A UNIVERSAL DESIGN FOR LEARNING STRATEGY TO ENHANCE THE TEACHING OF WORD PROBLEMS IN A MULTILINGUAL MATHEMATICS CLASSROOM

by

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DECLARATION

"I, MIRRIAM MATSHIDISO MOLEKO, declare that the doctoral degree research thesis or interrelated, publishable manuscripts/published articles, or coursework doctoral degree mini-thesis that I herewith submit for the doctoral degree qualification, PhD (Mathematics Education) at the University of the Free State is my independent work, and that I have not previously submitted it for a qualification at another institution of higher education."

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M.M. MOLEKO,

JANUARY 2018

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****

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DEDICATION

This thesis is dedicated to **Thato (Thatohatsi – my daughter)** who inspires me to do more, become great and to push beyond boundaries! Through this beautiful soul, I do not only know but also understand that *boldness, determination, faith* and *risky undertakings* are also some of life's ingredients of success!

****

LIST OF ABBREVIATIONS AND ACRONYMS

CAPS	Curriculum Assessment Policy Statement
CAST	Center for Applied Special Technology
CCS	Concrete Cognitive Strategies
CDA	Critical Discourse Analysis
CER	Critical Emancipatory Research
CT	Critical Theory
DoBE	Department of Basic Education
HoD	Head of Department
LoLT	Language of Learning and Teaching
MoI	Medium of Instruction
MWP	Mathematics Word Problem
PAR	Participatory Action Research
SLT	Social Learning Theory
UD	Universal Design
UDI	Universal Design for Instruction
UDL	Universal Design for Learning

SUMMARY OF THE STUDY

ABSTRACT

Mathematics word problems have always been a challenging concept in mathematics, not only on the part of the learners but also on the part of the teachers, albeit for different reasons. Learners often struggle to solve mathematics word problems and many teachers find it challenging to teach this genre of mathematics, for various reasons. Mathematics word problems are problems presented in text form and thus require learners to be proficient in the language of learning and teaching to understand and solve them. However, most learners do not master this genre of mathematics as they are not proficient in English, which is the medium of instruction in most cases. The challenge of teaching mathematics word problems is further aggravated by the fact that most South African schools are multilingual (i.e. a number of different languages are spoken in class, not only the language of learning and teaching) and research has shown that teaching in such contexts is complex. Mathematics teachers in multilingual classrooms, therefore, face challenges that hinder the teaching process. This necessitates the need to formulate a universal design for learning strategy to enhance the teaching of mathematics word problems in multilingual mathematics classrooms. Universal design for learning is an educational framework that has proven to be effective in terms of the teaching of learners in diverse classrooms, including multilingual classrooms. This study seeks to formulate universal design for learning guidelines in an effort to assist in the effective teaching of mathematics word problems in multilingual mathematics classrooms.

A participatory action research approach was adopted to generate the empirical data and ensure that the voices of all the stakeholders were captured. The study involved mathematics teachers, English and mathematics teachers, mathematics literacy teachers, Grades 10, 11 and 12 mathematics learners, a mathematics head of department and the principal to explore the following research question: How can we utilise the aspects of universal design for learning to develop an effective teaching strategy for mathematics word problems in a multilingual mathematics classroom? The data were generated through meetings, forums, lesson observations, document analysis

(learners' homework and class work) and teacher-to-teacher observations as well as reflective discussions. Critical emancipatory research was adopted as the lens that underpins this study. The adoption of critical emancipatory research was informed by its requirement that all the people concerned (including the marginalised) should be included in the research process and that their voices should be heard, respected and acknowledged as contributing to the broader goal of the research study, which is to bring about a change in their situation (teaching of mathematics word problems in a multilingual mathematics classroom). Critical discourse analysis was adopted as the tool to analyse the discourses in this study. The selection of critical discourse analysis was inspired by the fact that it enables the researcher to analyse not only text data but also any visual cues and behaviour displayed by the participants. My observations also assisted in establishing the deeper meaning of the claims.

Six major themes emerged from the data analysis of this study, justifying the need for teachers to look carefully into their teaching practices and adapt new ways of teaching in an effort to optimise learning and enhance the solving of mathematics word problems. These themes thus suggest the teaching implications for teaching of this mathematics genre. Firstly, the research findings indicate that learners lack the necessary reading skills to comprehend mathematics word problems. Secondly, learners also lack the mathematical vocabulary and register needed to comprehend and solve mathematics word problems. Thirdly, the learners' inability to visualise mathematics word problems makes it difficult to procedurally solve these problems. Fourthly, ambiguity also causes a lack of understanding, resulting in the failure to solve mathematics word problems. Fifthly, the teachers' inability to assist learners in developing effective problem-solving skills, especially in terms of solving mathematics word problems, was a major concern. Lastly, the teachers' negative attitude towards the use of learners' home languages as a possible resource to aid learners in the solving of mathematics word problems was mentioned as a challenge that had to be addressed. The challenges that emerged from the study had implications for teaching and thus required the teaching of mathematics word problems to be approached differently to enable learners to solve these problems. A universal design for learning strategy that encourages the application of the three principles, namely multiple means of representation, multiple means of action and expression as

well as multiple means of engagement was, therefore, recommended in order to enhance the teaching of mathematics word problems and to encourage teachers to be reflective about their practices and adapt them accordingly to remove learning barriers.

Keywords: mathematics word problems, multilingual mathematics classrooms, universal design for learning, participatory action research, critical emancipatory research, critical discourse analysis

OPSOMMING

Wiskunde-woordprobleme was nog altyd ‘n uitdagende konsep. Beide leerders en onderwysers ervaar dit as ‘n uitdagende genre in wiskunde, alhoewel hul redes hiervoor verskil. Leerders sukkel dikwels om wiskunde-woordprobleme op te los en baie onderwysers vind dit moeilik om die begrip aan die leerders te verduidelik. Woordprobleme is wiskundeprobleme wat in teksvorm aangebied word en dus van die leerders vereis om die onderrigtaal magtig te wees ten einde die probleme te verstaan en oplossings daarvoor te vind. Baie leerders sukkel egter met hierdie genre van wiskunde aangesien hulle nie Engels, wat gewoonlik die onderrigtaal is, magtig is nie. Die uitdaging om woordprobleme aan te bied word vererger deur die feit dat die meeste Suid-Afrikaanse skole veeltalig is (’n verskeidenheid tale word dus in die klas gepraat, nie net die onderrigtaal nie). Navorsing het ook getoon dat onderrig in hierdie omgewing kompleks is. Wiskunde-onderwysers in veeltalige klaskamers staar dus uitdagings wat die onderrigproses kan benadeel in die gesig. Dit noodsaak die ontwikkeling van ’n universele-ontwerp-vir-leer-strategie om die onderrig van woordprobleme in veeltalige klaskamers te bevorder. Die universele ontwerp vir leer is ’n opvoedkundige raamwerk wat suksesvol blyk te wees ten opsigte van die onderrig van leerders in diverse klaskamers, insluitend veeltalige klasse. Hierdie studie poog dus om riglyne van universele ontwerp vir leer te formuleer in ’n poging om die effektiewe onderrig van woordprobleme in veeltalige klaskamers te bevorder.

Deelnemende-aksienavorsing is gebruik om die empiriese data te versamel en te verseker dat die stemme van die belanghebbendes vasgevang word. Die studie het wiskunde-onderwysers van twee gekose skole betrek ten einde die volgende navorsingsvraag te beantwoord: Hoe kan ons die onderrig van wiskunde-woordprobleme in veeltalige klaskamers bevorder deur die gebruik van ’n universele-ontwerp-vir-leer-strategie? Die data is gegenereer deur vergaderings, klaskamerwaarnemings en dokumentontleding (leerders se huis- en klaswerk) en deur reflektiewe gesprekke. Kritiese emansipatoriese navorsing is gebruik as die lens wat die studie onderlê. Die rede hiervoor is dat dit vereis dat alle persone (insluitend die gemarginaliseerdes) wat ’n belang

by die saak het, ingesluit word in die navorsingsproses en dat hul stemme gehoor, gerespekteer en erken moet word as bydraend tot die groter doel van die studie, naamlik om ŉ verandering in die situasie teweeg te bring. Kritiese diskoersanalise is gebruik as die hulpmiddel om die gesprekke in die studie te ontleed. Die rede vir die gebruik van kritiese diskoersanalise is dat dit die navorser in staat stel om nie net teksdata te ontleed nie, maar ook enige visuele gebare en optrede wat die deelnemers mag toon. Die navorser se waarnemings het ook die vasstelling van die dieper betekenis van die stellings bevorder.

Uit die data-analise kon ses hooftemas bepaal word. Hierdie temas regverdig die noodsaaklikheid dat onderwysers hul onderrigpraktyke moet ontleed en nuwe metodes aanleer in ŉ poging om onderrig en die oplos van wiskunde-woordprobleme te bevorder. Hierdie temas stel dus die onderrigimplikasies voor ten opsigte van die onderrig van hierdie genre van wiskunde. Eerstens dui die bevindings daarop dat leerders nie oor die nodige leesvaardighede beskik om woordprobleme te begryp nie. Tweedens beskik leerders ook nie oor die wiskundige woordeskat en -register om woorprobleme te verstaan en op te los nie. Derdens bemoeilik die leerders se onvermoë om woordprobleme te visualiseer hul taak om dit volgens prosedure op te los. Vierdens veroorsaak dubbelsinnigheid ook onbegrip, wat daartoe lei dat die probleme nie opgelos kan word nie. Vyfdens is die onderwysers se onvermoë om leerders te help met effektiewe probleemoplossing, veral ten opsigte van woordprobleme, ook gemeld as ŉ uitdaging wat die hoof gebied moet word. Laastens is die negatiewe houding van die onderwysers ten opsigte van die gebruik van leerders se huistale as ŉ moontlike hulpmiddel vir leerders tydens die oplossing van woordprobleme, ook aangedui as een van die uitdagings wat die hoof gebied moet word. Dit sal onderwysers noodsaak om hul onderrigpraktyke te hersien ten einde leerders se probleemoplossingvaardighede, veral ten opsigte van woordprobleme, te bevorder.

LIST OF APPENDICES

APPENDIX A1: Ethical clearance

APPENDIX A2: Application letter to conduct a research study

APPENDIX A3: Permission letter to conduct a research study

APPENDIX A4: Teacher consent

APPENDIX A5: (English): Assent letter

APPENDIX A5: (Sesotho): Assent letter

APPENDIX A6: Consent letter – Principal

APPENDIX A7: Consent letter – HoD

APPENDIX A8: Consent letter – Mathematics literacy teacher

APPENDIX A9: Consent letter – Video recording/taking pictures

APPENDIX A10: Information session programme

APPENDIX B: Language editing certificate

APPENDIX C: Similarity index document

LIST OF TABLES

Table 2.1: UDL principles

Table 2.2: UDI principles and examples

Table 3.1: Mathematical Shapes and formulae for calculating the Area

Table 4.1: Framework for creating motivational spaces

Table 4.2: Participants involved in the PAR project

Table 7.1: Differences between arithmetic sequence and geometric series

LIST OF DIAGRAMS

Diagram 3.1: Word problem diagrammatic representation

Diagram 3.2: Evolution of access

Diagram 7.1: Venn diagram

LIST OF FIGURES

FIGURE 3.1: Right-angled triangle

FIGURE 3.2: Circle

FIGURE 3.3: Square

FIGURE 3.4: Yarn

FIGURE 3.5: Rectangular prism

FIGURE 5.1: Word problem and the steps taken to solve the problem

FIGURE 5.2: Parabolic graph – upward shift

FIGURE 5.3: Parabolic graph and its reflection

FIGURE 5.4: Word problem and questions

FIGURE 5.5: $3 \le x \le 5$ representation on Cartesian plane: Correct and incorrect solutions

FIGURE 5.6: Word problem with a picture alongside

FIGURE 5.7: The graphs, $y = Sinx$ & $y = Sinx + 2$, plotted on the same Cartesian plane

TABLE OF CONTENTS

DECLARATION ii
ACKNOWLEDGEMENTS iii
DEDICATION v
LIST OF ABBREVIATIONS AND ACRONYMS vi
SUMMARY OF THE STUDY vii
OPSOMMING x
LIST OF APPENDICES xii
LIST OF TABLES xiii
LIST OF DIAGRAMS xiv
LIST OF FIGURES xv

CHAPTER 1: ORIENTATION AND BACKGROUND 1
1.1 INTRODUCTION AND BACKGROUND 1
1.2 CRITICAL EMANCIPATORY RESEARCH 4
1.3 PROBLEM STATEMENT 5
1.4 RESEARCH QUESTION 6
1.5 RESEARCH OBJECTIVES 6
1.6 PRIMARY AND SECONDARY QUESTIONS 7
1.6.1 Primary question 7
1.6.2 Secondary research questions 7
1.7 RESEARCH DESIGN AND METHODOLOGY 8
1.7.1 Instrumentation 10
1.7.2 Research participants 111
1.8 CRITICAL DISCOURSE ANALYSIS 12
1.9 VALUE OF THE STUDY 133
1.10 ETHICAL CONSIDERATIONS 14
1.11 LAYOUT OF THE CHAPTERS 15
1.12 CONCLUSION 16

CHAPTER 2: THEORETICAL FRAMEWORK AND OPERATIONAL CONCEPTS INFORMING THE STUDY 177
2.1 INTRODUCTION 177

2.2 THEORETICAL FRAMEWORK 177
2.2.1 Historical background of critical emancipatory research 188
2.2.2 Critical emancipatory research objectives 200
2.2.3 Nature of reality 200
2.2.4 Relationship between the researcher and the participants 211
2.2.5 Role of the researcher 222
2.2.6 The role of critical emancipatory research in achieving the results of this study .233
2.3 DEFINITION OF THE OPERATIONAL CONCEPTS 244
2.3.1 Universal design for learning 244
2.3.1.1 Historical background 244
2.3.1.2 Moving from universal design to universal design for learning 266
2.3.1.3 Definition of universal design for learning in the context of this study 277
2.3.1.4 Universal design for learning – three groups of brain networks 277
2.3.1.5 Universal design for learning principles 290
2.3.1.6 Universal design for learning objectives 300
2.4 UNIVERSAL DESIGN FOR INSTRUCTION 311
2.5 MATHEMATICS WORD PROBLEMS 355
2.6 MULTILINGUAL CLASSROOMS 36
2.7 CONCLUSION 377

CHAPTER 3: LITERATURE REVIEW TOWARDS FORMULATING A UNIVERSAL DESIGN FOR LEARNING-BASED TEACHING STRATEGY FOR MATHEMATICS WORD PROBLEMS IN A MULTILINGUAL MATHEMATICS CLASSROOM ..388

3.1 INTRODUCTION 388
3.2 BACKGROUND 388
3.3 CHALLENGES PERTAINING TO THE TEACHING AND LEARNING OF MATHEMATICS WORD PROBLEMS IN MULTILINGUAL CLASSROOMS 399
3.3.1 Lack of reading skills 400
3.3.2 Lack of a mathematical vocabulary and register 422
3.3.3 Lack of visualisation skills 466
3.3.4 Lack of understanding caused by ambiguity 48
3.3.5 Teachers' lack of development of skills in terms of teaching mathematics word problems in multilingual classrooms 511
3.3.6 Teachers' negative attitude towards the use of home languages 533
3.4 SOLUTIONS TO THE IDENTIFIED CHALLENGES 555

3.4.1 Reading skills employed to enhance understanding of the mathematics word problems 555
3.4.2 Strategies to improve learners' mathematical vocabulary and register 577
3.4.3 Strategies to promote visualisation of word problems 600
3.4.4 Strategies to overcome misconceptions of word problems caused by the presence of ambiguous words 622
3.4.5 Empowering strategies to enhance teachers' ability to assist learners in developing effective word problem-solving skills 644
3.4.6 Using home languages as resources to promote learning and understanding of mathematics word problems 655
3.5 CONDITIONS CONDUCIVE TO THE EFFECTIVENESS OF THE SOLUTIONS 677
3.5.1 Conditions conducive to enhancing learners' reading skills and understanding..677
3.5.2 Conditions favouring the improvement of learners' mathematical vocabulary and register 6868
3.5.3 Conditions conducive to overcome lack of understanding caused by the presence of ambiguous words 69
3.5.4 Conditions conducive to the implementation of the strategies to enhance word problem text visualisation 711
3.5.5 Conditions conducive to teachers' empowering strategies to teach word problems in multilingual mathematics classrooms effectively 722
3.5.6 Conditions conducive to effecting the use of home languages as supportive resources in the teaching of word problems 744
3.6 POSSIBLE THREATS TO THE SUCCESSFUL IMPLEMENTATION OF THE STRATEGIES 755
3.7 EVIDENCE OF THE SUCCESSFUL IMPLEMENTATION OF THE UNIVERSAL DESIGN FOR LEARNING STRATEGY 777
3.7.1 Improved reading skills 777
3.7.2 Improved knowledge of the mathematical vocabulary and register 778
3.7.3 Improved skills to eliminate ambiguities 7979
3.7.4 Improved visualisation skills 800
3.7.5 Improved problem-solving skills 811
3.7.6 Improved understanding of word problems reinforced by the use of home languages 833
3.8 SUMMARY 844

CHAPTER 4: RESEARCH DESIGN AND METHODOLOGY 877
4.1 INTRODUCTION 877
4.2 DEFINING QUALITATIVE RESEARCH 88

4.3 DEFINING PARTICIPATORY ACTION RESEARCH 8989
4.4 PARTICIPATORY ACTION RESEARCH PRINCIPLES 9090
4.4.1 Epistemological stance of participatory action research 911
4.5 THE RELEVANCE OF PARTICIPATORY ACTION RESEARCH TO THIS STUDY ..922
4.6 CREATING MOTIVATIONAL SPACE TO INSPIRE AND MAXIMISE PARTICIPATION 944
4.7 THE PARTICIPANTS IN THE STUDY 955
4.7.1 Primary reasons for the selection of the participants and the school 98
4.8 INSTRUMENTATION 99
4.8.1 Lesson observation or demonstration 9999
4.8.2 Teacher-teacher observations 1000
4.8.3 Individual meetings 1000
4.8.4 Reflective session 1000
4.8.5 Document analysis 1011
4.9 DATA COLLECTION PROCEDURE 1011
4.10 ENSURING TRUSTWORTHINESS IN THE DATA GENERATION PROCESS 1033
4.11 UNFOLDING OF THE INTERVENTION 1044
4.11.1 Conditions before the commencement of the intervention 1044
4.11.2 Formulation of a dedicated team 1055
4.11.3 Information session 1077
4.11.4 Team-building exercise 1077
4.11.5 Meetings with the participants 108
4.12 DATA ANALYSIS AND INTERPRETATION 10909
4.12.1 Critical discourse analysis 10909
4.12.1.1 Textual analysis 1111
4.12.1.2 Social analysis 1111
4.12.1.3 Discursive practice 1122
4.13 Relevance of critical discourse analysis in this study 1122
4.14 ETHICAL CONSIDERATIONS 1133
4.15 CONCLUSION 1155

CHAPTER 5: DATA PRESENTATION, ANALYSIS AND INTERPRETATION OF A UNIVERSAL DESIGN FOR LEARNING STRATEGY TO ENHANCE THE TEACHING OF WORD PROBLEMS IN A MULTILINGUAL MATHEMATICS CLASSROOM 1166
5.1 INTRODUCTION 1166

5.2 CHALLENGES JUSTIFYING THE FORMULATION OF THE STRATEGY 11717
5.2.1 Lack of reading skills............11717
5.2.2 Learners' limited mathematical vocabulary and register 1244
5.2.3 Learners' inability to visualise the word problems............129
5.2.4 Ambiguous word problems............133
5.2.5 Teachers' inability to teach word problems 1366
5.2.6 Teachers' negative perception regarding the use of learners' home languages in classrooms 1400
5.2.7 Summary 1466
5.3 THE CONSTITUENTS OF AN APPROACH USED IN RESPONSE TO THE IDENTIFIED CHALLENGES 14747
5.3.1 Engaging learners in reading exercises 14747
5.3.2 Refining the learners' mathematical vocabulary 1522
5.3.3 Strategies to help scaffold visualisation skills 1555
5.3.4 Strategies to overcome the lack of understanding caused by the presence of ambiguous words 1611
5.3.5 Strategies to empower teachers to teach word problems in multilingual classrooms effectively 1644
5.3.6 Promoting the effective use of home languages within classrooms............167
5.4 CONDITIONS CONDUCIVE TO THE SUCCESSFULL IMPLEMENTATION OF THE STRATEGY............1733
5.4.1 Factors supporting the enhancement of learners' reading skills 1733
5.4.2 Factors contributing towards the improvement of learners' mathematical vocabulary and register 17878
5.4.3 Contributing factors towards successful elimination of ambiguity............18080
5.4.4 Conditions conducive to the successful implementation of strategies that improve visualisation of the word problems 1822
5.4.5 Contributing factors towards the enrichment of the teachers' skills to teach word problems in multilingual classrooms............1855
5.4.6 Favourable conditions towards the use of learners' home languages 1866
5.5 ANTICIPATED THREATS TO THE IMPLEMENTATION OF THE STRATEGY 1888
5.5.1 Lack of participation, unavailability of material written in other languages and stakeholders' unwillingness to participate 18888
5.6 EVIDENCE INDICATING THAT THE STRATEGY WAS EFFECTIVE............1911
5.6.1 Evidence of learners' improved reading skills............1922
5.6.2 Evidence of learners' improved mathematical vocabulary and register 1933

5.6.3 Evidence of the successful implementation of the strategies employed to address the ambiguity of the word problems..1955

5.6.4 Evidence of the successful implementation of the strategies employed to enhance learners' visualisation skills 197197

5.6.5 Evidence of improved teachers' capacity to assist learners in developing problem-solving skills 199

5.6.6 Successes attributed to the effective use of learners' home languages in the classrooms 2011

5.7 CONCLUSION 2033

CHAPTER 6: Discussion of the results of the study and recommendations..................2044

6.1 INTRODUCTION...2044

6.2 JUSTIFICATION FOR THE NEED TO FORMULATE A UDL STRATEGY TO ENHANCE THE TEACHING OF MATHEMATICS WORD PROBLEMS IN A MULTILINGUAL MATHEMATICS CLASSROOM 2044

6.2.1 Learners have difficulty reading and understanding the language used in the word problems..2055

6.2.2 Learners lack an understanding of the mathematical vocabulary and register .2055

6.2.3 Learners lack the skill to visualise word problems..20606

6.2.4 Words with multiple meanings cause word problems to be ambiguous..........20707

6.2.5 Teachers find word problems challenging and thus cannot teach learners effectively 20808

6.2.6 Most teachers view the use of home languages negatively and are reluctant to use them in class despite knowing that learners lack English proficiency 20909

6.3 SOME OF THE COMPONENTS OF THE UDL STRATEGY IDENTIFIED AS IMPORTANT IN ADDRESSING THE IDENTIFIED CHALLENGES.........................2100

6.3.1 Strategies employed to improve learner reading skills 2100

6.3.2 Strategies employed to improve learners' mathematical vocabulary and register 2111

6.3.3 Strategies employed to improve learners' problem visualisation skills 2111

6.3.4 Strategies employed to eliminate words which cause ambiguity.......................2122

6.3.5 Strategies to empower teachers to teach word problems effectively in multilingual classrooms...2122

6.3.6 Strategies to promote effective use of home languages..................................2133

6.4 SOME OF THE ENABLING FACTORS FOR THE COMPONENTS OF THE UDL STRATEGY TO WORK THAT WERE REGARDED AS ESSENTIAL IN ENHANCING THE TEACHING OF MWPS IN MULTILINGUAL MATHEMATICS CLASSROOMS 2144

6.4.1 Improving learners' reading skills and understanding......................................2144

6.4.2 Improving learners’ mathematical vocabulary and register 2144
6.4.3 Eliminating ambiguities from the word problems 2155
6.4.4 Implementation of strategies to improve learners’ visualisation of word problems 21515
6.4.5 Improving teachers’ skills to teach word problems in multilingual classrooms .21616
6.4.6 Effective use of learners’ home languages when teaching the MWPs............21717
6.5 SOME OF THE THREATS THAT COULD IMPEDE THE SUCCESSFUL IMPLEMENTATION OF THE STRATEGY THAT WERE CONSIDERED IMPORTANT 21717
6.5.1 Lack of participation in class activities...21717
6.5.2 Unavailability of mother tongue material 21818
6.5.3 Stakeholders’ unwillingness to cooperate 21919
6.5.4 Influence of linguistic structures in meaning and interpretation 21919
6.6 SOME OF THE IDENTIFIED SUCCESS INDICATORS OF THE IMPLEMENTATION OF THE UDL STRATEGY..21919
6.6.1 Improved reading skills of learners..2200
6.6.2 Improved mathematical vocabulary and register of learners 2200
6.6.3 Elimination of ambiguity 2211
6.6.4 Enhanced visualisation skills of learners 2211
6.6.5 Improved teaching skills...2222
6.6.6 Effective use of learners’ home languages in the classrooms 2233
6.7 SUMMARY 2233
6.8 IMPLICATIONS OF THE STUDY RESULTS ON THE PRACTICE.........................2244
6.8.1 Epistemological implication 2244
6.8.2 Methodological implication...2244
6.8.3 Pedagogical implication 2255
6.9 ASPECTS FOR FURTHER RESEARCH AND RECOMMENDATIONS 22626
6.10 CONCLUSION 22626

CHAPTER 7: THE PROPOSED UNIVERSAL DESIGN FOR LEARNING-BASED STRATEGY TO ENHANCE THE TEACHING OF MATHEMATICS WORD PROBLEMS IN MULTILINGUAL MATHEMATICS CLASSROOM 22727
7.1 INTRODUCTION..22727
7.2 PROPOSED UDL-BASED STRATEGY FOR TEACHING WORD PROBLEMS IN A MULTILINGUAL MATHEMATICS CLASSROOM..22727
7.3 MULTIPLE MEANS OF REPRESENTATION 22929

7.3.1 Provide options for perceptions 22929
7.3.2 Provide options for language, mathematical expressions and symbols 23030
7.3.3 Provide options for comprehension 2311
7.4 MULTIPLE MEANS OF ACTION AND EXPRESSION 2355
7.4.1 Provide options for physical action 23636
7.4.2 Provide options for expression and communication 23838
7.4.3 Provide options for executive functions 23939
7.5 PROVIDE MULTIPLE MEANS OF ENGAGEMENT 2400
7.5.1 Options for recruiting interest 2400
7.5.2 Options for sustaining effort and persistence 2411
7.5.3 Options for self-regulation 2411
7.6 SUMMARY 2422
7.7 LIMITATIONS OF THIS STUDY 2422
7.8 CONCLUSION 2433

REFERENCES 2455
TRANSCRIPTS 27777
APPENDICES 277

CHAPTER 1

ORIENTATION AND BACKGROUND

1.1 INTRODUCTION AND BACKGROUND

The purpose of the study was to utilise the aspects of universal design for learning (UDL) to develop an effective teaching strategy for mathematics word problems (MWPs) in a multilingual mathematics classroom. MWPs refer to mathematical exercises in which content is presented in the form of a story (Kasule & Mapolelo, 2013:265). MWPs are usually included in the school mathematics curriculum because of their potential role in promoting realistic mathematical modelling and problem solving. They also serve to make learners aware of how and when they can combine classroom mathematical knowledge and knowledge from everyday life when solving problems (Sepeng, 2013:170). Palm (2009:3) refers to MWPs as problems that encompass pure mathematical tasks that are “dressed up” in real-world situations, and necessitate learners to “undress” them to solve the problems.

Learners should be numerically literate to solve these types of mathematical problems successfully. This means they must not only be able to reason with numbers but also to identify, understand, interpret, create, communicate and compute numbers (UNESCO, 2004:13). Furthermore, learners must be able to use text to detect missing information, construct number sentences and set up calculation problems to solve MWPs (Fuchs, Seethaler, Powell, Fuchs, Hamlett & Fletcher, 2008:56). Although MWPs help learners to use their mathematics knowledge in solving problems and realising the applicability of this knowledge in their daily lives (Seifi, Haghverdi & Azizmohamadi, 2012:2923), most learners struggle to master this genre of mathematics.

A study that was conducted in Canada revealed that a major challenge regarding the teaching of MWPs was that learners were not proficient in reading and also lacked literacy skills, which negatively affected their understanding (Bohlmann & Pretorius, 2008:42). In Australia, it was found that learners’ failure to understand and solve MWPs successfully was caused by the use of ambiguous words in the problem statements since the meaning of these words depends on the setting. A word has a different meaning when it is used in the mathematics classroom than when it is used in our daily lives. These ambiguous words caused grave misunderstandings among learners (Meiers, 2010:5). Furthermore, research also indicates that some teachers in Malawi, Botswana and Zimbabwe are unable to teach MWPs to learners, resulting in learners not knowing how to convert words into mathematically computable numerals (Chitera, 2011:1006; Kasule & Mapolelo, 2013:265). In a large number of cases in Malawi, Canada and Asia, teachers' inability to teach MWPs effectively in multilingual mathematics classrooms resulted in learners not knowing how to make meaning of word problems. Thus, learners failed to identify the appropriate arithmetic operations to solve the problems (Chitera, 2011:1009; Polotskaia, Savard & Freiman, 2015:253). In South Africa, one of the major challenges was teachers who restricted learners from using their home languages as a resource to help them understand mathematical concepts. This kept the learners from developing a deeper understanding of word problems and limited their mathematical proficiency that would enable them to transfer meaning to their second language (Nkambule, 2009:4).

In addition to the above, multilingualism was found to be one of the main reasons why learners did not perform well in MWPs. According to Barwell (2009:32), the teaching of mathematics in multilingual classrooms is complex since it requires teachers to bring together learners with educational needs who, when taken exclusively, would call for different interventions. In agreement with Barwell's notion, Chitera (2009:1) adds that teaching and learning of mathematics in a language that is not the first language of the learners or the mathematics teacher, is complex and can create dilemmas of code switching, mediation and transparency for teachers (Adler, 2001:100-101). Based on these intricacies, Essien (2013:8) emphasises that it is crucial that teachers are equipped with the understanding and the skills they need to deal with the challenges posed by the teaching of mathematics in multilingual classrooms and to effectively support learners in these classrooms.

In an attempt to address the abovementioned challenges, a number of strategies have been explored and applied, including the development of the mathematical vocabulary through the use of various reading strategies (Kersaint, Thompson & Petkova, 2014:83).

In order to improve learners' visualisation skills, their ability to produce quality representations was scaffolded through the provision of explicit structures that enabled them to "see" the problem (Mulligan, 2011:32). This approach enhanced the learners' understanding of the problems and, consequently, enabled them to solve these problems. To address the misunderstandings caused by ambiguous words in MWPs, teachers provided the learners with opportunities to engage in various exercises, not only to grow accustomed to mathematical forms of expression but also to realise how these mathematical forms of expression could be used to communicate about mathematics in everyday life (according to the different mathematical contexts) (Skolverket, 2011:62).

Teachers' capacity development was recommended in an effort to ensure that they effectively guide and scaffold learners through teaching that facilitates language progress and learning (Hansson, 2012:103). Although these strategies have been effective to some extent, the teaching of mathematics in multilingual classrooms remains a challenge that needs to be addressed (Essien, 2013:8). Essien (2013:8) further notes that teachers must be equipped with the necessary understanding and skills to enable them to deal with and support learners in multilingual mathematics classrooms.

Essien's belief that mathematics teachers must be equipped to deal with the complexity of teaching in multilingual classrooms captured my attention and urged me to explore the matter further with the quest to providing possible solutions thereof. However, it was of the utmost importance that I first had to determine what had already been done in this regard to make a significant contribution to the topic. Various scholars have explored the teaching of learners in multilingual mathematics classrooms with a specific focus on the complexities of teaching in such classrooms (Chitera, 2009; Nkambule, 2009). Others explored the relationship between languages and discourses (Hansson, 2012; Setati & Adler, 2000), thus exploring the meaning of mathematics instruction in these types of classrooms (multilingual classrooms). Some researchers have explored the implementation of language-in-education policies in multilingual mathematics classrooms (Halai & Karuku, 2013), while others have explored the teaching strategies that would overcome the challenges in multilingual mathematics classrooms created by the language of instruction (Naidoo, 2015). However, none of these studies explored specific ways in which the teaching of MWPs could be enhanced in multilingual mathematics classrooms; more so, through the use of the UDL strategy, which had been proven to be a suitable strategy in addressing diversity in classrooms (Van Jaarsveld & Ndeya-Ndereya, 2015:205-206).

The UDL is “an approach to teaching that consists of the proactive design and use of inclusive instructional strategies that benefit a broad range of learners, including students with disabilities” (Scott, McGuire & Embry, 2002:1). It is basically an educational framework that guides the development of flexible learning environments to accommodate individual learning differences (Ndeya-Ndereya, 2016:2). It is underpinned by the following principles: multiple means of representation, multiple means of action and expression and multiple means of engagement (Courey, Tappe, Siker & Lapage, 2013:10). Therefore, a UDL framework encourages and enables teachers to “cater” for all learners, regardless of the different characteristics (e.g. prior knowledge, learning styles, background, etc.) they bring into the classrooms to engage with content from different perspectives and through varied levels of communication proficiencies (Webb & Hoover, 2015:539). A UDL approach positively influences and contributes to learners’ understanding of mathematics problems through a curriculum that is presented in multiple formats and of which the content is made accessible and appropriate for learners with diverse characteristics, such as different backgrounds, learning styles and talents (Ndeya-Ndereya, 2016:2).

1.2 CRITICAL EMANCIPATORY RESEARCH

In developing the UDL-based teaching strategy, the current study adopts critical emancipatory research (CER) as a lens that underpins the study. CER promotes egalitarianism, while aiming at enhancing humankind, communal values and parity by respecting the participants (Nkoane, 2012:98) of a research study. Within a CER framework, the participants are equal to the researcher (Mahlomaholo, 2009:225-226) and their opinions and contributions are continuously recognised, heard and valued (Dold & Chapman, 2011:512). Therefore, the engaging nature of CER encourages participation of all stakeholders (Moleko, 2014:17). In this study, the successful implementation of a

UDL-based strategy is realised through the engagement and participation of all stakeholders, including the learners, who are usually excluded from conversations and decision-making processes. The transformation that this study intends to bring about would be realised through the engagement of the “actual persons” who experience the challenge, as well as through their thoughtful ideas and efforts to address their challenge(s). Through CER, the learners will also have a voice in making decisions on how to be taught and in the identification of the activities and strategies that could aid their comprehension of MWPs. CER thus provide a platform for the challenge(s) regarding the teaching and learning of MWPs in multilingual mathematics classrooms to be looked into, from different perspectives, and for the deeper meaning of these challenges to be established (Mahlomaholo & Nkoane, 2002:2).

This lens requires the inclusion and participation of all the stakeholders in this research study (Deeper, 2012:9). In other words, through CER, the participants will be present and engage in all stages of the research, thus owning the outcomes of the research process (Jordan, 2003:187). CER was deemed apt for this study because it advocates values similar to those of participatory action research (PAR), which is the methodology adopted in this study to generate data. These values advance the agenda of educational transformation, emancipation and empowerment. Unlike other approaches, such as positivism and phenomenology, which place the researcher above the participants, CER encourages the researcher and the participants to operate on the same power level. Furthermore, in positivism and phenomenology the outcomes of the research are owned by the researcher (Jordan, 2003:190); however, CER elevates the status of the “researched” as co-researchers, requires their engagement in all the stages of the research and fosters ownership of the outcomes of the research brought about by mutual collaboration.

1.3 PROBLEM STATEMENT

Word problem solving in mathematics is an important aspect of learning mathematics and mathematical thinking. Previous research has shown the benefits of MWPs in mathematics education and in enabling learners to connect with real life (Vula &

Kurshumlia, 2015:44). Unfortunately, in everyday work, learners exhibit difficulties solving word problems, even when they may be skilled in performing other mathematics tasks. Some of the challenges that learners have difficulties with when solving MWPs include, among others, a lack of English proficiency, which is a requirement for mastering this mathematical genre. Language plays a significant role in mathematics learning; therefore, it is imperative that learners' language proficiency should be developed to cope with MWPs. Even though MWPs seem to be challenging for learners, teachers also find this mathematical genre challenging, which poses serious challenges for them to teach it effectively (Pearce, Bruun, Skinner & Lopez-Mohler, 2013:4; Seifi et al., 2012:2923). Adler (2001:4) avows that the teaching of MWPs is even more complex in a multilingual mathematics classroom in which most of the learners are still learning the language of learning and teaching (LoLT) as well as the mathematical language. Essien (2013:199) states that in a multilingual mathematics classroom of learners whose home language is not the LoLT, and who are not yet proficient in the LoLT, teachers usually find themselves confronted with a "triple" challenge of maintaining balance between attention to mathematics, attention to the LoLT and attention to mathematical language. The teaching of MWPs in multilingual mathematics classrooms is further made complex by the fact that many teachers have not been trained to deal with the complexities of teaching in such classrooms (Essien, 2013:163). It is against this brief background that the study sought to utilise the aspects of UDL to develop an effective teaching strategy for MWPs in a multilingual mathematics classroom.

1.4 RESEARCH QUESTION

The following comprehensive main research question was posed, based on the above discussion and problem statement:

How can we utilise the aspects of universal design for learning to develop an effective teaching strategy for mathematics word problems (MWPs) in a multilingual mathematics classroom?

1.5 RESEARCH OBJECTIVES

Based on the aim of this study, which is to utilise aspects of universal design for learning (UDL) to develop an effective teaching strategy for mathematics word problems (MWPs) in a multilingual mathematics classroom, the following specific objectives were formulated:

1.5.1 To document challenges embedded in the teaching of MWPs in multilingual mathematics classrooms.

1.5.2 To formulate components of the UDL strategy to respond to the documented challenges which are embedded in the teaching of MWPs in multilingual mathematics classrooms.

1.5.3 To identify the instructional conditions suitable for the implementation of the UDL-based strategy to enhance the meaningful learning of MWPs in multilingual mathematics classrooms.

1.5.4 To identify possible threats that could impede the successful implementation of the UDL-based strategy that is intended to enhance the meaningful learning of MWPs in multilingual mathematics classrooms.

1.5.5 To design the UDL-based strategy needed to elevate learners' performance and the meaningful learning of MWPs in multilingual mathematics classrooms.

1.6 PRIMARY AND SECONDARY QUESTIONS

1.6.1 Primary question

The primary research question for this study was: How can we utilise the aspects of universal design for learning to develop an effective teaching strategy for mathematics word problems in a multilingual mathematics classroom?

1.6.2 Secondary research questions

The secondary research questions for this study were as follows:

1.6.1.1 What challenges do mathematics teachers encounter or experience in the teaching of mathematics word problems in multilingual classrooms?

1.6.1.2 How can the aspects of a universal design for learning be infused in a mathematics word problem instruction to support learner performance in a multilingual classroom?

1.6.1.3 Under which conditions can the aspects of universal design for learning be successfully implemented?

1.6.1.4 What kind of threats may impede the successful implementation of the identified aspects of a universal design for learning in the teaching of mathematics word problems in multilingual classrooms?

1.6.1.5 What are the success indicators of the effective implementation of the identified aspects of universal design for learning framework in the teaching of mathematics word problems in multilingual classrooms?

1.7 RESEARCH DESIGN AND METHODOLOGY

The study adopted the participatory action research (PAR) approach to generate data. According to Dworski-Riggs and Langhout (2010:216), PAR is a research approach that seeks to promote social justice by creating conditions that encourage empowerment. When PAR is undertaken, researchers are expected to strive to render transparency and deconstruct the power hierarchies that are traditionally embedded in research (Anderson et al., 2015:183). PAR has been adopted in this study as it complements or matches CER (Tsotetsi, 2013:26), which is the lens underpinning this study. Both advocate for the creation of spaces in which the participants can be empowered through shared debates, and where everybody involved is an equal stakeholder (Lincoln, Lynham & Guba, 2011:102). Furthermore, PAR was adopted in this study because it promotes collaboration of the stakeholders involved in an attempt to solve a problem. Therefore, everybody involved was assigned a task to perform in an attempt to bring about a change in the situation (Pain & Francis, 2003:46). According to Eruera (2010:1 of 9), as PAR creates conversational spaces for critically deliberating issues without fear, all participants, including the marginalised, have the power to be listened to and to be heard (Dold & Chapman, 2011:512). In the context of this study, PAR afforded the participants involved equal opportunities to make contributions towards developing an effective UDL-based teaching strategy for MWPs in a multilingual mathematics classroom.

PAR requires the inclusion of all stakeholders in all stages of the research project. This means that the identification of the problem is achieved through a collaboration of all the stakeholders, and consequently, the solutions to address the problem are achieved through a collective effort. In this study, I identified the problem, which was deliberated upon by all the participants involved who, in turn, all deemed it to be a challenge that needed to be addressed.

According to Eruera (2010:2), PAR is a cyclical “learning by doing” process that entails planning, action, observation and reflection phases. Ebersöhn, Ellof and Ferreira (2007:128) define PAR as a cyclical process that entails five stages, namely diagnosis, action planning, implementation, evaluation and outcomes. During the “diagnostic stage”, the participants, as a collective, determine and agree on the nature of the problem that needs to be addressed. During this stage, the following questions may be explored by the group as justification that the problem exists and requires attention:

- What is the problem?
- How do you know that it is a problem?
- How do you feel about the problem?
- Why does the problem exist?
- What are the consequences of the problem?
- What would you like to happen?
- What do we need to learn to better understand the problem or find a possible solution?
- What resources do we already have that could help us to understand the problem?
- What other resources or sources of information may we need?

In the next stage, which is the “planning” stage, the participants (the researcher and the co-researchers) “map” or plan steps to address the identified problem and the evaluation methods to determine how successful the solution was (developing a strategic plan). During the “take action” stage, the steps that have been agreed upon, are implemented. It is during the “evaluation” stage that the researcher and the co-researchers gather data to determine whether a change has occurred (evaluate the results of the action). During the “outcomes” stage, the researcher and the participants analyse the data, discuss the findings, and reflect on the findings to determine to what extent the “action” has helped solve the problem.

The abovementioned stages of PAR are also reverberated in the UDL strategy, as this strategy requires the teachers to understand their learners in order to “identify” the challenges they have in terms of learning the specific content (identify the problem). Following this, the teachers need to devise ways in which to cater for all the learners and to maximise their learning experiences in the classroom (plan strategies to be implemented). The teachers then have to implement these strategies they have devised to cater for the different needs of the learners in their classes (implement strategies that have been devised). These strategies must then be assessed to determine if a change has occurred or not (evaluate strategies). The UDL reflection phase requires teachers to reflect on their teaching and to determine to what extent the “implemented strategies” have helped solve the identified problems (reflection).

1.7.1 Instrumentation

The data were generated through meetings (group and individual meetings), dialogues and lesson observations. In addition, a reflection session was conducted to ensure a common understanding among the participants and to make sure that the data were not taken out of context. The data were audio-recorded and video-recorded with the permission of the participants. The participants signed a consent form for their conversations to be recorded (see Appendices A4, A5, A6, A7 and A8). The data were generated and organised according to the agreed-upon constructs. The teachers’ lesson plans with worked out word problems, as well as the question papers, were also used as additional forms of generating data in this study.

The free attitude interview (FAI) technique was used to conduct the meetings. Tlali (2013:28) considers FAI to be a powerful tool to be used when a transformative lens such as CER is used. FAI heightens the quality of the data and lures the participants into contributing their views freely (Tlali, 2013:28). When FAI is used, an open-ended initial question is usually posed to initiate the discussions. The nature of this question (open-ended) is intended to ensure that the participants feel less constrained to think in a particular way. When FAI is applied, people talk as in a normal day-to-day conversation (Buskens, 2011:1). According to Tsotetsi (2013:161), the advantage of using FAI is that people tend to say more than they would have said in responding to a closed questionnaire. The CER advocacy of the creation of spaces for people to freely talk, thus makes it possible for FAI principles to be operationalised. The role of the researcher when FAI principles are used, is, among others, to respect the opinions of the participants and show an interest in what they are saying and to allow them to express their feelings freely. This is what the researcher must maintain in order to avoid bias.

According to Buskens (2011:2-3), the FAI technique may be conducted between two or more people (a group). Since the conversations take place in the form of normal day-to-day conversations, the participants are free to intervene or interject by asking questions or seeking clarity, just as the researcher would also do. In this study, the FAI was conducted during both the one-on-one sessions with the participants and when all the participants met as a group.

1.7.2 Research participants

The following participants, who are all directly involved in mathematics, took part in the study: five mathematics teachers, three mathematics literacy teachers, four mathematics and English teachers, one mathematics head of department (HoD), one principal, as well as three Grade 10 learners, four Grade 11 learners and three Grade 12 learners. Eruera (2010:1) states that the PAR approach requires active research participation and ownership by people who are motivated to identify and solve the issues that concern them. Furthermore, Eruera (2010) recommends that these people should be involved in all the stages of the research process aimed at addressing the issues that concern them.

In line with this recommendation, the participants in this study were persons who were concerned about the problem that was identified. The participants' subject expertise was also taken into consideration (e.g. lecturers, subject advisor, HoD and teachers). Learners were selected in order to share their learning experiences and ideas, and to identify possible strategies that could be effective in terms of improving their own learning of MWPs in multilingual mathematics classrooms. In addition to the abovementioned, the inclusion of all stakeholders, through the voice of the masses, ensured the democratisation of the process of implementing strategies to enhance the teaching and learning of MWPs in multilingual mathematics classrooms.

1.8 CRITICAL DISCOURSE ANALYSIS

Critical discourse analysis (CDA) was used to analyse the data in this research study. According to Bloor and Bloor (2007:2), CDA is an interdisciplinary approach that encompasses the analysis of text and talk in all disciplines of the humanities and the social sciences. CDA was used in this study because it "matches" CER (the theoretical framework that underpins this study) in that both seek to find solutions to the problem at hand (Bloor & Bloor, 2007:12). CDA further afforded the researchers and co-researchers in this study an opportunity to analyse the data at the textual, social and discursive practice levels (Van Dijk, 2009:459). Both CER, as the lens couching this study, and PAR, as the methodology adopted in this study to generate data, require the inclusion of the views of all the participants, expressed in any form with which they are comfortable. Likewise, CDA, through its three dimensions of analysis, namely textual analysis (description), processing analysis (interpretation) and social analysis (explanation) (Janks, 2009:1 of 24), encourages and allows for points of view to be expressed in various forms. From the above, it is clear that CDA complements CER and PAR in that it provides opportunities for the views that have been expressed to be interpreted and analysed in different dimensions and contexts, and to take into account factors that inform certain frames of thinking, reasoning and doing. Over and above this, CDA was adopted in this study because it endorses the notion that all participants, including the researcher as a participant, should be included in the process of analysing data (Chilisa, 2012:253) in all the stages.

In this study, I used verbatim reporting of responses, as is encouraged by Monyatsi, Steyn and Kamper (2006:219) in studies of this nature, in which the voices of the participants need to be heard. The recorded data were transcribed. The data were then categorised according to the themes as informed by the research objectives of the study (e.g. data that reflected challenges were grouped together, followed by data that constituted components of the solutions, followed by the conditions conducive to effective implementation of the strategy, etc.). I analysed the data at a textual level and a social level and also highlighted the discursive practices. These levels of analysis were used in an effort to assist me to understand meanings and the perspectives of the participants in context. Furthermore, these levels were used to demonstrate certain thoughts the participants held or carried, which led to certain behaviour, actions and the way in which they reported matters from where they were standing (from their perspective). The application of CDA was also used to interpret some visual cues that suggested or portrayed certain thoughts the participants held and their feelings about the issues at hand. These were important to consider since some of the words were not literally uttered; however, through these visual cues, I was able to see how the participants felt about the issues that were discussed. I thus also drew from those cues to interpret and to make sense of the data. I was mindful of the fact that the participants' perspectives should not be misinterpreted. Therefore, to avoid such misinterpretations, member checking of the participants' spoken words was conducted. This made it possible for me to portray and report the data in context.

1.9 VALUE OF THE STUDY

The study aimed to contribute significantly to the teaching and learning of MWPs in a multilingual mathematics classroom. The strength of this study lies in the use of CER (the theoretical framework underpinning the study), which is not commonly used in studies of this nature. CER requires the inclusion of all relevant stakeholders in the research project, including the marginalised “groups” that are usually excluded in the decision-making processes, thereby ensuring that the different points of view are explored from various angles and that the deeper meanings of these views are established. Also, the UDL strategy will provide a comprehensive guide towards the successful teaching of MWPs in multilingual mathematics classrooms. This strategy will enable the teachers to cater for the needs of all the learners regardless of the characteristics (prior knowledge, learning styles, background, etc.) they bring into the classrooms. Thus, this strategy contributes towards the inculcation of inclusive practices to meet and address the needs of diverse learner populations. Although the study is primarily concerned with the teaching of MWPs in multilingual mathematics classrooms, it could also be extended to other topics in mathematics and inform other studies seeking to enhance the teaching and learning of other content in multilingual classrooms. The value of this study further lies in the inclusion of all stakeholders in formulating such a UDL-based strategy to enhance the teaching of MWPs in a multilingual mathematics classroom. The participants gained confidence and developed a sense of ownership in what they consider to be their strategy, in which they fully contributed to its accomplishment and realisation.

1.10 ETHICAL CONSIDERATIONS

Scholars such as McMillan and Schumacher (2001:196) and Van Niekerk (2009:119) consider ethics significant since it deals with beliefs regarding what is morally good or bad, right or wrong, and proper or improper. In line with these notions, I requested permission to conduct the study from the principal of the school (**see APPENDIX A2)**, which was granted (**see APPENDIX A3**). The study would be conducted at two schools; however, I realised that the participants at the one school were much more enthusiastic than those from the other school. The first meeting during which I was expected to explain what the research project was all about, was postponed twice at the school with the less enthusiastic participants, which was not the case with the other school. This made me focus on one school, thus deviating from the original plan of conducting the study at two schools. The principal of the school in which the study was finally conducted, was contacted. All partners involved were given consent forms to sign (see Appendices A4, A5, A6, A7 & A8) as Maree and Van der Westhuizen (2007:42) recommend. I explained the contents of the form and instructed the participants to ask for clarity when they did not understand anything. I made it clear on the consent forms that no participant was pressured or coerced to take part in the research project. I assured them of anonymity with regard to the information they were to provide. I also made it clear that their participation was voluntary and that their basic human rights would be respected and protected at all times. The participants were further assured that confidentiality would be maintained, no personal information or identities would be disclosed and they would be kept abreast of the progress made. Since the study involved learners, the assent forms were completed on their behalf by their parents (see Appendix A5). I also indicated that the data would be stored for a period of six months, after which it would be destroyed.

Ethical clearance was sought from the University of the Free State and granted. The ethical clearance number of the study is **UFS-HSD2016/1194** (see Appendix A1).

1.11 LAYOUT OF THE CHAPTERS

Chapter 1: This chapter provides an overview of the whole study. The problem statement, aim of the study, research design and methodology, ethical considerations, value of the study, as well as the layout of the whole research are presented.

Chapter 2: This chapter highlights the theoretical framework underpinning the study. The chapter also elucidates the operational concepts used in this study.

Chapter 3: This chapter focuses on the literature review. The chapter highlights the following: the challenges encountered when teaching MWPs in multilingual mathematics classrooms, the solutions recommended to address these challenges, the conditions under which the strategies worked, the threats to the successful implementation of the strategy and the success indicators of the successful implementation of the strategy

Chapter 4: This chapter outlines the research design and methodology used to generate the empirical data.

Chapter 5: This chapter is about the analysis of the data and the presentation and discussion of findings on the aspects of the UDL strategy to develop an effective teaching strategy for MWPs in a multilingual classroom

Chapter 6: This chapter discusses the findings, conclusion and recommendations for implementing the aspects of the UDL strategy to enhance the teaching of MWPs in a multilingual mathematics classroom

Chapter 7: This chapter provides the guidelines for teaching MWPs in a multilingual mathematics classroom using the aspects of the UDL framework

1.12 CONCLUSION

This chapter provided the background to the study. The purpose of the study is captured clearly in this chapter. Furthermore, the problem statement, aim and research objectives, which are derived to assist in responding to the main comprehensive research question, were highlighted. The research design and methodology, the value of the study and the ethical considerations were also highlighted in this chapter.

The core of this chapter was to highlight the challenges which teachers encounter when teaching this mathematical genre and to indicate the need to consider a strategy that would respond to the challenges and thus enhance the teaching of this mathematical genre in a multilingual mathematics classroom. In this study, I propose the utilisation of the aspects of the UDL framework, which has proven to be an effective teaching framework when dealing with the issues of diversity and to promote inclusive practices within the teaching and learning contexts.

The next chapter focuses on the theoretical framework underpinning this study, as well as outlining and defining the operational concepts which the study has been anchored in.

CHAPTER 2

THEORETICAL FRAMEWORK AND OPERATIONAL CONCEPTS INFORMING THE STUDY

2.1 INTRODUCTION

The aim of the study was to utilise the aspects of the universal design for learning (UDL) to develop an effective teaching strategy for mathematics word problems (MWPs) in a multilingual mathematics classroom. In other words, the study sought to bring about transformation in how MWPs were taught in a multilingual mathematics classroom. In order to achieve the aim of the study, this chapter presents critical emancipatory research (CER) as the theoretical framework that underpins the study, and will discuss its origin and evolvement, objectives, nature of reality, the role of the researcher, the relationship between the researcher and the researched and its epistemological stance as justification for its use in this study. This chapter further highlights the role of CER in the attainment of the successes and the indicators of success of this study. The operational concepts, which serve as the pillars of the study, are also explicated and lucidly defined so that they are understood within the context of this study. Finally, a summary of this chapter and the main points of the next chapter are presented.

2.2 THEORETICAL FRAMEWORK

A theoretical framework is a set of theories put together to provide support for illuminating, viewing or considering a phenomenon (Lassa & Enoh, 2000:3). According to Rockinson-Szapkiw (n.d.:2 of 4), the role of the theoretical framework is that of linking the researcher to existing knowledge, thus demonstrating how the research fits into what is already known (the relationship between existing knowledge and new research). Furthermore, it demonstrates how one’s research makes a contribution to the field (intellectual goals).

A theoretical framework affords the researcher an opportunity to move from merely describing the phenomena to generalising numerous characteristics of the phenomena. According to Moleko (2014:11), it is crucial that the researcher clearly indicates that a theoretical framework is used in a research study as the theoretical framework provides central and predominant views. Moleko (2014:11) further advises that a theoretical framework must be relevant to the goals of the study. For instance, if the purpose of a study is to bring about emancipation and empowerment, it is important that a theoretical framework that advocates such values underpins the study. It is through the theoretical framework that the readers understand the researcher's thoughts on certain issues, by what those thoughts have been informed and the language that underpins those thoughts. For example, a positivist will make conclusions about a certain issue as informed by the amount of numerical data that supports the specific issue. However, a phenomenologist will make conclusions on a particular issue as informed by the coded data generated through "intense interrogation" of the researched. A theoretical framework or line of inquiry informs the problem statement, the rationale behind the study, the research questions, the selection of instruments, the choice of research methods, as well as the interpretations and discussions of the findings of the study (Pajares, n.d.:1).

In this study, CER as the theoretical framework underpinning the study, will assist in the establishment of the perspectives through which the researcher can view the problems related to the teaching and learning of MWPs. CER is adopted in this study to present the researcher's point of view on matters relating to the topic at hand.

2.2.1 Historical background of critical emancipatory research

CER stems from critical theory (CT), which was developed to oppose certain aspects of positivism, the most profound lens used by researchers in the nineteenth century. August Comte is known for coining the thoughts on positivism (Pring, 2000:90). Other names that are historically linked to positivism, are Locke, Hume and Bacon. These philosophers questioned all knowledge claims that went beyond that which could be observed. The Frankfurt School was the first group to develop CT, thus opposing the idea of science being the only way of establishing the truth (Higgs & Smith, 2006:68). They believed that knowledge is constructed by human beings through their lived experiences, which science does not recognise. This school contended that the search for knowledge must be founded upon the goal to foster and to transform the value of human life. Habermas (one of the critical theorists), in support of the rationale behind the critical theory, also criticised the view held by positivists, namely that all knowledge was based on things that could be experienced and measured, and that knowledge beyond that does not exist (Moleko, 2014:15). His belief was also founded upon the view that empiricism ignores the "fact" that knowledge is generated and constructed by human beings. According to Steinberg and Kincheloe (2010:140), knowledge that is constructed by human beings (within the context of critical theory) serves to decrease human anguish in the world.

This led to the birth of CER, which holds similar ideologies to those of critical theory, as a way of improving people's lives through emancipation. CER accentuates issues of social and power structures, thus liberating and empowering human subjects (Stahl, 2004:2). For the research to be carried out, CER necessitates the inclusion of all stakeholders throughout the research process (Campanella, 2009:2). According to Mahlomaholo (1998:9), CER was developed to ensure that the oppressed and the underprivileged are acknowledged and listened to, since what is spoken will be entrenched in ideology. Myer (2004:111), in accord with Freire, Macedo and Ramos (2007:33), states that CER is the most suitable theoretical framework to be adopted when addressing issues of social justice and transformation. It accentuates that the agency for change should rest with the persons in the relevant community, working closely with the researcher towards the goal of social transformation (Mertens, 2010:8).

More than a decade ago, Hooks (1994:11) regarded teaching as a service and a way of ploughing back, and as such, encouraged teachers to generate strategies for teaching learners effectively. Hooks further suggested that in such a process, teachers and learners should recognise one another as "whole" human beings, striving towards the construction of knowledge, not only of books but also of how to live in society. In line with Hook's notion, Shangase (2013:11) avows that since not all learners learn in the same way, a paradigm shift and a change in the way teachers think, write and speak are necessary. Therefore, in the context of this study, CER calls for transformation in classrooms, not only in **how** to teach but also in **what** to teach learners with different characteristics. It calls for teachers to be conscious of their teaching practices and to continuously adapt their teaching practices to cater for diverse learner populations.

2.2.2 Critical emancipatory research objectives

CER is intended for the establishment of platforms for engagement, empowerment and revolution for the oppressed (Lincoln, Lynham & Guba, 2011:102). It is also intended to encourage people from various communities to work with a researcher towards the goal of communal transformation (Mertens, 2010:8). Through the application of CER as a lens, unwelcome incidences and the root causes thereof in societies can be identified (Watson & Watson, 2011:68) and dealt with accordingly. The adoption of CER fosters changing the status quo of the participants, overcoming injustice and alienation, and promoting emancipation (Stahl, 2004:4). From an educational vantage point, CER affords stakeholders, especially those who previously have been excluded from existing structures of the school, the opportunity to understand how classroom practices unfold (Kincheloe & McLaren, 2011:5). The adoption of CER enables the research practice to move beyond the ameliorative changes of local action, towards the transformative potential of a greater shared force for change (Ledwith, 2007:606). Therefore, in the context of the classroom setting, CER calls for a collective change in terms of activities that must be designed cooperatively, allowing all learners access to these activities as well as enhancing their engagement in these activities.

2.2.3 Nature of reality

Ontologically, CER assumes that there are compound realities or truths in establishing and addressing the problem (Tsotetsi, 2013:7). Within the CER framework, it is possible that the "problem", according to one person, might not be a "problem" to others at all. Therefore, manifold realities about the so-called problem can be attached beyond what an individual believes to be the problem. In line with this, within a CER framework, the researcher is not the only one who has the knowledge to enhance the teaching of MWPs in a multilingual mathematics classroom using a UDL teaching strategy, but the participants are also considered knowledgeable since they have lived experiences and could relate with the subject at hand. According to Mertens (2010:32), CER accepts the existence of multiple realities that are shaped by social, cultural, economic, ethnic, gender and disability values. Moleko (2014:17) agrees, and adds that there can never be only one neutral or absolute truth because the construction of knowledge depends on multiple perspectives from the “mass”, and not from an “individual”. Mahlomaholo (2009:225) asserts that an “absolute truth” can never be established since CER provides different sides to a story. He further notes that the CER framework requires the researcher to be analytical in order to find the deeper meaning and also to look into all sides of the story. As a result, multiple truths regarding the subject are established as a diverse group of people, who see and do things differently, are involved in the process. In contextualising the nature of reality of CER, one realises that CER affords both teachers and learners an opportunity to look at the given word problem from different angles to reach a mutual, deeper understanding of what is required to solve it.

2.2.4 Relationship between the researcher and the participants

Epistemologically, CER undertakes a cooperative relationship between the researcher and the participants. According to Mahlomaholo (2009:225-226), this means that, within a CER framework, the participants are equal to the researcher. In this framework, the participants’ voices are not only heard but also acknowledged and respected on the same level as that of the researcher (Dold & Chapman, 2011:512). In order to maintain autonomy, a sense of responsibility and harmonious relationships, Campanella (2009:5) advises that the researcher and the participants should always be aware of the power differences between them. Kemmis (2006:472) regards the relationship between the researcher and the participants within a CER framework as a “close” relationship – one that leads to open dialogue, which he refers to as “communicative spaces”. According to Moleko (2014:18), this close relationship embraces transparency and openness, since the platform generated by CER at all times advocates an agenda of peace, freedom, hope, social justice and equity in all its forms (McGregor, 2003:4).

The relationship between the researcher and the participants within a CER framework, has often been referred to as a “lover model” (Lincoln, 2001:127). According to Lincoln (2001:127), the relationship between the researcher and the participants exists in a state of communal concern, where they not only care for and trust one another but also value their friendship and experiences, even after the accomplishment of the research goal. In addition to this, Moleko (2014:18) notes that the participants and the researcher, while carrying out the project, get to tap into and embrace one another's experiences, realising their significance in the success of the project. In the context of this study, CER ensures that the learners are also regarded as equally important stakeholders and that a mutual relationship exists between the learners and the teachers. When activities must be designed, learners' opinions are significant as the designs must cater for the needs of the learners. CER thus promotes the idea of classrooms being turned into spaces where teachers and learners are "partners" who work together in achieving the objectives of the lesson.

2.2.5 Role of the researcher

According to Mahlomaholo (2009:226), the researcher's role within a CER framework is to engage the participants in the research project with the aim of creating spaces for shared debates. This will not only empower the participants but also emancipate their thoughts to transform their situations, thus meeting the need(s) of a real-life situation. Jordan (2003:186) further adds that the researcher operating in this framework has to help the participants (especially the people who are confronted with the problem) to take control of their situation by bettering it so that they can own the outcomes of a research project of their own efforts. Since the nature of CER is participative and collaborative (Shangase, 2013:15), the role of the researcher becomes one of creating platforms for deliberations on the strategies to be employed and the roles to be assigned, thus ensuring that all the stakeholders participate. In line with this, Ledwith (2007:111) regards the role of the researcher as having to engage the participants in the process of transformation by working together. According to Reyes, Brackett, Rivers, White and Salovey (2012:701), such mutual transformation can only be realised when the people involved express warmth towards, respect for and interest in one another. In the context of this study, the researcher has to ensure that all the participants collectively take part in the process of designing strategies to address the challenges of teaching and learning MWPs in multilingual mathematics classrooms.

2.2.6 The role of critical emancipatory research in achieving the results of this study

The indicators of success in this study provide examples of achievements through the utilisation of the aspects of a UDL strategy to enhance the teaching of MWPs in a multilingual mathematics classroom. The success of this strategy was achieved through collaboration and could be traced back to multiple ways in which the teachers could teach this mathematical genre, as highlighted in Chapter 7. Through this strategy, the teacher is encouraged to use multiple means of representation, multiple means of action and expression and multiple means of engagement in an attempt to teach this mathematical genre effectively. CER, as the lens couching this study, required the participants to work collaboratively through shared debates on a platform in which they were all free to talk about the issue of concern. This mode of research lens, thus enabled the participants to co-construct ideas and to generate meaning and understanding out of the conversations that unfolded, which led to the results of this study. The platform created for all the people to participate, thus made it possible for the conversations to take place, stimulated mutual participation and engagement and facilitated an exchange of ideas that resulted in co-constructed meanings. The voices of all the participants were deemed significant and thus they enabled the team to derive “guidelines” for teaching MWPs in a multilingual mathematics classroom. It is of great significance to realise that the effects of the strategy may not necessarily only be measured by what “can be realised at the end” (overall results); however, it can also be identified from the processes involved and how people grow and develop a change of perspective in approaching the issue. This point is captured clearly by Objective 5 of this study, which is constituted by the voices of the people who highlight what works or what could work and how certain measures could be put in place to ensure that the strategy becomes effective. This serves as evidence of growth and a sense of developing a change of perspective, which can be traced from the participants’ spoken words. This also signifies a developed sense of ownership by the participants taking charge of their situation and thus recommending what would work in their context. Thus, through CER, the participants were able to share and tell stories in their own words. Some of the principles that are advocated by CER, as outlined in Chapter 2 (e.g. empowerment, emancipation and transformation), are evident in Chapter 5 through the participants' spoken words, as analysed and also evidenced in Chapter 4 in the description of the manner in which the data were generated.

One of the reasons why I opted to conduct a study of this nature (a study that is underpinned by CER as a theoretical framework), was to create a platform for the participants to *empower themselves* to be able to carry out this research project even when I (the researcher) would no longer be part of the project. The unfolding of the conversations in Chapter 5 serves as evidence that the participants gained knowledge in the process and that they were able to identify and suggest what could work for them. One of the principles of CER is the inclusion of the people, especially the ones who are directly affected, in order for them to voice what they believe could be the solution to their own situation. This notion is made vivid in the study and supported by the fact that I was not the only one who formulated the strategy but all the participants involved contributed, as CER advocates. The results of this study were achieved through collaboration, as well as respect for and acknowledgement of the different perspectives and inclusion of all the people, especially the marginalised.

2.3 DEFINITION OF THE OPERATIONAL CONCEPTS

The following sections (2.3.1-2.6) provide brief definitions of the operational concepts of this study, namely universal design for learning (UDL), mathematics word problems and multilingual classrooms. This is done in an effort to enhance the reader's understanding of how these concepts are viewed and operationalised in the context of this study.

2.3.1 Universal design for learning

This section will briefly discuss the historical background, principles and significances of UDL.

2.3.1.1 Historical background

The origin of UDL can be traced to the universal design (UD) movement of the 1990s. The UD concept was coined by the architect Ronald Mace, a wheelchair user, who opposed an orthodox approach of designing for the average user and provided a design foundation for more accessible and usable products and environments (Burgstahler, 2011:1 of 8). He proposed a new idea, namely that physical environments should be proactively designed to meet the needs of the diverse individuals who access these spaces. He further advised that the design fields, such as architecture, landscape, interior and product development, realistically should examine the needs of diverse consumers. The term "universal design" was coined to echo this approach of proactively integrating inclusive design features, while minimising the need for individual retrofitted accommodations (Aslaksen, Bergh, Bringa & Heggem, 1997:5 of 34).

Mace and other visionaries defined UD as "the design of products and environments to be usable by all people, to the utmost degree possible, without the need for adaptation or specialised design" (Story, Mueller & Mace, 1998:2). Later on, UD became popular with architects and designers who struggled to access certain buildings and streets in the United States of America (USA). Mace's recommendation was that the design "model" should be such that it meets the needs of diverse consumers or audiences. The main characteristic of UD is that it "proactively builds in features to accommodate the range of human diversity" (McGuire, Scott & Shaw, 2006:173).

After Mace's recommendation, it was suggested that building structures should be modified. These modifications included automatic doors, curb cuts, entry ramps and more. Later on, after these modifications had been affected, people realised that the modifications benefited not only persons with disabilities but also many others. Even today, these modified amenities still benefit a range of people with diverse needs. For instance, when people are carrying heavy loads, and their hands are full, they can use lifts and automatic doors without being assisted. Similarly, commuters in noisy airports and students in quiet libraries equally rely on closed captioning on television, even though these amenities were originally conceived as part of accommodating persons with disabilities.

2.3.1.2 Moving from universal design to universal design for learning

Some time later, following the successful execution of UD principles in the field of architecture, the UD philosophy found "fertile ground" in the field of education. School teachers and university professors alike have espoused UD as a conceptual and theoretical foundation on which to build a framework of teaching and learning that is inclusive and equitable and guides the creation of accessible teaching and learning materials (Schelly, Davies & Spooner, 2011:18). The logic behind that was, if the aim of UD were to remove barriers from the physical environment, then the aim of UDL would be to eliminate barriers from the learning environment. According to Davies, Schelly and Spooner (2013:195), David Rose (one of the founders of UDL) came into the picture after Ron Mace had coined the concept of UD, clarifying the focus of UDL by stating that "UDL puts the tag 'disabled' where it belongs — on the curriculum, not on the learner". In addition to Rose's clarification of the focus of UDL, the Council for Exceptional Children (2011:1) then stated that "the curriculum is disabled when it does not meet the needs of diverse learners".

After UD principles had been implemented in the field of architecture, educationists witnessed the successes that were being achieved. Educationists at institutions of higher education further observed that the hurdles faced by students with disabilities – for example, study materials that are not in electronic format, uncaptioned video and PDF files that do not contain any real text and, therefore, cannot be searched or read aloud by text-to-speech software – were often the same hurdles encountered by students who possessed different learning styles, used the latest computer technologies or whose native language was not English. In light of the above, many students can benefit from UDL – hence the word "universal" in "universal design for learning" (ACESS Project, 2011:3 of 4).

According to David Gordon, a director at the Center for Applied Special Technology (CAST), UDL is about providing options. He further states that "options are essential to learning, because no single way of presenting information, no single way of responding to information, and no single way of engaging students will work across the diversity of students that populate our classrooms" (ACESS Project, 2011:3 of 4). Alternatives reduce barriers to learning for students with disabilities, while "enhancing learning opportunities for everyone" (Sukhai & Chelsea, 2016:133).

Considering the above, it is clear that UDL does not support any single teaching practice; instead, it integrates current best approaches to engage learners and challenges them to think critically. It is a broader framework that comprises, among others, the teaching and learning theories, principles of teaching and best teaching practices (Ndeya-Ndereya, 2016:2). A UDL further helps teachers meet the learning needs of a diverse learner population through integration of "teaching" modalities, designs and technologies. Many people regard a UDL as a good teaching framework, since it allows teachers to consider learners' differences, preferences and needs at the onset of planning and teaching, rather than after the teaching has taken place (Israel, Ribuffo & Smith, 2014:12 of 38). A UDL advocates the presentation of information in ways that adapt to the learner, instead of requiring the learner to adapt to the information.

2.3.1.3 Definition of universal design for learning in the context of this study

Many scholars have, from various angles, provided definitions of the term "universal design for learning" (UDL). However, in the context of this study, the definition provided by Scott, McGuire and Embry is considered to be more appropriate, since it captures and reflects what this study is about. Therefore, according to Scott, McGuire and Embry (2002:1), UDL is an approach to teaching that consists of the proactive design and the use of inclusive teaching strategies that benefit a broad range of learners or learners with diverse needs.

2.3.1.4 Universal design for learning – three groups of brain networks

Although the UDL concept was implemented at educational institutions, neuroscientists were also interested in the concept and conducted research with the purpose of determining how learning occurs in the brain, as informed by UDL principles. From the findings of this research, they discovered three groups of brain networks, namely **recognition, strategic** and **affective networks** (Grabinger, Aplin & Ponnappa-Brenner, 2008:65). For Rose and Meyer (2006:vii), recognition networks make it possible to

receive and **analyse** information – the **“what”** of learning. Strategic networks make it possible to **generate patterns** and **develop strategies for action** and **problem solving** – the **“how”** of learning. The affective networks **fuel motivation** and **guide the ability to establish priorities, focus attention** and **choose action** – the **“why”** of learning.

After the neuroscientists had identified those networks, the CAST group developed a corresponding, comprehensive and detailed teaching framework (informed by the three brain networks) to classify the three UDL principles, namely **multiple means of representation, multiple means of action and expression** and **multiple means of engagement** (CAST, 2011). **Table 2.1** explicitly explains the three UDL principles according to the CAST group and the neuroscientists.

Table 2.1: UDL principles

Principle I: Provides Multiple Means of Representation (the “what” of learning)	**Principle II: Provides Multiple Means of Action and Expression** (the “how” of learning)	**Principle III: Provides Multiple Means of Engagement** (the “why” of learning)
This principle simply explains and recognises that learners, as a result of different backgrounds, socialisation skills, age and language, to name a few, differ in the ways they perceive and comprehend the information they are presented with. E.g., auditory learners perceive and understand information differently than visual learners do. All	This principle recognises that learners differ in the ways they navigate a learning environment and how they express what they know. For example, individuals with significant movement impairments (e.g. cerebral palsy), who struggle with strategic and organisational abilities (executive function disorders), who have language barriers, etc., all	This principle recognises that learners differ markedly in the ways in which they can be engaged or motivated to learn. There are various sources that can influence individual differences, including neurology, culture, personal relevance, subjectivity and background knowledge, along with a variety of other factors presented in

Principle I: Provides Multiple Means of Representation (the “what” of learning)	**Principle II: Provides Multiple Means of Action and Expression** (the “how” of learning)	**Principle III: Provides Multiple Means of Engagement** (the “why” of learning)
these factors thus require alternative ways of approaching content in order for learners to understand the information. Some learners simply grasp information faster or more efficiently through visual or auditory means rather than printed text. Also, learning and transfer of learning take place when multiple representations are used because they allow learners to make connections within, as well as among, concepts. Briefly, there is not one means of representation that would suit all learners. Therefore, this principle encourages the use of *multiple representations* to accommodate all learners.	approach learning tasks differently. Some learners may be able to express themselves better in writing than orally, and vice versa. It should also be recognised that action and expression require a great deal of strategy, practice and organisation, and learners differ with regard to these as well. From this, it should be noted that there is not one means of action and expression that will optimally suit all learners. *Therefore, providing options for action and expression is essential.*	these guidelines. Some learners are highly engaged by spontaneity and novelty, while others are disengaged, even frightened, by those aspects, preferring strict routine. Some learners might like to work alone, while others prefer to work with their peers. In reality, there is not one means of engagement that will optimally suit all learners in all contexts. Therefore, *providing multiple options for engagement is essential.*

2.3.1.5 Universal design for learning principles

According to Jimenez, Graf and Rose (2007:45), UDL provides students with multiple means of representation, expression, action and engagement in the classroom, as highlighted in Table 1 above. These researchers further state that when UDL principles are applied in class, they can assist teachers immensely in recognising barriers to learning, strategically addressing such barriers and monitoring learners' progress.

Within the UDL framework, there are nine principles of universal design for instruction (UDI). These are the most important principles that teachers have to keep in mind when teaching to accommodate all learners, regardless of the characteristics they bring into the classroom. These principles, however, can never be put in a specific order (e.g. principle 1, principle 2, etc.) since they overlap. For instance, if the teacher wants to engage learners in class activities, the information has to be presented in an explicit manner, the learners must be given opportunities to express themselves in the manner they understand the problem, and the activity must be designed in such a way that it would stimulate the learners to actually want to attempt to solve the problem. In this instance, one would realise that in trying to teach learners effectively and to ensure their understanding of the concepts, the teacher has to present the information in a format accessible to learners, allow the learners to make use of alternatives to express or demonstrate their learning, and stimulate the learners' interest and motivation for learning; hence, a single principle cannot be operationalised without the operationalisation of the others.

2.3.1.6 Universal design for learning objectives

UDL is an approach meant to endorse effective teaching. According to Ndeya-Ndereya (2016:6), UDL aims to:

- improve accessibility to learning for all learners;
- maximise learning for everyone;
- inculcate the practice of inclusive curriculum and instruction (validated by research);

- reduce barriers to learning; and
- increase learners' success.

Taking into consideration the origins of UDL, as well as its objectives, operationalisation in education and the multiple definitions provided by various scholars, the meaning of UDL in the context of this study is an educational framework that guides the teaching of diverse learners through multiple representation, expression, action and engagements.

2.4 UNIVERSAL DESIGN FOR INSTRUCTION

As highlighted in Section 2.3.1.2, UDL is a broader concept that is made up of, among others, teaching and learning theories and the best teaching practices. The UDL framework also comprises the UDI principles, which reflect teaching that consists of a proactive design and the use of inclusive instructional strategies that benefit a broad range of learners (Scott, McGuire & Embry, 2002:1), as briefly highlighted in Table 2.2.

Table 2.2: UDI principles and examples

Principle	Definition	Example(s)
Principle 1: Equitable use	Instruction is designed to be useful to and accessible by people with diverse abilities. Provides the same means of use for all students; identical whenever possible, equivalent when not.	Provision of class notes online. Comprehensive notes can be accessed in the same manner by all students, regardless of hearing ability, English proficiency, learning or attention disorders, or note-taking skill level. In an electronic format, students can utilise whatever individual assistive technology is needed to read, hear or study the class notes.

Principle	Definition	Example(s)
Principle 2: Flexibility in use	Instruction is designed to accommodate a wide range of individual abilities. Provides choice in methods of use.	Use of varied instructional methods (lectures with a visual outline, group activities, use of stories or web board-based discussions) to provide different ways of learning and experiencing knowledge.
Principle 3: Simple and intuitive	Instruction is designed in a straightforward and predictable manner, regardless of the student's experience, knowledge, language skills or current concentration level. Eliminate unnecessary complexity.	Provision of a grading rubric that lays out expectations for exam performance, papers or projects clearly; a syllabus with comprehensive and accurate information; a handbook guiding students through difficult homework assignments.
Principle 4: Perceptible information	Instruction is designed so that necessary information is communicated effectively to the student, regardless of ambient conditions or the student's sensory abilities.	Selection of textbooks, reading material and other instructional supports in digital format or online, so students with diverse needs (e.g. vision, learning, attention, English language learners) can access material through traditional hard copy or with the use of various technological supports (e.g. screen reader, text enlarger, online dictionary, etc.).

Principle	Definition	Example(s)
Principle 5: Tolerance for error	Instruction anticipates variation in the individual students' learning pace and prerequisite skills.	Structuring a long-term course project so that students have the option of turning in individual project components separately for constructive feedback and integration into the final product; provision of online "practice" exercises that supplement classroom instruction.
Principle 6: Low physical effort	Instruction is designed to minimise nonessential physical effort to allow maximum attention to learning. Note: This principle does not apply when physical effort is integral to essential requirements of a course.	Allowing students to use a word processor for writing and editing papers or essay exams. This facilitates editing of the document without the additional physical exertion of rewriting portions of text (helpful for students with fine-motor or handwriting difficulties or extreme organisation weaknesses, while providing options for those who are more adept and comfortable composing on the computer).
Principle 7: Size and space for approach and use	Instruction is designed with consideration for appropriate size and space for approach, reach, manipulation and use regardless of a student's body size, posture, mobility and communication needs.	In small class settings, use of a circular seating arrangement to allow students to see and face speakers during the discussion — important for students with attention deficit disorder or who are deaf or hard-of-hearing.

Principle	Definition	Example(s)
Principle 8: A community of learners	The instructional environment promotes interaction and communication among students and between students and faculty.	Fostering communication among students in and out of class by structuring study groups, discussion groups, email lists or chat rooms; making a personal connection with students and incorporating motivational strategies to encourage student performance through learning students' names or individually acknowledging excellent performance.
Principle 9: Instructional climate	Instruction is designed to be welcoming and inclusive. High expectations are espoused for all students.	A statement in the class syllabus affirming the need for class members to respect diversity to establish the expectation of tolerance and to encourage students to discuss any special learning needs with the instructor; highlights diverse thinkers who have made significant contributions to the field or shares innovative approaches developed by students in the class.

These principles will be used in this study to help conceptualise the best teaching practices for MWPs in a multilingual mathematics classroom and also assist in framing the results of the study. Their operationalisation in the classroom in the teaching of MWPs in multilingual mathematics classrooms is demonstrated in Chapter 5.

The following section provides a definition of mathematics word problems (MWPs).

2.5 MATHEMATICS WORD PROBLEMS

A decade and a half ago, Greer, Verscheffel and De Corte (2002:271) defined MWPs as "text that describes a situation assumed familiar to the reader and poses a quantitative question which subsequently requires an answer to be derived through mathematical operations performed on the data provided in the text, or otherwise inferred". MWPs refer to mathematical exercises of which the content is presented in the form of a story (Kasule & Mapolelo, 2013:265). Oliveira, Meskill, Judson, Gregory, Rodgers, Imperial and Casler-Ailing (2015:103) define MWPs as texts that are stretches of interconnected forms (e.g. clauses or sentences), written in a particular linguistic code (e.g. the English language) and register (e.g. mathematical terminology), and with a unique internal organisation (i.e. a textual structure) that can be coherently interpreted by readers who bring with them expectations, interests, viewpoints and prior reading experiences. According to these scholars, MWPs consist of two levels, namely the micro level (e.g. word usage, tense and reference) and the macro level (e.g. narrative organisational structure). These features set MWPs apart from other mathematical concepts, thus making MWPs a separate genre, characterised by a fixed set of features that need to be interpreted in specific ways. More than a decade ago, Bernardo (1999:194) noted that MWPs were the most challenging problems in mathematics education that learners had to solve. According to Seifi et al. (2012:2923), MWPs are real-world problems that require knowledge of mathematics for solutions and are not usually presented as equations that are ready to be solved, but rather as word or pictorial representations that must be interpreted symbolically, manipulated and solved.

Therefore, in the context of this study, MWPs are regarded as real-life story problems requiring an understanding of the text and language used to be converted into simple, computable and solvable equations with symbols and numerals. These problems require an intense analysis and interpretation to make it easier to be solved numerically.

2.6 MULTILINGUAL CLASSROOMS

Chitera (2009:vii) defines a multilingual classroom as a classroom with learners who bring with them a variety of main languages. She further states that this does not imply that all learners or teachers in the class are multilingual. For Adler (2001:4), a multilingual classroom consists of learners who are still learning English as the language of learning and teaching (LoLT) and simultaneously are learning mathematics, both as a discipline of knowledge and as a language. Essien (2013:9) avows that in the South African context, a multilingual classroom is one where the LoLT is an additional language, and where there is a presence of (home) language(s), all or most of which are in use or present a potential to be used in the classroom teaching process. In support of and acknowledging Essien's definition of a multilingual classroom, Nkambule (2009:3) adds that usually, the learners in many of these classrooms are not yet fluent in the LoLT, which is most often English. Halai and Karuku (2012:23) define a multilingual classroom as a context in which learners from varied linguistic backgrounds are taught from the same curriculum materials, by the same teacher, in the same classroom, and at the same time. They further declare that learners in this setting usually experience a challenge of linguistic alienation.

Setati and Adler (2000:247) add that in the South African context, English is the dominating language, despite the new progressive language in policy (LIP) that officially recognises the use of all 11 languages. Hansson (2012:107) regards multilingual mathematics classrooms as complex and thus requiring teachers to balance the different needs of the learners. He further notes that in this setting, the teacher should play an active role when teaching but, at the same time, let the learners themselves construct their knowledge. Taking into account the definitions by the various researchers mentioned above, in the context of this study, a multilingual mathematics classroom refers to a class where the learners and the teachers overall have various levels of proficiency in at least two languages, and there is a potential for those languages to be used in class at the same time.

2.7 CONCLUSION

This chapter discussed CER as the theoretical framework that underpins the study. In line herewith, this chapter discussed the origins and evolvement of CER as well as its objectives, nature of reality, the role of the researcher, and the relationship between the researcher and the researched, as justification for its adoption. This chapter also indicated the role of CER in achieving the objectives of this study. The operational concepts, which had served as the pillars of this study, were also explained and placed in context.

The following chapter will present a literature review on the teaching of MWPs and the challenges thereof that necessitate the application of a UDL strategy.

CHAPTER 3

LITERATURE REVIEW TOWARDS FORMULATING A UNIVERSAL DESIGN FOR LEARNING-BASED TEACHING STRATEGY FOR MATHEMATICS WORD PROBLEMS IN A MULTILINGUAL MATHEMATICS CLASSROOM

3.1 INTRODUCTION

The aim of the study is to utilise the aspects of universal design for learning (UDL) to develop an effective teaching strategy for mathematics word problems (MWPs) in a multilingual mathematics classroom. This chapter, based on the aim of the study, presents a literature review on the challenges pertaining to the teaching and learning of MWPs in multilingual classrooms, and looks at some of the UDL strategies used around the world (in other countries) as promising practices to address these challenges. Conditions favouring the implementation of the UDL strategies, the threats to their implementation as well as evidence to show the effectiveness of the strategies, are also provided in this chapter.

3.2 BACKGROUND

Having been a mathematics teacher for quite a number of years, I have witnessed the significant role that MWPs play in enabling learners to conceptualise mathematical concepts, and helping them realise the connection between mathematical language and the real world. In accord herewith, Gerofsky (1999:iii) describes MWPs as problems that establish a foundational narrative in mathematics education from the earliest years of learners' primary schooling. She further states, "Obliquely, word problems let us know what mathematics is allowed to be about, how we are to go about doing mathematics, and particularly, what is the uneasy relationship between the world of mathematics and our lived lives." Gerofsky (1999:60) avows that MWPs are coded in such a way that they refer to the world of mathematical objects and processes through tangential references to the world of human experiences. MWPs, among mathematics problems, *mostly* deal with relating real-world situations to mathematical concepts and, as such, are the "problems" that help learners use their mathematics knowledge in solving their daily problems (Seifi et al., 2012:2923). This means that MWPs can never be detached from human experiences. In solving these problems, learners get to tap into the real world in constructing meaning and knowledge, based on their prior knowledge. In support of this claim, Ladele (2013:ii) acknowledges that the teaching of MWPs is challenging and advises that, as such, teachers should be empowered to address the difficulties and misconceptions learners may have when solving these problems, thus echoing Bernardo's (1999:194) sentiment of more than a decade and a half ago.

In line with the above claims, the following sections are aligned to the five objectives of this study as noted in Chapter 1 (see Section 1.5). The first section highlights the challenges regarding the teaching and learning of MWPs in multilingual classrooms. The second section focuses on the components of the strategies that were employed to address the challenges. The third and the fourth sections look into the conditions conducive to the effectiveness of the strategies, as well as the possible and plausible threats to the successful implementation of the strategies respectively. Lastly, evidence of the successful implementation of the strategies is presented, followed by the conclusion.

3.3 CHALLENGES PERTAINING TO THE TEACHING AND LEARNING OF MATHEMATICS WORD PROBLEMS IN MULTILINGUAL CLASSROOMS

Learners' inability to solve MWPs is a challenge that teachers in mathematics education are aware of and concerned about. According to Bernado (1999:149), MWPs are the most challenging problems learners have to solve in mathematics education. In accord with this notion, Gooding (2009:31) believes that the challenge of teaching and learning MWPs and a trend towards poor performance of learners in this specific mathematical genre have raised interest in most people involved in mathematics education. The challenge of teaching this mathematical genre is further aggravated by the fact that most teachers also find it challenging to teach it since it requires English language proficiency, which most of them lack (Seifi et al., 2012:2923). Furthermore, previous research has shown the significant role of MWPs in mathematics education and in enabling learners to connect with real life (Vula & Kurshumlia, 2015:44). However, despite this role, many teachers find the teaching of MWPs quite challenging, and many learners still find it difficult to solve MWPs (Pearce, Bruun, Skinner & Lopez-Mohler, 2013:4). Based on this, the following paragraphs will discuss various challenges around the teaching and learning of MWPs in multilingual mathematics classrooms.

3.3.1 Lack of reading skills

An intense and detailed study conducted by Braselton and Decker in 1994, revealed that learners must be able to read and comprehend mathematics text before they can apply any mathematical skill to solve MWPs successfully. However, as these researchers proceeded with their research study, they realised that most learners did not comprehend mathematics text when they were reading. They concluded that reading in a mathematics class, unlike reading in other classes, is an intricate amalgam of words, numbers, letters, symbols and, sometimes, graphics (Braselton & Decker, 1994:276), which is the reason why learners struggle to comprehend the language text of MWPs.

In accord with this, Gooding (2009:31) also discovered that most learners in multilingual mathematics classrooms cannot read and understand the language used (LoLT) in mathematics word problems. Learners who are instructed in a language that they do not understand completely, such as a second or third language, often commit language-related errors (e.g. reading and comprehension) when solving MWPs (Oviedo, 2005:267). This, according to Bohlmann and Pretorius (2008:42), makes it particularly difficult for teachers to teach MWPs to learners properly. According to Gooding (2009:31), learners who cannot read and do not understand the LoLT, encounter challenges that make it even more difficult for them to obtain the correct solutions to the given problems. These learners are usually the least successful at solving MWPs, performing worse than learners with only mathematics difficulties or learners experiencing neither reading nor mathematics difficulties (Fuchs & Fuchs, 2002:565).

A study conducted by Bernado (1999:415) revealed that learners who were instructed in an LoLT that was not their first language, displayed an inability to show correspondence between the order of the stated events and the order of arithmetical data referring to the different events in the problem texts, because they could not read properly. That affected not only the solution performance of those learners but also led them to incorrect solutions to the problem.

In line with the above, mathematics teachers realise, in many instances, that learners are able to solve algorithms successfully when these algorithms are illustrated **numerically**, but they cannot solve such procedures when they are expressed **in words** due to lack of understanding of the LoLT (Seifi et al., 2012:2923). According to Huang and Normandia (2008:401), the reason why learners commit more mistakes when solving word problems than solving comparable number problems, is that word problems demand mathematical calculations along with other types of knowledge, including linguistic knowledge, to understand the problems, which learners lack in many instances. Seifi et al. (2012:2923) further note that the teachers' task of helping learners understand word problems is challenging because often teachers find word problems as difficult to solve as the learners themselves. Based on this, prominent researchers, Webb, Campbell, Schwartz and Sechrest (1966:314), call these types of mathematics problems "demon problems".

Morton and Qu (2013:88), in their study, which was intended to document the teachers' perspectives on the learners' poor performance in MWPs, revealed text difficulty as one of the challenges which most learners encountered. In the context of this study "text difficulty" means that word problems are complex as they involve more than one step and also require other kinds of knowledge, such as linguistic knowledge (as mentioned in the previous paragraph), numerical skills and computations, which are knowledge bases for solving problems of this nature. The findings of another study that was conducted in South Africa on the teaching of MWPs revealed that teachers' lack of proficiency in the LoLT when teaching probably would not result in positive gains as far as learners' performance was concerned (Sepeng & Madzorera, 2014:224). Sepeng and Madzorera (2014:217) further note that the said lack of proficiency does not only limit the process of understanding the given MWPs but also makes it difficult for learners to decide what operation(s) need(s) to be performed when they are reading.

Taking into consideration all of the above, it is clear that skilful reading is essential in understanding the problem text and, consequently, identifying the appropriate strategies to solve a problem. However, according to Sepeng (2013:172), many learners still encounter challenges in terms of reading and writing mathematics because teachers have not done much to counter this challenge.

3.3.2 Lack of a mathematical vocabulary and register

Ní Ríordáin, Coben, and Miller-Reilly (2015:9) note that mathematics is a language, albeit different from other languages. The difference, among other things, lies therein that no one uses it as a *first language* such as other natural languages (e.g. Sesotho, Setswana and English). In his seminal work, Halliday (1975:65) describes mathematical language as a language defined by the mathematical register. The mathematical register includes words, phrases, symbols and abbreviations, as well as ways of speaking, reading and writing that are specific to mathematics (Nkambule, 2009:19). One characteristic of the mathematics register is that it encompasses the distinctive vocabulary used in mathematics (Beeli-Zimmermann, Hector-Mason & Griffiths, 2015:13). Ní Ríordáin et al. (2015:13) add that the mathematics register is more than just vocabulary and technical terms as it also encompasses words, phrases and methods of arguing within a given state of affairs or situation, conveyed through the use of natural language. This means that the grammar and vocabulary of the specialist language are not a matter of panache, but rather a method for expressing very diverse “things” (Moschkovich, 2012:22). Taking all of this into consideration, it is clear that every language has its own, distinct mathematics register, incorporating ways in which mathematical meaning is expressed in that particular language. Setati (2005:9) notes that the successful solving of mathematical problems largely depends upon mastery of the mathematics register. However, according to Meaney (2005:129), learners are mostly constrained in the ways they can develop or redefine their mathematical understanding. Therefore, in the context of this research study, mathematics register means a set of meanings that fits a specific role in a language, together with the words and structures that express those meanings.

More than two decades later, Moschkovich (2002:190), referring to Haliday’s definition of mathematical language, noted that success in MWP solving necessitated a learner to be acquainted with the mathematics vocabulary and register. According to Vula et al. (2015:34), learners find the understanding of the problem, especially some of the words (mathematical vocabulary and register) used in these word problems particularly challenging when trying to solve MWPs. Not understanding the mathematical vocabulary and register, presents difficulties in word problem solving, thus causing misapplication of appropriate mathematical operations.

For example, a study by Irujo (2007:1 of 6) indicated that learners were usually exposed to only one numerical way of solving problems and that more emphasis was not placed on word problems. The consequence of this type of teaching was evident in a lesson in which the learners were given a word problem containing the mathematical expression "six divided by twelve" and were expected to find the answer. Most of the learners were not able to find the correct solution to the problem since they interpreted the problem as six divided by twelve = $\frac{12}{6} = 2$, instead of $\frac{6}{12} = 0.5$. The teacher realised that most of the learners confused the expression "**divided by**" with "**divided into**", which does not have the same meaning. The study thus indicated that learners' failure to understand how certain mathematical registers and vocabulary are used in different expressions, can potentially lead to incorrect solutions (Irujo, 2007:1 of 6).

Reynders (2014:2) notes that mathematics contains a specialised vocabulary and register that can convey definite ideas in written or spoken form. This being the case, the challenge is that learners have inadequate knowledge of the mathematical vocabulary and register, and this consequently leads to an inability to convert text expressed in English into mathematical language (Orton, 1996:120). Vula et al. (2015:35) declare that learners cannot be successful in terms of solving MWPs if they do not have the knowledge and comprehension of the essential mathematical vocabulary and register. Amen (2006:4 of 34) maintains that the understanding of the mathematical vocabulary influences the comprehension of lessons, tasks and various tests, especially in solving word problems, and that a lack of understanding of the mathematical vocabulary and register can affect the capability to solve MWPs. Findings by Brethouwer (2008:5) further reveal that learners who do not have sufficient knowledge of the mathematical vocabulary and register, cannot develop a deeper understanding of the mathematical concepts embedded in MWPs.

Sepeng and Madzorera (2014:224) believe that the role of the mathematical vocabulary is to assist in the building, expressing and communicating of mathematical notions and meanings through problem solving. The above descriptions of mathematical language indicate, among its significances, the role that the mathematical vocabulary and register play in ensuring the understanding of MWPs, and also in assisting the learners to comprehend MWPs. Boulet (2007:8) agrees with this notion and adds that the entirety of mathematical meaning is often rooted in mathematical words, symbols and numerals.

Although mathematical language plays a significant role in the conceptualisation of MWPs, Sepeng and Madzorera (2014:18) point out that technical terms, symbols and non-technical terms, as well as words with compound meanings, are the components of mathematics that habitually pose serious challenges to learners' comprehension in terms of solving mathematical word problems. The components of mathematical language that Sepeng and his associate regard as posing challenges in the comprehension of MWPs, indicate the complexity of solving word problems. This means that, included in the requirements to solve MWPs successfully, learners also need to be proficient in the mathematical vocabulary and register, which will enable them not only to understand the problem but also to conceptualise it and, ultimately, to solve it.

In her study, Langeness (2011:11-12) reveals three case scenarios that indicate the challenges in terms of solving mathematical word problems when learners do not have adequate knowledge of the mathematical vocabulary and register.

Scenario 1: Learners who were exposed to only one way of referring to the "function in mathematics" failed to solve the problems when an alternative reference to the "function in mathematics" was used.

For example, words referring to the function of **addition** include **"sum", "add", "plus", "and", "combine"** and **"increased by"**.

Therefore, learners who were only exposed to the words **"add"** and **"plus"** when referring to the function of addition, could not solve similar problems that required similar application of the operational sign, when words that they were unfamiliar with, such as **"and", "combine"** and **"increased by"**, were used instead. In this scenario, it should be noted that the learners' limited knowledge of the mathematical vocabulary and register can have a negative impact on their performance in terms of problem solving.

Scenario 2: The study also revealed that polysemy – diversity of meanings – can have a negative impact on learners' performance in terms of solving word problems. Polysemous words are words used in common speech and mathematics, but with different meanings.

For example, learners may come to class already knowing words such as "quarter", "remainder" and "place" since they are commonly used in their daily lives. However, these words have a different meaning when used in the context of the mathematics classroom.

Learners may know the various meanings of the word "right*"* in everyday use, such as the ***right* answer** or **one's *right* hand;** however, in the context of mathematics **"right"** has a totally different meaning, e.g. **right-angled** triangle, which means a triangle with an angle of 90 degrees, as shown in Figure 3.1.

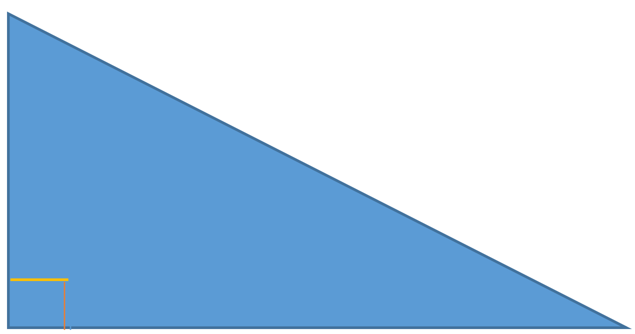


Figure 3.1: Right-angled triangle

In this category, it should be noted that specific words that are used in everyday life, can pose a challenge in terms of solving MWPs, particularly when learners do not know the meaning of those words in the context of mathematics.

Scenario 3: The same mathematical word can be used in more than one way within the field of mathematics itself.

For example, the word **"round"** can refer to the **shape of a circle** as shown in Figure 3.2 or the function of **rounding a number** to the nearest tenth, as shown below.

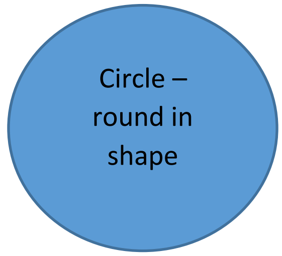


Figure 3.2: Circle

9.5 – rounding it off to the nearest tenth is equal to 10

Similarly, the word ***square*** can refer to a **shape as shown by Figure 3.3** or a **number times itself** as shown below.

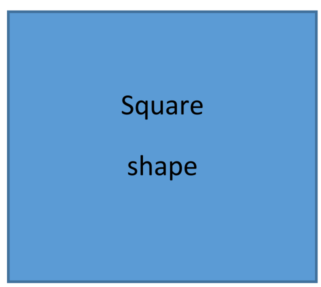


Figure 3.3: Square

The square of three is nine / $3^2 = 9$

Even though knowledge of the mathematical vocabulary and register is essential in word problem solving (Moschkovich, 2002:193), research indicates that learners still lack sufficient knowledge of the mathematical vocabulary and register to solve MWPs successfully.

3.3.3 Lack of visualisation skills

Being able to read and recognise keywords in MWPs is central to learners' understanding of MWPS; however, to solve MWPs successfully, learners must be able to visualise the problem (Cruz & Lapinid, 2014:1). This means that the ability to visualise the MWPs that must be solved can potentially lead to the selection of the appropriate operations, which subsequently can lead to obtaining the correct solution(s) to the problem. Teahen (2015:28) avows that without the ability to visualise word problems and the mathematics content involved in MWPs, learners will continue struggling to solve word problems. Findings from a study conducted in China revealed that most learners, due to their lack of visualisation skills, misinterpreted the questions and failed to obtain the correct solutions (Yeo, 2009:23).

In a similar study conducted previously in America, Cruz et al. (2014:4) revealed that the difficulties in translating MWPs into numerically solvable equations were a result of learners' inability to visualise the problem. Amnueypornsakul and Bhat (2014:111) avow that MWPs test critical aspects of reading comprehension together with the generation of solutions that agree with the word problem text. In other words, when learners read, they also have to visualise the situation or the problem portrayed by the text and thus figure out ways to solve the problem.

In a study conducted in America (Langaness, 2011:39), learners were given the following word problem to solve during a mathematics lesson:

Grandmother is teaching us how to sew. She has 30 yards of yarn, and she wants to give each of us 4 yards. How many of us can she give yarn to, and how much will she have left?

Figure 3.4: Yarn

The learners were given some time to discuss this problem and determine how it should be solved. During this discussion time, the teacher moved around to check how the groups were responding to the problem. Listening to their conversations, she realised that most of the learners were struggling not only to identify the actual problem that they had to solve but also the method to use to solve the problem. The teacher's observation was that the learners struggled because of an inability to create a clear picture of what the problem depicted, and an inability to identify what they have to visualise to solve the problem. It was only when the teacher drew a picture of the yarn and started "cutting" it into sections of four yards, that learners were able to visualise the problem well enough even to recognise what it required them to do.

From this scenario, it is clear that when learners read a mathematical word problem and are unable to think about it abstractly, conceptualise and form a vivid mental image of it, they may find it quite challenging to solve the problem. The role of the teacher in this regard would be to assist the learners in developing the necessary skills to form appropriate mental images (internal visualisation) by providing them with drawings or manipulatives (external visualisation) to serve as a guide in the process of generating problem pictures (PPs) in their minds. This will help the learners discover what the problem requires and identify appropriate ways to solve it. Although visualisation of the problem serves as another key component in solving MWPs, Morton et al. (2013:88) aver that learners still lack the ability to visualise the problem and to translate natural language into correct mathematical relationships that would lead to the attainment of the correct solutions.

3.3.4 Lack of understanding caused by ambiguity

Semanticists view ambiguity as semantic property. According to them, ambiguity involves elements such as the association of expressions in a language with something else (things or events in the world, mental representations, sets of possible worlds, and more). According to Degani and Tokowicz (2010:1292), translation ambiguity occurs when a word in one language can be translated into another language in more than one way. However, there are many reasons for ambiguity in translation. Prior, Kroll and Macwinney

(2013:460) note that, in some cases, language homonymy has the potential to lead to multiple translations. In their study, they found that certain words with multiple meanings caused confusion and hampered the learners' comprehension of the text of word problems, particularly where learners could not contextualise these words. For example, the English word "glass" has two meanings – **the material** and **the drinking vessel**. The English word "cook" can mean either the action, that is, a verb, or the person, that is, a noun.

Reynders (2012:30) highlights that polysemous words may cause translation ambiguities and also cause learners not to understand clearly what the mathematical problems require them to do. She noted words such as **"set", "power", "similar"** and **"difference"** as words in ordinary English that have a specific meaning in the context of mathematics. These words could cause confusion, particularly in instances where learners do not know the meanings within the mathematical context. Reynders further states that when learners are given MWPs containing polysemous words, misinterpretation is likely to occur because learners may already know a particular meaning of the word in a different context. Therefore, limited knowledge regarding the different meanings of words in different contexts may lead to the misinterpretation of the words within the mathematical lesson context and, ultimately, to obtaining incorrect solutions to the problem.

The term **"volume"** in the context of mathematics means the amount of space that a substance or object occupies, or that is enclosed within a container, and is usually measured in cubes or cubic units. Volumes of the different shapes (e.g. cone, cylinder, pyramid, etc.) can be determined or calculated in various ways.

Example:

The volume of the rectangular prism in (Figure: 3.3) is calculated as follows:

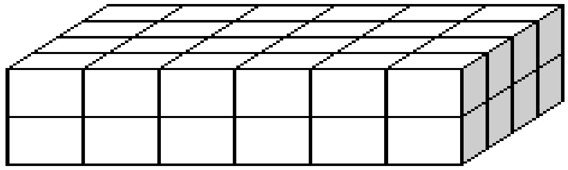


Figure 3.5: Rectangular prism

V – volume

l – length

b – breadth

h – height

Calculation:

$$V = l \times b \times h$$
$$V = 6cm \times 4cm \times 2cm$$
$$V = 48cm^3$$

In the context of technology and physics, volume could also mean the quantity or power of sound or the degree of loudness. Volume can also be associated with the loudness of the television or the radio at a learner's home. It is significant that learners have a clear distinction of a term according to the different contexts it can be used in.

Although many of us strongly believe in the precision that mathematics provides, the reality is that ambiguity and vagueness are commonly reported in mathematical conversations and text problems (Barwell, Leung, Morgan & Street, 2005:142). They pose challenges which, among others, confuse learners in terms of what to solve and how to solve it, as well as being unable to identify the appropriate operations – all of which often lead to the attainment of incorrect solutions. Based on this, it is imperative to address the issues of ambiguity in an endeavour to ensure success in terms of solving MWPs.

3.3.5 Teachers' lack of development of skills in terms of teaching mathematics word problems in multilingual classrooms

Bayazit (2013:1920) defines a problem as a notion that stirs apprehension in an individual and therefore leads him or her to pursue a solution by means of his or her own knowledge and experience. He outlines the three main characteristics of a problem as follows: i) the person who comes across it, will find it challenging; ii) it awakens a need in the individual who first comes across it, to solve it; and iii) the individual would not have had any preparation pertaining to the solution of the problem when he or she is challenged with it. In line with Bayazit's description of a problem, Tanrıseven (2000) concludes if a problem is a situation that generates uncertainty in the human mind, problem solving is the process of eliminating these uncertainties. Based on this, one realises that problem solving requires both cognitive and affective processes, as one not only needs to devise various alternative ways to cope with the uncertainty but also has to select the best alternative and apply it in an appropriate manner to cope with the uncertainty.

Taking the above description of a problem into consideration, it is clear that problem solving, as an integral component of mathematics, is a complex process that requires learners to be well guided in terms of successfully solving problems. The following question arises: What is the role of the teacher in developing the learners' problem-solving skills? In their study exploring the place of problem solving and mathematical thinking, Ersoy and Guner (2015:120) signify the vital role that a teacher plays in the process of problem solving as that of being a guider and an enabler. In certain instances, the teacher has to write all the strategies on the board, demonstrating the selection and application thereof, thus promoting the use of different problem-solving strategies. In addition to this, Langaness (2011:18) describes the role of the teacher as that of providing learners with direct teaching of the problem-solving process to enable them to solve MWPs. According to Langaness (2011:18), when learners are taught and engaged in problem-solving processes, they develop an understanding of word problems. Teachers are further expected to scaffold the problem-solving process to help learners write and solve the mathematical problems.

Although the teacher's role in the problem-solving process is significant, Seifi et al. (2012:2923) indicate that most teachers also find it difficult to solve MWPs, which has a negative impact on their teaching of MWPs. Therefore, these teachers are unable to scaffold learners' understanding of mathematical word problems. In a study conducted in South Africa, Sepeng et al. (2012:218) found that although problem solving was defined as the ability to read, process and solve mathematical situations, teachers were unable to nurture this ability in their learners since they had difficulties in terms of solving MWPs themselves. As a result, the learners could not develop personal connections and understanding of the mathematical concepts embedded in the word problems.

In a study conducted in Malawi, Chitera (2009:50) discovered that the teachers experienced limitations in terms of teaching mathematics, including MWPs, after they had completed their training. Chitera (2009:50) argues that teachers are not sufficiently trained by institutions of higher education to cope with the demands of teaching mathematics in multilingual mathematics classrooms. She avows that teachers are not exposed to richer and more multifaceted language practices in teacher training programmes. She further highlights that, among other things, teachers are not aware that each language has its own way of expressing mathematics concepts, and they lack understanding in terms of how language can be incorporated into the teaching of mathematics, despite the fact that they had completed their teacher training. She also indicates that often teachers are not even aware of the intricacies of teaching mathematics in multilingual classrooms where most learners are still learning the LoLT. This makes it even more challenging for the teachers to help develop learners' skills in terms of solving MWPs.

Example:

i) Given the equation $2b+8=20$, determine the value of b.

Instead of the problem being given in this form:

ii) What two numbers have a sum of 20 and a difference of 8?

It should be noted that, when a problem is given in the second form (as a word problem), it requires some skills to be applied to be solved, for example, an understanding of the language, vocabulary and register, visualisation, and computation skills, to mention a few. However, in its simplest form, in (i), only the application of operational signs and computations are required for this problem to be solved.

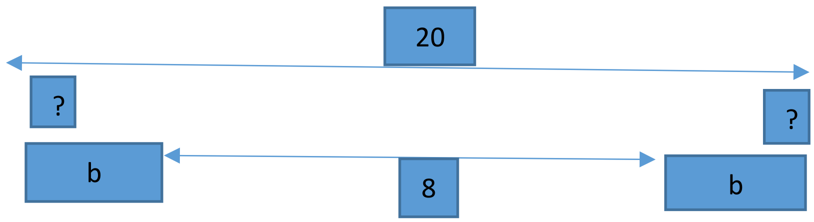


Diagram 3.1: Word problem diagrammatic representation

Sepeng and Madzorera (2014:218) refer to this type of training as one that provides only a "mechanical way" of solving the problems, which, in turn, does not cultivate learners' understanding of MWPs. Based upon this, they recommend that teachers should be capacitated in terms of solving MWPs (which is currently the skill that most teachers lack) to enable them to help learners develop "cognitive and affective" skills in terms of solving MWPs.

3.3.6 Teachers' negative attitude towards the use of home languages

The history of the use of Afrikaans and English as the dominant LoLT dates as far back as 1948, when there was a comparatively "loose policy" on the use of mother-tongue teaching (Hartshorne 1992:197). Legislation was passed after 1948 and then resources, essential to establish Afrikaans and English as self-sufficient official LoLTs in South African schools, were provided (Adler, 2001:7). During the Apartheid period, Afrikaans and English were the only languages vested with official status. In 1996, the new South African constitution was adopted under democracy and, in addition to Afrikaans and English, nine African languages, namely Setswana, Sepedi, Sesotho, Tshivenda, siSwati, Xitsonga, isiNdebele, isiZulu and isiXhosa, were added to make up the 11 official languages of South Africa. These languages were added not only to be recognised as existing in South Africa but to elevate and endorse their status in order to be recognised in teaching and learning, just like Afrikaans and English (Constitution of the Republic of South Africa 1996). According to Molotja (2008:9), learners and teachers elevate the “previously disadvantaged” languages to their rightful status by using them in instances where the LoLT (English) fails to facilitate the process of meaning-making.

A study conducted in Kenya revealed several benefits of the use of home languages in the teaching and learning of mathematics, such as its being a tool that, since it is their first language, encourages and enables learners to express themselves confidently in class (Khejeri, 2014:81). In this study, most teachers who were interviewed indicated that the use of learners’ home languages promotes participation of all learners in the lesson and also makes it easier for learners to grasp difficult concepts. Although the use of the learners’ home languages provides solid benefits, as highlighted above, some studies revealed that often mathematics teachers are still reluctant to provide their learners with an opportunity to use their home languages in class. A study conducted a decade ago in South Africa revealed one of the main reasons behind teachers’ negative attitude towards the use of learners’ home languages in class (Setati, 2008:111). It was found that teachers were more concerned about the teaching that would best give learners access to *social class, power, higher education* and *employment*, which they believed could be accomplished by teaching learners only in English, than in using their home languages to enhance understanding of difficult concepts (Setati, 2008:111). The research further demonstrated that teachers even “felt guilty” to do code switching, fearing that they may deprive learners an opportunity to acquire proficiency in English (Sepeng, 2015:656).

Other research in America and South Africa revealed that teachers use coercive measures to force learners to speak the “foreign” languages that are used as LoLT in the classrooms (Alidou & Brock-Utne, 2005:87). Sepeng (2015:663) found that in most instances, the use of a foreign or unacquainted language as LoLT leads to teachers using traditional and teacher-centred teaching methods, which do not create ideal circumstances for effective learning to occur. A study in Malawi revealed that most teachers are in favour of the use of colonial languages because of the belief that these languages offer more benefits to the learners, and that they are commonly used elsewhere in the world (Chitera, 2009:15). However, according to Nkambule (2009:4), teachers who do not allow learners to use their home languages as “resources” to augment their understanding of subject content, deprive learners of the many benefits that come with the use of their own languages. Based on this, Setati, Molefe and Langa (2008:17) call for a pedagogical strategy that would also allow for the use of learners' home languages optimally, consciously and transparently (visibly) to solve real-world MWPs, while simultaneously not depriving learners epistemological access to knowledge in the modern world. Naidoo (2015:182) emphasises that it is important that teachers identify teaching strategies that may overcome challenges created by the language of teaching, thereby improving learners' performance and mathematical ability.

3.4 SOLUTIONS TO THE IDENTIFIED CHALLENGES

The following sections outline the various UDL strategies, revealed in the literature, that are used to address the abovementioned challenges. Some of these strategies were applied in a number of countries with best practices in terms of teaching MWPs in multilingual mathematics classrooms – all in an effort to help learners comprehend and solve these complex problems.

3.4.1 Reading skills employed to enhance understanding of mathematics word problems

The ability to read is key in mathematics education, particularly when dealing with MWPs, as it has the potential to generate an understanding of the problem text. In other words, learners must be mathematically literate to master MWPs. Mathematicians define mathematical literacy as learners' ability to comprehend and deal with numbers amenably (National Council of Teachers of Mathematics, 2000). Mathematical literacy is attained only when learners can decipher numbers, apply and translate mathematical representations and abstract symbols, and use representations to model and interpret physical, social and mathematical phenomena as part of solving problems and constructing mathematical meaning (Cope, 2015:11).

According to Franz (2015:4 of 8), it is important that learners think flexibly about the concepts before attempting to solve a problem. He conducted a study that explored the challenges learners encountered when solving MWPs and found that the inability to read was one of the biggest challenges that learners faced. In his study, Franz also observed instances where learners could not make sense of what they were reading. For example, the concept of **the whole** could mean **one object** or **multiple members that make a whole**. Franz found that learners who were unable to contextualise the word "whole" in their reading, misinterpreted the given problem and, as a result, failed to solve the problem correctly. He suggested that learners must be provided with many reading and writing experiences on mathematical concepts to build their knowledge in terms of understanding the use of words in different contexts. He further noted that explicit teaching that allows learners to explore compound solutions, to communicate using mathematically correct academic language, and to deepen their understanding of mathematics, is imperative.

A study by Barton and Heideman (2002:vi), which was conducted in the United States of America, revealed that learners who are explicitly taught strategies for reading mathematics texts, develop significantly in their understanding of mathematical problems. The study further revealed that such reading strategies do not only improve learners' reading but also enable them to comprehend and apply the text, making it possible for them to solve MWPs. Therefore, they emphasise that there is a place and a need for the development of learners' reading skills in an effort to enhance their understanding.

In South Africa, researchers found UDL active reading strategies, which they regarded as "concrete cognitive strategies" (CCS) to be helpful in supporting learners who were struggling to read and to solve MWPs. These strategies included accessing prior knowledge, making and revising predictions, using visual cues and text features, making inferences or extrapolations, asking questions, making mental images, monitoring comprehension, and summarising (Brenner, 2009:5; Kamil, Borman, Dole, Kral, Salinger & Torgeson, 2008:1). According to Franz (2015:5), the active reading strategy of using visual cues supports learners in reading mathematical texts. In order for the visual cues to fully support learner comprehension of MWPs, explicit teaching of the relationship among text, graphics, equations and mathematical problems is required (Franz, 2015:5). Another strategy that serves as a powerful tool to support reading of mathematical text is "asking questions" (Kamil et al., 2008:1). This simply means that learners, as they read through the question, keep asking questions on the content in an effort to elucidate the concepts and to determine what they need to do to answer the question correctly. The question-answer relationship is another strategy that was recommended as it teaches learners how to decode the types of questions they are being asked (Fard & Nikou, 2014:302). Through this strategy (question-answer relationship), learners ask themselves about the relationship between the text and the specific question or mathematical problem, and after that, determine whether the problem is evident on paper, based entirely on background knowledge or "mental pictures" (they have created in their heads) or an amalgamation of both (Fenty, McDuffie-Landrum & Fisher, 2012:34). According to Franz (2015:5), the question of reasonableness is an important concept in mathematics and, as such, learners need to develop a sense that their solutions to the given problems are reasonable.

3.4.2 Strategies to improve learners' mathematical vocabulary and register

Mathematics can be described as a cultural activity as it uses its own language to communicate and has its own vocabulary, representations and symbols (Reynders, 2012:30). It is viewed not only as developing competency in completing procedures, solving word problems and using mathematical reasoning but also in developing socio-mathematical norms, such as presenting mathematical arguments and participating in mathematical discussions (Moschkovic, 2002:192).

Previous studies that investigated learners' understanding of a variety of mathematics terms revealed that learners often do not understand many of the words that are commonly used in mathematics. For example, the word "similar" means "proportional" in mathematics; yet, in ordinary English, it means "alike". The confusion, therefore, arises where one discovers that what is similar in ordinary English, is not necessarily the same in mathematics, and vice versa (Reynders, 2012:30). Taking this into consideration, it is clear that learners must know the mathematical vocabulary and register, and be able to distinguish between ordinary words in English, and when they are used in mathematical contexts. In support of this notion, Boulet (2007:9) adds that there is a need for learners to become fully conversant with the mathematics vocabulary and register to solve MWPs successfully. Moore-Harris (2005:18) avows that vocabulary development in mathematics is important and its successful development can be realised through reading strategies. Many words in mathematics have specific meanings that are quite different from their meanings when used in casual conversations (Reynders, 2014:2). This is made evident in the examples below:

Fraction: In mathematics, fractions are represented by values such as $\frac{1}{2};\frac{1}{4};\frac{5}{8}\dots\frac{a}{b}$ where a is the **numerator** and b the **denominator**. However, in ordinary English, "fraction" means just a portion of something.

Area: In mathematics, "area" means the amount of flat space that a figure covers. Depending on the figure, there are various formulae that can be used to calculate the area of a specific shape, as shown in Table 3.1.

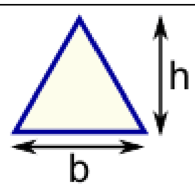
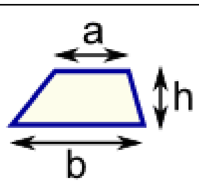
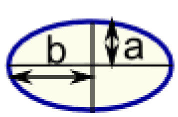
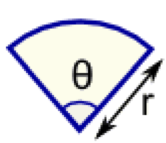
Shape or figure	How the area is calculated
	Triangle Area = ½ × b × h b = base h = vertical height
	Trapezium Area = ½(a+b) × h h = vertical height
	Ellipse Area = πab
	Sector Area = ½ × r^2 × θ r = radius θ = angle in **radians**

Table 3.1: Mathematical shapes and formulae for calculating the area

In general terms, “area” means a **certain section of land or space**. **Example (sentence)**: This particular ***area*** of Cape Town is beautiful.

Ní Ríordáin et al. (2015:14) encourage the use of strategies that develop mathematical vocabulary and register acquisition in the mathematics classroom in an effort to assist learners in using terms appropriately and contextualising them aptly. In doing so, teachers are expected to make sure that learners have frequent opportunities to read and speak, using precise mathematical language to develop fluency (Franz, 2015:5). Furthermore, teachers should consider the need for translation and interpretation, just as a reading teacher would when helping learners with a difficult or new text. Supported practice in reading mathematical texts is also essential in developing understanding. Furthermore, Miller and Koesling (2009:69) avow that immediate remediation is important in reducing potential learner confusion in class.

In a study that was conducted in South Africa, simple clarification of mathematical language, the use of familiar words in questions and a cognisant engagement of learners in learning new vocabulary and register in class, were found to be effective strategies that promoted the understanding of the vocabulary of mathematics. Other approaches included the writing of new words and leading learners to articulate these words, limiting the number of new words introduced in one lesson, allowing learners the multiple uses of a new word to ascertain understanding, encouraging the correct use of vocabulary and language, and teacher revoicing (Essien, 2013:20-21). Some of the strategies that proved to be effective, included pre-reading activities and conversations that access and build on learners’ background knowledge, text annotations that put a “gloss” on crucial vocabulary or provide the necessary contextual information without paraphrasing the text for learners, and activities during and after reading that allow learners to engage in knowledge building with their classmates. Central to all of the above, however, is the teachers’ understanding that texts are approached differently for different purposes, and that learners need opportunities to approach texts with these varied purposes in mind.

In an endeavour to improve the learning of mathematics vocabulary, some scholars have encouraged learners being taught the definitions of technical terms. In accord with this notion, Sepeng and Madzorera (2014:218) write that such a practice places learners at a vantage point when it comes to word identification, as it leads to the selection of suitable strategies and algorithms when approaching problem solving in word problems. However, Monroe and Orme (2002:139) warn that, when specialised vocabulary is used, teachers must take care to maintain a proper balance between facts on the one hand, and language on the other.

3.4.3 Strategies to promote visualisation of word problems

According to Teahen (2015:14), the rise of cognitive science promotes the process of visualising mathematical problems and requires this process to be recognised as an essential part of learning that needs to be embraced, cultivated and encouraged in deepening the understanding of mathematical concepts. In line with this, Mulligan (2011:23) emphasises that it is important that learners' ability to produce quality representations should be scaffolded by providing learners with the structure that enables them to solve problems. In accord with this, English and Halford (1995:57) add that "the essence of understanding a concept is to have a mental representation or mental model that faithfully reflects the structure of that concept". Therefore, it is reasonable to indicate that the mastery of creating mental images has the potential to enable problem solvers to make meaning of the problem and also to solve the problem. In support, Poch, Van Garderen and Scheuermann (2015:282) note that learners who are provided with visual aids, such as diagrams, to solve MWPs, have more clarity about what problems entail and, consequently, these learners are able to solve the given problems.

Content analysis of the teachers' responses in a study conducted in the United States of America, revealed that both internal and external visualisation were the predominant strategies that teachers used to help learners generate clear pictures of the problems they had to solve (Seifi et al., 2012:2925). In another of their studies, where 52 teachers were interviewed, the results indicated that the learners' difficulties, according to the teachers, were mostly caused by their inability to i) make both internal (mental) and external (drawings) representations, ii) understand the word problems, iii) make a plan, and iv) define related vocabularies. As a solution to this problem, the teachers then suggested that learners be guided on how to identify a pattern within a word problem, draw a picture and reword the problems in such a way that they could understand the meaning that is embedded in the problem text (Seifi et al., 2012:3).

The use of diagrams was found to be the best strategy to represent a word problem, particularly as the learners work towards an advanced level of MWPs (Poch et al., 2015:282). According to Jitendra and Star (2011:15), a diagram is a tool that helps to represent relations between the different elements described in the text. Its strength lies in the fact that, like any other visual representation, it can be used as a cognitive tool to help learners understand a problem situation, recognise what needs to be solved (be able to realise what is missing), select the appropriate strategy and, subsequently, solve the problem following the identified appropriate procedures (Poch et al., 2015:282). It provides a visual referent that learners can use in a metacognitive manner to monitor progress and to self-correct where necessary.

Although the use of a diagram serves as a vital cognitive tool in assisting the learners in visualising and solving the problem, Vaughn, Wanzek, Murray and Roberts (2012:15) avow that learners still need to be taught a method that incorporates the use of diagrams when solving problems. This means that learners still do not predominantly use diagrams when solving word problems, despite the benefits thereof. Therefore, it is necessary that learners should be taught a method that would encourage them to use and to incorporate diagrams when they are solving MWPs. In addition to this, Poch et al. (2015:289) advise that in order for learners to solve MWPs successfully using diagrams, teachers need to explicitly and systematically teach the learners about diagrams, including the different types of diagrams, and how and why they are used.

From the above, it is clear that proper use of a diagram not only advances learners' knowledge of solving MWPs but also builds their confidence and enhance their problem-solving performance (Swan, 2005:4). This means that the appropriate use of a diagram can be a positive and valuable contribution in terms of advancing learners' conceptual and procedural understanding of MWPs.

3.4.4 Strategies to overcome misconceptions of word problems caused by the presence of ambiguous words

De Jong and Harper (2005:105) avow that borrowed words from everyday English can cause problems for learners who are doing mathematics in a second language. In most instances, these words are ambiguous as they have a specific meaning in the mathematics register and a different meaning in their everyday English use (Yushau & Bokhari, 2005:4 of 18). The non-mathematical meanings of terms such as average, degree, even, odd, operation, and so forth, can influence learners' mathematical understanding significantly and cause confusion, thus leading to incorrect procedures in terms of solving the problems.

From the above, it is clear that the challenges that come with the presence of possible ambiguous words cause confusion when not appropriately contextualised within multilingual mathematics classrooms, and therefore, need to be addressed. Ní Ríordáin et al. (2015:14) agree with the above statement and caution if second language learners cannot acquire the correct meaning, it is likely that they would experience difficulties in terms of solving MWPs and thus would be unable to obtain the correct solutions; hence the need to address this challenge. In an attempt to address the misconceptions that are caused by translation ambiguity, Skolverket (2011:62) suggests that teachers should provide learners with opportunities to grow accustomed to mathematical forms of expressions. They should also teach the learners how these expressions, according to the different mathematical contexts, could be used to communicate about mathematics in their daily lives. Another way of addressing this challenge, suggested by Temple and Doerr (2012:288), is to create hybrid spaces in classrooms where learners can integrate their home-based discourses with disciplinary discourses, resulting in learning taking place at the intersection of the two, manifesting itself as a hybrid epistemological and discursive construct.

Clarkson (2007:193) writes that learners process thoughts more easily in their home languages because home languages facilitate semantic processes. In his study, he explored the actual use of translation into learners' "home languages in the language learning process". His study revealed that thinking in one's home language converts the input into terms that are more familiar. This enhances learners' confidence in their ability to understand and thereby reduces feelings of uncertainty. According to Nkambule (2009:22), the use of home languages also serves as a means of maintaining concentration long enough for meaning to be put together and assimilated. Clarkson (2007:193) notes that when learners think in their home languages, the "concepts are likely to come alive because the learner's network of associations is usually richer than in the second language". A study conducted by Kazima and Adler (2006:47) in Malawi, explored the meaning that learners assigned to the word "probability" in a given MWP before the term is explained to the learners. The findings revealed that the learners brought with them a range of meanings, such as "certain", "likely", "unlikely" and "impossible". Based on this, the researchers contended that the learners' interpretations and translations of words in many instances were influenced by linguistic structures in their home languages and by the interaction between the two languages in the Malawian society (i.e. the home language and the LoLT). Kazima et al. (2006:47) thus concluded that the meanings of mathematical words could not be considered separately from how learners understood those words in their home languages. This was made evident by the manner in which they demonstrated their understanding of the word "probability" in class *before* they were taught the meaning of that word in a mathematical context. Based on these findings, it is clear that giving an example of a word requires not only knowing the meaning of the word but also understanding the concept and the context in which the word is used.

In a study in the United States of America, a teacher was observed during a lesson where prior knowledge was activated and the learners practised new material (Temple et al., 2012:301). The strategy that the teacher used to familiarise learners with new words and words that they already knew was to use the words in such a manner that the different meanings, depending on the context they were used in, were portrayed. During this episode, the researchers realised that the teacher predominantly used elicitations, recasts, repetitions, metalinguistic feedback and clarification requests to provide feedback. While some of these did not encourage the learners to explore or to explain their reasoning, they nonetheless assisted the learners in recalling, clarifying and applying previously learned conceptual and linguistic knowledge. Simultaneously, this created various opportunities for the learners to practise using technical vocabulary precisely and in well-formed sentences, which may be viewed as supporting the development of fluency in the mathematical register.

Fitzpatrick and Metta (2002:6 of 6) identified probing as one of the interactional strategies which a teacher can use to overcome translation ambiguity when teaching. According to Franke, Webb, Chan, Ing, Freund and Battey (2009:381), this strategy is particularly effective for engaging learners in building new mathematical understanding in terms of the careful use of words that might cause ambiguity. Furthermore, this interactional strategy can be effective in developing learners' ability to work in an integrated manner with the manifold semiotic systems of the mathematical register.

3.4.5 Empowering strategies to enhance teachers' ability to assist learners in developing effective word problem-solving skills

One of the roles that the teachers in multilingual mathematics classrooms are expected to perform is to support second language learners' acquisition of the LoLT through language-orientated teaching. According to Essien (2013:1), teachers are usually presumed to understand the convoluted relationship between language and learning, which, in the case of mathematics, is the language subject teaching, which is reform orientated. Reform-orientated mathematics teaching is teaching characterised by an emphasis on communication and collaboration (Moschkovich, 1999:11). In line with this, the multi-level structural equation strategies using the Trends in International Mathematics and Science Study (TIMSS) (2003), revealed the significance of both the teacher and the learners taking responsibility for ensuring that the MWPs are solved successfully (Hansson, 2012:103). This means that the teachers' content knowledge of MWPs is essential in empowering the learners; however, the learners are equally expected to have knowledge of dealing with MWPs for successful teaching and learning to occur. Hansson further notes that teachers need to be capacitated so that they can effectively guide and scaffold learners with teaching methods that facilitate language progress and learning. In accord with Hansson's notion, Tsotetsi (2013:42) also deems professional teacher development significant for teacher empowerment in terms of the realisation of effective teaching.

Research conducted by Essien (2012:57) in a multilingual classroom in Tanzania revealed some of the dilemmas (similar to those revealed in Barwell's study in 2008) that necessitate teacher capacitation in terms of teaching MWPs in multilingual classrooms. Essien' recommendation, based on the identified dilemmas, is that teachers need to pay attention to issues such as how learners make sense of the given MWPs, which is often determined by how they understand the particular usage and structure of the language, how the use of everyday language shapes the learners' mathematics learning, how learners express mathematical thinking in their own language and how language is used in textbooks in comparison with how the teacher uses language in the classroom. Essien (2010:41) also notes that training institutions need to introduce modules that would specifically deal with and aim at creating awareness of the relationships between mathematics and language, especially with regard to the teaching of learners in multilingual settings.

Chitera (2011:1007), in accord with Essien's notion of teacher training, believes that for teachers to teach MWPs effectively, they need to be empowered so that they could assist learners to perform in many different ways, using tools of different kinds, particularly discourses that enable them to solve the MWPs. Harwell (2003:87) also emphasises the need for teachers to be fully capacitated on how to teach MWPs and urges that this should be addressed as a matter of "urgency" to improve learner performance. Furthermore, Nuangchalerm (2012:66) indicates that teachers' pedagogical knowledge needs to be enhanced, and deems teacher professional development a vital tool in addressing teachers' inability to assist learners in developing effective problem-solving skills.

3.4.6 Using home languages as resources to promote learning and understanding of mathematics word problems

Morgan (2007:241) declares that all mathematics classrooms are multilingual, and by using a form of pedagogy that switches between different home languages and the language of teaching, learners can access both mathematical ideas and powerful ways of thinking and speaking. In line with Morgan's declaration, research studies have been conducted in support of the use of learners' home languages as resources and tools for mathematics teaching and learning (Niesche, 2009:704; Setati et al., 2008:16). Jäppinen (2005:162) notes that it is important that opportunities are created where learners can learn mathematics in their home language so that they can develop language skills in their first and second languages. Israel and Thomas (2013:542) claim that mathematics is learned better when teachers use learners' home language(s) as a resource. In accord with this, Chitera (2009:11) urges teachers to regard home languages as a resource for teaching and learning since home languages offer the learners a cognitive advantage in learning mathematics. However, Nkambule (2009:4-5) cautions that although the learners' home languages prove to be good resources for the teaching and learning of MWPs, attention still needs to be paid to how learners' home languages can be used to facilitate mathematics learning. Therefore, the strategies on how to use home languages must be carefully explored. Nkambule further notes that for such strategies to work, it is imperative that learners gain epistemological access without losing access to English, which is presently seen as a necessary condition for gaining access to social goods.

According to Sibanda (2015:5), teachers need to ensure that learners become proficient in the language of assessment to register success in the assessment. If learners' proficiency in both the LoLT and in their home language improves, they can potentially perform above average. However, this presupposes that the teacher takes responsibility for enabling these learners to develop their language skills even when they learn mathematics. However, Setati (2008:107) cautions against the overuse of learners' home languages in the multilingual classroom. She contends that such overuse may keep the learners from acquiring proficiency in English, which is a global requirement. She further notes that learners want access to English because they are concerned with access to social goods and society, and are positioned by the social and economic power of English. Clarkson (2009:153-158) recommends three practical ways whereby teachers can "tactfully" promote the use of learners' home languages in multilingual classrooms, which must be done to stimulate their learning and understanding of MWPs without limiting the acquisition of the LoLT. Firstly, teachers can map the representation of the languages in their classroom and record each learner's competency in a specific language. This will help the teachers to become familiar with their learners' capabilities, thus attending to the needs of individual learners. Secondly, teachers must encourage learners to work in their home language when solving mathematical problems. This will prompt the learners to expand their knowledge of the mathematical register and vocabulary of the specific language. Thirdly, teachers can use open-ended questions to stimulate the growth of a rich language milieu as well as autonomous thinking. Learners usually enjoy thought-provoking classroom discussions when attempting to answer open-ended questions.

According to Halai and Karuku (2013:29), engaging learners' home communities as a resource for their mathematics learning, could also serve as another way of supporting learners' learning processes in a multilingual setting. They cited parents visiting classes as an example where parents could act as teacher assistants with the request to use their first language. The researchers further suggested the use of literature from the learners' home language as one way in which teachers can assist learners to connect mathematical ideas in both the language of teaching and the learners' home language of teaching.

3.5 CONDITIONS CONDUCIVE TO THE EFFECTIVENESS OF THE SOLUTIONS

The following sections describe the conditions favouring the implementation of the strategies highlighted above.

3.5.1 Conditions conducive to enhancing learners' reading skills and understanding

A study conducted in South Africa revealed that learners' comprehension of word problem text could improve if teachers frequently encourage learners to read the question on their own and provide the answer they think is represented by the word problem text (Nkambule, 2009:78-79). In cases where learners seem not to have read the question well, it is important that the teacher requests the learners to carefully re-read the question to understand it fully. The study further indicates that probing and directing the learners to the responses provided by others can potentially lead to a situation where the learners have a mutual understanding of the mathematical expressions. The study further revealed that the kind of environment where learners are requested to read on their own and are probed and directed to generate a shared understanding not only improves the learners' reading skills but also stimulates classroom discussions that lead towards a deeper understanding of the concepts and encourage learners to think.

Hlalele (2012:267) suggests that teachers use real-life situations for the learners to "model" them mathematically. He contends that mathematics is inseparable from organised life; and therefore, teachers' practices should not be alienated from real-life experiences. According to Hlalele (2012:267), this would ensure that learners carefully read the scenarios before working out the problem procedurally, which he regards as a vital condition for improving learners' reading skills. In America, the use of novelty, flexibility and creativity of response was found to be significant in helping learners improve their reading skills (Danesh & Nourdad, 2017:34). It was also determined that these strategies contributed positively to deepen and broaden learners' understanding of words and numbers found in MWPs.

3.5.2 Conditions favouring the improvement of learners' mathematical vocabulary and register

In an extensive study conducted in South Africa by Vula et al. (2015:35), it was claimed that for learners to solve MWPs successfully, they need to know the meaning of the mathematical register and vocabulary. They further stated that if learners knew and understood the meaning of the key terms, they could simply learn mathematical concepts and develop the necessary skills in mathematics. In the United States of America, learners' understanding of the mathematical vocabulary and register was found to have a positive influence on the comprehension of lessons, tasks and numerous tests, especially in solving word problems, as well as affecting their capabilities to solve MWPs (Amen, 2006:9 of 34).

In Australia, teaching learners the language of mathematics in many different ways was found to have a positive impact in terms of enabling the learners to develop a better understanding of MWPs and to choose the appropriate steps to solve MWPs successfully.

Taking time to write words related to problems and discussing their meaning in the context of the problem, afforded learners more opportunities to know what to do and how to go about solving MWPs (Solomon, 2009:2). Furthermore, teachers who provided learners with regular opportunities, helped them to re-engage in varied activities that subsequently developed and enriched their knowledge (Riccomini, Smith, Hughes & Fries, 2015:239).

A study conducted in the United States of America revealed that continuously observing and assessing how learners go about solving word problems and providing feedback of assessment, enabled the teacher to point out misconceptions about the given problem (Murchan, Shiel & Vula, 2012:17). This subsequently improved learners' performance in solving word problems and motivated and orientated them to work in further activities. This process helped to direct learners in the learning process and empowered them to gain essential skills that would lead to better performance in word problem solving.

Franz (2015:3) believes that for learners to solve MWPs successfully, they need to have a command of the specific register and vocabulary of mathematics. Moreover, Franz notes that teachers who provide learners with opportunities to communicate mathematically, make it possible for learners to develop fluency. In addition to this, the use of words on the wall, which are usually centrally located in the classroom, enables learners to recall mathematical definitions and vocabulary entailed in MWPs since learners can see and read these words in the classroom at any time. Biddle (2007:489) asserts that words on the wall do not only empower learners but also empower teachers since these words could be reviewed, applied and discussed whenever necessary.

3.5.3 Conditions conducive to overcome lack of understanding caused by the presence of ambiguous words

An American study revealed the significance and the role of language conception in solving MWPs. In this study, Salma and Rodigues (2012:6) maintain that language conception is critical in the teaching and learning of mathematics, since an understanding of mathematical concepts and the ability to solve mathematical problems largely depend on the language used during the course of teaching and learning. Salma et al. (2012:6) thus declare that performance in mathematical word problems correlates with language proficiency. Therefore, learners need to understand the language and know how to use it to translate words correctly. Furthermore, learners need to know and understand the context in which the translated words are applied before they can make sense of the word problems they are solving. Such an understanding of the mathematical vocabulary and register, informed by the correct interpretation and translation within the specific context, would make it possible to obtain the correct solutions.

A study that was conducted in Australia revealed that it is possible, in English, to skim a story or novel, use swift reading techniques or even miss sentences and descriptive paragraphs, and still understand what one has read in terms of the message and the morale of the story (Frobisher, 1996:133). However, non-fiction, which is the category that MWPs fall under, generally cannot be read in a superficial manner without losing essential details (Reynders, 2014:22). In other words, to avoid translation ambiguity and to arrive at the correct solutions, careful reading is essential. Learners need to read “between the lines” without skipping or missing any word since every word is essential in making meaning of the problem text. In their seminal work, De Corte, Verschaffel and De Win (1985:7) mention that to avoid translation ambiguity, learners must have well-developed semantic schemata for these types of problems and the ability to solve them conceptually. For Pape (2004:187), careful reading, comprehension of the text of the given MWP and the correct contextualisation of the text, not only address the issue of translation ambiguities but also enable learners to successfully convert text into numerically solvable equations.

A Malawian study revealed that often the meaning of a mathematical concept expressed in words differs from the meaning of the same concept expressed in symbols. For instance, **the number a is five less than the number b,** which the learner may mistakenly restate as **a = 5 – b**, when it should be **a = b – 5** (Chitera, 2009:33). In this case, translation ambiguity would not occur if learners performed linear translation, and not translate mathematical concepts word by word. However, learners need to have a holistic understanding of the given word problem.

A study that was conducted in Tanzania revealed that teachers who prepare tasks using the learners’ home language and their everyday life experiences, enhanced the learners’ understanding of problems, and this understanding eliminated ambiguities (Gorgorio & Nuria, 2001:11). Therefore, the use of learners' home language, together with their everyday life experiences, aids learning and eases the need for translation (Chitera, 2009:33).

3.5.4 Conditions conducive to the implementation of the strategies to enhance word problem text visualisation

Makina (2010:25) notes that visual imagination is a crucial element in the process of successfully solving MWP texts. She further claims that visualisation is an important aspect of mathematical understanding, insight and reasoning that, in turn, improves learners' critical thinking. According to Novotná, Eisenmann, Přibyl, Ondrušová and Břehovský (2014:2), graphical representation – in other words, drawing a picture – is one instrument that can be used to help learners visualise the problem. In their study conducted in Australia, they found the use of graphical representation effective in assisting learners to visualise the problem text. They realised when graphical representations are used, learners usually write down what has been given and often also what they want to determine. This type of drawing is called an *illustrative drawing* since it illustrates the problem to be solved. Sometimes a solution can immediately be seen once the drawing is completed. However, in most instances, learners have to manipulate this drawing (e.g. add suitable auxiliary elements) before they can attempt to solve the problem, with the help of the modified drawing. This type of drawing is called a *solution drawing* as it helps learners to clearly depict what they need to do to solve the problem.

Franz (2015:5 of 8) notes that visualising the problem is a good strategy to support learners in reading. The relationship between text, graphics, vertical equations and mathematical problems is usually portrayed in the drawing, indicating a need for explicit teaching on the visualisation of the problem. Franz further notes that explicit teaching regarding visual cues can significantly support learners' comprehension of the word problem and consequently lead to the possibility of correctly solving the problem.

According to Teahen (2015:i), for visualisation of a word problem to be effective, learners need to connect the text with their prior knowledge and experiences to create meaningful mental images. De Koning and Van der Schoot (2013) state that visualisation of the word problem by connecting text to prior knowledge and experiences serves as an effective way to enhance comprehension of the problem texts.

Furthermore, Teahen (2015:26) also notes that in order to produce a representation, learners must be able to read the information and access prior knowledge linked to the context by firstly “determining” what they already know, that is, to visualise what they understand the content to be. Creating mental visual images enables learners to make links to prior knowledge and experiences, and to stimulate deeper comprehension of what they are reading (Drapper 2010:2). Sadoski and Paivio (2013:50), in accord with Draper’s notion, add that “[w]ithout the activation of mental representations, no meaning can be present”. De Koning and Van der Schoot (2013:266) deduce that learners who have been trained in the use of mental imagery have a better reading comprehension compared to learners who have not received any training in a comprehension strategy.

3.5.5 Conditions conducive to teachers’ empowering strategies to teach word problems in multilingual mathematics classrooms effectively

Essien (2013:78) avows that it is imperative that institutions that prepare and supervise teachers provide them with effective teaching strategies. These strategies should not only focus on directly facilitating learners’ learning processes and experiences but also provide teachers with the opportunities to evaluate and reflect upon their own practices and skills as one means of maintaining and improving their professional development.

According to Kind (2009:172), pedagogical content knowledge includes teachers’ knowledge of representations, analogies, examples and demonstrations to make subject matter comprehensible to learners. It includes knowledge of specific topics that learners might find easy or challenging, as well as possible conceptions or misconceptions that learners might have relating to the topic. This means that the effective teaching of MWPs does not only rely on teachers’ knowledge of the content but also depends upon the pedagogies for teaching the learners effectively. One of the responsibilities of teacher training institutions should be to ensure the realisation of such effective pedagogies.

According to Ntloana (2009:2), effective professional development programmes of teachers stand at the centre of proposals for improving the quality of teaching and transformation of education. This means that the teachers who are undergoing teacher training must be provided with training that fully empowers them to teach learners effectively. Furthermore, these training programmes need to empower teachers with transformational teaching strategies that will enable learners to solve MWPs. Bernado (1999) views MWPs as a vital component of mathematics education, albeit difficult to solve.

Ramatlapana (2009:153) regards teaching as a complex activity that requires both knowledge of the content and of pedagogy. In order for effective teaching of MWPs to occur, Ramatlapana believes that teachers must have appropriate content knowledge of the subject, coupled with the skills to convey that knowledge to the learners. Therefore, effective teaching of MWPs has the potential to have a positive impact on how learners learn MWPs and also to contribute significantly to learners' mastery of the concepts.

According to Tsotetsi (2013:9), the successful teaching of MWPs depends upon the creation of spaces for teachers to share good practices and content knowledge. This means that teachers need to come together and share with one another knowledge gained, and ways in which MWPs can best be taught to the learners. This will expand and enrich the teachers' knowledge of teaching MWPs and, consequently, make it easier for learners to grasp the content and learn MWPs.

In Kenya, teachers jointly planned lessons and observed one another's teaching in actual lessons. This enabled the teachers to improve their teaching approaches based on the feedback they received from the observer teachers. Cooperative teaching enables teachers to utilise one another's strengths and also enables them to complement one another's knowledge and expertise (Steyn, 2011:224). The collaborative approaches aided teachers' understanding of the MWPs and also made it possible for them to show the relevance of what they taught learners through their day-to-day classroom practices (Begum, 2012:383; Murtaza, 2010:219).

3.5.6 Conditions conducive to effecting the use of home languages as supportive resources in the teaching of word problems

Essien (2013:9) notes that a holistic approach is required to address multilingual learners' educational needs. Essien further notes that such an approach should recognise and embrace multilingualism as a resource, rather than an obstacle to the teaching and learning of mathematics. This approach will, therefore, ensure that teachers use the LoLT to teach mathematics, but at the same time allow other languages to be used as resources in teaching and learning to enhance deep learning and promote the understanding of MWPs.

Setati (2005:448) states that it is important that teachers strike a balance between the use of English, which is the LoLT, and the use of learners' home languages in multilingual mathematics classrooms. She further notes that when communicating mathematically in class while maintaining this balance, the teacher's role should include, among others, i) managing the interaction between ordinary language and mathematical language, ii) managing the interaction between procedural and conceptual discourses and, finally, iii) managing the interaction between learners' main language and the language of learning and teaching (where the LoLT is different from the learners' home languages). Therefore, the management of all of these will make it possible for learners' home languages to be used productively in support of the LoLT, subsequently making the teaching of MWPs much more effective.

Essien (2013:57) believes that it is the teacher's ability to facilitate the use of the learners' home languages and English while paying attention to several important issues that will make the use of both languages beneficial to the learners. These issues include i) how learners make meaning of mathematics, which is determined by how they understand the specific usage and structure of the language, ii) how the use of everyday language shapes mathematics learning, iii) how learners express mathematical thinking in their own language and iv) how language is used in the textbooks in contrast to how the teacher and the learners use language.

3.6 POSSIBLE THREATS TO THE SUCCESSFUL IMPLEMENTATION OF THE STRATEGIES

From the discussion above, it is clear that a significant body of research has agreed to the important role that the use of learners' home languages plays in teaching and learning (Moschkovich, 2012:18; Setati, Chitera & Essien, 2009:65). These studies have contended for the use of learners' home languages as support in the teaching and learning of mathematics. Furthermore, these studies also agree that to facilitate multilingual learners' participation and success in the teaching and learning of MWPs (which is a significant component of ME), the learners' home languages should be recognised as authentic languages of mathematical communication. The use of the learners' home languages in these studies is through code switching, used predominantly to afford explanation to learners in their home languages.

Although a considerable amount of research seems to be in favour of the use of learners' home languages as a resource to aid the teaching and learning of mathematics, some studies have shown that most teachers do not support this at all (Sepeng, 2011; Setati, 2008). These teachers still prefer to "stick" to only the LoLT (English) when teaching, despite knowing that they are teaching learners who are not yet proficient in the LoLT. Halai and Karuku (2013:27) avow that as a result of being instructed in English only, learners withdraw from participating in class activities when they feel that their linguistic backgrounds are not recognised and valued. Furthermore, a study by Planas (2012:15) indicates that although many teachers in multilingual classes are not proficient in the LoLT themselves and thus use a simplified language, they expect learners to use the official language correctly, which does not promote learner participation.

I knew from experience that learners come to class with, among other things, different educational backgrounds, which do not only affect how they socialise with others but also how they learn. For instance, to learn MWPs, some learners (especially those with a low socio-economic status) prefer the teacher to explain the problem sentence by sentence and word for word and, after that, to be given a similar problem to solve on their own. Other learners, who are proficient in the LoLT, can read and understand the problem without necessarily relying on the teacher's explanation of each sentence, word for word.

Kazima (2007:171) notes that learners bring different cultural practices into the classroom, which are relevant for their mathematics learning and comprehending of MWPs, but are never addressed. Therefore, to focus only on how the language shapes the learning of MWPs (which is what most teachers do) does not give a full understanding of the challenges that learners face.

UNESCO's (2003:75) position paper on education in multilingual contexts contends for the need to strike a balance between allowing learners to use their mother tongue as a cognitive tool in the classroom, and in providing them access to global languages of communication through education so that they can successfully compete nationally and globally. However, research indicates that not only teachers view the use of learners' home languages negatively, but learners also do not want to be taught in their home languages. Setati (2008:111) found that both teachers and learners prefer English as the LoLT for mathematics. Her research shows that preference for the use of English above other languages is because of the "hegemony of English" and the desire or need to gain access to social goods, for example, access to higher education and employment. According to (Nkambule, 2009:3), most people still view English as a language that defines a person's level of education, which is why most teachers restrict learners from using other languages.

The Language in Education (LiE) policy encourages the use of the 11 official languages (Department of Education, 1997). It also guarantees people's access to justice in the language of their choice. The LiE policy further gives parents the right to choose the medium of instruction for their children. This means that the entire primary and secondary education can, "in principle", be conducted in the children's home languages as the media of instruction (Halai & Karuku, 2013:23). Even though the LiE policy promotes the use of learners' home languages, English is still viewed as the language of "access and power" and, therefore, the knowledge of the native languages does not pay off in the linguistic "marketplace" (Kamwangamalu, 2009:138). The fact that English is still viewed as the language of power is also made evident by the materials that are used to teach. Most of these are written in English and not in other languages. According to Halai and Karuku (2013:23), the limited teaching material in the mother tongue or unavailability thereof also poses a threat to the implementation of UDL strategies to teach MWPs effectively. On the basis of this, Setati (2008:113) contends that this ostensibly inclusive LiE policy is not effective enough in bequeathing the home languages' privileges, prestige, power and material gains that have for so long been associated with English.

3.7 EVIDENCE OF THE SUCCESSFUL IMPLEMENTATION OF THE UNIVERSAL DESIGN FOR LEARNING STRATEGY

The following sections highlight some of the successes accomplished by the implementation of UDL strategies to address the various challenges pertaining to the teaching and learning of MWPs.

3.7.1 Improved reading skills

Teachers who provide their learners with the skills to use summary strategies for reading and mathematics reinforce the learners' conceptualisation of mathematical problems. These summary strategies enable the learners to determine what is important, abridge the information by removing unnecessary details and putting it in their own words. Barton and Heidema (2002:11) contend that mathematics depends heavily on conceptual understanding and that effective reading skills can potentially develop the learners' understanding of mathematical concepts in terms of how they build on one another as well as how they are related. Learners who are frequently engaged in reading strategies develop an understanding of word problems and become familiar with the use of text. Learners find word text even easier to comprehend when they are taught how to read and identify key information, rather than to look for clue words (Bruun, 2013:57). This means that improved reading skills can potentially aid learners' holistic understanding of MWPs.

According to Hagaman, Casey and Reid (2016:44), when teachers explicitly integrate procedures for reading and paraphrasing into the lesson, the learners' comprehension of MWPs is enhanced. The study also revealed that teachers' reading of the word problem, highlighting the important information or rewording the word problem and omitting extraneous information, increases the understanding of the problem text, especially for learners with decoding and comprehension difficulties. Moreover, creating an environment where learners are expected to read on their own and are probed and directed to generate a common understanding, improves not only their reading skills but also stimulates classroom discussions, encouraging the learners to think deeply about the word problem (Nkambule, 2009:78-79). If learners ask questions about the word problem they have to solve while they are reading through the problem, the mathematical concepts and what the problem requires to be solved are elucidated (Kamil, Borman, Dole, Kral, Salinger & Torgeson, 2008:18).

3.7.2 Improved knowledge of the mathematical vocabulary and register

If learners are frequently exposed to the mathematical vocabulary and register, they tend to understand word problems and the different contexts where the vocabulary and registers can be used better. According to Reynders (2014:21), when learners frequently engage with the mathematical vocabulary and register, they eventually become experienced problem-solvers who can fill gaps and comprehend ambiguities that other learners who are not frequently introduced to various mathematical vocabulary and registers, cannot comprehend. Furthermore, if learners frequently do exercises that engage them with various mathematical registers and vocabularies, they develop the ability to make meaning through reading "between the lines". In their seminal work, De Corte et al. (1985:8) mention that teachers who develop learners' vocabulary and register create opportunities for learners to develop mathematical competencies in solving MWPs and also develop their semantic schemata of MWPs. Ní Ríordáin et al. (2015:13) note that the process of learning mathematics encompasses inexorably the mastery of the mathematics register. Meaney (2005:129) states that sufficient knowledge of the mathematical register allows learners to communicate their mathematical findings suitably and that learners without this fluency are limited in ways they can develop or redefine their mathematical understanding. Ní Ríordáin et al. (2015:13) avow that teachers who develop learners' mathematical vocabulary and register advances the learners' analytical, descriptive and problem-solving skills in a language and a structure by which they can explain a variety of experiences. These scholars further aver that the mastery of the mathematical register develops the learners' ability to listen, question, discuss, read, record and participate in mathematics.

Learners who are frequently exposed to mathematical register and vocabulary, which are important concepts in learning mathematics and mathematical thinking, and who are required to speak the language of mathematics in class, become successful in learning MWPs (Vula et al., 2015:34). A study conducted by Reyenders (2014:25) revealed that learners who are taught mathematics registers and vocabulary are not confused when they have to solve word problems. Other research studies have also shown the significant role of teaching mathematical vocabulary and registers to learners. These studies indicate that learners' chances of success in terms of solving MWPs increase if they are carefully taught mathematical registers and vocabulary (Vula et al., 2015:35). Teachers who continuously pay attention to mathematical vocabulary and register development, not only increase the learners' knowledge of the mathematical vocabulary and register but also nurture the development of learners' cognitive growth in terms of comprehending MWPs (Capraro & Capraro, 2006:23). Research has further shown that the mastery of the mathematical vocabulary and register as cognitive features makes it possible for learners to become fluent (e.g. repeated readings and assonances) and to read the language structure (e.g. verb, subject/noun agreement and morphemes) with understanding (Lane et al., 2008:58). Therefore, the mathematical vocabulary and register can potentially reinforce the conceptualisation of MWPs.

3.7.3 Improved skills to eliminate ambiguities

Lemke (1990:27) notes that learners come to the classroom with knowledge and when they hear a familiar word, they often link it to what they have previously heard and experienced. In accord with Lemke's notion, research also indicates that if a commonly used English word is used in a technical domain, learners hearing the word for the first time in class may integrate the technical usage as a new facet of the features of the word they already know. A study conducted by Makar and Confery (2005:31-32) revealed that when attention is paid to learners' use of non-standard words, the learners' comprehension levels on the specific topic increases and they find the topic easier. Kaplan, Rogness and Fisher (2014:9) assert that when learners are often made aware of ambiguities when they are taught MWPs, they end up being able to distinguish between colloquial meanings and technical meanings. For instance, Kaplan et al. (2014)

discovered that making learners aware of the ambiguities associated with the use of the word “random” from time to time enabled them to use it correctly in the appropriate context.

According to Galligan (2016:28), teachers who constantly highlight ambiguities when teaching and provide learners with the freedom to “build their voices” in class make it possible for learners to overcome the challenge of lexical ambiguities. Teachers should, therefore, be cognisant of the subtle differences in language. In certain instances, words used in a mathematics classroom may have different meanings and grammatical functions than when used in common English. Teachers, therefore, need to support learners’ using technical language when explaining concepts and encourage learners to make the connections between everyday meanings and meanings in a mathematical context. This consequently enables learners to differentiate between the different contexts in which a word can be used.

The study by Galligan (2016) further revealed that teachers who ask learners to first explain the meaning of a word before they provide them with the technical definitions thereof can effectively address the challenge of lexical ambiguity as they now can determine the learners’ prior knowledge and thus ensure that learners can distinguish between prior knowledge and new information. The teachers’ use of words in contexts where colloquial meanings coincide with technical meanings enhances the learners’ understanding of the use of words and builds a solid foundation (Kaplan et al., 2014:11). Over and above these, the teachers’ careful use of the mathematical vocabulary and register when teaching makes it possible for the learners to learn and to also be careful in terms of using words in different contexts (Rangecroft, 2002:37).

3.7.4 Improved visualisation skills

Learners who are taught how to visualise word problems through the process of delineating the steps find it easier to solve word problems as visualisation assists in building visual representation (Makar et al., 2005:31). Furthermore, breaking down the word problem into its constituent parts and subdividing the problems into schemas, make learners aware of the structural similitudes and differences in problems and their solutions to better identify and solve similar types of problems. This supports and monitors problem solving and addresses the flexible use of alternative strategies to solve the problem (Jitendra, Star, Rodriguez, Lindell & Someki, 2011:1).

Teachers who continue to guide learners in the visualisation process of MWPs, enable them to gain a clear understanding of the problem, to link the information to a solution and to have a holistic view of the problem that must be solved. The use of manipulatives helps making the problem more concrete and provides opportunities and suggestions for multiple ways to think about and to solve the problem (Hoong, Yvonne, Subramaniam, Zaini, Chiew & Karen, 2010:21). Furthermore, learners who are frequently encouraged to draw a diagram to visualise the problem, develop a conceptual and procedural understanding of the problem, leading to the attainment of the correct solution (Poch et al., 2015:282). Visualisation, therefore, makes abstract ideas more perceptible and encourages treating them almost as if they were material entities (Sfard, 1991:6).

Hanging the "process diagram chart" in the classroom during a lesson provides the learners with a visual demonstration of the concepts within the word problem and the procedure that will be followed in that part of the lesson. The chart can, therefore, be used as a way of transmitting the discourse to the learners (Chitera, 2009:164-165). According to Teahen (2015:17), teachers who allow and encourage learners to generate their own representations to illustrate concepts embedded in MWPs, develop the learners' understanding of mathematical concepts significantly. Furthermore, the study also reveals that both the internal representations (mental images) and the external representations (drawings) significantly maximise achievements in problem solving (Teahen, 2015:i). This means that teachers who allow and encourage learners to generate their own representations to illustrate concepts embedded in MWPs significantly, develop the learners' understanding of mathematical concepts.

3.7.5 Improved problem-solving skills

Teachers who are well capacitated in terms of teaching MWPs, know how to teach word problems in such a manner that learners find it easy to comprehend. For example, these teachers divide MWPs according to arithmetic operations; in other words, the problems are divided according to the different operational signs +, - , x and ÷, the numeric volume and whether the problem requires a single step or multiple steps to be solved. Furthermore, these teachers are able to guide learners in terms of solving the word problems, thus making sure that the learners get rid of irrelevant information that is not part of obtaining the solutions. These teachers also develop the learners' mathematical vocabulary and register, which are used to present MWPs and to empower the learners to use complex syntax to make meaning of the given problems (Kavkler, Magajna & Babuder, 2014:30).

According to Brown, Skow and the IRIS Center (2016:1), when teachers have gained adequate teaching skills in terms of teaching MWPs, they focus on the errors the learners commit in the retrieval of facts (e.g. a properly chosen, but miscalculated operation), and procedural errors (e.g. being unsure that an appropriate procedure is chosen and correctly executed). In this manner, teachers can identify misconceptions and knowledge gaps that need to be addressed. These teachers can also identify errors relating to the decoding of words, vocabulary and register errors, and errors associated with the transformation of oral descriptions in the appropriate mathematical symbols and correct operation. Informal, formal and summative assessment strategies help teachers engage in accurate, reflective decision making in terms of the strategies they can employ to aid learners' comprehension (TOEFL Institutional Testing Programme, 2002:2 of 18). This reflective phase usually uncovers difficulties or problems which, if not addressed, may impede progress toward self-improvement in teaching.

Part of becoming an effective teacher in terms of teaching MWPs is the teacher's ability to incorporate real-world activities with activities in the classroom. These real-world activities foster a realistic approach towards mathematical "modelling" and solving of the problem. According to Moleko (2014:96), the incorporation of real-life activities stimulates the learners' interest and also increases participation in class, which ultimately enhances the learners' understanding of the concepts.

3.7.6 Improved understanding of word problems reinforced by the use of home languages

Ball (2010:2) encourages the use of the learners' mother tongue as cognitive resources for learning mathematics in school. However, Essien (2013:8) cautions that while allowing the learners' home languages to be used, teachers need to strike a balance between allowing learners to use their home languages as a cognitive tool in the classroom and, as mentioned earlier, providing them with access to global languages of communication through education so that they can successfully compete nationally and globally. Balancing the use of learners' home languages as resources for learning, together with the use of the LoLT, makes it possible for learners to understand the content and, at the same time, learn the LoLT.

Code switching in a multilingual classroom plays a vital role in facilitating textual meanings for learners who have limited proficiency of the language of those texts, particularly in the grades immediately following the switch to a new medium of instruction (Ferguson, 2003:39). Code switching can also be used when there are noticeable learning difficulties in the learner's understanding of the MWPs, in which case the teacher can switch to a language in which the learner is more proficient to recap, elaborate on or clarify the problem. Code switching may also be used to re-formulate the teacher's instructions or the learners' words. Furthermore, code switching can be used to qualify the key components of a phrase or a sentence in a problem (Halai, 2011:127), thus enabling and deepening the learners' understanding of the concepts.

In a study conducted in South Africa, Setati (2005:447) examined the relationship between the language(s) used, the mathematics discourses and the cultural models that had been developed. The teacher in Setati's study switched between English and Setswana, which was the learners' home language. The teacher's use of Setswana produced conceptual discourses, in other words, discourses in which the reasons for solving a problem in particular ways and using particular procedures to solve such mathematical problems became explicit topics of conversation. The teacher's use of English produced procedural discourses, in other words, discourses that focused on the procedural steps taken to solve a particular problem.

Studies conducted by Chitera (2011:44) and Halai et al. (2011:28) in Malawi and Tanzania respectively revealed that the need for translation in a multilingual mathematics classroom is inevitable and that a translation strategy can be used for the following purposes: i) to emphasise an important point; ii) to enable learners who might not understand what is being said in the medium of instruction to participate in the lesson; and iii) to overcome the lack of some expressions in a given language. These studies further revealed that translation could also play a significant role in solving "word" problems, which requires more than just cognitive skills. Chitera (2009:42) and Halai and Karuku (2013:28) attest to the successful use of translation in the context when teachers use it cautiously and ensure that it will not lead to a mistranslation of the intended mathematical meanings.

3.8 SUMMARY

In this chapter, attention has been paid to CER as the theoretical framework underpinning this study by highlighting the following important elements: its historical background, its objectives, the nature of reality, the relationship between the researcher and the participants, and the role of the researcher. These aspects were explained with the aim justify the selection of CER as the appropriate theoretical framework for this study. The operational concepts for this study were also explained and defined comprehensively in this chapter to enhance the reader's understanding of the study. This chapter further paid attention to the five objectives of this study, as outlined in Chapter 1, in an effort to align existing literature with these objectives.

The first section of the literature review outlined the challenges pertaining to the teaching and learning of MWPs. These challenges include the following:

- Learners' inability to read, thus not understanding the language of the word problems.
- A lack of knowledge of the mathematical vocabulary and register.
- An inability to visualise the problem and selection of the correct operation.
- A lack of understanding caused by the presence of ambiguous words in word problems.

- Teachers’ inability to help learners develop effective skills in terms of solving MWPs.
- Teachers’ negative attitude towards the use of home languages.

The next chapter provides a discussion on the implementation of various UDL strategies to enhance the abovementioned challenges in terms of the teaching and learning of MWPs.

The conditions favouring the implementation of the strategies, and the threats that could impede the successful implementation of the strategies, were also outlined in this chapter. Lastly, the chapter provided evidence of the success of the implemented strategies. It should be noted that although the strategies were effective to some extent, the literature indicates that the challenges regarding the teaching and learning of MWPs persist and need to be addressed. This study is, therefore, aimed at finding better and more innovative ways in which to address the persisting challenges. In order to find better and innovative ways, this chapter indicated clearly that the solution lies in a collective effort in which the voices of all the people concerned are recognised.

This chapter further indicated the significance of devising various strategies to ensure that all learners in multilingual classrooms receive improved instruction on how to solve MWPs. What also became clear from this study, is the need for teachers to be intentional about their teaching practices in an effort to transform their classrooms.

Teachers, therefore, need to move away from teacher-centred approaches as these approaches lead to learners feeling excluded and segregated. These approaches neither support the main goal of basic education nor contribute positively to effective teaching. Teachers should also move away from teaching practices that simply accommodate learners, to teaching practices that advocate social justice and UDL principles. Ladner and Burgstahler (2015) echo the same sentiment and illustrate this notion, as shown in Diagram 3.2.

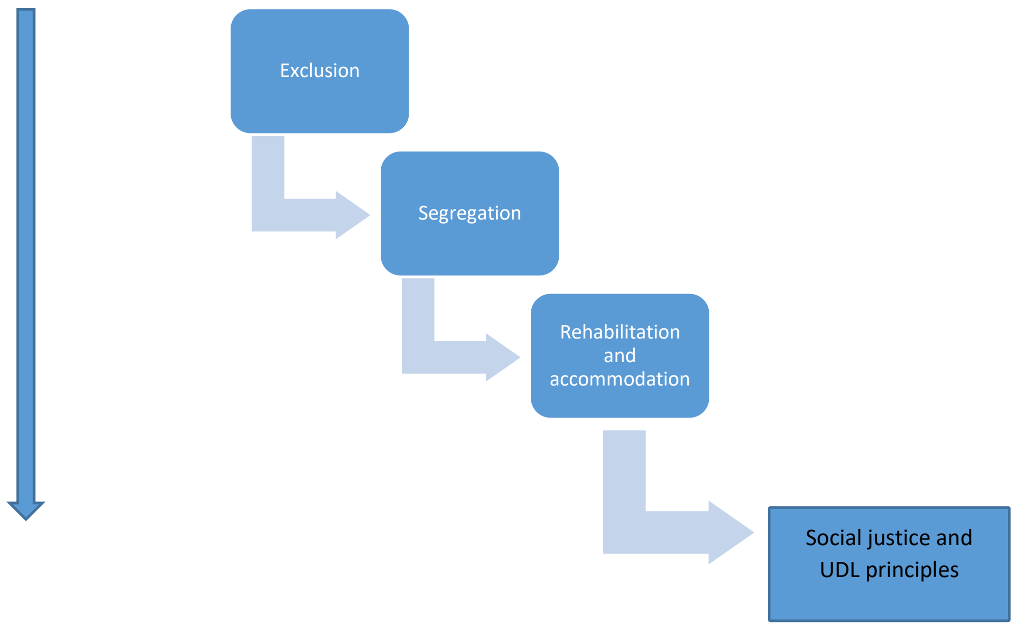


Diagram 3.2: Evolution of access (Ladner & Burgstahler, 2015)

The next chapter focuses on the methodology employed to generate data.

CHAPTER 4

RESEARCH DESIGN AND METHODOLOGY

4.1 INTRODUCTION

The purpose of this study was to utilise the aspects of a universal design for learning (UDL) to develop an effective teaching strategy for mathematics word problems (MWPs) in a multilingual mathematics classroom. In an effort to achieve this, as well as to respond to the research question (see Chapter 1, Section 1.4), this chapter focuses on the research methodology and design.

In Chapter 2, a description of the theoretical framework couching this study was provided. One of the justifications for the use of CER as the lens underpinning the study was its agenda to promote human empowerment, transformation and emancipation. In line with this, CER, in this study, is used to promote the empowerment of participants by creating spaces for sharing, intense engagements and deliberations. These engagements are aimed at leading towards a successful formulation of a UDL-based teaching strategy for MWPs in a multilingual mathematics classroom and ultimately enable the learners to understand this mathematical genre. Once empowered, these people (the co-researchers or participants) should be able to continue carrying out the project and sustain it on their own, even after the study has been completed and the researcher is no longer part of the project.

In line with the aim of this study, and guided by the theoretical framework underpinning this study, participatory action research (PAR) was chosen to generate the empirical data. The selection of PAR for this study was informed by its agenda, namely to critique unproductive practices, challenge the status quo of the participants, promote human empowerment and emancipation in transforming the prevailing situation (i.e. an issue of concern that needs to be addressed as collectively agreed upon) and bring about the desired change (Hlalele, 2014:104). PAR advocates the inclusion and comprehensive engagement of all stakeholders concerned, including the marginalised (the learners), which usually is not the case with other research methodologies. In line with this, it is important to note that PAR, unlike positivism and phenomenology, requires the inclusion and engagement of all the stakeholders, not only during the data-gathering stage but in all stages of the research project.

This chapter starts by defining qualitative research and PAR. An outline of the PAR principles is also provided in this chapter as well as a discussion on the relevance of PAR in this study. The categories of the participants involved in this study are provided as well as the instrumentation and the data generation procedure. Issues of trustworthiness, how the intervention was carried out, as well as the ethical considerations are all considered.

It should be noted that even though there are quite a number of benefits in using PAR as a methodology to generate data, there are challenges that usually arise when it is applied, which should be addressed. These challenges include, among others, a lack of participation, a lack of trust, a lack of self-confidence, power hierarchies and status issues, to mention only a few. This is especially true of participants who compare themselves to other participants in the group, and then feel insignificant. In line with this, I deemed it fitting to highlight some of the strategies applied to address these challenges. The chapter thus provides a justification for the need to create motivational spaces, to maximise the participants' level of participation, to develop a sense of trust and to inspire confidence.

4.2 DEFINING QUALITATIVE RESEARCH

According to MacDonald (2012:34), qualitative research incorporates the methods of observing, documenting, analysing and interpreting the characteristics, patterns, attributes and meanings of the human phenomena under study. Curry, Nembhard and Bradley (2009:1442) view qualitative methods as most suitable for use in studies aimed at understanding complex social processes, capturing essential aspects of a phenomenon from the perspective of the participants in the study and uncovering beliefs, values and motivations that underlie individual behaviour. Therefore, the purpose of a qualitative methodology is to understand and describe in detail rather than to predict and control, as in a quantitative study (Gratton & Jones, 2010:25). Qualitative methods focus on the totality of human experiences and the meanings endorsed by individuals living the experience, which result in a broader understanding of and a deeper insight into complex human behaviour (Neuman, 2011:101). Based on this, researchers who engage in qualitative research seek deeper meanings of what is regarded as the "truth", while aiming to study things in their natural setting (Hlomuka, 2014:46). This is done in an attempt to make sense of or interpret phenomena in terms of the meanings that the people concerned attach thereto. Likewise, Rubin and Babbie (2010:37) maintain that qualitative researchers aim to discover the world through someone else's eyes in a discovery and exploratory process that is experienced intensely.

4.3 DEFINING PARTICIPATORY ACTION RESEARCH

PAR is a form of qualitative inquiry that is predominantly considered as democratic, equitable, liberating and life-enhancing (Huffman, 2013:6). Huffman (2013:6) further notes that a good PAR study not only advances knowledge but also requires people to work towards a positive transformation of social and material conditions. Tsotetsi (2013:142) regards PAR as a democratic approach as it requires not only the inclusion of all persons concerned but also their voices to be heard (Dold et al., 2012:512). Democracy through PAR is achieved by enabling various voices to shape the research process, hence allowing the research to be more responsive to a diverse set of interests. On the other hand, equity through the PAR approach is achieved by enabling polyvocal facilitation.

According to Johnson and Martínez-Guzmán (2013:406), PAR aims to articulate knowledge production and transformative action and assumes interdependence between action and research (knowledge). Ideally, its purpose is to impart social change with a specific action (or actions). Within the framework of PAR, the participation of individuals or groups involved is deemed vital. It is also aligned with the emancipation and empowerment goals of PAR, whereby research is not only for producing knowledge but also for promoting a research praxis that contributes to issues of social justice as well as strengthening the capacity of individuals to play the role of an actor in their own lives (Miller & Rose, 2008:106). The philosophical foundations of PAR are congruent with a "postmodern tradition that embraces a dialectic of shifting understandings" in which "objectivity is impossible" and "manifold or shared realities exist" (Moleko, 2014:98).

> [PAR epitomises] the concept that people have a right to determine their own development and recognise the need for local people to participate meaningfully in the process of analysing their own solutions, over which they have (or share, as some would argue) power and control, in order to lead to sustainable development. (MacDonald, 2012:36)

According to Mertens (2010:30), PAR requires and enables the participants to take ownership of the research by being involved in determining how they could implement the activities best. PAR, through frequent dialogues and deliberations, has the potential to form public spaces where the participants and the researcher can reform their knowledge on how to address issues that influence daily life. According to Kidd, Kenny and McKinstry (2015:181), PAR supports the inclusion of lived experience perspectives. It further aims to establish recovery by ensuring that people not only gain but also retain hope, as well as ensuring a mutual understanding of abilities and disabilities, engagement in an active life, personal autonomy, social identity, meaning and purpose in life. Therefore, I, as the researcher of the current study, view PAR as an approach that aims to move towards social change through a participatory framework that cogitates the contexts of people's lives.

4.4 PARTICIPATORY ACTION RESEARCH PRINCIPLES

MacDonald (2012:39) states that, even though the development and definitions of PAR are different according to the different authors, the common principles and characteristics of PAR resonate. According to Tsotetsi (2013:182), PAR is democratic in nature, thus facilitating the participation of all the people involved in the research project. McDonald (2012:39) views PAR as equitable since it embraces equity among all the people. PAR is also liberating in the sense that it provides freedom from domineering and debilitating conditions. Furthermore, Nelson and Ochocka (1998:885) claim that PAR is life-enhancing, thus enabling the expression of full human potential. McDonald (2012:39) avows that PAR advances social practice through alteration, agreement on participation, alliance, establishing self-critical communities and involving people in theorising about their practices. In acknowledgement of these principles, Eruera (2010:1) concludes that PAR is a powerful research methodology that can be used to address intricate social issues.

PAR also encourages the establishment of spaces for critically discussing issues without fear. By doing so, all participants, including the marginalised and oppressed, are empowered to express their opinions and to be listened to (Dentith, Measor & O'Malley, 2012). According to Tsotetsi (2013:142), the strength of PAR depends on collective efforts, participation, indigenous knowledge, education and collective action. Furthermore, PAR has as one of its strengths an element of engagement with the communities in a cooperative association from the commencement on matters which the communities are determined and committed to address (Sanginga, Kamugisha & Martin, 2010:697). Communities will now be able to determine any "serious" issue and thus take responsibility to address it. In the context of the current research study, this means that learners and parents who are usually excluded from conversations and decision making, will also be included and their voices captured in an endeavour to address the problem.

4.4.1 Epistemological stance of participatory action research

Epistemologically, PAR requires a shift from the usual conventional or traditional research, such as positivism, to promote recognition of all the participants in an attempt to address complex human and social issues (Eruera, 2010:1). This means that within this mode of enquiry, the researcher and the participants assume positions of being co-researchers and working together to address the issue of concern (Mahlomaholo & Netshandama, 2010:75). The strength of PAR as a research approach lies largely in the inclusion and empowerment of the minority and often marginalised groups (Tsotetsi, 2013:141). According to Baum, MacDougall and Smith (2006:854), PAR differs from conventional research in three ways. Firstly, it focuses on research of which the purpose is to enable action. Secondly, PAR pays careful attention to power relationships, advocating for power to be deliberately shared between the researcher and the researched in such a way that they both operate on the same level. Thirdly, it contrasts with less dynamic approaches that remove data and information from their contexts. It is more apt to be used in transformational studies in which the agenda of human empowerment and inclusion is key, as in this study.

4.5 THE RELEVANCE OF PARTICIPATORY ACTION RESEARCH TO THIS STUDY

In the current study, I acknowledge that PAR is not the only approach or methodology that can be applied to generate data, but other methodologies also exist, for example, qualitative, quantitative and pragmatic research. The use of PAR in the current study, however, was informed by its agenda to bring about change through collaborative efforts between the researcher and the participants. It should be noted that the researcher, within the PAR framework, is also a participant whose status is equivalent to that of the other participants ("the researched"). In this framework, the participants' status is elevated in the sense that, firstly, their participation is required in all stages of the research process and, secondly, they are called "co-researchers" (not "the researched") since their engagement and contributions are equal to those of the researcher. This is one feature of PAR that distinguishes it from other traditional research approaches such as positivism and phenomenology. Hlalele (2014:104), therefore, regards PAR as a powerful approach and encourages its adoption in transformative research studies as it assists in addressing the drawbacks that have often been "suffered" by conventional research by not adequately integrating and taking cognisance of the subjects in the research process.

PAR was also adopted in the current research study for its agenda of instilling a sense of collective research ownership (Hlalele, 2014:103) and for the inclusion of all people, with no exception to those who are usually marginalised and excluded from the decision-making processes (the learners) (Moleko, 2014:57). The adoption of PAR was also considered for the production of knowledge through social participation (including the marginalised) to achieve the purpose of this study, which is to utilise aspects of a UDL strategy to develop an effective teaching strategy for MWPs in a multilingual mathematics classroom. I believe that a collective effort is essential in formulating a balanced, productive and functional strategy. Furthermore, it must be noted that within the PAR framework, different people with different ranks, levels of knowledge, commitment and confidence work together; they undertake different tasks and play different roles in an endeavour to realise the ultimate goal. In this study, such a characterisation was deemed valuable since the belief is that people have different lived experiences that should not be overlooked in generating knowledge and bringing about change in the arena of teaching and learning. PAR was, therefore, adopted in this study for its recognition of human value (McDonald, 2012:34).

According to Azaiza, Hertz-Lazarowitz and Zelniker (2010:271), PAR aims to create a space for power sharing among the participants. In this sense, PAR provides opportunities for the participants to equally share their knowledge, experiences and expertise in a climate where all contributions are treasured, regardless of the contributor. Moleko (2014:57) notes that PAR, as a methodology, embraces and instills basic human values that contribute towards the building of more cohesive humanities. Furthermore, PAR encourages the practitioners not to "take the glory" for themselves, but to acknowledge that other people are also capable of "doing things" and that their efforts can also contribute significantly to bringing about the desired change.

PAR was further adopted in this study because of its emancipatory nature as it seeks to emancipate the participants to take charge of their own lives and situations. PAR instills confidence in these people so that they can realise the potential, skills and talents they possess to transform their situations. Furthermore, PAR also makes the researcher responsible to "unleash" the participants' full human potential to assist them in realising their roles and capabilities. In line with this, Campanella (2009:2) advises that human beings must be regarded as "capable speaking beings", and not as mere objects that cannot think or do anything for themselves. In support of Campanella's sentiment, Moleko (2014:57) asserts that people can only be emancipated if they are engaged in discussions where they can express their views freely, within a platform that does not limit their social development or determination.

PAR and CER both advocate liberation, emancipation and empowerment of the people, which can only be realised in spaces where an agenda of peace, hope, freedom, social justice and equity in all its forms is espoused (Nkoane, 2013:2). PAR and CER are, therefore, complementary. This means that the existence of one calls for the existence of the other. In other words, "CER is theory" and "PAR is CER in action", meaning that PAR operationalises CER (Tsotetsi, 2013:ix). PAR was also adopted in some previous studies almost similar to this one. It was adopted mainly for its empowering and transformative nature. Furthermore, it was adopted for complementing CER, which was also the lens underpinning those studies (Hlomuka, 2014; Hlalele, 2014; Shangase, 2013; Tladi, 2013).

4.6 CREATING MOTIVATIONAL SPACE TO INSPIRE AND MAXIMISE PARTICIPATION

I hold the view that the creation of a space conducive to an agenda of peace, hope, freedom, social justice and equity is not the sole responsibility of the researcher, but equally so, that of the participants ("the researched"). This space allows participants to contribute without fear of judgement, knowing that their contributions are appreciated and that they are going to contribute significantly to the successful formulation of the envisaged strategy. The creation of the motivational space was used in this study to maximise the participation and confidence of the participants. The principles constituting the framework for creating motivational spaces as advocated by Wlodkowski (2003; 2008) were used to achieve this. Table 4.1 highlights the actions to be taken to create motivational space that makes it easy for participants to freely engage in discussions.

Table 4.1: Framework for creating motivational spaces (Wlodkowski, 2003; 2008)

Action	Description
Establishing inclusion	Creating a learning atmosphere where participants are connected to one another and also respect one another
Developing attitude	Creating a favourable disposition towards the learning experience through personal relevance
Enhancing meaning	Creating challenging, thoughtful learning experiences that include the participants' perspectives and values

Action	Description
Engendering competence	Creating an understanding that participants are effective in learning something they value

4.7 THE PARTICIPANTS IN THE STUDY

The participants in this research study comprised mathematics teachers, mathematics literacy teachers, English teachers, the mathematics head of department (HoD), learners in Grades 10, 11 and 12 and the principal. Table 4.2 indicates the participant group, the number of participants involved, the teachers' background in mathematics teaching and the participants' experience and involvement in mathematics.

Table 4.2: Participants involved in the PAR project

Participant group	Number (n) of participants	Background in mathematics teaching	Experience and involvement in mathematics
English and mathematics teachers	4	They teach both English and mathematics and are aware of the challenges the learners experience in both the subjects.	Six years experience teaching mathematics and English.
Learners	10	They are still attending school and face the challenges of teaching and learning on a daily	Grades 10, 11 and 12 mathematics learners (three learners in Grade 10, four learners in Grade 11 and three learners in Grade 12).

Participant group	Number (n) of participants	Background in mathematics teaching	Experience and involvement in mathematics
		basis. They are in a better position to explain their experiences in terms of the teaching and learning of MWPs.	These learners were drawn into the research since they are at a level at which they are able to explain clearly how they are taught. I chose learners in the three grades (10, 11 and 12) since I believed that they would assist in answering the research questions.
Mathematics teachers	5	They meet and teach learners daily and are familiar with the challenges that learners in multilingual mathematics classrooms encounter in the process of teaching and learning MWPs. They also have ideas on how learners should go	They have extensive teaching experience. (10-20 years in the field). They also have experience of assessing learners' activities.

Participant group	Number (n) of participants	Background in mathematics teaching	Experience and involvement in mathematics
		about solving the different types of MWPs.	
Principal	1	Teaches learners mathematics daily and is familiar with the challenges that learners in multilingual mathematics classrooms encounter in the process of teaching and learning MWPs.	He has extensive teaching experience (10-20 years in the field).
Head of department (mathematics)	1	Teaches learners mathematics daily and is familiar with the challenges that learners in multilingual mathematics classrooms encounter in the process of teaching and learning MWPs.	He has extensive teaching experience. (10-20 years in the field).

Participant group	Number (n) of participants	Background in mathematics teaching	Experience and involvement in mathematics
Mathematics literacy teachers	3	They meet and teach learners daily and are familiar with the challenges that learners in multilingual mathematics classrooms encounter in the process of teaching and learning MWPs in mathematics literacy. They also have ideas on how learners should go about solving the different types of MWPs.	

4.7.1 Primary reasons for the selection of the participants and the school

The participants and the school in this study were selected mainly for the following reasons:

- The participants were selected based on their interest in the issue of concern and their willingness to participate in the study and bring about change.

- The school is cosmopolitan and the language structure would thus better illustrate the challenges of teaching MWPs in multilingual mathematics classrooms.
- The school has, in terms of teaching experience, “new” and “old” mathematics teachers.
- Lastly, the school was selected on the basis that it was easily accessible and not far from where I stay.

It should be noted that within the transformative paradigm mode, the principle of inclusivity is espoused at all times and, therefore, all people who endeavour to participate are welcomed. Eruera (2010:7) advises that the people who take part in a PAR project should be those who will benefit from the outcomes of the research project directly. In line with this notion, the selection of the participants in this study was based on the following: interest in the issue of concern (identified problem), the contribution the person could make to the research project and the kind of influence the person has in the research project.

4.8 INSTRUMENTATION

In the following sections, a brief description of some of the tools that were used to generate data in this study is provided. An account for the use of each tool is also provided.

4.8.1 Lesson observation or demonstration

Chitera (2009:79) deems lesson observation significant because it gives the observer an opportunity to see and receive immediate information on the focus of an “inquiry”. Based on this immediate information, the resolutions the observer makes about the foci of attention during lesson observation affect and influence the extensiveness and analytical sufficiency of cumulative observations across a set of trials. In this study, some of the lessons that were presented in the classrooms were discussed and *demonstrated* by the teachers and the learners (see Chapter 5). The conversations enabled the researcher and the participants involved to make claims about the manner in which the MWPs were taught in “violation” of the aspects of the UDL framework to identify the aspects of the UDL broader framework that could assist in developing an effective UDL-based teaching strategy for MWPs in a multilingual mathematics classroom.

4.8.2 Teacher-teacher observations

> Self-reflection and self-critique are vital for educators. We need to understand that we do not have all the answers because we are ever-evolving beings, working on understanding ourselves and the people around us. (Anonymous)

The above statement portrays the need for educators to reflect on their practices and to critique unproductive practices. In alignment with the quote above, the teachers were also provided with the opportunities to reflect upon their own practices as well as what they have observed during the “peer observations” they engaged in to indicate productive practices as well as the lessons learned thereof.

4.8.3 Individual meetings

The research approach (PAR) I opted for does not encourage the practice of administering the questionnaires with close-ended questions, which prescribe the specific questions that the participants should respond to. Even though we had to gather together in the meetings to have discussions around the teaching of MWPs in a multilingual classroom, there were times when some participants could not be available at the specified times. I thus arranged other time slots that were convenient for these participants to also make their contributions. These conversations were also tape recorded.

4.8.4 Reflective session

We had a reflective session where we reflected upon how we started with the research project. The goal of the research project was highlighted during this session. We discussed briefly what had emerged as the challenges pertaining to the teaching of MWPs. We recapped what the solutions were, as well as what we considered to be the conditions conducive to the implementation of the strategies and the threats thereto to be anticipated and circumvented. This reflective session served as one way of ensuring that the words of the participants and the interpretation thereof were captured accurately. This was deemed significant for ensuring credibility, which Whittemore, Chase and Mandle (2001:530) refer to as a conscious effort to establish confidence in an accurate interpretation of the meaning of the data. It was also done to ensure that the results of the research reflect the experience of the participants or the context in an authentic way.

4.8.5 Document analysis

I conducted document analysis (learner homework and class work) in this study. Some of the examples that were reflected upon during the meetings and the forums were taken from the activities that were given to the learners in class. The homework and the class work were also used to make sense of the problems the learners were faced with in terms of this mathematical genre. Some of the “worked out” problems in the books served as examples (evidence) that enabled the participants to understand the challenges clearly, although some challenges were mentioned verbally both by the teachers and the learners.

4.9 DATA COLLECTION PROCEDURE

Lategan (2005:25) contends that creative research must involve new procedures and inventions, and should take a much less structured approach than conventional research; as such, it cannot always be pre-planned. In line with Lategan’s argument, I, therefore, adopted the free attitude interview (FAI) technique to be creative in our research. FAI is a technique that is mainly used in qualitative studies in which the transformative lenses are used as in the context of this study (in which CER is used as a transformative lens). FAI requires only one comprehensive question to be asked in which the researcher and the participants have to explore their own minds (Tlali, 2013:28; Tsotetsi, 2013:161). Tlali (2013:28) deems FAI suitable to be used in transformative studies since it provides opportunities for the participants to seek clarity and to explore and probe the questions. In this way, FAI fosters a deeper understanding, provides manifold views to explore the phenomenon and thus heighten the quality of the data.

The choice of the FAI technique in this study was informed by its “non-directive” nature, which opens the space for the participants to intervene or interpose. In this way, the researcher and the participants are able to evaluate and negotiate the issues of “reliability and validity”, which are accentuated in positivist and phenomenological paradigms (Meulenberg-Buskens, 2011:2). Mahlomaholo and Netshandama (2010:11) deem FAI significant to be applied in transformative studies since it allows for the participants to collectively produce knowledge and to be humane without isolating and undermining the integrity of the other participants involved in the study. Deducing from this, it is clear that FAI upholds the social justice principles of human dignity and respect for human rights and their freedom. FAI was further chosen in this study because it advocates similar principles as the ones advocated by CER, which is the lens underpinning this study. When the principles of FAI are applied, Mahlomaholo (2009:228) advises that the discourses should be followed up by a reflective summary, which then encourages them to think about their discussions and arguments carefully. A reflective summary also assists in ensuring that all the participants derive a common understanding of the matter and that all the important issues are captured and understood in context.

In this study, the application of FAI provided the space for the participants to engage freely and to be considerate of their own opinions and situations and those of others. In line with this, Kemmis (2008:127-128) and Swantz (2008:33-34), in their discussions of critical PAR and PAR respectively, indicate the coherence that exists between PAR and FAI in ensuring that the participants’ own views of the nature of their problems are fundamental to research and that the need requires to be holistically and deeply understood.

In line with the FAI requirement of having one main comprehensive, open-ended research question, the research question for this study was phrased as follows: “How can we utilise the aspects of UDL to develop an effective teaching strategy for MWPs in a multilingual mathematics classroom?” According to Tlali (2013:140), a comprehensive question, such as this one, is usually complex and too broad to be responded to in one session or meeting. This is because the responses given by the participants are usually manifold and complex and further lead to a myriad of clarity-seeking questions and follow-up questions before the obtainment of the summarised version. In this study, I used research objectives instead of sub-research questions and to illustrate the application of FAI in this study, for each one of the five objectives an open-ended question was asked within the meaning of the comprehensive question. Therefore, these questions were not of a "high level" and as comprehensive as the main question of the study. According to Steinberg and Kincheloe (2010:145), these questions need to be couched from views that were expressed by the participants. Tlali (2013:141) regards these questions as follow-up and clarity-seeking questions, which are usually intended to converge the discussions and viewpoints towards the aim of the study.

4.10 ENSURING TRUSTWORTHINESS IN THE DATA GENERATION PROCESS

According to Morse, Barrett, Mayan, Olson and Spiers (2002:2), there is a need in research for assurance that interpretations are trustworthy and reveal some "truth" external to the researcher and the participants' experiences. These scholars avow that "without rigour, research is worthless, becomes a fiction, and lose its utility". In line with this, some efforts were made to ensure that the interpretations of the data were correctly captured. Whittemore et al. (2001:522) state that authenticity is closely connected to trustworthiness, which is an important aspect in research for assisting the researchers to attempt to remain true to the phenomenon despite multivocality of an interpretive perspective. Denzin and Lincoln (2005:209) caution that the involvement of the researcher can influence the ability to speak authentically for the experience of the other participants and, therefore, it is important to "let research participants speak for themselves".

According to Whittemore et al. (2001:522), what makes qualitative research difficult to deal with the issues of "validity and trustworthiness", is the fact that qualitative research requires the researchers to incorporate consistency, subjectivity as well as creativity. For this reason, the contemporary dialogue that centres on the difficulty of establishing "validity" criteria in qualitative research continues. Drawing from this, it is reasonable to indicate that it is even more challenging to deal with issues of trustworthiness in a qualitative study in which PAR is used as an approach since the researcher is also a participant at the same time. Even though this is the case, it is still important to indicate how this aspect is handled in research to produce a research study that reflects the meanings and experiences that are lived and perceived by the participants

In this study, data were generated through the use of tape recorders. Some of the word problem questions were extracted from the previous question papers and the teachers' lesson plans. These questions and worked-out problems were captured by a camera to serve as evidence. Some of these examples are illustrated in Chapter 5. All meetings were tape recorded in the languages the participants were comfortable to use, namely Sesotho and English. The Sesotho verbatim transcriptions were translated into English and were included as part of the discussions in Chapter 5. The translated transcripts were discussed with the participants as a part of member checking to verify and ensure the correct interpretation thereof in trying to reveal some "truth" external to the participants' experiences. Member checking was also done to ensure that the involvement of the researcher did not influence the ability of the other participants to speak authentically about their experiences. This practice was deemed important to pursue in line with the principle of fairness of CER to ensure that the researcher did not "twist" or misinterpret the words of the participants. In this way, the participants were respected in the sense that they were seen as "capable speaking beings" (Campanella, 2009:2), who could also interpret what was discussed just like the researcher. This practice placed the participants in a similar, advantageous position as that of the researcher (Mahlomaholo & Netshandama, 2010:77).

4.11 UNFOLDING OF THE INTERVENTION

The following sections describe the unfolding of the intervention, starting with the description of the conditions before the commencement of the intervention.

4.11.1 Conditions before the commencement of the intervention

The school where the research was conducted was one of the schools that were categorised as "poor-performing schools", and its pass rates in mathematics were not good. According to Siqueira and Gurge-Giannetti (2011:79), poor school performance can be defined as a school achievement below the expected for a given age, cognitive skills and schooling. Before the data were collected from the school, the principal indicated that the learners did not generally perform well and that the school was categorised as a “non-performing” school. For the past three consecutive years, the school did not perform well, especially in mathematics and science. They obtained less than 50% in mathematics and science, which was worrying for them. The issue of the language of learning and teaching (LoLT), which the learners seemed to be struggling with, was also highlighted by the principal and the HoD for mathematics as one of the contributing factors towards this poor performance. It was also highlighted that learners struggled a lot with the word problems and this was mainly attributed to a lack of English proficiency. On the basis of these brief discussions, I saw indeed there was a need to conduct a study in that school. I believed the aspects of UDL to be utilised to develop an effective teaching strategy, although it was only meant for the word problem genre, could influence how teaching would be carried out and ultimately be adopted in the other subjects as well to improve the overall performance of the learners therein.

4.11.2 Formulation of a dedicated team

According to Eruera (2010:3), the community can identify a problem and approach the researcher for his or her expertise to assist them in addressing it. Likewise, Tsotetsi (2013:154) notes that when the researcher has identified an area of concern or challenge, he or she can approach the community to offer an idea or a solution and so gain the involvement of the community. In this study, I identified the problem regarding the teaching of word problems in a multilingual mathematics classroom. I approached the HoD and the principal regarding this issue and the idea to research the identified issue was welcomed. However, for ensuring that the process went smoothly, a dedicated team had to be established to ensure that the activities would be planned and carried out as planned. The principal, the mathematics HoD, one learner, one mathematics teacher and I were entrusted with coordinating the project. The composition of this team structure was in line with the PAR and CER principle of inclusivity to ensure that all the categories of the people involved in the project were represented. The principal notified the participants about the meetings and forums and also communicated the times at which the meetings would take place as well as the venues for the meetings. Although the initial plan was to involve as many people as possible in the project, due to time constraints and other activities the school was subjected to, I ended up working only with the participants from the school and excluded other possible partners (such as the mathematics tutors, lecturers and non-governmental organisation I had also wished to interact with during the research project). The fact that the principal and the HoD were excited about the project and thus took it upon themselves to drive it, made it easy for the meetings to take place on time and for everybody who took part to buy into the idea. During the interactions, I also realised that the teachers respected and had much trust in their principal, which made it easy for them to buy into the idea of a research project of this nature as their principal was excited about it and led it. The participants were told to feel free to raise issues without any fear since we were all operating at the same "power levels", even though our statuses were not the same. The words uttered by me during the first meeting, *"everybody is entitled to their opinions and therefore no one should feel that their opinion is of less value. You are free to make inputs and your inputs will always be valued*", made everybody feel at ease.

The first meeting took long since we had to discuss how we were going to carry out the activities. The participants' roles were also clarified in this meeting. This was in line with the PAR principle of transparency, which encourages that the people who are involved in the research project should know what the project is all about so that they can develop ownership in it. The participants asked questions regarding aspects they did not understand. At the end of the session, it was agreed that the HoD would send the programme to all the people involved to notify them about the meetings that would be taking place and how the meetings would be conducted. The PAR principles were communicated to all the participants to align them with these principles throughout the research project, especially during the meetings. These principles included, among others, active participation, respect for other people's views, working cooperatively, giving equal chances for individuals to participate and make contributions without being judged and refraining from making others feel inferior about the contributions they make. The participants were encouraged to uphold these principles at all times. At the beginning of each meeting, the person chairing the meeting started with these principles to set the tone for the discussions and to encourage the participation of all the stakeholders involved.

Although everybody seemed to have an understanding of what the research project was all about, the participants deemed it significant and agreed on having a short session where they could learn more about the UDL strategy since most of them were hearing about it for the first time. Another proposal was made for the participants to have a session during which they could get to know one another. This was suggested to instil a sense of trust, unity and closeness.

4.11.3 Information session

An information session was conducted for the participants to learn more about the UDL framework. The purpose of the information session was to educate the participants on the principles of UDL and their application thereof. The participants deemed the session significant in order for them to know what UDL entails. It was important for the session to be conducted before we started with the meetings so that the participants could make significant contributions by drawing from the UDL principles explained in the subsequent discussions. I conducted the session, and the participants were given an opportunity to ask questions when they did not understand something or they needed clarity. The outline of what was covered during the information session is attached to this document (see Appendix A10).

4.11.4 Team-building exercise

In an endeavour to achieve the purpose of this study, the participants and I became intentional in the activities in which we engaged. We believed that to generate a huge quantity of data that would assist in responding to the comprehensive research question for this study, the participation of all the people involved was essential. Therefore, in trying to improve and increase participation, a team-building exercise was deemed significant before the meetings could commence. The participants were aware of the fact that people are different in many ways (e.g. culture, background, personalities, age, statuses, experiences, etc.) and that getting them to work together as a team would not be an easy task. It was suggested that everybody should engage in a team-building exercise that was aimed at drawing the participants together and making them feel at ease as well as assist them in bettering their relations. Furthermore, the team building was meant to inculcate a spirit of trust, openness and caring for one another. Over and above all, it was meant to "fuel" and help sustain the participants' motivation to continue their involvement in the study.

One of the activities that were part of the team-building exercise was the turning point moment, which all participants engaged in. In this activity, the participants were paired and given 15 minutes each to share with their partner about a "turning point in their lives". This activity was meant to open a platform to enable the participants to learn about one another and to develop a sense of understanding. The participants shared the "awful" experiences that changed their lives. From that moment on, they started realising that everybody once had an issue that needed to be addressed at some or other stage in their lives and that even though their adversities were not similar, everybody faced hardships that make us all human. At the end of the session, I saw people giving each other warm, intense and comforting hugs, realising that elements of trust and caring for one another were slowly emerging. In affirming my observation, one participant commented as follows:

> *Shoo! I did not realise how much we all are faced with the different struggles and yet we continue with our lives as if nothing is happening. I have really learned a lot from this exercise. At least now I have a better understanding of what some of us are going through. Shoo!*

4.11.5 Meetings with the participants

As indicated in Section 4.9, the research question for this study is too comprehensive, complex and broad to be responded to in one session or meeting. Therefore, in an attempt to respond to this question, we considered Tlali's (2013:140) advice that we should have several meetings to respond to such a question. On the basis of this, we agreed to have a series of meetings in which we discussed issues in line with the objectives of this study. In the second meeting, we discussed the challenges and solutions to the identified challenges. In the third meeting, we discussed the conditions conducive to the strategy to work, the threats that might impede the implementation of the strategy and the indicators of the successful implementation of the strategy. In the fourth meeting, we had a reflection session with all the participants. In that session, we also applied the FAI principles, as we did in the other meetings. In order to ensure that the questions were not as complex as the main research question, we then asked simple questions, which were not as comprehensive as the main question of the study, in line with each research objective. These questions were couched from the views that were expressed by the participants' responses and perspectives (Steinberg et al., 2010:145). The reflection session provided a space for the participants to go over the remaining issues and was also used as a platform to confirm shared views and discrepancies. This form of practice, according to Tlali (2013:141), is important since it is in keeping with the versatility of the CER formats couching this study.

During the discussions, there were times when there was a need to ask follow-up and clarity-seeking questions based on the participants' responses. The application of the FAI principles facilitated a smooth process of reconciling the discourses with the main question and thus dissipated misconceptions and mystifications inherent in the responses (Biesta, 2010:43). These types of questions were also necessary to address contradictions and to strike a balance of the innate power relation struggles that might emerge because of diverse differences (Liasidou, 2008:486-489).

4.12 DATA ANALYSIS AND INTERPRETATION

The following sections briefly explain the data analysis and interpretation in the context of this study. The rationale for using critical discourse analysis (CDA) as an analytical tool is also explained.

4.12.1 Critical discourse analysis

In the current study, CDA was used as an analytic tool to analyse and interpret the empirical data. According to Fairclough (2013:183), CDA conveys the critical custom in social analysis and contributes to critical social analysis with a specific focus on discourse and the relationships between discourse and other social elements (power, ideologies, institutions, social identities, etc.). Fairclough (2013:183) further indicates that CDA is both normative and explanatory critique in nature. Furthermore, CDA is normative critique in the sense that it does not merely define existing realities but also assesses them according to the extent to which they match up to values that are deemed contentiously vital for just or decent societies (e.g. certain basics for human well-being). It is explanatory critique in the sense that it does not simply describe and evaluate existing realities but also seeks to explain them, for instance, by showing them to be effects of structures, mechanisms or forces suggested by analysts, the reality of which they seek to assess. In other words, inequalities in wealth, income and access to various social goods might be explained as an effect of mechanisms and forces associated with capitalism or particular varieties of capitalism. Bloor and Bloor (2007:2) avow that CDA is a vital analysis tool that encompasses the analysis of text and talk in all disciplines of humankind and the social sciences. It is a type of discourse analytical tool that is predominantly used to expose social power abuse, dominance and inequality that are enacted, reproduced and resisted by text and talk in the social and political milieu (Van Dijk, 2008:85). The main goal of CDA is to expose and to resist social inequality (Widdowson, 2004:89). The use of CDA is deemed significant in studies that require text and talks to be deeply understood "in context". To successfully understand the meanings in these talks, Van Dijk (1993:250) recommends the use of CDA since it allows data analysts to look at the structures involved, and the strategies and other properties of text, talk and communication that have a tendency to yield existing variations of power relationships.

In an effort to conceptualise the central use of CDA in the analysis of text and talk, Tsotetsi (2013:162) states that CDA is different from other territories or approaches in discourse analysis because of its focus on the eradication of dominance and inequality. CDA, unlike other domains in discourse analysis, is predominantly engrossed in and enthused by pressing social issues and enhancing understanding of the discourses relating to social issues. Based on this, the role of critical discourse analysts is to take a clear socio-political position, thus spelling out their point of view, philosophies and purposes, both within their discipline and within society at large. According to Moleko (2014:76), CDA affords analysts an opportunity to look at data from various angles and to understand it and to interpret it in context. This means that CDA not only makes it possible for data to be understood in context but also for the meaning to be provided in context. I, therefore, regarded CDA as a form of appreciative analysis framework in the sense that it appreciates and take cognisance of the different talks made by different people who come from different backgrounds and whose viewpoints are influenced by the diverse experiences and expertise. By providing the autonomy for data to be analysed on different levels, namely the textual analysis level, the social level and discursive practice level, CDA seeks to accommodate as many talks as possible in an endeavour to gain a deeper understanding of the issue of concern. This increases the chances of devising "functional strategies" to address the problem.

The sections below provide brief descriptions of the CDA three levels of analysis which were adopted in this study.

4.12.1.1 Textual analysis

Textual analysis focuses on the participants' spoken words regarding their views and experiences. In the context of this study, the textual analysis focuses on the participants' spoken words as transcribed to indicate their views and experiences regarding the teaching of MWPs in a multilingual mathematics classroom. The analysis also took into consideration the nonverbal cues and visual (facial) expressions of the participants as they expressed their views. Through textual analysis, I was able to clarify the spoken words, visual cues and expressions, which enriched the interpretation of the data.

4.12.1.2 Social analysis

According to Liasidou (2008:488) and Sheyholislami (2009:4), social analysis helps the researchers examine the overall societal structures such as societal behaviour and arrangements. These tend to be expressions of attitudes and values which the communities hold in high regard. In this study, social analysis assisted in exposing the behaviour and attitudes that were informed by the societal structures for the participants to do and say "things" in the manner they did.

4.12.1.3 Discursive practice

The use of the discourse or cognitive analysis of CDA was to mediate between the social or discursive space and eventually represent the people's thoughts of their common practices (Fairclough, 1995:61). Discursive practice is concerned with how text is produced and interpreted by the participants (researcher and co-researchers) to interpret the structure of discourse practice. It reflects how discourse at the local, institutional and societal levels is organised or shaped. The goal is to illustrate how text is interpreted and reproduced or transformed. The analysis at this level focuses more on the process (Fairclough, 1995:61). In this study, through the text, the discursive practices are highlighted to illustrate the people's thoughts and their communal practices.

4.13 RELEVANCE OF CRITICAL DISCOURSE ANALYSIS IN THIS STUDY

In this study, CDA was adopted to accommodate as many voices as possible and to ensure that the data were understood in different contexts. Furthermore, it was used to expose issues of power relations, dominance and inequality, which are the actions that eventually lead to others feeling excluded. Thus, CDA served as a "consciousness tool" that made people cautious about their actions and practices since they can make others feel excluded and, consequently, regard their viewpoints as unimportant. The use of CDA assisted in ensuring that the voices of all the participants, as well as their feelings regarding certain issues, could be captured and interpreted within context. Further to this, it afforded all the participants equal opportunities to participate in a platform in which all the inputs were regarded as important, regardless of whose input it was. In this sense, it is reasonable to indicate that CDA calls for the demystification of the power that might limit others from fully participating in the research project. In other words, it is possible that some participants in a research project may have power and status that intimidate other participants and thus limit their participation. CDA, therefore, requires the participants to be aware of these unequal power relations and dynamics and thus deconstruct in an endeavour to allow everybody to make inputs without being afraid or intimidated. CDA advocates similar principles as those advocated by CER. They both strive to examine the origin of the problem at hand to determine ways in which the problem could be solved (Bloor & Bloor, 2007:12). Furthermore, they both espouse the idea of collaborative working in addressing the problem at hand. Over and above, CDA was employed in this study as an “ideal” data analysis tool for its focus on social challenges (Van Dijk, 2008:86).

4.14 ETHICAL CONSIDERATIONS

McDonald (2012:45-46) delineated a number of ethical principles that researchers must consider when conducting PAR. The researcher must, first of all, ensure that all relevant stakeholders have been consulted and that the principles guiding the work are accepted before commencing with the research project. All participants must be allowed to influence the work, and those who wish to withdraw their participation may also do so at any stage of the research. Furthermore, the development of the work must remain noticeable and open to suggestions from others throughout the research process. The researcher must also ensure that permission is obtained before making observations or examining documents produced for other purposes, as there is shared ownership of the research. Descriptions of others’ work and points of view must be negotiated with all those who participated in PAR before publishing any of the work. Finally, the researcher must accept responsibility for maintaining confidentiality throughout the research process.

More than 15 years ago, O’Brien (1998:2 of 17) mentioned that within the PAR framework, the decisions regarding the direction of the research and probable results are co-operative. It is, therefore, essential that the researchers are informed about the nature of the research process from the beginning, including all personal biases and interests, while ensuring that there is equal access to information produced by the process for all the participants. It is also important, according to McDonald (2012:46), that the researcher and the initial design team create a process that maximises the opportunities for involving all the participants.

In light of the above, I wrote a proposal to the committee at the University of the Free State for my title to be registered. Ethical clearance was applied for and granted (clearance number: **UFS-HSD2016/1194**) (see Appendix A1). I sought permission to conduct research at the school where the research was ultimately conducted. A permission letter was granted (see Appendix A3). In the first official meeting with the principal and the other participants (highlighted in Table 4.2), I was given an opportunity to explain in detail what the research project was all about and its significance. I also explained the roles of the stakeholders in the research project. I asked the participants to ask questions when they did not understand something. The participants seemed to be happy about the research project. Judging from the enthusiastic comments they made and the questions they were asking to ensure that they were going to participate in the project, they fully understood what it was all about. Towards the end of the discussions, the HoD for mathematics commented as follows: “Ke thabile mme hao tlile. Ke tshepa hore sena se tlo re tswela molemo*!”* (**ET:** *I am glad you came, Madam. I hope this will be beneficial to us.*) She also wanted to know if I was going to share the results of the study with them in any way since they had previously been involved in other studies and the results had never been shared with them. I assured them I would share the results of the study with them. I showed the participants the letter the principal had given me, granting me permission to conduct the study. The principal emphasised the conditions as stipulated in the letter that the research would not disturb the school activities. I showed the participants the consent forms and also read their contents. I asked the participants to feel free to ask questions when they did not understand something. I further made it clear that they were at liberty to withdraw at any stage of the research without negative consequences.

The parents were invited to the next meeting with the principal and the mathematics HoD. The principal gave me an opportunity to explain to the parents what the research project was all about. I also indicated the significance of including their children in the research project. The parents seemed to have no objection to the participation of their children in the project. However, I read the contents of the assent letter they had to complete on behalf of their children since they were minors, to serve as proof that they had given permission for their children to participate in the research project. The form was also presented in a Sesotho version (see Appendix A5: English and Sesotho versions) so that all the parents could fully understand what it was about. The parents were asked to take part in the research process, even though most of them indicated that they were working and that they would not be able to attend the sessions. Eventually, all the forms were signed and gathered. The principal kept the consent forms safely and gave me copies of the signed consent forms as well as the forms signed by the parents on behalf of the learners. The participants were assured that the information would be highly confidential and that anonymity was assured. The consent forms were received before the commencement of the research process (Van Niekerk, 2009:119). The participants were informed that the data would be captured accurately without bias (Denzin & Lincoln, 2000:140)

4.15 CONCLUSION

In this chapter, PAR as a methodology was discussed and clear reasons for its adoption and suitability in this study provided. A clear association between CER, which is the theoretical framework underpinning this study, PAR and CDA was highlighted. The chapter also discussed the participants involved in the study and their roles. The instruments used to generate the data were also explained in this chapter. FAI was described, and its principles highlighted to demonstrate how the research question was responded to. A justification for its application in this study was also provided. The chapter explained how the issues of validity, reliability and trustworthiness were addressed in the study. The intervention process was explained, and its role highlighted. The use of CDA as an analytical tool was highlighted, with its three levels of data interpretation, namely textual analysis, social practice and discursive practice. The chapter also provided the relevance of CDA as a tool for analysing the data as well as the ethical process in respect of this study.

The next chapter focuses on the data analysis and interpretation of the findings of this study.

CHAPTER 5

DATA PRESENTATION, ANALYSIS AND INTERPRETATION OF A UNIVERSAL DESIGN FOR LEARNING STRATEGY TO ENHANCE THE TEACHING OF WORD PROBLEMS IN A MULTILINGUAL MATHEMATICS CLASSROOM

5.1 INTRODUCTION

The purpose of this study was to utilise the aspects of universal design for learning (UDL) to develop an effective teaching strategy for mathematics word problems (MWPs) in a multilingual mathematics classroom. The preceding chapters presented the theoretical framework underpinning this study and the methodology applied to generate the empirical data. The data generation instruments and procedures applied in this study were outlined, all in pursuance to respond to the main research question of this study, as highlighted in Chapter 1 (see Section 1.4). The underlying principles of the theoretical framework that was adopted, as well as the elements of critical discourse analysis (CDA), the technique that was followed in the data analysis phase of the research study, were discussed. All of these have aided to establish the context and background for this research, which aimed to develop an effective teaching strategy for MWPs in a multilingual classroom. This chapter thus outlines the process of data analysis employed in this study and presents the findings that led to the formulation of a UDL-based strategy for teaching the MWP genre.

In order to arrange the presentation of the data analysis in this chapter, the five objectives informing this study, as outlined in Chapter 1 (see Section 1.5), serve as the organising principles. The constructs will be formulated in line with those in Chapter 3 (literature review) to make sense of the generated data and to draw links. The literature reviewed is thus used to frame the data presentation and analysis through CDA as an analytic technique. Since the study intended to formulate a UDL-based strategy to develop an effective teaching strategy for MWPs in a multilingual mathematics classroom, the broader UDL framework principles will also be reflected upon to indicate how they feature and were operationalised in the implementation of the envisaged strategy.

The extracts captured in this chapter are direct quotations of the utterances made by the participants and they are not edited. The English translations are indicated by the abbreviation “ET”.

5.2 CHALLENGES JUSTIFYING THE FORMULATION OF THE STRATEGY

In this section, the challenges that justify the need for a UDL-based strategy to enhance the teaching of MWPs in a multilingual mathematics classroom, as identified by the participants, are examined. As already mentioned, the five objectives of this study, as outlined in Chapter 1 (see Section 1.5), are used as the organising principles to juxtapose the empirical data with the existing literature (Chapter 3). In addition, the constructs are formulated to make sense of the data and the theoretical framework underpinning this study, and the reviewed literature is used to frame the findings through CDA.

5.2.1 Lack of reading skills

According to Freitag (1997:16) reading is an important skill teachers should nurture in their learners in order for them to master it. Reading with understanding is even more important in enabling learners to reflect on the problem critically and give the correct answers to the questions in appropriate language. In agreement with this statement, Al-Mansour and Al-Shorman (2011:69) state that the ability to read is important because it exposes one to the use of the language and a variety of words, thus improving one’s vocabulary and knowledge. If learners understand what they read (when reading word problems), their chances of correctly solving the problems are improved.

Even though skilful reading seems to be playing a significant role in assisting learners in comprehending and solving word problems, the extracts below, from a meeting where the challenges were discussed, indicate that most of the learners lack reading skills and thus cannot solve word problems successfully. Examples from some of the lessons presented were also cited by the participants in their discussions. The participants commented as follows:

> *Mr Morake: Bana bana ha ba utlwisisi English language. Ha o ba file palo e ngotsweng ka mantswe ha ba kgone ho e tlisa di variableseng. For example,*

ha ore ho bona eeehh… d is three less than the number a, ha ba e expressa bona hangata ba ngola d = 3 – a instead of writing d = a – 3.

English translation (ET): *These learners lack English proficiency. When you have given them a word problem, they cannot translate it into variables. For example, eeehh… when you have given them the problem, d is three less than the number a, they express it as d = 3 – a, instead of writing d = a – 3.*

Ms Ntuli: "Bana bana hangata ha ba utlwisise ntho eo ba e balang…"

ET: *In most instances, learners do not understand what they are reading…*

Bokang: Nna ke nahana hore mathata ke English. Ho hlaha mantswe a thata moo ao ***re*** *sa a utwisising. O thole hore English entse e re hlola ho tloha ka period ya yona.*

ET: *I think English is problematic for most of us. We encounter difficult words which we do not understand in many instances. We struggle with English language even when we are taught it as a subject.*

The statement by Mr Morake, *"Ha o ba file palo e ngotsweng ka mantswe ha ba kgone ho e tlisa di variableseng"* (**ET:** "*When you have given them a word problem, they cannot translate it into variables*"), indicates that he is aware that the learners do not understand English. He attributes the lack of English proficiency to the fact that the learners are unable to convert the words into variables. According to Mr Morake, the ability to translate word problems into variables largely depends on the ability to read, which is a skill that most learners do not have. Drawing from the teacher's words, learners' reading skills are essential in assisting them to translate words into variables, which then makes it easier for them to work out the problems. However, the teachers seem not to be developing the learners' reading skills in their teaching, even though they are aware that most learners struggle with reading, which is made evident by the fact that they (the learners) are unable to translate the words into variables.

According to Israel (2014:7 of 38), UDL encourages multiple means of representation to support the ways in which the learners assign meaning to what they are reading (i.e. what they learn). This means that learners' reading skills need to be developed by the teachers in their teaching in ways through which they may enable learners to understand word problems, assign meaning to what they are reading and recognise what the text entails. This finding resonates with the finding in Section 3.3.1 regarding a study conducted by

Braselton and Decker (1994:276). The behaviour the teacher is portraying from his utterance is common in teaching and learning set-ups. The teachers, in many instances, become aware of certain aspects of learning that serve as learning barriers to the learners. However, since they know the scope of the work that they have to do, they choose to rather stick to that and do nothing about the real challenge, which is an impediment for learners to learn effectively. In this case, the teachers are aware that the learners lack reading skills (which are significant in mastering MWPs). However, just because they are not appointed to teach language and they do not qualify as language experts, they choose not to make an effort to address the linguistic challenges that prevent learners from understanding the word problems. This behaviour is contradictory to CER, which promotes caring as a vital value in society. CER espouses the notion that once the problem has been identified, the solutions must be thoughtfully and promptly devised and actions be implemented to address the identified problem.

Furthermore, Mr Morake's example indicates that one-to-one linear translation when reading is not always correct, as the order of mathematical concepts when it is expressed in words differs from when the concepts are expressed in symbols; hence the example, *"Ha ore ho bona eeehh d is three less than the number a, ha ba e expressa bona hangata ba ngola d = 3 – a instead of writing d = a – 3".* This is also supported in the literature (see Section 3.5.3) indicating that learners who lack reading skills and also lack an understanding of the English language structure, cannot correctly interpret the word problem, which results in their obtaining incorrect answers. Mr Morake's comment indicates the relationship that exists between reading and understanding. Learners' inability to correctly convert words into variables, which makes problem solving easier, is caused by their not understanding what they are reading. In agreement with the teachers' statements, Bokang has also confirmed that reading without understanding is one challenge they (the learners) encounter when solving word problems. According to Bokang, the fact that they do not understand English (lack English proficiency) and frequently come across difficult words they do not know the meaning of, makes it even more challenging to master this genre of mathematics. Vula et al. (2015:34) also support the claim that learners who lack English proficiency find MWPs challenging to solve (see Section 3.3.2). The word *"re"* (**ET:** *"we"*) from Bokang's statement above indicates that *most* learners do not understand English language and, therefore, experience difficulties when they have to solve MWPs. Furthermore, Bokang also states that the reason why they fail to master MWPs is because they are still struggling to master English as a subject at school. This was highlighted as one of the causes for not mastering MWPs, which exacerbates "poor" performance in this mathematical genre as opposed to the literature, which only points out a lack of reading skills as a challenge. This confirms the importance of properly teaching English as a subject in school since it serves as the base from which learners, in the context of this study, have to draw knowledge in terms of learning the language and thus mastering MWPs. However, this is not the case as Bokang indicates that most learners do not perform well in the subject English. What is portrayed in this case is the interrelation that exists between the subjects taught in schools. In Bokang's sense, the mastery of the English language could have a positive influence on their mastering of MWPs, which requires proficiency in English.

According to Long and Dunne (2014:135), it is important for learners to be engaged in the underlying mathematical structures that serve as foundations for them to understand complex concepts. The common knowledge that we have acquired so far, is that to build a house that would withstand even the severest storms, its foundation needs to be strong. A similar lesson is also drawn from Bokang's utterance that the "English subject" should serve as a strong foundation and an "enabling agent" for learners to master MWPs. However, this is not the case, drawing from the statement, "*O thole hore English entse e re hlola ho tloha ka period ya yona"* (**ET:** *"We struggle with English language even when we are taught it as a subject"*). The literature also supports the fact that English language proficiency is a requirement for laying a strong foundation for learners to master MWPs. According to Neville-Barton and Barton (2005:15), learners who are not proficient in English suffer considerable disadvantages that, most of the time, are not recognised, neither by the learners nor by their teachers. These scholars thus expose the myth that mathematics is language-free in nature.

Some of the participants commented on this challenge as follows:

> *Tseko: Bothata ke hore re le bana re botswa ho bala. Ha o bona di statements o se ontse o nyahama tlaaar obe o bile o skipa potso eo because di thata di problems tseo.*

> ***ET:*** *The problem is that we are lazy to read. When you see the statements, you immediately become discouraged and decide to skip those problems since they are difficult to solve.*
>
> *Kamo: Hape ka exameng le ha o di tlotse ontse oka nna wa pasa hobane ho hlaha tse few feela mos!*
>
> ***ET:*** *One can still pass the exam having not written these problems since a few of them appear in the tests and exams.*
>
> *Ms Moeketsi: Hao ka sheba bana ba maths lit ba tlwaetse mofuta ona wa dipalo and baya di bona ho feta bana ba pure maths.*
>
> ***ET:*** *Learners who are doing mathematical literacy usually do not struggle with these problems since they are familiar with them as compared to learners who do "pure mathematics".*

Tseko's statement above indicates that MWPs are not stimulating for the learners since they require more reading on their part (this might probably be caused by the fact that the teachers do not often engage learners in these types of problems). Drawing from Tseko's statement, it is evident that learners have preconceived ideas about MWPs. They were somehow made to believe that these types of problems are difficult to solve, hence the statement, "*o se ontse o nyahama tlaaar obe o bile o skipa potso eo because di thata di problems tseo".* As a result of this belief, learners have a tendency of not attempting to answer these types of problems when they are given during assessments. What seems to be giving the learners a leeway to skip these problems is the fact that only a few marks are usually assigned to this genre of mathematics. The small percentage mark awarded to this mathematical genre does not put pressure on learners to "push harder" in terms of striving to understand these problems. This was made evident by the statement uttered by Kamo when he said, *"Hape ka exameng le ha o di tlotse ontse oka nna wa pasa hobane ho hlaha tse few feela mos!"* One can also draw a sense of a negative attitude that is attributed to the notion held by the learners that MWPs are difficult to solve, which subsequently lead to no effort made to solve these problems in examinations. It is usually the case, even in our societies, that many people give up on "situations" they believe and perceive to be too difficult for them to solve, especially when they know that the consequences of not working towards addressing those situations may not affect them negatively. An "attitude of making no attempt" to solve MWPs, which are perceived to be difficult, is also notable in this instance, particularly because the repercussions are not detrimental.

What also came to the fore in Ms Moeketsi's utterances, is the fact that learners who are doing "pure mathematics" are not as well versed in word problems as those who are doing mathematical literacy, which causes them (pure mathematics learners) to become disinterested in this genre of mathematics. This comparison, to a certain extent, insinuates that the manner in which the "pure mathematics" teachers are teaching is different somehow from how the mathematics literacy teachers teach. According to Simmers (2011:2), one of the reasons for learners not performing well in mathematics is because they do not have an interest in the subject. This requires mathematics teachers to teach in such a manner that they would stimulate learner interest in the subject, thus tapping into the affective domain, which is complex since it is rooted in the emotional life of the learner and reflects the learners' beliefs, attitudes, impressions, desires, feelings, values, preferences and interests (Jarméus, Sundberg, Masog, Andersson, Rosenqvist & Koulouvari, 2012:36).

Also drawing from Ms Moeketsi's words, learners who do mathematics literacy are often exposed to word problems, which somehow may not necessarily be the case with learners who do "pure mathematics". Such a lack of exposure seems to cause learners to not make an effort to master this mathematical genre. Again, drawing from Ms Moeketsi, one gets the idea that the way teachers teach has an impact on how learners learn and eventually perceive the concept that is taught to them. The fact that mathematics literacy learners are exposed mainly to these types of problems, as stated by Ms Moeketsi, is evident in that, to a certain extent, these learners do not have a choice but to make an effort to understand mathematical concepts that are presented in this form. This, however, is not the case with the learners who do "pure mathematics". The act of "not making an effort" seems to be a result of learners being aware that even if they do not attempt to solve these problems, they can still pass since word problems carry a lower percentage in the assessments and therefore, the consequences of not being able to solve these problems are not dire.

Teacher influence is known and evident in schools. For instance, most of the time, if teachers dislike a specific content, their learners will also dislike that particular content, and vice versa. It also often happens that if teachers do not like specific content, they do not emphasise it when they are teaching, or they do not make an effort to teach it in a stimulating manner. They merely cover the specific content because they must provide evidence to the higher authorities (e.g. HoD, subject advisors, etc.) that the topic has been dealt with. In some extreme instances, teachers even choose not to teach the content they dislike at all. This type of practice deprives learners of the opportunity to comprehensively learn mathematics. The extracts above somehow capture this notion that there are instances where the teachers seem to have control in deciding what content to cover. However, this places the learners at the "receiving end"; in other words, the learners are in a position where they cannot control the content they are presented with or the manner in which the content should be presented or taught to them. CER, as the theoretical framework underpinning this study, as well as CDA, in this context, makes it possible for such power imbalances to be exposed. Furthermore, CER and CDA make it possible for the learners' voices to be captured to portray by what their lack of reading skills is caused(giving the deeper meaning of the challenge), which consequently necessitate the teachers to teach them in such a manner that they address this need.

The discussions above indicate that a lack of reading skills is a challenge that hampers learners' mastering of MWPs. A lack of English proficiency, difficult words that learners come across while reading, laziness to read and teaching strategies that do not stimulate reading interest were highlighted as some of the causes for the lack of reading skills, which keep learners from learning how to solve these problems effectively. The fact that in many instances learners do not bear dire consequences for not attempting to solve these problems, causes learners to make no effort to learn these problems. Furthermore, a lack of familiarity with these type of problems makes it difficult for learners' reading skills to advance. This suggests the need for teachers to address this challenge.

5.2.2 Learners' limited mathematical vocabulary and register

The language of mathematics has its own vocabulary and register. Ni Ríordáin et al. (2015:13) avow that the mathematics register includes more than only vocabulary and technical terms but also, for example, styles of meaning, modes of argument and mathematical practices. It also has a number of dimensions of intricacy, such as how mathematics texts are planned or how classroom mathematical discourse positions students. The mathematical vocabulary and mathematical register play a significant role in enhancing learners' understanding of mathematical content. According to Krick-Morales (2006:1), to solve MWPs successfully, learners must be conversant with the mathematics vocabulary and register. Vula et al. (2015:34) note that a particularly challenging part of solving MWPs is the comprehension of the problem itself, particularly the words (mathematical vocabulary and register) that are used in some problems. Therefore, a lack of understanding of the mathematical vocabulary and register can potentially present challenges towards successfully solving word problems, causing misapplication of opposite mathematical operations. In line with this, the participants, during the meeting where the challenges were identified, pointed out learners' limited mathematical vocabulary and register as one of the major reasons why they cannot solve MWPs successfully. The participants commented as follows:

> *Ms Masombuka: …eeeer…mohlala, haba le hae ha ho buuwa ka* ***Volume*** *ba inahanela TV kapa radio moo teng ba tlamehang ho theola kapa ba nyolla volume, empa ha bale sekolong volume ese e bolela ha hong. Mohlala, if hothwe,* ***calculate the volume of the prism*** *eleng hore it is totally different from what they already know from home. Hoka ba thata ho bona to work out this problem ha ba sa tsebe hore volume e bolelang ka classeng.*
>
> ***ET:*** *For example, in the home context, volume may be linked with TV or radio, the act of amplifying or decreasing the loudness, whereas at school, specifically in mathematics, volume means something different from what they already know about volume. For example, if the question says* ***calculate the volume of the prism,*** *then it may be difficult for them to work out this problem if they do not know what volume means in that context.*
>
> *Mr Phatudi: Ho se tsebe mantswe le ho a arohanya according to the different contexts ke hona hoo ele bothata. There is no way ngwana a ka "saksidang" asa utlwisise matswe ana a sekgowa.*

> ***ET:*** *The major problem is that learners do not know the meaning and application of certain words in different contexts. There is, therefore, no way in which learners could successfully solve these problems if they do not understand these words.*

The statements above indicate that the learners cannot solve mathematical word problems successfully because they do not understand "certain keywords" that make up or form part of the sentences of the word problem. According to the participants, these "keywords" are significant not only in making sense of the statements but also in guiding the learners to the specific area of content that is being referred to or which they need to focus on in terms of solving the problem (i.e. drawing their attention to a particular context). Ms Masombuka's statement, "*ha ho buuwa ka volume ba inahanela TV kapa radio moo teng ba tlamehang ho theola kapa ba nyolla volume, empa ha bale sekolong volume ese e bolela ha hong. Mohlala, ha ba calculeita volume of the prism eleng hore it is totally different from what they already know from home",* illustrates clearly that the learners come to class already knowing the meaning of certain words in particular contexts, which are applied differently in a mathematical context. This point agrees with the studies conducted previously, as highlighted in Section 3.3.2. Thus, the failure to understand these mathematical terms may cause learners to be unsuccessful in terms of solving the MWPs, as pointed out by Mr Phatudi, who emphasised in his statement that "*[t]here is no way ngwana a ka "saksidang" asa utlwisise matswe ana a sekgowa"* (**ET**: *"a learner could succeed without comprehending these English words"*). This means that learners need to understand the context in which certain words are used to interpret the questions correctly and solve the problems. The example that Ms Masombuka presented regarding the calculation of the *prism's volume* in the classroom, and the word *volume* being associated with loudness of the radio and television is a clear indication that learners come to class already knowing the meaning of certain terms that are also applicable in mathematical context, albeit with a different meaning. This echoes the sentiments shared by Moleko (2014:39), namely that learners need not be viewed as "clean slates", but as people who have lived experiences and who are also knowledgeable on certain aspects. Listening to the narratives of the two participants, one could draw the conclusion that in teaching MWPs, teachers tend not to teach in a manner that enables or encourages learners to master the mathematical vocabulary and register, on which, according to Setati (2005:9), as highlighted in Section 3.3.2, the success in terms of solving these problems largely depends. This type of teaching, which does not promote the understanding of the mathematical vocabulary and register, keep learners from being able to differentiate and contextualise the "concepts". Furthermore, this type of teaching, which does not make it possible for learners to distinguish the applicability of certain words in different contexts, is an indication that the teachers present word problems in ways that are divorced from the everyday experiences of most of the learners.

In relation to the above, some of the learners referred to the problems they had been given in their homework and class work books in the meeting when the challenges were identified. They outlined the following as some reasons for their limited mathematical vocabulary and register:

> *Rorisang: Re rutwa ka staela se iwane feela and o thole hore ka exam ho botswa ka staela seseng.*
>
> ***ET:** We are usually introduced to a single way of being questioned, which differs from how questions are asked later in the exam.*
>
> *Tshepo: [Referring to classwork book] Mohlala ka classeng ho tlabe ho thwe calculate the interest yearly ebe ka testeng hothwe annually. Bothata jwale e tlabe ele hore o tla be osa tsebe hore annually entse e bolela yearly.*
>
> ***ET:** For example, sometimes the question requires that we calculate the interest yearly and later in the exam we come across a new term, namely annually. The problem then would arise in terms of solving the problem since one would not know that "annually" also means "yearly".*
>
> *Mr Morake: "Honale moo marking center re neng re debeita ka hore some learners mona ho finance ba tlwaetse re bua ka "reducing balance method". That year examiner a be a disaeta hore a sebedise lentswe "diminishing balance method"! Wa tseba a ntsha bana ba bangata jwang tjhunung! Bana ba bona a different thing ka lebaka feela la lentswe leo as opposed to reducing balance method eo ba e tlwaetseng. Lentswe leo la tjhentjha approach ya bana altogether. Ke ha bana ba etsa approach e fapaneng.*
>
> ***ET:** There was a time at the marking centre when we debated about this issue when we found that learners knew the term "reducing balance method" for solving problems that involved money depreciation. However, the examiner that year used the term "diminishing balance method" instead of "reducing balance method". That term caused many learners to change the approach. They approached the problem differently.*

From the above statements by a teacher and two learners, it is clear that a limited mathematical vocabulary has the potential to cause “barriers” in learning and understanding. Learners who only know a limited number of words for a specific aspect may encounter serious challenges when they come across new words (with similar meanings) in an examination or test. This may result in the learners misinterpreting the question and, consequently, limit their chances of solving the problem correctly. A similar finding to this one was revealed in Section 3.3.2, namely that polysemy or diversity of meanings can have a negative impact on learners’ comprehension of word problems. It also hampers their performance, particularly when they have a limited knowledge of the mathematical vocabulary when solving word problems. According to Rorisang, teachers tend to use only specific words that are used in specific textbooks, without providing the learners with other words with similar meanings that may be used as substitutes or interchangeably with the ones they often use in class; this then causes the learners to be “stereotyped”. It came to the fore in this study that learners usually become confused by these words, especially when they encounter them for the first time in tests or examinations.

Rorisang’s words in the above extract indicate that restricting learners to a limited mathematical vocabulary and register has the potential to hamper their understanding of the word problems (concepts) in the long run, particularly in instances where the learners have to solve similar problems that are merely phrased differently. Tshepo’s example of calculating the interest rate *yearly* and later on, in the examinations, being asked to calculate the interest rate *annually* demonstrates the challenge that is posed by learners’ limited exposure to mathematical vocabulary and register. The UDL principle (simple and intuitive), according to Shaw, Scott and McGuire (2001:1 of 2), requires teaching to be designed in a manner that enables the learners, regardless of their experience, knowledge, language skills or current concentration level, to cope and understand. However, Rorisang’s words indicate clearly that teachers somehow do not align with this principle in their teaching by not teaching learners the mathematical vocabulary and register explicitly and thus not expanding their understanding of the mathematical vocabulary and register to facilitate their understanding of the various word problems.

Taking into account the discussion above and Mr Moraka's statement, "*lentswe leo la tjhentjha approach ya bana altogether. Ke ha bana ba etsa approach e fapaneng*" (**ET:** *"that term caused many learners to change the approach. They approached the problem differently"*), it is clear that not knowing the meaning of certain words or not knowing their substitutes with similar meanings may cause learners to think that the problem is a "new" problem, which they have never come across before. As such, the problem may seem to call for a different or new approach, while the problem does not at all require a different approach from what they have been taught.

When learners are taught in a particular way, it is more likely that they would expect to be assessed in the manner in which they were taught. This expectation is also depicted by the learners in the above extracts. They expect to be assessed through the use of the words they were introduced to and that are often used in the classrooms, which is not the case, as derived from their utterances. Teaching learners certain words and assessing them later on using different words (which they may be encountering for the first time) is an unfair practice according to CER. This kind of practice tends to create confusion and affects the problem-solving process.

The above discussions indicate that the learners have a limited mathematical vocabulary and register. This is caused by not being explicitly taught these words by the teachers. It is also evident from the above discussions that when learners are not familiar with certain key terms, they are prone to fail to master word problems. The above discussions also indicate that the teachers introduce learners to specific words that are not used later in the assessments; this causes confusion and leads to failure to solve the word problems. CER and CDA, which both advocate the inclusion of the marginalised (i.e. learners, in the context of this study) in the discussions, thus have made it possible for the voices of the learners to be captured in the conversations to enable the participants to develop an understanding of what they regard to be the problem and its cause. Based on this, the participants collectively agreed that learners' limited mathematics vocabulary and register is a problem that needs to be addressed.

5.2.3 Learners' inability to visualise the word problems

Learners' ability to recognise words is central to reading word problems. The ability to visualise the problems while they are reading them can potentially lead to successful problem solving. Therefore, the use of a visualisation strategy could improve learners' ability to visualise word problems, enable them to demonstrate their thinking and also assist them to connect with the text as they are solving the problems (Mierzwa, 2014:2). Visualisation of the problem (making mental pictures and drawings) also has the potential to lead to the selection of appropriate operations and the attainment of the correct solution(s) to the problem. Teahen (2015:28) maintains that without the ability to visualise, learners may find it difficult to solve word problems. Visualisation of the problems is important in clarifying the word problems, thus making it easier for learners to recognise how they must go about solving the problem. Although the participants believed that visualisation is important when solving word problems, they acknowledged that it is a skill that most learners lack and the teachers fail to develop as they teach them. They commented as follows during the meeting, while also drawing an example from the previous question paper:

> *Ms Masombuka: Bana ha ba filwe palo in the form of a story ba tshwanela ho e solva eya ba hlola because they just cannot deal with abstract things. They cannot make up the picture ka hlohong ya seo ho bolelwang ka sona*
>
> ***ET:** When learners are given problems in the story form, they find it difficult to solve since they are not able to deal with abstract things. They usually fail to make a sound picture of what the problem is about in their heads.*
>
> *Pholoho: Bo teacher ha ba elellwe hore ha osa kgone ho etsa picture ka hlohong ya ntho eo o e botsitsweng ho thatha ho e ngola fatshe. For example, let's say the question says "without sketching the graph, describe the shift of the function, $f(x) = Sinx + 2$". Ho kaba thata ho araba potso eo ha o sa tsebe ho visualaiza hape le ha osa tsebe hore $Sinx + 2$ e bolelang.*
>
> ***ET:** Teachers are not aware that it is difficult to write down a problem on paper when one cannot make a sound mental picture of it. For example, let's say the question says "without sketching the graph, describe the shift of the function, $f(x) = Sinx + 2$".It could be difficult to answer such a question when one cannot visualise and also does not know what $Sinx + 2$ means.*

The above extracts indicate that learners do not have the skill to visualise mathematical word problems (internal visualisation). According to Ms Masombuka's claim, namely that "*[t]hey cannot make up the picture ka hlohong*" (**ET:** *"…in their heads…"*), most learners are unable to form mental pictures of the given word problems while reading the problems. Ms Masombuka further claimed that learners fail to solve word problems because word problems require them to think abstractly, which, at times, may be difficult for most of the learners to do. Although the literature also highlights this as a challenge (see Section 3.3.3), this study, however, revealed that the reason for this challenge is that MWPs usually depict real-life situations, with which, at times, the learners may not be familiar.

Learners often find it easier to solve MWPs if the problems are presented in the form of variables, but not in story form. The latter form requires the learners to, first of all, *imagine* the problems while they are reading them (scenarios), thus creating mental pictures of what the problems are depicting and, after that, illustrate that picture or image in the form of variables that subsequently make it easier to solve the problems. Taking these into account, it is clear that problems presented in story form require one to be able to read, to understand what the problem entails, including what is required to be solved, to create a mental picture of what is required "inside the head" and to translate the text into "easy-to-solve" variables (work out the problem on paper to demonstrate understanding it). Based on this, it is clear that word problems are complex to solve since they demand an amalgam of certain skills, including visualisation. From the statements uttered, it is also clear that the failure to internally visualise the problem makes it difficult for learners to translate the text into numerically solvable equations, as also revealed by Cruz et al. (2014:4) in Section 3.3.3. The following statement was made by the teacher: "*ha ba filwe palo in the form of a story ba tshwanela ho e solva eya ba hlola because they just cannot deal with abstract things. They cannot make up the picture ka hlohong ya seo ho bolelwang ka sona"* (**ET:** "*When learners are given problems in the story form they find it difficult to solve since they can not be able to deal with abstract things. They usually fail to make a sound picture of what the problem is about in their heads"*). This indicates, to a certain extent, that the teachers do not make an effort in their teaching to teach in a manner that promotes visualisation of the word problem concepts. They just give problems and expect the learners to work them out, knowing that they cannot visualise these problems.

In addition to this, Pholoho noted that the learners' inability to visualise can make it difficult for them to provide written solutions to the given word problems – "…*ha osa kgone ho etsa picture ka hlohong ya ntho eo o e botsitsweng ho thatha ho e ngola fatshe"*) (**ET:** *"…when you cannot make a sound mental picture, it is difficult to write it down"*). This means that the process of "working out" the given word problem (showing the steps in writing) cannot be detached from visualising the problem (creating mental pictures). Therefore, if learners cannot read properly or read without understanding, it is possible that they either visualise the problem incorrectly or cannot visualise the problem at all. Both cases, however, lead to learners not being able to show the correct steps to obtain the correct solution in writing. From Pholoho's statement, "*Ho kaba thata ho araba potso eo ha o sa tsebe ho visualaisa hape le ha osa tsebe hore* $Sinx+2$ *e bolelang",* it is clear that the process of visualising the problem requires one to have prior knowledge and an understanding of the specific mathematical content at hand.

The inference one could draw from Pholoho's statement is that somehow the teachers do not provide the interpretations of the specific mathematical content and also do not provide the meanings of this mathematical content. The example cited by Pholoho, $f(x)=Sinx+2$, is about the "upward shift" of the basic Sine graph, which is mainly determined by the positive sign (+) and the number two (2) (the number of the units the graph will be shifted) on the right-hand side of the equation. Teachers who do not describe and interpret the changes that occur to the basic trigonometrical graphs (e.g. Sine, Cosine and Tangent graphs), in this case, the Sine graph, make it particularly difficult for the learners to respond to the questions related to the graph (i.e. they are not able to describe and interpret the graph). This means that even if the question does not require the learners to first plot the graph on the Cartesian plan to describe the shift, but requires that they "imagine" how the graph will be transformed, it will become difficult since they are unable to visualise the problem. They therefore, are not able to "have the picture" (in their heads) of the new graph and, therefore, cannot even provide the new coordinates that would be caused by the "shifting" of the graph without sketching the graph on paper. The unique lesson drawn from Pholoho's statement in addition to the literature in Section 3.3.3 is that when the mathematical content (the drawing of the Sine graph) is taught without the teacher explaining clearly what the content is about and what its constituents (e.g. + and 2) mean, this has a negative impact on the learners regarding how they make meaning of these "constituencies"; they consequently fail to develop the visualisation skills necessary to assist them to procedurally solve these problems.

According to Polya (1945:3), for learners to solve word problems successfully, they need, first of all, to understand the problem. Secondly, they have to devise a plan to solve the problem and, consequently, carry out the plan. A similar sentiment is also portrayed by Pholoho's statement that for the learners to solve word problems successfully, they need to first understand the problem and have a clear picture of the problem "in their heads" to enable them to devise a plan to carry out, thereby solving the problem. Deducing from Pholoho's statement, the inability to visualise the problem limits their conceptual understanding and thus makes it difficult for the problem to be solved procedurally since the problem cannot be "internally visualised" and, therefore, is not well understood. Even in real life, some of the problems continue to exist because they (problems) are not well understood and, therefore, the plans to be carried out to address these problems do not produce the desirable results. The UDL advocates that teachers should teach in a manner by which they eliminate unnecessary complexities (Thompson, Johnstone & Thurlow, 2002:3). However, this principle seems to be violated in the teaching of word problems. The learners seem to be taught in a manner that does not promote the development of the visualisation skill, which consequently makes it difficult to understand the problem and to devise the plan to solve the problem. CER endorses the notion that the problem must be understood for sound solutions to be devised for a problem to be addressed; however, Pholoho's utterance clearly indicates the opposite of the situation. Word problems are not taught in a manner that promotes visualisation, which facilitates the procedural understanding of word problems.

The above extracts show that the learners are unable to visualise word problems. Furthermore, the teachers make no effort to promote visualisation skills in learners. The teaching of word problems is presented in a mere traditional manner whereby learners are given problems and are expected to provide answers only. Learners are not asked to apply their prior knowledge; prior knowledge is not even elicited from the learners. Thus, this kind of practice reflects a convergent right-answer thinking and predictability, which do not contribute positively towards learner growth and comprehension of word problem concepts. CER and CDA in this regard have afforded the participants an opportunity to describe the challenge how they see it, from various angles, which have facilitated a deeper meaning of what was deemed to be the challenge and its causes. Based on the above, the participants have unanimously agreed that the learners' inability to visualise the word problems is a challenge that needs to be addressed.

5.2.4 Ambiguous word problems

Although many of us may strongly believe in the accuracy we envisage mathematics provides, the reality is that ambiguity and vagueness are also reported in mathematical conversations and mathematics textbooks (Barwell, 2005a:118; Barwell et al., 2005:142). These ambiguous statements pose challenges, which, among others, confuse learners to the extent that they are unsure of what must be solved in a given word problem or how to solve it. This leads to learners failing to choose the appropriate operations, and consequently, they obtain incorrect solutions. The challenge posed by the ambiguous statements was also mentioned by the participants:

> *Mr Phatudi: Let us look at this example. I want to show you what I am talking about: (taking out the previous question paper to show to the people who were present in the meeting)*
>
> ***A school will have to replace some of its equipment in 6 years' time. The principal calculates that the equipment costs R44 500.00. The school will establish a sinking fund to pay for equipment. Apart from the constant quarterly payments, the school makes an additional once only deposit of R6300 into the fund which yields interest at 6.85% p.a. compounded quarterly. The amount will be contributed towards buying the new equipment. Determine the value of the outstanding amount.***
>
> *To me, this statement sounds vague. Ho nale ntho esa utlwahaleng ka statement sena. [**ET:** "Something is not clear from this statement"] Is R44 500 the cost of the equipment ya sekolong [**ET:** "…of the school"] or is it of the equipment that has to be replaced in future or what? The sinking fund! It*

> *is established to pay for which equipment, new or old? Kannete baheso statement sena se vague for nna. [**ET:** "Truly speaking, this statement is vague to me"] No wonder why bana ba ileng ba fosa palo ena ka exam [**ET:** "…why learners got function wrong in the exam"] [Mr Phatudi "interrogating" the question to indicate its vagueness.]*

Mr Phatudi referred to one of the word problems that was asked in a previous examination paper as a practical example of the challenges that are posed by ambiguous statements and to indicate the negative effect that vague questions pose. He attributed the learners' failure to solve the problem correctly to the vagueness of the question, hence the statement, "*No wonder why bana ba ileng ba fosa palo ena ka exam."* According to Mr Phatudi, some of the key or guiding words that could have given meaning to or made the given problem clearer, were omitted, hence the question, *"The sinking fund! It is established to pay which equipment, new or old?"* In this regard, the important words ***old*** or ***new*** would have explicitly indicated to the learners if the sinking fund was going to be established for current or for future equipment. Thus, the omission of these words contributed in making the given problem ambiguous. Furthermore, it was also not clear whether the amount of R44 500,00 referred to the value of the current equipment of the school or to the cost of acquiring new equipment, which in this context might have been made clear by the word "will" since the statement was referring to the future cost. Although the example was cited from the past examination question to indicate how the omission of certain words may cause the word problem to be ambiguous, the manner in which Mr Phatudi interrogated the question, indicates that a similar challenge was bound to occur when teaching the learners. This means that the teachers who teach without taking into account and making learners aware of the negative effects of the omission of certain words in the problem may cause learners not to find the correct answers. For instance, the failure to highlight the significance of the presence of the word "will" and indicate that it denotes "future" may create confusion and hamper the problem-solving process. The words *new or old* may assist in clarifying that the sinking fund is to be established to pay the present equipment (described by the word *old*) or the future equipment (which will be described by the word *new*) in this context. Although the literature, similarly to the findings of this study, indicates that the ambiguous statements limit the understanding of the word problem (see Section 3.3.4), it also came out clearly in this study that teachers who do not highlight and emphasise the role of the “simple” but significant words in the teaching of MWPs may create ambiguities through their teaching, which may hamper the learning process.

Ms Masombuka commented as follows regarding the use of words with different meanings:

> *Hare sheba hona statementeng sena [referring to the problem provided by Mr Phatudi] lentswe “compounded” le ka ferekanya ngwana haholo ha a sa le tsebe hore le bolelang. Ngwana e mong aka seke a utlwisisa hore ho hlokahala hore interest e calculeitwe quarterly. Haholo jwang haeba a sa le tlwaela. Re le matitjhere, hare ele hloko taba ena ya mantswe a ka “causang” mathata.*
>
> ***ET:*** *When we look at the statement again, the word “compounded” may confuse learners. especially if they do not know its meaning. Another learner may not understand, that in this problem, the interest needs to be “calculated” quarterly. As teachers, we are not aware that these words can potentially cause problems.*

According to Ms Masombuka, polysemous words can potentially cause learners to misinterpret the question, particularly in instances where learners have not yet been exposed to the use of various terms with similar meanings. Ms Masombuka further believes that a learner with limited vocabulary may perceive the problem as ambiguous. This concurs with the literature in Section 3.3.4, namely that a lack of understanding of words with diverse meanings (polysemous words) can cause learners to misinterpret the question and to answer it incorrectly, hence the comment, “*Hare sheba hona statementeng sena lentswe ‘compounded’ le ka ferekanya ngwana haholo ha a sa le tsebe hore le bolelang”* (**ET:** “*When we look at the statement again, the word ‘compounded’ may confuse learners especially, if they do not know its meaning”*). The word “compounded” in this context also means calculate. Therefore, the failure to understand the meaning of the word that is used in the sentence may potentially cause confusion and result in the learners not understanding the problem holistically.

The statement, “*Re le matitjhere, hare ele hloko taba ena ya mantswe a ka “causang” mathata*” (**ET:** *“As teachers, we are not aware that these words can potentially cause problems”*), indicates that the teachers teach mathematics without making sure that the learners also learn the mathematical terms and understand them. It also indicates that the teachers teach without indicating the words with similar meanings to the learners. This creates problems for the learners, particularly when they encounter such a term for the first time in the test or examination. This resonates with Reynders' (2012:30) findings (see Section 3.3.4) that a lack of understanding the meaning of certain words causes confusion and misinterpretations since learners may already know a particular meaning of the word in a different context.

From the discussions above, it is clear that the omission of key or guiding words is one of several factors that can cause ambiguity in mathematical word problems that, in turn, may cause the reader to misinterpret the question. Furthermore, the ambiguity of MWPs can also be caused by unfamiliar words (to the learners) with the same meaning as that of the frequently used terms. This can potentially lead to misinterpretation of the word problem and incorrect solutions. The effects of ambiguity are notable even in real life; for instance, in court, people may find themselves in difficult situations and their cases cannot be defended due to statements they provided, which were regarded as ambiguous and thus misinterpreted. The multi-facetted stance of CER and CDA thus makes it possible for the participants to discuss the challenge and demonstrate its consequence in a manner in which they feel comfortable and from their own perspectives. In light of the above, the participants have unanimously agreed that it is unrealistic for teachers to just teach MWPs and expect the learners to learn the terminology on their own without formally teaching them. Therefore, ambiguity was identified as a barrier towards the teaching and learning of MWPs, and the teachers were sensitised to the fact that they should address it when teaching MWPs.

5.2.5 Teachers' inability to teach word problems

MWPs form an integral part of mathematics education for various reasons. According to Bates and Wiest (2004:17), MWPs provide questions that challenge learners to apply mathematical thinking to various situations, and they are an efficient means of relating this thinking to the real world. These scholars further state that word problems are either readily available in mathematics texts or can be written in a short period of time, which makes them useful to time-conscious teachers. Therefore, MWPs are significant in the sense that they bring an element of reality to the classrooms, serving an important role in advancing learners' understanding of mathematical concepts as well as enabling learners to relate mathematical concepts to real-life situations. The teachers' understanding of this mathematics genre is key in developing learners' understanding of this genre as they have to guide their learners to solve these problems step by step. They also have to teach the learners how to analyse and interpret these problems by applying teaching strategies that enhance learners' understanding of MWPs.

Although the teachers' expertise is of great significance in the development of learners' understanding of MWPs, it has become quite clear in the meeting where the challenges related to the teaching of MWPs were highlighted that teachers often find these problems as difficult to solve as their learners. The participants commented as follows:

> *Mr Simelane: Di word problems tsona di thata bo tsona! Jwale hee it becomes a major problem ha titjhere le yena antse anale problem ka tsona di word problems tsena.*
>
> ***ET:** Word problems are naturally difficult! So, what makes them even more challenging for learners is when the teacher also finds them difficult to solve.*
>
> *Mr Phatudi: Exactly! Di word problems ha di thata feela for bana empa hape le ho matitjhere.*
>
> ***ET:** Word problems are not only difficult for learners but for the teachers as well.*
>
> *Ms Masombuka: Hei le rona di ya re sokodisa maan!... Ha o bone ba bang ba bile ba di ngala basa di rute bana.*
>
> ***ET:** They also struggle with them! No wonder others ignore and do not teach learners these problems*

From the above extracts, it is clear that in many instances, both teachers and learners find MWPs difficult to solve. Evidence of this is also found in the literature, namely that most teachers find this mathematical genre difficult to teach too (see Section 3.3.5). Research has revealed that in teaching and learning situations where both the teacher and the learner are "non-native" users of the language of instruction, the teacher struggles as much as the learners (Kioko, 2015). This is one of the main reasons linked to the learners' inability to master this mathematics genre. The teachers are the "sources of knowledge" and "subject experts" whom learners rely on to acquire knowledge and establish an in-depth understanding of the subject (mathematics, in this case). Therefore, the teachers' inability to solve these problems themselves has a negative bearing on their learners. This was made clear by Mr Simelane's statement: "*Di word problems tsona di thata bo tsona! Jwale hee it becomes a major problem ha titjhere le yena antse anale problem ka tsona di word problems tsena"* (**ET:** *"Word problems are naturally difficult! So, what makes them even more challenging for learners is when the teacher also finds them difficult to solve*"). Deducing from Mr Simelane's comment, it is clear that teachers who find it difficult to comprehend and solve MWPs themselves, cannot scaffold learners' understanding of these problems. It is also true that the teachers' inability to master this genre of mathematics creates a lack of confidence and motivation on their part to teach it to the learners. This became clear in Ms Masombuka's comment, "*Ha o bone ba bang ba bile ba di ngala basa di rute bana"* (**ET:** *"No wonder others ignore and do not teach learners these problems"*). Drawing from Ms Masombuka's words, teachers who find MWPs challenging have a tendency of not teaching them to the learners.

A Sesotho idiomatic expression says, "tsela e botswa ho ba tswang ho yona" (***ET:*** *"the road is searched from those who have travelled it"*). This means that to navigate the journey, it becomes easier when one has gotten instructions from those that are acquainted with the road and its "ups and downs". The same can be said about the learners in this regard. They could master MWPs if they could receive meaningful guidance from the teachers, who were also once learners. However, deducing from the statement, "*titjhere le yena antse anale problem ka tsona*" (**ET:** "*teacher also finds them difficult to solve"*), it is clear that the learners are not benefiting as they should from the teachers who also seem to be grappling with this mathematical genre. In this instance, the principle of fairness that is advocated by CER is thus not exercised. This is because the learners continue to struggle, just like the teachers; however, the teachers still "get rewarded or remunerated", even though justice is not done in this case because the teaching of word problems to the learners (as recipients of knowledge) is not done sufficiently. In other words, the teacher not being able to teach the word problems effectively does not affect his or her "pocket"; however, the learners experience the knowledge gaps that keep them from mastering this mathematical genre and, in the long run, affect their higher grades.

The issue of teachers who find MWPs challenging to teach, as highlighted in this study, is also noted in Section 3.3.5. However, in addition to that, although it has never been explicitly mentioned during the discussions, I realised that one of the reasons why some teachers “do not like” teaching this genre of mathematics is to avoid being exposed as not being well versed in this mathematics genre. They, therefore, do not want to be embarrassed in front of their learners by showing this “weakness” on their part. The decision not to teach this mathematics genre thus becomes a “defensive mechanism” for the teachers to hide their “weakness”. Learners usually look up to teachers as the “source of knowledge”, and teachers do not want their learners to lose confidence in them. Also drawing from Ms Masombuka’s words, teachers seem to hold their “reputation and dignity” in high regard, which is why they would rather not teach MWPs so that their “reputation and dignity” could be retained regardless of depriving learners the mathematical knowledge that is important in shaping their mathematical thinking.

This kind of a behaviour is also noticeable in society. Once a person has decided that he or she does not like a particular task or perceives it as difficult to perform, the person will do everything in his or her power to avoid doing the task in order not to be exposed as not having the ability to perform the relevant task. In this study, teachers have also portrayed a similar “mentality” by not teaching this genre to protect their dignity and thus maintaining their “status quo”. Unfortunately, by “choosing” not to teach MWPs to the learners, the teachers deprived their learners of the opportunity to learn how to solve mathematical word problems, which, according to Bates et al. (2004:17), is significant in enabling them to apply mathematical thinking in real-life situations. Furthermore, the learners (who were not taught the MWPs properly), at some point, have to progress to the next level or grade with insufficient knowledge, and when they are confronted with the mathematics genre, they struggle to cope due to the knowledge gaps that have been created.

A UDL requires the instruction to be designed to accommodate a wide range of individual abilities and for teachers to provide choice in the methods of use (Shaw et al., 2001:1 of

2). However, drawing from the statement, (**ET:** *"They also struggle with them! No wonder others ignore and do not teach learners these problems"*), it became clear that no effort was made by the teachers to teach MWPs effectively so that their methods provided multiple ways of learning, experiencing and exploring this mathematical genre.

The discussions above prove that MWPs are not only difficult to solve for learners but teachers too find them challenging to solve and teach. This concurs with the findings in a study conducted by Seifi et al. (2012:2923), which indicate that most teachers find MWPs difficult to solve and teach (see Section 3.3.5). It is, therefore, not reasonable to expect learners to master MWPs when teachers, who are supposed to be "the source of knowledge", also find this mathematical genre difficult. These teachers, therefore, cannot scaffold the learners' problem-solving skills. The issue of power relations has become clear in this context in the sense that the teachers seemed to have control of what they "chose" to teach, while the learners have remained at the receiving end and do not have the power to dictate what they should be taught and how it should be taught. CER and CDA in this regard enable us to identify an element of unequal power relations that put others (learners) in a disadvantageous position. CER and CDA also reveal the different resultant behaviour that is informed or caused by particular thoughts and beliefs, thus help us understand why certain actions take place.

5.2.6 Teachers' negative perception regarding the use of learners' home languages in classrooms

The use of home languages, or not, is still raising heated debates, particularly within educational spaces. Some agree that learners should also be taught in their mother tongue, while others are completely against the idea. In the current study, the same debate has sparked during the meeting when the challenges pertaining to the teaching of MWPs were to be identified. Some teachers regarded the use of "English *only*" in the classrooms as a drawback for many learners, while others regarded it as one way of equipping learners with the skill to express themselves in the world, where English is considered a "requirement" for engagement and interaction between diverse people.

Some of the participants commented as follows in the meeting when the challenges were discussed:

> *Mr Morake: There is a teacher sekolong sane sa Moriting. [**ET: "...**at Moriting School."] Titjhere enwa o tswa hodimo kwana Africa [**ET:** "This teacher comes from North Africa."] I was told hore [that] he is doing very well in Life Sciences. O ruta bana ka sekgowa feela hobane ha a tsebe Sesotho. Ho ruta bana ka English feela ho ya thusa hobane ba kgona le ho mamela attentively. [**ET:** "He teaches learners in English only, since he cannot speak Sesotho. It really helps to teach learners only in English because they listen attentively."]*

> *Mrs Khumalo: Taba ke hore bana bana ba lebeletswe ho communiceita ka sekgowa ha ba se ba le tertiary le moo batla beng ba sebetsa teng.*

> ***ET:** The fact is that these learners are expected to communicate in English when they get to tertiary and also in the places where they will be employed.*

The extracts above indicate that some teachers still prefer the use of English *only* in the classroom over the use of learners' home languages, and this is also confirmed in the literature (see Section 3.3.6). Mr Morake's example of a teacher at Moriting School who only speaks English in the classroom, was an indication of how strongly he supports the use of English *only* in the classroom. According to Mr Morake, the use of English only has the potential to make learners listen attentively to what they are being taught. Although Mr Morake did not say it out loud, I deduced, judging by Mr Morake's facial appearance and his comment, that he either also uses only English in his classes or had seen someone else teaching in English only with success. He attested to the successes of such a practice; hence the statement, "*Ho ruta bana ka English feela ho ya thusa hobane ba kgona le ho mamela attentively"* (**ET:** *"It really helps to teach learners only in English because they listen attentively"*). Listening to Mr Morake, I realised that he commended the sole use of English in class when teaching since it tends to cause learners to listen to the teacher attentively. This does not mean that when teaching learners only in English that the learners will understand what they are being taught, but English may only draw learners' attention in class, deducing from Mr Morake's words. Again, drawing from Mr Morake's utterance, he seemed to believe that teaching learners in English was the only way to draw their attention and to engage them effectively in class. He seemed not to be concerned about the other forms of drawing learners' attention and engaging them in class activities. The manner in which he explained this and his facial expression when making his point be heard made it evident that he discourages the use of home languages when teaching. CER and CDA play a significant role by affording the researcher and the participants an opportunity to analyse not only the text and spoken words but also the facial expressions and visual cues in an endeavour to make sense of the data.

Mrs Khumalo also shared the same sentiment, namely that learners should be taught in English only. She commented from the perspective of someone who was looking at the "bigger picture". According to her, for learners in future to express themselves in English, which is considered to be a "universal language", they first need to learn how to speak and express themselves at school level in the classroom. Mrs Khumalo believes that communicating in English at school level would serve as preparation for learners' future engagements, where they will have to communicate with other people at institutions of higher education as well as at their workplaces, hence the comment, *"…bana bana ba lebeletswe ho communiceita ka sekgowa ha ba se ba le tertiary le moo batla beng ba sebetsa teng"* (**ET:** *"these learners are expected to communicate in English when they get to tertiary and also in the places where they will be employed"*). Mrs Khumalo's sentiment regarding the sole use of English in class implied that the only way to enable the learners to express the mathematical concepts well, even when interacting with other people (who come from different backgrounds – a diverse population), was through the use of English *only* in the classroom.

Although some of the teachers seemed to prefer the use of English rather than the learners' home languages, as highlighted by Mr Morake and Mrs Khumalo in the above extracts, the disadvantages of using English only were also mentioned. The participants described these as follows:

> *Ms Masombuka: Empa jwale re etsa jwang ha bana ba sa utlwisisi ka English? The thing is… bana bana English entse e ba thatafalla.*
>
> ***ET:*** *But then, what do we do when learners do not understand in English? English is difficult for these learners!*
>
> *Mr Phatudi: Nna ke bona ho sa thuse ho continuwa ka English empa bana ba sa utlwisisi.*

> ***ET:*** *It does not really help to continue teaching in English when learners cannot understand.*
>
> *Mr Moraka: Ha re ruta feela ke English bana ba bang ba tshaba ho bua hobane batla tsheuwa.*
>
> ***ET:*** *When taught only in English, some learners become afraid to express themselves since they may be laughed at.*

Ms Masombuka seemed not to be totally against the sole use of English; however, she considered the “total use” of it in certain instances as a barrier to learners who are not proficient in English. Her main concern, which is informed by her teaching experience, is that most learners in multilingual mathematics classrooms are not proficient in English and are still learning it, and therefore, often find it difficult to understand certain concepts when taught in English only. This confirms one of the findings in a study that was conducted in Malawi, that the teaching of mathematics in multilingual mathematics classroom is complex since most of the learners are still learning the language of learning and teaching (LoLT) (see Section 3.3.5). According to Ms Masombuka, the teacher in this kind of set-up (multilingual set-up) has two options when teaching the learners, namely either to teach in English only or to teach in English, but also use the learners’ home language(s) to facilitate an understanding of the concepts. Mr Phatudi also had some reservations regarding the sole use of English in class. He believed that teaching learners in English only was not productive, particularly in instances where they did not understand the concepts when they were taught in English and still needed explanations in their home languages to facilitate understanding and bring about clarification. These teachers were also aware of the benefits of teaching learners only in English; however, they were also mindful of the fact that these learners were “still learning the LoLT” while also learning the mathematical language and thus saw fit for the home languages to be used as “learning resources” to assist these learners to conceptualise the word problems. In this way, the teachers aligned themselves to the CER principle of fairness, which requires the teachers to explore manifold ways they can use to enable the learners to understand this mathematical genre.

Mr Moraka mentioned another consequence of the exclusive use of English only in the classroom, namely that learners might feel uncomfortable expressing themselves in a second language. This might cause the learners to withdraw from participating in the activities in trying to protect themselves from being “ridiculed and laughed at”. Mr Moraka’s sentiment indicated that although there might be good intentions with the sole use of English in the classroom, such as to fully equip and empower the learners to develop fluency, it might also cause limitations in terms of learner participation. Moleko (2014:88) asserts that without learner participation, it can be difficult for the teacher to identify in time the areas that learners find difficult or any misconceptions that they might have in order to put strategies in place to enhance the learners’ understanding of MWPs. The statement uttered by Mr Moraka cautioning about the sole use of English, which may, in turn, limit participation, is in line with the CER principle of inclusivity and active participation in the sense that people should not be limited in participating actively and fully. Thus, any form of obstacle which may keep the learners from participating should be dealt with and eliminated. In this case, the use of English, even though it comes with many benefits, where it limits participation and becomes a “barrier” towards learning needs to be dealt with.

Drawing from the extracts above, it has become clear that mathematics is taught to learners without the teachers considering the negative effects of the unproductive usage of different languages in the teaching and learning of MWPs. The social constructivists hold the perspective that first language speakers usually find themselves in a sociocultural context other than their daily context when they are in the mathematics classroom because language as a key concept now becomes “mathematics language”, which consequently affects learning (Duit & Treagust, 1998:18; Johnstone, 1993:120-121). Drawing from this, it is clear that even English first language speakers are challenged in the classroom in which they also have to learn mathematics language. Therefore, if the English language can affect the English first language speakers in this way, it is obvious that the impact of the sole use of English on non-native speakers is even more severe since these are the learners who are *not proficient in English* and are *still learning English* and who *also have to learn the mathematics language*. This indicates that if the teachers are not aware of these dynamics, it may be difficult for the learners to master this mathematics genre, which necessitates English proficiency. The tension that seems to exist between the teachers (those in favour of the exclusive use of English and those who want the home languages to be used as well) indicates to some extent that the teachers are not using the languages in a more “productive” way to facilitate learning; they have failed to strike a balance and to maintain productive teaching while drawing from the different languages when teaching learners.

It is common knowledge that people often refrain from doing “things” they are not confident to do, especially when they have to do these things in the presence of others whom they regard to know more than they do. The act of refraining from participating in class activities in this instance may also be attributed to the fact that those who lack confidence to do certain things in the presence of the others, may be afraid to be ridiculed. The opposite of this perception holds – if a person is confident about doing something, then the person will be able to do it with ease in front of others, without fear of being ridiculed. This “act of refraining” seems to be prevalent also in the multilingual mathematics classroom as highlighted by the participants. The learners who participate freely in class activities, as indicated by Mr Morake, seem to be those who are proficient in English and those who are not, tend to refrain from fully participating in class activities. A UDL advocates the creation of a climate that promotes openness and respect and also promotes information access (Salzberg, Baum, Price, Morgan & Keeley, 2006:4). This means that the teacher must ensure that all the learners feel free to participate in classroom discussions. However, drawing from the above extracts, some of the teachers make it difficult for learners to participate by using only English when teaching. Therefore, the learners who are not proficient in English cannot fully participate and they are subsequently deprived access to mathematical knowledge.

The above discussions indicate the imbalances in terms of the use of languages in class and the impact thereof. The teachers seem to teach in a manner that does not promote the acquisition of the LoLT. This leads to a situation in which the learners are not able to progress to the maximum since they are not equipped with the LoLT. Further to this, the negative impact of not mastering the LoLT manifests in learners not being able to demonstrate “critical thinking actions”, which Paul (2004:463) asserts can only be achieved and demonstrated through sound linguistic ability, which most learners seem to lack. Again, the use of the learners’ home languages seems to be associated to a certain extent with the practice that is “limiting” to the potential of the learner, even though others perceive it as a resource that may be used to promote, among other aspects, an in-depth understanding of MWPs as well as learner participation in classroom activities. It was on the basis of these tensions that the participants saw the need to generate ways in which the LoLT and the home languages could be used profitably and cautiously so that the teachers could strike a balance and teach MWPs effectively in multilingual mathematics classrooms.

5.2.7 Summary

The above sections highlighted the challenges related to the teaching of MWPs, necessitating the need to address these challenges. It is important to note, from the above discussions, that learners bring into the classroom various characteristics that have an impact on the learning process; this requires teachers to reflect on their teaching practices. Furthermore, these characteristics must be embraced and not perceived in a negative way since they form part of every classroom. The role of the teacher then becomes that of ensuring that the needs of all learners are addressed or taken care of. Caring is one of the core values in education (and also advocated by CER) that needs to be exercised. One way to portray elements of caring in educational settings, especially at school level, is to teach learners effectively to the extent that they can demonstrate an understanding of what they have learned. Furthermore, the above sections also highlighted the significance of language, used in classrooms, in enhancing learners’ understanding of what they are learning. Taking the above into consideration, it is clear that a strategy to address these challenges effectively is required to enhance the teaching and learning of MWPs in multilingual mathematics classrooms. The following sections outline the components of the solutions devised to address the challenges identified by the participants in this research study.

5.3 THE CONSTITUENTS OF AN APPROACH USED IN RESPONSE TO THE IDENTIFIED CHALLENGES

The previous sections highlighted the challenges identified by the participants in the meetings that were held. This section will highlight the solutions to the challenges, as identified by the participants.

5.3.1 Engaging learners in reading exercises

Reading is a skill that plays a major role in mastering MWPs. Learners who are skilful in terms of reading are able to make sense of MWPs. In accord with this, Zare and Mobaraken (2011:98) note that reading is a vital skill and the most significant skill for second language learners to master MWPs. These scholars further note that reading can never be separated from mathematical activities and tasks, and on the basis of this, it is crucial for teachers to advance learners' reading skills. According to McNamara (2009:1), learners need domain-specific knowledge to understand the content embedded in word problems. Domain knowledge refers to knowledge about the topic of the text. In order for the reading process to be successful, one needs to understand what the text entails, which would be made possible by understanding the vocabulary in context.

In light of the above, in a meeting when the solutions to the challenges were discussed, the participants highlighted some of the strategies they used in class to engage the learners in reading. Some lessons were presented to demonstrate and justify what they considered to be the solutions to the identified challenges. The participants commented as follows:

> *Ms Buthelezi: There is no way this children can master the word problems if they do not know how to read. Reading is important!*
>
> *Mr Maduna: True, mme, these learners ba tlameha ho rutwa ho bala kamehla. Ke ka ho bala feela ba ka mastarang mmetse ona.*
>
> ***ET:** True, madam! Learners have to be taught how to read frequently. They can only master this specific type of mathematics through reading.*
>
> *[All the participants agreed…mhh…mhh...mhh]*

The above extracts indicate the significance of reading in mastering MWPs. According to the participants, effective reading is key in terms of solving MWPs, and if learners lack the skill to read, they may find it challenging to approach this mathematics genre. According to Ms Buthelezi, it is vital that learners are encouraged to read since reading serves as a major prerequisite for mastering this mathematics genre, hence the comment, "*There is no way these children can master the word problems if they do not know how to read. Reading is important!"* The expression *"no way"* in Ms Buthelezi's statement indicates how strongly she feels about the significance of reading in successfully approaching MWPs. This means that the learners' reading skills need to be prioritised in the teaching of MWPs so that they are able to confront this mathematical genre. Mr Maduna also agreed and confirmed that reading is an important activity when learners are dealing with word problems. However, he admitted that reading is a skill that teachers should nurture in learners. This means that teachers must serve as "coaches" and thus ensure that they engage the learners in reading exercises. In doing so, they should guide them on how to read effectively so that the learners may develop the relevant reading skills. This is in line with Vygotsky's theory of learning as a social activity, specifically the zone of proximal development (ZPD), which implies that in the process of teaching and learning, a teacher as a "more knowledgeable other" is expected to shape and guide the learners (Vygotsky, 1987:209). Mr Maduna further indicated that the process of teaching learners how to read should be done frequently. The word "kamehla" (***ET:*** *"always"*) from Mr Maduna's statement means that teachers must frequently provide learners with opportunities to engage in mathematical concepts that would assist in developing their reading skills (since these problems require learners to read). Drawing from Mr Maduna's statement and taking into consideration that mathematics uses, among others, representations, symbols, variables and numbers, it is important that teachers should teach the mathematical statements that are represented in the form of variables in such a manner that they could also present similar questions in textual form. This may serve as one way of engaging learners with the concepts in a different form, which would require them to first reading the problem carefully and understanding it before translating it into simpler and solvable variables. This speaks to the UDL principle of multiple means of engagement. In this way the teacher provides the learners with the opportunities to engage with the concepts in different ways and learn how the mathematical concepts can be expressed in various ways. This also resonates with the CER principle of flexibility to ensure that in the process of teaching and learning, the learners get to develop a deeper understanding of the concepts which will, in turn, awaken their "sense of autonomy".

Another comment on how the learners were engaged in reading exercises was made by Ms Masombuka, who narrated and provided a lesson demonstration of a word problem she had given the learners in class. She reflected as follows:

I gave my learners the following problem in class *[Reading the question]*, "Five is less than three less than a number". I asked them to illustrate the statement on the number line. Most of them read it as follows: $5 < 3 < x$. However, I outlined the steps that helped them understand.

[Below are the steps through which Ms Masombuka took her learners.]

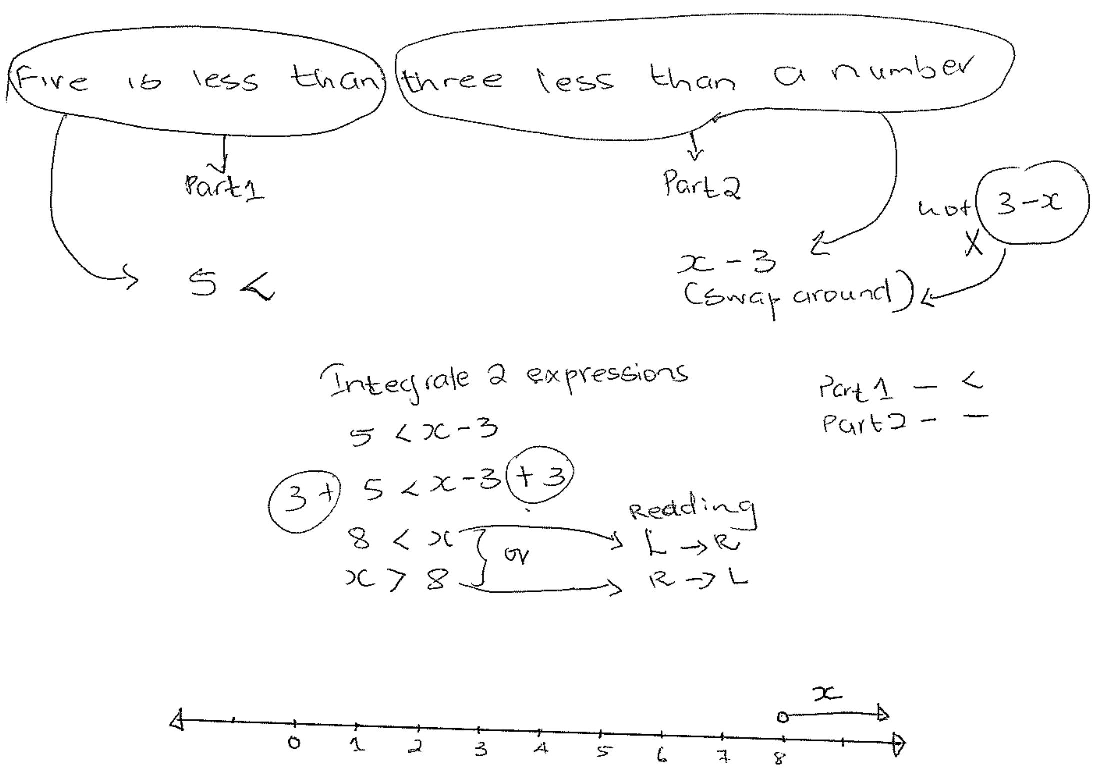


Figure 5.1: Word problem and the steps taken to solve the problem (picture taken from the teacher's notes)

From the above example, Ms Masombuka took us through the steps she performed to enable her learners to understand the “inequality” problem. She worked out the problem and explained the different steps as shown above; she first broke down the statement into two separate parts (parts 1 and 2), namely *“Five is less than”* and *“three less than a number”* respectively. She did that to draw the learners’ attention to the two concepts embedded in the two parts (which denote a similar meaning, but are represented differently) so that the learners could get a clear distinction between the concepts. She explained to the learners that *“is less than”* (part 1) from the given problem indicates that the learners would be working with an equation and, therefore, *“is less than*” (from part 1) was going to break that equation into the left and the right sides. She further indicated that “*a number*” in that sentence represents any “unknown number” and such an unknown number can be represented by the letter $''x''$. She highlighted that “*five is less than*” can be numerically and symbolically represented as $''5<''$ and “*three less than a number*” can be represented as $x-3$, in which x would be used to represent an “unknown” number. She explained that from the given word problem, “is” (from the first part), which is next to five (5), necessitated the application of the sign $''<''$. Since there was no “is” next to three (3) (in part 2), she had to use the “subtraction” concept, which also denotes “less”. Again, she explained the expression “*three less than a number*” by indicating that it should be expressed in a “turn-around” manner, that is, as $x-3$ and not as $3-x$. In order to clarify this concept, she used a real-life example explaining it as follows: “you have three rands less than what I have”. She referred to “what I have” in her example as any amount which she might have and mentioned “R5” as a specific example to draw on. In line with that, she illustrated the statement as $R5-R3=R2$. In this way, she became “flexible” in her teaching by providing a choice in methods of use, thereby representing information in multiple formats, which is what UDL advocates. The use of a real-life scenario indicates that the teacher drew from content-based language teaching, which Crandall and Tucker (n.d.:3 of 22) describe as an approach to language instruction that integrates the presentation of a topic from subject matter. The use of “five is less than three less than a number” ($5<x-3$), was an integration of the two statements, namely “five is less than” and “three less than a number”, which was made possible by differentiating between the meanings embedded in the two statements (parts) and

showing how to represent the two expressions based on the meanings thereof. She finally provided the solution as $8 < x$ and indicated that this also means $x > 8$. In this way, she demonstrated to the learners how to read from left to right and vice versa. Deducing from this illustration, the teacher made it clear that when reading from left to right, “eight is less than x” requires the sign “less than” ($<$) to be used, and when reading the same expression from right to left, it requires the “greater than” ($>$) sign to be used. What can be drawn from the procedure the teacher illustrated when she taught the problem is that for the word problem to be converted into symbols, learners needed to have a holistic understanding of what the word problem is all about when reading. This confirms the finding from a Malawian study that revealed that the meaning of a mathematical concept expressed in words often differs from the meaning when the same concept is expressed in symbols (see Section 3.5.3).

The exercise the teacher carried out showed how she taught this word problem to the learners and indicated the significance of carefully engaging the learners in the reading exercises by ensuring that the concepts are clearly explained while reading and they are understood in the context in which they are used. This means that the reading skills of the learners must be developed for them to be able to make sense of what the word problem is all about. In her lesson demonstration, the teacher demonstrated the significance of breaking the complex word problem into simpler parts. She made the learners understand, through careful reading, the meaning and concepts embedded in the two parts of the problem. This form of strategy for reading a complex word problem, thus developed the learners’ understanding of a mathematical problem significantly. Barton et al. (2002:iv) echoes a similar sentiment, namely when productive reading strategies are applied to improve learners’ reading skills, the learners tend to develop an understanding of word problems (see Section 3.4.1).

The above discussions indicate the significance of improving the learners’ reading skills by applying various skills. Improving the learners’ reading skills leads to an enhanced understanding of mathematical text as well as concepts embedded in the word problem(s). In order to come to this conclusion, the nature of inclusivity and flexibility of CER and CDA thus made it possible for such a demonstration and interpretation to be carried out. CER and CDA advocate the creation of "relaxed" spaces where the participants can provide the solutions to the problem in a manner with which they feel comfortable. In line with this, the teacher was able to provide insight into what, according to her, constitutes a solution towards the enhancement of the learners' reading skills by demonstrating the steps as shown above.

5.3.2 Refining the learners' mathematical vocabulary

A lack of mathematical vocabulary was identified as one of the main reasons why learners were not performing well, specifically in this genre of mathematics. The learners who were participants in this study confirmed that a lack of mathematical vocabulary was one of the biggest challenges they encountered when attempting to solve MWPs. However, in the meeting that was held to determine solutions to the identified challenges, some of the ways in which a lack of mathematical vocabulary could be addressed, were described. The participants deliberated as follows:

> *Ms Masombuka: Ho ka thusa haholo ho dula re bontsha bana hore mantswe a itseng a bolelang maemong a fapaneng! Ha ke etse mohlala ka lentswe lena **"function"**; mmetseng reka le sebedisa ho hlalosa equation. Empa Baelojing le ka bolela mosebetsi wa organ e itseng. In some instances, I provide the terms that can be used in place of the one that is used at that time.*
>
> ***ET:** It will help a lot to keep on indicating what certain words mean in different contexts! Let me make an example of this word "function". In maths, we can use it to describe an equation. However, in biology, it may imply an activity that a particular organ performs.*
>
> *Mr Twala: Ehe mme! Ho tshwana le hare dila ka **"probability"**. Let's say re re ho ngwana a fane ka **probability of A, kapa probability of A only** jwalo jwalo. Ho bohlokwa hore re hlalose hantle baneng distatements tsena.*
>
> ***ET:** True, madam! This is the same as when we deal with the "probability" concept. Let's say we ask a learner to give the **probability of A,** kapa **(ET: or) probability of A only etc.** It is important to explain these two statements to the learners clearly.*

The above extracts indicate the significance of teaching learners the mathematical vocabulary because, in doing so, the teachers expose learners to different mathematical words that can be used "interchangeably". It is important that learners are made aware of the different meanings that specific words have in various contexts. This supports Boulet's (2007:9) stance, as indicated in Section 3.4.2, that there is a need for learners to become fully conversant with the mathematics vocabulary and register to solve MWPs successfully. According to Ms Masombuka, it is important to not only teach but also highlight the use of a similar term in different contexts. This facilitates the teaching that ensures that the necessary information is communicated to the learners effectively, thus making information perceptible or distinguishable, which is what the UDL promotes. Furthermore, this is to ensure that the correct meaning in context is established and that the calculations are carried out accordingly. According to Ms Masombuka's statement, "*Ho ka thusa haholo ho dula re bontsha bana hore matswe a itseng a bolelang maemong a fapaneng!"* (**ET:** *"It will help a lot to keep on explaining what certain words mean in different contexts!"*), this means that teachers need to pay attention to the meaning of the mathematical terms that are used. Teachers should not focus only on the content they are teaching but ensure that they guide and facilitate the correct application of the mathematical terms and highlight the differences in use where the same word may be used in another context. This will prevent confusion that may arise as a result of the use of these words.

In her explanation, Ms Masombuka made reference to the term **"function"** as an example to show that the word has or carries a different meaning in mathematical context as opposed to the meaning it carries in biology (different meanings in different contexts). In mathematics, a "function" in simpler terms refers to an equation, which explains the "input, process and output" concept; however, in biology, it describes the work of a specific organ in the body. In line with this notion, it is clear that a word in ordinary English may not necessarily be interpreted and explained the same way in mathematics, as also highlighted by Reynders (2012:30) in 3.4.2. The words "*ho dula re*…" (***ET:*** *"to always…"*) from Ms Masombuka's statement imply that the teacher, in avoiding misinterpretation and misapplication, must frequently distinguish between the meanings of words in various contexts. This will make learners aware that words have more than one meaning and that they must use the relevant meaning, according to the context, to solve a problem correctly. The statement "*In some instances, I provide the terms that can be used in place of the one that is used at that time*", indicates that the words which can be used interchangeably also need to be taught to the learners. This would ensure that the learners are not only constrained to use specific terms that are used under a particular topic. However, careful use of the words is important, also that the relevant words are used, as guided by the Curriculum Assessment Policy Statement (CAPS) document. This is important for assessment purposes because it will expand and broaden the learners' knowledge of the mathematical vocabulary to use without diverging from the CAPS document and the stipulations thereof. This type of teaching is empowering in the sense that learners get to learn more about a variety of the key mathematical terms and their applicability in various contexts. This is in line with CER, which espouses the notion of the creation of classrooms as spaces in which learners could be provided with teaching that is "content-rich" to enable them to empower themselves.

Teachers who, in their teaching of mathematical vocabulary, frequently distinguish between the meanings of words in various contexts, make it easy for learners to recognise the words and apply them correctly in context. In this way, the teachers remove learning "barriers" that may be caused by the vocabulary the learners do not understand and so impede the learning of MWPs. In line with this, the UDL advocates the removal of the barriers to teaching and learning and, therefore, in this regard, "emphasises" that the teachers should address the mathematical vocabulary which may impede the learners' understanding of the MWPs.

Mr Twala also highlighted some of the phrases that contain key terms in word problems and suggested that these phrases need to be explicitly taught and explained to enhance learners' understanding. This sentiment is shared by Sepeng and Madzorera (2014:218), who maintain that learners need to be taught the definitions of technical terms since such a practice assists learners to recognise or identify words that would lead to the selection of suitable strategies and operations that facilitate smooth word problem solving (see Section 3.4.2). Further to this, Mr Twala cited an example that supported his stance in relation to learners being taught the vocabulary explicitly in his statement, *"probability of A, kapa probability of A only".* The statement, *"Ho bohlokwa hore re hlalose hantle baneng distatements tsena"* (**ET:** "*It is important to clearly explain to the learners these two statements"*), indicates that the two phrases, namely "probability *of A*" and "probability *of*

A only" differ. The word ***only*** in the second phrase distinguishes it from the first one (probability of A) and thus requires a particular approach or certain steps to be carried out, which may not necessarily be the case, in the example of probability of A. Therefore, an understanding of the difference in meaning between the two phrases, as well as the distinction between them, is important in obtaining the correct solutions. In light of the example cited by Mr Twala, it is of vital significance to indicate that MWPs, unlike "ordinary" English passages the reader can only "scan through" and still get the overall meaning embedded in the "paragraph", necessitates thorough reading (i.e. "reading between the lines") without omitting certain words. This notion is also supported by the literature discussed in Section 3.5.3, namely that it is possible to read a story or novel in English superficially and still make sense of what you are reading. However, MWPs require thorough reading to be solved correctly.

From the above discussions, it is clear that mathematical vocabulary plays a significant role in mastering MWPs. In accord with this notion, Saville-Troike (quoted in Laplante, 1997:70) states that "vocabulary knowledge in English is the most important aspect of oral English proficiency for academic achievement". Since mathematics is a language that has its own technical terms, it would be unfair to expect learners to understand these terms on their own without being formally instructed on using them correctly. Teachers, therefore, need to teach the mathematical vocabulary and to empower the learners with the variety of terms that are used. The above discussions suggest the role of teachers in multilingual mathematics classrooms as that of "experts" that can assist in developing the learners' mathematical vocabulary over and above the teaching of the content. The discussions also indicate the essentiality to ensure that the learners understand the vocabulary so that they can apply it in a proper manner in other contexts as well.

5.3.3 Strategies to help scaffold visualisation skills

Mathematical visualisation is the process of creating images or constructing mental representations and using such images effectively for mathematical discovery and understanding (Scriven & Paul, 2005:2). Scriven et al. (2005:2) note that critical thinking is a mode of thinking about subject content or problems in which the "thinker" enhances the quality of his or her thinking by skilfully taking charge of the structures inherent in thinking and striking intellectual standards upon them. According to Makina (2010:24), "learners are not born with the power to think critically, nor do they develop this ability naturally. It is a learned ability, the development of which needs to be facilitated". In support of Makina's claim, Campbell, Watson and Collis (1995:177) encourage teachers to develop the learners' visualisation skills so that they can become mathematically competent and critical learners. Campbell et al. (1995:177) further encourage the development of related visual images and intuitive skills in all the developmental processes.

In line with the above, Presmeg (1986:42) states that the learning of mathematics requires generalised and abstract thought and, therefore, one is expected to have the ability to perceive clear mental pictures. Adding to this, Presmeg avows that all mathematical problems involve reasoning or logic for the solution thereof and the presence of visual images is important in developing learners' visualisation skills.

The participants also deemed visualisation skills as important in solving the MWPs. The following extracts highlight what the participants had to say with regard to the measures that must be put in place to promote visualisation of the word problems. The extracts below are data-generated from the reflection session that took place. The participants have reflected on these aspectsas follows:

> *Lihle: Mr Phatudi usually draws the graphs on the chalkboard and ask the questions that… that are related to the graph. You will hear him saying, "if ex (x) is two at this point what will the value of y be?" You will also hear him asking, "what happens if two is added to the equation on the right?"*

> *Bonolo: Sometimes he gives us problems… for example, "y is equal to x squared". He gives us the drawing sheets so that we can plot the two graphs on the same Cartesian plane and gives us the different colours to use when we draw the graphs.*

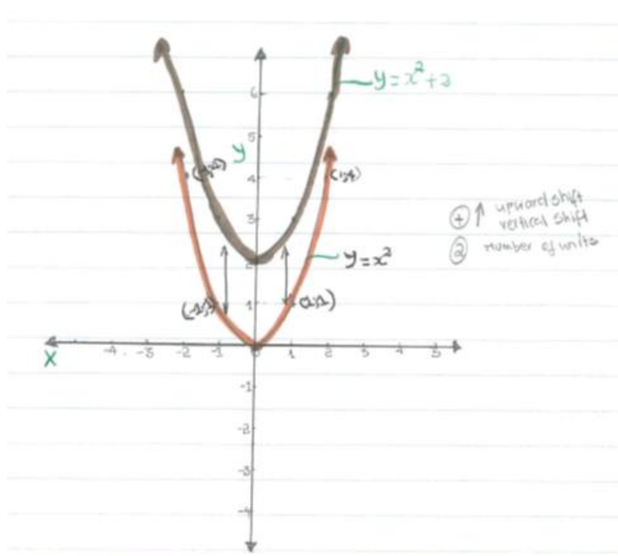
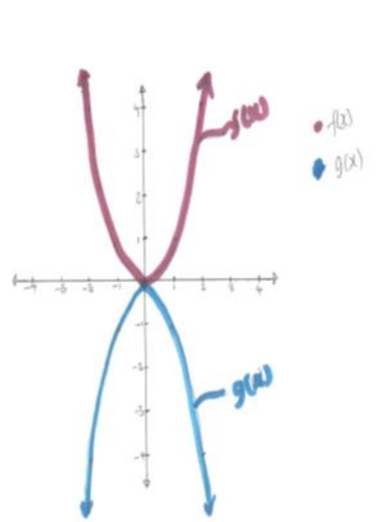


Figure 5.2: Parabolic graph – upward shift (picture taken from learner's notes)



$f(x) \Rightarrow (+a) - g(x) \Rightarrow (-a)$

Figure 5.3: Parabolic graph and its reflection *(picture taken from learner's notes)*

The above extracts, together with the pictures of the graphs that were taught to the learners by the teacher, indicate some ways in which Mr Phatudi (teacher) teaches the learners how to draw parabolic graphs. Lihle indicated that the teacher uses the chalkboard to sketch the graph. While drawing the graph, Mr Phatudi asks the learners questions that are related and would assist in the drawing of the graph. Drawing from Lihle's reflection of how they are taught to plot the parabolic graph, it came out that the teacher reads the problem and explains what the problem is about by highlighting the important features to note. He first ensures that the problem is understood and asks the learners questions related to the graph while simultaneously demonstrating how the graph should be plotted on the chalkboard. In this way, the teacher uses the chalkboard as a descriptive tool to show what the graph looks like. Even though Lihle did not say this verbally, it was clear from her description that Mr Phatudi also elicited their prior knowledge and used words that helped them imagine a picture. The statement, *"You will hear him saying; if x is two at this point what will the value of y be?"* indicates that the teacher does not teach the concepts in a "straight forward" manner; however, he wants the learners to make their own discoveries about the relationship between the x and y values. Discovery learning is built upon the constructivist view that holds the notion that the learner creates understanding through personal experience and interaction with external stimuli. Bruner's theory emphasises learning on discovery, which helps learners learn the varieties of problem solving by transforming information for better use, which assists the learners to learn how to go about every task of learning (Bruner, 1966a:87). This means that discovery learning requires teachers to teach learners to acquire information in a way that makes that information more readily viable in problem solving. Discovery learning thus promotes active learning and participation of the learners in the classroom. The description of the type of teaching that is provided by Bonolo echoes the epistemological stance of discovery learning theory, namely putting emphasis on the engagement of learners in class activities and allowing the learners to participate using their prior knowledge to create understanding.

The statement*, "You will also hear him asking, what happens if* 2 *is added to the equation on the right",* means that Mr Phatudi started by plotting the basic parabolic graph and ensuring that the learners understand it. This then served as prior knowledge for plotting the basic parabolic graph with "two units shift", that is, $y = x^2 + 2$. In order to check whether they could explain the changes that would take place when the new graph was to be plotted, the teacher added 2 as a constant in the equation. In order to make the concepts perceptible, for instance, the graph of parabola when "a", which is the coefficient of x^2, is positive and negative, the teacher used one Cartesian plane and asked the learners to plot the two graphs on the same Cartesian plane. In order not to confuse the graphs and to see the “shift” vividly, the teacher encouraged the learners to use different colours to distinguish which one has a negative coefficient of $x^2(-a)$ and which one has a positive coefficient of $x^2(+a)$. In this way, the teacher used the UDL principle of “flexibility in use” by using teaching resources that promote visualisation and make information perceptible. This provided learners with a different way of learning and experiencing knowledge. The practice of the principle of UDL flexibility in use demonstrates the teacher’s efforts in creating a classroom that reflects caring and inclusive practice values, which CER espouses. In this way, classrooms becomes a symbol of hope and a change that is needed for all the learners.

The other participants suggested the strategies that could be used to promote visualisation. They suggested as follows:

> *Mr Twala: Ka nako engwe ho ya thusa ho tla ka dintho tse tshwarehang classeng hore bana bana ba bone seo o buang ka sona. For example, ha o trita topic ya di shapes o katla ka khatbodo otlo bontsha bana di daemenshense live! Ba di bone straight ka mahlo!*
>
> ***ET:** In certain instances, it helps to bring to the classrooms the manipulatives so that the learners could physically see what we are talking about live.*
>
> *Mr Phatudi: Working together with the learners in order to draw the diagrams that depicts the problem also helps.*

In an effort to assist learners with visualisation, Mr Twala suggested that manipulatives be brought into the classroom (*“…dintho tse tshwarehang…”*) (**ET:** *“…tangible tools…”*) so that the learners may develop an in-depth understanding of the concepts through “seeing them in reality”. Sometimes the pictures in textbooks are not as clear as the tangible materials the teachers may bring to class. Therefore, when teachers bring concrete materials to class, learners may have a clearer picture of what the problem entails and develop a better understanding of the content, especially when they are engaged in the activities that involve the concrete materials brought to class. This confirms the Chinese proverb that says, *Tell me, I forget; involve me, I understand*. This means for the teacher to scaffold the learners’ visualisation skills through the use of diagrams, pictorials and manipulatives, the teacher needs to engage the learners in activities in which they can participate and discover the concepts on their own.

In addition to all this, Mr Phatudi deemed the use of diagrams in the classroom to depict the word problems as significant. He further recommended that the teacher should draw the diagram together with the learners. This would enable the teacher to scaffold the learners' understanding of word problems in terms of solving them and obtaining the correct solutions. The use of diagrams and manipulatives as visual aids and resources to improve learners' visualisation skills is also supported in the literature (see Section 3.4.3). However, in this study, a collaboration between the teacher and the learners when using manipulatives and sketching the diagrams was considered to be significant. In other words, learners should be regarded as important contributing partners in the construction of knowledge. Mr Phatudi's statement, "*Working together with the learners in order to draw the diagrams that depict the problem also helps*", indicates that even though the drawing of the graphs may be helpful to the learners to enhance their visualisation skills, it is still important for the teacher to teach the learners how to incorporate the use of diagrams when solving the problems. The notion of collaborative working is also espoused by CER. In this way the classrooms become spaces where collaborative working is exercised between the teachers and the learners. The notion of using diagrams as assistive tools in promoting visualisation is also supported by Vaughn et al. (2012:15) (see Section 3.4.3).

Considering the above discussions, the implications of both CER and CDA within the classroom context is that teachers need to see learners as equal participants in terms of making contributions and generating knowledge. It is, therefore, necessary for the activities to be designed such that they enable the learners to participate, thus making it possible for knowledge to be constructed jointly. Such a collaboration, which CER advocates in the process of knowledge construction, promotes the empowerment of learners and teachers and ensures that the learners' voices are also incorporated and heard during the classroom discussions. Furthermore, the discussions above indicate the significance of using diagrams, visual boards and manipulatives as assistive tools to help promote visualisation of the word problems.

5.3.4 Strategies to overcome the lack of understanding caused by the presence of ambiguous words

Saktel and Shrawankar (2012:71) note that most often learners face problems in understanding the correct meaning of the word problem sentence. According to these scholars, the correct sense of a word is obtained from the context of the sentence. These scholars further note that a sentence that comprises ambiguous words may not be understood in context. On the basis of this, it is important to remove and deal with ambiguous words that may lead to learners not understanding the word problems given. Ní Ríordáin et al. (2015:14) avow that statements that are not ambiguous enable learners, especially second language learners, to acquire the correct meaning (see Section 3.4.4).

To ensure that word problems are not ambiguous, the participants highlighted some of the strategies that could be employed during the meeting when the solutions to the challenges were identified. The participants commented as follows:

> *Ms Zwane: Ha re kgutlela exampoleng ela eo ntate ae entseng mona ya **"probability"** ha o sa e hlalosa hantle hore e bolelang, bana ba ka ofa dikarabo tse ikelang kwana. Thlaloso ya mantswe ana a tshwanang le bo probability e bohlokwa since anale di tlhaloso tse ngata.*
>
> ***ET:** Referring to the example that the father has provided earlier on the "probability" concept; when you do not explain the concept well in terms of what it means, then learners may give you wrong answers. An explanation of a word, such as probability, is important since it has diverse meanings according to the different contexts.*
>
> *Dibuseng: Di statements tseo re di fuwang di lokela ho ya straight to the point hobane ha di se jwalo di ya ferekanya and dija nako.*
>
> ***ET:** The statements that are given to us need to go straight to the point, otherwise they may bring confusion and waste time*
>
> *Lerato: Dipotso tsa di word problems di lokelwa hore di tjhekuwe ke di specialists hore di ya utlwahala kapa tjhe! When the questions are clear, they save time for us.*
>
> ***ET:** The word problem questions need to be evaluated by specialists to check whether they are understandable or not!*
>
> *Ms Masombuka: To avoid confusion and misinterpretation, I teach and explain concepts in English. When I realise that they do not catch what I am*

saying, then I use Sesotho to enable them understand the concepts and the differences.

The participants' views above indicate that some of the words used in word problems may have more than one meaning and thus cause ambiguity. As a result, the learners may misinterpret the given statements by using the incorrect meaning in the specific context. To overcome this challenge Ms Zwane recommended that the teachers should explain the "key" terms when they are teaching, provide their synonyms and show how they can be applied in different contexts. In addition, Dibuseng indicated that word problems must be evaluated by language specialists before they can be administered for assessment purposes. Dibuseng also indicated that word problems that are not ambiguous "save" the learners examination time since they are clear and can be easily understood and contextualised. According to Dibuseng, the role of a language specialist in this regard is to ensure that the problems are not ambiguous, which promotes the conceptual and procedural understanding of the problems. This is true, since generally one can attest to instances in which learners, at times, fail to finish writing the examinations due to questions that were supposedly unclear and required the learners to read them repetitively, thus wasting time.

Ms Masombuka drew from her own teaching as a reference, indicating how she dealt with words that might cause the problems to be ambiguous. She highlighted the use of Sesotho in instances in which the learners do not understand the concepts when they are taught and the problems explained in English. The use of Sesotho as a "resource" when teaching is a practice that is supported by Clarkson (2007:193), who claims that learners process thoughts more easily in their home languages since the home languages facilitate semantic processes (see Section 3.4.4). This practice is in line with the CER principle of inclusivity to ensure that all the learners understand the content, regardless of the difference in linguistic proficiencies. This is also in line with the UDL principle of simplicity and intuitiveness which requires word problems to be formulated in a manner that is not ambiguous and would not cause distractions that may impede understanding. Since the learners come to class knowing a range of meanings, it is important that the teachers develop the learners' interpretations and translations of words that in many instances, are influenced by linguistic structures in their home languages and the interaction between the two languages, namely the home language and the LoLT.

On the basis of this, it is reasonable to conclude that the meanings of mathematical words cannot be considered in isolation from how learners understand these words in their home languages. The use of home language as a resource for teaching thus ensures that the necessary information is communicated effectively and clearly to avoid misconceptions that may cause the word problem to be ambiguous; this speaks to the UDL principle of perceptible information. In this way, the teacher creates a hybrid space in the classroom where learners can incorporate their home-based discourses with disciplinary discourses, thereby resulting in learning that takes place at the intersection of the two, manifesting itself as a hybrid epistemological and discursive construct, as Temple et al. (2012:288) also avow in Section 3.4.4.

The extracts above indicate that teachers need to indicate the application of the word(s) in different contexts to make the learners aware of the differences. Furthermore, the extracts advise that there should be careful reviewing of the construction of word problems by language and subject experts to avoid ambiguity that may cause confusion. The implication of CER and CDA, in line with this, is that the clarification of the words in different contexts need to be provided to ensure that the answers provided relate to the topic at hand and in the relevant context. Through CDA, we are able to understand the implications of the use of language within the classroom and to understand the connotations in context. The above discussions indicate that when learners are clear about the statements provided, they do not become confused, and therefore, they are able to conceptualise the problem and solve it procedurally. On the same note, it should be understood in this context that language has to be represented in a manner that would make the given problem clear and simple to understand and respond to.

On the basis of the above discussions, it is reasonable to indicate that word problems that are not ambiguous make it possible for the learners to understand, conceptualise the MWPs and ultimately choose the appropriate operations to solve MWPs correctly. Again, considering the above discussions and meanings embedded within the given extracts, it is clear that the learners' interpretations and translations of words are usually influenced by the linguistic structures in their home languages and the interaction between the languages they speak and the medium of instruction (i.e. the home language and LoLT). It is also clear that the meanings and associations of mathematical words cannot be considered independent from the learners' understanding of these words in their home languages. It is, therefore, important to note that providing word examples necessitates not only knowing the meaning of the words but also an understanding of the subject content, concepts and the context in which the words are used.

5.3.5 Strategies to empower teachers to teach word problems in multilingual classrooms effectively

Formal teaching includes the construction of understanding through the use of language to enable the learners to make, among others, increasingly sophisticated generalisations (Creese, 2005:146). In this way, language becomes a tool for interpretation (Duit et al., 1998:33). Since many second language learners who do mathematics struggle to master the mathematical concepts because they lack proficiency of the LoLT, Essien (2013:3) deems it significant and emphasises that the teachers need to be empowered to teach mathematics in multilingual classrooms confidently.

In line with the above, some of the strategies to empower teachers to teach MWPs in multilingual mathematics classrooms were highlighted by the participants. They remarked as follows:

> *Ms Nkosi: I usually refer to an exponent as a power when I teach the concept of exponents. I never thought there was anything wrong about that until I attended a workshop where I learned that it was incorrect to say that. I wondered then, how many teachers were still teaching learners like that.*
>
> *Mr Nzuza: The only way for us as teachers to be able to effectively teach learners in these classrooms, which comprise of learners from different cultures, is to be trained effectively. I think itraining izo sisiza kakhulu [**ET: "…** It will help us a lot"] especially on how to teach these learners!*
>
> *Mr Simelane: Indeed training is important, especially on the usage of language in the classroom as well as how to teach content. It would be advantangeous if we could work with the language specialist.*

The extracts above indicate that teachers need training to teach MWPs effectively to learners, especially in multilingual classrooms. The issue of training was stressed by the participants since they considered the teaching of MWPs in multilingual mathematics classrooms to be complex as it requires the mastery of mathematical language, content and pedagogy. In emphasising the need for teachers to be trained, Ms Nkosi drew from her language practice by showing how she uses language in her teaching and how her language practice has a negative impact on learning. She cited the exponential example to support the fact that teachers need to be empowered on how to teach mathematics effectively in multilingual mathematics classrooms.

From her example, Ms Nkosi uses the term “power” to denote an exponent, which is incorrect. However, during the workshop she attended, she realised the mistake she has been making in her teaching all along. The long-term effects of Ms Nkosi’s language practice in that case was that she taught an exponential concept in a manner in which the concept was verbally uttered “incorrectly”, but written correctly, for instance, “two to power three” (verbally uttered) written as (2^3) . The mathematics teachers often refer to a “base – exponent” as a “base to power”, which is incorrect. The statement, “*I wondered then how many teachers were still teaching learners like that”,* confirmed the fact that the mistake was “still taught” to the learners; this supported the notion of the need for teacher capacity and language practice development. The issue of teacher training and development was also recommended by Essien (2013:57), as highlighted in Section 3.4.5, due to the complex nature of multilingual mathematics classrooms. According to Ms Nkosi, it is of paramount significance that teachers’ mathematical vocabulary and register should be advanced so that they would be able to meet the needs of the learners.

Mr Nzuza also shared a similar sentiment as Ms Nkosi’s; hence, he commented, “*The only way for us as teachers to be able to teach learners effectively in these classrooms, which comprise of learners from different cultures, is to be trained effectively*…”. The fact that the classrooms are comprised of learners whose home languages are not used as the media of instruction is reason enough for the teachers to be trained and empowered to carry out their teaching within classrooms of this nature, which literature indicates is complex (see Section 3.3.5). This notion supports Tsotetsi’s (2013:42) stance that professional teacher development is significant for teacher empowerment in terms of the realisation of effective teaching (see Section 3.4.5). Ms Nkosi's suggestion of further teachers' training as "the only way" for them to teach effectively in multilingual classrooms demonstrates a caring attitude, as CER advocates. The teacher deems the training significant in empowering them to be able to nurture and meet the needs of learners within multilingual mathematics classrooms.

The current study adds a critical aspect to the previous studies – that mathematics teachers need to work closely with "language specialists" for them to draw some lessons on language practices. The language specialists Mr Simelane was referring to in this case are the teachers who teach English as a subject. Mr Simelane considered the English teachers to be the immediate resources that could aid their teaching by shaping them in the appropriate use of English language, which serves as the medium of instruction. This reflects an element of collaboration, which CER promotes in terms of addressing what is considered to be a problem. A practice of this nature whereby teachers work collaboratively, also speaks to the UDL principle of a community of "learners" that promotes interaction and communication among the teachers and, consequently, deepens an understanding of how effective teaching could be designed. This indicates that "training only" is not enough; however, a constant interaction among the teachers is necessary in assisting them to put into practice the lessons they draw from the language practitioners. The application of CDA as a tool for interpreting data, in this case, assists us in understanding the participants' thoughts better when highlighting what they deem to be significant or the solution in terms of empowering the teachers to teach MWPs effectively in multilingual classrooms. CDA also enables us to understand and interpret people's feelings and thoughts through text, verbal and nonverbal communication, and so forth.

The above discussions indicate the significance of improving teachers' pedagogical knowledge as well as the understanding of language use when teaching learners in multilingual mathematics classrooms. The participants deemed further training as key to enhance the teachers' skills on how to teach and improve the language practice. The discussion also indicates that such training provided to the teachers enables them to cope with the demands of teaching mathematics in multilingual mathematics classrooms, which requires that the learners effectively learn the subject content and the mathematical language while still learning the LoLT. The teachers' recommendation of further training supports the saying, *"learning is a continuous process and one can never cease to learn and improve"*.

5.3.6 Promoting the effective use of home languages within classrooms

The social-constructivists perspective on language in the teaching and learning of mathematics is that both the first language speakers and those whose language is different from the LoLT face challenges when they are in the mathematics classroom. This is because language as a key concept in culture becomes "mathematics language", which can, in turn, affect learning (Duit et al., 1998:18). Although both the groups of learners become challenged in class, Creese (2005:147) unequivocally states that those whose language differs from the LoLT are even more challenged since they must concurrently learn a new language and achieve academic competence at the same time. In light of these perspectives, it is imperative for teachers to use language in a profitable manner to support and meet the needs of all the learners in the class. A similar notion was echoed by the participants in a meeting where the strategies to address some of the highlighted challenges were deliberated upon. The participants commented as follows:

> *Mr Phatudi: Taba ena e "two-way": English ekaba problem hape ekanna ya sebe problem. Re nale mantswe arona a sebediswang feela mmetseng and therefore aka etsa bana ba kgone ho utlwisisa hobane English re e sebedisa feela for sentence construction. Empa hare sebedisa English feela, re tlabe re ba thatafalletsa le ho feta hobane jwale bantse ba ithuta puo ena eo eseng leleme la habo bona.*

> ***ET:** This issue is a "two-way matter": English could be a problem and not a problem at the same time. We have words which are only used in mathematics and these words could help learners understand since we use English only for sentence construction. If we use English only in our classrooms, then we could be making it difficult for them to cope because they are still learning this language, which is not their mother tongue.*

> *Mr Twala: Nna ke nahana hore tshebediso ya di home languages le English ka classeng e bohlokwa.*

> ***ET:*** *I think the use of both the home language and English in class is important.*

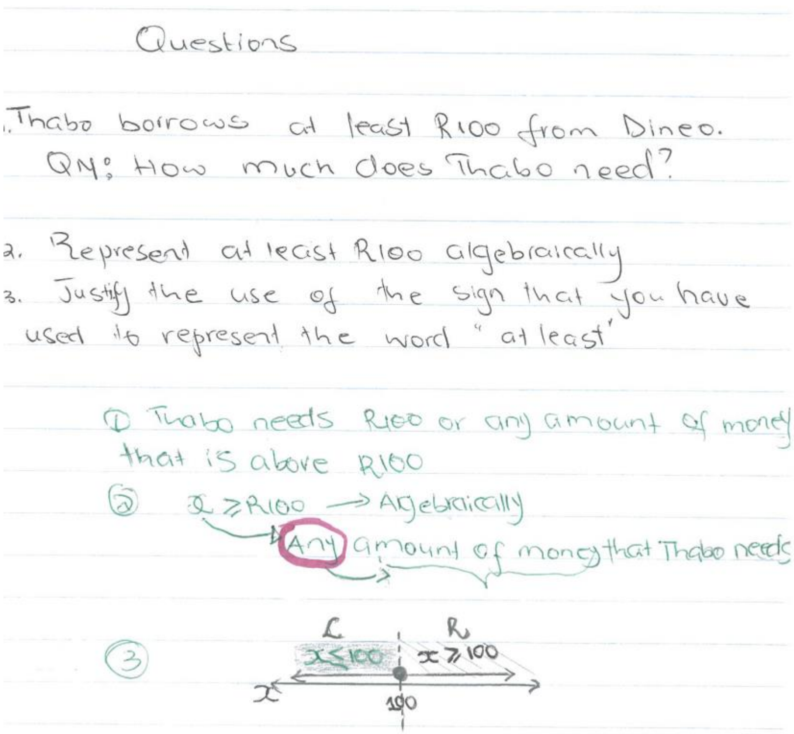
The participants' comments above indicate that tension still prevails in terms of which language(s) should be used to teach learners in class. Some teachers still embrace the sole use of English, while others recommend the use of both English and the learners' home languages. However, the teachers are aware that English is still widely recognised as the language of communication by many people from different backgrounds and different settings, including educational settings. In line with this sentiment, Essien (2013:5) notes that fluency in English is usually perceived as an "emblem of educatedness". A similar perception can also be identified in our societies, in which some of the people are regarded as highly educated and given respect because they are fluent in English. However, the sole use of English may be a disadvantage to some learners since most of them are still learning English and cannot speak it fluently. On the other hand, depending too much on the use of learners' home languages could have a negative impact on the learners' ability to express themselves fluently in English, hence the comment, "*Taba ena e 'two-way', English ekaba problem hape ekanna ya sebe problem"* (**ET:** "*The word 'two-way' indicates that there are still tensions and different opinions on the sole use of English when teaching in class").*

According to Mr Phatudi, when the learners' home languages are also used in class, specific words in the (English) mathematical register could be used in conjunction with the learners' home languages to facilitate the learners' understanding of the content. Hence the statement, "*Re nale mantswe arona a sebediswang feela mmetseng and therefore aka etsa bana ba kgone ho utlwisisa hobane English re e sebedisa feela for sentence construction"* (**ET:** *"…We have words which are only used in mathematics and these words could help learners understand since we use English only for sentence construction"*). Mr Phatudi further believes that learners can still understand and solve mathematical problems, even if they are taught predominantly in their home languages, as long as they are well versed in the key mathematical terms or concepts. His argument is that mathematics has its own "unique words" that are specifically used only in the context of mathematics to describe certain aspects. These words can assist the learners to understand mathematical concepts, even if the home languages are taught. The fact that mathematics has its own unique words, which are significant and can facilitate mathematical understanding, is also stated by Nkambule (2009:19) in Section 3.3.2. Therefore, if learners understand those words, they can still understand the topics that are presented to them even if the topics are presented in their home languages and not necessarily entirely in English. Although Mr Phatudi seemed to support the use of both languages (English and learners' home languages), he emphasised that English only should not be used in class since most of the learners lack English fluency and are still learning the language (Adler, 2001:4).

Mr Twala also believes in the use of both English and the learners' home languages in the mathematics classroom. He regarded the practice of code switching as quite important in terms of assisting learners in understanding MWPs. Mr Twala also believes that learners can still grasp the concepts, even when they are presented in their home languages, as mathematics has its own vocabulary and register. This was also deemed significant by Mr Phatudi, who was of the opinion that learners must not be taught in English only.

Furthering the discussion around this issue, Ms Nkosi commented as follows:

> *Ms Nkosi: Le teng bana ha ba rutwa fela ka Sesotho kapa ISizulu re yaba bolaya hobane di potso ka exam di tla be di ngotswe ka English. Hape eka ba disadvantage in other areas.*
>
> ***ET:** We "kill" these learners when we teach them in Sesotho or Isizulu because the exam questions are usually posed in English. The sole use of home languages would further disadvantage them in other areas.*
>
> *Tau: Like a tjholo hore English ke bothata, hodima hole jwalo motho ontse osa utlwisisi maths jwale hee ekaba problem e kgolo hare ka rutwa ka English feela.*
>
> ***ET:** As he has already alluded to the fact that English is problematic and that we still do not understand maths, it would be more challenging for us to learn mathematics if we could only be taught in English.*
>
> *Mr Nzuza: The problem I gave them in class reads as follows…[reading from note book].*


Questions

1. Thabo borrows at least R100 from Dineo.
QN: How much does Thabo need?

2. Represent at least R100 algebraically
3. Justify the use of the sign that you have used to represent the word "at least"

① Thabo needs R100 or any amount of money that is above R100
② $x \geq R100$ → Algebraically
Any amount of money that Thabo needs
③ L R
$x \leq 100$ $x \geq 100$
x 100

Figure 5.4: Word problem and questions (picture taken from the teacher's notes)

Ms Nkosi alerted to the sole use of learners' home languages as it may bear negative consequences on the learners' part. She mentioned the writing of examinations as one area of concern since the mathematics question papers are set in English only. She further expressed concern that negative consequences may also arise in some areas where the use of and proficiency in English are of key importance. According to Ms Nkosi, the use of English in the classroom may assist in developing fluency and in preparing learners for examinations and other areas. Ms Nkosi, however, believed that the teachers need to strike a balance between when to use the learners' home languages and when to use English. A valuable aspect from Ms Nkosi's argument is that mathematical sentence construction does not depend on the key terms only, as Mr Phatudi's had connoted. Ms Nkosi also emphasised that it is essential that the learners understand English "in totality" without relying on key terms only to determine or formulate or make sense of what the given problem(s) entails.

The practice of code switching in the classrooms was also supported by Tau, one of the learners. He claimed that most learners do not understand both **English and**

mathematics and that the sole use of English as the medium of instruction would create even more challenges for learners since they cannot master both of them at the same time, hence the comment, "*English ke bothata, hodima hole jwalo motho ontse osa utlwisisi maths jwale hee ekaba problem e kgolo hare ka rutwa ka English feela"* (**ET:** "*English is problematic and that we still do not understand maths, it would be more challenging for us to learn mathematics if we could only be taught in English*")*.* Therefore, the use of learners' home languages in this regard could serve as a resource in bridging the gap between lack of fluency in English and the language of mathematics. This is in line with CER, which advocates hope and social justice. Home languages in this regard are used as the "enabling" resources to facilitate understanding and assist the learners in the acquisition of the LoLT, while also learning mathematical concepts. The understanding the home languages brings about, therefore, gives hope to the learners in learning the word problems. The practice of using the home languages as resources for learning serves as one form of social justice in the classroom in which many learners are not proficient in the LoLT and are still learning it.

Mr Nzuza highlighted how he taught an inequality concept in which the learners were engaged with the phrase *"at least"*. From his narration, it seems to be of great significance for teachers to first recap the meaning of the phrase "at least" to assess whether the learners understand what it means. The teacher did this to establish if the learners understood the meaning of the phrase in different contexts. He elicited as much information as possible from the learners to ensure that he got an idea of how much the learners knew about that phrase. Through this exercise, he was able to explore the learners' ideas in their home languages. The practice of determining learners' prior knowledge, gaps and misconceptions serves as one other UDL principle that promotes good teaching. Teachers who follow this practice seek to be proactive in their teaching so that they can address the barriers towards learning on time and to subsequently design their teaching such that it meets the needs of the learners. After establishing that the learners know the meaning of the phrase "at least", Mr Nzuza then indicated (to the learners) how the phrase is usually used on "the streets" and in a mathematical context. He further indicated that it is not sufficient for the explanations to be provided verbally; however, it is important that they should be translated into mathematical symbols (algebraically) and for such illustration to be represented. The teacher thus created a learning environment for learners in which they could develop mathematical understanding of the different contexts in which the term “at least” is applied as well as symbolic representation. The teacher, therefore, assisted the learners to negotiate the meaning of the phrase “at least‟, not only in their home languages but also in mathematical language, using algebraic representation. The number line was used as an alternative way of expressing or illustrating the word problem algebraically and served as one powerful UDL visual cognitive tool that made information perceptible, simple and intuitive for the learners.

From the above discussions and the questions asked (in Figure 5.4), one can deduce that the teacher asked the learners to explain the meaning of the phrase because he was aware of the different connotations attached to the phrase. On the streets, the phrase “at least” is usually used to denote “the situation that an individual is confronted with, which is perceived to be better than the one that is faced by another person”. Hence, we hear everyday statements such as “at least wena onale dijo, o reng kanna ke senang le sente?” (***ET:*** *“at least you have food. What about me who do not even have a cent?”*) Again, this phrase “at least” can be used to denote “little” in languages such as Sesotho and Xhosa. In Sesotho the phrase “at least” is usually used as “bonyane” and in Xhosa as “bucinci”, of which in both the languages the phrase denotes “little”. However, if a word problem is provided, as shown above where the phrase “at least” is used, then the symbol (≥), which denotes greater or equal to, is used instead of the symbol (≤), which denotes less or equal to. In this context, one could draw that even though the word “less” can easily be associated with “little”, which is embedded in the phrase in the learners’ home languages as well as in English, its symbolic representation requires the greater or equal to sign to be used. CDA, in this instance, assists by ensuring that the contexts are distinguished clearly and the correct meanings are captured and attached according to the specific contexts.

The above extracts indicate that the use of learners’ home languages is equally important in the teaching and learning of MWPs as it aids learners’ comprehension of word problems. This is supported by Ntshangase (2011:5), who unequivocally states that learning mathematics in English only has a negative impact on the cognitive development of English second language learners. This means that the learners' home languages can be used as resources that can assist the learners to gain epistemological access. In line with this notion, Lotz-Sisitska (2009:57) contends that teachers should find new ways of thinking about teaching MWPs so that they can, in turn, enable all learners gaining such epistemological access.

The notion of using learners' home languages, in the context of this study does not refer to "*the total use of*", that is, the use of home languages for the full mathematics period, but only for clarifying concepts and enhancing learners' understanding of the concepts. This is in accord with Setati (2008:107), who discourages the overuse of learners' home languages in multilingual classrooms and claims that such a practice keeps the learners from acquiring proficiency in English, which is a global requirement (see Section 3.4.6). The participants thus regarded the use of learners' home language(s) as a resource that offers cognitive advantages to the learners. The use of learners' home languages was also deemed significant in enhancing and deepening an understanding of mathematics concepts. The use of home languages, therefore, makes it possible for learners to develop conceptual understanding since they have a richer network of associations in their home languages than in their second language (English), which is a point that was also noted by Clarkson (2007:193) in Section 3.4.4.

5.4 CONDITIONS CONDUCIVE TO THE SUCCESSFULL IMPLEMENTATION OF THE STRATEGY

Although the solutions to the problems can be identified, it is imperative to also identify the factors or the conditions conducive to the solutions to work. This indicates that the successful implementation of the strategy depends on a number of factors. The following sections highlight these factors.

5.4.1 Factors supporting the enhancement of learners' reading skills

Reading skills are significant in facilitating the growth and development of learners. It helps learners to decode a text and analyse, explain and express their own ideas regarding the concepts about which they are reading. It is, therefore, imperative for the learners to develop a strong ability to understand written material to cope with the MWPs. However, for learners' reading skills to be enhanced, certain factors play a significant role. In line with this, some of the factors that support the enhancement of the learners' reading skills were identified by the participants. An interesting example was captured from the rough work in the learners' homework or classwork books. The participants commented as follows:

> *Mr Twala: Ntho e ka thusang ke ha bana ba ka rutwa ho bala. Re bafe le di word problems tse ngata tse fapaneng hore ba tlwaele ho bala.*
>
> ***ET:*** *What would help, is when learners could be taught how to read. We need to also give them many different word problems to solve so that they can be used to reading.*
>
> *Mr Nzuza: Matitjhere a sekgowa a lokela hore thusa. Mohlomong ka di studies ba rute bana ka mekgwa ya ho bala ebe rona re ba etsetsa mehlala ya tshebediso ya mantswe a itseng ka thuso ya matitjhere a sekgowa.*
>
> ***ET:*** *English teachers may assist in this regard. They can teach learners how to read and then we (mathematics teachers) can provide them with the examples on the use of certain words through the assistance of English teachers.*

The above extracts indicate that learners need to engage in quite a number of reading exercises for their reading skills to improve. This is in agreement with the literature in Section 3.5.1. The extracts also indicate that the learners must be provided with a variety of word problems so that they can be exposed to numerous ways in which the problems could be phrased or expressed, hence the statement, "*Re bafe le di word problems tse* ***ngata tse fapaneng****..."* (**ET:** *"We need to also give them many different word problems to solve so that they can be used to reading")*. According to Mr Twala, this would get them accustomed to various forms of text that they need to know and understand to master MWPs. This is one finding that came from this study as a condition conducive to improving reading skills.

Furthermore, it is important for the mathematics teachers to collaborate with the English teachers. The English teachers' role in this collaboration would be to assist learners with reading techniques and clarification of various forms of text, while the mathematics teachers would teach learners about the application of these various forms of text within a mathematical context (e.g. word problems). Mr Nzuza deemed the relationship between the mathematics and the English teachers significant since it enhances text application knowledge and facilitates an understanding of various forms of text. The mathematics teachers seemed to be aware of the significance of reading, which is why they recommended the involvement of the English teachers, as the language experts, to help bridge the language (English) knowledge gaps. The mathematics teachers acknowledged complementing their colleagues in the joint process of teaching and shaping a well-informed envisaged type of learner whose reading skills are good enough to master word problems. In this regard, we see teachers identifying the resources (English teachers – human resource) they already have in their school to assist them in terms of achieving their goal. The teachers thus "tinker" the usual teaching of MWPs by the mathematics teachers by bringing in English teachers to produce collaborative teaching aimed at addressing the lack of reading skills. Furthermore, from the extracts above, it is clear that collaboration could be made possible only if both sides buy into the idea of working together and are willing to participate. The teachers' collaboration in teaching MWPs, taken from the participants' comments, is the contribution by this study in terms of improving learners' reading skills towards mastering MWPs. The teachers seemed to be aligning themselves with the CER principle of collaborative working for teachers to share good practice and to build and empower one another.

Adding to the discussions around the factors that could contribute towards the enhancement of learners' reading skills, the participants commented as follows:

> *Pitso: I think re ka thuseha haholo ha matijhere aka re fa nako ya ho bala ba be bare botsa ka di groups hore re understanda eng from the word problem.*
>
> ***ET:** I think we could benefit a lot if teachers could give us time to read on our own. They should also find out from the groups in class what they understand regarding the given word problems.*
>
> ***Mr** Phatudi: I usually read the question slowly, piece by piece while explaining the concepts at the same time. As I read I also write the symbolic notations to illustrate the concepts.*
>
> *Mr Phatudi: […!] now o nkgopotsa example enngwe so… mola ho topic ya di inequalities. [**ET:** "You remind me of a particular example from the Inequality*

topic"]! If you ask learners to indicate $3 \leq x \leq 5$ *on the Cartesian plane baya e fosa ba e bontsha so [showing how the learners work out the problem] [**ET:** "…they get it wrong and illustrate it like this [showing how the learners work out the problem"]… which is wrong. It becomes worse when, now the same problem is written in words. So ke dumellana le wena mam ha ore reading etla thusa haholo. [So, I agree with you, Madam, when you say, effective reading helps.]*

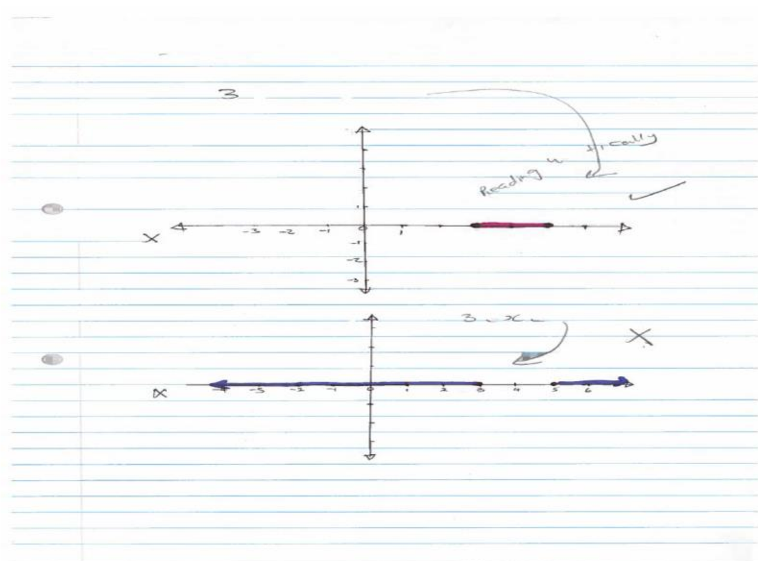


Figure 5.5: $3 \leq x \leq 5$ **representation on Cartesian plane: Correct and incorrect solutions (picture take from learner's homework/class work book – from the note section)**

Pitso's comment indicates that teachers need to assess learners' comprehension of what they have read and claim to understand. One way of doing so, is by dividing the learners into small groups and then give each group word problems to solve. However, before beginning to solve the problems, each group should read the word problem and then give their interpretation of the given word problem (explain what they think the problem is about). This would allow the teacher an opportunity to probe and elicit answers from the learners, through which the teacher can identify the gaps and misconceptions caused by the learners' lack of reading skills. Thus, in the process of improving learners' reading skills, teachers must assess the learners' interpretation of the word problems carefully as one form of evaluating their understanding through reading. This would give the teacher an opportunity to recognise any misconceptions quickly and address these accordingly to prevent further confusion. Giving learners an opportunity to express what they understand from their reading while the teacher simultaneously tries to identify the gaps, serves as a diagnostic type of assessment, which helps teachers to classify learners' mistakes. This is in line with the UDL principle of tolerance for error. This encourages the teachers to turn the classrooms into discursive spaces in which the learners can freely participate, engage in discussions and try solving the problems through committing errors and learning from such experiences.

During the conversations around the issue of engaging the learners in reading, Mr Phatudi remembered a typical word problem from the linear programing topic that involve the equality signs ≤, ≥, and so forth, which the learners usually struggle with and fail to answer correctly . Making reference to the example "x is greater or equal to three (3) and x is less or equal to five (5) ($3 \leq x \leq 5$)", Mr Phatudi demonstrated how he taught his learners this concept. He stressed that it was important for the teacher to read "out loud" while pausing to explain the concepts. The statement *"piece by piece while explaining the concepts at the same time",* indicates that the teacher needs to break the statement into smaller phrases and explain each phrase. This assists learners to develop an understanding of the word problem. It is also important that one writes down the important points while reading, drawing from the word problem since they assist in understanding what the word problem is all about and what it requires for the learners to solve it. Mr Phatudi made reference to the example he provided, namely $3 \leq x \leq 5$ to indicate that it means $x \geq 3$ and $x \leq 5$. It is important that the teacher draws the learners' attention that from the given example, $3 \leq x$ (reading from left to right) means $x \geq 3$ when reading from right to left. This should be emphasised since this would enable learners to realise that when reading from right to left, the sign "less or equal to" ($\leq$) changes to "greater or equal to" ($\geq$). Adding to this, the teacher needs to indicate that the expression $3 \leq x \leq 5$ denotes all the numbers (represented by the letter x) starting from 3 to 5. The numbers 3 and 5 are also included in the "set" since the symbol ($\leq$) is used. From this point one realises that it is significant for the teacher to draw the learners' attention to the fact that $3 \leq x \leq 5$ is different from $3 < x < 5$ and that in the latter expression the letter x denotes all the values that are between 3 and 5 and, therefore, 3 and 5 are not included as in the former expression.

All this needs to be clarified as the teacher is reading so that the learners can become accustomed to how the text is expressed and also how it should be represented symbolically. The use of a *number line* (as a non-language alternative) to represent the two expressions serves as complementary and thus makes the information perceptible according to UDL. The practice of explaining the concepts enables verbalisation using mathematical language and correct representation. Adler (2002:4) deems verbalisation as a powerful tool for thinking and for teaching since it affords learners an opportunity to unveil what they are thinking or what they know.

5.4.2 Factors contributing towards the improvement of learners' mathematical vocabulary and register

Developing the language of mathematics is an important aspect of teaching mathematics to learners and this process endures throughout the learners' mathematics education. Since the understanding of mathematical vocabulary affords learners access to concepts and mathematical instruction in the areas of language, it is, therefore, imperative for teachers to teach mathematical vocabulary effectively (Riccomini, Smith, Hughes & Fries, 2015:235-236). In order for the teaching of the mathematical vocabulary to be successful, it is important to be mindful about the factors that facilitate the teaching of mathematical vocabulary to improve the learners' mathematical vocabulary. Some of the factors to be mindful of were identified by the participants. They commented as follows:

> *Ms Masombuka: These mathematical terms need to be stressed when teaching learners. We need to also allow learners to talk within the classrooms.*
>
> *Tseko: O ka etsa nthonyana e kang pamphletenyana e hlalosang di terms tsa bohlokwa mabapi le concept e itseng. Ebe o fana ka mantswenyana a sehlotshwana ho hlalosa term e itseng. Sena se tla thusa bana haholo le ho increasa vocabulary ya bona.*
>
> ***ET:*** *You can design a pamphlet that consists of words that explain the important different terms that relate to a particular concept.*
>
> *Lerato: Ntho e ka thusang hape ke ha re ka ba le ntho e kang di tutorials teng moo bana ba ka buang ka dipuo tsa bona to enrich their understanding. Le vocabulary baka e thuta yona haholo hona ditutorialeng moo.*

> ***ET:*** *It would help to have tutorial sessions in which learners could speak in their home languages in order to enrich their understanding. They could also learn mathematical vocabulary during the tutorials.*

Teachers need to explicitly teach and emphasise specific terms when they are teaching MWPs to develop learners' in-depth understanding of the mathematical vocabulary. This aspect is also supported by the literature that teachers who teach learners and provide them with regular opportunities help them to re-engage in varied activities that subsequently develop and enrich their knowledge (see Section 3.5.2). Explicit teaching of the mathematical vocabulary and register serves as a good way of providing learners with alternative access to key vocabulary and mathematical language. Furthermore, teachers also need to provide the learners with opportunities to "communicate mathematically" in the classroom. This kind of interaction will assist learners with becoming familiar with mathematical terms and promote the unique internal organisation (textual structure) that can be logically interpreted, as highlighted in Section 3.5.2. The practice of providing learners with opportunities to communicate mathematically is in line with CER, which advocates the creation of opportunities for learners to communicate freely (mathematically).

What could also enhance the learners' understanding of the vocabulary and register of mathematics, as noted by Tseko, is compiling a glossary of words regarding a specific concept, with a clear definition of each word within the context of the application of the specified term. This would not only promote learners' "familiarity" with the terms but would also increase their vocabulary since they would be frequently exposed to these types of problems. The compilation of a glossary serves as a contribution this study makes in addition to teaching learners the specific meanings of the key terms. The glossary will, therefore, serve as a point of reference for learners to be reminded of the specific terms, even when the teacher is not present. This is in line with the CER principle of empowerment, which encourages the teachers to turn classrooms into spaces where learners could learn freely and thus be empowered to be able to cope even when the teacher is not around. The provision of the glossary serves as one way in which the teachers could inculcate and develop learner autonomy.

Furthermore, from the extracts above, it is clear that the implementation of tutorial sessions could also be of great assistance in broadening the learners' mathematical vocabulary. During tutorials, learners have the opportunity to work without the presence of the teacher (a figure of authority). Here, learners can talk and participate freely, using the languages they are comfortable with. Tutorials can, therefore, be used to enrich the learners' understanding of the mathematical terms and also serve as platforms where learners can practice how to communicate mathematically, thus improving their mathematical vocabulary and register. Tutorials therefore serves as one UDL strategy for sustaining effort and persistence.

5.4.3 Contributing factors towards successful elimination of ambiguity

According to Saktel and Shrawankar (2012:71), word ambiguity removal is a task of eliminating ambiguity from a word; through this process, the correct sense or meaning of the word is identified from ambiguous sentences. These scholars further note that sometimes people face problems in understanding the correct meaning of the sentence since the sentences are comprised of ambiguous words. Based on this, it is important for ambiguity to be eliminated and the contributing factors towards the successful elimination of ambiguity to be identified.

The participants highlighted some of the contributing factors towards the successful elimination of ambiguity in the sentences. An example was also drawn from the question paper. They commented as follows:

> *Ms Nkosi: The word problems need to be revised before the question papers are given to the learners.*
>
> *Tseko: It would help if we are taught the real-life scenarios whereby we can be shown how a particular word can be used in different situations.*
>
> *Mr Twala: Re tshwanetse hore re kgothaletse bana ho bala carefully [**ET:** "…We need to encourage learners to carefully read] and to also consider the important words."] Hapehape bana ba tshwanetse ho etsuwa aware hore haba bala ba utlwisise the question in total… [**ET:** "Furthermore, the learners need to be encouraged to read and understand the question in full."] Hapehape ho kathusa hore bana ba fuwe di scenarios le di pictures tsa tsona at the same time like in this instance [showing an example]*

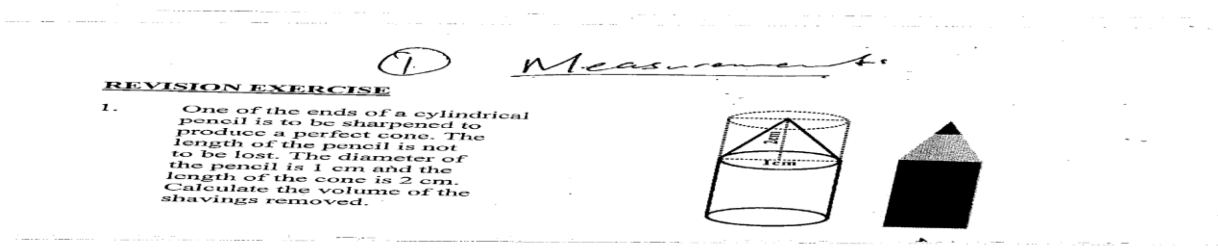


Figure 5.6: Word problem with a picture alongside (picture taken from the question paper)

According to Ms Nkosi, language practitioners need to review word problems to ensure that the statements are clear and will not cause confusion. Should the reviewer find that there are problems with the statements, it should be rephrased in such a way that the ambiguity is removed. This notion is also embedded in the UDL concept that advocates that language must be used carefully and that ambiguity must be removed since it creates barriers for learning. However, learners also need to be encouraged to read the word problems carefully before attempting to answer and this is in accord with the literature in Section 3.5.3, which states that translation ambiguity can be avoided by careful reading. This would ensure that they do not miss any word that may be significant in solving the relevant problem, as was previously illustrated by the example “determine the probability of A, which is different from determining the probability of A only”. According to Mr Twala, “only” is the indicator of how the problem must be approached and should the learner not “see” that word, the solution might be incorrect. Furthermore, illustrations of the implied context in the form of pictures or diagrams may also assist in eliminating ambiguities. This would ensure that the learners read the problem and understand it within the correct context. This serves as one UDL alternative of visual information representation that enhances perception.

To avoid translation ambiguity on the learners’ part, they should be encouraged to avoid “linear reading” while simultaneously translating word for word between the mathematical concepts. They should be encouraged to read carefully between the lines to have a holistic understanding of the given word problem. The words “in total” from Mr Twala’s comment means to have a holistic understanding of the given problem. This concurs with one of the findings from the study conducted in Malawi (see Section 3.5.3). The incorporation of real-life scenarios in the teaching of MWPs also assists in eliminating ambiguities, as learners are exposed to various situations where the different meanings of a particular word could be explored and learned. In this way, the teacher gets to show how a particular word, which carries different meanings according to the different contexts, can be applied in different contexts. The use of the scenarios in this instance serves as an alternative for learners to access the language and the concepts.

5.4.4 Conditions conducive to the successful implementation of strategies that improve visualisation of the word problems

Visualisation of the problem is a good strategy to support learners in reading, understanding and making sense of the problem. However, for the learners to visualise the problem, they need to connect the text with their prior knowledge and experiences to create meaningful mental images (Teahen, 2015:i). Visualisation is a very important activity in mathematics even though it is complex (De Guzman, 2002:3). In line with this claim, it is important for the conditions conducive to the successful implementation of the strategies that improve learner visualisation of the word problems to be identified. The participants identified the conditions as follows, drawing from some of the lessons that took place:

> *Ms Nkosi: …so for them to understand the problems we need to come up with tangible things so that they can be able to see what we are talking about. We can bring circles of the different sizes, strings and rulers or tapes to determine the value of Pie, which will be 3.14, and it will be the same in all the different shapes.*
>
> *Mr Phatudi: A clear example eo re ka e etsang mona keya [**ET: "…**that we can make here is this of…"] the Sine graph. Let's say the question says: Draw the graph [showing the problem on paper]* $f(x) = Sinx + 2$ *and describe the shift. The learners can be able to describe the shift if ba e bona. [**ET:** **"**…if they see it…"]*

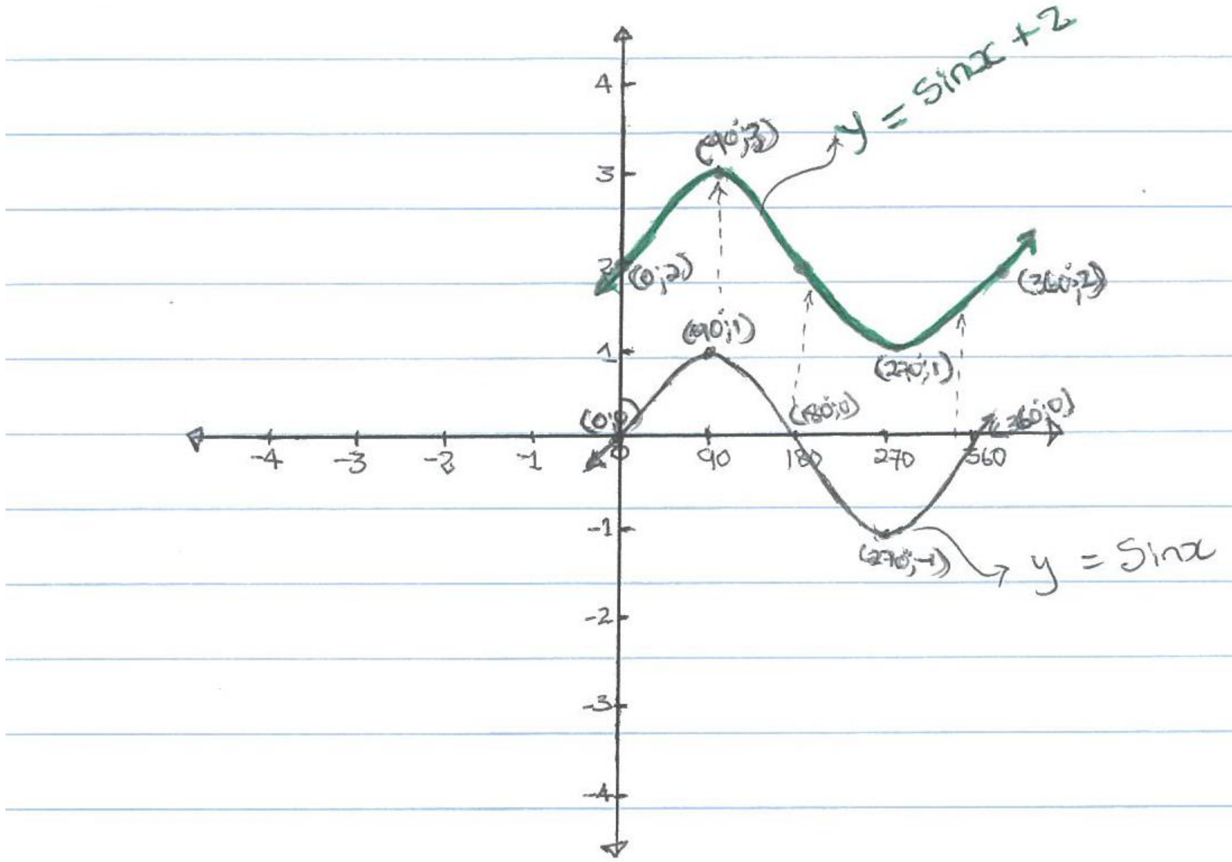


Figure 5.7: The graphs $y = Sinx$ & $y = Sinx + 2$ plotted on the same Cartesian plane (picture from the teacher's notes)

Teachers bringing real instruments to class is one of the factors that can improve learners' visualisation skills significantly and enhance their understanding of the content. The example Ms Nkosi provided shows how manipulatives can be used to clarify mathematics content visually. For example, to determine the value of Pie (π) the area of the circumference must be divided by the length of the diameter. The learners will have to record the areas of the circles of the different magnitudes and also record the diameters. When they use the formula $\pi = \frac{c}{d}$ they will notice that the quotient becomes the number that is closer to 3.14. The teacher will then explain to the learners that when accurate measurements are established, the value of Pie will be 3.14, which is the number that is closer to the ones they have obtained. Thus, according to UDL, involving the use of physical objects in teaching and promoting learner engagement in the activity make the content more perceptible and generate learners' understanding of such content through visualisation.

Graphical representation was also highlighted as one of the positive contributing factors that can significantly aid learners' visualisation of the word problems. According to Mr Phatudi, learners may be able to answer the questions related to the given word problem if the graph is drawn clearly. The example that Mr Phatudi used, namely $y = Sinx + 2$, represents the shift of the basic Sine graph. However, this shift can be distinctly described when the teacher guides the learners in terms of plotting the coordinates of the basic Sine graph and then ask them to plot the coordinates of the basic Sine graph with the addition of two units. The learners can then see the difference between the two graphs (in terms of the shift), which they will then have to explain as well. The learners can also be asked to plot the two graphs on the same Cartesian plane to see the differences clearly. The basic Sine graph will, therefore, serve as a point of reference in terms of answering the questions on the "new Sine graph", namely $y = Sinx + 2$. The use of graphical representation as a tool to enhance the learners' visualisation skills is also supported by the literature (see Section 3.5.4). However, in this study, such a strategy would work best if the teacher (as a "more knowledgeable other") guides the learners step by step instead of plotting the graph on the board while the learners are merely watching without being engaged in the activity. Therefore, the current study embraces a more learner-centred approach, together with scaffolding by the teacher, to enhance the learners' comprehension of the content through visualisation. Such a practice promotes collaborative working, which is a principle advocated by CER.

The above extracts make it clear that to plot the diagrams to demonstrate what is required (the shift), reading skills and understanding what the question requires are necessary to have prior knowledge and understand the vocabulary and mathematical register. Furthermore, by creating visual images (internal visualisation) learners can make associations with their prior knowledge and experiences, and thus stimulate a deeper understanding of what they are reading. The use of graphs, therefore, assists in varying the information display and thus provides the learners with varieties for perception.

5.4.5 Contributing factors towards the enrichment of the teachers' skills to teach word problems in multilingual classrooms

Mathematics teachers dealing with learners whose first language is not the language of instruction need to be aware of the intricate process of not only learning a second language but also the even more complex process of learning (mathematics) in a "foreign language" (Essien, 2010:34). Drawing from this statement, it is of vital significance that the teachers are well trained in order to be able to deal with the challenges of teaching in multilingual classrooms effectively. After lengthy discussions, the participants also highlighted a few conditions that could contribute to the enrichment of the teachers' skills in terms of teaching MWPs to learners in multilingual mathematics classrooms. The participants commented as follows:

> *Mr Simelane: We need to receive training e tla re thusang ho feisana le maemo ana. [**ET:** "…that will assist us to face this situations."]*
>
> *Mr Nzuza: [Interrupting] … Haholo jwang hobane rona mona bana ba rona ba sokola haholo ka sekgowa.*
>
> ***ET:** Particularly because our learners struggle to speak English.*
>
> *Ms Zwane: The peer observations also help. Matitjhere re kgona ho ahana through tsona.*
>
> ***ET:** Through peer observations, teachers are able to build each other.*

The participants identified the training of teachers on how to teach learners in multilingual mathematics classrooms as a key factor in enhancing their teaching skills, especially teaching of MWPs in multilingual mathematics classrooms. The teachers felt "inadequate" in terms of the skills needed to meet the needs of learners in multilingual mathematics classrooms. The implementation of teacher training was also deemed essential in empowering the teachers to cope with a variety of challenges that arise with the teaching of mathematics in classrooms of this nature (see Section 3.5.5). Mr Nzuza emphasised the need for such training due to the challenges that he came across in his classes. At the school where he teaches, English is the learners' second language, which necessitated such a training to empower the teachers to support their learners fully. Mr Nzuza also highlighted the fact that even though there was a need for training on the teaching of learners in these classrooms (focus on pedagogy); it was of vital importance that the teachers' content knowledge should also be enriched through professional development. The participants thus echoed a sentiment similar to that of Tsotetsi's description of successful teaching of MWPs, namely that its success depends on the creation of spaces for teachers to share good practices (pedagogy) and the possession of proper and sufficient content knowledge (Tsotetsi, 2013:9), as highlighted in Section 3.5.5.

Peer observation was also identified as one of the most significant elements that contributes positively towards the enrichment of the teachers' skills, specifically in teaching MWPs to learners in multilingual mathematics classrooms. Through peer observation, the teachers observe one another's teaching methods and provide constructive feedback that enables them to grow and to teach learners in multilingual mathematics classrooms effectively. The word "ahana" (**ET:** "*build"*) in the statement, "*Matitjhere re kgona ho ahana through tsona",* indicates that teachers do not have to work alone when striving to bring about change in the form of successful teaching and learning of MWPs in multilingual mathematics classrooms. The notion of collaborative teaching is also supported by Begum (2012:383) and Murtaza (2010:219), as highlighted in Section 3.5.5.

5.4.6 Favourable conditions towards the use of learners' home languages

The use of home languages as resources to support teaching and learning is important. However, Setati (2005:448) advises that teachers should strike a balance between the use of English, which is the LoLT, and the use of learners' home languages in multilingual mathematics classrooms.

In line with this sentiment, the participants highlighted the conditions favourable for using the learners' home languages productively when teaching MWPs as follows:

> *Ms Nkosi: We need to carefully use the learners' home languages. Rentse re etse hloko hore ba se lahlehelwe ke monyetla wa ho ithuta English [**ET**: "We need to be careful to ensure that they do not miss out on an opportunity to learn English."]*

Mr Phatudi: Indeed! We need to use the home languages profitably, still remembering that they will be required to know how to speak English when they get to tertiary."

Mr Nzuza: Ho bohlokwa ho ela hloko puisano ya bana in class, eeeehh… hore ba bua jwang ka phaposing as compared to outside, how they use everyday language, ba bontsha jwang their thinking regarding the concepts and le hore the textbooks di informa jwang their language.

ET: *It is important to be aware of the learners' conversations in class in terms of how they speak as compared to when they speak outside the classroom; how they use everyday language, how they demonstrate their thinking regarding the concepts as well as how the textbooks inform their language.*

The use of learners' home languages was also highlighted as an "enabling" strategy in aiding learners' comprehension of the problems. The participants were well aware that the total use of the learners' home languages would disadvantage the learners and deprive them of an opportunity to learn English. They were also aware that English proficiency is a prerequisite at institutions of higher learning and also for global access, hence the comment *"we need to use the home languages profitably, still remembering that they will be required to know how to speak English when they get to tertiary".*

The participants suggested that a balance needs to be maintained between the use of the learners' home languages and English as the medium of instruction. Furthermore, according to the participants, the learners' interaction in class when they communicate mathematically needs to be carefully managed so that the home languages are used as resources to aid learner comprehension. This indicates that the home languages play an important role of providing learners with a cognitive advantage. Chitera (2009:11) concurs with this notion and urge teachers to view home languages as resources for teaching and learning since they offer learners a cognitive advantage in learning mathematics (see Section 3.4.6). According to the teachers, multilingualism, as the predominant characteristic of classrooms in South Africa, should be embraced rather than being viewed as an obstacle. Since most of the learners in these classrooms are still learning English and are not proficient, the use of learners' home languages is, therefore, eminent in ensuring that they develop proper understanding of the concepts in an effort to increase their chances of success. This notion concurs with the literature in Section 3.5.6.

The participants also highlighted a number of significant issues to which teachers need pay attention in balancing the use of English and the learners' home languages. These issues included paying attention to how learners make sense of the word problems through communication, which is, to a certain extent, determined by how they understand a specific usage and structure of the language; secondly, how the use of everyday language affects mathematics learning; thirdly, how learners express mathematical thinking in their own language; and lastly, how language is used in the textbooks as opposed to how the teacher and the learners use language. Setati (2005:448) echoes all of these in Section 3.5.6.

The above sections highlighted favourable conditions for the strategy to be implemented successfully. The following sections highlight possible threats to the successful implementation of the strategy and also highlight the measures that were put in place in an effort to circumvent the threats, according to the participants.

5.5 ANTICIPATED THREATS TO THE IMPLEMENTATION OF THE STRATEGY

In the previous sections, discussions on the identified challenges, components of the strategy as well as the conditions conducive to the successful implementation of the strategy in the current study, were presented. In the following section, the discussion is focused on some of the threats that could impede the operationalisation of the strategy, and the steps that can be taken to circumvent the threats.

5.5.1 Lack of participation, unavailability of material written in other languages and stakeholders' unwillingness to participate

When a strategy is to be developed, it is important to anticipate the threats that may impede the implementation of the strategy. The participants are in a position to anticipate possible threats that may hamper the operationalisation of the strategy so that the mechanisms can be put in place to circumvent them. In line with this, Tsotetsi (2013:29) states that "by identifying the possible threats, the aim is to bring change and emancipate the participants, most importantly the excluded individuals". Therefore, the multi-facetted stance of CER makes it possible for the participants to discuss the anticipated threats from the different angles.

In light of the above, the possible threats to the successful implementation of the strategy were highlighted by the participants as follows:

> *Ms Zwane: Bana ba bang ba dihlong tsa ho bua ka classeng. Jwale ha ba rutwa ka English feela ba thola tuu hobane ba tshaba ho tsheuwa!*
>
> ***ET:*** *Some learners are shy to express themselves orally in class. When they are taught entirely in English, they keep quite during the class for fear of being laughed at.*
>
> ***Tseko:*** *It is easy to speak English in class when there is an element of respect because nobody can feel scared to talk.*
>
> ***Bonolo:*** *I think it would be good if we can also be allowed to show what we have learnerd using other media. Ehhh... for example, we can use diagrams and manipulatives.*
>
> *Ms Masombuka: …hare na material o ngotsweng ka dipuo tsa malapeng ho sapota English materials tseo re nang le tsona.*
>
> ***ET:*** *We do not have the material that is written in the different home languages to support the English materials we have.*
>
> *Mr Nzuza: If the stakeholders are not willing to to join hands then re kaba le bothata ba ho achieva sepheo sa rona ka leano lena [**ET:** "…then we will have a problem in achieving our goal through this strategy"]. Incentives may also assist for motivation purposes.*

The sole use of English in class, though it may be to the learners' advantage to learn English language and effectively communicate mathematically, can hinder learners' level of participation, especially participation of shy learners. This may cause these learners to withdraw from participating in class activities for the mere reason that they are afraid of being laughed at or ridiculed when they cannot express themselves fluently in English. This is also the concern raised by Ms Zwane, namely that the sole use of English in the classroom may pose a threat to learners. Deducing from Ms Zwane's utterance, it is clear that the use of home languages can promote learner participation in class, while the sole use of English, as the second language of the learners, may impede participation. This confirms the stance of Setati et al. (2009:65) and Moschkovich (2012:18) that the use of learners' home languages when used as support in the teaching and learning of mathematics facilitate multilingual learners' participation, as highlighted in Section 3.6.

It is common knowledge that society often "judges" people on their use of English, for instance, a person who is fluent in English, is considered "highly educated", and vice versa. This is also supported by Nkambule (2008:3), who claims that most people still view English as a language that defines how educated a person is, which is why most teachers still restrict learners from using other official languages. When learners withdraw from participating in class activities, the teachers' task of assessing the learners' understanding of the content becomes difficult. Furthermore, the lack of participation may limit class interaction and also cause learners to miss out on the benefits of learner-centred undertakings, which include being exposed to multiple ways of analysing, interpreting and solving word problems.

In an endeavour to address this challenge, the participants recommended the use of learners' home languages as authentic resources to clarify concepts to deepen learner understanding of the content and promote deeper learning. Another recommendation was that the teachers should provide opportunities for the learners to use their home languages to express themselves when they encounter difficulties in expressing themselves in English, which is their second language. However, the learners also need to be made aware of the significance of "practising" to communicate mathematically using English and the mathematical vocabulary and register. Establishing ground rules, such as respecting one another, was also recommended as a solution to this challenge. Respect in this context implied, among other things, "not laughing at others" as well as acknowledging that every person is different and that this element of diversity brings into the classroom "richness" in terms of teaching and learning. This is one UDL way of teachers creating a positive learning atmosphere for learners to engage freely in class activities. It was further suggested that learners should be granted an opportunity to express themselves in ways they find comfortable (e.g. using diagrams, concrete materials, audio, etc.). This also draws from the UDL principle of multiple action and expression, which encourages a multiplicity of ways in which learners can demonstrate what they have learned.

Limited teaching material in the mother tongue or the unavailability thereof was also identified as one of the threats to the implementation of the strategy to teach MWPs effectively. Ms Masombuka attributed the unavailability of mother tongue teaching material to the privileges that have been given to the English language over other languages. This threat was also highlighted in the study conducted by Halai and Karuku (2013:23), as highlighted in Section 3.6. However, to circumvent this threat, further training on the use of language conducted by the language and the mathematics subject content specialists was identified. Teamwork among the mathematics teachers and the English teachers, in order to assist one another, was also encouraged as a practice that should be tried and explored on an ongoing basis. This practice, which encourages teacher collaboration, is in line with CER, which espouses teamwork among the teachers.

Furthermore, the unwillingness of the stakeholders to cooperate in the implementation of the strategy was also identified as a possible threat. For instance, if the English teachers, as language experts, choose not to support the mathematics teachers, the strategy might be impeded, with the result that the envisaged results might not manifest as anticipated. Likewise, mathematics teachers who do not acknowledge the importance of further training and see it as waste of time may also hinder the attainment of the promising effects of the strategy. However, in order to address this challenge, it was recommended that the goal of the strategy should be communicated clearly to all the stakeholders (i.e. teachers, parents, learners and learning facilitators) in an effort to ensure that they buy into the idea. Furthermore, a form of incentive was recommended to keep the teachers and learners motivated while the strategy is being implemented.

5.6 EVIDENCE INDICATING THAT THE STRATEGY WAS EFFECTIVE

This section outlines the success indicators or evidence that the strategy was successfully implemented. The discussion is done in line with the objectives of this study, as outlined in Chapter 1.

5.6.1 Evidence of learners' improved reading skills

Barton et al. (2002:11) unequivocally state that mathematics depends mainly on conceptual understanding, and effective reading skills develop the learners' understanding of mathematical concepts. It was in line with this statement that the following comments were made by the teachers during the reflection session. The teachers drew some of the examples from the lesson observations and recorded progress in the learners' work (homework or class work books) in line with the strategies applied.

> *Ms Zwane: Ho kgothaletsa bana kamehla hore ba bale ka tataiso ya ka ho thusitse haholo. Ke bona ba se ba sena monyeme wa ho bala jwaloka pele. Hape ho fokotsa le diphoso tse ngata tseo be di commitang.*
>
> ***ET:** Encouraging learners all the time to read has helped a lot. I realised that they are now more keen to read than they were before. Frequent reading also reduces many errors which learners commit. Simplifying the language as well as incorporating the reading skills in our teaching may also assist a lot.*
>
> *Mr Twala: Lenna ke bona ho bala ka hloko ho thusa haholo. Ho ba thusa le ho presenta di shaded areas cartesian pleining haba filwe di inequalities.*
>
> ***ET:** I also realised that careful reading helps a lot. It enables them to present the shaded areas correctly on the Cartesian plane when they are dealing with inequalities.*

The above comments show that the strategies the teachers used had positive effects. Encouraging learners to read on their own frequently and guiding them at the same time in class when reading word problems improved their reading skills; hence the statement, "*ho thusitse haholo*" (**ET:** "*has helped a lot*"). Nkambule (2009:78-79) also encourages the use of this strategy, indicating that creating an environment where learners are expected to read on their own and are probed and directed to generate a common understanding, not only improves their reading skills but also stimulates classroom discussions and encourages learners to think deeply about the word problem, as noted in Section 3.7.1. Furthermore, engaging the learners in frequent reading of the different word problems improves the learners' attitude and morale, as implied in the following statement: "*Ke bona ba se ba sena monyeme wa ho bala jwaloka pele"* (**ET:** *"I realised that they are more keen to read than they were before"*). Through frequent reading, learners also developed the skill to read the problems holistically, which enabled them to illustrate the shaded areas correctly on the Cartesian plane. This also signifies that the learners' understanding of the language and their interpretation of the statements have improved. The statement, "*Ho fokotsa diphoso tse ngata tseo ba di commitang"* (**ET:** *"It reduces many errors which learners commit"*), indicates that the learners who are frequently engaged in reading exercises do not commit too many reading mistakes.

Deducing from the above, it is reasonable to indicate that reading is a skill that can be mastered over a period of time. It is an activity in which one should frequently engage to develop mastery thereof. Integrating procedures for reading into the lesson is important. Furthermore, creating an environment in which learners are expected to read on their own and are probed and directed to generate a common understanding improves reading and not only stimulates classroom discussions but also encourages learners to think deeply about what they are reading (word problem). The above discussion further indicates that through the right support, including the application of effective reading strategies, the teachers could significantly enhance the learners' reading skills. A simplified language could also play a vital role in improving the learners' reading skills. In this way, in line with the UDL principles, the teacher provides alternatives for heightening an understanding of the word problem and the concepts embedded in it. Supporting second language learners in multilingual mathematics classroom where the LoLT is not their first language is a caring practice, which CER promotes within classrooms. Such a caring practice breeds hope in learners and ensures that equality is attained within the classrooms in which the learners come from different backgrounds.

5.6.2 Evidence of learners' improved mathematical vocabulary and register

Saville-Troike (quoted in Laplante, 1997:70) states that vocabulary knowledge in English is the most important aspect of oral English proficiency for academic achievement. Taking into account the fact that mathematics has a large vocabulary and register, it is important for the teachers to teach learners these terms effectively. In line with this, the participants highlighted some of the successes related to an improved mathematical vocabulary and register as follows:

Ms Masombuka: Ke ile ka bona diphethoho tse ngata kamora hore ke addresse vocab ya some of the concepts. Ke bone ho addresa vocab ho tlisitse phapang e kgolo feela. Hape le ho hlalosa mantswe a sebediswang strictly mona mmetseng … bo function, determining the domain and range and so on and so on...ke bone ho thusa haholo. Ke bona ba kgona ho araba dipotso.

ET: *I realised changes after addressing the lack of vocabulary regarding certain concepts. Addressing vocabulary brought about a huge change. Furthermore, providing explanation of the words that we use in mathematics, such as function, determining the domain, and range, etc., has helped a lot. They can now answer the questions.*

Mr Morake: Glossary e important because e thusa bana hore ba hopole mantswe. Teng ha titjhere a e kgothalletse di results tsa yona dintle haholo.

ET: *Glossary is important because it helps learners to remember the words. Its good results are more evident when the teacher frequently encourages its use.*

Lerato: Di problems tsena di hlaka haholo feela ha bo teacher ba re explainetse mantswe ao a thata. Re be se rekgona le ho di solva.

ET: *These problems become clearer when the teachers explain the difficult words which they are comprised of. Consequently, we are able to solve them.*

The teacher reflected on the changes she noticed after addressing the issue of mathematical vocabulary. She realised the difference in terms of the learners' understanding of the vocabulary compared to the period prior to the implementation of the strategies. From the statement, "*Ke ile ka bona diphethoho tse ngata kamora hore ke addresse vocab ya some of the concepts"* (**ET:** *"I realised changes after addressing the lack of vocabulary regarding certain concepts"*), it is clear that the teacher noticed some significant improvements in the learners' knowledge of mathematical vocabulary in some of the concepts. The clarification of the mathematical register was also of great assistance in the improvement of the learners' understanding thereof. Furthermore, for the learners to remember the vocabulary and register, compiling a glossary was deemed important. Drawing from the teacher's utterances above, the glossary served to remind the learners of the meaning of the different words and the specific application of each word.

The statement, "*Teng ha titjhere a e kgothalletse di results tsa yona dintle haholo"* (**ET:** *"Its good results are more evident when the teacher frequently encourages its use"*), explains that the strategy works best when the teacher is regularly involved in the whole process by continually encouraging the learners to use the glossary. The learner also attested to the significance of explaining the mathematical vocabulary and register. The words *"di hlaka haholo feela"* (**ET:** *"they become clearer"*) indicate that the teacher's explanations of the mathematical vocabulary and register make the problem clearer and more understandable. This concurs with the literature, namely that teachers who constantly pay attention to mathematical vocabulary and register development, do not only "upsurge" learners' knowledge of the mathematical vocabulary and register but also cultivate the development of learners' cognitive growth (Capraro & Capraro, 2006:23) and strengthen their conceptualisation of MWPs. The learner's statement, "*Re be se rekgona le ho di solva"* (**ET:** "*We are then able to solve them"*), indicates that the clarification of the mathematical vocabulary and register makes the problem much more explicit and thus enables the learners to be analytic and describe what the problem requires and empower them to develop the ability to engage in problem solving. This is in accord with the findings in Section 3.7.2, namely that the clarification of the mathematical vocabulary and register provides learners with analytical, descriptive and problem-solving skills within a language and a structure through which they can explain a wide range of experiences.

5.6.3 Evidence of the successful implementation of the strategies employed to address the ambiguity of the word problems

Although many people believe in the precision that mathematics provides, the reality is that ambiguity and vagueness are commonly reported in mathematical talks and text problems (Barwell et al., 2005:142). This is because in certain instances, the words used in a mathematics classroom may have different meanings and grammatical functions than when used in common English. Therefore, teachers need to be mindful of these ambiguities and remove them so that learning does not become obstructed by these ambiguities. In line with these sentiments, the participants reported the successes of some of the strategies applied to address lexical ambiguity as follows:

Pholoho: Hare fuwe palo ebe teacher o fa group engwe le engwe monyetla wa ho hlalosa e thusa haholo hobane o kgona hore correkta hare sa hlalose seo potso ese batlang le ho re thusa hare misintepreta question.

ET: *It helps when a teacher grants every group in the class an opportunity to explain what they understand from the given problems. The teacher is able to correct us when we are not correctly explaining what the problem requires and also assists us to interpret the question.*

Ms Moeketsi: Ha ke ruta bana di word problems ke stressa le hona ho clarifaya mantswe aka etsang statement se seke sa hlaka. Ho etsa jwalo ho tebisa kutlwisiso ya bona.

ET: *I usually stress and clarify words that may cause the word problems not to be clear. That deepens their understanding.*

Ms Ntuli: It is imperative again to encourage learners to use the appropriate mathematical terms and not the ordinary language. For example, the words "at least" must be explained hore le bolelang in an ordinary language and also mathematically. So, that way, bana ba understand the application of the word in different settings.

The above extracts indicate that teachers who provide learners with opportunities to work in groups, reinforce participation in the classroom. In these groups, learners can discuss the problem from various angles and reach a consensus on what is required to solve the problem. This means that the learners agree, based on their interpretation of the given problem. Allowing the learners to work in groups is one UDL form of support to sustain effort and persistence. This practice also aligns with CER, which encourages the participation of all learners in an endeavour to construct knowledge and as many ideas as possible as well as to generate meanings thereof. During the time when the learners provide feedback, the teacher is able to analyse what each group is saying, thus identifying gaps and misconceptions possibly resulting from ambiguity. The teacher also has the opportunity to assist the learners in interpreting the question correctly and placing it in context, hence the words, "*o kgona hore correkta hare sa hlalose seo potso ese batlang le ho re thusa hare misintepreta question"* (**ET:** *"is able to correct us when we are not correctly explaining what the problem requires and also assists us not to misinterprete the question"*). The literature in Section 3.7.3 also confirms the success of this strategy by indicating that teachers who constantly highlight ambiguities when teaching and provide learners with the freedom to "build their voices" in class make it possible for learners to overcome the challenge of lexical ambiguities. Furthermore, in this study, it was revealed that learners may be given a list of words, which can be defined differently according to various contexts, and then be requested to provide the different definitions and indicate how these words could be applied in each identified context.

The teacher highlighted that frequently explaining possible ambiguous words during teaching assists in clarifying the meanings of the words. This serves as one UDL way of eliminating distractions and perceived threats that may be posed by the presence of the ambiguous words, which may, in turn, impede the conceptualisation of the word problems. In the process, learners become acquainted with the fact that words can be used in various contexts. Consequently, the teacher's careful use of the mathematical vocabulary and register when teaching helps learners to learn and be careful in terms of using words according to the different contexts. Kaplan et al. (2015:9), in Section 3.7.3, agree with this notion and assert that learners who are constantly made aware of the ambiguities during the teaching of MWPs, learn the appropriate use of words according to the different contexts. They also learn not to confuse the meanings of words used in the context of the mathematics classroom with how they are used in daily conversations – *"an ordinary language*". The example the teacher used to illustrate this was the use of the word "at least", which in "ordinary street language" (i.e. in our daily informal conversations) would mean "better", but in mathematics, means "greater or equal to" ($\geq$).

5.6.4 Evidence of the successful implementation of the strategies employed to enhance learners' visualisation skills

According to English et al. (1995:57), "the essence of understanding a concept is to have a mental representation or mental model that faithfully reflects the structure of that concept". In line with this notion, Mulligan (2011:23) emphasises the significance of ensuring that the learners' ability to produce quality representations is scaffolded by providing learners with the structures that enable them to solve problems. The successes of the strategies employed to enhance the learners' visualisation of the problem were highlighted as follows:

Ms Ntuli: Usually ha ke ruta bana di word problems, ke bala le bona the problem eo ba e fuweng. I also draw a picture ya ntho eo ho buuwang ka yona. For example, ha potso e batla ba deskraebe the type of a shift e bileng teng ya the...the... ha rere $f(x) = Cosx + 2$*. I draw it on the board so that they can see it.*

ET: *I usually read together with the learners the given problems. I also draw the picture of what the problem is all about. For example, when a question requires that the learners describe the type of a shift that occurrs in a graph where* $f(x) = Cosx + 2$*.*

Tshepo: Ha re filwe equation, for example $f(x) = 2CosA$ *ho bonolo ho bale picture ya graph ka hlohong ha e droyilwe and tithjere abe a e hlalosa ka di signs ho re graph e tlo shebela kae!*

ET: *When we are given the equation, for example,* $f(x) = 2CosA$ *…, it is easy to generate a mental picture of such a graph when it is drawn and the teacher explains it through the use of signs to indicate where it will face.*

Ms Moeketsi: I advise learners to use the different colours of the pens or highlighters when we deal with the concept of "probability". Those colours are able to help learners to differentiate and to see the point of intersection.

The first extract above indicates the significance of taking learners through reading the problem and generating a diagram that depicts the situation highlighted in the problem. This enables the learners who do not understand the text to capture the meaning in the form of the diagram. It is not always easy for learners to describe the shift of the graph $f(x) = Cosx + 2$, as the teacher indicated above, unless the graph is plotted on the Cartesian plane where it can be seen clearly. The teacher can show the learners the plotting of the graph step by step, while at the same time explaining what the graph entails. This allows the learners to experience the movement visually and, consequently, to describe the shift. The use of graphs thus serve as a UDL visual alternative and thus provide learners with options for perception. The use of diagrams is also supported by Poch et al. (2015:282) as the best strategy to represent a word problem, particularly as the learners work towards an advanced level of MWPs (see Section 3.7.4).

The meaning deduced from the second extract above is that in the case of a graph $f(x) = 2CosA$, it is easy for the learners to provide answers related to that graph when they already know what kind of graph it is (i.e. the Cosine basic graph), what it looks like, what two (2) as a coefficient of $CosA$ refers to in the graph, and so forth. According to the learner, if they have sufficient prior knowledge and have been exposed to basic graphs and their adaptations, they would find it easier to answer questions related to the given equation, without necessarily having to draw the graph. This concurs with the literature in Section 3.5.4 that visualisation of a word problem is effective, when learners can connect the text with their prior knowledge and experiences to create meaningful mental images.

According to Ms Moeketsi, teachers who encourage learners to use different colours when dealing with massive amounts of data, such as in the concept of probability, enable the learners to better interpret the data. The learners can also identify commonalities in the data and the point of intersection, which would be highlighted by the different colours. The use of the different colours, as one UDL information organiser, offer learners options for comprehension. Furthermore, the use of apparatus that learners can touch and see proved to aid the learners' comprehension of the content. Manipulatives help the teacher to "make practical" the given problem, and this way the teacher makes learning personally relevant and valuable. This thus serves as one UDL way of recruiting interest. The use of manipulatives in improving the learners' understanding of the content is also supported in the literature (see Section 3.7.4).

5.6.5 Evidence of improved teachers' capacity to assist learners in developing problem-solving skills

Ntloana (2009:2) avows that effective professional development programmes of teachers stand at the centre for improving the quality of teaching and transformation of education. This means that the teachers who are undergoing teacher training need to be provided with training that fully empowers them to teach learners effectively. Tsotetsi (2013:9) avers that the successful teaching of MWPs depends upon the creation of spaces for teachers to share good practices and content knowledge. This means that teachers need to come together and share with one another knowledge gained and ways in which MWPs can best be taught to the learners. In line with this, the participants highlighted some of the successes regarding the empowerment of teachers as follows:

> *Mr Phatudi: Training e thusitse matitjhere a mangata haholo. Ha re kopanela le matitjhere a mang e thusa ho increasa knowledge ya subject le kamoo content e ka presentuwang betere kateng.*
>
> ***ET:*** *Training has empowered many teachers. Working together with the other teachers helps increase the knowledge of the subject content, as well as how the content could be presented.*
>
> *Ms Masombuka: Ha re le fully equipped re kgona le ho prepara thouroghly. Le ka classroomung re kgona le ho etsa mehlala e thusang bana ka dintho tseo ba di tsebang already.*
>
> ***ET:*** *Being fully equipped enables us to prepare thoroughly. We are able to provide examples which assist learners to understand aspects that they already know.*
>
> *Ms Moeketsi: Training ya tshebediso ya puo mona ho di word problems e thusa le tithjere ho re a kgone ho guida bana fully.*
>
> ***ET:*** *Training on the use of language in this concept of word problems assists a teacher in order to guide learners fully.*

The training that was put in place to empower the teachers with the skills to teach MWPs effectively had positive effects. According to Mr Phatudi, the training benefited most of the teachers, as can be deducted from the words, "*Training e thusitse matitjhere a mangata haholo"* (**ET:** "*Training has empowered many teachers"*). What contributed to the positive effects of the training, is that the teachers collaborated on issues that had a negative impact on their teaching practices; therefore, the training provided a platform for them to closely engage and interact with one other regarding these issues. In the process, the teachers' knowledge of teaching MWPs increased and they were enlightened regarding the effective ways (approaches) of teaching MWPs; hence the statement, *"e thusa ho increasa knowledge ya subject le kamoo content e ka presentuwang betere kateng" (**ET:** "it helps increase the subject knowledge and how the content can be better presented").* Effective training, coupled with meaningful participation by the teachers, empowered them and instilled "a sense of readiness" to teach. Furthermore, such training enabled the teachers to prepare lessons that are prolific to the learners. This means that effective training can positively influence how teachers prepare their lessons. Training of this kind also exposes teachers to various angles of content, which deepens their understanding thereof and enables them to tap into reality by identifying relevant examples that support and clarify abstract content. Subsequent to this, the teachers enable the learners to integrate real-life problems with what they are taught in the classroom. This makes learning meaningful since learners get to realise the applicability of the concepts they are learning in class (which, in certain instances, are the concepts they already know) to real-life settings, hence the statement, *“re kgona le ho etsa mehlala e thusang bana ka dintho tseo ba di tsebang already” (**ET:** “We are able to provide examples which assist learners to understand aspects which they already know”).* In this way, the teacher makes learning personally relevant and valuable by using examples the learners can relate to. Moleko (2014:96), also supports the integration of real-life activities in the teaching of MWPs and avows that such a practice stimulates learners’ interest and increases participation in class, which consequently enhances learners’ understanding of the concepts (see Section 3.7.5).

Furthermore, training on the use of language (English terms according to the different contexts) empowers the teachers to guide the learners in approaching and answering MWPs. This means that well-thought, well-structured training, which is designed “intentionally”, has the potential to empower the teachers to effectively scaffold learners’ understanding. The training that provides teachers with an opportunity to interact on issues pertaining to the teaching of word problems further provides them an opportunity to uncover the difficulties or problems that serve as impediments towards their practices. Such training thus serves as a reflective phase, which in turn, makes teachers aware of the tools they can use as “progress ingredients” towards self-improvement in teaching.

5.6.6 Successes attributed to the effective use of learners’ home languages in the classrooms

Jäppinen (2005:162) states that for the learners to perform well in mathematics, opportunities where learners can learn mathematics in their home languages need to be created, so that they can develop language skills in their first and second languages. Israel et al. (2013:542) unequivocally avow that mathematics is learned better if teachers use learners’ home language(s) as resources. In line with these notions, the successes associated with the use of the learners' home languages as resources for teaching and learning were reflected upon by the participants as follows:

> *Lerato: Ha re dummellwa ho sebedisa Sesotho ka classeng ho betere haholo hobane rekgona ho utlwisisa seo re se rutwang.*
>
> ***ET:*** *Our understanding increases more when we are allowed to use Sesotho in the classroom.*
>
> *Bokang: Sesotho se kgona ho hlakisa mantswe ao re sa a utlwisiseng.*
>
> ***ET:*** *The use of Sesotho clarifies words that we do not understand.*
>
> *Ms Zwane: Bana ba shy ba kgona ho participeita ha re ba alawa ho sebedisa puo ya ha bo bona*
>
> ***ET:*** *Shy learners are able to participate when we allow them to use their home language.*

The recognition of the use of Sesotho in class facilitated an understanding of what the teachers were teaching about. Therefore, Sesotho as a home language in this regard becomes an "enabler". Furthermore, the use of the learners' home language expedites the clarification of words that confuse the learners, and raises participation levels in the classroom. Allowing the learners to use their home languages in classroom promotes learner engagement in class activities as they can comfortably express themselves in their home language, without fear of being ridiculed or laughed at. The learners' home languages can, therefore, be used to facilitate the process of learner engagement and will result in all learners being accommodated in the classroom setting. This concurs with the studies conducted by Chitera (2011:44) and Halai et al. (2011:28), which revealed that translation in a multilingual mathematics classroom is inexorable. Translation could be used, among other reasons, to emphasise an important point, to enable learners who might not understand what is being said in the medium of instruction to participate in the lesson and to overcome the lack of some expressions in a given language, as highlighted in Section 3.7.6.

This enables the teacher to determine the gaps and misconceptions any of the learners may have, and not only those who are fluent in English (the medium of instruction). The use of learners' home languages as cognitive tools is also recommended by the literature.

For instance, Ferguson, (2003:39) notes that code switching in a multilingual classroom plays a vital role in facilitating textual meanings for learners who have a limited proficiency of the language of those texts, as highlighted in Section 3.7.6.

5.7 CONCLUSION

In this chapter, the challenges justifying the formulation of a UDL-based strategy to develop an effective teaching strategy for MWPs in a multilingual mathematics classroom were discussed. The strategies that were implemented to address these challenges were outlined, as well as the conditions under which the strategies were effective. Threats that could hinder the successful implementation of the strategy and steps taken to counteract them were also discussed. Finally, indicators of success and evidence that the strategy was successfully implemented were presented.

In the next chapter, the focus is on the discussion of the results of the study as well as recommendations.

CHAPTER 6

Discussion of the results of the study and recommendations

6.1 INTRODUCTION

The aim of the study was to utilise the aspects of universal design for learning (UDL) to develop an effective teaching strategy for mathematics word problems (MWPs) in a multilingual mathematics classroom (Section 1.4). Based on this, the findings, conclusion and recommendations on the formulation and implementation of such a strategy will be presented in this chapter. The sections in this chapter are aligned with the objectives of this study, as outlined in Chapter 1. Findings on the challenges that justified the need for such a strategy will be discussed first. Secondly, the solutions to these challenges will be presented. After that, a discussion on conditions conducive to the successful implementation of the strategy, as well as the threats thereto, will follow. Finally, limitations of the study and recommendations for future research will be presented. The study focused on the following research question:

How can we utilise the aspects of universal design for learning to enhance the teaching of mathematics word problems in a multilingual mathematics classroom?

The findings of this study are essential in answering the abovementioned research question and informing the formulation of the envisaged framework, which is presented in the next chapter.

This chapter reports on the combined findings from the literature review in Chapter 3 and the empirical data of this study (Chapter 5).

6.2 JUSTIFICATION FOR THE NEED TO FORMULATE A UDL STRATEGY TO ENHANCE THE TEACHING OF MATHEMATICS WORD PROBLEMS IN A MULTILINGUAL MATHEMATICS CLASSROOM

The following sections justify the need to enhance the teaching of MWPs in multilingual mathematics classrooms.

6.2.1 Learners have difficulty reading and understanding the language used in the word problems

According to Sepeng and Sigola (2013:331), word problems in mathematics often pose a serious challenge to learners because they require that learners read and comprehend the text of the problem, identify the question that needs to be answered and create (conversion of text into numerical format) and solve a numerical equation. The findings of this study indicate that reading skills and English proficiency are significant in enabling learners to master MWPs. However, many learners have difficulty reading and understanding content that is embedded in a word problem due to a lack of reading skills and not being well versed in English, the medium of instruction (see Sections 3.3.1 and 5.2.1). Findings also indicate that English second language learners encounter severe reading challenges since they are still learning English language and, at the same time, learning the language of mathematics. A lack of reading skills thus affect learners' understanding of the content because they do not yet know the key terminologies needed to solve the problem and the also lack English proficiency. The results from the literature and the empirical data in this study further concur that a lack of reading skills and an inability to understand written text keep learners from mastering (effectively solving) MWPs. Adding to this, the results of this study indicate that teachers do not develop learners' reading skills in their teaching. This necessitates teachers to explore ways to enhance learners' reading skills to enable them to read with understanding. In their investigative study, Vilenius-Tuohimaa, Aunola and Nurmi (2008:409) avow that mathematics performance and reading skills are closely related. This means that the learners' ability to read with understanding can potentially increase their performance in mathematics, which is why teachers have to devise effective ways in an effort to improve learners' reading skills.

6.2.2 Learners lack an understanding of the mathematical vocabulary and register

The significance of understanding the mathematical vocabulary and register when solving word problems has been explored by several authors (Boulet, 2007:10; Riccomini et al., 2015:235-236). They found that knowledge of the mathematical vocabulary and register greatly assists learners in comprehending MWPs. However, the literature (see Section 3.3.2) and the empirical data (see Section 5.2.2) indicate that most learners fail to solve MWPs because they are not well versed in the mathematical vocabulary and register. According to Reynders (2014:2), a lack of understanding of the mathematical vocabulary and register leads to the misapplication of appropriate mathematical operations. One of the findings of this study indicated that a lack of understanding not only keeps learners from making sense of word problems but also hampers their ability to conceptualise and contextualise word problems. In support of the findings of this study, Vula et al. (2015:34) confirm that for most learners, a lack of mathematical vocabulary is a serious challenge in solving MWPs. It came to the fore in the results of this study that the teachers do not pay attention to mathematical vocabulary and register, which is one of the critical aspects towards the mastery of MWPs. The results of this study also revealed that teachers fail to assist learners to distinguish between the language they use for social purposes and a special kind of language proficiency they need to perform mathematical tasks. This perpetuates the challenge the learners have regarding mathematical vocabulary, and this, in turn, has an impact on the learning of this mathematical genre. Therefore, teachers have to devise ways in which to help learners develop and advance their mathematical vocabulary and register to master not only the mathematics content but also this mathematical genre.

6.2.3 Learners lack the skill to visualise word problems

According to Boonen and Jolles (2015:1), the ability to visualise the problem is key in ensuring that the visual representation structure of the given word problem is clear. Visualisation, therefore, contributes significantly to the building process of learners' comprehension of the concepts embedded in the word problem. The visual identification and representation of the problem structure facilitate the correct understanding of the text of the word problem and helps distil the mathematical operation(s) that should be performed. Although visualisation of the word problem is significant in assisting the learners to solve MWPs effectively, findings from the literature (see Section 3.3.3) and the empirical data of this study (see Section 5.2.3) reveal that most learners have difficulty in visualising word problems. Consequently, the inability to visualise the problem caused learners to misinterpret the problem and to provide incorrect solutions. The findings also show that the difficulties in translating MWPs into numerically solvable equations were a result of learners' inability to visualise the problem. It was also revealed in this study that learners who cannot create a mental picture (visual internalisation) of the problem, cannot portray the picture on paper (external visualisation). It also came to light in this study that teachers do not teach MWPs in multilingual mathematics classrooms in a manner that develops the learners' visualisation skills. Based on this, Boonen et al. (2015:2) emphasise the need for learner support in terms of developing visual representation skills and call for teacher intervention in doing so. From the above, it is clear that teachers need to generate effective ways to develop learners' word problem visualisation skills.

6.2.4 Words with multiple meanings cause word problems to be ambiguous

People usually link what they are hearing to what they have heard and experienced previously, in other words, prior knowledge. For instance, if a commonly used English word is used in a technical domain, learners hearing the word for the first time in class may integrate the technical usage as a new facet of the features of the word they already know. Therefore, according to Kaplan et al. (2014:1), the use of domain-specific words that are also used in common English may confuse learners and lead to incorrect associations between words that have different meanings from those they know.

In affirming the above, it was revealed in literature and the participants' comments (see Sections 3.3.4 and 5.2.4 respectively) that in certain instances, language homonymy in word problems leads to multiple translations. This means that certain words may have multiple meanings, causing ambiguity in word problems that confuse the learners. It was also discovered in the study that learners come to class already knowing the meaning of certain terms that are also used in mathematical content. Therefore, if these words are not clarified and their differences in terms of how they can be applied in different contexts not explained, learners may fail to conceptualise and contextualise the problem.

This study also revealed that if some of the keywords that should form part of the given word problem were omitted, it would cause ambiguity that, in turn, would result in learners being unable to make sense of the problems. This omission of keywords of important terms further leads to learners' inability to identify the appropriate operations to solve the problem correctly. Another aspect that surfaced during the discussions was that teachers can also contribute to ambiguity through the language they use when teaching. If they do not construct sentences carefully and apply clear language that allow learners to follow the instructions, the ambiguity challenges may be exacerbated. It is, therefore, important that ways to avoid ambiguity in word problems be put in place.

6.2.5 Teachers find word problems challenging and thus cannot teach learners effectively

The literature (Section 3.3.5) and the empirical data (Section 5.2.5) reveal that many teachers find MWPs as difficult to solve as learners, and that has a negative impact on their teaching. Since they cannot teach MWPs effectively as a result of their inability to solve MWPs themselves, the teachers are unable to scaffold learners' understanding or nurture their ability to read, process and solve mathematical situations. The correlation between *knowing* and *teaching* is depicted in the findings of this study, namely when teachers are *not well versed* in a particular topic, they "automatically" *find it challenging to teach*.

One reason for teachers' inability to teach MWPs effectively was attributed to teacher training that places more emphasis on the procedures to be carried out when solving numerical mathematical equations than on solving corresponding problems that are expressed in the form of text. The empirical data also reveal that the teachers' inability to teach MWPs creates a lack of confidence and motivation on their part, and, therefore, they often refuse to teach MWPs or only teach aspects that they find easy to teach. Unfortunately, this leaves learners with insufficient knowledge (knowledge gaps). Considering these challenges, Barwell, Barton and Setati (2007:114) deduce that the teachers' task is becoming crucial, and they need to be equipped with the understanding and skills they need to deal with and support learners, especially in multilingual mathematics classrooms. Taking this into consideration, it is clear that there is a need for teachers to be empowered to teach MWPs effectively.

6.2.6 Most teachers view the use of home languages negatively and are reluctant to use them in class despite knowing that learners lack English proficiency

The literature review and findings from the empirical data (see Sections 3.3.6 and 5.2.6 respectively) reveal that many teachers still value English over other languages. The findings also reveal that most teachers still prefer the sole use of English in the classroom, rather than learners' home languages. The teachers usually "feel guilty" when using the home languages since they also believe that English signifies a person's level of education. In other words, teachers seem to be more concerned about teaching that would give learners access to *social class, power, higher education and employment*, which they believe could be attained through the sole use of English, than about the learners' lack of proficiency in English, which hampers their understanding of the content. The findings further reveal that as a result of the language restrictions in the classroom, namely that learners should stick to the sole use of English, most learners refrain from participating in the activities. Furthermore, a deeper understanding of the content can not be reinforced since most learners struggle to understand the English terms as they are still learning the language. It was also found that the sole use of English tends to have teachers use traditional and teacher-centred teaching methods that limit productive learner interactions. Although it is vital that learners become fluent in English as it is a globally recognised language, the fact that learners make sense of the given English MWPs in line with how they understand the usage and structure of words and sentences in their home languages can never be overlooked. This means that the use of everyday language shapes the learners' mathematics learning and how they express mathematical thinking in their own language. Therefore, if learners are denied an opportunity to express mathematical terms in their home languages in class, teachers will be unable to identify fully the interpretation challenges that are caused by the influence of the learners' home languages on how they express mathematical thoughts in the medium of instruction, namely English.

6.3 SOME OF THE COMPONENTS OF THE UDL STRATEGY IDENTIFIED AS IMPORTANT IN ADDRESSING THE IDENTIFIED CHALLENGES

The following sections outline some of the important components of the strategy that were devised to address the identified challenges.

6.3.1 Strategies employed to improve learner reading skills

The literature and the findings from the empirical data (see Sections 3.4.1 and 5.3.1 respectively) indicate the significance of encouraging learners to think about the concepts openly before attempting to solve any given word problem. The findings also indicate that for learner reading skills to be improved, teachers need to provide learners with as many reading and writing experiences (learner engagement in reading exercises) in various mathematical concepts as possible. This is one way of building their knowledge and developing their skills and understanding of the use of the mathematical terms in various contexts. It is also of vital importance that learners should be taught active reading strategies and that these are implemented effectively in mathematics classrooms to develop the learners' understanding of mathematical problems. "Asking questions" was identified as a powerful tool to support the reading of mathematical text. Through this strategy, learners keep asking questions about the content as presented in the form of text as they read to help clarify the concepts and consider what the question requires. Regular provision of word problems was also found to be helpful in ensuring learner engagement with the exercises that stimulate thorough reading of word problems. The study also revealed that it is important for teachers to break down the sentences into smaller ones and to clarify the meaning of each part before integrating all the parts. Adding to this, the study revealed that it is important for teachers to read, and write what they are reading, and to demonstrate that on the board. In this way, the learners are able to learn how words are used, how statements are read (from *left to right* or from *right to left*) and how they are represented algebraically. For example, "a is less or equal to 10", which can be represented as $a \leq 10$, can also be represented as $10 \geq a$ (when reading from right to left).

6.3.2 Strategies employed to improve learners' mathematical vocabulary and register

The literature and the findings from the empirical data (Sections 3.4.2 and 5.3.2 respectively) reveal that frequently teaching learners the mathematical vocabulary and register is a good strategy to assist them in becoming conversant with the mathematics vocabulary and register. Demonstrating how mathematical terms apply to different contexts and emphasising the differences according to specific contexts were also found to be beneficial in improving learners' mathematical vocabulary and register. Clarification of mathematical language, the use of words they are accustomed to in questions and a cognisant engagement of learners in learning new vocabulary terms and register in class were also found from this study to be effective strategies in promoting learners' understanding of mathematics vocabulary. The use of the mathematics glossary was found to be another good strategy to assist learners in improving their mathematical vocabulary and register. Overtly focusing learners' attention on the linguistic features of word problems significantly helps learners explore and clarify the technical meanings of terms. The findings from this study also indicate that learners' mathematical vocabulary and register improve when they are frequently given opportunities to define mathematical terms in ways that make sense to them.

6.3.3 Strategies employed to improve learners' problem visualisation skills

The literature review and the empirical data of this research study (see Sections 3.4.3 and 5.3.3 respectively) revealed that problem visualisation skills can be improved through the use of manipulatives and diagrams. The manipulatives and diagrams clarify the essence of the problems and serve as visible or tangible objects to which learners can relate. The use of diagrams was found to be a powerful strategy to help learners generate mental pictures of the given word problems. Being able to create mental images has the potential to enable learners to make meaning of the given problems and solve the problems. The findings also revealed that for visualisation of the problem to improve, it is important that the learners should be guided in terms of how to identify a pattern within a word problem, draw a picture of the given word problem and reword the problem. This would enable learners to understand the meaning that is embedded in the problem text "in their heads". Frequent encouragement and incorporation of a diagram when solving word problems were found to be essential and the teachers were encouraged to teach this method to their learners. Teacher and learner collaboration in analysing the word problem and drawing the corresponding diagram were revealed as powerful strategies to enhance learners' visualisation skills.

6.3.4 Strategies employed to eliminate words which cause ambiguity

From the literature discussed in Section 3.4.4 and the participants' comments in Section 5.3.4, it is clear that although words with more than one meaning may cause ambiguity in MWPs, teachers can overcome this problem. This could be done by explaining to the learners and providing synonyms of how these words can be used in various settings**.** Again, teachers need to provide learners with ample opportunities to familiarise themselves with mathematical forms of expression and learn how these can be used to communicate about mathematics in daily life, according to the different mathematical contexts. The creation of hybrid spaces in classrooms, where learners can integrate their home-based discourses with subject-disciplinary discourses, and where learning occur at the intersection of the two, manifesting itself as a hybrid epistemological and discursive construct, was found to be another good strategy to address ambiguity. "Focus" and "probe" were also found to be effective strategies for engaging learners in the construction of new mathematical understandings of the careful use of words that might present an element of ambiguity. The empirical data of the research study also indicates that it is important that subject specialists review word problems thoroughly before they are presented to learners.

6.3.5 Strategies to empower teachers to teach word problems effectively in multilingual classrooms

The literature and the findings from the empirical data (see Sections 3.4.5 and 5.3.5 respectively) indicate that mathematics teachers have to undergo further training that must be tailor-made to address the complexities that exist in the teaching of mathematics in multilingual classrooms, as well as enhance the teachers' pedagogical content knowledge. Mathematics teachers further need to receive training in the use of language and work collaboratively with their colleagues, namely the English teachers, at school level. Essien (2013:9) also supports the idea of improving the English language proficiency of second language teachers (and that of the learners) in dealing with the challenges of teaching learners in multilingual mathematics classrooms, especially in classes where the learners' home language is not the language of learning and teaching (LoLT). However, Essien perceives this as a one-dimensional approach to the issue and. therefore, recommends that a more holistic approach should be considered to address the needs of the learners in multilingual mathematics classrooms fully.

The study revealed that for the teachers to scaffold learner understanding fully and to ensure that the learners master MWPs, they need to pay attention to i) how learners make sense of the given MWPs, which is often determined by how they understand the particular usage and structure of the language; ii) how the use of everyday language shape the learners' mathematics learning; iii) how learners express mathematical thinking in their own language; and iv), how language is used in the textbooks in comparison to how the teacher uses language in the classroom. The findings indicate that the teachers can only succeed in scaffolding the learners if they are empowered in terms of teaching learners in multilingual mathematics classrooms and have sufficient knowledge of the use of language in mathematics.

6.3.6 Strategies to promote effective use of home languages

During the deliberations, it became clear that there was tension between the two groups of teachers, namely those in support of the sole use of English and those who were in support of using home languages as well. The solution that was eventually decided upon was that teachers need to employ code switching where necessary. The literature and the findings from the empirical data (see Sections 3.4.6 and 5.3.6 respectively) encourage the "tactful" use of home languages to engender discussions in multilingual classrooms. In promoting the effective use of the learners' home languages, the teacher should encourage the learners to work in their home language when solving mathematical problems. The teacher can also use open-ended questions to stimulate the growth of a rich language milieu and autonomous thinking. Furthermore, engaging learners' home communities as resources could serve as another way of supporting learners in the process of learning as they learn concepts in English while enhancing their understanding through the use of their home languages.

6.4 SOME OF THE ENABLING FACTORS FOR THE COMPONENTS OF THE UDL STRATEGY TO WORK THAT WERE REGARDED AS ESSENTIAL IN ENHANCING THE TEACHING OF MWPS IN MULTILINGUAL MATHEMATICS CLASSROOMS

The following sections highlight some of the enabling factors towards the effectiveness of the UDL strategy.

6.4.1 Improving learners' reading skills and understanding

The literature and findings of this study (see Sections 3.5.1 and 5.4.1 respectively) indicate that constant and frequent learner engagement in reading exercises was helpful in acquainting learners with various text forms. The involvement of the English teachers as "language practitioners" was also identified as a vital factor in ensuring that they provide learners with the necessary reading skills. Providing the learners with real-life scenarios (in text form) regularly, inspired the learners to read with understanding and also enabled them to make connections (as they read). In other words, as they read about real-life situations, they make associations with things they already know, which makes it easier to understand the content. This strategy was deemed significant in showing learners how to apply what they learn to real-life situations, thereby connecting mathematics with real life.

6.4.2 Improving learners' mathematical vocabulary and register

From the literature review (see Section 3.5.2) and the findings of the study (see Section 5.4.2), it is clear that teachers who frequently teach learners the mathematical vocabulary and register improve the learners' understanding. Furthermore, teachers need to provide learners with opportunities to communicate mathematically in the classroom. Interaction of this kind assists learners in getting accustomed to the mathematical terms as well as promoting the unique internal organisation (textual structure) that can be interpreted logically. Again, through these interactions, the teachers can identify misconceptions around certain terms and clarify them on time. Compiling a glossary for specific content assists learners in capturing the correct meaning of terms within a specific context. Tutorials also serve as practice grounds, in a collegial environment, for communicating mathematically using the mathematical vocabulary and register. Words on the wall, which are usually centrally located in the classroom for everybody to see, are also helpful in enabling the learners to recall the mathematical definitions and vocabulary.

6.4.3 Eliminating ambiguities from the word problems

Both the literature and the empirical data (see Sections 3.5.3 and 5.4.3 respectively) indicate that learners need clear statements to understand what must be solved in a specific problem. Therefore, a careful review of word problem statements assists in the identification of words that could cause ambiguity and allow rephrasing of the specific problem statement (see Section 5.4.3). Learners must also be encouraged to read through MWPs carefully and put the key terms in context so that the embedded meaning is not lost. To avoid translation ambiguity, the teachers need to encourage the learners not to read and translate simultaneously (translating word for word between the mathematical concepts), but to read between the lines so that they can develop a holistic understanding of the given word problem. This means that the learners need to have semantic schemata for these types of problems to recognise and avoid ambiguity. The study also revealed the significance of giving word problems with diagrams or pictures alongside them to “channel thinking” or “converge thinking” in the right direction.

6.4.4 Implementation of strategies to improve learners’ visualisation of word problems

In Sections 3.5.4 and 5.4.4 (literature and empirical data respectively) it was revealed that tangible objects assist learners in clarifying concepts, thereby enhancing their understanding of the specific content. However, it is important that when tangible objects are used, the teacher thoroughly explains the concepts to the learners, thereby scaffolding their understanding. Because the learners can now visualise the problem, the concepts are clearer as well (see Section 5.4.4). Graphical representation was also highlighted as a contributing factor in learners' visualisation of word problems. Through graphical representation, learners can see the changes that take place when plotting the graphs (what is happening) clearly, and this enables them to respond appropriately to the questions. It was further discovered from the empirical data that for visualisation of the problem to be successful, learners must be taught to connect the text with their prior knowledge and experiences. This enables learners to create meaningful images that portray not only the given problem but also what is required to solve the problem (internal visualisation). The empirical data (see Section 5.4.4) indicate that when learners mentally create meaningful images, they (the learners) can produce illustrative drawings on paper (external visualisation).

6.4.5 Improving teachers' skills to teach word problems in multilingual classrooms

Training teachers on how to teach learners in multilingual mathematics classrooms came up as a requirement for improving the teachers' skills in teaching MWPs (see Sections 3.5.5 and 5.4.5) and also in increasing their confidence in teaching this mathematical genre. Since the training also included linguistic aspects, the teachers felt a need to be better equipped to teach MWPs and meet the needs of learners in multilingual mathematics classrooms (see Section 5.4.5). In order for the teachers to fully meet the needs of the learners in multilingual classrooms, their pedagogical knowledge as well as content knowledge also had to be enhanced. Such enhancement was deemed vital to benefit not only the teachers but also the learners as they would be taught by well-equipped and confident teachers.

Through peer observations, the teachers were able to observe one another's teaching methods and provide constructive feedback that would help them to teach learners in multilingual mathematics classrooms better. The collaborative approaches aided teachers' understanding of MWPs and enabled them to show the relevance of the content they taught learners through their day-to-day classroom practices.

6.4.6 Effective use of learners' home languages when teaching the MWPs

The literature and the findings of the study (see Sections 3.5.6 and 5.4.6 respectively) indicate that learners' home languages can be used to the learners' benefit if they are used for clarification purposes in an effort to deepen the learners' understanding of the content. Home languages were also found to be beneficial in terms of encouraging participation. However, the findings revealed that the use of learners' home languages needs to be carefully managed as resources to aid learner comprehension of the content. Furthermore, the productive use of learners' home languages necessitates the following to be taken into consideration: i) how learners make sense of mathematics when communicating, which is determined by how they understand a specific usage and structure of the language; ii) how the use of everyday language informs mathematics learning; iii) how learners express mathematical thinking in their own language; and iv) how language is used in the textbooks in comparison with how the teacher and the learners use language.

6.5 SOME OF THE THREATS THAT COULD IMPEDE THE SUCCESSFUL IMPLEMENTATION OF THE STRATEGY THAT WERE CONSIDERED IMPORTANT

The following sections will shed light on some of the anticipated threats to the successful implementation of the strategy.

6.5.1 Lack of participation in class activities

The literature review and the findings of the study (see Sections 3.6 and 5.5.1 respectively) indicate that a lack of learner participation is inevitable in multilingual classrooms when English is used more often than learners' home languages. When home languages are "banned" and the sole use of English is promoted, many learners may withdraw from participating in class activities for fear of being ridiculed or laughed at, should they not be able to express themselves well in English.

In an attempt to address this threat, the findings (see Section 5.5.1) indicate that teachers need to provide learners with as many opportunities as possible to express themselves, not only in English but also in their home languages where possible. This includes expression through recording, dramatisation, written journals, portfolios, and so forth. Furthermore, the teachers must deliberately, intentionally, purposefully and carefully use learners' home languages as stimuli to spark debates and encourage participation. Code switching was also recommended to increase learner participation in class. According to Essien (2013:59), code switching as a linguistic resource promotes learner participation and enables learners to harness their home language as a learning resource.

6.5.2 Unavailability of mother tongue material

The unavailability of teaching material written in learners' home languages was found to be another threat towards the successful implementation of the strategy. The literature review and the empirical data (see Sections 3.6 and 5.5.1 respectively) revealed that most teaching materials are still compiled in English only and, therefore, do not provide more clarity in a language that learners may find more understandable than English. These materials are usually not "reader-friendly" and thus require teacher intervention to make the content more comprehensible.

In order to address this, further training on the use of language, conducted by language and subject content specialists (English and mathematics teachers respectively), was identified as a possible solution to this threat. English teachers, as language practitioners, would assist in ensuring that the teaching material is simple and comprehensible, while the mathematics teachers would empower the learners with content in accordance with what the English teachers would have suggested as good practice to teach the problems that are in text format.

6.5.3 Stakeholders' unwillingness to cooperate

The empirical data (see Section 5.5.1) reveal that teachers may be unwilling to cooperate as they find it difficult to work together. For example, the mathematics teachers may regard their pedagogical content knowledge sufficient to teach learners MWPs effectively and may, therefore, not see the need to work collaboratively with the English teachers. In the process, they disregard the English teachers' role as "language specialists". On the other hand, the English teachers may not fully understand their role in assisting the mathematics teachers and, therefore, may not cooperate as expected. However, in addressing this, it was recommended that the goal rationale for teamwork between the mathematics and the English teachers should be clarified so that both sides may realise how they could assist each other in the process of teaching of MWPs.

6.5.4 Influence of linguistic structures in meaning and interpretation

The literature and the empirical data findings indicate that learners bring a mixture of languages into multilingual classrooms (see Sections 3.6 and 5.5.1). The linguistic structures emanating from the different languages usually influence the learners' interpretations and translations of text. The implication is that the meanings of mathematical words cannot be considered separately from the meanings that learners attach to those words in their home languages. The role of the teacher in this instance becomes challenging in the sense that the teacher has to make sure that common understanding and interpretation of the various words within specific contexts are established. However, addressing vocabulary in this instance was found to be helpful in ensuring that learners become aware of the different meanings and interpretations of the words used in common language and in the context of mathematics.

6.6 SOME OF THE IDENTIFIED SUCCESS INDICATORS OF THE IMPLEMENTATION OF THE UDL STRATEGY

The following sections highlight some of the indicators of the successful implementation of the UDL strategy.

6.6.1 Improved reading skills of learners

The literature and the empirical data (see Sections 3.7.1 and 5.6.1 respectively) indicate that the learners' reading skills improved when they were frequently engaged in reading, guided by the teacher. By frequently being engaged in the reading of various word problems, the learners eventually became used to these types of problems and were then able and willing to read the text in full, instead of just scanning it or not even attempting to read it at all. As the learners became accustomed to reading as a result of being frequently exposed to different reading activities or materials, their reading skills, analysis and interpretation of the word problem statements improved significantly. The empirical data (see Section 5.6.1) reveal that improved reading skills consequently reduce the number of errors since learners now understand what the word problem entails.

6.6.2 Improved mathematical vocabulary and register of learners

The literature and the empirical data (see Sections 3.7.2 and 5.6.2 respectively) indicate that the clarification of the mathematical vocabulary and register improved the learners' understanding of the application of specific terms in different contexts. The use of a mathematical glossary benefited the learners immensely since it consisted of the important terms of various concepts that the learners needed to know and understand (see Section 5.6.2). The glossary served to remind the learners of the meaning of the different words and the specific application of each word. When teachers constantly pay attention to the development of learners' mathematical vocabulary and register, they not only increase the learners' knowledge but also support the development of the learners' cognitive growth (see Section 3.7.2). When learners are frequently engaged with mathematical registers and vocabulary and are given multiple opportunities to communicate mathematically in the classrooms, they become successful in learning MWPs. Furthermore, the findings revealed that improved learner vocabulary reinforces and supports learners' conceptualisation of MWPs.

6.6.3 Elimination of ambiguity

The literature review and the results of this study (see Sections 3.7.3 and 5.6.3 respectively) indicate that it is important that teachers provide learners with opportunities to communicate mathematically in class. This gives the teacher an opportunity to listen to the learners and, through their conversations, identify possible causes of ambiguity or what learners portray to be causing ambiguity. When teachers continue to clarify these words by explaining them in context, the learners are made aware of the ambiguities and also learn how to avoid them (see Sections 3.7.3 and 5.6.3). When teachers carefully use the mathematical vocabulary and register when teaching, the learners also become careful in using the vocabulary according to the different contexts, thus avoiding ambiguities. They also learn not to confuse the meanings of words used in the classroom context with the meanings of the words in common language. Giving word problems to the learners with pictures or diagrams alongside the text assists in converging the learners thinking in the "right direction" and enable the learners to apply the concepts in the right context.

6.6.4 Enhanced visualisation skills of learners

Diagrams assist in improving learners' understanding of the problem since it depicts what learners' mental pictures should look like while they are reading (internal visualisation) (see Sections 3.7.4 and 5.6.4). Using different colours when dealing with large amounts of data enables learners to organise and differentiate the data (see Section 5.6.4). The colours also help learners to notice commonalities about the data easily and discover patterns. On the other hand, the use of manipulatives assists learners with concretisation of the problem and provides them with opportunities and suggestions for different ways to think about and solve the problem. Furthermore, learners who are regularly encouraged to draw a diagram to visualise word problems, develop a conceptual and procedural understanding of the problem, which both lead to the attainment of the correct solution. The empirical data (see Section 5.6.4) further indicate that breaking down the word problem into smaller parts serves as a good strategy to scaffold learners' understanding of the problem and stimulate visualisation. Therefore, dividing word problems into schemas allows the learners to visualise the problems, thus discovering the structural resemblances and differences in problems and their solutions. This better equips them to identify and solve similar types of problems.

6.6.5 Improved teaching skills

Teacher training on the teaching of MWPs as well as teaching of learners in multilingual classrooms empowered the teachers to do just that, namely to teach MWPs effectively in multilingual mathematics classrooms (see Sections 3.7.5 and 5.6.5). What also contributed to the success of the training was teacher collaboration and participation in issues that had an impact on their teaching practices (see Section 5.6.5). The training provided the teachers with a platform to engage and interact closely regarding these issues. In the process, the teachers' knowledge on the teaching of mathematical word problems increased. The teachers also became empowered with strategies to teach MWPs effectively in multilingual mathematics classrooms. The training offered the teachers opportunities to learn how to prepare lessons and activities for learners in multilingual classrooms effectively. Strategies, such as integrating real-life problems into the lessons and using examples, provided the learners with positive learning experiences and also made learning meaningful since the learners could realise the applicability of the concepts they were learning in class to situations in real life (see Sections 3.7.5 and 5.6.5). Teachers who are empowered could develop the learners' mathematical vocabulary and register as well as empower them to use intricate syntax to make meaning of the given problems. These teachers could also identify errors relating to the decoding of words, vocabulary and register, as well as errors akin to the transformation of oral descriptions in the appropriate mathematical symbols and correct operation. The training further equipped the teachers with the skills to engage in accurate reflective decision-making processes that inform the strategies that could be employed to aid learners' comprehension of MWPs.

6.6.6 Effective use of learners' home languages in the classrooms

The use of learners' home languages in the classroom helps with the clarification of words that keep learners from fully comprehending word problems (see Sections 3.7.6 and 5.6.6). The use of learners' home languages also assists in increased participation levels in classroom activities. Furthermore, home languages make it easy for learners who are unable to express themselves or are afraid to speak in English, to engage in class activities. When the learners' home languages are used as resources for learning to support the medium of instruction (English), learners tend to understand the content better. Code switching, though it has its own demerits, was found to benefit both the teachers and the learners since it ensured that the key ideas were conveyed swiftly and easily, not only in English but also in other languages that both the teacher and the learners understood. Code switching in a multilingual classroom plays an essential role in facilitating textual meanings for learners who have limited proficiency in the English language.

6.7 SUMMARY

The literature confirms that MWPs are the most challenging problems learners have to solve in mathematics education. It is, therefore, important that the learners are adequately supported to understand this genre of mathematics. This means that the teaching of MWPs needs to be enhanced so that it could address the challenges that learners may have regarding this genre of mathematics. The biggest challenge so far regarding the teaching and learning of MWPs is the language barrier, which has a negative impact on learner comprehension. Adding to this, many teachers find MWPs as challenging to comprehend as their learners, which has a negative impact on their teaching practices. Furthermore, the fact that this genre is taught in multilingual classrooms (which come with their own challenges and complexities, as described in Chapter 1) makes effective teaching of MWPs even more challenging.

Sections 6.2.1 to 6.2.6 of this chapter highlighted in brief the main findings regarding the challenges that justified the need for the formulation of a UDL-based strategy to enhance the teaching of MWPs in multilingual mathematics classrooms. The main findings pertaining to the solutions to the challenges, the conditions conducive to the successful implementation of the strategies, as well as the threats and indicators of success were also highlighted in this chapter. These findings were drawn from both the literature and the empirical data generated in this study. The findings revealed in this chapter indicate that the objectives of the study, as outlined in Chapter 1 (Section 1.5) were achieved.

6.8 IMPLICATIONS OF THE STUDY RESULTS ON THE PRACTICE

The current study was conducted over a period of two years. During this period, I implicitly immersed myself into the research process and the following forms of knowledge: pedagogical knowledge, content knowledge, pedagogical content knowledge, knowledge on how to deal with diversity and the application of inclusive teaching strategies; their direct implications to other fields of knowledge are a product of this study. The teaching implications of the study require the teachers to be reflective about their teaching practices, anticipate diversity and put in place the strategies that would enable every learner to learn MWPs in multilingual classrooms in a better way. The guidelines derived from this study may also serve as a guide in the other fields.

6.8.1 Epistemological implication

Prior to this research, several related and almost similar studies have been conducted locally and internationally. However, the present study adds useful insight into the use of the aspects of a UDL teaching strategy, which its application assists in creating inclusive learning spaces for all learners to effectively learn, regardless of the characteristics they bring into the classroom. This strategy thus serves as a guide on how to teach MWPs effectively in multilingual classrooms.

6.8.2 Methodological implication

The uniqueness of this study methodologically was to generate data that would lead to an understating of what the challenges pertaining to the teaching of MWPs in multilingual mathematics classrooms were, which is an issue that was also covered in some previously conducted studies of a similar nature. However, in this study, learners, who are usually marginalised in the decision-making process, were involved in the research project and their voices also constituted the results of this research. This inclusion, therefore, enabled the learners to explain how they were taught and also provided them with an opportunity to indicate how they should be taught. A slogan for people with disabilities says, “Nothing about us without us.” This means that for the issues pertaining to this group of people to be addressed, they also need to be involved in the discussions. In this study, even though it was not about disability, we also aligned ourselves with this slogan, in that when formulating a functional UDL-based strategy to enhance the teaching of MWPs in multilingual mathematics classrooms, we need to involve the learners whose strategy is, to some extent, intended to assist them to learn MWPs effectively. The choice of this methodology was informed by its inclusive nature, which afforded all the participants an opportunity to make contributions. In this way, the participants (including the learners) were able to bring about solutions to their situation. Adding to this, PAR was chosen because of its empowering nature, which afforded everyone to be enlightened about the issues revolving around the teaching of MWPs and what could be done to address the challenges thereof. This means that the project would continue even when the researcher would no longer be around due to the knowledge and experience they would have gained throughout the research project.

6.8.3 Pedagogical implication

The study provides some evidence of the positive influence of a UDL-influenced teaching strategy to facilitate MWP instruction in a multilingual classroom. Therefore, on the basis of the findings of this study, it may be reasonable to recommend that teachers apply the aspects of the UDL framework with a specific focus on the three main principles. The first principle requires the information to be provided through numerous means of representations and demonstrations to provide learners with different ways of acquiring, processing and integrating information and knowledge. The second principle necessitates the provision of multiple means of action and expression to provide learners with options for navigating and demonstrating what they have learned. Lastly, the third principle requires the provision of numerous means of engagement to stimulate individual learner interests, challenge them appropriately and motivate them to learn, thus tapping into their affective domain.

6.9 ASPECTS FOR FURTHER RESEARCH AND RECOMMENDATIONS

Considering how this research had ben conducted and the fact that it was done at one school, I suggest that a considerable number of schools should be clustered in order to observe what could come out in relation to this study. Furthermore, other aspects related to the teaching of MWPs in multilingual classrooms need to be explored further so that there could be as many strategies that could be derived as possible to address the teaching of MWPs in multilingual mathematics classrooms, which are considered to be complex spaces to teach in. These include, among others, exploring the effects of multilingualism on learner performance, added higher education institutional interventions to be put in place to address challenges posed by the existence of multilingual classrooms, and so forth. The effects of some of the strategies recommended in this study need to be explored further.

6.10 CONCLUSION

This chapter provided the main findings, as informed by the literature review (see Chapter 3) and the empirical data (see Chapter 5). The findings were presented in line with the objectives of this study, as outlined in Chapter 1. The summary of the findings, the limitations of the study and aspects for future research and recommendations were covered in this chapter.

The following chapter presents the envisaged UDL strategy for implementation.

CHAPTER 7

THE PROPOSED UNIVERSAL DESIGN FOR LEARNING-BASED STRATEGY TO ENHANCE THE TEACHING OF MATHEMATICS WORD PROBLEMS IN MULTILINGUAL MATHEMATICS CLASSROOM

7.1 INTRODUCTION

The purpose of this study was to utilise the aspects of universal design for learning (UDL) to develop an effective teaching strategy for mathematics word problems (MWPs) in a multilingual mathematics classroom. In line with the purpose of this study and on the basis of the findings, the reviewed literature and the objectives of this study, this chapter proposes a unique comprehensive UDL-based strategy to enhance the teaching of MWPs in a multilingual mathematics classroom. In this chapter, therefore, the guidelines for teaching MWPs in a multilingual mathematics classroom in line with the UDL principles are presented. These guidelines should assist teachers in teaching MWPs effectively in multilingual mathematics classrooms.

7.2 PROPOSED UDL-BASED STRATEGY FOR TEACHING WORD PROBLEMS IN A MULTILINGUAL MATHEMATICS CLASSROOM

The study mainly focused on the application of the UDL strategy to enhance the teaching of MWPs in multilingual mathematics classrooms. However, its inferences and application go beyond the teaching of MWPs in multilingual mathematics classrooms. Therefore, the strategy can inform the teaching of the other mathematical concepts (not only the MWPs) as well as the teaching of the concepts in other subjects within the school curriculum. My personal experience supported by the literature indicates that MWPs are complex and that learners find them challenging to master. Literature further adds that the teaching of MWPs becomes even more intricate in multilingual classrooms in which the learners are still learning the LoLT and, simultaneously, mathematics (Essien, 2010:33). However, UDL has been found to be an effective strategy to help teachers address the needs of all the learners, regardless of the characteristics (e.g. background, prior knowledge, learning styles, etc.) they bring into the classrooms, including the ones who do not have English proficiency. There is a paucity of research on the use of a UDL strategy to enhance the teaching of MWPs particularly in multilingual mathematics classrooms. This study, therefore, bridges this gap by providing a comprehensive guide for mathematics teachers to follow in teaching MWPs in multilingual mathematics classrooms guided by the aspects of the UDL strategy. The proposed UDL-based strategy, therefore, serves as an instrument that comprises the “pointers” for effective teaching of MWPs in multilingual mathematics classrooms. Therefore, through this proposed strategy, teachers will be empowered to teach mathematics in diverse classrooms and thus accommodate a wide range of learners, including those with limited English proficiency when teaching MWPs in multilingual mathematics classrooms.

On the basis of the literature and the empirical data, the study proposes the UDL-based strategy to teach MWPs effectively in multilingual mathematics classrooms; this strategy is guided mainly by three principles, namely multiple means of representation, multiple means of action and expression and multiple means of engagement. Since UDL is a broader framework, the proposed UDL-based teaching strategy for teaching MWPs in multilingual mathematics classrooms will also draw from the other aspects of the UDL broader framework. The discussion will be based on the theoretical point of view, with a specific reference on the findings in the previous chapters to explain the implications for teaching MWPs in multilingual mathematics classrooms. The strategy will address the key identified challenges that justified the need for the formulation of this UDL-based strategy, which serves as a major contribution this study is making to the body of knowledge.

As already alluded to, the proposed UDL-based strategy is based on three principles, namely multiple means of representation, multiple means of action and expression and multiple means of engagement. The following sections present the guidelines in line with these principles.

7.3 MULTIPLE MEANS OF REPRESENTATION

The principle of multiple means of representation requires that the content is presented in varied formats or modalities. This should be done to ensure that information is perceptible to the learners irrespective of their level of English proficiency. Multiple means of representation suggest easier mechanisms for learners to access information and to comprehend it. In line with this, the following sections (7.3.1 to 7.3.3) highlight ways in which multiple means of representation can be provided to learners by the teachers when teaching MWPs in multilingual mathematics classrooms. The information presented in these sections is informed by the objectives of this study as well as the findings of this study and thus serve as a guide for what needs to be done by the teachers in line with this principle. Multiple means of representation requires the following options to be provided to the learners when teaching MWPs: ***options for perceptions, options for language, mathematical expressions and symbols and options for comprehension.*** The following sections provide the descriptions of these options as well as the examples to demonstrate the operationalisation of this principle (multiple means of representation) in terms of the various options to be provided.

7.3.1 Provide options for perceptions

How learners make sense of the word problems differs based on their different understandings, conceptualisations as well as perceptions. This requires that the teachers vary the ways in which information could be presented. This is based on the premise that learners access information differently and, therefore, this principle encourages the provision of flexible and manifold ways to present information. For example, the teacher may not only rely on the use of the chalkboard to present information to the learners but PowerPoint, as a visual aid instrument (presentation graphic), may also be used to supplement the teaching process (see Section 6.3.3). Such presentation graphics may assist by increasing visual impact, spontaneity and interactivity and also improve learner focus. Presentation graphics of this nature assists in heightening learner visualisation skills.

Since learners lack the reading skills that facilitate understanding of the mathematical concepts, teachers, therefore, need to devise ways in which to simplify and make the word problems comprehensible. Teachers thus need to customise the display of information to provide options for perceptions (CAST, 2011:5)

Example: Learners may be given a problem in the form of text. A picture that highlights some of the features mentioned in the text may be provided alongside it to make the problem more perceptible. This will assist learners in realising what the problem entails or requires to be solved. This strategy would further assist in converging learner thinking in the "right direction".

One aspect of problem solving that is emphasised is that teachers must ensure that the learners understand what they are supposed to do to solve the problem (Perveen, 2010:9). Varied representation of the problem that makes it easier for learners to understand the given problems may, therefore, assist learners in knowing and understanding what they are asked to find or show (determine the unknown).

7.3.2 Provide options for language, mathematical expressions and symbols

As already highlighted in the previous chapters, mathematics has its own register, rules, grammar, syntax, vocabulary, word order, synonyms, negations, conventions, abbreviations, sentence structure and paragraph structure (Ní Ríordáin et al., 2015:13). It is, therefore, important for the learners to understand mathematical language so that they may not struggle to work out the given problems. Supporting this notion, Chitera (2009:32) avows that part of learning mathematics is gaining control over the mathematical language so that one is able to talk like a mathematician. Therefore, when learners are provided with opportunities to communicate mathematically in class, they eventually become empowered in terms of decoding the text, mathematical notations and symbols.

As previously alluded (see Section 3.3.2), mathematics is a language that is different from the language learners use at home and on the streets. Therefore, it is important that words that are used daily and take on different meanings in class be clarified so that learners might be familiar with these words and their usage in various contexts; for example, words such as ***set, table, function*** and ***domain*****.** These words are used every day by the learners at home and on the streets. They are also applicable in mathematical content. Therefore, if these words are not explicitly explained, they may cause confusion and obstruct learners' understanding of the content.

Consider the following example: **"three from seven leaves four".**

This statement can be interpreted and presented numerically as **"7 – 3 = 4"** and not **"3 – 7 = -4"**. Therefore, if the learners do not understand the meaning of the words used in the statement, such as ***from*** and ***leaves****,* and do not understand the ***syntax*** and ***structure*** of such a mathematical expression, then the learners may not obtain the correct solution.

It should also be of greater significance to realise that the learners' interpretations and translations of words are usually influenced by the linguistic structures in their home languages and by the interaction between their home languages and the LoLT (English). This means that the mathematical word meanings cannot be considered separate from how learners understand those words in their home languages and also in how they use those words in other settings. As such, this necessitates the teachers to promote understanding across the languages to ensure that the correct understanding and meaning in a particular context is established (see Section 6.3.2).

Consider the phrase ***"at least"***; mathematically speaking it implies **"greater or equal to"** and symbolically it can be represented as follows: **(≥)**. This phrase may be understood and interpreted differently by Sotho-, Xhosa- and Zulu-speaking learners drawing on their home languages for meaning, for example, "buncinci" in Isixhosa, "bonyane" in Sesotho and "okungenani" in IsiZulu. If this phrase is not defined correctly and put in context, the learners may misinterpret it in class (mathematical context) to imply "less" (English term).

7.3.3 Provide options for comprehension

MWPs mostly deal with concepts relating to real-world situations and thus help learners use their mathematics knowledge in solving their daily problems (Seifi et al., 2012:2923). This means that the problems which learners have to deal with are ***mostly*** those they already know of or familiar with. For example, learners may be given a word problem based on the concept of “building and measurements”. In this instance, they already know what a house looks like and also know most of the features of the house. With that information they already know in mind, they may then have to solve the problems related to measurements using the formulae taught in class.

Example: If they are told that the length of the one side of a room is ***8 m*** and the breadth is ***4 m***, they can easily determine the perimeter of the room when they know the characteristics of a room (i.e. it has four sides of which two are parallel). Therefore, the perimeter would be calculated as follows:

$$\begin{aligned}
&Perimeter = length + breadth + length + breadth\\
&P = l + b + l + b\\
&P = 2l + 2b\\
&P = 2(8) + 2(4)\\
&P = 16 + 8\\
&P = 24
\end{aligned}$$

OR

$$\begin{aligned}
&P = l + b + l + b\\
&P = 8 + 4 + 8 + 4\\
&P = 12 + 12\\
&P = 24
\end{aligned}$$

What is of vital significance from the above given problem is that for this type of problem to be solved successfully by the learners, the teacher needs to activate or check the learners’ prior knowledge of the concept or content (e.g. properties of the different four-sided figures, grouping of the like terms when performing addition, application of the operational signs, etc.) that is being dealt with. This would ensure that the learners are well guided in terms of solving the problem.

In terms of guiding information processing, visualisation and manipulation, the teacher may use the diagram to help the learners process information. The different colours may assist with the improvement of visualisation and make the picture of what the problem entails more clear. This promotes understanding and enables the learners to solve the word problems (see Section 6.3.3). An example of a mathematical concept that could be used in this case would be the “probability” concept.

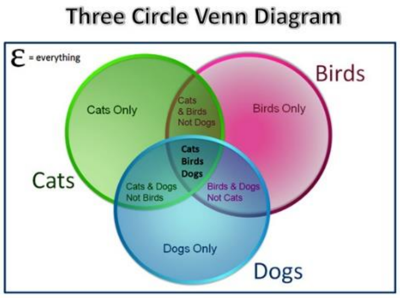


Diagram 7.1: Venn diagram

From the above diagram, learners may be able to see the **point of intersections** as made vivid by the different colourings. The different colourings assist with the organisation of the data and consequently improves the learners’ visualisation skills. To enhance learner understanding of the word problems, the teacher can also highlight patterns, critical features, main ideas and relationships that underlie the problem(s). In this way, the learners’ comprehension of the content and text could be enhanced.

Example: In a village that has a total number of ten houses, the first house has one tree, the second house three trees and the third house five trees…

In this type of word problem, the teacher may highlight the pattern (describing the relationship between the terms) as one way of enhancing learner understanding of the concept. The teacher may also give the general formula to be used and then explain how the alphabets or symbols in the formula relate with one another.

Example: In a formula $T_n = a + (n-1)d$

There are two ways to indicate the relationship between the terms in the problem given above. In order to get the next term, we have to add two (+2) and to get the previous term we have to subtract two (-2). Once the learners are aware of this relationship between the terms, they can determine the next term(s) on their own; e.g. $T_4, T_5, T_6 \ldots$

1, 3, 5, $T_4, T_5, T_6 \ldots$**!** Adding the unknown terms to a sequence in which the learners are already aware of its progression would then make it simpler for them to expand the pattern even further. After working out the unknowns, through the substitution method, thus obtaining the succeeding terms, the learners can make generalisations and derive a general term formula that applies for that sequence, for example: $T_n = a + (n-1)d$

When learners are given the series, **2, 8, 32, (** $T_3 x \frac{T_2}{T_1}), (T_4 x \frac{T_3}{T_2}), \ldots$ **…!** , to get the next term they have to multiply the present term with 4. To get the previous term they need to multiply the present term with $\frac{1}{4}$. Once all these have been established, learners can highlight a pattern, recognise the relationship that exists between the terms and make generalisations as well as determine the general formula following the above highlighted procedures (as in the previous example). Through this strategy, the learners may also be able to identify the relationship that exists between the operational signs (e.g. + and - as well as ÷ and ×).

The teacher may further provide the two general term formulae for arithmetic sequence and geometric series respectively and then explain their critical features to indicate their distinctions as shown in Table 7.1.

Table 7.1: Differences between arithmetic sequence and geometric series

Arithmetic sequence	**Geometric series**
Next term = current term plus difference e.g. $T_3 = T_2 + d$	Next term = current term multiplied by common ratio e.g. $T_3 = T_2 x \frac{T_2}{T_1}$
Common difference – d	Common ratio - r
$T_n = a + (n-1)d$ [general term formula]	$T_n = ar^{n-1}$ [general term formula]

The above discussions highlight ways in which the teachers can assist learners to understand the word problems. The guidelines provided above may be followed in order to improve learners' reading and visualisation skills. Information that is presented in varied modes improves the learners' understanding of the concepts. From the above discussions, in line with the provided examples, we can deduce that the teachers could provide multiple means of representation in class by providing various *options for perception, options for language, mathematical expression and symbol* as well as *options for comprehension.* According to CAST (2011), the provision of multiple means of representation through these options enables teachers to produce **resourceful and knowledgeable** learners.

7.4 MULTIPLE MEANS OF ACTION AND EXPRESSION

The previous chapters demonstrated the significance of affording the learners opportunities to communicate mathematically. This enables learners to develop the mathematical vocabulary and register through their conversations within the multilingual constructivism classroom. Drawing from the previous chapters, including the findings of this study, what emanated to be profound as part of this envisaged framework is that when the learners are given problems to solve, it is important for the teachers to provide them with time to carefully read the word problems and "process" the given information so that they can fully understand it. Such an understanding helps them know what is required of them to solve the given problems. Understanding the given problems is not sufficient, however. The learners also need to demonstrate what they have learned or what they understand about the problems. Since learners are different (in terms of personalities and abilities), it is, therefore, important for the teachers to provide learners with multiple opportunities to demonstrate what they have learned.

The principle of multiple means of action and expression emphasises that multiple options should be provided for the learners to demonstrate an understanding of what they have learned or an understanding of what they have read through their own interpretation. The following sections, therefore, highlight ways in which multiple ways of action and expression could be provided for learners in multilingual mathematics classrooms to demonstrate understanding of what they have learned. The discussions in the subsequent sections are informed by the objectives of this study as well as the findings as documented in Chapter 6. Under this principle (multiple means of action and expression), three options, namely ***options for physical action, options for expression and communication*** and ***options for executive functions*** are discussed.

7.4.1 Provide options for physical action

Mathematics requires much problem solving. This is the process by which learners demonstrate steps and routes that lead to the attainment of solutions. The findings revealed that during this process, learners have the responsibility to engage in the step-by-step solving of the problems, which have to be produced on paper. This means that the problem-solving activities which the learners are mostly engaged in require that they demonstrate what they have learned or demonstrate an understanding of what they are reading when dealing with MWPs (when reading the provided scenarios) through following certain procedures on paper. Problem solving in the context of MWPs also requires learners to think about what they are reading and thus make sense of it. This type of reading inspires visualisation of situations "in their heads" before they can write on paper what they think or believe to be the correct procedures to follow when solving the given problems. Based on this, the teacher needs to emphasise the need for careful reading (reading between the lines) and a thorough analysis of the given problems (see Section 6.4.3).

The action of reading and carefully analysing the problem enables the learners to see what is given and what is not given (known and unknown respectively). This then requires the teachers to provide the learners with "ample" time to read and process the given information on their own. Learners may be granted opportunities to use their home languages to interpret the questions so that they could establish what the problems entail or establish what they are required to solve (determine the unknown). Home languages may assist learners to comprehend the problems and to develop their mathematical proficiency to transfer meaning to the second language (English), which is the medium of instruction. Learners may also be placed in groups so that after reading the problem, they could analyse it together in their small, intimate groups. This type of interaction invigorates conversational engagements and stimulates the learners to communicate mathematically, thus developing their mathematical vocabulary and register in the process. Furthermore, it affords explorations and discovery of diverse interpretations and construction of meanings.

Through this type of interaction and engagement, the teachers can establish sources of misconceptions and misinterpretations leading to the attainment of incorrect solutions to the problems. Working in small groups further enhances learners' self-confidence and empowers and enables the learners to share with the rest of the class (bigger group). During the learner interaction (in their small groups), the teacher may identify which method(s) of responding to questions learners prefer. This is crucial for ensuring and maximising learner participation and engagement in class activities. The teacher may, consequently, capitalise on learner participation by highlighting the gaps and misconceptions while the discussions are taking place (see Section 6.6.3).

Another way of providing options for physical action is when a teacher optimises access to assistive tools in the classroom. This means that the teacher may allow learners to use manipulatives. These are concrete tools which the learners can relate to and which they can see. Since learners learn by doing, teachers can use these tools to reinforce mathematical concepts. Teachers may also use these tools for hands-on, small-group activities and for letting learners explore and develop reasoning and problem-solving skills. Words and charts on the wall may also be used as assistive tools to remind learners of the various concepts and enhance their understanding of the mathematical vocabulary and register. Engagement with these assistive tools makes information more perceptible and understandable (see Section 6.4.2).

Example: If learners are dealing with the concept "shapes and measurements", the teacher may bring to class boxes of different shapes to explain those various mathematical object shapes practically. The teacher may also demonstrate the different dimensions the different objects have. The different formulae to be used to calculate the particular aspects of each object may be provided to the learners with explicit explanations on how to use them to calculate those aspects (see Section 3.4.2, Table 3.1)

7.4.2 Provide options for expression and communication

The previous chapters highlighted ways in which options for expression and communication could be provided to the learners. These options are significant in teaching and learning since from time to time learners have to demonstrate what they have learned. Teachers need to recognise that learners express themselves effectively in different manners. Therefore, as an educator, multiple means for learners to express themselves and share their learning progress should be provided. In line with this, the teacher can provide options for expression and communication through building fluency with progressed levels of support for practice and performance (CAST, 2011:5). In order for the levels of fluency to be successfully built, the teachers need to provide the learners with various opportunities to express what they have learned and to communicate mathematically. Learners may be provided with tutorials that are conducted by the other learners (peers – senior learners who have done the subject before and have passed it) to help support learner comprehension of MWPs. Tutorials thus would serve as intimate spaces or platforms and “practice grounds” in which learners get to work together and communicate mathematically in a relaxed manner. This platform can be used by learners to learn the vocabulary and register, communicate mathematically, revise and receive clarity on the work that has already been taught by the teachers. This, in turn, fuels them with the necessary confidence to express what they have learned in any form.

The teachers can also provide learners with reading skills. Not only would this improve the ability to read text but it would also enhance comprehension of text and vocabulary which, in turn, may heighten their confidence to demonstrate what they have learned verbally or using any form of media (e.g. pictures, graphs, tables, etc.). The literal and explicit teaching of vocabulary and register may serve as one way of empowering learners in terms of expressing themselves and communicating mathematically. Giving learners homework that supports the intended goal to be achieved and allowing for feedback to be given in various forms is one strategy teachers can use to optimise and stimulate learner participation in class.

7.4.3 Provide options for executive functions

Teachers need to help learners manage and organise information so that they can make sense of it. In order to do this, they can provide external organisational aids and scaffolds to help keep learners organised. Examples may include the use of graphic organisers and templates for data collection and organising information, embedding prompts for categorising and systematising and providing checklists and guides for note taking. Guiding of the appropriate goal setting (CAST, 2011:5) requires the teachers to ensure that they give learners exercises that encourage learning of the mathematical vocabulary and register (language learning). The lesson plans should be designed in a manner that support the intended goal. The activities should also be carefully planned in a manner that assists in the achievement of the intended goal.

The teachers also have to undergo capacity developmental training so they could be equipped with the skills to teach effectively in multilingual mathematics classrooms. In addition to these, the teachers need to undergo training on pedagogy and content development to enhance their pedagogical content knowledge. The developmental training would help ensure that support planning and strategy development are fulfilled. Through the developmental training, the teachers would further gain knowledge on how to effectively guide and scaffold learner understanding of MWPs. In terms of enhancing the capacity for monitoring progress, teacher peer observations may have to be conducted for the teachers to give feedback to one another and thus build one another. The teachers may provide constructive feedback to their peers, based on their performance in class (classroom practices) (see Section 6.6.5).

The above discussions, coupled with the examples provided, indicate that multiple means of action and expression in class when teaching MWPs in multilingual mathematics classrooms could be provided by the teachers in various forms. This could be done through the provision of *options for physical action, options for expression and communication* as well as *options for executive functions.* Providing multiple means of action and expression through these options help produces strategic and goal-directed learners (CAST, 2011).

7.5 PROVIDE MULTIPLE MEANS OF ENGAGEMENT

Multiple means of engagement require teachers to develop various ways in which to help learners engage with the content. This could be done to ensure that learners stay engaged in the activities and also engage easily with the content. In line with this, the following sections (7.5.1 to 7.5.3) highlight ways in which multiple means of engagement could be provided to learners by the teachers when teaching MWPs in multilingual mathematics classrooms. The information presented in these sections is informed by the objectives as well as the findings of this study and thus serves as a guide for what could be done by the teachers in line with this principle to teach MWPs effectively in multilingual mathematics classrooms.

7.5.1 Options for recruiting interest

Learners differ significantly in what fascinates their attention and engages their interest. Even the same learners will differ over time and circumstance. This means that their "interests" change as they develop and gain new knowledge and skills, as their biological environments change and as they differentiate into self-determined beings. It is, therefore, essential to have alternative ways to recruit learners interest in ways that reflect the important inter- and intra-individual differences among them. Engaging learners in class activities is important in ensuring that the learners understand the concepts or content. It is also of vital significance that learners be kept engaged in the activities so that they do not lose interest. In order to optimise the level of engagement in the class activities, the teachers have to design activities that optimise individual choice and autonomy. The use of real-life situations or scenarios may assist immensely in optimising relevance, value and authenticity (see Section 6.6.5).

Clarification of concepts or mathematical terms as well as clarification of the application of such concepts or terms in various contexts assist in improving learner understanding of the concepts, thus minimising distractions towards learning. Furthermore, when the words that are used on daily basis as well as their application in the classroom activities are explained and a clear distinction is drawn in terms of application in the two contexts, that may assist with the elimination of ambiguities and also minimise threats and distractions towards teaching and learning (see Section 6.6.3).

7.5.2 Options for sustaining effort and persistence

As alluded to earlier, it is essential for teachers to ensure that learners are always kept engaged in the activities. This necessitates that teachers ensure learner-sustained effort and persistence. Teacher collaboration may immensely assist in supporting learners and making sure that learner-sustained effort and persistence are attained. Teachers may empower one another through the provision of constructive feedback to make the teaching of MWPs in multilingual mathematics classrooms a success. The provision of tutorials may also "come in handy" in ensuring sustained effort and persistence. Through the tutorials which are carefully planned and implemented, learners' understanding of the concepts may improve immensely. Through the implementation of tutorials, learner communities may be formed. These communities may assist in fostering a sense of working together, helping one another and learning from one another, which ultimately ensures sustained effort and persistence (see Section 6.4.2). For some learners, the option of working collaboratively with other learners is an effective way to sustain engagement in protracted activities. Therefore, the distribution of mentoring through peers can greatly increase the opportunities for one-on-one support. When carefully structured, such peer cooperation can significantly increase the available support for sustained engagement.

7.5.3 Options for self-regulation

The provision of multiple means of engagement also means to provide options for self-regulation. This necessitates that teachers promote expectations and beliefs that enhance motivation as they teach. Teachers can achieve this through the lessons they design, activities they engage learners in and how they go about presenting their lessons. The lessons, therefore, have to be presented in such a manner that they elevate the frequency of self-reflection and self-reinforcements. This is because the learners have to be engaged in problem solving frequently so that they can develop mastery of MWP. The teachers may further provide options for self-regulation through facilitating personal coping skills and strategies. Teachers may, therefore, teach learners reading strategies to assist them to cope in terms of reading the texts. This would enable learners to engage in MWPs individually without the presence of the teacher for guidance. Teaching learners mathematical vocabulary and register also assist in empowering the learners to effectively communicate mathematically. Learners thus engage with ease when they are confident about what they know. Furthermore, the teachers need to do self-assessments and reflection to assess their own teaching practices. This would assist them to know better ways in which to support their learners in multilingual mathematics classrooms.

7.6 SUMMARY

The above discussions, together with the highlighted examples, indicate that multiple means of engagement in class when teaching MWPs in multilingual mathematics classrooms could be provided by the teachers in numerous forms. The teacher could do this by providing *options for recruiting interest, options for sustaining effort and persistence* and *options for self-regulation.* Providing multiple means of engagement through these options help produces purposeful and motivated learners (CAST, 2011).

7.7 LIMITATIONS OF THIS STUDY

The first limitation of the study is that it was conducted at only two secondary schools in the Free State; therefore, the results of this study cannot be generalised. However, the results of this study may be applicable to contexts similar to those outlined in this study. Furthermore, even though the results were meant to show the teaching of MWPs in multilingual mathematics classrooms, readers may find similarities between their contexts and the contexts outlined in this study. The second limitation is that the study focused mainly on instances where both the teachers and the learners speak the same home language or the teacher is conversant with the learners' home languages in the classroom, in addition to the LoLT. Therefore, other scenarios, such as the teacher not knowing the learners' home language or the class being comprised of learners with different home languages, were not explored. Lastly, the intention was to understand the challenges pertaining to the teaching of MWPs in multilingual mathematics classrooms from the participants; therefore the voices of the people in this study are key to highlighting these challenges. There may be other challenges that are not outlined in this study; however, that is no indication that they are not applicable. The challenges identified in this study were deemed significant by the participants. It should be noted that the experiences of people are different and as such, people will report what they know, how they think and believe they know best, based on their experiences.

I could not go to the classrooms to video-record the sessions live as I initially anticipated, due to the request made by the principal for the research to be conducted when the classes have ceased and also due to receiving the conditional approval clearance letter later when the classes had already stopped. Therefore, the participants reflected upon how they taught the lessons (and also how they were taught the lessons), that is, what transpired when the lessons were taught. Therefore, some of the strategies that were recommended could not necessarily be tried and tested. However, since this is an ongoing project, which involves different cycles, the agreement was that in the next cycles, some of these strategies could be implemented carefully so that their effects could be traced. I am still in touch with the school to ensure that some of these strategies are implemented and monitor what emerges over and above these recommendations. It is reasonable in this sense to indicate that the articles that are to be written from this work should reflect upon the implementation of these strategies and the effects thereof.

7.8 CONCLUSION

On the basis of the findings in Chapters 5 and 6 as well as considering what the literature review entails, this chapter proposed a UDL strategy to enhance the teaching of MWPs in multilingual mathematics classrooms. The three principles of UDL, which are informed by the neuroscientist research, namely multiple means of representation, multiple means of action and expression and multiple means of engagement, were highlighted in this chapter. Furthermore, this chapter delineated how each principle could be operationalised in line with the findings of this study (Chapters 5 and 6) as well as the literature review. As explained in the section on the limitations of the study, the information that appear under these principles is not the only applicable information; however, this information was guided by the findings of this study through a participatory action mode.

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TRANSCRIPTS

APPENDIX A:

Hare sheba hona statementeng sena [referring to the problem which was provided by Mr Phatudi] lentswe "compounded" le ka ferekanya ngwana haholo ha a sa le tsebe hore le bolelang. Ngwana e mong aka seke a utlwisisa hore ho hlokahala hore interest e calculeitwe quarterly. Haholo jwang haeba a sa le tlwaela.

ET: *When we look at the statement again, the word "compounded" may confuse a learner, especially if they do not know its meaning. Another learner may not understand that in this problem the interest needs to be calculated quarterly.*

Mr Morake: *Bana bana ha ba utlwisisi English language. Ha o ba file palo e ngotsweng ka mantswe ha ba kgone ho e tlisa di variableseng. For example, ha ore ho bona eeehh… d is three less than the number a, ha ba e expressa bona hangata ba ngola d = 3 – a instead of writing d = a – 5*

ET: *These learners lack English proficiency. They cannot translate the word problems into variables. For example, eeehh…when you have given them the the problem, d is three less than the number a, they express it as d = 3 – a instead of writing d = a – 5.*

Mr Simelane: *Di word problems tsona di thata bo tsona! Jwale hee it becomes a major problem ha titjhere le yena antse anale problem ka tsona di word problems tsena.*

ET: *Word problems are naturally difficult! So what makes them to even more challenging for learners is when the teacher also finds them difficult to solve.*

Mr Phatudi: *Exactly! Di word problems ha di thata feela for bana empa hape le ho matitjhere.*

ET: *Word problems are not only difficult for learners but for the teachers as well.*

Ms Masombuka: *Hei le bona di ya ba sokodisa maan! Ha o bone ba bang ba bile ba di ngala basa di rute bana.*

ET: *They also struggle with them! No wonder some of them even choose not to teach learners.*

Ms Ntuli: *Bana bana hangata ha ba utlwisise ntho eo ba e balang…*

ET: *In most instances learners do not understand what they are reading…*

Bokang: *Nna ke nahana hore mathata ke English. Ho hlaha mantswe a thata moo ao re sa a utwisising. O thole hore English entse e re hlola ho tloha ka period ya yona.*

ET: *I think English is problematic for most of us. We encounter difficult words which we do not understand in many instances. We struggle with English even when we are taught it during the specific class.*

Mr Morake: *There is a teacher sekolong sane sa Moriting. [**ET:** "..at Moriting school"] Titjhere enwa o tswa hodimo kwana Africa [This teacher is from North Africa]. I was told hore [that] he is doing very well in Life Sciences. O ruta bana ka sekgowa feela hobane ha a tsebe Sesotho. Ho ruta bana ka English feela ho ya thusa hobane ba kgona le ho mamela attentively". [**ET:** "He teaches learners only in English since he cannot speak Sesotho. It really helps to teach learners only in English because then they can listen attentively"].*

Mrs Khumalo: *Taba ke hore bana bana ba lebeletswe ho communiceita ka sekgowa ha ba se ba le tertiary le moo batla beng ba sebetsa teng.*

ET: *The fact is that these learners are expected to communicate in English when they get to tertiary and also in the places where they will be employed.*

Ms Masombuka: *"Empa jwale re etsa jwang ha bana ba sa utlwisisi ka English?" the thing is, bana bana English entse e ba thatafalla.*

ET: *But then what do we do when learners do not understand in English? English is difficult for these learners.*

Mr Phatudi: *Nna ke bona ho sa thuse ho continuwa ka English empa bana ba sa utlwisisi.*

ET: *It does not really help to continue teaching in English when learners cannot understand.*

Mr Morake: *Ha re ruta feela ke English bana ba bang ba tshaba ho bua hobane batla tsheuwa*

ET: *Some learners fear to be laughed at when they express themselves in English.*

Tseko: *Bothata ke hore re le bana re botswa ho bala. Ha o bona statement o se ontse o nyahama tlaaar obe o bile o skipa potso eo because di thata di problems tseo.*

ET: *The problem is that we are lazy to read. When you see the statements you immediately become discouraged and decide to skip those problems since they are difficult to solve.*

Kamo: *Hape ka exameng le ha o di tlotse ontse oka nna wa pasa hobane ho hlaha tse few feela mos!*

ET: *One can still pass the exam having not written these problems since a few of them get to be asked.*

Ms Moeketsi: *Hao ka sheba bana ba maths lit ba tlwaetse mofuta ona wa dipalo and baya di bona ho feta bana ba pure maths.*

ET: *Learners who are doing mathematical literacy usually do not struggle with these problems as compared to learners who do "pure mathematics".*

Ms Masombuka: *…eeeer…mohlala, haba le hae ha ho buuwa ka Volume ba inahanela TV kapa radio moo teng ba tlamehang ho theola kapa ba nyolla volume, empa ha bale sekolong volume ese e bolela ha hong. Mohlala, if hothwe calculate the volume of the prism eleng hore it is totally different from what they already know from home. Hoka ba thata ho bona to work out this problem ha ba sa tsebe hore volume e bolelang ka classeng.*

ET: *For example, in the home context, volume may be linked with the radio, the act of amplifying and decreasing the loudness, whereas at school specifically in mathematics volume means somethings different from what they know from home about volume. For example, if the question says calculate the volume of the prism which different from what they already know, then it may be difficult for them to work out this problem if they do not know what volume means in that context.*

Mr Phatudi: *Ho se tsebe mantswe le ho a arohanya according to the different contexts ke hona hoo ele bothata. There is no way ngwana a ka succeedang asa utlwisise matswe ana a sekgowa.*

ET: *The major problem is that learners do not know the meaning and application of certain words different contexts. There is therefore no way in which learners could successfully solve these problems if they do not understand these words.*

Rorisang: *Re rutwa ka staela se iwane feela and o thole hore ka exam ho botswa ka staela seseng.*

ET: *We are usually introduced to a single way of questioning which differs from how questions are asked later in the exam.*

Tshepo: *Mohlala ka classeng ho tlabe ho thwe calculate the interest yearly ebe ka testeng hothwe annually. Bothata jwale e tlabe ele hore o tla be osa tsebe hore annually entse e bolela yearly.*

ET: *For example, sometimes the question requires that we calculate the interest yearly and later in the exam we come across a new term, namely "annually". The problem then would arise in terms of solving the problem since one would not know what annually means.*

Ms Buthelezi: *There is no way this children can master the word problems if they do not know how to read. Reading is important!*

Mr Maduna: *True, mme, these learners ba tlameha ho rutwa ho bala kamehla. Ke ka ho bala feela ba ka mastarang mmetse ona.*

ET: *True, madam! Learners have to be taught how to read frequently. They can only master this specific type of mathematics through reading.*

[All the participants agreed…mhh…mhh]

Tshepo: *[Refering from classwork book] Mohlala ka classeng ho tlabe ho thwe calculate the interest yearly ebe ka testeng hothwe annually. Bothata jwale e tlabe ele hore o tla be osa tsebe hore annually entse e bolela yearly.*

ET: *For example, sometimes the question requires that we calculate the interest yearly and later in the exam we come across a new term, namely annually. The problem then would arise in terms of solving the problem since one would not know that annually also mean yearly.*

Mr Morake: *Honale moo marking center re neng re debeita ka hore some learners mona ho finance ba tlwaetse re bua ka "reducing balance method". That year examiner a be a disaeta hore a sebedise lentswe "diminishing balance method"! Wa tseba a ntsha bana ba bangata jwang tjhunung! Bana ba bona a different thing ka lebaka feela la lentswe leo as opposed to reducing balance method eo ba e tlwaetseng. Lentswe leo la tjhentjha approach ya bana altogether. Ke ha bana ba etsa approach e fapaneng.*

ET: *There was a time at the marking center when we debated about this issue in which we found that learners knew the term "reducing balance method" for solving problems which involved money depreciation. However, the examiner that year used the term "diminishing balance method" instead of "reducing balance method". That term caused many learners to change the approach. They approached the problem differently.*

Ms Masombuka: *Bana ha ba filwe palo in the form of a story ba tshwanela ho e solva eya ba hlola because they just cannot deal with abstract things. They cannot make up the picture ka hlohong ya seo ho bolelwang ka sona.*

ET: *When learners are given problems in the story form they find it difficult to solve since they are be able to deal with abstract things. They usually fail to make a sound picture of what the problem is about in their heads.*

Mr Moraka: *Ha re ruta feela ke English bana ba bang ba tshaba ho bua hobane batla tsheuwa.*

ET: *When taught only in English, some learners become afraid to express themselves since they may be laughed at.*

Ms Masombuka: *Ho ka thusa haholo ho dula re bontsha bana hore mantswe a itseng a bolelang maemong a fapaneng! Ha ke etse mohlala ka lentswe lena "function"; mmetseng reka le sebedisa ho hlalosa equation. Empa Biologing le ka bolela mosebetsi wa organ e itseng. In some instances, I provide the terms that can be used in place of the one that is used at that time*

ET: *It will help a lot to keep on indicating what certain words mean in different contexts! Let me make an example of this word "function". In maths, we can use it to describe an equation. However, in biology, it may imply an activity which a particular organ performs.*

Mr Twala: *Ehe mme! Ho tshwana le hare dila ka "probability". Let's say re re ho ngwana a fane ka probability of A, kapa probability of A only jwalo jwalo. Ho bohlokwa hore re hlalose hantle baneng distatements tsena.*

ET: *True, madam! This is the same as when we deal with the "probability" concept. Let's say we ask a learner to give the probability of A, kapa (**ET:** "or") probability of A only, etc. It is important to clearly explain to the learners these two statements.*

Dibuseng: *Di statements tseo re di fuwang di lokela ho ya straight to the point hobane ha di se jwalo di ya ferekanya and dija nako.*

ET: *The statements which are given to us need to go straight to the point, otherwise they may bring confusion and waste time*

Ms Nkosi: *Le teng bana ha ba rutwa fela ka Sesotho kapa ISizulu re yaba bolaya hobane di potso ka exam di tla be di ngotswe ka English. Hape eka ba disadvantage in other areas.*

ET: *We "kill" these learners when we teach them in Sesotho or Isizulu because the exam questions are usually posed in English. The sole use of home languages would further disadvantage them in other areas.*

Tseko: *It would help if we are taught the real-life scenarios whereby we can be shown how a particular word can be used in different situations.*

Mr Twala: *Re tshwanetse hore re kgothaletse bana ho bala carefully [**ET:** "…We need to encourage learners to carefully read"] and to also consider the important words. Hapehape bana ba tshwanetse ho etsuwa aware hore haba bala ba utlwisise the question in total… [**ET:** "Furthermore, the learners need to be encouraged to read and understand the question in full."] Hapehape ho kathusa hore bana ba fuwe di scenarios le di pictures tsa tsona at the same time like in this instance [Showing an example]*

Ms Zwane: *Ho kgothaletsa bana kamehla hore ba bale ka tataiso ya ka ho thusitse haholo. Ke bona ba se ba sena monyeme wa ho bala. Hape ho fokotsa le diphoso.*

ET: *Encouraging learners all the time to read has helped a lot. I realised that they are now more keen to read than they were before. Frequent reading also reduces the errors. Simplifying the language as well as incorporating the reading skills in our teaching may also assist a lot.*

Mr Twala: *Lenna ke bona ho bala ka hloko ho thusa haholo. Ho ba thusa le ho presenta di shaded areas cartesian pleining haba filwe di inequalities.*

ET: *I also realised that careful reading helps a lot. It enables them to present the shaded areas correctly on the Cartesian plane when they are dealing with inequalities.*

Ms Ntuli: *It is imperative again to encourage learners to use the appropriate mathematical terms and not the ordinary language. For example, the words "at least" must be explained* *hore le bolelang in an ordinary language and also mathematically. So that way, bana ba understand the application of the word in different settings*

Mr Morake: *Honale moo marking center re neng re debeita ka hore some learners mona ho finance ba tlwaetse re bua ka reducing balance. That year examiner a be a desaeta hore a sebedise lentswe diminishing! Wa tseba a ntsha bana ba bangata jwang! Bana ba bona a different thing ka lebaka feela la lentswe leo as opposed to reducing balance method eo ba e tlwaetseng. Lentswe leo la tjhentjha approach ya bana altogether. Ke ha bana ba etsa approach e fapaneng.*

ET: *There was a time at the marking center when we debated about this issue in which we indicated that learners know the the term reducing balance method for solving problems which involve money depreciation. However, the examiner that year used the term diminishing instead of reducing balance. The introduction of this new term caused many learners to think that the problem required a different approach from the one they were used to and most of them got it wrong as a result.*

Mr Simelane: *When we frequently engage learners in this type of problems, e tla ba etsa ba tlwaele le hona ho bona ditaele tse ngata tseo dipotso di ka botswang ka teng [**ET:** "…they will become familiar with these problems and eventually recognise multiple ways in which questions could be asked"].*

Mrs Medupe: *Reading will also help these learners to distinguish between the language on the streets and the language in the classroom… for example the word "at least" can be used on the streets and also in the classroom and be di ba le meelelo e fapaneng […and have different meanings].*

Mr Phatudi: *[…!] now o nkgopotsa example engwe so… mola ho topic ya di inequalities. [**ET:** "You remind me of a particular example from the Inequality topic"] If you ask learners to indicate* $3 \le x \le 5$ *on the Cartesian plane baya e fosa ba e bontsha so […they get it wrong and illustrate it like this] [producing evidence] [insert a pic] which is wrong. So ke dumellana le wena mam ha ore reading etla thusa haholo. [So, I agree with you madam when you say, effective reading helps]*

Ms Masombuka: *Ho ka thusa haholo ho dula re bontsha bana hore matswe a itseng a bolelang maemong a fapaneng! Ha ke etse mohlala ka lentswe lena "function"; mmetseng reka le sebedisa ho hlalosa equation. Empa Biologing le ka bolela mosebetsi wa organ e itseng.*

ET: *It will help to keep on explaining what certain words mean in different contexts! Let me make an example of this word "function". In maths, we can use it to describe an equation. However, in biology, it may imply an activity which a particular organ performs.*

Mr Twala: *Ehe mme! Ho tshwana le hare dila ka "probability". Let's say re re ho ngwana a fane ka probability of A, kapa probability of A only jwalo jwalo. Ho bohlokwa hore re hlalose hantle baneng distatements tsena.*

ET: *True, madam! This is the same as when we deal with the "probability" concept. Let us say that we ask a learner to give the probability of A, kapa probability of A only, etc. It is important to clearly explain to the learners these two statements.*

Ms Masombuka: *Bana ha ba filwe palo in the form of a story ba tshwanela ho e solva eya ba hlola because they just cannot deal with abstract things. They cannot make up the picture ka hlohong ya seo ho bolelwang ka sona.*

ET: *When learners are given problems in the story form they find it difficult to solve since they can not be able to deal with abstract things. They usually fail to make a sound picture of what the problem is about.*

Miss Lengau: *Visualisation is a skill seo eleng hore bana ba bangata ha bana sona. Mme taba kgolo jwale ke hore wena jwale ka titjhere o thusa bana bana jwang hore ba develope skill sena!*

ET: *Visualisation most learners lack. Therefore, it remains your responsibility as a teacher to determine how you will assist learners in developing this skill.*

Lindiwe: *Ho thata ho etsa picture ya ntho eo o e balang ka hlohong ha o sa utlwisise ntho eo o e balang. Ke nahana hore ho important hore our teachers bare rute ka tsela etla etsang hore re ithute ho bona se batluwang ka hlohong.*

ET: *It is difficult to create a mental picture of what you are reading inside your head, particularly when you do not understand what you are reading. I think it is important for our teachers to teach us in ways which will make it possible for us to learn how to visualise the unknown inside our heads.*

Mr Twala: *Ka nako engwe ho ya thusa ho tla ka dintho tse tshwarehang classeng hore bana bana ba bone seo o buang ka sona. For example ha o trita topic ya di shapes o katla ka khatbodo otlo bontsha bana di dimenstionse live! Ba di bone straight ka mahlo!*

ET: *In certain instances, it helps to bring to the classrooms the manipulatives so that the learners could physically see what we are talking about.*

Mr Phatudi: *Working together with the learners in order to draw the diagrams that depicts the problem also helps*

Ms Zwane: *Ha re kgutlela exampoleng ela eo ntate ae entseng mona ya "probability" ha o sa e hlalosa hantle hore e bolelang, bana ba ka ofa dikarabo tse ikelang kwana. Thlaloso ya mantswe ana a tshwanang le bo probability e bohlokwa since anale di tlhaloso tse ngata.*

ET: *Referring to the example which the father has provided on the "probability" concept, when you do not explain it well in terms of what it means, then learners may give you wrong answers. An explanation of words such as probability is important since it has diverse meanings.*

Dibuseng: *Di statements tseo re di fuwang di lokela ho ya straight to the point hobane ha di se jwalo di ya ferekanya and dija nako.*

Dipotso tsa di word problems di lokelwa hore di chekuwe ke di specialists hore di ya utlwahala kapa thje!

The statements given to us have to go straight to the point because they can be confusing if they are not clear and may at times waste time for the people who perform them.

The word problem questions need to be evaluated by the specialists to check whether they are understandable or not!

Ms Nkosi: *Le teng bana ha ba rutwa fela ka Sesotho kapa sezulu re yaba bolaya hobane di potso ka exam di tla be di ngotswe ka English. Hape eka ba disadvantage in other areas.*

ET: *We "kill" these learners when we teach them in Sesotho or Isizulu because the exam questions are usually written in English. The sole use of home languages would further disadvantage them in other areas.*

Tau: *Like a tjholo hore English ke bothata, hodima hole jwalo motho ontse osa utlwisisi maths jwale hee ekaba problem e kgolo hare ka rutwa ka English feela.*

ET: *As he has already alluded to the fact that English is problematic, in addition to the fact that we do not understand Maths, it would even be more complicated if we could only be taught in English.*

Pholoho: *Ha osa kgone ho etsa picture ka hlohong ya ntho eo o e botsitsweng ho thatha ho e ngola fatshe. For example… lets say the question says: without sketching the graph, describe the shift of the function $f(x) = \sin x + 2$. Ho kaba thata ho araba potso eo ha o sa tsebe ho visualaisa hape le ha osa tsebe hore $\sin x + 2$ e bolelang.*

ET: *It is difficult to write down a problem on paper when you cannot make a mental picture of it. For example…lets say the question says: without sketching the graph, describe the shift of the function $f(x) = \sin x + 2$. The inability to visualise may cause difficulty for learners to respond to the question.*

Mr Phatudi: *Let us look at this example. I want to show you what I am talking about: (taking out the previous question paper to show to the people who were present in the meeting).*

Question:

A school will have to replace some of its equipment in 6 years' time. The principal calculates that the equipment cost R44 500. The school will establish a sinking fund to pay for equipment. Apart from the constant quarterly payments, the school makes an additional once only deposit of R6300 into the fund which yields interest at 6.85% p.a. compounded quarterly. The amount will be contributed towards buying the new equipment. Determine the value of the outstanding amount.

*To me this statement sounds vague. Ho nale ntho esa utlwahaleng ka statement sena. [**ET:** "Something is not clear from this statement"]. Is R44 500 the cost of the equipment ya sekolong [**ET:** "of the school"] or is it of the equipment that has to be replaced in future or what? The sinking fund! It is established to pay for which equipment, new or old? Kannete baheso statement sena se vague for nna. [Truly speaking this statement is vague to me]. No wonder why bana ba ileng ba fosa palo ena ka exam [**ET:** "…why learners got it wrong in the exam"]. [Mr Phatudi "interrogating" the question to indicate its vagueness]*

Ms Nkosi: *Language is important when communicating in class. However, it can be tricky when you yourself as a teacher cannot speak the language of the learners that you are teaching and yet you still have to teach them effectively. […]. In addition to knowing the language, I think how to teach learners in a certain way a particular concept is important. […] I really think that training will help us a lot to master the teaching of MWPs*

Mr Nzuza: *The only way for us as teachers to be able to effectively teach learners in these classrooms which comprise of learners from different cultures is to be trained effectively. I think itraining izo sisiza kakhulu [**ET:** "It will help us a lot"] especially on how to teach these learners!*

Mr Simelane: *Indeed training is important especially on the usage of language in the classroom as well as how to teach content.*

Mr Phatudi: *Taba ena e "two-way", English ekaba problem hape ekanna ya sebe problem. Re nale mantswe arona a sebediswang feela mmetseng and therefore aka etsa bana ba kgone ho utlwisisa hobane English re e sebedisa feela for sentence construction. Empa hare sebedisa English feela, re tlabe re ba thatafalletsa le ho feta hobane jwale bantse ba ithuta puo ena eo eseng leleme la habo bona.*

ET: *This is a "two-way matter", English could be a problem and not a problem at the same time. We have words which are only used in mathematics and these words could help learners understand since we use English only for sentence construction. If we use English only in our classrooms, then we are making it difficult for them to cope because they are still learning this language which is not their mother tongue.*

Mr Twala: *Nna ke nahana hore tshebediso ya di home languages le English ka classeng e bohlokwa.*

ET: *I think the use of both the home languages and English in class is important.*

Mr Twala: *Ntho e ka thusang ke ha bana ba ka rutwa ho bala. Re bafe le di word problems tse ngata tse fapaneng hore ba tlwaele ho bala.*

ET: *What would help is when learners could be taught how to read. We need to also give them many different word problems to solve so that they can be used to reading.*

Ms Nkosi: *The word problems need to be revised before the question papers are given to the learners.*

Mr Twala: *Re tshwanetse hore re kgothaletse bana ho bala carefully [**ET:** "…We need to encourage learners to carefully read"] and to also consider the important words. Hapehape bana ba tshwanetse ho etsuwa aware hore haba bala ba utlwisise the question in total… [**ET:** "Furthermore, the learners need to be encouraged to read and understand the question in totality"].*

Ms Nkosi: *…so for them to understand the problems we need to come up with tangible things so that they can be able to see what we are talking about. We can bring circles of the different sizes, strings and rulers or tapes to determine the value of Pi which will be 3.14 and it will be the same in all the different shapes.*

Mr Phatudi: *A clear example eo re ka e etsang mona keya [**ET:** "…that we can make here is this of…"] the Sine graph. Let's say the question says, Draw the graph [showing the problem on paper]* $f(x) = Sinx + 2$ *and describe the shift. The learners can be able to describe the shift if ba e bona on the board.*

Mr Nzuza: *Matitjhere a sekgowa a lokela hore thusa. Mohlomong ka di studies ba rute bana ka mekgwa ya ho bala ebe rona re ba etsetsa mehlala ya tshebediso ya mantswe a itseng ka thuso ya matitjhere a sekgowa.*

ET: *English teachers have to assist us. They can teach learners the various ways to read and then we can provide them with the examples on the use of certain words through the assistance of the English lecturers.*

I think re ka thuseha haholo ha matijhere aka re fa nako ya ho bala ba be bare botsa ka di groups hore re understanda eng from the word problem.

ET: *I think we could benefit a lot if teachers could give us time to read and also find out from the groups in class what we understand about the given word problems.*

Ms Masombuka: *These mathematical terms need to be stressed when teaching learners. We need to also allow learners to talk within the classrooms.*

Tseko: *O ka etsa nthonyana e kang pamphletenyana e hlalosang di terms tsa bohlokwa mabapi le concept e itseng. Ebe o fana ka mantswenyana a sehlotshwana ho hlalosa term e itseng. Sena se tla thusa bana haholo le ho increasa vocabulary ya bona.*

ET: *You can design a pamphlet which consists of words which explain the important different terms which relate to a particular concept.*

Mr Simelane: *"We need to receive training e tla re thusang ho feisana le maemo ana." [**ET:** "…that will assist us to face this circumstances"].*

*[Interrupting] …*Mr Nzuza: *Haholo jwang hobane rona mona bana ba rona ba sokola haholo ka sekgowa.*

ET: *Particulaly because our learners struggle to speak English.*

Ms Zwane: *The peer observations also help. Matitjhere re kgona ho ahana through tsona. [**ET:** "Through peer observations, teachers are able to build each other"].*

Ms Zwane: *Bana ba bang ba dihlong tsa ho bua ka classeng. Jwale ha ba rutwa ka English feela ba thola tuu hobane ba tshaba ho tsheuwa!*

ET: *Some learners are shy to express themselves orally in class. When they are taught in English only, then they keep quite during the class for fear of being laughed at.*

Ms Masombuka: *…hare na material o ngotsweng ka dipou tsa malapeng ho sapota English materials tseo re nang le tsona.*

ET: *We do not have the material that is written in the different home languages to support the English materials that we have.*

Mr Nzuza: *If the stakeholders are not willing to to join hands then re kaba le bothata ba ho achieva sepheo sa rona ka leano lena. [***ET***:…then we will have a problem in achieving our goal through this strategy.]*

Ho kgothaletsa bana kamehla hore ba bale ka tataiso ya ka ho thusitse haholo. Ke bona ba se ba sena monyeme wane wa pele ha ba bala.

ET: *Encouraging learners all the time to read has helped a lot. I realised that they are now more keen to read than they were before.*

Lenna ke bone ho thusitse haholo. Ke bone ba se ba kgona ho presenta di shaded areas Cartesian planeng haba filwe di inequalities. [O tsebe pele hwane ha one o ba file palo ena bane ba o fa karabo e so [… showing an answer on the paper] which is wrong, but nou baya utlwisisa.

I also realised that reading helped them a lot. They can now present the shaded areas on the Cartesian plane when given the inequalities. [Previously, when you had given them a problem like this one [showing a problem on paper] they would answer it as follows: [showing an answer on paper] which is a wrong answer, but now they seem to understand.

Ha ke sa bona diphoso tse ngata tseo ba neng ba di commita. Ke bona ba se ba di nepanyana moraong tjena.

ET: *They do no longer commit as many errors as they used to. They get them right lately.*

Lerato: *Ntho e ka thusang hape ke ha re ka ba le ntho e kang di tutorials teng moo bana ba ka buang ka dipuo tsa bona to enrich their understanding. Le vocabulary baka e thuta yona haholo hona ditutorialeng moo.*

ET: *It would help to have tutorial sessions in which learners could speak in their home languages in order to enrich their understanding. They could also learn mathematical vocabulary during the tutorials.*

Ms Nkosi: *We need to carefully use the learner's' home languages. Rentse re etse hloko hore ba se lahlehelwe ke monyetla wa ho ithuta English. [***ET**: "We need to be careful to ensure that they do not miss out on an opportunity to learn English"].*

Ke bona diphethoho tse ngata kamora hore ke addresse vocab ya some of the concepts. Ke bone ho addresa vocab ho tlisitse phapang e kgolo feela. Hape le ho hlalosa mantswe a sebediswang strictly mona mmetseng … bo function, determining the domain and range and so on and so on...ke bone ho thusa haholo. Ke bona ba kgona ho araba dipotso.

ET: *I see a change after addressing lack of vocabulary regarding certain concepts. Addressing vocabulary brought about a huge change. Furthermore, providing explanation of the words that we use in mathematics such as, function, determining the domain and range etc. has helped a lot. They can now answer the questions.*

Glossary e important because e thusa bana hore ba hopole mantswe. Teng ha titjhere a e kgothalletse di results tsa yona dintle haholo.

ET: *Glossary is important because it helps learners to remember the words. Its good results are more evident when the teacher frequently encourages its use.*

Di problems tsena di hlaka haholo feela ha bo teacher ba re explainetse mantswe ao a thata. Re be se rekgona le ho di solva.

ET: *These problems become clearer in terms of understanding when the teachers explain the difficult words which are comprised thereof. Consequently, we are able to solve them.*

Mr Phatudi: *Indeed! We need to use the home languages profitably still remembering that they will be required to know how to speak English when they get to tertiary.*

Mr Nzuza: *Ho bohlokwa ho ela hloko puisano ya bana in class, eeeehh… hore ba bua jwang ka phaposing as compared to outside, how they use everyday language, ba bontsha jwang their thinking regarding the concepts and le hore the textbooks di informa jwang their language.*

ET: *It is important to be aware of the learners' conversations in class in terms of how they speak as compared to when they speak outside the classroom, how they use everyday language, how do they demonstrate their thinking regarding the concepts as well as how the textbooks inform their language.*

Hare fuwe palo ebe teacher o fa group engwe le engwe monyetla wa ho hlalosa e thusa haholo hobane o kgona hore correkta hare sa hlalose seo potso ese batlang le ho re thusa hare misintepreta question

ET: *It helps when a teacher grants every group in the class an opportunity to explain what they understand from the problems given to them. The teacher is able to correct us when we are not correctly explaining what the problem requires and also assists us to interpret the question.*

Ha ke ruta bana di word problems ke stressa le hona ho clarifaya mantswe aka etsang statement se seke sa hlaka. Ho etsa jwalo ho tebisa kutlwisiso ya bona.

ET: *I usually stress and clarify words which may cause the word problems not to be clear. That deepens their understanding.*

It is imperative again to encourage learners to use the appropriate mathematical terms and not the ordinary language. For example the word "at least" must be explained hore le bolelang in an ordinary language and also mathematically. So that way, bana ba understand the application of the word in different settings.

Usually ha ke ruta bana di word problems, ke bala le bona the problem eo ba e fuweng. I also draw a picture ya ntho eo ho buuwang ka yona. For example ha potso e batla ba describe the type of a shift e bileng teng ya the...the....ha rere f(x)= Cosx +2

I usually read together with the learners the given problems. I also draw the picture of what the problem is all about. For example, when a question requires that the learners describe the type of a shift that occurred in a graph where f(x)= Cosx +2

Ha re filwe di equation, for example f(x) = 2 CosA ho bonolo ho bale picture ya graph ka hlohong ha e droyilwe and tithjere abe a e hlalosa ka di signs ho re graph e tlo shebela kae!

ET: *When we are given the equations, for example f(x) = 2 CosA, it is easy to generate a mental picture of such a graph when it is drawn and the teacher explains it through the use of signs to indicate where it will face.*

I advise learners to use the different colours of the pens when we deal with the concept of "probability". Those colours are able to help learners to differentiate and to see what is common.

Ha re dummellwa ho sebedisa Sesotho ka classeng ho betere haholo hobane rekgona ho utlwisisa seo re se rutwang.

ET: *Our understanding increases more when we are allowed to use Sesotho in the classroom.*

Sesotho se kgona ho hlakisa mantswe ao re sa a utlwisiseng.

ET: *The use of Sesotho clarifies words which we do not understand.*

Bana ba shy ba kgona ho participeita ha re ba allowa ho sebedisa puo ya ha bo bona.

ET: *Shy learners are able to participate when we allow them to use the home language.*

Training e thusitse matitjhere a mangata haholo. Ha re kopanela le matitjhere a mang e thusa ho increasa knowledge ya subject le kamoo content e ka presentuwang betere kateng.

ET: *Training has empowered many teachers. Working together with the other teachers helps increase the knowledge of the subject content as well as how the content could be presented.*

Ha re le fully equipped re kgona le ho prepara thouroghly. Le ka classroomung re kgona le ho etsa mehlala e thusang bana ka dintho tseo ba di tsebang already

ET: *Being fully equipped enables us to prepare thoroughly. We are able to provide examples which assist learners to understand aspects which they already know.*

Training ya tshebediso ya puo mona ho di word problems e thusa le tithjere ho re a kgone ho guida bana fully.

ET: *Training on the use of language in this concept of word problems assists a teacher in order to fully guide learners*

APPENDIX A1: Ethical clearance letter

UNIVERSITY OF THE FREE STATE
UNIVERSITEIT VAN DIE VRYSTAAT
YUNIVESITHI YA FREISTATA

UFS·UV
EDUCATION
OPVOEDKUNDE

Faculty of Education **14-Mar-2017**

Dear **Mrs Mirriam Mohlakoana**

Ethics Clearance: **A Universal Design for Learning strategy to enhance the teaching of word problems in a multilingual mathematics classroom**

Principal Investigator: **Mrs Mirriam Mohlakoana**

Department: **School of Education Studies (Bloemfontein Campus)**

APPLICATION APPROVED

With reference to you application for ethical clearance with the Faculty of Education, I am pleased to inform you on behalf of the Ethics Board of the faculty that you have been granted ethical clearance for your research. Your ethical clearance number, to be used in all correspondence is: **UFS-HSD2016/1194.** This ethical clearance number is valid for research conducted for one year from issuance. Should you require more time to complete this research, please apply for an extension. We request that any changes that may take place during the course of your research project be submitted to the ethics office to ensure we are kept up to date with your progress and any ethical implications that may arise. Thank you for submitting this proposal for ethical clearance and we wish you every success with your research.

Yours faithfully

Dr. MM Nkoane
Chairperson: Ethics Committee

………

Education Ethics Committee
Office of the Dean: Education
T: +27 (0)51 401 9683| F: +27 (0)86 546 1113 | E: NkoaneMM@ufs.ac.za
Winkie Direko Building | P.O. Box/Posbus 339 | Bloemfontein 9300 | South Africa www.ufs.ac.za

APPENDIX A2: Application letter to conduct a research study

UNIVERSITY OF THE FREE STATE
UNIVERSITEIT VAN DIE VRYSTAAT
YUNIVESITHI YA FREISTATA

FACULTY OF EDUCATION

P.O.Box 339

E-mail: molekomm@ufs.ac.za

+27(0)51 401 3103

071 6931 078

July 2016

…………………………………………………………………………………………………

The Principal

Thabo Mofutsanyana Education District

Private Bag X817

Witsieshoek

9870

Dear Sir

Re: Application to conduct research at your school

I am a doctoral student at the University of the Free State and I hereby request permission to conduct research at your school. The research will be in a participatory action research form. The focus of the study will be on the teaching of mathematics word problems in a multilingual mathematics classroom, thus seeking ways in which this mathematical genre can be better taught to the learners.

Yours sincerely

…………………………………..

M.M. Moleko (Ms)

APPENDIX A3: Permission letter to conduct a research study

education
Department of Education
FREE STATE PROVINCE

SENIOR SECONDARY SCHOOL

PHUTHADITJHABA
9866

PHUTHADITJHABA
9866
TEL: 058 713
FAX:058 713

@hotmail.co.za

Enq. Principal

2016 / 07 / 12

Dear Madam

Re: Request to conduct research at our school

I hereby acknowledge to have received your request to do research at our school in connection with word problems in Mathematics. I therefore hereby grant you permission to conduct this research and interact with learners and teachers of Mathematics under the following conditions:

- ✓ The interaction with teachers and learners should not be during teaching time.
- ✓ Parents of the learners sign consent forms.
- ✓ The school be given feedback on the findings of the research.

Hoping you will find the above in order.

Yours Faithfully

(Principal)

APPENDIX A4: Teacher informed consent form

Researcher
Matshidiso Moleko
Center for Teaching and Learning

Contacts: 076 7590 637
Email: tshidimolekot@gmail.com

Study Leader
Prof DJ Hlalele
8 New Education building
School of Education Studies
QwaQwa UFS
Contacts: 058-718 5003
E-mail: hlaleleDJ@qwa.ufs.ac.za

Date: /06/2016

INFORMED CONSENT

Dear Participant: Teacher

Request for your participation in the research project

I hereby request your assistance in conducting the research study, which is entitled: A UNIVERSAL DESIGN FOR LEARNING STRATEGY TO ENHANCE THE TEACHING OF WORD PROBLEMS IN A MULTILINGUAL MATHEMATICS CLASSROOM.

This study seeks to formulate a universal design for learning (UDL) strategy to enhance the teaching of mathematics word problems (MWPs) in multilingual mathematics classrooms. A UDL includes the purposeful design of learning environments and educational practices aimed at meeting the needs of a broad range of learners. Word problems are the most challenging problems learners have to solve in mathematics education. In order for learners to solve such problems they need to be numerically literate. This means that they need to be able to reason with numbers, identify, understand, interpret, create, communicate and compute numbers, which is what they find challenging in many instances. MWPs are not only challenging for learners but they also at times are difficult to teach. This is particularly true in multilingual mathematics classrooms where the learners' home languages are usually not supported and encouraged to be used as resources to aid learners' comprehension of the mathematical content. Taking into account the challenges that come with the teaching of MWPs, I therefore deemed this study significant in order to help address the existing challenges in an attempt to improve learner performance specifically in this type of mathematical genre.

Thus on the basis of the above background, the research question for this study is **"how can we utilise the aspects of universal design for learning to develop and effective teaching strategy for mathematics word problems in a multilingual mathematics classroom?** In this study, I adopt a participatory a action research (PAR) approach in trying to respond to this research question. PAR requires the inclusion of all the relevant stakeholders in addressing an issue of concern. Thus considering this PAR requirement and also knowing your role in education as a teacher, I therefore request your assistance in conducting this research study. With this study, the challenges of teaching and learning mathematics word problems will be identified and solutions devised in order to address the challenges. The ultimate aim of this study is to formulate a universal design for learning based teaching strategy for teaching this type of a mathematical genre which I believe could only be an outcome of a collaborative effort.

Please note that your participation in the study is voluntary and your basic human rights will be respected and protected at all times. We will maintain confidentiality, non-disclosure of personal information and identity, inform you at all times of the processes involved in the research study. You also have the right to leave or discontinue participation should you feel uncomfortable at any stage.

Your participation will add great value to this study.

Yours thankfully

Matshidiso Moleko (Researcher)

Please read the sentences below and attach your signature:

1. I fully understand the nature and purpose of the research study
2. I therefore give full consent to participate and do so freely without any cohesion
3. I hereby give permission for the use of information obtained during the study and the use of the findings thereof.

…………………….. **………………**

Signature (Participant) **Date**

APPENDIX A5 (English): Assent letter

Matshidiso Moleko
Center for Teaching and Learning

Contacts: 076 7590 637
Email: tshidimolekot@gmail.com

Prof DJ Hlalele
8 New Education building
School of Education Studies
UFS, Qwaqwa
Contacts: 058-718 5003
E-mail: hlaleleDJ@qwa.ufs.ac.za

Date: /06/2016

ASSENT LETTER

Dear Parent/Guardian

I am currently doing research with the University of the Free State in an attempt to develop an effective strategy to enhance the teaching of mathematics word problems in a multilingual mathematics classroom. Since your child is a minor, you as the parent/legal guardian are therefore kindly requested to provide permission for your child to be part of this research project. Confidentiality, anonymity and legality issues about this project will be discussed with you, the teachers and principal as it is imperative that you fully understand the nature and purpose of this study. You are free to withdraw your child from this study at any stage.

This project will comply with the rules and regulations of conducting a research.

If you would like any additional information, you are welcome to contact me on 051 401 3103 or at the following e-mail address: tshidimolekot@gmail.com.

If you would like your child to participate in this research, please sign below to give a consent.

Thank you.

____

M.M. Moleko

Please read the sentences below and append your signature :

4. I fully understand the nature and purpose of the research study

5. I therefore give full consent for my child to participate and do so freely without any cohesion

6. I hereby give permission for the use of information obtained during the study and the use of the findings thereof.

Name of learner ____

Grade ____

Signature of parent/guardian ____

Date ____

APPENDIX A5 (Sesotho): Formmo ya tumello hoy a motswadi/mohlokomedi

Matshidiso Moleko
Center for Teaching and Learning

Contacts: 076 7590 637
Email: tshidimolekot@gmail.com

Prof DJ Hlalele
8 New Education building
School of Education Studies
UFS, Qwaqwa
Contacts: 058-718 5003
E- mail:hlaleleDJ@qwa.ufs.ac.za

Date: /06/2016

Madume ho motswadi

Ke etsa diphuputso ho tswa Univesithing ya Foreistata mabapi le leano la ho ntlafatsa ka moo matitjhere a dipalo aka rutang dipalo tsa mantswe ka boqhetseke ka phaposing eo ho buuwang dipuo tse fapaneng ka teng. Ka lebaka la hobane ngwana wa hao o sa le ka tlase dilemong, o kopuwa ke hona ho mo dumella hore e be karolo ya porojeke ena ya diphuputso. Lekunutu la taba tse tla buuwa mona le maemo a tsa se molao mabapi le projeke ena a tla buua le wena, mesuwe/mesuwetsana le mosuwehlooho. Ho bohlokwa ke hona hore o utlwisise maemo le sepheo sa diphuputso tsena. O na le bolokolohi ba ho hula ngwana wa hao neng kapa neng diphuputsong tsena.
Projeke ena e tla ikamahanya le melao yohle ya ho etsa diphuputso.
Ebang o batla dintlha tse ding ho feta mona, o ikopanya le nna dinorong tsena: 051- 401 3103 kapa atereseng ena ya i-meil: tshidimolekot@gmail.com
Ebang o dumella ngwanahao hore a be le seabo projeke ena, o ka tlatsa tse latelang.

Ke a leboha.

____

M.M. Moleko

Ka kopo bala tse latelang mme o tekene:

4. Ke utlwisisa ka botlalo semelo le sepheo sa phuputso ena

5. Ka hona, ke fana ka tumello e felletseng hore ngwana wa ka a ka nka karolo phuputsog ena ntle le ho qobellwa
6. Ke fana ka tumello ho sebedisweng ha tshedimosetso e tlang ho fumanwa phuputsong ena le diphetong tsa teng

Lebitso la ngwana :...

Kereiti :...

Boitshaino ba motswadi/mohlokomedi :...

Letsatsi:……………………………………………

APPENDIX A6: Informed consent

Matshidiso Moleko
Center for Teaching and Learning

Contacts: 076 7590 637
Email: tshidimolekot@gmail.com

Prof DJ Hlalele
8 New Education building
School of Education Studies
UFS, Qwaqwa
Contacts: 058-718 5003
E-mail: hlaleleDJ@qwa.ufs.ac.za

Date: /06/2016

INFORMED CONSENT

Dear Participant: Principal

Request for your participation in the research project

I hereby request your assistance in conducting the research study, which is entitled: A UNIVERSAL DESIGN FOR LEARNING STRATEGY TO ENHANCE THE TEACHING OF WORD PROBLEMS IN A MULTILINGUAL MATHEMATICS CLASSROOM.

This study seeks to formulate a universal design for learning (UDL) strategy to enhance the teaching of mathematics word problems (MWPs) in multilingual mathematics classrooms. A UDL includes the purposeful design of learning environments and educational practices aimed at meeting the needs of a broad range of learners. Word problems are the most challenging problems learners have to solve in mathematics education. In order for learners to solve such problems they need to be numerically literate. This means that they need to be able to reason with numbers, identify, understand, interpret, create, communicate and compute numbers, which is what they find challenging in many instances. MWPs are not only challenging for learners but they also at times are difficult to teach. This is particularly true in multilingual mathematics classrooms where the learners' home languages are usually not supported and encouraged to be used as resources to aid learners' comprehension of the mathematical content. Taking into account the challenges that come with the teaching of MWPs, I therefore deemed this study significant in order to help address the existing challenges in an attempt to improve learner performance specifically in this type of mathematical genre.

Thus on the basis of the above background, the research question for this study is **"how can we utilise the aspects of universal design for learning to develop an effective teaching strategy for mathematics word problems in a multilingual mathematics classroom?** In this study, I adopt an participatory action research (PAR) approach in trying to respond to this research question. PAR requires the inclusion of all the relevant stakeholders in addressing an issue of concern. Thus considering this PAR requirement and also knowing your role in education as a principal (mathematics teacher), I therefore request your assistance in conducting this research study. With this study, the challenges of teaching and learning mathematics word problems will be identified and solutions devised in order to address the challenges. The ultimate aim of this study is to formulate a universal design for learning based teaching strategy for teaching this type of a mathematical genre which I believe could only be an outcome of a collaborative effort.

Please note that your participation in the study is voluntary and your basic human rights will be respected and protected at all times. We will maintain confidentiality, non-disclosure of personal information and identity, inform you at all times of the processes involved in the research study. You also have the right to leave or discontinue participation should you feel uncomfortable at any stage.

Your participation will add great value to this study.

Yours thankfully

Matshidiso Moleko (Researcher)

Please read the sentences below and attach your signature:

a. I fully understand the nature and purpose of the research study

b. I therefore give full consent to participate and do so freely without any cohesion

c. I hereby give permission for the use of information obtained during the study and the use of the findings thereof.

………………………..	**…………………**
Signature (Principal)	**Date**

APPENDIX A7: Informed consent

Matshidiso Moleko
Center for Teaching and Learning

Contacts: 076 7590 637
Email: tshidimolekot@gmail.com

Prof DJ Hlalele
8 New Education building
School of Education Studies
UFS, Qwaqwa
Contacts: 058-718 5003
E-mail: hlaleleDJ@qwa.ufs.ac.za

Date: /06/2016

INFORMED CONSENT

Dear Participant: Mathematics Head of Department (HoD)

Request for your participation in the research project

I hereby request your assistance in conducting the research study which is entitled: A UNIVERSAL DESIGN FOR LEARNING STRATEGY TO ENHANCE THE TEACHING OF WORD PROBLEMS IN A MULTILINGUAL MATHEMATICS CLASSROOM.

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Please note that your participation in the study is voluntary and your basic human rights will be respected and protected at all times. We will maintain confidentiality, non-disclosure of personal information and identity, inform you at all times of the processes involved in the research study. You also have the right to leave or discontinue participation should you feel uncomfortable at any stage.

Your participation will add great value to this study.

Yours thankfully

Matshidiso Moleko (Researcher)

Please read the sentences below and attach your signature:

d. I fully understand the nature and purpose of the research study

e. I therefore give full consent to participate and do so freely without any cohesion

f. I hereby give permission for the use of information obtained during the study and the use of the findings thereof.

……………………………….. **………………**

Signature (HoD) **Date**

APPENDIX A8: Informed consent

Matshidiso Moleko
Center for Teaching and Learning

Contacts: 076 7590 637
Email: tshidimolekot@gmail.com

Prof DJ Hlalele
8 New Education building
School of Education Studies
UFS, Qwaqwa
Contacts: 058-718 5003
E-mail: hlaleleDJ@qwa.ufs.ac.za

Date: /06/2016

INFORMED CONSENT

Dear Participant: Mathematics Literacy teacher

Request for your participation in the research project

I hereby request your assistance in conducting the research study which is entitled: A UNIVERSAL DESIGN FOR LEARNING STRATEGY TO ENHANCE THE TEACHING OF WORD PROBLEMS IN A MULTILINGUAL MATHEMATICS CLASSROOM

This study seeks to formulate a universal design for learning (UDL) strategy to enhance the teaching of mathematics word problems (MWPs) in multilingual mathematics classrooms. A UDL includes the purposeful design of learning environments and educational practices aimed at meeting the needs of a broad range of learners. Word problems are the most challenging problems learners have to solve in mathematics education. In order for learners to solve such problems they need to be numerically literate. This means that they need to be able to reason with numbers, identify, understand, interpret, create, communicate and compute numbers, which is what they find challenging in many instances. MWPs are not only challenging for learners but they also at times are difficult to teach. This is particularly true in multilingual mathematics classrooms where the learners' home languages are usually not supported and encouraged to be used as resources to aid learners' comprehension of the mathematical content. Taking into account the challenges that come with the teaching of MWPs, I therefore deemed this study significant in order to help address the existing challenges in an attempt to improve learner performance specifically in this type of mathematical genre.

Thus on the basis of the above background, the research question for this study is **"how can we utilise the aspects of universal design for learning to develop an effective teaching strategy for mathematics word problems in a multilingual mathematics classroom?"** In this study, I adopt a participatory action research (PAR) approach in trying to respond to this research question. PAR requires the inclusion of all the relevant stakeholders in addressing an issue of concern. Thus considering this PAR requirement and also knowing your role in education as a Mathematics Literacy teacher, I therefore request your assistance in conducting this research study. With this study, the challenges of teaching and learning mathematics word problems will be identified and solutions devised in order to address the challenges. The ultimate aim of this study is to formulate a universal design for learning based teaching strategy for teaching this type of a mathematical genre which I believe could only be an outcome of a collaborative effort.

Please note that your participation in the study is voluntary and your basic human rights will be respected and protected at all times. We will maintain confidentiality, non-disclosure of personal information and identity, inform you at all times of the processes involved in the research study. You also have the right to leave or discontinue participation should you feel uncomfortable at any stage.

Your participation will add great value to this study.

Yours thankfully

Matshidiso Moleko (Researcher)

Please read the sentences below and attach your signature:

g. I fully understand the nature and purpose of the research study

h. I therefore give full consent to participate and do so freely without any cohesion

i. I hereby give permission for the use of information obtained during the study and the use of the findings thereof.

…………………………………………. **…………………..**

Signature (Mathematics Literacy teacher) **Date**

APPENDIX A9: Informed consent

VIDEO-RECORDING/TAKE PICTURES

Dear Participant: Mathematics teacher/Learner/Principal/Mathematics Head of Department/ Mathematics Literacy teacher

Request for permission to video- record the lessons

I hereby request your assistance in conducting the research study which is entitled: A UNIVERSAL DESIGN FOR LEARNING STRATEGY TO ENHANCE THE TEACHING OF WORD PROBLEMS IN MULTILINGUAL MATHEMATICS CLASSROOMS

This study seeks to formulate a universal design for learning (UDL) strategy to enhance the teaching of mathematics word problems (MWPs) in multilingual mathematics classrooms. A UDL includes the purposeful design of learning environments and educational practices aimed at meeting the needs of a broad range of learners. Word problems are the most challenging problems learners have to solve in mathematics education. In order for learners to solve such problems they need to be numerically literate. This means that they need to be able to reason with numbers, identify, understand, interpret, create, communicate and compute numbers, which is what they find challenging in many instances. MWPs are not only challenging for learners but they also at times are difficult to teach. This is particularly true in multilingual mathematics classrooms where the learners' home languages are usually not supported and encouraged to be used as resources to aid learners' comprehension of the mathematical content. Taking into account the challenges that come with the teaching of MWPs, I therefore deemed this study significant in order to help address the existing challenges in an attempt to improve learner performance specifically in this type of mathematical genre.

Thus on the basis of the above background, the research question for this study is **"how can we utilise the aspects of universal design for learning to develop an effective teaching strategy for mathematics word problems in a multilingual mathematics classroom?"** In this study, I adopt a participatory action research (PAR) approach in trying to respond to this research question. PAR requires the inclusion of all the relevant stakeholders in addressing an issue of concern. Thus considering this PAR requirement and also knowing your role in education as a mathematics teacher/Learner/Principal/Head of Department/Mathematics Literacy, I therefore request your assistance in conducting this research study. With this study, the challenges of teaching and learning mathematics word problems will be identified and solutions devised in order to address the challenges. The ultimate aim of this study is to formulate a universal design for learning based teaching strategy for teaching this type of a mathematical genre which I believe could only be an outcome of a collaborative effort.

In order to be able to interpret certain aspects and to ensure that nothing important is left behind, I would like to video record the classroom lessons. Therefore I would like to ask for your permission to do so. Please note that the recording will only be used for data analysis and interpretation purposes and that your pictures will not be exposed in any way. This information will be treated with high confidentiality and measures will be put in place to protect your identity.

Please note again that your participation in this study is voluntary and that your basic human rights will be respected and protected at all times. We will maintain confidentiality, non-disclosure of personal information and identity, inform you at all times of the processes involved in the research study. You also have the right to leave or discontinue participation should you feel uncomfortable at any stage.

Your participation will add great value to this study.

Yours thankfully

Matshidiso Moleko (Researcher)

Please read the sentences below and attach your signature:

j. I fully understand the nature and purpose of the research study

k. I therefore give full consent to participate and do so freely without any cohesion

l. I hereby give permission for the use of information obtained during the study and the use of the findings thereof.

…………………..   ……………………

Signature   **Date**

APPENDIX A10

Information session programme

Information session 1:		
Activity	**Notes**	**Duration**
Welcome • Explaining the purpose of the information session • Introduction of the participants involved	Facilitator Participants	09:00-09:05
Introduction	Diversity and its implication for teaching and learning	09:05-09:15
Education on universal design for learning 1. Brief history of universal design (UD) 2. Origins of universal design for learning (UDL) 3. Definition of UDL 4. How does it work? 5. Why is it significant? 6. Aims and objectives of UDL	Facilitator giving education on the concept of UDL	09:15 -10:00
Comfort Break		
Nine principles of UDI	Description and implications for teaching Discussions on the application of UDI principles – examples provided	10:15-10:45
Conclusion	Questions and comments	10:45-11:00

APPENDIX B: Language editing certificate

CERTIFICATE OF LANGUAGE EDITING

Dr. L. Hoffman

Kroonstad

BA, BA(Hons), MA, DLitt et Phil

Member of South African Translators' Institute – No. 1003545

Cell no: 079 193 5256 Email: larizahoffman@gmail.com

DECLARATION

To whom it may concern

I hereby certify that the English language of the following thesis meets the requirements of academic publishing. This thesis was linguistically edited and proofread by me, Dr. L. Hoffman.

Title of thesis

A UNIVERSAL DESIGN FOR LEARNING STRATEGY TO ENHANCE THE TEACHING OF WORD PROBLEMS IN A MULTILINGUAL MATHEMATICS CLASSROOM

Candidate

Mirriam Matshidiso Moleko

Lariza Hoffman
Kroonstad
9 March 2018

APPENDIX C: Similarity index document

Thesis

ORIGINALITY REPORT

%13	%10	%5	%5
SIMILARITY INDEX	INTERNET SOURCES	PUBLICATIONS	STUDENT PAPERS

PRIMARY SOURCES

1	etd.uovs.ac.za Internet Source	%1
2	cjar.nipissingu.ca Internet Source	%1
3	twas.assaf.org.za Internet Source	%1
4	www.alm-online.net Internet Source	<%1
5	www.apexlearning.com Internet Source	<%1
6	accessproject.colostate.edu Internet Source	<%1
7	www.hamline.edu Internet Source	<%1
8	Submitted to University of KwaZulu-Natal Student Paper	<%1
9	Codruta Temple. "Developing fluency in the mathematical register through conversation in	<%1