THE RELATIONSHIP BETWEEN TEACHERS' MATHEMATICAL KNOWLEDGE AND THEIR CLASSROOM PRACTICES: A CASE STUDY ON THE ROLE OF MANIPULATIVES IN SOUTH AFRICAN PRIMARY SCHOOLS

BY

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ABSTRACT

The use of manipulatives to enhance conceptual understanding of mathematics is a critical component of the primary school mathematics curriculum in South Africa. Manipulatives are concrete or visual objects that are specifically designed to represent mathematical ideas, concepts and/or procedures. Whether or not manipulatives are used in the teaching of mathematics in the primary school classroom, and how they are used, if at all, depends on the teachers' knowledge and understanding of mathematics and their conceptions about classroom practice and the role of manipulatives therein. In this study, teachers' mathematical knowledge is defined as knowledge of both content and pedagogy, whilst classroom practice refers to the interaction among teachers, students and content.

The present study therefore explores the use of manipulatives in the teaching of mathematics in primary school classrooms. The study examines the role of manipulatives in shaping both the teachers' knowledge of primary school mathematics and their classroom (pedagogical) practices.

Critical theory is used as the underlying theoretical framework for the study and helps to frame the key constructs of the study, namely; teacher knowledge, mathematical manipulatives and classroom practice.

The study uses a multiple-case study qualitative approach designed with unstructured interviews (employing the free attitude interview technique) with four grade six mathematics teachers from each of four primary schools as the main data collection tools. Additional data were gathered through observation of lessons conducted in three of the four primary schools, group discussions and curriculum documents analysis. A Participatory Action Research (PAR) approach was chosen to collect data in respect of teachers' own knowledge, experiences and thinking about their mathematical knowledge, classroom practice and the use of manipulatives. The study employs the socio-cognitive approach to discourse analysis as a strategy to analyse the data obtained.

The study's main findings suggest that teacher knowledge of mathematics is more crucial in the effective use of manipulatives than perhaps any other single teacher attribute. Effective use of manipulatives is essentially characterised as the abstraction of mathematical concepts and relationships embedded in those manipulatives. To successfully do this highly cognitive mathematical task teachers are forced to draw heavily on their own knowledge of mathematics. Any other factors such as teacher beliefs, teacher pedagogy, etc. can only serve as a support base for teacher knowledge. The study concludes that teachers can only abstract mathematical concepts and make connections between them effectively if they themselves have sufficient knowledge of those mathematical concepts and their relationship. Furthermore, the study suggests that over time and with relevant professional teacher development support, the use of manipulatives may have the potential to shape/reshape teachers' mathematical knowledge.

This study concludes that the influence of manipulatives on teachers' mathematical knowledge and their classroom practices can be explained and understood within the context of the tensions and opportunities that arise in and from a teaching practice where teachers use manipulatives.

Based on the findings, the study then recommends a comprehensive professional teacher development programme for primary school teachers that provides hands-on experiences with manipulatives and promotes the reorientation of classroom practice through reflection and co-learning by the teachers alongside their learners.

ABSTRAK

Die gebruik van konkrete hulpmiddels ten einde die konseptuele begrip van wiskunde te versterk is ’n kritieke komponent van die wiskundekurrikulum in die laerskool in Suid-Afrika. Hulpmiddels is konkrete of visuele voorwerpe wat spesifiek ontwerp is om wiskundige idees, konsepte en/of prosedures te verteenwoordig. Die gebruik van konkrete hulpmiddels in die laerskool se wiskundeklaskamer asook hoe dit gebruik word, hang af van die onderwyser se kennis en begrip van wiskunde en hul idees oor klaskamerpraktyk en die rol van hulpmiddels daarin. In hierdie studie is onderwysers se wiskundige kennis gedefinieer as kennis van die inhoud en die pedagogie, terwyl klaskamerpraktyk verwys na die interaksie tussen onderwysers, leerders en die vakinhoud.

Die huidige studie verken dus die gebruik van hulpmiddels in die onderrig van wiskunde in laerskoolklaskamers. Die studie ondersoek die rol van hulpmiddels in die vorming van onderwysers se kennis van laerskoolwiskunde en hul klaskamer-(pedagogiese) praktyke.

Kritiese teorie word gebruik as die onderliggende teoretiese raamwerk vir die studie en help om die sleutelkonsepte van die studie, naamlik onderwyserkennis, wiskundige konkrete hulpmiddels en klaskamerpraktyk, te formuleer.

Die studie gebruik ’n kwalitatiewe veelvuldige-gevallestudiebenadering wat ontwerp is met ongestruktureerde onderhoude met vier graad 6 wiskunde onderwysers van elkeen van die vier laerskole as die hoofdataversamelingsinstrumente. Addisionele data is versamel deur die waarneming van lesaanbiedinge in drie van die vier laerskole, groepsbesprekings en analiese van kurrikulumdokumente. Voorbereidende Aksie Navorsing (VAN) was gekies om data te versamel ten opsigte van die onderwysers se eie kennis, ondervinding en denke rondom wiskundige kennis, klaskamerpraktyke en die gebruik van hulpmiddels. Die studie het die sosio-kognitiewe benadering tot gespreksanalise as ’n strategie gebruik om die data wat bekom is, te analiseer.

Die studie se hoofbevindings stel voor dat onderwyserkennis van wiskunde belangriker is in die doeltreffende gebruik van hulpmiddels as miskien enige ander

enkele kenmerk van ’n onderwyser. Doeltreffende gebruik van hulpmiddels word basies gekenmerk as die abstraksie van wiskundige konsepte en verhoudings wat in daardie hulpmiddels vasgelê is. Om hierdie hoogs kognitiewe wiskundige taak suksesvol te doen, word onderwysers gedwing om te steun op hul eie kennis van wiskunde. Enige ander faktore, soos die onderwyser se houdings, pedagogie, ens., kan alleenlik dien as ’n ondersteuningsbasis vir onderwyserkennis. Hierdie studie kom tot die gevolgtrekking dat onderwysers alleenlik wiskundige konsepte kan abstraheer en effektief verbindings tussen hulle kan maak as hulle self voldoende kennis van hierdie wiskundige konsepte en hul verhoudings het. Verder stel die studie voor dat, mettertyd en met voldoende professionele ondersteuning van onderwyserontwikkeling, die gebruik van konkrete hulpmiddels die potensiaal kan hê om onderwysers se wiskundige kennis te vorm/hervorm.

Hierdie studie het dus tot die slotsom gekom dat die invloed van hulpmiddels op onderwysers se wiskundige kennis en hul klaskamerpraktyke verduidelik en verstaan kan word binne die konteks van die spanning en geleenthede wat binne en buite’n onderrigsituasie waar onderwysers konkrete hulpmiddels gebruik, ontstaan.

Gebaseer op hierdie bevindings beveel die studie dus ’n omvattende professionele onderwyserontwikkelingsprogram vir laerskole aan wat praktiese ervaring met konkrete hulpmiddels bied en wat die heroriëntasie van klaskamerpraktyk deur nadenking en mede-leerervarings tesame met hul leerders voorstaan.

DECLARATION

Student number: 2007104793

I, Manthlake Julia Maboya, hereby declare that the study on *The Relationship Between Teachers' Mathematical Knowledge and Their Classroom Practices: A Case Study on the Role of Manipulatives in South African Primary Schools* is my own, and that it has not been submitted for a degree or examination at any other university and that all the sources I have used or quoted have been acknowledged by complete references.

__________________________ __________________________

Signature Date

DEDICATION

TO

My husband, Paseka

My sisters Mamaboloka and Liatla,

My children Makhabu, Nkoehatsi and Thekiso,

My son-in-law Monde Gxoyiya and my daughter-in-law, Mabatho,

My grandchildren, Nolwazi, Ogone, Khaya, Dioka and Thekiso (Jr.)

Your love, support and patience during all these years have always given me the courage and strength to pursue my dreams and to never give up. I hope that this has been an inspirational experience for all of you to also pursue your dreams with the same level of dedication, vigour and hard work. I owe this degree to my family; it belongs to all of you, Bathapama le Basikili!

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The journey towards the completion of this thesis was not an easy one. It was a journey characterised by many sleepless nights, relapses and very tight timelines. It was only upon reflecting on this trajectory that I realise how rekindling, rewarding and fulfilling the experience has been. It never occurred to me, until I had at least, to write this acknowledgement, that I had actually come to the end of the journey.

First and foremost I would like to thank the Almighty God for giving me life, strength, guidance and support to pursue this study until its finalisation. I am also grateful to my late parents, Thabo and 'Maletebele Lechesa, from whom I draw my inspiration, perseverance and the love for education.

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LIST OF ACRONYMS USED IN THIS THESIS

ANA	Annual National Assessments
CAPS	Curriculum and Assessment Policy Statement
CCK	Curriculum and Content Knowledge
CDA	Critical Discourse Analysis
CER	Critical Emancipatory Research
CES	Chief Education Specialist
C-P-A	Concrete-Pictorial- Abstract
DCES	Deputy Chief Education Specialist
DfEE	Department for Education and Employment
DHET	Department of Higher Education and Training
FAI	Free Attitude Interview
FET	Further Education and Training
FfL	Foundations for Learning
FSDoE	Free State Department of Education
GET	General Education and Training
ICTs	Information and Communication Technologies
ILLS	Instructional Leadership through Lesson Study
JC	Junior Certificate
JPTD	Junior Primary Teachers Diploma
KCS	Knowledge of Content and Students
KCT	Knowledge of Content and Teaching
KQ	Knowledge Quartet
LCD	Lowest Common Denominator
LoLT	Language of Learning and Teaching
LTSM	Learning and Teaching Support Materials
MEC	Member of Executive Council

MKT	Mathematical Knowledge for Teaching
MLMMS	Mathematical Literacy, Mathematics and Mathematical Science
MQI	Mathematical Quality of Instruction
NCTM	The National Council of Teachers of Mathematics
NEEDU	National Education Evaluation and Development Unit
NEPI	National Education Policy Investigation
NNS	National Numeracy Strategy
NSC	National Senior Certificate
NSE	Norms and Standards for Educators
NSNP	National School Nutrition Programme
OECD	Organisation for Economic Co-operation and Development
PAR	Participatory Action Research
PCK	Pedagogical Content Knowledge
PLC	Professional Learning Communities
PTD	Primary Teachers Diploma
PUFM	Profound Understanding of Fundamental Mathematics
RSA	Republic of South Africa
SA	Subject Advisor
SADTU	South African Democratic Teachers Union
SASA	South African Schools Act
SCK	Specialised Content Knowledge
SGB	School Governing Body
SII	Study of Instructional Improvement
SMT	School Management Team
TCA	Theory of Communicative Action
TELT	Teacher Education and Learning to Teach
TIMSS	Trends in International Mathematics and Science Study
UFS	University of the Free State

UK	United Kingdom
U.S.A.	United States of America

CHAPTER 1: ORIENTATION AND BACKGROUND TO THE STUDY

1.1 INTRODUCTION

“Although the policy context that surrounds education changes like a series of hurricanes blowing across the Gulf of Mexico, the substantive nature of what happens in classrooms stays pretty much the same.” (Stigler & Hiebert 2009:32).

The teaching and learning of Mathematics has always been a complex and challenging endeavour in South Africa and anywhere in the world. It is commonly accepted that knowing mathematics is essentially about both the acquisition of procedural and conceptual skills. Silver in Siegler and Alibi (2001:346) posits that ‘competence in domains such as mathematics rests on children developing and linking their knowledge of concepts and procedures’ (Siegler & Alibi 2001:346).To me, this feature is more salient in mathematics knowledge probably because of the abstract nature of mathematics. Teaching mathematics with understanding is therefore about helping learners to make significant connections between conceptual and procedural knowledge and striking a balance between the two types of knowledge. Uttal, Scudder and DeLoache (1997: 37) argue that ‘mathematics teachers face a double challenge. Symbols may be difficult to teach to children who have not yet grasped the concepts that they represent. At the same time, the concepts may be difficult to teach to children who have not yet mastered the symbols’. This situation compels teachers and mathematics education researchers to search for better strategies and techniques to help learners understand abstract mathematical concepts and ideas.

The intermediate phase (Grades 4 - 6) in the South African schooling system represents a transition phase from the lower primary school phase to the senior primary phase. In relation to Mathematics, this is a critical phase in that it lays the foundation for the introduction of algebra, one of the main branches in the discipline of Mathematics. Algebra is described as a generalised form of arithmetic, where symbols, letters and signs are used in place of or together with numbers (Chabongora 2012: 3). The teaching of mathematics for conceptual understanding becomes critical in the Intermediate Phase in preparing learners for more abstract mathematics in the Senior Phase and beyond.

In its continued search for better strategies and approaches for the effective teaching and learning of mathematics in primary schools, the Free State province of South Africa has introduced the use of manipulatives for mathematics teaching and learning. The establishment of mathematics laboratories in primary schools in 2011 has created a dilemma for both teachers and policymakers alike, raising serious questions as to how these mathematics laboratories have changed the primary school mathematics classrooms of the Free State. Specifically, researchers and policy makers have to be concerned about whether the introduction of mathematics laboratories has brought about any substantive changes to instruction and instructional practices of the mathematics teachers in the schools at all. If so, how is such change constructed and enacted by the teachers? How have they received, supported and sustained the policy changes in this context? It is these questions, among others, that have energised me to pursue the present research as a case study of the use of manipulatives in the mathematics laboratories in South Africa.

Further evidence of the need to improve South African learners' mathematics understanding and by implication their mathematical achievement, comes from the low levels of performance of the learners in the international, regional, and national mathematics assessments and tests respectively. The poor performance of South African learners in Mathematics is well documented (DBE 2010: 56; DBE 2011a: 98; DoE 2009: 87; OECD 2008: 20). For instance, at the international front, South Africa's Grade 9 learners scored the third lowest of all the participating countries with a score of 352, which is below the low-performance benchmark of 400 in the 2011 Trends in Mathematics and Science Study (TIMSS) (Reddy et al. 2012:4). At national level, the aggregate Grade 12 Mathematics scores for the past four years have been very low, with the percentage of learners performing at 40% and above hovering in around the 20% range (The DBE National Diagnostic Report on Learner Performance 2011a:98). More often, poor understanding of concepts is cited among the multiple causes for this poor performance. The DBE diagnostic report on the 2011 Grade 12 Mathematics results (2011a:99) noted that many candidates struggled with concepts in the curriculum that required deeper understanding. The results of the Annual National

Assessments (ANA) tests that were written in Numeracy/Mathematics and Literacy/Languages in 2011 by all South African primary school learners in Grades 1 – 6, as well as the Grade 9 learners, show similar trends. The average scores in Numeracy (Grade 3) and Mathematics (Grade 6) in the ANA results were 28% and 30% respectively (DBE 2011b: 20). The Free State learners' scores were 26% and 28% for Numeracy and Mathematics respectively, both figures below the national average. Undoubtedly, this has direct implications for the instructional methods and the type of experiences presented to learners to improve their conceptual and procedural understanding of Mathematics.

A number of instructional strategies, including strategies such as problem solving, collaborative learning, teaching for social justice, ethno-mathematics, etc. have been proposed and researched at different times, and some have even been tried in various mathematics classrooms globally. Although these strategies vary in terms of approaches, orientations and emphases, their common aim is to improve mathematics teaching and learning. The use of manipulatives to improve mathematics instruction belongs to all these sets of initiatives and strategies. Uttal *et al.* (1997:38) argue that the idea that children learn best through interacting with concrete objects has sparked much interest in the use of mathematics manipulatives. Manipulatives refer to all concrete objects that are specifically designed to help children learn mathematics and by implication, to help teachers enhance their teaching of mathematics. It is commonly assumed that concrete objects allow learners to establish connections between their everyday experiences and the abstract mathematical symbols, concepts and ideas. For instance, by dividing an orange equally among friends, children might develop a better understanding of the concept of fractions. While manipulatives are generally reputed to be worthwhile for enhancing the teaching and learning of mathematics, the realisation of such benefits depends on how manipulatives are being used by the teachers and learners, and how the concomitant changes in classroom instruction are received. This study seeks to understand how mathematics manipulatives are perceived, received and used to promote instruction and instructional change by primary school teachers in South Africa. In other words, what kind of mathematics teaching, classroom practices and mathematical knowledge do they help to develop in mathematics teachers?

Although much research has been conducted on the use of manipulatives in mathematics classrooms, little has been done on teachers' experiences with manipulatives and how those experiences shape their knowledge and classroom instruction. Mewborn and Cross (2007:260) conjecture that teachers' beliefs about the nature of Mathematics influence their beliefs about what it means to learn and do Mathematics, and these beliefs in turn influence instructional practices. These practices dictate the opportunities that students have to learn mathematics. Research on the use of manipulatives has mainly been dominated by the relationship between teacher variables and student achievement. However, it has been suggested that teachers' instructional practices may serve as a mediator of the relationship between these two constructs. For instance, there is general agreement that underlying beliefs guide a teacher's adoption and use of instructional techniques. This study puts teachers, and therefore teaching, at the centre of this curriculum initiative by specifically looking at how primary school teachers receive and use manipulatives, and how the use of manipulatives helps to improve, if at all, the teachers' mathematical knowledge for teaching and their mathematics classroom practices.

1.2 SIGNIFICANCE OF THE STUDY

This case study is significant in several ways. Firstly, at a personal level, the driving force behind this work comes from my interest in curriculum studies in general and in the Mathematics curriculum in particular. Within the Mathematics curriculum, my passion has always been on the professional development of mathematics teachers. Throughout my professional career, as a high school Mathematics teacher, a college Mathematics lecturer, a Mathematics subject advisor and ultimately as a senior manager and policy maker within the provincial department of education, I can relate to the teachers' struggles not only with the teaching of Mathematics, but also with the implementation of many of the changes in the newly developed Mathematics curriculum. Such struggles by the teachers may militate against the policy intentions aimed at improving teacher's knowledge, classroom practices and student learning if left unattended.

This study is also about awakening my consciousness with regards to my role as a curriculum change agent. Through the study, I also seek to develop my capacity to become (self) critical and (self) reflective and, most importantly, to hear the teachers' voices on the new curriculum innovations, especially the use of manipulatives in mathematics.

By adopting the critical theory lens for this study, I am able to ask questions about the utility and intentions of many interventions, such as the introduction of mathematics laboratories that are claimed to be inherently beneficial. I am able to ask critical questions about whose interests are served by curriculum change, for example: how do teachers influence the changes and practices, and what are the consequences with regard to their knowledge, beliefs and practices? It is this critical stance that helps to redefine my identity, to be unapologetic about my subjectivity, and to adopt a dialectical approach especially on interventions that are claimed to transform teaching practices and empower teachers. This study is therefore about re-examining my place within the curriculum processes both as a teacher and as a policymaker.

Secondly, the case itself is significant at the system level. The use of mathematics manipulatives in this case study is a critical indicator of the significant shifts in teachers' practices from those dominated and directed by teachers to those where learners engage with both physical and intellectual material. Knowledge gained through this study will allow teachers, researchers, other curriculum designers and teacher educators to gain in-depth understanding of the complexities of classroom instruction and become aware of various embedded mediating factors, both internal and external, that might either hinder or facilitate change in the teachers' practices of mathematics instruction. Focusing on such hidden elements will certainly assist the system in developing responsive intervention programmes in order to improve mathematics teaching and learning in a more sustainable manner.

It is hoped that insights from the present case study will assist the Department of Education with strategies that will transform mathematics laboratories to become learning sites for both learners and teachers in order to continually improve mathematics teaching and learning.

Thirdly, although much of the mathematics research on the use of manipulatives is located in the psychological paradigm, this study is located in the sociocultural paradigm. Its knowledge contribution to the research field in this domain will be in terms of bringing in the sociocultural dimension of the teaching practice. Moreover, as I have indicated earlier, much research in this field has mainly been on examining direct relationships, i.e. between teacher variables and student achievement, and manipulative use and student learning respectively. This research study will examine the indirect relationship between teachers' instructional practices (using manipulatives) and teacher variables such as teachers' Mathematical Knowledge for Teaching (MKT) and beliefs with the aim of contributing to a better understanding of the use of manipulatives within a complex classroom system.

1.3 BACKGROUND

This section looks at the status of Mathematics education in South Africa over two major time periods of curriculum reform in South Africa, i.e. the pre-1994 era and the post-1994 era, from the perspective of teacher roles and identities in an attempt to understand curriculum in mathematics classrooms practice. This will be viewed from a) the legislative and policy reforms in the education system in South Africa, b) major initiatives in mathematics education at the system level that were developed and implemented during the post-apartheid era to support the teaching and learning of Mathematics, and c) performance of South African learners in Mathematics to establish the impact of the above on mathematics teachers' instructional classroom practices. It is commonly acknowledged that changes arise from theoretical and philosophical underpinnings, what Vithal and Volmink (2005: 4-5) refer to as curriculum roots.

The history of education in the apartheid era in South Africa, a function of South Africa's segregationist social and discriminatory education policies, as well as its philosophy of Fundamental Pedagogics that underpinned such policies, is well documented (Vithal & Volmink 2005:5; Skovsmose 1998: 196; Parker & Adler 2005:62; Parker 2008: 59; OECD 2008: 204). The long term effects of these policies as manifested in discriminatory laws and practices were more pronounced in the Mathematics curriculum than in any other discipline of the school curricular. Literature supporting this view abounds (D'Ambrosio 1985, Khuzwayo 2005) and this is perhaps best articulated by Khuzwayo (2005: 309) who argues that 'South Africa is a country where the disparities in mathematics education represent a history of unjust social arrangements'. School Mathematics was used as a strategic tool to maintain and reproduce 'white supremacy' and therefore black subordination in South Africa. As a result, for Blacks in South Africa mathematics education has never been a right in terms of both access and quality. For instance, Black learners were denied access to Mathematics and many learners could not take Mathematics as a subject through to high school as many Black schools did not offer Mathematics at senior secondary level. In addition, Mathematics was taught then as an abstract, meaningless subject only to be memorised, and was meant to further the marginalization of Blacks (Khuzwayo 2005: 311).

Fundamental Pedagogics, as widely asserted, justified authoritarian teaching practices and promoted approaches that blocked and hindered the development of critical, reflective and innovative teaching. As noted by Khuzwayo (2005: 314), 'Neither the learner nor the teacher was seen to be in a position to challenge mathematics or mathematics knowledge but the ultimate goal was for the pupils and teachers to experience it as truth'. It is not surprising that Mathematics teaching was synonymous to 'telling' and 'transmission' of isolated and unrelated facts, algorithms and procedures. Mathematics classrooms were characterised by authoritarian teaching styles and reprimand, dominated by teacher-centred 'chalk and talk' methods, thus limiting learner engagement with mathematical concepts and ideas. Learning Mathematics was highly individualistic and meant memorizing, drilling and reciting decontexualised facts, procedures and algorithms, without any conceptual understanding at all. As a product of the apartheid system myself, I vividly recall how we used to meaninglessly sing, recite and drill multiplication tables in a chorus. The institutions preparing teachers for African schools often did not even offer Mathematics as a specialisation area (OECD 2008: 204). Assessment was equally traumatising, as noted by Graven in Graven (2002:21), almost synonymous with tests and examinations. I also recall how we used to stand against the wall every morning in an arithmetic classroom to 'pour' out the multiplication tables that we had memorised. Equally so, fundamental pedagogics also had a significant bearing on teachers who remained subservient and their teaching which was highly authoritative.

The end of the apartheid era in South Africa saw radical reforms in curriculum and classroom changes that would strive to give more, if not all students access to a better education, including the learning of Mathematics (Tirosh & Graeber 2003:645). This period in the history of South Africa is characterised by three major waves of curriculum reform, i.e. Curriculum 2005 (C2005), the Revised National Curriculum Statement (RNCS), and the Curriculum and Assessment Policy Statement (CAPS). Much of the reforms in the latter two were on the structure and terminology of these curriculum versions while the approaches remain the same as those of C2005. The impetus for these reforms in South Africa mainly came from the world-wide swing towards a constructivist perspective (Vithal & Volmink 2005: 6; Graven 2002:23). In South Africa this came across as a prescriptive methodology, replacing any existing set of ideas mathematics teachers might have had about the teaching of the subject.

Curriculum 2005 was launched in 1997 and implemented in phases from the beginning of 1998. In C2005, the subject Mathematics was replaced with the broader Learning Area Mathematical Literacy, Mathematics and Mathematical Sciences (MLMMS) within which Mathematics is defined as: '…the construction of knowledge that deals with qualitative and quantitative relationships of space and time. It is a human activity that deals with patterns, problem solving, logical thinking etc., in an attempt to understand the world and make use of that understanding. Such understanding is expressed, developed and contested through language, symbols and social interaction' (DoE, 1997a:2). Embedded in this definition is emphasis on a more social constructivist, learner-centred, and integrated approach to mathematics teaching and learning. Such emphasis represents, as noted by Graven (2002:24),a radical shift away from the previous teacher-centred approach towards a more learner-centred approach, from a performance-based approach to a competence-based approach, and from an absolutist paradigm which views Mathematics as a body of 'objective truth' to the contested nature of mathematics knowledge.

While there may be various interpretations of the notion of learner centred teaching, I found the definition by Brodie and Pournara (2005:33) more appropriate with regard to this study. They claim that substantive learner centred teaching involves engagement with learners' ideas through setting up tasks and classroom interactions which allow learners to engage in mathematical thinking and which enable teachers to help build and develop learners' ideas. Groupwork is one of the most popular strategies used to achieve learner centred teaching and has become almost synonymous with OBE classrooms in South Africa. Adler (2002: 3) also notes that most teachers in South Africa adopted forms/strategies such as group work and, by doing so, increased the possibilities of learning from talk (using language as a social thinking tool). Learners are expected to participate in oral and written work, communicating mathematically with their peers and their teacher to explain mathematical processes and solutions, describe and justify conjectures and present mathematical ideas and arguments, etc.

This sharp break with the shackles of fundamental pedagogics placed mathematics teachers in a dilemma, especially those teachers who were trained in the earlier behaviourist-influenced tradition. Recasting the role of the teacher from being a transmitter of knowledge to a facilitator of environments and experiences from which learners will learn seems to be a complex and daunting experience. The new curriculum was a novel system for all educators compounded by the fact that lesson content was no longer prescribed, leaving the development of learning programmes and learning material to the discretion of the teachers. A number of workshops were conducted to support the implementation of the new curriculum. Smith (2001), in her inquiry into mathematics teachers' experiences of policy change in South Africa, notes the following comments by teachers:

- 'There are those who continue to teach in their old ways, despite their attendance of workshops' (Smith 2001: 74).
- 'Teachers are told they are facilitators, however, they have not been taught to facilitate' (Smith 2001:75).
- 'Teaching mathematics necessarily incorporates drilling exercises and cannot solely be experienced, as is the perception ....Another thing that really worries me, I mean we have been, I was a product of where they threw the drilling of mathematics out and we had to experience and I know that a whole lot of my generation could not spell, we do not know our tables because of the system that we had' (Smith 2001:79)

- Although groupwork is important and perceived as meaningful, the learner as an individual remains important. "....they have moved away from individuals so that your stronger child is now carrying your weaker child' (Smith 2001:79)

However, one teacher in the study commented thus: 'Positive things about OBE is the new way of assessment, which is not only assessing academic performance, but other variants of skills and of achievements are also going to be assessed ' (Smith 2001: 79). These anecdotes indicate that some teachers still teach in the traditional way probably because of poor training or because of their own views of Mathematics. Whatever the reasons, these stories tell us that very little seem to have changed in our mathematics classrooms. With regard to teacher support , Christie in Smith (2001: 72) argues that not only was Curriculum 2005 imposed top-down, just like the apartheid curriculum, but it also seriously lacked sufficient teacher support, development and outcomes based on pedagogy preparation, offering only 'emergency training and materials'. This has resulted in different, often contradicting, interpretations of the new curriculum and its approaches.

Engelbrecht, Harding and Phiri (2010: 7-10) conducted a study to examine the mathematical preparedness of the 2009 intake of university students. These were the first cohort of students to have received school education within the OBE system. The following observations were noted:

- Students had a positive outlook and had confidence in their abilities.
- Students had a poor ability to 'write' Mathematics.
- There was a notable deterioration in general mathematical skills.
- There was also deterioration in content knowledge.

These two scenarios seem to confirm the findings of the committee that was appointed during the year 2000 to review C2005, being the following:

- Children's inability to read, write and count at the appropriate grade levels.
- Shift away from explicit teaching and learning to facilitation and group work

- Teachers did not know what to teach (DoE2000a:12).

Clearly, the deep shifts of philosophy and pedagogy implied in the new Mathematics curriculum pose serious concerns regarding the impact of the above reforms on instructional practices in mathematics classrooms. There is acknowledgement, as noted by the OECD report (2008:297), that the achievement of ‘deep change’ in educators’ practice takes time and needs many supportive elements. The top-down approach compounded by tight time-lines and inadequate preparation of teachers and resourcing, posed daunting challenges for the teaching force and teacher educators.

The context within which these changes were implemented needs to be recognised. Much has been written about the realities of mathematics education in South Africa which pose numerous challenges in terms of resources and adequately trained teachers. It is often acknowledged that any system is as good as its human resources. The National Mathematics and Science Audit report of 1997 published by Edusource revealed that more than 50% of professionally qualified mathematics teachers had no formal subject training and that the problem of inadequate training was particularly identified in the General Education and Training (GET) phase of the schooling system in South Africa (DoE 2001:12). This is further exacerbated by the reality that in South Africa, where very few students graduating with Mathematics choose teaching as a career. As noted by Makgato and Mji (2006: 254), the consequence of this is a vicious cycle of not many students taking Mathematics and Science related subjects at universities, resulting in an under-supply of mathematics educators in South Africa. This has resulted in some schools not offering Mathematics and Science any longer. The OECD report (2008:298) revealed that two thirds of South African teachers are between 35 and 50 years of age. This implies that most of the teachers in the system were trained during the apartheid era, often trapped in the shackles of Fundamental Pedagogics. These and other factors could have contributed to different and often inappropriate ways in which the new curriculum, including that of Mathematics, has been implemented in South Africa.

In addition to these factors, most black schools do not have the necessary resources such as textbooks, and classroom space, on which the new curriculum heavily relies, a situation which could have made it difficult for teachers to implement the constructivist approaches of the new curriculum. The Language in Education Policy (LiEP) (DoE 1997b) also has a bearing on the teaching of Mathematics within this curriculum reform. In most black schools, English is only introduced as a Language of Learning and Teaching (LoLT) in Grade 4, making it difficult to communicate mathematically, a central tenet of groupwork. The study conducted by Setati and colleagues found that Mathematics and Science teachers in both urban and non-urban schools felt much more pressure than their secondary colleagues to teach in English because their learners are still in the early stages of learning English (see Adler 2002: 3). These scenarios raise serious concerns about curriculum support to the effective implementation of these changes, as well as the preparedness of mathematics teachers with regard to the constructivist teaching approaches of the new curriculum, especially in primary schools.

To address these challenges, a number of initiatives and programmes have been developed at national and provincial levels, as well as at higher education institutions in South Africa. The National Strategy for Mathematics, Science and Technology (DoE 2001:14) for 2005-2009 was launched by the Ministry of Education in South Africa in 2001 to:

- Raise participation and performance by historically disadvantaged learners in Senior Certificate Mathematics and Physical Science,
- Provide high quality Mathematics, Science and Technology education for all learners taking the GET and FET certificates, and
- Increase and enhance human resource capacity to deliver quality Mathematics, Science and Technology education.

One key initiative within the National Strategy for MST has been the establishment of Dinaledi (Sotho word for 'stars') schools in 2001, targeting 102 schools at that moment as centres of excellence in mathematics and science, adopted as a strategy to promote Mathematics, Science and Technology in disadvantaged areas. By the year 2008, the target number of 500 schools had been reached. As noted in the 2007 national economic strategy, it was hoped that Dinaledi schools would double the number of Mathematics and Science high school graduates to 50 000 by 2008 (OECD 2008:94).

Another initiative at national level to address the challenges in mathematics education in South Africa is the Foundations for Learning (FfL) campaign (2007 - 2011) defined as an intensive four year campaign by the DoE aimed to improve the basic skills of learners in Grades 1-6 (DoE 2008). All primary schools were expected to increase the average learner performance in Literacy/Language and Numeracy/Mathematics to no less than 50% by 2011. The minimum expectations are that all teachers in Grades 1-6 will teach Numeracy/Mathematics skills for at least 30 minutes per day, including 20 minutes of written exercises and 20 minutes of mental arithmetic exercises. In addition, learners will be assessed annually through national standardised tests developed by the DoE (OECD 2008:172).

There are also a number of initiatives at provincial level aimed at addressing mathematics education challenges. For instance, in the Free State province of South Africa, the Member of the Executive Council (MEC) for Education launched a 'Maths for All' campaign in the year 2011 with the key focus of a) increasing the take-up of Further Education and Training (FET) Mathematics subject, b) strengthening the quality of mathematics teaching and learning in Free State schools, c) promoting and developing interest in Mathematics as a subject of choice, and d) increasing the number and quality of passes in Mathematics. This campaign has been supported *inter alia* by a number of curriculum resources such as the establishment of mathematics laboratories in more than 200 primary schools. The target of the department is to expand the mathematics laboratory project to a total 800 schools by 2014.

1.4 STATEMENT OF THE PROBLEM

Mathematical understanding is essential for primary school learners. Various approaches to enhance conceptual understanding in mathematics characterised many reforms in the Mathematics curriculum since the dawn of democracy in South Africa. These reforms in curriculum and in the approaches to the teaching and learning of Mathematics have often been supported through continuous teacher development programmes specifically designed to address the new approaches that emerged in South Africa over time. However, the main question remains as to whether these reforms in Mathematics and professional teacher development in new approaches, including the use of manipulatives, did bring about any change in mathematics teaching, classroom practices and teachers' mathematical knowledge. An in-depth study of teachers' discourse and practices in 6 elementary schools in South Africa in a project involving teachers' participation in a curriculum change and professional development found that although the teachers forged a complex practice with a significant shift in their social relations from isolation to collaboration, there was little substantive instructional change across all teachers' practices (Marneweck in Adler, Ball, Krainer, Lin and Novotna 2005: 373). This implies that in the end, teachers did not seize the opportunity to offer qualitatively better learning experiences for learners and for themselves. For this to happen, Siu, Siu and Wong (1993) have put forth a call for a new kind of mathematics teacher, the 'scholar teacher', one who is truly prepared to address the wealth of issues that arise in these changing times. Implied in this call are redefined views about a) a mathematics teacher, b) teacher knowledge acquisition as it impacts on and is impacted upon by teaching practices, and c) curriculum changes in Mathematics.

Manipulatives have been proposed as tools in mathematics classrooms because these tools can help students to learn Mathematics with understanding. The use of concrete objects or manipulatives in various mathematical strands has been a critical and necessary factor in the National Curriculum Statement (NCS) of Mathematics in South Africa (DoE 2002; DBE 2011c, DBE 2011d). This implies that there is some degree of recognition of the importance of manipulatives as an instructional strategy in not only enhancing children's learning of Mathematics but also in developing and nurturing their conceptual understanding of Mathematics. However, an underlying assumption is that teachers do possess the necessary skills to effectively use these manipulatives as an instructional strategy in their classroom practice. Hartshorn and Boren (1990: 3) note that teachers' training on the use of manipulatives critically influences their effectiveness. In this study I begin from the premise that the ability to effectively use manipulatives continues to be one of the neglected areas in the South African Mathematics curriculum, as it is often left to chance. Research on the use of manipulatives suggests that teachers' classroom practices in the use of manipulatives critically influences their effectiveness. Kelly (2006:188) contends that teachers need to know when, why and how to use manipulatives effectively, as well as to have opportunities to observe, first hand, the impact of allowing learning through exploration with concrete objects.

Research in this area has mainly focused on the direct relationship between teacher variables and student learning. What has not been sufficiently researched is the mediating role of teachers' instructional practice on the relationship between teacher variables and student learning.

As a policymaker and advocate for the use of manipulatives in the mathematics laboratories within the Free State schools, I remain curious about the realisation of their potential benefits as well as about challenges for teachers and for teaching in particular in relation to their use. In this study, as mentioned above, I view the introduction of mathematics laboratories as more of a call for pedagogic changes, and hence changes in the culture of teaching (teacher learning), teaching differently as it were. In this regard, Remillard and Bryans (2004:4) postulate that in order to teach differently, teachers need opportunities to learn mathematics in new ways and to consider new ideas with regard to teaching and learning.

This is why the broad purpose of this study is to explore the use of manipulatives as an opportunity for teachers and not just learners, to learn and also to explain the effects or lack thereof of manipulative use on mathematics teachers' own knowledge and classroom practices. In this study, I wanted to answer the research questions below from a critical stance.

1.5 RESEARCH QUESTIONS

1. How does the use of manipulatives in the teaching of primary school mathematics help to (re)shape the teachers' own mathematical knowledge for teaching?

2. How does the use of manipulatives help to (re)shape the teachers' own mathematical classroom practices?

3. How can we explain the influence of manipulatives or lack thereof on teachers' knowledge for teaching and classroom practices?

With regard to the first question, I further explore the following sub-questions:

a) What mathematical knowledge do teachers have?

b) How do teachers view Mathematics and mathematical knowledge?

c) How do teachers view the acquisition of mathematical knowledge?

With regard to the second question, I further explore the following sub-questions:

a) What instructional practices do teachers use?

b) What are the teachers’ views and beliefs about effective mathematics teaching?

c) What are the teachers' perceptions about the use of manipulatives?

With regard to the third question, I further explore the following sub-questions:

a) Do teachers’ pedagogical practices change when manipulatives are used in mathematics teaching?

b) Do teachers’ knowledge for teaching change when manipulatives are used in mathematics teaching?

c) What other factors mediate between teacher characteristics and instructional practices?

1.6 OBJECTIVES OF THE STUDY

As stated in this chapter, the use of concrete objects or manipulatives in various mathematical strands has been a critical and necessary factor in the National

Curriculum Statement (NCS) of Mathematics in South Africa. The Free State Department of Education has seized the opportunity of this policy prescript by establishing mathematics laboratories with concrete manipulatives in more than 200 primary schools across its five districts in an attempt to enhance the teaching and learning of Mathematics. However, what has not been fully explored is when, why and how these manipulatives will be effectively used by teachers in particular to enhance not only student learning but also to enhance the teachers' own instructional practices**.**

Research on the use of manipulatives has focussed mainly on the link between their use and students' mathematical learning. There is a relative dearth of research regarding how manipulatives come to transform pedagogy. This study is premised on the assumption that if manipulatives use is able to impact positively on students' mathematical learning, the use of manipulatives could also present opportunities for changes in pedagogical practices of mathematics teachers. The focus is on how manipulatives can act as a catalyst to transform pedagogical practices in Mathematics. This is in line with the argument advanced by Hardman (2005: 2) that a novel tool can provoke conflict within the context into which it is introduced, leading to the transformation of the practices within that context.

My belief is that to sustain any curriculum interventions teachers, through self-direction, reflections, learning and participation in a mathematics community can transform their own practices rather than simply follow a set of instructions on the use of manipulatives.

It is for this reason that the main objectives of this study are to:

I. Explore how the use of manipulatives in the teaching of primary school Mathematics help to (re)shape the teachers' own mathematical knowledge for teaching;
II. Explore how the use of manipulatives help to (re)shape the teachers' own mathematical classroom instruction;
III. Explain the influence of manipulatives or lack thereof on teachers' knowledge for teaching and classroom practices.

1.7 DELIMITATIONS OF THE STUDY

The study focused on primary schools in South Africa. Supporting curriculum implementation in all primary and secondary schools in the Free State is part of the researcher's professional work. In addition, the researcher is responsible for the Continuing Professional Teacher Development programme of the Free State Department of Education.

The research focused on four primary schools in the Mangaung Township of the Free State. However, through the Participatory Action Research (PAR) approach as well as the Learning Community (LC) structures in which mathematics teachers from other primary schools in Mangaung participated, the researcher managed to indirectly reach many other primary schools.

1.8 LIMITATIONS OF THE STUDY

This study has at least two limitations that relate to the utility of its recommendations as well as its research methodology respectively. Firstly, the research site is four primary schools with mathematics laboratories. Mathematics laboratories, as we have them in the Free State primary schools, consist of manipulatives, a situation that facilitates the availability of and easy access to manipulatives. It is widely acknowledged that the majority of schools in South Africa, especially in disadvantaged communities, are highly under-resourced. Research literature on manipulatives shows that the use of manipulatives depends on their availability. For, as rightly observed by Hartshorn & Boren (1990: 3), teachers can only use manipulatives if they are available. The findings of this study as well as the related recommendations might only be applicable to schools that are well resourced in terms of mathematics manipulatives. To minimize this limitation, I have been cautious in the choice of my case study, i.e. the exploration of the use of manipulatives in the mathematics laboratory. The choices are discussed further in chapter three.

Secondly, the study used Participatory Action Research (PAR), which requires real and active participation of appropriate stakeholders. It is commonly accepted that such participation will lead to greater effectiveness, ownership, efficiency, equity, transparency and sustainability. However, the notion of PAR can be alien and can therefore also limit participation. To decrease this limitation, prior to the formal research study, I conducted workshops using video clips showing communities taking responsibility for their own development. The need for teachers to act as social agents in order to transform their situatedness was underscored in all the workshops.

The other potential limitation relates to my personal interest in the study as articulated in section 1.2 above on the significance of the study. In this study, the possibility that my personal interest might influence data and hence compromise the results of the study was given consideration. Through careful choice of PAR as my data collection approach, the establishment of PAR structures and professional guidance by my supervisor, it is hoped that this potential limitation will not compromise the execution of the study and its results. Measures such as reflective interviews and PAR group discussions as discussed in chapter three, meant to collectively clarify data and review the researcher's interpretations helped to minimize these potential limitations.

1.9 FEASIBILITY OF THE STUDY

Clusters of schools in the Free State have organised themselves into Professional Learning communities (PLCs) managed by mathematics lead teachers from participating schools. These structures, facilitated by the mathematics lead teachers, were used as part of the research design and methodology, and made it easy for the participants to be actively involved in the study and to gather the necessary data needed for the study. This study made use of PLCs that have been established in the Mangaung cluster of schools.

1.10 THESIS OUTLINE

Chapter 1: Orientation and Background to the study

In Chapter One (this chapter) I introduced the study and presented a motivation for the study by highlighting its significance both at a personal level and a system level. I have also contextualised the study by presenting the status of school Mathematics education in South Africa. The aim of the study, i.e. to explore the use of manipulatives to enhance the teaching and learning of Mathematics, as well as the research problem and research questions were also articulated.

Chapter 2: Literature review on constructs that relate to my study including the theoretical framework on which the study is grounded.

In Chapter Two I reviewed a range of literature relevant to the focus of my study and that helped me to appropriate the current study within the existing literature. I also reflected on and adopted critical theory as the underlying theoretical framework for this study.

Chapter 3: Research methodology and design.

Chapter Three involves an outline of the research design, research methodology and the research method that I used to examine the research problem at hand. This also includes data gathering, data analysis strategy and ethical considerations.

Chapter 4: Data presentation and interpretation.

In Chapter Four I presented a discussion of the qualitative results according to my key theme as well as my sub themes.

Chapter 5: Presentation of findings and analysis.

Chapter Five provides a summary of the research and the main findings, as well as a detailed analysis thereof.

Chapter 6: Findings, recommendations and general conclusions
Chapter six provided a summary of the main findings organised in respect of my research questions. I also reflected on the gaps identified as well as the limitations of the study and provided recommendations at different levels of the system. I finally presented my general conclusions informed by my findings.

1.11 CHAPTER SUMMARY

In this chapter, I provided orientation and background to the study. As an introduction to the study, I started by firstly outlining the status of mathematics education in South Africa. This was done by reflecting on the poor performance of South African learners in mathematics and the introduction of mathematics laboratories in the Free State primary schools as an intervention to improve the teaching and learning of mathematics. I also presented a motivation for the study by highlighting the significance of the study at personal and system levels respectively. To contextualise the study, I provided a brief background on the status of Mathematics education in South Africa over two major time periods of curriculum reform in South Africa. This was particularly done from the perspective of teacher roles and identities in an attempt to understand curriculum in mathematics classroom practices.

The chapter also provided the problem statement, recognising that although the use of manipulatives has been a critical factor in the NCS of Mathematics in South Africa, the ability to effectively use manipulatives continues to be one of the neglected areas in the curriculum as it is often left to chance. The chapter outlined the problem as being whether reforms in Mathematics including the use of manipulatives, did bring about any change in mathematics teaching, classroom practices and teachers' mathematical knowledge. To address the problem, the chapter reflected on the research questions and the objectives of the study. I lastly reflected on the study's delimitations, limitations and its feasibility.

The next chapter presents the review of existing literature on constructs that relate to my study including the theoretical framework on which the study is grounded.

CHAPTER 2: LITERATURE REVIEW

2.1 INTRODUCTION

In this chapter I review a range of literature relevant to the present study. Firstly, I reflect on and adopt critical theory as an underlying theoretical framework to guide my research study in exploring the role of manipulatives in the teaching and learning of mathematics in primary schools and to answer the following research questions:

- How does the use of manipulatives in the teaching of primary school mathematics help to reshape the teachers' own mathematical knowledge for teaching?
- How does the use of manipulatives help to reshape the teachers' own mathematical classroom practice?
- How can we explain the influence of manipulatives or lack thereof on teachers' knowledge of mathematics and classroom practices?

Secondly, I reviewed four bodies of literature that were critical for shaping and conducting my study, namely:

i. Teachers' Knowledge for Teaching,

ii. Teachers' Classroom practice,

iii. Teachers' views about mathematics, mathematics teaching, and mathematics classroom practice,

iv. Mathematical manipulatives.

2.2 USING CRITICAL THEORY AS A LENS TO UNDERSTAND TEACHERS' KNOWLEDGE AND PRACTICE IN A TRANSFORMING SOUTH AFRICA

In order to understand and operationalise the aim and objectives stated above, I used Critical Theory as the framework. I found this framework more appropriate than positivism because unlike the latter, it highlights the theme of power and focuses on issues of change and transformation, which are at the very heart of this study (Crotty 1998: 4 – 9; Guba 1990: 19).

The process of using manipulatives to transform teachers' mathematical knowledge for teaching into effective classroom practice can best be analysed and understood from a perspective that sees reality as subjective and dependent on the individual teachers' constructions and recreations (Guba & Lincoln 1994: 107). It is not an easy task, if at all possible, to formulate a general, objective and all inclusive theory about how such a process occurs. Each teacher experiences this translation differently, depending on contextual factors that might be at play at any given time. To understand this process of translation one has to recognise that knowledge is always socially constructed in social interactions, for instance among teachers, between teachers and learners, and so on (Guba 1990:18). It is never an isolated experience that has universal applicability. Each teacher's classroom practice is guided by his/her perceptions and inputs. As a result, the teacher alone is able to reveal to me his/her felt experiences of how the process of translation of mathematical knowledge for teaching to classroom practice occurs through the use of manipulatives.

It is clear from the discussion above that the epistemological position I assume gives a lot of respect and power to the research participants unlike positivism which holds the view that things exist independently of consciousness and experience, and that they have truth and meaning residing in them (Crotty 1998: 4 - 9; Guba 1990: 19). I have adopted a broader framework of critical theory, underpinned by my epistemological stance of constructivism, as a theoretical lens used to frame my inquiry. The choice of this perspective helped me to explore the teaching and learning interactions and relations in a mathematics laboratory, and to reveal, explain and address mediations – cultural, social, historical, intellectual, ideological, etc. that may limit the opportunities and constrain the potential of mathematical manipulatives to enhance learning by both the teachers and learners.

2.2.1 The Origin of Critical Theory

Critical Theory, dating back to the 1920s and 1930s, has a very long, complex and multifaceted history that is usually traced back to the 1920s and 1930s. It is rooted in the Marxist theoretical perspective of which the main purpose was to critique and subvert domination in all its forms (Stinson, Bidwell, Jett, Powell and Thurman 2007: 620). It owes a profound debt to its originators, associated with the so-called ‘Frankfurt School’, in Frankfurt, Germany. The Institute was originally established during 1923 as the first Marxist-oriented research centre affiliated with a major German university. The tradition was developed by the first generation of critical theorists such as Benjamin, Adorno, Horkheimer, Marcuse, Lowenthal, and others (Kincheloe 2004: 46; McLaren 1989: 159). Jürgen Habermas, who developed social critique by establishing a Theory of Communicative Action (TCA), is often considered the most significant member of the second generation of the school (Bronner & Kellner 1989: 2).

Critical theorists begin with a perception of the world as being imperfect and unjust, as McLaren (1989: 166) articulates: ‘men and women are essentially unfree and inhabit a world rife with contradictions and asymmetries of power and privilege’. It is for this reason that a) the most defining features of critical theory are emancipation and transformation, b) the primary preoccupation of critical theory is with issues of power, justice and liberation of the oppressed from social practices and ideology that systematically mask power relations, and c) critical theory is on the side of the oppressed and the marginalized. Theorists in this tradition are concerned with the ways in which the interactive context between individual and society produce and reproduce a social system of domination, and therefore call for the reinterpretation of the world in order to transform it. Critical theory can be defined using a number of different but interconnected characteristics, as the practice of education in South Africa demonstrates.

For example, the study is located in South Africa, a country with highly unequal societies mainly due to the legacy of apartheid. It is furthermore acknowledged that a key challenge in South Africa remains the bringing about of equality within a public school system that operates within a highly unequal society (e.g. Action plan to 2014: Towards the Realisation of Schooling 2025, DBE 2011e). Critical theory thus becomes relevant as a theory that seeks to reveal contradictions, social inequalities and dominances that exist in social systems. Much has been written about the apartheid rule as well as its legacy of social injustices including inequalities, discrimination along racial, ethnic and gender lines (e.g. Adler 1994; Chisholm, 2012; Motala, Dieltiens, Carrim, Kgobe, Moyo and Rembe 2007; OECD 2008: 204; Parker & Adler 2005, Skovsmose 1998). These inequalities pervaded all spheres of life, including the education system. The particular education legacy of apartheid is a system fundamentally scarred by racial inequality, absurd levels of fragmentation, authoritarianism, and low skills-base (Adler 1994: 102). For example, in 1993, per capita expenditure in schools was four times less for an African pupil than for a white pupil (Edusource 1994 in Adler 1994: 102) and in African schools 72% of mathematics teachers and 70% of science teachers were under–qualified (Edusource 1993 in Adler 1994: 102). In 2003, it was estimated that almost 1 in 10 of the white cohort achieved an A aggregate in matric as compared to just over 1 in 1000 of the black cohort (Van der Berg 2007:859). Eighteen years into the democratic order; although inequalities are considerably smaller than those that existed under apartheid, inequalities resulting mainly from racial and ethnic fragmentations continue to pervade the provision of schooling and to have adverse implications for quality of and access to schooling in South Africa, especially for black learners.

In addition, the above tenets of critical theory have made it even more relevant to the study which aims at improving the practice of classroom teaching of mathematics because this is the discipline that holds a lot of power to those teachers and learners who are able to perform well in it. The learners of the teacher who can translate his/her mathematical knowledge for teaching into classroom practice more effectively, will benefit tremendously as better mathematical knowledge can enable them to secure better employment or become eligible for further education and training. The study of this translation thus is embedded into power issues where the 'better' teachers are those who have gained power over their own styles of teaching, meta-cognitively speaking.

Adler (2005: 163) argues that 'In South Africa, we continue to work in a socio-cultural and political context deeply scarred by apartheid education' and, as observed by Parker (2008: 98), the unequal distribution of knowledge and 'ability' is starker in the field of mathematics than in most other areas of the school curriculum'. This is corroborated by the Ministry of Basic Education in its Action Plan to 2014: Towards the Realisation of Schooling 2025 (DBE 2011e: 17) which acknowledged that 'the legacy of inequality with respect to many years of unequal capital expenditure remains stark; both as far as physical capital (such as school buildings and human capital (largely in the form of the training that teachers received in the past) are concerned'. This is more so because, as noted in the National Education Policy Investigation (NEPI), the majority of South African teachers have been trained in racially segregated colleges of education, most of which were 'academically isolated, small, poorly equipped and ineffective in the provision of quality teacher education, producing enormous variation in teacher qualifications' (Adler 1994: 103). This has led to variations in the quality of teachers where, according to Adler (1997:95), the typical white primary or secondary mathematics teacher is not only better qualified, but also works in very different conditions. Historically white, state aided schools are relatively well resourced. These differences in the quality and material conditions of teachers that advantage white teachers over black teachers raise serious questions about the taken-for-granted commitment of government to provide quality basic education to all learners.

Following the official end of apartheid in 1994, South Africa adopted a new constitution as well as new education policies (e.g. The 1995 White Paper on Education and Training, The South African Schools Act [SASA] 1996, The National Education Policy Act of 1996, etc.) all of which guarantee the fundamental right of all citizens to basic education, equity, redress, and the improvement of quality of schooling. For example, the underlying policy elements, and therefore areas of sustained concern for education in South Africa, are contained in the first Education White Paper (1995), namely access, success, quality, equity, and redress. The SASA (RSA, in its preamble, captured its rationale thus:

` ...this country requires a new national system for schools which will redress past injustices in educational provision, provide an education of progressively

> high quality for all learners and in so doing lay a strong foundation for the development of all our people's talents and capabilities, advance the democratic transformation of society, combat racism, and sexism and all other forms of unfair discrimination and intolerance, contribute to the eradication of poverty and the economic well-being of society, protect and advance our diverse cultures and languages, uphold the rights of all learners, parents and educators, and promote their acceptance of responsibility for the organisation, governance and funding of schools in partnership with the state; ... (Republic of South Africa 1996a).

Despite its progressive vision to provide a uniform funding system, the implementation of SASA has had unintended consequences that led to racial inequalities between schools servicing affluent communities and those servicing poor communities. Jansen and Taylor (2003: 20) argue that the political agreements that allowed for uncapped parental contributions to schooling have effectively washed out any gains from equity-based funding favouring black schools. This view is also supported by others (see Adler 1997:95; Motala *et al*. 2007:2; Van der Berg 2008:10) who argue that the fee structure of schools is one of the factors that continue to contribute to maintaining de facto disparities between schools. Jansen and Taylor (2003: 20) further note that in former white schools, tuition fees have in many cases increased by more than 100% since 1994, thereby creating a de facto class-differentiated education system in which mainly white learners are able to enjoy access. Clearly, this has created inequalities regarding indicators such as access, class size, quality and quantity of teachers, resource allocation, etc. For example, white schools use their fee-charging capability to not only employ additional teachers and thereby maintain low learner-educator ratios (OECD 2008: 22) but also to exclude learners from poor families who would ordinarily not afford to pay these exorbitant school fees. These paradoxes lead to the ideals of equity and quality in the schooling system being elusive.

2.2.2 Teaching and Learning in the Critical Paradigm

Linked to the above, critical pedagogy holds the view that knowledge (and, by extension, a teacher's professional knowledge) is not 'neutral' and 'objective' as often claimed (Darder 1991: 77, Kincheloe 2007: 12, Kincheloe 2004: 5; McLaren 1989: 169). Kincheloe (2004: 52) further argues that colleges of teacher education curricula devote more time to teaching technique, techniques examined outside of social, political and philosophical context. Kincheloe (2004: 5) states that 'little in the world and certainly little in the world of education is neutral' and further argues that knowledge is always produced as part of a web of power relationships (Kincheloe 2008: 10). In the critical paradigm, the act of knowing is a social practice with the intention to transform reality (teaching and learning). In reading the world, teachers get to understand and transform their own practice through individual and collective engagement with the learners and the 'others' outside the classroom walls with the intent to transform their practice (writing the world). This view recasts teachers as agents for social change, a role that requires of teachers to connect their classroom practice with the socio-political context in which learning takes place. Habermas in Kemmis & McTaggart (2005: 294) states that this makes it imperative for teachers to open communicative spaces in which people together can critically reflect on their lived experiences, character, conduct and consequences of their practice. Teachers construct their own knowledge through dialogical teaching using the language of critique and possibility, which Giroux (2004a: 36) advocates for, while ensuring classroom relations that encourage dialogue, deliberation and the power of learners to raise questions (Giroux 2004a: 43). In this way, the teacher becomes a teacher learner, subjecting his own teaching practice to scrutiny, critique and constant review by the collective in order to enhance learning. This is in line with Freire's 'problem-posing' education that regards dialogue as indispensable to the act of cognition, grounded on creativity, and stimulating true reflection and action upon reality (Giroux 2004b: 83). In this view, teachers and learners are not empty vessels, passively waiting to be filled with knowledge, but rather individuals with knowledge and life experiences, situated within their cultural, class, racial, historical and gender contexts (Breunig 2005: 117). Teachers in this view are seen as critical co-investigators in dialogue with learners and their peers on material that is connected to their own situatedness.

For Freire (2005: 51), human activity consists of both action and reflection, in other words, praxis that leads to the transformation of the world. Critical pedagogy views teachers not only as active participants in all the stages of the research process but also as co-researchers from the design stage up to the evaluation of the study. Warning against dehumanisation brought about by over emphasis on techniques, Kincheloe (2004: 2) posits that educators must first recognise that there is a problem and transform the reality (practice). This is further corroborated by Bhaskar in Francis and le Roux (2011: 301) who suggests an active concept of the subject, actively participating in shaping the course of a person's life in the process of ongoing choices of behavioural options in the face of social demands. Adams *et al*. in Francis and Le Roux (2011: 301) argue that teachers have a role to play in dismantling oppression and generating a vision for a more socially just future. The teacher becomes a researcher, who collectively with others, identifies and examine the current action (research problem) in order to change and improve on it. The teacher, together with learners, uses dialogue to identify and investigate generative themes from the concrete reality of the learner (Freire 2005: 106). Thematic investigation is expressed as an educational pursuit, a cultural action upon an individual's contextual reality (Freire 2005: 111). Freire (2005:106) further posits that this involves the investigation of people's thinking about and action upon the dimensions of reality, which is their praxis. There is a broad theoretical and empirical consensus that the influence of metacognition on the outcome of learning is strongly linked to the process of reflection. This casts the role of the teacher as both a theorist and a practitioner in the act of knowing. As a co-researcher the teacher participates in data collection from his own practice as he engages in dialogical teaching, in a sense objectifying his own practices by recording not only his but also the learners' reactions, impressions and judgements. The teacher also assumes the role of an artist (Shor & Freire 1987: 115) operating in the classroom (stage) where learners' voices are expressed in different visual and auditory modes. Data collected to answer the research question are critically reflected upon with the other participants with the view to allow the voice of the 'other' in remaking or relearning reality i.e. his teaching practice.

Critical pedagogy holds the view that pedagogy and hence praxis is never innocent (Giroux 2004a: 38). The first step towards the development of social consciousness as implied above, is for teachers to rethink the cultural and political baggage brought to each educational encounter (Giroux 2004a: 38) and to consciously deconstruct their false consciousness as they may pose as barriers to the development of ‘critical consciousness’. Kemmis and Smith (Kemmis 2007: 4) refer to ‘practice architectures’ that constitute mediating preconditions for practice. These ‘practice architectures’ are embodied in the teachers’ pre-existing beliefs, values, ideologies, interests and capabilities, relating to his own understandings and those of others about his/her teaching practice and the world. Freire argues that people need to engage in a praxis that incorporates theory, action and reflection as a means to work towards social change and justice, and he devised a literacy program based on this ideal as well as on the practical needs of his students. Freire (2005: 17) further posits that dialogue presents itself as an indispensable component of both the process of learning and teaching, and that the fundamental goal of dialogical teaching is to create a process of learning and teaching that involves theorising about the experiences shared in the dialogue process.

This calls for teachers to be reflective practitioners, possessing the capacity for critical inquiry and self-reflection. Teachers must have the ability to subject their pre-existing views and assumptions about their teaching practice to collective critique and scrutiny, to critically reflect on their own consciousness, to be open about and illuminate their own views and assumptions, to interact with others in a non-defensive language, to pose their false consciousness as a problem (self-doubt) and to develop consciousness which is less false. Positivists’ mechanistic view of reality does not perceive that the concrete situation of individuals conditions their consciousness of the world, and that in turn this consciousness conditions their attitudes and their ways of dealing with reality. For them, reality can only be transformed mechanically, without posing the person’s false consciousness of reality as a problem.

Freire framed the notion of empowerment within the concept of conscientization defined as learning to perceive social, political and economic contradictions and to take action against the oppressive elements of reality (Braa & Callero 2006: 12). Through active participation in the research processes, teachers are involved in decision-making processes at all stages of the research with the others. Shor and Freire (1987: 108) cast empowerment as a social act of transforming the society. This implies that the teacher in this view takes decision regarding his own practice, using his own teaching as theoretical resource, without relying on predesigned techniques.

Through critical discourse with others, the teacher becomes empowered to critically examine the world for inequalities and power structures that maintain social inequalities and subsequently transforms it. The role of the teachers in this paradigm is one of transformative intellectuals, described by Giroux (2010: 4) as requiring teachers to develop a discourse that unites the language of critique with the language of possibility so that social educators recognize that they can make a difference. Drawing from this view, teachers need to critically analyse, explore and illuminate the self, the learners and reality in order to identify and address any inherent power relations, biases, assumptions and structural inequalities that might militate against the transformation of their teaching practice. This demonstrates autonomy to act upon reality with the object to transform it. In this way, the teacher takes responsibility for his own development and the development of the others as a transformative intellectual. In contrast to the positivist view in which knowledge is 'objective', teachers engage in their teaching practice as mere technicians, whose role is just to deposit knowledge through their learnt methods, outside context. Izadinia (2011: 144) contends that letting teachers speak their minds and make their voices heard in the classroom may set the ground for them to become deeply and actively involved in all aspects of the learning-teaching process. In this view, teachers actively participate in the process, starting with the identification of themes from their own contexts, negotiating and co-determining their own aims and goals, procedures, structure, content and criteria for assessment. The view of teachers held in Critical Pedagogy is one of critical autonomous learners who can analyse, criticise and question not only materials they are studying and teaching, but also the context they live in (Izadinia 2011: 138).

At the same time, learners critically explore their consciousness in as far as it poses itself as a constraint to learning and to social transformation, and begin to deconstruct any false consciousness shaped by the social and economic circumstances in which they live. Deconstruction is primarily concerned with the unpacking of metaphors that have been solidified into truth. Breunig (2005: 118) argues that from an early age many students have been taught that to be a 'good student' requires one to be silent, passive and receiving knowledge transmitted by the teacher unquestioningly. Nowhere is this more evident than in dominant mathematics classroom practice. 'Tell and drill' remains the dominant practice in mathematics classrooms. Textbook-based teaching and rule-bound learning styles constitute pupils' mathematics diet (Adler 1994: 104). To a large extent this is exactly what the curriculum-in-use demands. Passing tests typically means being adept at a range of text-book exercises. Mathematics as a school subject now means getting answers to exercises as quickly as possible, and this is usually achieved by doing something to the numbers or symbols in the problem. Seldom are pupils required to communicate mathematically by getting the opportunity to explain and justify their manner of reasoning in solving mathematical problems. More profoundly, problems are presented meaninglessly and out of context. This is why, within a critical pedagogy paradigm, learners also have to critically reflect on their own epistemologies, subjectivities and biases as often reflected in their own and others' previously held assumptions about teaching and learning.

2.2.3 Mathematics teaching in the South African context and Critical Pedagogy

Drawing from the above discussion, it becomes imperative to understand teachers' knowledge, their practices and identities within the curriculum reform context in South Africa where the study is conducted.

Current curriculum reform and innovations, including those in mathematics education, present challenges, complexities and paradoxes to teaching and learning not only in South Africa, but worldwide. South Africa, as discussed elsewhere in this study, has witnessed major waves of curriculum reform (e.g. Outcomes Based Curriculum 2005, the National Curriculum Statement, and recently the Curriculum and Assessment Policy Statement) as well as the introduction of the Norms and Standards for Educators (NSE) (DoE 2000b), which outline the roles of educators as being mediators of learning, interpreters and designers of learning programmes and materials; leaders, administrators and managers; scholars, researchers and lifelong learners; community members, citizens and pastors; assessors; and subject specialists. In addition, the National Qualifications Framework Act: Policy on the minimum requirements for Teacher Education Qualifications (DHET 2011: 8–9) identifies five key competencies in respect of knowledge practices or learning that teaching as a complex activity requires. These types of knowledge practices or learning associated with the acquisition, integration and application of knowledge for teaching purposes are: a) Disciplinary learning that/which refers to subject matter knowledge, b) Pedagogical learning that incorporates general pedagogical knowledge and includes knowledge of learners, learning, curriculum and general instructional and assessment strategies, c) Practical learning that involves learning in and from practice, d) Fundamental learning that refers to learning to converse competently in a second official language, the ability to use Information and Communication Technologies (ICTs) competently, and the acquisition of academic literacy's, and e) Situational learning that entails knowledge of varied learning situations, contexts and environments of education (classrooms, schools, communities, districts, regions, countries and globally), as well as of prevailing policy, political and organisational contexts. The latter, as well as the seven roles as outlined in the NSE, in my view represent the embodiment of Freire’s liberating education or social justice education, making particular demands on teachers in terms of their classroom pedagogies and educational practices and, consequently, on their identities that are in line with the tenets of critical pedagogy, i.e. transformation, agency and empowerment. Adler, Reed, Lelliot and Setati (2002: 150) note that

> Teachers are expected to teach new knowledge in new ways, and so engage in ongoing learning in relation to their professional expertise. They are expected to produce learners with high level skills and integrated and flexible knowledge so that they may take their rightful place as informed and active citizens in their knowledge societies. Teachers are also expected to play a significant role in eradicating the social ills and inequalities that their learners bring to their classrooms.

This demonstrates the complexity of the role of teachers as key agents in the transformation of the education system. There seems to be some similarities between the roles of teachers in Freire’s liberating education and the roles of mathematics teachers as envisioned in both the NCS for mathematics and the NSE in South Africa. This is reflected firstly in the new curriculum for mathematics that puts greater emphasis on teaching for conceptual understanding, citizenship and on mathematical processes such as problem solving and reasoning skills, as well as and the ability to communicate mathematically (CAPS for Mathematics Grades 4-6, DBE 2011d, 4, 8–9). Secondly, it is also reflected in the view of mathematics as a human activity in the NCS as discussed in chapter one of this study (CAPS for Mathematics Grades 4–6, DBE 2011d: 8). Thirdly, it is reflected in the role of mathematics teachers as researchers, as scholars and as learners. All these orientations recast mathematics teachers as active participants in the construction of their own mathematical knowledge in order to continually improve their classroom practice. However, despite these similarities and the vision for transformation they represent, not much seems to have changed in many mathematics classrooms as well as in teacher identities in South Africa. This vision-reality tension raises important questions on whether these roles are realisable.

The tension can be explained in terms of the legacy of the apartheid curriculum which has also been characterised by authoritarianism and racial division, an over-reliance on textbooks, and a static view of the role of teachers as curriculum receivers (African National Council 1994; NEPI 1993 both in Adler 1994: 103). This is further supported by Ball, Cohen, Spillane and others in Jita and Vandeyar (2006: 40) who argue that the reform agenda represents a tall order for many of the classroom teachers whose experiences of mathematics and mathematics identities have been within the traditional approaches to school subject, which placed less emphasis on problem solving, discourse and reasoning. This implies that the new teacher identities may be difficult to realise as mathematics teachers, especially in poorer schools, remain recipients of knowledge passed down unquestioningly to them through curriculum materials. This view is corroborated by Adler (1997: 95) who, in her discussion of teachers as researchers in South Africa, notes that the majority of teachers are more used to following the prescriptions of education authorities than they are to working reflexively, reducing them to mere technicians implementing someone else's ideas. This is also exacerbated by pressure to complete the syllabus on time, especially in the context of the high stakes National Senior Certificate (NSC) examinations and more recently by the Annual National Assessments (ANA) written in mathematics and languages in Grades 1 to 6 as well as in Grade 9. Expecting teachers to perform these transformative roles amid the pressure to produce good results in examinations and other forms of assessment may prove to be elusive.

In addition, the reform curriculum in mathematics also requires that teachers use manipulatives in topics such as place value, fractions, shapes, etc. to promote meaningful learning of mathematical concepts and ideas. However, the effective use of manipulatives fundamentally depends on teachers' firm foundation of mathematical knowledge of these topics, as well as on their mathematical pedagogies. Much has been written about teachers' lack of mathematical content knowledge in South Africa's poorer schools (e.g. Adler 1994, 1997; DoE 2007; Bloch 2009; Taylor & Vinjevold 1999). For example, the report of the Ministerial Committee on Schools that work (DoE 2007b) noted that white secondary mathematics teachers have university degrees with a minimum of 1 year of tertiary mathematics. By contrast, qualified black secondary mathematics teachers are likely to have a three year college teaching diploma, with often extremely little Post-Secondary School mathematics. In this regard, Grootenboer and Jorgensen (2009: 256) contend that being strong in content knowledge offered a sense of confidence, which in turn was realised through teacher actions. This suggests that although the use of manipulatives may be new for many mathematics teachers (both black and white), inequalities in the quality of teachers give white teachers a competitive edge over teachers in poor schools regarding the use of manipulatives in their classroom practice. This situation, if not addressed through professional teacher development programmes that integrate the use of manipulatives to support teachers from poor schools, may contribute to reproducing inequalities that the reform mathematics curriculum purports to redress.

Communicating mathematically is one of the tenets of the reform curriculum in South Africa. This implies an acknowledgement of the role of language in the promotion of a deeper understanding of mathematics. In particular, Barwell, Bishop, Erduran, Halai, Iyamuremye, Nyabanyaa, Risvi, Rodrigues, Rubagiza and Uworwabayeho (2007: 43) note that it has become accepted that the use of the learners' home language can be a powerful tool in enabling learners to construct meaning across subjects, including mathematics and science. It needs to be recognised that the Language-in-Education Policy (LIEP) in South Africa (DoE 1997a) encourages multilingualism. However, many bi/multilingual classrooms in South Africa, including resources such as textbooks and curriculum material, continue to be characterised by the hegemony of English as the language of instruction and of initiating mathematical discourses despite it being the home language of the minority (Setati 2005: 462). Moreover, as noted by Adler (2002:3), the main language needs to be treated as a resource and a thinking tool rather than problem. Setati (2005: 450) reminds us that over and above it being a tool for communication and thinking, language has been, and continues to be used as a political tool to gain access to educational and material resources globally. This implies that the hegemony of English is a manifestation of the legacy of colonialism that continues to pervade many developing countries around the globe

The implementation of LIEP in classroom teaching and learning presents contradictions in relation to the assertion that language is a powerful tool for communication and thinking. Instead, for many learners and teachers, communication and articulation of mathematical ideas and concepts have not been easy due to a poor command of English. In addition, the inadequate communication ability of South African learners and teachers in the language of instruction has been identified as one of the factors contributing to both poor learner performance (e.g. Adler 2002; Howie 2003; van der Berg 2007) and to access to mathematics (Setati 2005: 462). This is particularly so in mathematics teaching in poorer schools. For example, the study conducted by Setati, Adler, Reed and Bapoo (2002: 144) found that language issues are complex in non-urban schools, where there is very limited English infrastructure in the surrounding community on which teachers can build. Exposure to English in these schools is via the teacher. The study also found that mathematics and science teachers in both urban and non-urban schools felt much more pressure than their secondary school colleagues to teach in English because their learners are still in the early stages of learning English. This further demonstrates inequalities between rural and urban schools due to English language use by non-English speakers and the rigidity with which English is used even where it is to the disadvantage of learners.

2.2.4 Justification for the Critical Theory Approach

This section provides a case for appropriating this study within a critical theory framework in general and within critical pedagogy in particular. In this study I argue that critical theory does offer powerful analytical, theoretical underpinnings and research tools to explore and understand mathematics laboratory interactions and relations within a complex classroom system in primary schools. Of the various possible frameworks to examine mathematics teaching and learning, critical theory has been adopted as a theoretical lens for the following reasons:

There is a growing interest among mathematics educational researchers and mathematics educators in exploring the social aspects of mathematics education. This is largely attributed to the 'social turn' that started in the mid-80s, as noted by Lerman (2006: 171). This 'social turn' marked a shift in the use of theoretical and methodological frameworks from disciplines such as sociology, social psychology, anthropology, political science and cultural studies rather than from cognitive psychology as has been the case for a great deal in this area (Valero 2008: 44). Although the disciplines of mathematics and psychology continue to be very influential in this area of research, Mathematics education is increasingly being recognised as a field of study in the social sciences. Since this study was exploring the social aspects of mathematics teaching and learning in a mathematics laboratory context, it makes sense to look to critical theory as a social theory for its theoretical basis.

There is also a growing interest in mathematics education research from a critical perspective including from a social, cultural, political, historical and other dimensions. For example, there has been a plethora of research studies in the past few decades relating to this trend, e.g. mathematics for social justice, critical mathematics, ethno mathematics and more (Vithal 2004; Skovsmose 1994; D'Ambrosio 1985; Valero 2008; Lerman 2006; etc.). These studies provide the necessary theoretical methodological frameworks for a critical stance in mathematics education research. This approach to research, as noted by Valero (2008: 44), has as a central concern 'the study of mathematics education as social practices related to the way in which power is distributed and structured in society'. This is precisely the viewpoint I adopted and from which I critically reflected on mathematics education in a laboratory setting. Vithal (2004: 1) argues that mathematics education from this perspective explores and deepens the link between mathematics education in its widest sense and concerns about democracy, equity and social justice. In relation to mathematics education, Valero (2008:50) argues that 'the general inequalities in society are reproduced in the ideological apparatus of the state, which include schools, and within them, mathematics classrooms'. Critical theory is a relevant research paradigm since its main concern is with issues of equality, domination and social justice embedded in social structures and power relations. To this end, many scholars in this paradigm argue that despite its claim to value neutrality, orthodox social science normally serves the ideological function of justifying the position and interests of the wealthy and powerful (e.g. Fals-Borda & Rahman 1991; Forester, Pitt & Welsh 1993; Freire 1982, all in Kemmis & McTaggart 2005: 273). What makes this framework more appealing is the fact that it addresses fundamental issues of justice, freedom and democracy, important values enshrined in the South African constitution aimed at the transformation of society in general and education in particular.

Habermas in Le Grange (2002: 37) argues that different knowledge traditions are linked with particular social interests. To this end, he distinguishes the following three types of interests that inform a research tradition: a) technical interest b) practical interest, and c) emancipatory interest (Le Grange 2002: 37). Usher in Le Grange (2002: 37) posits that the latter category of knowledge involves the unmasking of ideologies that maintain the status quo by denying individuals and groups access to knowledge or awareness about the material conditions that oppress or restrict them. Essentially such research tradition, contrary to both positivistic and interpretive traditions, has interest in research that changes the world in the direction of freedom, justice and democracy (Le Grange 2002: 37). This research study explored, through critical theory, how the use of manipulatives altered the teachers' a) mathematical knowledge for teaching, and b) classroom practice. My interest in the (self) emancipation of teachers from social, psychological and other forms of oppression is inherent in the research questions from the outset. The research questions further confirm my viewpoint regarding teachers' knowledge acquisition and their social agency in taking responsibility for transformation of their own practices. Critical theory is more appropriate as an underlying theoretical framework that informed this study. This is attributed to its emancipatory and transformational agenda as well as its tools that would help to awaken, nurture and develop teachers' social consciousness.

2.3 DEFINING AND DISCUSSING MATHEMATICAL KNOWLEDGE FOR TEACHING (MKT) AS AN OPERATIONAL CONCEPT

To further understand how teachers' knowledge and classroom practices can be emancipatory as discussed in the critical theory framework, I now turn my attention to some of the operational concepts anchoring the present study. I begin with the concept of Mathematical Knowledge for Teaching (MKT). There has long been acknowledgement among mathematics researchers, educationists and scholars alike of the vital role played by teachers' mathematical knowledge in the teaching of mathematics and in improving student outcome (Hill, Blunk, Charalambous, Lewis, Phelps, Sleep and Ball, 2008a: 431; Hill, Rowan and Ball 2005: 371). The study of teachers' subject matter knowledge and pedagogical skills necessary for teaching has been actively researched in the past decades.

The construct of MKT is a new type of teacher knowledge which, according to Hill *et al.* (2005: 373), refers to 'the mathematical knowledge used to carry out the work of teaching mathematics'. Ball, Lubienski and Mewborn (2001) posit that this new type of knowledge is built on two research approaches that dominated efforts to solve the problem of teachers' mathematical knowledge. One approach focused on the characteristics of teachers, while the other focused on teachers' behaviour. Ball et al. (2001:441) conjecture that the alternative approach, i.e. MKT, 'builds on both lines of prior work, on teachers and on knowledge, but shifts to a greater focus on teaching and on teachers' use of mathematical knowledge'

To have a better understanding of the development of the concept of MKT, I first locate it in the context of prior research on teaching and in particular on teacher knowledge. This has also assisted me to understand and explain some of the current teaching practices and the focus of some of the teacher development programmes in South Africa.

2.3.1 MKT and Prior research

Over a period of time, attempts have been made to understand the relationship between teacher knowledge and student achievement, and this has often been influenced by various perspectives on teachers' knowledge.

Prior research on teaching and on teacher knowledge has to a large extent been influenced by two major lines of thinking, namely the process-product and the educational production function perspectives respectively. The former perspective focused on teacher behaviours, i.e. what goes on (processes) in the classroom during the course of learning facilitation as the teacher helps students to learn, while the focus in the latter perspective was on educational resources which were viewed as 'inputs' to the educational system and included teacher characteristics such as teacher qualifications, experience and courses taken. The distinction between the two perspectives seems to centre on a) the construct to be used to predict student learning: whether to use 'teacher behaviours' or 'teacher characteristics', and b) on the focus of teacher knowledge. The process-product perspective focuses on pedagogical knowledge and skills independently of the subject-matter knowledge and the educational production function perspective focuses on the subject-matter knowledge to the exclusion of pedagogical knowledge. The latter distinction has clearly created a chasm between content and pedagogy, a situation which had and to a certain extent continues to have implications for other areas related to the work of teachers including their teaching practice, teacher education and development programmes, policy frameworks, etc.

Although research in both perspectives does acknowledge the role of teachers' knowledge in the improvement of student learning and in teaching, such research has not been able to describe and specify the kind of teachers' knowledge that constitutes effective teaching. Consequently, the two research perspectives could not specify the relationship between teachers' knowledge and their classroom practice. In addition to this limitation, both approaches use teachers' courses taken, qualifications etc., all of which are unsuitable to measure teachers' actual mathematical knowledge. My contention is that this specificity has become more urgent in the current mathematics reform movement in South Africa than before. The demands of the National

Curriculum in South Africa require that students learn mathematics in more meaningful ways, hence Cohen and Ball's proposal of fundamental revision of content and pedagogy (Cohen & Ball 1990: 234).

Concerns regarding the fragmentation between content and pedagogy are not new. For example, at the turn of the 20th century, John Dewey (1904/1964) articulated a fundamental chasm in the preparation of teachers – that of the 'proper relationship' of subject matter and method. He argued that this fragmentation of substance from method 'fundamentally distorted knowledge'. Ball and Bass (2000: 85) contend that this chasm continue to plague researchers, policy makers, teacher educators and teachers in the 21st century. They further argue that teachers' own knowledge of the subject, which affects what and how they teach, seems so obvious as to be trivialised. This 'taken for granted'-relationship had an impact on teaching, and it is suggested that this fragmentation in knowledge might result in fragmented teaching, which in turn will result in teaching methods that are not grounded in subject matter knowledge.

Shulman and colleagues (Shulman 1986: 6) also criticise various research paradigms on the study of teaching in the 1870s and 1980s for their conspicuous absence of pedagogy and content respectively. They question the paradigms for their assertion that one either knows content and that pedagogy is secondary and unimportant, or that one knows pedagogy and cannot be held accountable for content. In particular, Shulman and colleagues are concerned with prior researchers' lack of focus on subject matter knowledge, citing a host of questions that remain unanswered: Where do teachers' explanations come from; how do teachers decide what to teach, how to represent it, how to question students about it; and how to deal with problems of misunderstanding (Shulman 1986: 8). These concerns centre on what Shulman terms the 'missing paradigm' problem in the study of teaching, suggesting a call to reconceptualise the knowledge content that is needed for teaching.

In the mid-1980s, Shulman, Wilson, Grossman and Richert introduced the concept of Pedagogical Content Knowledge (PCK) to the field of research on teaching and teacher education (Shulman 1986). This represents a major breakthrough in the conceptualisation of what it means to know content for teaching. Pedagogical Content Knowledge was introduced as a special form of knowledge that a) bundles teacher knowledge into knowledge of subject, knowledge of students and knowledge of contexts (Shulman 1986,) and b) goes beyond knowledge of subject matter per se to the dimension of subject matter knowledge for teaching (Shulman 1986: 9; Hill, Ball and Schilling 2004: 12). This suggests that what teachers are able to do during their complex work of classroom practice depend fundamentally on this knowledge bundle and not on their course-taking as suggested by previous research. For me, this represents a significant step in recognising the complexity of the work of teaching and the multifaceted nature of teachers' knowledge.

Shulman and his colleagues' central concern has been the transition from expert student to novice teacher, in other words, how does the successful college student transform his or her expertise in the subject matter into a form that high school students can comprehend (Shulman 1986: 7). This essentially implies concern about the gap between content knowledge and general pedagogical knowledge as learned at college and practiced in the classroom situation. According to Ball, Thames and Phelps (n.d.: 1) a central contribution to the work of Shulman and his colleagues was to reframe the study of teacher knowledge in ways that included direct attention to the role of content in teaching. This was a radical departure from the research of the day, which focused almost exclusively on general aspects of teaching such as classroom management, time allocation and planning. The second contribution of the work was to leverage content knowledge as technical knowledge, key to the establishment of teaching as a profession. Shulman and (his) colleagues (Ball et al.n.d.:1) argued that high quality instruction requires a sophisticated professional knowledge that goes beyond simple rules such as how long to wait for students to respond. To characterise professional knowledge for teaching, they developed categories of teacher knowledge that were meant to underscore the important role of content knowledge and to situate content-based knowledge in the larger landscape of professional knowledge. To this end, they identified seven major categories of what they refer to as the knowledge base of teaching, which are summarised in Table 1 below.

- General pedagogical knowledge, with special reference to those broad principles and strategies of classroom management and organisation that appear to transcend subject matter

- Knowledge of learners and their characteristics

- Knowledge of educational contexts, ranging from workings of the group or classroom, the governance and financing of school districts, to the character of communities and cultures

- Knowledge of educational ends, purposes, and values, and their philosophical and historical grounds

- Content knowledge

- Curriculum knowledge, with particular grasp of the materials and programs that serve as "tools of the trade" for teachers

- Pedagogical content knowledge, that special amalgam of content and pedagogy that is uniquely the province of teachers, their own special form of professional understanding

Table 1: Shulman's Major Categories of Teacher Knowledge (Adapted from Ball *et al.* n.d.: 2)

The first four categories address general dimensions of teacher knowledge that were the mainstay of teacher education programmes at the time. The last three categories defined content-specific dimensions and together comprised what Shulman refers to as the missing paradigm in research on teaching. These are (a) content knowledge, (b) Pedagogical Content Knowledge (PCK), and (c) curriculum knowledge (Shulman 1986: 9). Among these categories, PCK has been of special interest to many scholars and researchers in the field of teaching because it identifies the distinctive bodies of knowledge for teaching (Shulman 1987). This special interest in PCK is justified particularly because of the lack of specificity in prior literature on teaching in relation to teachers' knowledge, a situation that has serious consequences on teachers' professional understanding of their work. In addition, interest in PCK is justified in view of a dire need to identify those specific teacher behaviours and strategies most likely to address the demands of the current reform in mathematics education.

Pedagogical Content Knowledge (PCK)

Shulman (1986: 7) defines PCK as:

> ... the most useful forms of representations of those ideas, the most powerful analogies, illustrations, examples, explanations, and demonstrations – in a word, the most useful ways of representing and formulating the subject that make it comprehensible to others . . . Pedagogical Content Knowledge also includes an understanding of what makes the learning of specific topics easy or difficult: the conceptions and preconceptions that students of different ages and backgrounds bring with them to the learning of those most frequently taught topics and lessons.

Shulman (1986: 9) conceives content knowledge as referring to the amount and organisation of knowledge per se in the mind of the teacher. This category of knowledge includes both facts and concepts in a domain, why facts and concepts are true and how knowledge is generated and structured in the discipline (Hill et. al 2005: 376; Hill et.al 2004: 13). In the case of mathematics, this type of knowledge is an important aspect of teachers' knowledge base. However, Powell and Hanna (2006: 377) assert that to teach a subject like mathematics effectively necessitates knowledge of mathematics that "goes beyond the knowledge of subject matter per se to the dimension of subject matter knowledge for teaching". This suggests that on its own, mathematics knowledge does not distinguish the knowledge possessed by a mathematics teacher from that possessed by a mathematician who pursues mathematics for its own sake only. This further suggests that in PCK, which ties together content knowledge and its pedagogy, the knowledge of mathematics for its own sake is not sufficient for effective mathematics teaching. Effective mathematics teaching not only requires going beyond knowledge of facts and concepts of a domain, but also understanding the structures of the subject matter. In short, it requires knowledge that will make mathematics teachable.

On pedagogical knowledge, Shulman and his colleagues argue that in addition to general pedagogical knowledge and knowledge of the content, teachers need to know about aspects such as the most useful forms of representation of ideas, what topics children find interesting or difficult, what errors and misconceptions learners bring to the classroom, etc. These pedagogical tasks seem to be at the heart of teachers' knowledge for teaching as espoused in Shulman's model of PCK, with emphasis on the understanding of students' cognitive structures in order to teach effectively. Unlike the earlier research work on teacher knowledge, this approach specifies pedagogical knowledge and its components in the context of particular mathematics content and topics, an element often overlooked in earlier research and by implication in policy related to teacher education and development. The concept implies that not only must teachers know content deeply, conceptually, and be familiar with the connections among ideas, but also must know the representations of and the common student difficulties with particular ideas. To develop this kind of knowledge, teachers need a combination of both pure mathematics content knowledge and a firm grasp on how to teach that content.

Central to the pedagogical knowledge as an aspect of the concept of PCK seems to be the knowledge of students, which calls for the understanding and knowledge of students' thinking in order to transform mathematics knowledge into mathematics knowledge for teaching. Hill *et al*. (2005: 376) argue that additional content related abilities such as to listen to students, to select and make use of good assignments, and to manage discussions of important ideas and useful work on skills, enable teachers to perform the tasks they must enact as teachers. Such abilities, specific to the work of teaching, have not been measured in the earlier research models. These key constructs, which might strongly mediate between teacher knowledge and learning outcome, are overlooked simply because they are either non-linear to student achievement and/or not easy to measure.

The following example in Ball and Bass (2000: 87) based on Deborah Ball's fifth grade class's work on decimal fractions, understanding place value and positional notation, illustrates such additional content related abilities required for effective teaching. In teaching decimal fractions Ball, an experienced teacher, knows that being able to anticipate what 5th graders know about decimal fractions depends in part on her knowledge of the number systems as well as on her understanding of the kinds of errors that 10 year olds make. She knows that 5th graders will often confuse .5 with .05 and that they draw this confusion, in part, from their conviction that 5 and 05 are the same number. First of all, regarding the content knowledge, this means that a 5th grade teacher needs to have a thorough understanding of base 10 number system and positional notation. However, when a 5th grader learns that the 5 in .5 represents the tenths, a position immediately to the right of the decimal point, he asks, 'Where is the 'oneths' place?'. The teacher needs to be able to hear that this likely emanates from a 10-year old's reasonable expectation that if there is a ones place immediately to the left of the decimal point, using symmetry, the oneths will be immediately to the right of the decimal point. Being able to hear this student is not enough, answering the question 'why isn't there a oneths place requires a certain explicit understanding of place value and of the multiplicative structure of the base 10 system that goes beyond being able to name the places (ones, tens, hundreds, etc.) or read numbers. Beyond being clear about mathematics, i.e. base 10 number system, helping a 5th grader to understand the missing 'oneths' requires of the teacher an intertwining of content and pedagogy, or PCK.

This kind of understanding is not something a mathematician would have; neither would it be part of a High School social studies teacher's knowledge. It is special to the teaching of elementary mathematics. PCK describes a unique subject-specific body of pedagogical knowledge that highlights the close interweaving of subject matter and pedagogy in teaching. Bundles of such knowledge are built by teachers over time as they teach the same topic to children of certain ages (Ball & Bass 2000: 87).

The third category, curricular knowledge, is defined by Shulman (1987: 8) as referring to teachers' 'tools of the trade' which are drawn from the curriculum and its associated materials. The curriculum, according to Shulman (1986: 10)

> '... is represented by the full range of programs designed for the teaching of particular subjects and topics at a given level, the variety of instructional materials available in relation to those programs, and the set of characteristics that serve as both the indications and contraindications for the use of particular curriculum or program materials in particular circumstances'.

Curriculum knowledge therefore involves teachers' awareness and understanding of how topics are arranged both within a school year and over longer periods of time, as well as ways of using a range of curriculum materials. This suggests that teachers' knowledge of these 'tools for the trade' involves more than following the curriculum but understanding it and being able to use these tools creatively in the best interest of the students, invoking innovations to tailor make these tools to suite the students' interests, needs and abilities. However, more often, authoritarian environments within which teachers operate may inhibit this kind of creativity and autonomy.

These tools of teaching, according to Shulman (1986: 10) present or exemplify particular content and remediate or evaluate the adequacy of student accomplishment. Of particular significance is the teachers' understanding of a full range of alternative 'tools' available for instruction to ameliorate particular student misconceptions or errors. In addition to this curriculum knowledge for a given subject within a grade, a professional teacher is expected to be familiar with the lateral curriculum as well as the vertical curriculum. The former refers to the teacher's ability to relate the content of a given topic or lesson to topics being discussed in other subjects. The latter relates to familiarity with topics that have been and will be taught in the same subject during the preceding and later years, and the materials that embody them.

As evidenced in the above discussion, Shulman's categories of teacher knowledge have provided a useful basis for many contributions to the debate around MKT. However, the ways of categorising teachers' knowledge do not explain how teacher knowledge affects classroom behaviour or in understanding effective teaching. This is corroborated by Ball *et al.* (2001) who argue that the question of how these knowledge components work together in real classroom situations and specifically, what makes mathematical knowledge usable for teaching is still unsolved. Seamus Hegarty in

Petrou (2007) argues that the effects of all the different areas of teacher knowledge can only be understood within the contexts of dynamic teaching situations. This suggests that since teacher knowledge firmly finds expression in the act of teaching, it can only be explained and understood through observation of such acts.

In addition to developing categories of teacher knowledge, Shulman (1986: 10) proposes that knowledge for teaching exists in three different forms. The first of these he terms propositional knowledge, much of which is taught to teachers and hence refers to theoretical knowledge held by teachers. The second form of knowledge, case knowledge, is a necessary complement to knowledge of propositions and develops through experience of teaching situations. It is knowledge of specific, well documented, and richly described events. The third form, strategic knowledge, refers to the way teachers act in the moment while teaching. It is the form of knowledge that comes into play as the teacher confronts particular situations or problems, whether theoretical, practical or moral. Strategic knowledge can, by definition, only be observed in teaching as it most visibly affects the learning of children. Although all knowledge forms are significant in terms of the how each of Shulman's categories may be organised, the last two forms of knowledge seem to be more relevant to the actual act/work of teaching.

Studies of teachers' knowledge structures have increased considerably during the 80s and 90s since Shulman (1986) introduced the notion of Pedagogical Content Knowledge. One such study is that of Liping Ma (Ma, 2010) in her book, 'Knowing and Teaching Elementary Mathematics', which still widely attracts interest on the issue of mathematics knowledge for teaching. From her study, in which she compared the Chinese and U.S. elementary teachers' mathematical knowledge, she produced a portrait of differences between the two groups. For example, she found that Chinese teachers' knowledge of elementary school mathematics is deeper than that of their American counterparts, despite the higher college degree attainment of American teachers. Ma used data from this study to develop the notion of 'Profound Understanding of Fundamental Mathematics' or PUFM, an argument for a kind of connected, curricularly-structured, and longitudinally coherent knowledge of core mathematical ideas (Ball & Bass 2003: 4). She describes the 'knowledge package' of the 72 Chinese elementary teachers she interviewed as consisting of a) key ideas that ‘weigh more’ than other ideas in the package, b) sequences for developing the ideas, and c) ‘concept knots’ that link crucially related ideas. Key in her notion of mathematical knowledge for teaching is a kind of culturally situated and curricular structuring of the content that readies it for teaching by identifying central ideas and their connections, as well as longitudinal trajectories along which ideas can be effectively developed. Comparing PUFM with earlier research perspectives in this field, Monk in Shechtman, Roschelle, Haertel, and Knudsen (2010a: 322) characterises it as good evidence of a thorough understanding of foundational school level mathematics as opposed to teachers’ credentials. Ma uses particular topics where Chinese teachers’ knowledge packages demonstrated connectedness, thoroughness, basic ideas, and multiple perspectives.

As a contribution to the notion of PCK, Ma (2010) uses the notion of PUFM to elaborate on the teachers’ knowledge package for teaching by using specific topics and diverting attention from simple knowledge of these topics to focusing on a) the depth which refers to large and powerful basic ideas, b) the breadth, which refers to multiple perspectives relating to a particular topic, c) thoroughness which is essential to weave ideas in a coherent whole, and d) connectedness, which related to the above three. This new notion of teachers’ knowledge packages represents a particularly generative form of and structure for pedagogical content knowledge, knowledge packages that constitute a refined sense of organisation and development of a set of related ideas in an arithmetic domain (Ball, Lubienski and Mewborn 2001: 449).

In addition to its contribution to the knowledge packages, Ma’s study also illuminates how Chinese teachers attain PUFM. Teachers interviewed on how they had acquired PUFM indicated that they acquired PUFM through colleagues and students, and by solving problems, teaching, and studying teaching materials intensively. The study revealed that Chinese teachers develop PUFM during their teaching careers – stimulated by a concern for what to teach and how to teach it, inspired and supported by their colleagues and teaching materials. Askey (1999: 7) notes that although the Chinese teachers Ma interviewed had a firm base of knowledge on which to build, their PUFM did not come directly from their studies in school, but from the work they did as teachers in their practice. Shechtman *et al.* (2010a: 322) note that coursework in American universities typically builds on and extends, but does not revisit, early mathematics and thus may not sufficiently address the mathematical content that school mathematics teachers mostly need.

As evidenced in the above discussions, Shulman's notion of PCK with its knowledge components required for teaching represents a significant development towards the study of Mathematical Knowledge for Teaching. However, scholars have raised concerns about the lack of specificity about these knowledge components and how they interplay among them in a real classroom situation, and what makes them usable for teaching (e.g. Ball, Thames & Phelps 2008).

On the other hand, Liping Ma's (2010) PUFM idea entails teacher knowledge packages that demonstrate connectedness, thoroughness, basic ideas and multiple perspectives in terms of specific mathematical topics. Through portraits of highly developed teacher knowledge Ma has complemented Shulman's model of PCK and has also significantly contributed to the understanding of the knowledge needed for teaching. However, her idea of PUFM was developed from teacher interviews rather than from teaching practice. Although both approaches sort of fill the research gaps that are left when the focus is only on teachers' credentials, the gaps in the two approaches seem to suggest that further research is needed to identify specific knowledge packages that relate directly to real teaching contexts.

2.3.2 The Link Between Mathematical Knowledge for Teaching (MKT) and Mathematical Classroom Practice

There is widespread consensus about the link between teacher knowledge and classroom practice in mathematics (Ball 2000; Ma 2010; Shulman 1986). As demonstrated in the research perspectives discussed above, the other aspects of teaching and of teacher's knowledge, as well as their interrelatedness, have either been blatantly ignored or trivialised. In sharp contrast to the above paradigms, another group of education scholars began to conceptualise teachers' knowledge for teaching differently, arguing that teacher effects on student achievement are driven by teachers' ability to understand and use subject-matter knowledge to carry out the task of teaching (Hill *et al*. 2005: 372).

Studies involving teachers' mathematical understanding as it relates to classroom practice can generally be classified into one of two groups: "deficit" and "affordance" approaches. In the "deficit" approach, the authors draw linkages between a teacher's lack of mathematical understanding and patterns in her mathematics classroom practice; while in the "affordance" approach the authors highlight the affordances strong mathematical (and related) understandings can create for classroom culture and practice.

Deficit studies published during the period 1990 to 1995 set the stage for policy makers' concerns about the mathematical quality of classroom work (Hill, Blunk, Charalambous, Lewis, Phelps, Sleep and Ball 2008a: 433). In every study in this genre, the researchers observed significant mathematical errors or imprecisions during classroom practice, ranging from inappropriate metaphors for mathematical procedures and incomplete definitions, to plain mathematical mistakes. Several analyses also went beyond errors to identify other patterns arising in the instruction of less knowledgeable teachers (Hill *et al.* 2008a: 431). On the other hand, "affordance" studies focus on the practice of teachers engaged in using a) new curriculum materials, b) new forms of teaching, and c) intensive professional development, to examine 'what higher-or high knowledge teachers can do with students and mathematics that others cannot' (Hill *et al.* 2008a: 434).

Two studies provide important addenda to these observations about the connections between mathematical knowledge and teaching. Borko, Eisenhart, Brown, Underhill, Jones and Agard (1992: 197), in their observation of a classroom lesson where a pre-service teacher, Ms Daniels, was teaching division of fractions, found that in spite of the teacher's extensive work in mathematics and her well preparedness, she nevertheless had significant difficulties explaining division of fractions in response to a student question, which called for a conceptual explanation. In another case study, Thompson and Thompson (1994: 19) conducted a teaching experiment involving a middle school teacher, Bill, who had strong and elaborate conceptualisations of rate. However, he had trouble speaking conceptually about rates during a tutoring session. These two cases, along with the "affordance" literature, suggest that there is knowledge used in the classrooms beyond formal subject matter knowledge, a contention also supported by Shulman's (1986) notion of PCK. More generally the affordance studies included attempts to identify what mathematics knowledge matters in the work of teaching - in many ways, making Shulman's theory specific to teaching mathematics.

Recently, researchers at the University of Michigan (Ball 2006; Hill, Rowan, & Ball 2005; RAND Mathematics Study Panel 2003) proposed a novel theory about teachers' Mathematical Knowledge for Teaching (MKT) with the aim to address the challenge of learning mathematics in ways that would make it teachable. The construct of MKT, that is, mathematical knowledge used to carry out the work of teaching (Ball *et al*. n.d.: 4; Shechtman *et al.* 2010a: 318), was built on prior research in the field (e.g. Dewey 1904/1964; Ma 2010; Shulman 1986, 1987). It includes both the mathematical knowledge that is common to individuals working in diverse professions and the mathematical knowledge that is specialised to teaching (Hill *et al*. 2008a: 430).

In their investigation of the research question: 'what do teachers need to know and be able to do to effectively carry out the work of teaching mathematics', the Michigan team adopted a 'bottom-up' approach (Ball *et al*. n.d.: 4). Through this approach, the team was able to provide a more detailed linkage between teacher knowledge and students' cognition and learning in the classroom. In studying MKT, case studies concerning specific topics such as multiplication, fractions, place value, and mathematics definitions have been researched well enough to illustrate what MKT looks like. The team's focus was not on what the teachers need to know, but on how teachers need to know that content (Ball *et al*. n.d.: 4).

For example, in 1996, Ball and Bass probe the interplay of mathematics and pedagogy. They set out examining the practice itself (Ball & Bass 2000: 89) to provide a practice-based description of what is called 'mathematical knowledge for teaching' (Hill & Ball 2004). They propose an analysis of 'core activities' of mathematics teaching, which include aspects such as figuring out what students know; choosing and managing representations of mathematical ideas; appraising; selecting and modifying textbooks, deciding upon alternative courses of action; steering a productive discussion and identifying the mathematical resources entailed by these teacher activities. Hill and Ball see teaching as a practice embedded with both regularities and endemic uncertainties where students, for example, quite often find topics such as probability, integers and fractions very difficult to understand. They contend that certain ways of approaching these topics – particular representations and methods of development – can mediate these difficulties. Although they acknowledge the valuable contribution of PCK, they also argue that no amount of PCK can prepare a teacher for all practice, for a significant proportion of teaching remains uncertain. Knowing mathematics for teaching must take account of both the regularities and the uncertainties of practice, and must equip teachers to gain knowledge from within the contexts of the real problems they have to solve (Ball & Bass 2000: 89). Lampert and Ball (1999: 38) argue that 'knowing teaching is more than applying prior understandings as it also depends fundamentally on being able to know things in the situation'.

Based on the first findings from their observations and the need to represent its hypothesis, the team proposed a refinement of Shulman's categories. In their proposed framework they illustrated the correspondence between their domain of MKT and Shulman's initial categories of Subject Matter Knowledge, in which the latter can be divided into two categories (Ball & Bass 2003: 6; Ball *et al.* n.d.: 5). As depicted in figure 1 below, the first category is called Common Content Knowledge (CCK) and refers to the mathematical knowledge and skills that are used in any setting, not necessarily in the setting of teaching, and includes the individual's ability to calculate an answer and to solve mathematical problems correctly. This is the mathematical knowledge that a well-educated adult, including a teacher, is expected to know. For example, people with CCK will be able to compute this subtraction problem and produce the correct answer:

291
~~307~~
- 168
  139

This knowledge is related to the content of curriculum but not a particular curriculum and it includes knowing when students have wrong answers (Ball *et al.* n.d.: 6)

The second category is that of Specialised Content Knowledge (SCK) and refers to knowledge that only teachers need to hold in their work (Ball, Hill, H and Bass 2005: 22). This is a distinct kind of content knowledge which, according to Ball and Bass (2003: 6) like PCK, is closely related to practice, but unlike PCK, does not require additional knowledge of students or teaching. It is distinctly mathematical knowledge, but not necessarily mathematical knowledge familiar to mathematicians. For example, let us consider the error in the subtraction problem below:

$$\begin{array}{r} 307 \\ -\ \underline{168} \\ 261 \end{array}$$

Any person with CCK will be able to spot the error. However, a teacher with SCK will go beyond error identification to analysis of the error, establishing the source of the mathematical error. With SCK, the teacher must not only be able to recognise the error but must also be able to establish as to what line of thinking could have produced this error (Ball *et al*. n.d.: 7) and evaluate alternative ideas. For instance, a teacher with SCK may want to probe into why the learner subtracted wrongly in the ones and the tens place respectively and subtracted correctly in the hundreds. SCK is at the centre of MKT and is mathematical knowledge that is used as teachers perform their moment-to-moment work, which is characterised by both regularities and irregularities. On the other hand, PCK is knowledge that teachers need to prepare in advance for their classroom teaching.

In addition, the team suggests that PCK as conceptualised by Shulman, can be divided into Knowledge of Content and Students (KCS) and Knowledge of Content and Teaching (KCT) (Ball *et al*. n.d.:6). KCS is '…..a type of pedagogical content knowledge that combines knowing about students and knowing about mathematics' (Ball *et al*. n.d.:9). This implies that teachers must know what students of a particular grade know and be able to anticipate the difficulties and obstacles they might experience, be able to hear and respond appropriately to students' incomplete thinking and finally choose appropriate examples and representations while teaching. In addition teachers must show awareness of students' conceptions and misconceptions about a mathematical topic.

They define KCT as a knowledge that represents a combination of knowing about mathematics and knowing about teaching (Ball *et al*. n.d.:9). It refers to teachers' decisions on the sequencing of activities and exercises, to their awareness of possible advantages and disadvantages of representations used while they teach, to their decisions to pause a classroom discussion for more clarification, and their decision to use students' opinion to make a mathematical remark (Ball *et al.* n.d.: 9). KCS and KCT as conceptualised by the team, are the same as the two central dimensions of PCK namely teachers' awareness of students' conceptions and misconceptions, and the use of representations, examples that teachers use in order to make the subject matter comprehensible to students.

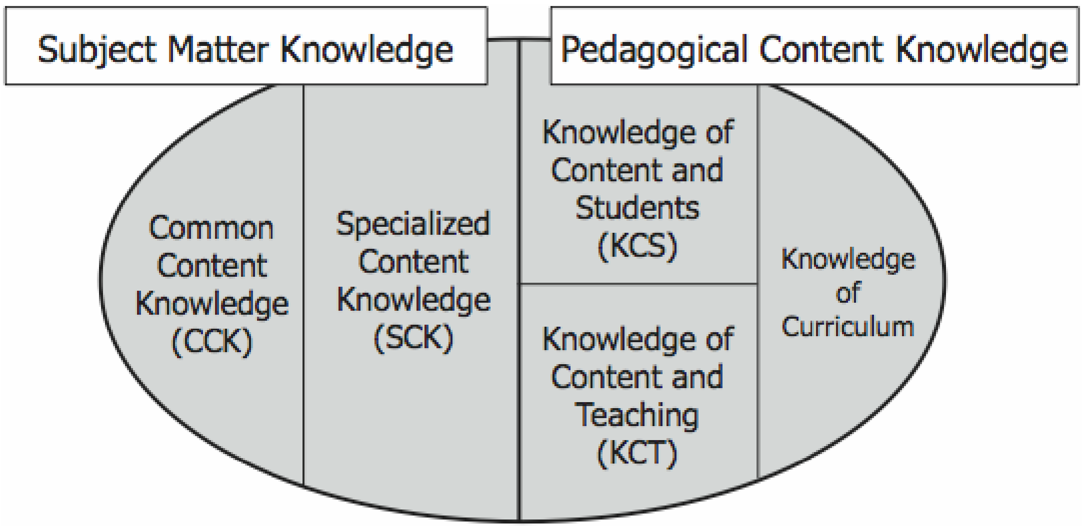


Figure 1: Domain map for mathematical knowledge for teaching (adapted from Hill, Ball & Schilling 2008b: 377)

Results from the lessons observed suggested that there is mathematical knowledge for teaching that is specialised for the work of teaching. This implies that there is additional knowledge of mathematics that teachers need to have in practice in order to be successful with students in classrooms. To illustrate briefly what it means to know mathematics for teaching, an example in the multiplication of whole numbers by Ball *et al.* (2005: 43) is presented in the computation below. In this scenario, respondents evaluate three different approaches to multiply 35 by 25 and determine whether any of these is a valid general method for multiplication.

Imagine that you are working with your class on multiplying large numbers when you notice that students have displayed their work in the following different ways:

Student A	Student B	Student C
35	35	35
X 25	X 25	X 25
125	175	25
+ 75	+ 700	150
875	875	100
		+600
		875

Figure 2: Item measuring Specialised Content Knowledge (adapted from Ball *et al.* 2005: 43)

Which of the examples above shows a method that could be used to multiply any two whole numbers?

In this example, all three responses are correct. This illustrates that teachers, in the course of their work, come across with not only with the errors students make but also with students' unfamiliar methods that may produce correct answers. In this instance, a teacher will be expected to provide feedback by evaluating and making judgement based on sound mathematical principles underlying the procedure. Most importantly, the teacher has to determine if the method is generalisable to other two digit whole numbers and provide justification. Most adults should be able to multiply 35 by 25 as this only requires common content knowledge, i.e. general step by step computation of the algorithm. However, this is not sufficient to deal with irregularities that unfold in the course of teaching mathematics. To evaluate alternative methods, examine their mathematical structure and principles, and justify and validate these students' responses would require additional knowledge, i.e. specialised content knowledge – distinctive knowledge that only teachers need to hold in their work (Ball & Bass 2003: 7). The present study is particularly interested in investigating such a specialised kind of knowledge that only teachers hold or need when using manipulatives. Such knowledge will enable them to be effective in their dynamic interaction with learners and environments such as those of manipulative use.

2.4 DEFINING AND DISCUSSING MATHEMATICAL CLASSROOM PRACTICE AS AN OPERATIONAL CONCEPT

It is widely acknowledged that mathematics knowledge for teaching is essential to student learning. What logically flows from this relationship is that teacher knowledge influences student learning through classroom practice. This has led to a presumption that teacher knowledge is essential for quality instruction (e.g. Hill *et al. 2*008a; Kulm 2008). In this regard, Hill *et al* (2008a: 431) argue that this presumed relationship, which has largely been based on logical claims rather than on direct observation, is not completely understood.

Classroom practice is described as an interaction or relationship among teachers, students, and content in which each of the three components is important. *Hill et al*. (2008a: 431) define classroom practice as a composite of several dimensions that characterise the rigor and richness of the mathematics content of the lesson, including the presence or absence of mathematical errors, mathematical explanation and justification, mathematical representation, and related observables. Substantial research has been conducted to establish the link between teacher knowledge and student achievement as well as between teacher knowledge and classroom practice (Shechtman *et. al* 2010a; Hill *et al.* 2008a). Different research programmes used different measures to examine if teacher knowledge is a predictor of student achievement and classroom practice.

To this effect, recent work has begun to examine another aspect of the chain of influence from teacher knowledge to student achievement, namely how teachers' MKT is related to their classroom practice (Shechtman *et. al* 2010a: 323). For example, case studies were conducted by Hill and her colleagues (Hill *et al.* 2008a) to examine the relationship between five teachers' MKT and the Mathematical Quality of their Instruction (MQI). In their conclusion, they noted that 'Although there is significant, strong, and positive association between levels of MKT and the MQI, we also find that there are important factors that mediate this relationship, either supporting or hindering teachers' use of knowledge in practice' (p. 431). In particular, they found that teachers with a low MKT were more likely to use imprecise mathematical definitions in class, choose inappropriate supplementary materials, and make more errors than teachers with a high MKT. However, Shechtman *et al*. (2010: 323) noted that one weakness of the analysis is that it ties MQI to the teacher's contributions with relatively less attention to the contributions of students and materials.

Other similar detailed case studies conducted by Roschelle and his colleagues widened the analytical lens by analysing key components to generate conjectures about where MKT is necessary, irrelevant or even a liability (see Shechtman et. al 2010a: 323). In this research study, Cohen, Raudenbush, and Ball argue that although MKT in action can be critical to learning, it is just one of a variety of potential learning resources or opportunities that may exist in the classroom (see Shechtman et. al 2010a: 319). Curriculum materials, interactive representational technology, student-student participation structures, and students' own prior attitudes and understandings are some of such other resources that may exist (Roschelle *et al.* 2009 in Shechtman *et. al* 2010a: 319). As Shechtman *et al.* (2010a: 319) argue, the qualities of such a system's learning opportunities and its emphases can vary widely among classrooms for reasons that may or may not be directly related to the teacher's MKT. This implies that, even if a teacher is well grounded in respect of MKT, other components of the classroom system that the teacher may not be in control of, play an important role in the quality of such a system.

Shechtman et al. (2010a) examined how teachers' mathematical knowledge relates to instructional and outcome variables. They used data collected in the context of two large scale randomized experiments collectively termed the 'Scaling Up SimCalc' studies. The 'Scaling Up SimCalc' project implemented three studies to address the broad research question: 'Can a wide variety of teachers use an integration of technology, curriculum, and professional development to increase student learning of complex and conceptually difficult mathematics?' Although these studies were implemented to examine the impact of interventions at middle school level, each consisting of an integration of the SimCalc software, curriculum, and professional development, they also provided an excellent opportunity to examine teacher knowledge in a classroom system that by design included a wide variety of resources for learning (Shechtman et. al 2010a: 319). The SimCalc setting had three of the four factors that Ball *et al.* conjectured as demanding teachers' MKT: a) providing new content, b) implementing reform, and c) managing change. On the relationship between the teachers' MKT and their instructional decisions during classroom practice, the research studies focused on three areas of decision making: a) teachers' topic focus, b) teachers' choice of simpler or more complex topics, and c) teachers' choice to spend more of fewer days in the computer lab. Findings that are relevant to my research study are the following:

- Inconsistent evidence for a link between teachers' MKT and student achievement,
- Instructional decisions are independent of MKT,
- MKT is learnable; teachers' knowledge grew in some ways through participation in the project,
- Teacher learning of MKT occurred most notably on less relevant, more novel content, and
- Teachers did not retain short term learning gains.

It needs to be noted that that prior to conducting the 'Scaling Up SimCalc' studies there was no literature that specifically addressed pedagogical challenges that teachers would experience when using the materials and as such the potential of such challenges was deduced from more general literature(Shechtman et al 2010a: 322). The most relevant, more general literature was about pedagogical challenges in teaching conceptually rich mathematics. For example, Hiebert and Grouws (in Shechtman et al 2010a: 325) argue that teachers must present and focus on connections among mathematics ideas and must engage students in struggling with the meaning of mathematical ideas. Obviously this requires depth of knowledge and it would be difficult to support students to make connections if teachers do not have sufficient MKT. Thus Shechtman *et al*. (2010a: 325) conjecture that MKT would be necessary in any mathematics teaching that includes a strong focus on conceptual understanding.

In another attempt to close the literature and research gap on teachers' knowledge with regard to classroom practice, Rowland, Huckstep and Thwaites (in Rowland, Thwaites and Huckstep 2003) used a grounded theory approach to data analysis of 24 video recorded lessons to develop a 'knowledge quartet' framework for the identification and discussion of primary teachers' mathematics content knowledge as evidenced in their teaching. They define the knowledge quartet as a comprehensive thinking tool about the ways that subject knowledge comes into play in the classroom. The framework consists of four broad dimensions through which teachers' subject content knowledge could be evidenced in practice. They conceptualised these dimensions as foundation, transformation, connection and contingency which will be briefly discussed below. They used a case study of a trainee, Naomi, who reflects upon her lesson of year 1 mathematics on subtraction a few hours after teaching it, to illustrate their 'knowledge quartet' framework in each of the four dimension.

The first dimension, foundation, is rooted in the foundation of the trainee's theoretical background and beliefs. It consists of trainees' knowledge, beliefs and understanding acquired in the academy, in preparation for their role in the classroom. The key components of this theoretical background are: knowledge and understanding of mathematics and knowledge of significant tracts of the literature on teaching and learning mathematics, together with beliefs concerning the nature of knowledge, the purposes of mathematics education, and the condition under which pupils will learn mathematics. In her lesson, Naomi intends to address 'difference' both conceptually and linguistically, by drawing from her acquired theory regarding the types of subtraction problem structures as proposed by Carpenter and Moser (see Rowland, Huckstep and Thwaites 2005: 267). The two problem structures that find expression in Naomi's lesson are the change-separate problem and the compare problem type respectively. In the former type, Naomi phrases the question thus: 'Connie had 13 marbles. She gave 5 marbles to Jim. How many marbles does she have left?' Linguistically, subtraction is the same as 'take away'. For a non-English speaker this might mean something different. In the latter type, the question becomes: 'Connie has

13 marbles and Jim has 5 marbles. How many more marbles does Connie have than Jim?' Two sets (Connie's marbles and Jim's marbles) are compared and the difference between the two sets becomes the answer. Subtraction is the same as difference.

The second category, transformation, concerns knowledge-in-action as demonstrated both in planning to teach and in the act of teaching itself. At the heart of this category is Shulman's observation that the knowledge base for teaching is distinguished by '... the capacity of a teacher to transform the content knowledge he or she possesses into forms that are pedagogically powerful' (Shulman 1987: 15). Of particular importance is the trainee's choice and use of examples presented to pupils, in connection with the acquisition of concepts and procedures, and also in the cause of mathematical enquiry. This dimension finds expression in the type and sequencing of questions, tasks, demonstrations, analogies and explanations Naomi chooses in her lesson. In her oral starter episode she chooses to sequence numbers thus: 8, 5, 7, 410, 821, 7, and 3, to be used in the 'number bond hat' involving the number bond 10. Rowland and colleagues (Rowland *et al.* 2003) cite reasons why they regard this as a well-chosen sequence. For example, the first and the third numbers are close to 10 and require little or no counting to arrive at the answer, 5 evokes a well-known double – doubling being an explicit National Numeracy Strategy NNS framework (DfEE 1999) while the choice 4 seemed to be tailored to one or more fluent children. This demonstrates that her choice of examples, they claim was well thought out as it was 'a) at first 'graded' b) included later an unusual/degenerate case, and c) highlighted a key structural property of addition, i.e. commutativity'.

The third category, connection, binds together certain choices and decisions that are made for the more or less discrete parts of mathematical content. Making connections among mathematical ideas is in essence about the acquisition of what Duckworth in Ma (2010: 121) refers to as intellectual 'depth' and 'breadth'. It concerns the coherence of the planning or teaching displayed across an episode, lesson or series of lessons, including the sequencing of topics for instruction, and an awareness of the relative cognitive demands of different topics and tasks. For example, the first connection that Naomi establishes is that between the subtraction structures, i.e.

change-separate ('take away') and comparative. The two involve very different procedures when carried out with manipulative materials.

The fourth category, contingency, concerns classroom events that were not anticipated or planned for, in particular the readiness to respond to children's ideas and a consequent preparedness, when appropriate, to deviate from an agenda set out when the lesson was prepared. This category consists of 'knowledge-in-interaction' (Thwaites, Huckstep, and Rowland 2005:170), as teachers do their work in a dynamic classroom system.

To support and expand the work done in content knowledge-based qualifications, investigators have recently developed instruments to measure teachers' mathematical knowledge, and have examined the consequences of varied levels of teachers' mathematical knowledge in classroom teaching and learning (Hill *et. al*. 2004). Hill *et al*. (2008a: 431) concede that the lack of specifics regarding how knowledge affects instruction left critical gaps in their theoretical knowledge. This study purports to establish if a relationship does exist between the knowledge held by teachers and their classroom practice, particularly when using manipulatives. The study also seeks to provide an explanation of the influence or lack thereof manipulative use in this relationship.

2.5 TRANSFORMING MKT INTO CLASSROOM PRACTICE: TEACHERS' BELIEFS ON THE TEACHING OF MATHEMATICS

Fundamental assumptions behind research on teachers' beliefs are that teacher behaviour are substantially influenced and even determined by teacher beliefs and that these behaviours, in turn, impact on student beliefs and behaviours (Li 1999: 64).
However, much of the early research on the effectiveness of mathematics teaching focused on teacher knowledge of mathematics as an aspect of teacher characteristics (e.g. Thompson 2004**)**. In this regard, Ernest (1989) argues that although knowledge is important, it alone is not sufficient to account for the differences between mathematics teachers. For example, two teachers may have similar knowledge, but while one teaches mathematics with a problem solving orientation, the other may be using a more didactic approach. Moreover, classroom practice is a complex and dynamic enterprise, characterised by interactions among a number of other constructs

that impact on teaching and learning. One such construct of classroom practice that has been identified for investigation is the mathematics teachers' belief system. Luft and Roehrig (2007: 40) concur with this assertion when they conjecture that beliefs are critical when it comes to understanding teachers' practice.

The key components of teachers' belief system, according to Ernest (1989) are the teachers': a) view or conception of the nature of mathematics, b) model or view of the nature of mathematics teaching, and c) model or view of the nature of mathematics learning. The last two views depend largely on one's conception of the nature of mathematics. Such views form the basis of the philosophy of mathematics, which may either be consciously or implicitly held philosophies. According to Ernest (1989) the teachers' view of the nature of mathematics is the most fundamental because it impacts on the other two closely related beliefs about mathematics teaching and learning, although it has also been suggested that the components are interrelated in many ways (e.g. Perry, Howard and Tracey 1999; Speer 2005; Thompson 1992). However, literature on mathematics teaching practices indicate that these are not the only aspects of teacher beliefs that need to be considered when looking for influences on the effectiveness of teaching (e.g. Gates 2006; Sztajn 2003). Skott (2001) considers micro aspects of the social contexts of mathematics classrooms such as teachers' beliefs about their students, about social contexts that are closely related to the students' motivation to learn, and their performance in mathematics (Philippou & Christou 2002; Zevenbergen 2003).

The idea that teachers' beliefs determine their approaches, their perceptions of teaching and learning, and what students learn is commonly accepted (e.g. Cross 2009; Ernest 1989; Wilson & Cooney 2002). This suggests that changes in approaches to the teaching of mathematics especially within mathematics reform programmes depend fundamentally on deep changes in the teachers' system of beliefs. Ernest (1989) argues that these changes in beliefs are associated with increased reflection and autonomy on the part of the mathematics teacher. He identifies the following elements as the most notable among the key elements on which the practice of teaching mathematics and the autonomy of the mathematics teacher depend: a) the teacher's mental contents or schemas, which include mathematics knowledge, beliefs concerning mathematics and its teaching and learning, and other factors, b) the social context of the teaching situation, particularly the constraints and opportunities, and c) the teacher's level of thought processes and reflection. Clearly, teacher knowledge alone is not enough to ensure successful implementation of reforms in mathematics.

Ernest (1989: 250) distinguishes three conceptions that are more prominent about the nature of mathematics: a) the instrumentalist view, b) The Platonist view, and c) the problem solving view. Instrumentalists view mathematics as a bag of tools made up of an accumulation of unrelated facts, rules and skills to be skilfully used by the trained artisan in the pursuance of some external end. Platonists view mathematics as a static but unified body of certain knowledge, a crystalline realm of interconnecting structures and truths, bound together by filaments of logic and meaning (Cai 2007: 266). Mathematics, from this point of view, is discovered, not created. From the problem solving view, mathematics is regarded as a dynamic, continually expanding field of human creation and invention, a cultural product. It is evident that these beliefs about the nature of mathematics do have consequences on teachers' actions in relation to teaching mathematics. For example, a teacher who holds an instrumentalist view would emphasise drilling of facts and procedures in his or her teaching, with less focus on conceptual understanding. Teaching in this instance becomes a matter of efficient transmission. The emphasis is on the strict adherence to content and strict following of text or scheme of work, with little or no consideration of instructional classroom activities such as the choice of representations, understating of student errors, etc. This will be more pronounced, for instance, in an education system that is characterised by high stakes examinations such as in South Africa. In this regard, Green in Beswick (2007:4) argues that the relative centrality of an individual's beliefs will differ from context to context. In addition to these categories, Ernest (1989) has proposed three teaching models to reflect the various roles a teacher might play in a classroom: Instructor model, explainer model, and facilitator model, each of which has clear links with the three categories of beliefs about mathematics, mathematics teaching and mathematics learning. For example, the intended outcome for an instructor often focuses on student skill mastery and correct performance; for the focus is on conceptual understanding with unified knowledge, and for the facilitator teacher, focus is on student confidence in problem posing and solving.

Over the last two decades, researchers have developed theories about teachers' beliefs and the way these beliefs impact teachers' classroom practice (e.g. Fang 1996; Thompson 1992). Recently, researchers have started to cross-culturally explore teachers' beliefs and look at how teachers' culturally constructed beliefs impact their teaching and the learning of their students (Cai 2002, Stigler & Hiebert 1999). For example, Fang (1996) in his review research on beliefs and practices synthesises the research on the relationship between beliefs and practice and suggests that beliefs tend to affect behaviours. Fang's findings are consistent with other educational researchers, who generally agree that beliefs are connected to actions in the classroom (e.g. Hashweh 1996; Wallace & Kang 2004). However, these and other authors indicate that pressing issues pertaining to beliefs and practice exist, such as the nature of the interaction between beliefs and practices. Luft (2001) embarked on a study involving both experienced and beginning teachers and found that the latter were more likely to change their beliefs when learning about inquiry but less likely to change their practices, while experienced teachers were less likely to change their beliefs and more likely to change their practices. The degree to which the inexperienced teachers were able to change was attributed to the formidable nature of the beliefs. The experienced teachers, on the other hand, had beliefs about teaching that were established and consistent with the goals of the professional development programme, which in turn influenced their decision to participate in the programme. The study by Luft and Roehrig (2007) aimed to examine the beliefs of science teachers about science teaching, revealed, among other findings, that the beliefs of science teachers can change and be modified, and that they are likely to do so within certain parameters.

A few studies have recently been conducted in an attempt to understand teachers' beliefs about effective teaching from cross cultural perspectives (Cai, Perry, Wong, and Tao Wong 2009: 4). For example, Marton, Alba, and Tse in Cai *et al.* (2009: 4) examined teachers' views on memorisation and understanding and found that western educators believe that memorization does not lead to understanding while for Chinese educators, memorization does not necessarily lead to rote learning; but instead can be used to deepen understanding. Stigler and Hiebert (1999) found that, Asian teachers teach mathematics in a coherent way because they believe that mathematics is a set of relationships between concepts, facts, and procedures. On the other hand they found that U.S. teachers view school mathematics as a set of procedures and skills. Stigler and Perry (1988) found that U.S. teachers tend to believe that young children need concrete experiences in order to understand mathematics. Chinese teachers believe that even young children can understand abstraction and that concrete experience only serves as a mediator for understanding (Stigler & Perry 1988).

Teachers' beliefs about teaching mathematics, according to Thompson (1992), can be revealed in the following aspects: desirable goals of the mathematics program, a teacher's role in teaching, appropriate classroom actions and activities, desirable instructional approaches and emphases, and legitimate mathematical procedures. Similarly, teachers' beliefs about the learning of mathematics cover the process of learning mathematics, what behaviours and mental activities are involved on the part of the learner, and what constitutes appropriate and prototypical learning activities (Thompson 1992). Speer (2005) summarises the views concerning teachers' beliefs about mathematics and teaching as identified by Kuhs and Ball (1986: 366) thus:

> The 'learner-focused' view centres on the learners' personal construction of mathematical knowledge through their active involvement in doing mathematics. The teachers' role is as a facilitator of student learning. The second view, 'content-focused with an emphasis on conceptual understanding,' focuses on the logical relations among mathematical ideas. 'Content-focused with an emphasis on performance' is similar to the previous one in its focus on mathematical content, but emphasizes rules and procedural mastery. The fourth view, 'classroom-focused,' emphasizes classroom activity that is structured, efficiently organized, where teachers present material clearly and students practice individually.

2.6 DEFINING AND DISCUSSING MATHEMATICAL MANIPULATIVES

2.6.1 History of Manipulatives

The use of concrete objects to understand mathematical concepts is not new**.** Even before humans could read and write and written numbers existed, there was a need to count. Ancient cultures used the human hand and its fingers to count but when numbers got bigger than ten or twenty (if toes were used) then other counting devices became necessary. Pebbles, sticks, stones, twigs, and cowries, etc. were used as physical objects to assist with counting. Later on, after early counting devices and their limitations in counting big numbers, the abacus entered the scene as a calculating machine.

The use of mathematical manipulatives has a long history. According to Uttal *et al.* (1997: 40), the idea that young children learn best through concrete objects is derived, at least in part, from the work of several educational theorists such as Piaget, Bruner, Montessori, Dienes, Froebel, Vygotsky and others. For instance, Moyer (2001: 175) noted that according to Piaget, children do not possess the mental maturity to grasp abstract mathematical concepts presented only verbally or symbolically and require various experiences with concrete materials and drawings for learning to take place. On the other hand, Bruner (1966) regarded learning as an active process in which learners construct new ideas or concepts based on their current and past knowledge. He underlined the role of physical objects in this process by posing three stages through which children represent their understandings: a) the enactive phase in which children manipulate familiar tools, b) the iconic phase where children create their own representations through drawings, and c) the symbolic phase where children are ready to move from the iconic representations to the standard symbolic notation. He believed that elementary school children's thinking focused on concrete properties that could be actively manipulated. This approach could, as noted by Bruner, 'empty the concept of specific sensory properties' and allow the student 'to grasp its abstract properties' (Uttal *et al.* 1997: 40). This involves the process of transitioning from the concreteness of materials to more abstract images of ideas.

Traditionally, manipulatives were regarded as useful because of their concreteness and therefore children were not required to reason abstractly or symbolically. The assumption was made that experience with particular objects will help children discover the abstract principles such objects embody (Uttal *et al.* 1997: 40). However, Dienes (1961), emphasises the use of manipulatives in order to provide a concrete referent for a concept, often at more than one level, instead of a referent for a given abstract idea or procedure. For example, the use of pattern blocks should go beyond the shape as representation to the identification and understanding of abstract ideas and concepts embodied in these concrete representations.

2.6.2 Definition of Manipulatives

Manipulatives can come in various forms and are commonly defined as physical objects that are used as teaching tools to engage students in the hands-on learning of mathematics (Boggan *et al.* 2010). Kennedy (1986: 6) defines manipulatives as 'objects that appeal to several senses and that can be touched, moved about, rearranged, and otherwise handled by children'. The two definitions suggest the educational nature of manipulatives and the active participation of children through the use of various perceptual senses, acknowledging that children learn in various ways. Ball (1992: 16) concludes that 'whether termed manipulatives, concrete materials, or concrete objects, physical materials are widely touted as crucial to the improvement of mathematics learning'.

Mathematical manipulatives can be classified as commercial and/or teacher-produced. Commercial manipulatives include objects such as pattern blocks, base ten blocks, interlocking cubes, Cuisenaire rods and many others. Teacher-made manipulatives, used in teaching place value for instance, include beans, sticks and stones, etc. Manipulatives are defined as tools designed to represent explicitly and concretely abstract mathematical ideas (Moyer 2001: 176; Moyer and Jones2004:1). This implies that in using concrete manipulatives, learners have to transcend the concreteness of these objects to learn the abstract concepts and ideas that are embedded in these objects. It is for this reason that, according to Goldsby (2009) the meaning of concrete needs to be defined so as to understand the role of concrete manipulatives and the concrete-abstract pedagogical sequence. Clements (1999: 47) argues that:

> Although manipulatives have an important place in learning, their physicality does not carry the meaning of the mathematical idea. They often can be used in a rote manner......Students may require concrete materials to build meaning initially, but they must reflect on their actions with manipulatives to do so.

This suggests that the use of highly attractive manipulatives, especially complex commercial manipulatives, may divert students' attention to their inherent properties and hence lead to less learning. In addition, Belenky and Nokes (2009: 103) argue that the relevant features that are central to deep understanding may be less salient and that the concrete details may distract students from these features. Using highly realistic situations and materials may cause the knowledge to be tied to particulars of the scenario, making transfer to other scenarios or into abstract terms more difficult.

2.6.3 Benefits of Using Manipulatives

The last three decades have seen a number of studies conducted within the paradigm of manipulative material usage, predominantly reporting on the attractive benefits of using mathematical manipulatives in the teaching and learning of a variety of mathematics topics. Belenky and Nokes (2009: 103) posit that using manipulatives has been hypothesised to be an effective pedagogical strategy because it a) reduces memory load, b) facilitates understanding by grounding new information in meaningful prior knowledge, and c) may increase students' motivation to learn and understand the instruction, task, or problem. In addition, Kosko & Wilkins (2010: 80) noted that there is also empirical evidence supporting the cognitive benefits of manipulative use as they relate to various aspects of communication including mathematical writing (Jurdak & Abu Zein) and mathematical discussion (Mercer & Sams; Hiebert & Wearne).

Many of the benefits associated with manipulatives use can be found in Bruner's 1973 series of detailed observations of children, which found that concrete materials can be used to develop deep understanding of certain mathematical concepts (Kosko &

Wilkins 2010: 79). The process described involves transition from manipulating concrete materials to creating images from the student's perception of the concept, and finally to the development or adoption of some form of symbolic notation representing the concept (Kosko & Wilkins 2010: 79).

2.6.3.1 Cognitive benefits

Manipulatives have been used to aid in learning abstract mathematical concepts and ideas. There is both empirical and theoretical evidence supporting the cognitive benefits of manipulative use (e.g. Belenky & Nokes 2009; Clements & McMillen 1996; Fuson & Briars 1990; Moch 2001; Moyer 2001; Sowell 1989; Suydam & Higgins 1977). For example, Clements and McMillen (1996) in their study found that the use of manipulatives increased scores on retention and problem-solving tests. Although in her study to investigate how and why middle school teachers use manipulatives, classroom dialogue was not the main aspect of the research, Moyer (2001) found that aspects of discussion were identified as part of manipulatives. This point is supported by Ojose and Sexton (2009: 4) who posit that manipulatives not only allow students to construct cognitive models for abstract mathematical ideas and processes, but also provides a common language with which to communicate these models to the teacher and other students. This implies that while communication is central to the promotion of mathematical understanding, the use of manipulatives enhance communication of mathematical ideas and concepts represented in the manipulatives. Moch (2001), in her study on *Manipulatives work,* found a correlation between the use of manipulatives and test scores. She found that even if manipulatives had been used for only 18 hours in seven weeks, the test scores improved by an average of 10 percentage points. Szendrei, in Miranda and Adler (2010: 17), argues that concrete materials help pupils develop and understand the concepts, procedures, and other aspects of mathematics. Comparing learners in respect of their concrete experiences Heddens (in Ojose & Sexton 2009: 5), conjectures that students who concretely experience and manipulate physical objects develop clearer mental images and can represent abstract ideas more completely than those with limited concrete experiences.

2.6.3.2 Affective benefits

Other research studies on manipulatives reported affective benefits associated with the use of manipulatives. For example, Goracke (2009) conducted a study on the use and impact of manipulatives on student attitude and understanding. She found that overall, students enjoy using manipulatives, not necessarily for the benefit of learning, but because it actively engages them in each lesson. She also found that students' attitude towards mathematics improved when greater manipulative use was infused into the lessons and that students felt more confident that they understood the material, which translated into better attitude. Ojose & Sexton (2009: 4) noted that manipulatives have an additional advantage of engaging students and increasing both interest and enjoyment of mathematics. This implies that the aesthetic value of the use of manipulatives may also promote active participation of learners as they learn mathematics.

Research literature on the effectiveness of manipulatives use on mathematics learning has not been consistent and equivocal. Some scholars have argued that concrete materials are not always beneficial (e.g. Uttal, Liu and DeLoache, 1999; Uttal *et al.*, 1997). This has often been explained in respect of the tension between the concreteness and the desired abstraction. Goldstone and Son (2005:74) noted that as an object's physical properties become more prominent, its ability to serve as a symbol decreases. This suggests that the effect of manipulatives use can only be realised if the connection between the concrete objects and the abstraction is mediated through *inter alia,* the use of less concrete materials such as virtual manipulatives. In another study, Resnick and Omanson noted that mathematical knowledge gained through the use of Dienes blocks did not correlate with students' ability to solve symbolic problems (Goldstone & Son, 2005:74). The implication is that the concreteness in manipulatives may not enhance the transferability of mathematical skills to symbolic contexts. The use of manipulatives has often been touted for engaging learners in hands-on experience and hence promoting active participation in mathematics classrooms. However Chi, in Belenky and Nokes (2009: 103), cautions that being active does not necessarily translate into engagement in cognitive processes that are associated with deep learning. This suggests that manipulatives may be time consuming or regarded as toys if learners do not engage in deep understanding of abstract mathematical ideas.

2.6.4 Effective Use of Manipulatives

The use of manipulatives in teaching mathematics has become almost as commonplace as the use of textbooks (Ojose 2009: 4). This situation, in addition to the much reported research findings that the use of manipulatives has a positive effect on student learning, could easily take the effectiveness of manipulatives for granted. The benefits of manipulatives as discussed above can only be realised if manipulatives are used effectively in the classroom.

The effectiveness of manipulatives use is determined by the extent to which manipulatives are utilised to achieve what they are intended for. It is not only the type of materials that are being used that is important but also how those materials are being used that determines effective use. Moyer, cited in Kamina and Iyer (2009: 2), argues that the reasons why manipulatives sometimes do not produce the required effect are because they a) are not used effectively in the classroom, and b) are poorly perceived. These include availability of resources, focused teacher professional development, communication, multiple representations and integration, etc. Kelly (2006: 188) delineates some of the necessary benchmarks for effective manipulative use. Firstly, it is important that teachers view and refer to manipulatives as tools to help students learn mathematics more efficiently and effectively rather than as toys. Secondly, manipulatives must be introduced in a detailed format with a set of behaviour expectations held firmly in place for students to begin to develop a respectful knowledge base about using manipulatives for mathematics learning. Thirdly, manipulatives need to be modelled often and directly by teachers in order to help students see their relevance and usefulness in problem solving and communicating mathematically. Finally, manipulatives should be continuously included as part of an exploratory workstation, or work time, once open explorations have been completed. Equally so, research studies have also found that the effective use of manipulatives requires certain conditions to be in place.

2.6.5 Teachers and the Use of Manipulatives

Manipulatives are tools used by the teacher and the students to enhance learning in general and to develop abstract mathematical ideas in particular. The latter process involves transition from manipulating concrete materials to creating images from the student's perception of the concept, and finally to the development or adoption of some form of symbolic notation representing the concept (Kosko & Wilkins 2010: 79). Ball (1992) found that children often fail to transfer what they learn from manipulatives to other forms of representation, including written and symbolic representations. The ability to flexibly represent mathematical concepts from one form of representation to the other is fundamental in mathematics and requires constant support, proper planning and time. John Dewey (in Kemmis & McTaggart, 2005: 300) believed that involvement in learning was always an intellectual activity that required both exploration and experimentation and provided opportunities to learn from unfolding experiences. This seems to suggest that exploring and experimenting with manipulatives can offer opportunities for learners and teachers to continually improve their understanding of mathematics.

If the use of manipulatives is to have any effect on student learning, then it would have had it by virtue of, first and foremost, having had an effect on teachers' knowledge and beliefs about manipulatives and their use. Adler *et al.* (2002) argue that resources themselves do not have an automatic educational meaning but that the meaning emerges through their use in the context of classroom practices and the subject [matter] being learned. This view puts the knowledge and experiences of teachers to use manipulatives at the centre. The idea is also supported by Moyer and Jones' (2004) year-long study with 10 middle school teachers who participated in training to examine how and why teachers use manipulatives. They found that the training led to an increased use of manipulatives but did not significantly change the way teachers taught. They also found that manipulatives were used to reinforce ideas already learned or as diversions rather than a way of helping children make sense of mathematics. These findings suggest that the lack of focused professional teacher development support that incorporates pedagogical knowledge of manipulatives in the teaching and learning of mathematics makes it difficult for teachers to effectively use manipulatives in their teaching. Teachers need to have knowledge of manipulatives and how to use them as representations of abstract mathematical ideas.

Teacher beliefs and background characteristics have been found to have a bearing on the effective use or otherwise of mathematical manipulatives. For decades, researchers have either encouraged or discouraged the use of manipulatives in mathematics classrooms. To a large extent, this has been attributed to teachers' views about manipulatives and their use. For example, in a study of 10 middle school teachers' views on the use of manipulatives, Moyer (2001) notes that teachers found the use of manipulatives to be fun and rewarding with students, but did not see the value of manipulatives as tools for learning mathematics. She attributes this to teacher beliefs such as 'real math' is not fun and that working with manipulatives cannot be taken serious. This implies that if a teacher believes that manipulatives are play objects, they will be regarded as time wasting especially where 'serious mathematics', that involve conceptual understanding, is involved. In fact, Moyer & Jones (2004:14) warn that:

> Teachers who view manipulatives as time wasting or secondary to the serious work of learning mathematics will inadvertently encourage their students to use manipulatives for play, rather than for mathematical learning or understanding.

In delineating some of the necessary benchmarks for effective manipulative use, Kelly (2006: 188) argues that it is important that teachers view and refer to manipulatives as tools to help students learn mathematics more efficiently and effectively, rather than as toys.

A number of research studies have established that long term use of manipulatives may increase student achievement in mathematics (e.g. Moyer & Jones 2004; Sowell 1989). This suggests that the successful use of manipulatives depends on the time of interaction with such manipulatives as well as the frequency of use of these manipulatives.

2.7 CLASSROOM PRACTICE AND THE USE OF MANIPULATIVES

Concrete materials are intended to enhance mathematical understanding. However, literature on manipulatives indicates that concrete materials do not automatically carry mathematical meaning for students (Thompson 1994: 3). Consequently, seeing mathematical ideas in concrete materials becomes one of the challenging cognitive goals of using concrete materials in the classroom. The teacher needs to be aware of multiple interpretations of materials in order to hear hints of those which students actually make (Thompson 1994: 5). It is also important that students can create multiple interpretations of materials. It is the teacher's responsibility to cultivate multiple viewpoints from which valid interpretations can be made. This will empower students and teachers to choose among manipulatives for the most appropriate relative to a current situation (Thompson 1994: 6). Adler *et al.* in Miranda and Adler, (2010) argue that even if the availability of manipulatives leads to significantly better practices, this will not happen in unproblematic and linear ways. Remillard and Bryans (2004: 4) argue that in order to teach differently, teachers need opportunities to learn mathematics in new ways and to consider new ideas about teaching and learning. To a large extent, those involved in current reform efforts have heeded the warnings of scholars about the impossibility of a teacher-proof curricular and the importance of teacher learning. To this end, in the current wave of curriculum development, some developers have taken up the task of designing curriculum materials that will not only provide teachers with guidance for classroom practice, but will also foster teachers' learning as they use them. Such curricular, as argued by Remillard would need to speak to teachers, rather than through them (Remillard & Bryans 2004: 4). This is what the present study seeks to investigate about the use of manipulatives by South African teachers: the possibilities for them to learn how to use manipulatives and change their classroom practice through the use of manipulatives in teaching.

Neal (2007: 4) argues that 'Any amount of materials will not replace quality interactions between adults and children which are vital in maximising learning opportunities'. This implies that the effectiveness of manipulatives is largely dependent on the quality of classroom practice, which in turn depends on the learning opportunities created by the teacher for students to actively participate in the use of manipulatives beyond their literal concreteness, and by seeing mathematics through them. This view is supported by Cobb and Yackel (1996: 185) who conjecture that the qualities of students' thinking are generated by or derived from the organisational features of the social activities in which they participate. They also conjecture that within the perspective that views learning as a process of both active individual construction and enculturation, processes of significance are considered to be integral to classroom mathematical practices established by a classroom community that might involve reasoning with physical materials, pictures, diagrams, computer graphics and notations. When attention shifts from collective to individual activity, the physical materials, symbols and notations that students use are viewed as constituent aspects of their activity rather than as external tools. As a consequence, the use of particular materials and symbols is considered to profoundly influence both the nature of mathematical capabilities that students develop and the processes by which they develop them (Cobb and Yackel 1996: 186).

Thompson (1994: 7) considers concrete materials to be appropriate for two purposes: firstly, they enable teachers and students to have grounded conversation as their use provide something concrete to talk about; secondly, concrete materials provide something that students can act upon. Since Bruner's (1973) descriptions of the interconnectedness of language and manipulative use, much literature focusing on this area appears to take the relationship of manipulative and language use as an assumed relationship (Kosko & Wilkins 2010: 80). 'Communication is an essential part of mathematics and mathematics education' (NCTM 2000: 60). Both written and oral forms of communication promote deeper understanding of concepts (NCTM 2000). Communication enables students to reflect upon concepts through interaction with others, thus creating opportunities to explain and to justify their reasoning and become more knowledgeable. This suggests that the effective use of manipulatives requires students to, among other actions, explore with manipulatives, reflect on their actions with manipulatives and communicate mathematical ideas. On the part of the teacher, it requires critical decisions not only about the choice of appropriate manipulatives but also the choice of tasks, contexts and questions that prompt students to reflect on various aspects of these learning materials, enabling students to transcend from their concreteness to understanding the abstract ideas they embody. In this regard, Ball (2003: 3) argues that teachers need to use representations skilfully, choose them appropriately and carefully map between a given representation, the numbers involved, and the operations and processes being modelled.

From the sociocultural perspective, it is widely accepted that language is important for learning and thinking and that the ability to communicate mathematically is central to mathematical learning and teaching (e.g. Setati 2008: 103; Steele 2001: 404). This is also acknowledged in a number of education systems such as the United States (National Council of Teachers of Mathematics (NCTM2000) and South Africa (National Curriculum Statement (NCS 2009; 2010) 'Through communication, ideas become objects of reflection, refinement, discussion and amendment (Steele 2001: 404). Mathematical concepts and ideas, through interaction with others, present students with opportunities to explain, justify and clarify their own understandings, thereby gaining deeper understanding of those concepts and ideas.

Thompson and Lambdin cited in Goldsby (2009: 3) consider concrete materials appropriate for two purposes: Firstly, they allow students to handle, observe, model, and internalise abstract concepts thereby allowing students to construct their own cognitive models for abstract mathematical ideas and processes. Secondly, they provide a common language with which to communicate these models to the teacher and other students. Manipulatives present students and teachers alike with communication spaces, where students make meaning by dialoguing about and describing mathematical ideas and thereby also making their thinking transparent, calling for teachers to observe and listen to them.

Certain teaching strategies and approaches in mathematics have been found to promote the effective use of manipulatives more than others. It has been hypothesised that being 'active' facilitates 'learning by doing' and increases attention and engagement. Activity-based approaches, as implied in the hands-on and manipulative nature of concrete materials, do contribute to their effective use. Belenky and Nokes

(2009: 103) note that engaging students in valuable and productive activities with manipulatives has been another way of aiding student learning. However, Chi, in Belenky and Nokes (2009: 103), argues that being active does not necessarily translate into engagement in cognitive processes that are associated with deep learning. This suggests that physical activities by students are not a sufficient condition for the understanding of abstract mathematical ideas, which is fundamental in the use of manipulatives. Teachers need to use approaches that intrigue and promote mental activity in order to effectively use manipulatives. For instance, the use of a metacognitive teaching approach has been found to promote mental activity. The study conducted by Belenky and Nokes (2009) found that when concrete materials are combined with prompts (metacognitive or problem focused) to reflect on what is being learned, students may be able to learn and understand abstract concepts. Goldstone & Pizlo 2009: 4 characterise such understandings as being both well-grounded and high-reaching. This implies that through efforts to prompt students to reflect on concrete manipulations and on their problem solving process students were able to learn abstract concepts.

2.7.1 Classroom Organisation and the Use of Manipulatives

One of the benefits of using manipulatives is that they afford learners and teachers the opportunity to have hands-on experiences with manipulatives – seeing, touching and manipulating them. This makes the availability of concrete materials a key requirement for their usage. Adler, in Miranda and Adler (2010: 16), introduces the notion of transparency, defined in terms of how the resources are contextualised and used. This implies that for the use of manipulatives to be effective, they need first and foremost to be concretely available for visibility and manipulation. The availability of manipulatives on its own is not a sufficient condition for their effective use. This is supported by Adler (2000) who cautions that '...., it should not be assumed that an increase of material resource will amount to better pedagogic practices (Miranda & Adler 2010: 16).

Coupled with the availability of manipulatives is the challenge regarding their organisation for use in the classroom. Johnson (2012: 5) posits that because there are so many different manipulatives, it is important to keep them organised and ready at a moment's notice. This requires that teachers well in advance need to thoroughly prepare for their lessons and have enough manipulatives for each student or groups of students, trying the lesson with manipulatives before presenting it to learners and anticipating issues that learners may run into with the lesson.

2.7.2 Teachers' Mathematical knowledge and the Use of Manipulatives

Cobb and Yackel (1996: 175) posit that grounding theory in practice reflects the view that the relationship between theory and practice is reflexive. Theory is seen to grow out of practice and to provide feedback to inform and guide practice. This is in contrast to more traditional styles of representation in which the basic tenets of theoretical positions are stated, and then implications are deduced for practice. Such a theoretical style was found to elevate theory over practice and thereby devaluing the relation between theory and practice (Schön 1983 cited in Cobb & Yackel 1996:176). Alternative styles of representation, i.e. grounding theory in practice, suggest a more collaborative relationship between teachers and researchers (Cobb & Yackel 1996: 186). The sociocultural perspective holds the view that learning occurs while participating in and contributing to the practices of the classroom community. This perspective links collective and individual processes.

Whilst tasks involving manipulatives are considered critical in the teaching and learning of mathematics, some scholars have argued that what has become more critical is teachers' ability to choose quality tasks and to guide students to deep mathematical understanding through these tasks. Common features of quality tasks have been expressed in several ways. For example, Jaworski (1994) writes about the demand for mathematical challenge while Zaslavsky (2005) writes about uncertainty. What is communicated in these and other similar research is that students' meaningful mathematical learning and understanding occurs when they experience difficulties in solving their tasks and resolve the difficulties through various measures. Zaslavsky (2005) note that quality tasks can enhance teachers' mathematical and pedagogical knowledge through reflecting upon their students' needs and difficulties.

2.8 STRATEGIES TO IMPROVE TEACHERS' CLASSROOM PRACTICE

Teachers' mathematical knowledge is crucial for improving the quality of classroom practice (Ball 1991; Ma 2010). The latter is a complex and challenging problem worldwide, one that preoccupies contemporary reformers and researchers alike. It is now widely accepted that just as classroom practice offers opportunities for student learning, so does it offer opportunities for teacher (re-) learning in order to meet the demands of the reform curriculum. Brown and Borko (1992) contend that mathematics teachers often deepen their knowledge and understanding of mathematical content and methods as they engage in the process of teaching mathematics. Ma (2010) concurs with this view when she points to teaching as one of the possible periods during which teachers' subject matter knowledge can be enhanced. This position is supported by a number of studies. For example, Zaslavsky and Peled (1996) found that there was a significant difference between the mathematical examples generated by experienced and those generated by prospective teachers with similar formal mathematical backgrounds.

It is for this reason that enhancing teacher learning opportunities to improve their knowledge, skills, and their teaching practice is at the heart of education reform efforts. To this end, research has increasingly identified the continuing development and learning of teachers as one of the critical mediators in the effectiveness of policy for teachers and teaching practice, including policy reforms such as the use of manipulatives in classroom practice (Desimone, Smith & Phillips: 2007; Cohen & Ball 2000:2). In fact, Sykes, in Desimone (2009), conjectures that education reform is often synonymous with teachers' professional development. However, a large body of literature has demonstrated that supporting teachers to meet the ambitions and complex visions of mathematics reform is difficult (e.g. Jaworski 1994; Kazemi & Stipek 2001). This suggests that strategies to improve teachers' knowledge, skills, and their teaching practice are essentially about professional development strategies. In turn, this also suggests the need to reconceptualise teacher learning opportunities so as to understand the key constructs that make them effective.

One way of explaining, understanding, and identifying the strategies for effective teacher learning opportunities is to focus on the critical features of the Professional Development activities. There has been an emergence of scholarship in professional development space including teacher learning and teacher change (e.g. Ball & Cohen 1999; Borko 2004; Darling-Hammond & McLaughlin 1995; Huffman, Thomas and Lawrenz 2003; Little 1993), characterised mainly by its theorising of 'high quality professional development'. Other studies have focused on exactly what and how teachers learn from professional development, or on the impact of teacher change on student outcomes, proposing a situative perspective on teacher learning (e.g. Borko 2004; Desimone, Porter, Garet, Yoon, and Birman 2002; Garet *et al.* 2001). Situative perspectives argue that, to understand teacher learning, we must study it within these multiple contexts, taking into account both the individual teacher-learners and the physical and social systems in which they are participants (Putnam & Borko 2000). These perspectives have in common a conception of the learning process as changes in participation in socially organized activity (Lave 1988; Lave & Wenger 1991). Such developments do certainly provide the necessary theoretical grounding for conceptualising professional development strategies, especially those meant to transform classroom practice. There is a consensus on the core features of high-quality professional development: a) content focus, b) active learning, c) coherence, d) duration, and e) collective participation (e.g. Birman, Desimone, Porter and Garet 2000; Desimone 2009; Elmore 2002; Garet, Porter, Desimone, Birman, and Yoon 2001; Putnam & Borko 1997). These core features have been used in this study to characterise activities entailed in professional development strategies intended to improve classroom practice. Among the five features referred to, Birman *et al.* (2000) identified duration and participation as the two structural features that set the context for professional development.

The amount of time spent by participants doing the activity, as well as the time limit allocated to the activity have also been found to positively impact on the effectiveness of professional development. Research by Birman *et al.* (2000) indicates that activities of longer duration have more subject-area content focus, more opportunities for active learning, and are more coherent with teachers' other experiences than do shorter activities. This implies that the effectiveness of professional development, including the core features of its activities as discussed below, depends on the number of hours that teachers spent on its activities. Garet *et al.* (2001) argue that professional development is likely to produce teachers' enhanced knowledge and skills if it is sustained and more intensive than shorter professional development. In addition, it has been found that even traditional forms of professional development may have better content focus, active learning and coherence if they are longer (Birman *et al.* 2000; Garet *et al.* 2001). For example, Birman *et al.* (2000) found that a professional development with a traditional format had high quality effective features and concluded that the characteristics of the activity and not the form of the programme are what matters. This implies that the length of the professional development does not on its own guarantee its effectiveness; rather, it may facilitate those activities (content focus, active learning and coherence) that give it its substance. Besides, professional development of longer duration promotes generative learning.

Collective participation refers to participation of teachers from the same department, subject or grade in sharing and reflecting on their experiences with others. Collective participation enables teachers to a) discuss concepts and problems that arise during the professional development activity, b) integrate what they learn with other aspects of their instructional content because they are likely to share common curriculum materials, course offerings and assessment requirements (Birman *et al.* 2000), and c) share professional culture in which teachers in a school or grade develop common understanding of instructional goals, methods, problems and solutions (Ball 1996 in Birman *et al.* 2000). According to Vescio, Ross and Adams (2008), the elements of collaboration that promote changes in teaching include strategies that open practice in ways that encourage sharing, reflection, and taking the risks necessary to change. Louis and Marks in Vescio *et al.* (2008) refer to open practice as reprivatisation of practice, which signifies a shift away from the traditional teacher culture that has been described as isolationist. This suggests a new culture that radically departs from the traditional culture where the development of teachers depended on the knowledge and theory of developers outside the teaching practice, to a culture that values and utilises the collaborative knowledge and experiences of teachers as experts on what is needed to improve their own practice and increase student learning. These conceptualisations clearly indicate that such a culture does not happen automatically, but needs support structures to facilitate such collaboration and reflections.

One model that has evolved in support of teachers' professional development is that of Professional Learning Communities (PLCs). PLCs at their best are grounded in generation of 'knowledge of practice (Cochran-Smith & Lytle: 1999), where teachers engage in collaborative inquiry and reflection to explore new ideas, current practice, and evidence of student learning. Although there is no universal definition of a PLC (Stoll & Louis 2007:2; Stoll, Bolam, McMahon, Wallace, & Thomas 2006:222), there is a consensus that PLCs have the capacity to promote and sustain professional learning in the school community as a means to collectively enhance student learning (Bolam *et al.* 2005; Louis *et al.* 1995 both cited in Stoll & Louis 2007:2). The most prominent features of effective PLCs seem to centre on collaboration, inquiry, critical reflection, and continuous improvement of both the teaching practice and student learning. For example, all the literature reviewed by Vescio *et al.* (2008: 86) support continuous teacher learning as an element of PLCs and noted that teachers involved in efforts to improve African-American students' literacy sought out scholarly literature on culturally relevant teaching (Hollins *et al.* 2004 cited in Vescio *et al.* 2008: 86) and that teachers searched for external/outside ideas to help them solve their teaching dilemmas (Berry *et al.* 2005 cited in Vescio *et al.* 2008: 86). This suggests the culture of teacher learning where teachers collaboratively search for solutions and best practices in order to transform their classroom practice.

In addition, the model assumes that the knowledge teachers need to teach well is generated when teachers treat their own classrooms and schools as sites for intentional investigation at the same time that they treat the knowledge and theory produced by others as generative material for interrogation and interpretation (Cochran-Smith & Lytle 1999: 272). This suggests an experimental type of approach in which those collaborative discourses need to be initiated and sustained through data that would serve as a basis for analysis, reflection and dialogue. Another model that has evolved in the past decade and that fits the 'experimental' type is that of a Lesson Study. Lesson study is an aspect of professional development that originated in Japan and relies on the observation of live classroom lessons by a group of teachers who collect data on teaching and learning and collaboratively analyse it (Lewis, Perry, Hurd and O'Connell 2006: 3). Stigler and Hiebert (1999) define lesson study as a school based collaborative professional development process by which Japanese teachers seek to improve the teaching and learning that occur in their classrooms. Stigler and Hiebert (1999) posit that teachers who participate in the lesson study see themselves as contributing to the development of knowledge about teaching as well as to their own professional development. They further characterise the Lesson Study strategy as: focused work on improving teaching, based on a long-term and continuous improvement model; a constant focus on student learning; a focus on direct improvement in teaching in context, and collaborative. Lesson study is a means to foster an enquiry stance within professional learning communities. Cochran-Smith and Lytle (1999) reiterate the role of both the individual and the community in which they work on the development of an inquiry stance towards professional knowledge. Scholars have generally characterised Lesson study as an iterative process in which teachers collaboratively reflect on and analyse their own practice for their own professional development (Chassels & Melville 2009; Fernandez, Cannon, and Chokshi 2003; Ono & Ferreira 2010), collectively referred to as 'plan-do-see' (Ono & Ferreira 2010: 64)

Professional development that focuses on content knowledge has been found to be effective in enhancing teachers' knowledge and skills. For example, teachers who participated in a national study conducted by Garet *et al.* (2001) reported that their knowledge and skills grew and their practices changed when they received professional development that was coherent, content focused and involved active learning. This is elaborated by Blank *et al.* (2007) in Darling-Hammond and Richardson (2009) who point out that professional development that focuses on student learning and helps teachers develop the pedagogical skills to teach specific kinds of content, has strong positive effects on practice. It needs to be recognised that content focus in this context, refers to content knowledge as it is used in teaching. Often, teachers are left to integrate on their own, content that they learn in professional development in their classroom practice. This calls for a professional development that enhances teachers' subject matter knowledge as well as the methods that help teachers to convey these understandings to learners. 'Teacher learning involves developing and integrating one's knowledge base about content, teaching and learning; becoming able to apply that knowledge in real time to make instructional decisions...' (Davis & Krajcik 2005: 3). Such a strategy seems to conform to Shulman's conception of mathematical knowledge for teaching, i.e. pedagogical content knowledge that emphasises issues that go beyond content knowledge and include quality of teacher interactions, tasks, questions, examples and representations of that content. This is an acknowledgement that content does matter to the extent that it is relevant in addressing both teacher and student learning. However, this is in contrast to professional development that focuses on a) content to the exclusion of skills necessary for effective classroom practices, and b) generic professional development that focus on teaching techniques without also emphasising on content. Improving and deepening teachers' knowledge of specific concepts and ideas in a specific subject, as well as specific methods rather than generic teaching techniques, should be at the centre of professional development initiatives if they are to be effective (Birman *et al.* 2000; Desimone 2009). This strategy highlights the importance of relevant content, that is, content that is embedded in practice and hence centred on both teacher and student learning.

Professional development that involves teachers' active learning including discussions, planning, and practice have also been reported to have increased teacher knowledge and skills, and changed classroom practice (Birman *et al.* 2000, Garet *et al.* 2001). This suggests active and hands-on participation by teachers in their own learning on matters that directly affect their craft, an approach that provides hands-on experience for teachers in the development of content knowledge and skill as discussed above to ensure that teachers take responsibility for improving their own classroom practice. Activities that include active learning by teachers are those that create opportunities for teachers to engage in meaningful analysis of teaching and learning, including opportunities to observe and be observed, plan classroom implementation, such as practising in simulated conditions and developing lesson plans, review student work, present demonstrations, lead discussions and write reports. This is in contrast to traditional professional development where teachers are being talked down to, and where teacher learning is abstract and does not allow teachers opportunities to meaningfully engage in knowledge and skills necessary for teaching. Active learning activities demonstrate how professional development may meaningfully engage teachers in (self) reflection activities as part of the development process. In addition, Darling-Hammond and Richardson (2009) contend that active learning allows teachers to transform their teaching and not simply layer new strategies on top of their own. The latter implies that teachers, as self-directive in their learning, are less likely to integrate into their existing experiences professional development activities that are imposed on them and that are not relevant to their contexts. This suggests a strategy that appreciates and values teachers' judgement, experience and expertise in transforming their own practice, instilling and developing in them a sense of agency and ownership of the development process.

Coherence of professional development with policies and other professional experiences has also been found to have a positive impact on teacher learning and classroom practice (Birman *et al.* 2000; Garet *et al.* 2001). Coherence refers to the extent to which professional development experiences are part of an integrated programme of teacher learning – activities that are consistent with teacher goals, build on earlier activities that are followed by additional activities, and encourage continuing professional communication among teachers in discussing their experiences with other teachers and administrators at the school. In contrast, traditional professional development has been found to be characterised by fragmented, once-off, inconsistent and often irrelevant activities. Professional development is said to be effective if it forms a coherent part of a wider set of opportunities for teacher learning and development (Birman *et al.* 2000; Garet *et al.* 2001). This strategy, which is consistent with the goals of addressing teachers' real and daily concerns as they enact the curriculum, promotes consistency and sustainability in teacher development. As argued by Birman *et al.* (2000), professional development is relevant to the extent that it meets the needs of teachers. This implies a strategy that values and builds on teachers' earlier experiences and learning, integrates teacher development into the life and culture of the school, and promotes consistency and sustainability.

2.9 THEORETICAL FRAMEWORK

This section provides a theoretical model (Figure 3) within which to contextualise the present study. The study seeks to investigate the use of manipulatives by South African teachers and the possibilities to improve both their mathematical knowledge for teaching and their classroom practice through the use of manipulatives. It builds on Hill and colleagues' descriptions of mathematical knowledge for teaching, to describe how teachers' use of mathematical manipulatives and their mathematical knowledge for teaching interact with one another to translate into effective classroom practice (Hill et. al 2005, 2004). In this framework, there are five operational concepts that are used to address the research questions, i.e. teachers' mathematical knowledge for teaching, teachers' mathematical classroom practice, teachers' views about mathematics, mathematical manipulatives and teachers' professional development.

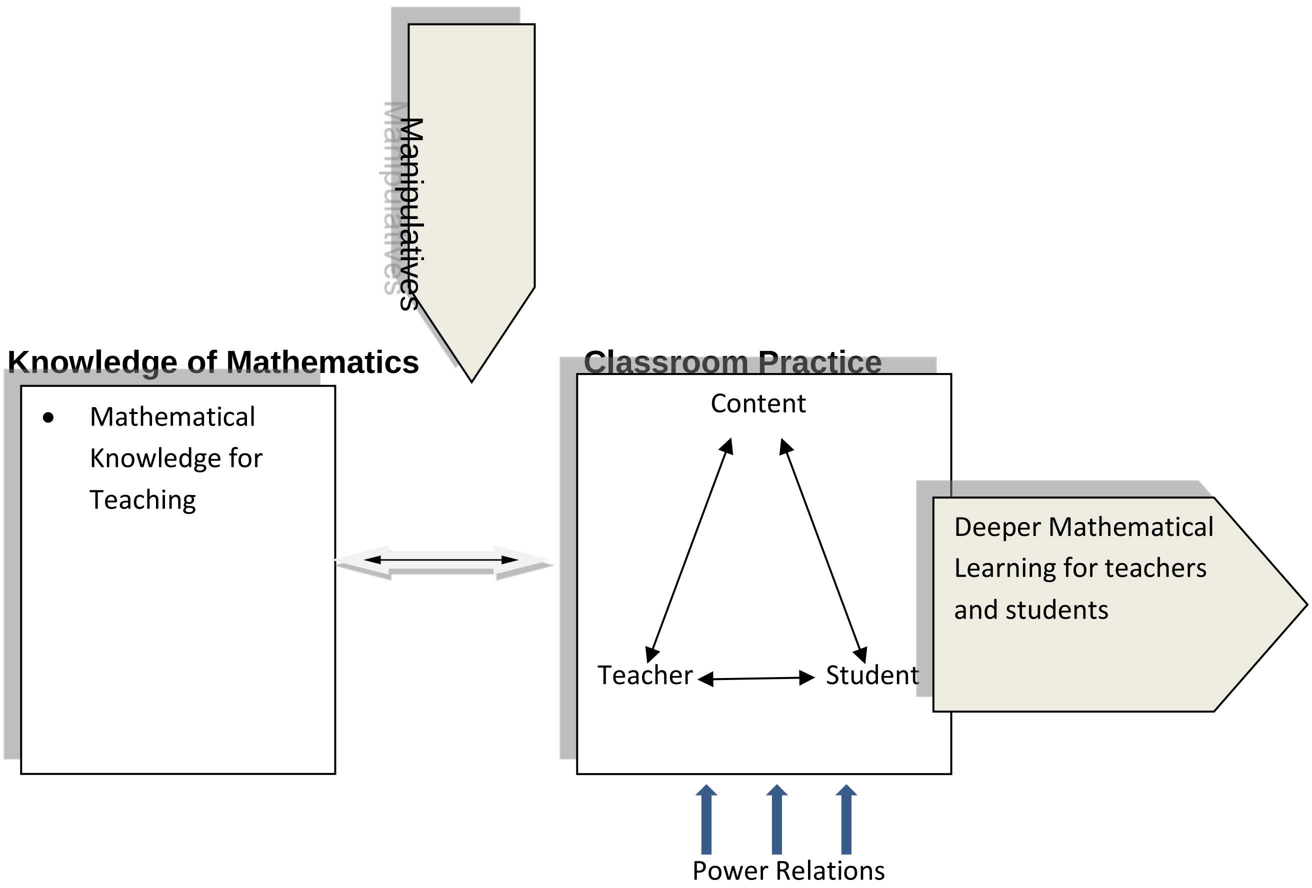


Figure 3: Theoretical Framework

Equally important to the model is how each of these constructs relates to the study as well as the interactions between and among them in making the framework more comprehensible.

The study seeks to investigate the use of manipulatives by South African teachers and the possibilities for them to improve both their mathematical knowledge for teaching and their classroom practice through the use of manipulatives. Mathematical knowledge that is embedded in practice undergirds the present study. The notion of mathematics knowledge for teaching, defined as 'the mathematical knowledge that teachers need to know to carry out their work as teachers of mathematics' (Ball, Thames & Phelps n.d.: 4) is exactly that kind of mathematical knowledge. The key component of MKT is a distinct mathematical knowledge referred to as Specialised Content Knowledge (SCK) that meets the mathematical demands of teaching specific topics as they emerge within the core task domains of teachers' work. The core activities, made up of predictable and recurrent tasks that teachers face in mathematics teaching, include aspects such as determining what students know; choosing and managing representations of mathematical ideas; steering a productive discussion, sizing up students' ideas, etc. Clearly, all these activities require specialised and ongoing use of mathematical knowledge, reasoning, insight, understanding and skills that require special content knowledge pertinent to the work of teaching. All these demands knowledge that goes beyond knowing correct mathematical procedures and algorithms, as rightly noted by Hill and Ball (2004: 331), teaching mathematics requires an appreciation of mathematical reasoning, understanding the meaning of mathematical ideas and procedures, and knowing how ideas and procedures connect. In the critical paradigm, the act of knowing is a social practice with the intention to transform reality (teaching and learning). To understand this process of translation one has to recognise that knowledge is always socially constructed in social interactions. The critical framework adopted in this study will help to reveal how power is distributed and structured in such classroom interactions. It will also be critical in helping to explain and address mediations – cultural, social, historical, intellectual, ideological, etc. that seek to transform mathematics classrooms.

There is widespread consensus about the link between teacher knowledge and classroom practice in mathematics (e.g. Ball 2000; Borko *et al.* 1992; Hill *et al.* 2008a; Ma 2010; Shulman 1986). Mathematical classroom practice is described as an interaction or relationship among teachers, students, and content in which each of the three components is important (Cohen & Ball 1992; Hill *et al.* 2008a: 431). Embedded in these interactions is a web of power relationships which make critical theory a more relevant research paradigm since its main concern is with issues of equality, domination and social justice that characterise social structures and power relations. Current reforms in mathematics teaching and learning from mechanical drill and memorisation to conceptual understanding of mathematics also make particular demands on teachers in terms of their classroom practice.

MKT, the key construct in this study, is essentially grounded and finds expression in what teachers do in their classroom practice. It is through these interactions that occur in the classroom that the core task domains of teachers' work as discussed above, can emerge. Hill *et al.* (2008a: 431) define classroom practice as a composite of several dimensions that characterise the rigor and richness of the mathematics of the lesson, including the presence or absence of mathematical errors, mathematical explanation and justification, mathematical representation, and related observables. Literature on research studies that examined the relationship between MKT and classroom practice found that there is positive association between the two constructs (e.g. Hill *et al.* 2008a: 430; Shechtman *et al.* 2010a:323**)**.This study theorises that efforts to improve teacher knowledge centre on teachers' ability to translate their knowledge into effective classroom practice and that the effects of teacher knowledge can only be explained and understood through observation of the contexts of the dynamic teaching situations. The chain of logic as theorised in this study is that a high quality of MKT will produce effective classroom practices and vice versa, which will in turn translate into higher levels of student achievement. This is the reason why the dimensions of MKT will be explored through direct classroom observations as well as primary records of teaching and learning, interviews, video tapes, learners' work, curriculum materials and teacher notes as the basis for making informed meaning and interpretations.

There is a sufficient body of literature to suggest that teachers' beliefs and conceptions about mathematics, teaching and learning do impact mathematics classroom practice (Ball 1988, Thompson, 1992, Fang, 1996). In fact, teachers' conception of the nature of mathematics determines their belief about its teaching and learning. For example, Ball (1988: 93) in her study on teachers' knowledge and beliefs about mathematics notes that the prospective teachers' ideas, explanations, and justifications about mathematics teaching and learning, were rooted in their assumption about the subject. Over the past two decades, researchers have developed theories about teachers' beliefs and the way these beliefs impact teachers' classroom practice (e.g. Fang 1996; Thompson 1992). Thompson in Ernest (1989) distinguishes three philosophies that are more prominent in the teaching of mathematics: a) the instrumentalist view – a set of unrelated facts, rules and skills used for some external end, b) the Platonist view – a static but unified body of certain knowledge and truths, and c) the problem solving view – a dynamic, continually expanding field of human creation and invention. All 3 these views about mathematics have consequences for classroom practice. For example, a teacher who holds an instrumentalist view would emphasise the drilling of facts and procedures at the expense of conceptual understanding. Ball (1988: 94) also notes that teachers' ideas about mathematics are a significant dimension of their subject matter knowledge. MKT by its nature emphasises substantive knowledge and deep understanding of mathematics and this would also depend on the assumptions held by teachers about mathematics. If mathematics is a set of truths, then there would not be any need for justification of mathematical knowledge. It is evident from the above that teachers' beliefs do have an impact on their knowledge and classroom practice, the key constructs of my research question. The framework adopted in this study has teachers' beliefs about mathematics as one of its dimensions which will be examined through interviews. The questions relate to the teaching and learning of mathematics. For example, what teachers think the teaching of mathematics entails.

Research literature on this relationship found that there are important factors that mediate this relationship, either supporting or hindering teachers' use of knowledge in practice (e.g. Hill *et al*. 2008a: 431; Shechtman, Haertel, Roschelle, Knudsen, and Singleton 2010b: 5). Some of the literature reviewed for the purpose of this study show that manipulatives, as teaching techniques, have been touted as crucial to the improvement of students' mathematics learning in general and in promoting conceptual understanding of abstract mathematical ideas in particular. Representation of mathematical ideas is one of the key constructs of MKT. Scholars have noted that in order to promote mathematical understanding, it is necessary to make connections between manipulatives and mathematical ideas explicit (e.g. Ball 1992; Driscoll 1981; Hiebert 1984 all cited in Ma 2010: 6). This implies that in using concrete manipulatives, learners have to transcend the concreteness of these objects to learn the abstract concepts and ideas that are embedded in these objects. This requires of teachers to explicitly decompress the mathematical concepts and ideas that are embedded in these manipulatives. Manipulatives also provide a common language with which to communicate these ideas to the teacher and other students (Ojose &Sexton 2009: 4). This implies that the strength of manipulatives lies in their effective use that hinges on teacher resources, as Ma (2010: 5) argues: 'The direction that students go with manipulatives depends largely on the steering of the teacher'. Clearly, to help students construct and articulate their understanding of Mathematics using these representations makes particular content and pedagogical demands on teachers, demands that resonate with MKT.

In order to understand and explain the dimensions of MKT as discussed above, the potential of manipulatives to facilitate and generate student responses and elicit the core activities of MKT become central in the current study. The realisation of this potential largely depends on teachers themselves as they interact with students and manipulatives in the broader context of their practice. This can only be possible when firstly, teachers apply the use of manipulatives as a resource and hence an opportunity for them to learn, and secondly, where teachers are supported through professional development programmes that integrate the use of manipulatives. In light of the above, the current study theorises that the effective use of mathematical manipulatives has the potential to improve teachers' mathematical knowledge for teaching and also change their classroom practice. The study also recognises that aspects of social injustice as embedded in power relations may limit the opportunities and constrain the potential of mathematical manipulatives to enhance learning by both the teachers and learners.

The dimensions of MKT will be explored through observation of classroom teaching that integrates manipulatives, focusing on how teachers a) make mathematical representation of concepts and ideas, b) make explicit underlying mathematical concepts, ideas and principles, c) manage any novel methods used by students, and d) facilitate mathematical discourse. Teachers with MKT are able to provide a conceptually based justification for mathematics solutions, problems, algorithms, etc.

2.10 SUMMARY OF LITERATURE REVIEW

In this chapter, I reviewed a range of literature in order to appropriate the current study within the existing research literature. I firstly reflected on Critical Theory as the underlying theoretical framework that couched this study. I illuminated how power and democratic values of social justice, equity, freedom and impact on transformation, emancipation and empowerment, all of which are at the very heart of this study.

I also gave a literature account on how theory on teacher knowledge has been problematised, and thus evolved over time, in an attempt to close literature and research gaps that exist on the relationship between teacher knowledge and classroom practice. Research developments on efforts to bring together teacher knowledge of mathematics and mathematics classroom practice, the main constructs in the research question, were brought to the fore. Contributions to a theory on mathematics teacher knowledge and practice by various scholars such as Lee Shulman (PCK), Liping Ma (PUFM), Tim Rowland, Peter Huckstep and Anne Thwaites (knowledge quartet framework) were discussed to indicate how they culminated into and helped to shape theory on MKT, the main construct of my research question. The Michigan team, proponents of MKT, build their theory on classroom practice, that is, from what Ball and Bass (2000: 90) refer to as a ‘job analysis’ approach in trying to understand the mathematical entailments of practice. Knowing mathematics and being able to use it in the course of teaching is the defining feature MKT, a Specialised Content Knowledge (SCK) that refers to knowledge that only teachers need to hold in their work. Although research on MKT has been classroom situated, its concern has largely been with the direct relationship between MKT and student learning or achievement. Consequently, other researchers and scholars have begun work on

MKT as it relates to classroom practice (e.g. Cohen *et al.*; Roschelle *et al.*; all in Shechtman et. al 2010b), the central issue in this study. They concluded that although a positive correlation exists between MKT and classroom practice, there could be other important factors that mediate this relationship by either supporting or hindering teachers' use of knowledge in practice (Hill *et al.* 2008a: 431). The possibility of using manipulatives to mediate the relationship, one of the key constructs in my research question, was also discussed in this chapter.

To this end, I also reviewed literature on manipulatives with regard to the history, definition, benefits, the use thereof, and more. Of particular importance is the theoretical account of the relationship between the use of manipulatives and the other two major constructs, that is teacher knowledge and classroom practice. The theoretical framework provided in this chapter pulls together the three major constructs to illustrate the main argument of the study, i.e. the possibilities for teachers to learn and also change their classroom practice through the use of manipulatives in teaching.

The next chapter presents the methodology employed in this research to answer my research question.

CHAPTER 3: RESEARCH METHODOLOGY AND DESIGN

3.1 INTRODUCTION

This chapter provides a detailed description of the research methodology applied to answer the following research questions that guide this study:

a) *How does the use of manipulatives in the teaching of primary school mathematics help to reshape the teachers' own mathematical knowledge for teaching?*

b) *How does the use of manipulatives in the teaching of primary school mathematics help to reshape the teachers' own classroom practices?*

c) *How can we explain the influence of manipulatives or lack thereof on teachers' knowledge for teaching and classroom practices?*

My primary goal in this study was to employ the use of manipulatives as a window to a) examine and understand the teachers' mathematical knowledge and classroom practices respectively, b) explain the influence of manipulatives or lack thereof on transforming teachers' knowledge for teaching and classroom practices, and thereby c) to ultimately contribute to research on teacher knowledge and improved teaching practices through an empirically generated framework on the use of manipulatives.

To realise this goal, I decided on a particular methodology and a plan of action that would enable me to gather as much data as possible to answer my research question.

The chapter begins with a brief description and justification of the research paradigm and approach adopted in this study. That is followed by an elucidation of the research design including details on ethical considerations, identification and selection of sites and participants, data collection methods and instruments, data analysis strategy, the researcher's role as well as the criteria for quality.

3.2 RESEARCH PARADIGM AND APPROACH

3.2.1 Research Paradigm

Since the ground breaking work of Thomas Kuhn, approaches to methodology in research have been seen to reside in 'paradigms' and communities of scholars (Cohen, Manion & Morrison, 2011: 5; Heron & Reason, 1997: 1; Somekh, Burman, Delamont, Meyer, Payne and Thorpe 2005: 2). In the preceding chapters of the study, I did declare my critical stance in this study as well as my emancipatory interest, both of which had implications for my research paradigm. A paradigm, in its most common or generic sense, is defined as the basic belief system or worldview that guides action, including action taken in connection with a disciplined inquiry (Guba, 1990: 17; Guba & Lincoln, 1994: 105). Guba and Lincoln (1994: 112) further argue that we cannot dismiss our paradigm assumptions because 'implicitly or explicitly, these positions have important consequences for the practical conduct of inquiry, as well as for the interpretation of findings and policy choices'. They offer four overall research paradigms in qualitative research, namely positivism, post-positivism, critical theory and constructivism. It was therefore important that I declare my research paradigm as it grounds the choice of my actions, interpretation and use of the findings, throughout the study. Mathematics is perceived as one of the most powerful social means for planning, optimising, steering, representing and communicating social affairs created by mankind (Keitel, 2006: 11)

3.2.1.1 Paradigm and Assumptions

In this study, there are certain assumptions that I made from the outset that structure and shape my actions. Consistent with the critical stance that I adopted in the study and as evidenced in the research question, my interest in this research study was of an emancipatory and transformative nature. It was directly aimed at a) the transformation of teachers' own knowledge of mathematics b) the transformation of teachers' own classroom practices into practices that make mathematics accessible and comprehensible to all learners, and c) indirectly aimed at the emancipation of teachers from systems of power relations that inhibit their creativity and their opportunities to learn and improve both their own knowledge of mathematics and their own classroom practice. My contention in this study is that the latter is fundamental in achieving the other two aims. This is in line with Guba and Lincoln's (1994: 113) description of the aim of an inquiry that characterises the critical paradigm:

> The aim of inquiry is the critique and transformation of the social, political, cultural, economic, ethnic, and gender structures that constrain and exploit humankind, by engagement in confrontation, even conflict. The criterion for progress is that over time, restitution and emancipation should occur and persist. Advocacy and activism are key concepts. The inquirer is cast in the role of instigator and facilitator, implying that the inquirer understands a priori what transformations is needed.

This further implies that researchers in the critical paradigm are interested in emancipation, social justice and transformation. As Usher in Le Grange (2002: 37) notes:

> This kind of interest involves the unmasking of ideologies that maintain the status quo by denying individuals and groups access to knowledge or awareness about the material conditions that oppress or restrict them.

This is in line with Foucault's conception of knowledge and power as inseparable as well as his idea that power does not come from above but from below in every social interaction (Foucault 1980). Access to such knowledge can lead to empowerment, liberation and emancipation only if factors that stand between the teachers' ability to control and direct their own behaviour are illuminated and are accordingly confronted.

In addition to social interaction, Moreira (2002: 70) notes that Vygotsky considers the ability to control and direct one's own behaviour, something that is made possible through internalisation, as an essential aspect of development. According to Vygotsky, this can be achieved by encouraging teachers to reflect upon their own thinking in order to enable them to develop their reflective and metacognitive processes.

3.2.1.2 Researcher Assumptions

Kincheloe and McLaren (1994:139–140) identify seven basic assumptions that most critical theorist researchers accept:

First, that all thought is fundamentally mediated by power relations which are historically and culturally situated and constructed. This implies that in this study I hold the view that mental processes and hence learning can only be understood if we understand the factors, in particular power relations that mediate them. This view is corroborated by Langemeyer and Nissen (2005: 188) who argue that it is through such mediators that thoughts and actions can be viewed as creations, appropriations and uses of cultural forms, and as part of the wider social practice. This explains why, in this research study, I also examine power relations as they relate to what teachers 'think' about manipulatives.

Second, that facts or 'truth' can never be isolated from the domain of values or removed from some form of ideological inscription. This view is supported by scholars (e.g. Crotty, 1998: 4–9; Guba, 1990: 19) who advance that knowledge is value laden. This explains why, in this study, I ontologically hold the view that reality is not neutral, that it is subjective and hence value laden. I see teacher knowledge and classroom practices as a function of social, political, historical, cultural and other factors which become reified into the classroom contexts. Kemmis (2007: 9) refers to different faces of unsustainability that are built into some of the practice architectures that shape our lives, enabling and constraining our collective possibilities for praxis – morally committed action oriented and informed by traditions of thought and action.

Third, the relationship between concept and object, and between the signifier and the signified, is never stable or fixed and is often mediated by the social relations of capitalist production and consumption. Liasidou (2012: 175) contends that current schooling acts as a political site, reproducing social hierarchies and legitimizing the existing power relations in a way that is attuned to the corporate modes of production. In this regard Kincheloe (2007) warns that pleasure is a powerful social educator, and the pleasure produced by capital teaches a very conservative political lesson. This implies that such pleasure, if not critically analysed, may lead to complacency and thus lead to reduction of humanity, untypical of political activism. My epistemological position in this study is transactional and subjectivist. I view knowledge as a social act and that the knowledge we hold is a product of our interactions and our relationships to others, which are inherently dynamic. This explains why, in this study, knowledge about teachers' use of manipulatives and their classroom practices was acquired through personal interaction with and participation among teachers and not only through perceptions, literature or curriculum materials.

Fourth, that language is central to the formation of subjectivity. This implies that language is not only a communication tool but also a cultural tool that shapes and conveys our subjectivity. Sinha (2000: 8) argues that ‘language is a condition, a ground and a support for subjectivity …’ This explains why I mainly used dialogical methods to answer the research question proposed in this study.

Fifth, that certain groups in any society are privileged and the oppression that characterises contemporary societies is most forcefully reproduced when subordinates accept their social status as natural, necessary or inevitable (i.e. hegemony). This assumption justifies why conscientisation i.e. awakening of socio-political activism is a key aspect in the research paradigm for challenging the status quo and for ensuring social transformation.

Sixth, that oppression has many faces, focusing on only one form of oppression at the expense of others often eludes the interconnections among them. This explains why in this study, I examined the phenomenon under study from multiple data sources and used the PAR model to collect data and thus allowing participants to become human instruments.

Seventh, that mainstream research practices are often implicated in the reproduction of systems of class, race, and gender oppression. Methodologically, my position in this study is both dialogical and dialectic. In order to explore and exchange meanings, my research methodology involves establishing direct discourse with the participants and also examining and analysing their classroom discourses as they interact with their students and content. In this way, their voices regarding their own experiences, thinking and actions became the primary method of multiple data sources that were used in this study. It is mainly through recording and interpreting these discourses and observations that deeper understanding of contradictions, power relations and other counter forces that inhibit the realisation of teacher creativity and emancipation were unravelled and confronted accordingly.

3.2.2 Research Approach

3.2.2.1 Qualitative Research

Informed by my research paradigm and aims, this study is inherently qualitative. There are as many definitions of qualitative research as there are scholars. In their latest definition of qualitative research, which best fits my paradigm; Denzin and Lincoln (2000: 3) advance that:

> Qualitative research is a situated activity that locates the observer in the world. Qualitative research consists of a set of interpretive, material practices that make the world visible. These practices transform the world. They turn the world into a series of representations, including field notes, conversations, photographs, recordings, and memos of the self. At this level, qualitative research involves an interpretive, naturalistic approach to the world. This means that qualitative researchers study things in their natural settings, attempting to make sense of, or interpret phenomena in terms of the meanings people bring to them.

In this study, I chose an approach that goes beyond simple description and interpretation of the world, that is, beyond what Freire refers to as ‘reading the world and reading the word’ to a critical emancipatory approach, what Freire regards as ‘reading and writing the world’.

This approach is relevant to my study because it a) is an inherent feature of my paradigm, b) allows for in-depth inquiry of phenomena in its social and political context, and c) allowed me not only to answer the ‘How’ and ‘Why’ questions in my main research questions, but also offered me the possibility to transform the material conditions of teachers and their classroom realities.

3.2.2.2 Features of Qualitative Research

There seems to be some convergence among scholars on a) the common features that characterise qualitative research, and b) the features of the Participatory Action Research (PAR) model (described in the section below) adopted in this study, features which best describe my research approach throughout the inquiry process (e.g. Creswell, 2007: 45, 47; Maykut & Morehouse, 1994: 43–47; Merriam, 2002: 4–5).

Natural setting: In qualitative research – a situated activity, researchers often collect data in the field at the site where participants experience the issue under investigation, that is, where human behaviour and events occur. To provide a full and in-depth understanding and interpretation of a phenomenon, the qualitative researcher needs to take account of the context in which such phenomenon is produced. Maykut and Morehouse (1994: 45) argue that personal meaning is tied to context and that the natural setting is the place where the researchers are most likely to discover, or uncover, what is to be known about the phenomenon of interest. Qualitative researchers seek questions that stress how social experience is created and given meaning (Denzin & Lincoln 2000: 10) from the perspective of the participant. In this study knowledge about mathematics teaching was gathered through observing real mathematics classroom settings. Mathematics teaching and relevant data such as time allocation, teaching strategies, classroom size, learner responses, questioning methods, etc. were observed and recorded directly from the real context. Such rich context bound information could not have been generated in a laboratory setting often associated with quantitative research.

Researcher as key instrument – In qualitative research, the researcher in person uses all his/her skills, experience, background, and knowledge as well as biases as the primary, if not the exclusive, source of all data collection and analysis (Maykut & Morehouse, 1994: 26). Understanding or making meaning is the goal of qualitative research and human instrument seems an appropriate means of data collection and analysis. The human-as-instrument researcher postures as an instrument for collection, and analysis enables the human investigator to gain deeper understanding of human experiences and situations in more responsive ways. Lincoln and Guba (1985: 193) justify this posture by arguing that a person, that is a human-as-instrument, is the only instrument that is flexible enough to capture the complexity, subtlety, and constantly changing situation that is the human experience. The collection of data for this study was personally done by co researchers as human-instruments. Data from interviews and classroom observations were collected by me and the members of the PAR advisory team (explained below). In-depth interviews through the Free Attitude Interview technique and observation methods allowed us to have personal contact with and attachment to the participants and the school community at large. This position helped us to establish the much needed rapport between the researchers and the participants. In addition, this helped us to ask clarifying and follow up questions in order to understand in-depth, the meanings that the participants make of the use of manipulatives. Most importantly, our presence in person helped to address other issues not necessarily related to the study. For example, in two of the schools visited, the team was able to intervene on matters relating to the high number of learners experiencing barriers to learning that was reported by the respective principals. All of the above would not have been possible with the use of qualitative methods only such as questionnaires, telephonic interviews, etc. that detach the researcher from the participants and their contexts.

Multiple methods – Qualitative research uses multiple methods of data collection such as interviews, observations, documents analysis, etc. The choice of multiple sources of data helps to provide a rich and holistic understanding of the phenomenon under study. As Baxter and Jack (2008: 554) note, each data source is a piece of the 'puzzle', with each piece contributing to the researcher's understanding of the whole phenomenon. This requires what Geertz in Denzin and Lincoln (2000: 17) calls 'thick description' of particular event, rituals, and customs. In this study, multiple data sources in the form of in-depth interviews, focus group discussions, classroom observations and documents analysis were used. This study sought to examine and make meaning of the use of manipulatives from the perspective of teachers. Multiple data sources helped to provide a thick description of the experiences of participants in respect of the use of manipulatives, essential in a qualitative study.

Reflexivity – Researchers 'position themselves' in a qualitative research study, they convey their background, how it informs their interpretation of information in a study and what they have to gain from the study. Reflexivity is a process of critical reflection both on the kind of knowledge produced from research and how that knowledge is generated (Guillemin & Gillam, 2004: 273). Locating reflexivity in research, Mason in Guillemin and Gillam (2004: 274) posits that reflexive research means that the researcher should constantly take stock of his actions and role in the research process and subject these to the same critical scrutiny as the rest of the ‘data’. In this study, the focus group discussions were used to take stock of our actions and to align them with the objectives of the study as well as its paradigm. Every discussion group session started with such reflections so as to ensure relevancy of our actions and coherence of the study. Through reflexivity, we were able to infuse the PAR research design into the case study mode, thus maintaining the necessary coherence throughout the study.

3.3 RESEARCH DESIGN

Research design, as defined by Yin (1994: 18) is the logic that links the data to be collected (and the conclusions to be drawn) to the initial question of the study. In relation to the research question and my critical stance, which are the main drivers of my inquiry, I chose a PAR model for my qualitative inquiry in order to understand the use of manipulatives in the teaching of primary school mathematics, and how they reshape teachers’ knowledge of mathematics and teachers’ classroom practices.

The sections that follow provide some background to and justification for the choice of a Participatory Action Research model that guided the process of my research design, ethical considerations and the description of my data collection processes.

3.3.1 PAR: Identification and description

3.3.1.1 PAR: Definition and justification

In this study, I chose the PAR model to collect data in respect of teachers’ own knowledge, experiences and thinking about their mathematical knowledge, classroom practice and the use of manipulatives. PAR is a social process followed in settings such as those of community and education. It helps people individually and collectively, to understand how they are formed and reformed as individuals and in relation to one another in a variety of settings to improve practice - teaching and learning (Kemmis & McTaggart, 2005: 280). It is often associated with commitment to social, political, and economic development responsive to the needs and opinions of ordinary people (Kemmis & McTaggart, 2005: 273). Teachers in general, at least in South Africa, are regarded as marginalised 'others' whose voices are not always heard on matters that affect them. This makes PAR an appropriate method in this study. I chose PAR first and foremost because of my critical emancipatory stance in the study and its transformative nature. As Freeman and Vasconcelos (2010: 8) argue:

> We conceive of critical theory as a participatory approach that engages constituents or stakeholders in a reflective and critical reassessment of the relationship between overarching social, economic, or political systems and everyday practices.

The explicit aim of PAR is to foster and promote empowerment (Maguire, 1987; Tandon, 1988; Fals-Borda & Rahman, 1991; Park, 1993 etc. all in Dickson & Green, 2001: 244). There is a clear convergence between my critical stance and PAR. The second reason why I chose PAR is because of its overall purposes, which include three types of potential change: the development or expansion of critical consciousness of co-researchers; improvement in the lives of those involved, as they define change or improvement; and transformation of fundamental societal structures and relationships (Maguire, 1987 in Brydon-Miller & Maguire, 2009: 82). Thirdly, PAR suits this kind of research study because my research questions that can only be adequately answered and understood by collaboratively working with teachers as co-researchers, possessing shared ownership of the research project and action orientation.

Power is at the heart of critical emancipatory studies. The different kinds of action research, of which PAR is one, differ in respect of their interest and purpose, as well as in their location of power**.** The distinction of three kinds of action research below, based on Habermas's theory of knowledge constitutive interests provides a better understanding of the concept of PAR (Kemmis & McTaggart, 2005: 297, Kemmis & McTaggart, 1986 in Kemmis, 2007:7). These are a) technical action research guided by an interest in improving control over outcomes. In this action research tradition, characteristic of positivistic social sciences, power and control are centralised in the researcher. The researcher determines the research problem, methodologies to be used and the interpretation of data to the total exclusion of the subjects. b) Practical action research guided by an interest in educating practitioners so that they can act more wisely and prudently, is typical of hermeneutic and interpretive social science. The voice of the 'other' is also taken into account, but power is still largely with the researcher and c) critical action research guided by an interest in emancipating people and groups from irrationality, injustice and harm or suffering. In this case, the role of the 'other' is further amplified and participants are viewed as co- researchers, acting collectively as 'we' or 'us' in the decision making processes of the entire research. This research study falls in the latter facet of action research, where the aim of the study goes beyond understanding and transforming teacher knowledge and classroom practices, to also transforming what Kemmis (2007: 9) refers to as social formations, i.e. the discourses (sayings), the doings and the relations in which the practices are grounded. In this way, power is distributed among the co participants who collectively participate in the entire research process. This is an affirmation of the notion that ordinary people can understand and change their lives through research, education and action, using PAR to openly challenge existing structures of power and creating opportunities for the development of innovative and effective solutions to the problems facing our schools and communities (Brydon-Miller & Maguire, 2009: 81).

PAR is defined as a fourth generation of action research that emerged in the connection between critical emancipatory action research and participatory action research that had developed in the context of social movements in the developing world (Kemmis & McTaggart, 2005: 272; Torres, 1992 in Somekh *et al*. 2005: 89). This implies that advocacy and activism as key concepts (Guba & Lincoln, 1994: 113) require a bottom-up approach in which teachers themselves engage in activist/emancipatory work as practitioners in their own classroom practices.

My research study is both educative and emancipatory in nature, as it seeks to collectively engage teachers in examining and addressing those factors that constrain them from transforming their mathematical knowledge and classroom practices. This position is further corroborated by Somekh *et al.* (2005: 8) who note that researchers who acknowledge the educative nature of carrying out research are likely to adopt more participatory methods and may place less emphasis on seeking objective data. They will also focus more on providing feedback of preliminary findings to enable practitioners to learn from research knowledge as it is generated. Constructing research as ‘educative’ has ethical implications and effects in terms of the quality of outcomes, for example through its ability to fine-tune findings to the field of study and increase their impact on practice perhaps with less emphasis on generalisations.

3.3.1.2 PAR: Structures

Participatory action research model consists of a spiral of self-reflection cycles of planning, acting and observing, and reflecting which are overlapping, fluid, open and responsive. The criterion of success in PAR is not whether participants have followed the steps faithfully but rather whether they have a strong and authentic sense of development and evolution in their practices, their understandings of their practices, and the situations in which they practice. Each of the steps outlined in the spiral of self-reflection is best undertaken collaboratively by co-participants in the participatory action research (Kemmis & McTaggart, 2005: 277).

To operationalise the PAR project, it was necessary to establish research structures consisting of a:

a) PAR Advisory team of 7 members, which consisted of the provincial mathematics Chief Education Specialist (CES), the provincial mathematics Deputy Chief Education Specialist (DCES), 2 district mathematics Subject Advisors (SAs), 2 mathematics Lead Teachers (LTs) and myself. The brief of the team was to steer the initial development of the PAR project.

b) PAR focus group of 41 intermediate phase teachers who volunteered to participate in the study as co-researchers. The group consisted of teacher participants as co-researchers from 15 schools in Mangaung. The group endorsed and provided input on proposals such as consent forms, the project plan, the interview and observation guides, etc., from the advisory team. The group also provided most of the data for the study and collaborated in the analysis of data and the refinement and presentation of the research findings through their reflections and comments.

c) PAR core participants consisted of 4 participants from the PAR focus group. These co-researchers volunteered to be interviewed individually and to be class visited for observation.

3.3.2 Ethical Considerations

Ethical dilemmas and concerns are part of everyday practice when doing research. Guillemin and Gillam (2004: 261) describe these occurrences in qualitative research (and in PAR) as ‘ethically important moments’. The ability to discern such moments can be considered the most basic element of ethical decision making (Haverkamp, 2005: 148). In this section I describe the ethical concerns and decisions that I made during the course of this research study.

In line with the qualitative and participatory nature of the study, I was the human instrument for data collection.

3.3.2.1 Permission to access the schools

Extensive data were collected from four schools that were identified as investigation sites for this research study. The external researcher may become the broker or mediator between local communities and institutions of larger society (Dickson & Green, 2001: 244). In this regard, I was responsible for organising access to investigation sites. The first ethical consideration that I made was to respect the autonomy of the respective schools involved, the authority of the accounting officer of the department as well as the authority of the principals of the respective schools. **This was also coupled with serious consideration of my position as a senior manager in the Department of Education in the Free State, a position which inherently raised po**wer issues. I familiarised myself with the policy of the FSDoE regarding the conduct of research in the schools, as the first step in the research plan. Based on the policy requirements, I wrote a letter to the Superintendent General of the Free State Department of Education requesting permission to gain access to the schools that have been identified as investigation sites for the study (Appendix 1). The PAR advisory team subsequently visited the schools where we met with the principals and the School Governing Bodies (SGBs) of each of the four schools. During our meetings with the relevant persons at the respective schools, we shared the purpose of my study and all the processes that relate to the study. The reason behind this was to ensure that the school community understands and embraces the purpose of the study for joint ownership of the research project. We also clarified our role in the study *vis-à-vis* our position as senior officials in the department. I subsequently wrote a letter to the principals to formally request permission to conduct the study in their schools. The purpose of the study and the declaration not to disrupt the teaching programme of the school were explicitly outlined in the letter (Appendix 2).

3.3.2.2 Informed consent and voluntary participation

The study involved four mathematics teachers whom we interacted with and collected data from through multiple methods. As noted by Guillemin and Gillam (2004: 271) research involving humans has inherent ethical tensions. Through my research skills and expertise I was able to bring this matter to the attention of the PAR advisory team and we subsequently drafted a consent letter for participants. The need for ethical considerations and the draft consent letter were shared with the PAR focus group for their input. For this reason, we considered the interaction with all participants as ‘delicate situations’ (Kellehear in Guillemin & Gillam, 2004: 271). This is more so because a) the study is about mathematics teaching, a subject in which most teachers are not confident, b) classroom observation is the main data collection method – a contentious issue in some schools where teachers are not positive about being observed by others, and c) the positions of the members of the advisory team within the department of education – which might raise power issues. Due to the sensitivity of the study, respecting the participants’ autonomy to make their own choices, as well as their integrity as human beings (not subjects) and as professionals became important ethical principles that guided the study. The most fundamental ethical consideration we collectively made was to seek informed and voluntary consent from the teachers to participate in the study (Appendix 3). In the consent letter, participants were made aware of their right to withdraw their participation in the study at any time, without penalty or prejudice, should an issue arise that makes them uncomfortable.

The PAR advisory team also made a number of follow-up visits to the schools to meet with the four participants before we started with data collection. We shared with the participants, more in detail than initially in the PAR group meetings, the purpose of the study, the processes and methods of data collection, data analysis, etc. The idea was to make sure that the participants consent to what they fully understand, and that they co-own the project. Our subsequent interactions with the schools were more of a collaborative and collegial nature, where we started to act as participants and as insiders in the activities of the schools more than as researchers. This assisted us not only in clarifying our role in the project as co-participants but also to concretely demonstrate our participating and transformative role, a situation that put the participants more at ease during the course of the study/throughout the study.

3.3.2.3 Confidentiality of data, anonymity, privacy and safety of participants

The participants were reassured that data collected during the inquiry would remain confidential. However, the participants were made aware of their right to the data in the form of copies of verbatim transcripts as well as to the research report. As a result, all notes and audio cassettes used during the discussions, interviews and observations were locked up and stored safely with only the primary researcher having access.

Participants' information shared during the discussions, interviews and observations was kept private and the research results were presented in an anonymous manner. Both the school and the participants were informed that in the case where the school, the participants or learners were quoted, pseudonyms would be used and that any identifying details that may compromise the school or the participants' confidentiality and privacy would be totally removed from the research report.

3.3.3 Procedure for the Selection of Sites and Participants

3.3.3.1 Sites selection

In this study, we chose the use of manipulatives in mathematics teaching by four primary school teachers who work in schools with mathematics laboratories as our unit of analysis. Since the intention of the study was to understand the meaning of a phenomenon from the participants' perspective, it was important for us to select sites and participants from whom the most could be learned. Patton (2003: 3) advises that it is important to select cases that are 'information-rich' and illuminative in order to gather appropriate data. These are cases from which one can gather a great deal of data on issues of interest needed for an in–depth research study. We decided on the criteria for the selection of both the sites and the participants. For the sites, we collectively decided to select schools that met the following broad criteria which will be elaborated on in the following section: a) schools within the Mangaung Metropolitan Municipality, b) schools with mathematics laboratories, and c) schools that participated in the Instructional Leadership through Lesson Study (ILLS) programme of the Department of Education.

In selecting the sites for my investigation, we made a thorough consideration of the limited resources at our disposal, easy and regular access to the sites and the richness of data that we needed for the in-depth study. Based on these considerations, we purposefully chose four schools within the Mangaung Metropolitan Municipality. Firstly, because these schools are within a radius of 15 km from the head office where the PAR advisory team is located, making it easier to regularly interact with the participants and to collect data from the sites. Secondly, the schools in the area are divided into two clusters, that is, cluster 1 and 2, with schools in each cluster located within a radius of not more than 800 m from each other. In addition, all the schools are located within a radius of not more than 7 km from the most central school that we identified as the PAR focus group meeting site. This allowed easy and regular interaction with all participants on one day, mostly during the afternoons between 14:00 and 17:00. Thirdly, through the members of the PAR advisory team's personal engagement with schools in the continuing professional development programmes in the district, and because of their proximity to the head office, we had already established a sound rapport with these schools. This facilitated easy entry to the schools, more so because our data collection was conducted during the national industrial action by the South African Democratic Teacher Union (SADTU) which declared among others, total disengagement with the Department of Education officials.

This research was conducted in four primary schools with mathematics laboratories in the Motheo Education District, which is located in the Mangaung Metropolitan Municipality of the Free State province in South Africa. Our choice of primary schools that participated as sites for investigation was made from the list of schools that have mathematics laboratories. These schools are among the schools that benefitted from the mathematics laboratory project by the Free State Department of Education in the district. Each of these schools has a mathematics laboratory. The project was aimed at, inter alia, harnessing manipulatives to support and strengthen the teaching and learning of mathematics in general and to strengthen conceptual understanding of mathematics of both teachers and learners in particular. Mathematics laboratories and their description (Appendix 4 & 5) are special classrooms that have been converted to comfortably host a minimum of at least 40 learners (50 – 60 m^2). These classrooms are furnished with special equipment, e.g. tables of different geometric shapes, chess board printed carpets and other special mathematical ambiences. Each classroom has 10 fixed tables that can each accommodate a group of four learners and is equipped with 10 computers, a data projector, a pull-down projector screen and a white board. Manipulatives are the defining feature of these laboratories as these mathematics laboratories are richly equipped with concrete manipulatives. These manipulatives cover content areas of measurement, number and number operations, space, shape and patterns such as interlocking cubes, multi base blocks, fraction charts, tangrams, geometric shapes, etc. At the time of this study, mathematics laboratories had already been established at the schools for one full year hence availability of manipulatives and a level of familiarity with those manipulatives by teachers at least, were assumed.

My choice was also based on the list of schools that participated in the Instructional Leadership through Lesson Study (ILLS) programme of the FSDoE (short list). The programme was run by the University of the Free State (UFS) during the 2012 and 2013 academic years, and consisted of modules on Instructional Leadership, Lesson Study and Mathematics Content enrichment. The advantage that these schools have over other schools is that teachers in these schools have been engaged in Professional Learning Communities (PLCs) through collaborative reflections, discourses, analysis, dialogue and improvement of their own professional growth.

3.3.3.2 Participants selection

The overall purpose of this study was to investigate the use of manipulatives in the teaching of mathematics in Grade 6, which is the last level of the Intermediate Phase (i.e. Grades 4–6) and the preparatory grades for the Senior Phase (i.e. Grades 7–9). In particular, the study sought to understand the experiences with the use of manipulatives of four mathematics teachers from the selected schools. I took advantage of two cluster meetings of mathematics teachers that were organised by Intermediate Phase subject advisors during the second week of May 2013. Through these cluster meetings in the area of Mangaung, I was able to share the objectives of my study with teachers and also to request volunteers to participate in the study and to sign up for participation. On the basis of the responses that I received, we compiled the first list of 41 teachers from 19 schools who volunteered to participate in the study. These were Intermediate Phase teachers spread thus; 28 teachers from 9 schools in cluster 1 and 13 teachers from 10 schools in cluster 2. The purpose of the exercise was to identify teachers who would form the PAR focus group for my study.

From the attendance register and in consultation with subject advisors, principals and the UFS manager of the ILLS programme, we were able to further identify those who teach mathematics in Grade 6, those who participated in the ILLS programme and/or those with PLCs that were functional. Based on these criteria, we compiled the second list of 6 teachers (2 teachers from 2 schools in cluster 1 and 2 teachers from 2 schools in cluster 2) who met all the criteria. However, due to the limited time frame of the study and the travelling implications for the participants, we considered an additional criterion on the final list of our primary participants, i.e. those whose classrooms would be observed, in order to remain with four teachers. These were teachers who had taught mathematics in the Intermediate Phase continuously for the past 5 years as we wanted to recruit teachers with reasonable competence and confidence in their teaching of the subject. We compiled the final list of 4 participants as the primary participants from 4 schools who met the last criterion. The template for the profile of the participants, affected classes and schools was generated and populated during the first visit (Appendix 6**)**.

3.3.3.3 Schools' profiles

Of the four schools selected, three are headed by female principals. Typical of many of the township schools, the majority of the learners in the four schools are from disadvantaged backgrounds and the literacy levels of their parents is low. Schools A and B are located in relatively new settlements whose formalisation is still in progress, despite starting in the early 2000s. Both schools were built after 1994 and have a unique structure, that is, different from traditional township schools that were built earlier. Families in these areas are mostly from rural and/or farm areas outside of the Mangaung Township. The schools present Grade R to Grade 7 with only one school extending to Grade 8. In all the schools, the predominant Home Language is also the Language of Teaching and Learning (LoLT) in the Foundation Phase. The LoLT in all the other grades is English. The profiles of the schools, all given pseudonyms, are given below.

(a) School A

The school is situated in a semi urban area and has a population of about 1 315 learners, 39 teachers and 8 members of the School Management Team (SMT) including the principal. The school was occupied in the year 2000 and presents Grade R to 7. The predominant home language is Sesotho, which is the LoLT in the Foundation Phase i.e. Grades R – 3. The LoLT for the other grades, i.e. Grades 4 – 7, is English. School A is a partial Section 21 school, a no-fee paying school and is categorised as a Quintile[1] 3 school. The school benefits from the National School

[1] Quintile ranking of South African schools is determined nationally in order to allocate resources to schools. The ranking from 1-5 is based on the poverty level of the community in which the school is located. A Quintile 1 school is the poorest school whilst a quintile 5 school is the least poor.

Nutrition Programme (NSNP), a programme which makes provision for a daily meal for all learners. Parental support is rated at 70% and the school survives on donations in various forms such as school uniform, vegetables, etc. The school boasts big sporting grounds and sufficient space for learners to play. Classrooms in the school are print rich, especially in the lower grades. Teachers at the school are described as diligent, dedicated and always willing to go an extra mile. The school's mathematics performance in the 2012 ANA tests for Grade 6 was at an average pass rate of 38.14%, an average which is above the national and the province averages respectively. However, only 19.59% of the learners in Grade 6 achieved 50% or higher in mathematics.

(b) School B

School B is situated in a relatively newly formalised settlement area established around the year 2000. The school has a population of about 1 437 learners, 50 teachers and 8 members of the School Management Team (SMT) including the principal. The school presents Grade R to 7 and caters mainly for learners from disadvantaged communities. The school's LoLT is Sesotho for the Foundation Phase, i.e. Grades R – 3, and English for the remaining grades, i.e. Grades 4 – 7. The school is a Section 21 school, a no-fee paying school and is categorised as a Quintile 2 school. The school also benefits from the NSNP. Parental level of support is minimal and can be ranked at 40%.The school boasts big sporting grounds and sufficient space for learners to play. Classrooms in the school are print rich, especially in the lower grades. The school's mathematics performance in the 2012 ANA tests for Grade 6 was at an average pass rate of 31.04%, an average which is above the national and the provincial averages respectively. However, only 6.21% performed at 50% and more.

(c) School C

The School is situated in a semi urban area and has a population of about 1 130 learners, 34 teachers and 7 members of the School Management Team (SMT) including the principal. School C starts from Grade R to 7 and caters mainly for learners from disadvantaged communities. The school's mathematics performance in the 2012 ANA tests for Grade 6 was at an average pass rate of 26.54%, an average which is below that of national and the province respectively. Only 0.88% of learners performed at 50% and more.

The school boasts a beautiful garden which also gives it a distinct feature and adds to an environment conducive to learning. The school is overcrowded with an average number of 45 learners per class. The FSDoE has provided the school with mobile classrooms to address the overcrowded conditions. Although the school was predominantly Setswana speaking, in recent years the demographics of the school have shown gradual changes to almost 50% Setswana and 50% Sesotho speaking learners. However, the school's LoLT is Sesotho for the Foundation Phase, i.e. Grades R – 3, which is different from the home language of Sesotho speaking learners. The LoLT for the Intermediate Phase, i.e. Grades 4 – 6, is English.

School C is a partial Section 21 school, a fee paying school and is categorised as a Quintile 4 school. The school does not benefit from the NSNP. Notwithstanding, the majority of the learners are from low income families who mostly live on a government social grant. The literacy level of the parents are moderate and the parents are described as highly cooperative and showing a high level of interest in their children's education.

(d) School D

School D is situated in a semi urban area and has a population of about 850 learners, 29 teachers and 5 members of the School Management Team (SMT) including the principal. There is little overcrowding in the classrooms except for Grades 5 and 6 with an average of 55 learners. The school offers Grade R to 8 and caters mainly for learners from disadvantaged communities. Notwithstanding, the school does not benefit from the NSNP because of its Quintile 4 ranking, and categorisation as a non-Section 21 fee paying school (R100 per learner/ year). The school does not have sports facilities.

About 50% of the learners stay in informal settlement areas that are far from the school and they have to walk long distances to attend school. The parents have a low literacy level and their level of participation in school activities is at about 50%. The school's home language, as well as the LoLT for the Foundation Phase, i.e. Grades R – 3 is Setswana, and English for all the other grades, i.e. Grades 4 – 8. Teachers at the school are generally very hard-working, dedicated and highly motivated. The school's mathematics performance in the 2012 ANA tests for grade 6 was at an average pass rate of 37.08%, an average which is above that of national and the province respectively. However, only 10.71% of learners performed at 50% and more.

3.3.3.4 Participants' profiles

The profiles of participants, all referred to using pseudonyms, are presented below:

Mr Makau is a mathematics teacher at school A. He is in his late 40s and has been in the teaching profession for 21 years. Mr Makau holds a Senior Primary Teachers Diploma in which he specialised in Mathematics and Natural Science. For the past 18 years he has taught Mathematics to different grades in the Intermediate Phase, namely Grade 6 to 8. He has also taught Natural Science to Grades 4–7 for 8 years. His Grade 6 class consists of 37 mixed ability learners mostly from disadvantaged backgrounds.

Ms Dikgomo is a mathematics teacher at school B. She is in her mid-40s and has been a teacher for 13 years. Ms Dikgomo holds a Junior Primary Teachers Diploma and an Honours (postgraduate) degree in Education. During her 13 years of experience as a teacher, she taught Mathematics to different grades in the Intermediate Phase for 7 years. She has also taught Natural Science to Grade 6. Her Grade 6 mathematics class consists of 43 mixed ability learners mostly from disadvantaged backgrounds.

Mr Kopung is a mathematics teacher at school C. He is in his late 40s and has been a teacher for 24 years. Mr Kopung holds a Primary Teachers Diploma in which he specialised in Mathematics and Natural Science. For the past 24 years that he taught Mathematics in different grades in the Intermediate Phase, he taught Grade 6 Mathematics for 24 years. He has also taught Natural Science to Grade 6 for six years at the school. His Grade 6 class consists of 39 mixed ability learners mostly from disadvantaged backgrounds.

Ms Bohata is a mathematics teacher at school D. She is in her late 40s and has been a teacher for 26 years. She holds a Primary Teachers Diploma in which she specialised in Mathematics and Natural Science. For the past 26 years that she has been teaching, she only taught mathematics to different grades in both the Intermediate and the Senior Phase respectively. She has taught Grade 6 Mathematics for 4 years. In addition to Mathematics, she is currently teaching Life Skills to Grade 4. Her Grade 6 class consists of 57 mixed ability learners mostly from a disadvantaged background.

3.3.4 Data Collection Processes, Methods and Instrument Design and Techniques

Data collection is defined as the process of gathering information to answer the research question. The data collection strategy is determined by the research question of the study and by determining which source(s) of data will yield the best information with which to answer the question (Merriam, 2002: 12).

This section sets about to describe the self-reflection cycles of planning, acting and observing, and reflecting as PAR data collection processes, as well as the methods and instruments that were used in generating such data.

3.3.4.1 Description of the data collection processes

Data for this study were collected over a period of 12 weeks, between April 2013 and June 2013. The data generation process in PAR consists of a spiral of self-reflection cycles of planning, acting and observing, and reflecting. These self-reflection cycles are overlapping, fluid, open and responsive to the dynamics of the study. In order to answer the research question, we decided to collect data *firstly* on what teachers say and think about their own knowledge, experience and understanding of (i) mathematics knowledge, (ii) mathematics teaching - classroom practice, and (iii) using manipulatives in the teaching of mathematics. *Secondly,* on what teachers do in their classroom practice as they (i) use their knowledge of mathematics, and (ii) use manipulatives in the teaching of mathematics. *Thirdly*, on how teachers relate and interact with learners, content and contexts in the teaching of mathematics while using manipulatives. What matters in this research, as Ball (1988: 45) reminds us, are the qualitative dimensions of teachers' knowledge, thinking and experiences – what they know and how they think about it. This is based on the premise that a teacher's knowledge of mathematics is a key determinant of the teacher's capacity to use manipulatives and to promote conceptual learning among learners. As Ma (2010: 26) argues, the way in which manipulatives would be used depended on the mathematical understanding of the teacher using them. In addition, to answer the broad research question in which the teachers' MKT is one of the key constructs, it became imperative for us to first and foremost, explore the four teachers' understandings and ways of thinking about mathematics and the teaching using manipulatives. The purpose was neither to judge, nor to compare and measure their mathematical knowledge. The purpose was rather to enable us to describe and have a general impression about how these teachers teach and think about particular topics. Data collected in this step helped us to understand the mathematical knowledge of the four teachers from their own perspective. This also helped us in identifying particular trends in respect of their own understanding of these topics and in understanding how they present these topics to learners.

(a) Initial Planning Cycle

The planning cycle started with the brainstorming discussion that was held between the members of the PAR advisory team. In this meeting, we reflected on the recent Intermediate Phase intervention programmes that were aimed at the improvement of mathematics teaching and learning in the district. In particular, we analysed and reflected on the error analysis workshops that took place during the 1st term of the 2013 academic year. For example, at the error analysis workshops teachers were requested to analyse errors by firstly, identifying the error, secondly, finding possible reasons for the error, and thirdly, developing intervention strategies to correct the errors and misconceptions. We particularly observed common patterns in how teachers identify possible reasons for the errors or misconceptions. Responses were mostly generic, at surface level and mainly in the form of 'learners cannot do ...' and/or 'learners lack the basic knowledge of concepts'. Consequently, the interventions that they developed were inappropriate and mostly failed to address fundamental mathematical concepts and ideas that undergird the concepts and procedures at hand. We became concerned as to whether teachers do have sufficient knowledge of mathematics to really identify such mathematical concepts and ideas.

It is at this point that I shared my study and its particular relevance to teacher knowledge and classroom practices with the team. I also shared my critical stance, the PAR model, selected theory on MKT, what it entails and how it affects teaching and learning, with the team. We ultimately agreed that teachers' knowledge of mathematics, how they hold it and how they use it in their classroom practice are significant factors that determine the quality of teaching and learning. It is at this point that we all agreed and recommended that in order to improve the quality of mathematics teaching and learning, we essentially need to focus on teachers' knowledge in the act of their practice and begin to challenge those factors that constrain the development of teacher knowledge and improved classroom practices. The team endorsed the research proposal and willingly committed to participate in the research project. It is from this brainstorming session that we decided to establish the PAR structures, to share my study, its purpose and benefits with the Intermediate Phase (Grades 4-6) Mathematics teachers at their monthly cluster meetings in the district, and to also request them to participate in the study. A draft operational plan was developed to guide the process. The team also drafted the consent forms as well as the interview and observation guides for consideration by the PAR focus group.

In the first cycle of acting and observing of the data collection process we used PAR focus group discussions, in-depth interviews with PAR core participants and classroom observations as baseline data sources. The first two data sources focused on the following objective of the study: to examine and understand what teachers say and think about their own knowledge, experience and understanding of (i) mathematics knowledge (ii) mathematics teaching - classroom practice, and (iii) using manipulatives in the teaching of mathematics. On the other hand, the classroom observations focused on the study objective: to observe, examine and understand what teachers do in their classroom practice as they (i) use their knowledge of mathematics, and (ii) use manipulatives in the teaching of mathematics. In a way classroom observations were used to consolidate the baseline data on teachers' sayings.

(b) Initial focus group meeting

The first step of the data collection process took the form of cluster meetings of which the purpose was more of an advocacy nature. The meeting took the form of group discussions which were all audio-recorded and field notes were also taken. The group discussions were attended by 41 Intermediate Phase teachers from 19 schools in cluster 1 and Cluster 2. In these cluster group discussions, the advisory team needed no introduction as they have been interacting with mathematics teachers in the cluster on a regular basis. The advisory team shared the reports, the analyses and the recommendations in respect of the recently held error analysis workshops with the team. Teachers were given the opportunity to reflect on the report and its recommendations, particularly teachers' mathematical content knowledge disposition. To generate discussions, I handed out copies of an article entitled "Teachers are clueless' (The Times 6 May 2013). The article was based on a report on the status of teaching in the Foundation Phase of the South African schooling system as described by the National Education Evaluation and Development Unit (NEEDU) that was published on 2 May 2013. I also shared with teachers how scholars, researchers and educators responded to a similar image painted about the profession in which George Bernard Shaw said: 'He who can does. He who cannot, teaches' (Shulman, 1986: 4). Rather than being defensive, scholars such as Shulman began to problematise teacher knowledge and developed the concept of PCK which has helped to reshape of our understanding and practice of teacher preparation and further development.

This was a major breakthrough in the study as this issue escalated to an analysis based on social and political perspectives, calling for teachers to act as social and political agents to change the perceptions created about teaching. At this point, the meeting agreed that in mathematics, this could be achieved mainly through improvement in their own subject knowledge and classroom practices by teachers themselves. I shared with the meeting the intention to conduct a study on the use of manipulatives as a window for investigating how teachers' mathematical knowledge influences or is influenced by classroom practice (Relationship). Other details regarding the study, e.g. purpose, benefits, nature, methodology, PAR structures, etc. were also shared with the teachers. It is in these meetings that the advisory team extended an invitation to teachers to participate voluntarily in the study as co-researchers and members of the PAR focus group. The PAR focus group also made inputs on the criteria for the selection of the four PAR core participants. The advisory team made it clear at the outset that selection for participation is based on teachers' willingness to participate. Four teachers were accordingly identified; two from each of the two clusters.

The four teachers' school principals were accordingly requested permission to allow the teachers to participate in the study. In addition to the letters, the advisory team visited each of the schools to formally present their proposal to the principal and the SMTs of the respective schools. A telephonic conversation was also made with the four teachers after the meeting, to ensure that they understand what the study entails and their roles in the study. The roles of the PAR focus group were outlined and were endorsed by the group. The remaining members of the group were invited to voluntarily sign up for participation as the members of the PAR focus group and were accordingly informed about their roles and responsibilities. Subsequently, all41teachers signed up; 28 teachers from 9 schools in Cluster 1 and 13 teachers from 5 schools in cluster 2. Although all the PAR focus group discussions were held in the afternoon, the principals of all the participating schools were accordingly informed about the study and were provided with the schedule of the PAR focus group meetings. The principals of the four PAR core participants were also informed about the dates of the PAR focus group discussions with the respective teachers.

(c) PAR focus group discussion

The second wave of my data collection process took the form of another PAR focus group discussion. The discussions focused on the following objective of the study: to examine and understand what teachers say and think about their own knowledge, experience and understanding of (i) mathematics knowledge (ii) mathematics teaching - classroom practice, and (iii) using manipulatives in the teaching of mathematics. This was an interpretive phase of the data collection process whose purpose was to make a general sense of, or interpret the phenomena of the use of manipulatives, teacher knowledge and patterns of classroom practice in terms of the meanings teachers themselves bring to them – from teachers' own perspectives. In particular, the discussions were meant to establish baseline data around the common tasks that teachers perform in the course of teaching mathematics using manipulatives. As Ma (2010: xxx) argues, 'knowledge from teachers rather than from conceptual frameworks might be 'closer' to teachers and easier for them to understand'. The discussions were all audio recorded and field notes were also taken. Data were collected through focus group discussions, using one exploratory question designed to probe teachers' feelings, thoughts and knowledge about their use of manipulatives mathematics in the context of common things that they do in the course of teaching. The question was posed in the context of a classroom scenario of using manipulatives (Appendix 7**).** For example, the scenario involved the mathematics of multiplication of two digit whole numbers. Teachers discussed how they would respond to these scenarios if they were to occur in their classrooms and why. In particular, we focused mainly on teachers' own understanding, feelings, experiences and thoughts about the use of manipulatives in teaching mathematics. Out of these discussions, some common themes emerged as aspects or dimensions of teaching, e.g. timetabling, Language of Learning and Teaching (LoLT), teacher motivation, etc. that were incorporated into the list of dimensions later used in the subsequent data collection processes. Data collected from the focus group discussion were used as baseline data that were later used to answer the research questions.

(d) Classroom observations

The third major level of data collection took the form of two classroom observations of each of the four PAR core participants. The initial observations took place before the intervention programme (discussed in the section below) and the second observations took place after the intervention programme. The purpose of the initial observations was to gather baseline data while the second set of classroom observations were conducted to gather data after the intervention programme so as to examine the nature of change, if any, which occurred in both the teacher knowledge and classroom practices. In both cases data were collected by describing the mathematical knowledge that teachers use in the course of teaching a specific topic through the use of manipulatives rather than the general mathematical knowledge.

The PAR focus group had decided on the domains of teachers' MKT and classroom practice that we sought to observe within each topic. These domains were included in the observation guide within the broad dimensions that sought to understand approaches to school mathematics. These dimensions included the lesson design, introduction, lesson aim, activities (including the quality of teacher-student interactions, tasks, questions, examples and representations of that content), concepts and procedures taught, teaching approach, teaching strategies, teaching materials, questions and assessment tasks. Such description can only be provided by gaining deeper understanding of teachers' knowledge of mathematics and how they use such knowledge in the course of teaching. To generate such data, we looked for issues relating to the participants' MKT as they emerged from the use of manipulatives and not judge their teaching competence. This was done by particularly examining how these teachers and their learners interpret and interact with one another and with manipulatives through watching and listening as the social act unfolded in the natural setting - the real classroom situation. Participants selected their own topics that were taught in mathematics laboratories.

(e) Individual interviews

The fourth wave of data collection took the form of one-on-one interviews with each of the PAR core participants. In order to learn and build a deep insight about these teachers' use of manipulatives, their mathematical understanding and how they approach and think about mathematics, it was important for us to have face-to-face interaction with them. Kahn and Cannell in Marshall and Rossman (1995: 80) describe an in-depth interview as 'a conversation with a purpose'. It is for this reason that we decided to use in-depth interviews as my primary source of data despite other available options. Qualitative in-depth interviews go beyond simple surface talks to the depth of the conversation, which involves what Maykut and Morehouse (1994: 80) describe as rich discussion of thoughts and feeling. This method was appropriate not only in yielding qualitatively rich information about each of the four teacher's knowledge of mathematics and their approach to teaching, but also in further building the necessary rapport with the participants. Through the use of interviews we were able to a) give recognition to the voice of these teachers as valuable data source, b) understand and answer my research question from the insider perspective, c) probe further into those mathematical ideas that underlie procedures and concepts in mathematics, and d) understand other contextual factors (beliefs, perceptions, etc.) that impact on teachers' knowledge and practice. This method was different from the quantitative methods that have been used in similar studies on teachers' MKT. For example, Hill and her colleagues in Hill *et al.* (2008a) used pencil-and-paper assessments to measure teachers' MKT while Rowan and his colleagues (Rowan, Camburn and Correnti 2004) used instructional logs as the primary data collection instrument in their Study of Instructional Improvement (SII). Such methods, while useful for other purposes, would have limited the richness of information that I gained from the in-depth interviews that I conducted for the purpose of the present study. Representing the perspectives of these teachers quantitatively would have stripped the teachers' experiences of its meaning and context.

It is against the above backdrop that for each of the two observations two in-depth interviews (pre and post observations) were conducted with each of the four teachers. Arrangements were made with the participants to conduct interviews with them after school hours and the interview lasted from 60 to 90 minutes per teacher, depending on the interviewee. I also made arrangements with the principals to organise one venue for the interview. All the interviews were tape recorded and later transcribed.

The pre-observation interviews were conducted to establish, from the teachers' perspective, how they were going to use manipulatives in teaching particular topics that they had prepared for the classroom observation. The main purpose of the pre-observation interviews was to make a specific sense of, or interpret the phenomena of the use of manipulatives, teacher knowledge and patterns of classroom practice in terms of the meanings teachers themselves bring to them – from each of the participant's own perspectives. Prior to the first observations, the core team visited each of the four schools twice in one week, spending 1– 2 hours per school during the school hours with each of the main participants and even taking part in their classroom activities. The first visits were meant to gather respective participants' profiles and those of their schools and help participants to be comfortable with the observations and research process in general. Analysis of documents such as the CAPS document, learners' written work and textbooks was also done during the first visits as part of the pre-observation data collection. The data collected related to specific mathematics topics that were to be taught during the classroom observations to supplement baseline data. Field notes were compiled during these visits to add to the profiles of the respective schools. During the second visits we conducted pre observation interviews with participants in order to probe further some of the themes that emerged during the PAR focus group discussions and to share the expectations regarding the classroom observations. These pre observation interview sessions helped to further develop interpersonal relationships with the participants. Pre observation interview sessions were vital to the process of observation that was to follow and to the setting of a joint collaborative agenda that is fundamental in a participatory action research.

I also conducted a post observation interview with each participant. These were more of reflective interviews that were conducted with the purpose to review and provide clarification on data collected during the classroom observation. The interviews were tape recorded and transcribed for analysis. Post (observation) interviews were held to probe into the classroom observations and the video recordings and to also allow teachers to check if my interpretation of their responses was correct. These interviews helped me to clarify, confirm and further probe into some moments during the classroom teaching that seemed to be significant yet not explicit enough.

(f) Intervention programme

After the initial classroom observations, an intervention programme was designed to address some of the challenges that emerged during both the classroom observations and the interviews. In particular, the programme addressed challenges relating to content areas of the multiplication of multi digit whole numbers and fractions, and the use of manipulatives in their (content areas) teaching. It needs to be recognised that this study is located in the broader ‘maths for all’ intervention project of the FSDoE aimed at the improvement of the teaching and learning of mathematics. Teacher development is one of the key thrusts of the project. As part of our responsibilities as managers in the FSDoE, we conducted a teacher development workshop for Grade 6 mathematics teachers for the two clusters. Although teachers had previously received some training on the use of manipulatives, the training had been a once off intervention conducted by an outside service provider. This prompted us not to assume that teachers had been adequately trained in the use of manipulatives. To answer the research questions in this study, we also needed to provide teachers with professional development opportunities to strengthen their mathematical content knowledge and to learn how to use manipulatives. This approach is supported by Kelly (2006:188) who contends that teachers need to know when, why and how to use manipulatives effectively, as well as to have opportunities to observe, first hand, the impact of allowing learning through exploration with concrete objects. As theorised in this study, teacher development is one of the strategies that mediate between classroom practice and the use of manipulatives.

The intervention took the form of a one-day workshop of 6 hours that was on a Saturday. The workshop took place in between the two major data collection cycles of the study. The purpose of the workshop was to provide teachers with opportunities to use manipulatives in selected topics, the idea being that if they use manipulatives, they will begin to think deeply about them. The intervention programme was designed in two sessions of three hours each. Each session covered two hours of content knowledge on the multiplication of multi digit whole numbers and fractions respectively and one hour on the use of manipulatives in teaching these topics. The purpose was to provide teachers with professional development opportunities that integrate manipulatives with content in order to strengthen their mathematical content knowledge and to learn how to use manipulatives. A variety of manipulatives were used including Cuisenaire rods. The purpose was to expose teachers to various manipulatives so as to emphasise the importance of choosing the appropriate manipulative.

(g) 2nd classroom observations

The fourth major level of data collection took the form of the second classroom observations of the four teachers' classroom teaching. The purpose of the observation was to gather data with regard to the participants' actual use of manipulatives in the teaching of particular topics of their choice after the intervention. Data were collected through the same observation guide as that used in the first observations. However, the second classroom observations sought to establish how all the dimensions of teaching unfolded after the intervention, focusing on specific topics that were taught as per the work schedule and CAPS.

As with the initial observations, the second classroom observations were coupled with both the pre and post observation interviews with the same purpose as in the initial observations. The observations gave us a cross-sectional view of the teachers' practices and understandings at a given point in time and were not used to generalise the teachers' practices of mathematics teaching as a whole.

3.3.4.2 Data collection instruments and techniques

Data collection in qualitative study research is typically extensive and drawing on multiple sources of information (Creswell, 2007:100). The purpose of data collection in qualitative research is to capture rich data that would facilitate deeper and holistic understanding of the phenomenon being investigated. The most useful ways of gathering these forms of data are participant observation, in-depth interviews, group interviews and the collection of relevant documents (Maykut & Morehouse, 1994: 46).

The following data collection instruments were used for the study; a) interviews, b) observations, and c) documents analysis, all of which will be discussed below.

(a) Interviews

Interviews were conducted at two levels a) Individual interviews with the PAR core participants, and b) Group interviews with the PAR focus group. The purpose of in-depth interviewing as advanced by Seidman in Mwingi (1999: 50) is neither to get answers to questions, nor to test hypotheses, and certainly not to evaluate. The purpose is to understand the experiences of other people and the meaning they make of such experiences, in other words to know what is in their hearts. This justifies the technique I used as described below.

At both levels, I used the Free Attitude Interview (FAI) technique as proposed by Ineke Buskens (1996: 1), which is also defined as a non-directive controlled depth interview. The technique involves asking one exploratory question to initiate discussions in a social conversation. For this reason, one exploratory question (Appendix 8) which was "How do you use manipulatives to teach mathematics in your classroom and why?" was asked. The exploratory question was informed by my research question. To contextualise the exploratory question, participants were presented with a classroom scenario in which a Grade 6 mathematics teacher successfully used multi base blocks for area representation to teach multiplication of multi digit whole numbers. However, the teacher had difficulty to explain and justify the rule of multiplication of decimal fractions**.** The technique also involves asking clarifying questions such as ''Could you explain a little bit more'; 'What do you mean when you say...?' etc., based on the information provided by the interviewee. Buskens (1996: 5) argues that a 'real' clarifying question will refer to an internal framework. Such questions allow for rich data as they encourage participants to elaborate more on their responses and make meaning of their responses. The technique also involves the use of a reflective summary (e.g. 'Did I understand you to mean .....?', 'you have the feeling that ...?') by the interviewer in his or her own words. The reflective summary was used to help participants to structure their information and also, as posited by Bohloko and Mahlomaholo (2008:374), not to waste time on unimportant aspects, but to focus on the essential issues.

The choice of FAI as a technique was appropriate for the study for the following reasons: a) it allowed participants to talk freely about their own knowledge, thinking and experiences about the use of manipulatives, b) it provided rich data from participants' responses to clarifying questions as well as to the reflective summaries, c) it allowed participants to also ask questions, thereby dismantling the power relations between the interviewer and the interviewee, and d) also helped, through the exploratory question, to keep the discussions within the framework of the research question. In conducting the interviews, I first made sure that atmosphere is relaxed and that participants talk freely about their knowledge, feelings and understanding on how they would use manipulatives to teach particular topics and why. In addition to the provision of rich data through in-depth interviews, the FAI technique was particularly helpful in counteracting any power differentials between the interviewer and the interviewees.

(b) Observations

Observation is defined as a purposeful, systematic and selective way of watching and listening to an interaction or phenomenon as it takes place (Kumar, 2005: 119). Based on the premise that knowledge is a social act and that the knowledge we hold is a product of our interactions and our relationships to others, I decided to focus on the interactions that occur in the act of teaching with manipulatives. For this reason, I used observations as the method of data collection to answer my research question. This method was found to be the best fit for both the purpose and the nature of my study. How these teachers and their learners interpret and interact with one another and with manipulatives could best be understood through watching and listening as the social act unfolded in the natural setting - the real classroom situation. My interest in gathering rich and full information about the live interactions and not only about perception of individuals made observations more relevant than questioning. I also decided to observe the situation more as a participant than as a researcher, through direct contact with the environment and actively participated in the activities of the cluster and those of the selected schools. I visited the schools on several occasions prior to the actual data collection processes. At a more interpersonal level, I engaged with the teachers on general issues regarding their profession and their subjects, and participated in their mathematics activities. This helped me to enhance the rapport that I had already established with the participants at the cluster level, a situation which helped to develop a more comfortable level of interaction. As a participant observer, I gained deeper understanding of the interactions in context and from an insider perspective, and most importantly I was able to reduce the power differential between myself and the participants.

It was also important for me to not only see and hear what I observed but also to record my observations so as to have a full description of the situation. Observation involves the act of systematically noting and recording events and behaviours (expressive of deeper values and beliefs) in the social setting (Marshall & Rossman, 1995: 79). The advantage of being present as a participant in the whole activity was that I was able to take notes on what I saw and heard while I was observing. I did not initially have any predetermined categories or a strict checklist of what I intended to observe. I wanted to have a holistic picture of the interactions and to ensure that as I record my observation, I do not miss any part of the interactions.

Two participant observations were conducted with each participant to examine and describe the MKT that teachers employ in the course of teaching and the using manipulatives. The observations were conducted over a period of four days, observing two teachers' classrooms per day. In both observations classroom activities/observations were video recorded and field notes were written during the observations and then transcribed immediately after the observations. Prior to each observation, I discussed with the teachers the aims and intentions of the research project, outlined expectations of the observations. I also emphasised the fact that the observation is not meant to judge their teaching competence but to help identify those salient features of MKT that we need in order to enhance theory on the use of manipulatives. The teachers were also informed that the lesson would be video recorded and that the video would be discussed at the focus group level. The day after the lesson I wrote a narrative description of the lesson, giving a textual account of what happened during the lesson. These narrative descriptions were written from the field notes, memos and with reference to the video recording.

Focus group discussions were conducted after each observation to reflect on the video recorded data of classroom observations. The purpose of the discussions was to allow members of the focus groups to reflect and analyse the recordings.

(c) Document analysis

Three sets of documents were selected for analysis: a) the CAPS document for the Intermediate Phase mathematics, b) learner written work, and c) textbooks. The choice of documents was guided by the research objectives as well as by literature review on the state of mathematics teaching in South Africa. On the latter, in chapter two I discussed the influence of the apartheid curriculum that was characterised by authoritarianism, over-reliance on textbooks and a static view of the role of teachers as curriculum receivers to whom knowledge is passed down unquestioningly through curriculum materials. It was therefore important to also analyse the ways and extent to which these curriculum materials are used to (dis)empower teachers. All these documents were read and analysed. The analysis of documents was helpful in a number of ways. For example, learners' books illuminated how teachers responded to learner errors and novel solutions, textbooks analysis showed how activities are structured and how mathematical concepts are explained and. the CAPS document reflected the sequencing of topics, key mathematical ideas, etc.

3.3.4.3 Data analysis

This section provides the description of the process of how data collected in this study were analysed. Qualitative data analysis involves organising, accounting for and explaining the data, in short, making sense of data in terms of participants' definitions of the situation, noting patterns, themes, categories and regularities (Cohen, Manion & Morrison, 2011: 537).

(a) Data analysis framework: Critical discourse analysis

Data collected in the study were analysed within the broader framework of Critical Discourse Analysis (CDA). Critical discourse analysis emerged in the late 1980s spearheaded by Fairclough, Wodak, van Dijk, and others (Blommaert & Bulcaen, 2000: 447). Scholars are in agreement that power is a central concept in Critical Discourse Studies, CDS (Rogers, 2004: 3; Wodak & Meyer, 2001: 8; etc.). CDS are generally interested in the social production of inequality, power, ideology, authority, or manipulation (Van Dijk in Blommaert & Bulcaen, 2000: 450). In particular, I used the socio-cognitive approach to discourse analysis as proposed by Van Dijk (1993, 2009) to analyse data collected in this study. According to Van Dijk (2009: 64) the socio-cognitive approach can be characterised; firstly, as the study of mental representations and the process of language users when they produce and comprehend discourse and participate in verbal interaction, as well as in the knowledge, ideologies and other beliefs shared by social groups. Secondly, as an approach that examines the ways in which such cognitive phenomenon are related to the structures of discourse, verbal interaction, communicative events and situations, as well as social structures, such as those of domination and social inequality.

In line with my critical stance, this approach is relevant in that it a) recognises reality as subjective and dependent on the individual’s cognition in the construction of meaning, and b) seeks to examine and expose how mental processes mediate in the (re)production and comprehension of unjust and abusive discourse structures, and in the resultant social structures of domination and inequality. This approach is also relevant to the focus of my study, namely the interactions (verbal and non-verbal) that occur in the act of teaching with manipulatives, which involves teachers, students and content. These interactions, which involve mental processes such as decision-making, meaning making, knowledge (re)creation, etc. can best be analysed and understood from this cognitive dimension. Education is seen as a major area for the reproduction of social relations, including representation and identity formation, but also for possibilities of change (Blommaert & Bulcaen, 2000: 451).

(b) Data analysis processes

At the first level of my data analysis, I generated transcripts of all video and audio recorded data from classroom observations and interviews respectively for all participants. According to Cohen et al. (2000: 537) the advantage of transcriptions is that they can provide important detail and an accurate verbatim record of the interview. However, the disadvantage is that contextual factors and non-verbal aspects may be overlooked. To counter this, I used the field notes and memos that I took during the observations and interviews, occasionally referring to the video recordings, to provide some insights which helped me to contextualise and provide further meaning and interpretation of the transcripts. The second level involved the development of a framework for representing and analysing my data, involving what Myers in Bondarouk (2004: 66) calls the hermeneutic movement of understanding. This movement is recognised as a metaprinciple (for interpretive studies) upon which the others expand and suggest that researchers should come to understand a complex whole from preconceptions about the meanings of its parts and their relationships. Denzin, on the other hand posits that Hermeneutics is the work of interpretation and understanding and that knowing refers to those embodied, sensuous experiences that create the conditions for understanding.

To pull all data together from all sources, i.e. from the whole, I used the three objectives that couch the aim of my study to organise the data into major categories, i.e. to the parts consisting of a) the use of manipulatives b) teachers' mathematical knowledge for teaching, and c) teachers' mathematical classroom instruction. This method resembles what Bondarouk (2004: 98) refers to as the categorisation of the text units in the transcripts according to the research model. This categorisation of text and talk as embedded in discourses, where the latter are ways of representing, includes what Luke in Rogers (2004: 56) refers to as 'systematic clusters of themes, statements, ideas and ideologies'. At the third level, I identified particular constructs from my literature review, i.e. back to the whole, relating to each category (objective) and used such constructs as subheadings within each category. For example, as illustrated in my framework, under the category of MKT, I had 10 constructs including teaching a topic, connectedness, correctness, explicitness, responding to learners' mistakes, generating a representation of a certain topic, and responding to a novel idea raised by a learner. I first illustrated how each subheading relates to the objective under which it is located. This helped me to appropriately locate each piece of my empirical data in the subheading, to minimise overlapping of data, and to ensure relevance, focus and coherence in my study at all times. To discuss and analyse my empirical data, I started by looking for similarities and differences between what literature says (theory) and empirical data within each construct. This is what Myers in Bondarouk (2004: 66) views as going back to the whole (finalising general relationships and functions in the initial theoretical concept).

The hermeneutic movement of understanding (the study of interpretation of written text) first develops here from the parts (explication of the idea from the transcripts) to the whole (explication of the context about organisational background, participants in the research and their interactions). The parts (categorisation) are again organised from the whole and then back to the whole (raising the text units to the level of the research constructs). Once again, we then return to the parts (characterisation of the linguistic features of the text units and refining every component in the research model) and finally back to the whole (finalising general relationships and functions in the initial theoretical concept).

3.4 THE RESEARCHER'S ROLE

At our first cluster meeting I started by clarifying my role to the participants as that of a co-participant.

3.4.1 A Subjective Inquirer

Drawing from my critical stance in this study, my role was that of a subjective inquirer. In line with the qualitative nature of the study, wherein the researcher is a key instrument (Creswell, 2007: 45; Marshall & Rossman, 1995: 59), I was personally involved in doing field work, as a human instrument for data collection through various sources. I personally interacted with the school communities in general and with participant teachers in particular. This was beneficial in many ways: it enhanced my rapport with the schools; it allowed me to have deeper understanding of the teachers' experiences in context as well as allowing me to approach the inquiry from an insider's perspective. The researcher's role was rather one of an active participant in the conversation instead of being a 'speaking questionnaire' (Potter and Wetherell in Bondarouk, 2004: 96).

3.4.2 A transformative Intellectual

A transformative intellectual plays a major role in helping practitioners link and interpret their own practice to a more critical ideological critique to unearth the sources of oppression in an institutional and societal analysis of power (Giroux, 1985). Giroux in Guba and Lincoln (1994: 115) notes that in the researcher's voice in the critical theory paradigm is that of a 'transformative intellectual'. In line with my critical stance, my role in this study was that of a change agent. My interest in this study extended beyond just understanding how teachers experience the use of manipulatives. My role was to examine teachers' experiences in context (social, cultural, etc.) in order to illuminate and gain insights into those hidden factors that stand against the emancipation of teachers.

3.4.3 A Passionate Participant

The inductive methods of critical theory require the researcher to be what Guba and Lincoln (1994: 112) refer to as a 'passionate participant' as opposed to being a detached observer. First and foremost the study was motivated by a passion for mathematics and my values as embedded in my paradigm. My subjective relationship with mathematics teaching was demonstrated throughout the study. I was not just an observer in the study but also an actor in the process of gathering data about the teaching of mathematics during the study. I was subjective in developing knowledge in this interaction as proposed by Guba and Lincoln (1994). This implies that in the course of doing fieldwork, I also used my experience and knowledge to interpret and make meaning of the phenomenon under study. During the PAR focus group discussions and the classroom observations, I also used my experience to complement the teachers' sayings and doings. As Labaree (2002: 113) rightly pointed out: 'the participant observer cannot be immunised from their respondents, acting like a detached recording instrument that merely synthesises the data and disseminates the findings'. My immersion was mainly motivated by a moral obligation to share my experiences while I also learn from participants. I also passionately believed that the teachers can transform their knowledge for teaching and their classroom practice.

3.5 CRITERIA FOR QUALITY

This section outlines the criteria I used to determine the quality of the study as well as the measures I put in place to ensure quality. Researchers are often required by those who evaluate their research (ethics committees, external and internal reviewers, funders, etc.) to detail the criteria for evaluating the quality of their research, as well as measures that the researcher has employed to ensure that they meet each of those criteria. Conventional researchers, often associated with the positivist tradition, have used the criteria of validity (internal and external) and reliability to justify the success and hence the value of their own research processes, as well as to frame the evaluation standards for evaluating the processes of other researchers. Lewin (2005: 216) defines validity as referring to whether or not the measurement collects the data required to answer the research question and reliability as concerned with the stability or consistency of measurements, that is, whether the same results would be achieved if the test or measure was repeated. Marshall and Rossman (1995: 143) refer to these requirements as canons that stand as criteria against which the trustworthiness of the research project can be evaluated, to which all research must respond. Lincoln and Guba in Marshall and Rossman (1995: 143) regard these canons as questions establishing the 'true value' of the study, its applicability, consistency, and neutrality, to which all research must respond.

Scholars and researchers have always included the question of what, where and how to evaluate and determine the success and worth of a research study in their debates on the qualitative/quality dichotomy (Denzin and Lincoln, 2000; Lincoln& Guba, 1985; Guba, 1990; Guba & Lincoln, 1994; Seale, 2003; Lather, 1991; etc.). More often than not, scholars have reported on negative reactions to qualitative research in mainstream academic community whereby it is perceived to be unscientific, politically motivated and suspicious (Lather, 1991; Denzin & Lincoln, 2000: 8, etc.). These reactions have largely been driven by different theoretical and philosophical dispositions of different researchers resulting in various and often contradicting criteria used to characterise and define success in the research processes, in locating success (process or the product) and in different strategies all in an attempt to meet the requirement. This is in effect a consequence of what is often referred to as a legitimation crisis that involves a serious rethinking of such terms as validity, generalisability, and reliability (Denzin & Lincoln, 2000: 19).

The attempt by qualitative researchers to create some overarching system for specifying quality have resulted in the proliferation of concepts such as trustworthiness, credibility, catalytic validity, etc. to replace positivistic terms of validity and reliability. For example, the work of Lincoln and Guba reflects these more recent shifts. Lincoln and Guba (1985) argue that establishing the trustworthiness of a research report lies at the heart of issues conventionally discussed as validity and reliability so that four questions have, from within the modernist paradigm, been asked of research reports, namely their truth value, applicability, consistency, and neutrality. However, trustworthiness of qualitative research, as noted by Shenton (2004: 63), is often questioned by positivists, perhaps because their concepts of validity and reliability cannot be addressed in the same way as in naturalistic work, characteristic of qualitative research. Advocates of critical social inquiry have located the validity of the inquiry in its capacity to effect change, thus seeking catalytic validity (Lather, 1986) as a central determination of the success or quality of inquiry. On the other hand, McTaggart in Melrose (2001: 165) has rejected the dominant discourse of validity, which hinges on the quest for generalisation and quest for causality (prediction and control of events), as being of any interest to action researchers. McTaggart does, however, regard PAR as valid if it meets the criteria of defensibility, educative value, political efficacy, and moral appropriateness.

Consistent with both my paradigm and approach in this study, I used the purpose of Critical Emancipatory Research (CER) to frame the criteria for quality in this research. The purpose of CER is to empower the powerless to become relevant to their conditions of exclusions and marginalisation, to become useful in terms of transforming their station in life and to foster adherence and advancement of values such as democracy and social justice in the manner that meets the methodological expectations of the community of scientists (Mahlomaholo & Nkoane, 2002: 74)

Freeman and Vasconcelos (2010: 8) argue that enlightenment and emancipatory action are developed by people within a particular socio-historic context for a specific purpose, and so the value and success of the purpose, process, and structure of a critical theory. It is for this reason that I located the quality of this study in the realisation of its transformative and emancipatory agenda. Hence the success of this research was determined by the extent to which this study has been able to advance its social and political goal of equity, social justice, freedom, peace and hope, in mathematics education, all of which give recognition and full human dignity to all, especially the powerless and the marginalised.

3.5.1 Social Justice

The quality of the study was evaluated by the extent to which it achieves social justice in the teaching and learning mathematics. Meulenberg-Buskens (1997: 1) advances that:

> Quality in social science research could refer to the degree to which it yields useful and valuable information, to the degree to which it enhances values such as democracy and social justice, and to the degree to which it empowers powerless people.

The extent to which the study promoted social justice was used as a criterion to determine its quality. Social justice in this study was made operational by including the powerless in matters that affect them, thereby empowering them and educating them to become transformative intellectuals who view knowledge as a social construction and understand teaching as a political activity.

As explained in chapter two, the legacy of the apartheid mathematics curriculum was characterised by segregation, inequalities, authoritarianism and other social ills. This has resulted in an inadequate teacher knowledge base, low confidence levels in teaching the subject, as well as isolation and marginalisation of most black mathematics teachers in SA, not only those who participated in the study, leaving them powerless. This has further resulted in disparities in mathematics performance which also show social class and race divisions.

Participating in the study forced them to collaboratively reflect with other teachers on their own experiences, feeling, thinking and inadequacies in respect of their own knowledge of teaching and classroom practices. Both the PAR and the dialogical method that characterised my data collection processes empowered participants by affording them the platform to talk openly and freely about their situation, making their voice the central data source. Throughout the study, the empowerment of participants was demonstrated in many ways. In my interaction with these teachers I have witnessed a high level of confidence in articulating their own situatedness, and in teaching the subject. During the PAR focus group discussions and the intervention programme they were reflective, innovative and autonomous in their professional judgement on issues of mathematics teaching.

Making teachers more conscious about the necessity of social change, that is, the awakening of socio-political activism in teachers is one of the critical aspects of social justice education. The PAR focus group was established to enable teachers to engage in collective action-reflection, that is, praxis, to progressively co-construct their own mathematics knowledge and practices in order to ultimately transform the world. The latter process involved teachers taking a political stand and action in changing the world towards a socially just order. In an attempt to be closer to the principles of CER, the use of PAR as my data collection approach helped to pursue the values of freedom, democracy and equity below.

3.5.2 Equity

The extent to which the study was successful in promoting equity was used as a criterion to determine its quality. Skovsmose and Borba (2004: 222) have argued that equity is basic to the quality of critical research. Equity was judged by the extent to which all teachers in the study, and by extension all learners, irrespective of their gender, ethnicity, class, socio economic background, and language proficiency access mathematics as well as how power was equitably distributed. Allexsaht-Snider and Hart (2001: 93) posit that equity in mathematics requires a) equitable distribution of resources to schools, students, and teachers, b) equitable quality of instruction, and c) equitable outcomes for students. This implies that teachers' knowledge of mathematics and their classroom practice are both aspects of teaching for equity, concerned with providing learners from diverse backgrounds with equal quality opportunities to learn.

The de-concentration of power in the researcher was the first point of entry in the study. The PAR processes used in this research study were aimed at the democratisation of both knowledge and power, thus promoting equity. As the researcher, I firstly chose the PAR model in order to depower myself and to equitably distribute power among the participants to enhance the person power of teachers. Participants were regarded as equal subjects who were empowered through participation in the PAR focus group to understand and change their own situations. This was demonstrated by a high sense of ownership of the project and level of commitment by teachers. Attendance of PAR focus group discussions and the intervention programmes was high even where short notice was given. The recent workshop for the PAR group was held over a long weekend and yet the attendance was at its peak. The study was also able to diffuse the power of the teachers in the classroom. Mathematics classrooms that were visited in the second round of classroom observations were characterised by high learner activity and dialogue (explanations, reasoning, and arguments) with minimal directing by the teacher. Learners were even free to develop their own mathematics problems, instead of relying only on teacher set problems.

I chose Mathematics Knowledge for Teaching (MKT) as one of the constructs in this study because of its defining feature, making mathematics accessible and comprehensible to learners. Mathematics is intrinsically powerful and by implication, it has the potential to empower those who acquire it. Mathematics is perceived as one of the most powerful social means for planning, optimising, steering, representing and communicating social affairs created by mankind (Keitel, 2006: 11). By developing teachers' knowledge of mathematics teaching I was able to create opportunities for the progressive advancement of democracy, providing equitable access to mathematics by all teachers and learners, irrespective of their gender, race, ethnicity and class. As Malloy (2002: 17) argues, the crux of democratic access to mathematics is our understanding and researching new ways to think about mathematics teaching and learning that has a moral commitment to the common good, as well as to individual needs. Data collected from classroom observations and group discussions showed that teacher knowledge of mathematics and how to teach have improved. This improvement has also translated in the improvement in learner performance as evidenced in learner books. It also needs to be recognised that the representation of female teachers in the study was higher than that of male teachers, thus bringing about gender equity in terms of access to mathematics.

3.5.3 Freedom

The extent to which the study was useful in bringing about freedom to the marginalised was used as a criterion to determine its quality. Freedom in this study is explained as freedom from all forms of oppressive power relations and structures that constrain and exploit humankind. This is a relevant criterion in a system which is characterised by authoritarianism, over-reliance on textbooks and a static view of the role of teachers as curriculum receivers as illuminated in the previous chapter. Giroux (2001: 80) argues that 'Domination and oppression are worked into the traditional setup, through which a culture of silence is formed by eliminating the paths that lead to a language of critique'. This implies that freedom from forms of domination and oppression can only be achieved by breaking the culture of silence, creating opportunities for collective critical reflection on reality from a socio-political perspective.

Throughout the PAR focus group discussions, opportunities were created for teachers to freely and openly engage in discussions about what they know and experience in their own classrooms regarding the use of manipulatives. An atmosphere conducive to the teachers feeling free to express themselves as equal partners in the study was created. For example, the facilitator in the intervention programme on the use of manipulatives in the teaching of fractions, was challenged for using inappropriate language such as ‘three over eight’ instead of three eighths (3/8). This was a demonstration of freedom from the authority of the hierarchy. The study also created opportunities for teachers to challenge some of the manipulatives for not providing appropriate representations of mathematical ideas. For example, during the intervention programme teachers raised an issue about the interlocking cubes as not an inappropriate representation of the Lowest Common Denominator (LCD) when adding and subtracting fractions. This demonstrated freedom from conformity and cultural materials. The thick description of data in this study was made possible by teachers’ freedom of speech and thought, which was not going to be possible had we used qualitative methods.

Freedom of thought and expression was also evident in the last classroom observations where learners were freely engaging with 3-D shapes, relating them to their life experiences. The teachers allowed learners to freely explore with manipulatives thus limiting their power and authority in the classroom. One learner in school C even remarked, at the end of the lesson, that: ‘re e fumane secret ya Euler!” meaning ‘we have discovered Euler’s secret!’ This demonstrated that lack of freedom from the authority of mathematical formulae, among others inhibits creativity, self-determination and self-affirmation.

3.5.4 Hope

The extent to which the study was useful in bringing hope to the situation of the marginalised was used as a criterion to determine its quality. Hope is central to Critical Theory (Gur-Ze’ev, 2005: 18). The criterion of hope was made operational by judging how the study was successful in creating opportunities for teachers to transform the situation in which they felt helpless, worthless and demotivated. Dehumanisation as a consequence of an unjust order should not be a cause for despair but for hope (Freire, 2005: 91) and for the pedagogy of possibility as theorised by Giroux (1988). This involved the possibilities and hope for transformation and emancipation, both of which provided scope for change and revolution in the unjust material conditions of society.

In this research I examined mathematics teachers’ knowledge, experiences and thinking about the use of manipulatives from a critical perspective, namely social, cultural, political, historical and other dimensions. The aim was to recreate and transform the teaching task, recasting teachers as transformative intellectuals who are capable of re-constructing their own knowledge and practice through dialogical teaching using the language of critique and possibility which Giroux (2004a: 36) advocates for, while ensuring classroom relations that encourage dialogue, deliberation and the power of learners to raise questions (Giroux, 2004a: 43). This is contrary to the deficit model which devalues teachers and renders them hopeless and voiceless. As Freire (2005: 91) rightly argues, hopelessness is a form of silence, of denying the world and fleeing it.

Knowledge about the use of manipulatives in the study was gathered from the teachers own experiences, knowledge, feelings and thinking about teaching. This in itself fostered respect for and confidence in teachers as the creators of knowledge, thereby recognising them as capable human beings. Most importantly, giving them hope as human beings capable to change their situation and that of their learners. Using their own experiences, teachers were able to identify which manipulatives were suitable for specific mathematical topics and why, thus providing valuable insights to the study. As demonstrated in the data from group discussions, teachers in the study took responsibility for the transformation of their situatedness. In our last group discussion teachers collectively acknowledged that they are responsible for learners' low achievement in mathematics and by themselves committed to conduct extra tuition for learners during the winter holidays to support the learners. Freire (2005: 91) advances that: 'Hope is rooted in men's incompletion, from which they move out in constant search – a search which can be carried out only in communion with others. This sense of agency, which emanated from their situation of desperation, gave them a sense of hope that they are capable of transforming the situation'.

3.5.5 Peace

The extent to which the study promoted peace to the situation of the marginalised was used as a criterion to determine its quality. Kellner (2005: 66) posits that a transformed democratic education must also address problems of war and conflict and make human rights education, peace education and the solving of conflicts through mediation an important part of a democratic curriculum. In other words, the power of mathematics education as mediation must also strive towards peaceful coexistence of different communities irrespective of their diversity. School mathematics was not only used as a strategic tool to maintain and reproduce ‘white supremacy’ and therefore black marginalisation in South Africa, it was also used to sort individuals and societies into social strata, that is into the’ have’s’ and the ‘have not’s’. This is more so because those who have access to mathematics have better life chances, better career and job opportunities than the ‘have not’s and this can be traced back to mathematics classrooms. Disparities, conflicts and domination, characteristic of such class stratification, often resulted into poverty, hunger, wars and other related social ills.

Peace is brought about by social inclusivity in respect of sufficient and equal distribution of resource in general and equal access to mathematics in particular. Mathematics education founded on the principles of social justice, equity, freedom and hope leads to equal access to opportunities, power and prosperity which all culminate in peaceful co-existence and world order. Freire’s insistence on dialogue and egalitarian teacher-student relations, provide the basis for peace education pedagogy (in Bartlett, 2008: 5). One of the goals of the study, the democratisation of mathematics knowledge and skills, has the promotion of peace as one of its long term effects. Critical reflection, analytical, problem solving, and dialogical skills are all essential for conflict management and resolution, which have as its ultimate intention the promotion of peace.

3.6 CHAPTER SUMMARY

In this chapter I have given an outline of the methodological and paradigm frameworks on which the study is based. I have also declared, discussed, and justified my critical emancipatory stance which grounded my study. I have used the seven basic assumptions that most critical theorist researchers accept, as proposed by Kincheloe and McLaren (1994: 139-140) to clarify my subjectivity and my ontological, epistemological and methodological position in the study.

I have discussed and justified the choice of qualitative research as an appropriate method for the study and its alignment to the critical emancipatory nature of my study. In particular, I have illustrated how qualitative methods were operationalised to generate a thick description of how teachers understand and attach meaning to their own knowledge and experiences about the use of manipulatives in their teaching of mathematics. I have also discussed the relevance of both my paradigm and qualitative methods in uncovering and addressing power relations, domination and social injustices. I have adopted PAR as my research strategy and this was also discussed in the chapter.

Brief descriptions of multiple data sources in the form of in-depth interviews, focus group discussions, classroom observations and documents analysis that were used and well as their operationalisation in the data collection process were presented. I also gave particular attention to Free Attitude Interview (FAI) as the data collection strategy that is compatible with my critical emancipatory stance.

The chapter also outlined and described the socio-cognitive approach to discourse analysis as proposed by Van Dijk (1993, 2009) as the data analysis strategy used in the study. It also elaborated on the criteria of social justice, equity, freedom, hope and peace that underpin CER to determine the quality of the study in an attempt to meet the methodological expectations of the community of scientists. The next chapter will look at how data generated in this chapter were analysed and presented and will also discuss the findings of the study.

CHAPTER 4: DATA PRESENTATION AND INTERPRETATION

4.1 INTRODUCTION

This chapter provides a detailed and comprehensive presentation and interpretation of empirical data collected during the study to answer the following questions:

a) How does the use of manipulatives in the teaching of primary school mathematics help to (re)shape the teachers' own mathematical knowledge for teaching?
b) How does the use of manipulatives help to (re)shape the teachers' own mathematical classroom practice?
c) How can we explain the influence of manipulatives or lack thereof on teachers' knowledge for teaching and classroom practices?

Data presented in this chapter were gathered through qualitative methods and mainly from primary sources including interviews with the four teachers, specific classroom descriptions, video recorded lessons, and documents such as learners' written work, teachers' files and curriculum materials. Some of data collected were in Sesotho which were later translated into English.

This data presentation and interpretation is done per case taking the form of chronicled stories of each of the four core participants in the study. Initially the study had targeted to chronicle the stories of four teachers from each of the four selected schools. However the fourth participant, Mr Makau's lessons could not be observed because on one occasion he was busy preparing for interviews and on the other occasion he was absent from work, was one of the challenges in conducting the study. Consequently, the number of core participants was reduced from the initial four to only three participants. It needs to be noted that the reduction of the number of core participants did not have an adverse impact on the quality of data as the PAR approach allowed for sufficiently rich and thick descriptions the three cases.

The chapter starts with a detailed story representing what Geertz in Denzin and Lincoln (2000: 17) calls ‘thick description’ of the cases. Each case commences with some background based on each teacher’s biography. Drawing on the literature review, data are organised in terms of broad issues in relation to the teachers’ a) knowledge about mathematics, b) knowledge of mathematics, c) classroom practices d) use of manipulatives and e) reflections and personal development. In each case, a segment of the lesson and/or the interview is presented to illustrate a particular claim and a brief interpretation of data is also provided. This is followed by a summary of what each story is about. In its conclusion, the chapter provides a summary of the interpretation of my empirical data, lifting pertinent issues to be considered as themes for analysis in the subsequent chapter.

This last section of the chapter gives a summary account of the characterisation of each participant’s teaching episodes in the mathematics laboratory space. Ball and her colleagues’ MKT framework, in particular the sub domains of a) representing, b) explaining, c) questioning, d) sensitivity to learners’ ideas, and e) restructuring tasks, was used as categories for data presentation. To this end, some critical moments where the participants either omitted or created opportunities for learning were understood as manifestations of tensions and contradictions that arose in the context of their classroom teaching and were selected. There is an acknowledgement, among socio-cultural theorists at least, that contradictions and tensions are dynamic forces of change (Engeström in Karaagac & Threlfall 2004: 142; Russel in Hardman 2005: 3), which underscore teacher agency as theorised in Critical Pedagogy and therefore a strategy to support teacher development.

4.2 THE STORY OF MS DIKGOMO

4.2.1 Background

My first encounter with Ms Dikgomo at a more personal level as with the other participants in the study was during a mathematics cluster meeting that was conducted in the beginning of the study. The cluster meeting focused particularly on schools that have mathematics laboratories in the one district of the province. Although she showed interest and participated actively in the discussions about teacher knowledge, classroom practice and the use of manipulatives, she was initially hesitant to participate in the research project. Of the four key participants in the study, she was the last to sign up, after I had engaged her separately and explained to her the purpose of the research project and what benefits it might have for the teaching of mathematics in general.

Ms Dikgomo, in her mid-40s, is a mathematics teacher at school B, where she has been a teacher for the past 6 years. She holds a three year Junior Primary Teachers Diploma (JPTD) in which she was trained as a generalist teacher for the whole foundation phase curriculum, i.e. Grades 1–3, catering for 7–9 year olds. However, she has been teaching mathematics at various grades in the Intermediate Phase (Grades 4–7, ages 10–13) and is currently teaching mathematics to three of the four Grade 6 (12 year olds) classes at her school. Explaining how she got to teach Mathematics at the school, she described that when she arrived at the school, she used to help a teacher who was teaching mathematics at the school then. This is how she was spotted as having some ‘clue’ of mathematics. The principal then allocated her to teach mathematics classes in the intermediate phase. When asked to describe her feelings about being assigned to teach the intermediate mathematics classes, she took a deep breath and conceded that:

> Yo! It was scary at first but I was fortunate to be mentored by a good teacher, Mr...XX.., who provided me with support by sharing ideas on mathematics teaching. It really helped to ease the pressure on me.

It seems understandable why Ms Dikgomo felt the way she did because she had not been trained to teach the Intermediate Phase in the first place, let alone to teach mathematics at that level. The importance of mentoring and support by another mathematics teacher becomes clear from her description.

4.2.2 Ms. Dikgomo's Knowledge about Mathematics

4.2.2.1 Views about mathematics

It is widely assumed that what teachers do in class is to a greater extent influenced by their personal beliefs and views about teaching and learning (e.g. Ernest 1989; Fang 1996; Hashweh 1996; Kang & Wallace 2004; Thompson 1992). Some of these views include ideas regarding the nature of mathematical knowledge, the purposes of mathematical knowledge, and the conditions under which mathematics is best learned. Although much of the data about Ms Dikgomo's beliefs about mathematics and its teaching and learning were obtained from the interviews, some segments of her lesson presentation also confirmed her beliefs about mathematics in general.

In the interview she pointed out that her favourite area in mathematics is the teaching of multiple operations. Asked why she enjoys this area of mathematics, she replied thus:

> It is, because in this area, once the children know the rules i.e. BODMAS, then it is easy to get the answer. Ha ba ka tseba feela hore o qala kae, ha hona ntho e tla ba hlola (if they can only know where to start, there is nothing that will be difficult for them)

Clearly, from this conversation, Ms Dikgomo seems to regard mathematics as a set of rules and procedures that are instrumental towards getting the correct answer. This is in line with the instrumentalist view where mathematics is seen as a set of unrelated facts, rules and skills used for some external end (Ernest 1989: 250).

It is not surprising that she views mathematics the way she does. Reflecting on her own primary mathematics experiences, she recalls how she learned BODMAS:

> My teacher used to say when you do multiple operations, put them in brackets in order to get the correct answer. He used to say we should test if we'll get the same answer in the case of multiple operations.

Two major points arise from this conversation with Ms Dikgomo regarding her views about mathematics and about (mathematics) teaching in particular. Firstly it is clear that mathematics is about mastering the rules of operations to get to the correct answer and secondly, teaching mathematics is therefore also about teaching these rules to the learners. Interestingly, this is mostly how mathematics is taught in many primary schools, not only in South Africa but across the world (Cai *et al.* 2009; Ma 2010; Rowland *et al.* 2005). The power of prior experiences as a learner of mathematics in her own learning to teach mathematics also becomes evident from the conversation. Citing an example of her own mathematics is illustrative of this power of prior experiences with mathematics – which have become somewhat of an influence in her own knowledge and teaching of the subject many years later. This power of prior experiences also encourages the perpetuation of a disempowering method of teaching and learning mathematics in that learners are not taught to discover and understand the rules of BODMAS for themselves.

During the lesson, her utterances and instructions to the learners served to further illustrate and emphasise her views about mathematics. Consider the following lesson segment:

Lesson segment: 1

The context is a mathematics lesson on the introduction of multiple operations. Ms Dikgomo asked learners to name the four basic operations.

T: What are the four basic operations?'

Ls: Addition, multiplication, subtraction and division.

T: But we still have others (operations) where we have to use eh..... the rule that will help us to get the other operations that we use. Can somebody remind us, what is that rule so that we should get other operations?

L_1: The BODMAS rule Ma'am

T: What are those two operations?'

L_2: 'Brackets and of'

T: So these are the ones we are going to focus on. To those four we add two other operations so we have six. Mathematicians have developed the BODMAS rule so that we don't have different answers. I'm going to show you how if each group will come up with different answers it will be chaos. That means ho tla ba le moferefere neh! (There will be chaos, isn't it!) '

(The teacher shows a video clip of motorists not obeying the rules on the screen to reiterate the consequences of not obeying the rules).

LS: (Watched the video clip in fascination)

T: Mathematics is about following the rules; if we don't there will be chaos. To avoid chaos, there must be a common answer.

In this lesson segment, Ms Dikgomo is very clear about the need for rules in mathematics as would be the case in real life, the need to follow the rules in order to avoid chaos, and more importantly the role of these rules in ensuring that we all arrive at the "correct answer'. This overemphasis on the common answer further illustrates the view she holds of mathematics as consisting of discrete procedures and rules. In this view, she overlooks the need for understanding of the rationale behind the algorithms. The mathematical attitude that is portrayed here is that mathematics is a neat and linear subject where different perspectives to mathematics are seen as chaotic rather than an opportunity to understand the underlying principles behind the algorithm. To Ms Dikgomo there seems to be no need for discourse and debate around mathematical solutions and the problem solving strategies themselves. The video clip provides further emphasis of her point and is probably the most powerful tool for driving this perspective to the learners which they are likely to remember long after the mathematics lesson has been completed.

4.2.2.2 Views about mathematics teaching and learning

Ms Dikgomo indicated that she was attracted to teaching as a profession since she finished her high school education, and decided upon teaching as her first career choice. Asked why she chose teaching, she replied thus: ‘I was driven by the desire to impart knowledge to others, especially when working with children because of the passion I have for them (children), and I wanted to see them succeed in life’. The quest to impart knowledge (mathematics) shows her understanding of teaching as transmission of knowledge from the knower to children. In this view, children are seen as entirely dependent on her own knowledge to be nurtured through parent-like practices in order for them to succeed in life. Her classroom practice in general, part of which I have extracted in the foregoing section, does suggest that Ms Dikgomo views teaching as imparting knowledge i.e. providing children with a set of rules, such as BODMAS, for step-by-step computations. Teaching mathematics therefore is generally about imparting the rules and procedures to the learners.

She views effective learning of mathematics as the mastery of the step-by-step procedures, the ability to remember and reproduce the rules and facts, and the attainment of the correct and common answer as quickly as possible. In the section below, I examined another segment from the first lesson to illustrate, once more, her emphasis on teaching for the correct answer.

Lesson segment: 2

(Ms Dikgomo writes the sum from the textbook on the chalkboard)

T: I want you first to use the cubes to do the sum; you have 5 minutes to complete the sum’

(Ms Dikgomo moves from one group to the other, checking the answers as the learners are writing their solutions in their books. Once the group had finished, they reported back by giving the answer and explained how they got to the answer.)

L_3: We start in the brackets; we multiply 20 by 3 we get 60 and we add 10 then 4 to get 74.

T: What about the others, what is your answer?

Other groups responded by just repeating what the first group had said, basically reporting back by just stating the BODMAS rule. Throughout, the teacher was just comfortable with their answer of 74. There was no deliberate effort by the teacher to verify if the learners understand the procedures and the concepts involved. It is clear that Ms Dikgomo views learning mathematics as synonymous with memorising and chanting the steps towards the correct answer. While Ms Dikgomo could be congratulated for creating opportunities for the learners to use cubes and engage in some classroom discourse about their problem solving and answers to the question, the major challenge lies in the fact that to her, the major issue was following the correct rules and getting the right answer. An opportunity to broaden the discourse about mathematics was therefore missed in this class. Explaining how they got the answer was limited to only restating the BODMAS rule instead of engaging in debate with each other and possibly finding even more approaches to solving a given problem.

4.2.3 Ms Dikgomo's Knowledge of Mathematics

4.2.3.1 Knowledge of the curriculum

To further understand Ms. Dikgomo's knowledge of mathematics, I posed a question about what she would describe the most important aspects she is trying to accomplish during the year with her Grade 6 learners. Ms Dikgomo elaborated on each of the four content areas as follows:

a. <u>Number, operations and relationships</u>: According to her the outcome of this content area includes knowledge of the number system, understanding of concepts, e.g. number, fractions, percentages as well as the application of basic operations addition, subtraction, multiplication and division to manipulate numbers in and out of context, etc. Asked why she thinks it is important to learn this content area she replied that 'learners need to be equipped with knowledge and skills to deal with everyday challenges relating to money, counting etc.'

b. <u>Measurement</u>: Ms Dikgomo pointed out that through this content area she wants her learners to have 'knowledge of units of measurement, terminology, skills of measuring and estimating time, temperature, perimeter, area, etc.' She wants her learners to know measurement in order to deal with 'everyday challenges relating to time, distance, area, etc.'; to be able to use appropriate mathematical language for communication and to learn accuracy as a value.

c. Patterns: This content area, according to her, involves knowledge of geometric and numerical patterns, generating a pattern and completing the sequence, skills to make informed predictions and entrenching the beauty of mathematics. The importance of this content area is to enable learners to make informed decisions and future projections/predictions.

I compared what Ms Dikgomo shared with me above with the prescripts of the Curriculum and Assessment Policy Statement (CAPS) for the Intermediate Phase Mathematics. I found that although in some aspects she was a bit generic, she seems to have a fair knowledge and understanding of the policy in terms of the content areas prescribed for the grade, the objectives and the outcomes of each content area. Clearly, Ms Dikgomo is aligned with the policy statement in terms of what aspects are important for the learners to engage with and the reasons therefore. However, alignment with the policy statements does not always translate to the kind of practice that is suggested by that same policy. This is the disjuncture that is often observed by researchers between policy and practice (Cohen and Ball 1990**).** The practice suggested by CAPS for instance would require that learners understand the operations so that they can "deal with everyday challenges relating to money, counting etc." as Ms Dikgomo argues in her description of the important areas of focus for her teaching. Finding the correct answer through the use of rules may not be the best way to provide learners with the skills for everyday problem solving. This is indicative of the disjuncture between policy and Ms Dikgomo's practice. This is more so because her approach tends to encourage rote learning, it suppresses and devalues knowledge that learners may already have about solving mathematical problems. The teacher dominates and the learners tend to be relegated to the status of passive recipients in their own learning encounters. It needs to be noted that this situation may (unfairly) suit the teacher especially considering the learners' ages.

4.2.3.2 Knowledge of mathematics

Just as with other participants, most of the data on her knowledge of mathematics were gathered from the scenarios that were presented to her regarding a) teaching a particular topic, b) the handling of student errors, c) generating representations and d) responding to learners' novel ideas (Appendix 7).

a) Approaches to teaching a topic

Her response regarding how she would go about teaching a topic on area showed that she has some knowledge of the topic in respect of the basic definition of the concepts, the formula and its application in calculating area. Her approach starts with the formula for area where she indicated that learners will learn more about the outer part of the classroom first so that they should be able to better grasp the concept of area, which she justified thus:

> Because when they have that idea of measuring perimeter it will lead them to measure the area. Then because we have done multiplication they will have to multiply the two (length and breadth) to calculate the area of that particular classroom.

Once more, this example showed that she expected learners to merely know the formula (rule) so that they can learn the step-by-step procedure of getting the correct answer. It is also interesting to note that Ms Dikgomo does not make any mention of the surface area. Rather she puts more emphasis on the outer part, i.e. the boundary as can be observed from the diagrams. There is an apparent insufficient understanding of the concept of area by Ms Dikgomo. Reliance on the length and breadth at this introductory stage of the teaching of the concept of area may entrench existing misconceptions of the learners regarding the concept. For instance it may lead to their confusion of concepts of area and perimeter. Although she indicated that learners will be given the task to measure and record the area of their rooms, the task is used to merely confirm the formula and not to promote deeper understanding of the concept of area. This appears to be the perpetuation of the influence of Ms Dikgomo's learned teaching approaches from her teacher (what she said about her teacher in section 4.2.2.1). Once again, Ms Dikgomo's approach provided within it some clear opportunities for development of conceptual understanding by the learners. However, her own understanding of what is mathematics and mathematics teaching entail seems to betray these opportunities for in-depth learning. While she is fairly strong in her knowledge of mathematics, her reaction to the scenario on the relationship between perimeter and area showed some knowledge gaps of substantive mathematics, what Ma (2010: 121) refers to as the ‘depth and breadth’ of the subject knowledge in her notion of PUFM . She was clearly confused by these two concepts, even though she knows and can apply the formula for each one.

b) Handling of student errors

In handling students’ errors, Ms Dikgomo identified the lining of zero’s as the mistake committed, and lack of understanding of the concept of place value as the root cause of the problem. She described how she would start the lesson by grounding her learners in the concepts and the mathematical principles that underlie the algorithm involved in the multiplication of 123 by 645. She would begin by explaining concepts, for example she explains that numbers are made up of digits, that the value of the respective digits is determined by their (digits’) respective place in the place value chart consisting of units, hundreds, thousands etc.

	TTH	TH	H	T	U
123		X	6	4	3
X 645			6	4	5
			6	1	5
123 X 5 =		$^{2}4$	9	2	0
	7	3	8	0	0
123 X 40 =	7	9	2	3	5
123 X 600 =					

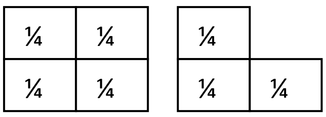
Table 2: Place value in multiplication of whole numbers (three digit by three digit)

Although place value plays an important role in trying to remedy the error, Ms Dikgomo does not make any mention of the distributive law which helps to explain the algorithm on the left of the table above.

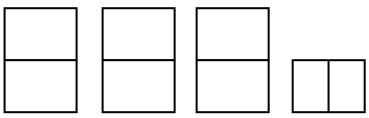
As indicated in lesson segment 2, Ms Dikgomo's overemphasis on common answers explains why she was somehow comfortable with her learners' mere restatement of the BODMAS rule. In most cases she seemed to be anxious to get to the common answer rather than to ensure that learners have a deeper understanding of the procedures and the mathematical ideas underlying those procedures. This is the point of concern about missed opportunities for learning mathematics in her classroom procedure.

c) Generating representations

Information on how Ms Dikgomo uses representations in her teaching was obtained from her response regarding what real world model she would use for teaching division by fraction $1\frac{3}{4} \div \frac{1}{2}$. She showed good understanding of the problem to explain how many halves there are in $1\frac{3}{4}$. She made a diagram using two chocolate bars that are divided into quarters. The one full chocolate bar has four quarters and the other bar has only have three quarters.



Altogether there are seven quarters and these quarters are grouped in twos to represent halves. By doing this there are halves and a remainder of one quarter, which she then divides into two parts as shown below:



1 + 1 + 1 + ½ (The number of halves)

To determine how many halves there are in 1 ¾ she adds up 1 + 1 + 1 + ½ = 3 ½. However, she experienced difficulty in explaining how she got the number of halves in a quarter chocolate bar which she just refers to as half of a quarter, meaning 'dividing into half' rather than 'dividing by one half'. This makes a strong case for the use of language to accurately describe what we do or want to do. Once again, we can see how knowledgeable Ms Dikgomo is in general terms through her ability to represent the problem pictorially. Although Ms Dikgomo mentioned other teaching aids such as charts and interlocking cubes, she only used one form of representation, i.e. a diagram, to model the problem. As discussed earlier, her knowledge seems to stop just at the point when deeper understanding seems to be required. Telling the learners a story, for example, might have supplemented the diagrams and promoted a better understanding of division by a fraction. To create such a story board, however, requires a much deeper understanding of the concepts and the problem at hand.

Lesson segment: 3

In the lesson on multiple operations, Ms Dikgomo wrote the second sum (36 ÷ 9) + (18 ÷ 3) = from the textbook on the chalkboard:

T: Use the cubes to do the sum in your groups, quickly!

(As with the first task, Ms Dikgomo moves from one group to the other, checking the answers as the learners write their solutions in their books)

L_6: Ten teacher!

T: How did you get ten?

L_6: We said 36 divide by 9 is 4 (showing with his fingers) and we said 18 divide by 3 we get eh..... (Looking at the other group members and wanting them to confirm)

L_s: Six Teacher! (Other learners from the group shouted.)

T: Where is the 10, show me how you got it with the cubes?

(The group pointed at the stacks of cubes long ones of 9s and short ones of 3s.)

Ls: One, two, three, four ...54. (Counting cubes in each stack.)

T: No, I want you to show me 10, you said the answer is 10!

Altogether the cubes added up to 54 contrary to the solution ‘10’. It was clear that even though learners correctly applied the rule, i.e. by working out the division operation inside the brackets first, they seem not to understand what was required in the problem. The latter was demonstrated by learners’ counting of single cubes instead of counting the stacks or groups of cubes, i.e. 4 stacks of 9 cubes each and 6 stacks of 3 cubes each, which add up to 10 stacks or groups of cubes. It was clear from this lesson segment that learners needed support not only to follow the rule and do the computation, but also to have a deeper understanding of the concept of division.

At this point the researcher intervened by using her experience to assist the process. As explained in Chapter 3 of the study the researcher’s immersion in the lesson was motivated by her role as a participant observer in the study. The researcher intervened by creating a word problem that represented the problem from a real life situation and using interlocking cubes to model the problem:

> One group of learners in the table have 36 red sweets and the other group has 18 yellow sweets. They want to pack the red sweets into stacks of 9 each and the yellow sweets into stacks of 3 each. Altogether how many stacks of sweets do we have per table?

This word problem helped learners to make meaning of the problem in context before they jumped into computing the sum. It also helped learners to understand that altogether the number of stacks is 10, even though the stacks differ in size.

Once more, Ms Dikgomo’s mathematical understanding seemed to stop just at the point where deeper understanding and conceptualisation was required to assist the learners with their representations.

d) Handling students’ novel ideas or solutions

Asked how she would respond to a scenario where a learner presented a ‘novel’ discovery that as the perimeter of a closed figure increases, the area also increases, she immediately accepted the theory: ‘Because we all know that perimeter is the measurement of the outer part of a closed figure, when we increase it the inner part of the figure becomes big as well’. The phrasing of her response begins by appealing to some common knowledge or rules which is an indication of limited exploration of ideas and hence lack of deep understanding of the concepts of perimeter and area.

In her attempt to help learners understand the problem, she explained how she would use shapes made up of strings of 6 cm length and 4 cm breadth to show the relationship between the two concepts. This would give a perimeter of 20 cm and an area of 24 cm$^{2.}$ She would then increase the length of the strings to 8 cm length and 5 cm breadth to increase the perimeter to 26 cm and hence the area to 40 cm^{2}.

However, the task given by Ms Dikgomo was limited to measuring the sides and using the formula to get both perimeter (20 cm) and the corresponding area (24 cm^{2}). She could have allowed learners for example to explore various measurements that represent the same perimeter of 20 cm such as 9 cm length and 1 cm breadth; 8 cm length and 2 cm breadth; 7 cm length and 3 cm breadth so as to illustrate that the same perimeter can cover different areas. After having had various dimensions of the same perimeter, she could have created the opportunity for learners to investigate and compare the areas when increasing the perimeter to 24 cm for example, with various dimensions, e.g. 11 cm length and 1 cm breadth and ultimately establish if the claim can be generalised. Her approach seems to have been based largely on computation using the correct formula – and thus measuring and substituting different values for the length and breadth.

4.2.4 Ms Dikgomo’s Classroom Management

4.2.4.1 Planning

Information on planning was obtained from the interviews and from the teacher’s file that contains the lesson plans and other curricular materials. The lesson aim, objectives and outcomes were well documented in the lesson plan and in most areas correlated with the teachers’ guide.

Although there was correspondence between the objective in the lesson plan and what was communicated during the lesson presentation, the objective was not specific on exactly what learners should be able to do at the end of the lesson. For example, the teacher stated that: ‘Today we are going to concentrate on multiple operations. This means we are going to use multiple operations in one sum’.

Lesson planning in this case, and probably in the school as a whole, does not seem to be a well thought out process. As Ms Dikgomo remarked during the interview after the (professional development) intervention programme:

> So it gave me the opportunity ya hore ha ke prepare lesson; ke batle hore eintleke what is my goal for this lesson; ke batla ho atjhiva eng - ke batla ba tsebe eng bana ba and before that o ne o tlabe o etsa prep just ho pliza HOD ya hao. (So it [the intervention programme] gave me the opportunity to prepare the lesson – to actually know what the goal of the lesson is, what do I want to achieve, what do I want the children to know. Before then one would prepare the lesson just to please one’s Head of the Department).

Her 80 minute (double period) lesson was divided into three sections: a) a 5-10 minute starter where the aim of the lesson was introduced and learners were asked some oral questions based on the work that was done previously, b) the main lesson where learners were given tasks to complete in groups, and c) the plenary session in which groups were presenting their findings. The main lesson and the plenary sessions depended on the number and degree of difficulty of the tasks given to learners. For example, in the first lesson two tasks were given. The first task: 10 + (20 x 3) + 4 = lasted for 25 minutes while the second task: (36 ÷ 9) + (18 ÷ 3) = lasted for 45 minutes. Ms Dikgomo was more generous with time to allow learners to engage with the problems at hand. This is in spite of her emphasis for them to work out the answer “quickly” - a retort she repeated every time she assigned them a task to do.

4.2.4.2 Resource management

Information on resource management was gathered during the lesson presentation. The lesson took place in the mathematics laboratory which is equipped with various manipulatives and mathematical charts on the walls. Both lesson plans indicated apparatus or manipulatives to be used, e.g. the second lesson indicated that interlocking cubes were going to be used.

When learners entered the classroom, packets of manipulatives were already on their tables, which showed that the use of manipulatives was planned in advance. In presenting the 1st lesson, learners working in groups of eight, were given a set of 200 interlocking cubes each and were instructed to count in groups of 10s. The counting lasted for some time and the reason for counting was not at all clear. This raises serious question about Ms Dikgomo's thoroughness in lesson preparation and lesson planning. The class was then instructed to use interlocking cubes to solve the problem that was taken from the textbook and written on the board.

4.2.5 Ms Dikgomo's Classroom Practice

In the following section, I present a summary of the major elements of M. Dikgomo's classroom practice.

Most of the data regarding Ms Dikgomo's classroom practice were gathered during her first lesson of 80 minutes on multiple operations with brackets. Her second lesson was on the introduction of the fraction concept and it only took 40 minutes. Both lessons were video recorded and each were preceded and followed by interviews. The Grade 6 class consisted of 43 mixed ability learners mostly from disadvantaged backgrounds and the lesson took place in a mathematics laboratory. Learners were seated in groups of about 8 during both lessons.

4.2.5.1 Facilitating learning

The lessons took place in a mathematics laboratory which as I have already explained is very rich in resources including manipulatives and printed materials, which appeared to create a very stimulating environment for the learners. The class was lively and learners were actively involved throughout the lesson. Although there was no deliberate instruction for learners to work in groups, group work seemed spontaneous. As learners entered the classroom, they went straight to their respective tables. Asked whether learners were assigned to specific groups the teacher responded that she ensures that they are mixed in terms of their abilities all the time:

> ‘ha nka lemoha hore group eo e nida ho splitwa kea splita. Ke cheka hore na ke mang a kgonang ho tlo ba thusa ka something – because ha baka dula ba le fife moo – problem e tlo ba teng – le ha nka miksa ka a le mong feela a tla ba gaeda mara ba se aware hore ke etsa jwalo (If I realise that a particular group needs to be split, I do so. I bring in someone who can assist them with something, even if it’s one learner to guide them because if there are five struggling learners in a group, there is going to be a problem).

The first sections of the lessons mainly focused on the aim and the introduction was characterised by whole class instruction. Ms Dikgomo stood in front of the class as she explained the aim of the lesson and asked a few oral questions to start her lesson. This flowed into group instruction as learners were given a group task which was copied from the textbook and written on the board. Ms Dikgomo started moving from one group to the other. This was done by merely checking the answers as the learners wrote their solutions in their books. In almost all the groups all the learners were bending towards one learner (probably the leader) and helping with the counting. Any group that had finished the task showed by putting up their ha**nds** and were given the opportunity to report back. Feedback was in the form of individuals from groups giving the answer and explaining how they got the answer. The latter was by way of saying where they started (e.g. in brackets) and followed by other operations in the sum. There was no evidence of learners working independently, on their own initiative and pursuing the problems from different dimensions in all the tasks.

Maintaining discipline and order in the classroom seemed to occupy most of Ms Dikgomo’s time during the lesson. During the interview she remarked that her learners lose concentration:

> kena le class ya bana ba so – ha ke qala ke jika feela keya tjhokbotong ba ya bapala – ha se ke kgutla hape they forget what we are doing – so it’s so strenuous on me – because ke tshwanetse ke dule ke omana ke batla attention ya elwa le elwa (I have a class of learners who once you go to the chalkboard, they start playing and when you go back to them, they have forgotten what we were doing. So it is strenuous on me, because I have to always scold them to get the attention of each one of them).

Asked as to why she thought concentration was so important when teaching the topic she replied that learners need to remember the steps, which involve division, multiplication and subtraction in one sum. Ms Dikgomo's overemphasis on discipline is not surprising. Her view of mathematics as consisting of rules and procedures justify her firm stance on orderliness. Full concentration is a condition for rules to be unquestioningly learned and to be successfully applied, perhaps much more than the need for learners to concentrate so that they could work accurately.

4.2.5.2 Supporting learning

Ms Dikgomo seems to believe in active participation of learners for them to learn successfully. For example, after introducing the research team to her class, she pleaded with the learners to participate actively and freely as they normally do and that they should not be intimidated by the presence of the visitors. She created opportunities for her learners to do mathematics by physically and freely handling manipulatives.

Her choice of tasks showed how she supported learning by gradually moving from simple to more complex tasks. She started with a task that involved addition and multiplication, which most learners could easily compute. The second task was on addition and division, where learners had to demonstrate their deeper understanding of the concept of division in order to appropriately model both the process and the product with interlocking cubes.

Although she was aware of those learners that were struggling, there was no evidence of deliberate remediation by the teacher to help struggling individuals within the groups. This is further demonstrated by the fact that neither the tasks nor the support were differentiated, all the groups were given the same tasks as though they were a homogeneous group.

As indicated in the discussions above, all the tasks that were given to learners were taken directly from the textbook. This limited Ms Dikgomo from being flexible in coming up with creative representations that would have better supported learning.

4.2.6 Ms Dikgomo's Assessment and Evaluation Practices

Information pertaining to Ms Dikgomo assessment and evaluation practices was obtained during the lesson presentations. In both lessons, the researcher was able to observe assessment strategies that involved individual and/or group verbal responses, writing on the board and explanation of findings.

Her assessment practices are characterised by group assessment where she uses oral and/or written questions. In most cases learners responded in a chorus, thereby limiting Ms Dikgomo's opportunity to establish whether individuals within the groups do have a thorough understanding of the subject matter. Ms Dikgomo's statement about group assessment seems to suggest that she is aware of the limitations of her assessment practices:

> hape ntho e mislidang ke hore a le mong ha a ka tjho answer and then class e ya echo kaofela o nahana hore ba tshwara ka pele — so nna ke be ke tsamaya ka concept ya hore ba understand kaofela – so ha ba ngola classwork individually – kere ok, ke tlo le tshwaya one by one – then ha batla ho nna ke bone hore joo! – ba out – ba hole totally ho nna (Again, what is misleading is that when one learner gives the answer, the whole class says the same answer, giving the impression that they grasp quickly. So I take it for granted that they all understand. It is only when they do classwork and I mark them individually that I realise that they are far from understanding what I taught them).

On a number of occasions, especially where oral questions were asked, she either ignored learners' responses or left them hanging and thereby missed the opportunity to use assessment to inform her subsequent instructional decisions. In some instances, it was not clear as to what Ms Dikgomo wanted to achieve with the questions that were asked, which might be why she ignored the responses. For example, in the lesson on multiple operations where Ms Dikgomo commenced by outlining the aim of the lesson, she asked the following oral questions:

Lesson segment: 4

T: What are the four basic operations?

LS: Addition, multiplication, subtraction and division.

T: How do we get the product of two numbers, what must we do to find the product?'

L_4: We add.

(The error was left unattended and no follow up was made to try to understand either the learner's misconception or its source).

T: Do we agree?

(There was a sign of uncertainty while a few learners responded in a chorus)

LS: No!

(While the learners were confused and anticipated to hear something from the teacher, she pointed at another learner *without following up on the misconception nor its source*).

L_5: We multiply.

These oral questions were immediately followed by tasks that were given to the learners to complete, instead of using the opportunity to substantively link the topic with other mathematical topics, principles or ideas. The lesson on multiple operations was introduced in isolation, leaving learners not properly grounded in the topic except to just follow the rule. For example, there was no deliberate link to the topics that had already been taught in the 1st term, such as number sentences to serve as an introduction to algebraic expressions, word sums, properties of whole numbers, area, etc. to locate multiple operations within the bigger picture.

4.2.7 Ms Dikgomo's Use of Manipulatives

Ms Dikgomo used manipulatives as a teaching strategy in both lessons that were observed, probably because the lessons were conducted in the mathematics laboratory where there are a variety of manipulatives. Data on the use of manipulatives were therefore gathered during both the interviews and the lesson observations. Reflecting on her own experiences when she was a learner, she emphasised the value of the practical application of mathematics in promoting understanding thus:

> At primary school we used to practically measure the distance from the classroom to the office and the teacher also showed us how to estimate by counting with our feet. He would ask us to bring different containers from home and compare measurements using containers of different shapes and sizes.

On the other hand, she reported her secondary school experiences of learning mathematics thus: '....I feel like I regressed a lot at high school as compared to my mathematics at primary school level'. She explained that she did mathematics up to Standard 10 (i.e. Grade 12), where she took mathematics at Standard Grade (lower level). She lost hope and never understood mathematics at this level. She further remarked that her mathematics teacher used to stand at the door and dictate formulae to them (students) and, because there was nobody at home to assist her with mathematics homework she failed mathematics in the final Grade 12 examination.

Despite the hopeless scenario she painted about her high school experiences, she explained her experiences at the teacher education as follows:

> It was exciting as we did mathematics in a more practical way, using various apparatus which promoted more understanding. My college years rekindled the love for mathematics when I had already lost hope. If you understand mathematics you stay motivated and you also want to share with others.

The quote above demonstrated Ms Dikgomo's perceptions of the affordances of manipulatives for practical learning. What was a bit of a concern is how she seems to get trapped in the simplistic use of manipulatives, a situation which leaves manipulative use at the level of their concreteness. Manipulatives were mainly used for illustrating or confirming a rule or simply for counting. This was obvious as every time the learners touched manipulatives, they would spontaneously start counting. The lesson segment on division shows that if learners are meaningfully supported with other representations such as a story, a word problem, etc., they would be able to use manipulatives to engage in deeper mental activity, mathematical discourse, reasoning, etc. Such an approach supports the idea that knowledge is socially constructed and also depicts collaborative learning which is participatory in nature.

In her pre-observation interview Ms Dikgomo indicated that learners get excited when they use manipulatives and that manipulatives promote concentration and allow learners to talk freely in class, irrespective of their abilities.

Post intervention programme

Ms Dikgomo's second 40 minute lesson was on fractions. The aim of the lesson was to introduce the fraction concept and its representation. The lesson took place after the research team had engaged with participants at various forums, that is, group discussions, intervention program, classroom visits and interviews. Compared to the first lesson that was observed, some episodes in the second lesson showed changes in Ms Dikgomo's teaching as illustrated in the following discussions. What was particularly different in her approach was how she meaningfully engaged her learners in a task in which they had to define a fraction. The lesson segment below, in which she established her learners' prior knowledge of fractions, lasted for about 3–5 minutes.

Lesson segment: 5

T: Good morning, class.
Ls: Good morning, teacher.
T: Today we are going to do fractions.
T: Who can tell me, what is a fraction hmmm...?
L1: Teacher! Teacher! (Raising her hand higher.) It is half, teacher (after being pointed at by Ms Dikgomo).
T: Oh, a fraction is a half! Class, do you agree with her?
Ls: Yes, teacher! (There are signs of uncertainty as learners were all looking at the teacher and seemed to be expecting her to give a ruling on the matter.)
T: Are you telling me that all fractions are halves?
Ls: No, teacher, no!
L2: A quarter, teacher!

Ms Dikgomo used similar follow up questions and learners kept on responding by giving the names of fractions they could remember – a third, three quarters, a fifth. It was evident from this lesson segment that Ms Dikgomo wanted to establish her learners' understanding of the concept of a fraction. She drew on her learners' informal knowledge of the concept of a fraction to inform her subsequent actions.

In her main activity, which lasted for about 20 minutes, she directed her learners to use interlocking cubes and pieces of paper to demonstrate each of the fractions they had listed.

Lesson segment: 6

T: What do you have on your table?
Ls: Interlocking cubes and papers teacher!
L1: And a ... ehhhh (Brushing his head)...sekere (pair of scissors) teacher!
T: In each table, I want you to use the cubes or papers to show me a half, a third, a quarter and a fifth. You said these are fractions! Work in twos, groups of twos, with a person next to you!
Ls: (Working enthusiastically and loudly on the task.)
T: You have paper and cubes, use what you like!

In the above episode, Ms Dikgomo created the opportunity for her learners to concretely experience particular fractions drawn on their own knowledge. Learners were able to make concrete representations of their fractions mainly through the part-whole approach.

In the last part of her lesson, which took 15 minutes, learners presented their representations of specific fractions. In giving direction as to how learners should present their respective tasks, Ms Dikgomo instructed as follows:

Instruction 1:

> Those who used cubes tell us how many cubes you used altogether. Those who used paper, le sheidile akere? Ba bang ba khatile neh? (You have shaded, isn't it? Others have cut, isn't it?) Tell us out of how many parts did you shade or cut your fraction.

Instruction 2:

> Make sure that when you present a fraction that you chose, for example a 'half', you must also tell us how you know that this is a 'half'. You must also show us how we write the fraction, not in words, but as a number. Akere le entse difrakshene tse ding eseng hafo fela? (Isn't it that you have made other fractions, not only half?). Please, le seke la re bolella ka hafo kaofela ha lona. Difrakshene di ngata! (Don't tell us about half, all of you. There are many fractions.)

The few learners who chose to use paper folding instead of paper cutting struggled to understand the first instruction as most of them had not shaded their fraction representations. In response, Ms Dikgomo changed her instruction to 'give us the total number of parts you folded'. What is strikingly apparent in her instruction was how she introduced the notion of a 'whole'. She carefully created opportunities for her learners to do their own 'wholes' using various tools [emphasising that wholes differ, e.g. in respect of the number of objects in that whole or the area shaded and this will give different number of objects or the area representing 'half']. She also directed her learners to represent the various fraction examples numerically. Her question: 'How do you know this is a half?' was a conceptual question that called for deeper understanding of a fraction concept that goes beyond the surface characteristics of a fraction model.

What was also different from the first lesson was that she did not take centre stage; her learner-centred approach was demonstrated in many ways. For example, she drew on her learners' intuitive knowledge about fractions to formalise their knowledge of the fraction concept. She also gave learners the freedom of choice in respect of the tools and the fractions they represent.

4.2.8 Ms Dikgomo: The Story of How Beliefs about Mathematics Influenced Her Classroom Practice and the Use of Manipulatives

The picture painted in this section is of a story of how Ms Dikgomo's beliefs about the nature of mathematics pervaded her classroom practices and the use of manipulatives. As mainly derived from her first lesson, Ms Dikgomo views mathematics, as a set of rules and procedures that are instrumental towards getting the correct answer. This is in line with the instrumentalist view where mathematics is seen as a set of unrelated facts, rules and skills used for some external end. Her views about mathematics seem to have also influenced the power relations in her classroom. Her high regard for the BODMAS rule is indicative of the authority and domination of mathematical rules over her and by extension, over her learners. Such domination proved to be disempowering as it translated into the following of rules without understanding mathematical principles that underlie the procedures.

In her instrumentalist view of mathematics, Ms Dikgomo focused on procedures, i.e. on steps to follow in multiple operations and on the correct answer at the expense of conceptual understanding. This demonstrated a tension between one of the NCS specific aims of mathematics teaching, i.e. to develop deep conceptual understanding in order to make sense of mathematics (DBE 2011d: 13) and Ms Dikgomo's focus on procedural knowledge. Her view of mathematics seemed to have an influence on her strong tendency to practise and drill steps with emphasis on remembering and getting the correct answer. The influence was also apparent in her overemphasis on discipline. Instead of using language as a tool to help explain mathematical concepts, ideas and terminology, she used language as a tool to regulate her learners' behaviour. As mentioned in the section above, this overemphasis by Ms Dikgomo's is not surprising as it is in line with her drill and practice methods, methods that require full concentration.

Her view of mathematics and its resultant drill and practice teaching methods seemed to have also impacted on her use of manipulatives. In her lesson on BODMAS, she particularly used manipulatives to illustrate and confirm the rule as well as for simple counting. This was apparent every time the learners touched manipulatives as they would count spontaneously and thus relegating manipulatives to mere physical activity. Central to literature on the use of manipulatives is the ability to transcend the concreteness of manipulatives to learning the abstract concepts and ideas that are embedded in these objects (e.g. Ball 1992; Driscoll 1981; Hiebert 1984 and others, all in Ma 2010: 6; Iliada, Gagatsis & Delivianni 2005). However, the lesson segment on division shows that if learners are meaningfully supported with other representations such as a story, analogies from their environment or a word problem they can use manipulatives to engage in deeper mental activity, mathematical discourse and reasoning.

4.3 THE STORY OF MS BOHATA

4.3.1 Background

My first encounter with Ms Bohata at a more personal level, just like the other participants in the study, was at mathematics cluster meetings that were conducted at the beginning of the study that focused particularly on schools that have mathematics laboratories. She showed interest and participated actively in the discussions about teacher knowledge, classroom practice and the use of manipulatives, and she, without hesitation, signed up to participate in the research project.

Ms Bohata is a mathematics teacher at school D. She is in her late 40s and has been a teacher for 26 years. She holds a Primary Teachers Diploma in which she did Mathematics and Natural Science as two of her subjects. For the past 26 years she has been teaching only mathematics in different grades in both the Intermediate and the Senior Phase respectively. She has taught Grade 6 mathematics for 4 years. In addition to mathematics, she is currently teaching Life skills to Grade 4. Her Grade 6 class consists of 57 mixed ability learners mostly from disadvantaged backgrounds.

Her desire was to see African children progressing and, as she pointed out; teaching is where you are able to see the result of the seed you sow:

> Most children do have a problem with mathematics and I joined teaching to make it a point that Black children succeed in mathematics and become better people in the technical world and in life in general. I was also inspired by my high school teacher in how she related to us as students. She was more of a mother to us; she showed us love and care even in terms of our social problems. For me teaching became a platform/career where I could become a social worker, a nurse, a teacher, etc.

Ms Bohata indicated that she was not trained as a mathematics teacher (specialisation) but developed a love for mathematics through the support she got from the Master Mathematics programme that was offered by her subject advisor. About her high school mathematics she remarked:

> I never did mathematics at matric (Grade 12); I only did it up to Junior Certificate (JC) level (an equivalent of the current Grade 9). I did mathematics only up to JC because we were always reminded that mathematics is not for the faint hearted. We were scared off from mathematics because everybody believed that mathematics teachers were the strictest in school, they were hard workers.

As to how she got to teach mathematics she replied thus:

> ‘No one was prepared at the school to take mathematics as everybody regarded it as a difficult subject. The principal then requested me to teach only mathematics in the school, from Grade 7 to 8’.

It is apparent that the principal had trust and confidence in Ms Bohata to teach mathematics to the highest grades. While other teachers were allotted more than one subject to teach, she was only responsible for teaching mathematics at those levels.

4.3.2 Ms Bohata's Knowledge of Mathematics

4.3.2.1 Knowledge of mathematics curriculum

When asked as to how she would describe the most important things she is trying to accomplish across the year with her Grade 6 learners in mathematics, Ms Bohata demonstrated good knowledge of the curriculum prescribed for the grade she is teaching. For example, she mentioned measurement as one of the content areas in Grade 6. She pointed out that in this content area her learners learn how to measure area, perimeter, volume, angles, etc. Children also learn the skill of converting units, e.g. from grams to kilograms, centimetres to kilometres, estimation of time, temperature, distance and the terminology involved. She wanted her learners to know measurement in order to differentiate objects, to learn punctuality, estimation and accuracy and to deal with everyday challenges relating to time, distance, area and temperature.

4.3.2.2 Ms Bohata's knowledge of mathematics

Data on Ms Bohata's knowledge of mathematics were gathered mainly through two classroom observations that were conducted during the month of June 2013. During these classroom observations, which took place in a mathematics laboratory, lessons were video recorded, immediately transcribed and observation notes taken. On both occasions, Ms Bohata presented lessons on 3D shapes. Learners entered the classroom hastily and all went to their respective tables where they were seated around tables and faced the white board located at the front of the classroom.

a) Connections among mathematical topics, concepts and ideas

As soon as the learners had settled at their tables, Ms Bohata greeted the class and outlined the objective of her lesson on 3D shapes thus: 'Today we are going to do different kinds of 3D shapes'. She then posed some oral questions as an introduction to the lesson and also to link the lesson topic to the learners' pre-knowledge. In this section of the lesson, which lasted for about 15 minutes, she requested learners to give examples of 2D shapes and to tell how many sides each of those shapes has. She drew a table on the board in which she recorded the learners' responses.

Ms Bohata started her lesson by outlining the lesson objective as ‘to do different kinds of 3D shapes’. Her intention was probably to ensure that learners know exactly what the lesson is all about and what they are expected to know at the end of the lesson. However, her lesson objective remained ambiguous as she did not clearly articulate what is entailed in ‘doing different kinds of 3D shapes’, i.e. whether it was about recognising, visualising, naming, describing, classifying etc. the 3D objects as prescribed in Section 3.2 of the Intermediate Phase CAPS document under Grade 6.

In her introduction of the lesson, she asked relevant oral questions that established a clear connection between what learners already know and the new topic. What is also commendable is that she did not only ask for the names of the shapes, she also wanted to establish if the learners understand the properties of 2D shapes. It only became apparent as the lesson progressed that the topic was about the different kinds of prisms as a category of 3D shapes or objects. However, Ms. Bohata seemed to be using 3D shapes and prisms interchangeably as can be noted from the table, among others. As will be noted in the sections below, this misconception pervaded other aspects of her lesson as well.

Scholars seem to be in agreement that conceptual knowledge of mathematics goes beyond mere knowledge of facts, principles and procedures; it involves knowledge and understanding of relationships among them (e.g. Eisenhart, Borko, Underhill, Brown, Jones and Agard 1993; Ma 2010; Ball 1988). Ms. Bohata had grounded her lesson on the knowledge of 2D shapes and their properties. She also made connections between the topic and the idea of patterns to promote meaningful understanding of and substantive logic behind the properties of 3D shapes. However, in both cases Ms Bohata could neither make the connections explicit nor provide reasons for the connections. She could have used the opportunity to explain, for example how polyhedrons (a family of 3D shapes) are made up of polygons (a family of 2D shapes) and how the properties of the latter have a bearing on the names and the properties of 3D shapes. For example, a pentagonal prism has a pentagon as its base and has 5 lateral faces. Ms Bohata’s inability to explicitly locate 3D shapes within the bigger picture raises questions as to whether the connections were deliberate or perhaps taken from curriculum material as is. This demonstrated her insufficient knowledge of the topic she was going to teach.

b) Handling learners' responses

There is a recognition, among scholars at least, that responding to children's ideas (anticipated or unanticipated) is one of the tasks teachers perform in the classroom (Ball 1988; Hill *et al*. 2008a; Ma 2010; Rowland *et al*. 2003, 2005; Shulman 1986). In the lesson segment below, Ms Bohata asked learners to give her examples of 2D shapes and one learner's response unsettled Ms Bohata a little bit. This is probably because the response did not seem to match her expectations.

Lesson Segment: 7

The context was a lesson where Ms Bohata introduced 3D shapes and had asked learners to give her examples of 2D shapes. Five learners had already given her five examples and she was looking for additional examples.

T: Give another shape, a 2D shape!

L5: Oval, teacher!

T: (Really looking unsettled and hesitant to write the response down.) Rectangle!

(She wrote rectangle on the list of examples that had already been provided by the learners and totally ignored the 6th learner's response)

In another section of the lesson Ms Bohata went on asking almost the same oral question regarding the prism name and the 2D shape it is made from and then had the whole class repeat the prism names. Ms Bohata was comfortable with the responses until she pointed at another group, sitting at a round table: 'How many sides do you have? The group responded that they had seven sides and that they had made a circle. Ms Bohata responded thus: 'It is a circle.... okay. Leave that one, fast! Make a triangular prism; just do a triangle for us!'

Ms Bohata's learners generally demonstrated the knowledge of the names of the 2D and 3D shapes and their corresponding properties. This seemed to be only limited to what she anticipated. However, teachers need to be ready and prepared, when appropriate, to deviate from a set agenda when the lesson was being prepared (Rowland, Huckstep and Thwaites 2004: 123). In both cases, Ms Bohata blatantly brushed aside the learners' unanticipated responses. For example, in the one segment a learner gave an example of an oval but instead of giving a response to the learner's answer, she ignored the learner and wrote her own example of a 2D shape, i.e. rectangle. It needs to be recognised that both responses were correct and actually called for Ms Bohata to be unambiguous about the category of 2D and 3D shapes her lesson focused on. This gave the impression that Ms Bohata wanted learners to respond according to her own expectations, i.e. what she knows and anticipates. Liping Ma (2010: 3) conjectures that what teachers expect learners to know is an indictment of their own knowledge. Her handling of the two responses raised questions about the depth and breadth of the teacher's knowledge. By ignoring the responses, she missed the opportunity to underscore another important element of the definition and/or properties of 2D shapes and 3D objects pertaining to flat and curved surfaces. She could have used the models to show the difference between a model with curved surfaces and a model with flat surfaces. Her ignorance of these responses also creates the impression that the activity might not have been well planned otherwise the inclusion of a circular object and an oval shape could have enriched the lesson further rather than be regarded as nuisances.

c) Multiple representations and contexts to complement manipulatives

The use of representations to model mathematical concepts, procedures and ideas has been touted as one of the teaching strategies to help make mathematics accessible and comprehensible to learners (e.g. Ball 1988a:22; Hill *et al.* 2008a:431; Shulman 1986). What logically flows from this claim is that the use of multiple representations and contexts will help make mathematics more accessible and comprehensible.

As evidenced in the discussions above and in the remainder of this section, Ms Bohata made use of manipulatives throughout her lessons. To complement manipulatives, she also made use of a table not only to display data but also to teach the subject matter. Ball (2003: 3) argues that teachers need to use representations skilfully, choose them appropriately and carefully map between a given representation, the numbers involved, and the operations and processes being modelled. The tables above helped Ms Bohata to carefully guide learners to make connections between the 2D shapes, their properties (sides) and their corresponding prisms, prism names and the prism properties. In this way, Ms Bohata made a deliberate attempt to make the knowledge of the properties of various kinds of prisms accessible and comprehensible to learners.

Ms Bohata also made use of the environment in the learners' context, i.e. the mathematics laboratory as representations of 3D objects. In the main activity of her lesson, she started with another set of oral questions that were meant to further strengthen and enhance the connection between 2D shapes and 3D objects in the mathematics laboratory environment.

Lesson Segment: 8

T: If you look around here in the mathematics laboratory, can you show me objects that have the shapes that you mentioned?

L1: A square, teacher, and points at the computer screen on the table

T: Is this a square? (Also pointing at the computer screen), No it is not a square!

(Points at another group.) That table!

L2: (Pointing at a square table.) This is a square, teacher!

T: Yes, it is a square. Why do you say it is a square, what are the properties of a square?

L3: Four sides are equal.

T: Other properties?

L4: Four vertices.

T: (Ignores the response.) One of the properties of a square is that it has 4 right angles at the corners! Another shape!

L5: (Pointing at the door.) A rectangle, teacher.

T: Why do you say a door is a rectangle?

L6: Two sides are equal.

The lesson continued with other shapes, i.e. triangle, octagon, pentagon and hexagon. It became clear that learners were able to correctly visualise and recognise the 2D shapes in various objects in the mathematics laboratory. However, Ms Bohata's success in making mathematics more accessible and comprehensible by using multiple representations could have been limited by her insufficient knowledge of the topic itself. .

d) The use of correct mathematical language

The use of correct and standard mathematical language and terminology by both the teacher and the learners is equally important. This did not seem to be of concern to Ms Bohata as she continued to arbitrarily use mathematical terminology and language involved in the topic. For example, as mentioned in the above section, she used the words 3D shapes and prisms interchangeably, giving the impression that the two are synonyms. Had she used the correct language and terminology, she could have easily been able to deal with the two learners' responses mentioned earlier on regarding an oval shape as another type of a 2D shape and a circle as a base of a cylinder (3D object/shape).

In another section of the lesson, one learner associated a door with a rectangle and a follow up question by Ms Bohata was: 'Why do you say a door is a rectangle?' This is the point where Ms Bohata could have made a clear distinction between a 'rectangle' as a noun for a flat shape and 'rectangular' as an adjective that describes the shape of an object. This would have prepared the learners to use the correct terminology, e.g. with regard to the prism names in the subsequent sections of the lesson.

Another incorrect use of mathematical terminology was apparent in the lesson segment above where Ms Bohata ignored the response that a square has four vertices. Instead of getting to the bottom of the misconception, she replaces vertices with right angles or corners, thereby giving the wrong impression that a vertex is synonymous with a right angle. This is another missed opportunity where she could have taken the opportunity to explain terminology such as vertex, edge, line, angle and other associated concepts.

e) Comprehensive knowledge of manipulatives and mathematical ideas embedded in them

The activity above was followed by an activity in which learners extracted the properties of 3D objects in respect of the number of faces, edges and vertices from their models to complete the table below. She started by directing them to open boxes of manipulatives on their tables and provided a brief explanation of parts, e.g. zoom struts and nodes in the box and how they are used to make structures. She instructed each group to build a prism similar to the shape of their tables using manipulatives.

Learners made different structures to represent different prisms. Learners were asked to name their respective prisms (which were written in the table under the 3D shape column) and to also match it with the list of 2D shapes already listed in the table. As and when groups presented their respective prisms, they also gave the properties of those prisms as embedded in their respective structures, i.e. the number of faces, vertices and edges that make up the structure as shown in the table below:

2D SHAPE	SIDES	3D SHAPE	FACES	EDGES	VERTICES
Triangle	3	Triangular Prism	5	9	6
Square	4	Cube	6	12	8
Hexagon	6	Hexagonal Prism	8	18	12
Pentagon	5	Pentagonal Prism	7	15	10
Octagon	8	Octagonal Prism	10	24	16
Rectangle	4	Rectangular Prism	6	12	8

Table 3: Properties of 3D shapes (faces, edges and vertices)

Lesson Segment: 9

In this lesson segment Ms Bohata guided learners to physically experience the mathematics embedded in the manipulatives beyond just making a model. Responses from this section were used to complete the table below.

T: In a cube, how many struts did you use, the zoom struts?

Ls: (Counting) 12, teacher!

T: Look at our table on the board, where do we put the 12 or is it already there?

Ls: No, teacher. (Chorus)

T: I want you to write 12 struts in your books next to cube. Now where is the pentagon? (The group that had made a pentagonal prism brings the structure to the front). How many struts did you use, ehhhh... I mean, how many zoom struts?

Ls: (Counting) 15, teacher!

T: In our table on the board now, where is the pentagon (moving a pointer along the pentagon row to the last three columns) do we write 15 or is it already there?

Ls: 15 is there.

T: Where, under which column in our table? (She draws another outer column with the heading 'struts.)

Ls: 15 is there, teacher, under edges!

T: Use your own prism now, count the zoom struts you used and tell me if the number is already there

Ls: (There was now a lot of noise from discussions and counting as all groups now enthusiastically work on their own prisms, showing excitement!)

The presentations continued with each group representative showing that the number of struts is already there under edges. As learners were presenting, Ms Bohata completed the column 'struts' with their data on the total number of struts used. Once the column was complete, she again asked: 'Do we really need this column? The learners in a chorus responded: 'No, teacher!' and she asked: 'Why? Why is it not necessary?' One learner, not really sure if what she was going to say was what Ms Bohata wants: 'It is here, teacher (pointing at the 'edges' column)'. The discussions and feedback continued until the whole class was convinced that it was not necessary as the total number of struts is already represented by the number of edges. The same approach was followed in the subsequent part of the lesson to show that the number of nodes and that of vertices are the same as shown in the new columns (bolded) below:

2D Shape	Sides	3D Shape	Faces	Edges	Vertices	Struts	Nodes
Triangle	3	Triangular Prism	5	9	6	9	6
Square	4	Cube	6	12	8	12	8
Hexagon	6	Hexagonal Prism	8	18	12	18	12
Pentagon	5	Pentagonal Prism	7	15	10	15	10
Octagon	8	Octagonal Prism	10	24	16	24	16
Rectangle	4	Rectangular Prism	6	12	8	12	8

Table 4: Relationship between properties of 3D shapes and struts and nodes

It is clear from the above that Ms Bohata afforded learners the opportunity to physically use manipulatives to construct 3D models. It needs to be noted that Ms Bohata, in her approach to this section of the lesson chose not to start with the definition of concepts and terminology. Instead, she created opportunities for learners to physically experience with the building of structures that model various concepts such as the prism, base, vertex, edge, faces and sides. Her learners were able to link the 2D shape names with their corresponding prism names without necessarily having to memorise the names. In the same way, her learners also comfortably used the prism names as they could link the name with their 2D shapes. Through this activity, Ms Bohata afforded her learners the opportunity not just to name the properties of the prisms but to physically see and count the faces, edges and vertices. She could easily guide the learners to realise that the edges and the vertices are represented by the zoom struts and the nodes respectively. These physical and visual experiences made it easy for learners to complete the above table.

To complete data on the faces, Ms Bohata counted the faces with the learners. However, this was confusing as the counting was a bit haphazard because unlike with vertices and edges, she could not have anything in the structure itself to represent the faces or to associate the faces with. At this point the researcher acting as a participant observer in the study, intervened by asking the learners if in a triangular prism for example, there were any other shapes. Learners looked at the triangular prism model, which the researcher was holding in front of the class and realised that in the triangular prism there were triangles (on top and beneath) and rectangles (on the sides). To clear the confusion when counting the number of faces in a triangular prism, learners were asked to count the number of triangles, that of rectangles and lastly add the totals up to get the total number of faces. It is at this point that Ms Bohata's approach took a different turn as demonstrated in the lesson segment below to explain the faces:

Lesson Segment: 10

T: Now tell me what shapes you see in your structure (adding a new column in the table – shapes in the prism). Look at your prism and tell us which shapes make up your prism and how many. Yeh... who wants to come?

L1: (Looking more absorbed and excited). Here, teacher, a rectangular prism.

T: What shapes, show us!

L2: (Picks up their structure and jumps to the front). This rectangle prism.... no, nx! This rectangular prism (turning it around) has a rectangle and a square.

T: (Writing rectangle and square in the new column) How many? Count them.

L2: (Pointing at the shapes and counting). One, two rectangles, yes, two rectangles! 1,2,3,4 squares!

T: (Writing in the table) 2 rectangles and 4 squares! This is what we have in a rectangular prism.

Ls: (From another group) Teacher, teacher! A pentagon!

T: Come, what shapes?

L4: (Turning the prism and pointing at the openings as he counts quietly.) Two pentagons and five rectangles!

T: Is he right? Let's see! (Taking the prism from the learner) One, two pentagons and one, two, three, four, five rectangles! Correct neh? Good! (Writing the totals in the table below).

Other groups presented their findings to complete the new column (bold) in the table below:

2D SHAPE	SIDES	3D SHAPE	**SHAPES**	FACES	EDGES	VERTICES
Triangle	3	Triangular Prism	**2 Triangles 3 Rectangles**	5	9	6
Square	4	Cube	**2 squares 4 Rectangles**	6	12	8
Hexagon	6	Hexagonal Prism	**2 Hexagons 6 Rectangles**	8	18	12
Pentagon	5	Pentagonal Prism	**2 Pentagons 5 Rectangles**	7	15	10
Octagon	8	Octagonal Prism	**2 Octagon 8 Rectangles**	10	24	16
Rectangle	4	Rectangular Prism	**2 Rectangle 4 Squares**	6	12	8
20 sided	20			-------	-------	-------

Table 5: Shapes (name and number) that make up 3D shapes

Ms Bohata provided affordances that helped her learners to understand fundamental mathematical concepts that underlie the properties of the prisms which in turn contributed to their conceptual understanding of these properties. Learners were able to easily complete the remaining data in the 4th column, by physically manipulating their structures. To conclude the lesson, the researcher asked the learners as to how many faces are in a prism made from a 20 sided 2D shape. The learners replied in a chorus: '22 Ma'am! Ms Bohata then gave learners homework to complete the remaining data on the 20 sided 2D shape.

f) Abstracting mathematics from the concreteness of manipulatives

Ms Bohata's second lesson was observed a week after the intervention programme which took the form of a workshop. I need to mention that Ms Bohata's reflections during the workshop showed that her classroom practice and approach to mathematics had taken a new turn. She used her lesson on 3D shapes to demonstrate how she was able to engage learners in meaningful mathematical activities through manipulatives. It is not surprising that, in her second lesson that I observed she was more confident and her approach was more robust and flexible.

Ms Bohata's lesson was a continuation of the first lesson that I observed before the intervention programme. After orally revising information in the table she immediately drew her learners' attention to some relationships between and among the columns. She used the opportunity to help learners to easily move from concrete mathematics to more abstract mathematics:

Lesson Segment: 11

T: Look! (Pointing at the twos in the new column and underlining them as she moves her pointer down the column.) What do you see? Do you see any pattern?

Ls: All have 2, ehhhh (holding her mouth).....the number ...two shapes.

T: Which two shapes are in a triangular prism.....? What two shapes do we have? Can you see a pattern?

Ls: Two triangles.

T: (Points at the prism name and the corresponding 2 shapes). Can you see, triangular prism – two triangles, pentagonal prism – two pentagons! What about a hexagonal prism?

Ls: Two hexagons, teacher!

T: Now, in 2 minutes, tell me if we can say the same about the other prisms!

Ms Bohata successfully and creatively used the knowledge of patterns to further explore the properties of prisms. Learners looked a bit lost in the beginning, probably because they were not used to deep learning of mathematics. The integration of patterns may have also confused them because they were doing shapes and not patterns. However, this proved to be a productive activity as learners were engaged both physically and mentally. Ms Bohata wanted them also to know where the 2 comes from and through that learners managed to realise that the two shapes are the one on top and the one below (base). The following lesson segment illustrates how

Ms Bohata used leading questions to entrench the concept of the base and to explain why we need two bases:

Lesson Segment: 12

T: Can you close a triangular prism with a rectangle? (Demonstrating in front of the class)

Ls: No, teacher. (In a chorus)

T: Why, why can't we use a triangle?

Ls: Ha e no kwalega, teacher! (It won't close properly.)

T: Why will it not close properly, why esa kwalege sentle?

Ls: Ha e tshwane le base teacher (It is not the same shape as the base.)

T: What shape can close properly here? (Pointing at the top of the triangular structure)

Ls: (In a chorus). Ke triangele, teacher! (It is a triangle teacher!)

T: So, gore e kwalege sentle (So, for it to be properly closed), the base and the top must be of the same shape?

Ls: (In a chorus). Yes, teacher!

T: Now can you see where the 2 comes from, can you see why we have two triangles?

Ls: Yes, teacher, base le sekwagelo sa prism! (The base and the lid of the prism).

Although her learners responded in a chorus throughout the above lesson segment, Ms Bohata managed to carefully use leading questions and terminology that learners are familiar with to build the concept of a base and the logic behind having 2 bases that are of the same shape in a prism.

To further explore the relationships amongst the data in the table, Ms Bohata guided her learners to identify other patterns as illustrated in the segment below:

Lesson Segment: 13

T: (Pointing at the number of sides and the same number under shapes). What do you see here? Hmm.....? (Pointing at the triangular prism) 3 here and 3 here! (Pointing at the cube) 4 here and 4 here! What about the hexagonal prism?

Ls: Five, teacher.

T: What about five?

Ls: Five sides and five rectangles.

T: (Underlines the numbers 3 & 3; 4 & 4 in the table). For a pentagonal prism what number will appear twice?

Ls: 5 teacher, yes 5!

T: Five sides and what?

Ls: 5 rectangles (not very sure).

T: Now can we say the same about other prisms?

Ls: (Some pointing at the table on the board as if they were counting). Yes, hexagon, hexagon, 5 sides and 5 hexagons, teacher!

T: Good, now you know where these (pointing at the number of rectangles in the table) come from. They come from what?

Ls: (pointing at the sides of a triangular structure). From here, the sides!

T: Yes, the number of the sides of the base determines the number of rectangles in a prism!

It was amazing to see how enthusiastic the learners were while doing this activity. In an earlier discussion with Ms Bohata, she indicated how learners are easily distracted when using manipulatives:

> And ha ba tloha feela ba etsa a soccer ball – ba tlabe ba di kopantse ba entse soccer ball – ha o re o sheba di table kaofela di tletse soccer ball – ka di frame works tseo – ke yona ya pele e tlang ka di hloohong tsa bona – ha baqala ba di

tshwara feela se ba di connecta ba etsa bolo. (Just at the outset they will be making soccer balls. When you look around all tables will be filled with soccer balls. With those frameworks, that is the first thing that comes to their minds, just when they touch them, they start to connect and make soccer balls)

Some researchers have noted that manipulatives are often regarded as play objects and as time wasting (e.g. Kelly 2006; Moyer 2001; Moyer & Jones 2004). However, the claim above depicts the significance and influence of the learners' immediate background to learning. The soccer ball appeared to be the most influential perhaps because of the perception that manipulatives are for playing just as the soccer ball is. The teacher seems to have used this knowledge to influence her learners' thinking about shapes to other things/structures. It needs to be recognised that the claim above was also made by other participants in the study. However, during the two lesson observations in Ms Bohata's classroom, this could not be verified. This could probably be attributed to the nature and quality of tasks that learners were engaged in. As is clear from the activities that Ms Bohata exposed her learners to, the activities were exciting and at the same time mentally challenging.

The above was further illustrated in the conclusion of her lesson where she flexibly and creatively led her learners to identify the patterns within and between the columns. Ms Bohata uses the table above to prepare her learners for more abstract mathematics:

Lesson Segment: 14

T: (Pointing at the table). In our next lesson, I want you to tell me if there is anything you notice about the numbers in the columns 6 and 7. Let's look at these (Pointing at the column 'edges'). She reads out loudly with learners: 9, 12, 15, 18 ... The next column; 6, 8, 10, 12, 16... Do you still remember multiples of numbers?

Ls: Yes, teacher!

In the interview I had with Ms Bohata after the lesson, I wanted to establish what her intentions are with the task above. She excitedly indicated how mathematically rich the table she used is. Her intention was to use the same table in the next lesson to support learners to identify the patterns and to generate rules. These rules would help them to determine the number of faces, edges and vertices without the support of manipulatives. This demonstrates Ms Bohata's bold approach and determination to engage her learners in more challenging and abstract mathematics.

4.3.3 Ms Bohata's Classroom Practice

4.3.3.1 Teaching approach

Throughout the lesson, Ms Bohata's approach has been characterised by facilitation of learning rather that direct teaching. For example, in both lessons, she was moving from one group to the other, explaining and giving support as learners were doing activities. She was consistently guiding her learners in their activities to understand the concepts and processes involved in the learning of 3D shapes. In general, her class was highly activity based. Active classrooms are about active engagement of learners in activities that will assist them to construct mathematical concepts, activities that require reasoning and creativity, gathering and applying information, discovering, and communicating ideas (Ball 1993; Lampert 1991). In the interview I conducted with her she remarked:

> I don't enjoy classrooms where children are just sitting and listening to me, I get bored. I also like the fact that in mathematics, children ask questions that challenge you as a teacher.

Perhaps her own experience of mathematics as a student as described below also explains her approach better.

> I had a good mathematics lecturer. We used to work as a team of students. We enjoyed sharing and explaining mathematics to other students. At college our lecturer created opportunities for us to do mathematics on our own, discovering things for ourselves and coming up with different ways of doing mathematics. Emphasis was mainly on the use of concrete objects, manipulatives to entrench the notion of teaching from concrete to abstract.

Although these seem to be progressive views about mathematics, they were not particularly apparent in Ms Bohata's own classroom practices, especially opportunities for learners to explain and reason out their responses. Ms Bohata's lecturer seemed to have had the conception that teacher-centred learning environments were oppressive on their (Ms Bohata and others) part of the learners. Also, the lecturer seemed to have had the view that such oppressive learning environments were not sustainable and as such needed to be transformed. It is apparent that the lecturer who Ms Bohata holds in high esteem (a good mathematics lecturer) viewed emancipation as characterised by social construction of knowledge through critical and participatory learning. In view of the fact that Ms Bohata was at college then, being prepared for future practice as a teacher, it can be argued that the transformation of mathematics learning environments may not be divorced from emancipation of learners and teachers alike.

4.3.3.2 Assessment and evaluation practices

As evidenced in the above discussions, oral questions characterised Ms Bohata's lessons. What was apparent in her oral questions was that her questions were not only confined to the 'what', she also made attempts to find the reasons behind the learners' responses. For example, in almost every lesson segments in the above sections, she asked 'Why do you say so?' However, she did not really ask questions that assess deeper understanding of mathematical ideas and concepts. Her questions were very shallow and rhetoric at times. For example, in a lesson segment where learners were to identify objects in the mathematics laboratory, she asked questions that required simple recall of the properties of 2D shapes when in actual fact she wanted them to refer to properties of 3D objects.

Lesson Segment: 15

T: If you look around here in the mathematics laboratory, can you show me objects that have the shapes that you mentioned?

L1: A square, teacher (points at the computer screen on the table).

T: Is this a square? (Also pointing at the computer screen). No it is not a square!

(Points at another group) That table!

L2: (Pointing at a table with a square shaped top). This is a square, teacher!

T: Yes, it is a square. Why do you say it is a square, what are the properties of a square?

L3: Four sides are equal.

This gave the impression that the questions were not well thought out - raising concerns regarding thorough lesson preparation in order to cater for assessment of learners' different cognitive levels. The same questions were asked in the introductory section of the lesson, i.e. the 2D shape and their properties. For example, the choice of questions that relate to the objects' lateral faces and the bases rather than the sides of the 2D shapes could have further clarified the properties of the prisms.

4.3.4 Ms Bohata: The Story of the Use of Manipulatives in (re)shaping Mathematical knowledge

The case of Ms. Bohata tells a story of how manipulatives can help (re)shape teachers' mathematical knowledge. Mathematical knowledge that was apparent in both lessons was investigated using the following two MKT categories of Ball and her colleagues a) common content knowledge (CCK), and b) Specialised Content Knowledge (SCK). While the former category is held by teachers in common with other educated people, the latter category is typically needed for teaching, both categories are essential as they support teachers in their mathematical work in the classroom.

Ms Bohata was able to draw on her CCK in her lessons in order to elicit the examples, names, meaning and properties of 2D and 3D shapes, polygons and prisms in particular. However, when she asked learners to give examples of 2D shapes, learners also gave an 'oval' and a 'circle' as examples. Ms Bohata had to draw on her CCK in order to decide if the examples were correct or not. However, what was apparent in her first lesson was her decision to ignore the responses, which seemed to suggest some gaps in her common content knowledge. She missed the opportunity to explain the different classifications of 2D and 3D shapes and some key properties of those shapes. The working definition of examples of polygons she used fell short of key properties such as straight lines, closed shape and plain figure. As a result, she missed the opportunity to establish a working definition of examples of prisms through the use of manipulatives and/or 3D objects from learners' environment. Ms Bohata's inability to explicitly locate prisms within the bigger picture of 3D shapes raised questions as to whether the connections she made between the polygons and prisms were deliberate or perhaps taken from some curriculum material as is. Once again, to explicitly establish the connections between the properties and names of 2D shapes and those of 3D objects. This demonstrated her insufficient common knowledge of the topic she was going to teach.

There were also moments when Ms Bohata needed to appeal to her SCK in her lessons. First and foremost, her handling of the two responses above raised questions about the depth and breadth of her knowledge of mathematics. In her use of multiple representations of 3D objects, Ms Bohata was able to a certain extent, to operationalise her SCK of mathematics to make the necessary connections between polygons and prisms, thus making the knowledge of the properties of various kinds of prisms accessible and comprehensible to her learners. Her constant usage of definitions that are not mathematically accurate also appealed to her SCK. On numerous occasions, Ms Bohata used the terms 3D and prisms interchangeably, the misconception which pervaded other aspects of her lesson. She missed another opportunity to explain different classifications of 3D objects of which prisms would be one. The ability to identify and elicit good and relevant examples of 3D objects in the environment also requires SCK. Although Ms Bohata only referred to examples of 3D objects in the mathematics laboratory environment, she did not explicitly use the opportunity to emphasise the properties of 3D objects. For example, when asked to identify the shapes approximated by objects in the lab, learners pointed to the computer screen and the tables, all of which have rounded edges and corners. Ms Bohata needed her SCK to put emphasis on accurate definitions of vertices, edges, and faces. This was another opportunity missed, where her SCK could have been influenced by the use of manipulatives and the objects in the lab.

Strong elements of unequal power relations that exist in Ms Bohata's classroom interactions with her learners were prominent in her first lesson. This was particularly evidenced by how she handled learners' responses. Such moments were reflective of the fact that authority rests exclusively with her. By ignoring and dismissing learners' responses, she actually used her authority to deny learners the opportunity to access and comprehend mathematical knowledge and concepts in geometry.

4.4 THE STORY OF MR KOPUNG

4.4.1 Background

As described earlier, I first met Mr Kopung during a mathematics cluster meeting that was conducted at the beginning of the study. The cluster meeting focused particularly on schools that have mathematics laboratories in the one district of the province. He showed interest and participated actively in the discussions about teachers' knowledge, classroom practice and the use of manipulatives. He accepted the request to participate in the research project. Of the four key participants in the study, he showed the most commitment to and interest in the study, as well as in being observed. He even invited the research group to participate in his lessons during the winter school holidays, when he provided extra tuition to learners, a practice which is generally uncommon with most teachers in South Africa.

Mr Kopung is a mathematics teacher at school C. He is in his late 40s and has been a teacher for 24 years. Mr Kopung holds a Primary Teachers Diploma (PTD) in which he did Mathematics and Natural Science for the Intermediate Phase level, i.e. Grades 4–7, catering for 10–13 year olds. Whereas the PTD is a three year qualification for full time candidates, Mr Kopung completed his PTD qualification in six years, as he was a part time candidate. He taught at a farm school at the same time as he was studying to become a qualified teacher. Of the four participants in this study, Mr Kopung is the only one to have attempted mathematics at university level, even though he later discontinued after failing Calculus on first year level.

Mr Kopung has been teaching mathematics for the past 24 years, to different grades, in the Intermediate Phase. At the time of the study, he was teaching Mathematics and Natural Science in Grade 6 (12 year olds), to 38 learners with mixed abilities and who come from disadvantaged backgrounds.

When asked about how he got to teach mathematics at the school, Mr Kopung attributed it to his commitment, hard work, experience and knowledge of mathematics from high school, as well as the performance of his learners:

> I was good in mathematics at the high school level and was trained as a mathematics teacher (specialisation) but developed the love for mathematics through my teaching experience. We used to have common marking centres for Standard 5 (equivalent of the current Grade 7) at a cluster level and because of the outstanding performance of my learners and the shortage of mathematics teachers; I was identified by other teachers and their principals as the best mathematics teacher.

As described earlier, Mr Kopung began his teaching career in a farm school setting, a complex situation wherein one class consisted of multiple grades. In those years, it was every South African farm school teacher's dream to secure employment at a township school (as opposed to the farm school conditions). In that context, therefore, the outstanding performance of Mr Kopung's learners may have helped to earn him a position within the township school. Since his recruitment to the township school, he has been teaching mathematics at the highest level, i.e. at Grade 7, the final grade of the primary schooling system. This may be an indication of the confidence that his principal and the other teachers have in him. What also stands out is that despite the historic and deliberate effort by the apartheid system of education, the less privileged teachers like Mr Kopung could still work hard, and facilitate participatory learning environments that accommodated diverse skills, and levels of development of learners.

4.4.2 Mr Kopung's Knowledge about Mathematics

4.4.2.1 Views about mathematics

Mr Kopung views mathematics as a subject that does not allow passiveness; he strongly believes that learners have to be actively involved in mathematics lessons. As he explained during the interview:

> I also like the fact that mathematics is a challenging subject; it makes you to think and to be broad minded. It is not like History where you cannot dispute the fact that Jan van Riebeck arrived in South Africa in 1652, it is just like that! In Mathematics there are always different ways of approaching a problem, looking at mathematics problems from different perspectives. Mathematics keeps you on your toes, and with the introduction of the Annual National Assessments and the common tasks, you cannot relax. You want to sharpen your skills, teaching methods and knowledge in mathematics in order to be competitive and keep abreast of the development in the subject.

Mr Kopung believes that problem solving is the heart of mathematics. To him, mathematics is a dynamic and continually expanding field of human creation and invention. What is particularly striking about his views of mathematics, perhaps in contrast with the other participants in the study, is that he sees mathematics as challenging to both the learners and the teachers and therefore requires different approaches and continuous professional development.

4.4.2.2 Views about mathematics teaching and learning

Mr Kopung's views about mathematics teaching and learning seemed to correlate with his views about mathematics. As he argued, mathematics does not allow for passiveness. This indicates that he views mathematics teaching as the creation of opportunities for learners to be actively involved in the learning process in general and in challenging mathematical processes underlying problem solving in particular. His example of History (as had been taught to him during the apartheid years) also demonstrated his understanding of mathematics teaching as going beyond telling learners facts but engaging them in mathematical processes of argumentation, explanation, proof, verification, and justification.

His views about mathematics also seemed to be consistent with his views about good mathematics teaching as demonstrated by how he characterised one teacher Mr **XX**, from whom he and other mathematics teachers at the school draw their inspiration, and whom they emulate as a good mathematics teacher:

> He is hard working, knowledgeable and passionate about mathematics. He uses different approaches to the same problem, for example when teaching fractions; he uses the idea of a family to explain the idea of an LCM when adding and subtracting fractions with different denominators. He knows high school mathematics and always makes connection of mathematical ideas of high school.

Similarly, Mr Kopung conceived of mathematics learning in the same terms where problem solving and being actively involved were important. Asked about his own experiences of mathematics at school he elaborated thus:

> Memorisation of multiplication tables, counting using all sorts of things – stones, sticks. Being called to the chalkboard to show and explain every step of the sum. I think this is important in maths for children to understand the steps, methods and procedures they follow to arrive at a solution. We used to love that and that is also how we learned multiples of numbers.

It is also important to note that although Mr Kopung sees problem solving as central to mathematics teaching and learning, he also has a place for memorisation in his mathematics classes. In a follow up interview when asked if memorisation is still relevant, Mr Kopung argued this way about memorisation:

> Yes, Ma'am! The CAPS requires learners to do mental maths in every lesson and I've seen how this has helped them especially with multiplication. When you do word problems with multiplication, it helps them to finish quickly.

My own interpretation of his argument is that memorisation of multiplication tables enhances fluency in mental mathematics. As he explained, when learners solve problems, they have time to focus on understanding the problem and not on multiplication facts. His views seem to be consistent with the Chinese educators' view that memorization does not necessarily lead to rote learning; but instead can be used to deepen understanding (Marton et al. in Cai *et al*. 2009: 4).

4.4.3 Mr Kopung's Knowledge of The Curriculum

As with the other participants, Mr Kopung seemed to be fairly conversant with the general mathematics content areas of Grade 6 as prescribed in the CAPS policy document for the Intermediate Phase. In the interview question in which he was asked to explain, in writing, the entailments of the Grade 6 mathematics syllabus he mentioned the following five content areas that form part of the Grade 6 curriculum: a) Data Handling, b) Measurement, c) Number and Operations, d) Space and Shape, and e) Patterns. Asked what he hoped to achieve with the content areas he mentioned, he responded as follows, mentioning only three areas:

a) Measurement: Mr Kopung listed the following topics that need to be covered under this content area i) perimeter, ii) area, iii) volume, iv) mass, and v) length. In each of these topics he wanted his learners to know how to measure accurately, as well as to estimate and use correct units of measurement. He indicated that these skills are important because one needs them in life.

b) Patterns: According to Mr Kopung, this content area involves the identification of geometric and number patterns. Learners need to develop the skill to spot and complete patterns. They must also be able to generate a general rule that will apply to all the terms in a sequence, for example, if learners are given an input and an output they must be able to formulate an equation or formula. This is important for learners because, if they know the rule, it saves them time because they can just substitute.

c) Space and shape: Mr Kopung intimated that in this content area, learners need to know 2D and 3D shapes and their properties, symmetry and transformations. According to him, learners need to develop the skills to identify, construct, name, describe and compare shapes. Learners also have to know different forms of transformations, especially in nature.

Other data regarding Mr Kopung's knowledge of the curriculum were also gathered during the 1st activity of his lesson presentation, in which he expected learners, inter alia, to compare and define a rectangle and a square respectively. The CAPS for Grade 6 mathematics in South Africa recommends the following four ways of comparing shapes: a) Checking whether they have straight or curved sides (closed with curved sides only, closed with curved and straight sides, closed with straight sides only), b) Closed shapes according to the number of sides, c) Looking at the length of their sides (square & rectangle) and d) Looking at the size of their angles. However, in his definition of the two 2D shapes, it was apparent that Mr Kopung used only one of the four ways to compare the shapes, i.e. the length of their sides. His description of the two shapes, i.e. a) A rectangle has two opposite sides that are equal; b) A square has all sides that are equal, omitted other key features relating to the sides (curved or straight), the angle sizes and the shapes (closed or unclosed). It needs to be noted that the two descriptions are not different from those of a parallelogram and a rhombus respectively. Consideration of these CAPS recommendations could have also helped Mr Kopung to guide his learners in better understanding the circle concept, as discussed in the section below. These omissions demonstrated the gaps in Mr Kopung's knowledge of the CAPS specifics in geometry.

4.4.4 Mr Kopung's Classroom Practice

Although two lessons of Mr Kopung were observed during the first two consecutive weeks in the month of June in 2013, data regarding his moment to moment decisions that informed his classroom practice were gathered mainly during the first lesson observation. This was due to the fact that the first lesson was conducted during a double period which lasted for 90 minutes while the second lesson lasted for only 45 minutes, a single period. Furthermore, the second lesson was conducted during the winter examinations period and was not as rich in detail because the focus of the lesson was on revision for examination purposes.

4.4.4.1 Specifying learning goals

Once the learners were settled, Mr Kopung greeted the class and introduced the research team to the learners. He started his lesson by outlining the learning goal to the class thus: 'Today we are going to talk about Euler's formula in relation to 3D shapes; we are going to verify Euler's formula.' He went on to project a slide with the following prerequisite concepts and skills on the white board: a) 2D shapes b) 3D solids, c) edges, vertices and faces of a solid and d) Euler's formula.

There is widespread acknowledgement that the skill of specifying learning goals is a foundational skill needed for studying and improving teaching (e.g. Morris, Hiebert & Spitzer 2009: 495). Clarity about learning goals requires, among others, the ability to explicitly specify and unpack the concepts and skills to be learned into their constituent parts. However, in this lesson, there was no explicit clarity on the expected learning (skills and concepts) as was apparent in Mr Kopung's ambiguous articulation of the learning goal. He referred to 'talking' about and 'verifying' the formula as if they are synonymous. Clarity about the concepts and skills to be learned depends on the ability to unpack the learning goal into its constituent parts. Although there was an attempt by Mr Kopung to unpack the learning goal into its constituent parts, the latter were also not explicit enough in relation to exactly what learning was expected for this section. For example, with regard to the constituent '3D solids' mentioned above, it was not very clear as to what skills or concepts were to be learned that would ultimately build towards the learning goal. Perhaps this could also be attributed to ambiguity about the learning goal itself. Such clarity is essential in the planning of activities that address these sub concepts or constituent parts, in evaluating students learning, and in looking for evidence that his learners understood each of the constituent parts. In this case, however, it only became clear during the second activity that Mr Kopung wanted his learners to construct 3D models, to name 3D shapes and to describe the properties of '3D solids', in particular, prisms.

4.4.4.2 Making explicit links between the teaching goals and classroom activities

Unpacking of the learning goals is not only essential in the planning of activities, it is also critical in the teacher's choice and sequencing of those activities. Mr Kopung's choice of activities on 2D shapes was helpful in establishing the necessary connections between the learners' prior knowledge and the new topic, as well as between geometric concepts. He began the first activity by asking a few oral questions on the properties of 2D shapes as evidenced in the lesson episode below:

Lesson Segment: 16

This lesson segment occurred in the context in which Mr Kopung asked learners to identify and name different shapes of tables in the classroom.

T: What different shapes do you see? (Standing next to a square shaped table). Tell me, the table that I'm standing next to, what shape do you see?

L1: A square, teacher.

T: How many sides does a square have?

L2: 4 sides.

T: Right, a square, 4 sides! (Standing next to a rectangular shaped table.) Now this table, what shape do you see?

L3: A rectangle, teacher!

T: A rectangle! How many sides does a rectangle have?

L4: 4 sides, teacher!

T: So, do you say it's also a square, hmm?

Ls: No, teacher, no! It is a rectangle! (Almost in a chorus)

T: So what is the difference between a square and rectangle?

L5: A square has 4 equal sides and a rectangle has only 2 sides that are equal!

T: Which of those two sides are equal?

L5: The upper one and the bottom part (pointing at the shorter sides)

T: Do you agree, class?

Ls: No, teacher!

L5: The left side and the right side! (Pointing at the longer sides)

T: Do we agree?

Ls: (Hands go down slowly and there is a sense of uncertainty.)

T: Come on, think properly! Do we agree?

L6: Both of them are equal but are not the same sizes.

T: Any other group, what do you say?

L7: Bottom part and the upper part are the same and the left and right sides are the same, so a rectangle is not the same as a square

T: In mathematics, we say the bottom side is opposite the upper side and the left side is opposite the right side. So we say the opposite sides of a rectangle are equal whereas in a square?

Ls: All sides are equal!

T: (Writes on the board). a) In a rectangle two opposite sides are equal; b) In a square all sides are equal

While this introduction was meant to establish and build on the learners' prior knowledge before they were introduced to a new topic, Mr Kopung deliberately placed due emphasis on the conceptual understanding of geometrical concepts. His teaching, in this case, showed emphasis on concept development of the properties of 2D shapes rather than on mere recalling of the properties. The choice of this simple activity on what learners already know, demonstrated the pedagogical principles that underpin his teaching, i.e. starting from the known and the simple and move to the unknown and the complex.

Although, in the activity above, Mr Kopung allowed his learners to identify 2D shapes in the environment, he did not make sufficient links between the 3D objects and real life in his second activity. For example, he only made reference to objects, i.e. tables in the classroom in relation to 2D shapes. He could perhaps have also made an explicit link between the rectangle and a rectangular prism, and a square and a cube. Mr Kopung also managed to engage his learners in some informal reasoning about what is similar/common and different between the two concepts, i.e. a square and a rectangle, an initiative which he could have escalated to the next level to show that a square is a special type of a rectangle. A similar approach with cubes and cuboids would have allowed the opportunity for further exploration of the relationship between 2D and 3D shapes.

4.4.4.3 Teaching methods and techniques

Knowing both content and how to teach is one of the domains of Deborah Ball and her colleagues' MKT framework. MKT is about the tasks that are entailed in carrying out the work of teaching. Such tasks are the function of decisions and choices made by the teacher regarding appropriate representations, sequencing of tasks, examples and non-examples to illustrate mathematical ideas and concepts, and questions to ask. Emanating from Mr Kopung's own views about mathematics, which I described earlier on, one would expect him to use teaching and learning strategies that are more challenging and engaging.

a) Questioning technique

The lesson segments used in this section would seem to suggest that Mr Kopung's teaching method to elicit and identify shapes is dominated by the questioning technique. For example, he used oral questions to establish his learners' prior knowledge of the properties of 2D shapes and to introduce the main activity of his lesson. His use of appropriate questions helped him not only to do some quick spot checks of evidence of what his learners understood at a particular point in time during the lesson, but also to facilitate the development of the concepts. Although he used appropriate questions, albeit to a limited extent, to stimulate his learners' thinking processes, he may have missed the opportunity to make visible learners' mathematical thinking processes regarding how they think about a circle, for example. This could have stimulated higher level mathematical processes such as reasoning, argumentation, and analysis. For example, in the 1st activity Mr Kopung asked learners to identify the shapes approximated by the tables in the classroom:

Lesson segment: 17

T: What shape of table do you see and why?
L1: Circle.
T: How many sides are there in a circle?
Ls: No sides.
T: No sides?
L2: One side.

While the learner was correct to name a circle as an example of a 2D shape, Mr Kopung ignored the responses in relation to the shape and thereby missed the opportunity to engage his learners in a productive mathematical debate around a circle. As Ms Bohata did in her lesson activity on 2D shapes, Mr Kopung also left the learners' responses unattended to, without giving or evaluating a mathematical explanation of the circle. This gives the impression that the two participants regard 2D shapes as synonymous with polygons only. Any response relating to 2D shapes that is not a polygon is simply ignored or not sufficiently addressed. This also gives the impression that Mr Kopung used the technique to elicit what he wants to hear and what he is comfortable with.

b) Explanations as technique

Mr Kopung used explanations of mathematical concepts and terminology to facilitate learning during the lesson. This was apparent especially where learners seemed to be struggling or where emphasis was needed to correct some misconceptions. Mr Kopung's decision to suspend a classroom discussion for more clarification and to use students' opinion to make a mathematical remark was apparent in his 1st activity. For example, as already alluded to, Mr Kopung took the time to establish if learners really understood what is different and similar between a rectangle and a square. His explanation of and emphasis on the term 'opposite' showed his knowledge of how his learners think and what possible misconceptions or errors they might commit in describing the characteristics of the two shapes. It is also critical that teachers explain mathematical terms clearly and accurately using appropriate examples and non-examples that are appropriate to the learners' level to promote their understanding of concepts and procedures. This was one of the missing pieces in Mr Kopung's classroom practice.

Mr Kopung wanted his learners to use the correct terminology such as 'opposite sides' where learners used the phrases 'bottom and upper sides' and 'right and left sides' to refer to the sides of a rectangle that are equal. In explaining the relative position of the sides, the phrase 'sides that face each other' could have been more appropriate to the learners' level before introducing the term 'opposite sides'. In this way learners would have also realised that there are two pairs of sides that face each other, which in turn could have helped to include 'two pairs of opposite sides are equal' in the description of a rectangle.

Although Mr Kopung used explanation as a technique to illuminate some concepts, his learners were hardly given the opportunity to explain their own responses. A decision to use students' opinion to make a mathematical remark is also one of the domains of MKT and was not apparent in his 1st activity. Two episodes were noted where learners' opinions warranted some explanations as to what they meant and why. For example, two learners responded that a circle has one side and a circle has no sides respectively but Mr Kopung did not give them any feedback on their responses. Another example is where one learner tried to explain his understanding of the sides of a rectangle that are equal thus: 'these are sides that are equal but not the same size'. In both examples, Mr Kopung ignored the learners' responses which in actual fact could have been explored further by allowing the learners to explain what they meant and why. For example, the learner's response in the latter example, which meant that equal pairs of sides that are not equal in size, was neither explained nor discussed. In the two episodes, Mr Kopung missed another opportunity which was necessary to stimulate mathematical debate and which could have made the learners' understanding and thinking more visible.

c) Representation as a technique

Throughout the lessons, Mr Kopung used different mathematical representations such as skeletons of 3D shapes from struts and nodes, diagrams of 3D shapes, a table for data presentation and a formula to facilitate better comprehension of 3D solids. The use of multiple representations would seem to support the Concrete to Pictorial to Abstract (C-P-A) approach as advocated by a number of scholars (e.g. Dindyal 2006: 182; Kosko & Wilkins 2010: 79; Pape & Tchoshanov 2001:125). The approach helps to promote the transition from manipulating concrete materials to creating images from the student's perception of the concept, and finally to the development or adoption of some form of symbolic notation representing the concept.

Mr Kopung used a variety of models in the 2nd activity to construct skeletons of different prisms using concrete materials. There is a general acknowledgement among scholars that concrete materials or manipulatives may also be used to facilitate deep understanding of concepts (e.g. Kosko & Wilkins 2010; Szendrei in Miranda & Adler 2010: 17; Heddens in Ojose & Sexton 2009: 5) including geometrical terms and concepts. In this regard, Belenky and Nokes (2009: 103) note that the relevant features that are central to deep understanding may be less salient and that the concrete details may distract students from these features. This implies that such relevant features, which might be obvious to the teacher, can only promote deep understanding if they are interpreted and made explicit to the learners through discussions. Comparing the use of manipulatives as a teaching strategy by US and Chinese teachers respectively, Ma (2010: 26) notes that Chinese teachers prefer to engage in discussions after the use of manipulatives. The use of concrete materials to model 3D shapes dominated the 2nd activity of Mr Kopung's lesson. He directed learners to use struts and nodes to construct models of prisms, e.g. cubes, rectangular prisms, triangular prisms, hexagonal prisms, etc. Through this activity, he was able to allow his learners to physically experience the shapes and their properties. Learners worked in small groups to construct the models and there was evidence of mathematical discussions among the learners. For example, in one group that was directed to construct a triangular prism, an argument ensued after one learner had constructed a pyramid, insisting that it was a triangular prism because all faces were made up of triangles. Instead of allowing and listening to the debate, Mr Kopung just told the group to use the triangle as a base. In another group there were arguments about the size of struts to be used in a single model. Upon hearing the arguments, Mr Kopung stopped everybody and said: ‘Construct a 3D shape using long blue struts and blue nodes’.

Mr Kopung used diagrams of prisms which he projected from a computer to the screen. The diagrams had in them some pointers to the edge, vertex and lateral faces and base. As and when learners presented data from their respective models, he projected the relevant diagram. In this way, learners were afforded the opportunity to visualise the prisms. However, there was no deliberate attempt to relate the diagrams to the models in respect of their characteristics. As a result, some learners struggled to reconcile the terms edge and strut, vertex and node and face and flat shapes in their models. As can be noticed from the table below, which was used to present data, there is no deliberate effort to link the 2D shapes with the 3D shapes, which also could have facilitated the understanding of faces as consisting of lateral, base and top views.

Mr Kopung began the second part of his main lesson by assigning a group task in which he directed learners to use struts and nodes to construct models of the listed prisms and to count the number of faces, vertices and edges to complete the table below.

PRISM NAME	FACES	EDGES	VERTICES
Triangular Prism			
Cube	6	12	8
Hexagonal Prism			
Pentagonal Prism			
Octagonal Prism			
Rectangular Prism			

Table 6: Complete the number of faces, edges and vertices

Learners actively participated in the activity as they began to extract data regarding the number of faces, vertices and edges from their models. However, as indicated above, learners struggled to reconcile the concrete and the abstract terms used, e.g.

struts and edges, a matter which was trivialised as it seemed too obvious to the teacher.

Once the groups had completed the table, Mr Kopung directed them to extract data in the table and present it in the form of a formula, in this case, Euler's formula. He directed learners to open the GSP5 & Shell computer program and to look for Euler's formula in the program. After he had opened the computer on his table, he directed his learners to the page that displayed a slide titled: 'The history of Euler' and instructed one learner from the front table to read the history of Euler from the computer screen. Mr Kopung wrote the formula on the white board and went on to explain the formula. He reminded his learners not to forget to use Euler's formula by counting the number of edges and vertices as this is the main objective of this activity. His insistence on Euler's formula created an impression that the formula is the only relationship among the features of the prisms that learners could establish and the only general rule that can be generated. Learners' knowledge and skills from the content area of patterns, functions and algebra, that is part of the Intermediate Phase Mathematics curriculum in South Africa, could have been used to allow learners to identify and describe the relationship between these features, and to generate a rule for each relationship.

It needs to be recognised that Euler's formula is not part of the Intermediate Phase mathematics curriculum in South Africa. During the interview which took place immediately after the lesson observation, Mr Kopung responded thus regarding his choice of Euler's formula: 'I want my learners to do challenging problems, to be ahead.' He further confirmed that this was an enrichment activity and that through the history of Euler; he wanted his learners to know that mathematics was done by people like themselves. After the formula was explained to the learners, he immediately spelt out the following instructions and reminders:

1. Step 1: construct a cube using long blue struts and nodes.
2. Class, we are going to prove the Euler's Formula.
3. Euler's Formula states that E=F+V-E =2
4. F stands for faces, V for vertices and E for edges!

5. Step 2: Count the number of faces and record it in the table.
6. Step 3: Count the number of vertices and record it in the table.
7. Step 4: Count the number of edges and record it in the table.
8. Step 5: Substitute the number of faces, edges and vertices from the table in the formula.
9. Verify if it gives you 2.

It was apparent in this activity that Mr Kopung wanted his learners not only to construct the models but also to abstract his predetermined mathematical relationships embedded in the models. For example, Euler's formula gives the relation between the number of vertices V, edges E, and faces F as V-E+F = 2 in polyhedrons. As Cobb, Yackel and Wood (1992: 12) observe, '... the external representation can be seen to serve as the medium through which the expert attempts to transmit his or her mathematical ways of knowing to students'. The model should not be used as a means of presenting readily apprehensible mathematical relationships but should instead be aspects of a setting in which the teacher and students explicitly negotiate their differing interpretations as they engage in mathematical activity (Cobb et al 1992: 6).

During the above activity of the main lesson, Mr Kopung displayed a strong tendency to tell and show rather than affording his learners space to interpret the models and data embedded in them (models) and to debate their findings or solutions. He drew learners' attention to the board as he explained and displayed the formula on the board.

Lesson segment: 18

This lesson segment occurred in the context of the main activity in which Mr Kopung himself used an example of a cube to find out if the formula works for a cube. He started with a set of steps to follow in order to 'prove' Euler's formula.

T: Let us prove that E = F + V – E = 2 (writing the formula on the board).

Ls: (Looking attentively and with anticipation at the board).

T: Step 1: how many struts do you have?

Ls: 12, teacher!

T: (swiftly substituting 12 in the formula on the board) Step 2: how many nodes do you have?

Ls: 8, teacher!

T: (substituting 8 in the formula on the board) Step 3: how many faces do you have?

Ls: 4, teacher!

T: No, count them. How many faces are in a cube?

Ls: 5, teacher (putting one hand on top of the cube model)

T: (pointing at the learner who said 5). Come with your cube, let's see, how you did count the faces

L: (pointing at the lateral faces and the top face) Here, teacher, 1, 2, 3, 4, 5, they are 5!

T: (pointing at the base). No! There are 6 faces. Step 5: now let's go to the formula... let's see! (Substituting 6 in the formula on the board). What do we have now? (Pointing at formula after substituting all the variables)

Ls: (In a chorus) 6 + 8 – 12, teacher!

T: How much is 6+8?

LS: 14.

T: Subtract 12 from 14….what is the answer?

Ls: It is 2! (again in a chorus).

T: It means we've proved Euler's formula! Now I want you to use other shapes to prove Euler's formula.

It was apparent from the above that the learners were still not sufficiently grounded in the concepts they were supposed to use in the formula. What Mr Kopung referred to as an ‘enrichment activity’ may have been included prematurely in the lesson. Although it is acknowledged that mathematical formulae are important as symbolic representation of concrete mathematics, accurate and correct statement of formula statements are equally important. Here, Mr Kopung wrote the formula E = F + V – E = 2. The use of the first E which he explained as Euler is incorrect and inaccurate, making the whole formula inaccurate and confusing. This is more so because there is another E symbol in the formula. This is further compounded by his use of the equals sign to indicate that Euler is equal to F + V – E and is also equal to 2. There is no doubt that this will seriously impede learners’ developing conceptions and understanding of the sign. Again, in his explanation of the formula, there was a serious omission that this formula is only true in particular 3D shapes, i.e. in polyhedrons and that it cannot be generalised.

During the last part of the main activity, learners presented their respective groups’ model in respect of the number of faces, vertices and edges which they used to complete the table. All the group presentations followed the same pattern, i.e. the name of the prism modelled, the number of faces, vertices and edges recorded in the table, substituting in the formula and working out the formula. More focus was on the learners’ presentation of correct data in the table and on their computational skills in working out F + V – E than on explaining mathematical ideas and processes underlying their findings. Mr Kopung and his learners seemed to be equally excited about getting the right answer ‘2’. It was at this point that the researcher intervened to try and introduce mathematical debates in the presentations, forcing learners to bring their understanding and thinking to the open. Learners were asked to explain what they have learned and understand of a 3D object and one learner responded: ‘We learnt about 3D shapes’. Another learner responded: ‘A 3D shape is a shape that has a flat surface’. Asked if they all agree with the latter, another learner remarked: ‘The flat surface is a 2D shape and a 3D shape can stand on its own’. Learners were further asked probing questions on 2D shapes and 3D objects and this is how they responded:

Lesson segment: 19

T: Can you call a cupboard a 3D object?

LS: Yes.

T: Why?

LS: Because it can stand on its own.

T: Can you call these tiles (pointing to the floor) 3D objects?

LS: No.

T: Why?

L1: Because a tile has a flat surface.

It was evident from this lesson segment that learners' understanding of the concept of 3D needed further exploration. For example, the last response above suggests that in a 3D object there are no flat surfaces. During an interview that followed the lesson observation the researcher tried to establish how Mr Kopung would present the difference between a 2D shape and a 3D object. Here is how he responded: 'A 2D shape (drawing a rectangle on a piece of paper) has a flat surface and a 3D object can stand on its own, like a cupboard'. It was clear from his response that this was a working definition both the teacher and the learners were using in his classroom. This further demonstrated the misconception that 3D objects are made up of flat surfaces only as in prisms and pyramids. Mr Kopung could have used the examples and non-examples of objects in the environment to show that 3D shapes could consist of a) only flat surfaces as in prisms and pyramids, b) both flat and curved surfaces as in a cone or a cylinder, and c) only curved surfaces as in a sphere.

The second classroom observation of Mr Kopung's lesson, which lasted for 45 minutes only, was a continuation of the main activity of the first lesson although the two lessons were a week apart. It must be noted that not all lessons are conducted in a mathematics laboratory. According to the timetable of the school, teachers use the lab three times a week and for this reason they do prioritise those topics that need to be conducted in the lab. The 2D and 3D topics are some of the priority topics. The second lesson was approached differently and this could probably be due to the gaps that were identified during the previous lesson and highlighted during the intervention programme that preceded the second lesson.

Mr Kopung used the modified table from the previous lesson that was observed to continue the activity on the relationship between 2D shapes and prisms. This time around, probably due to limited time, he had asked learners to bring a variety of containers including match boxes, juice cartons, cereal boxes, and jewellery boxes and also used diagrams from the textbook. He displayed the table that was completed in the previous lesson and reminded the learners about the number of faces, vertices and edges per prism. After explaining to the learners that every object has a base on which it was build, he asked the following questions, gave instructions for each object and directed the learners to complete the table below:

I. Choose any object and tell me what prism it is, the name of the prism!

II. Do you all agree?

III. What is the shape of the base?

IV. Write your answers next to the name of the prism.

V. How many sides does your base have?

(When the table is completed)

VI. What do you notice about the numbers in the 3rd column and the numbers in the other columns?

VII. Don’t hurry to give me answers, look carefully before you respond!

VIII. Check if there is a rule that we can apply in all prisms.

Prism Name	Base	Sides of the base	Faces	Edges	Vertices
Triangular Prism		***3***	***5***	***9***	***6***
Cube		***4***	***6***	***12***	***8***
Hexagonal Prism			***8***	***18***	***12***
Pentagonal Prism		***5***	***7***	***15***	***10***
Octagonal Prism			***10***	***24***	***16***
Rectangular Prism			***6***	***12***	***8***

Table 7: Determining the shape of the base given the number of faces, edges, vertices and the sides of the base

It was interesting to note how involved learners were in the real mathematics of looking for relationships between the base and the other features of the prism, despite the time limit. Realising that not all prisms were represented in the set of objects at the disposal of learners, Mr Kopung directed them to use diagrams of 3D shapes in the textbook. It was fascinating for learners to discover the rules by themselves. For example, learners could easily realise that to get the number of faces you can just add 2 to the 3rd column. However, typical of learners at this level, they struggled to articulate the link between the 3rd column and the columns on edges and vertices. This could probably be because learners find it easier to work with addition than with multiplication. Mr Kopung intervened to make learners realise that the two columns contain multiples of 3 and 2 respectively. It is after his guidance and intervention that learners managed to switch from simple addition to multiplication. One learner could not hide his excitement about generating the rules to the extent that he remarked at the end of the lesson: 're e fumane secret ya Euler!" meaning 'we have discovered Euler's secret!' This demonstrated how lack of freedom from the authority of mathematical formulae and highly commercial manipulatives, among others, may inhibit mathematical fun, creativity, self-determination and self-affirmation.

4.4.4.4 Classroom organisation/ management

Data on classroom management were gathered from the first lesson. Mr Kopung's 90 minute-long class period was organised in three phases, i.e. from whole class discussion to group activity and back to whole class discussion. Mr Kopung spent about 30–35 minutes on whole class activity in which he established learners' pre-knowledge and introduced the main activity. This was followed by another 30–35 minute - long group activity in which learners worked in groups of 3–4 for most of the teaching time, a feature that seems to be a social norm in Mr Kopung's classroom. The remainder of the class period, about 20–25 minutes was devoted to whole class discussions where learners explained and presented their solutions.

As mentioned in the above section, during his introduction of the main activity of the lesson, Mr Kopung displayed a strong tendency to tell and show. For example, he stood in front of the class and drew learners' attention to the board as he displayed Euler's formula on the board. His introduction of the main activities to the class was confined to telling the learners what each letter in the formula stood for and showing the learners the steps to follow. The step included counting and substituting in the formula the number of faces, vertices and edges using a cube as an example. Most of the time there was limited mental activity by learners whose involvement was only limited to orally answering questions relating to the properties of a cube, e.g. the number of struts and nodes used to construct a cube and the number of faces, edges and vertices. It was not apparent to the researcher or to the learners as to why the formula was important, what underlying mathematical ideas were embedded in it and how it was derived. During this section of the lesson, Mr Kopung's focus seemed to be more on displaying his own expert knowledge of the formula than on engaging his learners in meaningful mathematical activity during whole class discussions.

It was only after showing his learners the procedure of verifying Euler's formula that Mr Kopung was actively involved with his learners during the small group activity. He assigned a group task in which he directed learners to use struts and nodes to construct model of particular prisms and to follow the steps that he had demonstrated to them, i.e. counting the number of faces, vertices and edges and to feed those data in the table and ultimately in the formula. He went to groups that seemed to be struggling insisting 'follow the steps. If you are done, go to the next step'. To those who seemed to be making some progress he insisted: 'go to the next step, yes do that, follow the steps according to what is on the board, you read the instruction, follow the steps'. For the major part of the activity, Mr Kopung was moving from one group to another and occasionally shouting: 'Just follow the rules'. Although small group work is commendable and perhaps desirable for more effective management of classroom structures and meaningful activities, Mr Kopung focused on procedures which seemed to dominate his classroom practice and thereby limiting the opportunity of debating the mathematics involved. Warning against this kind of teaching strategy, Cobb, Yackel and Wood (1992: 11) posit that, being increasingly explicit and spelling out what students are supposed to learn, bring with it the danger that mathematics will become excessively algorithmatised at the expense of conceptual meaning. What was also apparent during these small group activities is that learners seemed to agree on solutions without engaging in any mathematical debates. This seemed to be inconsistent with the mathematical argumentation, explanation, proof, verification, and justification that seemed to characterise Mr Kopung's view of mathematics learning. Learners mostly worked collaboratively in the construction of models and seemed to work independently when asked questions that required deeper mathematical ideas. For example, this was apparent when learners were asked the difference between a square and a cube.

4.4.4.5 Sensitivity to learners

Being sensitive and able to meet the individual learning needs of the learners is one of the tenets of both Shulman's PCK and Deborah Ball and her Michigan team's MKT. There are a number of assumptions that Mr Kopung made about his learners' knowledge and thinking which suggested his knowledge of children and their thinking, and demonstrated his sensitivity to his learners' needs. In the first activity, he rightly assumed that his learners might confuse the description of a square with that of a rectangle. For this reason, he focused his activity on clarifying what is similar and different in the two 2D shapes. He also anticipated that his learners might not know the concept of opposite sides hence his emphasis on 'opposite sides' in the description of a rectangle. However, his non-attendance to learners' response to the sides of a circle, i.e. 'No sides' and 'one side' which is a common misconception or error made by children of this age, raises serious questions about his own knowledge and deep understanding of geometry.

Mr Kopung's assumption that learners will automatically translate concrete objects, e.g. struts and nodes, also demonstrated insensitivity to the learners. He did not take measures to make explicit the link between these concrete materials with mathematical concepts of edges and vertices. The use of examples of 3D shapes from the learners' environment is highly encouraged in the teaching of geometry (Cobb, Yackel and Wood 1992: 7). However, Mr Kopung's use of examples from the environment was only limited to what was available in the classroom environment. For example, Mr Kopung only used tables in the mathematics laboratory as examples from which learners had to approximate the 2D shapes. These are special kind of tables that learners would not ordinarily encounter in their environments. Besides being uncommon in the learners' environments, the tables were designed with safety features such as rounded corners and edges, which made them not to be the most suitable examples of prisms to use.

4.4.4.6 Assessment and evaluation

Mr Kopung used various forms of informal assessment during the lessons observed. For example, he used baseline assessment in which he used verbal questions prior to the main lesson to establish his learners' prior knowledge on 2D shapes. As the name suggests, information from his kind of assessment was supposed to help him establish his learners' readiness for the new topic. Through baseline assessment, Mr Kopung managed to establish his learners' knowledge of the properties of 2D shapes in relation to their (sides) relative size and number and their readiness to encounter knowledge of 3D objects.

During the lessons, Mr Kopung used formative assessment by asking verbal questions and observations to gauge the learning process of his learners. Formative assessment has constant feedback to learners as one of its distinguishing features (DBE 2011d: 293). The use of rich and challenging tasks and the high quality of classroom discourse and questioning are some of the broad characteristics of formative assessment (Black & William in Hodgen 2007). As the name suggests, formative assessment is used for teaching as it provides information that informs or directs subsequent teaching, including teaching methods. On a number of occasions during the lesson, lack of feedback to learners' responses was apparent. For example, in the activity where a learner in one group insisted that a pyramid was a triangular prism because all faces were made up of triangles, no feedback was given to the group or to the class. This was an important moment in which the teacher could have effectively engaged learners on their mathematical ideas underlying their conceptions of a pyramid and a prism. Instead, Mr Kopung told the group to use the triangle as a base. Clearly, his focus was more on following the steps to get the final product, i.e. the model done than on the mathematical processes involved which are critical in supporting their learning. There is no doubt that this focus on procedures compromised the high quality of classroom discourse that could have emerged from the group interactions. Such discourse could have helped to engage learners in higher order thinking and thus in promoting informal reasoning connectedness.

Comparing teaching and the kind of mathematics encountered by students in different TIMSS countries' classrooms, Stigler and Hiebert (1997: 55) contend that the nature and level of students' learning are probably influenced by the nature of their mathematical experiences in the classroom.

4.4.5 Mr Kopung: The Story of The Use of Manipulatives in (re)shaping Classroom Practice

Mr Kopung's story tells how the use of manipulatives can be a potential catalyst to transform his work of teaching which involved a) representing, b) explaining, c) questioning, and d) responding to learners' ideas.

The story tells how concrete manipulatives, if used in conjunction with other representations, influenced Mr Kopung's teaching to a more concrete, pictorial and abstract approach. Mr Kopung's teaching in both lessons was characterised by the use of multiple representations, i.e. models of prisms, drawings of prisms, and tables to establish the patterns and relationships between and among the properties of those prisms, and an equation to symbolically represent those relationships to facilitate and support the learning of the properties of 3D objects by his learners. Through those multiple representations as a teaching strategy, he afforded his learners the opportunity to experience different types of prisms and their properties concretely, visually, numerically and symbolically. The latter two represented ways of abstracting the mathematical concreteness as embodied in the manipulatives and to establish a connection between concepts within the topic and among topics, i.e. data handling and patterns, functions and algebra content areas in particular. The way in which teachers represent mathematical ideas, concepts and processes has been shown to impact on how well children learn (Iliada, Gagatsis & Delivianni 2005).

The case also represents a story of how concrete manipulatives foster collaborative groupwork, one of the cornerstones of the new National Curriculum Statement in South Africa. Manipulatives in the schools' mathematics laboratory, like any other resources, are not in abundance. Mr Kopung, through his collaboration and group work approach that formed the integral part of his lessons, afforded each learner the opportunity to share and use those limited manipulatives albeit mainly at concrete level. The story also illustrates how the promotion of meaningful and productive groupwork can be realised through clear and explicit articulation of and connection to learning goals.

Active and critical learning as one of the principles that underpins the Grade R–12 NCS (DBE 2011d: 4) to promote conceptual understanding requires inter alia, teaching that is characterised by rich debates and argumentation. The case of Mr Kopung also tells a story of how the use of manipulatives has the potential to generally promote communication and mathematical debates, reasoning and argumentation in particular. What were also apparent in Mr Kopung's lessons were moments in which the use of manipulatives elicited opportunities for mathematical communication, debates and argumentation. There was evidence of mathematical discussions and argumentation among the learners, e.g. about a pyramid and a triangular prism as they constructed 3D models. The potential was, however, curtailed by Mr Kopung's insistence on his predetermined knowledge and his rigidity in respect of what he wanted his learners to elicit. He consequently missed out on opportunities to engage his learners in mathematical debates and argumentation among themselves that would have made their thinking visible.

His mathematical work was also dominated by questioning, the strategy he used to direct his learners to elicit the knowledge, concepts and skills in relation to the names, definitions and properties of 2D shapes and prisms. Mr Kopung's approach to teaching across the two lessons could typically be described as one where the teacher uses manipulatives to drill and practise what has been learned rather that to explore manipulatives and to creatively work with them. Perhaps this could be due to the contradiction faced by Mr Kopung emanating from the new move in the South African NCS from a teacher-centred approach to a more learner-centred approach. The contradiction plays out in the tension between his authority in respect of his own mathematical knowledge and skills that he appeals to in his teaching, and active and critical learning as proposed in NCS. He mainly used low cognitive order questions requiring simple recall of knowledge, concepts and skills. The CAPS for Mathematics Grades 4–6 (2011: 10) recommends a transition from simple description of 2D shapes and 3D objects to classification and more detailed description of shapes and objects. This transition could only be realised through questions of high cognitive order that go beyond the naming of the shapes and the counting of sides i.e. simple drill and practice, to the why questions that require learners to explore manipulatives and to creatively work with them. Mr Kopung’s questions were predetermined as was clear from the common and linear structure of those questions, e.g. ‘What shape do you see?’, ‘How many sides does it have?’ The story of Mr Kopung is about how the exploratory and creative use of manipulatives can influence teachers’ questioning technique and hence their classroom practice.

The two lessons also illustrated opportunities for Mr Kopung to stimulate the learning of deeper mathematical ideas and concepts using appropriate examples and non-examples to explain mathematical terms clearly and accurately. The rigid structure of commercial manipulatives that were used by Mr Kopung, e.g. struts to represent straight lines, could have also limited his efforts in explaining the concept of a circle *vis a vis* polygons. This, as pointed out in the above section, was one of the missing pieces in Mr Kopung’s lessons. Reticence to learners’ responses was characteristic of Mr Kopung’s teaching. For example, in an episode involving a circle as an example of a 2D shape, he asked learners as to how many sides a circle has and acknowledged their responses, i.e. ‘no sides’ and ‘one side’ but set them aside. This demonstrated a tension between what he had set out to teach and the learners’ responses. When selecting and sequencing questions for the lesson, it is obvious that Mr Kopung had in mind particular shapes that he hoped to elicit from his learners. This shows that using manipulatives has the potential to elicit responses that sway his teaching from his set agenda, requiring him to restructure his agenda.

Mr Kopung subjected himself and his learners to the authority of mathematics formulae (Euler's). Traces of unequal power relations were also demonstrated where he insisted on his predetermined knowledge and what he wanted his learners to elicit. His rigidity, domination and authority was disempowering to him in that he could not sway his teaching from his set agenda.

4.5 CHAPTER SUMMARY

In this chapter, I presented a thick and detailed empirical data collected from the participants through qualitative methods and mainly from primary sources including interviews with the teachers, specific classroom descriptions, video recorded lessons and curriculum materials. The chapter provided data presentation in the form of chronicled stories of each of the three core participants in the study in respect of my key theme as well as my sub themes.

Data interpretation in the form of both spoken words that were later transcribed into text as lesson segments; and non-verbal interactions as communicative events and situations that were captured as my field notes were also presented in the chapter. In pursuance of the study aim and the research question, the chapter reflected on a combination of both the MKT and CDA frameworks which were used to analyse both mathematics teaching and the power relations between the learners and the teachers respectively. Through both frameworks, spoken words and non-verbal interactions in respect of the participants were interpreted so as to foster a better understanding and also compare them to theoretical data gathered from literature in chapter two to determine if there is conformity and corroboration or not. The following key issues were drawn from data interpretation in this chapter a) questioning practices, b) choice of tasks, c) connections among mathematical topics, concepts and ideas, d) use of multiple representations and contexts to complement manipulatives, and e) mathematical communication in relation to manipulatives use.

The next chapter presents the discussion of the findings organised in respect of the themes that emerged in chapter four as well as cross-case analysis of the findings of the study.

CHAPTER 5: PRESENTATION OF THE FINDINGS AND CROSS-CASE ANALYSIS

5.1 INTRODUCTION

This chapter sets out to provide a cross-case analytical account of the data that were presented in the previous chapter, and the key findings of my study. This is done through a comprehensive analysis of the data, to examine the common and the diverse across the three cases. Furthermore, by using the narratives in chapter four, I venture to answer in more specific terms, the research questions that frame the study.

The chapter begins with the development of constructs by extracting critical themes that emerged from the interpretation of data in chapter four. The next step is to establish common threads among the three cases as well as the differences that stand out in respect of the following themes: a) questioning practices, b) choice of tasks, c) connections among mathematical topics, concepts and ideas, d) use of multiple representations and contexts to complement manipulatives, and e) mathematical communication in relation to manipulatives use. In doing the cross-case analysis, I also looked at the literature review and made connections between what the literature says (i.e. the theory) and my empirical data within each theme. For this reason, critical moments and subsections thereof during the lesson observations and interviews are used as illustrations to either confirm or dispute the propositions from the literature review. This is what Klein and Myers in Bondarouk (2004: 66) views as going back to the whole, i.e. finalising general relationships and functions in the initial theoretical concept. I used the critical theory framework as a lens through which the teachers' voices and interactions (which are the primary data sources) were analysed. It is mainly through recording and interpreting these discourses and observations that deeper understanding of contradictions, power relations and other counter forces that inhibit the realisation of teacher creativity and emancipation were unravelled and confronted accordingly.

5.2 COMMONALITIES

To begin with, all the teachers in the study used the same Curriculum and Assessment Policy Statement (CAPS) for the Intermediate Phase Mathematics. This is a curriculum framework that was implemented for the first time in the (intermediate) phase in 2013 in all South African schools. All three teachers had been exposed to some form of a CAPS workshop (training) which was designed to orient the teachers about the use of the CAPS materials. The teachers displayed a fair and general knowledge of the Grade 6 curriculum in respect of the topics that were to be covered and the concepts and skills that learners are expected to learn in the grade. The schools were also using new CAPS aligned mathematics textbooks with teacher guides that are nationally approved. It is important to note that the FSDoE had provided each learner with the new CAPS aligned textbook to further encourage the implementation of CAPS in 2013.

On the face of it, all three teachers seemed to be familiar with the reform-oriented mathematics curriculum and its learner-centred approaches to mathematics teaching and learning. Although group work dominated all the classrooms I observed, there was no deliberate effort from the teachers to encourage learners to share their ideas among themselves in the respective groups except when learners volunteered to present their solutions to the whole class. The National Curriculum Statement recommends, among others, that learners communicate mathematically, pose and solve problems, reason logically, provide explanations and be critical and creative thinkers (CAPS for Mathematics Grades 4–6, DBE, 2011**)** in order to promote conceptual understanding in mathematics**.** Most of these elements of a learner-centred approach, as recommended in the new curriculum, were conspicuous by their absence in the lessons observed, an indication that the new approach may have been embraced rather superficially by the teachers. Perhaps this tension can be understood and explained by drawing on Ball (1993), Cohen (1990) and Spillane (2000) as cited in Jita and Vandeyar (2006:40), who argue that the reform agenda represents a tall order for many of the classroom teachers whose experiences of mathematics and mathematics identities have been within the traditional approaches to school subject that place less emphasis on problem solving, discourse and reasoning. There was a strong teacher-centred tendency in the lessons I observed. The lessons were characterised by teacher-talk and a culture of silence among the learners who almost never posed questions and were rarely called upon to explain their methods and answers to problems.

Such non-dialogical pedagogy is somewhat problematic in many ways. It is reflective of unequal power relations that exist in the classrooms where authority rests exclusively with the teacher. Clearly, such pedagogy inhibits not only dialogue, deliberation and the power of learners to raise questions but also the ability of learners to share with and learn from one another and to clarify both their questions and thinking. In addition, such pedagogy is disempowering to the teachers themselves in that it impedes them from being critical co-investigators in dialogue with learners and their peers on material that is connected to their own situatedness. Such pedagogy is also indicative of practices that are dehumanising and disempowering to the learners, where learners become mere recipients of knowledge, a situation which poses constraint to learning and to social transformation. This situation is in contradiction with Freire's 'problem-posing' education, which regards dialogue as indispensable to the act of cognition (Giroux, 2004b: 83).

Most lessons were also dominated by whole class questioning by the teacher and chorused responses from learners, with the result that there seemed to be little or no individual attention to the learners. This was confirmed by Ms Dikgomo's statement about the group responses, where she decries the value of such assessment practices:

'hape ntho e mislidang ke hore a le mong ha a ka tjho answer and then class e ya echo kaofela o nahana hore ba tshwara ka pele — so nna ke be ke tsamaya ka concept ya hore ba understand kaofela (What is misleading is that when one learner responds and the others chorus the same response, one thinks that they all understand)

Such questioning practices are also indicative of the violation of the principle of individualised learning, in which due recognition to individual learners' diverse understandings is given. Such practices also inhibit teachers from treating learners with respect and dignity as human beings in their own right but rather as a group.

The schools had functional mathematics laboratories, furnished with, among others, commercial manipulatives, geometric shaped tables and mathematics posters. The new laboratory activities had been well received by the three schools and were integrated into the respective schools' timetables. The laboratories were used on a rotational basis by different classes to allow access to all the mathematics teachers in each school. The three teachers also seem to hold the view that concrete manipulatives are important tools that promote learner activity and help learners to better understand mathematics. In the focus group interview where I had asked an exploratory question to probe for teachers' feelings, thoughts and knowledge about their use of manipulatives, the three teachers showed a positive disposition towards the use of manipulatives in mathematics teaching. In the context of the introduction of the new NCS, such a positive disposition was a positive signal for the implementation of mathematics laboratories as one of the interventions that were aimed at improving the quality of mathematics teaching and learning in primary schools in the Free State. It is important to note, however, that in all the schools the teachers only used commercial manipulatives that were found in the laboratories. No other manipulatives, i.e. teacher or learner made, were used. This seems to conform to the assertion that teachers, especially in poorer schools, remain recipients of knowledge passed down unquestioningly to them through curriculum materials. Adler (1997: 95) in her discussion of teachers as researchers in South Africa, notes that the majority of teachers are more used to following the prescriptions of education authorities than they are to working reflexively, making them more of 'mere technicians' implementing someone else's ideas.

What is also common about the schools is the fact that all of them are located in the semi-urban areas of the Mangaung Metropolitan Municipality and cater for learners from low income communities and working class families that are characterised by low to moderate literacy levels. Teachers described parental involvement and participation in the school community life as generally low. As prescribed by the NCS in South Africa, the Language of Teaching and Learning (LoLT) in the Intermediate Phase in all the schools is English (a second or third language for the teachers and learners at these schools). This is something that I observed to be a challenge for both teachers and learners as they worked through the curriculum and manipulatives.

There was overcrowding in almost all the classes where teachers worked with 43, 38 and 57 learners respectively, all of which were above the average teacher-pupil ratio of 1:30 in the Free State. It needs to be noted that in South Africa, teacher-pupil ratio is determined by the MECs for education in each province annually RSA (1998) [Employment of Educators Act 76 of 1998, updated in November 2011, section 5, subsection 1b].

In all the lessons observed, the teachers started by stating the goal or objectives of the lesson to their learners. I took note, however, that the teachers articulated their lesson objectives in rather ambiguous terms, using phrases such as ‘to do 3D shapes’, ‘to talk about Euler’s formula’ and ‘to concentrate on multiple operations’. This was also compounded by the fact that there were often no clear and explicit linkages between the lesson goals or objectives and the manipulative used during the lessons.

5.3 HOW LEARNING AND TEACHING OPPORTUNITIES WERE CREATED OR NOT CREATED THROUGH THE USE OF MANIPULATIVES

Facilitating students’ construction of mathematical understanding involves selecting fruitful tasks, asking good questions and judging which student ideas should be pursued. All this demands explicit analytical knowledge, the same kind of understanding entailed in constructing direct explanations (Ball, 1988a: 47). Embedded in their conception, manipulatives are meant to improve mathematics learning. Ball (1992: 16) concluded that ‘whether termed manipulatives, concrete materials, or concrete objects, physical materials are widely touted as crucial to the improvement of mathematics learning’. Similarly, Ma (2010: 5) argues ‘The direction that students go with manipulatives depends largely on the steering of the teacher’. In this section below, I further explore some of the emerging themes regarding the use of manipulatives in the three teachers’ classrooms.

5.3.1 Questioning Practices

I begin the discussion by examining the use of manipulatives and the teachers' questioning practices in class. Facilitating students' construction of mathematical understanding involves inter alia, asking good questions (Ball 1988a:47). The three teachers frequently used questioning as a strategy to a) establish learners' prior knowledge, or b) to link prior knowledge with new topics, and/or c) to prompt learners' explanations and d) to follow up on learners' initial explanations (although the latter happened fairly rarely). In doing so, teachers used different types of questioning practices, i.e. general questions, specific questions, probing questions and leading questions, which were directed either to the whole class or to individual learners.

The kind of questions and their nature, together with the responses that those questions elicit from learners, are important to consider especially in the light of the NCS that seeks, inter alia, to develop deep conceptual understanding of mathematics. Scholars have noted that in order to promote mathematical understanding, it is necessary to make connections between manipulatives and mathematical ideas embedded in them explicit (e.g. Ball, 1992; Driscoll, 1981; Hiebert, 1984 and others, all in Ma, 2010: 6). Accordingly, in all the lessons that were observed, teachers used different questioning practices to make such connections, though at different levels.

Ms Dikgomo

Ms Dikgomo used questioning as a technique during her whole class teaching, mostly to elicit her learners' prior knowledge. She asked such questions as contained in lesson segment 1 (see chapter four, sub-section 4.2.2.1 for details): 'What are the four basic operations?' and 'But we still have others (operations) where we have to use eh..... the rule that will help us to get the other operations that we use. Can somebody remind us, what is that rule so that we should get other operations?'

Ms Dikgomo posed the 'what' type of questions that are generally of a lower cognitive order and required only short recall answers from the learners. The second question, which was a leading question, demonstrates how the teacher channelled the learners with regard to what she wanted them to say. These questions and the responses were not even closely related to the manipulatives that she was using. Ms Dikgomo seemed to be more anxious to get the right answers from her learners than to ensure that they understand the concepts involved. Her questions were mostly directed at the whole class and through questions such as ‘Do we agree?’ she seemed to invite such whole class chorus responses that characterised her classroom.

In the main activity of the first lesson (see chapter four, sub-section 4.2.2.2 for details) discussed earlier, she hardly asked any questions at all. As illustrated in lesson segments 2 and 3 respectively, she wanted her learners to compute 10 + (20 x 3) + 4’ and (36 ÷ 9) + (18 ÷ 3) respectively using the interlocking cubes. The only questions she asked as the learners presented their answer of 74 and 10 respectively were: ‘What about the others?’, ‘How did you get 74?’, ‘How did you get 10?’ and ‘where is the 10?’

The first three questions were of higher cognitive level and had the potential to uncover the mathematical reasoning underlying the procedure. The ‘where’ question on the other hand, related to the use of manipulatives as it required of learners to demonstrate how they applied the rule. However, the learners in this case responded by just restating the BODMAS rule, i.e. ‘We start in the brackets; we multiply 20 by 3 we get 60 and we add 10 then 4 to get 74’. The other groups in the class also responded by just repeating what the other learners had said, basically chorusing their responses. Throughout the lesson, Ms Dikgomo appeared to be quite comfortable with the short answers such as that of 74 in this example. There was no deliberate effort by the teacher to verify if the learners understand the procedures and the concepts involved.

As illustrated in lesson segment 3, it was critical for Ms Dikgomo to understand how her learners think about the mathematical operations they used by asking questions that elicit their thought processes. This was apparent when the learners were asked ‘where is the 10?’ They started to count single cubes instead of counting the stacks or groups of cubes, i.e. 4 stacks of 9 cubes each, and 6 stacks of 3 cubes each, which add up to 10 stacks or groups of cubes. It was clear from this lesson segment that learners needed support not only to follow the rule and do the correct computation, but also to have a deeper understanding of the concept of division. However, Ms Dikgomo was more anxious to get to the common answer rather than to ensure that learners have a deeper understanding of the procedures and the mathematical ideas underlying those procedures. To this end, she resorted to single and lower cognitive order questions that merely elicited short, recall and memory questions about the rule and procedures embedded in the rule rather than allow learners to explain their solutions beyond just recall by using interlocking cubes.

In her second lesson, Ms Dikgomo directed her learners to represent the various fractions numerically. The type of question she posed, e.g. 'How do you know this is a half?' was undoubtedly a conceptual question that called for deeper understanding of the fraction concept. This was an open question that went beyond the surface characteristics of a fraction model. However, her questions each time just stopped short of probing into her learners' explanations.

It is clear from both lesson observations of this teacher that there were various opportunities to uncover learners' thinking processes and the underlying mathematical ideas, understandings of concepts, their methods and solutions, which are all critical for conceptual understanding. However, as I have discussed earlier many of the opportunities were missed. This seems to confirm the findings by Franke et al. (2009: 390) that single questions, whether specific or general, are not always sufficient to uncover enough details of the thinking processes behind students' strategies. The inability to seize the opportunity could be attributed to Ms. Dikgomo's over emphasis on procedural knowledge and on getting the correct answers.

Ms Bohata

Ms Bohata began her first lesson by establishing her learners' prior knowledge of 2D shapes. To do that, she posed a combination of lower and higher cognitive order questions such as 'Is this a square?', 'How many sides does it have? 'Why do you say it is a square?' and 'What are the properties of a square?' Beginning with simple and lower cognitive order questions, Ms Bohata managed to get her learners to recall the names and number of sides of 2D shapes. The lower order questions posed basically resonate with the concreteness of the manipulatives used. Her next set of questions, such as ‘Why do you say this is a ...?’ were of higher cognitive order and appealed to her learners’ deep understanding of the properties of 2D shapes. Those questions required conceptual understanding of the properties of the shapes in order to make a meaningful link between her learners’ prior knowledge and the new knowledge about 3D objects. However, as discussed earlier (see chapter four, sub-section 4.3.2.2b for details), her own limited knowledge of the salient features that are central to deep understanding of the properties and definitions of particular 2D shapes, might have limited her opportunities to elicit learners’ deep thinking about the shapes.

As I have discussed in the previous chapter, the responses that Ms Bohata elicited from her learners through these questions were limited only to the number of sides of the figures. The responses by two learners who cited respectively, a circle and an oval as examples of a 2D shape and 3D object, further illustrates the missing piece in Ms Bohata’s questioning practices. As conjectured by Ball (1988) facilitating students’ construction of mathematical understanding, involves inter alia, judging which student ideas should be especially pursued. By ignoring the learners’ responses, Ms Bohata missed the teaching opportunity to uncover the thinking and understandings behind the learners’ responses. This could have been an opportunity to explore the learners’ definition and properties of 2D shapes and 3D objects in relation to flat and curved surfaces. Through her questioning, she could also have skilfully guided the learners to use models to show the differences between a model with curved surfaces and a model with flat surfaces. Her failure to pursue these responses from the learners created the impression that the questions were not well thought out to include a circular object and an oval shape in the lesson itself.

In her second lesson, which was a continuation of the first lesson but after the intervention programme, Ms Bohata mainly used leading questions and prompts to support her learners in identifying relationships between 2D shapes, edges, faces and vertices of prisms in table **5** (see chapter four, sub-section 4.3.2.2f for details). She wanted to develop her learners’ understanding of the ‘2’ that was common denominator in the column on the number of shapes that formed each prism. To do this, she used both leading questions and examples in her learners’ language to develop the concept of a base as analogous to ‘sekwagelo’ (lid) and the logic behind the ‘2’. In this way, she managed to create the opportunity to scaffold her learners and help them to move from concrete mathematics to deeper and abstract mathematics. To encourage learners to generate a rule, she made use of an open question in lesson segments 11and 13 respectively: 'Can we say the same about other prisms?' The question itself aroused interest and called for learners to inquire, investigate, reflect on and verify their findings to see if those findings could be generalised in the form of a general rule.

What was striking in the lesson after the intervention was not only the use of various types of questions to develop her learners' conceptual understanding, but also how she shifted between the types of questions and how the questions were pitched at different levels, depending on the level of complexity of the task at hand. Scholars are in agreement that questions that teachers pose have the potential to scaffold learners' engagement with the task, shape the nature of the classroom environment, and create opportunities for learning high level mathematics (e.g. Kazemi & Stipek, 2001; Stein, Remillard, & Smith, 2007).

Mr Kopung

As with the other two participants in the study, questioning practices dominated Mr Kopung's lessons. In the introduction sections of his lessons, he used oral questions extensively to establish his learners' prior knowledge of the names and properties of 2D shapes and 3D objects respectively. Mr Kopung used questions such as 'What different shapes do you see?'; 'How many sides does a square have?', and 'How many faces do you have?', all of which were of low cognitive demand and related to either the concreteness of the models that learners had constructed or to what learners could remember. He used those questions to elicit recall and memory responses from his learners and this helped him to do some quick spot checks on what his learners understood at a given point in time during the lesson.

What was also notable in his questioning practice was the rhetoric and rigid manner in which he posed those questions in relation to each example of 2D shapes. This seemed to suggest that Mr Kopung's questions were predetermined, as suggested by the common and linear structure of the questions. The result was that the learners provided chorus responses to his questions. Chorusing was also promoted through questions such as: 'Do you agree class?' which also characterised segments of Mr Kopung's lessons. Even in cases where he used some 'why' questions, his structured and rhetoric way of posing those questions could not help him to elicit his learners' mathematical thinking, a key component in conceptual teaching. In this way, Mr Kopung may have inadvertently limited the opportunities for his learners to respond independently, to be creative and to receive individual attention, as most of his questions were directed to the whole class. The use of manipulatives, as directed by his questioning practices, was relegated to a mere drill and practice exercise.

Mr Kopung occasionally posed questions that were cognitively demanding as illustrated in lesson segment 16: 'What is the difference between a rectangle and a square?', and 'Do you say a rectangle is a square?' Such questions had the potential to stimulate learners to reflect and think deeply about the two shapes. In another episode within the same lesson segment 16, Mr Kopung wanted to develop his learners' understanding of the concept of 'opposite sides'. After one learner had referred to opposite sides as 'bottom and upper sides' and 'left and right sides', he remarked thus in an attempt to elicit other learners' views: 'Come on, think properly! Do we agree?' It was apparent from the lesson segment that Mr Kopung is quite aware of the potential of the interactions to uncover learners' thinking processes and to facilitate the development of deep understanding of some geometric concepts. However, Mr Kopung did not often pursue his learners' ideas or follow up on his questions, both of which could have elicited his learners' deeper thinking about the concepts at hand. He seemed to be content with the chorusing and simple definition of shapes as demonstrated by his whole class questioning practice. As a result, he may have missed many opportunities to probe his learners' understandings.

For example, Mr Kopung missed another opportunity to stimulate his learners' thinking processes when in lesson segment 17 he asked learners: 'How many sides are there in a circle?' Learners responded: 'No sides' and 'One side' respectively. Although this was potentially a thought provoking question, he once again, missed the opportunity to make visible his learners' mathematical thinking processes on how they think about a circle. The potential of the question was limited by his lack of follow up on the learners' responses to stimulate higher level mathematical processes such as reasoning, argumentation, and analysis, which could have been enabled through the use of manipulatives.

SUMMARY

It needs to be recognised that the questioning practice of the three teachers were dominated by single questions of lower cognitive level. Such questions elicited simple recall and memory responses from learners, which only remained at the literal concreteness of manipulatives. Chorusing and one directional questioning by the teachers were also some of the common features of their practice.

Ms Dikgomo’s preoccupation with rules and procedures towards the correct answer seemed to have contributed immensely to her approach. As a result, in her lesson on multiple operations, the questions and the responses elicited were not even closely related to the manipulatives that she was using. Even where her questions related to the manipulatives and could have elicited deeper understanding of the fraction concept from her learners, e.g. ‘How do you know this is a half?’ her questions each time just stopped short of probing into her learners’ explanations.

Ms Bohata and Mr Kopung posed questions that related to manipulatives in their respective lessons on 2D shapes and 3D objects. However, because of the lower cognitive levels of their questions, the responses they elicited from the learners remained at the concreteness of manipulatives, eliciting simple recall of the names of shapes and the counting of sides. What was also apparent in their questioning practices is their ignorance of learners’ responses, responses that presented opportunities to uncover their learners’ thinking and promote a deeper understanding of geometric shapes and concepts had they been sufficiently pursued. By ignoring learners’ responses, both teachers also missed crucial opportunities to explain salient features of 2D and 3D shapes.

In her second lesson Ms Bohata asked leading questions and provided prompts to support her learners in identifying relationships between 2D shapes, edges, faces and vertices of prisms. To do this, she used both leading questions and examples in her learners' language to develop the concept of a base as analogous to 'sekwagelo' (lid) and the logic behind the '2'. In this way, she managed to create the opportunity to scaffold her learners and help them to move from concrete mathematics to a deeper understanding of abstract mathematics embedded in manipulatives. Mr Kopung's practice was characterised by rhetoric and rigid questions that were linear in structure. This seemed to suggest that Mr Kopung's questions were predetermined and, as a result, elicited responses that he expected such that any response that deviated from his expectation was simply ignored. The result was that the learners provided chorus responses to his questions.

5.3.2 Choice of Tasks

Facilitating students' construction of mathematical understanding involves inter alia, selecting fruitful tasks (Ball, 1988). Mathematical tasks are considered to be of key importance to the learning of important mathematics as they allow learners to interact with mathematical ideas, concepts and procedures. This is why the selection of tasks is regarded as the most significant decision affecting student learning (Lappan & Briars, 1995). Literature on manipulatives suggests that concrete materials do not automatically carry mathematical meaning for students (e.g. Moyer, 2001; Moyer & Jones, 2004; Thompson, 1994; Uttal et al., 1997.). Tasks that involve the use of manipulatives must necessarily be tasks or activities that support learners to transcend the concreteness of these objects in order to learn the abstract concepts and ideas that are embedded in the objects. As Simon and Tzur (2004: 93) conjecture, the goal for student learning influences both the choice of tasks and hypotheses about the learning process. In the lessons observed, teachers used tasks and activities that involved the use of manipulatives differently and therefore created different mathematical opportunities for both learning and teaching. I now further explore these mathematical tasks from each teacher's classroom:

Ms Dikgomo

In her first lesson, Ms Dikgomo engaged her learners in the task of using interlocking cubes to compute the following sums, which were taken directly from the textbook:

a) 10 + (20 x 3) + 4 =   and
b) (36 ÷ 9) + (18 ÷ 3) =

Her overemphasis on the BODMAS rule and procedures seemed to have informed her choice of the two tasks at the expense of meaning making. The two highly structured tasks in their format could be less challenging for learners and perhaps did not even warrant the use of manipulatives. This is more so because the two tasks involved brackets, which remove the confusion about the order of operations. Over reliance on the textbook with its structured tasks seemed to have limited her creativity and freedom to represent the problem openly, e.g. in a story or a word problem. In this way, learners would have meaningfully learned the procedure by exploring with manipulatives. The use of highly structured textbook exercises may have contributed in disempowering both the teacher and her learners by limiting their freedom of thought, creativity and self-determination. This seems to confirm what Adler (1994: 104) defines as textbook-based teaching and rule-bound learning styles that constitute pupils' mathematics diet in South Africa, as well as her argument in Adler (1997: 95) that the majority of South African teachers tend to rather follow the prescriptions of education authorities than to work reflexively, reducing them to 'technicians' implementing someone else's ideas.

Although Ms Dikgomo moved from one group to the next during the lesson, her focus was mainly on checking the correct answers and not necessarily the procedures followed to get to the answers. This was demonstrated by the fact that once a particular group of students had completed their calculation, they had to report back by giving the answer and explain how they got the answer. In the latter, learners simply reinstated the BODMAS rule. For example in the task 10 + (20 x 3) + 4 =   the learners reported 'we start in brackets; we multiply 20 by 30 we get 60 and we add 10 then 4 to get 74'. There was no deliberate effort by the teacher to verify if the learners understood the procedures in respect of when to use them and why they work. In this regard, Ball (1988: 7) conjectures that 'knowledge of mathematical procedures entails knowing when to use them and understanding why they work'. Learners were not even afforded the opportunity to reflect on their actions with manipulatives as they were all in a hurry to finish and give answers. In this regard, Moyer (2001:178) contends that students may require concrete materials to build meaning initially, but they must reflect on their actions with manipulatives to do so. For Ms Dikgomo learning mathematics seems to be synonymous with memorising and chanting the steps towards the correct answer. This is further demonstrated by the fact that neither the tasks nor the support were differentiated, all the groups were given the same tasks as though they were a homogeneous group. As illustrated in lesson segment 3, the learners struggled with the division concept, and there was no deliberate remediation by the teacher to help struggling group members.

In her second lesson on fractions, Ms Dikgomo meaningfully engaged the learners in a task in which they had to define a fraction and also respond to the question 'How do you know that this is a half?' She also gave learners a task that allowed them to do their own 'wholes' to highlight the mathematical idea that wholes differ in respect of the number of objects or shaded areas in that whole, and this will give different number of objects or areas representing 'half'. Sociologists propose that open approaches to learning not only give access to a depth of subject understanding but also encourage personal and intellectual freedom that should be the right of all people in society (Ball, 1993; Willis, 1977 both in Boaler, 2002: 254–255). In the same vein, an open approach to mathematical tasks gives learners the opportunity to explore and learn about the fraction concept through the use of manipulatives. What was different from the first lesson was that Ms Dikgomo did not take centre stage; her learner-centred approach was demonstrated in many ways. As illustrated in lesson segment 6, she successfully drew on her learners' intuitive knowledge about fractions to formalise their fraction concept. She also gave learners the freedom to choose the tools and the fractions to represent. All this could be attributed to Ms Dikgomo's creativity and her distribution of power wherein she deliberately allowed learners to take ownership of their own learning.

The shifts between teacher-centred and learner-centred pedagogy, as illustrated in the first and the second lessons respectively, is indicative of shifting patterns of control. Unlike the first lesson where Ms Dikgomo took centre stage, she created an opportunity for learners to be in control of their own learning in the second lesson.

Such an approach enabled her learners to freely explore with manipulatives and take ownership of their own learning process.

Ms Bohata

Ms Bohata's two lessons were on prisms as examples of 3D objects. Her goal, though ambiguously phrased - as 'to do different kinds of 3D shapes' - was for her learners to construct models of different types of prisms as examples of 3D objects, and to learn, understand and describe the properties of those prisms. To realise her lesson goals, she selected tasks in which learners constructed models of different prisms and extracted the properties of those 3D objects with respect to the number of faces, edges and vertices from their models to complete table 3. What was striking about her approach was the support she gave her learners on using the manipulatives themselves before they began constructing their respective models. She started off by providing a brief explanation of the various parts, e.g. zoom struts and nodes in the box and how they are used to make structures. This approach seems to support Ojose and Sexton (2009: 4) who conjecture that manipulatives do not only allow students to construct their own cognitive models for abstract mathematics, but also provides a common language with which to communicate these models to the teacher and other students. Ms Bohata levelled the playing field by ensuring that her learners make meaning of the tools they were using to understand the mathematics involved. This approach seemed to be in agreement with the assertion of Uttal et al. (1997: 38) that 'for children to gain understanding using manipulatives, they must identify the mathematical concept being learned with the manipulative'. As illustrated in lesson segments 9 and 10 respectively, Ms Bohata's learners were able to complete the task mainly due to the attention that she paid to the details of the task that she assigned.

Ms Bohata then assigned her learners the task of building a prism model similar to the shape of their tables, using the manipulatives**.** The construction of models was not an end to itself but a means to conceptual understanding of the properties of those models, as can be seen from data that were provided by learners in tables 3, 4 & 5 they had to complete. Ms Bohata's learners generated knowledge of the properties of each prism from each model. Clearly, the task went beyond mere physical construction of prism models to mental activities that required understanding of mathematical concepts and ideas underlying the names, the properties and the definitions of those models. Embedded in the tables were sub-tasks that kept learners interested, focused and enthusiastic. This could be attributed not only to Ms Bohata's skilful choice of the task but also to the logical sequencing of the activities within the task itself. For example, learners made different structures to represent different prisms, named their respective prisms (which were written in the table under the 3D shape column), matched the names with the list of 2D shapes and provided data regarding the number of vertices, edges and faces of those prisms. The foregoing discussion suggests that the selection of tasks is an important consideration for any teacher, but the sequencing of the tasks and sub-tasks may even be more important for providing extended opportunities for learners to engage in real mathematics and to understand the basic reasoning behind the tasks.

Mr Kopung

The first task that Mr Kopung engaged his learners in was that of identifying and naming different shapes of tables in the classroom. Mr Kopung's choice of activities on 2D shapes was helpful in establishing the necessary connections between the learners' prior knowledge and the new topic, and between geometric concepts. Although the task was mainly a drill and practice activity, what stood out in the task was how Mr Kopung deliberately placed emphasis on the conceptual understanding of some geometrical concepts. He managed to engage his learners in some informal reasoning about what is similar and different between the two concepts, i.e. a square and a rectangle, an initiative which he could have escalated to the next level to show that a square is a special type of a rectangle. A similar approach with cubes and cuboids could have allowed him the opportunity to further explore the relationships between 2D and 3D shapes. In this task, Mr Kopung also paid special attention to one of the salient features that define geometric shapes, hence his emphasis on 'opposite sides' in the description of a rectangle. His teaching, in this case, placed emphasis on concept development of the properties of 2D shapes rather than mere recalling of the properties. However, not only was the choice of the task important in this case, but his knowledge of how learners think about the properties of shapes and how that knowledge informed points of focus within the task itself was also significant in this lesson.

Mr Kopung also assigned his learners a group task in which he directed them to use struts and nodes to construct model of prisms, to count the number of faces, vertices and edges to complete table 6, which already had a list of prism names, and to apply the data from the table in Euler's formula. Learners actively participated in the activity as they began to abstract data regarding the number of faces, vertices and edges from their models and display them in the table. However, learners struggled to reconcile the concrete and the abstract terms used, e.g. struts and edges; with vertices and nodes. This is one instance that illustrates the need to unpack the learning goals, and determine the detail how these goals can be realised through the use of manipulatives.

Once the learners had completed the table, Mr Kopung wrote the formula on the white board and went on to explain the formula. His introduction of Euler's formula before learners could make sense of the data in the table, and perhaps be guided to establish the relationships among the properties of shapes, made the sequencing of his tasks rather problematic. This section of the task was characterised by a step by step procedure in applying Euler's formula, relegating the task to just an algorithmic exercise that does not lead to real mathematics. In this way, Mr Kopung limited his own teaching opportunities by not incorporating other topics such as patterns, functions and algebra to promote deeper understanding of the properties of prisms. He may also have missed the opportunity to engage his learners in the real mathematics of analysing data, identifying and describing the relationship between those properties and of generating and verifying their own rules. It seems as if Mr Kopung in this activity, only wanted to display his own knowledge of the formula and also to abstract his own predetermined mathematical relationships embedded in Euler's formula. This seems to confirm the observation by Cobb, Yackel and Wood (1992: 12) that, '... the external representation can be seen to serve as the medium through which the expert attempts to transmit his or her mathematical ways of knowing to students'.

Summary

Ms Dikgomo's choice of tasks on multiple operations was characterised by less challenging and highly structured textbook exercises thereby limiting her freedom of thought, reflexivity and creativity. Her learners found it difficult to understand the concept of division and this could be attributed to the decontexualised nature of tasks that her learners could not relate to. In this regard, Dawe (1995: 243) warns that the connections between symbols on paper and their representation of real-life must be explicitly made. Ms Dikgomo's lesson on fractions was more learner-centred as she did not take centre stage. This demonstrated a shift, from the first lesson, in patterns of control through her choice of an open mathematical task which gave learners the opportunity to explore with manipulatives by drawing on their intuitive knowledge about fractions to formalise their fraction concept. She also gave learners the freedom to choose the tools and the fractions to represent.

Ms Bohata gave her learners the task to construct geometric models but started off by providing a brief explanation of the various parts, e.g. zoom struts and nodes in the box and how they are used to make structures. The support she gave her learners on using the manipulatives before they began constructing their respective models helped to provide a common language for communicating these models to the teacher and other students. As a result of the attention that she paid to the details of the task, her learners were able to complete and communicate the task with greater ease. Her skilful choice and logical sequencing of the task and sub-tasks helped her learners to move beyond the mere physical construction of prism models to mental activities that required understanding of mathematical concepts and ideas underlying the names, the properties and the definitions of those models. Embedded in the task itself were subtasks that kept learners interested, focused and enthusiastic most of the time, perhaps also ensuring that manipulatives are not relegated to toys.

The task that Mr Kopung gave to his learners started off with the formula, i.e. symbolic representation in which they had to substitute data relating to the properties of the prisms. The sequencing of his sub- tasks was found to be rather problematic in that no deliberate attempt was made to unpack the learning goals and how they are to be realised through the use of manipulatives. As a result, the task was characterised by a step by step procedure in applying Euler's formula, relegating the task to just an algorithmic exercise before learners could make sense of the data itself. This gave the impression that he wanted to display his own knowledge of the formula also to abstract his own predetermined mathematical relationships embedded in Euler's formula. This seems to confirm the observation by Cobb, Yackel and Wood (1992: 12) that, '... the external representation can be seen to serve as the medium through which the expert attempts to transmit his or her mathematical ways of knowing to students'.

5.3.3 Connections Among Mathematical Topics, Concepts, and Ideas

The knowledge of specialised content knowledge (SCK) is central to the teacher's ability to make connections between and among mathematical topics, concepts and ideas. Liping Ma (2010: 121), in her discussion of the notion of PUFM, cited Duckworth's observation that intellectual 'depth' and 'breadth' is a matter of making connections. This knowledge package is also in line with the third category of Rowland and his colleagues' framework, the Knowledge Quartet (KQ). According to them, the third category, connection, binds together certain choices and decisions that are made for the more or less discrete parts of mathematical content (Rowland et al., 2004: 123). Teaching for conceptual understanding is fundamentally about making connections among mathematical topics, concepts and ideas, and presenting mathematics as a coherent discipline. The lack of opportunities in U.S classrooms for students to discuss connections among mathematical ideas and to reason about mathematical concepts constituted one of the most prominent findings of the TIMSS (Stigler and Hiebert, 1999). This finding is no different for the South African context, as illustrated in the discussion that follows.

Ms Dikgomo

Ms Dikgomo's first lesson offered her the opportunity to make connections between the operations that were named in lesson segment 4, e.g. between addition and multiplication, and between multiplication and division. She also had the opportunity to establish links between multiple operations and the BODMAS rule. However, the type of questions she posed to the learners and her eagerness to get correct answers might have limited the opportunities for her to make such connections. As a result of this lack of explicit connection, the learners slavishly computed the sums that were given in lesson segments 2 and 3 respectively using concrete manipulatives. As argued by Pape & Tchoshanov (2001: 124), representations must be thought of as tools for cognitive activity rather than products of the end result of a task. The lack of explicit connections relegated manipulatives into tools for developing drill and practice skills. In this way, the learners got the correct answers without much conceptual understanding of the procedures they followed. As noted by Hill & Ball (2004: 331), teaching mathematics requires an appreciation of mathematical reasoning, understanding the meaning of mathematical ideas and procedures, and knowing how ideas and procedures connect.

The first challenge in this case, as illustrated in lesson segments 2 and 3, the teacher selected unchallenging sums that already had brackets in them and that could be computed easily without the aid of manipulatives. As already mentioned elsewhere in the study, Ms Dikgomo's main concern seemed to be for her learners to get the same and correct answer as quickly as possible. Her utterance: 'I want you first to use the cubes to do the sum; you have 5 minutes to complete the sum' clearly illustrates the point. The connection between the lesson goal and manipulatives was also not made clear. Tasks that involve multiple operations without brackets could have helped to realise the connection between multiple operations procedures and the BODMAS rule through manipulatives. Most importantly, as conjectured by Ball (1988: 7) 'knowledge of mathematical procedures entails knowing when to use them and understanding why they work'. Ms Dikgomo may have also missed the opportunity to make the connections between the topic and other relevant topics in the Grade 6 such as area, perimeter, etc.

<u>Ms Bohata</u>

Ms Bohata started her lesson by outlining the lesson objective: 'to do different kinds of 3D shapes'. Her intention was probably to ensure that learners know exactly what the lesson was about and what they were expected to know at the end of the lesson.

However, her lesson objective remained ambiguous as she did not clearly articulate what is entailed in 'doing different kinds of 3D shapes', i.e. whether it was about recognising, visualising, naming, describing, classifying, etc. the 3D objects as prescribed in section 3.2 of the Intermediate Phase CAPS document for Grade 6. Perhaps this also explains why explicit and clear connections between the lesson goal, the lesson tasks and the use of manipulatives were not apparent.

Ms Bohata grounded her lesson on prisms and their respective properties on her learners' prior knowledge of 2D shapes and their properties. In doing so, she made connections between and among topics, concepts and ideas in a number of ways. She established a link between concepts within the geometry topic by guiding her learners to use their knowledge about 2D shapes and their properties to make and name different prism structures, and to extract the properties of those prisms. Learners were asked to name their respective prisms (which were written in table 3 under the 3D shape column) and to also match it with the list of 2D shapes already listed in the table. As and when groups presented their respective prisms, they also gave the properties of those prisms as embedded in their respective structures, i.e. the names and number of 2D shapes, and the number of faces, vertices and edges that make up the structure.

Ms Bohata also made connections between topics, i.e. properties of 3D objects, data handling and numeric patterns to promote meaningful understanding of and substantive logic behind the properties of 3D shapes. However, in both cases Ms Bohata could neither make the connections explicit nor provide reasons for the connections. Once more, she may have missed the opportunity to explain, for example, how polyhedrons (a family of 3D shapes) are made up of polygons (a family of 2D shapes) and how the properties of the latter have a bearing on the names and the properties of 3D shapes. For example, a pentagonal prism has a pentagon as its base and has 5 lateral faces. The major reason behind the connections is to ensure coherence of mathematics as a discipline. Ms Bohata's inability to explicitly locate prisms within the bigger picture, i.e. the family of 3D shapes was apparent when she ignored the responses of the two learners who cited respectively, a circle and an oval as examples of a 2D shape and 3D object. This also illustrates the missing piece in Ms

Bohata's SCK and PUFM of the topic at hand, constructs that are characterised by both horizontal and vertical knowledge of the topic.

Mr Kopung

In his introduction of the first lesson, Mr Kopung outlined his lesson goal by listing the following prerequisite concepts and skills to be learned: a) 2D shapes, b) 3D solids, c) edges, vertices and faces of a solid, and d) Euler's formula. In so doing, he established the connections between mathematical topics and concepts. In lesson segment 16, Mr Kopung engaged his learners in an activity where they had to identify and name different 2D shapes. Through this activity, he managed to establish a link between his learners' prior knowledge about 2D shapes and the new topic. What stood out in Lesson Segment 16 is how Mr Kopung deliberately placed due emphasis on the conceptual understanding of geometrical concepts such as a square and a rectangle. By emphasising the difference between a square and a rectangle, Mr Kopung was also able to further make a connection between concepts within the same topic. All the above scenarios illustrated how Mr Kopung managed to present mathematics as a coherent and connected system as proposed by Ball and Bass (2000).

Despite the above illustrations, there were a number of episodes where Mr Kopung missed opportunities to make and strengthen mathematical connections. In introducing the task that involved the use of concrete manipulatives, Mr Kopung did not establish any explicit link between the components of concrete manipulatives and the mathematical concepts, skills and ideas embedded in the purpose of the task. This resulted in his learners struggling to reconcile the terms edge and strut, vertex and node, and face and flat shapes in their models, a situation that could have limited his learners' conceptual understanding. Scholars have noted that in order to promote mathematical understanding, it is necessary to make connections between manipulatives and mathematical ideas explicit (e.g. Ball, 1992; Driscoll, 1981; Hiebert, 1984 and others, all in Ma, 2010: 6). Mr Kopung may have assumed that these connections would be obvious to the learners.

Mr Kopung's insistence on following the rules was apparent in the main activity of his first lesson. After spelling out the steps to follow, he went to the groups that seemed to be struggling, insisting that they 'follow the steps. 'If you are done, go to the next step', those who are making progress 'go to the next step, yes do that, follow the steps according to what is on the board, you read the instruction, follow the steps'. Moving from one group to the other and occasionally shouting: 'Just follow the rules' limited his own teaching opportunities for making the connections between the topic on 3D objects and other topics such as data handling and patterns, to promote deeper understanding of the properties of prisms.

Summary

It needs to be noted that the three teachers articulated their lesson objectives in rather ambiguous terms, a situation that could have made it difficult for them to make explicit linkages between the lesson goals or objectives and the manipulative used during the lessons.

Ms Dikgomo did not make explicit links among mathematical topics, concepts and ideas in her lesson on multiple operations thus presented mathematics as a discrete discipline. Her insistence on learners to complete the task quickly and to get the same and correct answer may have sent the message that it is not necessary to make the connections as long as the answer is correct. This lack of explicit connection forced her learners to slavishly compute the sums that were given to them using concrete manipulatives, and thus relegated manipulatives into tools for developing drill and practice skills

Ms Bohata and Mr Kopung managed to establish a link between their respective learners' prior knowledge about 2D shapes and the new topic on 3D objects. Ms Bohata also managed to link her topic on 3D objects with other Grade 6 topics on data handling and patterns, although inexplicitly so.

Mr Kopung introduced his first lesson goal by established the connections between the topic and the prerequisite concepts and skills to be learned, i.e. a) 2D shapes b) 3D solids, c) edges, vertices and faces of a solid, and d) Euler's formula. He also placed emphasis on the conceptual understanding of geometrical concepts such as a square and a rectangle and thereby managed to make a connection between concepts within the same topic. However, in his main activity of the same lesson, he did not make the connections between the topic on 3D objects and the other topics, such as data handling and patterns, to promote deeper understanding of the properties of prisms. This could mainly be attributed to his insistence on learners to follow the rules as illustrated in his utterance: 'Follow the steps. If you are done, go to the next step', 'those who are making progress go to the next step, yes do that, follow the steps according to what is on the board, you read the instruction, follow the steps'. As in the case of Ms Dikgomo, this resulted in his learners slavishly computing the formula and thus relegating manipulatives into tools for verifying someone else's ideas rather than as tools for constructing their own abstract models of the characteristics of prisms and their relationships.

5.3.4 Use of Multiple Representations and Contexts to Complement Manipulatives

As with other domains of MKT, the use of representations, including physical objects, is intended to make mathematics accessible and comprehensible to learners. However, transition from manipulating concrete materials to creating images from the learner's perception of the concept, and finally to the development or adoption of some form of symbolic notation representing the concept (Kosko & Wilkins, 2010: 79), has become the main challenge regarding the use of concrete representations.

To this end, Bruner, 1964 in Drews (2007: 20) underlined the role of physical objects in this process by posing three stages through which children represent their understandings: a) the enactive phase in which children are involved in some form of action by manipulating physical tools, b) the iconic phase where children create images through their own representations through drawings, pictures or images and c) the symbolic phase where students are ready to move from the iconic representations to the standard language or symbolic notation. The sequence follows Bruner's (1966) learning model based on three levels of engagement with representations, i.e. active (e.g. manipulating concrete materials), iconic (e.g. pictures and graphs), and symbols (e.g. numerals). Students are expected to abstract mathematical procedures that are analogous to symbolic procedures. Through the use of analogy, transformation and simplification, new understandings are built from existing knowledge (Pape & Tchoshanov, 2001; 123). Learners have different learning styles, i.e. visual, kinaesthetic, etc. and using multiple representations may be a strategy to accommodate those different learning styles and thereby giving access to mathematics to all learners.

Ms Dikgomo

In her first lesson, Ms Dikgomo used multiple representations in different forms and in different ways. She used a visual representation in the form of a video picture to emphasise the need for rules in both mathematics and in a real life context, i.e. a motorist not obeying the rules and causing an accident. The video clip provided further emphasis of her point and was probably the most powerful tool for driving this perspective to the learners, which they are likely to remember long after the mathematics lesson was completed.

This was followed by a symbolic representation, e.g. 10 + (20 x 3) + 4 = and (36 ÷ 9) + (18 ÷ 3) that modelled multiple operations. She then directed her learners to use concrete objects, i.e. interlocking cubes in her first lesson to compute the above sums in an attempt to concretely model the multiple operations, the procedure and the BODMAS rule embedded in the symbolic notation.

Ball (2003: 3) argues that teachers need to use representations skilfully, choose them appropriately and carefully map between a given representation, the numbers involved, and the operations and processes being modelled. What was apparent in Ms Dikgomo's lesson was not only the use of different modes of representation but also a different sequence from that proposed by Brunner and others. She used the abstract to concrete approach to teaching in the lesson, an approach that would seem to contradict the concrete-pictorial- abstract advocated by Brunner and others. If such an approach is used, the likelihood is high that concrete materials will merely be used to verify the rule by simply counting to get to the answer. In that way, the opportunity to model procedures and the operations in order to understand the mathematics underlying those procedures and operations may be compromised. Cramer and

Karnowski in (Kosko & Wilkins, 2010: 80) posit that when describing different forms of representations, identify manipulatives as concrete representations that should be followed by pictorial representation, and then verbal and written representations. They further contend that the latter two forms of representation are critical for linking informal mathematical knowledge to abstract representations and understandings.

In the task that she assigned her learners, Ms Dikgomo started with the symbolic notation that represented multiple operations. Here, she seemed to have little or no influence on the phrasing of the representation as it was taken directly from the textbook as a finished product. The activity directly followed the oral questions, e.g. 'What are the four basic operations? How do we get the product of numbers? The questions sought to elicit learners' recall of prior knowledge of mathematical concepts and language. However, Ms Dikgomo did not explicitly link her learners' pre-knowledge to the subsequent task for which her learners used manipulatives to compute the symbolic notation. In addition, there was no clarity given to the learners as to how the task needed to be done, especially with interlocking cubes. This disconnect could have also contributed to her learners' inability to model the operations, concept formation, language acquisitions and procedure that she wanted her learners to understand. Instead, concrete manipulatives, i.e. interlocking cubes, just became tools to verify the BODMAS rule. This may have been influenced by her belief about mathematics teaching and learning, that is characterised by overemphasis on rules and procedures. Disconnection with prior knowledge was apparent in the lesson as she only used concrete manipulatives to verify the symbolic notation.

Manipulatives were used differently, however, in Ms Dikgomo's second lesson. In teaching the fraction concept, she started with learners exploring concretely with manipulatives and paper folding. Unlike in the first lesson, it seemed as if her use of multiple representations was not a coincidence, but that it formed part of her lesson aim which she articulated thus during the post interview:

> Ke aimile hore ngwana a understande ho iketsetsa le ha a sa bone – le ha dintho tseo di le siko ka pela hae a kgone ho etsa fraction on his or her own a sa bone di interlocking ke hore picture eo e dule ka minding (My aim is for

learners to understand and formulate a mental picture of the fraction concept on their own, even without the use of concrete manipulatives).

Ms Dikgomo wanted to establish her learners' understanding of the concept of a fraction and she drew on her learners' informal knowledge of the concept of a fraction to inform her subsequent actions. In her main activity, which lasted for about 20 minutes, she directed her learners to use interlocking cubes and paper folding to demonstrate each of the fractions they already know. She knew the importance of the concept of a 'whole' and used the part-whole approach to teaching the fraction concept. To further support learners in their understanding of the fraction concept, she posed the question: 'How do you know this is a half?' This was a conceptual question that called for deeper understanding of the fraction concept that goes beyond the surface characteristics of a fraction model. Ultimately, she carefully directed her learners to formally represent the various fraction examples numerically. This seemed to be in line with how classroom mathematics scenarios using concrete materials were defined as typically beginning with exploration, followed by a more systematic manipulation (Pape & Tchoshanov, 2001: 123). These researchers suggest that students be provided some time to explore the materials without direction, and then finally be funnelled into the written, symbolic procedures.

Ms Bohata

Ms Bohata made use of concrete, visual and abstract modes of representation throughout her lessons. Through the construction of various prism models, she afforded her learners the opportunity to physically experience prisms and their properties. To ensure that her learners experience these physical tools beyond their concreteness, she also created opportunities for her learners to abstract the properties of prisms from the models. This she carefully managed to do by complementing manipulatives with the use of tables, which she not only used to collect and display data, but also to teach the subject matter. As illustrated in Lesson Segment 10, Ms Bohata directed and guided her learners to use their concrete models as concrete referents when completing the table. She deliberately guided the learners to realise that the edges and the vertices are represented by the zoom struts and the nodes respectively. These physical and visual experiences made it easy for learners to complete table 5 as directed. Ball (2003: 3) argues that teachers need to use representations skilfully, choose them appropriately and carefully map between a given representation, the numbers involved, and the operations and processes being modelled.

Through the use of tables as representations, Ms Bohata carefully guided her learners to identify patterns by making connections between the 2D shapes, their properties (sides) and their corresponding prisms, prism names and the prism properties. In this way, Ms Bohata made a deliberate attempt to make the knowledge of the properties of various kinds of prisms accessible and comprehensible to learners. Her learners also used the patterns observed to generate informal rules about prisms, e.g. the number of faces is equal to the sum of the sides of the base and 2, and the number of sides of the base is equal to the number of lateral faces (Lesson Segments and 13 respectively). This was illustrated by the ease with which learners could apply this rule to determine the number of faces in a 20 sided based prism, drawing on data in the table. Moving learners from the concrete to the abstract mathematics seems to have been enabled not only by the careful sequencing of the representation modes but also by flexibly moving from one mode to the other. This seems to support the observation by Pape and Tchoshanov (2001: 125) who contend that any intensive use of only one particular mode of representation does not improve students' conceptual understanding and representational thinking. Ms Bohata provided affordances that helped her learners understand fundamental mathematical concepts that underlie the properties of the prisms which in turn contributed to their conceptual understanding of these properties.

Ms Bohata's use of the learners' environment was, however, limited in that she only used objects in the mathematics laboratory as representations of 3D objects. Her success in making mathematics more accessible and comprehensible by using learners' contexts as representations could have been limited by her lack of creativity and the formality of the mathematics laboratory space.

Mr Kopung

Throughout the lessons, Mr Kopung used different mathematical representations such as skeletons of 3D shapes from struts and nodes, diagrams of 3D shapes, a table for data presentation and a formula to facilitate better comprehension of the relationship amongst the properties of 3D solids. The use of multiple representations, which seemed to support the Concrete to Pictorial to Abstract (C-P-A) approach as advocated by a number of scholars (e.g. Dindyal, 2006: 182; Kosko & Wilkins, 2010: 79; Pape & Tchoshanov, 2001:125), helped to promote the transition from manipulating concrete materials to creating images from the student's perception of the concept and its properties, and finally to the development or adoption of some form of symbolic notation representing the concept and its properties.

Mr Kopung's use of concrete models and the table was nearly the same as that of Ms Bohata. What was different and perhaps important to note, was how Mr Kopung used Euler's formula as a form of symbolic notation representing the relationship amongst the faces, vertices and edges of prisms. His use and prioritisation of Euler's formula created the impression that the formula is the only symbolic representation of relationship amongst the features of the prisms. Cobb and Yackel (1996: 186) conjecture that the use of particular materials and symbols is considered to profoundly influence both the nature of mathematical capabilities that students develop and the processes by which they develop them. The process of analysing data, identifying relationships and generating the rule out of such relationships was misconstrued to mean mere following of instructions and substitution.

While there is acknowledgement that mathematical formulae are important as symbolic representation of concrete mathematics, accurate and correct statement of formula statements is equally important. This makes it imperative that such representations, whenever used to represent mathematical concepts, ideas and procedures, be accurate and correct at all times. In his lesson, Mr Kopung exposed his learners to Euler's formula which he wrote on the board as $E = F + V - E = 2$. The use of the first E, which he explained as Euler, is incorrect, making the whole formula incorrect and confusing. This is more so because there is another E symbol in the formula. This is further compounded by his use of the equals sign to indicate that Euler is equal to F + V – E, and is also equal to 2. There is no doubt that this will seriously impede learners' developing conceptions and understanding of the sign and the relationships among the properties of prisms. Again, in his explanation of the formula, there was a serious omission that this formula is only true for particular 3D shapes, i.e. in polyhedrons, and that it cannot be generalised. These gaps in his usage of the formula as a symbolic representation could be attributed to Mr Kopung's eagerness to showcase his own knowledge and to be assertive, considering that he is the only one to have attempted mathematics at university level, even though he later discontinued after failing Calculus at first year level.

Additionally, Mr Kopung made use of diagrams of prisms which he projected on the screen from a computer. The diagrams had in them some pointers to the edge, vertex and lateral faces and base. As and when learners presented data from their respective models, he projected the relevant diagram. In this way, learners were also afforded the opportunity to switch from concrete to visual modes of prism representation, thus accommodating learners' different learning styles.

Summary

It is important to note that teachers mainly used commercial manipulatives that were found in the laboratories, to almost total exclusion of either teacher or learner made manipulatives, or to contexts that relate to learners' real life experiences. Such an approach may impede active and meaningful construction of knowledge especially when learners come from low income and working class families who may not be familiar with such manipulatives. The approach seems to contradict Freire's problem-posing pedagogy which contends that learners are not empty vessels, passively waiting to be filled with knowledge but rather are individuals with knowledge and life experiences.

In her lesson on multiple operations, Ms Dikgomo used visual, symbolic and concrete representations to model the BODMAS rule and its application. However, she displayed unskilful use of manipulatives by starting from the pictorial, to the abstract and then to the concrete representation, as opposed to the concrete-pictorial-abstract sequence (Cramer and Karnowski in Kosko & Wilkins, 2010: 80). As a result, her learners struggled to move from their informal mathematical knowledge of counting to abstract representations and understandings of division, involving ‘groupings’. In this way, interlocking cubes became tools just to verify the BODMAS rule instead of cognitive tools to help understand the mathematical ideas underlying the procedure. As argued by Pape & Tchoshanov (2001: 124), representations must be thought of as tools for cognitive activity rather than products of the end result of a task. In her second lesson on fractions, Ms Dikgomo started by allowing her learners to concretely explore the concept of a fraction before she led them to symbolic representations. In addition, her approach was more open in that she allowed learners to use their intuitive knowledge to model and represent the concept of ‘half’ from their own ‘wholes’, i.e. different number of objects and shaded areas. In this way, she created the opportunity for her learners to freely explore and learn about the fraction concept through the use of manipulatives of their choice, using their own knowledge as a basis without rushing them into symbolic representations. This represents a case of the shift in patterns of control from teacher centred to learner centred where the interest of the learner is put before that of the teacher.

Mr Kopung and Ms Bohata made use of the concrete, visual and abstract modes of representation throughout their lessons. The sequence of representations, i.e. Concrete – Pictorial - Abstract was also evidence of how the tables and the symbolic modes of representation were used to complement the manipulatives. Flexible use of various representations, e.g. from pictorial and/or symbolic back to concrete, was also apparent in both cases and thus increasing and supporting access to abstract mathematics by learners with different learning styles. This seems to support the observation by Pape and Tchoshanov (2001: 125) who contend that any intensive use of only one particular mode of representation does not improve students’ conceptual understanding and representational thinking. In both cases, leading learners from the concrete to the abstract mathematics seems to have been enabled not only by the careful sequencing of the representation modes, but also by flexibly moving from one mode to the other.

Ms Bohata carefully managed to complement manipulatives with the use of tables, which she not only used to collect and display data but also to teach the subject matter. She carefully guided her learners to identify patterns from data in the tables and to make the knowledge of the properties of various kinds of prisms accessible and comprehensible to learners. Her learners also used the patterns observed to generate informal rules about prisms and this let them to easily apply their own determined rule to determine the number of faces in a 20 sided based prism, drawing on data in the table. Each time she directed and guided her learners to use their concrete models as concrete referents when completing the tables and when formulating a rule. Ball (2003: 3) argues that teachers need to use representations skilfully, choose them appropriately and carefully map between a given representation, the numbers involved, and the operations and processes being modelled.

As mentioned before, Mr Kopung additionally made use of prism diagrams with pointers to the edge, vertex and lateral faces and base which he projected on the screen from a computer. As and when learners presented data from their respective models, he projected the relevant diagram. In this way, learners were also afforded the opportunity to switch from concrete to visual modes of prism representation, thus accommodating learners' different learning styles. However, Mr Kopung's symbolic representation of the properties of prisms in the form of Euler's formula was problematic in many ways. He introduced the formula before learners could fully understand the relationships among the properties of prisms. He also represented the formula inaccurately. His use and prioritisation of Euler's formula created the impression that the formula is the only symbolic representation of relationship among the features of the prisms thus limiting his learners' capability to generate their own formulae. These gaps in his usage of the formula as a symbolic representation could be attributed to Mr Kopung's eagerness to showcase his own knowledge and assertiveness at the expense of the learners' interest and understanding. This makes it imperative that whenever representations are used to represent mathematical concepts, ideas, and procedures, they be accurate and correct at all times. Without such accurate representation learners' developing conceptions and understanding of the sign and the relationships among the properties of prisms as embedded in the formula, may be seriously impeded. In his second lesson, Mr Kopung guided and allowed learners the freedom to generate their own rules (formulae) from data in the table. One learner could not hide his excitement about generating the rules that he even remarked at the end of the lesson: 'Re e fumane secret ya Euler!' meaning 'We have discovered Euler's secret!' This demonstrated how lack of freedom from the authority of mathematical formulae and highly commercial manipulatives, among others, may inhibit mathematical fun, creativity, self-determination and self-affirmation, all of which are fundamental to mathematical learning and understanding.

5.3.5 Mathematical Communication in Relation to the Use of Manipulatives

Abstracting mathematical ideas and concepts from the concreteness of manipulatives has become one of the critical components of the use of manipulatives as a teaching strategy. The use of manipulative, as suggested by Marshall & Paul (2008: 340), should not be seen purely as a means to an end, i.e. the development of traditional arithmetic skills, but really as a catalyst for deepening mathematical understanding. To achieve this, they suggest, the skilful teacher will need to encourage the students to talk about, discuss and explain their understandings gleaned from exploring with mathematical manipulatives. In this way, language becomes a tool to bridge the gap between the concrete and the abstract. Creating opportunities for learners to engage in mathematical discussion of explanation, argumentation, justification etc., helps learners to uncover and clarify their mathematical ideas, concepts and procedures that are embodied in concrete manipulatives. Equally so, such opportunities allow teachers to understand their learners' thinking, their meaning making processes, their needs and interest and be responsive to this by adjusting both their teaching practices and their knowledge.

<u>Ms Dikgomo</u>

In her pre-observation interview Ms Dikgomo indicated that learners get excited when they use manipulatives, that manipulatives promote concentration and allow learners to talk freely in class irrespective of their abilities. However, what was not apparent in her lessons is how she capitalised on these advantages, especially learners' free talk. As mentioned earlier in this chapter, in spite of the opportunities provided by manipulatives, her over-emphasis on the mastery of rules and procedures in order to get common and correct answers overshadowed the need for conceptual understanding. She overlooked the need for understanding of the rationale behind the algorithms. To her, the algorithm of the BODMAS rule became an end to itself, thus limiting her opportunities to allow her learners to communicate mathematically while using concrete manipulatives.

In one episode of lesson segment 4 (see chapter four section 4.2.6 for details), Ms Dikgomo posed the question ‘How do we get the product of two numbers, what must we must do to find the product?’ and one learner responded: ‘We add’. The learner’s response could have been a trigger for conceptual explanation and justification. However, the error was left unattended and no follow-up was made to try to understand either the learner’s misconception or its source. This is not surprising because for Ms Dikgomo a) mathematics is a neat and linear subject which can easily be mastered by only following correct procedures, b) a different perspective to mathematics is regarded as chaotic rather than an opportunity to understand the underlying principles behind the algorithm, and c) discipline and orderliness are needed to enforce rules and procedures in mathematics. All these seem to suggest that there is no need for discourse and debate around mathematical solutions and problem solving strategies in her class.

Both the low level questions and the unchallenging tasks seemed to limit the opportunities for learners to actively participate in mathematical discussions and meaning making in Ms Dikgomo’s first lesson. This was illustrated during the session where learners presented their solutions to the task that involved the application of the BODMAS rule. Explaining how learners got the answer was limited to only restating the BODMAS rule instead of engaging in conversation with each other and finding even more approaches for solving the problem. Again, Ms Dikgomo may have missed the opportunity to allow her learners to communicate their thinking and understandings of mathematical ideas, concepts and procedures to her and their fellow learners. This was further exacerbated by lack of differentiation in the tasks given. All learners were given the same task and this also limited the opportunities to broaden the discourse about different procedures and approaches.

Ms Bohata

On a number of occasions, Ms. Bohata also seems to have missed opportunities to engage her learners in meaningful mathematical discussions. In the episodes where Ms. Bohata asked her learners to give examples of 2D shapes and 3D objects respectively, her learners responded 'oval' and 'circle' respectively. Both responses could have prompted explanation, argumentation and justification. However, Ms Bohata missed the opportunity to engage her learners in mathematical discussions as she simply brushed the responses aside and instead insisted that they give examples that she wanted. As illustrated in her reaction: 'It is a circle.... okay. Leave that one, fast! Make a triangular prism; just do a triangle for us!' Her handling of those two responses which limited her opportunities to engage her learners in mathematical debates seems to be due to her knowledge gap regarding the topic itself and/or her traditional approaches which placed less emphasis on discourse and reasoning. The latter is supported by the claim that the reform agenda represents a tall order for many of the classroom teachers whose experiences of mathematics and mathematics identities have been within the traditional approaches to school subjects, which placed less emphasis on problem solving, discourse and reasoning (Ball, 1993; Cohen, 1990; Spillane, 2000 all in Jita and Vandeyar, 2006: 40). Scholars have referred to the latter as new mathematical learning practices that students need to master in addition to mathematics itself in the reform-oriented curriculum (Boaler, 2002; Cohen & Ball, 2000; 2001), practices that are rarely given particular attention and therefore are seldom taught.

In traditional mathematics classrooms, learners are required to produce correct answers whereas in reform oriented classrooms they often need to go beyond correct answers and explain their methods and the approaches they have used in keeping with the reform agenda of the NCS in South Africa. To be successful in the classroom, students need to master not only mathematics but also particular learning practices (Boaler, 2002: 243) including mathematical discourse, explanations and reasoning. These learning practices can only lead to successful participation of learners in reform mathematics classrooms teaching if learners are supported. For teachers whose experiences of mathematics have been within the traditional approach to provide such support, the development of more appropriate learning opportunities has become imperative.

In her second lesson, Ms Bohata wanted to develop her learners' understanding of the '2' that was a common denominator in the column on the number of shapes that formed each prism in Table 5. To do this, as illustrated in lesson segment 12, she used both leading questions and examples in her learners' language to develop the concept of a base as analogous to 'sekwagelo' (lid) and the logic behind having two bases in a prism. This illustrates that mathematical discourse as one of the new learning practices does not happen automatically; there must be a deliberate effort by the teacher to create opportunities for learners to learn mathematical discourse and to scaffold learners until they master the practice.

Mr Kopung

During the main activity of his lesson (see chapter four, sub-section 4.4.4.3 (c) for details) Mr Kopung displayed a strong tendency to tell and show rather than affording his learners space to interpret the models and data embedded in them (models) and to debate their findings or solutions. He drew his learners' attention to the board as he explained the steps to be followed in order to use data emanating from the prism models and to apply Euler's formula. Warning against this kind of approach, Cobb et al. (1992: 6) posit that the model should not be used as a means of presenting readily apprehensible mathematical relationships but should instead be aspects of a setting in which the teacher and students explicitly negotiate their differing interpretations as they engage in mathematical activity.

There were important episodes in lesson segment 16 where differing interpretations emerged as opportunities for mathematical debates, argumentation, logical reasoning and justification. In lesson segment 16, Mr Kopung asked his learners if a rectangle is a square, based on the fact that they both have four sides. In the same lesson segment, he wanted his learners to explain their understanding of 'opposite sides are equal' and learner 6 responded: 'Both of them are equal but are not the same sizes'. In both cases, opportunities that triggered learners to explore manipulatives and to discuss, explain, and justify their thinking were imminent. However, these opportunities were not fully exploited and thus resulted in Mr Kopung missing the opportunity to meaningfully engage his learners. He did manage to engage his learners in some informal reasoning about what is common and different between the two concepts though, i.e. a square and a rectangle. However, the initiative could have escalated to the next level by showing that a square is a special type of a rectangle had he allowed his learners the space to engage in mathematical discussions and explanations to justify their thinking. This could be attributed to his tendency to resort to chorusing as illustrated in his frequent response: ‘Do we agree?’, ‘Any other group, what do you say?’

Similarly, in another episode in lesson segment 17, Mr Kopung asked learners: ‘How many sides are there in a circle?’ Learners responded: ‘No sides’ and ‘One side’ respectively. This question was appropriate in that it helped to uncover knowledge about how learners think about a circle. This question had a great potential to elicit differing views and presented the opportunity for learners to explain, debate and justify their views. Instead of taking advantage of the opportunity, Mr Kopung left the learners’ responses unattended without giving or evaluating the mathematical explanation of the circle. Once more, Mr Kopung may have missed the opportunities to make visible his learners’ mathematical thinking processes about the circle.

In one group that Mr Kopung had directed to construct a triangular prism, an argument ensued after one learner had constructed a pyramid instead, insisting that it was a triangular prism because all faces were made up of triangles. Instead of allowing and listening to the debate, Mr Kopung just told the group to use the triangle as a base. Similarly, in another group there were arguments about the size of struts to be used in a single model. Upon hearing the arguments, Mr Kopung stopped everybody and said: ‘Construct a 3D shape using long blue struts and blue nodes’. It became evident from these episodes that when there are disagreements, Mr Kopung did not seize the opportunity to listen and allow debates. Instead, he resorted to his authority to divert learners to what he wanted them to do. The impression created is that disagreements and debates were not welcome in his class and perhaps did not even have a place in mathematics.

Summary

As mentioned in the second paragraph of section 5.2 of the study, all the lessons observed were characterised by teacher talk and a culture of silence among the learners who almost never posed questions and were rarely called upon to explain their methods and answers to problems. This could be attributed to the use of English as LoLT in the Intermediate Phase, which I observed to be a challenge with both teachers and learners as they worked through the curriculum and manipulatives. In all three cases there were important episodes in the respective teachers' lessons where differing interpretations emerged as opportunities for mathematical debates, argumentation, logical reasoning and justification and yet these opportunities were missed. This is not surprising given the fact that teacher-centred approaches mostly characterised their teaching and the dominance of closed approaches to questions and tasks given to learners.

In the introduction of her first lesson, Ms Dikgomo missed the conceptual explanation and justification opportunity when a learner associated the 'product' concept with addition. She left the error unattended without any efforts to try to understand either the learner's misconception or its source. Where learners were requested to explain how they arrived at their answers in a multiple operations exercise, the discourse remained at a very elementary level. Follow-up questions such as 'Why is it important to start with brackets; 'How would you do it if there were no brackets; 'How would you go about if you had a different operation inside the brackets? etc. could have extended the discourse. This illustrates the potential that further questions instead of single questions may go a long way in making mathematical discourses a culture in our classrooms. The culture of silence was further exacerbated by the lack of differentiation in the tasks given. This may also limit the opportunities to broaden the discourse about different procedures and approaches, and the sharing of ideas among the learners. All these seem to suggest that there is no need for discourse and debate around mathematical solutions and problem solving strategies in her classroom.

Ms Bohata, in her first lesson, missed the opportunity to engage her learners in mathematical discussions by simply ignoring the responses and using her authority to channel learners to give the responses that she wanted. She also uses time constraints as an excuse as illustrated in one of her reactions to one learner's response: 'It is a circle.... okay. Leave that one, fast! Make a triangular prism; just do a triangle for us!' Her handling of the learners' responses that were not according to her expectations limited her opportunities to engage her learners in mathematical debates. This seemed to be due to her knowledge gap regarding the topic itself and/or her traditional approaches which placed less emphasis on discourse and reasoning. In her second lesson, consciously or unconsciously, she carefully scaffolded her learners. This illustrates that mathematical discourse, as one of the new learning practices, does not happen by itself, there must be a deliberate effort by the teacher to create opportunities for learners to learn mathematical discourse and to scaffold learners until they master the practice. This approach typifies what Boaler, (2002: 253) characterises as the complex support that teachers may need to provide to students.

Mr Kopung, in his first lesson asked his learners if a rectangle is a square, based on the fact that they both have four sides. In the same lesson segment, he wanted his learners to explain their understanding of 'opposite sides are equal' and learner 6 responded: 'Both of them are equal but are not the same sizes'. In both cases, opportunities that triggered learners to explore manipulatives and to discuss, explain, and justify their thinking were imminent yet these opportunities were not fully exploited. In another lesson segment an argument that ensued after one learner had mistaken a pyramid for a triangular prism was simply dismissed. Instead of allowing the debate and listening, Mr Kopung just told the group to use the triangle as a base. All these are illustrations of how Mr Kopung missed the opportunities that were already there to engage his learners meaningfully in the lessons. This could be attributed to his tendency to resort to his authority as well as his eagerness to assert and showcase his knowledge of Euler's formula.

5.4 CONCLUSION

In this chapter, teaching in mathematics laboratories was discussed under the following five themes that played themselves out during the task of teaching by the three teachers, namely a) questioning practices, b) choice of tasks, c) connections among mathematical topics, concepts and ideas, d) use of multiple representations and contexts to complement manipulatives, and e) mathematical communication in relation to manipulatives. I made use of the five themes to discuss how each of the three teachers provided or failed to provide the opportunity to their respective learners to enhance the learning of mathematics using manipulatives while also looking at the literature review in comparison with my empirical data within each theme. A summary of affordances and constraints to mathematical learning in mathematics laboratories in this study was also provided within each of the themes in respect of each teacher.

In the next chapter a detailed description of the recommended strategy framework and guidelines on how mathematical laboratories and manipulatives in them can be effectively utilized to enhance mathematical learning will be presented. The framework will be informed by the contributory factors that are embedded in the affordances and constraints to mathematical learning as discussed within each theme, as well as by issues raised as commonalities.

CHAPTER 6: FINDINGS, RECOMMENDATIONS AND CONCLUSIONS

6.1 INTRODUCTION

The use of manipulatives is not a new phenomenon in the teaching and learning of mathematics in the South African education system. However, the establishment of mathematics laboratories in the Free State Primary schools in 2011, consisting of concrete and visual manipulatives, raised questions about their (laboratories) effectiveness. The major objective of the current study was to explore how teachers make use of manipulatives in mathematics laboratories. The study was also aimed at investigating whether the use of manipulatives has the potential to bring about any substantive changes to teachers' mathematical knowledge and classroom practices, and to seek explanations for the influence of manipulatives or lack thereof on teacher knowledge and their classroom practice in primary schools. To realise these objectives, teachers' experiences with the use of manipulatives, the meanings that these teachers attach to their experiences and how these experiences shape the nature of changes (if at all) in both their mathematical knowledge and classroom practices were explored. With particular focus on three case studies, the study sought to answer the following research questions:

a. How does the use of manipulatives in the teaching of primary school mathematics help to reshape teachers' own mathematical knowledge?
b. How does the use of manipulatives help to reshape the teachers' own mathematical classroom practice?
c. How can we explain the influence of manipulatives or lack thereof on teachers' mathematical knowledge and classroom practices?

This study presented case studies of three in-service teachers who teach Grade 6 mathematics in three primary schools with mathematics laboratories in the Motheo Education District, which is located in the Mangaung Metropolitan Municipality of the

Free State province of South Africa. The case study method was chosen for the study because it; a) provides a framework to investigate a current, real life issue that occurs in the course of real classroom teaching and learning, b) allows for access to an in-depth understanding of how teachers experience the use of manipulatives.

Guided by the principles of CER, data were gathered through qualitative methods and from multiple primary sources including in-depth interviews, focus group discussions, observations and document analyses. In-depth interviews with the three core participants and group discussions with the forty one PAR focus group members were conducted to examine and understand what teachers say and think about their own knowledge, experience and understanding of (i) mathematics knowledge (ii) mathematics teaching - classroom practice, and (iii) using manipulatives in the teaching of mathematics. Additional data were gathered through the three teachers' classroom observations, pre and post observation interviews in order to observe, examine and understand what teachers do in their classroom practice as they (i) use their knowledge of mathematics, and (ii) use manipulatives in the teaching of mathematics. Analysis of documents such as the CAPS document, learners' written work and textbooks was also done during the first visits as part of the pre-observation data collection. The data collected related to specific mathematics topics that were to be taught during the classroom observations to supplement baseline data.

The unavailability of the fourth participant, Mr Makau whose lessons could not be observed because on one occasion he was busy preparing for a job interview and on the other occasion he was absent from work for personal reasons, was one of the challenges in conducting the study. Consequently, the number of core participants was reduced from the initial four to only three participants. It needs to be noted that the reduction of the number of core participants did not have an adverse impact on the quality of data as the PAR approach allowed for sufficiently rich and thick descriptions of the three cases.

This study intended to explore how teachers make use of mathematics manipulatives and the role of the use of manipulatives in the relationship between teachers' knowledge and their classroom practice in primary schools in the Free State. This chapter starts off with the aim and objectives of the study followed by a synopsis of each of the preceding five chapters of the study. The chapter also presents the findings organised in respect of the research questions and their respective constructs. Within each research question, conclusions will be presented. The recommendations of the study as well as the conclusions are provided at the end where the former endeavours to also highlight research gaps that could not be addressed by the study and the possibilities for future research in the area.

6.2 AIM OF THE STUDY

The aim of the study was to explore how teachers make use of mathematics manipulatives and the role of the use of manipulative in the relationship between teachers' knowledge and their classroom practice in primary schools in the Free State. Pursuant to this aim and the research question, the study sought to understand how teachers use manipulatives to transform mathematics learning and this was done from the perspective of teachers' classroom practice, which can only be understood by exploring what informs it. Part of what informs teachers' practices is the teachers' knowledge of content and of the teaching of mathematics, i.e. pedagogy. It is for this reason that the study of the strategies for effective utilisation of manipulatives in mathematics laboratories primarily involved examining the relations between teacher knowledge, classroom practice, the use of manipulatives and the possibilities for the latter to reshape teacher knowledge and classroom practice.

6.3 SUMMARY OF THE STUDY

Chapter 1 of the study provides background and orientation as a way of introducing the study to the readership. The chapter reflected briefly on the problem statement, the study question, the aim of the study, i.e. to explore the use of manipulatives to enhance the teaching and learning of mathematics, and the study objectives. To justify the need for a strategy, the chapter also highlighted the significance of the study both at a personal level and a system level. The chapter also presented the status of school mathematics education in South Africa, in an attempt to contextualise the study. Furthermore the reader was also introduced to the method and design, as well as the theoretical framework that were chosen to couch the study.

In **Chapter 2** a range of literature was searched in order to appropriate the constructs of the current study within the existing research literature. The chapter reflected extensively on Critical Theory as the underlying theoretical framework that couched this study, as well as on how power and democratic values of social justice, equity and freedom impact on transformation, emancipation and empowerment, all of which are at the very heart of this study. Five operational concepts of the study pertaining to teachers' mathematical knowledge for teaching, mathematical classroom practice, views about mathematics, use of mathematical manipulatives and professional development, were also discussed with a view to enhance convergence of thought of the readership. Literature searched indicates that although much research has been conducted on the use of manipulatives in mathematics classrooms, little has been done on teachers' experiences with manipulatives and how those experiences shape their knowledge and classroom practice.

Chapter 3 presents an outline of the chosen research methods and design in order to generate a thick description of how teachers understand and attach meaning to their own knowledge and experiences about the use of manipulatives in their teaching of mathematics. The chapter presents a declaration, discussion, and justification for my critical emancipatory stance which grounded the study. The choice of my research paradigm, qualitative methods, PAR model and data collection strategy, their appropriateness to the study and their compatibility with the critical emancipatory nature of my study was discussed and justified. It also provided an explanation and description of the sociocognitive approach to discourse analysis as proposed by Van Dijk (1993, 2009) as the data analysis strategy used in the study. The chapter also elaborated on the criteria that underpin critical emancipatory theory to determine the quality of the study in an attempt to meet the methodological expectations of the community of scientists.

Through **Chapter 4,** the study presents a thick and detailed empirical data collected from the participants through qualitative methods and mainly from primary sources including interviews with the teachers, specific classroom descriptions, video recorded lessons and curriculum materials. The chapter provides data presentation in the form of chronicled stories of each of the three core participants in the study, as well as data interpretation in the form of both spoken words that were later transcribed into text as lesson segments, and non-verbal interactions as communicative events and situations that were captured as my field notes. In pursuance of the study aim and the research question, a combination of both the MKT and CDA frameworks were used to analyse both mathematics teaching and the power relations between the learners and the teachers respectively. Through both frameworks, spoken words and non-verbal interactions in respect of the participants were interpreted so as to foster a better understanding and also compare them to theoretical data gathered from literature in chapter two to determine if there is conformity and corroboration or not.

In Chapter 5 the findings of the study are presented in respect of the following five themes that emerged in chapter 4 during the task of teaching by the three teachers in mathematics laboratories a) questioning practices, b) choice of tasks, c) connections among mathematical topics, concepts and ideas, d) use of multiple representations and contexts to complement manipulatives and e) mathematical communication in relation to manipulatives. The chapter also makes use of the five themes to provide further analysis of the stories in chapter 4 by discussing how each of the three teachers provided or missed the opportunities to enhance the learning of mathematics using manipulatives. It also provides a summary of affordances and constraints to mathematical learning in respect of each participant per theme.

Finally, **Chapter 6** presents summaries of findings, conclusions on the lessons learned and recommendations. The summary of the findings is organised in respect of the research questions. The chapter also highlights the gaps in and limitations of the study as indications of possible future research in the area.

6.4 RESEARCH FINDINGS

The findings from the study are presented under four main topics which are aligned to the three research questions.

6.4.1 Teacher knowledge and Mathematics embedded in manipulatives

The abstraction of mathematics embodied in manipulatives is a central task to their use. Furthermore, the task entails a high level cognitive process that requires knowledge that goes beyond simple and common knowledge of topics, to deep substantive and connected mathematical knowledge and understanding pertinent to the work of teaching. The latter reflects the disposition of teachers as empowered and capable of being in control of their own knowledge as well as its transformation. Analysis of data revealed several key observations on the use of manipulatives that relate to teachers' mathematical content knowledge:

The study found that when teachers do not draw on their own knowledge of mathematics they are unable to facilitate both the abstraction and understanding of concepts represented in the models. Ms Bohata and Mr Kopung used models of 2D shapes and 3D objects to help learners to comprehend geometric concepts and the properties of various geometric shapes. They were both prompted to draw on their own knowledge of geometry to facilitate both the abstraction and understanding of concepts embedded in the models. Ms Bohata was forced to use her knowledge of geometry to respond to the learners who cited the 'oval' and the 'circle' as examples of 2D shapes. Those responses were correct and called on her knowledge of different classifications of 2D shapes. However, she disregarded her learners' responses, thus created the impression that the responses were wrong. On the other hand, Mr Kopung left the learners' responses regarding the number of sides in a circle unattended, which also created the impression that a circle does not have sides. Both cases are reflective of limited exposure to deeper mathematical knowledge that the teachers needed to draw on. This is supported by evidence from empirical data in section **4.3.1** of the study in which Ms Bohata acknowledges that she does not have any teaching qualification in mathematics and that her study of mathematics content knowledge did not go beyond Junior Certificate, an equivalent of grade 10. This seems to be also consistent with empirical data in section **4.4.1** in which Mr Kopung, despite the fact that he was trained as a mathematics teacher (specialisation) at college; he discontinued his university mathematics after failing first year level calculus.

To enable learners with such abstraction, teachers needed a specialised kind of content knowledge, which was found to be limited in the three case studies. This situation forces teachers themselves to explore, on an ongoing basis, mathematics with manipulatives in order to see and interpret the mathematics in the manipulatives. Without such exposure to the mathematics embedded in manipulatives, teachers themselves will undoubtedly find it difficult to see mathematical ideas in manipulatives, and even more difficult to explicate them to the learners. The exploration of mathematics through the use of manipulatives by teachers themselves may foster teachers to learn such specialised knowledge, provided teachers expand their own ideas about their roles as learners alongside their learners. In this way, teachers engage in the process of emancipation from limited forms of knowing to becoming reflective practitioners who are able to stand outside themselves and regulate their own learning and knowledge.

Establishing connections among mathematical ideas, concepts and procedures is also an integral component of the process of extracting mathematics from manipulatives. Scholars seem to be in agreement that conceptual knowledge of mathematics goes beyond mere knowledge of facts, principles and procedures; it involves knowledge and understanding of relationships among them (e.g. Eisenhart et al., 1993; Ma, 2010; Ball, 1988). As mentioned in the above discussion, teaching with manipulatives requires of teachers to establish the connection between the ideas and concepts represented by those manipulatives. In her discussion of PUFM Ma cites Duckworth's observation that intellectual 'depth' and 'breadth' is a matter of making connections (Ma, 2010: 121).

Data in this study have demonstrated moments where teachers' deeper understanding of mathematics was called upon in order to make connection between ideas, concepts and procedures but did not make them explicit. In section **4.2.3.1** of the study, empirical data illustrated how Ms Dikgomo's knowledge of mathematics just seemed to stop at the point where deeper understanding of mathematics was required. In her lesson on multiple operations, she wanted her learners to model the procedure and to explain how they got the answer. Her content with learners' correct answers and reinstatement of the BODMAS rule was indicative of her view about mathematics as consisting of rules to be followed to get the correct answer. This is consistent with empirical data in section **4.2.2.1** in which she explains that she enjoys teaching multiple operations because in this topic, once the children know the rules i.e. BODMAS, then it is easy to get the answer. Her own limited knowledge of the relationship between the concepts (e.g. division) and the procedure involving the BODMAS rule was apparent when she could not help her learners to model their solutions. Instead of counting the stacks or groups of cubes (division concept), learners counted single cubes despite getting the correct answer. Both Mr Kopung and Ms Bohata, in their respective lessons on 2D and 3D objects hardly made mention of the concepts of area and volume to show the connection between mathematical concepts. Ms Bohata made connections between 2D shapes and 3D objects as well as between the topics i.e. geometry and patterns to promote meaningful understanding of and substantive logic behind the properties of 3D shapes. However, in both cases Ms. Bohata could neither make the connections explicit nor provide reasons for the connections. She thus missed the opportunity to explain, for example how polyhedrons (a family of 3D shapes) are made up of polygons (a family of 2D shapes) and how the properties of the latter have a bearing on the names and the properties of 3D shapes. These cases show the knowledge gap that played itself out as a tension between the demand for a conceptual explanation and teacher knowledge. It is in and from the context of such tensions and discomfort that teachers may learn and transform their knowledge of mathematics through self-reflection. Seizing such teachable moments is reflective of how teachers are capable of being freed from lower forms of thinking and teaching.

Explicit identification of mathematical ideas and concepts that are embedded in manipulatives requires learners and teachers to communicate, to explain and to share mathematical ideas and understandings among themselves. Fennema and Romberg in Chabongora (2011) posit that the ability to communicate or articulate one’s ideas is an important goal of education and also a benchmark for understanding. This not only helps teachers to determine their learners’ understandings but it also helps learners to illuminate and clarify their own understanding in order to meaningfully construct their own knowledge of those abstract concepts. To enhance mathematical learning and to support learners, teachers need to listen and to evaluate learners' emerging and incomplete ideas. This task requires teachers to make use of their knowledge of mathematics in order to respond appropriately and to probe further into those ideas that warrant to be pursued.

Data in this study have shown that teachers were inclined to simply ignore or dismiss learners' ideas especially those that required conceptually based justifications. Such responses and lack of feedback from teachers may be indicative of their limited knowledge of mathematics. Data on Mr Kopung and Ms Bohata show that conceptually rich questions such as 'is a square a rectangle?' and ideas such as those relating to 'the number of sides in a circle' (See sections **4.4.4.2** and **4.4.4.3** respectively for details) were either brushed aside or left hanging in the air without being evaluated. Similarly, Ms Dikgomo missed the opportunity to elicit deeper understanding of the fraction concept from her learners, e.g. 'how do you know this is a half?', her questions just stopped short of probing into her learners' explanations each time (See section **4.2.8** for details). Listening to and interpreting learners' emerging and incomplete ideas by probing deeper into their understanding as they explore with manipulatives may present teachers with opportunities to improve on and develop their own knowledge of mathematics. Such a humane and respectful approach inspires hope for deeper forms of understanding.

6.4.2 The use and choice of representations

The use of representations, including concrete manipulatives, is intended to make abstract mathematical concepts accessible and comprehensible to learners. Transition from informal understanding as represented in the concreteness of manipulatives to finally formal understanding as represented in the symbolic notations has become the main challenge regarding the use of concrete representations. Both literature and the present study have shown that the effective use of manipulatives requires of teachers to choose them carefully and to sequence multiple modes of representations in order to complement them.

The study has shown that the approach to the teaching of mathematics that starts from the abstract to the concrete does not enhance mathematical learning. Both Mr

Kopung and Ms Dikgomo started their activities with symbolic notations (formal and abstract) and used manipulatives at the end just to confirm Euler's formula and the BODMAS rule respectively (see sections **4.4.4.3 c** and **4.2.2.1** for details). In this way, manipulatives were relegated to tools for drilling and practicing procedures instead of being cognitive tools. Ms Dikgomo's learners modelled the procedure involving the BODMAS rule by slavishly counting interlocking cubes without relating them to mathematical concepts and ideas underlying multiple operations. Consciously or unconsciously, both teachers changed their approaches in their second lessons and started their activities by exposing their learners to concrete manipulatives prior to supporting them to transit to abstract notations. This is evidence of how teachers are capable of changing and transforming their classroom practices, indicative of the potential to learn and correct (self-regulation) their practice from their own teaching context with manipulatives.

The study also found that in all but one lesson (Mr Kopung's second lesson) observed, the teachers used commercial manipulatives that were found in the laboratories to the exclusion (almost) of either teacher or learner made manipulatives or to contexts that relate to learners' real life experiences. Piaget (1973) argues that children actively construct knowledge of the world by continually interacting with and adapting to their environment. Teachers need to also know and make use of representations from their learners' contexts by also taking into cognisance what learners find interesting and motivating. Ms Bohata decries the 'distractive' nature of such unfamiliar manipulatives:

> And ha ba tloha feela ba etsa a soccer ball – ba tlabe ba di kopantse ba entse soccer ball – ha o re o sheba di table kaofela di tletse soccer ball – ka di frame works tseo – ke yona ya pele e tlang ka di hloohong tsa bona – ha baqala ba di tshwara feela se ba di connecta ba etsa bolo. (Just at the outset they will be making soccer balls. When you look around all tables will be filled with soccer balls. With those frameworks, that are the first thing that comes to their minds, just when they touch them, they start to connect and make soccer balls).

Data on school profiles have shown that the three schools mainly cater for learners from poor backgrounds (see section 3.3.3.3 for details) which suggest that the commercial manipulatives may be unfamiliar to most learners and hence may require longer time for learners to first get acquainted with them. In this way, teachers will be demonstrating their commitment to equitable distribution of opportunities to learn mathematics to all learners irrespective of their backgrounds.

The pressure to cover the syllabus seemed to play itself out in the tension between the time allocated and the need to enhance meaningful learning. Data from Mr Kopung's second lesson show that due to limited time (45 minutes), he decided to use a variety of containers such as match boxes, cartons of juice, cereal boxes, and jewellery boxes which he had asked his learners to bring to the mathematics laboratory. This provided evidence that such tension may enable teachers to be creative and rethink how they may use other concrete objects from the learners' environment along with the commercial objects. This is indicative of the potential that exists for teachers to change their classroom practice by capitalising on children's contexts and adopting teaching approaches that start from familiar to unfamiliar contexts.

6.4.3 Handling learners' responses

To make mathematics accessible and comprehensible to learners, teachers need to know aspects such as the kind of errors and misconceptions learners bring to the classroom as they interact with the content and materials. Mathematics laboratories provide space for learners to break away from the classroom routine and to explore mathematics through the use of manipulatives. For this reason, exploration with and reflection on manipulatives has become one of the objects of using manipulatives.

The study has found that while exploring with manipulatives, teachers are confronted with emerging mathematical tasks that are often unprecedented, emanating from learners' responses. The current study has also demonstrated that as and when learners explore with manipulatives, learners make different interpretations (right or wrong) and often teachers become evasive to such responses more so if they were unprecedented. This is an illustration that teachers often want learners to give responses that they (teachers) want and know. Liping Ma (2010:3) conjectures that what teachers expect learners to know is an indicator of their own knowledge.

Data in this study have shown moments where teachers' knowledge of children and thus knowledge of what they might bring to class was not capitalised on. As discussed elsewhere in this study, both Mr Kopung and Ms Bohata in their lessons on geometric shapes and concepts were often confronted with their learners' responses that they had not anticipated. In responding to the learner who cited a circle as an example of a 2D shape, Ms Bohata insisted that the learner give her the example that she wanted. As illustrated in her reaction: 'It is a circle.... okay. Leave that one, fast! Make a triangular prism; just do a triangle for us!' Her handling of the learner's response shows that she did not anticipate the response; instead, she had her own list of examples that learners had to conform to. The response, which can be attributed to both her limited knowledge of geometry and knowledge of children respectively, limited her opportunities to engage her learners in mathematical debates and to open up opportunities for her own learning. Teachers with knowledge of their children are often better positioned to meet the mathematical demands of teaching with manipulatives. Such knowledge further helps teachers to figure out what mathematical ideas and errors learners might bring as they work with manipulatives and how to deal with them. Understanding of students' cognitive structures and knowledge of content has the potential to help teachers to effectively use manipulatives. Teachers with knowledge about students, i.e. experience with students, familiarity with common student errors and their thinking, will be able to handle learners' responses with precision and the necessary attention to details of children's thinking. This represents a socially just approach that affords fair opportunities for learners to access sophisticated knowledge meaningfully and effectively.

6.4.4 The use of language to express mathematical ideas

Language is a powerful communication and thinking tool in mathematics and as noted by Ball and Bass (2003), it is not only the primary medium of communication but also a foundation of mathematical reasoning. Language is particularly central as a tool for reflection on and interpretation of manipulatives and enables learners to share and interact with others, to explain, justify and become more knowledgeable.

One of the findings of this study is that almost all the classrooms were characterised by teacher talk and a culture of silence among the learners who almost never posed questions and were rarely called upon to explain their methods and answers to problems. This represents a tension between the official curriculum and the enacted curriculum, where the former advocates for learner- centred approaches to teaching. Although this could be attributed to a number of factors, limitations in the use of English as LoLT (see section **3.3.3.3** for details on the schools' profiles) and teachers' background that is within the traditional approaches to teaching seem to be central to the tension (see section **3.3.3.4** for details on the teachers' profiles).

Data in the study have also shown how one teacher created opportunities for her learners, including the usage of concepts in learners' own home language, to engage in mathematical discourse by carefully scaffolding them until they mastered the task. In her second lesson, Ms Bohata managed to carefully use leading questions and terms that learners are familiar with to build the concept of a base and the logic behind having 2 bases that are of the same shape in a prism (see section **4.3.2.2** for details). Scholars refer to new learning practices that were ushered in by the introduction of the reform curriculum that are often taken for granted and mathematical discourse is one of them. This illustrates that mathematical discourse does not happen by coincidence, it is a learning practice that can be supported and learnt by both teachers and learners with time in the process of exploring with manipulatives. Ms Bohata used her learners' home language to enable her learners with something concrete and familiar to discourse about. This is consistent with the assertion by Thompson (1994: 7) who considers concrete materials to be appropriate because they enable teachers and students to have grounded conversation as their use provide something concrete to talk about. This represents a case where the use of concrete manipulatives can engender mathematical discourse through carefully selected questions and tasks, including the use of learners' home language to explain some concepts. In addition, deeper understanding is facilitated through learners using multiplicity of perspectives grounded in their own backgrounds and languages.

The use of correct and standard mathematical language, terminology and notations by both teachers and learners is an equally important factor in the use of manipulatives. Data on Mr Kopung and Ms Bohata have shown that the two teachers continued to arbitrarily use mathematical terms and language involved in the topic. Both teachers frequently used the terms prism and 3D shapes interchangeably. There was also confusion in the use of terms such as ‘rectangle’ and ‘rectangular shape’. Data also demonstrate incorrect usage of a mathematical notation, i.e. Euler’s formula, by Mr Kopung. Such inappropriate use of language and notation might restrict teachers’ capacity to promote conceptual learning through manipulatives and engender learners’ misconceptions and errors in mathematics as and when they use manipulatives and other modes of representation.

6.4.5 Planning

The study has shown that manipulatives are a key element of teaching and learning in mathematics laboratories, making them a primary basis from which learners and teachers build their mathematical knowledge. This makes manipulatives an integral part of teachers’ lesson plan in respect of the choice and sequencing of tasks, questions, examples and other aspects of planning. The study suggests that specific lesson aims and objectives need to reflect on how manipulatives will be used to achieve those lesson aims and objectives, otherwise manipulative use might be just be a coincidence.

The study found that the teachers articulated their lesson objectives in rather ambiguous terms using phrases such as ‘to do 3D shapes’, ‘to talk about Euler’s formula’ and ‘to concentrate on multiple operations’, making it difficult for them to make explicit links between the lesson objectives and the manipulatives used during the lessons. Data in this study show that Ms Dikgomo asked questions that were not even closely related to the manipulatives that she was using in her first lesson (see chapter four, sub-section 4.2.2.1 for details). These are some of the illustrations of moments were the task of lesson design required teachers to have knowledge of content and of teaching, defined by Cohen and Ball (2000) as mathematical knowledge that interacts with the design of instruction.

Teachers often use questions and tasks in their interaction with learners, curriculum and manipulatives to support learning. The choice and sequencing of such tasks and questions form part of teachers' planning. The study has shown that choice and sequencing of questions and tasks can serve as scaffolds for learners to make the necessary transition from abstract to concrete mathematics. However, this study has also shown that in order for teachers to support learners to transcend the concreteness of manipulatives, teachers need to choose tasks that are challenging and that need learners to explore with manipulatives. Teachers also need to choose tasks and sub tasks that are appropriately sequenced to carefully scaffold learners. The study found that single questions of a general nature are not sufficient to abstract mathematics from manipulatives. Teachers need to follow up on learners' responses in order to probe further into their thinking. Such an approach is empowering to the teachers as it deepens the teachers' own understanding of their learners' thinking.

6.4.6 Explaining the influence of manipulatives or lack thereof on teachers' knowledge and classroom practices

Influence on teacher knowledge

The establishment of mathematics laboratories in the three schools has been well received and they have since been integrated into the respective schools' timetables. Teachers showed a positive disposition towards the use of manipulatives as tools that promote understanding in mathematics. The availability of a wide range of manipulatives offers teachers the opportunity to explore mathematical concepts and ideas that are embedded in these manipulatives. Such explorations of mathematics may create opportunities for them to expand their mathematical knowledge and understanding. However, this depends largely on how the teachers themselves define their roles. Teachers who see themselves as lifelong learners may seize the opportunity to learn along with their learners while those teachers who adopt an expert stance might miss the opportunity.

The study demonstrated that questioning practices that focus on probing and high cognitive order questions do have the potential to elicit learners' mathematical thinking and understanding as they use manipulatives. Such questioning practices encourage learner explanations and thus offer opportunities for teachers to learn deeper mathematics. Ms Bohata and Mr Kopung posed questions that presented opportunities to explain salient features of 2D and 3D shapes such as 'why do you say this is a rectangle?' Such questions would have uncovered their learners' thinking and deep understanding of geometric shapes and concepts had they been sufficiently pursued. Both teachers missed the opportunity to learn and expand their own knowledge of the definition of geometric shapes that focuses on number of sides only to the exclusion of the critical and salient features that are central to deep understanding of the properties and definitions of particular geometric shapes.

A key finding of this study suggests that unchallenging tasks that focus on simple procedures and rules may not support learners to transcend the concreteness of manipulatives. Tasks that involve the use of manipulatives must necessarily be tasks that support learners to transit from informal knowledge to abstract concepts and ideas that are embedded in the objects. The choice of challenging tasks facilitates construction of mathematical knowledge by learners and teachers alike. Uncertainties and tensions arise as teachers struggle to come to terms with challenging tasks and this itself is a niche for them to learn more about mathematical concepts, ideas and relationships involved. Ms Dikgomo, in her lesson on multiple operations, selected unchallenging tasks that already had brackets and that could be computed easily without the aid of manipulatives. In this way, she missed the opportunity to expand her own learning of how mathematical ideas such as area, perimeter, etc. are related to the topic on multiple operations.

Influence on teachers' classroom practice

Another important finding of this study suggest that listening to learners' explanations emanating from the exploration of manipulatives gives teachers insight into learners thinking. Such a disposition is essentially an opportunity for teachers to know their learners and their thinking. Knowing learners and their thinking helps teachers in the choice and sequencing of questions, tasks, examples, and representations all of which are essential components of classroom practice. Mr Kopung asked his learners if a rectangle is a square, based on the fact that they both have four sides. He subsequently took the time to establish if learners really understood what is different and common between a rectangle and a square. His explanation of and emphasis on the term ‘opposite’ showed his knowledge of how his learners think and the possible misconceptions or errors they might commit in describing the characteristics of the two shapes. This demonstrates that opportunities to explore manipulatives, to discuss, explain, and justify their thinking, which were created, were also opportunities for Mr Kopung to approach his classroom practice differently.

This study has also shown that an open approach to manipulative use enables learners to freely explore with manipulatives and take ownership of their own learning process. Creating opportunities for learners to freely explore with manipulatives has the potential to promote free construction of mathematical knowledge from those manipulatives. Such an approach enables teachers to shift their control patterns which in turn create opportunities for teachers to think freely, to be reflexive and creative. Such teacher disposition is an indication of the opportunities for teachers to learn open approaches to their classroom practice. Ms Dikgomo used an open approach in her use of manipulatives to teach fractions, an approach in which she distributed power to her learners by not taking centre stage. She created opportunities for learners to explore with manipulatives by drawing on her learners’ intuitive knowledge about fractions to formalise their fraction concept. She also gave learners the freedom to choose the tools and the fractions to represent. Her case demonstrates how a shift in patterns of control, created opportunities for her to expand and use her knowledge and understanding of a learner-centred pedagogy.

Manipulatives, just like any other resource are not in abundance and their use therefore foster collaboration among learners. However, this study has demonstrated how teachers continually impeded learners from working collaboratively, i.e. explaining and sharing ideas among themselves. This shows that the use of manipulatives create opportunities for learners to work collaboratively and thus the potential for teachers to learn and use approaches that encourage mathematical discourse, talking, explanations and sharing of mathematical ideas among learners. The latter has the potential to also influence the classroom culture towards that of a mathematics community characterised by ongoing mathematical communication. One of the findings of this study is that manipulatives do have the potential to generate productive mathematical discourse**.** However, this depended largely on the type of questions and tasks that teachers pose as well as on how teachers responded to learners' ideas. As evidenced in both Ms Bohata and Mr Kopung's cases, opportunities for learners to explain and be given conceptually rich responses were missed each time. Responding to learners' responses is also an opportunity created by teachers themselves to learn classroom approaches that promote mathematical communication.

What has changed in teachers' classroom practices?

Although the above sections demonstrate that teacher-centred approaches still dominate school mathematics classrooms, the study has also shown in several ways small steps towards a reconceptualised use of manipulatives. As mentioned elsewhere in the study, the three teachers showed a positive disposition towards the use of manipulatives in mathematics teaching. This was a positive signal towards creating opportunities for learners to experience mathematics through manipulatives and thus advancing their knowledge of mathematics and concepts. Data also showed some improvements in the teachers' practices as they use manipulatives, especially after the intervention programme. Ms Bohata mainly used leading questions, prompts and examples in her learners' language to develop her learners' understanding of geometric concepts and connections amongst those concepts. This was indicative of her understanding that manipulatives do not teach and that learners may not readily acquire mathematics embedded in them. She also recognises the need to scaffold her learners and help them to move from concrete mathematics to deeper and abstract mathematics.

Allowing learners some freedom in their selection of manipulatives and in exploring with manipulatives is a minor step in encouraging responsibility for their own learning. Data show that teachers began to limit their power and authority and that of the subject (mathematics) when using manipulatives. This was demonstrated in the lesson where Mr Kopung's learners were able to discover Euler's formula. His case demonstrated his reconceptualization of manipulatives as cognitive tools rather that as tools for drilling and practicing procedures. Ms Dikgomo allowed her learners freedom to choose the tools and the fractions they represent. Her open approach in her use of manipulatives was indicative of how she distributed power to her learners by not taking the centre stage. Her case demonstrates how a shift in patterns of control created opportunities for her to expand and use her knowledge and understanding of a learner-centred pedagogy.

The study also showed signs of teachers' emancipation from conformity to cultural materials. This was demonstrated where teachers challenged some of the manipulatives for not providing appropriate representations of mathematical ideas. For example, during the intervention programme teachers raised an issue about the interlocking cubes as not an inappropriate representation of the LCD when adding and subtracting fractions. This demonstrated the teachers' freedom from conformity and the authority of curriculum materials, both of which render teachers as passive curriculum receivers to whom knowledge is passed down unquestioningly. This also shows a positive step towards teachers' level of accountability and flexibility in their use of manipulatives.

Although the section above shows some isolated illustrations of positive changes in how teachers use manipulatives, these are minor steps towards a reconceptualised use of manipulatives. It remains to be seen if and how these new dispositions towards the use of manipulatives will be sustained in their mathematics classrooms.

6.4.7 Summary of key findings

I. The extraction of mathematics from manipulatives is one of the key elements in the effective use of manipulatives and requires of teachers to have a substantive and connected knowledge of mathematics. Without such knowledge, facilitating the transition from concrete to abstract mathematics becomes problematic.

II. The exploration of mathematics through the use of manipulatives by teachers themselves may allow teachers the opportunity to learn from manipulatives and thus expand their own mathematical knowledge.

III. Listening to and interpreting learners' ideas help teachers to understand their learners' understandings and may thus present teachers with opportunities to improve on their own knowledge of mathematics and how to teach it.

IV. Tensions that emerge in and from the context of practice between the cognitively demanding tasks and teacher knowledge present a potential for teachers to transform their knowledge of mathematics through self-reflection.

V. Teachers' knowledge of content seems to be at the root of much of the construction of classroom practice.

VI. Handling learners' responses enable teachers to anticipate what experiences, misconceptions, errors etc. learners may bring to class and thus provide opportunities to teach and even reconfigure their classroom practice from a more informed position.

VII. The use of correct and accurate mathematical language and notation is equally if not more important than the mathematical ideas they represent.

VIII. Learning mathematics through manipulatives that are unfamiliar to learners may lead to the learners needing more time to first get acquainted with them. Compounded by the curriculum coverage pressures, teachers may be forced to rethink how they may use other concrete objects from the learners' environment along with the commercial objects. This is indicative of the possibility for teacher learning in respect of their classroom practice.

6.5 CONCLUSION

The use of manipulatives beyond their concreteness as espoused in this study is characterised by conceptual mathematics teaching and learning and is in alignment with the vision of the reform curriculum in South Africa. The successful enactment of the new curriculum is in essence a strong support for the use of manipulatives. This depends to a larger extent on teachers' appropriate orientation towards the new curriculum and its learner-centred approaches. As indicated elsewhere in the study, most lessons observed were also dominated by whole class questioning practices and chorusing from learners, characteristic of the traditional approaches to classroom teaching. This resulted in a practice that paid little or no attention to individual learners. This study also found that such questioning practices do not promote mathematics learning by individual learners. This was confirmed by Ms Dikgomo's statement about the group responses, where she decries the value of such assessment practices (see section 5.2 par. 4 in chapter 5 for details). This acknowledgement by Ms Dikgomo is indicative of the potential for teachers to learn and change their classroom practices for the better.

The influence of manipulatives on teachers' mathematical knowledge and on teachers' classroom practice can therefore be explained and understood within the context of the tensions and opportunities that arise in and from a teaching practice where teachers use manipulatives. This means that such influence can only be realised in the practice of teaching wherein contradictions and tensions are created through the use of manipulatives. The use of manipulatives is characterised by moments of tension and discomfort especially when teachers' mathematical knowledge is challenged. For example, data from section 4.3.2.2 (in lesson segment 7) where Ms Bohata asked learners to give examples of 2D shapes were indicative of the tension between her own knowledge of the topic and the conceptual explanation she had to provide to learners. Such a tension presented a compelling context and entrance for her to critically reflect on her own knowledge of the classification of 2D and 3D shapes and some key properties of those shapes. As I have argued elsewhere in this study, it is in such moments that teachers, if they seize the opportunity, may learn and transform their knowledge of mathematics through self-reflection.

6.6 RECOMMENDATIONS FOR FUTURE RESEARCH, POLICY AND PRACTICE

The section presents some of the recommendations that emanated from the study. The recommendations have implications at different levels of the system, i.e. macro, meso and micro levels. These include recommendations for future research:

6.6.1 Recommendations for future research

i) The current study is based on data generated from a small fraction of topics of the Grade 6 mathematics curriculum. In addition, the study recognises that teachers might find it easier to use manipulatives in some topics that in others. With the ground work that has been done through this study, further research that examines the influence of and the use of manipulatives in other primary school mathematics topics is recommended. A larger number of participants could also be recruited to investigate patterns across the population of mathematics teachers in the Free State province.

ii) The present study was conducted in mathematics laboratories. Further research is recommended to investigate if there is a real shift in teachers' mathematical knowledge and classroom practices in their mathematics classrooms as opposed to the mathematics laboratories.

iii) Assessment is an important aspect of the mathematics curriculum. Teachers' integration of manipulatives in their assessment practices also needs to be investigated further.

6.6.2 Implications for policy and practice

i) The pressure to cover the curriculum came out indirectly during the study observations. As a consequence, exploration with manipulatives was often not given sufficient time in the lessons observed. At a policy level the study recommends that the amount of time that is allocated to mathematics be reviewed to accommodate the use of manipulatives.

At the level of either the Province or the District, the study recommends that the FSDoE should provide ongoing support to teachers through a training programme that has a strong focus on Pedagogical Content Knowledge (PCK) and that integrates active and hands-on use of manipulatives.

ii) With the introduction of the new curriculum, a number of concepts that relate mainly to mathematical classroom practice and approaches were introduced in the official curriculum. In my interactions with the teachers I observed that they are grappling with the operationalisation of most of those concepts. The study recommends that teachers be provided with intensive training on the operationalisation of these new curriculum concepts including concepts such as ‘conceptual understanding’, ‘collaborative work’, ‘mathematical discussion’ and ‘mathematical reasoning’.

iii) The development of a glossary of mathematical terms, concepts and notations by teachers is recommended as one of the outcomes of the training on new curriculum concepts.

iv) I also noticed that there are pockets of good practices among the schools, including in the schools that participated in the study. The study therefore recommends that schools must create platforms where teachers can share their good practices in relation to manipulatives use to take advantage of existing knowledge and expertise within the system.

v) It was important to note that in many of the mathematics laboratories, teachers used only commercial manipulatives to the exclusion of manipulatives in and from their learners’ own environments. The study therefore recommends that teachers need to integrate manipulatives from their learners' own environments, alongside the commercial manipulatives.

vi) A number of opportunities for teacher learning that were offered, in the context of using manipulatives, were also noted and yet such opportunities were for the most part not fully exploited. It is for this reason that the study recommends that teachers rethink their role as reflective practitioners and co-learners alongside their learners.

6.7 FINAL THOUGHTS

The teaching and learning of mathematics has always been a complex and challenging endeavour worldwide and the situation is not different in South Africa. This situation has compelled stakeholders such as teachers, researchers, and governments to continue to search for better strategies to improve the teaching and learning of mathematics. Mathematics laboratories (equipped with manipulatives) have been introduced in the Free State primary schools to enhance mathematics teaching and learning. The assumption appears to be that the use of manipulatives will change teaching and classroom practice and, consequently enhance mathematics learning. The current study sought to explore how teachers make use of manipulatives in mathematics laboratories, to investigate whether the use of manipulatives will bring about any substantive changes in teachers' mathematical knowledge and classroom practices, and to seek explanations for the influence of manipulatives or lack thereof on teacher knowledge and their classroom practice in primary schools in the Free State.

The study has found that teacher knowledge of mathematics is more crucial in the effective use of manipulatives than perhaps any other single teacher attribute. Effective use of manipulatives is essentially characterised as the abstraction of mathematical concepts and relationships embedded in those manipulatives. To successfully do this highly cognitive mathematical task teachers are forced to draw heavily on their own knowledge of mathematics. Any other factors such as teacher beliefs, teacher pedagogy, etc. can only serve as a support base for teacher knowledge. The study concludes that teachers can only abstract mathematical concepts and make connections between them effectively if they themselves have sufficient knowledge of those mathematical concepts and their relationship.

One of the findings of this study suggests that over time and with relevant professional teacher development support, the use of manipulatives may have the potential to re (shape) teachers' mathematical knowledge. Data from this study have shown moments of tension and discomfort that occurred when teachers' mathematical knowledge was challenged. I have argued, in the sections above, that it is in and from the context of such tensions and discomfort that teachers may learn and transform their knowledge of mathematics through self-reflection. Considering that at the time of the study the introduction of these manipulatives was only in its second year, with time the potential of those manipulatives to transform teachers' knowledge may be realised. The study has also noted that, depending on how teachers define their roles, the availability of a wide range of manipulatives also offers teachers the opportunity to explore mathematics with manipulatives. Such explorations of and reflection on manipulatives may create opportunities for them to learn and to expand their mathematical knowledge and understanding.

The study has also found that where opportunities for teacher learning are created such as in the context of teaching with manipulatives, a mutual relationship may exist between teacher knowledge and classroom practice. The implication is that if manipulatives are conceived of and used as learning tools for teachers, their use may help to strengthen the relationship between teacher knowledge and classroom practice. Data in this study have shown that teacher knowledge is also accountable for aspects of classroom practices such as choice of tasks, questions, handling learners' responses, etc. This study concludes that a mutual relationship seems to exist between teacher knowledge and classroom practice where opportunities for teacher learning are created by the use of manipulatives.

The questioning practices of the three teachers were dominated by single questions of a lower cognitive level. Such questions elicited simple recall and memory responses from learners, which only remained at the literal concreteness of manipulatives.

Chorusing and one directional questioning by the teachers were also some of the common features of their practice. The facilitation of highly cognitive processes as demanded by the use of manipulatives was often not successfully implemented due to such low cognitive level questioning practices. Such situations could not create the tension and discomfort that usually provide opportunities for deep reflection and learning for both teachers and learners. Through the use of simple questions of lower cognitive level, teachers lost the opportunity to strengthen the relationship between teacher knowledge and their classroom practices.

As a policy maker I have learnt of the importance of the extraction of mathematical concepts and relationships embedded in manipulatives as a critical component of any mathematics teacher development programme (pre-service and in-service). I have also learnt that teachers need to be continually trained and conscientised on the value of creating opportunities for their own knowledge to be challenged, to be reflective and to be teacher learners. Through this study I came to realise that teachers are not sufficiently exposed to the setting of cognitively demanding tasks and questions, which are necessary for creation of tensions and discomfort. Such exposure will, with time, inculcate the culture of learning opportunities for teachers. As a policy maker I have also learnt that training that emphasises the use of laboratories as learning sites for teachers has the potential to strengthen the relationship between teacher knowledge and classroom practices.

As a researcher, I have also learnt that the use of manipulatives as a teaching strategy must not be left to chance. Efforts need to be made to intensively investigate the use of manipulatives and to ensure that they also promote and support the development of teachers' mathematical knowledge and their classroom practices. Through this study I have discovered the importance of contextualising and adapting the use of manipulatives to local conditions. I now value the teachers' voices more and the importance of PAR as an approach that promotes teacher empowerment and emancipation for their own development. I have also come to appreciate the importance of social agency as the cornerstone of any transformation agenda, including transformation of teaching and learning in the mathematics classroom.

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8. APPENDICES

8.1 APPENDIX 1

Letter to the Superintendent General

XX DONUM TOWNHOUSES

P. J SCHOEMAN STREET

LANGENHOVEN PARK

BLOEMFONTEIN

9300

THE SUPERINTENDENT GENERAL

FREE STATE DEPARTMENT OF EDUCATION

CR SWART BUILDING

BLOEMFONTEIN

Dear Sir

Re: Request to be granted permission to conduct a research study in primary schools in the Free State

A research study will be conducted by the undersigned at primary schools in Mangaung that have mathematics laboratories. The purpose of the study is to investigating whether the use of manipulatives will bring about any substantive changes to teachers' mathematical knowledge and classroom practices, and to seek explanations for the influence of manipulatives or lack thereof on teacher knowledge and their classroom practice in primary schools in the Free State.

Participants in the study will include, amongst others, learners, members of the school management team and mathematics educators in the schools. To avoid the disruption of the teaching programme, major activities of the study will take place during weekends. Two classroom observations will take place per participant during second term of 2013 and arrangements will be made two months in advance.

It is against this backdrop that permission is hereby requested to conduct research at schools in Mangaung. The study will benefit the schools as well as the entire schooling system in the Free State.

Participation of the teachers and schools is entirely voluntary and they will be under no obligation to take part in this study. If they choose to take part, and an issue arises which makes them uncomfortable, they may at any time stop their participation with no further repercussions.

If you wish to discuss anything about the research, please feel free to contact me directly to discuss it with me. Kindly note that you are also free to contact my study supervisor whose details are as follows:

Prof. L.C. Jita
P.O. Box 339
Bloemfontein
9300.
Tel: 051 401 7522
Fax: 086 269 9453
Email: jitalc@ufs.ac.za

Yours sincerely

MJ Maboya (Mrs)

Signature: .... Date: ....

8.2 APPENDIX 2

Letter to the principals

XX DONUM TOWNHOUSES

P. J SCHOEMAN STREET

LANGENHOVEN PARK

BLOEMFONTEIN

Dear Mr Kopung (not real name)

RE: REQUEST FOR PERMISSION TO CONDUCT MY RESEARCH STUDY AT YOUR SCHOOL

My name is Mantlhake Julia Maboya of the Free State department of education. I am currently working on a PhD programme with the University of the Free State. The programme involves the conduct of a research study whose title is 'The relationship between teachers' mathematical knowledge and their classroom practices: A case study on the role of manipulatives in South African primary schools'.

I would like to request permission to conduct the research study at your school and to allow me to observe some grade 6 lessons in your school's mathematics laboratory. To ensure that data gathered during the study will be used solely for study purposes, information in discussions will be kept private and the research results will be presented in an anonymous manner to protect your identity. You are further guaranteed that the documents as well as video and audio tapes used will be safely locked and destroyed after the study.

I am confident that the department and your school will benefit greatly from this study whose objectives are to:

I. Explore how the use of manipulatives in the teaching of primary school Mathematics help to reshape the teachers' own mathematical knowledge for teaching;
II. Explore how the use of manipulatives help to reshape the teachers' own mathematical classroom instruction;

III. Explain the influence of manipulatives or lack thereof on teachers' knowledge for teaching and classroom practices.

While I greatly appreciate your school's participation in this important study and the valuable contribution that will make, participation is entirely voluntary and your school is under no obligation to take part in this study. If you do choose to take part, and an issue arises which makes you uncomfortable, you may at any time stop your participation with no further repercussions.

If you or your teachers (or learners) experience any discomfort or unhappiness with the way the research is being conducted, please feel free to contact me directly to discuss it, and also note that you are free to contact my study supervisor whose details are:

Prof. L.C. Jita
P.O. Box 339
Bloemfontein
9300.
Tel: 051 401 7522
Fax: 086 269 9453
Email: jitalc@ufs.ac.za

You are kindly requested to indicate your approval by providing your signature in the space below.

Yours sincerely

MJ Maboya (Mrs) Mr Kopung (Principal)

Signature: .... Signature: ……………………………

Date: .... Date: ……………………………………

8.3 APPENDIX 3

Letter of consent to the teachers

XX DONUM TOWNHOUSES

P. J SCHOEMAN STREET

LANGENHOVEN PARK

BLOEMFONTEIN

Dear Participant

INFORMED CONSENT:

Dear Participant

My name is Mantlhake Julia Maboya of the Free State department of education. I am currently working on a PhD programme with the University of the Free State. The title of my thesis is 'The relationship between teachers' mathematical knowledge and their classroom practices: A case study on the role of manipulatives in South African primary schools'.

This study seeks to investigate whether the use of manipulatives will bring about any substantive changes to teachers' mathematical knowledge and classroom practices, and to seek explanations for the influence of manipulatives or lack thereof on teacher knowledge and their classroom practice in primary schools in the Free State. We would like you to participate with us in this research because of the expertise you will bring into the study.

The possible risks to you in taking part in this study are safety at the meetings venue, emotional trauma that might arise from discussions, hunger due to the long hours spent in meetings and confidentiality and we have taken the following steps to protect you from these risks:

- Police, medical and psychological services will be on standby for any threatening situations that might arise at the venue
- The gates of the meeting venue will be locked for the safety of participants
- Catering will be provided in the case where the meetings take longer than 3 hours

- All meetings will be held during the day
- The documents and video and audio tapes will be safely locked and destroyed after the study
- Participants' information in discussions will be kept private and the research results will be presented in an anonymous manner to protect all participants' identity.

I am confident that all of us will benefit greatly from this study. As a co researcher you will share your experiences with the use manipulatives and will also be exposed to the conduct of action research in your own classroom. The recommendations emanating from the study will be used to develop intervention programmes that are responsive to your needs. This will in turn benefit your learners as they will understand mathematics better.

While I greatly appreciate your participation in this important study and the valuable contribution you can make, your participation is entirely voluntary and you are under no obligation to take part in this study. If you do choose to take part, and an issue arises which makes you uncomfortable, you may at any time stop your participation with no further repercussions.

If you experience any discomfort or unhappiness with the way the research is being conducted, please feel free to contact me directly to discuss it, and also note that you are free to contact my study supervisor whose details are:

Prof. L.C. Jita
P.O. Box 339
Bloemfontein
9300.
Tel: 051 401 7522
Fax: 086 269 9453
Email: jitalc@ufs.ac.za

Should any difficult personal issues arise during the course of this research, I will endeavour to see that a qualified expert is contacted and able to assist you.

Yours sincerely,

____

Mantlhake Julia Maboya (Ms)

Please fill in and return this page to the undersigned and keep the letter above for future reference

----

Study: 'The relationship between teachers' mathematical knowledge and their classroom practices: A case study on the role of manipulatives in South African primary schools'.

Researcher: Ms. M. J MABOYA

Participant's Name and Surname: ______________________________________

Gender: _______________________________

Contact number: ___________________________

- I hereby give free and informed consent to participate in the abovementioned research study.
- I understand what the study is about, why I am participating and what the risks and benefits are.
- I give the researcher permission to make use of the data gathered from my participation, subject to the stipulations he/she has indicated in the above letter.

8.4 APPENDIX 4

Mathematics Laboratory Photo

8.5 APPENDIX 5

Mathematics Laboratory Description

A. Seating arrangement	**Narrative**
Learners have assigned seats	
Seating arrangement is random	
Seats arranged in semi –circles	
Seats arranged in circles	
B. Walls	
Learners’ artwork	
Laboratory Rules of behaviour	
Illustrations of mathematical concepts posted	
Number line	
Fraction charts	

Graphs or charts	
C. Manipulatives	
Packed in cupboards	
Easy access by learners	
Variety of manipulatives used	
Only Commercial manipulatives in the lab	
Commercial and other manipulatives in the lab	

Make notes of other observations in the lab:

8.6 APPENDIX 6

Profiles of the Participants, Class and School

PART A: PARTICIPANT'S PROFILE

Item	Description	Number of years
1.	Experience in Teaching	
2.	Experience in teaching Mathematics	
3.	Experience in teaching Mathematics at Intermediate Phase level	
4.	Experience in teaching Mathematics in Grade 6	

5.	Other subjects taught	Number of years
5.1		
5.2		
5.3		

6.	Professional Qualifications (P.T.D; etc.)	Major Subject	
6.1			
6.2			
6.3			

7.	**Academic Qualifications (B.A, etc.)**	**Major Subjects**	
7.1			
7.2			
7.3			

8.	Gender- Mark with (X)	Male	Female

9.	Age – Mark with (X)			
	Less than 30	30 - 39	40 – 49	50 - 60

PART B: Baseline Data of the Grade 6 class

1.	Description of class (Gr. 6A or 6B etc.)	
2.	No of Learners in class	

3.	Describe learners of this class’s ability in Mathematics

4.	Briefly describe the Socio Economic Background of learners in this class

5.	How often do you use the Mathematics Laboratory	Per week	Per Month

PART C: SCHOOL PROFILE

1.	Geographic Location (Mark with X)	Urban	Rural	Semi-urban
2.	School classification	Section 21	Non-section 21	Partial Section 21
3.	Quintile Ranking			

Item	Description	Number
4.	Total Number of learners	
5.	Total Number of teachers	
6.	Total Number of SMTs including Principal	

7.	LoLT	Foundation Phase	Intermediate Phase

8.	Predominant Home Language	

9.	Is the school part of NSNP (Mark with X)	Yes	No

10.	Socio Economic Background(Mark with X)	Low income earners	Middle income	High Income
11.	Literacy level of Parents (Mark with X)	Low	Moderate	High

8.7 APPENDIX 7

Teaching Primary Mathematics Scenarios

B1. Choose TWO examples from the topics that the teachers have identified above as being important for their learners.

a) What would you say a pupil would need to understand or be able to do before they could work on this? Why is____ important for this? Is there anything hard here that might be difficult for your pupils?

b) How could you tell if your students were "getting it''?

(Probe for what it means to "know" or "understand"-----or whatever word they use----something in mathematics).

B2. I would like to get your ideas on some topics that are often considered for students at this level. I want to tap on your rich experiences as a primary school teacher. Hopefully we can accumulate a range of ideas on how primary school teachers approach these challenges and then compile them to help other teachers, especially the new teachers

<u>Responding to student difficulties: Place value</u>

a) Suppose you are trying to help some your students learn to multiply large numbers. You notice that when they try to calculate:

123

<u>X 645</u>

615

492

<u>738</u>

1845

Instead of this:

123

<u>X 645</u>

615

4 920

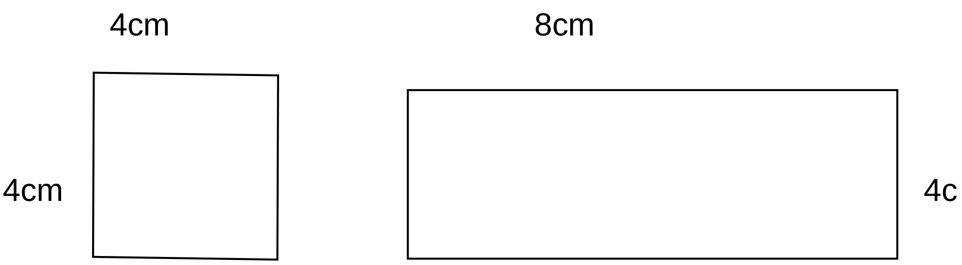
<u>73 800</u>

79 335

What would you do if you notice that several of your learners were doing this?

<u>Responding to your learners novel ideas: Perimeter/Area proof</u>

b) Imagine that one of your students comes to class very excited. She tells you that she has figured out a theory that you never told the class. She explains that she has discovered that as the perimeter of a closed figure increases, the area also increases. She shows you this picture to prove what she is doing:



Perimeter = 16 cm.   Perimeter = 24 cm.

Area = 16 cm^2.   Area = 32 cm^2.

How would you respond to this student?

c) In teaching, this will happen that something may come up where you're not sure yourself about whether the mathematics is correct or not. I'm interested in how you think you'd react to that.

d) Suppose you had the opportunity to use various apparatus to help students with this question,

- What apparatus would you use? And

- How would you use them?

- What difference would their use make in this case?

<u>Generating representations: divisions and fractions</u>

a) Do you remember how you were taught to divide fractions? How would you solve a problem like this one?

$1 \frac{3}{4} \div \frac{1}{2}$

b) Many people find this hard. In your view, what makes this especially difficult?

c) Something that many teachers try to do is to relate mathematics to other things. Sometimes they try to come up with real-life situations or story problems to show the application of some particular piece of content.

- What would you say would be a good situation or story or model for $1 \frac{3}{4} \div \frac{1}{2}$?
- Would this be a good way to help students learn about division by fraction?

8.8 APPENDIX 8

Exploratory Question

“How do you use manipulatives to teach mathematics in your classroom and why?”

NB! Clarifying questions were asked, such as:

‘‘Could you explain a little bit more?”

“What do you mean when you say......?”

“You have the feeling that ...?” etc.