RISK EFFICIENCY OF OPTIMAL WATER ALLOCATION WITHIN A SINGLE AND MULTI-STAGE DECISION-MAKING FRAMEWORK

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DECLARATION

I, Primrose Madende, hereby declare that this dissertation submitted for the degree of *Magister Scientiae Agriculture* in the Faculty of Natural and Agricultural Sciences, Department of Agricultural Economics at the University of the Free State, is my own independent work, and has not been previously submitted by me to any other university. I also hereby cede copyright of this work to the University of the Free State.

Primrose Madende
April 2017
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DEDICATION

This thesis is dedicated to my beloved husband, son and my parents, Pauline and Walter Madende to whom I will always be grateful for this life opportunity and support.

I also dedicate this thesis to the aspiring girl child for every dream can become a reality if you just believe that "*it can be done*" and that we are all "*Masters of our Own Destinies"*

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"The real winners in life are the people who can look at every situation with an expectation that they can make it work or make it better."

Barbara Pletcher

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ABSTRACT

The main objective of this research was to compare the results obtained from modelling irrigation water allocation decisions within a single-stage decision-making framework with the results obtained within a multi-stage sequential decision-making framework under a full water quota and a restricted water quota.

A unified irrigation decision-making framework was developed to model the impact of the interaction between water availability, irrigation area and irrigation scheduling decisions as multi-stage sequential decisions on gross margin variability. An Excel ® risk simulation model that utilises evolutionary algorithms embedded in Excel® based on the Soil Water Irrigation Planning and Energy management (SWIP-E) programming model was developed and applied to optimise irrigation water use. The model facilitates the simulation of the economic consequences resulting from changes to the key decision variables that need to be optimised through gross margin calculations for each state of nature. Risk enters the simulation model as crop yield risk through different potential crop yields in each state of nature and stochastic weather which determines irrigation management decisions. Water budget calculations were replicated to include 12 states of nature within a crop rotation system of maize and wheat. The risk simulation model was applied in Douglas, a typical location of an irrigation farm.

The results showed improved risk management within a multi-stage decision-making framework as indicated by higher gross margins and reduced variability due to improved irrigation scheduling decisions under both a full and restricted water quota scenario. Close to potential yields, if not full potential yields were achieved within both decision-making frameworks. However, a significant reduction in per state irrigation water use resulted within a multi-stage decision-making framework sequentially resulting in improved gross margins. A full irrigation strategy with reduced areas was followed under a restricted water quota with reduced gross margins resulting owing to lower gross incomes. The resulting impact of risk aversion on gross margin risk was insignificant within a multi-stage decision-making framework, whilst a more evident impact within a single-stage decision-making framework was indicated by a significant increase in minimum gross margins.

The resulting monetary value of modelling irrigation decision within a multi-stage sequential decision-making framework was R11 149 and R14 413 under a full and restricted water quota respectively for a risk averse decision-maker. The resulting value of a multi-stage decision-making framework assuming risk neutrality was significantly lower at R4 261 and R7 019 for a full and restricted water quota respectively. Results indicate that the interaction between different decisions made at different times during the growing season as represented with a multi-stage decision-making framework, becomes much more important under restricted water supply conditions taking risk aversion into account.

The cost of a water restriction within a single-stage and multi-stage decision-making framework of R218 319 and R215 561 respectively resulted under a risk neutral framework. Under risk aversion, a slightly lower cost of a water restriction of R212 513 and R209 249 was generated within a single-stage and a multi-stage decision-making framework respectively. The lower costs for a water restriction within a risk framework owes to the fact that risk averse decision-makers already make conservative decisions hence a water restriction will have a relatively limited impact on such a decision-maker.

The overall conclusion is that, ignoring modelling irrigation decisions as sequential decisions within a multi-stage decision-making framework overlooks the risk reducing impact of the true nature of irrigation decisions. As a result, water use dynamics are not explicitly accounted for with the gross margin risk and the value of a water restriction over-estimated. The main recommendation from this research is hence that, agricultural water allocation policies should be formulated based on crop water optimisation models that consider the multi-stage decision-making framework within which irrigation decisions are made to ensure that the impact of any given policy on water use management is not over-estimated. Further research should focus on testing the global optimality of the solutions of the risk model with alternative evolutionary algorithm techniques and also reformulation of the model within a mathematical programming environment.

Key words: Single-stage decision-making framework, Multi-stage decision-making framework, water use dynamics, sequential irrigation decisions, simulation, evolutionary algorithms, water restriction, risk decision-making

CHAPTER 1
INTRODUCTION

1.1 BACKGROUND AND MOTIVATION

Agriculture is considered the foundation and one of the prominent pillars of developing economies, with South Africa (SA) not being an exception. Despite contributing only about 3% to total Gross Domestic Product (GDP) of SA, the success of the sector remains of paramount socio-economic significance in creating employment opportunities, earning foreign currency, social welfare, ecotourism and is considered a backbone of food security (Statistics South Africa, 2014). While maize and wheat production in South Africa is highly variable as the crops are produced under diverse environments, the harsh global El Nino conditions experienced during the 2015/16 planting season resulted in a devastating 50% reduction in South Africa's maize yield and area in comparison to the average of the past 5 years (USDA, 2016). Similarly, wheat production was also significantly low due to the El Nino–induced drought during the 2015/16 growing season. The drought has seen five of the nine provinces in South African declaring drought emergencies in 2016 coupled with the hiking of the maize price to record levels (Grain SA, 2016). The dwindling water resources owing to recurring droughts and erratic rainfall patterns renders the improvement of irrigation water management decisions greater priority given that agriculture consumes approximately 60% of SA's already scarce water resources (DWA, 2013). The sustainability of irrigation farming that is already under pressure due to the drought induced water restrictions is thus more accentuated.

Owing to the biological nature and climatic dependence of agriculture, irrigation farming is considered to be inextricably dependent on time and uncertain in nature (Blanco and Flichman, 2002). The interaction between different decisions made at different times during the growing season becomes much more important under limited water supply conditions. Crop type and area decisions are made at the beginning of the growing season when the climatic conditions of the entire growing season are still unknown to the decision-maker. For a given water availability scenario, the area decisions determine whether a deficit irrigation strategy will be followed as the area decision determines the amount of water that could be applied on a per hectare basis.

Irrigation water scheduling decisions on the other hand are made sequentially throughout the growing season as the uncertain weather conditions unfold given the crop area decision already made. The sequential decisions made by irrigation farmers facilitate the adjustment of irrigation water schedules for each consecutive stage depending on the currently prevailing weather conditions. Thus, the decision-maker is able to manage production risk by taking cognisance of new information from unfolding weather states. Research efforts by Botes, Bosch and Oosthuizen (1996) to evaluate the value of irrigation information for decision-makers under both limited and unlimited water supply conditions concluded that irrigation scheduling decisions improved as more irrigation information was taken into account, especially under limited water supply conditions.

Factors other than water availability and crop water demand may further complicate irrigation water allocation decisions (Venter, 2015). The introduction of time of use (TOU) tariffs forces decision-makers to consider improving their decision-making to reduce their irrigation costs. Irrigation water allocation is based on the marginal factor cost (MFC) of an input and the TOU nature of the Ruraflex electricity tariff implies that the MFC of using an additional unit of electricity will be different for different times of the day and days of the week. Multi-stage sequential decision-making thus enables irrigation farmers to incorporate such exogenous factors into their decisions.

The question, however, is not whether irrigators should adopt a sequential decision-making framework or not. Rather, the problem is that currently applied methodologies to model irrigation water allocation decisions do not acknowledge the fact that multi-stage sequential decisions allows irrigators to manage their risks better. Consequently, researchers might over-estimate the impact of water restrictions since their modelling framework does not allow for irrigation water allocation decisions to be made within multiple-stages throughout the growing season. Representing the true nature in which irrigation farmers make decisions is complex as you have to consider the impact of irrigation water allocation decisions on the stock of field water supply dynamically throughout the growing season. The latter mentioned necessitates the inclusion of daily water budget calculations. Assumptions also need to be made on the available information on which irrigation water allocation decisions are based. Typically, such information is not certain hence the inclusion of risk into the analyses is imperative.

1.2 PROBLEM STATEMENT AND OBJECTIVES

Irrigation water allocation decisions at farm-level are currently modelled within a single-stage decision-making framework and therefore misrepresents the actual manner in which irrigators make irrigation water allocation decisions in reality. The unavailability of a modelling framework that represents irrigation decisions within a multi-stage decision-making framework results in researchers, water managers at water user associations and policy makers being unsure of the impact of better representing irrigation water allocation decisions on the main decision variables and hence the value of limited water resources. Consequently, decision support under limited water supply conditions is hampered.

Considerable research efforts have been commissioned in South Africa on crop water use management under both limited and unlimited water supply conditions. Botes *et al*., (1996*)* applied a Simulation-Complex (SIMCOM) model to determine the value of irrigation information for decision-makers with neutral and non-neutral risk preference under both limited water supply and unlimited water supply conditions. Results indicated that risk attitudes have an impact on the expected yields and the amount of irrigation water applied. However, the interaction between crop, area planted and water availability on the ability to supply enough irrigation water on a per hectare basis to produce a non-stressed crop was assumed away by keeping area irrigated constant.

Grové and Oosthuizen (2010) developed an expected utility optimisation model to optimise water allocation between multiple crops under stochastic weather conditions. The decision variables of the model include choice of crop type, area planted and irrigation schedule. Multiple irrigation schedules (1296) were included into the optimisation model for each crop in an effort to consider the intra-seasonal dynamics of water allocations within a multi-crop setting. Only three different states of nature were included in the model to reduce the dimensionality problem. Similarly, several research efforts on deficit irrigation (DI) accentuated on how the level of risk aversion will determine the level of DI preferred by an irrigator (Botes, 1990; Grové *et al.,* 2006; Grové, 2006). However, none of the research considered sequential decision-making resulting in misrepresenting the risk framework irrigators make decisions with dynamics of water use only being approximated or overlooked.

Stochastic dynamic programming (SDP) is a frequently preferred method by international researchers (Rhenals and Bras, 1981; Bryant, Mjelde and Lacewee, 1993; Bras and Cordova, 1981; Burt and Staunder, 1971; Kennedy, 1988; Alamdarlo, Ahmadian and Khalilian, 2014) to represent the dynamic nature of irrigation water allocation decisions. Locally Gakpo, Tsephe, Nwonwu and Viljoen (2005) used SDP to optimise irrigation water allocation under a capacity sharing (CS) arrangement. A linear programming (LP) model was firstly used to optimize farm water use during the immediate season. The gross margins calculated from the LP model were then inputted in the SDP to optimize the water use in storage in the farmers' CS over the entire planning horizon. The marginal value product of water determined with the SDP model hence indicated the value of an additional unit of water to be used in the future contrary to the value of value of an additional unit of water to be used immediately determined with the LP model. SDP facilitated inter-year irrigation water allocation decision-making over a number of years depending on the states of water availability. The SDP model was used to optimise water quantity and select the best water management strategy with the aim of maximizing expected gross margin over the entire planning zone. The researchers however were unable to include area decisions for multiple crops as the area under production was predetermined. Also, the number of states included in the model was limited to reduce the dimensionality problem with no short term sequential decision such as weekly decisions considered.

Recently Venter and Grové (2016) demonstrated the use of non-linear programming to optimise inter-seasonal water allocation between two crops taking cognisance of time of use electricity tariffs. The research was the first of its kind to successfully account for water dynamics through the optimisation of a daily water budget using non-linear programming. The model allows for changes in irrigation area and daily irrigation amounts to optimise the inter-seasonal water allocation. The daily water budget was also linked to an electricity energy accounting module to enable an evaluation of the financial implications of considering time of use electricity tariffs. Risk neutrality was assumed since the model would become too complex and unable to overcome the exorbitant computational requirements that render infeasible the optimal solution. Furthermore, the model is structured within a single-stage decision-making framework where area and irrigation scheduling decisions are made within the same timeframe. Global optimality of the solutions could not be guaranteed as the solver is only able to find local optima.

Haile (2017) showed that complex simulation models could be solved using evolutionary algorithms (EA). The advantage of EAs is that the complexity of the model does not render the solution infeasible. However, EAs do not guarantee optimality but near optimal solutions.

The review of South African literature shows that no unified framework exists within a South African context to model the interaction between water availability, irrigation area and irrigation scheduling decisions as multi-stage sequential decisions.

The main objective of this research is to compare the results obtained when modelling irrigation water allocation decisions within a single-stage decision-making framework with the results of a multi-stage sequential decision-making framework under a full water quota and a restricted water quota. Comparing the results of the two decision-making frameworks for the two alternative water quotas will allow for the determination of the impact of modelling irrigation water allocation decisions within a multi-stage sequential manner on:

- Total gross margin risk and irrigation management decision variables (irrigated areas and irrigation water use) under a full and restricted water quota.

  The results from an optimisation model cast within a single-stage decision-making framework will provide the area irrigated, irrigation schedule and associated irrigation costs that will maximise the certainty equivalent given any state of nature could unfold. The results will be used together with the crop yields in each state of nature to determine the distribution of gross margin variability. Subsequent optimisations will alter the single-stage irrigation schedule on a weekly basis given a specific state of nature is unfolding and the future weather variability is risky. The area irrigated determined during the first stage together with the state specific irrigation schedules, associated costs and crop yields will be used to determine the distribution of gross margin variability for the multi-stage sequential decision-making framework.

- The monetary value that will result from the improved modelling of irrigation water allocation decisions for risk averse decision-makers under a full and restricted water quota.

  The monetary value of the multi-stage sequential decision-making framework was calculated as the difference in the certainty equivalents (CE) generated within a single-stage decision-making framework to that generated within a multi-stage decision-making framework.

- The monetary cost of valuing the impact of restricted water use resulting from ignoring the improved modelling of irrigation water allocation decisions for risk averse decision-makers. The monetary cost of restricted water use is calculated as the difference in the CE generated under a full water quota and a restricted water quota for risk averse decision-makers.

1.3 STUDY AREA

The research was conducted in Douglas, a town situated close to the convergence of the Vaal and Orange Rivers in the Northern Cape Province. Douglas is a typical location of an irrigation farm where farmers source irrigation water from the Vaal River and Orange River. Douglas receives an average rainfall of approximately 211mm per annum with most rainfall occurring mainly during autumn. The semi-arid and arid environment leads to the reliance on irrigation farming along the river's fertile lands which supports the production of quality agricultural products. Maize and wheat are dominant crops that are planted under irrigation under seasonal crop rotation systems. The farming units in the Northern Cape significantly vary in size with a typical irrigation farm of approximately 412 hectares (BFAP, 2012). Douglas receives the lowest rainfall (0mm) in June and the highest (57mm) in March. The monthly distribution of average daily maximum temperatures shows that the average midday temperatures for Douglas range from 18.4°C in winter and increase to 32.9°C in summer. Clovelly and Hutton soils are the two main types of soil found in the district.

1.4 STUDY OUTLINE

The thesis is organized in five main chapters. The first chapter presents the background of the study and a motivation on why the study is relevant. The problem statement was constructed from this background enabling the researcher to design objectives of the study. The following chapter, chapter 2, provides a review of the components of the soil water budget and the relationship of the components with crop water stress and crop yield. Thereafter, a discussion of the currently existing solution procedures to solve dynamic irrigation water allocation problems is provided. Chapter 3 describes the research model developed in terms of its formulation and application and sources of data. Chapter 4 presents the researcher's findings and conclusions from the analysis done. Finally, chapter 5 outlines the summary and recommendations based on findings of the research.

CHAPTER 2
LITERATURE REVIEW

2.1 INTRODUCTION

Chapter 2 commences with an overview of how crop water use relates to the soil water budget status. A discussion of the components of a soil water budget and how these components relate to crop moisture stress and crop yield is provided. The subsequent section classifies dynamic problems and discusses the available solution procedures for irrigation water use dynamic problems. Thereafter, a discussion of the implication of energy accounting on dynamics of irrigation water scheduling is provided. The final section of the chapter discusses dynamic modelling applications in South Africa.

2.2 CROP WATER USE

Soil water availability to crops is dependent on the water status of the soil water budget. Knowledge of the status of the soil water budget at any given crop growth stage is hence critical to schedule the timing and amount of irrigation events to avoid crop moisture stress. Accounting for the daily state of the soil water balance requires an assessment of the incoming and outgoing water flux into the root zone of the soil daily. The following section discusses the components of the root zone soil water budget and how these components relate to crop water stress and crop yields.

2.2.1 SOIL WATER BUDGET COMPONENTS

Components of the soil water budget can be represented by means of a container whose water content fluctuates depending on water inflows into the container and outflows from the container. Figure 2.1 adopted from Allen, Pereira, Raes and Smith (1998) represents the components of a soil water budget. Evapotranspiration, run-off and deep percolation represent the outgoing water flux from the root zone while irrigation, rainfall and capillary rise represent incoming water flux into the root zone as represented with arrows in Figure 2.1.

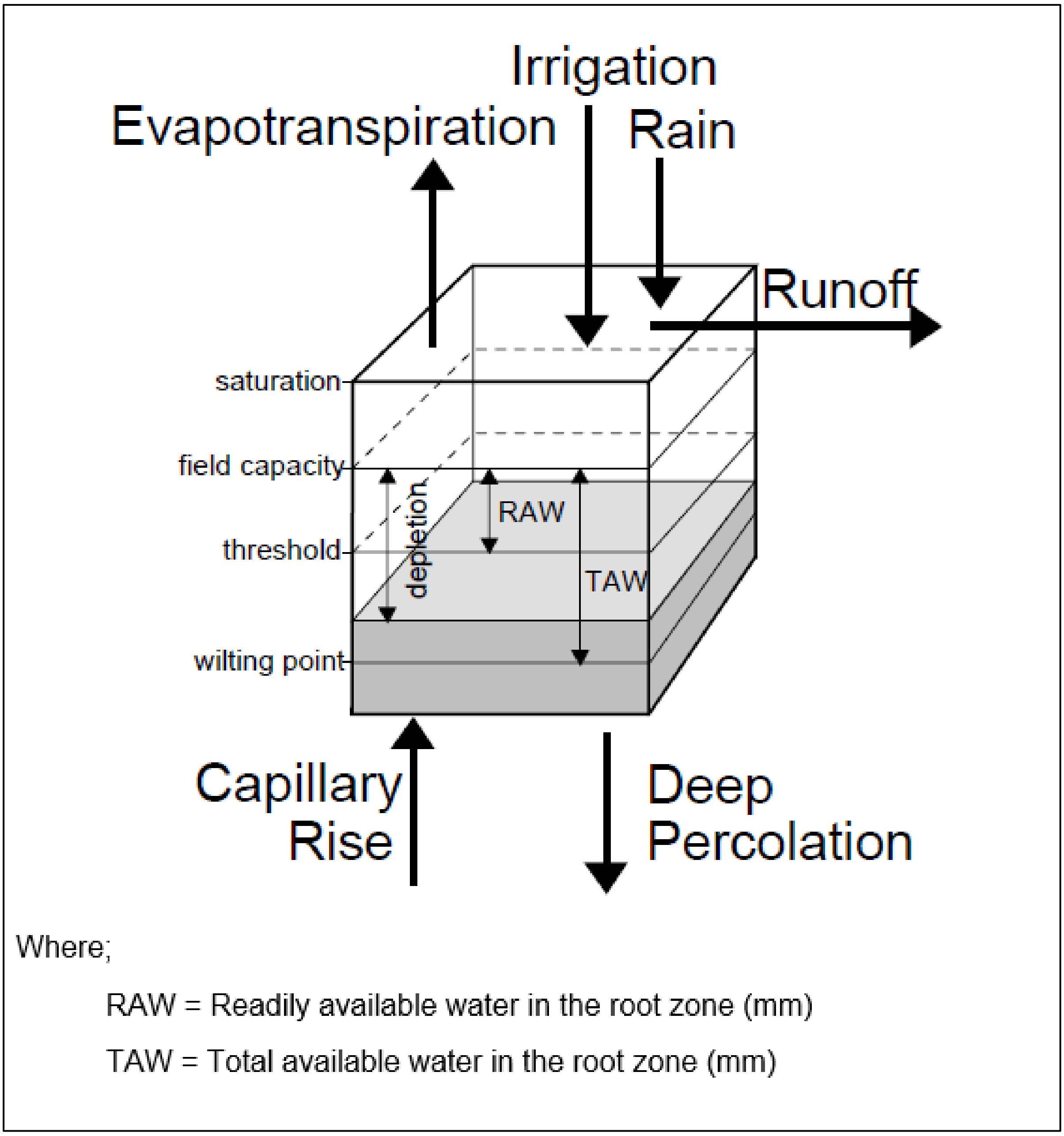

Figure 2. 1: Soil water budget components representing water fluxes within the root zone

The upper limit of soil water budget is referred to as field capacity (FC). Soil water content at FC represents the total water available (TAW) in the root zone that can be potentially utilised by crops. TAW hence represents the root zone water holding capacity (RWCAP) which determines the maximum amount of water that can be contained in the root zone. Water content might however temporarily exceed RWCAP following a rainfall or irrigation event as indicated by the saturation water content level. In such instances, soil water is assumed to be lost through evapotranspiration, deep percolation beneath roots and surface run-off to adjust soil water content to FC (Allen *et* al., 1998). As long as soil water content is below RWCAP, no water is lost from the soil through deep percolation and run-off. If water uptake by crops progresses without water deficits being replaced through irrigation or rainfall, the lower limit of soil water content known as the wilting point (WP) is reached where crops can no longer uptake any water from the soil.

Theoretically, water in the soil is available for plant uptake until WP. However, the rate of uptake of the water from the soil by the crops decreases as actual root water content (RWC) drops below a certain level of the TAW resulting in crops experiencing water stress before WP. Readily available soil water (RAW) hence represents an average fraction of the TAW that is easily extractable from the root zone by crops before experiencing water stress. Soil water stress conditions are induced as soon as RWC depletes below a threshold level where RAW is depleted. The daily state of the water budget can hence be expressed in terms of soil water depletion at the end of each day according to the following equation;

$$Dr_i = Dr_{i-1} - (R - RO)_i - IR_i - CR_i + ETa_i + BR_i \qquad \text{Equation (2.1)}$$

Where;

$\mathrm{Dr_i}$ Root zone depletion at the end of day *i* (mm)

$\mathrm{Dr_{i-1}}$ Root zone water content at the end of the previous day *i*-1 (mm)

R_i Rainfall on day *i* (mm)

RO_i Soil surface run-off on day *i* (mm)

IR_i Irrigation application on day *i* (mm)

CR_i Capillary rise due to root development on day *i* (mm)

ETa_i Actual crop evapotranspiration on day *i* (mm)

BR_i Water draining below the root zone by deep percolation on day *i* (mm

Root zone depletion determine root water shortages relative to the FC. The minimum root zone depletion is hence zero when soil water content is at FC. As water content in the root zone depletes as out fluxes offset influxes, the root zone depletion increases and reaches its maximum when no water is extractable from the soil through evapotranspiration. Root zone depletion can therefore not exceed TAW.

Irrigation events are scheduled when or before the RAW is depleted to compensate water depletions and increase RWC above the threshold level to avoid crop water stress. The root zone depletion should be equal or less than RAW to facilitate crop water stress-free conditions. An irrigation event scheduled in one stage will hence affect the state of the water budget in the next stage. Irrigation application should however not exceed root zone depletion to avoid deep percolation or run-off which has a negative implication on resulting irrigation costs. Water deficits in crops and the resulting water stress on plants influences crop evapotranspiration and crop yield (Kallitsari, Georgiou and Babajimopoulos, 2011). The following section discusses the relationship of the soil water budget components to soil water stress conditions.

2.2.2 SOIL WATER STRESS

Crop moisture stress is induced under non-standard conditions when root zone depletion exceeds RAW ($Dr > RAW$). Water stress conditions limit the amount of water lost from the root zone through evapotranspiration resulting in ETa reducing below potential or maximum levels (ETm). The magnitude of crop water stress can hence be quantified by assessing the extent by which ETa falls short of ETm (Rao, Sarma and Chander, 1988; Kallitsari *et al.*, 2011). The reduction of ETa under water stress conditions can be represented by a crop stress coefficient Ks. Ks is calculated using the following equation (Allen *et al*., 1998);

$$K_s = \frac{TAW - Dr}{TAW - RAW} \qquad \text{Equation (2.2)}$$

Where;

TAW Total Available water in the root zone (mm)

Dr Root zone depletion (mm)

RAW Readily available water in the root zone (mm)

Ks represents a non-dimensional transpiration reduction factor between zero and one that is dependent on the amount of water available in the root zone. If the actual RWC ($TAW - Dr$) is more than the threshold soil water content level before RAW is depleted, Ks is equal to one. Ks reduces as actual RWC falls below RAW. Figure 2.2 indicates the effect of soil water stress on ETa as represented by Ks.

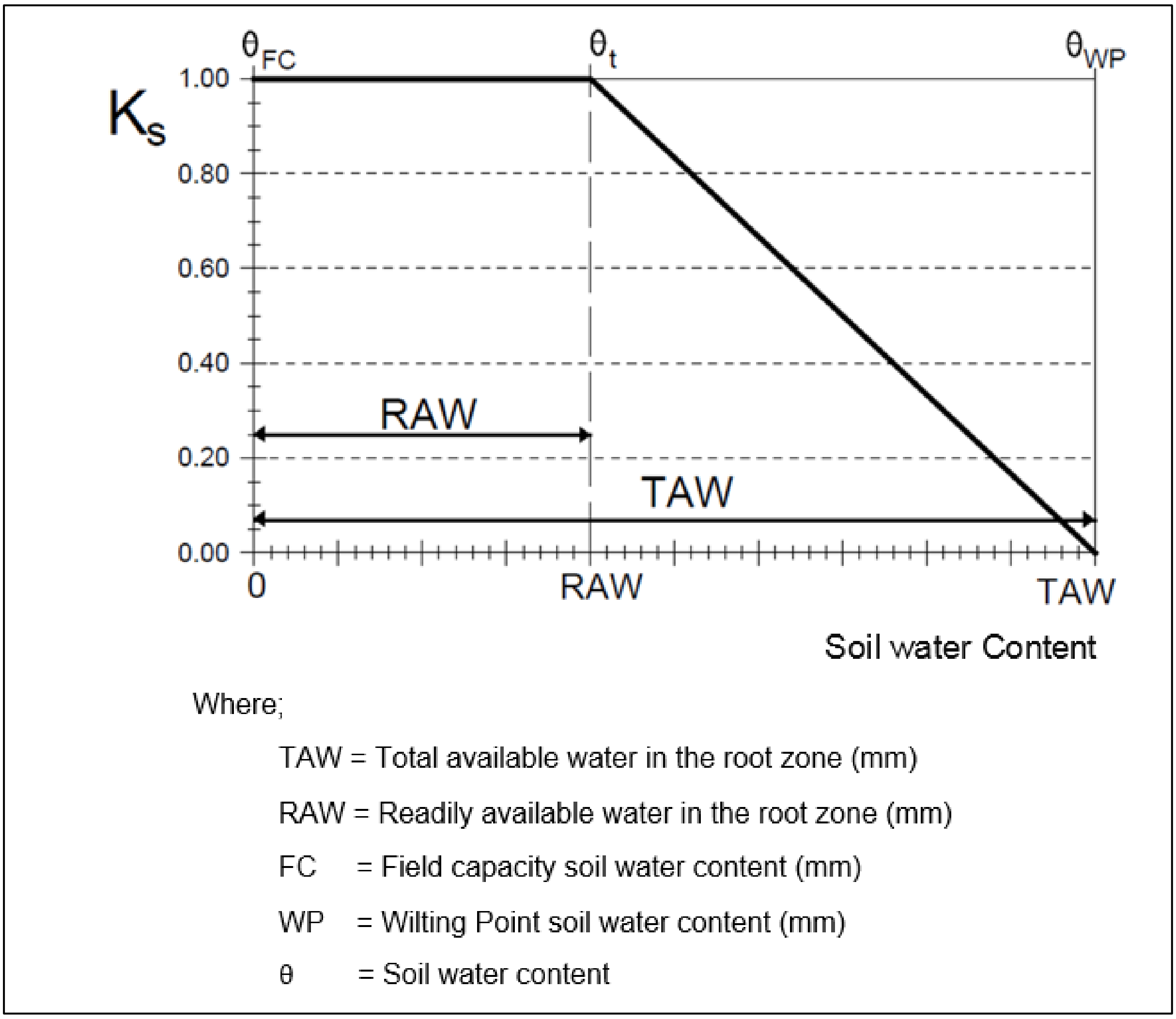

Figure 2. 2: Effects of soil water stress on actual crop evapotranspiration (ETa) as represented by a crop stress coefficient Ks

At any given crop development stage, a crop does not experience water stress as long as ETa is equal to ETm as represented by a maximum Ks equal to one between θ_{FC} and θ_t. θ_t represents a threshold RWC level before RAW is exhausted. Crop water stress free conditions are enforced by maintaining water content in the soil above θ_t. As water is extracted from the soil beyond θ_t, ETa reduces below ETm as represented by a proportional reduction of Ks until WP. In essence, if soil water content satisfies crop water requirements, ETa=ETm and if soil water is in deficit, ETa<ETm. Crop water stress can influence crop growth and the subsequent yield. A discussion of the relationship between soil moisture stress and crop yield is provided in the following section.

2.2.3 YIELD-MOISTURE STRESS RELATIONS

The effect of crop water stress on yields can be evaluated through the quantification of the relative evapotranspiration deficit (1-(ETa/ETm)). Representation of functional relationship between crop yield and consumptive water uses estimated by ETa known as a crop water production function is however complex. Accounting for the effects of crop water stress in different periods (weekly, monthly or crop growth stages) of the growing season complicates the crop water use/yield relations regardless of the linear relationship between ETa and crop growth (Rao *et al.*, 1988; Jensen, 1968; Doorenbos & Kassam, 1979). The independent effects of crop water stress in each period are dependent on the yield response factor (Ky) to water deficits during a specific stage. A multiplicative water production function is hence applicable to combine the effects of crop water stress on yield for the different periods. The Stewart multiplicative formula represents a simple heuristic multiplicative form of crop water production function models (Stewart *et al.*, 1977). The formula is based on the linear relationship between relative evapotranspiration deficits and relative yield decrease presented by Doorenbos and Kassam (1979). Stewart multiplicative yield response function is presented by the following equation (De Jager, 1994);

$$Y_c = ym_c \times \Pi_{g=1}^{4} \left(1 - ky_{c,g}\left(1 - \left(\frac{\sum_i ETa_{c,g}}{\sum_i ETm_{c,g}}\right)\right)\right) \qquad \text{Equation (2.3)}$$

Where;

Y_c	Actual yield for crop *c* (t/ha)
ym_c	Maximum (potential) yield for crop *c* (t/ha)
$ky_{c,g}$	Yield response factor for crop *c* in growth stage *g*
$ETa_{c,g}$	Sum of daily actual crop evapotranspiration for crop *c* in growth stage *g* (mm)
$ETm_{c,i}$	Sum of daily maximum crop evapotranspiration for crop *c* in growth stage *g* (mm)

The multiplicative crop water production form suggests that crop water deficits in different crop growth stages may reduce the resulting crop yield, in a multiplicative manner. Ky is a crop and growth stage specific factor that quantifies the reduction in relative yield in response to reduced ETa as a result of soil water deficits.

Determining the yield-moisture stress relationship facilitates effective scheduling of timing and amounts of irrigation water.

2.2.4 SUMMARY AND CONCLUSION

The daily state of the water budget represents the stock nature of water resources as water fluxes in one period influences the availability of water in the next period. Irrigation scheduling decisions are hence made considering the stock nature of water resources. Computation of a daily water budget routine is necessary to determine the timing and amount of irrigation water needed to compensate deficits to avoid crop water stress. Crop water stress resulting from soil water deficits in the root zone is reflected by a reduction in ETa below ETm which is quantified by a crop water stress coefficient Ks. The reduction in ETa subsequently impacts the resulting crop yield. In conclusion, irrigation decisions are complicated decisions that are considered taking into account the dynamics of soil moisture depletion. The amount of irrigation water applied in one period affects the availability of extractable water by plants in the next time period since water can be stored in the soil. Application of a dynamic solution procedure is hence imperative when solving irrigation scheduling problems to account for irrigation water use dynamics.

2.3 CLASSIFICATION AND RESOLUTION OF DYNAMIC PROBLEMS

Sustainable irrigated agriculture relies substantially on the effective and efficient management of the supply and quality of natural resources such as water and soil. According to Young (1996), analysis of water use management in agriculture with the use of mathematical programming has been centred on deterministic, partial equilibrium and static models. However, the amplified need to take into cognisance the dynamic, intertemporal nature of irrigated agriculture in evaluating water allocation policies and water use strategies in the presence of exacerbated water scarcity supplies has served as an excellent impetus to the extension of these models to also develop dynamic models (Yakowitz, 1982). Some of the factors that result in dynamic concerns include the possibility of current production actions influencing productivity of future actions, a need to adjust over time to exogenous factors, exhaustible resource base and / or future uncertainty (McCarl and Spreen. 1997).

Water resources are considered to be a stock of natural capital as current water allocation decisions will affect the availability of future resources and subsequently, future returns. Optimisation of dynamic problems hence seek to determine the optimal time path of a given function and often deals with stock-flow (state and control variables) relationships among the variables at consecutive points in time. The section below highlights definitions of some key concepts when considering the application of dynamic models and the different classes of dynamic models.

2.3.1 TIME AND MODELS

Economic models are categorised as static or dynamic models given their representation of time. Dynamic models explicitly take time into account and they comprise of decision variables that are dependent on time (Bellman, 1957). Dynamic models consist of a sequence of operations, changes of state, activities and interactions resulting in an optimal solution over time. In contrast, static models comprise of decision variables that are independent of time as the model is conceptualised without time as an entity (Blanco and Flichman, 2002). Considering the biological nature of agricultural production, a significant time lag exists between initial production decisions and realisation of output. A dynamic analysis enables the decision-maker to consider the future consequences of the decision to be made presently. Dynamic models are thus considered to give more realistic solutions in irrigated agriculture as they express the intertemporal dependence nature of decisions in comparison to static models.

Within economic models, the main distinguishing factor between dynamic models and other models is the intertemporal nature of dynamic optimisation models. Intertemporal is generally defined as an economic term describing how an individual's current decisions impacts the options that are available in the future (Kennedy, 1986). Theoretically, a reduction of consumption in the present could significantly increase the levels available for consumption in the future, and the opposite is true. The optimisation for an intertemporal dynamic model is performed over all the time periods included in the analysis known as the planning horizon. The planning horizon can be set as infinite or finite depending on the specific problem.

As aforementioned, DP can be applied in deterministic or stochastic time settings. Deterministic and stochastic dynamic models are classified under the intertemporal optimisation models and these models represent one of the three dichotomies within dynamic programming as classified by Nagypal (1998). Discrete or continuous time and finite or infinite horizon represent the other two dichotomies of DP. Time may be continuous or discrete for finite horizons while time is continuous for infinite horizons. The section below elucidates on the classes of dynamic models.

2.3.2 CLASSIFICATION OF DYNAMIC OPTIMISATION MODELS

Intertemporal optimisation models solve dynamic problems by performing an optimisation on the entire planning horizon defined. An intertemporal decision can be generally defined as a decision made in the present that has an influence on the options available in the future. Irrigation management decisions on when and how much to irrigate are considered intertemporal as a decision in one period will affect the availability of water in the next period. It is important to note that this study considers short run intertemporal modelling as decisions are made on a weekly basis in contrast to the yearly decisions considered in other studies. Intertemporal dynamic optimisation models include deterministic and stochastic dynamic models as discussed below.

A dynamic model is considered to be deterministic if future information of all the parameters included in the model is assumed to be completely and perfectly known by the decision-maker (Blanco and Flichman, 2002). By implication, complete certainty is assumed when optimising a deterministic dynamic model.

In contrast to the assumption of complete certainty considered for a determinist dynamic model, a stochastic dynamic model incorporates probabilistic elements. Stochastic models optimises the objective function when randomness is present. These models are employed to solve dynamic problems under uncertainty. Stochastic models can be further categorised into single decision and sequential decisions dynamic models. The most prominent difference between these two solution methods is that single decision dynamic models find a single optimal decision over the entire planning horizon while sequential decision models determine an optimal sequence of decisions (multi-stage) (Hannah, 2014). Stochastic models seek to find a single optimal solution under uncertainty. Knowledge of the future is represented with probabilities of states of nature in a single decision stochastic model hence the optimal decision is on average basis given any one of the given states of nature has occurred.

Agricultural production is undeniably a dynamic, stochastic phenomenon that requires the decision-maker to take more information into account as it becomes available over the planning horizon. An indication that uncertainty impacts the optimal decision rule has resulted when analysing dynamic, stochastic systems hence sequential decision models represent sequential decision-making with the gradual incorporation of information as and when it becomes available to the decision-maker (Antle, 1983). In essence, sequential decision-making entails multi-stage decision-making.

2.3.3 SOLVING DYNAMIC PROBLEMS

The possible methodologies that can be employed for water resource management decisions in agriculture include simulation, dynamic programming and multi-period linear programming (Boeljhe and White, 1969). Though linear programming and simulation optimisation methods have been widely employed to solve dynamic solutions, the results achieved with simulation optimisation models are only near optimal solutions. The DP algorithm was introduced by Bellman (1957) and has since been the subject of continuous research efforts in agricultural water resource management. Substantial research efforts have been commissioned on DP programming which led to the invention and application of techniques for implementing DP to water resource management problems such as discrete dynamic programming, differential dynamic programming, state incremental dynamic programming and Howard's policy iteration method (Yakowitz, 1982). In an effort to explain methods to solve dynamic problems, solution methods are grouped into non-sequential and sequential dynamic optimisation models. Given the clarification of classes of dynamic models discussed in the previous section, deterministic and single decision stochastic models are classified under non-sequential dynamic optimisation models. Both models take into account all the periods involved in the planning horizon to determine a single optimal decision with no modification or without taking further information into account afterwards (Blanco and Flichman, 2002). In contrast, sequential decision models as defined, represent dynamic decision-making that solves inter-temporal problems in a sequential manner taking into account additional information as it becomes available and hence will be grouped under sequential dynamic optimisation.

2.3.3.1 Non-sequential dynamic optimisation

Yaron and Dinar (1982) applied a DP model to determine optimal irrigation strategies during peak seasons considering farm restrictions and shadow prices of water. A predetermined area of one hectare was considered for the optimisation. The DP model optimises the total quantity of water for one hectare over a growing season and then optimises the allocation of that water over time. The state of the hectare plot on day *t* and the decision taken on day *t* will influence the state of the plot and the decision to be made on day *t+1*. The DP model also assumes certainty about weather conditions hence deterministic in nature. The model only includes two discrete state variables. A major limitation of this study is the deterministic DP framework applied.

Rao *et al.* (1988) developed a deterministic DP technique to solve the water allocation problem under limited water supply conditions. The researchers modelled seasonal and weekly irrigation schedules for cotton under limited water supply conditions. The mathematical formulation was based on a dated water production function which is a mathematical relationship between ET and the associated yield and also on weekly soil water balance. Growth stages and weeks were the two decision time periods used in solving the allocation problem subject to water delivery and soil-water storage constraints. Growth stage optimal water allocations were obtained through a DP model that maximized the dated water production function. The water was then sequentially re-allocated at the second level to meet weekly water deficits within each stage. Rao *et al.* (1988) managed to develop a procedure that schedules limited irrigation water for short periods such as weekly intervals.

Locally, Botes *et al.* (1995) employed a comprehensive dynamic approach to value irrigation information for decision-makers with neutral and non-neutral risk preferences under conditions of both unlimited and limited water supply. The optimal solution was however only considered on an annual basis ignoring the possible impact of a real-time, multi-year and sequential analysis on the optimal solution. No updating of additional information was facilitated during the planning horizon to allow sequential decision-making.

2.3.3.2 Sequential dynamic optimisation

Over a decade ago, researchers were challenged to develop and apply stochastic dynamic planning methods in a bid to manage water resource allocation in agriculture (Backeberg and

Oosthuizen, 1995). Substantial progress in the development and application of stochastic dynamic programming has hence been stimulated with the increased risk encountered by farmers and aggregated scarcity of water supplies. There are two widely applied mathematically equivalent methods to solve sequential stochastic dynamic optimisation namely stochastic dynamic programming (SDP) and discrete stochastic programming (DSP). These models break down multiple decision problem into a sequence of sub-problems which can result in the model being too large hence the curse of dimensionality problem associated with dynamic programming. In practice, researchers might need to limit the possible state and decision variable for the model resulting in a solution that is a mere estimate of the optimal solution. In an effort to address the curse of dimensionality problem associated with DP, Blanco and Flichman (2002) developed a methodology based on a recursive stochastic programming method (RSP) to solve stochastic dynamic problems. In a sequential decision problems, a decision-maker faces a sequence of decisions with a decision for the next iteration being influenced by the decision made at the previous iteration. Sequential decision-making can be simply illustrated through decision trees. Below is a general tree diagram representing a sequential decision problem.

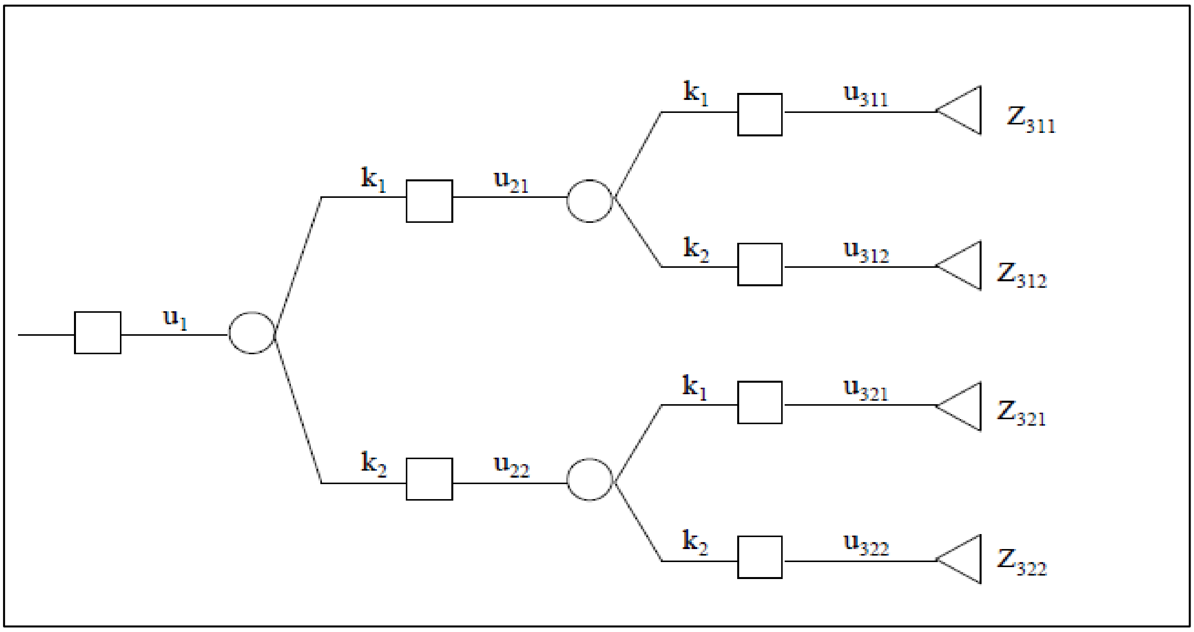

Figure 2. 3: A three stage (u_i) decision making problem where a square represents a decision node at each stage given the possible two states of nature (k_i) that could unfold at each stage represented by circles and the final results (Z_i) represented by triangles

As indicated in Figure 2.3, decision (u_1) is made at the initial state of the system. The decision made in the next stage (u_2) is dependent on the state of nature occurring (k_1 or k_2). The state of the system or nature is defined by a specific combination of discrete values of state variables (Gakpo *et al.* 2005). A square on a decision tree typically denotes a stage when a decision must be made while a circle denotes a chance node representing an event the decision-maker cannot control (Kikuti, Cozman & Filho, 2010). The following section discusses the application of sequential dynamic optimisation models.

2.3.3.2.1 Stochastic dynamic programming

Bryant *et al.* (1993) developed an intra-seasonal SDP model that optimally allocated predetermined number of irrigation intervals between two competing crops taking into account stochastic weather patterns. The main objective of the research was to maximise the expected net returns over the entire planning horizon defined as a single year given the specified number of irrigation events. A specified amount of water only sufficient to irrigate one of the fields could be pumped over a five-day irrigation cycle. The research considered 15 decision stages (15 potential irrigations) over 25 states of nature with a decision in stage *t*+1 being influenced by decision made in stage *t*. Three decisions were considered for each decision stage whether to irrigate crop 1, irrigate crop 2, or irrigate neither of the two crops. The stochastic sequential model allows water to be shifted between competing crops as the season progresses. However, similar to other dynamic programming studies, the area to be irrigated was predetermined. Similarly, Rhenals and Bras (1981); Bras and Cordova (1981); Burt and Stauder (1971) applied SDP model framework to allocate irrigation water over the growing season for a fixed area.

In South Africa, Gapko *et al.* (2005) applied a SDP to optimise water allocation under capacity sharing arrangements. A linear programming (LP) model was used to optimize farm water use during the immediate season while a SDP model was used to optimise water use over the entire planning zone.

2.3.3.2.2 Discrete stochastic programming

As developed by Dantzig (1955), discrete stochastic programming (DSP) has also been considered to be capable of solving sequential decision problems under uncertainty in farm management. The application technique has since been extended through considerable theoretical research efforts (Cocks, 1968; Rae, 1971a). DSP allows the formulation of problems in a linear programming framework. A decision tree is also typically used to represent sequential decision-making in a DSP solution. Application of DSP is however also limited due to the curse of dimensionality as noted with DP. Nevertheless, DSP's ability to take into consideration diverse activities and constraints common in the agricultural environment renders it an advantage compared to DP with regards to the ease of applicability.

Rae (1971b) applied DSP in farm management with the main aim of elucidate and build on to the empirical application of the DSP methodology. The main aim of the research was to evaluate the sequential problem solving ability of the DSP technique. To reduce the states of natured included in the model, the weather variables were simply classified as good, normal or bad. An increase in the expected utility for a fresh-vegetable holding resulted by employing SDP compared to that obtained by using a deterministic dynamic model. The author acknowledges that including more states of nature and allowing decisions to be formulated frequently as employed in this study could have improved the SDP model with specific reference to the efficient use of information received by the decision-maker.

A DSP model was also applied by Jacquet and Pluvinage (1995) to analyse the effects of climatic variability on production choices for cereal-livestock farms in France. Similar to other applications of DP, states of nature have to be limited to avoid the model exploding in size. Four states of nature (excellent, good, mediocre and catastrophic) that correspond to a climatic situation with special reference to rainfall were included in the model. The models employed also considered a two-stage decision process where stage one decision is made with only the probability of occurrence of each state of nature known to the decision-maker. The second stage decision is then made with the knowledge of random variable values (climatic scenario). The results of the study indicated the significance of utilising such an approach to analyse how climatic variations can influence choices or decisions made by farmers. DSP allowed the researchers to simultaneously take risk linked with climatic variability into account since model formulation is not based on average results.

All of the aforementioned studies concluded that DSP is easily applicable to the agricultural environment in comparison to DP. In addition, DSP allows the incorporation of risk analysis when optimising problems within a dynamic framework. However, this modelling technique also suffers from the curse of dimensionality problem hence the need to always limit the number of state and stage variables. A need to consider recursive stochastic programming that overcomes this curse of dimensionality problem hence exists as discussed in the following section.

2.3.3.3 Recursive stochastic programming

Recursive models are considered to belong with the family of dynamic models as the different decision stages are explicitly represented (Blanco and Flichman, 2002). The main difference between the recursive models and inter-temporal dynamic models lies in their optimisation procedures. In contrary to the backward recursion procedure of DP, RSP models solve complex problem by means of forward recursion. In addition, the optimisation is achieved for each individual time stage with the optimisation of the next stage dependent on the previous iteration's optimisation in contrast to optimising over the entire planning horizon as with intertemporal dynamic models. This results in the RSP method overcoming the curse of dimensionality problem associated with inter-temporal optimisation models.

Day (1961) developed the RSP method to present a process of an adjustment between real-life situations and optimal situations through gradual adaptation of changes of exogenous parameters. The approach was then extended by Blanco and Flichman (2002) to solve sequential stochastic dynamic problems analysing the sustainability of irrigation systems in a Tunisian region. The RSP methodology is based on the notion that the decision-maker's uncertainty about the future is higher than DP would anticipate. Given the decision-maker's knowledge to the future is significantly limited, it is difficult to fully anticipate the state of nature to unfold hence he must opt for a sub-optimal decision for the first iteration. The decision-maker can now adjust the decision for the second iteration taking into account the new information available. RSP involves a series of sequential optimisations as presented in the diagram below.

As indicated by Figure 2.4, stage u_1 decision is made in the first optimisation on stage 1 by taking into account all the information available at that moment. The decision taken at moment 2 will be a subject of the state of nature (k_i) that has occurred. At moment 2, the decision can be revised and optimised taking into account additional information that will be now available to the decision-maker.

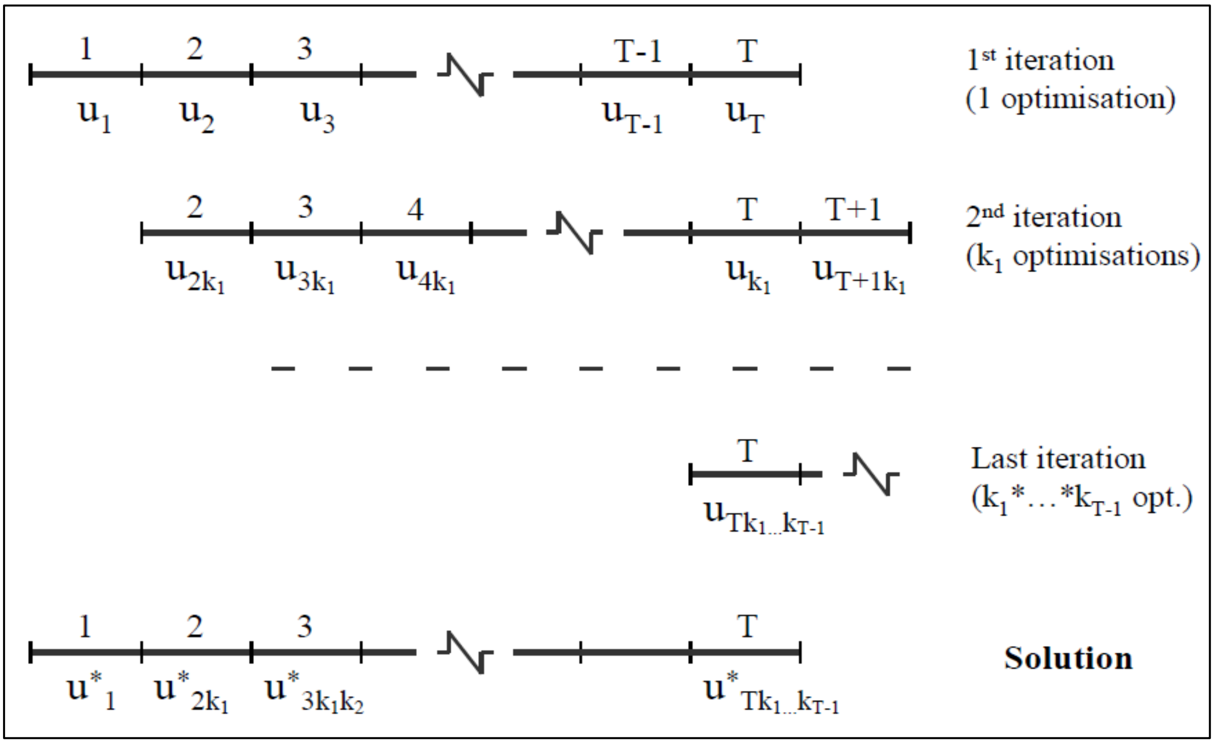

Figure 2. 4: Sequential solution procedure for a recursive stochastic programming method given u_i decision stages and k_i possible states of nature

This recursive decision-making will occur till the last stage (T) as represented in Figure 2.4. The main advantages of this model include its ability to have large number of state variables represented in the model, the ability of the model to introduce exogenous changes to some of the parameters including but not limited to stochastic resource availability and the ability to optimise both short term and long-term planning horizons.

2.3.4 SUMMARY AND CONCLUSION

Taking dynamics of water use into account when optimising irrigation water scheduling at farm level is imperative for efficient and effective agricultural water management. Ignoring the dynamics of water application will rather result in efficient and effective planning not management of irrigation water resources. DP has been increasingly employed as a technique to solve dynamic, inter-temporal decision problems in agricultural water resource management and the procedure has received considerable theoretical and application research attention over the past decade as noted in literature. Deterministic dynamic models ignore the expected cost of uncertainty thus optimal solutions are not risk efficient. Sequential decision-making is also not considered for deterministic and single decision stochastic models because a deterministic model assumes complete knowledge of all decision variables. In addition, a single-stage decision is considered for single decision stochastic model. The strength of DP lies in the ability of the model to break down multiple decision problems into a sequence of sub-problems allowing the optimisation of diverse problems. Furthermore, uncertainty and integer restrictions can be easily included in a DP model. DP allows sequential decision-making for stochastic dynamic models which is critical given the stochastic and dynamic nature of agricultural production activities. Nevertheless, the association of DP with the curse of dimensionality problem and the lack of a general algorithm has resulted in the development of a recursive programming methodology that overcomes the curse of dimensionality problem. RSP models solve complex problem by means of forward recursion allowing each time stage to be optimised individually with the optimisation of the next stage dependent one the previous iteration's optimisation. Thus, the recursive solution procedure overcomes the curse of dimensionality problem associated with inter-temporal optimisation models without limiting the number of states of nature considered and decision variables to a finite discrete set. Application of recursive programming is hence of paramount importance to better account for these dynamics.

The conclusion is that the applicability of techniques currently applied to solve dynamic problems is limited due to the curse of dimensionality. The stage and state variable are kept minimal to avoid the explosion of model in size resulting in dynamics only being approximated and risk being over-estimated. Though dynamic programming facilitates sequential decision-making, the curse of dimensionality limits the applicability of the models for complex dynamic problems. The application of a recursive stochastic technique that overcomes the curse of dimensionality to better account for dynamics of water use and explicitly representing risk is hence imperative for improved irrigation water management. Explicitly modelling dynamic irrigation decisions entails the application of more complex dynamic models. The results generated within a framework that accounts for water use dynamics hence represent important realities of irrigation decision-making that are important to take into account if models are utilised as water allocation decision tools.

2.4 IRRIGATION COSTS AND DYNAMICS

Electricity costs are regarded as one of the most significant components of total variable inputs for irrigation farming for crops such as maize and wheat as it is vital for pumping water from the source to the farms. The choice of the tariff plan that best suits the production activities of the irrigation farmers as designed by Eskom, does not only have an implication on the resulting electricity costs but can also impact irrigation water scheduling decisions hence the dynamics of water use. Eskom has designed several tariff options for irrigation farmers (Eskom, 2015/16 tariff booklet). However, only the impact of Landrate and Ruraflex tariff options on water use dynamics will be discussed below as they are utilized by the majority of irrigation farmers.

2.4.1 RURAFLEX

Ruraflex tariff is a time of use (TOU) tariff plan consisting of both variable and fixed costs. The variable costs of Ruraflex depend on the TOU differentiated active energy charges. The fixed charges that are applicable regardless of usage include the service charge, reactive energy charge, network access charge and administration charge. The tariff caters for rural consumers whose dual and three phase supplies have a Notified Maximum Demand (NMD) from 25kVA and a supply voltage greater than or equal to 22kV. The TOU tariff discourages the straining of the national electricity system during high demand or peak periods of consumption by charging a higher energy charge during these periods. Contrastingly, it creates an incentive for using electricity during the off-peak periods and low demand seasons by charging lower energy charges. Ruraflex provides irrigators with an opportunity to use electricity efficiently to counterpart to a certain extent the escalating tariffs. The season and the time of the day are the two main distinguishing aspects for the different tariff rates for the TOU tariff (Eskom, 2015/16 tariff booklet). The seasonal aspect differentiates the charges according to high and low demand while the daily aspect differentiates according to the time of the day. The daily aspect is effectively categorized into three periods which are off-peak, standard and peak periods. Contrasting from the previous years, the daily time periods allocation during the high demand season is now different from that of the low demand season. Below is an illustration of the three periods for each season.

Figure 2.5 illustrates the time of the day applicable to each period for each season. The peak, standard and off-peak periods entail periods of high, medium and low energy costs respectively. To benefit from the TOU tariffs, farmers have to schedule their irrigation in such a way that irrigation occurs mostly during off-peak and standard time slots.

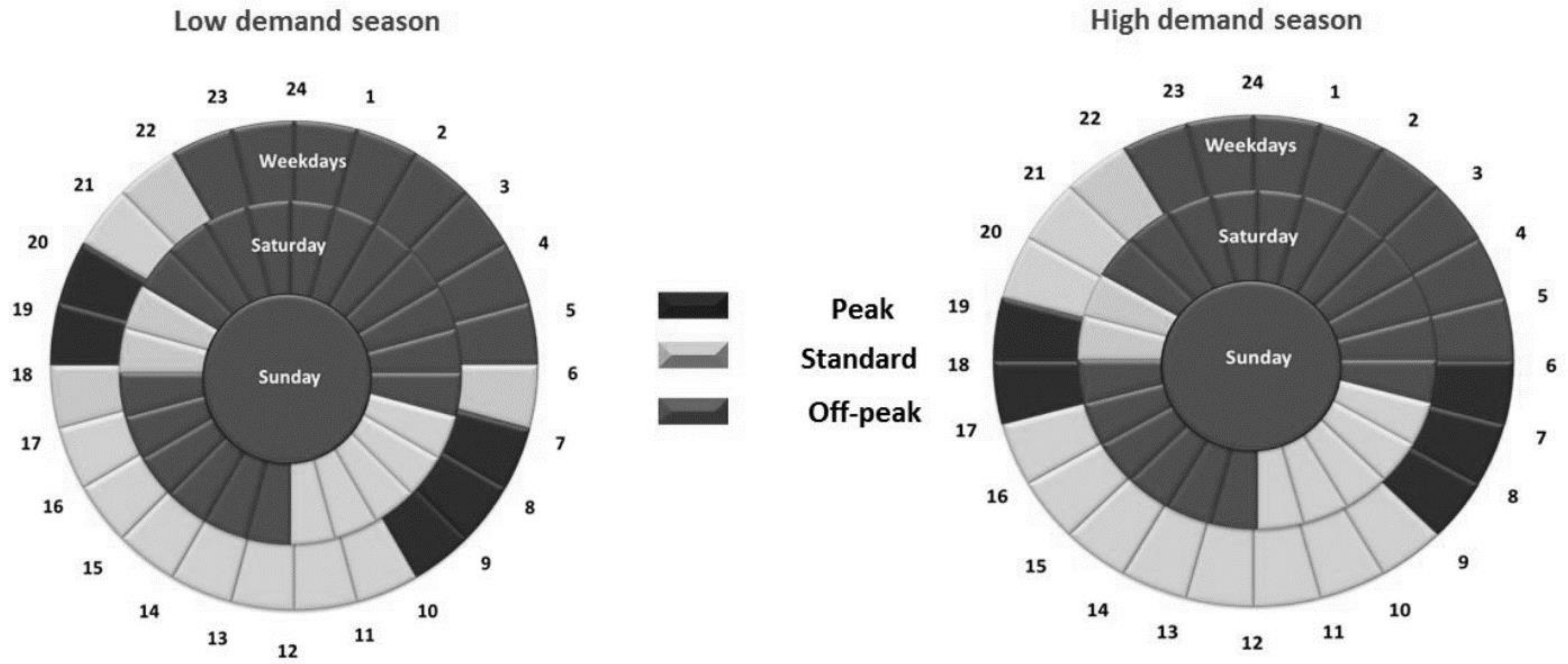

Figure 2. 5: Distribution of Ruraflex's peak, standard and off-peak time of use periods within a low and high demand season

Source: Eskom 2015/16 Tariff booklet

The TOU option hence complicates water allocation decisions as the decision-maker might have to schedule an irrigation event on a specific day of the week and time of the day in an effort to curb electricity costs.

2.4.2 LANDRATE

Similar to the Ruraflex tariff option, the Landrate tariff option comprises of both fixed and variable charges. However, the tariff plan is characterized by a single active charge that depends on the supply size. Likewise, the network access, service and administration charges constitute the fixed cost component of the tariff plan. The tariff is divided into six ranges which mainly differ according to the metered supply phase and the corresponding kilovolt-amperes as illustrated below.

Table 2. 1: Classification of Landrate tariff charges based on kVA supply

Landrate 1	single-phase **16 kVA** (80 A per phase) dual-phase **32 kVA** (80 A per phase) three-phase **25 kVA** (40 A per phase)
Landrate 2	dual-phase **64 kVA** (150 A per phase) three-phase **50 kVA** (80 A per phase)
Landrate 3	dual-phase **100 kVA** (225 A per phase) three-phase **100 kVA** (150 A per phase)
Landrate 4	single-phase **16 kVA** (80 A per phase)
Landrate Dx	single-phase **5 kVA** (limited to 10 A per phase)

Source: Eskom 2015/16 tariff booklet

The Landrate tariff ranges vary with the metered supply phases, the Notified Maximum Demand (NMD) and the amperes supplied per phase as illustrated in Table 2.1. Supplies that constantly utilize at least 1000kWh monthly are closely associated with Landrate 1, 2 and 3 while Landrate 4 is suitable for those that utilize below 1000kWh on a monthly basis (Burger *et al*., 2003). Landrate Dx is more suitable for non-metered low usage supplies.

The single active energy charge implies that the decision on the timing of the irrigation event has no implication on the resulting variable costs. The Landrate tariff option thus has no impact on the dynamics of irrigation water use. In the wake of the paradigm shift in the management of irrigation farming from a biological objective to maximize yields to an economic objective to maximise profits or benefits (English et al., 2002), the incentive of using the TOU Ruraflex tariff choice doesn't only impact irrigations costs but also the timing of irrigation events.

2.5 SOUTH AFRICAN APPLICATIONS OF DYNAMIC MODELLING APPROACHES

Considerable research has been conducted on optimal allocation of irrigation water in South Africa (Grové, 2006; Grové, 2008; Grové & Oosthuizen, 2010; Haile *et al*., 2014; Venter & Grové, 2016; Botes *et al*., 1990; Haile, 2017). Research efforts attempted to model the interdependency of water applications between irrigation applications in different time periods. The water allocation models however primarily focused on the depth of irrigation amounts without explicitly considering timing of irrigation events. Hence, only four research efforts significantly accounted for the dynamic, intertemporal nature of irrigation decisions through mathematical programming and simulation optimisation (Grové & Oosthuizen, 2010; Venter & Grové, 2016; Botes *et al*., 1990; Haile, 2017). What follows is a description of the aforementioned research efforts that considered water use dynamics in more detail.

2.5.1 MATHEMATICAL PROGRAMMING

A mathematical programming approach is applied by researchers to solve irrigation scheduling decisions problems as an alternative to DP. Grové and Oosthuizen (2010) developed a non-linear mathematical programming model to economically evaluate the effect of deficit irrigation (DI) under multiple crops taking increasing production risk of DI into account. The dynamic problem of optimising water use between multiple crops was approximated by including a large number of discrete activities to represent alternative water distribution strategies throughout the growing season. Multiple irrigation schedules were hence included in the optimisation model to dynamically optimise water use within a multi-crop setting. Maximising certainty equivalents for a defined range of risk aversion coefficients facilitated the incorporation of risk preferences into the model. Three different states of nature were included in the model to avoid the curse of dimensionality problem and on average optimal solutions were determined. The applicability of the mathematical model to solve complex dynamic problems with more states of nature is hence limited. Though daily water budgets were computed, water use dynamics were only approximated.

Venter and Grové (2016) developed and applied a non-linear mathematical programming model that linked the timing of irrigation events with the electricity tariff choice for improved energy management. A Soil Water Irrigation Planning and Energy management (SWIP-E) model was developed. The SWIP-E programming model is based on the SAPWAT optimisation (SAPWAT-OPT) (Grové, 2008) model that optimises a daily soil water budget for a single crop. The research effort successfully extended SAPWAT-OPT to facilitate optimisation of a daily water budget within an inter-seasonal water allocation setting. Optimal irrigation hours required per irrigation cycle were allocated to the different TOU periods within the Ruraflex tariff. Irrigation hours were allocated first to the off-peak time periods then the standard periods and lastly the peak-periods within the two-day cycle to manage the resulting variable electricity costs. In essence, a decision-maker was forced to irrigate during a certain time of the day and a certain day of the week taking into account the crop water requirements. Research results indicated that the Ruraflex tariff was more profitable as higher net present values resulted from reduced variable electricity costs. The optimal solution was however based on average data of 49 states of nature with only a single state on average basis considered to avoid the curse of dimensionality. The relationship between timing of irrigation events and energy costs was successfully modelled. However, the model failed to incorporate risk by including different states of nature hence risk neutrality for decision-makers was assumed.

2.5.2 SIMULATION OPTIMISATION

Botes *et al.* (1996) developed a simulation optimisation to optimise irrigation scheduling decision with the main aim of estimating the value of irrigation information under dynamic plant growth conditions. A simulation-complex (SIMCOM) model was developed and applied to maximize expected incomes given the different irrigation information scenarios. The complex method also known as the constrained simplex or the Nelder-Mead simplex algorithm achieves the maximum by moving away from low objective function values rather than moving in line towards the maximum. A crop growth simulation model was linked to an economic model to optimise irrigation scheduling for maize under conditions of limited and unlimited water supply. A crop simulation model was applied in an effort to realistically represent the stochastic, dynamic nature of irrigation scheduling decisions. The crop growth model firstly initializes soil, crop and weather variables which are then read into an irrigation scheduling subroutine. A decision to irrigate is made depending on the selected irrigation information strategy and weather data for the next three days. Six irrigation strategies considered in the model resulted from different combinations of information on soil water, plant growth and weather. A fixed irrigation amount of 10mm was applied when an irrigation event is triggered. Equal probabilities of occurrence of each state of nature included in the model was assumed. Research findings indicated that incorporation of additional information on soil water level as determined by a daily account of ETa improved irrigation scheduling decisions. Additional information for the next three days was used to make an irrigation decision prior to those three days. Though the model accounted for water use dynamics, the model is not applicable to schedule real-time irrigation scheduling decisions as the additional information will be unknown to the decision-maker prior to decision-making.

Recently, Haile (2017) developed an integrated bio-economic simulation optimisation model through the integration of the Soil Water Management Program (SWAMP) crop growth model with an economic model. The main aim of research was to evaluate the impact of optimal irrigation practices on stochastic efficiency, water use efficiencies and environmental. The developed SWAP-ECON model was linked to an evolutionary algorithm (EA) to determine the benefits of applying an optimal irrigation scheduling strategy. Application of an EA facilitated the generation of a feasible irrigation schedule that was used in the SWAMP model to simulate crop yield. SWAMP utilises daily simulations of the water budget to determine daily changes in water content of a multi-layer soil and seasonal impact on crop yield. SWAMP-ECON model can solve complex stochastic water allocation optimisation problems under salinity taking production risk into account. Seven states of nature were included in the simulation model. However, water was assumed to be a state-general input. Thus, the optimal irrigation strategy was determined such that it will maximise utility irrespective of the state of nature that occurs. Therefore, no adaptive decision-making was included in the model.

2.5.3 SUMMARY AND CONCLUSION

Formulation of procedures to optimise irrigation scheduling is complex given the stochastic and dynamic nature of such decisions. Modelling dynamic crop water optimisation problems hence requires one to explicitly take time into account if true water dynamics are to be accounted for. Though the computation of a daily water budget facilitates scheduling of irrigation events taking the status of the soil water balance into account, limiting state and stage variables in mathematical programming models only results in approximating such dynamics. The applicability of models to complex dynamic problems whose optimal solution is highly dependent on explicitly taking time into account is hence limited. Mathematical programming models are relatively easy to solve though explicitly representing irrigation water use dynamics within such a solution procedure is difficult. Simulation optimisation approaches were able to take into account a considerable number of states of nature given their ability to deal with high-dimensional non-linear or mixed integer optimisation problems. However, only near optimal solutions were generated as evolutionary algorithms are not based on optimality conditions.

Simulation optimisation models are more complex to solve in comparison to mathematical programming models as external algorithms are applied. The main shortcoming of the SA applications is that no sequential decision-making was considered to facilitate real-time adaptive behaviour as more water budget and weather information unfolds. The conclusion is therefore that irrigation water use dynamics were only approximated.

2.6 OVERALL CONCLUSIONS

From the literature review conducted, the following conclusions were made;

- Irrigation water allocation decisions are complicated decisions that are made sequentially in multi-stages taking into consideration the stock nature of field water supply dynamically throughout the growing season. An irrigation decision in one period will impact water availability to crops during the next time period since water can be stored in the soil. Under limited water supplies, area planted and irrigation decisions are made in multi-stages to allow an interaction between crop, area planted and water availability. Application of a dynamic solution procedure is necessary when solving irrigation scheduling problems to account for irrigation water use dynamics within a multi-stage decision making framework.
- Dynamic programming and stochastic dynamic programming have been substantially applied to solve dynamic, inter-temporal decision problems in agricultural water use optimisation research efforts. However, the association of dynamic programming with the curse of dimensionality limits applicability of the solution procedure to complex dynamic problems when area planted and irrigation scheduling decisions need to be considered in multi-stages when allocating limited water supplies. A recursive stochastic solution procedure that solves problems through forward recursion hence provides an alternative solution procedure to solve complex dynamic problems without the curse of dimensionality limitation.
- Currently available crop water use optimisation solution techniques within the South African context lack complexity with irrigation decisions modelled within a single-stage decision making framework where sequential adaptive decision-making is not considered. Thus, dynamics of irrigation water use and the associated production risk are only approximated if not overlooked. A new line of solution techniques that utilises evolutionary algorithms to optimise complex simulation models offers an alternative technique to solve complex multi-stage irrigation decisions. Evolutionary algorithms are capable of solving complex dynamic models without the complexity of the model rendering the solution infeasible.

- Time of use energy tariffs represents an exogenous factor that can further complicate irrigation scheduling decisions. Time of use tariffs forces decision-makers to consider improving their decision-making to reduce their irrigation costs. As a result, the timing of irrigation events is influenced which subsequently influences water use dynamics. Energy accounting is thus necessary when modelling irrigation scheduling decisions.

CHAPTER 3
METHODOLOGY AND DATA

3.1 INTRODUCTION

Achieving the main objective of this research requires some form of agricultural water use optimisation. The optimisation procedure used in this research deviates from the normal mathematical programming approaches typically used in South Africa as it uses the evolutionary algorithms embedded in Excel® to optimise water use. Evolutionary algorithms use random realisations of the decision variables as a basis to evolve to a better solution. Thus, the “optimal” solution is not achieved when optimality conditions are satisfied and therefore only near optimal solutions are possible. Special care was taken to ensure that the solutions that were generated were the best possible solutions.

Applying evolutionary algorithms to optimise agricultural water requires the development of a model that is able to simulate the economic consequences resulting from changes to the key decision variables that need to be optimised. The key decision variables in this research are the areas irrigated for maize and wheat as well as the respective irrigation schedules for the crops. The timing of irrigation events is vital as it has an impact on crop yield and irrigation costs. The first part of Chapter 3 is therefore devoted to a discussion of the calculation procedures to simulate the impact of changes in timing and magnitude of irrigation events on crop yield and the pumping hours necessary to apply the water. The crop yield estimations and the pumping hours provide the necessary link between the irrigation decisions and the quantification of the economic consequences thereof which is discussed next. Comparing the economics of the single-stage and multi-stage sequential decision-making frameworks requires the quantification of the risk. The procedures to include risk into the simulation model follows the economic module discussion which is then followed by a discussion of the specific procedures used to optimise water use for the two decision-making frameworks. Lastly, the data requirements are discussed.

3.2 SIMULATING TIMING OF IRRIGATION EVENTS

Accounting for water dynamics when scheduling irrigation decisions requires the computation of a daily soil water budget to avoid crop stress at any crop growth stage taking cognisance of the stock nature of water resources. An irrigation decision scheduled in one stage will affect the soil water balance and availability of water for the next stage. The representation of the stock nature of water through the computation of a daily water budget and the key output variables of the simulation model is discussed next.

3.2.1 DAILY WATER BUDGET COMPUTATION

The main objective of scheduling irrigation events is to determine when to irrigate and how much to irrigate depending on the state of the soil water balance. By calculating the soil water balance of the root zone on a daily basis, the timing and the depth of future irrigation events can be planned. Assessing the state of the components of the daily water budget as they relate to crop water stress and the resulting yield is thus vital. Crop evapotranspiration is useful for determining crop water requirements as the amount of water lost through evapotranspiration represents the amount of water required by the crop to compensate water loss. Evapotranspiration can hence be determined by measuring various components of the soil water balance (Allen, Pereira, Raes and Smith, 1998).

The effect of soil water content on evapotranspiration is conditioned primarily by the magnitude of the water deficit and the type of soil. Crop evapotranspiration under standard conditions denoted by ETm represents evapotranspiration from crops that are excellently managed, grown under optimum soil water conditions on large fields and achieving full potential yields. ETm is easily determined using the following equation;

$$ETm_{c,i} = Kc_{c,i} \times ETo_{c,i} \qquad \text{Equation (3.1)}$$

Where;

$Kc_{c,i}$ Crop coefficient for crop *c* on day *i*

$ETo_{c,i}$ Reference evapotranspiration for crop *c* on day *i* (mm)

Kc values range from zero to one and they differ with the stages of crop development. ETo is a climatic parameter estimated using climatic data and a grass reference crop expressing the atmospheric evaporation power. Scheduling of irrigation events is not necessary under standard conditions as crops do not experience water stress. However, under non-standard conditions, the actual evapotranspiration (ETa) by crops might differ from ETm due to non-standard conditions such as water stress, diseases, pests and low soil fertility. The computation of crop evapotranspiration under non-standard is hence complex.

Under non-standard conditions, one should determine the root water content (RWC) level where crops do not experience water stress to successfully simulate the timing and amount of irrigation events. The following calculation procedures were used to determine the status of the soil water balance to facilitate the simulation of irrigation events. Eta was calculated by the following equation;

$$ETa_{c,i} = min \left| \begin{matrix} ETm_{c,i} \\ ETm_{c,i} \left(\frac{RWC_{c,i}}{TAW_{c,i} - RAW_{c,i}} \right) \end{matrix} \right. \quad \text{Equation (3.2)}$$

Where;

$ETm_{c,i}$ Maximum crop evapotranspiration for crop *c* on day *i* under standard conditions (mm)

$RWC_{c,i}$ Actual root water content for crop *c* on day *i* (mm)

$TAW_{c,i}$ Total available water for crop *c* on day *i* (mm)

$RAW_{c,i}$ Readily available water for crop *c* on day *i* (mm)

Eta retains the minimum value generated between the ETm value and the product of ETm and Ks which represents evapotranspiration under water stress conditions. The minimum function indicates that ETa cannot exceed the potential or maximum evapotranspiration of a given crop generated under standard conditions. The following equation is used to calculate TAW;

$$TAW_{c,i} = (\theta_{FC} - \theta_{WP})\, RD_{c,i} \quad \text{Equation (3.3)}$$

Where;

θ_{FC} Soil water content at field capacity (mm)

θ_{WP} Soil water content at wilting point (mm

$RD_{c,i}$ Root depth for crop *c* on day *i* (m)

The magnitude of TAW is dependent on the type of the soil and the rooting depth. The soil type determines the amount of water that a soil can hold against gravitational forces while the rooting depth determines the drainage of the water below the roots if FC is reached. The RAW is calculated according to the following equation;

$$RAW_{c,i} = p_{c,i} * TAW_{c,i} \quad \text{Equation (3.4)}$$

Where;

$p_{c,i}$ Average fraction of TAW that can be depleted from the root zone before crop *c* experiences moisture stress on day *i*

The value of *p* is dependent on depth of the roots and the evaporation power of the atmosphere and ranges between zero and one. The greater the ETm, the larger the p value. The actual RWC is calculated as follows;

$$RWC_{c,i} = \min \left| \begin{matrix} RWC_{c,i-1} - ETa_{c,i-1} + R_{c,i-1} + IR_{c,i-1} + TR_{c,i} \\ RWCAP_{c,i} \end{matrix} \right. \quad \text{Equation (3.5)}$$

Where;

$R_{c,i-1}$ Rainfall for crop *c* on day *i* (mm)

$IR_{c,i-1}$ Irrigation amount for crop *c* on day *i* (mm)

$TR_{c,i}$ Additions to the RWC due to root growth for crop *c* on day *i* (mm)

$RWCAP_{c,i}$ Root zone water holding capacity for crop *c* on day *i* (mm)

The RWC accounts for all the incoming water flux into the root zone on a daily basis. The actual capacity of the soil water cannot exceed RWCAP hence the minimum value generated between the total water influx and the RWCAP is retained. If the water influx into the root zone exceeds RWCAP, the soil drains the water below the roots. Water content below the root (BRWC) for each day is hence determined with the following equation;

$$BRWC_{c,i} = min \left| \begin{matrix} BRWC_{c,i-1} + BR_{c,i} - TR_{c,i} \\ RWCAP_{c,i} \end{matrix} \right. \quad \text{Equation (3.6)}$$

Where;

$BR_{c,i}$ Water draining below the root zone for crop *c* on day *i* (mm)

$TR_{c,i}$ Additions to the RWC due to the root growth for crop *c* on day *i* (mm)

RD_{max} Maximum root development (m)

Where TR and BR are determined by;

$$TR_{c,i} = max \left| \begin{matrix} (RD_{c,i} - RD_{c,i-1})/(RD_{max} - RD_{c,i-1}) \ BRWC_{c,i-1} \\ 0 \end{matrix} \right. \quad \text{Equation (3.7)}$$

$$BR_{c,i} = max \left| \begin{matrix} RWC_{c,i-1} + BR_{c,i} - TR_{c,i} \\ 0 \end{matrix} \right. \quad \text{Equation (3.8)}$$

BRWC depends on the water movement into and out of the root zone and cannot exceed the RWCAP. If RWC is below RWCAP, the soil does not drain and BR can be equal to zero. TR is dependent on root development and BRWC hence it is only above zero during the development growth stage of the crop and it is zero in all the other crop growth stages. To initiate the water balance in the root zone for the first day, an initial soil water depletion of 50% is estimated for RWC and BRWC in the simulation model.

3.2.2 KEY OUTPUT VARIABLES

Crop yield and pumping hours were identified as key output variables necessary to link changes in irrigation schedules to the economics. Both these output variables are direct functions of the irrigation schedule that is used. The calculations of the key output variables are discussed next.

3.2.2.1 Crop yield

The Stewart multiplicative relative evapotranspiration formula was applied to quantify the actual yield of each crop by taking the effect of water deficits in different crop growth stages into account (De Jager, 1994).

$$Y_c = ym_c \times \Pi_{g=1}^{4} \left(1 - ky_{c,g}\left(1 - \left(\frac{\Sigma_g ETa_{c,g}}{\Sigma_g ETm_{c,g}}\right)\right)\right) \qquad \text{Equation (3.9)}$$

Where;

Y_c Actual yield for crop *c* (t/ha)

ym_c Maximum (potential) yield for crop *c* (t/ha)

$ky_{c,g}$ Yield response factor for crop *c* is growth stage *g*

$ETa_{c,g}$ Sum of daily actual crop evapotranspiration for crop *c* in growth stage *g* (mm)

$ETm_{c,i}$ Sum of daily maximum crop evapotranspiration for crop *c* in growth stage *g* (mm)

The Stewart yield response function does not directly relate crop yield to total water application. Rather, the response of the crop to water deficits is determined by crop yield response factors (Ky) which relate the relative reduction in yield (1-Ya/Ym) to relative evapotranspiration deficit (1-ETa/ETm). ETa is determined in the daily water budget. Crop response factors represent the sensitivity of crops to water stress hence they are crop specific and also vary at each crop growth stage. If irrigation decisions are scheduled taking into account the daily state of the soil water balance and avoiding crop water stress, the actual yield generated is close, if not equal to the potential yield.

3.2.2.2 Pumping hours

Precise quantification of daily pumping hours associated with irrigating a specific amount of irrigation water is essential given the Ruraflex time of use electricity tariffs are differentiated by time of the day and the day of the week (Venter, 2015). Necessary pumping hours are closely linked to the delivery capacity of the irrigation systems which is calculated using the following equation (Burger *et al.*, 2003):

$$RPH_{c,i} = \frac{\frac{IR_{c,i}}{\eta_s} A_c \times 10}{Q} \qquad \text{Equation (3.10)}$$

Where;

$RPH_{c,i}$ Required pumping hours to irrigate crop *c* on day *i* (Hours)

Q Flow rate (m^3/h)

η_s Pumping efficiency

$IR_{c,i}$ Irrigation application for crop *c* on day *i* (mm)

A_c Area planted for crop *c* (hectare)

For a given pivot size, the flow rate will determine how many millimetres of water the irrigation system can apply within a 24-hour period.

3.3 QUANTIFYING ECONOMIC IMPLICATIONS OF IRRIGATION EVENTS

The crop yields and the pumping hours from the previous section are used in the economics module to quantify the economic implications of different irrigation schedules.

3.3.1 GROSS MARGIN CALCULATION

The total gross margin was calculated using the following equation;

$GM_c = \sum_c PI_c - \sum_c YDC_c - \sum_c ADC_c - \sum_c IDC_c$ Equation (3.11)

Where;

PI_c Total production income for crop c (R)

YDC_c Total yield dependent costs for crop c (R)

ADC_c Total area dependent costs for crop c (R)

IDC_c Total irrigation dependent costs for crop c (R)

As indicated in Equation (3.11), the total yield dependent, area dependent and irrigation dependent costs incurred during production are deducted from the gross income to obtain the gross margin (GM) for the farm business. The components of the gross margin are discussed below.

3.3.1.1 Production income

The production income is a function of the yield produced, the size of area utilized for production and the price for the given crop. Equation 3.12 below represents the production income.

$PI_c = Y_c \times p_c \times A_c$ Equation (3.12)

Where;

Y_c Actual yield for crop c (ton/ha)

p_c Price for crop c (R/ton)

A_c Area utilized for production of crop c (ha)

The areas and prices for each crop are included as input parameters in the model. The actual crop yield used to calculate production income is determined with the daily water budget simulation.

3.3.1.2 Yield dependent costs

Yield dependent costs entail all production costs that change as the yield produced change. The cost reduction method developed by Grové (1997) form the basis of the yield dependent costs calculations. The calculation of yield dependent costs is represented by the following equation:

$YDC_c = A_c(vym_c - (ym_c - Y_c)vy_c)$ Equation (3.13)

Where;

vym_c Total yield dependent costs at maximum yield for crop c (R/ton

ym_c Maximum (potential) yield for crop c (ton/ha)

Y_c Actual yield for crop c (t/ha)

vy_c Scaling factor for less than proportional reduction in yield dependent costs for crop c (R/ton)

The total yield dependent costs indicated in Equation 3.13 are calculated using the economic data obtained from enterprise budgets for each crop. The scaling factor enables the calculation of the less than proportional reduction in yield dependent costs if the actual crop yield is less than the maximum potential yield. The following equation is used to calculate the scaling factor;

$$vy_c = \frac{(vym_c - vY_c)}{(ym_c - Y_c)} \qquad \text{Equation (3.14)}$$

Where;

vY_c Total yield dependent costs at actual yield for crop c (R/ton)

The scaling factor represents the fact that yield dependent costs reduce in a disproportional manner as actual yield reduces below maximum potential yield. If for instance, the yield dependent costs per ton of crop c is R500, a reduction of actual crop yield below maximum yield with a ton will not necessarily result is actual yield dependent costs that are R500 lower than the maximum potential yield dependent costs.

3.3.1.3 Area dependent costs

All production costs that change subject to a change in the size of the area under production are known as area dependent costs. The following equation is applicable for the calculation of area dependent costs for each crop;

$$ADC_c = A_c \times va_c \qquad \text{Equation (3.15)}$$

Where:

A_c Area under production of crop c (ha)

va_c Area dependent costs for crop c (R/ha)

As noted before, the area decision is made at the beginning of the production season. The area dependent costs are calculated using the information obtained from the enterprise budgets for each crop. Production costs such as seeds, fertilizers and pesticides costs are amongst the costs that are influenced by the size of the area planted.

3.3.1.4 Irrigation dependent costs

Irrigation dependent costs are costs incurred during crop production that are related to irrigation as represented by Equation 3.13 below.

$$IDC_c = EC_c + LC_c + RMC_c + W_c \qquad \text{Equation (3.16)}$$

Where;

EC_c Total electricity costs for crop c (R)

LC_c Total labor costs for crop c (R)

W_c Total water costs for crop c (R)

RMC_c Total repair and maintenance costs for crop c (R)

Electricity costs are incurred when pumping irrigation water from the source to the field. Total electricity costs comprise of both the variable electricity costs component and the fixed costs component. The charges for both the fixed and variable cost component are dependent on the choice of the electricity tariff plan. The Ruraflex tariff plan was utilized for this study given that the tariff option has an implication on dynamics of water use and is widely utilized by farmers within the study area. Total electricity costs calculations are represented by the formula below (Venter, 2015);

$$EC_c = \sum_{i,t}(ta_{i,t})kWPH_{c,i,t} + \sum_{i,t}(rc_i + dc_i)kWPH_{c,i,t} + \sum_{i,t} tra_{i,t}kvarPH_{c,i,t} + fec$$

Equation (3.17)

Where;

$ta_{i,t}$ Active energy charge for day *i* in timeslot *t* (R/kWh)

rc_i Reliable energy charge for day *i* (R/kWh)

dc_i Demand energy charge for day *i* (R/kWh)

kW Kilowatt requirement (kW)

$PH_{c,i,t}$ Pumping hours on day *i* in timeslot *t* to irrigate crop c (Hours)

$tra_{i,t}$ Reactive energy charge day *i* in timeslot *t* (R/kVARh)

$kvar$ Kilovar (kVAR)

fec Fixed electricity costs (R)

The active, reactive, reliable and network demand energy charges are the different types of variable electricity charges applicable for the Ruraflex tariff option as stipulated by Eskom's charging system (Eskom tariff booklet, 2015/16). The fixed cost component of the Ruraflex tariff option comprises of the network access charge, service charge, administration charge and reactive energy charge. The pump capacity or flow rate, the total pressure or dynamic head and pump and motor efficiencies are essential factors that affect the kilowatt requirement. Likewise, the flow rate, total irrigation water applied and the area under production are the vital factors that influence the required irrigation pumping hours. The calculation of the kVAR is dependent on the distinctive power factor for each pump. The fixed electricity costs which are payable regardless of using the electricity are determined by the tariff plan choice. Irrigation events were scheduled within a two-day cycle.

The incentive of utilising TOU Ruraflex tariff relies on a decision-maker being able to manage their decisions by scheduling irrigation events during off-peak periods where a lower active energy charge is applicable. After the total pumping hours needed within a production season are determined for each crop, the pumping hours allocated for each two-day irrigation cycle are restricted to the total hours available within an irrigation cycle and within the TOU period. The following equations were used to enforce a restriction of pumping hours allocated to each TOU period within a two-day irrigation cycle;

$$PH_{c,i,"off-peak"} = min \left| \begin{matrix} RPH_{c,i} \\ aph_{c,i,"off-peak"} \end{matrix} \right. \quad \text{Equation (3.18)}$$

$$PH_{c,i,"standard"} = min \left| \begin{matrix} RPH_{c,i} - PH_{c,i,"off-peak"} \\ aph_{c,i,"standard"} \end{matrix} \right. \quad \text{Equation (3.19)}$$

$$PH_{c,i,"peak"} = min \left| \begin{matrix} RPH_{c,i} - PH_{c,i,"off-peak"} - PH_{c,i,"standard"} \\ aph_{c,i,"peak"} \end{matrix} \right. \quad \text{Equation (3.20)}$$

Where;

$aph_{c,i,t}$ Available pumping hours during a time of use period (off-peak, standard and peak) when irrigating crop c on day *i* (hours)

$PH_{c,i,t}$ Required pumping hours allocated to a time of use period (off-peak, standard and peak) on day i while irrigating crop c (hours)

The simulation model assumes that irrigation events could occur every consecutive day. Thus, the model implicitly assumes that the irrigation hours could be spread over a two-day period to make better use of the time differentiated electricity tariff structure of Ruraflex. Irrigation hours are firstly allocated to the off- peak period of the first day of the cycle. If the irrigation hours needed for each cycle are greater than the hours available during the off-peak period of day one, the next irrigation event is carried out during the off-peak periods of the second day of the cycle. If all the hours available during the off-peak periods of both days are exhausted, the hours available during the standard periods are then used. Irrigation events will only be carried out during peak periods if the total hours available during off-peak and standards periods have been utilized. The simulation mode therefore allows for irrigation events to be scheduled during a specific time period and a specific day of the week to achieve minimum electricity costs.

The labour costs that are related to irrigation crop production are calculated according to the following formula suggested by Meiring (1989);

$$LC_c = \sum_i \frac{RPH_{c,i}}{24}\, lh\, lw \qquad \text{Equation (3.21)}$$

Where;

Lh Labour hours needed per 24 hours of irrigation for a given center pivot size (hours)

lw Labour wage rate (R/hour)

The system size and the type of the task being carried out determine the amount of labor that is required per hour for irrigation purposes. For this reason, the labor costs are deemed variable. The labor demand is determined for every 24 hours the irrigation system is under operation. The total pumping hours needed and the wage rate are also important for the calculation of total labor costs as indicated in Equation 3.21. Meiring (1989) also proposed the formula below that was used to calculate the repair and maintenance costs that form part of the irrigation dependent costs;

$$RMC_c = \sum_i RPH_{c,i}\, rt \qquad \text{Equation (3.22)}$$

Where;

rt Repair and maintenance tariff per 1000 hours pumped for an irrigation system (R/1000hours)

The pump usage directly influences the repair and maintenance incurred for the pump. The longer the time the pump is in use, the higher the repair and maintenance costs that are likely to be incurred. As indicated in Equation 3.22, the tariff is determined for every 1000 hours the irrigation system is pumping water. The water costs are also an important function of the total irrigation dependent costs. Below is an equation that was used to calculate the water costs;

$$WC_C = \sum_i IR_{c,i} A_c wt \qquad \text{Equation (3.23)}$$

Where;

wt Water tariff (R/mm)

The water tariff determines the monetary value of the water used for irrigation purposes which differs per water user association. Total amount of irrigation water used is vastly influenced by the amount of water needed to compensate evapotranspiration losses and the size of area to be irrigated.

Given the simulated water budget and the subsequent economic implications, the next section presents the inclusion of risk into the model to facilitate the simulation of risk implications of irrigation events.

3.4 SIMULATING RISK IMPLICATIONS OF IRRIGATION EVENTS

Risk enters the simulation model as crop yield risk through different potential crop yields in each state of nature and stochastic weather which determines irrigation management decisions. The fact that irrigation events influence the stock of field water supply that the crop use to satisfy evapotranspiration requirements necessitates a replication of the daily water budget calculations for each state of nature to simulate the impact of changes in the irrigation schedule on the key output variables. Replicating the water budget calculations for each state of nature allows for the determination of an irrigation schedule that will maximise utility irrespective of the occurrence of state of nature. Weather data was available to define 49 states of nature. However, only 12 states of nature were considered in this research to reduce the dimensionality of the simulation model.

Next, the procedure for identifying the 12 weather states through cluster analysis is discussed, followed by a discussion of the procedures used to identify a unique potential crop yield for each state of nature. The last section is devoted to calculation procedures that were used to include risk aversion into the model.

3.4.1 IDENTIFICATION OF WEATHER STATES

Weather data over a period of 49 years was used to identify representative weather states that could be included in the simulation model to quantify the impact of weather risk on irrigation management decision-making. The 49 possible weather states were reduced to 12 representative states through the use of cluster analysis (CA). CA is a recognized statistical classification tool designed to classify multivariate dataset into some number of clusters whose members are more similar to one another than to members of other clusters (Gong and Richman, 1994; Marzban and Sandgathe, 2005). The difference between the weekly average ETm and the weekly average rainfall for both crops represented the weekly average irrigation requirement variable used to cluster the states of nature. Research done by Marzban and Sandgathe (2005) with the main aim of evaluating the applicability of different CA methods concluded that the Ward's method performs the best among hierarchical methods. The Ward's hierarchical cluster method proposed by Ward (1963) that generates clusters by minimizing within-group sum of squares was hence applied. A dendogram presented in Figure 3.1 was extracted to determine the number of clusters and the representative states of nature. States of nature with limited differences on the weekly irrigation requirements were clustered together. The shorter the height difference between the stems of each state, the lesser the differences between them. Weekly irrigation requirement in state of nature 38 greatly varies from the other states as indicated by the highest stem which is the furthest from other states. The distance between every pair of clusters is computed, and the two closest clusters are merged into a single cluster at each iteration. The procedure is then repeated with the new set of clusters. The number of clusters, therefore, begins with N the sample size, and is systematically reduced to one, that is, the entire dataset. The horizontal line within the dendogram represents the cluster distance at which the cluster tree was cut with 12 clusters resulting. The cluster tree was cut at the level indicated to ensure minimum variations within each cluster.

The variation within each cluster are represented in Figure 3.2 depicting the representative state of nature for each cluster. In each cluster, the state of nature with the minimum sum of squared differences with all the states was chosen to represent that cluster. According to Figure 3.2, significant differences between clusters arose from negative weekly average irrigation requirements representing weekly average rainfall that is greater than weekly average ETm for the first half of the production season. The first half of production season represents maize production periods in summer with considerable rainfall levels and limited irrigation requirements. The considerable rainfall during week 10 and 12 received in state of nature 38 is also evident in Figure 3.2. The second half of the production season represents wheat production during winter periods with limited rainfall experienced hence the positive rainfall requirements. Winter rainfall is generally limited in the study area thus the variation on irrigation requirements will also be lesser than in summer.

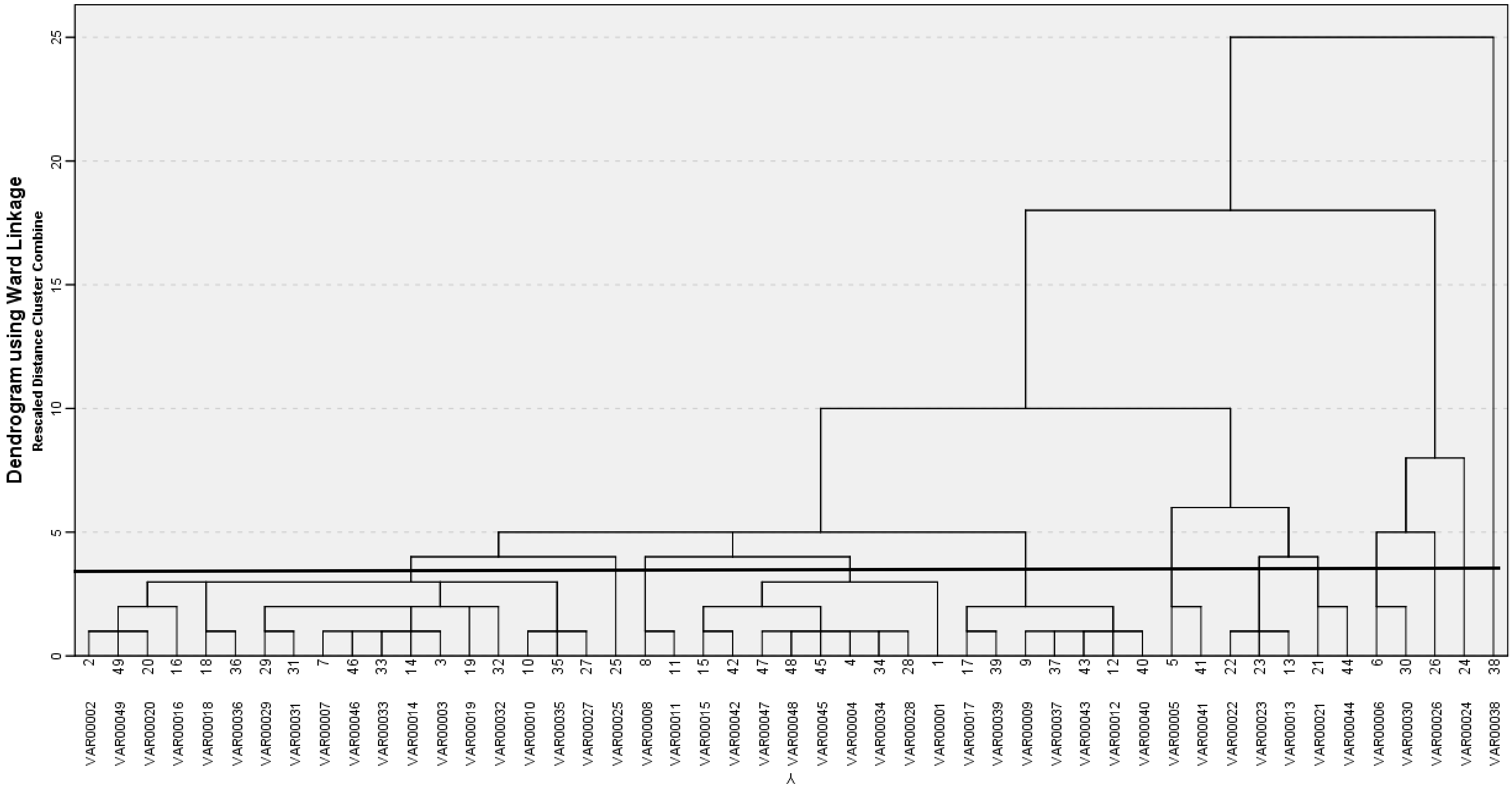

Figure 3.1: Dendogram representing the resulting 12 clusters created based on average weekly irrigation requirement data

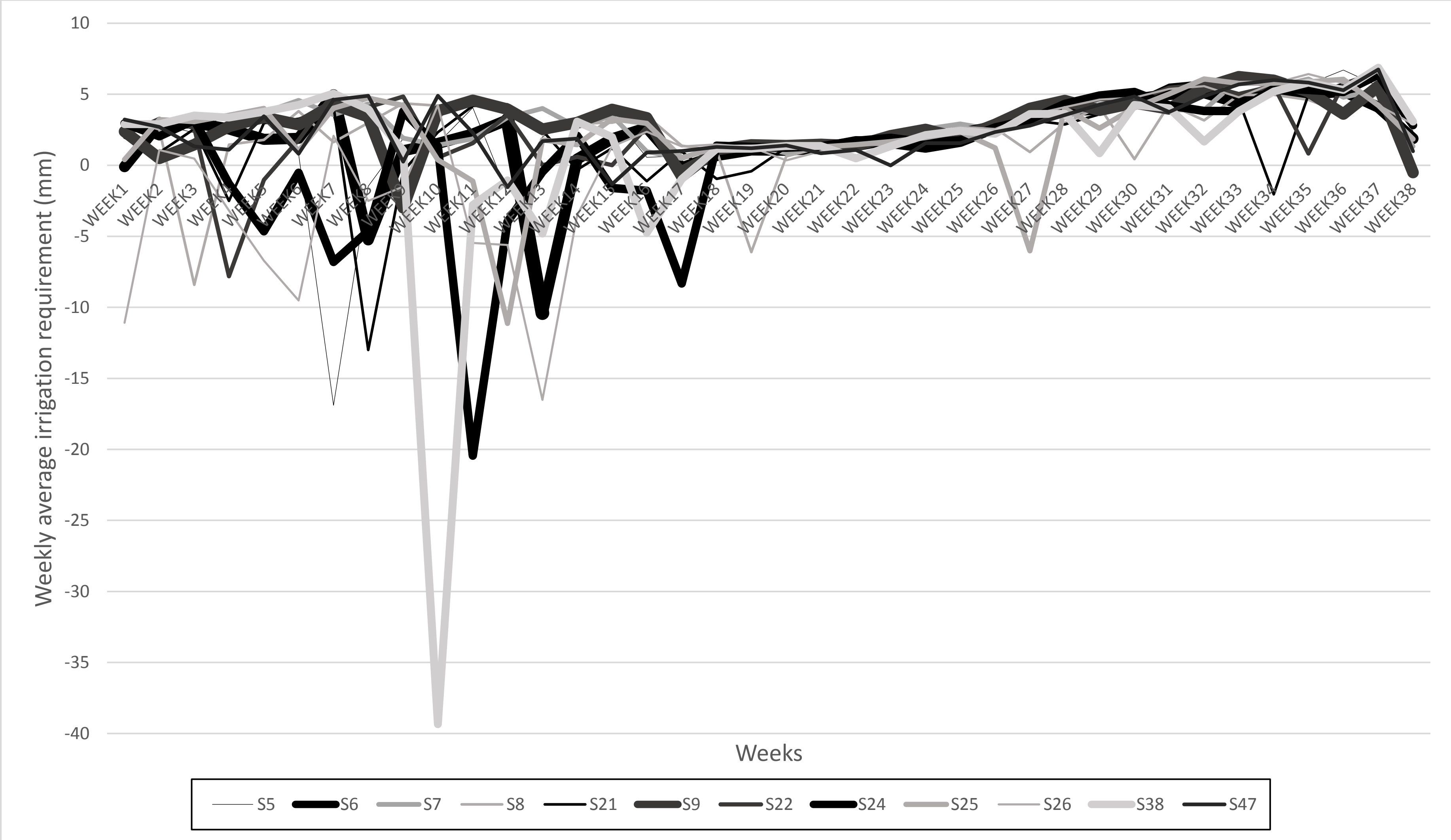

Figure 3.2: Variation of weekly average irrigation requirements for the 12 representative states of nature from each cluster

3.4.2 MAXIMUM POTENTIAL YIELD FOR EACH STATE OF NATURE

A yield index calculation was applied to calculate the maximum potential yield for each crop in each state of nature using the following equation;

$$\mathrm{ym}_{c,s} = \frac{\sum_{c,s}(\mathrm{ETm}_{c,s})}{\sum_{c,s}(\mathrm{ETm}_{c,s} * \mathrm{p}_s)} * yp_c \qquad \text{Equation (3.24)}$$

Where;

$\mathrm{ETm}_{c,s}$ Maximum potential evapotranspiration for crop c for state of nature s (mm)

p_s Probability of state of nature s to occur

yp_c Potential crop yield for crop c (ton/ha)

The potential yield of the research area was adjusted up or down based on the fraction of the specific year's ETm value relative to the average ETm of all the states of nature included in the model. The probability of each state of nature to occur was determined in accordance to the CA results. The number of states in each cluster was used to calculate the probability of each representative state of nature. For instance, if a cluster had three states of nature, the probability of any of the three states to occur will be equal to 0.0612 (3/49) where 49 represents the total number of states included in the historical data.

3.4.3 INCLUSION OF RISK AVERSION

Subjective expected utility theory (SEU) is considered the main theory used as a decision-making guide under risk (Rabin and Thaler, 2001). The SEU allows the decision-maker to assimilate risk levels and the corresponding utility levels. Due to the complex interpretation of utility owing to its ordinal nature, ranking of risky alternatives has shifted from using utility to using stochastic efficiency with respect to a function (SERF) which is based on the idea that using utility to rank risky alternatives is similar to ranking alternatives according to certainty equivalence (CE) (Harder, Richardson & Khalilian, 2004). Harder *et al*. (2004) defines CE as the specific amount measured with the same unit of measurement as the key output variable that has the same utility as the expected utility of the risky alternative. The decision-maker will hence be indifferent between CE and the utility derived from the risky alternative. CE therefore presents an easier way of ranking risk as the alternative with the highest CE is chosen at a specified level of risk aversion. According to Harder *et al*. (2004), the expected utility function of the decision-maker (EU) and the level of absolute risk aversion determine the CE. CE is hence calculated using the following negative exponential utility function if constant absolute risk aversion (CARA) is assumed (Babcook, Kwan and Feinerman, 1993);

$$CE = \frac{-In(EU(x))}{r_a(x)} \qquad \text{Equation (3.25)}$$

Where EU is determined using the following formula;

$$EU(x) = \sum_s P_s \left(e^{-r_a(x)GM_s}\right) \qquad \text{Equation (3.26)}$$

Where;

P_s Probability of occurrence of state of nature s

$r_a(x)$ Absolute risk aversion level

GM_s Gross Margin for alternative state of nature s (R)

The $r_a(x)$ was calculated according to the relationship between $r_a(x)$ and a standardized measure of risk aversion ($r_s(x^s)$) as indicated below;

$$r_a(x) = \frac{r_s(x^s)}{\sigma_x} \qquad \text{Equation (3.27)}$$

Where σ_x represents the standard deviation of the unstandardized data. The minimum and maximum level of $r_s(x^s)$ according to the plausible range for $r_s(x^s)$ between 0 and 2.5 determined by Grové (2010) were used to determine the impact of risk aversion.

3.5 OPTIMISATION PROCEDURE

Excel ® macros were used to command the simulation procedures within both a single-stage decision framework and multi-stage decision framework. The model was applied for two water supply conditions namely full water quota and restricted water quota for the Ruraflex electricity tariff under risk neutrality and risk aversion. The impact of the TOU Ruraflex tariff option on the dynamics of water supply and the fact that the tariff option is utilised by the majority of the farmers in the study area motivates why Ruraflex is the only tariff considered for the analysis. The Excel ® Solver was used to optimise the model using the evolutionary algorithm embedded in Excel ® due to the complexity of the model. A Macro was programmed in Excel VBA (Visual Basic for Applications) to allow the Excel Solver to automatically solve repetitive commands in the model for both water supply scenarios and also to reduce the amount of time consumed optimizing the problem due to the repetitive commands. Two macros were programmed with one solving the risk model within a single-stage decision-making framework and the other solving within a multi-stage decision framework for both a full water quota and restricted water quota supply scenario for a risk neutral and risk averse decision-maker.

Evolutionary algorithms are based on random generations of the decision variables that need to be optimized. As a result, enforcing limits could become difficult. Next, the procedures employed for constraint handling are discussed followed by a discussion of the macros that were used to solve the model under the single and multi-stage sequential decision-making frameworks.

3.5.1 CONSTRAINT HANDLING

Two distinct procedures were employed to place constrain the decision variables. The first procedure generates values between a lower and an upper limit and is easily enforced. Enforcing a constraint on the seasonal water allocation is more difficult since total irrigation water demand is determined by the interaction between area planted and the irrigation schedule. A penalty function was developed for the seasonal water allocation limit to inform the evolutionary algorithm if an unfavourable solution is generated.

3.5.1.1 Lower and upper limits

Under a full water quota, no constraint was enforced for area as full area can be irrigated given the availability of sufficient irrigation water. However, an area constraint was included for the limited water quota scenario. The following equation was used to restrict the area planted for each crop under a limited water quota scenario under a crop rotation system.

$$0 \geq A_C \leq 30.1 \qquad \text{Equation (3.28)}$$

The daily irrigation amounts could either be zero or take a value between a lower limit of 5mm and 24mm based on a two-day cycle and an irrigation system capacity of 12mm/day. IF functions were used to ensure that irrigation events smaller than 5mm were equated to no irrigation.

3.5.1.2 Penalty function

The main purpose of the penalty function is to inform the evolutionary algorithm that a solution was generated that is deemed infeasible. A penalty is calculated and subtracted from the GM if water use exceeds the amount of water allocated. The following exponential equation was used to calculate the penalty;

$$y = (ab^x - 1)s \qquad \text{Equation (3.29)}$$

Where;

y Penalty

a 1

b 100

x $\frac{water\ limit - water\ usage}{100}$

s Scaling factor of 1000

The properties of the function are such that the penalty becomes exponentially larger as the water allocation is exceeded.

3.5.2 SINGLE-STAGE DECISION FRAMEWORK MACRO

Within a single-stage decision framework the assumption is made that the area irrigated and the irrigation schedule for the whole season are determined at the beginning of the season when the weather for the rest of the season is unknown. As a result, the optimisation will determine the irrigation schedule that will maximise the CE irrespective of the state of nature occurring. Figure 3.3 explains the general purpose of the macro that was used to optimise the area irrigated and the irrigation schedules for the two crops while assuming a single-stage decision-making framework.

The first section of the macro allows the initialisation of ETm and Rain for each state of nature for the two crops. Initialisation is vital to ensure that the initial data for these two parameters that significantly influence the amount of irrigation water is retained in the model before any optimisation can proceed. Also, given that the model is run more than once to ensure the best near optimal solution is achieved, retaining the initial data before each optimisation is imperative. Before the Excel Solver can begin solving, the base sheet where all calculations are done is defined together with the cell to be optimised. Furthermore, the constraint sets the optimisation is subject to are also defined. Solver implements a GA technique procedures to achieve the near optimal solution. The optimisation is repeated as long as the generated solution is greater than the previous solution. The optimal solution is achieved when the same CE is achieved. The optimality of the solution is tested by rerunning the optimisation but with a different mutation rate. The single-stage decision framework model runs for approximately 30 minutes. It is important to note that the macro sections are similar for both water supply scenarios. However, the macro for a limited water quota scenario includes an additional constraint of area planted not to exceed full area achieved under a full water quota given the limited seasonal irrigation water amount.

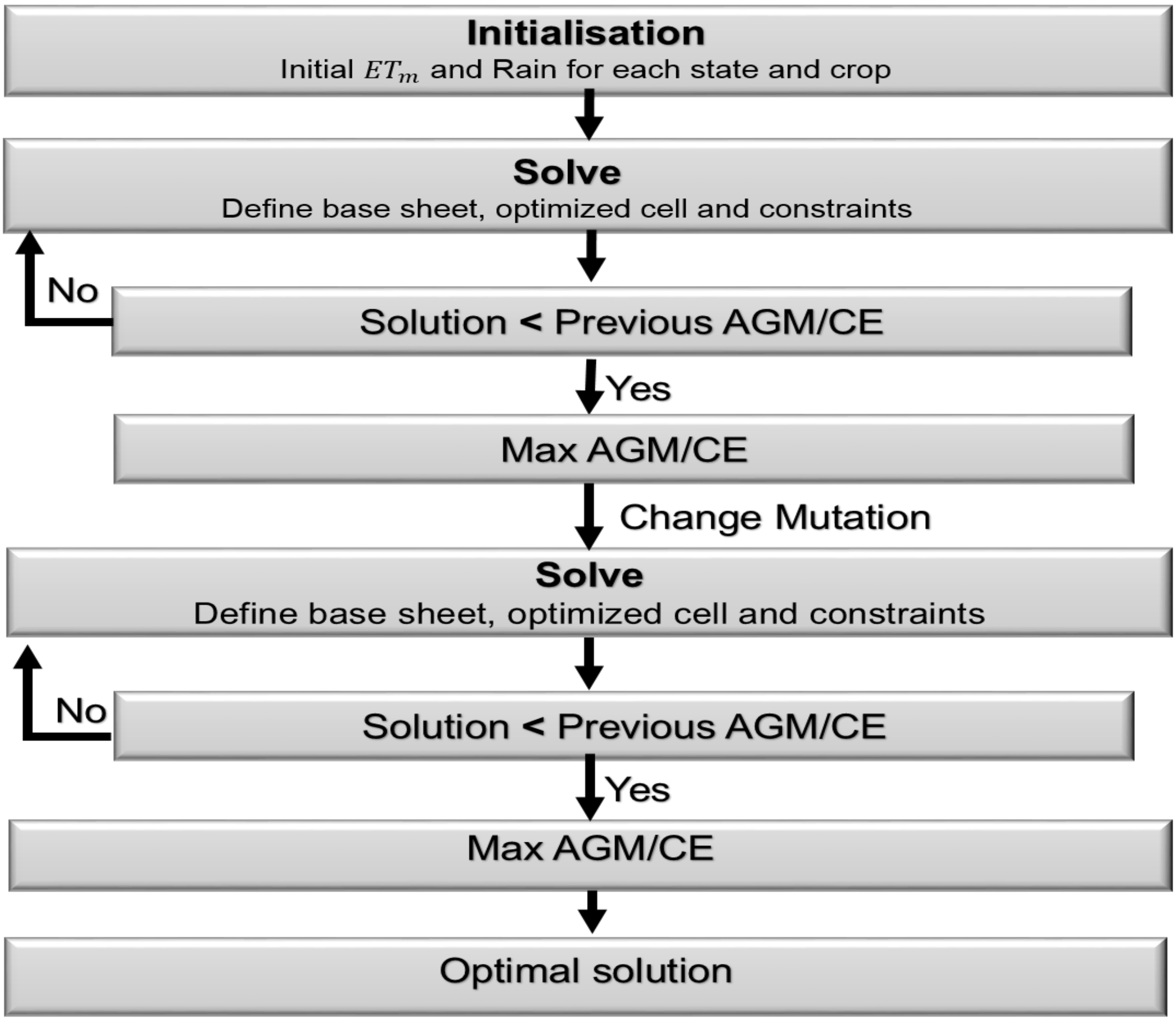

Figure 3. 3: Schematic representation of components of the single-stage decision-making framework solution macro

3.5.3 MULTI-STAGE DECISION FRAMEWORK MACRO

The multi-stage decision-making framework macro was programmed to facilitate recursive programming that allowed the gradual adaptation to different rainfall and ETa levels in each state of nature on a weekly basis. Within a multi-stage decision-making framework, the area decision is made in the first stage and the irrigation scheduling decisions are made in the second stage of decision-making. The multi-stage decision framework macro is a further development of the base macro to implement some sort of adaptive behavior of real time information as it becomes available. The model is based on the recursive stochastic programming (RSP) methodology developed by Blanco and Flichman (2002) as an alternative of solving complex dynamic problems that overcomes the curse of dimensionality. In contrast to the procedure followed in dynamic programming, RSP solves dynamic problems by means of forward recursion to facilitate an adjustment between real-life situations and optimal situations through gradual adaptation of changes of exogenous parameters. Important to note, the first stage of the multi-stage decision framework model to make area decisions is the same as that for the single-stage decision-making framework model. The second stage of the multi-stage decision framework facilitates sequential irrigation scheduling decisions as the model is updated with real-time data of rainfall and ETm in successive weeks. Modelling decisions within a multi-stage decision framework results in a specific irrigation schedule determined for each state of nature in contrast to the single irrigation schedule generated within a single-stage decision framework.

The components of the multi-stage decision-making framework macro are represented in Figure 3.4. As noted in Figure 3.4, the first component of the macro includes the defining of all the states included in the set of states to allow the updating to occur in each state defined. Likewise, the macro also allows the initialisation of ETm and rain to also retain the initial data before solving the model. The optimisation for the multi-stage decision-making framework requires a separate optimisation for each state of nature hence it was imperative to define the state assumed to occur for each optimisation. A cumulative distribution of plant days (CPD) was used to set the fixing ranges for each updating for each crop. The CPD range was set to be greater than the first day of planting for each crop (t1) and end on the last day of planting for each crop. The updating for both crops was however set to end when the value in the CPD range was equal to zero hence the last value in the CPD was set at zero. ETm and rainfall information was updated for each state with the data of the state assumed to occur.

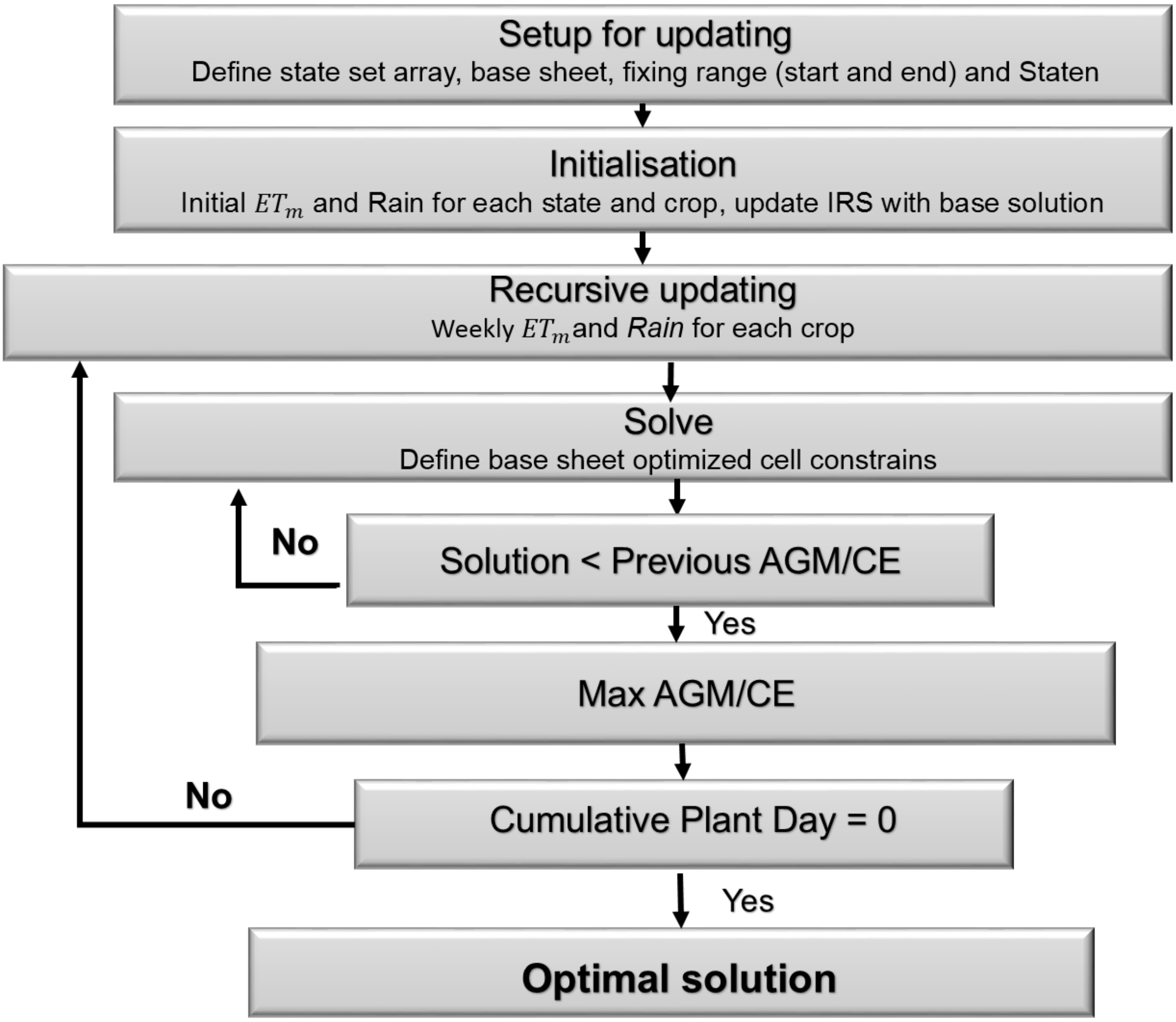

Figure 3. 4: Schematic representation of components of the multi-stage decision-making framework solution macro

The multi-stage updating macro commands the optimisation of the model after each weekly update of ETm and Rain data of all the other states with that of the state assumed to occur till CDP is equal to zero. The recursive updating procedure occurs after each optimisation until the CPD is equal to zero. The optimisation was repeated until the maximum CE is achieved. The multi-stage decision framework model runs for approximately 8 hours for each of the 12 states of nature. After completing all 12 recursive optimisations, the GMs resulting from each optimisation were combined to generate the distribution of gross margin risk associated with the multi-stage decision-making framework.

3.6 DATA REQUIREMENTS AND INPUT PARAMETER CALCULATIONS

Secondary economic, agronomic and irrigation dependent data was used to set up the Excel SWIP-E model. Some input parameters were calculated separately before they were used in the model. The following sections discussed the sources of the secondary data used and input parameter calculations.

3.6.1 ECONOMIC INPUT DATA

Some of the economic data utilised in the model was obtained from the cost guide published by Griekwaland-Wes Korporatief (GWK) (2016). Input data such as the crop price, area and target yield for both maize and wheat were extracted from the enterprise budgets of the cost guide. The maximum potential yield for each crop in each state of nature was calculated using the yield index method. The cost reduction method developed by Grové (1997) form the basis of the yield dependent costs calculations. A scaling factor was used to calculate the actual yield dependent costs according to a method proposed by Venter (2015). The economic data used in the model for both maize and wheat is represented below;

Table 3. 1: Economic input data for maize and wheat, 2016

	MAIZE	**WHEAT**
Crop price (R/ton)	4 840	4 800
Area dependent costs (R/ha)	9 077.34	6 298.34
Target Yield costs (R/ton)	11 710	9 939.10
Scaling factor (R/ton)	1 106.53	1 034.5

Source: GWK cost guide, 2016 and own calculations

3.6.2 AGRONOMIC INPUT DATA

Agronomic input data includes weather related, soil, water allocation, root growth and yield response factors data. The agronomic data is used for water budget calculations in the model. Weather related input parameters such as the ETO, Kc and rainfall were obtained from SAPWAT3 (South African Procedure for estimating irrigation Water requirements) (Van Heerden, 2015). Weather data extracted from the V13D weather station was used in SAPWAT3 to estimate the daily ETO, Kc and rainfall for each crop for a growing period of 120 days and

148 days for maize and wheat respectively over a period of 49 years. As aforementioned, the ETm was calculated as a function of the ETO and Kc for each state of nature. A soil with a water holding capacity (WHC) and a depth of 130mm/m and 1.2m respectively is used in the model. To initialise the BRWC and RWC for the first day of the water budget, a 50% initial soil water depletion was assumed. The yield response factors (Ky factors) and the length of growth stages (Ky days) as proposed by Doorenbos and Kassam (1979) were used in the model. The length of Kc and Ky days and yield response factors for each growth stage are presented in Table 3.2 for both crops.

Table 3. 2: Length of Kc and Ky days and yield response factors for the different crop growth stages for maize and wheat.

GROWTH STAGES					
		Initial	**Crop development**	**Mid-season**	**Late season**
Length of Kc-days	**Maize**	21	26	63	10
	Wheat	28	47	63	10
Length of Ky-days	**Maize**	50	15	45	10
	Wheat	91	17	30	10
Ky-Factors	**Maize**	0.4	1.5	0.5	0.2
	Wheat	0.2	0.6	0.5	0.1

3.6.3 IRRIGATION DEPENDENT INPUT DATA

The electricity tariffs, water tariff, labour wage rate, irrigation system design and repair and maintenance data were important inputs in the model for the calculation of irrigation dependent costs. The Ruraflex tariffs obtained from Eskom (2015/16) were used to calculate the electricity costs given the total pumping hours and the kilowatt usage. The Ruraflex tariff option chosen is based on transmission zone ranging between 300km and 600km, a voltage of less than 500V and a monthly utilised capacity ranging between 100kVA and 500kVA. Both the fixed and the variable cost components of both tariff options were used in the calculations. The tariffs used are presented in Table 3.3. The peak, standard and off-peak time of use periods entail periods of high, medium and low energy costs respectively. During the weekdays, total available hours for the peak, standard and off-peak periods are 5hours/day, 11hours/day and 8hours/day respectively for both low and high demand season. A total of 17hours/day and 7hours/day are available on Saturdays during the off- peak and standard time slots respectively with no peak periods available for both seasons as well. An entire day of off-peak period is available on Sundays. The reactive energy charge is only applicable during the high demand season hence only applicable for wheat. A minimum wage determined by DOL (2014) of R12.41per hour was used in the model accounting for 0.58 labour hours for every 24 hours. The repair and maintenance tariff of R0.413217 expressed per 1000 hours pumped base on a method proposed by Meiring (1989) was used.

Table 3. 3: Ruraflex Electricity tariffs applicable to the Douglas area, 2015/2016

<table>
<tr><th colspan="4">Variable Electricity Costs Tariffs</th></tr>
<tr><td rowspan="6">Active Energy charge (c/kWh)</td><td rowspan="3">High Demand (June- August)</td><td>Off-Peak</td><td>48.83</td></tr>
<tr><td>Standard</td><td>89.91</td></tr>
<tr><td>Peak</td><td>296.80</td></tr>
<tr><td rowspan="3">Low Demand (September- April)</td><td>Off-Peak</td><td>42.28</td></tr>
<tr><td>standard</td><td>66.63</td></tr>
<tr><td>Peak</td><td>96.81</td></tr>
<tr><td colspan="3">Reliability Service Charge (c/kWh)</td><td>0.38</td></tr>
<tr><td colspan="3">Network Demand Charge (c/kWh)</td><td>24.16</td></tr>
<tr><td rowspan="2">Reactive Energy Charge (c/kVArh)</td><td colspan="2">High Demand (June-August)</td><td>8.16</td></tr>
<tr><td colspan="2">Low demand (September- April)</td><td>0</td></tr>
<tr><th colspan="4">Fixed Electricity Costs Tariffs</th></tr>
<tr><td colspan="3">Network access Charge (R/KVA/month)</td><td>17.02</td></tr>
<tr><td colspan="3">Service Charge (R/Account/day)</td><td>56.93</td></tr>
<tr><td colspan="3">Administration Charge (R/POD/day)</td><td>26.39</td></tr>
</table>

A volumetric based water tariff of R0.716 based on the Van der Kloof water user association was used. The water allocation for the Douglas area is 1 000mm/ha. The design data of a 30.1 ha center pivot with a delivery capacity of 12mm/day used in the model was obtained from Myburgh (2014) as indicated in the table below.

Table 3. 4: Irrigation system design parameters of the infield irrigation system

Center pivot size (ha)	30.1
Delivery capacity (mm/day)	12
Flow rate (m^3/h)	150.5
Center pressure (m)	24.1
Efficiency (%)	0.775
Kilowatt pump (kW)	24.9
Kilovar (kVAR)	14
Kilovolt-ampere (kVA)	50

Source: Myburgh (2014)

CHAPTER 4
RESULTS AND CONCLUSIONS

4.1 INTRODUCTION

In this chapter, the results of the risk simulation model applied to achieve the objectives of the study are presented in three main sections. The first section presents the results of the gross margin variations and the responses of a decision-maker within a single-stage and multi-stage decision-making framework taking risk into account under a full water quota and a restricted water quota scenario. The second section presents results of the estimated value or benefit of considering a multi-stage decision-making framework within the two alternative water supply scenarios and risk preferences of the decision-maker. Thereafter, the results of the cost of a water restriction are presented in the third section. All the results presented are for a combined inter-seasonal production of maize and wheat.

4.2 MODELLING IRRIGATION DECISIONS

The manner in which decision-makers make area and irrigation water scheduling decisions can play a significant role on the resulting distribution of expected gross margins and the risk they face. Irrigation decisions are generally modelled within a single-stage decision-making framework (SSDF) where area and irrigation scheduling decisions are considered as a single decision. However, in reality, irrigation farmers make decisions within a multi-stage decision-making framework (MSDF) where area decisions are considered in the first stage and sequential irrigation scheduling decisions in the second stage to facilitate the optimal interaction of water availability, area and irrigations schedules. The results of the decisions made within a SSDF and a MSDF are presented below for the two alternative water supply scenarios. Special attention was given to the resulting gross margin (GM) variability and the responses of the decision-maker within each decision-making framework. The standardized measure of risk aversion $r_s(x^s)$ was set at 0 and 2.5 for a risk neutral and a risk averse decision-maker respectively in accordance to the minimum and the maximum of the plausible range for

$r_s(x^s)$ according to Grové (2008). Cumulative distribution functions (CDF) are used to quantify the level of gross margin risk faced by a risk neutral (RN) and a risk averse (RA) decision-maker within the two alternative decision-making frameworks for both a full water quota (FQ) and restricted water quota (RQ) scenario. The resulting gross margins (GMs) for each of the 12 states nature under each water supply scenario are used to graph the CDF.

4.2.1 SINGLE-STAGE DECISION FRAMEWORK

The resulting GM variabilities and the responses of the decision-maker under the two alternative water supply scenarios and risk preferences within a SSDF are presented below.

4.2.1.1 Gross margin variability

The resulting GM variations within a SSDF and how these variations alter under a FQ and a RQ water supply scenario taking risk preferences into consideration are depicted in Figure 4.1. As highlighted in Figure 4.1, a similar distribution of GMs results under both water supply scenarios and risk preferences with a notable lower tail. A distribution with a range and a standard deviation of 468 885 and 113 248 respectively is generated under a FQ scenario and 500 410 and 113 508 respectively under a RQ scenario for a RN decision-maker. Considering an extreme level of risk aversion resulted in a relatively lower range and standard deviation of 429 029 and 99 997 respectively under a FQ scenario and 409 454 and 90 664 respectively under a RQ scenario. The lower range and standard deviation under risk aversion indicate a smaller likelihood of the actual GMs to differ from the expected GMs. The lower tail comprises of expected GMs with a cumulative distribution probability of 30% and lower while the upper tail comprises of expected GMs with a cumulative distribution probability of more than 30%. A relatively larger range of 413 138 is noted under a RQ for the lower tail in comparison to a range of 356 760 noted under a FQ if risk neutrality is assumed. Also, an upper tail range noted under a RQ of 87 272 is smaller than that of 112 125.under a FQ.

However, under risk aversion, a shift of the CDFs to the left results in a reduction of the range of expected GMs for the lower tail more significantly under a RQ. In addition, an increase of the range of the upper tail is noted under a RQ in contrast to the reduced range under a FQ. The more evident improvement of GMs on the lower tail for the RQ scenario entail a significant sacrifice of the GMs on the upper tail under risk aversion within a RQ scenario hence the significant shift of the upper tail to the left.

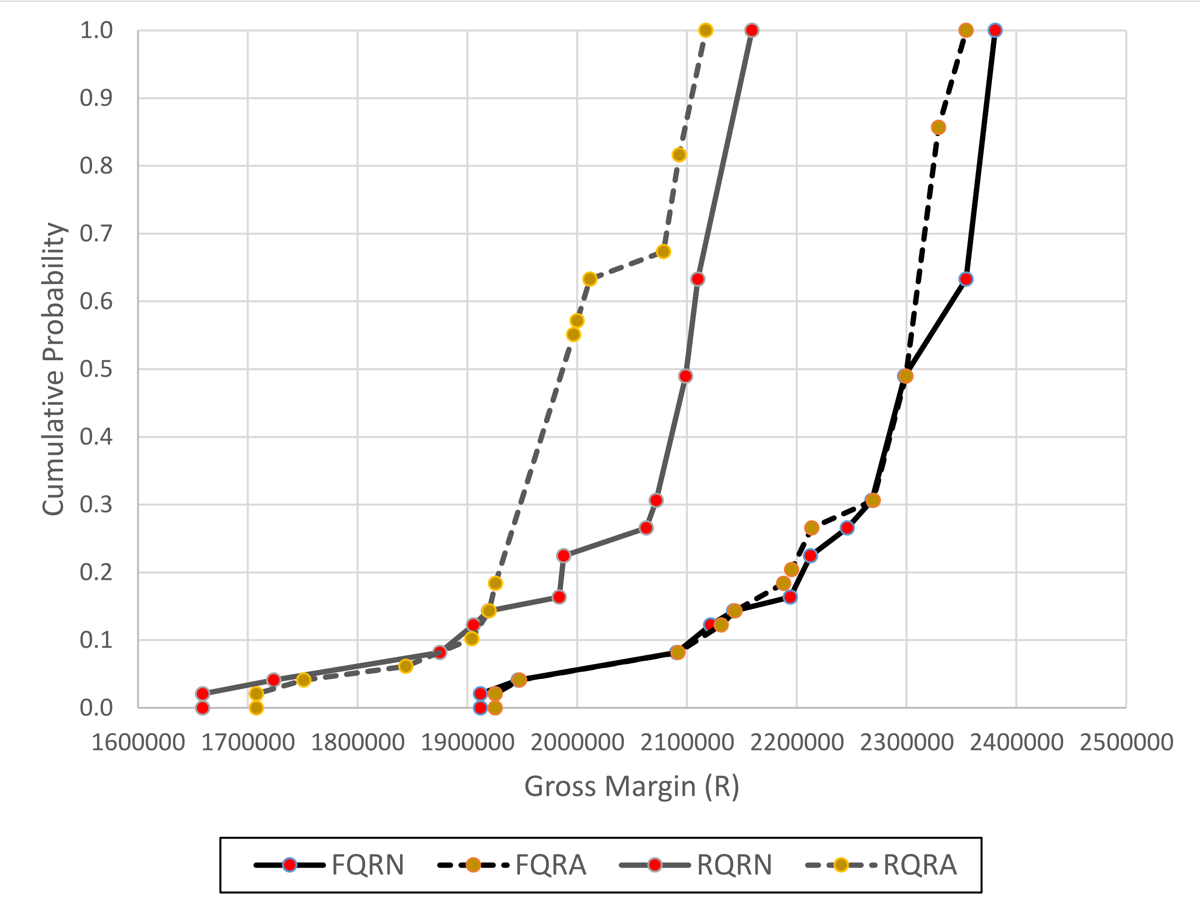

Figure 4. 1: Gross margin variability for a risk neutral (RN) and risk averse (RA) decision-maker within a single-stage decision-making framework for a full water quota (FQ) and a restricted water quota (RQ) scenario

4.2.1.2 Responses

The resulting variations of the GMs within a SSDF are attributed to the response of the decision-maker with regards to areas planted, irrigation water use and the resulting crop yields under the two alternative water supply scenarios and risk preferences. Given a single average best irrigation schedule is applied within a SSDF over all 12 states of nature, an over or under irrigation may result in other states hence a subsequent impact on the resulting yields and irrigation dependent costs. The resulting yield for each state thus has a significant impact on the resulting GM given that only production income (PI) and yield dependent costs could differ for each state as the irrigation schedule was kept constant across all states implying similar irrigation dependent costs. The resulting PI and yield dependent costs will determine how the level of GMs differs for each state within a SSDF. The responses of a decision-maker under each water supply scenario taking risk into account are presented next.

4.2.1.2.1 Full water quota

Table 4.1 indicates the responses of a risk neutral and risk averse decision-maker in terms the amount of irrigation water applied and the corresponding yields under a FQ scenario. Under a FQ scenario, maize and wheat are produced under full areas of 30.1 hectares and 30.1 hectares respectively regardless of the risk preference. In addition, the actual average yield of 17t/ha for maize was equal to the average potential yield while the actual average yield of 7.99t/ha for wheat was almost equal to the average potential yield of 8t/ha as indicated in Table 4.1.

However, taking risk aversion into account under a FQ scenario resulted in a reduction of the average actual yield of maize to 16.81t/ha while the average actual yield for wheat was kept constant. Also, to note, the total amount of irrigation water used under a FQ scenario reduced from 9055 for a risk neutral decision-maker to 8860m^3 for a risk averse decision-maker. The reduction in the average maize yield and the amount of irrigation water used owes to the extreme level of risk aversion used for the analysis with the improvement of the lower tail of the GM distribution considered imperative.

As noted in Figure 4.1, a distribution of higher minimum and lower maximum GMs is generated for both water supply scenarios when risk aversion is considered. Under extreme risk aversion levels, substantial emphasis is placed on an improved lower tail of the CDF hence improved minimums are preferred. A slight improvement in GMs for states of nature forming the lower tail of the risk averse CDF is hence noticed, while GMs of states forming the upper tail are reduced. Almost, if not, full potential yields were achieved in each state for both crops for a risk neutral and a risk averse decision-maker regardless of the reduced water use under risk aversion.

Table 4. 1: Optimized irrigation water use, crop yields and the total gross margin for maize and wheat in each state of nature within a single-stage decision-making framework for a risk neutral and risk averse decision-maker under a full water quota scenario, 2016

SINGLE-STAGE DECISION FRAMEWORK (FULL WATER QUOTA)								
	Risk Neutral				**Risk Averse**			
State	**Irrigation (m^3)**	**Maize yield (t/ha)**	**Wheat yield (t/ha)**	**Total Gross Margin(R)**	**Irrigation (m^3)**	**Maize yield (t/ha)**	**Wheat yield (t/ha)**	**Total Gross Margin (R)**
1	9055	15.58	8.23	2121460	8860	15.58	8.31	2131069
2	9055	16.03	7.58	2090756	8860	16.03	7.58	2091906
3	9055	17.61	7.99	2380657	8860	17.14	7.99	2329141
4	9055	17.39	7.29	2245992	8860	16.86	7.29	2187930
5	9055	17.11	8.32	2354040	8860	17.10	8.32	2354287
6	9055	16.83	8.00	2268532	8860	16.83	8.00	2269682
7	9055	16.21	8.24	2212365	8860	16.21	8.24	2213516
8	9055	14.51	8.12	1946196	8860	14.51	8.12	1947347
9	9055	16.23	8.09	2194099	8860	16.23	8.09	2195249
10	9055	13.98	8.38	1911772	8860	13.98	8.49	1925257
11	9055	16.10	7.87	2142381	8860	16.10	7.87	2143531
12	9055	17.24	7.81	2298310	8860	17.23	7.81	2299460
Average	**9055**	**17**	**7.99**	**2291835**	**8860**	**16.81**	**7.99**	**2271690**

The insignificant, if any, deviations in yields achieved between the two alternative risk preferences as indicated in Table 4.1 entail an over irrigation if risk neutrality is assumed. Though, a reduction in the amount of irrigation water used is coupled by a slight increase in the yield of wheat for states of nature 1 and 10, an increase in GMs under risk aversion is substantially attributed to reduced irrigation dependent costs given the reduction in irrigation water use. Also, a slight decrease in maize yield for states of nature 5 and 12 is noted under risk aversion though higher GMs still result as a result of the reduced water use. Nevertheless, a reduction in maize yields for states of nature 3 and 4 which form part of the upper tail of the risk averse CDF resulted in reduced GMs regardless of the reduced irrigation water use. The negative effect of the reduced PI due to the lower yields for states of nature 3 and 4 is thus greater than the positive impact of reduced irrigation dependent costs on GM under risk aversion. Given the responses under a FQ for the two alternative risk preferences, the responses under a restricted quota scenario are discussed in the next section.

4.2.1.2.2 Restricted water quota

Irrigation decisions made within a SSDF under a restricted water quota scenario resulted in a full area production of 30.1 hectares for maize for both a risk neutral and a risk averse decision-maker. However, wheat production is reduced to 20 hectares and 20.7 hectares for a risk neutral and risk averse decision-maker respectively. The reduction in area under production for wheat resulted in relatively lower GMs generated owing to the lower gross incomes. A distribution of GMs under a RQ scenario is hence represented by cumulative distribution functions lying to the left of the FQ cumulative distribution functions as indicated in Figure 4.1. The responses of a risk neutral and a risk averse decision-maker with regards to areas, water use and resulting yields under a RQ scenario are represented in Table 4.2. According to Table 4.2, average actual yields of 16.96t/ha and 16.2t/ha for maize and 7.88t/ha and 7.99t/ha for wheat are generated for a risk neutral and a risk averse decision-maker respectively when a water restriction is enforced. The resulting average yields for both crops are slightly lower than the average potential yields of 17t/ha and 8t/ha for maize and wheat respectively apart from maize yield for a RA decision-maker which is significantly lower than the average potential yield.

Table 4. 2: Optimized irrigation water use, crop yields and the total gross margin for maize and wheat in each state of nature within a single-stage decision-making framework for a risk neutral and risk averse decision-maker under a restricted water quota scenario, 2016

SINGLE-STAGE DECISION FRAMEWORK (RESTRICTED WATER QUOTA)								
	Risk Neutral				**Risk Averse**			
State	**Irrigation (m^3)**	**Maize yield (t/ha)**	**Wheat yield (t/ha)**	**Total Gross Margin (R)**	**Irrigation (m^3)**	**Maize yield (t/ha)**	**Wheat yield (t/ha)**	**Total Gross Margin (R)**
1	8355	15.58	7.85	1874834	8350	15.58	8.30	1925698
2	8355	16.03	7.58	1905485	8350	16.03	7.58	1919710
3	8355	17.52	7.91	2159168	8350	15.88	7.99	1996650
4	8355	17.36	7.29	2071895	8350	15.75	7.29	1904165
5	8355	17.11	8.01	2109910	8350	16.60	8.32	2093048
6	8355	16.83	8.00	2062835	8350	16.83	8.00	2078531
7	8355	16.21	8.14	1987600	8350	16.21	8.24	2011695
8	8355	14.51	7.96	1723523	8350	14.51	8.12	1751041
9	8355	16.23	8.09	1983695	8350	16.23	8.09	2000069
10	8355	13.98	8.01	1658757	8350	13.98	8.43	1707693
11	8355	15.89	7.87	1919139	8350	15.08	7.87	1843990
12	8355	17.23	7.76	2098877	8350	17.23	7.81	2117148
Average	**83505**	**16.96**	**7.88**	**2073516**	**8350**	**16.2**	**7.99**	**2013050**

In addition, irrigation water use slightly decreased when risk aversion is considered. Important to note is the significant variations of yields per state to facilitate the generation of higher minimums and lower maximum as preferred by a risk averse decision-maker. Though the slight decrease in water use in each state contributed to improve GMs under risk aversion, higher GMs significantly resulted from the increase in yields for wheat as noted for states of nature 1,3,5,7,8,10 and 12 as indicated in Table 4.2. The higher GMs at the lower tail of the risk averse CDF thus owe to the higher gross income that resulted from improved wheat yields. On the contrary, a reduction in maize yield for states of nature 3,4,5 and 11 resulted in lower GMs generated for the upper tail of the risk averse CDF. For states of nature 3 and 5, the reduction in PI due to lower maize yields was greater than the increase in gross incomes due to higher wheat yields hence the resulting lower GMs.

In light of the noted responses of decision-makers and the corresponding variations in GMs under a RQ and a FQ within a SSDF, the conclusion is that risk aversion does not significantly impact the resulting area and yields under a full water quota. However, irrigation water use is significantly reduced in effort to increase the minimum outcomes. Under a RQ, a significant difference between yields generated for a risk neutral and for a risk averse decision-maker are noted hence implying the noteworthy impact of risk aversion on responses of a decision-maker when water supply is restricted. Though water is considered a risk reducing input, the extreme level of risk aversion resulted in a reduction of irrigation water applied in an effort to improve minimum expected values.

4.2.2 MULTI-STAGE DECISION FRAMEWORK

The resulting GM variabilities and the responses of the decision-maker under the two alternative water supply scenarios and risk preferences within a SSDF are presented in the following section.

4.2.2.1 Gross margin variability

The resulting GM variations within a MSDF and how these variations alter under a FQ and RQ water supply scenario taking risk preferences into consideration are depicted in Figure 4.2.

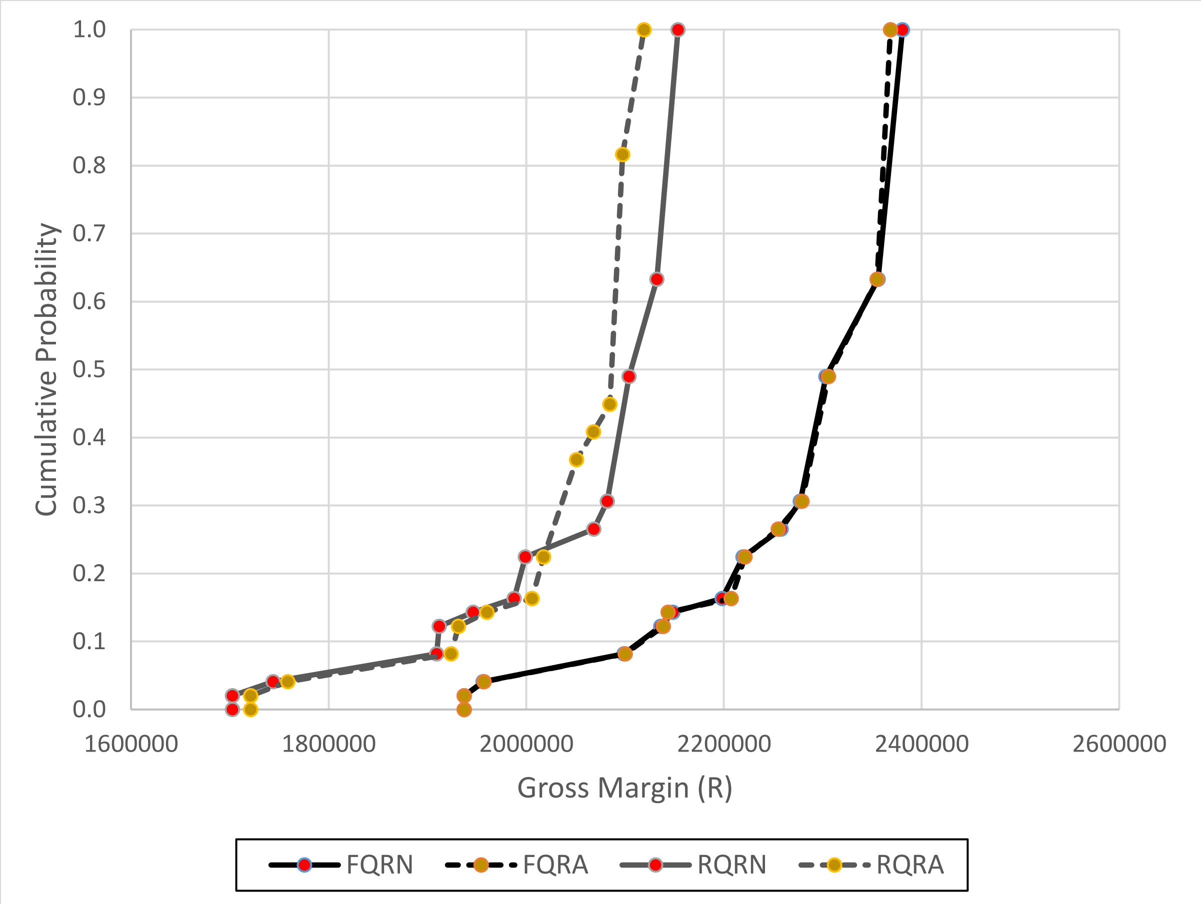

Figure 4. 2: Gross margin variability for a risk neutral (RN) and risk averse (RA) decision-maker within a multi-stage decision-making framework (MD) for a full water quota (FQ) and a restricted water quota (RQ) scenario

A similar distribution of expected GMs is noted for both a FQ and a RQ scenario within a MSDF as depicted in Figure 4.2. A distribution with a range and a standard deviation of 443 379 and 108 829 respectively is generated under a FQ scenario while 451 072 and 104 812 respectively is generated under a RQ scenario for a risk neutral decision-maker. Considering an extreme level of risk aversion resulted in a relatively lower range and standard deviation of 431 195 and 105 066 respectively under a FQ scenario and 397 954 and 84 660 respectively under a RQ scenario. Similarly, the lower tail of the CDFs comprises of expected GMs with a cumulative distribution probability of 30% and lower while the upper tail comprises of expected GMs with a cumulative distribution probability of more than 30%. A relatively larger range of 379 545 is noted under a RQ for the lower tail in comparison to a range of 340 160 noted under a FQ if risk neutrality is assumed. Also, an upper tail range noted under a RQ of 71 526 is smaller than that of 103 219 under a FQ for a risk neutral decision-maker. However, under risk aversion, a shift of the CDFs to the left results in a significant reduction of the range of expected GMs for the lower tail under a RQ while a slight increase of the range is noted under a FQ. Furthermore, an increase of the range of the upper tail is noted under a RQ in contrast to the reduced range under a FQ. The impact of risk aversion indicated by the shifts of the CDF to left is more visible within a SSDF compared to under a MSDF implying a lesser significant impact of risk aversion within a MSDF.

4.2.2.2 Responses

As aforementioned, the resulting variations in the GMs within a MSDF are also attributed responses of a decision-maker in terms of areas planted, irrigation water use and the resulting crop yields under the two alternative water supply scenarios and risk preferences. Given a state specific irrigation schedule is generated for each state of nature within MSDF, irrigation dependent costs and the subsequent yields have a significant impact on the resulting GM for each state. The responses of a decision-maker under a FQ and a RQ scenario within a MSDF are discussed next.

4.2.2.2.1 Full water quota

The areas under production for both maize and wheat generated within a SSDF correspond to those generated within a MSDF as the first stage of the MSDF is the same as single-stage within a SSDF. Full areas of 30.1 hectares for maize and 30.1 hectares for wheat are hence planted regardless of the risk preferences. Table 4.3 indicates the responses of a risk neutral and risk averse decision-maker in terms the amount of irrigation water applied and the corresponding yields under a FQ scenario. The actual average yields of 16.99t/ha for maize and 7.99t/ha for wheat were almost equal to the average potential yields of 17t/ha and 8t/ha for maize and wheat respectively if risk neutrality is assumed as indicated in Table 4.3. A slight reduction of actual average maize yield to 16.95t/ha under risk aversion is noted while the average actual yield for wheat remains constant for the two alternative risk preferences. In addition, a reduction of the total amount of irrigation water used per state under FQ reduced from 8389.40m^3 for a risk neutral decision-maker to 8128m^3 for a risk averse decision-maker. The reduction in the average maize yield and the amount of irrigation water used owes to the extreme level of risk aversion used for the analysis with the improvement of the lower tail of the GM distribution considered imperative. As expected under risk aversion, a distribution of slightly higher minimum GMs and lower maximum GMs is generated for a risk averse decision-maker as indicated in Figure 4.2. A significant reduction of irrigation water applied for all the states of nature result under risk aversion. The relatively higher GMs generated under risk aversion hence resulted from reduced irrigation dependent costs as the yields in each state vary slightly between the two risk preferences as highlighted in Table 4.3. A reduction in irrigation water in an effort to increase the minimum expected values however also resulted in a reduction maize yield for states of nature 3, 4, 5 and 11. Lower GMs thus resulted for the aforementioned states due to the lower PI, regardless of the reduced irrigation dependent costs. A noteworthy trend of relatively higher GMs resulting from reduced irrigation water use under risk aversion can be deduced from Table 4.3 with the exception of states of nature 3, 4 and 5 where a water reduction is coupled by a reduction in maize yield.

Table 4. 3: Optimized irrigation water use, crop yields and the total gross margin for maize and wheat in each state of nature within a multi-stage decision-making framework for a risk neutral and risk averse decision-maker under a full water quota scenario, 2016

MULTI-STAGE DECISION FRAMEWORK (FULL WATER QUOTA)								
	Risk Neutral				Risk Averse			
State	**Irrigation (m^3)**	**Maize yield (t/ha)**	**Wheat yield (t/ha)**	**Total Gross Margin (R)**	**Irrigation (m^3)**	**Maize yield (t/ha)**	**Wheat yield (t/ha)**	**Total Gross Margin (R)**
1	7935	15.58	8.31	2135987	7470	15.58	8.31	2138647
2	7570	16.03	7.58	2098793	7380	16.03	7.58	2100003
3	9080	17.61	7.99	2380657	8855	17.49	7.99	2368518
4	6885	17.39	7.29	2257702	6660	17.35	7.29	2255024
5	8710	17.11	8.32	2355928	8660	17.10	8.32	2355136
6	7335	16.83	8.00	2277437	7160	16.83	8.00	2278755
7	7800	16.21	8.24	2219198	7420	16.21	8.24	2221358
8	7195	14.51	8.12	1956223	7045	14.51	8.12	1957247
9	8290	16.23	8.09	2198311	7545	16.23	8.09	2207307
10	6610	13.98	8.49	1937277	6625	13.98	8.49	1937323
11	7930	16.10	7.87	2148447	7665	16.04	7.87	2143541
12	8200	17.23	7.81	2303080	7720	17.23	7.81	2305874
Average	**8389**	**16.99**	**7.99**	**2296096**	**8128**	**16.95**	**7.99**	**2292377**

4.2.2.2.2 Restricted water quota

As noted under a FQ scenario, the areas under production for both maize and wheat generated within a SSDF correspond to those generated within a MSDF as the first stage of the MSDF is the same as single-stage within a SSDF. Full area production of 30.1 hectares for maize for both a risk neutral and a risk averse decision-maker is hence applicable while 20 hectares and 20.7 hectares is applicable for wheat for a risk neutral and risk averse decision respectively under a RQ scenario. The lower areas for wheat resulted in a distribution of lower GMs under a RQ scenario represented by cumulative distribution functions lying to the left of the FQ cumulative distribution functions as indicated in Figure 4.2.

The responses of a decision-maker within a MSDF under a RQ scenario are presented in Table 4.4. The actual average yields of 16.99t/ha for maize and 7.89t/ha for wheat were almost equal to the average potential yields of 17t/ha and 8t/ha for maize and wheat respectively if risk neutrality is assumed as indicated in Table 4.4. A reduction of the actual average maize yield to 16.62t/ha under risk aversion is noted while the average actual yield for wheat remains constant for the two alternative risk preferences. In addition, a reduction of the total amount of irrigation water used per state under FQ reduced from $8015m^3$ for a risk neutral decision-maker to $7831m^3$ for a risk averse decision-maker.

A noteworthy reduction of irrigation water use in each state resulted in lower irrigation dependent costs hence the relative higher GMs at the lower tail of the risk averse CDF. However, a reduction in water use states of nature 3,4 and 5 resulted in lower maize yields due to under irrigation and subsequently resulted in lower GMs for the upper tail of the risk averse CDF. As similarly noted under a FQ within a MSDF, the negative effect of reduced gross income due to lower yields was greater than the positive impact of reduced irrigation dependent costs due to reduced water use.

The conclusion is that, the overall increase in GMs within a MSDF is attributed to the lower irrigation dependent costs incurred given the reduced per state irrigation water use. The updating of additional water budget information as is becomes available with irrigation decisions made sequentially reflects the true risk that a decision-maker faces and hence the higher expected GMs resulting from improved irrigation water scheduling.

Table 4. 4: Optimized irrigation water use, crop yields and the total gross margin for maize and wheat in each state of nature within a multi-stage decision-making framework for a risk neutral and risk averse decision-maker under a restricted water quota scenario, 2016

MULTI-STAGE DECISION FRAMEWORK (RESTRICTED WATER QUOTA)								
	Risk Neutral				**Risk Averse**			
State	**Irrigation (m^3)**	**Maize yield (t/ha)**	**Wheat yield (t/ha)**	**Total Gross Margin (R)**	**Irrigation (m^3)**	**Maize yield (t/ha)**	**Wheat yield (t/ha)**	**Total Gross Margin (R)**
1	8055	15.58	8.31	1911931	7435	15.58	8.31	1931407
2	7695	16.03	7.58	1909415	7485	16.03	7.58	1924023
3	8300	17.61	7.71	2153585	8195	16.97	7.72	2097356
4	6690	17.39	7.29	2082059	6665	17.15	7.29	2068059
5	8475	17.10	8.32	2132228	8385	16.23	8.32	2050947
6	7220	16.83	8.00	2068317	7050	16.83	8.00	2084659
7	7700	16.21	8.24	1999125	7230	16.21	8.24	2017553
8	6820	14.51	8.12	1743713	6930	14.51	8.12	1758734
9	7550	16.23	8.09	1987802	7330	16.23	8.09	2005927
10	7105	13.98	8.49	1702513	6720	13.98	8.49	1721175
11	7675	16.10	7.87	1946124	7715	16.10	7.87	1960157
12	8050	17.23	7.80	2103902	7765	17.23	7.80	2119129
Average	**8015**	**16.99**	**7.89**	**2080535**	**7831**	**16.62**	**7.89**	**2055024**

Having quantified the risk faced within a SSDF and a MSDF and how responses of a decision-maker within each decision-making framework, the following section presents the estimated value or the worth of a MSDF.

4.3 THE VALUE OF A MULTI-STAGE DECISION-MAKING FRAMEWORK

The value of switching to a MSDF is estimated by comparing the certainty equivalents (CE) of a SSDF to that of a MSDF for a both risk neutral and a risk averse decision-maker under a full and restricted water quota as presented in Table 4.5.

Table 4. 5: The value of a multi-stage decision framework for a risk neutral and risk averse decision-maker under full and restricted water quota scenarios, 2016

	FULL WATER QUOTA		**RESTRICTED WATER QUOTA**	
	Risk Neutral	**Risk Averse**	**Risk Neutral**	**Risk Averse**
Single-stage framework Certainty equivalent (R)	2 291 835	2 077 144	2 073 516	1 864 631
Multi-stage framework Certainty equivalent (R)	2 296 097	2 088 293	2 080 535	1 879 044
Value(R)	**4 261**	**11 149**	**7 019**	**14 413**

As indicated in Table 4.5, the value of a MSDF is R4 261 and R11 149 for a risk neutral and a risk averse decision-maker respectively under a FQ. The value is determined by subtracting the CE generated within a SSDF from that generated within a MSDF. Subtracting the CE for a less preferred alternative from the CE for a preferred alternative at a specified level of $r_s(x^s)$ yields a utility weighted risk premium, which is defined as the minimum sure amount that must be paid to a decision-maker to justify a switch between a preferred and a less preferred alternative (Hardaker *et al.*, 2004). The value hence represents the minimum sure amount that has to be paid to a decision-maker to justify a switch from making irrigation decisions within a SSDF to a MSDF. For a rational decision-maker, the CE under risk aversion is expected to be lower than the expected value but greater than the minimum value as noted in Table 4.5. The gain realized within a MSDF under a FQ is attributed to the improved irrigation water management taking additional water budget information into account as sequential irrigation decisions are made over the course of the production season. With the improved risk management within a MSDF as highlighted in the first section, the certain minimum amount that both a risk neutral and a risk averse decision-maker can receive increases within a MSDF. The resulting value of MSDF for the extreme level of risk aversion considered is R6 887 more than that generated under risk neutrality under a FQ as indicated in Table 4.5. The noteworthy increase in the value of MSDF owes to the risk reducing nature of the decision-making framework hence more favorable for a risk averse decision-maker.

As illustrated in Table 4.5, the rand value of switching from a SSDF to a MSDF is R7 019 and R14 413 for a risk neutral and a risk averse decision-maker respectively under a RQ. In other words, the minimum amount of money that a risk neutral and a risk averse decision-maker has to receive to consider a MSDF is R7 019 and R14 413 respectively. As afore-noted under a FQ scenario, the value of a MSDF is greater when risk aversion is taken into account. Nonetheless, a greater value for irrigation decisions made within a MSDF results if water supply is restricted compared to a full water quota scenario for both a risk neutral and risk averse decision-maker as highlighted in Tables 4.5. In addition, the value of a MSDF for a risk averse decision-maker under a RQ is R7 394 greater than that generated under risk neutrality and is also greater than the difference between the value under risk neutrality and risk aversion of R6 887 noted under a FQ. Generally, efficient and effective irrigation management is more beneficial under restricted water conditions as the responses of a decision-maker will have an impact on the resulting GMs hence the greater value of a MSDF under a restricted water quota.

The conclusion therefore is that, making irrigation decisions within a MSDF is worthwhile considering its resulting value when the CEs of two alternative decision-making frameworks are compared. Also, the utility weighted risk premium of a MSDF is more significant under restricted water supply hence it is imperative to consider such a framework given the worsening water resource scarcity problem. Modelling of irrigation decisions within a MSDF also results in a true reflection of the value of the CE.

4.4 THE COST OF A WATER RESTRICTION

To determine the cost of a water restriction, the CEs generated within the two alternative decision-making frameworks under a full quota and a restricted quota water supply scenario are compared for a risk neutral and a risk averse decision-maker. The results of the cost of a water restriction are presented in Tables 4.6. Given the certain amount that a risk neutral decision-maker receives reduces significantly when a water restriction is enforced, the cost of a water restriction is represented by the difference between the CE generated under a full water quota and that under a restricted water quota for each decision-making framework. As illustrated in Table 4.6, the cost of a water restriction for a SSDF of R218 319 is greater than that of R215 561 faced within a MSDF for a risk neutral decision-maker. By implication, the cost of a water restriction is over-estimated within a SSDF. The true quantification of risk is hence imperative if the true cost of a water restriction is to be determined.

Likewise, a greater cost of a water restriction of R212 513 results within a SSDF in comparison to R209 249 generated within MSDF under risk aversion as depicted in Table 4.6. An overestimation of the cost of a water restriction is hence also noted for a risk averse decision-maker. Nevertheless, the costs for a water restriction when risk aversion is considered are lower than under risk neutrality for both a SSDF and a MSDF. The lower costs can be attributed to the fact that risk averse decision-makers already make conservative decisions hence a water restriction will have a relatively lesser impact on such a decision-maker.

The conclusion is that, the results of the costs of a water restriction confirm the notion under which the aim of this study was constructed that suggested that applying a SSDF considering area and irrigation water scheduling decisions as one decision will lead to an under or over estimation of the cost of a water restriction. In this case, the cost of a water restriction is over-estimated within a SSDF which might result in imprecise modeling of irrigation decision tools especially under restricted water scenarios taking risk preferences into cognisance.

Table 4. 6: Certainty equivalents for each water supply scenario for the two alternative decision-making frameworks for a risk neutral and a risk averse decision-maker, 2016

	RISK NEUTRAL			**RISK AVERSE**		
	Full quota	**Restricted quota**	**Cost (R)**	**Full quota**	**Restricted quota**	**Cost (R)**
Single-stage framework Certainty equivalent (R)	2 291 835	2 073 516	**218 319**	2 077 144	1 864 631	**212 513**
Multi-stage framework Certainty equivalent (R)	2 296 097	2 080 535	**215 561**	2 088 293	1 879 044	**209 249**

CHAPTER 5
SUMMARY AND RECOMMENDATIONS

Chapter 5 provides a summary of each of the four chapters and the recommendations thereof in accordance to the conclusions drawn from the research.

5.1 INTRODUCTION

5.1.1 BACKGROUND AND MOTIVATION

Recurrent droughts and inconsistent rainfall patterns that have been experienced in South Africa over the past few production seasons have further threatened the sustainability of agriculture, especially irrigated agriculture given the already scarce water resources. Precise decision-making by irrigators has since become more imperative for sustainable irrigation farming given the drought-induced water supply restrictions.

Irrigation farmers face complex management decisions that include decisions on the type of crop to produce, the area under production and the water scheduling decisions that are made at different stages during the growing season. Crop type and area decisions are once-off decisions made at the beginning of the growing season which will greatly determine water needs and the irrigation strategy thereof. Irrigation water scheduling decisions are however made sequentially in multi-stages throughout the growing season as uncertain weather conditions unfold given the crop area decision already made. Sequential decision making facilitates adaptive decision behaviour by the decision-maker depending on currently prevailing weather conditions. As a result, decision-makers improve their production risk management as taking additional information into account will guide forthcoming irrigation decisions. In addition, multi-stage sequential decision-making enables decision-makers to incorporate exogenous factors that complicate irrigation water allocation decisions such as time of use electricity tariffs into their decisions. The interaction between these different decisions made in different stages during the growing season becomes much more important under limited water supply conditions.

Currently applied methodologies to model irrigation water allocation decisions in the South African context do not acknowledge the fact that multi-stage decision making facilitates improved production risk management. As a result, the impact of water restrictions on irrigation farmers might be over-estimated. To represent the true nature of irrigation decisions, one should consider the impact of irrigation water allocation decisions on the stock of the field water supply dynamically throughout the growing season.

5.1.2 PROBLEM STATEMENT AND OBJECTIVES

Irrigation water allocation decisions at farm-level are currently modelled within a single-stage decision-making framework and therefore misrepresents the actual manner in which irrigators make irrigation water allocation decisions in reality. The unavailability of a modelling framework that represents irrigation decisions within a multi-stage decision-making framework results in researchers, water managers at water user associations and policy makers being unsure of the impact of better representing irrigation water allocation decisions on the main decision variables and hence the value of limited water resources. Consequently, decision support under limited water supply conditions is hampered.

Significant research has been commissioned nationally on crop water use optimisation for both limited and unlimited water supply conditions. The significance of taking more irrigation information into account has been acknowledged especially under limited water supply. However, most researchers assume fixed irrigated area ignoring the impact of the interaction between crops, area planted and water availability on the ability to supply adequate irrigation water per hectare. In addition, applied models are structured within a single-stage decision-making framework where area and irrigation decisions are made within the same time period. Also, a limited number of states of nature have been considered in research efforts to limit the curse of dimensionality problems. Optimal irrigation strategies that maximize utility irrespective of the state of nature unfolding were hence determined with dynamics of water use and risk only approximated or overlooked. The review of South African literature also indicated that a unified irrigation decision-making framework that models the interaction between water availability, irrigation area and irrigation scheduling decisions as multi-stage sequential decisions is unavailable.

The main objective of this research is to compare the results obtained when modelling irrigation water allocation decisions within a single-stage decision-making framework with the results of a multi-stage sequential decision-making framework under a full water quota and a restricted water quota. Comparing the results of the two decision-making frameworks for the two water quotas will allow for the determination of the impact of modelling irrigation water allocation decisions within a multi-stage sequential manner on:

- Total gross margin risk and irrigation management decision variables (irrigated areas and irrigation water use) under a full and restricted water quota.

- The monetary value that will result from the improved modelling of irrigation water allocation decisions for risk averse decision-makers under a full and restricted water quota.

- The monetary cost of valuing the impact of restricted water use resulting from ignoring the improved modelling of irrigation water allocation decisions for risk averse decision-makers.

5.2 LITERATURE REVIEW

A literature review was conducted to guide the development of the solution procedures and model utilised in the research to optimise irrigation water use. Four main conclusions were drawn from the literature that are considered to be of the utmost importance when developing models to optimise agricultural water use.

Firstly, irrigation water allocation decisions are complicated, since such decisions are made taking the stock nature of field water supply dynamically into account throughout the growing season. Soil water availability to crops is dependent on the water status of the soil. The latter necessitates the computation of daily water budget to account for water fluxes and facilitate scheduling of irrigation events before soil moisture stress conditions are triggered that have an impact on crop growth and the resulting crop yield. Under limited water supplies, area planted and irrigation decisions are made in multi-stages to allow an interaction between crop, area planted and water availability to ensure sufficient irrigation water is supplied per hectare basis to avoid crop moisture stress. Irrigation scheduling decisions are dynamic decisions made sequentially, given a water application decision made in one period will impact water availability to crops during the next time period. Irrigation scheduling decisions are hence complex dynamic problems that need to be solved with a dynamic solution procedure that facilitates adaptive-decision behaviour throughout the growing season within a multi-stage decision making framework.

Secondly, trends of applications of dynamic modelling approaches reveal that dynamic programming approaches have been mostly applied to solve stochastic dynamic agricultural water use problems. Dynamic programming allows the model to break down multiple decision problems into a sequence of sub-problems allowing the optimisation of diverse problems in a sequential manner. However, if the number of state and stage variables included in the model are large, the model will explode in size deeming the solution infeasible, a dynamic programming limitation known the curse of dimensionality. In addition, water optimisation decisions are modelled in a single-stage decision-making framework where area planted and irrigation decisions are made simultaneously. The applicability of dynamic programming to solve complex dynamic problems where irrigation scheduling decisions are made subsequent to the area decision is hence limited. An alternative solution procedure (Blanco and Flichman, 2002) known as recursive stochastic programming that solves complex dynamic problems without the limitation of the curse of dimensionality should hence be considered when modelling water allocation problems. Recursive stochastic programming facilitates a sequential adjustment between real-life situations and optimal situations through gradual adaptation of changes of exogenous parameters through forward recursion. The decision-maker adjusts the decision for a given decision stage taking into account the new information available from the previous stage. The recursive solution procedure of Blanco and Flichman (2002) is hence a viable alternative to solve complex sequential irrigation scheduling decisions within a multi-stage decision-making framework.

Thirdly, a review of literature indicated that mathematical programming and simulation optimisation approaches have been applied by researchers to solve irrigation water application decision problems as alternatives to dynamic programming. Results from the water use optimisation studies considered in literature that fairly accounted for water dynamics within the South African context reveal that, the currently available solution techniques lack complexity. Mathematical programming models require simplification to limit the state and stage variables included in the model with no adaptive-sequential behaviour considered to avoid the curse of dimensionality. An attempt to incorporate risk into a mathematical programming model resulted in the model becoming infeasible (Venter, 2015). The applicability of mathematical programming models to complex dynamic problems whose optimal solution is highly dependent on explicitly taking time into account with area and irrigation decisions being made consecutively within a risk framework is hence limited. Simulation optimisation approaches were able to take into account a considerable number of states of nature given their ability to deal with high-dimensional non-linear or mixed integer optimisation problems. Application of evolutionary algorithms to optimise complex simulation models has been recently adopted in South Africa. Evolutionary algorithms provide an alternative solution technique that overcomes the complexity of modelling irrigation decisions without deeming the solution infeasible. The creditability of such a procedure was recently demonstrated by Haile (12017). The availability of alternative solution procedures such as evolutionary algorithms provides an opportunity for researchers to model more complex representations of irrigation scheduling management decisions.

Finally, time of use electricity tariffs are considered an exogenous factor other than water availability and crop water demand that may further complicate irrigation water allocation decisions. The Ruraflex time of use tariffs that charges different tariff for different time periods might force a decision-maker to schedule an irrigation event during a specific time of the day and a specific day of the week. The time of use tariff option will hence impact the decision on the timing of irrigation events and subsequently affecting irrigation water use dynamics. Energy accounting should thus be incorporated in irrigation water allocation models.

5.3 METHODOLOGY

An Excel ® simulation model based on the Soil Water Irrigation Planning and Energy management (SWIP-E) programming model (Venter, 2015) was developed and applied. Firstly, a daily water budget is computed to determine the timing and magnitude of irrigation events and the resulting crop yields and total irrigation hours as key output variables. The model facilitates the simulation of the economic consequences resulting from changes to the key decision variables that need to be optimised through gross margin calculations for each state of nature. The crop yield estimations determined in the water budget influences the gross margin through the gross income while the total pumping hours are accounted for within the irrigation dependent costs. Any adjustments in the water budget will hence alter the gross margin depending on the response of crop yields and total irrigation hours. The allocation of irrigation hours needed for each cycle within the time of use periods (TOU) of the electricity tariff was essential to account for the impact of the TOU Ruraflex tariff on water use dynamics as an exogenous factor.

An extension of the model to incorporate risk was essential to facilitate a comparison of the economic consequences within the single-stage and multi-stage sequential decision–making frameworks. Inclusion of risk in the model facilitates the quantification of the uncertainty in resulting yields and irrigation costs under stochastic weather induced variabilities. To determine an optimal irrigation schedule that optimises utility regardless of the state of nature occurring, replication of the water budget calculations for each of the representative 12 weather states was imperative. A cluster analysis classification tool was utilised to identify the 12 representative states of nature from the possible 49 states obtained and extracted from SAPWAT3 software. Risk preferences were incorporated into the model by means of maximization of the certainty equivalents assuming constant absolute risk aversion.

The Excel ® Solver is used to optimise the risk simulation model using the evolutionary algorithm embedded in Excel ®. Two Excel ® macros were programmed in Excel VBA (Visual Basic for Applications) to command the simulation procedures within each decision-making framework and to limit the amount of time consumed optimizing the problem given the repetitive commands. The single-stage decision-making framework macro commanded the optimisation of the base model where the area decision and an irrigation schedule are determined in a single-stage with only knowledge of the possible states of nature that could unfold. The multi-stage decision-making framework macro facilitated weekly sequential irrigation scheduling decisions considering additional rainfall and ETm information over the course of the production season. The first stage of the multi-stage decision-making framework is the same as the single-stage considered within a single-stage decision-making framework. A recursive stochastic programing methodology is adopted at the second stage of the multi-stage decision-making framework macro to facilitate sequential irrigation decisions as each state of nature unfolds on a weekly basis.

5.4 RESULTS AND CONCLUSIONS

5.4.1 SINGLE-STAGE DECISION-MAKING FRAMEWORK

The distribution of gross margins within a single-stage decision-making framework is represented with cumulative distribution functions. A similar distribution of gross margins with a noteworthy lower tail resulted under both full and restricted water quota supply scenarios within a multi-stage decision framework. Considering an extreme level of risk aversion resulted in the lower tail of the cumulative distribution function for both water supply scenarios shifting slightly to the right and the upper tails shifting to the left. The shifting resulted from increased minimum gross margin values with reduced maximum gross margins under risk aversion.

The resulting variations of the gross margins within a single-stage decision-making framework are attributed to the responses of the decision-maker with regards to areas planted, irrigation water use and the resulting crop yields under the two alternative water supply scenarios and risk preferences. Within a single-stage decision framework, the crop will be either over or under irrigated depending on the state of nature that occurs with the subsequent impact on the resulting yields and irrigation dependent costs. Full areas of 30.1 hectares and 30.1 hectares are produced for maize and wheat respectively regardless of the risk preference under a full quota with potential yields achieved for both crops. However, taking risk aversion into account under a full quota scenario resulted in a reduction of the average actual yield of maize to 16.81t/ha while the average actual yield for wheat was kept constant at 7.99t/ha. The total amount of irrigation water used under a full quota also reduced from $9050m^3$ under risk neutrality to $8860m^3$ under risk aversion. The reduction in the average maize yield and the amount of irrigation water used owes to the extreme level of risk aversion. Improved minimum gross margins within the lower tail of the cumulative distribution function results from reduced irrigation dependent costs while a slight decrease in maize yield resulted in lower gross margins within the upper tail.

Under a restricted water quota, irrigation decisions made within a single-stage decision-making framework assuming risk neutrality resulted in full areas being produced for maize regardless of risk preferences. A reduction of production area of wheat to 20 hectares and 20.7 hectares is however noted for a risk neutral and risk averse decision-maker respectively. A full irrigation strategy with reduced areas is hence followed under a restricted water quota with a reduced gross income resulting in lower gross margins. The resulting average yields for both crops under restricted water supply for both risk preferences are lower than the average potential yields of 17t/ha and 8t/ha for maize and wheat respectively. Though the total amount of irrigation water slightly decreased under risk aversion, the improvement of minimum gross margins at the lower tail is attributed to a notable increase in wheat yields. In contrast, reduced maize yields for states of nature comprising the upper tail of the cumulative distribution function resulted in reduced maximum gross margins.

The overall conclusion of the impact of modelling irrigation decisions within a single-stage decision framework where the area and irrigation scheduling decisions are made in one stage is that the impact of risk aversion is limited under a full water quota. Though irrigation water application significantly reduced in effort to increase the minimum gross margins, yield variations were limited between alternative risk preferences. However, significant yield variations resulted under a restricted water quota when risk aversion was considered implying the noteworthy impact of risk aversion on responses of a decision-maker when water supply is restricted. Though water is considered a risk reducing input, the extreme level of risk aversion resulted in a reduction of irrigation water applied to improve minimum expected values.

5.4.2 MULTI-STAGE DECISION-MAKING FRAMEWORK

Gross margins within a multi-stage decision-making framework are also represented with cumulative distribution functions with a similar distribution of gross margins resulting under both water supply scenarios. A distinctive irrigation schedule is determined for each state of nature hence the irrigation water application and subsequent irrigation dependent costs varied per state. Areas under production for both crops generated within a multi-stage decision-making framework corresponds to those generated within a single-stage decision-making framework as the first stage of decision-making is the same within both frameworks. The impact of risk aversion is indicated by the shifts of the cumulative distribution graph's lower tail to the right and upper tail to left. Important to note is that the shift of the lower tails to the right under risk aversion is more significant within a single-stage decision-making framework than within a multi-stage decision-making framework. The impact of risk aversion within a multi-stage decision-making framework is thus limited.

Under a full water quota, maize and wheat are produced under full areas for both risk preferences with almost potential yields achieved for both crops. The total irrigation water applied over all states of nature reduced from $8389.40m^3$ for a risk neutral decision-maker to $8128m^3$ for a risk averse decision-maker with limited yield variation between the two risk preferences. As expected under extreme levels of risk aversion, improved minimum gross margins within the lower tails results under both water supply scenarios. A reduction in irrigation water use under risk aversion resulted in lower irrigation dependent costs and subsequently higher gross margins for the lower tail of the cumulative distribution function. However, lower yields were generated in states of nature comprising the upper tail due to reduced irrigation water application under risk aversion resulting in lower gross margins. The positive impact of reduced water use on gross margin is hence offset by the negative impact resulting from yield reduction.

Under a restricted water quota, yields under production for both crops also correspond to those generated under a single-stage decision-making framework. The full irrigation and reduced area strategy is hence also followed under a restricted water quota for wheat production. Reduced area under wheat production resulted in lower gross margins with a cumulative distribution functions lying to the left of the full water cumulative distribution functions. A risk averse decision-maker responds under a full water quota scenario by reducing irrigation water from $8015m^3$ under risk neutrality to $7831m^3$. Higher gross margins at the lower tail under risk aversion hence resulted from a reduction in irrigation water use in each state, consequently resulting in lower irrigation dependent costs. However, a reduction in water resulted in lower maize yields due to under irrigation in some states of nature and subsequently resulted in lower gross margins within the upper tail of the risk averse cumulative distribution function.

The main conclusion is that the overall increase in gross margins within a multi-stage decision-making framework owes to the lower irrigation dependent costs incurred given the reduced per state irrigation water application. The updating of additional water budget information as is becomes available with irrigation decisions made sequentially reflects the true nature of irrigation decisions.

5.4.3 THE VALUE OF A MULTI-STAGE DECISION-MAKING FRAMEWORK

The value of switching to a multi-stage decision-making framework is estimated by comparing the certainty equivalents of a single-stage decision-making framework to that of a multi-stage decision-making framework. The value represents the minimum sure amount that a decision-maker must be compensated with to justify a switch from making irrigation decisions within a single-stage to a multi-stage decision-making framework. The value of a multi-stage decision-making framework is R4 261 and R11 149 for a risk neutral and a risk averse decision-maker respectively under a full water quota scenario. The minimum rand value a rationale decision-maker must receive to consider a switch from a single-stage to a multi-stage decision-making framework under a restricted water quota is R7 019 and R14 413 for a risk neutral and a risk averse decision-maker respectively. Sequential decision-making within a multi-stage decision-making framework results in improved irrigation water management for each state of nature. The consequent lower irrigation dependent costs thus result in a higher certainty equivalents.

The noteworthy value of a multi-stage decision-making framework owes to the risk reducing nature of the decision-making framework with sequential irrigation decisions considered as more water budget information is taken into account. The value of modelling irrigation decisions within a multi-stage decision-making framework is greater when water supply is limited as improving irrigation water scheduling is generally more beneficial when water supply is limited.

The conclusion therefore is that, modelling irrigation decisions within a multi-stage decision-making framework is worthwhile considering its resulting value when the certainty equivalents of two alternative decision-making frameworks are compared. Modelling decisions within a multi-stage decision-making framework is more significant under restricted water supply scenarios as greater value of the framework is realized when water supply is limited.

5.4.4 THE COST OF A WATER RESTRICTION

The cost of a water restriction was determined by comparing the certainty equivalents generated within the two alternative decision-making frameworks under a full and a restricted water quota. The cost of a water restriction generated within a single-stage decision-making framework is R218 319 while a cost of R215 561 was generated within a multi-stage decision-making for a risk neutral decision-maker. Under risk aversion, the cost of a water restriction within a single-stage decision-making framework of R212 513 is greater than that of R209 249 generated within a multi-stage decision-making framework. The costs for a water restriction when risk aversion is considered are lower than under risk neutrality within both decision-making frameworks. The lower costs can be attributed to the fact that risk averse decision-makers already make conservative decisions hence a water restriction will have a relatively limited impact on such a decision-maker.

The main conclusion is that modelling irrigation decisions within a single-stage decision-making framework over-estimates the value of a water restriction to irrigation farmers. Care should hence be taken when modeling irrigation decisions as the misrepresentation of the true decision-making framework of irrigators might mislead water allocation decisions.

5.5 RECOMMENDATIONS

In light of the results and conclusions determined from the study, the following recommendations were made.

- Caution is necessary when formulating agricultural water allocation policies based on crop water optimisation models that ignore the multi-stage decision-making framework within which irrigation decisions are made. Ignoring modelling irrigation decisions as sequential dynamic decisions results in over-estimating the impact of any given policy on water use management. It is hence critical to analyse farm-level profitability within a framework that better represents farmers' decision-making to provide policy decision-makers with improved decision-making tools.

- It is recommended that the optimality of the solutions of the risk model be tested with alternative evolutionary algorithm techniques. Excel ® Solver struggled to reach an optimal solution under a limited water supply taking risk aversion into account without assigning initial areas for both crops. The possibility of reformulating the model within a mathematical programming environment should be investigated. Such a reformulation could be solved to a global optimal solution using a global solver such as BARON (Branch and Reduce Optimisation Navigator) (Sahinidis, 1996).

- Evolutionary algorithms allow for the development of more complex models. The possibility of integrating the short-run optimisation model with a long-run component of investment analysis should hence be investigated.

- Historical data was used to provide an indication of future weather events. Probabilistic weather forecasts may provide better information about future weather conditions and should be incorporated into the model.

- The modelling framework is general and allows for the analysis of other agricultural problems such as contracting prices during different time periods.

- The levels of risk aversion included in this analysis was limited. Only the maximum and the minimum level of $r_s(x^s)$ of 0 and 2.5 respectively according to the plausible range for $r_s(x^s)$ determined by Grové (2010) were used to determine the impact of risk aversion.

Further research procedures need to incorporate more levels of risk aversion to improve the risk analysis.

- The model could also be expanded to optimize intra-seasonal water allocation with crops such as maize and groundnuts produced in the same production season competing for water resources.

ALAMDARLO HN, AHMADIAN M and KHALILIAN S. 2014. Application of stochastic dynamic programming in water allocation, Case Study: Latian Dam. *World Applied Sciences Journal*, 30 (7): 838-843.

ALLEN RG, PEREIRA L, RAES D and SMITH M. 1998. *Crop evapotranspiration: Guidelines for computing crop water requirements*. FAO Irrigation and Drainage Paper No 56. FAO. Rome. Italy.

ANTLE JM. 1983. Sequential decision-making in production models. *American Journal of Agricultural Economics,* 65:282-290.

AOKI M. 1967. *Optimisation of Stochastic Systems*. Academic Press: New York.

BABCOCK BA, CHOI EK and FEINERMAN E. 1993. Risk and probability premiums for CARA utility functions. *Journal of Agricultural and Resource Economics,* 18: 17-24.

BACKEBERG G.R and OOSTHUIZEN, L.K. (1995). *The economics of irrigation: Present research and future challenges.* Report to the Water Research Commission. WRC Report No TT 71/95. Pretoria: The Water Research Commission.

BELLMAN RE. 1957. *Dynamic Programming.* Princenton University Press, Princenton, New Jersey, USA.

BUREAU FOR FOOD AND AGRICULTURAL POLICY (BFAP). 2012. *The effect of electricity costs on irrigation farmin*g. National Agricultural Marketing Council, RSA.

BLANCO M and FLICHMAN G. 2002. *Dynamic optimisation problems: Different resolution methods regarding agricultural and natural resource economics.* Proceedings of the International Conference on risk and uncertainty in Environmental Economics: Risk and Uncertainty in Environmental Economics, Wageningen University, March 2002.

BOEHLJE MD and WHITE TK. 1969. A production-investment decision model of farm firm growth. *American Journal of Agricultural Economics,* 51:546-563.

.

BOTES JHF, BOSCH DJ and OOSTHUIZEN LK. 1995. The value of irrigation information for decision-makers with neutral and non-neutral risk preferences under conditions of unlimited and limited water supply. *Water SA*, 21(3): 221- 230.

BOTES JHF, BOSCH DJ and OOSTHUIZEN LK. 1996. A simulation and optimisation approach for evaluating irrigation information. *Agricultural Systems*, 51: 165- 183.

BOTES JHF. 1990. *An economic evaluation of wheat irrigation scheduling strategies using stochastic dominance*. (Afrikaans). M.Sc. Agric. thesis. Department of Agricultural Economics, University of the Free State, Bloemfontein, South Africa.

BRAS RL and CORDOVA JR. 1981. Intra-seasonal water allocation in deficit irrigation. *Water Resource Research*, 17(4): 866-874.

BRYANT KJ, MJELDE JW and LACEWELL RD. 1993. An Intra-seasonal dynamic optimisation model to allocate irrigation water between crops. *American Journal of Agricultural Economics*, 75 (4): 1021-1029.

BURGER JH, HEYNS PJ, HOFFMAN E, KLEYNHANS EPJ, KOEGELENBERG FH, LATEGAN MT, MULDER, SMAL HS, STIMIE CM, UYS WJ, VAN DER MERWE FPJ, VAN DER STOEP I and VILJOEN PD. 2003. *Irrigation Design Manual.* ARC-Institute for Agricultural Engineering.

BURT OR and STAUBER MS. 1971. Economic analysis of irrigation in sub humid climate. *American Journal of Agricultural Economics*; 53(1): 33-46.

COCKS KD. 1968. Discrete stochastic programming. *Management Science*, 15:72- 79.

DANTZIG G. 1995. Linear programming under uncertainty. *Management Science,* 1:197-206.

DAY RH. 1961. *Recursive Programming and Supply Predictions.* Agricultural supply functions. Iowa State University Press, Ames, Iowa, (108-125).

DE JAGER JM. 1994. Accuracy of vegetation evaporation ration formulae for estimating final wheat yield. *Water SA*, 20(4): 307-314.

DEPARTMENT OF LBOUR (DOL). 2014. *Minimum wage for farm worker sector with effect from 1 March 2014.* Department: Labour, Pretoria, South Africa.

DOORENBOS J and KASSAM AH. 1979. *Yield Response to Water*. Irrigation and drainage Paper 33. Rome, IT: Food and Agriculture Organisation (FAO).

ENGLISH MJ, SOLOMON KH and HOFFMAN GJ. 2002. A paradigm shift in irrigation management. *Irrigation and Drainage Engineering*, 128(5): 267-277.

ESKOM. 2015/16. *Tariffs and Charges booklet.* Available online at: http://www.eskom.co.za/content/2015_16DraftTariffBookver3.pdf.

GAKPO EFY, TSEPHE J, NWONWU F and VILJOEN MF. 2005. Application of Stochastic Dynamic Programming (SDP) for the optimal allocation of irrigation water under capacity sharing arrangements. *Agrekon*, 44(4): 436-451.

GONG X and RICHMAN MB. 1995. On the application of cluster analysis to growing season precipitation data in North America East of the Rocks. *Journal of Climate*, 8:897-931.

GRAINSA. 2016. *Crop Estimates.* Available online at: http://www.grainsa.co.za/pages /industry reports/crop-estimates.

GRIEKWALAND-WES KORPORATIEF (GWK). 2016. Input costs guide. March 2016.

GROVÉ B and OOSTHUIZEN LK. 2010. Stochastic efficiency analysis of deficit irrigation with standard risk aversion. *Agricultural Water Management*, 97 (6):792-800.

GROVÉ B, OOSTHUIZEN LK and BACKERBERG GR. 2006. *A dynamic risk optimisation model for evaluating profitable and feasible water management plans.* International association of Agricultural Economists Conference, Gold Coast, Australia, 12-18 August 2006.

GROVÉ B. 1997. *Modellering van die ekonomiese effekte van wisselvallige waterbeskikbaarheid vir besproeiingsboere in die Wintertongebied met inagneming van minimum binnestroomvloeivoorsiening.* (Afrikaans). M.Sc. Agric Dissertation. Department of Agricultural Economics, University of the Free State. Bloemfontein. South Africa.

GROVÉ B. 2006. *Generalised Whole-farm Stochastic Dynamic Programming model to optimise agricultural water use.* Report to the Water Research Commission. WRC Report No 1266/1/06. Pretoria: The Water Research Commission.

GROVÉ B. 2006. Stochastic efficiency optimisation analysis of alternative agricultural water use strategies. *Agrekon*, 45(4): 406-420.

GROVÉ B. 2008. *Stochastic Efficiency Optimisation Analysis of Alternative Agricultural Water Use Strategies in Vaal harts over the Long- and Short-Run*. Ph.D. thesis. Department of Agricultural Economics, University of the Free State, Bloemfontein, South Africa.

HAILE BO, GROVÉ B, BARNARD J and VAN RENSBURG LD. 2014. Exploring genetic algorithms for improved salinity management using SWAMP. *6th South African National Commission on Irrigation and Drainage Symposium.* Glenburn Lodge, 18-20 November 2014.

HAILE BO. 2017. *An economic analysis of salinity management with evolutionary algorithms in Vaal harts.* Unpublished Ph.D. thesis. Department of Agricultural Economics, University of the Free State, Bloemfontein, South Africa.

HANNAH LA. 2014. *Stochastic Optimisation notes.* Available online at: http://www.stat.columbia.edu/~liam/teaching/compstat-spr15/lauren-notes.pdf.

HARDAKER JB, RICHARDSON JW, LIEN G and SCHUMANN KD. 2004. Stochastic efficiency analysis with risk aversion bounds: a simplified approach. *The Australian Journal of Agricultural and Resource Economics,* 48(2): 523-270.

JENSEN ME. 1968. Water consumption by agricultural plants. In: Kozlowski TT (ed) Water deficits and plant growth, volume 2. Academic Press, New York, 1-22.

KALLITSARI C, GEORGIOU P and BABAJIMOPOULOS C. 2011. *Evaluation of Crop Water production Functions under Limited Soil water Availability with SWBACROS model*. Department of Hydraulic, Soil Science and Agriculture Engineering, Faculty of Agriculture, Aristotle University of Thessaloniki, Thessaloniki, Greece.

KENNEDY JOS. 1986. *Dynamic Programming. Application to Agriculture and Natural Resources.* Elsevier.

KENNEDY JOS. 1988. Principles of dynamic optimisation in resource management. *American Journal of Agricultural Economics,* 2: 57-72.

KIKUTI D, COZMAN FG and FILHO RS. 2010. *Sequential Decision-making with partially ordered preferences.* Escola Polit´ecnica, Universidade de S˜ao Paulo, Brazil. Elsevier.

MARZBAN C and SANDGATHE S. 2006. Cluster analysis for verification of precipitation fields. *Weather and Forecasting,* 21: 824-838.

McCARL BA and SPREEN TH. 1997. *Applied mathematical programming using algebraic systems*. Texas A & M University.

MEIRING JA. 1989. *'n Ekonomiese evaluering van alternatiewe spilpuntbeleggingstrategieë in die Suid-Vrystaat substree met inagneming van risiko*. (Afrikaans). M.Sc. Agric. Dissertation. Department of Agricultural Economics. University of the Orange Free State. Bloemfontien. South Africa.

MYBURGH C. 2014. Verbal communication with the author by Venter M. Valley, Douglas, South Africa.

NAGYPAL E. 1998. *Dynamic Optimisation user's guide.* Economics 212. Available online at: http://web.stanford.edu/group/econ/uploads/pmwiki/Math.Dynamic OptUserGuide.pdf.

RABIN M and THALER RH. 2001. Anomalies: risk aversion. *Journal of Economic Perspectives*, 15: 219-232.

RAE AN. 1971b. An empirical application and evaluation of discrete stochastic programming in farm management. *American Journal of Agricultural Economics*, 53: 625-638.

RAE AN.1971a. Stochastic programming, utility and sequential decision problems in farm management. *American Journal of Agricultural Economics,* 53: 448-460.

RAO NH, SARMA PBS and CHANDER S. 1988. Irrigation scheduling under a limited water supply. *Agricultural Water Management*, 15: 165–175.

RAO NH, SARMA PBS and CHANDER S. 1988. A simple dated water production function for use in irrigated agriculture. *Agricultural Water Management*, 13: 25-32.

RAO NH, SARMA PBS and CHANDER S. 1992. Real-time adaptive irrigation scheduling under a limited water supply. *Agricultural Water Management*, 20: 267-279.

RHENALS AE and BRAS RL. 1981. The irrigation scheduling problem and evapotranspiration uncertainty. *Water Resource Research*, 17(5): 1328-1338.

SAHINIDIS NV. 1996. *BARON:* A general purpose global optimization software package. *Journal of Global Optimization*, 8:201–205.

STATISTICS SOUTH AFRICA. 2014. *Agricultural Statistics*. Statistics South Africa. Pretoria, South Africa. Available online at: http://www.stassa.gov.za.

STEWART JI, DANIELSON RE, HANKS RJ, JACKSON EB, HAGAN RM, PRUITT WO, FRANKLIN WT and RILEY JP. 1977. *Optimizing crop production through control of water and salinity levels in soil*. Utah Water Research Lab. Logan, Utah.

USDA (UNITED STATES DEPARTMENT OF AGRICULTURE). 2016. *Commodity Intelligence Report*. May 2016. Available online at: https://pecad.fas.usda.gov /highlights/2016/05/SouthAfricaElNino.

VAN HEERDEN PS. 2015. SAPWAT3. Verbal communication with the author. Bloemfontein, South Africa.

VENTER M. 2015. *Modelling the economic trade-off of irrigation pipeline investments for improved energy management*. M.sc Agric thesis. Department of Agricultural Economics, University of the Free State, Bloemfontein, South Africa.

VENTER M and GROVÉ B. 2016. Modelling the economic trade-offs of irrigation pipeline investments for improved energy management. *Water SA*, 42(4): 542-550.

WARD JH. 1963. Hierarchical grouping to optimise an objective function. *Journal of American Statistical Association*, 58: 236-244.

YAKOWITZ S. 1982. Dynamic programming applications in water resources. *Water Resource Research*, 18(4): 673–696.

YARON D and DINAR A. 1982. Optimal allocation of farm irrigation water during peak seasons. *American Journal of Agricultural Economics*, 64(4): 681-689.

YOUNG RA. 1996. *Water economics*. In Water Resources Handbook, ed. by L. Mays, McGraw-Hill, New York.

APPENDIX A
FULL WATER QOUTA
SINGLE-STAGE DECISION-MAKING FRAMEWORK MACRO

```
Sub SolveBaseModel


Dim score As Variant

Dim max As Variant

Dim n As Integer

'=============================================================================

'initialise states

states = Array("state1", "state2", "state3", "state4", "state5", "state6",
"state7", "state8", "state9", "state10", "state11", "state12")

For n = 0 To UBound(states)


 Sheets(" INITIAL (ETM,RAIN,RAM)").Select


    'Crop1 etm

    'Range("B3:B122").Select

    Range(Cells(3, n + 2), Cells(122, n + 2)).Select

    Selection.Copy

    Sheets(states(n)).Select

    Range("C8").Select

    Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone,
SkipBlanks _

        :=False, Transpose:=False


    'Crop2 etm

    Sheets(" INITIAL (ETM,RAIN,RAM)").Select

    Range(Cells(124, n + 2), Cells(271, n + 2)).Select

    Selection.Copy

    Sheets(states(n)).Select

    Range("C137").Select

    Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone,
SkipBlanks _

        :=False, Transpose:=False
```

```
    'Crop1 rain
    Sheets(" INITIAL (ETM,RAIN,RAM)").Select
    Range(Cells(276, n + 2), Cells(395, n + 2)).Select
    Selection.Copy
    Sheets(states(n)).Select
    Range("F8").Select
    Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone,
SkipBlanks _
        :=False, Transpose:=False

     'Crop2 rain
    Sheets(" INITIAL (ETM,RAIN,RAM)").Select
    Range(Cells(397, n + 2), Cells(544, n + 2)).Select
    Selection.Copy
    Sheets(states(n)).Select
    Range("F137").Select
    Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone,
SkipBlanks _
        :=False, Transpose:=False
Next
'===========================================================================
Sheets("State1").Select

n = 1
max = 1
score = 0
Range("BM8:BM65582").ClearContents
Calculate

SolverReset
    SolverOk SetCell:="$G$305", MaxMinVal:=1, ValueOf:=0, ByChange:= _
        "$BH$8:$BH$141", Engine:=3, EngineDesc:="Evolutionary"
   SolverAdd CellRef:="$BH$8:$BH$141", Relation:=1,
FormulaText:="$BJ$8:$BJ$141"
   SolverAdd CellRef:="$BH$8:$BH$141", Relation:=3,
FormulaText:="$BI$8:$BI$141"
```

```
 SolverOptions PopulationSize:=30, MutationRate:=0.5, RandomSeed:=1,
MaxTimeNoImp:=300

    Do Until score = max

    SolverSolve userFinish:=True


 Range("G305").Select

    Application.CutCopyMode = False

    Selection.Copy


    score = Range("G305").Value

    max = Range("BN8").Value

    'Range("BM8").Select

    Cells(8 + n - 1, 65).Select

    Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone,
SkipBlanks _

        :=False, Transpose:=False

   n = n + 1

 Loop

score = 0

  SolverReset

    SolverOk SetCell:="$G$305", MaxMinVal:=1, ValueOf:=0, ByChange:= _

        "$BH$8:$BH$141", Engine:=3, EngineDesc:="Evolutionary"


    SolverAdd CellRef:="$BH$8:$BH$141", Relation:=3,
FormulaText:="$BI$8:$BI$141"

    SolverAdd CellRef:="$BH$8:$BH$141", Relation:=1,
FormulaText:="$BJ$8:$BJ$141"

    SolverOptions PopulationSize:=30, MutationRate:=0.075, RandomSeed:=1,
MaxTimeNoImp:=300


     Do Until score = max

    SolverSolve userFinish:=True


  Range("G305").Select

    Application.CutCopyMode = False
```

```
    Selection.Copy


    score = Range("G305").Value
    max = Range("BN8").Value
    'Range("BM8").Select
    Cells(8 + n - 1, 65).Select
    Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone,
SkipBlanks _
        :=False, Transpose:=False
   n = n + 1
 Loop
 Range("$BH$8:$BH$141").Select
    Application.CutCopyMode = False
    Selection.Copy


Range("BQ8").Select
Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks
_
        :=False, Transpose:=False
Range("CM8").Select
Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks
_
        :=False, Transpose:=False
End Sub
```

APPENDIX B
FULL WATER QUOTA
MULTI-STAGE DECISION-MAKING FRAMEWORK MACRO

```
Sub SolveStateModel


states = Array("state1", "state2", "state3", "state4", "state5", "state6",
"state7", "state8", "state9", "state10", "state11", "state12")


Dim score As Variant

Dim max As Variant

Dim n As Integer


Dim fixat As Variant


Dim dynachange As Range

Dim dynamin As Range

Dim dynamax As Range

Dim crop1Last As Integer


Sheets("State1").Select

CPDdays = 0

fixat = Range("BP8").Value

'============================

'zero = to state 1

staten = 0

'============================

Range("$BI$8:$BI$141").Value = 0

'============================================================================

'initialise states

For n = 0 To UBound(states)


 Sheets(" INITIAL (ETM,RAIN,RAM)").Select

    'Crop1 etm

    'Range("B3:B122").Select
```

```
    Range(Cells(3, n + 2), Cells(122, n + 2)).Select

    Selection.Copy

    Sheets(states(n)).Select

    Range("C8").Select

    Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone,
SkipBlanks _

        :=False, Transpose:=False


    'Crop2 etm

    Sheets(" INITIAL (ETM,RAIN,RAM)").Select

    Range(Cells(124, n + 2), Cells(271, n + 2)).Select

    Selection.Copy

    Sheets(states(n)).Select

    Range("C137").Select

    Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone,
SkipBlanks _

        :=False, Transpose:=False


    'Crop1 rain

    Sheets(" INITIAL (ETM,RAIN,RAM)").Select

    Range(Cells(276, n + 2), Cells(395, n + 2)).Select

    Selection.Copy

    Sheets(states(n)).Select

    Range("F8").Select

    Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone,
SkipBlanks _

        :=False, Transpose:=False


     'Crop2 rain

    Sheets(" INITIAL (ETM,RAIN,RAM)").Select

    Range(Cells(397, n + 2), Cells(544, n + 2)).Select

    Selection.Copy

    Sheets(states(n)).Select

    Range("F137").Select

    Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone,
SkipBlanks _
```

```
        :=False, Transpose:=False
Next
'===========================================================================
Sheets("State1").Select

Do Until fixat = 0

Range("$BM$8:$BM$100").ClearContents

Range("$BQ$8:$BQ$141").Select
    Application.CutCopyMode = False
    Selection.Copy

Range("BH8").Select
Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks
_
        :=False, Transpose:=False

crop1Last = Range("BD5").Value

If fixat <= crop1Last Then
'Cells(8 + ((fixat - 1) / 2), 61).Value = 5
Calculate

For s = 0 To UBound(states)
nn = s + 1

    'Crop1 etm

Worksheets(" INITIAL (ETM,RAIN,RAM)").Activate
Range(Cells(3, staten + 2), Cells(3 + fixat - 2, staten + 2)).Select
Selection.Copy
Sheets("state" & nn).Select
Range("C8").Select
```

```
    Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone,
SkipBlanks _
        :=False, Transpose:=False


        'Crop1 rain
    Sheets(" INITIAL (ETM,RAIN,RAM)").Activate
    Range(Cells(276, staten + 2), Cells(276 + fixat - 2, staten +
2)).Select
    Selection.Copy
    Sheets("state" & nn).Select
    Range("F8").Select
    Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone,
SkipBlanks _
        :=False, Transpose:=False
Next s


Sheets("State1").Select


Set dynachange = Range(Cells(8 + ((fixat - 1) / 2), 60), Cells(143, 60))
Set dynamin = Range(Cells(8 + ((fixat - 1) / 2), 61), Cells(143, 61))
Set dynamax = Range(Cells(8 + ((fixat - 1) / 2), 62), Cells(143, 62))


dynachange.Select
dynamin.Select
dynamax.Select


Else


'Cells(8 + ((fixat - 82 - 1) / 2) + 1, 61).Value = 5
Calculate


For s = 0 To UBound(states)
nn = s + 1
Sheets("state" & nn).Select


    'Crop2 etm
```

```
    Sheets(" INITIAL (ETM,RAIN,RAM)").Select

    Range(Cells(124, staten + 2), Cells(124 + fixat - 200 - 2, staten +
2)).Select

    Selection.Copy

    Sheets("state" & nn).Select

    Range("C137").Select

    Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone,
SkipBlanks _

        :=False, Transpose:=False



     'Crop2 rain

    Sheets(" INITIAL (ETM,RAIN,RAM)").Select

    Range(Cells(397, staten + 2), Cells(397 + fixat - 200 - 2, staten +
2)).Select

    Selection.Copy

    Sheets("state" & nn).Select

    Range("F137").Select

    Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone,
SkipBlanks _

        :=False, Transpose:=False


Next s

Sheets("State1").Select


Set dynachange = Range(Cells(8 + ((fixat - 82 - 1) / 2) + 1, 60),
Cells(143, 60))

dynachange.Select

Set dynamin = Range(Cells(8 + ((fixat - 82 - 1) / 2) + 1, 61), Cells(143,
61))

dynamin.Select

Set dynamax = Range(Cells(8 + ((fixat - 82 - 1) / 2) + 1, 62), Cells(143,
62))

dynamax.Select


End If

n = 1
```

```
max = 1
score = 0

Do Until score = max
    SolverReset

    SolverOk SetCell:="G$305", MaxMinVal:=1, ValueOf:=0, ByChange:= _
        dynachange.Offset(0, 0), Engine:=3, EngineDesc:="Evolutionary"

    SolverAdd CellRef:=dynachange.Offset(0, 0), Relation:=3,
FormulaText:=dynamin.Offset(0, 0)
    SolverAdd CellRef:=dynachange.Offset(0, 0), Relation:=1,
FormulaText:=dynamax.Offset(0, 0)

    SolverOptions PopulationSize:=30, MutationRate:=0.5, RandomSeed:=1,
MaxTimeNoImp:=300
    SolverSolve userFinish:=True

 Range("G305").Select
    Application.CutCopyMode = False
    Selection.Copy

    score = Range("G305").Value
    max = Range("BN8").Value

        'Range("BM8").Select
    Cells(8 + n - 1, 65).Select
    Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone,
SkipBlanks _
        :=False, Transpose:=False
   n = n + 1
 Loop

 Range("$BH$8:$BH$143").Select
    Application.CutCopyMode = False
    Selection.Copy
```

```
Range("BQ8").Select
Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks
_
        :=False, Transpose:=False

CPDdays = CPDdays + 1
fixat = Cells(8 + CPDdays, 68).Value
Loop
End Sub
```

APPENDIX C

LIMITED WATER QUOTA

SINGLE-STAGE DECISION-MAKING FRAMEWORK MACRO

```
Sub SolveBaseModel


Dim score As Variant

Dim max As Variant

Dim n As Integer


''initialise states

states = Array("state1", "state2", "state3", "state4", "state5", "state6",
"state7", "state8", "state9", "state10", "state11", "state12")

For n = 0 To UBound(states)


 Sheets(" INITIAL (ETM,RAIN,RAM)").Select


    'Crop1 etm

    'Range("B3:B122").Select

    Range(Cells(3, n + 2), Cells(122, n + 2)).Select

    Selection.Copy

    Sheets(states(n)).Select

    Range("C8").Select

    Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone,
SkipBlanks _

        :=False, Transpose:=False


    'Crop2 etm

    Sheets(" INITIAL (ETM,RAIN,RAM)").Select

    Range(Cells(124, n + 2), Cells(271, n + 2)).Select

    Selection.Copy

    Sheets(states(n)).Select

    Range("C137").Select

    Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone,
SkipBlanks _

        :=False, Transpose:=False
```

```
    'Crop1 rain
    Sheets(" INITIAL (ETM,RAIN,RAM)").Select
    Range(Cells(276, n + 2), Cells(395, n + 2)).Select
    Selection.Copy
    Sheets(states(n)).Select
    Range("F8").Select
    Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone,
SkipBlanks _
        :=False, Transpose:=False

     'Crop2 rain
    Sheets(" INITIAL (ETM,RAIN,RAM)").Select
    Range(Cells(397, n + 2), Cells(544, n + 2)).Select
    Selection.Copy
    Sheets(states(n)).Select
    Range("F137").Select
    Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone,
SkipBlanks _
        :=False, Transpose:=False
Next

'===========================================================================
Sheets("State1").Select

n = 1
max = 1
score = 0

Range("BM8:BM65582").ClearContents
Calculate

SolverReset
    SolverOk SetCell:="$G$305", MaxMinVal:=1, ValueOf:=0, ByChange:= _
        "$BH$8:$BH$141,$BH$2:$BH$3", Engine:=3, EngineDesc:="Evolutionary"
```

```
   SolverAdd CellRef:="$BH$8:$BH$141", Relation:=1,
FormulaText:="$BJ$8:$BJ$141"

   SolverAdd CellRef:="$BH$8:$BH$141", Relation:=3,
FormulaText:="$BI$8:$BI$141"


   SolverAdd CellRef:="$BH$2:$BH$3", Relation:=1,
FormulaText:="$BJ$2:$BJ$3"

   SolverAdd CellRef:="$BH$2:$BH$3", Relation:=3,
FormulaText:="$BI$2:$BI$3"


 SolverOptions PopulationSize:=30, MutationRate:=0.5, RandomSeed:=1,
MaxTimeNoImp:=300


    Do Until score = max


    SolverSolve userFinish:=True


 Range("G305").Select

    Application.CutCopyMode = False

    Selection.Copy


    score = Range("G305").Value

    max = Range("BN8").Value

    'Range("BM8").Select

    Cells(8 + n - 1, 65).Select

    Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone,
SkipBlanks _

        :=False, Transpose:=False

   n = n + 1

 Loop

score = 0

  SolverReset


    SolverOk SetCell:="$G$305", MaxMinVal:=1, ValueOf:=0, ByChange:= _

        "$BH$8:$BH$141,$BH$2:$BH$3", Engine:=3, EngineDesc:="Evolutionary"
```

```
    SolverAdd CellRef:="$BH$8:$BH$141", Relation:=3,
FormulaText:="$BI$8:$BI$141"

    SolverAdd CellRef:="$BH$8:$BH$141", Relation:=1,
FormulaText:="$BJ$8:$BJ$141"


   SolverAdd CellRef:="$BH$2:$BH$3", Relation:=1,
FormulaText:="$BJ$2:$BJ$3"

   SolverAdd CellRef:="$BH$2:$BH$3", Relation:=3,
FormulaText:="$BI$2:$BI$3"


    SolverOptions PopulationSize:=30, MutationRate:=0.075, RandomSeed:=1,
MaxTimeNoImp:=30


     Do Until score = max


    SolverSolve userFinish:=True


  Range("G305").Select

    Application.CutCopyMode = False

    Selection.Copy


    score = Range("G305").Value

    max = Range("BN8").Value

    'Range("BM8").Select

    Cells(8 + n - 1, 65).Select

    Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone,
SkipBlanks _

        :=False, Transpose:=False

   n = n + 1

 Loop


 Range("$BH$8:$BH$141").Select

    Application.CutCopyMode = False

    Selection.Copy


Range("BQ8").Select
```

```
Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks
_
        :=False, Transpose:=False
Range("CM8").Select
Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks
_
        :=False, Transpose:=False
End Sub
```