions, risk averse farmers tend to specialise more in crops with high gross margins per cubic metre water applied.

5.1.2.3 PECAN scenario

The PECAN scenario has a very similar derived demand for irrigation water as FLOOD when all the price ranges are considered (Figure 5.7). Three differences are notable. Firstly, the transition from the relatively inelastic to the relatively elastic region of the curve is not as abrupt as for FLOOD. Secondly, the elastic region is relatively more inelastic when compared to FLOOD. Lastly, the willingness to pay for irrigation water is much higher than in the other scenarios due to the higher profit margins of pecan nut production. Figure 5.8 is used to show the derived demand for irrigation water at low price ranges.

In contrast with the other scenarios, Figure 5.8 shows that capital availability impacts on the derived demand curves even at high levels of water availability. However, for the risk neutral case the C150 and C300 curves tend to follow each other till water allocations are reduced to about 1.2 times current water allocations. When water allocations are reduced beyond the 1.2 fraction the impact of capital availability tends to increase. For the risk averse scenario the impact of capital availability is more significant when less than 1.45 of the current full water allocation is allocated. Figure 5.8 also shows a different relationship between risk aversion and the willingness to pay for irrigation water when compared to the other farm development scenarios.

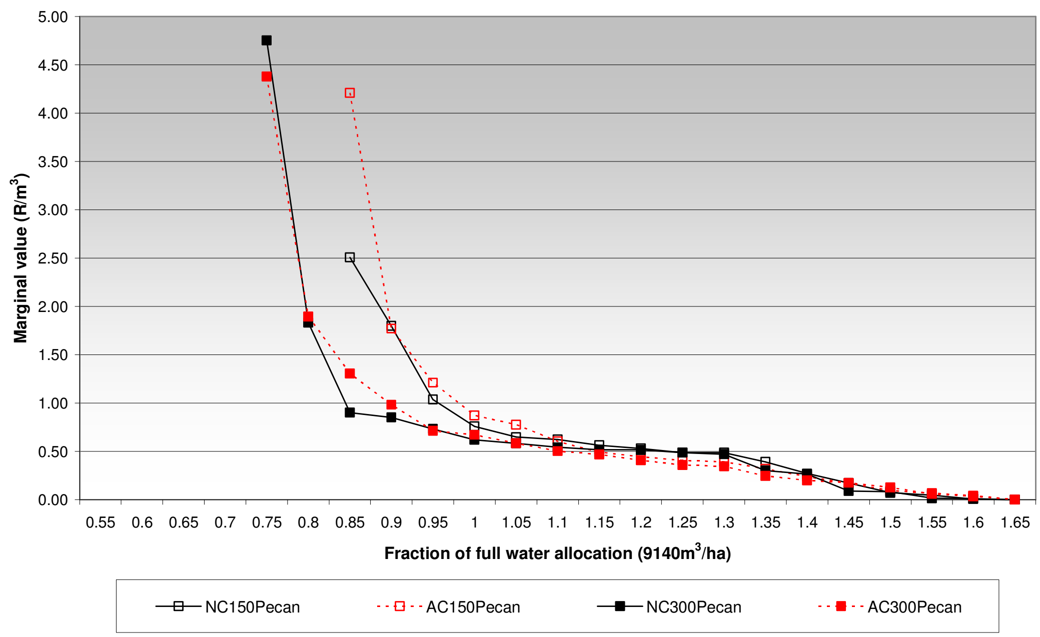

Figure 5.7: Irrigation water derived demand for the PECAN farm development scenario with two levels of starting capital (C150 and C300) and two levels of risk aversion (N and A).

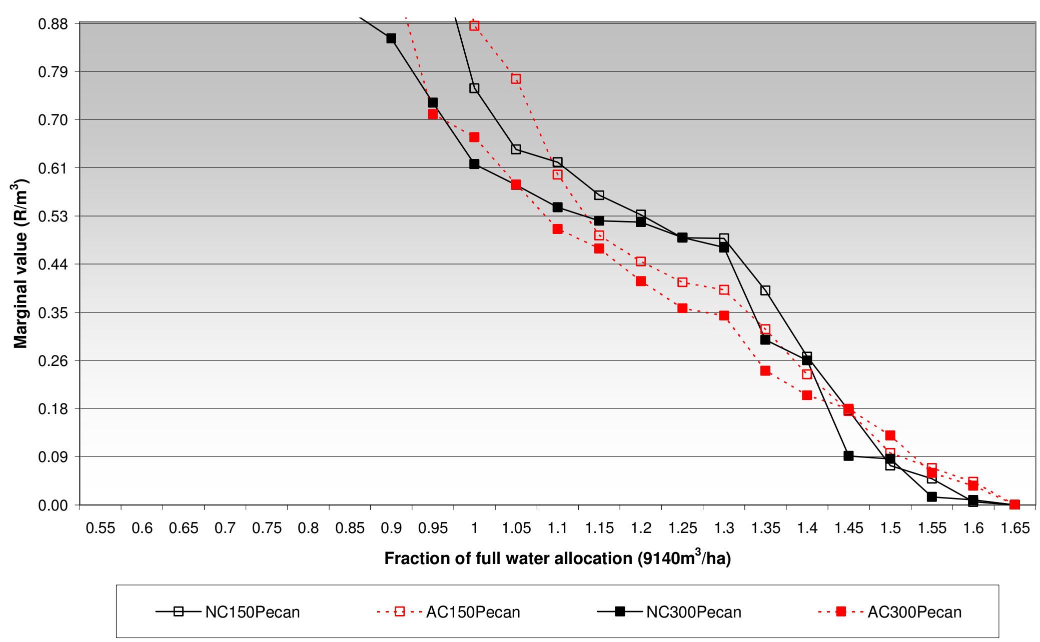

Figure 5.8: Lower price range irrigation water derived demand for the PECAN farm development scenario with two levels of starting capital (C150 and C300) and two levels of risk aversion (N and A).

Both FLOOD and PIVOT showed that risk aversion causes irrigation farmers to value irrigation water more than a risk neutral farmer as water is becoming scarcer and less when more water is allocated. The level at which an irrigator views water as being scarce is a function of its impact on cash flows and therefore starting capital availability. When PECAN is considered the derived demand curves show that irrigators are valuing irrigation water more than risk neutral farmers if water is scarce (impacting severely on cash flows) and when irrigation water availability is nearing abundancy. More specifically, the derived demand curves of the risk averse scenarios are above their risk neutral counterparts if water allocations exceed more or less 1.4 times current water allocations and when less water than a fraction of more or less 1.1 is allocated. When water is severely limiting the financial feasibility of PECAN, the crop mix other than the portion of pecan nuts, tends to favour groundnuts as is the case with FLOOD and PIVOT. Again, the cash flow requirements force the crop mix to become relatively more specialised in crops with high gross margins per unit applied water. At the other extreme water is also valued higher than risk neutral farmers if more than 1.4 times the current water allocation is allotted. During this phase the crops that compete for water during summer tends to stabilise. However, an increase in the area of wheat is observed. Although wheat has a low gross margin, it also has a low variability thereof. In this phase, it seems as if water is applied to reduce risk rather than to increase profit margins and therefore a risk averse farmer will value water more than a risk neutral farmer.

5.1.3 PRICE RESPONSIVENESS OF IRRIGATION WATER DEMAND

Section 5.1.2 discussed the derived demand for irrigation water for three alternative farm-developing scenarios. The characterisation of the derived demand of irrigation water is a necessary first step to compute the responsiveness of the quantity of water demanded to price increases, which is the main objective of this section. The responsiveness of the quantity of water demanded to price increases is measured by the own-price elasticity of demand which gives the percentage change in quantity demanded to a one percent change in the price. In this analysis, the arc formula is used because no parametric demand function was estimated. The arc formula computes the average elasticity between two prices (Tomec and Robinson, 1990). Since elasticity is expressed in percentage terms it is unitless and therefore allows for easy comparison between different scenarios.

Scheierling *et al.* (2004) demonstrated that even though price elasticity of demand for irrigation water may be inelastic large reductions in quantity demanded are possible if prices are increased from zero. Table 5.1 is used to determine the impact of a water tariff increase from zero to R0.0877/m^3 on the quantity of irrigation water demanded for the alternative farm development scenarios.

Table 5.1: Impact of price increase from zero to R0.0877/m^3 on quantity irrigation water demanded for the three farm development scenarios (FLOOD, PIVOT, PECAN) with two levels of starting capital (C150, C300) and two levels of risk aversion (A, N).

	Farm development scenario					
	FLOOD		PIVOT		PECAN	
	Starting capital		Starting capital		Starting capital	
	C150	C300	C150	C300	C150	C300
Risk neutral (N)						
Water reduction (m^3)	228981	233475	352988	359080	109779	130169
Percentage water reduction (%)	20.60	21.01	32.78	33.35	9.58	11.36
Elasticity	-0.11	-0.12	-0.20	-0.20	-0.05	-0.06
Risk averse (A)						
Water reduction (m^3)	322435	388290	383991	417448	95578	84435
Percentage water reduction (%)	29.95	34.94	36.85	40.06	8.34	7.37
Elasticity	-0.18	-0.21	-0.23	-0.25	-0.04	-0.04

Table 5.1 shows that the estimated price elasticity for all the alternative scenarios is inelastic. Starting capital availability has little impact on the estimated price elasticity for a specific farm development scenario. If one recalls that the results from the previous sections showed that the impact of starting capital only becomes significant when cash flows come under pressure at relatively higher price ranges. Therefore, starting capital availability has relatively little impact on the elasticities for the price range considered. However, the impact of risk aversion on price elasticity is significant for FLOOD and to a lesser extent for PIVOT. For these two scenarios risk aversion results in an increase in the price elasticity estimates. When the risk neutral case for FLOOD is considered an increase in the water tariff from zero to R0.0877/m^3 will result in a 20% reduction in quantity irrigation water demanded even though irrigation water demand is inelastic (-0.11). Due to the highly inelastic demand for irrigation water for PECAN under risk neutrality, the quantity of irrigation demanded is reduced minimally. For risk averse farmers the price elasticity for FLOOD and PIVOT ranges between -0.18 and -0.2 with corresponding ranges of reduction in the quantity of irrigation water demanded of 30% - 40%. Thus, the results confirm the findings of Scheierling *et al.* (2004) which showed that large reductions in the quantity of irrigation water demanded is possible in the presence of inelastic demand. Based on their results they conclude that pricing policy will be effective in bringing about reductions in water deliveries. A word of caution is necessary to put their results and the results obtained in Table 5.1 into perspective. In each case the reduction in the quantity of irrigation water demanded was calculated from a zero initial price and therefore from the maximum quantity of irrigation water demanded. When farmers already pay for their irrigation water the reductions in quantity demanded will necessarily be different. Another factor that may influence the effectiveness of water pricing as an instrument to conserve water is that water demand in South Africa is rationed to a maximum specified quota. The result of rationing water use is that the

demand for irrigation water becomes truncated at the quota level. Because farmers in Vaalharts are rationed to a maximum of 9 140m^3/ha the derived irrigation water demand curves presented in Section 5.1.2 are not a true reflection of the irrigation water demanded by the farmers since the derived demand curves are not truncated.

Several factors will determine the effectiveness of water price increases on the quantity of irrigation water demanded when water is rationed. If water is charged at its scarcity value (marginal value) the effectiveness of a water pricing policy will be determined by the price elasticity at that point. Important to note is that the price elasticity is influenced by the shape of the irrigation water demand curve as well as the position on the curve (Appels, Douglas, Dwyer, 2004). Thus, the level of the quota as well as the shape of the irrigation water demand curve influences the efficiency of pricing on the quantity of irrigation water demanded. When water tariffs are low in relation to the scarcity value of irrigation water price increases will not be effective in reducing the quantity of irrigation water demanded. Table 5.2 is used to determine whether an increase in the water tariff will cause a reduction in the quantity of irrigation water demanded for the three alternative farm development scenarios at current quota rationing.

Table 5.2: Absorbed scarcity rents for three alternative farm development scenarios (FLOOD, PIVOT, PECAN) with two levels of starting capital (C150, C300) and two levels of risk aversion (A, N) at current water quota of 9 140m^3/ha.

	Farm development scenario					
	FLOOD		PIVOT		PECAN	
	Starting capital		Starting capital		Starting capital	
	C150	C300	C150	C300	C150	C300
Water tariff (R/m^3)	0.0877	0.0877	0.0877	0.0877	0.0877	0.0877
Risk neutral (N)						
Shadow value (R/m^3)	0.1759	0.1682	0.0989	0.0960	0.7592	0.6207
Absorb scarcity rent (R/m^3)	0.0882	0.0805	0.0112	0.0083	0.6715	0.5330
Response increase (%)	100.53	91.81	12.82	9.47	765.71	607.72
Risk averse (A)						
Shadow value (R/m^3)	0.2409	0.1205	0.0701	0.0709	0.8726	0.6698
Absorb scarcity rent (R/m^3)	0.1532	0.0328	-	-	0.7849	0.5821
Response increase (%)	174.69	37.34	-	-	894.94	663.71

Table 5.2 shows that the willingness to pay for irrigation water is greater than the water tariff of R0.0877/m^3 with a quota rationing of 9 140m^3/ha for all the scenarios with the exception of PIVOT under risk aversion. As a result, some level of price increase will have no effect on the quantity demanded since price elasticities are zero. Thus, only if prices are set above a threshold, denoted by the shadow value of water, will a pricing policy be effective in curtailing irrigation water demand (De Fraiture and Perry, 2002). The absorbed scarcity rents indicate that water charges need to double for FLOOD and increase by 9.5% to 13% for PIVOT before risk

neutral farmers will alter the quantity of irrigation water demanded. For the risk neutral PECAN scenario, the water tariff needs to increase by a factor of 7.7 and 6.1 respectively for C150 and C300 in order to have any effect on the quantity of irrigation water demanded. In general, risk aversion caused the absorbed scarcity rents of FLOOD and PECAN to increase. The exception is FLOOD C300 where a decrease in the scarcity rent is modelled. The reason is that irrigators with C300 have not yet started to value their irrigation water more than the risk neutral farmers, which is the case for the other scenarios. As noted before, for PIVOT the water tariff is greater than the willingness to pay for water and risk averse farmers will be conscientious about their water use.

The results show that quite significant increases in the water tariff of R0.0877/m^3 is necessary to foster behaviour that will reduce the quantity of irrigation water demanded for FLOOD and PECAN. Interesting to note is that the PIVOT scenario is more responsive to price increases even though it is more efficient than flood irrigation. This result highlights the importance of the impact of financing new irrigation on willingness to pay for irrigation water.

5.1.4 CONCLUSIONS

The NPV water availability tradeoff curves highlight the importance of relative profitability of alternative water use strategies and cash flows on the ability of the farm to sustain water curtailments. This is especially true if one considers that irrigation water is a necessity for farms to survive financially in Vaalharts due to the infeasibility of dryland production alternatives. Thus, if water is curtailed beyond a certain point farms will go bankrupt. Although pecan nuts are more profitable than producing annual crops under centre pivot irrigation, reductions in water availability will severely impact on these farms due to the delayed income generated by pecan nuts. The importance of cash flows is also evident from the shifts in the NPV water availability curves to the left which indicate that the farms will be able to sustain larger water curtailments. The conclusion is that a threshold of water availability is necessary for profitable farming and that the farms ability to generate cash flows will significantly impact on the farm’s ability to sustain water curtailments.

The derived demand for irrigation water indicated that risk aversion and starting capital availability significantly impacted on the derived demand curves under limited water supply conditions. More specifically, risk aversion and starting capital availability cause an increase in the elasticity of the irrigation water demanded if cash flows start to be limiting. An important conclusion from the PIVOT scenario is that more efficient irrigation technology may not increase the ability of the farmer to pay for water if the lumpy technology needs to be financed. As a result the PIVOT scenario is more responsive to price increases when compared to the other scenarios, which require significant price increases for a response. The estimated price elasticity of the quantity of irrigation water demanded for all the scenarios is very low. Important to note is that the elasticity estimates are influenced by the shape of the demand curve as well as the position on the curve. Since a threshold quantity of irrigation water is necessary, the estimated elasticity values will be lower when compared to a demand curve that allows zero irrigation water demanded at high prices. The conclusion is that care should be taken when interpreting elasticity estimates from literature without knowing the conditions under which they were derived. The conclusion is also made that increasing water charges will have mixed results depending on the specific farm situation.

5.2 SHORT-RUN

The results presented in this section were generated with the utility maximisation model that is used to optimise water use at farm-level while considering increasing production risk of deficit irrigation. Two water supply scenarios are considered. The first scenario represents a situation of full water allocation (FA) where 9 140 m^3/ha is available to irrigate 76 ha under pivot irrigation. With the second scenario, water allocation is limited to 80% (LA) of the full water allocation. Two water use strategies are considered. The first strategy does not allow deficit irrigation and is referred to as full irrigation (FI) while the second allows deficit irrigation (DI). The optimised stochastic efficiency frontiers of the alternative water use strategies under FA and LA will be discussed first, followed by a discussion of the implied risk aversion towards the optimised water use strategies for the two water supply scenarios. Since significant differences exist between the implied risk aversion coefficients of each alternative, the optimised farm plans for each alternative are also subject to a SERF analysis with constant standard risk aversion.

5.2.1 OPTIMISED STOCHASTIC EFFICIENCY ANALYSIS

Figure 5.9 shows the objective function values that were optimised for the two alternative water supply scenarios and the two irrigation strategies. The values of $r_a(x)$ that were used during optimisation were chosen such that the *ex post* calculations of $r_s(x^s)$ do not exceed 2.5.

The result that the DI strategies for both the water supply scenarios are stochastically more efficient than their FI counter parts, is striking. With the DI strategies the crops are deficit irrigated to some extent in critical time periods when canal capacities are limiting in order to increase the area planted. As a result, the total gross margin of the farm is increased in spite of possible increases in the variability of gross margins. The stochastic efficiency frontiers of the alternative scenarios indicate that risk aversion has a significant impact on the optimised *CE's*. The reduction in the *CE's* from risk neutrality to the most extreme level of $r_a(x)$ considered is about R209 000 for FADI and LADI which is greater than the reductions for FI when water allocations are reduced. For FAFI and LAFI the reductions in *CE's* are respectively R182 000 and R162 000. To gain a better understanding of the impact of a water curtailment of 20% the utility weighted risk premiums between FA and LA were studied further for each of the water use strategies. The utility weighted premium gives the minimum sure amount that has to be paid to a decision-maker to justify a switch between a preferred and a less preferred alternative (Hardaker *et al*., 2004). Results of the analysis are shown in Figure 5.10.

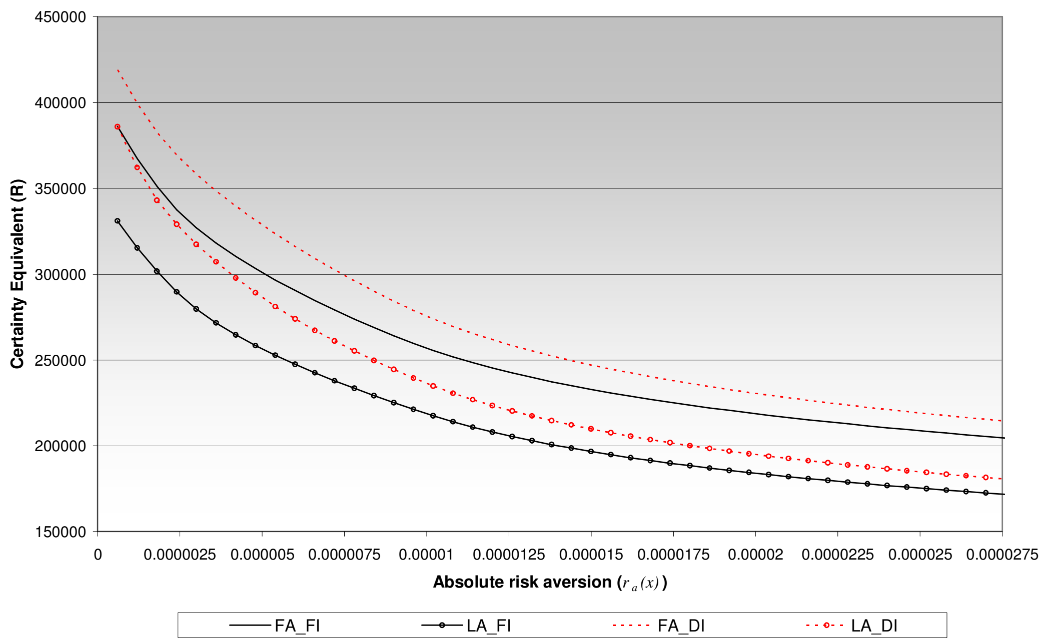

Figure 5.9: Constant absolute risk aversion stochastic efficiency frontiers under full (FA) and limited (LA) water supply conditions for full (FI) and deficit irrigation (DI) strategies.

A positive premium indicates that farmers will not willingly conserve water and that they need to be compensated to conserve water irrespective of the water use strategy employed. The level of compensation as well as the impact of increasing absolute risk aversion on these levels of compensation is significantly different between the scenarios considered.

When water allocations are reduced, farmers who practice FI need to be compensated more than farmers who practice DI if the baseline is FAFI. However, increasing levels of absolute risk aversion cause the compensation paid to LAFI to decrease while the compensation paid to LADI increases with increasing levels of absolute risk aversion. Due to this inverse relationship, the difference between compensation paid to LAFI and LADI necessary to foster a switch from FAFI decreases with increasing levels of absolute risk aversion. LADI is also compared to a baseline of FADI to account for farmers who are already practicing DI. An interesting relationship between the utility weighted premiums and increasing absolute risk aversion is determined for FADI vs. LADI in Figure 5.10. Increasing absolute risk aversion from neutrality to

$r_a(x)$=0.000005 causes the premiums to increase whereafter it starts to decrease as absolute risk aversion increases further.

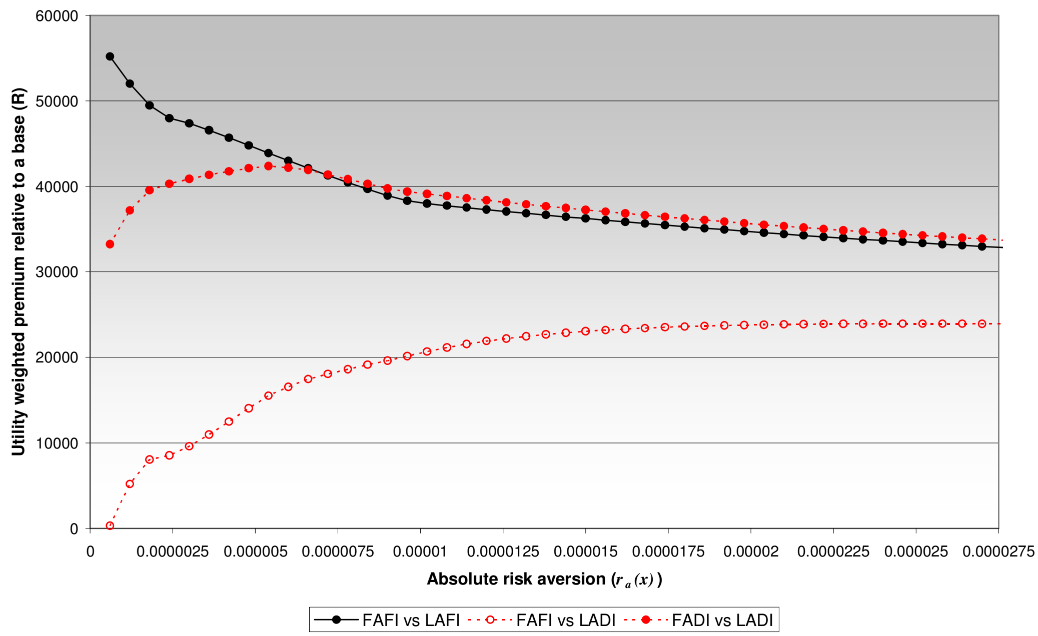

Figure 5.10: Utility weigted premiums between full (FA) and limited (LA) water supply for full (FI) and deficit irrigation (DI) strategies.

From the discussion above it is clear that DI has the potential to reduce the impact of water curtailments due to higher expected total gross margins resulting from irrigating larger irrigation areas even though gross margins are more variable. In Chapter 3 it is shown that assuming constant absolute $r_a(x)$ to compare risky alternatives may imply quite different degrees of implied risk aversion as measured by $r_s(x^s)$ if the variability of the alternatives that are being compared is different. In the following section, it is shown that the relationship between the premiums and absolute risk aversion can be explained by the implied risk aversion towards the alternatives.

5.2.2 IMPLIED RISK AVERSION TOWARDS ALTERNATIVE WATER USE OPTIMISATION STRATEGIES

The implied risk aversion towards the alternative water use optimisation strategies were studied by graphing the *ex post* calculations of $r_s(x^s)$ against the $r_a(x)$ values used in the optimisation model. The tradeoffs are shown in Figure 5.11.

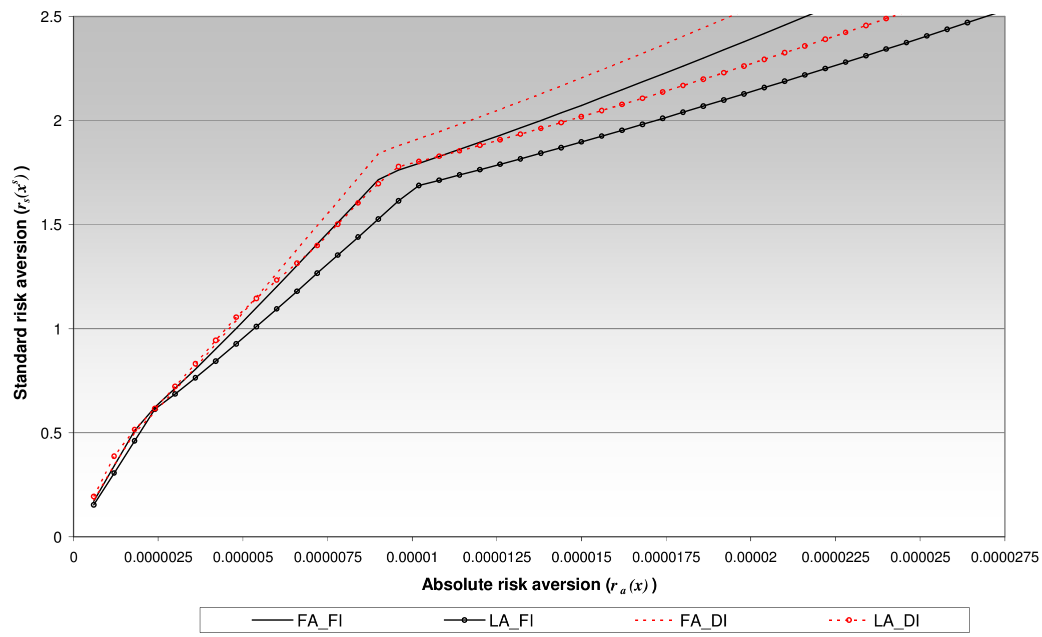

Figure 5.11: Implied risk aversion towards optimised scenarios under full (FA) and limited (LA) water supply conditions for full (FI) and deficit irrigation (DI) strategies.

The relationship between $r_a(x)$ and $r_s(x^s)$ for a specific scenario portrayed in Figure 5.11 shows three linear segments with different slopes. The first is from risk neutrality to about $r_a(x)$=0.0000025, the second from the last mentioned to about $r_a(x)$=0.00001 and the last segment stretches beyond $r_a(x)$=0.00001. The changes in the slopes of the alternatives are associated with structural changes in the composition of the crops in the crop mix or changes from one irrigation strategy to another. Along a specific segment a continuous tradeoff is modelled between crops and irrigation strategies.

Figure 5.11 shows that for a specific water supply scenario (FA or LA) the implied risk aversion towards DI is greater than FI for all the values of $r_a(x)$. At low levels of absolute risk aversion the difference between the alternatives are rather small. A larger value of $r_s(x^s)$ for a specific $r_a(x)$ value implies that the strategy will exhibit larger variability. When LAFI is compared to FAFI, the values of $r_s(x^s)$ for LAFI is consistently lower than FAFI. Such a result is possible since lower water availability causes the expected values to decrease as well as the standard deviations even though the crop mix might be proportionally the same as when water is not limited. In the specific case lower $r_s(x^s)$ values for LAFI compared to the baseline of FAFI resulted in decreasing compensation to increasingly risk averse farmers (FAFI vs. LAFI in Figure 5.10). When LADI is compared to a baseline of FADI the values of $r_s(x^s)$ for LADI is marginally greater than the values associated with the baseline from risk neutrality to about $r_a(x)$=0.000005. After this point, the $r_s(x^s)$ values of LADI start to become increasingly lower than FADI as absolute risk aversion increases. Interesting to note is that the range of $r_a(x)$ over which $r_s(x^s)$ of LADI is greater than the baseline corresponds to the range of $r_a(x)$ for which increasing compensation is necessary while decreasing compensation is necessary if the $r_s(x^s)$ of LADI is less than the baseline (FADI vs. LADI in Figure 5.10). Thus, it seems as if the utility weighted premiums are decreasing with increasing levels of absolute risk aversion if the implied risk aversion towards the baseline is greater than the alternative with which it is compared and visa versa. However, the same conclusion cannot be made when LADI is compared to a baseline of FAFI. Important to note is that the *CE's* are influenced by the expected outcome and the risk premium. The expected values that are optimised for a specific scenario decrease with increasing levels of absolute risk aversion. If the rate at which the expected values decrease as absolute risk aversion increases (slope of the tradeoff curve) is dissimilar between the alternatives that are compared, the conclusion may not hold. Evaluation of the expected value absolute risk aversion tradeoff curves indicated that the curves for LADI and FAFI are not parallel. The slope of LADI is greater causing the expected values of LADI to decrease more than FAFI as absolute risk aversion levels increase. Thus, the result that the utility weighted premiums between LADI and FAFI are increasing (FAFI vs. LADI in Figure 5.10) as absolute risk aversion is increased might be explained by the result that the expected values of LADI decrease faster than FAFI and not because of decreasing implied risk. The conclusion is that both the changes in the expected value and the variability of a risky prospect will determine the compensation necessary to induce a farmer to change his actions.

The fact that the difference between the $r_s(x^s)$ values of any combination of alternatives is increasing with increasing levels $r_a(x)$ highlights the importance of appropriately scaling $r_a(x)$ to calculate *CE's*. Next, the alternative farm plans that were optimised for each of the $r_a(x)$ values are subject to a SERF analysis with constant standard risk aversion.

5.2.3 STOCHASTIC EFFICIENCY ANALYSIS OF THE OPTIMISED WATER USED STRATEGIES WITH CONSTANT STANDARD RISK AVERSION

The procedure that is developed in Chapter 4 to conduct stochastic efficiency with respect to a negative exponential utility function with constant $r_s(x^s)$ is applied in this section to the optimised water use plans derived in the previous section. The stochastic efficiency frontier for each water supply water use strategy combination is determined by identifying the maximum *CE* for a given level of $r_s(x^s)$. The results of the analyses are shown in Figure 5.12 and Figure 5.13.

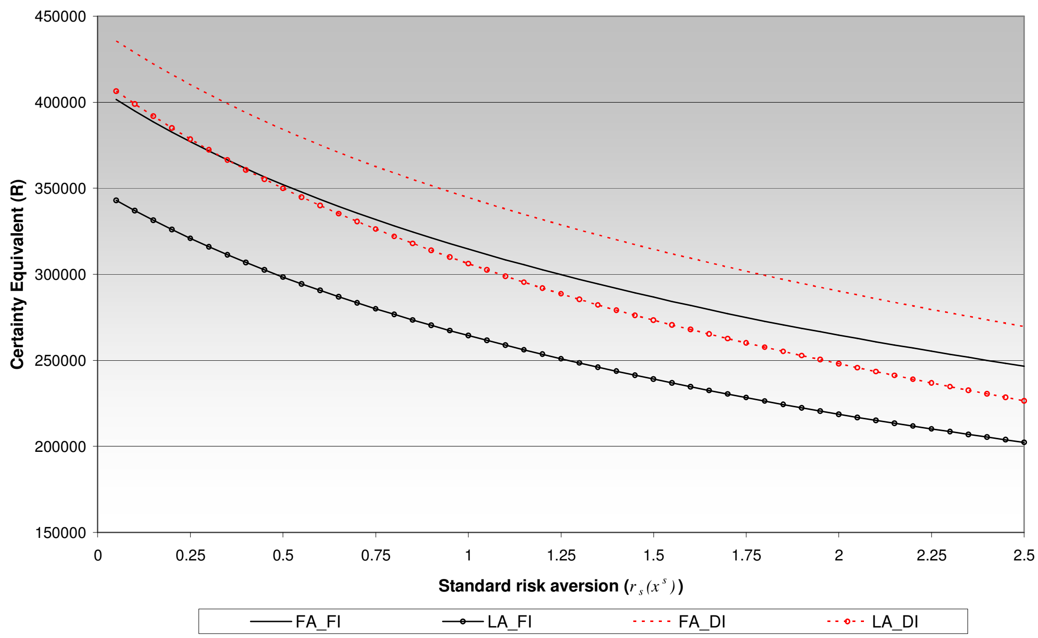

Figure 5.12: Standard risk aversion stochastic efficiency frontiers under full (FA) and limited (LA) water supply conditions for full (FI) and deficit irrigation (DI) strategies.

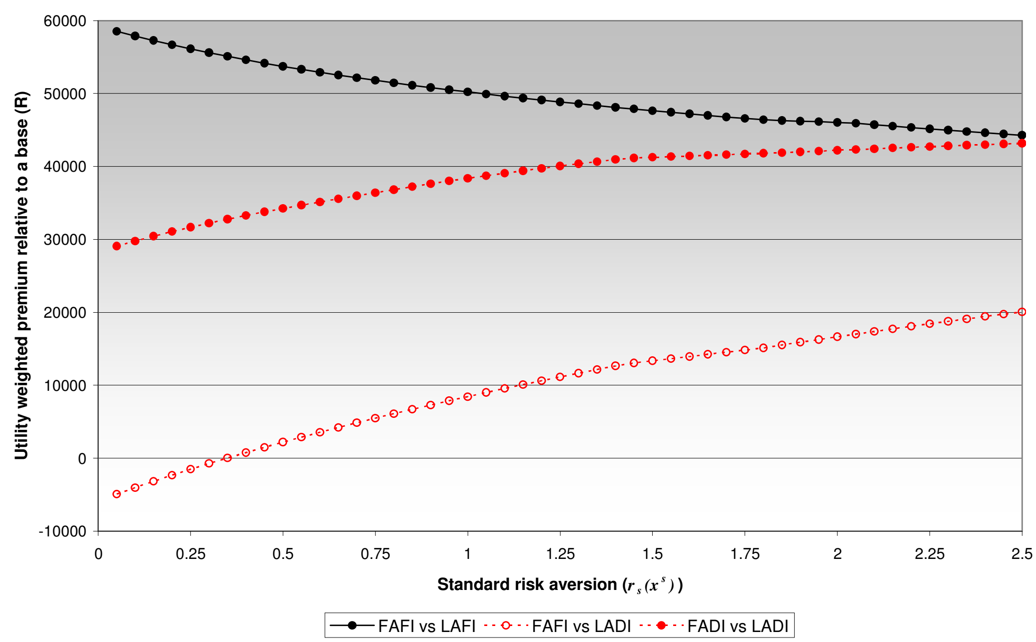

Figure 5.13: Standard risk aversion utility weighted premiums between full (FA) and limited (LA) water supply for full (FI) and deficit irrigation (DI) strategies.

No difference in the rankings of the alternatives is observed when the stochastic efficiency frontiers are compared to those derived from the utility maximisation model with $r_a(x)$ as the measure of risk aversion. The only exception is that the results in Figure 5.12 indicate that for values of $r_s(x^s) < 0.35$ LADI dominates FAFI. The reduction in the *CE's* from risk neutrality to $r_s(x^s)$ =2.5 is about R166 000 and R180 000 respectively for FADI and LADI which is greater than the corresponding reductions of R155 000 and R141 000 respectively for FAFI and LAFI. Thus, the reduction in *CE's* for each scenario is less than the corresponding reduction shown in Figure 5.9.

The negative exponential utility weighted premiums with constant risk aversion are shown in Figure 5.13 to determine the impact of increasing risk aversion on the level of compensation necessary to foster water conservation.

The utility weighted premiums calculated with $r_a(x)$ are quite different from those calculated with $r_s(x^s)$. It should be noted that direct comparisons are not desirable because the procedure employed in this section automatically scales $r_a(x)$ of the alternatives such that $r_s(x^s)$ is constant. Negative premiums are calculated for $r_s(x^s) < 0.35$ when LADI is compared to FAFI, which indicates that the stochastic efficiency frontier of LADI is above FAFI. The impact of increasing standard risk aversion on the premiums is also more consistent than before because the premiums either increase or decrease as $r_s(x^s)$ increases. When DI is considered increasingly risk averse farmers need to be compensated more than less risk averse farmers to adopt DI under limited water supply conditions irrespective of whether the baseline is FAFI or FADI. On the contrary, a negative relationship between increasing standard risk aversion and the calulated premiums for FI exists and as a result more risk averse farmers need to be compensated less than less risk averse farmers in order to conserve water.

5.2.4 CONCLUSIONS

An important conlusion is that deficit irrigation is stochastically more efficient than full irrigation under limited water supply conditions due to the larger areas irrigated. Important to note is that larger irrigated areas imply that water that is saved by deficit irrigation is used to irrigate these areas and therefore water diversions are not reduced by deficit irrigation. Although deficit irrigation increased the gross margins the increase is unable to compensate for the loss in total gross margin due to reduced water allocation. Larger $r_s(x^s)$ values are also associated with deficit irrgation meaning that the variability of gross margins under deficit irrigation will be higher. The conclusion is that farmers will not willingly choose to conserve water through deficit irrigation. Furthermore, deficit irrigation will not save water if increasing areas irrigated is an important factor determining the overall profitability of the strategy.

The result that the constant absolute risk aversion utility weighted premiums exhibit both an increasing and decreasing relationship with increasing levels of risk aversion when LADI is compared to FADI, highlights the limitation of the absolute risk aversion measure. The fact that the relationship could be explained by changes in $r_s(x^s)$ demonstrate the importance of considering standard risk aversion as an alternative to the absolute risk aversion measure. An important conclusion in terms of the procedures used is that the standard risk aversion utility weighted premiums that are calculated are not optimal. It is merely an evaluation of the farm plans that are optimal in terms of a specific $r_a(x)$. No attempt is made in this research to develop an optimisation model that will optimise farm plans that are optimal in terms of $r_s(x^s)$.

CHAPTER 6
SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

This chapter provides a summary of each chapter, while the final section is devoted to recommendations for water conservation policy and further research.

6.1 INTRODUCTION

6.1.2 BACKGROUND AND MOTIVATION

South Africa is currently undergoing a phase of water allocation reform. While South Africa's National Water Act and National Water Policy provide the legislative and policy framework for water allocation, they do not provide detailed strategies and approaches to promote equity, sustainability and efficiency in water use, or a process to roll this out across the country. A complicating factor is that half of the water management areas are experiencing water deficits, while the country as a whole is in surplus. However, estimates are that the country's status will change in the near future from a water-scarce to a water-stressed status. The current reality is that in many instances it is not practical or economically viable to transfer water from surplus to deficit areas, resulting in localised water scarcities. Furthermore, the potential options for supply augmentation are limited and attention will have to be given to managing the increasing demand for water as an alternative to reconcile imbalances between water requirement and availability through the use of water conservation and demand management principles (Backeberg, 2006). Water conservation and demand management relate to measures to improve the efficiency of water use and the reallocation of water from lower to higher benefit uses within or between water-use sectors. The implementation of water conservation and demand management would have some serious implications for irrigated agriculture, since it accounts for 62% of all water used in South Africa, with Government arguing that in many instances the water use is highly inefficient (DWAF, 2004b). The water conservation and demand management strategy for the agricultural sector makes it clear that the irrigated agriculture sector will be targeted as a source of water that can be made available to competing water users and the environment through the implementation of water conservation and demand management principles within the sector. Central to the strategy is the use of a pricing strategy as a powerful tool to reduce water demand and enhance water use efficiency (DWAF, 2004b).

Much research has been funded in South Africa in view of the development of decision support systems to enhance efficient water use. These efforts have mostly concentrated on the development of models to estimate crop water requirements so as to enhance irrigation planning

(Crosby and Crosby, 1999) and simulation models to enhance real-time irrigation scheduling whereby water applications are minimised to achieve maximum crop yields (Annandale *et al.*, 1999). English *et al.* (2002) argue that a paradigm shift is necessary to manage agricultural water use in future. The new paradigm would mean that irrigation applications would be based on economic efficiency principles rather than the application of irrigation water to achieve maximum crop yield. Optimising water use based on economic principles implies taking into consideration all the relevant costs and revenues and the opportunity cost of water (scarcity value) while allowing the crop to sustain some level of water stress resulting in yield reductions due to deficit irrigation. Many farm-level variables, such as irrigation technology, crop water requirements, crop yield response to water deficits, infrastructural constraints limiting water supply, credit availability, and input and output prices of crops, will determine the opportunity cost of water. Generally the interaction among these variables at farm-level is not well understood. A clear need exists for decision support that is able to integrate relevant information from different sources to achieve optimal water use at farm-level. Providing integrated decision support is complicated by the fact that some irrigators are risk averse and are operating in a deregulated market environment characterised by increased price volatility, and that deficit irrigation will decrease expected crop yields while increasing the variability thereof.

6.1.2 PROBLEM STATEMENT AND OBJECTIVES

Water managers are currently unsure about the effectiveness of alternative water conservation and demand management instruments such as increasing water charges and the promotion of alternative water conserving management practices that hamper water conservation and demand management in the agricultural sector. This uncertainty stems from a lack of understanding of the interaction of farm-level variables that influence optimal water use and the profitability of alternative water management options within the dynamic and stochastic environment within which farmers have to make decisions. A lack of models that are able to model these interactions satisfactorily while taking cognisance of the dynamics within irrigated agriculture, the development of the farm firm and the risks of agriculture further hamper the identification of feasible and profitable alternatives that will conserve water in the irrigated agricultural sector.

Evaluating alternative water conservation options has both a long- and a short-run dimension. Over the long run, irrigators need to decide on the adoption of appropriate irrigation technology and the cultivation of perennial crops with delayed income streams. Once the irrigation technology and area allocated to perennial crops are fixed, irrigators need to decide upon the allocation of limited water supplies amongst multiple crops within a season. Most researchers and decision-makers acknowledge the importance of taking risk into account when conducting profitability and feasibility analyses. However, most researchers choose to assume risk away due to a lack of data to quantify risk, increased modelling time, the expertise necessary to conduct risk analyses, and the difficulty of choosing realistic absolute risk aversion levels. Choice of absolute risk aversion levels is especially difficult, since the invariance property of arbitrary linear transformations of the utility function do not apply to arbitrary rescaling of the outcome variable (Raskin and Cochran, 1986). By implication some form of rescaling of the absolute risk aversion coefficient is necessary to represent risk aversion consistently. The problem is that there are inconsistencies between the alternative methods used to scale absolute risk aversion levels.

The main objective of this research is to develop models and procedures that will allow water managers to evaluate the impact of alternative water conservation and demand management principles in irrigated agriculture over the long-run and the short-run while taking risk into account.

Specific objectives are to develop:

- A generalised whole-farm stochastic dynamic linear programming model to evaluate the impact of price incentives to conserve water when irrigators have the possibility to adopt more efficient irrigation technology or cultivate high-valued crops.

- An expected utility optimisation model to economically evaluate deficit irrigation within a multi-crop setting as a strategy to conserve water while taking into account the increasing production risk of deficit irrigation.

- A procedure to standardise the choice of Arrow-Pratt absolute risk aversion coefficients for application with stochastic efficiency analysis techniques.

6.1.3 RESEARCH AREA

The research is conducted in the Vaalharts irrigation scheme located east of the Ghaap plateau, on the border between the Northern Cape and North West provinces. The area covers about 36 950 ha and is one of the largest irrigation areas in the world. The average rainfall is 442 mm per annum, occurring mostly in the form of irregular heavy thunderstorms during summer. The evaporative demand is high, which necessitates irrigation. The scheme is supplied with water abstracted from the Vaal River at the Vaalharts Weir about 8 km upstream of Warrenton. Canals are used to convey the water to the scheme and to distribute the water to 680 irrigation farmers once it reaches the scheme. Farm sizes are small, with 74% of farms under 100 ha. The irrigation types used predominantly are flood and pivot irrigation. Cash crops are by far the most important, and the most commonly found cash crops are wheat/barley, maize, groundnuts and cotton. Lucerne and pecan nuts are the most important permanent crops grown in the region.

6.2 LITERATURE REVIEW ON CROP WATER USE OPTIMISATION

A literature review was conducted to guide the development of procedures and models to optimise agricultural water use. Three aspects were identified from the literature as being of the utmost importance when developing models to optimise agricultural water use.

Firstly, a linear relationship exists between evapotranspiration and crop yield, while the relationship between applied water and crop yield is non-linear. Optimising agricultural water use is complicated, since irrigators only have control over the amount of applied water used to satisfy evapotranspiration demands, which necessitates the modelling of the non-linear relationship between applied water and crop yield. Recent trends in applied research on the optimisation of crop water use reveal that researchers are increasingly focussing on modelling the non-linear relationship between applied water and crop yield using the non-uniformity with which irrigation systems apply water linked to the Stewart multiplicative relative evapotranspiration formula. However, modelling procedures to simulate the impact of non-uniform applications on crop yield have only recently been adopted by South African researchers. Strikingly, the South African literature revealed a large number of optimisation studies that have followed the old paradigm of applying water to achieve maximum crop yields, thereby ignoring deficit irrigation possibilities. Results from optimisation studies incorporating deficit irrigation show that failure to model the non-linear relationship between applied water and crop yield results in underestimation of the potential benefits of deficit irrigation if it is profitable to deficit irrigate the crop.

Secondly, irrigation water is differentiated from other agricultural inputs in that crop yields are influenced by the stock of field water supply (irrigation water stored in the root zone, effective rainfall, and soil water carryover) rather than the specific amount of water applied during a particular period of time. By implication, irrigation decisions in different time periods will influence water availability for crop production in subsequent time periods, which highlights the importance of modelling the interdependencies between water use in different crop growth stages. The importance of modelling the interdependencies is furthermore highlighted by the fact that water deficits in different crop growth stages will impact differently on final crop yield. Dynamic programming or simulation-optimisation procedures whereby sophisticated search procedures are linked to crop growth simulation models are typically used to optimise the interdependencies between water management decisions. A problem with these approaches is that simplified assumptions need to be made to keep the models tractable, since adding more detail quickly results in models that are too large when the whole farm is considered. Furthermore, dynamic programming applications of water use optimisation between multiple crops typically boil down to a multi-tiered approach. At the first tier, optimal production functions are derived for use at the second tier to optimise water use between multiple crops. Cognisance should be taken of intraseasonal water supply assumptions and the economic theoretical principles that determine optimal water use when the multi-tier approach is followed. Economic theory suggests that seasonal crop water production functions assume technical efficiency and optimal distribution of irrigation water over the growing season. However, when multiple crops compete for limited water supplies where the intraseasonal water supply is limited, economic theory suggests that technical efficiency of water applications to a single crop will not be met due to the increased scarcity value of water when water is limited in a specific time period. Under such water supply conditions, incorporating information on optimal production functions in a second-tier optimisation model to allocate water optimally between competing crops will result in non-optimal solutions.

Thirdly, deficit irrigation may increase yield risk. The literature review revealed that reductions in applied water will lead to increasing reductions in expected crop yields and will most likely increase yield variability as the crop is deficit irrigated. English *et al.* (2002:272) furthermore argue that when the opportunity cost of water is taken into account and it is optimal to reduce water applications and at the same time increase the area irrigated, any losses that may be incurred will be amplified by the increased area under irrigation. Results from optimisation studies indicate the importance of increasing the profitability of deficit irrigation by increasing the area irrigated. Such a strategy will increase the irrigator’s exposure to risk. The importance of realistically modelling the interaction between crop growth, field water supply and weather is also highlighted by the review. The potential to use rainfall more efficiently has a significant impact on the adoption of deficit irrigation strategies by risk-averse decision-makers. Any information that will increase the potential to use rainfall more efficiently, such as improved localised weather forecasts, will improve the adoption of deficit irrigation strategies. However, use of deficit irrigation in areas where rainfall is low may cause risk-averse farmers to adopt full irrigation. Thus, evaluation of the feasibility of deficit irrigation is area specific.

The review of the South African literature on crop water use optimisation indicated that no single study has been able to simultaneously model the non-linear relationship between applied water on crop yield, the interdependencies between irrigation decisions in different time periods, risk and the opportunity cost of water. A procedure is proposed whereby the dynamic problem of optimising water use between multiple crops while taking cognisance of the intraseasonal water supply limitations is approximated with linear programming (Bernardo *et al.,* 1987). The approximation is based on the inclusion of a large number of discrete activities representing alternative ways of distributing water over the growing season in a linear programming model. Information for the activities is simulated with a crop-growth simulation model. The simplicity of the approach is appealing, because incorporating the non-linear relationship between applied water and crop yield while taking cognisance of the impact of water deficits in different crop growth stages, as well as production risk and other farm-level constraints, is straightforward. The creditability of such a procedure was recently demonstrated by Scheierling *et al.* (2004) who applied the procedure to determine the price responsiveness of the demand for irrigation water deliveries and consumptively used water.

6.3 CHOICE OF RISK AVERSION LEVELS FOR STOCHASTIC EFFICIENCY ANALYSIS

Decision-making under risk requires that the decision-maker integrates his/her subjective views on the variability of a specific outcome variable (risk quantification) and his/her preferences for those outcomes (utility). Since the preferences of decision-makers are not always known, stochastic efficiency criteria have been developed. Stochastic efficiency criteria allow some ranking of risky alternatives when the decision-makers' preferences for alternative outcomes are not exactly known. Usually some assumptions are made with respect to preferences, which translate into evaluating risky alternatives over a range of risk aversion levels to establish an efficient set of alternatives containing all the alternatives preferred by a decision-maker to whom the assumptions apply. However, there is still the problem of deciding upon appropriate ranges of risk aversion. The main objective of this part of the research is to establish plausible ranges of risk aversion for use with stochastic efficiency analysis techniques.

Choice of appropriate ranges of absolute risk aversion is difficult, since arbitrary scaling of the outcome variable changes plausible representations of risk aversion by the Arrow-Pratt absolute risk aversion coefficients. McCarl and Bessler (1989) indicated that both the level of wealth and the dispersion of the risky prospect influence creditable representations of absolute risk aversion levels. Therefore, assuming the same levels of absolute risk aversion may imply vastly different levels of risk aversion when alternatives with varying levels of variability are compared relatively. Thus, some form of scaling of Arrow-Pratt absolute risk aversion coefficients is necessary before they may be applied. The notion of scaling risk aversion coefficients is not new and several researchers have developed procedures to scale absolute risk aversion coefficients. However, there is little consistency amongst the alternative scaling methods.

Babcock *et al*. (1993) developed a procedure whereby the choice of absolute risk aversion level is made in such a way that the risk premium as a percentage of the size of the gamble is equal across alternatives that are compared. By implication the value of the absolute risk aversion coefficient is determined for each alternative based on the dispersion of the data. Evaluating the research of Raskin and Cochran (1986) revealed that a prerequisite for using their first theorem to scale absolute risk aversion is that all the alternatives must exhibit constant relative risk (same coefficient of variation) to yield answers consistent with the procedure developed by Babcock *et al*. (1993). Ferrer (1999) applied a procedure developed by Nieuwoudt and Hoag (1993) to standardise absolute risk aversion by expressing the data as a percentage of the range $(x_{max}-x_{min})$. The applicability of the procedure is limited, since it will only yield consistent answers when compared with the procedure developed by Babcock *et al*. (1993) if the risky alternatives that are compared consist of two equally likely outcomes. Thus, the relationship with the procedure of Babcock *et al*. (1993) is highly circumstantial. In this research, a new procedure has been developed whereby absolute risk aversion is scaled between alternatives based on the standard deviation of the risky prospects – hence it is referred to as standard risk aversion. This standardisation procedure has been developed by means of exploring a particular property of constant absolute risk aversion utility functions, namely that risk aversion is unaltered if a constant is added to or subtracted from each random outcome. By implication, constant absolute risk aversion imposes the requirement that alternative distributions may be compared with the same absolute risk aversion coefficient if the standard deviations of the distributions are equal. Any distribution may be transformed such that the resulting standard deviations after the transformation are all equal to one. This result is achieved if each payoff is divided by the standard deviation of the distribution of payoffs. Once the distributions are standardised such that the standard deviations of all the distributions are equal, the same level of standard risk aversion may be used with a utility function that exhibits constant absolute risk aversion to represent risk aversion. This newly formalised method is shown to be consistent with the standardisation procedure of Babcock *et al.* (1993). Furthermore, standard risk aversion is also shown to be equivalent to the risk aversion parameter used in applied MOTAD studies.

The above procedure is applied in this research to standardise values of absolute risk aversion used in secondary studies with the objective to establish a plausible range of risk aversion levels for use with stochastic efficiency analysis techniques. After evaluating research studies that apply the procedure developed by Babcock *et al.* (1993), elicited values of absolute risk aversion or derived absolute risk aversion values from applied MOTAD studies a range for standard risk aversion between zero and 2.5 is established to represent risk aversion.

Important to note is that constant absolute risk aversion is assumed in this research. Thus, the impact of wealth on risk aversion is ignored. Recent research by Meyer and Meyer (2005) argues in favour of using relative risk aversion as a measure for comparison.

6.4 RISK QUANTIFICATION AND CROP WATER USE OPTIMISATION MODEL DEVELOPMENT

Stochastic budgeting procedures are used to generate appropriately correlated inter- and intra-temporal matrixes of gross margins necessary to incorporate risk into the long-run and short-run water use optimisation models. Before conducting the stochastic simulations, yield risk and price risk need to be quantified.

Crop yield risk of alternative irrigation schedules is quantified by combining simulated crop yield indices with subjectively elicited crop yield distributions. SAPWAT (Crosby and Crosby, 1999) is a locally available model that uses a simple cascading water budget to estimate crop water requirements based on internationally accepted principles (FAO-56, Allan *et al.*, 1998) and is regarded as the standard in estimating crop water requirements in South Africa. The model is provided with a database that includes weather variables for three alternative states of nature and crop coefficients for a variety of crops grown in South Africa. A shortcoming of the model is that it does not include procedures to model the impact of the non-uniformity with which irrigation systems apply water on crop yield. The model is therefore further developed by incorporating into the model certain procedures developed by Li (1998) to calculate crop yield indices (percentage of maximum potential) with the Stewart multiplicative relative evapotranspiration formula, taking non-uniform irrigation applications into account. The simulated triangularly distributed crop yield indices for each irrigation schedule are combined with subjectively elicited irrigation system-specific crop yield distributions associated with full irrigation to quantify yield risk of deficit irrigation. Output price risk is characterised as empirical distributions of historical price information. GAMS code is developed to simulate the multivariate probability distributions based on procedures developed by Richardson *et al.* (2000) and to construct the necessary inter- and intra-temporal risk matrixes of stochastic gross margins for the respective programming models.

The structure of the dynamic linear programming model that is used to optimise water use over the long run (15 years) follows a disequilibrium known life type of specification (McCarl and Spreen, 2003) and was developed by Grové (2006a). The objective of the model is to maximise the present value of after-tax cash surpluses at the end of the planning horizon, plus terminal values for any activity beyond the planning horizon, minus a MOTAD risk premium. The normative approach proposed by Rae (1970) is used to calculate terminal values for each activity as the present value of future net revenue discounted from infinity for an assumed replacement cycle. Terminal values ensure that capital investments with cash flow streams beyond the planning horizon are not penalised. Terminal values are calculated for cropping activities, borrowing activities, irrigation investments and production loans, since all these activities may extend past the planning horizon. For details of the calculation procedures and a description of the GAMS code used to generate appropriate terminal values, the reader is referred to Grové (2006a).

Special care is taken to model cash flows. A cash surplus in any given year exists if the sum of production income, money in the bank account (including interest earnings) and any salvage income is more than the sum of all overhead expenses, loan repayments, living expenses and tax liabilities. Taxable income is a function of production income, operating expenses, salvage income, overheads, interest and depreciation deductions, as well as any losses transferred from the previous year. The dynamic linear programming model has the unique ability to defer tax payments until a positive taxable income is calculated. A link is established between different years through the bank account. Cash surpluses from the previous year can be used to purchase new irrigation technology and/or to finance operating expenses with any surplus deposited in a bank account. The model furthermore allows for the use of production loans as a means to finance production cost, and borrowed capital to finance irrigation investments. The amount of money that might be borrowed in any given year is limited by the credit facilities and the amount outstanding.

The model is structured as such that an investment in an irrigation system is made first, before any cropping activities can take place. The model includes five alternative crops (pecan nuts, lucerne, maize, groundnuts and wheat) and three alternative irrigation technologies (flood, small-pivot, and large-pivot). Only full irrigation strategies are included in the model due to the increased size of the programming model over multiple years when deficit irrigation activities are included in the model. Several factors, including irrigation system capacity, conveyance capacity, the capacity of the canal off-takes and the total water allocation, can limit the amount of water that can be applied to a crop within a specific time period.

A non-linear mathematical programming model is developed to evaluate the impact of limited water supply conditions on the decision regarding optimal water allocation amongst multiple crops while explicitly taking the production risk of deficit irrigation into account. Only pivot irrigation activities for maize, groundnuts and wheat are considered. The basic structure of the dynamic linear programming model in terms of land availability, resource use and irrigation water supply is retained for the short-run model specification. The objective function of the model follows the direct expected maximisation non-linear programming specification as presented by Boisvert and McCarl (1990), with the exception that certainty equivalent is maximised. Thus the stochastic efficiency frontier with respect to a negative exponential utility function with constant absolute risk aversion is presented by the objective function values.

In this research it is argued that absolute risk aversion coefficients should be chosen such that standard risk aversion is constant amongst the alternatives that are evaluated and that the maximum value of standard risk aversion should not exceed 2.5. While it is easy to scale absolute risk aversion for predefined alternative strategies to be compared, it is not possible to specify the relationship within a mathematical programming model. At best the relationship between absolute risk aversion and standard risk aversion can only hold *ex post* to the optimisation, because standard risk aversion is dependent on the standard deviation of the optimal farm plan. The optimised farm plans for each of the absolute risk aversion values considered during the short-run optimisation are therefore also evaluated by conducting a stochastic efficiency with respect to a function analysis with constant standard risk aversion values. In order to conduct the stochastic efficiency with respect to function analyses with constant standard risk aversion, a procedure is developed that will allow one to directly specify standard risk aversion as the risk aversion parameter in a constant absolute risk aversion utility function.

6.5 LONG-RUN AND SHORT-RUN MODELLING RESULTS

6.5.1 LONG-RUN RESULTS AND CONCLUSIONS

The stochastic dynamic linear programming model is used to evaluate the impact of price incentives to conserve water when irrigators have the option to adopt more efficient irrigation technology or cultivate high-value crops. More specifically the model is used to study the impact of risk aversion and the availability of starting capital on the derived demand for irrigation water and the associated expected net present value for three alternative farm developing scenarios. Scenario FLOOD presents the base case where the decision-maker is only allowed to produce maize, groundnuts, wheat and lucerne with flood irrigation. Scenario PIVOT is the same as scenario FLOOD, but allows for the possibility of the adoption of centre pivot irrigation. The final scenario, PECAN, is also similar to the base scenario, but allows for the production of pecan nuts under flood irrigation.

The net present value water availability trade-off curves indicate that scenarios PIVOT and PECAN are more profitable than FLOOD over the whole range of water availabilities. However, when PIVOT and PECAN are compared, PIVOT dominates PECAN when water is increasingly curtailed due to the fact that the net present value of PECAN is reduced more than with any of the other scenarios when water is curtailed. When water is curtailed the expected income of the pecan farm is reduced and as a result fewer pecan nut trees with delayed income streams are established and more cash crops are grown to sustain cash flows. PIVOT is also able to sustain higher levels of water curtailments, because pivot irrigation is more efficient when compared to flood irrigation. Thus, the threshold amount of water necessary to operate profitably is less than the other two scenarios. The net present values of all the scenarios are lower when risk is considered. However, risk seems to be of less importance in terms of expected net present value differences when water is severely limiting the financial feasibility of the farming operation. Starting capital does not affect the shape of the trade-off curves, but significantly affects the ability of the farmer to sustain higher levels of irrigation water allocation reductions as indicated by shifts in the trade-off curves to the left when starting capital was increased. The conclusion is that the relative profitability of alternative water use strategies and cash flows, which determine the ability to adopt modern irrigation technology and to establish high-value crops, plays a significant role in the farmer's ability to sustain water curtailments.

Each scenario's derived demand for irrigation water is studied to determine the impact of risk aversion and the availability of starting capital on the shadow values of irrigation water. All the derived demand curves are similar when the total price range is considered and are characterised by a relatively elastic derived demand at very low prices and relatively inelastic derived demand at higher price ranges. However, the transition from the relatively elastic to the relatively inelastic phase for PECAN is not as abrupt as for the other scenarios. Risk aversion also causes the derived demand curves to enter the relative inelastic phase more rapidly. Evaluation of the derived demand curves at low price ranges shows that the derived demand curve for PIVOT lies to the left of FLOOD whereas PECAN lies to the right of FLOOD. Thus, with a specific water allotment, the willingness to pay for water will be highest for PECAN and lowest for PIVOT. The fact that the willingness to pay for water for PIVOT is lower than for FLOOD is surprising when one considers that pivot irrigation uses less water and achieves higher and less variable crop yields when compared to flood irrigation. However, pivot adoption is only possible if the technology is financed and therefore the willingness to pay for water is lower. Generally, the availability of starting capital causes the derived demand curves to shift increasingly to the left and as a result higher capital availability is associated with lower willingness to pay for water. Both FLOOD and PIVOT show that risk aversion causes irrigation farmers to value irrigation water more than a risk-neutral farmer, as water becomes scarcer when more water is allocated. The level at which an irrigator views water as being scarce is a function of its impact on cash flows and therefore also the availability of starting capital. When PECAN is considered, the derived demand curves show that risk averse irrigators, value irrigation water more than risk-neutral farmers if water is scarce (impacting severely on cash flows) and when irrigation water availability is almost abundant. It almost seems as if water is used to reduce risk rather than to increase expected income. The conclusion is that one should be cautious in assuming that higher gross margins per unit of applied water will necessarily result in greater willingness to pay for water. Results demonstrate that financing lumpy irrigation technology and risk aversion will impact significantly on willingness to pay for irrigation water.

Arc elasticity estimates indicate that the elasticity of the quantity of irrigation water demanded is low. Important to note is that the elasticity estimates are influenced by the shape of the demand curve, as well as the position on the curve. Therefore it is difficult to determine the impact of risk aversion and the availability of starting capital on elasticity estimates. Although elasticity estimates are low, fairly large reductions in the quantity of irrigation water demanded are possible if water tariffs are increased from zero. Such a result is supported by Scheierling *et al.* (2004). However, irrigation water allotments are rationed and the effectiveness of water tariff increases on water conservation should rather be studied at the point of truncation. When water tariffs are low in relation to the scarcity value of irrigation water, price increases will not be effective in reducing the quantity of irrigation water demanded. Calculated absorbed scarcity rents indicate that water tariffs need to increase by a factor of approximately six for PECAN and more or less double that for FLOOD in order to foster water conservation through water tariff increases. Given the assumptions made and the prices assumed, PIVOT will be responsive to even small water tariff increases.

The overall conclusion is that risk aversion and the individual farming situation will have an important impact on the effectiveness of water tariff increases when it comes to water conservation. Adopting more efficient irrigation technology may not improve the farmer's ability to pay for water if the lumpy technology needs to be financed. Failure to take risk into account will furthermore cause researchers to either over- or underestimate the shadow value of water, depending on whether water is valued by the irrigator as relatively abundant or scarce. Care should also be taken when interpreting elasticity estimates from literature without knowing the conditions under which they were derived.

6.5.2 SHORT-RUN RESULTS AND CONCLUSIONS

The non-linear programming model that maximises certainty equivalent is used to optimise water use amongst multiple crops for a full irrigation and deficit irrigation strategy while considering the increasing production risk of deficit irrigation. A full water allocation scenario where 9 140 m^3/ha is available to irrigate 76 ha under pivot irrigation and a scenario where water allocation is limited to 80% of the full water allocation are considered.

The optimised stochastic efficiency with respect to negative exponential utility function efficiency frontiers indicate that the deficit irrigation strategies for both the water supply scenarios are stochastically more efficient than their full irrigation counterparts. Thus, it is profitable to use the water that is saved by deficit irrigation to irrigate larger areas in spite of increased production risk. However, increasing levels of absolute risk aversion cause the certainty equivalents of the deficit irrigation strategies to decrease more than the full irrigation strategies for the range of risk aversion levels considered. Utility-weighted premiums are used to determine the minimum sure amount that has to be paid to a decision-maker to justify a switch from a full water allocation full irrigation scenario to a less preferable alternative when water is curtailed by 20%. Positive premiums are calculated, which indicate that farmers will not willingly conserve water and that they need to be compensated to conserve water irrespective of the water use strategy employed. More interesting is the relationship between the utility-weighted premiums and absolute risk aversion. Results indicate that irrigators who practise full irrigation need decreasing levels of compensation with increasing levels of risk aversion, while irrigators who practise deficit irrigation need increasing levels of compensation with increasing levels of risk aversion if water is curtailed. The level of compensation that needs to be paid for deficit irrigation is, however, less than that for full irrigation. Deficit irrigation is also compared to a baseline scenario of full water allocation with deficit irrigation to take into account that some irrigators may already be practising deficit irrigation. Results indicate that the level of compensation increases to a maximum, after which it starts to decrease with increasing levels of absolute risk aversion. Thus, it is possible to conclude that risk aversion has no impact on the level of compensation necessary, because it is possible to calculate the same utility-weighted premium for decision-makers with very different risk preferences. However, in this research, it is argued that assuming constant absolute risk aversion to compare risky alternatives may imply quite different degrees of standard risk aversion.

The relationship between absolute risk aversion and standard risk aversion for a specific scenario shows three linear segments with different slopes where the changes in the slopes of the alternatives are associated with structural changes in the variability of the optimised farm plans. The implied risk aversion towards deficit irrigation is consistently greater than full irrigation for all the values of absolute risk aversion when a specific water supply scenario is considered that indicates that deficit irrigation is associated with greater gross margin variability. When the standard risk aversion levels of the pairs of alternatives that are used to calculate the utility-weighted premiums are evaluated, it seems that the utility-weighted premiums are decreasing with increasing levels of absolute risk aversion if the implied risk aversion towards the baseline is greater than the alternative with which it is compared and visa versa. However, one should caution against generalising such a statement, because the results also indicate that the rate at which the expected values change as risk aversion is increased will impact on the relationship. The conclusion is that both changes in the expected value and the variability of a risky prospect will determine the compensation necessary to induce farmers to change their actions.

To complete the analysis of deficit irrigation the optimised farm plans are subject to a stochastic efficiency with respect to an exponential utility function with constant standard risk aversion. The stochastic efficiency rankings amongst the alternatives are very similar to the results obtained when constant absolute risk aversion is assumed. However, results indicate that almost risk-neutral irrigators will prefer deficit irrigation to full irrigation even though water is curtailed by 20%. The reduction in certainty equivalents for each scenario is less than when constant absolute risk aversion is assumed, which may suggest that the range of absolute risk aversion coefficients used might be too large. The impact of increasing standard risk aversion on the utility-weighted premiums is shown to be more consistent than for those calculated with constant absolute risk aversion, since the premiums either increase or decrease as standard risk aversion increases. Careful evaluation of the efficiency frontier indicated that not all the optimised farm plans form part of the efficiency frontier. Thus, optimality in terms of constant absolute risk aversion does not imply optimality in terms of constant standard risk aversion.

The main conclusion is that although deficit irrigation is stochastically more efficient than full irrigation under limited water-supply conditions, irrigation farmers will not willingly choose to conserve water through deficit irrigation and need to be compensated to do so. Furthermore, deficit irrigation will not save water if the water that is saved through deficit irrigation is used to plant larger areas to increase the overall profitability of the strategy. The importance of considering standard risk aversion to discriminate amongst the alternatives is emphasised by the fact that the increasing and decreasing relationship between the utility-weighted premiums and increasing levels of absolute risk aversion could be explained by changes in standard risk aversion.

6.6 RECOMMENDATIONS

The main objective of this section is to make recommendations pertaining to water conservation policy and future research.

6.6.1 WATER CONSERVATION POLICY

Caution is necessary when formulating water conservation policy based on farm-level profitability analyses that ignore the mutual interaction among water legislation, water policy administration, technology, hydrology, human value systems and the environment. Only through a better understanding of these interactions will the policy goals of equitable access to water and sustainable and efficient use of water for optimum social and economic development be achieved. A new generation of decision support models is required in South Africa that will, on the one hand, illustrate the effect of alternative water policies on the economic efficiency of irrigation farming, and on the other hand, quantify the effect of irrigation farmers' actions on water resources within catchments. Such a view is supported by Whittlesey and Huffaker (1995).

Results from the deficit irrigation analysis indicate that water saved by deficit irrigation is used to irrigate larger areas. By implication the same amount of water is diverted, but through increased irrigated areas the total amount of consumptively used water has increased, resulting in less return-flow and/or deep percolation. Less return-flow may cause water allocation problems downstream if the return-flow has already been allocated to other water users. Less deep percolation, on the other hand, may have a positive or negative impact on water quality. Water is the primary transport medium of many agricultural pollutants such as nitrates. Reduced deep percolation will reduce groundwater pollution emissions. However, if a reduction in pollution emissions is accompanied by a reduced dilution capacity of the aquifer, the impact on groundwater quality will depend on the specific hydrological conditions of the area. Another environmental problem that is associated with deficit irrigation is that it may impact directly on the productivity of the soil through the build-up of total dissolved solids (salinity). In areas where salinity is a problem, the impact of deficit irrigation on soil productivity may cause deficit irrigation to be infeasible over the long-run if not managed.

The interaction between water conservation and its impact on the environment described above demonstrates the importance of taking a holistic approach when evaluating alternative strategies to conserve water, requiring well coordinated policy formulation.

6.6.2 FUTURE RESEARCH

Implementation of the National Water Act (Act 36 of 1998) has caused water managers, water-user associations and farmers to be confronted with new problems in their quest to implement the correct strategies and policies to achieve equitable access to water and to ensure sustainable and efficient use of water for optimum social and economic development. Application and further development of the models and procedures developed in this research may shed light on some of the problems confronting water managers. Herewith are some examples:

1. Salinity may compromise the long-run sustainability of deficit irrigation, and procedures should be developed to incorporate salinity into the modelling framework developed in this research to study the interaction between deficit irrigation, salinity, leaching requirement and water conservation. Water conservation requires more efficient use of irrigation water, while a leaching requirement is necessary to sustain crop production by leaching salt from the soil.

2. A cost benefit analysis should be conducted to determine whether the cost of metering actual water use will exceed the benefits of deficit irrigation where farmers need to prove that water allocations are not being exceeded if larger areas are planted with a deficit irrigation strategy. Many water user associations use an indirect measurement of water use that relies on determining the area planted and an estimate of the irrigation requirements of the crop under full irrigation. When farmers irrigate larger areas with deficit irrigation there will be a discrepancy between the actual and the estimated water use. The burden of convincing the water authority that the water use right is not being exceeded lies with the irrigator. The question is whether the cost of metering actual water use will exceed the benefits of deficit irrigation.

3. In many regions in South Africa, water supply is not regulated but is stochastic in nature, which necessitates the development of procedures to incorporate stochastic water supply into the models to facilitate decision support under stochastic water supply conditions.

4. The economic viability of alternative institutional water allocation arrangements should be evaluated with the generalised stochastic dynamic linear programming model to provide policymakers with the necessary information to decide whether or not such an arrangement should be institutionalised. Recently Pott, Hallowes, Mtshali, Mbokazi, Van Rooyen, Clulow and Everson (2005) proposed an institutional arrangement based on fractional water allocation and capacity sharing in South Africa.

5. Integrating the models developed in this research with information on climate change may provide policymakers with important information regarding the impact of climate change on the profitability of irrigation farming. The effects of climate change on water supply and crop water demand that directly influence the profitability of irrigation farming were recently demonstrated during a workshop held by the National Department of Agriculture (NDA, 2007).

6. Alternative means of levying water charges based on wealth distribution should be investigated, using the models to identify the most equitable alternative to achieve full cost recovery and water conservation and demand management. Although government policy clearly indicates that water user associations should move towards full cost

recovery and water conservation and demand management by increasing water charges, there is uncertainty regarding whether charges should be levied on a unitary or two-part basis, or on a volumetric or area basis, and whether subsistence farmers should be subsidised (Backeberg, 2005).

7. The models developed through this research may provide the means to meet the increasing need for profitability and financial feasibility studies due to the large number of irrigation schemes that are being revitalised and the prevailing sense of uncertainty regarding the economic impact of possible water curtailments during compulsory licensing of water use in water-stressed catchments.

8. SAPWAT should be further developed by incorporating the procedures developed in this research to quantify the effects of non-uniform water applications on crop yield, enterprise budgets, irrigation cost estimates and resource accounting activities so as to establish a comprehensive irrigation planning tool. The need for water user associations to provide comprehensive irrigation planning and irrigation scheduling advice has increased due to a decentralised management policy where water user associations will be held increasingly responsible for achieving water conservation and demand management while being faced with deteriorating extension services in South Africa (Backeberg, 2005).

A new procedure to represent risk aversion is also developed by means of this research, and the consistency with which decision-makers choose between risky alternatives as measured by this new method should be studied further. Procedures to develop a mathematical programming model that is able to search over alternatives with varying levels of variability based on constant standard risk aversion should be explored. More specifically, the use of genetic algorithms to model the highly non-linear relationship between constant standard risk aversion and scaled absolute risk aversion for use with constant absolute risk aversion utility functions should be investigated.

The long-run optimisation analysis does not include deficit irrigation strategies which merits further investigation of the linkages between the long-run and short-run decisions. Further research is also necessary to determine to what extent water markets can encourage more efficient water use at farm-level.

REFERENCES

ALLEN RG, PEREIRA L, RAES D and SMITH M. 1998. *Crop evapotranspiration: Guidelines for computing crop water requirements*. FAO Irrigation and Drainage Paper No 56. FAO. Rome. Italy.

ANNANDALE JG, BENADÉ N, JOVANOVIC NZ, STEYN JM and DU SAUTOY N. 1999. *Facilitating irrigation scheduling by means of the Soil Water Balance model*. Research Report to the Water Research Commission. WRC Report No 753/1/99. Pretoria: Water Research Commission.

ANNANDALE JG, STEYN JM, BENADÉ N, JOVANOVIC NZ, and SOUNDY P. 2005. *Irrigation scheduling using the Soil Water Balance (SWB) model as a user-friendly irrigation scheduling tool*. Research Report to the Water Research Commission. WRC Report No TT251/05. Pretoria: Water Research Commission.

APPELS D, DOUGLAS R and DWYER G. 2004. *Responsiveness of demand for irrigation water: A focus on the Southern Murray-Darling Basin*. Productivity Commission Staff Working Paper, Melbourne, Australia.

ARROW KJ. 1971. *Essays in the theory of risk bearing*. Amsterdam: North Holland.

ASCOUGH GW. 2001. *Procedures for estimating gross irrigation water requirements from crop water requirements*. M.Sc. (Eng) thesis. School of Bioresources Engineering and Environmental Hydrology. University of Natal, Pietermaritzburg.

BABCOCK BA, CHOI EK and FEINERMAN E. 1993. Risk and probability premiums for CARA utility functions. *Journal of Agricultural and Resource Economics*, 18: 17-24.

BACKEBERG GR. 1984. *Besproeiingsontwikkeling in die Groot-Visriviervallei*. Ongepubliseerde M.Sc. (Agric)-verhandeling. Universiteit van Pretoria, Pretoria.

BACKEBERG GR. 2004. Research management of water economics in agriculture – An open agenda. *Agrekon*, 43(3): 357-374.

BACKEBERG GR. 2005. Water institutional reforms in South Africa. *Water Policy*, 7: 107-123.

BACKEBERG GR. 2006. Reform of user charges, market pricing and management of water: Problem or opportunity for irrigated agriculture. *Irrigation and Drainage*, 55: 1-12.

BADENHORST J. 2003. Agricultural Extension Officer, National Department of Agriculture. Jan-Kempdorp. Personal interview.

BERNARDO DJ. 1985. *Optimal irrigation management under conditions of limited water supply*. Ph.D. dissertation. Washington State University.

BERNARDO DJ, WHITTLESEY NK, SAXTON KE and BASSETT DL. 1986. *Optimal irrigation management under conditions of limited water supply*. Cooperative Extension College of Agricultural and Home Economics, Washington State University, Pullman.

BERNARDO DJ, WHITTLESEY NK, SAXTON KE and BASSETT DL. 1987. An irrigation model for managing limited water supplies. *Western Journal of Agricultural Economics,* 12: 164-173.

BOISVERT RN and McCARL B. 1990. *Agricultural Risk Modeling Using Mathematical Programming*. Bulletin. Department of Agricultural Economics, Cornell University, Agricultural Experiment Station, New York State College of Agriculture and Life Sciences.

BOTES JHF. 1990. *An economic evaluation of wheat irrigation scheduling strategies using stochastic dominance*. (Afrikaans). M.Sc. Agric. dissertation. Department of Agricultural Economics, University of the Orange Free State, Bloemfontein.

BOTES JHF. 1994. *A simulation and optimization approach to estimating the value of irrigation information for decision-makers under risk*. Ph.D. thesis. Department of Agricultural Economics, University of the Free State, Bloemfontein.

BOTES JHF, BOSCH DJ and OOSTHUIZEN LK. 1995. The value of irrigation information for decision-makers with neutral and non-neutral risk preferences under conditions of unlimited and limited water supply. *Water SA*, 21(3): 221-230

BOTES JHF, BOSCH DJ and OOSTHUIZEN LK. 1996. A simulation and optimization approach for evaluating irrigation information. *Agricultural Systems*, 51: 165-183.

BREEDT H, LOUW AA, LIEBENBERG F, REINDERS FB, NELL JP and HENNING AJ. 2003. *Agricultural water use plan for the Nandoni dam: Phase 1 concept water use plan.* Third Draft, Unpublished Report, Agricultural Research Council, Pretoria.

BROOKE A, KENDRICK D, MEERAUS A. and RAMAN R. 1998. *The General Algebraic Modelling System (GAMS). User's Guide*. Boyd and Fraser Publishing Company, Danvers, Massachusetts.

BROTHERTON IA and GROENEWALD JA. 1982. Optimale organisasie op ontwikkelde besproeiingsplase in die Malelane-Komatipoortstreek. *Agrekon*, 21(2): 22-29.

COCHRAN MJ and RASKIN R. 1987. Interpretations and transformations of scale for the Pratt-Arrow absolute risk aversion coefficient: Implications for generalized stochastic dominance: Reply. *Western Journal of Agricultural Economics*, 12: 231-232.

CONRADIE B. 2002. *The value of water in the Fish-Sundays Scheme of the Eastern Cape.* Research Report to the Water Research Commission. WRC Report No 987/1/02. Pretoria: Water Research Commission.

CROSBY CT and CROSBY CP. 1999. *SAPWAT - a Computer Program for Establishing Irrigation Requirements and Scheduling Strategies in South Africa*. Report No. 624/1/99. Water Research Commission.

DAGPUNAR J. 1988. *Principles of random variate generation*. New York: Oxford Science Publications.

DE FRAITURE C and PERRY C. 2002. *Why is irrigation water demand inelastic at low price ranges?* Paper presented at the conference on Irrigation Water Policies: Micro and Macro considerations. Agadir, Morocco.

DE JAGER JM. 1994. Accuracy of vegetation evaporation formulae for estimating final wheat yield. *Water SA,* 20(4): 307-314.

DE JUAN JA, TARJUELO JM, VALIENTE M and GARCIA P. 1996. Model for optimal cropping patterns within the farm based on crop water production functions and irrigation Uniformity. *Agricultural Water Management*, 31: 115-143.

DEPARTMENT OF WATER AFFAIRS AND FORESTRY (DWAF). 1996. Water Law Principles. Discussion Document, Pretoria.

DEPARTMENT OF WATER AFFAIRS AND FORESTRY (DWAF). 1997. White Paper on a National Water Policy for South Africa, Pretoria.

DEPARTMENT OF WATER AFFAIRS AND FORESTRY (DWAF). 2004a. The National Water Resource Strategy, Pretoria.

DEPARTMENT OF WATER AFFAIRS AND FORESTRY (DWAF). 2004b. Water Conservation and Water Demand Management Strategy for the Agricultural Sector, Pretoria.

DOORENBOS J and KASSAM AH. 1979. *Yield Response to Water*. Irrigation and Drainage Paper 33. Rome, IT: Food and Agriculture Organisation (FAO).

ENGLISH M and RAJA SN. 1996. Review: Perspectives on deficit irrigation. *Agricultural Water Management*, 32(1): 1-14.

ENGLISH MJ, SOLOMON KH and HOFFMAN GJ. 2002. A paradigm shift in irrigation management. *Journal of Irrigation and Drainage Engineering*, 128(5): 267-277.

FERRER SRD. 1999. *Risk preferences and soil conservation decisions of South African commercial sugarcane farmers*. Unpublished PhD thesis. University of Natal, Pietermaritzburg.

FREUND R. 1956. The introduction of risk into a programming model. *Econometrica,* 21: 253-263.

FUASTI S and GILLISPIE J. 2006. Measuring risk attitude of agricultural producers using a mail survey: how consistent are the methods? *The Australian Journal of Agricultural and Resource Economics*, 50: 171-188.

GILLET CG, NIEUWOUDT WL and BACKEBERG GR. 2005. Water markets in the Lower Orange River Catchment of South Africa. *Agrekon*¸ 44(3): 363-382.

GROVÉ B. 2006a. Stochastic efficiency optimisation of alternative agricultural water use strategies. *Agrekon*, 45(4): 406-420.

GROVÉ B. 2006b. *Generalised whole-farm stochastic dynamic programming model to optimise agricultural water use.* Report to the Water Research Commission. WRC Report No 1266/1/06. Pretoria: The Water Research Commission.

GROVÉ B, NEL FS and MALULEKE HH. 2006. Stochastic efficiency analysis of alternative water conservation strategies. *Agrekon*, 45(1): 50-59.

GROVÉ B and OOSTHUIZEN LK. 2002. An economic analysis of alternative water use strategies at catchment level taking into account an instream flow requirement. *American Water Resource Association*, 38(2): 385-395.

HAILE BO, GROVÉ B and OOSTHUIZEN LK. 2003. *Impact of capital on the growth process of a sugarcane farm in Mpumalanga*. 41st Annual Conference of the Agricultural Economics

Association of South Africa, Agribusiness, Profits and Ethics: Pretoria, South Africa, October 2003.

HAMILTON JR, GREEN GP and HOLLAND D. 1999. Modeling the reallocation of Snake river water for endangered salmon. *American Journal of Agricultural Economics*, 81(5): 1252-1256.

HANCKE HP and GROENEWALD JA. 1972. Die effek van bronbeskikbaarheid op optimum organisasie in besproeiingsboerdery*. Agrekon*, 11(3): 9-16.

HARDAKER JB, HUIRNE RBM and ANDERSON JR. 1997. *Coping with risk in Agriculture*. Wallingford, Oxon, UK.

HARDAKER JB, HUIRNE RBM, ANDERSON JR and LIEN G. 2004. *Coping with risk in Agriculture*. CABI Publishing, CAB International, Oxford, UK.

HARDAKER JB, RICHARDSON JW, LIEN G and SCHUMANN KD. 2004. Stochastic efficiency analysis with risk aversion bounds: a simplified approach. The Australian *Journal of Agricultural and Resource Economics*, 48(2): 523-270.

HART CE and BABCOCK BA. 2001. *Rankings of risk management strategies combing crop insurance products and marketing positions*. Working paper 01-WP 267. Center for Agricultural and Rural Development. Iowa State University, Iowa.

HAZELL PBR. 1971. A linear alternative to quadratic and semi-variance programming for farm planning under uncertainty. *American Journal of Agricultural Economics*, 53(1):53-62.

HEROLD CW and BAILEY AK. 1996. *Long Term Salt Balance of the Vaalharts Irrigation Scheme*. Report to the Water Research Commission. Report No 420/1/96. Water Research Commission, Pretoria.

HEY J. 1979. *Uncertainty in microeconomics*. New York: New York University Press.

HURLEY TM, MITCHELL PD and RICE ME. 2004. Risk and the value of Bt corn. *American Journal of Agricultural Economics*, 86(2): 345-358.

JORDAAN H. 2006. *Forward pricing behaviour of Maize producers in Price Risk Management: The case of Vaalharts*. Unpublished MSc Agric Thesis, Department of Agricultural Economics, University of the Free State.

JORDAAN H, GROVÉ B, JOOSTE A and ALEMU ZG. 2006. Measuring the price volatility of certain field crops in South Africa using the ARCH/GARCH approach. *Agrekon*, 46(3): 306-322.

JORDAAN H, GROVÉ B, STEYN M, BENADE N, ANNANDALE JG and POTT A. 2006. *Development of a decision support system to evaluate the profitability of alternative irrigation schedules taking risk into account*. SANCID symposium, 15 – 17 November 2006, Blyde River Canyon, Swadini Resort.

LECLER NL. 2004. *Performance of irrigation and water management systems in the Lowveld of Zimbabwe.* PhD thesis. School of Bioresources Engineering and Environmental Hydrology. University of Natal, Pietermaritzburg.

LI J. 1998. Modeling crop yield as affected by uniformity of sprinkler irrigation system. *Agricultural Water Management*, 38: 135-146.

LOMBARD JP and KASSIER WE. 1990. *'n Strategiese besluitnemingsmodel vir die evaluering van landbougrondtransaksies in die Wes- en Suid-Kaap*. Ph.D- theses, Department of Agricultural Economics, University of Stellenbosch, Stellenbosch.

LOUW DB. 2002. *Evaluasie van die finansiële lewensvatbaarheid van boerdery ondernemings in die Vaalharts Watergebruiksvereniging beheergebied met die klem op die impak van watertariewe*. Ongepubliseerde navorsingsverslag. Sentrum vir Internasionale Landboubemarking en Ontwikkeling (CIAMD).

LOUW DB and VAN SCHALKWYK HD. 1997. The true value of irrigation water in the Olifants River Basin: Western Cape*. Agrekon*, 36(4): 551-600.

MANTOVANI EC, VILLA LOBOS FJ, ORGAZ F and FERERES E. 1995. Modeling the effects of sprinkler irrigation uniformity on crop yield. *Agricultural Water Management*, 27: 243-238.

MARÉ HG. 1995. *Marico-Bosveld Staatswaterskema: Stogastiese waterbronontledings gekoppel aan ekonomiese evaluasies*. BKS Ing. Verslag P5425/95/1. Departement van Landbou, Direktoraat Besproeiingsingenieurswese, Pretoria.

McCARL BA. 1987. Interpretations and transformations of scale for Pratt-Arrow absolute risk aversion coefficient: Implications for generalized stochastic dominance: Comment. *Western Journal of Agricultural Economics*, 12: 228-230.

McCARL BA and BESSLER DA. 1989. Estimating an upper bound on the Pratt aversion coefficient when the utility function is unknown. *Australian Journal of Agricultural Economics*, 33(1): 56-63.

McCARL BA and SPREEN TH. 2003. *Applied mathematical programming using algebraic systems*. Texas A&M University.

MEIRING JA. 1993. *Die ontwikkeling en toepassing van 'n besluitnemingsondersteuningstelsel vir die ekonomiese evaluering van risikobestuur op plaasvlak*. Ph.D- theses, Department of Agricultural Economics, University of the Free State, Bloemfontein.

MEIRING JA and OOSTHUIZEN LK. 1993. Die meting van besproeiingsboere se absolute risiko-vermydingskoëffisiënte met behulp van die intervalmetode: Die implikasies van skaalaanpassings*. Agrekon*, 32(2): 60-73.

MEIRING JA, OOSTHUIZEN LK, BOTHA PW and CROUS CI. 2002. *IRRICOST user's guide*. Report to the Water Research Commission. WRC Report No 894/2/02. Pretoria.

MEYER J. 1977. Choice among distributions. *Journal of Economic Theory*, 14: 326–336.

MEYER J. 2001. *Expected utility as a paradigm for decision making in agriculture*, in R.E. Just and R.P. Pope (Eds), A Comprehensive Assessment of the Role of Risk in US Agriculture, Kluwer, Boston, pp. 3–19.

MEYER DJ and MEYER J. 2005. Relative risk aversion: How do we know? *The Journal of Risk and Uncertainty*, 31(3): 243-262.

MITCHELL PD, GRAY ME and STEFFEY KL. 2004. A composed-error model for estimating pest damage functions and the impact of the Western Corn Rootworm Soybean variant in Illinois. *American Journal of Agricultural Economics,* 8(92): 332-344.

MOTTRAM R, DE JAGER JM, JACKSON BJ and GORDIJN RJ. 1995. *Irrigation water distribution management using linear programming.* Technical Report to the Water Research Commission. WRC Report No TT 71/95. Pretoria.

NATIONAL DEPARTMENT OF AGRICULTURE (NDA). 2005. *Abstract of agricultural statistics*. Agricultural Information Services. Pretoria.

NIEUWOUDT WL, GILLITT CG and BACKEBERG GR. 2005. Water marketing in the Crocodile River, South Africa. *Agrekon*, 44(3): 383-401.

NIEUWOUDT WL and HOAG DL. 1993. *Standardizing Arrow-Pratt absolute risk aversion to the range and scale of the data*. Unpublished paper, University of Natal, Pietermaritzburg.

OOSTHUIZEN HJ. 1995. *Berekening van die impak van die Marico-Bosveld Staatswaterskema op die nasionale ekonomie van Suid-Afrika en finale gevolgtrekkings oor die rehabilitasie van die skema*. Departement van Landbou, Direktoraat Landbou-ekonomie, Pretoria.

ORETGA JF, DE JUAN JA and TARJUELO JM. 2005. Improved irrigation management: The irrigation advisory service of Castilla – La Mancha (Spain). *Agricultural Water Management*, 77(1-3): 37-58.

ORTEGA JF, DE JUAN JA, TARJUELO JM and LOPEZ E. 2004. MOPECO: an economic optimization model for irrigation water management. *Irrigation Science*, 23: 61-75.

PETERSON JM and DING Y. 2005. Economic adjustments to groundwater depletion in the High Plains: Do water-saving irrigation systems save water? *American Journal of Agricultural Economics*, 87(1): 147-159.

PEREIRA LS, OWEIS T and ZAIRI A. 2002. Irrigation management under water scarcity. *Agricultural Water Management*, 57(3): 175-206.

POTT A, HALLOWES JS, MTSHALI S, MBOKAZI S, VAN ROOYEN M, CLULOW A and EVERSON C. 2005. *The Development of a Computerised System for Auditing Real Time or Historical Water Use From Large Reservoirs in Order to Promote the Efficiency of Water Use*. Report No. K5-1300. Water Research Commission, Pretoria.

POTT A, BENADÉ N, VAN HEERDEN P, GROVÉ B, ANNANDALE J and STEYN M. 2007. *Technology Transfer and Integrated Implementation of Water Management Models in Commercial Farming*. Report No. K5-1481. Water Research Commission, Pretoria.

PRATT JW. 1964. Risk aversion in the small and in the large. *Econometrica*, 32(1): 122-136.

RABIN M and THALER RH. 2001. Anomalies: risk aversion. *Journal of Economic Perspectives*, 15: 219–232.

RAE AN. 1970. Capital budgeting, intertemporal programming models, with particular reference of agriculture. *Australian Journal of Agricultural Economics*, 14(1): 39-52.

RASKIN R and COCHRAN MJ. 1986. Interpretations and transformations of the scale for the Pratt-Arrow absolute risk aversion coefficient: Implications for generalized stochastic dominance. *Western Journal of Agricultural Economics*, 11: 204-210.

RECA J, ROLDAN J, ALCAIDE M, LOPEZ R and CAMACHO E. 2001. Optimisation model for water allocation in deficit irrigation systems I. Description of the model. *Agricultural Water Management*, 48: 103-116.

RICHARDSON JW, KLOSE SL and GRAY AW. 2000. An applied procedure for estimating and simulating multivariate empirical (MVE) probability distributions in farm-level risk assessment and policy analysis. *Journal of Agricultural and Applied Economics*, 32(2):299-315.

RICHARDSON JW, SCHUMANN K and FELDMAN P. 2004. *SIMETAR Simulation for Excel to Analyze Risk©*. Department of Agricultural Economics, Texas A&M University.

SCHEIERLING SM, YOUNG RA and CARDON GE. 2004. Determining the price-responsiveness of demands for irrigation water deliveries versus consumptive use. *Journal of Agricultural and Resource Economics*, 29(2): 328-345.

SCHÜZE N, DE PALY M, WÖHLING T and SCHMITZ GH. 2005. *Global optimization of deficit irrigation systems using evolutionary algorithms and neural networks*. ICID 21st European Regional Conference 2005- 15-19 May, 2005. Frankfurt (Oder) and Slubice- Germany and Poland.

SCOTT KA, LOUW AA, LIEBENBERG F, BREEDT H, NELL JP and HENNING AJ. 2004. *Agricultural water use plan for the Inyaka dam: Phase 1 concept water use plan*. Second Draft, Unpublished Report, Agricultural Research Council, Pretoria.

SECKLER D, BAKER R and AMARASINGHE U. 1999. Water scarcity in the twenty-first century. *International Journal of Water Resource Development,* 15(1/2): 29-42.

SEPASKHAH AR and GHAHRAMAN B. 2004. The effects of irrigation efficiency and uniformity coefficient on relative yield and profit for deficit irrigation. *Biosystems Engineering*, 87(4): 495-507.

SHANGGUAN Z, SHAO M, HORTON R, LEI T, QIN L and MA J. 2002. A model for regional optimal allocation of irrigation water resources under deficit irrigation and its applications. *Agricultural Water Management*, 52: 139-154.

SOUTH AFRICA (Republic). DEPARTMENT OF WATER AFFAIRS AND FORESTRY 1998. National Water Act. Act No 56 of 1998. 20 August 1998.

STEWART JI and HAGAN RM. 1973. Functions to predict effects of crop water deficits. *Journal of Irrigation and Drainage Division, ASCE*, 99(4): 421-439.

STREUTKER A. 1977. The dependence of permanent crop production on efficient irrigation and drainage at the Vaalharts Government Water Scheme. *Water SA*, 3(2): 90-103.

TOMEK WG and ROBERTSON KL. 1990. *Agricultural product prices*. 3rd Edition. Ithaca, NY: Cornell University Press.

TSIANG SC. 1972. The rationale of the mean-standard deviation analysis, skewness preference, and the demand for money. *American Economic Review*, 62: 354-371.

VAN HEERDEN PS. 2001. PICKWAT Independent consultancy. Personal interview.

VAN HEERDEN PS, CROSBY CT and CROSBY CP. 2001. *Using SAPWAT to estimate water requirements of crops in selected irrigation areas managed by the Orange-Vaal and Orange-Riet water users associations*. Technical Report to the Water Research Commission. WRC Report No: TT 163/01. Pretoria: Water Research Commission.

VAN ROOYEN CJ. 1979. Waterbeskikbaarheid en Arbeid as Veranderlike Hulpbronne in Beplanning vir Optimale Organisasie in Besproeiingsboerdery. *Agrekon*, 18(1): 9-17.

VAUX HJ and PRUITT WO. 1983. *Crop water production functions*, in D.I. Hillel (Editor), Advances in Irrigation. Vol. II Academic Press, New York, pp. 61-97.

VEDENOV DV and BARNETT BJ. 2004. Efficiency of weather derivatives as primary crop insurance instruments. *Journal of Agricultural and Resource Economics*, 39(3): 387-403.

VILJOEN MF, DUDLEY NJ, GAKPO EFY and MAHLAHA JM. 2004. *Effective local management of water resources with reference to the middle Orange River*. WRC Research Report No: 1134/1/04. Water Research Commission, Pretoria.

VILJOEN MF, SYMINGTON HM, and BOTHA SJ. 1992. *Verwantskap tussen waterbeperkings en finansiële gevolge in die Vaalrivierwatervoorsieningsgebied met spesiale verwysing na besproeiingsboerderye in die Vaalhartsgebied*. Verslag aan die Waternavorsings-kommissie. WNK Verslag No: 288/2/92. Pretoria: Waternavorsingskommissie.

VILJOEN MF, SYMINGTON HM, BOTHA SJ, and DU PLESSIS LA. 1993. *Vaalhartsstaatswaterskema: Ondersoek na die finansiële en ekonomiese uitvoerbaarheid van die vergroting van die sekondêre en tersiêre besproeiingskanale*. Departement Landbou-ekonomie. Universiteit van die Oranje-Vrystaat, Bloemfontein.

WEINBERG M, KLING CL and WILEN JE. 1993. Water Markets and Water Quality. *American Journal of Agricultural Economics*, 75:278-291.

WHITTLESEY NK and HUFFAKER RG. 1995. Water policy issues for the twenty-first century. *American Journal of Agricultural Economics*, 68(5): 1199-1203.

WILLIS DB. 1993. *Modeling economic effects of stochastic water supply and demand on minimum stream flow requirements*. Unpublished Ph.D. thesis. Department of Agricultural Economics, Washington State University, Pullman.

APPENDIX A

GAMS CODE TO SIMULATE MULTIVARIATE DISTRIBUTIONS: EMPIRICAL AND TRIANGLE

```
*=========================================================================================
* START: (A)  GENERATE AND CORRELATE UNIFORMLY DISTRIBUTED RANDOM NUMBERS               *
*             Note: The procedures are based on Richardson, Klose and Gray (2000)      *
*=========================================================================================
*         (A1) INITIALISE SET IDENTIFIERS AND PARAMETERS                                *
*-----------------------------------------------------------------------------------------
 SETS
*----
 i            correlated entities
              /Maize,G_nuts,Wheat,P_maize,P_g_nuts,P_wheat,AW_maize,AW_g_nuts,AW_wheat
              YI_maize,YI_g_nuts,YI_wheat/
 t            years in the simulation
              /y1/
 r            labels for the random numbers
              /1*100/
y(i)          crop yield entities
              /maize,g_nuts,wheat/
aw(i)         applied water entities
              /AW_maize,AW_g_nuts,AW_wheat/
yi(i)         yield index entities
              /YI_maize,YI_g_nuts,YI_wheat/
;
alias(i,j,k);
alias(t,it,mt);

 Parameters
*----------
* reserved parameter: user supplied
 v(i,j)         correlation matrix
 autocor(i)     one year lagged correlation coefficient

* reserved parameters: code specific
  C(I,J)        Cholesky decomposition of correlation matrix (v)
  VC(I,J)       cc' = V check matrix
  WB(I,J)       Whistle blower check matrix

  IV(i,t,it)    inter-temporal correlation matrix
  IC(I,T,it)    Cholesky decomposition of inter-temporal correlation matrix
  IVC(I,t,it)   inter-temporal icic' = iV check matrix
  IWB(I,t,it)   inter-temporal Whistle blower check matrix

  ISND(R,I,t)   Independent standard normal deviates
  CSND(R,I,t)   Correlated standard normal deviates
  ACSND(r,i,t)  auto-correlated standard normal deviates

* output parameter
  CUD  (R,I,t)    intra-  and  inter-temporally  correlated  uniformly  distributed  random
numbers
;

*-----------------------------------------------------------------------------------------
*         (A2) DATA INPUT SPECIFICATION: INTRA AND INTER-TEMPORAL CORRELATIONS          *
*-----------------------------------------------------------------------------------------
*  give intra-temporal correlation matrix
*----------------------------------------
TABLE V(I,J) intra-temporal correlation matrix
             P_Maize    P_Wheat   P_G_nuts     Maize     Wheat     G_nuts
P_Maize       1.000      0.723     -0.050    -0.239    -0.424     -0.365
P_Wheat                  1.000      0.426    -0.203    -0.263     -0.716
P_G_nuts                            1.000    -0.101    -0.166     -0.174
Maize                                         1.000     0.003      0.619
Wheat                                                   1.000     -0.058
G_nuts                                                             1.000
;
```

```
*  Create lower triangular of correlation matrix
*-------------------------------------------------
   V(J,I)
         $(NOT V(J,I))
         = V(I,J);

options v :4:1:1;
display v;
*----------------------------------------------------------------------------------------
*  give one year lagged correlation coefficient
*-----------------------------------------------
parameter autocor(i) one year lagged correlation coefficient
/
P_Maize            0
P_Wheat            0
P_G_nuts           0
Maize              0
Wheat              0
G_nuts             0
/
;

* create inter-temporal correlation matrix from autocor(i)
*---------------------------------------------------------

*  Create diagonal values equal to one
IV(I,T,IT)
          $(ord(t)=ord(it))
            = 1;
*  create off-diagonal values based on one year lagged correlation coefficient
IV(I,T,it)
          $(ord(t)=ord(it)+1)
           = autocor(i);
*  Create upper triangular of inter temporal correlation matrix
IV(I,T,IT)
          $(NOT IV(I,T,IT))
           = IV(I,IT,T);
*----------------------------------------------------------------------------------------
*        (A3) FACTORISE CORRELATION MATRIX USING CHOLESKY DECOMPOSITION               *
*             Source: Principles of random variate generation                         *
*                     John Dagpunar (1988)                                            *
*                     Pages 157-158                                                   *
*             Note: SIMETAR uses upper triangle whereas this procedure uses the lower  *
*                   triangle                                                          *
*----------------------------------------------------------------------------------------
* intra-temporal decomposition
*------------------------------

   C(i,j)
         $(
          ord(i)=1
          and
          ord(j)=1
          )
          = SQRT(V(i,j));

   Loop((i, J)
              $(
                (ord(J) > 1)
                 and
                 ord(i)=1
                ),
                  C(J,i) = V(i,J)/1
        );

   LOOP(I
         $(ORD(I)>1),
         C(I,I) = SQRT(
                       V(I,I) - SUM(K
                                     $(ORD(K) < ORD(I)),
                                     SQR( C(I,K) )
                                    )
                      )
                       $(
                           (V(I,I) - SUM(k
                                          $(ORD(k) < ORD(I)),
                                          SQR(C(I,K))
                                         )
                           )
                          >0
```

```
                       );

         LOOP(J
               $(ORD(J) > ORD(I)),
               C(J,I) = (
                          (V(I,J) - SUM(k
                                         $(ORD(k) < ORD(I)),
                                          C(I,k)*C(J,k)))/C(I,I)
                          )
                           $(C(I,I)> 0);
               );
        );
*  Print out
   DISPLAY C;

*  Check the decomposition matrix CC'=V
*----------------------------------------

*  Sum over both the columns, m, and not over column row because you multiply
*  with the transpose of C
   VC(I,J) = SUM(k,
                 C(I,k)*C(J,k)
               );
*  Print out
   DISPLAY VC;

*  Whistle Blower check of the decomposition matrix
*  Note: Values should equal 0
   WB(I,J) = V(I,J)-VC(I,J);
*  Print out
   DISPLAY WB;

*-----------------------------------------------------------------------------------------
* inter-temporal decomposition
*------------------------------

   IC(i,t,it)
         $(
           ord(t)=1
           and
           ord(it)=1
           )
          = 1;

   LOOP((it, T)
               $(
                 ORD(T) > 1
                 and
                 ord(it)=1
                ),
                  IC(i,T,it) = IV(i,it,T)/1
        );

   LOOP(IT
          $(ORD(IT)>1),
           IC(i,IT,IT) = SQRT(
                              IV(i,IT,IT) - SUM(MT
                                                 $(ORD(MT) < ORD(IT)),
                                                   SQR(IC(i,IT,MT))
                                               )
                              )
                               $(
                                 (IV(i,IT,IT) - SUM(MT$
                                                     (ORD(MT) < ORD(IT)),
                                                      SQR(IC(i,IT,MT))
                                                  )
                                 )
                               >0
                              );

        LOOP(T$
               (ORD(T) > ORD(IT)),
               IC(i,T,IT) = (
                             (IV(i,IT,T) - SUM(MT
                                                $(ORD(MT) < ORD(IT)),
                                                 IC(i,IT,MT)*IC(i,T,MT)))/IC(i,IT,IT)
                             )
                              $(IC(i,IT,IT)> 0);
             );
        );
*  Print out
```

```
   OPTION ic :6:2:1;
   DISPLAY IC;

*  Check the decomposition matrix icic'=iv
*----------------------------------------

*  Sum over both the columns, m, and not over column row because you multiply
*  with the transpose of ic
   IVC(I,t,it) = SUM(Mt,
                     IC(i,t,Mt)*IC(i,it,Mt)
                    );
*  Print out
   DISPLAY IVC;

*  Whistle Blower check of the decomposition matrix
*  Note: Values should equal 0
   IWB(I,t,it)=IV(I,t,it)-IVC(I,t,it);
*  Print out
   DISPLAY IWB;
*-----------------------------------------------------------------------------------------
*        (A4) GENERATE CORRELATED UNIFORMLY DISTRIBUTED RANDOM NUMBERS                   *
*             Source: Richardson, Klose and Gray (2000)                                  *
*-----------------------------------------------------------------------------------------

*  Generate independent standard normal deviates equal to number of iterations
*------------------------------------------------------------------------------
   ISND(r,i,t) = NORMAL(0,1);
*  Print out
   DISPLAY ISND;

*  Correlate standard normal deviates
*-----------------------------------
   CSND(R,I,t) =  SUM(J,
                      C(I,J)*ISND(r,J,t)
                     );
*  Print out
   DISPLAY CSND;

*  Adjust csnd for auto-correlation
*---------------------------------
   ACSND(R,I,t) =  SUM(it,
                        IC(I,T,it)*CSND(R,I,it)
                      );
*  Print out
   options ACSND :4:2:1;
   DISPLAY ACSND;

*  Integrate area under the normal distribution to make it uniformly distributed
*--------------------------------------------------------------------------------
   CUD(R,I,t) = ERRORF(ACSND(R,I,t));

*   Assign random numbers to yield index and applied water
*---------------------------------------------------------

* assume 100% correlation between subjective yield and yield index
loop((yi,y)$yitype(y,yi),
            cud(r,yi,t) =  cud(r,y,t);
    )
;
* assume 100% inverse correlation between yield index and applied water
Loop((yi,aw)$yiawtype(yi,aw),
            cud(r,aw,t) =  1-cud(r,yi,t);
    )
;
*-----------------------------------------------------------------------------------------
*                                         END                                            *
*              GENERATE AND CORRELATE UNIFORMLY DISTRIBUTED RANDOM NUMBERS               *
*-----------------------------------------------------------------------------------------
*=========================================================================================
* START: (B)  SIMULATE MULTIVARIATE PROBABILITY DISTRIBUTIONS                            *
*             TRIANGLE & EMPIRICAL                                                       *
*             Source: Hardaker, Huirne and Anderson (1997)
*=========================================================================================
*        (B1) INITIALISE SET IDENTIFIERS AND PARAMETERS                                  *
*-----------------------------------------------------------------------------------------
 SETS
*----
* User specified
w         soil water holding capacity
          /sw140/
m         irrigation management options
```

```
*        /read from file/
*Note:  The  two  sets  above  were  used  to  identify  alternative  management  options  and
*       soils.
*      These sets are not code specific but are linked to code specific identifiers
*      the alias statement. Thus, the generality of the code is preserved.

* code specific sets
 rl            reserved:  min  and  max  user:  obs1*obs100  indicate  input  distribution
identifiers
          /min,obs1*obs100,max /
 o(rl)    user: obs1*obs100 indicate input distribution identifiers
*                obs1 = minimum, obs2 = most likely and obs3 = maximum of triangle
*                obs1*obs100 indicate empirical observation labels for empirical
          /obs1*obs100/
 pe       distribution identifiers
*         emp identifies data for the empirical distribution
*         tri identifies data for the triangle distribution
          /prob,emp,tri/
 mm(rl)   empirical distribution - pseudo minimum and maximum
          /max,min /
 il       empirical parameters used to interpolate empirical distribution
          /Pi,Pi1,Xi,Xi1 /
 dtype(pe,i) map correlated entities to distribution type (empirical or triangle)
          /emp.maize,emp.wheat,emp.g_nuts,emp.P_maize,emp.P_wheat,emp.P_g_nuts
           tri.AW_maize,tri.AW_g_nuts,tri.AW_wheat,tri.YI_maize,tri.YI_g_nuts
           tri.YI_wheat
          /

*Link code specific identifier 1 to m (management options)
Alias(i1,m);
*Link identifier 2 to w (soil water holding capacity
Alias(i2,w);
;
* code specific alias
ALIAS(O,O1);

 Parameters
*----------
* reserved parameter: user supplied
 Data(i1,i2,rl,pe,i) data used to characterise risk for empirical and triangle;

* reserved parameters: sort data from small to large
  SORT(i1,i2,RL,PE,I)    Temporary table use to store sorted data
  SORTMIN(i1,i2,pe,I)    Temporary table to store minimum of sorted values

* reserved parameters: empirical distribution
  CP(i1,i2,O,PE,I)              Assigned  cumulative  probabilities  for  the  empirical
distribution
  N(i1,i2,I)             Number of empirical observations
  EMIN(i1,i2,PE,I)       Empirical minimum value
  EMAX(i1,i2,PE,I)       Empirical maximum value
  PEMAX(i1,i2,I)         Probability of the empirical maximum value
  CUMPAR(i1,i2,R,IL,I,t) Parameters used to interpolate empirical distribution

* output parameter
  fx(r,i1,i2,i,t) randomly drawn output from empirical and triangle distributions;
;
*reserved scalars: empirical distribution
SCALAR SMALL small value used to determine pseudo minimum and maximum /0.0001/;

*-------------------------------------------------------------------------------------
*        (B2) DATA INPUT SPECIFICATION: PARAMETERS FOR EMPIRICAL AND TRIANGLE          *
*-------------------------------------------------------------------------------------

*  NOTE: Sample dataset for the first three elements of sets m (1296 elements) and
*        o (100 elements)
Table data(i1,i2,rl,pe,i) data used to characterise risk for empirical and triangle
                              Maize      G_nuts       Wheat     P_maize

N-N-N-N .sw140.obs1.emp       7.041       2.151       5.255     701.000
N-N-N-N .sw140.obs2.emp       7.259       2.255       5.328     828.000
N-N-N-N .sw140.obs3.emp       7.491       2.400       5.455     875.000
N-N-N-20.sw140.obs1.emp       7.041       2.151       5.255     701.000
N-N-N-20.sw140.obs2.emp       7.259       2.255       5.328     828.000
N-N-N-20.sw140.obs3.emp       7.491       2.400       5.455     875.000
N-N-N-40.sw140.obs1.emp       7.041       2.151       5.255     701.000
N-N-N-40.sw140.obs2.emp       7.259       2.255       5.328     828.000
N-N-N-40.sw140.obs3.emp       7.491       2.400       5.455     875.000
                       +    P_g_nuts     P_wheat    AW_maize   AW_g_nuts

N-N-N-N .sw140.obs1.emp    2121.000    1268.000
```

```
N-N-N-N .sw140.obs1.tri                          468.593     396.757
N-N-N-N .sw140.obs2.emp    2544.000    1355.000
N-N-N-N .sw140.obs2.tri                          519.844     447.694
N-N-N-N .sw140.obs3.emp    2855.000    1419.000
N-N-N-N .sw140.obs3.tri                          576.069     579.193
N-N-N-20.sw140.obs1.emp    2121.000    1268.000
N-N-N-20.sw140.obs1.tri                          465.449     393.223
N-N-N-20.sw140.obs2.emp    2544.000    1355.000
N-N-N-20.sw140.obs2.tri                          516.420     443.844
N-N-N-20.sw140.obs3.emp    2855.000    1419.000
N-N-N-20.sw140.obs3.tri                          572.393     573.155
N-N-N-40.sw140.obs1.emp    2121.000    1268.000
N-N-N-40.sw140.obs1.tri                          462.306     389.688
N-N-N-40.sw140.obs2.emp    2544.000    1355.000
N-N-N-40.sw140.obs2.tri                          512.996     439.994
N-N-N-40.sw140.obs3.emp    2855.000    1419.000
N-N-N-40.sw140.obs3.tri                          568.717     567.118

                      +    AW_wheat    YI_maize   YI_g_nuts    YI_wheat

N-N-N-N .sw140.obs1.tri     494.262       0.999       0.944       0.954
N-N-N-N .sw140.obs2.tri     573.060       0.999       0.997       0.967
N-N-N-N .sw140.obs3.tri     632.613       0.999       0.997       0.990
N-N-N-20.sw140.obs1.tri     494.262       0.999       0.940       0.954
N-N-N-20.sw140.obs2.tri     573.060       0.999       0.997       0.967
N-N-N-20.sw140.obs3.tri     632.613       0.999       0.997       0.990
N-N-N-40.sw140.obs1.tri     494.262       0.999       0.936       0.954
N-N-N-40.sw140.obs2.tri     573.060       0.999       0.997       0.967
N-N-N-40.sw140.obs3.tri     632.613       0.999       0.997       0.990
*                         ----------end of sample data-----------
;
*---------------------------------------------------------------------------------------
*        (B3) SORT TABLE DATA FROM SMALL TO LARGE                                      *
*           Note: The code is slow if a large number of values are used to characterise*
*                   risk with the empirical distribution. The user is advised to sort  *
*                   the data before using this code                                    *
*---------------------------------------------------------------------------------------

*  Create table with same contents as the input data
   SORT(i1,i2,RL,PE,I)
                    = DATA(i1,i2,RL,PE,I);

*  Loop over the observations
   LOOP((i1,i2,o,pe)
*                   $(sum(i,SORT(i1,i2,o,PE,i)))
                   ,
*         Determine the minimum value and write the value to Table DATA
          SORTMIN(i1,i2,pe,I) = SMIN(O1
                                  $(SORT(i1,i2,O1,pe,I)),
                                  SORT(i1,i2,O1,pe,I)
                                );
          DATA(i1,i2,O,pe,I)
                          $DATA(i1,i2,O,pe,I)
                          = SORTMIN(i1,i2,pe,I);
*         Create new SORT table without the previous minimum value
          SORT(i1,i2,O1,pe,I)
                          = SORT(i1,i2,O1,pe,I)
                                                $(

SORT(i1,i2,O1,pe,I)>SORTMIN(i1,i2,pe,I)
                                                  );
        );

Execute_unload "data",data;

*---------------------------------------------------------------------------------------
*        (B4) SIMULATE TRIANGLE DISTRIBUTION                                           *
*---------------------------------------------------------------------------------------

*   assign value to fx if the random number is <= the critical value
   fx(r,i1,i2,i,t)
            $(
              dtype("tri",i)
              and
              Cud(r,i,t)LE (
                             (Data(i1,i2,"obs2","tri",i)-Data(i1,i2,"obs1","tri",i))
                                                         /
                             (Data(i1,i2,"obs3","tri",i)-Data(i1,i2,"obs1","tri",i))
                            )
              )
          =   Data(i1,i2,"obs1","tri",i)
```

```
               +sqrt(
                      Cud(r,i,t)
                          *
                      (data(i1,i2,"obs3","tri",i)-Data(i1,i2,"obs1","tri",i))
                                                *
                      (Data(i1,i2,"obs2","tri",i)-Data(i1,i2,"obs1","tri",i))
                    );


*   assign value to fx if the random number is > the critical value
   fx(r,i1,i2,i,t)
            $(
              dtype("tri",i)
              and
              Cud(r,i,t)> (
                          (Data(i1,i2,"obs2","tri",i)-Data(i1,i2,"obs1","tri",i))
                                                          /
                          (Data(i1,i2,"obs3","tri",i)-Data(i1,i2,"obs1","tri",i))
                          )
              )
          =   Data(i1,i2,"obs3","tri",i)
              -sqrt(
                     (1-Cud(r,i,t))
                          *
                     (Data(i1,i2,"obs3","tri",i)-Data(i1,i2,"obs1","tri",i))
                                                    *
                     (Data(i1,i2,"obs3","tri",i)-Data(i1,i2,"obs2","tri",i))
                   );

execute_unload "fx",fx;

*----------------------------------------------------------------------------------------
*        (B5) SIMULATE EMPIICAL DISTRIBUTION                                            *
*----------------------------------------------------------------------------------------
*  Assign cumulative probabilities to empirical values
*------------------------------------------------------

*  Determine number of empirical observations with the absolute of the sigmodal values
 N(i1,i2,I)=SUM(O,ABS(SIGN(DATA(i1,i2,O,"EMP",I))));

*  Determine the minimum and maximum of the empirical observations
 EMIN(i1,i2,"EMP",I) = SMIN(O
                            $DATA(i1,i2,O,"EMP",I)
                            ,
                             DATA(i1,i2,O,"EMP",I)
                           );
 EMAX(i1,i2,"EMP",I) = SMAX(O
                            $DATA(i1,i2,O,"EMP",I)
                            ,
                             DATA(i1,i2,O,"EMP",I)
                           );
*   Assign  cumulative  probabilities  to  empirical  observations  excluding  pseudo  min  and
max
 LOOP(O,
        DATA(i1,i2,O,"PROB",I)
                             $DATA(i1,i2,O,"EMP",I)
                             = ORD(O)*1/N(i1,i2,I)-1/N(i1,i2,I)/2
     );
*  Determine the maximum probabilities for the maximum observed data
*  Note: the maximum is not = 1 since probabilities for the pseudo max's have not been
*       assigned
 PEMAX(i1,i2,I) = SMAX(O
                       $DATA(i1,i2,O,"EMP",I)
                       ,
                        DATA(i1,i2,O,"PROB",I)
                      );

*  Determine the pseudo minimum and maximum of the empirical observations
 DATA(i1,i2,"MAX","EMP",I)
                         $(DATA(i1,i2,"OBS1","EMP",I))
                         = EMAX(i1,i2,"EMP",I) + SMALL ;

 DATA(i1,i2,"MIN","EMP",I)
                         $(DATA(i1,i2,"OBS1","EMP",I))
                         = EMIN(i1,i2,"EMP",I) - SMALL ;

*  Assign probabilities of 0 and 1 to pseudo minimum and maximum
 DATA(i1,i2,"MIN","PROB",I) = 0
                              $(DATA(i1,i2,"OBS1","EMP",I));
 DATA(i1,i2,"MAX","PROB",I) = 1
                              $(DATA(i1,i2,"OBS1","EMP",I));
```

```
*  Print data with 2 decimals, first set in row and next two sets in the column
 OPTION DATA :7:1:2;
*  Print
 DISPLAY N, EMIN,EMAX,PEMAX,DATA;

* Lookup the parameters used to interpolate
*-------------------------------------------

*  Loop over each random number and observations for each correlated entity
 LOOP((i1,i2,R,RL,I,t)
*           Under the condition that:
            $(
*              the data in the table is ne 0 excluding the minimum observation
               ( DATA(i1,i2,RL,"PROB",I) OR SAMEAS("MIN",RL) )
*               and the lookup probability is LT the generated random number
                AND DATA(i1,i2,RL,"PROB",I) LE CUD(R,I,t)
*               and the higher probability is GT the generated random number
                AND DATA(i1,i2,"MAX","PROB",I) GT CUD(R,I,t)
              ),
*               write the lookup values of the lower and higher probabilities
                CUMPAR(i1,i2,R,"Pi",I,t) = DATA(i1,i2,RL, "PROB",I);
                CUMPAR(i1,i2,R,"Pi1",I,t) = DATA(i1,i2,RL+1, "PROB",I);
*               assign a value of 1 if the upper probability results in a zero
*               due to missing observation
                CUMPAR(i1,i2,R,"Pi1",I,t)
                                         $(DATA(i1,i2,RL,"PROB",I) EQ PEMAX(i1,i2,I))
                                         = DATA(i1,i2,"MAX", "PROB",I);
*               write the lookup values of the lower and higher observation
                CUMPAR(i1,i2,R,"Xi",I,t) = DATA(i1,i2,RL, "EMP",I);
                CUMPAR(i1,i2,R,"Xi1",I,t) = DATA(i1,i2,RL+1, "EMP",I);
*               assign a value of 1 if the upper observation results in a zero
*               due to missing observation
                CUMPAR(i1,i2,R,"Xi1",I,t)
                                         $(DATA(i1,i2,RL,"PROB",I) EQ PEMAX(i1,i2,I))
                                         = DATA(i1,i2,"MAX", "EMP",I);
       );
*  Print table
   DISPLAY CUMPAR;

*  Interpolate to find correlated random entities from empirical distributions
*---------------------------------------------------------------------------

*  Do the interpolation only for entities that have data
   FX(R,i1,i2,i,t)$CUMPAR(i1,i2,R,"Pi1",I,t)
*           interpolate between minimum and maximum
      =(
        (CUD(R,I,t) - CUMPAR(i1,i2,R,"Pi",I,t))
                                *
        (CUMPAR(i1,i2,R,"Xi1",I,t) - CUMPAR(i1,i2,R,"Xi",I,t))
                                /
        (CUMPAR(i1,i2,R,"Pi1",I,t) - CUMPAR(i1,i2,R,"Pi",I,t))
       )
        + CUMPAR(i1,i2,R,"Xi",I,t);

*  Print fx with 2 decimals, first set in row and next two sets in the column
   options fx :6:1:2;
   DISPLAY fx;

*  create GDX file with the simulated results
execute_unload "fx",fx;

*------------------------------------------------------------------------------------
*                                        END                                        *
*                    SIMULATE MULTIVARIATE PROBABILITY DISTRIBUTIONS                *
*                                TRIANGLE & EMPIRICAL                               *
*------------------------------------------------------------------------------------
*====================================================================================
* START: (C)  EXTRACT INDIVIDUAL PARAMETER FROM OUTPUT                              *
*====================================================================================
*        (C1) INITIALISE SET IDENTIFIERS AND PARAMETERS                             *
*------------------------------------------------------------------------------------
 SETS
*----
* User specified
pr(i)   price entities as a sub-set of correlated entities
        /P_maize,P_g_nuts,P_wheat/

*Note: Sub-sets for crop yield y(i), gross irrigation aw(i) and crop yield index yi(i)
*      are already identified under (A1)
```

```
 Parameters
*----------
* reserved parameter: user supplied
Yield_y(r,m,w,y) random crop yield - ton per hectare
Price_y(r,m,w,y) random crop prices - rand per ton
YieldIndex_y(r,m,w,y) random crop yield index - fraction
Gir_y(r,m,w,y) random crop gross irrigations - mm.ha;

*NOTE: Output parameters are specified for each crop (y) and not correlated entities (i)

*---------------------------------------------------------------------------------------
*        (C2) EXTRACT PARAMETERS AND CREATE GDX FILES                                  *
*---------------------------------------------------------------------------------------

* crop yield
*-----------
  Yield_y(r,m,w,y)= FX(R,m,w,y,"y1");
  execute_unload "Yield_y", Yield_y;

* crop prices
*------------
  loop((y,pr)
           $(prtype(y,pr)),
                           Price_y(r,m,w,y)= FX(R,m,w,pr,"y1")
       );
  execute_unload "Price_y", Price_y;

* crop yield index
*-----------------
  loop((y,yi)
           $(yitype(y,yi)),
                          YieldIndex_y(r,m,w,y)= FX(R,m,w,yi,"y1")
       );
  execute_unload "YieldIndex_y", YieldIndex_y;

* gross irrigations
*------------------
  loop((y,aw)
            $(awtype(y,aw)),
                           Gir_y(r,m,w,y)= FX(R,m,w,aw,"y1")
      );
  execute_unload "Gir_y", Gir_y;
*---------------------------------------------------------------------------------------
*                                         END                                          *
*                           EXTRACT INDIVIDUAL PARAMETER FROM OUTPUT                   *
*---------------------------------------------------------------------------------------
```

APPENDIX B

NUMERICAL EXAMPLE OF SERF ANALYSIS WITH CONSTANT STANDARD RISK

Original				Normalised			
Alternative distributions (i)				Alternative distributions (i)			
A	A+200	A/3	A/3+200	A	A+200	A/3	A/3+200
X				$x^s = x/\sigma_x$			
100	300	33	233	0.63	1.90	0.63	4.43
200	400	67	267	1.26	2.53	1.26	5.06
300	500	100	300	1.90	3.16	1.90	5.69
400	600	133	333	2.53	3.79	2.53	6.32
500	700	167	367	3.16	4.43	3.16	6.96
Standard deviation (σ_x)				Standard deviation (σ_s)			
158	158	53	53	1	1	1	1
$r_a(x_i) = r_s(x_i^s)/\sigma_{xi}$				$r_s(x_i^s) = r_a(x_i)\sigma_{xi}$			
0.01	0.01	0.03	0.03	1.58	1.58	1.58	1.58
$U(x) = e^{-r_a(x)x}$				$U(x^s) = e^{-r_s(x^s)x^s}$			
0.36788	0.04979	0.36788	0.00091	0.36788	0.04979	0.36788	0.00091
0.13534	0.01832	0.13534	0.00034	0.13534	0.01832	0.13534	0.00034
0.04979	0.00674	0.04979	0.00012	0.04979	0.00674	0.04979	0.00012
0.01832	0.00248	0.01832	0.00005	0.01832	0.00248	0.01832	0.00005
0.00674	0.00091	0.00674	0.00002	0.00674	0.00091	0.00674	0.00002
$EU(x)$				$EU(x^s)$			
0.11561	0.01565	0.11561	0.00029	0.11561	0.01565	0.11561	0.00029
$CE(x)=-ln(E[U(x)])/r_a(x)$				$CE(x^s)=-ln(E[U(x^s)])/r_s(x^s)$			
215.75	415.75	71.92	271.92	1.36	2.63	1.36	5.16
				$CE(x)=CE(x^s)\sigma_x$			
				215.75	415.75	71.92	271.92

APPENDIX C
COMBINED GRAPHS FOR LONG-RUN RESULTS

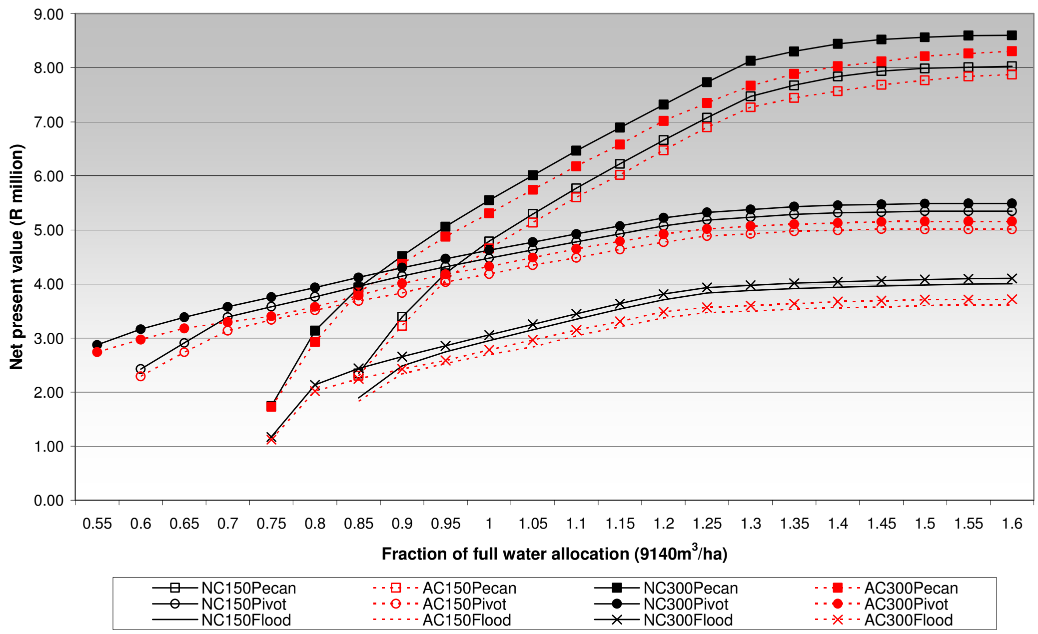

Figure C1: Net present value water availability tradeoffs for three alternative farm development scenarios, two levels of starting capital and two levels of risk aversion.

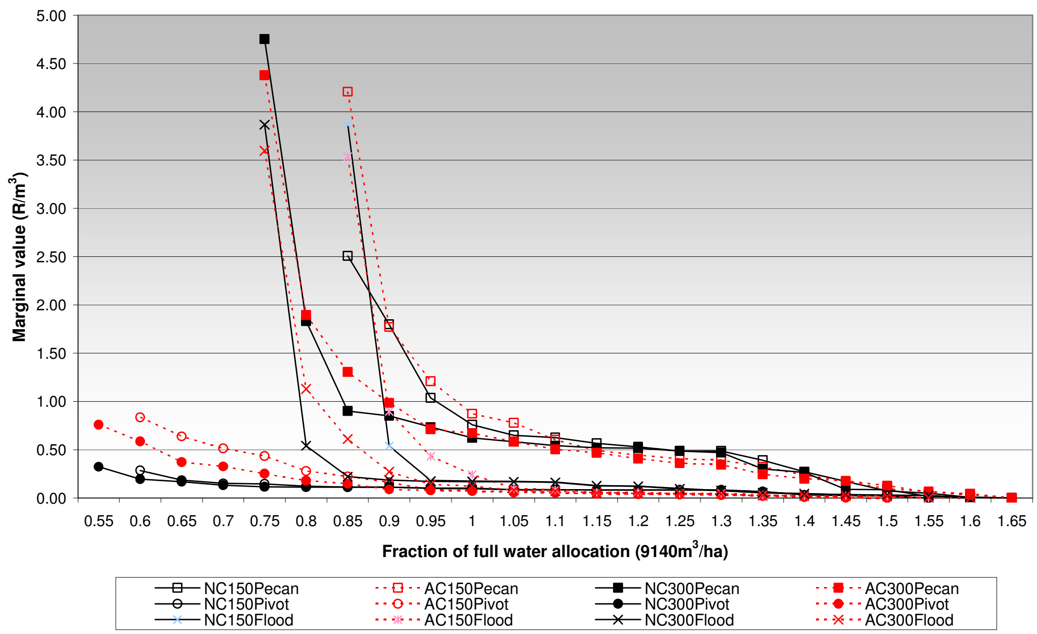

Figure C2: Irrigation Water derived demand for three alternative farm development scenarios, two levels of starting capital and two levels of risk.

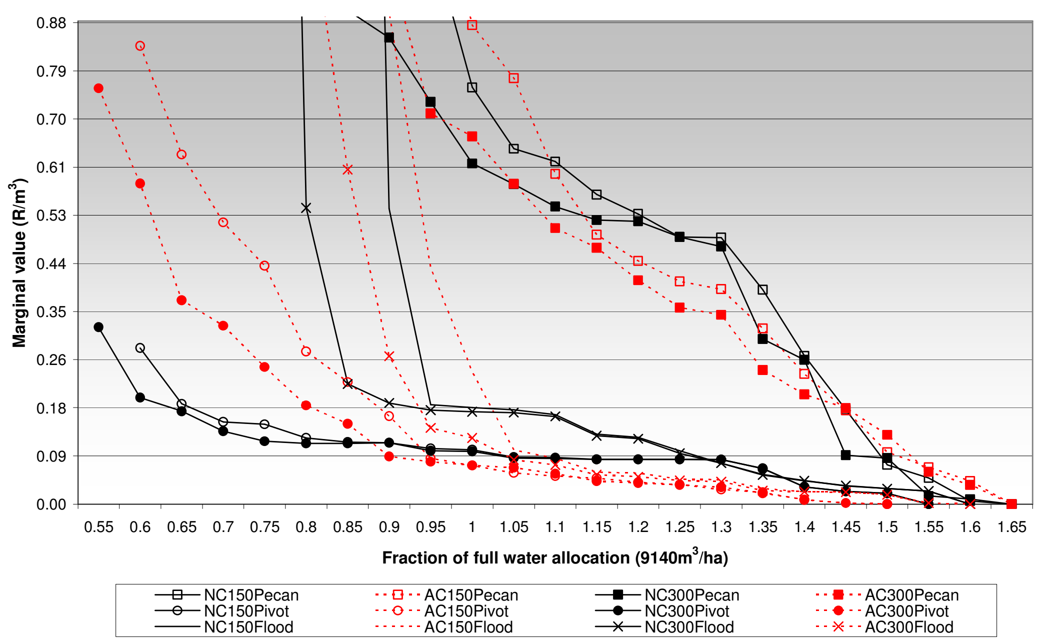

Figure C3: Irrigation Water derived demand at low price ranges for three alternative farm development scenarios, two levels of starting capital and two levels of risk.