THE IMPACT OF A CLASSROOM INTERVENTION ON UNIVERSITY STUDENTS' LEARNING IN A MATHEMATICS AND STATISTICS-RELATED SUBJECT

By

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DECLARATION

I declare that the thesis hereby handed in for the qualification Philosophiae Doctor degree in Higher Education Studies at the University of the Free State, is my own independent work and that I have not previously submitted the same work for a qualification at another University/faculty.

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DEDICATION

....to my dear and wonderful parents, Daan and Ria Delport, and my twin sister, Mardi

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ABSTRACT

A growing number of underprepared students are entering higher education institutions and students' underachievement in mathematics is indeed of great concern. Numerous research studies have pointed out that the academic domain of underpreparedness among such students entails a lack of mathematical ability and effective study skills. Research studies also point out that education should focus on "learning how to learn" and that educators in South Africa should turn back to their primary responsibility, which is to teach learners necessary thinking skills. Tertiary institutions, therefore, have the obligation to train future professional students, and the responsibility lies with lecturers to help students become aware of study skills and learning strategies with regard to mathematics-related subjects. A comprehensive look into learning theories and the learning of mathematics and statistics provided the researcher with insight with regard to these aspects. The researcher therefore deemed it necessary to conduct a research study with the aim to meet the need for improving students' academic performance in such subjects. The study reports on the introduction of a classroom learning strategy that was designed to improve students' academic performance in a mathematics and statistics-related subject at the Central University of Technology, Free State.

The study is located within a quantitative paradigm, with some enhancement by means of qualitative information. The study followed a non-equivalent pre-test post-test design involving an experimental group and control group of students. A quasi-experimental approach was used to determine whether the post-test performance of students who were exposed to the classroom learning strategy (experimental group) was higher than that of students who received no classroom learning strategy (control group) in the module *Business Statistics/Statistics II*. With regard to the qualitative mode of study, the researcher conducted a nominal group setting to determine the developmental experiences students found most useful after the implementation of the classroom learning strategy intervention. The quantitative analysis of students' post-test performance showed increases in students' academic performance in the module *Business Statistics/Statistics II*. The results that emerged from the nominal group technique setting also support the effectiveness of the researcher's proposed classroom learning strategy intervention, as it had a positive effect on students' attitudes regarding a mathematics and statistics-related subject.

Keywords: underprepared students, mathematical ability, effective study skills, classroom learning strategy intervention.

SAMEVATTING

'n Steeds groeiende getal onder-voorbereide studente betree die hoër onderwyslandskap en studente se onder-prestasie in wiskunde is inderdaad 'n bron van kommer. Talle navorsingstudies dui daarop dat onder-voorbereiheid onder sodanige studente 'n gebrek aan wiskundige vermoëns en effektiewe studievaardighede insluit. Navorsingstudies wys voorts daarop dat onderrig op "leer hoe om te leer" moet fokus en dat opvoedkundiges in Suid-Afrika na hul primêre verantwoordelikheid moet terugkeer, naamlik om leerders met die nodige denkvaardighede toe te rus. Hoër onderwysinstellings is dus genoodsaak om professionele studente vir die toekoms op te lei, en die verantwoordelik berus by dosente om hierdie studente te help om bewus te raak van studievaardighede en leerstrategieë met betrekking tot wiskunde-verwante vakke. 'n Volledige studie oor leerteorieë en die leer van wiskunde en statistiek het die navorser van die nodige insig in dié spesifieke vakgebiede voorsien. Die navorser het dit nodig geag om 'n navorsingstudie uit te voer ten einde die behoefte vir die verbetering van studente se akademiese prestasie in sulke vakke aan te spreek. Die studie gee die bevindinge weer van 'n klaskamer leerstrategie wat ontwerp is om studente se akademiese prestasie in 'n wiskunde en statistiek-verwante onderwerp aan die Sentrale Universiteit vir Tegnologie, Vrystaat, te verbeter. Die studie is kwantitatief van aard, maar word deur 'n kwalitatiewe komponent ondersteun. 'n Nie-ekwivalente groep voortoets-natoetsontwerp met 'n eksperimentele- en kontrolegroep van studente is gebruik. 'n Quasi-eksperimentele benadering is gebruik om te bepaal of die natoets prestasie van studente in die module *Business Statistics/Statistics II* wat aan die klaskamer leerstrategie blootgestel is (eksperimentele groep), hoër was as die studente wat nie aan die klaskamer leerstrategie blootgestel is nie (kontrole groep). Die navorser het, met betrekking tot die kwalitatiewe been van die studie, van 'n nominale groep tegniek sitting gebruik gemaak om die ontwikkelende ervarings wat studente die nuttigste gevind het ná die implementering van die klaskamer leerstrategie intervensies, te bepaal. Die kwantitatiewe analise van studente se natoets prestasie het toenames in die studente se akademiese prestasie in die module *Business Statistics/Statistics II* getoon. Die resultate wat voortvloei uit die nominale groep tegniek sitting ondersteun die effektiwiteit van die navorser se voorgestelde klaskamer leerstrategie intervensie, aangesien dit 'n positiewe effek op studente se houdings ten opsigte van 'n wiskunde en statistiek-gebaseerde vak gehad het.

Kernwoorde: ondervoorbereide studente, wiskundige vermoë, effektiewe studievaardighede, klaskamer leerstrategie intervensie.

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LIST OF ACRONYMS

AEC	Australian Education Council
ANCOVA	Analysis of covariance
ANOVA	Analysis of variance
ATKV	Afrikaanse Taal- en Kultuurvereniging
CME	Concerned Mathematics Educators
CUNY	City University of New York
CUT	Central University of Technology
DA	Deep approach
DM	Deep motive
DoE	Department of Education
DoE&S	Department of Education & Science
DS	Deep strategy
DST	Department of Science and Technology
HE	Higher Education
HEI	Higher Education Institution
HEIs	Higher Education Institutions
HG	Higher Grade
ITS	Integrated Tertiary Software
LG	Lower Grade
LTM	Long-term memory
NCTM	National Council of Teachers of Mathematics
NGT	Nominal Group Technique
NPHE	National Plan for Higher Education
NQF	National Qualification Framework
NRC	National Research Council
ODR	Office of District Report
R-SPQ-2F	Revised Study Process Questionnaire
RSA	Republic of South Africa

SA	Surface approach
SAPA	South African Press Association
SAQA	South African Qualifications Authority
SAS	Statistical Analysis Software
SG	Standard Grade
SM	Surface motive
S-R	Stimulus-response
SS	Surface strategy
TIMSSR	Third International Mathematics and Science Study-Repeat
UFS	University of the Free State
UOT	University of Technology
ZPD	Zone of proximal development

CHAPTER 1: ORIENTATION AND BACKGROUND

1.1 INTRODUCTION

This study is concerned with university students' underachievement in a mathematics and statistics-related subject. The researcher attempted to develop a classroom learning strategy, aimed at improving students' academic performance in a mathematics and statistics-related subject at the Central University of Technology (CUT), Free State. The primary purpose of the research was to determine if the use of a classroom learning strategy intervention could positively affect students' academic performance and learning approaches in a mathematics and statistics-related subject.

The persistent problem of poor academic performance of many students at primary, secondary and tertiary level, particularly in mathematics, is disturbing. The mathematics pass rate is a great concern worldwide, and even more so in South Africa. The Third International Mathematics and Science Study-Repeat (TIMSSR) in 1999 revealed that students from South Africa achieved the lowest performance among the 38 countries that participated in this study [Howie (in Maree, Louw & Millard 2004:25)]. Although these figures are disconcerting, they came as no surprise to those involved in undergraduate higher education.

Research by Ferrini-Mundy and Gaudard (1992); Frith, Frith and Conradie (2006) and Hooper (2006) indicate that first-year students' mathematical skills have progressively become a focus in mathematics educational research. Bohlmann and Pretorius (2002:196) purport that the conceptual complexity and problem-solving nature of mathematics make extensive demands on the reasoning, interpretive and strategic skills of students. The poor schooling system in mathematics results in tertiary students who underachieve in mathematics and need to be supported in the necessary skills required to master this subject. This means that students must overcome a surface approach to learning, which is associated with rote learning or memorising, and follow a deep approach to learning, to achieve academically well in mathematics. According to White and Kitchen (1991:523), students with moderate to severe developmental lag also need to be taught how to process and store information. These students are in dire need of a repertoire of learning approaches, strategies and methods in order to cope with the demands of tertiary education (De Boer & Van Rensburg 1997:160). Abrams and Jernigan (in Potgieter & Webb 2004:313) further add that the responsibility for providing effective intervention strategies lies with higher education institutions to help them with the retention of underprepared students.

To address this concern, study skills as part of an academic assistance program should be designed, in which students develop an inventory of study strategies (Cukras 2006:194). Academic proficiency includes effective study strategies, therefore the focus in this research is to familiarise underprepared students in mathematics and statistics with skills relative to learning how to learn.

In Chapter 1 the researcher presents an introductory overview of students' underpreparedness in mathematics. As such, underpreparedness is defined, factors that contribute to underpreparedness in mathematics are outlined and the importance of mathematics in HE is spelt out. The researcher also presents a view on statistics, as a learning area within mathematics instruction, and highlights the importance of statistical thinking. This particular study aims to improve the mathematical and statistical performance of third-year students in a multi-campus university environment. The problem statement is outlined, whilst the research questions, research aims, objectives, and some theoretical statements related to the study, are stated. The research design and methodology to be implemented are discussed, which includes aspects such as the literature review, research design, population and sampling, data collection, data analysis, reliability and validity, ethical considerations and evaluation of the proposed learning method for the CUT as a multi-campus university. Certain concepts which the researcher regards to be the most important elements of the study are also defined in this chapter.

1.2 BACKGROUND

A growing number of underprepared students are entering higher education, which is evident from increasing numbers of students who enrol in remedial, developmental and extended curriculum programmes, as well as a rise in dropout rates. In South Africa, the majority of students are underprepared for higher education, as recent statistics reveal that approximately one in every three students entering higher education in South Africa will have dropped out by the end of their first year of study Groenewald (in De Klerk, van Deventer & van Schalkwyk 2006:149). Figures from the 2001 publication of the National Plan for Higher Education (NPHE) set the drop-out rate at about 25% for "first-time entering" students (DoE 2001:3).

1.2.1 Underpreparedness defined

Miller, Bradbury and Pedley (1998:104) argue that, in educational literature, the term "underpreparedness" is widely used and "represents a distinct group of students". Dzubak (2005:1) describes the "underprepared" as "...a diverse group of students with different levels of aptitude, as well as different educational and socio-economic backgrounds". According to Anderson (2004:3), the underprepared student in HE is delineated as the student who enrols with considerable academic and societal difficulty associated with weak academic skills.

1.2.2 Underpreparedness in Mathematics

The underpreparedness of students in mathematics is a contributing factor to student's dropping out of HEIs, as confirmed by Moore (in Brussouw 2007:139). Maree and Schoeman (in Steyn & De Boer 1998:125) point out that underachievement in mathematics is noticeable for all learners in South Africa, especially black learners who perform more poorly in mathematics than do learners from other population groups. A reason for this may be that the education system in South Africa is embedded in a

Western culture, which includes aspects that are unfamiliar to most African cultures (De Boer & Van Rensburg 1997:159).

Sapa (*News-Education* 2009:2) reported that the 2008 mathematics results did not necessarily reflect real improvement in mathematics education in South Africa. The Concerned Mathematics Educators (CME) said in a statement that "[t]he mathematics targets achieved by the Education Department should not be misconstrued as indicators of real improvement in mathematics education in South Africa" (*News-Education* 2009:2). Co-ordinator of the group, Aslam Mukadam, said that although the department had met its mathematics targets in eight out of the nine provinces, the CME questioned what benefits this had in terms of the quality of passes. Mukadam said the CME believed that the final mathematics exam was of a lower standard and therefore widened the gap between school and university for top students. Mukadam (2009:2 of 5) argues that "[i]f this standard is going to be used as a benchmark for future examinations, it will not adequately prepare young students to study mathematics-related courses at university level." Mukadam (2009:4 of 5) purports that even students who passed matric mathematics above the 50 percent mark were not necessarily adequately prepared to cope with mathematics-related courses at tertiary institutions. Mr Johan de Koker (departmental head of the civil engineering technology department at the University of Johannesburg), emphasised in an article entitled "Ingenieurs-opleiers wantrou hoë wiskunde-punte" that the mathematical comprehension of learners is not on the same level as that indicated by matric symbols (*Die Volksblad* 2009a).

Mukadam (2009:4 of 5) argues that approximately 60 percent of the students failed mathematics in the year 2008. For Mukadam (2009:4 of 5), "[t]his stands in stark contrast to the extremely high pass rate of 78.8 percent in mathematical literacy. It is therefore easy to conceive that students will now opt for what is perceived as the simpler alternative to mathematics, viz. mathematical literacy".

According to the *Sowetan* (2007:1), one of our greatest failures over the past thirteen years is that our education system is not up to scratch. In his State of the Nation speech in February 2009, President Mbeki said that our matric mathematics pass rate had hardly improved in ten years (*Sowetan* 2007:1).

According to Alet Rademeyer (*Die Volksblad* 2009e:1), learners in mathematics struggle even with "...the conversion of simple fractions, decimals, percentages, and also rounding off of numbers". She is of the opinion that these learners have not yet developed more complex cognitive skills.

1.2.3 Factors that contribute to students' underpreparedness in mathematics

Why do so many students experience problems with mathematical subjects? Many mathematics teachers and researchers in the education community have been puzzled by this question for many years and the debate still continues around the underpreparedness of students in mathematics. This is evident from many previous research studies, which include numerous articles and publications.

Maree, Louw and Millard (2004:25) argue that historically disadvantaged students in South Africa often underachieve in mathematics due to a number of factors. According to Sharwood (1998:251), pre-technician students "lack a solid grounding in mathematics and science... due to ...inadequate schooling" characterized by "underqualified teachers" and also notes that mathematics is a particularly high-risk subject in South Africa. The then minister of education, Ms Naledi Pandor argued in an article (*Die Volksblad* 2009b) entitled "Leerlinge nie reg vir universiteit", that the changing learning context at universities is a cultural shock for many first-year entering students.

Over the past decade, numerous research studies have been conducted to investigate ways in which students' performance in mathematics could be improved, and some factors associated with underpreparedness have been identified. This is evident as literature indicates that the academic domain of underpreparedness entails a combination of a lack of English proficiency, mathematical ability and effective study skills (Robinson 1996:1 of 7).

A much needed and well-focused series of publications researching the problem regarding underpreparedness in mathematics as well as the reform of these courses, have been reported about in recent years (Paras 2001:66-73; Du Preez, Steyn & Owen 2008:49-62; Steyn & De Boer 1998:125-137; Botha, McCrindle & Owen 2003:132-134; Engelbrecht & Harding 2003:17-20; Nenty & Polaki 2005:67-77; Potgieter & Webb 2004:313-321; Polaki & Nenty 2001:41-52; Pretorius & Bohlmann 2003:226-236; Maree, Louw & Millard 2004:25-34; Bohlmann & Pretorius 2002:196-206; Chance 2002 (1 of 21); Boaler 1998:129-141; Taylor 1999:95-107; Yusof & Tall 1999:67-82; Anthony 2000:3-14; Takahashi, Watanabe & Yoshida 2000:129-136).

1.2.3.1 *Mathematical ability and skills*

Prof. Brahm Fleisch (*Die Volksblad* 2008a) emphasises the incidence of poor mathematical skills among primary learners. In his book *Primary Education in Crisis*, the findings of his research indicated that learners do not know how to add a large quantity of numbers or do long multiplication, and use a method in which they either make use of their fingers to add numbers, or draw circles on paper through which they perform addition or subtraction operations. In a study by Paras (2001:66-73) at the University of Natal, the focus was on *why* students are failing mathematics education. The findings of the study indicated that the majority of students do not understand the basic mathematical notations, which again is a demonstration of the poor school system. In an article entitled "Massas eerstejaars gaan druip" (*Die Volksblad* 2009c), Prof. Jonathan Jansen predicted a massive first-year failure rate amongst first-year students for the June assessment at universities in South Africa. Prof. Jansen further argued that the problem could be ascribed to a lack of order, structure and routine, as well as an indication level of school learning at many South African schools.

Du Preez, Steyn and Owen (2008:49-62) take the argument of identifying factors that contribute to underpreparedness further, by suggesting a programme of developmental support in mathematical and non-mathematical skills. The results of their study indicated an overall improvement in students' pre-calculus skills. The results also indicated that students whose entry level preparedness for mathematics was weak, improved significantly after they had been exposed to additional developmental learning facilitation strategies.

From an open-ended question probing students' feelings about mathematics and the nature of their mathematical difficulties, in a study by Pretorius and Bohlmann (2003:226-236), the following problems emerged: Students had difficulty in understanding what is being asked, followed the wrong approach to studying mathematics (for example memorisation) and experienced difficulty with the terminology of mathematics (Pretorius & Bohlmann 2003:230).

Steyn and De Boer (1998:125-131) used mind mapping as a study tool for underprepared students in mathematics and science. The results of their study indicated that, in most cases, the use of a mind mapping strategy resulted in an improvement of grades and that students experienced an empowerment in their academic disposition.

1.2.3.2 *English proficiency*

In a reading intervention programme for mathematics students, conducted by Pretorius and Bohlmann (2003:226-236), the findings revealed a robust relationship between reading ability and mathematics performance.

1.2.3.3 *Study skills*

According to Mr Japie Gouws, managing director of the ATKV (*Die Volksblad* 2009d:10), education should focus on "learning how to learn". He further states that instructors in South Africa should turn back to their primary responsibility, i.e. to teach learners the necessary thinking skills.

According to Raab and Adam (2005:93), study skills include the ability to acquire, record, remember and use information, which is a shortcoming amongst first generation students. Study skills are a very important factor that contributes to the success and academic proficiency of the academically underprepared student. According to Gal and Garfield (1997:18), these skills are generic and can be learned through all subjects, including mathematics and statistics.

Literature conclusively demonstrates the positive effect that the incorporation of study skills have on student success (Nolting 1997; Weinstein 1999; Springer, Stanne & Donovan 1999; Anthony 2000; Hagedorn, Sagher & Vali 2000; Taylor & Mander 2003; Felder & Brent 2004). With regard to mathematics instruction, Taylor and Mander (2003:224) remark that "strategies that were once thought to be acquired by osmosis" should become an "explicit component of university study". A community college in California revealed that "a lack of study skills" gave a 100% response to which the respondents "agreed to strongly agreed" (The Office of District Research Report 2001:2). The high percentage of this contributory factor was confirmed in a study regarding underpreparedness that was conducted at the University of the Free State in the recent past (Brussouw 2007:146). A study by Anthony (2000:3-14) identified some factors influencing first-year students' success in mathematics. The students and lecturers who participated in this study rated poor study techniques as a more influential factor in failure than inadequate mathematical background knowledge. The study also supports research findings which suggest that, for many students, poor performance is largely due to ignorance about the study skills required, or the inability to apply these skills appropriately, rather than to a lack of ability (Manalo, Wong-toi & Henning 1996:189). According to Pretorius and Bohlmann (2003:235), students' past experience of mathematics has usually been confined to manipulating symbols and numbers and dealing with isolated examples of concepts, rather than gradually building up understanding of concepts from text.

A study by Cukras (2006:194-197) investigated some study strategies that maximise learning for underprepared students. The investigation made use of a realistic classroom setting at Bronx Community College, which serves the most academically at-risk students enrolled at the City University of New York (CUNY). The investigation focused on the use of self-selected strategies and their importance in mastering material in difficult content areas, which in this case, was history and psychology. Although there was no counterbalanced design, the evidence was strong that implementing a study plan and monitoring were the most important aspects for self-regulated learning, because they consistently correlated with, and were predictive of, test performance. The finding of this study also appears to validate the gestalt theory, which demonstrates that the whole is greater than its parts (see 2.7.1). The employment of a study plan therefore enables students to take control of the entire learning process (Cukras 2006:197).

1.2.4 The importance of mathematics in higher education

The need to understand and use mathematics in everyday life and the workplace has never been greater, and will continue to increase [National Council of Teachers of Mathematics (in Nenty & Polaki 2005:67)]. Mathematical competence is important in both the natural sciences and social sciences and is a skill that facilitates the understanding of key concepts. In the most recent publication of *Quest* (a magazine of the Academe for Science in South Africa), Prof. Kobus Maree (*Die Volksblad* 2008c:10) describes the underpreparedness in mathematics as a national disaster. In his article *Why maths counts*, he proclaims that mathematics is by far the most important subject in fields that require people with the necessary skills for economic growth in the country.

Tertiary institutions have an obligation to train future professionals, and the responsibility rests on lecturers to be well-informed about the content knowledge, conceptual understanding and skills development of prospective first-year students, which is of specific relevance in view of the imminent changes in government funding formulae for tertiary institutions (Potgieter, Rogan & Howie 2005:121-123). Moreover, the South African government is committed to promoting and developing mathematics as a high priority, as mathematics is considered essential for students who venture into fields of marketing and management. It is therefore desirable that tertiary institutions offering courses in mathematics remain vigilant for the subject to be taught and expanded successfully. By assisting students become aware of study skills and learning methods in mathematics, or by challenging ideas that students have previously taken for granted, might lead to a more positive trend in mathematics achievement amongst students in the future.

1.2.5 A brief discussion on Statistics

1.2.5.1 *Statistics vs Mathematical literacy*

Burrill (2005:59) purports that, given that statistics is often taught as part of the mathematics curriculum, one of the issues any framework must address is the difference between mathematics and statistics. According to Blurtit (2009:2 of 6), the Wikipedia defines statistics as "...a mathematical science pertaining to the collection, analysis, interpretation or explanation, and presentation of data, and also with prediction and forecasting based on data". Statistics is applicable to a wide variety of academic disciplines, from the natural and social sciences to the humanities, government and business. On the other hand, mathematics is the academic discipline, that with its' supporting body of knowledge, involves the study of concepts such as quantity, structure, space and change. Mathematics evolved from counting, calculation, measurement, and the systematic study of the shapes and motions of physical objects, through the use of abstraction and logical reasoning (Blurtit 2009:2 of 6).

According to delMas (in Hulsizer & Woolf 2009:1 of 2), the teaching of mathematics is metaphorical in nature, in the sense that students learn mathematical ideas and then apply these ideas to a range of problems. However, although statistical reasoning operates on a similar plane, it is essentially different for several reasons. First of all, statistics is dependent on data, and secondly, context shapes statistical inquiry (Cobb & Moore 1997; Gal & Garfield 1997; Moore 1998; Wild & Pfannkuch 1999). Hulsizer and Woolf (2009:1 of 2) argue that researchers cannot separate the processes and reasoning involved in statistical inquiry from the content and context in that inquiry, and that students need to be aware of the reasons and justifications for their methodological and analytic choices and also recognise that there may be more than one approach or solution to a problem.

Thirdly, mathematics is simply a set of procedures in statistics (Gal & Garfield 1997; Nicholson, Ridgeway & McCusker 2006). Finally, the goal of statistics education is for students to be able to reason about research questions and data as opposed to computing a set of answers, and students need to learn that statistics involves the interplay of the abstract and the concrete. Hulsizer and Woolf (2009:1 of 2) argue that students need to recognise the similarities and difference in approaches when applying statistical processes to real-world problems.

According to Gordon (1995:1), students who enter a statistics course with previous coursework in mathematics often tend to use surface approaches to learning, which are ineffective when attempting to learn statistics (see 2.4.1). Ben-Zvi and Garfield (in Hulsizer & Woolf 2009:1 of 2) assert that a focus on formulas, equations and computations does not lead to the development of statistical literacy, thinking or reasoning.

A much needed and well-focused series of publications researching the teaching of undergraduate, or introductory statistics courses and making recommendations for the reform of these courses, have been written about in recent years (Hogg 1999; Garfield Hogg, Schau & Whittinghill 2002; Moore 2001). In order to improve undergraduate programme and statistics courses, Hogg (1999) suggests using continuous quality improvement. According to Garfield *et al.* (in Gordon 1998:40), the desired outcomes of introductory statistics courses should not only include statistical learning and understanding, but also students' willingness to persist in their learning and application of skills, as well as positive attitudes and beliefs about statistics. As one can see, these are challenges and opportunities for statistics education today. Gordon (1998:57) states that "[a]lthough the traditional lecture format and delivery of information model that Cobb (1993) deplores remains the mainstay of some statistics courses, alternative and diverse ways of thinking about statistical learning have been emerging".

1.2.5.2 *Statistical thinking*

According to Snee (1999:255), statistical research, practice, and education are entering a new era, one that focuses on the development and use of statistical thinking. In numerous texts and papers, the phrase "statistical thinking" is utilised in the title. While having our students "think statistically" sounds desirable, it may not be immediately obvious to many instructors what this involves and whether or not statistical thinking can be developed through direct instruction (Chance 2002:1 of 21). Burrill (2005:59) argues that researchers are finding that students lack the ability, at all levels, from elementary to tertiary, to reason and think statistically. Many research reports and experiences indicate that students often master technical skills but are unable to use these skills in meaningful ways. What is so unique to statistical thinking, beyond reasoning and literacy, is perhaps the ability to see the process as a *whole* (Chance 2002:5 of 21). According to Burrill (2005:59), questions that should frame this discussion include: "How do we carefully structure the curriculum? How can we make statistics education more inviting? What do we know about teaching and learning statistics and what do we need to know?"

1.2.5.3 *Research on the development of statistical thinking among students*

According to Wild and Pfannkuch (1999:223), the desire to imbue students with "statistical thinking" has led to the recent upsurge of interest in incorporating real investigations into statistics education. Research conducted by Gordon (1998) confirms the importance of statistical thinking. The report focuses on psychology students' feelings about learning statistics and their interpretations of statistics. An empirical case study was done at a major metropolitan Australian university, on over 250 students who completed a written survey which included questions on their attitudes towards learning statistics and their conceptions of statistics. Their conceptions of statistics were analysed from a perspective developed from phenomenography [Martin & Booth (in Gordon 1998:40)].

The results of the study indicated that a minority group of students expressed a greater willingness to participate in the statistics course and reported more thoughtful and personally meaningful conceptions of statistics. The researcher is of the opinion that these students followed a *deep approach to learning* (see 2.4.2). This group of students who expressed a greater willingness also achieved higher grades (Gordon 1998:56). However, the majority of students were studying statistics unwillingly and was seen by many as boring or difficult. According to Gordon (1998:56), students reported learning mechanical procedures or mastering decontextualised statistical concepts and methods. In other words, these students followed a *surface approach to learning* (see 2.4.1).

Lerman (in Gordon 1998:43) argues that the valuing of decontextualised, intellectual processes, divorced from personal elements, is expressive of oppressive discourse. According to Gordon (1998:43), it is this privileging of abstract thought, such as academic mathematics, that is disempowering for some students. The study by Gordon also indicated that a lack of awareness of the functionality of *statistical skills* and processes made it difficult for students to experience statistical thinking as personally meaningful (see 2.4.2; 2.7.4.1 and 3.3.2).

Gordon (1998:56) purports that the results of her study "...could alert researchers and teachers of statistics to look at the 'big picture' or system that surrounds statistics service courses and the qualitative variation in students' awareness of statistics – the diverse meanings of statistical knowledge to students". The findings of the study raise challenges for supporting the learning of "occasional users" of statistics in higher education [Nicholls (in Gordon 1998:40)]. According to Gordon (1998:40), these challenges include providing students with learning experiences which enable them to reinterpret statistics as personally meaningful knowledge rather than as a culturally "necessary" body of skills and concepts. This echoes Biggs and Tang's (2007) view on a deep approach to learning (see 2.4.2).

A further direction for research, indicated in a study by Gordon (1998), is the investigation of organisational frameworks that could promote active learning and help students develop positive attitudes to learning statistics (Gordon 1998:57). Cobb (1993:par.82) asserts that "[l]earning must be active if it is to build a student's sense of responsibility for the process; lecture-based courses undermine the student's sense of responsibility for learning". The teacher is neither producing a course for the student, nor producing a student for an employer. According to Cobb (in Gordon 1998:57), both these models make our students passive consumers rather than active constructors of their education.

Chance (2002:1 of 21) also emphasises the aspect of statistical thinking in statistical development. In her research paper, several suggestions are given for direct instruction aimed at developing "habits of mind" for statistical thinking in students as well as assessing students' ability to think statistically.

According to Chance (2002:5 of 21), this way we can attempt to develop students into novice statisticians and encourage them to appreciate this "wider view" (Wild 1994) of statistics. Wild (in Chance 2002:5 of 21) also argues that we may be able to develop "mental habits" that will allow non-statisticians to better appreciate the role and relevance of statistical thinking in future studies. Chance (2002:6 of 21) argues that we may not be able to directly teach students to "think statistically", but we can provide them with experiences and examples that foster and reinforce the type of strategies we wish them to employ in novel problems.

According to Chance (2002:17 of 21), we can specifically address the development of statistical thinking in all our students. Furthermore, we can hasten the development of these ways of approaching problems and applying methods in beginner students, by providing exposure to and instruction in the types of thinking used by statisticians. As Chance (2002:18 of 21) puts it, it is through repetition and constant reinforcement that these habits of statistical thinking develop into an ingrained system of thought. As students often "revert" to some of their old habits, these new habits of statistical thinking need to be continually emphasised in follow-up courses, particularly in other disciplines, to further develop statistical thinking.

1.2.6 The importance of the study

According to Chance (2002:2 of 21), "number crunching" must no longer dominate the landscape of the introductory statistics course. Instead, we have the luxury of allowing our students to focus on the statistical process that precedes the calculations and the interpretation of the results of these calculations.

Chance (2002:5 of 21) reiterates that statistical methods are too often seen as tools that are applied in limited situations, as it allow students to form a very narrow view of statistical application: pieces are applied in isolation as specified by the problem statement (rote/surface learning). Chance (2002:8 of 21) argues that, instead, instruction should encourage students to view the statistical process in its entirety and also to view statistics in the context of the world around them.

Wild and Pfannkuch (1999:224) argue that research in education, as well as experience in the quality arena, has shown that the thinking and problem-solving performance of most people can be improved by suitable structured frameworks. The researcher aims to incorporate these thought processes which are involved in solving real-world problems, using statistics with a view to improve such problem solving. The development of a theoretical structure is an important goal of instruction and the researcher is of the opinion that the thinking and problem-solving performance of students can be improved by suitably structured frameworks.

As most first-year students do not find elementary statistics an easy subject, beginner students need a more selective introduction to statistical thinking [Moore (in Wild & Pfannkuch 1999:251), as they often lack intellectual maturity as well as contextual knowledge needed for full statistical problem solving. The proposed classroom learning strategy will thus consist of the development of a framework for the thinking patterns that are involved in problem solving, the pre-requisite strategies for problem solving, as well as the integration of statistical elements within the problem solving (see Chapter 4).

There is a lack of documented research reflecting on an effective learning strategy in a mathematics and statistics-related subject. This particular study will try to meet this need for improving students' performance in a mathematics and statistics-related subject, in a multi-campus university environment, by proposing an effective learning method for first-year as well as third-year students at the CUT. With the above discussion in mind, the problem statement for this research needs to be affirmed.

1.3 PROBLEM STATEMENT

The debate and speculation about possible reasons for students' underpreparedness in mathematics and statistics will continue, but will not solve the problem. According to Du Preez, Steyn and Owen (2008:59), possible reasons which include an inadequate school system, poorly qualified school teachers, inadequate content of school syllabus, negative socio-economic background, language problems and cultural background differences will continue. The researcher is aware of first-year students' lack of understanding of fundamental mathematical concepts, but is more concerned about the consisting problem area that includes the lack of effective learning strategies amongst these students. The researcher sought to reverse the negative trend regarding underpreparedness in mathematics by means of a particular learning strategy intervention. The researcher is of the opinion that without such a learning strategy intervention, underprepared students will continue to be low achievers and regards this research as important to support academic skills by means of a learning method to improve underprepared students' academic performance in a mathematics and statistics-related subject. Brussouw (2007:157) states that, since students are sometimes not willing to undergo study skills training because of the stigma attached to counselling, the teaching of study skills should be integrated into mainstream teaching. Prosperity for South Africa's people requires that they increasingly apply scientific and technological expertise. If one relies on the strength of their scholastic achievements alone, the number of students who are able to complete their studies at tertiary institutions and universities is much too low to satisfy the demands of a developing country such as South Africa (De Boer & Van Rensburg 1997:1 of 12).

Today's students are representative of a wider spectrum, which includes different cultural backgrounds and many previously disadvantaged students. Many of the students that are enrolled for the Management Diploma at the CUT, enter higher education with a matric certificate, but without sufficient mathematical knowledge. According to Anthony (2000:3), most universities offer a range of first-year mathematics courses to cater for students with varying background knowledge and career aspirations. Although the assumption in the past was that students entering a standard first-year mathematics course had recently completed a complementary mathematics course at secondary school, it is increasingly evident that students in current calculus and algebra courses come from a wider spectrum (Anthony 2000:3). Many of these students are characterised by weak academic, linguistic and mathematical ability, as well as insufficient mathematical backgrounds, which immediately place them in a disadvantaged position.

The researcher regards an effective learning method intervention at multi-campus universities as a priority in the South African higher education arena. She is currently employed at the School for Information and Communication Technology of a South African multi-campus university, and has been lecturing mathematics and statistics-related subjects for the past eleven years. The researcher has witnessed and experienced the consequences and frustration of students' negative attitudes towards mathematics as well as statistics, and surface approaches to learning (see 2.4.1) when studying the subject. Studies and research to improve these practices in a mathematics and statistics-related subject at multi-campus universities are very limited and there exists a need amongst multi-campus universities, especially the CUT, to improve these practices.

The problem statement investigated in this study therefore relates to the lack of an effective learning strategy amongst first-year and third-year students in a mathematics and statistics-related subject. Therefore, the researcher aims to establish an effective learning strategy in such a subject for the CUT as a multi-campus university. The CUT will be used as the unit of analysis.

1.4 RESEARCH AIMS

The aim of the research is to determine whether the proposed classroom learning strategy intervention could have any positive impact on students' academic performance in a mathematics and statistics-related subject and, additionally, how it can be implemented and enhanced.

1.5 RESEARCH QUESTIONS

By mentioning the background to the problem statement, as well as the aim of the research, the researcher poses the main research questions that inform the study:

1. *Does the implementation of a classroom learning strategy intervention positively affect students' academic performance in the module Business Statistics/Statistics II?*

2. *Which developmental experiences did students find most useful after the implementation of the proposed classroom learning strategy intervention?*

In addressing the main research questions, answers to the following secondary research questions were pursued:

1. How does learning take place with regard to some important learning theories? (Chapter 2).
2. How is knowledge constructed in the process of learning mathematics and statistics? (Chapter 3).
3. How can students' cognitive processes be enhanced in order to improve their mathematical as well as statistical thinking. (Chapter 3).
4. How will the proposed learning strategy that students would be exposed to, be constructed? (Chapter 3).

5. What is the effectiveness of the proposed learning strategy? (see Chapters 5,6 and 7).

1.6 RESEARCH HYPOTHESES

For the purpose of this study, the empirical study tested the following research hypotheses:

H_0 : The post-test score of students who received the learning strategy (experimental group) is equal to the post-test score of students who received no learning strategy (control group) in the module *Business Statistics/Statistics II.*

H_a : The post-test score of students who received the learning strategy (experimental group) is higher than the post-test score of students who received no learning strategy (control group) in the module *Business Statistics/Statistics II.*

1.7 RESEARCH OBJECTIVES

The aim of the study was realised by pursuing the following research objectives:

1. Explore how learning takes place with regard to some important learning theories (Chapter 2).
2. Explore how knowledge is constructed in the process of learning mathematics and statistics (Chapter 3).
3. Explore how students' cognitive processes can be enhanced in order to improve their mathematical as well as statistical thinking (Chapter 3).
4. Construct the proposed learning strategy that students would be exposed to (Chapter 4).
5. Determine the effectiveness of the proposed learning strategy (Chapters 4, 5 and 6).

1.8 RESEARCH DESIGN AND METHODOLOGY

The following section entails a review of the various stages of the research design and methodology to be implemented. This review is based on the preceding discussions regarding the problem statement, the research questions, aims, objectives and theoretical statements and is discussed in more detail in Chapter 6 of the study.

1.8.1 Research design

As the aim of this research is to improve a current practice, this empirical research study should be regarded as evaluative in nature, as evaluative research focuses on evaluating the merit or worth of a particular practice at a given site (McMillan & Schumacher 2001:20). The study is located within a quantitative paradigm, with some enhancement by means of qualitative observations of students' problem-solving techniques. The study followed a non-equivalent pre-test post-test control group design involving an experimental group and a control group of students, which took place within a field experiment setting. A quasi-experimental approach was used in the attempt to answer the question whether the post-test performance of students who received the learning strategy (experimental group) was higher than students who received no learning strategy (control group) in the module *Business Statistics/Statistics II.* The researcher taught two groups of students, and it was impossible to assign students randomly within each class to each of the two methods. Therefore, the researcher kept each class intact and applied a different method in each of these classes. In this study, a non-equivalent control group quasi-experimental design was employed by using already established groups of subjects. The researcher used these intact classes and gave the subjects a pre-test in the module *Business Statistics/Statistics II* at the beginning of the second semester of the 2009 academic year. The learning strategy was taught to the experimental group of students, after which the post-test was given to both groups of students. For the purpose of this study, the researcher therefore relied on numerical data (tests scores) to test the relationship between the variables as well as to test the formulated hypothesis as to whether the proposed classroom learning strategy had any impact on underprepared students' performance in the module *Business Statistics/Statistics II*. The quantitative research design has been classified into an experimental design, as experimental studies establish probable causality [Maree (in McMillan & Schumacher 2007:255)]. An experimental approach was selected as the problem is clearly one of causation and presumably, the application of a learning strategy intervention (the independent variable) can be manipulated easily.

1.8.2 Population and sampling

In 2009, a third-year group of students at the CUT of the Free State was selected for this research project. The population consisted of 97 third-year students who were enrolled for the National Diploma: Cost and Management Accounting and the National Diploma: Internal Auditing at the CUT. The sample that was selected for this research was 97 third-year students taking *Business Statistics/Statistics II* as a compulsory subject. The subjects formed part of 2 intact classes of third-year students and use was made of non-probability sampling. The main group of 97 students was divided into two groups. Students (n=23) that attended the full- time lecture served as the experimental group and were subjected to the proposed learning strategy intervention. Students (n=74) that attended the part time lecture served as the control group and received traditional instruction (see 3.4.1). Students in this group were considered the "control" group as they did not receive the classroom learning strategy intervention. The students were all registered as part time students on campus and attended *Business Statistics/Statistics II* classes twice per week over a period of 6 months during the second semester of the 2009 academic year. As this was obviously a convenient sample, the research did not have many implications regarding access to the participants as the subjects used were already available, as in the case of a field experiment. The Researcher, as a full-time lecturer at the School for Information and Communication Technology at the CUT, lectured this particular subject.

1.8.3 Variables

As the intervention was assigned to the experimental group of students by the researcher, the independent variable was defined as the learning strategy intervention. For the purpose of the study, the dependent variable was represented by students' academic performance in the mathematics and statistics-related subject *Business Statistics/Statistics II*. As the question of interest was whether the post-test performance of students who received the learning strategy was higher than the post-test performance of students who received no learning strategy, the post-test score served as the dependent variable.

Possible extraneous variables that may compromise the results of this study are race, gender, age, previous mathematical background, class attendance, mark obtained in the first-year semester subject *Business Calculations*, as well as the pre-test score of the results in the module *Business Statistics/Statistics II*. The extraneous variables were accounted for by means of biographical data that was obtained from the CUT's data system. The researcher controlled these extraneous variables by building them into the research design and by measuring the variables and analysing their relationships with the dependent variables.

1.8.4 Instruments

In order to determine the effect of a learning strategy intervention on students' academic performance in the module *Business Statistics/Statistics II*, the data was collected by means of self-developed instruments (tests) intended to yield highly reliable and valid scores. The researcher developed two tests based on the syllabus for this semester subject and the format of these tests focused on the curriculum content. The components of these tests are discussed in Chapter 6 (see 6.6.7). The results (scores) of these tests were used in the study to assess students' post-test performance in the module *Business Statistics/Statistics II*. All results were compared to determine the effect the learning strategy intervention had on the subjects.

The researcher also complemented the more formal quantitative tests with additional qualitative information, in order to obtain richer data on the students. By making use of a reflection diary, the researcher aimed to take notes of students' problem-solving approaches and techniques. The researcher observed students' problem-solving behaviour during classes and reflected on the activities, students' problem-solving strategies as well as the general implementation of the proposed classroom learning strategy intervention. The researcher also made use of the nominal group technique (NGT) (see 6.6.7.3) in order to gauge developmental experiences students found most useful after the implementation of the proposed classroom learning strategy intervention.

1.8.5 Data collection

Both groups of students attended a theory lecture that was offered twice a week. The duration for each theory lecture was 80 minutes. During the theory lectures, the lecturer explained the work to students from the prescribed textbook. The researcher utilised the national prescribed syllabus for the module *Business Statistics/Statistics II* and kept strictly to the study guide. Both classes received exactly the same academic instruction by the same lecturer, covered the same content of work, and used the same prescribed textbook. Both groups of students also wrote the same tests on the same day in one venue.

Descriptive, quantitative biographical data (race, gender, age, previous mathematical background, and mark obtained in the first-year module *Business Calculations*) were obtained from the CUT Student Records Database at the beginning of the 2009 academic year. The researcher entered this biographical data on a database for the purpose of data analysis. The researcher also kept record of students' class attendance in the module *Business Statistics/Statistics II*, and entered the results into the same database.

The quantitative data from students' scores in the pre-test and post-test were obtained by the researcher during the second semester of 2009 and entered into the same database in which the results were to be analysed. Students were assessed during August and September of the 2009 academic year.

The first class test in the semester module *Business Statistics/Statistics II* served as the pre-test and was administered to the subjects in both groups in August before the implementation of the classroom learning strategy intervention. The learning strategy intervention was implemented after the first test and continued for a period of three weeks. The students in both groups were assessed for a second time (post-test) during September, after three weeks of the learning strategy intervention. The intervention continued for another four weeks, after which students were assessed for the third and final time during the exam, which took place at the end of October 2009. Only the results of the pre-test and post-test were taken into consideration for this study. All students were assessed on the same day, in the same venue, at the same time with regard to the pre-test as well as the post-test.

In the qualitative mode of the study the researcher collected information with regard to students' problem-solving approaches and techniques by means of a reflection diary. The researcher observed the students during the theory classes of the second semester and reflected on the activities, students' problem-solving approaches, as well as the general implementation of the learning strategy intervention. In order to determine the developmental experiences students found most useful after the implementation of the proposed learning strategy intervention, the researcher decided to conduct a nominal group setting with the experimental group of students. This setting took place during the last theory lecture, at the end of the semester module *Business Statistics/Statistics II.* The responses yielded by the nominal group setting were scored and coded and the resulting outcomes reported on in Chapter 7.

1.8.6 Data analysis

Raw data taken from the CUT's biographical data base and assessment results were recorded, statistically analysed and interpreted. Descriptive statistics were used to help explain and allow reflection on the demographic composition and performances of both groups of subjects. The statistical software package SAS was used for data analysis. The data were analysed by the Statistical Analysis Division of the Information and Communication Technology Services Department at the University of the Free State. According to the research plan (see Appendix A), the processing of the data included various detailed descriptive statistics, where the researcher aimed to look for similarities and differences, for groupings, patterns and items of particular interest. Some of these descriptive statistics included the mean, standard deviation, median, maximum, minimum and number of observations. The researcher presented frequency tabulations for each of the following categorical variables, namely gender, race and previous mathematical background. Detailed descriptive statistics, both for the total group of students and for the experimental versus control group separately, were presented for each quantitative variable, namely: Business Statistics results (both the pre-test and post-test scores), age, and Business Calculations (BCL) results.

To test for any relationship between the variables, the researcher made use of univariate analysis. The dependent variable (post-test score in the module *Business Statistics/Statistics II*) was analysed using one-way ANOVA fitting, one variable at a time, the dependent variable (group membership) and each of the confounding variables.

In order to determine if students' post-test performance in the module *Business Statistics/Statistics II* was in any way related to students' age, BCL results, and class attendance, the researcher made use of regression analysis. The Pearson product-moment correlation coefficient was computed to ascertain whether there are statistically significant correlations between these variables.

The researcher also made use of multivariate analysis in which the dependent variable (post-test score in the module *Business Statistics/Statistics II*) was analysed using analysis of covariance (ANCOVA). The analysis of covariance model contained the independent variable (group membership) and all possible confounders (gender, age, race, previous mathematical background, BCL results, and *Business Statistics/Statistics II* results: pre-test). The researcher calculated F-statistics and associated P-values for each variable in the model. A significance level of 5% was used throughout the study in testing for significance.

1.8.7 Reliability and validity

The reliability of this research depended on the reliability of the measuring instruments and the choice of the correct statistical procedure (Kerlinger 1986:405; Maas 1998:25-26). The consistent way in which all the data were processed and analysed by means of statistical packages undoubtedly contributed to the reliability of the study.

For the purpose of this study, the researcher critically examined the procedures for collecting the data in order to assess to what extent it is reliable and valid. To ensure internal validity, the researcher established standard conditions of the setting in which the research had been conducted and of data collection. All students were assessed on the same date, at the same time and in the same venue.

In order to ensure validity, the researcher developed strategies to rule out the plausibility that something other than the presumed cause accounted for the observed relationship (Polit & Beck 2008:287). The interpretation of the results depended largely on whether the groups differ in respect of some characteristics that might reasonably be related to the independent variable. Therefore, the researcher compared both groups regarding certain characteristics such as gender, race, age, and previous mathematical background. To ensure high internal validity in this study, the experiment required strict control of all sources of confounding and extraneous variables in order to make sure that these differences did not account for the results of this research. The extraneous variables were controlled in the research design through statistical measurement by building them into the design.

The Hawthorne effect may have been another threat to the internal validity of the research, as subjects may show an increase in positive behaviour simply because they know they are receiving special treatment, which in this research was the classroom learning strategy intervention. For this reason, the researcher therefore deemed it best for subjects to be unaware that they were being studied, or that they had received the classroom learning strategy intervention.

With regard to the external validity of the research, the results could not have been generalisable to all third-year students at other universities, as the research was conducted at one specific academic institution, viz. the University of Technology (CUT), and as no randomisation was used.

1.8.8 Ethical considerations

Approval for conducting the research was obtained from the Dean of the Management Faculty at the institution before any data were collected in the 2008 academic year. The findings of the study would be reported anonymously and the research was not intended to harm any student in any way.

The researcher did not inform students that they were part of a research project for the following reasons: The researcher is of the opinion that knowledge of participation in this specific research study, may invalidate the results. Providing such information could cause students to act unnaturally, which in turn, could influence the results and findings of the research. As the Hawthorne effect could have reduced the internal validity of this research study, it therefore seemed best for subjects to be unaware that they were being studied.

1.9 CONCEPT CLARIFICATION

The following concepts are clarified in order to ensure clarity and to provide a better understanding of their contextual use:

Approach to learning can be defined as "the process adopted prior to the outcome of learning" (Fourie 2003:123).

A **deep approach** to learning arises from an intention to understand, an active conceptual analysis and, if carried out thoroughly, generally results in a deep level of understanding (Ramsden 1988:19; Kember 1996:342-343).

Academic performance refers to the progress that is reported when a particular student has managed to accomplish academic tasks.

Constructivism emphasises that human learning is active and that knowledge is actively constructed within the social context of the classroom community (Worlley & Proctor 2005:4 of 22).

Critical thinking, with regard to mathematics, "is thinking about what is being asked in a given problem, determining what operations and procedures are used in a mathematics problem, with help from a mathematics teacher, and sharpening the analytical skills of learners to improve their mathematics" [Chang (in Makina 2010:27)].

Problem solving is "the identification and application of knowledge and skills that result in goal attainment" [Martinez (in Snowman, McCown & Biehler 2009:248)].

A **Problem solving strategy** is the "plan" used to solve a problem (Santrock 2009:332).

The **classroom learning strategy intervention** is defined as the facilitation of a learning strategy which is derived from a constructivist perspective, with the emphasis on the construction of mathematical meaning.

The **pre-test post-test non-equivalent control group design** is commonly used in **quasi-experimental designs** and involves a experimental group and a control group (Struwig & Stead 2001:10).

The **quasi-experimental design** is used when random assignment of subjects to experimental and control groups is practically impossible (McMillan & Schumacher 2006:273).

The **revised two-factor study process questionnaire (R-SPQ-2F)** is used to assess students' deep and surface approaches to learning (Biggs, Kember & Leung 2001:133).

The **nominal group technique (NGT)** is designed to facilitate collaborative decision making and is often used in educational settings (Jones 2004:23).

1.10 THE SIGNIFICANCE OF THE STUDY

The significance of the study concerns the development of a classroom learning strategy, which is aimed at improving students' academic performance in a mathematics and statistics-related subject – thus contributing towards better mathematical and statistical understanding and pass rates.

1.11 CHAPTER LAYOUT

Chapter 1 provides an orientation and background of the study at hand. A short discussion on underpreparedness in mathematics is included, whilst the research problem, questions, aim, objectives, theoretical statements, research design and methodology, as well as the demarcation of the study, are outlined.

Chapter 2 provides a broad perspective on learning. This includes the definition of learning, how learning takes place in the human brain, the importance of learning, as well as the processes that are involved in the learning process.

Chapter 3 deals extensively with a broad perspective on the learning of mathematics. The researcher presents a discussion on the nature of mathematics, learning theories in mathematics, how students learn mathematics, as well as how mathematical knowledge is constructed. The theoretical overview of the learning of mathematics aims to provide a clear understanding of the importance of the proposed classroom learning strategy intervention and how it can assist in improving students' academic performance in a mathematics and statistics-related subject.

Chapter 4 is concerned with the classroom learning strategy intervention. The proposed classroom learning strategy takes into account important perspectives gained in the literature review which led to a framework that holds the potential of addressing the features of the proposed classroom learning strategy. The researcher also explains her own problem-solving strategy, which formed part of this classroom learning strategy intervention.

Chapter 5 is concerned with a pilot study in which students' academic performance as well as their approaches to learning will be addressed, by the implementation of a classroom learning strategy intervention for a first-year mathematics and statistics-related subject at the CUT. The purpose of the pilot study is to improve the success and effectiveness of the empirical investigation and, by undertaking the pilot study, the researcher wishes to orientate herself towards the project she has in mind. The researcher also explains the classroom learning strategy intervention by means of a practical example from the prescribed textbook.

Chapter 6 describes the empirical investigation undertaken as a means of providing the necessary perspectives and understanding needed in the formulation of the envisaged learning method. The chapter commences with the research problem, namely the lack of an effective classroom learning strategy for third-year students at a tertiary institution. Subsequently the research design and methodology employed in each phase of the research project are outlined. The rationale for employing a non-equivalent pre-test post-test control group design in the research is also explained.

In Chapter 7 the data obtained from the empirical analytical research are presented and discussed. The findings are illustrated by means of tables as well as more advanced statistics, in a few instances.

Chapter 8 is devoted to an overview of the study, referring to the specific research questions, research aim and how the researcher went about addressing each question. Limitations are outlined and the need for further studies/research is also explained. Recommendations, based on the evaluation of the proposed learning strategy, are also made.

1.12 SUMMARY

This chapter provided a broad introductory perspective on underpreparedness in mathematics and statistics with specific reference to the importance of a learning strategy and study skills that could improve students' mathematical and statistical performance. In Chapter 1 the researcher presented an introductory overview of students' underpreparedness in mathematics. As such, underpreparedness was defined, factors that contribute to underpreparedness in mathematics were outlined and the importance of mathematics was spelt out. The researcher also discussed statistics as a learning area within mathematics and presented a discussion on statistical thinking. The problem statement was outlined, whilst the research questions, research aims, objectives, and some theoretical statements related to the study were stated. The research design and methodology to be implemented were discussed, which included aspects such as the literature review, research design, population and sampling, data collection, data analysis, reliability and validity, ethical considerations and evaluation of the proposed learning method for the CUT as multi-campus university. Certain concepts which the researcher regarded to be the most important elements of the study were also defined in this chapter.

A thorough literature study into how learning takes place, and more specifically, how mathematical knowledge is constructed, serve as directives in proposing a classroom learning strategy to third-year students in a mathematics and statistics-related subject.

The aim of this classroom learning strategy is to improve students' academic performance in such subjects, and hopefully, to improve the CUT and other multi-campus universities' pass rates. This will eventually help the CUT and other South African multi-campus universities to consolidate students' current learning strategies and implement a framework which is aimed at improving the mathematical and statistical performance of third-year students.

The following chapter focuses on learning.

CHAPTER 2: LEARNING THEORIES

Learning takes place through the active behavior of the student: it is what he does that he learns, not what the teacher does.~ Ralph W. Tyler (1949) ~

2.1 INTRODUCTION

The focus of this study is to introduce students to a classroom learning strategy intervention in a mathematics and statistics-related subject. As this particular study aims at improving students' mathematical and statistical performance in a multi-campus university environment, the researcher presented an introductory overview of students' underpreparedness with regard to mathematics in higher education in Chapter 1. As such, underpreparedness was defined; factors that contribute to underpreparedness in mathematics were outlined; and the importance of mathematics was spelt out.

As a prelude to discussing any type of learning strategy, it is important to understand the concept of learning and how learning takes place. Therefore, the researcher will focus on the following topics in this chapter, namely learning; approaches to learning; as well as some important learning theories in higher education. As the aim of this chapter is to encourage a wider understanding of human learning, the researcher will focus specifically on cognitive learning theories. By discussing these learning theories, the researcher aims to provide a mechanism for understanding the implications of events related to learning. These theories also provide useful information with regard to academic aspects such as problem solving [Tennant (in Jarvis & Parker 2005:101)].

2.2 BACKGROUND

According to Illeris (2004:84), the capacity to learn is part of the human potential for life fulfilment, individual development, as well as survival. In other words, the capacity to learn is as such basically of a libidinal nature. Jarvis and Parker (2005:14) argue that “[l]earning is lifelong, life-wide, and it plumbs the depth of human existence in the world. We are always both being-in-the-world and becoming, developing, growing, maturing”. The capacity for learning thus provides benefits for the individual as well as for society, since it contributes to the development of many different activities and occupations for individuals, as people develop the capacity to enrich their lives through learning (Mitchell 1999:9). Learning also plays a key role in society, as it transmits accumulated knowledge of a particular culture to new generations by making possible new discoveries and inventions that build on past developments.

According to Nicholls (2002:20), “[t]here is a vast literature on human learning from a variety of perspectives”. Biggs and Tang (2007:20) argue that, for the past decade, learning has been the subject of research by psychologists who were more concerned with developing the “One Grand Theory of Learning” than about studying the context in which people learned, such as schools and universities. According to Biggs and Tang (2007:28), it is only in fairly recent years that researchers into learning have studied learning as it takes place in institutions by students. Akiba and Alkins (2010:62) argue that the notion of learning has been one of the most central foci of a wide variety of efforts pertaining to educational research, practice and policy.

In an attempt to fashion meaningful educational experiences for an increasingly diverse student population, the topic of learning has also become more important to teachers, lecturers, as well as other practitioners. Woolridge (1995:50) argues that Knowles (1973), among others, points out that understanding how a person learns is a major requisite for a successful education programme. According to Child (2007:160), “[t]he importance of studying learning processes is self-evident, since one of the central purposes of the teacher’s task in formal educational settings is to provide well-organized experiences so as to speed up the process of learning, thus enabling pupils to make reasoned choices in solving life’s problems”. Nicholls (2002:20) purports that “[a]s a lecturer it is essential that you present information, ideas, concepts and knowledge in a way students can learn”.

As one of the core functions of universities is to facilitate student learning, higher education in South Africa is challenged to promote the academic success of students through quality teaching and learning. In reality, however, higher education in South Africa is challenged to develop effective and independent learners, as the learners these days are simply unable to engage in typical university tasks successfully (Amos 1999:177). Weinstien and Mayer (1986:315) emphasise the importance of students' understanding of their own learning by saying that "[i]t is strange that we expect students to learn yet seldom teach them about learning". Consequently, if students are to be successful within higher education, their academic literacy needs to be developed through engagement with learning in the mainstream disciplines themselves. According to Dison, Quinn, Nelson and Collett (1996:29), academic literacy is the term that is used for the development of those cognitive processes which are at the heart of students' ability to succeed at university. Starfield (1994) argues that other levels of knowledge; such as how a discipline poses and solves problems; how it conceives of and defines knowledge; what forms of explanation and argument are permissible; and how new knowledge is produced, should also be part of the curricula, other than focusing on mere content curriculum [Starfield (in Amos & Fischer 1998:18)].

According to Zhang, Soergel, Klavans and Oard (2008:3), meaningful learning is sense-making and, through meaningful learning, the learner assimilates new pieces of information into an existing relevant aspect of his or her knowledge structure (see 2.4.2). The researcher, however, believes that students' way of learning is often fragmented and the relationships between new information are often obscure (see 2.4.1). In order for learners to make sense of the new information they learn, they need to understand the relationships among the pieces of information, identify patterns, and build on their previous knowledge to create an updated understanding (Zhang *et al.* 2008:2). Warburton (2003:44) presents the view that "deep learning is a key strategy by which students extract meaning and understanding from course materials and experiences". According to Hall, Ramsey and Raven (2004:489), "Developing deep approaches to learning is claimed to enhance students' engagement with theory subject matter and result in improved analytical and conceptual thinking skills" (see 2.4.2).

Illeris (2004:18) purports that all learning has a content of skill or meaning and that the acquisition of this content is primarily a cognitive process. According to Zhang *et al.* (2008:2), research in cognition and learning provides important insights for understanding and sense-making. In this regard, sense-making refers to "the processes of relating new information to previous knowledge, creating structures, fitting data into structures to create representations, and thus arriving at an understanding of a situation or phenomenon" (Zhang *et al.* 2008:2). Amos and Fischer (1998:19) argue that cognitive development literature provides support for the understanding of student learning difficulties within the higher education context. Nicholls (2002:20) suggests that "developmental psychology gives insight into human behaviour and the thinking processes adopted by individuals". For the purpose of this study, the researcher will therefore focus on cognitive development, as it provides a feasible explanation of learning (Killen & Hattingh 2004:74).

Schunk (1996:13) argues that a basic issue in the study of learning concerns the process whereby learning occurs. According to Zhang *et al.* (2008:2), several models to capture the processes involved in sense-making have been proposed, but they do not include detail either of the conceptual changes that occur as the learner's mental representation develops, or of the underlying cognitive mechanisms that produce these changes. Amos and Fischer (1998:17) also argue that students do not necessarily lack the inherent abstract cognitive capability necessary for success in the higher education context, but rather have not yet learnt to mobilise the particular processes embraced in the ground rules of each discipline. Therefore, academic development needs to influence and impact on the learning-teaching situation in such a way that students can learn to mobilise the required cognitive processes entailed in the ground rules of each discipline they seek to study (Amos & Fischer 1998:19). Nicholls (2002:20) purports that these thinking processes can influence the learning outcomes of teaching and learning situations.

According to Reddy, Ankiewicz and Swardt (2005:15), a conceptual framework for a clearer understanding of how learning in education takes place may be sought in appropriate learning theories.

As a prelude to discussing this, the researcher will firstly define learning and distinguish between surface and deep approaches to learning. The researcher will also distinguish between behavioural and cognitive learning theories and elaborate on the cognitive learning theories in educational research. Understanding some general assumptions about these theories will help to provide a better grasp of the concepts underlying human learning as well as how theoretical principles are derived. The grasp of a solid theoretical background will also allow for the selection of appropriate instructional strategies (Reddy *et al.* 2005:15).

2.3 DEFINING LEARNING

Educational definitions of learning traditionally tend to focus on learning as the process by which people acquire skills, knowledge, understanding and attributes (Mahar & Harford 2004:5). Andrews and Haythornthwaite (2007:396) define learning as "knowledge acquired by systematic study; the possession of such knowledge".

According to Mahar and Harford (2004:5), learning is a complex concept that is defined differently depending on the context in which it is being discussed. For many, learning is a change in behaviour (Ware & Johnson 2000:103; Phillips & Soltis 2004:94). Learning is defined as "all relatively permanent changes in potential for behaviour that result from experience" (Lefrancois 2000:5). To capture the meaning of learning more precisely, Lieberman (2004:34) redefines learning as "a change in our *capacity* for behaviour, as a result of particular *kinds* of experience". Santrock (2009:231) defines learning "as a relatively permanent influence on behaviour, knowledge, and thinking skills, which comes about through experience".

More recently, learning has been becoming to be understood as a set of cultural, social, and institutional processes that occur throughout an individual's life ("life-long learning"). According to Illeris (in Jarvis and Parker 2005:87), all learning is based on two fundamental assumptions: "All learning includes two essentially different types of process, namely an external interaction process between the learner and his or her social, cultural and material environment, and an internal psychological process of acquisition and elaboration in which impulses are connected with the result of prior learning. Second, that all learning includes three dimensions, namely the cognitive dimension of knowledge and skills, the emotional dimension of feelings and motivation, and the social dimension of communication and cooperation – all of which are embedded in a societal context". According to Mahar and Harford (2004:6), the notion of learning as a social process is becoming accepted among educational researchers, policy-makers and practitioners.

2.4 APPROACHES TO LEARNING

Anthony (2000:3) argues that, in recent years, there has been considerable research examining university students' academic involvement and approaches to learning. According to Fourie (2003:123), the term "approaches to learning" can refer to "the process adopted prior to the outcome of learning" as originally proposed by Marton and Saljö in 1976. However, if students are asked by means of questionnaires how they usually go about learning, their approach to learning can also refer to "predispositions to adopt particular processes" (Fourie 2003:123). Anderson (2000:3) states that the way students conceive of learning relates to the way they approach their studies and consequently affects the quality of their learning outcomes. Kiguwa and Silva (2007:355) argue that "[a]n achieving approach may be linked either to surface-achievers who systematically learn selected details by memorizing to obtain high grades, or to deep-achievers who are organized and who plan for both meaning and high marks (usually these are the better students)".

The origins of student approaches to learning originated in Sweden and are attributed to Marton and Saljö's studies (1976a; 1976b) of approaches to learning, namely surface learning and deep learning (Botha, Fourie & Geyser 2005:60; Biggs & Tang 2007:20). In conceiving ways of improving teaching, Biggs and Tang (2007:22) see the concepts of surface and deep approaches to learning as very helpful. In Marton and Saljö's studies, students were given a piece to read and were asked questions afterwards. The first group of students followed a surface approach to learning – they remembered disjointed facts and did not comprehend what the author was trying to say. On the other hand, the second group of students followed a deep approach to learning – they went below the surface of the text to interpret its meaning, saw the big picture and how the facts and details made the author's case (Biggs & Tang 2007:20).

2.4.1 A surface approach to learning

Segers, Dochy and Cascallar (2003:191) describe a surface approach to learning as "an intention to complete the task with little personal engagement, seeing the work as unwelcome external imposition". According to Biggs and Tang (2007:22), a surface approach to learning arises from an intention to get the task out of the way with minimum trouble and without any exhilaration or enjoyment. Biggs and Tang (2007:29) also refer to a surface approach to learning as learning activities that are on too low a level to achieve the intended learning outcomes, for example by memorising facts in order to give the impression of understanding.

To do the task properly, higher levels of activities are required, but with a surface approach to learning, students make use of low cognitive-level activities. Applied to academic learning, examples of a surface approach to learning include rote learning and memorization. As Segers *et al.* (2003:191) purport: "a surface approach to learning is often associated with routine and unreflective memorization and procedural problem solving, with restricted conceptual understanding being an inevitable outcome".

Van Weert and Tatnall (2005:112) argue that a student who follows a surface approach to learning "only considers aspects of the subject matter in isolation; there is no commitment to seek out relationships between these aspects beyond what is required in assessment tasks". Students following this approach to learning focus on what Marton (in Biggs & Tang 2007:23) calls the "signs" of learning: the words used, isolated facts, and items treated independently of one another. These students tend to be unreflective on the purpose or strategy of learning and focus on words, text or formula, rather than meanings (Prosser & Trigwell 1999:91). The student thus has no personal interest or sense of relevance in what is being learnt. Also, whatever is learnt can easily be forgotton.

2.4.2 A deep approach to learning

Biggs and Tang (2007:24) argue that, when students feel a need to engage the task appropriately and meaningfully, they follow a deep approach to learning. A deep approach to learning arises from an intention to understand, an active conceptual analysis and, if carried out thoroughly, generally result in a deep level of understanding (Segers *et al.* 2003:191; Ramsden 1988:19; Kember 1996:342-343). According to van Weert and Tatnall (2005:112), a deep approach to learning "involves relating the content to one's own experience, developing an understanding of the underlying laws, rules, processes, and thus bringing onto oneself, new insights and perspectives".

Prosser and Trigwell (1999:84) argue that a student who follows a deep approach to learning is aware of more aspects of his or her learning situation than a student who follows a surface approach to learning. These students try to use the most appropriate cognitive activities for handling the task and make use of activities that are appropriate to achieving the intended learning outcomes. Intended outcomes at university should be high level, requiring students to reflect, hypothesise and apply (Biggs & Tang 2007:24). Students who feel this need-to-know automatically try to focus on underlying meanings, main ideas, themes, principles, or successful applications. Students thus develop a personal interest in the subject matter so that there is a desire to seek relationships between, and beyond, different aspects of the subject matter [Biggs (in van Weert & Tatnall 2005:112)]. For these students, learning is a pleasure; they have positive feelings and show interest in handling a task.

Apart from the learning strategies adopted, the student who follows a deep approach to learning will also organise information in a "holistic" way by integrating the whole and the parts (Fourie 2003:123). However, a sound foundation of relevant prior knowledge is required, as students needing to know will naturally try to learn the details and also make sure that they understand the big picture. According to Biggs and Tang (2007:24), there are four factors that encourage students to adopt a deep approach to learning, which are:

1. An intention to engage the task appropriately and meaningfully, which may arise from an intrinsic curiosity or from a determination to do well.
2. The appropriate background knowledge.
3. The ability to focus at a high conceptual level starting from first principles. This requires a well-structured knowledge base.
4. A genuine preference, and ability, for working conceptually rather than with unrelated details.

Fourie (2003:124) argues that "guiding students towards the adoption of a deep approach to learning should be an important aspect of teaching in higher education". Because of the importance of interdisciplinary thinking and holistic insight, deep learning is particularly relevant in the context of education and sustainability (Warburton 2003:44). Numerous calls have been made for educators to adopt strategies that produce such results.

In the next section, the researcher reviews the role of theory in the study of learning. The researcher briefly describes the key learning theories often cited in educational research and then highlights the learning theories that emphasise cognitive processes.

2.5 LEARNING THEORIES

Lefrancois (2000:25) defines learning theories as "attempts to systematize and organise what is known about human learning. They are useful not only for explaining but also for predicting and controlling behaviour, and they may also lead to new information". Learning theories originate as the ideas of philosophers, psychologists, educationists, teachers and lecturers in the quest to find out more about how learning takes place.

According to Lefrancois (2000:7), theorists begin with certain assumptions about human behaviour and, as a result, they develop tentative explanations for what they observe.

According to Taylor (2003:14), "[l]earning theories assist us in identifying how different individuals may manage, delay, progress through, or retreat from developmental tasks". Lefrancois (2000:8) argues that theories trying to explain how humans learn should logically lead to suggestions for arranging experiences in such a way that behaviour will change in desired ways. Gredler (1992:7) maintains that learning theories provide a mechanism for understanding the implications of events related to learning in both formal and informal settings. They are therefore useful in the design and evaluation of classroom practices and educational products. Thomas (in Lefrancois 2000:9) states that theories also serve to suggest which observations are most important, as well as which relationships among these observations are most meaningful (see 2.4.2).

Facilitators need to decide what skills and knowledge learners must acquire and how their learners should be taught. A knowledge of learning theories thus enables teachers and lecturers to examine their own beliefs about the teaching and learning process (Santrock 2009:7). These beliefs are based on an awareness of the learning theories from which they can choose. Teachers and lecturers will then be placed in a position to make valid decisions about the way they teach, in addition to providing information on a particular aspect of planning, such as facilitating problem solving (see Chapter 3), or enhancing motivation (Snowman, McCown & Biehler 2009:2).

According to Bigge (1982:7), no theory can be found to be absolutely superior to all others, but university teachers can develop learning theories of their own that, because of their internal harmony and educational adequacy, they can support. As Lefrancois (2000:9) purports, a theory "must be judged mainly in terms of its usefulness". As many of these theories are complementary, they can be applied equally to student learning.

2.5.1 Twentieth-century learning theories

Twentieth-century systematic learning theories are classified into two broad families, namely, S-R (stimulus-response) conditioning theories of the behaviouristic family and cognitive theories of the gestalt-field family, which emphasise cognition in learning (Lefrancois 2000:21; Santrock 2009:231; Bigge 1982:9). The vast difference between the two most prominent families of contemporary learning centres upon the behaviouristic assumption that human beings are passive or reactive and the gestalt-field assumption that they are interactive in relationship with their environments (Bigge 1982:49). Both behavioural and cognitive theories agree that differences among learners and the environment can affect learning, but they diverge in the relative emphasis they give to these two factors.

2.6 BEHAVIOURAL LEARNING THEORIES

In the first half of this century, behaviourism was a powerful force in psychology with the consequence that many historical views of learning represent behavioural theories. Boghossian (2006:713) argues that, although the foremost learning theory today is constructivism (see 2.9), the educational landscape has been dominated by behaviourism for more than twenty years.

According to Littlejohn and Foss (2009:57), the behaviouristic learning theories were developed during the 1950s and 1960s – a time when learning theories reflected behavioural psychology. According to Jordan, Carlile and Stack (2008:21), "behaviourists define learning as a relatively permanent change in behaviour as the result of experience". Littlejohn and Foss (2009:57) argue that "[a] major commonality of these theories was their emphasis on the stimulus characteristics of the communication situation". According to Jordan *et al.* (2008:21), classical behaviourists believe that all learning conforms to observable scientific laws governing behavioural associations and patterns. In other words, the learner simply responds to external stimuli in a deterministic manner.

Schunk (1996:13) argues that behavioural theories stress the role of the environment, specifically how stimuli are arranged and presented and how responses are reinforced. Behavioural theories thus imply that teachers should arrange the environment in such a way that students can respond properly to stimuli. Perry (1999:1 of 3) purports that, from a behaviourist perspective, the transmission of information from teacher to learner is essentially the transmission of the response appropriate to a certain stimulus.

In the next section, the researcher will first focus on the conditioning learning theory of Skinner.

2.6.1 Skinner's operant conditioning theory

According to Snowman *et al.* (2009:190), behavioural learning theories culminated in the work of B.F. Skinner. Skinner adopted a carefully structured approach called "operant conditioning" in the 1930s, in which the behaviour of the subject determines the response to the subject's own actions (Jordan *et al.* 2008:24). Skinner's operant conditioning learning theory is based on the assumption that features of the environment (stimuli, situations, events) serve as cues for responding. In Skinner's (1953) view, a response to a stimulus becomes more likely to occur as a function of the consequence of responding. The likelihood that people will respond to environmental cues is determined by the consequences of behaviours, where reinforcing consequences increase behaviour, while punishing consequences decrease behaviour (Santrock 2009:236). For Skinner (1953:72), "the only defining characteristic of a reinforcing stimulus is that it reinforces". According to Jordan *et al.* (2008:25), Skinner saw the development of behaviour as occurring through the effects of positive and negative reinforcement as well as punishment and extinction, where reinforcement is the observed effect of a reinforcer (Lefrancois 2000:99). According to Santrock (2009:236), "reinforcement is a consequence that increases the probability that a behaviour will occur". Reinforcement selects behaviour to increase in frequency and to be maintained in the behavioural repertoire (Garton 2004:123). Lieberman (2004:35) argues that "[i]n reinforcement, the consequence that follows a response is desirable and the effect is to strengthen it – the use of a reward to increase studying, for example".

According to Santrock (2009:236), "[o]perant conditioning is a form of learning in which the consequences of behaviour produce changes in the probability that the behaviour will occur". Boghossian (2006:716), however, is of the opinion that learners undergo some form of conditioning, where the goal of the conditioning is to produce a behavioural result. In other words, when an important event follows a response rather than a stimulus, the result is often a change in the response's probability to re-occur (Lieberman 2004:35). For example, if one's parents gave one a sports car every time one received a distinction in a course, the amount of time one spent studying would very likely increase. Perry (1999:1 of 3) argues that learners will continue to modify their behaviour until they receive some positive reinforcement. In other words, when the stimuli are present, reinforcement strengthens responses and thus increases their future likelihood of occurring. As Snowman *et al.* (2009:192) argue: "reinforcement takes place when consequences strengthen a preceding behaviour".

The generality of operant conditioning principles has been challenged by animal research on instinctual drift, autoshaping and superstitious behaviour. Cotton (1995:11-12), however, advocates that "in stimulus-response psychology we are little more than mindless puppets to be manipulated by those in authority". For Skinner (1976:156-157), no mention of cognitive processes is necessary. In this regard Schunk (1996:99) purports, "as a theory to explain human learning, the operant model has been found wanting due to its failure to consider mental concepts in explaining behavior".

2.6.2 A critical overview with regard to the behavioural learning theories

According to Schunk (1996:12), the behavioural theorists contend that explanations for learning need not include thoughts and feelings - not because these inner states do not exist, but rather because the explanation for learning lies in the environment and the individual's history. For the behaviourist, "knowledge does not depend upon introspection", where the focus is on "external observation of lawful relations between and among outwardly observable stimuli and the responses that follow" (Boghossian 2006:715).

Jordan *et al.* (2008:21) argue that "[a]lthough behaviourists do not deny that learners think, they mainly choose to ignore inaccessible mental processes and focus on observable behaviour". What may have reinforced the behaviourist exclusion of cognitive activity, is the use of animals in early behaviourist experiments, which confined observations to external behaviour (Lefrancois 2000:193). Not only is behaviourism linked to power and control and does it have connotations of animal testing, but it is also associated with an outmoded industrial training model that fails to take account of people's ability to take actions for themselves (Jordan *et al.* 2008:33).

Perry (1999:2 of 3) argues that, with regard to instruction, behaviourist teaching methods tend to rely on so-called "skill and drill" exercises to provide the consistent repetition necessary for effective reinforcement of response patterns. According to Perry (1999:2 of 3), behaviourist teaching methods have proven most successful in areas where there is a "correct" response or easily memorised material (see 2.4.1). Their efficacy in teaching comprehension, composition and analytical abilities is therefore questionable. Jordan *et al.* (2008:34) argue that, with regard to higher learning, "behaviourist techniques may not be effective in promoting deep learning, which is related to personal understanding and meaning-making" (see 2.4.2).

The researcher strongly believes that the "skill and drill" mode which is often associated with behaviourism deprives learners from following a deep approach to learning (see 2.4.2). As mathematics and statistics-related subjects require comprehension, analytical as well as problem-solving abilities, the researcher regards behaviouristic methods of instruction as inadequate.

Against the backdrop of behaviourism and conditioning theories there were some ongoing efforts that made different assumptions about and postulated different principles of human behaviour. According to Tennant (in Jarvis & Parker 2005:101), the emergence of cognitive psychology in the 1960s was made possible by the failure of behaviourism to translate mental constructs into behaviourist terms and by a new interest in information processing models of human cognition. One of these was the cognitive learning theories, which the researcher will discuss in the next section.

2.7 COGNITIVE LEARNING THEORIES

According to Santrock (2009:231), "cognition" means "thought", and refers to the processes involved in thinking (Lieberman 2004:23). According to Bigge (1982:171), the term "cognitive" is derived from the Latin verb *cognoscere*, which means "to know". Jordan *et al.* (2008:36) argue that "[c]ognitivism involves the study of mental processes such as sensation, perception, attention, encoding and memory", and that "learning results from organizing and processing information effectively". Lefrancois (2000:193) purports that cognitive psychology's principal interests are in higher mental functions such as perception, concept formation, memory, language, thinking, problem solving and decision making.

For Bigge (1982:172), learning within cognitive-field theory, is "an interactional process within which a person attains new insights or cognitive structures or changes old ones". Bigge (1982:173) further maintains that cognitive structure refers to the person's perception of the psychological aspects of the personal, physical, and social world. Cognitive learning theories focus on the processes involved when mental structures change, so that the capacity to demonstrate a change in behaviour (learning) occurs. According to Lefrancois (2000:189), "[c]ognitive approaches to learning are characterized by a preoccupation with topics such as understanding, information processing, decision making, and problem solving".

Schunk (1996:13) argues that cognitive theories acknowledge the role of environmental conditions as facilitators of learning, in which the university teacher's explanations and demonstrations of concepts serve as environmental inputs for students.

One of the early theories that included cognitive principles was the gestalt psychology. According to Lefrancois (2000:189), "[g]estalt psychology can be viewed as the basis of contemporary cognitive psychology".

2.7.1 Gestalt learning theories

Lefrancois (2000:176) is of the opinion that the gestalt psychology movement began in Germany around the time of World War I (1914-1918) and that it led to a new direction in learning theory that opposed behavioural learning theories. The position of gestalt psychology was formally first introduced by the German philosopher-psychologist Max Wertheimer in 1912 (Peterson 2006:17; Petermann 2007:10). By the summer of 1913, the gestalt theory of Max Wertheimer appears to have become an identifiable system of thought (King & Wertheimer 2007:107).

According to Mitchell (1999:71), "gestalt" is a German word meaning "form" or "whole", since these psychologists maintain that humans perceive sensations or concepts as whole units rather than isolated pieces. According to Bigge (1982:9), the gestalt-field theorists view learning as "a process of gaining of changing *insights*, outlooks, expectations, or thought patterns". They define learning in terms of reorganisation of perceptual or cognitive fields so as to gain understanding (Bigge 1982:12). The gestalt theory thus emphasises the importance of mental processes and is based on the belief that subjects react to unitary, meaningful wholes, not to individual and separate elements (see 2.4.2; 2.4.1). Schunk (1982:55) maintains that the gestalt theorists disagree with the behaviourists about the role of consciousness by proclaiming that it is only through conscious awareness that meaningful perception and insight can occur.

According to Lefrancois (2000:179), the primary beliefs of the gestaltists are that "the whole is greater than the sum of its parts", and that "people solve problems through insight". Gestalt psychologists claim that the analysis of learning into separate stimulus-response results in a distortion of the concept, favoured by the behaviourists (Mitchell 1999:72).

As a result of various experiments, Wertheimer (in Mitchell 1999:72) concluded that one's perception of the whole, such as the perception of motion, cannot be obtained from studying the specific stimulus elements of the whole in isolation. In other words, the whole is meaningful and loses meaning when it is reduced to individual compositions (Schunk 1982:55). The implication of Wertheimer's findings is that the whole concept possesses properties that differ from those of its separate elements, and that these properties only emerge when the whole is perceived as a complete concept (Mitchell 1999:72). According to Child (2007:175), this emphasises the essence of the gestalt view, in which objects or events are viewed as organised wholes. Mitchell (1999:72-73) states that the emergence of a whole concept in the human mind results from the cognitive organisation of its perceived elements, while learning tends to be a meaningful organisation of perceived elemental stimuli. In other words, learning occurs through perception.

2.7.1.1 *Insight*

According to Wolfgang Kohler (1927), another leader of the gestalt psychology, learning takes place through *insight* (Lieberman 2004:254; Lefrancois 2000:177). For Bigge (1982:96), the key word of gestalt-field psychologists in describing learning is *insight* as they regard learning as a process of developing new insights or changing old ones. Wertheimer (in Child 2007:171) calls the sudden and immediate behaviour when people suddenly perceive the "idea" of how to solve a problem, *insight*. Schunk (1982:55) views human learning as insightful and argues that insight occurs when people suddenly "see" how to solve a problem. If the elements, which can also be physical tools necessary to solve a problem, are present and available, the learner will mentally manipulate these elements until a mental connection is made between them, which can then be implemented to solve a problem (Mitchell 1999:73).

The gestalt theorists, for example, contend that - even though students may repeat the multiplication tables until they appear to have memorised them by rote - what they have actually done is to get the *feel* of some pattern that is present in the tables. The following example in mathematics illustrates insightful learning: What is the answer to $\sqrt{x^2}$? Students have perhaps never put the insight into words, but know that when x is equal to or greater than zero, $\sqrt{x^2} = x$ or $\sqrt{3^2} = 3$. When verbalised, your insight would be something like, "The square root of anything squared is that thing" (Bigge 1982:98).

2.7.1.2 *Reflective thinking*

Gestalt-field psychologists also interpret thinking to be a reflective process within which persons either develop new insight, or change existing tested generalised insights or understanding. John Dewey characterises reflective thinking as the "active, persistent, and careful consideration of any belief, or supposed form of knowledge in the light of the grounds that support it and the further conclusions to which it tends" [Dewey (in Bigge 1982:105)].

2.7.1.3 *Applying gestalt learning theory in the classroom*

Mitchell (1999:77) argues that, when learning, students can highlight, frame and contrast key concepts to make visual stimuli stand out more clearly. Students can also be helped by making them aware of the structure of the learning content as well as the relationships between its elements, which will in turn help them to conceptualise the learning material as an organised body of knowledge, in other words, a "gestalt". According to Child (2007:75), learning can also be promoted by university teachers if they organise the learning material in such a way that learners would perceive it as a *whole* (see 2.4.2) and not just as a jigsaw puzzle in which the pieces fit together to create a finished pattern (see 2.4.1).

According to Lefrancois (2000:186), gestalt theory has proven to be highly thought provoking and has contributed significantly to the development of later cognitive theories. From many years' experience in higher education, the researcher has witnessed time after time how students learn mathematics or statistics in an isolated or fragmented way. The researcher regards the gestalt learning theory as very important, since it places emphasis on the "whole" perceived relationship between elements and that the "whole" is perceived as a complete concept. Not only is the gestalt theory important in promoting learning material in an organised fashion, but it also assists in the process of reflective thinking (see 2.7.1.2).

In the next section, the researcher will discuss information processing theory as a component of cognitive learning theory as well as how the sensory memory, working memory and long-term memory function to process information, all of which have important implications for classroom teaching.

2.7.2 Information processing theories

The primary focus of information processing is on memory, in other words, the storage and retrieval of information (Huitt 2003:1 of 3). According to Lefrancois (2000:194), information processing refers to "how information is modified and changed". According to Snowman *et al.* (2009:212), information processing theory "seeks to understand how people acquire new information, how they store information and recall it from memory, and how what they already know, guides and determines what and how they will learn". Galotti (in Santrock 2009:231) and Schunk (1996:188) reached the conclusion that information processing theories therefore focus on how learners process information through attention, perception, encoding, storage and retrieval of knowledge.

According to the information processing approach, learners develop a gradually increasing capacity for processing information, which allows them to acquire increasingly complex knowledge and skills [Halford (in Santrock 2009:270)]. Shuell (in Schunk 1996:151) argues that information processing is involved in all cognitive activities such as perceiving, rehearsing, thinking, problem solving, remembering, forgetting and imaging. Information processing approaches have been applied to the study of learning, memory, problem solving, visual and auditory perception, cognitive development and artificial intelligence (Schunk 1996:151).

According to Child (2007:190), the role of memory in the process of learning is central to most aspects of learning and teaching. Lefrancois (2000:257) argues that "there will be no evidence of learning without something having happened in memory" and that "[s]tudying memory is, in effect, another way of studying learning". Santrock (2009:277) defines memory as "the retention of information over time". Lefrancois (2000:259) defines memory as "the availability of information and implies being able to retrieve previously acquired skills or information". According to Lieberman (2004:38), memory is "the process by which we code, store, and retrieve information about our experiences". In the researcher's view, memory can thus be seen as the ability to store, code and recall previously learned information or past experiences by means of repetition over time.

Information processing theorists challenge the idea that all learning involves forming associations between stimuli and responses (Snowman *et al.* 2009:212). Mitchell (1999:6) argues that every time one learns something new, information processing occurs. According to Huitt (2003:1 of 7), the most widely accepted theory in the information processing approach to cognition, is the "stage theory" which is based on the work of Atkinson and Shriffin (1968). According to Berryman, Smythe, and Lamont (2002:120), the stage theory model of human information processing is an influential model for cognitive development. The focus of this model is on how information is stored in one's memory, and Atkinson and Shriffin postulate the existence of three memory stores (Lieberman 2004:332). Santrock (2009:282) argues that, "[a]ccording to the Atkinson-Shriffen model, memory involves a sequence of sensory memory, short-term memory, and long-term memory stages".

The next section describes how information enters the cognitive system through the sensory stores, before moving into the short-term memory (working memory), where it is processed and finally stored in the long-term memory.

2.7.2.1 *Sensory memory*

The stimuli that people are continually exposed to from their environment, are what people process when they learn and remember things (Snowman *et al.* 2009:215). According to Lieberman (2004:336), this processing begins with the senses that are linked to the sensory memory, for example, reading, hearing, smelling, and feeling heat or cold. The sensory memory can thus be seen as the information storage area in the human mind that briefly retains stimuli from the environment, until they can be processed further (Santrock 2009:282).

According to Huitt (2003:3 of 7), there are two major concepts for obtaining information for the short-term memory. First, if a stimulus has an interesting feature, individuals are more likely to pay attention to a stimulus. Second, if the stimulus activates a known pattern, individuals are even more likely to pay attention. Lecturers can take advantage of this principle by having students call to mind relevant prior learning before they start with any new concept in a subject. Most students should, for example, be familiar with the formula (1) of a straight line $\boxed{y = mx + c}$ from their instruction during their school careers. In the module *Business Statistics*, the formula (2) which is used to calculate the regression line in linear regression is introduced to students, which is $\boxed{y = a + bx}$. Before the lecturer explains the new formula to the students, she writes the formula, which they are familiar with, on the chalkboard.

(1) $\boxed{y = mx + c}$

(2) $\boxed{y = bx + a}$ [Take note that $bx + a = a + bx$.]

Underneath it, she writes the new formula (2). In this way, the new stimulus (formula) activates a known pattern among the students and they should be able to grasp the newly introduced concept with less difficulty.

The next information storage system in the human mind is the so-called short-term memory.

2.7.2.2 *The short-term memory (working memory)*

The information storage area in the human mind that holds information in the mind while one consciously works with it, is the working memory, which relates to what one is thinking about at any given moment in time (Huitt 2003:3 of 7). According to Jordan *et al.* (2008:50), the short-term memory is limited in both capacity and duration. The working memory screens information and decides what to do with stimuli received, in other words, what information can be disregarded or retained by means of rehearsal or be transferred to the long-term memory (see 2.7.2.3).

According to Huitt (2003:4 of 7), it is necessary to point out important information to students, because of the variability of how much each student can work with. As the working memory can hold only small amounts of information for limited periods of time, too many items can become crammed into a limited space (Lieberman 2004:320). Mitchell (1999:7) refers to this process as "bottleneck". This limitation of the working memory can, however, be overcome. According to Huitt (2003:4 of 7), there are two major concepts for retaining information in the short-term memory, namely "organisation" and "repetition".

(a) *Organisation*

Lieberman (2004:438) argues that one of the most important determinants of how well we remember verbal material is the extent to which the words are structured or organised. The gestalt theory (see 2.7.1) and research also reveal that well-organised material is easier to learn and recall [Katona in (Schunk 1996:167)]. According to Lefrancois (2000:281), to organise is to arrange according to some system. The concept of "chunking" or "combining" information in a meaningful fashion, is an issue related to organisation (see 2.4.2). According to Santrock (2009:280), "chunking is a beneficial organizational memory strategy that involves grouping, or 'packing', information into 'higher-order' units that can be remembered as single units". Snowman *et al.* (2009:227) argue that "[t]he main purpose of chunking is to enhance learning by breaking tasks into small, easy-to-manage pieces". By classifying and grouping bits of information into organised chunks, learning is enhanced, since organised material improves memory because items are linked to one another systematically (Snowman *et al.* 2009:218). Material to be learned can also be organised by making use of a hierarchy, into which pieces of information are integrated (Santrock 2009:290).

(b) ***Repetition***

Lefrancois (2000:280) argues that rehearsal is the principal means of maintaining information in the short-term memory (see 2.7.2.2). It is also one means by which information is transferred to long-term memory (see 2.7.2.3). According to Santrock (2009:278), rehearsal is "the conscious repetition of information over time to increase the length of time information stays in memory". Although "rote rehearsal" (see 2.4.1) is a technique we all use to try to learn something (Huitt 2003:4 of 7), simply memorising something does not lead to learning; in other words, to relatively permanent change. As Santrock (2009:278) purports: "[r]ehearsal does not work well for retaining information over the long term because it often involves just rote repetition of information without imparting any meaning to it". The researcher strongly believes that students who make use of rote rehearsal also tend to follow a surface approach to learning (see 2.4.1).

2.7.2.3 *The long-term memory*

According to Jordan *et al.* (2008:51), "[l]ong-term memory (LTM) is the permanent repository of accumulated information". The long-term memory is the permanent information store in the human mind and differs from the working memory in both capacity and duration (Lieberman 2004:320). The capacity of the long-term memory is vast and durable, whereas the working memory is limited to approximately seven chunks of information for a matter of seconds (Lefrancois 2000:270). As people cannot store every single item of information in the environment, they have to select or *encode* those aspects they think are relevant.

(a) *Encoding*

According to Berryman, Smythe, Lamont and Joiner (2002:121), "encoding" is an important process in cognitive development and refers to the process of transforming information from the environment into a lasting representation. Santrock (2009:271) views encoding as "the process by which information gets stored in the memory". Schunk (1996:167) defines "encoding" as "the process of putting new or incoming information into the information processing system and preparing it for storage in the long-term memory".

In the researcher's view, students often fail to encode important aspects of a new subject because they do not know what information to encode, or simply because they do not know how to encode it. Usually encoding is accomplished by making new information meaningful and integrating it with known information in the long-term memory. Some factors that influence encoding are organisation, elaboration and schema structures (Snowman *et al.* 2009:217). The latter two factors are discussed in the next paragraphs.

(b) *Elaboration*

Santrock (2009:278) defines elaboration as "the extensiveness of information processing involved in encoding". According to Schunk (1996:168), "elaboration" is the process of expanding upon new information by adding to it what one knows. Schunk (1996:179) sees elaboration as the process of adding to information being learned in the form of examples, details, inferences, or anything that serves to link new and old information. As elaboration is a form of rehearsal, it facilitates learning [see 2.7.2.2 (b)]. This process of elaboration assists "encoding" and retrieval as they link the "to-be-remembered" information with other knowledge, which makes newly learned information easier to access in the expanded memory network (Santrock 2009:279).

(c) *Schema structures*

According to Bartlett (in Lieberman 2004:404), people remember new material through mental structures called *schemas*. According to Snowman *et al.* (2009:222), "[m]any cognitive psychologists believe that our store of knowledge in long-term memory is organized in terms of schemas". Santrock (2009:284) defines a schema as "information – concepts, knowledge, information about events – that already exists in a person's mind".

When students learn material, they make use of schemas which highlight important information as they attempt to fit information into the schema's spaces. According to Bartlett (in Davey 2004:231), a schema is "a mental framework or organized pattern of thought about some aspect". Schunk (1996:169) and Jordan *et al.* (2008:43) argue that any well-ordered sequence can be represented as a schema. According to Lieberman (2004:411), "[s]chemas help us to interpret and store our experiences, and they guide us in deciding what aspects of a scene require particular attention". Schemas thus assist the process of encoding, because they elaborate new material into a meaningful structure (see 2.4.2). As a schema is especially helpful for learning to occur, the lecturer can assist students to develop schemas by paying attention to how students use their background knowledge, as well as by helping them to use it as accurately and completely as possible in order to process new information (Snowman *et al.* 2009:223).

According to Schunk (1996:166), the associative structures of the long-term memory are propositional networks, or interconnected sets comprising nodes or bits of information. Propositions are interrelated when they share a common element, which allows people to solve problems. Propositions are the basic units of knowledge and meaning in the long-term memory and are defined by Lieberman (2004:413) as "the smallest unit of knowledge that can be meaningfully regarded as true or false". For example, "3 + 4 = 7" is a proposition. The way knowledge is stored in the long-term memory is based on the concepts of declarative knowledge and procedural knowledge, which are two types of propositional knowledge (Santrock 2009:283).

(i) *Declarative knowledge*

According to Santrock (2009:283), declarative knowledge involves "...the conscious recollection of information". According to Paris, Lipson and Wixson (in Schunk 1996:173) declarative knowledge includes facts, beliefs, opinions, generalisations, theories, hypotheses and attitudes. Consider the following example: When considering the operation to add fractions in mathematics, one must first have the same denominators in order to add fractions. Being able to state the rule is therefore a form of declarative knowledge, or "knowing how" (Santrock 2009:283).

Declarative knowledge may be represented in a variety of forms, which include the following: Concept maps, graphs, tables, rules, problem-solving strategies, learning strategies, schemas, and frameworks (Huitt 2003:5 of 7). According to Schunk (1996:178), meaningfulness, elaboration, and organisation enhance the potential for declarative information to be effectively processed and retrieved [see 2.7.2.3 (b); see 2.7.2.2 (a)].

(ii) *Procedural knowledge*

Procedural knowledge is knowing how to do something (Pollock & Cruz 1999:155; Lieberman 2004:383). Therefore, learners need to adapt their behaviour to changing conditions and then act according to these conditions (Mitchell 1999:9). With regard to the above-mentioned fraction example in mathematics, the conditions for adding fractions are different from the multiplication of fractions and depend on the denominators. The ability to add them correctly thus depends on recognising these conditions and responding appropriately. If the denominators are the same when adding fractions, you merely add the numerators and if the denominators are not the same, you must find a common denominator and then add the numerators. It is thus important to know the rule governing adding fractions to adapt to the different conditions and correctly perform the addition, which shows that procedural knowledge therefore depends on declarative knowledge (Mitchell 1999:9). The following diagram, namely Diagram 2.1, explains the three stages in which procedural knowledge occurs:

The declarative stage

Learners acquire declarative knowledge about the procedure, for example the denominator rule for addition of fractions. Performing the action is painstaking at this stage and occupies most of the learner's working memory space (Mitchell 1999:9).

↓

The associative stage

The learner still thinks about the steps to perform the operation, as he/she practises and receives feedback, but this enables him/her to combine the steps and perform the action smoothly (Mitchell 1999:9). With additional practice the learner then moves to the automatic stage.

↓

The automatic stage

The learner is now able to perform the process with little conscious thought or effort (Lieberman 2004:367). According to Berryman *et al.* (2002:121), automatisation is an important process in cognitive development. Lieberman (2004:367) describes automatisation as "the process by which a controlled process becomes automatic". These processes are automatic as they do not require a lot of attention. Automaticity thus refers to "the ability to process information with little or no effort" (Santrock 2009:271). The time and effort needed to perform tasks are dramatically reduced as automaticity develops. According to Berryman *et al.* (2003:121), automatisation is useful because it frees mental resources for solving other problems. When learners must consciously focus on basic operations, such as the adding or multiplication of fractions in mathematics, not enough working memory space remains to perform complex operations such as problem solving (see 2.7.2.2). In order to overcome this limitation of the working memory, it is thus important to practise basic skills to the point of automaticity.

Diagram 2.1: The three stages of procedural knowledge

Mitchell (1999:10) argues that before learners can apply procedural knowledge, they must identify conditions in a problem or exercise by practising procedural knowledge in context, which involves problems and exercises in realistic settings. With regard to mathematics, learners should, for example, learn mathematical operations by repeatedly solving realistic problems.

2.7.2.4 *Applying information processing theory in the classroom*

Santrock (2009:276) advocates that, when something new is presented or explained to learners, they need to be given some time to process this new information before the stimulus is changed again. Lecturers should allow students to think about something and ask only one question at a time, otherwise the memory trace for succeeding questions may be lost before students can attend to them. (Mitchell (1999:10) states that, when new mathematical concepts or calculations are presented to students, the lecturer should keep the explanations and descriptions short and slow enough to prevent overloading learners' working memories (see 2.7.2.2).

According to Snowman *et al.* (2009:219), organisation [see 2.7.2.2 (a)] and schemas [see 2.7.2.3 (c)] can be encouraged in learners when the lecturer highlights key points while explaining new mathematical concepts. Another example is organising textual material with heads and subheads (Lefrancois 2000:280). In developing complex networks and schemas, learners can be encouraged to explore relationships among ideas (Snowman *et al.* 2009:228). According to Snowman *et al.* (2009:227), new ideas should always be connected with previous learning. For example, if a student learned how to multiply fractions in mathematics, he/she can compare the process with the process of adding fractions. The lecturer can also teach students how to present information in an organised fashion, how to categorise (chunk) related information [see 2.7.2.2 (a)] and how to encode certain information [see 2.7.2.3 (a); (Santrock 2009:279; Snowman *et al.* 2009:227; Child 2007:190; Lefrancois 2000:280)].

2.7.2.5 *Concluding remarks*

In the preceding paragraphs, the researcher discussed information processing theory as a component of cognitive learning theory and how the sensory memory, working memory and long-term memory function to process information which has important implications for classroom teaching. The previous section also included a discussion pertaining influences on encoding and retrieval of information. In addition, the researcher highlighted the fact that learning requires the learner to engage in certain cognitive processes.

Snowman *et al.* (2009:212) argue that information-processing theory became a popular approach to the study of learning, because it provided psychologists with a framework for investigating the role of the nature of the learner – which behaviourism (see 2.6) ignored. With the information-processing theory, learners were now seen as highly active interpreters and manipulators of environmental stimuli, instead of being viewed as relatively passive organisms who responded in fairly predictive ways to environmental stimuli (Snowman *et al.* 2009:212). Some of these processes include paying attention, finding meaning in the stimuli received through the senses, retaining information through practice and making connections in the long-term memory.

According to Berryman *et al.* (2002:120), information processing theory is more like a framework for characterising a group of theories, rather like specific theories such as those of Piaget (see 2.10.1), or Vygotsky (see 2.11.1). However, the significance of information processing theory has grown concomitantly with a retreat from the Piagetian framework in cognitive development (Groen & Kieran 1983:352). The gestalt psychology (see 2.7.1) has also had an important historical influence on contemporary information processing views.

The next section extends the cognitive analysis by discussing cognitive learning theories and complex cognitive processing. The researcher will commence with a review of examples of early cognitive perspectives on learning, namely Bruner's theory of cognitive growth and Ausubel's meaningful reception learning. The discussion that follows also moves beyond that presented in the preceding paragraphs by covering other cognitive learning theories and topics relevant to learning that involve the operation of complex (higher-order) cognitive processes.

2.7.3 Bruner's theories of learning

Bruner's work came at a time when psychological thought was dominated by behaviourism (see 2.6). According to Hollyman (2000:2 of 4), Bruner was instrumental in the move from behaviourism to cognitivism (see 2.9) in the mainstream psychology of the 1950s and 1960s. Hollyman (2000:1 of 4) argues that Bruner has been a leader in the establishment of cognitive psychology as an alternative to the behaviourist learning theories that dominated psychology in the first half of the twentieth century.

2.7.3.1 *Discovery learning*

One of Bruner's contributions from the 1960s was the concept of discovery learning (Snowman *et al.* 2009:237). According to Jordan *et al.* (2008:57), Bruner adopted Piaget's ideas about active learning to form the basis of his principles of instruction and discovery learning. From there, Bruner moved to a more sophisticated constructivist position in "The culture of Education" (Bruner 1996), which examines the social importance of language and culture in meaning-making (Jordan *et al.* 2008:58).

Discovery learning stands in contrast to a direct-instruction approach, in which the instructor directly explains information to learners (Santrock 2009:439). In the education system, this means that an instructor would engage students in active dialogue and guide them when necessary so that they would progressively build their own knowledge base, rather than be "taught". In other words, students would gain a better understanding if they were allowed to pursue concepts on their own. Smith (2002:4 of 6) argues that Bruner makes the case for education as a knowledge-getting process: "To instruct someone … is not a matter of getting him to commit results to mind. Rather, it is to teach him to participate in the process that makes possible the establishment of knowledge. We teach a subject not to produce little living libraries on that subject, but rather to get a student to think mathematically for himself, to consider matters as an historian does, to take part in the process of knowledge-getting. Knowing is a process not a product" (Bruner 1966:72).

2.7.3.2 *Bruner's three stages of cognitive growth*

In 1964, Bruner formulated a theory of cognitive growth that postulates: "The development of human intellectual functioning from infancy to such perfection as it may reach is shaped by a series of technological advances in the use of mind" (Bruner 1964:1). In contrast to the approach of Piaget (see 2.10.1), Bruner looked at environmental factors as well as experiential factors and suggested that intellectual ability developed in stages through step-by-step changes in how the mind is used (Smith 2002:2 of 6). As a structural theorist, Bruner believes that knowledge is most effectively gained by personal discovery and is then classified enactively, iconically or symbolically. Bruner (1966) argues that, for people to become mature thinkers, they must acquire three major intellectual skills for representing the world (Lefrancois 2000:197). Bruner thus identified three stages of cognitive growth, namely the enactive stage, the iconic stage and the symbolic stage (Jordan *et al.* 2008:58).

According to Child (2007:130), *enactive* representations are the earliest stage of development postulated by Bruner. Young children rely extensively upon enactive modes to learn in their very early years. For example, children learn to walk through their own actions. Although this mode is present in people of all ages, it is more dominant when a person is young (Hollyman 2000:3 of 4). This dominance can be illustrated by the way a young person can often learn to play a musical instrument more quickly than an older person does.

During the next stage of childhood years, the *iconic* representation normally becomes dominant. In Bruner's view, we actually recall a composite sound by combining previous similar experiences, which Bruner calls the iconic representation (Child 2007:83). According to Lefrancois (2000:196), "the iconic representation involves the use of mental images that stand for certain objects or events". Killen and Hattingh (2004:75) argue that during the iconic mode, "learners gain the capacity to form internal pictures, images or 'icons' and use words to represent real objects and to facilitate thought". This mode of thinking also facilitates quite sophisticated activities, such as problem solving (see Chapter 3) in areas such as mathematics (Killen & Hattingh 2004:75).

The *symbolic* mode of learning becomes more dominant later, usually around adolescence. During this stage, students can understand and work with concepts that are abstract. Knowledge, when in the symbolic stage, is mostly in the form of arbitrary words, mathematical symbols and other symbol systems (Overbaugh 2004:3 of 5).

According to Lefrancois (2000:197), the enactive, iconic, and symbolic representations develop sequentially. For Bruner, developmental growth involves mastering each of these increasingly more complex modes, from enactive to iconic to symbolic. When faced with new material, Bruner's theory is efficacious in following a progression between the three stages. This holds true, even for adult learners. By mastering each of these modes, the learner becomes more skilled in translating from mode to mode (Child 2007:131).

2.7.3.3 *Categorisation*

Lefrancois (2000:197) argues that one of the principal concerns of Bruner's theory, is how people build up and use representations. In sharp contrast to the beliefs of Piaget (see 2.10.1) and other stage theorists, Bruner's ideas are based on categorisation and, according to him, a learner is capable of learning any material as long as the instruction is organised appropriately (Lefrancois 2000:200). According to Bruner (1960), "to perceive is to categorize, to conceptualize is to categorize, to learn is to form categories, to make decisions is to categorize". Bruner's theories therefore introduced the idea that people interpret the world largely in terms of its differences and similarities, which is a significant contribution to the understanding of how individuals construct their unique models of the world.

Like Bloom's taxonomy, Bruner suggests a system of coding in which learners form a hierarchical arrangement of related categories (Lefrancois 2000:209). As each successive higher level of category becomes more specific, Bloom's understanding of knowledge acquisition as well as the related idea of instructional scaffolding (see 2.11.1.4) is echoed. Downs (1994:173) advocates that it is the cognitive coding system that makes perception possible: "As we encounter ideas, events, or experiences, our mental response is to conceptualize them through the process of categorization, which is perception" (Downs 1994:173). As Lefrancois (2000:209) states: "[a] curriculum directed towards conceptual change presents problems and puzzles, challenges old ideas, and leads to the continual construction and reorganization of knowledge".

2.7.3.4 *Bruner's constructivist theory*

Bruner was one of the founding fathers of constructivism. According to Overbaugh (2004:2 of 5), Bruner's theory of constructivism falls into the cognitive domain and was influenced by the earlier theoretical research of both Vygotsky (see 2.11.1) and Piaget (see 2.10.1). According to Bruner, people remember things "with a view towards meaning and signification, not toward the end of somehow 'preserving' the facts themselves" (Hollyman 2000:1 of 4). For Bruner, learning is an active process in which learners construct new ideas or concepts based upon their existing knowledge. By relying on their cognitive structures, learners select and transform information, construct hypotheses, and make decisions.

According to Jordan *et al.* (2008:59), Bruner proposed a theory of instruction in 1966, "which is based on structuring and sequencing material in accordance with cognitivist ideas of mental processing". As a constructed entity, this view of knowledge is consistent with constructivism (see section 2.9). Learners' cognitive structures provide meaning and organisation to experiences and allow them to "go beyond the information given" (*Psychology* 2009:1 of 2). Bruner (1973) illustrated his theory in the context of mathematics for young children by means of the following example: "The concept of prime numbers appears to be more readily grasped when the child, through construction, discovers that certain handfuls of beans cannot be laid out in completed rows and columns. Such quantities have either to be laid out in a single file or in an incomplete row-column design in which there is always one extra or one too few to fill the pattern. These patterns, the child learns, happen to be called prime. It is easy for the child to go from this step to the recognition that a multiple table, so called, is a record sheet of quantities in completed multiple rows and columns. Here is factoring, multiplication and primes in a construction that can be visualized" (Bruner 1973).

Although Bruner's constructivist theory is a general framework for instruction based upon the study of cognition, constructivism is a very broad conceptual framework in philosophy and science and Bruner's theory represents only one particular perspective in this regard (see 2.9).

2.7.3.5 *Applying Bruner's theory to the classroom*

According to Jordan *et al.* (2008:58), "Bruner (1966) proposed a theory of instruction, which is based on structuring and sequencing material in accordance with cognitivist ideas of mental processing". Bruner also developed a curriculum model, which is based on a spiral (Jordan *et al.* 2008:58). According to Smith (2002:3 of 6), Bruner (1960:11-16) advocated that a theory of instruction should address four major aspects:

1. Predispositions towards learning: In other words, what experiences move the learner toward a love of learning in general, or of learning something in particular? Some factors that can contribute to this may be motivational, cultural and personal. Early teachers as well as parents' influence and social factors may also contribute to predispositions towards learning. This notion of *readiness for learning* underpins the idea of a spiral curriculum.

2. The ways in which a body of *knowledge* can be *structured* so that it can be most readily grasped by the learner: When one understands the fundamental structure of a subject, it makes it more comprehensible. In the structuring of knowledge, Bruner viewed *categorisation* as a fundamental process (see 2.7.3.3).
3. The most effective *sequences* in which to present new material to be learned: Sequencing in any subject, or lack of it, can make learning easier or more difficult.
4. *Reinforcement*: The nature of rewards and punishments should be selected and paced appropriately.

A major theme in the theoretical framework of Bruner is that learning is an *active* process in which learners construct new ideas or concepts based upon their *existing knowledge* (*Psychology* 2009:1 of 2). According to Child (2007:107), this means that new information would be classified and understood based on knowledge already gained. For Bruner (in Snowman *et al.* 2009:237), true learning involves "figuring out how to use what you already know in order to go beyond what you already think" (Bruner 1983:183). Lefrancois (2000:209) argues that in terms of this theory, learners engage in discovery learning, obtaining knowledge by themselves. Learners select and transform information, construct hypotheses, and make decisions by relying on a *cognitive structure*. However, in order for discovery learning to occur, learners require background preparation in the form of cognitive structures that provide meaning and organisation to experiences that allow the individual to "go beyond the information given" (Santrock 2009:237). As far as instruction is concerned, the lecturer should therefore try and encourage students to *discover* principles by themselves, where the task of the lecturer is also to translate information to be learned into a format appropriate to the student's current state of understanding (Lefrancois 2000:209).

Bruner's theory can be applied to subject matter by using a *spiral curriculum*, in which learners build on previous construction of knowledge to formulate more useful associations and authentic meanings (Lefrancois 2000:209). The notion of *readiness for learning* underpins the idea of the spiral curriculum (Jordan *et al.* 2008:58). In his book

The Process of Education, Bruner (1960:11-16) advocates that "[a] curriculum as it develops should revisit these basic ideas repeatedly, building upon them until the student has grasped the full format apparatus that goes with them" (Bruner 1960:13).

This means that the learner should be exposed to subject matter in such a way as to ensure both *overlap* with previous material (repetition and revision) and a steady *progression* in the complexity of the material – hence a spiral effect (Child 2007:453). Bruner (1960:11-16) stresses the importance of structure in learning by arguing that "[t]he teaching and learning of structure, rather than simply the mastery of facts and techniques, is at the center of the classic problem of transfer.... If earlier learning is to render later learning easier, it must do so by providing a general picture in terms of which the relations between things encountered earlier and later are made as clear as possible" (Bruner 1960:12). The lecturer can also provide input and structure. According to *Psychology* (2009:1 of 2), good methods for structuring should result in simplifying, generating new propositions, and increasing the manipulation of information. If the curriculum is organised in a spiral manner, the student will continually build upon what he or she has already learned.

2.7.3.6 *Concluding remarks*

Much of Bruner's theory can be linked to child development research, especially that of Piaget (see 2.10.1.1). Piaget related each mode to a specific period of childhood development (see 2.10.1.1), while Bruner saw each mode as dominant during each developmental phase (Smith 2002:3 of 6). Bruner's model of human development as a combination of enactive, iconic and symbolic skills has influenced psychological and educational thought over the past fifty years. According to Smith (2002:4 of 6), Bruner has had a profound effect on education. In this regard, Howard Gardner commented: "Jerome Bruner is not merely one of the foremost educational thinkers of the era; he is also an inspired learner and teacher. His infectious curiosity inspires all who are not completely jaded. Individuals of every age and background are invited to join in. In his words, '[i]ntellectual activity is anywhere and everywhere, whether at the frontier of knowledge or in a third-grade classroom'. To those who know him, Bruner remains the Complete Educator in the flesh..." (Gardner 2001:94).

2.7.4 Meaningful reception learning

David Ausubel (1963; 1968; Ausubel & Robinson 1969), one of the first learning psychologists to adopt the cognitive view of learning, favoured the implementation of meaningful reception learning. Ausubel (1968:83) purports that "the acquisition of subject-matter knowledge is primarily a manifestation of reception learning". That is, the principal content of what is to be learned is typically presented to the learner in more or less final form. Under these circumstances, the learner is simply required to comprehend the material and to incorporate it into his/her cognitive structure so that it is available for either reproduction, related learning, or problem solving at some future date.

2.7.4.1 *Meaningful learning*

Child (2007:453) argues that the concepts of meaningful learning and advanced organisers are at the heart of Ausubel's theory. The primary concept in Ausubel's theory is meaningful learning, as contrasted with rote learning (see 2.4.1). Ausubel's theory supports the concept of deep learning (see 2.4.2). According to Ausubel (Ausubel, Novak & Hanesian 1978), meaningful learning occurs when a learner encounters clear, logically organised material and consciously tries to relate the new material to ideas and experiences stored in long-term memory (see 2.7.2.3). Novak and Gowin (1984:7) purport that, in rote learning (see 2.4.1), new knowledge may be acquired simply by verbatim memorisation and arbitrarily incorporated into a person's knowledge structure without interacting with what is already there. According to Biggs and Tang (2007:106), building a well-structured knowledge base involves what Ausubel (1968) calls "reception learning". What this means is the reception of declarative knowledge and structuring it meaningfully (see 2.7.2.3). According to Driscoll (in Snowman *et al.* 2009:219), the basic idea behind meaningful learning is that the learner actively attempts to associate new ideas with existing ones.

According to Novak and Gowin (1984:4), the construction of knowledge begins with our observations of events or objects through the concepts we already possess. In other words, material must be related to existing knowledge in order for it to be meaningful (Child 2007:453). Schunk (1996:198) states that learning is meaningful when new material bears a systematic relation to relevant concepts in the long-term memory (see 2.7.2.3). In other words, new material expands, modifies, or elaborates information in the memory. Schunk (1996:198) also adds that, for individuals to learn meaningfully, they must choose to relate new knowledge to relevant concepts and propositions they already know (see 2.7.2.3).

Meaningful learning is related to deep learning, as students who follow a deep approach to learning engage in tasks appropriately and meaningfully (see 2.4.2).

2.7.4.2 *Advance organisers*

A major instructional mechanism, proposed by Ausubel (1963), is the use of advance organisers. According to Leonard (2002:5), "advance organizers are abstracts, outlines, and introductions of a large body of content that help structure and organize the content to be taught". Santrock (2009:429) argues that "[a]dvance organizers are teaching activities and techniques that establish a framework and orient students to material before it is presented. Ausubel (1963:81) argues that "these organizers are introduced in advance of learning itself, and are also presented at a higher level of abstraction, generality, and inclusiveness; and since the substantive content of a given organizer or series of organizers is selected on the basis of its suitability for explaining, integrating and interrelating the material they precede, this strategy simultaneously satisfies the substantive as well as the programming criteria for enhancing the organization strength of cognitive structure".

2.7.4.3 *Concept maps*

Concept maps were originally designed to present both a structure and a method to find out how students see the structure [Novak (in Biggs and Tang 2007:115)]. Santrock (2009:313) defines a concept map as "a visual presentation of a concept's connections and hierarchical organization". Showman *et al.* (2009:286) argue that concept mapping "is a technique that helps students identify, visually organize, and represent the relationships among a set of ideas". According to Novak and Gowin (1984:15), concept maps are intended to represent meaningful relationships between concepts in the form of propositions (see 2.7.2.3).

Biggs and Tang (2007:115) purport that concept maps can be used by students for organising their ideas or for clarifying difficult passages. According to Biggs and Tang (2007:117), concept maps present an overall picture, a holistic representation of a complex conceptual structure and can be used by students in their own studying. When students make use of concept maps, it allows them to see the whole perceived relationship (*gestalt*). This emphasises the importance of the gestalt learning theory (see 2.7.1). In a learning experience, concept maps help students to explicitly structure their thinking. According to Novak and Gowin (1984:15), a concept map can also provide a kind of visual road map showing some of the pathways we may take to connect meaning of concepts in propositions, in other words, give an indication of how the student sees the way in which individual concepts relate to one another (see 2.7.2.3).

Ausubel's research (1968; 1978) shows that the use of organisers promotes learning over that which occurs without organisers, especially regarding lessons designed to teach how concepts are related (see 2.4.2). Ausubel argues that knowledge is organised into hierarchical structures in which subordinate concepts are not only related to one another, but arranged under higher-level superordinate concepts (Mitchell 1999:22). This means that we tend to remember key ideas associated with a particular cognitive structure and retain the structure, even if we gradually forget some details. According to Mitchell (1999:22), the structure provides scaffolding that supports retention of the information as an organised body of knowledge and also functions as a frame within which to interpret new knowledge or relearned forgotten knowledge.

2.7.4.4 *Applying meaningful learning in the classroom*

Biggs and Tang (2007:107) argue that learning activities for reception learning (meaningful learning) can be managed by the university teacher by means of lecturing, tutorials, concept mapping, or teaching study skills to students. According to Snowman *et al.* (2009:255), instructors should present information in an organised fashion and in meaningful contexts. Mitchell (1999:22) and Santrock (2009:315) maintain that meaningful learning thus requires instructors to present the material in ways that encourage learners to make sense of it by relating it to what they already know and not just by memorising it in a rote fashion (see 2.4.1).

According to Child (2007:453), instructional materials should attempt to integrate new concepts with previously presented information through comparisons and cross-referencing of new and old ideas. The emphasis should be on understanding relationships among ideas, relationships between ideas and prior knowledge (Snowman *et al.* 2009:255). Therefore, meaningful learning will be retained longer, be better integrated with other knowledge, and be more readily available for application.

Santrock (2009:429) advocates that instructors make use of advance organisers when they begin a lesson to help students see the "big picture" of what is to come and how information is meaningfully connected. The researcher believes that when the material to be learned is not well organised and learners lack the knowledge needed to be able to organise it well for themselves, the lecturer can show the learners how to make use of frameworks within which new information can be assimilated (Santrock 2009:429). The lecturer can first present the most general ideas of a subject and then progressively differentiate in terms of detail and specificity.

According to Mitchell (1999:23), learners can apply meaningful learning by organising content in logical ways, for example by presenting key concepts, outlining the content, formulating questions that establish a specific learning set, indicating connections between parts of contents, and including summaries at the end of lessons (Child 2007:453; Snowman *et al.* 2009:226-228; Santrock 2009:292-295). The key concepts are to be broken down in small organised steps, be sequenced logically and new concepts have to be linked to familiar ones. According to Snowman *et al.* (2009:408), the student can manage his/her learning activities through collaborative learning groups, interactive work in class, or by attending peer-assisted study sessions (see 2.11.1.1).

According to Novak and Gowin (1984:7), the theory proposed by Ausubel (1963; 1968) is the best learning theory, because it focuses on concepts and propositional learning as the basis on which individuals construct their own idiosyncratic meanings. Ausubel's theory also has commonalities with gestalt theories (see 2.7.1) as well as with those that involve schemas as central principles [see 2.7.2.3(c)].

2.8 SOCIAL COGNITIVE THEORY

Schunk (1996:102) argues that social cognitive theory highlights the idea that much human learning occurs in a social environment. People learn about the usefulness and appropriateness of behaviours by observing models and the consequences of modelled behaviours. Consequently they act in accordance with their beliefs concerning the expected outcomes of actions.

Wildemeersch (1999:39) defines social learning as "combined learning and problem-solving activities which take place within participatory systems such as groups, social networks, movements and collectives, operating within 'real life' contexts and thereby raising issues of social responsibility". According to Landsberg (2005:228), this collaboration among learners goes beyond obtaining information from experts and/or just working with someone, but also involves the manner in which people work as a team to accomplish shared and clear goals.

According to Snowman *et al.* (2009:270), Albert Bandura (1977; 1993; 2001; 2006) is generally considered to be the driving force behind social cognitive theory. The next section therefore focuses on Bandura's social cognitive theory and the researcher highlights the modelling processes which play a prominent role in learning from a social cognitive perspective (Schunk 1996:103). The researcher eventually continues this discussion with reference to other important cognitive learning theories.

2.8.1 Bandura's social cognitive theory

Weiten, Lloyd, Dunn and Hammer (2009:48) are of the opinion that Bandura is one of several theorists who have added a cognitive flavour to behaviourism since the 1960s. According to Bigge (1982:155), Bandura's social learning theory consists of a blending of behaviouristic reinforcement theory (see 2.6), as well as purposive cognitive psychology (see 2.7) and aims at a balanced synthesis of cognitive psychology with the principles of behaviour modification.

According to Bandura's social learning theory (1977), learning centres in the reinforcement process, which consists of people developing self-activated, cognitive mediated expectations through grasping the consequences of both direct and observational experience (Bandura 1977:22; Bigge 1982:161; Child 2007:463; Snowman *et al.* 2009:271). Bandura (1977:22) stated that "[r]einforcement provides an effective means of regulating behaviours that have already been learned".

Bandura's work was primarily linked to the type of interaction processes that are called *imitation*. According to Jordan *et al.* (2008:60), Bandura proposed that imitation of others is a cognitive efficient means of learning. This Canadian phychologist's theory is two-pronged, as he argues that children learn by observing others and that they will learn from "model" behaviour if that leads to positive outcomes (Bandura 1977:22). According to Kincheloe and Horn (2008:52), social modelling, the self-system, and self-regulation are three very important concepts in Bandura's social cognition theory. These concepts are discussed in the next paragraphs.

2.8.1.1 *Imitation and modelling*

According to Child (2007:460), "[t]he study of motivation and learning as a consequence of social interaction and imitation has become a fruitful area of investigation within social cognitive theory". Bandura believes that learners learn behaviour by observing (imitating) the actions of important people in their lives (Mitchell 1999:56). Lefrancois (2000:123) defines imitation as "a type of emitted behaviour that occurs as a function of observing a model". Bandura (1977:22) presents the view that "most human behaviour is learned observationally through modeling: from observing others one forms an idea of how new behaviours are performed".

In Bandura's view (1977:13), "individuals use language and symbols to translate their observations of socially modeled behaviours into guides for future actions" (Kincehloe & Horn 2008:52). Learners store these observations in the form of mental images and then imitate them, which is therefore another way of learning. Bandura (2006:2) argues that "[a]lthough much social learning is fostered through observation of real-life models, advances in communication have increased reliance upon symbolic models. In many instances people pattern their behaviour after models presented in verbal or pictorial form". According to Bandura (2006:2), members of technologically advanced societies would spend much of their time groping for effective ways of handling situations that arise repeatedly, without the guidance of textbooks that describe in detail how to behave in particular situations.

Child (2007:460) argues that modelling occurs when an individual's behaviour changes as a result of observing another person's behaviour. According to Jordan *et al.* (2008:60), "[M]odeling is part of all learning ... and involves imitative rather than original behaviour, but it can be seen in a constructivist light – that is, people adapt modeled behaviour as a mental framework for their own purposes". As mathematical problem-solving ability is a difficult skill for many students to master, it has often been studied by social cognitive researchers who are interested in the effects of modelling (Snowman *et al.* 2009:293).

According to Bandura (1977:12; 1977:79), successful learning requires two other components, which are the learner's sense of self-efficacy and the learner's system of self-regulation. According to Snowman *et al.* (2009:293), "[s]ocial cognitive theory holds that self-efficacy and self-regulation processes should be positively related to each other and that both should be positively related to achievement".

2.8.1.2 *Self-efficacy*

According to Bandura (1995:2), self-efficacy is "the belief in one's capabilities to organize and execute the courses of action required to manage prospective situations". Schunk (1996:131) defines self-efficacy as "a belief about what one is capable of doing"; in other words, a personal belief about how successfully one can deal with a prospective difficult situation, such as a test, an interview or a contest. Bandura (1993:118) argues that "[e]fficacy beliefs influence how people feel, think, motivate themselves, and behave". If a person possesses a high degree of self-efficacy he/she will learn successfully through observation and imitation (Mitchell 1999:60). This will also have an influence on the learner's choice of tasks, quality of performance and persistence.

Zimmerman, Bandura, and Martinez-Pons (1992:664) argue that "numerous studies have shown that students with a high sense of academic efficacy display greater persistence, effort, and intrinsic interest in their academic learning and performance". Snowman *et al.* (2009:271) argue that "[a] student's high self-efficacy for mathematical problem-solving leads the student to work on mathematical problems rather than painting a picture. High self-efficacy also makes the student likely to persist in the face of difficulty". On the other hand, learners with particularly low self-efficacy frequently express pessimistic views toward learning (Akiba & Alkins 2010:64), for example, "I will never be able to pass maths".

According to Snowman *et al.* (2009:273), Bandura believed that self-efficacy is more influential than expected rewards or punishments or actual skills because it is based on a belief that one can or cannot produce the behaviours that are required to bring about a particular outcome. Students with the same mathematical skills may, for example, have different attitudes to mathematics and perform differently in tests of mathematical problem solving, because of differences in their self-efficacy beliefs (Bandura 2001; Martin 2004).

By exposing students to adult models (see 2.8.1.1) and by observing similar peers (see 2.11.1.1) performing a task can instil a sense of self-efficacy in learners and also increase students' self-efficacy for learning (Schunk 1996:134; Santrock 2009:473). This idea can be applied in the mathematics classroom when the lecturer selects certain students to complete mathematics problems on the chalkboard. By demonstrating success, the peer models may help raise students' self-efficacy for performing well. Students with learning difficulties who have mastered the skills may also serve as excellent models. The lecturer can, for example, place students who have been working hard and are gradually developing mastery, with students who are still having difficulties.

2.8.1.3 *Self-regulation*

Kincheloe and Horn (2008:52) define self-regulation as "the individual's ability to control his or her behaviour". According to Snowman *et al.* (2009:272), self-regulation "involves the consistent and appropriate application of self-control skills to new situations". According to Snowman *et al.* (2009:279), self-regulated learning refers to "any thoughts, feelings, or actions that are purposely generated and controlled by a student to maximize learning of knowledge and skills for a given task and set of conditions". Santrock (2009:257) argues that "[s]elf-regulated learning consists of the self-generation and self-monitoring of thoughts, feelings, and behaviours in order to reach a goal". The self-regulated learner is also referred to as self-directed, autonomous, or strategic (Snowman *et al.* 2009:279).

According to Bandura (1977:13), self-regulated learners are able to set and regulate their own standards of performance and to establish a system whereby they are able to regulate their progress through self-observation and self-judgement. Mitchell (1999:60) purports that, once learners have established their own systems and standards of evaluation, they are capable of responding effectively in order to promote their own learning progress, and will spend more time trying to improve their performance until they are satisfied with their achievements.

According to Child (2007:260), students' perceptions of themselves as learners and the self-value or esteem generated from this image will affect their approach and level of performance in solving problems. According to Snowman *et al.* (2009:273), students who believe that they are capable of successfully performing a task, are more likely to use self-regulating skills than students with low levels of self-efficacy. Research by Schunk (2008) and Schunk, Pintrich and Meece (2008) has found that high-achieving students are often self-regulatory learners (Santrock 2009:257).

2.8.1.4 *Applying social theory in the classroom*

In social learning theories, the essential components of learning are a model, reinforcement of the modelled behaviour and the learner's cognitive processing of the modelled behaviour (Mitchell 1999:60). Snowman *et al.* (2009:300) argue that, with regard to education in the classroom, both the teacher and the learners can serve as live models for a variety of social and academic behaviours.

Schunk (1996:115) argues that teachers serve as models when they help students acquire skills. According to Lefrancois (2000:129), students might be taught something new by being shown what to do (modelling effect). Learning a new behaviour in mathematics might, for example, be learning how to solve algebraic equations. Much of this kind of learning is accomplished by observing someone else's behaviour, as students would observe how the teacher is doing the new calculation in mathematics. According to Santrock (2009:255), the modelled demonstrations are typically incorporated into lessons in which students are taught diverse skills such as solving mathematical problems. Live models thus have the advantage that the desired behaviour can be demonstrated in front of the learners as well as that the learners gain opportunities to ask questions (Mitchell 1999:61).

According to Child (2007:463), the presentation of the modelled behaviour serves as a predictor of reinforcement for the observer in conceptual learning. What is important though, is that the behaviour to be learned must be interesting to the observer and must be portrayed at a level of complexity that the learner can understand (Santrock 2009:255). In mathematics instruction, the lecturer should not only talk about the pleasure gained when a problem can be solved, but also demonstrate his or her own enjoyment and interest when solving problems in front of the learners (Snowman *et al.* 2009:315). This is important for instruction, as social learning theories maintain that individuals pay attention to events in the environment that predict positive consequences, which have a functional value for the learner and will be observed closely (Bandura 1977:17).

According to Snowman *et al.* (2009:293), modelling is seen as an effective means of enhancing self-efficacy, teaching students how to use self-regulation skills, and increasing achievement. According to Santrock (2009:257), self-monitoring is also an excellent strategy for improving learning and one that instructors can use to help students learn to do effectively. Snowman *et al.* (2009:301) argue that students can be encouraged by means of the process of self-monitoring by requiring them to keep a log or a journal in which they stage goals, note how they prepare for an upcoming test, and access the extent to which they have achieved one or more goals.

2.8.1.5 *Concluding remarks*

A philosophical shift in the 1960s and 1970s from behaviourism (see 2.6) to various forms of cognitivism (see 2.7) induced exciting changes. According to Lefrancois (2000:193), the shift to cognitivism also saw a shift from an emphasis on animal research to a renewed emphasis on human research. Cognitive psychology renewed its interest in concept formation, complex problem solving, and the connection between cognitive structures and behaviour. According to Jordan *et al.* (2008:51), "[c]ognitivism presents a scientific approach to learning and offers a coherent understanding of the processes involved". However, the focus on learning with regard to cognitivism ignores social processes and embodiment. Jordan *et al.* (2008:51) present the view that cognitivism can be seen as a progressive step toward an approach that combines cognitive processes with the element of individual and shared meaning-making. That, of course, is constructivism, which the researcher will elaborate on in the next section.

2.9 CONSTRUCTIVISM

In the 1950s and 1960s no one had heard of "constructivism", as the dominant psychology was "stimulus response" theory (see 2.6.2), which held that what was going on in someone's mind was unscientific, because it speculated about matters that were essentially unknowable (Davis *et al.* 1990:104). According to Boghossian (2006:715), behaviourism (see 2.6) is diametrically opposed to constructivism. The difference between the behavioural and constructivist learning theories can be defined as follows: "Constructivists argue that there are multiple realities constructed by individuals. The human mind does not copy reality from outside directly, rather, it constructs reality" [Driscoll (in Boghossian 2006:715)].

Constructivism, as a development within cognitive psychology, emphasises that human learning is active and constructive and stresses the importance of the contextual character of human cognition [Van den Broeck, Opdenakker & Van Damme (in Howie & Plomp 2006:84)]. Confrey (1990:108) describes constructivism as "essentially a theory about the limits of human knowledge, a belief that all knowledge is necessarily a product of our own cognitive acts". Worlley and Proctor (2005:4 of 22) present the view that in constructivism, knowledge is not passively received by the student as it is passed on verbally from the lecturer, but rather actively constructed within the social context of the classroom community.

Constructivist learning theories argue that learners actively construct frameworks of understanding, by using both the knowledge they already possess, and new information that is presented to them (Mahar & Harford 2004:8; Jordan *et al.* 2008:55). Snowman *et al.* (2009:237) argue that "[t]o constructivists, meaningful learning is the active creation of knowledge structures from personal experience". According to Leonard (2002:2), "the Chinese philosopher, Lao-tse, simply defined the essence of active learning when he said: 'If you tell me, I will listen. If you show me, I will see. But if you let me experience, I will learn'". Thus, for effective learning, individuals engage their prior ideas and re-work them as new information comes along. This makes learning an active process in which learners develop new ideas and concepts based upon their existing knowledge (Von Glaserfield 1989). Knowledge is thus received by reflecting on our experiences and by fitting new information together with what we already know.

According to Biggs and Tang (2007:28), constructivism focuses particularly on the nature of learning activities the student uses. Van den Broeck *et al.* (in Howie & Plomp 2006:84) argue that constructivism and self-regulation of learning processes underline the responsibility of the learner for his own learning processes (see 2.8.1.3). In other words, knowledge and skills are constructed by students themselves during the learning process. Lefrancois (2000:336) argues that "[t]he constructivist learner is a self-motivated, mastery-orientated learner, driven by a need to know, to organize, to understand, to build meaning". In the constructivist theory the emphasis is placed on the learner rather than on the instructor, and on guiding rather than telling (Snowman *et al.* 2009:242).

In the next section, the researcher will focus on the two major strands of the constructivist perspective, which are cognitive constructivism and social constructivism.

2.10 COGNITIVE CONSTRUCTIVISM

According to Jordan *et al.* (2008:57), the work of Jean Piaget and Jerome Bruner (see section 2.7.3) is most often associated with cognitive constructivism.

2.10.1 Piaget

Born in Switzerland in 1896, Jean Piaget (1896-1980) began a career that would have a profound impact on both psychology and education. According to Kenny (1996:1 of 16), Piaget was the founding father of a branch of psychology that tries to unravel the mysteries of the human mind, how it grows and how it comes to know. Today, Piaget is generally acknowledged to be one of the founding fathers of psychology – as the English sociologist Anthony Giddens aptly puts it: "...the influence of Jean Piaget's work has been not far short of that of Freud." [Giddens (in Illeris 2004:27)].

Kenny (1996:1 of 16) argues that when Piaget was first discovered in the United States – about 1940 – he was classified as a child psychologist. According to Cohen (2001:31), "Many psychologists believe Jean Piaget (1896-1980) was the greatest child psychologist of the twentieth century".

As the author of a theory that postulated four stages in the development of intelligence, Piaget was discovered once more twenty years later, and rediscovered for the third time, as the progenitor of constructivism (Kenny 1996:1 of 16). For Piaget, constructivism arose out of a profound dissatisfaction with the theories of knowledge in the tradition of Western philosophy (Von Glaserfeld 1995:6). According to Ernst (1994:1), Piaget's constructivism has its roots in an evolutionary biological metaphor, according to which the evolving organism must adapt to its environment in order to survive. According to Hilton (2002:64), constructivism has its roots in Jean Piaget's groundbreaking research on cognitive development in children. Piaget was probably the most influential theorist of learning and his work on children's cognitive development has garnered much attention within the field of education. Although the work of Piaget is mostly associated with children's cognitive development, the researcher is of the opinion that Piaget's stages in the development of intelligence can also be applied to young adult students. In general, one is never too old to learn.

Piaget trained as both zoologist and mathematician in Geneva, where he and his co-workers investigated the development of organised mental logical structures. According to Lefrancois (2000:215), Piaget's description of changes in structure "is a description of the stages of human cognitive development". According to Chapman (1972:144), cognitive structures are built through the child interacting with his environment and are enlarged through further experience.

In Piaget's cognitive constructivist approach, "students construct knowledge by transforming, organizing, and reorganizing previous knowledge and information" (Santrock 2009:349). According to Piaget, learning is provoked by situations, whether by a psychological experiment; or by a teacher with respect to some didactic point; or by an external situation. It is provoked in general, as opposed to spontaneously. In addition, it is a limited process – to a single problem, or to a single structure [Piaget (in Steffe & Tzur 1994:9)]. This view of learning as being provoked by situations is compatible with the understanding of human beings as interactive organisms.

Piaget also believed that a stimulus was a stimulus only when it was assimilated into a structure and it was the structure which set off the response. As these conceptual structures setting off the response were the products of development, learning could be understood only in terms of development. Seffe and Tzur (1994:10) argue that a particular structure might be provoked or learned in the case of mathematics if it is based on an existing, simpler developmental mathematical structure.

According to Clark (1999:2 of 7), Piaget's theory has two major parts, namely the "ages and stages" component that predicts what learners can and cannot understand at different ages. The second part is his theory of development that describes how learners develop cognitive abilities.

2.10.1.1 *Piaget's stage theory*

One contribution of this Swiss scholar's theory concerns the developmental stages of children's cognition, from which he concluded that children go through distinct stages of cognitive development (Lefrancois 2000:215). According to Atherton (2009:1 of 1), a *stage* refers to a period in a learner's development in which he or she is capable of understanding some things, but others not. For Piaget, the mental development of any learner consists of a succession of four stages, namely, the sensorimotor stage, the symbolic or preoperational stage, the concrete-operations stage, and the formal operations stage (Snowman *et al.* 2009:64; Santrock 2009:41; Child 2007:90; Lefrancois 2000:215; Feldman 2004:183). Lefrancois (2000:321) argues that "each of these stages is marked by characteristic abilities and errors in problem solving, results from activities and abilities of the preceding period, and is a preparation for the next stage". Feldman (2004:224) argues that the four stages of Piaget's theory of cognitive development are not knowledge per se, but rather the sets of mechanisms by which knowledge can be achieved.

In the next section, the researcher will describe the different stages of Piaget's cognitive development theory with the emphasis on their importance to mathematical development. As the focus of this study is on adolescents aged 17 and above, and not on children, the researcher will provide a detailed description of the last stage of Piaget's cognitive development. According to Santrock (2009:66), the formal-operational stage describes cognitive development in middle school, high school, and beyond. However, the researcher deems it necessary to elaborate on the concrete-operational stage, as this stage precedes the formal operational stage.

Table 2.1: Piaget's theory of cognitive development

Stage		Characteristic
Sensori-motor		Infants:
0-2 years	-	experience the world through movement and senses
	-	learn that objects continue to exist even when not in view
	-	begin to represent behaviours through mental imagery or language.
Pre-operational		Children:
2-6 years	-	build a mental model of the world
	-	are still egocentric, only seeing the world from their own point of view
	-	can perform mental operations such as addition only when objects are present.
Concrete		Children:
operational	-	generate rules and principles based on their actions on the world
6-12 years	-	are able to understand only rules of which they have had direct experience
	-	cannot yet use rules to generalise to situations not yet experienced
	-	develop an ability to see other points of view.
Formal		Young people:
operational	-	are able to reason in a purely abstract and scientific way
> 12 years	-	are able to generate hypotheses about the world
	-	are increasingly able to construct models that explain most experiences

Source: Jordan *et al.* (2008:119)

- *Concrete operational stage (7-11 years)*

According to Snowman *et al.* (2009:66), "Piaget described the stage from approximately seven to eleven years as that of concrete operations". Feldman (2004:207) is of the opinion that "concrete operations is also described as logical in the adult sense, conforming to most of the requirements of rational thought". Santrock (2009:69) argues that learners at the concrete operational stage can perform operations, and logical thought replaces intuitive thought when reasoning can be applied to specific concrete examples. Santrock (2009:45) furthermore states that classification skills are present at this stage, but abstract problems go unsolved. Snowman *et al.* (2009:66) argue that learners in the concrete operational stage are often more capable of learning advanced concepts than most people realise.

Ojose (2008:27) argues that, during the third stage of Piaget's theory, the child shows remarkable cognitive growth, while children's development of language and acquisition of basic skills accelerate dramatically. At this stage, children utilise their senses in order to know and can now consider two or three dimensions simultaneously instead of successfully (Ojose 2008:27). Snowman *et al.* (2009:66) argue that during the concrete operational stage, "[s]chemes are developing that allow a greater understanding of such logic-based tasks as conservation, class inclusion and seriation". According to Child (2007:96), conservation is crucial for reasoning at the concrete stage of operations. Child (2007:96) argues that learners would, for example, have to be aware that - no matter how one presented the problem 2 + 4 + 3 or (4 + 2 + 3) - it would still add up to the same quantity. According to Lefrancois (2000:224), learners at this stage also develop the ability to deal with numbers, through the processes of classification and seriation. During this stage, two logical operations, namely *seriation* and *classification,* develop. Both of these are essential for understanding number concepts.

According to Ojose (2008:27), *seriation* is the ability to order objects according to increasing or decreasing length, weight, or volume. *Classification*, on the other hand, involves grouping objects on the basis of a common characteristic (Ojose 2008:27).

Burns and Silbey (2000:55) argue that "hands-on experiences and multiple ways of representing a mathematical solution can be ways of fostering the development of this cognitive stage". The hands-on activities provide learners with an avenue to make abstract ideas concrete, which allows them to get their hands on mathematical ideas and concepts as useful tools for problem solving.

According to Osjose (2008:28), mathematical instructors should engender learners with opportunities to present mathematical solutions in multiple ways, for example symbols, graphs, tables, and words which can be used as tools for cognitive development at this stage. Providing learners with various mathematical representations also acknowledges the uniqueness and diversity of learners and provides multiple paths for making ideas meaningful (Osjose 2008:28).

According to Feldman (2004:224), most people tend to function more with a concrete operational mind in most areas of activity than a rational, formal operational one. Snowman *et al.* (2009:69) present the view that, for students to grasp formal operational reasoning, they should be instructed in the concrete operational stage while making use of concrete operational schemes.

- *Formal operational stage (11 years and up)*

According to Feldman (2004:212), the formal operational stage is the most controversial of Piaget's stages and "represents the end point in the development of major, overall systems for understanding experience and interpreting information". The formal operational stage is achieved during adolescence and sets the constraints and establishes the rules that provide the basis for all subsequent thought development (Feldman 2004:212). Feldman (2004:212) adds that "[f]ormal operations are the culmination of the process of development of cognitive structures and the pinnacle of human reasoning".

For Piaget, learners are at the stage of formal operations if they reach the point of being able to generalise and engage in mental trial and error by thinking up hypotheses and testing them in their heads (Snowman *et al.* 2009:66). In the Piagetian account, formal thought is the highest and most powerful form of reasoning available to human beings (Feldman 2004:212). Learners develop the ability to think abstractly and metacognitively (see 3.8.3) and are also able to think hypothetically during this final stage of Piaget's theory. Lefrancois (2000:225) argues that adolescents, in contrast to children described in the previous three stages, are potentially capable of dealing with the hypothetical. During this stage, skills such as logical thought, deductive reasoning, and systematic planning also emerge (Santrock 2009:46). According to Santrock (2009:66), "Piaget's term hypothetical-deductive reasoning embodies the concept that adolescents can develop hypotheses about ways to solve problems (see 3.8) and systematically reach a conclusion". Without the necessity of perceptive data, the learner begins to develop abstract thought patterns through which reasoning is executed by using pure symbols (see 3.6). According to Killen and Hattingh (2004:76), a learner in this stage may be capable of formal operations in mathematics.

As formal mathematics is a highly precise and logical body of knowledge, written procedures greatly increase calculation efficiency and provide long-lasting records. By forming hypotheses and deducing possible consequences, the learner is capable of constructing his own mathematics during this stage. Child (2007:99) argues that, without having to refer to a concrete situation by the mathematics instructor, the formal operational leaner can solve mathematical operations such as $x + 5x = 12$.

According to Bringuier, Fischer and Pipp (in Feldman 2004:212), Piaget did not believe that development came to an end with the achievement of formal operations, but that the structures could be infinitely elaborated, extended, differentiated, compounded and transformed. In other words, the basic operating system remains in place for the rest of the individual's lifetime. In this respect the formal operational stage can thus be seen as different from the three preceding it (Feldman 2004:212).

Van Wagner (2005:1 of 1) purports that, in Piaget's view, early cognitive development involves processes based upon actions and later progresses into changes in mental operations. According to Lefrancois (2000:229), the learner is given a new set of mental "tools" with which to process information at each stage of development, while the teacher's task is to match appropriate content to the developmental stage of the learner. From his observations, Piaget also concluded that children are not less intelligent than adults, they simply think differently (Van Wagner 2005:1 of 1).

Piaget's stage theory is applicable only to conceptual development and he sees his subjects as being in interaction with the outside world. Consequently such knowledge derives from this activity (DeVries 2000:206). According to Lefrancois (2000:229), the work of Piaget has led to the realisation that attempts to teach the learner at a level beyond his stage of development, will culminate in rote-learning (see 2.4.1). This clearly indicates that one must teach according to the learner's age and stage by bearing in mind the functional concept of "readiness".

2.10.1.2 *Piaget's theory of cognition (theory of cognitive development)*

According to Mitchell (1999:21), *cognitive development* refers to the way in which the mental abilities of an individual develop from birth and also to the way knowledge is accumulated in an individual person's mind. Cognitive development is a cumulative process and cognitive frameworks depend on what preceded them (Child 2007:103). In cognitive development, new knowledge is continually being added to existing knowledge and new thought structures are continually being developed and organised in the mind. Cognitive growth thus occurs when new mental representations are created or modified to fit the demands of reality (Hilton 2002:64).

In order to address the question, "How does knowledge develop?", Piaget initiated a programme of research which is often referred to as "the master plan" (Smith 2002:515). During his research, Piaget developed an interest in the intellectual development of children and was almost exclusively concerned with the cognitive aspect of learning (Illeris 2004:27). In the next section, the researcher will elaborate on key principles that are important with regard to Piaget's theory of cognition.

(a) *Equilibration through adaptation*

According to Illeris (2004:29), Piaget's learning theory is centred on the concept of learning as a process of equilibration. Lerman (2001:42) is of the opinion that "[l]earning is then seen as cognitive reorganisation by the individual". Piaget saw this process as the "central problem of intellectual development" [Piaget (in Dawson-Tunik, Fischer & Stein 2004:257)]. In this process, the individual strives to maintain a steady equilibrium in his or her interactions with the surrounding world by means of a continuing adaptation, which is an active adjustment process by which the individual adapts him-/herself to his/her own needs. In other words, the individual is driven to organise his/her schemes in order to achieve the best possible adaptation to his/her environment. Piaget called this process *equilibration* (Snowman *et al.* 2009:62).

According to Snowman *et al.* (2009:62), Piaget postulates that human beings inherit two basic tendencies, namely organisation and adaptation. Organisation refers to "the tendency of all individuals to systematize or combine processes into coherent (logically interrelated) systems" (Snowman *et al.* 2009:62). Piaget (1954) argues that, as the learner seeks the construct 'understanding of the world', the developing brain creates schemas (Santrock 2009:40; Van Wagner 2005:1 of 1). Snowman *et al.* (2009:62) posit that adaptation is "the process of creating a good fit or match between one's conception of reality (one's schemes) and the real-life experiences one encounter".

Piaget maintains that adaptation is accomplished by two subprocesses, namely assimilation and accommodation (Lefrancois 2000:210; Snowman *et al.* 2009:62). Leonard (2002:1) argues that these two developmental learning processes espoused by Piaget are the key processes in the stages of human development. A continuing interaction between these two types of processes that are occurring in parallel and are continually balancing each other out, allows this adaptation to take place. These two very important processes are discussed in the next two sections.

(b) ***Assimilation***

According to Illeris (2004:29), the individual's adaptation of his or her environment occurs through assimilation. Assimilation involves the learner interpreting environmental events within the context of already existing cognitive structures (Leonard 2002:1). Illeris (2004:34) purports that, in assimilative learning, the learner adapts and incorporates impressions from his or her surroundings as an extension and differentiation of previously established cognitive structures. Learners thus assimilate new information by filtering and interpreting this new information in terms of their existing knowledge. Consider the following example:

Suppose students have learned during their school careers that the product of a and b is ab. Now, at university, they encounter the following formula, $\bar{x} = \frac{\sum fm}{\sum f}$, which is the formula that one uses to calculate the mean with regard to measures of central tendency. The situation presupposes that the student has already acquired a certain knowledge of mathematics, in this case, $fm = f \times m$. In other words, a cognitive scheme was developed to hold and structure the acquired mathematical knowledge. When the lecturer explains a mathematical process that has not previously been dealt with in class, the student acquires the new knowledge by means of assimilative extension of his/her mathematical scheme.

Illeris (2004:34) argues that assimilative learning, in its pure form, is characterised by a steady and stable progressive development in which the learning products are constructed, integrated and stabilised. This includes the majority of knowledge and skills learning that is traditionally aimed at in our education system, in which systematic attempts are made to comprehensively extend the knowledge and skills structures that exist within the various subjects (Illeris 2004:35).

(c) *Accommodation*

Leonard (2002:1) argues that "accommodation involves the process of changing the child's cognitive structure, to make sense of the new events occurring in the environment". In other words, when impressions from the environment cannot fit into the existing schemes, accommodation takes place. Santrock (2009:40) purports that, for Piaget (1954), "accommodation occurs when children adjust their schemas to fit new information and experiences".

Illeris (2004:35) argues that, in accommodative learning, previously established cognitive structures are altered through dissociation and reconstruction, as learning elements acquired earlier are released from their original context to subsequently form part of new structures. Accommodation thus implies a going beyond, or a transcendence of, the readiness already developed (Illeris 2004:30).

The accommodative processes can be short and sudden when the necessary preconditions are present, but can also be a lengthy process. The learner can thus understand immediately how something works (*insight*) (see 2.7.1.1), but can also struggle with a problem or a difficult relationship and gradually, or step by step, develop a new comprehension or find a solution. According to Illeris (2004:30), this highlights the fact that "even in relation to the clearest structures, for example, the field of mathematics and logic, there will be individual ways of perceiving the subject".

It often happens in mathematics education that students are called upon to make accommodations beyond their capacities. When the lecturer illustrates a mathematical procedure to students on the board, they imitate the lecturer and practise the fact or procedure until it is automatic (see 2.7.2.3). Students are, however, often given extensive written assignments which involve exercises predicted on the assumption that "practice makes perfect". According to Baroody and Gisburg (1990:58), the problem is that such a direct instruction approach often overlooks the crucial developmental process of assimilation and accommodation and the key developmental issue of readiness. Students do not all have the same readiness to learn a mathematical concept or skill and exercises may not be appropriate for everyone in the class.

When new topics are introduced to students, the problem is compounded before the students have had a chance to assimilate more basic lessons. As Lefrancois (2000:229) states: "[m]aterial presented to learners should not be so difficult that it can't be understood (assimilated) nor so easy that it leads to no new learning (no accommodation)". Because of this, the student eventually gets caught in a downward spiral because new topics often build upon previous lessons. It is thus essential, when a new topic is introduced, that the student's existing knowledge is adequate and organised in order for accommodation to take place [see 2.7.2.2 (a)]. As Snowman *et al.* (2009:69) purport, instructors "should nurture the process of cognitive growth at any particular stage by presenting lessons in a form consistent with, but slightly more advanced than the students' existing schemes. The objective here is to help students assimilate and accommodate new and different experiences as efficiently as possible".

(d) ***Reflective abstraction***

The basic mental operations involved in mathematics learning are regarded by Piaget as products of spontaneous reconstruction. According to Piaget, children develop mathematical concepts independently and spontaneously. In an article in the *Scientific American* (Nov. 1953), Piaget writes: "...a child of six and a half or seven often shows that he has spontaneously formed the concept of number even though he may not yet have been taught to count. Given eight red chips and eight blue chips, he will discover by one-to-one matching that the number of red is the same as the number of blue, and he will realize that the two groups remain equal in number regardless of the shape they take" [Piaget (in Chapman 1972:142)].

According to Steffe and Tzur (1994:10), Piaget made it clear that the mathematical operations and basic structures he identified were reconstructed by children as a product of their interactions within their physical, social, and cultural environment. Piaget explained the development of cognitive mathematical structures by relying on the concept of reflective abstraction.

Piaget (in Steffe 1990:172) characterises *reflective abstraction* as "the mode of abstraction that derives its knowledge from actions and from the subject's operations". According to Goodson-Espy (2005:1), reflective abstraction refers to "the metacognitive awareness of the subject concerning his or her activities and the organization of cognitive structures". Thus defined, reflective abstraction is necessarily constructive and enlarges and enriches the structure from which it starts [Piaget (in Steffe 1990:172)]. This is a key concept in Piaget's theory of cognition and a process of interiorising our physical operations on objects [Kenny 1996:10 of 16; Piaget (in Davis *et al.* 1990:8)].

Steffe (1990:172) maintains that constructivism emphasises the highly complex processes of abstraction that underlie mathematical understanding. If the meaning of a mathematical symbol is essentially an action, then it follows that mathematical concepts will be learned by means of abstraction based upon actions as well as sensory impressions.

According to Kenny (1996:10 of 16), Piaget elaborated the notion of reflection on mental operations, and provided a model for how it operates in conjunction with abstraction and generalisation. As a tool in helping students develop advanced mathematical thinking, Dubinsky (1991) argues for the use of Piaget's reflective abstraction concepts. However, one has to place careful emphasis on developing appropriate mathematical learning tasks in order to induce these reflective abstractions carefully.

In studies that sought to apply Piaget and Von Glaserfeld's ideas concerning reflective abstraction, Cifarelli (1988) and Goodson-Espy (1998) made use of specific, observable problem-solving actions to define levels of reflective abstraction. The levels of reflective abstraction to describe a learning process were defined as follows: *Recognition*, *Representation*, *Structural Abstraction*, and *Structural Awareness* (Goodson-Espy

2005:2). According to Goodson-Espy (2005:2), the differences between these levels are described in terms of whether a solver can:

- recognise having solved a similar problem before;
- re-use previous solution methods on a problem;
- develop novel strategies for a problem that the solver has not used previously;
- anticipate sources of difficulty and promise during the solution process when using a previously applied method;
- anticipate sources of difficulty and promise during the solution process when using a new solution method;
- mentally run through methods used previously;
- mentally run through potential solution methods; and
- demonstrate conscious awareness of problem-solving activities and decisions.

Goodson-Espy (2005:2) argues that, at the highest level (structural awareness), the problem created by the solver has become an object of reflection. These structures are considered by the students as objects and the students are able to make judgements about them without resorting to representing solution methods physically or mentally. Students become increasingly flexible in their thinking as they retain higher levels of reflective abstraction. According to Goodson-Espy (2005:2), an important feature of Cifarelli's levels is that they are a step toward describing whether a solver is conscious or unconscious of certain concepts during his/her problem-solving activity.

2.10.1.3 *Some critique with regard to Piaget*

According to Santrock (2009:48), Piaget's theory has not gone unchallenged. Bodrova and Leong (1996:27) argue that "Piaget placed thinking at the centre of child development". Mahar and Harford (2004:7) argue that, although Piaget's work has been very influential in curriculum development, his work is now seen to lack several important dimensions, as it fails to take into account adult learning. In addition, it does not take into account learners' social relationships in the wider world. However, as

DeVries (2000:190) purports: "his main goal was epistemological – to explain how knowledge develops, not how the child develops".

According to Lefrancois (2000:230), critics point out that Piaget seems to have drastically underestimated the ages at which young children are capable of certain important behaviours. In addition, Piaget left little doubt that he considered formal operations to be generally characteristic of most older adolescents, as well as most adults. Lefrancois (2000:230) argues that, "[i]ronically, it seems while Piaget underestimated the abilities of young children, he may have overestimated those of older children and adolescents". Santrock (2009:49) argues that some cognitive abilities can emerge later than Piaget thought: "[M]any adolescents still think in concrete operational ways or are just beginning to master formal operations". Many studies have failed to find evidence of formal operations among adults (Dulit 1972). By the same token, cross cultural studies have generally been hard-pressed to find much evidence of thinking beyond concrete operations in many cultures (Gelman 1978). However, as Child (2007:100) purports: "[c]hildren from very different social and ethnic backgrounds with varying degrees of verbal talent still appear to give the developmental pattern described by Piaget".

Snowman *et al.* (2009:69) argue that the safest conclusion than can be drawn from the literature with regard to Piaget's theory of cognitive development, is that learners who are in the process of developing the schemas that will govern the next stage of cognitive functioning can, with good-quality instruction, be helped to refine those schemas. This means that instructors should nurture the process of cognitive growth at any particular stage by presenting lessons in a form consistent with, but slightly more advanced than the learner's existing schemas (Snowman *et al.* 2009:69).

2.10.1.4 *Concluding remarks*

Piaget presumed that conceptual growth occurs because the child, while actively attempting to adapt to the environment, organises actions into schemas through the process of assimilation and accommodation [Piaget (in Deans 1972:242); Child 2007:89].

The child can assimilate new information if he has a schema available with which he can assimilate this new information. If the element of novelty is too great for assimilation to take place, the child will, however, have to adjust to the new situation by modifying existing and relevant schemas. Santrock (2009:41) argues that, as old schemas are adjusted and new schemas are developed, the learner organises and reorganises the old and new schemas. Through this alteration of existing structures new ones emerge and the child's conceptual equipment grows. "Eventually, the organization is fundamentally different from the old organization; it is a new way of thinking" (Santrock 2009:41).

The three concepts of organisation, adaptation and schemas are all related. With regard to this, Showman *et al.* state: "[i]n their drive to be organized, individuals try to have a place for everything (accommodation) so they can put everything in its place (assimilation). The product of organization and adaptation is the creation of new schemes that allow individuals to organize at a higher level and adapt more effectively" (Snowman *et al.* 2009:62).

Piaget's work provided mathematics instructors with crucial insights into how children learn mathematical concepts and ideas. Piaget thus explained mathematical development in this sense as a result of children interacting in their environments as self-regulating and autonomous organisms. According to Cohen (2001:31), "Piaget also had enormous influence on education as his ideas suggested children had to learn at their own pace". Kozulin (1995:67) advocates that, despite its revolutionary innovations, Piagetian theory left many questions unanswered or answered in ways that were not satisfactory. The first problem with regard to the Piagetian cognitive approach is the absence of the sociocultural aspect of learning. In the second place, the learning process Piaget proposed appears as a direct interaction of the child with the environment. From this perspective, it appears as if human mediators seem to be excluded from the exchange (see 2.11.1.1).

Later directions in cognitive theories of learning often draw from the work of Lev Vygotsky (1896-1934). According to Mahar and Harford (2004:7), this Russian psycholinguist's introduction of social factors has been very influential in theories of learning, as he takes Piaget's notion that development leads learning further, but approaches it from the opposite direction by arguing that, in fact, learning leads development. According to Illeris (2004:23), the Russian cultural historical approach represented by Vygotsky is preferred by some in opposition to the Piagetian approach, because it combines cognitive understanding with a social perspective.

2.11 SOCIAL CONSTRUCTIVISM

Santrock (2009:349) states that social constructivism emphasises the social contexts of learning. In addition, he adds that knowledge is mutually built and constructed (Bodrova & Leong 1996; Gauvain 2008). According to social constructivists, the construction of meaning is a social process in which the learner constructs meaning at individual and social levels as he/she learns (Snowman *et al.* 2009:239).

From a social constructivist position, the essence of learning is viewed not as a passive process of assimilation, but as an active process of achieving understanding, internalisation and appropriation. According to Gauvain and Perez (in Santrock 2009:349), "involvement with others creates opportunities for students to evaluate and refine their understanding as they are exposed to the thinking of others and as they participate in creating shared understanding".

According to Jordan *et al.* (2008:59), social constructivism is most closely associated with the work of Albert Bandura (see 2.8.1) and Lev Vygotsky. The researcher will elaborate on these aspects in the next section.

2.11.1 Vygotsky

Schunk (1996:213) maintains that perhaps no theorist has influenced modern constructivist thinking more than Vygotsky. As influenced by Piaget, many developmental psychologists argue that children become more capable of higher-level thinking, as they overcome cognitive conflict through the internal processes of assimilation, accommodation, and equilibrium (Snowman *et al.* 2009:75). In other words, cognitive development makes social development possible. Vygotsky, however, believed just the opposite was true, since he saw social interaction as the primary cause of cognitive development (Snowman *et al.* 2009:75). Vygotsky, like Piaget, views learners as active organisers of their experiences but, in contrast, Vygotsky's sociocultural theory emphasises the social and cultural dimensions of development (Child 2007:104; Salkind 2004:279; Leonard 2002:205; Sigelman & Rider 2008:210).

According to Moll (in Taylor 2003:14), "[o]ne of the major tenets of Vygotsky's theory is that there is a functional relationship between the effects of culture on cognitive development and biological growth". In Vygotsky's view, social interaction leads to changes in learners' thinking and then in their behaviour (Salkind 2004:279). Salkind (2004:279) argues that, as behaviour is rooted in the social context in which it occurs, both thought and behaviour vary depending on the cultures in which they take place. In order to elaborate the social dimension of psychological functioning, Vygotsky developed his well-known notion of a "zone of proximal development" (Haenen 2001:159). According to Leonard (2002:205), "[t]he zone of proximal development is an important aspect of social development theory, for it defines how children can reach their full potential as mature, intelligent, and socially responsible adults". Haenen (2001:159) adds that Vygotsky placed the interaction with adults and more competent peers at the very heart of this zone (see 2.11.1.1).

Taylor (2003:14) states that Vygotskian theory consists of three major components, namely "(1) the internalization of auxiliary cultural means, (2) the interpersonal, or social process of mediation, and (3) a child's knowledge is formed within the zone of proximal development, a cognitive space defined by social relational boundaries".

2.11.1.1 *Mediation*

According to Snowman *et al.* (2009:219), Vygotsky favoured mediated learning and developed mediation as a way to assist learners in developing cognitive processes. Vygotsky (1978) developed his theory of mediated learning from his study of literature and his experience as a teacher. In his quest to understand how learners develop cognitively, Vygotsky studied those cognitive functions that had not yet matured but were in the process of maturation (Mitchell 1999:35). He began to search for ways to help learners with learning difficulties and believed that, as learners can overcome these difficulties, they would be able to fulfil their full potential. Snowman *et al.* (2009:75) purport that Vygotsky, unlike Piaget, believed that learners gained significantly from the knowledge and conceptual tools handed to them by those who were more intellectually advanced. These psychological tools might have been language, mathematics, and approaches to problem solving (Snowman *et al.* 2009:239).

Vygotsky (1978) argued that, in order for social interactions to produce advances in cognitive development, they had to contain a process called mediation (Snowman *et al.* 2009:75). According to Amos and Fischer (1998:20), Vygotsky's theory of cognitive development in particular highlights mediation as the process whereby a more experienced person structures and conducts an interaction with another, less experienced person concerning a particular task through the medium of language. According to Vygotsky's theory (1978), the fundamental process of learning takes place through the child's interaction with a more knowledgeable person (Salkind 2004:279). The more knowledgeable person might be an adult, for example a parent or a teacher, or a peer (Santrock 2009:51). According to Shayer (2003:482), "the main source of mediation for adolescents is their peers, rather than 'scaffolding' by adults".

Kozulin and Presseisen (1995:67) argue that in the mediated learning situation, the more competent person places himself between the learner and the environment. Gal'perin (in Haenen 2001:159) argues that in this process, learning is *internalised* by the learner within a specific context and is the key to understanding how and why people develop higher mental processes. During the process in which what is learnt in the external world with the help of another is internalised, internal transformations take place, allowing the student to learn to mobilise the cognitive processes necessary for success [Vygotsky (in Amos & Fischer 1998:20)]. The task of mediators is therefore to arrange, schedule and organise appropriate stimuli on the basis of what they intend to achieve in relation to their culture.

According to Haenen (2001:159), Vygotsky views psychological functions and the means of mediating them as emerging from the child's social interaction with adults and peers. This idea is formulated in Vygotsky's often cited "general genetic law of cultural development". According to Vygotsky (1978:57), "[e]very function in the child's cultural development appears twice: first, on the social level, and later, on the individual level; first, between people (interpsychological) and then inside the child (intrapsychological)". DeVries (2000:191) argues that "[t]he key to understanding this passage is to emphasize that it is cultural development that first appears on the social plane, where cultural development involves learning the characteristics of the particular culture".

Mediated learning thus results from the interactional processes between developing human beings and experienced human beings (Mitchell 1999:44). The collaboration or mediation with another person in the zone of proximal development thus leads to development in culturally appropriate ways. According to Garton (2004:123), "collaboration in particular, and social interaction more generally, allows for exploring the *how*".

The learner thus acquires specific patterns of behaviour and strategies to deal progressively more efficiently with his or her environment, through the process of mediated learning experience (Mitchell 1999:44). Through mediated learning, learners can change the way they think and develop the efficient thinking skills that are necessary to become an autonomous and independent learner (Mentis, Dunn-Bernstein & Mentis 2007:p.*x*).

As mediated learning results from the interactional processes between developing human beings and experienced human beings, these processes are important with regard to Vygotsky's theory of mediated learning. In the next section, the researcher elaborates on three ways in which these interactional processes become clear.

(a) ***Learning partners***

Biggs and Tang (2007:126) argue that a great benefit for both student and lecturer is to require of students to form a partnership with another student or a small group of students – especially in large classes. When students form learning partnerships, they hear of different interpretations they themselves had not thought of. Such partnership allows for general mutual support, as students need someone to talk to where they share concerns, seek clarification over assignment requirements, or to check their own insecure interpretations of procedure or of content [Saberton 1985 (in Biggs & Tang 2007:126)].

This elaboration of known content facilitates deriving standards for judging better and worse interpretations and a reflective awareness of how one arrives at a given position. Examples are: "How did that student arrive at the right answer?"; "How did I get my answer?"; "What is the difference between the way I calculated my answer and the person sitting next to me?". According to Abercrombie [in Biggs and Tang (2007:140)], the reflective aspect of learning is thus sharpened, because students readily identify with one another's learning in a way they do not do with top-down teacher-directed learning.

Spivey (in Steyn, du Toit & Hay 2000:3) argues that similarities in understanding, achieved when people with similar backgrounds, experience, and knowledge approach texts in similar context with similar purposes and perspectives may be called "shared meanings". This means that meaning is mediated by the differences in perspectives among the participants. Steffe [in Steffe & Gale (1995:493)] argues that "the very process of taking the perspective of others can generate modifications in subjective knowledge". According to Biggs and Tang (2007:133), the creation of learning partnerships provides a continually accessible resource for discussing, reciprocal questioning and mutual support in an otherwise anonymous environment.

(b) ***Peer groups / tutoring***

Jordan *et al.* (2008:70) argue that "[i]n peer groups, individuals learn to interact, behave and conform in socially acceptable ways. They acquire social roles, responsibilities and identities, which are developed through relationships and group participation". According to Gabriel and Montecinos (in Jordan *et al.* 2008:76), "...peer groups engaged in learning tasks achieve a wider range of information and deeper meaning-making than individuals can achieve on their own". Jordan *et al.* (2008:76) contend that "[p]eer groups can challenge individual views by providing alternative perspectives, and they can also generate the drive to resolve differences so that social relationships are maintained".

Mather (in Amos 1999:181) argues that tutors are ideally positioned to influence first-year students', since they will perceive senior students as being successful in the system and an authority, of sorts, on how to achieve the desired outcome. According to Santrock (2009:353), fellow students can also be effective tutors, where one student teaches another. Senior students who serve as tutors are thus ideally positioned to play the role of *mediator*. The ideal tutorial is described by Pastoll (1992) as one in which there are peers rather than authority figures. Radloff and Murphy (1992:21) argue that a learning environment needs to be created, in which the tutor acts as a facilitator rather than the fountain of all knowledge.

(c) *Cooperative learning*

According to Snowman *et al.* (2009:115), cooperative learning is closely related to peer tutoring. According to Snowman *et al.* (2009:115), "[t]he general idea behind cooperative learning is that by working in small, heterogeneous groups and by helping one another master the various aspects of a particular task, students will be more motivated to learn, will learn more than if they had to work independently, and will forge stronger interpersonal relationships than they would by working alone".

Snowman *et al.* (2009:115) further argue that "[c]ooperative learning is a generally effective instructional tactic that is likely to be particularly useful with Latino, Black, and American Indian students", as these cultures value a communal orientation that emphasises cooperation and sharing. Research by Kagan (1998:106) as well as Lotan and Whitcomb (1998:1), has shown that cooperative learning leads to dramatically improved academic achievement and higher-order thinking skills (especially for the lower-achieving learners).

Research by Slavin (1995) shows that cooperative groups are most effective when students each have assigned responsibilities and all must attain competence before any students are allowed to progress. In order to attest to the recognised impact of the social environment during learning, the emphasis is currently on using peer groups for learning in such fields as mathematics [Cobb (in Schunk 1996:217)].

2.11.1.2 *The zone of proximal development (ZPD)*

The concept of the zone of proximal development is a key facet of Vygotsky's theory (1978), as he distinguished between the actual development and the potential development of the child. According to Snowman *et al.* (2009:77), Vygotsky referred to the zone of proximal development as the difference between what a child can do on his own and what can be accomplished with some assistance. Vygotsky (1978:86) defined the zone of proximal development (ZPD) as "the distance between the actual development as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers". According to Mitchell (1999:37), actual development is determined by what a learner can do unaided by an adult or instructor, where potential development, in contrast, is what a learner can do through problem solving under adult guidance or in collaboration with more capable peers.

Vygotsky (1978:87) argues that the zone of proximal development "defines those functions that have not yet matured but are in the process of maturation", accounting for "not only the cycles and maturation processes that have already been completed but also those processes that are currently in the state of formation". According to Mahar and Harford (2004:7), this zone represents an individual student's potential level of learning if helped by a teacher or caregiver. According to Jarvis and Parker (2005:95), Vygotsky's (1978) idea of transition to the zone of proximal development may be seen as a parallel to Piaget's accommodative learning [see 2.10.1.2 (b)].

2.11.1.3 *Creating the zone of proximal development*

Fischer and Van der Riet (1997:20) state that "it is through integrating academic development in the curriculum that the opportunity for creating the zone of proximal development occurs and thus the competencies inherent in academic literacy can be attained".

Vygotsky (in Snowman *et al.* 2009:77) maintains that students with wider zones are likely to experience greater cognitive development when instruction is pitched just above the lower limit of their ZPD, than students with narrower zones, because the former are in a better position to capitalise on the instruction. Some instructional methods which are likely to help students more through their ZPD include modelling, the use of rewards as well as punishments, feedback, categories, rules for organising and understanding ideas, and questioning (Santrock 2009:51; Snowman *et al.* 2009:77). Students' behaviour becomes more internalised and more automatic as they reach the upper limit of their

ZPD, where learning occurs and actually encourages development. As Snowman *et al.* (2009:77) purport: "[a]s students approach the upper limit of their ZPD, their behaviour becomes smoother, more internalized, and more automized". However, learning will be accompanied with difficulty beyond the upper limits of one's ZPD (Mitchell 1999:38). For example, when learners do not understand how numbers work in mathematics, they will be frustrated by how to multiply if teachers only use abstract examples. It is therefore important that instructors (teachers/lecturers) should adapt their instruction to each learner's ZPD, so that it will be of greatest benefit in furthering the learner's cognitive development (Mitchell 1999:38).

Mitchell (1999:38) advocates that the zone of proximal development can be applied through instruction in the following ways:

- To find a basis for determining students' zones of proximal development, the lecturer can assess students by means of questioning (DeVries 2000:196).
- The lecturer must find appropriate ways to present tasks to students. This can be done through the process of shared understanding (see 2.11.1.1), in which the lecturer and students have a common understanding of the task, which marks a starting point for development through joint problem solving. The lecturer can embed the task in a meaningful context, for example, relate mathematical problems to the students' lives, instead of presenting mathematical problems in the abstract. Dialogue that helps students analyse the problems they face can also be used [Tappan (in Snowman *et al.* 2009:77)].
- According to Santrock (2009:51), instructional support - which is accomplished by applying the concept of scaffolding - can also be provided (see 2.11.1.4). Such assistance enables students to complete tasks they are not able to accomplish or complete independently (Child 2007:106).
- Some learners' development may be slowed when working by themselves. This emphasises an important implication of the ZPD, according to which social interaction is necessary for facilitating development (Snowman *et al.* 2009:75). In order for students to develop fully, it means that they must work with more

skilled partners [see 2.11.1.1 (a)] who can systematically lead them into more complex problem solving.

The conditions that will necessitate effective social interaction between learning partners are *scaffolding*, which the researcher will elaborate on in the next section.

2.11.1.4 *Scaffolding*

Child (2007:106) argues that a major application in Vygotsky's ideas involves the concept of instructional scaffolding. This concept refers to the process of controlling task elements that are beyond the learner's capabilities so that the learner can focus on and master those features of the task that he or she can grasp quickly [Bruning, Schraw, & Ronning (in Schunk 1996:216)]. According to Jordan *et al.* (2008:64), scaffolding suggests support that is gradually withdrawn when learners have constructed their understanding and can act independently. In other words, through scaffolding, learners move through the zone of proximal development, as this concept includes assistance that allows learners to complete tasks they are not able to complete independently (Santrock 2009:351). This enables the learner to eventually complete tasks on his/her own.

According to Santrock (2009:51), a more skilled person (a teacher or advanced peer) adjusts the amount of guidance to fit the learner's current performance. The teacher or peer might do most of the work in a learning situation, but as the learner becomes more competent, the teacher or peer then gradually withdraws the scaffolding so that learners can perform independently (Jordan *et al.* 2008:64; Santrock (2009:351).

Snowman *et al.* (2009:77) state that "[t]he purpose of scaffolding is to help students acquire knowledge and skills they would not have learned on their own. As students demonstrate mastery over the content in question, the learning aids are faded and removed". Some scaffolding techniques include modelling, categories, rules for helping students organize, and understanding ideas [Gallimore & Tharp; Ratner (in Snowman *et al.* 2009:77)]. Snowman *et al.* (2009:77) argue that helping students solve problems by giving them hints and asking leading questions are an example of scaffolding. Snowman *et al.* 2009:289) further maintain that modelling by the teacher is one of the most common forms of scaffolding, as the teacher provides students with concrete examples by demonstrating *how* to solve problems (see paragraph 2.8.1.1). When learners struggle with a problem, the modelling technique by the teacher provides learners with access to their teacher's thinking.

What is important though, is to ensure that the scaffolding keeps learners in the ZPD, which is altered as they develop capabilities and are challenged to learn within the bounds of the ZPD (Mahar & Harford 2004:7). For cognitive development to take place, social interaction is therefore required and learning is restricted to a certain range from what the learners can do alone, to what they can do with help at any given age.

2.11.1.5 *Concluding remarks*

Cobb, Wood and Yackel (1990:126) argue that Vygotsky clearly made a profound contribution to our understanding of intellectual development by attempting to relate cognitive and social phenomena. In the process of social interaction, individuals create interpretations of situations that fit in with those of others for the purposes at hand, by negotiating and internalising meanings, resolving conflicts, mutually taking others' perspectives and constructing consensual domains for coordinated activity (Santrock 2009:51). As individuals attempt to make sense of situations while interacting with others, these compatible meanings are continually modified by means of interpretative processes.

In developing the academic literacy of students, Vygotsky's ideas (1978) (in Amos & Fischer 1998:22) contribute much to the understanding of how students can learn to mobilise their cognitive operational capacities in relation to specific situations and tasks to become academically literate. Vygotsky's theory of social learning has been expanded upon by contemporary psychologists such as Albert Bandura (see section 2.8.1).

2.12 SUMMARY

Chapter two is concerned with learning. The researcher explained the concept of learning, as well as the historical development of psychology of learning as a field of study. Important cognitive learning theories which emphasise the development of the learner's thinking processes were also presented. Behavioural learning theories were briefly discussed and, in contrast to these theories, the gestalt learning theory advanced by the German learning psychologists was also presented. The gestalt psychologists maintain that humans perceive sensations or concepts as whole units rather than isolated pieces. Thinking is interpreted as being a reflective process.

The researcher explained the information processing theory as a component of cognitive learning theory. In addition, it was defined how the sensory memory, the working memory and the long-term memory function to process information. The concept of encoding and retrieval of information was explained. Additionally, certain cognitive processes were explained, which include paying attention, finding meaning in the stimuli received through the senses, retaining information through practice, and making connections in the long-term memory.

The researcher extended the cognitive analysis by discussing Bruner's theory of cognitive growth and Ausubel's meaningful reception learning. This discussion moved beyond that presented in the preceding paragraphs by covering other cognitive learning theories and topics relevant to learning that involve the operation of complex (higher-order) cognitive processes. Bruner's theory informs educators how learning material could be structured, as well as sequenced material in accordance with cognitivist ideas of mental processing (see 2.7.3.5). Another important aspect of Bruner's learning theory is the use of a spiral curriculum by means of which learners build on previous construction of knowledge to formulate more useful associations and meanings (see 2.7.3.5).

The researcher also discussed meaningful learning, as it plays a vital role in promoting deep learning. The primary concept in Ausubel's theory is meaningful learning in which learners organise content in logical ways, such as concept maps (see 2.7.4.2). As human learning occurs in a social environment, the researcher elaborated on social cognitive theory (see 2.8). A discussion of the social cognitive theory of Bandura was also provided (see 2.8.1). A distinctive feature of Bandura's social cognitive theory is the central role it assigns to self-regulatory functions as well as the modelling processes, which play a prominent role in learning (see 2.8.1).

Finally, constructivism as well as the two major strands of the constructivist perspective - namely cognitive constructivism and social constructivism - was explained. The theories of Jean Piaget and Lev Vygotsky were discussed, as they focus on the development of thinking skills. The researcher presented a discussion of Piaget's stage theory as well as Piaget's theory of cognition. The key processes in the stages of child development, namely assimilation [2.10.1.2 (b)] and accommodation [2.10.1.2 (c)] were outlined. The researcher also discussed the process of reflective abstraction, as the development of cognitive mathematical structures relies on the concept of reflective abstraction [see 2.10.1.2 (d)].

Vygotsky's theory combines cognitive understanding with a social perspective and highlights mediation, which can be applied through the concept of scaffolding and peer collaboration (see 2.11.1.1). With regard to Vygotsky, the researcher emphasised the fact that it is possible to develop higher cognitive processes in a learner through mediation (see 2.11.1.1), which has its origin in social interaction. A thorough overview of the zone of proximal development, as well as social interaction in the zone of proximal development was presented (see 2.11.1.2). The condition that is necessary for effective social interaction between learning partners, namely scaffolding, was also discussed (see 2.11.1.4).

The researcher presents a comparison of behaviourism, cognitivism and constructivism in Table 2.2.

Table 2.2: A comparison between behaviourism, cognitivism and constructivism

Theory	Mental Activity	Learning Processes	Role of Instructor
Behaviourism	Irrelevant	Stimulus-response Reinforcement External event	Controls environmental stimuli
Cognitivism	Perception Attention Processing	Memory Surface and deep learning Encoding internal event	Applies cognitive principles to facilitate cognitive processes
Constructivism	Meaning-making	Retuning schemas and mental constructs Internal event	Supports meaning-making Challenges existing ideas

Source: Jordan *et al.* (2008:55)

In the next chapter, the researcher will focus on learning theories in mathematics and statistics, and on metacognitive learning theories, as these theories focus on the learners' awareness of how to learn and how to control and evaluate their own progress. The researcher will also review problem-solving techniques with regard to mathematics and statistics.

CHAPTER 3: THE LEARNING OF MATHEMATICS

We can know nothing that we have not made.~[Vico (in Ernst 1994:1)]~

3.1 INTRODUCTION

As indicated in Chapter 1, the purpose of this study is to determine the impact of a classroom learning strategy intervention on university students' academic achievements in a mathematics and statistics-related subject. Mathematical proficiency (see 3.2.1) forms an integral part of mathematics and statistics-related subjects and, as such, an introductory overview on students' underpreparedness in mathematics literacy was presented in Chapter 1. The researcher referred to the crisis mathematics is facing in higher education, whilst the research questions and aims in addressing the problem statement at hand were outlined.

Chapter 2 was concerned with the concept of learning in order to gain a better understanding of how students learn and how their cognitive processes can be enhanced. The concept of learning was defined, whilst some important learning theories (applicable to HE) were also outlined.

However, before the classroom learning strategy can be proposed, it is necessary to present a thorough theoretical overview of mathematics as a learning discipline, as well as how the learning of mathematics differs from that of other subjects. As mathematics forms an integral part of mathematics and statistics-related subjects, aspects such as the unique nature of mathematics, what it means to think mathematically, as well as some important learning theories in mathematics, need to be addressed. Therefore, Chapter 3 is concerned with various aspects regarding the learning of mathematics. As problem solving represents a key area in the learning of mathematics, the researcher will present a thorough overview of problem solving in mathematics as well as statistics. The researcher deems a proper discussion on these aspects necessary in order to come to a full understanding of the intervention she wishes to propose. The research in this chapter is mainly derived from a constructivist perspective on learning, which is based on the view that learning is contingent upon the activity and involvement of the student.

3.2 BACKGROUND

Before any discussion can commence on the learning of mathematics, it is important to understand the importance of mathematical proficiency, mathematical thinking, and how the lack of mathematical skills contributes to students' underperformance in mathematics. These issues are addressed in the following paragraphs.

3.2.1 Mathematical proficiency

According to Dossey, Mullis, Lindquist and Chambers (in Davis, Maher & Noddings 1990:1), most students, even at age 17, do not possess the breadth and depth of mathematics proficiency needed for advanced study in secondary school mathematics. However, the question is what exactly is meant by mathematics proficiency?

According to Milgram (2007:44), a key component of mathematical proficiency is "the ability to understand, use, and as necessary, create definitions". According to the curriculum standard that emerged from the National Council of Teachers of Mathematics (2000; 2006), mathematical understanding requires the ability to use problem solving, mathematical reasoning, and communicate mathematical thinking coherently and clearly to others (Kramarski 2008:83). In *Adding it Up*, Kilpatrick, Swafford and Findell (2001:16) present the notion of mathematical proficiency consisting of the following five interwoven strands:

1. Conceptual understanding – the comprehension of mathematical concepts, operations and relations.
2. Procedural fluency – the skill in carrying out procedures flexibly, accurately and appropriately.

3. Strategic competence – the ability to formulate, represent and solve mathematical problems.
4. Adaptive reasoning – the capacity for logical thought, reflection, explanation and justification.
5. Productive disposition – habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy [National Research Council (in Kilpatrick *et al.* 2001:116)].

Takahashi, Watanabe, and Yoshida (2006:130) argue that good teaching practices should promote the development of these strands of mathematical proficiency. According to the NRC (in Kilpatrick *et al.* 2001:116), these strands are "interwoven and interdependent"; therefore, good teaching practices cannot focus on just one or two of these strands. According to Kilpatrick *et al.* (2001:118), the central notion that strands of competence must be interwoven to be useful, reflects the finding that having a deep understanding requires that learners connect pieces of knowledge (see 2.4.2).

The NRC (2001:313) views teaching as "interactions among teachers and students around content" and adapt the teaching triangle model developed by David Cohen and Deborah Ball (see Figure 1). This model clearly shows mathematics teaching as the interaction among teachers, students, and mathematics; all taking place in contexts (NRC 2001:314).

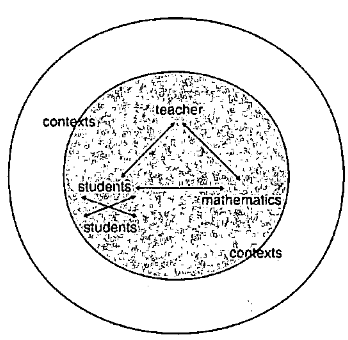

Figure 3.1:

The teaching triangle model

Source: Takahashi, Watanabe, and Yoshida (2006:130).

Kenny (2005:v) argues that "the ability to master and demonstrate mathematical knowledge came to be seen as the result of a process that involves teaching for understanding, student-centered learning, concept-building rather than memorization of facts, and the ability to communicate mathematical understanding to others".

In "Adding It Up", Kilpatrick *et al.* suggest that "[h]igh-quality instruction, in whatever form it comes, focuses on important mathematical content, represented and developed with integrity. It takes sensitive account of students' current knowledge and ways of thinking as well as ways in which those develop. Such instruction is effective with a range of students and over time develops the knowledge, skills, abilities, and inclinations that we term mathematical proficiency" (NRC 2001:315).

With regard to the lack of mathematical proficiency among many students, Davis *et al.* (1990:2) present the view that this problem should be addressed at an even deeper level: one in which the questions are centred around such issues as a consideration of the nature of mathematics, what it means to think mathematically, what it means to think like a mathematician, and what it means to engage in mathematical activity. Davis *et al.* (1990:2) argue that if we regard doing mathematics as involving complex processes that call for the use of heuristics and analysis, then another kind of learning activity becomes appropriate, and another mode of inquiry is needed. The researcher therefore deems it necessary to present a thorough discussion with regard to the topics mentioned above.

3.2.2 Mathematical thinking

According to Razali and Tall (1993:219), "a plausible way in which students may become more successful is to become consciously aware of more successful thinking strategies and this must be done in context designed to impose less cognitive strain". Over the past 20 years, cognitive psychologists have made significant strides in understanding students' mathematical thinking [see Baroody 1987a; Carpenter, Moser & Romberg 1982; Davis 1984; Fuson 1988; Gelman & Gallistel 1978; Ginsburg 1983;

1989; Hiebert 1986; Lesh & Landau 1983; Resnick & Ford 1981; Steffe, van Glaserfeld, Richards & Cobb 1983 (in Baroody & Ginsburg 1990:51)].

Despite current practices in mathematics education, mathematics teachers still embrace procedures, methods, skills, rules and algorithms in an unthinking, non-reflective fashion [Willis (in Worlley & Proctor 2005:4 of 22]. As Sanchez and Blancarte (2008:1 of 2) purport: "teachers usually propose for their students tasks without a meaningful context". Wild and Pfannkuch (1990:224) argue that this procedural activity of rule and rote practice does not lead to conceptual understanding, nor does it demonstrate or provide a foundation for the understanding of core concepts (see 2.4.1; 3.3.3.1).

To overcome this teaching method, researchers propose that university teachers must be well informed about the importance of developing mathematical thinking in their students. Maher and Davies (1990:89) argue that it is very significant that teachers be aware of students' thinking about a mathematics problem. By paying attention to the mathematical thinking of students engaged in active mathematical construction, and trying to make sense of what students are doing and why they are doing it, is a prerequisite to gaining insight into the nature of the development of students' representations. The knowledge university teachers gain from students' thinking thus makes it possible to challenge and extend students' thinking and appropriately modify or develop activities for students. When we look carefully at how students learn mathematics, the mathematics classroom can become a learning environment for both the university teacher and the student.

3.2.2.1 *Assumptions of mathematical thinking*

Goos, Galbraith and Renshaw (1999:45) list five assumptions about doing and learning mathematics that appear crucial to creating the culture of the community of mathematical inquiry. These assumptions are:

1. Mathematical thinking is an act of sense-making, and rests on the processes of specialising and generalising, conjecturing and justifying.
2. The processes of mathematical enquiry are accompanied by habits of individual reflection (see 2.7.1.2 and 3.5) and self-monitoring (see 2.8.1.3 and 3.8.3.2).
3. Mathematical thinking develops through scaffolding of the processes of enquiry (see 2.11.1.4).
4. Mathematical thinking can be generated and tested by students through participation in equal-status peer partnerships (see 2.11.1.1).
5. Interweaving of familiar and formal knowledge helps students to adopt the conventions of mathematical communication (see 3.8.1).

3.2.2.2 *Critical thinking skills*

According to Scriven and Paul (in Makina 2010:24), critical thinking is a mode of thinking about any subject content or problem in which the thinker improves the quality of his or her thinking, by skilfully taking charge of the structure inherent in thinking and imposing intellectual standards upon them. According to Makina (2010:24), learners are not born with the ability to think critically, that it is rather a learned ability, and its development needs to be facilitated. Santrock (2009:320) argues that "[i]f a solid basis of fundamental skills (such as literacy and math skills) is not developed during childhood, critical thinking skills are unlikely to mature in adolescence".

Critical thinking, with regard to mathematics, is "thinking about what is being asked in a given problem, determining what operations and procedures are used in a mathematics problem, with help from a mathematics teachers, and sharpening the analytical skills of learners to improve their mathematics" [Chang (in Makina 2010:27)]. Clement and Lochhead (in Makina 2010:27) argue that someone with critical thinking skills in mathematics is able to understand the logical connections between ideas; identify, construct and evaluate arguments; detect relevance and importance of ideas, and reflect on the justification of his or her own beliefs and values.

Birnbaum and Berger (in Silver 2001:3 of 3) distinguish the following critical thinking skills with regard to mathematics:

1. The ability to recognise information when it is presented in an unfamiliar format.
2. The ability to solve problems.
3. The ability to recognise when it is appropriate to use certain strategies and recognise when it is necessary to change those strategies.
4. Spatial understanding.
5. The ability to use representation (symbols) to solve a problem presented in an abstract form.
6. The ability to recognise relationships between numbers and shapes and link various pieces of information.

The researcher believes that critical thinking should be prominent in the learning of mathematics. Van den Berg (2004:279) points out that in an increasingly complex and specialised society, it is imperative that individuals think critically. According to Landsberg (2005:104), critical thinking skills are essential to the process of filtering, assimilating and finding new meaning in the torrents of information faced daily. According to Makina (2010:26), "[c]ritical thinking plays a crucial role in evaluating new ideas, selecting the best ones and modifying them, if necessary, by providing the tools for the process of self-evaluation". It can thus be asserted that critical thinking involves the ability to engage in reflective and independent thinking. According to Makina (2010:27), good critical thinking can thus be seen "as the foundation of mathematics since it enhances general thinking skills, promotes creativity and is crucial for self-reflection". Looking into critical thinking skills with regard to mathematics seemed relevant, as the aim of the research is to improve students' academic performance in a mathematics and statistics-related subject by means of a classroom learning strategy intervention. Proficiency in critical thinking is expected to enhance students' academic performance.

3.2.3 Poor study skills as an important factor in students' underperformance in mathematics

According to Dawkins (2006:1), the majority of people who are doing poorly in a mathematics class fall into three main categories: The first category consists of the largest group of students who just do not have good study habits and/or do not really know how to study a mathematical subject. The second category consists of the students who spend hours each day studying and still do not do well. The researcher is of the opinion that students in this category often suffer from inefficient study habits. The third and final category consists of those students who are simply not spending enough time studying at all.

According to Barnes (2005:43), the inadequate use of strategies for solving mathematical problems has been well documented. According to Child (2007:207), the study problems and strategies of students in higher education have attracted most attention from researchers and writers (Collins and Kneale 2000; Lashley 1995). The rapid increase in student numbers in recent years has also exacerbated these problems. According to Child (2007:207), "[s]tudents are often not aware of their study shortcomings and even when they do recognize that they have problems they do not readily seek out advice".

A study by Anthony (2000) sought to identify specific factors which are seen as having an important influence on students' levels of success in first-year mathematics courses. From the results of the study, it became evident that both students and lecturers rated poor study techniques as a more influential factor in failure than inadequate mathematics background knowledge (Anthony 2000:8). This factor supports research findings which suggest that, for many students, poor performance is largely due to ignorance about the study skills required, or the inability to apply these skills appropriately, rather than the lack of mathematical ability [Manalo (in Anthony (2000:8)].

The previous section was devoted to developing a background to mathematical understanding and mathematical thinking. Mathematical proficiency plays an integral part in mathematics and statistics-related courses. Since poor critical thinking and study skills are important factors that contribute to students' underpreparedness in mathematics, one can conclude that the need therefore exists to improve students' critical thinking and study skills in mathematics and statistics-related courses by means of a classroom learning strategy intervention.

3.3 MATHEMATICS

3.3.1 The importance of mathematics

According to Moyo (2010:133), an excerpt from a recent review of "Mathematical Sciences Research at South African Universities" pointed out that "...the importance of a sound foundation and research base in the mathematical sciences is absolutely critical for national development in science, engineering, commerce, and technology. Without this foundation, attempts at establishing a knowledge-based economy and generating innovation are doomed to failure. The development and transformation of human resources in science, engineering, and technology depend in a fundamental way on mathematical understanding, and a significant presence in each field of very highly skilled mathematicians" (Department of Science and Technology [in Moyo 2010:133]).

The importance of mathematics extends beyond the academic domain. Competence in mathematics is essential for functioning in everyday life, as well as for success in our increasingly technological workplace. As basic arithmetic skills are required for everyday computations and sometimes for job applications, young people who come to adulthood without mathematical skills are likely to find it difficult to function in society (Kirch, Jungeblut, Jenkins & Kolstad 1993). According to Milgram (2007:42), "our society could not even function without the application of a very high level of mathematical knowledge".

The application of mathematics, as well as statistics, is evident all around us and is the underpinning of our entire civilisation. In the information age of today, mathematics is a universal language used as a powerful tool by banks and insurance companies, surveyors and architects, builders and engineers, scientists and technologists (Baron 1972:25). Computations are constantly running, calculations are being made, and formulas are being proved. Whether you are receiving change from the attendant in the supermarket, balancing your checkbook or budgeting every month, mathematics is being used. Not only is mathematics used for counting, simple arithmetic and simple statistical reporting, but it is also a cumulative science in which new results are built upon and depend on earlier results.

According to Burger and Starbird (2005:xii), "[t]he realm of mathematics contains some of the greatest ideas of humankind – ideas comparable to the works of Shakespeare, Plato, and Michelangelo. These mathematical ideas helped shape history, and they can add texture, beauty, and wonder to our lives".

3.3.2 What is mathematics?

Most people perceive mathematics as a "fixed, static body of knowledge" as its subject matter includes "the mechanistic manipulation of a variety of numbers and algebraic symbols and the proving of geometric deductions" (Romberg & Kaput 1999:4). According to Burger and Starbird (2005:xi), many people view mathematics as "a set of formulas to be applied to a list of problems at the end of textbook chapters". However, "[t]he once accepted view of mathematics as basic arithmetic skills has given way to a broader view that emphasises mathematics as general processes, or ways of thinking and reasoning (NCTM 1989), as an important form of communication (DES 1982), and as a science of patterns (AEC 1991)" (Herrington, Sparrow, Herrington & Olivier 2002:2 of 4). Since statistics uses and investigates number patterns, it can be deduced that mathematics also forms the basis of statistics as a subject. .

The Department of Education (2002:2) views mathematics as "a human activity that involves observing, representing and investigating patterns and quantitative relationships in physical and social phenomena and between mathematical objects themselves. Through this process, new mathematical ideas and insights are developed. Mathematics uses its own specialised language that involves symbols and notations for describing numerical, geometric and graphical relationships. Mathematical ideas and concepts build on one another to create a coherent structure. Mathematics is a product of investigation by different cultures – purposeful activity in the context of social, political and economic goals and constraints".

For Burger and Starbird (2005:xi), mathematics is "a network of intriguing ideas – not a dry, formal list of techniques" and also a "living breathing, changing organism with many facets to its personality. It is creative, powerful, and even artistic". Romberg and Kaput (1999:xi) also emphasise the shift away from traditional instruction toward mathematics as a human activity. Romberg and Kaput (1999:5) view mathematics as "a human activity that reflects the work of mathematicians – finding out why given techniques work, inventing new techniques, justifying assertions, and so forth". Thurstone's description of mathematics justifies this perspective of human activity as he rightfully states: "[m]athematics isn't a palm tree, with a single long straight trunk covered with scratchy formulas. It's a banyan tree, with many interconnected trunks and branches – a banyan tree that has grown to the size of a forest, inviting us to climb and explore it" (Thurstone 1990:7).

3.3.3 The nature and uniqueness of mathematics

According to Fennema and Romberg (1999:ix), mathematics educators as well as mathematicians have provided new insights into the nature of mathematics. Their work has identified those processes, concepts, and ideas within the discipline that are crucial to successful survival in the 21st century.

Baron (1972:24) argues that, in order to understand and appreciate the nature of mathematics at any given time, it is necessary to take into consideration not only the formal structure in which the logical components are presented, but also the network of conceptual thinking in which it is embedded and the kind of applications which it finds. In considering the nature of mathematics, Baron (1972:25) purports that we must concern ourselves with the developing whole – not only with the formal accepted logical framework, but also with the methods of work and the thought processes of mathematicians.

3.3.3.1 *Conceptual understanding in mathematics*

Morris and Mather (2007:253) argue that, if students are to be successful in mathematics, they need to understand the concepts underlying basic skills, in other words, they need to gain conceptual knowledge. As problem solving is the centrepiece of mathematics instruction in most mathematics classrooms, students must also acquire and apply strategic knowledge for solving maths problems (Morris & Mather 2007:253).

The importance of conceptual understanding in mathematics has been well documented, as opposed to low-level procedural knowledge [see 2.7.2.3 (c); Hiebert & Carpenter (in Worlley & Proctor 2005:2 of 22)]. According to Worlley & Proctor (2005:8 of 22), conceptual knowledge in mathematics involves more than just a recall of facts for common examples. According to Kilpatrick *et al.* (2001:118), conceptual understanding refers to "an integrated and functional grasp of mathematical ideas". Students who have conceptual understanding know more than isolated facts and methods, understand why a mathematical idea is important, and also the kinds of contexts in which it is useful (see 2.4.1). Kilpatrick *et al.* (2001:118) argue that students with conceptual understanding organise their knowledge into a coherent whole, which enables them to learn new ideas by connecting those ideas to what they already know (see 2.7.1.3). These students can draw on related networks and adjust and adapt procedural knowledge to help solve unfamiliar problems [see 2.7.2.3(c)]. However, knowledge will be fragmented and effective problem solving will be impeded without a conceptual framework.

According to Kilpatrick *et al.* (2001:118), students who learn with understanding make a connection between facts and methods, which makes it easier to remember and use. Kilpatrick *et al.* (2001:118) argue that students are unlikely to remember a method incorrectly if they understand it. These students monitor what they remember, and also try to figure out whether it makes sense (see 3.8.3.1). Students with conceptual understanding attempt to explain the method to themselves and correct it if necessary.

According to Kilpatrick *et al.* (2001:118), evidence of conceptual understanding among students is often looked for in students' ability to verbalise connections among concepts and representations. However, conceptual understanding needs not be explicit, as students often understand before they can verbalise that understanding.

The researcher regards conceptual understanding in mathematics as well as statistics, as extremely important, as it also promotes a deep approach to learning (see 2.4.2). The aim of the researcher's classroom learning strategy intervention, is to improve students' academic performance in a mathematics and statistics-related subject, as well as for students to adopt a deep approach to learning (see Chapter 1).

3.3.3.2 *Mathematics is a sequential and cumulative subject*

According to Beck (2009:177), mathematics is a highly organised body of knowledge with interrelationships that require an order and sequence in its study. Because mathematics follows a sequential learning pattern, its learning differs from that of other subjects, and therefore, requires a different study system.

According to Nolting (2002:20), mathematics is a sequential subject that requires students to think how concepts apply to problems and connect with each other. What is meant by a sequential learning pattern, is that the material learned in mathematics on one day is used the next day and the day thereafter, and so forth. Moursund (2006:3 of 4) also presents the view that mathematics is a cumulative, vertically structured discipline, where one learns by building on the maths that one has previously learned. Nolting (2002:22) compares the learning of mathematics to building a house. Like a house, for which you first lay the foundation, then build the walls and finally the roof, mathematics must be learned in a specific order. Each chapter in a mathematical course is a foundation block for the next chapter, and the mathematical knowledge you have gained at the beginning of a course determines your success in mathematics. In other words, mathematical ideas must be built on prior learning and previously learned concepts in mathematics.

It can thus be asserted that the effective sequencing of knowledge in mathematics logically supports the building of concepts before procedures. Students must understand the mathematical concept to which it pertains, before practising a procedure. This view supports constructivism as a learning theory for mathematics, as new knowledge builds on prior knowledge in mathematics (see 2.9).

3.3.3.3 *Mathematics as a language that uses symbols*

According to Beck (2009:176), mathematics is more than a computational system, but also a language that uses symbols to condense thinking. Nolting (2002:24) argues that mathematics can be seen as a foreign language. In order to excel in any foreign language, you have to practise it every day. Just like any foreign language, mathematics has unfamiliar vocabulary words or *terms*, which must be put in sentences called *expressions* or *equations*. When students explain a mathematical equation, they are "talking" mathematical symbols and translating them into words and sentences. Beck (2009:177) argues that the symbolism itself gives mathematics ideas diverse uses, but that its abstractness makes the learning of mathematics difficult.

As the aim of the study was to improve students' academic performance in a mathematics and statistics-related subject by means of a classroom learning strategy intervention, the researcher deemed it necessary to present an overview of the nature of mathematics. Mathematics involves conceptual thinking, which is crucial if students are to follow a deep approach to learning (see 2.4.2). The learning of mathematics also requires students to follow a sequential or cumulative learning pattern, in which new knowledge builds upon prior knowledge. Students therefore have to understand and know previously learned concepts in mathematics as well as statistics. As mathematics and statistics-related courses involve concepts characterised by many symbols and formulas, students have to understand what certain symbols or formulas mean, and practise these every day to become "mathematics proficient".

3.4 LEARNING THEORIES IN MATHEMATICS

According to The National Council of Teachers of Mathematics (2007:xi), "[t]he learning of mathematics is the central goal of mathematics education". In research, various topics on the learning of mathematics have been explored, including how learners construct mathematical knowledge and effective methods of mathematics instruction. Schunk (1996:268) argues that mathematics has been an especially fertile area for cognitive research (see 2.7).

According to Martin (in NCTM 2007:1), "[l]earning theory and research on learning can provide a foundation on which we can better understand student learning". Ernst (1994:2) purports that the import of any theory of learning mathematics consists in facilitating interventions in the process of its teaching and learning.

3.4.1 Traditional teaching and mathematics learning practices

In the majority of traditional classrooms, the learning of mathematics is seen as "mastering a predetermined body of knowledge and procedures" (Goos *et al.* 1999:36). Lambdin and Walcott (in NCTM 2007:4) argue that "drill and practice", which had long been one component of mathematics instruction, became its primary focus in the early years of the twentieth century. The dominant view in mathematics education thus assumed that the teaching and learning of mathematics only required the effective transmission of mathematical knowledge. With this view in mind, traditional teaching and learning mathematics practices have mainly relied on teacher exposition followed by student practice.

3.4.1.1 *The role of the teacher*

According to Barnes (2005:50), the traditional formal and authoritarian approach to teaching mathematics has dominated South African classrooms for many years. According to Keast (1999:54), traditional mathematics teaching is based on an authoritative figure (usually the teacher) giving out information in a non-contextual way without relevance to the life of most of the students. Goos *et al.* (1999:36) argue that, in the majority of contemporary classrooms, the teacher would present the subject matter in small, easily manageable pieces and also demonstrate the correct procedure. The teacher's role has been to demonstrate how a manipulation is to be carried out or to explain how a concept is defined. However, new curriculum approaches in South Africa encourage teachers to facilitate, instead of "telling" (Brodie 2007:3).

3.4.1.2 *The role of the student*

Students, on the other hand, are expected to memorise facts (see 2.4.1) and to practise procedures until they have been mastered (Romberg & Kaput 1999:4)]. In traditional mathematics instruction, learning is based on remembering and correctly applying often complex and unconnected algorithms. Keast (1999:54) argues that the examples, exercises and problems are usually contrived and bear no relevance to or reflect few of the issues relevant to young people. Worlley and Proctor (2005:4 of 22) critically argue that students in traditional mathematics classrooms are indoctrinated to accept what they do not understand and have a passive involvement in an environment that requires unthinking responses. This view is obviously related to a surface approach to learning (see 2.6.2).

Although this approach might appear very reasonable, numerous research studies have shown that such mathematics instruction can leave students with imperfect understanding and flawed beliefs about mathematics (Goos *et al.* 1999:37). As Burger and Starbird (2005:xi) purport: "[f]or many, mathematics is the torture of tests, homework, and problems, problems, problems". Just the word "problems" suggests unpleasantness and anxiety. Because of this, a large proportion of students who are learning mathematics in the traditional way, feel incompetent and helpless in mathematics education.

According to Keast (1999:54), the traditional mathematics classroom is predominantly arranged in ways that encourage students to work individually where opportunities are often rare to discuss and talk through issues to form knowledge. However, social interaction (see 2.8 and 2.11) remains an integral part of learning (Barnes 2005:48). According to Brodie (2007:3), "[t]he new curriculum in South African schools calls for learners to participate in mathematics lessons and to express their mathematical ideas".

3.4.1.3 *Traditional curriculum activities*

Worlley and Proctor (2005:4 of 22) present the view that standardised tests are used in most western countries that measure low-level mathematical skills and strategies rather than conceptual or higher order thinking (see 2.4.2 and 3.3.3.1). The traditional mathematics curriculum consists of rule following and rote memorisation (see 2.4.1), where belief of success in low-level procedural knowledge is aligned to achieving in mathematics. This type of mathematics curriculum is ingrained in our schools and higher education institutions, in which "both rote acquisition of knowledge and mindless procedures are rewarded rather than true conceptual understanding and reflective deployment of strategic know-how" [Wood *et al.* (in Worlley & Proctor 2005:5 of 22)].

In these procedural activities, successful mathematics performance does not necessarily demonstrate, nor provide a foundation for the understanding of core mathematics concepts (see 3.3.3.1). The problem is that "doing mathematics" does not necessarily lead to understanding mathematics (Wild & Pfannkuch 1990:224). Rule and rote practice are related to "doing", and research has emphasised that a large proportion of school mathematics is "doing" (Mousley 1999; Swan 1990). Goos *et al.* (1999:37) argue that "[w]hen student activity is limited to imitating the techniques prescribed by the teacher, they can create the appearance of mathematical competence by simply memorizing and reproducing the correct way to manipulate symbols, and may even come to believe that producing the correct form is more important than making sense of what they are doing" (see 2.4.2).

According to Romberg and Kaput (1999:5), "[t]raditional school mathematics has failed to provide students with any sense of the importance of the discipline's historical or cultural importance, nor any sense of its usefulness". The traditional mechanistic approach to mathematics instruction of basic skills and concepts isolates the discipline from its uses and from other disciplines. Romberg and Kaput (1999:4) argue that, in the traditional process of symbol manipulation, only the deployment of a set routine is involved, with no room for ingenuity or flair, no place for guesswork or surprise, no chance for discovery, and in fact, no need for the human being. No wonder many students dislike the mathematics they are confronted with in some subjects at higher education institutions and fail to learn it well.

3.4.1.4 ***A shift away from traditional mathematics practices***

According to Baroody and Hume (in Barnes 2005:45), mathematics instruction should focus on understanding, encourage active and purposeful learning, foster informal knowledge, link formal instruction to informal knowledge, and encourage reflection and discussion.

The overwhelming body of research tells us to promote learning with understanding (*deep learning*), and should therefore be one of the fundamental goals of mathematics education (Hiebert & Carpenter 1992). Lin and Cooney (2001:3) argue that "[o]ne of the primary goals of teaching mathematics is to enable students to make sense of mathematics". Romberg and Kaput (1999:6) argue that, if mathematics is to serve students' needs to make sense of experience arising outside of mathematics instruction and mathematics itself, it must be firmly rooted in and connected to that experience.

According to Goos *et al.* (1999:36), "[t]he last decade has seen the emergence of an international reform movement in mathematics education that has promoted notions of communications, collaborative interaction and group problem solving – goals and practices that stand in contrast to those of traditional instruction". Current constructivist learning theories in mathematics suggest that students actively construct knowledge consensual with social and cultural settings, and are not passive receivers of knowledge (see 2.11). As Herrington *et al.* (2002:2 of 4) purport: "modern teaching practices involve strategies such as problem solving, investigations, practical activity, group work, projects, and applications of relevant technologies". According to Goldin (1990:31), mathematical learning occurs most effectively through guided discovery (see 2.7.3.1), meaningful application (see 2.7.4.4), and problem solving, as opposed to imitation (see 2.8.1.1) and reliance on the rote use of algorithms for manipulating formal symbols. These changing views of mathematics induced exiting changes in the teaching and learning of mathematics. In the next section, the researcher therefore reviews constructivism as a learning theory in mathematics.

3.4.2 Constructivism in mathematics

According to Santrock (2009:399), "[e]ducators currently debate whether math should be taught using a cognitive, conceptual, and constructivist approach or a practice, computational approach". Some theorists (Cobb, 1994; Lampert, 1990) have already contended that constructivism represents a viable model for explaining how mathematics is learned (see 2.9). As Confrey and Smith (1994:135) purport: "[c]onstructivism is widely used to support reform efforts in mathematics education". Numerous publications offer a discussion of constructivist views on the learning and teaching of mathematics [Davis *et al.*, Steffe and von Glaserfeld (in Ernst 1994:2)]. According to Burton (1999:24), "[c]onstructivism has been a potent influence on the thinking about how people come to know mathematics and, in many parts of the world, curricula have been influenced by its perspectives".

According to Noddings (1990:7), constructivism may be characterised as a cognitive position (see 2.10). As a cognitive position, constructivism holds that "all knowledge is constructed and that the instruments of construction include cognitive structures that are either innate or are themselves products of developmental construction" [Piaget (in Davis *et al.* 1990:7)]. According to Davis *et al.* (1990:7), this interpretation of constructivism is the one held by most constructivists in mathematical education.

Foote, Vermette and Battaglia (2001:24) argue that, from a constructivist point of view, learning is a process that engages the learner in sense-making activities that are shaped by prior knowledge (see 2.10.1.2); occur through social interaction (see 2.11.1.1); are closely tied to particular contextual situations, and involve the use of numerous strategies (see 3.8.2), of which many necessitate higher-order thinking. According to Foote *et al.* (2001:24), activities are structured so that learners create and control the development of their own learning from beginning to end and encounter problems that their current rules cannot solve (see 2.7.3.5).

Jaworski (1994:155) argues that constructivism in mathematics education developed in a very theoretical way throughout the 1980s. According to Burton (1999:24), one effect of constructivism over the 1970s and 1980s was a growth in interest in problem solving (see 3.8) as a means of engaging learners in mathematics. Santrock (2009:399) argues that some proponents of the cognitive approach argue against memorisation and practice in teaching mathematics and instead emphasise constructivist mathematical problem solving. Santrock (2009:399) argues that, in this approach, "effective instruction focuses on involving children in solving a problem or developing a concept and in exploring the efficiency of alternative solutions". This view of effective instruction is also true for the university student.

According to Worlley and Proctor (2005:4 of 22), the theoretical orientation of current practices in mathematics education is mostly founded in the developmental psychological research conducted by Piaget (see 2.10.1), as well as practices related to a Vygotskian sociocultural approach (see 2.11.1). The Piagetian perspective acknowledges a student's mathematical development as exclusively psychological, and the social aspect of classroom only as a catalyst for learning. However, the sociocultural approach of

Vygotsky foregrounds the social activities of learning. A middle ground between these two researchers is described as "emergent perspective" (Cobb 1999:309).

Worlley and Proctor (2005:4 of 22) argue that one of the central tenets of Cobb's (1999) emergent perspective is that "learning is both a process of active individual construction, where students construct new knowledge through reflecting upon their physical and mental actions, and the social process of classroom mathematical practices, both operating collectively and with equal significance".

The next section describes research that is attempting to coordinate a constructivist view of the learning of mathematics. The focus of this research has been on univerisy students' construction of mathematical and statistical knowledge, as the primary interest has been on the processes by which students create mathematical meaning in the course of classroom social interactions. This research has been influenced in general by Piaget's constructivist epistemology (see 2.10.1) as well as Vygotsky, who emphasises the importance of social interaction in the sound learning of mathematics (see 2.11.1).

3.4.3 The cognitive aspect in the learning of mathematics

The following are key Piagetian principles for teaching and learning:

3.4.3.1 *Learning is an active process*

In his early theory of learning, Piaget stressed the notion of the subject's active construction in learning mathematics, noting:

"Mathematics is, first of all and most importantly, actions exercised on things, and the operations themselves are more actions, but well coordinated among themselves and only imagined instead of being materially executed. Without a doubt it is necessary to reach abstraction, and this is even natural in all areas during the mental development of adolescence, but abstraction is only a sort of trickery and deflection of the mind if it doesn't constitute the crowning stage of a series of previously uninterrupted concrete actions" [Piaget (in Maher & Alston 1990:149)]. Voigt (1994:291) argues that with reference to Piaget, "learning mathematics is viewed as structured by the individual's attempts to resolve what the individual finds problematic in the world of his/her experience".

Child (2007:109) argues that "[a] particular contribution of Piaget's theory to the educational scene is in drawing attention to the child as an active participant in concept-learning processes" (see 2.10.1.2). According to Schunk (1996:272), mathematical knowledge is not passively absorbed from the environment, but rather is constructed by individuals as a consequence of their interactions (see 2.11). As Davis *et al.* (1990:2) purport: "learning mathematics requires construction, not passive reception, and to know mathematics requires constructive work with mathematical objects in a mathematical community". Constructing knowledge means that "students are active participants in a learning process by seeking to find meaning in their experiences" (Boghossian 2006:714). Graven and Venkat (2007:67) argue that, since South Africa's first democratic elections in 1994, there has been major educational reform. Apart from the implementation of new curriculums in schools and higher education, "mathematics is acknowledged for its important role in supporting learners to become active participants in the new democracy" (Graven & Venkat 2007:67). This view emphasises the importance that learners or university students should learn how to think.

Dawkins (2006:2) argues that one cannot learn mathematics by just going to class and watching the instructor teaching and working out problems. This means that a student has to attend classes and pay attention while the work is explained in these classes. Although this may sound like the traditional method of learning, the emphasis is placed on the student's active involvement in the learning process. Problems need to be worked out on a regular basis, even if the lecturer does not assign any homework. The student thus needs to be constantly and *actively* involved in the learning process. When students are actively engaged, intellectually and emotionally, meaningful learning occurs (see

2.7.4). In other words, when they encounter moderately novel situations that excite their natural curiosity.

3.4.3.2 *Learning should be whole, authentic, and "real"*

Piaget (in Illeris 2004:69) asserts that "[o]bviously for intelligence to function, it must be motivated by an affective power. A person won't ever solve a problem if the problem doesn't interest him. The impetus for everything lies in interest, affective motivation. Affective life, like intellectual life, is a continual adaptation, and the two are not only parallel but interdependent, since feelings express the interest and the value given to actions of which intelligence provides the structure. Since affective life is adaptation, it also implies continual assimilation of present situations to earlier ones – assimilation which gives rise to affective schemes or relatively stable modes of feeling or reacting – and continual accommodation of these schemes to the present situation." (Piaget 1951:205-206).

For Piaget (in Verhoef and Broekman 2005:273), the development of knowledge takes place by experiencing reality. Piaget made it clear that meaning is constructed as learners interact in meaningful ways with the world around them. Therefore, students are more likely to learn if they are engaged in meaningful activities. As opposed to isolated skill exercises, these whole, real, and authentic activities should be interesting to the student, and should be emphasised in Piagetian classrooms. Santrock (2009:401) advocates that the mathematics problem-solving activities might centre on the student.

According to Hoerl (in Chance 2002:17 of 21), students' understanding and retention could significantly be enhanced by using tangible case studies to introduce and motivate new topics, and by striving for overall understanding of key concepts before acquiring fine skills to apply numerical tools. The researcher makes use of this technique quite often, as it engages students' attention almost immediately when case studies and real-life examples are used (Santrock 2009:50). For example, when the concept of a frequency distribution is introduced to students in an elementary statistics-related subject, the university teacher can make use of students' test marks as raw data to illustrate measures of central tendencies to students. By doing so, students will be exposed to a "real-life" example which has relevance to their own lives. The researcher is also of the opinion that when students understand the "bigger" picture, they do not seem to struggle as much with the finer detail, as the "big" picture already makes sense to them.

Chance (2002:7 of 21) argues that one can provide more structure in the learning process through experiencing situations and learning how to ask relevant questions. University students are sometimes curious to know how the rest of the class performed in a certain written test. The university teacher can provide students with the raw test scores that were obtained in a certain test and, for example, ask students to work out the average for the test, and construct a histogram of the distribution of scores. Not only will students practise an example of measures of central tendency with regard to a statistics-related subject, but they will be exposed to a real-life example that makes sense to them.

The researcher often allows students to discuss some experience they have with statistics outside of class during the term. For example, students may be struck by an interesting statement in the media that they now view differently with their statistical debunking glasses on (Chance 2002:11 of 21). In this way, students can be led to appreciate the role of statistics in the world around them. By making use of such real-life experiences, students also have a higher degree of ownership and engagement in the whole learning experience (see 3.3.2 and 3.4.3.1).

3.4.3.3 *From the concrete to the abstract*

Building on students' knowledge is an important aspect of maths education. According to Foot *et al.* (2001:25), knowledge consists of past constructions. This means that individuals create new knowledge by reflecting on their past physical and mental actions [Piaget (in Foot *et al.* 2001:25)]. As individuals interact with the environment, they continue to use prior knowledge to tune and reshape understanding (Foot *et al.* 2001:25).

Constructivist theories of mathematics learning contend that learners progress developmentally from concrete to abstract thinking [Crowley 1987; Lunkenbein 1985 (in Baroody & Ginsburg 1990:57)]. According to Child (2007:109), concept formation has been one of the special provinces of the Swiss psychologist, Jean Piaget (see 2.10.1). The work and research of Piaget has led him to postulate a theory of qualitative changes during cognitive development from birth to adolescence which take place in a definite, inevitable sequence of maturational steps starting with biological mechanisms and culminating in a highly developed system of abstract operations (Child 2007:110).

When university students do mathematics, their intuitive knowledge begins with the apparent, which may be highly concrete, spotty and unsystematic. However, in time, they master informal knowledge; this time a bit more abstract but not entirely complete and systematic. Finally they acquire formal knowledge with this base, which is relatively abstract, complete, and coherent (see 2.10.1.1). Child (2007:110) argues that, when new topics are introduced to the learner, one should proceed from the concrete and practical to the more difficult abstract. Knowledge, in any domain, then gradually becomes relatively complete, systematic and logical. Child (2007:110) argues that when concepts are cumulative, the order of representation must be worked out carefully so as to build up schemas in a logical and orderly sequence.

By making connections between these various modes of representation, students then progress toward relational understanding of mathematical concepts. Worlley and Proctor (2005:9 of 22) present the view that "mathematics learning is about refinement and abstraction of ideas and concepts". Bruner's theory of instruction (1964) and Piaget's theory of construction, underpin these beliefs. The four major features of Bruner's theory are described in Chapter 2 (see 2.7.3.5).

a) Assimilation and accommodation

According to Foot *et al.* (2001:25), constructions come about through assimilation and accommodation (see 2.10.1.2). As Foot *et al.* (2001:25) purport: "[k]nowledge is not static; it continues to change as we encounter information that embellishes or contradicts it. 'Old' learning continues to effect 'new' learning". Cognitive theory also proposes that learners cannot immediately comprehend abstract instruction and for them to learn mathematics in a meaningful manner, they must be given the opportunity to assimilate it [see 2.10.1.2(a)]. As assimilation and interest go hand in hand, students often do not make the effort to assimilate new information unless it makes some sense and is important to them. Baroody and Ginsburg (1990:56) argue that the cognitive principle of assimilation implies that understanding cannot be imposed upon students and that it evolves as they actively try to make sense of the world. If mathematical symbols, computational algorithms (step-by-step procedures), and so forth are connected to students' existing, personal, counting-based knowledge of mathematics, it can make sense to them.

Santrock (2009:402) advocates that instructors should make enough information available for students to be able to come up with a method for solving maths problems, but withhold enough information so that students must stretch their minds to solve the problems. When a student has a misconception about solving a problem in mathematics, the constructivist lecturer will correct the misconceptions the student has when trying to solve the problem by having the student explore whether the result is satisfactory (Noddings 1990:13). If the results are not satisfactory, the university teacher should allow the necessary change in the student's procedural rules. This notion highlights the importance of the zone of proximal development in Vygotsky's theory of learning (see 2.11.1.2).

According to Noddings (1990:15), we need to know what our students are thinking, how they go about solving problems, and what they can do with the material we present to them. According to Child (2007:102), instructors should have some awareness of the range of possibilities in the concept formation of learners, which implies "knowing how the child thinks and understanding both the limitations and the potential of child thought" (Lefrancois 2000:229). For example, the building up from concrete to abstract reasoning is reflected in mathematics which begins with practical aspects before attempting deductive work (see 2.10.1.1).

Assimilation is prevented when there is a gap between a student's formal instruction and existing knowledge [see 2.10.1.2 (a)]. This gap between a student's relatively concrete informal mathematics and relatively abstract formal instruction for which he/she are often not ready, is a key reason for learning difficulties [Ginsburg, 1989; Hiebert, 1984 (in Baroody & Ginsburg 1990:57)]. According to Wild and Pfannkuch (1999:225), learning is much more than collecting information, as learning involves synthesising the new ideas and information with existing ideas and information into an improved understanding. This emphasises the importance of Piaget's existing mental structures and assimilation (see 2.10.1.2).

It is therefore crucial that the opportunity to assimilate mathematical knowledge must be given to students - to construct accurate and complete mathematical understandings. According to Baroody & Ginsburg (1990:63), this approach is important for fostering self-regulation (see 2.8.1.3) and a positive disposition toward mathematical learning and problem solving (see 3.8), as well as meaningful learning (see 2.7.4.1).

b) The role of practical experience in encouraging active assimilation and accommodation

According to Child (2007:102), building up schemas requires practical experience of concrete situations to encourage active assimilation and accommodation (see 2.10.1.2). Chen (2003:2 of 3) argues that direct experience, making errors and looking for solutions are vital for the assimilation and accommodation of information. The way in which information is presented is also important. According to Santrock (2009:50), the educational implication of Piaget's view is that learners learn best by making discoveries (see 2.7.3.1), reflecting on them (see 3.5), and discussing them (see 2.8 and 2.11), rather than blindly imitating the instructor (see 3.4.1.2) or doing things by rote (see 2.4.1).

c) *The role of reflection and cognitive conflict in enhancing the process of cognitive development and conceptual change*

According to Foot *et al.* (2001:25), "[m]eaningful learning occurs through reflection and resolution of cognitive conflict and, thus, serves to negate earlier, incomplete levels of understanding". In other words, cognitive development is promoted when there is a moderate degree of discrepancy between the learner's cognitive structure and some new event which he/she encounters (Richardson & Sheldon 1988:249; Mintzes, Wandersee, & Novak 1997:272). According to Richardson and Sheldon (1988:249), Piaget stresses cognitive conflict as promoting the equilibration process in his later work. Marx, Heron and Franklin (2004:49) argue that many researchers of constructivism view cognitive conflict as an important factor in conceptual change (see 3.3.3.1). For constructivists, deep learning occurs in periods of confusion and surprise and over long periods of time. It is these conflicts that cause learners to dissect, evaluate and reflect upon prior knowledge (Foot *et al.* 2001:25). Conflicts can be among peer students, among students and instructors, and between a student's present understanding and new information being learned (Marx *et al.* 2004:49). The role of cognitive conflict has relevance with regard to the classroom learning strategy intervention as it is an important factor in the process of conceptual change which will eventually result in deep learning.

3.4.3.4 *Concluding remarks*

With regard to mathematics education, Piagetian learning theory leads one to examine mathematical concepts for their operational bases [Steffe, von Glaserfeld, Richards and Cobb (in Confrey 1999:6)]. Burton (1999:ix) argues that "Piaget's developmental psychology with its theory of stages, and his methodology and epistemology have both inspired and constrained the devevelopment in the field of mathematics education".

Looking into the cognitive aspects of Piaget's theory in the learning of mathematics seems relevant, since higher education encourages students to be active participants in the learning process (see 3.4.3.1), promotes learning that is whole, authentic, and real (see

3.4.3.2), and encourages students to build on prior knowledge in order to move from the concrete to the abstract (see 3.4.3.3).

However, over the past two decades, there has been a shift away from the individualistic theories of learning that specify a strict sequence of stages (see 2.10.1.1), toward recognition of the importance of the social context of learning (see 2.11). Included in this has been the impact of the social learning theory of Lev Vygotsky (see 2.11.1).

3.4.4 The social aspect in the learning of mathematics

Beck (2009:176) purports that when it was found that memorisation and manipulation resulted in poor retention, little understanding, and almost no application in daily problems, a movement began to teach mathematics as a social and practical subject instead of just a skill. In recent years, the social interaction involved in the construction of the mathematics of children has been brought to the fore in order to specify its constructive aspects [Bauersfeld; Yackel *et al.* (in Steffe & Tzur 1994:8)]. According to Voigt (1994:275), there is a growing body of research that supports the relevance of social activities to learning mathematics.

For social constructivists, the construction of meaning is a social process in which the learner constructs meaning at individual and social levels as he or she learns (see 2.11). Learning is thus seen as an active process which is intimately associated with human interaction. Van Oers (in Steffe & Tzur 1994:11) argues that one of the basic tenets of the Vygotskian approach to education is the assumption that individual learning is dependent on social interaction (see 2.11.1.1). As Vygotsky (1978:88) purports: "[h]uman learning presupposes a special social nature by which children grow into intellectual life of those around them". According to Steffe and Tzur (1994:8), students' mathematical interaction also includes enactment or potential enactment of their operative mathematical schemes.

3.4.4.1 *The sociocultural perspective in the learning of mathematics*

In addition to knowledge construction, mathematical competence also depends on sociocultural influence [Cobb (in Schunk 1996:274)]. Shin (1997:36) argues that "one of the main trends in mathematics education is the sociocultural perspective that stems from Vygotsky". The sociocultural perspective can be considered complementary to constructivism, which resulted from Piagetian theory (see 2.10.1). Shin (1997:36) argues that "[i]n the sociocultural perspective, it is assumed that cultural and social processes are integral to mathematical activity". Van Boxtel (2004) argues that sociocultural theories emphasise the collaborative construction of knowledge, the mediational role of tools and the historical and cultural settings in which knowledge construction occurs (see 2.11.1.1). The classroom context is integral to the cognitive activity, incorporating the physical and social environments (Worlley and Proctor 2005:13 of 22).

The student's mathematical experiences as well as beliefs about what it means to know and do mathematics, are also influenced by the classroom community, as students negotiate the constraints in the classroom (see 2.11.1.1; Cobb 1999; Wood, Cobb & Yackel 1995). This assumption of constructivism values both the cognitive as well as the social perspective and recognises them as complementary (Wood *et al.* 1995). Santrock (2009:402) argues that, instructors should build opportunities into the math curriculum for students to use and improve their communication skills that engender discussion, argument and compromise (see 2.11.1.1).

Biehler (in Wild & Pfannkuch 1999:260) purports that recent learning theories in mathematics education regard learning as a process of enculturation (see 3.4.3), as learning to participate in a certain cognitive and cultural practice, in which the lecturer fills the role of a mentor and mediator (see 2.11.1.1). According to Biehler (in Wild & Pfannkuch 1999:261) this is especially true with regard to statistical thinking, which may be better thought of as a "statistical culture"-including value and belief system. Biehler argues that the kind of collaborative and communicative processes that are stimulated in the classroom would be very important for statistical enculturation. Another important feature is the extent to which teachers' behaviour and knowledge represent an authentic model of this culture.

When people with different backgrounds discuss the same problem, it often happens that the challenging of something they "know" and take for granted can remove an obstacle and lead to new insight. Wild and Pfannkuch (1999:229) argue that we tend to solve problems by following "precedents". Sometimes, an innocent question loosens a previous preconception, which leads us to see the problem in a new way. This notion emphasises the importance of collaborative learning and also has relevance to the classroom learning strategy intervention in which students are encouraged to solve problems together (see 2.11.1.1).

3.4.4.2 *Meaning in mathematics*

The existence of mathematical objects is synonymous with meaning. According to Thom (in Steffe 1990:168), "[t]he real problem which confronts mathematics teaching is not that of rigor, but the problem of the development of 'meaning', of the 'existence' of mathematical objects". The answer of mathematical meaning depends on various researcher's perspectives. Voigt (1994:277) purports that several lines of research in cognitive psychology emphasise the individual's sense making processes and the subject's cognitive development (see 2.7; 2.7.4 and 2.10.1.2). Cognitive research indicates that it is essential to distinguish between meaningful learning (see 2.7.4.1) and rote learning (see 2.4.1) and that it is not enough to absorb and accumulate information. Therefore, university students should be encouraged to follow a deep approach to learning (see 2.4.2).

According to Voigt (1994:277), "[m]athematical meaning is taken as a product of social processes, in particular as a product of social interactions. From this point of view, mathematical meanings are primarily studied as emerging between individuals, not as constructed inside or as existing independently of individuals".

a) Internalisation as an important process in constructing mathematical meaning

According to van Oers (in Steffe & Tzur 1994:11), internalisation is a fundamental process in learning as it constructs meaning in mathematics. Voigt (1994:291) argues that, with reference to Vygotsky, "the given environment seems to direct the individual's learning of mathematics. On the one hand, the individual is the actor (subject), and the mathematical knowledge is constructed by the actor. On the other hand, the individual is the object of cultural practices, and given mathematical knowledge is internalized". According to Voigt (1994:291), Vygotsky assumes that the characteristics of adult-guided interactions are internalised by the learner in development: "all higher mental functions are internalized social relationships" (Vygotsky 1981:163-164).

b) The zone of proximal development

Vygotsky (in Steffe & Tzur 1994:11) stressed that cultural meanings can become intermingled with personal sense through an educational process through one of his major concepts – the zone of proximal development (see 2.11.1.2). In the development of knowledge, Vygotsky (1978) stressed the role of competent other persons in the zone of proximal development (see 2.11.1.2). Goos *et al.* (1999:39) argue that we need to appreciate what Vygotsky meant by the zone of proximal development in order to understand how learning occurs, and how people come to appreciate the cultural tools that transform their relationships with one another and with the world. This notion of the zone of proximal development highlights the productive role peer tutors (see 2.11.1.1) can play in scaffolding (see 2.11.1.4) the learning of their fellow students (Goos *et al.* 1999:39).

3.4.4.3 *Applying social constructivism in the mathematics classroom*

Noddings (1990:17) argues that one way we can induce the engagement that is essential if students are to perform construction, is to increase the amount of time students spend working together. There are sound cognitive reasons for allowing students to work together (see 2.8 and 2.11). Vygotsky (in Noddings 1990:17) suggested that children gradually internalise the talk that occurs in groups and posited group interaction as one of the sources in the development of mental operations. This line of thinking called "social constructivism" puts great emphasis on the processes of communicating and negotiating in communities (see 2.11).

Mathematical concepts cannot be developed in the absence of language, where this language refers to "the students' need for opportunities to talk about, share solutions and strategies to problems and explain mathematical ideas to each other and the teacher to help them work through and clarify their thinking" (Worlley and Proctor 2005:11 of 22). Child (2007:110) also presents the view that language is most important for the internalisation of concepts (see 3.4.4.2). Goos *et al.* (1999:40) argue that an advantage of peer groups is that, "removed from the direct influence of the teacher, the students take personal responsibility for the ideas that they are constructing, so the authorship of mathematical knowledge is vested in themselves and their partners". This remark emphasises the importance of internalisation in the learning of mathematics (see 3.4.4.2).

As interactions provide opportunities for language, and reflection and learning as participants make sense of one another's mathematical ideas, the constructivist approach sees social interactions as an essential component of the acquisition of knowledge (Cobb *et al.* 1991; von Glaserfeld 1995). Speech can thus be seen as a social mode of thinking, as it provides both a means for sharing thoughts, as well as a tool for the joint construction of thinking. Van Boxtel (2004:133) argues that "from this perspective the learning and understanding of concepts is distributed over persons and tools, whereas the meaning of concepts are jointly constructed through communication, from which they can be appropriated be each individual".

3.4.4.4 *Concluding remarks*

According to Lerman (2001:44), Vygotsky's theory comprises at least three important factors:

1. It offers a coherent single framework for learning throughout life that applies equally to young children and to mature adults.
2. It attempts to integrate affect and cognition in focusing on meaning as its unit of analysis.
3. It offers a method for rooting knowledge and action in socio-historical-cultural settings (Lerman 2001:44).

It can thus be seen that social interaction constitutes a crucial source of opportunities to learn mathematics in the process of constructing mathematical knowledge. From this perspective mathematical learning can be seen as both an interactive, as well as a constructive activity in which meanings are negotiated (Cobb *et al.* 1990:127).

The preceding paragraphs have made three central points with regard to the social aspect in the learning of mathematics: the sociocultural perspective in the learning of mathematics emphasises the collaborative construction of knowledge in which students should be given opportunities to use and improve their communication skills (see 3.4.4.1); the process of internalisation forms an integral part in the construction of meaning in mathematics (3.4.4.2), through which students should be provided with opportunities to talk and share solutions and strategies to approach problems with one another through the medium of language; and that these meanings become intermingled through the educational process of the zone of proximal development in which peer tutors or more competent persons, such as the university teacher, can play a role in scaffolding. Each of these central points directs attention to how mathematical knowledge is constructed, which bears relevance to the classroom learning strategy the researcher wishes to propose.

3.5 REFLECTION

Reflection forms a crucial part in the process in which we construct knowledge (see 2.7.1.2). In Confrey's (1990:109) view, constructivism should not only emphasise the role of the constructive process, but also allow one to emphasise that students are at least partially able to be aware of those constructions and can then modify them through their conscious *reflection* on that constructive process.

3.5.1 Reflection defined

Landsberg (2005:104) defines reflection as a "meaning-making process that moves a learner from one experience into the next with deeper understanding of its relationships with and connections to other experiences and ideas". Reflective thinking thus makes the continuity of learning possible and ensures the progress of the individual and, ultimately, society. According to Landsberg (2005:104), reflection needs to happen in a community with others and requires attitudes that value the personal and intellectual growth of the self and others.

According to Illeris (2004:46), one practises *afterthought* when one reflects on something or gives further thought to something, which may be an event or a problem. With regard to the research in this study, reflection should be thought of in the cognitive sense of afterthought. Thus, it can be asserted that the constructive process is involved in all acts of perception and cognition and that students can gain a measure of access to the constructive process through the process of reflection.

According to Illeris (2004:46), the reflective process in mathematics is essential, in which the construct becomes the object of scrutiny. This makes sense as mathematics is a form of human language which is built from human activity, for example counting, folding, ordering, comparing, etc. Such a language is created when we reflect on such activities: learning to carry them out in our imaginations and to name and represent them in symbols and images. "Reflection thus functions as the 'bootstrap' by which the mathematician pulls himself up in order to stabilise the current construction and to obtain the position from which the next construct can be carried out, where reflection is seen as the 'objectification' of a construct" (Illeris 2004:46).

3.5.2 Development of students' reflective processes

Confrey (1990:116) suggests three stages in the construction of mathematical knowledge in order to promote a student's awareness of his or her problem solving. Confrey (1985) indicates that for students to modify and adapt their construction, they must: (1) encounter a situation that they experience as personally problematic; (2) act to resolve the problem; (3) assess the success of their action in resolving the problematic, or determine what problem remains. In learning mathematics, the teacher can make use of three categories of questions, which correspond roughly to these three stages of construction posited in Confrey (1985). In order to increase students' awareness of their own strategies and methods, the following three levels of questioning are therefore suggested:

3.5.2.1 *The first level: The interpretation of the problem*

The questions asked in the first level involve the request to reread or restate the question. For example, "What are we doing?", "What is the problem?" or "What does this problem say?". These type of questions would often focus the student's attention on the language the student was using. Although these questions may appear deceptively simple to an observer, it may be difficult for the student to repeat the problem or describe it in the appropriate fashion. The questions on this level serve a subtle and essential role as they can make the student seem unaccustomed to speaking mathematically. However, Confrey (1990:116) argues that this level of questioning can have a significant impact on students' success in solving a problem.

University teachers can make use of this questioning technique during practical classes when students work out exercises from the prescribed textbook. What is verbalised as repetition, and what may often sound to the listener to be the multiple rereadings of the problem, can have the effect of curtailing the amount of time the student needs to attempt to solve the problem and can represent significant cognitive processing on the part of the student (Confrey 1990:116).

3.5.2.2 *The second level: Cognitive strategies*

While working on a problem, the student can be asked what he or she is doing. The level of precision of the student's statements as a standard is used, requesting slightly more. Confrey (1990:116) argues that, on this level, the student is not allowed to introduce mathematical terminology or formulae without these being explained to them. In other words, students should be given an opportunity to explain how they perceive the problem.

When students solve problems from the exercises in the prescribed textbook during practical classes, they often struggle with these problems and ask for assistance. The university teacher can, for example, ask students why they choose a certain formula instead of another. Another example is that students often make computational errors while working with their calculators. The university teacher could then ask the students to show or explain to him/her how they arrived at their answer.

3.5.2.3 *The third level: Justification of strategies*

After students have identified what the problem is about and how they are going to go about solving it, the student is now given an opportunity to defend themselves. The explanation by students should fit their interpretation of the problem and the methods and strategies they have constructed. Questions such as "Why?", "What does that tell you?" and "Why not?" can be used at this level.

When university students solve problems during practical classes, they should be encouraged to be aware of the problem-solving strategies they are using. The questioning technique described above allows students to justify their problem-solving strategies and also to be engaged and aware of the strategies they employ when solving problems.

According to Confrey (1990:116), these three questioning strategies can be used when the primary goal is to develop the students' reflective processes. Also, when students are aware of what they are doing, and not just blindly imitating problems by rote, they follow a deep approach to learning (see 2.4.1 and 2.4.2).

3.6 SYMBOLS

3.6.1 The importance of symbols in mathematics

Novak and Gowin (1984:17) present the view that the aspect of learning that is distinctly human is our remarkable capacity for using written or spoken symbols to represent perceived regularities in events or objects around us. As Child (2007:84) purports: "[p]eople's ability to deal with problems in the absence of material evidence or mental images and to reach higher levels of complexity in appreciation of and response to the environment (in contrast to animals) is due entirely to the use of symbolic languages". According to Child (2007:84), these include number systems, musical notation, etc., as well as the spoken and written word. However, the role of symbolic notation in mathematics is so commonplace that the role of symbols is often by-passed in discussion.

According to Schunk (1996:193), prominent symbol systems include language and mathematical notation, through which the systems allow one to understand abstract concepts [for example, the variable x in the equation ($2x + 8 = 10$), and to alter symbolic information as a result of verbal instruction]. For Child (2007:84), language plays a vital role in this process. For example, for the mathematician, the symbol α is not just something that looks like an "a", but represents the symbol "alpha". According to Heuer (2005:51), a symbol such as $\sum$ is also known as a logogram, and stands for a whole word but has no sound-symbol relationship for students to decode.

According to Cobb (2002:171), people's activity with symbols is an integral aspect of their mathematical reasoning, rather than an external aid to it. Cobb (2002:171) argues that "[a]s a consequence, the process of learning to use symbols in general, and conventional mathematical symbols in particular, is cast in terms of participation". With this view in mind, the use of symbols is then seen not so much as something to be mastered, but as a constituent part of the mathematical practices in which students come to participate.

According to Cobb (2002:171), the development of ways of symbolising is acknowledged to be central to the process of mathematising from a number of theoretical perspectives. In constructivist terms, the tools and symbols students use profoundly influence both the course of their mathematical learning and results, and the increasingly sophisticated mathematical ways of reasoning that they develop [Kaput 1991; Thompson 1992 (in Cobb 2002:171)]. Van Oers (in Cobb 2002:171) explicitly frames mathematics as an activity that is mediated by the use of tools and symbols. According to Hung (2000:64), "[s]uch a process suggests that when students reflect on mathematical statements and symbols, there can be more than one mathematical interpretation of the statements and symbols". Hung (2000:64) argues that students should also learn to argue, defend, and debate their interpretations.

According to Hersh (in von Glaserfeld 1994:6), "symbols are used as aids to thinking just as musical scores are used as aids to music". The music comes first, the score comes later. If we see symbols the way Hersh did, it follows that a string of mathematical symbols will thus remain meaningless until we associate specific mental operations with the symbols, as recommended on the previous page. Baron (1972:38) argues that it is only possible to represent and preserve long trains of reasoning by means of specialised symbols. In other words, mathematical symbols possess meanings in mathematics. Cobb (2002:171) argues that "the ways that symbols are used and the meanings that they come to have are seen to be mutually constitutive".

Symbols provide for an economy of thought through which many of the elements of mathematics can be stacked away in a well-organised form to make verification possible. Child (2007:84) argues that we order our experiences through the acquisition of concepts, whether from the simplest discrimination and classification of objects to the complex mathematical or scientific concepts. According to Child (2007:109), "[m]uch of our experience is assimilated in the form of concepts and expressed in a symbolic form, as in verbal and mathematical modes". Child (2007:109) argues that we can thus organise our percepts and employ symbol forms to represent these experiences, by the processes of forming categories and discriminating between the critical attributes of objects and events.

According to Maharaj (2007:34), the KwaZulu-Natal Department of Education (2000, 2001 & 2002) and the Gauteng Department of Education (2000, 2001, 2002 & 2003) identified a number of recurrent issues evident in learners' work that lead to poor performance. According to Maharaj (2007:34), two of these issues are a poor understanding of mathematical terminology and concepts and the inability to recall and apply formulae. Kenney *et al.* (2005:6) stresses the importance of recognising the potential for enormous confusion that symbolic representation can create. As Barton and Heidema (2002:15) purport: "In reading mathematics text one must decode and comprehend not only words, but also signs and symbols, which involve different skills. Decoding words entails connecting sounds to the alphabetic symbols, or letters .... In contrast, mathematics signs and symbols may be pictorial, or they may refer to an operation, or to an expression. Consequently, students need to learn the meaning of each symbol much like they learn 'sight' words in the English language. In addition they need to connect each symbol, the idea it represents, and the written or spoken term that corresponds to the idea".

Romberg and Kaput (in Fennema & Romberg 1999:7) argue that in mathematics instruction, "its systems of signs and symbols must be learned and experienced as genuine, functional languages – for expressing, communicating, reasoning, computing, abstracting, generalizing, and formalizing – that the student experiences as serving his or her real needs". Not only should students know the concepts and procedures for some parts of mathematics, but also understand how mathematics is created and used. According to Romberg and Kaput (in Fennema & Romberg 1999:7), this view of mathematics is integrative: "It sees everything as part of a larger whole, with each part sharing reciprocal relationships with other parts" and "it is a philosophy that simultaneously stresses erudition and common sense, integration through application, and innovation through creativity. Most importantly, it stresses the creation of knowledge". Against this broad and ambitious view of mathematics, "traditional mathematics appears thin, lifeless, and isolated" [Romberg & Kaput (in Fennema & Romberg 1999:7)].

3.6.2 Formulae in mathematics

According to Dawkins (2006:2), one needs to understand how to use formulae in mathematics. As many formulae have restrictions to them, one needs to know how to use them correctly. General formulas require students to identify the parts in the problem that corresponds to certain parts in the formula. This is an extremely important statement, as it can often be very difficult to use formulas, if one does not understand how a formula works, as well as the principle behind it. If students try to memorise formulas, but they do not know how they work, or the principle behind them, the memorised formula will be worthless.

Consider the following example: Say we wish to calculate the simple interest accrued on R300 for 3 years at 4 % interest per year. The formula $I = P \times R \times T$ is the appropriate formula to use in solving this problem. Each of the symbols I, P, T and R has been chosen to represent an appropriate concept, for example I = Interest, P = Principal, T = Time and R = Rate of Interest. The way these symbols are arranged evokes the appropriate arithmetical operations by which one can obtain the answer to the problem. Also see Chaper 4 on how symbols and formulae can be organised by means of a concept map.

3.6.3 Symbols in the process of reflection

Symbols make an important contribution in the process of reflective activity. Skemp (1972:200) argues that, although it was years ago and in a quite different context, Freud pointed out that the process of making an idea conscious was closely associated to the use of symbols. As no one can see or hear someone else's verbal thoughts or mental images, concepts are elusive and inaccessible mental objects. Even if we do not write or speak symbols, they can be made visible and audible. As symbols are much more concrete objects of thought than the ideas which they represent and to which they are attached, we can use them in our thinking. By reflecting on one of our own schemas, and then making public the attached symbols, we can enable someone else to organise his own thoughts according to the same schema. Skemp (1972:201) purports that communicating our mental processes to other people and becoming aware of them ourselves are closely connected. Explaining our ideas to someone else can sometimes make us realise that we understand them less well than we thought we did, which will eventually lead to greater understanding.

From years' experience as an educator at a higher education institution, the researcher is aware that many students do not realise the importance of symbols or formulas when they study for a mathematics or statistics-related subject. The preceding paragraphs attempted to present the importance of symbols and formulas in the learning of mathematics and statistics based on literature. Looking into the importance of symbols and formulas in mathematics therefore seemed relevant, as symbols, and the use of formulas form an integral part in the learning of mathematics as well as statistics. Due to the importance of symbols and formulas, the researcher will devote special attention to these concepts with regard to the classroom learning strategy intervention she wishes to propose, as the aim of the research is to improve students' learning strategies in a mathematics and statistics-related subject.

3.7 PROBLEM SOLVING

Silver (1985:55) argues that "[a]ttempts to create models of problem-solving ... have become extremely popular in recent years among cognitive psychologists and mathematics educators". According to Schunk (1996:237), problem solving represents a key area for exploring the operation of complex cognitive processes. Regardless of perspective, researchers and practitioners agree that problem solving is important and that students need to develop problem-solving skills (Schunk 1996:238). In the following section, the researcher overviews problem-solving processes as well as an information processing (cognitive) view on problem solving, along with various strategies of solving problems.

3.7.1 Problem solving defined

Before a discussion on problem solving can commence, it is important to probe into the definition of a problem. According to Santrock (2009:331), "[p]roblem solving involves finding an appropriate way to attain a goal". The problem may, for example, be to compute a solution or to answer a question. According to Martinez (in Snowman *et al.* 2009:248), problem solving is "the identification and application of knowledge and skills that result in goal attainment". Garofalo and Lester (1985:169) view problem solving as a process involving the highest capabilities – visualisation, association, abstraction, comprehension, manipulation, reasoning, analysis, synthesis, generalisation – each needing to be "managed" and all needing to be "coordinated".

Polya (in Chapman 1972:133) purports that "solving problems is a practical art, like swimming or ski-ing or playing the piano; you can learn it only by imitation and practice. If you wish to become a problem solver you have to solve problems".

3.7.2 Insight in problem solving

According to Schunk (1996:239) problem solving often is thought to involve insight, or the sudden awareness of a likely solution (see 2.7.1.1). The gestalt psychologists postulated that much human learning is insightful and involves a change in perception (see 2.7.1). Initially, learners think about the steps necessary to solve a problem; they integrate these steps in various ways until the problem is solved, which happens suddenly and with insight. When Wertheimer (1945) distinguished rote memorisation from insight in humans, he emphasised the fact that most situations have some organisation, which highlights the notion of rote memorisation as an insufficient method of solving problems. Some students have difficulty in organising information and generalising what they have learned to similar examples and problems [Lerner (in Steele 2004:39)]. According to

Steele (2004:62), some students master skills when taught in isolation, but might not see how those same strategies are relevant to a subsequent, more difficult problem in the next unit of the study. By discovering the organisation of the situation, and by arranging and rearranging these elements, learners eventually gain insight into a solution and problems are solved (see 2.7.1.3 and 2.7.2.2). According to Kilpatrick *et al.* (2001:118), "competence in an area of inquiry depends upon knowledge that... is represented mentally and organised (connected and structured) in ways that facilitate appropriate retrieval and application". As organisation improves retention, promotes fluency, and facilitates learning related material, learning with understanding is thus more powerful than simply memorising (see 2.4.2; 2.7.1.3; 2.7.2.2 and 3.3.3.1). Teachers could therefore aid in problem solving by arranging elements of a situation so that students would be more likely to perceive how the parts relate to the whole (see 2.7.1.3 and 4.2).

3.8 PROBLEM SOLVING IN MATHEMATICS

Schoenfeld (1980:15) argues that the problem-solving process is one of the most important aspects of mathematics with which teachers should be concerned. As Davis and McKillip (1980:80) purport: "[t]he ability to solve problems is one of the most important objectives in the study of mathematics".

According to Silver and Thompson (1984:529), "many students are not capable of solving relatively straightforward mathematics problems, and most students fail to solve somewhat complex problems". On the other hand, why is it that students succeed in solving concrete problems that are focused and simple, but not large, complex conceptual problems? This question is answered by a description of Polya in "[t]he random walks of George Polya" [Polya (in Alexanderson 2000:24)]: "It was not given to him to solve very difficult problems or to build vast conceptual structures. Yet he could perceive the significance, the beauty, and the promise of a rather concrete, not too large problem, foresee the possibility of a solution, and work at it with intensity. And, when we had found the solution, he kept on working at it with loving care, till each detail became fully intuitive and the connection of the details in a well-ordered whole fully transparent" [Polya (in Alexanderson 2000:24)].

A class in which the students are helping the teacher work out problems and contributing actively to their solutions, is more likely to be dynamic and motivating than one that follows the classical "show and drill" mode (Schoenfeld 1980:15; see 3.4.1). Schoenfeld (1980:16) argues that we should not think of problem solving as a separate skill to be taught in a separate course, but rather that every hour in the classroom presents opportunities for showing students how to think mathematically.

Schoenfeld (1985:14) identifies four categories of knowledge/skills that are needed to be successful in mathematics, which are the following:

1. Resources – propositions of knowledge/skills needed in mathematics.
2. Heuristics – strategies and techniques for solving problems.
3. Control – decisions about when and what resources and strategies to use.
4. Beliefs – a mathematical "world-view" that determines how someone approaches a problem.

These four categories are important in the sense that they characterise someone's problem solving skills. As the aim of the researcher's study is to improve students' mathematical performance by means of a classroom learning strategy intervention, the researcher regards students' problem-solving skills as highly important. In the next section, the researcher discusses each of these four categories in detail.

3.8.1 Resources (knowledge/skills) and problem-solving skills

In *Adding It Up*, Schoenfeld (2007:64) describes one of the strands of mathematical proficiency as "strategic competence", which is the ability to formulate, represent and solve mathematical problems. According to Schunk (1996:270), successful mathematical problem solving depends on students' processing knowledge and problem-solving skills.

For Burris (2005:1 of 6), problem solving involves a variety of skills, which include the following:

- Knowing the basic arithmetic skills
- Knowing when to incorporate these skills into new contexts.
- Being able to perform the skills mentioned above.

However, Bley and Thornton (2001:37) advocate that, because students can carry out the operations in isolation, does not mean that they know when to apply them or how to make interpretations. A study by Hembree (1992:242) revealed that "[d]irect significant links were found between problem solving and various measures of basic performance, especially skills in basic mathematics".

Schoenfeld (2007:64) argues that students should be able to use the mathematical knowledge they have. According to Milgram (2007:51), problem solving in mathematics "requires real knowledge of the subject". As problem solving entails domain specificity and domain independence, one needs a lot of domain specific knowledge to solve problems within a domain. Jessup (2009:63) defines domain specific knowledge as "knowledge pertaining to a specific subject or area and gained through previous study, domain knowledge can take the form of either procedural or declarative knowledge" (see 2.7.2.3 (c). Domain-specific knowledge is thus the content in a specific area of content on which a learner focuses thinking skills. Therefore, the more a learner knows about an area of knowledge – a domain – the more effectively the learner will be able to think.

According to Moore (in Wild & Pfannkuch 1999:251), data always have a context, and students must learn by rather simple examples and experiences to pursue a synthesis of context knowledge and statistical knowledge. Lack of knowledge obviously constrains thinking (see 3.8.1). According to Wild and Pfannkuch (1999:229), what we "know" is not only our greatest asset, but also our biggest curse because the foundations of what we "know" are often not soundly based. According to Wild and Pfannkuch (1999:225), statistical investigation is used to expand the body of "context" knowledge. They argue that the ultimate goal of statistical investigation is learning in the context sphere.

First of all, a distinction has to be made between computation (use of rules procedures, and algorithms) and concepts (problem solving and use of strategies).

3.8.1.1 *Computational problems*

One of the sources of computational difficulties is poor declarative knowledge of number facts [see 2.7.2.3 (c)]. Schunk (1996:270) purports that basic addition, subtraction, multiplication, and division facts involving simple numbers are not well established in students' memories (see 2.7.2.1). For example: 4 x 8 = ? is a cue to retrieve this fact from the long-term memory (see 2.7.2.3) and until facts become established, children count or compute answers. One of the goals of computation instruction is for students to become skilled in using efficient procedures. The following are examples of some computational problems students' encountered in the subject *Business Calculations* as part of the classroom learning strategy intervention:

Example 3.1

$18 - (-5) = 23$

Source: Croucher (2002:M.7)

or to solve x and y in a simultaneous linear equation like

$$3x + 4y = 33$$
$$2x - 3y = 5$$

Source: Croucher (2002:M.73)

3.8.1.2 *Concepts*

As part of the classroom learning strategy intervention, the researcher explained the concept of solving simultaneous linear equations to students that took Business Calculations as a subject during the first semester of the 2009 academic year. The following example from the prescribed textbook is used as an illustration of a concept.

Example 3.2

A customer buys 5 apples and 6 pears at a fruit stall and is charged R3,10. Another customer buys 2 apples and 5 pears and is charged R2,15. How much are the apples and pears each?

Source: Croucher (2002:M79)

Solution:

The problem above involves two equations with two unknowns.

Let x be the price of 1 apple.

Let y be the price of 1 pear.

Then: $5x + 6y = 3.10$ (1)

$2x + 5y = 2.15$ (2)

We have two simultaneous equations in the variables x and y. Suppose we want to eliminate x from the equations, we first have to multiply equation (1) by 2, and equation (2) by 5. Then it follows that:

$10x + 12y = 6.20$ (3)

$10x + 25y = 10.75$ (4)

If equation (4) is then substracted from equation (3), it follows that:

$$
\begin{array}{rl}
 & 10x + 12y = 6.20 \\
\text{minus} & \underline{10x + 25y = 10.75} \\
 & -13y = -4.55
\end{array}
$$

Therefore: $\dfrac{-13y}{-13} = \dfrac{-4.55}{-13}$

$y = 0.35$

If we substitute $y = 0.35$ into equation (1), it follows that:

$$
\begin{aligned}
5x + 6(0.35) &= 3.10 \\
5x + 2.10 &= 3.10 \\
5x &= 3.10 - 2.10 \\
5x &= 1.00 \\
\frac{5x}{5} &= \frac{1.00}{5} \\
x &= 0.20
\end{aligned}
$$

Hence, the solution to the equations is $x = 0.20$ and $y = 0.35$. Therefore, each apple costs 20 cents and each pear 35 cents.

The problem above uses a mathematical computation not really more difficult than those required in the first illustration (see 3.8.1.1). The latter problem, however, does not explicitly tell students what to do. If students recognise the problem format, they will be able to solve the problem. Even if students are not directly told how to solve the problem above, recognition of the problem format and knowledge of procedures will lead them to perform the correct operations.

Initially, learners represent the skill as declarative knowledge [see 2.7.2.3 (c)] in a propositional network (see 2.7.2.3) and, through mental rehearsal and overt practice, facts concerning the different steps are committed to memory (see 2.7.2.2). As learners insert the steps they have memorised into this general heuristic, the declarative representation (see 2.7.2.3) changes into a domain-specific procedural representation and eventually becomes automated [see 2.7.2.3 (c)]. Learners quickly recognise the problem pattern and then implement the procedure without much conscious deliberation at the automatic stage [see 2.7.2.3 (c)]. As learners develop these skills, they are able to execute it rapidly and achieve greater accuracy in their answers.

Students often reveal that they have no understanding of the conceptual relationships indicated by the symbols in the formulas they have learned by heart, when asked to solve a simple problem that is in some way different from the familiar ones in the textbook. Von Glaserfeld (1995:5) advocates that to solve problems that are not identical to those presented in the preceding course of instruction requires conceptual understanding (see 3.3.3.1). When faced with novel problems, only students who have acquired such a conceptual repertoire have a chance of success. Much writing has stressed the social component in the development of conceptual knowledge (Von Glaserfeld 1995:11). Hunt (1977:78) suggests that conceptual level may serve as the basis for "optimizing the teaching/learning process".

It is thus important to be able to perform computations in order to understand how to solve a problem. However, one can be computationally proficient (see 3.8.1.1) but not be able to conceptualise problems (3.8.1.2), which emphasises the fact that mathematical proficiency requires both learning computation as well as problem solving together. According to Giordano (1992:88), heuristic strategies "are designed to help learners conceptualize problems and organize their responses to problems".

3.8.2 Heuristics

Santrock (2009:331) argues that students need to develop strategies to solve problems, such as heuristics. Burris (2005:3 of 6) argues that the "plan" used to solve a problem is often referred to as a problem-solving strategy. According to Santrock (2009:332), heuristics are strategies or rules of thumb that can suggest a solution to a problem.

Schunk (1996:240) advocates that general heuristics are most useful when one is working with unfamiliar content, but less effective when one is working within a familiar domain, because as domain-specific skills develop (see 3.8.1), students increasingly use established procedural knowledge [see 2.7.2.3 (c)] for that content. However, a heuristic will be more systematic for many students than their present problem-solving approaches, and may lead to better solutions.

According to Suydam (1980:43), evidence in the research strongly concurs that problem-solving performance is enhanced by teaching students to use a variety of strategies or heuristics (Blake 1977; Graham 1978; Pennington 1970; Webb 1979; Wilson 1968). Robinson (in Suydam 1980:38) argues that good problem solvers use a formal strategy more often than poor problem solvers do, who tend to rely more frequently on a random trial-and-error strategy. According to Schoenfeld (1980:9), "many students can learn to use heuristics, with the result being a demonstrable improvement in their problem-solving performance". Schoenfeld (1980:17) argues that "[i]f we really expect students to use a heuristics strategy, we must teach it with the same degree of seriousness we would devote to any other mathematical technique".

Rhee (2007:885) advocates that "[s]tudents need a framework, a heuristic that puts the classroom process into a larger structure of a problem-solving process". According to Graham (2006:207), schematic frameworks set out the main stages or steps involved in solving problems. They also emphasise principal steps and their interrelationships and hides detail which can be uncovered subsequently when analysed as (sub)processes.

In *How to solve* it, which is probably the most famous of all works on mathematical problem solving, Polya (1945) used a framework as a thinking tool. Experience in the quality arena and research in education have shown that the thinking and problem-solving performance of most people can be improved by suitable structured frameworks [Pea 1987:91; Resnick 1989:57 (in Wild & Pfannkuch 1999:224)].

3.8.2.1 *Polya's heuristic*

Schoenfeld (2007:64) purports that the starting point for any discussion of problem-solving strategies is the work of the mathematician George Polya. With the pioneering first edition of his book *How to Solve It* in 1945, Polya opened up the study of problem-solving strategies (Rhee 2007:883). According to DeGuire (1980:70), "George Polya is a master problem solver and teacher of problem solvers". Polya is regarded by many "as the person who codified the verbal processes in problem-solving" (2007:52). The core of the book was devoted to a "short dictionary of heuristic". According to Rhee (2007:882), Polya showed generations of maths teachers and students that problem-solving can be learned through the heuristic of simple questions that stimulate curiosity and creativity.

According to Mason (1999:195), "Polya used mathematical problems as a context in which to illustrate heuristics". According to Polya (1945:112-113), the aim of heuristic is to study the methods and rules of discovery and invention. Polya purports that "modern heuristic endeavors to understand the process of solving problems, especially the *mental operations typically useful* in this process" (Polya 1945:129-130). Polya's (1945:129-130) list of mental operations involved in problem solving is as follows:

1. Understand the problem
2. Devise a plan
3. Carry out the plan
4. Look back

- *Understand the problem*

"When saying 'understand the problem' Polya seems to mean simply read or observe the problem, a first cut" (Metcalfe 2006:126). Understanding the problem involves asking questions such as:

"What are you asked to find or show?"
"What information is given?"
"What is the unknown?"
"What are the data?"
"Is there enough information given? If not, what is missing?"
"Is there any extra information given? If so, what information is not needed?" (Metcalfe 2006:126)

According to Rhee (2007:891), a student must understand the problem first, before he/she can solve it. Although this thought might seem obvious to many, many students attempt to solve a problem without first understanding it (Rhee 2007:891).

- *Devise a plan*

According to Rhee (2007:891), "[n]o problem can be solved without understanding its essential nature". After understanding the problem, the task is to devise a plan of execution. Polya emphasised that knowledge builds upon itself: "We know, of course, that it is hard to have a good idea if we have little knowledge of the subject, and impossible to have it if we have no knowledge. Good ideas are based on past experience and formerly acquired knowledge [Polya (in Rhee 2007:892)]. According to Rhee (2007:892), Polya suggested reasoning by analogy. With his second step, Polya advises the need to seek analogous problems that have been solved (Metcalfe 2006:126). In devising a plan, students can ask themselves questions such as: "Do I know a related problem? What is the unknown?" (Rhee 2007:895).

In devising a plan, Schunk (1996:240) argues that one tries to find a connection between the known and the unknown. While carrying out the plan, each step needs to be checked to ensure it is being properly implemented and when one looks back, the solution needs to be examined.

- *Carry out the plan*

In the third step of Polya (1945), it is usually a relatively simple process to carry out the plan, once a problem has been carefully analysed and a plan is devised – if the plan is a suitable one for the given problem. It often happens that the original plan fails and another plan has to be devised. Students may need to modify their original strategy or select a new one. However, it is important for students to realise that not every problem will be solved within the very first attempt. Students should rather view a failed attempt as a learning experience. Cooperative learning partners (see 2.11.1.1) can be used to encourage and engage students as an aid in trying to help them to avoid becoming frustrated or discouraged.

The use of small group cooperative learning was reviewed by Garfield (1993) and Mohammed Yusof and Tall (1999) and showed that a mathematics course encouraging cooperative problem solving and reflection changed students' classroom attitudes from negative to a desired positive. According to Yusof and Tall (1999:68), there is evidence that presents the view that a supportive problem-solving environment can be of help in changing student's attitudes [Shoenfeld 1985, 1987; Davis and Mason 1987, 1988; Rogers 1988 (in Yosof & Tall 1999:68)].

- *Look back*

In the fourth step, it is important to check the solution, once an answer or solution is found. All steps and calculations should also be checked again. Students often fail to look back at the analysis and solution of a problem as a result of their haste to go on to another problem or some other work (Davis & McKillip 1980:91). Davis and McKillip

(1980:91) suggest that students should take a few moments "after a problem has been solved to reflect on the plan used to solve it".

Wild and Pfannkuch (1999:242) argue that problem-solving tools and "worry" or "trigger" questions can be taught to students, instead of relying solely on an apprenticeship model. Students may make use of the following self-questioning technique in the "looking back" process:

"Is the answer reasonable?"
"Does the answer fit the data in the problem?"
"Can I verify my answer my making use of another method of solution?"
"Does my answer fulfill all conditions or requirements of the problem?"
"Is there more than one answer?" (Davis & McKillip 1980:91)

As students review or reflect on a problem, they learn from their mistakes. Another aspect that Dawkins (2006:11) introduces, is the "error" factor. When students find errors in their homework or exams, they need to try to understand what the error is and what they did wrong. Reusser (in Barnes 2005:46) also points out the importance of understanding learner errors that occur while solving mathematical tasks. Students need to look for something about the error that they can remember in order to prevent them making the same error again. If students cannot see the error for themselves, the lecturer, peer students or tutor can help them find it. Students often make "rushed" errors, when they are in a hurry and do not pay attention to what they are doing. By slowing down, students can avoid making these unnecessary arithmetic or computational errors. According to Reusser (2000:21), errors should be viewed as "learning opportunities and as challenges to clarify conceptual misconceptions", rather than being seen as indicators of failure (Barnes 2005:46).

Dawkins (2006:11) argues that if students continually make repeated errors regarding a specific topic or type of problem, they probably do not really have a good grasp of the concept (see 3.8.1.2) behind the type of problem. Students need to go back to where the work is explained and seek help if they still do not understand the concept. According to Dawkins (2006:11), students can keep an "error list", in which they write down errors that they keep on making, with the correct method/solution opposite each error. By reviewing the list after a problem has been completed, students can see whether they have made any of their "common" errors. Students should therefore be encouraged to reflect on their mathematical experience and talk about their attempts to solve problems (see 2.7.1.2 and 2.11.1.1). The lecturer should also encourage students to consider the effectiveness of their solutions and determine where things may have gone wrong.

From the discussion above, it is clear that Polya's heuristic provides a foundation with regard to the study strategy the researcher wishes to propose. In the next sections, the researcher reviews several other heuristics, after which she proposes her own learning strategy (see Chapter 4).

3.8.2.2 *The "IDEAL" heuristic of Bransford and Stein (1984)*

According to Dewey's scheme for the logic of inquiry, the prototypical system of delivering mathematical facts leaves out the necessary first step in problem solving: the identification of the problem (Dewey 1933; 1938). For Schoenfeld (1985:101), it is interesting that Polya (1957) also omits this very important first step in problem solving. According to Schoenfeld (1985:101), the expert mathematician may take this first step for granted. However, for the student meeting the formal systems that mathematics offers and the historically accrued problem-solving contexts for which mathematics has been found useful, the first step is a giant one, which requires support. This very important first step was given support by a similar heuristic, known as IDEAL, which was formulated by Bransford and Stein in 1984:

I = Identify the problem

D = Define and represent the problem

E = Explore possible strategies

A = Act on the strategies

L = Look back and evaluate the effects of your activities

Coon and Mitterer (2008:342) argue that to apply the "ideal" thinking strategy one should first *identify* the problem, clearly *define* it, and then *explore* possible solutions and relevant knowledge. After that, one should *act* by trying a possible solution and finally, *look back* at the results and learn from them.

From the discussion above, it can be seen that the "IDEAL" heuristic of Bransford and Stein (1984) is very similar to that of Polya (see 3.8.2.1). The only difference between these two heuristics, is the first step, namely, the "identification of the problem". The researcher also regards this step as extremely important with regard to the learning strategy she wishes to propose.

3.8.2.3 *Maccini's "STAR" strategy*

In a quest to improve students' performance regarding algebra problems (Maccini & Hughes 2000; Maccini & Ruhl 2000), Maccini's model can be used. According to Dehn (in Morris & Mather 2007:251), Maccini's instructional strategy is used on abstract representations and solutions for problems involving addition, subtraction, multiplication and division of integers. Maccini's "STAR" strategy utilises manipulatives and a systemic sequence of steps (Morris and Mather 2007:326; Maccini & Gagnon 2006:3 of 9):

S = Search the word problem (Read and ask yourself questions, for example, what facts do I know? What do I need to find? Write down all the facts.

T = Translate the words into an equation or represent in picture form (Choose a variable; Identify the operations; Represent the problem by making use of concrete, semi-concrete, and abstract representations).

A = Answer the problem by using clues.

R = Review the solution (Reread the problem; Ask yourself if the answer make sense.)

Maccini's "STAR" strategy is very similar to Polya's heuristic (see 3.8.2.1) and the "IDEAL" heuristic of Bransford and Stein (see 3.8.2.2). By searching the word problem, one has to read (Polya) and identify the problem (IDEAL), and devise a plan (Polya). In the second step of the "STAR" strategy, one should translate the words into an equation, in other words, define and represent the problem and explore possible strategies (IDEAL). The third step of Maccini's STAR strategy suggests ways to answer the problem; in other words to carry out the plan (Polya), or to act on the strategies (IDEAL). The final step in the "STAR" strategy is to review the solution; in other words, to look back (Polya) and to evaluate the answer (IDEAL) (see 3.8.2.8).

3.8.2.4 *Poblem-solving strategies suggested by Burris (2005)*

Burris (2005) suggests the follow problem-solving strategies:

- Draw a picture or a diagram. When pictures are drawn, they may illustrate relationships between given information and facts that are not as easily seen when they are in word or numerical form. According to Pyke (2003:406), "students' use of symbols, words, and diagrams to communicate about their ideas each contribute in different ways to solving tasks and reflect different kinds of cognitive processes invested in problem solving".
- Solve an equivalent problem. In some cases, it is easier to solve a related or equivalent problem than it is to solve a given problem.

- Solve a simpler problem. If a student finds it difficult to solve a given problem, it may be possible to formulate and solve a simpler problem, as the process used in finding the solution for the simpler problem can give insight (2.7.1.1 and 3.7.2) into the more complex given problem.
- Look for patterns. In many problem-solving situations, patterns are useful, especially in solving real-world problems. According to the National Council of Teachers of Mathematics (2000:91), "[p]atterns are a way for young students to recognize order and to organize their world".
- Make use of logical reasoning. Burris (2005:3 of 6) advocates that logical reasoning and careful consideration are sometimes all that is required to solve a mathematics problem. Mathematics can and should make sense after all.

The problem-solving strategy suggested by Burris therefore focuses on the process of searching for a particular strategy. It therefore closely relates to the second step with regard to Polya's heuristic, which is to "*devise a plan*" (see 3.8.2.1 and 3.8.2.8).

3.8.2.5 *Problem solving hints provided by Dawkins (2006)*

Dawkins (2006:8) provides the following hints when problems are actually to be solved or worked out:

- Read the problem - By reading the problem, students will get an idea of what is required of them to do. Many students become lost at this point as they skim the problem and assume that they know what is going on and what they are required to do.
- Read the problem again - When student read the problem for the second time, notes should be made of what the givens are in the specific problem and what needs to be found.
- Clearly note what you are asked to find - Students should be clearly aware of what they are asked to find and keep that in mind all the time.

- Clearly note what you know - Students should write down all the information that is given to them.
- Draw a diagram - It may be appropriate sometimes to draw a diagram and label what is known and what needs to be found. According to Dawkins (2006:8), diagrams often suggest the solution technique.
- Devise a Plan - Students should try to figure out what they are going to need to work out the problem. The identification of appropriate formulas may help them in addition. See if there are any intermediate steps/answers that will be needed in order to arrive at the final answer.
- Work out the problem - Follow the appropriate steps to arrive at the final answer.
- Review the problem - Reconsider the problem, once you are satisfied that you have arrived at the correct answer. Reflect on the concepts/methods/formulas that were identified and utilised in the problem-solving technique. Try to understand why these concepts/methods/formulas were used on the specific problem.

Dawkin's hints closely relate to Polya's heuristic: By reading through the problem, and reading through it again, one can identify and understand the problem. When students note what they know, what they need to find, and draw a diagram, they are still busy with the "understanding" part of the problem. The last three steps, namely "devise a plan", "work out the problem", and "review the problem" are in agreement with the second, third and fourth steps as suggested by Polya (see 3.8.2.1).

Although the study hints provided by Dawkins (2006) are not a specific heuristic as such, they have importance and relevance to the study as they show commonalities with the heuristic of Polya (see 3.8.2.1), the "IDEAL" heuristic of Bransford & Stein (see 3.8.2.2), and Maccini's "STAR" strategy (see 3.8.2.3 and 3.8.2.8).

3.8.2.6 *A comparison of some popular heuristics in problem solving*

The importance of problem-solving strategies presented its case in mathematical problem-solving based on literature (see 3.8.2). In the following table, the researcher gives a comparison among the different problem-solving strategies mentioned in the literature above (see 3.8.2). By looking into these differences between several problem solving strategies, the researcher attempts to provide an overall picture with regard to these strategies.

Table 3.1 A comparison between some popular heuristics in problem solving

Polya	Bransford and Steyn (IDEAL)	Maccini's STAR strategy	Burri's problem-solving strategy	Dawkins	Morris and Mather
Understand the problem	Identify the problem	Search the word problem		Read the problem.	Understand the problem.
				Note what is asked to find. Note what is known.	Note what is known and unknown)
			Draw a picture or diagram.	Draw a diagram and devise a plan.	
	Define and represent the problem	Translate words into an equation. (Identify the operations)			Define and represent the problem.
Devise a plan	Explore possible strategies		Solve an equivalent or simpler problem. Look for patterns.		
Carry out the plan	Act on the strategies	Answer the problem	Make use of logical reasoning.	Solve the problem.	
Look back	Look back and evaluate the effects of your activities	Review the solution		Review the problem.	

3.8.2.7 *Techniques for solving "word" problems*

According to Davis and McKillip (1980:88), word problems are important, and they illustrate and provide practice in applying mathematics to everyday situations. According to Barnett, Sowder and Vos (1980:97), "[m]athematical word problems are more compact and conceptually dense than ordinary prose. An ordinary paragraph of prose usually contains one major idea, but mathematical word problems often squeeze several important ideas into a single sentence". This concept is illustrated in example 3.6 (see 3.8.1.2).

Maccini and Gagnon (2006:7 of 9) argue that students often have difficulties deciding how to approach mathematical word problems. It is often difficult for students to solve mathematical word problems, as they have difficulty in translating abstract linguistic representations, making effective procedural decisions, and carrying out specific plans. Students thus find it difficult to translate a problem from its linguistic representation into a mental representation. According to Schunk (1996:271), translation requires good declarative knowledge (see 2.7.2.3) as well as procedural knowledge (see 2.7.2.3). If the knowledge is better organised in the long-term memory (see 2.7.2.3), students will translate problems better, as the organisation reflects the underlying structure of the subject matter [Romberg & Carpenter (in Schunk 1996:271)].

3.8.2.8 *Morris and Mather's self-questioning technique*

Morris and Mather (2007:253) propose the following "self-questioning technique" for representing algebra word problems:

- Have I read and understood each sentence?
- Have I got the whole picture or a representation of the problem?
- Have I written down my representation of the problem? For example: known(s), unknown(s), type of problem, equation.
- What should I look for in a new problem to see it is the same kind of problem?

Morris and Mather (2007:253) argue that the following can be used as "self-questions" for solving algebra word problems.

- Have I written an equation?
- Have I expanded the terms?
- Have I written out the steps of my solution? For example, have I collected similar terms, isolated unknown(s), and/or solved for unknown(s)?
- What should I look for in a new problem to see if it is the same kind of problem?

This "self-questioning" technique could be very helpful to university students when they solve mathematical word problems.

3.8.3 Control

According to Harlow (in Chapman (1972:126), "children can learn how to learn, when the teacher supplies the initial motivation by teaching the association". The learning process involves the learning method, in which the student has been taught to recognise a perceptual clue or cues which lead to the solution of a class problem. The student also learns through a method of problem solving, when he or she is asked to attempt a mathematical problem and at the same time records his or her thought processes.

The process of learning how to think cannot be accomplished without metacognition (see 2.7). According to Gunstone (1994), all learners are metacognitive to a degree, and the role of the instructor is to enhance learners' metacognitive abilities by helping them to develop appropriate knowledge about metacognitive strategies in order for them to take control of their own learning.

3.8.3.1 *The importance of metacognition in the learning of mathematics*

According to Schunk (1996:270), metacognition is a type of knowledge that is crucial in the learning of mathematics, as it concerns how to manage the problem-solving process.

According to Child (2007:207), the term "metacognition" was introduced by Flavell in 1976. Snowman *et al.* (2009:225) argue that, one way to grasp the essence of metacognition is to contrast it with cognition. According to Snowman *et al.* (2009:225), "[t]he term cognition is used to describe the ways in which information is processed – that is, the ways it is attended to, recognized, encoded, stored in memory for various lengths of time, retrieved from storage, and used for one purpose or another". For Restivo (1999:128), "[c]ognition arises situationally out of the natural rituals of everyday interactions and conversations". According to Restivo (1999:125), "[c]ognition is lived; sensory and motor processes, perception, and action are not independent".

Landsberg (2005:120) defines metacognition as "the awareness one has about one's own thinking processes and the ability to use this knowledge to monitor and control one's cognitive processes". Child (2007:207) argues that when a person is self-consciously examining his or her mental processes, becoming aware of problems and adjusting accordingly in order to improve effectiveness, this is known as metacognition – learning to learn. According to Snowman *et al.* (2009:225), metacognition refers to "our knowledge about those operations and how they might best be used to achieve a learning goal".

Cognitive studies of problem solving have documented the importance of metacognition – "which is knowledge about one's own thinking and ability to monitor one's own understanding and problem-solving activity" (Kilpatrick *et al.* 2001:118). In monitoring one's execution of a solution to include discovering errors in computation, control is involved. What is required in mathematical problem solving, is that students first accurately represent the problem to include the given information and the goal, and then select to and apply a problem-solving production [Mayer (in Schunk 1996:270)].

The National Research Council (2005) recommends that maths instruction should support students' use of metacognitive strategies (in Santrock 2009:401). According to Santrock (2009:401), "[s]tudents can engage in metacognitive self-monitoring to determine their progress in solving individual math problems and their progress in a math course".

According to Fuson, Kalchman, and Bransford (2005:239), "[m]etacognitive functioning is also facilitated by shifting from a focus on answers, that is, finding where the error is, why it is an error, and correcting it".

3.8.3.2 *Metacognition applied in the classroom*

In the next section, the researcher presents a discussion on how metacognition can be applied in the classroom.

a) Modelling

Prior to any learning activity, the university teacher can assist university students by pointing out strategies to deal with problems, which will allow students to shape their progress and thought processes during that activity. Self-regulation strategies, such as self-questionning and self-monitoring, help students gain access to cognitive processes that facilitate learning; guide students as they apply the processes within and across domains, and regulate their application and overall performance in completing a task (Montague 2008:37).

Modelling by the university teacher is one of the instructional techniques that probably has the greatest impact on students, since students learn best by imitating the adults around them (see 2.8.1.1). Steele (2004:64) emphasises the importance of modelling procedures in mathematics instruction and contends that "modeling should emphasise step-by-step procedures with clear explanations of each step". Students should be taught how to approach problems and use strategies that enable them to select and organise relevant information and form appropriate links to be learned (see 2.7.2.2 (a); 3.3.3.1 and 4.2). The university teacher can demonstrate a variety of strategies to help students attend to, encode, remember, and retrieve information (see 2.7.2.4). By showing students how to answer questions or how to solve a mathematical problem, they will immediately know what is expected from them. What is very important, is that students should be given opportunities to practise by using these strategies.

b) Put the textbook aside

Dawkins (2006:3) provides very useful and general hints for the studying of mathematics, but the researcher regards one of these as extremely important: "Do homework without notes and book" (Dawkins 2006:4). From years of experience as an higher education educator, the researcher is aware that, by way of practice students do many exercises from the prescribed textbook. Although this practising of examples is very important in any mathematical or statistical subject, it often allows students just to "model" examples from the textbook. Chance (2002:13 of 21) emphasises the dependency students develop on knowing which section of the book a question comes from. This statement is extremely important, as students learn to apply procedures when directed, but are at a loss as to where to begin when presented with a novel question after the course in statistics has been completed. According to Chance (2002:13 of 21), students need to be given questions that are more open-ended and encouraged to examine the questions from different perspectives to build understanding. This will help students see that the focus is on translating the question of interest, not just on the calculations.

The researcher believes that, if students work out exercises and problems with the textbook next to them, they do not have the opportunity to reflect on their learning process. However, if they put the book aside, and try to solve problems without continually consulting the textbook, students will have a better idea of what they are struggling with and what concepts they still do not understand. The researcher gave this study hint to her students during the research study, and the results were surprising. In informal conversations with students during their practical classes, the students told the researcher that they wished someone had told them this earlier in their schooling, and that they apply this technique to most of their subjects now. According to the researcher's students, learning, to them, means having the textbook readily available to them while studying, to keep modelling from it.

c) Continual Practice

According to Dawkins (2006:4), students should not limit themselves to do just the homework that is assigned by the lecturer. If students do more homework, they will have a better understanding of their own problem-solving skills. Dawkins (2006:4) further argues that the only way to really learn how to solve problems, is to do many problems. The more students work, the better prepared they will be for assessments that follow. Dawkins (2006:4) also emphasises perseverance. It should be made clear to university students that it is quite normal not to understand newly introduced topics. Some topics need to be worked at continually before they are completely understood, and it can sometimes take up to several readings of a certain topic, before it makes sense. Work that students struggle with, can make sense all of a sudden if the student perseveres at it (see 2.7.1.1).

d) Self-questioning in the process of reflection

Students can also evaluate their progress and thought processes by reflecting (see 3.5) on the strategies that they have used after the learning activity has been completed. Self-generation of questions (see 3.5.2; 3.8.2.8) should also assist students to take conscious control of their own studying and facilitate comprehension in which they are encouraged to pause frequently and reflect on the main concepts or events (Montaque 2008:37). When students explore the consequences of their choice with regard to the learning strategies they use, they will become aware of their own learning behaviour, which is another way of promoting metacognition. According to Miller, Butler and Lee (in Steele 2004:65), self-monitoring strategies improve students' independent work-habits and achievement levels.

3.8.4 Beliefs pertaining to mathematics

According to Schoenfeld (1998:19), beliefs are seen as "mental constructs that represent the codifications of people's experiences and understandings" and that shape their perception and cognition in any set of circumstances (Pehkonen 1999:3). According to Bar-Tal (in Leder, Pehkonen and Torner 2002:3), "[b]eliefs have been viewed by social psychologists as units of cognition. They constitute the totality of an individual's knowledge, including what people consider as facts, opinions, hypotheses, as well as faith".

Pehkonen (1999:2) argues that beliefs seem to be formed and changed in the social environment; persons compare these beliefs with their new experiences and with the beliefs of other individuals. Furthermore, beliefs influence learning, and learning affects beliefs [Nisbet and Ross (in Pehkonen 1999:7)].

When learners do not understand formal instruction they often conclude that mathematics is not supposed to make sense. This emphasises the fact that beliefs can have a powerful impact on how students go about learning and using mathematics [Reyes 1984; Schoenfeld 1985 (in Baroody & Ginsburg 1990:62)]. According to Baroody & Ginsburg (1990:62) the feelings and beliefs about mathematics fostered in the primary school years can undermine the learning and use of mathematics for years or even a lifetime. When students have learned to believe that mathematics is foreign to their thinking, they abandon common sense and overlook their own practical knowledge. This habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy, highlights the fifth strand of mathematical proficiency, which is productive disposition (see 3.2.1).

3.9 CONCLUSION

Chapter three is concerned with the learning of mathematics and statistics. The researcher explained the importance, nature and uniqueness of mathematics. With regard to learning theories in mathematics, the researcher presented a discussion of constructivism, as it represents a viable model for explaining how mathematics is learned (see 3.4.2). The researcher elaborated on Piaget's constructivism and emphasised the role of active learning, assimilation, and that the learning of mathematics should be holistic, authentic and real (see 3.4.3). As mathematical competence also depends on sociocultural influence, a discussion of Vygotsky's social constructivism was also presented (3.4.4). This social interaction constitutes a crucial source of opportunities to learn mathematics.

The researcher reviewed reflection as a component of constructivism as well as how students' reflective processes can be developed (see 3.5.2). The importance of symbols with regard to mathematical notation was explained as well as how symbols make an important contribution to the process of reflective activity (see 3.6).

As problem solving represents a key area for exploring the operation of cognitive processes in mathematics, the researcher provided an overview of problem-solving processes as well as an information processing view of problem solving, along with various problem-solving strategies (see 3.8.2). The researcher extended problem solving by presenting a discussion on problem-solving skills, heuristics, control and beliefs in mathematics, as well as thinking skills needed in mathematics. This discussion is extremely important because of its relevance to the study strategy the researcher wishes to propose.

The next chapter outlines the classroom learning strategy intervention the researcher has designed and how it relates to the content contained in the literature study.

CHAPTER 4: THE CLASSROOM LEARNING STRATEGY

4.1 INTRODUCTION

The purpose of this study is to determine the impact of a classroom learning strategy intervention on university students' academic performance in a mathematics and statistics-related subject. The previous two chapters were devoted to developing an understanding of learning theories in higher education, as well as the learning of mathematics and statistics.

With regard to learning theories in Chapter 2, it became clear that students should follow a deep approach to learning, and that information should be organised in a "holistic" way for students to see the "big" picture (see 2.4.2). The gestalt learning theories discussed in Chapter 2 emphasise the notion that learning tends to be a meaningful organisation of elements (see 2.7.1). In order to help students become aware of the structure of learning content, as well as the relationships between its elements, students can be helped by highlighting, framing and contrasting key concepts to make visual stimuli stand out more clearly (see 2.7.1.3). The importance of structuring learning material is also emphasised by Bruner's theory (see 2.7.3.5), as well as Ausubel's theory, in which learners organise content in logical ways such as concept maps (see 2.7.4.2).

Self-regulatory functions play a prominent role in learning, and Bandura's social cognitive theory highlights the notion that human behaviour is learned observationally through modelling. This means that the university teacher can serve as a live model when he/she helps students acquire new skills and shows students how to solve mathematics and statistics problems (see 2.8.1.4). With regard to cognitive constructivism, Piaget made it clear that students should be taught in accordance with the student's age and developmental stage by bearing in mind the functional concept of "readiness". The university student's conceptual knowledge grows through the alteration of existing mental structures in which new ones emerge through the process of adaptation, which is accomplished by two subprocesses: assimilation and accommodation (see 2.10.1.2). Vygotsky's theory highlights mediation in the development of higher cognitive processes, which can be applied through the concept of scaffolding and peer collaboration (see 2.11.1.1). This means that students should be encouraged to work with a more skilled learning partner to facilitate higher order thinking [see 2.11.1.1 (a)].

With regard to the learning of mathematics and statistics discussed in Chapter 3, constructivism was discussed again, as it presents a viable model with regard to the learning of mathematics (see 3.4.2). When looking into Piaget's constructivism, it became clear that active learning, assimilation, and the view that the learning of mathematics should be whole, authentic and real, play an important role in the learning of mathematics (see 3.4.3). The importance of the social aspect in the learning of mathematics became apparent again in the discussion that was presented with regard to Vygotsky's social constructivism (see 3.4.4). The discussion presented in Chapter 3 on problem-solving processes as well as an information processing view on problem solving, is extremely important because of its relevance to the classroom learning strategy the researcher wishes to propose (see 3.7 and 3.8). The various problem-solving strategies and heuristics discussed in Chapter 3 (see 3.8.2) are also important as they provide a framework for the researcher's own problem-solving strategy (see 4.3).

The main objectives of the proposed learning strategy intervention are the following:

- To focus all lessons on building students' conceptual understanding of mathematics and statistics (see 3.3.3.1; Von Glaserfeld 1995:5).
- To teach students efficient strategies for solving mathematics and statistics-related problems (see 3.8.2).
- To introduce students to the researcher's own problem-solving strategy (see 4.3).
- To encourage students to follow a deep approach to learning (see 2.4.2 and 3.2; Biggs & Tang 2007).

- To teach students how to solve mathematics and statistics-related problems without modelling from the textbook.

In Chapter 4, the researcher explains the classroom learning strategy intervention by means of a practical example from the prescribed textbook that students' used during the course of their study at the CUT for the 2009 academic year. First, the researcher gives a practical example of how certain mathematical and statistical concepts can be structured in a meaningful fashion by means of a concept map.

4.2 A PRACTICAL ILLUSTRATION OF HOW LEARNING MATERIAL CAN BE ORGANISED BY MEANS OF A CONCEPT MAP

In the pilot study that was conducted in this research (see Chapter 5), students in the experimental group were exposed to the proposed classroom learning strategy intervention. The aim of the proposed learning strategy intervention was to determine if it had any positive impact on first-year students' academic performance in the module *Business Calculations*, which is a mathematics and statistics-related subject.

In order to help students become aware of how to structure learning content in a meaningful fashion, the use of a concept map was explained and illustrated to students in the experimental group. This formed part of the learning strategy intervention during the first semester of the 2009 academic year. With regard to the pilot study that was conducted in this research, the post-test comprised simple interest, compound interest and annuities. As the post-test followed after the implementation of the classroom learning strategy intervention, the researcher presented practical illustrations of concept maps with regard to these concepts before the post-test was written. The examples used in this illustration were obtained from the students' prescribed textbook (Croucher 2002) in the module *Business Calculations*.

Concept maps also formed part of the classroom intervention eventually implemented in the main empirical research study, but this time with third-year students enrolled for the module *Business Statistics/Statistics II*. Obviously the concept maps used in that study were different from the ones presented in this chapter.

Example 4.1 A concept map of Simple Interest

Symbols to know:

P = principal (or present value)

R = rate of interest (expressed as a fraction, usually as a rate per annum)

T = time (usually in years)

I = total simple interest (where $I = S - P$)

S = amount or maturity value of the principal

Formulas:

Simple interest: $I = P \times R \times T$

Time: $T = \dfrac{I}{P \times R}$

Rate: $R = \dfrac{I}{P \times T}$

Maturity value: $S = P + I$

Present value: $P = \dfrac{S}{1 + RT}$

Source: Croucher 2002:M107

Example 4.2 A concept map of Compound Interest

Symbols to know:

P = principal (or present value)

i = rate of interest (expressed as a fraction or decimal, usually as a rate per annum)

n = number of periods for which interest is accumulated

S = accumulated value at the end of n periods

Formulas:

Total compound interest earned: $S - P$

Time: $$n = \frac{\log\left(\frac{S}{P}\right)}{\log(1+i)}$$

Rate: $$i = \left(\frac{S}{P}\right)^{\frac{1}{n}} - 1$$

Accumulated value: $S = P(1+i)^n$ where $(1+i)^n$ is the accumulation factor

Present value: $P = S(1+i)^{-n}$ where $(1+i)^{-n}$ is the present value factor

Total compound interest earned = $S - P$

Nominal rates of interest: Remember to adjust both the interest rate (i) and number of periods (n) in the formulae.

Interest rate is divided by the number of periods
Number of time periods is multiplied by the number of periods

Number of time periods:

Semi-annually (twice or half yearly)	=	2
Monthly	=	12
Quarterly (every 3 months)	=	4

Source: Croucher 2002:M122

Example 4.3 A concept map of Annuities

Symbols to know:

A = present value of an annuity
R = amount of the annuity payment made per period
i = rate of interest per payment period (the periodic interest rate)
n = total number of payments (i.e. compounding periods)
S = future value of the annuity

Formulas:

Future value of an annuity: $S = R \times \frac{(1+i)^n - 1}{i}$

Present value of an annuity: $A = R \times \frac{[1-(1+i)^{-n}]}{i}$

If an interest rate is quoted as a rate per annum but the interest is not compounded annually, then the values of n and i need special attention. Suppose the interest is compounded k times per annum. Then we let:

n = number of interest periods = $k \times$ (number of years)

$$i = \text{interest rate per period} = \frac{\text{interest rate per annum}}{k}$$

Source: Croucher 2002:M144-145

In the next example, the researcher presents a comparison, by means of a concept map, with regard to simple interest, compound interest, and annuities. By comparing and contrasting these three concepts, students were able to see how the formulas change between simple interest, compound interest as well as annuities. Students were also able to see the different symbols that are used with regard to each of these concepts.

Example 4.4 A comparison with regard to Simple Interest, Compound Interest, and Annuities

Simple Interest	Compound Interest	Annuities
Symbols: P = principal (or present value) R = rate of interest (expressed as a fraction, usually as a rate per annum) T = time (usually in years) I = total simple interest (where $I = S - P$) S = amount or maturity value of the principal	**Symbols**: P = principal (or present value) i = rate of interest (expressed as a fraction or decimal, usually as a rate per annum) n = number of periods for which interest is accumulated S = accumulated value at the end of n periods	**Symbols**: A = present value of an annuity i = rate of interest per payment period (the periodic interest rate) n = total number of payments (i.e. compounding periods) S = future value of the annuity R = amount of the annuity payment made per period
Formulas: Simple interest: $I = P \times R \times T$	**Formulas**: Total compound interest earned: $S - P$	**Formulas**:

Time: $T = \frac{I}{P \times R}$ Rate: $R = \frac{I}{P \times T}$ Maturity value: $S = P + I$ Present value: $P = \frac{S}{1 + RT}$	Time: $n = \frac{\log\left(\frac{S}{P}\right)}{\log(1+i)}$ Rate: $i = \left(\frac{S}{P}\right)^{\frac{1}{n}} - 1$ Accumulated value: $S = P(1+i)^n$ where $(1+i)^n$ is the accumulation factor Present value: $P = S(1+i)^{-n}$ where $(1+i)^{-n}$ is the present value factor Nominal rates of interest: Remember to adjust both the interest rate (i) and number of periods (n) in the formulae.	Future value of an annuity: $S = R \times \frac{(1+i)^n - 1}{i}$ Present value of an annuity: $A = R \times \frac{[1-(1+i)^{-n}]}{i}$ If an interest rate is quoted as a rate per annum but the interest is not compounded

	Interest rate is divided by the number of periods Number of time periods is multiplied by the number of periods <u>Number of time periods</u>: Semi-annually (twice or half yearly) = 2 Monthly = 12 Quarterly (every 3 months) = 4	annually, then the values of n and i need special attention. Suppose the interest is compounded k times per annum. Then we let: n = number of interest periods = $k \times (\text{number of years})$ i = interest rate per period = $\frac{\text{interest rate per annum}}{k}$

The illustrations above were devoted to developing an understanding of how certain concepts in mathematics and statistics can be organised by means of a concept map. By framing and contrasting key concepts, university students can become aware of the structure of a specific learning content, as well as the relationships between its elements (see 2.7.1.3). The importance of structuring learning material, such as in the illustrations above, also emphasises Bruner's theory (see 2.7.3.5), as well as Ausubel's theory, according to which learners organise content in logical ways, such as concept maps (see 2.7.4.2). As the structuring of learning material is fundamental to the process of learning, it will also ensure that the learning process is meaningful (see2.7.4).

In the next section, the researcher elaborates on problem-solving strategies as part of the classroom learning strategy intervention.

4.3 THE PROBLEM-SOLVING STRATEGY AS PART OF THE CLASSROOM LEARNING STRATEGY INTERVENTION

The importance of problem-solving strategies in mathematics and statistics was discussed in Chapter 3 (see 3.8.2). In the following table, the researcher complements the comparison between the different problem-solving strategies mentioned in Chapter 3 (see 3.8.2.6), with her own problem-solving strategy, namely the "RIEQTSR heuristic".

Table 4.1 A comparison between some popular problem-solving strategies in mathematics.

Polya	Bransford and Steyn (IDEAL)	Maccini's STAR strategy	Burri's problem-solving strategy	Dawkins	Morris and Mather	RIEQTSR Heuristic
Understand the problem	Identify the problem	Search the word problem		Read the problem.	Understand the problem.	Recognise and Identify a problem
				Note what is asked to find. Note what is known.	Note what is known and unknown	Extract information
	Define and represent the problem	Translate words into an equation. (Identify the operations)	Draw a picture or diagram.	Draw a diagram and devise a plan.	Define and represent the problem.	
			Solve an equivalent or simpler problem. Look for patterns. Make use of logical reasoning.		Note what is known and unknown.	
Devise a plan	Explore possible strategies					Question and Translate
Carry out the plan	Act on the strategies	Answer the problem		Solve the problem.		Solve the problem
Look back	Look back and evaluate the effects of your activities	Review the solution		Review the problem.		Reflect on the problem

In the following section, the researcher presents the problem-solving strategy that was employed during the classroom learning strategy intervention. The problem-solving strategy is based on the popular heuristic of Polya (see 3.8.2.1), but also takes into consideration other important steps in problem-solving strategies as mentioned in Table 4.1. By carefully looking into the different steps of each of these heuristics, the following heuristic can be drafted. The following section entails a description of the researcher's own problem-solving heuristic, namely "the RIEQTSR heuristic".

R	=	Recognise a problem
I	=	Identify the problem
E	=	Extract information
Q	=	Question
T	=	Translate
S	=	Solve the problem
R	=	Reflect on/review the problem

- Recognise and identify the problem

In the first step of the heuristic, the student has to read through the problem and *recognise* (understand) that a problem exists. After the student recognises the existence of a problem, he/she can now *identify* what the problem actually entails.

- Extract information

In the second step of the heuristic, the student should *extract* relevant information from the problem, and clearly note what he/she is asked to find. This step can thus be seen as an extra step between "understand the problem" and "devise a plan" as proposed by Polya (see 3.8.2.1).

- Question

In the fourth step of the heuristic, the student needs to ask him/herself questions, for example: What information is given (known)? What information is missing? Which formulas should I use? This step has similarities to the second step "devise a plan" as proposed by Polya (see 3.8.2.1).

- Translate

The student should now *translate* the words in the problem into the appropriate symbolic notation. The student should substitute the given values in the problem into the appropriate symbols which form part of the appropriate formula he/she chooses. This step shows similarities with the second step of Polya, namely "devise a plan" (see 3.8.2.1)

- Solving the problem

After the student has substituted the given values into the symbols of the appropriate formula or equation, the student is now able to *solve* the problem. With regard to Polya's heuristic, this step is "carry out the plan" (see 3.8.2.1).

- Reflect on the problem

The student reads through the problem again and *reviews* his/her answer. Polya referred to this step as "look back" (see 3.8.2.1). The student makes sure he/she has understood the problem and that his/her computations are correct. The student can also make use of questioning techniques (see 3.8.2.8) to justify his/her answer in this step.

The researcher gives a practical illustration of her own heuristic in Chapter 5 (see 5.5.5).

4.4 CONCLUSION

In Chapter 4, the researcher extended her discussion on the learning of mathematics and statistics further by focusing on the proposed classroom learning strategy intervention. The researcher used a practical example from the textbook to illustrate how learning material in mathematics and statistics can be organised and structured by means of a concept map (see 4.2), in order to promote deep and meaningful learning (see 2.4.2 and 2.7.4).

As problem solving represents a key area in the learning of mathematics, the researcher drafted a table with a few popular heuristics in mathematical problem solving (see 3.8.2.6 and 4.2). After comparing these heuristics and problem-solving strategies, the researcher's own problem-solving strategy emerged, namely "the RIEQTSR heuristic" (see 4.3).

To capture the key features of the classroom learning strategy, some key features that are associated with the learning of mathematics and statistics are reported on in the following table. The key features in this table present its case based on the literature provided on learning theories and the learning of mathematics in the preceding two chapters, namely Chapter 2 and Chapter 3. The key components are described from two viewpoints, namely: the traditional learning method most students made use of in the learning of mathematics and statistics, and the researcher's proposed classroom learning strategy.

Table 4.2 A summary of the learning strategy intervention

Traditional method of study	The classroom learning strategy intervention
• Lecture-centred fashion	• Lecture-centred fashion, but more opportunities for students to construct their own knowledge and skills during the learning process (see 2.9 and 3.4.2).
• Practise exercises from prescribed book at home, by modelling from the textbook.	• Practise exercises from prescribed book during tutorial classes.
• Students work individually in a non- collaborative environment.	• Students should be encouraged to form a partnership with a learning partner during the tutorial classes. The creation of learning partners provides a continually accessible resource for discussing, reciprocal questioning and mutual support in an otherwise anonymous environment (see 2.11.1.1). • Students should be taught how to make summaries of each chapter by means of concept maps (see 4.2). • Students should learn how to organise new mathematical and statistical content in a logical way in which it makes sense to them. For example, students should learn how to identify key concepts, outline content, indicate connections between parts of content, and to make summaries at the end of each chapter (see 4.2). • Students should learn how to question themselves on new concepts in mathematics and statistics (see 3.5.2 and 3.8.2.8). • Students should be encouraged to keep track of their own learning, by making notes while practising and learning new mathematical and statistical concepts (see 3.8.2.5).

	• Students should be able to reflect on their learning and problem-solving strategies (see 3.5 and 3.8.2). • Students should be encouraged to work with peer students or a "learning buddy" in a collaborative environment. This social interaction is emphasised for facilitating development in the ZPD and also to help students to adopt a deep approach to learning (see 2.11.1.3). • Students should be taught how to extract information from texts and how to encode this information (see 2.7.2 and 3.8.2). • Students should learn how to connect parts of statistical concepts into a whole in order to see the "big picture" (see 2.4.2; 2.7.1 and 3.3.3.1) • Students should be encouraged to make use of problem-solving strategies, such as the "RIEQTSR" heuristic when solving problems from the textbook (see 4.3). • The expectations are that students will eventually follow a deep approach to learning (see 2.4.2).

The next chapter focuses on the empirical investigation undertaken in the study. The investigation commences with a pilot study.

CHAPTER 5: PILOT STUDY

5.1 INTRODUCTION

Chapter 5 is concerned with a pilot study in which students' academic performance as well as their approaches to learning were addressed, by the implementation of a classroom learning strategy intervention for a mathematics and statistics-related subject at the Central University of Technology, Free State.

The purpose of a pilot study is to improve the success and effectiveness of an investigation [Strydom (in De Vos, Strydom, Fouche, and Delport 2005:210)]. According to Strydom (in De Vos *et al.* 2005:205), a pilot study is one way in which the researcher can orientate herself towards the project she has in mind.

The pilot study followed a non-equivalent pre-test post-test control group design involving an experimental group and a control group. The learning strategy intervention was designed with the aim of improving third-year students' academic performance in a mathematics and statistics-related subject at the CUT. However, the researcher deemed it necessary to first conduct a pilot study with first-year students during the first semester of the 2009 academic year. The researcher then implemented the learning strategy intervention with third-year students during the second semester of the 2009 academic year.

For more than two decades now, concerns have been voiced about the profile of the school-leaving learner applying to enter universities in South-Africa, with an increasing number of students in the educational system who experience serious and persistent problems in interpreting and performing academic tasks. Steyn and De Boer (1998:125) purport that one of the outstanding features of underprepared students, is their inadequate schooling in mathematics and natural sciences. Due to this disadvantage, this remark is especially true for black learners in South Africa. Steyn and De Boer (1998:126) indicated that black learners perform more poorly in mathematics than do learners from other population groups. This means that underachievement in mathematics is especially noticeable among learners in South-Africa (Maree and Schoeman 1997:127). De Boer and van Rensburg (1997:160) argue that underachieving students in mathematics are in dire need of a repertoire of learning approaches, strategies and methods to cope with the demands of tertiary education.

According to Yusof and Tall (1999:67), the traditional methods of teaching mathematics at university often seem to lead students into a 'deficit mode' of rote-learning material to pass examinations (see 2.4.1). As a result, these procedural forms of thinking and working often prove to be resistant to change [Sierpinska, Schoenfeld and Williams (in Yusof & Tall 1999:67)]. Students learn the "product of mathematical thought" rather than the process of mathematical thinking [Skemp (in Yusof & Tall 1999:67)]. In agreement with Yusof and Tall (1999:67), Steyn and De Boer (1998:127) argue that one of the obstacles that the underprepared student must overcome is a surface approach to learning, which is associated with rote-learning (see 2.4.2).

The researcher presented a thorough discussion on approaches to learning in Chapter 2 (see 2.4). Biggs and Tang (2007:24) purport that "[w]hen students feel a need to engage the task appropriately and meaningfully, they follow a deep approach to learning". At higher tertiary institutions, outcomes measured by assessments should be high level, requiring students to reflect, hypothesise and apply. Students who feel this need-to-know automatically try to focus on underlying meanings, main ideas, themes, principles, or successful applications (see 2.4.1 and 3.3.3.1).

According to Raab and Adam (2005:93), underprepared and first generation students often lack effective study skills. Many of these students are not independent learners. To help students become self-regulated learners, Cukras (2006:194) suggests that study skill courses as well as academic assistance programs should be designed to address this concern.

According to Abrams and Jernigan (in Potgieter & Webb 2004:313), the responsibility lies with higher education institutions to provide effective intervention strategies to help with the retention of underprepared students. However, there seems to be a lack of research that fully encompasses an effective learning strategy for a mathematics and statistics-related subject at South African higher education institutions; or else, if such a strategy exists, it has not been implemented. In light of the above, the researcher realised the need for students to be supported with the necessary skills to master a mathematics and statistics-related subject she was teaching. Therefore, the researcher deemed it necessary to conduct a pilot study in which first-year students' academic performance as well as approaches to learning could be addressed, by the implementation of a particular classroom learning strategy intervention for a mathematics and statistics-related subject at the CUT. According to Bell (2005:147), a pilot study can serve as a measure to eliminate any faults or possible shortcomings of the instrument so that respondents in the main study will experience no difficulties in completing it. The main purpose of this pilot study was to determine whether the use of a classroom learning strategy intervention could positively affect first-year students' academic performance as well as students' learning approaches in the module *Business Calculations* at the CUT.

5.2 PROBLEM STATEMENT

By discussing the background to the problem statement, the two main research questions that informed the study were:

1. *Does the implementation of a classroom learning strategy intervention positively affect students' academic performance in the module Business Calculations?*

2. *Does the implementation of a classroom learning strategy intervention positively affect students' approaches to learning in the module Business Calculations?*

5.3 RESEARCH HYPOTHESES

In an effort to investigate the research problem, the study has tested the following research hypotheses:

The first research hypothesis that has been addressed was:

H_{01} : The post-test score in *Business Calculations* of the experimental group is equal to the post-test score of the control group.

H_{a1} : The post-test score in *Business Calculations* of the experimental group is significantly higher than the post-test score of the control group.

The second research hypothesis that was addressed was:

H_{02} : The mean difference score on the revised two-factor study process questionnaire (R-SPQ-2F) for the experimental group is equal to the mean difference score of the control group.

H_{a2} : The mean difference score on the revised two-factor study process questionnaire (R-SPQ-2F) for the experimental group is greater than the mean difference score of the control group.

5.4 IDENTIFYING THE VARIABLES

The pilot study tested the independent variable of a classroom learning strategy intervention on students' academic performance in the module *Business Calculations*, as well as the effect of the intervention on students' approach to learning. *Business Calculations* involves mathematical and financial calculations as well as some elementary statistics as a means of assisting in decision making.

5.4.1 Independent variables

For the purpose of this study, the independent variable was defined as the classroom learning strategy intervention. The particular learning strategy intervention is defined as the facilitation of a particular learning strategy which is derived from a constructivist perspective (see Chapter 4), with emphasis on the construction of mathematical knowledge and the processes by which learners create mathematical meaning (Biggs 1972:230). The first aim of the classroom learning strategy intervention was to improve students' academic performance in a mathematics and statistics-related subject. The second aim of the classroom learning strategy intervention was to motivate students to follow a deep approach to learning (see 2.4.2).

5.4.2 Dependent variables

For the purpose of this study, the first dependent variable was represented by the average of the post-test score of students' performance in a first-year mathematics and statistics-related subject, namely *Business Calculations*. Students' performance was measured by means of tests and exam scores in the module *Business Calculations*. As one of the questions of interest was whether the average post-test score is greater for the treatment group than it is for the control group, the researcher defined the average post-test score as follows: $Average = \frac{\sum(P_1 + P_2)}{2}$ where P_1 = Post-test 1 and P_2 = Post-test 2

The second dependent variable was represented by the mean difference score from the responses obtained from the R-SPQ-2F Questionnaire, which assesses students' approaches to learning. Students were assessed twice on this questionnaire, namely before the learning strategy intervention and thereafter. The difference between the pre-test and post-test with regard to this questionnaire was calculated for each student in the module *Business Calculations*.

5.4.3 Confounding/extraneous variables

Possible extraneous variables that might have compromised the results of this study were race, gender, age, "FTE status" and previous mathematical background. "FTE" refers to whether the student was entering the CUT for the first time (F), whether the student was transferred (T) from another HEI, or entering (E) from another programme in the same HEI. Therefore, a student who enrolled the previous year or changed courses, was considered a *not* "first-time entering" (N) student. A not "first-time entering" student may also refer to a student who had enrolled at the CUT previously, but is continuing after some years with the same course at the CUT (see 5.5.4.1). The extraneous variables were acquired by means of collecting biographical data from the CUT's data system. These variables were built into the design by measuring them and by analysing their influence on the dependent variables (McMillan & Schumacher 2006:118). In this research, the first dependent variable was the average post-test score of students' performance in the module *Business Calculations*. The second dependent variable was the difference score of the responses obtained from the R-SPQ-2F Questionnaire.

5.5 RESEARCH DESIGN AND METHODOLOGY

Given the primary research questions, this empirical pilot study is regarded as evaluative in nature, as the aim of evaluative research is to improve a current practice. The pilot study investigated the effect of a learning strategy intervention on students' academic performance in the module *Business Calculations*. Secondly, the study investigated the effect of a learning strategy intervention on students' approaches to learning. The study followed a non-equivalent pre-test post-test control group design involving an experimental group and a control group (Leedy & Ormrod 2001:236). This study was located within a quantitative paradigm, with some enhancement by means of qualitative observations of student's problem-solving approaches. A quasi-experimental approach was therefore used in answering the question whether the *Business Calculations* test and exam results, as well as learning approaches of students who had been exposed to the proposed learning strategy intervention, were any different from those of students who were not exposed to the learning strategy intervention.

For the purpose of this study, the researcher relied on numerical data (scores obtained from tests and the exam, as well as the R-SPQ-2F Questionnaire) to test the relationship between the variables as well as to test the formulated research hypotheses, i.e. whether the average post-test score in *Business Calculations* of the experimental group is higher than the average post-test score of the control group; and secondly, whether the mean difference score on the R-SPQ-2F Questionnaire for the experimental group is greater than the mean difference score of the control group.

As this research involved the systematic collection of observable and measurable data as well as the statistical analysis of the data, the quantitative paradigm was considered appropriate for this study.

The pilot study, which is located within a quantitative paradigm, was enhanced by means of qualitative observations of students' problem-solving approaches. The researcher made use of a reflection diary in which she recorded students' learning approaches to learning, problem-solving strategies as well as the implementation of the classroom learning strategy intervention.

As the problem statement is clearly one of causation, an experimental approach was selected. A non-equivalent pre-test post-test design involving two intact classes was used to establish probable causality (McMillan & Schumacher 2006:273), because the intervention of the learning strategy (the independent variable) was easily manipulated. For the purpose of this study, the researcher purposefully manipulated one variable (the classroom learning strategy intervention) and measured its consequences on the dependent/'outcome' variables. The experimental group of students received the proposed learning strategy intervention, while the control group received only traditional lecturing and discussion (see 3.4.1).

5.5.1 Population and Sampling

A group of first-year students at the CUT was selected for this research project. A sample of 139 first-year students was selected from the total population (N=177) of students who were enrolled for the National Higher Certificates in Financial Information Systems, and Accountancy, at the CUT. The students from both courses formed part of two intact classes of students who took *Business Calculations* (BCL11AB) as a compulsory module. The students were all registered as full-time students on campus and attended BCL classes three times per week over a period of six months during the first semester of the 2009 academic year.

The students (n=50) who were enrolled for the National Higher Certificate in Financial Information Systems served as the experimental group and were taught the proposed classroom learning strategy intervention. The students (n=89) who were enrolled for the National Higher Certificate in Accountancy served as the control group and received traditional instruction.

A non-probability sampling method was employed, as the researcher did not make use of a random selection of participants. The researcher made use of convenience sampling, and more specifically wholeframe sampling, as the subjects were available and formed part of the lecturer's (also the researcher) classes, and since the whole class participated in the study (McMillan & Schumacher 2006:125).

5.5.2 Data collection

Both groups of students attended two theory lectures twice a week and one tutorial once a week. The duration of each theory and tutorial lecture was 80 minutes. During the theory lectures, the lecturer explained the work to students and, during the tutorials, the students worked out exercises from the prescribed textbook. The researcher utilised the national prescribed syllabus for the module *Business Calculations* and strictly kept to the study guide. Both classes received exactly the same academic instruction (with different approaches) by the same lecturer, covered the same work content, and used the same prescribed textbook.

Descriptive, quantitative biographical data (race, gender, age and previous mathematical background) were obtained from the CUT Student Records Database at the beginning of the 2009 academic year. The researcher entered this biographical data on a database for data analysis purposes.

In order to gauge students' approaches to learning, the researcher administered the R-SPQ-2F Questionnaire to students before and after the learning strategy intervention. In the beginning of February 2009, the researcher administered the R-SPQ-2F Questionnaire to students during a theory class in the module *Business Calculations*. Participation in the questionnaire was voluntary and the responses were kept confidential. A total of 57 completed questionnaires were collected at the first and second testing, representing approximately 41% of the population of BCL students.

The students completed the R-SPQ-2F Questionnaire during a theory class at the beginning of the BCL semester module at the end of January 2009. The students also completed the same questionnaire during May, after the learning strategy intervention. The responses to the R-SPQ-2F Questionnaire items were scored and coded for each student and the resulting data were entered on a database for analysis purposes. The results were later statistically analysed to determine if there were any differences regarding students' approaches to learning.

The quantitative data from students' scores in the pre-test and both post-tests were obtained by the researcher during the first semester of 2009 and entered into a database in which the results were analysed. Students were assessed during February, April and in the exam in May. The first class test of the BCL semester subject served as the pre-test and was administered to the subjects in both groups in February before the learning strategy intervention. The test was compulsory for all students, as this test mark was used as part of their course mark in the module *Business Calculations*. A total of 139 test answer sheets were submitted, and after the tests were marked, the marks were all entered onto the Integrated Tertiary Software (ITS) system of the CUT.

The learning strategy intervention (see Chapter 4) was implemented with the experimental group of students after the first test and continued for a period of six weeks. The control group, however, received traditional instruction. During these six weeks, the researcher encouraged students in the experimental group to follow the proposed learning strategy and exposed them to good study habits. After each lesson, the researcher summarised the work by means of a concept map, and taught students how to study that specific content area from the prescribed textbook (see 4.2).

The students were assessed for a second time in the main test (first post-test) which was administered to the subjects in both groups during April, i.e. after six weeks of implementing the classroom learning strategy intervention. The test was compulsory for all students, as this test mark was used as part of their course mark in the module *Business Calculations*. A total of 139 test answer sheets were collected and, after the tests were marked, the marks were all entered onto the ITS system of the CUT. The researcher continued with the intervention for another three weeks with the experimental group of students, after which all students were assessed for the third and final time during the exam (second post-test), which took place at the end of May 2009.

All students were assessed on the same day, in the same venue, at the same time with regard to the pre-test as well as both post-tests. The researcher developed all the tests as well as the exam paper, which were moderated and based on the curriculum activities of the module *Business Calculations* for the semester. The researcher also gave instructions to all the students in English and marked all answer sheets herself.

In order to obtain richer data on the students, the researcher complemented the more formal quantitative tests with additional qualitative information. In this study the researcher made use of a reflection diary in which she took notes of students' problem-solving approaches and techniques. The aim of the reflective diary was to gain insight into students' problem-solving behaviour. The researcher observed the students during the tutorial classes in the first semester and reflected on their activities, problem-solving strategies, as well as the general implementation of the proposed classroom learning strategy intervention.

5.5.3 Measuring instruments

In order to determine the effect of the proposed classroom learning strategy intervention on students' academic performance in the module *Business Calculations*, the quantitative data was collected by means of three self-developed instruments (tests and exam) intended to yield highly reliable and valid scores. The researcher also used the R-SPQ-2F Questionnaire by Biggs, Kember & Leung (2001:133) to gauge students' approaches to learning.

In order to investigate students' academic performance in the module *Business Calculations*, the first class test served as the pre-test and the main test and exam served as the post-tests. The results (scores) of the tests and exam were used in the study to assess the pre- and post-test performances of students in the module *Business Calculations*. The average post-test score was calculated by computing the average of the first post-test (post-test 1) and the second post-test (post-test 2). All results were compared to determine the effect the classroom learning strategy intervention had on the subjects. The measuring instruments are discussed in the next section:

5.5.3.1 *The self-developed instruments*

The researcher developed two tests as well as an examination paper based on the curriculum activities for the semester module *Business Calculations*. The components of the tests and exam were as follows:

(a) *The pre-test*

A first semester test paper comprising 25 multiple-choice items was used in this study and served as the pre-test (see Appendix B). The majority of items were obtained from the prescribed textbook that students had to work from, with only one or two items from other literature sources. The test focussed on students' conceptual knowledge and each of the 25 questions represented a certain cognitive characteristic of important concepts in the module *Business Calculations*. Each question had five options that students could choose from, while the distracters in the multiple-choice test instrument were based on the mistakes students used to make as identified from the researcher's qualitative observations during tutorials. The students were assessed on the following concepts:

- Some basic mathematical concepts, which include whole numbers, fractions, decimals, exponents, scientific notation and logarithms.
- Financial calculations, which include percentages, commission, discounts, profit and loss, and stamp duty.
- Algebra, which includes algebraic terms, algebraic expressions, simple linear equations, simultaneous linear equations, and business problems using simple algebra.
- Ratios and proportions, which include profit ratios, efficiency ratios and liquidity ratios.

(b) ***The first post-test***

A second semester test paper comprising 25 multiple-choice test items developed by the researcher, was used for this study and served as the post-test (see Appendix C). The majority of items in the test were obtained from the prescribed textbook with a few items from other literature sources. The post-test covered three chapters and comprised conceptual questions regarding the following:

- Simple interest
- Compound interest
- Annuities

(c) ***The second post-test***

The exam semester paper comprising 25 multiple-choice test items, served as the second post-test (see Appendix D). As the time-interval between the pre-test and post-test was relatively short (three weeks) during which the intervention took place, the researcher used the exam results as a second post-test. The exam paper covered the whole syllabus and comprised the content of ten chapters from the prescribed textbook. The students were assessed on the same concepts mentioned above with regard to the pre-test and the first post-test, as well as on the following new concepts:

- Visual presentation of data
- Measures of central tendency
- Measures of dispersion

Content validity in this study was established by including only selected questions that are significant in a specific content domain from the prescribed textbook. The items in both tests and exam fairly represented the content domain that students were assessed on. Content validity for the items in both tests and exam were strengthened by asking another lecturer and also a statistician to review the items for clarity and completeness in covering most assessment and grading practices used (Bell 2005: 118; Salkind 2003:116).

5.5.3.2 *The revised two-factor Study Process Questionnaire: R-SPQ-2F*

In order to gauge students' approaches to learning, the researcher administered the R-SPQ-2F Questionnaire to all students before and after the learning strategy intervention. This instrument was developed by Biggs, Kember and Leung, and is used to assess deep and surface approaches to learning (Biggs, Kember & Leung 2001:133).

The R-SPQ-2F Questionnaire comprises 20 Likert-type scale test-items which test deep and surface approaches to learning (see Appendix E). Each of these scales consists of 10 items, while the deep and surface motive and strategy scales consist of 5 items each. The items in the questionnaire requested the students to react to the statements by choosing one of five options, which ranged from "this item is never or only rarely true of me" (scored 1); through "this item is sometimes true of me" (scored 2); "this item is true of me about half the time" (scored 3); "this item is frequently true of me" (scored 4); to "this item is always or almost always true of me" (scored 5).

According to Biggs, Kember and Leung (2001:145), the R-SPQ-2F Questionnaire is an ideal tool for teachers to use in evaluating and researching their own classes' learning approaches. The R-SPQ-2F Questionnaire has acceptable Cronbach alpha values for scale reliability and both deep and surface approach scales have well identified motive and strategy subscales [Biggs, Kember & Leung (2001:133)].

To assess the internal consistency of the R-SPQ-2F Questionnaire, Cronbach's alpha statistic was calculated. The Cronbach alpha values were computed for each component in the questionnaire in order to determine the scale and subscale reliabilities (Biggs, Kember & Leung 2001:141). According to Biggs, Kember and Leung (2001:142), the values reached acceptable levels, indicating that the subscales can be interpreted as internally consistent and considered as acceptable. However, Coakes and Steed, as well as Pallant (in Embi 2007:15), argue that alpha values above 0.70 are sufficient to demonstrate reliability. According to Biggs, Kember and Leung (2001:141), the Cronbach alpha values for the subscales (DM, DS, SM, SS) and the two latent constructs (DA and SA) are as follows:

Table 5.1 The Cronbach alpha values of the R-SPQ-2F Questionnaire.

Constructs	**Cronbach alpha value**
Deep approach (DA)	0.73
Surface approach (SA)	0.64
Subscales	**Cronbach alpha value**
Deep motive (DM)	0.62
Deep strategy (DS)	0.63
Surface motive (SM)	0.72
Surface strategy (SS)	0.57

Source: Biggs, Kember and Leung(2001:142)

5.5.4 Analysis of data and interpretation of results

For the purpose of processing the data obtained, the statistical software package SAS was used. Initially, descriptive statistics were used to help explain and allow reflection on the performances of the two groups of subjects. Frequency tabulations (number of students and percentage of students per category, both for the total group and for the experimental versus control group separately) for the following is presented:

- Gender – categorical variable
- Race – categorical variable
- Previous mathematical background – categorical variable
- FTE status – categorical variable

Based on the frequency distributions of the categorical confounding variable "race", categories of this variable might be combined for the purpose of the analysis described below.

Descriptive statistics (mean, standard deviation, median, minimum, maximum, and number of observations), both for the total group and for the experimental group versus control group separately, is presented for each quantitative variable, namely

- *Business Calculations* results: pre-test, post-test 1, post-test 2, and for the average of the two post-test results – continuous variables
- Age – continuous variable

To test for any relationships and differences in each variable with regard to students' demographic profiles (gender, age, FTE status and mathematical background) between the two groups of students, the researcher made use of univariate analysis. The average of the two post-test results in *Business Calculations* was set as the dependent variable. The dependent variable was analysed using one-way ANOVA fitting; one variable at a time; the independent variable (Group), and each of the confounding variables.

Regression analysis was also used in order to determine whether students' average post-test performance was in any way related to students' age. To ascertain whether there are statistically significant correlations between the average post-test score in the module *Business Calculations* and students' age, the Pearson product-moment correlation coefficients were calculated and the significance thereof ascertained. According to Lind, Marchal and Mason (2002:460), "the coefficient of correlation describes the strength of the relationship between two sets of interval-scaled variables".

Multivariate analysis was also used in which the dependent variable (*Business Calculations* results: average of post-test results) was analysed using analysis of covariance. The analysis of covariance model contained the independent variable (Group) and all potential confounders (gender, age, race, previous mathematical background, FTE status, and *Business Calculations* results: pre-test). F-statistics and associated *P*-values were calculated for each variable in the model.

In the next section of this chapter, the researcher reports on the findings of the pilot study, which are illustrated by means of tables. The student characteristics and demographic information are presented in Table 5.2 to Table 5.11, while some descriptive statistics are contained in Table 5.12 to Table 5.15 regarding the pre- and post-test results in the module *Business Calculations*. The results of the SPQ-R-2F Questionnaire regarding students' approach to learning are summarised in Table 5.16 and Table 5.17. The researcher also provides a thorough discussion of each of these results. The chapter concludes with the most important findings and results of the pilot study of the empirical investigation.

5.5.4.1 *Descriptive statistics of student characteristics and demographic information*

Table 5.2 to Table 5.6 show the population characteristics of the total group of *Business Calculations* students with the following variables: race, gender, age, FTE status and mathematical background.

Table 5.2: Frequency distribution of "Race" of the population of *Business Calculations* students

Race	Population of Business Calculations Students	
	Frequency	**Percentage (%)**
Black	127	91.37
Coloured	7	5.04
Asian	1	0.72
White	4	2.88
Total	**139**	**100**

Based on Table 5.2 with regard to students' race, the following profile can be drafted: The majority of students that took *Business Calculations* were black (91.37%). The conclusion can thus be drawn that the group of BCL students was almost homogeneous with regard to race.

Table 5.3: Frequency distribution of "Gender" of the population of *Business Calculations* students

Gender	Population of Business Calculations Students	
	Frequency	Percentage (%)
Female	76	54.68
Male	63	45.32
Total	**139**	**100**

From Table 5.3 it can be seen that there were more female students (54.68%) than male students (45.32) from the population of students that took *Business Calculations* as a subject.

The following table contains information about a student's FTE status. "First-time entering" means that no time has elapsed between passing Grade 12 and current enrolment.

First-time entering (F): A "first-time entering" student is mostly a learner who is studying for the first time at a tertiary institution.

Not first-time entering (N): A student who was transferred from another HEI, or entering from another programme in the same HEI.

Table 5.4: Frequency distribution of "FTE status" of the population of *Business Calculations* students

FTE Status	Population of Business Calculations Students	
	Frequency	Percentage (%)
First-time entering	118	84.89
Not first-time entering	21	15.11
Total	**139**	**100**

From Table 5.4 it can be seen that the majority of students who took *Business Calculations* were "first-time entering" students. In other words, 84.89% of the students finished Grade 12 the previous year (2008) and enrolled at the CUT in the academic year 2009. Only a small percentage of students (15.11%) were not "first-time entering" students. This high percentage of almost 85% for "first-time entering" students also made the population of *Business Calculations* almost homogeneous with regard to their "first-time entering" status.

Table 5.5: Frequency distribution of "Mathematical Background" of the population of *Business Calculations* students

Math Background	Population of Business Calculations Students	
	Frequency	Percentage (%)
Mathematics	89	64.03
None	50	35.97
Total	**139**	**100**

Table 5.5 shows that 64% of the population of students who took *Business Calculations* had a mathematical background up to Grade 12 level. Almost 36% of the students did not take mathematics up to Grade 12 during their school careers.

Table 5.6: Statistics regarding "Age" of the population of *Business Calculations* students

Age	N	Mean	Std Dev	Min	Max	Median
	139	19.70	2.46	17	37	19

Table 5.6 shows that the average student that took *Business Calculations* was 19.70 years old. The youngest student in the group was 17, while the oldest student was aged 37.

The next section (Table 5.7 – Table 5.11) describes the difference between the experimental and control group of students with regard to the demographical characteristics of each student in the module *Business Calculations*.

Table 5.7: Frequency distribution of "Gender" of the population of *Business Calculations* students

Gender	Population of Business Calculations Students			
	Experimental Group (n=50)		Control Group (n=89)	
	n	%	n	%
Male	14	28	49	55.06
Female	36	72	40	44.94
Total	**50**	**100**	**89**	**100**

Based on the preceding section on students' characteristics and demographic information, the following profile can be drafted: Table 5.7 regarding gender shows that there were more female students (72%) than male students (28%) in the experimental group of students. The control group, on the other hand, had more male students (55.06%) than female students (44.94%).

Table 5.8: Frequency distribution of "Race" of the population of *Business Calculations* students

Race	Population of Business Calculations Students			
	Experimental Group (n=50)		Control Group (n=89)	
	n	%	n	%
Asian	1	2	0	0
Black	43	86	84	94.38
Coloured	3	6	4	4.49
White	3	6	1	1.12
Total	**50**	**100**	**89**	**100**

With regard to race (see Table 5.8), both groups of students comprised a majority of black students, with a percentage rate of 86% for the experimental group and 94.38% for the control group. Only one Asian student formed part of the study and was in the experimental group. The experimental group comprised 6% coloured students and the control group 4.49%. The percentage rate for white students was higher in the experimental group (6%) than in the control group (1.12%), the experimental group of students had only three white students, while the control group had one white student

Table 5.9: Frequency distribution of "FTE status" of the population of *Business Calculations* students

FTE status	Population of Business Calculations Students			
	Experimental Group (n=50)		Control Group (n=89)	
	n	%	n	%
First time	45	90	73	82.02
Not	5	10	16	17.98
Total	**50**	**100**	**89**	**100**

From Table 5.9, it can be seen that the majority of students in both groups were "first-time entering" students. The experimental group comprised 90% of "first-time entering" students, while the control group comprised 82% of "first-time entering" students.

Table 5.10: Frequency distribution of "Mathematical background" of the population of *Business Calculations* students

Mathematical background	**Population of Business Calculations Students**			
	Experimental Group (n=50)		Control Group (n=89)	
	n	%	n	%
Mathematics	34	68	55	61.80
None	16	32	34	38.20
Total	**50**	**100**	**89**	**100**

Table 5.10 shows that 68% of students from the experimental group and 61.80% students from the control group had a mathematical background up to Grade 12 level. The experimental group comprised 32% students with no mathematical background, and the control group 38.20% students with no mathematical background. From these figures it can be seen that there were more students in both groups with a mathematical background up to Grade 12, than students with *no* mathematical background.

Table 5.11: Descriptive statistics of "Age" of the population of *Business Calculations* students

Group	**N**	**Mean**	**Std Dev**	**Min**	**Max**	**Median**
Experimental	50	19.60	2.09	17	27	19
Control	89	19.75	2.66	17	37	19

From Table 5.11 it can be seen that both groups of students had the same mean age, namely 19.60 for the experimental group and 19.75 for the control group of students. The minimum age was 17 for both groups of students. The oldest student in the experimental group was aged 27.

5.5.4.2 *Analysis of pre- and post-test performances*

The following section entails a summary of the results of the pre-test and post-tests in the module *Business Calculations*. Descriptive statistics (mean, standard deviation, median, minimum, maximum, and number of observations), both for the total group and for the experimental versus control group separately, are presented.

Table 5.12: Pre- and post-tests descriptive statistics of the total group

Variable	N	Mean	Std Dev	Min	Max	Median
Pre-test	139	62.26	16.59	28	100	60
Post-test 1	139	56.86	17.21	20	100	60
Post-test 2	139	67.14	18.02	28	100	68
Average	139	62.01	14.15	36	95	62

Table 5.13: Pre- and post-tests descriptive statistics of the Experimental and Control group

Group	Variable	N	Mean	Std Dev	Min	Max	Median
Exp	**Pre-test**	50	64.88	18.97	32	100	60
	Post 1	50	59.60	18.88	30	100	60
	Post 2	50	66.96	17.33	36	100	68
	Average	50	63.30	15.83	38	95	63.50
Control	**Pre-test**	89	60.79	15.01	28	96	60
	Post 1	89	55.31	16.10	20	80	50
	Post 2	89	67.24	18.50	28	100	68
	Average	89	61.29	13.16	36	90	61

The pre-test results show that both groups of students performed at an average level, with a mean score of 64.88% for the experimental group and a mean score of 60.79% for the control group. The experimental group of students thus performed 4.09% better in the pre-test than the control group of students. The first post-test shows decreases of 5% in scores for both groups of students with a mean score of 59.60% for the experimental group and a mean score of 55.31% for the control group.

5.5.4.3 *Analysis of Association*

The researcher also examined the relationship between achievement in *Business Calculations* and the confounding variables. As this analysis is related to the dependent variable, achievement was firstly described in terms of descriptive statistics such as means and standard deviations (see 5.5.4.2).

Analysis of variance (ANOVA) was performed in order to determine whether there was any difference in students' post-test performance with regard to each of the following variables: group, gender, mathematical background, and FTE status. The average of post-test 1 and post-test 2 served as the dependent variable, while group, gender, mathematical background, and FTE status served as the independent variables. The results of the ANOVA are given in the following table:

Table 5.14: F-statistics and associated P-values of the one-way ANOVA

Source	DF	F value	*p* value
Group	1	0.64	0.4241
Gender	1	0.02	0.8754
Math background	1	0.16	0.6884
FTE status	1	3.28	0.0724

From the table above it can be seen that the results were not significant. In other words, students' performance in the post-test (average between post-test 1 and post-test 2) were independent of each of the confounding variables, namely: group membership, gender, mathematical background and FTE status.

5.5.4.4 *Multivariate Analysis*

The effect of the dependent variable (average of post-test 1 and post-test 2) was analysed using analysis of covariance (ANCOVA). The analysis of covariance is based on the inclusion of covariates into the model, and allows one to account for inter-group variation associated not with the "treatment" itself, but with covariates. The analysis of covariance model contains the independent variable (Group) and all potential confounders (gender, age, race, previous mathematical background, FTE status, *Business Calculations* results: pre-test). The results are given in the following table:

Table 5.15: F-statistics and associated P-values of the one-way ANCOVA

Source	DF	F value	*p* value
Group	1	0.11	0.7385
Age	1	0.74	0.3925
Race	3	2.16	0.0964
Gender	1	0.19	0.6665
Math background	1	0.88	0.3502
FTE status	1	1.93	0.1672
Pre-test	1	16.98	<0.0001

The results in Table 5.15 confirm no significance between students' post-test performance and each of the covariates. However, the significance was very high [$p<0.0001$] between students' pre-test performance and post-test performance in the module *Business Calculations*. In other words, students who performed well in the pre-test also performed better in the post-test and vice versa for both groups of students. The pre-test performance for both groups of students was thus highly correlated with their post-test performance. As the curves of the pre-test and post-test for both groups of students remained the same, this implies that the post-test scores were not significantly better than the pre-test scores .

5.5.4.5 *Analysis of the R-SPQ-2F Questionnaire*

From a total of 139 students that were selected for the research study, only 51 students (37%) completed the approach to learning questionnaire. In order to see whether there was a difference in approach to learning before the study strategy intervention and thereafter, 33 (37%) students from the control group, and 18 (36%) students from the experimental group were used when the responses from the pre- and post-tests were analysed. The results of the mean scores regarding the R-SPQ-2F Questionnaire are summarised in Table 5.17.

Table 5.16: Pre- and post-test mean scores of the students who completed the R-SPQ-2F Questionnaire

Scales on questionnaire	Experimental Group (n=19)			Control Group (n=38)		
	Pre-test Mean	Post-test Mean	Mean difference score	Pre-test Mean	Post-test Mean	Mean difference score
DA	32	32	0	33	33	0
SA	26	25	-1	25	26	1
DM	17	17	0	17	16	-1
DS	16	16	0	16	16	0
SM	12	11	-1	11	12	1
SS	14	13	-1	14	14	0

With regard to a deep approach to learning, the results in Table 5.16 show no increase or decrease after the study strategy intervention for both groups of students. With regard to a surface approach to learning, the experimental group shows no difference between the pre-test and the post-test, while the control group shows an increase of the mean score between the pre-test ($\bar{d} = 25$) and the post-test ($\bar{d} = 26$).

In order to answer the second research hypothesis (see paragraph 5.3), the scores students obtained in the pre-test and post-test of the revised two-factor Study Process Questionnaire (R-SPQ-2F) were used in a paired t-tests. Each test's p-value was calculated for both groups of students.

Table 5.17: P-values for the difference score between the pre-test and the post-test for students who completed the R-SPQ-2F Questionnaire

		Paired Differences		
	Pre-test – Post-test	Mean difference score	t	*p*-value
Exp Group	Deep Approach	0.05556	0.035	0.972
	Surface Approach	1.16667	0.688	0.501
Control Group	Deep Approach	0.36364	0.526	0.603
	Surface Approach	-0.93939	-0.687	0.497

The results indicated that there was no significant difference between the pre- and post-test results of the experimental group's approach to learning. Therefore, the second hypothesis is also rejected.

As the response rate was very low (37%) for both groups of students, no accurate conclusion could really be drawn. From the responses that were obtained from the questionnaire, it became very clear that many students misunderstood the purpose of the questionnaire, although the lecturer did explain it to them. Some students made a mockery of the questionnaire by choosing the "Strongly Agree" option with regard to all the questions, without reading the questions first. By choosing this response to all twenty questions, students would indicate that they follow both a deep and surface approach to learning. Other students completed only half of the questionnaire, which made their questionnaires impossible to use for data analysis. The results from the questionnaire therefore indicated that students failed to judge questions accurately, with an over-appraisal of a deep approach to learning prior to the study strategy intervention. This did not match students' performance in the pre-test. This over-confidence in students who thought they follow a deep approach to learning is of real concern. All of these factors contribute to the fact that the R-SPQ-2F Questionnaire cannot be seen as a true reflection of students' approaches to learning in this study.

5.5.5 *Qualitative analysis of students' approaches to learning*

In order to obtain richer data on the students' approaches to learning, the researcher complemented the more formal quantitative tests with additional qualitative information (see 5.5.2 and 5.5.3). In this study the researcher made use of a reflection diary in which she noted students' problem-solving approaches and techniques. The aim of the reflective diary was to gain insight into students' problem-solving behaviour. The researcher observed the students during the tutorial classes in the first semester and reflected on the activities, students' problem-solving approaches as well as the reaction to the general implementation of the proposed classroom learning strategy intervention.

In the next section, the researcher will describe the difference with regard to students' problem-solving strategies; namely between students who received *no* learning strategy intervention (the control group), and students who received the learning strategy intervention (the experimental group). Students' problem-solving strategies are illustrated by means of an example from the prescribed textbook (Croucher 2002), which focuses on simple interest, compound interest and annuities. These concepts all formed part of the learning strategy intervention during the first semester of the year 2009. Following the classroom learning strategy intervention, students in *Business Calculations* wrote a post-test on simple interest, compound interest and annuities. Consider example M6.6 from the prescribed textbook (Croucher 2002:M127)

Example 5.1

A company secretary has an investment opportunity in which a lending institution offers her an interest rate of 4% compounded quarterly. If she decides to invest an amount of R6000 under the scheme for 8 years, calculate:

(a) the accumulation factor;

(b) the accumulated value after 5 years;

(c) the total compound interest earned.

Solution

In this case, P = R6000. Since the interest is compounded 4 times per year, the interest rate per 3-month period is:

$$i = \frac{0.04}{4} = 0.01$$

The total number of interest rate periods over 8 years is:

$$n = 4 \times 8 = 32$$

$$\begin{aligned}\text{The accumulation factor} &= (1+i)^n \\ &= (1+0.01)^{32} \\ &= (1.01)^{32} \\ &= 1.374\,940\,68\end{aligned}$$

$$\begin{aligned}\text{The accumulated value} = \text{S} &= P(1+i)^n \\ &= \text{R}6000 \times 1.374\,940\,68 \\ &= \text{R}8249.64 \text{ (to the nearest cent)}\end{aligned}$$

$$\begin{aligned}\text{Total compound interest earned} &= \text{S} - \text{P} \\ &= \text{R}8249.64 - \text{R}6000 \\ &= \text{R}2249.64\end{aligned}$$

5.5.5.1 *Students that formed part of the control group*

During one of the tutorial classes, students who formed part of the control group were asked to solve the above problem from the prescribed textbook. The students in the control group were allowed to make use of their own problem-solving strategies to solve the problem. They were allowed to consult the prescribed textbook as well as their class notes. The researcher observed the group of students closely and found that many of the students in this group solved new problems by simply modelling similar examples from their class notes and textbook (see 3.8.1.1). According to the students in this class, this is the way they normally learn mathematics or statistics. They try to solve new problems by referring to the textbook to see how similar examples were solved.

The method most students employed in this group included low-level mathematical skills and strategies rather than conceptual or higher order thinking (see 3.3.3.1). These students clearly followed a surface approach of rule following and rote memorisation (see 2.4.1). This type of approach also showed no reflective use of strategic know-how. In these procedural activities, successful academic performance does not demonstrate, nor provide a foundation for the understanding of core mathematical or statistical concepts. This clearly demonstrates that "doing mathematics" does not necessarily lead to understanding mathematics (see 3.3.3.1).

- ***The researcher's observation of how students who receive no learning strategy intervention would typically solve the problem.***

1. Students read through the problem, but do not know what is asked/expected of them. Students attempt to extract information from the problem, but do not know which formulas, as well as the symbols for each of the concepts in the problem, to use. In other words, students cannot translate the words in the problem into appropriate symbol notation. For example, in this case, P = R6000. Some students did know that the symbol for the principal value is P. However, the student who does not know the correct symbol (which is associated with the principal amount of money given) might assume that S = R6000. This clearly indicates the importance of what is given/asked, as well as the importance of the associating symbols with regard to the appropriate formulas (see 3.6.1).

2. Since the interest is compounded 4 times per year, the interest rate per 3-month period is:

$$i = \frac{0.04}{4} = 0.01$$

Some students in the control group did not know how to convert interest per year, to interest compounded quarterly. Some students even multiplied 0.04 by 4, i.e. (0.04×4). As many students did not know what *compounded quarterly* means, they only took the interest rate as 4 %, and substituted 4 into the formula, instead of 4 % converted into a decimal.

[take note that $4\% = \frac{4}{100} = 0.04$]

3. The total number of interest rate periods over 8 years is:

$n = 4 \times 8 = 32$

Some students in the control group only took the time as 8 [n = 8]. The words "*compounded quarterly*" again did not make any sense to them, or maybe they read the question so cursorily that they did not even notice it.

4. Some students in the control group also chose the wrong formulas for calculating the accumulation factor and accumulated values here: They used the present value factor $(1+i)^{-n}$ instead of the accumulation factor $(1+i)^{n}$. This might be because they did not contrast the formulas on a concept map (see 4.2), or realise that there is actually a difference between these formulas. During one of the classes, one of the students actually came to the researcher and asked her "Why do some formulas have a positive n in the exponent, while others have a negative sign in front of them? Is there a printing error in the textbook?". This question clearly indicates that the student did not realise that the formula with the positive exponent $(1+i)^{n}$ is used when working out the accumulated value (S), and that the formula with the negative exponent $(1+i)^{-n}$ is used when calculating the present value (P).

5. Students solved the problem. However, as wrong formulas were chosen, and wrong numbers had been substituted, students' answers were invariably incorrect.
6. Students did not reflect on the problem.

5.5.5.2 *Students who formed part of the experimental group*

During the other tutorial class, the students in the experimental group were not allowed to consult the prescribed textbook or any notes in an attempt to solve this problem. Instead, the researcher taught the students her own RIEQTSR problem-solving strategy (see 4.3).

- ***The researcher's observation of how students who received the study strategy intervention would typically solve the problem***

1. Student made use of the researcher's "RIEQTSR" heuristic (see 4.3).

2. Students read through the problem, recognised what this problem is about and identified it as a problem that involves compound interest (see 3.2.2.2).

3. Students extracted information from the problem and clearly noted what they were asked to find (see 4.3).

4. Students questioned themselves (see 3.8.2.8 and 4.3). For example: "What information is given? What information is missing? Which formulas should I use when working with compound interest?". Students knew which formulas are used for each of these concepts and wrote them down from the list of formulas provided at the back of the textbook, as they recognised them. The use of a *concept map* structured these concepts in a meaningful fashion (see 4.2).

   (a) the accumulation factor = $(1+i)^n$

   (b) the accumulated value $S = P(1+i)^n$

   (c) the total compound interest earned $= S - P$

5. Students translated the words into their appropriate symbol notation. They were able to substitute the values into the appropriate symbols which form part of the formulas (see 3.6).

P = R6000. Since the interest is compounded 4 times per year, the interest rate per 3- month period is:

$$i = \frac{0.04}{4} = 0.01$$

The total number of interest rate periods over 8 years is:

$$n = 4 \times 8 = 32$$

4. Students solved the problem, by substituting the values of the symbols into the appropriate formulas:

The accumulation factor $= (1+i)^n$

$= (1+0.01)^{32}$

$= (1.01)^{32}$

$= 1.374\ 940\ 68$

The accumulated value = S $= P(1+i)^n$

$= R6000 \times 1.374\,940\,68$

$=$ R8249.64 (to the nearest cent)

Total compound interest earned $=$ S – P

$=$ R8249.64 – R6000

$=$ R2249.64

5. Students reflected on this problem and also reviewed their answer.

5.5.5.3 *Students' approaches to learning while studying for the post-test*

In the following section, the researcher describes students' approaches to learning while studying for the post-test. The findings of students' approaches to learning are based on the researcher's observations during the practical classes as well as comments made by some students during these classes. The researcher diarised these findings and comments for the duration of the classroom learning strategy intervention.

5.5.5.3.1 The researcher's observations regarding how students who received *no* learning strategy learned.

- Many students in this group followed a surface approach to learning, which arose from an intention to get the task completed with minimum trouble (see 2.4.1). Students apparently made use of low cognitive-level activities and acquired new knowledge simply by rote learning and memorisation (2.7.4.1 and 3.4.1.2). In an attempt to solve problems, students made use of words used, isolated facts, and treated items independently of one another (see 2.4.1 and 2.7.4.1). Learning thus became a drag for the student, without any enjoyment in doing the task.
- Students lacked conceptual understanding (see 3.3.3.1). Knowledge was fragmented and effective problem solving was impeded without a conceptual framework. Students learned to apply procedures when directed, but were lost when presented with a new problem (see 3.8.3). Students seemed to be computationally proficient, but not able to conceptualise problems (see 3.8.1.1 and 3.3.3.1).
- Students lacked context knowledge, which constrained their thinking (see 3.8.1; Wild & Pfannkuch 1999:229). Students lacked domain specific knowledge, and were therefore unable to solve problems, as problem solving entails domain specificity and domain independence (see 3.8.1).
- Students saw elements as individual compositions (see 2.4.1).

- Students imitated and relied on the rote use of algorithms for manipulating formal symbols. Students memorised formulas, but not the principle behind them (see 3.6).
- As the working memory can hold only small amounts of information for limited periods of time, too many items can become crammed into a limited space (see 2.7.2.2). This happens when the student feels "I cannot take in any more information, it is too much". Students apparently did not know how to encode new information, which has implications with regard to their long-term memory (see 2.7.2.3).
- Students' existing knowledge was not adequate and organised, therefore assimilation was prevented (see 2.10.1.2 and 3.4.3.3].
- Students found a way of mapping a new problem onto a problem that had already been solved so that the previously devised solution could be applied or adopted (see 3.4.3.3).
- Students followed a traditional approach to learning (see 3.4.1).

5.5.5.3.2 The researcher's observations regarding how students who received the learning strategy learned.

The researcher concluded from her observations that

- Students made use of the researcher's RIEQTSR (recognise, identify, extracting information, question, translate, solve, reflect) heuristic technique with regard to problem solving (see 4.3).
- The researcher observed students during the tutorial classes and found that students in the experimental group who received the learning strategy intervention, followed a deep approach to learning and felt a need to engage the task appropriately and meaningfully with an intrinsic curiosity and determination to do well (see 2.4.2). When students solved problems, they made use of the most appropriate cognitive activities, which required them to reflect, hypothesise and apply. Students in this group focused on underlying meanings, main ideas, themes, principles and useful applications. They worked from first principles, which require a well-structured knowledge base

as well as appropriate background knowledge (see 2.7.4.1 and 3.4.3.3). Students thus had the reception of declarative knowledge and structured it meaningfully (see 2.7.4.1 and 4.2).

- Students had the ability to work conceptually rather than with unrelated details, because it was observed that they followed a deep approach to learning (see 2.4.2 and 3.3.3.1).
- Students made use of concept maps, which provided structure and represented meaningful relationships between concepts in the form of propositions. The concept map (see 4.2) presents an overall picture, a holistic (gestalt) representation of a complex conceptual structure. They organised the material to be learned in such a way that they perceived it as a whole and understood how the different concepts are connected as a whole (see 2.7.1).

An issue related to organisation is the concept of chunking or combining information in a meaningful fashion (see 2.7.2.2). Learning was thus enhanced, as organised material usually improves memory because items are linked to one another systematically, and there is a relationship between ideas. Students made use of schemas which highlighted important information and assisted encoding, because they elaborated new material into a meaningful structure. The process of encoding allowed students to transform information into a lasting representation in the long-term memory (see 2.7.2.3). The lecturer taught students how to categorise (chunk) related information and how to encode certain information (see 2.7.1; 2.7.3 and 3.8.2).

- Students were taught how a body of knowledge can be structured, which makes understanding of a subject more comprehensible (see 2.7.3.4). According to Bruner, categorisation is a fundamental process in the structuring of knowledge (see 2.7.3.2 and 2.7.3.4). This also ensures that the learning process is meaningful (see 2.7.4).
- Students were encouraged to extract relevant information from a problem statement, and to identify and analyse elements of a problem. Information made sense to the student, and the student could assimilate [see 2.10.1.1 (a) and 3.4.3.3] new information by interpreting and filtering new information in terms of his/her existing knowledge. Accommodation took place as students'

existing knowledge was adequate and organised [see 2.10.1.2 (b)]. Students also made use of reflective abstraction as a tool to develop advanced mathematical thinking [see 2.10.1.2 (c)].

- Students followed a constructivist approach to learning (see 2.9 and 3.4.2) and were actively involved and engaged in the learning process.
- In order to facilitate cognitive development, the researcher emphasised social interaction and placed students with more experienced peers. Through teacher scaffolding (see 2.11.1.4), and mediated learning experiences (see 2.11.1.1), learners moved through the zone of proximal development (see 2.11.1.2) and became independent learners, as meaning was mediated by the different perspectives among the students. The researcher used modelling as a form of scaffolding and provided students with concrete examples of *how* to solve problems (see 2.11.1.4). Mathematical knowledge was therefore constructed by individuals as a consequence of their interactions and meaning was created by individuals through their interactions with one another during practical classes (see 2.11.1 and 3.4.4).
- Students made use of questioning techniques in order to increase their awareness of their own strategies and problem-solving methods, as well as the development of their reflective processes (see 3.5.2 and 3.8.3.2).
- Students gained control over their own learning and the management of their own problem-solving process through the process of metacognition (see 3.8.3.1). The researcher applied metacognitive knowledge by encouraging students to organise relevant information and form appropriate links to be learned (see 3.8.3.2 and 4.2).

5.5.5.4 *Additional information with regard to the qualitative observations*

When the researcher asked students "*How do you feel about studying a mathematical subject?*" and "*How do you perceive this subject so far?*" and "*What makes this subject difficult for you?*", the following problems emerged:

- Students with no mathematical background, in both the experimental as well as the control group, said that they felt at a disadvantage and also inadequate regarding the subject *Business Calculations*.
- Some students admitted that they followed the wrong approach when studying mathematics, for example rote learning.
- Some students said that they felt lost when doing problems on their own, while, when in class, the work seemed easy as long as the lecturer explained it.

The pervasive lack of appropriate study skills is clearly illustrated in the following comment by one of the students:

> "When the lecturer explains the work to us on the board, everything seems so easy, but when I'm at home, trying to solve problems on my own, I'm lost, I don't know where to start..."

Through several informal conversations with students during lectures, it became very clear that the proposed learning strategy intervention had provided them with new study skills regarding a mathematics and statistics-related subject. A large proportion of students, especially those with no mathematical background, told the researcher that the learning strategy intervention helped them in ways they had not thought of before. Some students even applied elements of the method to some of their other subjects, as they found the learning strategy to be helpful. From this qualitative point of view, it therefore seemed that students appeared to benefit from the proposed learning strategy.

5.5.6 *Summary of findings*

1. The first research question of the pilot study was:

Does the implementation of a classroom study strategy intervention positively affect students' academic performance in the subject Business Calculations?

The pre-test results showed average results for both groups of students (see Table 5.13). The pre-test comprised much revision from work that had been covered in school. However, the first post-test results showed marked decreases in scores for both groups of students, with only a few students in both groups showing an increase in marks. This might be ascribed to the fact that the first post-test covered concepts that were new to students and which they were not familiar with. Although the results of the second post-test showed marked improvements for both groups of students, the researcher cannot claim that the improvement of scores resulted specifically from the proposed learning strategy intervention. The first research hypothesis (see 5.3) is therefore rejected, meaning that the post-test score in *Business Calculations* of the experimental group was not significantly higher that the post-test score of the control group.

Students in the experimental group were classmates with students in the control group and saw these students on a regular basis at other lectures (McMillan and Schumacher 2006:140). Many of these students were accommodated in the same hostels on campus and formed their own study groups. Although students in the experimental group did not know that they were part of an experiment, and were not aware that the study "hints" or notes they received during classes were actually part of a particular learning strategy intervention, some students could still have exchanged notes with some of their friends in the control group. There could also have been other contributing factors, such as increased exposure to mathematics discourse during the semester, or students taking examinations more seriously and therefore studying harder.

The results of the one-way ANOVA (see Table 5.14) further confirm that students' performance in the post-test (average between post-test 1 and post-test 2) was independent of each of the confounding variables, namely: group membership, gender, mathematical background and FTE status. The results of the one-way ANCOVA (see Table 5.15) also confirm no significance between students' post-test performance and each of the covariates. The significance, however, was very high between students' pre-test performance and post-test performance in the module *Business Calculations* as measured in this pilot study. In other words, students who performed poorly in the pre-test tended to perform poorly in the post-test as well, while students who performed well in the pre-test, tended to perform well again in the post-test (see Table 4.15).

2. The second research question of the pilot study was:

Does the implementation of a classroom study strategy intervention positively affect students' approaches to learning in the module Business Calculations?

As the response rate was very low (32%) for both groups of students with regard to the R-SPQ-2F Questionnaire, no accurate conclusion could really be drawn. From the responses that emanated from the questionnaire, it became very clear that many students misunderstood its purpose, although the lecturer did explain it to them. Some students made fun of the questionnaire and completed it incorrectly, while others completed only half of the questionnaire. The results indicated that students failed to judge questions accurately, with an over-appraisal of a deep approach to learning prior to the study strategy intervention that did not match students' performance in the pre-test. This over-confidence of students that they do follow a deep approach to learning is of real concern. All of these factors therefore made a contribution to the fact that the R-SPQ-2F Questionnaire cannot be seen as a true reflection of students' approaches to learning in this study. Therefore, the second research hypothesis (see 5.3) is also rejected, i.e. the mean difference score on the R-SPQ-2F Questionnaire for the experimental group was not significantly higher that the mean difference score of the control group.

What the results above suggest is that the learning strategy intervention did not guarantee mathematical success, or high marks in the relevant subject. However, the study strategy might have been beneficial to students with *no* mathematical background in the sense that it gave them the necessary study skills in respect of a mathematics and statistics-related subject. On the other hand, students with some mathematical background may have been used to their own way of studying mathematics, or did not find it necessary to change their already established and ingrained study habits.

5.6 RELIABILITY AND VALIDITY

For the purpose of this study, the researcher critically examined the procedures for collecting data in order to assess to what extent the design is likely to be reliable and valid.

The Alpha Cronbach of the structured R-SPQ-2F Questionnaire was adequate (see Table 5.1) and the consistent way in which all the data were processed and analysed by means of statistical packages undoubtedly contributed to the reliability of the study.

Internal validity is the extent to which differences in the dependent variable are accounted for by differences in the independent variable and not by any extraneous or third variables (Maas 1998:24; Kerlinger 1986:300). The extraneous variables were controlled in the research design through statistical measurement by building them into the design.

Leedy and Ormrod (2001:104) define external validity of a research study as "the extent to which its results apply to situations beyond the study itself – in other words, the extent to which the conclusions drawn can be generalised to other contexts". A concern with regard to the external validity of this research is the extent to which the research results are generalisable (Saunders, Lewis & Thornhill 2003:102). As this research was conducted at one specific University of Technology (the CUT), and no randomisation was used, the results may not be generalisable to all first-year students at other universities.

5.7 ETHICAL CONSIDERATIONS

McMillan and Schumacher (2001:198) suggest that, for research to be conducted through an institution, in this case the CUT, approval for conducting the research should be obtained from the institution before any data are collected. Permission for this research study was obtained from the Dean of the Management Faculty at the CUT in 2008.

According to McMillan & Schumacher (2006:143), informed consent implies that the subjects have a choice about whether to participate in research. The researcher was of the opinion that knowledge of participation in this specific research might invalidate the results and regarded it as acceptable that the subjects did not know that they had been participants. McMillan and Schumacher (2006:143) purport that "some educational research is quite unobtrusive and has no risk for the subjects. As the Hawthorne effect might have reduced the validity of this study, it seemed best for subjects to be unaware that they were being studied in order to maximise the internal validity (McMillan and Schumacher 2006:140).

5.8 CONCLUSION

Although the results of the pilot study were not significant, the researcher recognised that many students enter university without the necessary study skills regarding a mathematics and statistics-related subject.

The integration of a learning strategy intervention as part of the learning process has a broader focus than mere reinforcement of practising exercises through tutorial classes, which was the traditional study method followed by the control group of students. The pilot study has contributed greatly towards a better insight into the refinement of a study strategy intervention as well as the implementation of the next phase of the study.

With regard to the refinement of the research study, the researcher decided to make no changes with regard to the classroom learning strategy. Although the quantitative results of the pilot study were not significant, it became clear from students' remarks and the researcher's own observations during the tutorial classes, that the classroom learning strategy has a positive impact on students' approaches to learning. However, no valid conclusion could be made from the responses that were derived from the R-SPQ-2F Questionnaire. Therefore, the researcher decided rather to make use of the nominal group technique setting in the main research study.

CHAPTER 6: RESEARCH DESIGN AND METHODOLOGY

6.1 INTRODUCTION

In the first chapter the researcher presented an orientation and background of the study at hand. A short discussion on underpreparedness in mathematics was included, whilst the research problem, questions, aims, objectives, theoretical statements and research design and methodology were outlined. The problem statement (see 1.3) centres on the challenge of addressing underpreparedness in mathematics among students and gives rise to the formulation of the main research question, namely: *Does the implementation of a classroom learning strategy intervention positively affect third-year students' academic performance in the module Business Statistics / Statistics II?*

In order to answer this question and consequently determine the effectiveness of the learning strategy intervention, the main research question is informed by five subsidiary questions (see 1.4). Based on a comprehensive literature review, the following subsidiary research questions are addressed in Chapter 2 and Chapter 3, namely:

1. How does learning take place with regard to some important learning theories? (Chapter 2)
2. How is knowledge constructed in the process of learning mathematics and statistics? (Chapter 3)
3. How can students' cognitive processes be enhanced through a constructivist perspective in order to improve their mathematical as well as statistical thinking. (Chapter 3)
4. How will the proposed learning strategy that students would be exposed to, be conducted? (Chapter 4)
5. What is the effectiveness of the proposed learning strategy? (Chapters 5, 6 and 7)

In Chapter 2 the researcher provided a thorough theoretical overview on the concept of learning. Behavioural learning theories were briefly discussed (see 2.6), while the gestalt learning theory was also presented (see 2.7.1). The researcher also presented some important cognitive learning theories which emphasise the development of the learner's thinking processes (see 2.7). The researcher explained information processing theory as a component of cognitive learning theory (see 2.7.2) and extended the cognitive analysis by discussing Bruner's theory of cognitive growth (see 2.7.3) and Ausubel's meaningful reception learning (see 2.7.4). The researcher discussed Bandura's social cognitive theory (see 2.8.1) and highlighted the modelling processes, which plays a prominent role in learning from a social cognitive perspective (see 2.8.1.1). Finally, constructivism was explained as well as the two major strands of the constructivistic perspective, which are cognitive constructivism and social constructivism. The researcher emphasised the theories of Jean Piaget with regard to cognitive constructivism (see 2.10), and the theories of Lev Vygotsky with regard to social constructivism (see 2.11).

In Chapter 3 the researcher provided a thorough theoretical overview of the nature and importance of mathematics as a learning discipline (see 3.3.1 and 3.3.3). Some important learning theories in mathematics were also presented so as to come to a full understanding of how a learning strategy intervention in a mathematics and statistics-related subject may be implemented and how it can assist in optimising students' academic performance (see 3.2; 3.3 and 3.4). As the main purpose of this study was to improve students' academic performance in a mathematics and statistics-related subject at the Central University of Technology, Free State, by means of a classroom learning strategy intervention. The researcher also made mention of various learning strategies employed by other researchers (see Chapter 3).

In Chapter 4, the researcher detailed her own classroom learning strategy intervention.

In Chapter 5, the researcher reported on a pilot study conducted with first-year students at the CUT. The aim of the pilot study was to determine the impact of the researcher's classroom learning strategy on first-year students' academic performance in a mathematics and statistics-related subject at the CUT. The purpose of the pilot study was to improve the success and effectiveness of the proposed classroom learning strategy intervention.

Chapter 6 describes the research design and methodology for the final empirical investigation used in this study. The chapter commences with the research problem, whereafter the research questions, research aims and hypotheses are spelt out. The research design and methodology are outlined, whilst the data are presented and discussed. The results of the pre and post-test regarding students' performance in the module *Business Statistics/Statistics II* are also presented.

6.2 BACKGROUND TO THE RESEARCH PROBLEM

Over the past decade, numerous articles have been written about the crisis in higher mathematics education. Many of these articles focus on factors that might have an impact on underperformance in mathematics, as well as offering some interventions for the underprepared student in mathematics (Paras 2001:66-73; Du Preez, Steyn & Owen 2008:49-62; Steyn & De Boer 1998:125-131; Botha, McCrindle & Owen 2003:132-134; Engelbrecht & Harding 2003:17-20; Nenty & Polaki 2005:67-77; Potgieter & Webb 2004:313-321; Polaki & Nenty 2001:41-52; Pretorius & Bohlmann 2003:226-236; Maree, Louw & Millard 2004:25-34; Bohlmann & Pretorius 2002:196-206; Boaler 1998:129-141; Taylor 1999:95-107; Yusof & Tall 1999:67-82; Anthony 2000:3-14; Takahashi, Watanabe & Yoshida 2000:129-136). These authors and teachers have made conscientious efforts to provide very useful strategies and interventions for the crisis mathematics is facing in higher education (see Chapter 1).

Mathematical skills have progressively become a focus in mathematics education research (Ferrini-Mundy and Gaudard 1992; Frith, Frith and Conradie 2006). There is also strong evidence that implementing a study plan and monitoring were the most important aspects for self-regulated learning, because these consistently correlated with, and were predictive of, good test performance (Cukras 2006:194-197). However, very little information as to how a learning strategy at a higher education institution might impact students' academic performance in a mathematics and statistics-related subject, is available. Reasons for this may be that lecturers assume that students know how to study for a mathematics and statistics-related subject, or because previously disadvantaged students or cultural diversity have taken the blame for underachievement in mathematics over the past decade. As underprepared and first generation students often lack effective study skills (see 1.2.3.3 and 3.2), the researcher deemed it necessary to address this concern through the implementation of an effective learning strategy intervention.

Currently there is very little research that fully encompasses an effective learning strategy for a mathematical subject at South African higher education institutions. Furthermore, no research thus far has investigated whether the implementation of a learning strategy can improve students' academic performance in a mathematics and statistics-related subject. However, the researcher managed to obtain an article on the following topic: "How to study mathematics" (Dawkins 2006:1-11). The article focuses on some general hints regarding the studying of mathematics, hence it still does not provide a specific learning strategy or practical example regarding the studying of a mathematics-related subject.

6.3 PROBLEM STATEMENT

By discussing the background to the problem statement and emphasising the lack of study skills among underprepared students, the main subsidiary questions that inform this empirical study are:

1. *Does the implementation of a classroom learning strategy intervention positively affect students' academic performance in the module Business Statistics / Statistics II?*

2. *Which developmental experiences did students find most useful after the implementation of the proposed classroom study strategy intervention?*

6.4 RESEARCH HYPOTHESIS

For the purpose of this study (and also in investigating the research problem) the empirical study tested the following research hypothesis:

H_0 : The post-test score of students who received the learning strategy (experimental group) is equal to the post-test score of students who received no learning strategy (control group) in the module *Business Statistics/Statistics II.*

H_a : The post-test score of students who received the learning strategy (experimental group) is higher than the post-test score of students who received no learning strategy (control group) in the module *Business Statistics/Statistics II.*

6.5 IDENTIFYING THE VARIABLES

This research study tested the effectiveness of a classroom learning strategy intervention on students' academic performance in the module *Business Statistics/Statistics II.* The study also sought to discover the developmental experiences students found most useful after the implementation of the proposed classroom study strategy intervention.

6.5.1 Independent variables

The independent variable is defined as the classroom learning strategy intervention. The researcher manipulated and assigned the intervention to two groups of students. Polit and Beck (2008:755) argue that the independent variable "is believed to cause or influence the dependent variable; in experimental research, the manipulated (treatment) variable".

For the purpose of this research, the classroom learning strategy intervention was defined as the facilitation of a particular learning strategy which is derived from a constructivist perspective, with the emphasis on the construction of mathematical knowledge and the processes by which learners create mathematical meaning (see Chapter 4; Biggs 1972:230). The aim of the classroom learning strategy intervention was to improve students' academic performance in the module *Business Statistics/Statistics II.* The learning strategy is discussed in Chapter 4.

6.5.2 Dependent variables

For the purpose of this study, the dependent variable was represented by students' post-test performance in a mathematics and statistics-related subject.

Students' achievement was measured by means of two tests scores in the module *Business Statistics/Statistics II*. As the question of interest is whether the post-test performance of students who received the learning strategy (experimental group) is higher than the post-test performance of students that received no learning strategy (control group) in the module *Business Statistics/Statistics II*, the post-test score served as the dependent variable.

6.5.3 Confounding/extraneous variables

McMillan (2008:38) argues that extraneous variables affect the dependent variables but are either unknown or may not be controlled by the researcher. Possible extraneous variables that might have compromised the results of this study were race, gender, age, previous mathematics background, attendance, mark obtained in the first-year semester subject *Business Calculations*, and the pre-test score of the results.

According to McMillan and Shumacher (2006:255), a very unique characteristic of experimental research, is that there is a determined effort to ensure that no extraneous variables provide plausible rival hypotheses to explain the results. The extraneous variables were acquired by collecting biographical data from the CUT's data system. The confounding variables were built into the design by measuring them and by analysing their influence on the dependent variable, which in this research was the post-test score of students in the module *Business Statistics/StatisticsII* (McMillan & Schumacher 2006:118).

6.6 RESEARCH DESIGN AND METHODOLOGY

Given the primary research question, this empirical research study is regarded as evaluative in nature, as the aim of evaluative research is to improve a current practice. According to McMillan and Schumacher (2001:20), evaluative research focuses on evaluating the merit or worth of a particular practice at a given site. McMillan (2008:16) purports that evaluation research is directed toward making decisions about the effectiveness or desirability of a programme with the goal to make judgments about alternatives in decision-making situations. In this research the evaluation research was focused on a specific learning strategy intervention and involved judgments about such questions as: Did the classroom learning strategy intervention have any effect on students' academic performance in the module *Business Statistics / Statistics II*? According to McMillan and Schumacher (2006:15), evaluation research can add to existing knowledge about a specific practice and stimulate further research and methodological development.

The research took place within a post-positivist paradigm and followed a critical realist ontology (Maree 2007:65). The researcher recognises objectivity as an ideal that can never be achieved within this paradigm. However, research within a post-positivistic paradigm is conducted with a greater awareness of subjectivity (Maree 2007:65). According to Seale (in Maree 2007:65), "post-positivism is a useful paradigm for researchers who maintain an interest in some aspects of positivism such as quantification, yet wish to incorporate interpretivist concerns around subjectivity and meaning, and who are interested in the pragmatic combination of qualitative and quantitative methods". Research studies that take place within a post-positivism paradigm "focus on establishing and searching for evidence that is valid and reliable in terms of existence of phenomena rather than generalisation" (Maree 2007:65).

6.6.1 The non-equivalent pre-test post-test control group design

This study investigated the effect of a learning strategy intervention on students' academic performance in the module *Business Statistics/Statistics II*. The methodology employed in this research is partly based on an approach used by Maree, Louw and Millard (2004:25-34). Two control assessments were given to students, with the first version serving as the pre-test and the second as the post-test. The study followed a non-equivalent pre-test post-test control group design involving a treatment group and an experimental group. This study is located within a quantitative paradigm, with some enhancement by means of qualitative observations of students' problem-solving techniques. A quasi-experimental approach was used in answering the question whether the post-test performance of students who received the learning strategy (experimental group) was higher than that of students who received no learning strategy (control group) in the module *Business Statistics/Statistics II.*

6.6.2 The rationale for using quantitative research in this study

Charles and Mertler (in Ivankova, Creswell and Clark 2007:255) argue that in quantitative research, the researcher relies on numerical data to test the relationship between the variables. For the purpose of this study, the researcher relied on numerical data (pre-test and post-test scores) to test the relationship between the variables as well as to test the formulated hypothesis, i.e. whether the post-test performance of students who received the learning strategy (experimental group) is higher than that of students who received no learning strategy (control group) in the module *Business Statistics/Statistics II.*

McMillan and Schumacher (2001:15) define quantitative research as an inquiry into an identified problem, based on testing a theory, measured with numbers, and analysed using statistical techniques, aimed at determining whether the predictive generalisations of a theory hold true. Maree (2007:145) defines quantitative research as "a process that is systematic and objective in its ways of using numerical data from only a selected subgroup of a universe (or population) to generalise the findings to the universe that is being studied". According to Ivankova *et al.* (2007:255), "the quantitative researcher tests the theories about reality, looks for cause and effect, and uses quantitative measures to gather data to test the hypothesis (or questions)" about the variables. Wimmer and Dominick (2000:106) also purport that quantitative researchers generally follow a deductive model in data analysis and that "hypotheses are derived prior to the study, and relevant data are then collected and analysed to determine whether the hypotheses are confirmed or not confirmed".

According to Schurink (1998:241), the main aims of quantitative research are to objectively measure the social world, to test hypotheses and to predict and control human behaviour. For Ivankova *et al.* (2007:255), "[t]he goal of quantitative research is to describe the trends or explain the relationships between the variables". Neill

(2007:1 of 2) argues that quantitative research bears some advantages as the researcher tends to remain objectively separated from the subject matter and that the use of numbers allow for greater precision in reporting results. Apart from that, quantitative research also permits the use of powerful methods of mathematical analysis (Wimmer & Dominick 2000:49).

As the main purpose of this study is to describe or explain, it is quantitative in nature (Neuman 2000:22; Rosier 1988:107). As this research involved the systematic collection of observable and measurable data as well as the statistical analysis of the data, the quantitative paradigm was considered appropriate for this study. Thus, in light of the above, the quantitative modes of inquiry employed in this study were advantageous in the sense that they:

- enabled the use of formal instruments such as tests;
- allowed the identification of variables and measurement of relationships (see 6.5); and
- permitted the use of powerful methods of mathematical analysis, thus allowing for greater precision in reporting the results (see 6.6.8).

6.6.3 The rationale for enhancing the quantitative research with qualitative observations

The empirical study, which is located within a quantitative paradigm, was enhanced by means of qualitative observations of students' problem-solving strategies. The researcher made use of a reflection diary in which she noted students' learning approaches, problem-solving strategies as well as the implementation of the classroom learning strategy intervention. The researcher also decided to conduct a nominal group setting with the experimental group of students at the end of the semester module *Business Statistics/Statistics II.* The researcher conducted this setting to determine the developmental experiences students found most useful after the implementation of the proposed classroom learning strategy intervention.

According to McMillan & Schumacher (2001:428), "qualitative research is interactive face-to-face research, which require relatively extensive time to systematically observe, interview, and record processes as they occur naturally". This type of research is conducted in a natural setting and involves a process of building a complex and holistic picture of the phenomenon of interest and generates rich, detailed and valid data that contribute to in-depth understanding of the context. The goal of a study based upon a qualitative process of inquiry is to understand a social or human problem from multiple perspectives. When qualitative data is analysed, one seeks to discover patterns such as change over time or possible causal links between variables.

The qualitative modes of inquiry employed in this study were advantageous in the sense that they:

- increased the researcher's depth of understanding of students' problem-solving strategies and behaviour;
- allowed the researcher to become the instrument of observation;
- made it possible for the researcher to examine the concerns and issues surrounding students' problem-solving strategies.

6.6.4 The quasi-experimental design employed in this study

The quantitative research design has, among others, been classified into an experimental design, as experimental studies establish probable causality (Ivankova *et al.* 2007:255). McMillan (2008:11) argues that, in experimental research, the investigators have control over one or more factors (variables) in the study that may influence the subject's behaviour, and that by manipulating a factor one can then see what happens to the subject's responses as a result. According to McMillan (2008:11), "the purpose of manipulating a factor is to investigate its causal relationship with another variable".

An experimental approach was selected, as the problem is clearly one of causation and, presumably, the intervention of a learning strategy intervention (the independent variable) would be manipulated easily. McMillan and Schumacher (2006:253) regard researcher-controlled interventions, or direct manipulation of a treatment as a very distinct feature of experimental research. They define the term *experiment* as "a way of learning something by varying some condition and observing the effect on something else" (McMillan & Schumacher 2006:253). According to McMillan and Schumacher (2006:254), the experimental design is presented as the "gold standard" for determining cause and effect in educational research and has been developed to answer a specific kind of research question, namely the cause-and-effect question. With regard to this study the research question was: "Does the implementation of a classroom learning strategy intervention positively affect students' academic performance in the module *Business Statistics / Statistics II*?"

Maree (2007:149) purports that another characteristic that distinguishes an experimental design from other designs is that there is some control, in that some participants are used as a control group by not receiving the treatment. For the purpose of this study, the researcher therefore purposefully manipulated one variable (called the treatment or controlled variable which in this research was the classroom learning strategy intervention) and measured its consequences on the dependent variable (which in this research was represented by students' post-test score in the module *Business Statistics / Statistics II).*

The reason why a quasi-experimental design was employed in this study instead of a true experimental design, was that random assignment of subjects to experimental and control groups was practically impossible, which is often the case in educational research (McMillan & Schumacher 2006:273). Struwig and Stead (2001:10) argue that in quasi-experimental research, "participants are not randomly assigned to groups" which was the case in this research study. Another reason why the researcher decided to employ a quasi-experimental design, was that the researcher could not control certain confounding variables (Struwig & Stead 2001:10). Although a quasi-experimental design is not truly experimental in nature, it does provide reasonable control over most sources of invalidity and is usually stronger than true-experimental designs (McMillan & Schumacher 2006:273). Take note that even though there was random assignment of the intervention to classes, students were not randomly assigned to classes.

According to Struwig and Stead (2001:10), the pre-test post-test non-equivalent control group design is commonly used in quasi-experiments. The quasi-experimental design closely approximates most desirable experimental designs, and is also commonly used in educational research (McMillan 2008:230). According to McMillan (2008:231), this design is often used when subjects are available in existing, or "intact", groups such as classes. In this design, participants are randomly assigned to two groups, the experimental group and the control group (Struwig & Stead 2001:9). For the purpose of this study, both classes A and B wrote a pre-test at the beginning of the second semester of the 2009 academic year. The classroom learning strategy intervention was taught to class A and continued for a period of seven weeks. Both classes A and B wrote a post-test at the end of the second semester. The design can be portrayed by means of the following figure:

Class		Pre-test		Intervention		Post-test
A	→	O	→	*X*	→	O
B	→	O	→		→	O
				7 weeks		

Figure 6.1: Pre-test Post-test Control and Experimental Group Design
Source: McMillan and Schumacher 2006:274

6.6.5 Population and sampling

A third-year classroom at the Central University of Technology (CUT) of the Free State was selected for this research project. A sample of 97 third-year students was selected from the total population of students who were enrolled for the National Diploma: Cost and Management Accounting (BRNDTJ) and the National Diploma: Internal Auditing (BRNDAJ) at the CUT. The students who were enrolled for the National Diploma - Cost and Management Accounting - took *Business Statistics* (BSS22AB) as a compulsory subject in their third year of study. The students who were enrolled for the National Diploma - Internal Auditing - took *Statistics II* (STC22AB) as a compulsory subject in their third year of study. Although the student groups were enrolled for different diplomas, both the modules *Business Statistics* and

Statistics II were presented by the same lecturer, used the same prescribed textbook and studied the same content. In other words, if a student was enrolled for *Business Statistics*, he could attend the *Statistics II* classes or vice versa, because the content of both modules was exactly the same.

Both the aforementioned programmes carry 120 SAQA credits and are pitched on the NQF level 6. Both of these programmes have an admission requirement of a National Higher Certificate in Accounting. The prerequisite for both programmes is that the student may only follow the second-, third- or fourth-year level of study on condition that the respective first-, second- or third-year levels have been completed successfully. Also, students have to pass the first-year mathematics and statistics-related subject *Business Calculations* (BCL11AB) in order to take *Business Statistics/Statistics II*. The duration for both programmes is one semester. The students in both modules *Business Statistics* and *Statistics II* were all registered as part-time students on campus and attended one of two eighty-minute theory lectures once per week over a period of 12 weeks during the second semester of the 2009 academic year.

Two eighty-minute theory lectures were presented every week in the module *Business Statistics / Statistics II*. One theory lecture was presented during the day, and the other lecture part-time. As some students worked on a full-time basis and others not, students could therefore choose which lecture they wanted to attend. What was explained during the first theory class, was repeated again in the second lecture. The students who worked on a full-time basis as well as some students who preferred evening classes, attended the theory lecture on a Tuesday evening. Students who preferred day classes attended the theory lecture on a Thursday afternoon.

During the theory lectures, the lecturer explained the work to students. The researcher utilised the national prescribed syllabus for both of these programmes and kept strictly to the study guide. Both programmes shared the same syllabus, covered the same content from the same prescribed textbook. Both classes also received exactly the same academic instruction by the same lecturer and also wrote the same tests.

The students (n=23) who attended the day time lecture served as the experimental group and were taught the proposed classroom learning strategy intervention. The students (n=74) who attended the part-time (evening) lecture served as the control group and received traditional instruction. The students from this group were considered the "control" group because they did not receive the proposed intervention [McMillan (2008:230)].

A non-probability sampling method was used, as the researcher did not make use of a random selection of participants. Gravetter and Forzano (2003:118) argue that in non-probability sampling, the odds of selecting a particular individual are not known because the researcher does not know the population size or the members of the population. The researcher made use of convenience sampling, and more specifically wholeframe sampling, as the subjects were available and formed part of the lecturer's (also the researcher) classes, and as the whole class participated in the study. According to Struwig and Stead (2001:111), "[a] convenience sample is chosen purely on the basis of availability". According to McMillan (2008:117), it is quite common for the population to be the same as the sample, in which case there is no immediate need to generalise to a larger population. McMillan (2008:117) argues that in educational research, especially with experimental studies, a group of subjects is used that has not been selected from a larger population.

6.6.6 Data collection

Descriptive, quantitative biographical data (race, gender, age, previous mathematical background, and mark obtained in *Business Calculations*) were obtained from the CUT Student Records Database. The researcher kept an attendance register in which students' attendance was noted in all theory classes. The researcher entered this biographical data on a database for comparison and analysis purposes.

In order to investigate students' academic performance in the module *Business Statistics/Statistics II*, the first class test served as the pre-test, and the main test served as the post-test. The scores students obtained in both tests were used in the study to assess the pre- and post-test performances of students. The researcher compared all results to determine the effect the classroom learning strategy intervention had on the subjects. Students were assessed during August and September 2009.

At the beginning of the second semester, the lecturer gave formal instruction to both groups of students for a time period of 3 weeks, without any learning strategy intervention as yet. After these 3 weeks of formal instruction, the first class test was administered to the subjects in both groups during August 2009.

The first class test in August served as the pre-test and covered five chapters, of which three chapters were revision of the first-year subject *Business Calculations*. The test was compulsory for all students, as this test mark was used as part of their course mark in the module *Business Statistics / Statistics II*. After the tests were marked, the marks were all entered onto the ITS system of the CUT. A total of 97 test answer sheets were submitted.

The learning strategy intervention was implemented with the experimental group immediately after the pre-test and continued for a further period of three weeks. The main test served as the post-test and was administered to the subjects in both groups during September after the learning strategy intervention with the experimental group. It covered three chapters. After the tests were marked, the marks were all entered onto the ITS system of the CUT. A total of 97 test answer sheets were submitted.

The researcher continued for another four weeks with the learning strategy intervention, after which the students were assessed for the final time during the exam, which took place at the end of October 2009. All students were assessed on the same day, in the same venue, at the same time, in both tests as well as the exam. Please note that the exam results were not taken into account in this study.

With regard to students' learning and problem-solving strategies in the qualitative mode of the study, the researcher collected information by means of a reflection diary. The researcher observed students' behaviour during the theory classes and reflected on students' problem-solving strategies as well as the general implementation of the learning strategy intervention. The researcher also decided to conduct one nominal group setting with the experimental group of students, during the last theory class, at the end of the semester module *Business Statistics / Statistics II* in 2009. The setting was used to determine the developmental experiences students found most useful after the implementation of the proposed classroom learning strategy intervention. Eight students attended this session. The researcher recorded the students' responses and entered the numeric scores on a spreadsheet for analysis purposes. The results generated by the nominal group setting are discussed in Chapter 7.

The measuring instruments are discussed in the next section:

6.6.7 Measuring instruments

In order to determine the effect of the proposed classroom learning strategy intervention on students' academic performance in *Business Statistics / Statistics II*, the quantitative data was collected by means of self-developed instruments (tests) intended to yield highly reliable and valid scores. The researcher also made use of the nominal group technique (NGT) to gauge developmental experiences students' found most useful after the implementation of the proposed classroom study strategy intervention.

The researcher developed two tests based on the curriculum activities for the semester module *Business Statistics / Statistics II*. The components of the tests were as follows:

6.6.7.1 *The pre-test*

McMillan (2008:160) defines a test as "an instrument that requires subjects to complete a cognitive task by responding to a standard set of questions". As the study investigated the relationships with achievement, norm-referenced tests were used because these types of tests are most likely to provide data that will show a large variance (McMillan 2008:161). According to McMillan (2008:160), high variability is good when the researcher is looking for relationships between test results and other variables. The first semester test paper in the module *Business Statistics / Statistics II* served as the pre-test (see Appendix F). In August, during the second semester of

2009, the pre-test was administered to 97 students at the CUT. The aims of the test were to:

- Investigate how students perform in the subject before the proposed classroom learning strategy intervention.
- Analyse the results of the scores.

The pre-test comprised five chapters, of which three were revision of the first-year subject *Business Calculations*. The test instrument comprised ten questions that were obtained from the prescribed textbook that students had to work from. The test instrument was developed by the researcher in order to determine the extent of conceptions about core elementary statistics-related topics. The test focussed on students' conceptual knowledge and each of the ten questions represented a certain cognitive characteristic of important concepts in the module *Business Statistics/Statitstics II*. The pre-test comprised conceptual questions regarding simple interest, compound interest, annuities, elementary probability and the normal distribution. Students were assessed in terms of their ability to perform the following:

- Perform calculations involving simple interest
- Manipulate the simple interest formula
- Calculate compound interest
- Calculate the present and accumulated values of a principal of money
- Solve problems that involve transposing the compound interest formula
- Solve problems involving the future value of an annuity
- Calculate the present value of an annuity
- Calculate the probability of events
- Calculate conditional probabilities
- Use the general addition law for probabilities
- Apply Venn diagrams
- Apply probability tree diagrams
- Identify the properties of the normal distribution and normal curve
- Identify the characteristics of the standard normal curve
- Read z-score tables and find areas under the normal curve
- Find the z-score given the area under the normal curve

- Compute properties
- Apply the Central Limit Theorem
- Solve business problems that can be represented by a normal distribution
- Calculate estimates and their standard errors
- Calculate confidence intervals for the population mean

6.6.7.2 *The post-test*

The second semester test paper in the module *Business Statistics/Statistics II* served as the post-test (see Appendix G). The test comprised ten questions and was developed by the researcher. The items in the test were obtained from the prescribed textbook that students had to work from. The post-test covered three chapters and comprised conceptual questions regarding correlation, regression analysis, and time series and trend analysis. Students were assessed in terms of their ability to perform the following:

- Calculate the correlation and relationships between variables
- Draw and interpret a scatter diagram
- Calculate the product-moment correlation coefficient
- Calculate the rank correlation coefficient
- Test a correlation coefficient for significance
- Use linear regression
- Calculate the least-squares regression equation
- Calculate the goodness of fit of an equation
- Use the regression line for prediction
- Identify and use common methods of fitting secular trend lines to time series
- Make forecasts

6.6.7.3 *The nominal group technique*

The researcher made use of the nominal group technique (NGT) in order to gauge the developmental experiences that students in the experimental group found most useful after the implementation of the classroom learning strategy intervention. According to Strydom (in De Vos, Strydom, Fouche, and Delport 2005:419), the nominal group is typically a small-group data gathering technique used with up to ten members. The NGT is used "when the needs of problems ... have been prioritised by way of ... self-survey techniques" (Weyers 2001:116-120). According to Delbecq, VandeVen and Gustafson (in Jones 2004:23), the NGT is designed to facilitate collaborative and democratic decision making and is often used in educational settings to investigate a wide range of topics. As opposed to traditional voting methods in which only the largest group is considered, the NGT takes everyone's opinion into account.

The NGT was originally developed by Delbeck and VandeVen (1975) and is a possible alternative to brainstorming. According to Farrokh (1997), "the nominal group technique is a generic name for face-to-face group techniques in which instructions are given to group members" with regard to judgmental decision making in which creative solutions are sought (Dunham 1998). Waddell and Stephens [in Jones (2004:24)] argue that the key advantage of the NGT is the "incorporation of the ideas and desires of all the learners, regardless of their levels of assertiveness or extroversion". As the process in the NGT encourages the more passive group members to participate, it prevents the domination of discussion by a single person. At the end, the NGT results in a set of prioritised solutions or recommendations. What makes the NGT so appropriate to use is that it "produces a prioritized list of ideas in 2 hours or less" (Farrokh 1997).

When considering the NGT in its "pure" form, it consists of six steps, which are the following:

1. Each member of the group generates ideas silently.
2. Participants share ideas in a 'round robin' format.
3. The group discuss each idea that was generated, and duplicated items are eliminated from the list with all ideas that were generated.
4. Each member in the group proceeds to rank the ideas, 1^{st}, 2^{nd}, 3^{rd}, 4^{th}, and so on.
5. The numbers each idea receives are totalled, and the idea with the lowest total ranking (i.e. most favoured) is selected as the final decision.

6. Final voting is performed on the priority of items.

6.6.7.3.1 Advantages and disadvantages of using the NGT in educational research

Jones (2004:24) purports that the primary advantage of the NGT over other strategies is the enhanced opportunity for all participants to contribute ideas and to minimise the domination of the process by more confident or outspoken individuals. Other advantages of the NGT are that it avoids problems caused by group interaction. Some students are reluctant to suggest ideas because they are concerned about being critisised, while others are reluctant to create conflict in groups. As the NGT has the clear advantage of minimising differences and ensuring relatively equal participation, it overcomes both of these problems (Jones 2004:24). Another advantage of the NGT is that it saves a lot of time, produces a large number of ideas and also provides a sense of closure that is often not found in less-structured group methods. As the results are available immediately after the discussion session, it provides a greater sense of accomplishment for members as well as ease in interpreting results.

However, the NGT also has some disadvantages and limitations, which include the following: A major disadvantage of the NGT is that the method lacks flexibility by only being able to deal with one topic or one problem at a time (Jones 2004:24). As there is no opportunity for participants to think about the issue in depth or to generate additional ideas in their own time, idea generation during the meeting itself could be limited. As all participants must feel comfortable with the amount of structure involved in the NGT; there must be a certain amount of conformity on the part of the participants involved in the NGT. The possibility exists that students may not have expressed all of their perceived needs, due to the lack of anonymity because of some discomfort with the formal group process. Another disadvantage of the NGT is that opinions may not be covered in the voting process or ideas may be constrained, and the process may appear to be too mechanical. In terms of the number of respondents, the data may be limited in nature, as it often requires a follow-up survey or other quantitative methodology prior to making final decisions about an issue.

The researcher decided to make use of the NGT because of the following advantages:

- Some students in the experimental group were more vocal than others.
- Some students think better in silence.
- There was a concern about some students not participating in the discussion.
- Students do not easily generate enough ideas (Jones 2004:24).

6.6.7.3.2 The procedure

The researcher decided to conduct one nominal group setting with the experimental group of students at the end of the semester module *Business Statistics / Statistics II*. The setting was used to determine the developmental experiences students found most useful after the implementation of the proposed classroom study strategy intervention.

The researcher asked the experimental group of students for their participation in what was described as a small-group discussion during the last lecture of the semester module *Business Statistics / Statistics II*. The researcher explained the purpose and procedure of the discussion to all students. Only eight students attended this session.

The session was designed to follow the initial steps of the NGT process closely, in order to retain the integrity of the NGT process – particularly in relation to ensuring input from all the participants. The researcher divided the eight students present into two groups of four members each, and gave them the opportunity to introduce themselves to the rest of the group. The researcher explained the NGT to the students after which the question to be considered was probed: "How did you benefit from the "study method" in the semester module *Business Statistics / Statistics II*? Participants were encouraged to think as broadly as possible with regard to this question.

1. Silent individual generation of ideas – the researcher provided each student with a sheet of paper and asked them to write down all ideas that came to mind when considering the above-mentioned question. The students spent 15 minutes in silence, individually brainstorming all possible ideas and recording their ideas on paper.
2. Sharing of all participants' ideas – After 15 minutes of writing, the researcher invited the participants to volunteer one item from their list at a time, taking turns (in a round-robin fashion), until all ideas were listed on a flip chart. The

researcher made it very clear that no criticism was allowed, but rather encouraged clarification in response to some questions. At this stage there was no debate about items and the students were encouraged to write down any new ideas that might have arizen from what other students had shared. This process ensured that all participants had an opportunity to make an equal contribution and provided a written record of all the ideas generated by the group.

3. Group discussion of all generated ideas – Both groups discussed all the listed ideas on the flip chart in order to organise these ideas on the list as well as to remove duplications. All participants were given the opportunity to evaluate their own ideas individually and to vote for the best ones anonymously, after which each student's votes were shared with the rest of the group and tabulated. Both groups prepared a group report that displayed the ideas that received the most votes. The researcher then listed the ideas with the most votes on the chalkboard.
4. Preliminary vote to select the most important ideas – The researcher provided each participant with a sheet of paper and asked them to list the three topics on the chalkboard which they thought were the most important.
5. After the voting and ranking process, immediate results were available in response to the question probed.
6. The discussion concluded having reached a specific outcome.

6.6.8 Data analysis and reporting

The statistical analysis plan (see Appendix A) of this research was constructed by the Head of Department of the Mathematical Statistics Department at the University of the Free State (UFS). The data was analysed by the Statistical Analysis Division of the Information and Communication Technology Services Department at the UFS. For the purpose of processing the data obtained, the statistical analysis software package SAS was used. Initially, descriptive statistics were used to describe the experimental and control groups regarding the variables (Struwig & Stead 2001:158). In Chapter 7, the researcher presents frequency tabulations (number of students and percentage of student per category, both for the total group and for the experimental versus control group separately), for the following:

- Gender – Categorical variable
- Race – Categorical variable
- Previous mathematical background – categorical variable

The processing of the data included various detailed descriptive statistics (mean, standard deviation, median, maximum, minimum, number of observations) – both for the total group and for the experimental versus control group separately- will be presented for each quantitative variable, namely:

- *Business Statistics* results, both pre-test and post test – Continuous variable
- Age – Continuous variable
- BCL results – Continuous variable

The researcher made use of univariate analysis to test for any relationship between the variables. The dependent variable (post-test score in *Business statistics/Statistics II*) was analysed using one-way ANOVA fitting, one variable at a time, the dependent variable (group) and each of the confounding variables. According to Struwig and Stead (2001:162), "[t]he one-way ANOVA determines whether groups or treatments differ statistically significantly with regard to the group mean scores from one dependent variable".

The researcher further made use of regression analysis in order to determine whether students' post-test performance was in any way related to students' age, BCL results, and class attendance. To determine whether there are statistically significant correlations between the post-test score in the module *Business Statistics/Statistics II* and BCL results, as well as class attendance, correlations were calculated and the significance thereof ascertained. Pearson product-moment correlation coefficients (expressed as r-scores) were computed to see how the post-test scores relate to the BCL score as well as class attendance. Salkind (2008:75) argues that the Pearson correlation coefficient is used to examine the relationship between two variables, if both of those variables are continuous in nature. According to Struwig and Stead (2001:160), "[t]he Pearson product-moment correlation is used to determine the extent to which variation in one continuous variable explains the variation in another continuous variable".

Multivariate analysis was used in which the dependent variable (*Business Statistics/Statistics II* post-test results) was analysed using analysis of covariance (ANCOVA). According to Salkind (2008:276), "[a]nalysis of covariance is particularly interesting because it basically allows you to equalize initial differences between groups". In other words, ANCOVA allows the researcher to determine the effect of the learning strategy intervention with all potential confounding variables removed. The analysis of covariance model contained the independent variable (group) and covariates (gender, age, race, previous mathematical background, class attendance, BCL results, and *Business Statistics/Statistics II* results: pre-test). F-statistics and associated P-values have been calculated for each variable in the model.

6.7 RELIABILITY AND VALIDITY

For the purpose of this study, the researcher critically examined the research design in order to assess to what extent it is reliable and valid.

6.7.1 Reliability

The reliability of research depends on the reliability of the measuring instruments and the choice of the correct statistical procedure (Maas 1998:25-26; Kerlinger 1986:405). According to Struwig and Stead (2001:130), "[r]eliability is the extent to which test scores are accurate, consistent or stable", and "produces similar results under constant conditions on all occasions" (Bell 2005:117). The consistent way in which all the data were processed and analysed by means of statistical packages undoubtedly contributed to the reliability of the study.

6.7.2 Validity

According to Maas (1998:24) and Kerlinger (1986:300), internal validity is the extent to which differences in the dependent variable are accounted for by differences in the independent variable and not by any extraneous or third variables. In other words, "the quality of the researcher's evidence regarding the effect of the independent variable on the dependent variable" (Polit & Beck 2008:196). Did the proposed classroom learning strategy intervention really bring about improvements in students' performance in mathematics and statistics – or were other factors responsible for students' improvement in post-test scores? According to Polit and Beck (2008:287), "the researcher's job is to develop strategies to rule out the plausibility that something other than the presumed cause can account for the observed relationship". The extraneous variables were controlled in the research design through statistical measurement by building them into the design.

The researcher also established internal validity in this research by making sure that all the students were assessed on the same date, at the same time and in the same venue as these conditions may have had an influence on the internal validity of this research (McMillan & Schumacher 2006:141). According to McMillan and Schumacher (2006:140), another threat to the internal validity of experimental research is the Hawthorne effect, where some subjects may have increased positive or desirable behaviour simply because they know they are receiving special treatment. According to Polit and Beck (2008:755), the Hawthorn effect is "[t]he effect on the dependent variable resulting from subject's awareness that they are participants under study". Therefore, the researcher deemed it best for subjects to be unaware that they were being studied in order to maximise both internal and external validity.

Struwig and Stead (2001:136) define external validity as "the extent to which you can generalise the results of a study to other populations". As the research was conducted at one specific institution, the CUT, and because randomisation was not used, the results of this research may not be generalisable to all third-year students at other universities.

6.8 ETHICAL CONSIDERATIONS

McMillan and Schumacher (2001:198) suggest that, for research to be conducted through an institution, in this case the CUT, approval for conducting the research should be obtained from the institution before any data are collected. Permission for this research study was obtained from the Dean of the Faculty of Management at the CUT in 2008.

According to McMillan & Schumacher (2006:143), informed consent implies that the subjects have a choice about whether to participate in research. According to Strydom (in De Vos *et al.* 2005:59), however, subjects are sometimes not informed that they are part of a research project, for valid reasons. The researcher was of the opinion that knowledge of participation, in this specific research, may invalidate the results and regarded it as acceptable that the subjects did not know that they had been participants. Providing such information would cause subjects to act unnaturally, which in turn, would influence the results (Bryman 2000:112-113). McMillan and Schumacher (2006:143) purport that "some educational research is quite unobtrusive and has no risk for the subjects". As the Hawthorne effect could have reduced the validity of this study, it seemed best for subjects to be unaware that they were being studied in order to maximise both internal and external validity.

6.9 OVERVIEW

In this study the research problem centres on addressing students' underachievement in mathematics and statistics by means of a classroom learning strategy intervention in a third-year university classroom. As such, Chapter 6 describes the empirical investigation undertaken as a means of providing the necessary perspectives and understanding needed in the implementation of a learning strategy intervention.

The chapter commences with the research problem, namely to determine whether the use of a classroom learning strategy intervention had any impact on third-year students' academic performance in the module *Business Statistics / Statistics II.*

The empirical study which is mainly located within a quantitative paradigm, was enhanced by means of qualitative observations of students' problem-solving strategies, as well as the nominal group technique setting. Whereas the quantitative data included numerical scores obtained from test instruments, the qualitative data comprised additional information that the researcher gathered by means of a reflection diary as well as the results emanating from the NGT setting. Given the primary research question, this empirical research study is regarded as evaluative in nature, as the aim of evaluative research is to improve a current practice. According to McMillan & Schumacher (2001:20), evaluative research focuses on evaluating the merit or worth of a particular practice at a given site. McMillan (2008:16) argues that evaluation research is directed toward making decisions about the effectiveness or desirability of a program with the aim of making judgments about alternatives in decision-making situations. In this research the evaluation research was focused on a specific classroom learning strategy intervention and involved judgments about the question: Did the classroom learning strategy positively affect students' academic performance in the module *Business Statistics/Statistics II*?

In the remainder of the chapter the research project is thoroughly discussed in terms of the following: Objectives of the instruments, justification for the modes of inquiry employed, population and sampling, structure of the test instruments and the nominal group technique, data analysis and issues related to validity and reliability, are addressed.

The findings from the post-test statistical analysis, as well as qualitative observations from students' problem-solving strategies and also the results obtained from the R-SPQ-2F questionnaire enabled the researcher to implement the proposed classroom learning strategy intervention, which could hopefully improve students' academic performance in the module *Business Statistics / Statistics II*.

The researcher also conducted one NGT setting with the experimental group of students at the end of the semester module *Business Statistics / Statistics II*. This setting provided the researcher with insight with regard to the developmental experiences students found most useful regarding the implementation of the proposed classroom study strategy intervention.

In the next chapter the research findings of the empirical investigation are presented and interpreted.

CHAPTER 7: RESULTS AND INTERPRETATION OF RESULTS

7.1 INTRODUCTION

The main purpose of this study was to determine the impact of a classroom learning strategy intervention on third-year students' academic performance in a mathematics and statistics-related subject at the CUT. The researcher therefore deemed it necessary to conduct a pilot study to measure the effect of the classroom learning strategy intervention (see Chapter 5).

Chapter 6 described the research design and methodology for the main empirical investigation performed in this study. The chapter commenced with the research problem (see 6.3), whereafter the research questions (see 6.1), research aims and hypotheses were spelt out (see 6.3). The research design and methodology were also outlined (see 6.6).

In Chapter 7, the results of the pre- and post-test regarding students' academic performance in the module *Business Statistics/Statistics II* are presented. The pre-test and post-test results enabled the researcher to compare results of students in the experimental and control group and to make valid conclusions about the effectiveness regarding the classroom learning strategy intervention.

7.2 ANALYSIS OF DATA AND INTERPRETATION OF RESULTS

The primary objective of the statistical analysis was to investigate the impact of a classroom learning strategy intervention on third-year students' academic performance in the module *Business Statistics/Statistics II*, while adjusting for the following potential confounding variables: gender, age, race, previous mathematical background, BCL results, and pre-test results for *Business Statistics/Statistics II*.

Frequency tabulations (number of students and percentage of students per category), both for the total group and for the experimental group versus control group separately are presented:

- Gender – categorical
- Race – categorical
- Previous maths background – categorical

Descriptive statistics (mean, standard deviation, median, min, max, number of observations, both for the total group versus the experimental group separately) are presented for each quantitative variable, namely:

- *Business Statistics/Statistics II* results, both pre-test and post-test
- BCL results
- Age

To test for any relationships and differences in each confounding variable with regard to students' demographic profile (gender, age, race, previous mathematical background, BCL results, and pre-test results in the module *Business Statistics/Statistics II*) between the two groups of students, the researcher made use of complementary statistical techniques which included the ANOVA and ANCOVA.

The researcher made use of univariate analysis in which the dependent variable (post-test results in the module *Business Statistics/Statistics II*) was analysed using one-way ANOVA fitting, one variable at a time, the dependent variable (group) and each of the confounding variables.

The researcher also made use of multivariate analysis in which the dependent variable (post-test results in the module *Business Statistics/Statistics II*) was analysed using analysis of covariance (ANCOVA). The analysis of covariance model contained the independent variable (group) and all potential confounders (gender, age, race, previous mathematical background, BCL results, and pre-test results in the module *Business Statistics/Statistics II*). F-statistics and associated P-values were calculated for each variable in the model. A significance level of 5% was incorporated throughout the whole study.

In the next section, the researcher reports on the findings of the research study, which are illustrated by means of tables. The student characteristics and demographic information are presented in Table 7.1 to Table 7.8, while some descriptive statistics are contained in Table 7.9 to Table 7.13 regarding the pre- and post-test results in the module *Business Statistics/Statistics II.* The results of the nominal group technique setting are summarised in Table 7.14 and Table 7.15. The researcher also provides a thorough discussion of each of these topics. The chapter concludes with the most important findings and results of the empirical investigation.

7.2.1 Descriptive statistics of student characteristics and demographic information

Table 7.1 to Table 7.3 show the population characteristics of the total group of students with the following variables: gender, race and previous mathematical background.

Table 7.1: Frequency distribution of "Gender" of the population of *Business Statistics/Statistics II* students

Gender	**Population of Business Calculations Students**	
	Frequency	**Percentage (%)**
Female	56	57.73
Male	41	42.27
Total	**97**	**100**

From Table 7.1 it can be seen that there were more female students (57.73%) than male students (42.27) in the population of students that took *Business Statistics/Statistics II* as a subject.

Table 7.2: Frequency distribution of "Race" of the population of *Business Statistics/Statistics II* students

Race	**Population of Business Statistics Students**	
	Frequency	**Percentage (%)**
Black	93	95.88
Coloured	2	2.06
White	2	2.06
Total	**97**	**100**

Based on Table 7.2 with regard to students' race, the following profile can be drafted: The majority of students that took *Business Statistics/Statistics II* were black (95.88%). The conclusion can thus be draw that the group of students was almost homogeneous with regard to race.

Table 7.3: Frequency distribution of "Mathematical Background" of the population of *Business Statistics/Statistics II* students

Maths Background	**Population of Business Calculations Students**	
	Frequency	**Percentage (%)**
Commercial mathsematics (SG)	9	9.28
Mathsematics (HG)	5	5.15
Mathsematics (LG)	2	2.06
Mathsematics (SG)	47	48.45
None	34	35.05
Total	**97**	**100**

Table 7.3 shows that almost 65% of the population of students that took *Business Statistics/Statistics II* had a mathematical background up to Grade 12 level. Thirty-five

percent of the students did not take mathematics up to Grade 12 level during their school careers.

The following table contains the continuous variables (age, BCL results, and class attendance) with regard to students' demographic profile in the module Business *Statististics/Statistics II.*

Table 7.4: Descriptive Statistics of "Age", "BCL results", and "Attendance" of the population of *Business Statistics/Statics II* students

Variable	N	Mean	Std Dev	Min	Max	Median
Age	97	22	3.15	18	38	21
BCL	97	52.08	15.60	15	87	54
Att* pre	97	84.61	24.53	0	100	100
Att* post	97	66.77	29.71	0	100	67

Att* = Attendance

Table 7.4 shows that the average student that took *Business Statistics/Statistics* was 22 years old. The youngest student in the group was 18, while the oldest student was 38 years old. The total population had an average of 52.08% in the first-year semester module *Business Calculations* (BCL11AB). The results in Table 7.4 shows an average of 84.61% class attendance with regard to the pre-test in the module *Business Statistics/Statistics II*, while the class had a 66.77% class attendance with regard to the post-test. From these figures, it can be seen that there was a decrease in class attendance with regard to the post-test in the module *Business Statistics/Statistics II.*

The following section (Table 7.5 to Table 7.7) describes the difference between the experimental and control group of students with regard to the demographical characteristics of each student in the module *Business Statistics/Statistics II.*

Table 7.5: Frequency distribution of "Gender" of the population of *Business Statistics/Statistics II* students

Gender	**Population of Business Calculations Students**			
	Experimental Group (n=23)		Control Group (n=74)	
	n	%	n	%
Male	10	43.48	31	41.89
Female	13	56.52	43	58.11
Total	**23**	**100**	**74**	**100**

Based on the preceding section in respect of students' characteristics and demographic information, the following profile can be drafted: Table 7.5 regarding gender shows that there were more female students (56.52%) than male students (43.48%) in the experimental group as well as in the control group of students (58.11% vs 41.89%).

Table 7.6: Frequency distribution of "Race" of the population of *Business Statistics/Statistics II* students

Race	**Population of Business Calculations Students**			
	Experimental Group (n=23)		Control Group (n=74)	
	n	%	n	%
Black	23	100	70	94.59
Coloured	0	0	2	2.70
White	0	0	2	2.70
Total	**23**	**100**	**74**	**100**

With regard to race (see Table 7.6), both groups of students comprised a majority of black students, with a percentage rate of 100% for the experimental group and 94.59% for the control group. The control group comprised 2.70% coloured as well as 2.70% white students, but the experimental group had none from these two races.

Table 7.7: Frequency distribution of "Mathematical background" of the population of *Business Statistics/Statistics II* students

Mathsematical background	**Population of Business Calculations Students**			
	Experimental Group (n=23)		Control Group (n=74)	
	n	%	n	%
Comercial Maths	1	4.35	8	10.81
Maths (HG)	0	0	5	6.76
Maths (LG)	0	0	2	2.70
Maths (SG)	11	47.83	36	48.65
None	11	47.83	23	31.08
Total	**23**	**100**	**74**	**100**

Table 7.7 shows that 52.17% of students from the experimental group and 68.92% students from the control group had a mathematical background up to Grade 12 level. The experimental group comprised 47.83% students with no mathematical background while the control group had 31.08% students. From these figures it can be seen that there were more students (68.92%) in the control group with Grade 12 level mathematical background than there were in the experimental group (52.17%).

From the tables above, it can thus be seen that the students in the experimental group (who received the study strategy intervention) had very similar profiles to the students in the control group. A further description of the population of *Business Statistics/Statistics II* students involved a comparison of the continuous variables (age, BCL results, and class attendance) between the experimental group and the control group with regard to the students' demographic profiles.

Table 7.8 contains the continuous variables (age, BCL results, and class attendance) with regard to the experimental and control groups of students' demographic profiles in the module Business *Statististics/Statistics II.*

Table 7.8: Descriptive statistics of "Age", "BCL results", and "Attendance" of the population of *Business Statistics/Statistics II* students

Group	Variable	N	Mean	Std Dev	Min	Max	Median
Exp	**Age**	23	22.26	3.84	19	38	21
	BCL	23	48.57	16.75	0	73	53
	Att 1*	23	78.35	23.82	33	100	67
	Att 2*	23	74.13	17.22	33	100	67
Control	**Age**	74	21.92	2.92	18	33	21
	BCL	74	53.18	15.18	0	87	54.50
	Att 1*	74	86.55	24.58	0	100	100
	Att 2*	74	64.49	32.39	0	100	67

Att 1* = Students' class attendance with regard to the pre-test

Att 2 *= Students' class attendance with regard to the post-test

From Table 7.8 it can be seen that both groups of students had more or less the same mean age, namely 22.26 for the experimental group and 21.92 for the control group of students. The minimum age was 19 for the experimental group and 18 for the control group. In the experimental group, the maximum age was 38, while the oldest student in the control group was 33 years of age. Both groups of students had a median age of 21.

Students in the experimental group showed an average mark of 48.57% in the first-year semester module *Business Calculations* (BCL11AB). The control group, on the other hand, showed an average of 53.18%. The control group thus performed 4.61% better than the experimental group of students with regard to the first-year semester subject BCL. The results in Table 6.8 show that 78.35% of students in the experimental group attended all classes before the pre-test, while 86.55% of the control attended all classes. The control group thus reflected an average of 8.2% higher class attendance than the experimental group of students. Both groups of students showed a decrease in class attendance after the pre-test. Seventy-four percent of students in the experimental group attended all classes, while 64.49% of students in the control group attended all classes before the post-test in the module *Business Statistics/Statistics II.* However, the decrease in class attendance percentage for the control group (22.06%) was much higher than it was for the experimental group of students (4.22%).

In the next section, the researcher reports on the analysis of students' pre-test and post-test performance in the module *Business Statistics/Statistics II.*

7.2.2 Analysis of the pre- and post-test performance

The following section entails a summary of the results of the pre-test and post-tests for both groups of students. Within these tables, the key results obtained by the test conducted by the researcher in 2009 are documented. The descriptive statistics in Table 7.9 provide an overview of the overall performance of both groups of students in the module *Business Statistics/Statistics II.* Descriptive statistics (mean, standard deviation, median, minimum, maximum, and number of observations), both for the total group of students and for the experimental versus control group separately, is presented in Table 7.10.

Table 7.9: Pre- and post-tests descriptive statistics of the total group.

Variable	N	Mean	Std Dev	Min	Max	Median
Pre-test	97	58.13	14.50	23	91	57
Post-test	97	71.60	18.51	20	100	74

The results in Table 7.9 show a average pre-test performance of 58.13% for the total group of students that took *Business Statistics/Statistics II.* The total group of students eventually showed an average increase of 13.47% with regard to the post-test. In the next table, the researcher compares the results of the pre-test and post-test between the experimental and control groups of students in the module *Business Statistics/Statistics II.*

Table 7.10: Pre- and post-tests descriptive statistics of the Experimental and Control group.

Group	Variable	N	Mean	Std Dev	Min	Max	Median
Exp	**Pre-test**	23	58.70	16.18	31	87	57
	Post-test	23	71.30	15.30	43	97	74
Control	**Pre-test**	74	57.96	14.05	23	91	55.50
	Post-test	74	71.69	19.50	20	100	74

The pre-test results in Table 7.10 show that both groups of students performed at an average level, with a mean score of 58.70% for the experimental group and a mean score of 57.96% for the control group. The experimental group of students thus performed 0.74% better in the pre-test than the control group of students. The post-test showed a marked increase of about 13% in scores for both groups of students, with a mean score of 71.30% for the experimental group and a mean score of 71.69% for the control group.

7.2.3 Analysis of Variance (ANOVA)

The researcher also examined the relationship between achievement in *Business Statistics/Statistics II* and the demographic characteristics of the students. As this analysis is related to the dependent variable (post-test performance), achievement was firstly described in terms of descriptive statistics such as means and standard deviations (see Table 7.9 and Table 7.10). To ascertain whether there were any statistically significant differences between the categories of each of the demographic variables in terms of the dependent variable (post-test scores), ANOVA's were conducted.

Analysis of Variance (ANOVA) was performed in order to determine whether there was any difference in students' post-test performance with regard to each of the following variables: group, gender, mathematical background and race. The post-test in the module *Business Statistics/Statistics II* served as the dependent variable, while group, gender, mathematical background, and race served as the independent variables. The results of the ANOVA are given in the following table:

Table 7.11 F-statistics and associated P-values of the one-way ANOVA

Source	DF	F value	*p* value
Group	1	0.01	0.9312
Gender	1	1.30	0.2572
Maths background	1	0.37	0.8325
Race	1	1.61	0.2054

From the table above it can be seen that the results were not significant. In other words, students' performance in the post-test was independent of each of the following variables, namely: group, gender, mathematical background, race and age. The ANOVA test thus confirmed that there was no significant difference between each of the variables with regard to students' post-test performance in the module *Business Statistics/Statistics II.*

7.2.4 Regression analysis

The researcher made use of regression analysis in order to determine if students' post-test performance was in any way related to students' age, BCL results, and class attendance. To ascertain whether there are statistically significant correlations between the post-test score in the module *Business Statistics/Statistics II* and age, BCL results, as well as class attendance, correlations were calculated and the significance thereof ascertained. Pearson product-moment correlation coefficients (expressed as *r*-scores) were computed to see how the post-test scores relate to students' age, BCL results, as well as class attendance. The results are given below:

Table 7.12 F-statistics and associated P-values of the one-way ANOVA

Source	DF	r^2	*p* value
Age	1	0.033347	0.0734
BCL results	1	0.008522	0.3685
Attendance Test 1	1	0.001556	0.7013
Attendance Test 2	1	0.001833	0.6771

From Table 7.12 it can be seen that the results of the statistical analysis software confirmed that no significant correlation existed between students' post-test performance in the module *Business Statistics/Statistics II* and students' age, BCL results as well as class attendance in the module *Business Statistics/Statistics II.*

7.2.5 Multivariate analysis

The dependent variable (post-test results in the module *Business Statistics/Statistics II*) was analysed using analysis of covariance (ANCOVA). The analysis of covariance model contained the independent variable (group membership) and all potential confounding variables (gender, age, race, previous maths background, BCL results, and pre-test results in the module *Business Statistics/Statistics II*). The F-statistics and associated P-values are presented for each variable in the model in the following table.

Table 7.13 F-statistics and associated P-values of the one-way ANCOVA

Source	DF	F value	*p* value
Group	1	0.05	0.8252
Age	1	2.78	0.0994
Race	2	2.06	0.1336
Gender	1	0.40	0.5287
Maths background	4	0.41	0.8018
Test 1	1	0.57	0.4520
Attendance Test 1	1	0.06	0.8059
Attendance Test 2	1	0.19	0.6672
BCL results	1	0.76	0.3861

The results in Table 7.13 confirm that there are no significant differences between students' post-test performance and each of the covariates. In order to determine the effect of each of the independent variables, each of the covariates was removed from the model. The conclusion remained the same, i.e. there was no relationship between students' post-test performance and each of the independent variables.

7.2.6 Results of the nominal group technique

The researcher decided to conduct one nominal group technique setting with the experimental group of students at the end of the semester module *Business Statistics / Statistics II*. The setting was used to determine the developmental experiences students found most useful after the implementation of the proposed classroom study strategy intervention.

The researcher asked the experimental group of students for their participation in what was described as a small-group discussion during the last lecture of the semester module *Business Statistics / Statistics II*. The researcher explained the purpose and procedure of the discussion to all students during the session. As the students did not know that they had been subjects and in an experiment, the researcher never mentioned the word "intervention". Students were only asked to discuss their developmental experiences, if any, that they found most useful after the researcher had exposed them to a specific learning strategy and provided them with good study habits. The researcher told students that she was curious to know if they had benefited from the learning strategy or not, and that it would be interesting to know how students felt about it. Eight students attended this session.

After removal of duplications and combination of overlaps, a total of 5 topics were nominated by the group. The topics and number of votes they received are listed in the table below:

Table 7.14 Topics and votes from the setting

The study strategy reduced my time of study	19
The study strategy changed my attitude towards statistics	27
I enjoy statistics classes more now	20
The learning strategy made more sense than the textbook alone	24
The study strategy made me want to learn more and do better	30

The table below displays the topics and votes in *ranked* order according to the topic that received the most votes in the NGT setting.

Table 7.15 Topics and votes from the setting in ranked order

The study strategy made me want to learn more and do better	30
The study strategy changed my attitude towards statistics	27
The learning strategy made more sense than the textbook alone	24
I enjoy statistics classes more now	20
The study strategy reduced my time of study	19

From Table 7.15 it can be seen that the study strategy had a very encouraging effect on students' approach to learning, as it made students want to learn more and do better in the module *Business Statistics/Statistics II.* The learning strategy also changed students' attitudes toward statistics, and students said that it made more sense than the textbook alone. Topics that received the fewest votes from the group were that they enjoy statistics classes more now, after the learning strategy intervention, and that the study strategy has reduced their time of study.

7.3 SUMMARY OF FINDINGS

1. The first research question of the study was:

Does the implementation of a classroom learning strategy intervention positively affect students' academic performance in the module *Business Statistics/Statistics II*?

The pre-test results in the module *Business Statistics/Statistics II* showed average results for both groups of students (see Table 7.10). Both groups of students showed marked increases in the post-test of the module *Business Statistics/Statistics II.* Because both groups of students showed marked increases in the post-test, the researcher cannot claim that the improvement in scores was due to the implementation of the classroom learning strategy intervention. The research hypothesis (see 6.4) is therefore rejected and the conclusion drawn that the post-test score in *Business Statistics/Statistics II* of the experimental group was not significantly higher that the post-test score of the control group. Although students in the experimental group were exposed to the learning strategy, they were unaware of the fact that they were subjects in the research study and formed part of the experiment. For this reason, students in the experimental group may have shared information about the learning strategy with students in the control group. Another factor that might have compromised the results in the study is the fact that the experimental group of students was very small (n=23).

2. The second research question of the study was:

Which developmental experiences did students' find most useful after the implementation of the proposed classroom study strategy intervention?

From the responses that was elicited from the NGT setting and reflection diary, the conclusion can be drawn that the learning strategy made students want to learn more and do better in the module *Business Statistics/Statistics II.* In other words, it seems that the learning strategy had a positive effect on about 32% of the experimental group of students' attitude towards the module *Business Statistics/Statistics II.*

In the next chapter, the researcher provides a conclusion, makes recommendations and indicates limitations of the research study.

CHAPTER 8: CONCLUSION, RECOMMENDATION AND LIMITATION OF THE STUDY

8.1 INTRODUCTION

This study reported on the introduction of a classroom learning strategy intervention that was designed to improve university students' academic performance in a mathematics and statistics-related subject. The study also reviewed factors that influence deep learning and discussed ways in which university teachers can encourage students to adopt and use deep learning strategies.

This chapter commences with an overview of the study, referring to the specific research questions and aim, as presented in Chapter 1. It also explains how the researcher went about addressing each of the subsidiary questions. The significance of the study is outlined, the limitations are presented and the need for further research is also explained

8.2 AN OVERVIEW OF THE STUDY

The problem statement (see paragraph 1.3) centres on the challenge of addressing underpreparedness in mathematics among students and gave rise to the formulation of the main research question, namely: *Does the implementation of a classroom learning strategy intervention positively affect students' mathematical and statistical performance in the module Business Statistics / Statistics II?.*

In order to answer this question and consequently determine the effectiveness of the learning strategy intervention, the main research question was informed by five subsidiary questions (see paragraph 1.4). Based on a comprehensive literature review, the first three of these research questions have been addressed in Chapters 2 and 3, namely:

1. How does learning take place with regard to some important learning theories? (Chapter 2)
2. How is knowledge constructed in the process of learning mathematics? (Chapter 3)
3. How can students' cognitive processes be enhanced through a constructivist perspective in order to improve their mathematical as well as statistical thinking? (Chapter 3)
4. How was the proposed learning strategy, that students would be exposed to, constructed? (Chapter 4)
5. What is the effectiveness of the proposed learning strategy? (Chapters 5, 6 and 7).

In the following paragraphs, the researcher explains how she addressed each of these subsidiary questions with reference to the literature provided in this empirical investigation.

8.2.1 How does learning takes place with regard to some important learning theories? (Chapter 2)

In order to address the first subsidiary question, the researcher conducted a thorough discussion with regard to learning theories based on literature (see Chapter 2). By looking at different learning theories, the researcher aimed at gaining a better understanding of how students learn. Before students can be taught any learning strategy, or learn how to learn, one needs to understand "how" learning takes place.

Chapter 2 provides a discussion of students' approaches to learning (see 2.4), since the purpose of this study is for students to eventually follow a deep approach to learning (see 2.4.2). The discussion on learning theories (see 2.5) enabled the researcher to decide what skills and knowledge university students must acquire and how they should be taught. As thinking processes play an important role in learning, the researcher elaborated on cognitive learning theories as they emphasise the development of students' thinking processes (see 2.7). The Gestalt learning theory supports the concept of deep learning (see 2.4.2) as it maintains that humans perceive sensations or concepts as whole units rather than isolated pieces, which relates to a surface approach to learning (see 2.4.1). Information processing theory, as a component of cognitive learning theory, is important as it explains how the memory encodes and retrieves information in the learning process (see 2.7.2).

The researcher extended her discussion on cognitive learning theories by discussing Bruner's theory of cognitive growth (see 2.7.3) and Ausubel's meaningful reception learning (see 2.7.4). Bruner's theory is important as it informs educators how learning material could be structured, as well as about the sequencing of material in accordance with cognitivist ideas of mental processing (see 2.7.3.5). The use of a spiral curriculum, through which learners build on previous constructions of knowledge to formulate more useful associations and meaning is another important aspect of Bruner's learning theory (see 2.7.3.5).

As meaningful learning plays a vital role in promoting deep learning, the researcher gave a presentation with regard to Ausubel's theory. Scrutiny of this theory, made it evident that university students should be encouraged to organise content in logical ways, such as concept maps (see 2.7.4.2 and 4.2). The researcher elaborated on social cognitive theory, as human learning occurs in a social environment (see 2.8) and discussed the social cognitive theory of Bandura (see 2.8.1). A distinctive feature of Bandura's social cognitive theory is the central role it assigns to self-regulatory functions as well as the modelling processes, which play a prominent role in learning (see 2.8.1).

Finally, constructivism as a learning theory was explained (see 2.9). The theories of Jean Piaget and Lev Vygotsky focus on the development of thinking skills. Vygotsky's theory combines cognitive understanding with a social perspective and highlights mediation which can be applied through the concept of scaffolding and peer collaboration (see 2.11.1.1). With regard to Vygotsky, the researcher emphasised the fact that it is possible to develop higher cognitive processes in a student through mediation (see 2.11.1.1), which has its origin in social interaction. A thorough overview of the zone of proximal development as well as social interaction in the zone of proximal development was presented (see 2.11.1.2). The condition that is necessary for effective social interaction between learning partners, namely scaffolding (see 2.11.1.4), was also discussed.

8.2.2 How is knowledge constructed in the process of learning mathematics and statistics? (Chapter 3)

How can students' cognitive processes be enhanced through a constructivist perspective in order to improve their mathematical as well as statistical thinking? (Chapter 3)

Chapter 3 is concerned with the second and third subsidiary research questions. Chapter 3 commences with the importance, nature and uniqueness of mathematics as a learning discipline. With regard to learning theories in mathematics, it becomes clear that constructivism presents a viable model for explaining how mathematics is learned (see 3.4.2). Piaget's constructivism emphasises the role of active learning, assimilation, and that the learning of mathematics should be holistic, authentic and real (see 3.4.3). A discussion on Vygotsky's social constructivism provides a clear understanding of why mathematical competence also depends on sociocultural influence (see 3.4.4). This social interaction among university students constitutes a crucial source of opportunities to learn mathematics.

The importance of symbols with regard to mathematical and statistical notation is explained, as well as how symbols make an important contribution to the process of problem solving (see 3.6). As problem solving represents a key area for exploring the operation of cognitive processes in mathematics, problem-solving processes as well as various problem-solving strategies become relevant during the discussion of the learning of mathematics and statistics (see 3.8.2). The researcher extends problem solving by presenting a discussion on problem-solving skills, heuristics, control and beliefs in mathematics, as well as the thinking skills needed in mathematics. This discussion is extremely important because of its relevance to the classroom learning strategy the researcher proposes in Chapter 4.

8.2.3 How was the proposed learning strategy, that students would be exposed to, constructed? (Chapter 4)

In Chapter 4, the classroom learning strategy unfolds. The researcher presents the learning strategy by bringing together the perspectives gained in the literature review (see Chapter 2 and Chapter 3) with a framework that holds the potential of addressing the problem statement (see 1.3). By comparing some popular heuristics in mathematical problem solving by means of a table (see 3.8.2.6), the researcher's own "RIEQTSR" heuristic emerged (see 4.3). The key features of the proposed classroom learning strategy are outlined, thereby addressing the fourth subsidiary research question as presented above. The key features of the learning strategy are based on important perspectives gained from the literature with regard to the learning of mathematics and statistics.

8.2.4 Reflecting on the effectiveness of the proposed learning strategy? (Chapters 5, 6 and 7)

The pilot study (see Chapter 5), as well as the empirical investigation (see Chapter 6 and Chapter 7), are concerned with the final subsidiary question. These chapters describe the design and methodology employed in the empirical investigation undertaken as a means of determining the effectiveness of the proposed classroom learning strategy intervention. It also explains the rationale for employing a quasi-experimental pre-test post-test design in the research study. The results of the pilot study are presented and discussed (see 5.5.4). The researcher makes use of tables as well as more advanced statistics to illustrate the findings. These include regression analysis, ANOVA and ANCOVA (see 5.5.4). The results included the pre- and post-test results in the module *Business Calculations* as well as the results that emanated from the approach to learning questionnaire (see 5.5.4.5).

A thorough discussion of how students who formed part of the control group and experimental group solve problems was also presented (see 5.5.5). The importance of the researcher's proposed classroom learning strategy becomes relevant during this discussion. The revised two-factor study process questionnaire in the pilot study should have provided insight into students' approach to learning in a mathematics and statistics-related subject but failed to do so, due to human error. Qualitative observations, however, provided insightful results. Because of the failed attempt to gain results by means of the approach to learning questionnaire, a nominal group technique setting was carried out in the main research investigation (see 6.6.7.3). The NGT setting was conducted so as to come to a better understanding of how the classroom learning strategy intervention had influenced students, and to allow the researcher to gain insight into which developmental experiences students found most useful (see 6.6.7.3).

In Chapter 7 the results of the empirical investigation are presented and discussed (see 7.2). The researcher makes use of tables as well as more advanced statistics to illustrate the findings. These include regression analysis, ANOVA and ANCOVA. The results included the pre-test and post-test results in the module *Business Statistics/Statistics II* as well as the results that emerged from the nominal group setting (see 7.2.6). The quantitative analysis of students' post-test performance, as indicated by the ANOVA, showed that students in the experimental group did not perform significantly better than students in the control group (see 7.5). However, the researcher is of the opinion that the results derived from the NGT setting, as well as students' remarks which the researcher wrote down in her reflection diary, support the effectiveness of the researcher's proposed classroom learning strategy intervention. During this discussion, students were asked which developmental experience they found most useful after the implementation of the proposed classroom learning strategy (see 7.8). The topic that received the most votes during this discussion, was that the researcher's proposed classroom learning strategy "made students want to learn more and do better" (see Table 7.15). Apart from this, it seems that the classroom learning strategy also had a positive effect on students' attitudes regarding a mathematics and statistics-related subject, as it "changed students attitudes toward statistics", and students "enjoy statistics classes more". The proposed classroom learning strategy also "made more sense than the textbook alone".

Although the quantitative results of the pilot study, as well as the main study, are not significant, the researcher is continuing with her classroom learning strategy in the mathematics and statistics-related courses she is presenting at the CUT. Students that took *Business Statistics/Statistics II* in the second semester of the 2010 academic year also made use of the researcher's learning strategy, and the effectiveness of the strategy proved to be the same in respect of the attitudes of this new group of students' attitude towards the subject. A small group of students that took *Business Statistics/Statistics II* in the academic year 2010, thanked the researcher at the end of the semester module for "the way she presented the mathematics and statistics-related subject". Remarks were made such as "Can't you offer all our subjects ma'm, because you make learning seem so easy"; and "we hated maths in school, but now that we know how to study for the subject, we enjoy it"; and "we wish someone told us about this learning strategy during our school careers".

8.3 LIMITATIONS

The interpretation of the results obtained from this study took place within the context of the research design. There are three limitations to the study, all of which are caused by the fact that the design was not a true experimental, per-test, post-test design. It being a quasi-experimental design the researcher had no control over the time spent on the intervention, the size of the groups, as well as the interaction between experimental and control group participants.

- The first limitation of this study is caused by the setting within which the research was done. The quasi-experimental pre-test post-test design took place within a field experiment setting. According to Kerlinger (1986:369), "[a] field experiment is a research study in a realistic situation in which one or more independent variables are manipulated by the experimenter under carefully controlled conditions as the situation will permit". A field experiment setting implies that the researcher does not have control over such factors as the time frame within which an intervention may run because tests are set by the institution at predetermined dates. The time that elapsed between the pre-test and post-test, for both the pilot study (six weeks) and main research study (seven weeks), was very short (see 5.5.2 and 6.6.6). The intervention could only take place in these short time periods between the pre- and post-tests. As this field experiment operated with less control, it places a limitation on this study and can be seen as a factor "that is often a severe handicap" (Kerlinger 1986:369). Despite this weakness of field experiments, they have their unique virtues as well. According to Kerlinger (1986:370), "[t]he variables in a field experiment usually have a stronger effect than those of laboratory experiments" and "[t]he more realistic the research situation, the stronger the variables". Kerlinger (1986:370) regards this statement as one advantage of doing research in educational settings. The research done in this study took place in a realistic educational setting, which contributed to the external validity of the research, "since the more realistic the situation, the more valid are generalizations to other situations likely to be" (Kerlinger 1986:370). Although results of the quantitative study were negative and the validity of these results are not argued, the qualitative observations and the results of the NGT may carry much weight because they were obtained within a realistic and true educational setting, but with eight students.

- A second limitation of this study is that the groups that were used in the quantitative design were small. Again this was due to the fact that the researcher had to contend with the field experimental setting which the research situation offered. The small number of students in the experimental group could have had an effect on the outcome of the statistical analyses and be the reason why no significant results were found regarding the effectiveness of the classroom learning strategy intervention.

- A third limitation of this study (again due to the field experimental setting of the research) was that the knowledge gained by the classroom learning strategy intervention might have become known to students in the control group, as students in the experimental group were not "isolated" from students in the control group. Students in the experimental group saw students in the control group on a daily basis, shared rooms in the same hostels on campus, and were friends and/or classmates. The researcher is therefore of the opinion that students in the experimental group may have shared the proposed learning strategy with students in the control group, which could have been a reason why students in the control group also performed better in their post-tests.

8.4 RECOMMENDATIONS

The findings of this research can provide other universities with some broad guidelines or indicators with regard to a learning strategy for third-year students that take a mathematics and statistics-related subject.

For future research, the R-SPQ-2F Questionnaire can be used again to assess students' approaches to learning. Maybe if students are aware that they form part of a research study, or if a person with more authority has students complete the questionnaires, they will take it more seriously and random completion of questionnaires will not occur.

The researcher encourages other educators in mathematics and statistics-related subjects to follow a constructivist style of teaching that will promote deep learning and at the same time encourage students to participate actively in the learning process. Educators are also encouraged to teach students "how to learn", by means of a classroom learning strategy, and not just blindly "model" exercises from textbooks which are readily available to them. Educators can encourage students to make use of concept maps (see 4.2) and allow students to "memorise" the "RIEQTSR" problem-solving strategy to make use of during assessments (see 4.3).

8.5 CONCLUSION

This study is concerned with university students' underachievement in mathematics and statistics. The researcher attempted to develop a classroom learning strategy, aimed at improving students' academic performance in a mathematics and statistics-related subject at the CUT. The primary aim of the research was to encourage students to make use of a specific learning strategy to learn a mathematics and statistics-related subject, and eventually follow a deep approach to learning. This study has contributed greatly towards better insight into the learning of mathematics and statistics-related subjects. The researcher is of the opinion that the proposed classroom learning strategy has the potential to improve students' conceptual understanding in a mathematics and statistics-related subject, encourage students to follow a deep approach to learning, as well as to improve students' problem-solving abilities in such subjects. Educators in mathematics and statistics-related subjects can benefit from this study by realising the value and the need for empowering their students with learning strategies in such subjects. The researcher hopes that other mathematics and statistics educators in South Africa will be able to apply this learning strategy in their own classrooms.

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APPENDIX A

RESEARCH ANALYSIS PLAN

"Impact of a learning strategy intervention on first-years' academic performance in the module Business Statistics"

Statistical Analysis Plan

Version 1 **Date: 15 March 2010**

1. Objective of statistical analysis

The primary objective of the statistical analysis is to investigate the impact of a learning strategy intervention on first-years' academic performance in the module Business Calculations, while adjusting for the following potential confounders: Gender-categorical, Age-Continuous, Race-categorical, Previous math background– categorical, FTE status (categorical).

2. Descriptive analysis

Frequency tabulations (number of students and percent of students per category, both for the total group and for the experimental versus control group separately) for the following will be presented:

- Gender-Categorical
- Race- categorical
- Previous math background – categorical
- FTE status – categorical

Based on the frequency distributions of the categorical confounding variable Race, categories of this variable might be combined for the purposes of the analysis described below.

Software: SAS Proc FREQ.

Descriptive statistics (mean, SD, median, min, max, number of observations, both for the total group and for the experimental versus control group separately) will be presented for each quantitative variable, namely

- Business Calculations results: pre-test, post-test 1, post-test 2 and for the average of the 2 post-test results - Continuous
- Age-Continuous

Software: SAS Proc MEANS.

3. Univariate Analyses

The dependent variable is the following:

- Business Calculations results: average of post-test results - Continuous

The dependent variable will be analysed using using one-way ANOVA fitting, one variable at a time, the dependent variable (Group) and each of the confounding variables.

Software: SAS Proc GLM.

4. Multivariate analysis

The dependent variable (Business Calculations results: average of post-test results) will be analysed using analysis of covariance. The analysis of covariance model will contain the independent variable (Group) and all potential confounders (gender, age, race, previous math background, FTE status, Business Calculations results: pre-test). F-statistics and associated P-values will be calculated for each variable in the model.

Software: SAS Proc GLM.

"Impact of a learning strategy intervention on third-years' academic performance in the module Business Statistics"

Statistical Analysis Plan

Version 2 **Date: 19 February 2010**

1. Objective of statistical analysis

The primary objective of the statistical analysis is to investigate the impact of a learning strategy intervention on third-years' academic performance in the module Business Statistics, while adjusting for the following potential confounders: Gender-categorical, Age-Continuous, Race-categorical, Previous math background– categorical, Class attendance - continuous, BCL results- continuous, Business statistics results (pre-test) – continuous.

2. Descriptive analysis

Frequency tabulations (number of students and percent of students per category, both for the total group and for the experimental versus control group separately) for the following will be presented:

- Gender-Categorical
- Race- categorical
- Previous math background– categorical

Based on the frequency distributions of the categorical confounding variable Race, categories of this variable might be combined for the purposes of the analysis described below.

Software: SAS Proc FREQ.

Descriptive statistics (mean, SD, median, min, max, number of observations, both for the total group and for the experimental versus control group separately) will be presented for each quantitative variable, namely

- Business Statistics results, both pre-test and post-test - Continuous
- Age-Continuous
- BCL results-continuous
- Class attendance

Software: SAS Proc MEANS.

3. Univariate Analyses

The dependent variable is the following:

- Business statistics results: post-test - Continuous

The dependent variable will be analysed using using one-way ANOVA fitting, one variable at a time, the dependent variable (Group) and each of the confounding variables.

Software: SAS Proc GLM.

4. Multivariate analysis

The dependent variable (Business statistics results: post-test) will be analysed using analysis of covariance. The analysis of covariance model will contain the independent variable (Group) and all potential confounders (gender, age, race, previous math background, class attendance, BCL results, Business statistics results: pre-test). F-statistics and associated P-values will be calculated for each variable in the model.

Software: SAS Proc GLM.

APPENDIX B

BUSINESS CALCULATIONS PRE-TEST

/pp

Central University of Technology, Free State	DATUM: 25 Feb 2009 TYD: 15:00 DATE: 25 Feb 2009 TIME: 15:00 VAK: Besigheidsberekeninge KODE: BCL11AB SUBJECT: Business Calculations CODE: BCL11AB **Assessering – Toets 1** **Assessment – Test 1** SKRYFBEHOEFTES/STATIONERY: Meervoudige-keuse antwoordblaaie & calculators Multiple-choice answersheets & sakrekenaars

OPGESTEL DEUR / COMPILED BY
Ms. D. H. Delport

****

ONDERRIGPROGRAM(ME)/INSTRUCTIONAL PROGRAMME:	KODE/CODE:
National Higher Certificate: Accounting /	BRHSAB
National Higher Certificate: Financial Information Systems	BRHSFA

INSTRUKSIES/INSTRUCTIONS:

Duur van vraestel: 1 Uur **Maksimum punte: 10 vrae x 2 = 20 punte**
Duration of paper: 1 Hour **Maximum marks: 20 questions x 2 = 20 marks**

HIERDIE VRAESTEL BESTAAN UIT 6 BLADSYE.
THIS PAPER CONSISTS OF 6 PAGES.

1. The value of $\frac{1}{2}+\frac{5}{8}+\frac{3}{12}$ (in fraction form) is:

(a) 1,375 (b) $\frac{9}{12}$ (c) $\frac{9}{24}$ (d) $\frac{11}{8}$ (e) none of the above

2. The value of $\left(1+\frac{1}{8}\right)\div\frac{1}{4}$ (in fraction form) is:

(a) $\frac{9}{32}$ (b) $\frac{9}{2}$ (c) 4,5 (d) 0,28 (e) none of the above

3. The decimal 0.0025 expressed as a fraction (in lowest terms) is:

(a) $\frac{2,5}{100}$ (b) $\frac{25}{10000}$ (c) $\frac{1}{400}$ (d) 0,25% (e) none of the above

4. The fraction $\frac{5}{80000}$ expressed as a decimal is:

(a) 0,0000625 (b) $6,25^{-05}$ (c) 0.625 (d) 625 (e) none of the above

5. The percentage $21\frac{1}{4}\%$ converted to a fraction is:

(a) $\frac{85}{4}$ (b) $\frac{17}{80}$ (c) 21,25 (d) $\frac{21}{4}$ (e) none of the above

6. The value of $10-\{9-1-[2(6-1)-3(3-1)]+5\}+7$ is:

(a) 2 (b) 13 (c) 8 (d) -5 (e) none of the above

7. The value of $4^{\frac{3}{2}}$ is:

(a) 11 (b) $\frac{3}{2}$ (c) 6 (d) 8 (e) none of the above

8. The value of $\left(2^2\right)^{-3}$ is:

(a) $\frac{1}{64}$ (b) -12 (c) 64 (d) -64 (e) none of the above

9. Use logarithms to calculate the value of $\frac{\sqrt[3]{200}}{2}$

(a) 0,466 (b) 4,793 (c) 0,283 (d) 11,67 (e) none of the above

10. The characteristic of the number 0,0279 is:

(a) 1 (b) 2 (c) 0 (d) -2 (e) none of the above

11. The number 0.000445 expressed in scientific notation is:

(a) $4{,}45 \times 10^2$
(b) 445×10^2
(c) $4{,}45 \times 10^4$
(d) $4{,}45 \times 10^{-4}$
(e) none of the above

12. Evaluate the following $\left(\sqrt[3]{64}\right)\left(\sqrt{36}\right)$:

(a) 8 (b) 24 (c) 4 (d) 6 (e) none of the above

13. A broker charges a fixed fee of R100 plus 2,5% on the amount of all investments made. If a customer invests a total of R3000 through the broker, what is the total brokerage payable?

(a) 3100 (b) 75 (c) 3250 (d) 175 (e) none of the above

14. A store normally sells a certain brand of fridge for R700 but is currently offering it for sale at R577,50. What percentage discount is the store offering on the fridge?

(a) 122,50 % (b) 75 % (c) 17,5 % (d) 32,50 % (e) none of the above

15. A store buys shoes for R100 from the supplier and sells them for R180 (not including GST). The profit as a percentage of the cost price is:

(a) 80 % (b) 180 % (c) 0,80 % (d) 100 % (e) none of the above

16. An electrician makes to a company switchboard and presents a final bill for R260,70. How much is the GST component of the bill ?

(a) 286,77 (b) 26,07 (c) 23,70 (d) 284,40 (e) none of the above

17 A bedside table has a retail price of R176 including GST. Calculate the retail price exluding GST.

(a) 160 (b) 193,60 (c) 158,40 (d) 192 (e) none of the above

18. The solution for x in the following equation $\frac{x-1}{x+1} = 2$ is:

(a) 3 (b) -3 (c) 1 (d) 2 (e) none of the above

19. The following simultaneous linear equation is given:

$$2x + 3y = 13$$
$$x - 3y = -7$$

The value of x is:

(a) 5 (b) 1 (c) -5 (d) 2 (e) none of the above

20. The value of y in the equations (in question 19) is:

(a) -6 (b) -3 (c) 3 (d) -1 (e) none of the above

21. The ratio 150,75 : 201 expressed in lowest terms is:

(a) 50,25 : 67 (b) 603 : 804 (c) 201 : 268 (d) 3 : 4 (e) none of the above

22. During a sales on woman's shoes, a store sold 568 pairs in 22 minutes. Express the number of pairs sold per hour.

(a) 25,82 pairs/hour
(b) 1549 pairs/hour
(c) 51,64 pair/hour
(d) 12496 pair/hour
(e) none of the above

23. An airline has total sales of R23500 000 and assets of R9400 000. The total asset turnover (expressed as a percentage) is:

(a) 2,5% (b) 250% (c) 40% (d) 400% (e) none of the above

24. A company has current assets of R1,4 million and a current-asset ratio of 0,80. Its current liabilities are:

(a) 1.12 million
(b) 571 429
(c) 0,57 million
(d) 1,75 million
(e) none of the above

25. A computer company has stock of R110 000 on 1 July 2001 and closing stock of R230 000 on 30 June 2002. Total sales during the financial year were R625 000. The stock turnover ratio is:

(a) 3,68 (b) 170 000 (c) 0,272 (d) 1,84 (e) none of the above

Formules

$$C = F + (S \times R)$$

$$C = S \times R$$

$$R = \frac{D}{L}$$

$$D = R \times L$$

$$DP = L - D$$

$$P = SP - CP$$

$$L = CP - SP$$

$$P_S = \frac{P}{SP}$$

$$P_C = \frac{P}{CP}$$

$$L_S = \frac{L}{SP}$$

$$L_C = \frac{L}{CP}$$

$$\text{Total asset turnover} = \frac{\text{sales}}{\text{total assets}}$$

$$\text{Return on investment} = \frac{\text{net profit}}{\text{total assets}}$$

$$\text{Stock turnover} = \frac{\text{cost of goods sold}}{\text{average stock}}$$

$$\text{Average stock} = \frac{\text{opening stock} + \text{closing stock}}{2}$$

$$\text{Debtor turnover} = \frac{\text{gross credit sales}}{\text{average gross accounts receivable}}$$

$$\text{Average collection period} = \frac{365}{\text{debtor turnover}}$$

$$\text{Current-asset ratio} = \frac{\text{current assets}}{\text{current liabilities}}$$

$$\text{Acid-test ratio} = \frac{\text{current assets} - \text{inventory}}{\text{current liabilities} - \text{bank overdraft}}$$

APPENDIX C

BUSINESS CALCULATIONS POST-TEST

/pp

Central University of Technology, Free State	DATUM: 29 April 2009 TYD: 15:00 DATE: 29 April 2009 TIME: 15:00 VAK: Besigheidsberekeninge KODE: BCL11AB SUBJECT: Business Calculations CODE: BCL11AB **Assessering – Hooftoets** **Assessment – Main Test** SKRYFBEHOEFTES/STATIONERY: Meervoudige-keuse antwoordblaaie & calculators Multiple-choice answersheets & sakrekenaars

OPGESTEL DEUR / COMPILED BY
Ms. D. H. Delport

****

ONDERRIGPROGRAM(ME)/INSTRUCTIONAL PROGRAMME:	KODE/CODE:
National Higher Certificate: Accounting /	BRHSAB
National Higher Certificate: Financial Information Systems	BRHSFA

INSTRUKSIES/INSTRUCTIONS:

Duur van vraestel: 1 Uur — **Maksimum punte: 10 vrae x 2 = 20 punte**
Duration of paper: 1 Hour — **Maximum marks: 20 questions x 2 = 20 marks**

HIERDIE VRAESTEL BESTAAN UIT 4 BLADSYE. (Voorblad ingesluit)
THIS PAPER CONSISTS OF 4 PAGES. (Cover sheet included)

1. The amount of simple interest earned on R35 000 deposited at a rate of 6,5 % per annum for a period of 5 years and 3 months is:

   (a) 11943,75 (b) 2275 (c) 12057,50 (d) 1194,38 (e) none of the above

2. A newsagent wants to have an amount of R25 000 in 6 years time when she retires. She can invest her money at a simple interest rate of 8,5 % per annum. How much money (to the nearest rand) should she invest now to achieve her aim?

   (a) 16560 (b) 16556,29 (c) 3840 (d) 16556 (e) none of the above

3. A bank lends a client R10 000, which must be paid at the end of 9 months, when R10281,25 will be due. The annual simple interest rate the bank is charging is:

   (a) 0,3125 (b) 3,75 (c) 1,37 (d) 11,42 (e) none of the above

4. After what period will a principal of R3000 amount to R3487,50 at 5 % simple interest per annum ?

   (a) 3 years and 25 months
   (b) 23,25 years
   (c) 3,25 years
   (d) 23 years and 25 months
   (e) none of the above

5. An amount of R50 000 is invested for 5 years in a bank that pays an interest rate of 7 % per annum, compounded semi-annually. The accumulation factor (rounded off to two decimals) is:

   (a) 70127,59 (b) 70500 (c) 1,41 (d) 70529,94 (e) none of the above

6. How much money should be deposited now into an 8 % per annum interest-bearing account, compounded quarterly, so that an amount of R25 000 will be accumulated in 6 years? [Use only 2 decimals in the accumulation factor]

(a) 15754,24 (b) 15500 (c) 15543,04 (d) 15750 (e) none of the above

7. Peter deposits R8000 in his savings account that pays 6 % inerest, compounded quarterly. The total amount of interest that he will earn after 5 years is:

(a) 2800 (b) 10800 (c) 10720 (d) 2720 (e) none of the above

8. Suppose that R2500 amounts to R3750 in 6 years with interest compounded monthly. What annual rate of compound interest has been used?

(a) 0,56 (b) 11,75 (c) 0,0056 (d) 6,78 (e) none of the above

9. A barrister is repaying a loan in annual instalments of R3500 for 8 years. If the interest rate is 7 % per annum compounded annually, the present value of the loan (to the nearest rand) is:

(a) 21900 (b) 20500 (c) 20900 (d) 22100 (e) none of the above

10. A chemist took out a loan in order to purchase an airconditioning unit for the pharmacy. He was required to repay the loan in monthly instalments of R75 for 3 years. Calculate the future value (to the nearest rand) of the loan if the interest rate was 5 %, compounded every month.

(a) 87 (b) 65 (c) 2907 (d) 236 (e) none of the above

$$I = P \times R \times T$$

$$S = P + I$$

$$S = P(1 + RT)$$

$$P = \frac{S}{1 + RT}$$

$$S = P(1 + i)^n$$

Accumulation factor $= (1 + i)^n$

Amount of compound interest $= P \times$ (accumulation factor $- 1$)

$$P = S(1 + i)^{-n}$$

Present value factor $= (1 + i)^{-n}$

$$i = \left(\frac{S}{P}\right)^{\frac{1}{n}} - 1$$

$$n = \frac{\log\left(\frac{S}{P}\right)}{\log(1 + i)}$$

$$S = R \times \frac{(1 + i)^n - 1}{i}$$

$$s_{\overline{n}|i} = \frac{(1 + i)^n - 1}{i}$$

$$S = R \times s_{\overline{n}|i}$$

$$A = S(1 + i)^{-n}$$

$$A = R \times \left[\frac{1 - (1 + i)^{-n}}{i}\right]$$

$$a_{\overline{n}|i} = \frac{1 - (1 + i)^{-n}}{i}$$

$$A = R \times a_{\overline{n}|i}$$

$$R = \frac{S}{\frac{(1 + i)^n - 1}{i}}$$

$$R = \frac{S}{s_{\overline{n}|i}}$$

APPENDIX D

BUSINESS CALCULATIONS EXAM PAPER

CENTRAL UNIVERSITY OF TECHNOLOGY, FREE STATE
SENTRALE UNIVERSITEIT VIR TEGNOLOGIE, VRYSTAAT
YUNIVESITHI E BOHARENG YA THEKENOLOJI, FOREISTATA

Central University of Technology, Free State

HOOF EKSAMEN / MAIN EXAMINATION

DATUM:	26 Mei 2009	**SESSIE:**	09:00
DATE:	26 May 2009	**SESSION:**	09:00
VAK:	Besigheidsberekeninge	**KODE:**	BCL11AB
SUBJECT:	Business Calculations	**CODE:**	BCL11AB

SKRYFBEHOEFTES / STATIONERY:

1. Multiple-choice answersheets / Meervoudige-keuse antwoordbladsye
2. Sakrekenaars / Calculators

EKSAMINATOR / EXAMINER:	Ms. D.H. Delport / Me. G. Magadi
MODERATOR:	Mr. D. Naude

INSTRUCTIONAL PROGRAMME:	**CODE:**	
National Higher Certificate: Accounting /		BRHSAB
National Higher Certificate: Financial Information Systems		BRHSFA

INSTRUKSIES/INSTRUCTIONS:

Duur van vraestel:	2 ure	**Maksimum punte:**	50
Duration of paper:	2 hours	**Maximum marks:**	50

BEANTWOORD AL DIE VRAE.
ANSWER ALL THE QUESTIONS.

THIS PAPER CONSISTS OUT OF 12 PAGES.

....
EKSAMINATOR/EXAMINER

....
MODERATOR

Determine the following:

$$\boxed{\frac{2}{3} \times \frac{5}{8} - \frac{1}{4}}$$

(a) $\frac{9}{20}$ (b) $\frac{1}{6}$ (c) $\frac{5}{48}$ (d) $\frac{25}{24}$ (e) none of the above

A family consisting of 2 adults and a number of children went to the cinema. Each adult paid R9 and each child paid R3,50. If the total bill was R39, how many children went?

(a) 4 (b) 3 (c) 2 (d) 6 (e) none of the above

The number 2645 expressed in scientific notation is:

(a) $2{,}645 \times 10^{-3}$
(b) $2{,}645 \times 10^{3}$
(c) 2645×10^{4}
(d) 2645×10^{0}
(e) none of the above

AMINATOR/EXAMINER

MODERATOR

The following expression $\boxed{\log x + 2\log y}$ rewritten as the natural logarithm of a single quantity is:

(a) $\log(x2y)$
(b) $\log(x+2y)$
(c) $\log\left(\frac{2x}{y}\right)$
(d) $\log\left(xy^2\right)$
(e) none of the above

A real estate agent advertises that he will charge the following commission (based on the selling price) on any property that he sells:

5,0 % on the first R50 000
3,0 % on the next R75 000
2,5 % on the next R75 000
2,0 % on the amount exceeding R200 000

Calculate the commission that he will be charged on property that sells for R185 000.

(a) 6625 (b) 1875 (c) 6250 (d) 29125 (e) none of the above

A painter, who was entitled to a trade discount of 20 % on all materials purchased, was sent a bill for R350. What was the list price of the goods that the painter purchased?

(a) 437,50 (b) 420 (c) 70 (d) 280 (e) none of the above

....
AMINATOR/EXAMINER

....
MODERATOR

7. A service station buys brake pads for R36 from the supplier and sells them for R41,40 (not including GST). The profit as a percentage of the cost price is:

(a) 15 % (b) 5,4 % (c) 13 % (d) 54 % (e) none of the above

A manufacturing plant has current assets of R120 000 of inventory, and current liabilities include R185 000 of bank overdraft. Current assets in total are R424 000 and current liabilities in total are R327 000. The acid-test ratio (to two decimal places) is:

(a) 1,06 (b) 1,30 (c) 2,14 (d) 1,49 (e) none of the above

A grandmother decides to invest R300 per month for 5 years at 15 % per annum, compounded monthly to pay her 13-year old granddaughter's university fees in 5 year's time. If each deposits takes place at the beginning of the month, what amount will be available at the end of the term? [Make use of all the decimals and round the final answer off to the nearest rand]

(a) 29000 (b) 24000 (c) 2022,71 (d) 26572,35 (e) none of the above

....
AMINATOR/EXAMINER

....
MODERATOR

uestion 10 – 12 is based on the following table:

e accompanying table contains the price (in cent/kg) and the quantity (in thousands of tons) of all exports of ural animal fibers from South Africa in 2000 and 2005.

		Price per unit		Quantity per unit	
m	Unit	2000	2005	2000	2005
erino	1 kg	45	50	70	80
mbs	1 kg	25	30	40	50
hair	250 g	35	40	100	120

The simple aggregate index for 2005 with 2000 as base year is:

(a) 114,29 (b) 105 (c) 87,5 (d) 1,14 (e) none of the above

The Laspeyres index of 2005 relative to a base year of 2000 is:

(a) 87,93 (b) 113,73 (c) 1,14 (d) 114,29 (e) none of the above

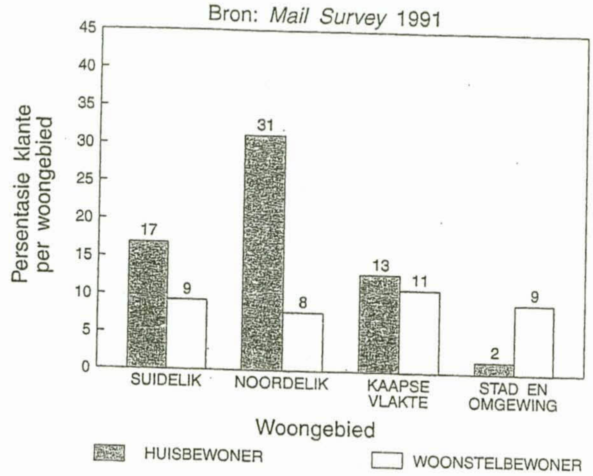
The Paasche index in 2005 relative to a base year of 2000 is:

(a) 87,79 (b) 1,14 (c) 114,29 (d) 113,81 (e) none of the above

....
AMINATOR/EXAMINER

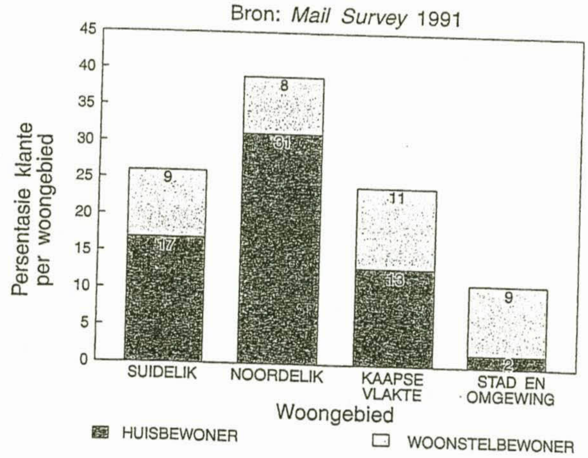
....
MODERATOR

3. The following graph is a:



(a) pie chart
(b) histogram
(c) multiple bar chart
(d) compound bar chart
(e) none of the above

. The following graph is a:



(a) pie chart
(b) compound bar chart
(c) multiple bar chart
(d) histogram

....
AMINATOR/EXAMINER

....
MODERATOR

(e) none of the above

nswer questions 15 – 19 by making use of the following data:.

he management of a department store asked the lift operator to record the number of people who traveled in ch ride of the lift starting from the ground floor. The operator did this over a period of 100 rides, with the sults shown below.

Number of people	Frequency
1 – <3	16
3 – <5	28
5 – <7	34
7 – <9	13
9 – <11	9

The sample size is:

(a) 34 (b) 2 (c) 100 (d) 5 (e) none of the above

The mean is:

(a) 2 (b) 5,42 (c) 100 (d) 34 (e) none of the above

The mode is:

(a) 3,35 (b) 0,63 (c) 34 (d) 5,44 (e) none of the above

8. The median is:

(a) 50 (b) 3,35 (c) 5,35 (d) 0,65 (e) none of the above

9. The first quartile is:

(a) 28 (b) 1,64 (c) 25 (d) 3,64 (e) none of the above

nswer questions 20 – 25 by making use of the following data:.

ccording to a book publisher, the number of enquiries received on 12 successive working days by a large okshop about a forthcoming novel was:

28 15 13 9 18 22 16 8 5 15 20 11

The sample size is:

(a) 12 (b) 28 (c) 15 (d) 180 (e) none of the above

The arithmetic mean is:

....
AMINATOR/EXAMINER

....
MODERATOR

(a) 10 (b) 18 (c) 15 (d) 180 (e) none of the above

The Range is:

(a) 28 (b) 180 (c) 15 (d) 23 (e) none of the above

The third quartile is:

(a) 18 (b) 19 (c) 20 (d) 3 (e) none of the above

The median is:

(a) 15 (b) 16 (c) 13 (d) 18 (d) none of the above

The standard deviation is:

(a) 458 (b) 23,36 (c) 4,83 (d) 6,45 (e) none of the above

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AMINATOR/EXAMINER

....
MODERATOR

Formules

$$C = F + (S \times R)$$

$$C = S \times R$$

$$R = \frac{D}{L}$$

$$D = R \times L$$

$$DP = L - D$$

$$P = SP - CP$$

$$L = CP - SP$$

$$P_S = \frac{P}{SP}$$

$$P_C = \frac{P}{CP}$$

$$L_S = \frac{L}{SP}$$

$$L_C = \frac{L}{CP}$$

$$\text{Total asset turnover} = \frac{\text{sales}}{\text{total assets}}$$

$$\text{Return on investment} = \frac{\text{net profit}}{\text{total assets}}$$

$$\text{Stock turnover} = \frac{\text{cost of goods sold}}{\text{average stock}}$$

$$\text{Average stock} = \frac{\text{opening stock} + \text{closing stock}}{2}$$

$$\text{Debtor turnover} = \frac{\text{gross credit sales}}{\text{average gross accounts receivable}}$$

$$\text{Average collection period} = \frac{365}{\text{debtor turnover}}$$

$$\text{Current-asset ratio} = \frac{\text{current assets}}{\text{current liabilities}}$$

$$\text{Acid-test ratio} = \frac{\text{current assets} - \text{inventory}}{\text{current liabilities} - \text{bank overdraft}}$$

$$I = P \times R \times T$$

$$S = P + I$$

$$S = P(1 + RT)$$

$$P = \frac{S}{1 + RT}$$

$$S = P(1 + i)^n$$

Accumulation factor $= (1 + i)^n$

Amount of compound interest $= P \times$ (accumulation factor $- 1$)

$$P = S(1 + i)^{-n}$$

Present value factor $= (1 + i)^{-n}$

$$i = \left(\frac{S}{P}\right)^{\frac{1}{n}} - 1$$

$$n = \frac{\log\left(\frac{S}{P}\right)}{\log(1 + i)}$$

$$S = R \times \frac{(1 + i)^n - 1}{i}$$

$$s_{\overline{n}|i} = \frac{(1 + i)^n - 1}{i}$$

$$S = R \times s_{\overline{n}|i}$$

$$A = S(1 + i)^{-n}$$

$$A = R \times \left[\frac{1 - (1 + i)^{-n}}{i}\right]$$

$$a_{\overline{n}|i} = \frac{1 - (1 + i)^{-n}}{i}$$

$$A = R \times a_{\overline{n}|i}$$

$$R = \frac{S}{\frac{(1 + i)^n - 1}{i}}$$

$$R = \frac{S}{s_{\overline{n}|i}}$$

$$\bar{x} = \frac{\Sigma fm}{\Sigma f}$$

$$\bar{x} = \frac{\Sigma x}{n}$$

$$\text{Mo} = L + \frac{d_1}{d_1 + d_2}(i)$$

$$\tilde{x} = L + \frac{\left(\frac{n}{2} - C\right)}{f}(i)$$

$$Q_1 = L + \frac{\left(\frac{n}{4} - C\right)}{f}(i)$$

$$Q_3 = L + \frac{\left(\frac{3n}{4} - C\right)}{f}(i)$$

$$\text{Mean deviation} = \frac{\Sigma|x - \bar{x}|}{n}$$

$$\text{Mean deviation} = \frac{\Sigma f|x - \bar{x}|}{\Sigma f}$$

$$s = \sqrt{\frac{\Sigma x^2 - n\bar{x}^2}{n - 1}}$$

$$s = \sqrt{\frac{\Sigma(x - \bar{x})^2}{n - 1}}$$

$$\text{Simple aggregate index} = \frac{\Sigma p_n}{\Sigma p_o} \times 100$$

where Σp_n = sum of the current period prices
Σp_o = sum of the base period prices

$$\text{Laspeyres index} = \frac{\Sigma p_n q_o}{\Sigma p_o q_o} \times 100$$

where $\Sigma p_n q_o$ = equivalent expenditure incurred in the current period
$\Sigma p_o q_o$ = expenditure incurred in the base period

$$\text{Paasche index} = \frac{\Sigma p_n q_n}{\Sigma p_o q_n} \times 100$$

where $\Sigma p_n q_n$ = expenditure incurred in the current period
$\Sigma p_o q_n$ = equivalent expenditure incurred in the base period

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....
MODERATOR

APPENDIX E

THE REVISED TWO-FACTOR STUDY PROCESS QUESTIONNAIRE

The revised two-factor study process questionnaire (R-SPQ-2F)
Source: Biggs, Kember and Leung (2001:148-149)

This questionnaire has a number of questions about your attitudes toward your studies and your usual way of studying *Business Calculations* (BCL11AB).

Please fill in the appropriate circle alongside the question number. The letters alongside each question stand for the following response. Please answer each item.

A - this item is never or only rarely true of me
B - this item is sometimes true of me
C - this item is true of me about half the time
D - this item is frequently true of me
E - this item is always or almost always true of me

Student Number:

1. I find that at times studying gives me a feeling of deep personal satisfaction.

A	B	C	D	E

2. I find that I have to do enough work on a topic so that I can form my own conclusions before I am satisfied.

A	B	C	D	E

3. My aim is to pass the course while doing as little work as possible.

A	B	C	D	E

4. I only study seriously what's given out in class or in the course outlines.

A	B	C	D	E

5. I feel that virtually any topic can be highly interesting once I get into it.

A	B	C	D	E

6. I find most new topics interesting and often spend extra time trying to obtain more information about them.

A	B	C	D	E

7. I do not find Business Calculations very interesting so I keep my work to the minimum.

A	B	C	D	E

8. I learn some things by rote, going over and over them until I know them by heart even if I do not understand them.

A	B	C	D	E

9. I find that studying academic topics can at times be as exciting as a good novel or movie.

A	B	C	D	E

10. I test myself on important topics until I understand them completely.

A	B	C	D	E

11. I find I can get by in most assessments by memorizing key sections rather than trying to understand them.

A	B	C	D	E

12. I generally restrict my study to what is specifically set as I think it is unnecessary to do anything extra.

A	B	C	D	E

13. I work hard at my studies because I find BCL interesting.

A	B	C	D	E

14. I spend a lot my free time finding out more about interesting topics which have been discussed in the BCL classes.

A	B	C	D	E

15. I find it is not helpful to study topics in depth. It confuses and wastes time, when all you need is a passing acquaintance with topics.

A	B	C	D	E

16. I believe that lecturers shouldn't expect students to spend significant amounts of time studying material everyone knows won't be examined.

A	B	C	D	E

17. I come to most classes with questions in mind that I want answering.

A	B	C	D	E

18. I make a point of looking at most of the suggested readings that go with the lectures.

A	B	C	D	E

19. I see no point in learning material which is not likely to be in the examination.

A	B	C	D	E

20. I find the best way to pass examinations is to try to remember answers to likely questions.

A	B	C	D	E

APPENDIX F

BUSINESS STATISTICS/STATISTICS II PRE-TEST

/pp

Central University of Technology, Free State

DATUM: 18 Aug 2009 TYD: 17:15
DATE: 18 Aug 2009 TIME: 17:15

VAK: Besigheidstatisitiek KODE: BSS/STC22AB
SUBJECT: Business Statistics CODE: BSS/STC22AB

Assessering – Toets 1
Assessment – Test 1

SKRYFBEHOEFTES/STATIONERY:
Calculators
Sakrekenaars

OPGESTEL DEUR / COMPILED BY
Ms. D. H. Delport

****

ONDERRIGPROGRAM(ME)/INSTRUCTIONAL PROGRAMME:	KODE/CODE:
N DIP: BESTUUR / MANAGEMENT	BBNDBU
N DIP: KOSTE- EN BESTUURSREKENINGKUNDE / COST AND MANAGEMENT ACCOUNTING	BRNDCO
N DIP: MUNISIPALE ADMINISTRASIE / MUNICIPAL ADMINISTRATION	BONDMP

INSTRUKSIES/INSTRUCTIONS:

Duur van vraestel: 1 ½ UUR **Maksimum punte: 35**
Duration of paper: 1 ½ HOURS **Maximum marks: 35**

Question 1

1.1 A newsagent wants to have an amount of R25 000 in 6 years time when she retires. She can invest her money at a simple interest rate of 8,5 % per annum. How much money (to the nearest rand) should she invest now to achieve her aim?

(4)

1.2 An amount of R50 000 is invested for 5 years in a bank that pays an interest rate of 7 % per annum, compounded semi-annually. Calculate the accumulated value? [Round the accumulation factor off to two decimal places]

(4)

1.3 A chemist took out a loan in order to purchase an airconditioning unit for the pharmacy. He was required to repay the loan in monthly instalments of R75 for 3 years. Calculate the present value (to the nearest rand) of the loan if the interest rate was 5 %, compounded every month.

(5)

[13]

Question 2

2.1 A coin is tossed 3 times. What is the probability of obtaining at least 1 tail?

(4)

2.2 Suppose a coin is tossed, a six-sided die is rolled and a card is drawn randomly from a deck of 52 cards (13 diamond cards is in the deck of cards). The events are as follows:

A = coin is a head
B = die is a two
C = card is a diamond

What is the probability that all three events A, B and C to occur ?

(4)

2.3 Let $P(B) = 0{,}5$ and $P(A \mid B) = 2$

Find the following: $P(A \cap B)$ (2)

[10]

Question 3

The length of time customers queue for service at a supermarket checkout follows a normal distribution with a mean of 8 minutes and a standard deviation of 2,50 minutes. A random of 25 customers is chosen. Find the following probabilities that their mean waiting time in the queue is:

3.1 less than 7,00 minutes (2)

3.2 more than 7,00 minutes (2)

3.2 between 7,00 and 9,00 minutes (3)

[7]

Question 4

A fast-food outlet wishes to determine how long its customers entering the queue in the store have to wait for their order to be taken. A random sample of 60 customers was observed. Their mean waiting time was 7,20 minutes with a standard deviation of 2,50 minutes.

Find a 99 % confidence interval for the mean waiting time in the queue for all customers.

[5]

Total = [35]

$$I = P \times R \times T$$

$$S = P + I$$

$$S = P(1 + RT)$$

$$P = \frac{S}{1 + RT}$$

$$S = P(1 + i)^n$$

Accumulation factor $= (1 + i)^n$

Amount of compound interest $= P \times$ (accumulation factor $- 1$)

$$P = S(1 + i)^{-n}$$

Present value factor $= (1 + i)^{-n}$

$$i = \left(\frac{S}{P}\right)^{\frac{1}{n}} - 1$$

$$n = \frac{\log\left(\frac{S}{P}\right)}{\log(1 + i)}$$

$$S = R \times \frac{(1 + i)^n - 1}{i}$$

$$s_{\overline{n}|i} = \frac{(1 + i)^n - 1}{i}$$

$$S = R \times s_{\overline{n}|i}$$

$$A = S(1 + i)^{-n}$$

$$A = R \times \left[\frac{1 - (1 + i)^{-n}}{i}\right]$$

$$a_{\overline{n}|i} = \frac{1 - (1 + i)^{-n}}{i}$$

$$A = R \times a_{\overline{n}|i}$$

$$R = \frac{S}{\frac{(1 + i)^n - 1}{i}}$$

$$R = \frac{S}{s_{\overline{n}|i}}$$

$$(A_1 \text{ or } A_2 \text{ or } A_3 \ \ldots \text{ or } A_n) = P(A_1) + P(A_2) + P(A_3) + \ldots + P(A_n)$$

$$P(A_1 \text{ and } A_2 \text{ and } A_3 \ldots \text{ and } A_n) = P(A_1) \times P(A_2) \times P(A_3) \times \ldots \times P(A_n)$$

$$P(A \text{ or } B) = P(A) + P(B)$$

$$A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$(A \text{ and } B) = P(A) \times P(B)$$

$$(A) + P(B) = 1$$

$$(A|B) = \frac{P(A \text{ and } B)}{P(B)}$$

t least one person has the characteristic) = 1 − P(*no* person has the characteristic)

$$z = \frac{x - \mu}{\sigma}$$

$$x = \mu + z\sigma$$

$$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

$$\text{Standard error of the mean} = \frac{\sigma}{\sqrt{n}}$$

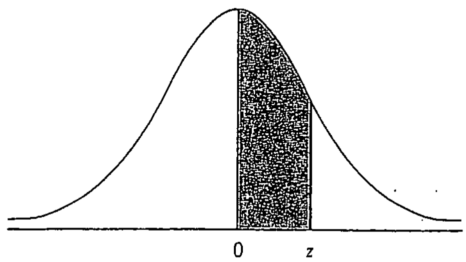
$$\left(\bar{x} - 1.96\frac{s}{\sqrt{n}}, \bar{x} + 1.96\frac{s}{\sqrt{n}}\right)$$

$$\text{Standard error of the mean} \doteqdot \frac{s}{\sqrt{n}}$$

$$\left(\bar{x} - 1.96\frac{\sigma}{\sqrt{n}}, \bar{x} + 1.96\frac{\sigma}{\sqrt{n}}\right)$$

For a 90% confidence interval for μ, we replace the 1.96 by 1.645.
For a 99% confidence interval for μ, we replace the 1.96 by 2.58.

Tables



This table gives the area between 0 and $+z$.

Table 1 AREAS UNDER THE STANDARD NORMAL CURVE

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09	z
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359	0.0
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753	0.1
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141	0.2
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517	0.3
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879	0.4
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224	0.5
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549	0.6
0.7	.2580	.2611	.2642	.2673	.2703	.2734	.2764	.2793	.2823	.2852	0.7
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133	0.8
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389	0.9
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621	1.0
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830	1.1
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015	1.2
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177	1.3
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4192	.4306	.4319	1.4
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441	1.5
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545	1.6
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633	1.7
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706	1.8
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767	1.9
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817	2.0
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857	2.1
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890	2.2
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916	2.3
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936	2.4
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952	2.5
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964	2.6
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974	2.7
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981	2.8
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986	2.9
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990	3.0
3.1	.4900	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993	3.1
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995	3.2
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997	3.3
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998	3.4
3.5	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	3.5
3.6	.4998	.4998	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	3.6
3.7	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	3.7
3.8	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	3.8
3.9	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	3.9

APPENDIX G

BUSINESS STATISTICS/STATISTICS II POST-TEST

/pp

Central University of Technology, Free State	DATUM: 6 Okt 2009 TYD: 17:15 DATE: 6 Okt 2009 TIME: 17:15 VAK: Besigheidstatisitiek KODE: BSS/STC22AB SUBJECT: Business Statistics CODE: BSS/STC22AB **Assessering – Toets 2** **Assessment – Test 2** SKRYFBEHOEFTES/STATIONERY: Calculators Sakrekenaars

OPGESTEL DEUR / COMPILED BY
Ms. D. H. Delport

****

ONDERRIGPROGRAM(ME)/INSTRUCTIONAL PROGRAMME:	KODE/CODE:
N DIP: BESTUUR / MANAGEMENT	BBNDBU
N DIP: KOSTE- EN BESTUURSREKENINGKUNDE / COST AND MANAGEMENT ACCOUNTING	BRNDCO
N DIP: MUNISIPALE ADMINISTRASIE / MUNICIPAL ADMINISTRATION	BONDMP

INSTRUKSIES/INSTRUCTIONS:

Duur van vraestel: 1 ½ UUR **Maksimum punte: 35**
Duration of paper: 1 ½ HOURS **Maximum marks: 35**

Question 1

Suppose for a given set of data we have the following information for two variables x and y:

$n = 20$
$\sum x = 280$
$\sum y = 160$
$\sum (x-\bar{x})(y-\bar{y}) = -225$
$\sum (x-\bar{x})^2 = 450$
$\sum (y-\bar{y})^2 = 305$

1.1 Find the product-moment correlation coefficient and interpret your answer. (5)
1.2 Calculate the coefficient of determination. (2)

[7]

Question 2

Two consumer organizations X and Y, were each asked to rank 8 brands of mobile phone for value for money. The results were as follows:

Brand of mobile phone	A	B	C	D	E	F	G	H
Ranking by X	5	2	7	3	4	1	6	8
Ranking by Y	2	4	1	6	3	8	7	5

Calculate the rank correlation coefficient. [5]

Question 3

In a series of trials, a car was driven over a fixed distance of 200 km at 5 different constant speeds, and the number of litres of fuel used in each trial was recorded. The results were:

Trial	1	2	3	4	5
Speed (km/h)	30	40	50	60	70
Fuel (L)	16	18	20	18	16

1.1 Find the least-squares regression line. (5)
1.2 Predict the fuel consumption if the speed is 80 km/h. (2)
1.3 In a certain factory, the rank correlation coefficient between the number of industrial accidents in each of 12 sections last year and the number of employees in that section was found to be $r_s = 0.63$. Is this value of r_s significant? What conclusions can be drawn ? (5)

[12]

Question 4

A retail outlet reports the following sales (in R'000) during the Christmas holiday season for each year between 2000 and 2008

Year	Sales (R1000)
2000	52
2001	37
2002	85
2003	84
2004	49
2005	110
2006	113
2007	62
2008	136

4.1 Calculate the 3-year moving averages for the data above. (6)
4.2 Calculate the trend line by making use of the zero-sum method. (5)

[11]

Formulas

$$r = \frac{S_{xy}}{\sqrt{(S_{xx})(S_{yy})}}$$

$$r^2 = \frac{(S_{xy})^2}{S_{xx} S_{yy}}$$

$$r_s = 1 - \frac{6\Sigma d^2}{n^3 - n}$$

$$S_{xx} = \Sigma(x - \bar{x})^2 = (n - 1)s_x^2$$

$$S_{yy} = \Sigma(y - \bar{y})^2 = (n - 1)s_y^2$$

$$S_{xy} = \Sigma(x - \bar{x})(y - \bar{y})$$

$$z = r\sqrt{n - 1}$$

$$\hat{y} = \alpha + \beta x$$

$$\hat{y} = a + bx$$

$$b = \frac{S_{xy}}{S_{xx}}$$

$$a = \bar{y} - b\bar{x}$$

$$b = \frac{\Sigma xy}{\Sigma x^2}$$

$$a = \bar{y}$$

Table 3 CRITICAL VALUES FOR THE RANK CORRELATION COEFFICIENT

n	Critical value
5	0.9000
6	0.8286
7	0.7450
8	0.7143
9	0.6833
10	0.6364
11	0.6091
12	0.5804
13	0.5549
14	0.5341
15	0.5179
16	0.5000
17	0.4853
18	0.4716
19	0.4579
20	0.4451
22	0.4241
24	0.4061
26	0.3894
28	0.3749
30	0.3620

Source: Adapted from W. J. Conover, *Practical Nonparametric Statistics*, New York, Wiley, 1971, p. 390