Estimation of Representative Transmissivities of Heterogeneous Aquifers

By

Gideon Steyl

Thesis submitted in the fulfilment of the requirements for the degree of

Philosophiae Doctor

In the

Institute for Groundwater Studies

Faculty of Natural and Agricultural Sciences

At the

University of the Free State

Promoter: Prof. G.J. van Tonder

November 2011

Declaration

To my best knowledge and understanding, the thesis contains no material which has been previously published or written by another person except where due references has been given.

I, Gideon Steyl declare that; this thesis hereby submitted by me for the Doctorate of Philosophy degree in the Faculty of Natural and Agricultural Sciences, Institute for Groundwater Studies at the University of the Free State, is my own independent work. The work has not been previously submitted by me or anyone at any university. Furthermore, I cede the copyright of the thesis in favour of the University of the Free State.

Acknowledgements

I would like the following people that have assisted me in completing this research.

1. Prof. A. Roodt fellow researcher and current head of Department Chemistry, for having the vision to allow me to complete my research in a new field unhindered. Also for his support and understanding that new roads need to be made and the quest for new adventures.
2. To my wife Liza (Elizabeth) Steyl for enduring my strange sense of fun in doing my second Ph.D. To my two daughters, Anneke and Elizabeth Beatrix Steyl, I would also like to express my gratitude since you made it possible for me to do this.
3. My mother, Hester Cornelia Spies, without your support over the past 36 years I could not have obtained all the degrees and reached my current position. I hope that the Lord holds a special blessing for you.
4. Prof. G. van Tonder ("Beste Professor") for the support and understanding that new roads have to be taken with old dogs and hopefully we can contribute to a better South Africa.
5. Prof. J. Botha and Dr. D. Vermeulen I would like to say thank you for all your contributions to my research. Prof. Botha for all the long conversations about more theoretical aspects than people could handle at IGS; it is fun to talk to someone I can relate too. To Dr. Vermeulen thank you for the geology lessons which made a chemist into a geohydrologist, but chemistry is still the foundation.
6. To all the staff in Chemistry and at the Institute for Groundwater Studies that have supported my efforts – thank you.
7. To my students in both Chemistry and at the Institute for Groundwater Studies, I would like to thank you for making my hands lighter and that I could assist you in obtaining your degrees.
8. Finally, to our heavenly Father who has found it in his way to allow me to do all of this work and setting me free to do what I must.

"*Studying, and striving for truth and beauty in general,*
is a sphere in which we are allowed to be children throughout life."

Dedication to Adriana Enriques, ca. Oct. 22, 1921.
Einstein Archives 36-588

"It is not in winning or losing, but in the spirit it is done."

– Old Japanese Proverb

Keywords

Hydraulic test

Transmissivity

Regional mean values

Geostatistics

Modelling

Table of Contents

Chapter 1 Introduction 1
1.1 Aquifer Systems in Africa 1
1.2 The South African Perspective 4
1.2.1 Geology 4
1.2.2 Climatic factors 6
1.2.3 Hydrogeological factors 9
1.3 Research Statement 11
1.4 Research Objectives 12
Chapter 2 Theory 13
2.1 Introduction 13
2.1.1 Flow and Storage in Fractures and Porous Medium 15
2.1.1.1 Validity Range of Darcy's Law 15
2.2 Difficulties Associated with Scientific Models in Practice 16
2.2.1.1 Observational-analytic Modelling 18
2.2.1.2 Stochastical Continuum Modelling 20
2.2.1.3 Inverse Modelling 21
2.3 Geostatistical Methods 22
2.3.1 Variogram 26
2.3.2 Parameter estimation 27
2.3.3 Analogy between hydrological and electrical flow 27
2.3.4 Unconfined Aquifers 29
2.4 Review of Hydraulic Tests and Possible Interpretations 31
2.4.1 Support Volume 31
2.4.2 Further issues 32
2.5 Testing and Heterogeneity – the Statistical Approach 33

2.5.1 Heterogeneous Media and the Theis Method 35

2.5.2 Stochastic modelling 37

2.5.3 Some other approaches 37

2.6 Hydrogeological Scaling 38

2.6.1 Geology and Testing 39

2.6.2 Geologic Setting 39

2.6.3 Scale-Up 39

2.7 Inversion Algorithm 41

2.8 Conclusion 42

Chapter 3 Model Applicability and Description 44

3.1 Introduction 44

3.2 Model description 47

3.2.1 Software 47

3.2.2 Model Construction 48

3.2.3 Results and Visualisation 49

3.3 Conclusion 49

Chapter 4 Conceptual Models 50

4.1 Introduction 50

4.2 A Theoretical Investigation into Hydraulic Conductivity Distributions 50

4.2.1 Hydraulic Conductivity Distribution under Natural Conditions 51

4.2.1.1 Estimating Hydraulic Conductivity as a Function of Discharge over an Area 55

4.2.2 Hydraulic Conductivity Distribution under Forced Gradient Conditions 60

4.3 Conclusion 66

Chapter 5 Case Studies 67

5.1 Introduction 67

5.2 The Effect of Random Selection on Regional Hydraulic Conductivity Estimates 67

5.3 The Effect of Directed Selection on Regional Hydraulic Conductivity Estimates 71

5.4 Estimating the Regional Hydraulic Conductivity from Field Data 77

5.5 Combining Measured and Published Hydraulic Conductivity Values 81

5.5.1 Krugersdrift Test Site Evaluation 82

5.5.2 Campus Test Site – The Riemann Evaluation 84

5.6 Conclusion 90

Chapter 6 Discussion 91

6.1 Introduction 91

6.2 Horizontal Heterogeneity 98

6.3 Vertical Heterogeneity 100

6.3.1 Different Scales 101

6.3.2 Fracture Connectivity 104

6.4 Estimation of Representative Transmissivities 105

6.4.1 Mean Parallel Flow: Representative Transmissivity value 108

6.4.1.1 Stochastic Model to estimate K(eff) 108

6.4.1.2 Deterministic model to estimate K(eq) 109

6.4.2 Convergent Flow: Representative T-Value 110

6.4.2.1 Geostatistical model: 2D 110

6.4.2.2 Deterministic model: 2D 111

6.4.3 CONVERGENT FLOW: REPRESENTATIVE T-VALUE 112

6.5 Conclusion 115

Chapter 7 Conclusions and Recommendations 116

Chapter 8 References 120

Chapter 9 Appendix 126

9.1 Software 126

9.2 Model Construction 126

9.3 Results and Visualisation 126

Chapter 10 Abstract / Opsomming 127

10.1 Abstract 127

10.2 Opsomming 129

Table of Figures

Figure 1-1 Groundwater regional map of Africa. Blue, green and brown represent respectively major, complex and shallow groundwater basins. Darker shading indicates higher recharge rates (WHYMAP, 2008, Steyl and Dennis, 2010). 3

Figure 1-2 Generalised geological map of South Africa (legend key see Figure 1-3) (CGS, 2000, Johnson et al., 2006). 5

Figure 1-3 Legend key for Figure 1-2, showing the main geological features and groups(CGS, 2000, Johnson et al., 2006). 5

Figure 1-4 Mean annual precipitation (left hand, mm) and mean annual rainfall concentration as a percentage of total (right hand, %) of South Africa (Schulze et al., 1997). 7

Figure 1-5 Mean annual temperature (left hand, ^{o}C) and mean annual evaporation potential (right hand, mm) of South Africa (Schulze et al., 1997). 8

Figure 1-6 Generalised areas as a function of rain seasons in South Africa (Schulze et al., 1997). 8

Figure 1-7 A regional map showing a subsection of the Karoo Supergroup with blue patterns indicating sills while green to red represents dykes in the area. 9

Figure 1-8 Estimated transmissivity values (m^2/d) for South Africa. Left-hand side constructed from reported borehole yield data points and right-hand side from the Groundwater Resource Directed Management (Dennis and Wentzel, 2007) Database. 10

Figure 1-9 Estimated storativity values for South Africa (GRDM). 10

Figure 2-1 Effects on Darcy's Law at small gradients. 16

Figure 2-2 The experimental semivariogram is a scatter plot of the semivariance against distance between sampling points (blue diamonds). A variogram from the exponential family is fitted to the data, the nugget effect for parameters are excluded in this analysis. 25

Figure 2-3 Description of theoretical semivariogram. If X = 0 and the semivariance does not equal zero then this is called the nugget. The maximum value, i.e., plateau is refered to as the sill, with the region of influence representing the value of x for which the theoretical semivariance is 95 % of the distance between sill and nugget. 26

Figure 2-4 Illustration of blocks in series and parallel with flow direction indicated by blue arrow. 28

Figure 2-5: Derivates responses for a number of field test at active well (Raghavan, 2004). 36

Figure 2-6 Outline to suggest scheme to estimate properties (Raghavan, 2004). 42

Figure 3-1 Grid extent of model with observation boreholes located at the crossed circle points. Grid points in the model system is 500 x 500 cells with each cell block representing a 10 m x 10 m unit. 48

Figure 3-2 Randomized distribution of hydraulic conductivity values in the matrix region, left hand figure with red square in it. A zoomed view of the red square in shown on the right hand side with the three distinct hydraulic conductivity zones (Green = 0.01 m/d; Blue = 0.1 m/d; White = 1 m/d). 49

Figure 4-1 Randomly distributed transmissivity values over a two dimensional surface (100 x 100 elements). 51

Figure 4-2 Histogram of transmissivity distribution values as represented for each random function in a 100 × 100 matrix (Figure 4-1). 52

Figure 4-3 Basic statistical analysis of (1, 2, 3) map distribution in both 100 × 100 and 1000 × 1000 sample areas. 53

Figure 4-4 Statistical analysis of (1, 10, 100) map distribution in both 100 × 100 and 1000 × 1000 sample areas. Y-axis scale in logaritmic units. 53

Figure 4-5 Statistical analysis of (1, 100, 10000) map distribution in both 100 × 100 and 1000 × 1000 sample areas. Y-axis scale in logaritmic units. 54

Figure 4-6 Random distribution of hydraulic conductivites for a specific test area. 56

Figure 4-7 First order (left hand side) and second order (right hand side) hydraulic head value results after a 6 year simulation period. 57

Figure 4-8 Difference map of first and second order hydraulic conductivity under simulated natrual flow. 58

Figure 4-9 Hydraulic conductivity map of the study area with a cavity in the central region. 59

Figure 4-10 Discharge from seepage wall over an extended time period of 6 years. Both the one order difference and two order difference scenarios are given for the same model. 60

Figure 4-11: Random number-generated maps. The left figure shows a random distribution of hydraulic conductivity values. In comparison, lineaments or line segments that could act as high conductivity zones are also represented on the right figure. 61

Figure 4-12 Hydraulic conductivity maps showing the presence of a high (left) and low (right) conductivity blocks in the middle of the randomly generated values. 62

Figure 4-13 Contours maps of drawdown at the end of six years of constant pumping at 100 m^3/d. Left hand side illustrate the contours in a random field while the right hand side indicates the effect of two low conductive zones on the cone of depression contours. 63

Figure 4-14 Contours maps of drawdown at the end of six years of pumping at a 100 m3/d. Left hand side illustrate the contours in a random field with a high hydraulic conductivity zone located in the

middle. The right hand side indicates the effect of a low hydraulic conductivity zone on the cone of depression contours. 63
Figure 4-15 Cooper-Jacob fit of simulated hydraulic test data for the random one order system. The estimated hydraulic conductivity for the hydraulic test is 0.0183 m/d. 64
Figure 5-1 Random distribution of hydraulic conductivites for a specific test area. 68
Figure 5-2 Effect of smoothing and filtering of original hydraulic conductivity map (filter set at 50 % of highest peak). 72
Figure 5-3 Effect of bias on the filter of hydraulic conductivity map (left filter set at 20 % and right 40 %). 75
Figure 5-4 Effect of bias on the filter of hydraulic conductivity map (left filter set at 50 %). Right hand indicates the change of filter from 40 to 50 %. 76
Figure 5-5A plan view of the regional water flow direction in the presence of a dyke structure. 78
Figure 5-6 Side on view of regional water flow in the presence of a dyke structure. Left-hand side indicates the situation if the water level is below the weathered zone of the dolerite dyke. Right-hand side indicates the situation if the water level is above the weathered zone. 78
Figure 5-7 Regional transmissivity map as constructed from GRDM data (Hughes et al., 2007). 81
Figure 5-8 Regional transmissivity map as constructed from climate change data. 82
Figure 5-9 Krugersdrift test site located within the Free State province, South Africa. The two areas where hydraulic test data were obtained is indicated in the bottom left hand section as A1 and A2 respectively. 83
Figure 5-10 Campus test site with borehole locations shown in the map. Blue dots indicate boreholes located in the matrix and black dots represent boreholes that intersect the bedding plane fracture zone (Riemann, 2002). 85
Figure 5-11 Hydraulic test observation points at the campus test site (Riemann, 2002). 87
Figure 6-1 Plan views of two typical flow regimes in an aquifer system, (A) natural uniform flow and (B) convergent flow. The black dots represent boreholes and the red dot a pollution source. 91
Figure 6-2 Relative two dimensional influence of hydraulic tests at the 4 boreholes with time, (a) short time and (b) longer time. Just from a visual inspection of the location of the boreholes it can be deducted that $T_1<T_2<T_3<T_4$ and (c) increase of volume material as seen by hydraulic test for a homogeneous aquifer and (d) for a heterogeneous aquifer. 93
Figure 6-3 Ratio of volume of fracture material to the total volume material with increase in distance from abstraction borehole. 93

Figure 6-4 High transmissivityzone of 1000 m^2/d situated in an aquifer with transmissivity of 20 m^2/d. The yellow dot shows the position of a borehole.. 94
Figure 6-5 The figure on the left shows the expected drawdown in the area after 4 320 days. On the right hand side a drawdown curve from which it can be seen that the high transmissivity block has nearly zero influence on the drawdown with an estimated transmissivity (24 m^2/d) value for the whole area. The influence of the no-flow boundary (decreasing water level) can clearly be observed in the latter part of the curve. 95
Figure 6-6 Inclusion of four high transmissivity zones in a model area.. 95
Figure 6-7 Influence of four high transmissivity zones on the drawdown and estimated representative transmissivity value. Fitted transmissivity value is 24 m^2/d. 96
Figure 6-8 Representative transmissivity value = 33 m^2/d if abstraction takes place in one of the four high transmissivity zones of 1000 m^2/d. The influence of the no-flow boundaries can clearly be seen at late times. 96
Figure 6-9 (A) Four boreholes drilled at different locations in a 1 km^2 block radius, with the red dot indicating the pollution source and (B) the thickness of the lines is representative of the transmissivity (T) values observed within the area. Borehole 1 has the lowest transmissivity value, while borehole 4 the highest transmissivity value.. 98
Figure 6-10 (a) Dark lines indicate the variable connectivity-length of the four boreholes in respect to the fractures and (b) fractures connecting the left side of the block to the right side. 98
Figure 6-11 Vertical section of borehole that intersects two main water-yielding fractures. 100
Figure 6-12 Method of defining point values of macroscopic quantities illustrated with porosity f (Hubbert, 1956)... 103
Figure 6-13 Hydraulic conductivity in the fractured bedrock of the Mirror Lake watershed and its vicinity as estimated over increasingly larger physical dimensions from (A) discrete-interval, single hole hydraulic tests, (B) cross-hole hydraulic tests, and (C) regional ground-water flow modelling (Hsieh, 1998). 105
Figure 6-14 General behaviour of effective (stochastic) and equivalent (deterministic) transmissivity estimates with an increase in aquifer volume.. 112
Figure 6-15 Drawdown and derivative plots for boreholes located in the general area of Bethulie (left) and Boshof (right). 114
Figure 6-16 Drawdown and derivative plots for boreholes located in the general area of Limpopo – gneiss (left) and Postmasburg – dolomite (right). 114
Figure 7-1 Illustrating the effect of transmissivity and recharge on steady state calibration methods... 118

List of Tables

Table 4-1 Evaluation of mean values for differing ranges of hydraulic coductivity values in an area as depicted in Figure 4-1. 52

Table 4-2 Summary of values for the discharge rate of the one and two order systems for the cavity model. 59

Table 4-3 The influence of conductive zones on the average hydraulic conductivity of different order systems. Randomly generated low conductivity zones (lines) were included (see Figure 4-11). 64

Table 4-4 The influence of high and low conductive block zones on the average hydraulic conductivity of different order systems. 65

Table 5-1 Mean values of randomly sampled points in a first order difference hydraulic conductivity map (0.01, 0.1, 1). 68

Table 5-2 Mean values of randomly sampled points in a second order difference hydraulic conductivity map (0.01, 1, 100). 70

Table 5-3 Smoothing matrix format as applied to first order difference map. 71

Table 5-4 Mean values of randomly sampled points in a first order difference biased hydraulic conductivity map (0.01, 0.1, 1). 73

Table 5-5 Mean values of randomly sampled points in a second order difference biased hydraulic conductivity map (0.01, 1, 100). 74

Table 5-6 Mean values of randomly sampled points in a first order difference biased hydraulic conductivity map. 75

Table 5-7 Hydraulic conductivity values compared to geological formation. 80

Table 5-8 Selected transmissivity values from hydraulic tests conducted at Site A1 and A2. 83

Table 5-9 Representative transmissivity values from Riemann (2002) for the September 2000 data set. 88

Table 5-10 Representative transmissivity values from Riemann (2002) for the September 2000 data set. 88

Table 5-11 Results of the evaluated aquifer parameter for the constant rate test UO5, July 2000, obtained from the fracture-piezometers. 89

List of Acronyms

ρ	Density (mass per unit volume)
s	Drawdown (length)
ν	Flux or Darcy velocity of the fluid
g	Gravitational constant
h	Hydraulic head
K	Hydraulic conductivity (m/d)
k	Intrinsic permeability
S	Storage coefficient
T	Transmissivity (m^2/d)
μ	Viscosity of a fluid at a specific temperature
CGS	Council for Geosciences (South Africa)
GLUE	Generalised likelihood uncertainty estimation
GRDM	Groundwater Resource Directed Management (Dennis and Wentzel, 2007)
IGS	Institute for Groundwater Studies, Free State University, Bloemfontein
TDS	Total dissolved solids (mg/l)
WHO	World Health Organization
WRC	Water Research Commission

Chapter 1 Introduction

Africa is an ever-changing human landscape, with social and political issues driving the development of the continent. In this regard the availability of freshwater is of notable concern, since the stability of a government depends heavily on its ability to provide services for basic human needs (CSA, 1996). Water resources management in this regard is gaining importance as a critical development issue (UNWater, 2006). Through management programs the population's environment can be improved. These include poverty reduction, agricultural productivity, industrial growth and sustainable growth in downstream communities (Davis et al., 2003). In Africa the World Health Organization (WHO) estimated that if access to basic water and sanitation services were improved, the health sector would save more than US$11 billion in treatment costs. People would gain 5.5 billion productive days each year due to reduced diarrheal disease (Tobin, 2008).

1.1 Aquifer Systems in Africa

Regional aquifers in Africa can differ substantially from one area to another and exist as alluvial, lacustrine, basaltic and sedimentary aquifers in coastal zones. The development of aquifer systems in Africa relies primarily on two major factors, *i.e.*, the tectonic and climatic environments. In terms of aquifer setting, two completely different geological domains exist. Firstly, the mobile belt of the Atlas range and secondly the African platform, which are separated by the South Atlas Fault Zone located between the southern Saharan Craton and smaller microplate mesetas to the north (Steyl and Dennis, 2010).

The folded zones of northwest and southern Africa only occupy about 3 % of the continent but sustains ca. 10 % of its population (Zektser and Everett, 2004). In South Africa the region between Durban and Cape Town consists of paleozoic limestones and paleozoic sandstones, quartzites and shales. In the Atlas fault zone fissured carbonate aquifers of mainly limestone and dolomite of the Jurassic and Cretaceous age contribute to coastal and interbedded clastic porous aquifers.

Major aquifers of the African platform are found over an extensive area, interacting with coastal regions. Six major aquifer systems can be identified on their respective lithology; continental sandstone, carbonate, sandstone-carbonate (variable), alluvial, basaltic and crystalline basement aquifers (Zektser and Everett, 2004).

The continental sandstone aquifer group contains the Nubian Sandstone Aquifer, the continental intercalaire of the Sahara, the Karoo and the Kalahari. The carbonate aquifer group consists of the Jabal Akhdar-Sirte (coastal basin) in northern Libya. The sandstone-carbonate variable composition is found in the North Sahara basin in Algeria and Tunisia (Margat, 1994). The sandstone-limestone hydrogeological complex occurs mainly in coastal basins in Mauritania, Senegal, Côte d'Ivoire, Cameroon, Gabon, Angola, Somalia and Mozambique (Zektser and Everett, 2004).

Alluvial aquifers of the Neogene and Quaternary age occur in the sedimentary and coastal basins and are commonly found in Tunisia and the Atlantic coast of Morocco. The alluvial aquifer group contains the Congo basin and the Nile River alluvial aquifer (Egypt and Sudan). The latter is often clayey with the thickness of the alluvial aquifer increasing northwards from a few meters at Cairo to *ca.* 1000 m at the Mediterranean Coast. The northern coastal section of the Egyptian Delta Aquifer is less productive and contains brackish or saline water (RIGW/IWACO, 1988, Hefny et al., 1991).

Sedimentary basins are an important resource for water in the narrow coastal zones which includes the coastal basins of Gabon, Congo, Zaire, Angola and Mozambique. The Gabon coastal basin covers an area of about 55 000 km^2 and is composed of a multilayered aquifer system (Figure 1-1). The aquifers consist of continental sediments, evaporites interbedded with carbonate rocks and marine sediments. Salinity, expressed as TDS (Total Dissolved Solids) varies from less than 500 mg/l to 5 g/l, the higher salt load increases with depth and usually occurs at depths greater than 400 m.

Parts of Democratic Republic of Congo, Congo and Angola are underlain by the Congo basin; since large amounts of surface water are available very little attention has been paid to the groundwater aquifer system. In the river systems of this basin a large number of dams have been constructed in the recent past. However, recently the availability of potable water has shifted the focus back to investigating groundwater sources as a bulk supply (Supply, 2008).

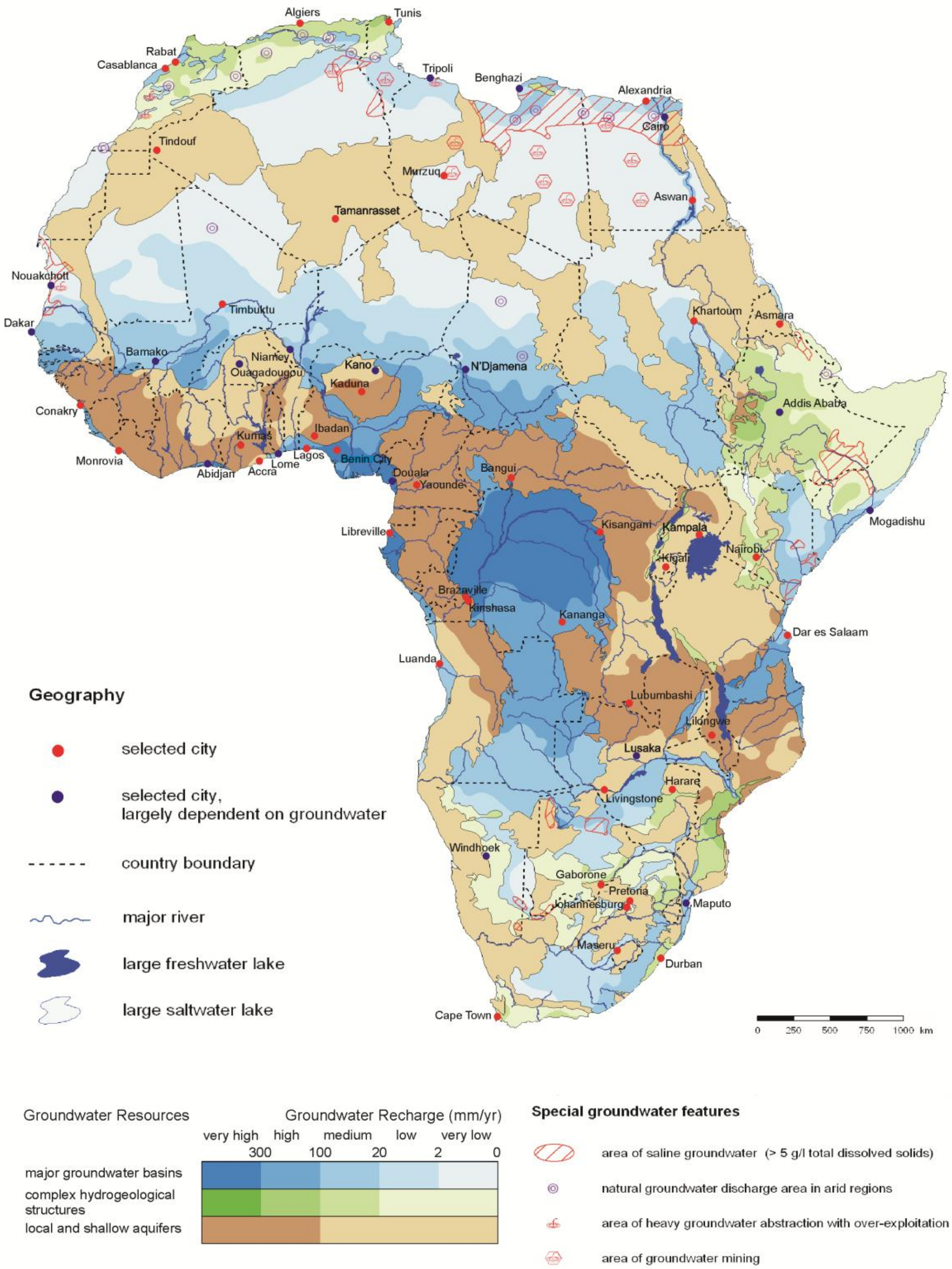

Figure 1-1 Groundwater regional map of Africa. Blue, green and brown represent respectively major, complex and shallow groundwater basins. Darker shading indicates higher recharge rates (WHYMAP, 2008, Steyl and Dennis, 2010).

1.2 The South African Perspective

South Africa can be classified as a developing country with an increasing water demand due to industrial and population growth (Robins et al., 2006). The country has a well-developed dam system and has expanded water supply systems to neighbouring countries such as Lesotho. Due to the ever expanding need for water and the arid climate of South Africa alternative water supply systems are required. One alternative is to focus on groundwater as a source of potable water for rural areas or small towns (DWAF, 2004). This is an attractive option, since reticulation networks are costly and would require construction over vast distances. In contrast local aquifers can be used and managed on a sustainable basis to provide potable water to a community (DWAF, 2004).

South Africa's groundwater resources are generally underutilised and less effectively managed than its surface waters (Robins et al., 2007). Characteristics that make groundwater attractive as a water resource for local communities is that the water stored in the aquifer can be abstracted as required. To a certain extent the groundwater, if managed correctly, can sustain a community during extended drought periods which is critical from a South African perspective (Robins et al., 2006). Due to the relatively low cost of drilling and pump installation, a water resource can be allocated close to the supply point. In general groundwater is of an acceptable quantity and quality to be used without further treatment; this significantly reduces costs and increases the range of usage for local communities. One of the key issues in South Africa is that reliable water resources are not evenly distributed through the country side. Two primary factors affect the distribution of groundwater resources in South Africa is the complex geology (Figure 1-2 and Figure 1-3) and variable local climate. Furthermore South Africa shares some of its aquifers systems with all of the neighbouring countries, i.e., Botswana, Lesotho, Namibia, Mozambique and Zimbabwe (Cobbing et al., 2008). This increases the responsibility on the South African side to effectively manage these respective trans-boundary aquifers.

1.2.1 Geology

As noted previously the geohydrology of South Africa tends to be complex; dominated by fractured aquifers (including karst limestone aquifers). The potential of many South African aquifers has not been fully developed and this is partly due to the lack of groundwater information(Robins et al., 2006).

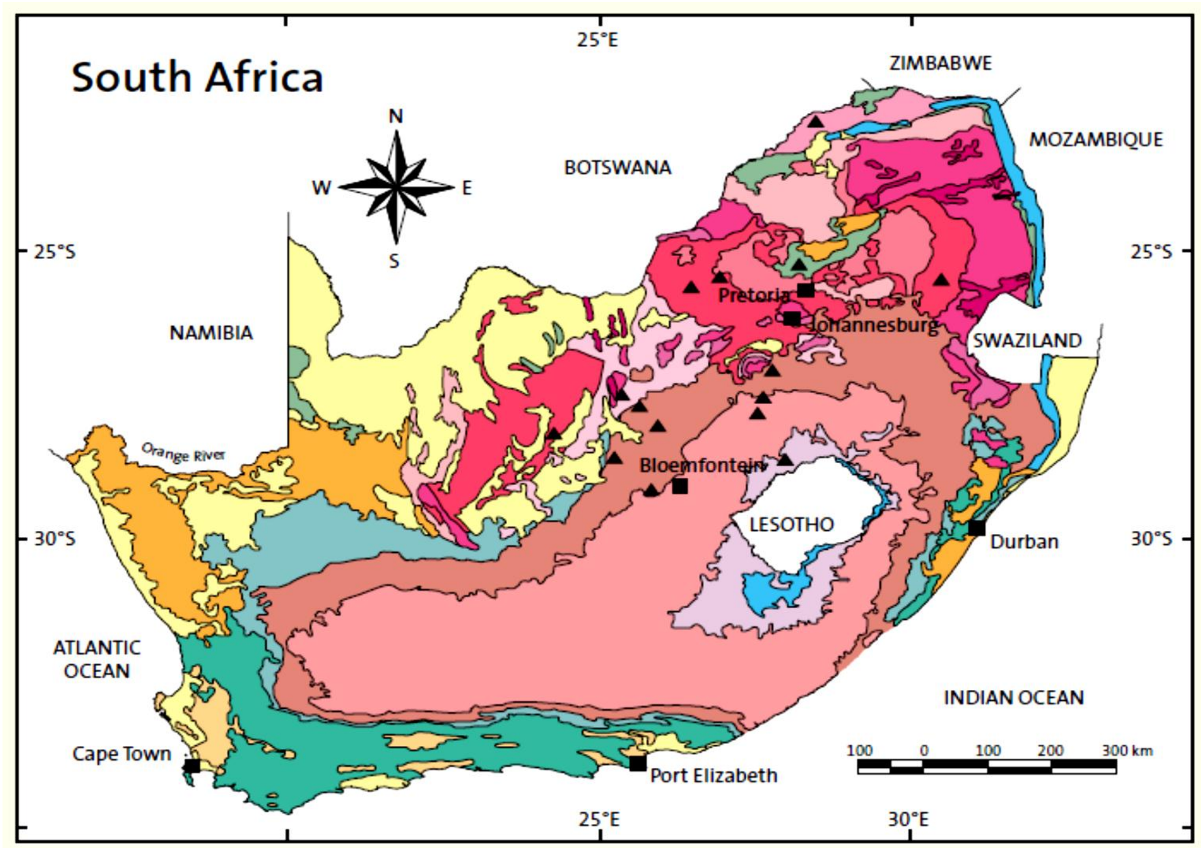

Figure 1-2 Generalised geological map of South Africa (legend key see Figure 1-3) (CGS, 2000, Johnson et al., 2006).

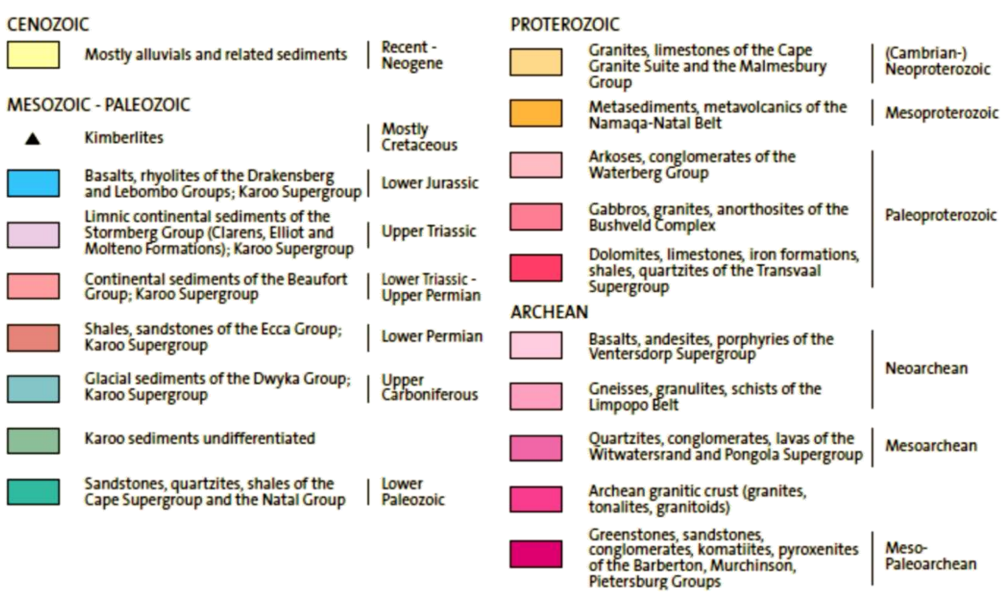

Figure 1-3 Legend key for Figure 1-2, showing the main geological features and groups(CGS, 2000, Johnson et al., 2006).

The investigation of aquifer systems is linked to the local geology of an area, and in this regard South Africa consists mostly (> 80 %) of the Karoo Supergroup (Figure 1-2). The Karoo Supergroup is dominated by hard, fractured rocks which might have dual porosity properties to a lesser or greater extent (Humphreys, 2000, Van der Linde and Van Biljon, 2000).The geometries of these aquifer systems is also complicated since intrusive structures and faulting commonly occur in these areas. In particular if drilling targets are required for water supply, it is advised to target the margins of dolerite intrusions. However, certain studies have shown that high yielding boreholes can also be drilled in the country rock (Burger et al., 1981). This is largely motivated by the baking affect that the dolerite intrusion would have had on the country rock. As the intrusive structure cooled, fracturing would have occurred and subsequently caused a preferred pathway to form along the dyke structure.

In respect to the crystalline basement (Granitic Plutons, Greenstone and Late Proterozoic sediments) specialized drilling techniques are typically required and the geohydrology of the aquifers are dominated by fractured rock systems.

Only four significant unconsolidated aquifers are present in South Africa, i.e., Kalahari, Atlantis, Langebaan and the Zululand/Mozambique aquifer underlying the St. Lucia world heritage site(Steyl and Dennis, 2010). The Cape Flats aquifer underlies *ca*. 630 km^2 and is an important source of water for Cape Town; however pollution has threatened its full development. The Mozambique/Zululand aquifer has a surface area of 7 000 km^2 and extends for 1 250 km along the coastline. It is in this area that the St. Lucia Wetland Park is situated; the existence of the nature reserve effectively protects this coastal region from exploitation. Due to forestry activities in the region, a reduction in water levels has been observed in the northern part of the Zululand aquifer. This has resulted in seawater intrusion along this area; however remedial action is currently underway to rectify this situation (DNWRP, 2004).

1.2.2 Climatic factors

The influence of climatic conditions on recharge and water usage have been investigated recently in WRC reports (Xu et al., 2007, Bredenkamp et al., 2007, Meyer, 2005, Dondo et al., 2010), these reports all indicate that the process of groundwater recharge in South Africa is a highly complex problem. To illustrate this point a few maps were compiled showing different processes. A recent thesis by Dr. Van Wyk (2010) on groundwater recharge highlighted the effect of rainfall volume versus intensity when it comes to the volume of recharge. In Figure 1-4 the mean annual precipitation and rainfall concentration percentages is shown. The average annual precipitation in South Africa is ca. 500 mm, which is significantly less than that observed for the world with a value of 860 mm. Firstly, the mean annual precipitation in South Africa increases from the arid west to the tropical east. High precipitation values are observed along the eastern section extending into the north of South Africa. Comparing this mean annual rainfall pattern with the mean annual rainfall concentration it can be deduced that the northern section of South Africa has significantly more intense rainfall events (thunder storms). In contrast the southern half of South Africa and the eastern coastal section has a moderate continuous rainfall pattern.

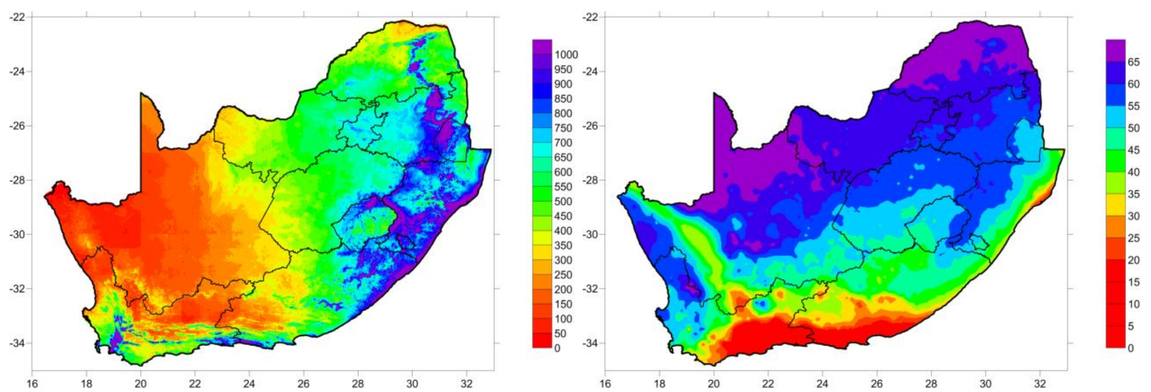

Figure 1-4 Mean annual precipitation (left hand, mm) and mean annual rainfall concentration as a percentage of total (right hand, %) of South Africa (Schulze et al., 1997).

It is expected that if episodic recharge does occur, it will generally be associated with high intensity rainfall events as noted previously (Van Wyk, 2010). Interestingly, the Karoo type aquifers are associated with high intensity rainfall areas which should increase recharge in these areas, however due to the low rainfall volumes it is expected that recharge would be in the order of a few millimetres per annum.

A second environmental factor that influences aquifer systems is the mean annual temperature and the associated mean annual evaporation potential (Figure 1-5). The average South African temperature range during the summer months is between 24 – 20 ^{o}C while during the winter time it ranges from 10 – 17 ^{o}C. The average temperature only decreases along the mountainous central belt and associated highlands, as indicated in Figure 1-5. This trend is not reflected in the evaporation potential with relatively low evaporation potentials along the eastern and coastal area (< 2 m) while inland and to the west the evaporation potentials are higher (> 2 m). This observation can be related to both the mean annual rainfall volume and mean annual temperature. It is expected that areas with high annual temperatures and evaporation potentials should be more reliant on groundwater sources, since aquifers are less vulnerable to these factors and thus represents a more constant source of water.

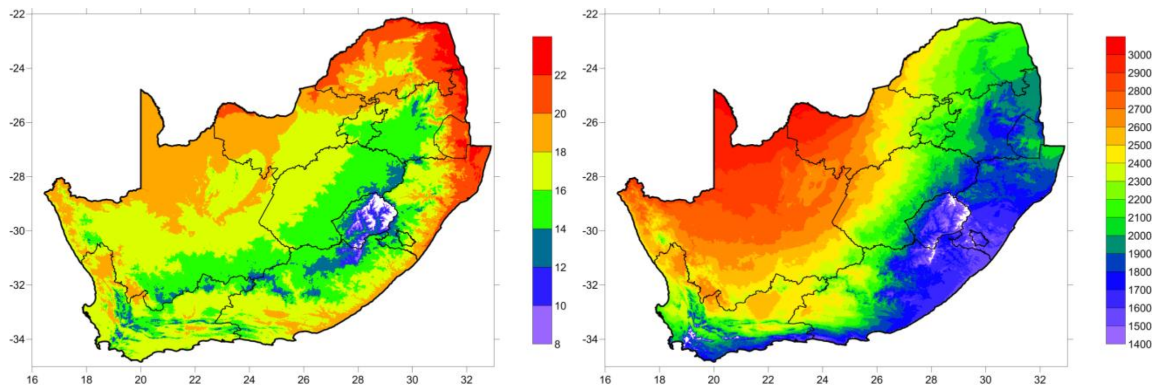

Figure 1-5 Mean annual temperature (left hand, °C) and mean annual evaporation potential (right hand, mm) of South Africa (Schulze et al., 1997).

Finally, the general rainfall patterns of South Africa are shown in Figure 1-6, illustrating the season progression and the major rainfall seasons. The seasonality of rainfall will also change the effective recharge observed in an aquifer, since lower evaporation potentials are dominant during the winter season allowing for pools to form which can recharge over a longer timeframe.

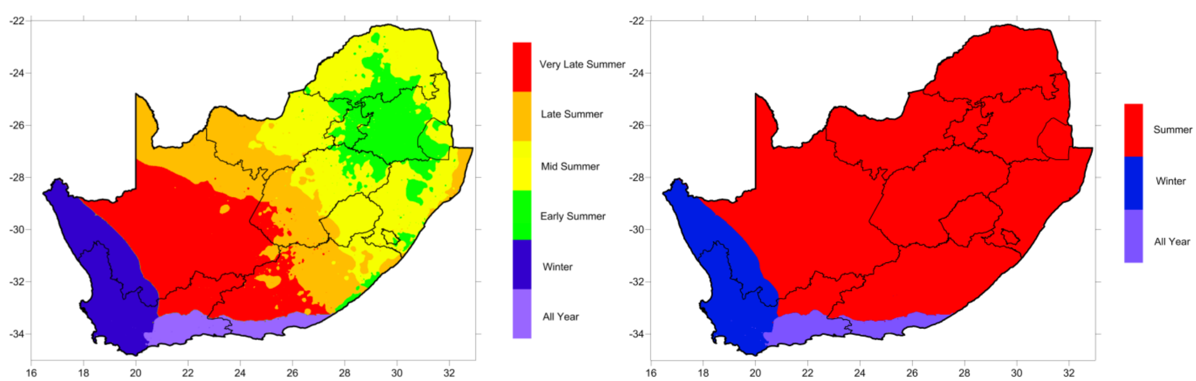

Figure 1-6 Generalised areas as a function of rain seasons in South Africa (Schulze et al., 1997).

In the next paragraph regional hydrogeological South African maps will be presented. The main focus is to supply a group of maps that will illustrate water resource directed management options and the impact it can have on South Africa's water resources.

1.2.3 Hydrogeological factors

The majority of South African aquifers are found in hard rock geological formations, as noted previously the geological structure would have an impact on the transport properties of the aquifer. In regard to bulk flow parameters various estimations of transmissivity and storativity values have been attempted, however no satisfactory results have been obtained that could be applied in groundwater management (Murray et al., 2011). One major headache is the presence of dolerite dykes and sills, which infiltrated the Karoo Supergroup (Figure 1-2) during the early Jurassic period. The irregular distribution of high and low transmissivity zones complicates the effective estimation of regional groundwater flow parameters (Figure 1-7).

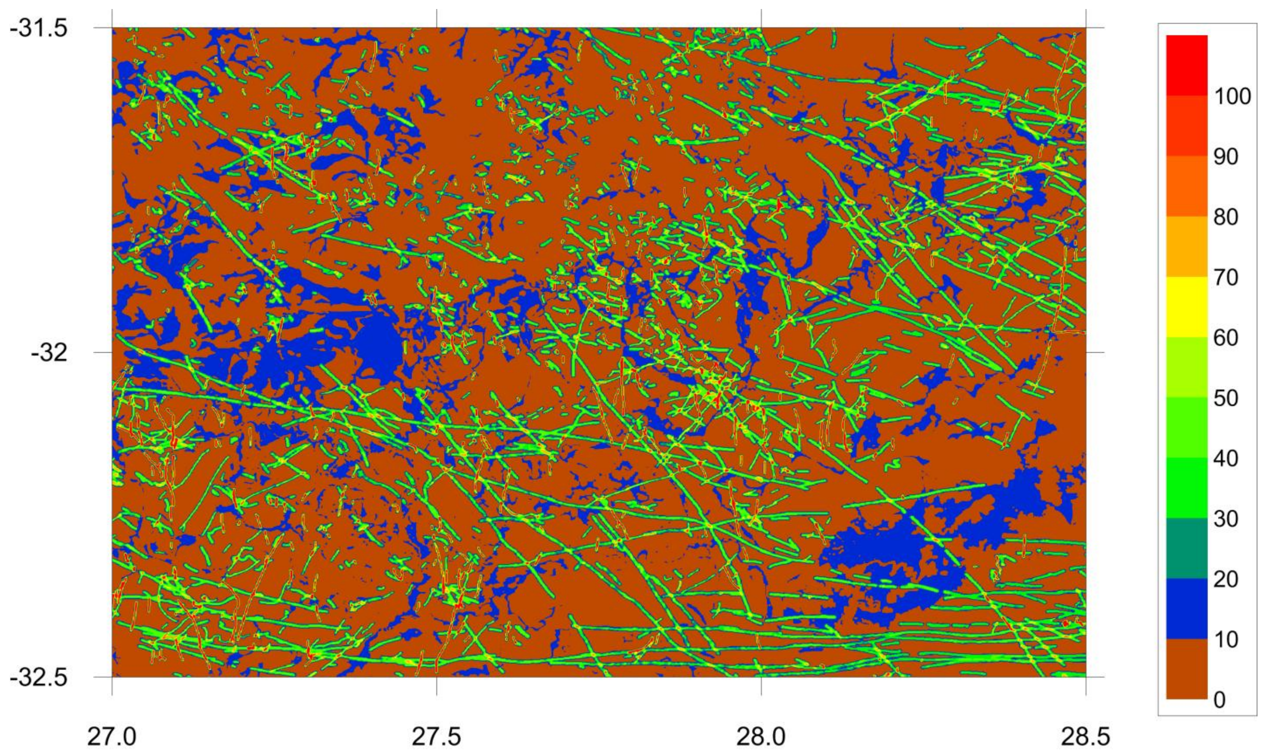

Figure 1-7 A regional map showing a subsection of the Karoo Supergroup with blue patterns indicating sills while green to red represents dykes in the area.

The estimation of transmissivity values have been done using various methods, i.e., conversion of borehole yield and hydraulic test data (Figure 1-8). It is clear from the figures that different transmissivity values were obtained, this stems from the underlying observations used in determining the values. The first set of transmissivity values were calculated from borehole yields which would indicate a long-term average scenario. The maximum transmissivity that could be observed for this map was 44 m^2/d. In contrast if hydraulic test data is used, maximum transmissivity values were observed in the range of 400 – 500 m^2/d. The discrepancy between the maximum values indicates that there might be a biasing factor in one of the methods or the methodology of obtaining the data itself.

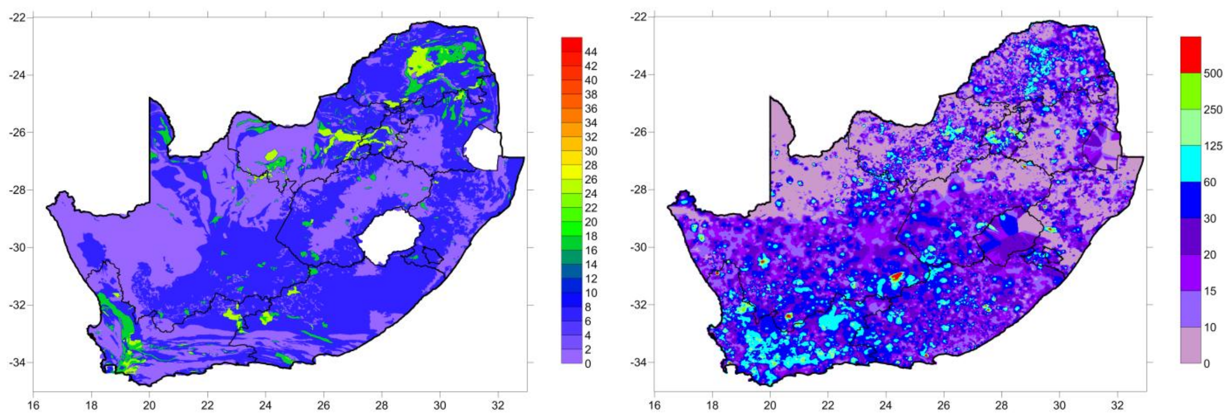

Figure 1-8 Estimated transmissivity values (m^2/d) for South Africa. Left-hand side constructed from reported borehole yield data points and right-hand side from the Groundwater Resource Directed Management (Dennis and Wentzel, 2007) Database.

Turning to storativity values for South Africa very little reported data exists which can be used for calculations. In general storativity values for the Karoo Supergroup is estimated to be in the range of 10^{-3} – 10^{-5}. Considering the map (Figure 1-9) this assumption for South Africa holds, since 95 % of South Africa's aquifers is located in the Karoo Supergroup formations.

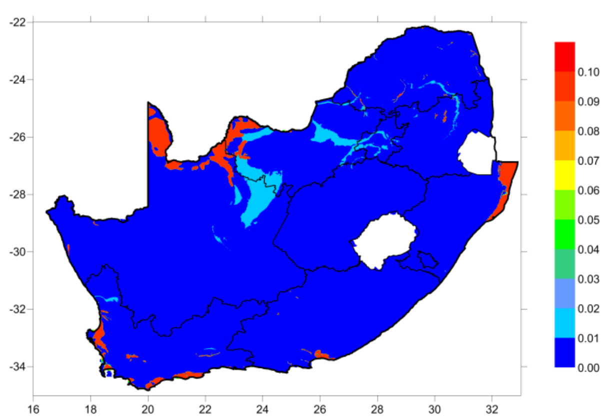

Figure 1-9 Estimated storativity values for South Africa (GRDM).

1.3 Research Statement

Groundwater resources in the past have played a vital role in the development and sustainability of rural South Africa communities. Due to reticulation costs, larger cities are investigating groundwater assets as an alternative water supply resource to assist in periods of drought or low water supply periods. One issue that hampers the development of a coherent strategy for developing this resource is the prospecting for adequate groundwater sources that is sustainable over the longer term.

Key factors that would assist in this endeavour are bulk flow parameters and the issue of scale. In general bulk flow parameters are heavily dependent on the geology of South Africa and the competency of the well test analyst in determining these parameters. A further complication is the synthesis of the obtained data into a management strategy as well as extrapolating the observed transmissivity values from a local to a regional scale.

In this thesis various case studies will be presented that range from small scale field investigations to large regional studies. The estimation of a representative transmissivity value for each of the study sites will be developed and eventually linked to a regional estimation value.

Thus the focal point of this study is to evaluate methodologies for use in converting bulk flow properties from a local perspective into parameters that could be applied on a regional level. In order to accomplish this objective, an investigation into estimation methods for bulk flow parameters was required.

A critical issue of this study was to determine methods that yielded representative transmissivity values in a heterogeneous aquifer setting. Furthermore, the conditions under which certain models could be applied that would add credence to the values obtained. A review of geostatistical methods will be presented and subsequently the calculation of an average value for a system.

1.4 Research Objectives

In order to obtain a balanced evaluation of methods to determine regional transmissivity values the following research objectives were set:

1. A literature review of current methods used in the groundwater industry to determine transmissivity values.
2. The construction of a conceptual model system to evaluate usage of different mean value calculation methods.
3. Investigate case studies to determine the effects of random sampling as compared to biased or directed sampling methods.
4. Discuss and evaluate different possible scenarios that could be encountered in the field as well as methods to estimate regional transmissivity values.
5. Present a possible methodology to indicate the way forward and requirements for geohydrological data in databases.

The following chapters will present each of the above objectives in a systematic manner. In every section an attempt will be made to comment on the applicability of each of the mean value calculation methods to estimate regional transmissivity or hydraulic conductivity values. The following chapter will present a short summary on the theory behind bulk flow parameters and the use of statistical methods in groundwater.

Chapter 2 Theory

2.1 Introduction

The basis of geohydrological investigations is that the core assumptions made by Darcy and subsequent researchers are upheld (de Marsily, 1986). Fluid flow through porous media is usually described with *Darcy's law*: The flux or the Darcy's velocity of the fluid (v) is proportional in magnitude to and coincidental with the negative gradient of the potential field or hydraulic head.

$$v = -k\frac{\rho g}{\mu}\nabla h$$

Equation 2-1

(ρ: density of the fluid, g: gravitational acceleration, ρg: specific weight of the fluid, k: permeability, μ: viscosity of fluid, h: fluid head)

Under the assumption that a fluid flows through a given cross section of a porous media without any obstruction by the sand grains, the *hydraulic conductivity* can be given as

$$K = k\,\frac{\rho g}{\mu}$$

Equation 2-2

Fluid properties in groundwater systems are usually independent of the head and may be assumed constant. Permeability is considered when discussing the transmissive capacity of the rock rather than hydraulic conductivity.

Transmissivity,T, is a concept related to permeability used in aquifer testing. It is the vertically averaged product of the hydraulic conductivity and the saturated aquifer thickness, b, in a two-dimensional reservoir:

$$T = \int_{o}^{b} K\,(z)dz$$

Equation 2-3

With the elevation of a point above the datum.

Storativity (S) is a measure of the ability of a reservoir to release or absorb fluid per unit surface area under a unit change in head. In an aquifer test, storativity is considered as the vertically averaged product of the sum of the fluid compressibility (c) and the pore compressibility (c_f) the porosity (ϕ) and aquifer thickness (b).

$$S = \int_{o}^{b} \phi(z)(c + c_f)dz$$

Equation 2-4

In general the hydraulic conductivity does not only depend on the fluid but also on the viscosity. It should be noted that the viscosity of water varies considerably with temperature. It is in this regard that one should be careful when dealing with water table aquifer systems in which temperatures and relative atmospheric pressures can change over a short time frame. Secondly, as noted in the introduction most of South African aquifers are characterised by fractured rock systems. There are two possible general methods for dealing with a conducting fractured medium within an aquifer system.

The first method relies on the idea of a continuous medium that is characterised by several conducting fractures. Thus each subset of fractures can be defined by a directional conductivity (hydraulic conductivity tensor) and successively the intensity and directions of flow can be combined to calculate the principal axis of anisotropy. The method of continuous medium is valid for a certain scale of observation since an average description of flow velocity and direction is used in the estimate. The estimation of the subset of fracture properties can be obtained by using a statistical measure of an aperture, distance and dip or by in situ methods where the hydraulic conductivity of each fracture set is measured. Both estimation methods suffer from the same disadvantages in that it is assumed that the fracture network is infinite and are homogenous in the principal properties under investigation.

The second method of modelling flow in a fractured medium is done under the assumption that a discontinuous medium is present. Thus, the elementary fractures or subset of fractures can be represented by an equivalent fracture of the same family (closed subgroup). The model is composed of nodal points where the hydraulic head is calculated and between the inter-nodal points a plane can be defined to compute the velocities. The most significant drawback of this method is that the property of each of the fractures should be known in space. This is however, not possible on a regional scale.

2.1.1 Flow and Storage in Fractures and Porous Medium

In general when applying the Darcian method of flow in a porous medium, it is assumed that the water flow is steady and invariant with time. However, if one considers transient flow, the fundamental properties of fractures appear such that it is clear that a dual or double porosity medium is present. In this instance both the hydraulic conductivity of the porous medium (K_m) and the hydraulic conductivity of the fracture (K_f) should be considered. In the steady state the dual porosity medium can be described by using an equivalent hydraulic conductivity and storativity values. However, under transient conditions fluid flow will be much greater in the fractures than in the porous medium ($K_f >> K_m$). In contrast the storativity of a fracture is much less than that for the porous medium that surrounds it ($S_m >> S_f$).

2.1.1.1 Validity Range of Darcy's Law

Darcy's Law is defined on a macroscopic scale which results in averaging effects occurring. However, at extreme values Darcy's Law is invalidated. The two extreme hydraulic gradients are represented at both low and high extremities.

Low gradients occur typically in fine cohesive material such as compacted clays (montmorillonite). If the gradient is too low the transmissivity in the medium is effectively zero (Figure 2-1). This can be represented by a threshold value (i_0). However, a second interval exists in which the relationship between the hydraulic gradient and the transmissivity is non-linear. The upper value of this non-linear boundary can be represented by a defined hydraulic gradient (i_1), see Figure 2-1. Hydraulic gradient values greater than i_1 corresponds proportionally to Darcy's Law ($i > i_1$).

In the instance where hydraulic gradients are too high, the proportionality between the gradient and the filtration velocity is negated. In order to make this transitional hydraulic gradient into a definable quantity, the Reynolds number in porous medium is used ($R_e = U\rho\sqrt{k}/\mu$) where U is the filtration velocity, k intrinsic permeability and μ viscosity. In practice Darcy's law is valid in a porous medium where R_e ranges from 1 – 10, i.e., where purely laminar flow occurs.

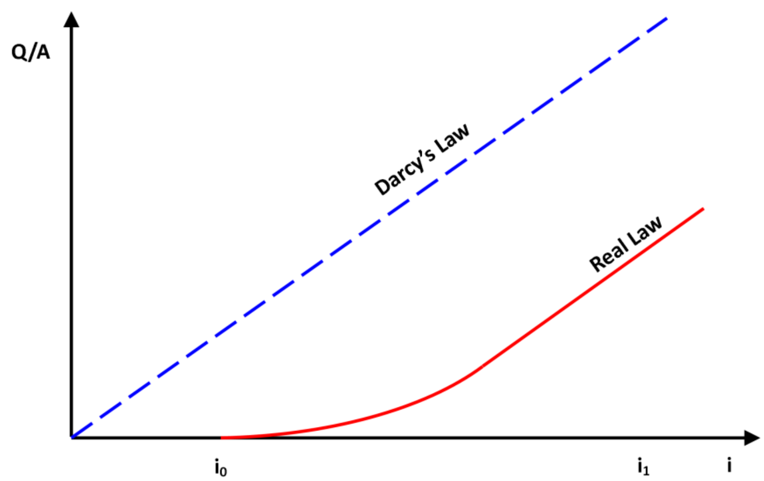

Figure 2-1 Effects on Darcy's Law at small gradients.

2.2 Difficulties Associated with Scientific Models in Practice

Although there are exceptions, the mathematical models associated with the conceptual models of physical systems are based on one or more partial differential equations. Take for example the one derived from Darcy's law in Equation 2-5, often used in groundwater flow investigations (Bear, 1972, Bear, 1979, Botha, 1996a)

$$\boldsymbol{S_0}(\boldsymbol{x},\boldsymbol{t})\boldsymbol{D_t}\boldsymbol{\phi}(\boldsymbol{x},\boldsymbol{t}) = \boldsymbol{\nabla}\cdot[\boldsymbol{K}(\boldsymbol{x},\boldsymbol{t})\boldsymbol{\nabla}\boldsymbol{\phi}(\boldsymbol{x},\boldsymbol{t})] + \boldsymbol{f}(\boldsymbol{x},\boldsymbol{t})$$

Equation 2-5

This equation contains in the terminology of Equation 2-5 two relational parameters S_0**(x,**t**)** (the storativity of the porous medium) and **K(x,t)**, the forcing function **f(x,**t**)** representing the strength of sources (+) and sinks (–) that may be present in the flow field and the observable piezometric head

ϕ(x,t**)**. Since Equation 2-5 is a partial differential equation one could use it to predict the behaviour of **ϕ(x,**t**)** at the positions **x** and time t, as is indeed done in practice. However, as is known from the theory of partial differential equations, this can only be done if the following conditions are met beforehand.

Step 1. The domain Ω spanned by **x** and its boundary $\partial\Omega$ must be known.

Step 2. The relational parameters, such as S_0**(x,**t**)** and **K(x,**t**)** in Equation 2-5, must be known at all points **x** in Ω and any time t for which the differential equation has to be solved.

Step 3. Any forcing functions that appear in the differential equation must be known.

Step 4. Boundary conditions, appropriate values of the dependent variable, e.g. **ϕ(x,**t**)** in Equation 2-5, must be known at all points along $\partial\Omega$ and again at any time t for which differential equation has to be solved.

Step 5. Initial conditions, appropriate values of dependent variable must be known across Ω for a suitable time, i.e., t_0= 0.

These five conditions will henceforth be referred to collectively as constraining parameters. Note that although these constraining parameters relate primarily to mathematical models based on differential equations, all mathematical models commonly used in science and technology today contain constraining parameters in one form or another.

A method frequently used (especially historically) to satisfy the constraining parameters in a scientific model of a physical system, is to simplify the constraining parameters, geometrically or otherwise, in such a way that the underlying mathematical model reduces to an analytical model. The method is particular useful when a physical system either displays a simple behaviour, or in a laboratory study where the constraining parameters can be adjusted to satisfy a suitable analytic model.

There is little doubt that analytical models play a significant role in the development of the modern technological age. In fact, the question may be asked whether scientists do not sometimes unnecessarily disregard analytical models. Nevertheless, there exist physical systems, which cannot be modelled adequately with analytical models. This is especially the situation in what may be called the environmental and biological sciences. Take for example the scientific model of an aquifer which is described by Equation 2-5 as a mathematical model. Although the application of remote sensing and similar techniques (Hoffmann, 2005, Meijerink, 2007), may change the situation in the not too distant future, there do not exist methods today to determine the constraining parameters for this model. Results derived from this and similar mathematical models therefore will always be in doubt, unless values of the parameters can be derived in one or another way. This would not be too much of a problem, were it not for the fact that such mathematical models often represent the only viable approach to study the evolution of physical systems crucial to the environment on earth.

Three approaches are commonly used to solve the problem of unknown constraining parameters in scientific models: (a) observational-analytic modelling, (b) stochastically continuum modelling, and (c) inverse modelling. These models are described separately below using Equation 2-5 as the basis for the mathematical model of groundwater flow. However, this should not be interpreted that the approaches only apply to models of groundwater flow or that these methodologies can only be applied separately. On the contrary, these methods can be applied separately or in combination to any scientific model subject to the problem of unknown constraining parameters.

2.2.1.1 Observational-analytic Modelling

Observational-analytic techniques refer to the use of suitable analytic models of the system under investigation to interpret observations on the system. The technique is fairly widely used in the interpretation of observations with existing theories and to derive values of the relational parameters in the mathematical model of a system. Witness, for example, the use of the Theis equation to derive values for the relational parameters S_0 and **K** in Equation 2-6 of a uniform infinite aquifer from hydraulic or hydraulic tests (Kruseman and De Ridder, 1991) and the interpretation of geophysical surveys (Kirsch, 2006).

$$s = \frac{Q}{4\pi T} W(u); \quad u = \frac{r^2 S}{4Tt}$$

Equation 2-6

A glance at Kruseman and De Ridder (1991) and related books creates the impression that the observational-analytic technique can yield accurate values of the relational parameters in a large number of mathematical groundwater flow models. The conclusion thus quickly arose that the technique can be applied with confidence to the modelling of flow in the subsurface of the earth—a view supported by the following arguments (Black, 1993, Raghavan, 2004):

(a) The relational parameters derived from a hydraulic test tend to constant values if the test is run long enough.
(b) Engineers and modellers often influence the pressure head distribution derived from a model to observed values through the use of concepts such as skin factor and well loss, even though these concepts when initially introduced were intended to account for specific physical phenomena.
(c) There is a need in both the oil industry and groundwater investigations for information to be evaluated and acted upon in real time particularly because of cost, safety and necessity considerations.
(d) The earliest investigations of groundwater flow phenomena centred on aquifers in relative uniform sedimentary deposits whose internal geometry very much resembles the simple and well known geometry of a porous medium and therefore can simply be neglected.

Modellers in the oil industry and geohydrology consequently became very complacent with the use of observational-analytic techniques. Nevertheless, there are a number of disadvantages associated with this approach. The first and most important of which is the neglect of the geometry of an aquifer or oil reservoir (Black, 1993, Botha et al., 1998). Models based on observational-analytic techniques are consequently often restricted to sets of fixed relational parameters and horizontal flow; thereby neglecting properties such as anisotropy, deformation, preferential flow paths and flow in the vertical direction. This may lead to a significant underestimation of the extent—hence the economic value of the resource (Raghavan, 2004). The technique consequently adds little to a better understanding of the aquifer or reservoir and neglects the major advances that have been made in geological modelling over the past 20 years. Moreover, the technique essentially ignores any ‘messages’ in hydraulic tests, thus reducing the role these tests could and should play in the development of mathematical models for subsurface flow (Raghavan, 2004).

2.2.1.2 Stochastical Continuum Modelling

Observations show that natural rocks often are highly fractured and heterogeneous and hence exhibit pronounced spatial variability (Neuman, 2005). The result is that relational parameters of mathematical models derived with observational-analytic modelling for such rocks exhibit a similar variability. This caused the introduction of what is known today as the stochastic continuum concept over the years (Neuman, 2005, Raghavan, 2004). The basis of this assumption is to assume that a constraining parameter can be represented as a random field of given statistics that is spatially stationary. Although the concept could in principle be applied to any of the constraining parameters in a mathematical model, there is a tendency (at least in the oil industry and geohydrology) to restrict it to the permeabilities of boreholes. One reason for this is that the assumption of statistical stationarity implies, geologically speaking that the parameter can be described statistically with a distribution that does not depend on the position where the parameter is measured. One could, therefore use a single distribution to generate suitable permeabilities for an aquifer, assuming that there exists at least one observed value.

There are at least three difficulties associated with the previous approach. The first is that it is not clear how one can extract a meaningful geological description of the medium borehole from a random field of permeabilities. The second is the question of scale dependence of the permeabilities (Hunt, 2006, Raghavan, 2004). As discussed by Hunt (2006) and Raghavan (2004), statistical homogeneity implies that permeabilities must be scale-dependent, i.e., depend on the volume of the aquifer or oil reservoir used in the determination of permeability values. However, this dependence is more of an artefact related to the geometry of the aquifer or oil reservoir that can be removed by using an appropriate geometric model of the aquifer or oil reservoir. This means that there does not exist a so-called effective permeability or other effective parameters for that matter, as follows from the assumption of stationarity.

2.2.1.3 Inverse Modelling

The main advantage of physical theories is that it allows one to predict the future behaviour of a physical system, given a complete description of the physical system, especially the relational parameters—a procedure variously called the modelisation problem, simulation problem, or the forward problem (Tarantola, 2005). However, the situation frequently arises in many branches of science and engineering that one has some information on a physical phenomenon or physical system and wants to determine values for the relational parameters in the mathematical model associated with the theory or scientific model. As one could expect this operation is commonly called the inverse problem.

A major difference between the forward problem and the inverse problem, from the mathematical point of view, is that the forward problem always has a unique solution (in deterministic physics), while the inverse problem has multiple solutions (in fact, an infinite number)—the so-called equifinality phenomenon (Beven, 2006). This means that one has to use special methods in handling the inverse problem. One approach to achieve this is to observe that the predicted values are generally not identical to the observed values, even in the case of the forward problem, for two reasons: measurement uncertainties and modelisation imperfections. These two very different sources of error generally produce uncertainties with the same order of magnitude, because as soon as new experimental methods are capable of decreasing the experimental uncertainty, new theories and new models arise that allow one to account for the observations more accurately. For this reason, it is generally not possible to set inverse problems properly without a careful analysis of modelisation uncertainties (Tarantola, 2005).

The way to describe experimental uncertainties is well understood and described in most textbooks on probability theory and statistics (Tarantola, 2005, Dekking et al., 2005). However, the proper way to put together measurements and physical predictions—each with its own uncertainties—is a matter in progress (Le and Zidek, 2006, Tarantola, 2005) and will not be discussed further here. A far more interesting aspect for the practical application of inverse problem is how to apply the inverse problem.

When solving inverse problems, scientists are often faced with two very different difficulties. The first is to find at least one model of the system that is consistent with the observations. The second difficulty arises in problems where finding at least one solution is doable, but how can one quantify the non-uniqueness of the result? Two different philosophies are in use today to try and solve this problem. The first carefully avoids using any a priori information on the model parameters that could 'bias' the inferences to be drawn from the data. The second philosophy, which is clearly Bayesian, asks the basic question: how does newly acquired data modify previous information? In other words, when starting with some a priori state of information on constraining parameters, what is the *a posteriori* state of information at which one arrive after 'assimilating' new data? Observations like these, lead Tarantola (Tarantola, 2006) to the concept of a Popperian-Bayesian approach, that will use all available a priori information to sequentially create models of the system, potentially creating an infinite number of possibilities. For each model, the forward modelling problem must be solved, predictions to actual observations, and some criterion used to decide if the fit is acceptable or unacceptable, given the uncertainties in the observations and the physical theory or model being used. The unacceptable models are the falsified models, and must be dropped. The remaining models represent the solution of the inverse problem that can be investigated further through the GLUE methodology of Beven (Beven, 2006) for example and data assimilation techniques.

In the following sections the analysis techniques commonly applied in geohydrology will be presented and discussed. This is done in order to assist in the calculation of average transmissivity or hydraulic conductivity values for an area. Initially, the focus will be on geostatistical methods followed by unconfined aquifers, hydraulic test interpretation and finally the effect of heterogeneous media on results obtained.

2.3 Geostatistical Methods

The term geostatistics is generally applied to a special branch of applied statistics. It was developed to treat problems that arise when conventional statistical theory is used to approximate changes in ore grade within a mine. Fundamental to geostatistics is the concept of regionalised variables, which has properties intermediate between a truly random variable and one that is completely deterministic. Regionalised variables are functions that describe natural phenomena that have geographic distributions, i.e., ground surface elevation, changes of grade within an ore body or water levels within an area. In contrast to random variables, regionalised variables are point wise continuous but cannot be determined or described by any definable deterministic function (Walder, 2008). Accordingly, a more formal definition can be given that describes the interchange between these parameters.

Definition 1:$\{Z(x)\}$, $x \in R^n$ is a random field if Z(x) is a random variable $\forall x \in R^n$. If $x = t \in R^n \rightarrow Z(t)$ is a random process.

Definition 2: A single realisation $\{z(x)\}$, $x \in R^n$ of a random field $\{Z(x)\}$, $x \in R^n$ is regionalised variable.

Definition 3: The covariance function of the random field$\{Z(x)\}$, $x \in R^n$ is defined as

$$C(x,h) = Cov\big(Z(x), Z(h)\big) = E\big\{\big(Z(x) - EZ(x)\big)\big(Z(x+h) - EZ(x+h)\big)\big\}$$

Where E represents the mean of the corresponding random value or the product of the random differences.

The above definitions is further simplified if the following two assumptions are made,

(i) First-order stationarity: $EZ(x) = EZ(x+h) = \mu = constant$. This result would specify that the mean of the random field is constant and that the mean value is the same at any point in the field.

(ii) Second-order stationarity: $C(x,h) = C(|h|)$. Indicating that the covariance between any pair of locations depends on the length of the distance vector (h), which would results in the second-order stationarity eliminating the directional effects.

Due to the inherent complexity of regionalised variables, it is generally impossible to determine all values within an area. More specifically the regionalised variables are presented as a sample of specific observations in an area, thus it is represented by a sub-set of the population. The size, shape, orientation and spatial arrangement of these observations constitute the support of the regionalised variable and if any of these observations change then an associated change in the characteristic of the regionalised variable will be observed.

In order to apply geostatistics in one, two or three dimensional space an estimate of the regionalised variable should be constructed. This is usually done by means of the semivariance (γ), which expresses the rate of change of a regionalised variable along a specific orientation. Approximating the semivariance requires an analysis technique similar to time-series analysis. Thus the semivariance is a measure of the degree of the spatial dependence between observations along a specific support (data points). If it is assumed that point measurements and equally spaced samples (z) are gathered then the semivariance (z_i) can be approximated as:

$$z_i = \sum_i^{n-h} \frac{(x_i - x_{i+h})^2}{2n}$$

Equation 2-7

The x_i is a measurement of a regionalised variable, X, taken at location i and x_{i+h} at an interval of h. The number of points (n) can be measured, such that the number of comparison between points is n – h. The importance of the semivariance is its use as an accurate and precise statistic to quantify the dissimilarity of a variable between a chosen central point (X_i) and several other points (X_2, X_3, ..., X_n) with increasing distances from the central point. For each pair of points ((X_iX_2), (X_iX_3), ..., (X_iX_n)), the value of the semivariance is plotted on the y-axis versus the distance between the points h on the x-axis, to give a scatter plot called the experimental semi-variogram. The relationship between the semivariance and distance from the central point will depend on the amount of regional dependence.

Once the experimental semivariogram has been plotted, a smoothed line of best fit called the theoretical semivariogram is fitted through the points with the restriction that it must start from a relatively low value at the central point, subsequently increase, but eventually plateau out at a constant value (Figure 2-2). If there is some regional dependence, then the values of the variable at the central point and those nearby will be similar thereby, giving relatively small semivariances. As the distance from the central point increases, the amount of dependence reduces, so the semivariances will tend to increase but also become more scattered. At this distance (and beyond) the two points are equivalent to having been chosen at random from the population, so each of the widely scattered semivariances will estimate the population variance for samples of n=2. Once the experimental semivariogram has been plotted, a smoothed line of best fit called the theoretical semivariogram is fitted through the points with the restriction that it must start from a relatively low value at the central point, subsequently increase, but eventually plateau out. The averaged value at the plateau gives a relatively good estimate of the population variance.

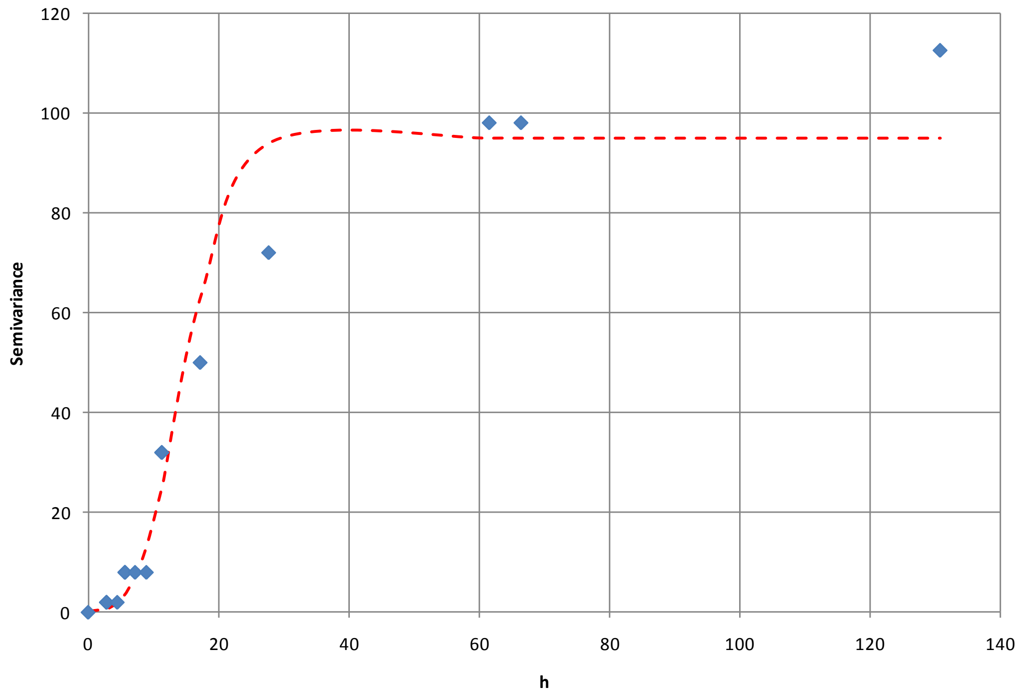

Figure 2-2 The experimental semivariogram is a scatter plot of the semivariance against distance between sampling points (blue diamonds). A variogram from the exponential family is fitted to the data, the nugget effect for parameters are excluded in this analysis.

The features of the theoretical semivariogram are shown in Figure 2-3. The semivariance at X = 0 is called the nugget or nugget effect. This is related to the spherical model of the semi-variogram: $\gamma(h) = C_0 + C\left[1 - e^{-h/a}\right] \;\; h > 0$, in which C_0 corresponds to the nugget effect. When the semivariance reaches its maximum height at the plateau, its value is called the sill. The outer limit of the region of influence surrounding the central point is defined as the value of X when the semivariance has reached 95% of the difference between the sill and the nugget.

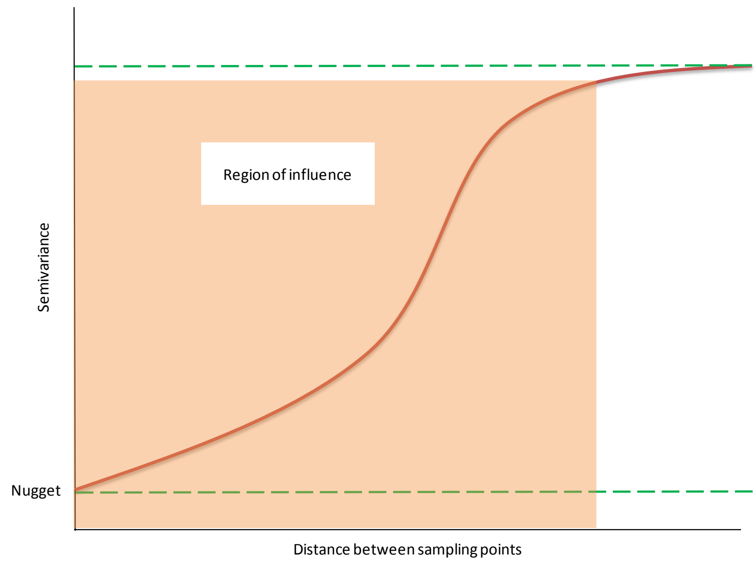

Figure 2-3 Description of theoretical semivariogram. If X = 0 and the semivariance does not equal zero then this is called the nugget. The maximum value, i.e., plateau is refered to as the sill, with the region of influence representing the value of x for which the theoretical semivariance is 95 % of the distance between sill and nugget.

One important application of the theoretical semivariogram is to predict the value of a variable at sites where it has not been measured. The width of the 95% confidence interval around the line of the theoretical semivariogram will depend on the amount of regional dependence. When there is no regional dependence, the line will rise rapidly and the 95% confidence interval around it will be relatively wide, because it is the smoothed average of many estimates made when n = 2. When there is regional dependence, the line will rise more slowly. Its 95% confidence interval will initially be very narrow because the regional dependence surrounding the central point will constrain the estimates of the semivariance to within a relatively small range. If a variable shows regional dependence and the point(s) at which you want to predict it lie within the regions of influence of known locations, it is possible to make quite precise estimates of its value. This is the basis of the method of interpolation called Kriging.

2.3.1 Variogram

Definition 4: Let $\{Z(x)\}$, $x \in R^n$ be a random field. A variogram of the random field is defined as

$$\gamma(x,h) = \frac{1}{2}E\{Z(x+h) - Z(x)\}^2.$$

Definition 5: If the variogram of a random field depends only on the length of the translation vector, the field is called intrinsic stationary, which means that $\gamma(x, h) = \gamma(|h|)$.

Some properties of the variogram of an intrinsic stationary field are $\gamma(0) = 0$ and $\gamma(h) \geq 0$. If the mean of a random field is known, i.e., $EZ(x) = \mu = constant$, then the following relationships between the variogram and covariance functions holds: $\gamma(h) = C(0) - C(h)$ and $C(h) = \gamma(\infty) - \gamma(h)$.

2.3.2 Parameter estimation

The variogram-based methods of estimation are widely used in classical geostatistics as a method to estimate unknown parameters. It starts with formulating a model for a particular application that involves both spatial and non-spatial exploratory analysis. Since $EZ(x) = \mu$ it can also be assumed that $\mu_i = \mu(x_i)$ and that the mean value can be expressed in the general form of $\mu(x) = \beta_0 + \sum_{j=1}^{p} \beta_j d_j(x)$, where $d_j(x)$ are spatial explanatory variables. Considering it from a modelling perspective, the mean and covariance structure together define a linear Gaussian model for the data. There is, however no implication that the data should follow a Gaussian distribution.

This in part describes the problem of estimating the transmissivity or hydraulic conductivity of an area. Given a domain such that only a few points are known implicitly in the domain (D), then it can be argued if a subdomain (S) is selected it can be unique (D $\notin$ S). This implies that a subdomain is not represented by samples in the larger domain. To counteract this effect the ergodicity hypothesis is adopted, i.e., the average of a process over time is equal to the average of that process over its statistical ensemble. This hypothesis strictly only applies for a homogeneous random stationary process, which in turn requires that the mean and covariance should not depend on time and space.

2.3.3 Analogy between hydrological and electrical flow

There is very often a strong erratic component in the transmissivity (spatially uncorrelated) values of an area, which might result in two boreholes located close to each other to have significantly different transmissivities. Due to spatial variability one can derive a representative average transmissivity value that could represent an area as a first order approximation. Using a deterministic approach, a set of uniform blocks can be placed in such a configuration that both parallel and serial flow could be simulated (Figure 2-4).

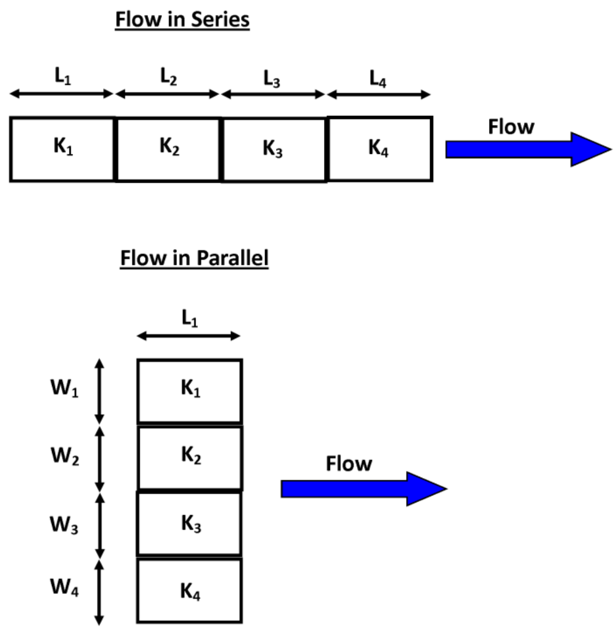

Figure 2-4 Illustration of blocks in series and parallel with flow direction indicated by blue arrow.

If blocks are placed in such a configuration that water flow is in series then the law of harmonic composition can be applied, i.e., $\sum L_i/K_{mean} = \sum L_i/K_i$. If flow is parallel to the blocks then the arithmetic composition can be used such that $K_{mean}\sum W_i = \sum W_i K_i$. Interestingly this is exactly the same as Ohm's law in electricity.

In the probabilistic approach the transmissivity can vary in all directions of space. Research in this field has led to the following observations (Matheron, 1967, Gelhar, 1976, Bakr et al., 1978, Gutjahr et al., 1978).

1. If the flow is uniform, the average transmissivity is independent of the spatial correlation of the transmissivity and the number of dimensions. The estimated values always range between the harmonic mean and the arithmetic mean of the local transmissivity values.
2. If the probability distribution function of the transmissivity is lognormal and flow is two dimensional and uniform, then the average local transmissivity is equal to the geometric mean.
3. If flow is non-uniform and constant in time, there is no law of composition.

It is from point three that most of the hydraulic test analysis is derived. In general analytical solutions are derived for confined systems which are invariant in time. However in the next section a closer inspection will be done for anisotropic unconfined aquifers.

2.3.4 Unconfined Aquifers

During a hydraulic test in an aquifer system the relative change in aquifer thickness can be as little as 1 % or greater than 80 %. If water levels drop significantly, a loss in transmissivity is observed since transmissivity is directly related to the product of the saturated thickness (D) of the aquifer and the hydraulic conductivity (T = KD). Since most of the well field analysis methods are based on calculating the transmissivity of the aquifer, artefacts can presumably appear in analysed transmissivity values.

Neuman (Neuman, 1973, Neuman, 1972, Neuman, 1974, Neuman, 1975) has investigated the phenomena of a fully or partially penetrating well pumping in an anisotropic unconfined aquifer. Furthermore, the delayed drainage of the unsaturated zone by gravity was included in the analysis. In the evaluation the anisotropy is defined is the interaction of the horizontal versus the vertical hydraulic conductivity. During the study it was assumed that the free surface or water table remains at a level (z = e) and that the boundary conditions at this surface is $K_z \frac{\partial s}{\partial z} = -\omega_d \frac{\partial s}{\partial t}, z = e, \forall r, t$. The proof of the analysis is similar to that of Streltsova (Streltsova, 1975, Streltsova and Tillotson, 1975, Streltsova, 1976). If the well is fully penetrating and screened along its entire length, then the drawdown in a piezometer is given by:

$$s(r,t) = \frac{Q}{4\pi T} \int_0^\infty 4yJ_0\left(y\beta^{\frac{1}{2}}\right)\left[u_0(y) + \sum_{n=1}^{\infty} u_n(y)\right] dy$$

$$u_0(y) = \frac{\{1 - exp[-t_s\beta(y^2 - \gamma_0^2)]\}tanh(\gamma_0)}{[y^2 - (1+\sigma)\gamma_0^2 - (y^2 - \gamma_0^2)^2/\sigma]\gamma_0}$$

$$u_n(y) = \frac{\{1 - exp[-t_s\beta(y^2 - \gamma_n^2)]\}tanh(\gamma_n)}{[y^2 - (1+\sigma)\gamma_n^2 - (y^2 - \gamma_n^2)^2/\sigma]\gamma_n}$$

Equation 2-8

The values for γ_0 and γ_n can be obtained as the roots of

$$\sigma\gamma_0 \sinh(\gamma_0) - (y^2 - \gamma_0^2)\cosh(\gamma_0) = 0, \quad \gamma_0^2 < y^2$$
$$\sigma\gamma_n \sin(\gamma_n) + (y^2 + \gamma_0^2)\cos(\gamma_n) = 0$$
$$(2n-1)(\pi/2) < \gamma_n < n\pi, \quad n \geq 1$$

Equation 2-9

The general parameters are defined as r is the distance from the piezometer to the well, Q the constant discharge rate, T transmissivity, J_0 the Bessel function of the first kind and zero order, $t_s = Tt/Sr^2$.

The interpretation is done on log-log paper using a three step process. The solution unfortunately is only an approximation obtained by linearisation. It does not take into account the reduction in the saturated thickness with time. Although the aquifer is assumed to be of uniform thickness, this condition is not met if the drawdown is large compared with the aquifer's original saturated thickness. A corrected value for the observed drawdown s then has to be applied. Jacob (1944) proposed the following correction factor **s'**= **s** - (s^2/2D), where **s'**= corrected drawdown, s = observed drawdown and D = original saturated aquifer thickness. According to Neuman (1975), Jacob's correction factor is strictly applicable only to the late-time drawdown data, which fall on the Theis curve.

The preceding paragraph highlights a specific problem that one has with unconfined aquifer systems. It is generally assumed that in fitting curve sets to data that the aquifer thickness is relatively constant. However in unconfined aquifers under aggressive pumping conditions this is clearly not true at which point the relation T = KD should be applied. Thus the observed transmissivity is related to the thickness of the aquifer or more specifically to the water level. A second observation that can be inferred from the above is that the hydraulic gradient should also play a role in the observed transmissivity although it is not clear what the relationship would be over an extended time period due to a change in the saturated thickness of the aquifer. This is typically observed during pumping in which the initial gradient close to the well is large and as time progresses the gradient converges to a constant value. The gradient change would thus also influence the calculated transmissivity of the well and/or area.

2.4 Review of Hydraulic Tests and Possible Interpretations

It is generally assumed in a hydraulic test that one is working with a single-phase flow of fluids to a well in a heterogeneous porous media (Raghavan, 2004). Wells play an important role in estimating aquifer properties in both hydrocarbon recovery and groundwater systems. Resolution of uncertainty in complex geological environments is essential for management of resources in both disciplines. The aim of these investigations is to forecast an aquifer's performance over the long term and methods of improving the yield of the system.

In order to construct a sensible model for an aquifer the following points should be considered.

- Geological model that is quantitative and that is based on interpretations of geology and geophysics and measurements based on core analysis, outcrops and petrophysics.
- A conceptual geological model should be determined for the local and regional scale.
- Coarse scale version of a fine-scale computational model that is suitable for rapid computations over a regional scale.
- Fine-scale computational model that is based on variables that are directly pertinent to fluid dynamics.

Computational models are focused on the roles of wells, particularly the connectivity of wells to the aquifer/reservoir and also connectivity between the wells. The measured interconnectedness of the aquifer appears to be dependent on the number and location of the wells tapping the system. Thus in evaluating the hydraulic test results a possible interpretation of the aquifer is made.

2.4.1 Support Volume

According to Raghavan (2004), support volume plays an important role in the matter of hydrogeologic scaling in the groundwater literature. The properties of the porous medium and its shape affect the volume of the aquifer investigated by hydraulic tests. Single well tests in the form of pressure build-up tests are used to evaluate reservoirs containing hydrocarbons in the petroleum system. Measurements at observation wells located at a distance from a pumping well are considered to be more reliable for aquifer parameter estimations than measurements at a pumping well itself.

The differences between petroleum reservoirs and the groundwater systems are:

- The nature of the fluids in the porous medium (the compressibility of fluids in a hydrocarbon reservoir is much larger than that of water filled aquifer).
- Well spacing (i.e. wells are usually at a considerable distance from each other in the petroleum reservoir) due to budget constraints.

Well spacing and compressibility of water enables the groundwater aquifer to be tested under steady flow conditions.

2.4.2 Further issues

The Theis method (Theis, 1935) imposes a single value for the properties of the aquifer volume that is sampled even if the geological formation may exhibit a continuous variation in properties. Thus a simplification of the real aquifer system is obtained. Similarly, methods such as Cooper-Jacob (Kruseman and de Ridder, 1994) also produce an average estimation of transmissivity and storativity for a specified system.

Hydraulic conductivity or transmissivity as well as porosity and storativity generally depend on the volume of the porous medium sampled. Transmissivity cannot be derived without recognising the factors that influence its estimation. The concept of a single effective transmissivity for a single or group of well(s) should be avoided in the evaluation of hydraulic tests since it is a self-defeating proposition when complex geological formations are considered (Raghavan, 2004).

2.5 Testing and Heterogeneity – the Statistical Approach

Models which do not consider heterogeneity are only used due to a lack of tools and concepts to describe the aquifer accurately in three dimensions. These kinds of models should only be used as a means of simplified parameter evaluation and assessment of short-term effects.

The issue of heterogeneity with respect to well testing and performance have been addressed by:

- Simulation of variations in properties of the porous medium applying deterministic (i,e. no randomness) methods and correlating the properties of the porous medium with properties derived by conventional means.
- Statistical methods.

Cardwell and Parsons (1945) method considers steady state, radial flow to a well in an aquifer in the form of a circle. The method shows that the equivalent transmissivity (T_e) is bounded by the weighted and harmonic means of the transmissivity of different aquifer regions.

$$\frac{\int_\Omega \frac{dx}{r^2(x)}}{\int_\Omega \frac{dx}{T(x)r^2(x)}} \le T_e \le \frac{\int_\Omega \frac{T(x)dx}{r^2(x)}}{\int_\Omega \frac{d(x)}{r^2(x)}}$$

Equation 2-10

Where $\Omega =$ region of aquifer bounded by the outer boundary and the well bore

$r(x) =$ distance from point **x** to the central well, and

$d(x) = r(x)drd\theta$ is an infinitesimal area

Hence they considered a deterministic approach to estimate the transmissivity of a porous medium. However, deterministic methods are not sufficient to predict uncertainty in well performance because of lack of knowledge of hydrogeological properties, an insufficient numbers of observations and the high degree of spatial variability. Statistical methods are based on the concept that the porous medium is statically homogenous, i.e., the porous medium consists of a single facies.

In geological terms, the assumption of statistical homogeneity implies that the porous medium consists of single facies (Anderson, 1997), and that variability is independent of location and shape. The logarithm of the permeability, k (known as the log permeability), is defined by and is presumed to be Gaussian.

$$Y = \ln(k)$$

Equation 2-11

In groundwater literature, the statistical properties of *Y* is described by its geometric mean,

$$k_G = exp\langle Y \rangle$$

Equation 2-12

Where angle brackets represent ensemble average and the two point correlation

$$C_y(x, y) = \sigma^2 \rho_Y\,(\mathrm{x}, \mathrm{y})$$

Equation 2-13

σ^2: Variance of Y and $\rho_Y\,(\mathrm{x}, \mathrm{y})$: Autocorrelation function of *Y*

The mean is usually assumed to be independent of location, that is, $\langle Y(\mathrm{x}) \rangle \equiv \langle Y \rangle$, and the two point covariance depends only on the separation vector, $\mathbf{s} = |\mathrm{x} - \mathrm{y}|$, and not the actual values of **x** and **y**. The system is isotropic if the covariance depends only on the magnitude of **s,** and the covariance is characterised by a unique measure known as a range or integral scale.

Matheron (1967), illustrated that the effective permeability, k_{eq}, of a two dimensional system under the assumption that the porous body is a lognormal, sub-isotropic and ergodic medium is given by

$$\ln(\, k_{eq}) = E\ [\ln k_G\,]$$

Equation 2-14

k_G : geometric mean of the permeability of the individual elements of the heterogeneous medium.

2.5.1 Heterogeneous Media and the Theis Method

Warren and Price (1961) derived a statistical method which considers flow to a well and concluded that if the porous medium is assumed to consist of a quilt or patchwork of heterogeneous porous elements that are randomly distributed in three dimensions, then such a medium may be represented by a porous rock with a conductivity or transmissivity equal to the geometric mean of the transmissivity of the individual elements. They have predicted lower estimates of transmissivity for the 3-D flow, and this is attributed to the numerical bias results when finite difference models are used to analyse well response. It is furthermore difficult to deduce results for three-dimensional flow if the well penetrates the aquifer completely especially by computational methods.

Interference tests in heterogonous two-dimensional formations analysed with the Theis method by Vandenberg (1977), appear to yield storativity estimates which vary over a much wider range than the transmissivity estimates. This indicates that transmissivity estimates, T, appear to be dependent upon the distance between the pumping and observation wells. The same theory was confirmed by Meier et al. (1998) by interpreting drawdown curves at many observations wells using the Jacob method (Jacob, 1946). Whenever a heterogeneous system is approximated by a homogenous one, anisotropy should be considered such that the estimate of transmissivity reflects a tensor. If this is not considered, the estimates of S may not be representative of the porous medium. This is significant because variations in S are obtained even in relatively homogeneous but anisotropic reservoirs when reservoir anisotropy is considered.

The problem with evaluating hydraulic test responses at the observation boreholes is that there are very few diagnostic features in the drawdown curve to reflect the underlying complex geology. Raghavan (2004) suggest that drawdown responses be evaluated simultaneously in all observation boreholes with a realistic model that incorporates the underlying heterogeneity to some extent.

Toth (1967) matched the different parts of the measured drawdown to different parts of the Theis curve and observed that the time rate of drawdown along the first limb is determined by the actual transmissivity of the aquifer from which water is pumped. He concluded that calculated equivalent transmissibility's are expected to decrease as the length of the hydraulic test increases, approaching a value that is determined by the hydraulic conductivity and thickness of the whole rock complex contributing to the well. The observations of Toth (1967), are illustrated by the Figure 2-5, below taken from Raghavan (2004). The derivative responses at active (pumping) wells for a number of fields are illustrated. The responses are from wells producing all types of fluids from a variety of geological environments.

Common features from three well responses, in Figure 2-5 are:

- The small values of the derivative curve at an early time suggest that the wells appear to be located in a highly permeable region.
- The rapid rise in the derivative curve at intermediate times appears to suggest a loss in connectivity between the well and the reservoir as the fluid is produced.
- Some of the curves appear to stabilize at later times, suggesting that the reservoir permeability is low at least when compared with the value corresponding to that of the near well.

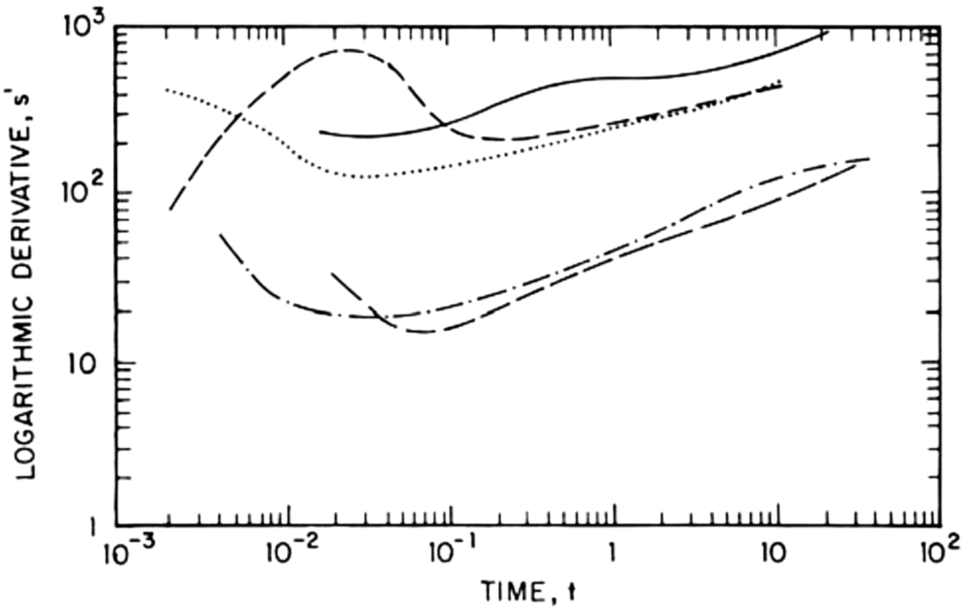

Figure 2-5: Derivates responses for a number of field test at active well (Raghavan, 2004).

2.5.2 Stochastic modelling

This method focuses on flow to wells in heterogeneous formations by assuming a random, but spatially stationary transmissivity field of given statistics. The method evaluates hydraulic test responses in heterogeneous formations assuming that the properties are a random function of the well coordinates.

Shvidler (1966) examined the role of permeability by assigning discrete values of permeability to the location of the wells, thus obtaining a network of values of permeability within certain accuracy.

Permeability, *k(**x**)* at a point x is an intrinsic property of the rock which doesn't change with time and is independent of the well location. Over the years a stochastic approach to modelling flow and transport has been developed to account for the spatial variability of conductivity and also account for the uncertainty concerning such variations.

To reconcile permeability values on a small scale, obtained from core and other static measurements to conductivities obtained from hydraulic tests, a power-averaging scheme where the pumping permeability is of the form were applied in Eq. 2-14.

$$k = \left[\frac{1}{V} \int_V^{\prime} k^{\omega} \, dv \right]^{1/\omega}$$

Equation 2-15

V: volume of interest over which the point of values of permeability are available

2.5.3 Some other approaches

Raghavan (2004) assumed the underlying heterogeneity may be handled by Gaussian models. The limitation of Gaussian models can be handled by multipoint geostatistics and fractals. Neuman et al. (1990) suggests that distinct geological units that are statistically homogeneous on a hierarchy of scales may be described by a fractal model. Raghavan (2004), states that the well responses (Figure 2-5) can be analysed by assuming a fractal model to represent the heterogeneity of the porous medium.

Oliver and Indelman (Oliver, 1990, Indelman, 2003) indicate that for a mildly heterogeneous condition, it should be possible to apply Theis (1935) solution. However, the interpretation of the permeability needs to be different. This approach does improve the interpretation of the test data shown in figure 2-5. The results mentioned above (Oliver, 1990, Indelman, 2003) do not help in interpreting test like those shown in Figure 2-5. A continuously increasing derivative indicates that the effective permeability decreases significantly with distance.

2.6 Hydrogeological Scaling

Freeze (1985) has suggested that heterogeneity may be incorporated into analyses of groundwater system by using stochastic methods. Raghavan pressure tests invariably yield transmissivity estimates that are much larger than those obtained from core analysis. Riva et al. (2001) confirmed that for a steady flow to a well that discharges at a constant rate, the transmissivity increases with support volume. The problem is that the values obtained from field tests are well above the limit of k_G predicted by stochastic theories.

Most classical concepts of hydraulic test analysis suggest that hydraulic conductivity increases linearly with the test radius, i.e., the transmissivity increases with increasing support volume for steady flow to a well that discharges at a constant rate.

According to Raghavan (2004), the effective value of *k* appears to change, and invariably appears to decrease with distance (i.e. losing of connectivity) in a single-well test (converging and diverging flow). It was also stated that the observation is supported by considering the derivative responses shown in Figure 2-5. As the derivative curve increases with time for $t > 10^{-1}$, there is no possibility to conclude that transmissivity increases with support volume. Neuman and Di Federico (2003) concluded that statistical homogeneity often inferred from standard geostatistical analyses may not provide a realistic indication of the hydrogeologic medium properties but instead present an artefact of the scale of investigation and method of inference.

2.6.1 *Geology and Testing*

There have been increases in the use of seismic and geological data when interpreting hydraulic tests. This often requires:

- The use of numerical model to predict the effect of the geological model upon the well test responses.
- The conditioning of geostatistically generated geologic models to reflect the well response.
- The history matching of the numerical model with observed well test data.

2.6.2 Geologic Setting

Caution should be used when a geological environment is modelled, since it is critical to consider whether these models are developed in a geological context. Geological models that are based on the assumption that aquifer may be described by stochastic methods beg the question as to whether such techniques are applicable to real-world conditions. Conceptual geologic models provide the information on the properties of regions of high conductivity (existence, distribution, connectivity) and offer boundaries within which the parameters will be estimated.

2.6.3 *Scale-Up*

Scale-up / up-scaling is the aggregation of fine scale cells around features that significantly influence fluid flow without increasing the number of cells too excessively due to computational limitations. Scale-up is usually carried out by attempting to concentrate nodes around features that significantly influence fluid flow.

Those involved in scale-up primarily concern themselves with two issues:

- Ensuring that the pressure-flow relationship is maintained at the fine-scale and coarse scale levels.
- Ensuring that breakthrough characteristics of all reservoir fluids are preserved at both scales.

Scale-up algorithms consider steady state or quasi-steady conditions. Scale-up process needs to be part of the optimisation algorithm where hydraulic or well test are concerned. Hydraulic or pressure tests should be analysed once the scale is determined since it will preserve the connectivity in the system. Scale-up requires that the differences in the scales over which the properties vary be quite distinct (large).

Scale-up techniques are of two kinds, local and global. Global techniques impose a global pressure drop across the model and evaluate patterns within the system to determine regions that may be aggregated. Transmissivity values in three principal directions at the fine scale yield a tensor representation at the coarse scale. Local techniques presume that the imposed pressure gradients across a small group of cells will not affect the scale of the parameter. Local techniques do no yield transmissivity representations in the form of a tensor. The transmissivities that are obtained by the scale-up algorithm by global techniques depend on boundary conditions that are used as well as the well location.

Raghavan (2004) determined that using a scale up criteria based on steady state procedures appears to be inadequate for capturing responses of the kind of interest to those who examine transient behaviour.

Factors governing aggregation and scale-up appear to be different:

- Aggregation and scale up processes are affected by the location of boundaries, discontinuities and connectivity.
- Approaches to scale-up assume a priori steady flow and this option is unavailable if we wish to examine transient flow.
- If the pressure-derivative curve is used as the measure to evaluate and ascertain the adequacy of the coarse-scale model, then this measure indicates that the degree of coarsening permitted is much less than that permitted by simply considering pressure drops.

Scale–up techniques will usually eliminate the underlying correlation that governs the porosity-permeability relationship and any small-scale feature that may be of interest. According to Raghavan (2004), it is impossible to deduce geological characteristics, except for large-scale, structural features from a model that has been scaled.

2.7 Inversion Algorithm

Jacquard and Jain (1965), were the earliest to attempt to adjust transmissivity and porosity of the reserve by perturbing the transmissivity and porosity fields in a systematic way. Raghavan concluded that a hydraulic test curve will not show a direct method of resolving small-scales values of conductivity or transmissivity. With a hydraulic test curve it is possible to incorporate these small-scale properties along with their complex arrangements of these effects in a rigorous and quantitative way.

According to Raghavan (2004), it is useful to recognize that three different models exist, i.e., model of reality, the conceptual model which can be subdivided into the quantitative model and the computational or numerical model. Each of these models reflects a particular scale to describe aquifer properties. It is virtually impossible to comment on anything about the underlying geological model on a quantitative level based on adjustments made at the computational level.

The scale-up process, at least as far as hydraulic well tests are concerned, has been shown to be part of the optimisation algorithm (Figure 2-6). Analysis of a hydraulic or pressure test should only be done once the scale that will preserve the connectivity of the well to the reservoir is determined.

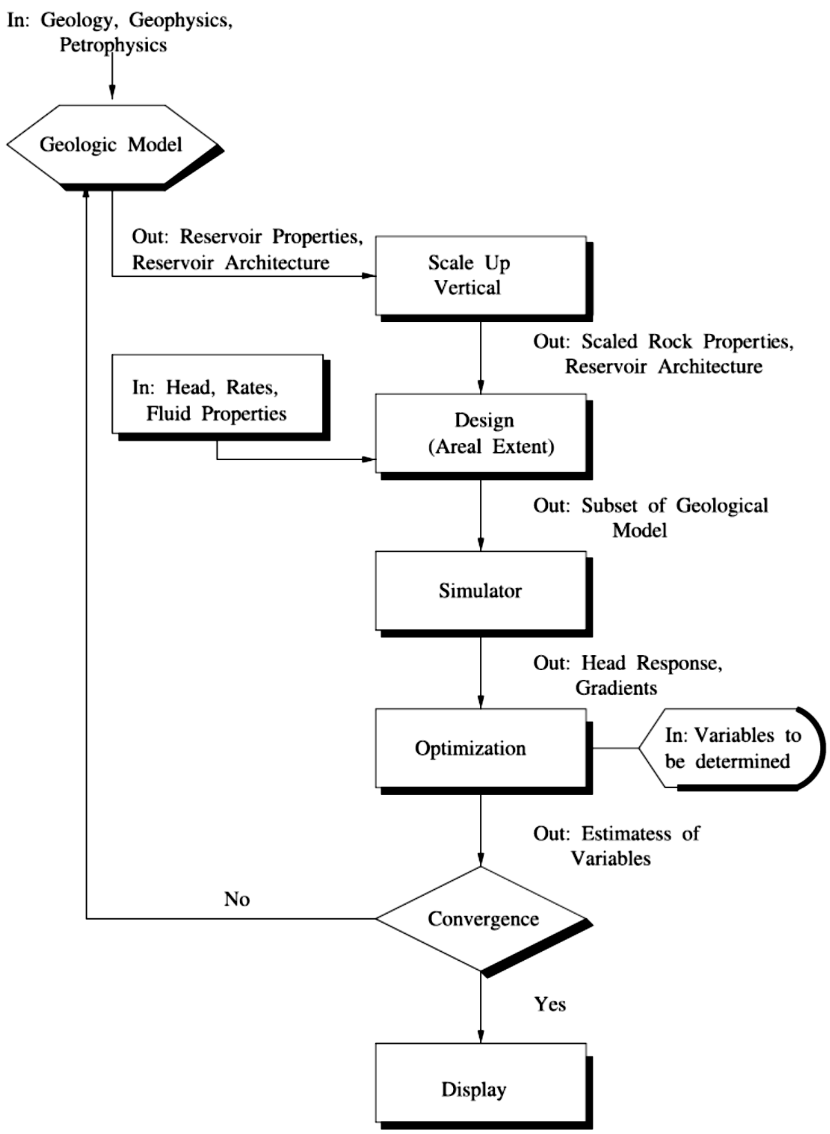

Figure 2-6 Outline to suggest scheme to estimate properties (Raghavan, 2004).

2.8 Conclusion

In this chapter a review of methods applied in geohydrology were presented. The most important bulk flow parameter estimation procedures were discussed as well as the assumptions that are related to the systems. The methods of Theis and Cooper-Jacob permit geohydrologists to determine transmissivity values from hydraulic tests. The major assumption in these methods is the concept of a homogeneous isotropic medium. Few diagnostic clues are directly available to deduce information on heterogeneity from pressure measurements unless a drastic change in transmissivity influences the test. Furthermore, the heterogeneity of the geology in an area greatly affects the determination of the transmissivity or hydraulic conductivity in that region. The use of multiple well tests in a study area can be used to construct an average hydraulic conductivity, although great care should be taken in evaluating the result.

In the following section a conceptual model of regional hydraulic conductivity values will be used. This will be done to highlight the effect that conductive zones have on estimated hydraulic conductivity values. A distinction between natural systems (no pumping) and forced gradient systems (pumping) will be made to evaluate further the effect on the estimation of a regional hydraulic conductivity value.

Chapter 3 Model Applicability and Description

3.1 Introduction

The sustainable use of groundwater in South Africa is hampered by the lack of suitable methods to control and manage the fractured hard-roc aquifers that forms the core of aquifers in the country. One reason for this is that there do not exist a unique theoretical basis of these aquifers. The result is that attempts to develop such methods are often based on the classical equation for density-independent flow in a vertically compressible saturated porous medium (Botha, 1996b).

$$S_0 D_t \varphi(\boldsymbol{x}, t) = \boldsymbol{\nabla} \cdot \boldsymbol{K}[\boldsymbol{\nabla \varphi}(\boldsymbol{x}, t)] + f(\boldsymbol{x}, t) \qquad (1)$$

where $\varphi(\mathbf{x}, t)$ is the piezometric head of the fluid in the medium, ∇ the gradient or nabla operator, $f(\mathbf{x}, t)$ the strength of any sources or sinks, and $\mathbf{x}$ and t the usual space and time coordinates. The parameters S_0 and $\mathbf{K}$, are commonly referred to as the specific storativity and hydraulic conductivity tensor of the porous medium. The parameter S_0 was introduced by Terzaghi (Terzaghi, 1925) to account for the vertical deformation of the porous medium, while $\mathbf{K}$ is a generalization of the scalar, K, introduced by Darcy (1856) to account for the ease with which water flow through a sand column, as demonstrated by the explicit expression of his law

$$\boldsymbol{q} = -\boldsymbol{K}\boldsymbol{\nabla}\varphi(\boldsymbol{x}, t) \qquad (2)$$

Where q is nown as the mass flux of fluid per unit area of the medium.

It follows from its definition above that **K** should depend on the properties of both the porous medium and the fluid. Such a dependence is not very useful from the physical point of view, as it implies that one cannot clearly distinguish between the influences of the medium and the fluid on the flow. A more useful representation would be one where **K** can be factored into two or more factors, which measure the influences of medium and fluid separately. Nutting (1930) has shown that this objective can be achieved, by factoring **K** as

$$\boldsymbol{K} = \frac{\boldsymbol{k}\rho g}{\mu} \qquad (3)$$

where ρ is the density of the fluid, μ its dynamic viscosity and g the acceleration of gravity and **k** a parameter, commonly called the intrinsic permeability of the porous medium, which accounts for the interactions between the fluid and the porous matrix.

The parameter S_0 is according to Botha and Cloot (2004) not a constitutive or relational parameter as Botha (Botha, 1994) calls them, in the sense that such parameters are used in physical theories. It should thus not come as a surprise to learn that there does not exist a universal physical interpretation of this parameter (Botha, 1996b), although Domenico and Schwartz (Domenico and Schwartz, 1990) argue that the form

$$S_0 = \rho g(\varepsilon\beta + \alpha) \qquad (4)$$

with ε and α the porosity and compressibility of the medium and β the compressibility of the fluid. However, to arrive at this expression, require a considerably number of assumptions, some of which are not very elegant theoretically (de Marsily, 1986).

Since it is not easy to determine the tensors **K** and **k** in the field, they are usually replaced in groundwater investigations with the scalars K and k, without negating the three-dimensional nature of the flow of the water. However, groundwater investigations centred historically mainly on aquifers situated in sedimentary rocks, where the flow is usually restricted by bedding planes in the rocks. This observation and the lack of sufficient observational and computational resources historically led geohydrologists to the view that they could view aquifers as two-dimensional horizontal planes, in other words view an aquifer as existing in two-dimensional space instead of the natural three-dimensional space.

There are essentially two methods can be used to reduce the dimensions of a natural phenomenon, which Botha (Botha, 1988) called the physical and mathematical reduction of dimensions. The physical reduction can be used whenever a natural phenomenon behaves in such a way that most of its activity is restricted to less than the usual three spatial dimensions, e.g. the vibrating string, or by restricting the dimensions artificially in a laboratory, as in Darcy's original experiments. Although the vertical thicknesses of aquifers are usually smaller than their horizontal dimensions, they are often not negligibly smaller. The dimensions of such aquifers can thus only be reduced mathematically. However, the procedure requires that flow in the aquifer satisfies very special boundary conditions at its top and bottom and that one replace parameters S_0 and K with their two-dimensional counterpart's storativity, or storage coefficient (de Marsily, 1986), and transmissivity defined as

$$S(\boldsymbol{x}', t) \equiv L(\boldsymbol{x}', t)S_0(\boldsymbol{x}', t) \text{ and } T(\boldsymbol{x}', t) \equiv L(\boldsymbol{x}', t)K(\boldsymbol{x}', t) \tag{5}$$

where L(**x**', t) is the thickness of the aquifer spanned by the coordinates

$$(\boldsymbol{x}', t) = (x, y, t)$$

Although the dimensionally reduced two-dimensional flow equation

$$S(\boldsymbol{x}', t)D_t\varphi(\boldsymbol{x}', t) \equiv \nabla' \cdot T(\boldsymbol{x}', t)[\nabla'\varphi(\boldsymbol{x}', t)] + L(\boldsymbol{x}', t)f(\boldsymbol{x}', t) \tag{6}$$

does not seem to differ mathematically very much from the full flow equation in Equation (1) with **K** replaced by K, Equation (6) their physical interpretations differ completely in the sense that while Equation (1) has a sound physical interpretation, Equation (6) is nothing more than an idealized realization of a real world aquifer. Nevertheless, this physical weakness of Equation (6) is also its strength, in that there is nothing in its definition that says one cannot use it as a phenomenological representation or model of a real world aquifer. In other words, adjust predictions of Equation (6) to observations of an aquifer, perhaps even on a real time basis. It is important to note that such an interpretation does not imply that the representations of S(**x'**, t), T(**x'**, t) and L(**x'**, t) must be constants. On the contrary, judging from the history of high energy physics where phenomenological models were for the first time applied in the exact sciences (at least to the knowledge of the author), there is sufficient reason to believe that the parameters will be variable functions of **x**' and t.

While the phenomenological interpretation can be criticized both philosophically and scientifically, there is nothing that prevents one to try to use it to develop a tool, which groundwater practitioners can use to manage and control hard rock aquifers more efficiently in the future. However, to achieve this objective will require that the approach be applied diligently and with insight on the part of the investigator, even to extent of abandoning some of the cherished ideas about aquifers, e.g. that the parameters S(**x**', t), T(**x**', t) and L(**x**', t) are constants in both space and time throughout the aquifer.

One question that arises at this point in the discussion is how one develops such a phenomenological model. Although the discussion in the thesis concentrates on large scale and time independent parameters, both of which contravene the physical essence behind a phenomenological model as outlined above, the approaches discussed in the present thesis in principle provide with some reservations excellent ideas on how to develop such a model. Three of these reservations include:

a. The idea that one can derive a single representative transmissivity for an aquifer. Rock properties do vary considerably across aquifers, even in the case of relatively homogeneous aquifers. Thus, it would be more appropriate to base the area considered in deriving a representative transmissivity on the geology of the area, rather than the extent of the aquifer.
b. The negligence of the possible dependence of a representative transmissivity on the time.
c. The negligence of the role that storativity plays in the evolution of an aquifer. For, as can be demonstrated by the numerical model or even the well-known Theis solution of (6) the hydraulic head distribution in an aquifer is more sensitive to variations in S than in T.

With the above in mind a detailed section on model construction will be presented in the next section to assist the reader in evaluating the methods applied in the subsequent chapters.

3.2 Model description

3.2.1 Software

Processing Modflow Version 5.5.4 (Chiang et al., 1998) was used as a front end for Modflow (Harbaugh et al., 2000). The PCG2 solver was used during all simulations. Random network generation and matrix calculations were performed using Matlab 2011a (Matlab, 2011).

3.2.2 Model Construction

Standard units used throughout the modelling section are meters and days. Where appropriate grid sizes was adapted to 100 × 100, 1000 × 1000 or 500 × 500 unit areas. A unit length was assumed to be 10 meters. Thickness of aquifer in the model was assumed to be 100 m. The simulation was run for the duration of 2160 days or 6 years. An hydraulic gradient (i = 0.001) was induced in the system with the up-gradient (left hand side of the model) starting at a 0 m head value while down-gradient (right hand side of model) section had a value of -0.499 m. The top and bottom boundaries were assumed to be a no flow zones. Specific storage in the model was set to 0.0001 m^{-1}.

A typical example of a model grid setup is shown in Figure 3-1 with the respective observation borehole positions. Onto these grid cells the randomized hydraulic conductivity is mapped to give a regional distribution of hydraulic conductivity zones, see Figure 3-2. The randomized hydraulic conductivity zones were generated in Matlab using a normal distribution randomizer to assign a value to each of the points in the grid matrix. The values were grouped into three distinct classes to mimic the distribution of dolerite dykes and sills in typical Karoo aquifer systems (Figure 1-7 versus Figure 3-2).

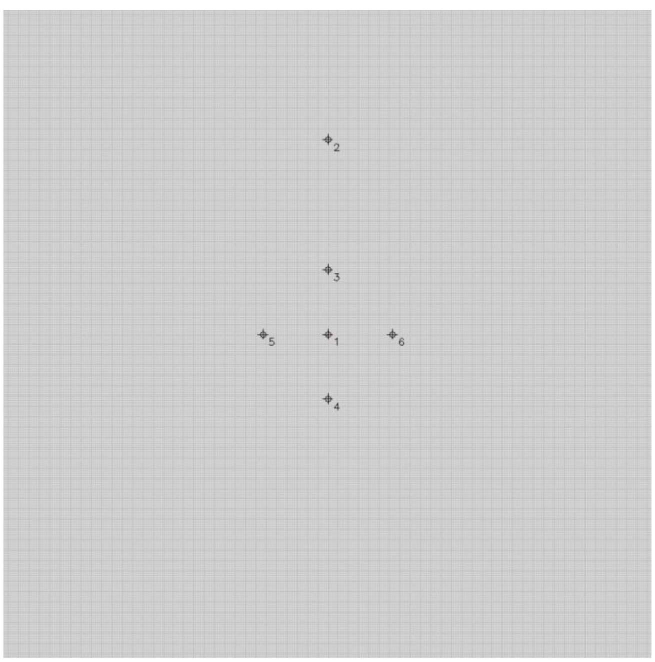

Figure 3-1 Grid extent of model with observation boreholes located at the crossed circle points. Grid points in the model system is 500 x 500 cells with each cell block representing a 10 m x 10 m unit.

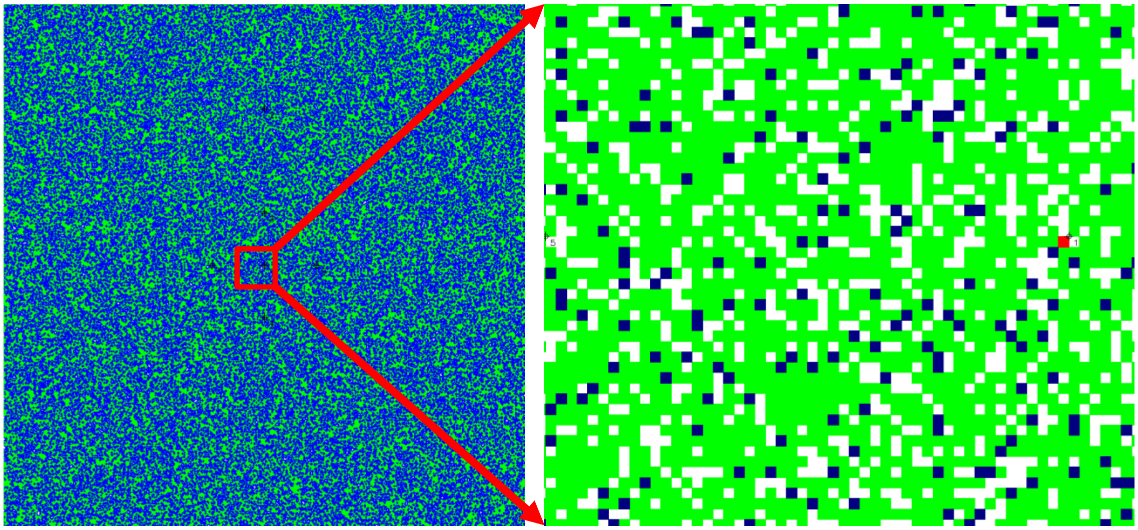

Figure 3-2 Randomized distribution of hydraulic conductivity values in the matrix region, left hand figure with red square in it. A zoomed view of the red square in shown on the right hand side with the three distinct hydraulic conductivity zones (Green = 0.01 m/d; Blue = 0.1 m/d; White = 1 m/d).

3.2.3 Results and Visualisation

The results obtained from each run was visualized by means of the built-in 2D-Visualizer in Processing Modflow (Chiang et al., 1998) subsequently data was prepared for the thesis using Surfer 9 (Software, 2010). Where required data was extracted from Processing Modflow and further processed in Excel 2010.

3.3 Conclusion

In the current chapter the construction of a basic Modflow system was presented. Particularly the assignment of these randomized hydraulic conductivity values to the individual cells were discussed. In the following chapters these principles will be used to illustrate the influence these random assignments of values have on the calculated hydraulic properties of the aquifer system.

Chapter 4 Conceptual Models

4.1 Introduction

In this chapter various case studies will be reported to illustrate the effect of hydraulic tests on stationary and forced gradient systems. To evaluate these systems a random selection of points will be investigated that can be assigned a discrete value. In addition to illustrate the effect of ranges of difference, a set of values will be chosen such that a low variation between values is observed and conversely a set with high value differences. These values can then be applied to a numerical system in which the effect of the distribution of hydraulic conductivity values can be determined. Subsequently, a constructed natural system, which does not include any abstraction, should be investigated to determine if the random distribution of hydraulic conductivity and the inherent variation in these values effect mean value estimates. Finally, forced gradient conditions should be investigated to evaluate if the variation in hydraulic conductivity adversely affects the estimation of hydraulic conductivities from hydraulic tests. Chapter 9 contains a general description of model constructions.

4.2 A Theoretical Investigation into Hydraulic Conductivity Distributions

Resource determination of an area is usually driven by the determination of the bulk flow parameters, such as hydraulic conductivity and storativity values. At this stage a decision usually is made on the basis of either maintaining the area under natural conditions (no pumping) or an abstraction (pumping) scenario is envisaged. In both instances water levels, hydraulic testing and distribution of the water resources (aquifer) are required. Since it is not possible to evaluate the total area for these parameters certain assumptions have to be made such that an average bulk flow parameter for an area can be determined. In wide-ranging situations a simple average of observation points is assumed to be sufficient.

It is to the above paragraph that the following sections are devoted too, i.e., what is in an average of an area. It is assumed that the storativity for the theoretical area is constant throughout and that the study site consists of a dual porosity medium. Two types of environments are possible for the system, firstly a natural system with no pumping in the area and secondly a pumped system.

4.2.1 Hydraulic Conductivity Distribution under Natural Conditions

In order to evaluate the effect of hydraulic conductivity distribution in a natural system a mathematical approximation of Figure 1-7 was constructed. A clear variation between the country rock and the dolerite intrusions can be observed. The simplest approach was to use a randomly distributed matrix (Figure 4-1) that contained discrete integer values that ranged from 1 to 3. These values were specifically chosen to mimic the system observed in Figure 1-7 and the hydraulic properties of the dyke structures ($K_f \rightarrow \infty$ m/d) are significantly larger than the country rock ($K_m = 10^{-3}$ m/d). A few transitional areas were also included since baking and sedimentation process occur in the vicinity of the dykes.

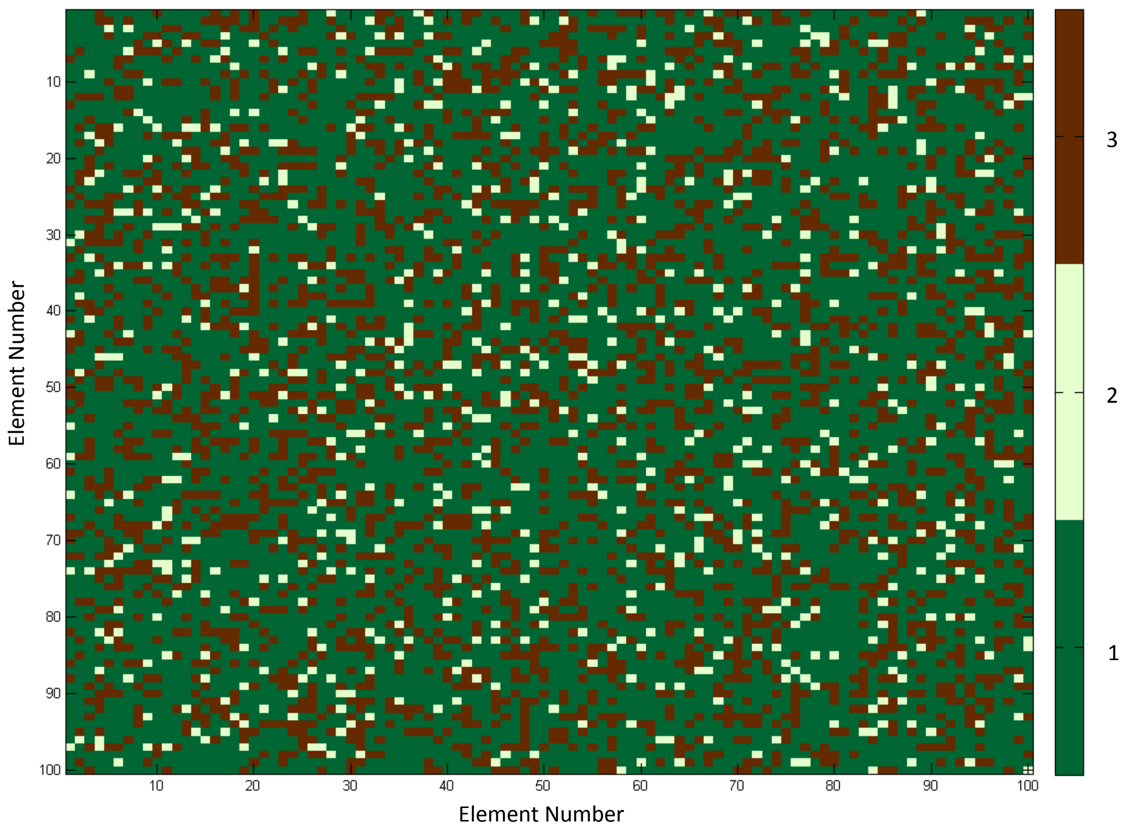

Figure 4-1 Randomly distributed transmissivity values over a two dimensional surface (100 x 100 elements).

Thus, because the hydraulic conductivity values of the whole area is known no uncertainty could exist in the analysis. In addition to the 100 × 100 randomly distributed matrix a second larger system (1000 × 1000 elements) was incorporated to evaluate the change in calculated averages of the system. In the current discussion the focus will be on the following mean values – arithmetic, geometric and harmonic mean. The standard deviation and minimum/maximum values were included to illustrate the changes observed compared to the average values obtained. A further interesting observation is that the absolute change in hydraulic conductivity between the country rock and the dolerite intrusions can be of different orders, i.e. differ by 1, 10 or 100. In Table 4-1 these values are listed and the associated histogram is shown in Figure 4-2. A graphical representation of the averages and associated minimum and maximum values are illustrated in Figure 4-3 – Figure 4-5.

Table 4-1 Evaluation of mean values for differing ranges of hydraulic coductivity values in an area as depicted in Figure 4-1.

	100x100	1000x1000	100x100	1000x1000	100x100	1000x1000
Min	1	1	1	1	1	1
Max	3	3	100	100	10000	10000
Arithmetic Mean	1.599	1.304	27.362	15.389	2663.400	1441.674
Geometric Mean	1.405	1.184	3.628	2.014	17.007	4.065
Harmonic Mean	1.269	1.116	1.427	1.186	1.505	1.190
Standard Deviation	0.878	0.706	43.369	34.695	4407.400	3507.570

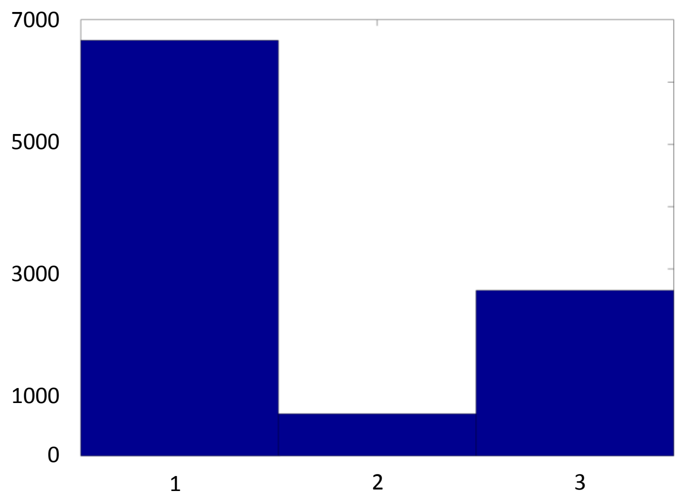

Figure 4-2 Histogram of transmissivity distribution values as represented for each random function in a 100 × 100 matrix (Figure 4-1).

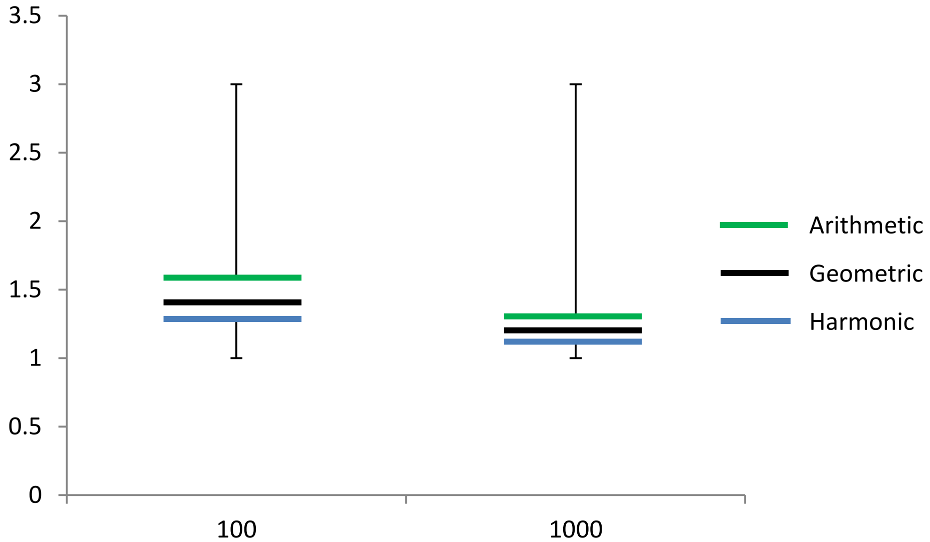

Figure 4-3 Basic statistical analysis of (1, 2, 3) map distribution in both 100 × 100 and 1000 × 1000 sample areas.

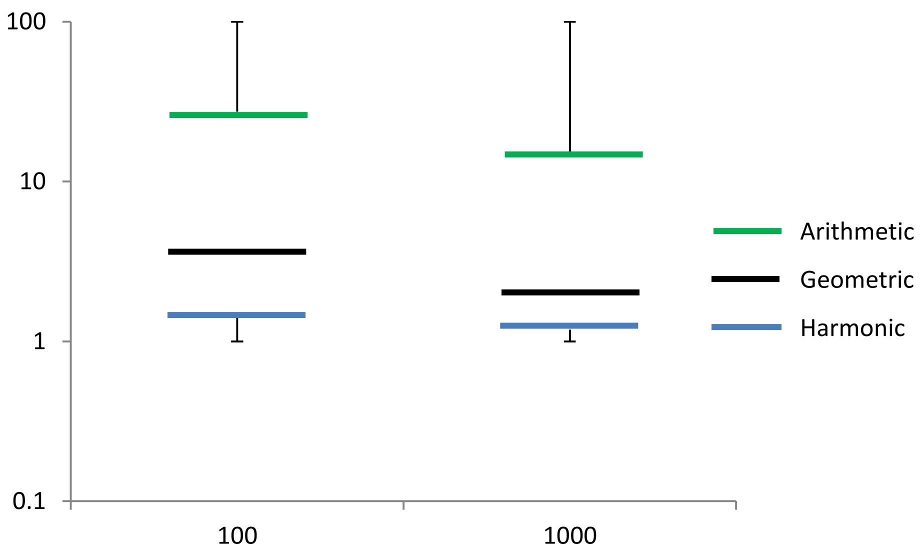

Figure 4-4 Statistical analysis of (1, 10, 100) map distribution in both 100 × 100 and 1000 × 1000 sample areas. Y-axis scale in logaritmic units.

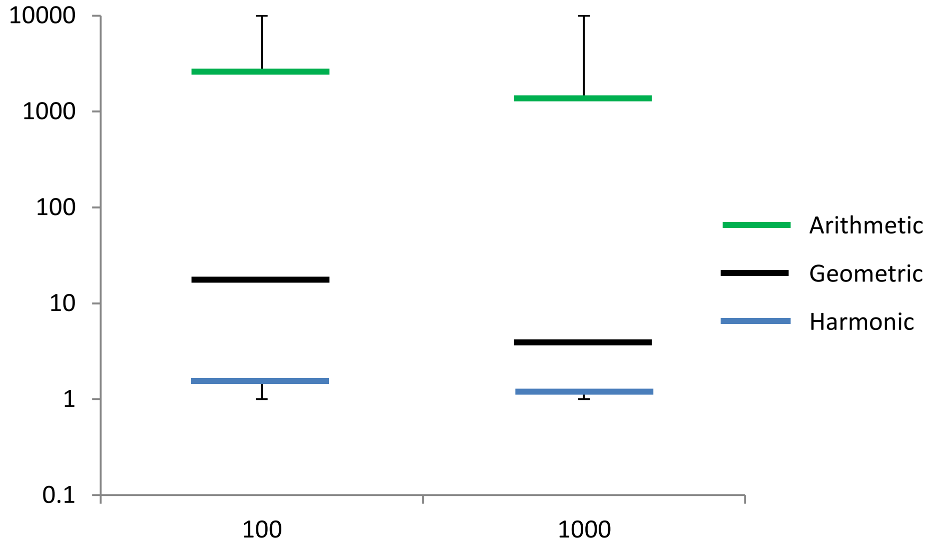

Figure 4-5 Statistical analysis of (1, 100, 10000) map distribution in both 100 × 100 and 1000 × 1000 sample areas. Y-axis scale in logaritmic units.

The following general observations can be made from Table 4-1, Figure 4-3 – Figure 4-5.

1. The relative difference between the highest and lowest hydraulic conductivity value plays a significant role in the mean values calculated for each system. The arithmetic mean over estimates the hydraulic conductivity in an area, while the geometric and harmonic mean tends to give lower order estimates.
2. In each instance as compared to the order of the hydraulic conductivity difference, the larger sample set gives a lower mean value. This is a result of the relative distribution of higher versus lower hydraulic conductivity zones as indicated in Figure 4-2 and it is directly reflected in the method (geometric, arithmetic or harmonic) of calculating these mean values.
3. The mean values as calculated for the change in order (left to right over Table 4-3) decreases in variation. With the harmonic mean being nearly constant, while the geometric mean has a steady increase as the order is increased in the system. The most aggressive growth in the mean value is observed for the arithmetic mean, which is adversely affected by higher hydraulic conductivity values.

As noted in Section 2.3.3 the natural flow system is controlled by zones of low hydraulic conductivity if parallel flow is considered over an area, thus it is reasonable to expect that the average hydraulic conductivity value for an area should be close to that of either the harmonic or geometric mean of the system. However, since natural flow is dependent on the local hydraulic conductivity and the hydraulic gradient (which can approach zero); it is expected that the harmonic mean would be a good approximation since it stays nearly constant throughout all of the order changes and dimensions of sample selection.

4.2.1.1 Estimating Hydraulic Conductivity as a Function of Discharge over an Area

In the above discussion the main focus was on the distribution of the hydraulic conductivities over an area and estimating the mean value. This section will attempt to illustrate a method of estimating the hydraulic conductivity by using Darcy's Law (Q = KiA). Due to memory limitations only a 500 by 500 element system (each element 10 m x 10 m) could be constructed with a total thickness of 100 m for the aquifer system (Chapter 9). A random distribution for the hydraulic conductivity values was used (Figure 4-6). A constant head boundary was placed on the left and right-most border of the theoretical area. Secondly, a hydraulic gradient of 10^{-4} was imposed on the system to simulate the natural flow from left to right as seen in Figure 4-6. A mass flux on either side and in the middle was setup which spanned the whole 5000 m of the model. The hydraulic conductivity values for the one order system is defined as the following set $\{0.01, 0.1, 1\}$, while the two order system is defined by $\{0.01, 1, 100\}$ for the lowest and highest values (Figure 4-6).

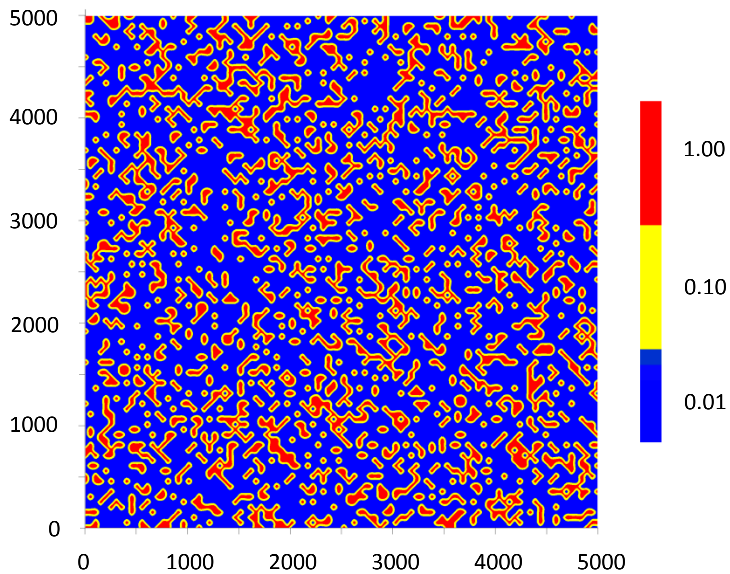

Figure 4-6 Random distribution of hydraulic conductivites for a specific test area.

With the above in mind an estimate of the mass flux was derived for the right, left and central zones of Figure 4-6. The respective values were 0.9196, 0.9359 and 0.9231 m^3/d for the first order difference while for the second order difference the values are 0.9806, 1.0010 and 0.9838 m^3/d. It is interesting to note that although no restraints were placed on the model, except the order differences in hydraulic conductivities, the approximate discharge volume values are relatively similar. This tends to confirm the influence of low K horizons on heterogeneous aquifer systems. Converting these values to hydraulic conductivities results in the following values of 0.0184, 0.0187 and 0.0185 m/d for the first order difference and for the second order difference the values are 0.0196, 0.0200 and 0.0197 m/d, respectively. A change in hydraulic conductivity can be observed, however the change is less than 10 % indicating that the low hydraulic conductivity value of the country rock dominates the natural flow regime. The arithmetic, geometric and harmonic mean of the hydraulic conductivity zones are defined for the one and two order systems as 0.2608, 0.0358, 0.0142 m/d and 24.8698, 0.1283, 0.0144 m/d. In both instances the harmonic mean gives the closest approximation of the hydraulic conductivity of the area.

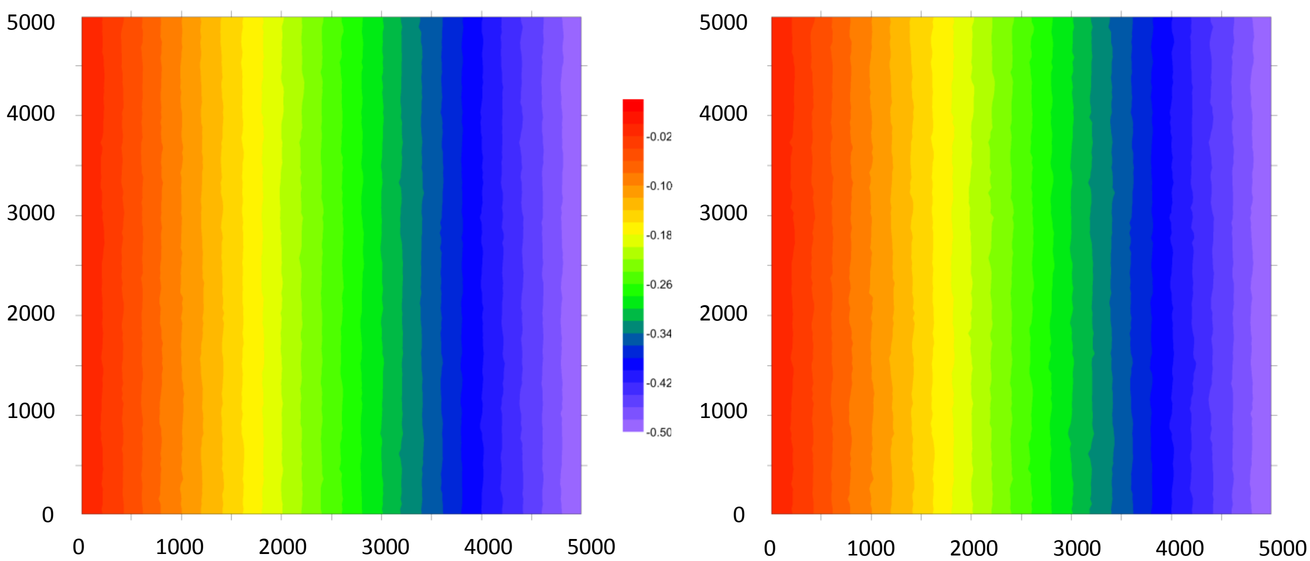

Figure 4-7 First order (left hand side) and second order (right hand side) hydraulic head value results after a 6 year simulation period.

In Figure 4-7 no clear difference can be observed for the respective hydraulic head values for the one and two order difference systems. However, if a difference image (Figure 4-8 left hand side) is constructed between the one and two order systems, a strong variation can be detected between zones. The random distribution of higher hydraulic conductivity areas can be observed, especially if the random distribution of hydraulic conductivity zones is overlain over the difference map (Figure 4-8 right hand side). The two order difference map was subtracted from the first order difference system, thus positive (yellow to red) values indicate an elevated hydraulic head in the one order system compared to the two order system. The inverse is also true in which the two order difference is greater than the one order systems resulting in a negative (green to blue) value. Thus, the blue regions indicate a build-up of water or hydraulic head since a large hydraulic conductivity difference between the country rock (0.01 m/d) and the simulated hydraulic conductivity of the dolerite intrusions (10000 m/d) is observed.

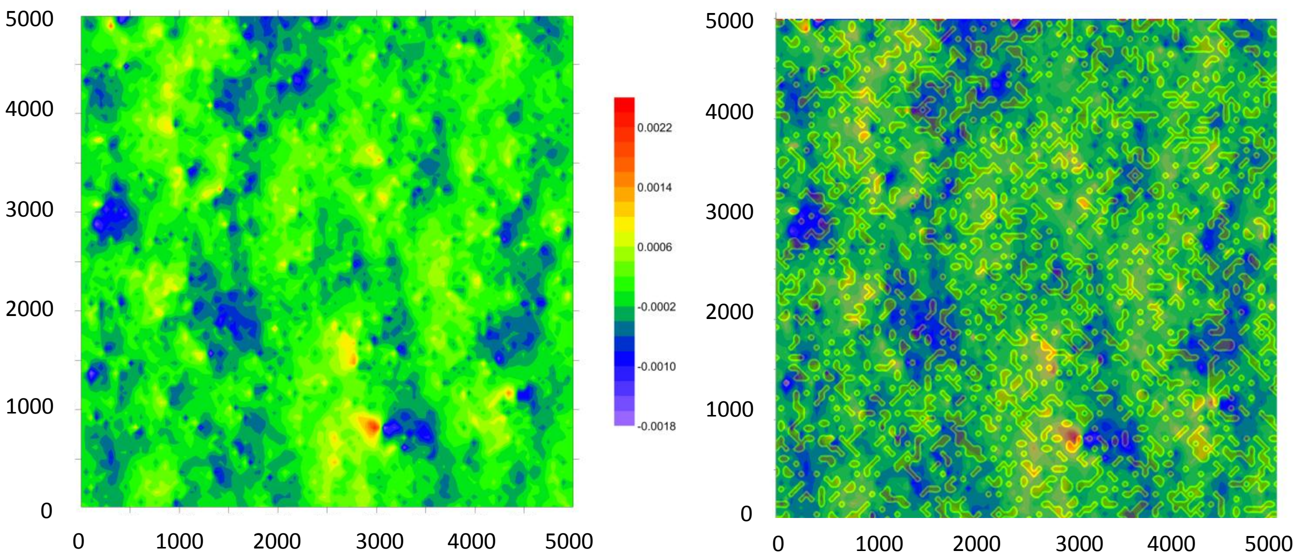

Figure 4-8 Difference map of first and second order hydraulic conductivity under simulated natrual flow.

Furthermore, visually comparing the distribution of light blue and dark blue zones to that of yellow and red zones, a higher number of blue zones are observed. This illustrates that the higher two order hydraulic conductivity areas, has a larger effect on the transport parameters, thus affecting the movement of water. It is interesting to note that no simple linear correlation exist which would indicate where head build up would occur in the system (Figure 4-8).

A final evaluation of the natural system is to estimate what would occur if a sink is placed in the middle of the area. A rectangular area (1000 m × 1000 m) was placed in the centre of the model and the water level within this area was kept at 50 m below surface level (Figure 4-9). In nature this scenario can represent a sheer bank in which groundwater falls to a river level.

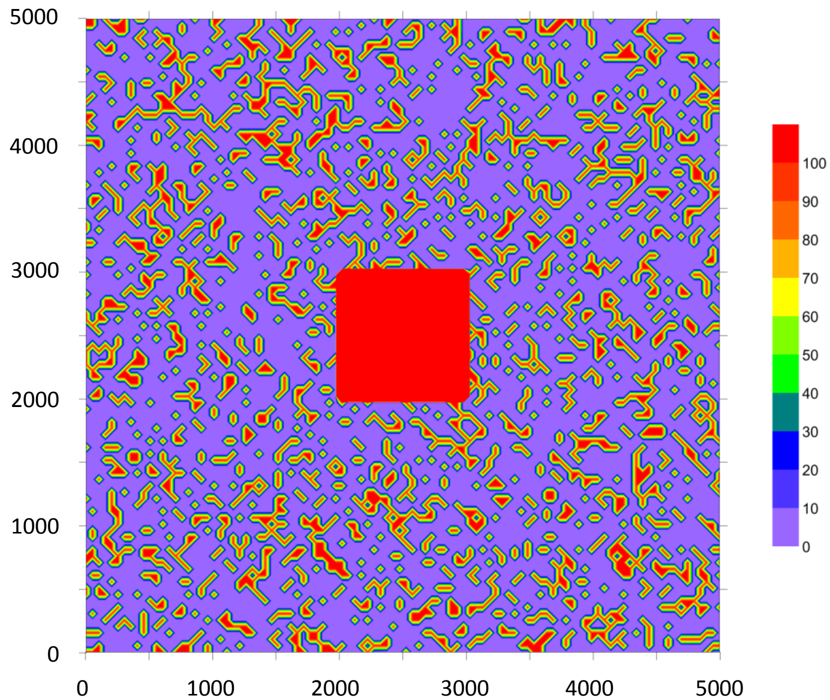

Figure 4-9 Hydraulic conductivity map of the study area with a cavity in the central region.

To assist in the evaluation the influx from one of the walls were computed over an extended time frame (6 years) and the results are reported in Table 4-2 and Figure 4-10. The initial discharge into the cavity for the one and two order systems is 2824 and 18970 m^3/d, respectively. It can be clearly observed from Figure 4-10 that both the one and two order solutions tend towards the same result (141 – 148 m^3/d); this would again indicate that the initial high hydraulic conductive zones are overshadowed by the influence of the country rock.

Table 4-2 Summary of values for the discharge rate of the one and two order systems for the cavity model.

Time (d)	Discharge 1 order (m^3/d)	Discharge 2 order (m^3/d)
2	2824	18970
12	1186	1222
112	437	454
360	267	277
720	204	212
1080	176	184
1440	160	168
1800	149	156
2160	141	148

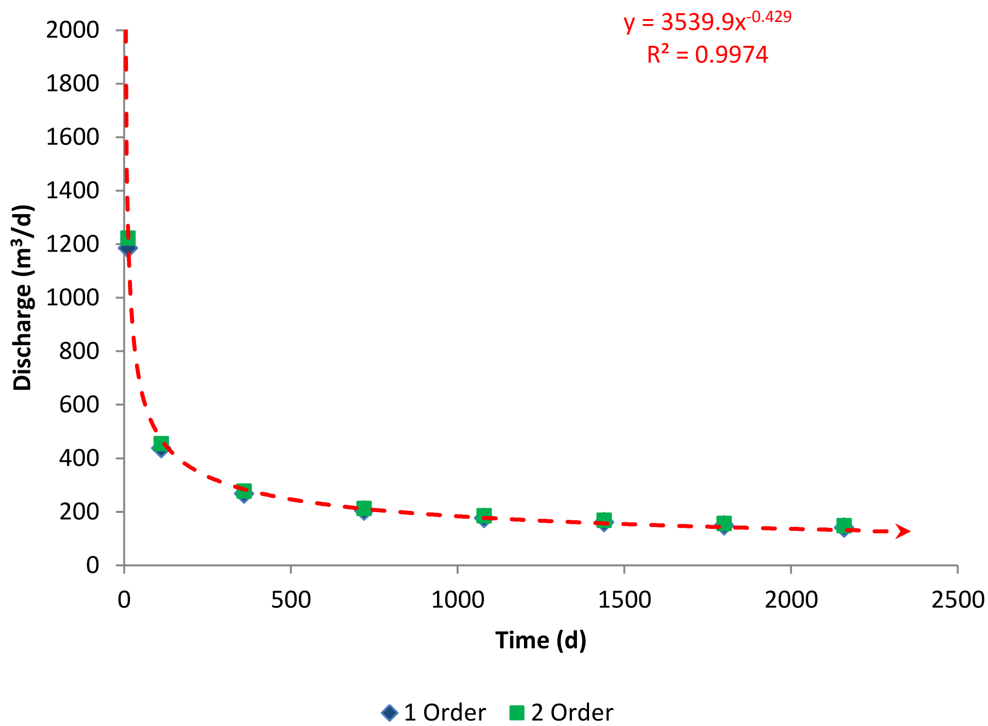

Figure 4-10 Discharge from seepage wall over an extended time period of 6 years. Both the one order difference and two order difference scenarios are given for the same model.

In this section the calculation of mean (arithmetic, geometric and harmonic) values were discussed in order to determine the most suitable average value for an area. The harmonic mean gave consistently the most stable results. It was shown that the geometric mean was susceptible to change if orders of magnitude were observed between the low and high hydraulic conductivities zones.

Subsequently, the natural calculated hydraulic conductivity of an area can be estimated by determining the harmonic mean. It can be reasoned that the hydraulic properties of the area is controlled by the lower hydraulic conductivity values instead of the geometric mean values as proposed by pure statistical approaches.

4.2.2 Hydraulic Conductivity Distribution under Forced Gradient Conditions

In this section a model was created that had 500 x 500 cells (1 cell = 10 m × 10 m) with a random hydraulic conductivity distribution as presented in Figure 4-11. A similar approach as that followed in the natural system was used, in that the influence of the change in the order of hydraulic conductivity values was of interest. The hydraulic conductivity values for the one order system is defined as the following value set $\{0.01, 0.1, 1\}$, while the two order system is defined by $\{0.01, 1, 100\}$ for the lowest and highest hydraulic conductivity values. In addition, a second set of scenarios that represent the presence of high or low hydraulic conductivity zones around the pumping well were included (Figure 4-12). This was done to determine the long-term effect that the final hydraulic properties of the system would have on the hydraulic conductivity estimate.

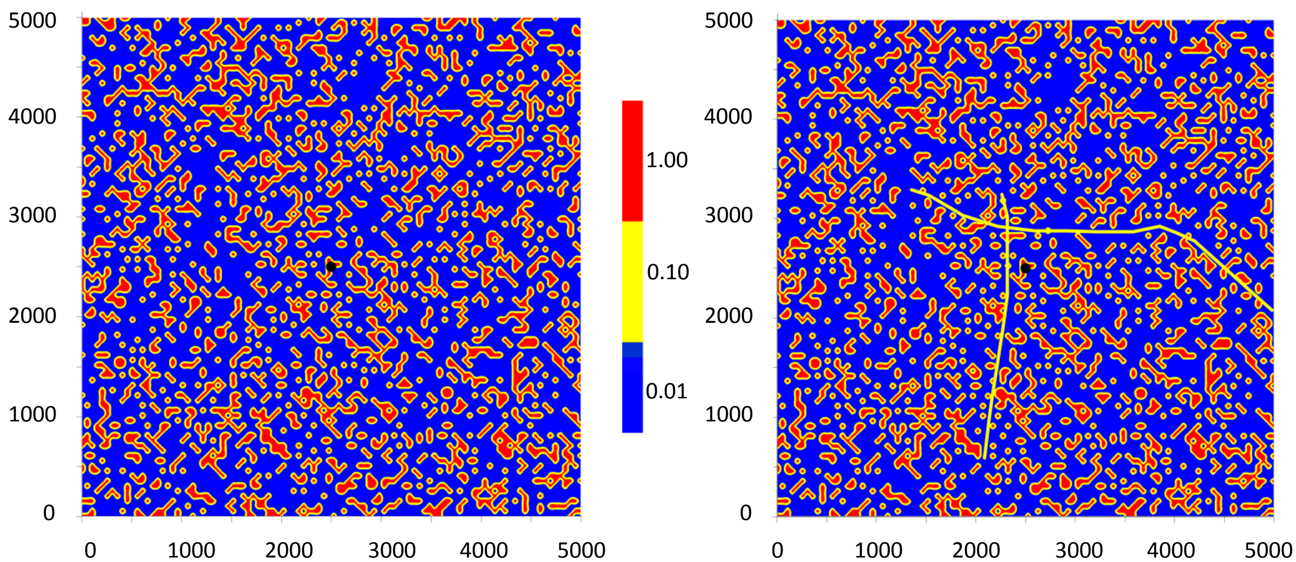

Figure 4-11: Random number-generated maps. The left figure shows a random distribution of hydraulic conductivity values. In comparison, lineaments or line segments that could act as high conductivity zones are also represented on the right figure.

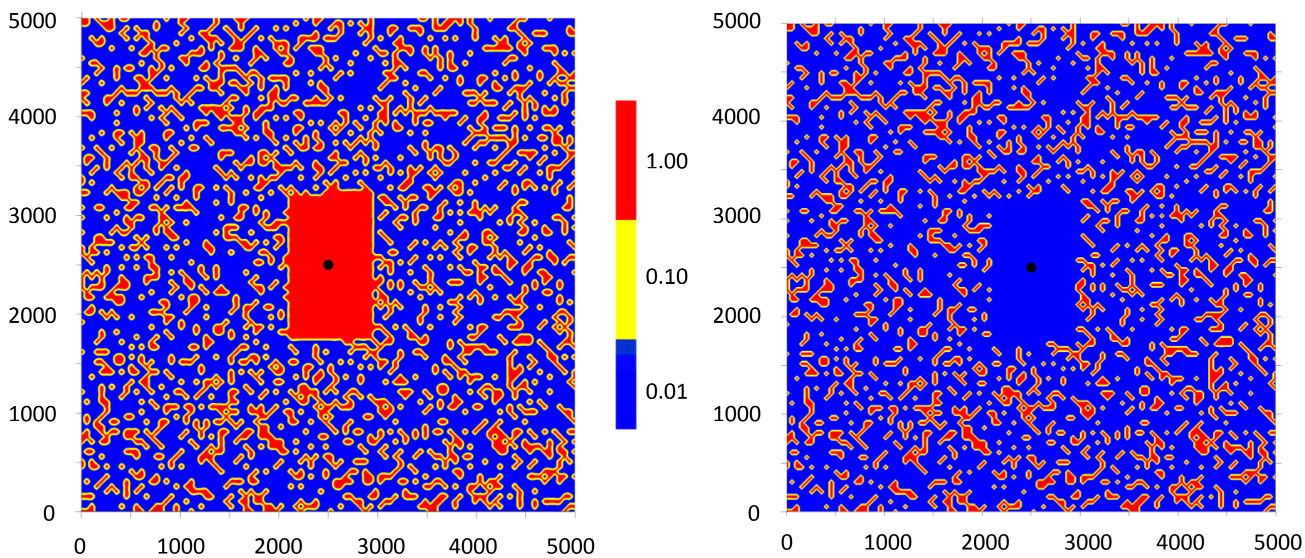

Figure 4-12 Hydraulic conductivity maps showing the presence of a high (left) and low (right) conductivity blocks in the middle of the randomly generated values.

In all four of the scenarios a pumping well was placed in the centre and set to pump at a rate of 100 m^3/d, over a six year time period. The final drawdown contours are presented in Figure 4-13 – Figure 4-14. It is clear from Figure 4-13 that the inclusion of low conductivity zones had no real apparent effect on the cone of depression of the system. However, upon comparison of the blocked zones the high hydraulic conductivity block has a significantly smaller drawdown in its centre when compared to the low hydraulic conductivity block.

The drawdown values over a time period in the borehole were recorded and fitted using the Cooper-Jacob straight line method (Figure 4-15). The fits were done at late time to give an estimate of the hydraulic conductivity of the system. Since the Cooper-Jacob method gives an estimated value based on graphical fit, it was expected that large variations in the hydraulic conductivity distributions would yield significantly different results. However, from Table 4-3 and Table 4-4, the method proofed itself relatively robust in estimating the hydraulic conductivity values.

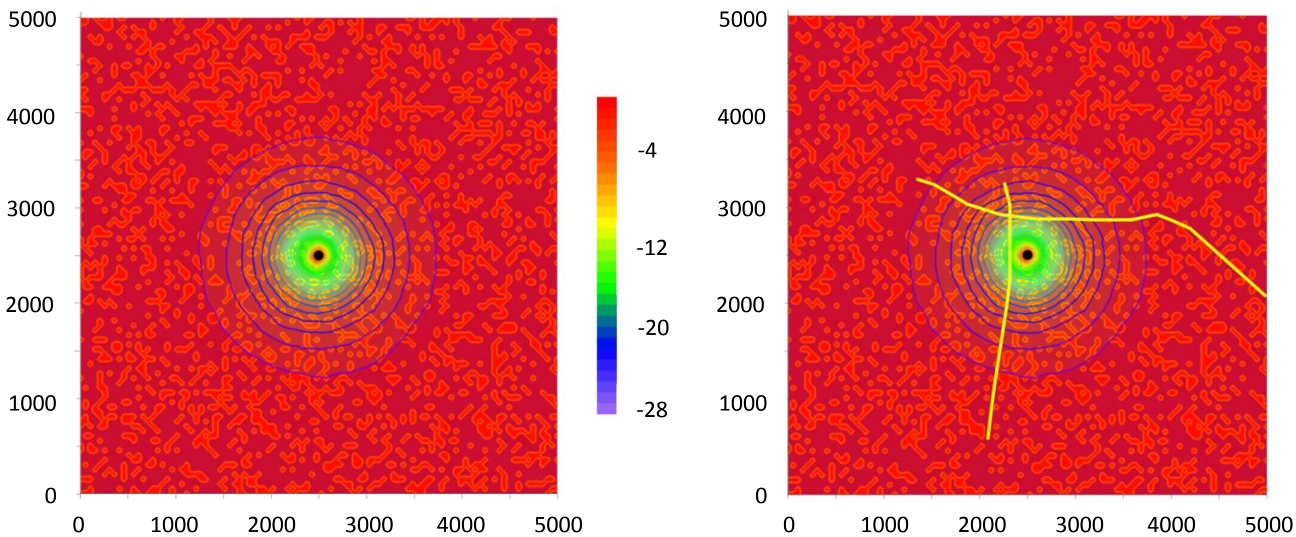

Figure 4-13 Contours maps of drawdown at the end of six years of constant pumping at 100 m^3/d. Left hand side illustrate the contours in a random field while the right hand side indicates the effect of two low conductive zones on the cone of depression contours.

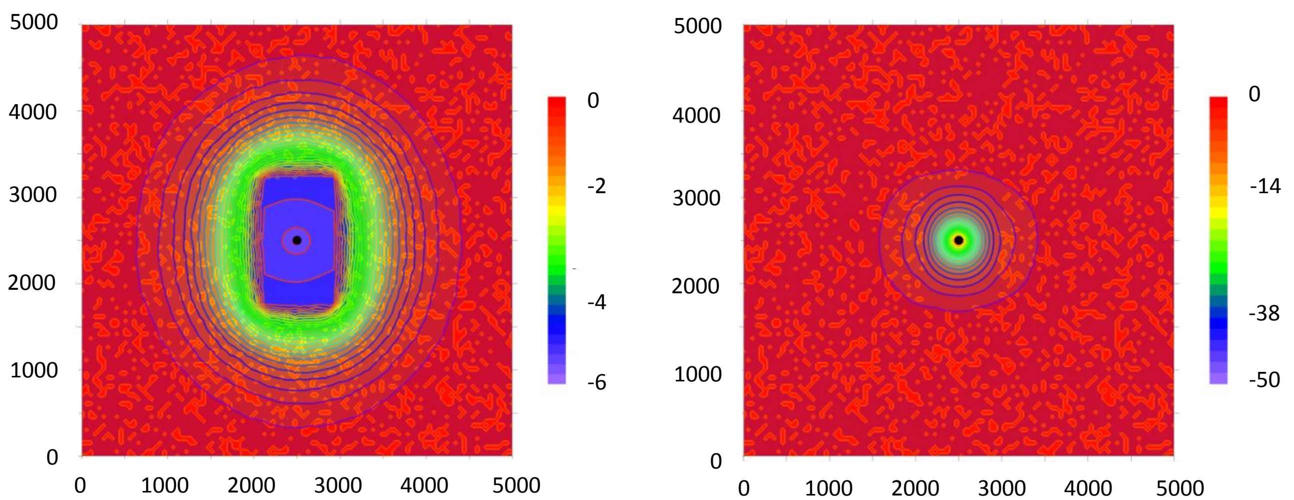

Figure 4-14 Contours maps of drawdown at the end of six years of pumping at a 100 m3/d. Left hand side illustrate the contours in a random field with a high hydraulic conductivity zone located in the middle. The right hand side indicates the effect of a low hydraulic conductivity zone on the cone of depression contours.

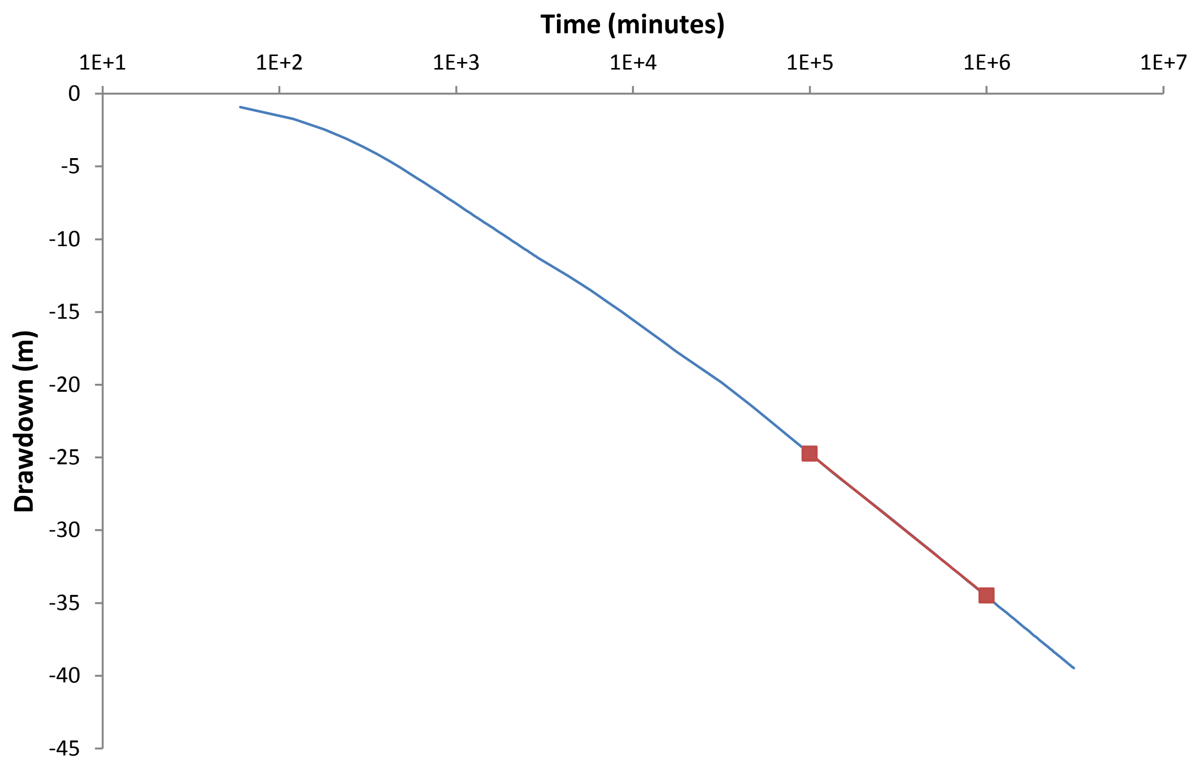

Figure 4-15 Cooper-Jacob fit of simulated hydraulic test data for the random one order system. The estimated hydraulic conductivity for the hydraulic test is 0.0183 m/d.

Table 4-3 The influence of conductive zones on the average hydraulic conductivity of different order systems. Randomly generated low conductivity zones (lines) were included (see Figure 4-11).

Order - Mean	Arithmetic	Geometric	Harmonic	Observed
1 order				
Random	0.2608	0.0358	0.0142	0.0183
1 line	0.2592	0.0355	0.0142	0.0192
2 lines	0.2582	0.0353	0.0142	0.0181
3 lines	0.2565	0.0350	0.0141	0.0174
2 order				
Random	24.8698	0.1283	0.0144	0.0203
1 line	24.7098	0.1262	0.0144	0.0203
2 lines	24.6084	0.1249	0.0143	0.0190
3 lines	24.4448	0.1228	0.0143	0.0177

Table 4-4 The influence of high and low conductive block zones on the average hydraulic conductivity of different order systems.

Block – Mean	Arithmetic	Geometric	Harmonic	Observed
Low$\{0.01, 0.1, 1\}$				
1 order	0.2485	0.0336	0.01395	0.0100
2 order	23.6515	0.1133	0.01409	0.0199
High$\{0.01, 1, 100\}$				
1 order	0.2971	0.0422	0.01497	0.0325
2 order	28.5546	0.1779	0.01514	0.0411

The results from Table 4-3 are interesting since in all instances the harmonic mean underestimates the hydraulic conductivity of the area by 20 – 30 %. In contrast the geometric mean overestimates the hydraulic conductivity by 80 – 100 % for the one order system and 500 – 600 % for the two order system. This clearly illustrates the effect of dual porosity systems on the estimation method for areal hydraulic conductivity mean values. It should be kept in mind that the difference in orders between the one and two system is $\{0.01, 0.1, 1\}$ and $\{0.01, 1, 100\}$, respectively. Thus in both instances the low hydraulic conductivity value for the area is 0.01. Compared to both the observed and the dominant background value, it is expected that the harmonic mean would give results which is more consistent with the actual regional hydraulic conductivity. Ironically, the arithmetic mean in all instances gives nonsensical values and will be discarded in the rest of this discussion.

In the instance of the high and low hydraulic conductivity block values (Table 4-4) the influence of large homogeneous structures on the bulk flow parameter were investigated. The harmonic mean gives relatively good estimates that range from 30 – 60 % errors. The geometric mean overestimates by 200 – 400 %, except in one instances (30 % error) and that is when the high conductivity block is used with the one order system. This is most likely a fortuitous event and is not reflected in the general analysis; nevertheless the geometric mean overestimates the hydraulic conductivity in an area.

Considering all of the systems in Table 4-3 and Table 4-4 an average value was calculated and the difference between each of the mean value methods was calculated. The harmonic mean gave errors that ranged from 6 – 60 %, with the best results obtained for the low hydraulic conductivity block (6 %) and the randomly distributed (20 – 30 %) systems. The high hydraulic conductivity block yielded the largest average underestimation difference of 60 %. The geometric mean gave average errors that ranged from 90 – 550 %, with the best results obtained for the randomly-distributed one-order systems. The remainder of the average geometric mean values were greater than 200 %, rendering the estimated hydraulic conductivity results useless. Thus, if a mean value of an area needs to be estimated it is most likely best to use the harmonic mean, with the knowledge that at least 30 % of the calculated values should be added to derive a realistic estimate of the hydraulic conductivity of the region.

4.3 Conclusion

In this section two distinct environments were investigated to determine what type of mean value should be used to determine a reasonable estimate of the hydraulic conductivity. It was assumed that the hydraulic conductivity in the whole area was known and that no uncertainty exists as to the location or variation of the distribution. Thus it was deemed necessary to use a numerical model system in which different possible scenarios could be tested. The scenarios tested included the idea that the relative difference in the hydraulic conductivity zones could dramatically affect the mean values calculated for an area. In addition the difference in hydraulic conductivities was set such that an order of difference could be observed, i.e. $\{0.01, 0.1, 1\}$ and $\{0.01, 1, 100\}$.

Firstly, a natural flow scenario (no pumping) was investigated in which a low gradient was used to determine the flow through a region. Since two sets of hydraulic conductivities were used the values obtained indicated that the harmonic mean, although it underestimated the hydraulic conductivity for the area yielded more accurate values. It was shown that the geometric mean was susceptible to change if orders of magnitude were observed between the low and high hydraulic conductivities zones. Thus it can be reasoned that the hydraulic properties of an area is controlled by the lower hydraulic conductivity values, instead of the geometric mean values as proposed by pure statistical approaches.

Secondly, a forced gradient scenario (pumping) was evaluated in which a pumping well was placed in the middle of the study area. Considering all of the systems investigated the harmonic mean gave errors that were far less than that observed for the other methods. Similar to the natural flow scenarios, the geometric mean of a forced gradient system was sensitive to large differences between hydraulic conductivity zones. It was determined that the harmonic mean underestimates the regional mean hydraulic conductivity, and may thus require adjustment to improve the approximation.

Chapter 5 Case Studies

5.1 Introduction

In the following sections an investigation into reported hydraulic conductivity or transmissivity values will be presented and discussed. In order to evaluate these systems the previously constructed random map (Conceptual Model section) will be used to illustrate the effect on hydraulic conductivity values from a totally random selection of points and secondly the influence of a directed search method on regional hydraulic conductivity values. Subsequently, field data obtained from the WRC (Murray et al., 2011) and other regional studies conducted at the Institute for Groundwater Studies (IGS) will be included to demonstrate the observed effects of estimating regional transmissivity values.

5.2 The Effect of Random Selection on Regional Hydraulic Conductivity Estimates

As noted in the preceding section the constructed random map (Figure 5-1) will be used with the first order difference hydraulic conductivity values of $\{0.01, 0.1, 1\}$. A random selection of points in the map was taken in order to mimic the effect of site exploration. The number of sampled points was steadily increased from 10, 100, 1000 and 10000 over the possible 250000 data points in order to illustrate the effect of sample size on observed values.

The values obtained were tabulated in Table 5-1 with four different samples being taken to illustrate the random behaviour of the system. As noted in the previous chapter the expected regional hydraulic conductivity values should be in the order of 0.0184 – 0.0187 m/d. The arithmetic mean is an order of magnitude larger than the expected regional hydraulic conductivity value. In addition, the increase in sample size does not improve the result but instead worsens the approximation. Furthermore, if sample 4 in Table 5-1 is considered the initial estimate with 10 samples sites is significantly worse than with 100 sample sites. This would indicate that not only does the arithmetic mean suffer from over estimating the regional hydraulic conductivity but that it also gives highly variable results as a function of sampling.

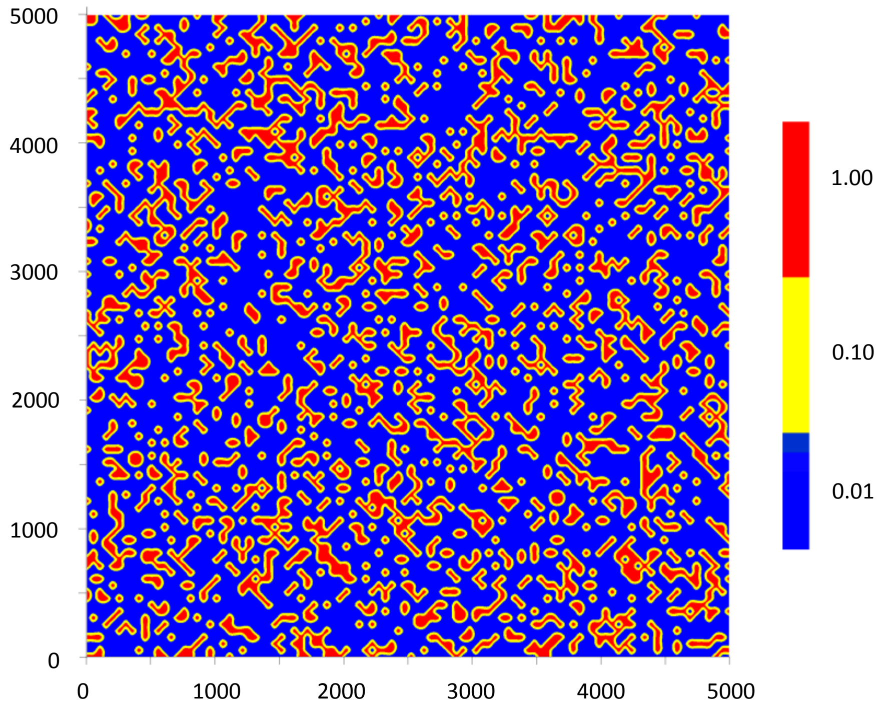

Figure 5-1 Random distribution of hydraulic conductivites for a specific test area.

Table 5-1 Mean values of randomly sampled points in a first order difference hydraulic conductivity map (0.01, 0.1, 1).

	1	2	3	4	Average
Arithmetic					
10	0.2080	0.2080	0.2170	0.5050	0.2845
100	0.2881	0.2404	0.2755	0.2161	0.2550
1000	0.2606	0.2542	0.2790	0.2837	0.2694
10000	0.2558	0.2532	0.2617	0.2685	0.2598
Average	0.2531	0.2389	0.2583	0.3183	0.2672
Geometric					
10	0.0251	0.0251	0.0316	0.1000	0.0455
100	0.0372	0.0309	0.0407	0.0309	0.0349
1000	0.0349	0.0343	0.0385	0.0408	0.0371
10000	0.0348	0.0345	0.0361	0.0372	0.0357
Average	0.0330	0.0312	0.0367	0.0522	0.0383
Harmonic					
10	0.0125	0.0125	0.0140	0.0198	0.0147
100	0.0140	0.0134	0.0151	0.0139	0.0141
1000	0.0140	0.0140	0.0145	0.0150	0.0144
10000	0.0141	0.0141	0.0143	0.0144	0.0142
Average	0.0137	0.0135	0.0145	0.0158	0.0143

The geometric mean has a far better performance in the estimation of the hydraulic conductivity values for the region. In general the values are overestimated by a 100 %. However, with an increase in sample sites, a steady growth of the hydraulic conductivity can be observed. As in the previous section, sample 4 shows a significant deviation from the other samples. In this instance the geometric mean recovers as sample sites are increased. If the average value for different sampled sites is compared no real systematic improvement can be observed. The average of all the samples results in an overestimation of a 105 %.

In the instance of the harmonic mean values, the values tend to be in the same range as the expected regional hydraulic conductivity (0.0184 – 0.0187 m/d). Interestingly, the harmonic mean underestimates the regional hydraulic conductivity by approximately 30 %. However as the number of sample sites are increased, an increase in estimated hydraulic conductivity is observed, the underestimation is reduced to 20 %. The harmonic mean has a far better analysis of sample 4, with an estimated value which is initially larger than the observed regional hydraulic conductivity but eventually reduces to the expected harmonic mean values for the other samples. If the average of the different samples is taken an average underestimation of 23 % is calculated. The harmonic mean values for the hydraulic conductivity values are more robust in estimating final values as compared to the other methods.

Subsequently, it is of interest to evaluate what would happen to a sample set that had two orders of difference in the hydraulic conductivity values $\{0.01, 1, 100\}$ as noted in the conceptual model chapter.The values obtained were tabulated in Table 5-2 with four different samples being taken to illustrate the random behaviour of the system. As previously noted in the conceptual model chapter the expected regional hydraulic conductivity values for the second order difference map should be in the range of 0.0196 – 0.0200 m/d.

The arithmetic mean is still an order of magnitude greater than the regional hydraulic conductivity value. The random samples taken in this section shows a greater variation in the samples, with sample 4 resulting in the best approximation. As the sample size increases an initial decrease in estimated hydraulic conductivity values is observed. However, with large sample sets a steady increase in the arithmetic value is observed which results in excessive estimates.

Table 5-2 Mean values of randomly sampled points in a second order difference hydraulic conductivity map (0.01, 1, 100).

	1	2	3	4	Average
Arithmetic					
10	0.4060	0.3070	0.3070	0.2080	0.3070
100	0.1945	0.2836	0.2557	0.3268	0.2652
1000	0.2577	0.2675	0.2702	0.2713	0.2667
10000	0.2616	0.2638	0.2596	0.2675	0.2631
Average	0.2799	0.2805	0.2731	0.2684	0.2755
Geometric					
10	0.0631	0.0398	0.0398	0.0251	0.0420
100	0.0269	0.0407	0.0372	0.0437	0.0371
1000	0.0352	0.0368	0.0378	0.0366	0.0366
10000	0.0359	0.0361	0.0357	0.0369	0.0361
Average	0.0403	0.0384	0.0376	0.0356	0.0380
Harmonic					
10	0.0166	0.0142	0.0142	0.0125	0.0144
100	0.0132	0.0149	0.0147	0.0146	0.0144
1000	0.0142	0.0143	0.0145	0.0142	0.0143
10000	0.0142	0.0142	0.0142	0.0144	0.0143
Average	0.0145	0.0144	0.0144	0.0139	0.0143

The geometric mean has significantly smaller estimate values for the hydraulic conductivity of the system, with the overestimated estimated values ranging from 0.0631 – 0.0269 m/d (0.0196 – 0.0200 m/d). This results in an overestimation that is in the order of 35 to 200 %. The variation in values between each sample is also great and no clear trend can be observed to evaluate possible methods of improving the outcome. If the average of the different samples is taken an average underestimation of 90 % is calculated. Increasing sample size also does not improve the estimation of hydraulic conductivity values, with samples 2 and 3 showing a decreasing trend. However Samples 1 and 4 has the opposite trend.

The harmonic mean values tend to underestimate the regional hydraulic conductivity value by 15 to 30 %. If the average of the different samples is taken an average underestimation of 30 % is calculated. In general all harmonic mean values for the system are in the same order and no significant variation is observed between samples.

The results obtained in this section indicate that random sampling of a region results in variation in the average observed hydraulic conductivity values. The arithmetic mean was shown to be sensitive to changes in both the first and second order difference systems. The geometric mean gave a better estimation of the regional hydraulic conductivity values, although it overestimated

5.3 The Effect of Directed Selection on Regional Hydraulic Conductivity Estimates

Due to the method of borehole site selection a certain amount of bias is observed in regional zone. Firstly, the topography and local geology of the area is considered and if possible any geological structures could be observed. Secondly, geophysical methods such as magnetic, gravimetric, electromagnetic and electrical resistivity surveys are employed to determine the most favourable borehole site locations. Finally, local knowledge can also greatly improve the changes of siting a high yielding borehole. These methods are by no means fool proof and low-yielding boreholes can be sited by these methods. However, these methods do improve borehole siting procedures and generally give favourable results. In addition to the above, most borehole contractors and private land users are only interested in high yielding boreholes (> 1 l/s) and typically backfill lower yielding borehole sites.

To cover this possibility, a biasing procedure needs to be used to evaluate different yielding scenarios in both a one and two order difference systems. In order to assist in this a magnetic approach was implemented. This seems reasonable given the predominance of dolerite intrusions throughout South Africa (Figure 1-7). A smoothing algorithm was used on both the first and second order difference maps. Smoothing was done using a 3x3 matrix with a specified weighting scheme (Table 5-3) to normalise the values obtained. In the instance of the first order difference map it was a quarter while in for the second order difference map it was 1/400.

Table 5-3 Smoothing matrix format as applied to first order difference map.

0.25	**0.5**	**0.25**
0.5	1	0.5
0.25	0.5	0.25

The obtained value was then evaluated against a threshold value to determine a biasing matrix. The biasing matrix was then applied to the original hydraulic conductivity map and all values not included in the biasing map were set to zero. This would in effect mimic the use of a skilled geohydrologist, which would ignore all other areas except where anomalies were observed in a region. A typical biasing matrix is spatially represented in Figure 5-2 compared to the original map of Figure 5-1. Thus only these regions will be considered as possible borehole drilling sites.

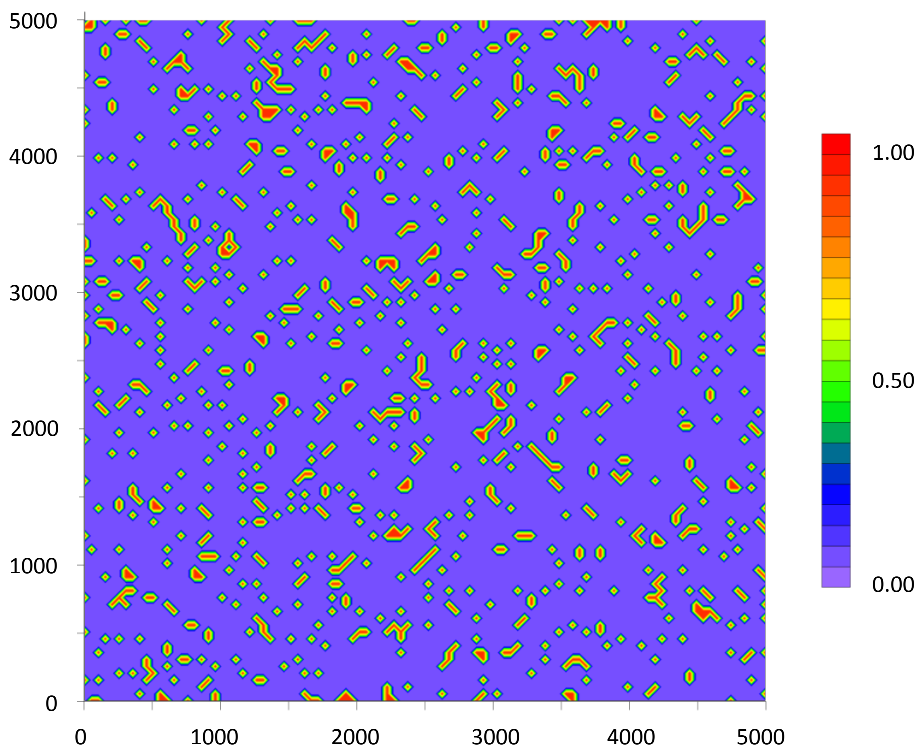

Figure 5-2 Effect of smoothing and filtering of original hydraulic conductivity map (filter set at 50 % of highest peak).

Figure 5-2 is then used as a reference map to select the hydraulic conductivity values from Figure 5-1, which in turn is then randomly sampled to give values to be used in estimating mean hydraulic conductivity values. These values are tabulated in Table 5-4 for a first order difference map and Table 5-5 for a second order difference map, respectively.

As noted previously the expected regional hydraulic conductivity values for the first order difference map should be in the order of 0.0184 – 0.0187 m/d. Compared to the values reported in Table 5-4, it can be clearly observed that the arithmetic mean does not even come close to representing the expected regional value. Similarly, the geometric mean fails to represent the general value and is in this instance useless. The mean estimate error is in the order of thousands of percentages for both the arithmetic and geometric mean.

Table 5-4 Mean values of randomly sampled points in a first order difference biased hydraulic conductivity map (0.01, 0.1, 1).

	1	2	3	4	Average
Arithmetic					
10	1.0000	1.0000	0.9100	0.7120	0.9055
100	0.8920	0.8614	0.9028	0.7939	0.8625
1000	0.8830	0.8938	0.8853	0.8619	0.8810
10000	0.8828	0.8841	0.8798	0.8814	0.8820
Average	0.9144	0.9098	0.8945	0.8123	0.8828
Geometric					
10	1.0000	1.0000	0.7943	0.3162	0.7776
100	0.6166	0.5248	0.6607	0.3981	0.5501
1000	0.5902	0.6194	0.6012	0.5433	0.5885
10000	0.5905	0.5946	0.5832	0.5875	0.5889
Average	0.6993	0.6847	0.6598	0.4613	0.6263
Harmonic					
10	1.0000	1.0000	0.5263	0.0461	0.6431
100	0.0910	0.0673	0.1099	0.0500	0.0796
1000	0.0835	0.0911	0.0876	0.0745	0.0842
10000	0.0838	0.0850	0.0822	0.0832	0.0836
Average	0.3146	0.3109	0.2015	0.0635	0.2226

In contrast to the arithmetic and geometric mean the harmonic mean give results which are better than expected. The harmonic mean hydraulic conductivity values for the regional area does show a dependence on sample size, although sample 4 has the best mean value approximation. The mean estimate error is in the order of 300 % for the harmonic mean, which clearly illustrates the influence of bias in the data. The absence of lower hydraulic conductivity values have skewed the results and in this instance not even the harmonic mean can give a reasonable assessment.

Considering the second order difference map, a similar approach to that of the first order difference map was followed. The biasing in this instance gives more pronounced effects. As noted previously the expected regional hydraulic conductivity values for the second order difference map should be in the order of 0.0196 – 0.0200 m/d. Firstly, the arithmetic mean values for the regional hydraulic conductivity does not even resemble an approximation. Similarly the geometric mean fails to estimate the regional hydraulic conductivity. The lowest approximation for the hydraulic conductivity value is 15 m/d which is three orders greater than the actual value. Both these mean value calculation methods diverge from the regional hydraulic conductivity value, and are thus is over sensitised to large values of hydraulic conductivity in the sample sets.

Table 5-5 Mean values of randomly sampled points in a second order difference biased hydraulic conductivity map (0.01, 1, 100).

	1	2	3	4	Average
Arithmetic					
10	80.0020	100.0000	90.0010	100.0000	92.5008
100	89.0011	89.0209	90.0109	89.0209	89.2635
1000	90.0069	87.4131	87.4171	89.0090	88.4615
10000	88.5908	87.8524	87.9128	88.2798	88.1590
Average	86.9002	91.0716	88.8354	91.5774	89.5962
Geometric					
10	15.8489	100.0000	39.8107	100.0000	63.9149
100	36.3078	39.8107	41.6869	39.8107	39.4040
1000	40.9261	33.1131	33.7287	37.6704	36.3596
10000	36.5426	34.3716	34.6258	35.3346	35.2187
Average	32.4064	51.8239	37.4631	53.2039	43.7243
Harmonic					
10	0.0500	100.0000	0.0999	100.0000	50.0375
100	0.0908	0.1108	0.1109	0.1108	0.1058
1000	0.1062	0.0876	0.0907	0.0979	0.0956
10000	0.0956	0.0905	0.0913	0.0919	0.0923
Average	0.0857	25.0722	0.0982	25.0751	12.5828

In comparison the harmonic mean does give results which are nonsensical, such as the first entries for sample 2 and 4. However, in general the calculated harmonic mean values are significantly closer that that obtained from the other methods. The mean estimate error is in the order of 300 % for the harmonic mean, which clearly illustrates the influence of bias in the data. The absence of lower hydraulic conductivity values have again skewed the results and in this instance not even the harmonic mean can give a reasonable assessment of the regional hydraulic conductivity values.

To further investigate the influence of the bias of the system a stepwise increase from 0 to 50 % was performed on the first order system. It was done in for 0, 20, 40 and 50 % filter values to determine if there is a cut off in which a reasonable estimate for the regional hydraulic conductivity could be estimated. The results are tabulated in Table 5-4 and the respective biasing maps are shown in Figure 5-3 – Figure 5-4.

Table 5-6 Mean values of randomly sampled points in a first order difference biased hydraulic conductivity map.

Percentage	0	20	40	50
Arithmetic				
10	0.2080	0.4150	0.6040	1.0000
100	0.2881	0.4780	0.7174	0.8920
1000	0.2606	0.4309	0.7838	0.8830
10000	0.2558	0.4361	0.7895	0.8828
Average	0.2531	0.4400	0.7237	0.9144
Geometric				
10	0.0251	0.0794	0.1585	1.0000
100	0.0372	0.0933	0.2951	0.6166
1000	0.0349	0.0789	0.3945	0.5902
10000	0.0348	0.0804	0.4024	0.5905
Average	0.0330	0.0830	0.3126	0.6993
Harmonic				
10	0.0125	0.0195	0.0246	1.0000
100	0.0140	0.0197	0.0397	0.0910
1000	0.0140	0.0188	0.0516	0.0835
10000	0.0141	0.0189	0.0524	0.0838
Average	0.0137	0.0192	0.0421	0.3146

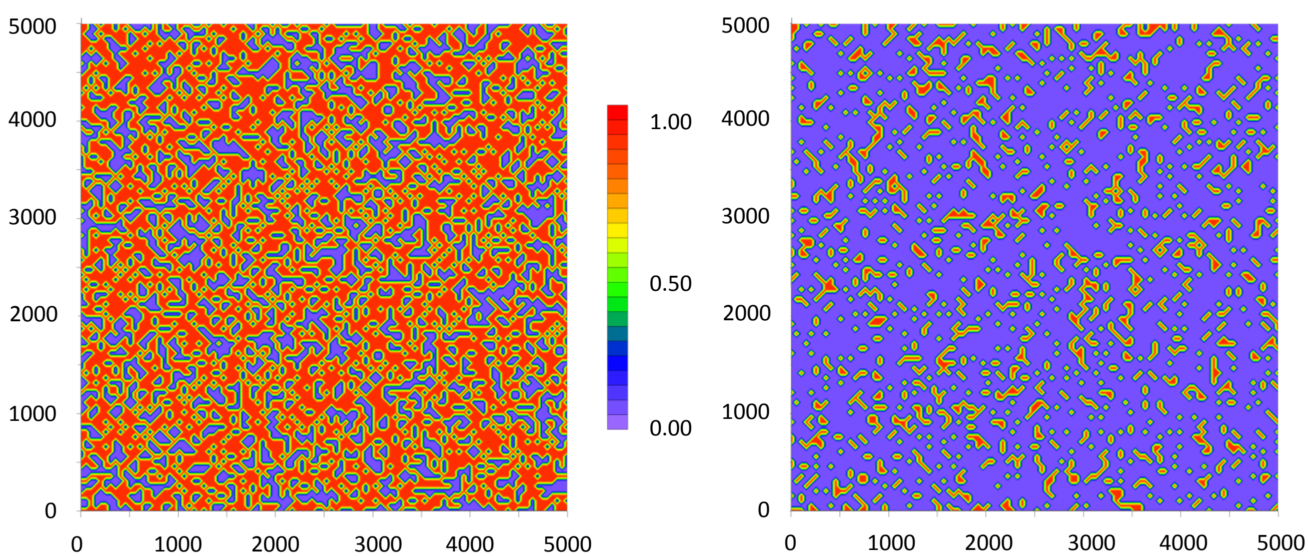

Figure 5-3 Effect of bias on the filter of hydraulic conductivity map (left filter set at 20 % and right 40 %).

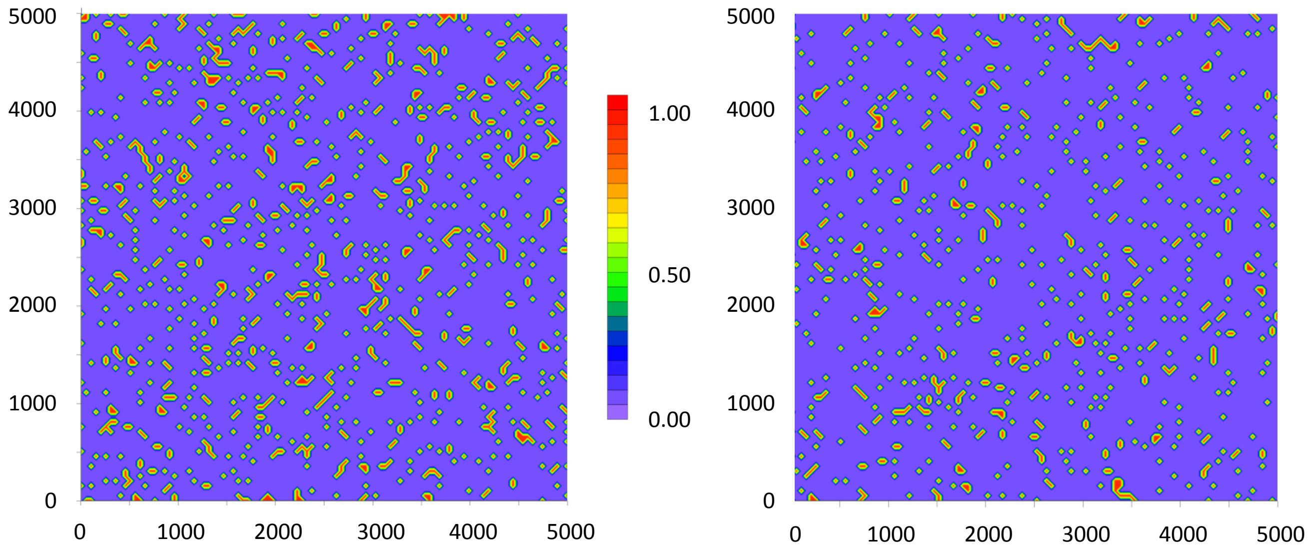

Figure 5-4 Effect of bias on the filter of hydraulic conductivity map (left filter set at 50 %). Right hand indicates the change of filter from 40 to 50 %.

In Figure 5-3 the difference between the 20 and 40 % filter can be clearly observed, with the red areas defining active sampling points. The 20 % biased map leaves most of the sampling area unchanged, i.e., some of the lower conductivity zones are included in the mean value calculations. An approximate doubling in the arithmetic mean value is observed for the 0 to 20 % filter values. The geometric mean is still below 0.1 m/d, but is roughly 4 times the expected value. The harmonic mean values are less than 10 % different from the expected hydraulic conductivity values. Although the estimation of the actual hydraulic conductivity for the region is obtained from the 20 % filter values, it does give a rough estimate to determine possible field values. A possible approximation of the field values can be obtained by including one fifth of lower yielding boreholes in the high yielding data set, thus resulting in a better estimate.

The change from 20 to 40 % is quite significant; a far lesser area in the original hydraulic conductivity map is left unchanged. This also results in a dramatic change in the mean values calculated for the system. The arithmetic and geometric mean is orders of magnitude different from the actual regional hydraulic conductivity value. The hydraulic conductivity as estimated by the harmonic mean has also deteriorated but still is only a 100 % greater than the expected value. This result indicates that although significant changes in the sampling method were implemented, it is still possible to calculate a reasonable estimate of the regional hydraulic conductivity.

The difference between 40 and 50 % filtering was not expected to have a great impact, however the change is significant. The difference map between 40 and 50 % is shown in Figure 5-4 (right hand side); the filtering removed a significant part of the sample sites in the original hydraulic conductivity map. This in part is the reason behind the jump in mean values, since the values used are mostly on high conductivity zones.

It is clear from this section that as much bias as possible should be removed from the system before a mean hydraulic conductivity value estimate for a region is calculated. To remedy this data from low yielding boreholes should be included as well as conducting hydraulic tests in areas where the country rock dominates the hydraulic parameters of the region. Furthermore, for a rough estimate it is expected that approximately 20 % of the hydraulic conductivity or transmissivity values should come from low yielding boreholes.

5.4 Estimating the Regional Hydraulic Conductivity from Field Data

The work presented here is based on data obtained from a WRC project on field measurements of transmissivity values (Murray et al., 2011). In this section geological formation data will be discussed and methods of obtaining a reasonable estimate of the hydraulic conductivity or transmissivity values. To further reduce the complexity of the system it will be assumed that in each value reported an aquifer thickness of 100 m is in effect, thus T = KD can be used to convert the transmissivity values (dependent on aquifer thickness) to hydraulic conductivity which is independent of aquifer thickness.

In addition to the above a short discussion on the conceptual model of dolerite dykes and sills will be included in this section. The rationale behind the discussion is the impact that these structures have on the transport of water in the Karoo aquifer systems. Firstly, dolerite dykes can act as conduits with high hydraulic conductivity zones (Figure 5-5). Secondly, the dyke structures can create an obstruction which can intercept or hinder the flow of water across the formation. Finally, it is rare that a dolerite dyke would not have weathered close to the surface and if the water level rises high enough a connection between the upstream and downstream aquifer systems is highly likely (Figure 5-6). The impact of this is that two aquifer systems can act in unison or can be totally disconnected from each other.

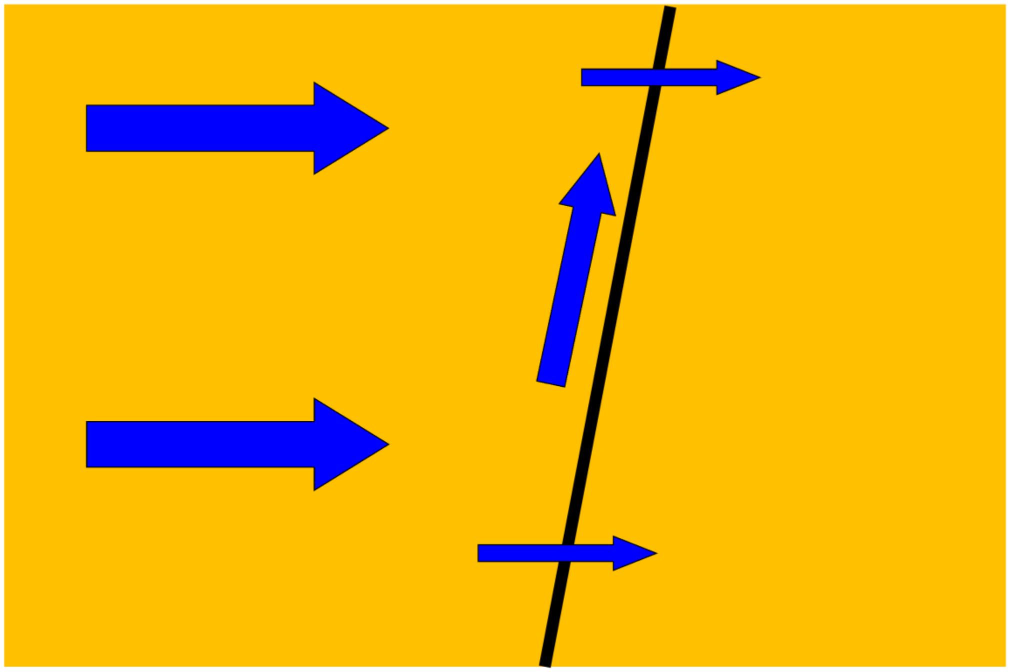

Figure 5-5A plan view of the regional water flow direction in the presence of a dyke structure.

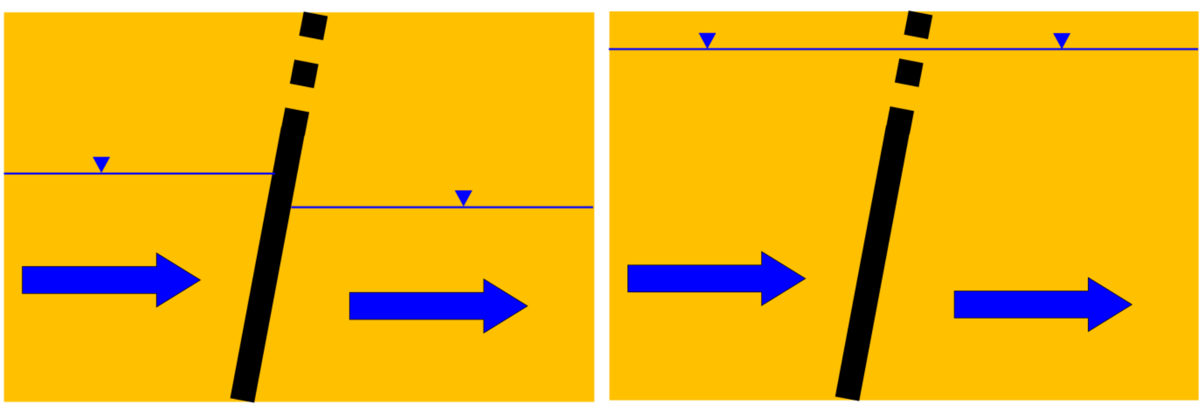

Figure 5-6 Side on view of regional water flow in the presence of a dyke structure. Left-hand side indicates the situation if the water level is below the weathered zone of the dolerite dyke. Right-hand side indicates the situation if the water level is above the weathered zone.

The scenarios presented in Figure 5-5 and Figure 5-6 can have a significant impact on the interpretation of regional hydraulic conductivity values. As presented in the Conceptual Model chapter the influence of high hydraulic conductivity zones in the vicinity of a regional low hydraulic conductivity area can change the behaviour of the estimated or measured bulk flow parameters. Although the dominant formation in the area has a lower hydraulic conductivity value, it is the high hydraulic conductivity zones which are targeted for water supply well fields. To illustrate this observation data obtained from the WRC project (Murray et al., 2011) and is collated in Table 5-7. Note transmissivity values have been converted to hydraulic conductivity values (D = 100 m). The “Lower K” values were calculated based on the harmonic mean of 33 % of the lowest hydraulic conductivity values of the sample population. The “Upper K” values were calculated from 33 % of the highest hydraulic conductivity values and the arithmetic mean of the sample population. Finally, the “Middle K” values were calculated based on the median of the remaining 33 % of the sample population values. This was done for the matrix, sills, dykes, sill margins and alluvium for the respective formation from which hydraulic test data was collected.

It is interesting to evaluate the collected data in Table 5-7 and compare it to the results obtained in Table 5-6. The data presented in Table 5-7 has a “Lower K” mean hydraulic conductivity value for the matrix in the order of 0.010 m/d with values that range from 0.003 – 0.039 m/d. The “Middle K” mean value for the matrix is 0.018 m/d with values that range from 0.006 – 0.073 m/d. It is remarkable that the “Lower K” and “Middle K” mean values do not differ dramatically, an effective increase of 100 % on estimated values. This could be indicative that these two sample sets do not have significant differences and most likely represent the same population, i.e., similar geological formations.

Thus an estimate of the regional hydraulic conductivity of the area can be estimated to be 30 % of the harmonic mean value (0.012 m/d) tabulated in Table 5-7.

Once the higher conductive zones are considered, the progression of mean values follows a similar trend as that observed in Table 5-6. However, it should be noted that the difference in the hydraulic conductivity between the country rock or matrix and the fracture network would be far greater than three orders in magnitude, i.e., 1×10^{-3} vs. 1×10^{3} m/d respectively. This could be the only explanation why an order of increase in the hydraulic conductivity is observed as one moves from the “Lower K” through to the “High K” values in the respective sequences. The observed change can in part be an artefact from the mean value calculation method (Table 5-5) in which each method emphasises a certain section of the data.

Table 5-7 Hydraulic conductivity values compared to geological formation.

Formation	Lower K (m/d) Range						Middle K (m/d) Range						Upper K (m/d) Range					
	Matrix	Dyke	Sill	Sill margin	Alluvium & intergranular	Alluvial basin	Matrix	Dyke	Sill	Sill margin	Alluvium & intergranular	Alluvial basin	Matrix	Dyke	Sill	Sill margin	Alluvium & intergranular	Alluvial basin
Adelaide-1	0.020	0.020	0.018	0.013	0.002	0.113	0.037	0.422	0.363	0.349	0.053	0.325	0.274	2.822	2.099	4.198	0.439	2.548
Adelaide-2	0.012	0.017	0.010	0.010	0.002	0.113	0.022	0.268	0.218	0.207	0.053	0.325	0.166	1.756	1.116	1.672	0.439	2.548
Adelaide-3	0.005	0.013	0.008	0.008	0.002	0.113	0.009	0.149	0.113	0.151	0.053	0.325	0.085	0.836	0.510	0.842	0.439	2.548
Adelaide-4	0.039	0.040	0.026	0.013	0.002	0.113	0.073	0.814	0.700	0.349	0.053	0.325	0.537	4.832	3.886	4.198	0.439	2.548
Adelaide-5	0.008	0.020	0.012	0.013	0.002	0.113	0.015	0.188	0.147	0.349	0.053	0.325	0.111	1.277	0.869	4.198	0.439	2.548
Burgersdorp	0.011	0.016	0.010	0.040	0.002	0.113	0.021	0.256	0.207	0.396	0.053	0.325	0.156	1.790	0.925	2.222	0.439	2.548
Clarens	0.009	0.022	0.014	0.025	0.002	0.113	0.017	0.204	0.161	0.073	0.053	0.325	0.122	1.102	0.703	0.545	0.439	2.548
Drakensburg	0.007	0.013	0.007	0.007	0.002	0.113	0.013	0.166	0.128	0.128	0.053	0.325	0.099	0.910	0.719	0.719	0.439	2.548
Dwyka-1	0.008	0.011	0.006	0.004	0.002	0.113	0.015	0.179	0.139	0.147	0.053	0.325	0.111	1.439	1.000	1.119	0.439	2.548
Dwyka-2	0.008	0.020	0.012	0.007	0.002	0.113	0.015	0.188	0.147	0.051	0.053	0.325	0.111	1.332	0.700	0.791	0.439	2.548
Dwyka-3	0.005	0.010	0.006	0.004	0.002	0.113	0.009	0.116	0.086	0.147	0.053	0.325	0.064	0.883	0.544	1.119	0.439	2.548
Ecca	0.010	0.025	0.015	0.026	0.002	0.113	0.019	0.225	0.179	0.179	0.053	0.325	0.135	1.213	0.700	0.950	0.439	2.548
Elliot	0.007	0.015	0.009	0.025	0.002	0.113	0.013	0.164	0.126	0.073	0.053	0.325	0.095	1.234	0.709	0.545	0.439	2.548
Katberg	0.013	0.021	0.013	0.011	0.002	0.113	0.024	0.274	0.223	0.147	0.053	0.325	0.175	1.695	0.990	1.582	0.439	2.548
Molteno	0.008	0.018	0.011	0.008	0.002	0.113	0.015	0.188	0.147	0.134	0.053	0.325	0.111	1.276	0.860	0.857	0.439	2.548
Msikaba	0.003	0.022	0.014	0.014	0.002	0.113	0.006	0.032	0.021	0.021	0.053	0.325	0.044	0.704	0.274	0.274	0.439	2.548
Pietermaritzburg	0.013	0.027	0.017	0.017	0.002	0.113	0.024	0.271	0.221	0.251	0.053	0.325	0.175	1.440	0.792	1.386	0.439	2.548
Prince Albert-1	0.007	0.012	0.007	0.018	0.002	0.113	0.013	0.155	0.119	0.145	0.053	0.325	0.091	1.373	0.843	1.586	0.439	2.548
Prince Albert-2	0.005	0.013	0.007	0.018	0.002	0.113	0.009	0.119	0.088	0.145	0.053	0.325	0.072	0.573	0.481	1.586	0.439	2.548
Tierberg 1	0.010	0.067	0.047	0.008	0.002	0.113	0.019	0.222	0.177	0.155	0.053	0.325	0.135	2.039	1.531	1.772	0.439	2.548
Tierberg-2	0.010	0.016	0.009	0.008	0.002	0.113	0.019	0.225	0.179	0.155	0.053	0.325	0.135	1.758	0.993	1.772	0.439	2.548
Tierberg-3	0.010	0.014	0.009	0.006	0.002	0.113	0.019	0.225	0.179	0.145	0.053	0.325	0.140	1.170	0.858	1.114	0.439	2.548
Volksrust	0.007	0.017	0.010	0.012	0.002	0.113	0.013	0.161	0.124	0.098	0.053	0.325	0.095	0.747	0.440	0.531	0.439	2.548
Vryheid-1	0.004	0.008	0.005	0.006	0.002	0.113	0.007	0.094	0.069	0.084	0.053	0.325	0.054	0.624	0.396	0.623	0.439	2.548
Vryheid-2	0.005	0.013	0.007	0.006	0.002	0.113	0.009	0.128	0.096	0.084	0.053	0.325	0.072	0.775	0.411	0.623	0.439	2.548
Vryheid-3 / -4	0.009	0.019	0.012	0.022	0.002	0.113	0.017	0.201	0.158	0.143	0.053	0.325	0.119	0.948	0.605	1.047	0.439	2.548
Waterford-1 / -2	0.014	0.020	0.013	0.019	0.002	0.113	0.026	0.302	0.248	0.226	0.053	0.325	0.190	2.417	1.342	1.988	0.439	2.548
Whitehill	0.009	0.018	0.011	0.012	0.002	0.113	0.017	0.184	0.143	0.483	0.053	0.325	0.130	1.220	0.869	1.221	0.439	2.548

In this section the problems in using documented data from field studies were highlighted as well as the method by which average values were calculated for the respective formations. The following section will incorporate field studies as well as the national groundwater database search of selected areas.

5.5 Combining Measured and Published Hydraulic Conductivity Values

In South Africa commercial exploitation of groundwater resources are regulated by law to such an extent that production boreholes are required to be registered at the Department of Water Affairs. A recent study conducted at the IGS centred on the application of this data to estimate regional transmissivity values (Figure 5-7). The values were obtained from Groundwater Resource Directed Management (GRDM) Database in which borehole yields were converted to transmissivity values following a method suggested by Hughes and Parsons (Hughes et al., 2007). Another set of data values were obtained from a current research project on climate change within the IGS under the funding of the Department of Water Affairs (Figure 5-8). The two sets of data will be used in conjunction with field results to discuss the evaluation method of estimating regional transmissivity values.

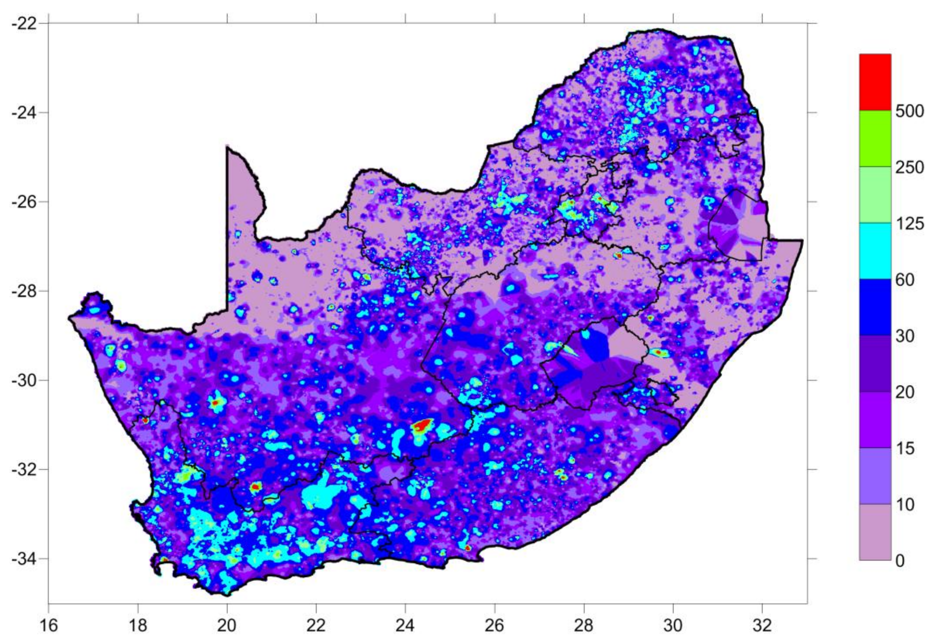

Figure 5-7 Regional transmissivity map as constructed from GRDM data (Hughes et al., 2007).

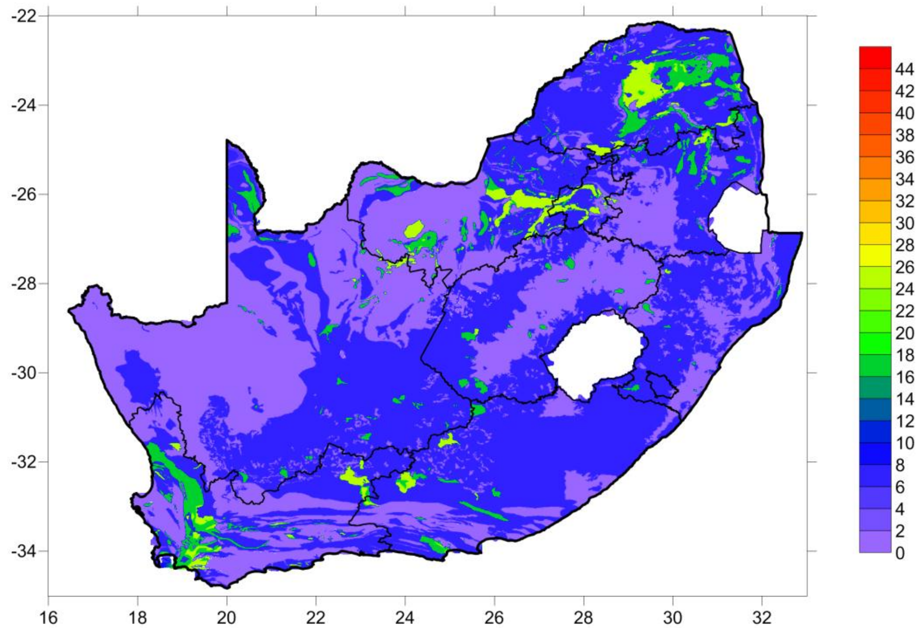

Figure 5-8 Regional transmissivity map as constructed from climate change data.

The difference in the estimated transmissivity values can be clearly observed from the respective scale bars. The maximum transmissivity values for the GRDM model approaches 500 m^2/d while in contrast the climate change model has a maximum of 46 m^2/d. The most significant difference between the two systems is that firstly geology was incorporated in the climate change model as well as a harmonic mean value based on hydraulic tests in the respective areas. Approximately an order difference exists between the two model systems; this is analogous to the difference between the arithmetic and geometric mean values as compared to the harmonic mean assessment from the previous sections.

5.5.1 Krugersdrift Test Site Evaluation

Since it is difficult to evaluate the efficacy of these methods at such a large scale, a recently developed test site near the Krugersdrift Dam was used as a starting point for the evaluation of estimated transmissivity values. Both regional approximations for the transmissivity values are known for the area as well as a set of hydraulic test results. The site location is presented in Figure 5-9 while the hydraulic test data is given in Table 5-8. The estimated regional transmissivity value for this area for the GRDM and climate change models are 15 – 25 and 5 m^2/d, respectively. In the instance of the GRDM model a differentiation in expected transmissivity values was observed, with site A1 having an approximate transmissivity of 25 m^2/d and site A2 a value of 15 m^2/d.

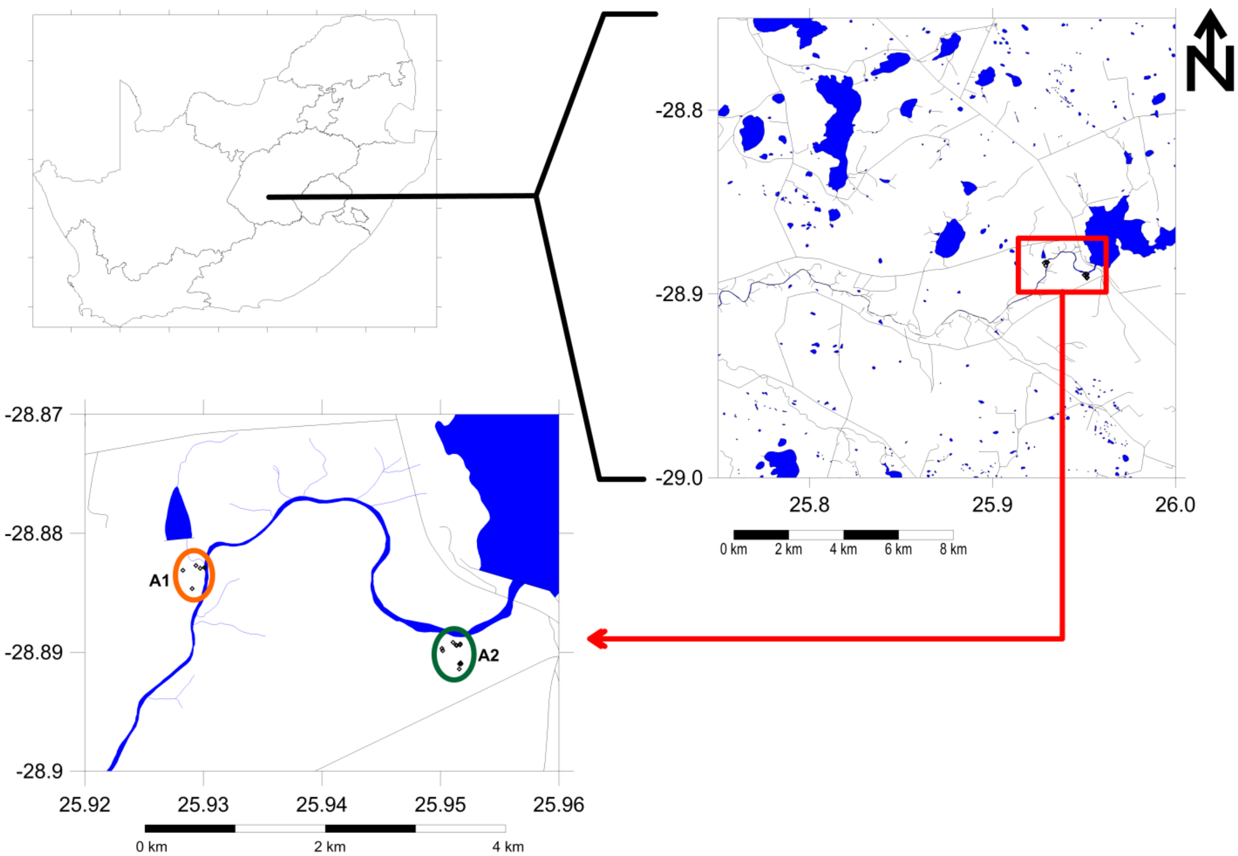

Figure 5-9 Krugersdrift test site located within the Free State province, South Africa. The two areas where hydraulic test data were obtained is indicated in the bottom left hand section as A1 and A2 respectively.

Table 5-8 Selected transmissivity values from hydraulic tests conducted at Site A1 and A2.

Borehole ID	T (m2/d)	Borehole ID	T (m2/d)
A1_1	142	**A2_4**	23
A1_2	0.3	**A2_5**	45
A1_3	164	**A2_6**	87
A1_4	130	**A2_7**	44
A1_5	138	**A2_8**	49
A1_6	0.7	**A2_9**	2
A2_1	65	**A2_10**	269
A2_2	110	**A2_11**	58
A2_3	42		

Considering the hydraulic test data from Table 5-8 a clear spread of transmissivity values for the respective sites can be observed. Site A1 has transmissivity values which range from 164.00 – 0.30 m^2/d, if a regional estimate of the transmissivity value for this area is based on the above observations a harmonic mean value of 1.25 m^2/d would be obtained. If scaled by 30 %, the mean would nominally increase the value to 1.63 m^2/d. This would result in an estimate well below that determined by the GRDM and climate change model, although the latter gave an approximation which is in the same order. Site 2 has considerably more data for the area with transmissivity values ranging from 269.00 – 2.00 m^2/d. If it is considered on its own a harmonic mean value of 15.95 m^2/d is obtained and the scaled value would be 20.74 m^2/d. The value calculated is three times greater than the climate change model but almost exactly the same as that of the GRDM model. Finally, considering all the data the for the two test sites a harmonic mean transmissivity value of 3.10 m^2/d is calculated that approximates the climate change model values.

It is clear from the above that the number of samples considered plays a critical part in the evaluation of the regional transmissivity of the area. Secondly, the inclusion of hydraulic test data with low transmissivity values is critical in determining a balanced regional bulk flow value. As noted previously a quarter of the values obtained should represent the matrix or host formation to give an effective reflection of the regional transmissivity.

5.5.2 Campus Test Site – The Riemann Evaluation

The campus test site has featured in a number of studies in which the IGS has taken part in over the last four decades. The site has been developed with the sole purpose of investigating hydrogeological properties in Karoo type aquifers (Botha et al., 1998, Dennis et al., 2010), as such a number of boreholes have been drilled in a relatively small area (Figure 5-10). Some studies (Van Tonder et al., 2001, Riemann, 2002) probed the influence of fracture zones on the observed transmissivity values obtained. A variety of physical methods were employed, i.e., slug, multi-rate, constant head and constant discharge tests. The results were analysed by various techniques and transmissivity values were reported for both the fracture and the surrounding matrix material.

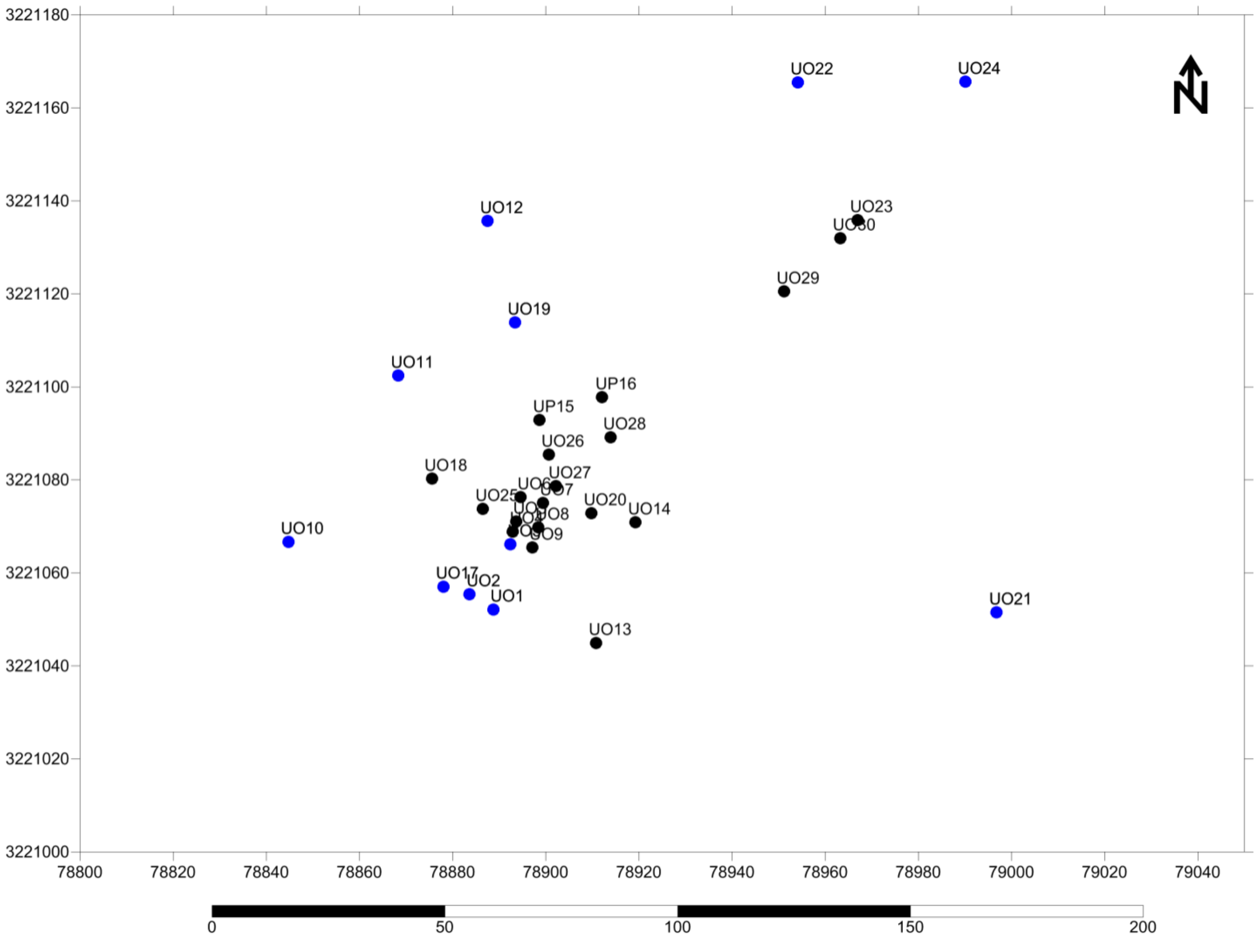

Figure 5-10 Campus test site with borehole locations shown in the map. Blue dots indicate boreholes located in the matrix and black dots represent boreholes that intersect the bedding plane fracture zone (Riemann, 2002).

Riemann (2002) stated that two types of models are typically applied in fractured-rock aquifers which are the single fracture model (Huitt, 1955, Snow, 1965, Wilkes, 1999, Witherspoon et al., 1980) and the double porosity model (Moench, 1984, Moench, 1988, Dougherty and Babu, 1984). It could be deduced from the discussion that each of these models suffered from specific types of shortcomings, most notably an accurate description of the fracture geometry and the interconnectivity network (Riemann, 2002). Due to these deficiencies, porous media methods are still commonly applied in fractured and dual porosity medium aquifers (Kruseman and De Ridder, 1991). To overcome these weaknesses in the analytical models, a numerical model was used to evaluate the aquifer system and determine an approximate transmissivity value (Chiang and Riemann, 2001).

The campus test site is underlain by a series of mudstones and sandstones from the Adelaide Subgroup (Beaufort Group). Mapping of geological outcrops around the campus site reveals the existence of extensional and shearing fractures. The most dominant type of fractures in these sediments are sub-horizontal bedding plane fractures, orthogonal and diagonal fractures with dominant north-west, north-east and east-west trends (Riemann, 2002).

A dominant black shale layer at approximately 13 m below surface level forms a low permeable layer that separates the top mudstone and bottom sandstone layers from each other. The three aquifer systems are present at the campus test site. The top phreatic aquifer occurs within the upper mudstone layers of the campus test site. This aquifer is separated from the middle and main aquifer, which occurs in a sandstone layer between 8 and 10 m thick, by a layer of carbonaceous shale with a thickness of 0.5 m to 4 m thick (Dennis et al., 2010). The bottom aquifer occurs in the mudstone layers which are more than 100 m thick and which is in turn underlain by a sandstone unit (Botha et al., 1998, Riemann, 2002). In each of the boreholes tested in this case study (UO23, 26, 27, 28, 29, 30) the fracture zone occurred from 20 to 24 m below surface. The aperture of the fracture was estimated to be in the order of 2 mm (Riemann, 2002), it should be highlighted that calcification does occur in these fracture zones. Thus the aperture would vary significantly in the area due to the calcification of the fractures zone as well as the irregular distribution of the horizontal bedding plane fracture.

Hydraulic test data analysis of the tested boreholes (UO5, 23, 26, 27, 28, 29, 30) were analysed by means of the Cooper-Jacob method (Kruseman and De Ridder, 1991) as well as a numerical model (NWMP, 2006, Barker, 1988). During the investigation of Riemann boreholes UO5 and UO26 were the main discharge points respectively. Initially none of the boreholes were equipped with piezometers, packers or equipment which could cause preferential flow. This scenario represented the first set of data points. Subsequently, piezometers were installed which focused either on the fracture zone or the matrix formation. In this instance this represents the second scenario in which preferential measurements are made.

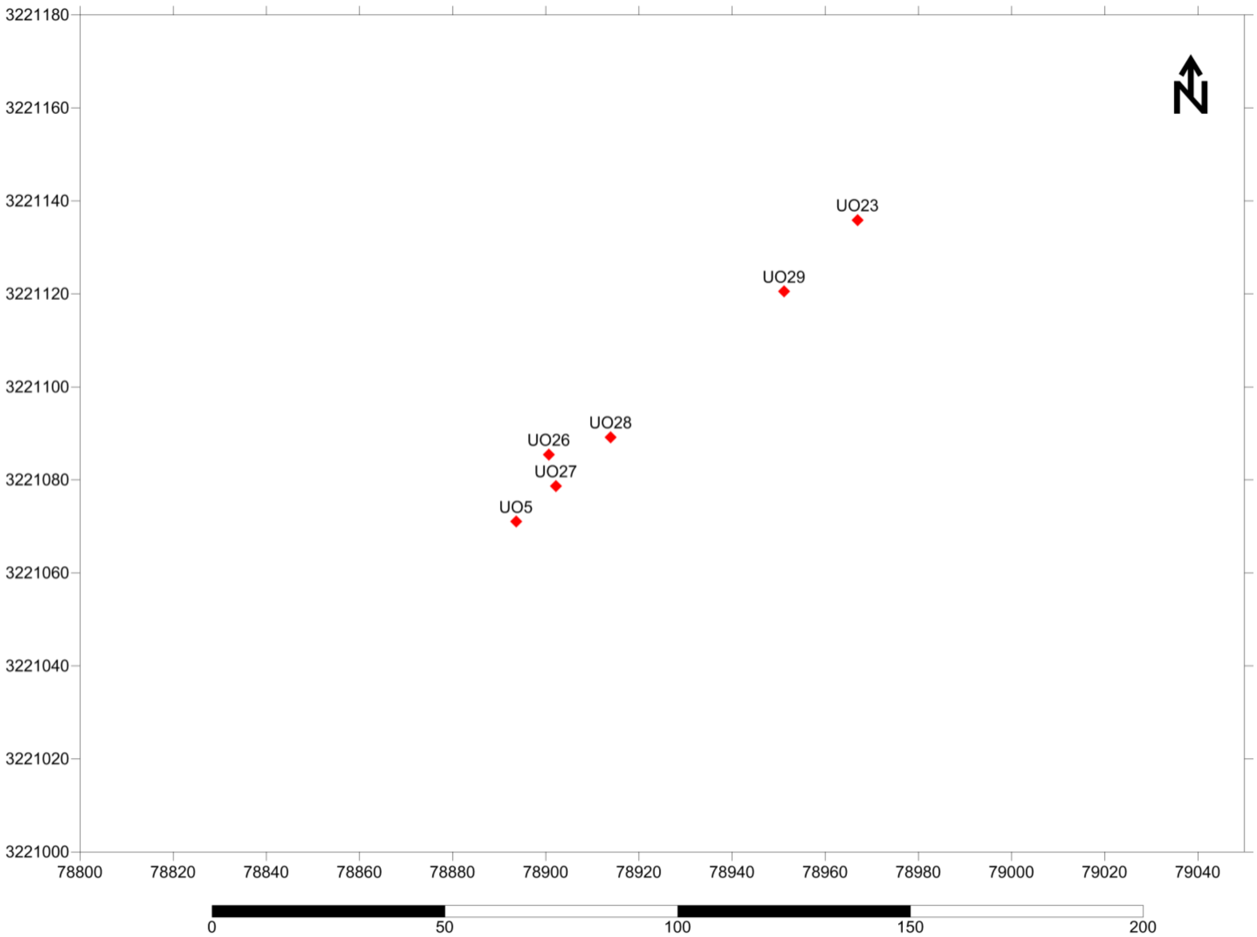

Figure 5-11 Hydraulic test observation points at the campus test site (Riemann, 2002).

Transmissivity values from the initial test phase (no piezometers) are shown in Table 5-9. It is interesting to note that the transmissivity values for both the Cooper-Jacob and Theis methods are nearly similar for the matrix while the estimated fracture zone transmissivity is nearly an order larger. The hydraulic test was performed over a 13 h time period with a constant discharge rate of 0.71 l/s. The boreholes were monitored continuously throughout the test and data fitted to obtain values reported in Table 5-9.

Considering the preceding results obtained, it is expected that the transmissivity of the country rock or matrix formation should be less than 10 m^2/d. However since hydraulic tests represent the mean value of the total aquifer system, the effect of a mean value between these two extreme values in the aquifer is observed. The result is that although a high transmissivity zone exists, the low transmissivity zones eventually control the bulk flow parameters in the aquifer.

Table 5-9 Representative transmissivity values from Riemann (2002) for the September 2000 data set.

Borehole	Cooper-Jacob	Theis	Comment
	T (m^2/d)	T (m^2/d)	
UO26	10.6	10.0	Discharge point
UO27	10.6	10.0	T-values (fracture zone + matrix)
UO28	10.5	10.0	
UO29	10.7	11.0	
Fracture	755.0		Cooper-Jacob II for fracture zone

Similar results were obtained if UO5 was the abstraction point, with only a small change in the estimated total transmissivity observed. However a significant change is observed in the projected transmissivity of the fracture zone (Table 5-10). This could most likely be ascribed to the sensitivity of the Cooper-Jacob II method to observation borehole data in calculating this value.

Table 5-10 Representative transmissivity values from Riemann (2002) for the September 2000 data set.

Borehole	Cooper-Jacob	Theis	Comment
	T (m^2/d)	T (m^2/d)	
UO5	13.4	13.0	Discharge point
UO27c	13.8	14.0	T-values (fracture zone + matrix)
UO23c	13.7	13.0	
Fracture	534.0		Cooper-Jacob II for fracture zone

Riemann (2002) calibrated a model using inverse modelling techniques. The estimated transmissivity values for the matrix and fracture zone were 9.5 m^2/d and 720 m^2/d, respectively. The obtained values are similar to those calculated from the respective hydraulic tests. Interestingly, if the harmonic mean of the two values is taken a mean value of 18.8 m^2/d is obtained which compares favourably with the transmissivity values obtained in Table 5-10.

The second constant discharge test with installed piezometers on the Campus Test Site was conducted at borehole UO5 with a discharge rate of 2 l/s for a period of 24 h. The piezometers in boreholes UO23, UO27, UO28 and UO29 were used to observe the water level in the fracture zone and the rock matrix, respectively. The water levels in boreholes UO6, UO7, UO20, UO26 and UO30 were also measured during the period of the test. The drawdown behaviour in the fracture-piezometers and the boreholes

(Table 5-11) are similar to the previously reported values in Table 5-10. This would indicate that although piezometers were installed only to observe the fracture zone with a high transmissivity value, the influence of the matrix can be detected (Riemann, 2002). Thus, the observed transmissivity value from the hydraulic test is a mean value of both the fracture zone and the formation matrix transmissivity value.

Table 5-11 Results of the evaluated aquifer parameter for the constant rate test UO5, July 2000, obtained from the fracture-piezometers.

Bore-hole	Cooper-Jacob	Theis	Comment
	T (m^2/d)	T (m^2/d)	
UO5	10.6	9.0	Discharge point
UO27c	10.7	11.0	T-values (fracture zone + matrix)
UO28c	10.6	11.0	
UO29c	10.6	11.0	
UO23c	10.6	10.0	
All BH	743.0		Cooper-Jacob II

In all instances in the above discussion the inclusion of bedding plane fracture did not substantially changed the observed transmissivity value. The difference between hydraulic testing an uncased borehole, which results in an average transmissivity value of both the fracture zone and the matrix, and the use of piezometers to exclude the matrix, does not reduce the influence of the matrix formation on the obtained transmissivity value. This would indicate that the fracture network is not extensive or interconnected, and that the water derived from the aquifer is supplied by the formation which has a higher storativity value than the fracture. Riemann (2002) reported various analysis techniques and experimental methods to evaluate hydraulic tests results. However, the transmissivity of the host rock or matrix significantly reduced the calculated transmissivity value. It is most likely that the host rock or matrix has a lower transmissivity value (5 – 8 m^2/d) than that reported by research on the campus study site (Riemann, 2002).

5.6 Conclusion

Random-biased sampling on a regional map showed the effect of directed drilling programmes on hydraulic conductivity estimates. The harmonic mean consistently underestimated the regional hydraulic conductivity; however this method was not prone to sample size and outlier values. In a second approach random biased sampling was done on the regional map, showing the effect of directed drilling programs on the hydraulic conductivity values obtained. The effect of biasing was clearly demonstrated in the mean values obtained and the harmonic mean gave the most sensible results of all the respective mean values. The more biasing that was introduced into the system, the more unlikely it was that an actual mean value for the regional hydraulic conductivity could be determined.

Since a purely theoretical approach was taken in the previous two sections and evaluation of documented average field data was considered. The field data contained three different mean values, i.e., low, middle and high hydraulic conductivity estimates. It was observed that the low hydraulic conductivity values had a remarkable similarity with the theoretical data sets of the previous sections. Nonetheless the higher hydraulic conductivity values of the high estimate indicated that the higher hydraulic conductivity zones are more than three orders greater than the less conductive zones.

Finally, comparisons between transmissivity mean value models and actual field data were done at a test site. The number of boreholes that were required to calculate a representative mean transmissivity value in an area depended on the number of low yielding boreholes that were included. The influence of distance on the transmissivity was also given with the two local sites in the area providing two different transmissivity mean values that differed by more than three times.

This section clearly illustrated that to determine a mean value for an area that is representative of a region; low transmissivity boreholes should be included in the evaluation. If this is not done then the mean value obtained is skewed and represents only the upper stratum of transmissivity or hydraulic conductivity values. This in effect has little use in estimating the regional mean transmissivity or hydraulic conductivity of an area.

Chapter 6 Discussion

6.1 Introduction

In this section the preceding chapters' observations will be combined and discussed in such a manner that a sensible conclusion about transmissivity or hydraulic conductivity can be determined. In general two flow systems are of interest to hydrogeologists. In the first instance (Figure 6-1 A) there is no stress applied to the aquifer (natural flow) and the second depicts a convergent flow field (Figure 6-1 B) when groundwater is abstracted from the aquifer. Natural flow is of interest when an estimate of groundwater fluxes towards is necessary, i.e. a river. In contrast, in the instance of the convergent case, the interest is to estimate the impact of abstraction on the water level or to determine a sustainable yield for a borehole or well-field. Figure 6-1 shows the position of four boreholes, which are located in different hydrogeological environments, and in addition the location of a waste site (denoted by a red dot). The location of different size fractures is indicated and will be used in the following sections.

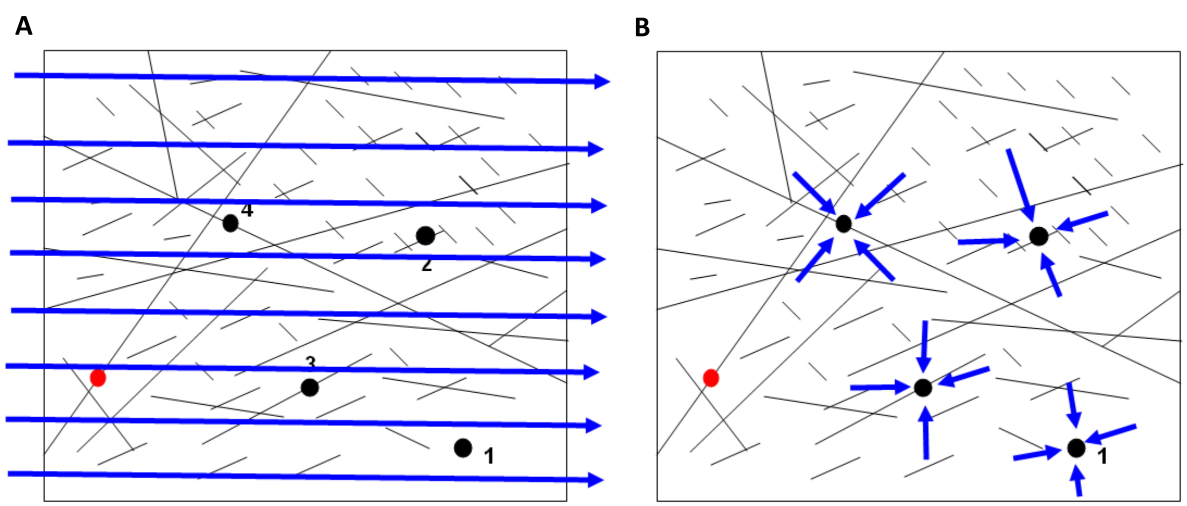

Figure 6-1 Plan views of two typical flow regimes in an aquifer system, (A) natural uniform flow and (B) convergent flow. The black dots represent boreholes and the red dot a pollution source.

In either scenario illustrated in Figure 6-1, the most important parameter to estimate is the hydraulic conductivity (K) or transmissivity (T). For natural flow (Figure 6-1 A), Darcy's Law is used to estimate the flux over the area (see Section 4.2). In the instance of convergent flow (Figure 6-1 B) the sustainable abstraction rate can be estimated with analytical or numerical methods (see Section 2.4). In either instance an estimate of transmissivity or hydraulic conductivity is required and the field methods that can be conducted to estimate these values are typically derived from measurements (slug tests, packer tests, borehole flow meter tests, borehole dilution tests and hydraulic tests).

The most commonly used method to estimate the transmissivity value for each borehole is to perform a hydraulic test with the subsequent analyses of the drawdown data with an analytical model. Assuming that the correct model was used and that the estimated transmissivity values are "correct" (Figure 6-1 A). The boreholes located in Figure 6-1 A are placed in such a method that borehole 1 is located in the country rock or matrix, borehole 2 has a limited fracture network, borehole 3 is a more extensive fracture network and finally that borehole 4 is in a well-connected fracture network. Then the following hypothetical values can be assigned to each of the individual boreholes; T_1 = 1; T_2 = 5; T_3 = 20 and T_4 = 100 m^2/d, respectively.

It then follows that:

- Average transmissivity values can be calculated for the area, i.e., arithmetic mean (33.5 m^2/d); the geometric mean (10 m^2/d) and the harmonic mean (3.5 m^2/d).
- If a lognormal distribution is assumed, i.e., the distribution follows a bell-shape (Gaussian) curve for the logarithmic transformed values, then the geometric mean is 10 m^2/d with standard deviation of 7.1. Typically stochastic modellers use the geometric mean and variation in their models to address uncertainty.
- Upon estimating the ratio of the volume of fracture-influenced material to the total volume of influenced material the estimated transmissivity value always decrease with the affected volume (or time during the hydraulic test). This ratio is a factor of the ratio between the fracture transmissivity (T_f) and the matrix transmissivity (T_m), e.g., Figure 6-2 C and D shows examples of how the volume of material can change with hydraulic test time for different transmissivity value of T_f and T_m. Figure 6-3 illustrates that the volume of fracture material divided by the total volume of material decreases with time and thus the representative transmissivity equals a decrease in value with time in a heterogeneous aquifer.

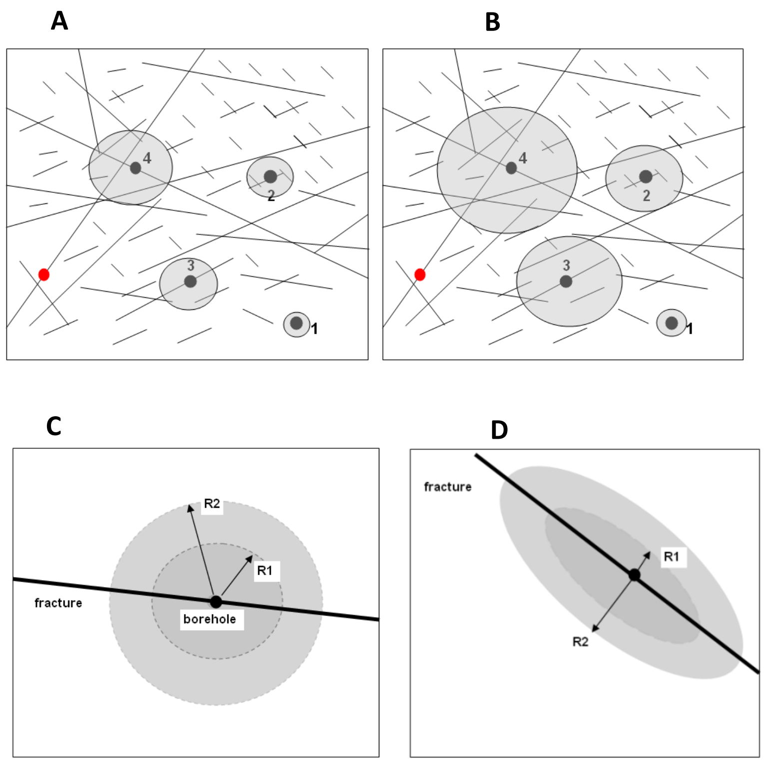

Figure 6-2 Relative two dimensional influence of hydraulic tests at the 4 boreholes with time, (a) short time and (b) longer time. Just from a visual inspection of the location of the boreholes it can be deducted that $T_1<T_2<T_3<T_4$ and (c) increase of volume material as seen by hydraulic test for a homogeneous aquifer and (d) for a heterogeneous aquifer.

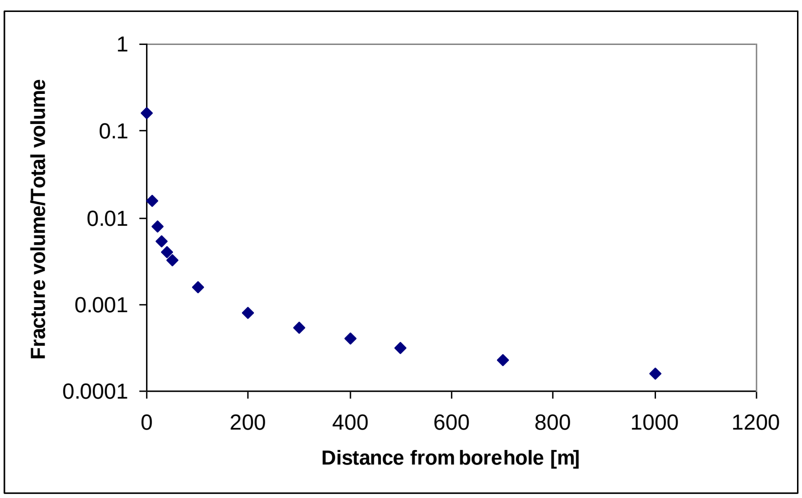

Figure 6-3 Ratio of volume of fracture material to the total volume material with increase in distance from abstraction borehole.

Next consider an oversimplified situation (Figure 6-4) in which a high transmissivity block (T = 1000 m^2/d) was included in an aquifer with a general background transmissivity (T = 20 m^2/d). It is further assumed that the storage coefficient is constant in the area (S = 0.01) and the area is surrounded by a no-flow boundary. Finally the water is abstracted from a borehole situated in the low transmissivity area (T = 20 m^2/d) at a rate of 100 m^3/d. The influence of the large transmissivity zone is extremely small on the representative transmissivity value, which implies that the change in drawdown is also extremely small (Figure 6-5).

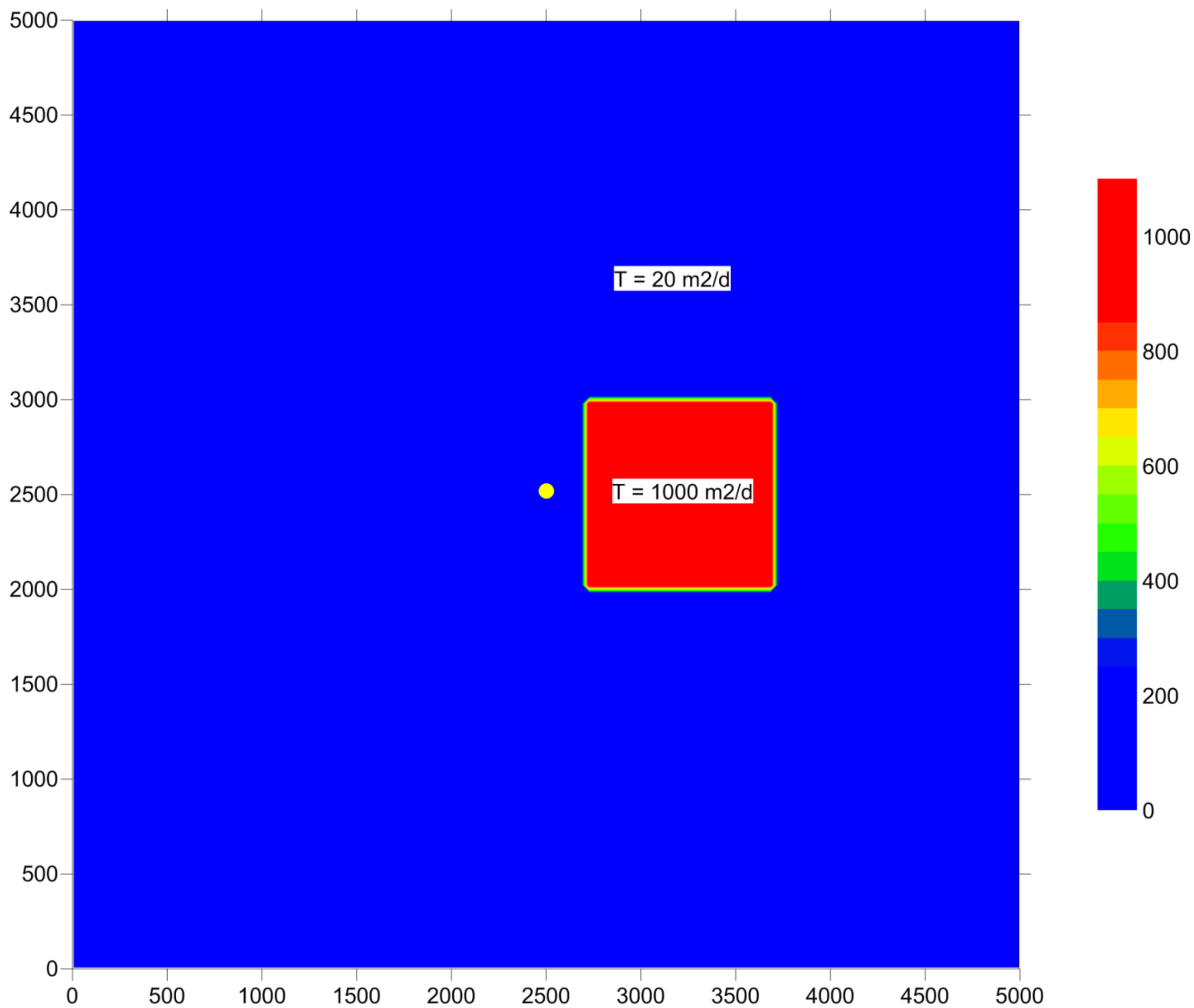

Figure 6-4 High transmissivityzone of 1000 m^2/d situated in an aquifer with transmissivity of 20 m^2/d. The yellow dot shows the position of a borehole.

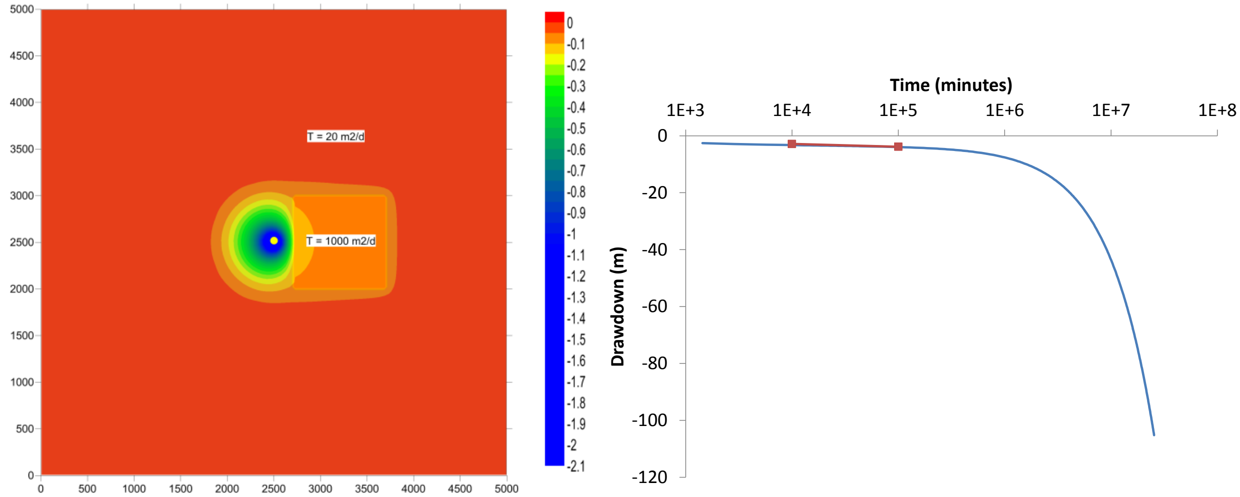

Figure 6-5 The figure on the left shows the expected drawdown in the area after 4 320 days. On the right hand side a drawdown curve from which it can be seen that the high transmissivity block has nearly zero influence on the drawdown with an estimated transmissivity (24 m^2/d) value for the whole area. The influence of the no-flow boundary (increasing drawdown or decreasing water level) can clearly be observed in the latter part of the curve.

Even if four high transmissivity zones (Figure 6-6) are included, the influence on the representative transmissivity is still very small (Figure 6-7).

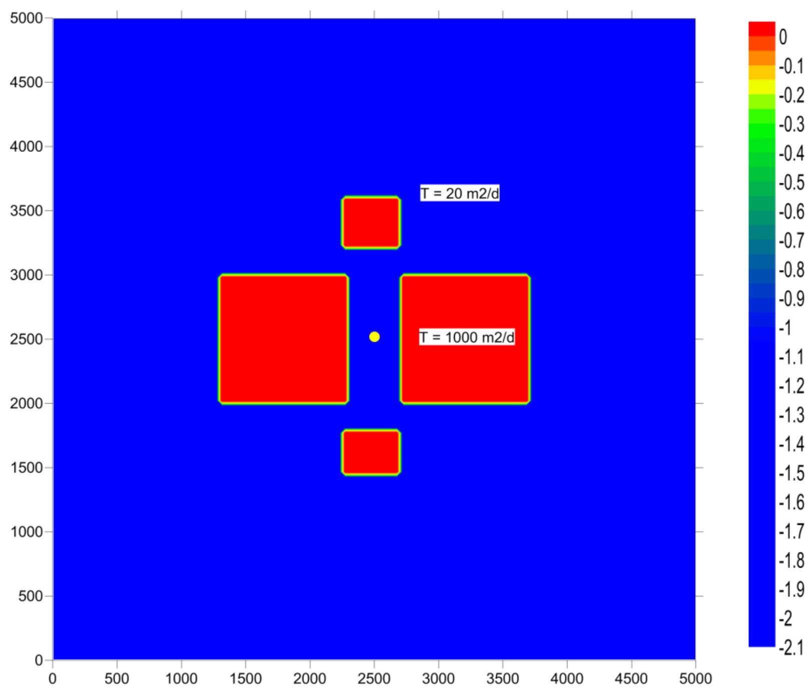

Figure 6-6 Inclusion of four high transmissivity zones in a model area.

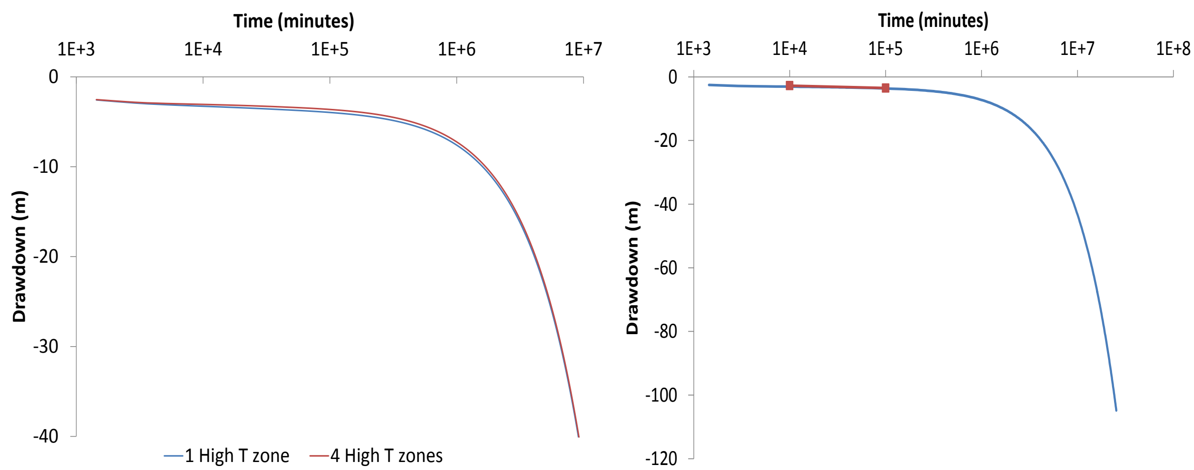

Figure 6-7 Influence of four high transmissivity zones on the drawdown and estimated representative transmissivity value. Fitted transmissivity value is 24 m^2/d.

If the borehole is moved to inside one of the high transmissivity zones of 1000 m^2/d, the representative transmissivity value is 33 m^2/d, which indicates a dramatic decrease in transmissivity with time (Figure 6-8).

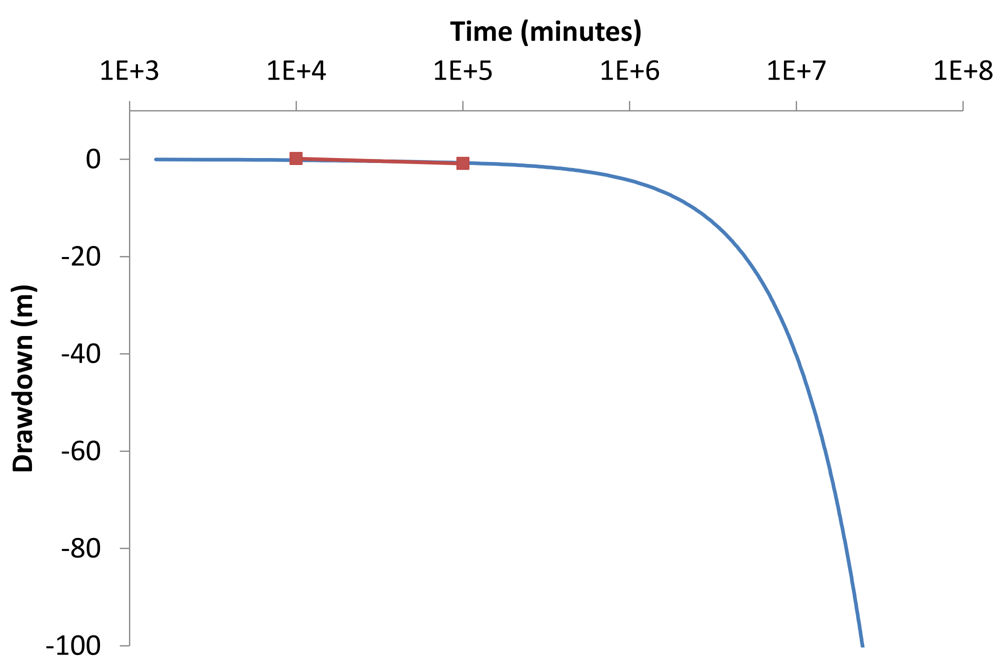

Figure 6-8 Representative transmissivity value = 33 m^2/d if abstraction takes place in one of the four high transmissivity zones of 1000 m^2/d. The influence of the no-flow boundaries can clearly be seen at late times.

In summary the following observation can be made if it assumed that only two points are known in the area:

- The borehole situated in the lower transmissivity zone of 20 m^2/d results in the following averages being observed for the geometric mean (20, 1000) = 141 m^2/d and harmonic mean (20, 1000) = 39 m^2/d, respectively, while the true or representative transmissivity for the area is 24 m^2/d.
- The borehole situated in the high transmissivity zone of 1000 m^2/d results in the following averages being observed for the geometric mean (20, 1000, 1000, 1000, 1000) = 457 m^2/d and the harmonic mean = 92 m^2/d, while the true or representative transmissivity for the area is 33 m^2/d.
- In the cases where abstraction took place in the preceding example, it is evident that even the harmonic mean considering only the highest and lowest transmissivity values (39 m^2/d) is still larger than the estimated representative transmissivity value (33 m^2/d) of the total aquifer as determined by curve fitting methods.

In addition, if it is assumed that all transmissivity values at all points in the area is known and that the four high transmissivity zones are used; then the geometric and harmonic mean of 29 m^2/d and 22 m^2/d can be calculated, respectively. Finally, if it is assumed that there is only one high transmissivity zone in the area (Figure 6-4) and all values are known. Then the geometric and harmonic mean values for the area is equal to 23 m^2/d and 21 m^2/d, respectively. In either instance these values differ from 24 m^2/d or 33 m^2/d depending on the scenario implemented from the proceeding examples.

Of interest to regional-scale resource assessments is understanding whether the representative transmissivity will substantially differ between confined aquifer and water table aquifers. To investigate this effect the horizontal and vertical heterogeneities need to be considered and accounted for in the solution.

6.2 Horizontal Heterogeneity

To illustrate the impact of horizontal heterogeneities on a system, different fracture networks were constructed (Figure 6-9 and Figure 6-10). Fracture characteristics shown on these maps include: fracture length, fracture aperture and fracture connectivity.

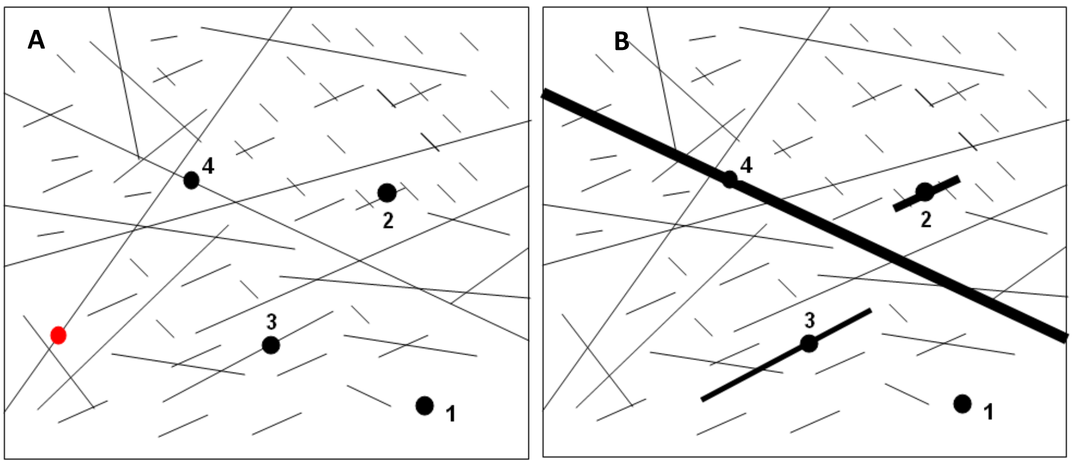

Figure 6-9 (A) Four boreholes drilled at different locations in a 1 km^2 block radius, with the red dot indicating the pollution source and (B) the thickness of the lines is representative of the transmissivity (T) values observed within the area. Borehole 1 has the lowest transmissivity value, while borehole 4 the highest transmissivity value.

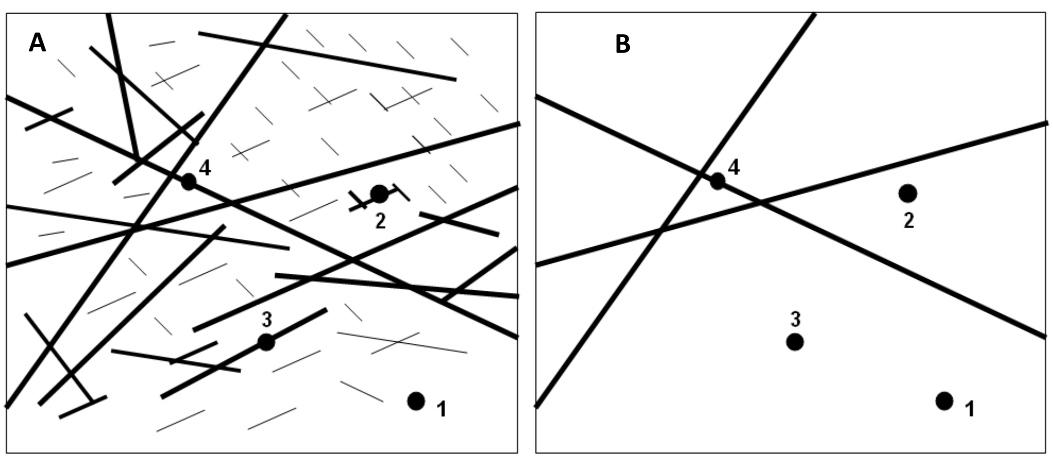

Figure 6-10 (a) Dark lines indicate the variable connectivity-length of the four boreholes in respect to the fractures and (b) fractures connecting the left side of the block to the right side.

A visual inspection of Figure 6-9 and Figure 6-10 indicates:

- As the thickness of the lines represents the apertures of the different fractures, each of the boreholes are drilled into a fracture with a different yield and thus a different transmissivity value:
    - Borehole 1 has the lowest transmissivity value because only the matrix was intersected.
    - The transmissivity values of the other boreholes are in the order: $T_2 < T_3 < T_4$.
- Any irrigation farmer will choose to have a borehole such as borehole 4 because of the more favourable fracture characteristics of this borehole compared to the other three.
- The "internal" boundaries of the fracture, i.e., the effective length (extent) of the fracture and the connectivity to other fractures are greater in borehole 4 thus borehole 4 is superior to the other boreholes in terms of sustainability (ignoring for the moment the depth of the main water strike).
- The transmissivity values in the vicinity of each borehole vary with orders of magnitude (aperture thickness).
- If boreholes 2, 3 and 4 are pumped, the representative transmissivity value of each borehole will decrease with time.
- The fracture that intersects borehole 4 is the most vulnerable to pollution.

Some related questions include: What will happen if borehole 1 is pumped? Will the representative (equivalent) transmissivity increase with time?

In geohydrology, two main questions need to be addressed:

(1) Under natural conditions and assuming that flow is from left to right when considering Figure 6-1 and Figure 6-2, what will be the groundwater flux across the outflow boundary? To estimate this flux, a geohydrologist will have to use some kind of average for the block in Figure 6-1 or divide the block into smaller parallel strips normal to the flow direction and estimate the representative transmissivity of each of these strips and then apply Darcy's Law.

(2) Under converging conditions (transient state abstraction from boreholes), what is the representative or average transmissivity value to use to estimate the drawdown in the borehole after a long time?

Before answering these two questions, one has to investigate the effect of the vertical dimension.

6.3 Vertical Heterogeneity

Figure 6-11 shows a borehole that is intersected by two main water-yielding fractures, i.e., Fracture 1 and 2. Both fractures have the same aperture and thus the same transmissivity value (close to the borehole).

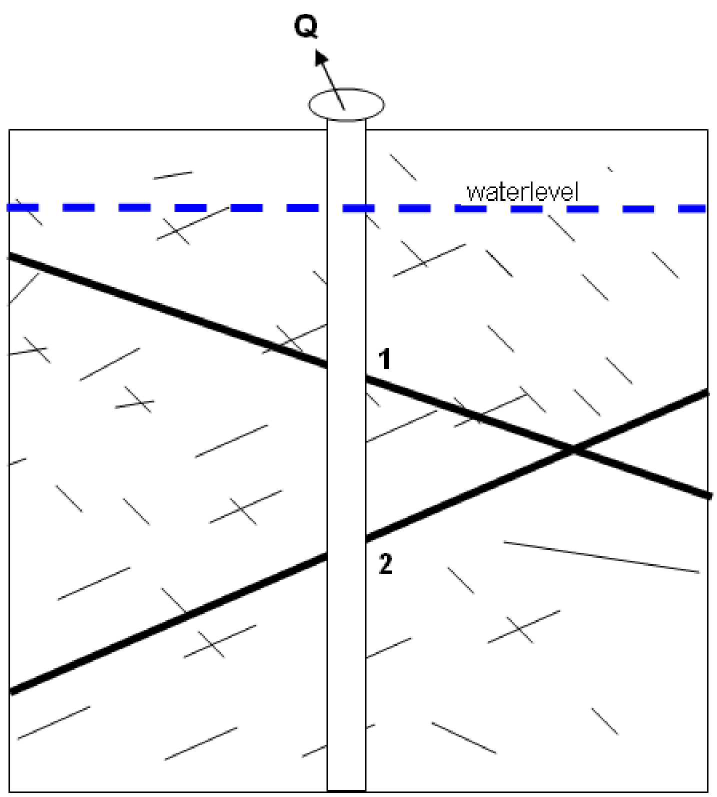

Figure 6-11 Vertical section of borehole that intersects two main water-yielding fractures.

From a visual inspection of Figure 6-11, the following can be deduced:

- If abstraction takes place, fractures 1 and 2 will deliver equal amounts of water to the borehole, thus $T_1 = T_2$ and $T_{total} = T_1 + T_2$ (for early times at least);

- Fracture 1 will be dewatered first and then $T_{total} = T_2$;

- If a farmer has to choose between one of the fractures he would likely choose fracture 2 because it delivers water to the borehole for a longer time than fracture 1. Secondly more drawdown is available;

- It is thus clear that transmissivity (representative) will always decreases with time.

6.3.1 Different Scales

Heterogeneity expressed as transmissivity or hydraulic conductivity is the single most significant feature in fractured-rock aquifers but unfortunately it can vary with orders of magnitude (even more than 12 orders) over very short distances. The estimation of a representative hydraulic conductivity which controls the average behaviour of groundwater flow within an aquifer at a given scale has been the focus of the past few decades (Sanchez-Vila et al., 2006).It is the goal of up-scaling to be able to move from different types of volume. Typically the scale volumes of interest can be defined as the small (< 0.5 m), intermediate (aquifer test scale) and the large (aquifer) scale systems. By using a representative hydraulic conductivity a parameter needs to be estimated that controls the average behaviour of groundwater flow within an aquifer at a given scale.

Three related concepts are defined**:**

- Effective hydraulic conductivity, which relates the ensemble averages of flux and head gradient.
- Equivalent conductivity, which relates the spatial averages of flux and head gradient within a given volume of an aquifer.
- Interpreted conductivity, which is derived from interpretation of field data.

In geohydrological studies that involve complex environmental questions, scale inconsistency is very common. The scale at which transport and flow phenomena in the porous media are best described is usually very different from, i.e., larger than the scale at which measurements are made and interpreted, but also very different from the scale (smaller) required for management decisions. When focusing on the modelling of groundwater flow and transport the following spatial scales are often distinguished: (Bierkens and Van der Gaast, 1998):

- The *pore scale* (10^{-6}–10^{-2} m); the scale at which flow and transport through porous media is described in terms of forces and mass fluxes within the fluid phase and the solid phase and between these phases. Groundwater flow for instance is described by the Navier-Stokes equations;

- The *core scale* (10^{-1}–100 m); the scale at which flow and transport are described in terms of continuity equations and simplified flux equations such as Darcy's law and Fick's law. The minimum scale at which these simplified flux equations are valid is called the representative elementary volume (REV). This is exactly the scale at which measurements of hydraulic properties are performed on samples from drilling cores;

- The *model block scale* (10^{1}–10^{2} m); the scale of blocks or elements of numerical flow and transport models;

- The *local scale* (10^{2}–10^{3} m); the scale at which groundwater flow and -transport is considered as three-dimensional. Examples of local scale groundwater problems are pollution and remediation studies around waste sites and the assessment of travel time distributions in protection areas around drinking water wells;

- The *regional scale* (horizontal dimension 10^{3}–10^{5} m); the scale at which the subsoil is divided into permeable layers (aquifers) and less permeable layers (aquitards). The groundwater flow through aquifers is considered to be mainly horizontal and the flow through the aquitards mainly vertical. The lowering of water tables due to groundwater abstraction is often modelled at this scale. The assumption that the flow in aquifers is predominantly horizontal is valid when the horizontal extent of the model domain is much larger than the vertical extent. This is usually the case for large model areas (horizontal 10^{3}– 10^{5} m) in alluvial and marine deposits (vertical extent of the model domain 10^{2} m).

There is however, one further problem with scaling an average value across all of these scale factors. If only the macroscopic scale is considered, a scale that is large in comparison with the grain or pore size of the porous medium, the flow through the medium can be defined as a continuous phenomenon through space. In general it can also be assumed for dual porosity aquifer systems that the property being observed is also dependent on location (Hubbert, 1956).

The following is a direct quote from the paper but due to its remarkable applicability to the problem at hand it was deemed necessary to reproduce it here (Hubbert, 1956).

"Suppose that we are interested in the porosity at a particular point. About this point we take a finite volume element ΔV, which is large as compared with the grain or pore size of the rock. Within this volume element the average porosity is defined to be

$$\overline{f} = \frac{\Delta V_t}{\Delta V}$$

where ΔV_r is the pore volume within ΔV. We then allow ΔV to contract about the point P and note the value of $\overline{f}$ as ΔV diminishes. If we plot $\overline{f}$ as a functionof ΔV (Figure 6-12), it will approach smoothly a limiting value as ΔV diminishes until ΔV approaches the grain or pore size of the solid. At this stage $\overline{f}$ will begin to vary erratically and will ultimately attain the value of either 1 or 0, depending upon whether P falls within the void or the solid space."

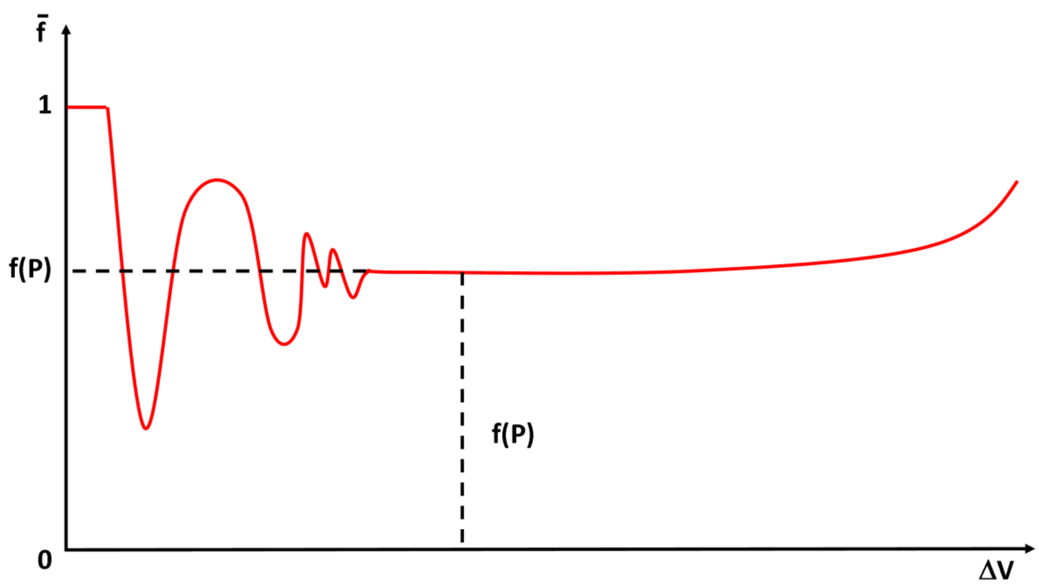

Figure 6-12 Method of defining point values of macroscopic quantities illustrated with porosity f (Hubbert, 1956).

"However, if we extrapolate the smooth part of the curve of $\overline{f}$vs ΔV to its limit as ΔV tends to zero, we shall obtain an unambiguous value of f at the point P. We thus define the value of the porosity f at the point P to be

$$f(P) = extrap \lim_{\Delta V \to 0} \frac{\Delta V_t}{\Delta V}$$

where "extrap lim" signifies the extrapolated limit as obtained in the manner just described."

In the paper it was stated that similar operations for a point value of any other macroscopic property can be obtained. The proof was extended to include lengths, volumes and areas. In a similar method transmissivity of hydraulic conductivity can be defined.

$$Q(P) = \lim_{\Delta T \to 0} \overline{Q}(\Delta T)$$

$$Q(P) = \lim_{\Delta K \to 0} \overline{Q}(\Delta K)$$

6.3.2 Fracture Connectivity

Berkowitz (1995) aimed to describe the significance of fracture network connectivity based on the observation, that seemingly dense fracture networks could have little or no interconnectivity. This would result in highly fractured rocks having small transmissivities and hydraulic conductance capabilities. In his work he further described what he called the backbone of a fracture network. This can be described as being the highways on a high definition road map. If you take away all the urban and industrial dead ends, you are left with the highways that connect point A to point B. The highways, backbone, are the fractures that represent the interconnected fractures that allow the movement of water within an aquifer.

"The estimation of the bulk hydraulic conductivity of the fractured rock at the Mirror Lake site over the increasingly larger dimensions indicates that the connectivity of the fractures is important in characterising the bulk hydraulic properties of the rock. If the highest conductivity fractures are not connected over any significant distance, then the hydraulic conductivity of the fractures that connect the more transmissive fractures are the ones that will control the bulk hydraulic properties of the rock at large dimensions. At other fractured rock sites with different geologic controls on fracture properties, the connectivity of fractures may yield different trends in the estimates of the hydraulic connectivity over increasingly larger physical dimensions." – Figure 6-13 (Shapiro et al., 2007)

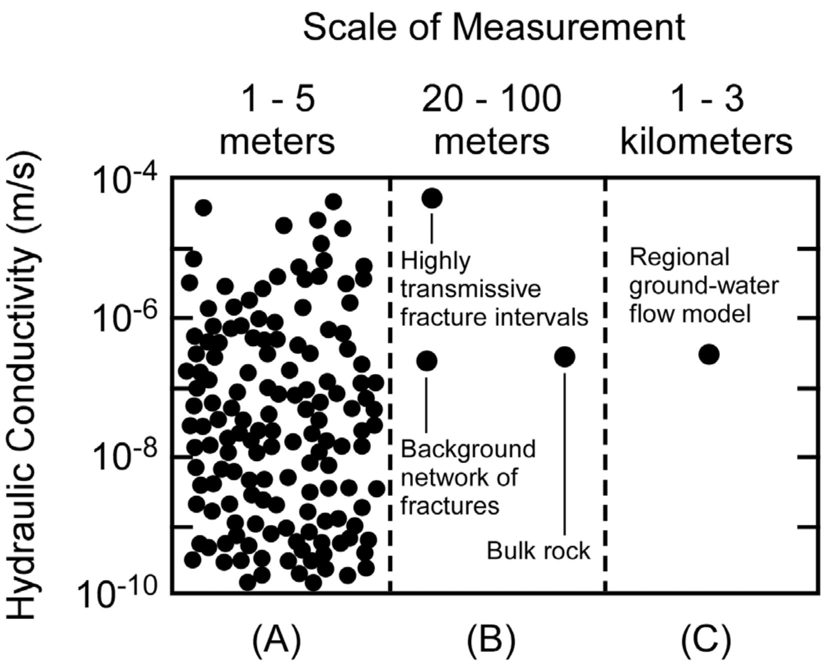

Figure 6-13 Hydraulic conductivity in the fractured bedrock of the Mirror Lake watershed and its vicinity as estimated over increasingly larger physical dimensions from (A) discrete-interval, single hole hydraulic tests, (B) cross-hole hydraulic tests, and (C) regional ground-water flow modelling (Hsieh, 1998).

6.4 Estimation of Representative Transmissivities

In practice, two methods are primarily used to estimate representative transmissivity (or hydraulic conductivity values), namely the 1) stochastic models (expected average value and variance) and the 2) deterministic models (analytical and numerical). Oil engineers are in favour of deterministic methods, while groundwater practitioners prefer the stochastic approach. In the stochastic approach, the representative transmissivity value is known as the effective transmissivity and in the deterministic way it is known as the equivalent transmissivity.

The geostatistical way (stochastic modelling) is used and advocated by a group of (mainly) groundwater experts and includes names like Neuman, Dagan, de Marsily, Sanchez-Villa, Carrera, Matheron, Journel, Gelhar, Kitanidis, Kinzelbach, Rubin, Freeze, Indelman and Gomez-Hernandez. The deterministic group includes mainly experts from the oil industry like Raghavan, Bourdet and Gringarten. Well known experts in the groundwater field who supported the deterministic group are Darcy, Theis, Meinzer, Cooper, Jacob, Todd, Bear and Hunt.

As discussed by Dagan (1997), the choice is always affected by how uncertainty is viewed. Deterministic approaches are based on viewing parameters as unique, but uncertain, local quantities at some given scale. Stochastic approaches are based on viewing reality as one among the ensemble of possible spatial distributions (realisations) of the parameter. In the random method the approach has been to model the natural logarithm $Y = \ln T$ of aquifer transmissivity (T) as a statistically homogeneous, multivariate Gaussian random field with a given variance and spatial correlation function.

From a theoretical standpoint, the groundwater community—which has invested a considerable amount of research in stochastic techniques—has provided very important findings on the impact of aquifer heterogeneity on well hydraulics. The main issue was: "What is the significance of estimated parameters of a heterogeneous aquifer when based on the 'best' fit between a homogeneous model and measured data?" Another related question was: "Is it possible to infer some statistical parameters, e.g. T, K and S, describing the heterogeneity from an aquifer test?" The answers have been on one hand disappointing. Inferring the degree of heterogeneity (variance, covariance function) from well testing is extremely difficult unless a large amount of interference tests are conducted (Noetinger and Gautier, 1998, Sánchez-Vila et al., 1999). On the other hand, these studies have shown that the parameters that are identified by well test interpretation can have a clear theoretical meaning: in many cases they are the equivalent transmissivities of the medium that would be obtained by up-scaling under uniform flow (Meier et al., 1998, Indelman, 2003).

Two related hydraulic conductivity parameters are defined as (i) effective hydraulic conductivity, K(eff), (used by the geostatistical group), which relates the ensemble averages of flux and head gradient and (ii) equivalent hydraulic conductivity, K(eq), (used by the deterministic group), which relates the spatial averages of flux and head gradient within a given volume of an aquifer.

Effective parameter estimation is based on averaging over the ensemble of realisations. Thus the definition of effective hydraulic conductivity is obtained from the generalisation of Darcy's law that results from relating the expected values of specific discharge and head gradient

$$\langle q \rangle = -K(eff)\ \nabla\langle h \rangle$$

where the angle brackets indicate ensemble averaging in the probability space of hydraulic conductivity, that is, averaging all the possible head and specific discharge fields that could be obtained with the ensemble of hydraulic conductivity fields. If the value that relates both expectations, K(eff), exists, it is a second-order tensor and is called the effective hydraulic conductivity. The tensorial nature of K(eff) may be a consequence of statistical anisotropy, boundary conditions, or domain geometry.

Equivalent parameter estimation relies on averaging in physical space. The resulting representative parameter is termed equivalent. Alternative terminologies that are often employed are block-averaged or volume-averaged parameters. Several authors refer to these as up-scaled parameters, since these are usually representative of some average behaviour observed over blocks larger than the support scale. Along these lines an equivalent hydraulic conductivity, K(eq), can be defined by means of an averaged version of the effective K definition given previously:

$$\mathbf{q} = -K(eq)\ \nabla\mathbf{h}$$

Here the letter in bold, refers to averaging in the spatial (volumetric) sense.

Considerable uncertainty has been voiced regarding the potential scale-dependence of hydraulic conductivity.

- Data from many sources seem to imply an increasing hydraulic conductivity, *K*, with increasing scale of measurement (Brace, 1984, Bradbury and Muldoon, 1990, Schulze-Makuch, 1996, Sánchez-Vila et al., 1999).
- Other apparently solid, theoretical derivations (Hunt, 2006) as well as numerical simulations (Hunt, 2006, Paleologos et al., 1996) appear to imply the contrary. Conclusions from some of the experiments have been contested for various reasons, such as whether the borehole intervals were sufficient, (Butler and Healey, 1998) whether turbulence was present in the bore-

holes (Lee and Lee, 1999) or whether juxtaposition of different measurement methods led to interpretive errors (Zlotnik et al., 2000). Thus it is possible that some of the experimental results are spurious.

6.4.1 Mean Parallel Flow: Representative Transmissivity value

6.4.1.1 Stochastic Model to estimate K(eff)

For the stochastic model, it is assumed that the logK value has a Gaussian distribution. In statistics the average of log K is known as the geometric mean.

The equivalent hydraulic conductivity of a flow system is that which would give the same flow rate with the given boundary conditions if the soil were uniform. It can be shown (Youngs, 1983) that the equivalent hydraulic conductivity *K(eq)* of a rectangular block of soil with variable hydraulic conductivity *K(x,* y, z) when flow takes place through it as a result of equipotentials being applied at two opposite faces, lies between the arithmetic mean K_a and the harmonic mean K_h of the conductivities of the soil elements making up the block. Thus

$$K_h < K(eff) < K_a$$

The geometric mean value K_g is often taken as the value to be used in flow computations, since $K_h < K_g < K_a$.

Alternatively the following equation can be used (Youngs, 1983)

$$K(eff) = [(K_a)^2 K_h]^{1/3}$$

For lognormal hydraulic conductivity fields:

$$K(eff) = K_g \exp[(\text{variance of K})/2]$$

Assuming that the univariate distribution of K is lognormal, another alternative equation is:

$$K(eff) = (K_a)^{n-1/n} (K_h)^{1/n}$$

Where n=flow dimension

6.4.1.2 Deterministic model to estimate K(eq)

Equivalent conductivity is heavily influenced by boundary conditions. The most restrictive boundaries to equivalent conductivities are the same than for the stochastic method:

$\mathbf{K_h < K(eq)_{ii} < K_a}$

Where ii = deviation direction from parallel flow.

This implies that the equivalent value for any direction is bounded between the harmonic and the arithmetic mean of the local K values within the block. The difference between this equation and the effective K one, is that now averages are taken in the physical rather than in the probability space.

A combination of limits can be used to obtain a back-of-the-envelope calculation of K(eq) in a given block. The idea is based on an extension of Matheron's equation (1967) to the real space, thus writing

$\mathbf{K(eq) = (K_a)^{n-1/n} (K_h)^{1/n}}$

While this equation is formally equal to the geostatistical one, the conceptual difference is, again, that averages are now taken within the block. Guérillot et al. (1990), used a simple combination of the K_{min} and K_{max} values for K(eq):

$\mathbf{K(eq) = (K_{min})^{0.5} (K_{max})^{0.5}}$

Since the work of Journel et al. (1986) many authors have used a combined analytical-numerical method to estimate equivalent parameters based on a power-averaging formula. Gomez-Hernandez and Gorelick (1989) present the discrete version of a spatial average p norm as an estimator of K(eq) for very large blocks (large enough so that the equivalent value can also be viewed as a pseudo-effective value), K_b according to

$$K(eq) \approx K_b = \left[1/N \sum_{i-1}^{N} K_i^p\right]^{1/p}$$

Where N = number of elements within the large block.

6.4.2 Convergent Flow: Representative T-Value

6.4.2.1 Geostatistical model: 2D

A common approach has been to model the natural logarithm Y = ln(T) of aquifer transmissivity (T) as a statistically homogeneous, multivariate Gaussian random field with a given variance and spatial correlation function. There are many cases when a two-dimensional approach can be considered a proper modelling choice.

Dagan (1989) considered a prescribed flow rate boundary condition at the well and by simple asymptotic considerations concluded that if a well in an unbounded aquifer pumps at a constant deterministic rate, then the T_{eff} near the well is equal to T_H, while T_{eff} far from the well is equal to its mean uniform flow counterpart, i.e., the geometric mean, T_G. Contrariwise, if hydraulic head is prescribed at the well, the pseudo-effective transmissivity at the well should become equal to TA, thus recovering the near-well limit of Matheron (1967). These limits have been verified by subsequent studies and were complemented by closed-form solutions for the space-dependent pseudo-effective conductivity for unbounded (Indelman and Abramovich, 1994) and bounded (Sanchez-Vila, 1997, Riva et al., 2001) domains.

An important assumption for the geostatistical model is that the T_f and T_m must not differ with an order of magnitude. Thus, the assumption is that the flow is predominantly radial in the weak sense. The weak sense is crucial, since this assumption clearly does not hold point wise in the vicinity of the borehole, wherein flow is controlled by the local T values. Away from the borehole, the hydraulic head gradient varies slowly in space, mainly driven by the distance to the borehole.

In the geostatistical model, the effective transmissivity is thus the geometric mean. Since the transmissivity increases proportionally with the support volume, geometric mean-based estimates are rarely correct as real hydraulic test data has shown.

6.4.2.2 Deterministic model: 2D

Cardwell and Parsons (1945) method considers steady state, radial flow to a well in an aquifer in the form of a circle. The method shows that the equivalent transmissivity (T_e) is bounded by the weighted and harmonic means of the transmissivity of different aquifer regions.

$$\frac{\int_\Omega \frac{dx}{r^2(x)}}{\int_\Omega \frac{dx}{T(x)r^2(x)}} \leq T_e \leq \frac{\int_\Omega \frac{T(x)dx}{r^2(x)}}{\int_\Omega \frac{d(x)}{r^2(x)}}$$

Where $\Omega =$ region of aquifer bounded by the outer boundary and the well bore

$r(x) =$ distance from point **x** to the central well, and

$d(x) = r(x)drd\theta$ is an infinitesimal area

Hence they considered a deterministic approach to estimate the transmissivity of a porous medium.

Figure 6-14 illustrates (Sanchez-Vila et al., 2006) the general behaviour of effective and equivalent transmissivity estimates with the stochastic and deterministic methods.

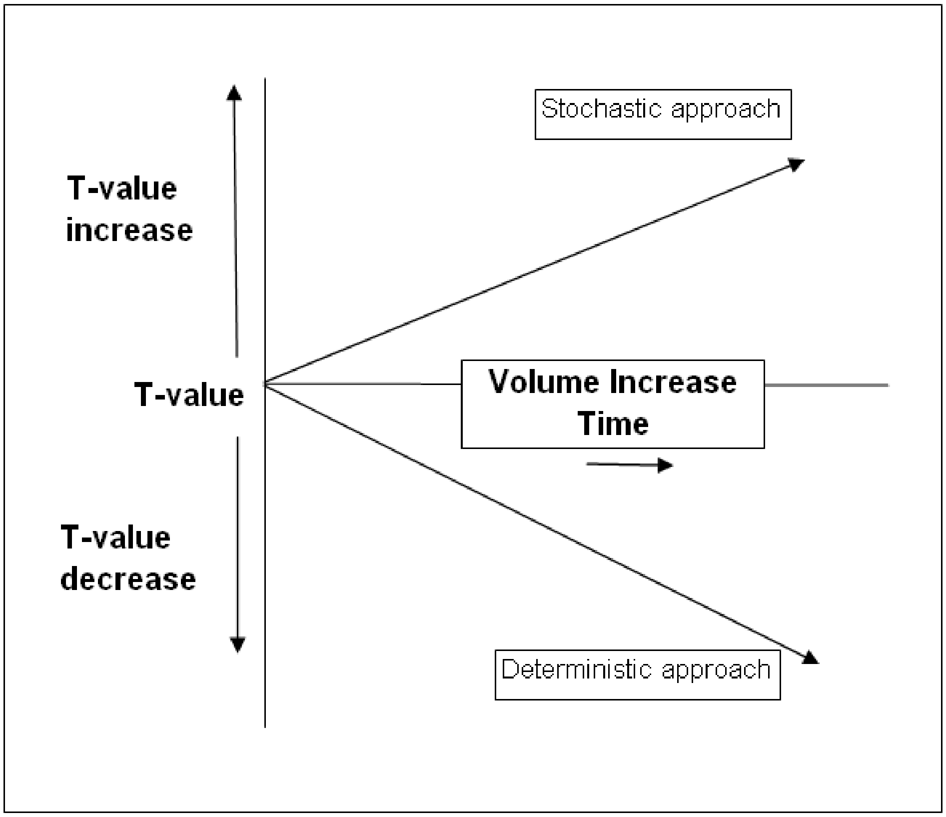

Figure 6-14 General behaviour of effective (stochastic) and equivalent (deterministic) transmissivity estimates with an increase in aquifer volume.

6.4.3 CONVERGENT FLOW: REPRESENTATIVE T-VALUE

The hydraulic tests presents a challenge to interpret when considering derivative plots in that the drawdown eventually stabilize with time which indicates that the derivative will tend towards zero. A second peculiarity is that derivative plots tend to use log-log scale axis. This results in a flattening out of the derivative (s') plot if infinite acting radial flow is encountered. For additional information about these techniques refer to Horne (1995) and Earlougher (1977).

Interpretation of more than 2 000 constant rate hydraulic tests have shown that the representative T always decreases with time, except if the borehole was situated close to a river or a constant head boundary.

A set of examples which include drawdown and derivative results will be presented to graphically illustrate the trends observed in South African aquifer systems (Figure 6-15 and Figure 6-16). It should be kept in mind that a decrease in derivative implies that the T value also decreases.

In Figure 6-15 both the Bethulie (left hand side) and the Boshof (right hand side) drawdown curves no significant change can be observed. However, upon considering the derivatives of these drawdown curves a number of variable time zones become evident. It is clear from the derivative plot that the observed transmissivity of the aquifer changes with time, since an increase and decrease in the derivative is detected. This is most probably caused by the presence of fracture zones that are dewatered during the hydraulic test of the aquifer system as would be expected from typical Karoo aquifer systems. In the Bethulie hydraulic test (Figure 6-15 right hand side) three clearly defined zones as indicated in the derivative plot can be detected. Contrasting this observation the Boshof hydraulic test (Figure 6-15 left hand side) has multiple zones with variable transmissivity regions.

In Figure 6-16 two different aquifer systems are considered, i.e., Limpopo gneiss and Postmasburg dolomitic aquifers. In the instance of the Limpopo gneiss a significant change in drawdown can be observed which can be divided into three zones as indicated by the derivative curve. In the latter part of the hydraulic test the fracture zone has not been dewatered sufficiently to show a loss in connectivity. The loss in connectivity can be clearly observed if one considers the Postmasburg dolomitic aquifer (Figure 6-16). In this instance the drawdown changes significantly in late time and a sharp decline and increase is detected in the derivative plot.

It is clear from the data presented in Figure 6-15 and Figure 6-16 that a change in transmissivity values can be observed during a hydraulic test. The resultant effect is that an average transmissivity value is calculated once curve fitting approach is employed. In addition transmissivity values change over time during a hydraulic test. This viewpoint is contradictory to the geostatistical approach which assumes that transmissivity values are time invariant.

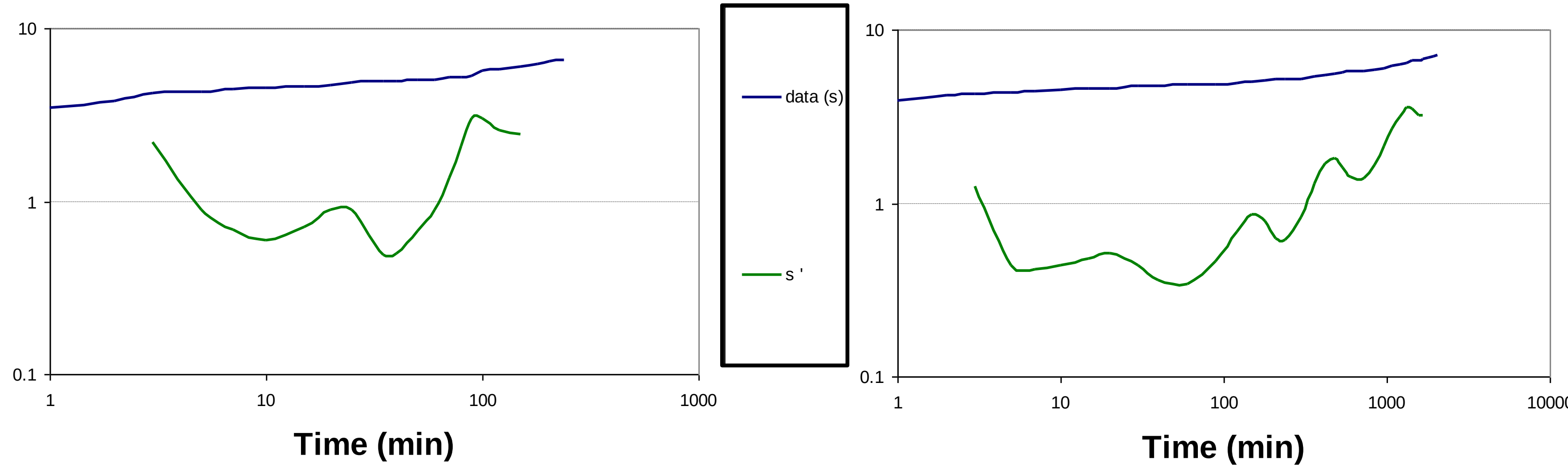

Figure 6-15 Drawdown and derivative plots for boreholes located in the general area of Bethulie (left) and Boshof (right).

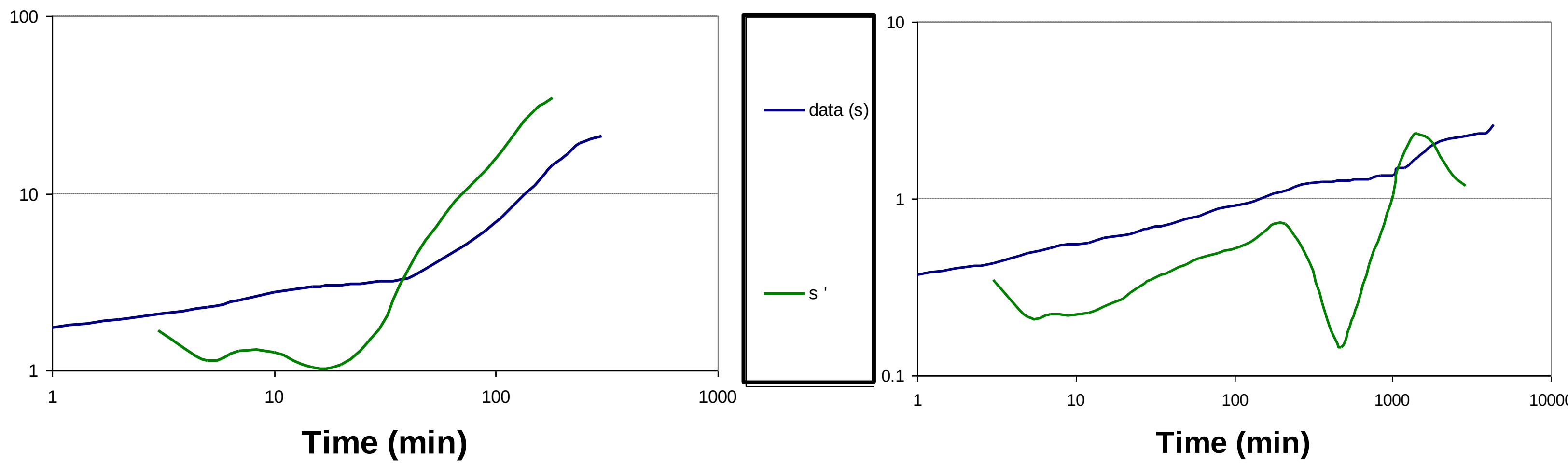

Figure 6-16 Drawdown and derivative plots for boreholes located in the general area of Limpopo – gneiss (left) and Postmasburg – dolomite (right).

6.5 Conclusion

In this chapter the impact of estimating the correct transmissivity value was illustrated from the perspective of local and regional points of view. The influence of high transmissivity zones on a pumping borehole in a low transmissivity zone was clearly illustrated in that no real significant effect could be observed. The estimation of regional transmissivity values are in effect dominated by the location of the borehole in the different transmissivity zones and the observed effective transmissivity value obtained. The impact of horizontal heterogeneities and different fracture networks were discussed and the influence these features have on the actual transmissivity value obtained, i.e., the influence of internal boundaries on hydraulic test data. Scale effects were also addressed from a regional perspective, with a focus on apparent scaling and the actual regional transmissivity value which should be obtained.

The estimation of representative transmissivity values were discussed as seen from a stochastic modelling perspective as well as from the deterministic point of view. A comparison between main stream groundwater and oil industry specialists were noted in which both groups share the fundamental training but differ on the methodology of determining the observed transmissivity values. Finally, the discussion was focused on determining a mean transmissivity value and what is actually meant by this average by each group.

In the following section a roadmap of practices will be given as it should be implemented from a South African perspective.

Chapter 7 Conclusions and Recommendations

South Africa has a variety of aquifer systems that range from porous media, fractured hard rock, dolomitic to dual porosity aquifers. By far the most dominant aquifer systems are located in the Karoo type sediments and are comprised of heterogeneous formations. This in a sense makes South Africa unique in that the water supply to rural communities is drawn from “weak” aquifer systems that are low yielding. The estimation of regional transmissivity values thus carries an additional burden in that if the regional transmissivity values for an area are overestimated, inadequate provision would be made for water supply to the local community. This in turn could cause negative economic and social impacts in these effected communities. Thus in effect knowing the regional transmissivity value could give an estimate of extractable volume of water in an area to establish a continuous water resource. It should be kept in mind that management of aquifer systems not only depends on bulk flow parameters but also on managing the water level in these systems.

In essence using geostatistical methods are not advised if regional transmissivity values are required from a South African perspective. The reason behind this statement is that the distribution of transmissivity values in an area does not follow the basic precepts that are required for these methods to work. In general the values are discontinuous in distribution and statistically skewed. Furthermore the presence of transmissivity areas or points that differ significantly in magnitude, i.e., transmissivity values which differ by more than two orders, can be located within one meter from each other. The explanation for this phenomenon is the presence of dolerite dykes. These create baked-fractured zones with exceptionally large transmissivity values compared to the extremely low transmissivity ranges of the surrounding country rock (shales, mudstone and siltstone).

As noted in previous chapters in which different scenarios were investigated, the calculation of a mean transmissivity value for a region is not a simple matter. Firstly, if databases are used a careful inspection of the reported data is required. In general, in South Africa it is the practice to only report high yielding boreholes, this in effect only represents anomalies from a statistical perspective and thus makes any attempt at estimating regional transmissivity values impossible. It was found in this study that if approximately 25 % to 35 % of reported boreholes in an area were to be low yielding that an estimated transmissivity value could be obtained that represent the regional transmissivity.

A second major point of concern is the lack of data concerning low yielding or "dry" boreholes, these boreholes should be at least tested and noted in a database before being sealed. The information obtained from these boreholes are in effect more valuable to management strategies than most of the current data out in the literature which reports moderate to high yielding boreholes.

Finally, if a mean value is to be calculated, it is strongly advised to use the harmonic mean which overemphasises the contribution of the low transmissivity values in a region. This would result in realistic estimates of available water in an area since the host rock dominates the bulk flow parameters in a region over an extended time period of abstraction. To illustrate in part this point, contemplate the following situation in which the governing flow equation for three-dimensional saturated flow in saturated porous media is mathematically described.

$$\frac{\partial}{\partial X}\left(K_X \frac{\partial h}{\partial X}\right) - Q = S_S \frac{\partial h}{\partial t}$$

Where K_X is the hydraulic conductivity along the major component axis which is assumed to be orthogonal and parallel to the major axes of the main hydraulic conductivity vectors. The piezometric head (h) and volumetric flux per unit volume is represented by the term Q. The specific storage coefficient is defined as the volume of water released or taken up from storage per unit change in head per unit volume of porous material.

Considering the steady state condition ($\frac{\partial h}{\partial t}$ = 0) and that the transmissivity can be expressed as the product of the hydraulic conductivity and the thickness of the saturated zone (T = KD). Then a simplification can be made such that under steady state conditions the following would hold true.

$$\frac{\partial}{\partial X}\left(T_X \frac{\partial h}{\partial X}\right) - Q = 0$$

Thus during the calibration of the numerical model only two main parameters needs to be adjusted, i.e., transmissivity and recharge. In general calibration is performed by specifying initial estimates of recharge/discharge and hydraulic conductivity, and solving the model for steady-state flow conditions. These estimated input parameters were then varied in successive simulations until the steady-state solution most closely matches the hydraulic heads of interest. After calibrating a model to this data the hydraulic tests were simulated with the model. This is defined as the standard procedure for calibrating a groundwater model (Anderson and Woessner, 1992).

As indicated the specified limits for the transmissivity of the respective areas are set. If this is taken into account with the knowledge that only high yielding boreholes are typically reported, then the implication is that the transmissivity values can be as much as two orders greater than that observed from the regional transmissivity value. Since this is the case, the general groundwater contours in an area can be simulated but due to the higher estimated regional transmissivity values a lower groundwater level would be observed (Figure 7-1).

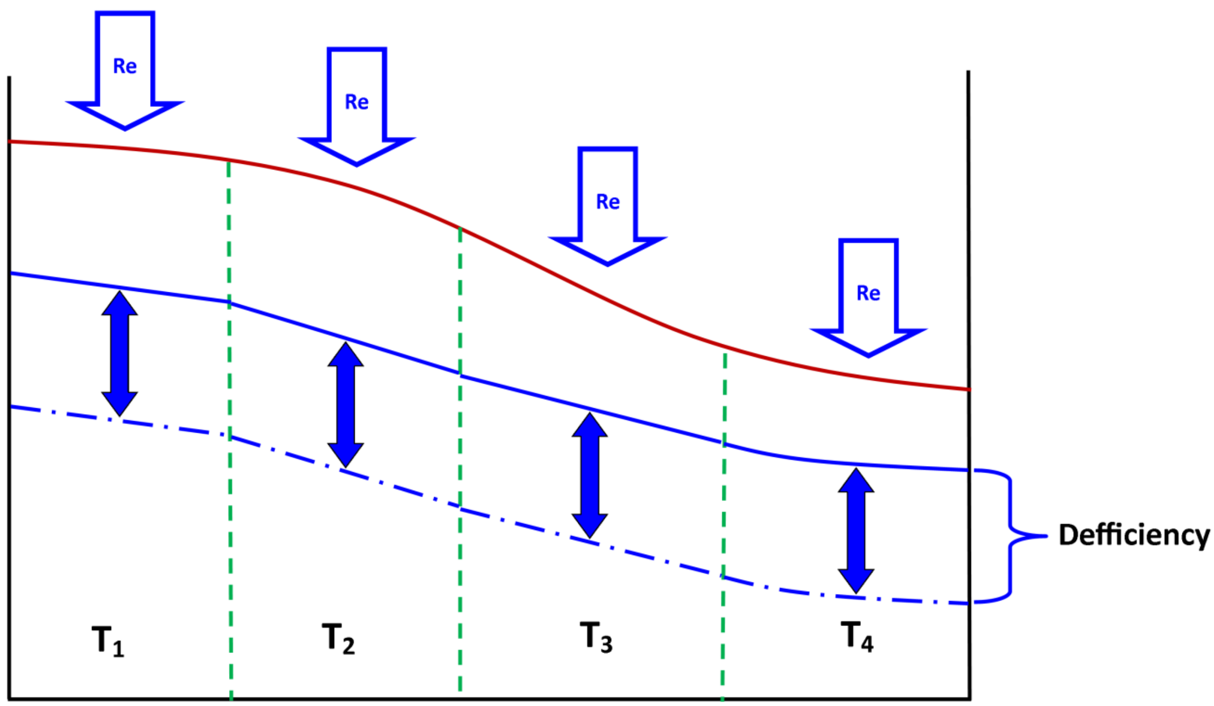

Figure 7-1 Illustrating the effect of transmissivity and recharge on steady state calibration methods.

To counter this reduced water level a higher recharge would be applied to offset this observation. This in turn would then result in a calibrated model in which the recharge would be significantly higher than what would naturally occur. The final knock-on of this modelled system would be that the regional recharge values as well as the transmissivity values would be overestimated resulting in a mismanaged aquifer system. A recent investigation by Van Wyk (2010) indicated that the estimated recharge for

South Africa is far less than the values currently accepted. In the instance of South Africa this could have disastrous consequences since the country experiences periodic droughts which impacts on water demand. Consequently, if models are used to estimate recharge values, a more robust method must be employed to determine actual regional transmissivity values that include low yielding boreholes as a first estimate. Finally, water levels should be monitored and managed to optimally control abstraction rates.

Chapter 8 References

ANDERSON, M. P. 1997. *Characterization of geological heterogeneity, in Subsurface Flow and Transport: A Stochastic Approach,* New York, Cambridge Univ. Press.

ANDERSON, M. P. & WOESSNER, W. W. 1992. *Applied Groundwater Modeling,* San Diego, CA, Academic Press, Inc.

BAKR, A. A., GELHAR, L. W., GUTJAHR, A. L. & MCMILLAN, J. R. 1978. Stochastic analysis of spatial variability in subsurface flow. Part 1: Comparison of one and three dimensional flows. *Water Resour. Res.,* 14**,** 263-271.

BARKER, J. A. 1988. A Generalized Radial Flow Model for Hydraulic Tests in Fractured Rock. *Water Resources Res.,* 24**,** 1796 – 1804.

BEAR, J. 1972. *Dynamics of Fluids in Porous Media.,* New York., American Elsevier Publishing Company, Inc.

BEAR, J. 1979. *Hydraulics of Groundwater. ,* New York, McGraw-Hill, Book Co.

BERKOWITZ, B. 1995. Analysis of fracture network connectivity using percolation theory. *Mathematical Geology,* 27**,** 467-483.

BEVEN, K. J. 2006. A manifesto for the equifinality thesis. *Journal of Hydrology,* 320**,** 18–36.

BIERKENS, M. F. P. & VAN DER GAAST, J. W. J. 1998. Upscaling hydraulic conductivity: theory and examples from geohydrological studies. *Nutrient Cycling in Agroecosystems,* 50**,** 193-207.

BLACK, J. H. Hydrogeology of fractured rocks—a question of uncertainty about geometry. *In:* BANKS, S. B. A. D., ed. Proceedings of the Hydrogeology of Hard Rocks Memoires of the XXIVth Congress of the IAH, 1993 Ås (Oslo), Norway. International Association of Hydrogeologists, 783–796.

BOTHA, J. F. 1988. Interpretation of Two-dimensional Models in Geohydrology. University of the Orange Free State, Bloemfontein: Institute for Groundwater Studies.

BOTHA, J. F. 1994. Models and The Theory of Groundwater Motion. Institute for Groundwater Studies: University of the Orange Free State, P.O. Box 339, Bloemfontein.

BOTHA, J. F. 1996a. Principles of Groundwater Motion. *Unpublished Lecture Notes.* Bloemfontein Institute for Groundwater Studies.

BOTHA, J. F. 1996b. Principles of Groundwater Motion. *Unpublished Lecture Notes.* University of the Orange Free State, P.O. Box 339, Bloemfontein 9300.

BOTHA, J. F. & CLOOT, A. H. J. 2004. Karoo aquifers, deformations, hydraulic and mechanical properties. *Water Research Commission of South Africa.* Pretoria: WRC.

BOTHA, J. F., VERWEY, J. P., VAN DER VOORT, I., VIVIER, J. J. P., COLLISTON, W. P. & LOOCK, J. C. 1998. Karoo Aquifers: Their Geology, Geometry and Physical Behaviour. *Water Research Council of South Africa.* WRC.

BRACE, W. F. 1984. Permeability of crystalline rocks: new in situ measurements. *J. Geophys. Res.,* 89**,** 4327–4330.

BRADBURY, K. R. & MULDOON, M. A. 1990. Hydraulic conductivity determinations in unlithified glacial and fluvial materials. *In:* NIELSON, D. M. & JOHNSON, A. I. (eds.) *Ground water and vadose zone monitoring.* ASTM STP.

BREDENKAMP, D. B., VOGEL, J. C., WIEGMANS, F. E., XU, Y. & JANSE VAN RENSBURG, H. 2007. Use of natural isotopes and groundwater quality for improved estimation of recharge and flow in dolomitic Aquifers. *In:* COMMISSION, W. R. (ed.). Pretoria.

BURGER, C. A. J., HODGSON, F. D. I. & VAN DER LINDE, P. J. 1981. Hidrouliese Eienskappe van Akwifere in die Suid-Vrystaat. *Bulletin 7. Instituut vir Grondwaterstudies*.

BUTLER, J. J. & HEALEY, J. M. 1998. Relationship between pumping-test and slug-test parameters: scale effect or artifact? *Ground Water,* 36**,** 305–313.

CARDWELL, W. T. & PARSONS, R. L. 1945. Average permeabilities of heterogeneous oil sands. *Trans. Am. Inst. Min. Metall. Pet. Eng.,* 160**,** 34–42.

CGS. 2000. *Generalised geology of South Africa*.

CHIANG, W.-H., KINZELBACH, W. & RAUSCH, R. 1998. Aquifer Simulation Model for Windows - Groundwater flow and transport modeling, an integrated program. *In:* BERLIN, G. B. (ed.). Stuttgart.

CHIANG, W. H. & RIEMANN, K. 2001. Guidelines for aquifer parameter estimation with computer models. *Water Research Council of South Africa.* Pretoria: Institute for Groundwater Studies.

COBBING, J. E., HOBBS, P. J., MEYER, R. & DAVIES, J. 2008. A critical overview of transboundary aquifers shared by South Africa. *Hydrogeology Journal,* 16**,** 1207-1214.

CSA 1996. Constitution of the Republic of South Africa. *In:* PARLEMENT (ed.). Pretoria: Republic of South Africa.

DAGAN, G. 1989. Flow and Transport in Porous Formations. Springer-Verlag, New York.

DAGAN, G. 1997. Stochastic modeling of flow and transport: The broad perspective. *In:* DAGAN, G. & NEUMAN, S. P. (eds.) *Subsurface Flow and Transpor.* New York: Cambridge Univ. Press.

DARCY, H. P. G. 1856. *Les Fontaines Publiques de la Ville de Dijon,* Dalmont, Paris.

DAVIS, R., HIRJI, R., RADSTAKE, F. & TUINHOF, A. 2003. Water Resources and Environment. *In:* BANK, T. W. (ed.) *Water Quality: Assessment and Protection.* 1818 H Street, N.W., Washington, D.C. 20433, U.S.A.

DE MARSILY, G. 1986. *Quantitative Hydrogeology - Groundwater Hydrology for Engineers*.

DEKKING, F. M., KRAAIKAMP, C., LOPUHAÄ, H. P. & MEESTER, L. E. 2005. *A Modern Introduction to Probability and Statistics. Understanding Why and How,* London, Springer-Verlag.

DENNIS, I., PRETORIUS, J. & STEYL, G. 2010. Effect of fracture zone on DNAPL transport and dispersion: a numerical approach. *Environ. Earth Sci.,* 61**,** 1531-1540.

DENNIS, I. & WENTZEL, J. 2007. Groundwater resource-directed measures (GRDM) software. *Water SA,* 33**,** 79-86.

DNWRP 2004. Internal strategic perspective: Usutu to Mhlathuze Water Management Area. *In:* PLANNING, D. N. W. R. (ed.). DNWRP, Pretoria, South Africa.

DOMENICO, P. A. & SCHWARTZ, F. W. 1990. *Physical and Chemical Hydrogeology,* New York, N.Y., John Wiley & Sons.

DONDO, C., CHEVALLIER, L., WOODFORD, A. C., MURRAY, R. & NHLEKO, L. 2010. Flow Conceptualisation, Recharge and Storativity Determination in Karoo Aquifers, with special emphasis on Mzimvubu - Keiskamma and Mvoti - Umzimkulu Water Management Areas in the Eastern Cape and KwaZulu-Natal Provinces of South Africa. *Water Research Commission of South Africa.* Pretoria.

DOUGHERTY, D. E. & BABU, D. K. 1984. Flow to a partially penetrating well in a double-porosity reservoir. *Water Resources Research,* 20**,** 1116-1122.

DWAF 2004. National Water Resources Strategy. *In:* FORESTRY, D. O. W. A. A. (ed.) First Edition, September 2004 ed. Pretoria: Department of Water Affairs and Forestry.

EARLOUGHER, R. C. J. 1977. *Advances in Well Test Analysis*, Society of Petroleum Engineers.

FREEZE, R. A. 1985. Historical correspondence between C. V. Theis and C. I. Lubin. *Eos Trans. AGU,* 66**,** 442.

GELHAR, L. W. Effects of hydraulic conductivity variations on groundwater flows. 2nd IAHR Congress, 1976 Lund, Sweden.

GOMEZ-HERNANDEZ, J. & GORELICK, S. 1989. Effective groundwater model parameter values: Influence of spatial variability of hydraulic conductivity, leakance and recharge. *Water Resources Research* 25.

GUÉRILLOT, D., RUDKIEWICZ, J. L., RAVENNE, C., RENARD, G. & GALLI, A. 1990. An Integrated Model for Computer Aided Reservoir Description: from Outcrop Study to Fluid Flow Simulations, Improved oil recovery. *Oil & Gas Science and Technology* 45**,** 71-77.

GUTJAHR, A. L., GELHAR, L. W., BAKR, A. A. & MCMILLAN, J. R. 1978. Stochastic analysis of spatial variability in subsurface flow. Part 2: Evaluation and application. *Water Resour. Res.,* 14**,** 953-960.

HARBAUGH, A. W., BANTA, E. R., HILL, M. C. & MCDONALD, M. G. 2000. MODFLOW-2000, the U.S. Geological Survey modular ground-water model user guide to modularization concepts and the ground-water flow process. *In:* SURVEY, U. S. G. (ed.). Denver, CO, Reston, VA.

HEFNY, K., FARID, M. S. & HUSSEIN, M. 1991. Groundwater assessment in Egypt. *Water Science,* 10**,** 10-21.

HOFFMANN, J. 2005. The future of satellite remote sensing in hydrogeology. *Hydrogeology Journal,,* 13**,** 247–250.

HORNE, R. N. 1995. *Modern Well Test Analysis: A Computer-Aided Approach* California, Petro Way

HSIEH, P. A. 1998. Scale effects in Fluid Flow through Fractured Geologic Media, in Sposito, Garrison. *Scale Dependence and Scale Invariance in Hydrology.* Cambridge, United Kingdom: Cambridge University Press.

HUBBERT, M. K. 1956. Darcy's law and the field equations of the flow of underground fluids. *Journal of Petroleum Technology,* 207**,** 222-239.

HUGHES, D., PARSONS, R. & CONRAD, J. 2007. Quantification of the Groundwater Contribution to Baseflow. . *In:* WRC (ed.) *Water Research Council of South Africa.* Pretoria.

HUITT, J. L. 1955. Fluid Flow in Simulated Fractures. *Amer. Inst. Chem. Eng. Journal,* 2**,** 259-264.

HUMPHREYS, W. F. First in, last out: should aquifer ecosystems be at the vanguard of remediation assessment? Contaminated Site Remediation: From Source Zones to Ecosystems, 4 – 8 Dec. 2000 2000 CSRC. Melbourne, Victoria.

HUNT, A. G. 2006. Scale-dependent hydraulic conductivity in anisotropic media from dimensional cross-over. *Hydrogeology Journal,* 14**,** 499–507.

INDELMAN, P. 2003. Transient well-type flows in heterogenous formations. *Water Resour. Res.,* 39**,** 1064.

INDELMAN, P. & ABRAMOVICH, B. 1994. Nonlocal properties of nonuniform averaged flows in heterogeneous media. *Water Resources Research,* 30.

JACOB, C. E. 1944. Notes on determining permeability by pumping tests under water-table conditions. *In:* SURVEY, U. S. G. (ed.).

JACOB, E. E. 1946. Drawdown test to determine effective radius of artesian well. *Trans. Am. Soc. Civ. Eng.,* 112**,** 1047–1070.

JACQUARD, P. & JAIN, C. 1965. Permeability distribution from field pressure data. *Soc. Pet. Eng. J.,* 5**,** 281–294.

JOHNSON, M. R., ANHAEUSSER, C. R. & THOMAS, R. J. 2006. *The Geology of South Africa*, The Geological Society of South Africa, Johannesburg and the Council for Geoscience, Pretoria.

JOURNEL, A. G., DEUTSCH, C. & DESBARATS, A. J. 1986. Power averaging for block effective permeability. *56th California Regional Meeting of the Society of Petroleum Engineers.*

KIRSCH, R. 2006. *Groundwater Geophysics. A Tool for Hydrogeology,* Berlin Heidelberg, Springer.

KRUSEMAN, G. P. & DE RIDDER, N. A. 1991. *Analysis and Evaluation of Pumping Test Data,* Wageningen, International Institute for Land Reclamation and Improvement, P.O. Box 45, 6700, The Netherlands.

KRUSEMAN, G. P. & DE RIDDER, N. A. 1994. *Analysis and Evaluation of Pumping Test Data,* Wageningen, The Netherlands, International Institute for Land Reclamation and Improvement, P.O. Box 45,6700 AA Wageningen, The Netherlands.

LE, N. D. & ZIDEK, J. V. 2006. *Statistical Analysis of Environmental Space-Time Processes*, Springer Science+Business Media, Inc.

LEE, J.-Y. & LEE, K.-K. 1999. Analysis of the quality of parameter estimates from repeated pumping and slug tests in a fractured porous aquifer system in Wonju, Korea. *Ground Water,* 37**,** 692–700.

MARGAT, J. 1994. *Groundwater operations and management from Groundwater Ecology*, Academic Press.

MATHERON, G. 1967. *Elements pour une the´orie des milieux poreux,* Paris, Masson.

MATLAB 2011. Matlab. Natick, MA: Mathworks Inc.

MEIER, P. M., CARRERA, J. & SANCHEZ-VILLA, X. 1998. An evaluation of Jacob's method for the interpretation of pumping tests in heterogeneous formations. *Water Resour. Res.,* 34**,** 1011–1025.

MEIJERINK, A. M. J. 2007. *Remote Sensing Applications to Groundwater,* Paris, United Nations Educational, Scientific and Cultural Organization.

MEYER, R. 2005. Analysis of groundwater level time series and the relation to rainfall and recharge. *In:* COMMISSION, W. R. (ed.). Pretoria.

MOENCH, A. F. 1984. Double-porosity models for a fissured groundwater reservoir with fracture skin. *Water Resources Research,* 20**,** 831-846.

MOENCH, A. F. The response of partially penetrating wells to pumpage from double-porosity aquifers. International Conference on Fluid Flow in Fractured Rocks, May 16-18, 1988 1988 Atlanta, GA.

MURRAY, R., BAKER, K. & MUSEKIWA, C. 2011. Transmissivity Map of the Main Karoo Basin: The delineation of high-yielding wellfield areas in Karoo Aquifers as future water supply options to local authorities. *In:* WRC (ed.) *Water Research Council of South Africa.* Pretoria: WRC.

NEUMAN, S. P. 1972. Theory of flow in unconfined aquifers considering delayed response of the water table. *Water Resour. Res.,* 8**,** 1031.

NEUMAN, S. P. 1973. Supplementary comments on "Theory of flow in unconfined aquifers considering delayed response of the water table". *Water Resour. Res.,* 9**,** 1102.

NEUMAN, S. P. 1974. Effect of partial penetration on flow in unconfined aquifers considering delayed gravity response. *Water Resour. Res.,* 10**,** 303.

NEUMAN, S. P. 1975. Analysis of pumping test data from anisotropic unconfined aquifers considering delayed yield. *Water Resour. Res.,* 11**,** 329-342.

NEUMAN, S. P. 2005. Trends, prospects and challenges in quantifying flow and transport through fractured rocks. *Hydrogeology Journal,* 13**,** 124 – 147.

NEUMAN, S. P. & DI FEDERICO, V. 2003. Multifaceted nature of hydrogeologic setting and its interpretation. *Rev. Geophys.,* 41**,** 1014.

NEUMAN, S. P., ZHANG, Y.-K. & LEVIN, O. 1990. Quasilinear analysis, universal scaling, and Lagrangian simulation of dispersion in complex geological media. *In:* CUSHMAN, J. H. (ed.) *Dynamics of Fluids in Hierarchical Porous Media.* San Diego, Calif.: Academic.

NOETINGER, B. & GAUTIER, Y. 1998. Use of the Fourier-Laplace Transformation and of Diagrammatical Methods to Interpret Pumping Tests in Heterogeneous Reservoirs. . *Advances in Water Resources,* 21**,** 581-590.

NUTTING, P. G. 1930. Physical analysis of oil sands. *American Association of Petroleum Geologists Bulletin,* 14**,** 1337-1349.

NWMP 2006. nSIGHTS 2.40 User Manual Version 2.1. . *ERMS #530161.* Carlsbad, NM: Sandia National Laboratories.

OLIVER, D. A. 1990. The averaging process in permeability estimation from well-test data. *SPE Form. Eval.,* 5**,** 319–324.

PALEOLOGOS, E. K., NEUMAN, S. P. & TARTAKOVSKY, D. M. 1996. Effective hydraulic conductivity of bounded strongly heterogeneous porous media. *Water Resour. Res.,* 32**,** 1333–1341.

RAGHAVAN, R. 2004. A review of applications to constrain pumping test responses to improve on geological description and uncertainty. *Reviews of Geophysics,* 42**,** RG4001.

RIEMANN, K. 2002. *Aquifer parameter Estimation in Fractured Rock Aquifers using a combination of. Hydraulic and Tracer Tests.* PhD, University of the Free State.

RIGW/IWACO. 1988. *Hydrogeological Map of Egypt, scale 1:2,000,000*.

RIVA, M., GUADAGNINI, A., NEUMAN, S. P. & FRANZETTI, S. 2001. Radial flow in a bounded randomly heterogeneous aquifer. *Transp. Porous Media,* 45**,** 139–193.

ROBINS, N. S., CHILTON, P. J. & COBBING, J. E. 2007. Adapting existing experience with aquifer vulnerability and groundwater protection for Africa. *Journal of African Earth Sciences,* 47**,** 30-38.

ROBINS, N. S., DAVIES, J., FARR, J. L. & CALOW, R. C. 2006. The changing role of hydrogeology in semi-arid southern and eastern Africa. *Hydrogeology Journal* 14**,** 1483-1492.

SANCHEZ-VILA, X. 1997. Radially convergent flow in heterogeneous porous media. *Water Resour. Res.,* 33**,** 1633–1641.

SANCHEZ-VILA, X., GUADAGNINI, A. & CARRERA, J. 2006. Representative hydraulic conductivities in saturated groundwater flow. *Reviews of Geophysics,* 44**,** RG3002.

SÁNCHEZ-VILA, X., MEIER, P. M. & CARRERA, J. 1999. Pumping tests in heterogeneous aquifers: an analytical study of what can be obtained from their interpretation using Jacob's method. *Water Resour. Res.,* 35**,** 943-952.

SCHULZE-MAKUCH, D. 1996. *Facies dependent scale behavior of hydraulic conductivity and longitudinal dispersivity in the carbonate aquifer of Southeastern Wisconsin.* Ph.D., University of Wisconsin.

SCHULZE, R. E., MAHARAJ, M., LYNCH, S. D., HOWE, B. J. & MELVIL-THOMSON, M. 1997. South Africa Atlas of Agrohydrology and -Climatology. *In:* COMMISSION, W. R. (ed.) *Water Research Commission Report.* Pretoria. South Africa.

SHAPIRO, A. M., HSIEH, P. A., BURTON, W. C. & WALSH, G. J. 2007. Integrated multi-scale characterization of ground-water flow and chemical transport in fractured crystalline rock at the Mirror Lake Site, New Hampshire. *In:* HYNDMAN, D. W., DAY-LEWIS, F. D. & SINGHA, K. (eds.) *Subsurface Hydrology: Data Integration for Properties and Processes.* American Geophysical Union Geophysical Monograph Series.

SHVIDLER, M. I. 1966. The source-type solution in the problem of unsteady filtration in a medium with random nonuniformity (in Russian). *Izv. Akad. Nauk. SSSR. Mekh. Zhidk. Gaza,* 4**,** 137–141.

SNOW, D. T. 1965. *A Parallel Plate Model of Fractured Permeable Media.* PhD, University of California.

SOFTWARE, G. 2010. Surfer 9. 9 ed. Golden, Colorado: Golden Software.

STEYL, G. & DENNIS, I. 2010. Review of coastal-area aquifers in Africa. *Hydrogeology Journal,* 18**,** 217-225.

STRELTSOVA, T. D. 1975. Aquifer test design for pumping in the vicinity of a river (Part I). *Civ. Eng. Dept., Univ. Birmingham*.

STRELTSOVA, T. D. 1976. Hydrodynamics of groundwater flow in a fractured formation. *Water Resour. Res.,* 12**,** 405-414.

STRELTSOVA, T. D. & TILLOTSON, A. 1975. Aquifer test design for pumping in the vicinity of a river (Part II). . *Civ. Eng. Dept., Univ. Birmingham*.

SUPPLY, U. W. 2008. P091092, Ministry of Energy, Democratic Republic of Congo.

TARANTOLA, A. 2005. Inverse Problem Theory and Methods for Model Parameter Estimation. Philadelphia.

TARANTOLA, A. 2006. Popper, Bayes and the inverse problem. *Nature Physics***,** 2.

TERZAGHI, K. 1925. *Erdbaumechanik auf bodenphysikalischer Grundlage,* Deuticke, Leipzig.

THEIS, C. V. 1935. The relationship between the lowering of the piezometric surface and the rate and duration of discharge of a well using ground-water storage. *Eos Trans. AGU,* 2**,** 519–524.

TOBIN, V. J. 2008. Water, Environment and Sanitation. *In:* UNICEF (ed.).

TOTH, J. 1967. Groundwater in sedimentary (clastic) rocks. *Symposium on Groundwater Hydrology.* San Francisco, Calif.: Am. Water Res. Assoc.

UNWATER 2006. Coping with Water Scarcity, A strategic issue and priority for system-wide action. *In:* FAO (ed.) *Land and Water Development Division.* Viale delle Terme di Caracalla 00100 Rome, Italy.

VAN DER LINDE, G. P. & VAN BILJON, W. J. Using Flow and Transport Modelling Efficiently for Pump and Treat Systems: Case Study in Fractured Aquifer. Contaminated Site Remediation: From Source Zones to Ecosystems, 4-8 December 2000 2000 CSRC, Melbourne, Victoria.

VAN TONDER, G. J., BARDENHAGEN, I., RIEMANN, K., VAN BOSCH, J., DZANGA, P. & XU, Y. 2001. Manual on Pumping Test, Analysis in fractured – Rock Aquifers. Bloemfontein: Institute of Groundwater Studies.

VAN WYK, E. 2010. *ESTIMATION OF EPISODIC GROUNDWATER RECHARGE IN SEMI-ARID FRACTURED HARD ROCK AQUIFERS.* PhD Full, University of the Free State.

VANDENBERG, A. 1977. Pump testing in heterogeneous aquifers. *J. Hydrol.,* 34**,** 45–62.

WALDER, O. 2008. *Mathematical Methods for Engineers and Geoscientists,* Berlin-Heidelberg, Springer-Verlag

WARREN, J. E. & PRICE, H. S. 1961. Flow in heterogeneous media. *Soc. Pet. Eng. J.,* 1**,** 153–169.

WHYMAP. 2008. *BGR & UNESCO, http://www.whymap.org/, cited 15 Dec 2008*.

WILKES, J. O. 1999. *Fluid Mechanics for Chemical Engineers*, Prentice Hall.

WITHERSPOON, P. A., WANG, J. S. Y., IWAI, K. & GALE, J. E. 1980. Validity of Cubic Law for Fluid Flow in a Deformable Rock Fracture. *Water Resour. Res.,* 16**,** 1016-1024.

XU, Y., WU, Y. & DUAH, A. 2007. Groundwater recharge estimation of Table Mountain Group aquifer systems with case studies. *In:* COMMISSION, W. R. (ed.). Pretoria.

YOUNGS, E. G. 1983. Soil physical theory and heterogeneity. *Agric. Water Manage.,* 6**,** 145-159.

ZEKTSER, I. S. & EVERETT, L. G. 2004. Groundwater Resources of the World and their Use. *In:* EDUCATIONAL, S. A. C. O., 7, PLACE DE FONTENOY, 75352 (ed.) *IHP-VI, Series on Groundwater.* Paris 07 SP, France.

ZLOTNIK, V. A., ZURBUCHEN, B. R., PTAK, T. & TEUTSCH, G. 2000. Support volume and scale effect in hydraulic conductivity: experimental aspects. *In:* ZHANG, D. & WINTER, C. L. (eds.) *Theory modeling, and field investigation in hydrogeology: a special volume in honor of Shlomo P. Neuman's 60th birthday.* Boulder, Colorado: Geol Soc Am Spec Pap

Chapter 9 Appendix

9.1 Software

Processing Modflow Version 5.5.4 (Chiang et al., 1998)was used as a front end for Modflow (Harbaugh et al., 2000). The PCG2 solver was used during all simulations. Random network generation and matrix calculations were performed using Matlab 2011a (Matlab, 2011).

9.2 Model Construction

Standard units used throughout the modelling section are meters and days. Where appropriate grid sizes was adapted to 100 × 100, 1000 × 1000 or 500 × 500 unit areas. A unit length was assumed to be 10 m. The thickness of aquifer in the model was assumed to be 100 m. The simulation was run for the duration of 2160 days or 6 years. An hydraulic gradient (i = 0.001) was induced in the system with the up-gradient (left hand side of the model) starting at a 0 m head value while down-gradient (right hand side of model) section had a value of -0.499 m. The top and bottom boundaries were assumed to be a no flow zones. Specific storage in the model was set to 0.0001 m^{-1}.

9.3 Results and Visualisation

Results obtained from each run were visualized by means of the built-in 2D-Visualizer in Processing Modflow (Chiang et al., 1998). The data was then prepared for the thesis using Surfer 9 (Software, 2010). Where required data was extracted from Processing Modflow and further processed in Excel 2010.

Chapter 10 Abstract / Opsomming

10.1 Abstract

The study describes the effect of calculating a generalised mean transmissivity or hydraulic conductivity value for a region or aquifer system as it pertains to South Africa. Resource determination of an area is usually driven by the determination of the bulk flow parameters, such as hydraulic conductivity and storativity values. At this stage a decision is usually made on the basis of either maintaining the area under natural conditions (no pumping), or an abstraction (pumping) scenario is envisaged. In both instances water levels, hydraulic testing and distribution of the water resources (aquifer) are required. Since it is not possible to evaluate the total area for these parameters certain assumptions have to be made such as that an average bulk flow parameter for an area can be determined. In wide-ranging situations a simple average of observation points is assumed to be sufficient.

A systematic research approach was followed in which a three-step process was used to evaluate methods of calculating these mean values.

In the first instance a conceptual model approach was used, and all bulk flow parameters were generated by means of matrices to represent the natural system. Three typically employed mean values (arithmetic, geometric and harmonic) were calculated for two different dimensional matrices, i.e., N x N (N = 100 and 1000) with different hydraulic conductivity zones. In addition the relative difference between these hydraulic conductivity zones were steadily increased to mimic observed parameters in the field, i.e. typical hydraulic conductivity of shale (K = 0.01 m/d) versus a fracture zone (K = 100 m/d). In all instances the harmonic mean performed the best and as the number of sample sets were increased, a reduction in mean values were observed. As part of the conceptual model approach, two typically encountered scenarios were investigated, i.e. natural flow and forced gradient conditions. Under these two scenario conditions the harmonic mean performed the best to estimate the actual observed hydraulic conductivity value.

Secondly, case studies were presented which highlighted the influence of sample size on observed parameters. Additionally, the effect of the differences between the low and high hydraulic conductivity

zones on the calculated mean value as a function of sample size, was also reported. In all of these case studies the harmonic mean was the closest in approximating the observed hydraulic conductivity. It is evident from this section that the number of host rock (formation) hydraulic conductivity values plays a critical part in the mean value calculation since it is general practice in South Africa not to report low yielding borehole hydraulic test values.

In the third step, the results were discussed in the context of a more general approach to the problem of calculating a regional mean hydraulic conductivity of transmissivity value. The estimation of representative transmissivity values were discussed as seen from a stochastic modelling perspective as well as from the deterministic point of view. A comparison between main stream groundwater and oil industry specialists were noted in which both groups share the fundamental training but differ on the methodology of determining the observed transmissivity values. The impact of horizontal heterogeneities and different fracture networks was discussed and the influence these features have on the actual transmissivity value obtained, i.e. the influence of internal boundaries on hydraulic test data. Scale effects were also addressed from a regional perspective, with a focus on apparent scaling and the actual regional transmissivity value which should be obtained.

The findings of this study are that in essence using geostatistical methods are not advised if regional transmissivity values are required from a South African perspective. The reason behind this statement is that the distribution of transmissivity values in an area does not follow the basic precepts that are required for these methods to work. In general the values are discontinuous in distribution and statistically skewed. Furthermore, the presence of transmissivity areas or points that differ significantly in magnitude, i.e. transmissivity values which differ by more than two orders, can be located within one meter from each other. The explanation of this phenomenon is the presence of dolerite dykes, which create baked-fractured zones with exceptionally large transmissivity values compared to the extremely low transmissivity ranges of the surrounding country rock (shales, mudstone and siltstone). In addition, the lack of data concerning low-yielding or “dry” boreholes is a major source of concern since it influences the calculated mean value to a high degree.

10.2 Opsomming

In hierdie studie word die berekening van 'n algemene gemiddelde transmissiviteit of hidroliese geleidingswaarde bespreek soos dit onder Suid-Afrikaanse toestande beleef word. Dit word gedoen omdat hulpbronbeplanning grotendeels gedryf word deur massavloeiparameters soos hidroliese geleidings en stoorwaardes. Gewoonlik word daar beplanning gedoen deur na die gemiddelde waardes te kyk en deur 'n besluit uit te voer of daar onttrekking in 'n gebied moet plaasvind, al dan nie. In beide gevalle word watervlakke, pomptoetse en verspreiding van waterbronne benodig. Aangesien net selektiewe datapunte beskikbaar is, moet 'n gemiddelde massavloeiwaarde bereken word en oor die algemeen word net 'n gemiddelde waarde van die datapunte bereken.

'n Sistematiese navorsingsbenadering is gebruik deur 'n driestapproses the volg wat verskillende gemiddelde waardeberekeningsmetodes gebruik om die gemiddelde waarde te bereken.

In die eerste geval is 'n konsepsuele model gebruik om massavloeiparameters te skep sodat dit natuurlike toestande naboots. Drie algemene gemiddelde waardes is gebruik, naamlik rekenkundige, geometriese en harmoniese gemiddelde om twee verskillende matriksstelsels te bereken ($N \times N$, $N = 100$ en 1000) met verskillende hidroliese geleidingsones. Verder is die relatiewe verskil tussen die hidroliese geleidingsones ook ondersoek aangesien dit natuurlike stelsels naboots, naamlik skalie ($K = 0.01$ m/d) versus fraktuursones ($K = 100$ m/d). Oor die algemeen het die harmoniese gemiddelde die beste gevaar soos die aantal waarnemingspunte verhoog is. 'n Verder deel van die konsepsuele metode is om die twee algemene stelsels wat in die natuur voorkom, te ondersoek, naamlik natuurlike vloei en onttrekkingstoestande. In beide gevalle het die harmoniese gemiddelde die beste gevaar om die hidroliese geleidingswaarde te bepaal.

In die tweede geval is toetsstudies gebruik om die invloed van monsternemingspunte op veranderlikes te bepaal. Verder is die invloed van die verskille tussen die hoë en lae hidroliese geleidingsones as 'n funksie van monsternemingspunte bepaal. Weereens het die harmoniese gemiddelde die beste gevaar, alhoewel dit duidelik was dat die aantal lae hidroliese geleidingswaardes 'n kritiese impak op die gemiddelde waarde het. Verder word dit ook beskou dat lae opbrengs boorgatdata gerapporteer moet word om hierdie verskynsel teen te werk.

In die derde stap word die resultate oor die algemeen bespreek en hoe gebiedsgebonde gemiddelde hidroliese geleidingswaardes bereken kan word. Die skatting van verteenwoordigende transmissiwiteits-waardes word bespreek van 'n stochastiese sowel as 'n deterministiese benadering. 'n Vergelyking tussen grondwater en olie-industriegebruike word ook uitgelig, aangesien dieselfde onderliggende metodes gebruik word maar verskillende resultate verkry word. Die invloed van horisontale heterogeniteite en fraktuurnetwerke is ook bespreek aangesien dit die hidroliese geleidingsvermoë van die sisteem beïnvloed.

Die bevindings van die studie is in kort dat geostatistiese metodes nie vir gemiddelde waardes van hidroliese geleiding in Suid-Afrika gebruik word nie. Die rede agter hierdie stelling is dat die verspreiding van waardes nie die algemene onderliggende geostatistiese raamwerk volg nie. Oor die algemeen is die waardes se verspreiding diskontinue en statisties assimetries. Verder is daar substansiële verskille in die relatiewe verskil in waardes binne 'n meter of twee van mekaar. Agter hierdie verskynsel is die voorkoms van dolerietgange en plate wat gebakte sones vorm met aansienlike hoë hidroliese geleidings-waardes in vergelyking met die omringende geologiese materiaal. Verder is die tekort aan data vir droë boorgate ook 'n bron van kommer aangesien dit gemiddelde waardeberekeninge beïnvloed.