Exploring Common Algebraic Expression Challenges in a Grade 10 Mathematics Classroom

by

Sophie Musi

B.Sc ED DEGREE (NUL); ACE (UFS); PG Dip (UFS)

Dissertation in fulfilment of the requirements for the degree

MAGISTER EDUCATIONIS (Mathematics Education)

UNIVERSITY OF THE FREE STATE
UNIVERSITEIT VAN DIE VRYSTAAT
YUNIVESITHI YA FREISTATA

Faculty of Education

University of the Free State

Supervisor: Dr M.M. Moleko

April 2022

DECLARATION

I declare that the dissertation, EXPLORING COMMON ALGEBRAIC EXPRESSION CHALLENGES IN A GRADE 10 MATHEMATICS CLASSROOM, hereby handed in for the qualification of Magister Artium at the University of the Free State is my sovereign work, and I have not previously submitted the same work for a qualification at/in another university/faculty.

I hereby cede copyright to the University of the Free State.

ACKNOWLEDGEMENTS

My sincere gratitude to:

- The Lord Almighty, for giving me strength and courage to continue with this project and never giving up. “How Great Thou Art”! Glory Be To Him.
- My supervisor, Dr. Moleko, for her selfless and immense support in guiding me throughout this project. This endeavour would not have been possible without her support, motivation, and patience. Your inspiring soft words of encouragement did wonders.
- My lovely children (Mpinane and Olivier). I want to extend my deepest gratitude to them for their prayers and unconditional support during this challenging time in my life. Their belief in me kept my spirit and motivation high.
- My mother (Mampho) who supported me and encouraged me not to give up on this project. I am grateful and fortunate to have a mother of your calibre and a prayer warrior. The sacrifices you made for me to complete this journey will forever be cherished.
- My brothers (Mojalefa and Keke) for always backing me up. I know I could always bank on your support!
- My sister-in-Law (Matumo), your support and believe in me is amazing. In you, I found a loving sister, and friend. Words cannot express my gratitude towards you.
- My friend (Keketso) for her undying support and unwavering belief that I can achieve so much. Had it not been for your motivation and prayers, I would never have completed this project. You are an amazing true friend!
- I also want to extent my appreciation to the Principal, Deputy Principal, teachers, and the learners of the school who helped me in the process of collecting data. None of this work would have been possible without their valuable participation in this project.

DEDICATION

This dissertation is dedicated firstly to my late daughter **DITEBOHO (DIAKO).** May Your Soul Rest in Eternal Peace Mokoena. Secondly to my beloved daughter **Mpinane**, my son **Olivier**, my mother **Mampho**, my brother **Mojalefa** and his wife **Matumo**.

It would not have been possible for me alone without their unconditional love, support and prayers especially given the serious challenges we experienced during the period of this study. You are my pillar of strength. Many thanks to you!

ABSTRACT

The purpose of the study was to explore the common algebraic expression challenges in a mathematics classroom and to suggest the best practices or solutions to address these challenges. The study was qualitative in nature, underpinned by a Critical Emancipatory Research (CER) research paradigm. An Action Research (AR) design was adopted as a research design. Data were collected from a team of two mathematics teachers with experience of more than ten years of teaching mathematics in the FET-phase, and 43 Grade 10 learners in one of the high schools in Motheo District. The lesson observations and focus group discussions were used as the primary data collection instruments. Data were generated through active engagement and discussion among the participants using the Free Attitude Interview technique (FAI). Data were analysed by using Braun and Clarke’s (2006) six stages of thematic analysis technique. The findings revealed the following common algebraic expressions challenges and teaching issues pertaining to Grade 10 mathematics; Gap between algebra and arithmetic, Inability to represent word expression in algebraic format, Teachers’ lack of pedagogical knowledge, Teachers’ inability to explain algebraic concepts in-depth, Learners’ inability to manipulate algebraic expressions and Improper use of mathematical vocabulary/expression to mention a few. The study revealed these challenges as sources of difficulty for learning and teaching algebraic expressions in Grade 10. The study thus advocates for the need for knowledge acquisition of these common challenges and awareness thereof, in order for teachers to successfully teach the algebraic expressions.

Keywords: Algebra, Algebraic expression, Approach, Common Challenges, Models

LIST OF ABBREVIATIONS

AR	Action Research
CAPS	Curriculum and Assessment Policy Statement
CAST	Center for Applied Special Technology
CER	Critical Emancipatory Research
DH	Departmental Head
DoE	Department of Education
ET	English Translations
FAI	Free Attitude Interview
FET	Further Education and Training
FOIL	First, Outer, Inner, Last
KAT	Knowledge of Algebra for Teaching
MMAE	Multiple Means of Action and Expressions
MME	Multiple Means of Engagement
MMR	Multiple Means of Representations
NSC	National Senior Certificate
PCK	Pedagogical Content Knowledge
PLCs	Professional Learning Communities
PST	Purposive Sampling Technique
RME	Realistic Mathematics Education
SA	Subject Advisor
SADC	Southern African Development Community
SCK	Subject Content Knowledge

SWOT	Strengths, Weaknesses, Opportunities and Threats
TA	Thematic Analysis
UD	Universal Design
UDL	Universal Design for Learning
ZPD	Zone of Proximal Development

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

CHAPTER 1 :
EXPLORING COMMON ALGEBRAIC EXPRESSION CHALLENGES IN A GRADE 10 MATHEMATICS CLASSROOM

1.1 INTRODUCTION

Algebra is a fundamental topic in the school mathematics syllabus around the world, as well as in South Africa. It is used generally in all branches of mathematics and science (Seng, 2010). Moreover, the demand for algebra at various levels of education is increasing and this is because algebra is used in companies to estimate their annual budgets, which involve annual expenditures. Various stores use algebra to predict the demand of particular products and subsequently, place their orders. Individuals also use algebra quite frequently in their everyday lives without even realising it (Mulungye, 2010). For instance, this can be when shopping; calculating annual taxable income; and bank interest and instalments to loans (Egodawatte, 2011). Therefore, algebra can assist all individuals to develop mathematical reasoning that is essential in all walks of life. Furthermore, it affects the decisions one makes in many areas, such as personal finance; travelling; how often one must train to reach a certain level of fitness; cooking, and many more (Libby, 2017). Based on what algebra can do, therefore, it can be concluded that a deep understanding of algebra is essential to improve not only the understanding of the various mathematical concepts, but also life in general.

In defining the nature of algebra, Aygor and Burhanzade (2015) consider it to be a branch of mathematics, which transforms information into general algebraic expressions and equations. The general algebraic expressions and equations are often represented, using numbers and symbols. For instance, $2x + 5$ (algebraic expression) and $y = 2x + 5$ (algebraic equation). Algebra is further regarded as the first mathematical aspect that needs extensive abstract thinking, which has shown to be a problematic skill for many learners, especially learners who do not understand how algebra can be used in real-life situations (Star et al., 2015).

In my teaching experience of mathematics, I have observed that many learners have difficulty in simplifying algebraic expressions. They demonstrate the ability to master the concepts in arithmetic, such as the four basic operations. They further display the ability to solve lengthy arithmetic problems, such as 20 - 4+(7+3) x 2, however, they are reluctant to use algebraic methods, such as adding, subtracting, multiplying and dividing algebraic fractions, as well as factorising algebraic expressions (Egodawatte, 2011). On a global scale, learners' errors and misconceptions in algebraic expressions are a significant concern and South Africa is no exception (Mhakure et al., 2014). According to Mulungye (2016), the above-mentioned factors (difficulty in simplifying algebraic expressions and reluctance in adding, subtracting, multiplying and dividing algebraic fractions, as well as factorising algebraic expressions) contribute to learners' low performance in mathematics.

Makonye (2015) avows that it is one of the most challenging concepts, which often contributes to learners' low performance. The algebraic expressions (which form part of algebra) seem to be a problem for most learners to comprehend and solve. They are often not taught in a manner that develops a conceptual understanding, which is essential for further application in various topics in which they serve as a pre-requisite (Osei, 1998). For instance, due to the lack of time that is required for planning active learning strategies, teachers tend to use traditional (lecture-based) methods. According to Canter, King, William, Metcalf and Pots (2017), this traditional 'one size fits all' method does not meet the requirements of present-day inclusive classrooms and diverse learners. Tsimane (2020) discourages the traditional teaching method, because it does not aid the development of conceptual understanding, which is the ultimate goal for meaningful learning of mathematical concepts in general.

According to Makonye (2015), some of the common challenges faced by Grade 10 mathematics learners, regarding algebraic expressions, include conjoining error, cancellation error and lack of closure property. Regarding conjoining error, for example, learners may write $3x + 4b = 7xb$. This misconception seemed to be caused by early years' experience of arithmetic, whereby summing up $3 + 4$ would produce 7, in relation to cancelation error. According to Makonye and Stepwell (2016), the error is committed, because learners consider like terms or expressions in the numerator and denominator to be common, even if they are not common. For example, $\frac{ax-b}{x}$ learners may write the answer as $(a-b)$, in relation to lack of closure property. Ncube (2016) believes that learners cannot accept $a+3$ as an answer and hence they perceive it to be incomplete. Thus, they continue to complete it and write it as $3a$.

Seng (2010) posited that some of the common challenges pertaining to teaching and learning of algebraic expression include the transition gap from arithmetic to algebra. Ferrer (2020) regards this transition gap from arithmetic to algebra as a fundamental cause of learning difficulties in mathematics in general. In addition, Mbewe (2016) mentioned misapplication of rules, lack of understanding of the concept of the variable and inadequate teacher pedagogical knowledge as other common challenges in the teaching and learning of algebraic expressions (Akyuz, & Yildiz, 2019). Inadequate teacher pedagogical knowledge deprives the teacher an ability to teach learners in such a manner that promotes conceptual understanding.

The above discussions highlight the common algebraic expression challenges. To address these, there is therefore a need to understand these challenges in-depth and to also identify the root causes so that the suitable and appropriate solutions could be devised. It is against this backdrop, that this study is aimed to identify the common challenges in the teaching and learning of algebraic expressions in a Grade 10 mathematics classroom, in order to devise, suggest or identify the best practices to overcome these challenges.

The common algebraic expression challenges in the context of this study refer to challenges, such as lack of teachers' pedagogical knowledge, concept of a variable, inability to represent word expressions in algebraic format, vocabulary issue, gaps between arithmetic and algebra, as well as some common misconceptions, such as conjoining error, cancellation error and misapplication of rules.

1.2 PROBLEM STATEMENT

Algebraic expressions play a significant role in the mathematics curriculum (Egodawatte 2011). In order for learners to progress and do well in mathematics, they need to be able to read and write expressions, and to be skilled in the calculations and operations of algebraic expressions (Ferretti, 2020). Although the algebraic expressions play a huge role in mathematics, in terms of developing the learners' mathematical reasoning skills and enabling learners to master the mathematical concepts, they are often taught in an unproductive manner (Mulungye, 2010).

More often, algebraic expressions are taught in a way that focuses more on drills and procedural understanding (Aygor & Burhanzade, 2015). They are also taught abstractly in a manner that detaches them from real- life situations. These unproductive practices result in learners not developing the conceptual understanding that is necessary for them to develop personal connections between mathematical concepts. These forms of practice also render the learners passive in the process of learning. Thus, they lack understanding of algebraic expressions, which is much needed as a prerequisite, in order to master topics, such as geometry, statistics, trigonometry, functions and calculus.

The learning preferences of different learners also contribute to the manner in which learners comprehend concepts related to algebraic expressions (Ncube, 2016). This is because algebra is abstract in nature and the use of pronumerals, numbers and operational signs (Seng 2010) confuses some of the learners and inhibits their learning, which give rise to some errors and misconceptions. Therefore, it is for this reason that the study investigated the common algebraic expression challenges in a Grade 10 mathematics classroom.

1.3 RESEARCH QUESTIONS

1.3.1 Main research question

The main research question of this study is:

How can the common algebraic expression challenges in a Grade 10 classroom be addressed?

1.3.2 Secondary research questions

In order to answer the main research question of this study, the following sub-questions were formulated:

- What are common algebraic expressions challenges in a Grade 10 mathematics classroom?
- Which strategies can be implemented to address the common algebraic expression challenges experienced by Grade 10 learners?

1.4 PURPOSE/AIM OF THE STUDY

The purpose of the study was to explore the common algebraic expression challenges in a Grade 10 mathematics classroom.

1.4.1 Research objectives

Therefore, the research objectives are:

- To identify the common algebraic expression challenges in a Grade 10 mathematics classroom.
- To provide the solutions to address the identified common algebraic expression challenges experienced by Grade 10 learners.

1.5 RESEARCH PARADIGM

In this study, Critical Emancipatory Research (CER) as the research paradigm was found to be suitable, because it encourages teamwork. In this study, the researcher and the participants were both required to work as a team in order to enhance the teaching of algebraic expressions in Grade 10 mathematics. CER affords people an opportunity to voice out their views about issue(s) that affect them and be respected to also voice out their views about the solutions to their problems (McGregor, 2003). Thus, participants' views in this research would be highly regarded as that of the researcher.

The objective of CER is to inspire people who experience the problem in order to understand their circumstance and find ways to overcome it (Jordan, 2003). Hence, because of its nature, which is to allow people to talk freely (Nkoane, 2012), CER will then enable the participants together with researcher to generate ideas and as a result gain deeper meaning of the problem and various views be taken into consideration (Mahlomaholo, 2009). According to Nkoane (2012), CER permits the researcher and the participants to have a quality conversation as equal partners that will enable them to get common understanding on different perceptions regarding their challenges. Thus, in the context of this study, CER will assist the researcher and the participants to better comprehend the common challenges they encounter during the instruction of algebraic expressions in Grade 10 mathematics and what best practices they can use to overcome these challenges.

1.6 DEFINITION OF OPERATIONAL CONCEPTS

The subsequent sections provide brief definitions of the operational concepts of this study. These concepts are important since they serve as anchors on which the study is based.

1.6.1 Algebra

According to Usiskin (1988), there are four conceptions of algebra as generalized arithmetic, the study of structures, the set of techniques applied for solving some problems, and the study of interactions among quantities. Sfard (1995) defines algebra is an art of general calculations. Kieran (2004) has the view that algebra is conceptualized in five ways, such as verbally guided manipulations; the study of structure; the study of functions, relations, and joint variation; generalization and formalization, as well as a modeling language. Moreover, Kieran (1996) classified algebra based on the typical activities in which learners are engaged in as generational activities, transformational activities, and global meta-level activities. In this study, the Keiran (1996) model was adopted since generational activities deals with the formation of expressions and equations, while transformational activities deal with collection of like terms, expansion, factorisation, substitution, addition and multiplication of polynomial expressions, exponentiation with polynomials, solving equations, simplifying expressions, working with equivalent expressions and equations, etc. Lastly, the global meta-level activities are said to be activities in which algebra is utilised as a tool, but which are not limited to algebra. Such activities comprise noticing structure, studying change, generalizing, analyzing relationships, justifying, problem solving, modeling, as well as proving (Keiran, 2004).

1.6.2 Algebraic expressions

Algebraic expressions are mathematical sentences, which do not consist of an equal sign ($=$), such as $3x-5$ (Erling, Ashmore & Kapur, 2016). According to Adnan et al. (2021), algebraic expressions are mathematical sentences, which incorporate letters, numbers and operation signs. For example, $3x^2-4x+5$, $(a-3b)(a+b)$, $\frac{x-4}{2x}$ are some the algebraic examples. Furthermore, Brown et al. (2011) perceives algebraic expressions as the expressions that involve numbers, letters or variables, and operational signs and that yield a number when substituting numbers for pronumerals. For example, given that $x=3$ in $3x^2-4x+5$ then the expression becomes 20 when substituting for x. Therefore, in this study, all the definitions provided above are considered.

1.6.3 Approach

According to Merriam Webster's Online Dictionary (2021), an approach is a way, or a means adopted, in tackling a task, problem, or situation. Kapur (2018) explains approach in the context of teaching as a pedagogical approach, which involves putting into practice the high- influence teaching strategies, which are based on evidence. Mazundar (2017) referred to approach as the way teachers in the middle and high school levels normally teach mathematics (abstract first approach). For instance, polynomials are taught in an abstract manner without referring to their applicability in a real-life situation. An approach in the context of this study refers to the way the researcher tackled the issue of exploring common algebraic expression challenges in the Grade 10 mathematics classroom, using the action research approach.

1.6.4 Common challenges

A *challenge* is something new and problematic, which needs great effort and willpower to handle (Collins Online Dictionary, 2022). If something is *common,* then it is found in large quantities or it occurs frequently. Therefore, in the context of this study, a common challenge is a problematic aspect, which happens often in the teaching and learning of algebraic expressions. Some of the common challenges pertaining to the teaching and learning of algebraic expressions in Grade 10 will be discussed in detail in Chapter 2.

1.6.5 Models

In the perspective of Sitabkhan et al. (2019), in mathematics, a model refers to any picture, drawing, or object that is used to represent targeted mathematical concepts. According to Evans and Field (2020), a model is the teacher's conception of the nature and choice of teaching actions and classroom activities, contributing to his or her personal approaches to the teaching of mathematics. It incorporates mental imaginings of typical classroom teaching and learning activities, as well as the principles fundamental to teaching orientations. In addition, O'rinov Nodirbek Toxirjonovich, and Abduvaliyevich (2020), believe that mathematical models are mathematical expressions of interest for the problem being studied and can be articulated using numerical expressions or formulas, figures, graphs or geometric representations, algebraic expressions and equations, tables, pictures, etc. However, the study adopts the explanation of Evans and Field (2020), which described a model as a teacher's nature and choice of teaching strategies and classroom activities he or she can apply in the classroom during instruction. The adoption of this explanation is because the purpose of the study was to explore the common algebraic expression challenges in a Grade 10 mathematics classroom. Therefore, it is crucial that teachers select appropriate teaching strategies and classroom activities (best practices) to reduce these challenges.

1.7 SOME THEORIES THAT UNDERPIN THE TEACHING OF ALGEBRAIC EXPRESSIONS

The following sections provide a brief explanation of some theories guiding this study.

1.7.1 Sfard's theory

According to Sfard (1995), the development of algebra embraces three main stages, which are a rhetoric, syncopated and symbolic stage. In the rhetoric stage, ordinary numbers were used to describe the solution to the problem without the use of symbols or special signs. In the next stage (syncopated), mathematicians began to utilise the abbreviations for unknown quantities. The abstract (symbolic) stage (the algebra we use today), is characterised by the use of letters or symbols. Furthermore, Ng (2020) affirms that in the rhetoric stage all mathematical arguments were represented in word format. In the syncopated stage, the author mentions that letters were introduced to represent unknown quantities. The introduction of letters was done in order to find solutions for equations with one or two unknowns. However, both unknowns were expressed in terms of one letter/symbol, for example, if Mojalefa's age is y then his father's age is $20 + y$. Moreover, in the symbolic stage, solutions to a group of general equations such as $ax^2 + bx + c = 0$ were determined. Thus, the theory that underpins algebraic expressions is Sfard's theory, which states that the historical development of algebra from rhetorical to symbolic, must be reproduced in the individual so as to comprehend algebra (Machaba & Mphuthi, 2016).

1.7.2 Realistic Mathematics Education (RME)

Realistic Mathematics Education (RME), as advocated by Kusumaningsih and Herman (2018), is known as a learning theory initially developed on Freudenthal's impression, arguing that mathematics is a human activity, which should be related to real-life practices. In his point of view, Freudenthal avows that learners could not be thought as submissive recipients. However, opportunities should be presented for them to develop mathematics reasoning through their everyday experience with the supervision of the teachers (Kusumaningsih & Herman, 2018). RME theory entails three instructive principles, which are reality, level and intertwinement. The reality principle advocates that mathematics learning should commence from meaningful problems. In relation to algebraic expressions, this refers to turning contextual problems into symbolic algebra problems, rearranging, and reconstructing symbolic problems within the mathematical world. The level principle emphasises that in the process of mathematics, learners encounter different levels of understanding. With regard to learning algebra, the principle provides a link between the formal and informal mathematical world in the form of mathematical models. In the viewpoint of the intertwinement principle, learners are offered rich algebra problems in which they can use different mathematical concepts. For example, in this view, not only can the algebra domain be used, but also geometry knowledge to comprehend and solve problems (Jupri et al., 2018).

Wagner and Parker (2013) explain algebraic expression as a combination of operations and variables, such as $a+1$, $3b$ or $w - r$. Since these expressions involve two distinct systems (letters and numbers), and pose as economical, they can also cause confusion within learners' understanding of algebraic expressions. According to its underpinning theory (RME), it is advisable that algebraic expressions be taught in a manner that takes learners through the rhetoric, syncopated and symbolic stages, as this will assist in eliminating the challenges caused by imposing the symbolic stage (Ferretti, 2020).

1.8 SOME OF THE BEST PRACTICES (MODELS) IN TEACHING AND LEARNING ALGEBRAIC EXPRESSION

The following section highlights some of the models that can be put in use to guide the teaching of algebraic expressions. These models/approaches will be discussed more in Chapter 2.

1.8.1 5E guided inquiry model

According to Garzon and Casinillo (2021), the 5Es in the 5E guided inquiry model stand for engage, explore, explain, elaborate, and evaluate. The model is said to be the instructional framework that creates the learning process of the inquiry-based approach and produces positive results in learners' achievement when used to teach algebraic expressions.

1.8.2 Universal Design for Learning (UDL) model

Universal Design for Learning (UDL) is an approach to the creation of learning experiences that integrates multiple means of engaging with content and people, representing information, and expressing skills and knowledge (CAST, 2015). UDL as a teaching pedagogy is central to three planning and instructional design principles, which are: multiple means of representation to afford learners numerous ways of information and knowledge acquisition, multiple means of engagement to tap into learners' interests, as well as challenging and arousing their motivation to learn (Cumming & Rose, 2021).

1.8.3 The modified Lesh's model

Lesh's model embraces that information in mathematics can be represented in concrete form, pictorial representation, real-life experiences, verbal symbols, and written symbols (Lesh, 2003; Johnson; 2018). Furthermore, Johnson (2018) modifies Lesh 's model to include pictures in motion by technology to relate physical representations to their still-picture representations.

1.8.4 Variation theory model

According to Marton (2015), and Marton and Booth (1997), the variation theory of learning refers to variation as a necessary element in teaching, which assists learners to be aware of what is to be learned. A number of studies on how variation theory can be put at work to studies is that how the concept is processed and what aspects are made possible to perceive in a lesson has an impact on what is made possible to learn (object of learning) (Huang & Yeping, 2017; Marton, 2015; Marton & Pang, 2013; Sun, 2011; Watson & Mason, 2006).

1.8.5 Problem-solving model

Lee (2016) refers to a mathematical problem-solving strategy as a strategy that makes reference to the methods and ideas that a person come up with when finding a solution to the problem and is significant to the success of finding the solution.

1.9 RESEARCH METHODOLOGY

The qualitative methodology that was adopted in this study was carried out in the form of Action Research (AR). AR is a research approach in which participants examine their own educational practices systematically and carefully, using the techniques of research (Newton & Burgess, 2016). In the first instance, teachers as practitioners, work best on problems they have identified for themselves. Secondly, teachers become more effective when encouraged to examine and assess their own work and then consider ways of working differently. In line with this, Reed (2010) asserts that action research is an organised approach that combines action and reflection, in order to advance one's practice. Furthermore, Kemmis and McTaggart (1992) argue that to carry out action research one has to plan, act, observe and reflect more cautiously, more systematically, and more thoroughly. Thus, action research involves organised investigations embarked on to find resolutions to a problem. Through this research methodology, we were able to identify common algebraic expression challenges and suggest working solution thereof.

According to Baskerville (1999), and Kemmis, McTaggart and Nixon (2014), action research consists of five stages namely: **Plan; Act; Observe; Reflect, and Re-plan**. In line with these steps/stages of AR, the following Table 1.1 displays what was done in this study

Table 1.1: Eight steps of action research

Plan	- In this stage during the focus meeting the researcher and participants firstly identified of common algebraic expression challenges in Grade 10 mathematics classroom was done. - Secondly, ways to address the common algebraic expression challenges were suggested.
Act	- The implementation of the suggested ways to address the common algebraic expression challenges, as well as implementing other ways which may not have been suggested, happened.
Observe	- The researcher sat in class and observed the teaching of algebraic expressions. The researcher identified whether the suggested ways of addressing the common algebraic challenges have been successful or not. The researcher further identified any other way(s) implemented to teach algebraic expressions. The observation form (see Appendix D1) was used to gather data based on what was observed.
Reflect	- Teachers and the researcher reflected on what worked and what did not work regarding the teaching of algebraic expressions. - After the lesson, the researcher interviewed the learners in order to establish the source of errors and misconceptions displayed during lesson observation.
Re-plan	- Re-plan to address what did not work in the next cycle/phase/stage.

1.10 THE PARTICIPANTS AND SAMPLING

A Purposive Sampling Technique (PST) was used to select the participants in this study. PST is a deliberate selection of participants, due to the knowledge and experience they possess (Tongco, 2007). Therefore, target participants included two mathematics teachers from one school in Motheo District, and 43 Grade 10 learners. These participants were recruited from the Motheo District in the Free State. These two teachers were chosen because of their long experience in teaching FET-phase mathematics and one of them receiving a high achievement award certificate for producing a 100% pass rate in the Grade 12 mathematics results in three consecutive years. The school has been chosen based on its long term overall good Grade 12 results. The Grade 10 learners were chosen on the basis of being the ones who were taught the content and were able to describe their learning experiences.

1.11 RESEARCH INSTRUMENTS

Data collection instruments included classroom lesson observations; focus group discussions, as well as interviews. Classroom observation is a process wherein a formal or informal observation of teaching is done, while it is taking place in a classroom or learning environment (Gitomer et al., 2014). Therefore, the researcher sat down and observed the classroom teaching and recorded the teacher and learners' actions and discussed these observations thereafter. The focus group is a gathering of people who have been chosen to take part in a facilitated discussion, in order to stimulate their insights about the research topic (Nyumba et al., 2018). The focus group meetings assisted the researcher to gain a deeper understanding of the challenges experienced, as well as gathering rich information on what can be done to address the challenges observed from various viewpoints. An interview is an important data collection technique, which involves verbal communication between the researcher and the participant (Fox, 2009). In this study, the interviews were held between the researcher and the participants (learners) to reflect on challenges encountered during classroom observation.

1.12 DATA COLLECTION PROCEDURES

Focus group discussions were held in the form of reflections and focus group discussions. In these meetings, the discussions were based on the common algebraic expression challenges relating to the teaching of algebraic expressions, as well as identifying the appropriate solutions to address the challenges. The discussions were audio and video recorded. During the meetings, the conversations were initiate and facilitated by the Free Attitude Interview (FAI) technique. As stated by Meulenberg-Buskens (2011), FAI is an appropriate technique that is used when more information needs to be drawn from participants. Thus, FAI provided an opportunity for open-ended questions to be verbal, than are found in surveys that require the answer, yes or no; true or false.

1.13 DATA ANALYSIS

To analyse the data, the thematic analysis technique was used. This technique employs systemic identification; organisation; and presenting insights into patterns of meaning throughout a data-set (Braun & Clarke, 2012). Therefore, this technique assisted the researcher to visualise and create a sense of collective meanings and experiences. Data analysis followed the six steps of thematic analysis. According to Kiger and Varpio (2020), the six steps of TA are:

1.13.1 Step 1: Familiarising oneself with the data

In this step the researcher familiarized him/herself with the whole set of data. This means that the researcher must read repeatedly or listen to recorded data. Getting familiar with data serves as a valuable and important step, since it is foundational for all other steps that followed (Kiger & Varpio, 2020).

1.13.2 Step 2: Generating initial codes

After one has become familiar with data, the coding process can now commence. In this process, the researcher took notes on the potential data items and thus generated codes. A code, as defined by Boyatzis (1998), is the highly basic section, or components of the unprocessed data or information that can be considered in a meaningful way, in relation to phenomenon. Thus, coding was done manually in this study.

1.13.3 Step 3: Searching for themes

According to Braun and Clarke (2006), this step involves examining data extracts that are coded and collated in Step 2 for themes of wider significance. In addition, Kiger and Varpio (2020) mentioned that during the developmental and organizational stage of themes, thematic maps are beneficial for visually showing cross-connections between concepts, as well as among main themes and sub-themes.

1.13.4 Step 4: Reviewing themes

This step is described by Braun and Clarke (2006) as a two-level analytical step. In the first level, the researcher views the coded data to make sure it properly fits. In the second level, the researcher takes a decision of whether individual themes fit within the data-set in a sensible manner, as well as whether the thematic map sufficiently symbolizes the full body of data. In order to achieve this job, the researcher read once more the complete data-set to revise themes and to re-code for supplementary data that falls beneath the themes that have been newly created or modified in this form, then improved the thematic map accordingly.

1.13.5 Step 5: Defining and naming themes

In this step, the researcher created a definition and narrative description of each theme, including its importance to the main study question. The researcher reviewed the names of the themes to be included in the final report, as well as making sure that they were correctly and sufficiently defined (Kiger & Varpio, 2020).

1.13.6 Step 6: Producing the report/manuscript

This step entails the writing the final analysis and description of findings. In the perspective of King (2004), this step is a continuation of the analysis and description of data done in the prior steps, and not a separate step altogether.

Thus, in the light of these steps, the entire transcribed text was thoroughly read to obtain an overall and comprehensive impression of the content and context, before the abstraction process of coding can begin.

1.14 ETHICAL CONSIDERATIONS

Permission was requested from the Department of Education (DoE) to conduct the study. Permission was also requested from the principal of the school where data was generated. An application for ethical clearance was done at the University of the Free State and the study was ethically cleared (Ethical Clearance No: UFS-HSD2020/2092) (see Appendix A1). According to Terry (2007), securing the informed consent of the participants is a pivotal part of conducting research, as this recognises the research participants. Thus, all participants (teachers and learners) were asked to sign the informed consent and assent forms respectively.

The consent forms were given to the teachers for them to sign. Since the learners who were participating in the study were regarded as minors, the researcher requested their parents to sign the assent forms in order to give permission for their children to participate in the study.

Confidentiality and anonymity were considered during the process of the study. The data that was generated was strictly confidential, unless the participants wished it to be made known. Therefore, the researcher ensured that the participants were not quoted by their names; codes and pseudonyms were used. It is therefore vital that the process of research takes into consideration all possible ethical considerations. This was done in an effort to protect participants from harm and to make sure that their dignity remained untarnished.

1.15 ENSURING TRUSTWORTHINESS

According to Guba (1981), four criteria could be used for ensuring trustworthiness of the qualitative research and these are credibility, transferability, dependability and confirmability. Therefore, in this study, audio and videotape recordings, transcriptions, and the documentation of minutes by the researcher and co-researcher influenced these criteria (credibility, transferability, dependability, and confirmability) for the findings and their interpretations. Moreover, the trustworthiness of a study concerns the level of trust and confidence in the data (Lemon, and Hayes, 2020), interpretation and methods used to ensure the quality of the study (Gunawan, 2015). In contrast to the positivistic approach where the method is structured and detailed, the AR approach permits the flexibility of the researcher and co-researchers, because it allows a wide-ranging individual contribution throughout all stages of the research process. In this study, to ensure the consistency and trustworthiness, member checking was considered, triangulation, detailed transcription, systematic planning, and coding were taken into account.

1.16 VALUE OF THE STUDY

It is hoped that this research could contribute significantly to the effective teaching and learning of algebraic expressions in Grade 10 by recommending the best practices for teaching algebraic expressions in order to reduce common algebraic expression challenges identified in this study. These best practices will provide an opportunity for diverse learners to actively learn the algebraic expressions and thus improve their performance in mathematics. As learners' performance improves, not only will the community benefit, but the Department of Education as well. The study could also empower the teachers with valuable lessons and skills on how to use best practices models to design effective lessons for their learners.

1.17 CONCLUSION

In this chapter, the introduction and background of the study have been outlined. A problem statement was presented along with the main research question and subsidiary questions. The objectives of the study were formulated and directed towards the achievement of the aim of the study. The chapter further outlined the research paradigm couching the study, as well as the theories underpinning the teaching and learning of the algebraic expression. Some of the best practices models of teaching and learning algebraic expressions were briefly highlighted. The research design and methodology were briefly described. Issues of trustworthiness, the value of the study and ethical considerations were indicated.

The next chapter is the literature review, which outlines in detail the research paradigm couching the study. It provides more information on the theories fundamental to teaching and learning of algebraic expression, as well best practices models that can be use in this regard.

CHAPTER 2 :
LITERATURE REVIEW

2.1 INTRODUCTION

The study sought to explore the common algebraic expressions challenges in a Grade 10 mathematics classroom. This chapter discusses the paradigm that couches this study, which is critical emancipatory research. Moreover, the choice of CER as a paradigm underpinning the study, is justified in the sub-headings namely, its origins, objectives, realism, the role of the researcher and the association between the researcher and the researched.

Definitions of operational concepts are provided in this chapter so that they can be understood as central pillars on which this study is anchored. Finally, an extensive review of related literature in relation to objectives of the study are covered so to inform the best practices at the later stage. These best practices derived from internationally, continentally and locally, will serve as basis for the teaching and learning algebraic expressions in Grade 10.

2.2 RESEARCH PARADIGM

Kivunja and Kuyini (2017) both have a view that in educational research the phrase ‘research paradigm’ defines a researcher’s way of viewing the world, or a researcher’s reflections on the beliefs she/he has about the kind of world she/he lives in or wishes to live in. In addition, Lassa and Enoh (2000) explained that paradigm is a set of theories arranged together to provide a foundation or a backup for explanation, viewing or designing phenomena. The role of the paradigm is to tie the researcher to current knowledge (Labaree, 2013). Hence, allowing the researcher to move from a simple description of an observed phenomenon to taking a broad view about numerous aspects of that phenomenon. It is thus noteworthy for the researcher to choose the paradigm that will couch their study, because it avails main standpoints and the way to the research (Groenewald, 2004). Based on this therefore, the paradigm that underpins this study is

Critical Emancipatory Research (CER). According to Groat and Wang (2001), CER is considered to be an approach that encompasses critical theory.

The following sections provide more details on CER as a paradigm couching this study starting with its origins.

2.2.1 Origins of critical emancipatory research

Critical Emancipatory Research (CER) was first developed by the Frankfurt school (Higgs, 2007). These authors opposed the positivist concept behind science as being the only way of attaining the truth. Their argument was that people create knowledge through their life experiences. Furthermore, they asserted that the most important search for knowledge should be based on an aspiration to enhance the quality of human life. In criticizing and challenging the primitive scholars, these scholars based their opinion of the truth with regard to what could be experienced and measured. In the same breath, as other critical authors, their claim was that positivists' idea ignores the fact that it is people who create knowledge in the perspective of critical theory and serve to reduce the people suffering in the world (Moleko, 2018). Therefore, there was an emergence of CER as a way of improving people's lives.

Engagement is the cornerstone of CER for all people with the inclusion of the oppressed, marginalized and those deprived of the freedom to take part in activities, which involve them. In the perspective of Mahlomaholo (2009), CER contends that society must be treated with respect, since it advocates for freedom, peace, team spirit, equity, and hope. Moreover, emancipatory approaches inspire a rich, diverse and innovative body of knowledge focused on participation, inclusion, and collaboration (Crawford, 2022). Therefore, based on these principles, both researcher and the participant identified common algebraic expressions and some strategies that could be implemented to address those challenges. Moleko (2018) also assert that everyone should be given the opportunity to be heard and respected. Therefore, there are two issues that CER promotes namely, "respect for the side-lined and the need for the side-lined voices to be heard" (Moleko, 2018). My take with respect to the side-lined, is that people individually view, feel and experience ideas differently hence, regardless of the prevailing circumstances, they should not be omitted in discussions. In my view, the issue of letting the marginalised speak for themselves is that everyone has something to voice out, hence, despite their status quo, they must be given a platform to voice out their concerns with issues affecting them.

According to Campanella (2009), CER aids appreciating peoples' capability of voicing out their thoughts and not as ordinary things, which can never contemplate or do anything for themselves. Respecting people makes them to speak freely and be listened to without being judged depending on their status.

Thus, the choice for CER in this study is based on its ability to encourage the development of a directive for action, and for what constitutes ethical practices, as well as a close relationship between the participants and the researcher. Furthermore, CER enforces social justice and improves the outline of prolonged epistemologies (Acharya, Belbase, Panthi, Khanal, Kshetree, & Dawadi, 2022). It provides with the opportunity for possible multiple solutions to be attained in an effort to enhance performance in relation to algebraic expressions, as it enforces collaboration among people in teamwork. Hence, through CER as a paradigm, the research question for this study could be answered in full.

2.2.2 Objectives of critical emancipatory research

According to Moleko (2014), CER focuses on the causes of dominance, rather than its signs. This is because CER is based on an anti-dominance philosophy and is a paradigm used to recognize and change the root causes of dominance (Ledwith, 2007). CER is intended to transform and advance social equality and social interactions by creating platforms of engagement, empowerment and radical change for the oppressed (Moleko, 2014). A multiple of authors mentioned that CER's objectives include transformation of participants' world and reality for the improvement of their lives by generating appropriate and treasured knowledge (Biesta, 2010; Nkoane, 2012; Mahlomaholo, 2012; Merriam & Ntseane, 2008; Piper & Piper, 2009). In the perspective of education, Kincheloe and McLaren (2011) indicate that CER gives the participants, especially those who were side-lined before in the arrangements of the school, the chance to gain understanding of how classroom activities unfold. Consequently, in the setting of the classroom, CER advocates for a joint change in terms of activities that must be planned in collaboration, while all learners are given access to these activities, as well as increasing their participation in these activities.

2.2.3 The role of the researcher

In the context of CER, the role of the researcher is to engage participants in the study so as to grant them power, transform and set them free from the not-so-useful performances and views and as an end result meet the needs of a real-life situation (Moleko, 2014). The role of the researcher is also to assist the participants to be in charge of their situation by improving it and moreover, owning the results of the study of their own work. According to Shangase (2013), CER is participative and collaborative in nature, therefore, the researchers' role is to create platforms for the debates on the strategies to be applied, and responsibilities to be allocated. In the same breath, Ledwith (2007) considers the role of the researcher to be able to engage the participants in the process of change by working together. However, Capanella (2009) asserts that it is also more important for the researcher to be sincere, and hence abide by ethical issues and eventually establish mutual reliance among the participants. Moleko (2014) affirms that scholars must be considerate, tolerant, and careful of the issues confronting the communities, and should permit the participants to conveniently voice out these issues. Furthermore, it is vital that scholars work with participants rather than working on them, giving them an opportunity to be more human and develop an ability to respect and listen to one another. Thus, sustaining responsibility and modesty amongst the participants.

In the context of this study, CER speak about the course of emancipatory practise for social justice and democratisation and transformation of society (Tlali, 2013), the researcher has to guarantee that all the participants jointly take part in the process of planning strategies to address the challenges of teaching and learning of algebraic expressions in Grade 10.

2.3 ALGEBRAIC EXPRESSIONS

An algebraic expression is a mathematical expression, which encompasses variables, constants and operational signs (Seng,2010); for example, $3x - \frac{7}{x} + 8, y^2$, $\frac{2(a+b)}{5b^3}$,$\sqrt{a^2 + b^2} - 4$ or $(x - 3)(3x + 7)$ just to show a few. Star et al. (2015) explain an algebraic expression as a symbol or a group of symbols for variables, numbers, and arithmetic operations at work to represent a quantity; for instance, the unknown amount of taxi fare can be represented as x rands, the unknown area of a rectangle can be given by $a(a - 2)$ if a represents length of a rectangle and $(a - b)$ represents the width.

According to Machaba and Mphuthi (2016), algebraic expressions are mathematical sentences which do not have an equal sign (=) (e.g., $4x + 5$, $2x + 3y$, $9x^2$). They further state that in an algebraic expression there are numbers or variables or a combination of numbers and variables with operation signs. Algebraic expressions play an important role in the mathematics curriculum and in mathematics in general. In order to progress and do well in mathematics, learners need to be able to read and write expressions, and to be skilled in computations and manipulations of algebraic expressions. Hence, Seng (2010) has a view that for learners to be able to simplify or manipulate algebraic expressions precisely, they need to understand the concept of variables and the algebraic terms well. Moreover, as stated in Brown et al. (2011), an algebraic expression involves numbers, parentheses, operation signs and pronumerals. It is further stated that it is an expression that becomes a number when numbers are substituted for the pronumerals. According to Erling et al. (2016), algebra and its expressions are time and again considered to be the language of mathematics, because they describe relationships.

Furthermore, some scholars have defined algebraic expressions as mathematical sentences, which incorporate letters, numbers and operational signs (Adnan et al., 2021). For example, $2x + y$, x^2 and $\frac{3a-7}{2}$ are all expressions. An algebraic expression is not an equation, because it does not have an equal sign; it is also not an inequality, because it does not contain an inequality sign. However, in the equation like $2x^2 - 8 = 3$, the left-hand side represents an algebraic expression. The right-hand side can also be regarded as an expression, since it can be expressed as $3x^0$. Moreover, in line with definition of

Brown et al. (2011), who highlight that an algebraic expression can be a numerical value resulting from substituting numbers for variables in an expression, the right-hand side in the equation $2x^2 - 8 = 3$ can be regarded as an expression. The following examples (Table 2.1) display application of algebraic expressions in different topics of mathematics, which in turn put emphasis on the importance of learners' understanding of algebraic expressions as fundamental. These examples (Table 2.1) indicate the different representations of the algebraic expressions across the mathematics topics, which then signify the need for reinforcing its understanding to ensure that learners can master the different mathematical topics.

Table 2.1: Mathematics topics that show application of algebraic expressions

Topic	**Example**
Exponents	$\frac{a^3.a^2}{a^7}$
Number patterns	$\boldsymbol{a + (n - 1)d}$ [right hand side part extracted from the equation $T_n = a + (n - 1)d$]
Functions	$\boldsymbol{3x + 1}$ [right hand side part extracted from the equation $f(x) = 3x + 1$]
Analytical geometry	$\boldsymbol{\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}}$ [right hand side part extracted from the equation $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$]
Trigonometry	$\boldsymbol{cos^2x + sin^2x}$ [right hand side part extracted from the equation $cos^2x + sin^2x = 1$]
Calculus	$f'(x) = \lim_{x-0} \frac{f(x+h) - f(x)}{h}$

(Taken from Mind Action Series Mathematics Grade 12 Textbook)

The most vital skill in algebra is understanding how to simplify and recognize algebraic expressions (Machaba & Mphuthi, 2016). According to Van de Walle et al. (2014), learners find it a worthless task to simplify equations and solve for x, because they find simplifying of algebraic expressions challenging and teachers not making it easy for them to understand. Van de Walle et al., (2014) further indicate that algebraic expressions and equations must be taught in a meaningful manner. Learners are mostly confused with the instruction 'simplify', and most of these learners have no idea that simplify means to rewrite a given expression into an easier one. Moreover, those who know the meaning of this instruction do not know how to manipulate the expression in order to simplify it. It is also worth noting that teachers need to teach this topic in a manner that will accommodate diverse learners' needs.

2.3.1 THEORIES THAT UNDERPIN THE TEACHING OF ALGEBRAIC EXPRESSIONS

In this study, focus is placed on two theories, namely Sfard's theory of transition from an operational to structural approach in computational mathematics (transition from rhetorical to symbolic algebra) and Realistic Mathematics Education (RME), which are some of the theories that are predominantly used to guide and inform the teaching of the algebraic expressions.

2.3.1.1 Sfard's theory

In the viewpoint of Sfard's theory, historical development of algebra, which is from a rhetorical to symbolic state must be reproduced in the individual to comprehend algebra (Machaba & Mphuthi, 2016). Various authors have shown that there are three phases in the development of algebra (Ng, 2022; Sfard, 1995; Windsor, 2009). These phases are **rhetorical, syncopated and symbolic algebra**.

2.3.1.1.1 Rhetorical phase

In the **rhetorical algebra,** symbols are not used; this phase is described by a verbal description. For instance, the question may be phrased like this,

> *"Think of a number, divide the number by two, and add three, the result is seven."*

The solution to the problem would also be represented in words. Ng (2022) and Hart (2021) also mentioned that in the rhetorical period there were no symbols and abbreviations to represent numbers, therefore, problems, calculations and solutions were written using words.

2.3.1.1.2 Syncopated Phase

Syncopated algebra is described using some abbreviations for the regularly recurring quantities and operations. In this phase, the concern was to solve equations e.g., linear equations, simultaneous equations, quadratic equations and cubic equations. For instance,

> *solving for the unknown in the equation $2x - 4 = 5$ as well as solving for x and y in the equations $x + y = 6$ and $x^2 - y^2 = 12$.*

In this phase, the focus was to find solutions to the unknowns, but not determine the general solution (Ng, 2022). In addition, although letters were used in the syncopated phase, they did not function to express generality (Kama, 2020).

2.3.1.1.3 Symbolic Phase

Symbolic algebra, which is the algebra we use today, is described by letters (Didis & Erbas, 2015, Winsor, 2009; Ng, 2022). The concern in this regard was to find the general representation of a solution, such as generalizing the nth term of a sequence ($T_n = ar^{n-1}$) or finding a general formula for linear functions ($y = mx + c$), quadratic functions ($y = ax^2 + bx + c$), and or formula for calculating speed ($S = \frac{D}{T}$). According to Hart (2021) introducing symbols made concepts that were already known much more comprehensible and standard. One more benefit of using symbols is that they make new ideas, concepts, facts, and connections viable. However, according to Ng (2022), symbolic expression is only beneficial when the symbols are supported by rich associations and meanings. This means that providing learners with rich context problems from real-life situations is helpful for them to understand algebraic expressions.

In the context of this study, this theory (Sfard's theory) will assist by providing the teachers with different ways in which the algebraic expressions can be represented and be solved. Therefore, the algebraic expressions can be presented in word format as in the rhetorical phase, as well as in symbols as in the syncopated phase or symbolic phase. This will enable the teachers to think of the varied ways in which they can teach algebra, and this will improve the teaching of algebraic expression.

2.3.1.2 Realistic mathematics education theory

Realistic Mathematics Education (RME), as advocated by Kusumaningsih & Herman (2018), is known as a learning theory ultimately developed on Freudenthal's(1977) impression, arguing that mathematics is a human activity, which should be related with real-life practices. In his point of view, Freudenthal avows that learners could not be thought of as submissive recipients. However, opportunities should be presented for them to develop mathematics reasoning through their everyday experience with the supervision of the teachers (Kusumaningsih & Herman, 2018). The RME theory is based on six instructive principles, which include activity, reality, level, intertwinement, interactivity, and guidance which will be discussed below shortly. The characteristics of RME, namely "real-world" context, models, learner production and construction, interactive and intertwinement, indicates that RME starts with real problems so that learners can use the previous experience directly. Also, RME can also help learners develop a comprehensive particular concept in mathematics, as well as apply mathematical concepts to new fields and the real-world problem. RME is one of the approaches, which addresses problems caused by traditional and abstract mathematics learning (Bray & Tangney: 2015).

2.3.1.2.1 Activity Principle

In the learning process, the activity principle of RME regards learners as active partakers. Mathematics is perceived as a human activity, hence learning mathematics means doing mathematics (Fauzana, Dahlan & Jupri, 2020). According to Van den Heuvel-Panhuizen and Drijvers (2014), this principle advocates that mathematics is acquired by 'doing' mathematics. This principle concurs with situations of permitting learners to be confronted with problem situations, which aid them to create different mathematical instruments and understandings by themselves (Van den Heuvel-Panhuizen, 2000).

2.3.1.2.2 Reality principle

The emphasis within this principle is that mathematics learning should commence from meaningful real-world problems. This refers to turning contextual problems into symbolic word problems, rearranging and reconstructing symbolic problems within the mathematical world (Kusumaningsih & Herman, 2018). In the context of Van den Heuvel-

Panhuizen and Drijvers (2020), this principle can be viewed in two ways. The first view is that, it can be seen to express the importance, which is attached to the goal of mathematics education with the inclusion of learners' ability to use mathematics education to solve problems in the real-life experiences (Ginting et al., 2018). The second view is that mathematics education should start from the real-life situations, which learners are familiar with so that they can attach meaning to the mathematical constructs they encounter when solving these problems. It is further argued that instead of starting with definitions and abstract teaching, in RME, teaching starts from rich context mathematical real-life problems (Hirza et al., 2014).

2.3.1.2.3 Level principle

The level principle emphasizes that in the process of learning mathematics, learners encounter different levels of understanding as they are involved through discourse with real-life problems (Van den Heuvel-Panhuizen & Drijvers, 2020). With regard to learning algebra, the principle provides a link between the formal and informal mathematical world in the form of mathematical models (Barnes, 2005). The use of models means that problems or notions in mathematics can be expressed in the form of models; both models of real-life circumstances and models leading to the nonconcrete level. In the level of concrete, learners can be viewed that they have mastered a concept if they recognise the object they have never known. Learners' capability in problem solving needed to be improved, especially the ability in improving problem-solving techniques and strategies, and the ability in problems synthesis. The use of an inappropriate model can make a boring class, while making it tough for the learners to apprehend the concept, and finally decline learners' inspiration in learning.

2.3.1.2.4 Intertwinement principle

Lastly, intertwinement refers to different topics incorporated to create an insight of a particular concept contemporaneously (Hirza, et al., 2014; Lerman, 2018). This means that the mathematics concepts should not be considered as isolated areas of the curriculum and not as individual activities. For instance, learning areas such as geometry, measurement and data handling are considered as heavily interacted.

2.3.1.2.5 Interactivity Principle

The interactivity means that learning process activities are built by the interaction of learners with learners, learners with teachers, learners with the environment and so on. Hence, learners are provided with the opportunity to work in collaboration, since interactivity is also a social activity (Kusumaningsih & Herman, 2018; Lerman, 2018). Hence, RME favours whole-class discussions and group work, which offer learners opportunities to share their strategies and discoveries with others. This enables group effort and cooperation among learners. In this way, learners can get ideas for improving their strategies. Likewise, interaction recalls reflection, which empowers learners to attain a higher level of understanding.

2.3.1.2.6 Guiding Principle

The guiding principle talks about rediscoveries of the already discovered mathematical concepts or theories. Teachers afford learners the opportunity to rediscover ideas, such as those that have been conceded by former mathematicians (Fauzana, Dahlan & Jupri, 2020). According to Hirza et al., (2014), one thing the teachers can do when guiding the learners, is choosing the best instructional approach. The contribution of learners means that problem-solving schemes or notions are primarily discovered by contributions from learners (Kusumaningsih & Herman, 2018). Thus, teachers' duty is to be proactive in guiding learners in learning to acquire concepts that are in accordance with mathematical principles.

In a similar way, this theory will assist in this study by emphasizing the importance of representing content matter in different ways to accommodate the needs of diverse learners. Since both Sfard's theory and RME are the theoretical frameworks underpinning this study, they link well with each other, because both advocate that learners need to be taught from rich context real-life, as well as representing information in different ways.

2.4 RELATED LITERATURE

This section provides a literature review on the teaching and learning of algebraic expressions in Grade 10. The section commences by highlighting the literature on some of the best practices/models/approaches that can be used to guide the teaching and learning of algebraic expressions. The section further exposes some of the errors, misconceptions and gaps regarding the algebraic expressions. The subsequent sections are therefore aligned with the study research questions as outlined in Section 1.3.

2.4.1 SOME OF THE BEST PRACTICES / MODELS / APPROACHES THAT GUIDE THE TEACHING OF ALGEBRAIC EXPRESSION

The following section highlights the literature on some of the best practices/models/approaches that can be used to guide the teaching and learning of algebraic expressions.

2.4.1.1 5E guided inquiry model

The 5E **guided inquiry model** is explained to be the instructional framework that creates the learning process of an inquiry-based approach where learning cycles referred to 5Es are implemented (Bybee et al., 2006; Tural et al., 2010). According to Garzon and Casinillo (2021), the 5E **guided inquiry model** produces positive results in learners' achievement when used to teach algebraic expressions. The 5Es represent ***e***ngaged, ***e***xplore, ***e***xplain, ***e***laborate and ***e***valuate. Since the goals of the 5E guided inquiry model focus on the active engagement of learners in knowledge acquisition and exposure to higher mathematical thinking level, it is therefore regarded as the most practical constructivists method, as it permits learners to analyse and synthesise knowledge in the constructivist classroom. This approach epitomises and embraces the RME theory which underpins this study, in that it requires learners to solve problems in the context of real-life experiences. Thus, it encourages learners to own and investigate real-life problems.

2.4.1.1.1 Engage

The first stage of the 5E Inquiry Model is called *Engage*. It is the stage where the teacher introduces a problem or a contradictory event in a familiar context that learners cannot explain with their existing knowledge, because it falls too short, or it is not suitable with their new understanding. This gives rise to a cognitive conflict, which in turn motivates learners. Thus, it provides an opportunity to trigger and stimulate the learners' previous knowledge. Strategies such as asking learners probing questions specifically designed to trigger designed to trigger recall or providing learners with relevant context can assist them use prior knowledge to promote the retention and integration of new knowledge (Ruiz-Martin & Bybee, 2022). Research has shown that helping learners to recall significant knowledge from previous lessons or their own experiences can ease the integration of new information (Hattan, Singer, Loughlin & Alexander, 2015; Moleko, 2021, Ruiz-Martin & Bybee,2022)

Hence, this stage of 5E Model can assist teachers in addressing the common algebraic expression challenges faced by grade 10 learners if they (learners) are engaged in the lesson activities.

2.4.1.1.2 Explore

In this stage, learners investigate phenomena, observe, and share their observations, propose explanations, and discuss their understandings. The teachers' role in this regard is to facilitate by guiding and scaffolding learners' thinking. The goal of this is to promote connections between learners' pre-knowledge and the new knowledge to be acquired (Ruiz-Martin & Bybee, 2022). To achieve this goal, learners are engaged in activities that follow a guide inquiry-based approach, which promote thinking and sense-making during the learning activities.

2.4.1.1.3 Explain

In this stage, the new information acquired in the *Explore* stage is formalised and gives a platform for teachers to introduce this new information or ideas formally and assist learners to arrange it in ways that will enable retrieval when needed. According to Garzon and Casinillo (2021), in this stage, learners express their learning back into the group to share with others.

2.4.1.1.4 Elaborate

The elaborate stage incorporates activities that demand learners to use the knowledge they have learned to solve new problems in new environments. These activities give learners a chance to apply new knowledge they acquired in multiple contexts (Garzon & Casinillo, 2021). Research suggest that exposing learners to multiple contexts advances deeper understanding. This may be because they (learners) are more likely to extract the significant features of ideas and to create their own representation of knowledge (Moleko, 2021)

2.4.1.1.5 Evaluate

At this stage, the learners' knowledge and skills are assessed by the activity that challenges their understanding. However, assessment can be done throughout the other stages of inquiry model as a formative assessment that provides opportunities for feedback throughout the whole learning process (Ruiz-Martin & Bybee,2022). It is recommended that the evaluation stage be integrated throughout the learning process (Bybee, 2015).

2.4.1.2 Universal Design for Learning (UDL) Model

Another best approach/model that can used to productively teach algebraic expressions, is Universal Design for Learning (UDL). According to Katz and Sokal (2016), UDL is an inclusive pedagogical framework that comprises of the set of principles for designing curriculum that offers diverse learners with equal opportunities to gain knowledge (TEAL, 2010). UDL is an approach to the creation of learning experiences that integrates multiple means of engaging with content and people, representing information, and expressing skills and knowledge in varied ways. UDL gives all individuals equal opportunities to learn and provides a blueprint for creating instructional goals, methods, materials, and assessments that work for everyone - not a single, one-size-fits-all solution, but rather flexible approaches that can be customized and adjusted for individual needs (Center for Applied Special Technology, 2013; TEAL, 2010). Multiple authors note the potential for pedagogies derived from the Universal Design for Learning (UDL) framework to benefit all students, including the students with disabilities in science (Basham & Marino, 2013). UDL is a framework for designing instructional techniques that minimizes, reduces, or eliminates learning barriers for all individuals. The main purpose of UDL is to use a variety of teaching methods to remove any barriers to learning and give all learners equal opportunities to succeed. It is about building in flexibility into the teaching that can be adjusted for every learner's strengths and needs.

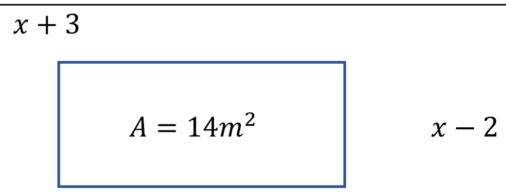
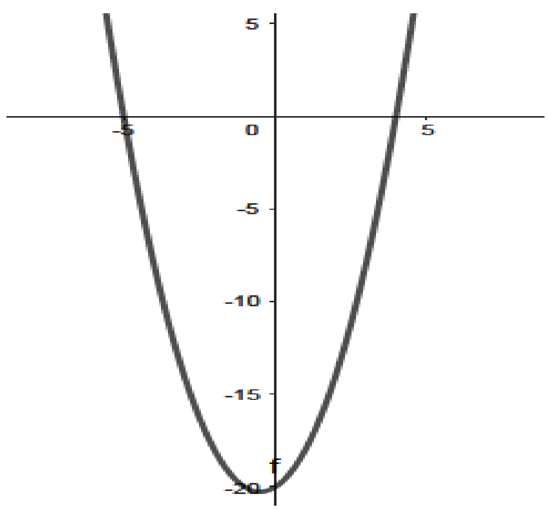
UDL framework is used more often in the initial state to address learners' preferences and needs, rather than at the latter stages, which call for adaptation or retrofitting. It is an educational practice that works in unison with differentiated instruction. It asserts that lessons and environments should be designed to be accessible to all learners, regardless of their learning needs. Differentiation recognizes that not all learners learn in the same way and therefore we need to actively plan for those differences. There are three principles of UDL, namely, Multiple Means of Representation (MMR), Multiple Means of Action and Expression (MMAE), and Multiple Means of Engagement (MME). These principles are linked with the brain networks in neuroscience as follows; MMR is linked with recognition brain network, MMAE is linked with strategic brain network, whilst MME is linked with the affective brain network (Al-Azawei et al., 2017). In this study, only one principle of UDL will be considered namely, Multiple Means of Representation (MMR).

2.4.1.2.1 Multiple Means of Representation (MMR)

Teachers need to present subject matter by multiple means. Therefore, it means that teachers have to design multiple approaches to explain a concept (e.g., text, discussions, diagrams, pictures, relating concept of learners' lives, etc.) This is aimed at assisting learners to comprehend and master the subject matter with minimum effort (McGuire, 2011). Hence, MMR provides options for perception, language and symbols, and comprehension. The idea is that students with various disabilities and needs may all require different ways of approaching concepts, while others may simple comprehend more proficiently in a pictorial or auditory presentation rather than in printed format. When presenting information, it is important to use different modes of content delivery so that all learners can be reached (McGuire & Scott, 2006). As an example, when teaching and using symbols and vocabulary, it is essential to note that what might assist one learner, may confuse another. Furthermore, it is important to actively engage learners in the learning process so that they can transform knowledge into useable fragments to attain understanding (Moleko, 2021). Achievement in all these is based on the scaffolding of information, stimulating or supplying of prerequisite knowledge, highlighting of patterns and main concepts, guiding of knowledge processing, visualization, and handling, and maximizing the generalisation of knowledge (CAST, 2011).

The following Table 2.2 is an example of multiple means of representation wherein one problem could be represented in multiple formats.

Table 2.2: Multiple means of representation of a problem

Representation	Example	Explanation
Word problem	"If Thapelo has a rectangular plot of length three more than x and width two less than x find the exact dimensions of the plot if the area of the plot is $14m^2$	The problem is stated and key points are given.
Diagram		Learners may represent the problem in a diagram (showing sides in the form of expressions).
Equation	$(x + 3)(x - 2) = 14$ $x^2 + x - 20 = 0$ $(x + 5)(x - 4) = 0$ $x = -5 \text{ or } x = 4$	The problem can be represented in an algeraic equation form whereby the left-hand side is an algebraic expression, which learners must be able to simplify.
Graph		The word problem can also be represented in a graphical form and thus produce a quadratic equation.

2.4.1.2.2 Multiple Means of Action and Expression (MMAE)

It is vital that in the learning process learners are able to express their understanding of the learning content. Hence, differentiated teaching approaches should be used to cater for learners' individual preferences (Johnson-Harris & Mundschenk, 2014). UDL admits that learners interact with the material in a different way, and that they should be afforded the opportunities to demonstrate what they have learned in ways suitable to their individual preferences (Dell et al., 2012). This therefore means, there should be *varied* methods of response (CAST, 2011). For instance, usage of tools and technology should be at optimal level and the expression and communication should allow interaction with physical manipulatives. Learners' expression should be allowed in the form of text, drawing, storyboards, speech, and other mediums to enable them to demonstrate their learning processes (Mavrovic-Glaser, 2017).

2.4.1.2.3 Multiple Means of Engagement (MME)

Various ways and actions should be applied to stimulate and motivate learners in order to actively engage them in the learning process (Moleko, 2021). MME provides options for sustaining effort, promotes expectations and beliefs that optimize motivation (McGuire, 2011), since it makes the teacher to be motivated towards delivering a meaningful lesson. Since MME assists with various ways and actions of learner engagement in the concept development, it enables the teacher to facilitate personal coping skills, as well as help learners to set meaningful goals towards mathematics, and other school-related subjects (Kartz, 2016). Furthermore, it enables learners to be able to assess themselves and reflect on their learning process. The idea is that what interest individual learners differs significantly. Hence, tasks have to be varied in such a way that they can be interesting to various learners. The learners should therefore be engaged in meaningful goal, tasks and activities.

Initial motivation and self-regulation skills of individual learners vary, thus external forces must provide ways to support these differences. There must be provision of reminders on goals, prompts for desired outcomes, differentiation within the activities, and varied degrees of freedom (Moleko, 2021). Students must be given a platform to communicate and collaborate. Moreover, cooperative learning should be encouraged and supported (CAST, 2011).

In line with the above, the UDL model requires the use of multiple representations in the teaching of algebraic expressions. It requires learners to be given varied opportunities to demonstrate what they have learned. Furthermore, it requires teachers to vary the engagement strategies in order to make content appealing to a diverse learner population.

2.4.1.3 The modified Lesh model

Johnson (2018) advocates for the use of the modified Lesh's model, which the author (Johnson) describes as to embrace concrete representation, pictorial representation, real-life experiences, verbal symbols, and written symbols (Lesh, 2003) as shown in Figure 2.1 below.

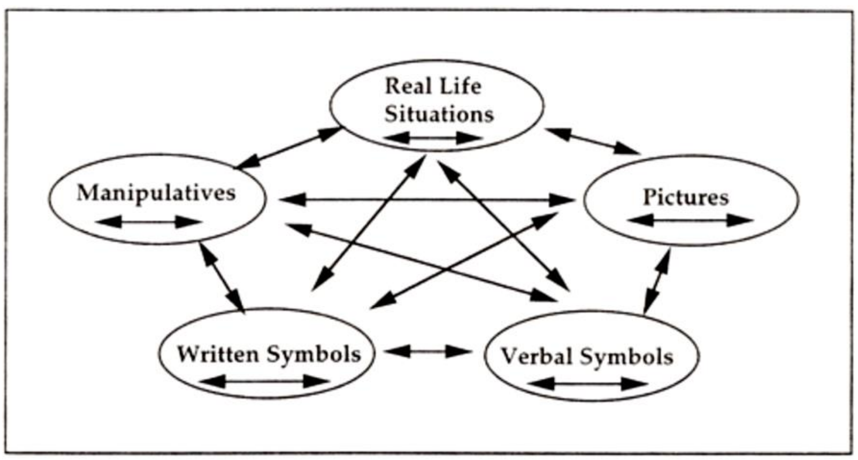


Figure 2.1: Lesh Translational Model

The author (Johnson) modified this model to include pictures in motion through the use of technology to relate physical representations to their still-picture representations. Hence, closing the gap of understanding the transference from one representation to another, possibly to a more abstract one. For example, the use of GeoGebra can help teachers to illustrate how parameters affect the shape of graphs and hence affords learners an understanding of the difference between parameter and variables. Thus, Figure 2.2 below shows the modified Lesh's model.

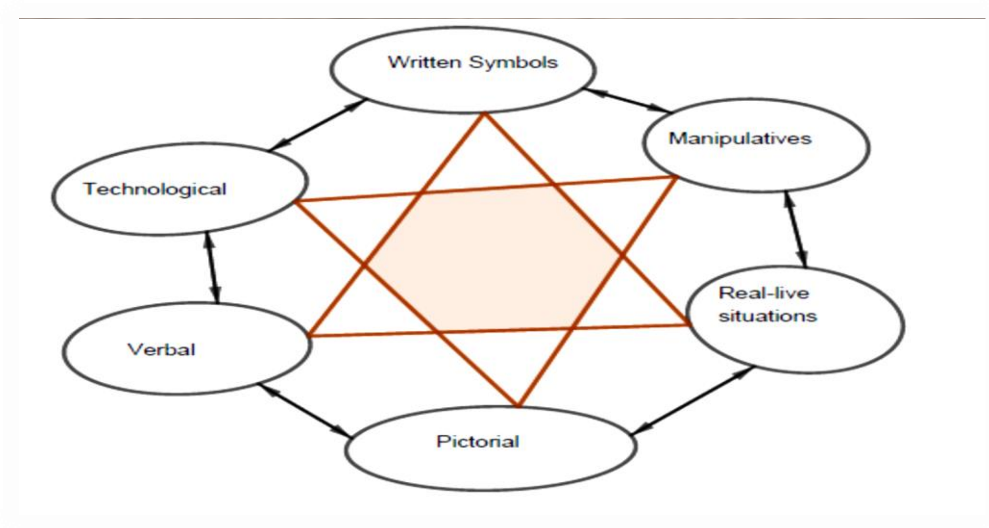


Figure 2.2: Johnson Mathematical Representation Model Air Game

Since conceptual knowledge is critical in attaining mathematical proficiency (Crooks & Alibali, 2014), physical manipulatives and graphical pictorial representations of mathematical ideas are regarded as helpful (Johnson, 2018). So, Johnson further states that the ability for teachers to use various representations and translate from one to another with relative ease can benefit learners significantly. In addition, Lamon (2001) attests that this ability is foundational in learners' mathematical proficiency. In line with the principles of RME the Lesh's model and the modified model embraces the use of real-life context during instruction, as it is fundamental for mathematical proficiency.

This model requires the algebraic expressions to be taught in a manner that promotes conceptual acquisition of knowledge, since it calls for multiple representation of concepts, which is also in line with UDL's principle (MMR).

2.4.1.4 Variation theory model

According to Marton and Booth (1997), the variation theory is a learning and experience theory that describes in a certain way how a learner might come to perceive, understand, or experience a given phenomenon or object of learning. According to Attorps et al. (2011), the variation theory embraces two main principles, which are firstly the object of learning, which in this study is simplification of the algebraic expressions. Secondly, the object of learning is put into practice and understood differently by learners. Ekawati, and

Lin (2014) explain the object of learning to be the understanding of a particular subject teachers desire learners to understand, which is more often aimed at short-termed educational goals and the general aspect, which relates to the ability to improve through the acquisition of the subject matter in question.

Lo (2012) mentioned that in relation to mathematics teaching methods, the variation theory calls for learners' centredness and active learning. Furthermore, Ekawati and Lin (2014), attest that the variation theory requires teachers to draw out and apply learners' existing knowledge as they plan classroom activities. They also emphasise on teaching in-depth the content by choosing examples that afford learners an opportunity to discern significant aspects of the content, as well as selecting teaching methods that are learner centred, focusing on patterns of variation like instruction by contrast.

2.4.1.5 Problem-solving model

Research has shown that problem-solving strategy is one of the fruitful strategies that can be used in mathematics to solve problems (Daulay, & Ruhaimah 2019; Shirali, 2014). According to Lee (2016), a mathematical problem-solving strategy is a strategy that makes reference to the methods and ideas that a person come up with when finding a solution to the problem and is significant to the success of finding the solution. Thus, the most comprehensible problem-solving strategies are Polya's problem-solving strategies (Nurkaeti, 2018; Lee, 2016), as shown in Table 2.3 below.

Table 2.3: Polya's Problem-solving Strategies

Step	**Problem-solving strategy**	**Note**
Step 1	Understanding the problem	Must clearly know what the question means, realise the key points and context of the problem, and be able to find the answer.
Step 2	Devising a plan	Clearly know the relationship between the key points of the problem, select a suitable approach and devise a plan for solving a problem, which is the most major task in the process of problem solving.
Step 3	Carrying out the plan	Practically calculate by yourself and find the answer.
Step 4	Looking back	Look back the entire process of problem solving, check the computation of the problem, discuss the meanings of the problem.

The implementation of Polya's problem-solving strategies, in conjunction with teachers' probing interventions has shown significant improvement in learners' performance in the study conducted by Lee (2016). Thus, the scholar claims that guiding learners to find the solutions for themselves with the use of a problem-solving strategy does not only assist learners to improve performance, but also helps them to gain deeper understanding of the concepts. According to Nurkaeti (2018), in solving the word problems learners are required to think high order thinking even though they are required to find something known or asked, the mathematical model used, and carry out calculation according to the provided mathematical model. Hence, the engagement of learners in this activity is in line with Polya's problem-solving strategy. Thus, Polya's problem-solving strategy is considered to be a suitable strategy to help learners to solve word problems (Lee, 2016). This is also in line with RME, which advocates for exposing learners to practical daily life problems, which are often represented in word problem format.

2.4.2 COMMON CHALLENGES/ISSUES PERTAINING TO THE TEACHING OF ALGEBRAIC EXPRESSIONS IN GRADE 10

The following sections highlight the common challenges pertaining to the teaching and learning of algebraic expressions.

The challenge in algebra is that most of the learners fail to understand the main concepts of algebra (Pramesti & Retnawati, 2019). Once learners fail to understand the key aspects of algebra, they have difficulties in mathematics. One of these key aspects is simplifying algebraic expressions (Garzon & Casinillo, 2021).

Algebra is a generalised form of arithmetic where letters and both operation and direction signs are used. The use of letters and signs, according to Forste (2007), makes algebra to be abstract and difficult to comprehend. This is because algebraic ideas are based on general ideas instead of real facts or events. Literature exposes some of the common algebraic expression challenges, which are encountered in Grade 10 mathematics classrooms in the subsequent sections.

2.4.2.1 Inadequate teacher pedagogical content knowledge

The key challenge in the teaching of algebraic expressions is the teachers' lack of pedagogical content knowledge (Akyuz, & Yildiz, 2019). The teacher's pedagogical knowledge assists a teacher in knowing learners' misconceptions and conceptions with regards to the subject matter at hand. Some teachers may not be able to anticipate (think ahead) that learners may give an answer as a value when they are supposed to leave an answer as an expression. Therefore, teachers may not plan ahead to include intervention strategies to rectify this challenge. As an example, learners with a conjoining tendency may continue to work out $7a + 3$ as $10a$ or 10 (Ncube, 2016), if they were asked to simplify an expression where the final step should be $7a + 3$ (Luciello et al., 2014; Muchoko et al., 2019). Learners do not accept $7a + 3$ as a solution (a final answer), (Egodawatte, 2011). In line with this, Yildiz and Akyuz (2017) attest that when teachers have strong subject matter knowledge, they take into consideration the learners' thinking capability as they plan their lessons. According to Yildiz et al. (2020), mathematics teachers and preservice mathematics teachers usually have partial or insufficient knowledge of learners in general. This means that they do not have sufficient knowledge of how learners learn and make sense of the algebraic concepts and this insufficient knowledge makes it difficult for them to address the learners' challenges, pertaining to learning algebraic expressions. According to Dalton et al. (2019), in South African classrooms, challenges faced include teacher training, which do not fully prepare teachers in terms of developing/advancing their knowledge of both content and teaching strategies to be able to effectively teach learners.

2.4.2.2 Concept of a variable

Research shows that learners have challenges regarding the different roles of literal symbols as placeholders, generalised numbers, unknowns or carrying quantity (Jupri et al., 2014). With a placeholder, literal symbols are seen as a hollow vessel in which a numerical value can be kept for retrieval in the later stage, for example, in $x^2 + 7x + 10$ or $2x - 5$, x is a placeholder, which means is a "blank spot" or "unknown" in which it can be replaced by the number and then get either a numerical sentence or a numerical expression (Madden, 2019). *As an unknown*, a literal symbol is used in a problem-solving process in which the goal is to find a solution of an equation; for instance, when asking learners to solve for x in $x^2 - 5x + 6 = 0$.

As a generalized number, a literal symbol will result in true statements, for instance, $3x + 4x = 7x$ or $a + b = b + a$ (Egodawatte, 2011). *As a varying quantity,* a literal symbol is used in a functional relationship either as an input argument or as the output function value, i.e., the linear relationship between kilometres travelled and the cost of traveling per kilometre $(y = 15x)$ if y represents the cost of travelling, then x represents the number of kilometres travelled. Stiphout et al. (2011) call this the symbol and structure sense. This is also confirmed by Isik and Kar (2012) that learners fail to realize the use of letters in place of the unknown. To a certain extent, this shows that teachers do not make substantial efforts to clarify symbols and letters used in algebraic expressions to enable learners to develop their understanding. According to the research done by Samuel et al. (2016), teachers are too fast when teaching manipulation of algebraic expressions and this often leaves learners with misunderstandings. According to Osei (2021), one of the misconception learners have about the variable x is to mistake the variable to be a multiplication sign.

2.4.2.3 Inability to represent word expression in algebraic format

Converting from verbal relational statements to algebraic format, causes problems and confusion for learners of all ages (Joffrion, 2005). Braselton and Decker (1994) and Moleko (2018) mentioned that learners must be capable of reading and understanding mathematical word problems before they can be able to solve them successfully. However, these researchers realised that learners face a challenge of not understanding mathematics word problems when they read, thus this makes it even more difficult to represent these problems algebraically. In line with this, Gooding (2009) confirms that learners who cannot read and do not understand the Language of Learning and Teaching LoLT, face challenges that make it even more difficult for them to successfully solve the given word algebraic expression problems. For instance, Osei (2021) mentions that learners may represent a word problem, such as “Five less than a number” as “$5 - x$” simply because the expression “less than” follows five in the word representation.

In addition, teachers also claim that the difficulty of solving algebraic expressions represented in word form is as a result of learners’ inability to identify keywords, define vocabulary, analyse long sentences and comprehend written context (Gafoor & Sarabi, 2015). Furthermore, Moleko (2021) indicates that some teachers argue that the clarity of concepts, text comprehension and terminology, mathematical language use, comprehending operations embedded in the text and vocabulary, as well as the structure of the word problem, are some of the factors affecting learners’ solving of algebraic word problems. For example, when asked to write an algebraic expression representing *“a product of two consecutive numbers”* learners write xy instead of writing “$x(x+1)$ or $(x-1)x$” and the reason for this is because x and y are consecutive in alphabetical order, but that does not make them the consecutive numbers.

2.4.2.4 Some common misconceptions when simplifying algebraic expressions

The study sought to explore the following misconceptions learners exhibit when simplifying algebraic expressions

a) Conjoining error

Learners have to recognize that an algebraic expression, such as $2x + 7$ is two sided, because it represents a calculation process, as well as being an algebraic object in its own right (Egodawatte, 2011; Ncube, 2016). This means that $2x + 7$ can be a solution from simplifying an algebraic expression or it can be an expression for which x can be substituted with a value to obtain the value of such an expression and according to Sfard (1991), this is called the process-object duality. Thomas and Tall (1991) regard the inability to switch between the process and object as the process-product obstacle. The learners who have this problem tend to oversimplify an expression $7x + 2$ and write it as $9x$. This is called **conjoining error** and it means learners combine letters and numbers meaninglessly. The learner perceived that the answer should not contain the $' + '$ and/or ‘ – ‘ sign.

b) The parsing obstacle

Another challenge is identified as an inability to untangle the order in which the algebraic expressions must be comprehended and manipulated; at times inconsistent with the order of natural language. This is called the parsing obstacle. For example, in dealing with $8 - 3y$ learners may read from left to right as $8 - 3$ giving 5 and consider the full expression to be equivalent to $5y$; in dealing with $y + 6$, learners may read it as y and 6, and interpret this as $6y$ (Jupri et al., 2014). In my view, this tell us that teachers fail to use multiple representation of information when teaching this aspect, such as showing learners that 8 and $3y$ are not like terms and if 8 represents the number of litres of milk used by a family of three people $(3y)$ then it is not possible to subtract one from the other. This means that teachers do not offer ways of customizing the content so to bring clarification that will enable learners to realise and understand that 8 and $5y$ are not the like terms and therefore they can never be added together or subtract one from the other to get the answer.

c) The “cancellation error”

In some expressions, such as $\frac{2a+3}{a}$ and $\frac{a+x}{b+x}$ learners tend to display challenges comprehending the meanings and forms of algebraic fractions. Some learners tend to carry out operations that are incorrect. According to Ncube (2016), this is because learners are familiar with cancellation of common factors without proper consideration of the situations that come with cancellation of common factors in the denominators. Makonye and Stepwell (2016) regard this error as **cancellation error**. Learners with this misconception tend to cancel the same variables in an improper situation. In this instance, learners cancel out a in the numerator and denominator and hence end up having 5 as an answer, i.e. $\frac{2a+3}{a} = 5$.

In the second case they cancel x in the numerator and denominator and have an answer as $\frac{a}{b}$

(Egodawatte 2011), i.e., $\frac{a+x}{b+x} = \frac{a}{b}$. To some extent, this shows that teachers do not provide enough guidance in manipulation of the algebraic expressions. Demby (1997) referred to this type of error as 'thoughtless and slapdash cancellation'. This is because learners just cancel out without considering whether the variable is a common factor or not.

d) Misapplication of rules

Furthermore, some learners have a challenge in expanding brackets, such as $3(4a+2)$. In this case, learners give an answer as $12a+2$ and this indicates failure to consider parentheses when applying operations (Aydin-Güç & Aygün, 2021). The learners expand one part of the bracket and leave out the other part. Other learners have a challenge of not knowing to what extent they should apply the pre-multiplier (Ncube, 2016), which in this case refers to a number or variable in front of the brackets, such as 3 in the above example. Thus, when simplifying the expression *5(2p + 3) + (2 + 5p),* they multiplied contents of both the first and second sets of brackets with the number 5. Hence, they obtained the solution as *10p + 15 + 10+ 25p*. In this regard, learners distorted the distributive law. In "situation 1", the distributive law states that $a(b+c)+d(e+f) = ab + ac + de + df$. In "situation 2", the distributive law states that $a(\,b+c) + (d+e) = ab + ac + d + e$. Therefore, the learner applied situation 1 instead of situation 2. According to Ncube (2016), learners use known rules in appropriate situations, but incorrectly adapt the known rules. This therefore, shows that teachers teach rules explicitly, but they do not emphasize the notion of adapting the known rules in different situations. For example, scaffolding *5(2p + 3) + (2 + 5p)* into two separate expressions, such as *5(2p + 3)* and *(2 + 5p*) and asking learners to add them may assist teachers to overcome these challenges. Egodawatte (2011) commented that when expanding the expression $(x-2)^2$, some learners just gave their answer as x^2-4 and when given the expression $(2x+b)^3$, learners give their answer as $8x^3+b^3$. In this case, learners fail to retrieve the correct expansion of a binomial, as was found by Mbewe (2013) in his study on misconceptions and errors in algebra.

This means that learners may have confused the exponential law, which states that $(ab)^n = a^n b^n$ and the distributive rule, which states that:

$$(a-b)^2 = (a-b)(a-b)$$

$$= a(a-b) - b(a-b)$$

$$= a^2 - 2ab - b^2$$

2.4.2.5 Vocabulary issues related to algebra

Some challenges in algebra, as mentioned by Star et al. (2015) and Joseph (2020) include vocabulary issues and mathematical language, whose lack of knowledge and understanding creates knowledge gaps. Table 2.4 below displays some of the vocabulary challenges encountered by learners in the teaching and learning of algebra (Thompson & Rubenstein, 2000).

Table 2.4: Algebra vocabulary issues encountered by learners in Mathematics and examples

Potential pitfall	**Example**
Some words are common in mathematics and everyday English while having a diverse meaning.	Origin, function, radical, range and imaginary.
Some mathematics words are shared with English and have similar meanings, but the mathematical meanings are more precise.	Continuous, limit, amplitude, dividend and slope.
Some words have several mathematical meanings.	Square, base inverse and degree.
Some mathematical terms are only found in a mathematics content.	Asymptote, integer and hyperbola.
Modifiers may change mathematical meaning in important ways.	Root and square root, inverse and inverse function.
Some words are shared with science and have different technical meanings in the two areas of study.	Solution, radical and variable.
Some mathematical phases must be learned and comprehended entirely.	One-to-one function.
Some mathematical terms sound like every day English words.	Sine or sign, cosine or cosign.
Some mathematical words are related, but learners confuse their diverse meanings.	Equation and expression, solve and simplify.

(Source: Thompson & Rubenstein, 2000)

Moreover, learners' inability to tell apart and understand algebraic terms. such as coefficients, constants, evaluate, simplify, expand, factorize and many more. also contribute to their low attainment in algebraic expression (Samuel et al., 2016).

In the perspective of Thompson and Rubenstein (2000), in classrooms, the language of mathematics plays at least three key roles, as it is a vital component of our instruction:

- Teachers deliver instruction through the medium of language. Thus, language of mathematics is our major means of communication.
- Learners develop understanding as they process concepts through language.
- Teachers diagnose and assess learners' understanding by listening to their oral communication and by reading their mathematical writings.

These authors further mention that unlike common English, which is used daily by learners in reading, watching television, conversation, and elsewhere, the language of mathematics is mainly limited to school, i.e., it is mostly at school that mathematics language is used, thus it is vital that teachers use it properly. Subsequently, teachers need to be sensitive to many issues associated to the language of mathematics and learners' increasing articulacy with it (Joseph, 2020). Therefore, it is important that learners understand and master the language of mathematics in order to perform well in this subject (Chval & Chávez, 2011). However, judging by the fact that learners still struggle with the terminology used in mathematics, this indicates that the teachers do not invest time in developing strategies that address the vocabulary issues. Gay (2008) notes that teachers do not even invest time to develop a sound mathematical language mastery themselves.

2.4.2.6 Gaps between arithmetic and algebra

Various scholars, such as Tabach and Friedlander (2008), Mbewe (2013) and Ncube (2016) claim that learners often display an arithmetic-algebra gap, which Seng (2010) and Ferrer (2020) regard as a fundamental cause of learning difficulties in mathematics in general. If learners possess a good arithmetic background, they are not likely to face challenges in algebra. This is because algebra knowledge is built upon the foundation of already acquired arithmetical knowledge. According to Kieran (2007), learners have been found to experience a great challenge in making a transition from arithmetic to algebraic thinking. According to Mbewe (2013), arithmetic strategies are intuitive, primitive and context-bound and often involve the basic operations, such as addition, subtraction, multiplication and division. Moreover, Mbewe noted that arithmetical problems are connected, hence learners can reason outright from the known to the unknown. On the other hand, algebraic problems are said to be disconnected, since reasoning is solely on the unknown. Consequently, arithmetical and algebraic reasoning tend to be different and thus it could be the source of learners' difficulties in the transition from arithmetic to algebra (Yildiz & Akyüz, 2019). The role of teachers is to close this transition gap from arithmetic to algebra. However, teachers fail to close this gap, because they still use the traditional methods of delivering content (Theobald et al., 2020), which subsequently deny learners opportunities to demonstrate what they know (Moleko, 2018). In addition, they employ these teacher-centred methods, because their pedagogical content knowledge is insufficient (Yildiz & Akyüz; 2019).

2.4.3 SOLUTIONS TO CHALLENGES

The following sections provide solutions to the identified challenges in the preceding sections above.

2.4.3.1 Enhancing teachers' pedagogical content knowledge

According to CAPS (2011), at each school a central point of organisation, planning and teaching should be inclusive. In addition to this, Donohue and Bornman (2014) affirm that it is only if all teachers have a comprehensive understanding of how to diagnose and address barriers to learning, and how to plan for a variety of learners, that this can happen. Inclusive mathematics education makes every effort to promote individual learning by adapting to learners' diverse abilities and create joint learning situations in the classroom community with collective goals of even-handed participation (Ayaya et al., 2020). Joint learning is understood as the pedagogical realizations of the broad notion of inclusion as a virtuous obligation to find ways to live and acquire knowledge together, while dealing effectively with otherness and which can include whole-class discussions (Karz & Sokal, 2016), as well as heterogeneous small-group activities in which all learners take part.

To improve teacher pedagogical knowledge content in teaching of algebraic expressions, teachers need to be assisted by professional developmental workshops (Shehu, 2009), where they can network with their peers to share good practices (Akyuz, & Yildiz, 2019; Begum, 2012; Murtaza, 2010). In addition to this, the Department of Education should assist teachers with in-service training (developmental workshops) to improve their skills, not only in teaching, but also in inclusive education (Yolk, 2021). In addition to the above lesson, study is said to be another useful strategy to improve teachers' pedagogical content knowledge (Lewis et al., 2012). Lesson study has been initiated in Japan and over the years has been introduced in other countries, including South Africa (Doig & Groves, 2011; Perry & Lewis, 2009). It is said to encourage teachers to create their own community inquiries to reflect upon their practices. Hence, it is described as a cycle of instructional improvement in which teachers, collaboratively formulate goals for learners' acquisition of knowledge and long-term improvement; design together a ''lesson'' planned to bring to action these goals by conducting the lesson in a classroom, with one team member teaching and others observing and gathering information on learning of learners and improvement; reflect on and discuss the information gathered during the lesson, utilizing it to improve the lesson, the unit, and instruction more generally; and if desired, repeat the process again and again (Olson et al., 2011). Tsotetsi (2013) advocates that one more strategy that can help teachers to improve their pedagogical content knowledge, is teacher collaboration. This collaboration provides teachers with a platform to share content ideas, best practices and challenges they encounter in the classrooms and discuss how best they can handle these challenges.

Moreover, Sheila (2016) perceived that to improve learner performance in the classrooms, teachers must be able to incorporate technology in their lesson so that they could achieve maximum outcomes. This means that teachers must possess a technological knowledge. Technological knowledge encompasses the knowledge of textbooks, chalk, and chalkboards, as well as more advanced technologies, which include internet digital video and other inclusive technologies (Wijers, 2001).

2.4.3.2 Developing an understanding of a variable concept

A variable is a letter or symbol that is used to represent any unknown number (Lim, 2010; TIMES, 2011). It is also a quantity that may change within the context of a mathematical problem or experiment. On the other hand, parameters are symbols that are thought of as constants can change so that a comparison of different scenarios and conditions can be made (Redish, 2018) for example in the equation $(x) = ax^2 + bx + c$, *a, b* and *c* are parameters and x is a variable. Therefore, according to Wijers, (2001), for learners to develop a real understanding of variables and parameters, it is essential that there should be a connection of the development of mathematical (algebraic) conceptions to "machine-procedures and technical skills"; this means that teachers must incorporate technology, such as Geogebra in their presentations, which will assist them to illustrate how parameters differ from variables. In addition to this Laurens, Batlolona et al. (2017) mention that to improve, learning should include learning aids, such as real and concrete modified materials so that learning is made easier for abstract concepts. These aids give learners an opportunity to discuss and actively learn about the variables and parameters. Erling et al. (2016) also confirm that actively engaging learners in activities, which enable them to identify the variables and constants in a real-life situation, will also assist learners to understand the notion of variables. Furthermore, Usry (2021) attests that when learners are motivated and actively involved with hands-on-algebra activities, real-life connections and algebraic-thinking skills are improved as learners utilize problem-solving techniques and technology. He further emphasizes that these types of activities should form an integral part of every introduction to the algebra course.

2.4.3.3 Enhancing learners' understanding of the algebraic expressions through mathematical problems that are represented in word format

According to Moleko and Mosimege (2020), mathematics word problems assist learners in skills development of understanding when and how to put at work classroom mathematical knowledge in their everyday life problems. The mathematics word problems also form part of realistic mathematic education, which contributes a rich context regarding real-life situation problems (Van den Heuvel-Panhuizen & Drijvers, 2014). The study conducted by Moleko and Mosimege (2020) proposes a learner-centred teaching planning approach so as to enhance learners' ability to represent the word problems in an algebraic format by drawing from learners' experiences during planning of instruction. This is in-line with Freudenthal 's view that learners must be presented with an opportunity to construct their own ideas by using their own methods (Zulkardi, 2002). Similarly, Hirza et al. (2014) reckon that mathematics teaching has to be focused to permit learners to reinvent mathematics concepts with the use of their own methods. Moleko and Mosimege

(2020) further highlighted the importance of teaching a mathematics vocabulary, since it assists learners to meaningfully comprehend word problems.

According to Kieran and Chalouh (1993), it is crucial that prior to learning to represent word problems algebraically, that the school children be afforded a platform to discuss them in easily comprehensive, day-to-day language, in order to develop conceptual understanding.

2.4.3.4 Addressing some common misconceptions when simplifying algebraic expressions

2.4.3.4.1 Addressing conjoining error

According to Tan (2015), the conjoining error can be addressed by encouraging learners to substitute values as the variables in the initial expression, as well as the solution expressions to check if they get the same answer. Moreover, according to Akyuz, and Yildiz (2019), teachers can also use substitution, the order of operations, and going backward strategies during the simplification of algebraic expressions.

For example, substitution for $x = 1$ in $2x + 7 = 9x$

$2(1) + 7 = 9(1) \rightarrow 9 = 9$ [meaning, the expression on the left is equal to an expression on the right]

this is only true for $x = 1$. When x is any other number except 1, the statement is not true. e.g., for $x = 2$ then $2(x) + 7 = 9x$ gives

$2(2) + \ 7 = 9(2)$

$11 = 18$ which is not true i.e., $(11 \neq 18)$

As a result, it is incorrect to say $2(x) + 7 = 9x$

For the order of operations, then a teacher can explain that $2(x) + 7 \neq 9x$, because before addition can be done, multiplication has to be dealt with and in this case multiplication of 2 and x gives $2x$ since x is an unknown value. Thus, to the product $2x$, 7 is added and the result is $2x + 7$.

An example of working backwards strategies, the teacher may show learners that $9x$ cannot be written as $2x + 7$, but rather as $2x + 7x$.

To improve understanding of algebraic expressions. Star et al. (2015), have the opinion that learners should be taught to use the structure of algebraic representation. This should be carried out by strategies, such as encouraging learners to utilize reflective questioning so that they can identify the structure of the expression as they simplify it (Hoch & Dreyfus, 2004; 2006).

2.4.3.4.2 Addressing the parsing obstacle

Parsing obstacle is an inability to untangle the order in which the algebraic expressions must be comprehended and manipulated, as well as inconsistency with the order of natural language. This also shows that learners lack understanding regarding like and unlike terms. So, one of the way the concepts of like and unlike terms can be tuaght meaningfully is to use concrete objects, such as the chocolates in the box as in Figure 2.3 below:

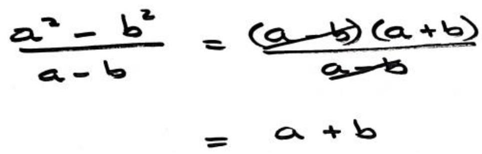
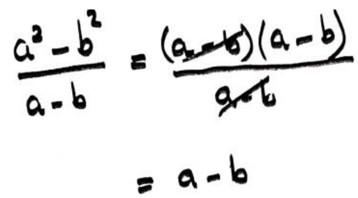
Figure 2.3: Box of chocolates

Based on Figure 2.3 above, a teacher may show the learners that there are different coloured chocolates in the box. If one is then asked to represent the chocolates based on their different colours then: the box contains 1 red chocolate, 1 pink chocolate, 1 yellow chocolate and 5 brown chocolates ($1r + 1p + 1y + 5b$). Therefore, based on the colour, there are $1r + 1p + 1y + 5b$ chocolates in the box where r, p, y, and b represents red, pink, yellow and brown respectively. Hence, having understood the concept of like and unlike terms from concrete representation, learners would be able to identify that $8 - 3y$ cannot be manipulated any further, because 8 and 3y are unlike terms. Jupri et al. (2018) also advocate for the use of RME principles to allow learners to gain more insight to the concept of like terms and unlike terms by suggesting the use of area and perimeter of a rectangle as a reminder and starting point to emphasise the concept of like terms.

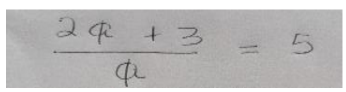
2.4.3.4.3 Addressing cancellation error

According to Dalton et al. (2012), providing multiple, flexible options for learner engagement to support affective learning, is beneficial to learners. This principle involves affording learners interesting learning opportunities that motivate and stimulate them, according to their personal backgrounds and interests. Hence, Star et al. (2015) encourage the use of worked examples to reduce some of the misconceptions that learners exhibit during instruction. These scholars feel that worked examples with guided promptings to select the correct solutions can assist learners to discover their mistakes and thus gain deeper understanding of the concepts. They also suggest that teachers develop a set of reflective questions to guide learners to gain more understanding of the algebraic concepts or to choose the correct strategy to use in determining the solution to a given problem. For instance, a teacher may provide learners with a problem correctly solved and incorrectly solved and use reflective questioning to guide learners to gain insight on how the problem should be solved.

Table 2.5: Addressing cancelation error

Given problem	Correctly solved	Incorrectly solved
Simplify; $\frac{a^2-b^2}{a-b}$	$\frac{a^2-b^2}{a-b}=\frac{(a-b)(a+b)}{a-b}$ $=(a+b)$ 	$\frac{a^2-b^2}{a-b}=\frac{(a-b)(a-b)}{a-b}$ $=(a-b)$ 

However, Makonye and Khanyile (2015) recommended that cancellation error can be eliminated by using probing as an intervention strategy so that learners may realise where they committed mistakes. The method of probing involves a dialogue between a teacher and a learner to foster reflection and critical thinking about the given problem.

For example, in the case of , the teacher may probe learners with questions to find out why they cancelled as they did in the expression $\frac{2a+3}{a}$. As learners are asked about their answers to algebraic problems, they question and explore their own understanding and improve intuitions on the weakness of their present knowledge.

2.4.3.4.4 Addressing misapplication of rules

According to Star et al. (2015), teaching learners through facilitated whole class discussion on how various methods can be applied for the same problem and whether some methods are suitable for simplifying algebraic expressions or not, is highly beneficial to learners. In addition, having learners compare and evaluate different methods for simplifying algebraic expressions by encouraging them to relate problem structures and solution strategies to find out the relationships between similar and different problems, strategies, and solutions, may assist in minimising misapplication of rules (Tabach & Friedlander, 2008). Another good strategy could be that of first teaching learners about the correct application of the rules, and then later giving them the related problems with incorrect/misapplication of the rules being applied to check if they can identify the misapplication and to ask them to correct the application of rules.

2.4.3.5 Promote the use of language that reflects mathematical structure

According to Thompson and Rubenstein (2000), using precise mathematical language is a fundamental component to understand structure and sets the foundation for the use of reflective questioning, multiple representation, and diagram use/understanding. During class presentations, teachers need to rearticulate algebra solution steps in precise mathematical language to explain logically the structure of a problem, operation, solution steps and strategies. In addition to this, mathematical language must be used to assist learners to analyse and voice out the specific features that build up the structure of algebraic representation (Star et al., 2015). Furthermore, mathematical language must also be used during introduction of a new topic (Joseph, 2020) or concept in an effort to encourage learners to describe the structure of algebra problems with accurate and appropriate terms (Chval & Chávez, 2011). Table 2.6 shows the examples of the informal language teachers use in an effort to assist learners to understand simplification of algebraic expressions and the correct mathematical language that should be used (Star et al., 2015).

Table 2.6: Inaccurate language vs accurate language

Informal language	**Formal mathematical language**
Take out the x.	- Factor x from the expression. - Divide both sides of the equation by x, with a caution about the possibility of dividing by 0.
Move the 5 over.	Subtract 5 from both sides of the equation.
Use the rainbow method. Use FOIL.	Use the distributive property.
Solve an expression.	- Solve an equation. - Simplify an expression. An expression can be simplified but an equation can be solved
A is apples.	- Let *a* represent the number of apples. - Let *a* represent the cost of the apples in Rands. - Let *a* represent the weight of the apples in kilograms.
Plug in the 3.	Substitute 3 for y.
To simplify, flip it and multiply. To divide a fraction, invert and multiply.	- To simplify, multiply both sides by the reciprocal. - To divide fractions, multiply by the reciprocal.
Do the opposite to each side.	- Use inverse operations. - Add the opposite to each side.
The numbers cancel out.	The numbers add to zero. The numbers divide to one.
Plug it into the expression.	Evaluate the expression.

Among other things, such as teaching techniques, instructional strategies, models, and use of particular representations, mathematical language should also be included in teacher training programs in order to develop teachers' knowledge (Akyuz & Yildiz, 2019). Teachers should also expose learners to whole class discussions, as well as group discussions so that learners could get an opportunity to improve their mathematics vocabulary. They can also provide learners with a list of terminology related (word Bank) to the concept of algebraic expressions.

The following seven strategies may be used by the teachers to improve learners' mathematical proficiency in the classroom:

1. Connect mathematics with learners' life experiences and existing knowledge (Barwell 2003; Secada & De La Cruz, 1996).
2. Create classroom environments that are rich in language and mathematics content (Anstrom 1997; Khisty & Chval 2002).
3. Emphasize meaning and the multiple meanings of words. Learners may need to communicate meaning by using gestures, drawings, or their first language while they develop command of the English language and mathematics (Moll, 1988, 1989; Morales et al., 2003; Moschkovich, 2002).
4. Use visual supports, such as concrete objects, videos, illustrations, and gestures in classroom conversations (Moschkovich 2002; Raborn 1995).
5. Connect language with mathematical representations (e.g., pictures, tables, graphs, equations) (Khisty & Chval 2002).
6. Write essential ideas, concepts, representations, and words on the board without erasing so that learners can refer to them throughout the lesson (Stigler et al., 1996).
7. Discuss examples of learners' mathematical writing and provide opportunities for learners to revise their writing (Chval & Khisty 2009).

In addition, before proceeding with a topic, Joseph (2020) suggests that mathematics teachers should teach the mathematical language in a mathematics classroom to enhance better performances of learners in mathematics. Literature further mentions that even though these strategies support learners to improve their mathematical vocabulary, most teachers tend to implement only those strategies they find easy to use and ignore others (Chval & Chávez, 2011). These researchers agree with the fact that incorporating these strategies in the classroom is a challenging and complex exercise, therefore, it requires teachers' collaboration in planning and implementing them, since they are catering for diverse learners' needs.

2.4.3.6 Addressing the gap between arithmetic and algebra

Kieran (2004) and Wijers (2001) both have a view that a translation gap from arithmetic to algebra may be minimized by introducing learners to algebra early at lower grades. According to Subramaniam and Banerjee (2004), the transition from arithmetic to algebra needs learners to understand that arithmetic expressions encompass a three-fold meaning such as: process, product and relation (Norton & Irvin, 2007; Stacey & Chick, 2004). Thus, a relational meaning of an expression is expressed when we say that $10 + 6$ denotes a number which is 'six more than ten' (or alternatively, 'ten more than six'). Thus $10 + 6$ and $12 + 4$ denote the same number, but express different relations. In algebraic form, if learners can be able to visualize $4(b + 1)$ in different forms, as well as including pictorial forms (Akyuz, & Yildiz, 2019), then teachers would have achieved application of multiple forms of representation and learner engagement in their classrooms. Moreover, as claimed by Muchoko et al. (2018), different scholars have several suggestions for teaching strategies that would increase learners' algebraic performances later at secondary school. These include strategies that emphasize understanding of fractions when learners are still at lower grades, proper usage of context (Tabach & Friedlander, 2008), as well as inclusive pedagogical strategies, such as group work where learners may collaborate with others and share ideas (Banerjee & Subramaniam, 2012).

2.5 CHAPTER CONCLUSION

This chapter has revealed the different approaches in which the teaching of algebraic expressions in Grade 10 could be carried out. It has revealed the learning challenges, which teachers have to be aware of and subsequently address when teaching the algebraic expressions in Grade 10. The solutions to the identified common algebraic expression challenges are also highlighted in this chapter. The paradigm couching this study is explained and the justification for its suitability is indicated. The nature of algebraic expression, as well as the best practices for teaching algebraic expressions are also defined and discussed extensively in this chapter.

The following chapter will discuss the methodology and research design that is adopted to guide the study and their principles and how it better suits this study rather than others.

CHAPTER 3 :
RESEARCH DESIGN AND METHODOLOGY

3.1 INTRODUCTION

The purpose of the study was to explore the common algebraic expression challenges in a Grade 10 mathematics classroom. In an effort to achieve this, as well as to respond to the research question, the focus of this chapter is on the research methodology and design. In line with the aim of this study, and guided by the research paradigm underpinning this study, an Action Research (AR) was chosen to generate the empirical data.

This chapter begins by outlining the definition of qualitative research and Action Research (AR) as a research design. The relevance and application of AR is also highlighted in this chapter. The selection of the participants in this study, as well as the instrumentation and the data generation procedure are outlined. Issues of trustworthiness, how the AR was carried out, data analysis, as well as the ethical considerations are all considered.

3.2 RESEARCH METHODOLOGY

According to Olds et al. (2005), qualitative research is regarded as the collection and analysis of textual data (focus groups, interviews, conversational analysis, observation, surveys, ethnographies), and by its importance on the context within which the study occurs. MacDonald (2012) echoes that qualitative research combines the methods of observing, keeping a record, analysing and interpreting the characteristics, patterns, traits, qualities and meanings of the human phenomena under study. In studies focusing at attaining knowledge regarding complex social developments, capturing essential aspects of a phenomenon from the viewpoint of the partakers in the study and unveiling beliefs, principles and inspirations that trigger people's behaviour, qualitative methods are perceived the most appropriate by scholars (Moleko, 2018; Mohajan, 2018).

The aim of qualitative research is not to generate general truths, but rather aims for wisdom (Moleko, 2018). Therefore, the purpose of qualitative research is to understand, describe, and interpret matters in a systematical way from the perception of the participants (Mohajan, 2018). Thus, qualitative research is used to investigate the behavior, perspectives, feelings, and individuals' experiences, and what lies at the central point of their lives. In the current study, qualitative research is used to help in explaining and interpreting the findings related to common algebraic expression challenges encountered by learners and teachers in mathematics class. The following sections highlight and provide the description of methodology and research design used to generate the data for the current study.

3.2.1 Defining action research

Action Research (AR) as stated by Bleijenbergha et al. (2020) was originally introduced by Kurt Lewin in 1946. They further explained that Lewin defined action research as a comparative research done over conditions and effects of different types of collective action, as well as research that is done to reach a collective action. Literature has revealed that, even though Lewin has introduced this term 'action research', he did not live long enough to develop his ideas clearly. Therefore, scholars interpreted his ideas individually and built upon them (Dickens & Watkins, 1999).

According to Bradbury (2015) and Kemmis (1988), action research is defined as a democratic and participative direction to the development of knowledge. In this regard, to bring about change, participants collaborate and together they identify problems and come up with intervention strategies, as well as putting those strategies into practice. This affords everyone who is involved in the research to voice out their ideas freely and are given an opportunity to be heard without feeling uncomfortable.

In the view of Burns (2010), AR is defined as a self-reflective critical and systematic approach to examine and evaluate your own teaching circumstances. Furthermore, Bleijenbergha et al. (2020) advocate that action research is a participatory research stratagem entailing of cycles of stages in which scholars and practitioners work together using academic literature and research methods in both solving actual organisational problems and moving forward scholarly knowledge. In line with this, action research, according to Carr and Kemmis (1983), involves cycles of self-critical and reflective processes where teachers gain knowledge about their own classroom environments and their teaching practices.

According to Carr and Kemmis (1983); Kemmis and McTaggart (1988), the practice of action research as a tool for professional development and refining classroom teaching and learning, is not new. Several studies have outlined the benefits for teachers-as-researchers to concentrate on teaching practices and skills in their own classrooms (Aldridge et al., 2021; Aldridge et al., 2012; Aldridge et al., 2004; Bell & Aldridge, 2014). A vital element of teacher action research is the reflection stage; in this stage teachers have the opportunity to review their own teaching experiences, so as to come up with solutions to the challenges that need to be addressed (Fullan, 1999). The involvement of teachers in action research, in the view of Carr and Kemmis (1983), mainly in relation to their personal teaching strategies and skills, provides a valuable form of professional advancement.

3.2.2 The relevance of action research

The use of AR in this study was informed by the need to bring about change in the classroom practices through collaboration between the researcher and the researched. It is worth noting that that the researcher is regarded as the researched within the AR framework. According to Elden and Levin (1991), participants in AR are not expected to be treated as objects, but rather as active participants and should be engaged in powerful participation and discourse. AR offers participants opportunities to equally share their ideas, experiences and expertise without fear of being judged (Moleko, 2018).

AR was also chosen, because of its ability to allow teachers to self-reflect critically on their own classroom teaching practices. As they explain the roles of AR, O'Connor, Greene and Anderson (2006) state that AR is a type of research that is meaningful and authentic in nature to the teacher-researcher, as it is carried out by the teacher in his/her own classroom environment. Therefore, AR permits teachers to own their teaching and materializes when teacher researchers anticipate classroom and instructional problems, design a plan, execute a plan, evaluate the outcomes and reflect on the process. With the use of an AR process, teachers not only learn about learners and colleagues, but they also learn about themselves as they search for ways to continually develop (Ferrance, 2000). Thus, teachers become more in control of their professional development when they carry out AR. The viewpoint of Hensen (1996) is that once teachers attain ownership of research, particularly AR, there are several ways in which learning can be acquired with the inclusion of putting into practice new strategies, evaluating current programs, escalating instructional reserves, engaging in professional growth and most crucially, assisting teachers with pedagogical knowledge development.

According to Sax and Fisher (2001), AR affords teachers with opportunities to ascertain changes they need to make in their lessons practices by availing teachers with the framework to build their own classroom projects. That is, it enables teachers to carry out their own research in the comfort of their classroom with their learners as co-researchers. More often, when teachers decide about their own AR projects, they apply a systematic approach to discover solutions to instructional problems or matters. Since the professional development implemented is systemic, interactive and ongoing, it becomes very influential. Furthermore, teachers' involvement in action research in a collective school environment, has been proven to have a measurable and direct impact on learner attainment, behaviour and equity, as well as the performance of colleagues and school front-runners (Reeves, 2008).

As indicated by Hong and Lawrence (2011), AR is emancipatory in nature, because it calls for practitioners to critically reflect on the structures and social arrangements that control parts of the population (Newton & Burgess, 2008), some of which teachers themselves might strengthen. Thus, in this study, the reflection stage assisted the researcher and the participants to critically reflect on the challenges encountered during instruction and to suggest possible solutions. This is in line with CER, which also seeks out to emancipate people by encouraging engagement within the research project and permitting participants to voice their challenges and be respected (Dold et al., 2011). Moleko (2014) attests that the only way to emancipate people is by engaging them in the discussions where they can freely express their opinion without limiting their determination or social growth. In this study, the use of AR offers teachers and learners a platform to identify their challenges and find strategies or solutions to improve the teaching of algebraic expressions.

3.2.3 Application of action research

According to Lewin (1946), AR is a cyclic back and forth between ever deepening monitoring of the how things stand with regard to the problem within the individuals, the organization and the system, and a sequence of research-informed action experiments. This means that action research involves a repetition of steps whereby researcher and co-researchers are engaged in identifying a problem, planning, implementation, reflection and evaluation and repeating the same steps until the issue is resolved or minimized. Originally, Lewin (1946) formulated AR stages to consist of analysis, fact-finding, conceptualization, planning, execution, more fact-finding or evaluation; and then a repetition of this whole circle of activities when the problem persisted which result in a coil of such circles (Lewin, 1946; Sanford, 1970). Figure 3.1 shows Lewin's model of action research.

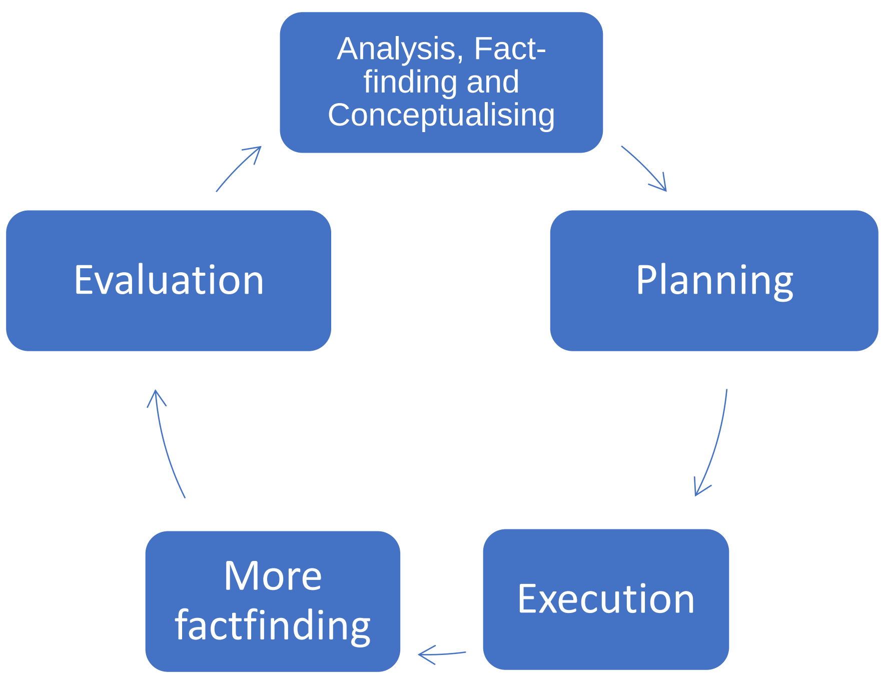


Figure 3.1: Lewin's Model of AR

According to Bleijenbergha et al. (2020), Lewin's model has been modified to show a sequential process that relates to a central point. In this model, the first stage is to identify a problem. That means the participants (researcher and co-researchers) collaborate and identify the problem, as well as formulate the research questions to the identified problem (Bradbury, 2015). This collaboration happens throughout all stages of action research, since AR is participatory in nature. The second stage: plan analysis, refers to participants using literature to analyse the problem and possible solution to the identified problem. In the third stage: plan of action, researchers and co-researcher design a plan of action to address the problem at hand. In the fourth stage: implementation stage, the researchers execute the plan of action by implementing the solution in order to end the problem. During the fifth stage, data is collected by the researchers to find out whether the problem has been resolved. The six stages are: theoretical implication and planning of new strategy, means that researchers collaborate to develop a theory based on the findings and plan a new strategy to overcome the problem if it persists, as well as planning the way forward.

Figure 3.2 shows the stages of AR as depicted by Bleijenbergha et al. (2020).

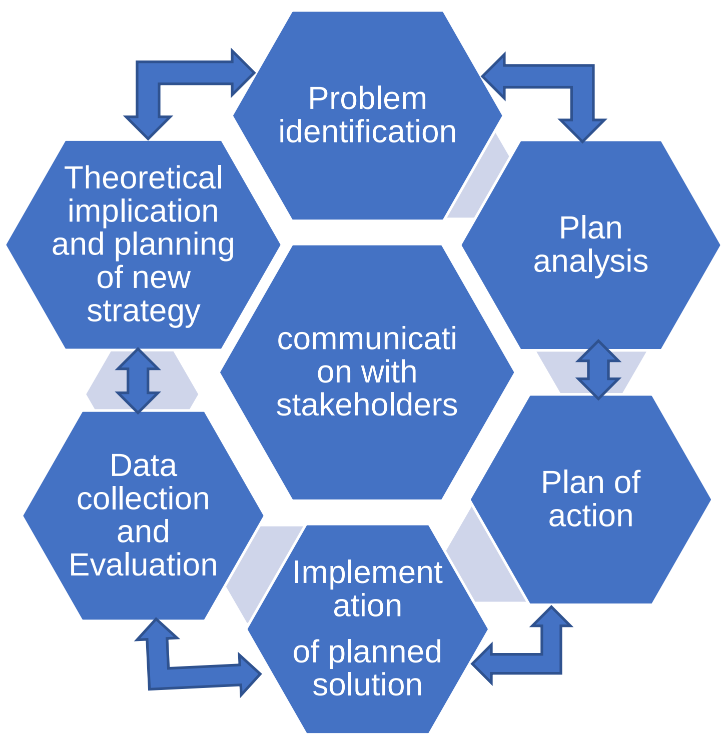


Figure 3.2: Stages of action research

The current study adopted the AR model by Bleijenbergha et al. (2020). This is entirely because it shows that during the process of AR for each stage all stakeholders will be engaged, and every communication will be through them as researchers and co-researchers. In addition, it was adopted mainly for its nature of empowering and transforming people's knowledge and, for complementing CER, which is a paradigm underpinning this study.

According to Saco (2004), AR is a tool that teachers use in their classrooms to discover and test new strategies, and AR is frequently considered to be a professional development opportunity because, in many cases, it is utilized by teachers to examine a new instructional strategy, assess a new curriculum program, or evaluate an existing pedagogical method. AR, therefore, initiates the desire for change whereby the teachers act as change agents, because these teachers have a skill or power to stimulate, facilitate and coordinate the change effort. Thus, implementation of an AR process, assisted informing day-by-day instruction and has transformed and improved teachers' views, choices and thinking, with regard to curriculum (O'Connor et al., 2006). Hence, according to Edward and Burns (2016), AR pursues both to better understand and advance an aspect of teaching and learning.

According to Dickens and Watkins (1999), the beginning of an AR project often occurs with a reflection when a group of people identify a concern or issue, and it is turned into a common objective. Participation, therefore, is obtained through this shared objective and need to do something to resolve it. The AR group will then construct the project with the researchers, as they are experts in their own field. This commences with reflection, identification, and planning, including clarification of the issue, who should be involved, how the research process should materialize (including research activities), where it could take place, and other similar issues. The next stages are action and observation, whereby the research process is streamed and looked over with an AR group from the community and feedback gathered according to its value, impact, and results.

In this study, a model of AR by Bleijenbergha et al. (2020) was adopted. It was adopted mainly for its empowering and transformative nature. Furthermore, it was adopted for complementing CER, which was also the lens underpinning those studies. The following steps of AR were followed to generate data in this study; Plan, Act, Observe, Reflect and Re-plan. Table 3.1 below outlines the stages of AR followed in this study and also describes what each stage entailed.

Table 3.1: AR Stages

Stage	Description
Plan	- In this stage during the focus meeting, the researcher and participants firstly identified the common algebraic expression challenges in a Grade 10 mathematics classroom that was done. - Secondly, ways to address the common algebraic expression challenges were suggested/identified.
Act	- The implementation of the suggested ways to address the common algebraic expression challenges, as well as implementing other ways which may not have been suggested, took place.
Observe	- The researcher sat in class and observed the teaching of algebraic expressions. The researcher identified whether the suggested ways of addressing the common algebraic challenges are working or not. The researcher further identified any other way(s) implemented to teach algebraic expressions. The observation form (see Appendix D1) was used to gather data based on what was observed.
Reflect	- Teachers and the researcher reflected on what worked and what did not work regarding the teaching of algebraic expressions.
Re-plan	- Re-plan to address what did not work in the next cycle/phase/stage.

3.3 THE UNFOLDING OF THE ACTION RESEARCH

The following sections explain how the AR unfolded starting with the conditions prior to the commencement of the study

3.3.1 Conditions before the commencement of the study

There were several strategies that were applied in an effort to improve learner achievement in mathematics, such as providing learners with extra classes, having one-on-one sessions with learners, enrolling learners in different mathematics competitions, such as Mathematics Olympiads and district competitions.

The school also has a mathematics laboratory, even though is incomplete. A mathematical laboratory is a place whereby learners get an opportunity to learn and explore mathematical concepts and ideas, as well as being able to prove or verify the mathematical theorems, rules and facts with the use of various activities and techniques. So, most of the equipment, which was supposed to be fitted in the laboratory was not fitted. For example, an overhead projector and the whiteboard were not mounted, and equipment such as geoboards, algebra tiles, building blocks and 3-d shaped objects to name some few, were unavailable. Hence, the fact that the laboratory was in an incomplete state might have contributed to lower performance in mathematics, particularly in algebra.

Learners' mathematics performance in Grade 10 has a lower percentage as compared to their performance in Grade 11 and Grade 12. That is, even though the school is a combined school where majority of learners are from Grade 9 within the school, other learners come from feeder schools. Feeder schools in this case are intermediate schools with the highest grade being Grade 9. Therefore, learners who finish Grade 9 must move to other schools to start Grade 10. Some of the learners in this grade (grade 10) come from Lesotho where they had been under a different education system. This can be because the school is in the area closer to the border line between South Africa and Lesotho in Free state, and as a result, based on this and other various reasons it is easier for parents to send their children to such a school.

Therefore, the transition of these learners poses a challenge for them to fit into the South African FET phase curriculum. The transition of most learners from Grade 9 to 10 has shown to be challenging, because some of these learners display low performance in mathematics and especially in algebraic expressions. The departmental head and the mathematics teacher also mentioned that as they analysed the results that the majority of learners are not performing well in algebra. Despite all these efforts, improvement in this regard is insignificant or non-existing. Thus, this made me realize the need to conduct this research. Table 3.2 outlines the activities that were carried out during the different stages of AR in this study.

Table 3.2: Plan of Action

AR stages	Actions / Activities	Responsible person	Instrument	Duration
Problem identification	The team met to discuss challenges pertaining to the teaching of algebraic expressions in Grade 10.	The participants together with the researcher	Focus group discussion	2 hours
Plan action of	- Class observations	Researcher	Focus group discussion	3 hours
Implementation	**Live lesson observations:** - I observed the live lesson presentations using the observation form in **(see Appendix D1)**, and one of the teachers was video recording the lesson.	Researcher observed the lesson. One of the teachers video recorded the lesson.	Lesson observations	**6 lessons in total (each lesson lasting for 40 minutes)**
Evaluation / Reflection	Focus group discussions were held to reflect on what worked and what did not work during the live lessons. The sessions were also provided in order for teachers to reflect generally on their experiences of teaching algebraic expressions.	Participants and the researcher.	Focus group discussion	**2 hours**
Interview sessions	Researcher conducted interview sessions with the learners after lesson observations to probe for more information and clarity	Researcher and learners	Interview schedule (drafted during the lesson observations)	**2 hours**

Re-planning	After identifying what did not work, we then re-planned on how the problem(s) could be addressed taking into consideration the newly suggested solutions.	Participants and the researcher	Reflection session	**1 hour**

3.4 DATA GENERATION

This section deals with the aspects of data generation in this study. It describes how data were generated in this study, starting with sampling and participants' selection.

3.4.1 Sampling and selection of the participants

According to Tomal (2010) and Sider et al. (2021), action research requires active research participation and ownership by people who are motivated to identify and address issues that concern them. Thus, a Purposive Sampling Technique (PST) has been used to select the participants in this study. PST is a deliberate selection of participants, due to the knowledge and the experience they possess (Tongco, 2007). Moreover, the importance of availability and willingness to participate should be taken into consideration, as well as the capability to converse experiences and opinions in an articulate, expressive, and reflective way (Etikan et al., 2016).

Ross (2010) notes that the foundation of effective user research is based on profiling the right participants, because the results of the research are only as good as the participants involved. He considers representative, well-spoken, and thoughtful research participants as important, since they provide instrumental feedback, however, even though participants of this caliber can be useful for a research project, finding and engaging them and getting them to show up for their sessions, is at times a huge challenge.

Table 3.3: Selected participants and their contributions

Participants	Experience	Contribution
Grade 10 learners	They were taught the subject throughout their school life up to present. These learners come from different feeder schools and different education systems as some come from Lesotho. Feeder schools in this case are intermediate schools with the highest grade being Grade 9. The learners in these schools must move to other schools to start Grade 10 when they have completed Grade 9. More than 80% of these learners come from Grade 9 and others are repeaters of Grade 10.	They know the challenges they encounter as they learn. So, they will share their problems and misconceptions they have regarding the algebraic expressions with the team and this will help to strengthen the intervention intended to overcome their challenges.
Mathematics teachers	They are teaching mathematics on a daily basis. They have far-reaching teaching experience as one of them has about 10 years and the others has experience of about 25 years. They also have experience as markers of mathematics' final papers in the marking centre.	They have knowledge of the challenges learners come across in class. They are to carry out the action plan by making it a reality.

3.4.2 Data generation procedures

As stated by Meulenberg-Buskens (2011), Moleko (2021), and Tsotetsi (2013), the Free Attitude Interview (FAI) technique is an appropriate technique to be used when more information needs to be drawn from participants. This section explains how the FAI principles were adopted and used in this study to collect data, starting by describing the FAI principles, and then outlining the significance of the comprehensive question in synergizing the discourses, participants' views and perspectives. FAI may be conducted between two people or as a group (Buskens, 2011; Sekonyela, 2021). In this study, a FAI was conducted for the group (learners and teachers) and used to gather information in a focus group discussion. Similarly, to CER, the paradigm couching this study, the flexibility of FAI, its focus on respect and interest in listening to the participants, made it suitable for collecting data in this study. The choice of FAI is also based on its ability to provide an opportunity for open-ended questions to be verbal than in surveys that require them to answers with a yes or, true or false. The advantage with regard to FAI, is that participants may say more than they would have said in responding to a closed questionnaire, because the nature of discussion enables the participants to feel free.

I, as a researcher, observed six live lessons in which the two mathematics teachers presented their lessons for the duration of 40 minutes per lesson (three lessons per teacher). As I was observing the lesson and noting down the important points (field notes), the lesson presentation was also video recorded by the co-researcher. This was very useful to supplement the observation, as some of the important aspects may be omitted during note taking.

The team, which is formed by me (researcher) and the participants, held several meetings in the form of reflections and focus group discussions for two hours each in three weeks. . In these meetings, the discussions were based on the challenges relating to the teaching of algebraic expressions, as well as identifying the appropriate solutions to address the challenges However, due to the limited time to meet learners because of conditions of COVID-19 where grade 10 learners were not available every day at school, only one cycle of AR was done. Also, some of the meetings were virtual, especially meetings between researcher and the teachers.

The discussions were audio and video recorded, and these deliberations were allowed to take place in Sesotho and English, depending on the comfortability of the participants in order for them to feel free to contribute and converse their views. Data quoted in Sesotho were translated to English with the help of the translator to ensure that the views and meanings of the text are not lost or misinterpreted. Both the unproductive and productive practices observed, were also highlighted and discussed. During the meetings, the conversations were initiated and facilitated by a Free Attitude Interview (FAI) technique.

3.4.3 Instrumentation

This section discusses instruments and tools that were used to collect data in this research. The choice and use of an instrument and/or tool was mainly determined by the nature of data to be collected. According to Krishnaswamy (2004), for an AR project to be successful, the research partners need to have an understanding of its extensive goals (Sider et al., 2021). He advises that the researcher's goals should be elucidated so that they can be related to those of the other members of the team. This means that without a clear sense of what the research project is trying to recognize, it will be tough to formulate a practical and effective action research project.

3.4.3.1 Lesson observation

In the perspective of Gitomer et al. (2014), observation is the process of studying classroom events to ascertain teaching strategies and learner interest. According to Lam (2001), classroom observation is a tool used to improve the quality of teaching and learning amongst teachers. Moreover, Olsen (2008) mentioned that observation can be utilised to get comprehension into organisation, planning, methods of presentation, and techniques for behaviour management, and diverse learners' differences. According to Cohen and Goldhaber (2016), observation offers the researcher the opportunity to collect live data from natural occurring communal circumstances. It is on this view that I as a researcher sat down in the classroom and observed lessons in order to gather information. Live data might give the researcher a clear perspective on how the participants constructed meaning, because in some cases, what people say might differ from what they do (Robson, 2002). Data obtained through observation is used for the purpose of the description of settings, activities and people; and the meanings of what is observed from the perspective of the participants. Observation can guide the researcher to profound understandings than interviews alone, because it provides knowledge of the context in which events occur and may enable the researcher to see things that the participants themselves are not aware of, or that they are unwilling to discuss (O'Leary & Wood, 2017; Patton, 1990).

3.4.3.2 Focus group

The study has also used a focus group as another instrument to collect data. According to Morgan (2002), a focus group is a research technique that collects data through group interaction on a topic chosen by a researcher. In line with this, Nyumba et al. (2018) also mentioned that a focus group is a gathering of deliberated people who are chosen to take part in a facilitated discussion, anticipated to stimulate their insights about the research topic. Morgan (2002) further explains that one of the many uses of a focus group is based on its ability to give a voice to the marginalized participants to be heard. However, Magill (1993) had a view that the value of a focus group goes way beyond listening to others, because it can serve as a foundation for empowering participants, as well as allowing participants to exercise a fair degree of control over their own dealings, which work hand-in-hand with CER as the lens couching this study or as a tool in action research (Morgan, 2002; Padilla, 1993).

In addition, Fontana and Frey (2005) holds the view that a focus group is a group interview where the researcher asks a set of targeted questions planned to stimulate collective opinions about a specific topic (Ryan et al., 2014). Thus in our focus group meetings, I as the researcher, asked the questions planned for the focus group interviews as outlined in Appendix D1. In this case, I acted as a facilitator by not only taking control of existing relationships, but also by creating a relaxed and comfortable environment for unacquainted participants.

In the view of Ryan et al. (2014), the focus group situation agrees to conversations that inspire elaborations, agreements, and disagreements amongst participants that disclose the range of reactions to a specific issue. Hence, the focus group meetings assist the researcher to get a deeper understanding of the challenges experienced and also gather rich information on what can be done to address the challenges from various viewpoints, because a focus group allows for various participants with different experiences (Colom, 2021; Morgan, 1996).

3.4.3.3 Follow-up interviews (one-on-one)

Even though the focus groups discussions were facilitated in such a way that a relaxed and comfortable environment was created (Nyumba et al., 2018), some of the participants were still not at ease to exchange their ideas. Then, we scheduled a follow-up interview with the participants to accommodate them and to provide them with a conducive environment for them to express their views in-depth. These conversations were tape recorded. For the learners, the following two general interview questions were asked, firstly, *what aspects of algebraic expressions do you find challenging?* And second, *what makes algebraic expression more difficult for you?* Furthermore, to gain more understanding with regards to learners' incorrect responses and misconceptions observed in class, the follow up interview sessions were organised. The follow up questions were based on classroom observations, particularly what the learners seemed to struggle with during the lessons. The questions were therefore based on learners' individual challenges and aimed at finding/determining the root cause of learners' error and misconceptions.

3.4.3.4 Reflection sessions

The reflection sessions were scheduled for the participants to reflect upon their practices. With the aim of the study highlighted, we reflected on the challenges of teaching algebraic expressions in Grade 10. We reflected on the solutions to the identified challenges and what we regarded as the conducive conditions to the implementation of UDL strategies as well as other best practices model discussed in section 2.4. The reflective session was also done to ensure that the participants' words were interpreted correctly.

3.5 ENSURING TRUSTWORTHINESS

According to Gunawan (2015), for a study to be trustworthy, it is if and only if the reader of the study perceives it to be so. Guba (1981) has proposed four criteria for ensuring trustworthiness of the qualitative research, which are credibility, transferability, dependability and confirmability. Credibility of the study means the quality of research is convincing and believable. This talks about selection procedures followed to select the participants. Whether participants were randomly chosen or purposefully chosen, as well as whether participants were given an opportunity to refuse to take part in the research (Lincoln & Guba, 1989; Shenton, 2004). Drawing from this, it is worth noting that it is even more challenging to deal with issues of trustworthiness in a qualitative research where AR serve as an approach and the researcher is also a participant in it. However, the importance of how this aspect is dealt with in this research to produce a study that reflects the meaning, ideas and lived experiences by participants, cannot be overlooked.

Therefore, during the first meeting, participants were informed that they are free to withdraw from the study at any time if they wished to, without fear of being negatively treated. Transferability of the study talks to the fact that if there are enough similarities between two situations, readers may be able to deduce that the results of the research would be the same or similar in their own situation. In other words, they transfer the results of a research to another context. Hence, according to Shenton (2004), this criterion needs the research to include issues, such as a number of participants, the area where the study will be conducted, the type of people who contributed to the data, data collection and methods, the number and length of data collection sessions and lastly, the period over which data was collected. As a result in this study, the target participants included two mathematics teachers from School A, and 43 Grade 10 learners. These participants were recruited from Motheo District in the Free State.

Gunawan (2015) has mentioned that in quality research, dependability is an evaluation of quality of the incorporated processes of data collection, data analysis and generation of theory. It is associated with consistency of the findings, thus dependability suggests that the results must be accurate and consistent. According to Shenton (2004), in addressing dependability, the following issues should be considered: the description of what was planned and executed on a strategic level in research design and its implementation should be provided, minutes of what was done during data collection should be provided and evaluation of the effectiveness of the process of inquiry undertaken (See table 3.2).

Confirmability as the last criterion, is established when credibility, transferability and dependability are all established. It has to do with the level of trust that the study findings are based on participants' narratives and words, but not on the researcher's preferences (Shenton, 2004).

3.6 DATA ANALYSIS

According to Daly et al. (1997), thematic analysis (TA) is a technique used to seek for themes that arise as being significant to the description of the occurrence. The process entails the recognition of themes through reading and re-reading of the data in an extremely careful manner (Rice & Ezzy, 1999). It is therefore, as stated by Fereday and Muir-Cochrane (2006), a form of pattern appreciation within the data, whereby developing themes become the classifications for analysis.

Thus, to analyse data, Thematic Analysis (TA) was used. This technique is all about systemic identification, organisation and presenting of insight into patterns of meaning throughout a data-set (Braun & Clarke, 2012). The technique was chosen firstly, because both TA and AR are concerned with working with data in a systemic way. Secondly, this technique was chosen, because of its flexibility to analyse a wide range of data types such as face-to-face interviews, focus groups' textual data, diaries, as well as online discussion forums (Terry et al., 2017). Lastly, the choice of this technique was based on the fact that the technique is the most suitable one to analyse data collected in the qualitative research, which this study is utilizing.

Thus, the following table shows the implementation of the steps of thematic analysis in the context of this study.

Table 3.4: Six steps of thematic analysis

Steps	**Descriptions**
1. Familiarising oneself with the data	In this step the researcher familiarized him/herself with the whole set of data. This means that the researcher read repeatedly and listened to recorded data. Getting familiar with data serves as a valuable and important step since it is foundational for all other steps that follow (Kiger & Varpio, 2020).
2.Generating initial codes	After the researcher has been familiar with data, the coding process commenced. In this process the researcher took notes on the potential data items and thus generated codes. A code, as defined by Boyatzis (1998), is the highly basic section, or components of the unprocessed data or information that can be considered in a meaningful way in relation to a phenomenon. Thus, coding was done manually in this study.
3. Searching for themes	According to Braun and Clarke (2006), this step involves examining data extracts that are coded and collated in step 2 for themes of wider significance. In addition, Kiger and Varpio (2020) mentioned that during the developmental and organizational stage of themes, thematic maps are beneficial for visually showing cross-connections between concepts, as well as among main themes and sub-themes.
4. Reviewing themes	This step is described by Braun and Clarke (2006) as a two-level analytical step. In the first level the researcher views the coded data to make sure it properly fit. In the second level the researcher takes a decision of whether individual themes fit within the data-set in a sensible manner, as well as whether the thematic map sufficiently symbolizes the full body of data. In order to achieve this job, the researcher read

	once more through the complete data-set to revise themes and to re-code for supplementary data that falls beneath the themes that have been newly created or modified in this form, then improves the thematic map accordingly.
5. Defining and naming themes	In this step the researcher creates a definition and narrative description of each theme, including its importance to the main study question. The researcher reviewed the names of the themes to be included in the final report, as well as making sure that they are correctly and sufficiently defined (Kiger & Varpio, 2020).
6. Producing the report/manuscript	This step entails the writing of the final analysis and description of findings. In the perspective of King (2004), this step is a continuation of the analysis and description of data done in the prior steps and not a separate step altogether.

3.7 ETHICAL CONSIDERATIONS

I wrote a proposal to two committees at the University. I was granted permission by the Ethics Committee and Committee for Title Registration, and I then continued to seek permission of entering the school premises from the Free State Department of Basic Education, which through its district office gave me permission (See Appendix A1). I indicated my field of research to the principal in the first meeting and showed them the permission letter to make them aware that the Free State Department of Basic Education was aware of the research process. I also made it clear that the participants could withdraw at any stage of the research without fear of being penalised.

All the participants were requested to take part on their free will and were treated with respect and dignity throughout. The participants were anonymised and their rights were respected. The conditions of anonymity were applied to the collection of data by means of cameras, tape recorders and voice-recorders, as well as to data collected face-to-face or through participant observation (Opdenakker, 2006).

The participants were requested to sign the consent forms. Each of the participants had a copy at hand as the forms were read aloud. All members signed at the first meeting except for learners below 18, who took the forms home so that their parents may sign them. In the meeting following this one, all forms were signed and put together. The participants were promised that the data would be anonymised and kept safely for a period not exceeding six months and thereafter be destroyed as is normal practice in research (Smith, 2003). The study was ethically cleared by the University Ethics Board and assigned the reference number UFS-HSD2020/2092 (see Appendix A1).

3.8 CONCLUSION

In this chapter, AR as a methodology was discussed and clear reasons for its adoption and suitability in this study, provided. The chapter also discussed the participants involved in the study and their roles. The instruments used to generate the data were also explained in this chapter. FAI was described, and its principles highlighted to demonstrate how the research question was responded to. A justification for its application in this study was also provided. The chapter explained how the issues of trustworthiness were addressed in the study. The intervention process was explained, and its role highlighted. The use of TA as an analytical tool was highlighted. The chapter also provided the relevance of TA as a tool for analysing the data, as well as the ethical process in respect of this study.

The next chapter focuses on the data analysis and interpretation of the findings of this study.

CHAPTER 4 :
DATA ANALYSIS

4.1 INTRODUCTION

The purpose of the study was to explore the common algebraic expression challenges in a Grade 10 mathematics classroom. The previous chapters presented the research paradigm underpinning this study, review of literature and the methodology employed to generate the first-hand data. The data generation instruments and procedures utilised in this study were outlined in fulfillment to the main research question of the study, as highlighted in Chapter 1 (see Section 1.4). This chapter presents, discusses, analyses and provides interpretation of data, using a thematic analysis technique. The chapter will highlight the common algebraic expression challenges in a Grade 10 mathematics classroom as identified by the participants through focus group/reflection sessions and also as observed from the classroom lessons. The chapter will further highlight the solutions which were identified and attempted to address the identified challenges. The objectives of the study were used as the organising principles. This chapter discusses the findings that reveal some intervention strategies that can be used to address the identified common algebraic expression challenges in a Grade 10 mathematics classroom.

Points to take note of in this chapter:

- The data were collected using the language the participants were comfortable to speak. Therefore, in some sections, the extracts are written in Sesotho, which is the language that was also used during data collection. The Sesotho extracts are however, translated in English. As highlighted in Chapter 3, the translation was carried out with the assistance of the language specialist. This was done in order not to misinterpret or misrepresent the data.
- ET is used as an abbreviation for English translation

4.2 COMMON ALGEBRAIC EXPRESSION CHALLENGES / ISSUES PERTAINING TO THE TEACHING AND LEARNING OF GRADE 10 MATHEMATICS

The subsequent sections discuss the common algebraic challenges in a Grade 10 mathematics classroom.

4.2.1 Inadequate teacher pedagogical content knowledge to teach algebraic expressions

According to Yildiz and Akyus (2019), the knowledge of how learners acquire understanding and where learners experience problems are essential elements of Pedagogical Content Knowledge (PCK) so that teachers can be in a position to provide alternative explanations and models. In the view of Black (2009), PCK is the knowledge of a teacher that he/she uses to unfold the mathematical topics and present the content in ways for learners to successfully learn mathematics. To be more specific, mathematics teachers' knowledge is essential to learners' mathematics achievement (Yildiz & Akyus, 2019). PCK is important because it provides teachers with ability to solve problems using a variety of methods in different contexts. Even though it is evident that teachers' PCK is very important for the good performance of the learners, the participants' responses during the focus group discussion, wherein the common algebraic challenges were identified in a Grade 10 mathematics classroom, indicate that both novice and experienced teachers lack knowledge of innovative, inclusive and flexible teaching methods to teach, especially, the diverse learner population of the Grade 10 algebraic expressions. The comments made were as follows:

> *Nna kannete ke latela mehlala ya textbook e be ke ba fa classwork hobane algebraic expression ya grade 10 e ngata ene pace setter e tla be e re o e etse in three weeks* **(Mr Letuka).**
>
> [The truth is, I just follow the textbook examples and give them classwork, because Grade 10 algebraic expression content is too much and on top of that, one is expected to finish it within three weeks.]
>
> *'Hape bana ba na ba botsoa, ha o re oba fa mosebetsi baa complainer, ba bang o tla bona hore ba kgethile ho etsa Maths ka baka la ho matha ka mora bakgotsi* **(Mme Ntlama).**

> [To add on that, these learners are lazy, because they complain when they are given some work to do. You can also notice that some have chosen to do mathematics, because they are influenced by their friends.]

Mr Letuka's utterances insinuate that he does not use a variety of teaching strategies during his lesson presentation. He confines himself to the examples that are provided in the textbook and follows them as they are to teach learners. This also shows that the teacher does not reflect on appropriate teaching strategies (i.e., topic specific strategies) that could assist him to teach algebraic expressions in a manner that is comprehensible to all the learners. The teacher thus uses a textbook approach as a "one-size-fits-all strategy", which according to the UDL model, does not cater for all the learners. According to Bray and Tangney (2015), a one-size-fits-all strategy cannot be effective because learners differ in the manner in which they assimilate information (see section 2.3.2.2). Hence, by choosing this one-fit-all strategy, the teacher violates principles of 5E guided inquiry model, which advocates for active engagement of learners in the learning process (see section 2.4.1). This, therefore, means that learners need to be exposed to different strategies that will cater for their various needs and improve their understanding.

What came out clearly in Mr Letuka's utterance is that the teachers do not make efforts to advance their PCK, because of the limited time (three weeks for the content to be completed) they have to teach the different learning areas as outlined in the Curriculum and Assessment Policy Statement (CAPS) document. Following the textbook and its examples, as outlined in the text book, thus seem to be a simple way of completing the tasks as quickly as possible and as expected of them. On the other hand, with regard to Mme Ntlama's utterances, "*Hape bana ba na ba botsoa, ha o re oba fa mosebetsi baa complainer…."* ["To add on that, these learners are lazy because they complain when they are given some work to do…"], the other reason why teachers do not bother to go an extra mile in terms of devising different teaching strategies, is because they teach learners who are "lazy" and always complain when they have to engage in problem solving. The laziness of the learners thus seems to be a discouraging factor for teachers to devise and apply the innovative, inclusive and flexible teaching strategies for teaching algebraic expressions. The teachers' PCK subsequently does not advance because of this. According to Guler and Celik (2021), PCK is central to excellence in teaching, as it reveals the special blend of the content and pedagogy that forms teachers' professional understanding of how certain topics, issues and questions may be sequenced, organised and represented for instruction. It also helps teachers to go deeper when they explain the concepts. Therefore, teachers who lack PCK become a barrier towards learning, since they are unable to unpack the concepts in a meaningful and comprehensible manner.

4.2.2 Teachers' inability to explain algebraic concepts in-depth

According to Yildiz and Akyus (2019), teachers need to have knowledge of putting at work multiple representations, justifying reasoning, making use of generalization and posing problems for teaching algebraic reasoning. Therefore, this requires teachers to have a Specialized Content Knowledge (SCK) of algebra. That means teachers must have a deep understanding of algebra so that they can be able to explain concepts thoroughly for the benefit of the learners. Moreover, these scholars suggest that for teachers to be able to teach algebraic expressions, they must know how to plan instruction for teaching mathematics, lead a discussion, and make decisions about the instruction. Furthermore, in order to develop learners' thinking ability, teachers can present the procedures of operations with algebraic expressions by connecting the arithmetic procedures based on the similarity of the structure of expressions. The extracts below are from the lesson presented by Mr Letuka, indicating his inability to explain the algebraic expressions in depth:

Algebraic expression/equation lesson presented by Mr Letuka: 2 examples extracted from the lesson.

Mr Letuka: "…an algebraic equation has an equality sign whereas an algebraic expression does not have an equality sign therefore ($2x - 7 = 2y$ *is an equation but* $3(x+5)$ *is an expression".*

…

Mr Letuka: When you are given an expression such as this one (pointing at the chalkboard) and you are required to simplify it. Then here you must determine the lowest common multiple. In order to do so, in this case you can multiply the first fraction by five over five, which is still the same as 1 and multiply the second fraction by 4 over 4, which is also the same as 1.

$$\frac{3x}{4} + \frac{2x}{5}$$

During lesson observation, when defining algebraic expressions to learners, the teacher did not elaborate much on how an algebraic expression is different from an algebraic equation. He only mentioned that in an algebraic equation there is an equal sign ($2x - 7 = 2y$), but in an algebraic expression there is no equal sign [$3(x+5)$]. Although this explanation is helpful in terms of assisting learners to distinguish between the algebraic expressions and algebraic equations, it does not give any conceptual explanation related to the mathematical meaning of the reason that makes the algebraic expression not to involve an equality sign. When it came to showing learners how to simplify $\frac{3x}{4} + \frac{2x}{5}$, Mr Letuka only mentioned to learners that they should rewrite the expression in such a way that the denominators are the same by finding the Lowest Common Denominator (LCD) $\frac{\square}{20} + \frac{\square}{20}$(in this case LCD is 20), and he guided the learners that they must first find the LCD by identifying the Lowest Common Multiple (LCM). He said that this can be done by multiplying the first fraction by 5 (i.e., multiply numerator 5 and denominator by 5) and the second fraction by 4 (i.e., multiply numerator 4 and denominator by 4). He then said that they should add the numerators and write the denominator as it is (i.e., 20). This means that they will have $\frac{15x}{20} + \frac{8x}{20} = \frac{23x}{20}$.

Mr Letuka did not refer the students back to the concept of equivalent fractions to assist them to understand the simplification of the expression that he gave them. Thus, in his teaching he violated the UDL (MMR principle), which requires teachers to elicit learners' prior knowledge to enable the learners to understand and build new concepts. Tapping into learners' previous knowledge about equivalent fractions could have assisted some learners and made them aware of the application of equivalent fractions during the addition and subtraction of algebraic fractions. This act of Mr Letuka violates the UDL model (see section 2.4.2), which advocates for the need to elicit learners' prior knowledge.

4.2.3 Learners' inability to apply the main algebraic concept

Teachers play an important role in the teaching of algebra, since they are the ones who make decisions about algebra instruction (Black, 2009). However, the study done by Marpa (2019) revealed that many learners are often challenged in terms of applying the main algebraic concepts, especially the classification of algebraic expressions, according to the multiplication of algebraic expressions, degree, and translation of mathematical phrases and sentences into mathematical symbols and equations. Therefore, an inability to apply the main concept of algebra can potentially present challenges to learners in terms of manipulating the algebraic expressions. During the focus group discussion, the participants echoed the same sentiments. They commented as follows:

> **Mr Letuka:"** *When asked to evaluate an expression such as* $243^2 - 242^2$ *without the use of a calculator, learners work it out as follows:"*

$$243^2 - 242^2$$
$$= 24(3^2 - 2^2)$$
$$= 24(9 - 4)$$
$$= 120$$

> **MMe Ntlama***: "Kapa ha ba fuwe* $29^2 - 27^2$ *hona le hore ba sebedise factorization of difference of two squares ba etsa mosebetsi o mongata wa ho multipler ba be ba fose karabo."*

> [Or if they are given $29^2 - 27^2$instead of applying factorization of the difference of two squares they work out the problem by using a lot of steps which sometimes yield a wrong answer.]

Mr Letuka: *"When you have given them an expression, they tend to cancel the terms and not the factors. For example,*

$\frac{3x-15x}{3x}$ *they will give an answer of* $1-15x$*."*

Based on Mr Letuka's first utterances above, learners seem to have difficulty to evaluate an expression such as $243^2 - 242^2$, since they are used to solving the difference between two squares using the variables (e.g. $x^2 - y^2$) and not the numbers. This therefore, might mean that learners are not exposed to this type of problems in various forms. They also think that the square applies for the last digit only as depicted in the example made by Mr Letuka above.

As highlighted in section 2.4.4, the variation theory model advocates content variation that must be taught in-depth by choosing examples that afford learners an opportunity to discern significant aspects of the specific content. However, in this regard the teacher seemed to have not adhered to the variation theory in order to develop a deep understanding of mathematical structure to represent this concept.

According to Mme Ntlama's sentiment above, instead of learners applying factorization of difference of two squares, they work out the problem and end up getting it wrong or they use a calculator, which they were asked not to use from the question's instructions. It also indicates that when dealing with factorization of the difference of two squares, teachers do not expose learners to the difference of two square involving numeric values. As a result, learners fail to apply the concept of algebra, because they cannot relate the numerical difference of two squares and algebraic difference of two squares.

The second statement of Mr Letuka shows that learners have a problem with simplification of algebraic fractions, because they tend to cancel the terms instead of factoring out the common factor first. The result *of* $1-15x$ shows that learners considered $3x$ as the only factor and cancelled it out. That is, they only noticed $3x$ in the numerator and denominator and cancelled it out. To some extent this implies that teachers do not emphasise the importance of identifying a common factor and taking it out before cancelling it out. Learners are therefore not exposed more often to a variety of such problems as part of addressing not only the incorrect way(s) of cancelling the common factors, but also the algebraic structure as necessitated by the variation theory (see section 2.4.4) .

The discussion further continued as follows:

> **Mme Ntlama:** *"O bone ha a simplifaya* $8a^2 + 3a^2$ *a re answer ke* $11a^4$ *ka nako yeo ntse ke hlalositse hore re kopanya di coefficients feela e seng di exponents."*
>
> [You can see that when you ask them to simplify $8a^2 + 3a^2$ they give an answer of $11a^4$ even after I explained that they should only add coefficients and not the exponents.]
>
> **Mr Letuka (adding to that):** *"Or you say to them when you have like terms you should just add the coefficients."*
>
> **Miss Ntlama:** *"…Sometimes we stick to one way of representing the concepts because we want to save time…".*

The teachers' utterances indicate that teachers really do not challenge themselves to use teaching strategies that would assist learners to comprehensively learn algebraic concepts. Mme Ntlama's statement, "…*ke hlalositse hore re kopanya di coefficients feela e seng di exponents"* [I explained that they should only add coefficient not exponents.] indicates that she used a chalk-and-tell method, which Bray and Tangney (2015) (see section 2.3.3.3.2) discourage, as it does not always result in productive learning. Mme Ntlama's statement indicates that she does not use various ways to make learners understand what $8a^2 + 3a^2$ means and represents. She thus violates a UDL principle of multiple means of representation, which requires various representations to be used in order to make the concept simple and intuitive (CAST, 2011). One way to simplify and make the concept comprehensible, according to UDL, is by using multiple means of representation, which requires content/concepts to be represented in multiple formats (Moleko, 2018) (see section 2.4.2). This makes it easy for learners to recognize the same problem represented in various forms However, judging from the teachers' comments above, it seems as if teachers only rely on one way of representing the concept (algebraic expression) and this subsequently makes it difficult for the learners to recognize the same concept when it is represented differently (Moleko & Mosimege, 2021).

Based on Miss Ntlama's comment, one gathers that the reason why teachers are not using multiple representations to present/represent the mathematical concepts, is because they see it as time consuming, since it requires teachers to do a lot in terms of preparation. However, Moleko (2022) discourages the perception that the use of multiple representation is time consuming. In fact, she espouses the use of multiple representation, since it does not only reinforce understanding, but also engages learners in deep learning, which is necessary to inspire conceptual understanding. Teachers often resort to methods that would save them time, regardless of their ineffectiveness.

Other comments from the focus meeting were as follows:

> **Mr Letuka:** *"O bone ha potso e re solve for x in the following equation* $\frac{4^x-1}{2^x-1}=17$ *ba sa kgone ho nahana hore the factorizing methods they learned in quadratic expression can be applicable."*
>
> [You will notice that when they are given a question like solve for x in the following equation $\frac{4^x-1}{2^x-1}=17$ they are unable to think that factorization methods they learned during quadratic factorization can be applicable.]

Based on the above comment by Mr Letuka, learners encounter difficulties when given an equation such as $\frac{4^x-1}{2^x-1}=17$ and asked to solve it, because they cannot recognize that such an equation can be factorized. This is because they are used to the standard structure of factorization where they factorise the quadratic equations in the form of $ax^2+bx+c=0$.

This shows that the manner in which the teachers teach learners how to factorize, does not enable them to recognize when factorization can be performed, especially when there is a change of structure in how the equation is written or presented. Thus, learners seem to realise the need for factorization when the exponents are numbers and not the variables, as in the example provided by Mr Letuka. To a certain extent, this shows that teachers do not provide learners with a variety of problems, which will enable learners to learn more about the different algebraic expression and equation structures to be able to solve the different equations, thus violating the variation theory.

4.2.4 Lack of understanding the variable concept

In the perspective of Yildiz and Akyus (2019), teaching of the concept of the variable concept is initially done in the context of generalization of patterns; thereafter, the teaching of algebraic expressions is done conceptually and procedurally in the context of operations with algebraic expressions related to these generalizations. Hence, *lateral symbols,* according to Jupri et al. (2014), are essential in algebra, however, learners have challenges about differentiating their different roles as placeholders, a generalised number, unknown or carrying quantity (see 2.5.1.2). This challenge became apparent during classroom observation when learners were asked to find the dimensions of a rectangle whereby the length and the width were expressed in terms of x and given the area of the rectangle (see an activity below)

Algebraic expressions lesson presented by Mr Letuka

Mr Letuka: Now people let us do the following activity…

Activity 4.2.4

Determine the dimensions of the following rectangle

$x - 3$

$x + 5$

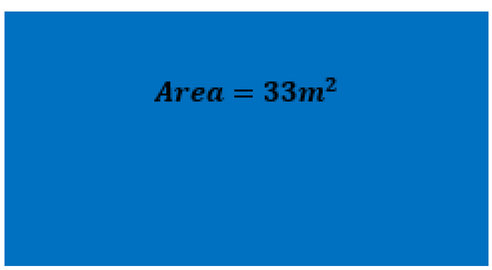


Figure 4.1: Activity 4.2.4

Most of the learners solved this as follows: $(x-3)(x+5) = 33 \therefore x^2 + 2x - 48 = 0 \therefore (x+8)(x-6) = 0 \therefore x = 6$ or $x = -8$ and left it there as a final answer without considering that the question required them to determine the dimensions of the rectangle, which means they need to select the appropriate value $(x = 6)$, substitute in the given dimensions and write down the exact dimensions, which are length = 11 and breadth = 3. Those who went on to the extent of finding the dimensions used both values of x to calculate the dimension, instead of discarding the negative value. In this instance, one gathers that for these learners, the variable x is just an unknown, so when they have solved it, it is enough for them. On the other hand, when they do not discard the negative values it shows they fail to apply the context in which the question was asked. That is the question requiring dimensions of a rectangle, which means they were asked to determine length and width of the rectangle and therefore, the valid answers should be both positive values. This shows that teachers, in most cases fail to expose learners to different forms of algebraic representations, such as a pictorial form as shown in Fig 4.1 above. The teachers also seem to fail to invest time in realistic mathematical modeling (see section 2.3.2.2) of various algebraic expressions to promote understanding. The same question could be formulated in the form of a word problem, thus depicting a real-life problem and judging by the learners' failure in activity 4.2.4, it seems as if they might even find it more difficult to attempt it, since many learners find word problems challenging to solve (Moleko, 2018). This also shows to a certain extent, that no substantial efforts are made in engaging the learners with different representations of the algebraic expressions. This limits the learners' ability to tackle the different types of the algebraic expressions given to them.

4.2.5 Inability to manipulate algebraic expressions

According to Ferretti (2020), majority of learners learn algebra rules by heart, with little or no conceptual understanding of their meaning. The algebraic terms, rules and properties seem to be worlds away from the thought processes of most learners. According to Marpa (2019), algebraic conception depends not on learners' knowledge of the formulas and understanding the calculations right, but rather on comprehending the concepts and operations, and development of mathematical thinking. Thus, for learners to have conceptual understanding, they must understand the following aspects of algebraic expressions; structure or rules of algebra or rules of arithmetic such as associative, commutative, transitivity and the closure property, which they seem not to know. To develop the learners' understanding of algebraic expressions, the teachers therefore need to pay attention to these aspects. Figure 4.2 has been extracted from one learner's book during classroom presentation and was observable during the teacher's lesson.

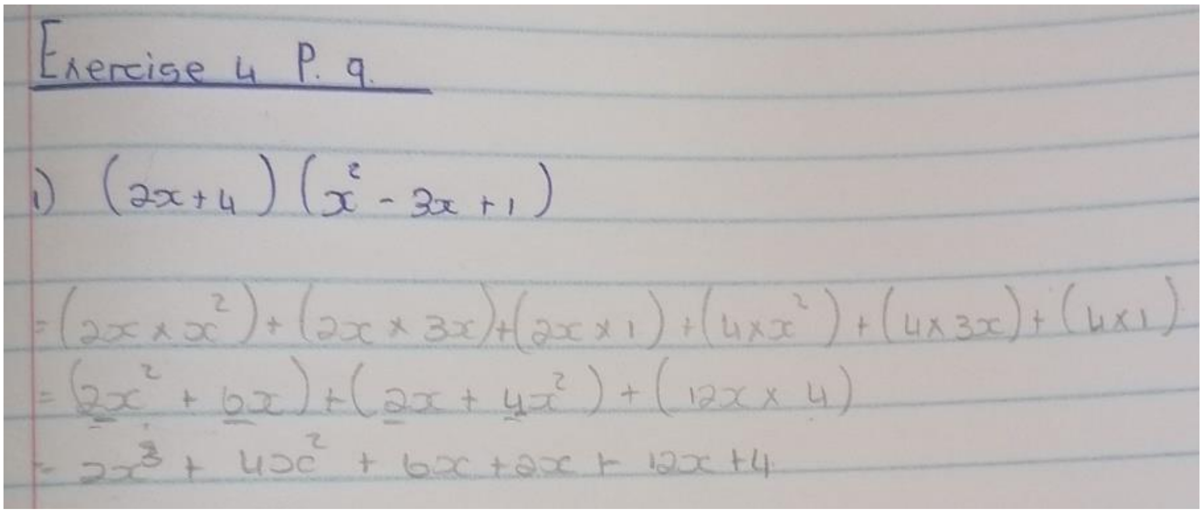


Figure 4.2: Extract A from learner's book

The learner applied the distributive property in a correct manner, however, the middle negative term in the trinomial has not been correctly used, because she used positive $3x$ instead of negative $3x$, and as a result she obtained a wrong answer. Even though she wrote the product of $(2x + x^2)$ as $2x^2$ in the second step, she wrote it correctly in the last step as $2x^3$.

The following conversation took place between the researcher and learner during a reflection session after the lesson had been presented. The researcher tried to establish what made the learner commit the errors that she had committed.

Researcher: *…You applied the distribution property well here, but you multiplied* $2x$ *by* $3x$ *not minus* $3x$ *, why did you do that?* **Halieo (learner):** *Was I supposed to multiply by* $-3x$*?* **Researcher:** *Yes, the negative sign is assigned to* $3x$*. The sign in front of a number is assigned to that number and wherever you carry that number you must also carry it.*

Halieo: *But other numbers I multiplied with them without their signs.*

Researcher: Those numbers were positive, tell me, is $2x = +2x$*?*

Halieo: *Yes, they are equal.*

Researcher: *Is,* $3x\ is\ equal\ to - 3x$*?*

Halieo: *No,* $3x\ is\ not\ equal\ to - 3x$.

Researcher: *So, you see you were supposed to multiply by* $-3x$*?*

The above conversation shows one of the learners' work when multiplying a binomial and trinomial. The learner is able to apply the distributive law of property, but instead of multiplying by $-3x$, has multiplied by $3x$. The challenge here is that the learner has a problem of understanding that the sign in front of the number is part of that number and they both make up a single term. Which means the learner lacks conceptual understanding of a concept "term".

Another extract of the learner's worked out problem that shows misuse of the commutative property, is shown in Figure 4.3.

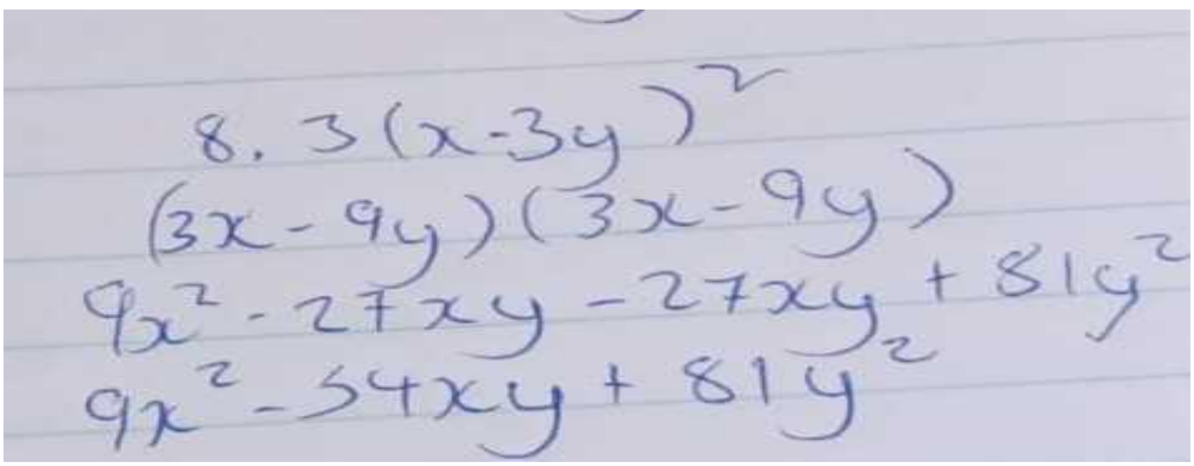


Figure 4.3: Extract B from learner's book

The following are the extracts of the conversation between the researcher and the learner during a reflection session after the lesson observation.

Researcher: *Can you please take me through your first step in working out this.*

Tefo: *I multiplied by 3 inside the first bracket and the second bracket.*

Researcher: *why did you do that?*

Tefo: *Because we are asked to square.*

Researcher: *But the square applies only for the bracket. That is,* $3(x-3y)^2 = 3(x-3y)(x-3y)$

Tefo: *Yes teacher, and that means I must multiply by 3 the first and the second bracket.*

The challenge portrayed through figure 4.3 is that the learner has used the pre multiplier inappropriately, which indicates that this learner does not understand that $3(x-3y)^2$ can be written as $3(x-3y)(x-3y)$ so that he can start by multiplying the first bracket by the pre-multiplier or multiply the two brackets first and end with multiplication by pre-multiplier. Which means that the learner may have a challenge with the commutative property.

Another mistake detected during the class lesson is show below (see Figure 4.4).

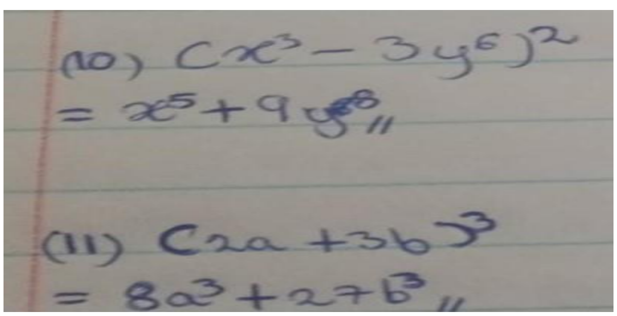


Figure 4.4: Extract C from learner's book

The extract below is the conversation that took place during the reflection session after the classroom observation:

Researcher: *I could notice that you squared the first and the last term in your workings, may you tell me why?*

Lintle: *It is because when we are given* $(x^2)^3$ *the answer is* x^6*.*

The mistake that is portrayed in figure 4.2.5 (c), is that the learner misused the exponential law here, which says when given a power raised to another exponent, you multiply the exponents. In this case, the learner was given a binomial and not a single termed expression and that exponential law cannot be applied.

In relation to this aspect, teachers from the focus group meeting reacted as follows:

> **Mme Ntlama:** *Sometimes you find that learners are not able to leave an answer as a binomial expression like $x + 7$, but they want to leave it as $7x$.*

The comments made by Mme Ntlama indicate that the learners have a problem of closure when they are simplifying algebraic expressions, because for them $x + 7$ cannot be regarded as an answer, because they perceive an answer to be in the form of a single term only, hence why they continue to simplify it further to be $7x$. This challenge was also observed by Seng (2010) in his study about error analysis of learners in simplifying algebraic expressions. The implication here is also that they lack conceptual understanding of manipulating like terms. This indicates that teachers may not have dealt with this aspect of like terms thoroughly.

4.2.6 Improper use of mathematical vocabulary/expression

The language of mathematics is an important element of mathematics teaching, as it plays at least three essential roles in the mathematics classroom (Gay, 2008; Thompson & Rubenstein, 2000). Firstly, teachers deliver instruction through the medium of language. As a result, the language of mathematics is our major means of communication.

Secondly, learners develop understanding as they process concepts through language.

And lastly, teachers establish and assess learners' understanding by listening to their oral communication and by reading their mathematical writings (see section 2.5.1.5).

According to Gay (2008), teachers are role models in the way they use mathematics vocabulary. This implies that teachers must possess a sound knowledge of mathematics vocabulary and use words/terms/expressions properly during a lesson presentation and when they name or explain an object or an action (Joseph, 2020; Planas, 2020). Although teachers' sound knowledge of vocabulary is so important, sometimes they seem to use words improperly during a lesson and this impacts negatively on learning. In the focus group, the teachers mention the following:

> **Mme Ntlama:** *"I am one of those teachers who refer to the exponent as the power… when introducing exponents, I would explain to learners that x^n is a power and x represents a base and n represents an exponent. However, when talking during instruction I normally say 'to the power of n', and this is wrong and confuses learners."*
>
> **Mr Letuka:** *"Bona, fraction ke ena $\frac{2}{3}p$ $(pointing\ at\ the\ fraction)$ (two-thirds of p), some teachers read this as "two over three p" and this is confusing learners because even $\frac{2}{3p}$ is read as 'two over three p'."*
>
> [Look, here is a fraction, …]

The comments made by Mme Ntlama, "*I am one of those teachers who refer to the exponent as the power…"* tells that she is aware of the mistake she is making, but does not make an effort to rectify it. She also acknowledges that it is wrong to say *"to the power of n"* when referring to x^n.

The utterances of Mr Letuka also show that the way teachers themselves use words in the class should be carefully considered, because it can mislead learners and cause confusion. Thus, teachers should be very vigilant on how to use mathematical terms in class, because according to Thompson and Rubenstein (2000), using precise mathematical terms is a fundamental component in the mathematics classroom (see 2.5.2.4). Therefore, teachers should not assume that all learners understand and know mathematics language (vocabulary). However, this assumption could be because teachers themselves do not invest time to master mathematics vocabulary (Gay, 2008) (see 2.5.1.5).

4.2.7 Gap between algebra and arithmetic

Seng (2010) asserts that there is a didactic cut in the child's thought in the transition from arithmetic to algebra and that learners struggle to operate with unknowns in the transition to algebraic thinking. In addition, Gningue (2016) affirms that the transition from using numbers to using symbols is much more problematic for many learners than it has been anticipated. The insufficient arithmetic knowledge acquired, which is the basis upon which knowledge of algebra is acquired, is the reason for learning difficulties to close the arithmetic-algebra gap learners seem to display. Proper employment of context has shown to be a fruitful bridge between arithmetic and algebra. The extracts below are comments made by teachers about the transition gap between arithmetic and algebra as a challenge:

> **Mme Ntlama:** *"Bothata mona ke hore bana ba na di concepts tse ba ithutileng with arithmetic numbers ba ba le bothata ba ho di sebedisa ha re fihla di variableseng. Mohlala feela, ha ba kopanya, batlosa, multiplication and division of algebraic fraction ba ba le bothata."*
>
> [The problem is that the learners have a problem of applying concepts they learned in arithmetic when now dealing with variables, for example when they add, subtract, multiply and divide algebraic fractions.]
>
> **Mr Letuka:** *E ne nna ha ke tsebe hore na bothata bona ba bana ba hore ba se utlwisise hore ha ba adder, ba subtracta, ba multiplier ba bile ba divider ha ba fihla ho algebra na nka bo hlola ka ho etsa jwang hobane ha ke utlwisise na bo bakwa keng.*
>
> [So, I do not know how can I overcome this problem of learners for not understanding how to carry on addition, subtraction, multiplication and division in algebra, because I do not understand what causes it.]
>
> **Mme Ntlama:** *And this transition gap ke yona e etsang hore le ha ba fumana karabo e le* $2x + 7$ *ebe ba tswela pele hore ke* $9x$ *kapa* $a + b$ *ba re karabo ke* ab*.*
>
> [And this problem I think is the one that causes learners to continue with addition instead of leaving the answer as $2\mathrm{x} + 7$ to say it is $9\mathrm{x}$ or $\mathrm{a} + \mathrm{b}$ to be ab.]

The utterances made by Mme Ntlama, "*… di concepts tse ba ithutileng with arithmetic numbers ba ba le bothata ba ho di sebedisa ha re fihla di variableseng…."* indicates that teachers are aware that learners face a transition gap from arithmetic to algebra. According to Mme Ntlama, learners are unable to solve the expression, which contains the variables (algebraic expressions) and she believes that this transition gap from algebra to arithmetic is the one that causes errors/misconception. Hence the statement, "*And this problem I think is the one that causes learners to continue with addition instead of leaving the answer as* $2x + 7$ *to say it is* $9x$ *or* $a + b$ *to be* ab*".*

Mr Letuka's utterances, "… *na nka bo hlola ka ho etsa jwang hobane ha ke utlwisise na bo bakwa keng"* [So I do not know how can I overcome this problem… because I do not understand what causes it], also confirms that he is aware of a gap between algebra and arithmetic, which is causing learners to commit errors and to develop misconceptions. Although he is aware of the problem, he seemed not to have an idea or plan of how to address it. Not knowing how to address the challenge means that the teacher continued to teach learners without coming up with interventions to incorporate in his teaching to address it.

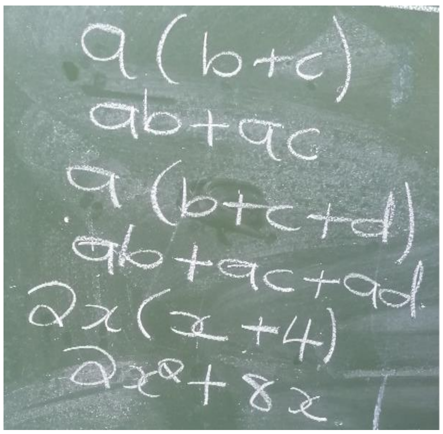
4.2.8 Confusion caused by teacher's inconsistent demonstrations

According to Kosko (2016), teachers' selection of questions and tasks during mathematical discussion and how the examples are solved through guided practice, strongly influence learners' involvement in and understanding of the concept under discussion. This means the teaching of mathematical examples should be carried out carefully, since learners follow the procedural steps as shown by the teacher. The extract below shows how Mme Ntlama presented the concept of algebraic expression for multiplication of monomials and binomials, multiplication of two binomials with exclusion of product of binomial and trinomial.

Algebraic expression lesson presented by Mme Ntlama

After explaining to learners what an algebraic expression is, Mme Ntlama made examples for the learners, which included multiplication of a monomial and a binomial, as well as binomial and a binomial: the following pictures show what she did,

Monomial and binomial



Product of two binomials

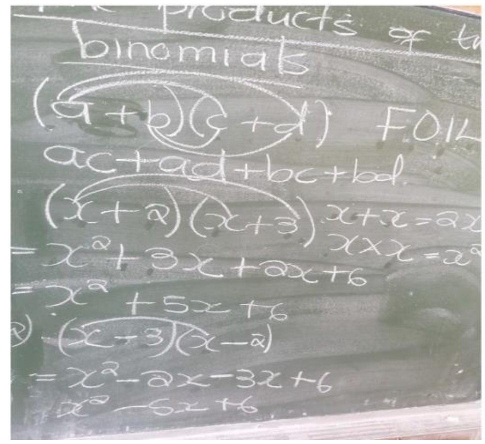


Figure 4.5: Mme Ntlama's illustrations of a monomial multiplied by binomial and binomial multiplied by binomial respectively

In the instance where Mme Ntlama was explaining the multiplication of a monomial with a binomial, the teacher did not use the arrows as she did in the example of multiplying two binomials. This inconsistency caused problems, especially for some "weak learners" who then forgot that both terms in the brackets were supposed to be multiplied by the number/term/variable outside the brackets. For instance, in the case of $2x\,(x + 4)$ learners wrote an answer as $2x^2 + 4$ instead of $2x^2 + 8x$ (see figure 4.6 below).

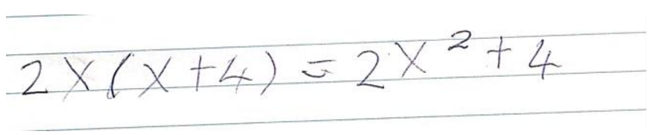


Figure 4.6: Learner answer

The researcher also noted that when multiplying a binomial and a binomial, Mme Ntlama used the curved lines as indicators for how the process of multiplication was to be carried out and told the learners that the method that she was using is called FOIL (First, outer, inner and last). However, when learners were supposed to multiply a binomial by a trinomial, some of the learners could not solve the problem, because it looked different from the two problems, which the teacher dealt with (see figure 4.6), (i.e. the product of a monomial and binomial and a product of binomial and binomial). This implies that the teacher did not apply the variation theory, because the examples she used did not align with the activities she tasked the learners with (see section 2.4.3). In her examples the teacher did not include the example of finding a product of a binomial and a trinomial, hence other learners encountered problems in this regard. The fact that the teacher did not use an example of a binomial multiplied by a trinomial, made it difficult for the learners to solve it. Some learners were therefore stuck, and others got the answer wrong, because they did not understand what they should do, since they were not able to identify first, outer, inner and last in this case. For instance, see Figure 4.7:

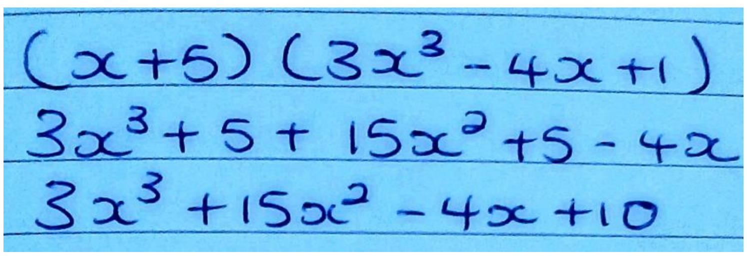


Figure 4.7: Incorrect learner answer

The following extract is a conversation by the researcher and one of the learners in a reflective one-on-one interview session that took place after the classroom observation.

Researcher: *Lerato, tell me, how did you work out this problem?*

Lerato: *I wanted to follow the example that we were given in class for using FOIL. I multiplied first, outer, inner and last values in that order.*

Researcher: *So, that is why you did not multiply the -4x by anything?*

Lerato: *Yes Ma'am, I just wrote it as it is, since I did not know what I should do with it.*

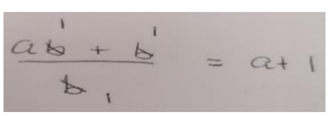
Researcher: *Ok, I see.*

To a certain extent, this shows that teachers do not pay attention to the way they select the examples. This also indicates that teachers lack knowledge to select appropriate activities that will enable learners to solve the related problems regardless of the different structures that each problem comes with. The fact that the teacher used curved lines when multiplying the two binomials and did not use it when multiplying a binomial with a monomial, caused learners not to vividly see the principle that should be followed when performing multiplication. According to Koskow (2016), the structure of concepts in conveying subject matter is important.

4.2.9 Teachers' inability to communicate mathematically correct in the classroom

The teacher must be able to communicate well when teaching in the classroom (Zayyadi et al., 2020). Teacher communication is an important component in achieving learning objectives, especially in improving pupils' learning outcomes (Sahabuddin, 2019). If the teacher's communication skills are not good and the teacher also cannot communicate mathematically correct, this can lead to learners' poor performance in the topic/subject. It is therefore important that teachers should refine their communication as this will assist learners in achieving better results in the subject. The extract below shows learners' challenges in simplifying algebraic fractions, which may have been caused by the teacher's communication during presentation.

> **Mme Ntlama:** *"We end up communicating wrong in the class because of trying to make learners perform well, for example, during factorization by grouping of* $x(a-b)+y(b-a)$*. I normally ask my students to switch and swap the signs for the second bracket as follows* $x(a-b)-y(a-b)$ *and then take out the common bracket to get* $(a-b)(x-y)$ *Then, you find that when they are supposed to work out* $x(a+b)-y(b+a)$ *they still want to apply switch and swap, then they get* $x(a+b)+y(a+b)$ *which is now incorrect."*

> **Mr Letuka:** *"Let me give you another example… when we have an expression like this*
> 
> *other teachers say 'you can see b on top and b below therefore all you have to do is to cancel the b's and get the answer as* $a+1$*.' Therefore, when learners come across a situation like* $\frac{a+b}{b}$ *they give an answer as* a *because they have cancelled the* b*'s which is now incorrect."*

Based on the statement made by Mme Ntlama, teachers resort to methods, which make simplification of algebraic expressions easier to get correct answers, but which does not assist learners with conceptual understanding, which is required for them to factorise the common factor. Instead of teaching the learners to factorise a -1 in this situation, the teacher instructs learners to change the operational sign in front of the bracket to the opposite operational sign and exchange the position of variables inside the brackets $x(a-b)+y(b-a)=x(a-b)-y(a-b)$. This is not wrong, but it does not provide conceptual understanding, but rather provides procedural knowledge. Therefore, this makes it difficult to solve the problem.

In the same way the method displayed by Mr Letuka seemed to work for the first fraction $\frac{ab+b}{b}$*,* but it could not work for the second fraction $\frac{a+b}{b}$*,* because there was no conceptual understanding of how to simplify these algebraic fractions and as a result this impedes learners' understanding*.* The utterance*, "you can see b on top and b below therefore all you have to do is to cancel the b's and get the answer as* $a+1$*"* again does not provide learners with a conceptual understanding of factorizing common factors before they can cancel out. The word *'cancel'* is also misleading, because it does not provide conceptual understanding and therefore gives learners the impression that when they see same letters in a numerator and denominator, they should cancel. That is why for the fraction $\frac{a+b}{b}$ they still found the solution as $a+1$*,* which is incorrect.

The extracts above indicate that the source of this challenge could be the way teachers talk during classroom instruction. This also indicates that offering simplified and conceptually poor versions of mathematical language can disadvantage and hamper learners' progress in gaining knowledge (Planas, 2021). And this requires teachers to be very careful of how they talk in the classroom and to what extend should they use the words, which are meant to aid learners' capability of solving problems, since problems can arise when learners attempt to apply misunderstood shortcuts (Saleh, Prahmana, Isa, & Murni, 2018). However, Planas (2021) refers to this as practices of lexical elaboration; meaning that explanations that give clues to the meanings of vital concepts or simple strategies of working out a problem. Moreover, she perceives them to be very fruitful and significant, especially to the learners who are in the process of learning the language of teaching and learning; therefore they should be used carefully in the classrooms.

However, learners tend to misuse these practices as shown in Figure 4.8 below:

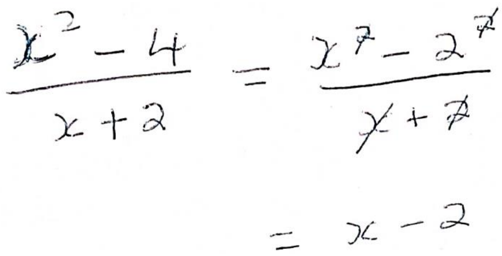


Figure 4.8: Cancellation error due to non-recognition of common factors

Studying the figure above, even though the final answer is correct, the learner just cancelled without really checking the common factor. In this case, a learner seemed to have a problem with factorisation of quadratic expression.

During a one-on-one interview session, the following extracts were recorded.

Researcher: *You got the answer correct, but I cannot follow your method, would you please take me through your method?*

Thabo: *I first wrote 4 in exponential form. Then I cancelled the common factors. That is, I cancelled 2 from x^2because x goes x times in x^2, and I also cancelled 2, because $4\ divide\ by\ 2\ = 2$. So after that I remained with the answer $x - 2$.*

Researcher: *I see, but when you look at the numerator, what type of quadratic expression is that?*

Thabo: *It is a difference of two squares.*

Researcher: *Can you factorise it first?*

Thabo: *Yes, we can, the factors are $(x - 2)(x + 2)$. Now I see that if I divide this numerator by the denominator, I will get $x - 2$ as an answer.*

The conversation above shows that the learner was not able to identify common factors for the given task. Therefore, s/he resorted to the quick method of cancelling without considering common factors, which in this case produced a correct answer even though the steps taken to obtain the answer were wrong.

The figure below shows cancellation error due to carelessness:

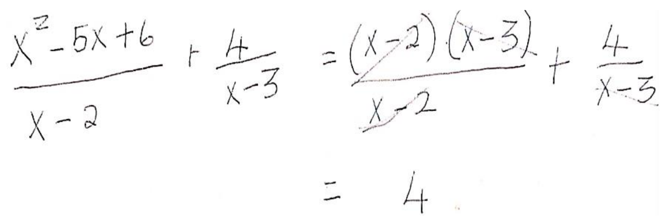


Figure 4.9: Cancellation error due to carelessness

The extract above indicates that in the first step the learner was correct to write factors of the numerator. Even though, the division by $x-2$ was also correct, division by $x-3$ was not correct. That is, the cancellation of $x-3$ was incorrect. The extract below is the conversation between the researcher and one of the learners, who committed this error:

Researcher: *can you explain to me how you cancelled in this addition of algebraic fractions.*
Thomas: *the common factor in the first fraction is* $x-2$ *and is cancelled.* $x-3$ *is a common factor between the first and second fraction, so I also cancelled it and obtained 4 as an answer.*
Researcher: *do you consider common factors between first and second fraction during addition or during multiplication?*
Thomas: *We consider them when we multiply.*
Researcher: *But here we are adding.*
Thomas: *Eh…no? My mistake…*

The reflective conversation above shows that sometimes learners make mistakes due to carelessness or lack of concentration on the task they are working on.

4.2.10 Inability to represent word expression in algebraic format

RME and UDL advocates for the use of real-life story problems (see sections 2.3.2.2 and 2.4.2). However, learners have a challenge of representing word expressions into algebraic expressions. The following extracts from the focus group meeting highlight some of the challenges identified by the teachers when dealing with this aspect.

> **Mme Ntlama:** *"Sometimes learners do not understand what they asked to do when a problem is word problem, example "write an algebraic expression showing a product of two consecutive numbers."*

Mr Letuka: *"Others have a problem of writing word problems into algebraic expressions, for example mathematical expressions like those indicated in the table below."*

Table 4.1: Some commonly used algebraic word expressions

	Word format	*Algebraic format*	*Learners' understanding and representation*	
a)	*A number 4 less than* x	$(x-4)$	*4 is less than x*	$4 < x$
b)	*A number 3 more than* x	$(x+3)$	*3 is more than x*	$3 > x$

Mme Ntlama: *"Nna ke dumela hore hona ho etswa ke hore le rona matichere hare ba ruti tsona hantle."*

[I believe that this is caused by us as teachers, because we do not teach them thoroughly.]

Mr Letuka: *"Fluence in mathematics language is contributing so much to this regard, because you find that a learner does not know the meaning of some of the words in the problem.*

According to the statement made by Mme Ntlama, learners are challenged when they are supposed to write algebraic expression from the word expressions. This could be because learners do not know or understand the mathematical language that is used. For example, learners may have a problem with understanding the meaning of "*consecutive numbers*" or "*a product of*". Mme Ntlama further mentions that this problem may be caused by teachers, as they do not do it thoroughly. This further implies that for the fact that teachers rush over this aspect, might be because they themselves are challenged with word problems. The comments made by Mr Letuka shows that learners are somehow challenged by mathematical word problems as indicated in the table 4.1 above. However, learners must be able to solve word problems, since is it a crucial learning outcome, which is regarded as an important skill in mathematics (Wardhani & Argaswari, 2022). This challenge impedes learners' performance regarding word problems. Hence, reports from end of year examination in Grade 12 revealed that learners' inability to write worded problems into algebraic expressions restrained them from providing accurate solutions. Mr Letuka further mentions that learners' problem in this regard is because of mathematics language and this indicates that teachers do not make an effort to equip themselves with enough strategies to assist learners to have enough knowledge for learning how to write word expressions into algebraic expressions. According to Moleko (2018), word problems are not only difficult for learners, but also for teachers and therefore it becomes difficult for these teachers to teach learners in a manner which will improve their understanding. As a result, the teacher becomes challenged to scaffold learners' problem-solving skills (see section 2.4.5).

4.2.11 Summary

The common algebraic expressions challenges and issues related to the teaching and learning of algebraic expressions were highlighted in the sections above, therefore, based on the challenges and issues identified there is a need to devise a strategy that will assist to address these challenges in order to enhance the teaching of algebraic expressions in Grade 10. The subsequent sections outline the components of the solutions devised to address the challenges identified by the participants in this research study.

4.3 THE SOLUTIONS IMPLEMENTED AND SUGGESTED TO ADDRESS THE IDENTIFIED CHALLENGES

The previous sections highlighted the challenges identified by the participants during the focus group discussion and during the class observations. The subsequent sections therefore, discuss the solutions to the identified challenges as implemented and suggested by the participants.

4.3.1 Enhancing teachers' pedagogical content knowledge

Pedagogical Content Knowledge (PCK) is important as it enables the teachers to understand how learners learn and to help these learners develop the ability to solve the mathematical problems (Ziyyadi et al., 2020). It is therefore vital that teachers develop the content and pedagogical knowledge in order for them to effectively teach the mathematical concepts. Shulman (1986) puts more emphasis that content knowledge on its own is not enough to successfully deliver knowledge, but also needs to be supplemented by pedagogical knowledge. This calls for teachers' PCK to be advanced so that they could learn more about the teaching strategies that they can use to effectively teach algebraic expressions.

In line with the above, in a meeting when the solutions to the challenges were discussed, the participants highlighted the following:

> **Mr Ntlama:** *"Maybe if the subject advisors can organise DH training workshops where they would be trained with teaching strategies, that will make it easier for them to deliver subject matter. And these DH then disseminate this information to the teachers they supervise, because they interact with them on daily basis."*
>
> **Mr Letuka:** *"Kapa bahle ba bitse matitjhere a Grade 10 ho ya ba hlahlella ka mekgwa e ka sebetsang ha bobebe."*
>
> [Or they can invite all Grade 10 teachers to the workshops to train them on strategies that they can use.]
>
> **Mme Ntlama:** *"Le ho attenda structures like PLC where teachers come together to discuss ideas and different approaches to deliver subject matter ho ka thusa."*
>
> [And to attend…can help.]

Mme Ntlama's suggestion that the training workshops be organised for teachers to be empowered with knowledge of the teaching strategies to be used, indicates that teachers are aware that pedagogical content knowledge can be improved if both teachers experienced, or novice teachers can be trained. Her statement also seems to encourage cooperation between Departmental Heads and the teachers they supervise. Mr Letuka also emphasised the fact that teachers should be trained in his statement when he said "*Kapa bahle ba bitse matitjhere a Grade 10 ho ya ba hlahlella ka mekgwa e ka sebetsang ha bobebe"* [Or they can invite all Grade 10 teachers to the workshops to train them with strategies that can be used]*.* Mme Ntlama's comment. *"Le ho attenda structures like PLC where teachers come together to discuss ideas and different approaches to deliver subject matter ho ka thusa",* indicates that teachers' attendance of the PLC meetings can be helpful, because that is where teachers come together to share ideas and teaching strategies. It is in these meetings where teachers, who are more experienced get to share their expertise. This is in line with Vygotsky's Zone of Proximal Development (ZPD), which advocates mediated learning through guidance of a more knowledgeable other. Mme Ntlama seemed to believe in mutual collaboration where knowledge could be shared amongst the peers to empower each other. Platforms, such as PLCs, are therefore good spaces for empowerment, since the platform provides teachers with opportunities to engage in meaningful discussions and sharing of the best practices. This is in line with CER, which advocates for a joint change in terms of activities that must be planned in collaboration by the teachers, as well as giving learners increased opportunity to take part in the learning activities (see section 2.2.2)

4.3.2 Strategies to improve explanation of the algebraic expressions

The perception of Guler and Celik (2021) is that for teachers to be able to teach algebra productively they must possess Knowledge of Algebra for Teaching (KAT). This type of knowledge assists the teacher to plan, present and explain the content of algebra in a meaningful manner. The need to improve explanation of the algebraic expressions was emphasised during the focus group discussion. The teachers thus highlighted how explanations could be improved as follows:

Mme Ntlama: *"To explain better, sometimes it is best to remind learners the concepts of addition and subtraction of fractions using numerical fractions before one can teach addition and subtraction of algebraic fractions."*

Mr Letuka: *"I also think teachers need to be trained on how to explain and teach manipulation of algebraic expressions, because even if a teacher knows the content of algebra, sometimes it is not easy to teach it or make the learners gain conceptual understanding."*

Mme Ntlama:" *Putting learners in groups so that they can share ideas with one another may also work."*

Mme Ntlama's utterances, *"To explain better, sometimes it is best to remind learners the concepts of addition and subtraction of fractions using numerical fractions before one can teach addition and subtraction of algebraic fractions",* confirms the perception of Muchoko et al. (2019) when they mentioned that knowledge of addition and subtraction of fractions is an essential prior knowledge for overcoming challenges in algebraic operations. Therefore, it is vital for a teacher to regard prior knowledge (in the process of explaining concepts to learners) as a basis of his or her instruction so that learners may make meaningful connections to the new concept. As indicated in section 2.4.4, the variation theory supports teachers' ability to draw out and apply learners' existing knowledge as they plan classroom activities. Muchoko et al. (2019) further state that it is crucial that learners have deep understanding of manipulation of concepts like fractions when learners are still in the lower grades. Mme Ntlama also has a view that group work will be beneficial, because it provides learners an opportunity to discuss with their peers their thoughts and ideas. It is during this group work that those explanations of the algebraic concepts will be provided from diverse perspectives, thus making learning meaningful.

The comment made by Mr Letuka that there is a need for teachers to be trained in topic specific knowledge of how to teach algebra, is in line with Yildiz and Akyus's (2019) utterances when they emphasize that it is important that teachers need to have conceptual knowledge for teaching algebraic concepts. This is displayed by the statement, "*I also think teachers need to be trained on how to teach manipulation of algebraic expressions…"* means that focus needs to be placed how to explain mathematical concepts including algebraic expression while teachers are still at tertiary level and are still being trained to become teachers. The training courses may therefore include the use of teaching strategies, specific representations, mathematical language, models, and instructional methods as forms of teaching that may be used as supplementary to improve explanation of the algebraic expressions (Yildiz & Akyus, 2019). Furthermore, this can be done through developmental workshops provided by the Department of Education (DoE) wherein some light would be shed to equip teachers with skills and strategies on how to explain the algebraic expressions well.

4.3.3 Addressing learners' lack of understanding the main algebraic concepts

Ferretti (2020) notes that in several cases a deep understanding of the meaning of algebraic terms could allow learners to resolve algebraic expressions without carrying out algorithmic procedures, which in most cases lead to errors. According to Osei (2020), learners must have a deep understanding of the basic concepts of algebra and of how they may apply these concepts as the application of these concepts is essential in most technical careers, which learners may want to follow after their high school learning. Regarding strategies to use in terms of developing understanding of main algebraic concepts, teachers' comments were as follows:

> **Mr Letuka:** *"Wa tseba ha e ne kare ha motho a ruta mohlomong a concept like difference of two squares a ba sa ntsa a bontsha bana hore even in numerical form it can be applied or any other form then it would be better."*
>
> [You know, when teaching a concept such as difference of two squares, one should show learners that even in numerical form it can be applied or any other form then it would be better.]
>
> **Mr Letuka:** ***"****Mohlala* [for example] *Factorise* $4a^2 - 36$, $2^{2x} - 1$, $225 - 144$"
>
> **Mme Ntlama:** *"That is true, even in simplifying exponents, it would help to expose learners to application of difference of two squares."*

The above extracts where "difference between two squares" was cited as an example, indicate the significance of using the different representations to reinforce understanding of the algebraic expressions to address the learners' lack of understanding the main algebraic concepts. In the above extracts, Mr Letuka's suggestion is that the concept of difference between two squares should be taught in such a manner that both the numbers and the variables are used. This means that the concept must be taught in such a manner that it is represented in different forms. According to Mr Letuka, the different forms of representations will aid learners' comprehension of the concept and this is supported by the UDL principle, MMR (see section 2.4.2). In line with UDL, the multiple representation principle does not only aid comprehension, but it also makes it easy for learners to recognize the information and analyse it. In line with the variation theory model, the use of different representation is crucial in terms of addressing the structure. Thus, its application in this instance, will make it easy for learners to recognize the structure across the given representations. Mme Ntlama also suggested that this concept of difference between two squares should be addressed in other mathematics topics as well. In this case she was referring to exponents. It was important for her to raise it, because the concept of difference between two squares is also applicable when dealing with exponents wherein its representation is also different e.g.:

$$25^x - 9 = 5^{2x} - 9$$
$$= (5^x - 3)(5^x + 3).$$

This therefore calls for teachers to apply the variation theory, which enables teachers to select appropriate examples and tasks depending on the object of learning (concept) (Ekawati & Lin, 2014), see section 2.4.4. For example, the variation theory requires teachers to give learners a variety of examples and problems involving the difference of two squares, such as $x^2 - y^2$, $x^2 - 16$, $125^3 - 124^2$, $36^x - 16$.

4.3.4 Reinforcing the understanding of variables and parameters

The mathematical description of a variable in the online dictionary by Merriam-Wester.com (2022) refers to "a quantity which during a calculation is assumed to vary or be capable of varying in value." While the parameter is mathematically defined by Merriam-Wester.com (2022) as a "quantity whose value is selected for the particular circumstances and in relation to which other variable quantities may be expressed." For example, in the following example $y = ax + b, x$ is the variable while a and b are parameters. In relation to developing the understanding of variables and parameters, teachers need to employ various strategies and teachers mentioned those strategies in the focus group discussion we had. The following are their comments during the discussion:

> **Mr Letuka:** *"For learners to understand the notion of variables and parameters it will be best if teachers may represent the information in pictorial form."*

> **Mme Ntlama:** ***"****Mohlala* [For example] *the teacher can come to class already having drawn diagrams showing the effect in the change of values of* a *and* b *for* $y = ax + b$ *or include the use of technology, such as GeoGebra to display the effects of the parameters."*

> **Mr Letuka:** *"I also think when dealing with number patterns we can use them to reinforce the concepts of variable and parameter, because for the linear pattern* $T_n = bn + c$*,* b *and* c *are parameters, so changing their values produces different number patterns where* n *is a variable, so learners may be asked to investigate different number patterns."*

The comments above encourage teachers to use different representation to reinforce understanding of the concepts of variable and parameters. Mr Letuka in the first instance, suggests the use of pictures to assist learners to understand the variable and parameter concepts. The use of pictorials is supported by the Lesh's translational model, which required the content to be represented in varied formats to facilitate understanding. Lesh's modified representation model encourages teachers to represent symbolic information in pictorial form, as well as the use of moving pictures (animation) where necessary to enforce conceptual understanding (see section 2.4.3). Mme Ntlama also recommended the use of technology, such as GeoGebra, which enables teachers to engage learners in meaningful learning (Mosia, 2016). Bouck et al. (2013) also support the use of technology, since it affords learners' access and enhances their understanding of algebraic expressions.

In line with a UDL model, learners must be given opportunity to access information in a different format and be engaged in a different way from a chalk-and-talk approach. Different representations will make concepts perceptible, simple and intuitive (Moleko, 2022). This way the learners will develop an understanding of variables and parameters. In the second instance, Mr Letuka's comments was about engaging learners in an investigation whereby they will investigate the effect of parameters. In line with the RME model, practice gives learners an opportunity to get a deeper understanding of these concepts, because according to Saleh et al. (2018), what they attain from the learning process, based on their own thinking pathways, will be logically accepted and produce a more meaningful learning.

4.3.5 Enhancing the learners' understanding of simplifying algebraic expressions

In this section we are going to analyse some of the strategies teachers suggested to address the issue of conjoining error, which results due to learners' inability to handle like and unlike terms, learners' discomfort to leave an answer as an expression (lack of closure), as well as parsing obstacle and expected answer obstacle. During classroom observation, the following was observed.

Lesson presentation by Mme Ntlama (addressing the issue of parsing obstacle)
Please do the following activities…
2. Simplify the following:
a) $2x-(5x-17)$
I notice that for question 2a) most of you left the answer as $14x$ *instead of* $17-3x$*. you must remember that 17 and* $3x$ *are not like terms. For example, let us say 17 represent the total number of cupcakes in a bag (showing learners a plastic bag containing cupcakes) and 3 friends each gets an unknown (x) number of cupcakes from the bag, then how many cupcakes remain in the bag?*

Figure 4.10: A bag of cupcakes

Tebello: *Ma'am I think the remaining cupcakes are* $17 - 3x$*, because we do not know how many each of the three friends got.*
Mme Ntlama: *Good, you are right, the remaining cupcakes are* $17 - 3x$*.*

Based on the observation from the lesson extract above, the teacher has emphasized that the two terms are not like terms. She even explained more by providing a real-life scenario of cupcakes, as in the figure 4.11 where she asked them to find the remaining cupcakes after three friends were given x cupcakes each. This is in line with Norman's (2018) perspective that, to enhance learners' understanding of manipulation of algebraic expressions, teachers need to first use concrete objects before manipulating these expressions symbolically. This will assist learners to conceptualize the concepts of "like and unlike" terms and consequently address the "*Lack of Closure* and *Name Process Dilemma"* difficulties. This also concurs with realistic mathematical modeling, which encourage the use of a real-life context to deepen learners understanding.

Lesson presented by Mr Letuka (Addressing the issue of expected answer obstacle)
Mr Letuka: Good people you can't say $2x + 7 = 9x \text{ or } 9$*. Remember that these two terms are said to be unlike terms, because 2x is a term made up of a constant 2 and a variable x, while 7 is a term made up of a constant number 7 only. Look at the following diagram and tell me your answer.*

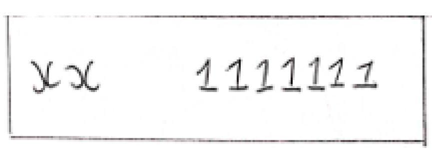


Figure 4.11: Practical exercise used by Mr Letuka to address the concept of like and unlike terms

Thato: It is $2x$ *and 7 ones*
Mr Letuka: So what is 7 ones?
Thato: It is 7. Ok …now I see, sir the answer is $2x + 7$

In the extract above Mr Letuka, explained to learners how the two terms differ and used a pictorial representation of the expression so that learners understand that they cannot give the answer as $9x$ or 9. Both UDL and Lesh's translation model support what Mr Letuka has done, because they both encourage the teacher to use pictures to enhance conceptual understanding of learners (see section 2.4.2.1 and 2.4.3 respectively)

Lesson presented by Mme Ntlama (Like and Unlike terms)

Mme Ntlama*: Let's say there are 2 bananas that can be represented by* $2x$ *and 7 apples by* $2y$*, now write down the sum of these fruits.*

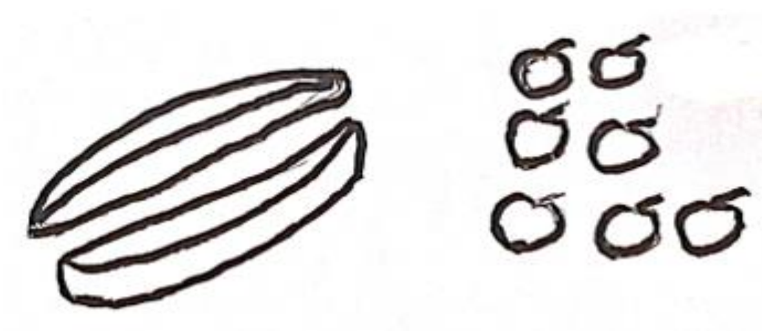


Figure 4.12: A diagram of fruit salad method

Halieo: *My answer is* $2x\ +7y$

Mme Ntlama: *Good Halieo, so why are you not saying it is* $9xy$*? I can see that Thomas has* $9xy$*.*

Halieo: *It is because we have 2 bananas and 7apples not 9 banana apples so we cannot write an answer as* $9xy$*. Therefore, the answer is* $2x + 7y$*.*

Mme Ntlama: *Now what about* $2x + 7$*? Can we simplify it any further?*

Thomas: *No Ma'am, we can't simplify it any further. The answer is* $2x + 7$*, because* $2x$ *and 7 are unlike terms.*

In the above lesson presentation, the teacher has used a pictorial form in order to emphasize the concept of like and unlike terms and as a result addressing conjoining error and/or parsing obstacle and expected an answer obstacle. The use of the fruit salad method assisted the teacher to make learners understand that $2x + 7$ cannot be simplified any further, because the terms in it are unlike terms and hence learners were put in the position to accept $2x\ + 7$ as an answer. The lesson observation for these teachers shows that the teachers believe that the issue of conjoining error can be addressed by learners' understanding of the concept of like and unlike terms in algebraic expressions.

Both teachers made use of a physical representation to deal with these challenges. Since conceptual understanding is critical to mathematical competence, hands-on manipulatives and graphic pictorial representations of mathematical thoughts are helpful in this regard (Johnson, 2018). This concurs with Lesh's representation model, which supports that teachers use various representations of information (see section 2.4.3). Furthermore, Gningue (2000) affirms that in teaching the concept of simplifying algebraic expressions, the use of manipulatives, such as algebraic tiles, base ten blocks, clips, counters and others, allows the concept of like terms and unlike terms to draw from various perspectives. This is because even though the materials are diverse, the same concepts are inherent to all of them.

4.3.6 The use of representations and real-life contexts to address algebraic expression vocabulary and reinforcing understanding

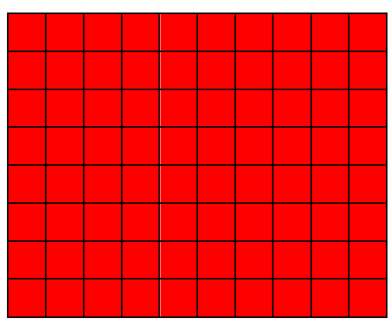
Language is critical for improving opportunities for learners to learn mathematics (Planas et al., 2018) Therefore, during a lesson it is important that teachers use mathematical language to assist learners to analyse and voice out the specific features that build up the structure of algebraic representation (Star et al., 2015) (see section 2.5.2.4). The lesson below captures this. This lesson shows how the algebraic concept could be dealt with using another topic, which subsequently helped the teacher address language issues and also address certain features of the algebraic expressions. The teacher used the concept of area to teach in a manner that assisted learners to see the specific features that build up the structure of algebraic representation and learned the mathematical language.

Algebraic expression lesson presentation by Mme Ntlama

Mme Ntlama provided learners with building blocks for which learners were asked to calculate the area. After reminding learners about the definition of area, Mme Ntlama asked the learners to calculate the area.

Mme Ntlama*: Take the red building block (rectangle shape) and calculate its area.*

8cm



10cm

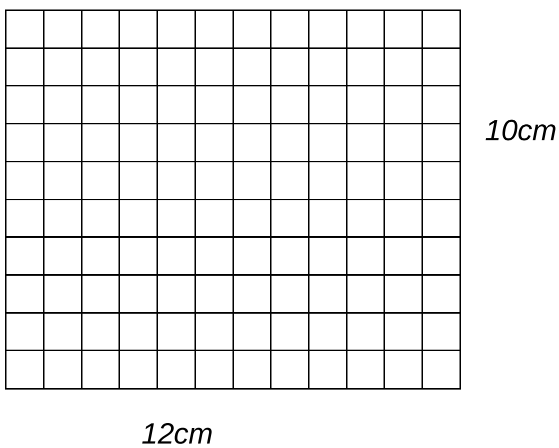
Keketso: *When I count the number of squares, I get 80 squares, therefore the area is 80 square units.*

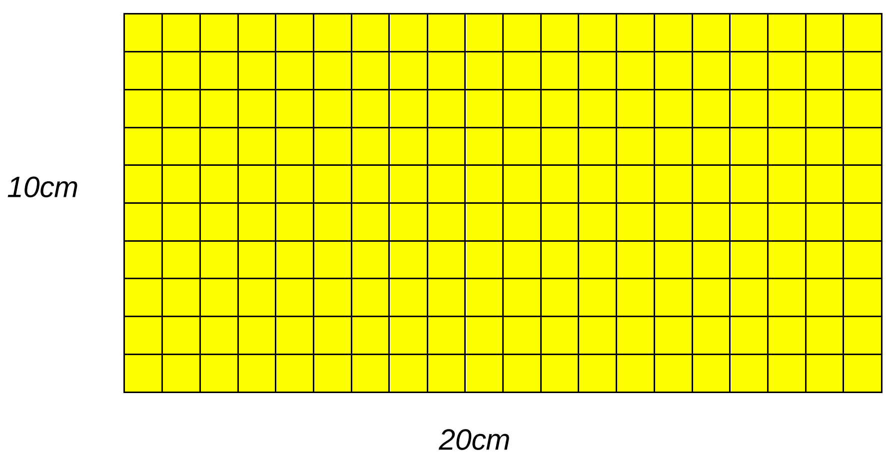
Mme Ntlama: *You are correct, now turn the building block over to the other side, you will find the dimensions there. Then use them and the formula for calculating the area of the rectangle to determine the area of that orange building block.*

Vuyani: *Ma'am, we get the answer by saying area of a rectangle equals length times breadth, which is 8 times 10 and we get 80 units square as an answer.*($A = lb = 8 \times 10 = 80$*units square)*

Tankiso: *So Ma'am it is the same as the answer we got for the other side.*

Mme Ntlama: *Now let us find the area of the white and the yellow building blocks*



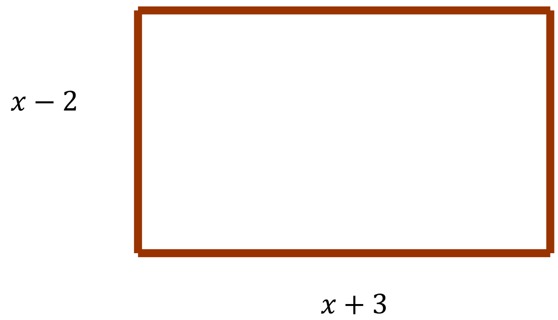


Mme Ntlama: *…..*

Now put the blocks together horizontally and find the area….

After discussing many ways that learners came up with in their arrangement of the building blocks and reaching the conclusion for the answer ….

Mme Ntlama: *Now let us find the expression for the area of the rectangle given that the* $width\ (w) = x - 2\ and\ length(l) = x + 3$



Mme Ntlama: *What type of an expression is that? Monomial, binomial, or trinomial? And why?*
Mme Ntlama: *What is the difference between your answers for the first two rectangles and the last one?*
Mme Ntlama: *Classify the dimensions of the given rectangle into monomial, binomial and trinomial.*
Mme Ntlama: *Write down the factors of the expression* $x^2 + x - 6$.
Mme Ntlama: *Classify the following expressions into monomial, binomial and trinomial* ($20a \div c$, $6y^3 - 4y \div 7$, $10p^2 + 5\frac{p}{2}$, $3x(x - 5) + 7$, $mn - 2n + 5m^2$)

In this lesson, the teacher reminded the learners how to find the area of a rectangle when given both dimensions in numerical form. She then moved to expressing the area of a rectangle, given both dimensions in algebraic form. Then, learners were able to multiply the dimensions and get an algebraic expression in the form of a trinomial expression for example, $(x - 2)\ \ (x + 3) = x^2 + x - 6$. In response to tell whether the resulting expression is a monomial, binomial or trinomial, some learners were able to get it right even though some did not. The teacher had to discuss with learners how to identify them. The learners were also asked to classify the two dimensions into monomial, binomial and trinomial. In trying to establish the correct description of these types of expression, the teacher gave learners other examples and asked them to classify them. E.g. $6y^3 - 4y \div 7$ and $10p^2 + 5\frac{p}{2}$. The use of the visuals in the lesson above shows that the algebraic expressions can be applied when calculating the area in a real-life context and this is the practice that epitomises the RME theory of teaching mathematics (see section 2.2). In addition, the teacher's use of visuals as another form of representation in the lesson, showed a different approach to teaching algebraic expressions, which is encouraged by the modified Lesh's model (see section 2.4.3) The activity tasked to learners to classify the expressions to some extent assisted in reinforcing their understanding of the terminology used in algebraic expressions.

The use of a real-life context in the teaching of algebraic expressions is encouraged by the RME theory. According to Adendorff (2019), RME encourages the teaching of mathematics to be from a rich real-life context, because it offers great opportunity to foster learner-centeredness and non-authoritarian methods of classroom instruction, since when learners consciously engaged in the process of finding mathematical concepts, using a concrete object, it then will give them a strong trace in recalling the finding (Saleh et al., 2018). This concurs with the study conducted by Johnson (2016), which revealed that the achievement and the improvement of learners' mathematical reasoning capability, taught by using methods influenced by RME, are better than those that are taught by using conventional learning. Moleko (2021) advocates that teaching mathematics using real-life contexts increases engagement, because learners are given a platform to apply the concepts they learned in their real-life experiences and hence become better persons in life.

Teachers may therefore use visuals, modeling, and other techniques to assist them when presenting key information (Short, 2017). Moreover, in the view of Johnson (2016), physical representations may be used, because they serve as tools to mathematical thought and communication. In line with MMR, some learners simply comprehend information faster or more effectively through visual or auditory means rather than printed text (Moleko, 2018). The use of representations did not only assist in addressing the mathematical language, but also enabled learners to see the specific features that build up the structure of algebraic expressions.

4.3.7 Bridging the gap between arithmetic and algebra

Teachers must formulate teaching strategies that are informed by learners' content gaps and they should also find remedies for the learners' misconceptions (Mosia, 2016). In line with this, therefore some of the suggestions were made to address the gap between arithmetic and algebra during the focus group discussion.

> **Mme Ntlama:** *"I think the use of real-life scenario would assist to close this arithmetic-algebra gap."*
>
> **Mr Letuka: "***Le ho sebedisa di-manipulatives objects kapa tsona ditshwantsho can be very helpful."*
>
> [And the use of manipulatives, as well as pictures…]
>
> **Mr Letuka:** *"I normally start by reminding learners about the fractions, how does it happen that we have a fraction, how do we add, subtract, multiply and divide numerical fractions before I can work with algebraic fractions with learners. For example, the fraction $\frac{2}{3}$ may represent that out of three oranges, Tebello has eaten two oranges and the fraction $\frac{5}{7}$ may represent that out of seven oranges, Michael has eaten five oranges. Therefore, to find the fraction representing the total oranges eaten by both of them, we need to add the two fractions.*
>
> *Then after this, I show them that when adding algebraic fractions, we will still follow the same method as we did with numerical fractions.*
>
> *E.g. ask learners to work out $\frac{2}{3}+\frac{5}{7}$ and guide them to work out $\frac{2}{x}+\frac{5}{y}$"*

The utterances from Mme Ntlama, "*I think the use of real-life scenario would assist to close this arithmetic-algebra gap*", indicates that it is important to teach learners by relating content with the "things" they experience in their daily life, because this will allow them to make deep connections to the concepts they are presented with. This is in line with the view of Kusumaningsih and Herman (2018), when they suggest teaching learners with the use of RME, which encourages teaching of mathematics from the real-life context (see section 2.3.2.2). Mr Letuka commented about the use of manipulatives and pictures to be utilized in order to improve learning. This comment calls for teachers to really consider and plan their teaching, such that it becomes meaningful to learners. Johnson (2016) attests that physical representations serve as tools for mathematical thought and communication, and therefore, they assist to lighten concepts in ways that support reasoning and understanding hence, he modified Lesh's translational model into Johnsons mathematical representation model (see section 2.4.4). The last comment by Mr Letuka suggests that teachers must use learners' prior knowledge to connect learners' arithmetic knowledge to algebraic concepts. In this way, it will be easier for learners to make connections, because they will be taught from what they already know in arithmetic. Tabach and Friedlander (2008), and De Groot and Boyajian, (2015) have the perception that proper application of context has been shown to be a fruitful bridge between arithmetic and algebra.

The example given by Mr Letuka (*ask learners to work out* $\frac{2}{3}+\frac{5}{7}$ *and guide them to work out* $\frac{2}{x}+\frac{5}{y}$*)* is in line with the variation theory model, which encourages teachers to use various structures to represent information so that learners get exposure to different structural formats of the content (see section 2.4.4). This act of Mr Letuka of teaching this concept by first considering learners' prior knowledge regarding fraction, also concurs with Ekawati and Lin (2014), when they attest that the variation theory model requires teachers to draw out and apply learners existing knowledge as they plan classroom activities.

4.3.8 Advancing teachers' ability to select appropriate activities (examples) to use during instruction

According to Canque et al. (2021), tiered activities are one way to address readiness effectively, so all learners study the same concept, but complete activities appropriate to their readiness levels. This may mean that if the teacher teaches class A and B, she or he may use different activities depending on the level of achievement of the learners. It also emphasizes the importance of teachers' knowledge of his or her learners so that he or she can make appropriate decisions on how to plan the lesson. The issue of selecting the appropriate activities to be used during the instruction was stressed during the focus group discussion. The teachers reflected and commented as follows:

Mr Letuka: *“Mohlomong Mme ha more experienced teachers can pair and share their expertise with novice teacher moo ho kgonehang.”*

[Perhaps if more experienced teachers can pair and share their expertise with novice teachers where it is needed.]

Mme Ntlama: *“Mokgwa o mong o ka sebetsang mona e kaba variation theory hobane e ruta matitjhere hore na ba ka kgetha jwang activities/ examples or exercises tseo bana ba ka di etsang.”*

[Another approach can be to use variation theory, because it teaches teachers how to select activities/examples or exercises they can use or assign to learners.]

The comment made by Mr Letuka “…*ha more experienced teachers pair and share their expertise with novice teacher…”* [If more experienced teachers can pair and share their expertise with novice teachers…] indicates that experienced teachers - since they had been in the field for a long time - have more expertise than novice teachers and they are able to guide the novice ones on how to select the appropriate algebraic expression activities. This is in line with Vygotsky’s zone of proximal development, wherein a more knowledgeable teacher supports the other one who is not more knowledgeable (Xi & Lantolf, 2021). Mr Letuka believes that the experience of the other teachers could serve a useful purpose of empowering others with the knowledge on how to select the appropriate activities (examples) to use during instruction.

The statement made by Mme Ntlama, “*mokgwa o mong o ka sebetsang mona e kaba variation theory…”* [Another approach can be to use variation theory…] tells that some teachers are aware of the variation theory and how to use it, because she continues to explain by saying, *“hobane e ruta matitjhere hore na ba ka kgetha jwang activities/ examples or exercises tseo bana ba ka di etsang”* [Because it teaches teachers how to select activities/examples or exercises they can use or assign to learners]. So, if the variation theory enlightens teachers about a selection of activities such as examples, classwork and homework for learners, it means teachers will be in a better position to select meaningful activities for the learners and learners’ knowledge of algebraic expressions may be advanced. Ekawati and Lin (2014) attest that the variation theory model can immensely assist teachers to teach content matter in-depth and provide many examples in which the same concept is used to give a firm foundation of conceptual understanding.

4.3.9 Communicating mathematically correct in the classroom

According to Planas et al. (2018), communication in the classroom has an interactive function, constructing positions for learners and teachers and framing relationships between them and to the mathematics. Therefore, it is vital that teachers communicate well with their learners during instruction. It is also important for them to communicate the content well in order to ensure that learners understand this content. In line with this the teachers commented as follows:

> **Mr Letuka:** *"I think if teachers can be organized some developmental workshops where teachers will be trained how to communicate properly with learners in the classroom, it will help."*
>
> **Mme Ntlama:** *"And the other thing can be the use of the lesson study, whereby teachers come together to plan a lesson, which will be presented by one of them."*
>
> **Mr Letuka:** *"E ka thusa ntho yeo ya lesson study hobane after presentation then they can discuss how the presenter communicated with learners."*
>
> [Lesson study can be helpful because…]
>
> **Mme Ntlama:** *"Le matichere ka bo oona ba tshwanetse ho ipha nako ya ho itukisa hantle le puo ya bona ka classing."*
>
> [And teachers must invest time to plan well their communication in the class.]

The extracts above indicate the significance of communicating well in class. The extracts show that teachers themselves think that developmental workshops are useful in assisting them to improve their communication skills in the mathematics classroom, based on Mr Letuka's statement, "*I think if teachers can be organized some developmental workshops where teachers will be trained on how to communicate properly with learners in the classroom, it will help"* and Mme Ntlama also agrees with him.

The teachers also have a common understanding that the lesson study can be useful in this regard. According to Olson et al. (2011), lesson study is described as a cycle of instructional improvement in which teachers come together to formulate goals for learners' acquisition of knowledge. In this regard, teachers plan the "lesson" together, one presents it and they reflect upon it together (see section 2.5.2.1). This will assist, because during the instruction, the teachers who are serving as observers will be able to identify instances where the algebraic concepts were not well communicated. The lesson study will thus provide the teachers with an opportunity to re-plan a lesson and to identify ways in which to improve it.

Furthermore, Mme Ntlama indicates that teachers should invest time to plan well so that their communication in the classroom should be meaningful. This means that planning of the teaching of algebraic expressions should not only be limited to planning the content only, but the plan should also be on how to communicate mathematically correct. This calls for teachers to plan the use of language in the class by choosing the suitable vocabulary for that particular lesson and even drawing up the mathematical register for that lesson (Moleko, 2021; 2018).

4.3.10 Enhancing learners' ability to express word expressions in algebraic format

In the view of Dewanto et al. (2017) and Wardhani and Argaswari (2022), word problems are mathematics problems that afford learners the platform to support them in relating the relationship between mathematics and real-life experiences. These scholars further stated that word problems can be used to evaluate the level of understanding of learners to the mathematical ideas. Sepeng and Madzorera (2014) avow that the word problems can be difficult for learners to express in simple algebraic format. On the basis of this, Moleko (2018) asserts that there is a need for teachers to pay attention in developing the learners' understanding of converting the word problems into simple algebraic expressions. In the light of the above, during the focus group meeting where the solutions to the challenges were discussed, the participants demonstrated some of the strategies they used in class to teach word problems and to show learners how to express them in algebraic format. The following extract details Mr Letuka's presentation from the focus group reflection session:

In the first case I gave the learners the following statements to represent in algebraic format:

a) A number five less than x

b) A number two more than x

In the second case, I gave the learners the following scenario [pointing at Figure 4.13] and asked them to represent it in an algebraic expression format:

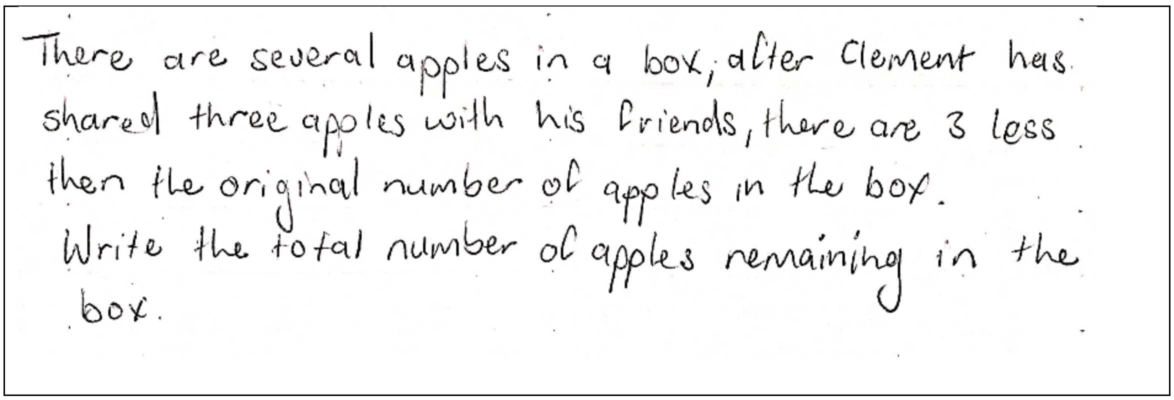

There are several apples in a box, after Clement has shared three apoles with his friends, there are 3 less then the original number of apples in the box.
Write the total number of apples remaining in the box.

Figure 4.13: A word problem

A noticeable number of learners gave answers to the first case as:

a) $5 < x$

b) $2 > x$

In the second case, few of the learners wrote the answer as $3 < x$ *and the majority got the answer correct.*

I had to explain to the learners that for a) the correct answer is $x - 5$*, because this means that* x *is an '**unknown number of objects'** and **5** of them are '**removed**' or this **'unknown number of objects'** is '**reduced by 5'**. I also explained to them that for* $5 < x$ *the meaning here is that the number 5 is less than* x *and the question would have read 5 is less than* x*, so the word '**is**' guides us to compare both numbers and thus write the expression as* $5 < x$*. Then for b) the correct answer is* $x + 2$ *since in this case, the number* x *is '**increased**' by 2. I further asked learners* *who got the second case correct which talks about Clement and his friends while they got the answer wrong in the first case to explain why that happened. Then their answer was that in the second case it was very clear that they were asked to find the remaining apples after Clement and his friends ate 3 of them. So, for them it was easy to find the remaining apples to be $x - 3$. I asked them to identify similar parts for the first case and second case.*

In the extract above, Mr Letuka took us through the approach he used to enable learners to gain an understanding of how and in which situation they should make use of an inequality sign. He explained to learners why their answers in inequality form were incorrect by using the example of a real-life situation. He drew learners' attention to the fact that x represented the unknown number of the object and thus this number is reduced by 5, hence the answer became $x - 5$. He also showed them why he was saying their answer $x < 5$ / $5 < x$ was wrong by highlighting that the statement given did not say 5 is less than x but "…5 less than x". So because between '5' and 'less' in the statement given, "is" is not included, then this does not represent an inequality. However, if the statement read as follows, "5 is less than x" then "$5 < x$" would be correct. Mr Letuka also mentioned that he referred learners to the second case where he had given them a word problem in the context of real-life. He further stated that learners had understood the question and majority of them were able to give the correct response. He said the learners were able to explain that since they were asked to find the remaining apples after three apples were eaten, it made it easier for them to write the expression. He again drew the attention of the learners to relate the question in first case and second case to make them aware of how the questions in first case are applicable in real-life. In this way he used a real-life context, which is what a RME theory advocates for (see section 2.3.2.2). The use of a real-life scenario provided learners with multiple means of representing information, which is in line with an UDL model. This also afforded learners an opportunity to apply Polya's problem-solving strategies (see section 2.4.5). Moleko (2018), Adendorff (2019), Wardhani and Argaswari (2022), all advocate that learners should be taught from a rich real-life context to advance their conceptual understanding. According to Ng (2022), word problem expressions are fundamental, since in the rhetorical stage of algebra, problems are expressed in longhand and no symbols are used. For example, figure 4.13 shows the problem presented in a rhetorical stage of Sfard's theory. Thus, it is crucial that learners are guided well through this stage to the syncopated stage, which now is where the letters are introduced to represent the unknowns (see section 2.3.2.1).

4.4 CONCLUSION

In this chapter, common challenges pertaining to the teaching of algebraic expressions in Grade 10, were discussed. Strategies that were implemented and suggested to address these challenges, were also outlined.

The following chapter focuses on the discussion of the results of the study, as well as recommendations.

CHAPTER 5 :

FINDINGS, CONCLUSIONS, AND RECOMMENDATIONS OF THE STUDY

5.1 INTRODUCTION

The purpose of the study was to explore the common algebraic expression challenges in a Grade 10 mathematics classroom. This chapter presents the findings of the study, based on the common algebraic expression challenges identified. The chapter also presents the findings based on the identified solutions. The chapter begins by highlighting the primary question and the subsidiary questions, as well as the objectives of the study. Thereafter, the chapter presents the summary of the study, findings based on challenges identified and furthermore reports on the components of the solutions to the identified challenges. The chapter provides recommendations, limitations and implications of the study. Lastly, the chapter provides a conclusion.

5.2 RESEARCH QUESTIONS OF THE STUDY

The study sought to respond to the following main research question:

How can the common algebraic expression challenges in Grade 10 be addressed?

The subsidiary questions were:

- What are common algebraic expressions challenges in a Grade 10 mathematics classroom?
- Which strategies can be implemented to address challenges pertaining to the teaching of algebraic expressions in a Grade 10 mathematics classroom?

5.3 PURPOSE AND OBJECTIVES OF THE STUDY

The main aim purpose of the study was to explore the common algebraic expression challenges in a Grade 10 mathematics classroom.

In pursuance of this purpose, the two objectives were formulated as follows:

- To identify the common algebraic expression challenges in a Grade 10 mathematics classroom.
- To provide the solutions to address the identified challenges experienced by grade 10 learners.

5.4 THE SUMMARY OF THE STUDY

The purpose of the study was to explore the common algebraic expression challenges in a Grade 10 mathematic classroom. Based on the purpose of the study, the most and suitable people affected by the challenges were the ones to be involved in sharing their experiences and therefore devise solutions to address the challenges. Therefore, this study was guided by CER as the theoretical framework that supported the arguments in this regard. As a researcher, I was given a chance to view participants involved in the research as capable human beings and not to consider them as molecules in the laboratory (Myer, 2004) through CER as a lens. In addition, through CER, I was provided a role of interpreting other people' interpretations and to make sense out of them. Moreover, CER enabled the researcher, and the participants, to be critical and to draw from all angles of the challenge, a deeper meaning related to it (Mahlomaholo, 2009). Since the participants involved were the ones affected by the challenges, the use of this framework was significant, as the participants were also expected to give solutions to their own experiences, and therefore be enlightened and empowered as well. The participants were all free to share their lived experiences and to suggest possible strategies to overcome the common challenges in algebraic expression in a Grade 10 mathematics classroom.

As documented in 2.3, the operational concepts were comprehensively defined. The literature related was consulted with the aim of studying common algebraic expression challenges in Grade 10 in SA and in other countries. The Action Research (AR) approach was used in order to generate the empirical data. As defined by Reason and Bradbury (2008, p.4), "Action research is a participatory process concerned with developing practical knowledge in the pursuit of worthwhile human purposes. It seeks to bring together action and reflection, theory and practice, in participation with others, in the pursuit of practical solutions to issues of pressing concern to people, and more generally the flourishing of individual persons and their communities". In line with this, AR afforded the researcher an opportunity to work with the participants to identify common algebraic challenges in a Grade 10 mathematics classroom and to suggest possible solutions to the identified challenges.

Thus, in the present study, data were generated through a focus group discussion and observation sessions of teachers, present in the classroom. Thematic analysis was used to analyse and interpret data as discussed in section 3.6. The generated data revealed a critical finding, namely that there is a need for a course on the pedagogy of teaching algebraic expressions to improve teaching and learning thereof. The finding further revealed that continuous developmental workshops need to be provided to empower the teachers with knowledge on how to productively teach the algebraic expressions, particularly using the existing models which are outlined in section 2.4. The other findings of the study are highlighted in the subsequent sections, as well as the recommendations, limitations, recommendations for future research and the conclusion to the study.

5.5 FINDINGS ON COMMON ALGEBRAIC EXPRESSIONS CHALLENGES TO THE TEACHING

The subsequent sections discuss the related findings of the study based on the common algebraic expression challenges identified in a Grade 10 mathematics classroom and their solutions.

5.5.1 Inadequate teacher pedagogical content knowledge

The findings of the study revealed the inadequate teacher pedagogical content knowledge to be one of the factors that contribute to the learners' poor performance in manipulating algebraic expressions. Consequential to teachers' lack of pedagogical content knowledge, teachers resort to teacher-centred methods of instruction, rather than teaching methods which promote learner-centredness teaching (see section 4.2.1). One trait of PCK is the teachers' knowledge of learners and how they learn. Therefore teachers who lack this knowledge, cannot plan and teach learners in a way that he/she can address diverse individual's needs. Furthermore, this impedes effective planning of lessons to deliver content material in a variety of ways, in order to accommodate diverse learners' needs (see section 4.2.1).

As documented in Chapter 2, several scholars (Black, 2009; Grit & Akyuz, 2017; Yildiz & Akyus, 2019; Ziyyadi et al., 2020) have also found teachers' lack of PCK to be a challenge that can cause them to be ineffective in the classroom. This is because when there is lack of pedagogical knowledge, teachers are not able to unpack content material in a meaningful way and scaffold the subject content in an appropriate manner. One of the traits of a good teacher who has strong PCK, can be traced in his/her knowledge of the learners and the ability to devise teaching strategies that cater for his/her learners' needs (Ziyyadi et al., 2020). However, the findings of this study revealed that teachers often display lack of knowledge of their learners and their learning processes and that this makes it hard for these teachers to devise the appropriate teaching strategies that cater for their needs. A study by Güler and Çelik (2018) also revealed a similar finding; that teachers have significant deficiencies with respect to *knowledge of learners* and *presentation of content* when it comes to algebra teaching knowledge.

Also, according to Dalton et al. (2019), in South African classrooms, challenges faced, include teacher training, which do not fully prepare teachers in terms of developing/advancing their knowledge of both content and pedagogy to be able to effectively implement a variety of teaching methodologies, which discourage amongst them, one-size-fits-all strategies. To this end, the study revealed that teachers who lack PCK are unable to connect the mathematical concepts with real-life scenarios, The modified Lesh’s model affirms that the ability for teachers to use various representations and translate from one to another with relative ease can benefit learners significantly (see section 2.4.3). The UDL model also encourages teachers to be able to represent information in various formats (see section 2.4.2.1). Moreover, teachers should be in a position to give learners an opportunity to be actively engaged in their learning by using learner-centred methods, such as MME and the 5E guided inquiring model. Furthermore, this concurs with CER, which calls for people to be given opportunity to freely talk about issues that affect them without being intimidated (see section 2.2).

5.5.2 Teachers’ inability to explain algebraic concepts in-depth

Another finding which emerged, is that some teachers are unable to explain algebraic concepts in-depth. For example, the teacher provided one form of explanation on how to describe or distinguish an algebraic expression (see section 4.2.2). Findings also indicated that teachers in most cases, use teacher-centred methods when explaining concepts that they are not well conversant with, instead of presenting the information in a manner that will allow learners to be engaged and discover the answers themselves. The UDL model for teaching stresses the need to activate learners’ prior knowledge in order for meaningful learning to take place. However, the findings of the study revealed that the teachers often fail to tap into learners’ previous knowledge when dealing with simplification of algebraic fractions and this made it difficult for learners to develop connections of the concepts and for meaningful learning to take place (see section 4.2.2). The study further revealed that the teachers could not relate the prerequisite knowledge of learners to the new concept at hand and therefore regarded learners as “empty vessels”. This act impedes learners’ progress and limits them from gaining conceptual understanding and making connections of the prerequisite knowledge and the new concepts. Yildiz and Akyus (2019) also found that both in-service and pre-service teachers lack pedagogical content knowledge to teach algebraic expressions and hence propose that both these teachers be trained to use various knowledge types in order to effectively teach algebraic expressions.

5.5.3 Inability to apply the main algebraic concept

When discussing the issue of inability to apply the main algebraic concept there were several issues that were mentioned by the teachers. The first one was that learners are not able to apply the concept of difference of two squares when given numerical expressions. The second one is that they fail to apply factorisation in other contexts, such as simplifying exponential expressions (whereby x is an exponent and base is a number) and solving exponential equations. This is because learners regard exponential expressions to be different from algebraic expressions, since teachers only introduce or do not integrate exponents as they give examples of algebraic expressions. Thus, learners are only used to algebraic expressions, whereby x is the base and not the exponent. Moreover, teachers fail to expose learners with different representations, structures and contexts, where factorisation can be used, for example in quadratic equations ($ax^2 + bx + c = 0$), in exponential equations and/or expressions ($9^x - 1 = 0$), and in factorisation by grouping ($ax - bx + ay - by$). This limits learners' understanding of different types of algebraic expressions (see section 4.2.3). The findings of the study thus revealed that the teaching of algebraic expressions is conducted without considering the variation theory, which emphasises the need to pay attention to structure of the mathematical concepts. Hence, learners could not realise that $x^2 - 1$ can be factorised/simplified the same as $2^x - 1$.

In-line with this finding, Muchoko et al. (2019) also mentioned that, in some instances, learners display challenges in identifying familiar concepts presented to them in a different form, apart from the structure that they are familiar with. Therefore, they remain focused on the forms they are familiar with in their thinking and hence provide a wrong solution or fail to complete the problem, as well as not even attempting to solve the problem at all. The other finding was that learners have a challenge of applying factorisation when simplifying algebraic fractions, such as $\frac{ax+b}{2x}$. This is because learners seem to have difficulties in understanding the meaning of "common factors", because once they see similar figures/numbers, they start cancelling without really understanding why they should cancel. The learners thus seemed to have not developed an understanding of what cancelling means and when they should cancel. Teachers seemed not to pay attention to addressing the concept of common factors and also highlighting the instances or conditions where cancelling should be applied.

5.5.4 Lack of understanding the notion of a variable

The findings indicate that the learners have a challenge of understanding the notion of a variable, because they displayed difficulty in finding the dimensions of a given rectangle by just determining the values of x and not the dimensions of a rectangle (see section 4.2.3). The finding here is that although learners were able to apply the formula for area of a rectangle and solve for x, they could not understand that solving for x is not enough to solve the problem; they still needed to determine dimensions by substituting the positive value of x back into the given dimensions, which were in the form of binomial expressions. In some cases, the learners were not even able to select the appropriate value of x that they should use, they used both values of x and failed to discard the negative value and this made them to obtain negative answers, which they submitted as their solutions. Now this implies that the learners were not able to understand the context of the question and submitted negative values as dimensions of the rectangle (see section 2.4.1.3). Although learners can easily determine the value of x when they are given a binomial equation or even a quadratic equation, they seem to have a problem to determine the x values when given diagrams e.g., a rectangle with dimensions. The fact that learners sometimes give negative numbers for the dimensions of a rectangle, to a certain extend show that they disregard the context within which they are solving the problems. This shows that teachers do not pay attention to giving examples and making learners aware of the context, which according to Moleko and Mosimege (2021) is an important thing to do.

5.5.5 Inability to manipulate algebraic expressions

Manipulation of algebraic expression has shown to be problematic for some learners, because some learners misused the commutative property as displayed in section 4.2.5. The finding revealed that learners displayed a *"neglect minus sign error"* (Aydin-Guc & Aygun, 2021) during the multiplication of algebraic expressions. This means that learners tend to ignore the minus sign in the given algebraic expression. This finding concurs with *the* error of *neglecting* the *minus,* stated by Das (2020), as well as Aydin-Guc and Aygun

(2021). Seng (2010) also found the same challenge where learners multiplied the pre-multiplier without considering the negative sign attached to it. The literature refers to this misconception to be caused by epistemological challenges of comprehending the concept of 'negative'.

The conjoining error was also revealed as one of the challenges learners encounter during manipulation of algebraic expressions. One of the findings informed by observation, is that one of the reasons why learners say $2x + 7 = 9x\ or\ 9,$ is because teachers do not explain to learners that $2x$ is a term that is made by 2 and x whereas 7 is a term made by the constant 7. Furthermore, it was revealed by teachers that learners sometimes tend to feel uncomfortable to leave an answer in an expression format like $2x + 7$ or $17 - 3x$ and as a result they commit cojoining error (see section 2.5.1.4.1 and 4.2.5).

Also, Zayyadi et al. (2020) have identified some of these challenges in their research, whereby learners display a challenge of conjoining/closure property, misuse of distribution property and commutative property, as well as parsing and an expected answer obstacle. This finding also concurs with the study done by Mbewe (2013) and Seng (2010) that learners misapply rules or apply inappropriate procedures in other circumstances during manipulation of algebraic expressions. Although the findings in this study are supported by the previous findings from the other studies, the current study attributed these challenges to teachers' lack of PCK, insufficient time to delve deeper into the concepts and teachers' lack of reflection to be able to identify learners' misconceptions and errors, so as to think of ways to address them.

5.5.6 Improper use of mathematical vocabulary/expression

Regarding the improper use of mathematical vocabulary, the finding of the study revealed that teachers sometimes do not use the mathematical vocabulary in a proper way during their teaching of algebraic expression. As highlighted in section 4.2.6, during the focus meeting, it was revealed that teachers are sometimes the source of the learners' misconceptions, since they use terminology that is not appropriate in the classroom, such as referring to the 'exponent' as 'power' (for instance referring to x^n as "x to the power of n"). So, this reference confuses the learners and makes it difficult for them to distinguish between an exponent and a power. The learners' inability to tell apart and understand algebraic terms contribute to their low attainment in algebraic expression (see 2.4.1.5). Plana (2020) thus cautions that significant learning challenges may result from low explanation to imprecise mathematical talk. Therefore, this suggests the need for teachers to be careful with the manner in which they talk in mathematics classrooms to avoid confusing learners and impeding learning.

5.5.7 Gap between algebra and arithmetic

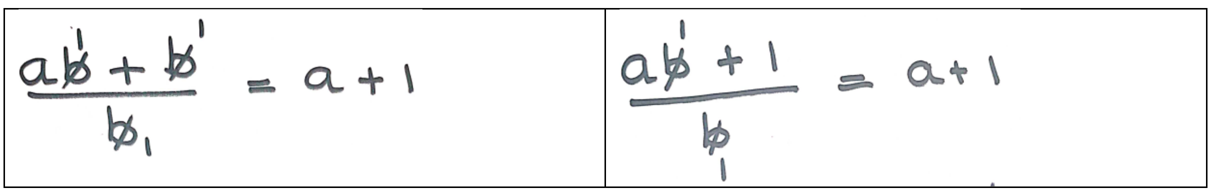
The findings regarding the gap that seemed to exist as learners transit from arithmetic to algebra was mentioned by the participants and they further stated that this gap is the primary cause of errors committed by learners in manipulating algebraic expressions and the misconceptions displayed by learners in this aspect. The other revelation is that even though the teachers are aware of this transition gap, they continue to teach learners without providing any intervention strategies to overcome this challenge. Thus, this also adds more problems to the common challenges learners have, as teaching of algebraic expressions get more advanced (see section 2.4.1.6 and 4.2.8). Muchoko et al. (2019) noted that the transition gap that learners seem to have is a result of inadequate pre-knowledge on concepts like variables, constants and fractions in algebraic expressions and hence this impedes learners to gain comprehension and mastery of algebraic concepts, which is vital for success in all aspects of mathematics.

5.5.8 Teachers' inability to select appropriate activities (examples) to use during instruction

Under this topic, the study revealed that in some instances, teachers have a problem of selecting appropriate examples to use in class (see section 4.2.8). The finding is that when the teacher does not give learners activities similar to the ones they were exposed to and/or the example used to introduce the concept, it causes problems for the learners. This challenge was found to inhibit learners' progress at a later stage when they are given advanced problems to solve. Therefore, it is crucial that during content matter, in-depth delivery emphasis be put on the choice of the example to afford learners opportunity to discern the critical feature of concepts (Ekawati & Lin, 2014). This means, teachers must provide learners with as many examples as possible in order to establish a firm foundation for the concepts delivered. In-line with this, Koskow (2016) confirms that the choice of questions allocated to learners in scaffolding the content should align with the mathematics at stake by the teacher. The use of inappropriate mathematics examples in a mathematics classroom, violates the variation theory model which demands the opposite to hold.

5.5.9 Teachers' inability to communicate mathematically correct in the classroom

Teachers' inability to communicate mathematically correct in the classroom emerged as one challenge, which needs to be addressed in order to effectively teach algebraic expressions in Grade 10. The findings indicated that teachers find themselves communicating with learners in such a way that will simplify a concept or make them understand the concept, but only to find that has caused some misconceptions. For example, figure 5.5 9 shows improper teacher communication in the classroom:



Scenario 1 scenario 2

Figure 5.1: A diagram showing the cancellation method

Scenario 1 in Figure 5.1 shows a cancellation method where a teacher is telling learners to cancel out the b's in an expression$\frac{ab+b}{b}$, since they see b on top and on the bottom of the fraction. This strategy works in this case, because b is a common factor of both numerator and denominator. However, this may not work in a case such as $\frac{\mathrm{a+b}}{\mathrm{b}}$ (see scenario 2 in Figure 5.5.9) where now they can see b on top and on the bottom but b is not a common factor of both numerator and denominator (see section 2.5.1.4.2 and 4.2.9). Therefore, in scenario 2, the learners have incorrectly applied cancellation and hence came to be an incorrect answer.

In addition, as documented in section 4.2.9, the lexical elaborations referred to by Planas (2021; 2020) are used by teachers in an effort to provide learners with clues to the meanings of important concepts. However, the finding is that these lexical elaborations in some cases, do not provide learners with conceptual understanding of concepts, but rather provide learners with procedural understanding. As a result, learners encounter problems later on when working on similar problems, which require them to apply those concepts. In addition to these findings, Planas (2021) has also commented that unfocused teacher utterances in the classroom accompanying other ways and forms of communication, remains an obstacle to the teaching of exact meanings of concepts.

5.5.10 Inability to represent word expression in algebraic format

Representing word problems in algebraic format was found to be challenging for learners. The finding that emerged in this regard shows that teachers experience a challenge of teaching learners who are unable to write algebraic expressions from the given word problems (see section 4.3.10). Similarly to the finding of this study, Marpa (2019) also found that the majority of learners in the study he conducted, were unable to translate word problems into mathematical form and thus unable to use correct mathematics. He further mentioned that the inability to translate word problems into algebraic expression and equations is the main contributing factor to poor performance. The current study further revealed that some of the teachers themselves also find it difficult to understand and analyse the word problems and to translate them into simple algebraic expressions.

5.6 FINDINGS ON THE SOLUTIONS FOR THE IDENTIFIED CHALLENGES

Findings on the components of the solutions to the challenges experienced, are presented in the following sections.

5.6.1 Enhancing teachers' pedagogical content knowledge

The generated data revealed that to improve teachers' pedagogical content knowledge, there must be teachers' training in the form of workshops organized by the subject advisors. These training workshops should give teachers a platform to share ideas on the strategies that can be implemented in teaching algebraic expressions in Grade 10. The other revelation was that at school level, the departmental heads must organize workshops within their schools whereby mathematics teachers come together and share best practices regarding algebraic expressions. The focus group forums also suggested that teachers be engaged in the PLCs meetings to improve their pedagogical knowledge. This is because in these meetings, teachers share ideas on the strategies that can be used to teach certain topics and these teachers are already attending PLC meetings, thus, are already reaping the fruits of such meetings. They also believe that PLC's meetings impact positively on the teaching and learning of mathematics. This also concurs with what is mentioned by Dufour and Marzano (2011), when they mention that PLC is the best strategy to improve learners' attainment by developing teachers' collaborative capability. They further claim that PLC is specifically intended to create conditions that aid teachers to become more knowledgeable and skillful in their practice. However, the study conducted by Osei (2020) revealed that the basic school mathematics teachers pose satisfactory algebra knowledge to teach mathematics, even though this knowledge is not improving over their years of experience. Osei's revelation thus, even confirms the need for continuous teacher development.

5.6.2 Strategies to address teachers' inability to explain the algebraic expressions in-depth

The findings in relation to teachers' inability to explain the algebraic expressions in-depth, as documented in section 4.3.2, is that there should be teachers' training, whereby teachers will be equipped with skills and knowledge of how to teach algebra. That is, continuous developmental workshop trainings should be put in place to assist teachers in teaching algebraic expressions. Another finding is that teachers may use team teaching or collaboratively plan lessons on this topic, whereby they would be able to share best practices. Teachers may also implement strategies such as the 5E guided inquiry model, which encourages learners to apply investigative approaches to be able to explain algebraic expression in-depth (see section 2.4.1). In the perspective of Garzon and Casinillo (2021), teaching algebraic expressions, using the 5E guided inquiry model, may benefit learners greatly, because this model allows learners to engage in learning, explore, explain, elaborate and evaluate the learning object. This 5E guided model can play a role in addressing teachers' inability to explain algebraic expressions, since by its nature it encourages the teachers to dig deeper and to understand the content in-depth, so that they can be in charge and in control of how they guide learners.

5.6.3 Strategies to address lack of understanding the main algebraic concepts

Under this topic, the participants revealed that to improve learners' application of main algebraic concepts, content should be represented in different formats (see section 4.3.3). The idea here is that when learners are exposed to different representations of concepts, they become more equipped and skilled in solving the problems they are tasked with. The use of multiple representations is also espoused by several researchers as a good strategy to reinforce understanding of the mathematical concepts (Al-Azawei, Parslow & Lundqvist, 2017; Moleko & Mosimege, 2021, 2020; Mpalami & Moleko, 2022). As documented in section 2.4.2.2, teachers must create platforms for learners to discuss and analyze solved problems by asking learners to define the steps taken in the simplification process and to explain the reasoning behind their decisions. This is the process that will assist learners to express their learning processes (Ndeya-Ndereya, 2016), but at the same time provide teachers with an opportunity to identify the gaps and misconceptions (Moleko, 2014). It is only when the teachers have correctly identified the gaps and misconceptions that they will in turn devise the appropriate strategies that will address them. The UDL model also supports the notion of providing learners with opportunities to express what they have learnt in different ways with an intent for teachers to know more and understand better the areas where the learners are experiencing the challenges. The variation theory was also mentioned as another strategy that can be implemented to advance learners' comprehension of main algebraic concepts. The variation theory enables teachers to select appropriate questions, activities or examples that will assist learners to gain an in-depth understanding of the content (Ekawati & Lin, 2014). In addition, the questioning sequence should be selected in such a manner that examples should be aligned with classwork activities, homework activities, as well as the test questions (Koskow, 2016). The use of real-life examples was also found to be useful to improve understanding of the main algebraic concepts. This is espoused by the RME model. Furthermore, the findings revealed the significance of explicit teaching of the algebraic key terms to enable learners to develop their understanding. This is supported by the UDL model, which requires teachers to clarify the mathematical vocabulary and symbols in order to improve learning and reinforce understanding.

5.6.4 Reinforcing the understanding of variables and parameters

Discussion regarding reinforcement of understanding variables and parameters revealed several approaches that can be used. As highlighted in sections 2.4.2.3 and 4.3.4, the use of pictorial representation was found to be a useful strategy to aid learners' understanding of variables and parameters. The use of technology, such as GeoGebra, was also found to be useful in terms of reinforcing understanding of the variables and parameters, since it (Geogebra) has the ability to provide a visual display of the effects of the parameters. The visual display thus makes the concepts to be perceptible. Bouck et al. (2013) also discovered that the teaching of algebraic expression using technology has a positive impact in improving learners' understanding of the mathematical concepts. Moreover, as documented in section 2.4.2.3, various scholars also confirm that actively engaging learners in activities, which enable them to identify the variables and constants in a real-life situation, will also assist learners to understand concepts deeper (Kartz, 2016; Moleko, 2021; Steffe & Ulrich, 2020). Another finding revealed the significance of giving learners opportunities to investigate the effect of parameters, because this will enable them to distinguish parameters from the variables. This promotes discovery learning, which is another form of an inquiry-based, constructivist learning theory that takes place in problem-solving situations where learners draw on their own experiences (Garzon & Casinillo, 2021).

5.6.5 Enhancing learners' understanding of simplifying algebraic expressions

The discourses based on learners' understanding of algebraic expressions have shown that the use of physical representations in the form of pictures or diagrams was found to be fruitful to assist learners' comprehension in dealing with the "like and unlike" terms. The participants reported to have gained a positive response when engaging learners in activities that show or emulate physical representation to illustrate the concepts of "like terms and unlike" terms (see section 4.3.5). As documented in section 2.4.2.4, Akyuz and Yildiz (2019) argue that, if learners can be able to visualize an algebraic expression in different forms, as well as including pictorial forms, then teachers would have achieved application of multiple forms of representation and learner engagement in their classrooms. Furthermore, the findings revealed that the rich context of real-life scenarios should be used to aid understanding of simplifying the algebraic expressions (see 4.3.5) and that teachers should adopt the principles of RME and apply them in their teaching to make it meaningful (see section 2.3.3.3.2). These principles advocate for teaching of mathematics concepts to be from the rich context of real-life experiences, as they argue that mathematics is a human activity. Therefore, learners should be engaged in real-life discourses, whereby concepts may not be explored in isolation of real life.

5.6.6 Using different representation to address algebraic language and to reinforce its understanding

The findings revealed that the use of visuals and real-life context examples, assist teachers to present concepts in a meaningful manner. This has also shown a different approach of presenting algebraic expressions, which now affords learners an opportunity to relate the concept to a real-life situation, which normally teachers do not display in their teaching. Out of this context, learners are able to talk about monomial, binomial, trinomial and factors and can now notice that a trinomial can be obtained as a product to calculate area. As documented in section 4.3.6, Johnson (2016) has conversed that physical representations may be used, because they serve as tools to mathematical thought and communication. Section 2.4.2.6 highlights that before proceeding with a topic, mathematics teachers should teach the mathematical language instruction in a mathematics classroom to enhance better performances of learners in mathematics (Joseph, 2020). The finding in this study, regarding the algebraic language, revealed that it is important for teachers to explicitly teach it. This will make it possible for learners to understand the algebraic concepts. In the study conducted by de Groot and Boyajian (2015), it was found that mathematical language of both learners and teachers changed during the set of lessons presented and revelations of deeper understanding were made in the structural properties of algebraic expressions of additions and subtraction and the relationship shown with numeric polynomial addition and subtraction where a place value system is at work. This finding is in accord with the finding in the current study, however, the current study further suggests the need for teachers to be careful of these changes during the lesson so as to avoid any confusion that these changes may spark and thus impede learning.

5.6.7 Bridging the gap between arithmetic and algebra

The finding under this heading, revealed that proper application of context can help to bridge the transition gap from the arithmetic to algebraic form. For example, as documented in section 4.3.7, teaching learners initially, using real-life scenarios and concrete objects, promote better understanding. Further revelation as documented in section 2.4.2.5, is that strategies that emphasise understanding of concepts of fractions at the lower grades and strategies, such as group work where learners are engaged in diverse sharing of ideas, should be implemented (Banerjee & Subramaniam, 2012). In addition, another finding is that it is important to exemplify with integers first and then connect algebraic expression to aid learners' comprehension in learning how to solve the algebraic expressions. According to Theobald et al. (2020), active strategies are the best strategies to narrow the transition gap from arithmetic to algebra. Hence, some of the best practices models, such as Lesh's translational model and the variation theory model, as well as RME, can be utilised to assist in bridging the transition gap from arithmetic to algebra, since these strategies encourage teachers to actively engage learners in the learning process, as well as using real-life context in teaching (see section 2.4.3. and 2.4.4, respectively).

5.6.8 Advancing teachers' ability to select appropriate activities (examples) to use during instruction

The findings of this study indicate that sharing of how to teach algebraic expressions by both the experienced teachers and novice teachers has a potential of increasing the ability of teachers with an appropriate choice of class activities. This therefore means that the schools must create platforms where teachers can discuss the appropriate algebraic expression examples to be used. This platform would provide the teachers with opportunities to learn from each other. In addition, the findings revealed the need to implement the variation theory whereby teachers' selection of classwork exercises and homework are correlating or aligned with the examples used to develop the understanding of the content. Scholars, such as Ekawati and Lin (2014), regard the variation theory as a useful way to assist teachers to teach content matter in-depth by providing many examples in which the same concept is used to give a firm foundation of conceptual understanding (see section 4.3.8). The variation theory was also deemed to be useful in addressing the structure of algebraic expressions.

5.6.9 Improving teachers' ability to communicate mathematically correct in the classroom

To improve the teachers' way of communicating mathematically correct in the classroom, the findings revealed that there should be developmental workshops whereby both novice and experienced teachers would undergo training regarding the teaching of algebraic expressions from experts (see section 4.3.9). This will enable the teachers to learn from experts how to communicate mathematically correct in class. This will also give teachers an opportunity to express ideas pertaining to algebraic expressions correctly. The other finding was that teachers may use a lesson study approach as another way in which teachers may come together and learn from one another. This approach (lesson study) is espoused by CER, which is the paradigm underpinning this study in that, it requires teachers to be regarded as capable human beings who possess the knowledge and skills, which they can use to transform their own practices. According to a lesson study approach, teachers from the same school or cluster (circuit) are expected to collaborate, plan the lessons together and then one of them present it (lesson), while others are observing (Tsotetsi, 2013) what is happening during the lesson, including the communication that is taking place. After presentation, these teachers may then come together to reflect on the communication that took place when the lesson was in progress. This type of collaboration serves as an empowering phase in which teachers would learn from the lived experiences of the others.

5.6.10 Enhancing learners' ability to express word expressions in algebraic format

With regards to enhancing learners 'ability to express word expressions in algebraic format, the findings revealed that often using a real-life scenario, assists learners to understand the question better and majority of them in this study were able to give correct answers. The findings also revealed that word problems should be broken down into smaller parts that are easily understood. This will make it possible for learners to understand what the problem entails and what it requires to solve them. The findings of the study revealed that it is easier for learners to solve the word problems and convert them into simple algebraic expressions, if they understand what the problem requires. The finding of the study thus stresses the need for teachers to use problem-solving models in order to enable learners to understand and solve the word problems. The finding of the study also emphasises the need for teachers to pay attention and explicitly teach the algebraic expression "terms" in order for learners to develop the understanding of the word problems and to be able to convert them into simple algebraic expressions that are easy to solve.

5.7 RECOMMENDATIONS OF THE STUDY

This study was purposed to identify the common algebraic expressions challenges in the Grade 10 mathematics classroom. To do that, various challenges were echoed amongst the participants and solutions were provided. Since algebraic expressions in Grade 10 are the fundamental concepts of most topics in FET-phase mathematics, poor attainment of learners in this aspect, creates a barrier to learning and hence deprives learners an opportunity to master other topics, which require the basic knowledge of manipulating algebraic expressions. To teach this concept in a meaningful manner, teachers need to acknowledge amongst others, the diversity of learners and thus make their presentations flexible, perceptible, and accessible to all learners. This means that the teacher-centred methods should not have a place in their classrooms.

One of the recommendations in this study, is for teachers to carefully examine learners' errors and misconceptions, such as conjoining error, cancellation error and others, as well as the misconception of like and unlike terms. Hence, they must devise ways to overcome them before they present the topic of algebraic expressions to learners.

The study further recommends the application of some of the best practices/models, such as the Universal Design for Learning (UDL) approach, 5E the variation theory model, modified Lesh's translational model and Polya's problem-solving model, as well as Realistic Mathematics Education (RME) to teach the algebraic expressions in a meaningful manner. These models would enable the teachers to present the algebraic expressions in multiple formats, thus simplifying the content and making it understandable. Furthermore, these models will enable the teachers to select the appropriate examples that would aid the learners' comprehension of the algebraic expressions. The models will further assist in making the algebraic expressions simple and intuitive.

Furthermore, the study recommends that there is a need for training of teachers at institutes of higher learning on pedagogy to teach algebraic expressions. Training should also be done in the form of workshops provided by the Department of Education so as to equip teachers with knowledge of the appropriate teaching strategies to teach this concept. Moreover, at circuit levels, PLCs must be active to provide teachers a platform to collaborate and share best practices to overcome the challenges they encounter in their practice. In addition, it is further recommended that teachers at cluster or school level, should use a lesson study whereby they collaborate and plan a lesson together. This will afford the teachers with opportunities to learn from one another.

5.8 LIMITATIONS TO THE STUDY

The data for this study was collected during the COVID-19 lockdown period. Therefore, the time to get access to the schools was limited, because the time tabling approach was on a rotational basis, which meant that on some days, learners were not at school. Due to this, the researcher had to engage in online meetings instead of face-to-face focus group meetings with the teacher participants. This posed some problems to find a suitable time for the online meetings.

The tittle of the study is based on exploring the common algebraic expressions in a Grade 10 mathematics classroom; thus, it made the study to be more focused on the challenges than the solutions. However, I have included the sections on how the challenges can be addressed, thus providing the solutions to the identified challenges. I did this, because in order for research to be meaningful, it has to bring solutions in order to transform the unfavourable conditions. The inclusion of the solutions' sections was also propelled by the paradigm underpinning the study (CER), which requires not only the challenges to be identified, but also the solutions to be identified by the people who are experiencing these challenges.

However, the solutions suggested and discussed in this study may not be the only ones available to address the identified challenges. Furthermore, because of the limited time and COVID-19 conditions, which were prevailing at the time, the study only produced one cycle of AR. Therefore ,some of the suggested intervention strategies were not put into practice. Even though this study was conducted at a high school, it should be noted that the findings cannot be applicable to all schools, since the circumstances in various schools are different. However, the findings of this study could be applicable in schools where similar challenges are experienced under similar conditions to those of the school under the present study.

5.9 RECOMMENDATIONS FOR FUTURE RESEARCH

Since the research was done in one school in the rural area and conducted during the Covid-19 season, it is recommended that the study be carried out with a considerable number of schools, which include schools in the urban areas in order to identify common algebraic expression challenges for diverse learner population. In addition, this study recommends that research be conducted, based on the best practices and modelling of the algebraic expressions. Furthermore, research should be conducted on various interventions that should be put in place to address the errors and misconceptions in algebraic expressions. Furthermore, the study recommends that more action research-orientated studies be conducted in order for the various interventions put in place, in an attempt to address the common algebraic expression challenges to be identified in various contexts.

5.10 CONCLUSION

This chapter has presented the findings of the study according to the objectives of the study, as mentioned in Chapter 1. The summary of the study was also presented in this chapter. The chapter further provided the recommendations made in light of the findings of the study. The limitations of the study and recommendations for future research were also outlined.

REFERENCES

Acharya, B. R., Belbase, S., Panthi, R. K., Khanal, B., Kshetree, M. P., & Dawadi, S. D. (2022). Critical Conscience for Construction of Knowledge in Mathematics Education. *International Journal of Education in Mathematics, Science and Technology*, *10*(4), 1030-1056.

Adendorff, S. A. (2019). International reflections on Realistic Mathematics Education, International Reflections on the Netherlands Didactics of Mathematics: Visions on and Experiences with Realistic Mathematics Education, Marja van den Heuvel-Panhuizen (Ed.). *Pythagoras*, *40*(1), 1-3.

Adnan, M., Abidin, Z. Z., Mustam, A. A., Maat, S. M., Sutama, S., Prihaswati, M., & Zulnaidi, H. 2021. The ability and analysis of Students' errors in the topic of algebraic expression. *Journal of Physics: Conference Series, 1988*(1), 012049.

Al-Azawei, A., Parslow, P., & Lundqvist, K. (2017). The effect of universal design for learning (UDL) application on e-learning acceptance: A structural equation model. *International Review of Research in Open and Distributed Learning: IRRODL*, *18*(6), 54-87.

Aldridge, J. M., Fraser, B. J., & Sebela, M. P. (2004). Using teacher action research to promote constructivist learning environments in South Africa. *South African Journal of Education*, *24*(4), 245-253.

Aldridge, J. M., Fraser, B. J., Bell, L., & Dorman, J. (2012). Using a new learning environment questionnaire for reflection in teacher action research. *Journal of Science Teacher Education*, *23*(3), 259-290.

Aldridge, J. M., Rijken, P. E., & Fraser, B. J. (2021). Improving learning environments through whole-school collaborative action research. *Learning Environments Research*, *24*(2), 183-205.

Althauser, K. L. (2018). The Emphasis of Inquiry Instructional Strategies: Impact on Pre-service Teachers' Mathematics Efficacy. *Journal of Education and learning*, *7*(1), 53-70.

Attorps, I., Björk, K., & Radic, M. (2011). The use of mathematics software in university mathematics teaching. In *The proceedings of the 7th Congress of the European Society for Research in Mathematics education* (pp. 2188-2197).

Ayaya, G., Makoelle, T. M., & van der Merwe, M. (2020). Participatory action research: A tool for enhancing inclusive teaching practices among teachers in South African full-service schools. *Sage Open*, *10*(4), 2158244020963576.

Aydin-Güç, F., & Aygün, D. (2021). Errors and Misconceptions of Eighth-Grade Students Regarding Operations with Algebraic Expressions. *International Online Journal of Education and Teaching*, *8*(2), pp.1106-1126.

Aygor, N., & Burhanzade, H. (2015). Students' performances in solving 2nd Degree Equations with one Unknown. *Procedia-Social and Behavioral Sciences*, *197*, 13-18.

Ayub, A., Razzaq, A., Aslam, M. S., & Iftekhar, H. (2013). A conceptual framework on evaluating SWOT analysis as the mediator in strategic marketing planning through marketing intelligence. *European Journal of Business and Social Sciences*, *2*(1), 91-98.

Baez-Leon, C., Palacios-Ceña, D., Fernandez-de-las-Peñas, C., Velarde-García, J. F., Rodríguez-Martínez, M. Á., & Arribas-Cobo, P. (2021). A qualitative study on a novel peer collaboration care programme during the first COVID-19 outbreak: A SWOT analysis. *Nursing Open*.

Banerjee, R., & Subramaniam, K. (2012). Evolution of a teaching approach for beginning algebra. *Educational Studies in Mathematics*, *80*(3), 351-367.

Barnes, H. (2005). The theory of Realistic Mathematics Education as a theoretical framework for teaching low attainers in mathematics. *Pythagoras*, *2005*(61), 42-57.

Baskerville, R. L. (1999). Investigating information systems with action research. *Communications of the association for information systems, 2*(1), 19.

Basham, J. D., & Marino, M. T. (2013). Understanding STEM education and supporting students through universal design for learning. *Teaching exceptional children*, *45*(4), 8-15.

Begum, S. (2012). A secondary science teacher's beliefs about environmental education and its relationship with the classroom practices. *International Journal of Social Sciences and Education*, *2*(1), 10.

Begum, S. (2012). Transition from lecture based to activity based teaching through whole school improvement in Gilgit-Baltistan Pakistan. *International Journal of Social Science and Education*, *2*(3), 378.

Bell, L. M., & Aldridge, J. M. (2014). *Student voice, teacher action research and classroom improvement*. Springer.

Biesta, G. (2010). Pragmatism and the philosophical foundations of mixed methods research. *Sage handbook of mixed methods in social and behavioral research*. Sage.

Black, J. W. (2009). Content knowledge and pedagogical content knowledge of algebra teachers and changes in both types of knowledge as a result of professional development. In *Proceedings of the Annual Meeting of the Georgia Association of Mathematics Teacher Educators* (Vol. 3, No. 1, pp. 11-23).

Bleijenbergh, I., van Mierlo, J., & Bondarouk, T. (2021). Closing the gap between scholarly knowledge and practice: Guidelines for HRM action research. *Human resource management review*, *31*(2), 100764.

Bouck, E. C., Meyer, N. K., Joshi, G. S., & Schleppenbach, D. (2013). Accessing algebra via MathSpeak™: Understanding the potential and pitfalls for students with visual impairments. *Journal of Special Education Technology*, *28*(1), 49-63.

Bradbury, H., & Reason, P. (2008). Issues and choice points for improving the quality of action research. *Community-based participatory research for health: From process to outcomes*, 225-242.

Bragg, L. A. (2017). Action Research on the Application of Variation Theory in Mathematics Teacher Education. *Mathematics Teacher Education and Development*, *19*(1), 121-136.

Braun, V., & Clarke, V., 2012. *Thematic analysis*. American Psychological Association.

Bray, A., & Tangney, B. (2017). Technology usage in mathematics education research– A systematic review of recent trends. *Computers & Education*, *114*, 255-273.

Brown, P., Evans, M., Hunt, D., Mcintosh, J., & Pender, B. (2011). Jacqui Ramagge. *The Improving Mathematics Education in Schools (TIMES) Project Construction a Guide for Teachers-Year*, 7-8.

Burgstahler, S., & Russo-Gleicher, R. J. (2015). Applying universal design to address the needs of postsecondary students on the autism spectrum. *Journal of Postsecondary Education and Disability*, *28*(2), pp.199-212.

Burgstahler, S. (2009). Universal Design in Education: Principles and Applications. *DO-IT*.

Burns, A. (2010). *Doing Action Research in Language Teaching: A Guide for Practitioners*. Routledge.

Buskens, I. (2011). The free attitude interview in context. *The Free Attitude Interview*.

Canque, M. S. (2021). Differentiated Instruction through Tiered Activities in Teaching Geometry to the Junior High School students. *Turkish Journal of Computer and Mathematics Education (TURCOMAT)*, *12*(6), 5373-5380.

Capellini, V. L. M. F., Maturana, A. P. P. M., Brondino, N. C. M., Peixoto, M. B. C. L. B. M. and Broughton, A. J. (2019). Evaluation of the Quality of Education Offered to Students with Special Needs in Public Schools in the City of Bauru, Brazil. *International Journal of Educational and Pedagogical Sciences*, *13*(4), pp.435-440.

Carr, W., & Kemmis, S. (1983). *Becoming critical: Knowledge through action research.* Deakin University Press.

Çelik, D., & Güler, M. (2018). Examination of Pre-Service Elementary School Mathematics Teachers' Knowledge for Algebra Teaching. *International Online Journal of Educational Sciences*, *10*(1).

Cohen, J. (2015). Challenges in identifying high-leverage practices. *Teachers College Record*, *117*(7), 1-41.

Cohen, J., & Goldhaber, D. (2016). Building a more complete understanding of teacher evaluation using classroom observations. *Educational Researcher*, *45*(6), 378-387.

Colom, A., 2021. Using WhatsApp for focus group discussions: Ecological validity, inclusion and deliberation. *Qualitative Research*, p.1468794120986074.

Conowal, T. A. (2018). *Cognitively Guided Instruction in Elementary Mathematics: Understanding Factors That Influence Classroom Implementation* (Doctoral dissertation). Kennesaw State University.

Cooks, N. M., & Alibali, M. W. (2014). Defining and measuring conceptual knowledge in mathematics. *Developmental Review*, 34, 344-377.

Cumming, T. M., & Rose, M. C. (2021). Exploring universal design for learning as an accessibility tool in higher education: A review of the current literature. *The Australian Educational Researcher*, 1-19.

Daulay, K. R., & Ruhaimah, I. (2019). Polya theory to improve problem-solving skills. *Journal of Physics: Conference Series, 1188*(1), 012070.

De Groot, C., & Boyajian, M. (2015). Like Terms: What's in a Name? *The Mathematics Teacher*, *108*(7), 506-511.

Deng, J., Yin, S., & Rao, P. (2021). Analysis of BIM Application Strategy in Engineering Project Based on Fuzzy Mathematics-SWOT. *Journal of Information Technologyin Civil Engineering and Architecture*, *13*(4), pp.113-119

Deppeler, J., Loreman, T., & Sharma, U. (2005). Improving inclusive practices in secondary schools: Moving from specialist support to supporting learning communities. *Australasian Journal of Special Education*, *29*(2), 117-127.

Dewanto, M. D., Budiyono, B., & Pratiwi, H. (2017, August). Students' error analysis in solving the math word problems of high order thinking skills (HOTS) type on trigonometry application. In *1st Annual International Conference on Mathematics, Science, and Education (ICoMSE 2017)* (pp. 126-131). Atlantis Press.

Dick, L. K., Appelgate, M. H., Gupta, D., & Soto, M. M. (2022). Continuous improvement lesson study: A model of MTE professional development. *Mathematics Teacher Educator*, *10*(2), 111-128.

Dickens, L., & Watkins, K. (1999). Action research: rethinking Lewin. *Management learning*, *30*(2), 127-140.

Didis, M. G., & Erbas, A. K. (2015). Performance and difficulties of students in formulating and solving quadratic equations with one unknown. *Educational Sciences: Theory & Practice*, *15*(4).

Doig, B. (2013). Mathematical Tasks and Learning Goals: Examples from Japanese Lesson Study. *Mathematics Education Research Group of Australasia*.

Doig, B., & Groves, S. (2011). Japanese lesson study: Teacher professional development through communities of inquiry. *Mathematics teacher education and development*, *13*(1), 77-93.

Donohue, D., & Bornman, J. (2014). The challenges of realising inclusive education in South Africa. *South African journal of education*, *34*(2).

DuFour, R., & Marzano, R. J. (2011). *Leaders of learning: How district, school, and classroom leaders improve student achievement*. Solution Tree Press.

Duval, R. (2006). A cognitive analysis of problems of comprehension in a learning of mathematics. *Educational studies in mathematics*, *61*(1), 103-131.

Edwards, E., & Burns, A. (2016). Action research to support teachers' classroom materials development. *Innovation in Language Learning and Teaching*, *10*(2), 106-120.

Egodawatte, G. (2011). *Secondary school students' misconceptions in algebra* (Doctoral dissertation). University of Toronto.

Ekawati, R., & Lin, F. L. (2014). Designing teacher professional development for mathematics teaching with variation theory. *Journal on Mathematics Education*, *5*(2), 127-137.

Elden, M., & Levin, M. (1991). Cogenerative learning: Bringing participation into action research. *Participatory action research*, 127-142.

Erling, E., Ashmore, K., & Kapur, K. (2016). Learning from Misconcetions: Algebraic Expressions (http://www.tess-india.edu.in/).

Etikan, I., Musa, S. A. and Alkassim, R. S. (2016). Comparison of convenience sampling and purposive sampling. *American journal of theoretical and applied statistics*, *5*(1), pp.1-4

Evans, D., & Field, A. P. (2020). Maths attitudes, school affect and teacher characteristics as predictors of maths attainment trajectories in primary and secondary education. *Royal Society open science*, *7*(10), 200975.

Evmenova, A. (2018). Preparing teachers to use universal design for learning to support diverse learners. *Journal of Online Learning Research*, *4*(2), 147-171.

Fauzana, R., Dahlan, J. A., & Jupri, A. (2020, April). The influence of realistic mathematics education (RME) approach in enhancing students' mathematical literacy skills. In *Journal of Physics: Conference Series* (Vol. 1521, No. 3, p. 032052). IOP Publishing.

Fereday, J. & Muir-Cochrane, E. (2006). Demonstrating rigor using thematic analysis: A hybrid approach of inductive and deductive coding and theme development. *International journal of qualitative methods*, *5*(1), pp.80-92.

Ferguson, B. T., McKenzie, J., Dalton, E. M., & Lyner-Cleophas, M. (2019). Inclusion, universal design and universal design for learning in higher education: South Africa and the United States. *African journal of disability*, *8*(1), 1-7.

Ferrance, E. (2000). Themes in education: Action research. *Brown University: Educational Alliance*, *34*(1), 1-33.

Ferrer, G. A. (2020). Error Analysis in the Operations of Algebraic Expressions of Grade 8 Students at Kalinga State University Laboratory High School. *International Journal of English Literature and Social Sciences, 5*(6), 2494-2504.

Ferretti, F. (2020). The Manipulation of Algebraic Expressions: Deepening of a Widespread Difficulties and New Characterizations. *International Electronic Journal of Mathematics Education*, *15*(1).

Fontana, A., & Frey, J. H. (2005). The interview. *The Sage handbook of qualitative research*, *3*, 695-727.

Foster, D. A. V. I. D. (2007). Making meaning in algebra: Examining students' understandings and misconceptions. *Assessing mathematical proficiency*, *53*, 163-176.

Fox, N. (2009). Using interviews in a research project. *The NIHR RDS for the East Midlands/Yorkshire & the Humber*, *26*.

Fullan, M. (2002). Principals as leaders in a culture of change. *Educational leadership*, *59*(8), 16-21.

Garzon, J.R. and Casinillo, L.F. (2021). 5E Guided Inquiry Model and Students' Attitude and Achievement in Algebraic Expressions. *Review of Socio-Economic Research and Development Studies*, *5*(1), pp.1-22.

Gay, A. S. (2008). Helping teachers connect vocabulary and conceptual understanding. *The Mathematics Teacher*, *102*(3), 218-223.

Ginting, M. S., Prahmana, R. C. I., Isa, M., & Murni, M. (2018). Improving the reasoning ability of elementary school student through the indonesian realistic mathematics education. *Journal on Mathematics Education*, *9*(1), 41-54.

Gitomer, D., Bell, C., Qi, Y., McCaffrey, D., Hamre, B. K., & Pianta, R. C. (2014). The instructional challenge in improving teaching quality: Lessons from a classroom observation protocol. *Teachers College Record*, *116*(6), pp.1-32.

Glaser, K.M., 2017. *Teacher Knowledge and Use of Universal Design for Learning* (Master thesis). Governor State University.

Gningue, S. M. (2000). *The use of manipulatives in middle school algebra: an application of Dienes' variability principles*. Columbia University.

Gningue, S. M. (2016). Remembering Zoltan Dienes, a maverick of mathematics teaching and learning: applying the variability principles to teach algebra. *International Journal for Mathematics Teaching and Learning*, *17*(2).

Goedde, A., Weaver, J., Patterson, N., Matney, G., & Nadler, J. Digital Tools to Promote Lesson Study: Using Lesson Study as a vehicle for TPACK professional development in teacher educator practice. *International Journal for Lesson & Learning Studies,* 10(2), 187-201

Groenewald, T. (2004). A phenomenological research design illustrated. *International journal of qualitative methods*, *3*(1), 42-55.

Guba, E. G. (1981). Criteria for assessing the trustworthiness of naturalistic inquiries. *Ectj*, *29*(2), 75-91.

Guler, M., & Celik, D. (2021). The effect of an elective algebra teaching course on prospective mathematics teachers' pedagogical content knowledge. *International Electronic Journal of Mathematics Education*, *16*(2), em0636.

Gunawan, J. (2015). Ensuring trustworthiness in qualitative research. *Belitung Nursing Journal*, *1*(1), 10-11.

Hart, S. (2021). Where do mathematical symbols come from?

Hattan, C., Singer, L. M., Loughlin, S., & Alexander, P. A., (2015). Prior knowledge activation in design and in practice. *Literacy Research: Theory, Method, and Practice, 64*(1), 478-497.

Hirza, B., & Kusumah, Y. S. (2014). Darhim, and Zulkardi,“. *Improving intuition skills with realistic mathematics education,” IndoMS-JME*, *5*(1), 27-34.

Hoch, M., & Dreyfus, T. (2004). Equations–a structural approach. I MJ Høines & AB Fuglestad (Red.). In *Proceedings of the 28th conference of the International Group for the Psychology of Mathematics Education* (Vol. 1, pp. 152-155).

Hoch, M., & Dreyfus, T. (2006, July). Structure sense versus manipulation skills: An unexpected result. In *Proceedings of the 30th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 3, pp. 305-312).

Hong, C. E., & Lawrence, S. A. (2011). Action Research in Teacher Education: Classroom Inquiry, Reflection, and Data-Driven Decision Making. *Journal of Inquiry and Action in Education, 4*(2), 1-17.

Isik, C., & Kar, T. (2012). The analysis of the problems posed by the pre-service teachers about equations. *Australian Journal of Teacher Education (Online)*, *37*(9), 93-113.

Johnson, B., & Christensen, L. (2012). *Educational research: quantitative, qualitative and mixed methods approaches,* 4th Ed. Sage.

Johnson, E. L. (2018). A New Look at the Representations for Mathematical Concepts: Expanding on Lesh's Model of Representations of Mathematical Concepts. *Forum on Public Policy Online, 2018*(1).

Johnson, H. L. (2016). Quantitative reasoning in mathematics education: Directions in research and practice. In *Reconceptualizing STEM Education* (pp. 163-180). Routledge.

Johnson, R. B, & Christensen, L., (2017). Educational research: *quantitative, qualitative and mixed methods approaches*, 6th Ed. Sage

Johnson-Harris, K. M., & Mundschenk, N. A. (2014). Working effectively with students with BD in a general education classroom: The case for universal design for learning. *The Clearing House: A Journal of Educational Strategies, Issues and Ideas*, *87*(4), 168-174.

Jordan, S. (2003). Who stole my methodology? Co-opting PAR. *Globalisation, societies and education*, *1*(2), 185-200.

Joseph, J. U. (2020). Understanding the Concept of Language of Mathematics for Effective Teaching and Learning of Mathematics in a School System. *International Journal for Educational and Vocational Studies*, *2*(10).

Jupri, A., Drijvers, P., & van den Heuvel-Panhuizen, M. (2014). Difficulties in initial algebra learning in Indonesia. *Mathematics Education Research Journal*, *26*(4), 683-710.

Jupri, A., Usdiyana, D., & Sispiyati, R. (2019). Designing an algebra learning sequence: the case of operations on algebraic expressions. *Journal of Physics: Conference Series*, *1280*(4), 042038).

Kapur, V. (2018). Deconstructing Teacher Education: Technology and the Intern. In *Dynamic Learning Spaces in Education* (pp. 51-68). Springer, Singapore.

Katz, J., & Sokal, L. (2016). Universal design for learning as a bridge to inclusion: A qualitative report of student voices. *International Journal of Whole Schooling*, *12*(2), 36-63.

Katz, J., 2013. The Three Block Model of Universal Design for Learning (UDL): Engaging students in inclusive education. *Canadian Journal of Education*, *36*(1).

Kellner, D. (2000). Habermas, the public sphere, and democracy: A critical intervention. *Perspectives on Habermas*, *1*(1), 259-288.

Kemmis, S. (1988). The action research. *Educational research, methodology, and measurement: An international handbook*.

Kemmis, S. (1993). Action research and social movement. *Education policy analysis archives*, *1*, 1-1.

Kemmis, S. (2001). Educational research and evaluation: opening communicative space [The 2000 Radford Lecture.]. *Australian Educational Researcher*, *28*(1), 1-30.

Kemmis, S. (2008). Critical theory and participatory action research. *The SAGE handbook of action research: Participative inquiry and practice*, *pp.* 121-138. Sage.

Kemmis, S., & McTaggart, R. (1988). The action research planner. Victoria. *Australia: Deakin University*.

Kemmis, S., McTaggart, R., & Nixon, R. (2014). The action research planner: Doing critical participatory action research.

Ketzenberger, K. E. (2013). Center for Applied Special Technology. *Encyclopedia of Special Education: A Reference for the Education of Children, Adolescents, and Adults with Disabilities and Other Exceptional Individuals*.

Kieran, C. (2007). Learning and teaching algebra at the middle school through college levels: Building meaning for symbols and their manipulation. *Second handbook of research on mathematics teaching and learning*, *2*, 707-762.

Kiger, M. E., & Varpio, L. (2020). Thematic analysis of qualitative data: AMEE Guide No. 131. *Medical teacher*, *42*(8), 846-854.

Kincheloe, J. L., & McLaren, P. (2011). Rethinking critical theory and qualitative research. In *Key works in critical pedagogy* (pp. 285-326). Brill.

King-Sears, M.E., Johnson, T.M., Berkeley, S., Weiss, M.P., Peters-Burton, E.E., Evmenova, A.S., Menditto, A. and Hursh, J.C., 2015. An exploratory study of universal design for teaching chemistry to students with and without disabilities. *Learning Disability Quarterly*, *38*(2), pp.84-96.

Koirala, H. P. (2005). The effect of mathmagic on the algebraic knowledge and skills of low-performing high school students. *International Group for the Psychology of Mathematics Education*, 209.

Kosko, K. W. (2016). Preservice elementary mathematics teachers decision making: The questions they ask and the tasks they select. In *Proceedings of the 38th annual meeting of the North American Chapter of the International Group for the Pyschology of Mathematics Education* (pp. 1341-1344).

Krishnaswamy, A. (2004). Participatory research: strategies and tools. *Practitioner: Newsletter of the National Network of Forest Practitioners*, *22*(3), 17-22.

Kusumaningsih, W., & Herman, T. (2018). Improvement Algebraic Thinking Ability Using Multiple Representation Strategy on Realistic Mathematics Education. *Journal on Mathematics Education*, *9*(2), 281-290.

Labaree, R. (2013). Organizing your social sciences research paper: Theoretical framework. *USC Libraries, University of Southern California*.

Lam, S. F. (2001). Educators' opinions on classroom observation as a practice of staff development and appraisal. *Teaching and Teacher Education*, *17*(2), 161-173.

Lassa, P. N., & Enoh, O. A. (2000). The nature and function of theoretical framework in educational research.

Laurens, T., Batlolona, F. A., Batlolona, J. R., & Leasa, M. (2017). How does realistic mathematics education (RME) improve students' mathematics cognitive achievement? *Eurasia Journal of Mathematics, Science and Technology Education*, *14*(2), 569-578.

Ledwith, M. (2007). On being critical: Uniting theory and practice through emancipatory action research. *Educational action research*, *15*(4), 597-611.

Lee, C. I. (2016). An appropriate prompts system based on the Polya method for mathematical problem-solving. *Eurasia Journal of Mathematics, Science and Technology Education*, *13*(3), 893-910.

Lemon, L. L., & Hayes, J. (2020). Enhancing trustworthiness of qualitative findings: Using Leximancer for qualitative data analysis triangulation. *The Qualitative Report*, *25*(3), 604-614.

Lerman, S. (2019). Researching Vygotsky, and Researching with Vygotsky in Mathematics Education. In *Technology and Psychology for Mathematics Education* (pp. 32-47).

Lewin, K. (1946). Action research and minority problems. *Journal of social issues*, *2*(4), 34-46.

Lewis, C. C., Perry, R. R., Friedkin, S., & Roth, J. R. (2012). Improving teaching does improve teachers: Evidence from lesson study. *Journal of teacher education*, *63*(5), 368-375.

Libby, J. (2017). *Math for Real Life: Teaching Practical Uses for Algebra, Geometry and Trigonometry*. McFarland

Lincoln, Y. S., & Guba, E. G. (1989). Ethics: The failure of positivist science. *The Review of Higher Education*, *12*(3), 221-240.

Lisarelli, G., & Baccaglini-Frank, A. (2021). On different strategies to solve problems involving algebraic expressions. In *9th ERME Topic Conference: Perspectives on conceptual understanding of flexibility and number sense in arithmetic*.

Lowrey, K. A., Hollingshead, A., Howery, K., & Bishop, J. B. (2017). More than one way: Stories of UDL and inclusive classrooms. *Research and Practice for Persons with Severe Disabilities*, *42*(4), pp.225-242.

MacDonald, C. (2012). Understanding participatory action research: A qualitative research methodology option. *The Canadian Journal of Action Research*, *13*(2), 34-50.

Machaba, F., & Mphuthi, G. (2016). *Grade 9 learners' conceptions of algebraic expression: a case study of one of the schools in the Mt Ayliff District.* Retrieved https://uir.unisa.ac.za/bitstream/handle/10500/22857/France%20Machaba,%20Gabriel%20Mphuthi.pdf?sequence=1

Magill, R. A. (1993). Modeling and verbal feedback influences on skill learning. *International Journal of Sport Psychology*.

Mahlomaholo, S. (2009). Critical emancipatory research and academic identity. *Africa education review*, *6*(2), 224-237.

Mahlomaholo, S. (2012). Grade 12 examination results' top 20 positions: the need for the creation of sustainable learning environments for social justice in all schools. *Journal for New Generation Sciences*, *10*(2), 46-62.

Makonye, J. (2015). Understanding of Grade 10 learner errors and misconceptions in elementary algebra. *Journal of Educational Studies*, *2015*(si-1), 288-313.

Makonye, J. P., & Khanyile, D. W. (2015). Probing grade 10 students about their mathematical errors on simplifying algebraic fractions. *Research in Education*, *94*(1), 55-70.

Marpa, E. P. (2019). Common Errors in Algebraic Expressions: A Quantitative-Qualitative Analysis. *International Journal on Social and Education Sciences*, *1*(2), pp.63-72.

Mazumdar, L. (2017). Mathematics Teaching-Balancing Abstract Verses Concrete. *Interwoven: An Interdisciplinary Journal of Navrachana University, 1*(2), 10-17.

Mbewe, T. L. (2013). *Misconceptions and errors in Algebra at grade 11 levael: The case of two selected secondary schools in Petauke District* (Doctoral dissertation). University of Zambia.

McGuire, J. M. (2011). Inclusive college teaching: universal design for instruction and diverse learners. *Journal of accessibility and design for all*, *1*(1), 38-54.

McGuire, J. M., & Scott, S. S. (2006). An approach for inclusive college teaching: Universal design for instruction. *Learning Disabilities Multidisciplinary Journal, 14*(1), 21.

Mcguire, J. M., Scott, S. S., & Shaw, S. F. (2006). Universal design and its applications in educational environments. *Remedial and special education*, *27*(3), 166-175.

Messinger-Willman, J., & Marino, M. T. (2010). Universal design for learning and assistive technology: Leadership considerations for promoting inclusive education in today's secondary schools. *NASSP Bulletin*, *94*(1), pp.5-16.

Meulenberg-Buskens, I. (2011). *Free attitude interview technique.* London: unpublished notes.

Mhakure, D., Jacobs, M., & Julie, C. (2014). Grade 10 students' facility with rational algebraic fractions in high stakes examination: Observations and interpretations. *Journal of the Association for Mathematics Education in South Africa*, *10*, pp.1-13.

Mhakure, D., Jacobs, M., & Julie, C. (2014). Grade 10 students' facility with rational algebraic fractions in high stakes examination: Observations and interpretations. *Journal of the Association for Mathematics Education in South Africa*, *10*, 1-13.

Moleko, M. M., & Mosimege, M. D. (2021). Flexible teaching of mathematics word problems through multiple means of representation. *Pythagoras-Journal of the Association for Mathematics Education of South Africa*, *42*(1), 575.

Moleko, M. M. (2014). *Enhancing the functionality of supplemental instruction for first-year mathematics students at a higher education institution* (Doctoral dissertation). University of the Free State.

Moleko, M. M. (2018). *A universal design for learning strategy to enhance the teaching of word problems in a multilingual mathematics classroom* (Doctoral dissertation). University of the Free State.

Moleko, M. M. (2021). Multiple Means of Engagement Strategies for Maximising the Learning of Mathematics in Pandemic-regulated Classrooms. *International Journal of Learning, Teaching and Educational Research*, *20*(8).

Morgan, D. L. (1996). Focus groups. *Annual review of sociology*, *22*(1), 129-152.

Morgan, D. L. (2002). Focus group interviewing. *Handbook of interview research: Context and method*, pp.141-159.

Mosia, M. S. (2016). *Improving teachers' technological pedagogical content knowledge for teaching Euclidean geometry using integrated information communication technologies software* (Doctoral dissertation). University of the Free State.

Muchoko, C., Jupri, A., & Prabawanto, S. (2019). Algebraic visualization difficulties of students in junior high school. *Journal of Physics: Conference Series, 1157*(3), 032108.

Mulungye, M. M. (2016). *Source of Students' Errors and Misconceptions in Algebra and influence of classroom Practice Remediation in Secondary Schools Machakos Sub-Country, Kenya*. Department of Educational Communication and Technology.

Murtaza, K. F. (2011). Quality Performance in Academic Programs. Retrieved http://www.piqc.edu.pk/casestudies/Khush_Funer_Murtaza_Quality_Performance_in_Academic_Programs_Education_Quality_Case_Study_PIQC.pdf

Ncube, M. (2016). *Analysis of errors made by learners in simplifying algebraic expressions at grade 9 level* (Doctoral dissertation). University of South Africa.

Ncube, M. (2021). *Exploring the effects of concept-based instruction in the teaching and learning of mathematics: a case of algebraic expressions* (Doctoral dissertation). University of South Africa.

Ndlovu, Z. A. (2015). *Exploring pre-service teachers' mental constructions of matrix algebra concepts: a South African case study* (Doctoral dissertation). University of KwaZulu-Natal.

Newton, P., & Burgess, D. (2016). Exploring types of educational action research: Implications for research validity. In *The Best Available Evidence* (pp. 33-46). Brill Sense.

Ng, S. F. (2022). The model method: Crown jewel in Singapore mathematics. *Asian Journal for Mathematics Education*, *1*(2), 147-161.

Nkoane, M. M. (2012). Critical emancipatory research for social justice and democratic citizenship. *Perspectives in Education*, *30*(4), 98-104.

Norman, Z. D. (2018). *Exploring the effect of service learning experiences on students' servant leadership and cultural sensitivity*. Gonzaga University.

Norton, S., & Irvin, J. (2007). A concrete approach to teaching symbolic algebra. In *Proceedings of the 30th annual conference of the Mathematics Education Research Group of Australasia* (Vol. 2, pp. 551-560).

Ntseane, P. G. (2011). Culturally sensitive transformational learning: Incorporating the Afrocentric paradigm and African feminism. *Adult Education Quarterly*, *61*(4), 307-323.

Nurkaeti, N. (2018). Polya's strategy: an analysis of mathematical problem solving difficulty in 5th grade elementary school. *Edu Humanities*| *Journal of Basic Education Cibiru Campus*, *10*(2), 140.

Nyumba, O., Wilson, T. K., Derrick, C. J., & Mukherjee, N. (2018). The use of focus group discussion methodology: Insights from two decades of application in conservation. *Methods in Ecology and evolution*, *9*(1), 20-32.

O'Leary, M., & Wood, P. (2017). Performance over professional learning and the complexity puzzle: lesson observation in England's further education sector. *Professional development in education*, *43*(4), pp.573-591.

O'Connor, K. A., Greene, H. C., & Anderson, P. J. (2006). Action Research: A Tool for Improving Teacher Quality and Classroom Practice. *Online Submission*.

O'rinov Nodirbek Toxirjonovich, M. A., & Abduvaliyevich, S. S. (2020). Mathematical modeling: from the classroom to the real world. *Solid State Technology*, *63*(2s).

Olds, B. M., Moskal, B. M., & Miller, R. L. (2005). Assessment in engineering education: Evolution, approaches and future collaborations. *Journal of Engineering Education*, *94*(1), 13-25.

Olsen, B. (2008). How reasons for entry into the profession illuminate teacher identity development. *Teacher education quarterly*, *35*(3), 23-40.

Olson, J. C., White, P., & Sparrow, L. (2011). Influence of lesson study on teachers' mathematics pedagogy. In *Lesson study research and practice in mathematics education* (pp. 39-57). Springer, Dordrecht.

Opdenakker, R. (2006). Advantages and disadvantages of four interview techniques in qualitative research. *Forum qualitative sozialforschung/forum: Qualitative social research, 7*(4).

Osei, C. M. (1998). *Student teachers' knowledge and understanding of algebraic concepts: The case of colleges of education in the Eastern Cape and Southern KwaZulu Natal, South Africa* (Doctoral dissertation). University of Witwatersrand.

Osei, W. (2020). *Algebra teaching knowledge of basic school mathematics teachers* (Doctoral dissertation). University of Cape Coast.

Padilla, R. V. (1993). Using dialogical research methods in group interviews. *Successful focus groups: Advancing the state of the art*, 153-166.

Patton, M. Q. (1990). *Qualitative evaluation and research methods*. SAGE Publications, inc.

Perry, R. R., & Lewis, C. C. (2009). What is successful adaptation of lesson study in the US? *Journal of Educational Change*, *10*(4), 365-391.

Planas, N. (2020). *Study of and work on mathematics teacher talk in classroom teaching-EUIN Working Paper.* University of Barcelona -GIPEAM

Planas, N. (2021). How specific can language as resource become for the teaching of algebraic concepts? *ZDM–Mathematics Education*, *53*(2), 277-288.

Planas, N., Morgan, C., & Schütte, M. (2018). Mathematics education and language: Lessons and directions from two decades of research. *Developing research in mathematics education*, 196-210.

Pramesti, T. I., & Retnawati, H. (2019, October). Difficulties in learning algebra: An analysis of students' errors. In *Journal of Physics: Conference Series* (Vol. 1320, No. 1, p. 012061). IOP Publishing.

Reed, R. A. (2010). *From learner algebraic misconceptions to reflective educator: three cycles of an action research project* (Doctoral dissertation). University of KwaZulu-Natal.

Reeves, D. B. (2008). *Reframing teacher leadership to improve your school*. ASCD.

Rice, P. L., & Ezzy, D. (1999). *Qualitative research methods: A health focus.* Oxford University Press.

Robertson, J. (2000). The three Rs of action research methodology; reciprocity, reflexivity and reflection-on-reality. *Educational Action Research*, 8(2), 307-326.

Canter, L. L. S., King, L. H., Williams, J. B., Metcalf, D., & Potts, K. R. M. (2017). Evaluating pedagogy and practice of universal design for Iearning in public schools. *Exceptionality Education International*, *27*(1).

Robson, M. (2002). *Problem-solving in Groups*. Gower Publishing, Ltd.

Rose, D. H., & Meyer, A. (2002). *Teaching every student in the digital age: Universal design for learning*. ASCD.

Ruiz-Martín, H., & Bybee, R. W. (2022). The cognitive principles of learning underlying the 5E Model of Instruction. *International Journal of STEM Education*, *9*(1), 21.

Ryan, K. E., Gandha, T., Culbertson, M. J., & Carlson, C. (2014). Focus group evidence: Implications for design and analysis. *American Journal of Evaluation*, *35*(3), pp.328-345.

Samuel, K., Mulenga, H. M., & Angel, M. (2016). An Investigation into Challenges Faced by Secondary School Teachers and Pupils in Algebraic Linear Equations: A Case of Mufulira District, Zambia. *Journal of Education and Practice*, *7*(26), 99-106.

Sanginga, P. C., Kamugisha, R. N., & Martin, A. M. (2010). Strengthening social capital for adaptive governance of natural resources: A participatory learning and action research for bylaws reforms in Uganda. *Society and Natural Resources*, *23*(8), 695-710.

Sax, C., & Fisher, D. (2001). Using qualitative action research to effect change: Implications for professional education. *Teacher Education Quarterly*, 71-80.

Sekonyela, L. (2021). Student Challenges with the University Access Program in South Africa. *Journal of Ethnic and Cultural Studies*, *8*(1), 239-270.

Seng, L. K. (2010). An error analysis of Form 2 (Grade 7) students in simplifying algebraic expressions: A descriptive study. *Electronic Journal of Research in Education Psychology*, *8*(20), 139-162.

Sfard, A. (1991). On the dual nature of mathematical conceptions: Reflections on processes and objects as different sides of the same coin. *Educational studies in mathematics*, *22*(1), 1-36.

Sfard, A. (1995). The development of algebra: Confronting historical and psychological perspectives. *The Journal of Mathematical Behavior*, *14*(1), 15-39.

Shangase, B. B. (2013). *Strategies for the implementation of further education and training learner attainment improvement plan* (Doctoral dissertation). University of the Free State.

Sheila, A. (2016). *Integrating Information Communication and Technology in Mathematics education at secondary level, a case of Nairobi County, Kenya* (Doctoral dissertation). Kenyatta University.

Shenton, A. K. (2004). Strategies for ensuring trustworthiness in qualitative research projects. *Education for information*, *22*(2), 63-75.

Shirali, S. A. (2014). George Pólya & problem solving... An appreciation. *Resonance*, *19*(4), 310-322.

Sider, S., Morvan, J., & Börner, M. (2021). Partnerships to support quality education in Haiti: a case study addressing the Sustainable Development Goals. *Compare: A Journal of Comparative and International Education*, 1-18.

Sitabkhan, Y., Davis, J., Earnest, D., Evans, N., Ketterlin-Geller, L., Lutfeali, S., & Srikantaiah, D. (2019). *Instructional strategies for mathematics in the early grades* (p. 4). A Working Paper Developed by the Mathematics Working Group.

Smith Canter, L. L., King, L. H., Williams, J. B., Metcalf, D., & Rhys Myrick Potts, K. (2017). Evaluating pedagogy and practice of Universal Design for Learning in public schools. *Exceptionality Education International*, *27*(1), 1-16.

Smith, D. (2003). Five principles for research ethics. *Monitor on psychology*, *34*(1), 56.

Stacey, K., & Chick, H. (2004). Solving the problem with algebra. In *The Future of the Teaching and Learning of Algebra The 12thICMI Study* (pp. 1-20). Springer, Dordrecht.

Star, J. R., Foegen, A., Larson, M. R., McCallum, W. G., Porath, J., Zbiek, R. M., & Lyskawa, J. (2015). *Teaching Strategies for Improving Algebra Knowledge in Middle and High School Students. Educator's Practice Guide. What Works Clearinghouse.* NCEE.

Tabach, M., & Friedlander, A. (2008). The role of context in learning beginning algebra. *Algebra and algebraic thinking in school mathematics*, *70*, 223.

Tall, D., & Thomas, M. (1991). Encouraging versatile thinking in algebra using the computer. *Educational studies in mathematics*, *22*(2), 125-147.

Terry P. B. (2007). Informed consent in clinical medicine. *Chest*, 131(2), 563-568.

Terry, G., Hayfield, N., Clarke, V., & Braun, V. (2017). Thematic analysis. *The SAGE handbook of qualitative research in psychology*, *2*, pp. 17-37.

Theobald, E. J., Hill, M. J., Tran, E., Agrawal, S., Arroyo, E. N., Behling, S., & Freeman, S. (2020). Active learning narrows achievement gaps for underrepresented students in undergraduate science, technology, engineering, and math. *Proceedings of the National Academy of Sciences*, *117*(12), 6476-6483.

Thompson, D. R., & Rubenstein, R. N. (2014). Literacy in language and mathematics: More in common than you think. *Journal of Adolescent & Adult Literacy*, *58*(2), 105-108.

Tirosh, D., Even, R., & Robinson, N. (1998). Simplifying algebraic expressions: Teacher awareness and teaching approaches. *Educational studies in mathematics*, *35*(1), 51-64.

Tlali, M. F. (2013). *Transformational learning of physical science through service learning for sustainability* (Doctoral dissertation). University of the Free State.

Tomal, D. R. (2010). *Action research for educators*. Rowman & Littlefield Publishers

Tongco, M. D. C. (2007). Purposive sampling as a tool for informant selection. *Ethnobotany Research and Applications*, *5*, 147-158.

Tsotetsi, C. T. (2013). *The implementation of professional teacher development policies: A continuing education perspective* (Doctoral dissertation)., University of the Free State).

Usry, J. (2021). *Algebra 1 Teachers' Perspectives of the Impact of Professional Development Training on Students' Academic Success* (Doctoral dissertation). Saint Leo University.

Van de Walle, J., Lovin, L. A. H., Karp, K. H., Jennifer, M., & Williams, B. (2014). *Teaching Student-Centered Mathematics: Developmentally Appropriate Instruction for Grades 3-5: Volume II*. Pearson Education Limited.

Van den Heuvel-Panhuizen, M., & Drijvers, P. (2020). Realistic mathematics education. *Encyclopedia of mathematics education*, 713-717.

Van Dijk, T. A. (2017). *Discourse and power*. Bloomsbury Publishing.

Wang, X. (2015). The Literature review of algebra learning: focusing on the contributions to students' difficulties. *Creative education*, *6*(02), 144.

Wardhani, T. A. W., & Argaswari, D. P. (2022). High school students' error in solving word problem of trigonometry based on Newman Error Hierarchical Model. *Infinity Journal*, *11*(1), 87-102.

Wicks, P. G., & Reason, P. (2009). Initiating action research: Challenges and paradoxes of opening communicative space. *Action Research*, *7*(3), 243-262.

Wijers, M. (2001). How to deal with algebraic skills in Realistic Mathematics Education. In *12th ICMI Study Conference: the future of the teaching and learning of algebra.* University of Melbourne.

Windsor, W. J. J. (2009). Algebraic thinking-more to do with why, than x and y. Proceedings of the tenth International Conference Models in Developing Mathematics Education. - Dresden : Hochschule für Technik und Wirtschaft, 2009.

Wotherspoon, T. (2002). The Dynamics of Social Inclusion. *Perspectives on Social Inclusion Working Paper Series*, 1-26.

Yıldız, D. G., & Akyüz, D. (2019). Examining Two Middle School Mathematics Teachers' Knowledge for Teaching Manipulation of Algebraic Expressions during Lesson Planning and Instruction. *Türk Bilgisayar ve Matematik Eğitimi Dergisi*, 1-1.

Yildiz, D. G., & Akyuz, D. (2020). Mathematical knowledge of two middle school mathematics teachers in planning and teaching pattern generalization. *Ilkogretim Online*, *19*(4).

Yildiz, P., Atay, A., & Çiftçi, S. K. (2020). Preservice Middle School Mathematics Teachers' Definitions of Algebraic Expression and Equation. *International Journal of Contemporary Educational Research*, *7*(2), 156-164.

York, J. (2021). The Relationship between Inclusive Classroom Practice and Student Self-Perception Outcomes.

Zayyadi, M., Nusantara, T., Hidayanto, E., Sulandra, I. M., & Sa'dijah, C. (2020). Content and pedagogical knowledge of prospective teachers in mathematics learning: commognitive framework. *Journal for the Education of Gifted Young Scientists*, *8*(1), 515-532.

APPENDICES

APPENDIX 1: UFS ETHICAL APPROVAL

UNIVERSITY OF THE FREE STATE
UNIVERSITEIT VAN DIE VRYSTAAT
YUNIVESITHI YA FREISTATA
UFS UV

GENERAL/HUMAN RESEARCH ETHICS COMMITTEE (GHREC)

03-Jun-2021

Dear Ms Sophie Musi

Application Approved

Research Project Title:
A Universal design for learning (UDL) in the teaching of algebraic expressions in grade 10

Ethical Clearance number:
UFS-HSD2020/2092/21

We are pleased to inform you that your application for ethical clearance has been approved. Your ethical clearance is valid for twelve (12) months from the date of issue. We request that any changes that may take place during the course of your study/research project be submitted to the ethics office to ensure ethical transparency. furthermore, you are requested to submit the final report of your study/research project to the ethics office. Should you require more time to complete this research, please apply for an extension. Thank you for submitting your proposal for ethical clearance; we wish you the best of luck and success with your research.

Yours sincerely

Dr Adri Du Plessis
Chairperson: General/Human Research Ethics Committee

205 Nelson Mandela Drive
Park West
Bloemfontein 9301
South Africa

P.O. Box 339
Bloemfontein 9300
Tel: +27 (0)51 401 9337
duplessisA@ufs.ac.za
www.ufs.ac.za

APPENDIX 2: TITLE CHANGE APPROVAL

UNIVERSITY OF THE FREE STATE
UNIVERSITEIT VAN DIE VRYSTAAT
YUNIVESITHI YA FREISTATA

UFS EDUCATION

17 June 2022

APPLICATION FOR TITLE REGISTRATION

Applicant: Musi, S
Student Number: 2013182882
Discipline: Mathematics Education
Study Code: Masters (EDMA8900)

Dear Ms Musi

Your request to change your title, was approved by the Committee for Title Registration. Your new title will be as follows:
"Exploring common algebraic expression challenges in a grade 10 mathematics classroom".

All of the best with your studies.

Yours sincerely,

Prof Patrick Mafora
Chair: CTR committee

Ms CS Duvenhage
Secretary: CTR committee

205 Nelson Mandela Drive | Park West, Bloemfontein 9301 | South Africa
P.O. Box 339 | Bloemfontein 9300 | South Africa | www.ufs.ac.za

APPENDIX 3: PERMISSION TO CONDUCT RESEARCH FROM FS DoE

education
Department of Education
FREE STATE PROVINCE

Enquiries: MZ Thango
Ref. Notification of research: S Musi
Tel. 082 537 2854
Email: MZ.Thango@fseducation.gov.za

District Director
Motheo District

Dear Mr. Molol

NOTIFICATION TO CONDUCT RESEARCH PROJECT IN YOUR DISTRICT BY S MUSI

The above mentioned candidate was granted permission to conduct research in your district as follows:

Topic: The teaching of algebraic expressions in grade 10: A Universal Design for a Learning approach.

1. **List of schools involved:** Louw Wepener Combined School.
2. **Target Population:** Forty Three Grade 10 Mathematics Learners, Four Grade 10 Mathematics Teachers, and Two FET Phase Mathematics Subject Advisors at the selected School.
3. **Period of research:** From the first week of February 2021 until 30 September 2021. Please note the department does not allow any research to be conducted during the fourth term (quarter) of the academic year nor during normal school hours. The researcher is expected to request permission from the school principals to conduct research at schools.
4. **Research benefits:** The value that the research may have for the Free State education department is that the application of universal design for a learning approach may improve learners' performance and the teachers will be exposed to a teaching strategy that will assist them to accommodate diverse needs of learners when teaching algebraic expressions in grade 10.
5. Strategic Planning, Policy and Research Directorate will make the necessary arrangements for the researchers to present the findings and recommendations to the relevant officials in the district.

Yours sincerely

Mr. M.M. Sithole
DDG: Corporate Services

15/01/2021
DATE:

RESEARCH NOTIFICATION: S. MUSI. 15 DECEMBER 2020. MOTHEO DISTRICT
Strategic Planning, Research & Policy Directorate Private Bag X20565, Bloemfontein, 9300 - Thuto House, Room 101, 1st Floor, St Andrew Street, Bloemfontein

www.fseducation.gov.za

Enquiries: MZ Thango
Ref: Research Permission: S Musi
Tel. 082 537 2654
Email. MZ.Thango@fseducation.gov.za

education
Department of Education
FREE STATE PROVINCE

1360 Jackob Street
Qibing
Wepener
9944

Dear Ms. S. Musi

APPROVAL TO CONDUCT RESEARCH IN THE FREE STATE DEPARTMENT OF EDUCATION

This letter serves as an acknowledgement of receipt of your request to conduct research in the Free State Department of Education

Topic: The teaching of algebraic expressions in grade 10: A Universal Design for a Learning approach.

1. **List of schools involved**: Louw Wepener Combined School.
2. **Target Population:** Forty Three Grade 10 Mathematics Learners, Four Grade 10 Mathematics Teachers, and Two FET Phase Mathematics Subject Advisors at the selected School.
3. **Period of research**: From the first week of February 2021 until 30 September 2021. Please note that the department does not allow any research to be conducted during the fourth term (quarter) of the academic year. Should you fall behind your schedule by three months to complete your research project in the approved period, you will need to apply for an extension. The researcher is expected to request permission from the school principals to conduct research at schools.
4. The approval is subject to the following conditions:

   4.1 The collection of data should not interfere with the normal tuition time or teaching process.

   4.2 A bound copy of the research document or a CD, should be submitted to the Free State Department of Education, Room 101, 1st Floor, Thuto House, St. Andrew Street, Bloemfontein.

   4.3 You will be expected, on completion of your research study to make a presentation to the relevant stakeholders in the Department.

   4.4 The ethics documents must be adhered to in the discourse of your study in our department.

5. Please note that costs relating to all the conditions mentioned above are your own responsibility.

Yours sincerely

Mr. M.M. Sithole
DDG: Corporate Services

15/01/2021

DATE:

RESEARCH APPLICATION BY S. MUSI, PERMISSION LETTER. 15 DECEMBER 2020 MOTHEO DISTRICT
Strategic Planning, Research & Policy Directorate Private Bag X20565, Bloemfontein, 9300 · Thuto House, Room 101, 1st Floor, St Andrew Street, Bloemfontein

www.fsdoe.fs.gov.za

APPENDIX 4: LETTER TO THE PRINCIPAL TO CONDUCT RESEARCH

The Principal

Dear Sir/Madam

Re: Application to conduct research in the school

I am a Masters student at the University of the Free State and I hereby request permission to conduct research in a school, in the Motheo Education District. The research will be in ‘ Action Research’ form at this schools and will last for three months. This will take place every week. My focus will be on the implementation of a universal design for learning in the teaching of algebraic expressions in grade10.

Yours sincerely,

S Musi

Researcher:	Principal:
Signature:………………………..	Signature:………………………..

APPENDIX 5: INFORMATION SHEET FOR ASSENT FROM PARENT/CHILD

Dear Participant

I am currently doing research with the University of the Free State on the study titled "Exploring Common Algebraic Expression Challenges in a Grade 10 Mathematics Classroom"

Since you are a Grade 10 learner studying mathematics, you are therefore requested to assist in this research in order to give it credibility. Participation is not compulsory and if you decide not to participate, that will not be held against you. Confidentiality, anonymity and legality issues about this project will be discussed with you, as it is imperative that you fully understand the nature and purpose of this study. You are free to withdraw from this study at any stage.

This project will comply with the rules and regulations of conducting a research.
If you would like any additional information, you are welcome to contact me on 0785841089 or at the following e-mail address: sophiamusi@gmail.com
If you would like to participate in this research, sign below by giving consent.
Thank you

____
S.Musi
Name ____
Signature ____
Date ____
Contact details

APPENDIX 6: INFORMATION SHEET FOR ASSENT FROM TEACHER

Dear Participant

I am currently doing research with the University of the Free State on the study titled “Exploring Common Algebraic Expression Challenges in a Grade 10 Mathematics Classroom”

Since you are an expert in teaching mathematics, you are therefore requested to assist in this research in order to give it credibility. Participation is not compulsory and if you decide not to participate, that will not be held against you. Confidentiality, anonymity and legality issues about this project will be discussed with you, as it is imperative that you fully understand the nature and purpose of this study. You are free to withdraw from this study at any stage.

This project will comply with the rules and regulations of conducting a research.
If you would like any additional information, you are welcome to contact me on 0785841089 or at the following e-mail address: sophiamusi@gmail.com
If you would like to participate in this research, sign below by giving consent.
Thank you

____

S.Musi

Name ____

Signature ____

Date ____

Contact details

APPENDIX 7: LESSON OBSERVATION FORM

Practices associated with models that inform the teaching of algebraic expressions			
Model	**Observed aspects**	**Yes/No**	**Notes**
Realistic Mathematics Education (RME)	• Reality principle • Level principle • Intertwinement		
Sfard's Theory	• Verbal description • Symbolic representation(e.g.$2y + 5$) • Generalised representation e.g. $ax^2 + ba + c$		
UDL (Multiple Means of Representation - MMR)	• Provide options for perception • Provide options for language, mathematical expressions, and symbols • Provide options for comprehension		
5E-Guided inquiry	• Engagement • Exploration • Explanation • Elaboration • Evaluation		
Modified Lesh's Model	• Concrete representation • Pictorial representation • Real-life experiences • Verbal symbols • Written symbols • Pictures in motion		
Variation theory	• Detect the common features of several examples related to the concept. • Contrast • Consider various solutions to a problem or various explanation for a phenomenon. • Associate the shape of the word with the sound and meaning of the word and		

	simultaneously manipulate the shape, sound, and meaning of the word.		
Polya's problem solving	• Understanding the problem • Devising a plan • Carrying out the plan • Answer verification		
Mathematical language	• Explanation of algebraic terms • Use of simple language to reinforce understanding • Using the appropriate mathematical terms/terminologies and mathematical language		
other			

APPENDIX 8: TRANSCRIPTS

Transcripts

Nna kannete ke latela mehlala ya textbook e be ke ba fa classwork hobane algebraic expression ya grade 10 e ngata ene pace setter e tla be e re o e etse in three weeks. ***(Mr Letuka)***

English translation (ET): The truth is, I just follow the textbook examples and give them classwork, because Grade 10 algebraic expression content is too much and on top of that, one is expected to finish it within three weeks.

'Hape bana ba na ba botsoa, ha o re oba fa mosebetsi baa complainer, ba bang o tla bona hore ba kgethile ho etsa Maths ka baka la ho matha ka mora bakgotsi. ***(Mme Ntlama)***
[ET: To add on that, these learners are lazy, because they complain when they are given some work to do. You can also notice that some have chosen to do mathematics, because they are influenced by their friends.]

Algebraic expression/equation lesson presented by Mr Letuka: 2 examples extracted from the lesson. *Mr Letuka: "…an algebraic equation has an equality sign whereas an algebraic expression does not have an equality sign therefore* $(2x - 7 = 2y$ *is an equation but* $3(x + 5)$ *is an expression".* *…* *Mr Letuka: When you are given an expression such as this one (pointing at the chalkboard) and you are required to simplify it. Then here you must determine the lowest common multiple. In order to do so, in this case you can multiply the first fraction by five over five, which is still the same as 1 and multiply the second fraction by 4 over 4, which is also the same as 1.* $\frac{3x}{4} + \frac{2x}{5}$

Mr Letuka: when asked to evaluate an expression such as $243^2 - 242^2$ *without the use of a calculator, learners work it out as follows:*

$243^2 - 242^2$

$= 24(3^2 - 2^2)$

$= 24(9 - 4)$

$= 120$

MMe Ntlama: kapa ha ba fuwe $29^2 - 27^2$ hona le hore ba sebedise factorization of difference of two squares ba etsa mosebetsi o mongata wa ho multipler ba be ba fose karabo. [ET: Or if they are given $29^2 - 27^2$ instead of applying factorization of the difference of two squares they work out the problem by using a lot of steps which sometimes yield a wrong answer.]

Mr Letuka: When you have given them an expression, they tend to cancel the terms and not the factors. For example,
$\frac{3x-15x}{3x}$ they will give an answer of $1 - 15x$.

Mme Ntlama: O bone ha a simplifaya $8a^2 + 3a^2$ a re answer ke $11a^4$ ka nako yeo ntse ke hlalositse hore re kopanya di coefficients feela e seng di exponents.
ET: You can see that when you ask them to simplify $8a^2 + 3a^2$ they give an answer of $11a^4$ even after I explained that they should only add coefficients and not the exponents.
Mr Letuka (adding to that): Or you say to them when you have like terms you should just add the coefficients.
Miss Ntlama: "…Sometimes we stick to one way of representing the concepts because we want to save time…".

Mr Letuka: O bone ha potso e re solve for x in the following equation
$\frac{4^x-1}{2^x-1} = 17$ ba sa kgone ho nahana hore the factorizing methods they learned in quadratic expression can be applicable.
ET: You will notice that when they are given a question like solve for x in the following equation $\frac{4^x-1}{2^x-1} = 17$ they are unable to think that factorization methods they learned during quadratic factorization can be applicable.

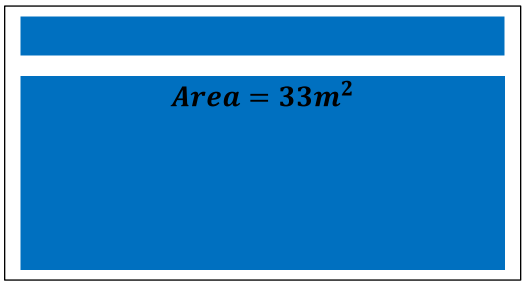
Algebraic expressions lesson presented by Mr Letuka

Mr Letuka: Now people let us do the following activity…

Activity 4.2.4

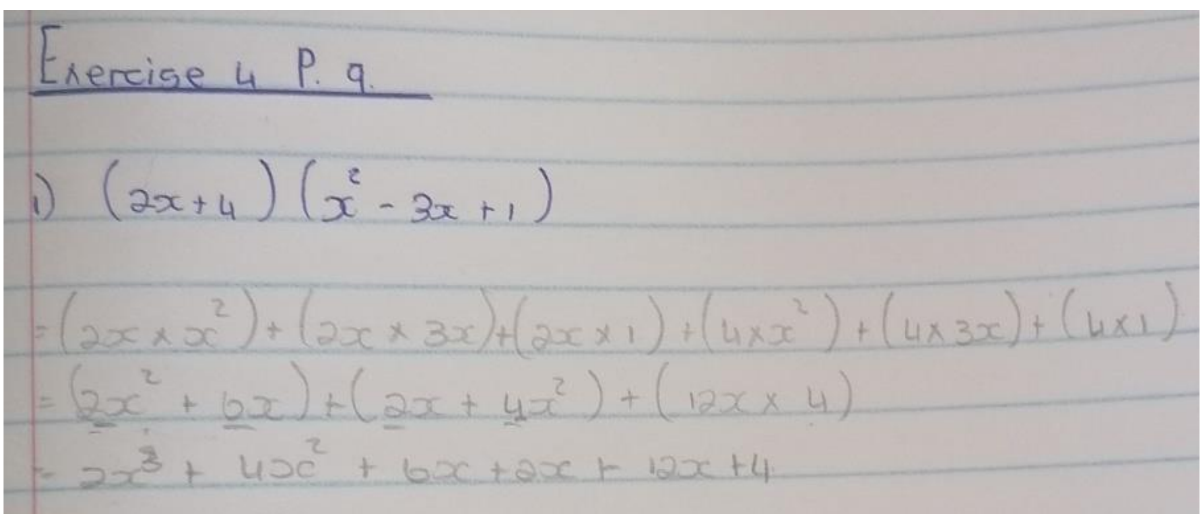
Determine the dimensions of the following rectangle

$x - 3$



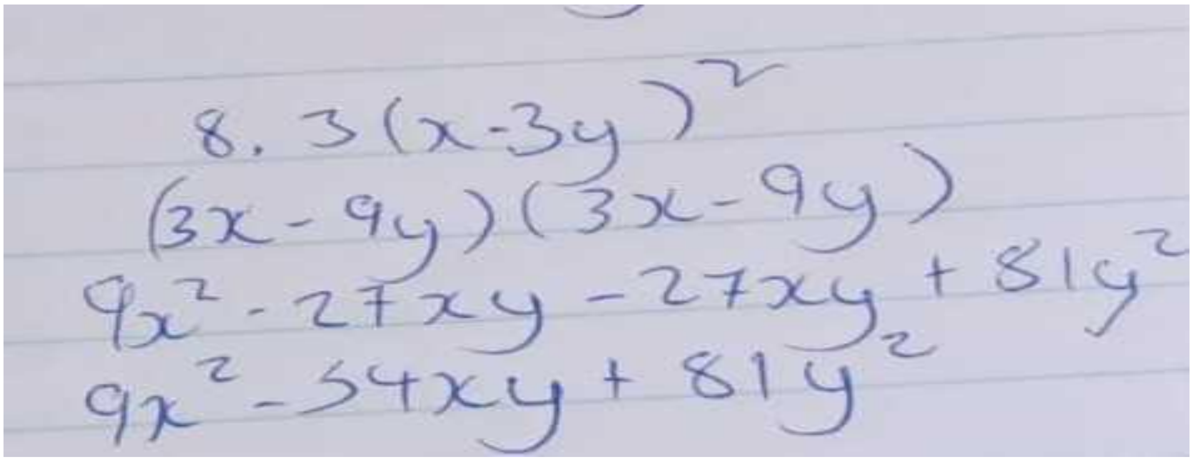
$x + 5$

Fig 4.2.4

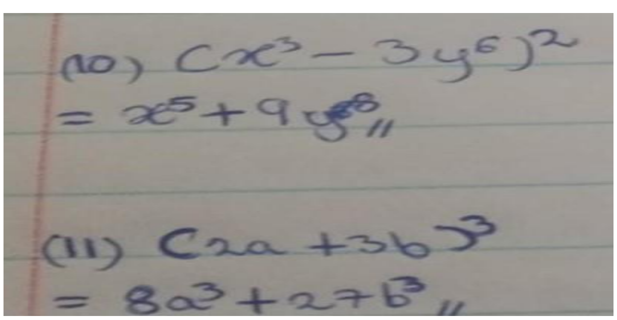


Researcher: "…You applied the distribution property well here, but you multiplied $2x$ by $3x$ not minus $3x$, why did you do that?"
Halieo (learner): Was I supposed to multiply by $-3x$?
Researcher: Yes, the negative sign is assigned to $3x$. The sign in front of a number is assigned to that number and wherever you carry that number you must also carry it.

Halieo: But other numbers I multiplied with them without their signs.
Researcher: Those numbers were positive, tell me, is $2x = +2x$*?*
Halieo: Yes, they are equal.
Researcher: Is, $3x\ is\ equal\ to - 3x$*?*
Halieo: No, $3x\ is\ not\ equal\ to - 3x.$
Researcher: So, you see you were supposed to multiply by $-3x$*?*



Researcher: Can you please take me through your first step in working out this.
Tefo: I multiplied by 3 inside the first bracket and the second bracket.
Researcher: why did you do that?
Tefo: Because we are asked to square.
Researcher: But the square applies only for the bracket. That is, $3(x - 3y)^2 = 3(x - 3y)(x - 3y)$
Tefo: Yes teacher, and that means I must multiply by 3 the first and the second bracket.



Researcher: I could notice that you squared the first and the last term in your workings, may you tell me why?
Lintle: It is because when we are given $(x^2)^3$ the answer is x^6.

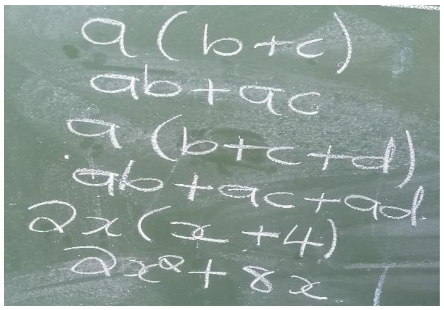
Mme Ntlama: Sometimes you find that learners are not able to leave an answer as a binomial expression like $x + 7$, but they want to leave it as $7x$.

Mme Ntlama: I am one of those teachers who refer to the exponent as the power,… when introducing exponents I would explain to learners that x^n is a power and x represents a base and n represents an exponent. However, when talking during instruction I normally say " to the power of n", and this is wrong and confuses learners.
Mr Letuka: Bona, fraction ke ena $\frac{2}{3}p$ $(pointing\ at\ the\ fraction)$ (two-thirds of p), some teachers read this as "two over three p" and this is confusing learners because even $\frac{2}{3p}$ is read as "two over three p"
ET: Look, here is a fraction, …

Mme Ntlama: Bothata mona ke hore bana ba na di concepts tse ba ithutileng with arithmetic numbers ba ba le bothata ba ho di sebedisa ha re fihla di variableseng. Mohlala feela, ha ba kopanya, batlosa, multiplication and division of algebraic fraction ba ba le bothata.
ET: The problem is that the learners have a problem of applying concepts they learned in arithmetic when now dealing with variables, for example when they add, subtract, multiply and divide algebraic fractions.
Mr Letuka: E ne nna ha ke tsebe hore na bothata bona ba bana ba hore ba se utlwisise hore ha ba adder, ba subtracta, ba multiplier ba bile ba divider ha ba fihla ho algebra na nka bo hlola ka ho etsa jwang hobane ha ke utlwisise na bo bakwa keng.
ET: So I do not know how can I overcome this problem of learners for not understanding how to carry on addition, subtraction, multiplication and division in algebra, because I do not understand what causes it.
Mme Ntlama: And this transition gap ke yona e etsang hore le ha ba fumana karabo e le $2x + 7$ ebe ba tswela pele hore ke $9x$ kapa $a + b$ ba re karabo ke ab.
ET: And this problem I think is the one that causes learners to continue with addition instead of leaving the answer as $2x + 7$ to say it is $9x$ or $a + b$ to be ab .

Mme Ntlama made examples for the learners, which included multiplication of a monomial and a binomial, as well as binomial and a binomial: the following pictures show what she did,

Monomial and binomial ***Product of two binomials***



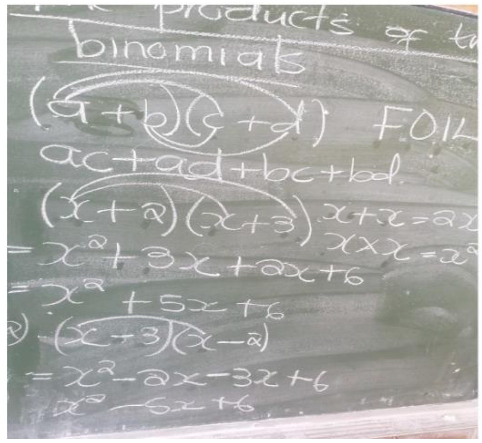
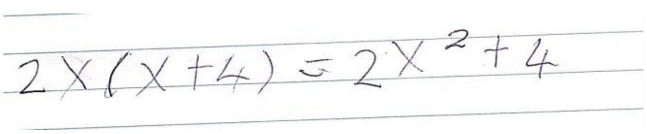
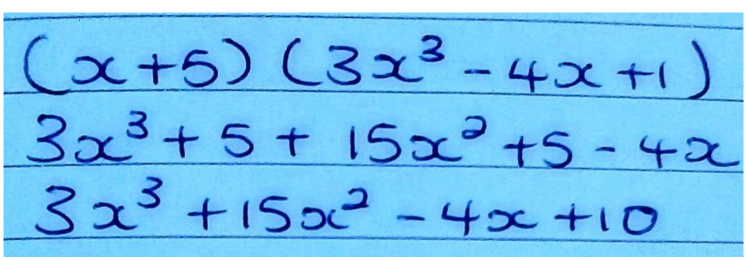


Figure 4.2.8.1: Mme Ntlama's illustrations of a monomial multiplied by binomial and binomial multiplied by binomial respectively





Researcher: Lerato, tell me, how did you work out this problem?

Lerato: I wanted to follow the example that we were given in class for using FOIL. I multiplied first, outer, inner and last values in that order.

Researcher: So, that is why you did not multiply the -4x by anything?

Lerato: Yes Ma'am, I just wrote it as it is, since I did not know what I should do with it.

Researcher: Ok, I see.

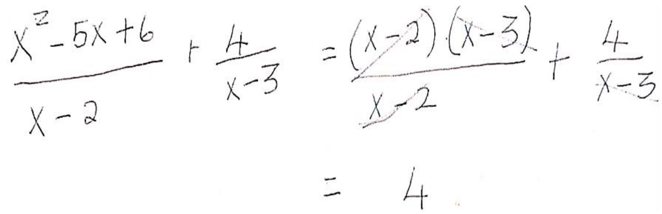
Mme Ntlama: We end up communicating wrong in the class because of trying to make learners perform well, for example, during factorization by grouping of $x(a-b)+y(b-a)$. I normally ask my students to switch and swap the signs for the second bracket as follows $x(a-b)-y(a-b)$ and then take out the common bracket to get $(a-b)(x-y)$. Then, you find that when they are supposed to work out $x(a+b)-y(b+a)$ they still want to apply switch and swap, then they get $x(a+b)+y(a+b)$ which is now incorrect.

Mr Letuka: Let me give you another example….when we have an expression like this $\frac{ab+b}{b} = a+1$ *other teachers say "you can see b on top and b below therefore all you have to do is to cancel the b's and get the answer as $a+1$". Therefore, when learners come across a situation like $\frac{a+b}{b}$ they give an answer as a because they have cancelled the b's which is now incorrect.*

$$\frac{x^2-4}{x+2} = \frac{x^2-2^2}{x+2}$$
$$= x-2$$

Researcher: You got the answer correct, but I cannot follow your method, would you please take me through your method?
Thabo: I first wrote 4 in exponential form. Then I cancelled the common factors. That is, I cancelled 2 from x^2 because x goes x times in x^2, and I also cancelled 2, because $4\ divide\ by\ 2\ = 2$. So after that I remained with the answer $x-2$.
Researcher: I see, but when you look at the numerator, what type of quadratic expression is that?
Thabo: It is a difference of two squares.
Researcher: Can you factorise it first?

Thabo: Yes, we can, the factors are $(x-2)(x+2)$. Now I see that if I divide this numerator by the denominator, I will get $x-2$ as an answer.



Researcher: can you explain to me how you cancelled in this addition of algebraic fractions.

Thomas: the common factor in the first fraction is $x-2$ and is cancelled. $x-3$ is a common factor between the first and second fraction, so I also cancelled it and obtained 4 as an answer.
Researcher: do you consider common factors between first and second fraction during addition or during multiplication?
Thomas: We consider them when we multiply.
Researcher: But here we are adding.
Thomas: Eh…no? My mistake……

Mme Ntlama: Sometimes learners do not understand what they asked to do when a problem is word problem, example “write an algebraic expression showing a product of two consecutive numbers”
Mr Letuka: Others have a problem of writing word problems into algebraic expressions, for example mathematical expressions like those indicated in the table below:

Table 4.2.10 showing some commonly used algebraic word expressions

	Word format	*Algebraic format*	*Learners’ understanding and representation*	
a)	*A number 4 less than x*	$(x-4)$	*4 is less than x*	$4<x$
b)	*A number 3 more than x*	$(x+3)$	*3 is more than x*	$3>x$

Mme Ntlama: nna ke dumela hore hona ho etswa ke hore le rona matichere hare ba ruti tsona hantle

ET: I belief that this is caused by us as teachers, because we do not teach them thoroughly.
Mr Letuka: Fluence in mathematics language is contributing so much to this regard, because you find that a learner does not know the meaning of some of the words in the problem.

Mr Ntlama: Maybe if the subject advisors can organise DH training workshops where they would be trained with teaching strategies, that will make it easier for them to deliver subject matter. And these DH then disseminate this information to the teachers they supervise, because they interact with them on daily basis.
Mr Letuka: Kapa bahle ba bitse matitjhere a Grade 10 ho ya ba hlahlella ka mekgwa e ka sebetsang ha bobebe.
ET: Or they can invite all Grade 10 teachers to the workshops to train them on strategies that they can use.
Mme Ntlama: Le ho attenda structures like PLC where teachers come together to discuss ideas and different approaches to deliver subject matter ho ka thusa.
(ET: And to attend…can help)

Mme Ntlama: To explain better, sometimes it is best to remind learners the concepts of addition and subtraction of fractions using numerical fractions before one can teach addition and subtraction of algebraic fractions.
Mr Letuka: I also think teachers need to be trained on how to explain and teach manipulation of algebraic expressions, because even if a teacher knows the content of algebra, sometimes it is not easy to teach it or make the learners gain conceptual understanding.
Mme Ntlama: Putting learners in groups so that they can share ideas with one another may also work.

Mr Letuka: Wa tseba ha e ne kare ha motho a ruta mohlomong a concept like difference of two squares a ba sa ntsa a bontsha bana hore even in numerical form it can be applied or any other form then it would be better.

ET: You know, when teaching a concept such as difference of two squares, one should show learners that even in numerical form it can be applied or any other form then it would be better.

Mr Letuka: mohlala (ET: for example) Factorise $4a^2 - 36$, $2^{2x} - 1$, $225 - 144$

Mme Ntlama: That is true, even in simplifying exponents, it would help to expose learners to application of difference of two squares.

Mr Letuka: For learners to understand the notion of variables and parameters it will be best if teachers may represent the information in pictorial form.
Mme Ntlama: Mohlala ,(ET: For example) the teacher can come to class already having drawn diagrams showing the effect in the change of values of a and b for $y = ax + b$ or include the use of technology, such as GeoGebra to display the effects of the parameters.
Mr Letuka: I also think when dealing with number patterns we can use them to reinforce the concepts of variable and parameter, because for the linear pattern $T_n = bn + c$, b and c are parameters, so changing their values produces different number patterns where n is a variable, so learners may be asked to investigate different number patterns.

Lesson presentation by Mme Ntlama (addressing the issue of parsing obstacle)

Please do the following activities…

2. Simplify the following;

a) $2x - (5x - 17)$

I notice that for question 2a) most of you left the answer as $14x$ instead of $17 - 3x$. you must remember that 17 and $3x$ are not like terms. For example, let us say 17 represent the total number of cupcakes in a bag (showing learners a plastic bag containing cupcakes) and 3 friends each gets an unknown (x) number of cupcakes from the bag, then how many cupcakes remain in the bag?

Figure 4.3.5.1 A bag cupcakes

Tebello: Ma'am I think the remaining cupcakes are $17 - 3x$, because we do not know how many each of the three friends got.

Mme Ntlama: Good, you are right, the remaining cupcakes are $17 - 3x$.

Lesson presented by Mr Letuka (Addressing the issue of expected answer obstacle)

Mr Letuka: Good people you can't say $2x + 7 = 9x$ *or* 9. *Remember that these two terms are said to be unlike terms, because 2x is a term made up of a constant 2 and a variable x, while 7 is a term made up of a constant number 7 only. Look at the following diagram and tell me your answer*

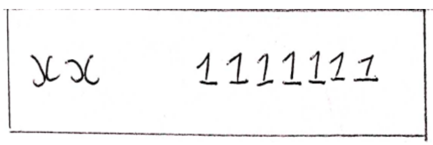


Figure 4.3.5.2 *Practical exercise used by Mr Letuka to address the concept of like and unlike terms*

Thato: It is $2x$ *and 7 ones*

Mr Letuka: So what is 7 ones?

Thato: It is 7. Ok …now I see, sir the answer is $2x + 7$

Lesson presented by Mme Ntlama (Like and Unlike terms)

Mme Ntlama: Let's say there are 2 bananas can be represented by $2x$ *and 7 apples by* $2y$, *now write down the sum of these fruits.*

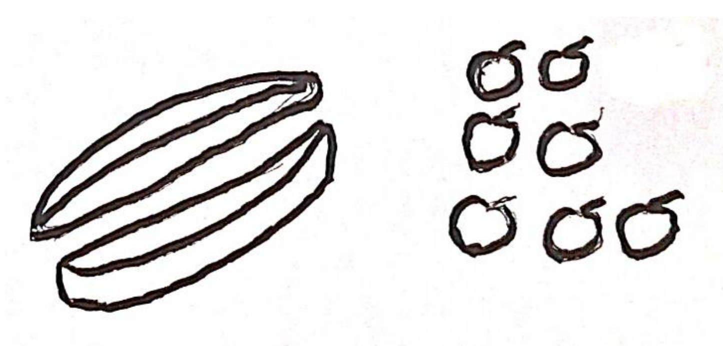


Figure 4.3.5.3 A diagram of fruit salad method

Halieo: My answer is $2x + 7y$

Mme Ntlama: Good Halieo, so why are you not saying it is $9xy$? I can see that Thomas has $9xy$.

Halieo: It is because we have 2 bananas and 7apples not 9 banana apples so we cannot write an answer as $9xy$. Therefore, the answer is $2x + 7y$.

Mme Ntlama: Now what about $2x + 7$? Can we simplify it any further?

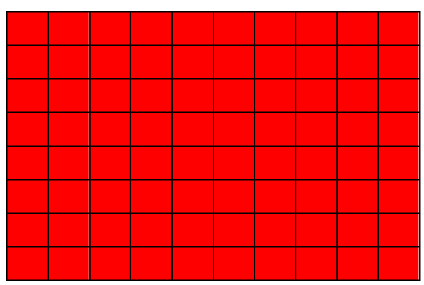
Thomas: No Ma'am, we can't simplify it any further. The answer is $2x + 7$, because $2x$ and 7 are unlike terms.

Algebraic expression lesson presentation by Mme Ntlama

Mme Ntlama provided learners with building blocks for which learners were asked to calculate the area. After reminding learners about the definition of area, Mme Ntlama asked the learners to calculate the area.

Mme Ntlama: Take the red building block (rectangle shape) and calculate its area.

8cm



10cm

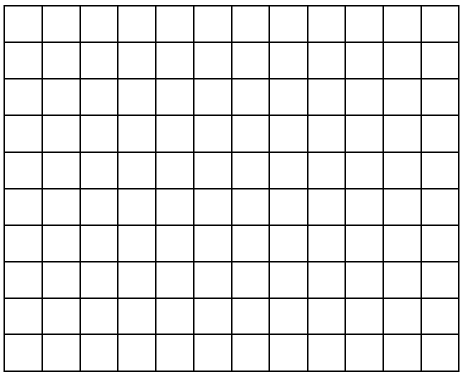
Keketso: When I count the number of squares, I get 80 squares, therefore the area is 80 square units.

Mme Ntlama: You are correct, now turn the building block over to the other side, you will find the dimensions there. Then use them and the formula for calculating the area of the rectangle to determine the area of that orange building block.

Vuyani: Ma'am, we get the answer by saying area of a rectangle equals length times breadth, which is 8 times 10 and we get 80 units square as an answer.($A = lb = 8 \times 10 = 80$*units square)*

Tankiso: So Ma'am it is the same as the answer we got for the other side.

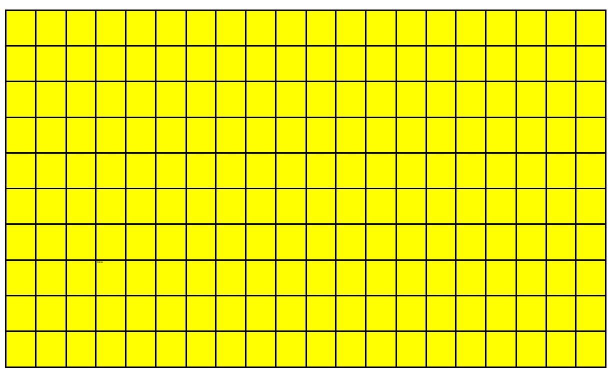
Mme Ntlama: Now let us find the area of the white and the yellow building blocks



10cm

12cm

10cm

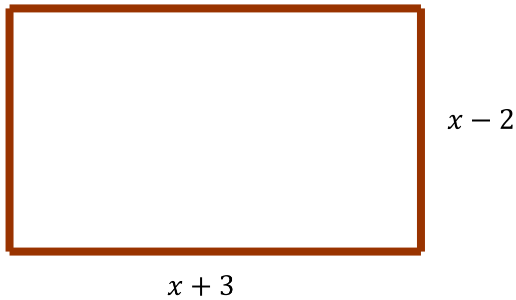


20cm

Mme Ntlama: ….. Now put the blocks together horizontally and find the area….

After discussing many ways that learners came up with in their arrangement of the building blocks and reaching the conclusion for the answer ….

Mme Ntlama: Now let us find the expression for the area of the rectangle given that the $width\ (w) = x - 2\ and\ length(l) = x + 3$



Mme Ntlama: What type of an expression is that? Monomial, binomial, or trinomial? And why?

Mme Ntlama: What is the difference between your answers for the first two rectangles and the last one?

Mme Ntlama: Classify the dimensions of the given rectangle into monomial, binomial and trinomial.

Mme Ntlama: Write down the factors of the expression $x^2 + x - 6$.

Mme Ntlama: Classify the following expressions into monomial, binomial and trinomial

($20a \div c$, $6y^3 - 4y \div 7$, $10p^2 + 5\frac{p}{2}$, $3x(x - 5) + 7$, $mn - 2n + 5m^2$)

Mme Ntlama: I think the use of real-life scenario would assist to close this arithmetic-algebra gap.

Mr Letuka: Le ho sebedisa di-manipulatives objects kapa tsona ditshwantsho can be very helpful.

ET: And the use of manipulatives, as well as pictures ….

Mr Letuka: I normally start by reminding learners about the fractions, how does it happen that we have a fraction, how do we add, subtract, multiply and divide numerical fractions before I can work with algebraic fractions with learners. For example, the fraction $\frac{2}{3}$ *may represent that out of three oranges, Tebello has eaten two oranges and the fraction* $\frac{5}{7}$ *may* *represent that out of seven oranges, Michael has eaten five oranges. Therefore, to find the fraction representing the total oranges eaten by both of them, we need to add the two fractions.*

Then after this, I show them that when adding algebraic fractions, we will still follow the same method as we did with numerical fractions.

E.g. ask learners to work out $\frac{2}{3}+\frac{5}{7}$ *and guide them to work out* $\frac{2}{x}+\frac{5}{y}$

Mr Letuka: Mohlomong Mme ha more experienced teachers can pair and share their expertise with novice teacher moo ho kgonehang.

ET: Perhaps if more experienced teachers can pair and share their expertise with novice teachers where it is needed.

Mme Ntlama: mokgwa o mong o ka sebetsang mona e kaba variation theory hobane e ruta matitjhere hore na ba ka kgetha jwang activities/ examples or exercises tseo bana ba ka di etsang.

ET: Another approach can be to use variation theory, because it teaches teachers how to select activities/examples or exercises they can use or assign to learners

Mr Letuka: I think if teachers can be organized some developmental workshops where teachers will be trained how to communicate properly with learners in the classroom, it will help.

Mme Ntlama: And the other thing can be the use of the lesson study, whereby teachers come together to plan a lesson, which will be presented by one of them.

Mr letuka: E ka thusa ntho yeo ya lesson study hobane after presentation then they can discuss how the presenter communicated with learners.

ET: "Lesson study can be helpful because…"

Mme Ntlama: Le matichere ka bo oona ba tshwanetse ho ipha nako ya ho itukisa hantle le puo ya bona ka classeng

ET: …And teachers must invest time to plan well their communication in the class.

In the first case I gave the learners the following statements to represent in algebraic format;

c) A number five less than x
d) A number two more than x

In the second case, I gave the learners the following scenario [pointing at figure 4.3.10] and asked them to represent it in an algebraic expression format:

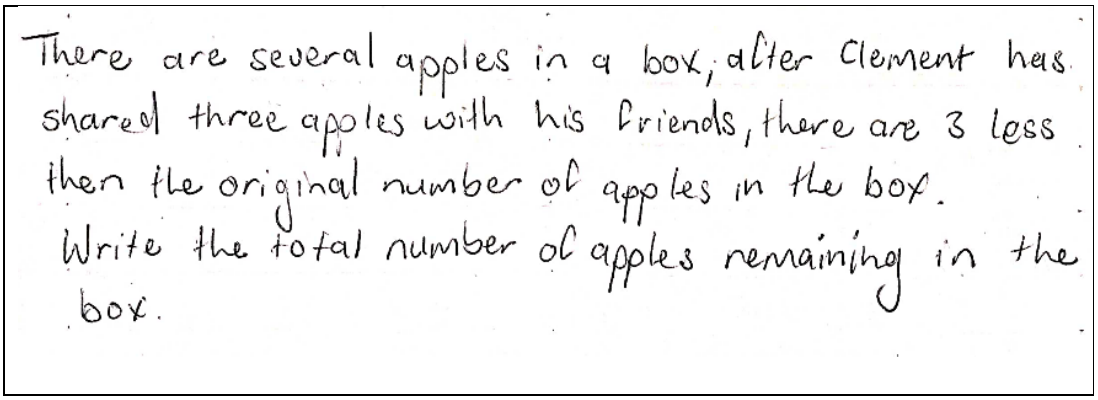

There are several apples in a box, after Clement has shared three apples with his friends, there are 3 less then the original number of apples in the box.
Write the total number of apples remaining in the box.

Figure:4.3.10 A word problem

A noticeable number of learners gave answers to the first case as:

c) $5 < x$
d) $2 > x$

In the second case, few of the learners wrote the answer as $3 < x$ *and the majority got the answer correct.*

I had to explain to the learners that for a) the correct answer is $x - 5$*, because this means that* x *is an '**unknown number of objects'** and **5** of them are '**removed**' or this **'unknown number of objects'** is '**reduced by 5'**. I also explained to them that for* $5 < x$ *the meaning here is that the number 5 is less than* x *and the question would have read 5 is less than* x*, so the word **'is**' guides us to compare both numbers and thus write the expression as* $5 < x$*. Then for b) the correct answer is* $x + 2$ *since in this case, the number* x *is '**increased**' by 2. I further asked learners who got the second case correct which talks about Clement and his friends while they got the answer wrong in the first case to explain why that happened. Then their answer was that in the second case it was very clear that they were asked to find the remaining apples after Clement and his friends ate 3 of them. So, for them it was easy to find the remaining apples to be* $x - 3$*. I asked them to identify similar parts for the first case and second case.*

APPENDIX 9: CERTIFICATE OF EDITING

Michelle Woolley

WRITER EDITOR PROOFREADER TRANSLATOR

Bachelor of Library and Information Science: B.Bibl.
Reference & Research Librarian
Bachelor of Arts **Honours** in **Translation Studies and Editing**

Associate Member of Professional EDITORS' Guild (PEG)

CERTIFICATE OF EDITING

This letter certifies that I have edited the Dissertation detailed below.

Title:

Exploring Common Algebraic Expression Challenges in a Grade 10 Mathematics Classroom

Author:

Sophie Musi

Regards
Michelle Woolley

Date: 18/01/2023

michellewoolley12@gmail.com
083 298 2077

Professional EDITORS' Guild

APPENDIX 10: TURN IT IN REPORT

Exploring Common Algebraic Expression Challenges in a Grade 10 Mathematics Classroom

ORIGINALITY REPORT

8%	5%	3%	5%
SIMILARITY INDEX	INTERNET SOURCES	PUBLICATIONS	STUDENT PAPERS

PRIMARY SOURCES

1	dergipark.org.tr Internet Source	<1%
2	www.researchgate.net Internet Source	<1%
3	Submitted to Bowling Green State University Student Paper	<1%
4	C Muchoko, A Jupri, S Prabawanto. "Algebraic visualization difficulties of students in junior high school", Journal of Physics: Conference Series, 2019 Publication	<1%
5	Submitted to Durban University of Technology Student Paper	<1%
6	Submitted to Midlands State University Student Paper	<1%
7	www.ejmste.com Internet Source	<1%

8	docplayer.net Internet Source	<1 %
9	researchspace.ukzn.ac.za Internet Source	<1 %
10	Michelle E. Kiger, Lara Varpio. "Thematic analysis of qualitative data: AMEE Guide No. 131", Medical Teacher, 2020 Publication	<1 %
11	www.tara.tcd.ie Internet Source	<1 %
12	Submitted to Hult International Business School, Inc. Student Paper	<1 %
13	www.teindia.nic.in Internet Source	<1 %
14	Submitted to Laureate Higher Education Group Student Paper	<1 %
15	Submitted to De La Salle University - Manila Student Paper	<1 %
16	Submitted to Universitas Pendidikan Indonesia Student Paper	<1 %
17	Paul Ernest. "The Knowledge, Beliefs and Attitudes of the Mathematics Teacher: a	<1 %

	model", Journal of Education for Teaching, 1989 Publication	
18	citeseerx.ist.psu.edu Internet Source	<1 %
19	Inge Bleijenbergh, Jorrit van Mierlo, Tanya Bondarouk. "Closing the gap between scholarly knowledge and practice: Guidelines for HRM action research", Human Resource Management Review, 2020 Publication	<1 %
20	Submitted to University of the Free State Student Paper	<1 %
21	Submitted to Helderberg College Student Paper	<1 %
22	edtechreview.in Internet Source	<1 %
23	repository.uinjkt.ac.id Internet Source	<1 %
24	journals.sagepub.com Internet Source	<1 %
25	ris.utwente.nl Internet Source	<1 %
26	www.coursehero.com Internet Source	<1 %

27	Submitted to Universiti Teknologi MARA Student Paper	<1%
28	scholarworks.uaeu.ac.ae Internet Source	<1%
29	www.scribd.com Internet Source	<1%
30	Submitted to University of Nottingham Student Paper	<1%
31	Submitted to American College of Education Student Paper	<1%
32	Submitted to Mancosa Student Paper	<1%
33	rune.une.edu.au Internet Source	<1%
34	core.ac.uk Internet Source	<1%
35	www.ukessays.com Internet Source	<1%
36	Submitted to University of Abertay Dundee Student Paper	<1%
37	Mustafa Guler, Derya Celik. "The Effect of an Elective Algebra Teaching Course on Prospective Mathematics Teachers' Pedagogical Content Knowledge",	<1%

	International Electronic Journal of Mathematics Education, 2021 Publication	
38	Submitted to Wright State University Student Paper	<1 %
39	Submitted to Middle East Technical University Student Paper	<1 %
40	Submitted to University of Sheffield Student Paper	<1 %
41	Submitted to University of the West Indies Student Paper	<1 %
42	ukzn-dspace.ukzn.ac.za Internet Source	<1 %
43	Submitted to Vaal University of Technology Student Paper	<1 %
44	W. D. KEAT. "MODELLING OF 3-D MIXED-MODE FRACTURES NEAR PLANAR BIMATERIAL INTERFACES USING SURFACE INTEGRALS", International Journal for Numerical Methods in Engineering, 11/15/1996 Publication	<1 %
45	theses.gla.ac.uk Internet Source	<1 %
46	www.enhancementthemes.ac.uk Internet Source	<1 %

47	Submitted to University of Cape Town Student Paper	<1 %
48	Submitted to Adtalem Global Education, Inc. Student Paper	<1 %
49	opensiuc.lib.siu.edu Internet Source	<1 %
50	stars.library.ucf.edu Internet Source	<1 %
51	Submitted to 61068 Student Paper	<1 %
52	Submitted to University of the Western Cape Student Paper	<1 %
53	eddieliku.blogspot.com Internet Source	<1 %
54	www.fortbendisd.com Internet Source	<1 %
55	www.studymode.com Internet Source	<1 %
56	Submitted to Oxford Brookes University Student Paper	<1 %
57	Submitted to Seattle Pacific University Student Paper	<1 %
58	Submitted to University of Hong Kong Student Paper	<1 %

59	Submitted to University of Sunderland Student Paper	<1%
60	www.khaleejtimes.com Internet Source	<1%
61	Submitted to Da Vinci Institute Student Paper	<1%
62	Theresia Laurens, Florence Batlolona, John Batlolona, Marleny Leasa. "How Does Realistic Mathematics Education (RME) Improve Students' Mathematics Cognitive Achievement?", Eurasia Journal of Mathematics, Science and Technology Education, 2017 Publication	<1%
63	project-house.net Internet Source	<1%
64	Submitted to Massey University Student Paper	<1%
65	Submitted to North West University Student Paper	<1%
66	Submitted to University of Central Florida Student Paper	<1%
67	Submitted to University of Witwatersrand Student Paper	<1%
68	Submitted to Washington-Lee High School Student Paper	

		<1%
69	etd.lib.metu.edu.tr Internet Source	<1%
70	etheses.bham.ac.uk Internet Source	<1%
71	Submitted to Sheffield Hallam University Student Paper	<1%
72	hdl.handle.net Internet Source	<1%
73	www.open.edu Internet Source	<1%
74	www.yumpu.com Internet Source	<1%
75	Submitted to Walden University Student Paper	<1%
76	ninetothrivedotnet.files.wordpress.com Internet Source	<1%
77	www.tandfonline.com Internet Source	<1%
78	Submitted to Kenyatta University Student Paper	<1%
79	Submitted to Newman College Student Paper	<1%

80	Ryan, Julie, Williams, Julian. "Children's Mathematics 4-15", Children's Mathematics 4-15, 2007 Publication	<1 %
81	Submitted to University of Bedfordshire Student Paper	<1 %
82	Submitted to University of Birmingham Student Paper	<1 %
83	Submitted to University of Limpopo Student Paper	<1 %
84	Wei Lu, Ying Fu. "Spectroscopy of Semiconductors", Springer Science and Business Media LLC, 2018 Publication	<1 %
85	1library.net Internet Source	<1 %
86	Submitted to De La Salle University Student Paper	<1 %
87	Katharine Javian. "Implementing Universal Design for Learning in Introduction to American Government", Cambridge University Press (CUP), 2023 Publication	<1 %
88	Submitted to Trinity Christian Academy Student Paper	<1 %

89	Submitted to University of Leeds Student Paper		<1 %
90	Submitted to University of Strathclyde Student Paper		<1 %
91	oaktrust.library.tamu.edu Internet Source		<1 %
92	repository.charlotte.edu Internet Source		<1 %
93	Anna Sfard. "On the dual nature of mathematical conceptions: Reflections on processes and objects as different sides of the same coin", Educational Studies in Mathematics, 1991 Publication		<1 %
94	Submitted to Loughborough University Student Paper		<1 %
95	Submitted to University of KwaZulu-Natal Student Paper		<1 %
96	Submitted to University of Venda Student Paper		<1 %
97	easierwithpractice.com Internet Source		<1 %
98	www.fisme.science.uu.nl Internet Source		<1 %

99	Submitted to University of Johannsburg Student Paper	<1 %
100	centaur.reading.ac.uk Internet Source	<1 %
101	dspace.lboro.ac.uk Internet Source	<1 %
102	repository.sustech.edu Internet Source	<1 %
103	Yu-Ling Tsai, Ching-Kuch Chang. "USING COMBINATORIAL APPROACH TO IMPROVE STUDENTS' LEARNING OF THE DISTRIBUTIVE LAW AND MULTIPLICATIVE IDENTITIES", International Journal of Science and Mathematics Education, 2008 Publication	<1 %
104	digitalcommons.wayne.edu Internet Source	<1 %
105	dokumen.pub Internet Source	<1 %
106	etheses.whiterose.ac.uk Internet Source	<1 %
107	rest.neptune-prod.its.unimelb.edu.au Internet Source	<1 %
108	scholarworks.waldenu.edu Internet Source	<1 %

109	timor-leste.iom.int Internet Source	<1 %
110	udsspace.uds.edu.gh Internet Source	<1 %
111	www.eera-ecer.de Internet Source	<1 %
112	www.essay.uk.com Internet Source	<1 %
113	www2.uwe.ac.uk Internet Source	<1 %
114	Kerry Lee, Swee Fong Ng, Rebecca Bull. "Learning and Solving More Complex Problems: The Roles of Working Memory, Updating, and Prior Skills for General Mathematical Achievement and Algebra", Elsevier BV, 2017 Publication	<1 %
115	Marja van den Heuvel - Panhuizen. "Can scientific research answer the 'what' question of mathematics education?", Cambridge Journal of Education, 2005 Publication	<1 %
116	Shankar Sankaran, Gita Sankaran. "Chapter 9 Thinking Inside the Box: Applying the Theory of Karma to Make Boundary Judgements in	<1 %

Systemic Interventions", Springer Science and Business Media LLC, 2017
Publication

117	bura.brunel.ac.uk Internet Source	<1 %
118	cdigital.uv.mx Internet Source	<1 %
119	erepository.uonbi.ac.ke Internet Source	<1 %
120	etheses.dur.ac.uk Internet Source	<1 %
121	laur.lau.edu.lb:8443 Internet Source	<1 %
122	partapcollege.org Internet Source	<1 %
123	ses.library.usyd.edu.au Internet Source	<1 %
124	ujcontent.uj.ac.za Internet Source	<1 %
125	www.thoengwit.net Internet Source	<1 %
126	Chien I Lee. "An Appropriate Prompts System Based on the Polya Method for Mathematical Problem-Solving", EURASIA Journal of	<1 %

Mathematics, Science and Technology Education, 2016
Publication

127	sadiwidodo.blogspot.com Internet Source		<1 %

Exclude quotes	On	Exclude matches	< 5 words
Exclude bibliography	On		