ENHANCING CONCEPTUAL DEVELOPMENT AND MATHEMATICAL UNDERSTANDING THROUGH ETHNOMATHEMATICAL RESEARCH AND APPROACHES

Inaugural Lecture presented by Prof Mogege Mosimege at the University of the Free State on 20 September 2018

Introduction

The debate over whether it is better to teach conceptual understanding or procedural understanding in mathematics has dominated the mathematics education discussions for a long time. As a result a number of research studies have been conducted to try and settle this contestation. Among these are Gleman and Williams (1997), Rittle-Johnson and Alibali (1999), Lim (2002), Mary and Heather (2006), and Arslan (2010). These and many other studies have not necessarily settled the debate, but have instead provided some form of evidence that conceptual understanding is as important as procedural understanding. In one of the recent studies Joersz (2017) concludes that a clear link between conceptual and procedural knowledge exists in the mathematics classroom with both having their place in student mastery. The University of Chicago School Mathematics Project (UCSMP) has, since 1983, conducted a Project that looks at building conceptual understanding of mathematics through a project called 'Everyday Mathematics'. The UCSMP describes the project as 'a research-based and field-tested curriculum that focuses on developing children's understanding and skills in ways that produce life-long mathematical power'. The evidence provided by this project and other related school mathematics projects has made them to come to a conclusion that when children are introduced to abstract concepts before they have a solid basis for understanding those concepts, they tend to resort to memorization and rote learning which ultimately affects their long term understanding of mathematical concepts. One of the reasons why I have been attracted to this Project is that they conclude that students' understanding of concepts is developed through:

(i) Real world examples and concrete objects (manipulatives)
(ii) Pictorial representation
(iii) Discussion of ideas and methods.

The use of real world examples does not only help in conceptual understanding of mathematical concepts, it also helps to convey to the learners that there is a bigger picture regarding the knowledge of mathematics. It gives them a sense and notion that this knowledge is not just for assessment purposes and to determine a mark, but that it has got a wider application beyond the narrow confines of the mathematics classroom.

What is Conceptual Understanding?

After spending many years as an elementary school educator, Jean Soyke (2016) draws from this experience and says that conceptual understanding in mathematics refers to the notion that a student is not just taught how to do mathematics, but also why it is necessary to learn mathematics. Curtis (2016) takes this further to indicate that conceptual understanding is ‘the ability to describe and model the context and concrete application of a mathematical idea. She elaborates on this and says that a student who understands the concept of multiplication, division, addition and subtraction is able to:

- Create a model with blocks or tiles
- Draw a representation, and
- Come up with a context (or word problem) that makes sense.

Conceptual mathematics understanding can also be regarded as deep knowledge of the underlying concepts of mathematics and how they relate to one another (Crooks and Alibali, 2014) and as knowledge that involves a thorough understanding of underlying and foundational concepts behind the algorithms performed in mathematics (Hope, 2006). The implication is that as a student you do not just memorize and master the algorithm, but you also understand why the specific algorithm will work under certain conditions and why it will not work in other conditions.

Kilpatrick, Swafford and Findell (2001) put emphasis on conceptual understanding as a critical component of deeper mathematical knowledge. They say that ‘students with conceptual understanding know more than isolated facts and methods. They understand why a mathematical idea is important and the kinds of contexts in which it is useful. They have organized their knowledge into a coherent whole, which enables them to learn new ideas by connecting those ideas into what they already know’. The challenge of conceptual understanding is not only a student problem. The same authors (Kilpatrick, Swafford and Findell, 2001) argue that ‘teachers may know the facts and procedures, but often have a relatively weak understanding of the conceptual basis for that understanding’.

Illustration of Conceptual Challenges and Mathematical Understanding

Sweetland and Fogarty (2008) give an illustration of the conceptual challenges experienced by a group of Elementary (Foundation Phase) teachers attending a mathematics workshop. The teachers were asked to work on the following problem:

(i) Fractions:

“What is 1/5 x 1/4?”

Within a few seconds every teacher in the room arrived at the correct answer of 1/20
Then the teachers were asked: “How did you get the answer?” Most teachers could easily explain this: “You multiply the numerators, and then you multiply the denominators”. The next comment and question from the facilitator was: “What you are giving me is the formula. You have described the steps you were taught to follow. I would like to challenge you to demonstrate why the formula works? In other words, Prove it”.

When you ask a student or a teacher to explain why a particular formula works, it gives them an opportunity to go deeper and get the meaning behind the manipulation. It forces them to move from procedural thinking to conceptual thinking. It forces them to try and understand why the formula works.

(ii) The Theorem of Pythagoras

Another activity that Sweetland and Fogarty refer to (2008) is that of the Theorem of Pythagoras. This well-known Theorem states that in a right-angled triangle (right triangle) the square on the hypotenuse is equal to the sum of the squares of the other two sides. Here the procedural understanding refers to the fact that you able to put in the numbers to show that $a^2 + b^2 = c^2$ where c is the hypotenuse and a and b are the other sides of the triangle. The conceptual understanding would require the student or teacher to clearly articulate and demonstrate that for any right angle triangle the area of the square made with the hypotenuse is equal to the sum of the areas made by the squares on the other two sides of the triangle.

This second example is also appropriate in this Lecture as it features prominently in some of the ethnomathematical work that will be referred to later.

The South African School Mathematics Curriculum

The Curriculum and Assessment Policy Statement (CAPS) for mathematics for the Further Education and Training (FET) Phase in South Africa identifies eight specific aims that teachers must take into account as they work with the learners to advance and enhance mathematical understanding (Department of Basic Education 2011, p. 8). All these specific aims are important for the teaching and learning of the mathematical content, however, the following four seem to be very important and central for the teachers to continuously reflect upon and find ways to integrate in their work:

(i) Mathematical modelling is an important focal point of the curriculum. Real life problems should be incorporated into all sections whenever appropriate. Examples used should be realistic and not contrived. Contextual problems should include issues relating to health, social, economic, cultural, scientific, political and environmental issues whenever possible
(ii) To show Mathematics as a human creation by including the history of Mathematics
(iii) To promote accessibility of all mathematical content to all learners. It could be achieved by catering for learners with different needs
(iv) To prepare the learners for further education and the world of work

These four specific aims require the teachers to find ways to ensure the content is made more meaningful to the learners inside and outside the mathematics classrooms. The specific aims also suggest that teachers have got to continuously find appropriate examples that relate the content to the real world and the world of work. The mathematical modelling aim, in particular, draws attention to the fact that mathematics cannot be taught only for the purpose of understanding the rules and procedures and equations without ensuring that they can find relevance and application in real life outside the classroom. This is in line with Boaler's argument (2001, p. 126) that if students only ever reproduce standard methods that they have been shown, then most of them will only learn that particular practice of procedure repetition, which has limited use outside the mathematics classroom.

Ethnomathematical Research

Ethnomathematical research and focus in mathematics education can be traced back to the seminal work of Ubiratan D'Ambrosio (Brazil), followed closely by the work of Paulus Gerdes (Mozambique). Both have contributed extensively and immenselyto the definitions of ethnomathematics and to the conceptual development of this area in mathematics education. Their definitions and ideas have subsequently been embraced, extended, and critiqued by other mathematics educators working in this area and other areas of mathematics education. One of the earlier definitions of ethnomathematics by D'Ambrosio states:

> [Societies] have, as a result of the interaction of their individuals, developed practices, knowledge and in particular, jargons...and codes, which clearly encompass the way they mathematise, that is the way they count, measure, relate, and classify and the way they infer. This is different from the way all these things are done by other cultural groups. [We are] interested in the relationship... between ethno-mathematics and society, where 'ethnos' comes into the picture as the modern and very global concept of ethno both as race and/or culture, which implies language, codes, symbols, values, attitudes, and so on, and which naturally implies science and mathematics practices.
>
> (D'Ambrosio, 1984)

Here D'Ambrosio looks at the cultural elements such as language, codes, symbols, values, and attitudes which characterise a particular practice. Such elements are largely unique from one cultural group to another.

In a subsequent definition, he defines the cultural groups as national tribal societies, labour groups, children of a certain age bracket, and so on (D'Ambrosio, 1985). '...we will call ethnomathematics the mathematics which is practised among identifiable cultural groups, such as national-tribal societies, labour groups, children of a certain age bracket, professional classes and so on (1985:45).

Following these two early definitions, D'Ambrosio (1989) then went further to divide into parts the components of ethnomathematics and explains that '...we use the term ethnomathematics (ethno + mathema + tics) for the art or technique of understanding, explaining, learning about, coping with, and managing the natural, social, and political environment through processes like counting, measuring, sorting, ordering and inferring - processes that result from well-identified cultural groups'.

The processes of counting, measuring, sorting, ordering, and inferring in D'Ambrosio's definition relate very closely to Bishop's (1988:22-23) six fundamental activities that he contends are characteristic of every culture. The six activities are counting, locating, measuring, designing, playing and explaining. Indeed these six fundamental activities by Bishop are identifiable in various cultures.

Gerdes (1994:20) who sadly passed away in November 2014, defines ethnomathematics as 'the field of research that tries to study mathematics (or mathematical ideas) in its (their) relationship to the whole of cultural and social life'. Gerdes (1996:915; 1997:343) goes on to indicate that as a research field, ethnomathematics may be defined as the 'cultural anthropology of mathematics and mathematical education.'

The definitions of ethnomathematics given above suggest that mathematical concepts and processes would be easier and better understood by the learner when they are related to socio-cultural contexts as well as real-life situations. This is likely to make the subject more accessible to most learners as they can relate it to many of the activities and experiences outside the classroom. When this happens, mathematics reduces from being an abstract subject that has no connection to what the learners experience in a real life situation to a subject whose content finds application and relevance to socio-cultural experiences.

Although Gerdes provides this definition, he also stresses the importance of seeing ethnomathematics as a movement and he provides a framework for understanding this notion of an ethnomathematical movement and ethnomathematicians – researchers involved in the movement (Gerdes, 1996:917) as follows:

(i) Ethnomathematicians adopt a broad concept of mathematics, including, in particular, counting, locating, measuring, designing, playing, and explaining (Bishop, 1988);

(ii) Ethnomathematicians emphasize and analyse the influences of socio-cultural factors on the teaching, learning and development of mathematics;

(iii) Ethnomathematicians argue that the techniques and truths of mathematics are a cultural product, and stress that all people – every culture and every subculture – develop their own particular forms of mathematics;

(iv) Ethnomathematicians emphasise that the school mathematics of the transplanted, imported 'curriculum' is apparently alien to the cultural traditions of Africa, Asia and South America;

(v) Ethnomathematicians try to contribute to and affirm the knowledge of the mathematical realisation of the formerly colonised peoples. They look for cultural elements which have survived colonialism and which reveal mathematical and other scientific thinking;

(vi) Ethnomathematicians in 'Third World' countries look for mathematical traditions, which survived colonisation, especially for mathematical activities in people's daily lives. They try to develop ways of incorporating these traditions and activities into the curriculum;

(vii) Ethnomathematicians also look for other cultural elements and activities that may serve as a starting point for doing and elaborating mathematics in the classroom;

(viii) In the educational context, ethnomathematicians generally favour a socio-critical view and interpretation of mathematics education that enables students to reflect on the realities in which they live, and empowers them to develop and use mathematics in an emancipatory way.

Mathematics educators working in the area of ethnomathematics have either explored one specific aspect given above or a component thereof, for instance research projects that have investigated how games may be used in the mathematics classroom, may be classified as dealing with a few of the aspects above.

Some Ethnomathematical Studies in Southern Africa

In South Africa most of the ethnomathematical studies have been conducted at Wits University under the guidance and supervision of Professor Paul Laridon. In 1996 Laridon, together with a number of mathematics educators and post graduate students undertook a study funded by the National Research Foundation on '*The place of Ethnomathematics in the Secondary School mathematics Curriculum in South Africa'*. (Purkey, 1998). Following this study, a number of students embarked upon post graduate studies in ethnomathematics. For instance Mogari (2002), Ismael (2002), Cherinda (2002), among others. Examples of other studies are Mosimege (2000), Dabula (2000), and others. Recent work in this area has been done by Mosimege (2012) and Mogari (2014). These studies are identified here as an example of the kinds of ethnomathematical studies that have been undertaken. They provide many examples of how indigenous activities can be used to introduce and advance conceptual development in mathematics.

In other Southern African countries, the work of Paulus Gerdes and some of his students and colleagues at the Ethnomathematics Research Centre in Maputo, Mozambique is noted. Included in the extensive and elaborate ethnomathematical studies that he has undertaken (Gerdes: 1985, 1988, 1994, 1995), is the book '*Geometry from Africa: Mathematical and Educational Explorations'* (1999). The importance of this book is its relevance to the Mathematics Curriculum and Assessment Policy Statement for the FET as it gives detailed mathematical analysis and examples related to the Theorem of Pythagoras, Hexagonal Weaving, Diagonally woven baskets, and Twisted decahedron.

An additional feature of the book by Gerdes (1999) which would be of great help to mathematics educators who may want to explore similar studies is the explanation of the research methodology used. Gerdes says the following about the methodology:

> We developed a complementary methodology that enables one to uncover in traditional, material culture some hidden moments in geometrical thinking. It can be characterised as follows. We looked to the geometrical forms and patterns of traditional objects like baskets, mats, pots, houses, fishtraps, and so forth and posed the question: Why do these material products possess the form they have? In order to answer this question, we learned the usual production techniques and tried to vary the forms. It came out that the form of these objects is almost never arbitrary, but generally represents many practical advantages and is, quite a lot of times, the only possible or optimal solution of a production problem. The traditional form reflects accumulated experience and wisdom. It constitutes not only biological and physical knowledge about the materials used, but also mathematical knowledge, knowledge about the properties and relations of circles, angles, rectangles, squares, regular pentagons and hexagons, cones, cylinders, and so forth.

Even though the methodology described by Gerdes above is largely applicable to ethnomathematical work that focuses on geometric shapes, it provides an opportunity to understand (and even trial) how one form of ethnomathematical work is undertaken. This is useful for mathematics educators who would be interested to explore ethnomathematical work in their classrooms related to Geometry.

Mathematical Analysis in Ethnomathematical Studies and the Connection to Mathematical Content

Many ethnomathematical studies have focused upon analysis of various indigenous activities to reveal mathematical concepts and principles that are associated with such activities. Gerdes (1995:8) has identified that ethnomathematical studies revolve around two forms of analysis: (i) mathematical traditions that survived colonization and mathematical activities in people's daily life and ways to incorporate them into the curriculum (ii) culture elements that may serve as a starting point for doing and elaborating mathematics in and outside school. In both forms of analysis, researchers use their mathematical understanding to interpret an indigenous activity and reveal a variety of mathematical concepts associated with the activity.

Example of Mathematical Analysis based on String Figure Games

The analysis of the String Figure Gate 6 (Mosimege, 2000) reveals a number of mathematical concepts. It is important to indicate that the list of mathematical concepts specified below is not exhaustive. It is possible that other mathematical concepts may be found through further analysis or literature on String Figures. The following mathematical concepts were found in the analysis of String Figure Gates in general:

(i) Identification of a variety of geometric figures after making the different String Figure Gates: triangles; quadrilaterals (depending on how the string was stretched, quadrilaterals also specified into squares and rectangles);

(ii) Specification of relationships between various figures and generalisations drawn from these relationships;

- triangles and quadrilaterals: $y = 2x + 2$
- quadrilaterals and intersecting points: $y = 3x + 1$;
- quadrilaterals and the number of spaces (spaces is given by the combination of triangles and quadrilaterals): $y = 3x + 2$

(iii) Symmetry: Symmetry in terms of performance of some steps in making the gates - performing an activity on one side similar to the one done on the other side; Exploration of the different types of symmetries and the related operations in the different gates - bilateral (reflectional) symmetry, rotational symmetry (different folds), radial symmetry, translational (repetitive) symmetry, antisymmetry; Various properties of symmetries; Disentangling the string along a specific line of symmetry which ensures that the string does not get entangled.

Examples of Analysis of Beadwork and Other Activities outside the Classroom

In the book '*Geometry from Africa: Mathematical and Educational Explorations'* Gerdes (1999) illustrates further how mathematical analysis of various indigenous activities may be done. The importance of this book is its relevance to CAPS for all the Phases, especially the FET Phase as it gives detailed mathematical analysis and examples related to the Theorem of Pythagoras, Hexagonal Weaving, Diagonally woven baskets, and Twisted decahedron.

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Concluding Thoughts

The integration of ethnomathematical approaches and studies in the teaching and learning of mathematics is almost certainly bound to change how learners view and understand mathematics. In does not only serve as a good basis for a deeper conceptual understanding, it also helps to create a connection between mathematics classrooms and the real world. It actually takes the learners (and educators) to look at real life applications from a socio-cultural perspective. When the learners are looking for examples and illustrations that are meaningful, they will look around them and say mathematics makes sense, it helps me to look around find meaning.

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