ENHANCING MATHEMATICS PEDAGOGICAL CONTENT KNOWLEDGE IN GRADE 9 CLASS USING PROBLEM BASED LEARNING

by

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DECLARATION

I Bedeshani Moses Mceleli, declare that the Doctoral Degree research thesis, **ENHANCING MATHEMATICS PEDAGOGICAL CONTENT KNOWLEDGE IN GRADE 9 CLASS USING PROBLEM BASED LEARNING**, that I herewith submit for Doctoral Degree qualification in Education at the University of the Free State is my independent work, and that I have not previously submitted for a qualification at another institution of higher education.

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B M Mceleli

June 2019

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LIST OF ABBREVIATIONS/ACRONYMS

AfL	Assessment for Learning’
ANA	Annual National Assessment
BETD	Basic Education Teacher Diploma
BETD	Basic Education Teacher Diploma
CAPS	Curriculum and Assessment Policy Statement
CBPAR	Community-Based Participatory Action Research
CCK	Common Content Knowledge
CDA	Critical Discourse Analysis
CER	Critical Emancipatory Research
CL	Critical Linguistics
CRT	Critical Race Theory
DBE	Department of Basic Education
EC	Eastern Cape
ECDOE	Eastern Cape Department of Education
ECRS	Eastern Cape Rural Schools
EFA	Education for All
ELRC	Education Labour Relations Council
FET	Further Education and Training
FM	Further Mathematics
FME	Federal Ministry of Education
FOIL	First-Outside-Inside-Last
FPAR	Feminist Participatory Action Research
GET	General Education and Training

IQMS	Integrated Quality Management System
IRE	initiate, response and evaluate
LCE	Learner-Centred Education
LCPA	Learner-Centred Pedagogical Approach
LCT	Learner-Centred Teaching
MCKT	Mathematics Content Knowledge for Teaching
MDAS	My Dear Aunt Sally
MKT	Mathematics Knowledge for Teaching
MoE	Ministry of Education
MoEAC	Ministry of Education, Arts and Culture
MoHETI	Ministry of Higher Education, Training and Innovation
MPCK	Mathematics Pedagogical Content Knowledge
MSSI	Mpumalanga Secondary Science Initiative
NCTM	National Council for Teachers of Mathematics
NDP	National Development Plan
NEPA	National Education Policy Act
NERDC	Nigerian Educational Research and Development Council
PAM	Personnel Administration Measures
PAR	Participatory Action Research
PBL	Problem Based Learning
PBLW	Problem Based Learning Workshop
PCK	Pedagogical Content Knowledge
PD	Professional Development
PLCs	Professional Learning Communities

PTD	Primary Teachers’ Diploma
PUFM	Profound Understanding of Emergent Mathematics
RME	Realistic Mathematics Education
SA	South Africa
SADC	Southern African Development Community
SAQA	South Africa’s Qualifications Authority
SBA	School-Based Assessment
SCK	Specialized Content Knowledge
SSA	Sub-Saharan African
SWOT	Strengths Weaknesses Opportunities Threats
TIMSS	Trends in International Mathematics and Science Study
UNAM	University of Namibia
USA	United States of America
ZPD	Zone of Proximal Development

ABSTRACT

This study was aimed at designing a strategy to enhance mathematics pedagogical content knowledge (PCK) of teachers teaching Grade 9 learners using a problem-based learning (PBL) approach. PCK is the blending of content and pedagogy into how particular topics are presented to learners. Four components of PCK, namely understanding of learners' misconceptions, understanding of content knowledge for teaching, understanding of pedagogical knowledge and understanding of curriculum knowledge were used to define a knowledge base needed for teaching mathematics. Furthermore, in the context of this study, PBL was used to enhance the above-mentioned PCK components through coordinated teams. PBL is defined as a learner-centred instructional method that utilizes real problems as a primary pathway for learning that develops learners' ability to analyse ill-structured problems to strive for a meaningful solution.

The study focused on how to enhance Grade nine mathematics teachers' PCK using problem-based learning. It explored the challenges that teachers face when teaching Grade nine mathematics in terms of mathematics pedagogical content knowledge (MPCK). These challenges included, but were not limited to the non-existence of a coordinated team to enhance MPCK for teaching the Grade nine curriculum; poor follow-up of learners' misconceptions; insufficient lesson preparation; insufficient use of curriculum materials when teaching; no integration of assessment and lesson facilitation; non-implementation of a learner-centred approach and poor mathematical knowledge for teaching.

The study generated a strategy to respond to these challenges. However, the major challenge was that the knowledge base needed for teaching mathematics is contextually bound and complex. Therefore, the study adopted Critical Emancipatory Research (CER) as a theoretical lens for the study, mainly due to its critical commitment to confront social oppression and challenge well-established ways of thinking that frequently limit teachers' potential. In this study, CER enabled co-researchers and I to consciously work together towards mastering critically challenging and changing systems that routinely oppress them. Through CER the study embraced multiple perspectives and negotiated meaning in formulating a strategy to respond to the identified challenges.

Guided by an epistemological stance that embraces the value of welcoming subjective views on knowledge production, participatory action research (PAR) created a platform for participants who later became co-researchers to engage in knowledge production activities with equality and tolerance of contrasting views. Through problem-based learning workshops (PBLW), anchored in PAR methodology, a team of eight Grade nine mathematics teachers, their classes, a principal, a mathematics subject advisor and I worked together at the research sites. The research team collectively identified challenges that teachers faced, and enacted negotiated solutions to improve the wisdom of practice when teaching Grade nine mathematics.

The generated data comprised photos, video recordings, audio recordings, learners' scripts, co-researchers' reflections, and lesson plans. Data were analysed using Critical Discourse Analysis (CDA). To understand the deeper meaning of the personal and subjective accounts of co-researchers' lived experiences in teaching mathematics, data were analysed and interpreted at three levels of CDA, namely text, discursive practice and social structure. Through CDA the study analysed problems experienced by teachers who teach mathematics. This was done for the purpose of proposing possible solutions and strategies that might be developed, adopted and adapted to effectively address the problems that teachers experienced.

Finally, to sustain the formulated strategy to enhance MPCK during and beyond the duration of the study, the conducive conditions for the strategy were investigated and enacted. The study further analysed and presented possible ways to circumvent threats and risks that could derail successful implementation of the strategy. The study was transformative in nature, which created the opportunity to operationalise and evaluate the success of the strategy prior to it being considered for recommendation. In conclusion, the study findings are revealed, indicators of success are identified, and recommendations are made. Some of the findings were that teachers worked in silos; their lessons were inadequately prepared; mathematics manipulatives were not judiciously utilized as the classroom discourse was teacher centred, starting with demonstration first and assessment later. Lastly, teachers' knowledge gap regarding mathematics knowledge for teaching resulted in a learning cul-de-sac.

Keywords: Mathematics pedagogical contend knowledge (MPCK), Problem based learning (PBL), Coordinated team.

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CHAPTER 1 : OVERVIEW OF THE STUDY

1.1. INTRODUCTION

This study was aimed at designing a strategy to enhance mathematics pedagogical content knowledge (PCK) of teachers teaching Grade 9 learners using a problem-based learning (PBL) approach. This chapter introduces this initiative with a brief background to contextualise the problem statement. It also provides brief outlines of the theoretical framework, study design, methodology and data analysis.

1.2. BACKGROUND TO THE STUDY

Pedagogical content knowledge is the blending of content and pedagogy into how particular topics are presented to learners (Shulman, 1987:8). Ngo (2013:82) views PCK as teachers' understanding of common students' errors within a topic. Although Peng (2013:84) argues that PCK is elusive and difficult to define, PCK entails teachers' ability to help learners comprehend mathematics concepts. Appleton (2008:525) also includes knowledge of curriculum as one of the PCK components. On the other hand, PBL is a learner-centred instructional method that utilizes real problems as a primary pathway for learning and enhancing students' ability to analyse ill-structured problems to strive for a meaningful solution (Ramsay & Sorrel, 2006:2). According to Laursen (2013:31), PBL components include, but are not limited to problems, a team-based approach and reflection on the appropriateness of the product results. The study used these PBL components to enhance Grade 9 teachers' mathematics pedagogical content knowledge (MPKC).

Drawing from the research conducted by Trends in International Mathematics and Science Study (TIMSS) between 2002 and 2011, Spaul (2013:17) posited that South African (SA) Grade nine learners have been outperformed by Grade eight learners from 21 other middle-income countries in mathematics. The research findings demonstrated that SA Grade nine mathematics learners were comparatively two years' worth of learning behind the average Grade eight pupil (Spaul, 2013:17). Linking learner performance to quality of teaching, the literature also claims that poor subject knowledge, and poor mathematics teaching and learning are serious problems in South African education (Diko & Feza, 2014:1457). A positive correlation between learners' performance and teachers' understanding of PCK components such as content knowledge confirms the view that "teachers cannot help learners with content that they do not understand themselves" (Venkat & Spaul, 2015:122; George & Adu, 2018:141). In the SA context, rural teachers seem to struggle with mathematics subject matter knowledge (Spaull, 2013:5). Consequently, "teacher's poor understanding of the concepts of ratio and number" resulted in incoherent, illogical and convoluted explanations, which made no sense to learners (Bansilal, Brijlall & Mkhwanazi, 2014: 36). The research also revealed that teachers had insufficient knowledge of teaching strategies and of students' misconceptions regarding quadratic functions (Sibuyi, 2012:71).

It seems that in Lesotho, a country in the Southern African Development Community (SADC) region, teachers also used ineffective teaching methods and religiously followed rules and procedures to teach fractions, instead of concept understanding (Marake, 2013:185). In Turkey and in Nigeria teachers were unable to identify and correct learners' misconceptions (Tansil & Kose, 2013:2; Zuya, 2014:121). According to Kilic (2011:19), teachers could not understand learners' reasoning when learners presented the following incorrect solutions for 4 divided by 0, namely $4 \div 0 = 0$, or $4 \div 0 = 4$. In the United States of America (USA), teachers lack mathematics knowledge (Ball & Bass, 2002:13) and had significant difficulty in explaining the "meaning of division with fractions" (Ball, 1990:453). Evidently, from the above scholarly discourse, poor teacher knowledge in terms of MPCK components does not seem to be only a SA problem, but is a common challenge in many countries around the world. It is self-evident that teachers cannot help children learn things they themselves do not understand (Ball, 1991:6). In supporting Ball's (1991:6) assertion, McNamara (1994:231) theorized that teachers would have difficulties to teach if they had no knowledge of and experience in terms of content knowledge.

As an antidote to the above challenges Ono and Ferreira (2010:65) reported about the project called Mpumalanga Secondary Science Initiative (MSSI) that intended to enhance teachers' teaching skills in mathematics. During reflection on the observed mathematics lessons in the MSSI project, participants agreed to first identify positive aspects in the observed lesson, and presented suggestions on the identified arears of improvement (Ono & Ferreira, 2010:67). In Namibia, teachers used questioning, prior knowledge and pair and group work as strategies to implement learner-centred education (LCE) (Amakali, 2017:686). This major shift in the Namibian education system resulted in the introduction of the Basic Education Teacher Diploma (BETD) programme founded on learner-centred pedagogical principles (Peters, 2016:39). In Nigeria teachers were encouraged to attend training workshops to improve their subject matter knowledge (Obilor, 2012:48; Zuya, 2014:117). It is also reported that pre-service teachers in California claimed that they felt like they were 'really learning how to teach mathematics' after having engaged in a training programme that blended subject-matter, that is, number sense, algebra and functions with pedagogical training (Morales, Anderson & McGowan, 2003:49).

However, these studies referred to above did not use a PBL approach in developing PCK. The use of the PBL approach to enhance PCK is reported to have enabled teachers to refine their reasoning ability in integrating their knowledge, curriculum and learners (Peterson & Treagust, 1995: 304). In addition, participants in the study conducted by Schmude, Serow and Tobias (2011:682) valued analysing students' work by exploring strategies and ideas that could help develop the student's understanding of mathematics. Moreover, the literature also revealed that mathematics scores increased significantly as problem-solving skills, critical thinking, creative thinking, and maths communication skills increased over the years of PBL implementation (Inman, 2011:55 & 99).

1.3. PROBLEM STATEMENT

In view of the background given above it is evident that PCK is still a challenge in many countries (Marake, 2013:185; Spaull, 2013:5; Tansil & Kose, 2013:2 & Zuya, 2014:121). The practice of teaching mathematics as a set of arbitrary and unrelated rules seems to continue unabated. Learners are expected to memorize mathematics procedures and as consequence, their mathematics concept understanding has been negatively affected. The study locates the problem within the aspects of mathematics content knowledge that embody its teachability, thus PCK (Ball, 1990:453). It appears that teachers have challenges in terms of mathematics content knowledge, mathematics pedagogical knowledge, understanding of learners' knowledge and mathematics curriculum knowledge (Ball & Bass, 2002:13). Therefore, in response to the preceding challenges, the study designed a strategy to assist teachers' collaborative emancipation by addressing the following research questions.

1.3.1. Research question

How can mathematics pedagogical content knowledge of teachers be enhanced when teaching Grade 9 learners using problem-based learning?

1.3.2. The aim of the study

The aim of the study was to design a strategy to enhance mathematics pedagogical content knowledge of teachers teaching Grade 9 learners using problem-based learning.

1.3.3. Objectives of the study

The objectives of the study were to:

- identify and analyse challenges that teachers teaching Grade 9 learners face regarding their mathematics pedagogical content knowledge;
- formulate components of a strategy to respond to challenges facing Grade 9 mathematics teachers regarding pedagogical content knowledge using problem-based learning;
- understand conditions for the successful implementation of the strategy to respond to challenges facing Grade 9 teachers in their mathematical pedagogical content knowledge using problem-based learning;
- anticipate possible threats in the design and implementation of the strategy to respond to challenges facing Grade 9 teachers in their mathematical pedagogical content knowledge using problem-based learning;
- understand and investigate the indicators of success in the implementation of the strategy to respond to challenges facing Grade 9 teachers in their mathematical pedagogical content knowledge using problem-based learning.

1.4. THEORETICAL FRAMEWORK

In developing a strategy, the study used Critical Emancipatory Research (CER). CER promotes social justice and democracy while aiming at enhancing humanity, social values and equity by showing respect to the participants (Nkoane, 2012:98). In CER the participants are treated as equals with the researcher and it is seen to be empowering and liberating (Mahlomaholo, 2009:225-226). Guided by CER, the researcher worked together with participants in developing the strategy to emancipate Grade nine mathematics teachers in terms of MPCK enhancement. CER furthermore encourages a relationship of mutual trust and respect between the researcher and participants. In CER the participants are recognised and valued, and thus treated with respect as fellow humans by the researcher, unlike in a positivist paradigm where they are treated as if they are mere impersonal objects in a natural science laboratory (Mahlomaholo, 2009:225-226).

CER facilitates politics to confront social oppression in rural schools and the researcher has to learn how to put his knowledge and skills at the disposal of the researched participants, for them to use in whatever way they choose (Oliver, 1992: 110). It is also argued that critical teachers (co-researchers in this case) must challenge their own well-established ways of thinking that frequently limits their potential and this could lead to critical consciousness that enables them to change systems that routinely oppress them (Tutak *et al.*, 2001:66). This study, therefore sought to empower mathematics teachers so that they might be able to reflect on and openly criticise their own classroom practice.

1.5. CONCEPTUALIZING OPERATIONAL CONCEPTS

Shulman's (1986:9) seminal presentation defined PCK as the 'special amalgam' between content knowledge and pedagogical knowledge mainly focusing on ways of making the subject comprehensible to others. By implication, pedagogical knowledge and content knowledge become components of PCK, including curriculum knowledge, which according to Shulman was referred to as 'tools of trade' (1987:8). Peng (2014:88) included knowledge of learners' understanding as one component of PCK.

The strategy focused on the following PCK components: content knowledge, pedagogical knowledge, understanding of students' misconceptions, and curriculum knowledge

PBL is not only instructional approach as Ramsay and Sorrel (2006:2) asserted, but an educational strategy or even a philosophy (Savin-Baden & Major, 2004: 5) that is grounded in a constructivist learning theory (Goodnough, 2006:302; Mcconnell, Parker & Eberhardt, 2013:221). In addition, PBL refers to collaborative learning in small groups (Murray-Harvey, Pourshafie & Reyes, 2013:115). In the proposed strategy we recommend the use of PBL with more focus on collaborative team work to enhance Grade nine teachers' MPCK.

1.6. OVERVIEW OF THE LITERATURE REVIEW

The operationalisation of the objectives of the study was done through reviewing the literature on good practices in terms of education policy frameworks, problem-based learning and the research findings. The literature reviewed is local, regional (the Southern African Development Community [SADC]), continental and global. Key concepts arose as constructs to be used in Chapter four to interpret the empirical data.

1.6.1 Justification for the need to develop a strategy to enhance PCK using PBL

SA education policies do not explicitly prescribe the application of PBL, although they seem to embrace its principles (Mahlomaholo, 2013a:67). Despite the claimed success of PBL in mathematics teaching and learning (Erickson, 1999:520), it seems that there is little research that has explored how the adoption of PBL impacts the development of PCK (Goodnough, 2006: 303). "In a few studies focusing primarily on teachers' use of PBL in their own classrooms, teachers reported changes in their enthusiasm for teaching, critical thinking skills, and classroom practices" (Weizman, Covitt, Koehler, Lundeberg & Oslund, 2008:31-32). Evidently, one of the fundamental principles of PBL is that learning takes place through dialogue among team members characterized by mutual respect (Barge, 2010:15). Despite team work being espoused in PBL principles, Mosia (2016: 115) reported that mathematics teachers who did not collaboratively teach Euclidean geometry failed to realize knowledge of teaching as a socially constructed endeavour. The non-existence of coordinated teams denied teachers the opportunity to use collective wisdom to untangle encountered problems in the teaching practice (Mosia, 2016:115).

1.6.2 Determining the components of the strategy to enhance PCK using PBL

The initial research meetings identified challenges in relation to teachers' inability to help learners comprehend mathematics concepts. In addressing these challenges, the research team was constituted from the school community, members of society, and education officials. In developing comprehensive working solutions for the identified problem, thus, enhancing the 'wisdom of practice' (Shulman, 1986:9), expertise from various sectors was needed. The components of the strategy focused on the establishment of a coordinated team to resolve problems regarding mathematics content knowledge for teaching (MCKT), pedagogical knowledge, learners' mathematics misconceptions and mathematics curriculum knowledge. Coordinated team work is in line with the narrative that the meeting of two agents or inter-subjectivities results in growth and "reciprocal beneficiation" (Mahlomaholo, 2012a:293). Coordinated team members attach value to learners' thinking when they collectively try to understand learners' ways of solving mathematics problems and in the process, they consequently create new knowledge (Gardee & Brodie, 2015:2). Through collaborative work, coordinated team members do not only share expertise on curriculum knowledge, which Shulman (1987:8) referred to as 'tools of trade,' but also share experiences about what Shulman (1986:9) called the most powerful forms of representation.

1.6.3 Conducive conditions to enable successful implementation of the strategy

The conditions for the successful implementation of the strategy include active collaborative group-work or team settings, real-life problems and a democratic environment in the classroom where both the tutor and the students have the same status in the dialogic arena (Armitage, 2013:13; Humelo-Silver, 2004:236; Krogh & Jensen, 2013:10; Mahlomaholo, 2013a:72). It is further claimed that PBL facilitators must possess communication and social skills, and must take a genuine interest in students' learning through using real-life problems (Coffin, 2013: 204). Furthermore, Stegeager, Thomassen & Laursen (2013:153) theorized that "the problem should express the students' 'astonishment' or 'cognitive disturbance' in the context of the relevant academic disciplines". It is also crucial to consider the cultural conditions, like teacher-centred approach, that may contradict PBL. PBL should be implemented in a piecemeal way, carefully balancing the innovative principles and the conservation of the old values and norms, especially when implemented in an institute with long standing traditions in teaching and learning (de Graaff, 2011: 125).

1.6.4 Identification of threats that might derail the implementation of the strategy

Noted threats that could hinder successful BPL implementation inter alia include resource limitations, the influence of tradition, and inappropriate change strategies (Li, 2013:177). A perception also exists that PBL is time consuming, impossible to implement in large classes, would likely increase teachers' workload, and is in conflict with the dominant culture of old non-democratic teacher-centred practices (Mahlomaholo, 2013a:80). PBL critics argue that it is less effective and it "may have negative results when students acquire misconceptions or incomplete or disorganized knowledge" (Kirschner, Sweller and Clark, 2006:84). To avoid these risks, however, proponents of PBL argue there should be thorough debriefing in concluding the learning experience to consolidate the learning experiences and to demystify learners' misconceptions (Savery, 2006:12). To reduce resistance to change from traditional to learner-centred approaches, to encourage teamwork among teachers, through promotion of a gentle change, which would allow educators to experience PBL, seemed more feasible (Li, 2013:182). On the other hand, Mahlomaholo (2013a:80) suggested the advocacy programme to cultivate a buy-in from stakeholders, which, by implication may influence the prioritization of PBL resources.

1.6.5 Demonstrating the indicators of success of the strategic framework

Successful implementation of PBL was evidenced through the improvement of mathematics PCK and learners' mathematical reasoning. Teachers need PCK to be able to understand the possible difficulties that their students may encounter in a specific topic (Karaman, 2012:59). The use of PBL demonstrated the possibility for teachers to acquire the necessary PCK to teach particular topics (Goodnough, 2006:303; Peterson & Treagust, 1995: 304). Ball and Bass (2003, 44) further claim that "just as students need to learn to reason mathematically, so, too must teachers develop and learn practices to support such learning". By implication, the improved learners' mathematical reasoning also indicated success regarding the use of PBL to enhance mathematics PCK.

1.7. RESEARCH DESIGN AND METHODOLOGY

Rather than merely describing what is happening at research sites or explaining it, this study went beyond that to design a framework and strategy to attempt to resolve particular real-life problems at the research sites (cf. Mahlomaholo, 2013b:4614). The study therefore adopted Participatory Action Research (PAR) (MacDonald 2012: 37) as a practical intervention to enhance mathematics PCK using PBL. PAR refers to a collective enquiry in social situations and taking action or effecting change in order to improve the rationality and justice of participants' own social practices (Green, George, Daniel, Frankish, Herbert, Bowie & O'Neill, 2003:419). PAR has an emancipatory stance through enabling people to "unshackle themselves from the constraints of irrational, unproductive, unjust and unsatisfying social structures which limit their self-development and self-determination" (Kemmis & Wilkinson, 1998:24). It is collaborative in nature, hence research that has adopted this methodology is done "with others rather than on or to others" (Cresswell, 2013: 25).

A team of eight Grade nine mathematics teachers, their classes, one principal and the subject advisor were assembled to work together at the schools in Joe Gqabi District (Mount Fletcher) in the Eastern Cape Province. The team engaged on the following stages of action research, namely reflection, planning, action and observation that followed each other in a spiral or cycle (Khan and Chovanec, 2010: 35; MacDonald, 2012: 37; Mc Taggart, 1994:315). The data were generated through meetings, workshops, discussions, reflections and observations using audio and video tapes.

The strategy was implemented in six schools in the rural area of Mount Fletcher in the Joe Gqabi Education District. For confidentiality and anonymity, the schools' names are not mentioned, and for co-researchers pseudonyms were used. To stimulate debate, reflective meetings were held after every lesson observation. The co-researchers' proposals in terms of how mathematics lesson plans in the strategic plan should be done shaped the research process. In line with the views of Cresswell (2013:25) and Kemmis and Wilkinson (1998:24), co-researchers were central to the study and their voices were heard, rather than being perceived as objects to be manipulated and regulated in a setting detached from the real world of their lived experiences and practices (cf. McGregor & Murnane, 2010:425). All participants were allocated roles and responsibilities. The coordinated team identified resources for that particular activity and time frames were also determined for activities to take place.

1.8. DATA ANALYSIS

Data were analysed (transcribed) through the use of Critical Discourse Analysis (CDA) (cf. van Dijk, 2001:352). To understand the deeper meaning of the personal and subjective accounts of co-researchers' lived experience in teaching mathematics, generated data were analysed and interpreted at three levels of Fairclough's (1995:97) dimensional conception of CDA, thus, text, discursive practice and social structure. The data comprised photos, video recordings, audio recordings, learners' scripts, co-researchers' reflections, and lesson plans.

1.9. ETHICAL CONSIDERATIONS

The study first sought full permission from the Eastern Cape Department of Education (ECDoE), and its findings as well as results will be made accessible to the public (see Appendix 1). However, the identities of participants, who later became co-researchers were concealed and remained confidential. This process included letters of consent and permission to participate from co-researchers and parents of participating learners (cf. Maree & van der Westhuizen, 2007:42). Co-researchers were informed about the nature of the study together with the benefits of the study and were informed about the right to terminate their participation in the study at any time should they wish to do so.

1.10. LAYOUT OF CHAPTERS

Chapter 1: This chapter focused on the introduction, background, problem statement, research question, aim and objectives of the study.

Chapter 2: This chapter focused on the literature review outlining the theoretical framework, operational concepts and related literature.

Chapter 3: This chapter presented the research design and methodologies and explained how the generated data were analysed.

Chapter 4: This chapter is devoted to the data analysis, as well as the presentation and interpretation of the results towards the strategy to enhance MPCK using PBL.

Chapter 5: In this chapter the conclusions, summary of the findings and recommendations of the study are presented.

1.11. CONCLUSION

This study was aimed at designing a strategy to enhance the mathematics pedagogical content knowledge (PCK) of teachers teaching Grade 9 learners using a problem-based learning (PBL) approach. This chapter presented a brief background to the study, contextualised the problem statement and discussed the study objectives. The theoretical framework, study design, methodology and data analysis of the study have been elucidated briefly, and the layout of the chapters of this report has been explained.

CHAPTER 2 : LITERATURE REVIEW ON THE STRATEGY TO ENHANCE MATHEMATICS TEACHERS’ PCK USING PBL

2.1 INTRODUCTION

This study was aimed at designing a strategy to enhance mathematics pedagogical content knowledge (PCK) of teachers in the Grade 9 class using a problem-based learning (PBL) approach. Chapter two presents the theoretical framework and conceptual discussions guiding the study in order to achieve its aim and objectives. The historical origin of the theoretical framework was traced. I then looked at the operational concepts together with related literature in terms of legislative imperatives and policy directives regarding mathematics PCK and PBL in the South African context, including one country from the Southern African Development Community (SADC), Africa and internationally. Due to the scope of the study we could only explore one country from each of the above-mentioned regions, namely Namibia, Nigeria, and the United States of America, in order to understand global trends in terms of challenges facing mathematics teachers.

2.2 THEORETICAL FRAMEWORK

The theoretical framework is a lens through which the world is viewed, including the assumptions that guide the way of thinking and actions taken by the researchers and participants Mertens (2010) cited in (Tsotetsi, 2013: 25). The theoretical framework identifies the researcher’s world views and thus delineates the assumptions and preconceptions about the areas being studied (Green, 2014: 35). It further helps the researcher in ensuring that the research process is coherent and guides the selection of relevant methodologies to achieve the research aims (Green, 2014: 35). In developing a strategy, this study used Critical Emancipatory Research (CER) as a guiding lens and as a perspective through which the strategy to enhance mathematics PCK using BPL is anchored.

2.2.1 The origin of critical theory

"CER has its philosophical roots in several traditions, among which Marx's analysis of socio-economic conditions and class structures; Habermas' notion of emancipatory knowledge, and Freire's transformative and emancipatory pedagogy" (Nkoane, 2012 99). Critical theory originated from a group of German intellectuals who came together in the late 1920s with the Frankfurt School (Sumner, 2003: 3; Mahlomahulo, 2009:225). It is rooted in Marxist perspectives, critique and subvert domination in all its forms (Stinson, Bidwell, Jett, Powell & Thurman, 2007: 620). However, it questions the assumptions made by Marxism and does not embrace the orthodoxy of Marxism tradition (McLaren, 1989: 190 & Sumner, 2003: 3). According to Sumner (2003: 4) "critical theory adopts an overtly critical approach to inquiry". It disputes the positivist view of objectively examining the systems of domination and inevitable hopes that it will bring about awareness of social injustices, motivating self-empowerment and social transformation (Stinson *et al.*, 2007: 620). Instead, it advances the cultivation of conscious suspicion and critical attitude at all levels, while, on the other side, it seeks human emancipation by liberating human beings from the circumstances that enslave them (Horkheimer, 1982: 244) and change systems that routinely oppress them (Tutak *et al.*, 2001: 66).

Moreover, Sumner (2003: 5 citing Latter, 1991) argues that critical theory is imbued with a concept of 'catalytic validity'. Catalytic validity is the degree to which the research process focuses participants towards knowing reality in order to transform it, while channelling its impact so that they ultimately gain self-understanding and self-determination through research participation (Sumner, 2003:5). Critical theory is traditionally concerned with the expressive aspects of power relations and to engage the marginalized so that they can rethink their socio-political role. As viewed by the literature, 'power' is not natural (Hlalele, 2014: 104), but a mutable political mechanism that could be arranged in other ways (Dworski-Riggs & langhout, 2010 in Hlalele, 2014: 104). As power relations are humanly designed, critical theory therefore creates an environment to restore the human dignity, brings hope and peace so that the marginalized and voiceless are able not to only take part in the research proceeding but to influence the process and its findings towards their context.

2.2.2 Objectives of CER

CER is a transformative research paradigm where the researcher does not arrogantly impose his or her knowledge and techniques, but respects and combines his or her knowledge with the knowledge of the researched while taking them as full partners and co-researchers that can advance the knowledge gap (Fals Borda, 1995 in Hlalele, 2014: 103). CER promotes social justice and democracy while aiming at enhancing humanity, social values and equity by showing respect to the participants (Nkoane, 2012: 98). In CER the participants are recognised and valued, and thus treated with respect as fellow humans by the researcher, unlike in a positivist paradigm where they are treated as if they are mere impersonal objects in a natural science laboratory (Mahlomaholo, 2009:225-226). Guided by the CER lens, the strategy intends to create a dialogic environment whereby mathematics teachers in Grade 9 classes may self-emancipate in terms of PCK enhancement.

CER facilitates politics to confront social oppression in rural schools, particularly in Grade 9 mathematics classes and where the researcher has to learn how to put his knowledge and skills at the disposal of the researched participants, for them to use it in whatever way they choose (Oliver, 1992: 110). Critical teachers (participants in this case) must challenge their own well-established ways of thinking that frequently limit their potential and this could lead to critical consciousness that enables them to change systems that routinely oppress them (Tutak *et al.*, 2001: 66). The literature also asserts that “CER advocates peace, hope, equality, team spirit and social justice; thus, CER is changing people’s hearts and minds, liberating and meeting the needs of real-life situations” (Tshelane & Tshelane, 2014: 288). This paradigm allows the researcher and participants or co-researchers to contextualize the challenges and develop most appropriate components of the solution. Freedman (2006: 88) believes that the emancipatory practice provides researcher and participants with an opportunity to engage and negotiate the meaning construction.

On the other hand, critical theory is vehemently against the “naturalist approach that suggests that the researcher should study the social world in its undisturbed state” (McCabe & Holmes, 2009: 1522), however, it advocates the view of empowering the powerless by transforming the existing social inequalities and injustices (McLaren, 1989: 186). In fact, “critical theorists begin with the premise that men and women are essentially unfree and inhabit a world rife with contradictions and asymmetries of power and privilege" (McLaren, 1989: 193). The above argument has a direct implication towards research in the social world, like in teaching. Mathematics PCK especially in South Africa has been influenced by the context (apartheid in this case), as it is believed that society's historical conditions are created and influenced by the asymmetries of power and special interests (Alvesson & Sköldberg, 2000 in Sumner, 2003: 4). Power relations can be made the subject of radical change through critical theory (Sumner, 2003: 4), hence a naturalist approach is not plausible and instead a critical emancipatory approach is more relevant. It is a fact that the country is at its 21st birthday of democracy, however, the ideology of apartheid oppression and marginalisation is still rife in its education system (Mahlomaholo, 2009: 224). McLaren (1989: 186) argues that the traditional view of teaching and learning as a neutral process from power and politics can no longer be credibly endorsed as the critical research has given primacy to the social, political and economic order to better understand the workings of contemporary schooling.

The inequalities that were created by colonialism of a special kind called apartheid in the South African society seem to perpetuate social injustice unabated (Nkoane, 2012: 98). On the other hand, there has been a long-standing brain drainage from rural villages to the cities, while the "rural resources of culture and energy become depleted" (Hlalele, 2014: 101). Empirical evidence exists that these inequalities manifest even in the education system as Spaull (2013: 6) argues that readily available data regarding learner achievement show that there are two different public-school systems in South Africa. In fact, education is not exonerated from the same fate as other poor services in rural areas (Hlalele, 2014: 101). The poor and marginalised, who are predominantly black children, are systematically channelled towards poor education while white children and few black elites are able to receive better education. For an example, TIMSS (2011 in Spoull, 2013: 6) show that Grade nine learners in the Eastern Cape (EC) were 1,8 years' worth of learning behind Gauteng at an average. Quality teaching and learning in rural contexts remain a pipe dream for all levels of the educational endeavour (Hlalele, 2014: 101). This can be attributed to the kind of teacher cadre found in the EC, as it logically is true that education cannot be better than its teachers. A study using CER as a guiding lens is sensitive to "those who were located in the periphery of society, excluded, relegated, marginalised and oppressed"

(Nkoane, 2012:100). In essence, the research process in this regard is focused on transforming both researcher and the participants' research site to advance democracy, liberation, equity and social justice (Nkoane, 2012: 100). While the researcher endeavours to interpret other people's interpretations and tries to make sense thereof, the research process is seen as the most humanising experience (Mahlomaholo, 2009:225).

2.2.3 Justification for the critical theory approach

CER is an appropriate theoretical framework that informs this study. This is attributed to its emancipatory and transformational agenda, as well as to its objective to engage the marginalised so that their voices can be heard and respected (Dold and Chapman, 2011 cited in Hlalele, 2014: 104). CER takes cognisance that the researched best understand their social ills and are best suited to come up with the appropriate sustainable solutions. In terms of CER, co-researchers "would be left owning the working strategy" (Tsotesti, 2014: 29). Once the research process had been completed, it was envisaged that Grade 9 mathematics teachers would continue using the strategic framework as they had been part of developing it, as CER advances the agenda of human emancipation regardless of status (Hlalele, 2014: 104). The teachers' voices are part of the strategy in the CER's engaging nature which allows for a deeper meaning and for multiple perspectives to be considered (Mahlomaholo, 2009: 34). Moreover, Hlalele argues that it enables "participants to identify possible threats and thus implement measures to evade them [participants] as part of changing their situation" (2014: 104). Therefore, this justifies my adoption of this paradigm as it advances social justice and gives hope to the marginalised (Tsotetsi, 2014: 29).

In essence the poor teachers' MPCK in Eastern Cape as one of the inequalities in South African education as has been tabled in the previous sections is not a neutral and apolitical phenomenon, but systematically and socio-politically designed. During the apartheid era school mathematics produced and maintained white supremacy and black subordination as they were denied the rights in terms of both access and quality (Maboya, 2014: 7). CER shines a critical light on the societal settings and reveals the dominating interests of the "wealthy elite who have succeeded in convincing most people that those elite interests are also the interests of society at large" and as such, all research serves certain class interests, which are seldom clarified (Sumner, 2003: 3). Critical theory as a framework of this study is found more appropriate as it focuses on the issues of change and transformation that are at the heart of this study (Maboya, 2014: 23). Despite the domination of the poor by the interests of the wealthy elite, Frankfort School argues that humans can change reality (Sumner, 2003: 3). CER serves critical-emancipatory interests and also demands researchers to confront the question of whose interests their research serves (Sumner, 2003: 3). The epistemological position we assume gives respect, dignity and power to the research participants in shaping the direction of the research process in their context.

2.3 DEFINITION AND DISCUSSIONS OF OPERATIONAL CONCEPTS

The aim of this section is to define and discuss the operational concepts underpinning this study. Mathematics PCK and the PBL approach will be discussed and contextualized to Grade 9 mathematics teachers in EC rural schools in order to develop a strategy to enhance mathematics PCK of Grade 9 teachers using the PBL approach.

2.3.1 Pedagogical Content Knowledge (PCK)

In defining PCK the literature draws heavily on what Shulman (1986: 7) called ‘the missing paradigm’ in teacher education as it is defined as the blending of content knowledge and pedagogy on how to present topics to learners (Shulman, 1987: 8). This knowledge base is only valid in the province of a teacher during their complex work in the classroom (Shulman, 1987: 8). PCK is specific to teaching and therefore, it separates subject teacher from subject expert (Kwong, Joseph; Eric & Khoh, 2007: 28). For instance, Ibeawuchi (2010: 12) argues that “mathematics educators differ from mathematicians not necessarily in quantity or quality of subject matter knowledge, but in how that knowledge is organised and used”. However, a mathematician may not have the capacity to transform their content knowledge into forms that are pedagogically powerful and yet adaptive to the variations in ability and learners’ background (Shuman, 1987: 15). On the other side mathematics teachers are able to adapt mathematics subject matter knowledge for pedagogical purposes (Marks, 1990: 7).

There is no universal agreement among researchers in terms of defining PCK, as the term PCK is widely used while its potential has not been fully realised and lacks clarity of definition (Hurrel, 2013: 57). However, this study does not intend to give the actual clarity of PCK definition, instead it intends to enhance the construct of PCK in Grade 9 mathematics educators. Nonetheless, Shulman (1986: 7) defined PCK as "the most useful forms of representations of those ideas, the most powerful analogies, illustrations, examples, explanations, and demonstrations – in a word, the most useful ways of representing and formulating the subject that makes it comprehensible to others". However, Peng (2013: 84) argues that PCK is elusive and difficult to define, while Loughran, Gunstone, Berry, Milroy, and Mulhall (2000 cited in Goodnough & Hung, 2009: 231) viewed PCK:

> *"as a mixture of interacting elements, including views of learning, views of teaching, understanding of content, understanding of students, knowledge and practice of children's conceptions, time, context, views of scientific knowledge, pedagogical practice, decision-making, reflection, and explicit versus tacit knowledge of practice, beliefs, or ideas, all of which interact and result in PCK".*

The common elements in researchers' view about PCK, however, is the ability to combine content, curriculum, and pedagogy while considering learners' misconceptions when presenting mathematical content and ideas so that students may comprehend and develop a deeper insight of mathematics concepts and procedures. Teachers need to know more than the notion of "this is how you get the correct answer" in mathematics but transcend to a richer, more complete understanding of the whys (Mecoli, 2013: 24). For an example, "teachers also need to understand students' thought process to help them understand questions such as: Why (- 3) X (- 5) = 15?" (Kar, 2017: 7). All these forms of knowledge amalgamated are components of PCK and therefore PCK could be viewed as the ability of a teacher to help learners comprehend mathematics content and understand the reasons behind mathematics procedures.

Regarding teacher education, Shulman (1986: 8) raised the following questions which are springboards for the PCK components: "Where do teachers' explanations come from? How do teachers decide what to teach, how to represent it, how to question students about it, and how to deal with problems of misunderstanding?" In an attempt to enhance PCK, the researcher must answer these questions. Although there is a plethora of literature regarding PCK containing different definitions of PCK, in Shulman's view teachers' explanations come from the content knowledge. What to teach is prescribed in the curriculum, how to represent it is drawn from pedagogy, and capacity to deal with problems of misunderstanding comes from understanding of common students' misconceptions. The strategy will focus on enhancing these PCK components, that is, content knowledge, pedagogical knowledge, understanding of students' misconceptions, and curriculum knowledge.

2.3.1.1 Mathematics content knowledge and beliefs

Subject matter knowledge is defined as:

> *"the teachers' knowledge of central facts, concepts, ideas and principles in mathematics, how they view these as being organised and relating to each other, and how they are able to make use of this knowledge in arriving at and evaluating correct claims, representations and solutions"* (Barnes, 2007: 18).

This category of knowledge includes both facts and concepts in a domain, why facts and concepts are true and how knowledge is generated and structured in the discipline (Hill; Ball & Schilling, 2004: 13). In the case of mathematics, this type of knowledge may include how mathematics is viewed, perceived and believed. Beliefs, however, will be dealt with later in this subsection. Powell and Hanna (2006: 377) assert that to teach a subject like mathematics effectively necessitates knowledge of mathematics that is more than knowledge of subject matter *per se* but subject matter knowledge for teaching. This suggests that knowing particular mathematical ideas and procedures like to invert and multiply when dividing by a fraction as mere fact or routine is insufficient for using those ideas flexibly in diverse classrooms that may not be easy to anticipate (Ball & Bass, 2003: 28). The teacher should not only understand that something is so, but why it is so (Ball; Thames & Phelps, 2008: 391).

Over and above PCK, which ties together content knowledge and its pedagogy, also enunciates that content knowledge for its own sake is not sufficient for effective teaching. However, effective teaching does not only require going beyond the ability to compute correctly and understand the conceptual structure of mathematics, but being able to teach it to students (Maher & Muir, 2013: 73). It requires specialized content knowledge (SCK) (Hill, Ball, & Schilling, 2008: 377) that will make subject matter teachable. Hill *et al.* (2008: 377-378) claim that SCK helps mathematics teachers to accurately teach particular tasks by providing explanations for mathematical ideas, rules and procedures including understanding of ways to examine mathematics solutions. Moreover, Maher and Muir (2013: 77) cited Ma (1999) who compared USA and Chinese teachers' understanding of multiplication algorithm and found 61% of USA teachers and 8% of Chinese teachers were not able to provide authentic conceptual explanations for the procedure. These teachers lacked what Ma (1999) called profound understanding of fundamental mathematics (PUFM) which is defined as deep, vast, and thorough knowledge of concepts and their interconnections (Ma, 1999: 120).

Contrary to PUFM, Davis and Renert (2013, 247) argue that mathematics knowledge needed by teachers is not clear-cut and they framed it "in terms of a learnable participatory disposition within an evolving knowledge domain". They then developed a notion of 'profound understanding of emergent mathematics' from Ma's construct that is PUFM. Their view was that mathematics knowledge for teaching is a "sophisticated and largely inactive mix of familiarity with various realizations of mathematical concepts and awareness of the complex processes through which mathematics is produced" (Davis & Renert, 2013: 247). The notion of profound understanding of emergent mathematics emphasised the issue of knowledge production by those who are engaged in the process of teaching and learning. They argue, for example, that experts are unable to explain their choices of interpretations, examples and analogies, but simply adapt their actions appropriately to the encountered circumstances (Davis & Renert, 2013: 247). It is also reported that the research was undertaken with teachers rather than on teachers (Davis and Renert, 2013:247). In this case teachers are not regarded as subjects for research but they participated as co-researchers. On the other side, the narrative of emergent mathematics resonates with Hurrel's (2013: 55) view that characterises PCK as a contextualised practical knowledge of teaching and learning of a particular classroom setting. Mathematics teaching for learners whose dominant language is different from the language of instruction, for an example, could be a complex task for teachers as they have to adapt to the context (Jacob & McConney, 2013: 95).

The value of mathematics content knowledge in teaching mathematics cannot be overemphasised. Ball, Lubienski and Mewborn (2001: 445) present an analytical result of the work of Ball (1988) that studies teachers' knowledge of multiplication of multi-digit numbers, focusing on the performance of the algorithm to probe teachers' understanding of place value. Nineteen prospective elementary and secondary teachers were interviewed. They were asked how they would help their students if they presented their work in the following manner when multiplying 123 by 445:

$$\begin{array}{r} \mathbf{1\ 2\ 3} \\ \underline{\mathbf{4\ 4\ 5}} \\ \mathbf{6\ 15} \\ \mathbf{4\ 9\ 2} \\ \underline{\mathbf{7\ 3\ 8}} \\ \mathbf{1\ 8\ 4\ 5} \end{array}$$

It is common for people to know the shortcut (algorithm) without learning the conceptual foundation of the procedure. In demonstrating challenges regarding poor conceptual knowledge, teachers tried to explain the algorithm, using language such as "lining up correctly or moving the numbers over" (Ball et a, 2001: 445) without a clear explanation and understanding of the role of the place value in multiplication. Knowing mathematics content for teaching requires more than knowing its facts and concepts like place value but an understanding of its principles, structures and the established rules of what is legitimate to do and say in a subject (Ball, Hoover & Phelps, 2008: 391). It is believed that a lack of sound knowledge of mathematics affects the instructional practice. In extreme cases, if the teacher has nothing in terms of content knowledge, then the teacher has nothing to teach (McNamara, 1994: 231). Clearly, one cannot teach what one does not know (Maher & Muir, 2013: 74). Inter alia, teachers must have sound knowledge of what they wish to teach, including presentation skills and techniques to make such knowledge comprehensible to learners (McNamara, 1994: 230).

While the literature advances that teachers need to deeply understand the mathematical ideas that are central to the grade they want to teach (Kar, 2017: 7), such knowledge base also encompasses teachers' beliefs about mathematics including their orientation towards the subject matter and perceptions of mathematics learning and teaching (Barnes, 2007: 18; Hauk, Toney, Jackson, Nair & Tsay, 2014: 19). Daniels, a teacher that participated in the study conducted by Borko, Eisenhart, Brown, Underhill, Jones and Agard (1992: 216) nonetheless was unable to provide a correct representation for division of fractions or to explain why the invert-and-multiply algorithm works. Her belief about mathematics teaching was that teaching mathematics is about just following rules and procedures without asking why. This is what Skemp (1978:9) called 'instrumental understanding' where one just follows mathematical rules without reasoning. However, in terms of 'relational understanding' teachers cannot have just the procedural knowledge of the appropriate grade level mathematics, but also must have mastered underlying principles of mathematical ideas (Skemp, 1978:9). "They need to know how to represent and connect mathematical ideas so that students may comprehend them and appreciate the power, and diversity of these ideas" (Kar, 2017: 7).

Moreover, what the teachers do in their classrooms, what they view to be important and peripheral in mathematics teaching and learning is influenced by their beliefs of what mathematics teaching and learning is all about, as Ernest (1989: 253) articulates that "mathematics teachers' beliefs have a powerful impact on the practice of teaching". This involves what a teacher considers to be desirable goals of a mathematics programme, her own role in teaching, what the students should do in the classroom, legitimate mathematical procedures and desirable instructional approaches. The research community echoes each other on the debate that teachers' beliefs, "regardless of whether they are consciously or unconsciously held, play a significant, albeit subtle role in shaping the teachers' characteristic patterns of instructional behaviour" (Thompson, 1984:124). The beliefs held by teachers "influence their perception and judgements, which, in turn, determine their behaviour in the classroom" (Philippou and Christou, 1998: 190). It could be arguable whether to first change the teachers' beliefs in order to change the teaching practice or the other way around. Nonetheless, whichever way one might choose, it is clear that teachers are likely to reject any teaching practice which is in conflict with their beliefs. This suggests that changing the teaching practice without challenging teachers' beliefs is likely to be an unsuccessful endeavour.

2.3.1.2 Knowledge of mathematics pedagogy

Pedagogical knowledge is the second kind of PCK that goes beyond the knowledge of the subject matter *per se* (Shulman,1986: 9). It is also posited that teachers need to "realize that knowing a mathematical answer and knowing how to teach for student understanding are different" (Mecoli, 2013: 25). This particular form of content knowledge embodies the aspects of content most germane to its teachability (Shulman, 1986: 9). Shulman's construct, that is, PCK, revolves around the understanding and the transformation of subject matter knowledge for teaching purposes (Peng, 2013: 86). The purpose for this component of PCK is to demystify mathematics concepts such as $5^0 = 1$. For learners to understand this, teachers need more than explaining the exponential rules and procedures, but the most powerful analogies, illustrations, examples, explanations and demonstrations (Shulman, 1987: 9). In essence, teachers cannot help children learn things they themselves do not understand (Ball, 1991: 6). It is further argued that the pedagogical instruction that views mathematics as a set of meaningless arbitrary rules does not help student develop a deeper insight of mathematics (Skemp, 1978: 9). Contrary, knowledge of instructional strategies constitutes knowledge of strategies employed for teaching the subject, not only rules and procedures without explaining why (Ijeh & Onwu, 2013: 364).

Pedagogical knowledge involves knowledge of teaching methods and strategies used in the classroom, including understanding how students learn when planning an instruction (Aksu, Metin & Konyal, 2014: 1366). It is common knowledge in the teaching fraternity that pedagogical knowledge involves classroom management, preparation, assessment of learners' work and more. This knowledge base is what Shulman (1987:8) referred to as broad principles and strategies that appear to transcend subject matter. It is the ability to teach the mathematics content effectively that "goes beyond knowing a body of content that is common to all mathematicians" (Holmes, 2012: 65). Teachers need to have in-depth knowledge regarding the application of teaching and learning methods (Aksu *et al.*, 2014: 1366). However, the value of PCK is more evident in teaching mathematics when teachers show some shaky background in this regard. Ball (1991: 8) interviewed prospective teachers on how they would respond to a student when asked what is seven divided by zero. Laura's answer (a teacher in Ball's work) reveals that she understands division by zero in terms of a rule. "She thinks of it as something one must remember, not something one can reason about" (Ball, 1991: 7). Her answer starts with an argument that zero is such a stupid number. One would wonder what would be likely to happen in her classroom, when students have to learn to understand 'stupid' numbers, or stupid mathematics. Laura further admits that she does not know how to explain, but she would tell the students that, "that's just the way it is, it's just one of those rules, like in English - sometimes the C sounds like K - you just have to learn it" (Ball, 1991: 7).

Due to teachers' lack of mathematics PCK, students have to memorize mnemonic devices or rules in order to solve mathematics exercises. For example, "students are still told to invert and multiply to divide fractions and to use 'My Dear Aunt Sally' (MDAS) to remember to multiply and divide before adding and subtracting in an expression" (Ball, Lubienski & Mewborn, 2001: 435). Nonetheless, teachers need to have a wide repertoire of ways of representing mathematical ideas that students can engage with, not only arbitrary algorithms (Jacob & McConney, 2013: 98). Apparently, this kind of teaching (the use of arbitrary algorithms) may be adopted due to lack of PCK, and the implications of the above debate is that even though teachers might be able to solve mathematics problems, they may not automatically be an expert in helping learners comprehend mathematics' concepts.

In the above-mentioned situation learners undoubtedly are going to have some mathematics misconceptions. This study is not trying to argue that educators should not use algorithms when they teach; however, the emphasis is on the need to explain the mathematical concepts underpinning them. Teachers, therefore, are expected to understand the conceptual advantages, relative strengths and weaknesses of particular representations (Ijeh & Onwu, 2013: 36). In order for a teacher to make a well-considered and comprehensible lesson presentation, it is crucial to "know the learners' conceptions about a particular topic, and also the possible difficulties they will experience during the teaching and learning of the topic" (Ijeh & Onwu, 2013: 366).

K

2.3.1.3 Knowledge of students' understanding and learners' mathematics misconceptions

Knowledge of students' understanding entails teachers' knowledge about students' understanding of a particular subject including their preconceptions and misconceptions of particular topics (Peng, 2014: 88; Hauk, Toney, Jackson, Nair & Tsay, 2014: 26). Teachers should not only know the subject matter, but should be able to understand it from the learner's perspective (Moseley, 2000 in van der Sandt & Nieuwoudt, 2003: 199). This teachers' ability to understand learners' misconceptions in a mathematics class is a valuable component of PCK which separates a mathematician from a mathematics teacher. On the other hand, understanding and expressing oneself in the language of learners, using the concepts they use is viewed as 'cognitive congruence' (Hung, Jonassen & Liu, 2008: 494). The understanding of this component of PCK that includes knowledge of how learners think - particularly about specific mathematics content - helps the teacher to identify learners' misconceptions and correct them (van der Sandt, 2007: 344).

A deeper insight of learners' common misconceptions in a particular topic helps the teachers to think on their feet in terms of how to correct them and helps learners develop a deeper insight regarding the concept taught. The teacher's explanation for learners should go beyond a definition of accepted truth or falsehood, but he/she must be able to explain why (Shulman, 1986:9; Skemp, 1976: 21). Baker and Chick (2006: 62) raised the following question from a teacher in their study regarding a subtraction item: "You notice a student working on this subtraction problem: What would you do to help this student?"

(438- 172 = 346)

$$\boxed{\begin{array}{r} 438 \\ \underline{-172} \\ \underline{346} \end{array}}$$

This question was intended to establish the teacher's understanding of the learner's thinking in this regard without just giving the correct answer. The teacher should be in the learner's shoes (cognitive congruence) and understand what went wrong from the learner's point of view and then adapt teaching materials and pedagogy accordingly to help learners with particular misconceptions (Hung, Jonassen & Liu, 2008: 494; Peng, 2014: 88). On the other side, this kind of knowledge is viewed as 'anticipatory thinking' (Hauk *et al.*, 2014: 26) which is a way of thinking about how learners may engage with content, processes, and concepts. This includes the awareness of responsiveness to students. Anticipatory thinking could help teachers to select appropriate curriculum materials that would mitigate against learners' misconceptions.

2.3.1.4 Mathematics curriculum knowledge

The curriculum is constituted by programmes designed for the teaching of particular subjects in a particular class or level including instructional materials available in relation to those programmes and topics (Shulman, 1986: 10). Curriculum knowledge is what Shulman called 'tools of the trade' (1987: 8). It is a tool box with materials, where the teachers draw from when planning to teach a particular mathematics topic. This includes, but is not limited to, texts, teaching resources, teaching aids, prescribed examinations, tests and schemes to teach mathematics (Tunner-Bisset, 2001 cited in van der Sandt, 2007: 345). Moreover, PCK components, including curriculum knowledge, are inextricably linked to each other as the teacher needs to have to think and know about the learners, the content she wishes to teach and the pedagogical approach to use as she draws from the tool box. In short, curriculum knowledge is viewed as the ability of "making judgments about the mathematical quality of instructional materials and modifying as necessary" (Holmes, 2012: 67). Consequently, teachers with sound curriculum knowledge carefully select, adapt and manipulate curricula appropriately to meet the needs of the individual students (Holmes 2012: 64; Ball & Cohen, 1996: 6). Their choice of curriculum material, however, is influenced by their beliefs about what is important, and their ideas about students (Ball & Cohen, 1996: 6).

Shulman (1986: 10) divides curriculum knowledge into two categories, that is, lateral and vertical curriculum knowledge. The former is viewed as being familiar with curriculum material that is studied by students even in other subjects in the same grade and at the same time, while the latter involves understanding of topics that have been and will be taught in the subject area during the preceding and later years respectively (Shulman, 1986: 10). On the other side, Hauk *et al.* (2014: 26) argue that vertical knowledge includes the understanding of the connective relationship of pre-requisite topics and potential future topics. They further claimed that “curricular thinking is ways of thinking about (strategies, approaches to) mathematical topics, procedures, and concepts as well as the relationships among them” (Hauk *et al.*, 2014: 26). Curriculum knowledge is inextricably linked to other components of the PCK. It is not just the topics done in a particular grade, but clear linkage of learners, content and teaching strategies as the teacher selects what to be taught in a particular lesson from the ‘tool box’. Putting it differently, the researchers further introduced a phenomenon called ‘horizon knowledge’ regarding curriculum knowledge (Ball, Thames & Phelps, 2008: 403; Hurrel, 2013: 58). Horizon knowledge is viewed as the capacity to make connections across related mathematical topics over the curriculum span and “articulate how the mathematics you teach fits into the mathematics which comes later” (Hurrel, 2013: 58). This kind of knowledge helps the teacher in planning mathematics lessons with an understanding of what have been taught to learners and building a foundation of concepts that would come later.

2.3.2 Problem-based Learning (PBL)

Earlier it has been alluded that this study had assumed CER as theoretical framework that would guide the development of a strategy to enhance MPCK using a PBL approach. In this section PBL as an operational concept is traced by presenting its historical background, origin, definition and effectiveness at a multidisciplinary level and in mathematics education in particular with the focus on PCK.

PBL is seen as a learner-centred instructional method that utilizes real problems as primary pathway for learning and enhances students’ ability to analyse ill-structured problems to strive for a meaningful solution (Ramsay & Sorrel, 2006:2). Savery (2006: 9) defined it as “an instructional (and curricular) learner-centred approach that empowers learners to conduct research, integrate theory and practice, and apply knowledge and skills to develop a viable solution to a defined problem”. The above researchers resonate on the notion that PBL is an instructional approach. Their argument regarding PBL involved encouragement of active learning through use of scenarios to engage students in the learning process (Savin-Baden & Major, 2004: 1), and therefore is viewed as a pedagogical approach based on problems (Rui, Rong-Zheng, Hong-Yu, Jing, Xue-Hong & Chuan, 2015: 223).

Other PBL proponents, however, understood it as a general educational strategy or even as a philosophy rather than merely a teaching approach (Savin-Baden & Major, 2004: 5). In teacher development "PBL provides a constructivist referent" that is grounded in a constructivist learning theory (Goodnough, 2006:302; Mcconnell, Parker & Eberhardt, 2013: 221). A philosophical assumption of PBL is that learning occurs when we solve many problems that we face every day (Hung, Jonassen & Liu, 2008: 488). It represents a paradigm shift from traditional teaching and learning philosophy, which is more often lecture-based to an active, self-directed learning style and a constructivist approach supported by teacher scaffolding (James, Khaja & Sequeira, 2015: 316). In essence, it is a constructivist philosophy as learners actively construct their knowledge (Canturk-Gunhan, Bukova-Guzel & Ozgur, 2012: 148) in the process of trying to solve ill-structured contextual problems. The above argument makes PBL more than just a pedagogical approach but a philosophy of learning where a small group of learners work collaboratively to solve a real-life problem or a scenario. This shifts from the learning view that treats students as tabula rasa to a view that takes cognisance of students' prior knowledge. Seemingly, PBL works as a general educational philosophy in an educational institution where the educational objectives, teaching, learning, and assessment methods including the organizational culture are restructured as a whole in accordance with the value of PBL (Li, 2013: 179).

It is further argued that PBL comprises curricula organised around problems rather than disciplines (Savin-Baden & Major, 2004: 6). Barrows (1996: 8) also attests that "the curricular linchpin in PBL … is the collection of problems in any given course or curriculum with each problem designed to stimulate student learning in areas relevant to the curriculum". Hillman (2003: 20) cites Fogarty (1997) who defines PBL as a curriculum model designed around real-life problems that are ill structured and open ended. Moreover, "essential characteristics of PBL comprised curricula organisation around problems rather than disciplines, an integrated curriculum and an emphasis on cognitive skills" (Savin-Baden & Major, 2004: 6). Subjects and courses are designed as a collection of problems that are used as the motive and chief focus of the learning activity (Hillman, 2003: 2). It is then assumed that while students endeavour to solve these problems, they develop skills and concepts of the field of study and these problems or predicaments are grounded in a specialist model, rather than on formal wisdom from professionals (Hillman, 2003: 2). Students do not necessarily depend on the teacher to deliver the content, but focus on a self-directed learning in their quest for a solution using skills and prior knowledge acquired from multiple disciplines. In PBL, the curriculum is organised and designed around carefully crafted problematic situations adapted from real-world issues (Aldred, Ttimms & Meredith, 2007, 230). It is further argued that learners develop critical thinking, problem-solving, and collaborative skills "as they identify problems, formulate hypotheses, conduct data searches, perform experiments, formulate solutions, and determine the 'best fit' of solutions to the conditions of the problem" (IMSA, 2001 in Aldred, *et al.*, 2007: 230).

Despite the different views about PBL, there are common features that encapsulate PBL, namely learner centredness, an active process of knowledge construction, collaborative and self-directed learning. In line with these attributes, Barrow's (1996: 5) description of PBL highlights the following features, namely learner-centredness, learning takes place in small groups, teacher's role becomes a facilitator, stimulus for learning is organised around problems and problems are a vehicle for developing problem solving skills. On the other hand, Hmelo-Silver (2004: 237) presented the PBL cycle as depicted in Figure 2.1 below.

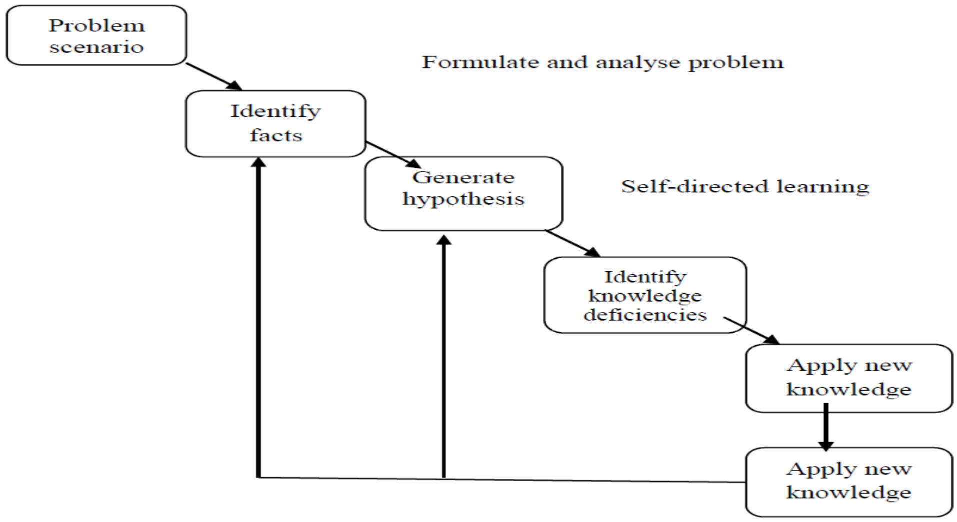


Figure 2.1: The problem-based learning cycle (Hmelo-Silver, 2004: 237)

In the PBL approach, learning begins with a problem to be solved as learners are first given a problem scenario as illustrated in Figure 2.1. Tutorial processes begin with the problem presentation and ends with student reflection (Hmelo-Silver, 2004: 242). Learners are expected to formulate and analyse the problem through identification of relevant facts or information in the scenario in order to generate hypotheses about possible solutions. In the process the teacher assists learners by posing questions (Barrows, 1986: 5) so that learners could identify the knowledge deficiencies, which encourage learners to search for new information. When learners are engaged at this stage of PBL, it is argued that that they are engaged in a self-directed learning after which they would test the newly researched information against the earlier generated hypothesis. The learning process takes place as learners work with problems while they develop skills and understanding of concepts (Sulaiman *et al.*, 2004: 58).

2.3.2.1 Historical background of PBL

PBL "was introduced in medical education as an alternative to traditional instruction, because graduates were found to have knowledge but lacked the required problem-solving skills to utilise this knowledge" (Batdi, 2014: 347). Barrows, who was a professor of neurology, proposed PBL as an alternative to traditional instruction that led to it being adopted for the first time by the McMaster University in Canada and subsequently it gained popularity in the USA and Europe (Rui *et al.*, 2015: 223). The literature also acknowledged the work of Don Woods in coining PBL while working with chemistry students in McMaster's University in Canada (Dirckinck-Holmfeld, 2009: 4). As this approach brought radical changes to the medical curriculum it implied that learning should focus on the problem, which was the patient and her complaints (Dirckinck-Holmfeld, 2009: 4). It was assumed that, while learners were engaged in analysing the problem, which would be a patient problem in the case of medical students, they would formulate questions, learning goals and learn in the process of solving the problem at hand.

Other universities inspired by the success of the McMaster medical curriculum followed PBL, for example the establishment of Aalborg University as centre for PBL in Denmark (Dirckinck-Holmfeld, 2009: 4). The adoption of PBL had expanded into elementary schools, middle schools, high schools, universities, and professional schools (Savery, 2006: 11). It is reported that Illinois Mathematics and Science Academy (IMSA) was leading the way; as a result, "middle and elementary schools soon joined the ranks of those implementing PBL" (Inman, 2011: 44-45). Moreover, PBL has gained prominence in a wide variety of disciplines including mathematics in particular (Erickson, 1999: 518). The literature further claimed that PBL had spread to numerous other fields of education although it has not been used extensively thus far in teacher education (Schmude, Serow & Tobias, 2011: 678). Based on the claimed success of PBL, this study relatively assumed that Grade 9 mathematics teachers' PCK could be also enhanced by using PBL.

2.3.2.2 PBL characteristics and conditions of implementation

PBL as an explicit approach to learning (Goodnough, 2006: 303) is in contrast with the notion where concepts are first presented in the lecture format, then followed by problems (Sulaiman, Atan, Idrus & Dzakiria, 2004: 58). As an alternative PBL rejected the learning of content and skills in a hierarchical list of topics (Hung, 2013: 31). Instead it advocated active learning, encouraged mutual exploration and academic debate (Rui *et al.*, 2015: 223). According to Sulaiman *et al.* (2004: 58) "the problem thus served as the organising centre and the stimulus for learning and represented the vehicle that developed students' creative and high-order thinking skills". From this approach the concepts, procedures and skills would be learned as learners try to untangle the problem. According to Sulaiman *et al.* (2004: 58), problems and scenarios mirrored real-world issues. Over and above, Barrows' (1996: 5) description of PBL tabled the following list of PBL characteristics, that is, learner-centeredness, teacher's role becomes a facilitator, and learning takes place in small groups.

2.3.2.2.1 Learner-centredness

Contrary to the learning of content and skills in a hierarchical list of topics (Hung, 2013: 31) the learners in PBL are presented with a problem under the guidance of facilitator. Learners must take responsibility for their own learning, by identifying what they need to know to better understand and manage the problem on which they are working (Barrows, 1986: 5). This includes, but is not limited to determining where they will get that information, for example, it may be from books, journals, faculty, on-line information resources, and so forth (Barrows, 1986*:* 5). The skills, concepts, procedures and abstraction are developed as learners untangle the given problem scenario. In essence, PBL in this regard is about what the learners can do as it orientates learners towards meaning construction over fact collection (Rhem, 1998: 1). Students within a PBL context direct the learning process and have a right to design learning objectives, select learning materials, and choose learning activities that are coined as high learning autonomy (Li & Du, 2015: 20).

PBL as a learner-centred approach "empowers learners to conduct research, integrate theory and practice, and apply knowledge and skills to develop a viable solution to a defined problem" (Savery 2006: 12). In the process and on completion of each problem, learners as engaged problem solvers, reflect on abstract knowledge that they have gained (Tamabara, 2015: 83). The knowledge gained from this experience or at the completion of the problem will then be applied in a new but similar situation. In general, learner-centred learning allows more learning autonomy to learners (Li & Du, 2015:18-19). However, the students' autonomy has an impact of the role of an educator in a PBL environment. It well is put by Li and Du (2015: 21) that:

> *"However, Teachers' attitudes towards student learning autonomy are highly influenced by their perceptions of their role. A teacher who positions himself as a traditional instructor tends to reduce student*

learning autonomy, while a teacher who sees himself as a facilitator is more willing to grant students more freedom to learn on their own".

2.3.2.2.2 Teacher's role becomes a facilitator

As it has been discussed earlier that PBL is a learner-centred approach (Li & Du, 2015:18), its posture creates an irreconcilable tension between the learners' autonomy and the educators' traditional role of imparting knowledge. To reduce this tension the teacher's role changes from traditional instructors to facilitators. "Within the context of PBL, teachers are not instructors; rather, they are expected to become facilitators to offer a supportive learning atmosphere and scaffold students' learning process" (Li & Du, 2015: 20). In a PBL environment, the teacher is viewed as a master learner, who is able to model good learning and thinking strategies, rather than a master of the content itself (Tambara, 2015: 100). According to Hmelo-Silver (2004: 245), the facilitator's role is to move the learners through the various stages of PBL and to monitor the group process. While on the other hand, it is theorized that the facilitator role "is better understood in terms of metacognitive communication" (Barrows, 1996). In the process the facilitator asks probing questions that they should be asking themselves to better understand the problem.

Moreover, the facilitator takes cognisance of Vygotsky's (1978:86) zone of proximal development (ZPD) when developing the scaffolds for students that are unfamiliar with PBL approach. ZPD is "the difference between what a learner can do without help and what a learner can do with help" (Vygotsky, 1978: 86). As articulated by Hmelo-Silver (2007: 100), scaffolding is a key element of cognitive apprenticeship, whereby students become increasingly better problem-solvers in a PBL environment. In line with this notion, learners are presented with opportunities to engage in complex tasks that would otherwise (without the 'scaffolds') be beyond their current abilities. However, it is emphasised that the facilitators' scaffolding role needs to progressively fade as the learners become more experienced with PBL (Hmelo-Silver, 2004: 244). At this stage the facilitator is expected to abandon direct instructions for learners to "assume greater responsibility for their own learning than they would otherwise do" (Tambara, 2015: 101). The facilitator needs to adopt the role of a resource providing aid, and a group mentor in a PBL context. The PBL teacher raises propping questions that would spark a debate and "inspire the learners to be involved in discussion, instead of directing learners in how to solve problems" (Tamabara, 2015: 102).

2.3.2.2.3 Learning takes place in small groups

PBL groups are made up of five to eight or nine students (Barrows, 1996: 5). The group members should be altered at the end of each curricular unit to form new groups in order to give them practice in working with a variety of people (Barrows, 1996: 5). Researchers claim that "PBL places students in small collaborative groups as a means of confronting them with alternative views of prior knowledge as well as with different problem-solving methods" (O'Shea, Verzat, Raucent, Ducarme, Bouvy & Herman, 2013: 96). Learners in small dialogue groups choose issues of mutual interest from a selection of material provided and then identify the underlying problem it evokes (Murray-Harvey, Pourshafie & Reyes, 2013: 115). This process allows learners to be able to take risks trying new tactics of solving the problem at hand and raise questions with their peers without fear of being judged. This arrangement is contrary to a situation where the mathematics teacher is at the centre in determining the correct or wrong answer or solution. The following summary presented by Armitage (2013: 9) depicts the safe learning environment in small groups:

> *"First, it invites students to dialogue in an open, safe environment with each other, an important aspect at the beginning of a programme of study. Second, it shows students there is 'no right answer', but rather a need to justify themselves in the gaze of their peers. This also provides an opportunity for students to become reflective and critical thinkers and illustrates that the ownership of opinions and knowledge is not solely the 'gift of the teacher' or of textbooks".*

On the other hand, real-world problems also act as stimulus that drives student learning and are a defining feature of PBL that takes place in small collaborative groups (Murray-Harvey *et al.*, 2013: 114). A specific principle that underpins how PBL works is collaborative learning within the small groups (Murray-Harvey *et al.*, 2013: 115). It is not disputed that group dynamics in terms of PBL implementation are central components to the creation of knowledge, however, feedback and reflection on the learning process are essential aspects of PBL (Armitage, 2013: 1). After completion of each problem, it is imperative that the new skills, concepts and procedures learned from the small groups should be consolidated and crystallised through a debriefing process.

2.3.3 PBL towards mathematics PCK

PBL has been discussed earlier particularly with regard to medical schools where it originated. This subsection is focused on PBL in relation to mathematics PCK. The debate about the spread of PBL to other disciplines has been advanced. The literature has supported the argument that PBL has shown success as compared to traditional methods of teaching, for example, the study conducted by Rui *et al.* (2015: 226-227) indicated the superiority of PBL over traditional methods, and as a result PBL students displayed professional competence and fared better in their professional role after completing PBL training. PBL also showed tremendous success in mathematics teaching. The work of Christine, Trinter, Tonya, Moon and Catherine (2015: 45) found evidence that mathematics learners exposed to PBL curricula showed characteristics of mathematical promise. The findings of Christine *et al.* (2015; 45-46) also suggest that students belonging to traditionally underrepresented groups (such as low socio-economic status) in the area of mathematics may demonstrate characteristics of mathematical promise when given appropriate learning opportunities.

"Mathematically promising students are defined as those who have the potential to become the leaders and problem solvers of the future" (Sheffield, 2003: 3). It is further argued that learners that are mathematically promising or talented often display a mathematical frame of mind; they are able to think logically and construct generalizations, and they exhibit mathematical creativity, curiosity, and perseverance (Sheffield, 2003: 4). The debate that is advanced by PBL proponents is that all mathematics learners that are engaged in the PBL approach display the characteristics of 'mathematically promising students', irrespective of their background (Christine *et al.*, 2015: 45-46). We want to argue that student teachers or in-service mathematics teachers also are learners in their own right when they are taught mathematics concepts. As earlier surmised, mathematics content knowledge, which includes concepts, is but one component of PCK. The implication of Sheffield's study (2003: 3) is that the teachers' content knowledge (which is a PCK component) could be enhanced when educators are engaged in the PBL approach when they are developed.

Some studies had a particular focus on using PBL to develop teachers' MPCK. The study conducted by Schmude *et al.* (2011: 682) focused on a number of PCK components such as student knowledge and pedagogical knowledge. Apparently, student teachers in Schmude's study valued "the actual goals of the scenarios, such as analysing the mathematical work of the student". They engaged in exploring strategies and ideas that could help develop the students' understanding of mathematics (Schmude *et al.*, 2011: 682). In essence, the MPCK of the teachers in the study mentioned (Schmude *et al.*) was enhanced, particularly their understanding of learners' conceptions and misconceptions, while they also explored some pedagogical strategies to help learners improve their comprehension. For example, in the study conducted by Schmude *et al.* (2011: 682) this what the teacher had to say as she was sharing her experience after having been part of a course that used the PBL approach:

> *"I valued how to improve students' learning by being able to recognise where the students are having difficulties and as a teacher, what steps to take to help the students succeed. By using real-life problems and seeing them occur, it makes it much easier to understand and learn how to fix the problems rather than just being taught about different approaches".*

Generally, teachers' MPCK demonstrate some improvement after they have been engaged in teacher development programmes using PBL, although in some cases, such as the study that was conducted by Martin *et al.* (2013:8), the results did not show any statistical significance; nonetheless, there were promising indicators that applying the PBL instructional method in a mathematics education course appeared to increase MPCK. A few studies were conducted regarding the enhancement of MPCK by using PBL, particularly in rural schools, but it seems that there is little research that has explored how the adoption of PBL impacts the development of academic staff's pedagogical knowledge (Goodnough, 2006: 303). "In a few studies focusing primarily on teachers' use of PBL in their own classrooms, teachers reported changes in their enthusiasm for teaching, critical thinking skills, and classroom practices" (Weizman, Covitt, Koehler, Lundeberg & Oslund, 2008: 31-32).

Consequently, this study focuses on formulating components of the strategy to respond to challenges facing Grade 9 mathematics teachers regarding PCK using PBL.

Nonetheless, there is a reported success in the implementation of PBL in mathematics teaching as the research indicated the increase of problem-solving, decision-making, modelling, and reasoning-process skills for learners who learn from teachers using this approach (Erickson, 1999: 520). This study intends to expand the research work to cover the use of PBL to enhance MPCK. Moreover, studies such as those of Brenner *et al.* (1997), Kieran (1993) and O'Callaghan (1998) cited in Erickson (1999: 520) claimed that in classrooms where PBL was used as a teaching approach, it enhanced conceptual understanding of specific mathematics content such as multiple representations for learning functions and increased levels of solving geometry problems. Although mathematics conceptual understanding is regarded as a PCK component, the study conducted by Erickson was not focused on enhancing PCK *per se*; for example, at times it compared the performances of students in classes using a PBL curriculum with students in traditional classes and found that students using the PBL outperformed students using traditional approaches. Drawing from this narrative, it is assumed that using PBL could also enhance MPCK.

2.4 RELATED LITERATURE

Earlier it has been declared that the aim of this study was to design a strategy to enhance the MPCK of teachers in Grade 9 using PBL approach. In order to achieve the aim of this study the related literature is traced in terms of good practice regarding MPCK and PBL in mathematics teaching across the globe. Secondly, this section relates literature reviewed to gain an understanding of challenges, to anticipate threats and to find ways to circumvent them. Finally, conditions favouring the implementation of the strategy to enhance MPCK of teachers, and indicators of success in the implementation are drawn from good practice to guide empirical evidence regarding the success of the strategy. Specifically, this section starts by discussing literature regarding challenges facing mathematics teachers.

Spaul (2013:17) paints a very bleak picture, which displays the incompetence of South African Grade nine learners that were compared with Grade eight learners from 21 other middle-income countries in mathematics. Grade nine mathematics learners were two years' worth of learning behind the average Grade eight pupil (Spaul, 2013:17). Linking learner performance to quality of teaching, the literature also claims that poor subject knowledge, poor mathematics teaching and learning are serious problems in South African education (Diko & Feza, 2014: 1457). In view of this poor state of affairs, the study specifically identified challenges regarding the following constructs: the need for a coordinated team to enhance MPCK using PBL, knowledge of learners, curriculum knowledge, content knowledge, and pedagogical knowledge. Other than the need for a coordinated team to enhance MPCK using PBL, the components of PCK are subdivided into subtitles that emerged in an attempt to unearth the broad construct of MPCK.

2.4.1 The need for a coordinated team to enhance MPCK using PBL

In this subsection, the focus is on the experiences regarding the effectiveness of coordinated teams in enhancing MPCK using PBL. The related literature in this regard is traced from the South African (SA) context, Southern African Development Community (SADC) context, African context and International context in order to place our debate within the scholarly discourse.

2.4.1.1 The need for a coordinated team to enhance MPCK using PBL in the SA context

Mahlomaholo (2013: 67) shed light on SA higher education policies, arguing that these policies seem to embrace the principles of PBL, although "there was no explicit intention of applying the PBL approach *per se* in their formulation". Critical cross-field outcomes stipulate that institutions for learning should produce learners who can identify and solve problems using critical thinking while "working effectively with others as member of a team, group, organisation, community" (Bender; Daniels; Lazarus; Naude & Sattar, 2006: 40). Moreover, the curriculum and assessment policy statement (CAPS) also aims at producing individuals that can work effectively with others as members of a team (DBE, 2011: 5). In the same vein policy directives assert that teachers as learning mediators should use problem-based tasks as teaching strategy and encourage group work (Brunton, 2003: A-49). While practitioners work with others in team-teaching create a democratic classroom with an atmosphere which is sensitive to culture, race and gender differences, as well as to disabilities (Brunton, 2003: A-52). Personnel Administration Measures (PAM) also encourage co-operation and collaboration of teachers in order to maintain good teaching standards (DBE: 2016: A-19).

In emphasising the collaborative work of a team, the Aalborg model of PBL includes group work and collaboration (Askehave *et al.*, 2015: 3). “Student groups also engage in close cooperation with their supervisor(s) and with external partners, e.g. businesses or other project groups” (Askehave *et al.*, 2015: 5). Evidently, the emphasis in PBL is on peer learning - members of the team have to collaborate with team mates to design solutions to the problems (Han & Teng, 2005: 3). As team members share knowledge they also “organize for themselves the process of collaborative learning” (Kolmos, Graaff & Du, 2009: 11-12). The research conducted by Han and Teng (2005: 3) illustrated that 75% of learners reported that working in a team had helped them in mathematics and that they had been able to contribute their ideas in their teams. According to Kolmos *et al.* (2009: 11) “[t]he team-learning aspect underpins the learning process as a social act where learning takes place through dialogue and communication”. In Aalborg, the culture of collaboration is maintained through learners’ dialogues that are characterized by mutual respect (Barge, 2010: 15). Seemingly, team work is the cornerstone for PBL, hence the research argues that for the team “to reach the project goal, the members of a team have to learn to co-operate effectively” (Graaff & Kolmos, 2003: 659). As we visited Aalborg University regarding our research (see Appendix 2), we learnt that all presentations predominantly emphasized group work, collaboration and team-based learning as the bedrock for PBL success (Dahms 2017: 16 & Kolmos, 2017: 2).

Research also suggests that a coordinated team needs many people to collaboratively plan to advance the learners’ educational interests (Qhosola, 2016: 57). Collaborative teaming is described as “an ongoing process whereby educators with different areas of expertise work together voluntarily to create solutions to problems that are impeding students’ success” (Knackendoffel, 2007: 1). Studies have shown that teachers’ cooperation with colleagues, reflection on educational practice and “provision of a supportive working environment that encourages collaboration may benefit teachers’

PCK development” (Evens, Elen, & Depaepe, 2015 :2). In support of the notion of a team, Mosia (2016: 61) demonstrated the devastating consequences of working in silos, whereby mathematics teachers were denied the opportunity to share skills and knowledge, which subsequently would have enhanced their MPCK. Jita and Mokhele (2014: 1) refer to collaborative structures for teacher learning as clusters and their findings reveal that “clusters seem to enhance teachers’ content knowledge and pedagogical content knowledge”. In essence team work gives teachers a platform to share problems and consequently get emancipated in their teaching practice.

A study that was conducted by Mosia (2016; 115) found that “[t]he absence of a team denied the teachers the opportunity to have a common or shared vision”. This non-existence of a coordinated team apparently still occurs despite policy imperatives, the success of PBL and assertions in literature that suggest that team work empowers teachers in handling classroom challenges. Mosia (2016: 115) reports that mathematics teachers who did not collaboratively teach Euclidean geometry failed to realize knowledge of teaching as a socially constructed endeavour. The non-existence of coordinated teams in SA schools is reported by Qhosola (2016: 138) as she argued that “had two teachers worked collaboratively, the performance of both classes could have been better”. The two teachers in Qholosa’s (2016:138) study were teaching the same subject, in the same grade but in different class groups, but failed to collaboratively work together while teaching at the same school. In line with Qhosola’s (2016: 145) finding that teachers worked in silos, she found that they were not aware of the power of collective capacity that could enhance their MPCK.

The literature shed some light on why there is a disjuncture between the envisaged policy imperatives and teaching practices. It is believed that teams are not viewed as important under the positivist stance of knowledge production because teachers are seen to be knowing it all (Mosia, 2016: 63). The soloist view of teaching suggests that learning mathematics is an individualistic activity and that interacting with others is not a necessary feature of teaching and learning (Ranamena, 2006: 14). Resulting from the rigid inspections during the apartheid era teachers rejected to be observed in their classrooms, instead they preferred to struggle in isolation “rather than open the door to support and collaboration” (Jita & Mokhele, 2014: 11-12). Furthermore, collaborative teams could not manage to secure time to meet regularly (Ono & Ferreira, 2010: 70). It is further argued that the “National Department of Education barred all workshops during the school term because of poor matriculation results" (Ono & Ferreira, 2010: 67). Apparently, any collaborative team work intended to enhance PCK was put in abeyance while the focus shifted towards improvement of matric results.

2.4.1.2 The need for a coordinated team to enhance MPCK using PBL in Namibia

In this subsection the inquiry focuses on Namibia, one of the countries that belong to the SADC region. The Ministry of Education (MoE) in Namibia formally adopted the system of school clusters to improve the management of education, and for professional support to teachers (Mendelsohn & Ward, 2007: 19). The National Subject Policy Guide for mathematics stipulates that:

> *The purpose of cluster subject group meetings is to improve efficiency, build capacity and empower teachers. Attending and participating in cluster subject activities can play a positive role in collaborative development and improving quality teaching and learning (MoE, 2008: 13).*

The purpose of clusters includes but not limited to improving teaching and learning, enhancing the professional performance of teachers and principals, and reducing the isolation of schools and teachers through sharing among schools (Mendelsohn & Ward, 2007: 19-20). It is posited that "[c]lusters provide a framework for collaboration between schools and teachers" (Mendelsohn & Ward, 2007: 19). Over and the above it has been assumed that clustering would improve teaching "through sharing resources, experiences, and expertise among teachers" (Pomuti & Weber, 2012:1 & Mosia, 2016: 1151). Besides their focus on management, these clusters provide a platform for teachers to learn "from each other's experience in dealing with … problems" (Mendelsohn & Ward, 2007: 16). Notwithstanding the none-overtly pronouncement of PBL in Namibian clusters, however, their features seem to have taken a PBL posture, as the research found that the basic premise of PBL is that learning starts from dealing with problems arising from professional practice (Fatokun & Fatokun, 2013: 664).

Notwithstanding their policy directives, many Namibian schools are not immune from the South African praxis of teachers working in silos when teaching mathematics

(Mendelsohn & Ward, 2007: 8). It appeared that most teachers experienced professional isolation due to the small size of schools where a teacher would be the only one teaching a particular subject (Mendelsohn & Ward, 2007: 8). “For instance, 83% of schools that offer Grade 7 mathematics have only one Grade 7 mathematics teacher” (Mendelsohn & Ward, 2007: 8). The professional isolation is most prevalent in schools that “are located in rural areas where they are geographically isolated from regular support services” (Mendelsohn & Ward, 2007: 8). Evidently, the conditions, such as being the only teacher for mathematics and geographical isolation, distance between schools, bad road conditions and the use of personal money to attend the cluster meetings (Mendelsohn & Ward, 2007: 10) militated against the existence of collaborative teams. The literature also reported that teachers did not like to attend cluster meetings because they did not learn much (Pomuti & Weber, 2012: 4). This was due to the authoritarian control that dominated the teaching practice and as a result teachers only “implemented the reform as a way of conforming to the orders from the head office” (Pomuti & Weber, 2012: 4).

2.4.1.3 The need for a coordinated team to enhance MPCK using PBL in Nigeria

In this subsection, the focus is on Nigeria to understand the good practice in terms of collaborative teams as a support base to enhance MPCK. Our search shows no evidence of policy position regarding clusters or collaborative mathematics teaching. However, once-off workshops that take five days as a cascade model for teacher development seem to be prominent in Nigeria (Nwagbara and Edet, 2013: 2). The policy components of the Universal Basic Education scheme show a significant importance attached to teachers’ in-service capacity development (Ogunrin, 2011:744). The National Policy on Education (NPE) declares that lifelong education will be the basis for the nation’s education policies to produce intellectually grounded and professionally committed teachers through in-service programmes offered in the form of seminars, workshops and conferences (Osuji, 2009: 298-299). From the above-mentioned policy directives, it seems that clustering or collaborative teaching was not central in enhancing MPCK.

The literature suggests innovations in the in-service programmes which, among others, include a peer tutoring and cluster-led teacher approach (Mkpa 2000, cited in Osuji, 2009: 300). The former manifests itself when colleagues seek professional assistance and guidance on any aspect of the discipline which is defective. “In this way, the area of professional competence of each colleague benefits the other, eventually leading to each member of staff growing academically and professionally” (Osuji, 2009: 300). While in the latter, teachers from a group of five or less schools in a local area come together to share experiences in certain subjects. Clusters enhance mutual assistance among teachers, and emancipation in terms of content and skills, without necessarily going to any training institution (Osuji, 2009: 300). It is further posited that a cluster is a group of neighbouring schools grouped around a larger nucleus school (Nwagbara, 2014:13). As teachers assemble in clusters “they become creative in problem-solving, effective utilization of available resources, through lesson study, preparation of lesson plans, production of teaching materials, classroom management and other pedagogical skills (Nwagbara, 2014:13).

The main purposes of clusters are to improve teaching, which in our case is MPCK, by sharing experience and expertise among teachers, and to harness resources, especially skills, from several small schools through effective networks and collaboration among the teachers (Nwagbara, 2014;13). The cluster programme provides a bottom-up approach and is needs driven. “The on-the-job needs of teachers are identified, the teachers are trained on-the-job, work cooperatively and collaboratively to share ideas, build a local resource network and take the lead in all teaching and learning activities” (Nwagbara, 2014;13). It is further proclaimed that clusters reduce stress for novice teachers and reduce “costs, time and risks involved in long travelling distances to attend workshops and seek support and focus on specific challenges unique to the local environment” (Nwagbara, 2014;14).

Contrary to research findings that showed interest in collaborative teams through clusters, it seems that Nigerian teacher development programmes do not embrace clusters. Research argues that once-off workshops of five days per year seem to be the commonest model for teacher development (Nwagbara, 2014:13). A review of research on basic education provision in Nigeria exhibited a mismatch between problem conceptualisation and training activities, hence, in-service practices fail to improve teachers’ instructional practices in the classroom (Akyeampong, Sabates, Hunt & Anthony, 2009: 50). The literature also proclaims that teachers complain about workshops being too fragmented and unrelated to teaching practices (Nwagbara and Edet, 2013: 2). The research only recommended that in-service mathematics re-training should be "strengthened by grouping of teachers working together to learn, share experiences, … rather than large scale workshops that have little impact on the teachers' practice" (Nwagbara and Edet, 2013: 6). Seemingly, collaborative teaching or clusters were not used as a strategy to emancipate teachers in Nigerian schools.

It appears that a research gap exists regarding coordinated teams to enhance MPCK in Nigerian schools. This study has earlier alluded that is seems that there is neither a policy position nor well researched praxis on collaborative mathematics teaching. The literature advances socio-economic reasons that seem to be an impediment towards in-service activities (Nwagbara and Edet, 2013: 7), albeit not collaborative teams *per se*. "It has to be noted that the period for retraining cuts into their vacations and weekends and often takes time away from a needed second income or interferes with family life" (Nwagbara and Edet, 2013: 7). Consequently, the literature proposed that mathematics teachers should be handsomely paid for participating in in-service programmes (Nwagbara and Edet, 2013: 7).

2.4.1.4 The need for a coordinated team to enhance MPCK using PBL in the United States of America (USA)

In this subsection, the study focuses on the USA to understand the good practice regarding coordinated team work. In terms of collaborative mathematics teaching, the literature suggests that little is known about USA policy stunts and policy context of public schools in influencing how teachers' social networks form, function, or change over time (Coburn, Mata & Choi, 2013: 312). Nonetheless, it appears that districts encourage teams of teachers at the school site to work together to make changes in their practice (Coburn, Mata & Choi, 2013: 311).

Despite the fuzzy policy position in terms of collaborative teaching, the literature refers to proponents of collaborative teaching, referred to as professional learning communities (PLCs) (Dufour, 2004b: 8). "The powerful collaboration that characterizes professional learning communities is a systematic process in which teachers work together to analyse and improve their classroom practice" (Dufour, 2004b: 8).

According PLCs proponents, teachers build their capacity when they collaboratively "help each other develop and implement strategies to improve current levels of student learning" (DuFour 2004a: 63). Within the PLCs model, teachers work collaboratively as a team, discuss and reflect on the organizational structure and the "organizational structure becomes a primary agent directly mediating teacher professional growth" (Graham, 2007:2). Graham (2007:1) investigated the relationship between PLCs activities and teacher improvement in a first-year middle school. The findings of Graham's (2007: 13) study suggest that,

> *"as individual teachers grew to trust and respect each other, and as conversations increasingly addressed substantive issues of teaching and learning, teachers were able to 'see through each other's eyes' such that each member of the team was able to benefit from the collective wisdom of all members".*

In essence, the opinions of participants in Graham's research also attest that PLCs enhances PCK as they argue that it helps knowing that that there is somebody down the hall that one can engage with if one is wondering how to approach something instructionally (Graham,2007: 9).

Despite the availability of evidence that exhibited positive effects of collaborative teaming in teaching mathematics (Murata, 2002: 67 & Jang, 2006: 178), the USA teachers strongly believe in individual teaching styles (Stigler, Thompson & Ji, 2013: 230). In the research conducted by Stigler *et al.* (2013: 230) teachers expostulated the establishment of collaborative teams and maintained that "developing a common lesson plan sounds nearly impossible". The teachers' assertion as they contended against working collaboratively with others was that they kept the lesson details in their heads without being documented nor shared with other fellow teachers (Stigler *et al.*, 2013: 230). The research also highlighted that 30-50% of teachers left the teaching profession within the first three to five years of service in the USA (Ball & Cohen, 2014: 318). Notwithstanding the regular complaint about lack of professional efficacy as one of the reasons for their departure, it became clear that new teachers often were "isolated from their more experienced colleagues" (Ball & Cohen, 2014: 318). As much as many teachers strived to help learners develop mathematical skills, they did so largely on their own.

2.4.1.5 The impact of non-existence of a coordinated team to enhance MPCK

In summary, the research reported a plethora of disadvantages when mathematics teachers work as individuals. Mosia (2016: 116) reported that “[i]t became evident that teachers did not even share their challenges, teaching methods, resources or teaching aids” due to non-existence of a coordinated team. According to Tsai (2004: 329):

> *“Teachers are challenged by the interplay between the reform vision of instruction and their own experience with more traditional pedagogy. Research suggests that teachers need to learn mathematics in a manner is consistent with the way we expect them to teach.”*

In a nut shell, mathematics teachers cannot be expected to teach mathematics in a collaborative way when they have not been exposed to collaborative teaching. Instead, they are likely to continue using the traditional approaches such as the banking concept. The banking concept is seen to hinder learners’ thinking “and allows for their passive reception of knowledge, which in turn may be an effort to disempower both teachers and students” (Tutak, Bondy & Adams, 2011: 67). It is articulated from the research findings above that any professional development without coordinated teams is unlikely to be effective.

2.4.1.6 The impact of a coordinated team to enhance MPCK

From the discussion above the need to establish coordinated teams as a platform to empower both the researchers and the co-researchers could no longer be ignored. As a researcher I ceased to be what Basov and Nenko (2011: viii) call a ‘lone wolf scholar’ with the understanding that when two agents or inter-subjectivities meet, they “grow and develop through mutual and reciprocal beneficiation” (Mahlomaholo, 2012, 293). Mahlomaholo (2012a: 293) further argued that “[b]oth of them individually and collectively ultimately possess more than their separate individual knowledges in this process”. The research suggests that the lack of teamwork results in ineffective pedagogical practices (Mosia, 2016: 63). In support of the need for a team, the literature enunciates that teachers find space to communicate, share and address issues, observe one another’s work and develop expertise in various aspects (Jita & Mokhele, 2014: 4) when there are teachers’ networks or clusters, or, coordinated teams in our case. They get rescued from a ‘guru’ mentality by sharing problems and learners' misconceptions which they encounter in their praxis. The absence of a coordinated team was not the only challenge in an attempt to enhance MPCK using PBL, but poor understanding of learners' misconceptions as discussed in the following section seemed to be a hindrance as well

2.4.2 The need to identify and follow up learners' misconceptions

This section focuses on the impact of understanding and follow-up on learners' misconceptions in enhancing MPCK using PBL. Teachers should know their learners' mathematics misconceptions in order to develop the pedagogical "conditions necessary to overcome and transform those initial conceptions" (Shulman 1986: 10). Ball (1997: 732) further argues that teachers must learn what their students know in order to effectively approach mathematical topics. It is further posited that "knowledge of students is as essential a resource for effective teaching as is knowledge of mathematics itself" (Ball, 1997: 732). Understanding learner's knowledge might guide teachers to scaffold learners' ideas and mediate the construction of new knowledge (Brodie, Lelliott & Davis, 2002: 557). Learners' misconceptions result from a naive concept of images that does not measure up to the concept definitions (Luneta & Makonye, 2010: 44). Common learners' misconceptions include, but are not limited to, "conjoining and premature closure, negatives and subtraction, multiplication and indices, the equality relationship and evaluating letters rather that accepting an open expression as a final answer" (Pournara, Hodgen, Sanders and Adler, 2016: 5). Errors such as conjoining persist to bedevil learners beyond Grade 9 as Pournara, Hodgen, Sanders and Adler (2016: 5) presented a table demonstrating the percentages of learners' responses showing conjoining errors. Accordingly, teachers should tailor the lessons plans and presentations in a way to address learners' difficulties in order to eliminate their misconceptions (Kılıç, 2011:18). In essence, teachers eliminate such misconceptions by probing questions (Kılıç, 2011:19) and acting in line with PBL.

TABLE 2: Percentage of responses showing conjoining error.

Item		Typical errors	% of all responses		
			Grade 9	Grade 10	Grade 11
1.2	Simplify $2a+5b$	$7ab$	53	50	42
1.4	Simplify $2a+5b+a$	$7ab$, $8ab$	25	18	14
3.2	Add 4 to $n+5$	$9n$	11	12	7
3.3	Add 4 to $3n$	$7n$	25	32	17
5.3	If $e+f=8$ $e+f+g=?$	$8g$	17	11	10

Figure 2.2: Conjoining error (Pournara et al., 2016: 5)

In this study we shall use misconceptions to encompass all learners' errors and misunderstanding of mathematics concepts. It is prudent for teachers to understand these learners' misconceptions in order to plan a pedagogical approach that would alleviate problematic patterns in mathematics concepts (Makgakga, 2014, 4). The literature philosophised that when teachers follow up on learners' misconceptions which they themselves cannot explain, they develop MPCK as a consequence (Herholdt & Sapire, 2014: 57). Over and above, the DoE (2011, cited in Sapire, Shalem, Wison-Thompson, & Paulsen, 2016: 1) provided the rationale that the introduction of Annual National Assesment (ANA) required teachers to use learner perfomance data diagnostically, a process that placed a new and complex cognitive demand on teachers' PCK. On the other hand, mathematical problems without readily available answers are central to the PBL approach as it embraces a radical shift from content coverage to problem engagement (Malan & Ndlove & Engelbrecht, 2014: 2).

The literature further postulates that probing focuses learners on what they have written or presented as mathematical answer by forcing them "to justify and make sense ot their utterings" (Makonya & Khanyile, 2015: 66). Probing as a strategy to follow up on learners' misconceptions involves discussion in which the learners are encouraged to reflect on their thinking while it is questioned (Makonya & Khanyile, 2015: 56). This process develops what Skemp (1976: 77) called "relational understanding", which demands learners to not only know what they do in mathematics but to also understand the reason behind their actions (Mdaka, 2014: 5). On the same wave length as Skemp's relational understanding, Mulugye (2016: 18) viewed conceptual understanding as “knowledge that is rich in relationships and it is a concept-oriented, relational approach”, which embraces knowing how and why. “In contrast, procedural understanding is a rule-oriented, instrumental approach. It is knowing how but not knowing why” (Mulungye, 2016: 18). In essence, probing learners’ thinking exhibits their conceptual understanding or misconception as they are not only required to show how but to also explain why they have acted in a particular way.

2.4.2.1 The need to identify and follow up on learners’ misconceptions in SA

This section explores the SA praxis in terms of how the identification and follow-up on learners’ misconceptions affect PCK enhancement using PBL. The PAM document maintains that the utilization of learners’ own experience should be considered and be utilized as a fundamental and valuable resource (DoE, 2016, A-18). According to Brunton (1996: A-5) teachers are mandated to recognise learners’ aptitudes, prior knowledge and experiences and encourage critical thinking. These policy prescripts enable teachers to identify learners’ zone of proximal development (ZPD) in order to meticulously appropriate instruction. Specifically, teachers as assessors should keep and interpret detailed diagnostic records of assessment (Brunton, 1996: A-49) to base learning on problems identified. Consequently, teachers draw from learners’ existing knowledge, skills and experience to thoroughly plan their lesson (Brunton, 1996: A-49). Figure 2.3 that follows, exhibits examples of common learners’ misconceptions provided by CAPS (DoE, 2010: 83).

Look out for the following **common misconceptions** where:

- learners multiply unlike bases and add the exponent.

Example:

$x^m \times y^n = (xy)^{m+n}$

- learners multiply like bases and add the exponents

Example:

$2^5 \times 2^7 = 4^{12}$ instead of the correct answer 2^{12}.

- learners forget, for example, that when squaring a binomial there is a middle term

Example:

$(x + y)^m = x^m + y^m$

- learners confuse adding the exponents and adding the terms

Example:

$x^m + x^n = x^{m+n}$ or x^{mn}

Figure 2.3: Common learners' misconceptions (DoE 2010: 83)

This indicates that teachers should focus attention on learners' misconceptions to appropriately plan their lessons (DoE, 2010: 83). PCK proponents suggest that understanding learners' mathematical misconceptions on topics taught empowers the teacher to anticipate learners' learning misconceptions and to be ready to give alternative models or explanations to demystify those misconceptions (Ma'rufi, Budayasa & Juniati, 2018: 2). Accordingly, the CAPS policy highlights learners' common misconceptions for teachers to understand learners' thinking as they plan and teach their lessons.

The other side of the coin is that PBL proponents also suggest that misconceptions are a necessary step in learning to apply new knowledge (Hmelo-Silver, 2004: 250). "By articulating incorrect knowledge, learners have the opportunity to revise their false beliefs when they are confronted with correct knowledge" (Hmelo-Silver, 2004: 250). PBL embodies a constructivist nature, hence knowledge is not viewed from an absolutist perspective, but actively constructed by learners based on their prior knowledge (Malan *et al.*, 2014: 2). In the SA context PBL seems to have the potential to put learners at the centre of the learning activity (Golightly & Muniz, 2013: 433).

Prior knowledge is an important part of cognition since cognition is influenced by learners' prior knowledge, background and environment (de Villiers, de Beer & Golightly, 2016: 506). In essence from a PBL perspective, understanding of learners' prior knowledge is fundamental to guide teachers not to present learners with trivial problems or extremely difficult problems, but appropriate problems just a little bit beyond their ZPD level.

The research findings also echo the same sentiments namely that understanding learners' thinking, whether presented in the form of errors or misconceptions, is fundamental towards designing effective lesson presentation and best practices for remediation (Herholdt & Sapire, 2014: 43-44; Gardee & Brodie, 2015: 2). It is also put on record that in SA there has been an invigoration focusing on identification and analysis of mathematics learners' misconceptions in recent years (Pournara, *et al.*, 2016: 2). The research conducted by Luneta and Makonye (2010:44) reported that learners' misconceptions emanated when learners tried to construct mathematical meanings using their prior knowledge. According to DBE (2013 in Herholdt & Sapire, 2014: 43), the ultimate aim of error analysis "is to improve learner achievement by focusing on remedial interventions targeting common errors and misconceptions evident in learners' responses to the national tests". It appeared that teachers need to have a good grasp of learners' level of mathematical understanding and mathematical content (Herholdt & Sapire 2014: 44) to span through learners' ZPD and learning trajectory. Literature also posits that when teachers develop interest to understand causes of learners' misconceptions "they may come to value learners' thinking and find ways to engage their current knowledge in order to create new knowledge" (Gardee & Brodie, 2015: 2).

Regardless of the existence of empirical evidence that amplifies the importance of understanding of learners' conceptions (Shulman, 1986: 9-10) SA teachers seem to be ignorant in this regard. Notwithstanding, the research postulates that a large number of SA teachers are unaware of their learners' mathematical misconceptions (Luneta & Makonye, 2010:36). Evidently, SA mathematics teachers seem not to understand the role of probing learners' misconceptions in teaching mathematics. Teachers in Mji and Makgato's (2006: 260) study exhibited obliviousness regarding the importance of probing learners' misconceptions. Learners that were involved in their study complained that their teachers worked too fast, were impatient with them and ended up shouting at learners and asking questions such as “How come you don’t understand such an easy sum?” (Mji & Makgato, 2006: 260). The anger and the frustration displayed by teachers in Mji and Makgato’s research suggest that teachers could not identify or probe or remediate learners’ misconceptions. The literature resonates the argument that SA teachers have a gap in terms of their learners’ mathematical knowledge (Adler, 2005:9). Apparently, teachers struggled to trace back mathematical ideas and their antecedents with their learners (Adler, 2005:9). Moreover, two case studies also revealed that teachers’ inability to appropriately interpret identified learners’ misconceptions resulted in inappropriate remediation of learners’ errors (Gardee and Brodie, 2015: 3).

2.4.2.2 The need to identify and follow up on learners’ misconceptions in Namibia

In this section the focus falls on Namibia to explore good practices in relation to the identification and remediation of learners’ mathematical misconceptions as articulated by legislative mandates and research findings. Shulman (1986: 9-10) long ago concluded that teachers needed to know mistakes that learners were likely to make when they encountered a particular mathematics topic. The National Institute for Educational Development (NIED) commissioned by the Minister of Education in Namibia put on record that learning should build, extend and challenge learners’ experiences and prior knowledge (MoE, 2014a: 5). This policy guideline mandates teachers to identify and correct learners’ misconceptions (MoE, 2014a: 54). The Learning Support Teachers’ Manual also emphasizes the centrality of understanding learners’ misconceptions (MoE, 2014b: 3). To understand learners’ thinking, “learning must start with finding out what the learners’ existing knowledge, skills and understanding of the topic are” (MoE, 2014b: 3). More activities should be built on and extend the learners’ knowledge (MoE, 2014b: 3). The National Promotion Policy Guide for junior and senior primary school phases makes provision for teachers to “clearly identify the learning difficulties and set out a plan of action (support programme) to remedy the learning difficulties” (MoE, 2015: 3).

In the same vein, the scholarly discourse affirms the above policy narrative that teachers should built on, extend and challenge learners’ prior knowledge and experience when they teach (Peters, 2016: 52). To be able to do this, teachers need to understand learners' mathematical understanding or misconceptions and how to connect learners' prior and future learning (Courtney-Clarke & Wessels, 2014: 3). It is also recognised that "learners' prior knowledge and skills, and construction of knowledge rather than passive participation of students" play a pivotal role in the learner-centred education philosophy (Kapenda, Torkildsen, Mtetwa & Julie, 2008: 3). In Namibia a Realistic Mathematics Education (RME) professional development case study starts by posing a realistic problem from the context invoking prior knowledge and experience (Peters, 2016: 260). A report on factors that cause poor mathematics performance in the National School Secondary Certificate (NSSC) assessment shows that learners are less likely to make common computational errors when their prior knowledge of procedures is based on concept understanding (Mateya, Utete & Ilukena, 2016: 160). In essence, teachers "need to first understand their students' prior Mathematics content knowledge" to develop an effective instructional approach (Akpo, 2012: 142).

Despite the profound proposal by Borasi (1987 in Nalube, 2014: 59) that views errors as a 'springboard for inquiry', it is observed that Namibian teachers fail to identify and analyse learners' misconceptions (Vatilifa, 2012: 121; Courtney-Clarke & Wessels, 2014, 2014: 2).

NV: Ok, so when you were teaching these fraction topics, what errors or confusions that you noticed learners. Tend to have when you were teaching them fractions or when they were learning fractions?

R3: My learners got confused, let us say you taught them and you did examples together, now you give them exercises, I remember when they were changing to equivalent fractions, they could not get it correct as they do not remember whether they multiply or add two functions together.

Figure 2.4: Teachers' inability to identify learners' misconceptions (Vatilifa, 2012: 121)

Evidently, the empirical evidence in Figure 2.4 above extrapolated from the interview between Vatilifa (2012: 121) and the focus group reveals that teachers are unable to identify learners' misconception regarding fractions. On the other hand, Hill, Ball and Schilling (2008: 390) logically reasoned that mathematics thinking behind learners' errors should probe prior knowledge focusing on how learners' calculations went astray. In contrast to Hill *et al.* (2008:390) the report that was presented by Courtney-Clarke & Wessels (2014: 2) suggests that Namibian primary schools are predominantly characterised by drill, practice and rote learning. Moreover, teachers that participated in the study that was conducted by Moru and Qhobela (2013:228), however, seemed to be unable to "identify the errors where students treated infinity as a number".

2.4.2.3 The need to identify and follow up on learners' misconceptions in Nigeria

In this section we focus on Nigerian experiences in relation to identification and remediation of learners' mathematics misconceptions from a legislative perspective and research findings. The study that was designed to identify students' right conceptions and misconceptions of six basic algebraic concepts in mathematics recommended that Federal and State Ministries of Education should train teachers on how to examine and identify their misconceptions (Idehen & Omoifo, 2016: 10). These recommendations included the organisation of seminars and workshops to list and discuss identified misconceptions in order to develop strategies to correct them (Idehen & Omoifo, 2016: 10). The rationale for using learners' misconception as instructional spring board could be attributed to Lee and Luft (2008 in Luneta, 2014: 75) who agitated that PCK does not only include knowledge of how learners learn specific content but "deep understanding of learners' misconceptions and errors associated with certain concepts, as well as remedial activities and enrichment tasks needed to challenge learners".

Furthermore, a report presented in the literature suggests that in Nigeria education researchers have tried to identify learners' mathematical misconceptions using misconceptions as pivotal in designing mathematics instruction (Zuya, 2014: 119; Sirajo, 2015: 44; Zuya & Kwalat, 2015: 103). Teachers' ability to create cognitive disturbance in helping learners recognize their misconceptions is viewed as a springboard in teaching and learning mathematics (Zuya, 2014:119). In affirming this narrative, the research theorization posits that teaching geometry should be done in a manner that minimizes learners' misconceptions (Zuya & Kwalat, 2015: 101).

Implicitly, mathematics teachers "should be able to identify and address such misconceptions when they arise" (Zuya & Kwalat, 2015:101). Accordingly, it is of vital importance to put to learners that misconceptions are not a sign of failure, instead they are a positive source for improvement (Sirajo, 2015: 44). Misconceptions, in our case referred to as good errors, according to Sirajo (2015: 44) reveal learners' metacognitive process and guide teachers in developing most appropriate prompts and teaching strategies.

Despite the existence of empirical evidence compelling teachers to identify and address learners' misconceptions when designing instruction, in Nigerian schools the opposite is true (Zuya, 2014: 121). Zuya's (2014: 121) inquiry reveals that teachers are unable to ask learners investigative questions. Instead, they ask instructional questions and according to Zuya (2014:121) these questions "cannot identify students' errors and misconceptions or assess students' thinking process". The study that investigated the adequacy of mathematics teachers in identifying learners' misconceptions and development of strategies to address them revealed that teachers could not identify learners' misconceptions with respect to angles in parallel lines (Zuya & Kwalat, 2015: 100). Even worse, these teachers also were unable to suggest specific ways that would help remove learners' misconceptions (Zuya & Kwalat, 2015: 112). On the other side of the coin, it appeared that teachers' beliefs about mathematics teachers militated against any efforts to identify and follow up on learners' misconceptions (Ladale, 2013: 4). In terms of the Nigerian teaching tradition, mathematics teaching is largely teacher-centred (Ladale, 2013: 4).

2.4.2.4 The need to identify and follow up on learners' misconceptions in the USA

In this subsection the focus is on the USA context and praxis on learners' misconceptions. The New Jersey Mathematics Curriculum Framework provides strategies for enhancing teachers' "ability to use non-threatening questions that elicit explanations and reveal misconceptions", which is pivotal to understand learners' thinking (Rosenstein, Caldwell & Crown, 1996:596). The curriculum framework also recognises the use of journals as a source of learners' problems that emerge from the classroom and learners' misconceptions (Rosenstein *et al.*, 1996: 596). It is also expected of teachers to “identify prerequisite skills for a given topic” (Florida Department of Education, 2013: 9). This process guides teachers’ prediction of common mathematics misconceptions on topics such as area and perimeter (Florida Department of Education, 2013:9). The California Department of Education (2015: 671) also emphasises the identification of learners’ misconceptions like over-generalisation and over-specializations in order to correct them. Teachers are urged to deeply probe learners’ written work to understand reasons why specific procedures were learned (California Department of Education, 2015: 678). “These discoveries will help teachers plan for and provide instruction to meet the needs of their students” (California Department of Education, 2015: 678).

The research suggestions also put forward that an understanding of learners’ thinking is a major component of Shulman’s PCK in mathematics teaching (An & Wu, 2012: 717). According to An and Wu (2012: 717) knowledge of learners’ thinking enables teachers to establish learners’ levels of understanding concepts, and identifying and correcting possible misconceptions. Knowledge of learners’ common misconceptions “is an essential element of tailoring your instruction to meet struggling learners’ math-specific needs” (Allsopp, Lovin & Ingen, 2018:8). The study that inquired about analysing misconceptions in grading homework revealed that through grading homework and following up on learners’ understanding, the effectiveness of participants’ teaching was improved (An & Wu, 2012: 746). In line with An and Wu’s findings (2012), teachers who have developed what Holmes, Miedema, Nieuwkoop and Haugen (2013: 28) refer to as the “highest depth of knowledge” level are able to identify and conceptually correct learners’ misconceptions. Seemingly, teachers’ ability to detect patterns of misconceptions may not be an intuitive process, as the research findings suggest that it can be acquired from the wisdom of practice and engagement of professional development.

A narrative presented by researchers that appears to be unanimously agreed upon, is that as part of PCK teachers should understand learners’ misconceptions (Shulman, 1986:10) and use them as springboards for inquiry to develop a plethora of instructional strategies (Borasi, 1994:2000). In contrast to classical research proposals as coined above, resent research findings report that USA teachers who participated in these inquiries still do not consider learners’ mathematics misconceptions as pivotal to their instructional praxis (Bitter & O’Day, 2010:2; An & Wu, 2012: 718). Instead of identifying misconceptions from learners' homework, they only "provide answers to students at the beginning of class for their self-correction or group correction" (An & Wu, 2012: 718). According to Homes *et al.* (2013:26) teachers had not been commonly exposed to how to detect the underlying misconceptions in learners' errors. The belief that learning mathematics depended on special ability (Rosenstein *et al.*, 1996: 11) seemed to be an impediment towards US teachers' considering misconceptions as an exhibition of learners' thinking. For example, a comparative study conducted by An (2004) in An & Wu, 2012: 721) revealed that only 7% of USA teachers as compared to 75% of Chinees teachers understood learners' misconceptions and appropriately planned their instruction accordingly. Over and above other issues, time constraints have been highlighted as a challenge as teachers struggle to cover all topics to comply with the pressures of the federal and state accountability systems (Bitter & O'Day, 2010:2). Apparently, these pressures force teachers to move to the next topic at the expense of considering new ways to demystify misconceptions for struggling learners (Bitter & O'Day, 2010:2).

2.4.2.5 The impact of failure to detect and follow up on learners' misconceptions

Shulman (1986: 9-10) had long advocated that teachers need to know mistakes that learners are likely to make when they encountered a particular mathematical topic. Notwithstanding Shulman's assertion, the manner in which teachers deal with misconceptions "can either enhance or limit learners' understanding of mathematics" (Gardee & Brodie, 2015: 2). The empirical evidence suggests that teachers who do not consider learners' misconceptions "tend to emphasize didactic lecturing to avoid the embarrassment of difficult questions" (Anakwue, 1997:49) and consequently limit learners' understanding of mathematics. When reacting to learners who display some misconceptions in their exercises these teachers use utterances such as "not quite, or, no we do it like that, go and correct it" (Marake, 2013:118). Figure 2.5. below exhibits how a teacher in Marake's research marked the learner's work. It was reported that the teacher lost her temper and threw down the learner's book.

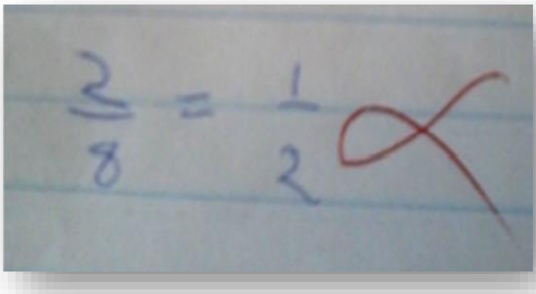


Figure 2.5: A learner's work marked by a teacher (Mareke, 2013: 140)

There were no comments written or feedback given and this particular learner apparently did not know why she got the answer wrong (Mareke, 2013:140). It is posited that learners' achievement improves when misconceptions are analysed to understand learners' thinking in order to appropriately adjust pedagogy (Herholdt & Sapire, 2016: 44). However, the opposite is true when teachers behave like in the case reported by Marake. These teachers fail to nip the 'blind spot' from the bud and perpetually reinforce misconceptions resulting in what we call a 'learning cul-de-sac'.

2.4.2.6 The impact of effective identification and follow up of learners' misconceptions

In summary, probing learners' misconceptions enable teachers and researchers to see what learners can do and cannot do (Pournara *et al.*, 2016:9). Since there is no 'quick fix' to resolve learners' misconceptions, Pournara *et al.* (2016:9) posit that teachers are provided with deeper insight into learners' thinking when the attention towards learners' ways of speaking about algebra is increased. On the other hand, an inquiry using grading of homework to follow up on learners' misconceptions also enhanced effectiveness of teaching (An & Wu, 2012:746). Evidently, for teachers to demystify learners' misconceptions, they should create cognitive conflict by probing learners to explain why they acted in a particular manner (Makonya & Khanyile, 2015: 66). It is proclaimed that "learner errors must not be superficially described" (Herholdt & Sapire, 2014: 57), rather they must be underpinned by understanding why, when and how learners often make particular errors in a particular topic (Herholdt & Sapire, 2014: 57). As teachers follow up learners' misconceptions, trying to understand why they made such errors, “they come to value learners’ thinking and find ways to engage their current knowledge in order to create new knowledge” (Gardee & Brodie, 2015:2). In line with Gardee and Brodie’s (2015: 2) assertion, teachers’ new knowledge develops from PBL in trying to understand learners’ erroneous thinking, emancipate them and their MPCK gets enhanced.

2.4.3 The need for mathematics curriculum knowledge

In this section, curriculum knowledge which is a broad component of PCK is delineated as sub-constructs, such as the use of curriculum materials, lesson planning and integration of assessment with lesson facilitation which are explicitly enacted in the classroom when teaching mathematics. Moreover, it is self-evident that mathematics teachers interact with the curriculum programme and curriculum resources when planning instruction (Remillard & Kim, 2017:65). In line with Remillard and Kim’s (2017:65) assertion, the abstract knowledge of the curriculum gets displayed when teachers make use of appropriate curriculum material when planning a lesson, as they choose the most effective analogies and curriculum materials.

Shulman (1986:10) views curriculum knowledge as “the full range of programs designed for the teaching of particular subjects and topics at a given level” which include a variety of relevant instructional materials. Apparently, Shulman’s notion of curriculum knowledge suggests knowledge of available and relevant curriculum materials including, but not limited to texts, teaching resources, teaching aids, prescribed examinations, tests and schemes to teach mathematics (Tunner-Bisset, 2001 cited in van der Sandt, 2007: 345). Remillard and Kim (2017:67) refer to the mathematics curriculum (or programme) as a “broad set of tools, including a teacher’s guide, student text, and additional supports, designed to guide and support instruction over time, often over several months or years”. On the other hand, Shulman’s (1986:8) view of curriculum knowledge also puts an emphasis on the grasp of programmes that serve as tools of trade for teachers and an understanding of how the subject matter might be organised into a programme of instruction, with attention to lateral and vertical curriculum connections. This understanding of a programme of study or syllabus also encapsulates the full understanding of curriculum goals (Tamabara, 2015: 47). For example, in the SA context CAPS clearly stipulates the purpose, principles and aims of the National Curriculum Statement which is a programme of study for mathematics Grades seven to nine (DBE, 2011: 4-5).

Remillard and Kim's (2017: 66) analysis assumes a participatory view of teachers' use of curriculum resources, which holds that teachers actively interact with such resources to design instruction. Drawing on a socio-cultural framework, they understand teachers' use of curriculum resources to be an example of the agent-tool relationship proposed by Vygotsky (Remillard and Kim, 2017: 66). From the debate in this regard, curriculum knowledge seems to be a broad concept (Remillard & Kim, 2017: 67), hence we decided to subdivide it into sub-concepts to elucidate its implementation in the teaching of mathematics, thus the use of curriculum materials, lesson planning and integration of assessment with lesson facilitation.

2.4.3.1 Insufficient utilization of mathematical resources when teaching

In this section this report elucidates mathematics resources as a sub-contract of curriculum knowledge to unearth the negative impact of none-utilizing of mathematics resources on teachers' MPCK. We start by briefly explaining mathematical resources with the intention to reveal the impact of their minimal utilization in mathematics classrooms in the SA context, SADC, Africa and internationally.

Mathematical resources are "any form of specific mathematical apparatus" such as images, ICT, tools or paper that could be used to provide mathematical teaching or learning aid (Drews, 2007: 21). "Obviously, the use of didactic materials/tools can play an important role in the discovery and expression" of mathematics relationships (Ahmed, Clark-Wilson & Oldnow, 2004: 318). The term 'manipulative' and teaching aids will be used interchangeable in this study to include all mathematics teaching and learning aids used to enhance teaching and learning of mathematics. Manipulatives are defined as "either virtual or tangible material that can be manipulated in shape or size as we use them to make meaning of our learning environment" (Miranda & Adler, 2010: 17). In terms of research in mathematics education, it appears that there is a positive correlation coefficient between the use of manipulatives and learners' conceptual knowledge (Golafshani, 2013:104). This line of thinking was also articulated in a report published by the Ontario Ministry of Education (2004:25) in Canada. The report proclaimed that learners working without manipulatives may struggle to understand the relationship between area and perimeter. “They may assume that as you increase the area of a figure, you necessarily increase its perimeter” (Ontario Ministry of Education, 2004: 25). In contrast, manipulatives like flat square tiles “provide an effective concrete way” to investigate the relationship between an area and a perimeter (Ontario Ministry of Education, 2004: 25). Given the following flat square tiles, the teacher may engage the learners in the following manner: “I have a garden that is 6m long by 2m wide, I want to expand the area of my garden without buying extra garden fence. Is that possible?” (Ontario Ministry of Education, 2004: 25).

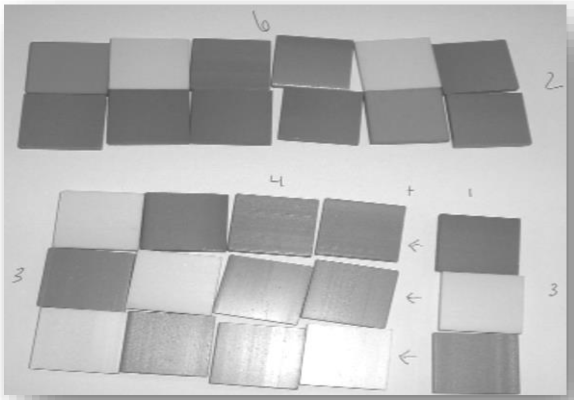


Figure 2.6: Manipulatives used as teaching aid (Ontario Ministry of Education, 2004:26)

As learners engage in discussing and re-arranging the tiles, they test their thinking and visualize mathematics solution (Ontario Ministry of Education, 2004: 25). Drawing from Bruner’s (1964) three modes of representing experiences, Drews (2007: 19-20) theorized that using “physical resources, models and images in mathematics teaching and learning relates well to the iconic modes of representation, with mental imagery and language supporting the understanding and use of symbols”. Moreover, Haylock and Cockburn (cited in Drews, 2007: 20) also proposed a connection network between ‘concrete, pictures, language and symbols’ as of significant importance to the understanding of mathematics concepts (see Figure 2.6).

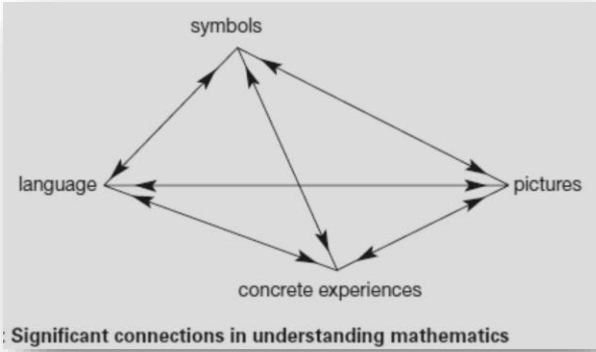


Figure 2.7: Connections in understanding mathematics (Drews, 2007: 20)

As the above learning theories echo each other, it seems that they unanimously agree that the use of mathematics manipulatives develops learners' understanding of mathematics concepts. Perhaps, the understanding of twoness of two as symbol and a number develops well from different representations as put explained in literature (Drew, 2007: 20). It is clear, more often than not, that manipulatives should be used by the learners to inspire their reasoning, "rather than by the teachers to demonstrate procedures" (Ontario Ministry of Education, 2004:27). The emphasis on use of manipulatives enables learners to develop deeper insights of mathematics concepts. Durmuş and Karakirik (2006:119) viewed teaching aids / manipulatives as cognitive tools that "might improve students' active involvements" in the classroom discourse in terms of learners' reflections on relations and the concepts investigated.

2.4.3.1.1 Insufficient utilization of mathematics resources in SA

This section describes the SA experiences in terms of policy mandates, learning theory and research findings in order to understand the effect on insufficient utilization of mathematics resources. SA policy directives are crystal clear in terms of optimal utilization of teaching aids when teaching, as provided in section 3.1.7 of the Personnel Administrative Measures (PAM) document that *inter alia* an educator must "establish a classroom environment which stimulates positive learning and actively engages learners in the learning process" (DBE, 2016: A-18). The Education Labour Relations' Council (Brunton, 2003: A-48) further mandates teachers as learning mediators to use

> *"media and everyday resources appropriately in teaching including judicious use of: common teaching resources like text-books, chalkboards, and charts; other useful media like overhead projectors, computers, video and audio; and popular media resources, like newspapers and magazines as well as other artefacts from everyday life".*

Moreover, the Education Labour Relations' Council (Brunton, 2003: A-48) provides that a teacher as learning mediator must "construct learning environments that are appropriately contextualised and inspirational", while being able to demonstrate sound knowledge of resources appropriate to teaching in a SA context. Teachers are also expected to select and prepare suitable textual and visual resources for learning (Brunton, 2003: A-48).

It has been earlier put on record that SA education policies do not explicitly prescribe PBL (see section 2.4.1.1.1.), although, they seem to embrace PBL embedded policies. As we inquire on PBL as learning theory in relation to utilization of manipulatives to enhance MPCK, our inquiry is not only based on the SA context *per se*. Despite the increase in literature sources on PBL applied in teacher education the research exhibits that in "[a]s far as can be determined, there are no review reports on implementation of PBL in teacher education" (Borhan, 2014:8). In view of Borhan's (2014:8) assertion this study attempts to close this gap in terms of optimal utilization of PBL learning materials to enhance MPCK. Nonetheless, it appears that the use of concrete materials like notes and coins, including the use of role play helps learners to make sense and meaning of financial dilemmas (Sawatzki, 2014: 559). Evidently, in the PBL classroom learners use manipulatives as powerful tools to represent and model mathematics problems (Jarvis, 2016: 24-25). These cognitive tools help learners to demonstrate their thinking (Jarvis, 2016: 36). According to Koszalka, Grabowski and Kim (2002: 6) learners are encouraged to gather information from all forms of available learning resources, such as "print-based materials, electronic and human resources" thus from peers, teachers and experts. The results of the study conducted by Koszalka *et al.* (2002: 16) reveal that resources enhance lesson planning, "strengthened the relationship between the overall problem scenario and learning activities, further instructions to 'coach' teachers in using PBL". On the other hand, Barge (2010: 20) also asserts that provisioning of the required materials for group projects improves collaboration among group members.

Apparently, “problem solving tasks appear to be connected to some form of tool, tangible, or manipulative” (Kelly, 2006: 186). On the other side, the existence of a problem inevitably characterizes PBL learning materials (Rajagukguk and Simanjuntak, 2015: 348). As part of PCK components, Shulman (1986: 10), regarded curriculum materials as “*materia medica* of the pedagogy, the *pharmacopeia* from which the teacher draws those tools of teaching that present or exemplify particular content”. In terms of ‘tools of the trade for teachers’ (Shulman, 1987:8) or ‘*materia medica*’ (Shulman, 1986:10), PBL uses a simple tool, a structured whiteboard to help students learn to untangle problems (Hmelo-Silver, 2004: 238). Specifically, it is posited that for learners to generate and understand the meaning of area and volume formulas they should respectively experience the praxis of covering and stacking manipulatives (Hwang, Su, Huang & Dong, 2009:229) as the teacher posed the problem in Figure 2.8

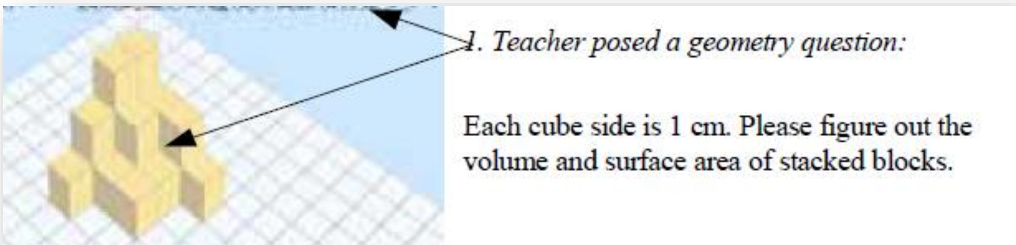


Figure 2.8: Geometry question for students (Hwang et al., 2009: 243)

Unlike physical boards that have limited physical space, the multimedia whiteboard has unlimited space that allows learners to use text, images or spoken words to express their thoughts (Hwang, *et al.*, 2009: 231). Evidently, the use of manipulatives promotes deeper insight in mathematics concepts (Laski, Jor’dan, Daoust & Murray, 2015:2) as learners validate or refute others’ solutions and thinking by manipulating geometric objects (Hwang *et al.*, 2009: 233). Over and above, manipulatives spark the debate between students and teachers as they evoke amusement in the teaching process (Kontaş, 2016: 11). Virtual manipulatives also provide abstract knowledge building experiences that are not available in the real world, thus, visual and auditory displays that cannot be originally obtained through the human senses, but through transduction (Winn, 1993: online).

The research conducted by SA scholars affirmed that the use of visual tools by teachers resulted in mathematics being taught to be easier to remember, interesting and fun (Naidoo, 2012:8). According to Naidoo (2012:8), the use of visual tools encourages learners to actively participate in negotiating meaning construction. It also has been proven that when manipulatives are effectively used, they deepen learners' grasp of mathematical concepts (Brijlall & Niranjan, 2015:363). Hlalele (2012:274) further recommended the use of manipulatives for SA mathematics teachers in rural schools as he argued that they (manipulatives) yielded a statistically significant reduction in mathematics anxiety. Manipulative use enhances learners' visualization of space and shape, as well as their knowledge of mathematical terminology and concepts (Brijlall & Niranjan, 2015: 364). Learners' who participated to Brijlall and Niranjan's (2015: 369) research proved that they used the following manipulative in Figure 2.9 and 2.4.4 to visualize and understand the application of trigonometrical calculations.

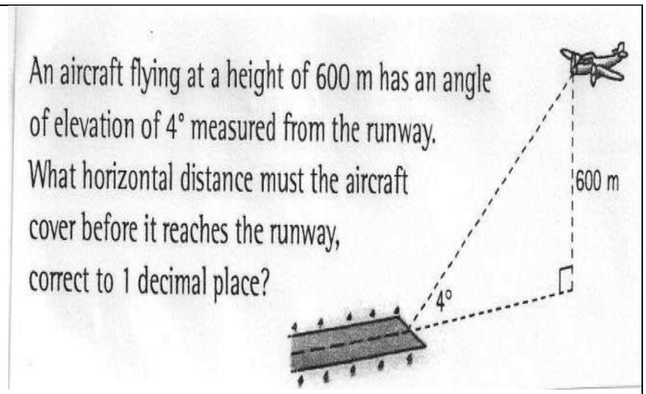


Figure 2.9: Geometry question for students (Hwang et al., 2009: 243)

The above models of mathematical concepts apparently helped learners to relate mathematical concepts to real-life situations.

In summary, policy directives (Brunton, 2003:A-48) demonstrated the value of manipulatives in teaching mathematics, while studies in PBL revealed that that the effective utilization of manipulatives in mathematics teaching increase the mathematics examination grades and improve cognitive flexibility of students (Kontas, 2016: 11). However, in terms of the SA context the research painted a different picture, which magnified the challenge of inadequate supply of learning support material in the area of trigonometry (Brijlall & Niranjan, 2015: 363-364) resulting in insufficient utilization of teaching aids. Despite the weakness in the SA context, manipulatives do not only enable learners to use any language, they also are comfortable to discuss mathematical ideas, but also tend to extend PCK for teachers in trigonometry (Brijlall & Niranjan, 2015: 363). Notwithstanding Brijall and Niranjan’s (2015: 363) assertion, SA teachers that were observed maintained that they did not have “proper resources to teach mathematics adequately” (Mudaly & Naidoo, 2015: 43). The challenge of a lack of resources apparently continued during the implementation of CAPS and as a result SA mathematics classroom remain without material resources (Mahajh, Nkosi & Mkhize, 2016: 379). According to Mahajh *et al.* (2016: 379) the non-availability of teaching aids in mathematics classes was exacerbated by the fact that teachers needed to be trained on how to develop their own resource materials. Researchers echoed each other that SA schools lack “learning material and other basic resources”, which in turn tends to be the main barrier to learning (Nygren, Sutnen, Blignaut, Laine & Els, 2012: 32). In view of what the literature has posited one can safely coin an argument that teachers apparently have failed to use manipulatives to enhance learning.

2.4.3.1.2 Insufficient utilization of mathematics resources in Namibia

In this section, we focus on Namibia to explore good practices that might be relevant to the study under review. The national mathematics subject policy guide for Grades five to twelve encourages mathematics teachers’ innovations in terms of producing their own teaching and learning materials (MoE, 2008: 6). The policy guide further proposes *inter alia* the inclusion of ICT to enhance learning while making teaching enjoyable and fun (MoE, 2008: 7). Moreover, the display of wall charts and artefacts is viewed as making learning interesting as learners have access to the displayed material over a period of time (MoE, 2008: 7). Apparently, the idea of displaying the same artefact over a period of time is believed to enhance memory and understanding. According to MoE (2008:7), teachers should select pictures that would stimulate learners to ask questions. Over and above, the policy also stipulates the duties of subject heads which include ensuring the availability of teachers' resources and learning materials while subject teachers are expected to consult the nearest resource centres (MoE, 2008:11-12).

Adding to policy aspirations, the inquiry that was conducted by Miranda and Adler (2010: 21) suggested that the use of manipulatives such as algebra tiles enabled participants to visualise characteristics of algebraic expressions that are not overtly seen. In an attempt to promote the use of manipulatives in Namibian mathematics classrooms, these researchers (Miranda & Adler, 2010:14) investigated how algebra tiles enhanced algebraic meaning. Three teachers that participated in the research were given the expression (x + 2)(x + 3) to explore. They set algebra tiles to determine the area of the shapes they formed (see Figure 2.10).

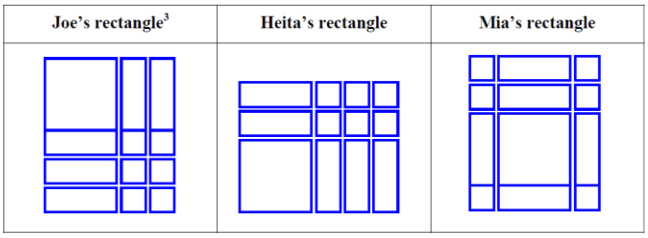


Figure 2.10: Algebra tiles enhancing algebraic meaning (Miranda & Adler, 2010: 19)

The researchers argued that Joe could not determine the area of the rectangle he formed and while he was puzzled, the group members used a different strategy to help him (Miranda & Adler, 2010: 19).

Mia:	What is the area of this 'big square'[pointing to the big blue square]?
Joe:	It's x squared.
Mia:	What about the blue strips? What is the area of each one?
Joe:	x?
Mia:	And how many do you have of those?
Joe:	Five?
Miranda:	And how many units [small squares] do you have?
Joe:	I have six of them.
Heita & Mia:	Now what is their total area, altogether?
Joe:	x squared, five x, and six, Ooohhh! [Tapping his own head with the right hand]

Figure 2.11: Solving a problem (Miranda & Adler, 2010: 20)

The above vignette exhibits the 'wow' effect that was facilitated by the use of manipulatives. According to Miranda and Adler (2010: 20), Joe laughed at himself when epiphany ensued, and he recognised the rectangle dimensions to determine the area. Referring to the First-Outside-Inside-Last (FOIL) algorithm, participants realized the power of manipulatives and they maintained that the way they taught algebraic expressions did not make sense to learners as compared to judicious use of manipulatives (Miranda & Adler, 2010: 20). The Ministry of Education in Namibia also commissioned the University of Namibia to conduct research about *inter alia* effective use of teaching aids in numeracy instruction to develop strategies for optimal use of manipulatives (Ministry of Education, Arts and Culture (MoEAC) and the Ministry of Higher Education, Training and Innovation (MoHETI) (2015: 9). The results illuminated that materials in the toolkit that were developed by co-researchers through action research easily facilitated the implementation of a learner-centred approach (MoEAC & MoHETI, 2015: 27).

Although the research findings show that the use of manipulatives contributes to learners' "visualization of space and shape" (Brijlall & Niranjan, 2015: 364), however, Namibia seemed to be one of many of African countries that do not use manipulatives in mathematics classrooms as a common practice (Miranda and Adler, 2010: 15). It appears that lack of resources is a major deterrent to utilization of manipulatives in many schools, which are mainly in rural areas (Chirimbana, 2014: 8). The literature has also highlighted that, *inter alia*, shortage of teaching resources has curtailed the implementation of learner-centred approach in Namibian schools (Kapenda, Torkildsen, Mtetwa & Julie, 2014: 8). Consequently, learners from rural schools persistently fail to attain minimum requirements for admission in tertiary institutions (Chirimbana, 2014: 8). Apparently, the envisaged change from colonial education towards social justice, equity and democracy (Chirimbana, 2014:8) has not yet been realized due to a lack of resources. This contradiction seems to deprive the rural masses of access to tertiary institutions. It appeared that manipulative use was not prescribed by any curriculum documents hence teachers "had little awareness of the need to use material resources for enhancing mathematics learning (Miranda & Adler, 2010: 16-17). Consequently, teachers were not obliged to use manipulatives (Miranda & Adler, 2010: 17).

2.4.3.1.3 Insufficient utilization of mathematics resources in Nigeria

Our focus in this section investigates the good practice and understanding of the impact of insufficient utilization of mathematics manipulatives in teaching mathematics. As far as can be determined, we could not find a policy that clearly articulates Nigerian mandates in terms of institutionalization of mathematics manipulative use in Nigerian schools. Nonetheless, districts across Nigeria apparently encouraged educators to familiarize themselves on how to properly use manipulatives as instructional tool (Ojo, Adelowo, Emefiene, Kalu, Adebayo & Ibrahim, 2015:225). Educators were encouraged to attend workshops to acquaint themselves about how to use manipulatives (Ojo *et al.,* 2015:225). Both anecdotal and empirical evidence revealed that students' performance of those teachers who used manipulatives improved and as a consequence, districts encouraged educators to attend workshops to acquaint themselves with judicious use of manipulatives (Ojose & Sexton, 2009: 5).

On the other side scholars provided considerable empirical evidence to support their argument that the use of mathematical manipulatives not only create an opportunity for learners to construct cognitive models for abstract mathematical ideas and processes, but it also creates an environment that empowers learners to communicate their ideas generated from these models to teachers and other fellow learners (Ojose & Sexton, 2009: 5). The proponents for effective utilization of manipulatives further maintained that as manipulatives actively engage learners, they tend to increase learners' "interest in and enjoyment of mathematics" (Ojose & Sexton, 2009: 5). A report from a quasi-experimental research that examined the effect of the use of area tiles in developing learners' interest in mathematics revealed that the use of manipulatives equally improved both female and male learners' interest in mathematics (Takor, Iji & Abakpa, 2015: 97-98). Another inquiry that aimed at determining the impact of simple improvised geometric manipulatives on mathematics achievement of high school learners exhibited that the experimental group outperformed the control group in geometry (Aburime, 2007:13).

In contrast with the above highlighted empirical evidence that suggests the use of manipulatives as inextricable tools of trade for a pedagogy that enhances MPCK, Nigerian teachers were reportedly ignorant about the importance of instructional material and they could not put to good use the available, albeit inadequate materials (Adebule & Ayoola, 2015:148). It appears that there is a positive correlation coefficient between the lack of instructional material and poor learner performance in mathematics (Aburime, 2007:7). It is also documented in literature that many teachers in Nigeria who have been observed do not use manipulatives (Ojo *et al.*, 2015: 226). Efforts to determine the reasons for not using manipulatives, rendered reports in which teachers complained about time constraints and they mentioned that manipulatives were too time-consuming (Ojo *et al.*, 2015:226). The demand for content coverage before learners wrote standardized tests seemed to be one of the impediments that caused teachers to renege from using manipulatives (Ojo *et al.*, 2015:226). Evidently, the lack of resources, time constraints and demands posed by standardized tests appeared to be deterrent towards any attempts tried by teachers to use manipulatives.

2.4.3.1.4 Insufficient utilization of mathematics resources in USA

The situation in the US context is investigated to understand good practice that could be used to enhance MPCK. The National Council for Teachers of Mathematics (NCTM) presented professional standards for teaching mathematics wherein the fourth and the fifth professional standards stipulated the requirements for using manipulatives in mathematics classes (NCTM, 1991: 2-3). Specifically, for the discourse to continue, the use of 'tools' such as concrete materials, models, pictures and diagrams should be encouraged (NCTM, 1991:2-3). The NCTM (1991: 3) went further to mandate the creation of learning environments and materials in a way that would facilitate mathematics learning. The Department of Education's policy in Florida declares that schools must utilize 50% of their budget allocation to purchase instructional materials (Education Fact Sheet, 2010-2011:81). These instructional materials *inter alia* include, but are not limited to, manipulatives, electronic media, and computer software that serve as the basis for instruction in mathematics (Florida Department of Education, 2018: 3). To ensure the optimal utilization of manipulatives in New Jersey schools, the governor and education commissioner for the state issued a memorandum instructing school principals to ensure that learners had stipulated manipulatives, such a counting chips and abacus during the administration of mathematics assessment (State of New Jersey, 2010, 4).

Literature claims that manipulatives have been at the centre of the discussions about improvement of mathematics education (Ball, 1992: 16). The narrative that understanding develops through hands-on activities has become part of the educational dogma, which suggests that using manipulatives helps learners (Ball, 1992: 17). However, researchers McNiel and Jarvin (2007, as cited in Marsh, 2016:36) presented an antithesis, that purports statically insignificant impetus of manipulatives on learners' mathematical performance. The research narrative purports that manipulatives may be an impediment to learners' abstract mathematical reasoning as they (manipulatives) provide multiple representations that could stimulate learners to focus on having fun and play at the expense of developing mathematical understanding (Marsh, 2016: 36). Baroody (1989: 4) had earlier expressed the same narrative, namely that simply using manipulatives does not have a magical power to enhance meaningful learning. Teachers' guides on how to use manipulatives also give an impression that learners are likely to automatically draw anticipated conclusions when using manipulatives (Ball, 1992: 17). Ball (1992: 17) further theorizes that this narrative proposes that the expected conclusions probably reside within the manipulatives themselves.

From the above views it appears that the literature contended that any assertion that seems to purport the existence of a 'magic wand' in manipulatives usage, might magically enlighten learners' understanding of mathematical concepts (Baroody, 1989:4; Ball, 1992:17; Marsh, 2016: 36). Instead, it steadfastly advocates for judicious use and appropriateness of manipulatives, which apparently enhance MPCK and learners' concept understanding as a consequence (Baroody, 1989:4; Marsh, 2016: 39).

It is well documented in terms of empirical evidence that effective use of manipulatives in mathematics classes improves learners' abstraction (Schaeffer, 2010: 26; Pon, 2013: 23; Carbonneau, Marley, & Selig 2013: 381 Laski & Jor'dan, Daoust, & Murray, 2015 :8). Pon (2013: 23) articulated three steps required for effective implementation of manipulatives. In exploration, as the first step, the learners explore how to use manipulatives to solve given problems, in the second step the teacher guides learners on how to use manipulatives through scaffolding, and the third step (abstraction) requires the teacher to facilitate the concrete features to mathematical concepts and algorithms (Pon, 2013: 23). In essence, manipulatives as tools of trade in the learning context are used to spark debate as learners explain their ways of getting solutions to problems while in the process the gap between the concrete and the abstract is bridged (Marsh, 2016:33). Overarchingly, the research findings suggest that manipulatives provide hands-on instructional techniques that inevitably enhance conceptual development (Schaeffer, 2010: 26). In line with proponents that agitate for effective use of manipulatives, it seems that learners that use manipulatives outperform their fellow counterparts that do not use manipulatives (Holmes *et al.*, 2013: 4).

Despite the wide availability of manipulatives in USA schools and repeated calls for their use, some teachers reject using manipulatives (Holmes, 2013:1). USA schools do not necessarily lack mathematics resources like teachers in other countries but they also do not use manipulatives (Pia, 2015:828; Furner & Worrell, 2017:7). Teachers contend that they do not have time to plan (Pia, 2015: 828) and have a "lack of knowledge of multiple uses of certain manipulatives" (Furner & Worrell, 2017: 7). Teachers also defend their stunts of rejecting manipulatives (Holmes, 2013:1). They emphasised a lack of training in how to teach using manipulatives as a hinderance and further maintained that manipulatives were incongruent with standardized assessments (Holmes, 2013:1). It is also a truism that educators generally believe manipulative usage is only important in the lower grades, hence less manipulatives usage occurs in higher grades than in lower grade levels (Furner & Worrell, 2017:12). Drawing heavily from Furner and Worrell's (2017: 12) report, it became evident that teachers viewed the use of manipulatives as 'fun maths', and a waste of time which is not necessary for teaching and learning serious mathematics. Secondly, free access to manipulatives by learners posed problems for teachers that were not confident with teaching abilities as learners tend to challenge their teachers' methods (Furner & Worrell, 2017:11). The literature postulated that the new environment with manipulatives may threaten teachers as their position of being the "all-knowledgeable person that students look to for the correct answer" will no longer exist (Furner & Worrell, 2017:14). For teachers to claim back their position of power in the mathematics classroom discourse, they perpetually stop using manipulatives.

2.4.3.1.5 The impact of non-utilization of manipulatives on MPCK

In advancing the need for manipulative use in the mathematics classroom Bale (2006: 39) makes it clear that learners who do not use manipulatives fail to understand the concept of division by fraction. Apparently, learners only followed the procedure without understanding the division by fraction. According to Bele (2006: 38), "A student used the following procedures to solve 1/5 ÷ 3:

Stage 1: 1/5 ÷ 3/1

Stage 2: 1/5 x 1/3

Stage 3: $\frac{1 \times 1}{5 \times 3}$

Stage 4: 1/15"

When the students were required to explain their understanding in terms of how they solved the problem, they presented the "Keep it, change it, flip it method" (Bale, 2006: 38). The procedure means that the first fraction is kept as it is, change the division sign to multiplication, and write the "multiplicative inverse of the second fraction, then multiply numerator by numerator and denominator by denominator" (Bale, 2006:38). When learners were asked why they would solve the problem as above, they said, "[j]ust because that's the way you do it" (Bale, 2006: 38). The research findings exhibited that learners only followed the procedure without understanding, due to the non-availability and non-use of manipulatives. However, when learners were given thinking tools or 'tools of the trade', in our case, manipulatives, they were able to "construct their knowledge about fractions and operations on fractions (Bale, 2006: 59). The assertion that learners relate easily to the use of visual aids and mathematics becomes fun when learners see real objects and pictures (Masilo, 2015: 102) affirms the need for manipulatives use in mathematics classes.

The available empirical evidence demonstrates that inadequate use or non-use of manipulatives results in learners' poor understanding of mathematics concepts (Holmes, 2013: 4) makes the call for effective use of manipulatives more relevant. The literature further posits that "lack of necessary instructional materials and resources reduces the students to mere passive participants in the learning process" (Adebule & Ayoola, 2015: 148). As espoused by the research, inadequate use of manipulatives encourages perpetually continuation of drilling practice (Schaeffer, 2010: 26) and reinforcement of the banking theory concept. It is also put on record that teachers maintain their position of power when they disregard manipulative use and focus on their traditional methods (Furner & Worrell, 2017:14). The other side of the coin is that classroom discourse that perpetuates injustice continues unabated as learners' voices are muffled. As prefigured by literature, the research findings suggest that manipulatives enable learners and teachers to concretely represent the abstract mathematics concepts (Masilo, 2015: 99). It is also self-evident that when learners are not using manipulatives, they apparently fail to understand mathematics concepts (Holmes, 2013: 1).

2.4.3.1.6 The impact of manipulatives on MPCK enhancement using PBL

Shulman (1987: 7) long ago identified the 'blind sport' in teacher education which, when ignored, makes our task of teaching almost insurmountable. Particularly important is the use of tools of trade for teachers (Shulman, 1987: 8), thus manipulatives in our case, to demystify abstract features of mathematics concepts through transduction as this seems to be to the advantage of teachers. In line with tenets that advocate for the judicious manipulatives use, it appeared that continuous utilization of manipulatives has proven to yield good results for teaching and learning, especially using learner-centred approaches such as PBL (Hmelo-Silver, 2004: 238; Hwang, Jia-Han,Yueh-Min, & Jian-Jie, 2009: 229). From this narrative, it seems that a concerted effort of effective utilization of manipulatives in mathematics teaching could enhance MPCK. Consciously, we recognized through empirical evidence at our disposal that manipulatives in themselves are no panacea to our mathematics problems, nor do they possess magic powers that may automatically enlighten learners' insights of mathematics concepts (Baroody, 1989:4; Ball, 1992:17; & Marsh, 2016: 36). In spite of that, however, research findings report promising advantages in the effective utilization of manipulatives in terms of giving multiple representation of mathematics concepts to learners (Hlalele, 2012: 274; Takor, Iji & Abakpa, 2015:97-98). Apparently, these multiple representations proliferate communications and engagement amongst learners and the development of multiple negotiated meanings of mathematical concepts (Marsh, 2016:33). Thus, to summarise: from the literature referenced above, the need for continually judicious utilization of mathematics manipulatives cannot be overemphasized.

2.4.3.2 Lesson planning as the enactment of curriculum knowledge in teaching mathematics

In this section of the study report the enaction of curriculum knowledge through lesson planning. In terms of curriculum alignment, lesson planning is viewed as an aspect of curriculum knowledge (Anderson, 2002: 256), although it is not curriculum knowledge per se. Anderson (2002: 256) presented a triangle model representing three primary components of curriculum, namely objectives, assessments, instructional activities and supporting materials. The sides of the triangle represent curriculum alignment and demonstrate the relationships between the above-mentioned components (Figure 2.12).

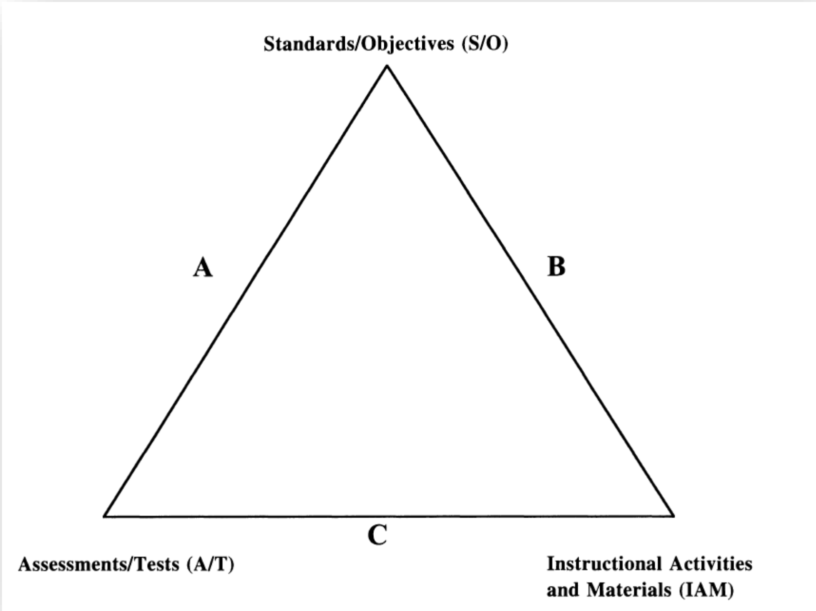


Figure 2.12: Curriculum alignment (Anderson, 2002: 256)

Specifically, curriculum alignment emphasizes that learning objectives or standards should be related to instructional processes and assessment (Martone & Sireci, 2009: 1334). In essence, teachers' decisions are influenced by the desired content or learning outcomes and "teachers' decisions in turn will translate into their instructional practice" (Porter, 2002:5). Shulman (1986: 10) did not clearly categorize lesson planning under the curriculum knowledge component of PCK. Nonetheless, his argument that curriculum knowledge is not only limited to understanding of a variety of available instructional materials but goes beyond and includes knowledge of the effectiveness and implications of programmes and materials for given contexts. Shulman (1986:10) suggests the inclusion of lesson planning. A year later, Shulman (1987: 15) specifically emphasised preparation, thus lesson planning in our case, as one of the important aspects for his model of pedagogical reasoning.

On the other side, the narrative that mathematics teachers interact with curriculum programme and curriculum resources when planning instruction (Remillard & Kim, 2017: 65) is consistent with the view that lesson planning is an aspect of curriculum knowledge, henceforth viewed as a sub-construct of curriculum knowledge. Adding to the scholarly debate, Shen, Poppink, Cui & Fan (2007: 252) specifically viewed lesson plans as instructional resources. This is in line with Remillard and Kim's (2017: 65) assertion that when teachers make use of appropriate curriculum material when planning lessons, they enact and display knowledge of the curriculum. It seems to be a truism that there is an inextricable interaction and participatory relationship between teachers, resources, the planned and the enacted curriculum (Edenfield, 2010:5; Mapolelo & Akinsola, 2015:507). In line with the narrative that teachers interact with instructional resources when teachers plan their lessons (Mapolelo & Akinsola, 2015:507), we argue that curriculum knowledge is enacted through lesson planning. This section therefore explores the enactment of curriculum knowledge through lesson planning to enhance MPCK using PBL.

Lesson planning is viewed as the teachers' road map which guides the teachers on what need to be learned, including strategies of how effective it could be done in a given time in the classroom (Malkova, 2012: 1). This involves the utilisation of coherent activities that intend to enhance learners' cognitive development or cognitive structures (Magano, 2009: 13). It includes, but is not limited to, learning trajectory (Mousley, Sullivan & Zeyenbergen, 2004:375), strategies of representing subject matter (Shen, Poppink, Cui, & Fan, 2007: 249), learners' prior knowledge, assessment opportunities, selection and preparation of resources (Jones, 2007: 69). In essence, lesson planning gives teachers opportunities to explore multiple aspects of PCK (Shen *et al.*, 2007:249). Evidently, aspects of Shulman's (1986: 9) PCK such as the most powerful analogies and most useful forms of representation seem to be pivotal parts of lesson planning. Specifically, preparation, thus lesson planning in this study, is coined as the "critical interpretation and analysis of texts, structuring and segmenting, developing of curricular repertoire, and clarification of purposes" (Shulman, 1987: 13).

Below, lesson planning is further discussed in terms of PBL. Using PBL for ongoing professional development and practice, the research suggests that teachers "develop the ability to apply their knowledge in real classroom settings (i.e., use clinical reasoning)" (Weizman, Covitt, Koehler, Lundeberg, Oslund, Low, Eberhardt, and Urban-Lurain 2008: 31). Furthermore, they share and discuss both videotaped and written descriptions of their teaching problems in order to revise plans according to the data generated (Weizman *et al.*, 2008:33). As teachers assess the effectiveness of their actions during their meetings and collectively decide on a possible way of action, they develop a lesson plan to untangle their teaching problems. In PBL there is no one correct way to teach, however, "using good problems to plan instruction with the focus on student thinking and reasoning is one strategy that holds promise" (Erikson, 1999: 516). Erickson's outline of a lesson plan includes the planned degree of guidance for learners, considering what learners could do without the teacher's help or guidance. The research submits that teachers could use ZPD to "bridge the gap between what a learner can do without help and what a learner can do with assistance" (Siyepu, 2013: 3) in their planning.

2.4.3.2.1 Inadequate lesson planning in teaching mathematics in SA

The above section has briefly presented the concept of lesson planning in relation to curriculum knowledge as a component of PCK. This section therefore focuses on the SA context to understand education policies and a review of related literature regarding mathematics lesson planning. The SA legislative framework has meticulously linked lesson planning to curriculum knowledge as Brunton (2003: H-49) puts it that learning programmes do not only contain work schedules, sequencing and the pacing of curriculum topics for each year, but also include "exemplars of lesson plans to be implemented in any given period". It is further put on record that analysing lesson plans is influenced by the understanding of appropriate selection, sequencing and pacing of content and vice versa (Brunton, 2003: A-55). In addition, the PAM document *inter alia* describes the duties of educators in relation to their obligation to plan lessons. As they plan their lessons, teachers are expected to take into account new approaches, techniques and resources in their field (DBE, 2016: A-18). They also need to utilize learners' own experiences as a fundamental and valuable resource while preparing a variety of strategies that recognise learning as an active process (DBE, 2016: A-18). Lesson planning should be inclusive in nature (DBE, 2011:5) with thorough preparation drawn from a variety of resources and experienced skills (Brunton, 2003: A-49). In terms of CAPS one of the enabling factors for teachers to appropriately plan mathematics lessons to pitch at the appropriate learners' cognitive level is the understanding of learners' level of proficiency in a particular mathematics topic (DBE, 2011: 154).

In addition to policy mandates regarding lesson planning the literature also posits that sufficient planning enables teachers to embrace learners' pre-existing knowledge in the lesson through using different resources (Qhosola, 2016:224). Extrapolating from curriculum proponents for lesson planning Bantwini (2010: 86) claims that teachers are required to develop proper lesson plans. Overarchingly, teachers need to clarify goals of mathematics lesson (Kodisang, 2015: 27). A lesson plan is expected to show lesson outcomes and specific content to be addressed in a particular context (Moloi, 2013: 13). Teachers that participated in the research that was conducted by Kodisang (2015: 92) highlighted the importance of lesson planning as an important guide to teaching and keeping the teacher focused on the envisaged goals of the lesson. Evidently, it seems that the quality of mathematics teaching depends on lesson planning (Volmnik 2010: 25). Lesson planning is not only pivotal to curriculum delivery, but it is also a framework to conceptualize instructional delivery (Ramaila & Ramnarain, 2014:450). Teachers that participated in Ramaila and Ramnarain's (2014: 454) study appeared to value lesson planning and believed that writing a lesson plan gave them an opportunity to explicate how the lesson should unfold.

In the SA context the policy of the DoE prescribes that all teachers should plan (Magano, 2009: 12). However, many teachers still fail to adequately prepare their lessons (Magano, 2009: 14). Inadequate lesson planning seems to be a factor inhibiting the ideal goal of the Education Department (Magano, 2009, 14). Based on literature, it appears that SA teachers neither prepare for lessons, nor do they develop adequate lesson plans. Moreover, it is also evident from the responses of teachers that participated in Malebese's (2016: 141) study that their lesson preparation did not meet the standards stipulated by DBE. Their lesson plans failed to accommodate different learning styles, cultural backgrounds and also did not clearly develop concepts through the use of prior knowledge and identification of relevant resources to be used (Malebese, 2016:141). The teachers' failure to have well planned lessons was reiterated by Maboya (2014: 159) where it was put that "the objective was not specific on exactly what learners should be able to do at the end of the lesson". Lesson planning was apparently not well thought through (Maboya, 2014:159) as one of the participants claimed that they would only prepare the lesson just to please their heads of department. Evidently, the above-mentioned teachers did not use the lesson plan as road map to guide both learners' and teachers' activities in the classroom, but only as a malicious compliance.

As clearly put above, the teacher community of practice in SA schools do not adequately plan their lessons. Evidently, there is a plethora of hinderances towards lesson planning, *inter alia*, lack of supervision from the schools has been highlighted as an issue and apparently resulted in a culture of teaching without lesson plans (Bantwini, 2010: 86). It was found that teachers had a negative attitude towards curriculum reforms. Curriculum reforms demanded proper lesson plans and required teachers to maintain files reflecting detailed daily classroom activities (Bantwini, 2010:86). Teachers felt threatened by the requirements of curriculum reforms as these seemed to demand more of teachers' limited time; hence, they argued that lesson planning was all about paperwork (Bantwini, 2010: 86). Teachers that participated in research that was conducted by Ramaila and Ramnarain (2014:7) bemoaned the lack of time for lesson planning due to overwhelming demands of assessment and marking. While others in the same study maintained that the current set of school textbooks was inadequate for lesson planning (Ramaila & Ramnarain, 2014:7).

2.4.3.2.2 Inadequate lesson planning in teaching mathematics in Namibia

In this section of the report, we want to focus on Namibia to explore good practices in relation to lesson planning as articulated by legislative mandates and research findings. Overarchingly, the national curriculum for basic education provides curriculum as a framework from which teachers are expected to develop their lesson plans (MoE, 2010: 1). In terms of the Department of Education's policy on curriculum, it is compulsory for every teacher to plan a written lesson preparation that clearly outlines "the date, time, theme and topic, teaching and learning materials, lesson objectives and basic competencies to be achieved" (MoE, 2009: 5). To ensure compliance with the policy mandates, teachers are provided with a template according to which they do daily or weekly written preparations (MoE, 2009: 5). The design of each lesson must demonstrate methods of assessment and show "how it will contribute to the structure of the learning experience" (MoE, 2010:37). The national promotion policy guide for junior and senior primary school phases also stipulates that lesson plans should portray learning support for learners that did not progress to the next grade focusing on the competences that were not achieved (MoE, 2015: 4). The national curriculum for basic education also acknowledges that it is advantageous for the subject teacher to follow the same class throughout the phase to understand curriculum overview for a phase, which consequently leads to better lesson planning since the teacher is familiar with both learners and their families (MoE, 2010:38).

Advocates for lesson planning maintain that "teachers should at all times be thoroughly prepared for each lesson" (Moses 2012: 5). It is also reported that the University of Namibia (UNAM) works in collaboration with senior secondary schools targeting mathematics graduate teachers in their first year of teaching (Kapenda *et al.,* 2008:129). The UNAM mentoring programme helped the protege teachers with lesson planning (Kapenda Not in ref list *et al.,* 2008:129). On the same wavelength, resent research findings support the view that syllabuses and textbooks are used by teachers toprepare and plan their lessons by linking the scheme of work with the lesson planned (Nambira, 2016: 36). For teachers to be regarded as competent in lesson planning and preparation, they need to exhibit in-depth content knowledge, pedagogical knowledge and the ability to connect the lesson plan with assessment and evaluation through consideration of learners' prior knowledge (Nambira, 2016: 37). The literature also alludes to experienced teachers with competent PCK who are more able to translate the contents "of the mathematics syllabus and breaking them into minute components that are cooperated into the lesson plan" (Nambira, 2016: 42). It is further argued that professional development to help teachers learn should engage teachers in detailed lesson planning (Peters, 2016:83). The students that participated in Albin and Shihomeka's (2017:316) research concluded that their positive scores in lesson presentations were attributed to their well-planned lessons. Overarchingly, a thoroughly prepared lesson enables quality teaching which promotes learners' interest (Moses, 2012: 5).

Regardless of policy directives that compel Namibian teachers to have written lesson plans (MoE, 2009: 5), it appeared that Namibian teachers did not embrace the praxis of lesson planning (Kapenda, Kandjeo-Marenga, Kasandra, & Lubben, 2002: 55 & Kasanda, 2015: 196). A study conducted by Hileni *et al.* (2002: 55) used lesson plans to establish the range of intended objectives for practical activities included in the lesson plans in Namibian secondary schools. Their findings suggest that teachers' lesson plans provide "only a limited picture of what actually goes on in classes during practical lessons" (Hileni *et al*, 2002: 60). The literature further posited that lesson planning was one of professional practices that continued to challenge teachers

(Kasanda, 2015: 196). The quality of lesson planning depended on teachers' mathematics qualifications and the region from where the teachers came from (Ngolo, 2012: 16). Implicitly, teachers who were unqualified to teach mathematics and whose majority came from rural areas struggled with lesson planning (Kela, 2017: 83). It appeared that lesson plan templates that were available in schools were not used or implemented due to shortage of trained personnel (Kela, 2017: 83). In summary, the above section presented a picture that demonstrated poor lesson planning in Namibian schools

2.4.3.2.3 Inadequate lesson planning in teaching of mathematics in Nigeria

This section focuses on an inquiry in terms of the good practice regarding lesson planning and understanding of the impact of inadequate lesson planning in schools in the Nigerian context. In terms of policy hierarchy in Nigeria, the Federal Ministry of Education (FME) and Nigerian Educational Research and Development Council (NERDC) develop the national curriculum and subject researched syllabuses based on the policy (Assaju, 2015: 172). The schools adjust the syllabus into schemes of work from which the teachers are expected to do a break-down of the scheme of work to arrive at daily lesson plans (Assaju, 2015:172). The legislative framework in Nigeria provides inspectors with the authority to assess teachers on how they use the scheme of work in planning their lessons (Federal Republic of Nigeria, 2005: 19).

In line with the policy mandates the research findings also emphasised the importance of lesson planning in determining what is taught in a particular period and how it is taught (Okwuedei, 2010:100). According to Okwuedei (2010:99), the curriculum is also seen from the perspective of a teaching plan. Henceforth, "[t]he plan for learning, therefore, should be visualized as what teachers do when they attempt to prepare their daily lessons (Okwuedei, 2010: 100). Fatade, Mogari and Arigbabu (2013:35) investigated the effect of PBL on senior secondary school students' achievements in (FM) in Nigeria. At the experimental school, mathematics graduate teachers observed how the researcher led "discussions in the Further Mathematics classroom using PBL in a scaffolding manner to suit the already prepared instructional lesson plan" (Fatade *et al.*, 2013: 35). As part of lesson planning other scholars also suggested the inclusion of a set of induction guidelines to arouse learners' interest (Okafor & Anaduaka, 2013: 250).

Notwithstanding the importance of lesson planning in guiding what transpires in a mathematics classroom, it appears that lesson planning is still a challenge for the observed Nigerian schools (Bot & Celeb, 2014:25). It became apparent that it was difficult if not impossible for Nigerian teachers "to plan mathematic lessons in such a way as to promote pupils' active involvement" (Aremu & Salami, 2012:6). Evidently, the findings of Bot and Celeb's (2014: 25) study exhibited difficulties in planning mathematics lessons due to overload with a high teacher/learner ratio of 1: 233 on average. In the light of the issue of lack of time required to plan, the review of mathematics programmes in schools had been highlighted as one of the challenges that militated against lesson planning (Ismaila, Shahrill & Mundia, 2015: 477). Another impediment that continued to bedevil mathematics teaching in the observed Nigerian schools was the teaching of mathematics by teachers who are not qualified, resulting to "poorly prepared lessons, avoiding some topics that may appear difficult for them to teach" (Bot & Celeb, 2014: 21). Apparently, the worse noted scenario was the observable confusion displayed by some experienced teachers who behaved like protégés when confronted with the task of lesson planning (Okwuedei, 2010: 98). As deduced from the above research reports, it could be proclaimed that mathematics teachers in Nigeria were either unable to develop lesson plans that could actively involve learners, or to develop adequate lesson plans.

2.4.3.2.4 Inadequate lesson planning in teaching of mathematics in the USA

In this section the we report on inquiries about USA policy mandates and research findings in terms of good practice to understand the impetus of inadequate lesson planning in mathematics teaching. In an attempt to institutionalize lesson planning, the Department of Public Instruction in North Carolina piloted high schools in a Microsoft programme that comprised online lesson plans (North Carolina Department of Public Instruction, 2012: 8). This pilot programme allowed teachers to access instructional resources such as lesson plans (North Carolina Department of Public Instruction, 2012: 7). On the other hand, Houston and Beech, (2002: 1) had earlier designed a handbook that provided teachers with a tool to improve instruction through effective lesson planning. With the use of the above-mentioned handbook, teachers were able to align the process of lesson planning with all instructional components to improve learners' achievement of district goals and the Sunshine State Standards (Houston & Beech, 2002: 1). To encourage authentic mathematics lesson planning, universal design for learning guidelines in New Jersey provided that supervisors or principals should review teachers' lesson plans and provide feedback (New Jersey Department of Education, 2013: 1). In Florida teachers were expected to submit weekly lesson plans to their supervisors, and instructional assistants must demonstrate the integration of the standards and the benchmarks into daily learning activities (Florida Education Department, 2015: 10-11).

Echoing the policy mandates in relation to lesson planning, the research findings exhibited the work of a mathematics education group which had over 15 years created detailed lesson plans for each session of each content course and worked to improve the lessons over time (Morris & Hiebert, 2017: 532). In generating data to understand how participants' experiences might impact their performance in lesson planning tasks, participants were requested to plan a single lesson for each given four mathematics topics (Morris & Hiebert, 2017: 535). The results of Morris and Hiebert's (2017: 553) study showed that participants were more competent in completing lesson planning tasks for topics covered in the mathematics content courses for elementary Pre-Services Teachers (PSTs) than for topics not covered in the courses. To improve their effectiveness in teaching and learning, the literature intellectualized that the design of classroom instruction should be grounded on careful lesson planning (Ding & Carlson, 2013: 306-361). Ding and Carlson (2013:361) further reported on the Institute of Education Science's (IES) recommendations which suggested the use of instructional principles as scaffolds to support teachers' lesson planning (Ding & Carlson, 2013: 361).

In contrast to the important role of lesson planning in teaching and learning due to the consideration of mathematical content of a lesson and of students' thinking it appeared that some of the observed teachers in US still struggle with lesson planning techniques in mathematics (Swearingen, 2014: 2). Apparently, teachers complained that they were unable to plan mathematics lessons, arguing that they had other subjects to teach too (Ding & Carlson, 2013:381). The literature also reports that protégés are most affected, to such an extent that they were unable to think on their feet and deal with unexpected learners' responses due to inadequate lesson preparation (Swearingen, 2014:2). Moreover, profiles of participants in Swearingen's (2014: 146) study revealed no prior lesson planning experiences. Echoing the narrative that suggested inadequate lesson planning in an interview, Lyle argued that he was struggling with developing lesson plans and was unable to plan for advanced learners (Lannin, Webb, Chval, Arbaugh, Hicks, Taylor & Bruton, 2013:420). Seemingly, viewing the curriculum as authoritative influenced teachers' limited selection of problem sets from the mathematics text book (Swearingen, 2014:154). On the other hand, elementary teachers as compared to their secondary counterparts firmly believed that detailed plans would hinder their ability to make connections across subjects and prohibit their teaching flexibility (Ding & Carlson, 2013: 360). In what seems to us an appropriate summary, failure to carefully consider key components of the lesson plan resulting in inadequate lesson planning might be due to teachers' beliefs (Ding & Carlson, 2013:360, Swearingen, 2014:154; Morris & Hiebert, 2017:544).

2.4.3.2.5 Impact of inadequate lesson planning

Generally, inadequate lesson planning seems to be a factor inhibiting the achievement of Education Department goals (Magano, 2009, 14). Specifically, inadequate lesson plans fail to accommodate different learning styles, and cultural backgrounds and also do not clearly develop concepts through use of prior knowledge and identification of relevant resources to be used (Malebese, 2016:141). Teaching without lesson plans results in teachers getting to class without objectives specifying exactly what learners should be able to do at the end of the lesson (Maboya, 2014:159). Without well thought through lesson plans, novice teachers were reportedly unable to think on their feet and deal with unexpected learners' responses (Swearingen, 2014:2.). Qhosola's (2016: 139) study noticed that inadequate lesson planning did not give clarity on how marginalized learners' prior knowledge could be effectively utilised and integrated in the process of acquiring the new knowledge. In addition, Mosia (2016: 124) presented empirical evidence showing how inadequate lesson plans failed to assess the learners' prior knowledge of geometry concepts. In line with Mosia's findings (2016: 124), the inclusion of assessment of what learners brought to class, in the lesson plan could have been used to build on what they already knew.

2.4.3.2.6 The impact of thoroughly planned mathematics lessons

It appears that the development and application of a thoroughly prepared lesson plan consequently improve teachers' wisdom of practice (Ho, Watkins & Kelly, 2001, cited in Mosia 2016: 56). Apparently, the development of a detailed lesson plan provides teachers with the opportunity to accommodate learners' pre-existing knowledge in the lesson (Moloi 2014:193 & Qhosola, 2016:224). Lesson plans that embrace learners' marginalized knowledge apparently harmonise the relationship between learners' background environments and mathematical concepts taught in the classrooms (Moloi, 2014:189). Detailed lesson plans that show lessons' outcomes and specific content to be addressed in a particular context (Moloi, 2013:13) guide teachers to keep focused on the envisaged goals of the lesson (Kodisang, 2015: 92). According to Qhosola (2016: 222), there are mandatory expectations for teachers to "conduct research as part of lesson preparation in order to identify the most suitable approach and tools necessary for a particular lesson". In resonance with Qhosola, the research agitates that meticulous collection of thoughts and appropriate manipulatives to exemplify what to be taught to learners from different backgrounds are the bedrock for lesson panning (Mosia, 2016: 19). Perhaps, during this painstaking process of lesson planning, teachers develop in aspects regarding mathematics traceability. It is in fact asserted that lesson planning opened teachers' minds in terms of how they could effectively present their lessons using different skills (Tsotesti, 2013: 100).

2.4.3.3 The need for integrated assessment with lesson facilitation

In this section the focus will turn to understanding how curriculum knowledge is manifested when assessment is integrated with lesson facilitation. According to Magnusson, Kracjik and & Borko (1999:109) assessment is viewed as a PCK component. It is reported that Shulman and Grossman models of PCK did view knowledge of assessment as a component of PCK until it was later considered by Magnusson and her colleagues (Fernandez, 2014:96, Danişman & Tanişli, 2017:17).

On the other hand, curriculum alignment demonstrates an inextricable relationship between objectives, instructional activities, and assessment (Anderson, 2002:256; Martone & Sireci, 2009: 1334). Moreover, Fernandez (2014: 89) also emphasized the inseparability of knowledge of the assessment process from the aims and purposes of the curriculum. The literature theorises that formal and informal assessment influences the PCK development (Park & Oliver, 2008:272). Evidently, learners' responses affect teachers' decisions to modify instructional strategies employed (Park & Oliver, 2008: 272). As teachers assess learners, they often encounter learners' questions that have no readily available answers in terms of subject matter knowledge (Park & Oliver, 2008: 272). Without necessarily rejecting the narrative that views knowledge of assessment as a distinct PCK component (Magnusson *et al.*, 1999:109), this study focuses on understanding curriculum knowledge exhibited through integrated assessment with lesson facilitation. In line with Anderson's (2002:256) and Martone and Sireci's (2009:1334) views we understand knowledge of assessment from the stance that aligns it to curriculum knowledge.

According to CAPS, "[a]ssessment is a continuous, planned process of identifying, gathering and interpreting information regarding the performance of learners, using various forms" (DoE, 2011:154). Yukon Department of Education (2015) specifically views assessment from a school-based premise and defines it as a systematic process of gathering information from many sources that help in making appropriate educational decisions. Both above definitions put emphasis on the role of assessment, thus to determine learning needs and how they can be addressed to enhance learning (Kesianye, 2015: 212). The DoE (2011:154) views assessment as a continuous process that gives learners regular feedback to enhance their learning experience and resonates with the view that considers it as an integral part of teaching (Kesianye, 2015:212). In essence, assessment should not be only about allocation of a pass or a fail mark, but it hinges around the improvement of learning outcomes and support of meaningful learning (Mark, 2013: 1). Overarchingly, assessment integrated with instruction seems to improve the teaching practice (Suurtamm, Kim, Díaz & Sayac, 2016: 3). As teachers use evidence generated through assessment, inferences are made at any stage of instruction (Suurtamm *et al.*, 2016:25). This suggests that teaching is often adjusted in line with learners' understanding of the tasks while the lesson progresses and the assessment is not used as a separate entity but an integral part of teaching (Kesianye, 2015: 213).

Furthermore, from a PBL perspective as previously articulated, learning begins with a problem to be solved as learners are first given a problem scenario (see section 2.3.2). It is emphatically put on record that teaching begins with the problem presentation and ends with student reflection in terms of PBL (Hmelo-Silver, 2004: 242). As learners formulate and analyse the problem through identification of relevant facts or information in the scenario in order to generate hypotheses about possible solutions, the teacher assists learners by posing questions so that learners could identify their knowledge deficiencies, which encourage learners to search for new information (Hmelo-Silver, 2004:336). The PBL philosophy posits that learners are always confronted with problems which provide a stimulus for learning (Gijbels, Dochy, Bossche & Segers, 2005:29). In terms of PBL “teachers should not offer direct answer but illuminate students when they have questions” (Li & Du, 2015:20). From a PBL premise, learners are presented with a problem first, which is the starting point directing their learning process (Barge, 2010: 7). To be more specific, Dahl and Kolmos (2015:65) theorizes that assessment in PBL should be aligned with group projects and intended learning outcomes (ILOs). From AAU experience, group assessment enables learners not only to respond to content but also learning processes thus, process competences (Dahl & Kolmos, 2015: 65). These process competences include, but are not limited to, reflection on one’s own thinking, reasoning and reflection, communication, production, cooperation, arguing, and negotiating (Dahl & Kolmos, 2015:65). Tan and Keat (2005:162-163) further attest that in PBL, learning mostly takes place through group interaction, hence assessment processes mainly focus on group work.

It is believed that PBL teaches clinical reasoning and promotes both self-assessment and peer assessment (Atwa & Al Rabia, 2014:1). In the former (self-assessment), learners judge their work by using evidence and a clear set of criteria (Dharma & Adiwijaya, 2018: 2) while in the latter (peer assessment), learners engage in an act of judging the quality of peers’ work (Atwa & Al Rabia, 2014:2; Aliasa, Masekb & Sallehc, 2014:310). As learners assess and judge the work of others, they gain a deeper insight in their own performances (Atwa & Al Rabia, 2014:2). Self- and peer assessment give a sense of ownership of the process and learners become responsible for their learning when assessment is treated “as part of the learning process, where mistakes are opportunities rather than failures” (Atwa & Al Rabia, 2014:2). Overarchingly, teachers need to thoroughly prepare and use crystal-clear assessment criteria to enable learners to carry out self- and peer assessment (Atwa & Al Rabia, 2014:2). Atwa and Al Rabia refer to these criteria as a checklist, while others call it a rubric (Clark, 2017:15; Hallinger & Bridges, 2007:119). According to Macdonald (2005:86) peer, self and collaborative assessment enables learners “to make judgements about how well they are learning not just how much they have learned”.

2.4.3.3.1 The need for assessment integrated with lesson facilitation in SA

In this sub-section, the study focuses on SA good practice about the impact of assessment integrated with lesson facilitation as provided by policy mandates and reported in research findings. In terms of CAPS policy, assessment is integral to teaching and learning (DBE, 2015:4). The DBE (2011:155) further puts it that the purpose of assessment is to continuously collect data on learners’ performance that could be utilized to improve learning. Seemingly the DBE (2011:155) suggests a break away from traditional assessment where the assessment was done at the end of a lesson. Assessment “should not be seen as separate from the learning activities taking place in the classroom” (DBE: 2011:155). As it is used to provide feedback to learners at the same time, it also informs planning for teaching, it could be done through observations, discussions and learner; teacher conferences (DBE: 2011:155). The philosophy underpinning integrated quality management system (IQMS) is to assess strengths and identify areas for development (DBE, 2003:3). The National Education Policy Act (NEPA) provides for an educator as an assessor to keep detailed records of diagnostic assessment and use them in the process of improving learning programmes (Brunton, 1996:A-49). As mediators of learning, teachers are mandated to use higher level questioning, problem-based tasks and appropriate use of group-work as teaching strategies (Brunton, 1996: A-49).

The literature reiterated that lesson presentations “should be motivated by meaningful problems and be integrated with regard to subject matter” (Van Staden & Motsamai, 2017:3). In the SA context, school-based assessment (SBA) is used as an engine of educational transformation which forms an integral component of teaching and learning (Van Staden & Motsamai, 2017:3). SBA creates a platform for presentation of meaningful problems to ensure, *inter alia*, learners develop deeper insight on basic mathematical concepts (Van Staden, & Motsamai, 2017:3). It is also reported that for assessment to help learners to construct understanding of mathematics, it should frequently use feedback (Umugiraneza, Bansilal & North, 2017:3). Seemingly, "scaffolding provided in the form of hints and prompts during assessment can support learners in attaining targets" (Umugiraneza *et al.*, 2017:3). In line with the opinions of proponents of integrated assessment, an argument advancing the alignment of assessment with learning targets of the curriculum "to ensure seamless teaching, learning and assessment" has been put on record (Kanjee & Sayed, 2013:444). In essence, detailed information collected by high-quality assessments enables teachers to make decisions regarding their teaching strategies (Kanjee & Sayed, 2013:444).

Research findings, however, document a big disjuncture between the envisaged outcome of SBA and the observed reality at school level. According to Kanjee and Sayed (2013:458)

> *"[t]he practical reality presents a different picture, as assessment practices of teachers are dominated by a discourse of recording and reporting of learners' scores, with limited focus on the use of assessment for addressing learners' learning needs"*

The observed reality suggests that SA educators continue to separate assessment from instruction, irrespective of the policy directives (Umugiraneza, Bansilal & North, 2017:10). Apparently, teachers' first choice was teacher-led instructions as they did not "engage in progressive methods such as classroom discussions, group work and practical examples in their classrooms" (Umugiraneza *et al.*, 2017:10). Recent research reports that "the 'assessment focused, measurement-driven' approach continues" unabated to bedevil any curriculum innovations (Kanjee & Sayed, 2013:443-444). Paradoxically, teachers continue to use measurement-driven assessment despite its negative effect on teaching and learning. It is noted that teachers lack training and support in effective use of classroom assessment; instead they spend overs two hours a week grading and recording tests and examinations (Kanjee & Sayed, 2013:443). Moreover, teachers seem to have no time to attend any professional development programmes (Umugiraneza, et al, 2017:11). Van Staden and Motsamai (2017: 11) presented evidence that suggests reasons such as "teachers' lack of knowledge in devising alternative forms of assessment or lack of adequate in-service training and support to empower teachers to develop a repertoire of assessment skills".

2.4.3.3.2 The need for integrated assessment with lesson facilitation in Namibia

In this section the study focuses on Namibian good practice in terms of a legislative framework and research findings regarding integrated assessment. An integrated planning manual provides that teachers should continually change and appropriately adjust their prepared lesson plans through assessment (MoE, 2015: 13). Furthermore, it is expected that "[g]ood teachers regularly spend time evaluating their teaching" (MoE, 2015:15). The Integrated planning manual suggests that self-evaluation in the classroom enables teachers to identify areas for improvement and confirms the strength of their practice (MoE, 2015:15) This process seems to support teachers' professional development and builds confidence (MoE, 2015:15). The provisions of the national mathematics subject policy guide agitate that continuous assessment and compensatory teaching are a critical part of the lesson plan (MoE, 2009:5). The integrated planning manual records exhibit that the purpose of assessment "is to help teachers improve their teaching and provide for a better learning experience for the learner" (MoE, 2015:16). Assessment in this regard is explicitly viewed as 'Assessment for Learning' (AfL) (MoE, 2015:16). The National Promotion Policy Guide for Junior and Senior Primary phases emphasises that assessment as an ongoing component of curriculum design is to evaluate the effectiveness of instruction (MoEa, 2015:3).

In Namibia and SA the introduction of continuous assessment was profoundly used "to ameliorate the negative impact of the once-off high-stakes examinations (Sayed & Kanjee, 2013: 378). The last decade saw a growing number of Sub-Saharan African (SSA) countries developing "classroom assessment practices that enhance learning" (Sayed & Kanjee, 2013:378). Education reforms in Namibia, for example, focused on education for all (EFA) by utilizing assessment to improve teaching and learning through integrated assessment (Sayed & Kanjee, 2013:378). According to Samson and Marongwe (2013:197), who outline the purpose of formative assessment, or assessment for learning (AfL), its purpose is to gather evidence that will guide the teaching and learning process. AfL may enable the teachers to adjust their instructional procedures (Samson & Marongwe, 2013:197). Specifically, continuous assessment contributes 35% to the final promotion mark for mathematics Grade 10 in Namibia and this suggests that teachers must embrace the importance of ongoing assessments and continual adjustments to their teaching practices (Samson & Marongwe, 2013:199). Overarchingly, the education reforms emphasise the use of set criteria to assess strengths and weaknesses against norm referencing assessment (Iipinge & Kasanda 2013:429).

Contrary to the policy intentions, the observed reality elucidates that "[a]cross many SSA countries, the effective use of classroom assessment practices to enhance learning has been relatively limited" (Sayed & Kanjee, 2013:378). Big classes seem to be an impediment towards effective implementation of ongoing continuous assessment that could really take care of individual learning styles (Iipinge & Kasanda 2013:438). Research also reports that most teachers in Namibia fail to effectively use assessment information to help learners to improve their learning due to the little attention paid to assessment integrated with instruction (Sayed & Kanjee, 2013:379). The administrators and teachers seemingly are faced by a number of challenges that militate against integrated assessment; among others, the dominance of summative assessment and huge discrepancies between continuous assessment and final exam marks (Sayed & Kanjee, 2013:379). On the other hand, teachers' insufficient knowledge of assessment rubrics and pressure to complete the curriculum also militate against the implementation of AfL (Iipinge & Kasanda 2013:438). Paradoxically, the prescribed assessment structure in Namibian schools of having a 'test' every Friday poses limits and hampers the implementation of integrated assessment which the policy mandates intend to achieve (Peters, 2016:363).

2.4.3.3.3 The need for assessment integrated with lesson facilitation in Nigeria

This section explores the Nigerian praxis in terms of implementation of assessment integrated with lesson facilitation. Our inquiry focused on policy mandates and research findings in this regard. "The National Policy on Education recommends the use of Continuous Assessment in the evaluation of pupils at schools' level" (Federal Ministry of Education ,2005:157). The Federal Department of Education advances that "[o]ne important merit of continuous assessment as a teaching strategy is its corrective role" (Apea, 1998: 7). The education policy posits that continuous assessment helps to identify weaknesses which are subsequently corrected early (Apea, 1998: 7). The provisions of the Nigerian education sector emphasized that continuous assessment will not only be used to appropriate instruction, but it will also be used to determine the advancement of learners from one class to another in primary school (Federal Ministry of Education, 2005: 298-299).

Literature also posits that what is taught is routinely influenced by assessment (Anyanwu, Onwuakpa & Ezenwanne, 2017:146). In line with the narrative that suggests that assessment drives the curriculum (Anyanwu *et al.*, 2017:146), school-based assessment (SBA), thus integrated assessment in our case, involves the interaction between the teacher and learner from the beginning to the end of the lesson (Ogbebor-Kigho, Onuka & Owolabi, 2017:293). "It is expected that through this approach, teachers would be able to integrate assessment results into instructional practice" (Adebowale & Alao, 2008:5). This interaction is aimed at making judgement and improving the effectiveness of the teaching and learning process (Ogbebor-Kigho *et al.,* 2017:293). Put differently, this interaction generates data to be used to improve the achievement of learning outcomes (Ogbebor-Kigho *et al.,* 2017:293) and appropriate instruction for individual learners (Adebowale & Alao, 2008:5). In a nut shell, the feedback provided by integrated assessment enables learners to understand what they should improve and guides teachers to adjust instruction (Adediwura, 2015:355).

Scholarly debates claim that assessment and evaluation in mathematics, are "either glibly treated or completely ignored" (Anakwue, 1997:211). Challenges such as negative attitudes toward the continuous assessment approach tend to hamper the proper implementation of continuous assessment (Adebowale & Alao, 2008:5). Evidently, mathematics assessment is inundated with paper-pencil tests, where learners work individually on problems (Adediwura, 2015:355). It is proclaimed that such tests fall short in determining learners' concept proficiency (Adediwura, 2015:355). The Federal Ministry of Education (2005:109) identifies overpopulated classrooms to be unconducive to the implementation of continuous assessment practice. In terms of FME, perhaps teachers feel overburdened by overcrowded classes. Apea (1989:129) also echoes the view that overcrowded classes are incompatible with continuous assessment, resulting in continuous assessment becoming “phoney as teachers are forced to set the minimum tests/assignments”.

2.4.3.3.4 The need for integrated assessment with lesson facilitation in USA

In this section the study reports on the US policy mandates and research findings in terms of good practice to understand the impetus of unintegrated assessment in mathematics teaching. According to the Californian Department of Education (2015: 671) teachers are mandated to establish their learners’ current understanding of mathematics concepts so that they can design mathematics instruction that will lead to mastery of mathematics standards (California Department of Education, 2015: 671).

The New Jersey Department of Education (1996:12) also provides two additional standards which explicitly address how linking assessment to learning and instruction fosters success in mathematics. The education policy specifically provides that formative assessment enables teachers to gather information about the learning process as it is happening and defines it (formative assessment) as an assessment for learning (AfL) (California Department of Education, 2015:671). Seemingly, formative assessment measures and unearths learners’ strengths and areas where growth is still necessary. This is done through constructive feedback and consequently developing learners’ self-regulation while enabling learners to reflect upon their mathematical learning (California Department of Education, 2015: 667). Overarchingly, integrated assessment seems the key that provides learners with mathematics instruction design to help them progress to higher levels of learning. Teachers can alter their lessons or instructional strategies to cater for learners’ needs when they are armed with this particular knowledge of assessment (California Department of Education, 2015:671).

In line with the policy directives espoused above, Troy (2011: 2) also views formative assessment as an assessment for learning (AfL), “meaning that it takes place expressly to inform instruction”. “The general goals of formative assessment is one of instruments that will both inform and improve teachers’ instructional planning and student achievement” (Heritage, 2010a:3). It takes place while learning is underway, not at the end of the sequence of learning as Troy (2011:3) espouses that it is embedded in the instructional process. In the same vein Heritage (2010:3) posits that

‘[F]ormative assessment is not an adjunct to teaching but, rather, integrated into instruction and learning with teachers and students receiving frequent feedback”. Heritage (2010a:4) further presents raw data illustrating her participants’ experiences towards implementing formative assessment in their classes.

Shawn: I used to do a lot of *explaining*, but now I do a lot of *questioning*. I used to do *a lot of talking*, but now I do *a lot of listening*. I use to think *about teaching the curriculum*, but now I think *about teaching the student*.

Figure 2.13: Raw data illustration Heritage, 2010a:4)

Shawn seems to be excited about the new experiences as learners seem to do a lot of talking, while the teachers listen to learners’ participation in the assessment process. Heritage (2017: online) describes Shawn’s experience as “less is more”. This implies that the learner develops a deeper understanding of mathematical concepts, when the teachers talk less and allow learners to do more reflection of their thinking and understanding of mathematical concepts elucidated through formative assessment. In fact, formative assessment is an integral part of the instructional process that makes learning become more active and a participatory process (Heritage, 2010a:16). In essence,

> *“formative assessment is conceptualized as a practice and a process centred on the idea of feedback loops in which both teacher and student use information to alter the gap so as to further learning” (Heritage, 2010a:6).*

Evidently, formative assessment is not about ‘I got it’ or ‘I did not get it’ and re-teach (Heritage, 2017: online), but it is conceptualized as a pedagogical process whereby learning is evaluated by both teacher and learners while occurring (Collins, 2016: 1).

In contrast to pockets of good practice in terms of formative assessment, thus integrating assessment in instruction in our case, it is reported that it has surprisingly received narrow treatment in the US (Heritage, 2010:3). Torrance and Pryor (1998, cited in Heritage, 2010:3) also report that formative assessment has received relatively little attention. Due to teachers' little or no experience with formative assessment, implementing it may become a frustrating process (Brink, 2017:3). Brookhart (2009, in Heritage, 2010:3) also lamented that the US puts more emphasis on tests, schedules and data reports at the expense of formative assessment. It is also enunciated that high school teachers may not be expected to implement formative assessment practices when they have not been given full opportunity to learn it (Brink, 2017:3). The evidence from Brink (2017:11) suggests that teachers from the observed "high school were accustomed to teaching, testing, and moving on with instruction". Brink (2017:21) further argues that teachers had little familiarity with formative assessment since it was a new concept. Evidently, Aaron, a teacher in Brink's (2017:94-95) inquiry, saw formative assessment as additive to his work.

2.4.3.3.5 The impact of none implementation of integrated assessment with lesson facilitation on MPCK

"Stereotypically summative assessments have little impact on learning as it is happening" (Wallace, 2013:3). In contrast with formative assessment, summative assessment is separated from instruction and the impact of teaching on learning is commonly measured well after a presumable learning process, when the "class has moved on to a new topic of learning" (Wallace, 2013:3-4). Teachers who rely on summative assessment apparently adopt a discursive practice of marking with tick and cross in mathematics learning (Adediwura, 2015:356). It seems that this discursive practice of marking with tick and cross in mathematics learning tends to limit learners' thinking when answering questions" (Adediwura, 2015:356). Accordingly, the "adjective *ongoing* reinforces that formative assessment is a classroom process that is enacted while the learning is occurring, not something done after the learning has taken place" (Council of Chief State School Officers, 2018:6). In line with Wallace's view, at this stage (when the lesson is over) there is little that could be done except to re-teach since the teacher may not be in a position to identify how learners got in a cul-de-sac. Heritage (2010, online) puts it on record that learners are not mail boxes in which instruction could be delivered. Instead, learners should know how they are to be assessed as no one can "be asked to hit a target if they do not know what the target is" (Brink, 2017: 24).

The point is that learners "suffer when their teachers do not possess the needed rich information about their learning strengths and weaknesses" regardless of who or what is to blame (Buhagiar & Murphy, 2008:179). As teachers fail to integrate assessment with instruction, they also loose the opportunities of being empowered in terms of developing the appropriate intervention strategies to overcome identified misconceptions through integrated assessment. In line with Shulman's (1986:6) assertion of a 'blind spot', when teachers do not know their learners in terms of their cognitive levels and mathematical misconceptions, teaching and learning is likely to get into a cul-de-sac. As a consequence, when teachers do not integrate assessment with instruction, their learners are likely to acquire only superficial knowledge. Such surface knowledge would not give details of "what students actually know and can do, let alone on what they can nearly do" (Buhagiar & Murphy, 2008:178). It is self-evident that such teachers are unlikely to offer appropriate scaffolding to their learners and as a result lose the opportunity to enhance their MPCK.

2.4.3.3.6 The impetus of integrated assessment with lesson facilitation on MPCK enhancement

The literature reports that formative assessments "produce significant and often substantial learning gains" (Black & Wiliam, 2010: 83). The scholarly debate is in resonance with the narrative that formative assessment, that is, assessment embedded in instruction gives teachers an insight about students' mathematical thinking (Kesianye, 2015:213). Integrated assessment enables teachers to use learners' natural ways of thinking as the classroom practice (Nagasaki & Becker, 1993:44). It is also reported that teachers' recognition of learners' "cognitive tendencies" in turn helps teachers to improve their pedagogical strategies and consequently contribute to their development (Nagasaki & Becker, 1993:46). In agitating for the need for assessment embedded instruction, Andrews, Ryve, Hemmi and Sayers (2014: 8) philosophized that formative assessment and building on students' thinking develop an appropriate teacher knowledge base. This suggests that teaching is often adjusted in line with learners' understanding of the tasks while the lesson progresses (Kesianye, 2015:213).

The need to integrate assessment with instruction cannot be overemphasised in enhancing MPCK and emancipating teachers in designing strategies to get themselves and their learners out of what we call a teaching and learning cul-de-sac. In terms of ZPD teachers have to discern their learners' potential to advance learning, so that the presented activities are neither too trivial nor too demanding. Heritage (2010: online) refers to Colvin' (2009) 'learning zone' as the cutting edge of learners' competences (Figure 2.14).

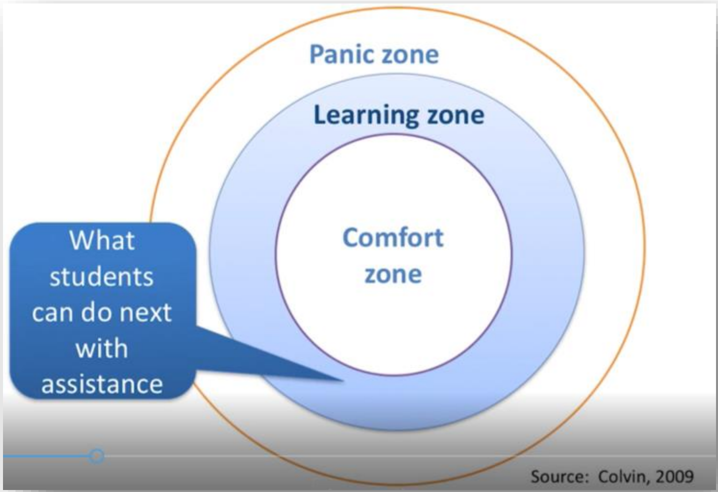


Figure 2.14: Learning zone (Heritage, 2010c: online)

The literature furnished evidence that *inter alia* one crucial assessment function is to accurately identify this zone (Buhagiar & Murphy, 2008:170). This identification of the zone enables teachers to appropriately develop mathematics lesson plans to rescue learners from a 'cul-de-sac' in order to realize learning trajectory. Formative assessment viewed from a cognitive perspective enables teachers to continually gather and interpret evidence to design instruction that builds on what Vygotsky (1978) called 'maturing functions' (cited in Heritage, 2010a:8). According to Heritage (2010a:8) formative assessment enables teachers and learners to consistently work in the ZPD, the area where learning takes place. For teachers to prosper in unearthing what is within the learners' reach, in order to provide them with appropriate experiences and to support and extend learning, they need to keep "a very close eye on emerging learning through formative assessment" (Heritage, 2010b: 8).

Teachers may also benefit from "establishing classroom routines and practices that support learning" (Chief State School Officers, 2018: 6). Andrews *et al.* (2014: 14) also posit that assessment embedded instruction apparently develops a discursive practice where one has to justify one's opinion during the classroom discourse. Formative assessment is not done on learners, but learners participate in the process and the practice, as it is argued that it takes place within a community of practice (Heritage, 2010: 9). "The norms established in the community are mutual support, trust, respect, and collaboration (Heritage, 2010b:9). As the community of practice interprets evidence and use feedback generated through formative assessment, teachers and learners "assume the roles of partners in the learning process" (Heritage, 2010: 9).

2.4.4 The need for mathematics pedagogical knowledge

In this section, pedagogical knowledge, one of Shulman's (1987: 8) components of PCK is explored to understand good practice and the impact of lack of this knowledge base in teaching mathematics. According to Shulman (1987:7), pedagogical knowledge refers to "strategies of classroom management and organization that appear to transcend subject matter". Kar (2017:5) claims that "[p]edagogical knowledge addresses the *how* of teaching", which is acquired through learning and Shulma's (1986:9) "wisdom of practice". Overarchingly, PKC is defined as an amalgam of a special kind between content and pedagogy which only uniquely exists in the province of teachers and in their own special form of professional understanding (Shulman, 1987:8; Ball *et al.,* 2008:391). This definition suggests that pedagogical knowledge is one of the major components of PCK. In Kar's (2017:5) view a part of teaching, namely pedagogical knowledge, deals with how particular topics and problems are appropriately presented to accommodate learners' diversity. Without separating pedagogical knowledge from the PCK, Shulman's (1986:9) elaboration of his 'special amalgam' includes but is not limited to, the most powerful from of representation of ideas, most powerful analogies, illustrations, examples, explanations, and demonstrations - "the ways of representing and formulating a subject that makes it comprehensible to others".

As Shulman (1986: 9) articulated that pedagogical knowledge goes beyond the knowledge of the subject matter per se, his emphasis highlights the aspects that embody subject's teachability (see section. 2.3.1.2). This kind of knowledge draws a line between a mathematics specialist and mathematics pedagogue (Shulman, 1987:8). In line with the understanding that no single most powerful representation exists (Shulman, 1986:9) and that no one size fits all (Heritage, 2010: online), this study focuses on two aspects of pedagogical knowledge, namely 'content knowledge that embodies the aspects of content most germane to its teachability and presentation that makes mathematics concepts comprehensible to others (Shulman, 1986:9). Paradoxically, it is philosophised that "less is more" (Heritage, 2010b: online). Heritage (2010b, online) presents a seductive argument that teachers need to teach less and assess more. Her argument articulates that, for effective teaching to take place we need to know the level of our learners' conceptual understanding in order to teach appropriately. As we engage in formative assessment, we are able to identify the cutting edge of learners' competences. To summarize, the debate suggests that inter alia a learner-centred pedagogical approach seems to resonate with the call for understanding of learners' ZPD and learning zone (see section 2.4.3.3.6).

2.4.5 The need to encourage a learner-centred pedagogical approach

Shulman's (1986: 9) assertion that "no single most powerful form of representation" exists suggests that teachers must have a variety of forms of representation centred on learners' learning needs and styles. There is a body of knowledge recognising that learners have "different ways and have different learning styles" (Zain, Rasid & Abidin, 2012:320). According to Malan *et al.* (2014:3), "[m]ost students are not homogeneous in their background, knowledge, experience, learning abilities or learning styles". In essence, the heterogeneous nature of learners in a mathematics classroom makes the call for a learner-centred approach most relevant and urgent.

"Learner-centred teaching is a unified approach that focuses on student learning rather than on what the teacher does" (Alsardary & Blumberg, 2009:401). Apparently, learner-centred teaching (LCT) does not use only one teaching method but a plethora of different types that change the classroom discourse by creating a learning environment shifting away from instruction delivery (Alsardary & Blumberg, 2009:401).

This shift from what is commonly known as the traditional approach or a teacher-centred method towards teaching was caused by dissatisfaction with the behaviour-oriented perspective in teaching and learning (Bingolbali & Bingolbali, 2015:2601). Consequently, terms such as student-centred learning, student-centred pedagogy, child-centred learning, student-centred education, learner-centred learning and student-centred teaching came into use (Bingolbali & Bingolbali, 2015:2601). Common among these terms is that learners are at the centre of the focus. This study uses a learner-centred pedagogical approach (LCPA) as a preferred term to encapsulate the centredness of the learners in teaching and to coin shared common meanings of above-mentioned terms.

Learners needs are at the centre of LCPA and they determine how the teacher should facilitate learning. The literature claims that in LCPA, the fundamental focus is “on what learners (not teachers) are doing” (Weimer, 2002: xvi). Specifically, the research highlighted that LCPA focuses on learners’ needs such as their abilities, interests, and learning styles (Robinson, 2012:7). Arguably, this pedagogical style is underpinned by the epistemological stance that puts learning at the core of teaching practice instead of teaching (Smyth, 2005:809). LCPA is based on a constructivist theory of learning (Da Costa Alipio, 2014:25) and socio-cultural theories of learning (Vale, Weaven, Davies & Hooley, 2010: 571) which both advocate for the utilization of learners’ experiences and prior knowledge as the spring board for teaching. When implementing this approach, teachers need to value a collaborative approach to teaching while learners’ wisdom and their contributions need to be respected (Moate & Cox, 2015:379). Democratic classroom discourse is but one condition that enables the implementation of LCPA as the power relations shift and put the teacher at the peripheral positioning (Moate & Cox, 2015:379). To cater for different learning needs and learning styles, LCPA practitioners, use differentiated modalities to facilitate learning and inspire learners’ deeper learning (Heritage, 2010b:, online) by creating a “learning environment encouraging students to actively engage in and take ownership of their learning experiences” (Moate & Cox, 2015:379). Flores (2010:75) also attests that mathematics learners in an LCPA learn mathematics through actively participating in the acquisition of their mathematics knowledge. In contrast to LCPA, teachers who use teacher-centred models of teaching rely on lectures as the primary means of instruction (Moate & Cox, 2015:379).

The heterogeneous nature of learners in a mathematics classroom makes the call for LCPA such as PBL more relevant to accommodate different learning styles. "PBL is a learner-centred approach where students engage with problems with whatever their current knowledge/ experience affords" (Savery, 2006: 12-13). In presenting the success of PBL in mathematics teaching the research exhibits that learners that are taught using PBL develop better mathematics critical thinking ability as compared to those taught in conventional ways (Widyatiningtyas, Kusumah, Sumarmo & Sabandar, 2015:37).

> *"Being learner-centred focuses attention squarely on learning: what the student is learning, how the student is learning, the conditions under which the student is learning, whether the student is retaining and applying the learning, and how current learning positions the student for future learning" (Weimer, 2002: xvi).*

The notion of focusing on what learners do and conditions under which learning takes place, guides teachers to adapt their didactical approach to suit different learners' learning styles. "Understanding of learning in this way might enable teachers to see possibilities for scaffolding learners' ideas and mediate new meaning" (Brodie, Lelliott & Davis, 2002: 557). Subsequently, learner-centredness seems to improve learning as learners "need to create their own meaning of content and not just memorize what the teacher says" (Alsardary & Blumberg, 2009: 401-402).

On the other side, PBL theory views learner-centredness as an instructional method that utilizes real problems as primary pathway for learning (Ramsay & Sorrel, 2006:2). According to this approach learners work in small groups of six to ten learners (Mme, 2011: 3). In essence, PBL is a pedagogical approach based on problems (Rui *et al,,* 2015: 223). PBL is one of many forms of active learning that create opportunities for learners to exhaust their capabilities in solving a problem without the teacher's help. Furthermore, the implementation of PBL in mathematics teaching increased problem-solving skills, decision making and reasoning-processes (Erickson, 1999:520). Over and the above, we have earlier presented an argument on BPL's learner-centredness posture (see section 2.3.2.2.1).

2.4.5.1 The need to encourage a learner-centred pedagogical approach in SA

This section focuses on good practice in the implementation of LCPA and understanding of the impact of classroom discourses that are dominated by a teacher-centred pedagogical approach. In our attempt to understand the good practice in this regard, the study interrogated legislative frameworks and research findings in terms of SA experiences. Brunton (2003: H-51) has emphatically put it on record that "[m]eaningful education has to be learner-centred" In addition, a learner-centred approach is an underlying philosophy of the new curriculum in SA (DBE, 2011: 9). According to the DBE (2011:4), a learner–centred method allows learners the opportunity to develop and employ critical thinking skills. As a radical paradigm shift from a traditional banking concept in a learner-centred approach, learners become active agents engaged in construction of their knowledge. Education policies provide that teachers should adjust their teaching strategies to match the developmental stages of learners, different learning styles and other differences among learners (Brunton, 2003: A-49). Inherently, to embrace learners' diverse needs, the learner-centred participatory mode of teaching and activity-based education is emphasized (Brunton, 2003: H-47). The PAM document also articulates the duties of educators and presents mandates that commit teachers "to establish a classroom environment which stimulates positive learning and actively engages learners in the learning process" (DBE, 2016: A-24). Inter alia, teachers should utilize the learners' prior knowledge and learners' experiences as a fundamental and valuable resource (DBE, 2016: A 24).

From a research point of view, LCPA seems to be appealing as it brings hope of intellectual liberation and breaking away from oppressive traditional approaches (Nykiel-Herbert, 2004:249). In contrast to traditional approaches, LCPA proponents claim that it immensely benefits learners when it is judiciously applied (Mhlolo, 2013: 1). In the case of LCPA, learners become active participants in their learning (Msimanaga, 2017:153) instead of being treated as depositories who are expected to regurgitate like a parrot, without any meaning attached to what was deposited. Specifically, Umugiraneza, Bansilal and North (2017:2) reasoned that LCPA enables learners in developing mathematical reasoning. Their emphasis is on learners making meaning of mathematics concepts, while the teacher is seen by learners "as someone who is there to help them make sense of mathematics" (Umugiraneza *et al.,* 2017:2).

In line with Shulman's (1986: 9) seminal definition of PCK that "there are no single most powerful forms of representation", the literature seems to emphasize the uniqueness of learners in the teaching and learning process (Msimanaga, 2017:68). In line with Shulman (1986: 90) and Mismanga (2017: 68) our focal point of LCPA is learners' needs in the teaching and the learning process as they learn in different styles. Evidently, co-researchers in the study that was conducted by Msimanga (2017: 53) stated that teachers' screen-checking of learners' understanding and tracking down learners' progress were of paramount importance in determining appropriate LCPA suitable to unique learners' learning needs.

Despite the availability of evidence illustrating that didactics focusing on learning and learners' needs seem to yield positive results in terms of learners' mathematics critical learning, SA teachers "continue to teach in predominantly teacher-centred ways" (Brodie *et al.,* 2002: 546). Inter alia, teachers' conceptual knowledge has been highlighted as one of aspects which restrained teachers from engaging learners in a LCPA (Brodie *et al.*, 2002:546). It appeared that teachers used a teacher-centred approach to musk their subject matter incompetence. In an LCPA, the risk for learners to raise awkward questions which may not have readily available answers is high, hence teachers with shaky MPCK would be more comfortable with a teacher-centred approach where they have control of what happens in the classroom (Chick, 1996:21). According to Chick (1996:21) South African teachers adopted an authoritarian role by "doing most of the talking … with most of the pupil responses taking the form of group chorusing". Furthermore, they believed that their traditional practices were better than what the new curriculum suggested hence they did not show any intention to change from teacher centred to LCPA (Brodie *et al.*, 2002: 547).

It must be noted here that in the post-apartheid era a new curriculum "characterised as learner-centred" was introduced with the view that its judicious application would immensely benefit learners (Mhlolo, 2013:1). In contrast to the new curriculum expectations, the researchers lesson observations exhibited that "lessons were generally content driven, teacher-centred and transmission based" (Mhlolo, 2013:2). A recent study conducted by Chirinda and Barmby (2017:1) also echoed a body of knowledge that furnishes evidence indicating "that South African teachers tend to implement traditional approaches in the classroom". As observed by the researchers, teachers use examples to demonstrate the new content focusing on algorithms application (Chirinda & Barmby, 2017:1). Implicitly, learners are not necessarily engaged in concept understanding but are expected merely to follow the teacher's methods when they do exercises after the lesson demonstration. In the SA context, LCPA seems to be "much harder to achieve in practice than it appears to be in policy" (Umugiraneza *et al.*, 2017:2). Secondly, Umugiraneza *et al.* (2017:2) observed that LCPA, "is of the most pervasive ideas; yet it is very hard for them to take root in the classroom". Apparently, LCPA requires an education system that have teachers with sound knowledge of mathematics content (Umugiraneza *et al.*, 2017:2). In the SA context the opposite is true (Bansilal, 2012: 117; Spaull, 2013:5).

2.4.5.2 The need to encourage a learner-centred pedagogical approach in Namibia

In this section, we focus on Namibia again to explore good practices in relation to LCPA as articulated by legislative mandates and research findings to understand the impact of its non-implementation of LCPA. Notwithstanding the evidence that exhibits the domination of teacher-centred classroom discourses in SA schools, the education policy in sub-Saharan Africa has gradually moved "away from prevailing pedagogical traditions towards learner-centred pedagogy" (Vavrus, Thomas & Bartlett, 2011:33). Curriculum reforms inter alia included some elements of a learner-centred pedagogy (LCP), "such as active learning and critical thinking" (Vavrus *et al.*, 2011:33). Centrally to LCP, "[l]earners are not passive recipients of information but are active agents engaging in constructing their own knowledge" (Zain, Rasid & Abidin, 2012:319).

In Namibia, the ministerial policy provides for a teaching and learning approach based on a paradigm called Learner-centred Education (LCE). Apparently, in Namibia LCPA is referred to as Learner-Centred Education (LCE). The National Promotion Policy Guide provides presuppositions regarding LCE, where is clearly put that all learners can learn and develop when placed under the right circumstances that recognise differences from person to person (MoE, 2015b:3). The call to improve the education system hinges around the need to better respond to learners' needs through a commitment to LCE as stipulated in Namibian policies (MoE, 2014:3). In terms of LCE, what learners do and their needs are placed at the centre of the classroom activities (MoE, 2014:3). The policy mandates learners to be put at the centre of teaching and learning. This implies that the classroom discourse must start with establishing existing learners' knowledge, skills and insight in a particular topic area (MoE, 2014:3). In terms of LCE, the focus is on learners' active participation in their own learning and that of their peers (MoE, 2014: 3). This is based on the narrative that learners bring to "school a wealth of knowledge and social experience gained continually from the family, the community, and through interaction with the environment" (MoE, 2015a: 37). The policies suggest that this wealth of knowledge should be used as the starting point for teaching and learning (MoE, 2015a: 37). The policies further dispute the notion that suggests learners sitting in groups means LCE is taking place but agitate that "[i]t is the activities in which learners participate that make lessons learner-centred (MoE, 2014: 46). In a nut shell, the recognition of learners' different learning styles and learners' needs must be at the centre in terms of LCE implementation (MoE, 2014:129).

The introduction of LCE demands changes in teachers' roles, thus moving away from "authoritarian, instruction-oriented teaching towards a more supportive learner-centred approach" and the envisaged change could be achieved through professional development (PD) (Peters, 2016;125). This study attempts to close this gap by developing a strategy to enhance MPKC using PBL. Apart from policy prescriptions, Namibian scholars also made inquiries about the implementation of LCE at school level (Amakali, 2017:679, Mutilifa & Kapenda, 2017:1260 & Peters, 2016:8). It appears that using various learner-centred activities attracts learners' interest (Mutilifa & Kapenda, 2017: 1260). The results that emerge form Mutilifa and Kapenda's (2017:1260) quasi-experimental inquiry, show that the experimental group outperformed the control group. Overarchingly, this quasi-experimental inquiry did not only use one learner-centred activity to make learners actively participate in the teaching and learning process, as Shulman (1986:9) once theorized that there is no single powerful form of representation. In the Oshikoko region a case study also was conducted to explore the teachers' perceptions and the strategies they used to develop learners' understanding through the implementation of LCE (Amakali, 2017:679). The empirical evidence presented by Amakali (2017:686) reveals that teachers have some knowledge of LCE, as the participants argue that the "use of prior knowledge is the best way to teach" as it enables teachers to access learners' ideas. Namalwa, a participant in the above-mentioned case study contends: "… like to test their knowledge first before I supplement what they already know, it is learner centred 'mos'" (Amakali, 2017:686). Evidently, teachers use questioning, prior knowledge and pair and group work as strategies to implement LCE (Amakali, 2017:686). It must be mentioned, however, that changing from a teacher-centred to a learner-centred approach appears to have been a major shift in the Namibian education system (Peters, 2016:8). This paradigm shift was institutionalized in Namibia, resulting in the introduction of the Basic Education Teacher Diploma (BETD) programme founded on learner-centred pedagogical principles (Peters, 2016:39).

LCPA proponents theorize that teachers with learner-centred experience tend to create learning environments for learners to encounter critical learning incidents that develop learners' critical thinking (Amakali, 2017:680). For example, the learning environment created by using algebraic tiles empowered teachers to learn different ways of helping learners interpret and solve algebraic problems (Miranda & Adler, 2010:25). In Namibia innovations to ensure learner-centred mathematics teaching are accepted by teachers (Miranda & Adler, 2010:15). In contrast to the expectations, however, it appears that many mathematics teachers have no inkling of what learner-centred lessons look like, and implementing it may only remain a pipe dream (Miranda & Adler, 2010:15). A recent study conducted by Anyolo, Karkkainen and Keinonen (2018:67) confirms that "teaching and learning methods that take learners as passive listeners in the classrooms" continue unabated due to teachers' resistance to change to LCE. In Namibia LCPA implementation is hindered by strong authoritarian traditions and teachers' conflicting views of knowledge acquisition (Vavrus *et al.*, 2011:35).

In defence of the teachers Peters (2016:8) and Amakali (2017:684) report that teachers have no practical experience of LCE since no professional development (PD) was conducted for all teachers. The teachers complained about difficulties they experienced when trying to implement LCE and argued that it was new and different from the way they were taught (Amakali, 2017:684). Teachers have their own notion of LCE as they believe that asking questions which mostly require 'yes' or 'no' answers is the actual implementation of LCE (Peters, 2016:87). In addition, it is explicitly stated that teachers have a misconception of the notion that LCE "puts the learner at the centre of teaching and learning" (Amakali, 2017:680). It appears that teachers relinquished their responsibility of teaching as they tried to embrace the view that learners must take charge of their own learning" (Amakali, 2017:680). From the scholarly debate reported above, one could perhaps argue that the teachers neither had clarity on LCPA nor enacted it. In line with no single powerful form of representation (Shulman, 1986:9) and the unifying approach focusing on learning (Alsadary & Blumberg, 2009:401) our LCPA focuses on learners' learning needs, not necessarily special needs as put in terms of inclusive education when resolving issues of learners with impaired learning abilities. Our view encompasses the recognition and adjustment of teaching and learning activities to accommodate diverse learning styles as the bedrock of LCPA implementation.

2.4.5.3 The need to encourage a learner-centred pedagogical approach in Nigeria

In this section the report deals with good practice in the Nigerian context in terms of the implementation of LCPA. The inquiry focuses on policy mandates and research findings in this regard in order to understand the impact of non-implementation of LCPA. In as far as it could be determined, the policy articulation regarding LCPA that could be found was minimal. The Federal Ministry of Education mandates the formal curricular design to draw and borrow good practices from the non-formal route (Makoju, Obanya, Nwangwu, Fagbulu, Aderogba, Ayuodele, Olapeju, Yusufu & Kalu, 2005:8). Among other strengths of the non-formal route are democratic values, and learner-centred education programmes, "tapping from and building on learners' experiences, relevant and immediately applicable to the needs of learners" (Makoju *et al.*, 2005:8). Put differently, the curricular design is emphatically mandated to put 'learner's needs' at the centre of education programmes (Makoju *et al.*, 2005:8). It appears that education praxis from non-formal practices continue "to appeal to the interest of the learner … to retain them in the learning experience" (Makoju *et al.*, 2005:66). The learning experiences and the information barrage have led to a generation of empowered learners who no longer are passive, who demand respect and recognition from their teachers and may even know more than their teachers (Makoju *et al.*, 2005:66).

Despite limited policy articulation regarding LCPA, the literature has agitated for the need to transform mathematics classroom discourse from teacher-centred to LCPA (Okafor & Anaduaka, 2013:251) The above-mentioned call for change advocates for making mathematics learning meaningful to learners through LCPA (Okafor & Anaduaka, 2013: 251). Evidently, Idogho's (2016: 38) assertion suggests that LCPA appears to be the most suitable pedagogy for individuals' development. In line with Idogho's assertion, the findings of what Aremu and Salami (2013:366) call pupil-centred activity-based instructional strategy reveal that it produced teachers who are able to teach using activity-based primary mathematics lessons. Activity-based instructional strategy is a kind of learner-centred instructional strategy which has been shown to be more effective than teacher-centred instructional strategies (Aremu & Salami, 2013:357). Proponents of LCPA support each other on narratives claiming that learner-centred practices underpinned by constructivist philosophies, tend to actively involve learners and "produce greater motivation and academic improvement than those who use more teacher-centred approaches like the transmissive or lecture methods" (Atomatofa, Okoye & Igwebuike, 2016:1472). A recent study which compares the traditional lecture method and learner-centred strategies shows that learner-centred strategies such as PBL and blended learning alleviated learners' misunderstandings about the nature of mathematics and improved their acquisition of algebra (Ojaleye & Awofala, 2018:496).

In addition, the literature argues that a flipped learning and teaching strategy may enrich the learners' mathematics learning in very specific ways (Abah, Anyagh & Age, 2017:79), and is beneficial to the learning of topics such as calculus (Abah *et al.,* 2017:81). The flipped model of instruction has adapted a learner-centred culture, with teachers using any content available, including "international content to maximize classroom time via active learning that is dependent on school level and subject matter" (Abah *et al.*, 2017: 80). In a nutshell, a flipped classroom is regarded as a "strategic reversal of the traditional classroom" like analogously flipping a coin (Abah *et al.*, 2017:79). In a flipped model easily playable commercially produced videos are mostly used to package instructional content which is sent to learners' smart phones to enable them to explore the learning content before the class commences (Abah *et al.*, 2017:79). Learners bring to the class, what was used to be done through homework after the instructional content has been extensively explored. This institutional content is then clarified in the classroom through active engagement in the classroom activities. According to the Flipped Learning Network (2014, cited in Abah *et al.,* 2017:70) four pillars of flipping learning represented by the letters F-L-I-P are

Flexible environment, Learning culture, Intentional content, and Professional educator. In essence, F-L-I-P provides learners with opportunities to bring to class their unresolved mathematics problems. Over and above other advantages, F-L-I-P creates a team spirit among learners as they challenge each other's presented solutions to problems coined in the instructional content that has been set prior to class (Abah *et al.*, 2017:80).

Despite the positive findings reported, it appears that other research findings in Nigeria reveal that primary school teachers are unable to use LCPA (Aremu & Salami, 2013:365). The implications of these research findings are that school mathematics is fully taught through teacher-centred procedures (Abah *et al.*, 2017:7). Learners are expected to religiously follow teachers' algorithms as influenced by what Aremu and Salami (2013: 356) call "do-it-as-I-have-done-it syndrome", which dominates Nigerian classrooms as a classroom discourse. In a nut shell the observation of class room discourse in African countries reveals high "prevalence of transmission pedagogy with lecturing and drilling being common teaching methods in schools" (Vavrus *et al.*,, 2011:37). As a basic philosophy of the Nigerian education system teachers have to talk while learners are directed to listen (Idogho, 2016:39). This philosophy of the Nigerian education system is based on the assumption that learning will take place when teachers speak clearly (Idogho, 2016:39). On the other hand, Okoli, Ogbondah and Ekpefa-Abdullahi (2015:132) contend that teacher training in Nigeria does not embrace constructivist learning, learner-centred instruction and integration. This comes as no surprise, since there is little or no policy provisions that seek to advocate for LCPA.

2.4.5.4 The need to encourage a learner-centred pedagogical approach in the USA

This section explores the US policy mandates and research findings in terms of good practice to understand the impetus of non-implementation of LCPA in mathematics teaching. According to the Department of Education in New Jersey information and understanding are viewed as "collective responsibility and property of all who come to school to learn" (Rosenstein, Caldwell & Warren, 1996:649). Learners are also included in the above community of practice; hence New Jersey policy makers and teachers are seizing the opportunity to understand and employ different pedagogical approaches to enhance learners' conceptual understanding of mathematics (Rosenstein *et al.*, 1996:649). Specifically, the need to develop an instructional repertoire for such learner-centred classrooms cannot be over-emphasized (Rosenstein *et al.*, 1996:649). In response to the call for LCPA, the state of California has aligned learner-centred learning and individual learners' needs to academic and social success (California Department of Education, 2015:672). Overarchingly, a significant number of schools in the USA demonstrate the ability to use learner-centred instruction in all content areas (US DoE, 2012: 7). The observed collaboration among teachers is focused on increasing learners' performance and implementation of learner-centred instruction (USA DoE, 2012:7). Michigan's Consolidated State Plan *Under the Every Student Succeeds Act* mandates Michigan state to implement strategies called Top 10 State over 10 Years, which allows districts to focus on learner-centred instruction to ensure deeper learning, access to high-quality, meaningful and challenging learning experiences (Michigan Department of Education: 2017:98). The Long Beach Unified School District (LBUSD), California's third largest school district, in partnership with the national non-profit organization seems to lead in the national initiative on learner-centred teaching and learning strategies that put learners at the centre (US Department of Education, 2015: 6).

From research findings it appears that higher education and many universities have witnessed a growing interest in student-centred learning, that is, LCPA in our case (Wright, 2011: 92). Teacher-centred instruction has been out-performed by learner-centred instruction at promoting almost every conceivable learning outcome (Kaput, 2018:11). There is also a belief that more often than not, LCPA provides a more effective learning environment and college teachers are consequently moving towards LCPA (Wright, 2011: 96). Evidently, LCPA seems to be a dominant praxis in North Carolina as it is used as prerequisite for teacher licensure assessment (Cunningham, 2008: 12).

The following model shows a number of activities that take place in learner-centred mathematics classroom in terms of its characteristics (Walters, Smith, Leinwand, Surr, Stein & Bailey, 2014:5). The model precisely demonstrates two main aspects of a learner-centred mathematics classroom, namely classroom environment and mathematics instruction. We heavily relied on this model as we developed our own understanding of LCPA. The aspects of classroom environment may not be limited to what Walters and other posit, but may include effective use of manipulatives, integrated assessment and understanding of learners' mathematical misconceptions, as we have alluded to in the previous sections of this study report. The second aspect of the model presented by Walters *et al.* (2014:5) puts mathematical reasoning at the top of mathematics instruction, thus, "to understand the 'why' as well as 'how'". This model seems to precisely coin the essence of LCPA to aspects that put learners needs at the centre of learning and teaching (see Figure 2.13) below.

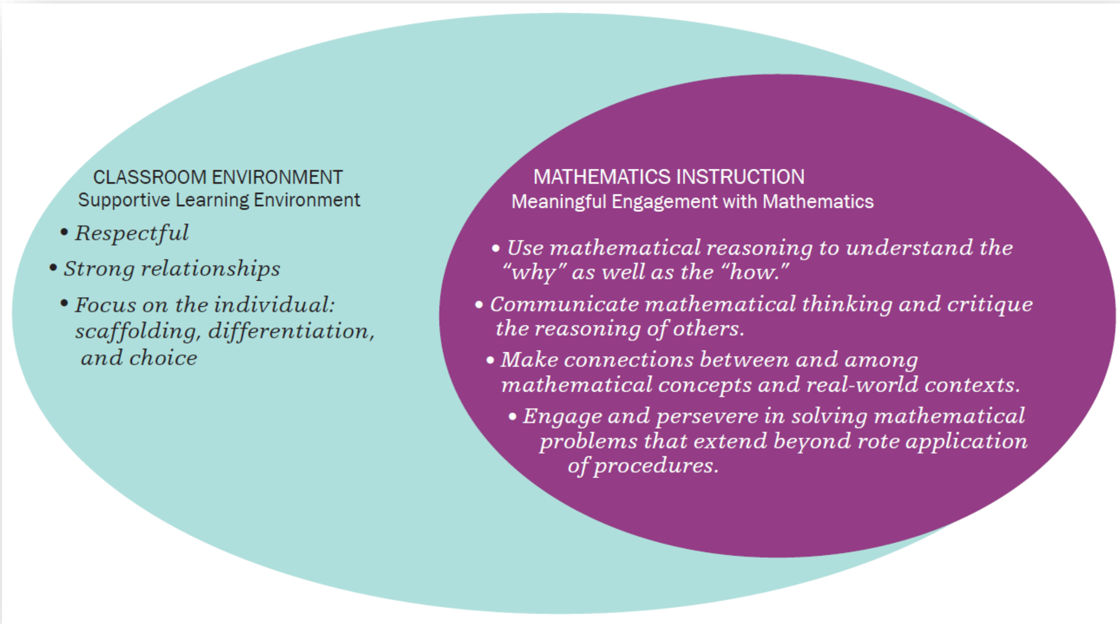


Figure 2.15: Characteristics of Student-Centred Mathematics Classrooms (Walters et al., 2014:5)

In addition to this scholarly debate, Walters *et al.* (2014:6) inquired about "different ways in which highly regarded high school mathematics teachers implement student-centred instructional practices". From their inquiry two prominent aspects emerged, namely the development of new mathematics rules and procedures, and reinforcement of prior mathematical learning (Walters *et al.*, 2014:12). The former refers to when learners are presented with a new mathematics concept or rule through a brief concept introduction and the teachers then guide the learners with "a sequence of examples and associated questions that, when completed, would lead to the new mathematical rule, procedure, or concept" (Walters *et al.*, 2014: 12). In the latter, learners are given the opportunity "to strengthen their understanding and practice applying mathematics content" (Walters *et al.*, 2014: 12). When reinforcing mathematical learning the teacher strategically discusses the solutions of the problems that were earlier given with the whole group to consolidate the understanding as groups and individuals substantiate their solutions (Walters *et al.*, 2014: 12). Students were provided with mathematics problems to solve, individually or in groups. The findings of the above-mentioned inquiry exhibit that learners in classes with more LCPA developed high levels of engagement and interest (Walters *et al.*, 2014: 36). These learners are also reported to have had high scores in the Programme for International Student Assessment (PISA) (Walters *et al.*, 2014: 36). Overarchingly, the results of the above-mentioned inquiry show that teachers can use LCPA despite existing challenges and difficulties by making sure that the mathematics concepts ensue in engaging the learners (Walters *et al.,* 2014: 37).

Despite the existence of irrefutable evidence putting it that "teacher-centred methods of teaching are not effective" (Aremu & Salami, 2013:356), in United States of America (USA) "teachers often employed" a teacher-centred approach (Polly & Hannafin, 2011: 120). In the mathematics lessons teachers continue to follow a teacher-based lecturing method which is consistent with traditional teacher-centred approach (Walters *et al.*, 2014.3). To mud waters, this prevalence of rote instruction continues unabated, despite its widely recognized flaws (Walters *et al.*, 2014:3). In confirming the dominance of teacher-centred approaches in American schools, Westberg (2014:8) further argues that

> *"[w]hile teacher-centred classroom structure is certainly the most common format in today's schools, we as teachers need to look at other teaching styles in order to engage our students and involve them in the learning process".*

The information shared above indicates an irreconcilable tension between the current practices and the urgent need for change. The research also enunciates that teachers seem to have misconceptions in terms of what LCPA means. The notion that learners must be active in the process of knowledge construction is mistakenly understood to imply an abdication of the teachers' role in the teaching and learning process (Mascolo, 2009: 2). The assertion that good mathematics teachers do not tell learners but facilitate learners' own developing mathematical powers (Cunningham, 2008:11) has far reaching consequences in terms of how teachers interpret their role in the classroom. This notion of teachers relinquishing their responsibility, flies in the face of LCPA implementation and it is sufficient to collapse any attempts to implement LCPA. It is also reported that teachers and learners tend to resist LCPA implementation (Weimer, 2002:xix) as it seems to disturb their social structure. Teachers tend to question the value of LCPA and argue that it lowers standards and panders to learners, while learners overtly or passively show "their preference for the way things used to be" (Weimer, 2002:xix). Evidently, from a body of knowledge that is readily available, the need for a paradigm shift from teacher a centred-approach to learner-centred approaches has been emphasised (Mhlolo, 2013:1, Vavrus et al, 2011: 31 & Westberg, 2014: 8). However, the teacher-centred approach seems to be highly prevalent in other school systems across the globe. The research confirms the assertion that a teacher-centred approach is not only a challenge in South African schools but it also manifested in other sub-Saharan countries (Vavrus *et al.*, 2011:35)

2.4.5.5 The impact of non-implementation of LCPA

Due to the dominance of the teacher-centred approach at a very tender age of learners' lives, many learners tend to "get the impression that mathematics is an abstract and difficult subject reserved for a selected few with 'magic' brains" (Okafor & Anaduaka, 2013: 251). In terms of how learners experience the teaching and learning process their view has not been negated but affirmed (Okafor & Anaduaka, 2013:251). It is also reported that the use of traditional teacher-centred approaches in mathematics has consequently made learners to "often view mathematics as a set of isolated procedures" (Celik, 2018: 1963). According to Nyaumwe, Bappoo, Buzuzi and Kasiyandima (2004:33), traditional approaches, which involve "teacher-centred instructional methods that do not make learners develop conceptual understanding of mathematics", have been criticised because they do not encourage problem-solving skills in learners. Instructional methods based mainly on teacher talk, do not involve much questioning, discussion or individual development of understanding (Umugiraneza, Bansilal & North, 2017:2). The challenge with a teacher-centred approach is the lack of focus on high order cognitive processes such as critical thinking and evaluation of outcomes but it is focused on "listening, memorising and recalling information" (Marre & Molepo, 2005:732). As a result, learners often acquire inadequate mathematics concept understanding and only develop superficial knowledge through "rote learning of basic concepts" (Marre & Molepo, 2005: 732).

Freire (1970 71) discusses a scenario where a teacher presented to the learners that "[f]our times four is sixteen" and concluded that the learners tended to record and repeat what the teacher says without perceiving the actual meaning of four times four. "Narration (with the teacher as narrator) leads to students to memorize mechanically the narrated content. Worse yet, it turns them into containers, into receptacles to be filled by the teacher" (Freire, 2005:71-72). In essence, learners are expected to simply swallow information without engaging with the subject content being taught (Ganyaupfu, 2013:30) with the intention to regurgitate it like a parrot when required to do so. Freire (1970:72) puts it that under this method,

> *"[e]ducation thus becomes an act of depositing, in which the students are the depositories and the teacher is the depositor. Instead of communicating, the teacher issues communiques and makes deposits which the students patiently receive, memorize, and repeat. This is the 'banking' concept of education, in which the scope of action allowed the students to extend only as far as receiving, filing, and storing the deposits".*

In this classroom discourse the teacher is the only one in control of both knowledge transmission and the pace at which learners are supposed to acquire knowledge. Furthermore, activity-based learning is not encouraged for learners to learn to solve real-life problems and as a result learners may get lost in terms of understanding (Ganyaupfu, 2013:30).

Furthermore, the contrast between the banking concept of education and problem posing education revealed the urgent need for educationists to break away from the traditional method of teaching. The latter, views dialogue as an indispensable act to unearth reality while the former, rejects dialogue (Freire, 1970: 83).

> *"Banking education (for obvious reasons) attempts, by mythicizing reality, to conceal certain facts which explain the way human beings exist in the world; problem-posing education sets itself the task of demythologizing … [b]anking education treats students as objects of assistance; problem-posing education makes them critical thinkers" (Freire, 1970: 83).*

In line with theorization contemplated above, the literature does not contend with the view that learners can no longer be taught with the same traditional methods that have previously failed them (Malan *et al.*, 2014: 2). As has been demonstrated by research teaching is not merely about dispensing rules and procedures for learners to mechanically memorize "but should also actively engage students as primary participants (Ganyaupfu, 2013:30). PBL as learner-centred approach suggests "a shift form content coverage to problem engagement" (Malan *et al.*, 2014:2). The radical movement from viewing students as passive recipients to critical thinkers as they are engaged with problems signifies a significant break with traditional approaches (Malan *et al.*, 2014:2). The rejection of an absolutist view of knowledge by PBL suggests that there is an urgent need to move towards a learner-centred approach to embrace multiple perspectives of learners' knowledge as corner stone for teaching and learning.

2.4.5.6 The impact of a learner-centred pedagogical approach (LCPA) implementation

It is believed that fundamental mathematics concepts are drawn from human experiences and existence (Ahmed, Clark-Wilson, & Oldknow, 2004:318). The view that there is a plethora of ways to communicate mathematical relationships to make them accessible to others (Ahmed *et al.,* 2004:318) is in line with the denouncement of the existence of a single powerful representation (Shulman, 1986: 9). It appears that embracing LCPA will help us to unshackle the views that chain us to only formalizing mathematics. Evidently, formalizing mathematics constrained us from valuing "common sense based on experience as a valid, indispensable and legitimate basis for checking the mathematical procedures and algorithms" (Ahmed *et al.*, 2004:318). Learners' communication of their common sense-based on mathematics experiences in a learner-centred classroom, may enable teachers to draw from a 'mathematics tool box' appropriate strategies to address the needs of learners. Arguably, LCPA proponents posit that a learner-centred classroom enables teachers to "transform mathematics classrooms into lively, engaging learning environments in which" learners make meaningful connections with their learning experiences and environment to the real world (Walters *et al.*, 2014:2). Apparently, in a learner-centred classroom the focus is on what learners do, hence teachers use what Alsardary and

Blumberg (2009: 401) call a 'unified approach' to accommodate different learning styles. A body of knowledge exists that suggests that LCPA is the same as PBL, the blended approach, and the flipped model which develop learners' critical thinking ability, increase their problem-solving skills, decision making and reasoning processes, attract learners' interest, alleviate learners' misunderstandings about the nature of mathematics, and learners out-perform those taught through a traditional teacher-centred approach (Widyatiningtyas *et al.,* 2015:37; Erickson, 1999: 520; Umugiraneza *et al.*, 2017:2; Mutilifa & Kapenda, 2017:1260; Atomatofa, *et al.*, 2016:1472; Ojaleye & Awofala, 2018:496; Abah *et al.,* 2017:79; Kaput, 2018:11; Walters, *et al.*, 2014:36). The implication of improving learners' performance and enhanced clarity on mathematics concepts may suggest that LCPA has a positive effect on teachers' PCK who practise LCPA both as classroom discourse and as discursive practice.

2.4.6 The need for mathematics content knowledge for teaching

In section 2.4.4 we focused on pedagogical knowledge, one of the PCK components drawn from Shulman's (1987:8) special amalgam of content and pedagogy. In this section the study focuses on content knowledge, the other PKC component in the above-mentioned special amalgam. Content knowledge is viewed as the organised amount of knowledge in a teacher' mind (Shulman, 1986:9). Teachers' understanding in this regard goes further to clarify why something is so (Shulman, 1986:9). In advancing Shulman's debate about PCK, Ball, Thames and Phelps (2008:395) further present what they call 'mathematics knowledge for teaching' (MKT). This kind of knowledge is a prerequisite needed for anyone to be able to teach mathematics (Ball *et al,*, 2008:395).

Drawing from Shulman's special amalgam, Ball *et al.* (2008: 399) further meticulously developed and presented domains of MKT as reflected in their model. This study will only focus on the common content knowledge (CCK) and specialized content knowledge (SCK) as other domains have been discussed earlier (see section 2.3.1). CCK is referred to knowledge of mathematics that could be used by everyone including teachers, as it is not only specific for teaching purposes (Ball *et al.*, 2008: 399). The implication of CCK is that teachers need to have this knowledge of mathematics like non-teachers who use mathematics for their interests. It is argued that a teacher would apply CCK like any other mathematician to determine whether the solution of a mathematics problem is correct or wrong (Mosia, 2016:59). Teachers must be able, for an example, to determine that $1/2 \div 1/6 = 3$ is correct. Other than recognizing the inaccurate definitions from textbooks, teachers must also be able to write accurate mathematics notations on the board (Ball *et al.*, 2008:399).

Secondly, SCK is knowledge of mathematics including skills uniquely existing in the knowledge area of a teacher, as it is not required for any other purpose except teaching (Ball *et al.*, 2008:400). Specifically, teachers that have acquired SCK should know that learners are likely to give 12 or 20, as answer when the equal sign is interpreted as a signal to add in the following problem $5 + 7 = \_\_ + 8$ (Ball *et al.*, 2008:393). In addition, the identification of misconceptions that learners have, including the understating and unpacking of $x^0 = 1$ to make meaning to learners is an example of SCK. It appears that teaching encapsulates decompressing compressed mathematical knowledge to reveal concept meaning to learners. Teachers need, for example, to unpack the process that leads to x^0, so that learners can understand that any number to the power zero is equal to one as a rule, but also develop a deeper meaning underpinning this notation. According to Ball *et al.* (2008:401), this SCK is only necessary for teaching, and not required in other settings. Teachers have to unpack algorithms and make particular content features visible and comprehensible to learners through unpacking compressed mathematical knowledge (Ball *et al.*, 2008:400). For teaching purposes, one needs to explain how to divide by a common fraction and justify why we invert and multiply when we divide by a fraction (Ball *et al.*, 2008:400). Apparently, the concepts like reciprocal and realization that it is easy to divide by one becomes appropriate and relevant. However, this kind of mathematics knowledge may not be relevant or used for any other settings other than teaching, hence it is referred to as SCK.

Moreover, the researcher presents a construct referred to as mathematics integrity (Wu, 2018:14), which seems to specify mathematics content knowledge for teaching (MCKT). Mathematics integrity is characterized by five principles, namely: precise concept definition; every mathematics statement is supported by reasoning; precise mathematics assertions; coherent presentation of mathematics topics, and purposeful presentation of mathematics topics (Wu, 2018:14). Precise concept definition eliminates misunderstanding as it clearly clarifies “exactly what the concept does or does not say” and enables the possibility of logical deductions (Wu, 2018:15). Furthermore, supporting every mathematics assertion with meaningful reasoning enables learners to realize that mathematics is learnable (Wu, 2018:17); it is not a set of unrelated meaningless rules. In terms of mathematics integrity teachers need to be able to explain the reasoning of say a negative multiplied by negative is equal to positive, and why we change division to multiplication and convert the fraction when we divide by a fraction. In essence, mathematics concepts are coherent, as an embroidery, where skills and concepts are logically interwoven (Wu, 2018:18). The division concept is essentially the same for whole numbers and fractions (Wu, 2018:18). It is also elucidated that mathematics concepts are taught for a particular mathematics purpose (Wu, 2018:19).

2.4.6.1 PBL and the development of mathematics knowledge for teaching

In this section the study explores the influence of PBL in developing mathematics knowledge for teaching. PBL has been discussed earlier (see section 2.3.3 & section 2.4.4), nonetheless, this section focuses on PBL in relation to mathematics teachability. The outstanding feature of PBL is that learning and teaching experiences begin with problems (Merritt, Lee, Rillero, & Kinach, 2017:4; Holmes & Hwang, 2016:449; Barge, 2010:7; Hung, 2013:32; Gram, Jæger, Liu, Qing & Wu, 2013:764). The narrative that in the learning problem is encountered first, suggests that the application of the knowledge drawn from solving the problem enables to enthuse learners, “aid in retention, and assert that knowledge used is better remembered” (Schmude, Serow & Tobias, 2011:678). In terms of PBL this implies that integration, the acquisition of new skills and knowledge construction begin from the process of untangling the problem (Höhle, 2005:99).

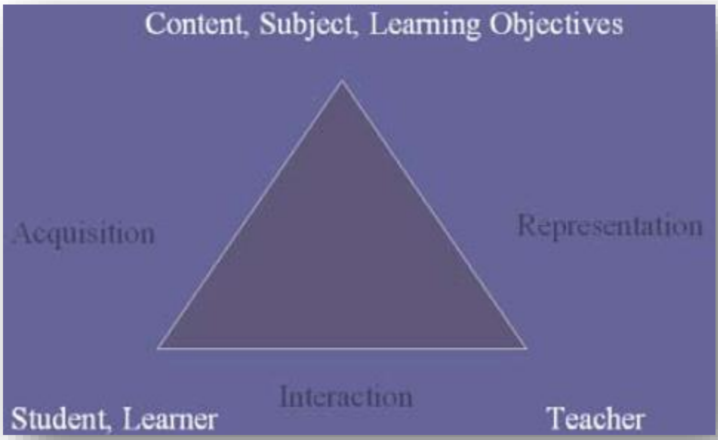


Figure 2.16: Characteristics of Student-Centred Mathematics Classrooms (Walters et al., 2014:5)

According to Laursen (2004 in Höhle, 2005:99) the above triangle model (Figure 2.14), illustrates how the teacher represents subject and learning objectives through interacting with learners. It also shows how learners acquire skills and knowledge through interaction with the teacher (Höhle, 2005:9). These problems are not necessarily solved by a simple algorithm due to their complexity (Hmelo-Silver & Barrows, 2006:24). They require active learners who work collaboratively in small groups "to investigate, pose questions, gather information, and carry out the work necessary to resolve the problem" (Merritt *et al.*, 2017:4). In Aalborg the University has established what is called group rooms to create an environment for PBL implementation (Höhle, 2005:104; Li & Henriksen, 2011:664). Group rooms are like offices that can accommodate a small number of students and they are situated next to the teachers' rooms to enable students to get help at all times of the day.

In addition, it appears that the PBL environment tends to create a more motivating atmosphere that encourages learning of the necessary information for the problem to be solved (Holmes & Hwang, 2016:449). Put differently, in the PBL environment learners do not avoid challenges in mathematics learning but become what Holmes and Hwang (2016:449) call 'challenge seekers', as the group dynamics tend to provide learners with cognitive scaffolding. In essence, learners are provided with the opportunity to learn how mathematics theories are derived (Hmelo-Silver & Barrows, 2006:23). For an example, learners could investigate the relationship between the exterior angle of a triangle and the opposite interior angles of a triangle. In the process of investigating the given theorem or mathematics concept and procedure, the PBL environment allows them to develop skills related to “what questions to ask, how to make predictions from theories, and how theories and rules can be tested” (Hmelo-Silve, & Barrows, 2006:23). While learners rely on their peers in a group, including mathematics manipulatives provided (Abdullah, Tarmizi & Abu, 2010:375), they also have massive opportunities to learn mathematical processes associated with communication, representation, modelling, and reasoning (Abdullah *et al.*, 2010:371). According to Barge (201:10) feedback and reflection are the most important outcomes of the PBL model drawn from peer and supervisor’s critique, thus, the teacher in our case.

Another feature of PBL that seems to also stand out is LCPA (Mustaffa, Ismail, Tasir & Said, 2014:2). On the same wavelength, it is also asserted that PBL promotes learner-centred learning and teaching processes (Schmude, Serow & Tobias, 2011:678). On the other hand, Hmelo-Silver and Barrows (2006:24), view it as “a premier example of a student-centred learning environment” in which learners are presented with an opportunity to collaboratively co-construct knowledge through productive discourse practices. Nonetheless, in this study report, the learner-centredness concept has been discussed extensively although it has not necessarily been explored as PBL pe se (see section 2.4.4). According to Mustaffa *et al.* (2014: 3), there is a need for further research in relation to the use of PBL in learning mathematics as it is not commonly practised in school. On the other side, Schmude *et al.* (2011) also argue that PBL has not been extensively used in teacher education thus far. In view of this apparent gap, this study is investigated the use of PBL to enhance MPCK for Grade nine mathematics teachers.

Abdullah *et al.* (2010: 375) conducted a study to explore the effects of PBL on mathematics performance and instructional efficiency comparing the affective products of learning between PBL and the conventional teaching strategy. The findings of their study reveal that PBL enables learners to solve problems and improve their verbal and written communication skills through working collaboratively with others (Abdullah *et al.*, 2010:375). It also appears that learners in a PBL environment outperformed their counterparts in traditional classroom discourse in terms of their ability to retain knowledge learned (Holmes & Hwang, 2016:449), and to apply knowledge to novel problems and real-world scenarios (Hmelo-Silver & Barrows, 2006:24; Holmes & Hwang, 2016:449). One of the most significant claims of the impact of PBL is the improvement of learners' "deeper understanding of mathematical concepts (Holmes & Hwang, 2016:449). In echoing other proponents of PBL, Hung (2013:32) maintains that knowledge acquired form a PBL set up, "is not just theoretical concepts and principles but a set of usable working knowledge". Teachers that use PBL also claim that it is a solution that encourages understanding of concepts when learning (Mustaffa *et al.*, 2014:8).

Perhaps the safe environment in which learners engage with problems in a PBL setting could be attributed to the claimed success of PBL. It is observed that the safe environment where learners are not worried about being judged enables them to speak more and "articulate their views safely while also hearing the views of their peers" (Holmes & Hwang, 2016:449). In essence, when the teachers establish the cutting edge of their learners' competences or ZPD, they are able to provide appropriate cognitive scaffoldings. It could be argued that participating in a dialogue and sharing views at the same time justify why one's view should be accepted as valid may facilitate solid construction of mathematics knowledge (Holmes & Hwang, 2016:449). In a PBL environment the teachers' role changes from being knowledge dispensers, but "create affordances for productive discourse" (Hmelo-Silver, & Barrows, 2006:24). In line with PBL principles, the teacher facilitates the learning process, pushing learners to think deeply by using modelling of the kind of questions learners need to be asking themselves (Hmelo-Silver & Barrows, 2006:24). Specifically, the teacher becomes an expert learner in PBL through modelling "good strategies for learning and thinking, rather than providing expertise in specific content" (Hmelo-Silver & Barrows, 2006:24). As an expert learner, the teacher acts as cognitive apprentice to provide cognitive scaffolds (Hmelo-Silver & Barrows, 2006:24) in decompressing complex mathematics concepts.

2.4.6.2 The need for mathematics content knowledge for teaching in SA

This section explores SA policy mandates and research findings in terms of good practice to understand the impetus in inadequate mathematics content knowledge for teaching. As part of teachers' competencies and skills provided by professional qualifications, teachers are expected to have basic knowledge of subjects they teach (DBE, 2016: B-44). Departmental prescriptions further provide for *inter alia*, the aims of teaching and learning mathematics to be understood as a human activity encouraging "deep conceptual understandings in order to make sense of mathematics" (DBE, 2011: 8). The conceptual understanding includes, but is not limited to using "knowledge of reciprocal relationships to divide common fractions" (DBE, 2011:17). The National Development Plan (NDP) advocates for teachers to have a sound knowledge of subjects they teach, particularly mathematics (NDP, 2013: 303). In a nut shell, the NDP (2013: 307) mandates the teacher development sectors to improve teachers' knowledge through building teachers' subject knowledge and providing training in effective teaching methods.

A positive correlation between learners' performance and teachers' comprehension confirms the view that "teachers cannot help learners with content that they do not understand themselves" (Venkat & Spaull, 2015:122; George & Adu, 2018:141). In line with the policy expectations, the research also affirms that teachers should not only have sufficient knowledge for a particular subject but should also know their learners' strengths and weaknesses (George & Adu, 2018:141). For teachers to be competent in MCKT, the research suggests they should "focus on developing teachers' capacity for mathematical explanations … knowing and doing mathematics in ways that are helpful for teaching mathematics" (Venkat & Spaull, 2015:128). In line with a focus on developing teachers' capacity in terms of MCKT, proportional reasoning has been identified as starting point (Venkat & Spaull, 2015:129). Proportional reasoning is described "as central to multiple mathematical topic areas: fractions, percentages, ratio, and many covariation situations". When proportional reasoning has not been frequently attended to in the early and middle years of schooling, it tends to create difficulties for later understanding of algebraic functions (Venkat & Spaull, 2015:129).

The study conducted by Mji and Makgato (2006: 206) revealed that SA mathematics teachers have challenges regarding mathematics content knowledge. According to Mosia (2016: 2), SA teachers seem to have a glaring inadequacy in terms of Euclidean geometry content knowledge. One of the learners who participated in Mji and Makgato's (2006: 206) study complained that they were memorizing and they did not understand. "*When we ask the teacher ... he does not know. What can we do?"* (Mji & Makgato, 2006: 206). Educators that participated in Mji and Makgato's (2006: 206) inquiry also acknowledged the shortcomings they had with respect to certain topics. A study that explored the teachers' understanding of the concepts of quadratic equations, patterns, functions (hyperbolic and quadratic), aspects of calculus and linear programming (Bansilal *et al.*, 2014:35) showed that mathematics teachers in KwaZulu Natal "do not know sufficient school mathematics – which needs to be addressed urgently" (Bansilal *et al.*, 201449). The literature resonates that poor subject knowledge, and poor mathematics teaching and learning are serious problems in SA education (Diko & Feza, 2014: 1457). Apparently, poor training of teachers resulted in a significant content knowledge gap (Umugiraneza, Bansilal & North, 2017 72).

2.4.6.3 The need for mathematics content knowledge for teaching in Namibia

This section is devoted to exploring Namibian policy mandates and research findings in terms of good practice to understand the impetus of inadequate mathematics content knowledge for teaching. The education policy in Namibia stipulates that teachers should not teach from the textbook, but they should be well-acquainted with the syllabus content (MoE, 2008:2). The syllabus and the scheme of work provided by the National Subject Policy Guide give guidance on how to teach and administer mathematics at schools (MoE, 2009:1). "Mathematics teachers should be creative and innovative to produce their own teaching and learning materials linked to practice" (MoE, 2009: 6). On the other hand, the subject heads are besieged by a mandatory task of improving teachers' competences through team building and continuous professional development (MoE, 2009: 10). Moreover, student teachers are placed at schools during school-based studies and observe class teachers when modelling good practice. Through critical reflection on their teaching practices juxtaposed with the observed good practice, student teachers evaluate their learning in order to improve their teaching ability and develop appropriate subject knowledge and concepts (MoE, 2009 14).

Namibian scholars also articulate the importance of MCKT in teaching mathematics. The University of Namibia (UNAM) has established a unit for continuous professional development to enhance teachers' content knowledge (Kasanda, 2015: 195). Teachers that participated in continuous professional development (CPD) listed some of the aspects they viewed relevant to their work to ensure effective teaching and learning, namely different assessment methods, setting relevant questions and solving equations (Kasanda, 2015: 195). Mathematics continuous professional development (MCPD) proved to be beneficiary as participants highlighted that it presented an opportunity to interact with others and provided materials to be used as reference in their classroom practice (Kasanda, 2015: 195). It has been also reported that Realistic Mathematics Education (RME) conducted thirteen workshops, and the majority of participants "made effective use of the opportunity to increase their content knowledge and to improve their teaching, by taking part as active, independent learners and problem-solvers" (Peters, 2016: 348). These research interventions were in recognition of MCKT as the prerequisite for teaching mathematics.

Nonetheless, evidences in Namibia indicate that mathematics teachers lack mathematics content knowledge (Kasanda, 2015: 196; Nambira, 2016: 35). Kasanda (2015: 193) reported the responses from principals who participated in their inquiry and their responses seemed to suggest that the advisory teachers failed to contribute in enhancing the pedagogical and content knowledge of the Mathematics teachers. The lack of subject content knowledge was identified as on the listed factors that contributed towards learners' poor performance in mathematics (Nambira, 2016:35-36). It was observed that unqualified mathematics teachers who mostly came from rural regions of Namibia, exhibited more incompetency in mathematics content knowledge (Nambira, 2016: 36). The incompetent teachers apparently are unable to logically sequence concepts for learners to understand, instead, they confuse learners and have trouble to present analogies and subsequent explanations (Fuma, 2018: 4). According to Fuma (2018:4) teachers that lack mathematics content knowledge, rely heavily on textbooks and are unable to pose questions which engage and stimulate learners' critical thinking. It seems that academic courses offered by colleges could not provide adequate subject knowledge as they failed to cover several crucial areas of professional competence (Peters, 2016:5). As a result, these teachers would constantly "commit errors such as mispronunciations, confuse explanations of fundamental concepts, and misappropriate units of measurements" (Fuma, 2018:5). Due to inadequate content knowledge, these teachers were unable to tangibly scaffold learners' understanding.

2.4.6.4 The need for mathematics content knowledge for teaching in Nigeria

In this section the study explores good practice in the Nigerian context in terms of MCKT. The inquiry focused on policy mandates and research findings in this regard in order to understand the impact of inadequate MCKT. As the Federal Department of Education introduced curriculum reforms in Nigeria, learners were expected to be taught by qualified teachers, fully armed with teaching skills and techniques (Awofala, Ola-Oluwa, & Fatade, 2012:6). This calibre of teachers with sound knowledge was expected to select appropriate and adequate facts for planning their lesson notes (Ayeni, 2010:144). According to Obilor (2012:44) teachers' mathematics knowledge manifests in a plethora of "representations including metaphors and illustrations" in the classroom discourse. The notion that one cannot give what he/she does not have suggests that mathematics teachers must acquire knowledge of the subject matter during training (Okafor & Anaduaka, 2013:249). Accordingly, it was suggested that teachers should attend training workshops to improve their subject matter knowledge (Obilor, 2012:48; Zuye, 2014:117).

Regardless of the call for professional learning to improve teachers' PCK and their ability to recognize and remediate learners' algebraic misconceptions (Ladale, 2013 ii), it seems that teachers had limited mastery of content knowledge. Since the curriculum reforms, teachers seem to struggle to understand some topics and terminology used in the curriculum (Awofala *et al.*, 2012:2). Apparently, the curriculum innovations have exacerbated the challenge of poor mathematics content knowledge. Furthermore, teachers that participated in Ladale's (2013: 55) study admitted that they did not understand word problems and consequently used few of them. According to Zuye (2014:121) teachers' response to a problem involving 4n+7, exhibited that they "matched the variable n with only a particular number, and hence had difficulty understanding the various uses of the variable concept". Evidently, the teachers' inability to understand mathematics problems, although elementary in nature, is a serious challenge for mathematics teaching (Zuye, 2014:121). It is further posited that teaching and learning of geometry seemed to be in jeopardy due to teachers' inadequate knowledge in this branch of mathematics (Zuya & Kwalat, 2015:112).

2.4.6.5 The need for mathematics content knowledge for teaching in the USA

This section explores US policy mandates and research findings in terms of good practice to understand the impetus of inadequate mathematics content knowledge in mathematics teaching. The Department of Education in New Jersey has long mandated schools and districts regarding continuous capacitation of teachers with inadequate content knowledge and limitations in terms of instructional repertoire (Rosenstein, *et al.*, 1996:532). In this regard, the New York State Education Department also adopted an intervention programme to enhance teachers' mathematics content knowledge (Louie, Sanchez, North, Cazabon, Melo & Kagle, 2011:2). The State Board of Education in California also included the need to improve teachers' knowledge of mathematical language (California Department of Education, 2015:689). It is also postulated that professional development that allows teachers to determine their learning trajectories as they focus on key problems of instructional practice seem to be most profitable in strengthening content knowledge and recognition of learners' misconceptions (Atkinson & Minnich, 2014:7). In elaborating the narrative that knowledge of content needed for teaching goes beyond simply "knowing" the content, Ball *et al.* (2008: 393) differentiated the task of finding the perimeter of a rectangle from analysing learners' unanticipated generalization about the relationship between perimeter and area. Identifying and understanding processes involved in remediation of learners' generalization about the relationship between perimeter and area is content knowledge needed for teaching. This special knowledge is different from only knowing how to find the perimeter of a rectangle. It also pronounces that teachers must constantly advance their MCKT for the enhancement of their learners' mathematics performance (Lee *et al.*, 2018: 76). The literature further proposes that to foster deep mathematical knowledge development, teachers should be engaged in learning mathematics through problem solving and work with peers and mentors in a reflective way (Masingila, Olanoff & Kimani, 2018: 431).

Despite the importance of MCKT in terms of developing teachers PCK and consequently learners' performance in mathematics (Lee *et al.,* 2018:76), it appears that US teachers lack precise definitions of concepts such as fractions, negative numbers, the meaning of division of fractions, decimals, constant rate, percent and slope (Wu, 2018:24). Wu (2018:24) takes it a step further and reveals the absence of precise reasoning of teachers regarding how to divide fractions. On the other hand, the research has put it on record that the improvement of teachers' MCKT has a reciprocal effect in enabling teachers to connect mathematical topics (McCoy, 2011:25). Notwithstanding McCoy's assertion, Wu (2018: 18) philosophised that the lack of coherence in mathematics teaching resulted in the division of fractions to be "still a much-feared concept at the moment". The research presents a scenario of a teacher (Ms Daniels) who could not answer a question regarding the division of fractions (Borko *et al.*, 1992:197). Elies, a learner in Ms Daniels's class asked the following question, "I was just wondering why, up there when you go and divide it and down there you multiply it, why do you change over?" (Borko *et al.*, 1992:197). Ms Daniels tried to answer using convoluted explanations and diagrams that did not work, and as a result she ultimately abandoned the question and moved on with the algorithm used for division by fraction (Borko *et al.*, 1992:198). Although she was a qualified mathematics teacher with a university degree, it seems that her course work did not help her in terms of conceptual understanding of division by fractions (Borko *et al.*, 1992:216).

MCKT has been flawed by a pervasive Textbook School Mathematics (TSM) over four decades (Wu, 2018:12). Arguably, this kind of flawed mathematics knowledge has been shared mostly in mathematics education circles until its claimed flaws ceased to be noticeable (Wu, 2018:12). It is argued that TSM does not regard precision as its main concern, for an example, the "definition of a^0 is presented - informally to be sure - as 'reasoning', and the result is that this motivation for a definition is commonly misconstrued as a proof of the theorem that for any $a > 0$, $a^0 = 1$" (Wu, 2018:12). Apparently, teachers tend to rely on TSM, which unfortunately does not provide precise definitions and precise reasoning why a^{-n} is written as $1/a^n$. According to McCoy (2011: 4), many learners are still subjected to mathematics classes taught by teachers with limited understanding of MCKT. Due to imprecisions of TSM, teachers get confused "between what a definition is and what a theorem is" (Wu, 2018: 26).

2.4.6.6 The impact of poor MCKT in relation to teaching of mathematics

An urgent need exists to address the challenge of a lack of basic content knowledge amongst teachers, particularly those in rural areas to avert the harm caused by these teachers to learners. Learners taught by incompetent mathematics teacher do not only lose confidence in the teacher but, importantly, they lose confidence in the subject as well (Okafor & Anaduaka, 2013:249). Teachers with insufficient conceptual understanding experience limited wisdom of practice (Lee, 2018:84) and are more likely to use inappropriate techniques that undermine learners' long-term learning trajectories (Spaull, 2013:29). Among others, these inappropriate techniques include, but are not limited to, just converting and multiplying when dividing by a fraction without understanding the concept of dividing by a fraction. Evidently, Thabo, a student participant in Mji and Makgato's (2006: 260) study protested that when he told the teacher that he did not understand, the teacher shouted at him telling him to use his brains. The teacher claimed not to understand how it happened that Thabo did not understand "such an easy sum" ((Mji & Makgato, 2006:260). The above scenario demonstrates poor MCKT, and as a result the poor teacher resorted to shouting, instead of using an alternative technique to help Thabo to understand. It is also posited that teachers with inadequate MCKT heavily depend on textbook explanations (McCoy, 2011:26) and fail to create an environment conducive to mathematics learning, instead they tend to suppress learners' interest in the subject (George & Adu, 2018:142).

2.4.6.7 Impact of sound MCKT in the teaching of mathematics

It is maintained that developing MKT has a number of benefits for the teaching process (Ball & Bass, 2003: 28). These benefits include, but are not limited to, enabling teachers to decompress mathematical concepts, skills, and procedures, while connecting mathematical ideas within and across mathematical domains. (Ball & Bass, 2003: 28). Evidently, MCKT prompts mathematics dialogue during the classroom discourse in ways that learners can understand. In line with Ball and Bass's argument, strong professional development seems to encourage teachers to self-reflect on their practice and "develop ways of engaging students in deeper inquiry and metacognition" (Atkinson & Minnich, 2014:7). Seemingly, MCKT develops teachers PCK and consequently learners' performance in mathematics, as teacher become more comfortable in their teaching as compared to those with inadequate MCKT (Lee *et al.*, 2018:76). Vemkat and Spaull (2015:122) also assert that "PCK rests firmly on a content knowledge base". Accordingly, a teacher with sound mathematics content knowledge for teaching (MCKT) “has the ability to teach to the understanding of the learners” through utilization of comprehensive lesson plans and mobilization of appropriate manipulatives (George & Adu, 2018:141).

2.5 CONDITIONS FAVOURABLE FOR ENHANCING MPCK USING PBL

The challenges, the need and components of the strategy to enhance MPCK using PBL have been discussed in section 2.4. This section presents conditions conducive to creating a favourable environment for successful introduction of the strategy. Every education system in each country has its peculiarities in terms of contextual factors that either impede or enable policy implementation (Tsotetsi, 2013:103). These contextual factors may also affect the implementation of the strategies presented in this study report. This section considers the following conditions for the optimal implementation of the solutions or components of the strategy: conditions that strengthen the functionality of the dedicated team; conditions conducive to lesson preparation, conditions conducive to LCPA implementation, and conditions that are conducive for continued teachers’ emancipation regarding mathematics content knowledge for teaching.

2.5.1 Factors favourable for the functionality of the dedicated team

According to college chatter Edward Everett Hale once philosophized: “Coming together is a beginning, keeping together is progress, working together is success” (Durban Girls College, 2018: 1). From this pragmatic statement it seems that the success of any team is working together. In essence a team creates the environment for one to achieve what could not be achieved by an individual without a team (Qhosola: 2016: 54). Other than being obsolete, like what Basov and Nenko (2011: viii) call a ‘lone wolf’, Mahlomaholo (2012: 293) theorized that two agents working together possess more knowledge than they would if they worked separately as individuals. As postulated by Everson, Funk, Kaufman, Smith, Nallamothu, Pagani and Hollingsworth (2018: 1026) the team work revolves around ongoing collaborative interaction between personnel. They also maintain that the success of the teamwork depends on how more readily team members support one another and make “it easier to provide input and to ask questions" (Everson *et al.*, 2018:1017). It is further postulated that when a team is established, team members need to repeatedly work in cohesive groups in order to become acquainted with each "other's preferences, personalities, strengths, and weaknesses" (Everson *et al.*, 2018: 1017). This view posits that when the team is newly established, the frequency of meetings is pivotal to enable team members to familiarize themselves with the team routines.

On the other side, Qhosola (2016: 201) mentions that other aspects which improve the quality of team work, inter alia, incudes but are not limited to commitment, open communication, collective leadership and establishment of team norms. According to Pearce and Herbik (2004, cited in Qhosola, 2016: 80), commitment to a team is a long-term promise which is made and kept by dedicating time, energy and other possible resources to ensure the functioning of a team. Open communications entail accommodation of contradictory opinions that inevitably develop the sense of equally for all the team members, which, according to Mosia (2016: 163), results in sustainability of optimal functionality of the team. Accordingly, tolerance of divergent views exhibits mutual respect, equality and humility among team members are virtues that create cohesive team spirit (Mosia, 2016:163). In ensuring shared leadership and collective responsibility to reduce the power domination (Qhosola, 2016:201), team members are given responsibilities to perform (Mosia, 2016:75). These roles and responsibilities, like chairing the meeting, taking minutes and organizing venues form part of what Qhosola (2016: 203) calls team norms. To summarize, it appears that commitment and open communication foster teamwork, while shared leadership and team norms seem to encourage team work. Furthermore, as articulated by scholars, shared leadership and team norms are other important factors that strengthen the team work.

The literature further highlights the attributes that describe the member characteristics of a desirable team spirit, namely "initiative, trust, openness, helpfulness, flexibility, and supportiveness" (Stevens & Campion, 1994:504). According to Tsotetsi (2013:105), these attributes also include,

> *"patience, hard work, the clear purpose of the team, open communication between the team and other teachers, clear roles and responsibilities, strong relationships between the team and other teachers, and a*

willingness to share information and listen to other people as well as participation".

In a nut shell, as extrapolated from the discussion above, these attributes attached to team members seem to embrace human values, like democracy, dignity, hope and social justice.

2.5.2 Conditions conducive to encouraging lesson preparation

Other than viewing lesson planning as a road map giving guidance on what and how learners need to learn (Malkova, 2012:1) lesson planning also presents teachers with opportunities to explore multiple aspects of PCK (Shen *et al.*, 2007: 249). PCK aspects that have been discussed in detail in section 2.4 include identification of learners' misconceptions, judicious utilization of manipulatives, and assessment integrated with lesson facilitation. According to Mosia (2016,19), lesson planning focuses on the meticulous collection of resources concerning what needs to be taught and deep thinking about what needs to be included in the lesson plan seem to encompass the above-mentioned PCK components. Overarchingly, coordinated teamwork also presents an opportunity for teachers to share their real classroom experiences and problems in terms of the above-mentioned aspects of PCK. In support of a collegial professional community, that is a coordinated team in our case, Shen *et al.* (2007: 248) assert that these communities of practice set conditions that enable teachers to reflect on and improve their teaching practices, including lesson planning. According to Shen *et al.* (2007:248), it appears that a coordinated team creates favourable conditions for lesson planning.

2.5.3 Conditions conducive to fostering LCPA implementation

It appears that teachers need to create an environment that promotes learning for individuals who often come from diverse backgrounds when planning mathematics lessons (Mosia, 2016:169). The narrative of formalizing mathematics denies teachers the opportunity to value learners' common sense (Ahmed *et al.*, 2004:318). Consequently, an alternative notion that embraces learning from the learners' view is submitted (Moloi, 2014: 271). This suggests that the focus on lesson planning should be on more than content, but on what learners can do, thus, LCPA in our case. As presented by Moloi (2014: 271), an important condition for the mathematics classroom in order to cater for learners' needs, is that learners' experiences and their prior knowledge should feature in both the lesson plan and lesson facilitation. Inevitably, learners become quiet when their marginalized knowledge is not used or recognised in the classroom (Moloi, 2014:271), while the opposite is true. This is in line with Heritage's (2010b: 8) perspective as she posits that for teachers to prosper in unearthing what is within the learners' reach, they need to use integrated assessment with lesson facilitation and follow up on learners' misconceptions.

Moreover, the research views manipulatives as cognitive tools that improve learners' active engagement in the classroom discourse (Durmuş & Karakirik, 2006, 119). Apparently, learners are able to validate or refute others' solutions and defend their thinking through use of manipulatives (Laski *et al.*, 2015: 2). As learners defend or refute others' thinking, probing questions and enabling prompts also create an environment that helps learners focus on compressed mathematics concepts needed to be demystified before learners could untangle complex problems. In a nut shell it appears that judicious utilization of manipulatives creates favourable conditions that encourage LCPA implementation. Furthermore, democratic classroom discourse creates conditions conducive to LCPA implementation as the power relations shift and put the teacher at the peripheral positioning (Moate & Cox, 2015: 379). Learners rely on their peers in a group to provide mathematics manipulatives (Abdullah, Tarmizi, & Abu, 2010:375), and have ample opportunities to learn mathematical processes associated with communication, representation, modelling, and reasoning (Abdullah *et al.*, 2010: 371).

2.5.4 Conditions conducive to teachers' continued emancipation on MCKT

Improving co-researchers' MCKT is one of the strategy components to enhance MPCK using PBL. From the epistemological stance that knowledge is socially constructed, the establishment of a platform for team members to tap from each other's strengths and reduce weaknesses becomes a vital condition for continued emancipation in terms of MCKT. PBL emphasises peer learning where team members work together in designing solutions to the problems (Han & Teng, 2005:3). Team learning is anchored in a philosophical underpinning that views learning as a social process, through dialogic communication (Kolmos *et al.*, 2009:11). The PAM document also encourages co-operation and collaboration of teachers in order to maintain good teaching standards (DBE: 2016: A-19). Evidently, teamwork, such as collaborative planning, has been proven to provide supportive working environments that may benefit teachers' enhancement of MCKT and PCK development (Jita and Mokhele, 2014:1; Evens, Elen, & Depaepe, 2015:2). Namibian education policies also confirm that cluster subject group meetings improve the quality of teaching and learning (MoE, 2008: 13) through sharing resources and expertise among teachers (Pomuti & Weber, 2012:1). Apparently, it helps to know that that there is somebody down the hall with whom you can engage if you are wondering how to approach something instructionally (Graham, 2007: 9).

2.6 FACTORS THAT MAY POSE THREATS TO THE STRATEGY IMPLEMENTATION

This section explores factors that may threaten the implementation of the developed strategy to enhance MPKC using PBL. Explicitly, the section focuses on discussing factors that might derail the implementation of the strategy and learn how to avoid them. This study identified the following factors that may threaten the implementation of the developed strategic framework, namely inherent threats regarding the establishment of the coordinated team, threats towards effective use of manipulatives and negative attitudes towards lesson planning.

a) Challenges in establishing a coordinated team

As reflected in section 2.4.1.1, lack of time for coordinated team meetings seems to hold a threat for effective operations of the coordinated team. On the other hand, the rigid inspections during the apartheid era made teachers reject anybody who wished to observe in their classrooms for fear of being judged (Jita & Mokhele, 2014: 11-12). Furthermore, conditions such as being the only teacher for mathematics in a school and geographical isolation, distance between schools, bad road conditions and the use of personal money to attend the cluster meetings militate against the existence of collaborative teams (see section 2.4.1.2).

b) Threats to effective use of manipulatives

As reflected in section 2.4.3.1.4 Furner and Worell (2017: 12) reported teachers' views regarding the use of manipulatives as a waste of time which is not necessary for teaching and learning serious mathematics. The environment with manipulatives threatens teachers' position of being the only ones to approve which mathematics answers are correct or not (see section 2.4.3.1.4). For teachers to claim back their position of power in the mathematics classroom discourse, they perpetually stop using manipulatives.

c) Threats to lesson planning

The research findings highlighted impediments towards lesson planning, such as lack of supervision from the schools, and teachers' negative attitude towards curriculum reforms (Bantwini, 2010:86). Teachers were threatened by curriculum reforms' requirements as they seemed to demand more of teachers' limited time, hence they argued that lesson planning was all about paperwork (Bantwini, 2010: 86). Teachers also bemoaned the lack of time for lesson planning due to overwhelming demands of assessment and marking (Ramaila & Ramnarain, 2014:7). Teachers further contended that they were unable to plan mathematics lessons arguing that they had subjects to teach other than mathematics (Ding & Carlson, 2013: 381).

2.7 INDICATORS OF SUCCESS

This section explores indicators of success in the implementation of the strategy to respond to challenges facing Grade 9 teachers in their MPCK using PBL. The success of this study would be realized when the teachers who participated in the study exhibit knowledge and skills to unearth learners' mathematics misconceptions, enactment and display of curriculum knowledge through judicious use of manipulatives, detailed lesson planning, facilitated in a LCPA to consider learners' needs, formative assessment to ascertain the learning zone, and understand mathematics content knowledge for teaching. Over and above it is prudent for teachers to collaboratively work together in resolving their problems regarding the wisdom of practice as they collectively work together to develop knowledge and skills in relation to the above-mentioned aspects.

2.7.1 Successful identification and elimination of learners' mathematics misconceptions

Following up on learners' misconceptions enables teachers to develop an understanding of learners' thinking (Herholdt & Sapire, 2014: 44) and consequently to learn new ways of making of mathematics comprehensible to learners. Other than re-teaching or quick fixes like replacing flawed answers with the correct ones, analysis of the underlying mathematical misconceptions based on learners' erroneous answers helps teachers to develop cognitive scaffolds such as enabling prompts. In the process of developing cognitive scaffolds, teachers' knowledge of practice gets enhanced, as they acquire new strategies to eliminate learners' misconceptions. This is in line with Gardee and Brodie's (2015: 2) narrative, proclaiming that valuing learners' thinking encourages teachers to "find ways to engage their current knowledge in order to create new knowledge". As teachers engage learners to create new negotiated knowledge, they become learners as well and as a consequence their MPCK get enhanced.

2.7.2 Successful enactment and display of curriculum knowledge through judicious use of manipulatives

The use of Shulman's (1987: 8) tools of the trade for teachers, that is, manipulatives in our case, seems to demystify abstract features of mathematics concepts through transduction. The power of manipulatives apparently seems to advantage teachers in terms of effective teaching and learning especially when using LCPA like PBL (Hmelo-Silver & Barrows, 2007: 238; Hwang, Jia-Han, Yueh-Min & Jian-Jie, 2009: 229). From this narrative, it seems that a concerted effort of effective utilization of manipulatives in mathematics teaching could enhance MPCK. Effective utilization of manipulatives presents multiple representation of mathematics concepts to learners and proliferate communications, engagement amongst learners and the development of multiple negotiated meanings of mathematics concepts (see section 2.4.3.1.6).

2.7.3 Successful enactment and display of curriculum knowledge through detailed lesson planning

Lesson planning guides teachers on what needs to be learned, including strategies of how it could be done effectively in a given time in the classroom (see section 2.4.3.2). It includes, but is not limited to, learning trajectory, selection, and preparation of resources. Multiple aspects of PCK are explored through lesson planning (Shen, Poppink, Cui, & Fan, 2007: 249). When teachers make use of appropriate curriculum material in lesson planning, they do not only enact their knowledge of the curriculum but also display their knowledge of the curriculum (see section 2.4.3.2).

2.7.4 Successful use of integrated assessment to ascertain learning zone

Assessment-embedded instruction gives teachers insight in students' mathematical thinking as they use learners' natural ways of thinking in the classroom (see section 2.4.3.3.6). In turn this practice improves their pedagogical strategies and consequently contributes to their development (Nagasaki & Becker, 1993: 46). The identification of the learning zone through assessment-embedded instruction enables teachers to develop mathematically appropriate strategies to rescue learners from a 'cul-de-sac' in order to realize the learning trajectory. This praxis enables teachers and learners to consistently work in the ZPD, the area where learning takes place (Heritage, 2010b: 8). Apparently, teachers prosper in unearthing what is within the learners' reach, when they keep a very close eye on emerging learning through assessment-embedded instruction (Heritage, 2010b: 8). Teachers arguably prosper in unearthing what is within the learners' reach, when they keep a very close eye on emerging learning through assessment-embedded instruction (Heritage, 2010b: 8).

2.7.5 Success in implementation of LCPA considering learners' learning needs

It is believed that fundamental mathematics concepts are drawn from human experiences and existence (Ahmed, Clark- Jeavons & Oldknow, 2004: 318). It appears that embracing LCPA will help us to unshackle the views that chain us to only formalising mathematics through demonstration first and later require learners to do exercises emulating the earlier demonstration. Evidently, formalising mathematics constrained us from "valuing 'common sense' based on experience as a valid, indispensable and legitimate basis for checking the mathematical procedures and algorithms" (Ahmed *et al.*, 2004:318). Learners' communication of their common sense-based mathematics experiences enables teachers to draw from the 'mathematics tool box' appropriate strategies to address the needs of learners. Arguably, in an LCPA environment, learners make meaningful connections between their learning experiences and the environment to the real world (Walters *et al.*, 2014: 2). The implication of improving learners' performance and enhanced clarity on mathematics concepts may suggest that LCPA has a positive effect on teachers' PCK that practise LCPA both as classroom discourse and a discursive practice.

2.7.6 Successful understanding of MCKT

SCK is knowledge of mathematics that uniquely exists in the domain of a teacher and is only needed for teaching purposes (Ball *et al.,* 2008: 400). Specifically, it includes the understanding of answers that learners are likely to give and how to remedy their misconceptions (Ball *et al.*, 2008: 393). Teachers have to unpack algorithms as well as making particular content features visible and comprehensible to learners through unpacking compressed mathematical knowledge (Ball *et al.*, 2008: 400). For teaching purposes, one needs to explain how to divide by a common fraction and justify why we invert and multiply when we divide by a fraction (Ball *et al.*, 2008: 400). Furthermore, teachers with sound MCKT are able to support every mathematics assertion by reasoning, which inevitably enables learners to realize that mathematics is learnable (Wu, 2018:17). They present mathematics topics in a coherent way, clearly demonstrating understanding of the division concept as essentially the same for whole numbers and fractions (Wu, 2018: 18). Seemingly, teachers become more comfortable in their teaching as compared to those with an inadequate MCKT (Lee *et* *al.*, 2018: 76). Accordingly, a teacher with sound MCKT displays the ability to develop comprehensive lesson plans and the mobilization of appropriate manipulatives (George & Adu, 2018:141). Moreover, they become expert learners and act as cognitive apprentices to provide cognitive scaffolds (Hmelo-Silver & Barrows, 2006:24) in decompressing complex mathematics concepts.

2.7.7 Successful implementation of coordinated team work in enhancing MPCK using PBL

Evidently, clusters, that is, coordinated teams in our case seem to enhance teachers' PCK (Jita & Mokhele, 2014: 1). Coordinated teams provide teachers' space to communicate, share and address issues, observe one another's work and develop expertise in various aspects (Jita & Mokhele, 2014: 4). It could be argued that coordinated teams act as 'guru teacher rescue operation', as they move teachers away from 'guru' mentality by sharing problems and learners' misconceptions they encounter in their praxis. Apparently, a collegial supportive working environment seems to benefit teachers' PCK development (Evens, Elen, & Depaepe, 2015: 2). In essence, team work gives teachers a platform to share problems and consequently get emancipated in their teaching practice.

2.8. Conclusion

In Chapter two the theoretical framework that guided the study was discussed, including operational concepts and related literature. In this chapter, the use of CER was justified. Components of PCK were explored in relation to mathematics teachers, and PBL was also extensively examined. Literature related to the objectives of the study was discussed with reference to examples experienced in countries from the SADC, Africa, and internationally in order to demonstrate the rationale for the study.

CHAPTER 3 : RESEARCH DESIGN AND METHODOLOGY ON THE STRATEGY TO ENHANCE MATHEMATICS TEACHERS' PCK USING PBL

3.1 INTRODUCTION

The aim of this study was to develop a strategy to enhance teachers' mathematics pedagogical content knowledge (MPCK) using a problem-based learning (PBL) approach. The research question was: How can the mathematics pedagogical content knowledge of teachers be enhanced when they teach Grade 9 learners using problem-based learning? The objectives of the study were to:

- identify and analyse challenges that teachers teaching Grade 9 learners face regarding their mathematics pedagogical content knowledge;
- formulate components of a strategy to respond to challenges facing Grade 9 mathematics teachers regarding pedagogical content knowledge using problem-based learning;
- understand conditions for the successful implementation of the strategy to respond to challenges facing Grade 9 teachers in their mathematical pedagogical content knowledge using problem-based learning;
- anticipate possible threats in the design and implementation of the strategy to respond to challenges facing Grade 9 teachers in their mathematical pedagogical content knowledge using problem-based learning;
- understand and investigate the indicators of success in the implementation of the strategy to respond to challenges facing Grade 9 teachers in their mathematical pedagogical content knowledge using problem-based learning.

Chapter three further presented the operationalization of the objectives of the study. I first outlined the research design and methodology of Participatory Action Research (PAR), which advocates for a collective enquiry in social situations and for taking action or effecting change in order to improve the rationality and justice of participants' own social practices (Green, *et al.*, 2003:419). In this chapter I further elaborated on the constitution of the research team.

3.2 PAR AS THE PREFERRED APPROACH

For this study we adopted participatory action research (PAR) as preferred research design, based on the belief that "it is not utopian to hope for education that emancipates students, teachers and societies from irrational forms of thinking" (Kemmis, 2006:463). This research design rejects the acceptance of social evils, but it advocates for the researched to be co-researchers and fully participate in the research process. Co-researchers are not treated as research objects. In enhancing MPCK through this research design, mathematics teachers as co-researchers identified challenges regarding their MPCK level and developed working solutions for these issues. It is well put that "action research can be empowering, liberating and emancipatory" (McTaggart, 1994:325). The use of PAR provides the marginalised with the opportunity to engage with their education (Bland & Atweh, 2007: 346).

Teachers are often blamed for the ills of an education system; their knowledge and skills are devalued and deemed inappropriate for educational 'reform' (Lerman, 1990a in Mceleli, 2004: 1) and "teachers' voices are often ignored, resulting in them feeling devalued and demotivated" (Oliver, de Lange & Wood, 2010:44). Contrary to this, in PAR the marginalised and the voiceless are able to position themselves as researchers and their voices are comfortably expressed (Bland & Atweh, 2007:346). In fact, the teachers' ability to influence change should not be underestimated, but teachers should lead the educational interventions as they have first-hand experience of what is taking place in the classroom (Oliver, de Lange & Wood, 2010:44).

3.2.1 Historical origin of PAR: Connection to CER and objectives of the study

According to Frisby, Crawford and Dorer (1997:10), PAR "is often used interchangeably with action research, emancipatory research, or participatory research"; however, they do not mean the same thing, as Nelson, Ochacka, Griffin and Lord (1998: 884) claim that participatory research and action research do have common assumptions, but differ in other respects. In essence, PAR "blends the traditions of participatory research and action research" (Nelson *et al.*, 1998:884). This study, however, will not be elaborating on the differences between participatory research and action research; it is going to trace the historical origin of both action research and participatory research and subsequently will show how they evolved and blended to form a new construct called PAR.

According to the literature, action research is traced from the work of Lewin (1946) that emerged after World War II (Baskerville & Woo-Harper, 1996:236; Frisby *et al.*, 1997:10). As a result, Kurt Lewin is often referred to as the "originator of action research" (Adelman, 1993:7). The action research perspective was developed as a means to abate "prejudice and discrimination against religious and ethnic minorities" (Glassman & Erdem, 2014:207). During the years 1933 and 1936, Lewin visited the United Kingdom (UK) and worked at the Tavistock Institute of Human Relations where, subsequently, action research found expression for the first time (Kemmis & McTaggart, 2007:272). Lewin was against research that did not lead to 'social action', as he put it that "[r]esearch that produces nothing but books will not suffice" (1946:35). He further argued that action research was the characteristic of research needed for social practice (Lewin, 1946:35). The literature claimed that Lewin's intention was to give a voice of the voiceless, oppressed and marginalized by helping them to "seek independence, equality, and co-operation through action research" (Adelman, 1993: 7). Over and above that, action research is defined as knowledge production "to guide practice, with the modification of a given reality occurring as part of the research process itself" (Oqust, 1978:145). Lewin (1946) concluded that "[n]o action [should be taken] without research; no research [should be conducted] without action" (cited in Adelman, 1993:8). Apparently, Lewin's work "gave impetus to the action research movements in many different disciplines" (Kemmis & McTaggart, 2007:272).

Nelson *et al.* (1998:884) proclaim that "Lewin proposed cycles of problem definition, fact finding, goal setting, action, and evaluation to simultaneously solve problems and generate new knowledge". In advancing Lewin's work the literature defines action research "as proceeding in a spiral of steps, each of which is composed of planning, action, observation and the evaluation of the result of the action" (McTaggart, 1994: 315). The identification of a social problem with the intention to solve it is the distinct character of action research. Action research is driven by the motive to take action in order to improve or bring about a desirable change of an imperfect concern (McTaggart, 1994:316). In the context of this study, co-researchers and I identified and analysed challenges that the co-researchers faced regarding mathematics pedagogical content knowledge in Grade 9, in order to take action by formulating components of a strategy to respond to the challenges they faced in Grade 9 mathematics regarding their MPCK by using problem-based learning. The debate in literature furthers advanced active participation by the participants or co-researchers "in the exploration of problems that they identify and anticipate" (Adelman, 1993:9). Action research had been used in different formats as influenced by different philosophical underpinnings.

3.2.1.1 Forms of action research

On the basis that PAR historically is traced from action research, I regard it as prudent to explore different forms of action research in order to clearly understand PAR. Xiao, Kelton and Paterson (2012: 324) and Kemmis (2001: 91-92) clustered action research forms in terms of their philosophical underpinnings and categorized the approach into three types, namely "(i) technical action research with empirico-analytic underpinnings; (ii) practical action research with interpretive underpinnings; and (iii) critical action research with critical theory underpinnings" (Xiao, *et al.*, 2012:324). It is clearly put that each philosophical underpinning has its own justification for being in terms of its quest for knowledge (Kemmis, 2001: 92).

In the case of technical action research that is influenced by empirical-analytic research, action research is interested in getting things done effectively. It is a form of problem solving oriented towards changing a particular outcome of practice and is regarded as successful when outcomes match aspiration (Kemmis, 2006: 95). The examples may include, but are not limited to "decreasing classroom behaviour problems or increasing the rate of production in factories" (Kemmis, 2006: 95). In my opinion, technical action research is rather inclined towards a positivist theory, as it is mainly concerned with the outcomes of the research process. The literature attests that technical action research "tends to be applied as a positivist approach" (Kagan, Burton & Siddiquee, 2016 draft: 5). In essence, technical action research involves the identification of a problem, and the intervention that is then tested as its goal, is the promotion of efficient and affective practice, while the relationship between the researcher and practitioner is largely technical and facilitatory (Kagan, Burton & Siddiquee, 2016 draft: 5). I want to argue further that 'action learning' belongs to this category of action research as its key objectives are "organizational efficacy and efficiency" (Kemmis and McTaggart, 2007: 274). It is also clearly put that action learning fundamentally creates a space for people to learn from each other's experience by bringing them together and the emphasis is "on studying one's situation, clarifying what the organization is trying to achieve, and working to remove obstacles" (Kemmis and McTaggart, 2007: 274).

On the other hand, practical action research underpinned by interpretive philosophy has an interest in wise and prudent decision-making (Kemmis, 2006: 95). Moreover, practical action research is a non-positivist approach, but is flexible as mutual understanding is sought between the researcher and the participants (Kagan, Burton & Siddiquee, 2017: 57). Classroom action research uses qualitative interpretive modes of inquiry done by teachers "with a view to teachers making judgment about how to improve their own practices" (Kemmis and McTaggart, 2007: 274) is an example of practical action research. "Practical action research is viewed as more relevant and authentic for teachers" (Manfra, 2009:38). Practical action research focuses on "teachers' self-understanding and judgement" (Kemmis and McTaggart, 2007: 274). This version encourages teachers to conduct classroom inquiry and form conclusions about the best practices. This may include teachers' self-understanding and judgement regarding their mathematical content knowledge. Its proponents claim that it can illuminate crucial aspects of teachers and their learners using reflection to generate new knowledge about teaching and learning (Manfra, 2009:38). When teachers have adopted this kind of research, they are able to redefine their professional knowledge landscape and craft knowledge (Manfra, 2009:37). It emphasises on the interpretations and reasoning of teachers and learners taking part in discussions about how to act appropriately in a classroom situation with which they are confronted (Kemmis, 2001: 92).

Centrally critical action research is different from the latter two forms of action research and it is also called emancipatory action research (Kemmis, 2001: 92). It is formed from the integration of action research and critical theory (Davis, 2008: 139). In essence, it is influenced "by critical theory from Habermas and the Frankfort School" (Mack, 2012: 421). It further goes beyond problem-solving by questioning how it has been socially and historically constructed in a society (Xiao, 2012:324). Critical action research as one of the forms of action is viewed as "validation and extension of action research or participatory action research processes that combines critical theory with the action research paradigm" (Davis, 2008:139). In essence, emancipatory action research or critical action research is based on critical theory that advances critical consciousness that manifests politically to promote change (Manfra, 2009: 39). Critical action research focuses on dismantling the power relations of the traditional power hierarchy between the co-researchers and the researcher. Critical action researchers think and believe that education can free people from structures of domination and also believe that critical action research can be a liberating and empowering act (Esposito & Venus Evans, 2007: 223). This view is based on the debate that critical action research deals with social justice, democracy, gender, ethnicity and other political tendencies that curtail and reduce co-researchers to be the research subjects. Mahlomaholo (2013: 320), for example, claims that this form of research focuses on advancing social justice, hope and peace, instead of concepts such as reliability, objectivity, generalizability, credibility and consistency.

Moreover, critical action research brings about collective self-reflection and social analysis with a view to practically taking an action to foster change in order to improve things (Kemmis and McTaggart, 2007:273). Emancipatory action research develops practitioners' self-critical consciousness and quest to "overcome felt dissatisfaction, alienation, ideological distortion, and the injustices of oppression and domination" (Kemmis, 2001: 92). In line with Kemmis and McTaggart (2007: 273) objectives of this study focuses on the identification and analysis of challenges that teachers face regarding MPCK in Grade 9 in order to formulate components of the strategy to respond to these challenges. The proponents of critical action research emphasise the collaboration of the research stakeholders with the researcher in the identification of the problem and collectively develop the research agenda to strategize towards solutions (Davis, 2008:139). Co-researchers in this study, among others, have identified the challenge of being unable to understand the thought process that leads to learners' misconceptions in a mathematics classroom. Other collectively identified problems in this regard that are encapsulated in the objectives of this study will be presented in details in chapter four.

3.2.1.2 Participatory research

On the other hand, participatory research is viewed as an “alternative philosophy of social research that is associated with social transformation” (Kemmis & McTaggart, 2007:273). Although it originated from human rights activism, it is rooted in “liberation theology and neo-Marxist approaches to community development” (Kemmis & McTaggart, 2007:273). Participatory research emerged from working with voiceless and oppressed people in developing countries (Nelson *et al.*, 1998:884). Apparently, (Freire (1970) in Nelson *et al.*, 1998:84) influenced the concept of adult education by engaging individuals in critically analysing their situation and organising actions to improve it. Participatory research differs from action research in the sense that action research is research that informs 'action', regardless of who makes the decisions. This kind of action research often is conducted by outside consultants, whose task merely may be to inform the decisions made by management (McTaggart, 1994: 314). In participatory research, however, the co-researchers or those affected by the issue being researched, inquire the matter in collaboration with the researcher, and then take action with the researcher to effect (social) change (Green *et al.*, 1995:4 in Frisby *et al.*, 1997:10).

For this study a participatory action research (PAR) design and methodological approach were adopted. The research team (researcher and co-researchers), therefore, collaboratively identified and came to an understanding of the conditions for the successful implementation of the strategy identified, namely problem-based learning, to respond to the challenges facing the co-researchers regarding their MPCK. Moreover, participatory research is based on the view that the oppressed or the voiceless should be fully engaged in the whole process of the investigation by participating in the research question development, in designing research instruments, and in collecting and reflecting on the data collected (Nelson *et al.*, 1998:884). In essence, Grade 9 mathematics teachers (co-researchers in this case) are better positioned to understand the conditions for successful implementation of the strategy. The literature declares that “[t]he term ‘participatory process’ emphasises the fact that research need not be ‘done on’ participants as objects, but can be a collaborative practice (Hawkins, 2015: 4).

Kemmis and McTaggart (2007:273) further identified three aspects that distinguished participatory research from conventional research, that is, the shared ownership of the research process, community-based analysis of social problems, and an orientation toward community action. Guided by participatory research, co-researchers in the first meeting of this study took over the process, and as a result, I struggled to influence their focus towards this study's research objectives. According to Kemmis and McTaggart (2007: 273) participatory research advances

> *"social, economic, and political development responsive to the needs and opinions of ordinary people, proponents of participatory research have highlighted the politics of conventional social research, arguing that orthodox social science, despite its claim to value neutrality, normally serves the ideological function of justifying the position and interests of the wealthy and powerful ".*

The above argument advances the emancipatory role of participatory research as it responds to the will and the needs of the oppressed, marginalized and voiceless (Kemmis and McTaggart, 2007:273). Contrary to conventional social research that embraces the positivist theoretical framework, participatory research embraces CER. Community-based analysis of social problems and an orientation to community action are attributes of participatory research that highlight the emancipation that characterises this research design. Co-researchers using this research design and methodology are likely to be able to identify threats such as, but not limited to, overcrowding and lack of resources that can militate against the successful implementation of the strategy. Subsequently, they are better positioned to identify ways to avoid threats and identify conditions for successful implementation of the strategy in their context.

3.2.1.3 Participatory Action Research (PAR)

The amalgamation of action research and participatory research gave birth to a new construct, namely PAR, or, as Nelson *et al.* (1998:884) put it, PAR "blends the traditions of participatory research and action research". PAR is a research process whereby the researched or participants take part in the decision-making in the research process, from problem identification all the way through to the data generation, analysis and formulating the research results (Anderson, McKenzie, Allan, Hill, McLean, Kayira, Knorr, Stone & Butcher, 2015:181). It is an interpretive, qualitative method that “attempts to fracture away from traditional social science methodologies” (Khan & Chovanec, 2010:34). PAR rejects positivism and embraces critical emancipatory research (CER). In terms of PAR the researchers no longer concern themselves with concepts such as validity, reliability and significance (McTaggart, 2006:313) when conducting research. Subsequently, the research goal is not hypothesis testing, but to learn in collaboration, that is, the researcher and the co-researchers and “move toward social change in order to improve the human condition” (Glassman & Erdem, 2014:15).

In the study reported here the co-researchers identified challenges regarding their understanding of learners’ mathematics misconceptions. Consequently, they designed a strategy together with the researcher to enhance the research team’s understanding of Grade 9 learners’ misconceptions. Not only was I taken up by PAR’s transformative agenda in terms of power relations, but the well-documented discourse that “[i]t seeks to build the knowledge, skills and abilities of participants, and to facilitate informed and collaborative responses for the common good” (Cuthill, 2010:22) convinced me. PAR rejects the notion that school communities lack knowledge; rather, “they do have valuable knowledge that can be utilised to resolve problems in their own contexts” (Mahlomaholo, 2013c:320). It is common knowledge that teachers are likely to reject a research process that suggests that they lack mathematical content knowledge. On the other hand, the promotion of self-critical awareness builds trust between researchers and participants (Moreno, 2015: 182) and as a result mathematics teacher are likely to take risks and display the gap regarding their MPCK components.

Historically, PAR originated in the search for alternative methods of providing aid in order to reduce dependency of Third-World countries due to the failure of international development to improve their situation (Frisby *et al.*, 1997:11). In the Third-World countries an urgent need existed for Nyerere’s ‘self-reliance’ theory, thus, for attaining “economic and cultural independence at a corporate level” (Nasongo & Musungu, 2009: 113). History tells us that the international efforts “were criticized for attacking the symptoms of poverty by providing food, shelter, and medical aid, a strategy that did little to build the human capital needed for self-determination and self-sufficiency”

(Frisby *et al.*, 1997:11). In essence, the material conditions dictated for the provision of an alternative research philosophy. The research attests that PAR proponents are faced with a task to present PAR an alternative philosophy of social research that embraces social transformation in the majority of countries as a community development movement (McTaggart, 1994:314).

The term 'PAR' evolved differently from different countries, for an example, in SA it was first discussed by Flanagan, Breen and Walker (1984) in a book entitled *Action research: Justified optimism or wishful thinking?* (Esau, 2013:3). It was given more prominence in the Faculty of Education at the University of the Western Cape in 1987, as Van den Berg and Meerkotter (1996), cited in (Esau, 2013:3) pronounced that "all action research had to be liberatory" and it might be "a powerful force in freeing South African teachers from the shackles of their socialization". Coincidentally, it "found a home in the South African anti-apartheid teaching fraternity, where the clarion call for 'People's Education for People's Power' motivated teacher activists to oppose apartheid education in their classrooms" (Esau, 2013:3). In Tanzania, Marja Lissa Swantz used the term 'participatory research' for the first time in the early 1970s to describe work of creating locally controlled development projects through the use of the knowledge and expertise of community members (Brydon-Miller, 1997:658). Similar efforts were described in Colombia, emphasizing social change, and in India, approximately at the same time, a similar approach to conduct community-based research was witnessed (Hall, 1997 in Brydon-Miller, 1997:658). In Australia and Europe researchers emerged during the 1980s who believed in research that "represents educational transformation and emancipation by working with others to change existing social practices and by using critical reflection and social criticism as key research processes" (Hawkins, 2015:4). It appears that PAR was introduced as more overtly critical and emancipatory research, visibly political and orientated to change.

However, the evolution of PAR as a transformative research philosophy is not immune to criticism. Sometimes it is viewed and branded as "consulting, masquerading as research" (Baskerville and Woo-harper, 1996:241). The view that PAR lacks scientific rigor has been advanced by its critics claiming that the existing unclear definitions could lead to confusion and authority problems among researchers (Cronholm & Goldkuhl, 2004:47). The positivist researchers who are used to conventional methods may not accept the legitimacy of PAR. The view that PAR is a soft research method and does not use hard data makes it vulnerable to researchers not familiar with the approach and methodology (MacDonald, 2012:41). Baskerville and Woo-harper (1996: 241), on the contrary, claim that for action research to maintain its rigour, it tenaciously clings "to its disciplined constructs of cyclical theoretical infrastructure, data collection and evaluation". There should be a clear cycle of activity, including a premise of pronounced theory under test and there should be an empirical data collection, for example, diaries (Baskerville and Woo-harper, 1996:241). In this study I identify with the proponents of PAR in agreeing that social science research is different from laboratory experiments. One cannot just treat the researched (co-researchers) like measuring the boiling point of a certain liquid under controlled conditions.

In essence, PAR focuses on voice and everyday experiences instead of figures (MacDonald, 2012:41). It involves merging the "research theory and praxis, thus producing exceedingly relevant research findings" (Baskerville & Woo-harper, 1996:235), and, therefore, involves both the researcher and co-researchers in the process. The proponents of PAR advance the view that the research process of PAR embraces mutual and collaborative inquiry intended to reach a shared understanding of a situation, "about what to do, and a sense that what people accomplish together will be valid and legitimate, not only for themselves but also for anyone who views the situation" (Hawkins, 2015:5). Gummesson (1988) cited in Baskerville & Woo-harper (1996:241) tabled the following differences between consulting and action research:

> *(i) researchers require more rigorous documentary records than consultants; (ii) researchers require theoretical justifications and consultants require empirical justifications; (iii) consultants operate under tighter time and budget constraints; (iv) the consultation is usually linear – engage, analyse, action, disengage while the action research process is cyclical.*

As a PAR scholar, I am convinced by its transformative research agenda, and understand that it intends to address social issues as the literature also claims that once the private and community sector agencies and academics are acquainted with the PAR approach, they endorse and support it. "As such, 'quality' within PAR strives to be both socially accountable and academically defined" (Cuthill, 2010:32). Cuthill

(2010: 32) further tables a report about the reputation of 'Sandstone University' that had undisputed academic rigour in terms of PAR usage as a research methodology. More specifically, trustworthiness and rigour in PAR are premised from a view that research never is value free (Padbett, 2008:10). In addition, to ensure rigour the data generation techniques should be anchored in triangulation, which includes, but is not limited to drawing from multiple sources of data, consulting with the co-researchers on the accuracy of the data, and maintaining an audit trail (Padbett, 2008:10). I accept it as normal for critics of PAR to question its credibility, as long as they use a positivist lens as their ontology, which belongs to a different school of thought than CER that harmonises with PAR.

3.2.2 Objectives of PAR

A plethora of objectives of PAR exits; however, I am going to focus on the few that seem to be most relevant to this study. McDonalds (2012: 38) mentioned the following as the purpose of PAR, namely, "to foster capacity, community development, empowerment, access, social justice, and participation". Social justice and participation in research encompass treating the researched as equals of the researcher, and establishing a research team composed of co-researchers and the researcher. PAR seeks to influence the social justice movement based on its participative nature that allows co-researchers "to critically scrutinise their understandings of, and appreciation for, justice, difference, diversity and human dignity" (Hawkins, 2015:6). By actively reflecting on their MPCK shortcomings and becoming more sensitive to the mathematics learners' misconceptions co-researchers became aware of social justice issues, such as the resources available to learners, and therefore they participate in the development of strategies for teaching for social justice. "The basis for this view is that people should not be treated as passive subjects; people should be treated as active agents" (Cronholm & Goldkuhl, 2004:48). Moreover, collaborative learning as social process gives room for participants to "uphold prior knowledge, and listen to and value the voice of each participant" (Hawkins, 2015:5). The literature also attests to it that PAR incorporates the critical reflection of "historical, political, economic, and geographic contexts in order to make sense of issues and experiences requiring action for changing or improving a situation" (McDonalds, 2012: 38).

Social justice and participation are central to the objectives of PAR which means, by implication that one cannot ignore political and economic influences that manifest at the research sites. The inequalities, for example, that were created by colonialism of a special kind called apartheid in the South African society, seem to perpetuate social injustice unabated (Nkoane, 2012:98). On the other hand, there has been a long-standing brain drainage from rural villages to the cities, while the "rural resources of culture and energy become depleted" (Hlalele, 2014:101). Empirical evidence shows that these inequalities are manifested even in the education system, as Spaull (2013:6) argues that readily available data regarding learner achievement show that there are two different public-school systems in South Africa. In fact, education is not exonerated from the same fate as other poor services in rural areas (Hlalele, 2014:101). The poor and marginalised who predominantly are black children, are systematically channelled to poor education, while white children and few black elite's children receive a better education. The Trends in International Mathematics and Science Study (TIMSS) (2011 in Spoull, 2013:6), for example, shows that Grade nine learners in the Eastern Cape (EC) were 1,8 years' worth of learning behind Gauteng learners at an average. Quality teaching and learning in rural contexts remain a pipe dream for all levels of the educational endeavour (Hlalele, 2014: 01). This can be attributed to the kind of teacher cadre found in the EC, as it is a logically sound supposition that education cannot be better than its teachers. Consequently, the call for social justice as espoused by PAR as its objective remains more relevant as informed by the above-mentioned contextual factors of the research sites concerned.

Other objectives of PAR are to foster capacity, community development, empowerment and, as a result, it "challenges the notion that our schools and communities lack knowledge, instead of demonstrating that they do have valuable knowledge that can be utilised to resolve problems in their own contexts" (Mahlomaholo, 2013c:320). Overarchingly, PAR focuses on powerless groups of individuals, thus, the exploited, the poor, the oppressed, and the marginalized, hence its processes are "potentially empowering, liberating, and consciousness-raising for individuals, as it provides critical understanding and reflection of social issues" (McDonalds, 2012:40). The research characterised PAR as research with the people not on them. It emancipates and develops people, and enables community members to conduct research through meaningful involvement in the research project "that is intended to effect community change; produce data for advocacy; and place a high value on experiential knowledge" (Ochocka, Moorlag & Janzen, 2010:4). As it upholds democratic values, that is, equity, it is liberating, by providing freedom from oppressive, debilitating conditions; and provides life-enhancing opportunities that "enable the expression of people's full human potential" (McDonalds, 2012:39). "Finally, PAR with an emancipatory agenda, though not a magical cure for all that ails education, can become a powerful tool supporting the transformation of our society in a very uncertain 21st century" (Esau, 2013: 8).

Through access, as one of PAR's objectives, PAR focuses on power sharing while building the relationship between the researcher and co-researchers who consequently share the ownership and control of research (Ochacka, *et al.*, 2010: 4). One of the critical objectives of PAR is to involve the researcher and the co-researchers to work together with the aim of examining problematic situations and to change them for the better (Kindons, Pain and Kesby, 2007:28). The co-researchers prioritise what they view as the most relevant aspects to be studied during the research process. In this study, the co-researchers seemed comfortable to start interrogating learners' mathematics misconceptions rather than their mathematics content knowledge. As PAR "links academic theory to practice through an iterative process of reflective learning involving diverse stakeholders" (Cuthills, 2010:22), it gives co-researchers access to research. Invariably, they were able to link learners' mathematics misconceptions to their mathematics content knowledge and ultimately also were comfortable to interrogate other components of MPCK, such as pedagogical knowledge and curriculum knowledge. Based on the view that the process of PAR is one of mutual and collaborative inquiry, what the co-researchers "accomplish together will be valid and legitimate" (Hawkins, 2015: 5). The co-researchers' access to research is underpinned by the view that "outsiders have roles as convenors, catalysts and facilitators" (Cuthill, 2010:22). This view is contrary to the delivery of knowledge by experts, but advocates for the co-production of new knowledge and shared understandings (Cuthill, 2010:22).

It must be emphasised, however, that the objective of PAR is not to change individuals "but to give the oppressed members of a community or social group the capabilities of critiquing their own praxis of the immediate" (Glassman & Erdem, 2014:213). The literature further claims that the praxis enables the oppressed masses to not only criticize their condition, but to problematize it with the intention of eventually overcoming it (Glassman & Erdem, 2014:213). In fact, PAR as research methodology does not only describe what is wrong or what needs to be fixed, but invites co-researchers to critically reflect on their MPCK with the intention of collaboratively developing a working strategy to overcome their limitations. An example of such a strategy, inter alia, may be, but is not limited to, a scenario of how a teacher would explain the exponential rule that any number raised to zero is equal to one, so that it would be mathematically sound and learners would understand it, rather than just saying that is the way it is done. When one considers the example given above, the argument that PAR is empowering and fosters capacity (McDonalds, 2012:38) seems to be more relevant.

PAR as a transformative research methodology seeks to encourage a dialogue process that is "non-hierarchical in nature; all participating partners are equally important as problem solvers, thinkers, and learners" (Glassman & Erdem, 2014:209). The literature claims that the researched participants are not mere subjects of research; instead, they actively contribute to the research process (MacDonald, 2012: 41). In essence, each co-researcher is allowed to present subjective understanding of various relationships through the democratization of research processes (Glassman & Erdem, 2014:209). PAR as it is an interventionist approach, could be viewed as a paragon of the post-positivist research like CER. My argument rests on the debate that action research "is experimental, yet multivariate. It is observational, yet interventionist" (Baskerville & Woo-harper, 1996:236). PAR is in harmony with CER as it encourages the research team to explore possible conditions to solve their problems that may include the use of PBL in enhancing their MPCK. PAR "moves social inquiry from a linear course and effects perspective on a participatory framework that considers the contexts of people's lives" (McDonalds, 2012:36). The research process actively involves researching with the people to create and explore change (Hawkins, 2015: 4) in a cyclic process of research, reflection, and action that challenges the "dominant positivist social science research as the only legitimate and valid source of knowledge" (McDonalds, 2012: 36).

3.2.3 Formats of PAR

This subsection discusses different forms of PAR. the literature disputes viewing action research as a single and monolithic research method but views it as a class of research approaches (Baskerville, 199:9).

3.2.3.1 Feminist Research

"The overt ideological goal of feminist research is to correct both the invisibility and the distortion of female experience in ways relevant to ending women's unequal social position" (Lather, 1986:68). Feminist Participatory Action Research (FPAR) is both a conceptual framework and a methodological approach that trigger critical understanding of women's multiple perspectives like double jeopardy for black women in particular and "works towards inclusion, participation, and action, while confronting the underlying assumptions researchers bring into the research process" (Reid, Tom & Frisby, 2006:18). In the case of black women, their situation includes, inter alia, oppression through patriarchy and race. Poverty rates for black women, for example, are substantially higher than for their male counterparts and other women in general. This is clearly demonstrated by the exposure of their subordinate status regimes, based on gender and the existing disparities in wealth, employment, and health that continue unabated along the gender lines (Houh & Kalsem, 2015:262). FPAR blends feminist theory and PAR by demanding from women to be directly involved "in all stages of the research process, including identifying the problems to be explored" and the rest of the research process (Reid *et al.*, 2006: 316). Although FPAR raises critical issues such as the voiceless-ness of women, and that they have been disenfranchised for a long time, those issues are not directly addressed by the objectives of this study, as those issues focus on women's issues. However, FPAR can be used as an analogy in this regard, as it also is critical in the debate concerning equity, social justice, freedom and hope.

"Some women also participated to connect with other women in the community as a strategy for reducing their social isolation" (Reid *et al.*, 2006:318). Becoming part of society seems to help women to feel they belong and to be able to share their atrocities. The same experiences women in the study conducted by (Reid *et al.*, 2006) reported, were shared by Grade nine mathematics teachers who felt helplessly isolated. They have since found a home to share their experiences with the co-researchers. This platform has been created by both PAR and PBL. Co-researchers had agreed unanimously to go to their individual classes to introduce the division concept that they had highlighted as a challenge in teaching division by fractions in Grade nine. When they came back to the following meetings, they reflected on their experiences which were further discussed to propose different approaches to the problems identified.

3.2.3.2 Critical Race Theory (CRT)

In an attempt to achieve the objectives of this study one cannot fail to consider the socio-political context of the site where the study was conducted. According to Kincheloe (1995 cited in Manfra, 2009:40), all research is essentially political in nature and refutes the claimed objectiveness that hides “true politics under rhetoric”. Politics, among others, include understanding of material conditions and educational access by different races. Critical action research assumes that “society is essentially discriminatory”; however, through purposeful action it can be reduced or changed (Davis, 2008: 140). This kind of research challenges the dominant discriminatory positivist research approaches that claim to take an apolitical posture. Instead, it embraces the critical theory approach by promoting a critical consciousness that is practical and politically conspicuous to foster change. It simply problematizes the status quo, such as the power hierarchy and authority. It disrupts and destabilizes “the characterization of traditional knowledge production and social science research as objective” and is apolitical (Houh & Kalsem, 2015:263). The research further enunciates that critical action research emphasises participation “and social analyses in the critical tradition that reveal the disempowerment and the injustice created in industrialized societies” (Kemmis and McTaggart, 2007:273). As influenced by the debate related above in terms of injustice in society and education, in particular the Critical Race Theory (CRT) which is a form of PAR, seems to be a relevant approach to critically explore the contextual factors and the conditions for successful implementation of the strategy to enhance MPCK using PBL.

CRT is a movement that had its origin in the law discipline and quickly moved to other disciplines like education. It is also viewed as a race-equity methodology that is intended to elucidate racial prejudice and challenge discriminatory racial hierarchies (Ford & Airhihenbuwa, 2010:30). In the context of the study reported here, it involved the political history and material conditions at the research sites as it awakened the critical lens in approaching teachers' challenges regarding their MPCK. All the schools involved in this study were in quintile one, which, by implication, were underprivileged - not by accident of history but mechanically designed thus by the former apartheid regime. The schools were relatively poor in terms of teacher quality, material resources and learners' backgrounds. In order to restore teachers' dignity and hope, "the research and the action must be participatory", with those who were affected by the actions being involved in the decision-making processes at all stages of the research process (cf. Houh & Kalsem, 2015:265). In essence, this research methodology advocates teachers' critical pedagogical actions, and increases social justice in the classroom (Mack, 2012:421).

Centrally to CRT is the understanding of the "influences of racism on both outcomes and research processes" (Ford & Airhihenbuwa, 2010:30). This understanding in terms of PAR assumed that co-researchers are endowed with unique knowledge and history that is indispensable to the "framing of research questions, design, data analysis, interpretation, and creation of meaningful products and action" (Torre, 2009:112). "Critical race theorists integrate critical analyses of their lived experiences and disciplinary conventions to advance knowledge on inequities" (Ford & Airhihenbuwa, 2010:31). Although CRT focuses on advancing social justice in the research process, such as critical consciousness regarding inequalities in educational access in terms of race, it also sharply focuses on people's lived experiences. PBL uses real-life problems or experiences to solve problems and in the process of solving real-life problems, learning of concepts and pedagogical knowledge occurs in the case of teachers. As in CRT, the synthesis of lived experiences "can enhance the relevancy of findings for communities and provide disciplines with fresh perspectives on old problems" (Ford & Airhihenbuwa, 2010:31).

CRT, as one of the PAR formats, was deemed most suitable for this study. The study took place in deeply rural black schools, of which it is commonly believed that South African black schools were deliberately disenfranchised. The knowledge transmission model of instruction in Africa had been reinforced by missionization and colonization of the Africans (Roberts, Brown & Edwards, 2015: 367). It is against this backdrop that

CRT and PBL seemed to supplement each other in terms of human emancipation. As it has been earlier debated, PBL integrates theory and praxis in an empowering manner (see Chapter two, section 2.3.2). Zintle, one of the co-researchers in this study, for example, affirmed that ever since she had joined the team of co-researchers, she had started to be reflective and to think critically about herself and her pedagogy and as a result she had stopped underestimating her students' ability. At first, she thought that there was nothing her students could bring to the mathematics classroom; later she could give her students any task to do and she was not afraid to take risks with them. The sharing of lived mathematics classroom experiences by co-researchers seemed to be a cornerstone of this study and influenced the strategy formulation thereof.

3.2.3.3 Community-Based Participatory Action Research (CBPAR)

CBPAR gives a different perspective on how to conduct research using PAR as a methodological approach to generate data. CBPAR centres on "participatory harvesting of local knowledge for improving social lives and livelihoods of the co-researchers" (Openjuru, Jaitli, Tandon & Hall, 2015: 221). The participants'/ co-researchers' lived experiences and their knowledge are a valuable premise for collectively understanding their challenges regarding MPCK. According to the Centre for Social Concerns (2008, cited in Openjuru *et al.*, 2015:221), CBPAR is a collaborative effort whereby both academic researchers and non-academic community members work together to generate social action for positive social change. This approach accommodated the involvement of parents of Grade nine mathematics learners, curriculum advisors, Grade nine mathematics learners, business people who were interested in mathematics teaching, Grade nine mathematics teachers and those who were not necessarily teaching Grade nine learners. It gave an opportunity to all community members who could contribute to the research process to enhance MPCK using PBL. Overarchingly, PAR embraces democracy and equitability, and acknowledges equity of people's worth by providing freedom from oppressive and debilitating conditions (MacDonald, 2012: 39). This life-enhancing experience provided by PAR "enables the expression of people's full human potential" (MacDonald, 2012: 39).

Grade nine mathematics teachers who were the pivot of the research team were driven by collaboration and self-criticism that presented new insights regarding their MPCK. The research team's interest was aroused by discovering that the team did not necessarily need an expert from somewhere else to help them with challenges they encountered in mathematics teaching and learning. However, their experiences were different to Freire's (1970) "cult of expertise" (in Lather,1986: 76). The pedagogy of the oppressed by Freire vehemently opposed "methods which impose a substantive focus and alienating methods on research subjects" and regarded such a methodology as a 'cult of expertise' that is part of the unequal relationships inherent in an oppressive social order (Lather,1986: 76). CBPAR opposed the delusion that reduces communities' intelligences and abilities, which advanced a view that propagated a cult of expertise, that there should be someone from elsewhere who is an expert to solve communities' problems. Accordingly, Glassman and Erdem (2014:209) rejected the cult of expertise view and conventional academic research that portrayed "researchers as experts with knowledge and ultimate problem solvers through their objective research tools" and advocated for the revolutionary components of PAR. In resonance with Lather' research, MacDonald (2012: 39) asserted that "collaboration; establishing self-critical communities; and involving people in theorizing about their practices" were fundamental principles of PAR. The focus in terms of MacDonald was community's direct involvement in the research process which was in essence CBPAR. The collaboration of the co-researchers brought about innovations on how to handle particular concepts in a mathematics classroom. Comprehensive examples of what the co-researchers brought to the research meetings are presented in the next chapter.

In addition, Freirian research was "designed to have an arousal effect", that reoriented co-researchers' perceptions of issues in ways that influenced subsequent attitudes and behavioural actions (Lather, 1986: 76). This empowering effect, that was drawn from the sense of belonging as the co-researchers developed an understanding that they were not the only ones experiencing some challenges in their mathematics teaching and learning. This humanising experience could be noted in Glassman and Erdem's (2014: 209) argument that "[t]he dialogue between the community members and the researcher/facilitators would provide the data for researchers to analyse from a number of different perspectives". Perhaps the sense of belonging and being valued as people who add value to the research process vivified the co-researchers' ideas about what kind of actions should be taken by the assigned research team members in trying to resolve their challenges. Profoundly, the democratization of the research process through CBPAR opened a room for all participants/co-researchers to be "equally important as problem solvers, thinkers, and learners" (Glassman & Erdem, 2014: 209).

In restoring communities' dignity, MacDonald (2012: 39) specifically cited the research that was conducted by Selenger (1997 in MacDonald 2012) who maintained that it should be understood that the problem originated in the community itself and therefore should be defined, analysed, and solved by the community. This includes the understanding that the ultimate goal of PAR is the radical transformation of social reality and improvement in the lives of the individuals involved; thus, community members are the primary beneficiaries of the research. The literature further agitates for the involvement and active participation of the community at all levels of the entire research process as CBPAR encompasses a range of powerless groups of individuals, namely the exploited, the poor, the oppressed, and the marginalized (MacDonald, 2012:39). Community participation creates a greater awareness in individuals of their own resources and that they can mobilize them for self-development leading to a more accurate and authentic analysis of social reality.

CBPAR seemed to be most relevant to my study as it encapsulated a number of issues raised by the co-researchers. This presumption is based on the view that the team of co-researchers recognised that the creation of objectives on their terms was an act of re-humanisation (Mahlomaholo, 2013c:320) and restoring people's dignity. In my own opinion, recognising community-based knowledge is at the core of CBPAR. CBPAR, for example, is "about the social and economic wellbeing of the marginalized, poor and excluded populations of this world, exploited industrial workers" (Openjuru *et al.*, 2015: 221). CBPAR values the importance of the indigenous knowledge in conducting research that is guided by a critical theory lens. Mahlomaholo (2013c: 320) claimed that using indigenous knowledge when engaged in research helped the participants to reject the theories and the body of knowledge that caused humiliation. On the other hand, the proponents of CBPAR assert that the "local or indigenous knowledge's role in speaking to and addressing these world problems is now a widely accepted view" (Openjuru *et al.*, 2015:221).

CBPAR seems to advance an agenda that is working towards a transformative social praxis and is influenced by CER as a philosophical underpinning. In describing this notion, which makes my argument necessary and urgent, Mahlomaholo (2009: 225) states the following:

> *CER sees the researched as other human being(s), as equal subjects like the researcher. It sees the researcher as being tasked with the role of interpreting other people's interpretations and trying to make sense thereof. Research is seen as the most humanising experience and one from which the researcher must emerge more human, more humane, more cautious, more respecting and more open-minded to signals and messages coming from a very diverse list of sources.*

For communities to take charge in the improvement of their oppressive situations, they need to develop a self-determination that requires both the demystification of ideologies that musk dominant and oppressive social relationships (Lather, 1986:73). In the case of Grade nine mathematics teachers, the dominant ideas were pushed down their throats by the departmental officials without involving them in the development of the intervention strategy.

Moreover, CBPAR provides room for everyone affected by the challenge to narrate personal experiences including failures and successes in order to develop a collective intervention strategy. The researcher learned from the co-researchers as to what seemed to be working in a particular community of mathematics teachers. Accordingly, the components of PBL, such as projects and real-life problems that are solved through a team-based approach, and reflection by the team on the appropriateness of the product are directly addressed by CBPAR as claimed by Laursen (2013:31). As elucidated above, our strategy was influenced by linking CBPAR and PBL.

3.2.4 PAR Steps and Stages

PAR moves from a premise that emphasises the investigation of actual practices, not abstract practices, whereby the research team is confronted with real, concrete problems (Kemmis & McTaggart, 2007:277) that do not necessarily have ready-made solutions. As researchers and co-researchers untangle the problems, they follow certain steps. In reality, the process might not be linear where the steps or stages of

PAR would sequentially follow on each other in a form of neat spirals of planning, acting and observing, and reflecting, but they overlap and more likely fluid, open, and responsive to experience (Kemmis & McTaggart, 2007:277). Nonetheless, the literature presents four cyclical steps of PAR that oscillate between diagnosis, action, measurement and reflection (James, Milenkiewicz & Bucknam, 2008:15). On the other hand, Kemmis and McTaggart (2007:287) exhibit the cyclical steps of PAR in the form of planning of change, acting and observing the process followed by reflecting on the subsequent change that further leads to re-planning for the next cycle of steps. These cyclical steps were first proposed by Lewin but they were not accepted as research demanded that research cycles should involve a definition of the problem, followed by fact finding, whereby the research team sets goals and initiate actions to simultaneously generate new knowledge and solve the problem while evaluating the action and the attainment of the set goals (Nelson *et al.*, 1998: 884).

The uniqueness on the cyclical steps of PAR from other research methods lies on the data generation through the use of reflections and co-researchers' personal experiences to inform subjective decisions (James *et al.*, 2008:14). In the implementation of these cyclical steps, the co-researchers are fully engaged in the investigation process through problem definition and co-researchers' personal experiences in order to develop a deeper insight about the nature of the problem investigated (Nelson et ai., 1998:884). Glassman and Erdem (2014:214) elaborated on the research steps and viewed the research process as "a cycle of continuous exploration and understanding, an ongoing cycle of action as praxis, research as co-scientization, and reflection leading to transformation of praxis - all within the context of vivencia, the lived experience". PAR process steps give credence to intuitively driven moments and epiphanies that are in contrast to the scientific view that is anchored in a positivist theoretical framework, which advocates for an absolute quantifiability of data collection for hypothesis testing (James *et al.*, 2008:8). James *et al.*'s steps seem to be relevant to the objectives of this study as they exemplify how to collectively conduct an inquiry to understand MPCK challenges facing Grade nine mathematics teachers.

3.2.4.1 Diagnosis

The diagnosis stage is usually the first stage of the research process which is characterized by the identification of a problem and raising questions that are rather fuzzy at the beginning of the research process and become clearer with time and understanding (James *et al.*, 2008:150). The central idea of this step is to study or inquire about a solution to important social issues working together with those directly experiencing those social or organizational issues. In Esau's study (2013:3) the participants' problem was not improving classroom practice; however, they were concerned with changing unequal relations in the social context. In this study the issue is the enhancement of Grade nine teachers' MPCK. Apparently, co-researchers would need to be part of diagnosing the problem, including critically reflecting on themselves so that they can fully take part in the research process that seeks to solve the problem. As espoused in the CBPAR, the community members that are the beneficiaries of the research process need to be part of defining the problem that affects them. As reflected in Kemmis and McTaggart's (2007:287) research, this stage ends up by developing a plan of action to solve the problem that is ultimately followed by the immediate orchestration of the plan.

In other research approaches that do not use PAR as a research methodology the diagnosis of a problem is done by an outside researcher who does not form part of the research audience, and the problem is solved based on statistical evidence followed by the development of a hypothesis that needs to be tested irrespective of both material and socio-political conditions of the researched audience. The literature attests that positivism uses a quantitative research methodology that relies on hypothetic-deductive reasoning, "since it is theory-led and tends to be confirmatory" (Ngulube, 2015: 27). In a quantitative study the researchers extract or collect data, analyse the data and present the results in numerical form rather than narratively (Donmoyer, 2008: 713). Quantitative research is also viewed as an antithesis of qualitative research (Donmoyer, 2008:713) as it relies on establishing statistical correlations between variables. However, its weaknesses are noted, especially that in the real world and in social sciences with the exception of laboratory experiments, it is extremely difficulty and even impossible to employ random assignment and tight controls. As a consequence, laboratory experiments are the best set up to conduct quantitative research; unfortunately, this does not always hold true for real-world situations. It is agreed that research “results generated from artificial settings such as laboratories lack ecological validity” (Donmoyer, 2008:715).

In view of the above elucidation, PAR appears to be best suited to conduct this study that seeks to enhance Grade nine teachers’ MPKC by using PBL. At the diagnosis level and problem definition stage, one could not have just told Grade nine mathematics teachers that they lacked MPCK. However, in terms of PAR, problem definition is introduced through co-researchers brainstorming about a social issue related to their praxis, such as the high failure rate of mathematics learners in Grade nine as compared to other classes in SA schools. Diagnosis involves the articulation of the issue including understanding of its importance in order to develop a plan (Coghlan & Brannick, 2005:22). As PAR exhibits commitment to democratic values such as transparency, openness, communication ethos and social justice, this step creates an opportunity for the research team to collectively identify and analyse challenges that teachers face regarding MPKC in Grade nine.

Finally, this step was coined by the development of an intervention plan to a commonly identified problem or problems. The literature clearly puts it that planning action is drawn from framing of the issue diagnosed and analysis of the context together with the understanding of the project (Coghlan & Brannick, 2005:23). This context analysis leads to the development of a plan that includes the formulation of the strategy components to respond to the challenges collectively diagnosed. In this study the components of the plan included the development of the strategy to respond to challenges facing Grade nine mathematics teachers regarding MPCK using PBL. Informed by the CBPAR adopted by this study, the context analysis included, inter alia, critical understanding of conditions for the successful implementation of the strategy to respond to challenges facing Grade nine teachers together with the indicators of successful implementation of the strategy.

3.2.4.2 Action

This step of PAR is about the execution of the plan developed in the previous step as put down by the research plans, and the interventions are implemented (Coghlan & Brannick, 2005:23). This stage is the actual praxis as the co-researchers simply go ahead and actually do what they have planned although the plan may not have envisaged all circumstances in which it is enacted (Kemmis, Mctaggart & Nixon, 2014:105). It is advised that the researcher should monitor and record everything that happens as they put the plan into action so as to be able to have a solid base for reflection and a re-planning cycle (Kemmis *et al.*, 2014:105). CER as philosophical underpinning for PAR and theoretical framework of this study, embraces the subjective interpretation of the situation by the co-researchers concerned as they practically implement planned and re-planned actions depending on what transpired on the ground. The research team guided by this theoretical framework collaboratively considered contextual factors and conditions that could militate against the implementation of the planned action and develop strategies to mitigate against adverse conditions towards enacting the plan.

On the other hand, the debate about what constitutes action in terms of PAR seems to be problematic. The literature claimed that the expectations about what constitutes action differed considerably as smaller actions that are achievable at a personal and local level may go unrecognized (Reid *et al.*, 2006: 317). This may include, but not limited to the teacher's ability to reflect what had happened in his/ her mathematics classroom with the aim to enhance MPCK. Moreover, Reid *et al.* (2006:317) defined action as:

> *a multi-faceted and dynamic process that can range from speaking to validate oneself and one's experiences in the world to 'the process of doing something', such as taking a deliberate step towards changing one's circumstances.*

In the above citation it is recognised that action, especially in research guided by CER, does not necessarily need any generalization, but rather a change in individuals' social lives. "By maintaining commitment to local contexts rather than the quest for truth, PAR liberates research from conventional prescriptive methods, and seeks to decentralize traditional research" (MacDonald, 2012:36). PAR gave the research team in this study an opportunity to take an action through recording and sharing of their success stories as indicators of success in the implementation of the strategy.

The action stage is an important step of PAR, as it is both used to generate data and effect an intervention to the problems collectively identified. The data are generated through engagement with others in the PAR cycles and it should be noted that the acts that are intended to generate data are themselves interventions (Coghlan & Brannick, 2005:99). Simply put, acts are both an intervention and a tactic to generate data. The workshop organised to empower the co-researchers and I (the researcher) in a particular mathematics pedagogical approach or content, for example, in actual fact served two purposes, namely generating data and intervening to address the challenges that had been collectively identified. The literature claims that all actions are interventions and hold political implications for the education system (Coghlan & Brannick, 2005:99). The researcher, therefore, is not neutral, he is directly involved, both by influencing and being influenced by the research process. In line with Habermas's theory of 'communicative actions' (Cecez-Kecmanovic & Janson, 1999:7) the interaction of the research team members is both formal and informal.

Communicative action is what happens when people interrupt what they are doing and raise questions about what is happening (Kemmis *et al.*, 2014:34). Questions are frequently asked when there is cognitive disturbance as people encounter doubts "about the validity and legitimacy of their understandings about what is going on" (Kemmis *et al.*, 2014:34), and having a feeling that something does not augur well with their understanding. People, in the case of teachers, may raise questions like: Is that how it usually is done and what kind of mathematics is that? At this stage, people have entered a space of communicative action. The research portrayed that as people stop and inquire about what is happening, they get to a unique form of action, which is different from the usual strategic action of getting things done, that characterises much of our lives but get into a communicative action mode (Kemmis *et al.*, 2014:35). The above awaking process and realization of self-consciousness in terms of CER help co-researchers to identify possible threats and devise strategic measures to evade them (Hlalele, 2014:104). In essence, CER centres on discursive power relations whereby they (power relations) are practised through communication (Nkoane, 2012:99).

It is further documented that as part of a liberating process, expression of freedom and social justice, people make conscious and deliberate efforts to achieve intersubjective agreements about an issue (Kemmis *et al.*, 2014:34). The co-researchers are not pushed to reach consensus about resolving a particular issue, but they develop a mutual understanding of each other's different viewpoints. Evidently, this stage of PAR, as it enacts the plan, puts the CER into practice; for example, advancing the agenda for equity, peace and hope (Mahlomaholo, 2009:226) is a liberating process that changes people's thought processes in handling their needs of real-life situations (Tshelane & Tshelane, 2014:288) by curtailing the dominant and oppressive thoughts. In particular, the emphasis on democratic approaches and equality principles in resolving matters of social interest puts theory into praxis (Tshelane & Tshelane, 2014:288). Moreover, the reflections on all occasions need to be recorded (Coghlan & Brannick, 2005:99) to inform an evaluation or re-planning cycle. The recorded journal may include, among others, observations, meeting resolutions, new plans and the evaluation of the action in terms of understanding the success indicators.

3.2.4.3 Observation, measurement and evaluation

This stage of PAR focuses on observing the impact of the action taken with a view to measuring the achievement of the intended intervention and the evaluation of knowledge produced. It examines both intended and unintended outcomes of the enacted action to determine if the original diagnosis fitted, or whether the enacted action matched the diagnosis, and also evaluates whether the "action was taken in an appropriate manner" and with an understanding of what informs the next cycle of diagnosis, planning and action (Coghlan & Brannick, 2005:23). As the previous stage had generated data, the research team at this stage critically observed the generated data and sifted through it to see whether things went as they were planned (Kemmis *et al.*, 2014: 107). In the process of examining the action outcomes, the conditions of successful implementation of the strategy to enhance MPCK were clarified and understood as the strategy had been put into praxis. It is further proclaimed that as the data are organised, the data also are analysed and interpreted in efforts to explain what happens to oneself, as it is also articulated that this stage aims to put together "a narrative account of what happened" (Kemmis *et al.*, 2014:107). These narrative accounts included the collective discussions of the different understandings of problems experienced at the research sites.

Moreover, the intersubjective narratives, interpretations and understanding of the indicators of success in the implementation of the strategy are inextricable from the context of the research sites in terms of material conditions, socio-political posture and "cultural discursive arrangements in the semantic space shared with others involved and affected" (Kemmis *et al.*, 2014:107). Seen through a CER lens, it is understood that dominant discourses may be used to distort reality and, therefore, unmasking such potential distortions plays a pivotal role in CER (Nkoane, 2012:100). In reality no one has an absolute way of presenting action outcomes; instead a democratic consensus is best suited to accommodate different views. However, the meaning-making that is underpinned by a positivist view relies on coding and calculating the frequency of a particular behavioural occurrence to draw a conclusion. The failure of positivist forms of social science research to embrace a transformative agenda is clearly exposed by its systematic reproduction of power relations that dominated the subordinate groups within a capitalistic society (Jordan & Kapoor, 2016: 137). On the contrary, in CER meaning-making "is about making sense of other people's interpretations and understanding their world informed by their experiences" (Nkoane, 2012:100). PAR, as it embraces the transformative agenda of CER it breaks the shackles of the dominating discourses, restores people's dignity and gives hope to marginalised people.

Tilting towards the end of this stage, the co-researchers would have noticed how their practice had enabled and constrained their capacity to make changes they intended to make in the social space (Kemmis *et al.*, 2014:107). The humane experience brought about by PAR encouraged mathematics teachers to consciously observe, measure and evaluate the action outcomes. Moreno (2015:182) states that PAR is a process through which people who currently are poor and oppressed, progressively transform their environment through their own praxis. In particular, the awakening experience or tipping point of change for the oppressed people as they begin to raise questions and critiques about actions they had once believed to be fundamental for their survival is regarded as conscientization (Glassman & Erdem, 2014,213). Conscientization results in people stopping to be recipients of knowledge, but developing a deep awareness about their socio-cultural realities which shaped their lives, including their capacity to transform reality (Glassman & Erdem, 2014,213). Ultimately this awakening and liberating experience leads to disindoctrination, as people recognize that the socially imposed knowledge maintains the status quo such as social injustice, and does not necessarily serve the public interests (Glassman & Erdem, 2014,213).

PAR takes a deliberate stance to evaluate the action outcomes under the idea of interwoven co-existence and inter-existence of the people, “not simply being there without having anything to do with each other” (Moreno 2015:183), but influencing each other. The notion of people’s co-existence, called vivencia, was derived from Ortegay Gasset’s ideas (1957, cited in Moreno, 2015:183). It is through actual experience of something that we intuitively apprehend its essence; we feel, enjoy and understand it as reality, and we thereby place our own being in a wider, more fulfilling context (Moreno 2015:183). On the other hand, Glassman and Erdem (2014: 212) viewed vivencia as a “full experience of an event with its all possibilities, lived through direct participation. In other words, vivencia cannot be observed; it can only be lived, felt, and experienced”. At this stage of PAR the co-researchers’ experiences, as they were personally recorded through field notes and audio taped, were verbally expressed in the research team meetings in a democratic atmosphere. The research emphasised that to ensure fairness, relevance and accuracy one’s account of what happened, one needs to go over one’s account with others and avoid speculation when discussing one’s observation with others (Kemmis *et al.*, 2014:108). Invariable, the co-researchers and I realized that we were co-inheritors of findings and the truth arrived at.

3.2.4.4 Reflection

The research team at this stage of PAR already had an account of what happened in the previous stages and it was now charged with the responsibility to deeply reflect on it (cf. Kemmis et a.l, 2014:108). The reflection process entails exploration of one’s beliefs, thoughts and actions in a deliberate and critical narrative way that is part of the PAR cycle (Morales, 2016:160). The literature lists three forms of reflection, namely content reflection, process reflection and premise reflection (Coghlan & Brannick, 2005:26).

> *Content reflection is where you think about issues, what is happening, etc. Process reflection is about strategies, procedures and how things are being done. Premise reflection is where you critique underlying assumptions and perspectives (Coghlan & Brannick, 2005: 26).*

Overarching, reflection is viewed as a meta-cognitive process that talks to a high level of mental conception of what an individual does and thinks (Morales, 2016:160). In a PAR environment, these reflection formats take place in an atmosphere where co-researchers are also able to offer their own interpretations of the research findings through engaging in a dialogue (Jordan, 2008: 599). The co-researchers' voices and reflective feedback give credibility to the research findings by sharing knowledge and perspectives that would usually not be accessible to an outside researcher (Jordan, 2008: 599-600). In particular, vivencia breaks down the elitist research paradigms "especially those that create abstract propositions about those they speak of" (Moreno, 2015:183).

Although it is important to think about how what you intended to do, turned out, one needs to reflect with a sense of historical moments and in the context of the action that occurred (Kemmis *et al.*, 2014:108). As the research team reflects back, the objectives of the study are juxtaposed with the outcomes of the enacted action. In this study the understanding of indicators of success and conditions for the successful implementation of the strategy to respond to challenges facing Grade nine teachers in their MPCK using PBL, for example, has become a point of reference as the research team collectively reflects on what happened in the implementation of the strategy. These historical moments include but are not limited to what has been demonstrated by co-researchers' efforts to change and how the architectural practice in their context enabled and constrained them (Kemmis *et al.*, 2014:109). The reflection stage of PAR is critical as it informs the research team whether to re-plan or strengthen what seems to be working in terms of solving their social problems. "The notion of emancipation is important here" (Hooley, 2005: 69), as the team needs to decide on the next stage without relying on the outside 'expert'. The researcher has to spend time reflecting on findings of the observations, negotiating the meaning with the co-researchers building a shared understanding.

In terms of CER, participation and reflection encourage the research team to critically examine their values and beliefs and through the process they become enlightened people (Hooley, 2005:69). The notion of communicative action becomes more pronounced to allow people to handle unwelcome news individually and collectively (Kemmis *et al.*, 2014:113). By nature, PAR does not always tell people what they want to hear, it sometimes exposes the worst scenarios when people are trapped in self-constraining beliefs and practices. The reflection stage of PAR sometimes brings unwelcome news about the nature and consequences of the way things are done in a community (Kemmis *et al.*, 2014:113). In the light of PAR individuals co-exist in a society as it is well documented that individualization is impossible without socialization and socialization is impossible without individualization (Habermas, 1992b, cited in Kemmis & McTaggart, 2007:280). The individual intersubjective interpretation of narratives needs to be handled with sensitivity as people share their feelings and lived experiences that cannot be described and judged by an outsider. It is further claimed that such unrestrained communication would ignite a debate resulting in unforced consensus that institutionalizes the common good (Bronner & Kellner, 1989:10). By nature, the PAR spiral cycles are not necessarily linear, as you pause and reflect individually and with others you might need to re-plan as informed by what has been discovered (Kemmis *et al.*, 2014:113). Centrally, the whole process of knowledge production is guided by philosophical assumptions that will be discussed in the following subsections, namely ontology and epistemology (Ngulube, 2015:127).

3.2.5 Ontology

Ontology and epistemology make up the paradigmatic base of research in a subject field (Ngulube 2015:127). Ontology is a branch of philosophy concerned with the nature of reality in terms of what constitutes reality and how existence can be understood (Gray, 2013:19). It involves “philosophical assumptions about the nature of knowledge, or the nature and existence of social reality” (Ngulube 2015:126). Precisely, it represents the worldview by defining the nature of the world and the individual's place in it, including possible relationships to that world and its parts (Guba & Lincoln, 1994: 107). It pertains to whether the reality is external to an individual or the product of individual consciousness (Poonamallee, 2009:71). The philosophical stunts that one chooses in terms of whether reality is seen as an objective phenomenon that exists externally to human beings or whether it is created by one’s own consciousness guides the research that is conducted. A positivist researcher believes that a single observable reality exists, independent of the experience of it and it is value-neutral, while a critical theorist “holds that reality is an ever-changing product of social processes” (Loewenson, Laurell, Hogstedt, D’Ambruoso & Shroff, 2014:1).

More specifically, the positivist view about the nature of reality is based on a belief that reality objectively exists out there which can be known through observable data by quantitatively measuring the relationships between variables (Koshy, Koshy & Waterman, 2010:12).

According to CER's ontological perspective, it is a congeries that has over time shaped reality, which includes but is not limited to social, political, cultural, economic, ethnic, and gender factors (Guba & Lincoln, 1994:110). The ontological stuns that leads to a knowledge extraction model of research fails to recognize the contextual issues, such as poverty, gender, race and power relations. Precisely, CER advocates for educational research that has a moral obligation to address social and political inequalities in order to realize social justice (Mack, 2010:11). Considering the research sites in this study, it was prudent for me as part of the research team to take note of the socio-economic realities created by the political history. These realities inter alia included poor school infrastructure, lack of resources and, in some cases, teachers that never specialized in mathematics in their training were made to teach mathematics. In a nutshell, this ontological perspective supports the acceptance of "the Other in its full distinctiveness, in its full difference" (Montero, 2000:135). Understanding of where the co-researchers come from in terms of their realities, without being judgmental created a space for sharing MPCK experiences in order to create co-researchers' shared reality.

Accordingly, meaningful relations are created and the domination of one by the other is curtailed through accepting the other (Montero, 2000:135), through engagement and free participation that develops trust. A critical ontological view acknowledges that the positionality of a person in terms of power and privilege influences his/her nature of reality, hence someone privileged might hold one version of reality as compared to the marginalized person (Mertens, 2015: 81). This study argues along the same lines as Mertens (2015:82,) who suggests that the nature of reality should be critically examined to allow intersubjective views. This ontological view puts power in the people who are directly affected by change as they develop strategies to improve their situation. Critical theorists believe that research is conducted for the emancipation of people to change the dominant discourses in a society (Mack, 2010:9). In terms of critical theory, the research should challenge the reproduction of inequalities (Mack, 2010:9). This concept of ontology comes into play when the co-researchers are encouraged to critically examine their own assumptions about the problems and the interventions (Mertens, 2015:82).

As the case was in this study, one needs to anticipate possible threats when designing and implementing the strategy. Taking cognisance of mathematics teachers' views in terms of what is mathematics and how it is taught, including the socio-economical influence and political history that shaped their social settings in the research sites was an important step. Essentially, CER acknowledges that reality is shaped by ethnic, cultural, gender, social, and political values, hence it focuses on realities that are mediated by power relations which are socially and historically constituted (Penterotto, 2005: 130). It is common knowledge that in the South African context mathematics was made accessible to whites, while blacks were denied access. One could argue that over time the segregation in this regard had eventually shaped co-researchers' realities in terms of MPCK. Gramsci' s theory of hegemony agitated that hegemonic patterns in a society are maintained because people are dominated both by coercion and by consent (Frisby *et al.*,1997:15). People may assume that the situations they find themselves in, are their own fault or are the natural order of things and therefore accept their situation (Frisby *et al.*, 1997:15). As we attempted to create our shared reality it was prudent to consider above factors that influenced our historical reality that we aspired to challenge and change.

Specifically, this research has an agenda to change co-researchers' lives, hence exploring their realities becomes critical. People's realities include but are not limited to their beliefs, indigenous knowledge and cultural influences which may either constrain or emancipate them. For an example, mathematics teachers adopted the use of 'safe-talk' in teaching mathematics under the apartheid era (Chick, 1996:10). Safe talk is the type of classroom discourse that promotes chorus responses from learners "to help the students to avoid the loss of face associated with being wrong in a public situation" and to help teachers avoid the disappointment associated with display of incompetence (Chick, 1996:10). Critically, CER, ontology and PAR present an opportunity for the co-researchers to unapologetically evaluate their praxis in order to transform it and invariably influence broader social transformation. At the heart of this study was the development of a strategy to contribute to the enhancement of Grade nine teachers' MPCK using PBL. Co-researchers needed to see themselves as being able to present information about themselves, to tell and interpret their stories as against the argument raised by Frisby *et al.* (1997:15) who claim that people usually do not regard themselves as being important enough to present information about their lives. Crucially, PAR “affirms people’s right to be listened to and understood” by bringing a new perspective to their situation that encourages them to take action to improve it (Frisby *et al.*, 1997:15) and develop an understanding of a new reality.

3.2.6 Epistemology

Epistemology is the philosophy of knowledge or how we come to know (Krauss, 2005: 758). According to Krauss (2005:758) the term epistemology was derived from the Greek word *episteme*. The research resonates with the definition of epistemology as Koshy *et al.* (2010:14) also view it as the theory of knowledge and how something can be known. Morgan and Smircich (1980: 493) argued that the different world views regarding ontology imply different grounds of epistemology about the social world. This implies that an objectivist view of the social world that views reality as a neutral and an absolute truth result in an epistemological stance that emphasises an objective form of knowledge that can be known through studying of the precise nature of laws, regularities and relationships among phenomena measured in terms of social facts (Morgan & Smircich, 1980:493).

In the research process a line is drawn between two schools of thought. Knowledge is either viewed as hard and capable of being transmitted in a tangible form, or it is softer, more subjective and “even of a transcendental kind based on experience and insight of a unique and essentially personal nature” (Poonamallee, 2009:71). The first-mentioned view argues that what comes from the dominant discourse is fixed and anything that does not comply with a predetermined criterion and rules is not regarded as knowledge in this regard. This kind of knowledge acquisition is influenced by the absolutist view of the social world view, which claims knowledge is out there as a real crystal-clear phenomenon which people have to acquire through a number of observations and set rules. The latter mentioned view of knowledge refers to a subjective experience which does not only regard the privileged, powerful and the researcher’s point of view as the only knowledge, but values the narratives and views of the researched, co-researchers and the marginalised. The literature denounced positivism as too orthodox for inquiry in the human sciences and argued that it proved to be obsolete resulting in the requirement of new visions (Lather, 1986: 63). In line with Lather' (1986: 63 view, this study has adopted a transformative epistemological view.

Critical theory's ontological view disputes the positivist grounds of knowledge and advocates for an epistemological stance that embraces the value of welcoming subjective views when people concretize their relationship with the social world (Morgan & Smircich, 1980: 493). This is in line with the Sesotho saying that knowledge does not reside in one house, many people have different ways of knowing. In terms of PAR, bringing them together helps so that we can tap into those different knowledges. This study has adopted a philosophical stance that is anchored in CER, hence I argue in line with the view that research should attempt to the address political, historical, cultural and socioeconomic situation of co-researchers. Critical theory holds a view that knowledge is not neutral but subjective, context bound and always political (Loewenson *et al.*, 2014: 20). Over and above, in PAR knowledge is built and developed out of collective comparison of subjective vivencia of that reality (Loewenson *et al.*, 2014:20). Proponents of this epistemological view promote PAR with a belief that people have a universal right to knowledge production and social transformation (Brydon-Miller, 1997:659).

Moreover, Habermas rejected the claim that "knowledge is produced by some sort of 'pure' intellectual act in which the knowing subject is himself 'disinterested'" (Carr & Kemmis, 1986:134). Instead, people are driven by subjective interests and their political material conditions in knowledge production. Driven by the subjective interests, the marginalised get empowered as they work to better their situation as against absolute knowledge accumulation (Guba & Lincoln, 1994:114), and in the process produce knowledge. The dialectical process "of historical revision that continuously erodes ignorance and misapprehensions and enlarges more informed insights" grows and changes knowledge (Guba & Lincoln, 1994:114). The critical epistemological view presents an opportunity to the co-researchers to critically examine their historical praxis regarding MPCK and in the process experience MPCK epiphanies in terms of how to change their hegemonic and constraining praxis. PAR allows co-researchers' voices as equal partners in identifying the challenges and interventions and to be heard as producers of educational knowledge in the research enterprise (Esau, 2013:3). Habermas views knowledge as an outcome of human activity that is motivated by natural needs and interests (Carr & Kemmis, 1986:134).

Once the knowledge has been critically looked at in terms of what Habermas calls ‘knowledge-constitutive interests’ (Carr & Kemmis, 1986:134), how it is socially constructed (Koshy *et al.*, 2010:12), and how it, in turn, shapes reality, the co-researchers do not only focus on improvement of their praxis , but also on broader social transformation by changing unequal relations (Esau, 2013:4). Within PAR the knowledge production occurs when people tell stories based on subjective accounts and interpretations of co-researchers’ lived experiences (Koshy *et al.*, 2010:12). The co-researchers’ subjective accounts of what they found to be working for them in terms of unshackling the ideologies that constrained them in teaching mathematics became knowledge. Accordingly, the new MPCK produced in the teaching of Grade nine mathematics and awakenings experienced by co-researchers enabled them to reject the oppressors’ reality, thus Freire’s (1970) notion of ‘self-depreciation’ (cited in Lesser & Blake, 2006: 160). In the oppressors’ opinions schools were places to reinforce the idea that students were incapable of learning mathematics (Lesser & Blake, 2006:160). However, one’s critical consciousness has been raised through lessons and skills enacted in praxis that led to transformation (Mack, 2010:10). In the whole process, as a researcher I was directly involved although not self-imposing my ideas as informed by the critical pedagogy that challenges structural inequalities and power domination of the marginalized groups. My role as a researcher had been examined and is discussed in the following sub-section.

3.2.7 Role of the Researcher

The researcher is at the centre of the research process; hence it is crucial that the role of a researcher should be clarified (Unluer, 2012:1). However, the researcher’s role is not immune to research paradigm (Postholm & Madsen, 2006:49). The philosophical assumptions or world views assist researchers in choosing the problems to study, the questions to ask and the theories to utilize in their production of valid knowledge (Ngulube 2015:127). In a nutshell the researcher’s role is informed by the theoretical framework chosen to be used when conducting a research. One can choose to be an insider, thus a complete member of the community involved in the study, or an outsider, that is, a complete stranger from the group being studied (Unluer, 2012:1). Other than initiating and guiding the research process, the researcher's role in the positivist paradigm is primary focused on capturing and objectively representing what has existed out in the world (Postholm & Madsen, 2006:49). The emphasis on neutrality results in the researcher's role to be seen "as a privileged processor of expert knowledge" (Lather,1986:73).

Contrarily, Latter (1986: 63) disputed the claims of neutral and interest-free knowledge and argued that it was logically impossible to have such claims. The emphasis on the neutral role of a researcher blinds one "to the way in which practice is constituted as a multiple reality that is perceived differently by different participants" (Kemmis & McTaggart, 2007:286). The researcher's role should be "reconceptualised as that of a catalyst who works with local participants to understand and solve local problems" (Lather, 1986: 73). This view resonates with the argument which advocates that expertise is shared in PAR and the role of the researcher is one of a 'facilitator' (Noonan, 2015: 196), working together with a diverse team of co-researchers. In line with Noonan's (2015: 196) view and other proponents of CER that regarded positivism as a Hippocratic delusion of relevance, I became an inside facilitator in the research team. The research documented a number of advantages in conducting an inquiry from within, inter alia, greater understanding of the politics of the institutions under study, knowledge of the culture being studied and an established intimacy with the co-researchers (Unluer, 2012:1) which promoted dialogue and trust.

Accordingly, Heron and Reason (2006 in Hawkins, 2015:6) mentioned three fundamental and interdependent issues to be considered by a researcher when beginning a research process. They argued that the first one is a thorough orientation and induction of the co-researchers so that they can own the process. In congruence with Freirian research that was designed to have an "arousal effect, to reorient participants' perceptions of issues which subsequently influenced their attitudes and behaviours" (Lather, 1986:73), the orientation process was viewed to cognitively empower participants (Hawkins, 2015:6). Secondly, it is also noted that the researchers must strive for the emergence of participative decision making by encouraging co-researchers to discuss the strategies to be enacted. Thirdly, to empower participants emotionally and interpersonally and to create a climate of mutual trust and warmth, the researcher should allow open and free expression (Hawkins, 2015:6). Contrary to the conventional forms of research methodology where authority is vested in the researcher, PAR aims to shift responsibility for the research process onto individuals and groups who are directly affected by these inequalities. From Hawkin's (2015:7) perspective the research project is understood to be participants' inquiry and they became co-researchers with the researcher that was a facilitator.

When I initiated the investigation, I invited as many participants as possible who might contribute to the formulation of the solution. In line with the ontological stance (multiple realities) underpinning this study, I facilitated interactive meetings with co-researchers to prioritise components of MPCK that needed to be investigated. Together, as a research team, we proposed a strategy to be enacted in response to challenges regarding MPCK. We further interpreted and analysed the generated data, and drew conclusions on what worked or did not work and decided on new cycles of actions to be taken. Driven by the epistemological stance which believes that the most influential knowledge is produced through collaborative actions and is embedded in social relationships (Hawkins, 2015: 6), I consciously reflected on the effect of power that was traditionally bestowed on me by continually encouraging and reminding the research team that all co-researchers' responses were equally important and should be treated with respect. Taking cognisance of Mahlomaholo's (2013:318) multiple perspectivism, the co-researchers and I agreed that the solutions to challenges facing the research team were unknown to all of us, but through collective diagnosis and understanding of the situation, solutions would emerge, albeit slowly.

However, one could not be totally immune from the difficulties involving a shift in the roles of both researchers and participants. It is claimed that the researchers' dilemma was two sided whereby the researchers needed to let go of control and co-researchers needed "to step up and become more engaged" (Nelson *et al.*, 1998:886). As a rule of thumb, I had to constantly remind myself and the research team that more important than the academic purposes of what we were doing (research), was the research team's way of critically reviewing its praxis with the aim of improving it. Accordingly, power disparities required the research facilitator to encourage co-researchers to devise strategies that empowered all to speak and to be accurately heard and be respected (Hawkin: 2015:7). According to Tuck *et al.* (2008 in Moreno, 2015:183), PAR meetings are described as contact zones, which are politically and intellectually charged spaces whereby differently positioned co-researchers are able to together experience and analyse power inequalities. It is further argued that fundamentally PAR is about the right to speak (Hall, 1993 cited in Nelson, Ochacka, Griffin & Lord, 1998:885). The researcher's role as an involved facilitator influenced the relationship between the researcher and the co-researchers.

3.2.8 Relationship between researcher and co-researchers

PAR aims to shift responsibility for the research process onto individuals and groups who are directly affected by these inequalities. In line with PAR, researchers set their expertise alongside the lay knowledge, skills and experiences of people who are the focus of their investigations. In this way the research process is conceptualized as an encounter, where equal partners meet, enter into dialogue and share different kinds of knowledge and expertise on how to address issues affecting a group or community. In this respect PAR is unequivocally committed to a politics of equity and social transformation (Jordan & Kapoor, 2016:138). In a transformative paradigm like the one adopted by this study, researchers and co-researchers take turns to share information regarding the purpose of the research (Mahlomaholo, 2013c:318). This humanising exercise developed professional, intimate relationships between the co-researchers and the researcher. In line with the proponents of CER, this research values the co-researchers as equally important as the researcher.

The voice of the co-researchers, their perspectives and meaning are not only recorded for the purposes of later interpretation, but as part of the fabric of the research methodology. This is in line with ccritical pedagogy which puts the voices of marginalised and disenfranchised at the centre of the research discourse (Nkoane, 2011:119). Researchers do not carry out transformation for participants but with them (Freire, 1970:49). This relationship focuses on empowerment of the people for whom the research is intended and their voices are no longer muffled as they are fully engaged in the research process (Lindsey, et.al., 1999 :1240). Such a relationship sees the researcher as being tasked with interpreting other peoples' interpretations and trying to make sense thereof; research is seen as the most humanizing experience (Mahlomaholo, 2009:225). Furthermore, strengthening the consensual relation in research, Nkoane (2013: 396) requires us to become totally immersed as equal partners in this intellectual journey. Through this approach, we value principles of democracy, social justice, sustained livelihood and empowerment of all.

In this relationship, the power is shifted from a top-down researcher-dominated initiative, and power relations are reversed through the explicit understanding that researchers are not individuals with ready answers "who come to do research so that they endorse their findings; rather, it is a collaborative journey to generate data" (Dube, 2016: 41). Participants become co-researchers in generating data, while the researcher is no longer the sole arbiter of what counts as knowledge Mumby (1993 cited in Dube, 2016: 41); instead the knowledge is generated through consensual relationship between the researcher and co-researchers (Nkoane, 2013: 396). Shedding light on this relationship, Nkoane (2013: 394) avers that social justice becomes a norm in this kind of relationship because it is about respect and addresses issues of equity, freedom, peace and hope. CER values the contribution of the participants, and as such, the research becomes transformational and a problem-solving platform.

3.2.9 Rhetoric/ Language

According to PAR and CER, because of the co-researchers' views in and contribution to the research process, the language is considered as an important tool to build "the relationship of mutual trust, humanity and care" (Qhosola, 2016:38). Depending on the theoretical framework that underpins one' study, the language tends to reveal power relations. Arguably, the use of certain language connotations and insinuations tends to reduce people to objects and subjects (Qhosola, 2016:38). From a participatory posture, the researched are an indispensable part of the research process. Their role in the research process elevates them to be equals of the researchers and therefore they are called co-researchers. In terms of PAR, generated local useful information is recorded in an accessible form (Tsotetsi & Mahlomaholo, 2015:49). By implication, co-researchers use their language to express their views equally. According to Strickland (2006, cited in Qhosola, 2016:38), the interpreter may be employed if needed to encourage meaningful conversations. CER neutralizes the researcher's powers, "thus becoming co-learners occupying the same status as the participants and partners in knowledge generation" (Tsotetsi & Mahlomaholo, 2015:49). On the other hand, CER enables co-researchers to tell their stories, and their knowledge is not viewed as weak knowledge but respected as valuable knowledge (Tsotetsi & Mahlomaholo, 2015:49). This study allowed co-researchers to use the preferred language in presenting their thoughts. Their views were respected in terms of shaping the study. As they experienced how intersubjective views were valued, they even proposed some tasks that were beyond the scope of this study, namely the setting of common assessment tasks and cluster moderations in each academic term (see appendix 5, appendix 6 & appendix 9).

3.2.10 Ethical consideration

I first solicited permission to conduct the research form the Eastern Cape Education Department's head office (see appendix 11). Having been granted permission by the Department of Education to conduct research in the Joe Gqabi District of Education, formally known as Mount Fletcher, and I also received ethical clearance from the University of the Free State (see appendix 7). During the initial meetings attended by invited participants who later became the co-researchers, I informed them about the ethical considerations involved in the study. A copy of the letter from the provincial Department of Education granting permission for the study to be conducted was presented to and discussed with all the co-researchers (see appendix 1). It was further clearly explicated that they had freedom to withdraw their participation from the study at any time, should they wish to do so. The study was conducted in a manner which would not cause harm to or threaten the lives of the co-researchers. All those who showed willingness to participate in the study were asked to sign consent forms. The data were generated through the discussions during problem-based learning workshops (PBLW), lesson observations and reflection on lived experiences. Data were captured by audio and video recordings. In addition, learners' scripts and reflection on the lesson presentation were used to generate data.

3.3 CONSTITUTING THE COORDINATING TEAM

The study focused on schools in the Joe Gqabi District of Education, particularly schools that belong to a small town called Mount Fletcher in the Eastern Cape (EC), which is a predominately rural and poor area. The Mount Fletcher town is located in the Elundini Municipality in the Joe Gqabi district municipality. Tekete (2012:1) tabled the research findings of the Department of Social Development (2004) that showed the Joe Gqabi district municipality as a poverty-stricken district with the following demographic features: 89% rural population; 10% farming population; and 1% urban. It is claimed that blacks living in poverty constitute about 80% of the population in the Elundini Municipality (Tekete, 2012:1). From this community, secondary teachers often travel from the nearby towns to the rural schools, that is, Matatiele, Mount Fletcher and Maclear during the early hours of the day. The location of the study area is fertile ground for the expression of critical emancipatory research (CER). A study using CER as a guiding lens is sensitive to “those who were located in the periphery of society, excluded, relegated, marginalised and oppressed” (Nkoane, 2012:100). In essence, the research process in this regard was focused on transforming both researcher and the participants’ research site, and advance democracy, liberation, equity and social justice (Nkoane, 2012:100). While the researcher endeavours to interpret other people’s interpretations and try to make sense thereof, the research process is seen as the most humanising experience (Mahlomaholo, 2009:225).

Co-researchers were invited through a formal invitation sent to HD Ndzelini circuit schools. The invitation reflected the title of the study and the co-researchers’ involvement in the project were clearly outlined. This was done to present a clear picture of what co-researchers were expected to do to attract the most suitable co-researchers. Specifically, on the consent form that was signed by co-researchers, ethical consideration aspects, such as confidentiality, anonymity and freedom of withdrawal from this study at any stage were spelt out in details. After follow-up calls had been made, co-researchers who indicated their willingness to participate in the study signed consent forms to indicate that no one had been coerced to be part of this research. Out of eight mathematics teachers that participated in the study, only two pairs were teaching in the same schools and the rest, thus, four teachers came from different neighbouring schools participated in this study. One mathematics subject advisor and one principal did not only sign the consent letters, but also participated in the study. Other principals allowed their teachers and learners to participate in the study. The co-researchers who responded to the invitation were invited to a meeting to get familiarized with one another and be introduced to the aim of the study. From the first meeting, we agreed to form a team with the aim of improving mathematics teaching practice.

3.4 CREDENTIALS AND ROLES OF THE RESEARCH TEAM

This section discusses co-researchers' credentials and the roles they played in the study as coordinating team members. This is a representation of a wider group of people affected by the identified need. For the purpose of this report only the coordinating team is highlighted in the section below, namely Grade 9 learners, mathematics teachers, subject advisor (learning facilitators) and the researcher.

3.4.1 The study coordinator

The role of the researcher has been explained earlier (see section 3.2.7). However, my role as study coordinator is explicitly described, focusing of the exact role that I played in this study. Other than initiating the study I become a team leader and coordinated a team of co-researchers. My role was also elucidated in terms of the CER theoretical framework as coaching the study and relating it to PAR. After receiving the ethical clearance, I convened the first preparatory meeting with prospective co-researchers who had been invited to participate in the study. I organised PBL workshops, conducted research with the research team, coordinated the activities of the research team, participated in collaborative planning meetings, recorded generated data during collaborative planning sessions and lesson observations, that were further analysed and interpreted together with the research team. Furthermore, I ensured the ethical clearance processes were adhered to.

3.4.2 Senior phase mathematics teachers

Teachers were invited to participate in the study so that they could be actively engaged in the efforts that were aimed at enhancing their MPCK using PBL. After signing the consent forms, they actively participated in the process of finding solutions to their problems and being co-constructors of knowledge. They were emancipated and developed a consciousness of how they had been teaching mathematics. Six of teachers that participated in this study only taught senior phase mathematics classes (Grades 8 and 9), namely, Rhulumente, Mbuyi, Njovane, Jones, Ntozine and Zintle. These names are pseudonyms that were used to conceal their identities. They were teaching in different schools, except for Rhulumente and Njovane who were teaching at the same school. Five of the teachers that only taught in the senior phase were qualified to teach at senior phase level, except for Mbuyi, who was only qualified to teach at primary phase. Mbuyi worked under Mohlakoana's principalship, the only principal who fully participated in this study. Mbuyi had taught mathematics for 16 years, although she was only qualified to teach at primary level. Rhulumente had taught at senior phase for seven years and then moved to the foundation phase where she was an HOD for 13 years until she resigned. Two years ago, that is in 2015, she joined the Department of Education again. Due to curriculum changes, she could be viewed as a novice teacher in the senior phase. Zintle was a seasoned mathematics teacher. She has taught mathematics for 29 years at senior phase and seemed to have valuable experience in terms of teaching practice. Ntozine had a primary teachers' diploma (PTD) and a Bachelor in Education Management (B.Ed). She had been teaching at primary phase for 11 years and had joined senior phase for the past six years. She had been promoted to Head of the Mathematics Department during the course of this study. We earlier explained that Njovane taught at the same school as Rhulumente. They both taught Grade nine classes, but for different groups. Njovane had been teaching Grade nines for six years and had a Bachelor of Science qualification. He also had a post-graduate certificate in education that made him qualified to teach in SA schools.

3.4.3 Further Education and Training (FET) phase mathematics teachers

FET teachers' participation in the study helped the coordinated team to gain a coherent understanding of mathematics as taught from Grade 9 to Grade 12, thus, together they had integrated vertical curriculum knowledge. Of the eight teachers that participated in the study, two were teaching in both the FET phase and the senior phase, namely Falafala and Nowele. Falafala, like Mbuyi, had a senior primary teachers' diploma and had been teaching mathematics for 21 years at senior phase. Falafala has been recently transferred to a new FET school where she was allocated grades nine and ten learners to teach. Nowele was a newly qualified teacher, with a Bachelor of Education degree. She had been teaching in both the senior phase and the FET phase for the past three years. Falafal and Nowele were the other pair that Teachers involved in this study had knowledge of working with learners from different socio-economic conditions, such as child-headed families. They developed an interest in understanding learners' backgrounds and presented parents and other stakeholders with quarterly results of analyses. They were able to meet parents/stakeholders whenever necessary to discuss learners' progress and possible interventions.

3.4.4 Principals

Of the six schools that participated in this study, Mohlakoana was the only principal that fully participated in the study. However, he taught English and life orientation, but not mathematics. He had been teaching English for 27 years although he had been a principal for seven years. When we visited his school for classroom observation, he was always keen to get feedback on what had transpired during the class room observation, and shared his views on how we could improve mathematics teaching. Other principals signed the consent forms and allowed both learners and teachers to participate in the study, without getting actively involved in the research process. Generally, principals helped us sort out disciplinary problems that we encountered at the research sites, especially during lesson observations. Drawing on the trust from the side of parents, principals were able to explain the meaning and the purpose of the study to sceptical parents and indicated that the study would do no harm to their children. As the activities of the study were also shared with them, principals created conditions conducive to the implementation of the components of the strategy to enhance MPCK using PBL.

3.4.5 Grade 9 learners

Since the learners were minors, for them to participate in the study we first solicited their parents' consent by means of letters. Consent forms were written in learners' home languages for parents to understand the contents, thus, *isiXhosa* and *seSotho*.

Learners contributed in the study through written tasks and communicating their thoughts to peers. After each on-site lesson observation, a maximum of three learners were requested to voluntarily reflect on the lesson proceedings. Their reflections were either taken from their written work or audio recorded to be later transcribed, analysed and interpreted. BL as learner-centred approach enabled learners to apply knowledge and skills to develop a viable solution to a defined problem (Savery 2006:12). In line with PBL and learner-centredness, learners engaged in problems given to them to solve, reflected on the lesson presentations, and proposed how they would like future lessons presented. Although learners were not the centre of focus *pe se*, their contribution exhibited both challenges and success indicators in terms of the enactment of the strategy to enhance MPCK using PBL. They further indicated that if they were teachers, they would allow those learners who seemed to have understood mathematics concepts to share how and why the methods of finding a solution worked.

3.4.6 Subject advisor

A mathematics subject advisor is responsible for the schools in the district through conducting diagnostic research and professional development workshops. Tau paid visits to schools to support mathematics teachers in presenting their classes and to ensure compliance with CAPS and departmental policies. In terms of social structure, he occupied a powerful position. Notwithstanding his position he contributed to the study by becoming an expert learner rather than a knowledge dispenser. His expertise and his social standing helped to sustain the developed strategy after it had been enacted and operationalized. Although he was mainly tasked with facilitating during our problem-based learning workshops (PBLW), he indicated that he was not a know-all and emphasized that he was also learning to become a better mathematics subject advisor through sharing experiences with the coordinated team. Tau had been teaching in the FET phase for six years and had been a subject advisor for eight years. His humbleness changed the power relations in the coordinated team and encouraged open sharing of problems.

3.4.7 Parents

Most of the learners' parents were unemployed and a few were eligible to receive social grants on the grounds of their age. They were from the rural community which is at the periphery of active economic participation. They signed consent forms permitting their children to participate in the study. Despite coming from marginalised backgrounds, they felt valued when the schools and members presented the purpose of the study and quarterly analysis results of their children's performance, particularly in mathematics. They allowed their children to attend extra classes as part of the implementation of the strategy.

3.5 COMMON VISION

Common vision is the coordination of both the existing and future efforts by pulling the mathematical science community towards the same direction to improve learners' success (Saxe & Braddy, 2015:3). It serves as a catalyst that draws collective wisdom from diverse stakeholders towards a particular goal (Saxe & Braddy, 2015:8). It appears that a common vision motivates organised members to all gear their efforts towards attainment of the common goal (Tsotetsi, 2013:74) thereby improving our teaching practice. Common vision further enables members of the coordinated team to brainstorm together and propose future actions (Saxe & Braddy, 2015:31). Daily activities of a coordinated team are clearly directed and guided when there is a common vision (Mosia, 2016:139). This guidance, according to Qhosola (2016: 217) unifies the team efforts and eliminates personal interests in the team's vision that may derail the whole process of implementing the strategy. Common vision invokes a team's consciousness about their current reality, providing them with a clear picture about the envisaged destination (Qhosola, 2016:217). The initial meetings with the participants that later became co-researchers were employed to discuss the vision and the need of establishing the coordinated team.

3.6 STRENGTHS WEAKNESSES OPPORTUNITIES THREATS (SWOT) ANALYSIS

A SWOT analysis identifies and evaluates the environment of an organisation in terms of internal strengths and weaknesses, including external opportunities and threats (Sammut-Bonnici & Galea, 2015:1). The identification of these factors (strengths, weaknesses, opportunities and threats) enables the development of a strategy that may take advantage of strengths that will counteract weaknesses and make optimal use of opportunities to eliminate threats (Tsotetsi, 2013:76). When the coordinated team identify the "areas in which they lack expertise, strategies can be developed to overcome weaknesses and thus increase the overall efficiency and efficacy of the planning process" (Thomas, Chie, Abraham, Jalarajan Raj & Beh, 2014:5). In accordance with Thomas *et al.*'s (2014: 5) view, the initial meetings with co-researchers realized the opportunities provided by team work and were able to identify areas of weakness in terms of their lived experiences regarding teaching of mathematics. Team members were relatively well qualified in terms of professional qualifications required for teaching mathematics in SA schools. Their qualifications were viewed as a strength in the team, especially the participation of the subject advisor. However, their actual allocation in terms of which grade to teach seemed to threaten their efficiency. The process of a SWOT analysis guided the team in developing priorities and in their strategic planning.

3.7 PRIORITIZATION OF CHALLENGES

The process of prioritization is a management discipline that guides strategic planning and implementation at all levels (Kerzner, 2013 in Malebese, 2016:118). The formulation of a strategic plan, *inter alia*, deals with choices between different options and strategies (Rondinelli & Cherif, 2009:2). Prioritization is done to ensure the achievement of "higher needs identified along the analysis and evaluation of problems" (Rondinelli & Cherif, 2009:2). It is a subjective process of attaching value to identified problems to determine which ones could be chosen for attention first in the decision-making approach (Rondinelli & Cherif, 2009:2). It is recommended that participants should negotiate to achieve consensus in terms of which problem to start with in the strategic plan. As priority, co-researchers in this study unanimously agreed on the establishment of a coordinated team that would be a platform for presentation and untangling problems. Specifically, the coordinated team would organize what we called problem-based learning workshops (PBLWs) to collectively discuss problems that emanated from co-researchers' teaching practice in terms of pedagogically related challenges and mathematics content gaps. Thirdly, the coordinated team resolved to include the on-site visits to enable the team to understand contextual factors regarding the identified problems.

3.8 STRATEGIC PLANNING

A strategic plan is a document that provides a blueprint on how to effectively and efficiently attain an organisation's goals, with detailed actions needed to be achieved (Maleka, 2014:15). On the other side, it is viewed as the specific details of actions to be taken, responsibilities attached to specific persons, indicating the "duration of the activity, resources needed as well as performance indicators" (Tsotetsi, 2013:156). Strategic planning cycles around the following phases: environmental scanning, strategy formulation, strategy execution, and evaluation (Maleka, 2014:16). The following table, Table 3.1 presents our plan of action as developed during the planning meetings.

Table 3.1: Plan of action

Activity	Responsibility	Monitoring	Evaluation	Time frame
<u>Preparatory phase</u> Initial planning meetings	Research coordinator	Research coordinator	Participants' attendance, brainstorming and development of team norms and action plan	One-hour meetings from 02/02/2017 to 23/03.2017 every fourth night (see appendix 10)
<u>Phase 1</u> Problem-Based Learning Workshop and Collaborative lesson planning	Subject advisor and co-researchers Co-researchers and research coordinator	Facilitation of the workshop. Presentation of co-researchers challenging topics and problems. Development of new lesson planning schedule. Two or three co-researchers meet and plan together depending on contextual factors	Discussion of success stories and challenges in mathematics teaching. Sharing of different approaches to mathematics concepts' representations Development of detailed lesson plans, juxtaposed with DBE lesson plans	2 hours
<u>Phase 2</u> Lesson observation	Research coordinator with available co-researcher & 3	Research coordinator and available co-	Discussion emerging components of MPCK, such as	60 minutes

	learners who volunteered	researchers. All eight co-researchers observed at least twice	concepts, misconceptions, skills and proposal for improvement on identified gaps	
<u>Phase 3</u> Lesson reflection	Co-researchers and 3 learners who volunteered	After every lesson observation, the lesson would be discussed	Co-researchers would indicate what worked well and proposed improvement and re-planning where there were gaps	30 minutes
<u>Phase 4</u> Assessment of the lessons	Co-researchers, 3 learners who volunteered and research coordinator	After every lesson observation the lesson would be discussed	Co-researchers and research coordinator would indicate what worked well and proposed improvement	30 minutes

As reflected in the above table, each activity plan constituted four phases, coherently structured in a manner that implied diverse dimensions. However, some phases were repeated in cyclical dimensions in line with Kemmis and McTaggart's (2007: 287) recommendations, who presented the cyclical steps of PAR in the form of planning of change, acting and observing the process, followed by reflecting on the subsequent change that further leads to re-planning for the next cycle of steps. These diverse dimensions were viewed from an ontological stance that embraced the otherness as we collaboratively worked with a team of co-researchers (see section 3.2.5). In accordance with the epistemological perspective, that in teaching and learning mathematics knowledge is accessed from transcendental experiences of uniquely subjectivism (Poonamallee, 2009:71), co-researchers came from different schools will a wealth of knowledge that was essentially valued to guide this study as new knowledge was produced through negotiated meaning.

3.8.1 Phase one: Problem-based Learning Workshop and Collaborative lesson planning

Having established the co-ordinated team of co-researchers and the team norms (see Appendix 10) after the preparatory phase we started with a workshop and planning sessions and identified the MPCK challenges and success stories that could be shared with team members. The team of co-researchers comprising the subject advisor, eight co-researchers and the research coordinator met during what we called problem-based learning workshops (PBLW) and collaborative planning meetings. The subject advisor, as a facilitator, allowed the co-researchers to raise issues pertinent to them regarding the teaching of mathematics. From the co-researchers' discussions, the team adopted common grounds for the teaching of mathematics which would encourage learners to construct knowledge in meaningful ways. A teaching and learning environment should be created where learners could learn from each other through engaging them in explaining mathematical reasoning to support their assertions. This learning and teaching environment would allow the co-creation of knowledge through active engagement in problem-based learning activities.

In a nut shell, the co-researchers realized that embracing a more learner-centred pedagogical approach could provide learners with an opportunity to be critical thinkers. It also emerged that manipulatives could be used to exemplify mathematics concepts as the coordinated team shared the powerful analogies to represent mathematical ideas (Shulman, 1986:9). Based on the PBLWs the coordinated team comprehended that mathematics should not be taught as a set of meaningless, unrelated rules, but every mathematics assertion should be supported by reasoning which would enable learners to apprehend that mathematics is learnable (Wu, 2018:17). Towards the end of this phase co-researchers had developed detailed lesson plans in line with DBE lesson plans (see Appendix 3). Collaborative lesson planning considered using learners' preconceptions as the springboard for learning through assessment-embedded instruction.

3.8.2 Phase two: Lesson observation

Lesson observation was conducted for all eight co-researchers that participated in the study. Each co-researcher experienced at least two lesson observations. When possible, the lesson observation was not only done by the research coordinator, but he was accompanied by an available member of the coordinated team. During the first round of lesson observations, the co-researchers generally presented without lesson plans and manipulatives. Predominantly, lesson presentations were teacher-centred, focusing on demonstrating mathematics procedures which learners would be later assessed on, and the co-researchers worked in silos. At this level, challenges were also diagnosed and audio-visually recorded for later utilization during the reflection on the lesson presented. In essence the first round of lesson observations confirmed the diagnosis and challenges identified in phase one, which exhibited teaching practices that generally were in contrast with good practice (see section 2.3.2.2.3). Challenges that were identified will be specified in Chapter four.

However, after having internalized the teaching practice adopted in phase one during collaborative lesson planning, co-researchers managed to let go focusing on procedural knowledge and embraced good practice in teaching mathematics. They critically listened to learners' views and made use of these views as the starting point for discussion. Learners were encouraged to use manipulatives or any other resources available to provide reasons for their thinking. Their contextualized lesson plan drawn from collaborative lesson planning meetings embraced aspects of good practice in teaching mathematics. Aspects such as assessment-embedded instruction were used to identify the cutting edge of learners' competences (Heritage, 2010c: online) in order to appropriately dovetail the pedagogical approach to suit the learning needs.

3.8.3 Phase three: Reflection on lesson presented

The reflection process was done by learners, co-researchers and the research co-ordinator after each lesson presented in order to reach a common interpretation of what transpired during the lesson presentation. Learners were given an opportunity to share with the class what they had learnt from the lesson presented. They were allowed to propose how best they thought the future lesson could be presented. According to Moloi (2013:128), reflective feedback "provides dual dimensions of extensive-intensive facets, wherein several observations are made to reach substantiated conclusions (extensive case) and vice versa (that is, intensive cases)". Multiple perspectives seemed to emerge during this phase; parties that were part of the lesson presentation came up with different views in terms of how the lesson worked out. Learners we able to argue their case, which, by implication, demonstrated success of the lesson in developing a deeper understanding of mathematics concepts. Both the researcher and the observed co-researchers would arrive at a negotiated decision in terms of the lesson success, considering both learners' verbal feedback and their written work.

3.8.4 Phase 4: Assessment and evaluation of the lesson

This phase focused on gathering a narrative account of what happened (Kemmis *et al.*, 2014:107). These narrative accounts were collectively discussed by the co-researchers and the research co-ordinator as they tried to make sense of other people's interpretations and understanding of their world informed by their experiences (Nkoane, 2012:100). The following excerpts exhibited subjective accounts about the lessons' success:

> *Researcher: What was different from today's lesson as compared to other lessons?*
>
> *Setleko: The difference today, Em… when the teacher teaches us, sometimes I do not understand and do not quite see how it is done. What we were doing today, it was for the first time for me to discover that it was so easy to solve mathematics problems when we work as group as compared when I worked alone.*

The researcher raised the same question to Sponono and Nodada (learners that were in Rhulumente' class). They had this to say:

> *Sponono: It is that when you have a certain opinion about how you see mathematics problem you were allowed to express it.*
>
> *Researcher: What did you like in today's class?*
>
> *Nodada: What I liked today, is that I know and I am able to tell others what was happening, I have heard what was taught and as I left the class, I understand the multiplication of binomials.*

The argument raised by the learner participants above, illustrated a shift in how they were usually taught. Rhulumente also presented her experiences regarding her lesson that started with mathematics problems rather than a formal presentation and demonstration of mathematics concepts to be followed by assessment.

> *Researcher: I really enjoyed the lesson presentation, especially that there was a lesson plan that was first discussed by a team, we would like to know, what was different from today's lesson as compared to other lessons?*
>
> *Rhulumente: What is different today, Um … the learners were more active and learners are the ones who were busy doing their work calculating some activities. I think the group working is so important to them.*

These accounts from the co-researchers and learners exhibited the shift from teaching practices that constrained learning to experiences providing an empowering environment created by collaborative planning and the sharing of expertise. At the end of this phase, the co-researchers noticed how their practice had enabled changes and fostered their capacity to make a difference in the social space (cf. Kemmis *et al.*, 2014:107).

3.9 DATA ANALYSIS THROUGH CRITICAL DISCOURSE ANALYSIS (CDA)

The study used critical discourse analysis (CDA) to analyse and interpret the data. CDA is traced back to Western Marxism trends that criticised social injustice and the unequal distribution of power in new capitalist societies (Bijeikienë, 2008:105). Wodak and Meyer (2008:4) view CDA as a network of scholars that emerged in the early 1990s. However, Fairclough (2013 in Mosia, 2016:102) claimed that CDA emerged as a sub-area of discourse analysis as early as the 1970s. It is clear that CDA has been used by multidisciplinary fields for a number of decades. Nonetheless, this study is not focused on the history of CDA, but on how it was used to analyse, interpret and explain enacted inequalities and dominance through text and talk (van Dijk, 2001:352). Moreover, there is no single homogeneous version of CDA, but a shared perspective of conducting linguistic, semiotic and discourse analysis (Bijeikienë, 2008:105). According to Fairclough and Wodak (1997, cited in Bijeikienë, 2008:105), this shared perspective draws on the consensus that the discursive events have a dialectical relationship which is shaped by social structures and also shapes them. CDA is used to analyse and interpret data through application of its principles and its agenda.

3.9.1 Principles of CDA and its agenda

CDA is one of the analytical research modes that critically examines discourse in terms of how social power is abused and inequality is enacted (van Dijk, 2001: 352). CDA "has an explicit political agenda" committed to intervene against domination and openly declared its emancipatory interests by exposing the ideological effects of a discourse (Jones, 2004:98-99). It resists social inequality (van Dijk, 2001: 352) in terms of its explicit position to understand and to expose the production and reproduction of unequal power relations between social classes through the ways in which they represent things and position people (Jones, 2004:99). Centrally, CDA exposes how texts conceal or overtly manifest power relations, legitimize dominance, control discourse, hegemony and ideology, and take a political stance by enabling CDA analysts to overtly align themselves with the discriminated and powerless in social struggles. CDA analysts intend to describe and interpret structures and properties of text, talk and communicative events that reproduce the concealment of dominance (van Dijk, 1993:250). In the process it to reveals political and ideological motivations of the practices and conventions in and behind texts.

CDA views power as a social power of different groups and institutions (Bijeikienë, 2008: 106). Van Dijk (2001: 354) specifically, defines social power in terms of control. A particular group (usually elite) for example, is claimed to have power if it is able to more or less control the minds and actions of other social groups (Van Dijk, 2001: 355). The eligibility of the power base is determined by the access to limited resources such as force, money, status, fame, knowledge and information (Van Dijk, 2001:355). Van Dijk (2001:356-357) in distinguishing different forms of control, mentioned control of context, control of topic and control of text and talk. Since power is not absolute, he argued that the dominated groups may resist or accept, condone and comply with power domination. At the heart of CDA is the urge to overtly resist and denaturalize power domination by revealing ideas and assumptions in texts. Van Dijk (2001:355) also noted that power domination may "be integrated in laws, rules, norms, habits, and even a quite general consensus", and as such the dominated group legitimizes such power, viewing it as natural and taking a form of Gramsci's hegemony. Paradoxically, power abuse and dominance may seem to be jointly produced when the dominated group views dominance as natural and legitimate (van Dijk, 1993:250). Through CDA we were able to understand why mathematics teachers chose to teach certain topics over others and what informed their pedagogical choices.

Moreover, Fairclough (1989: 43) explored various dimensional relations of language and power by distinguishing between ‘power in discourse’ and ‘power behind discourse’. ‘Power in discourse’ is demonstrated in cases when relations of power are exercised and enacted in a ‘face-to-face’ spoken discourse where participants are unequal, for an example a doctor and a medical student (Fairclough, 1989:43-44). During what Fairclough (1989:44) calls an ‘unequal encounter’ the doctor controls the discourse as he exercises his social power by controlling the student’s contribution through various linguistic means. This is no exception in many mathematics classrooms. For example, (Mceleli, 2004:15) exposed this kind of classroom discourse that took place in an Eastern Cape Rural Schools (ECRS) mathematics class.

> *ECRS mathematics teachers use some clues to stimulate the chorus responses from pupils. For example, when they teach the concepts such as sums or difference, they use clues such as the following: When we want to get the sum we use add …, then pupils say “addition”. When we want to get the difference, we use sub…, then learners in a chorus say “subtraction”.*

This kind of power in discourse suggests that the teacher is the only one who has the power to approve or disapprove mathematics procedures and answers. It is also clearly put that the doctors’ interruptions were consciously orchestrated to control the contributions of the students (Fairclough, 1989: 44).

On the other hand, the power behind discourse is linked to the notion of ideology, which is another significant corner-stone in CDA (Bijeikienë, 2008:107-108). The researchers echoed each other in viewing power behind discourse as a hidden power that shapes and constitutes various public discourses in terms of power relations that are not generally apparent to people (Fairclough, 1989:55; Bijeikienë, 2008:106). Power behind discourse “is meant to reach out of a particular speech event” (Bijeikienë, 2008:106) and to say more than just a speech by expressing and canvasing a particular ideology. In essence, “the whole order of discourse is put together and held together as a hidden effect of power” (Fairclough, 1989:55). In our view power behind discourse could be viewed as a concealed intention of the text producer. The power behind the discourse is implicitly presented in the practice of the media, rather than explicitly (Fairclough, 1989: 51). Apparently, the goal of “CDA is to demystify discourse by deciphering ideology” (Wodak, 2011:52). Ideologies function effectively when they are viewed as common sense, least visible and taken for granted by the general public (Fairclough, 1989:85), and as a result they become dominant.

3.9.2 Three dimensions of critical discourse analysis

Fairclough (1995:97) identified the three-dimensional conception of CDA, thus, text, discursive practice and social structure. These three levels of CDA are elaborated on below.

3.9.2.1 Critical discourse analysis at text level

The research acknowledges that texts are the evidence for the existence of discourse, which is a concrete realisation of abstract forms of knowledge that is not immune from interactive influence of sociolinguistic factors (Tenorio 2011:186). Text is analysed in terms of what it really means in a particular context. Mahlomaholo (2012:51) asserts that CDA is used to obtain deeper meaning of text which is more than just sentence structures. “Analysis of texts also includes linguistic analysis, and semiotic analysis of, for instance, visual images” (Fairclough, 2015:5), while Fowler and Kress (1979:196) argued that the unearthing of text meaning involves critical linguistics (CL). In essence, CL as sub-discipline of CDA considers linguistic choices made by the text producer and enunciates that these linguistic choices show a particular ideological stance towards a particular topic (Rashidi & Fam, 2011: 112).

Moreover, CL insists that all representations are influenced by the value-systems that are ingrained in the language used for representation (Fowler, 1996: 4) In actual fact the linguistic analysis at textual level involves thematic patterns, macro-proposition, lexicalization and rhetorical devices, which reveal a transformation of ideological and political interests into social reality (Min, 1997:161). The use of linguistic analysis exposes misrepresentation, distortion and discrimination in a variety of modes of public discourse (Fowler, 1996: 5). Fowler’s argument went further to claim that a text could be represented differently or in some other way and therefore might be portraying a different significance (1996:4). The use of CL as a sub-discipline of CDA revealed the discourse intentions subtly hidden in complex spoken sentence structures as co-researchers self-reflect on their experiences of, for example, teaching geometry in Grade nine. Through analysis of the language used by co-researchers during implementation sessions, we were able to understand the challenges of designing a strategy to enhance MPCK for teaching mathematics, for example, its functions in the teacher's multiple realities.

3.9.2.2 Critical discourse analysis at a discursive practice level

Fairclough (2004:119) views discursive practice as a way of being in the world, thus, rules, norms, and mental models of socially acceptable behaviour in specific roles or relationships are used to produce, receive, and interpret the message. Discursive practice includes spoken and unspoken rules, and conventions that govern how individuals learn to think, act, and speak. This kind of interaction is what Fairclough (1985: 740) called 'orderliness', thus "the feeling of participants that things are as they should be". It "draws on conventions that naturalize particular power relations and ideologies" (Woodside-Jiron, 2004:193). Accordingly, for one to be referred to as being a teacher, she must behave in terms of ideological norms associated with teaching (Fairclough, 1985,750). One is expected to talk like a teacher and see things like a teacher (Fairclough, 1985,750). Learners and the community tend to naturally adhere to this orderliness normatively associated with a subject position. In terms of the Gramscian concept of hegemony, ideas that are dominating seem to be neutral and should stay unchallenged as people tend to "forget that there are alternatives to the status quo" (Wodak & Meyer, 2008:8). This discursive practice only embraced hegemonic ruling class ideologies to discredit the mathematical conceptions, while rejecting indigenous mathematics ideologies and non-standardized mathematics procedures (Moloi, 2103:135). Our approach of using PBL to enhance MPCK, presented an alternative to the teacher-centredness status quo. PBL as an LCPA presented active learner participation in the classroom discourse that would disturb a conviction purported to be natural.

3.9.2.3 Critical discourse analysis at level of social structure

Social structure refers to a setting with a set of conventions that determines rights and obligations in terms of social standing (Fairclough, 1985:746). In this case, the text becomes more than just words; it discloses how those words are used in a particular social context (Fairclough, 2004:121). According to Halliday (1978, cited in Fairclough, 1985:746), people affirming their own statuses and roles on the social structure by their everyday acts of meaning. This social practice establishes and transmits shared systems of value and of knowledge (Fairclough, 1985:746). It appears that there is a constant power struggle between the discourse and social structure. Dialectically, social structures not only produce and shape discourse, but they are the product of discourse (Fairclough, 1989:38). At a micro level of social structure, thus, the classroom, the teacher's positioning in terms of power determines the discourse (Fairclough, 1989:38). On the other side, it is only when teachers and learners occupy these structural positions to continue to be part of the social structure so that the "discourse in turn determines and reproduces social structure" (Fairclough, 1989:38). Most prudently, the power relations between the discourse and the social structure determine conservativism or transformability. Put differently, a shift in power relations suggests possibilities of transformation, while power stability indicates conservation of social structure and the dominant social group keeps its position (Fairclough, 1989:40).

Specifically, the social structure analysis reveals the extent to which the text upholds or reproduces hegemonic discursive or social practices and how it stands in relation to certain prevalent conditions (Van Dijk, 1993:250). As this study uses CDA to understand mathematics classroom discourse at the research site, this analysis is done with the aim of understanding, exposing and resisting social inequality (Ruiz 2009:5). Thus, the objective of uncovering concealed power domination to open up opportunities for people to identify the dominant ideology and escape from such oppressive discourse (Fairclough, 1993:138). It is therefore evident in this study that a central narrative of CDA is on denaturalization of social power and presents a possible alternative to the status quo.

3.9.3 CDA and Critical theory

From the discussion above, it is clear that CDA is concerned with revealing concealed inequalities and power relations within society, while Dube and Hlalele (2018:76) submitted that CER-framed relations are a catalyst for change. In reconciling CDA and CER it appears that CDA complements and operationalises this theoretical framework, thus CER. It has been elucidated that when the power of the social structure is conserved results into continuation of inequalities (Fairclough, 1989:40). These inequalities and ideologies are represented as common sense when they have been naturalized (Fairclough, 1985:752). CDA, like CER, takes an “explicit position, and thus wants to understand, expose, and ultimately resist social inequality” (Van Dijk, 2015:466). Mahlomaholo (2009:224) posited that CER appeared to be an effective way to subvert distorted consciousness about oppression. According to Van Dijk (2015:466), CDA and CER are neighbouring disciplines. In essence, CDA apparently is one of the disciplines that react against dominant asocial and uncritical paradigms of the 1960s and 1970s (Van Dijk, 2015:466). In line with the CDA posture’s reaction towards domination, CER is empowering, liberating and its equity agenda advances social justice, peace, freedom and hope (Mahlomaholo, 2009:226). More specifically, “CER assumes that power relations are discursive” (Nkoane, 2012:99). This study used CDA to reveal power relations concealed in text, as well as discursive practices and social structures that might constrain teachers’ MPCK. CER coached this study to empower mathematics teachers in terms of their MPCK towards social justice and democratic principles.

3.10 CONCLUSION

In this chapter PAR has been historically traced and elucidated as research methodology to collaboratively generate data which were analysed through CDA. The credentials of co-researchers had been presented in terms of ethical considerations. Finally, strategic planning was tabled indicating how the strategy to enhance MPCK using PBL was enacted and evaluated.

CHAPTER 4 : ANALYSIS OF DATA, PRESENTATION AND INTERPRETATION OF RESULTS

4.1 INTRODUCTION

The aim of this study was to develop a strategy to enhance Grade nine teachers' mathematics pedagogical content knowledge (MPCK) using problem-based learning (PBL). This chapter presents a discussion of the data analysis, and then provides the interpretation of the generated data geared at strategies to enhance Grade nine teachers' MPCK using PBL. The analysis and interpretation of the empirical data will be discussed in terms of the objectives of the study. This is done in justification for the need to develop a strategy to enhance Grade nine teachers' MPCK using PBL. The analysis of the data that were generated was done according to the five objectives of the study, which acted as the still rods that framed the study. Each objective is unpacked in terms of identified relevant constructs that constitute it, which emerged from the literature review and formulate appropriate sub-headings. An appropriate opening discussion follows with the aim of setting out good practices in terms of policy-related issues, theory and previous research findings for each sub-heading.

The empirical data will then be presented in the form of written text, pictures and scenarios. The data will be interpreted and juxtaposed with good practice, that is, legislative frameworks, theory and previous research findings. The deeper meaning of the texts will be analysed using CDA at three levels, namely text, discursive practice and social structure (Fairclough, 1995: 97). In addition, Mahlomaholo (2012: 51) asserts that CDA is used to obtain deeper meaning of text which is more than just sentence structures. Over and above, CDA exposes social inequalities (Van Dijk, 2008:85) that may have constrained the emancipation of teachers in terms of their PCK. Moreover, the evidence is further interpreted through a CER lens in order to understand co-researchers' utterances from a view point that promotes social justice and democracy. This process will be repeated for all the objectives using PAR.

In terms of the objectives of this study, in this chapter the challenges experienced by teachers who teach Grade nine mathematics (expressions and division of fractions as an example) will be analysed. This is done with a view to establishing possible strategies that may be developed and adapted to address the challenges that are experienced by these Grade 9 mathematics teachers. Conditions under which the strategies are developed are thoroughly examined, because these conditions may pose threats and inherent risks that may impede the successful implementation of the strategies. It is for that reason that the strategies and solutions will be operationalized, assessed and evaluated prior to them being considered as sufficiently conclusive to serve as the successful response to the research question and to be used to enhance Grade nine teachers' MPCK.

4.2 THE NEED TO FORMULATE COMPONENTS OF THE STRATEGY TO ENHANCE MPCK USING PBL

In this section data related to key components that constitute the need to formulate the strategy to enhance MPCK using PBL are explored. Prior to the establishment of a coordinated team, members met for the first time and shared the problems experienced by mathematics teachers to propose possible solutions. The meeting identified four components of MPCK, that is, content knowledge, pedagogical knowledge, know learners and curriculum knowledge, under which the following challenges emerged: (i) none existence of coordinated team to enhance MPCK for teaching content areas for Grade nine curriculum; (ii) insufficient lesson preparation when teaching; (iii) non-implementation of learner-centred approach when teaching; (iv) insufficient use of teaching aids or curriculum materials when teaching; (v) poor follow up of learners' misconceptions; (vi) no integration of assessment and lesson facilitation, and (vii) poor mathematical knowledge for teaching. These challenges are now discussed in order to develop deeper understanding.

4.2.1 Non-existence of coordinated team to enhance MPCK for teaching content areas for Grade nine curriculum

The positive effect of good practice in terms of the existence of a coordinated team in teaching and learning mathematics was juxtaposed with the analysis and interpretation of the empirical evidence gathered from the research sites. Good practice inter alia involves the argument that co-teaching offers an opportunity for teachers to share their expertise (Sileo & van Garderen, 2010:14). Generally, a team creates a platform for individuals to combine their competencies and strengths in the process to overcome individual weaknesses. Collaborative teaching reduces teacher professional isolation and promotes collegiality (Murata, 2002:67). The presence of a coordinated team increases access to social and material resources (Jang, 2006:178). In PBL team members collaboratively analyse and investigate ways to solve an ill-structured problem (Trinter, Moon & Brighton, 2015: 27). In essence, PBL works in a collaborative learning process within the small groups (section 2.3.2.2.3). In Aalborg mutual respect appears to be a corner stone that sustains the culture of collaboration through dialogues (see section 2.4.1.1). Moreover, the Curriculum and Assessment Policy Statement (CAPS) for Grades seven to nine mathematics advocates for individuals to work effectively with others as members of a team to encourage an active and critical approach to learning, contrary to “rote and uncritical learning of given truths” (DBE, 2011: 4-5). South Africa’s Qualifications Authority (SAQA) also encourages aspects of team-work such as “to work effectively with and respect others” (SAQA, 2012: 9).

Contrary to good practice, the data generated indicated that there was no team work before the intervention of this study. This became evident during the teachers’ reflection in the first meetings that we organised to establish the coordinated teams. This is what Mbuyi said:

> *“Mbuyi: “Before, I attended these group sessions, I did not guide my learners to focus on the corners in order to identify the angles required.”*

The confession made by Mbuyi to her colleagues suggested that there was no team she could join before the intervention of this study. Other than her expression of the feeling of self-emancipation, the word *‘before’* clearly suggests there was no team before she joined the coordinated team established by this study.

Moreover, the data generated during visits and observation of lesson presentations also demonstrated practice that were different from the good practice espoused by the literature and policy mandates.

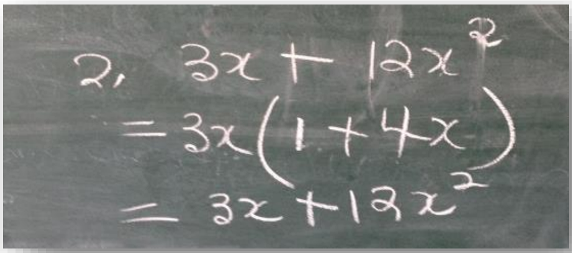


Figure 4.1: Rhulumente's chalkboard summary

Figure 4.1 represents a summary that was recorded from Rhulumente's class during the observation of the lesson presentation. The picture is a reflection of her chalkboard summary, when she showed her class how to factorize $3x + 12x^2$.

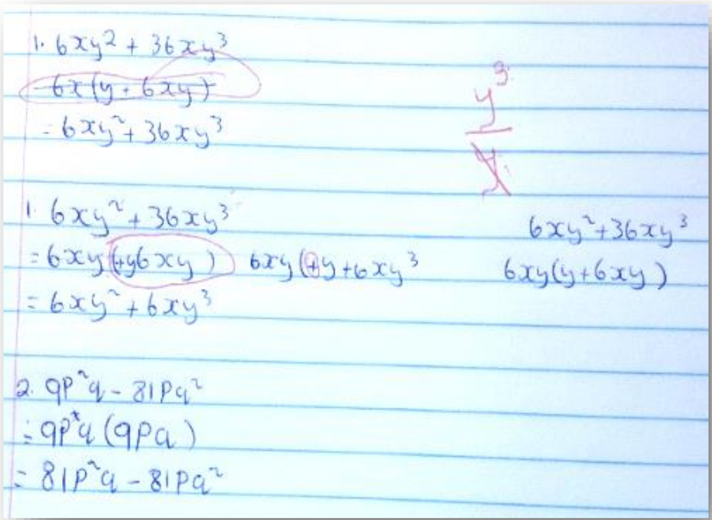


Figure 4.2: Learner's work in Rhulumente' class

Figure 4.2 represents an example of a learner's exercise at the end of the lesson presentation. Rhulumante and Njovane were working at the same research site teaching the same Grade nine class, but different class groups. During the observation period, they were teaching different topics. Rhulumente was teaching factorization using highest common factor, while Njovane was teaching algebraic equations. The following represents our experience when we observed Njovane's lesson presentation.

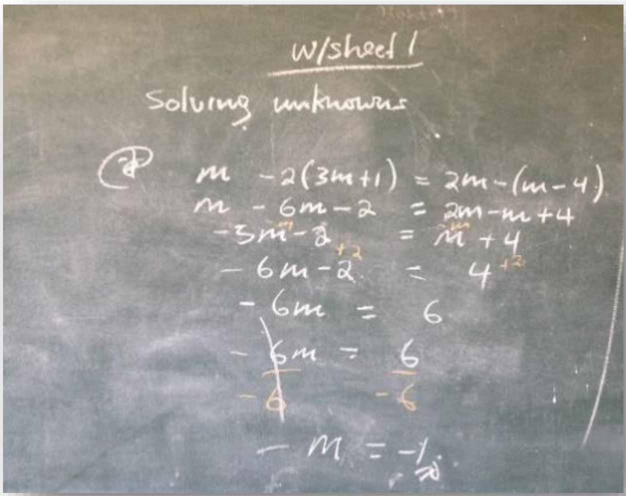


Figure 4.3: Mr Njovane's lesson presentation

Mr Njovane taught algebraic equation, which was a different topic from what Rhulumente presented.

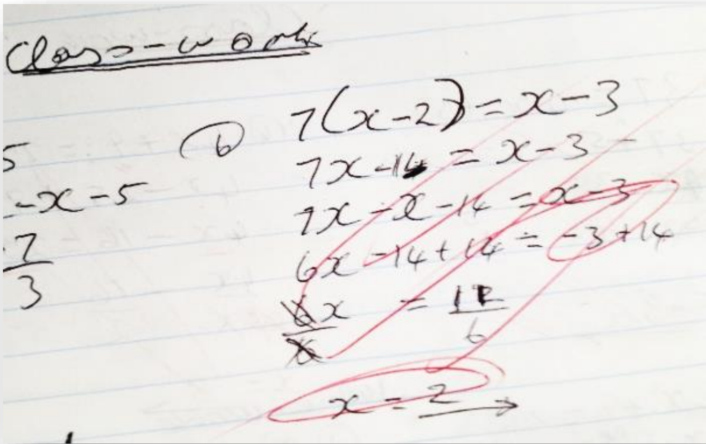


Figure 4.4: Example of learner's work in Mr Njovane's class

The above picture (Figure 4.4) depicts learners' work on given exercises after the lesson presentation. The CAPS document dictates that in the third term, the first three topics to be dealt with in Grade nine in terms of the work schedule are functions, algebraic expression, and algebraic equations (DBE, 2011: 118). Njovane was far ahead of Rhulumente as he was teaching algebraic equations, the third topic in term three as per the work schedule. This disjuncture, apparently, demonstrates the non-availability of a coordinated team and emphasises their working in isolation. The teaching arrangement as evidenced from the observation of Rhulument and Njovane demonstrates the prevalence of the culture of working in silos, and not as a team. Their situation is worse because they are teaching the same subject, same Grade in the school, but they work differently.

These two teachers, working at the same school, teaching learners the same subject in the same grade but in different classes, is a classic example of teachers who did not work as a coordinated team focusing on enhancing MPCK for the teaching and learning of algebra. Working individually on different topics denied them the opportunity to share individual expertise as they could, had they worked as a team on each topic. In addition, Mbuyi claims that before she joined the team, she did not have the wisdom of practice to enable her *“learners to focus on the corners in order to identify the angles required”.* Her claim exhibits how working in silos denied her the opportunity to work on her weaknesses. It is evident that Njovane was ahead of Rhulumente in terms of curriculum coverage. Subsequently, due to no collaborative teaching, Rhulument and Njovane did not have the opportunities to tap from each other’s competences and to support each other to counter their weaknesses.

Another area where they could have worked together to improve their performance is in marking learners’ work. The flawed marking in the learner’s work in Figure 4.4 probably could have been identified, had the teachers worked as a team. Due to flowed marking the learner might have been convinced that he got it correct. However, it is self-evident that $7x – x – 14 = x – 3$ is different form the initial equation, thus, $7(x-2) = x – 3$, in terms of keeping the left-hand side equal to the right-hand side, yet marked as correct by the teacher that works alone. Contrary to Murata’s (2002:67) view of promoting collegiality through collaborative teaching, collegiality could not be realized in this case; instead we observed perpetual professional isolation. Apparently, Rhulumente and her learners would not have been left behind had there been a team that could create opportunities to collaboratively discuss and analyse real problems and challenges experienced by colleagues. As they worked individually in isolation on different topics, opportunities to access social and material resources were inevitably limited.

We also want to argue that there was no effective teaching as Mrs Rhulumente's learners got all the given exercises wrong. When we reflected on Rhulumente's lesson presentation, she had this to say:

> *"I do not think they understood the lesson, I have to repeat it."*

She acknowledged that her learners did not understand the lesson and that she intended to repeat it. Due to the absence of a coordinating team, co-researchers could not work together effectively as members of a team to encourage active and critical approaches to learning.

Following is the record of what transpired when Rhulumente taught factorization of expression using the highest common factor:

She wrote $3x + 12x^2$ on the chalkboard and she said:

1. $3x$ is our ...... first (learners completed her sentence by also saying first term in chorus), and plus $12x^2$ is our se…… (Learners in a chorus said second term).
2. Ok, in order to factorize that expression we need to find the highest common multiple of $3x$ plus $12x^2$. (Learners continued to complete her sentences in a chorus form).
3. Who can tell us the highest common factor for $3x + 12x^2$? What is the highest common factor for $3x + 12x^2$? Yes sis' (pointing to one girl learner in the class)
4. 3x, ……. (The girl learner responded)
5. 3x (she affirmed the answer)
6. Is she correct?
7. Yes (the class responded in a chorus)
8. 3x is our ...... highest common factor, 3x (affirming the learners' answer).
9. 3x into …. 3x into 3x goes…. into how many times? (she inquired from the class)
10. One (Class responded in a chorus)
11. Uhm? (She inquired again).
12. One (The class responded again).
13. Once (She affirmed and corrected one by saying once).
14. 3x into 3x … once (She affirmed) can we write one?
15. No mam (The class responded).
16. Can we? Ok let's put that one
17. 3x into $12x^2$…Hee?
18. 4 times (The class responded in a chorus).
19. 4 times (she confirmed the learners' answer)
20. 3 into 12?
21. 4 times….Uhm? …..4 times… (The learners joined the teacher in a chorus).
22. x into x^2?
23. x^2 (Few learners responded in a chorus)
24. x into…… (She raised her voice)
25. xx x^2 (Few learners responded but were not sure about the answer).
26. X goes how many times into x^2?
27. Once (Few learners responded).
28. Heee?
29. Once (Few learners responded)
30. Once (she confirmed the answer)
31. So we write …… what must we write here? ..... Hmm?
32. x^2 (a few boys at the back responded).
33. x^2 (She inquired with high voice).
34. x (The class now changed the answer to x).
35. x , we write here….x….(she affirmed what should be written).

The above extract revealed that Rhulumente did not encourage an active and critical approach to learning. It seemed that her learners were involved in the lesson although they were not actually participating but chorusing by either repeating or completing the teacher's sentences. This kind of classroom practice where learners' responses in a chorus form is what Chick (1996: 21) calls 'safe talk'. From line 17 to line 35 learners seemed to have been confused. The teacher would raise her voice when she was not happy with the answer and subsequently, learners would change their initial answer. When she wished to know what was x into x^2, the learners responded by saying it was x^2, she raised her voice to show discontentment. However, learners were not given an opportunity to explain themselves regarding their first answer so as to encourage reasoning. Contrary to CAPS articulation (DBE, 2011: 4-5) that advocates for individuals to work effectively with others, however, learners in the above lesson presentation were deprived of the opportunity to think critically and working effectively with others. Non-existence of coordinating teams for teaching mathematics, made evident above, has far-reaching consequences for both teachers and learners.

The notion of teachers working in isolation seems to entrench the idea of teacher centredness as opposed to learner-centred teaching approaches. Rhulumente was the only one that was in control of the classroom discourse. As she demonstrated the requirement aspects to factorize an expression using the highest common factor, she used common multiples and common factors interchangeably.

> *In order to factorize that expression, we need to find the highest common multiple of 3x plus 12x2. Who can tell us the highest common factor for 3x + 12x2? What is the highest common factor for 3x + 12x2? Yes sis'…*

While her learners were still puzzled about whether they should use common multiples or common factors, she pointed at a girl learner to respond on the spot. The girl learner could not find space and time to apply her mind or to request more clarity between common multiple and common factor, but she had to conform to the accepted classroom discourse, where the teacher is in charge and is the only one who asks questions, not the other way around.

Due to this pattern that prevailed at the research sites, namely a lack of coordinated teams, teachers could not find an opportunity to reflect, critique, and be criticised by other team members. Unfortunately, learners had to be subjected to the same discursive practice where conformity to the power domination was socially accepted.

We wish to argue, in accordance with Qhosola (2016: 142), that working in silos could alienate teaching and learning but rather impose ideas that are not necessarily the only effective approaches. Due to learners' positioning in terms of power relations, their views were not respected. They had to follow the teachers' affirmed answer in a chorus form. In lines 14 to 16 when the teacher asked whether they could write one, learners responded negatively. Unfortunately, they were not given a chance to explain their view. Instead they were just ignored and the teacher continued to write one without justification, disregarding and disrespecting learners' views. The control was legitimately taken over by the teacher and this was socially accepted. Learners only participated when spoken to; the classroom discourse was not democratised to allow learners to initiate or ask questions. There was no team, the teacher had no access to multi-perspectives and intersubjective views from a team of colleagues, and as result her learners were denied the privilege to identify, evaluate and apply solutions from different perspectives. Had there been a team and collaboration, teachers could have been provided with opportunities to collaborate and assist one another in their teaching (Jang, 2006: 192) and inevitably enhanced their MPCK.

Furthermore, the absence of a coordinated team is looked at through a CER lens, and its dehumanising effects are unearthed. Fundamentally, CER has a moral obligation to denaturalize inequalities and marginalization in order to realize social justice (Tutak, *et al.*, 2011:70-71). CER serves critical-emancipatory interests through allowing multiple voices and gives hope to the marginalised (section 2.2.4). CER acknowledges that reality is shaped by power relations which are socially and historically constituted (section 3.2.5).

Contrarily, when looking at the discourse patterns above through the CER lens, the absence of a team perpetuates the reproduction of inequalities and injustice. In terms of the power relations that prevailed in Rhulumente's class, learners were not given an opportunity to raise questions or to justify their views. Her learners were struggling to comprehend the lesson and were also behind in terms of the CAPS work schedule as compared to Njovane's learners. All these learners from different class groups were expected to write the same trial and final mathematics examination. The implications were that Rhulemente's learners were likely to fail mathematics and as a consequence, be excluded from any future careers that are mathematically embedded. Such exclusion would defeat the emancipatory agenda of CER and perpetuate the reproduction of socially and historically constituted power domination, injustice and inequalities. As teachers work in isolation, they seem to be unaware of the power of collective capacity that generates multiple resources and pedagogical strategies to realize quality education. Their failure to work as a team may result in the lack of providing quality education which would be the violation of learners' rights to quality education.

The findings that emerged from this study showed that non-existence of a coordinating team for mathematics teaching seemed to perpetuate social injustice. These findings pointed out that mathematics teachers were not working together as a team, which by implication denied learners and teachers the benefits of being exposed to multiple perspectives provided by the team. Evidently mathematics teachers could not even find a space to share their challenges regarding their MPCK due to the non-existence of teams. This confirmed Murata's (2002:75) point of view that teams provide teachers with an "opportunity to learn from one another and to implement new ideas in the safe environment provided by the team structure". Moreover, our findings were in line with the opinions of Qhosola (2016: 142) who posits that working in silos might alienate teaching and limit the access to different ideas and approaches of subject expertise resulting in the entrenchment and reproduction of inequalities. Rhulumente's non-exposure to team work affected the way in which she handled her mathematics lesson. This is in line with the narrative that teachers teach mathematics in a manner consistent with how they have been taught (see section 2.4.1.5). Put differently, this apparently suggests that Rulumente could have handled her classroom discourse differently had she been exposed to the benefits of team work in teaching mathematics. From Mahlomaholo's theorization, it is also evident that there is a mutual and reciprocal benefit when agents work collectively as they both possess more knowledge as compared to their separate individual knowledges (see section 2.4.1.6). In conclusion, this study contributes to the argument that the enhancement of MPCK may not be effective if teachers fail to work together as a team so that they could complement each other and increase access to multiple pedagogics and material resources. We therefore regarded the absence of a team as the first challenge that had to be overcome if we were to adhere to the emancipatory agenda through PAR.

4.2.2 Poor follow up of learners' misconceptions

As a good practice, teachers must make sure that they know what their learners know, including their mathematics misconceptions in order to effectively approach a mathematics topic, overcome and transform those initial conceptions (see section 2.4.2). It appears that the examination of learners' errors serves better to help learners overcome identified problem-solving difficulties and "provides a starting point to address the errors through attention to the underlying erroneous thinking", rather than re-teaching in an attempt to fix them (Pournara *et al.*, 2016: 9). Cognitive congruence and anticipatory thinking guide teachers to select appropriate curriculum materials that would mitigate learners' misconceptions (see section 2.3.1.3). Probing questions eliminate misconceptions and enable teachers to tailor the lesson plans and presentations in a way to address learners' difficulties (see section 2.4.2). On the other side, in PBL, misconceptions are used as a necessary step in learning (Hmelo-Silver, 2004: 250). As learners articulate incorrect knowledge or misconceptions, they have the opportunity to revise their false beliefs when they are confronted with probing questions (see section 2.4.2.1). Learners' errors initiate activities among learners (Granberg, 2016: 46). Error analysis enables teachers to find the best ways to remediate the misconceptions (DBE, 2015: ii). In essence, utilization of learners' experiences is a valuable resource for teaching hence teachers are obliged to recognise learners' aptitudes and thinking (see section 2.4.2.1).

At the research sites, the opposite was true in terms of the data generated when juxtaposed with the good practice envisaged above. Ntozine presented the following table to for learners to complete (Table 4.1).

Table 4.1: Table to be completed by learners

X	1	2	3	4		10	***12***
Y	3	5	***7***	***9***		***15***	41

Learners went to the chalk board individually to complete the table given. The ***bold italic*** figures on the above table were the answers given by learners. In terms of the above function, learners put 15 as value of y corresponding to 10 and 12 as x value corresponding to 41 in the ordered pairs. However, Mrs Ntozine did not attempt to either follow up on the learners' thinking or to draw their attention to their answers, as recommended by Pournara *et al.* (2016: 9). Instead, she went further and requested them to determine the general rule of the above table. At first, we thought she was going to use the general rule to prove which values of y were correctly filled. Learners did not apply their minds to determine the general rule they just guessed. This is what they said:

Plus two…..times……divide…….

Apparently, learners just gave anything they thought was mathematically relevant to the task that was given to them. Unfortunately, there were no probing question to examine and understand their inputs but the teacher just showed them that they were wrong without making any follow up on their reasoning behind their answers. In the process of determining the general rule, there was no mention of those values of x and y, that is, 15 and 12 respectively. Learners copied the table as above assuming that it was correct. In a nut shell, Ntozine could not identify what her learners knew since both correct and wrong answers were guess work since learners were not given any opportunities to justify their answers. Over and above learners' wrong answers were not used as the springboard to inform appropriate teaching strategy.

Njovane also was not immune from the above teaching practice, that is, failing to follow up on learners' misconceptions or errors. After having marked the learners' work as indicated in Figure 4.5 below, he did not analyse errors, nor used them to dovetail his teaching in order to remediate them. He just marked them wrong without knowing what learners were thinking in term of their answers. He neither lifted learners' misconceptions to be discussed by the class nor questioned learners regarding the errors they committed. Instead he decided to re-teach the task that he had given to learners in an attempt to make corrections. Figure 4.5 below is a reflection of Njovane's chalkboard summary after re-teaching the problem that was earlier given to learners to solve.

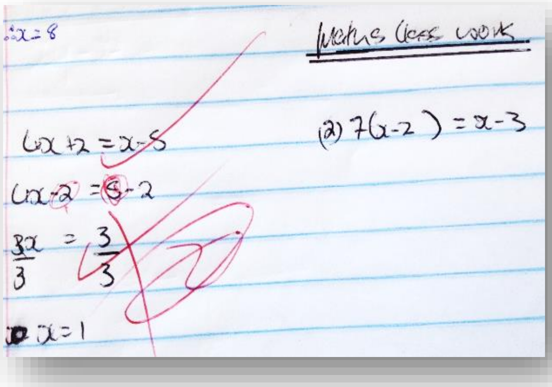


Figure 4.5: Learner's work from Njovane's class

In line two of the above learner's work Njovane marked it correct and encircled -2 on the left-hand side of the equation and marked the learner correct. The learner mistakenly left -2 that was used to eliminate +2 from the left side of the equation and changed the sign of 5. This completely changed the equation in line two from the initial equation. Njovane did not recognise the learner's error as a necessary step in learning and consequently used it as a springboard for learning and teaching. He therefore failed to probe the learners to justify their reasoning. Had he identified the misconception as enacted by the learner mentioned here, he could have provided the learners with enabling prompts to eliminate the complexity of the problem (Russo, 2016: 8) in terms of learners' ZPD. In the process of providing the learners with cognitive scaffolds, he could have self-emancipated and enhanced his MPCK while developing strategies of clarifying equation concept. Instead he decided to reteach the equations in the form of making corrections as reflected in Figure 4.6 below.

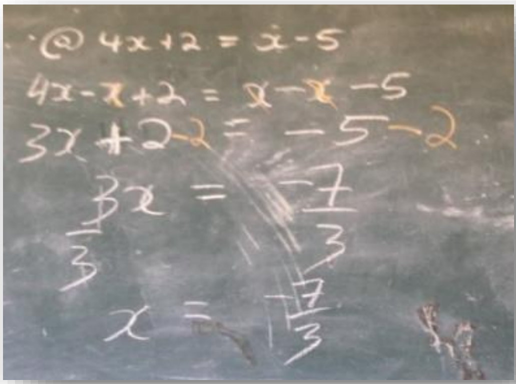


Figure 4.6: Njovane's chalkboard summary

Figure 4.6 depicts Njovane's corrections on a chalk board summary. When we reflected on his lesson presentation afterwards, we inquired in terms of how he generally handled his learners' errors. He responded:

> *"Em …, when they make mistakes sometimes, there are those who correctly follow the procedure well and make mistake at the answer …. when they have made mistakes, you are supposed to tell them so that they know that there is a mistake in their work".*

In actual fact Njovane pointed out that when learners made mistakes and argued that the teacher was supposed to tell learners, what their mistakes were. The issue of asking promting questions, so that learners might self-evaluate, identify their mistakes and justify why they had solved mathematics problem in a particular manner was never raised by Njovane. Instead he personally identified learners' mistakes and showed them by marking them wrong. He went further to provide the correct answers for learners to make corrections. He did not follow up on learners' reasoning behind their actions and in the process lost the opportunity to take advantage of his learners' misconceptions. It appeared that he could not anticipate his learners' misconceptions as he failed to guide learners through questioning their work or allowing other learners to discuss the errors committed. He only believed that a teacher should tell where the learner had committed an error and either should correct it by replacing the error with the correct answer or through re-teaching the task that was given to the learners for them to copy corrections. His statement that, "*When they have made mistakes, you* *are supposed to tell them so that they know that there is a mistake in their work",* suggests that he believes that when you tell learners where their misconceptions are and give them correct answers, learners automatically would understand the mathematics concepts on the topic taught and apparently would not repeat the same mistake.

In line with good practice and according to Shulman (1986: 10), Njovane, should have developed teaching conditions required to overcome and transform learners' misconceptions in solving algebraic equations. His learners seemed to misunderstand the concept of equations, particularly the behaviour of the signs in order to keep the balance. Njovane failed to establish what learners knew about equations and focused on correcting the sign errors made by learners. However, he neither followed up nor examined why his learners presented with such misconceptions. Had he examined the learners' misconceptions, they would have provided the starting point to resolve learners' misconceptions and subsequently sparked a debate among learners. Instead he focused on re-teaching the task rather than establishing learners' underlying erroneous thinking (Pournara *et al.*, 2016: 9). It appeared that learners did not understand why they should eliminate some terms from either side of the equation as the issue of keeping the balance in the equation was not emphasised through use of teaching aids to demystify the concept and avoid misconceptions. Both Ntozine and Jovana did not do error analysis in order to find the best ways to remediate errors as DBE mandated. Njovane only pointed out and corrected the errors; it was even worse with Ntozine's whose learners guessed the answers and eventually copied erroneous chalk board summaries without any intervention from the teacher.

Overlooking and disregarding learners' misconception when they completed a table of coordinates implied that Ntozine did not pay attention to learners' underlying erroneous thinking. From the linguistic units such as images of the chalkboard summary, learners' work and co-researcher utterances recorded in the study, it appeared that examination of learners' misconceptions was not a discursive practice. Apparently, the accepted norm was for the teacher to confirm or disapprove the learners' work, sometimes without justification. According to Mogashoa (2014: 105), whether in written or spoken form, text represents the speaker's beliefs, position and ideas. In line with Magashoa (2014:105), Njovane argued as follows:

“When they have made mistakes, you are supposed to tell them so that they know that there is a mistake in their work”.

His argument demonstrated his beliefs that it was only necessary to show and correct learners’ misconceptions through re-teaching the concept, not by probing them to explain their thinking. The research put it that anything that is either written or said “about the world is articulated from a particular ideological position” (Min 1997: 148). Njovane viewed himself as the only arbiter who could determine what was wrong or correct in his mathematics class and learners had accepted his position of authority as natural. This view was in line with the suggestion that embraced the process of turning the ideology of the powerful class into a universal belief through naturalization (Fairclough, 1989:129). In terms of the social structure, teachers are the most powerful individuals who are the only ones who determine what goes in mathematics classroom.

Contrary to CER, teachers were influenced by their deep-rooted belief that it was in their supreme power to correct learners’ misconceptions. Equality and social justice advocated by CER (Tshelane & Tshelane, 2014:288) were not part of the classroom discourse in either Ntozine or Njovane’s mathematics lesson presentations. In our view, it was really unfair and unjust for Ntozine to let learners continue to copy a wrong chalkboard summary without her pointing out errors for discussions. Ignoring learners’ erroneous thinking underlined their misconceptions, and did not challenge their intelligence in terms of understanding why mathematical concepts work in a particular way. However, Mahlomaholo (2009: 226) shed light on how good CER changes people’s lives by “liberating them from not-so-useful practices and thoughts”. Evidently, from empirical data generated from the research sites, teachers’ PCK was constrained by their belief in terms of disposition regarding power relations and subsequently their failure to follow up on learners’ misconceptions. It was prudent for learners’ voices to be un-muffled so that teachers could understand and develop pedagogical strategies to mitigate against the emergence of such misconceptions.

The findings of this study demonstrated that teachers did not follow up learners’ misconceptions. Instead, when learners displayed their mathematics misconception, they were marked wrong and at the most the teacher would re-teach the concept. This confirms viewpoint of Shuman’s (1986:10) ‘missing paradigm’ thus, PCK that enables teachers to develop teaching conditions required to overcome and transform learners’ misconceptions. The literature reiterates and advances an argument that teachers need to have an insight into children's mathematical thinking to be able to effectively guide them toward deeper understanding of challenging mathematical concepts (Gearhart & Saxe, 2004:305). It became evident that teachers did not even examine learners' misconceptions and did not do error analyses. Had teachers done an error analysis, they would have used learners' misconception as starting point in their lesson planning and inevitably created a platform for learners to debate their thinking underlying their errors. In closing, our findings were also in line with the departmental policy that error analysis helps educators to find the best ways to remediate errors (DBE, 2015: ii). However, the findings of this study further revealed that teachers did not know what their learners knew, and as a result, they would either re-teach, or ignore learners' misconceptions. Had they allowed learners to expose their erroneous thinking resulting in misconceptions, they could have changed their hearts and minds regarding learners' errors, subsequently their PCK would have improved and got liberated from their routinely constraining practice. This confirms Gardee and Brodie's (2015: 2) narrative that when teachers try new ways to understand their learners' erroneous thinking, their MPCK get enhanced.

4.2.3 No or insufficient use of teaching aids or curriculum materials when teaching

In this section the study explores the impact of the non-use of curriculum material in teaching and learning on the development of the wisdom of practice. The teaching materials which Shulman (1986: 10) refers to as 'tools of the trade' facilitate teaching efforts by exemplifying a particular content to learners and remediate the adequacy of learners' accomplishments. Concrete material enables both teacher and learners to have a grounded conversation (Thompson, 1994:8). It is argued by research that concrete materials are effective aids to learners' thinking and teaching; they further reduce the emphasis on mathematical procedure and encourage what Thompson (1994:9) calls "conceptually-oriented instruction". In accordance with Durmus and Karakirik's (2006:121) view that visual material provides an interactive environment for learners, we want to argue that teaching aids enable learners to pose and solve problems while connections between mathematical concepts and operations are formulated. In terms of the Integrated Quality Management System (IQMS) teachers are charged with the responsibility to create a positive learning environment (DBE, 2003: 2) to stimulate effective learning through active engagement of learners in the learning process (DBE, 2016: A- 18). The cognitive tools for this include, but are not limited to, text books, charts, artefacts, live videos, and other useful media (see section 2.4.3.1.1). Central to PBL, the above-mentioned cognitive tools, including human resources help learners to demonstrate their thinking when untangling problems (see section 2.4.3.1.1). Manipulatives seem to also strengthen relationships (Koszalka *et al.,* 2002:16) and improve collaboration among group members (Barge, 2010: 20) and proliferate communications resulting in multiple negotiated meanings of mathematics concepts (see section 2.4.3.1.6).

In contrast with best practice, our classroom observations and discussions at the meetings on the research sites revealed that mathematics teachers did not use mathematics teaching aids. It transpired from Mbuyi's class that there were no teaching aids to facilitate teaching efforts as Shulman (1986: 10) recommended. After her lesson presentation without teaching aids, we drew her attention to the importance of such aids. The following is a record of our interaction:

> *Researcher: I could see that your class is not print-rich, what is your view about the use of teaching aids when teaching mathematics?*
>
> *Mbuyi: :No … it's that they fall off from the wall, we usually hang them on the wall, but what we use to stick them does not hold. Learners also remove them that is why we do not have them.*

Despite her appeal and a number of reasons why she did not have mathematics teaching aids, it was evident that she did not use them. Her argument that the teaching materials fell off the wall did not hold water as she did not have them on the day of observation. She further blamed learners for removing the teaching aids from the wall. Apparently, her understanding of the teaching aid was only wall posters; as a result, she did not consider it important for her learners to have graph books for working with linear graphs, that is, $y = mx + c$. Non-use of teaching aids seemed to be prevalent practice across the research sites.

We also invited her principal, that is, Mohlakoana, to be part of our talk as we reflected on our experiences regarding Mbuyi's classroom observation.

> *The researcher: I have seen that learners when drawing graph, use their exercise books, they do not have graph papers.*
>
> *Mbuyi: (jumped in and responded before the principal could respond) "No … you know what, they were given … they usually in that pack (meaning the pack of stationery provided by the DoE for quintile one schools) it is just them who do not have the graph papers, otherwise they are usually available in that pack.*
>
> *Mohlakoana: (joined the conversation) The school is provided with packs and we issue them to learners as they are.*
>
> *Mbuyi: When you tell learners to bring the graph books, you found out that they no longer have them because we do not use them early (at the beginning of the year).*
>
> *The researcher: Then what do you do?… to me I think it is unfair to them, when they are supposed to draw the graph, they are first compelled to first draw the Cartesian plan.*
>
> *Mohlakoana: For the progress of the work, they must write on their own, we decided to punish them because it is the same even with text books, they do not use them, they do not want to use them.*

From this conversation, it is evident that the teacher did not have the tools of the trade, or '*materia medica*', thus, materials needed to make learning an effective experience (see section 2.4.3.1.1). Consequently, learners had to first draw the x and y axis on their exercise books, not on the graph paper. The implication of this was that learners' drawings were not accurate since because the graph paper with standard measurements was not available. Both the principal and Mbuyi blamed the learners for apparently losing their graph books with which they were supposed to have been issued. Moreover the principal emphasised the need for progress, thus, content coverage, irrespectively of whether learners learnt or not. He (the principal) further argued that they were punished for having lost the graph books that were claimed to have been issued to them. Mohlakoana also revealed that there was a culture among learners according to which they would refuse to take text books, claiming that they were afraid of losing them. The same conversation took place among Nowele, Falafala and I, during which it was confirmed that in their school the majority of learners also did not have text books.

Falafala: When we give learners homework or any task to do, we refer them to a particular page of the prescribed text book, only to be told that the text book has been stolen or left at home. Learners usually leave text books at home to reduce the risk of being stolen by those who did not take them when they are issued by the school.

The researcher: You mean that they deliberately leave them at home?

Nowele: Yes, they leave their text books at home.

Falafala: Yah, text books got lost, because many learners do not have text books, we usually encourage them to them, but to avoid owing the school at the end of the year, they do not take text books.

Nowele: Those who did not take text books, still form those who took them.

Falafala: But when we refer them to the text books in preparation for a test, they complain that they do not have text books, yet they earlier refused to take them.

It appears that in Nowele and Falafala's research site learners refused to take text books to avoid the consequence of losing them. The few that took a risk by taking the text books, kept them at home to secure them from being stolen by those who did not take books; consequently, teachers do not use this valuable resource in teaching and learning mathematics.

At the beginning of the fourth school term in 2016, we also realized Njovane's work books were clean, and had nothing written. As we inquired about this anomaly, this is what he had to say:

Jovana: Yes, they are supposed to be clean because, I did not have them, I only received them last week, they are new …

The researcher: Generally, what is you view regarding the use of teaching aids?

Njovane: I do not usually use teaching aids in expressions and equations … I do not need them in these topics.

Njovane argued that he does not need teaching aids for teaching expressions and algebraic equations, though they may be relevant in teaching other topics. On the other side, Ntozine protested that teaching aids were a scarce commodity.

When you need teaching aids you need to personally purchase them.

When we inquired whether she had ever requested the school management about the provisioning of mathematics teaching aids, she said that she had never approached the school management.

Moreover, Mbuyi's learners had to use their exercise books to draw a Cartesian plane before they could draw a function graph. The inaccurate hand-generated Cartesian planes had no standard measurements between x values on the x axis and y values on the y axis resulting in the graph drawn from this type of Cartesian plane losing mathematical meaning. We also invited the principal (Mhlakoana) of the school to our conversation regarding the non-availability of graph books/papers with the intention to solicit his support and buy-in regarding the use and the provisioning of teaching aids. When we inquired the reason why learners were not using graph papers, Mohlakoana argued that learners were provided by the Department of Education with packs with graph books included. Mbuyi further agitated that when learners were required to bring the graph books to class, she discovered that they no longer had them. When we raised the inconvenience that learners experienced to work without graph papers, Mohlakoana argued that the inconvenience was a correct punishment to learners who lost their graph books.

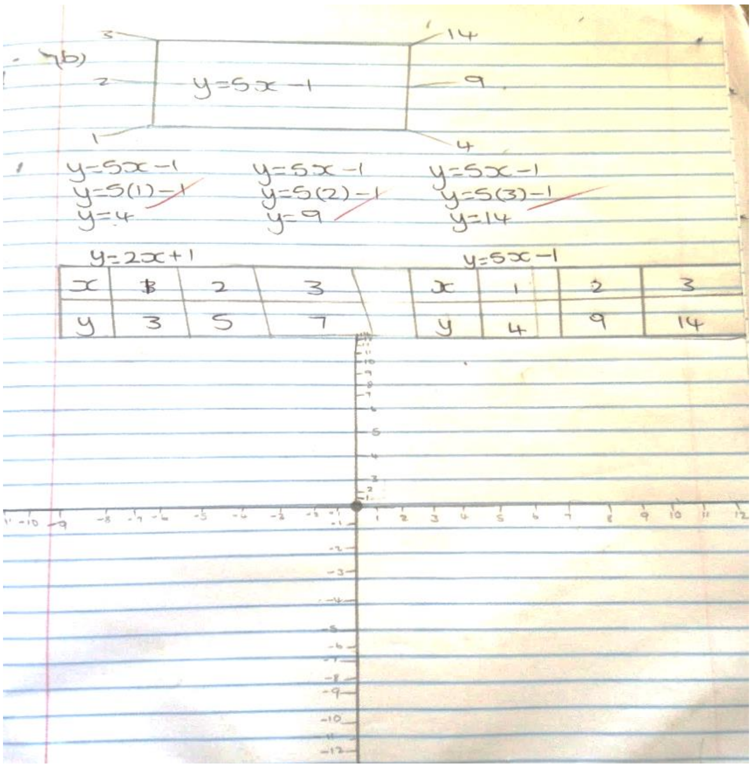


Figure 4.7: Learner's hand-generated Cartesian plane

The process of first drawing the Cartesian plane (see Figure 4.7), which is not necessarily the aim and the focus of the mathematics lesson, detracted learners from focusing on conceptually-oriented instruction (Thompson, Philipp & Boyd, 1994:7) The process curtailed the learners' accomplishment of the concept as the non-standard handmade Cartesian planes did not produce the desired outcome. Consequently, due to learners' delay in developing x and y axis for their graphs the problem-solving skill that could have ensued was constrained by the non-availability of graph papers. For an example, the different behaviour of the graph when the sign of the gradient changes that could have generated discussions and resulted in learners posing questions as they compared a plethora of graphs at their disposal could not emanate from Mbuyi's class. In line with Shulman's (1986: 10) argument that teaching aids are not limited to visual materials, but they also include software, however, the unavailability of such teaching aids denies learners the opportunity to use the input of different gradients and y-intercepts in order to observe and analyse the changing behavioural patterns of a graph on software. Students could not explore the y= mx + c graph beyond the teacher' emphasised procedure due to the non-use of curriculum material that would provide an interactive environment amongst learners.

Consistently, both co-researchers' practice and their utterances demonstrated that they did not attach much value to mathematics teaching aids. Their argument that teaching aids fell from the wall, that they were a scarce resource and were not relevant for certain topics clearly demonstrated a discursive practice that prevailed at the research sites. The claim that text books got stolen seemed to be an accepted excuse for not using teaching and learning materials. Ntozine viewed teaching aids as a financial cost to her; however, she never requested the school management to purchase such curriculum materials. Instead, she did not use mathematics teaching aids. From the raw data generated, it appeared that the co-researchers had a number of excuses for not using teaching aids. Mbuyi's principal saw nothing wrong with learners working without graph papers. In the light of Mbuyi's practice and her principal's views regarding the use of teaching aids, we want to argue that not using teaching aids was a discursive practice that was socially accepted. Through learners' positioning and power relations in a school's social structure, their learning and understanding of the function graph was not considered as of primary importance.

Instead the social set up of authority and power denied learners the interactive environment amongst themselves that would be provided by teaching aids as they could have manipulated them to formulate connections between mathematical concepts and operations.

Teaching mathematics may never be used as a punishment. Inconveniencing learners by depriving them of learning material that would make a better and deeper understanding of mathematics concepts possible is the worst kind of social injustice. Social injustice and domination are vehemently opposed by CER proponents. It is unfair towards learners to work without the enabling curriculum materials and as a consequence make it unnecessarily difficult to understand and learn mathematics. It is also unfair towards the teacher who has to struggle to engage learners in conceptually-oriented instruction because of a lack of enabling mathematics teaching aids. CER intends to raise consciousness about practices that rottenly constrained people's emancipation. Moreover, mathematical functions span vertically across the grade curriculum and have the biggest weight in terms of curriculum mandates, thus 35% in Grade nine CAPS (DBE, 2011: 11). The implication is that learners who have been denied deeper insight of functions in Grade nine are likely to struggle with mathematics in the future grades. As a result, the emancipatory agenda of CER is likely to fail.

The findings of this study showed that mathematics teaching aids were not used by mathematics teachers. The opportunities for learners to pose questions and the experience of grounded conversation provided by manipulatives were limited due to the teachers not using mathematical teaching aids, which, inevitably exacerbated teacher domination in the class. The findings of this study reject the implications of Fennema's study (1972: 639) which claimed that learners who did not use manipulatives outperformed those who used them, implying that learners who were not exposed to teaching aids developed better mathematics insight. Instead, our findings were in line with those of both Golafshani (2013:141) and Durmus and Karakirik (2006:121) who respectively argued that manipulatives represent abstract mathematical ideas and concepts visually, which help learners develop a deeper insight in mathematics. Moreover, a panel of experts argued that teaching aids such as square flat tiles provided an effective concrete way to explore the relationship between an area and the perimeter (Ontario Ministry of Education, 2004:25). Had the learners been exposed to the appropriate use of mathematics teaching aids, they could have come up with unconventional methods to demonstrate concept understanding that subsequently might have sparked a debate. It may be concluded that teachers who do not use manipulatives, lose the opportunity of concretely representing the abstract mathematics concepts to develop deeper insight of mathematics concepts (see section 2.4.3.1.5) and inevitably become better mathematics teachers in turn.

4.2.4 Insufficient lesson preparation before teaching

For a lesson plan to be successful it should address the objectives for student learning, teaching or learning activities and assessment strategies to check on learners' understanding (Milkova, 2012:1). Objectives must be realistic, measurable, and with a possibility of being achieved within the lesson period (Johnson, Uline & Perez, 2011 in Mosia, 2016:121). During lesson preparation teachers have the opportunity to think deeply about subject matter and to develop multiple pedagogical activities that will enable learners to comprehend the subject content (Shen, Poppink, Cui & Fan, 2007:249). It enables the teachers to ponder and study students' prior understandings, which makes it possible for teachers to anticipate students' reactions and solutions to the problems during the lessons (Doig & Groves, 2011:81). It takes a creative introduction of a new topic to stimulate interest and encourage thinking (Milkova, 2012: 2). Moreover, from a PBL perspective of learning teaching begins with a problem (Barge, 2010: 7) and therefore the lesson plan should present a series of problems, which engage learners in groups. According to the PAM document one of the profound responsibilities of teachers is to prepare quality lessons taking into account *inter alia* orientation, new approaches, techniques, evaluation and aids in their field (DoE, 2016: 18). Specifically, the PAM document articulates that teachers need to tap into "learners' experiences as a fundamental and a valuable resource" (see section 2.4.3.2.1). In their preparation teachers should use a variety of strategies to meet the curriculum outcomes while recognising that learning is an active process (DBE, 2016: A-18). Grade nine mathematics lessons of the DBE (2016, online) affirm the above debate in terms of good practices regarding lesson planning (see Appendix 3).

Despite the importance of lesson planning as elucidated above, the co-researchers' practice at the research site was inconsistent with best practice. They did not attach much importance to lesson planning, as is shown in Figure 4.8.

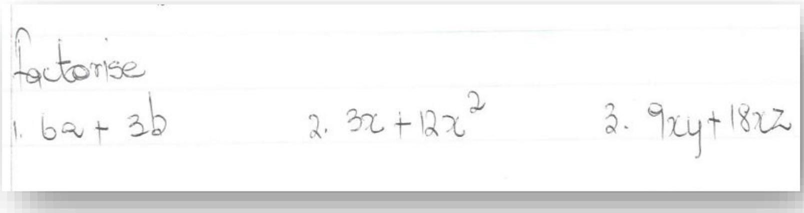


Figure 4.8: Rhulumente's lesson plan

Figure 4.8 demonstrates what Rhulumente regarded as a lesson plan. It only showed three expressions that she used to demonstrate factorization to her learners. There were no lesson objectives, no learners' activities and no indication of prior knowledge required for the lesson. Instead, the learners were given three exercises to do (Figure 4.9) after the formal presentation of the lesson.

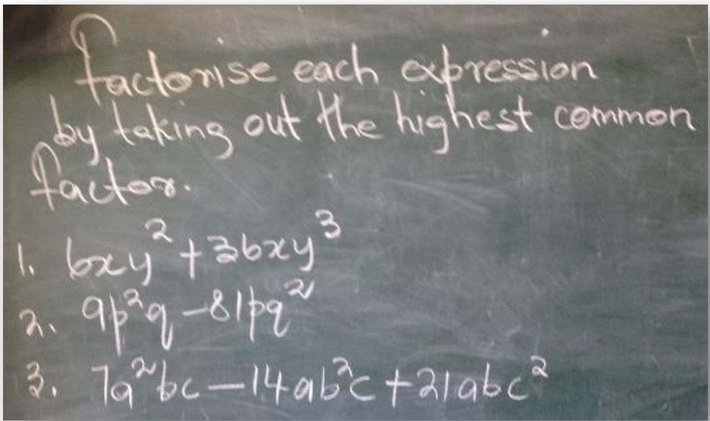


Figure 4.9: Learners' exercises given by Rhulumente

Miss Mbuyi could not show us the lesson plan as requested. Following is our conversation with her regarding the lesson plan:

> *Researcher : Do you have a lesson plan of what you were teaching today?*
>
> *Co-researcher: Ehh,…… is there a lesson plan?*

Researcher : Yes, as you were teaching.

Co-researcher: Today?

Researcher : Yes

Co-researcher: No I did not have it today

Researcher : Otherwise, is there anything that you use as a lesson plan when you teach?

Co-researcher: I do not have any that I could say I have done now, I do not want to lie….

Despite the policy statements and the research findings in terms of good practice as far as lesson planning is concerned the co-researchers either had no lesson plans or had totally inappropriate and insufficient lesson plans. Mbuyi did not have a lesson plan at all when we visited her class. She responded that she did not have a lesson plan that day when she was requested to show her lesson plan. When asked to produce any other lesson plan she might have used she admitted that she did not have any and did not want to lie about it, thus revealing her practice of teaching without lesson preparation. Nowele and Falafala also could not present any form of a lesson plan when asked for it.

The researcher: "If you do not even scribble your lesson plans, you mean that the manner in which you are going to solve mathematics problems with your learners is only seen in class?"

Falafala: "Oh no we do not have it, another thing we are spoilt by this thing of talking about the same thing for years and years, as result you feel that you no longer need to write it down, it is already in the mind"

Evidently, from the above extract, the culture of planning for mathematics lessons did not exist at this research site. They apparently believed that there was no need for lesson planning because they had been teaching the same content for a number of years. They argued that their lesson plans were in their heads, forgetting that they taught different learners with unique learning styles in those claimed number of years.

In Rhulumente's class there was no lesson plan and lesson objectives were not stipulated. Her notion of a lesson plan lacked what Milkova (2012:1) calls an explicit explanation of what learners need to learn and how the learning activities would be done. By implication, as Mbuyi did not have a lesson plan, she also did not have either measurable lesson objectives or assessment strategies as expected in a good lesson plan. It transpired that all the co-researchers referred to above, failed to use the opportunity that is usually presented by planning a lesson, namely thinking deeply about the subject matter and multiple methods that would best fit the topic and learner that they intended to teach. Contrary to the DBE (2016: online) lesson plan that clearly stipulates learners' required prior understanding, such as laws of exponents, commutative, associative and distributive properties that the teacher would build up when teaching algebraic expressions, Rulumente's lesson plan did not indicate prior knowledge required for factorization. Failure to prepare, or inadequate lesson preparation denied teachers what Doig and Groves (2011: 8) called an opportunity to anticipate learners' reactions and solutions usually given by lesson planning. Mis Falafala, for example, claimed that her lesson plans were in her mind and she only decided how to present it, or how to involve her learners, when she got into the classroom. As a result, she could not anticipate learners' misconceptions, and she had no planned activities that could stimulate interest and critical thinking. Due to insufficient lesson preparation she could not present learners with mathematics problems to solve to enable them to determine whether they had understood the lesson. In contrast with PBL, they started with formal lecturing (not problems) and a demonstration of how to work out a mathematics task, and the learners were later given exercises to work out, regurgitating the teacher's earlier demonstration.

The scrap of paper Rhulumente proffered as lesson plan illustrated how little value she attached to lesson preparation. Her notion of a lesson plan that comprised a list of mathematics tasks to be used to demonstrate a mathematical concept, for example, factorization convinced us that she did not attach much value to lesson preparation. The key activities to critically engage learners in factorization were not included in the lesson plan. This discursive practice unearthed the teacher's beliefs and ideas about learners that she intended to teach. In line with Mogashoa (2014: 105), who argued that those in power view their words as self-evident truths and dismiss ideas of powerless as irrelevant and inappropriate. The teachers at the research sites disregarded learners' preconceived ideas about the topics they intended to teach. They only concentrated on what they intended to say and do in the classroom and did not plan any activities that were intended to scaffold learners beyond their ZPD. The notion that people act out the social structure and affirm their own status in their everyday acts of meaning (Halliday, 1978 in Fairclough, 1985: 746) helped us to understand why learners' needs and ideas were ignored by the teachers. The teachers' social status and positioning in the process of lesson planning only advanced teachers' interests but not learners'. The teaching practice in this regard appeared to be about the content in their heads, not in relation to its teachability.

Mbuyi's argument that she did not want to lie, was a confirmation of the teaching practice that existed at the research site. Lucke (1996 cited in Mogashoa, 2014:105) views texts as moments of inter-subjectivity which demonstrate a social practice between human subjects. The inter-subjective moments that we observed with Mbuyi and her principal convinced us that the practice of teaching without lesson preparation was a norm and a rule that was legitimatized and socially accepted. As Mbuyi failed to produce lesson plans, she did not show any discomfort even in the presence of her school principal. It seemed that the principal knew about the discursive practice of teaching without lesson preparation. In fact, the principal did not utter a word about Mbuyi's teaching practice. This practice of non- compliance with policy regarding lesson preparation seemed to be legitimized and socially accepted as Mbuyi's principal did not seem surprised at all by her practice of teaching without lesson plans.

Learners' interests were not considered when there was no lesson plan or when the lesson plan had no clear, measurable objectives and learners' activities. Non-inclusion of learners' activities in the lesson plan undermines learners' intelligence and attaches no value to prior knowledge regarding the topic that the teacher intends to teach. The inequalities are perpetually reproduced when the teachers fail to consider learners' interests through inclusion of learning activities in the lesson plan. This is against the CER objective, which advances denaturalization of inequalities and power domination in order to give the voice to the vanquished and voiceless (Hlalele, 2014:104). The act of denying learners quality teaching and learning through inadequate lesson preparation maintains and reproduces the atrocities of the apartheid legacy such as discouraging black students to study mathematics (Mahlomahola, 2013:4691). Social justice, peace and hope cannot be realized when learners are disenfranchised from their fundamental right, namely the right to quality education, as their status of being second-class citizens in their own country in terms of access to mathematically inclined careers will then continue unabatedly.

The findings of this study demonstrated that prior to this study mathematics teachers who participated in the study either did not have or had insufficient lesson plans. This confirms Mosia's (2016: 125) findings that failure to adequately prepare a mathematics lesson results in ineffective teaching, which may lead to learners finding mathematics difficult to comprehend. Insufficient preparation contradicts the policy prescriptions such as those of the Education Labour Relations Council (ELRC) which dictate to teachers that they must in their preparation consider a variety of teaching approaches (DoE, 2003:C-67) such as PBL to teach mathematics. These empirical findings, namely non-preparation for lessons are an impairment of teachers' MPCK development. These co-researchers, who did not thoroughly prepare their mathematics lessons, denied themselves an opportunity of becoming better teachers in terms of PCK. A well-presented lesson is attributed to a well-planned lesson (Albin and Shihomeka, 2017:316).

4.2.5 No integration of assessment and lesson facilitation

In this section, the study focuses on understanding curriculum knowledge manifestations when assessment is integrated with lesson facilitation. According to the DBE, the data regarding learners' performance that are continuously generated through integrated assessment should be utilized to improve learning (see section 2.4.3.3.1). The DBE's (2011:155) policies further provide for integrated assessment to be used to provide feedback to learners, and at the same time, it also informs planning for teaching. Heritage (2010a:3) posits that this kind of assessment not only improves learners' achievements but also informs and improves teachers' instructional planning (see section 2.4.3.3.4). The assessment embedded in instruction gives teachers an insight about learners' mathematical thinking (see section 2.4.3.3.6) and helps them to appropriately adjust the instruction in relation to learners' cognitive levels (Ogbebor-Kigho, et al, 2017: 293). Evidently, assessing learners' thinking can strengthen classroom practice and improve learners' mathematical understanding (Gearhart & Saxe, 2004: 304; Umugiraneza *et al.*, 2017: 3). According to Gearhart and

Saxe (2004: 310), learners' knowledge of both mathematical concepts and procedures would develop when their teachers build their capacities to access learners' mathematical understanding. The cognitive scaffolds provided by enabling prompts during assessment support learners in attaining learning trajectories (see section 2.4.3.3.1). Seemingly, integrated assessment does not only inform the lesson planning, but enriches the teacher with the anticipated learners' misconceptions (Gearhart & Saxe (204: 309). The same applies to PBL - learners are also confronted with problems which provide a stimulus for learning (Gijbels, Dochy, Bossche & Segers, 2005:29). From a PBL perspective assessment enables learners not only to respond to content but also to learning processes, thus process competences (Dahl & Kolmos, 2015:65).

An observable trend of separating assessment from lesson presentation became evident during our classroom visits at the research sites. Rhulumente presented her lesson from the beginning to the end and then gave her learners a classwork activity to assess whether they had understood her lesson. Learners' participation in the lesson presentation was to complete the teacher's sentences in a chorus form (see section 4.2.1). We had earlier argued that, according to Chick (1996: 21), chorus responses were viewed as 'safe talk' meant to hide the teacher's incompetence and to prevent learners from asking awkward questions that the teacher may not be able to respond to. In our talk with her after the lesson presentation we asked her why she did not first establish learners' understanding of factors. She confirmed our observation by saying,

> *"I first demonstrate and then give them the problem".*

It was only after she had marked the classwork that she gave after the lesson that she discovered that her learners did not understand factorization. Other than wrong answers written by her learners in their exercise books, we further inquired about her views in terms of whether the lesson was a success or not. The following was her response.

> *"No, I found out they did not understand totally and I saw it necessary that I must repeat the lesson".*

Evidently, assessment was treated as a separate entity from lesson presentation as she claimed that she first demonstrated to her learners how factorization was done.

She only discovered that her explanation was not understood at the end of the lesson, when no adjustments could be made to suit the learners' cognitive levels. Putting it differently, if her learners could have given the correct mathematics answers at the end of her lesson presentation, she could have assumed that they understood the concept she presented.

Furthermore, Nowele's lesson plan demonstrated that she also viewed assessment as an aspect separate from teaching and lesson presentation (see appendix 4). Her lesson plan shows the steps to be followed and there no assessment activities were included in her lesson plan, except for the classwork exercises to be done by her learners at the end of her lesson presentation. She would only know whether they understood the simplification of expression at the end of the lesson.

When we inquired about why there were no assessment tasks before and during Njovane's lesson presentation, he argued that:

> *"I believe in first showing them, tell them the critical parts and then give them exercises so that they give me the feedback."*

Njovane also argued that he believed in first showing the learners how to work out mathematics concepts and later give them exercises to do. Njovane's argument demonstrated his belief and practice, thus, not integrating assessment in teaching. All the above co-researchers who have been mentioned here in relation to the separation of assessment from lesson presentation also did not consider the learners' prior knowledge. They did not identify the learners' competence in in order to adjust their own instructional strategies in terms of either providing enabling prompts for learners that needed cognitive scaffoldings or providing extended prompts for advanced learners.

Moreover, the practice of not integrating assessment and teaching was also prevalent in Ntozine's class. As we observed the trend of first demonstrating how to work out the mathematics concepts and then give exercises to learners afterwards, we could not figure out what she intended to do when we looked at learners' given class work. The following (Figure 4.10) is the classwork that was given to learners:

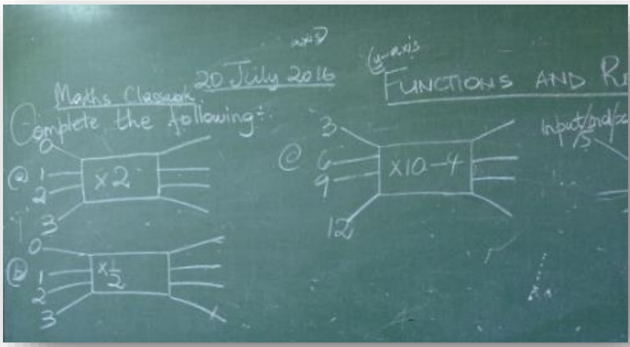


Figure 4.10: Mrs Ntozine's chalkboard summary

Ms Ntozine did not have a lesson plan that could guide us in terms of lesson objectives. As we inquired after the lesson presentation in terms of what she intended to assess and achieve, this was what she said:

> *"I want them to come out with an output".*

She did not assess learners on finding the general rule, which we viewed as an objective of the lesson and high order question that was supposed to be part of assessment. Other than the observable disjuncture between the assessment and lesson presentation, there was no integration of assessment with lesson presentation.

The same practice was prevalent in Mbuyi's class. When we asked her about her view about giving a problem for learners to solve or integrating assessment with lesson presentation, she raised the following question:

> *"Am I not supposed to first teach them so they could know the procedure and then give them a problem to solve afterward?"*

Mbuyi's question confirmed our observation, that she first presented the procedure on how to solve mathematics problems and then gave learners the problem with the expectation that they should follow the teacher's procedure. Moreover, what was more pronounced in her question, was that anything different from first presenting the lesson and assess later was rather unorthodox.

When we juxtaposed our vivincia on the research sites against the good practice regarding integrating assessment with lesson presentation we discovered a serious divergence. Evidently, the co-researchers, as noted above, did not only believe in separating assessment from lesson presentation, but also actually first presented their lessons and then gave learners exercises to test their understanding. They acted contrary to the education policy directives. Their failure to recognise the pivotal role of connection between assessment and instruction in improving mathematics understanding, as articulated by Gearhart and Saxe (2004: 304) and Umugiraneza *et al.* (2017: 3), denied their learners better understanding of mathematical concepts. Although Rhulumente did not table what went wrong with her presentation, she acknowledged that her learners did not understand her lesson and decided to re-teach it. As per the empirical data at our disposal we could argue that the learners' thinking to influence and strengthen classroom practice was not assessed. Instead, what transpired is what Heritage (2017: online) calls 'got it or did not get it and re-teach', which is different from assessment embedded in instruction that focuses on conceptualized pedagogical processes whereby the learning process is evaluated by both teacher and learners while it occurs (Collins, 2016: 1).

Learners were not given problems to stimulate their thinking and mathematical understanding as with the case in PBL (Gijbels *et al.*, 2005:29), but they were subjected to safe talk. Furthermore, the opportunity to enhance learners' ability to understand both mathematical concepts and procedures was lost due to the non-integration of assessment with lesson presentation. When Rhulumente decided to reteach the lesson, she still had no inkling in terms of what went wrong or why the learners did not understand what she had taught. She was likely to do the same presentation, hoping for a miracle to occur so that learners would finally understand those mathematical concepts. She did not use the opportunity to identify her learners' competences and their mathematical thinking in order to appropriately adjust her lesson presentation (Ogbebor-Kigho *et al.*, 2017: 293). Had she integrated assessment with her lesson presentation, she could have managed to receive feedback to be able to provide enabling prompts during the lesson presentation. There was nothing that informed her lesson planning that she intended to re-teach as she apparently did not anticipate learners' misconception to build on her intervention strategy. Learners were not provided with any feedback so that they could understand where they needed to improve; instead the teachers put a cross indicating that the class work tasks that were given at the end of the lesson were wrong. In line with

Adediwura's (2015:356) theorizes that the practice of marking with tick and cross marks in mathematics learning tends to limit learners' thinking. There were no explanations given to learners for the reasons of the negative marks; the least that was done, were corrections through re-teaching and learners were then expected to compare their answers with the corrections. Finally, what Dahl and Kolmos (2015: 65) call 'process competences' were not assessed as learners were never asked as to why they solved the given exercise in a particular way.

Meanwhile, as we tried to understand the interaction and social practices at the research sites, we were influenced by Mogashoa's (2014:105) use of relationships between text, talk and society. The opinions aired by the co-researchers were directly related to their classroom discourse. They did not only believe in separating assessment from lesson presentation but enacted it by demonstrating mathematical concepts and procedures before they could assess learners. Mbuyi protested by asking whether she was not supposed to first demonstrate the procedures before she could engage learners in assessments. Her protest indicated a deep-rooted discursive practice. For her to do otherwise would be to betray her conviction to the discursive practice and understanding of how assessment should be handled in relation to lesson presentation. It was a common practice among the co-researchers that were involved in this study to demonstrate first and assess later. As a result, we were convinced that separating assessment from teaching was socially accepted. Over and above, teachers saw themselves as the most powerful group in terms of knowledge possession and power relations in the classroom discourse. Although it was not overtly put, their hegemonic position in the classroom was enacted as they consciously viewed learners as empty vessels to be first given a demonstration before they could be assessed. They believed in what Freire (1993: 250) called the 'banking concept', hence they did not attempt to assess any understanding of mathematical concepts before having demonstrated how mathematical procedures were supposed to be done.

In contrast to CER, teachers at the research sites were the only elite group that had hegemony in the classroom discourse. We have earlier argued that CER has an emancipatory agenda which is rooted in Habermas's notion of emancipatory knowledge, and Freire's transformative and emancipatory pedagogy (Section 2.2). These fundamental values of CER intend to transform the classroom discourse from what Freire called pedagogy of the oppressed towards emancipatory pedagogy. In terms of the empirical evidence from the research sites, features of pedagogy of the oppressed seemed to be perpetuated. In Ntozine's class lesson presentation had nothing to do with assessment. The substitution in a flow diagram as in depicted in Figure 4.10 above was not linked to the formulation of a general rule. We could argue that learners were engaged in some activity but not learning anything to enable them to understand how the general rule is formulated in functions generation. Moreover, CER empowers participants with the ability to tell their stories (Tsotetsi & Mahlomaholo, 2015:49) including their thinking. Participants' knowledge is not regarded as weak knowledge, but as valid and respected (Tsotetsi & Mahlomaholo, 2015:49). However, the participants in the study, did not value learners' prior knowledge, hence it was not assessed. Both Mbuyi and Njovane had strong beliefs that disregarded learners' prior knowledge as they presumed that such knowledge could only be regarded as weak knowledge, although they did not overtly say so. Evidently, from both Rhulumente and Mbuyi's classes, failure to integrate assessment with lesson presentation seemed to have devastating results, which could prevent learners from studying mathematics in the future. The non-integration of assessment with lesson presentation seemed to have a potential to curtail the emancipatory and transformative agenda of CER, but to perpetuate social injustice and marginalization of the previously disadvantaged sector of SA people.

The findings of the study confirm the assertion that assessment separated from instruction only allows the evaluation of the impact of teaching on learning when the lesson presentation is over and on the spot intervention can no longer be done (Wallace, 2013:3-4). In line with Nkoane (2012:98) the acts of poor teaching practices that prevailed during the apartheid era, thus safe talk in this case, seemed to be continued unabated. In this instance, the lack of intergraded assessment with teaching on the part of the teacher, who was in the position of power to do so, unfairly disadvantaged learners (Van Dijk, 2006: 360). Consequently, teachers forfeited the opportunity to strengthened their classroom practices to improve their learners' mathematical understanding by using assessment as integral part of instruction (Gearhart & Saxe, 2004: 304). Had teachers integrated assessment with teaching, from lesson preparation to presentation, their MPCK could have been developed through learning new ways of resolving learners' misconceptions that could be elucidated by assessment-embedded instruction. This is in line with the view which asserts that teachers' recognition of learners' 'cognitive tendencies enhances their pedagogical strategies (Nagasaki & Becker, 1993: 46).

4.2.6 Non-implementation of learner-centred approach when teaching

In a learner-centred approach to teaching the learner becomes the active agents engaged in construction of their knowledge contrary to being passive recipients of information as they negotiate generated solutions "through sharing and exchange of ideas" (Zain *et al.*, 2012: 319). Zain *et al.* (2012: 320) further recognised that learners learned uniquely due to their endowment with different learning styles. A learner-centred approach embraces different learning styles through its focus on what the learners do and how they learn. Furthermore, in terms of learner-centred (Mme 2011: 3) theories of learning, such as PBL, teachers encourage learners' self-directed learning process or 'learning autonomy' (Li & Du, 2015:20), by raising questions that inspire learners' active involvement in discussions on how to solve problems (Tambara, 2015: 102). In a PBL environment, students self-determine the sources of information such as books, journals, faculty, on-line information resources, and so forth (Barrows, 1996*:* 5) as they determine the best fit of solutions to the problem (IMSA, 2001 in Aldred, Timms & Meredith, 2007:230). Evidently, from the research conducted by Erickson (1999: 520) the implementation of PBL in mathematics teaching increased problem-solving, decision making and reasoning-process skills for learners who learn from teachers using this approach. PBL as a learner-centred approach "empowers learners to conduct research, integrate theory and practice, and apply knowledge and skills to develop a viable solution to a defined problem" (Savery 2006:12). According to the DBE (2016:18) teachers must recognise that teaching is an active process. The SA education policy framework also advocates for learner-centred approaches through curriculum aims that intend to produce learners who are able to investigate, reason logically, analyse, represent, interpret information, communicate, pose and solve problems (DBE, 2011: 9). The DBE (2011:4) purports that a learner–centred method of teaching accommodates a deep approach to learning in an effort to allow learners the opportunity to develop and employ critical thinking skills.

During our visits to the research sites the dominance of a teacher-centred approach was prevalent in contrast to learner-centred pedagogy as advocated by good practice in terms of research, learning theories and policy frameworks. Njovane's lesson presentation was predominantly teacher-centred; learners were only involved through chorus responses after he had confirmed the answer to the question he had earlier posed. When he demonstrated how to solve: $m - 2 (3m + 1) = 2m - (m - 4)$, the following transpired:

> *"Before we can solve this now, we need to work out m, can you see m? M is an unknown, can you see that? We need to remove the brackets, how are we going to remove the brackets? -2, this negative 2 is going to multiply every term found inside the brackets, isn't so?"*

At intervals learners responded by saying yes and this kind of interaction transpired throughout the lesson presentation. As learners were asked if it wasn't' so they were expected to say, 'yes'. Njovene's teaching approach was distinctly teacher-centred, his learners were only participating through answering to rhetoric questions such as *"can you see m, can you see that, isn't so?"* All these questions did not prompt learners to think critically about the concepts at their disposal but to comply with the classroom discourse they found themselves in. And as such they would mechanically say yes without really engaging in a discussion with the teacher or other fellow learners regarding the concept of algebraic equation. When we interacted with Njovene and his learners after the lesson presentation, he argued that he believed in first explaining the concepts to learners before they could be given exercises to do on their own.

> *"I believe in first showing them, tell them the critical parts and then give them exercises so that they give me the feedback."*

His belief that learners need to be told first what they are supposed to do and let them regurgitate what they have been told influences the teaching approach that he adopted. Njovane's teaching was about him in terms of what he was doing not his learners. He used learners' exercises to get feedback. In essence the process of teaching was centred on him. Contrary to this, his learners had a different view about the way they were supposed to be taught. Khwezi, one of his learners, argued that:

“If I were a teacher, I would let one of the learners who seem to understand the concept to explain it to other learners, they may understand it better from one of them”.

These learners’ voices were a call for a need to be involved in lesson presentation in one way or another by critically participating and expressing their views and understanding of the concept. Learners’ lived experience about learning indicated that they learnt better from others. However, the teacher-centredness that prevailed in their class did not allow them an opportunity to express their views.

The teacher-centred approach did not only emerge in Njovane’s class but it was prevalent in the other research sites too. Rhulumente’s lesson presentation discussed in section 4.2.1 showed the dominance of the teacher during the lesson presentation. She first demonstrated how to factorize expressions using the highest common factor before she could give exercises for her learners to solve, with the expectation that learners would follow the same procedure.

At another research site, Ntozine completed the following table with her learners.

Table 4.2: Ntozine's table

X	1	2	3	4		10	***12***
Y	3	5	***7***	***9***		***15***	41

However, she did not give the task to her leaners to complete, instead she guided them on how to complete the table. She further developed a flow chart and asked learners to develop a general rule. Learners guessed the general, for example, they first said the general rule was “*plus two*”, in a chorused form, meaning that we need to add two to the value of x in order to get the value of y. Learners were not given the time to work as group on the problem, but they just guessed. Ntozine only showed them that what they guessed was wrong. After having recognised that ‘plus two’ was not working, some learners just said “*times*” and others said *“divide*”. The above classroom discourse was dominated by the teacher while learners were not meaningfully engaged in learning but were merely responding to cues given by the teacher.

From the empirical evidence thus far discussed, it appears that the teachers predominantly failed to engage learners as active agents in their knowledge construction, but threated them as tabula rasa who did not have any ideas to share with others. Teachers seemed to have abdicated their roles as learning mediators that required them to be sensitive to learners' needs (Potenza, 2002:1). In contrast to Zain *et al.* (2012: 320), who refuted passive assimilation of information, Njovane believed in first demonstrating how the problem was solved and he viewed learners as passive recipients of information. Besides, he regarded learners as all learning in the same way, and that is through listening to the teacher. The uniqueness of learners and their different learning styles were overlooked. This view of teacher-centredness denied learners' the opportunity of active involvement and self-directed learning where learners are able to pose questions and choose best learning materials that best suit their different learning styles. Evidently, from both Ntozine and Rhulumente's classes, students' mathematics problem-solving, decision making and reasoning processes did not improve. In Ntozine's and Rhulumente's classes learners guessed the solutions and completed the teacher's cues respectively. Apparently, learners taught through teacher-centred approaches have limited chances to pose questions, communicate and develop viable solutions to defined problems. Learners' reasoning process and the ability to choose information sources were seriously curtailed by teachers' domination in the teaching and learning process in the referred research sites. In essence, none-implementation of the learner-centred approach muffled learners' voices and restrained their abilities to negotiate with others their solutions to the given problem. The dominance of the teacher-centred approach; therefore, defeated the DBE's (2011:4) advocated learner centred method of teaching that accommodates a deep approach to learning in an effort to allow learners the opportunity to develop and employ critical thinking skills.

Our vivencia at the research sites in terms of the text recorded during our visits unequivocally demonstrated that there was no implementation of learner-centred approaches. Njovane's approach and belief of first presenting algebraic equation before he could engage learners through exercises was not an enigma. Instead we could argue that it was a legitimate rule as it was commonly prevalent across the research sites. Learners as well seemed to be familiar with the pedagogical approach of passively listening to the teacher's presentation and responding to teachers' cues in a chorus. The power relations that prevailed in the teacher-centred approach seemed to be natural as they were harmoniously accepted by all the stake holders, that is, teachers and learners alike. This was evident from Njovane's approach as he put it that

> *"before we can solve this now, we need to work out m" ;*

and Rhulument when she said

> *"In order to factorize that expression, we need to find the highest common multiple of 3x plus 12x2"..*

No learner tried to pose a question or request for clarity during the lesson presentation. This kind of discursive practice is what Fairclough (1985: 740) called 'orderliness', thus "the feeling of participants that things are as they should be". Learners and teachers seemed to be guided by certain unwritten norms and rules on how to interact during the lesson presentation.

Moreover, the power relations in terms of the classroom discourse clearly displayed the hegemony of teacher-centred pedagogy in the mathematics class. Learners were desperately copying the chalkboard summary in order to use it as a guide when their time came to reproduce or mimic what the teacher had said during the presentation. Teachers' social standing in the teacher-centred approach entrenched the teachers' dominance. Learners could not challenge clear mistakes made by their teachers because of their social standing in the classroom discourse that had a teacher-centred approach as a discursive practice.

> *"Remember that in the stem and leaf table, the stem represents units….. and the leaf represents units"*

The above extract was the mistake made by James in her presentation of a data handling lesson. However, learners conformed to the teachers' presentation as she requested their confirmation of her statement through the use of cues. Contrary to learners' earlier confirmation of James's statement, they presented the correct answers when it was their turn to present the given data in the form of a stem and leaf table. Their answers demonstrated that they knew that the stem represents tens in a stem and leaf table, not units, although they had earlier confirmed their teacher's statement that '*the stem represents units'*, by completing the teachers cues through a chorus. In terms of Gramsci's theory of hegemony, the research confirms that the hegemonic patterns in a society and organizations are maintained because people are dominated both by power imbalances and by consent of the dominated groups as they regard their situation to be a natural order of things (Frisby *et al.*, 1997:15). In line with Frisby *et al*'s. (1997:15) assertions, learners in James's mathematics class assumed that the discourse in their classroom was a natural order of things that could not be changed or challenged. Had learners challenged the teacher, both the discursive practice and the power relations that prevailed in their class could have been disturbed and inevitably the orderliness would be denaturalized.

Looking at the above discourse patterns from the research sites through the CER lens, the non-implementation of a learner-centred approach seems to entrench teachers' hegemony in the classroom and perpetuates the reproduction of inequalities. This is in contrast with CER, which seeks for human emancipation by liberating human beings from the circumstances that enslave them and change systems that routinely oppress them (Section 2.2.1). CER proponents seek to expose and challenge hegemony and traditional power assumptions held about relationships to advance social change (Given, 2008: 140). The treatment of learners as tabula rasa and the belief that learners should be first shown how to solve mathematics concepts before they could be given problems to solve, diminished the learners' active role and maintained the position of power of the teachers in the teaching and learning process. The teachers' domination muffled learners' voices to such an extent that they were unable to pose questions or correct teachers' mistakes; they merely had to conform to the prevailing classroom discourse. A teacher-centred approach which is a direct opposite of a learner-centred approach neither exposes nor questions hegemony and the power assumptions held about organisations such as schools. This behavioural positioning of communities in a social structure manifested through non-implementation of the learner-centred approach perpetually reproduces inequalities.

The results of the study pointed out that there was non-implementation of the learner-centred approach. Consequently, teachers failed to create the opportunity for learners to develop and employ critical thinking skills. Instead it perpetually reinforced the unrealistic view that purported mathematics to be reserved for the few who have magic brains (Okafor & Anaduaka, 2013: 251). The belief that learners were learning when the teachers dominated the class presentation, and learners' assumption that they were learning when they memorised without understanding fostered rote learning. Skemp (1976: 2) referred to rote learning as "rules without reason". For example, turning a fraction upside down and multiplying for division by a fraction (Skemp, 1976: 2) is one of the usually unquestioned rules in the classroom discourse dominated by a teacher-centred approach. According to literature this kind of classroom discourse encourages rote memorization which is mostly applicable to "animals that one does not credit with a thinking mind" (von Glasersfeld, 1994: 7). As an unintended consequence, caused by non-availability of learners' opportunities to pose questions in a mathematics class that does not embrace a learner-centred approach, learners are subjected to learn rules without reasons which is a practice that is in contrast with critical thinking and reasoning enshrined in CAPS mandates (DBE, 2011:4).

4.2.7 Poor knowledge of mathematics content for mathematics teaching

Departmental policies provide that mathematics should be understood as a human activity encouraging deep understanding of concepts and making sense of mathematics (DoE, 2011: 8). Developing MCKT enables teachers to decompress mathematical concepts, skills, and procedures, while connecting mathematical ideas within and across mathematical domains (Ball & Bass, 2003). Strong professional development seems to encourage teachers to self-reflect on their practice and "develop ways of engaging students in deeper inquiry and metacognition" (Atkinson & Minnich, 2014:7). Seemingly, MCKT develops teachers PCK (see section 2.4.5.7). Teachers with sound MCKT are able to teach to the understanding of the learners through utilization of comprehensive lesson plans and mobilization of appropriate manipulatives (see section 2.4.5.7). Specifically, the teacher becomes an expert learner in PBL through modelling "good strategies for learning and thinking, rather than providing expertise in specific content" (Hmelo-Silver, & Barrows, 2006:24). As an expert learner, the teacher acts as cognitive apprentice to provide cognitive scaffolds in decompressing complex mathematics concepts.

Data generated regarding teachers' MCKT seemed to be in contrast to good practice in terms of research, learning theories and policy framework.

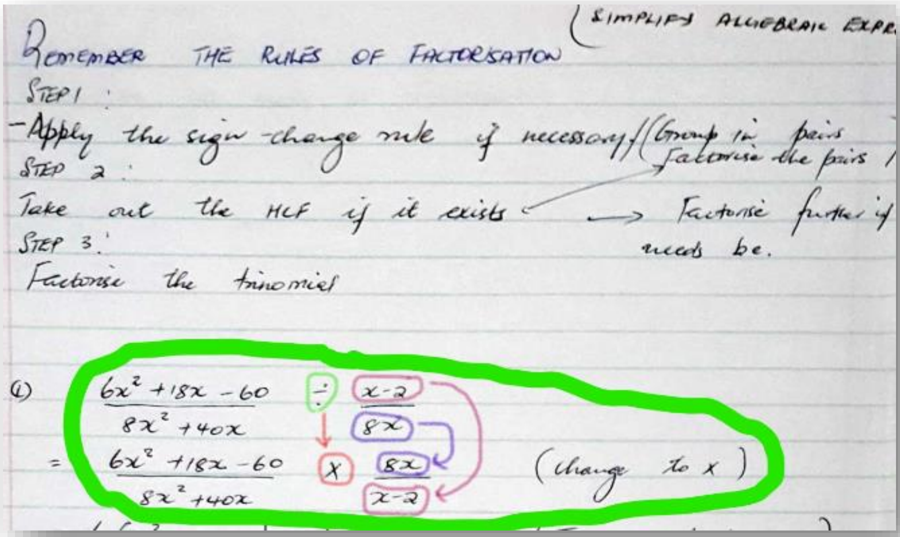


Figure 4.11: Excerpt from Nowele's lesson plan

The above excerpt (Figure 4.11) demonstrates Nowele's use of the algorithm for division by fraction in presenting algebraic expressions. As reflected in the figure above, (x – 2) has been made a denominator while (8x) has been made a numerator. In her lesson plan she emphasized that the division sign should be changed to a multiplication sign. Her division of fractions appears to be rule based, since there is no explanation why she changed the division sign to a multiplication and inverted a numerator to a denominator. Evidently, this lesson plan shows Nowele's computational skills in simplifying algebraic expressions, but does not display deep understanding of division concepts in terms of fractions. The same lesson was further observed in class. As part of her introduction she started by demonstrating the following task: $\frac{4}{9} \div \frac{8}{3}$ and presented it as follows: $\frac{4}{9} \times \frac{3}{8}$.

> *Nowele: "In division you change that division sign to a multiplication, which means you take denominator and make it a numerator, your numerator to be your denominator. In addition, and subtraction, we look for the LCM, but in multiplication and division we do not look for LCM. What we need to know in division of a fraction we change that divide to multiplication, are we in agreement?"*
>
> *Learners: "Yes" (in a chorus).*

After the lesson presentation we discussed her lesson and allowed her to make a reflection:

> *Researcher: Do you think your learners understood your lesson?*
>
> *Nowele: No they did not understand it, they did not …*
>
> *Researcher: What do you think was a challenge to them?*
>
> *Nowele: Let me say, these learners of mine, they are familiar with … I do not just demonstrate one example, I do not only do one example and then give them an exercise. The time was too limited, what I was teaching today is very complicated. I was supposed to demonstrate by doing more exercises.*
>
> *Researcher: Ok, thank you, tell me why do we change division sign to be a multiplication?*
>
> *Nowele: It is because we cannot divide a fraction by a fraction … if it is a fraction, divide means how many times does this go in this one, but when it is a division with a division we are unable, we need to first solve it … change it to be a multiplication because we cannot divide a division with a division.*

From the above conversation, it appeared that learners did not understand the concept of division by a fraction. Put differently, they did not connect division of whole numbers and the division of fractions. Their teacher believes that she has to demonstrate more examples so that learners could probably remember the mathematical procedure, which in our view is not mathematics concept in this case. The implication is that it is believed that they will finally see how it is done when more examples have been demonstrated. Secondly, the teacher also seems to only have procedural understanding, not conceptual understanding as she confidently argued that, "*we cannot divide a division by a division*".

Njovane's presentation on equations was transcribed (see Appendix 8). The following equation, as reflected in Figure 4.3., was used as an example to demonstrate what Njovane called solving the unknown: $m - 2(3m + 1) = 2m - (m - 4)$ (see section 4.2.1). As reflected in Appendix 8, learners were engaged in an activity that did not encourage any mathematical sense making or deeper concept understanding. They were mostly asked questions that had nothing to do with any learning but to respond 'yes'. The following except demonstrated the kind of question used by Njovane was extrapolated from his lesson presentation as transcribed in the attached Appendix 8:

1. Njovane*: Yes, we are solving the unknown, do we understand … we are solving the unknown equation (raising his voice)*

2. Learners*: (in a chorus) We are solving the unknown.*

9. Njovane: Before we can solve this now, we need to work out m, can you see m?

10. Learners*: Yes (learners responded in a chorus)*

11. Njovane*: m is an unknown, can you see that?*

12. Learners: *Yes*

13. Njovane*: We need to remove the brackets, how are we going to remove the brackets? -2, this negative 2 is going to multiply every term found inside the brackets, isn't it so ?*

14. Learners*: Yes*

21. Njovane*: -2 yes (he confirmed) can you see that we are through with the left side?*

22. Learners: *Yes*

The above excerpt from Appendix 8 exhibits the type of activity that learners were engaged in. This activity does not give a precise definition of the equation concept, but the teachers focuses learners on working out m or solving the unknown. In line 47, from Appendix 8, he had this to say

> *"So now we need to deal with … is to collect all numbers with m and put them on one side and also collect all numbers without m and put them on the other side. In which sides should we put m's, left or right?"*

In his statement he did not use the accurate mathematics language, such as terms with variables m to be put on one side of the equation. Instead he used language such as numbers without m. Mathematically, this kind of language is not accurate and does not necessarily mean what he intended to say. Furthermore, in line 49, from Appendix 8, Njovane advanced that there was a need to remove m from the right-hand of the equation, not using the language such as elimination to maintain the balance of the equation. He did not even explain why he was using the inverse of positive m in both sides of the equation. One doubts if Njovane's lesson presentation was in any way dealing with deep learning and understanding of the equation. He seemed to be focused on computational matters of the problem as he hardly mentioned the term equation except in line one. He did not precisely differentiate between the algebraic expression and equation. In line 63 of Appendix 8 he missed the opportunity of explaining or allowing learners to explain why they added two on both sides of the equation. The chalkboard summary of his lesson presentation also reveals that he did not take much heed of precision as reflected in Figure 4.3 (see section 4.2.1). He wrote the solution as – m = - 1 instead of m = -1 as the correct answer. Moreover, it also appears that the learners struggled to understand the equation concept as reflected in Figure 4.12 (see Figure 4.5 in section 4.2.2). Evidently, the marking of these learners' work presents doubts about the teacher's understanding of the concept of algebraic equations.

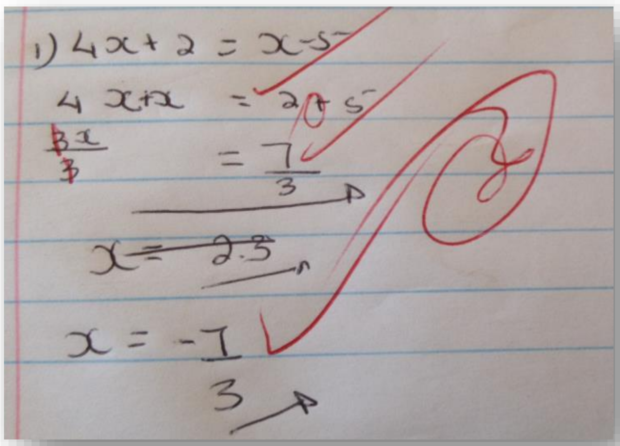


Figure 4.12: Learner's work after the lesson presentation

Over and above, our talk with him after his lesson presentation revealed that he was not solid in terms of his understanding of equations, particularly the aspects of keeping the balance between the left and the right-hand side of the equation. As he was requested to explain in simpler terms his understanding of the equation concept.

> *"The equation … generally, if I could explain, it is like solving the unknown … something that is not known, for instance you can put it in words, then represent it using numerical."*

This explanation of equation was not convincing as a definition that would be presented by a teacher who had a deeper MCKT of the concept. He gave an explanation that also left us confused about what he actually wanted to say in defining algebraic equation. His explanation of equation did to not come close to what Andrews *et al.* (2014:14) defined as "two expressions denoted as being of equal magnitude".

Despite the research claims that teachers should not only teach procedural knowledge but should also elucidate the reasoning that supports it (Wu, 2018:10), it appears that the participating co-researchers could neither make sense nor deeply understand the concepts. The convoluted explanation by Njovane did not make sense, while Nowele could only exhibit surface knowledge, mostly at a procedural level. Nowele could not go further and explain why she changed division to multiplication in contrast with Shulma's (1986: 9) assertions that a teacher should be able go further to clarify why something is so. Nowele could not relate or connect division of fraction with the division of whole numbers and it appeared impossible to divide a fraction by a fraction, as she argued that "*we cannot divide a division by a division*", unless a rule of changing division to multiplication is used, although no conceptual explanation was attached to it. From self-admission, they could not engage learners in deeper inquiry and metacognition. Admitting that learners did not understand is actually confirming poor MCKT which consequently has a negative influence on PCK.

Evidently, the interaction from line 9 to line 14 of Njovane's transcribed lesson presentation, did not engage learners in any metacognitive process where the teacher could act as cognitive apprentice or an expert learner. Instead, learners were asked questions that did not require any mathematical reasoning, but mostly required 'Yes' answers. Questions such as "*m is an unknown, can you see that*?", does not require a metacognitive process, but learners had to follow the established discursive practice and say yes. This practice of learners responding in chorus form is, according to what Chick (1996: 21) called 'safe talk', used by teachers to conceal their incompetence regarding MCKT (see section 2.4.4.1). Teachers from the research sites were unable to decompress compressed mathematics concepts. Nowele contended that the time was too limited for her to teach a complex topic, she needed to demonstrate more examples before her learners could understand, while Rhulumente opted for re-teaching. Specifically, Nowele was unable to decompress mathematics procedure entailed in an algorithm for division by fraction. From Figure:4.11 it could be argued that due to their poor MCKT, they did not develop and utilize comprehensive lesson plans and the mobilization of appropriate manipulatives to exemplify mathematics concepts involved (see Figure: 4.8 in section 4.2.4).

The generated data further were analysed and interpreted through CDA to gain a deeper understanding and meaning of what Hussain, Jote and Sajid (2015: 242) call text-talk-visual that is embedded in a social context. Accordingly, people use representational systems, such as words, images and gestures to "build relationships, knowledge, identities, and worldviews" (Rogers, 2011: 5). Ways of representing and ways of interacting include but not limited to using classroom discourse, such as initiate, response and evaluate (I-R-E) (Rogers, 2011: 12). McElhone (2012: 6-7) maintained that teachers initiated interaction by posing questions, learners would respond and the teachers evaluated the responses creating I-R-E iteration genre. The following excerpt extrapolated from Appendix 8, lines five to seven indicated the power relations that prevailed in the classroom discourse in the interaction between the teachers and the learners on the research sites:

> *Njovane: What is an equation, how do we see it, how do we differentiate the equation? Because? Because of what? This is what it is … its an equation not expression. Why am I saying so?*
>
> *Learners: Expressions have brackets …*
>
> *Njovane: (Interrupted learners) The equal sign ….*

In this instance, Njovane questions, learners respond in a chorus form and Njovane evaluates their response by approving or disapproving. When he approves, he repeats learners' responses but when he disapproves, he gives an alternative answer or raises his voice (see Appendix 8). Learners seemed to understand the unwritten norms and classroom rules, that they had to complete teachers' statements and respond by saying 'yes' as a discursive practice. Nowele presented that "*What we need to know in division of a fraction we change that divide to multiplication".* Her presentation purports that learners are part of her decision to change division to multiplication as she carefully uses 'we'. She did not explain why she changed division to multiplication, and by including them in the statement, made them part of her statements, so that they did not question it. She concealed her incompetence in terms of MCKT which was later revealed during the reflection discussion after her lesson presentation. As learners are frequently asked if they agree with the statement presented, this is apparently a way of coercing them to appear to be part of what is presented to them. This discursive practice is socially accepted as learners regard it as natural that when they are asked if they agree, they should automatically say yes. The positioning of the teacher under this interaction genre makes teachers very powerful in terms of being the only ones who determines what takes place in the classroom. The structural arrangement does not easily permit learners to ask questions but to respond to questions. Learners' voices are muffled, and they have to learn rules without questioning or without understanding but through memorization.

In this study we did not use coding and counting the number of words in order to make meaning. The study was guided by democratic values, social justice, and working with participants in transforming their social situation (Nkoane, 2012:99). The meaning-making is determined by social constructions (Nkoane, 2012:100), such as text-talk-visual generated by different communities. Considering Figure 4.11, due to the teacher's situation in terms of MCKT, the lesson plan did not justify how certain mathematics computations are done. Learners were subjected to such classrooms dominated by teachers. Lesson presentations were not premised on democratic values and the inequalities seemed to continue unabated. Probably, had the voices of the marginalized, that is, the learners, been given a space, learners could have raised questions that would compel teachers to search for more information regarding mathematics concepts they intended to teach. However, due to power imbalances, teachers are positioned in such a way that they are the only ones who determine what should take place in the classroom. Consequently, both learners' and teachers' social standing did not change. In a nut shell, teachers fail to get emancipated and become learners as well when searching for new knowledge and explanations they could not provide, as would have been the case when learners were in a democratic classroom with the prevalence of a culture of learners raising questions.

The findings of this study exhibited that teachers had insufficient MCKT, and at the least, their understanding was at a procedural level. They were unable to explain why they followed such mathematical procedures. It appeared that they did not have a deep conceptual understanding. They could not provide precise definitions of concepts like algebraic equation, instead they presented a convoluted explanation that did not clearly tell exactly what the concept says or does not say. Consequently, they mostly used safe-talk and I-R-E classroom discourses to conceal their limitations in terms of MCKT. Their self-admission that learners did not understand concepts that were taught also was an indication of inadequate MCKT as the research argues that there is a positive correlation between MCKT and learner performance (Venkat & Spaull, 2015 :122; Nambira, 2016: 35-36). Furthermore, it is also enunciated that teachers cannot provide what they do not have (George & Adu, 2018:14). By implication, learners' inability to comprehend these concepts taught indicates poor MCKT. The findings of this study confirm Mosia's (2016: 2) assertions regarding poor teachers' content knowledge particularly in Euclidean geometry. In line with the study that revealed insufficient school mathematics from KwaZulu-Natal teachers (Bansilal *et al.*, 2014: 49), the findings of this study also affirm teachers' incompetence on fundamental school mathematics, such as division by fractions. Njovane's inability to pose questions that stimulate critical thinking, but relied on questions that only required yes on no answers affirms Fuma's (2018: 4) assertion that teachers who lack MCKT seem to struggle in posing questions that create cognitive disturbance.

4.3 ANALYSIS OF FORMULATED STRATEGY COMPONENTS FOR CHALLENGES IDENTIFIED REGARDING MATHEMATICS PEDAGOGICAL CONTENT KNOWLEDGE

This section is focused on suggested strategy components to respond to the challenges facing Grade nine mathematics teachers' MPCK using PBL. Based on the actual classroom practices, this section presents an analysis of the components of the strategy in an attempt to resolve each challenge mentioned in section 4.2. The components of the strategy that may respond to the challenges identified in the previous section are the following: the establishment of a coordinated team, follow up on learners' misconceptions, lesson preparation, the use of curriculum material, integration of assessment with lesson presentation, a learner-centred pedagogical approach, and sound mathematics content knowledge for teaching.

4.3.1 Establishment of coordinated team

This section explores the impact of coordinate teams in emancipating teachers in terms of their MPCK. The data generated in this regard are juxtaposed against the good practice in order to understand the team power in developing the community of practice. A team presents its members with an opportunity to learn from each other (Mosia, 2016:137) by exposing team members to diverse strategies and different reflections. According to Mosia (2016: 137), teachers find space in a team to communicate, to share and address issues, to observe one another's work and to develop expertise in various aspects (see section 2.4.1.6). Team work resonates with the ontological stance that embraces a shared reality that allows the nature of reality to be critically examined by intersubjective views (Mertens, 2015: 81). Teams strengthen competencies and promote collegiality while, on the other hand, reduces professional isolation and help individuals to overcome weaknesses (section 4.2.1). PBL also encourages peer learning and members have to collaborate with team mates to design solutions to the problems (Han & Teng, 2005:3). In a PBL environment, there is collaborative learning among team members as team members share knowledge (see section 2.4.1.1). PBL also affirms the notion of team work as Graaff & Kolmos (2003: 659) argue that PBL is collaborative learning related to practice. The CAPS document also encourages team work and respect for others (section 4.2.1). Team work promotes an active and critical approach to learning contrary to "rote and uncritical learning of given truths" (DBE, 2011: 4- 5). In essence, a team creates a platform for individual mathematics teachers to share their frustrations and success stories regarding teaching mathematics in a safe environment without being afraid to be judged. The interaction of team members bringing multiple realities and their subjective worldviews about their lived experiences in the mathematics classroom allows them to choose what seems to work for different classroom contexts.

After a number of consultative meetings with stake holders from the research sites under investigation, a coordinating team was established in an attempt to find solutions to the challenges identified in section 4.2. When the co-researchers joined the coordinating team of Grade nine mathematics teachers in the cluster, they developed a platform to reflect on their experiences with the team. The following data were generated as evidence of the claim of finding solutions. Figure 4.13 depicts the scribbling which is evidence of the planning meeting between Nowele, Falafala and I.

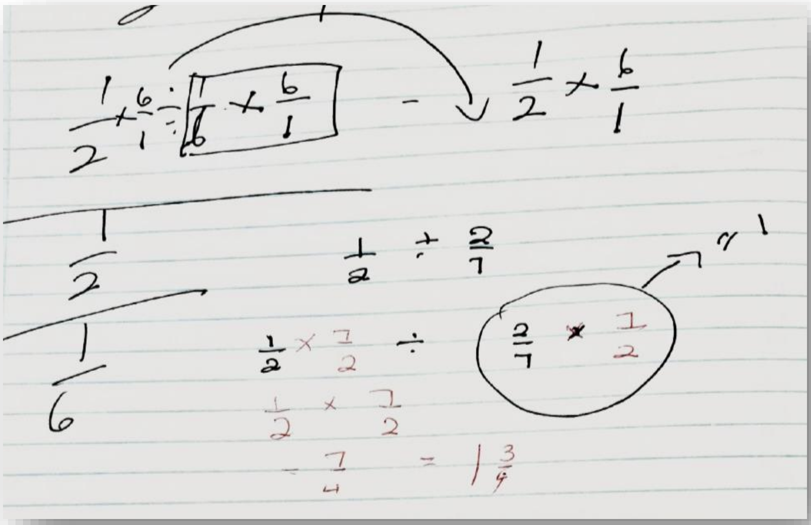


Figure 4.13: Outcome of a problem-solving meeting of a team

As we planned together, the team tried to come up with ways to help learners to understand the concept of division by a common fraction. As we discussed this problem, it appeared that it is very easy to divide by one because any number divided by one remains the same. The team raised the issue that we needed to use the reciprocal to eliminate the fraction that is a divider or a denominator. In this cooperating team we were guided by PAR and CER perspectives that posit that a researcher should be available to assist the participants, and not only act as an outsider with a view to extracting knowledge, but to actively participate in the research process without self-imposing. In line with CER we also participated in the lesson planning with the above-mentioned co-researchers. When both fractions, that is, the dividend and the divisor, were multiplied by the reciprocal of the divisor, namely $\frac{2}{7} \times \frac{7}{2}$ the answer of the divisor is one and we were left with the dividend multiplied by the reciprocal of the divisor. This process demonstrated the mathematical concept underlying the algorithm that seems as changing the division sign to multiplication and flipping the divisor fraction upside down when dividing by fractions. As reflected above we collectively multiplied both fractions by the reciprocal of $\frac{2}{7}$ that is, $\frac{7}{2}$. The fraction in the denominator became one and we were left with $\frac{1}{2} \times \frac{7}{2}$ on the numerator. Nowele also proposed that learners would understand the process better if the fractions were presented as in the case presented below:

$$\boxed{\frac{1/2}{1/6} \text{ as compared to } \frac{1}{2} \div \frac{1}{6}}$$

She argued that learners would better understand that one sixth is a denominator that needs to be eliminated through multiplication by its reciprocal. As reflected above, it appears that the planning session enabled co-researchers to experience what Mahlomaholo (2012: 293) calls mutual and reciprocal beneficiation.

Falafala was given a chance to present her thinking and experiences regarding collaborative planning.

> *Yah, it assists us, for one to go to class well prepared, more than we think we know what we do, you can find out, maybe you come out with an easy way to help learners understand mathematics concepts ... like it is much easy to divide anything by one."*

Apparently, Falafala's claim suggests that planning together changed her practice of thinking she knows what to do when she gets to her classroom without having prepared. Her words, "*Yah, it assists us, for one to go to class well prepared",* represent an indication that she has since changed her stance and considers the importance of thorough preparation and deep thinking to come out with easy ways to teach mathematics, such as division by one, before she goes to her class.

Below are the highlights of the issues raised by reflections of team members during the meetings and workshops of the coordinating team:

> *Rhulumente: "For me, I would really appreciate these small gatherings, I would truly appreciate them, worse senior phase is new to me, but not as such, I had taught at senior phase for nine years, I moved and taught at foundation phase for 13 years, I am now back at senior phase again, that is why I am saying I am going to be happy with our small gatherings, because it is important for me to know."*

Ntozine: "It is for my first time to teach grade nine, I have never taught grade nine, I had been teaching up to grade eight. I think that I am part of this team, I would understand the challenging grade nine concepts."

From the text that emerged from the discussions and co-researchers' reflections during co-researchers' meetings, the study provided a foundation for the establishment of the coordinating team. Both Rhulumente and Ntozine respectively expressed their appreciation and the value they attached to being part of the small gatherings for mathematics teachers. Their appreciation was a result of their experienced benefits provided by a team, as Schreiber and Valle (2013: 396) argued that complimentary benefits are imperative for team members. Rhulumente's utterances demonstrated the sigh of relief that after all her professional isolation the team now would help her close the gap in terms of her minimal experience with the class she was teaching. On the other hand, Ntozine's confession that she was a protégé, exhibited that she was looking forward to the benefits of understanding grade nine mathematics concepts better. Evidently, her statement: "*I think that I am part of this team*" suggests that there was an established team to which she finally belonged. Mbuyi also reflected about being part of the coordinated mathematics team.

Mbuyi: "I am happy that in our session we have people who teach both grade nine and ten, so that they can guide us on which areas we need to focus on."

Mbuyi's assertion presumes that the team does not have only members that teach in Grade nine, but also have members that teach Grade ten is an added advantage to her. Her presumption was that they might be more knowledgeable than she was and that might assist in broadening her curriculum spectrum. According to her statement, she would also understand what Shulman (1987) calls vertical curriculum knowledge. As she handled her Grade nine curriculum, she might have a view in terms of what her learners would need to know in Grade ten because of her participation in the dynamic team. She further continued to share her epiphanies with the team members.

Mbuyi: "Before, I attended these group sessions I did not guide my learners to focus on the corners in order to identify the angles required."

Mbuyi explicitly put it that she had developed new pedagogical strategies to enable her learners to correctly identify required angles. At the same time, she realised that her emancipation only happened when she joined the team of co-researchers. Put differently, when the team did not exist before the intervention of this study, she was stuck with the problem she could not solve or share with any one as she was professionally isolated.

However, this study may not present an exhaustive list of team members' 'vivencia', nonetheless, this what Jones had to say to share hers:

> *Jones: "When we meet like this you end up liking parts of mathematics that you initially did not like, because of the way it was taught to you."*

Furthermore, Jones was in agreement with the view that the team reduced weaknesses, as she claimed to be confident to go back to her school and teach topics, she initially was uncomfortable with before she became part of the team. The team environment created a space to present their challenges and problems, as Jones put it:

> *Jones: "When learners come from these lower grades, they are familiar with multiplying positive integers, and how would I help them understand the multiplication of negative integers?*

In the same vein Rhulumente also presented her problem with her learners in relation to division by fractions:

> *Rhulumente: "Colleagues my learners forget to change numerator to denominator when dividing by fraction."*

As team members were trying to resolve the ill-structured problems presented by their colleagues, Njovane proposed a profound strategy, namely that we needed to take all our unresolved mathematics problems to the PBL workshop that was initiated by this study:

> *Njovane: "Misconceptions among learners (occur in learners when we are in the class) should be dealt with deliberately in each workshop."*

Njovane's proposal influenced the team of co-researchers to behave like a community of practice, as they had to go to their various schools and practise the shared experiences from the coordinated team and periodically meet to share success stories and problems. Taking the advantage of the team, Zintle also raised the following question:

Zintle: "My learners have a problem with naming the correct angles when the angle is labelled by three letters of the alphabet, like angle ABC, especially when lines are cut by a transversal."

During the PBLW that followed, Zintle, for example, raised an issue regarding the inability of her learners to name the correct angles in activities that involved parallel lines that were cut by a transversal. Her argument was that learners were able to correctly point the angles on the board but when they had to write the angles they had chosen or pointed on the board, they tended to write or label a different angle. As part of the research team I intervened by raising a question:

The researcher: "Are we saying our learners have a problem in naming the angles? Unfortunately, we cannot blame them, they are learners, without necessarily blaming anyone, but there is something that we are not doing correctly, what then are we supposed to do, we cannot end by only narrating the said story, what can we do to make sure that they (learners) understand labelling angles."

Falafala: "I am used to use this one (pointing at angle) …why I am using this one, is $C\hat{A}D$ *because, sometimes this sign (pointing at* $\angle$*) they (learners) do not accurately write it, but that cap (pointing at the angle symbol above A in angle CAD) is that one that makes it clear in terms of which angle are we talking about. I use it most of the time."*

The researcher: "As we put the cap on top of the middle letter of alphabet, in angle CAD what does that mean?"

Falafala: Like as it is said angle CAD, the cap is on top of A, it means the angle that we are looking for is the one with the cap, at the vertex, what is inside the vertex is what we are looking for."

The establishment of a coordinated team as mentioned above created space for co-researchers to learn from each other as Ntozine attested that she was part of a team and this would support her to understand Grade nine concepts that she found challenging. Secondly, co-researchers were able to communicate their expertise as Falafala demonstrated to the team the strategy of helping learners accurately identify the correct angles when two lines are cut by a transversal. As Falafala's strategy was adopted by the team to be experimented at various research sites, her competencies in this regard were strengthened. From her $C\hat{A}D$ classroom experience, she noticed that in many cases, it was not that learners did not know the required angles, but when they named those required angles, they tended to mix the letters of the alphabet used to represent the angles or they arranged them the other way around and misrepresented the angles they intended to select. Falafala argued that she used the cap symbol on top of the middle letter to identify the vertex inside which there is an angle that is required instead of writing it as $\angle$ CAD.

In line with good practice, Rhulumente's and Ntozine's arguments suggested that the team overcame individual weaknesses, as they viewed themselves as novice teachers regarding mathematics teaching in Grade nine. Apparently, the establishment of the coordinated team exposed team members to different reflections and a plethora of strategies, hence Mbuyi argued that she valued to be part of a team of teachers that did not only teach Grade nine but who also taught Grade ten. In essence, her argument acknowledged the different expertise and intersubjective views provided by diverse team members (*cf.* Mertens, 2015:82). Evidently it is not disputable that the team provided a safe environment for mathematics teachers to share their frustrations and weaknesses with the intention to collaboratively learn without fear of being judged (Holmes & Hwang, 2016:449).

Jones comments suggested that she was no longer professionally isolated as she now gained confidence in teaching mathematics topics that she used to avoid before she joined the team. Zintle was also no longer left alone with problems she could not solve. Since they had become part of the team, they were no longer professionally isolated, as the team created a platform for presentation of lived experiences in terms of mathematics classroom discourse. In the same vein, Jones was also no longer professionally isolated as she was now able to share her frustrations regarding her learners who were not familiar with adding negative integers. As the team members became familiar with PBLW, it was easy for them not only to present their claimed success stories, but also shared with the team real problems they encountered in their mathematics classrooms. Rhulumente, for example, had a challenge regarding learners who did not seem to be able to follow the rule she taught them for dividing by fractions. In the meetings that we referred to as PBLW, there were no ready answers. Co-researchers would start negotiating the meaning of their proposed solutions to the presented problems.

During PBLW, Tau who was a facilitator in the planning sessions, shared his experience of working as teacher in handling mathematics problems. Tau (the curriculum advisor who helped in the team) argued that he had lost touch with General Education and Training (GET) mathematics concepts. But after attending a workshop on analysing learners' misconceptions, his GET mathematics concept knowledge had developed.

> *Tau: "You can see I also lost touch with the GET mathematics content, but I managed to attend one workshop for error analysis, that was one valuable workshop …. it is after teaching, you assess the learners, you identify the errors that the learners are doing and come up with the strategy to say if learners continue to do mistakes like these, what are the misconceptions, that is the first thing and then come up with the strategy on how would you correct those misconceptions."*

When the study looked deeper at co-researchers' spoken words and expressions of feelings regarding their experiences of working with colleagues, it appeared that there was an immediate need to establish the coordinated team. Tau's humbleness, although he was a subject advisor, demonstrated the value of team work especially that the team was at its initial stages of establishment. Prominently, his argument was a suggestion that we all need each other, and covertly proposed inevitable needs for the establishment of formal teams clustering neighbouring schools.

As evidenced during the consultative meetings Rhulumente proposed the establishment of a coordinated team as she argued that she would truly appreciate the co-researchers' small gatherings. In line with Mahlomaholo's (2012:51) recommendation, we conducted text-level analyses of CDA to obtain a deeper understanding of the text which entailed more than just sentence structures. The feeling of happiness expressed by Mbuyi suggested that co-researchers were treated with respect while working in a team which enabled them to identify weaknesses and to present intersubjective experiences in terms of possible solutions. From Njovane's suggestion it was also evident that the power relations had been changed as co-researchers were able to shape the direction of the team's work not depending on the researcher. The team's discursive practice embraced democratic values and reduced domination by the elite as reflected by Tau's humbleness. In essence, the positivist positioning of the researcher as the knowledge extractor was denaturalized and the researcher too became a learner in the process.

Team work is in line with CER, which advocates for the emancipation of co-researchers. The presence of a coordinated team presented an opportunity for co-researchers to plan together. Moreover, some of the programmes adopted by the team of co-researchers went beyond the scope of this study. Aspects, such as common summative assessment tasks to be used for learners' progression and cluster moderations were also included. In fact, it is well documented that in CER a researcher does not arrogantly impose his/her strategies but respects and combines his/her knowledge with co-researchers' knowledge and views them as partners (see 2.2.3.), hence co-researchers were at liberty to include some aspects they saw fit to enhance their MPCK. Moreover, the fact that team members were able to express their weaknesses, for example, Zintle regarding her inability to guide learners to identify required angles when parallel lines were cut by a transversal indicated that the team embraced elements of trust, humanity, respect and democracy. These values are promoted by CER (Nkoane, 2012:98). The team respected co-researchers, to such an extent that Rhulumente was able to expose her insecurities regarding teaching in Grade nine mathematics. The team was able to identify teaching practices that routinely constrained them, such as failure to analyse the reasoning behind learners' misconceptions.

The presence of a coordinated team became a spring board for self-emancipation of co-researchers as they found an opportunity to engage on the problem-based learning workshops (PBLW). We called them PBLW because, other than emancipating co-researchers regarding their teaching practises, they were mainly a platform to untangle ill-structured problems which came from co-researchers' real problems drawn from their teaching experiences. Their expression of the feelings of contentment regarding the team work demonstrated that working with their real problems experienced in their teaching practices was the most humanising professional development process.

The findings of this study confirm the view that team work enables collaborative design of solution to real contextual problems (Han & Teng, 2005: 3). In line with Mosia (2016: 137) the coordinated team created an opportunity for co-researchers to learn from each other. Team work further reduced professional isolation of mathematics teachers. Co-researchers were able to share resources such a mathematics workbook that were redistributed from where there was a surplus to the research site that had a shortage. The discussion of lessons planned collaboratively improved their teaching practice as they gained confidence in handling topics that they were previously uncomfortable with. In closure, the above epiphanies in terms of enacted experiences regarding the existence of a coordinate team confirmed narrative that collaborative structures of clusters enhance the effectiveness of teaching practices, thus, content knowledge and pedagogical content knowledge (Jita & Mokhele (2014: 1).

4.3.2 Identification of and follow-up on learners' misconceptions

In this section the data generated are juxtaposed with good practice to understand the impact of the PBL environment in enhancing MPCK when teachers identify and remediate learners' misconceptions. Inter alia teachers have a duty to consider learners' own experiences (DBE, 2016:18). Utilization of learners' own experiences is viewed as a fundamental and valuable resource (DBE, 2016:18) in guiding teachers about where to start as they plan and present their lessons. In attesting the above policy mandates, Makgakga (2014:7) pointed out that "it is imperative for teachers to teach mathematics using learners' errors and misconceptions as this will guide them on what learners grapple with". This view is not a lone voice as it is theorized that understanding learners' knowledge might guide teachers to scaffold learners' ideas and mediate the construction of new knowledge (Brodie, Lelliott & Davis, 2002: 557). In accordance with Brodie *et al.*'s opinions, teachers must understand why, when and how learners often make particular errors in a particular topic (see section 2.4.2.6). Re-teaching is no 'quick fix' to resolve learners' misconceptions, but paying more attention to learners' ways of speaking about algebra provides teachers with deeper insight into learners' thinking (see section 2.4.2.6). As teachers do follow-up and analysis of learners' misconceptions, they develop an understanding of learners' thinking (Herholdt & Sapire, 2014: 44) and consequently learn new ways of making mathematics comprehensive to learners. In response to learners' misconceptions teachers are able to adapt classroom discourse that may improve learner performance (Herhodt & Sapire, 2014: 44). From a PBL perspective teachers are expected to scaffold learners' process of learning by providing a supportive learning atmosphere

(Li & Du, 2015: 20). In this regard, teachers are not supposed to offer direct answers to learners' misconceptions but illuminate students' cognitive process (Li & Du, 2015) by raising questions that will act as enabling prompts.

In our lesson observations Njovane engaged learners to identify misconceptions, while Rhulimente used enabling prompts to help learners demystify their misconceptions. The following figure (Figure 4.14) depicts how learners worked out the problem in Njovane's class.

Figure 4.14: Learners' way of solving a problem

From the above task it was evident that learners were not sure about the meaning of brackets when solving a mathematics problem. As the co-researcher moved around the groups he interacted with a group without disturbing other groups. He requested the group to explain how they worked out the problem by asking the following question:

Njovane What do we do when we remove the brackets?

Emily (Learner): We put an addition sign.

Ntabeni (another learner): We multiply five by eleven.

Njovane: Why have you decided to add or multiply?

Seemane (another learner): We made a mistake, one of us insisted that we must add although we wanted to multiply, we then cancelled multiplication and put addition.

Njovane Eh …eh, why?

Evidently, in the above discussion, the co-researcher not only identified learners' misconceptions, but engaged them to explain it themselves. Apparently, learners were not shown what to do, however, they were confident to present multiple perspectives from the same group. Moreover, group dynamics seemed to have also played a bigger role as Seemane protested that they had earlier decided to multiply eleven by five to remove the brackets. Njovane' questions were aimed at trying to understand why they decided to add or multiply. He did not suggest any correct or wrong answer, instead he persuaded learners to justify their thinking. In the process he was in a position to understand why they acted in a particular manner and therefore could devise a remedy.

On the other side, in Rhulumente's class a group was given a problem to solve, namely, 5(x + 4).

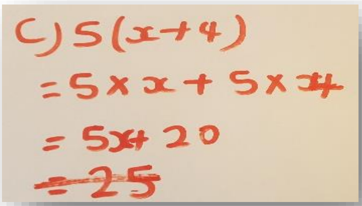


Figure 4.15: Example of problem solved by learners

Learners, in their calculations (Figure: 4.15), went on to add 5x + 20 resulting in 25 as their answer. This kind of misconception is referred to as 'lack of closure,' (see section 2.4.2) as this group of learners did not accept expressions such as 5x + 20 as the final answer. This kind of misconception also manifests when learners do not understand the concept of 'like terms'. When the group was required to explain how they got to 25 as answer, they just crossed 25. Apparently, learners developed an idea that when individuals are asked to explain their actions, that suggested that their answer might be wrong since they were not usually required to explain correct answers. Over and above that the research found that sometimes learners conjoined unlike terms and ignored the letter attached to the coefficient (see section 2.4.2). Nonetheless, the co-researchers continued to probe for how they got to 25 and why they crossed it out when they were asked to explain themselves. In this instance, the co-researchers taught learners that they did not have to change their views when they were challenged, but must explain the reason behind their actions.

As part of the lesson presentation Rhulumente also included the following problem, namely, (a + b)(a + b), to assess learners' understanding of the associative property of multiplication. As they worked out the problem, they could not recognise that ab and ba meant the same thing in multiplication. In terms of their ZPD, they could only solve the problem at the following level, thus:

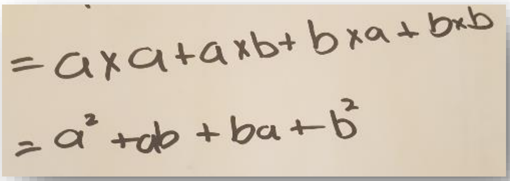


Figure 4.16: Students' solution to a problem

Rhulumente had to intervene. However, what was peculiar about her intervention, was that she did not just tell them the answer. She gave them other problems to demostrate the associative property, which were rather simpler. She enacted an 'enabling prompt' that helped learners to see that (a x b) is the same as (b x a) as they continued towards learning trajectory (section 2.4.1.2). She requested them to multiply 4 by 6 and 6 by 4. The group could see that it does not matter which number you started with in multiplication, the answer is the same, hence (a x b) is the same as (b x a). She recognised and used learners' experiences in handling associative property in the multiplication of numbers. This learners' prior knowledge (associative property in the multiplication of numbers) in this regard further was utilized as foundation to construct new knowledge, as the learners were faced with an unfamiliar problem of multipliying a by b. This experience of success to the group had a 'wow' effect, and as a result they could see that ab and ba are like terms, hence they finally solved the problem as follows:

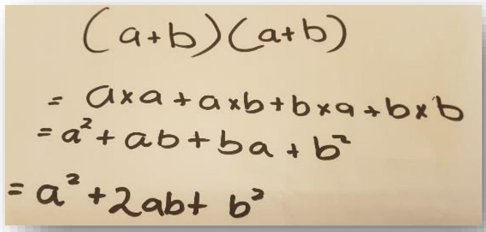


Figure 4.17: Learners' solution

Rhulumente managed to identify that learners had misconceptions regarding associative property. She could see that her learners did not understand that (5 x 6) is the same as (6 x 5), especially when it was represented in the form of (a x b) and (b x a). Having identified the stage where they could not go any further without aid (ZPD), she used enabling prompts to guide learners to realize that (a x b) is the same as (b x a).

The data generated illustrated that co-researchers consciously identified and followed up on learners' misconceptions. Evidently, co-researchers managed not to only identify learners' misconceptions but also used them to understand what learners grappled with (Makgakga, 2014:7). For Njovane, the learners' misconception became a source of engagement, while Rhulumente developed a strategy to overcome the misconception (Shulman, 1986: 10) by scaffolding learners' ideas and mediated the construction of new knowledge (Brodie *et al.*, 2002:557) through the use of numbers as learners earlier did not comprehend that (a x b) is the same as (b x a). With the understanding of knowledge of learners as an essential resource for effective teaching in teaching mathematics (Ball, 1997:732), they presented learners with a series of problems that would exhibit misconceptions such as conjoining (Pournara *et al.*, 2016: 5), lack of closure (Pournara *et al.*, 2016: 2) and poor understanding of associative property in multiplication of binomials. We could safely argue that learners' cognitive processes were illuminated (Li & Du, 2015: 20) as they were not given direct answers when they experienced challenging problems beyond their ZPD. Instead they were given enabling prompts as a strategy which came with a 'wow' effect to learners as they finally saw how trivial the associative property was, which earlier appeared as a hindrance to their learning trajectory. As co-researchers requested learners to explain themselves, they developed a deeper understanding of learners' thinking and revised their pedagogical approaches in order to improve learner performance (Herholdt & Sapire, 2014: 44).

In our quest to analyse the data generated through on site classroom observation we used CDA as tool of analysis. Specific features of text and talk were observed to have specific effects on managing strategic understanding (Van Dijk, 2006: 365). These features inter alia included visual representations (Van Dijk, 2006: 365) and tone of voice. Teachers who do not want to manipulate learners' thinking would use the same voice irrespective of whether the answer is correct or wrong (Brodie *et al.*, 2002: 552). As Njovane probed learners, he did not change the tone of his voice to suggest that he favoured a particular answer over the other, instead he allowed learners to continue expressing their thoughts regarding the removal of brackets. As the co-researchers embraced new discursive practices, their behaviour was different from the extracts given in section 4.2.1, before the implementation of the strategy. In section 4.2.1 Rhulumente raised her voice which we viewed as an indication of discontentment with the answer given by her learners.

As a community of practice co-researchers learnt from each other how to probe learners to explain their reasoning and how to develop cognitive scaffolds to help learners to demystify their misconceptions without just telling them what was viewed as the correct answer. Evidently, the classroom discourse seemed to have changed as co-researchers implemented the strategy to follow up on learners' misconceptions. Rhulumente was no longer raising her voice as a sign of discontentment with learners' views. As it also appeared in Njovane's class, the same group presented two different solutions for the same problem, thus, **5(11)** and they were prepared to defend their views without a fear to be judged. It was put on record that "discursive practice refers to rules, norms and mental modes of socially accepted behaviour in specific roles" in receiving and interpreting messages (McGregor, 2003:3). The new behaviour exhibited by the co-researchers of not just giving the correct answer when learners displayed misconceptions, suggested that ideologies that dominated their classroom discourse appearing as neutral and natural (Wodak & Meyer, 2008:8) had been challenged as co-researchers followed up learners' misconceptions.

As we further analysed the data at our disposal through a CER lens we were at least convinced that the co-researchers allowed multiple perspectives to be democratically discussed in their classes. This is in line with McCabe and Holmes's (2009:1522) views that CER advances emancipation by exposing dominating truths so that individuals can negotiate new modes of acting. Co-researchers revised their pedagogical approach from seeing themselves as the only providers of solutions in a mathematics class and used learners' misconceptions as part of learning. Evidently, the use of enabling prompts by Rhulumente indicated that learners' views were valued in her classroom. Co-researchers could realize that just giving correct answers to learners' misconceptions without engaging learners' cognitive congruence constrained their teaching. In line with the finding in literature that research from a CER perspective "is seen as the most humanising experience" (Mahlomaholo, 2009:225), the success experienced by learners when they were probed to explain themselves changed the lives of co-researchers as they expressed feelings of liberations from "not-so-useful practices and thoughts" (Mahlomaholo, 2009: 226). In a nut shell, follow-up on learners' misconceptions apparently comes with epiphanies to co-researchers resulting in reviewing their power-dominating methods which routinely constrained them.

In closing, the findings of this study apparently affirmed the finding from the literature which encapsulated that analysis of misconceptions enabled teachers to adapt their pedagogical approach for teaching the new concepts when they knew their learners' mathematical thinking (Makgakga, 2014:4). As learners conjoined 5x + 20, co-researchers not only knew and identified his learners erroneous thinking but went further to scaffold their thinking to demystify the misconception called lack of closure in expressions (Pournara *et al.*, 2016:5). This enactment seemed to confirm Shulman's (1986:10) assertion that teachers must know their learners' misconceptions in order to overcome them. It became crystal clear that analyses of misconceptions helped teachers to understand learners' thinking so as to be able to adjust the ways they engaged with learners. Viewing mathematics problems from learners' perspectives, thus, cognitive congruence, caused co-researchers to become master learners as well as they learnt how to illuminate learners' cognitive process. Co-researchers' vivencia regarding follow-up on learners' misconceptions also affirms that probing develops the humility for individuals to accept that their initial positions were not correct as they discovered their mathematical errors after having made “well thought-out judgement qualified by good reasons” (Makonye & Khanyile, 2015:57). This view is in line with Skemp’s (1978: 9) “relational understanding, by knowing not only what method worked but why” instead of using methods and rules without reasons.

4.3.3 Sufficient utilization of curriculum materials when teaching

In this section the research reports on the impact of utilizing curriculum materials when teaching mathematics. A positive learning environment or ‘learning space’ as referred to in terms of IQMS enables learners to pose questions among themselves (DBE, 2003:2) which in our view resonates with PBL. The PAM document mandates teachers to establish a classroom environment which stimulates positive learning and actively engages learners in the learning process through the use of curriculum materials (see section 2.4.3.1.1). In a PBL classroom learners should be provided with a plethora of learning material to choose from (Li and Du, 2015: 20). From a PBL perspective, both human and material resources improve collaboration, enhance lesson planning and strengthen the relationship among team members (see section 2.4.3.1.1). The research findings also showed that teachers used algebra tiles as a remediation for students who could not grasp mathematics concepts such as distributing property (Hubbard, Beverly, Handrick and Habluetzel, 2013: 4).

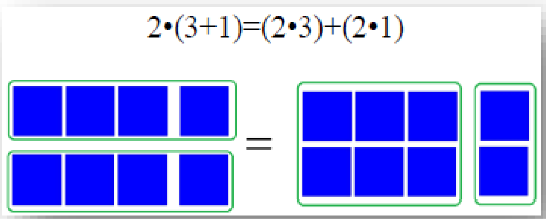


Figure 4.18: Algebra tiles

In the same vein, the literature (Hubbard *et al.*, 2013:4) also attests that “manipulatives can effectively reverse most arithmetic misconceptions” of student teachers before they become full-time teachers (Green, Piel & Flowers, 2008: 241). “In addition to providing a medium for experimentation and discussion, manipulatives can also provide a model or visual for complex concepts" (Ontario Ministry of Education, 2004: 25). According to Shulman (1986: 10) these teaching materials referred to as 'tools of trade' facilitate teaching efforts by exemplifying a particular content to learners and remediate the adequacy of students' accomplishments. Concrete material enables both teacher and learners to have a grounded conversation (Thompson, 1994:566). Teachings aids such as learning material are effective aids to learners' thinking and learning that further reduce the emphasis on mathematical procedures and encourage what Thompson (1994:557) calls "conceptually-oriented instruction". In accordance with Durmus and Karakirik's (2006: 121) view that visual material provides an interactive environment amongst learners, we want to argue that teaching aids enable learners to pose and solve problems while connections between mathematical concepts and operations are formulated.

In line with the good practice mentioned above, it appeared that the co-researchers in this study made convincing strides using manipulatives to improve their wisdom of practice. The following diagram (Figure 4.19) illustrates how Zintle presented the classroom with flash cards that students used to identify required angles.

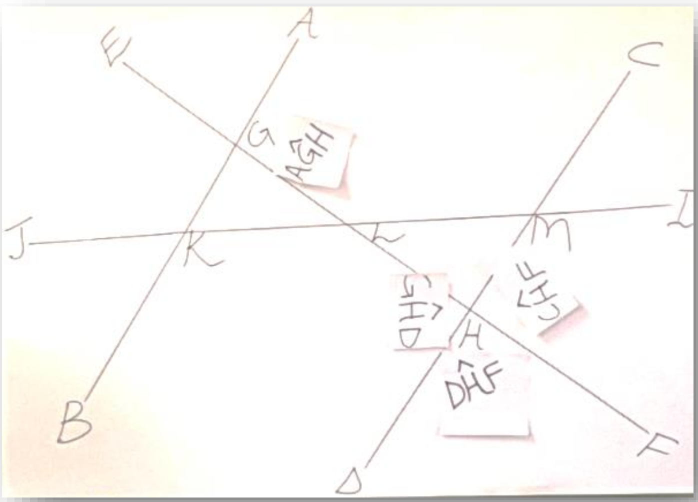


Figure 4.19: Flash cards used as teaching aid

The above picture suggests that co-researchers used visual teaching aids in presenting their lesson albeit in a subtle way. The cap on top of the letter G in angle AGH is an indication of the vertex where the angle is. Apparently, these paper cuttings helped learners to identify the correct angles. For an example, learners were able to place angle DHF on the correct angle. Evidently, these paper cuttings were not only used as manipulatives but were also used as the enabling prompts to help learners to accurately identify and correctly label the required angles.

Ntozine used chart papers to develop examples of congruent triangles (Figure 4.20).

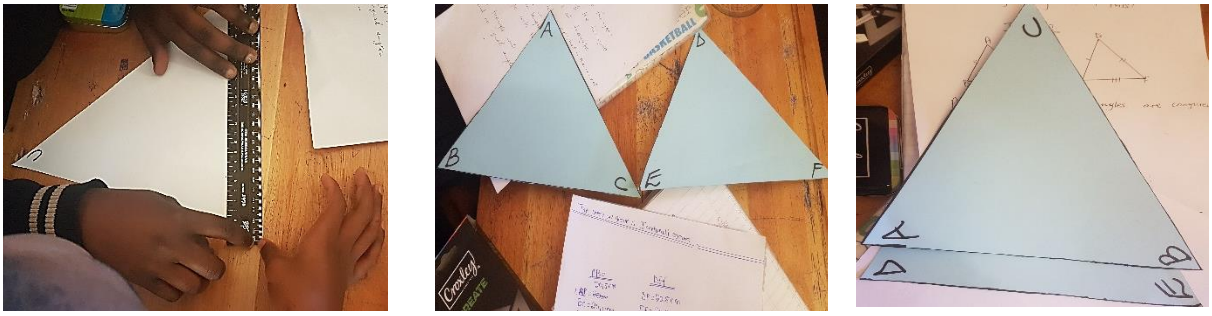


Figure 4.20: Examples of congruent triangles used as teaching aids

Ntozine allowed her students to first measure and record the magnitude of each side of the two triangles given to each group. Later the learners were asked to identify the patterns of their recordings and to observe the relationship of the given triangles. The following learners' recordings (Figure 4.21) demonstrate that the above triangles are equal in every respect and can fit on top of each other without leaving a space or overlapping.

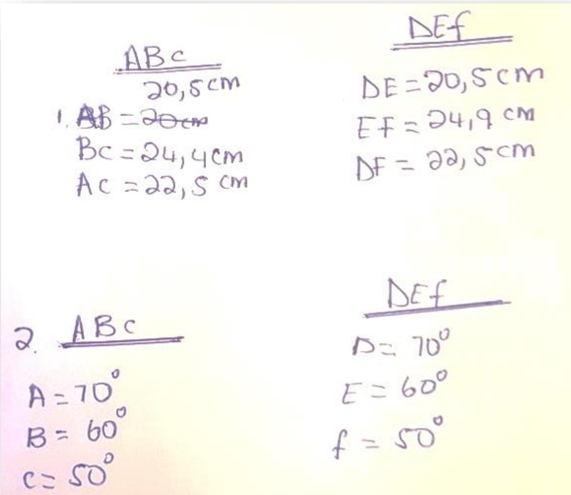


Figure 4.21: Congruency of triangles

The text from learners' recordings illustrates that learners were not first given the minimum requirements of congruent triangles. Instead, they were given manipulatives and the instructions to investigate the phenomenon, namely congruency of triangles. For an example, in triangle ABC, learners first wrote 20 cm as the magnitude of side AB and later changed it to 20,5 cm without the teacher's intervention, but through negotiated meaning as they also recognised the patterns that emerged from their records.

The following is a representation of the reflection we had with the co-researchers and learners who participated in a discussion on what transpired in the class:

> *Researcher: What was different in your class today as we used chart triangles to exemplify congruency?*
>
> *Ntozine: Learners were motivated by the group work as they shared the chart triangles and as result the lesson was nice*
>
> *Researcher: Would you have done it better without teaching aids?*
>
> *Ntozine: No … it becomes very easy with teaching aids, other than drawing triangles … it became easy I have seen that it just got too easy. For an example, next time when you need the minimum requirements that involve angles you just come with triangle with angles that are already measured."*

The above extract depicts an interactive classroom discourse. The chart triangles were manipulated to represent congruency and Ntozine's learners were motivated and ready to defend their solutions. Over and above it seemed that her learners had developed a deeper understanding of congruency. For an example, Moseoa (a learner in Ntozine's class) argued that a triangle that fitted exactly on top of another without overlapping were congruent. That suggested that even if he could be given different minimum requirements for congruency other than side, side, side equal to side, side, side (SSS = SSS) he could perhaps prove the congruency of the triangles he would be given.

Researcher: How would you explain congruency so that your younger sibling could understand it?

Vumile (a learner): When the sides of triangles are equal, those triangles are congruent.

Moseoa (a learner): Congruence is when angles ... em triangles that can exactly fit on top of each other, that are equal.

Evidently, the above images and discussion show that the co-researchers presented mathematics manipulatives for learners to use them as powerful tools which enabled them (learners) to understand the concept of congruency.

Furthermore, Zintle once raised a question in the PBLW that she was unable to help learners identify the correct angles when learners were given a task like in Figure 4.22 below. She had this to say:

"What could be the easiest way that could help learners to correctly identify the required angles?"

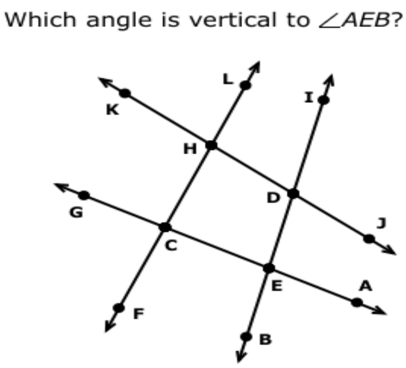


Figure 4.22: Learners required to identify congruent angles

After having been part of the reflections with other co-researchers' lived experiences in our PBLW regarding the use of manipulatives, and other ill-structured problems, Zitle's weaknesses were overcome as she also adopted a particular way to symbolize the angles. With her learners she experimented with the strategy of putting the cap sign or angle sign on top of the middle letter to indicate the vertex of the angle and to make sure that learners were able to identify the correct angles, for example, angle AGH in Figure 4.19. In essence, she managed to overcome her MPCK challenge, namely the inability to help learners comprehend the angle concept. Zintle also argued that manipulatives stimulated motivation of learners in her classroom:

> *"What was different in class as I use teaching aids is that I do not tell them what to do, but just give them problems so that they come up with solutions in groups."*

As was evidenced by Zintle's opinion, giving learners problems to solve had become more common use in the classroom discourse. On the other hand, when learners were asked to reflect about what was happening in their class, Ika (a learner) said:

> *"I liked that learners were disciplined; we were able to answer questions honestly and without being afraid of other learners."*

Manipulatives seemed to have enabled learners to honestly present their answers without fear of being mocked by their peers. In the process of solving given problems, learners took charge of their learning as the use of teaching aids created an interactive environment amongst learners. Manipulatives gave learners something to work with, thus, tools of the trade that exemplified particular content (Shulman, 1986:10). Learners developed confidence to go to the board and paste the given angle paper cuttings on top of the correct angle. Moreover, Ntozine's used of manipulatives created a positive environment as learners tried different ways of physically demonstrating that given triangles were congruent. They engaged in conceptually-oriented conversation (Thompson, 1994:9). While flipping over chart paper triangles, they posed a number of questions about the congruence concept. In line with the assertion that manipulatives provide visuals for complex concepts (Ontario Ministry of Education, 2004: 25), charts provided learners with a connection between the magnitudes on the triangle sides with the meaning of mathematics notation used to indicate corresponding equal sides. Other than focusing on measured corresponding sides, learners were also able to see that congruent triangles had equal corresponding angles.

In our quest to obtain a deeper meaning of the above text we used CDA (Mahlomaholo, 2012b: 51). Ntozine's response that learners worked on their own to resolve given problems revealed that the use of manipulatives helped learners to understand the concept. When her learners were asked how they would explain the meaning of congruence to their peers, they claimed that they would say congruence is when two triangles are equal in every respect and accurately fit on top of each other. These learners' definition of congruence demonstrated that manipulatives helped learners focus on concept understanding and reduced the emphasis on mathematical procedures (cf. Thompson, 1994:9). Moreover, the engagement of learners with manipulatives transformed the classroom discourse from a teacher-centred approach to a democratic engagement. It is also enunciated that linguistic choices revealed a particular ideological stance towards a particular topic (Rashidi & Fam, 2011:112). In the same line of thinking, Zintle claimed that when she used manipulatives, she did not lecture her learners "*but just give them problems so that they come up with solutions in groups".* Her argument, although not overtly put, shows that she usually used a particular conventional teaching method, however, through the use of manipulatives she managed to let go of her positional power and trusted that learners could work out the problems on their own.

As we further juxtaposed the generated data against CER, power was not viewed as a natural phenomenon, but as a political mechanism that is mutable and could be arranged in other ways (Hlalele, 2014: 104). Using manipulatives created an environment where learners were engaged in concept understanding by making connections between manipulatives and the abstract world of mathematics. The power relations in terms of listening to and following the teacher's way inevitably were disturbed. Learners developed hope and confidence in handling mathematics topics without being afraid of being judged as multiple perspectives were considered. Centrally, "CER is changing people's hearts and minds" by liberating them through advocacy for "hope, equality, team spirit and social justice" (Tselane, 2014: 288). In line with CER's liberating agenda, learners became free from fear as Ika (learner that participated in Zitle's class) clearly puts that they were not afraid of other learners. As we listened to what she was not saying, we also understood that she was not afraid of the teachers either, since she had a shield, namely manipulatives to use when defending her ideas. The most outstanding impact of manipulatives was that they enabled the learners to assume a position of power and to be free to raise their ideas and make their voices to be heard.

The findings of this study demonstrate that the use of teaching aids helps learners to develop a deeper insight of mathematics concepts like angles and congruence. In accordance with Green *et al.* (2008:241) manipulatives supported both teachers and learners to reverse misconceptions in terms of angle identification. This affirms Shulman's (1986: 10) view that tools of trade enhance teaching efforts by exemplifying a particular content to learners and remediate the adequacy of learners' accomplishments. However, the use of manipulatives particularly for secondary education level is not immune from critique, as Thompson (1994:3) argued that "[t]o see mathematical ideas in concrete materials can be challenging". The findings of this study reject Thompson's assertion; instead they are in line with the suggestion that manipulatives are likely to help learners make connections between the world of abstract mathematics and the real world (Drews, 2007: 20). Evidently, Ntozine claimed that:

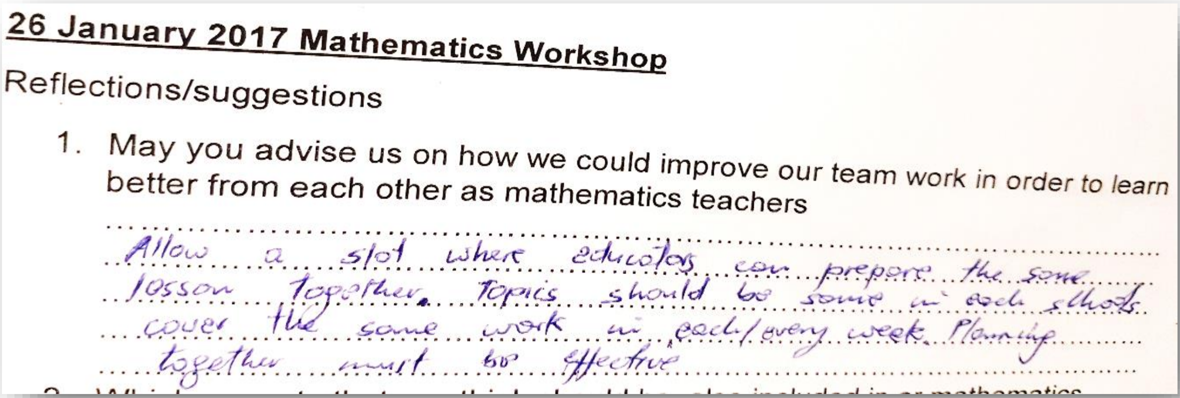
> *"it become very easy with teaching aids, other than drawing triangles".*

Ntozine's claim indicated using models helped learners to understand the concept of congruency other than routinely following the role that when SSS = SSS, therefore such triangles are congruent. This study's findings also confirmed Green *et al.*'s (2008) theorization that Manipulatives reversed both teachers' and learners' misconceptions as it was the case with Zitle in this study. The discussions generated amongst learners also confirmed that manipulative use provided a medium for experimentation and discussion (Ontario Ministry of Education, 2004: 25).

4.3.4 Detailed lesson planning

Lesson planning is prudent to ensure effective teaching and enhancement of MPCK in particular. As indicated (in section 4.2.2) a lesson plan should have a realistic, measurable objective that is time bound. The details for a lesson plan should include clear assessment strategies, learning activities and understanding of learners' knowledge (Milkova, 2012:1). Detailed planning enables teachers to embrace learners' pre-existing knowledge in the lesson through using different resources (see section 2.4.3.2.1). Lesson planning is not limited to the above aspects, but also includes the choice of pedagogical approach, the choice of curriculum materials and anticipated learners' misconceptions (section 4.2.2). It is common knowledge that lesson planning should take into account the policy mandate of the DoE. The policy mandate requires teachers to understand that teaching is an active process (DBE, 2016:18) that involves learners' participation as well. Understanding of learners' prior knowledge serves as a corner stone for lesson planning (Doig & Groves, 2011:81) to guide teachers in developing a series of assessment activities for learners. PBL also resonates with the notion of teaching that starts with problems (Fatokun & Fatokun, 2013: 664 & Hmelo-Silver, 2004: 242). From a PBL view of teaching, lesson planning should integrate assessment with teaching by developing a series of problems that will help learners comprehend mathematical concepts.

The data presented below demonstrate the value that the co-researchers attached to planning the lessons collaboratively.



26 January 2017 Mathematics Workshop

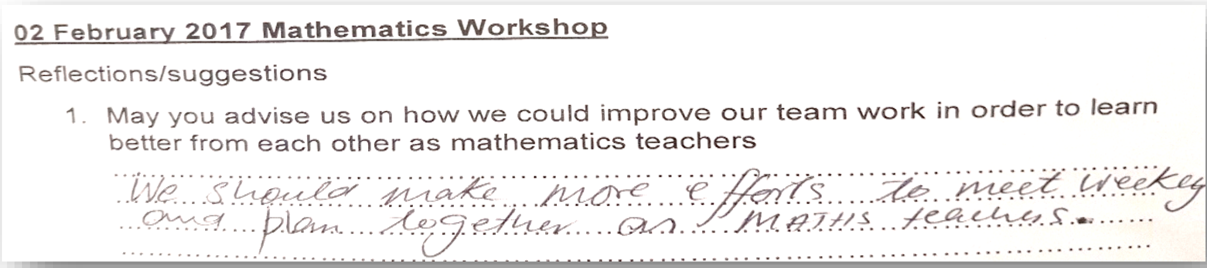
Reflections/suggestions

1. May you advise us on how we could improve our team work in order to learn better from each other as mathematics teachers

Allow a slot where educators can prepare the same lesson together. Topics should be same in each schools cover the same work in each/every week. Planning together must be effective

Figure 4.23: Teacher's reflection

Evidently, from the above extract from co-researchers' individually written reflections, it seemed that Zintle clearly suggested that planning together should be part of the coordinated team's task. Planning mathematics lessons together was a popular view as Jones in the extract below also indicated that weekly meetings should focus on collaborative planning.



02 February 2017 Mathematics Workshop

Reflections/suggestions

1. May you advise us on how we could improve our team work in order to learn better from each other as mathematics teachers

We should make more efforts to meet weekey and plan together as MATHS teachers.

Figure 4.24: Teacher's suggestion re collaborative planning

Drawing from the DBE's lesson plans that seem to cover all the aspects that are required in terms of good practice regarding lesson planning, the coordinated team adopted DBE's lesson plan framework (section 4.2.2), However, individual teachers were encouraged to consider their contexts as they refined the final lesson plans to be presented in their classes.

On different research sites, the following lesson plan was transcribed from Rhulumente's lesson plan. The same lesson plan presentation was also observed in class and what transpired will be discussed in the following section.

Table 4.3: Lesson plan

Date : 22/02/2017
Duration: 2 hours
Grade : 9
Topic : Algebraic Expressions: Multiplication of binomials
Objective: By the end of the lesson learners should be able to determine the product on two binomials.
Resources: Text books, marking pens, flip charts.
Prior Knowledge: laws of exponents, commutative property, associative property, distributive property, properties of integers, like and unlike terms.
Assessments: Understanding of distributive property and associative property.
Activity 1.
Learners would be required to solve the following tasks in groups:

Groups 1 and 2	Groups 2 and 3	Groups 5 and 6
• 5(10) • 6(6 + 4) • 5(x + 4)	• 5(7) • 5(8 + 3) • Y(y + 3)	• 5(11) • 5(3 + 4) • a(a + b)

Activity 2.

Groups 1 and 2	Groups 2 and 3	Groups 5 and 6
• (2 + 3)(4 + 6) • (x + 3)(x + 4)	• (2 + 3)(4 + 3) • (a + 3)(a + 2)	• (2 + 3)(8 + 3) • (a + b)(a + b)

Activity 3.
All groups to work on the following tasks:
Simplify: a. (p + 3q)(p + q)
b. (3x + y)(2x + 5y)
c. (x + 1)(x + 2)
Activity 4. : Consolidation
Learners work will be marked and presented on the board to identify misconception and consolidate the concept of multiplication of binomials

In contrast with what Rhulumente had earlier claimed to be her lesson plan (see section 4.2.2) where she only listed three expressions to be factorized, in the above lesson plan the objective of the lesson is clearly formulated. Specifically, the lesson plan indicates the expected learners' prior knowledge on which to build the new concept. As learners multiply binomials, they are expected to understand the laws of exponents and the distributive property. The teacher's lesson plan clearly demonstrates the intentions of making use of this valuable knowledge in order to establish the learners' ZPD. In Rhulemente's lesson plan, learners are given a series of problems while working in small groups as a form of formative assessment to ascertain the cutting edge of their competences or what Heritage (2010c: online) calls the 'learning zone'. In our view Rhulemente's lesson plan is in line with PBL. Instead of demonstrating how mathematics problems are solved, learners are confronted with mathematics problems. Her activities were sequenced in such way, that they covered the anticipated learners’ misconceptions. For an example, the task of multiplying (a + b)(a + b) did not only require the understanding of laws of exponents and distributive property, but also required the understating of associative property, where learners might not see that multiplying (a x b) is the same as multiplying (b x a). Although, the pedagogical approach was not mentioned, it could be deduced that she had adopted a PBL approach or an LCPA as she consolidated learners’ activities at the end of the learners’ activities on given problems.

Prominently, when this study inquired about the challenges facing mathematics educators, at its initial stages, Falafala was one of the educators who did not have mathematics lesson plans. She argued that she had been teaching the same class over a number of years, and inherently had the lesson plan in her head and apparently saw no need to prepare a lesson. However, the data presented in section 4.3.1 exhibited evidence of a lesson planning session in which she participated. For example, Figure 4.11 illustrated the debates that ensued between Falafala and other co-researchers during their lesson planning session (see section 4.3.1). During their debate they managed to consider mathematics concepts underpinning mathematics algorithms as they developed the best ways of making division by common fractions understandable to learners.

In terms of text as per Rhulumente’s lesson plan she seemed to value her work as reflected by the detailed planned activities. With the help of team members, she developed scaffolds of mathematics problems guided by her learners’ understanding of mathematics concepts. Thoroughly planned lessons helped co-researchers to focus on activities related to mathematics teachability. They developed strategies to make mathematics concepts easily comprehendible for their learners, the case of division by one which is the reason for using a reciprocal. In line with Shulman’s argument, one of the PCK components a teacher should know her learners. Moreover, a shift occurred in teachers’ teaching practice as they now regard collaborative planning as a pivotal component of their teaching. Co-researchers’ arguments clearly suggested that they needed each other for lesson planning each week before they could present their lessons in their various classes. In essence, they started to develop a transformative discursive practice, whereby as a coordinated team it would be strange for them to go to their classes without having met, planned together and shared their lived experiences regarding the presentation of collaboratively planned lessons. In their plans they valued learners' knowledge, since they did not regard learners as empty vessels. Consciously, their positioning as the only source of knowledge was disturbed; hence they were ready to plan series of assessment tasks and problems as part of their teaching strategy other than a demonstration plan of the concept and methodology followed by classwork. The guru mentality was gradually diminishing as reflected by Falafala's statement "*more than we think we know what we do"* (see section 4.4.2). Planning gave them a window to anticipate what would happen in their classroom as they considered a number of factors and in so doing chose the best analogies and curriculum materials to enhance their MPCK.

From Dube's (2016:35) argument, "emancipation comes within a context where there is domination". Both the researcher and researched, thus, co-researcher in our case, become consciously aware of the relationship where one dominates the other. Evidently, Rhulumente's lesson planning, as she began to value her learners' prior knowledge, indicated that she recognised the devastating results of the undemocratic relationship she had had with her learners, hence she was willing to let go. Her lesson plan indicated the patience she had with learners, as she included what could be seen as the most trivial problems for her learners. Closely looking at her enacted stuns, it clearly was in line with CER in terms of giving a voice to the marginalized. Apparently, those learners who had been alienated by a dominating relationship, may now also experience success and further develop courage to tackle more challenging mathematics problems. Lastly, her conscious inclusion of problems which were likely to expose the anticipated learners' misconceptions, with the view to let them explain their thinking seemed to advance democratic values in her mathematics classroom. She thought deeply about aspects and manipulatives to use in order to make mathematics concepts teachable, and gave hope in terms of learners' success.

In closing, this study contributed to the community of scholars by presenting empirical evidence which clearly indicated that lesson planning improved one's MPCK. The co-researchers' lesson plans had clear and measurable objectives and learning activities. Milkova's (2012: 1) asserted that a lesson that addresses objectives for students' learning is likely to be successful. Logically, a successful mathematics lesson cannot be divorced from effective MPCK. The findings of this study showed that as teachers focused on developing ways that would make mathematics concepts more teachable, they in turn got emancipated and acquired skills of not only teaching what they regarded as just rules, but also demystified mathematics concepts, as was the case with Nowele. Furthermore, the findings of this study confirmed earlier research findings that described a lesson plan as a professional space for teachers to think deeply about the content in terms of its teachability (Shen *et al.*, 2007:249). Evidently, from a DBE adopted lesson plan framework (see 4.2.2) and Rhulemente's lesson plan in particular, co-researchers had ample opportunity to think deeply about subject matter and make use of 'vertical knowledge' which, Hauk *et al.* (2014: 26) regarded as a connective relationship of prerequisite topics and potential future topics. Needless to emphasize the adoption of DBE's lesson plan by co-researchers in this study, but what was prominent, was the ability to carefully anticipate learners' misconceptions and use them as foundation for lesson planning. Our findings also confirmed the narrative that sufficient planning enables teachers to embrace learners' pre-existing knowledge in the lesson through using different resources (Qhosola, 2016: 224). In terms of Shulman's missing paradigm, these co-researchers' ability to understand learners' misconceptions as they planned lessons, indicated the development of their MPCK.

4.3.5 Integrated assessment with lesson facilitation

In this section, the study report is devoted to an exploration of pockets of good practice and to juxtapose them with generated data in order to understand the effect of implementing integrated assessment in enhancing MPCK. The philosophical purpose of IQMS is to assess strengths and identify areas for development (DBE, 2003:3). Furthermore, the National Education Policy Act (NEPA) mandates teachers to use detailed records of diagnostic assessment to improve learning programmes (Brunton, 2003: A-49). Teachers are urged to use higher level questioning, problem-based tasks and appropriate use of group-work as teaching strategies (see section 2.4.3.3.1). The literature also submits that data generated from high-quality assessments enable teachers to make useful decisions to appropriately adjust their instructional procedures (Kanjee & Sayed, 2013: 444; Samson & Marongwe, 2013:197; Adediwura, 2015:355). In essence, integrated assessment does not focus on norm referencing but evaluates strengths and weaknesses guiding learners to understand what they should improve (section 2.4.3.3.3). As teachers recognise learners' cognitive tendencies, they consequently improve their pedagogical strategies (see section 2.4.3.3.6). The identification of the learning zone through integrate assessment enables teachers to appropriately develop mathematics lesson plans that could rescue learners from a cul-de-sac in order to realize the learning trajectory (see section 2.4.3.3.6). Teachers manage to unearth what is within the learners' reach when they keep a very close eye on emerging learning and consistently working on ZPD through integrated assessment (Heritage, 2010:8). As postulated by Andrews *et al.* (2014: 14), assessment-embedded instruction apparently develops a discursive practice where one has to justify one's opinion during the classroom discourse. In terms of PBL, teachers should guide students towards solutions when they have questions and not provide direct answers (Li & Du, 2015:20). As learners are presented with a problem first, their solutions are used as the spring board to direct their learning process (Barge, 2010:7).

Our observations as we collectively implemented the strategy adopted in our session of PBLW indicated that co-researchers broke away from a traditional way of assessment while gradually moving towards integrating assessment with instruction. Moreover, assessment is interwoven with other mathematics activities and our classroom observation cut across every aspect of mathematical teaching that emerged, for example, in section 4.3.4 where we earlier presented and analysed generated data regarding the use of manipulatives. When Zintle was requested by the team to explain in her views how her class was different when she used manipulatives, she responded:

> *"What was different in class as I use teaching aids is that I do not tell them what to do, but just give them problems so that they come up with solutions in groups."*

As evidenced by Zintle's words, giving learners problems to solve became more pronounced in her classroom discourse. Apart from elucidating the effectiveness of manipulatives in helping learners to comprehend abstract concepts, her words revealed that she had created an environment where she could give her learners problems to work on. She made manipulatives available in her class, gave problem to her learners and allowed them to choose and use available thinking tools at their disposal. Giving problems to learners while lesson presentation is underway, suggests assessment embedded in instruction.

The following extract exhibits how Njovane's class also integrated assessment with instruction in his classroom.

> *Njovane: Tell the person next to you any triangle you know, you are given 2 minutes*
>
> *Group one : Isosceles triangle*
>
> *Nosipho from Group 2: Que … lateral triangle*
>
> *Njovane: Is there any other triangle?*
>
> *Sifiso: : Quadrilateral triangle (the class laughed)*
>
> *Themba : Equilateral triangle*
>
> *Group of learners in a chorus: Equilateral triangle……*

The above extract indicates that Njovane started with the assessment task for learners to do as way of establishing their prior knowledge. Learners were given space to think, negotiate and defend their views to their peers. The statement that says, "*Tell the person next to you*", suggested that learners were working in groups to solve the task at hand. Putting it differently, learners had to first share their thinking with others in the group before it could be presented as a solution. At a group level, any idea about or answer to the question presented was interrogated by group members where one was expected to explain why a particular response to the task would stand the test of time. It was clear for Njovane that his class was not sure about types of triangles. However, the discussion that ensued in terms of the properties of triangles strengthened their knowledge and remediated the misconceptions of those who were not certain. Moreover, the lesson plan was about the construction of special triangles, such as equilateral triangles. Following (Figure 4.25) is the learners' work after having been given a problem to accurately construct a special triangle without using a protractor.

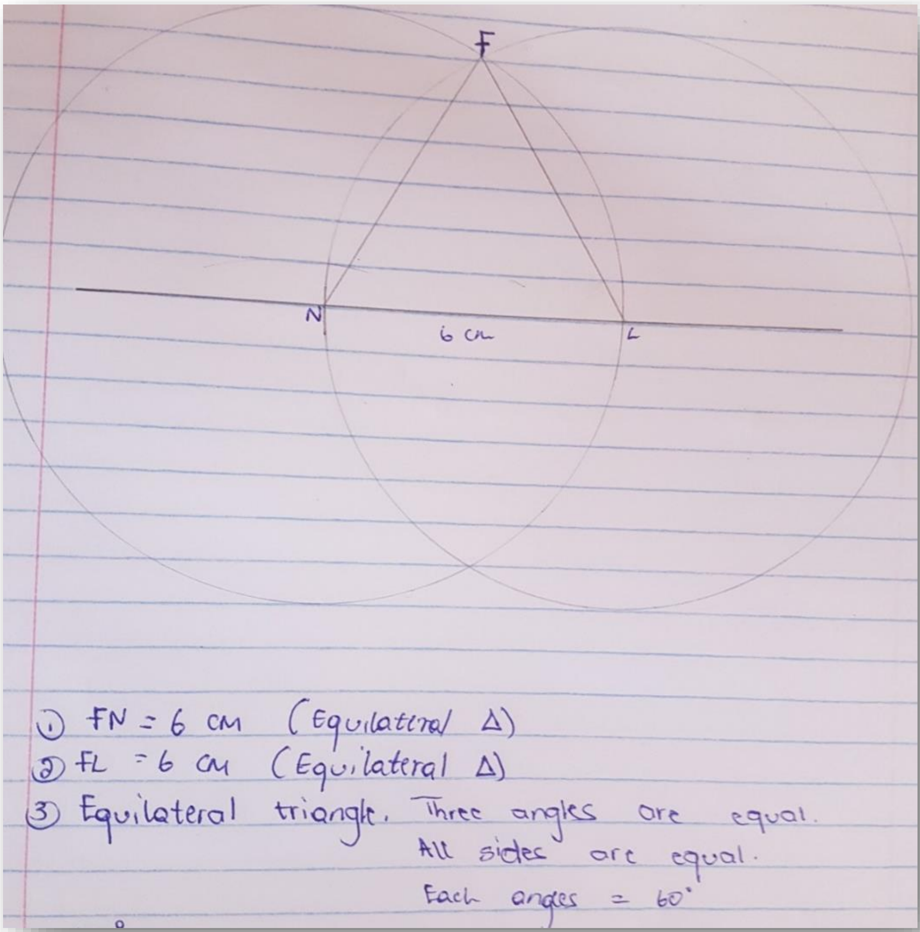


Figure 4.25: Construction of a special triangle

As reflected in the figure above, learners were given a work sheet with step by step instructions to accurately construct a special triangle. Njovane would only engage one group at a time, either on request by the group members or when he suspected that a particular group seemed to have a problem in terms of the process skills. As reflected above these learners' work suggested that they understood mathematical arguments involved when one needs to prove that triangle FNL is an equilateral triangle, although they gave equilateral triangle as the reason for saying side FN = 6 cm instead of arguing that they measured the sides. Nonetheless, integrated assessment helped Njovane to identify areas for development.

On the other side, the data that are presented in section 4.3.6 regarding how Rhulumente embraced the learner-centred approach also showed that Rhulumente used integrated assessment with instruction in her lesson presentation. As we observed teachers at the research sites, we could not only observe one particular aspect (solution) at a time. However, we analysed each solution separately even if it was simultaneously enacted in various aspects regarding mathematics teaching. The following extract indicates a shift in Rhulumente's beliefs regarding the teaching of mathematics:

> *Researcher: "Do you think your learners understood your lesson?"*
>
> *Rhulumente: "Yes (with confidence) … once learners are willing to solve mathematics problems usually it is that they understand, once they do not understand you would see, they reluctantly try and become lazy. I am confident with my learners, they surprised me, and they just worked out problems on their own. It looks like they need you to just throw them with mathematics problems."*

From the extract above, it seems that Rhulumente's learners were willing to solve problems. In accordance with Aalborg's opinion about PBL where the problem is viewed as the starting point in directing learning (Barge, 2010:7), it is evident from the extracts that the culture of starting a lesson with problems had been adopted by the research team members. Rhulumente's confidence (body language) also showed that the new teaching practice with which she had experimented, namely assessment integrated with lesson presentation apparently improved learners' mathematical understanding, in line with the notion presented by Umugiraneza *et al.* (2017: 3). It was a humanizing experience for her to understand that it was possible to work together with her learners to untangle mathematics problems without first presenting a demonstration and assess later.

Furthermore, our discussion with Jones after her lesson suggested that she sometimes used the traditional way of assessment although she was part of our community of practice.

> *Researcher: As you say that you sometimes present your lesson and later assess your learners because of time constraints, comparatively speaking which one you think works better for your learners, between assessing later and integrating assessment with instruction?*
>
> *Jones: I think it is the one that we used today*

Researcher: Yah…..

Jones: E ... m when learners are given the problem without telling them how to do it, they come up with their ways and you only get there as the teacher to help them, but it is time consuming.

From the above extract, it appears that Jones attested that integrated assessment improved the effectiveness of her teaching despite her complaining about time constraint. Learners, in this instance, came with their own ways and the teacher had to appropriately adjust the intervention to provide required help.

As we further juxtaposed the data generated with good practice, we engaged in a meta-cognitive process (section 3.2.4.4) with co-researchers reflecting and discussing our intersubjective interpretations of observations and lived experiences at the research sites. Zintle's words that:

"just give them problems so that they come up with solutions in groups"

is an indication that her lesson presentation did not separate assessment from instruction. In essence, her instructional approach started with assessing the level of learners' competences in order to focus her instructional approach on learners' misconceptions. In line with the DBE's (2011:155) policy mandates, her lesson presentation did not only integrate assessment with instruction, but used the feedback to inform her planning and teaching. It appeared that learners benefited from the integration of assessment with teaching as co-researchers confidently claimed that learners came up with their solutions. In line with (Graue, 1993:281) these benefits to learners and most effective results emanated from a more bound connection between assessment and instruction as was the case in the above extract (Figure: 4. 24) regarding Njovane's learners. Learners were able to use resources at their disposal, such as a text book to get guidance in terms of how to construct a special triangle without using a protractor. In line with Barge's (2010:7) findings, learners were presented with a problem first as part of the lesson presentation, which, according to Gijbels *et al.* (2005: 29), provides a stimulus for learning. From Rhulumente's response that as teacher *"they need you to throw them with mathematics problems*" also indicated some level of emancipation of the teachers who were now confident to assess both mathematical concepts and procedures.

Moreover, the view that one needs to help learners when they ask questions without giving them the direct answer (Li & Du, 2015: 20) emerged from co-researchers' classes as they only responded through enabling prompts to solve complex problems (Russo, 2016:8; Sullivan *et al.*, 2015:54). Furthermore, the integrated assessment that had been exhibited by co-researchers was part of their lesson planning. The literature proclaims that integrated assessment enriches the teacher with the anticipated learners' misconceptions (Gearhart and Saxe, 204: 309). As presented above, Njovane anticipated that his learners might have preconceptions and conceptions regarding properties of special triangles such as an equilateral triangle. With the understanding that his learners functioned at different cognitive levels, they were given an opportunity to learn from others as they worked as groups. As co-researchers patiently trying to understand the learners' solutions and ways, they themselves enhanced their MPCK.

From a CDA perspective, text reveals both opaque motivations and politics underlying arguments for or against a particular statement or value (Mogashoa, 2014:105). Jones's argument for integrated assessment revealed her consciousness about how she and her learners benefited from integrated assessment. As Mahlomaholo (2013:321) viewed research from CDA, PAR and CER perspectives "as an act of healing", co-researchers experienced the democratization of their classroom discourse as they embraced integrated assessment with instruction. In essence, the discursive practice's manipulative nature gets exposed by CDA (Tenorio, 2011:188). Apparently, it had been a norm for teachers to rush for time at the expense of learners. However, since we engaged in integrating assessment with instruction, discursive practice appeared to be gradually changing. It appeared that it was not easy for Jones to let go of her discursive practice as she claimed that assessment embedded instruction "*is time consuming".* Notwithstanding Jones's claims about assessment integrated with lesson presentation, teachers presented learners with problems as their intention was not to judge learners but rather to use assessment as the starting point to create a space for the discussion of multi-perspectives. In the process learners were allowed to explain their solutions to others in small groups. As the groups presented and defended their views in front of the whole class, the power relations tended to be democratic as the teachers also tried to understand learners' thinking.

From a CER perspective the research process “makes the research team even more resolved to craft alternative ways to show how powerful they are” regardless of past exclusion and marginalisation for a number of centuries (Mahlomaholo, 2013c:322). In line with Mahlomaholo (2013:322), the co-researchers in this study gradually embraced the values of justice and democracy as they allowed learners to come with their own ways of solving given problems. Learners’ views and preconceptions were used as the starting point for teaching. Learners’ experience of defending their views made them willing to handle any given problem. Evidently, from Rhulumente’s words, learners were willing to solve mathematics problems as they were no longer marginalised in the classroom discourse. Furthermore, CER advocates that knowledge and understanding be reached through grounding facts in their historical context (Lunn, 2009: 937). Co-researchers were able to compare the new experience of integrated assessment with their historical context, where teachers had to rush to finish the prescribed content and then assess separately or later. As they started to embrace the new approach, they felt empowered as they became more knowledgeable in terms of their learners’ levels of concept understanding.

The findings of this study suggest that lesson planning must integrate assessment with instruction. Our findings are in line with the DBE (2015: 4) policy mandates. Njovane was able to identify that his class was not sure about properties of special triangles through integrating assessment with instruction. These findings confirmed the view that the purpose of assessment is to continuously collect data on learners’ performance that would be utilized to improve learning (DBE, 2011:155). The confidence exhibited by Rhulumente as she gave them a series of problems affirmed the literature theorization that learners benefit when their teachers engage in an ongoing assessment (Gearhart & Saxe, 2004:309). Integrated assessment does not only establish learners’ cognitive levels but also unearth learners’ misconceptions which are the starting point for lesson presentation. Our findings also affirmed PBL stunts of presenting problems first as a starting point for teaching (Barge, 2010:7), as Zintle and Rhulumente claimed that learners should be thrown with problems. This view confirms the literature findings that a more bound connection between assessment and instruction gives the most effective results (Graue, 1993:281). Njovane’s learners, once they understood the properties of special triangles through integrated assessment, were able to experience success in construction of an equilateral triangle without using a protractor (see Figure 4.22).

4.3.6 Learner-centred pedagogical approach (LCPA)

In this section the report deals with the impact of a LCPA in emancipating teachers in terms of PCK enhancement. According to the DBE (2011:4) LCPA allows learners the opportunity to develop and employ critical thinking skills. As a radical paradigm shift from a traditional banking concept to LCPA, learners becoming active agents engaged in construction of their knowledge (Zain *et al.,* 2012:319). Through LCPA, teachers are able to change a mathematics classroom environment to a lively experience where learners would make meaningful connections between their learning experiences and the real world (Walters *et al.,* 2014: 2). In this enabling environment, learners are able to communicate their common sense-based mathematics experiences while enabling teachers to draw from a 'mathematics tool box' appropriate strategies to address the demonstrated learners' needs (see section 2.4.4.7). PBL theory views learner-centredness as an instructional method that utilizes real problems as a primary pathway for learning (Ramsay & Sorrel, 2006:2). According to this approach, learners work in small groups of five to eight or nine learners (Barrows, 1996: 5). In essence, PBL is a pedagogical approach based on problems (Rui *et al.*, 2015:223). PBL is one of many forms of active learning that give learners the chance to exhaust their capabilities in solving a problem without a teacher's assistance. Furthermore, the implementation of PBL in mathematics teaching increased problem-solving skills, decision making and reasoning processes (Erickson, 1999: 520). According to Savery (2006: 12) as a learner-centred approach, PBL has the ability to empower learners to apply knowledge and skills to develop solutions to a given problem.

Our lesson observations on the research sites revealed that co-researchers adopted a learner-centred pedagogical approach to present their lessons. Ntozine engaged learners in accurate construction of 90^0 and 45^0 and the following conversation transpired after the lesson.

> *Researcher: "I would like to know if there is anything different from your lesson presentation today, if any, since you are part our team?*

Ntozine: "What is different is that I am not telling them anything, I am giving them, thus, leaners to work on the mathematics tasks on their own, so that they come up with conclusion/solution, that is what is different from what I have been doing."

The researcher: "In your view, how do your learners take it when you look at them?"

Ntozine: "I could see that they understood construction - this one of 90^0, but they seem not to understand construction of 45^0, maybe they were already tired, probably, I think I should have done one angle today and not mix them".

Other than confirming her implementation of LCPA like others (Zintle and Jones) as discussed in section 4.3.5, she gave learners problems to solve without first demonstrating what she used to do before the intervention of this study, her focus was on learners' competences. Evidently, Ntozine managed to identify that learners did not do well in constructing 45^0 as compared to 90^0. She did not blame learners, but understood that she should not have mixed the accurate construction of these angles in the same lesson presentation.

On the other side, Rhulumente had divided her class into six small groups, thus group one to group six. She gave three different problems to the class in which two groups were given the same problem. The following figures represented how the groups were divided.

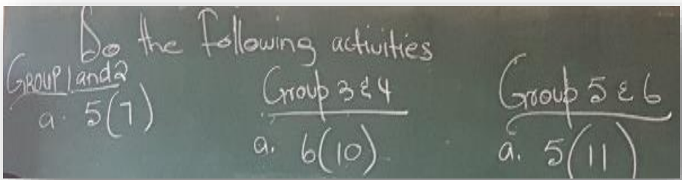


Figure 4.26: Learners' group work

The above tasks were introductory tasks intended to elucidate prior knowledge and to remind learners about how to solve mathematics problems.

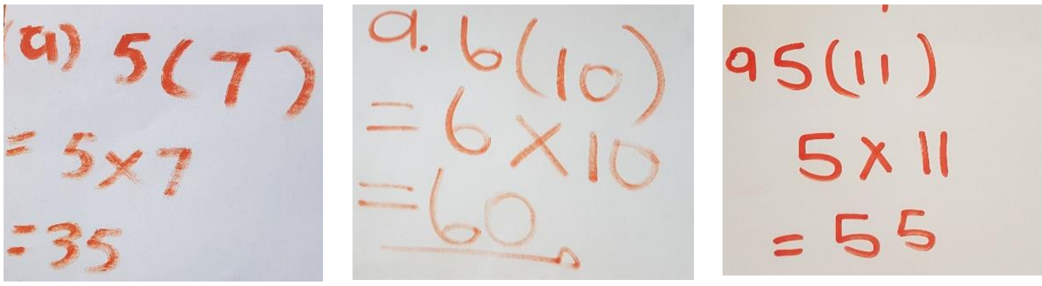


Figure 4.27: Problems learners had to solve

All the groups apparently understood the meaning of brackets and as a result they got the solutions correct. Consequently, the teacher expanded the numbers inside the brackets and allowed learners to work as groups.

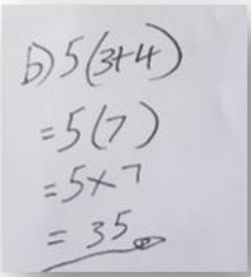
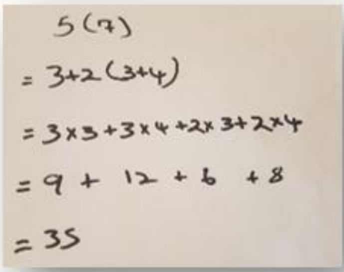
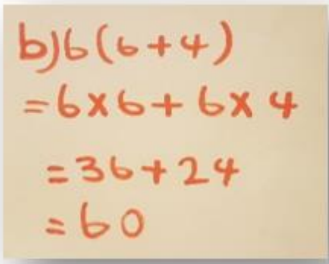
		

Figure 4.28: Group 4	**Figure 4.29: Group 3**	**Figure 4.30: Group 6**

Figure 4.28: How learners solved the equation

Group four on the left added (3+4) inside the brackets before they multiplied by five. Apparently group four remembered the BODMAS rule and as a result they went back to the original problem, thus 5(7) before they could solve the given task. However, group three in the middle and group six on the right, managed to link the present task with the previous one that they had recently done (Figure 4.27), hence they multiplied every number inside the brackets by every number outside the brackets and added the products. It seemed that groups three and six understood that the number outside the brackets should be distributed to every number inside the brackets. After having consolidated each group's work, the groups that had completed the task were given new problems.

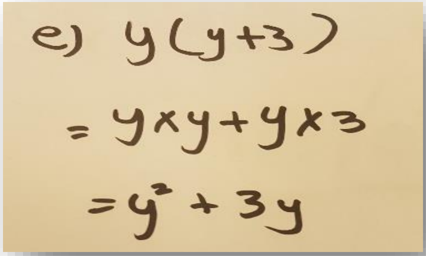


Figure 4.29: Group 3

Learners became motivated to engage in active learning, especially as they were not necessarily told which method to follow, but to only explain how they have worked out the solution. When the groups had finished the given problem, they would demand the next problem from the teacher. The groups did not bother about confirmation from the teacher in terms of their task being marked before they could engage on the next problem. The above activities indicated that when the teacher saw that her class was confident with the understanding of the distribution property, she went further to assess learners' understanding of exponents which are prerequisite knowledge needed to be applied in the multiplication of binomials. The following figure demonstrates how Rhulumente consolidated the problems that involved the understanding of distribution property. Her methodology seemed to be unorthodox; she did not follow the conventional ways like BODMAS. She clearly emphasized the understanding of distribution property of multiplication as a skill required to solve problems regarding the multiplication of binomials.

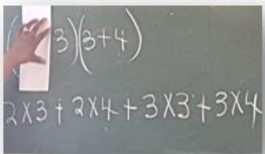
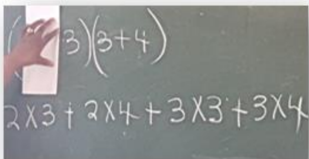
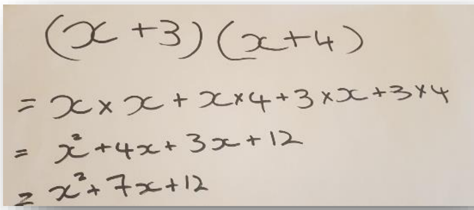
		
Figure 4.32: Distributive property	**Figure 4.33: Distributive property**	**Figure 4.34: Distributive property**

Figure 4.30: Rhulumente's method

As she covered 3 with a paper, she wanted her class to focus on multiplying every number inside the brackets by 2. Rhulumente did the same when she covered 2 in the first group of brackets. Over and above, learners had already mustered the distribution property of multiplication in the previous tasks that were given. Her patience with her learners as she consolidated the task was an indication of how important it was for learners to develop a deep insight about the concept. Her approach reduced the memorization of procedures like FOIL, which means that when you multiply binomials, you need to start with the multiplication of First terms, followed by Outside terms, then Inside terms and Last terms of the binomials. After the above mentioned built up, she gave the rest of the class the same problem while learners were still working in groups. Figure 4.35 shows how one of the groups actually worked out the problem.



$(x+3)(x+4)$

$= x \times x + x \times 4 + 3 \times x + 3 \times 4$

$= x^2 + 4x + 3x + 12$

$= x^2 + 7x + 12$

Figure 4.31: One group's solution for a problem

The learners' work shown in Figure 4.31 clearly demonstrated that Rhulumente's class understood the application of exponents and distributive property of multiplication when multiplying the binomials. When we juxtaposed the objective of her lesson with the empirical evidence at our disposal, we were convinced that she had achieved the lesson's objective.

Finally, we had a short session with three learners and Rhulumente to reflect on the lesson presentation.

> *Researcher: What was different from today's lesson as compared to other lessons?*
>
> *Setleko: The difference today, em … when the teacher teaches us, sometimes I do not understand and do not quite see how it is done. What we were doing today, it was for the first time for me to discover that it was*

so easy to solve mathematics problems when we work as a group as compared to when I worked alone.

Sponono: It is that when you have a certain opinion about how you see a mathematics problem you were allowed to express it.

Nodada: What I liked today, is that I know and I am able to tell others what was happening, I have heard what was taught and as I left the class, I understand the multiplication of binomials.

The argument raised by these learner participants illustrated a shift in how they were usually taught. Rhulumente also presented her experiences regarding her lesson which was started with mathematics problems rather than with a formal presentation and demonstration of mathematics concepts to be followed by assessment.

Researcher: I really enjoyed the lesson presentation, especially that there was a lesson plan that was first discussed by a team, we would like to know, what was different from today's lesson as compared to other lessons?

Rhulumente: What is different today, hm … the learners were more active and learners are the ones who were busy doing their work calculating some activities. I think the group working is so important to them.

Researcher: What do you think caused them to be more active?

Rhulumente: I think the approach ….

Researcher: Could you please elaborate on the approach?….

Rhulumente: Um … where by … Eh … I think the approach was the best because when we started our lesson, we first used what they are familiar with, that is the numbers before we included the variables ….

Researcher: What did you like or dislike in today's lesson presentation?

Rhulumente: Uhm….today, I liked everything… team work builds you as teacher, working together builds you and you are able to see where your mistakes are, you are able to see if there is a skill that you are lacking somewhere somehow. The team really develops you.

Researcher: Do you think your learners understood your lesson?

Rhulumente: Yes (with confidence) … once learners are willing to solve mathematics problems usually it is that they understand, once they do not *understand you would see, they reluctantly try and become lazy. I am confident with my learners, they surprised me, and they just worked out problems on their own. It looks like they need you to just throw them with mathematics problems.*

From Rhulumente's chalkboard summary, it was evident that she had grouped her learners into six groups. In line with the LCPA, the literature advocates for grouping of learners into manageable groups of five to eight or nine learners (Barrows, 1996: 5). Moreover, her pedagogical approach focused on what learners knew in order to guide them beyond the ZPD. This approach resonates with Zain *et al.*'s (2012: 319) conviction, according to which learners are viewed as active agents in the construction of knowledge. From our observation, the co-researcher used a series of problems as a basis for lesson presentation which in our view echoes the literature in which PBL is regarded as an approach which is based on problems (Rui *et al.*, 2015:223). Apparently, learners' critical thinking was encouraged by the co-researcher's instructional approach, as the DBE (2011:4) claims that LCPA provides the opportunity for learners to employ their critical thinking. As evidenced from Rhulumente's responses when she argued that with this approach, she was surprised by how learners solved problems and argued that she was now confident to just give them more mathematics problems. Learners, as participants in this study also confirmed that they welcomed the new experience whereby everyone was permitted to voice his/her views or opinions about how the problems should be solved in a small group. Over and above, learners seemed to also had experienced a certain level of success. It appears that teachers were able to draw from what Shulman calls a tool box with appropriate strategies to address learners' demonstrated needs. From learners' responses it seemed that LCPA changed the mathematics classroom environment to a lively experience where learners would make meaningful connections between their learning experiences and the real world, as Walters *et al.* (2014: 2) philosophized.

As we further looked at what transpired at the research sites through CDA, Nodada's reflection about his experience of LCPA suggests that the classroom discourse presented him with an opportunity to share his understanding of multiplication of binomials with others. In terms of a CDA text choices show a particular ideological stance towards a particular topic (Rashidi & Fam, 2011: 112). Nodada's choice of words shows that he felt a sense of belonging and involvement under LCPA environment. In line with CDA's position on unearthing a deeper meaning of text, we want to advance that Sponono's utterances suggested that she seized the opportunity to raise her opinions on how to solve mathematics problems without fear. It appears that Ntozine also managed to identify the learning zone for her learners while she self-reflected on both effectiveness and inefficiency of the strategy she employed, hence she argued that she should not have mixed the two angles in the same lesson presentation. In accordance with Turhan and Okan (2017: 220), the co-researchers appeared to have adopted a more humanistic view of teaching, "open to learning new things from their students, giving importance to their students' emotions and individual differences".

As CDA draws attention to power imbalances and injustices, hoping to influence people to take corrective actions (Fairclough, 1992:36), we could confidently argue that in Rhulumente' class, the dominating power relations had been denaturalized. The perception of teachers being the sole authority in the classroom had been altered and the relationships between learners and teachers were now calmer and non-antagonistic (Turhan & Okan, 2017:220-221). That learners' voices were no longer muffled, suggested that the class had adopted a new discursive practice under a LCPA. The turn taking was no longer controlled by the teacher, but it depended on the group members as they tackled the given problems.

Furthermore, Rhulumente's interaction with learners in group by group without stopping other groups suggested that she did not wish to embarrass them by displaying to the whole class that they did not know, but wished to establish what happened. She respected their ideas and wished to be taken through by the group in terms of their thinking as they strived to resolve the problem. She consciously considered the notion that she might have deficiency on some skills required to help learners in relational understanding of mathematics concepts. Rhulumente surely moved from her powerful position and as a consequence disturbed the social structure as she tried to understand her learners' thinking. Instead, she became a learner as well in an attempt to empower herself in terms of how to understand learners' underlying thinking in order to use it to enhance her pedagogical approach. She moved out of the old tradition of teaching and embraced what learners knew, other than the idea of transmitting knowledge to learners. Furthermore, it is enunciated that the spoken words are forms of social practice and users of language "may enact, confirm or challenge more comprehensive social and political structures and institutions" (van Dijk, 1997:30). In line with van Dijk's view, Rhulumente's argument about the LCPA indicates that she developed confidence as it allowed her to focus on what learners knew or were supposed to know.

We further viewed the implementation of the LCPA approach through a CER lens. CER creates conditions to subvert distorted consciousness (Mahlomaholo, 2009:224) and "provides a much needed paradigmatic change in the world of unjust society" (Nkoane, 2010:112-113). The lesson observations presented evidence which suggested paradigmatic change as co-researchers consciously managed to let go of the teacher-centred approach. The unjust experiences of learners such as being in the class but unable to understand mathematics concepts presented, due to institutionalized power domination were changed. In terms of PAR knowledge production occurs when people tell stories based on subjective accounts and interpretations of co-researchers' lived experiences (see 3.2.6). It appeared that justice was served as learners confidently expressed their feeling of satisfaction regarding their vivencia in LCPA. For example, the following was a reflection by one learner that participated in this study:

> *Setleko: "It was for the first time for me to discover that it was so easy to solve mathematics problems when we work as group as compared to when I worked alone".*

Moreover, in an LCPA, teachers seemed to respect learners' ideas and allowed for a democratic process for everyone to advance and negotiate one's ideas. In essence, PBL as a learner-centred approach allowed an opportunity for social structure to be challenged to enhance democratic participation. On the other hand, this approach empowered both teachers and learners that participated in this study. Rhulumente attested that this approach built and professionally developed teachers, especially when they worked as a team. Evidently, from co-researchers' lived experiences we can safely claim that LCPA contributed to CER's empowering agenda.

The findings of this research reveal that LCPA changes the classroom discourse from focusing on what teachers do to how actively learners learn. Our findings confirmed that learners under this approach become active agents in knowledge construction (Zain *et al.*, 2012: 319). As co-researchers grouped learners into small groups, learners in turn employed critical thinking in solving given mathematics problems. The co-researchers worked on challenging learners' decision-making and reasoning processes. The findings of this study also will advance the literature debate that as an LCPA, PBL empowers learners to apply their knowledge in solving problems (Savery, 2006:12). As attested by the co-researchers, this approach empowered teachers as well regarding their classroom practice, as evidenced in Mrs Rhulemente's reflections where she displayed confidence about the success of her lesson in terms of achieving its objective. The success of this approach implicitly suggests that the teachers' MPCK was enhanced as co-researchers overtly put it that they got emancipated and professionally developed. Finally, LCPA, allowed intersubjective views and granted the co-researchers an opportunity to develop confidence, and encouraged learners to not only use conventional procedures and algorithms when solving mathematics problems.

4.3.7 Understanding of mathematics content for teaching

This section explores the development of MCKT as component of the strategy to enhance MPCK using PBL. In emphasizing conceptual knowledge, education policy demands mathematics teachers to use their understanding of reciprocal relationships in dividing common fractions (see section 2.4.5.2). Teachers have to unpack algorithms, and make particular content features visible and comprehensible to learners through justification of why we invert and multiply when we divide by a fraction (Ball *et al.*, 2008:400). The process of respectful professional development should allow teachers to determine their learning trajectories focusing on key problems of instructional practice, strengthening MCKT and recognition of learners' misconceptions (see section 2.4.5.5). MCKT development enables teachers to decompress mathematical concepts, skills, and procedures, while connecting mathematical ideas within and across mathematical domains (Ball & Bass, 2003). Teachers with sound MCKT are able to teach to the understanding of the learners (see section 2.4.5.7) through providing learners with opportunities to learn how mathematics theories are derived (Hmelo-Silver & Barrows, 2006:23). In terms of mathematics integrity, teachers should support every mathematics assertion by reasoning and present mathematics concepts in a coherent way (Wu, 2018:17-18).

Our onsite observations and collaborative lesson planning sessions revealed that co-researchers relatively developed MCKT through participation in PBLW as evident in section 4.3.1. Co-researchers attended PBLW, where, other than the presentation of planned topics, co-researchers found space to raise their lived experiences and problems in terms of teaching practice. Other than PBLW, lesson planning meetings with two or three coordinated team members from the same school or neighbouring schools are used as cells to enhance MPCK.

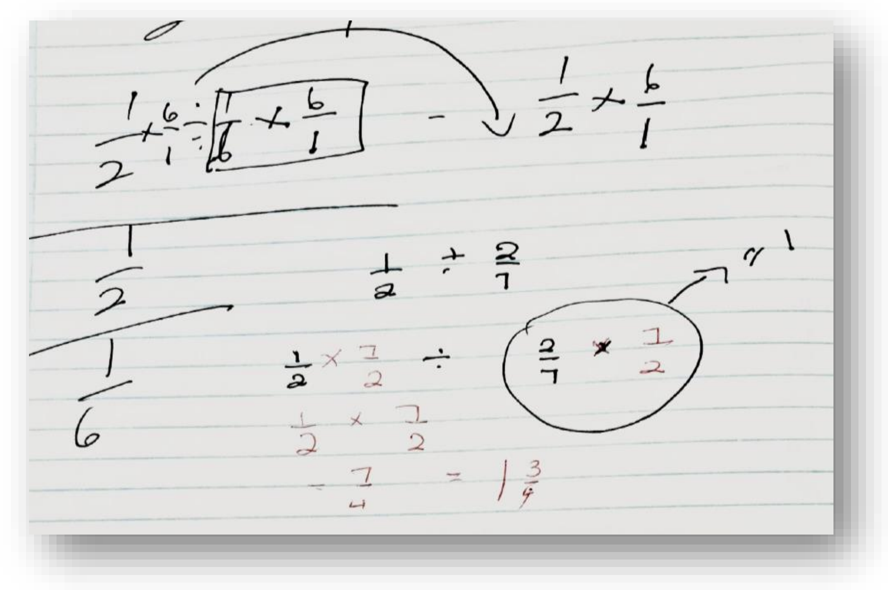


Figure 4.32: Teacher planning for a lesson

The above figure, as also reflected in section 4.3.1, indicates the interaction of co-researchers as they planned a lesson together. Nowele also proposed that learners would understand the process better if the fractions were presented as $\frac{1/2}{1/6}$ as compared to the following representation, namely $\frac{1}{2} \div \frac{1}{6}$. She further explained why the former representations would make sense to learners as compared to the latter.

> *"Learners would be able to see that the fraction $\frac{1}{6}$ is a divisor and is the one that needs to be eliminated through use of reciprocal".*

She argued that learners would better see that one sixth is a denominator that needs to be eliminated through multiplication by its reciprocal. Evidently, planning enabled co-researchers to choose the best forms of representation and focus on mathematics concepts teachability. Observable from the above is that the mathematics concept of division by fraction is viewed in line with what Hung *et al.* (2008:494) call cognitive congruence, as teachers consciously try to express the concepts in line with learners' language and thinking.

During the reflection session about our experience regarding team planning, Nowele submitted that:

> *"We gained something, we were taking it for granted that it is just a rule, if you divide by fraction you just multiply, eh, you just change the division to a multiplication sign, as we work together, we gained a lot in terms where it starts."*

As extrapolated form the above extracts, both Nowele and Falafala developed their own thinking about how to make mathematics concepts better understood by their learners. According to Nowele, collaborative lesson planning helped her to understand the genesis of what seems to be literally flipping the fraction upside down and changing the division sign to multiplication sign. Before her participation in a collaborative lesson planning exercise, she thought what seems to be changing dominator to be a numerator was just a rule.

In addition, Falafala argued that:

> *"You can find out, maybe you come out with an easy way to help learners understand mathematics concepts ... like it is much easier to divide anything by one."*

Falafala's reflection on collective lesson planning as a component of enhancing MPCK exhibited that the process helped to choose the forms of concept representation. The use of the reciprocal for her was not just a mathematics routine, but a purposeful endeavour to decompress mathematics concepts so that learners could see and understand mathematics reasoning underpinning mathematics algorithms and mathematics concepts.

We have earlier indicated that the developed components in terms of the strategy to enhance MPCK using PBL are not independent from each other, they are rather interwoven. For example, the data generated from classroom observation exhibited more than one component at times. Classroom observations also demonstrated a certain level of teachers' mastery of MCKT. Rhulumente meticulously decompressed the distribution property concept of multiplication as reflected in Figure 4.29 and Figure 4.30 (see section 4.3.6). In Rulumente's class, learners presented $\mathbf{a^2 + ab + ba + b^2}$ as their final answer as reflected in Figure 4.13 (see section 4.3.2). Apparently, they did not realize that (**ab)** is the same as (**ba)** in terms of the associative property of multiplication. Furthermore, when learners could not recognise like terms when they were given a task regarding multiplication of binomials, for example, **(ab**) and (**ba),** Rhulument did not opt to reteach, but provided cognitive scaffolds through the use of enabling prompts. The enabling prompts resulted in a 'wow' effect on learners as they finally realized the concept of associative property in multiplication (see section 4.3.2). The teacher seemed to have understood her learners' ZPD, thus, the level they could achieve without help and developed appropriate scaffolds to help them get to the learning trajectory.

Nowele's classroom observation also exhibited a change of teaching practice after having participated in the programme of PBLW and collaborative lesson planning. She gave learners examples of numbers divided by one and consolidated the task as follows:

> *Nowele: "Divide the number, the number divided by one, you get that number, if any number is divided by one the answer is still that number…so…, ok…, If we say a whole number, you divide a whole number by one the answer is that number, nothing changes even if it is not a whole number, even if it is a fraction things are the same…if you divide by one any number, the answer is that number."*

The above excerpt illustrates the effort made by the teacher to consolidate learners' activity by focusing them on the understanding that for any number divided by one the answer is that number. She emphasised that behaviour of one in terms of division is the same, including in fractions and variables as reflected in Figure 4.33.

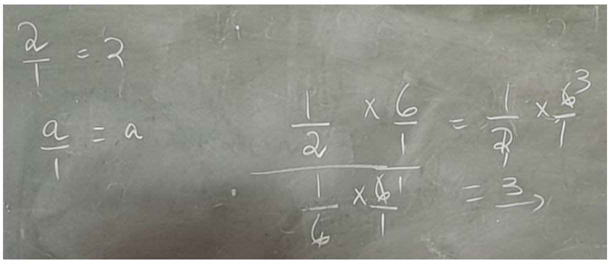


Figure 4.33: Nowele' chalkboard summary

She continuously reminded learners about how the number, thus, one, behaves in division. This constant reminder was a purposeful endeavour, as the same concept of division by one is a precursor also used to decompress the algorithm used in division by fractions.

She further used manipulatives displayed through a data projector to exemplify dividing by fractions as reflected in Figure 4.34.

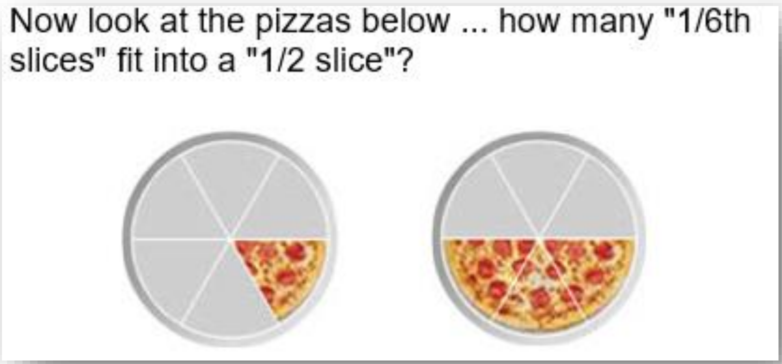


Figure 4.34: Fraction division manipulative

> *Nowele: How can you divide a fraction by a fraction? We have our example on the data projector. Look at the pizzas on the data projector, how many 1/6th slices fit into a 1/2 slice? We see two pizzas on the projector, do we all know a pizza?*
>
> *Learners: Yes (laughing)*
>
> *Nowele: Look at each pizza, how many slices does each have?*
>
> *Learners: Six*

Nowele: How do we represent or call each slice in terms of fractions?

Learners: One sixth

Nowele: Ok, how many times would one sixth fit in half of the second pizza?

Luvele: I think three times (a learner in Nowle's class)

From Nowele's interaction with learners, it appears that she understands that manipulatives do not automatically provide understanding in a magical form, but it needs to be mediated and be used as a tool to exemplify the concept. She further represented the task in Figure 4.33 in fraction notation (see Figure 4.33). Her representation was in line with what had been agreed upon during collaborative lesson planning (see section 4.3.1).

Nowele: Remember, we started with division by whole numbers, where we said any number divided by one the answer is that number. So now we are going to fractions, we divide half by one sixth (represented as in Figure 4.33). Which number that we can use to multiply one sixth to make one?

Mazwi: one (a learner in Nowele's class)

Nowele: Class, … he says one… but we are looking for an answer that is one, if we have one sixth, with what are we going to multiply one sixth in order to get one? Remember, if we multiply by one as Mazwi says, the answer will remain one sixth, but we want an answer that is one.

Tlokoa: Six (a learner in Nowle's class).

Nowele: Ah…Tlokoa says it is six (not quite convinced with the answer) … let us try it … is it correct … what answer are we going to get?

After the class tried to multiply $\frac{1}{6}$ by 6, they got one. Although the answer was correct, however, that is not what Nowele wished for, she then further reminded the class that 6 can be written as $\frac{6}{1}$ to introduce the concept of reciprocal. She further explained that if they multiplied the denominator, thus, $\frac{1}{6}$ in this case, by its reciprocal they should also multiply the numerator ($\frac{1}{2}$) by $\frac{6}{1}$ to keep the value of the fraction unchanged. In her explanation she included that when the denominator becomes one after multiplication by its reciprocal, division by fraction looks like just changing the division to multiplication and invert the denominator fraction, yet there is a lot of mathematics involved instead of the short cut. The concept of division by fractions was further applied in solving algebraic expressions (see Figure 4.34 below).

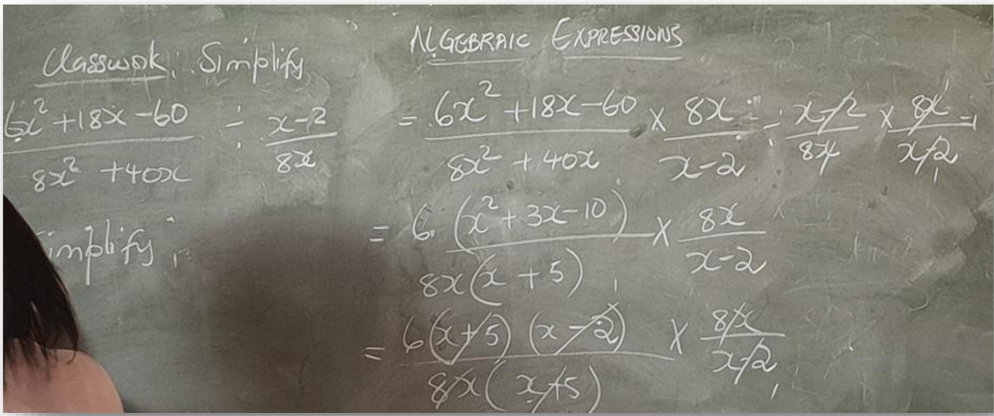


Figure 4.35: Nowele' application of division by fractions

As indicated in the data presented above, teachers that participated in this study seemed to have developed critical aspects of MCKT which influenced their teaching practice. From both the lesson planning stage and classroom presentations, co-researchers showed a deep understanding of using reciprocal in a division of fraction to decompress fraction division algorithm. The shortcut that appears to be changing division to multiplication and invert the fraction had been unearthed. Nowele evidently changed from viewing the above algorithm as a rule but understood the genesis behind it. In line with Hmelo-Silver and Barrows (2006:23) co-researchers provided learners with an opportunity to see how the division of fraction algorithm is derived through unpacking of the algorithm and made hidden mathematics content features visible and comprehensible to learners. Moreover, the co-researchers realized the importance of viewing mathematics content from its teachability perspective (Shuluman, 1986:9). Their representation of division in the form of $\frac{1/2}{1/6}$ instead of $\frac{1}{2} \div \frac{1}{6}$ signifies their understanding of what Shulman (1986: 9) calls the most useful forms of representation of concepts.

Rhulemente's lesson presentation exhibited the understanding of the cutting edge of her learners' learning zone and instead of re-teaching the lesson she decided to provide learners with enabling prompts to get to the learning trajectory. The coherence of mathematics concepts was meticulously revealed to learners as they were engaged with enabling prompts to demonstrate the associative property of multiplication. It created a ‘Wow” effect as learners realized that multiplying six by four is the same as multiplying four by six, a mathematical procedure applicable to (a x b) and (b x a). Specifically, in helping learners to realize coherence and connectedness of mathematics concepts, Nowele kept reminding learners to remember that the division concept is the same in whole number and in fractions. Co-researchers were able to explain why they multiplied by the reciprocal both numerator and the denominator. According to Wu (2017-18), supporting mathematics assertions by reasoning is but one the principles of mathematics integrity that demonstrate teacher’s mastery of MCKT.

The presented data further were analysed through CDA to understand the deeper meaning of the generated data. According to Fairclough and Wodak (2005:457) an interdiscursive version of CDA demonstrates innovation and change from a contextual perspective. The data generated in this study, clearly demonstrated how co-researchers developed innovations in terms of their teaching practices after having joined a co-ordinated team. Evidently, before Nowele and Falafala joined the team, they viewed changing division to multiplication and invert in division by fraction as an arbitrary rule. However, when they became part of the team, they realized the genesis of the algorithm and unpacked hidden mathematics procedures for learners.

Furthermore, Fairclough (1985: 740) describes orderliness as an understanding that “things are as they should be” or they should be as one would normally expect them to be. In terms of classroom discourse, that (things are as they should be) was regarded as a norm before the intervention of this study, for example I-R-E was significantly disturbed by teachers’ empowerment in MCKT. Nowele reluctantly allowed learners when they claimed that six was the number required to multiply one sixth in order to get one. Her allowing the class to test their view, thus multiplying by six, exhibited the disturbance of the previous discursive practice that embraced orderliness. The class and individual learners were able to laugh and present their answers to be tested without being automatically approved or disapproved by the teacher. As the teacher tried to let go of orderliness, we argue that presumably she has developed an understanding that every mathematics assertion should be supported by reasoning and allowed divergent views to be tested to establish mathematics reasoning. Due to her clarity on mathematics concepts she had no reason to be threatened in her social standing, but allowed democracy to prevail. This was a break away from what we have earlier argued that certain classroom discourses such as safe-talk and I-R-E are used by teachers to conceal their incompetence regarding MCKT.

Overarchingly, the data also were analysed through CER as the guiding lens of this study. In avoidance of putting the otherness in jeopardy critical theory refutes the assimilation of the singular into a concept (Duvenage, 2012:120). However, it agitates for democratic values, where thinking is based on reasons irrespective of one's social standing (Duvenage, 2012: 128). Evidently, they adopted classroom discourse where one needs to explain the reasons why they use a particular computation in solving mathematics problems indicates a break away from metaphysical thinking to democratic engagements. The probing and testing of mathematical assertions from both teachers and learners reduced the teacher' social standing and encouraged humility when one is proven wrong. The recognition of otherness, multi-perspectivity and negotiated meaning supported by reasons is complementary to CER's objectives. When teachers exhibited humbleness, in accommodating and testing learners' views which they consequently built on the reciprocal concept showed that the power relations have been denaturalized.

From the above debate, it seems that this study has contributed to academic discourses, by submitting that MCKT influences the enhancement of MPCK. Teachers, that have clarity of the decompressed mathematics procedures and concepts, do not only rely on algorithms but go further to exemplify and explain why a particular mathematics assertion is advanced and reveal the compressed mathematics concepts. Furthermore, they willingly risk and test other people's assertions against their own views without being afraid of being proven wrong. Rhulumente's MCKT of the associative property of multiplication enabled her to understand learners' ZPD and provided appropriate enabling prompts for cognitive scaffolding, without resorting to re-teach the lesson. This ability to think on her feet, evidently was made possible by her deeper understanding of the concept at her disposal. The findings of this study affirm narratives that mathematics integrity eliminates teacher dependency on text books (Wu, 2018:14), and enables teachers to provide mathematics reasoning for every assertion they submit. In line with Hmelo-Silver and Barrows (2006:23) our findings exhibit that when teachers allow learners to see how mathematics theories are derived, learners develop better understanding of mathematics concepts.

4.4 CONDITIONS CONDUCIVE TO ENHANCE MPCK USING PBL IN ACCORDANCE WITH THE FORMULATED STRATEGY

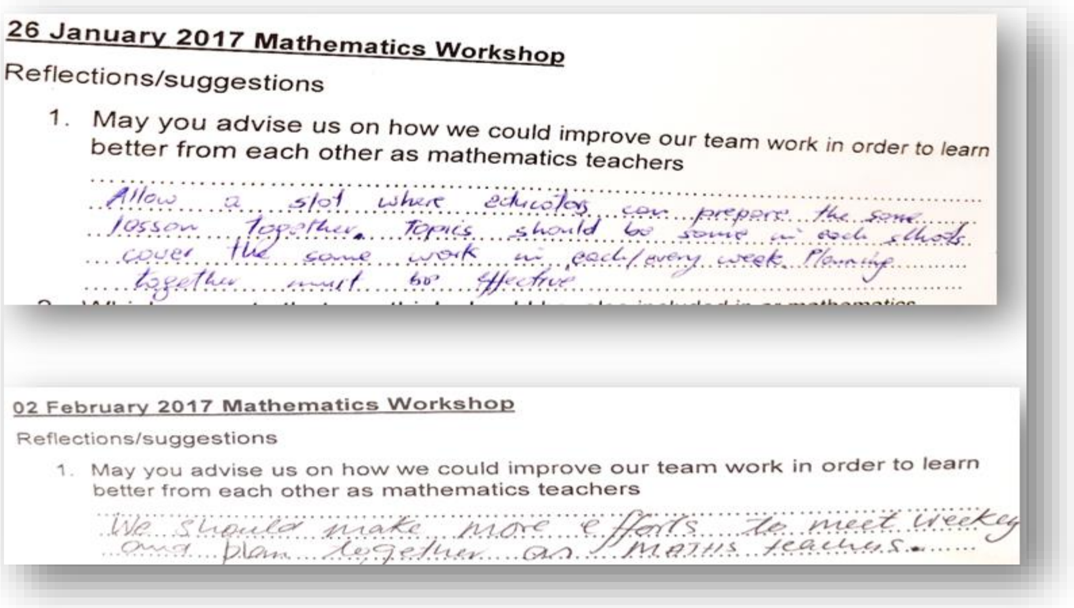
The previous section, 4.3, has outlined the components of the strategy to enhance MPCK using PBL. This section provides the conditions conducive to the implementation of the solutions discussed in 4.3 as they will be implemented beyond the duration of the study. These contextual factors may also affect the implementation of the strategies presented by this study. This section considers the following conditions for the optimal implementation of the solutions or components of the strategy: conditions that strengthen the functionality of the dedicated team; conditions conducive to lesson preparation; conditions conducive to LCPA implementation; and conditions that are conducive for continued teachers' emancipation regarding mathematics content knowledge for teaching.

4.4.1 Factors strengthening the functionality of the dedicated team

The team work revolves around ongoing collaborative interaction between personnel (see section 2.5.1). It is put on record that the success of the teamwork depends on how more readily team members support one another (Everson *et al.*, 2018:1017). It is further postulated that when a team is established, team members need to repeatedly work together in cohesive groups in order to become acquainted with each other's capabilities and challenges, including personality traits (see section 2.5.1). This view posits that when the team is newly established, the frequency of meetings is pivotal to enable team members to familiarize themselves with the team norms, such as commitment, open communication, collective leadership (Qhosola, 2016: 201) and tolerance of divergent views by exhibiting mutual respect among team members (Mosia, 2016:163).

4.4.1.1 *Commitment and open communication forester teamwork*

As reflected above, these conditions, namely commitment and open communication prevailed in the operations of our PBLW although they were not religiously followed. The data generated through operationalization of PBLW suggested that team members valued team work, respected each other and trusted other team members without fear of being judged. The reflections as presented by co-researchers in Figure 4.35 exhibit the prevalence of commitment as theorized by the literature to be one of the necessary conditions for effective team work.



26 January 2017 Mathematics Workshop

Reflections/suggestions

1. May you advise us on how we could improve our team work in order to learn better from each other as mathematics teachers

Allow a slot where educators can prepare the same lesson together. Topics should be same in each school cover the same work in each/every week. Planning together must be effective

02 February 2017 Mathematics Workshop

Reflections/suggestions

1. May you advise us on how we could improve our team work in order to learn better from each other as mathematics teachers

We should make more efforts to meet weekly and plan together as MATHS teachers.

Figure 4.36: Reflections after a workshop

Extrapolated directly from the text presented by the co-researchers, it became clear that they were committed to weekly meetings where they could share their challenges in our PBLW. Evidently, when we focus on the dates of the presented data, there was only a week between these meetings. We want to submit that these frequent weekly meetings were of great value to help team members to become familiar with group dynamics and team routines, especially when the coordinated team was newly established. This frequency of meetings is in line with the narrative which enunciates that frequent meetings develop group cohesiveness and understanding of preferences, strengths, and weaknesses (Everson *et al.*, 2018: 1017). During the initial meetings at which we established the team and determined their roles, Rhulumente contended that:

> *“Colleagues, can someone else other than me, please take the minutes, I am not good at taking the minutes, I like to talk and I get left out in recording the important decisions of the meeting.”*

Rhulumente’s plea that she should be given another role exhibits the open communication and the team’s acceptance of members’ weaknesses. As a result, the task of writing minutes was consequently given to Ntozine. Arguably, this group’s cohesiveness subsequently helped members to change from being a group to becoming a team.

Secondly, co-researchers were overtly open in communicating how they felt and presented their challenges without fear of being judged as reflected by Rhulumente’s contention against the responsibility that was initially given to her. Respect, human dignity, trust and democracy are values that influenced the team’s operations, although they were not all documented on what Qhosola (2016: 203) calls team norms. Tau (the FET mathematics subject advisor), who was given the responsibility to facilitate in most of our meetings unless he was not available, confessed his weakness regarding his poor content knowledge of Grade nine mathematics, since his functions were more relevant to FET mathematics, thus Grades 10 to 12.

> *“You can see I also lost touch with the GET mathematics content.”*

This statement suggests that he was open and honest about his weaknesses despite his position (mathematics subject advisor) and did not want to be viewed as a know-it-all who acted like a ‘lone wolf’. His humbleness helped to persuade the team that we were all learning in this process of interaction. As a result, the team seemed to understand that as a team we possessed more knowledge than each of us individually (Mahlomaholo, 2012a:293), indicating that the sum is greater than its separate parts. According to Mosia (2016:163), open communications also involve tolerance to contractions, while Tsotetsi (2013:105) added patience as one crucial value that characterizes team spirit. The coordinated team was patient with team members, which unintendedly delayed the process and the time lines regarding the submission of this study. We earlier reported that Jones was rather reluctant to let go of deep-rooted discursive practices of first demonstrating before she could give tasks to learners. As she contended the developments from our community of practice, she argues that:

> *Jones: E..hm when learners are given the problem without telling them how to do it, they come up with their ways and you only get there as the teacher to help them, but it is time consuming.*

To her, this approach was time consuming, although she also affirmed that it seemed to work in terms of learner performance. However, her beliefs regarding content coverage and completing the syllabus timeously were rather a deterrent in embracing change. Nonetheless, the team was patient with her and there was no judgement made against her.

4.4.1.2 Shared leadership and team norms encourage team work

Furthermore, as articulated by scholars, shared leadership and team norms are other important factors that strengthen the team work. Appendix 10 presents the details of the operationalized strategy which was further communicated to the principals of the participating schools. This attached Appendix 10 was extrapolated from the minutes of our meetings with co-researchers. As it gives scheduled times and details of activities of what should take place, it represents what Qhosola (2016: 203) calls team norms. It was communicated to the principals as well. This opened communication and eliminated any chance that co-researchers would be refused attendance. It further minimised clashes between teachers' responsibilities at school and the expected participation in the PBLW. As reflected in section 4.3.1, during our PBLW, one of Tau's responsibilities was to prepare presentations for our meetings, although it was not his sole responsibility. Other co-researchers were presenting as well, in the form of sharing how they handled mathematics topics in their schools. Ntozine was responsible for taking minutes of our sessions (see appendix 9). As part of the responsibilities given to the members of the research team, Falafala was tasked to set common assessment tasks which were moderated by Tau (see Appendix 5 & Appendix 6). In line with Tsotesti's (2013:105) and Mosia's (2016:75) recommendations clear roles and responsibilities were given to members of the research team to help the team to adhere to our seven-point plan as articulated in appendix 9 and appendix 10.

4.4.1.3 CER perspective on conditions that encourage teamwork

In terms of CER, people are treated equally regarding their contribution to teamwork, irrespective of their position or organisational rank (Zuber-Skerritt, 2001:11). In line with this narrative, co-researchers in this study were treated with respect as their views or weaknesses were not used to judge them, instead they were used as a spring board for collaborative planning. Moreover, Zuber-Skerritt (2001:11) presented a construct called "symmetrical communication" which is a communication that is not hierarchical, but transcends across the ranks and positions. From this notion, symmetrical communication carries a message that although people are different, each one "has knowledge, skills, capabilities or talents in a particular area which need to be identified and used effectively" (Zuber-Skerritt, 2001:11). To summarise, the data presented regarding the conditions conducive to the operationalization of a coordinated team; thus, commitment, team norms, shared leadership and open communication are indications that this study managed to create conditions that enhanced team work.

4.4.2 Conditions conducive to lesson preparation

It appears that coordinated teamwork creates opportunities for teachers to share their real classroom experiences and problems. This platform provides conditions conducive to enabling teachers to reflect and improve their teaching practices including lesson planning (see section 2.5.2).

4.4.2.1 A coordinated team creates favourable conditions for lesson planning

From the brief synopsis presented in section 4.4.2 above it appears that collegial professional communities, thus the coordinated team in our case, are one of the conditions conducive to enhancing lesson planning. The following data generated are presented as evidence to substantiate the above-mentioned claim.

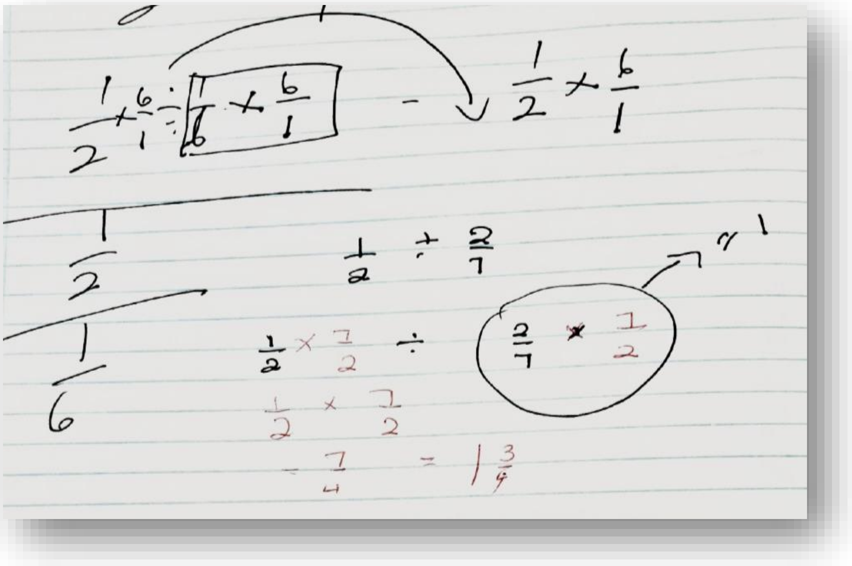


Figure 4.37: Co-researchers interaction in planning together

The above figure, also reflected in section 4.3.1 indicates the interaction of co-researchers as they planned a lesson together. Nowele also proposed that learners would understand the process better if the fractions were presented as in the case presented below.

$$\frac{{}^{1}/_{2}}{{}^{1}/_{6}} \text{ as compared to } \frac{1}{2} \div \frac{1}{6}$$

She argued that learners would better see that one sixth is a denominator that needs to be eliminated through multiplication by its reciprocal. The reflection done after our planning session suggested that the co-researchers experienced mutual and reciprocal benefits as Falafala confirmed that:

> *Yah, it assists us, for one to go to class well prepared, more than we think we know, what we do, you can find out, maybe you come out with an easy way to help learners understand mathematics concepts ... like it is much easier to divide anything by one."*

As extrapolated form the above excerpt, Falafala seemed to have developed thinking about how to make mathematics concepts understood by their learners. It seems that Falafala claimed that planning together changed her practice which is an indication that a coordinated team creates conditions which are conducive to detailed lesson planning. As reflected by Nowele's proposal, the coordinated team presented an opportunity for diverse views on planning a lesson that would create fertile ground for learners to develop a deeper insight in mathematics concepts.

Evidently, from Figure 4.13 it is clear that collegial professional communities create a condition that is conducive to enabling co-researchers to thoroughly prepare their lessons. Extrapolated from Nowele's excerpt as she claims that "*we were taking it for granted that it is just a rule, if you divide by fraction you just multiply*". Drawing from her argument, it became clear that the team work helped co-researchers not only to present division by fractions as rule, but also to understand the concept as they planned to unearth the genesis underpinning what they called 'just a rule'. Falafala, also realized the importance of a detailed mathematics lesson plan as she argued that planning together helped her to be well prepared before she went to class. Her words, that "*more than we think we know what we do"* were an indication that the team planning demonstrated to her that teachers should plan their lessons in line with learners' cognitive congruence and by coming up with what Shulman called the most powerful analogies of making mathematics concepts teachable to others. In the same vein Falafala realized that division by one could be used to give clarity to a concept of division by fractions. In coining our argument, a coordinated team is a valuable condition conducive to fostering detailed lesson planning. This confirms the findings of the literature, namely that team work presents teachers with opportunities to explore multiple aspects of PCK (Shen *et al.*, 2007:249). As evidenced above, team work did not only help teachers to thoroughly prepare their lessons but further emancipated them in terms of both their pedagogical knowledge and content knowledge. As Nowele confessed, changing the division sign to a multiplication sign when dividing by a fraction was nothing more than 'just a rule'. Evidently her claim that "*as we worked together, we gained a lot in terms of where it starts*" exhibits that her MPCK was to a certain extent enhanced.

4.4.2.2 CER view on how a coordinated team creates favourable conditions for lesson planning

In line with Critical Emancipatory Research CER, the research agenda is co-created by those involved in the process (MacCabe and Holmes, 2009 cited in Tshelane &

Tshelane, 2014:287). The debate that ensued amongst the co-researchers as they collectively planned mathematics lessons, portrayed some level of co-creation of strategies of representing mathematics concepts. The argument that learners would better understand division by fractions when one sixth is put as denominator not horizontally as reflected by Nowele's proposal, showed acceptance of multiple perspectives. Seemingly, CER's ontological stance supports the acceptance of the other in its full uniqueness (see section 3.2.5). Flagrantly, the team work allowed the use of different views and analogies to enrich the lesson planning. Put differently, some silent issues that might hinder learners' understanding of concepts are unearthed during collaborative planning. Apparently, teachers learn different ways of representing mathematics concepts from learners' cognitive congruence. In conclusion, our submission affirms the view that a collegial professional community, thus, a coordinated team in our case, set conditions that enable teachers to reflect and improve their teaching practices, and lesson planning in particular.

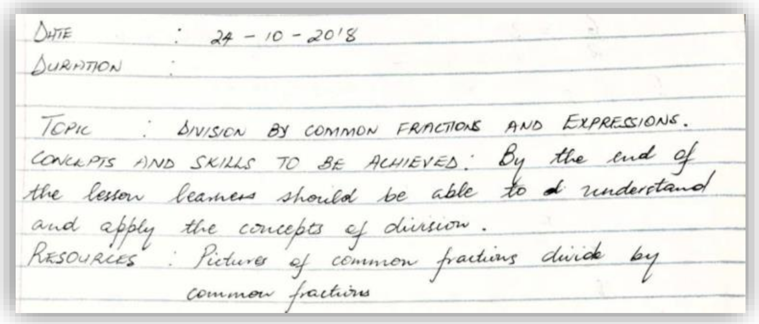
4.4.3 Conditions conducive to encouraging LCPA implementation

It was put on record that for teachers to prosper in unearthing what is within the learners' reach, they needed to use integrated assessment with lesson facilitation and follow up learners' misconceptions through appropriate utilization of manipulatives (see section 2.5.3). In line with the above-mentioned assertion lesson planning should be more than content but focus on what learners can do, thus LCPA in our case. As presented by Moloi (2014:271), an important condition in the mathematics classroom in order to cater for learners' needs, is the featuring of learner's experiences and their prior knowledge in both the lesson plan and lesson facilitation.

4.4.3.1 Judicious utilization of manipulatives encourages LCPA implementation

As discussed earlier in Section 4.3.3, the effective utilization of manipulatives enabled co-researchers to employ LCPA. It is also evident from learners that were part of their classes as they actively engaged in the meaning making regarding minimum conditions required for triangles to be congruent. Secondly, the following figure, Figure

4.35, presents a part of the final, contextualized individual lesson developed from the planning session meeting. In the lesson plan Nowele wrote her actual name and her actual school's name as the lesson was part of the daily work. From the copy given to us, we managed to conceal the real name in compliance with the ethical consideration of anonymity.



DATE : 24 - 10 - 2018
DURATION :
TOPIC : DIVISION BY COMMON FRACTIONS AND EXPRESSIONS.
CONCEPTS AND SKILLS TO BE ACHIEVED: By the end of the lesson learners should be able to ~~d~~ understand and apply the concepts of division.
RESOURCES : Pictures of common fractions divide by common fractions

Figure 4.38: Lesson plan developed during the planning session meeting

As reflected above, the lesson had a clear, measurable and achievable objective. It also included the manipulatives used to exemplify the concept of division by a common fraction. Our observation of lesson presentations at the research sites, exhibited that teachers' use of manipulatives enabled them to engage learners in active meaning making through the use of manipulatives as reflected in Figure 4.16 and Figure 4.17 (see section 4.3.3). In line with the above-mentioned findings, this study submits that judicious use of manipulatives is one of the conditions that encourage LCPA. As learners focus on the problem at their disposal, they use manipulatives as scaffolds without being entirely reliant on teachers' approval in terms of whether the solution they come up with, is mathematically correct or not. In essence manipulatives are used as tools to explain, why learners think and believe their solutions and procedures are mathematically sound.

4.4.3.2 Probing of learners' responses encourages LCPA implementation

As it were, probing questions and enabling prompts helped learners focus on compressed mathematics concepts needed to be demystified before learners could untangle complex problems. Evidently, the following engagement between Njovane and his learners attests to the narrative that probing questions assess what learners can do or cannot do while the lesson is underway, at the same time probing questions enable the teachers to timeously and appropriately adjust their instructions as informed by the learners needs.

> *Njonana: "Why have you decided to add or multiply?"*
>
> *Seemane (learner): We made a mistake, one of us insisted that we must add although we wanted to multiply, we then cancelled multiplication and put addition.*
>
> *Njovane: "Eh… eh, why?"*

The use of 'whys' in his questioning, gives learners the opportunity to present what they think. Without them displaying why they acted in a particular manner, he would not be aware of their challenges. And as a consequence, he would have administered a wrong 'medication' if this was a case of medicine, which might not help to resolve the actual problem, but rather exacerbate it. On the other hand, Rhulumente intervened on learners' misconceptions through the use of enabling prompts. When learners could not see that (a $\times$ b) is the same as (b $\times$ a) she gave them a simpler but similar task, for example 4 $\times$ 6 and 6 $\times$ 4 to enable them to see the associative property in multiplication (see section 4.3.2). In coining our debate, it also appears that the use of probing questions is one of the conditions conducive to put learners at the centre of the learning and teaching process.

4.4.3.3 CER perspective on conditions that foster LCPA implementation

As people develop an understanding of a new reality, PAR affirms the "people's rights to be listened to and understood" (Frisby *et al.*, 1997:15). In line with this narrative, people bring new perspectives to their situation that might encourage them to take action to improve it. In essence, as learners use manipulatives to explain their perspectives, their seemingly unorthodox methods draw the teacher's attention resulting in a series of why questions. Inevitably, the classroom discourse tends to be centred on learners' new thinking. In line with Moloi's (2014: 271) view when learners' knowledge is not marginalized, they become active in the process of negotiated meaning. The democratic values that develop in the process of negotiating the meaning, consequently change the power relations.

4.4.3.4 Conditions conducive to teachers' continued emancipation on mathematics content knowledge for teaching

Improving the co-researchers' MCKT is one of the strategic components in enhancing MPCK using PBL. From the epistemological stance that knowledge is socially constructed the establishment of a platform for team members to tap from each other's strengths and reduce weakness becomes a vital condition for continued emancipation in terms of MCKT. PBL emphasises peer learning as team members work together in designing solutions to the problems (see section 2.5.4).

Co-researchers embraced the above-mentioned conditions that foster continued development in MCKT. Evidently, planning together, sharing lived experiences regarding problems and success stories are conditions which are conducive to encourage continued enhancement of co-researchers' MCKT. Falafa's humanising expression of self-discovery evidently suggests that collaborative lesson planning encouraged development of MCKT.

> *"Yah, it assists us, for one to go to class well prepared, more than we think we know what we do, you can find out, maybe you come out with an easy way to help learners understand mathematics concepts ... like it is much easier to divide anything by one."*

She admitted that what she thought she knew was not suffice for teaching mathematics. Her reflection on collaborative lesson planning illustrates her attention to mathematics concepts and aspects pertinent to its teachability. Her understanding of developing ways of making mathematics concepts easily understandable by learners is attributed to collaborative lesson planning.

On the other hand, when co-researchers share problems that they encounter in their teaching practices, they consequently develop MCKT in the process of resolving shared problems. Mbuyi and Zitle presented their inability to enable learners to accurately identify and label angles when two parallel lines are cut by a transversal (see section 4.3.1). Jones claimed that:

When learners come from these lower grades, they are familiar with multiplying positive integers, and how would I help them understand the multiplication of negative integers?

Although the question was not put explicitly, she seemed to only understand multiplication of negative by negative as a rule. However, she could not support this assertion by any mathematics reasoning and apparently her deficiency impaired learners' relational understanding of multiplication of negative by negative. Prudently, collaborative lesson planning offered favourable conditions for sharing problems.

Falafala shared her success stories regarding what seemed to be a problem with colleagues:

I am used to use this one (pointing at an angle). Why I am using this one, is because sometimes this sign (pointing at ∠) they (learners) do not accurately write it, but that cap (pointing at the angle symbol above A in angle CAD) is that one that makes it clear in terms of which angle are we talking about. I use it most of the time.

This excerpt exhibited that she used the above-mentioned approach more often than not as it seemed to be working for her. Precisely, collaborative planning provided opportunities for teachers not only to share their problems, but it was a condition conducive to sharing success stories that could be proliferated across the team members.

4.5 FACTORS THAT THREATEN THE IMPLEMENTATION OF THE DEVELOPED STRATEGIC FRAME WORK

This section explores factors that threaten the implementation of the developed strategy to enhance MPKC using PBL. Explicitly, this section focuses on discussing factors that almost derailed the implementation of the strategy and how they were circumvented. As it were, the following factors threatened the implementation of the developed strategic framework; that is, inherent threats regarding the establishment of the coordinated team, threats towards effective use of manipulatives and negative attitudes towards lesson planning.

4.5.1 Inherent threats regarding the establishment of the coordinated team

As reflected in section 2.4.1.1, lack of time for coordinated team meetings seemed to be a threat towards effective operations of the coordinated team. On the other hand, the rigid inspections during the apartheid era made teachers reject anybody to observe in their classrooms, for fear of being judged (Jita & Mokhele, 2014: 11-12). Furthermore, conditions such as being the only teacher for mathematics in a school and geographical isolation, distance between schools, bad road conditions and the use of personal money to attend the cluster meetings militated against the existence of collaborative teams (see section 2.4.1.2).

This study also experienced similar threats regarding the effective implementation of coordinated team work. Co-researchers' biographic data (see section 3.3) gave a synopsis of the rural nature of the terrain of schools that participated in this study. A limited number of team members, namely four teachers worked in close proximity, namely; Rhulumente and Njovane, who taught the same grades but in different class groups in the same school, and Falafala and Nowele, who also taught in the same grades but different class groups at another school. The rest of the co-researchers were the only mathematics teachers in their schools.

During the PBLW co-researchers raised the issue that:

> *"The school does not pay our transport when attending these meetings as compared to other departmental workshops".*

The above excerpt illustrates that the use of personal financial resources to attend coordinated team meetings seemed to be a threat towards optimal participation. To circumvent this threat, the meeting resolved that over and above the consent forms that were issued to participating schools, our strategic framework should also be communicated to their school principals (see Appendix 10). The threat was circumvented by principals' buying in to the intervention strategy as it included issues that were not only beneficial to co-researchers' emancipation regarding their MPCK, but also issues such as common standardized examination papers and moderation processes which were of benefit to the schools as well.

4.5.2 Threats towards effective use of manipulatives

As reflected in section 2.4.3.1.4, Furner and Warrell (2017:12) reported teachers' views regarding the use of manipulatives as a waste of time which is not necessary for teaching and learning serious mathematics. The environment with manipulatives threatens teachers' position of being the only ones to approve which mathematics answers are correct or not (see section 2.4.3.1.4). For teachers to claim back their position of power in the mathematics classroom discourse, they perpetually stopped using manipulatives.

Our observations and interaction with co-researchers at the research sites to a certain extent revealed factors that tended to threaten the effective utilization of manipulatives. During the reflective session after the class visit Njovane contended that:

> *Uhm, teaching aids, I do not necessarily need teaching aids in expressions and factorization and as result I do not have them, but they are usually available for statistics.*

This excerpt exhibits the teachers' belief regarding the use of manipulatives in certain mathematics topics. His belief threatens any attempt of effective utilization of manipulatives on particular topics. Consequently, he deliberately avoided the use of manipulatives on such topics such as expressions and factorization, contrary to Miranda and Adler's (2010:20) research findings that suggested that manipulatives developed a deeper insight in understanding of expressions (see section 2.4.3.1.2). Notwithstanding these deep-rooted beliefs, which seemed to threaten effective utilization of manipulatives, the coordinated team work circumvented this threat as co-researchers shared their experiences of using manipulatives in developing an understanding of mathematics concepts, even those concepts which might seem to be trivial or easily understandable. For an example, Falafala shared her experience of using what she called a 'cap' in identifying the required angles (see section 4.3.1). This (Falafala's cap) was later adopted by Zintle in her class (see section 4.3.3).

On the other side, Ntozine contended that *"When you need them you need to personally purchase them"*. Her protest was a threat towards utilization of manipulatives in her class. Apparently, the non-availability of manipulatives in her school demanded her to buy some. However, when we inquired from her about whether she had reported the matter to the principal, we learnt that she never did. The team advised the co-researchers not only to think about sophisticated manipulatives, but to use the available resources at school to design mathematics manipulatives. Evidently, the use of simple chat papers by Zintle and Ntozine as reflected in Figures 4.16 and 4.17 respectively illustrates that the threat was circumvented.

4.5.3 Threats towards lesson planning

The research findings highlighted impediments towards lesson planning, thus, a lack of supervision from the schools, and teachers' negative attitude towards curriculum reforms (Bantwini, 2010:86). Teachers were threatened by curriculum reforms' requirements as they seemed to demand more from teachers' limited time, hence they argued that lesson planning was all about paperwork (Bantwini, 2010:86). Teachers also bemoaned the lack of time for lesson planning due to overwhelming demands of assessment and marking (Ramaila & Ramnarain, 2014: 7). Teachers further contended that they were unable to plan mathematics lessons arguing that they had other subjects to teach too (Ding & Carlson, 2013: 381).

On the research site it was observed that co-researchers' attitudes and beliefs tended to threaten effective lesson planning. When we inquired why co-researchers did not have proper and detailed lesson plans, Falafala argued that:

> *"We really do not have lesson plans, let us be honest, when we are going to attend moderations, we download them from the departmental website and put them in our files for compliance."*

Evidently, this excerpt clearly demonstrates co-researchers' attitudes, which might threaten detailed planning of mathematics lessons. The downloaded lesson plans were not used as a road map to guide teaching activities, but were merely used to maliciously comply with moderation requirements. This attitude towards lesson planning seemed to be a threat towards the value of a detailed lesson in mathematics teaching. Falafala further claimed that she had been teaching the same class over a number of years and she did not need to prepare because the lesson was in her head. While Nowele claimed that:

"No, no, no, we do not do any formal lesson preparation, I mean I sometimes scribble on a lose piece of rough paper, just on a paper that you do not seriously consider."

These opinions held a serious threat to detailed lesson planning. Co-researchers' attitudes and the value they attached to lesson planning seemed to put the development of detailed lesson plans in danger. Nonetheless, sharing of lived experiences and collectively agreed on team norms as reflected in Appendix 10 circumvented the threat towards detailed lesson planning.

1. Common planning
2. Common formal assessment tasks
3. Weekly meetings/ workshops to evaluate progress and sharing of challenging concepts
4. District moderation of formal tasks
5. Sharing of mathematics teaching resources
6. Class visits to observe the best practice in the implementation of the programme.
7. Extra classes to ensure coverage of term work schedule.

Figure 4.39: Excerpt from Appendix 10

The first collectively agreed upon resolution (see Appendix 10) emphasizes the aspect of common lesson planning. Implicitly, common lesson planning during PBLW eliminates the chances for any team member to go to class without a prepared mathematics lesson plan.

Moreover, co-researchers' reflections regarding PBLW, asserted that:

"The teamwork keeps us updated, because every team member is going to make it a point that targeted work is covered, unlike when there is no one who supervises you, in this programme you are supervised by your colleagues. You will personally be embarrassed when you come to the next meeting and discover that your colleagues have moved and you did not do anything, you will feel ashamed and then you will end up working now."

Extrapolated from the text in above excerpt, it appears that the teamwork created a safety net that encourages team members to be steadfastly guided by the team norms as a true foundation of their operations. No one in a team who would like to let down the team. Ntozine's words, namely, "*unlike when there is no one who supervises you",* suggested that the team also provided the supervision aspect which seemed to be lacking at school level. In a nutshell, threats to detailed lesson planning were circumvented by the team norm (see Appendix 10). The issue of lack of supervision in the schools, for an example, as articulated by Bantwini (2010:86), had been circumvented through collegial supervision during the PBLW.

4.6 INDICATORS OF SUCCESS IN THE IMPLEMENTATION OF A FORMULATED STRATEGY TO ENHANCE MPCK USING PBL

The previous sections 4.2, 4.3, 4.4 and 4.5 were in the process of developing the strategy to enhance mathematics pedagogical content knowledge of teachers teaching Grade 9 learners using problem-based learning. This section reports about indicators of success in the implementation of the strategy. In reporting the indicators of success in this study, a special reference is made in terms of good practice as discussed in section 4.1 and 4.2. Therefore, if the components of the strategy were able to resolve challenges that emerged at the research sites, it seems to be fair that they serve as indicators of success as highlighted in the following sections.

4.6.1 Successful exhibition of knowledge and skills to unearth learners' mathematics misconceptions

Following up on learners' misconceptions enable teachers' development of an understanding of learners' thinking (Herholdt & Sapire, 2014:44), and, consequently, they learn new ways of making mathematics understandable to learners. Other than re-teaching or quick fixing like replacing a flawed answer with the correct one, analysis of the underlying mathematical misconception regarding learners' erroneous answers, helps teachers to develop cognitive scaffolds such as enabling prompts. In the process of developing cognitive scaffolds, teachers' wisdom of practice gets enhanced, as they acquire new strategies to eliminate learners' misconceptions.

Evidently, the use of why questions and enabling prompts in this study, which helped learners to decompress compressed mathematics concepts, is an indicator of success in relation to what in Shulman's component of PCK is called knowledge of learners (see section 4.3.2). Elimination of learners' misconceptions, such as a lack of closure, exhibits success of the employed strategy. The co-researchers did not just mark learners' work wrong when learners had added 5 x to 20 to get 25, instead, they requested learners to explain their calculations. As learners tried to explain why they had worked in a particular manner, they did not only realize that adding unlike terms was mathematically incorrect, but they also understood why they arrived at a wrong solution. In essence, the concept of adding like terms was amplified by the learners' erroneous answer which was used as spring board for teaching and learning. Logically, learners' improved achievement and the elimination of miscomputation indicated success in terms of the employed strategy. It is put on record that before the intervention of this study, co-researchers neither identified nor followed up learners' mathematics misconceptions as reflected in Figure 4.5 (see section 4.2.2). Evidently, the understanding of learners' misconceptions regarding the associative property of multiplication enabled co-researchers to appropriately adjust pedagogy as they provided enabling prompts to help learners to achieve learning trajectories (see section 4.3.2).

The findings of this study are in line with Gardee and Brodie's (2015: 2) narrative, arguing that valuing learners' thinking encourages teachers to "find ways to engage their current knowledge in order to create new knowledge". As teachers engage learners to create newly negotiated knowledge, they become learners as well, and as a consequence their MPCK get enhanced.

4.6.2 Successful enactment and display of curriculum knowledge through judicious use of manipulatives

Judicious utilization of manipulatives indicates success in terms of enactment and exhibition of curriculum knowledge. Effective utilization of manipulatives presents multiple representations of mathematics concepts to learners and proliferates communications, engagement amongst learners, and the development of multiple negotiated meanings of mathematics concepts.

The observed judicious use of manipulatives seems to indicate success in terms of creating cognitive disturbance to learners. Learners did not memorize conditions for congruent triangles, but used manipulatives to prove congruency (see Figure: 4.20).

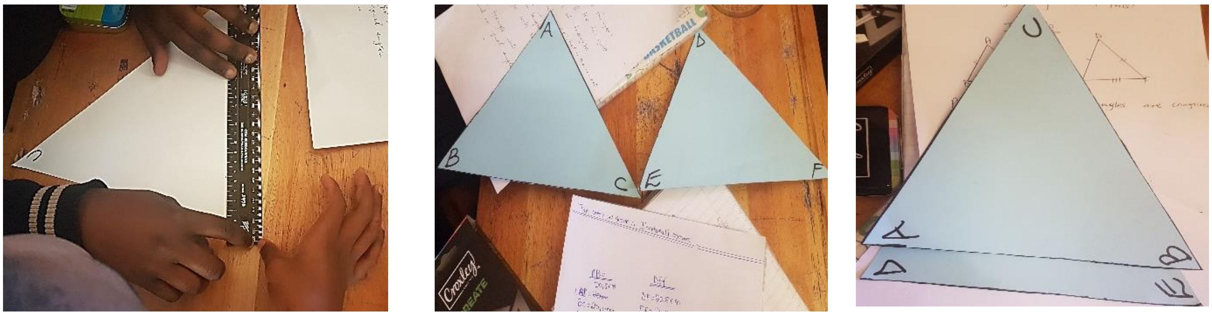


Figure 4.40: Using manipulatives to prove congruency

Learners' understanding of the congruency concept through the use of manipulatives indicates that teachers apparently succeeded in exemplifying an abstract concept. As exhibited in Figure 4.20, learners were able to manipulate tools of the trade at their disposal to satisfy the conditions for congruency. Learners' opinions were recorded regarding how they would explain congruency to a peer or a younger sibling. Moseoa (a learner in Ntozine's class) responded:

> *Congruence is when angles ... em triangles that can exactly fit on top of each other, that are equal.*

Moseoa's explanation indicates a deep understanding of the concept of congruency, other than satisfying minimum requirements. Seemingly, the judicious use of manipulatives in this study enhanced teachers' MPCK. These findings confirm the notion that manipulatives use presents multiple representations of mathematics concepts to learners (Takor, Iji & Abakpa, 2015:97-98). The teacher's positioning of being the sole provider of answers is denaturalized through engagement amongst learners and the development of multiple negotiated meanings of mathematics concepts as they manipulate the tools of trade to defend their views.

4.6.3 Successful enactment and display of curriculum knowledge through detailed lesson planning

When teachers make use of appropriate curriculum material in lesson planning, they do not only enact their knowledge of curriculum, but also display their knowledge of the curriculum (see section 2.4.3.2).

From the generated data in this study it appeared that particular attention had been paid to the fundamental aspects of lesson planning, such as lesson objective, prior knowledge of learners and relevant manipulatives as reflected in Figure 4.41 below.

Topic : Algebraic Expressions: Multiplication of binomials
Objective: By the end of the lesson learners should be able to determine the product on two binomials.
Resources: Text books, marking pens, flip charts.
Prior Knowledge: laws of exponents, commutative property, associative property, distributive property, properties of integers, like and unlike terms.
Assessments: Understanding of distributive property and associative property.
Activity 1.
Learners would be required to solve the following tasks in groups:

Groups 1 and 2	Groups 2 and 3	Groups 5 and 6
• 5(10) • 6(6 + 4) • 5(x + 4)	• 5(7) • 5(8 + 3) • Y(y + 3)	• 5(11) • 5(3 + 4) • a(a + b)

Activity 2.

Groups 1 and 2	Groups 2 and 3	Groups 5 and 6
• (2 + 3)(4 + 6) • (x + 3)(x + 4)	• (2 + 3)(4 + 3) • (a + 3)(a + 2)	• (2 + 3)(8 + 3) • (a + b)(a + b)

Activity 3.
All groups to work on the following tasks:
Simplify: a. (p + 3q)(p + q)
b. (3x + y)(2x + 5y)
c. (x + 1)(x + 2)

Figure 4.41: Excerpt from Rhulumete's lesson

Figures 4.41 illustrates detailed lesson plans developed by co-researchers as part of the team norms. It seems that lesson planning assisted them in building new knowledge from learners' prior knowledge. Coherent series of learners' activities as reflected in Figure 4.42 apparently indicated teachers' emancipation and development of consciousness regarding MPCK aspects that would make mathematics concepts understandable to learners. This practice was gradually built through coordinated team meetings using lived experiences to make learning meaningful as reflected in Figure 4.11 (see section 4.3.1), where participants discussed the most powerful analogies for concept representation as they collectively planned their lessons. Therefore, the planning of mathematics lessons in this study seemed to have covered the scope of curriculum alignment and apparently exhibited the enactment of curriculum knowledge.

4.6.4 Using assessment-embedded instruction to determine the learning zone

The identification of the learning zone through assessment-embedded instruction enables teachers to appropriately develop mathematics-appropriate strategies to rescue learners from a cul-de-sac and to realize their learning trajectory. This praxis enables teachers and learners to consistently work in the ZPD, the area where learning takes place (Heritage, 2010b:8). Apparently, teachers prosper in unearthing what is within the learners' reach, when they keep a very close eye on emerging learning through assessment-embedded instruction (Heritage, 2010b:8). Specifically, assessment-embedded instruction apparently develops a discursive practice where one has to justify one's opinion during the classroom discourse.

The data generated during research site visits exhibited that co-researchers seemed to embrace the praxis of using assessment-embedded instruction. Specifically, activity one extrapolated from Figure 4.42 above, demonstrates a series of planned assessment activities that were integrated with lesson facilitation. As evidenced in Figure 4.42, teachers who participated in this study broke away from the practice of first demonstrating to the learners and later giving them exercises to do. Apparently, Zintle's words that: *"I do not tell them what to do, but just give them problems so that they come up with solutions in groups",* indicate a certain level of success in classroom discourse that embraces assessment integrated with instruction. The following excerpt illustrates how Njovane's class also integrated assessment with instruction in his classroom.

Njovane: Tell the person next to you any triangle you know, you are given 2 minutes.

Group one : Isosceles triangle

Nosipho from Group 2: Que…lateral triangle

Njovane: Is there any other triangle?

Sifiso: : Quadrilateral triangle (the class laughed)

Themba : Equilateral triangle

Group of learners in a chorus: Equilateral triangle……

The above excerpt indicates that Njovane integrated the assessment in his lesson facilitation to establish the learning zone. As learners presented their thinking, he could identify the learning gaps and consequently adjusted the instruction to accommodate the learning needs. Evidently, when Rhulumente implemented LCPA, she used instruction that was integrated with assessment to assess learners' understanding regarding removing brackets; a skill needed when solving expressions as exhibited in Figure 4.31 (see section 4.3.6). She first assessed the learners on simple tasks, such as 6(10) and advanced to y (y +3) to assess the distributive property of multiplication, before she could require learners to multiply a binomial by a binomial such as (x + 3) (x + 4) as reflected in Figure 4.32 (see section 4.3.6). It appeared that the co-researchers' lived experiences illustrated above indicated success in terms of integrating assessment with lesson instruction. In the process co-researchers learned new ways of adjusting their lesson plans and instruction to accommodate learners' emerging learning tendencies. Implicitly, their ability to first assess learners' understanding of distributive property in multiplication using numbers before they could assess them on algebraic expression suggested that integrated assessment developed the teachers' MPCK as well. Apparently, the wisdom of practice had been enhanced through assessment-embedded instruction. The findings of this study are in line with the narrative propagating that teachers prosper in unearthing what is within the learners' reach, when they keep a very close eye on emerging learning through assessment-embedded instruction (Heritage, 2010b: 8).

4.6.5 Success in implementation of LCPA considering learners' learning needs

Arguably, in an LCPA environment, learners make meaningful connections between their learning experiences and the real world (see section 2.7.5). The implication of improving learners' performance and enhanced clarity on mathematics concepts may suggest that LCPA has positive effects on teachers' PCK that practise LCPA both as classroom discourse and as discursive practice.

On the research sites learners were given the following tasks to respond to as groups:

> *Ntozine: I want each group to define the following angles. We are going to do this activity within two minutes, you may please start writing … start writing what you know … write on the paper given to you … write in the paper, that I have just given to you, what is a right angle? What is an acute angle? What is an obtuse angle? What is a straight angle? And also, what is a revolution?*

In the extract above, Ntoizine grouped the learners and instructed them to communicate their definitions in writing, so that she could establish their prior knowledge as she intended to teach construction of special angles. Moreover, the following extract indicates that Ntozine's learners successfully managed to construct 90^0, but struggled with the construction of 45^0:

> *Ntozine: "I could see that they understood construction this one of 90^0, but they seem not to understand construction of 45^0, maybe they were already tired, probably, I think I should have done one angle today and not mix them."*

Notwithstanding the learners' challenges regarding the accurate construction of 45^0, it appeared that LCPA enabled her to develop some insights regarding the wisdom of practice; thus, MPCK in our case, as she argued that she thought she should have allowed learners to construct one angle at a time, as mixing angles seemed to confuse learners.

Furthermore, learners in Njovane' class were given a worksheet which instructed them to construct special angles. The following excerpt illustrates Milwa's (one of learners that participated in Njovane' class) experience regarding LCPA in her class.

> *Researcher: What was different in todays' lesson?*
>
> *Milwa : We usually work as individuals but of late we work as a group.*
>
> *Researcher: What was special with group work?*
>
> *Milwa : We helped each other, showing each other how to get the solution.*

Evidently, Milwa's claims indicated that the tasks were given to them to work out as groups, where they were able to help each other in terms of solving the problem. From the extract below, it became clear that LCPA helped learners to develop a deeper understanding of mathematics concepts.

Researcher: Do you think you understood what you were doing and would you be able to explain it to others?

Milwa : Yes (with confidence), I could

Researcher: Briefly, how would you explain?

Milwa : When you construct a special triangle, you must have a campus, sharp lead pencil, a ruler, then draw a straight line without a magnitude, put a lead pencil in the campus and measure the length you would use from a ruler using the campus with the pencil, for an example 6cm. Measure from zero to 6cm on the ruler and make sure that your campus is tight. Draw an arch on your straight line, then put the campus where the arch crosses the line and draw another arch the same line at 6 cm away from the first arch. Then write down the points on the first and the second arches. Check with the ruler to make sure the distance between the arches is exactly 6cm or you might have made a mistake. If it is accurate you write 6 cm. Then you take your campus again, open it at 6 cm and draw the first circle, thus point N, from the first arch and the second circle from the second arch, thus point L. Where the circles intersect above the line, write point F, then take your ruler to verify if the distance from N to F and from L to F is 6 cm. If it is 6 cm then take the ruler and join your points to form an equilateral triangle.

The recorded learner's explanation of how to accurately construct an equilateral triangle indicates the learner's clarity regarding the concept. Milwa did not only include material resources needed for the construction of the equilateral triangle, but she also gave details of how to manipulate such resources to double check the accuracy of her measurements. As she argued that one must check the distance between arches with the ruler, specifically, suggests that with LCPA mistakes are part of the learning process. The verification of whether the answer is correct or wrong is not only the task of the teacher, but an important role of the learner too.

Researcher: If you were a teacher, how would you teach? Which way would you use?

Milwa : Me?

Researcher: Yes

Milwa : How would I do? (Laughing) I would……

Researcher: Let me make an example, you have said that your teacher used to first demonstrate how to solve a mathematics problem and later gave you exercises to do, but today he did not show you. What did he do?

Milwa : He just gave us tasks to do

Researcher: Which one is better in your own view?

Milwa :To let us do mathematics problems without first being shown, so that we can think on our own how to solve the problem. If he observes that learners are struggling, it is then he can show, or any learner that seems to know it, should explains how she got it.

The statement, "*he just gave us tasks to do"* exhibits that the teachers did not first demonstrate the concepts, but gave learners the problems to solve. As learners began to embrace LCPA, they indicated its benefits in terms of allowing them to communicate their common sense-based mathematics experiences, which on the other side enabled teachers to draw from the 'mathematics tool box' appropriate strategies to address learners' challenges. Implicitly, improving learners' performance and enhance clarity on mathematics concepts, as in the case above, apparently suggests that LCPA has positive effects on teachers' PCK that practise LCPA both as classroom discourse and as discursive practice. The findings of this study confirm the view that LCPA enables teachers to lively engage learners to make meaningful connections of their learning experiences and environment to the real world (Walters *et al.*, 2014:2). In the process teachers become learners as well and further develop skills to untangle live experiences regarding mathematics teaching and learning.

4.6.6 Successful understanding of MCKT

Teachers with sound MCKT are able to support every mathematics assertion by reasoning which inevitably enables learners to realize that mathematics is learnable (see section 2.7.6). They present mathematics topics in a coherent way, clearly demonstrating division concepts as essentially the same for whole numbers and fractions. They exhibit relational understanding of mathematics, using most effective analogies which learners can relate to and making mathematics concepts understandable to others. Co-researchers seemed to embrace the above-mentioned aspects as earlier detailed (see section 2.7.6).

The collaborative development of detailed lesson plans indicated success in terms of the co-researchers' emancipation of MCKT, as they evidently used easily accessible manipulatives (see Figure 4.17) to exemplify mathematics concepts, such as congruency and division by fractions. Co-researchers' ability to explain the underlying reasons for their assertion and decompressing mathematics algorithms indicated success in terms of their MCKT development. Nowele became an expert learner when the class proposed that they should multiply $\frac{1}{6}$ by 6 in order to get one. She further proclaimed that 6 is the same as $\frac{6}{1}$. Her argument was used as a scaffold to introduce the concept of a reciprocal (see section 4.3.7).

Finally, the coordinated team shared experiences regarding why the answer was positive when a negative number was multiplied by a negative number. Zintle shared with the team the analogy that seemed to work for her class.

> *"Let us assume negative as an enemy, then positive as a friend. The enemy of my enemy is my friend, then negative multiplied by negative is positive; while the friend of my friend is my friend, that is, positive multiplied by positive is positive. The enemy of my friend is my enemy, that is negative multiplied by positive is negative."*

Zintle's wisdom of practice had a wow effect on the team members as they realized how her analogy simplified the reason why ($-\times - = +$). It appeared that the learners could easily relate to the above analogy and consequently they understood why negative multiplied by negative gives a positive answer.

The team further used the distributive property of multiplication to theorize how we could get a positive integer as an answer when multiplying negative integers, thus (-5 x -3 =?). Drawing from our understanding that the zero is the answer for the addition of additive inverses, thus, (-4 + 4 = 0) and that any number multiplied by zero is equal to zero we used the following example to theorize the reasoning underlying the assertion that negative multiplied by negative is equal to positive.

6 (4 + -4) = 6(0) = 0.

6 x 4 + 6 x -4 = 0

24 + …… = 0

24 + -24 = 0

The team agreed that whatever the answer of (6 x -4) might be, for the above-mentioned statement to be consistent with our mathematics understanding of additive inverses, 6 x -4 must be equal to -24. In order to be explicitly consistent with how an additive inverse behaves, 6 x -4 must be -24 in order to get zero when added to 24, thus (24 + - 24 = 0). Multiplying 6 by – 4 to get -24 is consistent and coherent with the mathematical understanding that views multiplication as repetitive addition.

We further tried the distributive property of multiplication when the integers were both negative.

-5 (3 + -3) = -5(0) = 0

-5 x 3 + -5 x -3 = 0

-15 + …… = 0

-15 + 15 = 0

At this stage we already knew the answer of multiplying a negative integer with a positive integer, thus, -5 x 3 = -15. To complete the above mathematics statement, that is, [-15 + (-5 x -3) = 0], we used the same approach, namely (-5 x-3) must be 15 in order to get zero when added to -15. From this endeavour the assertion that negative multiplied by negative is equal to positive was supported by mathematical reasoning. Co-researchers' active participation in PBLW did not only develop procedural understanding, but conceptual understanding referred to as relational understanding of mathematics (Skemp, 1978:9). They stopped viewing mathematics procedures as unrelated rules to be memorized, and developed an intuition that every mathematics assertion should be justified by mathematics reasoning for it to be comprehensible to learners.

4.6.7 Successful implementation of the coordinated team work in enhancing MPCK using PBL

In essence, team work gives teachers a platform to share problems and consequently get emancipated in their teaching practice. The PBLW sessions which Tua facilitated indicated success with the establishment of the coordinated team (see section 4.3.1). The empirical evidence reflected in Figure 4.13 above illustrates that the coordinated team was successfully established. It is important to note that these coordinated team meetings, referred to as PBLW, created a platform for lesson planning, sharing of good practices and resolving problems regarding mathematics content knowledge and lesson presentation. The debate that ensued during the lesson planning meetings as extrapolated from Falafala's claim confirmed that the PBLW provided co-researchers with an opportunity to share pedagogical knowledge (see section 4.3.1). The following figure, Figure 4.43, demonstrates how best practices were shared by the co-researchers during the PBLW sessions, as they always did their utmost to make the concepts understandable to learners.

Falafala: "I am used to use this one (pointing at angle) $C\hat{A}D$ *why am using this one, because, sometimes this sign (pointing at ∠) they (learners) do not accurately write it, but that cap (pointing at the angle symbol above A in angle CAD) is that one that makes it clear in terms which angle, are we talking about. I use it most of the time.*

Figure 4.42: Falafala's way of angle presentation

Co-researchers adopted and practised what seemed to be working. Evidently, Figure 4.16 exhibits how Zintle adopted and practised the skills she learnt from a PBLW session (see section 4.3.3). These co-researchers' interactions apparently had what Mahlomaholo (2012:293) calls reciprocal beneficiation. As they shared their lived experiences, they seemed to improve their MCPK while explaining to others the way they handled certain challenges regarding the presentation of mathematics concepts to learners. Those who earlier felt they had gaps in their knowledge regarding particular concepts, expressed the opinion that they were emancipated through

PBLW. Falafala, for example, attested how the coordinated team planning helped her regarding division of fraction by fraction as she claimed that

> *"more than we think we know what we do, you can find out, maybe you come out with an easy way to help learners understand mathematics concepts ... like it is much easier to divide anything by one".*

In conclusion: it appears that coordinated team work successfully anchored the strategy to enhance MPCK using PBL. During PBLW sessions co-researchers successfully planned together, shared their classroom experiences and problems which they resolved together with team members, and shared their success stories, which ultimately were adopted by other team members. In a nut shell, PBLW sessions seemed to have emancipated teachers in terms of their MPCK. The findings of this study affirm the notion that coordinated teams enhance teachers' PCK (cf. Jita & Mokhele, 2014:1). In accordance with Evens, Elen and Depaepe, (2015:2), this study confirmed that a collegial, supportive working environment benefits teachers' PCK development

4.7 CONCLUSION

In our research, we organized coordinated teams and four components of PCK into a pentagonal form, with PCK in the centre, as reflected in the figure below (Figure 4.44).

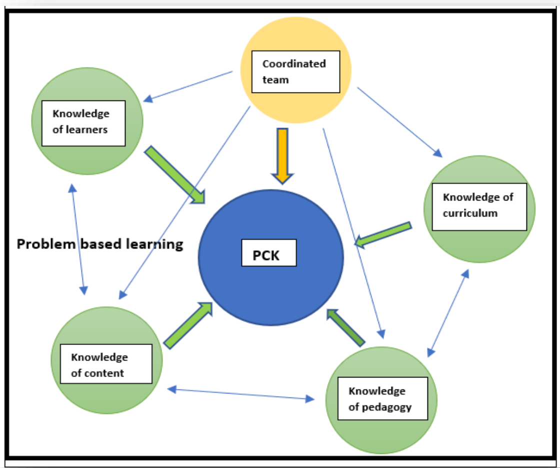


Figure 4.43: Model for a strategy to enhance PCK using PBL

This model is explicitly summarizing the employed strategy to enhance MPCK using PBL. It encapsulates our holistic approach of developing a strategy to enhance MPCK for Grade 9 teachers using PBL. At the centre, PCK is enhanced through interface and development of five vertices of the pentagon. The square represents a PBL environment in which these five vertices of the pentagon are developed. The arrows from the circle represent the coordinated team, indicating that all four PCK components on which this study focused, were developed through coordinated team work, during which problems and success stories were shared. As teachers get emancipated in terms of these four PCK components, their MPCK inevitably was enhanced.

CHAPTER 5 : FINDINGS, CONCLUSIONS AND RECOMMENDATIONSH

5.1 INTRODUCTION

The aim of this study was to develop a strategy to enhance Grade nine teachers' PCK using PBL. In this chapter of the study report a summary of the findings is provided in accordance with the study objectives, the components of the strategy, favourable conditions for its applicability, threats that could potentially derail its implementation and indicators of success in relation to the presented empirical evidence of its applicability. The recommendations are based on the findings. The conclusion also briefly reflects on the value of the study to the teaching and learning of mathematics.

5.2 BACKGROUND

SA education policies provide that teaching and learning of mathematics should develop deep conceptual understanding (DoE, 2011:17). In terms of the NDP teachers should have a sound knowledge of subjects they teach, particularly mathematics (NDP, 2012:303). In affirming policy provisions, teacher should over and above their content knowledge, also know their learners' strength and weakness (George & Adu, 2018:141) in order to appropriately adjust instruction. However, SA teachers seem to have a glaring inadequate MCKT (Mosia, 2016:2; Bansilal *et al.,* 2014:49; Makgato & Mji, 2006: 206). Consequently, learners have to memorize mathematics concepts without understanding (Makgato & Mji, 2006:206). In 2014, Grade 9 mathematics learners obtained 11% national average pass percentage (Bansilal, 2017:3). Poor mathematics teaching is attributed to a significant content knowledge gap (Umugiraneza, Bansilal & North, 2018:72). Teaching in SA is predominantly teacher-centred (Brodie *et al.*, 2002:546) due to poor teachers' conceptual knowledge (Brodie *et al.*, 2002:546). It appeared that teachers used a teacher-centred approach to musk their subject matter incompetence. The problem statement that emerged as a result of the above discussion will be presented in the next section.

5.2.1 Problem statement

Spaul (2013:17) painted a very negative picture displaying the incompetence of SA Grade nine learners when compared to the mathematics competence of Grade eight learners from 21 other middle-income countries. Grade nine mathematics learners were two years' worth of learning behind the average Grade eight pupil (Spaul, 2013:17). Linking learner performance to quality of teaching, it also became evident that poor subject knowledge, and poor mathematics teaching and learning are serious problems in SA education (Diko & Feza, 2014: 1457). In view of this it appeared that teachers have a serious challenge with MPCK, that is, what Shulman (1986:7) theorized as the ability to represent and formulate the subject in order to make it comprehensible to learners. Emanating from the discussion on the background it was evident that many teachers focused on memorization of mathematics algorithms and rules resulting in learners struggling to meaningfully relate mathematics problems to their lives.

5.2.2 Research question

The research question that I wanted to answer by means of this study was: How can mathematics pedagogical content knowledge of teachers teaching Grade 9 learners be enhanced by using problem-based learning?

5.2.3 The aim of the study

The aim of the study was to design a strategy to enhance mathematics pedagogical content knowledge of teachers teaching Grade 9 learners using problem-based learning.

5.2.4 The objectives of the study

The objectives of the study were to:

- identify and analyse challenges that Grade 9 teachers face regarding their MPCK;

- formulate components of the strategy to respond to challenges facing Grade 9 mathematics teachers regarding MPCK using PBL;
- understand conditions for the successful implementation of the strategy to respond to challenges facing Grade 9 teachers in their MPCK using PBL;
- anticipate and circumvent possible threats that could derail the implementation of the strategy to enhance the MPCK of Grade 9 teachers using PBL;
- understand and investigate indicators of success in the implementation of the strategy to respond to challenges facing Grade 9 teachers in their MPCK using PBL.

5.3 FINDINGS AND RECOMMENDATIONS

This section of the report presents a summary of the findings together with recommendations of the study.

5.3.1 Lack of a coordinated team to enhance MPCK for teaching content areas for grade nine curriculum

The findings that emerged from this study showed that there was no coordinated team for mathematics teachers, from which they could untangle mathematics teaching practice problems and inherently develop the wisdom of practice. As it were, a silo mentality even affected teachers that worked in the same school teaching the same Grade 9 learners, but different class groups. There was no team work for people to plan together and develop strategies to make mathematics concepts comprehensible to learners. Due to the non-existence of a coordinated team, mathematics teachers did not have a platform to share their challenges, teaching methods and teaching aids. The implications of the non-existence of a coordinated team did not only deny teachers opportunities to tap from each other's expertise but denied learners the benefits of being exposed to multiple perspectives and different material resources provided by the team members. Emanating from the initial meetings, it appeared that there was an urgent need for the establishment of a coordinated team. In realizing the challenges caused by the non-existence of a coordinated team, the following recommendations are advanced:

5.3.1.1 Recommended strategies for the formulation of the coordinated team

After several consultative meetings with various stakeholders interested in mathematics teaching and learning, it was recommended that the coordinating team be established by members who voluntarily agreed to participate in the study. The meetings culminated in the formation of the coordinated team dedicated to create and implement a strategy to enhance MPCK, with a special focus on Grade 9 mathematics. The team should ensure collaborative planning and reflection among the teachers and recognize learners' marginalized knowledge as valuable to influence their mathematical teaching strategies.

To summarise the study recommends that the team should embrace CAPS and SAQA mandates in terms of good practice. Furthermore, coordinated team work should be institutionalized to refocus both mathematics teachers and other stake holders interested in teaching and learning, as well as collaborative planning and reflection on lived experiences. At least once every ten days, the coordinated team should have one official session for both impact assessment and re-planning to mitigate against professional isolation and nine days for teaching to form what call (1 + 9). Evidently, the presence of a coordinated team became a spring board for self-emancipation of co-researchers as they found an opportunity to engage on the problem-based learning workshops (PBLW). Collaborative development of contextual working strategies of problems faced from lived experiences of mathematics teaching and learning, thus, the use of enemy analogy to make sense of multiplication of negative integers, contributed to knowledge production (see section 4.6.6). The process reduced the abstractness since it acknowledged the lived lives of both learners and teachers. However, this can only happen if the conditions are conducive to the establishment of a coordinated team.

5.3.1.2 Recommended conditions for a coordinated team

The study proposed the following as conditions conducive to the operationalization of a coordinated team, namely, commitment, team norms, shared leadership and open communication. The implementation of the strategy was possible, because co-researchers managed to develop a common vision and the norms that guided the operationalization of the strategy. The team members were given equal responsibilities, and democratic values, respect and social justice were adopted as the guiding framework for the team. The study further recommends the accommodation of a teacher-initiated programme or interventions during the meetings of the coordinated team to encourage ownership of the programme. Restoration of self-worth for team members, through collective leadership and open communication should become the bedrock of the coordinated team. Over and above, co-researchers became committed to the team, when they experienced growth and their views were recognized in the programmes of the team.

5.3.1.3 Threats and risks regarding the creation of the coordinated team

The study identified risk factors that could possibly threaten the implementation of the strategy. The identified risk factors included, but are not limited to the distance between the schools, the use of personal resources to attend PBLW, and a lack of time. We have earlier recommended that the strategy should be institutionalized to avail not only utilizing teachers' personal time, but official time be allocated for professional development. To avoid this threat, the strategic framework should be communicated to their school principals for buy-in (see Annexure G). These threats would be circumvented when the participating schools witnessed teachers' professional and personal benefits in terms of improved mathematics performance. These initiatives and benefits could encourage schools' support of their staff members to fully participate in the coordinated team's agenda. In line with Graham's (2007: 9) assertion that it helps knowing that that there's somebody down the hall that you can engage if you're wondering how to approach something instructionally, we also recommend the use of 'WhatsApp' to encourage communication among teachers to reduce the impact of professional isolation.

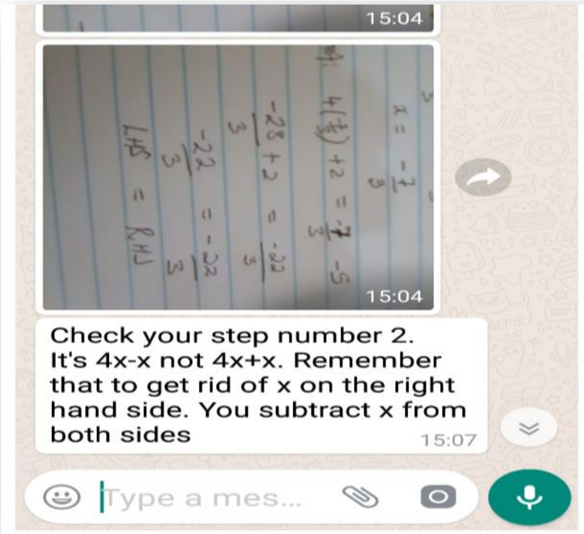


Figure 5.1: WhatsApp

Evidently, Figure 5.1 exhibits how co-researchers used WhatsApp to share and resolve their challenges. The coordinated team created a platform for mathematics teachers to trust each other and know whom to talk to when experiencing instructional problems. In essence, WhatsApp mitigated against the threat caused by the distances between the schools.

5.3.2 Understanding of learners, identification and follow up to their misconceptions

The findings of this study revealed teachers' poor understanding of learners' problems, hence they did not follow up on learners' misconceptions. Instead, they put a cross on learners' work to indicate that it was incorrect and decided to re-teach the mathematics topic. They neither examined learners' misconceptions nor conducted error analyses (see section 4.2.2). They did not even try to establish what their learners knew, but only believed in demonstration first and then evaluating the success of their instructional delivery.

5.3.2.1 Recommended strategy to identify and follow up on learners' misconceptions

The study recommends that for teachers to be able to avoid a teaching and learning cul-de-sac, they should follow up on learners working to understand what learners can do without any assistance. Evidently, when learners conjoin 5 x + 20 to make 25, co-researchers were able to use prompts as scaffolds for learners to realize like terms as reflected in Figure 4.15 (see section 4.3.2). Identification and remediation of learners' misconceptions made teachers to be expert learners as they tried to understand their learners' ways of solving mathematics problems. In the process they developed new and appropriate strategies to help learners comprehend mathematics concepts from learners' congruence perspective. Probing learners' thinking and allowing them to support their mathematical assertion by mathematical reasoning is recommended. This process develops humility in individuals to accept that their initial positions were not correct as they discover their mathematical errors. On the other side, teachers think deeply about the alternative ways, analogies and manipulatives that could be used to effectively bring concept clarity to learners. While engaged in deep thinking, they inevitably get their MPCK enhanced and develop new knowledge that would be used to resolve more or less similar problems in future. This new knowledge is what Shulman (1986: 9) called 'wisdom of practice'.

5.3.2.2 Recommended conditions conducive to identification and following up on learners' misconceptions

From the epistemological stance that knowledge is socially constructed, dialoguing with learners to understand their preconceptions and misconceptions is recommended. Peer learning, where team members work together in designing solutions to the problems would be most appropriate. Milwa (a learner from Njovane's class) had this to say when asked how they would have liked mathematics to be taught:

> *If I were a teacher, I would let one of the learners who seems to understand the concept to explain it to other learners, they may understand it better from one of them.*

These learners' voices were a call for a need to critically participate in knowledge production through expressing their views and understanding of the concept to their equals, thus, peers. Learners' lived experience about learning indicated that they learned better from others, hence peer learning is recommended. Furthermore, from these experiences, teachers would be in a position to share some of the challenges and problems that could not be resolved in class with coordinated team members. Falafala presented what worked for her class in terms of the strategy to help learners accurately identify the required angles when parallel lines were cut by transversals (see section 4.4.4). The study therefore recommends constant PBLW sessions to allow teachers to share success stories and classroom discourse problems that continue to bedevil them when they are working alone.

5.3.2.3 Threats and risks with regard to identification and follow-up on learners' misconceptions

Focusing on syllabus completion at the expense of learners' understanding of mathematics concepts seems to be a threat that requires concerted efforts to identification and follow-up on learners' misconceptions. Algorithms' compressed mathematics concepts act as a short cut without meticulously revealing all relational mathematics computations involved. Teachers tend to rely on algorithms for curriculum coverage. These threats would be circumvented when co-researchers experience professional growth acquired from PBLW in terms of their teaching practice and ability to demystify learners' misconceptions. Extrapolated from a reflection session during one of the PBLW sessions, Jones's comments revealed a certain level of growth since she had joined the coordinated team

> *Jones: When we meet like this you end up liking parts of mathematics that you initially did not like.*

Although not overtly put, her appreciation of the topics that she did not like before she joined the team could be viewed as professional development on her part. By implication, she would also teach these topics that she earlier did not like. On the other side, learners' improved performance was evidenced by Rhulumente's use of enabling prompts in scaffolding learners to understand associative property of multiplication (see section 4.3.2). The experience of success brought about a 'wow' effect to the group of learners when they realized how important it was to use what they already knew to solve similar problems. The study therefore recommends and submits that the identification of and follow-up on learners' misconceptions did not only help learners but also enhanced teachers' MPCK.

5.3.3 Insufficient utilization of curriculum materials when teaching

The findings of this study showed that mathematics manipulatives were not used by mathematics teachers. The opportunities for learners to pose questions and the experience of grounded conversation provided by manipulatives were limited due to the non-use of mathematical manipulatives. Inevitably, not using mathematical manipulatives reproduced teacher domination in the class. Overarchingly, teachers that do not use manipulatives, lose the opportunity of concretely representing the abstract mathematics concepts to develop deeper insight of mathematics concepts and inevitably becoming better mathematics teachers in turn. The inaccurate hand-generated cartesian plane had no standard measurements between x values on the x axis and y values on the y axis resulting in the graph drawn from this type of cartesian plan losing mathematical meaning (see section 4.2.3).

5.3.3.1 Recommended strategies to encourage sufficient utilization of curriculum materials when teaching

From an epistemological perspective that views knowledge as subjective people's experiences which do not only regard the privileged and powerful, but also values the narratives and views of marginalized, teachers should create an environment that enables learners to share available manipulatives. As reflected in Figure 4.19, paper cuttings helped learners to accurately identify the correct angles. Evidently, these paper cuttings were not only used as manipulatives but were also used as enabling prompts to help learners accurately identify and correctly label the required angles. Manipulatives enabled learners to have something to work with; thus, tools of trade that exemplified and defended their thinking. Figure 4.20 exhibited how learners tried different ways of physically demonstrating that given triangles were congruent (see section 4.3.3). Moseoa (a learner in Ntozine's class) had this to say to explain his understanding of congruency.

> *Moseoa (a learner): Congruence is when angles ... em triangles that can exactly fit on top of each other, that are equal.*

The study therefore recommends the use of manipulatives as powerful tools which enable learners to understand mathematics concepts. Using manipulatives makes the learning process engaging, meaningful and relevant for learners, because they acquire the knowledge, skills and tools to deal with the kind of problems they will encounter in future. Learners used manipulatives as tools to advance and defend their mathematics assertions.

5.3.3.2 Recommended conditions conducive to sufficient utilization of curriculum materials when teaching

The study recommends the creation of a conducive atmosphere for learners to use manipulatives, such as team work. From different team members, learners would see different perspectives in terms of how the manipulative relates to abstract mathematics knowledge with what they already know, either form real life or prior knowledge. Moreover, detailed lesson planning enables teachers to embrace learners' pre-existing knowledge in the lesson through using different resources. Evidently, detailed lesson plans clearly indicate manipulatives that would be required to exemplify and represent particular mathematics concepts (see section 4.3.4). Over and above, the coordinated team presents an opportunity for collaborative lesson planning and sharing of resources with success stories in helping learners understand concepts. During coordinated team planning meetings, team members shared problems regarding the shortage of manipulatives in their schools, or their obliviousness about relevant manipulatives for particular topics. For instance, Falala used manipulatives to help a team member regarding her inability to help learners correctly identify the required angles (see section 4.3.1). The study also recommends the inclusion of manipulatives as a standing agenda item during PBLW.

5.3.3.3 Factors threatening the promotion of sufficient utilization of curriculum materials when teaching mathematics

The environment with manipulatives threatens teachers' position of being the only ones to approve which mathematics answers are correct or not (see section 2.4.3.1.4). For teachers to claim back their position of power in the mathematics classroom discourse, they often stopped using manipulatives. Our observations and interaction with co-researchers at the research sites to a certain extent revealed teachers' belief that seemed to threaten the effective utilization of manipulatives. During the reflective session after the class visit Njovane contended that:

> *Uhm, … teaching aids, I do not necessarily need teaching aids in expressions and factorization and as a result I do not have them, but they are usually available for statistics.*

His belief threatens any attempt of effective utilization of manipulatives in particular topics. Consequently, he deliberately avoided the use of manipulatives in topics such as expressions and factorization. Notwithstanding these deep-rooted beliefs that seemed to threaten effective utilization of manipulatives, the coordinated teamwork circumvented this threat as co-researchers shared their experiences of using manipulatives in developing understanding of mathematics concepts, even those concepts which may seem to be trivial or easily understandable. For example, Falafala shared her experience of using what she called a 'cap' in identifying the required angles (see section 4.3.1). This (Falafala's cap) was later adopted by Zintle in her class (see section 4.3.3).

On the other hand, Ntozine contended that *"*When you need them you need to personally purchase them*"*. Her protest holds a threat for the utilization of manipulatives in her class. Apparently, the non-availability of manipulatives in her school demanded from her to buy some. However, when we inquired whether she had reported the matter, we learnt that she never did. The team advised the co-researchers not only to think about sophisticated manipulatives, but to use the available resources at school to design mathematics manipulatives. Evidently, the use of simple chat papers by Zintle and Ntozine as reflected in Figure 4.16 and 4.17 respectively illustrates that the threat was circumvented.

5.4 INADEQUATE LESSON PLANNING

The findings of this study demonstrate that prior to this study mathematics teachers that participated in the study either did not have or had insufficient lesson plans. Failure to adequately prepare a mathematics lesson results in ineffective teaching, which may lead to learners finding mathematics difficult to comprehend and as a result they had to re-teach some lessons. When we reflected on Rhulumente's lesson presentation, she had this to say: *"*I do not think they understood the lesson; I have to repeat it*".*

The empirical evidence, namely the lack of preparation and lesson plans, seemed to impair teachers' MPCK development. The co-researchers who did not thoroughly prepare their mathematics lessons denied themselves an opportunity of becoming better teachers in terms of MPCK.

5.4.1 Strategies recommended to foster sufficient lesson preparation

A study recommendation is that the coordinated team should design scheduled sessions for collaborative lesson planning to develop team members' MPCK. Logically, a successful mathematics lesson cannot be divorced from effective MPCK. It is further recommended that teachers should focus on developing ways that would make mathematics concepts teachable; in turn they get emancipated and acquire skills to not only teach what they regard as rules, but demystify mathematics concepts. For instance, during collaborative lesson planning Nowele drew the attention of the team towards aspects of mathematics concepts suitable to improve its teachability:

> *For learners to see which fraction is below the dividing line let us put one sixth below the dividing line, to show that we are dividing half by a sixth as compared to representing them like* $\frac{1}{2} \div \frac{1}{6}$.

Her argument made sense to the team: when the fraction below the dividing line has been eliminated, it will be clear to learners that the fraction above is divided by one, and therefore remains the same. The study further recommends that lesson planning should carefully anticipate learners' misconceptions and use them as foundation for lesson planning. In order to improve MPCK, the study recommends that during preparation, a team of teachers should share their individual understanding of an identified concept and how they think it could be best represented to develop learners' understanding. Evidently, Falafala's 'cap' for angle identification and Zintle's 'enemy' analogy for multiplication of negative integers were new strategies developed to make mathematics concepts comprehendible to others (see sections 4.3.1 & 4.6.6 respectively).

5.4.2 Conditions conducive to fostering sufficient lesson preparation

The study recommends the creation of an atmosphere conducive to collective wisdom through collaborative lesson planning. When teachers plan together, they tap from each other's expertise in terms of wisdom that has been acquired through practice in solving real problems encountered in teaching and learning of mathematics. The reflection done after our planning session suggested that co-researchers experienced mutual and reciprocal beneficiation as Falafala confirmed that:

> *Yah, it assists us, for one to go to class well prepared, more than we think we know what we do, you can find out, maybe you come out with an easy way to help learners understand mathematics concepts ... like it is much easier to divide anything by one.*

As extrapolated from the above excerpt, Falafala seemed to have developed thinking about how to make mathematics concepts understood by their learners. It seems that Falafala claims that planning together changed her practice, and this is an indication that the coordinated team creates conditions conducive to detailed lesson planning. The coordinated team presented an opportunity for diverse views on planning a lesson that would create fertile ground for learners to develop a deeper insight in mathematics concepts. When teachers plan together, focus on meaning making and how to help learners develop a deeper understanding of concepts. They developed and adopted new strategies from the coordinated team work and stopped teaching mathematics as just unrelated rules.

5.4.3 Factors that could threaten lesson preparation

Lesson planning requires a great deal of time, which teachers do not necessarily have, as they teach in many grades and other subjects at times. Consequently, teachers tend to complain about overload. On the other hand, the supervision of lesson plans is not done thoroughly especially where a mathematics teacher happens to be the only mathematics teacher in a school. To circumvent these risks, namely a lack of time, overload and a lack of supervision the study recommends collaborative lesson planning through coordinated team meetings.

Co-researchers' reflections regarding PBLW, asserted that:

> *The teamwork keeps us updated, because every team member is going to make it a point that targeted work is covered, unlike when there is no one who supervises you, in this programme you are supervised by your colleagues. You will personally be embarrassed when you come to the next meeting and discover that your colleagues have moved and you did not do anything, you will feel ashamed and then you will end up working now.*

Extrapolated from the excerpt above, teamwork created a safety net that encouraged team members to be steadfastly guided by the team norms as the true north of their operations. No one in a team who would like to let down the team. Ntozine's opinion, namely "*unlike when there is no one who supervises you",* suggests that the team also provides the supervision aspect which seems to be lacking at school level. On the other hand, the reciprocal benefits of collective wisdom encouraged teachers to plan for the topics that they would have avoided to teach if they were not part of the team. The coordinated team created time for collaborative planning and that reduced the burden of planning alone.

5.5 NON-INTEGRATED ASSESSMENT WITH LESSON FACILITATION

The findings of the study revealed that assessment was treated as a separate entity from lesson presentation. Teachers did not only believe in demonstration first and assessment later, they actually enacted their beliefs. Consequently, they failed to integrate assessment with lesson presentation. During the reflection session, they contended against assessment-embedded instruction, arguing that learners needed to be shown first how to solve mathematics problems. At the end of the lesson learners were given different problems based on the same concept, which were solved through following the method earlier demonstrated. Teachers lost the opportunity to establish learners' competences and did not use prior knowledge as the springboard for the introduction of new concepts. They only taught learners on the assumption that by virtue of being in Grade 9 they were supposed to understand topics that were presumably done in previous grades. They could only discover after the lesson that learners did not understand what was taught and it was too late for early interventions and adjustments that could have been done during lesson presentation.

5.5.1 Recommendations to foster assessment-embedded instruction

The study recommends lesson facilitation to be integrated with assessment and instruction. The purpose of assessment is to establish the cutting edge of learners' competences, particularly formative assessment. This kind of assessment is part of instruction; it is not used for making decisions regarding learners' promotion to the next grade. Fundamentally, it guides the interaction between the teacher and learners so that the instruction is appropriately adjusted for effective teaching and learning. The study recommends that assessment should also be for the purpose of improving learning and not only for grading purposes. This could be achieved when lesson facilitation is integrated with assessment. Integrating assessment with lesson facilitation does not only establish learners' cognitive levels, but also unearths learners' misconceptions which are the starting point for lesson presentation. For teachers to know what learners understand and think in terms of a particular mathematics concept, they should continuously gauge learners' capabilities during the lesson presentation.

5.5.2 Conditions conducive to encouraging assessment-embedded instruction

The study recommends peer assessment, group work and oral feedback as conditions conducive to encouraging assessment-embedded instruction. Frequent feedback received from assessment-embedded instruction is utilized to improve learning through timeous adjustment of instruction during lesson presentation, not when the lesson is over. When learners assess each other's work, the teachers receive a deep insight of learners' misconceptions and what they are capable of. In the group work the teachers do not only assess the product, but the process too, which is an impotent part of learning.

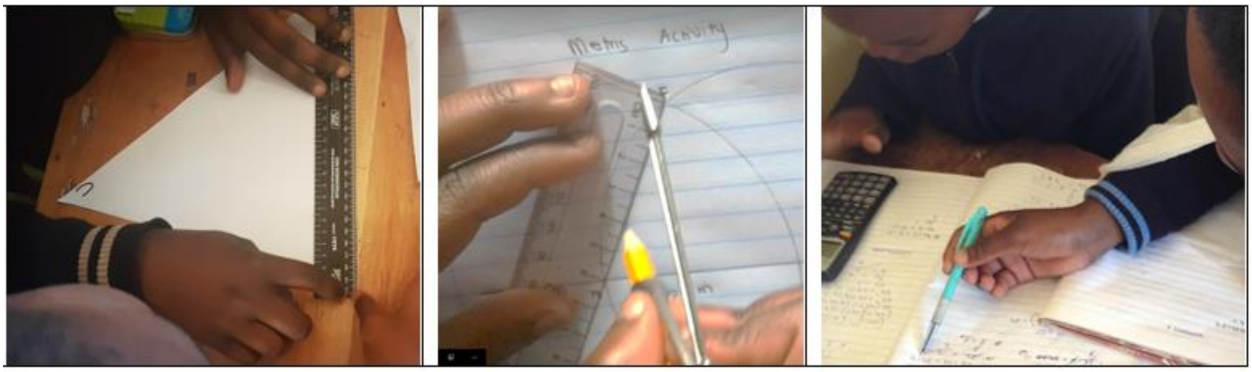


Figure 5.2: Group work

Evidently, when learners work together, the teacher finds the opportunity, not only to assess the product, but also the process and skills involved and needed to solve a particular mathematics problem. On the left and the right sides of Figure 5.2 learners collaboratively work together on processes required to solve a problem, while the middle picture of the same figure demonstrates the skills involved in an accurate drawing of an equilateral triangle. From the above synopsis, the teachers were able to assess learners' capability while the lesson is underway and consequently provided necessary scaffolds to help learners achieve the learning trajectories. In the same vein, the dialogue between learners as they assess each other during the process of solving the given problems helped the teachers to appropriately intervene. The study further concludes that discussion and dialogue provide possibilities for learners to engage with the feedback, not only to correct the incorrect answers, but also to understand the logic behind given answers and understand why their answers are correct or wrong. Group work created a condition conducive to assessment-embedded instruction.

5.5.3 Factors threatening the implementation of assessment-embedded instruction

More often than not, teachers believe in first demonstrating how mathematical problems are solved and assessed at the end of a lesson when little could be done to nip the problem from the bud. They tend to present a plethora of examples demonstrating the procedural approach on how particular mathematics tasks are solved, for instance, converting the fraction, and changing division to multiplication. When learners have memorized the procedures after having watched a variety of examples demonstrated, they reproduce the same procedures without deeper understanding of why the fraction was flipped over. To circumvent these beliefs and practices, teachers should participate in collaborative planning where they will be exposed to the processes of assessment embedded instruction and conducting dialogue. Evidently, the discussions that ensued from the planning session of PBLW exhibited that teachers shared experiences of what others regarded as arbitral rules that only can be learnt through memorization, thus, enemy analogy representing multiplication of negative numbers (see section 4.6.6**)**. Therefore, the study recommends consistent, structured and institutionalized collaborative planning meetings to expose teachers to multiple perspectives so that they could let go of their practice of separating assessment from lesson presentation.

5.6 NON-IMPLEMENTATION OF A LEARNER-CENTRED APPROACH TO TEACHING

The findings of the study showed that there was non-implementation of a learner-centred approach. Consequently, teachers failed to create the opportunity for learners to develop and employ critical thinking skills. Instead, they perpetually reinforced memorization of mathematics rules without a deeper understanding of mathematics concepts. The belief that learners are learning when the teachers dominate the class presentation, and learners' assumption that they are learning when they memorize without understanding foster rote learning. A lack of opportunities for learners to pose questions in a teacher-centred mathematics class forces learners to learn subject rules without understanding. Most questions asked in a teacher-centred approach do not prompt learners to think critically about the concepts at their disposal, but rather to comply with the classroom discourse they find themselves in. And as such they would mechanically say yes without really engaging in a discussion with the teacher or fellow learners regarding mathematics concept taught (see section 4.2.6).

5.6.1 Recommendations to encourage learner centred pedagogical approach

The study recommends a classroom discourse that embraces a learner-centred pedagogical approach (LCPA). LCPA focuses the teacher on how actively learners learn. Moreover, in an LCPA, teachers seem to respect learners' ideas and allow for a democratic process for everyone to advance and negotiate ones' ideas. In essence, PBL as a learner-centred approach allows an opportunity for social structure to be challenged to enhance democratic participation. As co-researchers, grouped learners into small groups, learners in turn employed critical thinking in solving given mathematics problems. When Siponono, a learner from Rhulumente's class, reflected on his experience about Rhulemente's lesson presentation, which was predominantly learner-centred, he had this to say:

> *Sponono: It is that when you have a certain opinion about how you see mathematics problem you were allowed to express it.*

From the above excerpt, learners were not only allowed to voice their views but their opinions were valued as part of mathematics learning. The co-researchers worked on challenging learners' decision making and reasoning processes. As attested to by the co-researchers, this approach empowered teachers as well in their classroom practices, as evidenced by Rhulemente's reflections in which she displayed confidence about the success of her lesson in terms of achieving its objective (see section 4.3.6). Finally, LCPA, allowed intersubjective views and provided a chance to the co-researchers to develop confidence, and encouraged learners to not only use conventional procedures and algorithms when solving mathematics problems. The study recommends the use of probing questions and enabling prompts to critically engage learners in the process of teaching and learning.

5.6.2 Recommended conditions that support a learner-centred pedagogical approach

The study recommends judicious utilization of manipulatives to encourage implementation of LCPA. Evidently, the effective utilization of manipulatives enabled co-researchers to employ LCPA (see section 4.3.3). As reflected in the above-mentioned section learners were able to measure and put on top of each other manipulatives at their disposal to list minimum conditions required for triangles to be congruent. Our observation of lesson presentations at the research sites exhibited that teachers' use of manipulatives enabled them to engage learners in active meaning making through the use of manipulatives as reflected in Figure 4.16 and Figure 4.17 (see section 4.3.3). In line with the above-mentioned findings, this study submits that judicious use of manipulatives is one of the conditions that encourage LCPA. As learners focus on the problem at their disposal, they use manipulatives as scaffolds without being entirely reliant on teachers' approval in terms of whether the solution they came up with, was mathematically correct or not. In essence manipulatives are used as tools to explain, why learners think and believe their solutions and procedures are mathematically sound.

Secondly, the study also recommends the use of probing questions and enabling prompts to support LCPA. Probing questions and enabling prompts helped learners focus on compressed mathematics concepts needed to be demystified before learners could untangle complex problems. Probing questions and enabling prompts further focused learners' uncertainty about particular mathematics concepts. Evidently, the following engagement between Njovane and his learners supported that narrative that probing questions gave clarity on what learners could do or could not do while the lesson was underway.

> *Njovane: Why have you decided to add or multiply?*
>
> *Seemane (learner): We made a mistake, one of us insisted that we must add although we wanted to multiply, we then cancelled multiplication and put addition.*
>
> *Njovane: Eh…eh, why?*

The use of 'whys' in his questioning, gave learners the opportunity to present what they thought. Without them displaying why they acted in a particular manner, he would not have known their challenges. On the other hand, Rhulumente intervened in learners' misconceptions through the use of enabling prompts. When learners could not see that (a × b) is the same as (b × a) she gave them a simpler but similar task, namely 4 × 6 and 6 × 4 to enable them to see associative property in multiplication (see section 4.3.2). In coining our recommendations, it also appeared that the use of probing questions was one of the conditions conducive to put learners at the centre of learning and teaching process.

5.6.3 Factors that could threaten the implementation of a learner-centred pedagogical approach

It has been articulated earlier that the use of manipulatives created conditions conducive to encouraging LCPA. On the other side of the coin, non-availability of manipulatives could threaten the effective implementation of LCPA. For instance, Mbuyi's argument that manipulatives fell from the wall was her reason why there were no manipulatives used in her class, while Ntozine contended that one should personally purchase manipulatives to have them (see section 4.2.3). Moreover, the teachers' belief about how mathematics was supposed to be taught seemed to threaten the use of LCPA as classroom discourse. Evidently, Njovan and Mbuyi's agitation that they were supposed to first demonstrate to learners so that they could reproduce what had been demonstrated revealed their disbelief in LCPA (see section 4.2.5).

To challenge these beliefs and practices, teachers should participate in collaborative planning where they will be exposed to how effective LCPA through the use of enabling prompts and probing questions to actively engage learners in the process of meaning making. Evidently, the discussions that ensued from the planning session and the enactment of using of 'cap' to identify required angles (see section 4.3.3), as well as forms of representing division by fraction in a way that was most suitable to its teachability (see section 4.3.1), challenged teachers' deep rooted beliefs in the teacher-centred approach. Finally, the use of enabling prompts simplified a complex problem for learners and they ultimately realized how associative works in multiplication of algebraic expressions without being directly given an answer by the teacher (see section 4.3.2). Therefore, the study recommends collaborative planning and enabling prompts to mitigate against factors that could threaten implementation of LCPA.

5.7 INSUFFICIENT KNOWLEDGE OF MATHEMATICS CONTENT FOR MATHEMATICS TEACHING

The findings of this study indicate that teachers have insufficient MCKT, and at the least, their understanding is at a procedural level. Notwithstanding, their procedural knowledge but they are unable to explain why they follow specific mathematical procedures. It appeared that they do not have a deep conceptual understanding. They could not provide a precise definition of a concept like algebraic equation, instead they presented a convoluted explanation that does not clearly tell exactly what the concept says or does or say. Consequently, they mostly use safe-talk and I-R-E classroom discourses to conceal their limitations in terms of MCKT. Their self-admission that learners did not understand concepts that were taught is also an indication of inadequate MCKT as the research argues that there is a positive correlation between MCKT and learner performance (Venkat & Spaull, 2015:122; Nambira, 2016:35-36). Njovane's inability to pose questions that stimulate critical thinking instead of questions that only require yes and no answers revealed a lack of MCKT, as he struggled to pose questions that created a cognitive disturbance (see section 4.2.7).

5.7.1 Recommendations to develop teachers' understanding of MCKT

The study recommends the establishment of a coordinated team to create a platform to share and resolve problems experienced in the teaching practice. It appeared that such a team provided a safe environment for mathematics teachers to share their frustrations, weaknesses and success stories with the intention to collaboratively learn without a fear of being judged. Evidently teams promote collegiality by reducing isolation (see section 4.2.1). As the team members became familiar with PBLW, it was easy for them to not only present their claimed success stories, but to also share them with the team - real problems as they encountered in their mathematics classrooms. During the reflection session about our experience regarding team planning Nowele submitted that:

> *We gained something, we were taking it for granted that it is just a rule, if you divide by fraction you just multiply, eh, you just change the division to a multiplication sign; as we work together, we gained a lot in terms of where it starts.*

As extrapolated form the above extract, team members developed thinking about how to make mathematics concepts be understood by their learners. According to Nowele, collaborative lesson planning helped her to understand the genesis of what seems to be literally flipping the fraction upside down and changing the division sign to a multiplication sign. As co-researchers decompressed mathematics meaning of division by fractions, they realized the most effective forms of concept representation. Using the reciprocal for them was not just a mathematics routine, but a purposeful endeavour to decompress mathematics concepts so that learners could see and understand mathematics reasoning underpinning mathematics algorithms and mathematics concepts. The study therefore recommends the establishment of PBLW as platform used to untangle teachers' mathematics content knowledge for teaching.

5.7.2 Recommended conditions to develop teachers' understanding of MCKT

From the epistemological stance that knowledge is socially constructed, the establishment of a platform for team members to tap from each other's strengths and reduce weakness becomes a vital condition for continued emancipation in terms of MCKT. PBL emphasises peer learning where team members work together in designing solutions to the problems (see section 2.5.4). Co-researchers embraced the above-mentioned conditions that foster continued development in MCKT. Evidently, planning together, and sharing lived experiences regarding problems and success stories are conducive conditions that encourage continued enhancement of co-researchers' MCKT. Fafafal's expression of self-discovery evidently suggests that collaborative lesson planning encouraged development of MCKT.

> *Yah, it assists us, for one to go to class well prepared, more than we think we know what we do, you can find out, maybe you come out with an easy way to help learners understand mathematics concepts ... like it is much easier to divide anything by one.*

She admitted that what she thought she knew was not sufficient for teaching mathematics. Her reflection on collaborative lesson planning illustrates her attention towards mathematics concepts and aspects pertinent to its teachability. Her understanding of developing ways of rendering mathematics concepts more understandable for learners is attributed to collaborative lesson planning. Seemingly, collaborative lesson planning presented excellent conditions for sharing problems.

5.7.3 Factors that could threaten strategies to develop teachers' understanding of MCKT

Lack of time for coordinated team meetings seemed to be a threat for effective operations of the coordinated team (see section 2.4.1.1). Furthermore, conditions such as being the only teacher for mathematics in a school, and geographical isolation, distance between schools, bad road conditions and the use of personal money to attend the cluster meetings militated against the existence of collaborative teams (see section 2.4.1.2). Co-researchers' biographic data (see section 3.3) provides a synopsis of the rural nature and the terrain of the schools that participated in this study. Team members were the only mathematics teachers in their schools, except Rhulumente and Njovane who taught in the same grades but in different class groups at the same school. Falafala and Nowele, also taught the same grades but different class groups in the same school. The rest of the co-researchers are the only teachers in their schools.

To circumvent this threat, the meeting resolved that over and above the consent forms that were issued to participating schools, our strategic framework also should be communicated to their school principals (see Annexure G). The threat was circumvented through principals' buy-in to our intervention strategy as it included issues that were not only benefits towards the co-researchers' emancipation regarding their MPCK, but also issues such as common standardized examination papers and the moderation process which were of benefit to the schools as well. Secondly, the benefits provided by team work encouraged co-researchers' commitment towards PBLW. Evidently, co-researchers' reflections regarding PBLW, asserted that:

> *The teamwork keeps us updated, because every team member is going to make it a point that targeted work is covered, unlike when there is no one who supervises you, in this programme you are supervised by your colleagues. You will personally be embarrassed when you come to the next meeting and discover that your colleagues have moved and you did not do anything, you will feel ashamed and then you will end up working now.*

Extrapolated from the text in the excerpt above, it appeared that the teamwork created a safety net that encouraged lesson planning and the sharing of lived experiences. No one in a team who would like to let down the team. Ntozine’s words, namely, “*unlike when there is no one who supervises you”,* suggest that the team also provides the supervision aspect which seems to be lacking at school level. In a nutshell, threats towards detailed lesson planning were circumvented by the team norms (see Appendix 10).

5.8 SUMMARY OF THE STRATEGY AND CONCLUSION

The aim of this study was to develop a strategy to enhance Grade nine teachers’ MPCK using PBL. At the centre of the study was the enhancement of MPCK through using PBL, with the intention to collectively resolve mathematics problems encountered by teachers through PAR. Implicitly, the strategy focused on collaboratively identifying teaching practice problems in relation to MPCK components, thus, knowledge of learners, knowledge of the curriculum, knowledge of content, and knowledge of pedagogy. Overriding, all these identified challenges were discussed, shared and resolved through coordinated team meetings that we referred to as PBLW. As we discussed individual co-researchers’ problems and shared success stories we reciprocally benefited and produced new knowledge of how to handle challenges that continued to restraint co-researchers before they joined the coordinated team. Developing each of these PCK components inevitably enhanced MPCK.

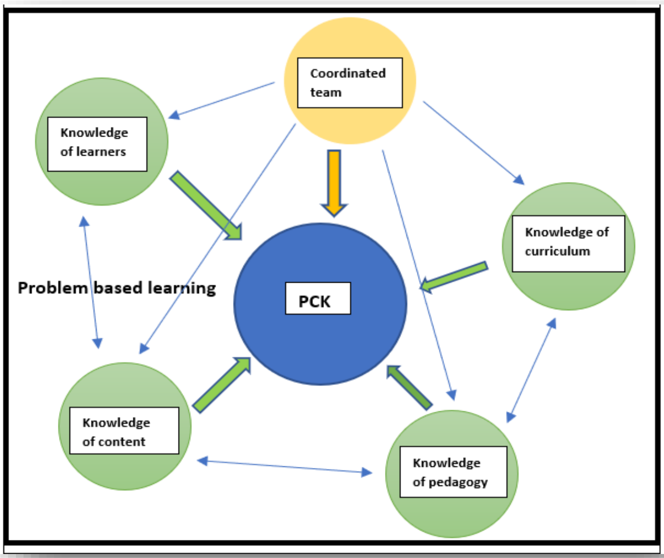


Figure 5.3: Model for a strategy to enhance PCK using PBL

This model is explicitly summarizing the employed strategy to enhance MPCK using PBL. It encapsulates our holistic approach of developing a strategy to enhance MPCK for Grade 9 teachers using PBL. At the centre, PCK is enhanced through the interface and development of five vertices of the pentagon. The square represents the PBL environment under which these five vertices of the pentagon are developed. The arrows in the circle, representing the coordinated team, indicate that all four PCK components that this study was focused on, developed through the coordinated team work, where problems and success stories are shared. As teachers get emancipated in terms of these four PCK components, their MPCK is inevitably enhanced.

This chapter presented the findings of the study, which exhibited that teachers worked in silos. The findings further revealed teachers' poor understanding of learners, that mathematics manipulatives were not used, insufficient lesson plans, assessment was treated as an entity apart from the lesson presentation, teacher-centred classroom discourse, and teachers' insufficient MCKT. These findings justified the formulations of strategy components to enhance MPCK using PBL. The study also reported on the implementation of the strategy components, including conditions conducive to the implementation, and how threats that could derail its implementation were circumvented. Finally, the study presented a summary of evidence demonstrating that collaborative team work tried to resolve individual co-researchers' problems, resulted in coordinated team members developing knowledge of handling those problems including the ability to deal with relatively similar problems in future.

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APPENDIX 1: LETTER OF APPROVAL FROM THE HEAD OF DEPARTMENT

Province of the
EASTERN CAPE
EDUCATION

STRATEGIC PLANNING POLICY RESEARCH AND SECRETARIAT SERVICES
Steve Vukile Tshwete Complex • Zone 6 • Zwelitsha • Eastern Cape
Private Bag X0032 • Bhisho • 5605 • REPUBLIC OF SOUTH AFRICA
Tel: +27 (0)40 608 4773/4035/4537 • Fax: +27 (0)40 608 4574 • Website: www.ecdoe.gov.za

Enquiries: B Pamla Email: babalwa.pamla@edu.ecprov.gov.za Date: 08 May 2015

Mr Bedeshani Moses Mceleli
P.O. Box 490
Matatiele
4730

Dear Mr Mceleli

PERMISSION TO UNDERTAKE A DOCTORAL STUDY: ENHANCING MATHEMATICS PEDAGOGICAL CONTENT KNOWLEDGE IN GRADE 9 CLASS USING PROBLEM BASED LEARNING

1. Thank you for your application to conduct research.

2. Your application to conduct the above mentioned research in four Junior Secondary Schools under the jurisdiction of Mount Fletcher District of the Eastern Cape Department of Education (ECDoE) is hereby approved based on the following conditions:

    a. there will be no financial implications for the Department;

    b. institutions and respondents must not be identifiable in any way from the results of the investigation;

    c. you present a copy of the written approval letter of the Eastern Cape Department of Education (ECDoE) to the Cluster and District Directors before any research is undertaken at any institutions within that particular district;

    d. you will make all the arrangements concerning your research;

    e. the research may not be conducted during official contact time, as educators' programmes should not be interrupted;

    f. should you wish to extend the period of research after approval has been granted, an application to do this must be directed to Chief Director: Strategic Management Monitoring and Evaluation;

    g. the research may not be conducted during the fourth school term, except in cases where a special well motivated request is received;

h. your research will be limited to those schools or institutions for which approval has been granted, should changes be effected written permission must be obtained from the Chief Director: Strategic Management Monitoring and Evaluation;

i. you present the Department with a copy of your final paper/report/dissertation/thesis free of charge in hard copy and electronic format. This must be accompanied by a separate synopsis (maximum 2 – 3 typed pages) of the most important findings and recommendations if it does not already contain a synopsis.

j. you present the findings to the Research Committee and/or Senior Management of the Department when and/or where necessary.

k. you are requested to provide the above to the Chief Director: Strategic Management Monitoring and Evaluation upon completion of your research.

l. you comply with all the requirements as completed in the Terms and Conditions to conduct Research in the ECDoE document duly completed by you.

m. you comply with your ethical undertaking (commitment form).

n. You submit on a six monthly basis, from the date of permission of the research, concise reports to the Chief Director: Strategic Management Monitoring and Evaluation.

3. The Department reserves a right to withdraw the permission should there not be compliance to the approval letter and contract signed in the Terms and Conditions to conduct Research in the ECDoE.

4. The Department will publish the completed Research on its website.

5. The Department wishes you well in your undertaking. You can contact the Director, Ms. NY Kanjana on the numbers indicated in the letterhead or email nelisakanjana@gmail.com should you need any assistance.

NY KANJANA
DIRECTOR: STRATEGIC PLANNING POLICY RESEARCH & SECRETARIAT SERVICES

FOR SUPERINTENDENT-GENERAL: EDUCATION

APPENDIX 2: CERTIFICATE OF ATTENDANCE IN AABORG UNIVERSITY

UNESCO

United Nations
Educational, Scientific and
Cultural Organization

AALBORG UNIVERSITY

Aalborg Centre for Problem Based Learning
in Engineering Science and Sustainability
under the auspices of UNESCO

Certificate

Aalborg March 28, 2017

BEDESHANI MCELELI

Participated in a PBL workshop at Aalborg University. The objectives of the workshop were:

- To gain knowledge of PBL as a system and the various elements in the system: problems, projects, courses, learning objectives, assessment, and student process skills (PBL skills) content.
- Participants to address critical issues such as: students' gab of knowledge and skills, cooperation among teachers, authority learning
- To experience The Aalborg University Model of Problem Based and Project Organised Learning by meeting students and academic staff

Centre Director Prof. Anette Kolmos

Anette Kolmos

APPENDIX 3: LESSON PLAN FROM DBE

MATHEMATICS LESSON PLAN
GRADE 9
TERM 1: January – March

PROVINCE:	
DISTRICT:	
SCHOOL:	
TEACHER'S NAME:	
DATE:	
DURATION:	2 Hours

1. **TOPIC: ALGEBRAIC EXPRESSION:** EXPAND AND SIMPLIFY ALGEBRAIC EXPRESSIONS **(Lesson 2)**

2. **CONCEPTS & SKILLS TO BE ACHIEVED:**

By the end of the lesson learners should know and be able to use the commutative, associative and distributive laws for rational numbers and laws of exponents to:

- Add and subtract like terms in algebraic expressions
- multiply integers and monomials by:
  - monomials
  - binomial
  - trinomials
- Divide the following by integers or monomials:
  - monomials
  - binomials
  - trinomials
- simplify algebraic expressions involving the above operations

3. RESOURCES:	DBE Book 1, Sasol-Inzalo Book1, textbook
4. PRIOR KNOWLEDGE:	• laws of exponents • commutative, associative and distributive properties

5. REVIEW AND CORRECTION OF HOMEWORK (suggested time: 10 minutes)

Homework provides an opportunity for teachers to track learners' progress in the mastery of mathematics concepts and to identify the problematic areas, which require immediate attention. Therefore, it is recommended that you place more focus on addressing errors from learner responses that may later become misconceptions.

6. INTRODUCTION (Suggested time: 10 Minutes)

Activity 1

Allow learners to do the following activities:

1. Calculate the following:
   a) $5(3 + 4)$
   b) $5 \times 3 + 5 \times 4$
   c) $6 \times 3 + (4 + 6)$
   d) $(6 + 4) + 3 \times 6$
   e) $3 \times (4 \times 5)$
   f) $(3 \times 4) \times 5$

2. Which properties are indicated by (a) and (b), (c) and (d), (e) and (f)

Note:
Learners should have noticed that the results of each pair of question are the same. This is because operations with numbers have certain properties, namely the distributive, commutative and associative properties.

Activity 2

Discuss with learners the multiplication and division laws of exponents by using the following problems:

a) $3x^3 \times 4x^2 = 12x^5$
b) $a^2b^3 \times a^7b^5 = a^9b^8$
c) $x^7 \div x^2 = x^5$
d) $\frac{d^2e^5}{d^7e^3} = \frac{e^2}{d^2}$

<table>
<tr><th colspan="2">7. LESSON PRESENTATION/DEVELOPMENT (Suggested time: 20 minutes)</th></tr>
<tr><th>Teaching activities</th><th>Learning activities (Learners are expected to:)</th></tr>
<tr><td>

Activity 1

Let learners do the following activities individually and have a class discussion.

Find the equivalent expressions by identifying and grouping like terms in the expression below:

a) $2b^2 - 4b + 1 - b^2 - b - 4b^2 - 2b + 6$
b) $x^2y + xy + 3xy - xy^2 + 2x^2y - 4xy$
c) $3x^2 + 13x + 7 + 2x^2 - 8x - 12$

Activity 2

Demonstrate the following problems to the learners:

Simplify by using the distributive property.

a) $3x^2 \times 5x^3$
b) $a(b + c)$
c) $-3(2x^2 - 3x + 4)$

Activity 3

a) Demonstrate the following problem to the learners:

$(6a + 9b) \div 3$

Solution:
= $(6a \div 3) + (9b \div 3)$
= $2a + 3b$

b) Show learners that this problem can also be written in a fraction form i.e. $\frac{6a+9b}{3}$

Solution:

$$= \frac{6a}{3} + \frac{9b}{3}$$

= $2a + 3b$

</td><td>

- work individually on the given activities
- participate in class discussions
- follow the demonstration of the teacher

8. CLASSWORK (Suggested time: 15 minutes) Sasol-Inzalo Workbook 1, pg. 129, no. 4, 6

Sasol-Inzalo Workbook 1, pg. 133, no. 7 (c, d, f, g,)

</td></tr>
</table>

APPENDIX 4: NOWELE'S LESSON PLAN

SIMPLIFY ALGEBRAIC EXPR

REMEMBER THE RULES OF FACTORISATION

STEP 1:

- Apply the sign-change rule if necessary (Group in pairs / Factorise the pairs)

STEP 2:

Take out the HCF if it exists → Factorise further if needs be.

STEP 3:

Factorise the trinomial

(1) $\dfrac{6x^2+18x-60}{8x^2+40x} \div \dfrac{x-2}{8x}$

$= \dfrac{6x^2+18x-60}{8x^2+40x} \times \dfrac{8x}{x-2}$ (change to x)

$= \dfrac{6(x^2+3x-10)}{8x(x+5)} \times \dfrac{8x}{(x-2)}$ (Take out the HCF)

$= \dfrac{6\cancel{(x+5)}\cancel{(x-2)}}{8\cancel{x}\cancel{(x+5)}} \times \dfrac{8x}{\cancel{(x-2)}}$ OR (Factorise)

$= \dfrac{6}{7}$

$\dfrac{6\cancel{(x+5)}\cancel{(x-2)}}{8x\cancel{(x+5)}} \times \dfrac{8x}{\cancel{(x-2)}}$

$= \dfrac{6}{8x} \times 8x$ (Simplify)

$= \dfrac{3}{\cancel{4x}_1} \times 8\cancel{x}^2$

$= 6_7$

Classwork:

(1) $\dfrac{x^2-3x-18}{x^2-6x} \div \dfrac{6x^2+18x}{(-\cancel{2}x)^2}$

(2) $\dfrac{16p^4-1}{4p^2-1} \div \dfrac{4p^2+1}{1}$

APPENDIX 5: COMMON ASSIGNMENT SET BY FALAFALA

MATHS ASSIGNMENT GRADE 9 TERM 1

QUESTION 1

Consider this set of numbers

$$\boxed{\begin{array}{l} -4;\ -\sqrt{9};\ 0{,}5,\ \frac{3}{2};\ \sqrt{3} \\ 58;\ 0{,}9;\ 0;\ \sqrt{1};\ -7;\ \frac{2}{3} \\ 15;\ \sqrt{25};\ 3;\ 333333\ldots \end{array}}$$

1.1. Write down all the numbers that are

(a) Natural numbers

(b) Whole numbers

(c) Integers

(d) Rational numbers

(e) Irrational numbers

QUESTION 2

2.1. Calculate the following integers

2.1.1. (a) $-(6)^2 + \sqrt[3]{-125} - (-4)^3$

2.1.2 (b) $\frac{(-2)^2}{4} - (-4) - \sqrt{49}$

2.1.3 (c) $-3^2 \times \sqrt{9+16} + \frac{-3 \times 2^3}{-2-6}$

(d) QUESTION 3

Calculate the following fractions

3.1. $\frac{1}{4} \div \frac{1}{3} \times \left(\frac{4}{5} + \frac{1}{10}\right)$

3.2. $-\left(-\frac{1}{2}\right)^3 - \left(-2\frac{1}{2}\right)^2$

3.3. $\sqrt[3]{-\frac{1}{8}} - \sqrt{2\frac{1}{4}}$

APPENDIX 6: COMMON ASSIGNMENT MODERATED BY TAU

Province of the
EASTERN CAPE
DEPARTMENT OF EDUCATION

PROVICE OF THE EASTERN CAPE
MT FLETCHER

GRADE 9

MATHEMATICS ASSIGNMENT

MARKS : 110

Instructions

1. This assignment consists of seven (7) questions.
2. Answer all questions.
3. Clearly show all your calculations.
4. A non-programmable scientific calculator may be used unless stated otherwise.

1

QUESTION 1

Consider the following numbers:

$\sqrt{4}$; 1 ; $\frac{1}{6}$; -5 ; $\sqrt{\frac{17}{16}}$; $\frac{8}{2}$; $-\sqrt[3]{-27}$; $\sqrt[3]{13}$; $\frac{22}{7}$; $\sqrt{5}$

1.1 Choose all the rational numbers

1.2 Choose all the irrational numbers

(10) **[10]**

QUESTION 2

Simplify the following:

2.1 $-(6)^2 + \sqrt[3]{-125} - (-4)^3$ (4)

2.2 $\frac{(-2)^2}{4} - (-4) - \sqrt{49}$ (3)

2.3 $-3^2 \text{ x } \sqrt{9+16} + \frac{-3\times 2^3}{-2-6}$ (4) **[11]**

QUESTION 3

Calculate the following:

3.1 $\frac{1}{4} - \frac{1}{3} \times (\frac{4}{5} + \frac{1}{10})$ (3)

3.2 $-\left(\frac{-1}{2}\right)^3 - \left(-2\frac{1}{2}\right)^2$ (4)

3.3 $\sqrt[3]{\frac{-1}{8}} - \sqrt{2\frac{1}{4}}$ (4)

3.4 $\sqrt[3]{\frac{1}{64}} + \left(-\frac{1}{3}\right)^3$ (4) **[15]**

QUESTION 4

4.1	A man borrowed R10 000 from his friend. Their agreement was that at the end of one year, he must repay the loan with 3% interest. How much altogether must he pay back?	(3)
4.2	A woman used R1 000 to join an investment club. At the end of one year, her money had grown to R1 240. What was the annual interest rate?	(3)
4.3	R5 000 is invested for 5 years at a compound interest rate of 7% p.a. Calculate the final amount and the compound interest earned. Round off the answer to two decimal places.	(5)
4.4	Johannes started to save money six years ago. The current value of his investment is R38 000. The interest for investment was 7% p.a. simple interest. How much did he invest six years ago?	(3)
4.5	Adelaide bought a printer for R15 000 on hire purchase. She agreed to pay 15% deposit plus 18% p.a. simple interest, by means of monthly payments over a period of 5 years. Calculate:	
4.5.1	The amount Adelaide will pay on hire purchase over 5 years.	(4)
4.5.2	Her monthly instalment.	(2)
4.5.3	The total amount paid for the printer.	(1)
4.5.4	The amount she should have saved if she bought the printer cash.	(1) **[22]**

QUESTION 5

5.1 Katso is a truck driver. He travels at an average speed of 80 km/h and covers a certain distance in 3 hours 20 minutes. At what average speed should he travel to cover the same distance in 2 hours 40 minutes? (5)

5.2 $200ml$ of liquid has a mass of $300g$.

5.2.1 What would the mass of a litre of this liquid be? (3)

5.2.2 Draw a bar graph of the mass of the liquid (in g) versus the volume (in ml) (3)

5.3 Jabulani has a fixed amount of money to buy sweets. If the sweets cost R2 each, she can buy 50 sweets. How many sweets will she be able to buy if the sweets cost R5 each? (4) **[15]**

QUESTION 6

6.1 Write the following in standard notation (decimal form)

6.1.1 $9{,}345 \times 10^{7}$ (1)

6.1.2 $4{,}389 \times 10^{-8}$ (1)

6.2 Simplify the following and write your answers with positive exponents where necessary.

6.2.1 $(a^{2})^{-2} \times (a^{3})^{2}$ (3)

6.2.2 $4(x)^{-3} \times (x^{2})^{4}$ (3)

6.2.3 $(3x^{2})(3x)^{-2}$ (3)

6.2.4 $\frac{2x^2}{(2x)^2}$ (3)

6.2.5 $\frac{-9y^2}{(-3y)^2}$ (3)

6.3 Simplify the following algebraic fractions:

6.3.1 $\frac{2-x}{4}+\frac{x+2}{2}$ (3)

6.3.2 $\frac{2x}{3}-\frac{3x}{2}-\frac{x+1}{6}$ (5) **[25]**

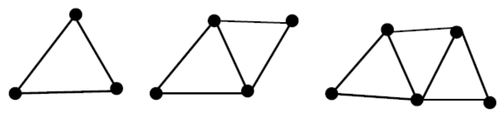
QUESTION 7

7.1 Describe the rule in your own words and determine the next three terms of the following patterns:

7.1.1 45 ; 50 ; 55 ; 60; ... (4)

7.1.2 $\frac{1}{2}$; $\frac{11}{2}$; $\frac{21}{2}$; $\frac{31}{2}$; ... (4)

7.2



Study the geometric pattern above. How many matches sticks are needed for:

7.2.1 The 5^{th} pattern. (2)

7.2.2 The 10^{th} pattern. (2) **[12]**

oooOOOooo

APPENDIX 7: ETHICAL CLEARANCE LETTER FROM UFS

UNIVERSITY OF THE FREE STATE
UNIVERSITEIT VAN DIE VRYSTAAT
YUNIVESITHI YA FREISTATA

UFS
UV

Faculty of Education
Ethics Office

Room 12
Winkie Direko Building
Faculty of Education
University of the Free State
P.O. Box 339
Bloemfontein 9300
South Africa

T: +27(0)51 401 9922
F: +27(0)51 401 2010

www.ufs.ac.za
BarclayA@ufs.ac.za

30 September 2015

<u>ETHICAL CLEARANCE APPLICATION:</u>

ENHANCING MATHEMATICS PEDAGOGICAL CONTENT KNOWLEDGE IN GRADE 9 CLASS USING PROBLEM BASED LEARNING

Dear BM Mceleli

With reference to your application for ethical clearance with the Faculty of Education, I am pleased to inform you on behalf of the Ethics Board of the faculty that you have been granted ethical clearance for your research.

Your ethical clearance number, to be used in all correspondence, is:

UFS-EDU-2015-025

This ethical clearance number is valid for research conducted for three years from issuance. Should you require more time to complete this research, please apply for an extension in writing.

We request that any changes that may take place during the course of your research project be submitted in writing to the ethics office to ensure we are kept up to date with your progress and any ethical implications that may arise.

Thank you for submitting this proposal for ethical clearance and we wish you every success with your research.

Yours sincerely,

Duvenhage

CS Duvenhage
Faculty Ethics Officer

UNIVERSITY OF THE FREE STATE
UNIVERSITEIT VAN DIE VRYSTAAT
YUNIVESITHI YA FREISTATA

APPENDIX 8: NJOVANE'S TRANSCRIBED LESSON PRESENTATION

1. *Mr Njovane: Yes, we are solving the unknown, do we understand…we are solving the unknown equation (raising his voice)*
2. *Learners: (in a chorus) We are solving the unknown.*
3. *Mr Njovane: You see how it goes m – 2 could you read for me……*
4. *Learnres: x*
5. *Mr Njovane: We do not have x here, there it is… do you see how is this… listen… this is an equation. What is an equation, how do we see it, how do we differentiate the equation? Because? Because of what? This is what it is…its an equation not expression. Why am I saying so?*
6. *Learners: Expressions have brackets…*
7. *Mr Njovane: (Interrupted learners) The equal sign….*
8. *Learners: The equal sign (responding in a chorus)*
9. *Mr Njovane: Before we can solve this now, we need to work out m, can you see m?*
10. *Learners: Yes (learners responded in a chorus)*
11. *Mr Njovane: m is an unknown, can you see that*
12. *Learners: Yes*
13. *Mr Njovane: We need to remove the brackets, how are we going to remove the brackets? -2, this negative 2 is going to multiply every term found inside the brackets, isn't so*
14. *Learners: Yes*
15. *Mr Njovane: But be careful of the negative sign, it changes everything... -2 multiplied by 3 is…*
16. *Learners: - 5*
17. *Mr Njovane: (repeating the learners' answer) -5 wow!!! -2 x 3 is ….*
18. *Learners: - 6 (they changed their initial answer)*
19. *Mr Njovane : (Confirmed the answer) -6m, -2 multiplied by + 1 what is the answer…*
20. *Learners: - 2*
21. *Mr Njovane: -2 yes (he confirmed) can you see that we are through with the left side*
22. *Learners: Yes*
23. *Mr Njovane: Where are we moving to now?*
24. *Learners: Right..*

25. *Mr Njovane: We will leave 2m as it is… there it is, negative 1 but 1 is not written…. If there is a standalone negative you should know that it is -1. Now let us move one -1 multiplied by m…. what is the answer? It is –m*
26. *Learners: (repeated the teacher's answer in a chorus) – m*
27. *Mr Njovane: -1 multiplied by -4 what I the answer?*
28. *Learners: Positive, positive*
29. *Mr Njovane: Yes, be careful about the sign, the answer is +4 yes*
30. *Learners: + 4*
31. *Mr Njovane: Look now you have the left-hand side and the right-hand side, we need to collect now… like terms, do you see like terms?*
32. *Learners: Yes*
33. *Which one are we going to start with on both sides, left as well as right?*
34. *Learners: m*
35. *Mr Njovane: Now we have this one… m – 6m, meaning that we have 1 – 6, what is the answer?*
36. *Learners: Minus…-5 (one group said minus, another said -5)*
37. *Mr Njovane: No…no…no…what is the answer…the correct answer?*
38. *Learners: -5m*
39. *Mr Njovane: (raising his voice) Negative 5m*
40. *Learners: -5m (repeating in a chorus)*
41. *Mr Njovane: - 5m – 2… yes, the equal sign again, here is 2m – m can we see that?*
42. *Learners: Yes*
43. *Mr Njovane: Meaning that it is 2 – 1, one is not written I have long been saying that, what is the answer?*
44. *Learners: 1 m*
45. *Mr Njovane: 1 m…we gona write m + 4 can you see it now?*
46. *Learners: Yes*
47. *Mr Njovane: So now we need to deal with… is to collect all numbers with m and put them one side and also collect all numbers without m and put them on the other side. In which sides should we m's, left or right?*
48. *Learners: On the left*
49. *Mr Njovane: On the left (he confirmed), meaning that we have to remove this m, so now when we move this m, we need to apply what?... the inverses… do we agree?*
50. *Learner: Yes*
51. *Mr Njovane: This is positive me can we see? What is the inverse of positive m?*
52. *Learners: Negative m*

53. *Mr Njovane: Meaning that we have -m on this side and on the other side let us put negative m. This is gona be m – m, we know that is it not so?*
54. *Learners: Yes*
55. *Mr Njovane: Is the any m remaining on the right-hand side?*
56. *Learners: No*
57. *Mr Njovane: So, what is left on the right-hand side, is?*
58. *Learners: 4*
59. *Mr Njovane: On this side we have negative 5m and we bring another one negative which is m, how many negatives do we have?*
60. *Learners: 6*
61. *Mr Njovane: 6 (he confirmed) yes, we have got – 6m – 2, again we still have that 2 now. Let us have a look now you see this 2? This 2 now is negative 2, we need to use an additive inverse on this negative 2, what is an additive inverse of this negative two? Its positive 2*
62. *Mr Learners: Positive two (Learners joined and completed his statement in a chorus).*
63. *Mr Njonava: When we add positive two here, on the left-hand side, on the right-hand side we must also add what? Positive two…*
64. *Learners: Positive two (Learners completed his statement in a chorus)*
65. *Mr Njonava: Do you see that positive two…yes…now let us move on, negative two plus two is the same as two minus two, are we in agreement?*
66. *Learners: Yes*
67. *Mr Njovane: What is the answer here…do we still have… the answer is zero*
68. *Learners: Zero (Learners completed his statement in a chorus… some learners said it is negative zero)*
69. *Mr Njovane: Zero neh…the answer is – 6m, new we got only 4 + 2, what is the answer?*
70. *Learners: 6*
71. *Mr Njovane; The answer is going to be 6, add then we use now, the mult…?*
72. *Mr Njovane and Leaners: The multiplication inverse (Learners joined in a chorus to complete his statement)*
73. *Mr Njovane: The inverse of this one is… division*
74. *Learners: Division*
75. *Mr Njovane: We have to divide now by… negative 6m (did not explain why and dividing by 6m is wrong)*
76. *Learners: 6 m*

77. *Mr Njovane: Let's just divide by 6…are we agreeing?*
78. *Learners: Yes (they also confirmed without asking why)*
79. *Mr Njovane: We do this on both side… are in agreement? On both side we divide by negative six, ca we see that? (without explaining why do it on both side)*
80. *Learners: Yes*
81. *Mr Njovane: So that now – 6 cancels – 6, what is left?*
82. *Learners: M*
83. *Mr Njovane: M that is positive, can you see that (but wrote negative m on the chalk board and learners did not notice that)*
84. *Learners: Yes*
85. *Mr Njovane: Now this one, 6 cancels 6…I mean 6 divided by – 6 what is the answer?*
86. *Learners: -1*
87. *Mr Njovane: The answer is negative one… this is the correct answer, you can also prove… when you got your answer you can prove it… by taking your one and put it in every part that has m in your, are we in agreement…*
88. *Learners: Yes.*

APPENDIX 9: MINUTES OF THE COORDINATED TEAM MEETINGS

CLUSTER MEETING MATHS

Date: 18th October 2016

Venue: [redacted] Secondary School.

AGENDA

1. Opening prayer
2. Cridentials
3. Establishment of the Cluster Maths Team.
4. Work schedule verification
5. Identification of problem areas
6. Planning
7. Closure

The meeting was opened with a prayer by Miss [redacted]. Mr Mceleli explained the purpose of the meeting. All the expected members were present except Miss [redacted] ([redacted]) and Mr [redacted] ([redacted]).

ESTABLISHMENT OF THE CLUSTER MATHS TEAM.

We agreed that the chairperson of the Team is Mr [redacted] ([redacted]) and the secretary is Mrs [redacted] ([redacted]). Mrs [redacted], Miss [redacted], M[redacted], [redacted] and Miss [redacted] are additional members.

WORK SCHEDULE VERIFICATION

Topics to be covered : Transformation

: Statistics

: 3-D objects

: Probability

Miss [redacted] ---- about to start the
Mrs [redacted] ---- Transformation as they were
still busy with revisions

Mr [redacted] ----- finished the transformation
topic and busy with probability

Miss [redacted] ------ busy with transformation
Mrs [redacted] topic and about to be finished

IDENTIFICATION OF PROBLEM AREAS.

- angles cut by a transversal
- Conversions of units
- Equations factorisation
- Confusion between LCM and LCD
- When doing Lesson plan, be aware of the misconceptions

PLANNING

- Plan the lessons for (i) transformation topic
- (ii) angles cut by a transversa

Lesson plans are available at DBE website.

- Mr [redacted] will plan a lesson on factorisation of Equations,
- Mrs [redacted] will plan a lesson of Transformatio
- Mr [redacted] will plan a lesson on Probability
- Mrs [redacted] will plan a lesson on Statistics
- Miss [redacted] will plan a lesson on 3 D objects
- Miss [redacted] will plan a lesson on parallell lines ~~angles~~ cut by a transversal line.

Transformation lesson plan ~~will~~ was ~~be~~ asked to be available on the following day (19.10.2016

The meeting was closed with a prayer by Miss [redacted].

MATHEMATICS PLANNING WORKSHOP

Date: 26 January 2017

Venue: [redacted] Secondary School.

AGENDA

1. OPENING
2. INTRODUCTION
3. PURPOSE
4. UNPACKING OF THE POLICY
5. ASSESSMENT TASKS
6. LESSON PLANNING AND MEDIATION
7. PROGRAMME OF ACTION
8. CLOSURE

The meeting was opened with a prayer by Mr [redacted]

Introduction : Mr [redacted] J.S.S.

- Mrs [redacted] J.S.S)
- Miss [redacted] J.S.S
- Miss [redacted] S.S)
- Mrs [redacted] SS)
- M [redacted]
- M [redacted]
- Miss [redacted] J.S.S
- Mr
- Mrs [redacted] J.S.S)

PURPOSE: is a teacher rescue programme in Mathematics

UNPACKING OF THE POLICY ......done by Mr. [redacted]

PLANNING

- Work Schedule Term 1
- Assessment Tasks – SBA
- Lesson Plan Mediation

Work Schedule – before you teach, check the policy document.

Assessment Tasks-(SBA) – quality of tasks.

- Assignment
- Test (Controlled)

Two tests should be administered before the controlled test.

Examiners Controlled Test

Mrs [redacted]
Mr [redacted] } Examiners

Mrs [redacted] ... Moderator (Controlled test)

Miss [redacted]
Mrs [redacted] } Assignment

Moderator .... ~~Mrs~~ Miss [redacted]

Assignment — Topics to be covered
Whole numbers
Integers
Common fractions
Decimal fractions
Exponents
Numeric and geometric patterns
Functions and relationships

Controlled Test : Cover Term 1 work

Assignment will be given to learners for a week but will be written in class on a due date.

APPENDIX 10: TEAM NORMS

Province of the
EASTERN CAPE
DEPARTMENT OF EDUCATION

Resource Centre Education Building * Hospital Road * Private Bag X1133 * MT FLETCER * 4770* REPUBLIC OF SOUTH AFRICA * Tel: +27 (0)039 2570960 Fax: 039 2570956 * Website: ecprov.gov.za * Enquiries: B M Mceleli* 0820665979 mceleli@yahoo.com 30/01/2017

To : The Principal

Subject: Mathematics Improvement Plan

This serves to inform you about our district plan to improve mathematics performance in grade nine class in order to ensure maximum mathematics enrolment in grade ten. This is against the background that learners who obtain level one in grade nine mathematics may be allowed to progress to the next grade but not enrol for mathematics subject in grade ten. Our intervention plan is as follows:

1. Common planning
2. Common formal assessment tasks
3. Weekly meetings/ workshops to evaluate progress and sharing of challenging concepts
4. District moderation of formal tasks
5. Sharing of mathematics teaching resources
6. Class visits to observe the best practice in the implementation of the programme.
7. Extra classes to ensure coverage of term work schedule.

Weekly meetings have been arranged as follows:

Date	Time	Venue
02/02/2017	13H00	District Office
16/02/2017	13H00	District Office
23/02/2017	13H30	District Office
02/03/2017	13H30	District Office
09/03/2017	14H00	District Office
16/03/2017	14H00	District Office
23/03/2017	14H00	District Office

This programme will be evaluated and reviewed at the end of term one and you are humbly requested to make necessary arrangements so that all your grade mathematics teachers attended this intervention programme.

Thanking you in anticipation.

B Moseli (0820665979)

....

P O BOX 490

MATATILE 4730

09 APRIL 2015

THE SUPERITENDENT GENERAL

EASTERN CAPE DEPARTMENT OF EDUCATION

ZWELITSHA

Dear Sir

Re: Request to be granted permission to conduct a research study in secondary schools in the Eastern Cape

The purpose of the study is to enhance grade 9 teacher's mathematics pedagogical content knowledge using problem based learning approach in rural schools in the Province of the Eastern Cape.

Participants in the study will include, amongst others, learners, members of the school management team and mathematics educators in the schools. To avoid the disruption of the teaching programme, major activities of the study will take place during weekends. Two classroom observations will take place per participant during third term of 2015 and arrangements will be made two months in advance. Participation of the teachers and schools is entirely voluntary and they will be under no obligation to take part in this study. If they choose to take part, and an issue arises which makes them uncomfortable, they may at any time stop their participation with no further repercussions. The schools in the circuit will be invited to voluntary participate in the research process.

It is against this backdrop that permission is hereby requested to conduct research at schools in Mount Fletcher District. The study will benefit the schools as well as the entire schooling system in the province of the Eastern Cape. Kindly note that you are also free to contact my study supervisor whose details are as follows:

Prof. G. M. Mahlomaholo
Mobile number: 0826042723/ 0748884375
Email: mahlomaholomg@ufs.ac.za

Yours sincerely

B M Mceleli (Mr) 0820665979