

**THE LEVEL OF METACOGNITIVE AWARENESS OF  
PRE-SERVICE MATHEMATICS TEACHERS  
AT A HIGHER EDUCATION INSTITUTION**

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*Honour to the One in whom we move and live and have our being. The awareness of His presence has sustained me throughout this journey.*

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I dedicate this dissertation in memory of my beloved father.

## SUMMARY

There are ongoing concerns about educational institutions not empowering learners with the knowledge, skills, and dispositions needed for school achievement, lifelong learning, and the workplace of the new millennium. In particular, South African learners have performed poorly in recent national and international assessments of mathematical proficiency. As a result, the Department of Basic Education has asserted the importance of enhancing the quality of Mathematics teaching and learning. Enhancing the ability to teach Mathematics has the potential to improve educational outcomes, as well as increase future employment and higher education opportunities for young South Africans.

The poor Mathematics results point to the need to enhance, among other things, learners' metacognitive awareness. Metacognitive awareness entails the knowledge and regulation of one's cognitive processes. Enhancing metacognition could not only support learners in solving mathematical problems, and so improve mathematical achievement, but could also enhance productive and lifelong learning in learners. Fostering metacognitive awareness within Mathematics learners involves first fostering metacognitive awareness in Mathematics teachers, who are responsible for facilitating quality Mathematics teaching and learning. However, research suggests that teachers generally do not teach or model metacognitive awareness to their learners, or display metacognitive adaptive competence in their own teaching practice.

The purpose of the study was to determine the level of metacognitive awareness of Mathematics pre-service teachers at a Higher Education Institution. Framed within a post-positivist/interpretivist paradigm, a mainly quantitative research approach with a minor qualitative enquiry informed the study. The Metacognitive Awareness Inventory (MAI) was distributed to fourth-year Mathematics pre-service teachers at a South African Higher Education Institution in order to determine their metacognitive awareness regarding *Knowledge of cognition* (comprising of *Declarative knowledge*, *Procedural knowledge*, and *Conditional knowledge*) and *Regulation of cognition* (comprising of *Planning*, *Information management*, *Monitoring*, *Debugging*, and *Evaluation*).

To enrich the findings of the quantitative analysis, the qualitative data generated from a think-aloud problem-solving session—where the pre-service teachers recorded their thought processes whilst solving a problem—was analysed to determine the extent to which their reported metacognitive awareness translated into successfully solving a Mathematics problem. In the quantitative findings on the MAI, the pre-service teachers reported a moderately high level of metacognitive awareness; in addition, they reported a higher level of metacognitive knowledge (*Knowledge of cognition*) than of metacognitive skills (*Regulation of cognition*). Findings from the think-aloud problem-solving session, meanwhile, point to an inadequate level of metacognitive awareness, indicating a gap between what the pre-service teachers report to do in the learning and problem solving of Mathematics and what they can actually do in a problem-solving context. There is historical precedent for this gap, as noted in the scholarship.

The close of the study highlights the need to enhance the metacognitive awareness and reflective practice of these Mathematics pre-service teachers by enhancing their metacognitive skills—*Monitoring, Debugging, and Evaluation*—and enhancing their problem-solving skills. It is further recommended that reflective problem-solving opportunities built around complex, novel problems be incorporated into Mathematics modules in teacher training, to facilitate prolonged and deliberate reflection. More broadly, it recommends that metacognitive reflective and problem-solving opportunities are provided for novice and underqualified teachers.

Such opportunities will aid prospective and current Mathematics teachers to become mathematically proficient and metacognitively aware themselves, to deal with novel scenarios in Mathematics and their teaching practice and to translate this metacognitive adaptive competence for their learners.

**Key words:** metacognitive awareness, mathematical proficiency, productive learning, Mathematics achievement, Knowledge of cognition, Regulation of cognition, Metacognitive Awareness Inventory, mathematical achievement, adaptive competence

# OPSOMMING

Kommer bestaan dat opvoedkundige instellings nie daarin slaag om leerders te bemagtig met die kennis, vaardighede en gesindhede wat nodig is vir skoolprestasie, lewenslange leer, en die werksomgewing van die nuwe millennium.

In die besonder, Suid-Afrikaanse leerders het swak presteer in onlangse nasionale en internasionale assessering van wiskundige vaardigheid. As gevolg hiervan, het die Departement van Basiese Onderwys die belangrikheid van die verbetering van die gehalte van wiskunde onderrig en leer daar gestel. Die verbetering van die vermoë om wiskunde te onderrig, het die potensiaal om opvoedkundige uitkomst te verbeter, sowel as om toekomstige werks en Hoër Onderwysgeleenthede vir jong Suid-Afrikaners te verhoog.

Hierdie swak wiskunde resultate dui op die nodigheid om, onder andere, metakognitiewe bewustheid by leerders te verbeter. Metakognitiewe bewustheid behels die kennis en regulering van 'n persoon se denkprosesse. Die verbetering van metakognisie kan nie net leerders in die oplossing van wiskunde probleme ondersteun, en so wiskunde prestasie te verbeter nie, maar kan ook produktiewe en lewenslange leer by leerders bevorder. Bevordering van metakognitiewe bewustheid in wiskundeleerders behels eerstens die bevordering van metakognitiewe bewustheid in wiskundeonderwysers, wat verantwoordelik is vir die fasilitering van gehalte wiskunde onderrig en leer. Navorsing dui egter daarop dat onderwysers oor die algemeen nie metakognitiewe bewustheid onderrig of modelleer aan hul leerders nie, of metakognitiewe aanpasbare bevoegdheid toon in hul eie onderrigpraktyk nie.

Die doel van die studie was om die vlak van metakognitiewe bewustheid van voornemende wiskundeonderwysers by 'n hoëronderriginstelling te bepaal. Geraam binne 'n post-positivistiese/interpretivistiese paradigma, 'n hoofsaaklik kwantitatiewe navorsingsbenadering met 'n mindere kwalitatiewe ondersoek het die studie toegelig. Die Metakognitiewe Bewustheidsvraelys (MAI) is toegedien aan die voornemende vierdejaar-wiskundeonderwysers by 'n Suid-Afrikaanse Hoër Onderwysinstelling om hul metakognitiewe bewustheid met betrekking tot *Kennis van Kognisie* (bestaande uit *Verklarende kennis*, *Prosedurele kennis*, en *Voorwaardelike kennis*) en *Regulering van Kognisie* (bestaande uit *Beplanning*, *Inligtingverwerkingsbestuur*, *Monitering*, *Remediëring*, en *Evaluering*) te bepaal.

Om die bevindinge van hierdie kwantitatiewe analise te verryk, is kwalitatiewe data, gegenereer uit 'n "Think Aloud" probleemoplossingsessie, ontleed—waar voornemende onderwysers hul denkprosesse aanteken tydens die oplossing van 'n probleem—om vas te stel in watter mate die voornemende wiskundeonderwysers se gerapporteerde metakognitiewe bewustheid neerslag vind in die suksesvolle oplossing van 'n wiskunde probleem.

In die kwantitatiewe bevindinge op die MAI, rapporteer die voornemende onderwysers 'n matig hoë vlak van metakognitiewe bewustheid en, bykomend, hoër metakognitiewe selfkennis (*Kennis van Kognisie*) as metakognitiewe vaardighede (*Regulering van Kognisie*). Bevindinge van die "Think Aloud" probleemoplossingsessie, egter, wys na 'n onvoldoende vlak van metakognitiewe bewustheid, wat dui op 'n gaping tussen wat die voornemende wiskundeonderwysers rapporteer om te doen in die leer en probleemoplossing van Wiskunde en wat hulle in werklikheid kan doen in 'n probleemoplossingskonteks. Daar is historiese presedent vir hierdie gaping, soos aangedui in die literatuur.

Die samevatting van die studie beklemtoon die noodsaaklikheid om die metakognitiewe bewustheid en reflektiewe praktyk van hierdie voornemende wiskundeonderwysers te verbeter deur die verbetering van hul metakognitiewe vaardighede, *Monitering*, *Remediëring* en *Evaluering*, en die verbetering van hul probleemoplossingsvaardighede.

Vervolgtik word aanbeveel dat daar in Wiskunde modules in onderwysopleiding, reflektiewe probleemoplossingsgeleenthede met komplekse, outentieke probleme ingebou word, wat die geleentheid bied vir langdurige en doelbewuste reflektering.

'n Verder algemene aanbeveling is dat metakognitiewe reflektiewe en probleemoplossingsgeleenthede vir beginner en ondergekwalfiseerde onderwysers daargestel word. Sulke geleenthede sal bydra om voornemende en huidige wiskundeonderwysers wiskundigvaardig en metakognitief bewus te maak, om dus nuwe scenario's in Wiskunde en hul onderwyspraktyk te kan hanteer, en om sodoende hierdie metakognitiewe aanpasbare bevoegdheid aan hul leerders oor te dra.

## DECLARATION

I, Henriëtte du Toit, declare that this dissertation, submitted for the degree of **MAGISTER EDUCATIONIS**, is wholly my own work, and that all sources consulted as part of the research process have been explicitly referenced throughout. I also certify that this document has not been submitted previously at the University of the Free State or at any other higher education institution. I hereby cede copyright of this study to the University of the Free State.

Furthermore, I wish to acknowledge and thank the National Research Fund for their contribution to funding the study. The study reflects my views and not theirs.



Henriëtte du Toit

2017



# EDITOR'S DECLARATION

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Australia

## TO WHOM IT MAY CONCERN

This letter is to confirm that I served as editor on Henriëtte du Toit's dissertation, titled *The Level of Metacognitive Awareness of Mathematics Pre-Service Teachers at a Higher Education Institution*, editing the document for language, grammar, punctuation, and academic style.

Regards,



Dr B.D. Kooyman

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12 December 2016

To whom it may concern

**Statistical analysis: Henriette du Toit Masters Project**

This is to confirm that, in the period April to December 2014, I have performed the statistical analysis of the data related to the Masters project of Henriette du Toit (UFS Student number 1986102158).

Sincerely

A handwritten signature in blue ink, appearing to read 'R Schall'.

(Prof R Schall)

Statistical Consultation Unit



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## LIST OF ACRONYMS

<b>ANA</b>	Annual National Assessments
<b>CAPS</b>	Curriculum and Assessment Policy Statement
<b>CDE</b>	Centre for Development and Enterprise
<b>CSSC</b>	Constructive, Self-regulated, Situated, and Collaborative
<b>DBE</b>	Department of Basic Education
<b>DHET</b>	Department of Higher Education and Training
<b>ILS</b>	Inventory Learning Style
<b>MAI</b>	Metacognitive Awareness Inventory
<b>MARSI</b>	Metacognitive Awareness of Reading Strategies Inventory
<b>MRTEQ</b>	Minimum Requirements for Teacher Education Qualifications
<b>MSLQ</b>	Motivated Strategies for Learning
<b>NEET</b>	Neither Employed nor in Education or Training
<b>NRC</b>	National Research Council
<b>NSC</b>	National Senior Certificate
<b>OECD</b>	Organisation for Economic Co-operation and Development
<b>SAQA</b>	South African Qualifications Authority
<b>SEMLI-S</b>	Self-Efficacy and Metacognition Learning Inventory–Science
<b>SRL</b>	Self-Regulated Learning
<b>TIMSS</b>	Trends in International Mathematics and Science Study
<b>ZPD</b>	Zone of Proximal Development

# CHAPTER 1

## INTRODUCTION

### 1.1 ORIENTATION

Due to globalisation, technological advances, the information explosion, and the socio-economic challenges of the new millennium, lifelong learning skills and an adaptive approach to situations are necessary for dealing with the novel, complex problems of this information-rich world (Cornford, 2000: 1; Timperley, 2011: 3). To adapt and succeed in the new millennium, it is vital that learners<sup>1</sup> and future leaders in politics, technology, business, and education can solve real-life problems effectively and efficiently. Solving these real-life problems requires a higher level of skills and knowledge. Consequently, there is a call for adaptive skills to enable the transfer of knowledge and skills to novel situations (Bransford, Brown & Cocking, 2000: 18, 19; Hartman, 2001a: 34; Lin, Schwartz & Hatano, 2005: 245–255). In the South African context, these generic skills for learners are mentioned in various documents: the Curriculum and Assessment Policy Statement (CAPS) (Department of Basic Education [DBE], 2010b: 8–9), the Minimum Requirements for Teacher Education Qualifications (MRTEQ) (Department of Higher Education and Training [DHET], 2015: 64), and the South African Qualifications Authority (SAQA) documents (South African Qualifications Authority [SAQA], 2012: 10) (see Section 1.2).

However, concerns are raised by politicians and educators that institutions are failing to adequately empower and support learners in acquiring the knowledge, skills, and disposition crucial for life beyond schooling and in the workplace (Centre for Development and Enterprise [CDE], 2013: 7–12; Cornford, 2000: 1–4; 2002: 357–358). The challenge, therefore, lies in making learning more authentic, useful, and contextualised to equip learners to solve the problems they are confronted with, both in and beyond schooling. Adaptive competence must be nurtured for dealing with this fast-evolving world and for applying existing knowledge to new scenarios and problems. In Education, the goal is to develop learners into lifelong metacognitive

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<sup>1</sup> Throughout the study, the term 'learners' is used to describe school learners and learners generally, while the term 'student' is used to describe those in higher education.

learners (Cornford, 2000: 5, 10; De Corte, 2010: 46; Desoete, 2007: 22; Hartman, 2001a: 34; Organisation for Economic Co-operation and Development [OECD], 2016a: 3; see Section 2.3.4.2).

Teachers are the ones expected to empower their learners with the disposition, knowledge and skills to succeed at school and in the workplace. Metacognitive reflective teachers possess adaptive competence to adapt and improve their performance in the classroom and within their profession. It is, therefore, important to cultivate reflective practices and adaptive skills in higher education, in addition to supporting the continuous professional development of teachers (Cornford, 2000: 7; Jindal-Snape & Holmes, 2009: 219; Larrivee, 2008: 341). Metacognition underpins the development of adaptive competence (Duffy, 2005: 300; Duffy, Miller, Parsons & Meloth, 2009: 241–242; Lin et al., 2005: 245; Timperley, 2011: 18) and facilitates the transfer of knowledge into skills to deal with novel scenarios (see Sections 1.3; 2.1; 2.3.1.2; 2.3.4.2).

Teachers are envisaged to be metacognitive reflective professionals who translate their knowledge and skills for their learners by modelling an awareness of cognitive processes and how to regulate these. Teachers, by deliberately, consciously, and habitually reflecting on their own feelings, thoughts, and actions in novel and problem-solving situations, empower learners to develop lifelong adaptive metacognitive skills. Consequently, teachers' metacognitive awareness—as adaptive competence and a key component in quality teaching and learning—is the focus of the study.

## **1.2 BACKGROUND**

The 2013 Organisation for Economic Co-operation and Development (OECD) Country Report on South Africa states that poor educational outcomes remain a critical problem, contributing to the high unemployment rate of 51% in the last quarter of 2012 among South African youth (OECD, 2013: 20). Moreover, in 2016, one third of young South Africans between ages 15 and 29 were identified as “Neither Employed nor in Education or Training” (NEET) whilst the population-wide unemployment figure was 26.5% (OECD, 2016b: 1).

This crisis in poor educational outcomes is additionally reflected in the continuing poor performance by South African learners in English, Mathematics, and Science in

international and national tests. The quality of basic and vocational education must be improved to produce skills which are required in the labour market (CDE, 2013: 10; OECD, 2013: 2). Mathematics teaching and learning especially is a key locus for concern, as Mathematics provides access to study and employment in scientific, medical, engineering, technological, and business professions, whilst basic numeracy skills are a requirement in most vocations and informal enterprises (CDE, 2013: 12; DBE, 2010a: 17).

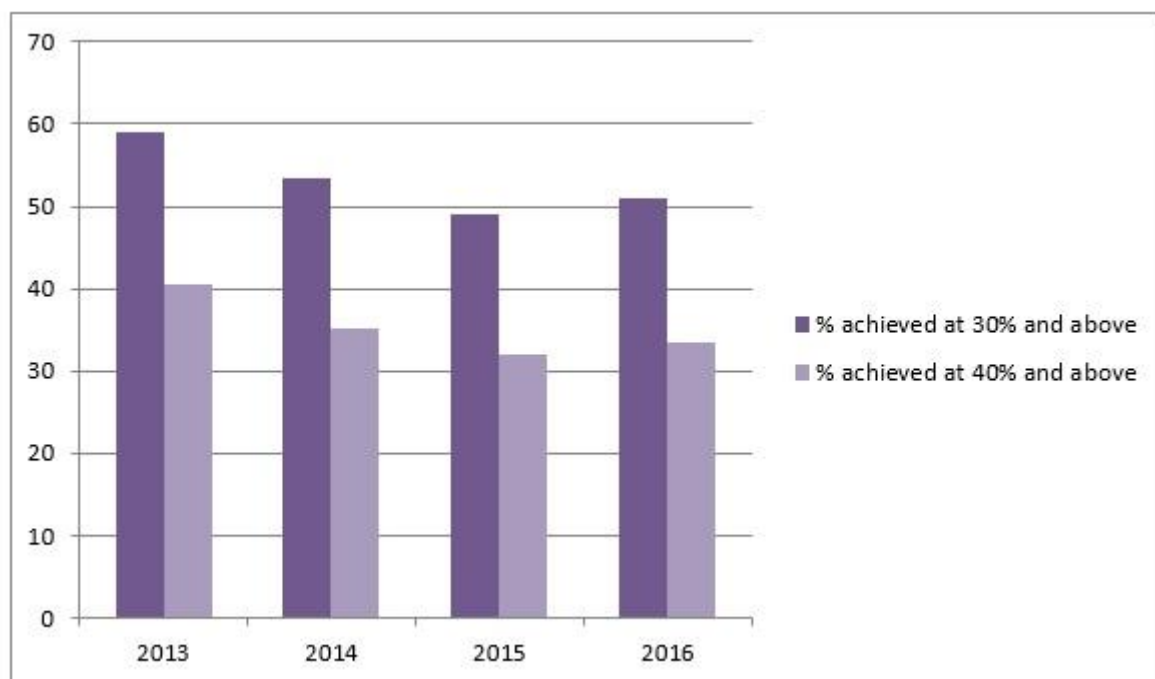
Internationally, the Trends in International Mathematics and Science Study (TIMSS)—an indicator of achievement in Mathematics and Science—ranked South Africa the lowest of the participating countries in their 2003 survey, with its score of 264 points for Grade 8 Mathematics well below the international average of 466 (National Centre for Education Statistics [NCES], 2004: 5, 7). Subsequently, in the 2011 TIMSS, South Africa obtained a score of 352, below the centre-point score of 500 (Mullis, Martin, Foy & Arora, 2011: 42–43, 470) and worse than any other middle-income country (CDE, 2013: 3). Finally, the 2015 TIMSS indicated an increase to a score of 372 (Trends in International Mathematics and Science Study South Africa [TIMSSSA], 2015: 6). It is worth noting that in 2011 and 2015, Grade 9 learners in South Africa, Botswana, and Honduras competed in the TIMSS against Grade 8 learners in the other countries (CDE, 2013: 4).

In 2014, the South African benchmarked results of Grade 9 Mathematics learners were well below expectations (DBE, 2015: 3). The Annual National Assessments (ANA) revealed less than 5 per cent of South African learners achieving 40 per cent or more in Mathematics in 2012 (CDE, 2013: 6). In her address following the release of the 2014 ANA results, the Minister of Basic Education, Ms Angie Motshekga, stated that South African learners in Grades 4, 6, and 9 displayed poor problem-solving skills in English, Mathematics, and Science and suggested that logic skills were not being engaged or taught sufficiently in these core subjects (Motshekga, 2014: 2). This point is raised in the 2011 CAPS document, which stipulated that the aims of teaching should not be limited to addressing the “how” of a matter, but should also cover the “when” and “why” in solving problems to help develop problem-solving and cognitive skills. This will help facilitate understanding and deeper learning, consequently equipping learners to use their learning in education and work-life (DBE, 2011a: 8).



Ultimately, this observation translates to the teaching of metacognition, which involves reflection on the how, when, and why of strategy use (see Section 2.2.4.1).

Further evidence of challenges facing Mathematics teaching and learning, and thus further justification for nurturing metacognition, can be found in South African learners' Grade 12 results. The low pass rate of Grade 12 Mathematics learners is alarming, as Mathematics is a prerequisite to university study in many core professions (CDE, 2013: 6). The National Senior Certificate (NSC) Examination Report 2016 indicates a slow improvement in Mathematics marks and the NSC pass rate from 2015–2016 (DBE, 2016b: 51–53). However, the overall declining trend from 2013 to 2016 is a concern.



**Figure 1.1: NSC Mathematics performance trends 2013–16 (DBE, 2016a: 151).**

The percentage of those candidates who passed Mathematics at 40 per cent has decreased from 40.5 % in 2013 to 33.5 % in 2016 (DBE, 2016a: 151). Importantly, the diagnostic report highlighted poor higher-order thinking skills among learners. Learners had trouble answering questions requiring higher-order thinking (i.e. analytical, evaluative, and problem-solving questions). This suggests a deficit of learning opportunities in problem-solving and extension exercises, presupposing a sound comprehension of basic concepts (DBE, 2016a: 5).

In the South African NSC examinations, expectations are set that learners should be able to answer questions on the higher-order thinking level, which in the Mathematics examination paper is proposed to include 15% problem solving and 30% complex procedures (DBE, 2011a: 53). Based on the low level of achievement among NSC Mathematics candidates from 2013–2016 and the ANA results in 2014 for Grade 9 Mathematics learners, it appears there is a general inability among South African learners to make effective use of these desired higher-order thinking skills, particularly in Mathematics. It can be argued, therefore, that in South Africa these higher-order skills are not being taught or developed sufficiently in lower grades, nor built upon in secondary school. Higher-order thinking skills—good problem-solving skills in particular—are a significant contributor to good performance in Mathematics (see Sections 1.3; 2.2.2; 2.3.2; 2.3.3; 2.3.4.2). Because successful problem solving is central to mathematical proficiency, the aim of Mathematics education is to develop competent problem solvers (see Section 2.3.4).

Moreover, this low level of achievement of Mathematics candidates at secondary school indicates South African education is not producing the skills needed for the current job market (OECD, 2013: 2). This suggests that South Africa will struggle to satisfy the workplace demands for employees with skills related to Mathematics and Science in future, particularly in scientific, technological, and business professions (OECD, 2013: 8, 9). Consequently, raising achievement in Mathematics by improving the pass rate in lower grades, as well as increasing the number of Grade 12 learners achieving high marks in Mathematics among other key subjects, is a key concern and goal emphasised by various stakeholders, including the government (DBE, 2015: 2, 31; OECD, 2013: 2, 8–9).

In 2010, the Department of Basic Education (DBE) identified enhancing Mathematics results as a major priority in their draft education document—*Action Plan to 2014: Towards the Realisation of Schooling 2025*—to improve the quality of education and learning (DBE, 2010a: 5–6). Six of the eight goals focused on improving Mathematics competency and achievement (Goals 1, 2, 3, 5, 8, and 9), indicating concern about the standard of Mathematics learning. Unfortunately, the targets set for the senior phase by the DBE in 2010 were not met, with Mathematics in Grade 9 specifically not showing any improvement (DBE, 2014: 10).

Consequently, in the DBE's latest education document, *Action Plan to 2019: Towards the Realisation of Schooling 2030* (DBE, 2015: 3), these goals were reinstated, with Goal 9 a key priority and the focus shifted towards improving the performance of Grade 9 Mathematics learners. There is also a renewed focus on teaching and professional development, as one of the five priority goals is to enhance teachers' practice by improving "the professionalism, teaching skills, subject knowledge and computer literacy of teachers throughout their entire careers" (Goal 16) (DBE, 2015: 3).

Quality learning and teaching, especially in Mathematics, is therefore a key educational goal in South Africa. However, it has been perceived that teachers of Mathematics in South African schools have a low standing within the global context (CDE, 2013: 3). Underperforming teachers in South Africa have shown a tendency to overestimate their own mathematical proficiency and their learners' performance relative to the curriculum, as well as underestimating their own learning and curricular deficits (Spaull, 2013: 21). As highlighted by Spaull (2013: 21), 89% of Grade 9 teachers in South Africa indicated in the 2011 TIMSS that they felt "very confident" teaching Mathematics, a sentiment undermined by the poor 2014 ANA results, whilst paradoxically, teachers in the best-performing countries were more moderate when estimating their own proficiency: for instance, in Finland just 69% felt very confident and in Singapore only 59% expressed this sentiment (Mullis et al., 2011: 314–315).

While such findings paint a challenging picture of overall national performance in Mathematics, it is important to emphasise that there are very capable and gifted Mathematics teachers and learners throughout South Africa, which is exemplified by a number of schools achieving 100% pass rates in recent years (DBE, 2016c: 2, 5–10). Nonetheless, Mathematics teachers in general are expected to raise the standard of Mathematics teaching and learning. This necessitates enhancing teachers' skills in numeracy and Mathematics (CDE, 2013: 11) and their abilities to deal with practical problems in the classroom relating to subject matter and classroom management. In addition, using practical real-life problems makes learning more authentic for learners and better prepares them for the demands of the workplace. Moreover, teachers are expected to manage classrooms, make decisions, and solve problems daily: all activities which entail metacognitive adaptive competence (Duffy, 2005: 300; Duffy et al., 2009: 241–242; Lin et al., 2005: 245). The

extent to which teachers are metacognitively aware of their own teaching abilities and learning deficits—and, moreover, can teach with and for metacognition—remains questionable (Duffy et al., 2009: 244; Kohen & Kramarski, 2012: 2; see Section 2.3.5). Consequently, this carries significant implications for the training of pre-service and in-service teachers, who must consciously and deliberately foster and develop these adaptive skills in their learners' learning and problem solving (Azevedo, 2009: 93; Cornford, 2002: 366; Duffy et al., 2009: 241–242; Kohen & Kramarski, 2012: 7; Van Der Walt & Maree, 2007: 238).

The Minimum Requirements for Teacher Education Qualifications (MRTEQ), which are the national standards used for graduate teachers, set expectations for teachers to be knowledgeable about their subject, possess good problem-solving skills, and be metacognitive reflective in their practice (DHET, 2015: 64; see Section 1.3). The basic competencies required of newly qualified teachers include knowing how to teach their subject, knowing what effective learning is and how to mediate it, identifying learning or social problems, and managing and creating a conducive classroom environment. In addition, beginner teachers should be able to reflect critically “on their own practice to constantly improve it and adapt it to evolving circumstances” (DHET, 2015: 64). This expectation for teachers to be adaptive and reflective informs good teaching practice (see Section 2.3.5). Furthermore, it is expected that pre-service teachers should be able to act on a certain level requiring them to solve problems and manage their learning successfully (SAQA, 2012: 10). Teachers are therefore expected to be metacognitively aware themselves and to teach these skills to their learners, as learners can hardly acquire knowledge or skills at school that their teachers do not already possess (Barber & Mourshed, 2007: 16). This requires metacognition as an adaptive competence—enabling teachers to reflect on their personal abilities and take appropriate action—to be fostered in pre-service and in-service programs (see Sections 2.3.5; 2.3.5.1). However, it is worth noting that half of the Council of Higher Education's evaluated in-service training programmes, primarily in Mathematics, were deemed inadequate in developing teachers' abilities to solve practical problems in the classroom (DBE, 2015: 35).

Continuing professional development—and hence lifelong learning—should enhance and develop teachers' skills to reflect and improve upon their practice. It should serve to develop awareness and willingness in teachers to reflect upon their own strengths

and weaknesses; in short, to be metacognitively aware (see Section 2.3.5). Teachers' evaluations of themselves, i.e. their metacognitive awareness, are important for generating improvement and change. Self-assessment of one's abilities, however minimally performed, may inform policy and actions (DBE, 2015: 34). Such self-assessment demonstrates a personal willingness to improve on practice as a professional, as the individual takes control of their own learning and fosters greater autonomy, which is essential to developing adaptive competence (Bransford et al., 2000: 18; Timperley, 2011: 8). However, as noted above, the extent to which teachers are metacognitively aware and able to teach with and for metacognition remains questionable both internationally and nationally (Duffy et al., 2009: 244; Grossman, 2009: 17; see Section 2.3.5) and will consequently be explored in the study.

The 2007 McKinsey educational report states that "the only way to improve the level of the outcomes that must be demonstrated is to improve instruction; therefore, teachers should be skilled to become effective instructors" (Barber & Mourshed, 2007: 26). The content knowledge possessed by teachers is a necessity, but alone is not a sufficient basis for successful teaching and learning (Spaull, 2013: 16). Adaptive experts are knowledgeable about their subject content and how best to teach and adapt this content, marking them as lifelong adaptive learners (Timperley, 2011: 6–7). Teachers must be effective facilitators of knowledge acquisition through an array of methods and approaches. However, the meeting of targets to improve learner achievement and teacher skills, as stated in the priority goals of the latest Action Plan, can only be envisaged within the larger context of education in South Africa.

The broader national education context and outcomes are impacted by numerous socio-economic factors and challenges, including poverty (OECD, 2013: 2; Spaull, 2015: 51). Such factors lie beyond the scope of the investigation. The study focuses on another vital aspect of enhancing teaching quality in Mathematics, which is the enhancement of teachers' metacognitive awareness.

In my experience as a teacher across both secondary and higher education contexts, I observed a number of trends in Mathematics teaching and learning firsthand. Pre-service teachers find it difficult to reflect on their teaching practicums, and also find it

challenging to explain or elaborate upon how they would solve mathematical problems; instead, they prefer to teach solving routine problems in a lecture style. Additionally, as a presenter of in-service teacher training sessions, I noticed teachers were unskilled in using different problem-solving methods and strategies. In my own classroom teaching and that of my colleagues, emphasis on reaching performance targets and completing a time-demanding, difficult school Mathematics syllabus often lead us to opt for step-by-step algorithmic routine problems, rather than spending time teaching learners how to think about solving problems. These observations led to questions about the teaching and learning of Mathematics, and whether pre-service teachers are able to reflect adaptively on how they learn and teach Mathematics.

In the South African context, research on metacognition in Mathematics teaching and learning is not well-published, as noted by Van Der Walt, Maree and Ellis (2008: 231). To the best of my knowledge, no previous study in a South African context has investigated the metacognitive awareness of Mathematics didactics students (fourth-year pre-service teachers) and additionally their metacognitive awareness in a problem-solving context. The study, therefore, contributes to addressing this lack in national research and provides recommendations to the educational community to enhance the metacognitive awareness of teachers and by inference learners.

Before describing the problem statement, research questions, and research design of the study, it is important to first establish the role of metacognition in teaching and learning.

### **1.3 METACOGNITION IN TEACHING AND LEARNING**

Metacognition has been widely researched and defined in various ways by Sperling, Howard, Staley and DuBois (2004: 118) among others (see Section 2.2.3). Metacognition generally refers to the ability to reflect upon, understand, and regulate one's thinking and learning processes, an understanding of the term dating back to Flavell's earliest definition (Flavell, 1976: 232).

Metacognition is operationalised by Flavell (1979: 909) into four categories: metacognitive knowledge, metacognitive experience, metacognitive skills and strategies, and metacognitive goals. Metacognition, in this study, distinguishes between two components, namely *Knowledge of cognition* and *Regulation of cognition*

(see Section 2.2.4). It is also important to differentiate between metacognitive knowledge (which refers to the what, how, when, and why of strategy use in learning and problem solving) and metacognitive skills (which refers to regulating strategy use in learning and problem solving) (see Sections 2.2.4.1; 2.2.4.3).

Metacognition is considered significant in improving learners' learning processes and consequently in demonstrating learning outcomes and expectations. The aim of education is to transform learners into lifelong metacognitive learners (Cornford, 2000: 10; De Corte, 2007: 22; Hartman, 2001a: 34). Productive learning is facilitated by metacognition as an adaptive competence, which enables people to transfer and use their learning, knowledge, and skills in novel scenarios across different domains and contexts (Bransford et al., 2000: 18, 19; Hartman, 2001a: 34; Lin et al., 2005: 244–245; see Sections 2.3.1.2; 2.3.4.2).

Educational researchers and educators accept metacognition as a key element of higher-order thinking and assert the importance of acquiring and teaching higher-order thinking skills (i.e. metacognitive, critical, and problem-solving skills) (Akyol & Garrison, 2011: 184; Anderson & Krathwohl, 2001: 57; Desoete, 2007: 718; Pugalee, 2001: 237; Schoenfeld, 2007: 60; Van Der Stel, Veenman, Deelen & Haenen, 2010: 219; Van Der Walt & Maree, 2007: 237; see Section 2.2.2). Research has indicated that enhancing learner metacognition results in successful learning and academic achievement in various domains, including Mathematics (Sperling, Richmond, Ramsay & Klapp, 2012: 1; see Sections 2.3.2; 2.3.3). Internationally, the National Research Council report, *How People Learn: Brain, Mind, Experience and School*, states that metacognition supports active learning, especially when individuals take control of their learning through reflection, setting goals, and monitoring progress to achieve their goals (Bransford et al., 2000: 18).

In Mathematics, good problem-solving abilities are a significant contributor to good performance in Mathematics (DBE, 2010b: 8–9; 2011a: 8, 53; Schoenfeld, 1992: 338; 2007: 60; see Section 2.3.4.1). The importance of teaching learners how to solve problems successfully finds ample support in scholarly literature, the South African policy documents, and reports on the poor results obtained by South African learners in Mathematics. These all point to the need for teaching problem solving. Metacognition is key in successful problem solving, along with other

attributes of mathematical proficiency such as affect, heuristics, and content knowledge (see Section 2.3.4.2), and facilitates the transfer from one phase to another in the four-phase problem-solving framework (Carlson & Bloom, 2005: 62–69; Pugalee, 2001: 239–243; see Section 2.3.4.4). Metacognition is also a key aspect of productive learning in Mathematics (see Section 2.3.4.3), with De Corte (2007: 22) asserting that metacognition as adaptive competence is the ultimate goal of Mathematics education.

It therefore follows that an individual's metacognitive awareness of his or her own thinking processes enhances productive learning and improves achievement (Schellings, Van Hout-Wolters, Veenman & Meijer, 2013: 980; White, Frederiksen & Collins, 2009: 178). Performance is enhanced by metacognitive knowledge (Pintrich, 2002: 225) and metacognitive skills (Van Der Stel & Veenman, 2010: 224; Van Der Stel et al., 2010: 228), and consequently the enhancement of metacognition in learners could improve academic achievement (Larkin, 2009: 149). This indicates that learners' metacognitive knowledge (see Section 2.2.4.1) and metacognitive skills (see Section 2.2.4.3) could and should be enhanced (see Sections 2.4.1–2.4.3).

Because metacognition can be enhanced, the premise is that metacognition should be taught (Desoete, 2008: 436; Hartman, 2001b: 150; White et al., 2009: 178) as the learning of metacognitive and cognitive skills enables individuals to process information effectively, apply knowledge and skills to new situations, and become lifelong learners (Cornford, 2000: 5; 2002: 357–358; Schraw, Crippen & Hartley, 2006: 116–117).

In teaching, metacognition is well-recognised as a means of improving teachers' skills and reflective practices, thereby improving their teaching practice (Duffy, 2005: 300–305; Jindal-Snape & Holmes, 2009: 219; Kohen & Kramarski, 2012: 7; see Section 2.3.5). Metacognition is therefore a key element in pedagogy, particularly the effective teaching of Mathematics. Moreover, Mathematics teachers should promote metacognition and self-regulation (Van Der Walt et al., 2008: 231). The third major finding of the National Research Council (Bransford et al., 2000: 21) stressed the development of metacognition and self-regulated learning as a means for teaching professionals to be effective and autonomous in their teaching practice and learning (Timperley, 2011: 8).



This research pertains primarily to the metacognition of school learners. Whilst South African research on the metacognitive awareness of undergraduate Mathematics teachers, as noted above, is scarce, a noteworthy study by Van Der Walt (2014: 1–22) investigated the level of metacognitive awareness and self-directedness in the Mathematics learning of prospective second- and third-year intermediate and senior phase Mathematics teachers. Although these undergraduate teachers reported a high level of metacognitive awareness in the study, it did not correlate with their learning achievement. It is suggested by Van Der Walt (2014: 1–22) that undergraduate Mathematics teachers, when assessing their own learning behaviour, might under- or over-estimate their level of metacognitive awareness or self-directedness.

Similarly, college students may be metacognitively aware of monitoring their learning and problem solving, but may not seem as successful in regulating learning and problem solving (Bjork, Dunlosky & Kornell, 2013: 417; Koriat, 2012: 297). These pre-service teachers might have knowledge of effective learning behaviour, yet fail to implement this knowledge in learning or problem solving (see Section 5.4.1).

Additional research has indicated that undergraduate students do not easily reflect (Grossman, 2009: 17; Jindal-Snape & Holmes, 2009: 219). Metacognitive reflection is difficult for pre-service and in-service teachers in particular because of situational factors (Duffy et al., 2009: 244; Kohen & Kramarski, 2012: 2, 6). Adaptive metacognition is key in dealing with these unique challenges of classroom variability (Lin et al., 2005: 245). Unfortunately, metacognition is not generally associated with teachers' professional development or pre-service teacher education (Duffy, 2005: 300, 308; see Sections 2.3.5; 2.3.5.1).

The abovementioned carries implications for teacher training and professional development, as metacognition is acquired with intentional, deliberate instruction and modelling over a prolonged period of practice and through conscious implementation (see Section 2.3.5.1). The premise for enhancing teachers' metacognition is that teachers will translate their own metacognitive knowledge and skills to their learners. The question of whether the development of metacognition takes place during a teaching and learning situation must be asked. This raises yet another question: Are

pre-service Mathematics teachers aware of metacognition, and more specifically, what is their awareness thereof in a problem-solving context?

## **1.4 PROBLEM STATEMENT AND PURPOSE**

As indicated in Section 1.2, it is evident from the poor performance of South African Mathematics learners that there is incongruence between the expectations set for learners and the performance of those learners. As a result, there is cause for concern. The DBE has identified the enhancement of Mathematics results as a key priority in their draft Action Plan to improve the quality of teaching and learning in South Africa (DBE, 2010a: 5–6). In the latest Action Plan, the improvement of teachers' skills is deemed integral to this undertaking, particularly on how to better teach Mathematics and solve problems (DBE, 2015: 3).

Teacher competence in teaching Mathematics is a key factor in addressing poor learner performance. Metacognition is one of the four attributes of mathematical proficiency and productive learning in Mathematics and thus impacts achievement (see Section 1.3). Teachers are expected to demonstrate metacognitive awareness as adaptive competence in solving Mathematics problems, as part of reflection on their teaching practice and in managing their own learning (DHET 2015: 64; SAQA, 2012: 10; see Section 1.2). As such, teachers are expected to be metacognitively aware themselves and, moreover, to teach for metacognition and enhance learners' metacognition, which in turn may lead to better academic achievement.

Therefore, the question was posed whether the development of metacognition takes place during the teaching and learning situation. Ensuing from the notion that teachers have the potential to translate their metacognitive knowledge and metacognitive skills for their learners—consequently enhancing the Mathematics performance of South African learners—the purpose of the study was to investigate the level of metacognitive awareness of pre-service Mathematics teachers.

## 1.5 RESEARCH QUESTIONS

Guided by the purpose statement provided above, the study investigated the following primary research question: **What is the level of metacognitive awareness of pre-service Mathematics teachers?**

To explore the primary research question, the following secondary research questions needed to be answered:

**Secondary research question 1:** How is metacognitive awareness conceptualised?

**Secondary research question 2:** What is the role of metacognitive awareness in Mathematics teaching and learning?

**Secondary research question 3:** What is the level of metacognitive awareness of pre-service Mathematics teachers on the Metacognitive Awareness Inventory (MAI)?

**Secondary research question 4:** What is the level of metacognitive awareness of pre-service Mathematics teachers in a problem-solving context?

The research questions are operationalised as follows:

- To review existing literature on the conceptualisation of metacognitive awareness;
- To review existing literature on metacognitive awareness in Mathematics teaching and learning;
- To measure and evaluate the level of metacognitive awareness of fourth-year pre-service Mathematics teachers quantitatively by using the MAI;
- To explore qualitatively the level of metacognitive awareness of pre-service teachers in a mathematical problem-solving context; and
- To provide recommendations based upon this research to the Mathematics Education community aimed at enhancing the level of metacognitive awareness of teachers.

Secondary research questions 1 and 2 entailed a literature review of existing scholarship. Aspects that needed to be explored were the conceptualisation of metacognition, the association between metacognition and learner achievement in Mathematics, the nature of Mathematics, aspects relating to the competent teaching and learning of Mathematics, the role of teachers' metacognition in their teaching practice, and teaching to enhance metacognition.

The quantitative secondary research question 3 employed a questionnaire, the Metacognitive Awareness Inventory (MAI) developed by Schraw and Dennison (1994), which was administered to pre-service Mathematics teachers to determine their level of metacognitive awareness.

The qualitative secondary research question 4 involved an investigation into pre-service Mathematics teachers' metacognitive awareness during a think-aloud, problem-solving session in relation to a four-phase problem-solving framework.

## **1.6 RESEARCH DESIGN**

As indicated above, a mainly quantitative approach was employed to explore secondary research question 3, whereas a qualitative approach was adopted for secondary research question 4, to enrich the findings of the third (see Section 3.3.2). In educational research, human behaviour is complex and context-bound. Creswell (2014: 2) asserts the usefulness of using both quantitative and qualitative methodologies in describing human behaviour. As metacognitive awareness is difficult to measure, multiple methods (on-line and off-line) are advocated (Desoete & Roeyers, 2006: 13; see Section 3.4.1), although a primarily quantitative approach is well-used in various studies (Schellings et al., 2013: 966), as was the case in the study.

### **1.6.1 Population and sample**

The population included all fourth-year pre-service Mathematics teachers enrolled at higher education institutions in South Africa. A convenience purposive non-probability sample of pre-service Mathematics teachers at a specific higher education institution was selected. The sample was convenient since I was their lecturer at that stage. It was purposive as the specific sample consisted of participants with the relevant

attributes for the study, and it was a non-probability sample as no choice regarding individual participants was made; rather, the whole fourth-year pre-service Mathematics teacher cohort was invited to participate in the study ( $n = 41$ ).

### **1.6.2 Data collection methods and procedures**

As indicated above, data was collected using both quantitative and qualitative methodologies.

In the quantitative part of the study, the MAI developed by Schraw and Dennison (1994) was administered to determine the level of metacognitive awareness of the pre-service teachers. It is a standardised questionnaire which measures the metacognitive awareness of adults and adolescents, and has been employed subsequently in various studies (Mevarech & Fridkin, 2006: 85–97; Sperling et al., 2004: 117–139; Van Der Walt, 2014: 9; Young & Fry, 2008: 1–10).

The supporting qualitative study employed a think-aloud method, with the aim to assess and describe the pre-service teachers' metacognitive awareness in a Mathematics problem-solving context (see Section 3.4.3.2). Think-aloud methods are a commonly accepted method of assessing a person's thinking processes (Pugalee, 2004: 29) and have been effectively used to assess metacognition in various studies (Desoete, 2007: 705–718; Desoete & Roeyers, 2006: 13; Meijer, Veenman & Van Hout-Wolters, 2012: 600; Schellings et al., 2013: 967–968).

During the data collection phase, the following steps were followed:

A literature review was conducted using national and international sources to answer secondary research questions 1 and 2 on the conceptualisation of metacognitive awareness and its role in Mathematics teaching and learning, as well as to identify a standardised measuring instrument that could be employed for collecting the quantitative data (Leedy & Ormrod, 2013: 92). The identified MAI, adapted to a South African mathematical educational context, was translated into Afrikaans, as the study was conducted at a parallel-medium higher education institution (see Section 3.4.3.1.3).

A pilot study was carried out using a convenient purposive sample of second-year pre-service teachers ( $n = 57$ ) at the same higher education institution where the main study

was conducted (see Sections 4.2.3.1). The pilot was undertaken to determine the validity and reliability of the adapted questionnaire translated into Afrikaans. The pilot group was representative of the sample in the main study, as both groups had been exposed to Mathematics Education instruction in their first year of study (see Section 3.4.3.1.3).

Qualitative data for the main study was obtained from the fourth-year pre-service teachers' (n = 41) written comments about their thinking processes during a Mathematics problem-solving session, prior to administering the MAI. Quantitative data for the main study was obtained by administering the MAI to the same purposive sample of fourth-year pre-service teachers (n = 41).

### **1.6.3 Quality criteria**

Reliability in quantitative research refers to the consistency and dependability of an instrument to measure the same construct or concept over time (Leedy & Ormrod, 2013: 92; Tavakol & Dennick, 2011: 53). The degree of reliability in a measure depends on the employment of the results (Ary, Jacobs & Sorenson, 2010: 248). Cronbach's alpha was used to determine the internal consistency of the questionnaire (see Section 3.4.3.3). The translated and piloted MAI was found to be highly reliable ( $\alpha = 0.94$ ) (see Section 4.2.3.1; Table 4.1).

In the main study, a high degree of internal consistency ( $\alpha = 0.89$ ) was found for the MAI as instrument (see Section 4.2.3.2). Moreover, the two-factor model, *Knowledge of cognition* and *Regulation of cognition*, was strongly supported ( $r = 0.54$ ,  $p < 0.05$ ) (see Sections 3.4.3.1.3; 4.3.1). This corroborated with the findings on the MAI in Schraw and Dennison's study (1994: 460–464) as well as in subsequent studies (Sperling et al., 2004: 124; see Sections 4.2.1; 4.2.2).

Reliability is a necessary but not sufficient condition for validity. Validity refers to the extent to which the instrument measures what it is intended to measure (Leedy & Ormrod, 2013: 89) and the degree to which meaningful and useful interpretations are supported by evidence and theory. These inferences are drawn from a specific instrument measuring particular concepts and constructs for a particular purpose in a particular situation (Ary et al., 2010: 225, 235; Creswell, 2009: 149). The original MAI was developed and standardised to measure

the metacognitive awareness of adolescents and adults, and may be useful in planning metacognitive awareness training and identifying low monitoring skills (Schraw & Dennison, 1994: 472; Young & Fry, 2008: 8). In this study, the purpose of the MAI was to measure the level of metacognitive awareness of pre-service Mathematics teachers. Inferences drawn from this instrument's quantitative scores afford limited possibilities for generalisation due to the small purposive sample ( $n < 100$ ) and non-parametric data (see Section 3.4.5).

Furthermore, generalisability is not the aim in qualitative research; rather, its value lies in the contribution of rich, thick descriptions and themes, especially those developed in a specific setting (Creswell, 2009: 193; 2014: 203–204). Data from the think-aloud session—the pre-service teachers' written statements on their mental processes in a problem-solving context—contributed to an understanding of their level of metacognitive awareness.

Qualitative validity means that the researcher reviews their findings for accuracy using certain procedures (Creswell, 2014: 201). Strategies employed to enhance the validity of the findings include the use of rich, thick descriptions to convey the findings by offering many perspectives about every theme (i.e. the 8 subscales on the MAI and the four phases of the problem-solving framework); clarifying the bias of the researcher; and employing a peer who reviewed and asked questions about the study (Creswell, 2014: 202).

Qualitative reliability (trustworthiness) indicates that the approach of the researcher is stable and consistent across different attempts to collect and analyse data (Creswell, 2014: 201). Quality reliability procedures were followed in the think-aloud problem-solving session by documenting all procedural steps. During the process of methodical coding, data were carefully related to the definition of the subscales and an audit trail of the data was maintained (Ary et al., 2010: 502–503; Creswell, 2014: 203).

#### **1.6.4 Role of the researcher**

From a post-positivist stance in the quantitative research, I aimed to be objective and impartial by focusing on facts when organising and analysing the data (Ary et al., 2010: 13–14). As the study has a qualitative component as well, from

an interpretivist stance, my role in data collection and analysis had to be identified to ensure credibility. My assumptions based on literature, experience, and perceptions of higher education shaped my personal experience and understanding of the educational context, which lead to enhanced awareness and knowledge which were valuable to the study. Previous experience gained while working with pre-service and in-service teachers, plus awareness of the issues and challenges they face, brought bias to the study which may have shaped my view and interpretation of the data. However, effort was made to ensure objectivity and trustworthiness throughout (Creswell, 2014: 206; see Section 3.4.3.3).

### **1.6.5 Data analysis and interpretation**

Descriptive statistics were used to organise and analyse the quantitative data collected from 41 pre-service teachers (Pietersen & Maree, 2010a: 239). Due to the small number of respondents and the use of non-random sampling on an ordinal scale, a non-parametric test, the *Spearman Rho* Coefficient, was used to determine the relationship between the two factors (*Knowledge of cognition* and *Regulation of cognition*) of the MAI (Pietersen & Maree, 2010a: 237; see Section 4.3.1). In the study, the correlation coefficient corroborated with that of the original MAI and the adapted MAI (see Sections 4.2.1; 4.2.2). Furthermore, individual tendencies were identified and interpreted by integrating the findings from the quantitative data with related theory, e.g. attributes of mathematical proficiency and aspects of productive learning (see Sections 2.3.4.2; 2.3.4.3).

A detailed analysis using a methodical coding process organised the qualitative data into meaningful categories. Data were coded according to items on the MAI representing metacognitive behaviours. Frequencies of metacognitive behaviours were calculated and presented in tabular form for the purpose of interpretation (Creswell, 2009: 189; Meijer, Veenman & Van Hout-Wolters, 2006: 209; see Appendix 6). In the analysis, items were further grouped under the subscales of the MAI as well as according to the four-phase problem-solving framework. Interpretations were made by integrating the findings from the qualitative data with related theory, as found in the literature (see Sections 4.4.2–4.4.4).



I aimed to provide a rich description of the written statements as these indicate the metacognitive knowledge and metacognitive skills of the pre-service teachers in relation to problem solving (Ary et al., 2010: 425–426). These descriptions offered a greater understanding of the pre-service teachers' roles in the problem-solving process and, consequently, enriched the quantitative findings of the MAI (see Sections 4.4; 4.4.1–4.4.4).

The qualitative think-aloud problem-solving session could not be correlated with the quantitative questionnaire as a whole, with metacognitive *skills* more overt in the practical solving of the Mathematics problem, whereas metacognitive *knowledge* was mostly implicit and referred mainly to the general learning of Mathematics as measured by the MAI. It should be noted, therefore, that inferences and comparisons between the quantitative data from the MAI and the qualitative data obtained from the problem-solving session in which the pre-service teachers participated are limited. In the study, the qualitative findings serve to enrich the quantitative findings by highlighting metacognitive problem-solving behaviours (see Section 4.5).

## **1.7 DELINEATING THE FIELD OF STUDY**

The context of this research is Higher Education. The field of Mathematics teaching and learning, particularly Higher Education teacher training in Mathematics, delineates the field of study. This viewpoint is considered in terms of how metacognitive awareness relates to higher achievement in Mathematics learning and problem solving. The role of metacognitive awareness as adaptive competence in teaching, as well as in mathematical proficiency and productive learning, takes a primary position in the study. In summary, the study's focus is the metacognitive awareness of pre-service teachers in Mathematics learning and problem solving.

## **1.8 DISSERTATION LAYOUT AND PRESENTATION**

Following this chapter, Chapter 2 provides the literature review for the study, conceptualising metacognitive awareness, the role it plays in teaching and learning Mathematics, and ways for teaching and enhancing metacognitive awareness in both teachers and learners. Chapter 3 is the methodology chapter, describing the empirical research approach, philosophical worldview, and research methods behind the study.

Chapter 4 analyses, interprets, and summarises the primary quantitative data to determine the level of metacognitive awareness of the pre-service teachers, as well as the qualitative data to enrich the findings of the quantitative data. Finally, Chapter 5 concludes by reiterating the key points and findings of the study, reinforcing the value of metacognitive awareness in the teaching and learning of Mathematics to both individual and national interests.

## **1.9 SUMMARY OF CHAPTER**

The purpose of this chapter was to focus attention on metacognitive awareness as a key component in teaching and learning to improve the overall outcomes of learners in teaching and learning. The importance and necessity of improving the quality of teacher education in South Africa was illustrated. In addition, the chapter established that metacognition is one of the four attributes of mathematical proficiency, whilst metacognitive adaptive competence improves teacher practice, which could improve learner outcomes, particularly in Mathematics.

An overview of the research process that underpins the study was provided. The purpose of the study, namely to determine the level of metacognitive awareness of pre-service Mathematics teachers, was outlined; research questions were stated; and the philosophical worldview, research approach, and research methodology which informed the study were discussed.

In Chapter 2, three main themes are explored in the literature. The origin and definition of metacognitive awareness, the role of metacognitive awareness in the teaching and learning of Mathematics, and teaching for metacognition are discussed, with the intention to explore and establish the need for metacognitive awareness in pedagogy and teacher training.

## **CHAPTER 2**

# **METACOGNITION AS A CONCEPTUAL BASIS AND ITS ROLE IN TEACHING AND LEARNING**

### **2.1 INTRODUCTION**

Novel situations—socially, economically, and technologically—demand innovative responses. In today’s society, it is necessary for individuals to develop a generally adaptive approach to deal effectively with new and challenging situations on a daily basis. Moreover, addressing the challenges posed by pervasive technological change, social demands, and the sheer volume of information in circulation—both individually and collectively—requires certain knowledge and skills. Adaptive ‘learning-to-learn’ skills facilitate transferability of existing knowledge to deal with the demands of new situations (Cornford, 2000: 5; Lin et al., 2005: 245), and as these situations are typically beyond the individual’s usual experience, reflection is triggered (Rogers, 2001: 42).

Lifelong learning has become a commonplace concept in professional and workplace environments. Citizens are expected to be responsibly engaged in socio-economic and information technology issues. This entails a journey of learning about these issues and finding solutions for them. It also involves thinking in new ways about how to approach these issues, requiring a manner of thinking that utilises a repertoire of skills which can be adapted and applied to new scenarios.

Therefore, in education, lifelong learning has become an important focus. Institutions are called upon to empower their learners with lifelong learning skills. A continual interest in education would consequently help learners to become more knowledgeable about cognition in general, and especially more aware of their own thinking (i.e. metacognitively aware), with the intention to further their learning at school and thereafter (Bransford, Brown & Cocking, 1999, cited in Pintrich, 2002: 119; Cornford, 2000: 6; Schraw et al., 2006: 130).

In addition, the incredible tempo at which information changes and increases necessitates change in the goals and methods of education. The focus in learning

and instruction is shifting from absorbing large quantities of information and developing basic skills towards achieving deeper understanding and developing the types of reasoning skills that will aid learners in obtaining and utilising new knowledge (Zohar & David, 2008: 60).

The ability to learn effectively implies that the learner needs skills which can be applied generally to new potential learning experiences; that is, 'learning-to-learn' skills which comprise cognitive and metacognitive strategies (Cornford, 2000: 5; 2002: 357). Metacognition, as an adaptive skill, facilitates productive learning and successful problem solving (De Corte, 2010: 45; Lin et al., 2005: 244–245; see Sections 2.3.4.2; 2.3.4.3). Adaptive metacognition involves adapting one's environment and oneself in response to novel situations. Furthermore, metacognitive awareness develops and enhances learners' learning processes and, as a result, their learning outcomes (Thomas & Mee, 2005: 221).

Metacognition influences academic achievement in learning and problem solving (Azevedo, 2009: 88, 93; Hartman, 2001a: 35; Schellings et al., 2013: 980) and particularly in Mathematics (Larkin, 2009: 149; Van Der Stel et al., 2010: 219; Van Der Walt, 2014: 8–9). Consequently, metacognitive skills, like critical thinking and problem-solving skills, are adaptive skills which are mandatory to be learnt and therefore taught to learners (see Sections 2.2.2; 2.4.3).

Teachers are expected to manage classrooms, make decisions, and solve problems daily. This requires metacognitive adaptive expertise, namely the ability and skills to reflect and adapt thoughts, feelings, and actions accordingly (Duffy, 2005: 300; Duffy et al., 2009: 241–242; Lin et al., 2005: 245). This carries significant implications concerning the training of pre-service and in-service teachers, who must consciously foster and develop these adaptive skills in their learners' learning and problem solving (Azevedo, 2009: 93; Cornford, 2002: 366; Kohen & Kramarski, 2012: 7; Van Der Walt & Maree, 2007: 238). Teachers are therefore expected to be reflective practitioners (Jindal-Snape & Holmes, 2009: 219; Larrivee, 2008: 341; Rogers, 2001: 37), who along with having specialist content knowledge should model and scaffold metacognitive awareness for their learners and teach metacognition explicitly, thus translating these skills for their learners

(Cornford, 2002: 366; Pintrich, 2002: 223; Thomas & Mee, 2005: 222; see Section 2.3.5).

## 2.2 THE CONSTRUCT METACOGNITION

In Sections 2.2.1 to 2.2.7, secondary research question 1, “How is metacognitive awareness conceptualised?”, is expounded. The background and development of the term “metacognition” in the scholarship is discussed below.

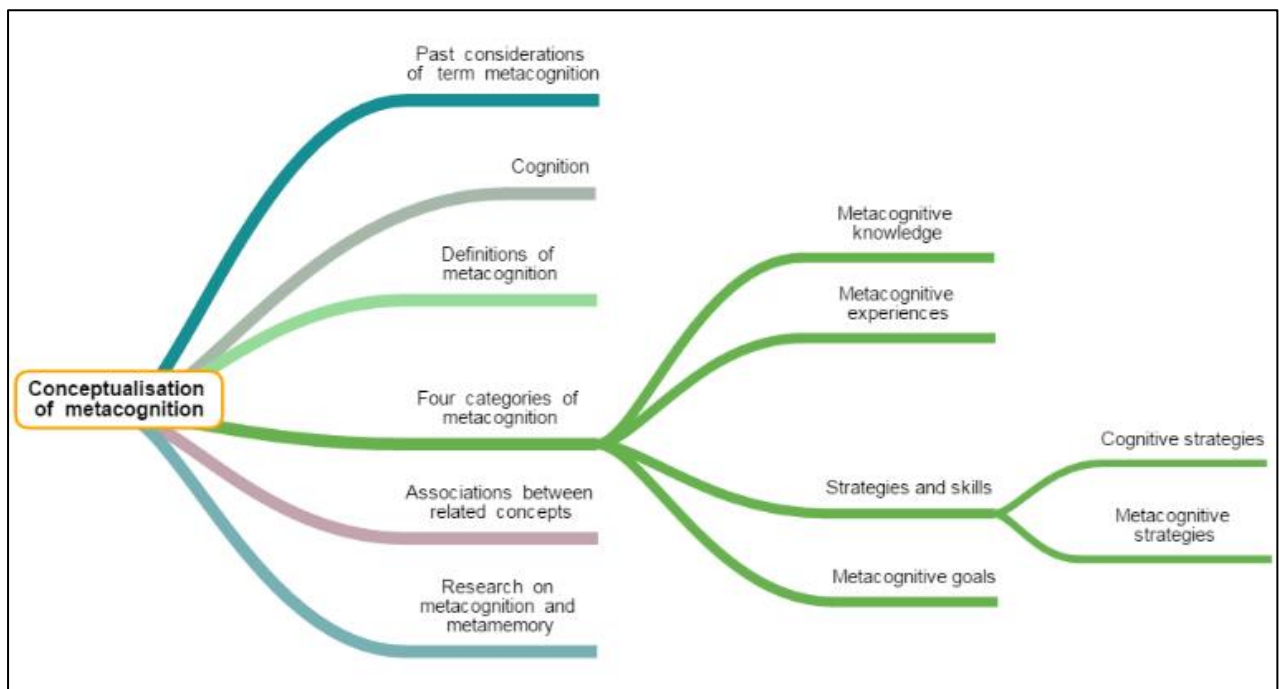


Figure 2.1: Conceptualisation of metacognition (see also Appendix 8)

### 2.2.1 Past considerations of the term metacognition

Metacognition was first mentioned in educational literature in the 1970s, when John Flavell (1976: 232) described metacognition as “knowledge concerning one’s own cognitive processes and products or anything related to them”. Subsequently, seminal works by Flavell (1976, 1979) and Brown (1987) laid foundations for the contemporary understanding of metacognition.

Flavell (1976: 232) further described metacognition as “the active monitoring and consequent regulation and orchestration of those processes in relation to cognitive objects or data on which they bear, usually in the service of some concrete goal or

objective". A theory proposed by Brown (1987) and used in subsequent studies (Larkin, 2009: 150; Schraw & Dennison, 1994: 461; Sperling et al., 2004: 118) would propound two components: *Knowledge of cognition* and *Regulation of cognition*. Schraw and Dennison (1994: 461) extended *Regulation of cognition* into subcomponents: *Planning, Information management, Monitoring, Debugging, and Evaluation*. Various other studies support this two-component view of the construct metacognition.

Generally, therefore, metacognition is defined by its component parts. However, the term "metacognition" as a construct is not exactly defined (Sperling et al., 2004: 118; Sperling et al., 2012: 2) as researchers have not reached consensus about the nature of those components and their exact relationship to each other. Neither have they reached consensus about the relation of metacognition to constructs such as cognition, self-regulation, self-regulated learning, and meta-memory (Dinsmore, Alexander & Loughlin, 2008: 392; Sperling et al., 2004: 118; Veenman, Van Hout-Wolters & Afflerbach, 2006: 4) and affective states like motivation and self-efficacy (Boekaerts, 1999, cited in Dignath & Büttner, 2008: 235; Schraw et al., 2006: 112).

Various terms are used to describe views of metacognition in literature. These include, among others, metacognitive reflection, adaptive meta-cognition, metacognitive awareness, self-reflection, transformative metacognition, and adaptive expertise (Brown, 1987; Cornford, 2000: 5; Flavell, 1976, 1979; Georghiades, 2004: 367; Jindal-Snape & Holmes, 2009: 21; Lin et al., 2005: 245; Pintrich, 2002: 219; Rogers, 2001: 37; Schraw & Dennison, 1994: 461; Veenman et al., 2006: 4). Metacognition as conceptualised and defined in literature—as well as metacognition in relation to the concepts of cognition, self-regulation, self-regulated learning (SRL), meta-memory, and affective states—is discussed over the following sections.

### **2.2.2 Cognition**

A brief discussion of cognition, higher-order thinking skills, and cognitive strategies will contribute to conceptualising metacognition. Literature is not clear on the nature of the interrelationship between cognition and metacognition. It is described as an interchangeable or circular process (Veenman et al., 2006: 6) and it is therefore

difficult to assess whether cognition or metacognition is taking place at a given moment.

An early definition by Flavell (1976: 232) indicates the relationship between cognition and metacognition. Metacognition is viewed as the monitoring and regulating of cognition with an objective in mind (see Section 2.2.1). Later views of the interaction between cognition and metacognition commonly regard metacognition—awareness of and reflection upon one’s own thoughts, feelings, and actions—as higher-order cognition.

According to Weinert (1987, cited in Georghiades, 2004: 371):

On the surface, it seems easy to distinguish between cognition and metacognition. Metacognitions are second-order cognitions: thoughts about thoughts, knowledge about knowledge, or reflections about actions. However, problems arise when one attempts to apply this general definition to specific instances. These problems concern whether metacognitive knowledge must be utilised, whether it must be conscious and verbalisable, and whether it must be generalised across situations.

Veenman et al. (2006: 5) support this view of a higher-order component factor overseeing and regulating the cognitive system whilst simultaneously being part of it. Cognition could therefore be regarded as the combination of lower and higher-order thinking skills and strategies. Lower-order thinking skills involve more routine procedures such as learning mnemonic or algorithmic procedures like solving a quadratic equation in Mathematics. Meanwhile, higher-order thinking skills are more complex cognitive skills such as problem-solving, decision-making, conceptualising, critical and creative thinking, and metacognitive skills like planning, monitoring, and evaluating.

These higher-order thinking skills enable a learner to explain, generalise, synthesise, hypothesise, interpret, and construct new knowledge (Anderson & Krathwohl, 2001: 44; Killen, 2010: 23). Bloom’s Revised Taxonomy refers to metacognitive knowledge as a dimension of higher-order thinking (Anderson & Krathwohl, 2001: 44; Killen, 2010: 23). Meanwhile, higher-order thinking—for

example, problem-solving skills and critical thinking skills—is required for successful learning and solving problems in Mathematics (DBE, 2011a: 8–9).

Challenging tasks engage learners in complex higher-order thinking (Killen, 2010: 23). In Mathematics, higher-order thinking is elicited particularly by novel, complex problem-solving tasks. Being able to solve problems is key in mathematical proficiency (see Section 2.3.4.2) and therefore key in mathematical achievement. The CAPS document for Grade 10–12 Mathematics states that Grade 12 learners are required to do 15% problem-solving tasks and 30% complex tasks in their final assessment (DBE, 2011a: 53), which means 45% of the tasks require higher-order processes. Learners should understand the *how*, *when*, and *why* of problem solving which equips them to use knowledge in lifelong learning and problem solving (DBE, 2011a: 8). This implies metacognitive knowledge as reflecting on thinking (see Section 2.2.4.1) and through this understanding of when and why to use appropriate problem-solving strategies, metacognitive skill is demonstrated (Hartman, 2001a: 33; see Section 2.2.4.3). Therefore, metacognition as higher-order thinking plays a key role in solving these complex and non-routine tasks, since metacognition is elicited by novel and challenging problem-solving situations (Meijer et al., 2006: 232; Sperling et al., 2004: 120).

Furthermore, cognition encompasses different cognitive strategies employed in information processing such as problem solving or text studying (Weinstein & Mayer, 1986, cited in Dignath & Büttner, 2008: 232–233). These cognitive learning strategies are processed at a cognitive level (see Section 2.2.4.3). Flavell (1979: 907) refers to actions (or strategies) as the cognitions or behaviours utilised to reach a goal or cognitive enterprise.

Lower-order thinking skills do not involve these cognitive activities and strategies. In Mathematics, lower-order thinking involves learning rules, definitions, algorithms, procedures, and formulae (see Section 2.3.4.2.1). Metacognitive and cognitive activities have a bearing on each other in a circular process (Veenman et al., 2006: 8; see Section 2.3.4.4). Consequently, they are not easy to distinguish from each other and metacognition often must be inferred from cognitive activities (Veenman et al., 2006: 6), as it is not verbalised or has become automated within the individual. Strategies, therefore, could be cognitive or metacognitive and



often could be inferred from and distinguished by the intent of the strategy. Flavell (1979: 909) states that cognitive strategies are elicited to make cognitive progress, whereas metacognitive strategies monitor cognitive progress. It follows that the mindful use of metacognition precedes effective use of cognitive learning strategies.

In summary, it can be said that employment of metacognition and cognition is an interchangeable process (see Sections 2.2.4.2; 2.2.5). Moreover, there is an interplay between cognitive and metacognitive strategies (see Section 2.2.4.3).

### **2.2.3 Definitions of metacognition**

There are various definitions of metacognition (Sperling et al., 2004: 118). Flavell (1976: 232) states:

Metacognition refers to one's knowledge concerning one's own cognitive processes or anything related to them ... Metacognition refers, among other things, to the active monitoring and consequent regulation and orchestration of those processes in relation to cognitive objects or data on which they bear, usually in the service of some concrete goal or objective.

Intuitive conceptual understandings of metacognition as “thinking about thinking” and “knowing about knowing” derive from the original definition of metacognition by Flavell (1979). Flavell (1979: 906) operationalised metacognition into four key categories: metacognitive knowledge, metacognitive experience, metacognitive goals, and metacognitive activation of strategies (Dinsmore et al., 2008: 393). He further separated metacognitive knowledge into person, task, and strategy variables (see Section 2.2.4.1). A framework suggested by Brown (1987) and employed in later studies (Larkin, 2009: 150; Schraw & Dennison, 1994: 460; Sperling et al., 2004: 118) proposes two components: *Knowledge of cognition* and *Regulation of cognition*.

*Knowledge of cognition* typically relates to what we know about our own cognition, or cognition generally. It comprises of three components of metacognitive awareness: *Declarative knowledge*, *Procedural knowledge*, and *Conditional knowledge*. *Regulation of cognition* includes five components: *Planning*, *Information*

*management, Monitoring, Debugging, and Evaluation* (Larkin, 2009: 150; Schraw & Dennison, 1994: 461; Schraw & Moshman, 1995: 352; Sperling et al., 2004: 118). *Knowledge of cognition* and *Regulation of cognition* are related and each makes a unique contribution to learning and problem solving (Schraw & Dennison, 1994: 466; Sperling, Howard, Miller & Murphy, 2002: 119; see Section 3.4.3.1.1).

Veenman et al. (2006: 5) confirm that in the literature the most regular distinction in metacognition is made between metacognitive knowledge and metacognitive skills. They describe metacognitive knowledge as the interaction between person, task, and strategy variables (see Section 2.2.4.1), whereas metacognitive skills refer to the regulating of one's problem solving and learning activities (see Section 2.2.4.3). Furthermore, Pintrich (2002: 219) mentions that this basic distinction between *Knowledge of cognition* (metacognitive knowledge) and *Monitoring and Regulation of cognition* (metacognitive control) parallels the two dimensions of the Revised Taxonomy Table of Bloom (see Section 2.2.2).

In a more recent definition, Akyol and Garrison (2011: 184) emphasise metacognition as a reflective construct. They state that metacognition is the awareness, willingness, and readiness to reflect on the thinking and learning process (Akyol & Garrison, 2011: 184). Their definition corresponds with the basic definition of metacognition as *Knowledge of cognition* and *Regulation of cognition*, but it further separates the latter into two distinct reflective and activity-based dimensions: *Monitoring of cognition* and *Regulation of cognition*. *Knowledge of cognition* is a general aspect of metacognition which is observable any time, whereas *Monitoring of cognition* and *Regulation of cognition* are viewed as more reflective and activity-based (Akyol & Garrison, 2011: 184).

Therefore, *Knowledge of cognition* is considered as pre-task reflection upon one's cognitive processes, i.e. an awareness of the self (knowledge about cognition, cognitive strategies, and affective states such as motivation and self-efficacy) and awareness of skills. *Monitoring of cognition* is reflection-on-action, i.e. awareness of thinking and learning processes and the willingness to reflect upon them. *Regulation of cognition* is the reflection-in-action dimension of the learning process: it is the use of strategies to reach goals (Akyol & Garrison, 2011: 184; see Section 2.2.4.3).

Consensus amongst researchers is that metacognition is generally defined by its component parts and consists of both *Knowledge of cognition* and *Regulation of cognition*. *Knowledge of cognition* (metacognitive knowledge) consists of three subcomponents—*Declarative*, *Procedural*, and *Conditional* metacognitive knowledge—whereas *Regulation of cognition* (metacognitive skills) consists of five components: *Planning*, *Information management*, *Monitoring*, *Debugging*, and *Evaluation* skills. This definition by Schraw and Dennison (1994), based on Flavell’s initial definition, will be used in the study. Similar is the definition of Akyol and Garrison (2011); however, it elaborates on metacognition as a reflective construct and, consequently, also has bearing on the teacher’s professional practice (see Section 2.3.5).

## **2.2.4 The four categories of metacognition**

In this section, the four categories of metacognition—namely metacognitive knowledge, metacognitive experiences, metacognitive strategies and skills, and metacognitive goals—are discussed.

### **2.2.4.1 Metacognitive knowledge**

Flavell (1979: 907) describes metacognitive knowledge as knowledge concerning cognitive matters. Many researchers expound this view of Flavell as the knowledge, awareness, and deeper understanding of one’s own cognitive processes (Desoete, 2009: 436), as well as the knowledge of having a large repertoire of cognitive strategies and knowing how, when, and why to use them (Zohar & David, 2008: 62). It also includes pre-task reflection on cognitive processes and affective states, such as motivation and self-efficacy (Akyol & Garrison, 2011: 185).

Flavell (1979) refers to three categories: knowledge of person, task, and strategy. The person category comprises knowledge and beliefs about others and oneself as cognitive beings (Flavell, 1979: 907). An example is believing you are better in arithmetic than word problems, or that you are better than your friends in Mathematics. It also involves metacognitive knowledge and insight about your own understanding or memory. For example, it is the awareness that what you

understand at this present moment, you might not be able to remember later, or that what you do not remember now, you might be able to recall later.

The task variable consists of knowledge concerning available information during a cognitive undertaking and how this information affects and constrains the way one deals with it, or knowledge about task demands and goals (Flavell, 1979: 907). For example, it may be knowledge that the available information in a problem statement is insufficient to find a solution to the problem. Metacognitive knowledge is therefore an understanding about how one could manage this lack of knowledge to reach the goal of finding a solution, and how likely one is to be successful. It also could be the knowledge that recalling the properties of a task is more demanding than a recognition task (Pintrich, 2002: 221) and therefore would require different strategies in approaching the task.

Finally, the strategy category refers to metacognitive knowledge about cognitive strategies or procedures for achieving various goals (Flavell, 1979: 907). It involves knowledge about general strategies for learning, thinking, and problem solving, not their actual use. For example, it would refer to the knowledge about various learning strategies that exist to memorise material (Pintrich, 2002: 220), not the application of a specific strategy. It is the knowledge of the mnemonic 'BODMAS' for the order of operations in a problem, but it is not about using the mnemonic.

Flavell's person, task, and strategy categories of metacognitive knowledge correspond with the view of metacognitive knowledge consisting of three subscales: *Declarative*, *Procedural*, and *Conditional knowledge*, as per later definitions (Larkin, 2009: 150; Schraw & Dennison, 1994: 460; Schraw & Moshman, 1995: 353; Sperling et al., 2004: 118). Flavell's operational definition includes *Declarative knowledge* (knowing that a person is good at Mathematics or knowing that a task is difficult) and *Procedural knowledge* (knowledge about how to use a strategy). Although *Conditional knowledge* is not explicitly mentioned by Flavell (1979), it is implied as knowing about when and why to use strategies.

*Declarative knowledge* of oneself or self-knowledge, therefore, is awareness of one's own individual knowledge, knowledge of one's own strengths and weaknesses, and knowledge about various cognitive strategies. It also encompasses knowledge about

one's beliefs about one's motivation and self-efficacy relating to tasks—i.e. the interest and value a task holds for the individual—and goals for completing a task. It is, ultimately, awareness of one's own knowledge base and expertise (Jacobs & Paris, 1987: 259; Schraw & Dennison, 1994: 460; Schraw & Moshman, 1995: 352; Pintrich, 2002: 220; Zohar & David, 2008: 61).

*Procedural knowledge* is the awareness of knowledge about *how* to use these strategies and procedural skills effectively (Jacobs & Paris, 1987: 259; Schraw & Dennison, 1994: 460; Schraw & Moshman, 1995: 352; Zohar & David, 2008: 61), not the actual use of the strategies. For example, it includes knowing how to plan cognition by setting sub-goals; how to monitor cognition by asking oneself questions about a text, or checking an answer to a Mathematics problem; and how to regulate cognition by rereading a problem or text that one does not understand, or correcting a calculation error in a Mathematics problem (Pintrich, 2002: 220).

*Conditional knowledge* is being aware of conditions which impact learning, such as why certain strategies are effective, when they are applicable, and when they are most appropriate (Jacobs & Paris, 1987: 259). It is knowledge about when and why strategies are used and the effectiveness of those strategies (Jacobs & Paris, 1987: 259; Schraw & Dennison, 1994: 460; Schraw & Moshman, 1995: 352; Zohar & David, 2008: 61).

In short, *Declarative knowledge* relates to knowing *about* things: it is an awareness that knowledge and strategies exist. *Procedural knowledge* relates to knowing *how* to do things. Finally, *Conditional knowledge* relates to the *why* and *when* aspects of cognition (Brown, 1987; Jacobs & Paris, 1987: 259; Schraw & Dennison, 1994: 460; Thomas & Mee, 2005: 222; Zohar & David, 2008: 61).

Metacognitive knowledge, therefore, is an awareness of one's own cognition and a reflection on the conditions (namely how, when, and why) to implement knowledge, processes, and strategies. The subsequent actual implementation is regarded as metacognitive skillfulness (see Section 2.2.4.3) to achieve a specific goal or outcome (see Section 2.2.4.4).

#### ***2.2.4.2 Metacognitive experiences***

Flavell (1979: 908) distinguished metacognitive knowledge from metacognitive experiences, noting that “Metacognitive experiences are any conscious cognitive or affective experiences that accompany or pertain to any intellectual enterprise”. Thomas and Mee (2005: 222), meanwhile, confirm that metacognitive experiences are “conscious experiences associated with cognitive undertakings/actions and metacognitive knowledge”.

As per an illustration by Flavell, during learning or problem solving you may suddenly feel that you are not understanding what you are reading, or sense that the problem you have just begun to solve will be simple. These feelings of difficulty or confidence are subjective experiences prompting awareness of one’s own thoughts and feelings (Flavell, 1979: 908). He also emphasised that metacognitive experiences are more likely to be elicited by situations requiring highly conscious thinking, and viewed metacognition as conscious and purposeful to achieve a specific goal (Editorial, 2012: 245). Meijer et al. (2006: 232) confirm that task difficulty should promote metacognition. For example, a situation which prompts metacognition is solving novel problems. A novel task may elicit feelings of inadequacy or difficulty, or alternatively elicit interest to attempt the task. Problem-solving scenarios thus elicit a conscious awareness of thoughts and feelings and the subsequent regulation of these thoughts and feelings. Consequently, this demands mindful planning and careful evaluation of the steps implemented to make progress and solve problems successfully. Efklides (2008, cited in Desoete, 2009: 436) confirms that metacognitive experiences make individuals aware of their own cognition and prompt control processes with the goal to self-regulate.

Flavell (1979) introduced the term meta-memory as the ability of an individual to monitor and manage the content of their memory (Editorial, 2012: 246) with metacognitive experiences being the impetus for monitoring. These metacognitive experiences are subjective experiences: feelings of knowing, feelings of familiarity, feelings (or judgement) of learning, feelings of confidence or doubt about the correctness of the solution provided, and feelings of difficulty (Schwartz & Efklides, 2012: 146). Meta-memory experiences influence decisions about how to study and how to improve learning efficiency.

Meta-memory is initially seen as a subcomponent of metacognitive knowledge, as knowledge about memory capabilities and strategies affects memory processes. Later research by Nelson and Narens (1990, cited in Veenman, 2011b: 199) on metacognitive experiences, especially feelings of learning and judgment of learning, emphasises the monitoring or evaluating of memory contents. Veenman (2011b: 199) further states that by including monitoring, meta-memory research has crossed the border between the two subcomponents of metacognition, namely metacognitive knowledge and metacognitive skills. Schwartz and Efklides (2012: 145) add that meta-memory informs learning and studying on a micro-level as it prompts learners to focus more on certain items, and also on a macro-level as metacognitive knowledge guides the monitoring and regulation of learning.

Therefore, when studying metacognition from a memory and learning perspective, Nelson and Narens' (1990, 1994, cited in Editorial, 2012: 246) meta-memory framework explains metacognition as a flow of information from the object level (cognition) to the meta-level (cognition about cognition), which in turn informs the object to attain the goal state. Having metacognitive thoughts and feelings about cognition is described as monitoring, whereas the response to the environment or adapting of behaviour is referred to as control, or regulation. In reading a text, for example, the meta-level evaluates cognition. It could involve realising that a given text is not understood and acting on it to improve cognition by choosing a strategy, such as rereading a key section of the text to reach the goal state of comprehension.

Regarding memory and learning efficiency, monitoring refers to the gathering and interpreting of information and the regulation thereof by any action, or intention to act, due to the monitoring (van Overschelde, 2008, cited in Editorial, 2012: 246). This corresponds with Akyol and Garrison's (2011: 184) view of *Monitoring of cognition* as reflection-on-action and *Regulation of cognition* as the reflection-in-action dimension of the learning process (see Section 2.2.3).

Ultimately, metacognitive experiences are elicited by the task and the progress one is likely to make towards reaching a goal. Metacognition is therefore associated with affective experiences like motivation, interest, and self-efficacy during learning and problem solving, and is the willingness and awareness to

reflect on these experiences (Akyol & Garrison, 2011: 184; Rogers, 2001: 42; Schraw et al., 2006: 112). Learners who evaluate their capability to learn or perform a task and expect to do well (self-efficacy) are more inclined to demonstrate a range of cognitive and self-regulatory strategies (Wolters & Pintrich, 1998: 43–44). Metacognitive experiences can therefore affect the three other categories, namely metacognitive knowledge, metacognitive goals, and metacognitive strategies and skills (Flavell, 1979: 908), as well as their associated affected states (see Section 2.2.5).

### ***2.2.4.3 Strategies and skills***

The effective use of (metacognitive) strategies and skills is discussed in this section. Strategies are actions or concrete activities designed to reach a learning goal more efficiently, and Flavell (1979: 907) holds that, depending on the goal, strategies can be either cognitive or metacognitive in nature. Strategies could also refer to heuristic strategies as used in problem solving (see Sections 2.3.4.2.2; 2.3.4.4) and to metacognitive strategies, cognitive strategies, or motivational strategies.

Cognitive strategies refer to the individual's cognitive processes during the encoding of information whilst performing problem solving or studying text, while metacognitive strategies refer to their knowledge of and control over their own cognitive processes (Weinstein & Mayer, 1986, cited in Dignath & Büttner, 2008: 233). Motivation strategies play an important part by motivating the learner (Kohen & Kramarski, 2012: 2; McCombs & Marzano, 1990, cited in Dignath & Büttner, 2008: 233).

This observation implies that metacognitive strategies monitor cognitive strategies; in other words, metacognition has a regulating or monitoring component overseeing cognition. For example, when planning which heuristics or steps to use during a problem-solving activity (metacognitive strategy), ordering the heuristics or steps in a certain sequence (cognitive strategy) is required. The intent of cognitive strategies is to make cognitive progress, while the intent of metacognitive strategies is to monitor the cognitive progress (see Section 2.2.2).



Skills refer to higher-order thinking skills, such as critical thinking skills, problem-solving skills, and metacognitive skills. Skillfulness refers to an ability which can be demonstrated. Metacognitive skills can be viewed as the voluntary control that individuals exert over their own cognitive processes (Desoete, 2009: 436) and the purposeful application of cognitive behaviours at a particular moment (Van Der Stel & Veenman, 2014: 117), as per *Regulation of cognition*. Task analysis or information management, planning, monitoring, and evaluation are manifestations of metacognitive skillfulness (Van Der Stel & Veenman, 2014: 117).

In summary, metacognitive skill is the ability to employ metacognitive strategies to achieve a metacognitive or cognitive goal. To illustrate metacognitive skillfulness in Mathematics, consider the following: a metacognitive strategy is operational when, for example, one evaluates an answer for reasonableness and consequently adjusts the answer (e.g. by changing the units) or decides to approach the problem from another perspective by using another heuristic (e.g. working backwards). In other words, through employing the metacognitive strategy *Evaluation* to reach the goal of solving the problem successfully, the ability to evaluate is demonstrated, which marks the demonstration of a metacognitive skill, *Evaluation*.

Although metacognition develops over an individual's lifetime (Kohen & Kramarski, 2012: 1; Veenman et al., 2006: 7) and most adults possess some level of metacognitive awareness (Kohen & Kramarski, 2012: 2), metacognitive skillfulness can be enhanced through training and consequently influence achievement (Akyol & Garrison, 2011: 189; Sperling et al., 2012: 1, 5; see Sections 2.3.3; 2.4.3). Children develop metacognitive awareness between the ages of 8 to 10 (Veenman et al., 2006: 8) and these skills may differ in quality and quantity (Van Der Stel et al., 2010: 221). Metacognitive strategies and skills are general or domain-specific in nature (Schellings et al., 2013: 986; Veenman et al., 2006: 8) and become more general following the age of 12 (Veenman, 2011b: 202). Therefore, both general and domain-specific skills could be enhanced.

As strategies become automated, a metacognitive skill—in the form of an ability to employ metacognitive strategies to achieve a metacognitive or cognitive goal—is developed. Monitoring skills develop before regulating skills. Moreover, young

children's abilities to transform monitoring into effective regulation seem to depend heavily on task characteristics and specific instruction (Editorial, 2012: 249).

#### **2.2.4.3.1 Cognitive strategies**

Cognitive strategies improve learning, memorising, and understanding. They include rehearsing, elaborating, organising, problem-solving strategies or heuristics, and critical thinking strategies (Anderson & Krathwohl, 2001: 56; Bjork et al., 2013: 423; Mayer & Wittrock, 1996, cited in Dignath & Büttner, 2008: 232–233; Schraw et al., 2006: 114–115; Sperling et al., 2004: 120).

General cognitive strategies are rehearsal strategies and refer to copying, underlining, or repeating information to oneself. Elaboration strategies involve paraphrasing or summarising and synthesising information. Organisational strategies entail constructing flow diagrams (Bjork et al., 2013: 423; Mayer & Wittrock, 1996, cited in Dignath & Büttner, 2008: 232–233; Sperling et al., 2004: 120). Cognitive strategies that support a deeper understanding include critical thinking and problem-solving strategies. Problem-solving strategies, or heuristics, are more complex and help learners to obtain a deeper level of understanding, for example through generating questions or constructing graphs, diagrams, and tables (see Section 2.3.4.2.2), whereas critical thinking strategies entail questioning what one reads when studying (Bjork et al., 2013: 423; Schraw et al., 2006: 113).

#### **2.2.4.3.2 Metacognitive strategies**

Metacognitive strategies include *Planning*, *Information management*, *Monitoring*, *Debugging*, and *Evaluation* (Schraw & Dennison, 1994: 460).

*Planning* refers to selecting metacognitive strategies and allocating resources appropriately (Schraw, 1998: 115). It includes setting goals, activating relevant prior knowledge, and allocating learning resources through practices such as time management (Kohen & Kramarski, 2012: 1; Schraw & Dennison, 1994: 474; Schraw et al., 2006: 114).

*Information management* refers to the strategy sequences used in processing information more efficiently (e.g. organising, elaborating, summarising, selective

focusing) (Schraw & Dennison, 1994: 475–476). It therefore refers to managing information with optimal efficiency (Kohen & Kramarski, 2012: 1). Learners could enhance their understanding and memory efficiency, and consequently their learning, by using techniques such as drawing mind-maps, tables, or diagrams.

*Monitoring* is assessing one’s learning or strategy use (Schraw & Dennison, 1994: 475). By reflecting on one’s own thought processes (Akyol & Garrison, 2011: 184; Carlson & Bloom, 2005: 54), current knowledge and skill levels are monitored (Kohen & Kramarski, 2012: 1). *Monitoring* strategies can therefore help to check one’s own comprehension and performance, through self-testing to name one example. Self-testing is a powerful metacognitive strategy to check comprehension. It is used by learners when they ask themselves questions while reading a text, or when they check their answers to Mathematics problems (Pintrich, 2002: 220). Learners might assess material to see how well they could remember it in the exam, or may decide to focus on and allocate more time to difficult sections, or might even learn easy sections first to ensure they know enough to pass the examination (Bjork, et al., 2013: 427). Learners’ judgments of learning and their consequent studying could be accurate or inaccurate (see Section 2.2.6). In Mathematics, problem-solvers reflect on how effective their strategies and plans are. They often ask questions like “Will this take me as far as I want to go?” and “How efficient will this approach be?” (Carlson & Bloom, 2005: 54).

*Debugging* refers to strategies used for correcting comprehension and performance errors (Schraw & Dennison, 1994: 475). It entails learners remediating their work by improving their understanding and correcting their mistakes (Kohen & Kramarski, 2012: 1). In Mathematics, for example, learners might reread something that they do not understand, or correct their calculation errors (Schraw et al., 2006: 114).

*Evaluating* entails analysing the effectiveness of a performance or strategy following a learning experience (Schraw & Dennison, 1994: 475). This means judging the progress and effectiveness of one’s learning—and, consequently, re-evaluating one’s goals and conclusions in response (Schraw, 1998: 115)—as well as reflecting upon performance against required standards and goals (Kohen & Kramarski, 2012: 1). Examples of this include re-evaluating goals and

conclusions and revising one's predictions (Schraw, 1998: 115). In Mathematics, for example, learners might evaluate the appropriateness of a solution and decide to debug or find an alternate solution (see Section 2.3.4.4.4).

#### ***2.2.4.4 Metacognitive goals***

Flavell (1979: 907) refers to goals (or tasks) as “the objectives of a cognitive enterprise”. Artz and Armour-Thomas (1998: 9) characterise goals as intellectual, social, and emotional outcomes that are expected to result from learning and teaching experiences. Metacognitive experiences can trigger cognitive or metacognitive strategies to attain cognitive or metacognitive goals (Flavell, 1979: 908). For instance, if a learner experiences disappointment (metacognitive experience) due to difficulty solving a problem, they can reread and hence rephrase the problem statement using their own words to understand it more clearly.

Alternatively, by asking questions such as “What am I doing?”, “Why am I doing it?”, and “How does it help me?” (Schoenfeld, 1992: 63), the individual becomes aware of their own thinking: a metacognitive strategy aimed at the metacognitive goal of assessing and monitoring one's own understanding, thoughts, and actions. This provokes another metacognitive experience, namely that the goal of problem solving is difficult to achieve. Consequently, this leads to knowledge about one's own ability, which is metacognitive knowledge (see Section 2.2.4.1).

As a result, cognitive strategies are elicited to make cognitive progress, whilst metacognitive strategies monitor cognitive progress (Flavell, 1979: 909). Meaningful and productive learning is facilitated by conscious awareness of and orientation towards a goal (De Corte, 1995: 71; 2000: 254). An orientation for mastery rather than performance encourages the employment of metacognition (Masui & De Corte, 2005: 366). Learners choose and set goals and consequently monitor and evaluate their progress to achieve those goals. Goal-setting, therefore, facilitates learning and problem solving.

## **2.2.5 Associations between related concepts: metacognition, meta-memory, self-regulation, self-regulated learning, and affective states**

As stated previously, in education, self-regulation—like metacognition—is generally defined by its component parts. Like metacognition, there is a lack of clarity about the relationships among self-regulatory constructs (Sperling et al., 2004: 118, 120). Azevedo (2009: 87) describes metacognitive monitoring, metacognitive regulation, and reflection as self-regulatory processes in learning. De Corte (1995: 259) initially referred to metacognition and self-regulation interchangeably, but in later literature referred to meta-knowledge as metacognitive and meta-volitional knowledge, and self-regulatory skills as metacognitive skills or volitional skills (De Corte, 2007: 21–22), hence recognising the affective components interacting with metacognition and self-regulation. Efklides (2012: 294) asserts that the self-regulation of learning is facilitated by both metacognitive monitoring and affect (see Section 2.3.4.2).

Self-regulation, therefore, is a related construct to metacognition. Self-regulation or self-regulated learning (SRL) refers to an active process in which learners develop and set their own learning goals, then actively monitor and control their own cognitive processes, behaviour, and affect in pursuit of these goals within contextual constraints (Pintrich, 2000, cited in Azevedo, 2009: 87; Schunk, 2005b: 85; Schunk & Zimmerman, 2008: vii). Self-regulated learning, therefore, refers to the learner's ability to understand and control their learning environments (Schraw et al., 2006: 116). Self-regulation and self-regulated learning are often used as interchangeable concepts, with self-regulated learning commonly used in educational terms. Metacognition is considered a subordinate or superordinate component of self-regulation.

The component parts of self-regulated learning are described by Schraw et al. (2006: 112) as cognition, metacognition, and motivation. Sperling et al. (2004: 118–120) refer to three well-accepted constructs associated with self-regulated learning—namely metacognition, academic strategy use, and motivation—and consider academic strategy use as processed on a cognitive level. Both these views corroborate, therefore, to position metacognition as a subordinate

component of self-regulation, with motivation and cognitive strategy use as complementary components. Each of these components plays a necessary and interdependent role in self-regulation (Schraw et al., 2006: 112). Self-regulated learning can thus be viewed as metacognitive monitoring and control (*Regulation of cognition*) of cognitive, situational, and motivational factors. These explanations illustrate that these constructs are interrelated and intertwined and can be inferred from each other during the observation or verbalisation of actions.

When metacognition is viewed as the superordinate construct to self-regulation, it is generally defined in terms of its two related component parts: *Knowledge of cognition* and *Regulation of cognition* (see Section 2.2.3). *Regulation of cognition* is generally viewed as the self-regulatory component of metacognition. It is further referred to as metacognitive skills, and seen as the self-monitoring and self-regulation of metacognitive knowledge and cognition (see Section 2.2.4.3).

From a cognitive point of view, the metacognitive theory of self-regulated learning makes a link between monitoring and control (Dunlosky & Rawson, 2012: 271). In educational terms, control is commonly referred to as regulation (Editorial, 2012: 246). Nelson and Narens' (1990, 1994, cited in Veenman, 2011b: 205) meta-memory model explains the link between monitoring and control when learners regulate their memory and learning; when task errors or poor performance are detected, a bottom-up process is activated where control processes (from the object level) inform monitoring (on the meta-level) (Dunlosky & Rawson, 2012: 272; Veenman, 2011b: 205). That is, information flows from the object level (e.g. when reading a problem statement) to the meta-level (which diagnoses comprehension problems and therefore monitors comprehension) and informs the object level on how to respond or adapt behaviour (regulate behaviour by rereading to improve comprehension). However, Veenman (2011b: 205), in extension of this model, proposes a top-down process which is triggered not by prior control processes—i.e. the detection of anomalies in the task—but as a set of conscious self-instructions to regulate task-performance.

## **2.2.6 Research on metacognition and meta-memory**

Research on metacognition has been undertaken in different domains since the 1970s when Flavell (1976, 1979) introduced the terms metacognition and meta-memory (Editorial, 2012: 246), with meta-memory as the first component of metacognition (Veenman, 2011b: 199). Subsequently, two main threads of research developed. The first was in developmental psychology—prompted by the work of Flavell (1979) and Brown (1987)—which has been of main concern for educational practitioners and researchers. The second was in cognitive psychology and investigated basic processes in metacognition—for example, meta-memory—as prompted by the work of Nelson and Narens (1990, 1994, cited in Koriat, 2012: 298; see Section 2.2.4.2).

However, in recent years research into metacognition and meta-memory has developed shared interests (Koriat, 2012: 298). By including monitoring activities, meta-memory research has crossed the border between metacognitive knowledge and metacognitive skills, i.e. metacognition (Veenman, 2011b: 199). Insight from both research groups could benefit learners by improving memory and learning efficiency, as well as the relationship between monitoring, regulation, and performance (Koriat, 2012: 298). Consequently, this section provides a brief overview of meta-memory research to explain the association between meta-memory and metacognition, and thus its influence on learning efficiency, which could indicate to teachers how to direct their learners and help them to better direct their studies autonomously.

Research addresses the interplay between monitoring and regulation, the implications of monitoring accuracy for effective learning and memory, and the relationship between monitoring, regulation, and performance (Koriat, 2012: 296). There is plenty of research on the ability of learners to monitor their performance. However, less is known about the ability of learners to implement the output of their monitoring towards the regulation of their cognition and behaviour and how this benefits performance (Koriat, 2012: 297), as well as how to develop self-monitoring and self-regulation using instructional intervention in the classroom (Editorial, 2012: 245).

Research on improving memory and learning efficiency indicates that metacognitive experiences (Judgment of Learning, Feeling of Knowledge) influence consequent learning (see Section 2.2.4.2). Current findings on metacognitive judgments indicate that metacognitive experiences dictate which items are felt to be difficult or easy (Efklides, 2011, cited in Schwartz & Efklides, 2012: 150). This could result in learners adapting their consequent learning strategies by restudying difficult items, monitoring themselves via self-testing, or focusing on easier items to ensure success.

In addition, metacognitive judgements by learners assessing their own learning could be accurate or inaccurate due to factors like overconfidence or stability bias (Schwartz & Efklides, 2012: 147). Overconfidence leads to underachievement in learning performance (Koriat, 2012: 296). According to research by Dunlosky and Rawson (2012: 277–278) in which undergraduate students studied key term definitions, it was found that overconfident students overestimated their own learning and retaining of content and hence stopped studying prematurely. Monitoring also impacts learning performance: when learners are overconfident, their test performance is weaker (Editorial, 2012: 247–249). Although monitoring is not always accurate and does not always translate to regulation (Koriat, 2012: 298), enhancing awareness about accurate monitoring could improve learners' consequent regulation (Dunlosky & Rawson, 2012: 279). For example, learners should be made aware that extrinsic cues in the learning environment (e.g. underlined key words when revising) may not necessarily transfer to remembering those words under test conditions.

Furthermore, faulty mental models and beliefs could influence learning (Bjork et al., 2013: 417). McCabe (2011, cited in Schwartz & Efklides, 2012: 145) investigated undergraduate students' metacognitive knowledge and found that these students more often endorsed unproductive and inefficient learning strategies. Providing metacognitive knowledge about factors and strategies that affect memory and learning may improve learning efficiency, for instance through self-testing and spacing (Schwartz & Efklides, 2012: 145).

Moreover, research on the contribution of metacognitive monitoring and regulation to effective learning and performance (Pieschl, Stahl, Murray & Bromme, 2012, cited in Koriat, 2012: 298) shows that these affect performance (see also Section 2.3.3). Monitoring is the prerequisite for regulation of learning (Editorial, 2012: 247) and



consequently better performance. However, monitoring is not necessarily adequate or conducive to regulation. In research with children, it was found that monitoring does not always translate into regulation and efficient behavioural strategies (Koriat, 2012: 298). Schraw et al. (2006: 114) state that even skilled adult learners might monitor themselves poorly under certain conditions.

Finally, comments about meta-memory research on learning support the dissociation of monitoring from regulation (Koriat, 2012: 297). More research is invited on the conditions and relationships between monitoring, regulation, and performance (Koriat, 2012: 297). Furthermore, self-regulation of learning is complex and not fully understood, and there are many intertwining factors influencing real-world learning (Efklides, 2012: 294; Schwartz & Efklides, 2012: 150).

### **2.2.7 Summary of the construct metacognition**

Metacognition as a multi-faceted construct has been defined in different ways (see Sections 2.2.1; 2.2.3). Flavell's (1979) original definition operationalises metacognition into four categories: metacognitive knowledge, metacognitive experiences, strategies and skills, and goals (see Section 2.2.4). It serves as a point of reference and is built upon by later researchers.

Schraw and Dennison's (1994) definition is widely accepted and conceptualises metacognition into two subcomponents: *Knowledge of cognition* and *Regulation of cognition*. The former refers to *Declarative, Procedural, and Conditional knowledge*, while the latter refers to the self-regulatory aspects of metacognition where the metacognitive skills of *Planning, Monitoring, Evaluation, Debugging, and Information management* inform the learning or problem-solving process. *Knowledge of cognition* (metacognitive knowledge) and *Regulation of cognition* (metacognitive skills) are related and each makes a unique contribution to learning and problem solving (Schraw & Dennison, 1994: 460).

Akyol and Garrison (2011: 184) expound metacognition as a reflective construct. Their definition corroborates with the two-component view of metacognition, but further separates *Regulation of cognition* into reflective and activity-based dimensions, namely *Monitoring of cognition* and *Regulation of cognition* (see

Section 2.2.3). On a meta-memory level, monitoring and control (or regulation) are linked as a two-way flow of information (Dunlosky & Rawson, 2012: 271).

Metacognition, therefore, operates interdependently in relation to other constructs like cognition, self-regulation, self-regulated learning, meta-memory, and affective states, to inform teaching, learning, and problem solving. The role of metacognition in teaching and learning is discussed in Section 2.3.

## 2.3 METACOGNITION IN TEACHING AND LEARNING

The application of metacognition and its implications for use in teaching and learning form the foundation of the study. Perspectives and research on learning and teaching are discussed in Sections 2.3.1 and 2.3.2. The role of metacognition in mathematical achievement and other factors pertaining to learning and problem solving in Mathematics are discussed in Sections 2.3.3 and 2.3.4. The role of the teacher in facilitating metacognition and Mathematics learning is discussed in Section 2.3.5. Finally, teaching for metacognition will be discussed across Sections 2.4.1 to 2.4.4. These sections contribute to exploring secondary research question 2: “What is the role of metacognitive awareness in Mathematics teaching and learning?”

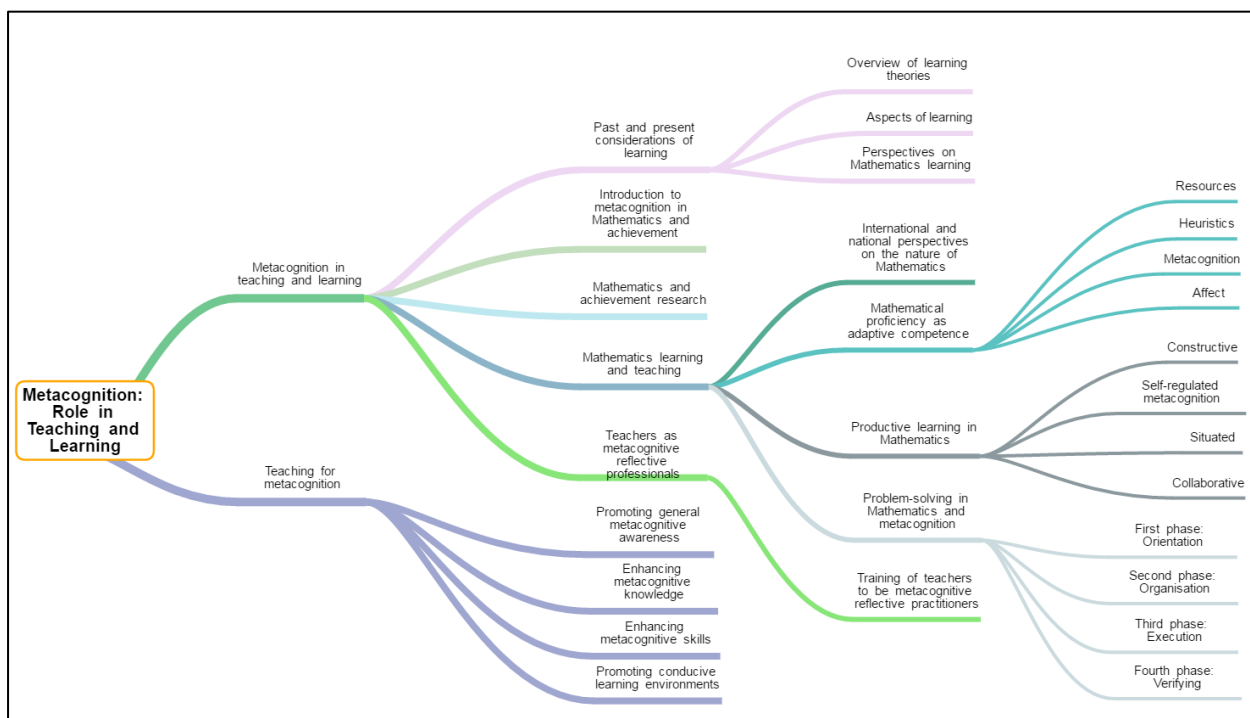


Figure 2.2: Metacognition in teaching and learning (see also Appendix 8)

### **2.3.1 Past and present considerations of learning**

Learning is complex and has evolved over the last century. A short overview of learning theories is provided in Section 2.3.1.1, followed by aspects of learning in Section 2.3.1.2 and perspectives on Mathematics learning in Section 2.3.1.3. Metacognition is a key aspect of these views.

#### ***2.3.1.1 An overview of learning theories***

Major research theories that contribute to an understanding of learning include behaviourism, cognitive psychology, constructivism, socio-constructivism, and more recently neurocognitive research. A brief overview of these concepts follows, as it is beyond the scope of this discussion to provide a comprehensive survey.

The **behaviourist** view of learning focuses on observable behaviour, where the setting of objectives and specific tasks facilitate the acquisition of knowledge and skills (Stavredes, 2011: 35). Behaviourism does not consider the functions of the mind. It contrasts with **cognitive psychology**, which focuses on information processing in understanding memory processes and the employment of cognitive strategies to effectively process information to ensure knowledge acquisition for long-term memory (Stavredes, 2011: 37, 49).

**Constructivism** describes learning as constructing knowledge by activating prior knowledge, beliefs, and attitudes (Stavredes, 2011: 41). Among constructivist theories, Jean Piaget (1936) and Leo Vygotsky (1978) have heavily informed current views of learning and instruction. According to Piaget's (1936) constructivist theory, developmental maturity must exist for learners to benefit from learning experiences, and the roles of context or language are not considered, whilst in Vygotsky's (1978) **socio-constructivist** theory, knowledge is constructed through language and groups in cultural contexts (McInerney & McInerney, 2010: 57–58). Vygotsky (1978) also emphasised the need for guided discovery and scaffolding in the zone of proximal development (ZPD), where learners learn problem solving by adult guidance or from more capable peers to improve upon functions in the process of maturing (Hartman, 2001b: 158; McInerney & McInerney, 2010: 57–58). In addition, he emphasised writing as a method to make internal thoughts visible and to generate

meaning (Pugalee, 2004: 28). Regarding instruction—and in contrast with behaviourism—constructivism promotes active learning in authentic contexts, with the teacher as facilitator to model and scaffold critical thinking and reflective strategies (Stavredes, 2011: 39–41).

**Neurocognitive** research on how the brain functions and cognitive processes is the latest development to shed light on productive learning. Neuroscience is beginning to provide evidence for learning principles that have emerged from laboratory research, indicating that learning alters the physical structure—and hence the functioning—of the brain (Bransford et al., 2000: 4).

### ***2.3.1.2 Aspects of learning***

Educational research on productive learning pertains to higher-order thinking (e.g. problem solving, metacognition, and critical thinking) and affect (e.g. motivation, self-efficacy, affective self-regulation, and attributions) besides knowledge and skills (Hartman, 2001a: 34). Key principles and factors influencing learning are proposed in a report (*How People Learn: Brain, Mind, Experience, and School*) by the National Research Council (Bransford et al., 2000). Productive learning entails learning with understanding—namely making sense, being actively involved, and setting goals—and is influenced by the prior knowledge, skills, beliefs, and concepts of the individual (Bransford et al., 2000: 8–12). Learning is further influenced by the learning and teaching context in which it transpires, where cooperation in problem solving and argumentation enhances cognitive development (Bransford et al., 2000: 25).

Metacognition supports active learning by learners when they take charge of their learning through setting goals and monitor their own progression to accomplish these goals (Bransford et al., 2000: 12, 18). Furthermore, as learning involves the acquisition of new information but also the updating and revision of previous information, metacognition and meta-memory are researched in this study (see Section 2.2.6). Koriat (2012: 297) holds that metacognitive monitoring and perhaps metacognitive regulation are involved in the revision of information. Hartman (2001a: 34) states that effective learning is active, meaningful, retained over time, and transferable to different domains and contexts. Transfer requires

adaptive competence, which is metacognition (Lin et al., 2005: 244–245). Well-chosen, specific learning experiences facilitated by metacognition enable people to transfer and use their learning, knowledge, and skills in novel scenarios (Bransford et al., 2000: 4).

Learning may, however, be inadequate or ineffective due to a poor knowledge base (Garner, 1990, cited in Hartman, 2001a: 34). Alternatively, learners might have the knowledge and skills to perform complex tasks, but do not successfully apply these. Reasons proposed for failing to implement—or inadequately implementing—knowledge and skills include the following: learners may not recognise that the scenario demands certain knowledge or skills, i.e. they may possess *Declarative (that)* or *Procedural knowledge (how)* but lack *Conditional knowledge* of *why* and *when* to apply and transfer it (Hartman, 1993, cited in Hartman, 2001a: 34; see Section 2.2.4.1); learners may possess inadequate knowledge of the link between the task and the strategies or general strategies (Hartman, 2001a: 34); learners may display poor cognitive monitoring (Garner, 1990, cited in Hartman, 2001a: 34); monitoring does not necessarily translate into regulation and therefore change of feeling, thinking, and doing (Koriat, 2012: 298); and finally affective components such as a lack of motivation or confidence (Hartman, 2001a: 34). Misconceptions and false beliefs about learning (Bjork et al., 2013: 417) might also impact implementation. Furthermore, the learning and teaching environment may not facilitate the effortful application of strategies (Hartman, 2001a: 34) and routine tasks may not elicit higher-order thinking. Moreover, the teacher plays a pivotal role in facilitating productive learning and problem solving (see Section 2.3.5).

In summary, productive learning is active and meaningful. It entails building knowledge (constructivism), considering the prior knowledge, skills, and beliefs of individuals and groups (the individual and situated nature of knowledge and learning), regulating learning by setting goals and monitoring progress individually (metacognition and self-regulation), and working collaboratively to make sense of scenarios (socio-constructive) (see Section 2.3.4.3).

### ***2.3.1.3 Perspectives on Mathematics learning***

Regarding Mathematics learning, the focus has shifted from behaviourism—which prevailed in the first half of the twenty-first century—towards cognitive/constructivist and situated/socio-cultural perspectives, as influenced by developmental psychology and advances in information processing and cognitive psychology research (Edwards, Esmonde & Wagner, 2011: 57). Consequently, in Mathematics Education research, the focus shifted from the tension between acquiring routine computational skills (behaviourists) and the importance of meaningful Mathematics learning towards developing skills and sense-making in Mathematics (constructivist/cognitive). It was followed by the importance of language and the socio-cultural context of Mathematics (situated learning and situated cognition) (Edwards et al., 2011: 57).

Cognitive/constructivist research studies Mathematics learning through information processing and constructivist approaches, which inform many Mathematics curricula (Schoenfeld, 2006, cited in Edwards et al., 2011: 60). Information processing/cognitive research has illuminated important aspects of mathematical cognition, among which are knowledge acquisition and memory, higher-order thinking skills, complex problem-solving, sense-making, and metacognitive processes (Edwards et al., 2011: 57, 59). This research on information processing, however, focuses on cognitive factors only and does not include the impact of motivation and affect on effective learning (Pintrich, 2004: 306). In this respect, different aspects were identified that influence mathematical learning and achievement (Edwards et al., 2011: 59), including affective components (see Section 2.3.4.2.4). Acquiring the adaptive competence to apply knowledge and skills meaningfully and flexibly to open-ended, real-life problems—as opposed to routine expertise, i.e. solving problems routinely and without understanding—is a key goal (De Corte, 2010: 45). This is referred to as mathematical proficiency (see Section 2.3.4.2).

Constructivism—with its emphasis on active learning, inquiry, and modelling activities—is considered a key cognitive paradigm in Mathematics learning during the last decades (Edwards et al., 2011: 57). However, a later socio-cultural perspective challenged the information-processing constructivist cognitive view,

describing it as information that is absorbed, processed, and stored in the mind whilst independent of social context and social interaction. In situated cognition and situated learning, mathematical knowledge is situated in context and culture, and refers to competence in life settings (De Corte, 2010: 40; Edwards et al., 2011: 59). Furthermore, research into the role played by discourse and language has provided insight into Mathematics learning, describing it as interactive, participatory, and situated/socio-cultural (Edwards et al., 2011: 65). De Corte (2010: 40) confirms a socio-constructivist view as a generally dominant view of learning, which considers the constructivist tenet of active and meaningful knowledge acquisition individually and in a social context. For Mathematics learning particularly, the CSSC approach describes four major characteristics of productive learning: Constructive, Self-regulated, Situated, and Collaborative (see Section 2.3.4.3).

In the South African educational context, the importance of metacognition in Mathematics teaching and learning—particularly the teacher’s own metacognition in facilitating Mathematics learning—is emphasised (Van Der Walt, 2014: 8–9; Van Der Walt & Maree, 2007: 237–238). The knowledge about cognitive and metacognitive strategies (*Knowledge of cognition*) for learning and remembering, and skills to adapt to fluid learning demands (*Regulation of cognition*), are the metacognitive aspects that facilitate learning (Hartman, 2001a: 34). Metacognition, as adaptive competence, is an important goal in lifelong learning and particularly in learning Mathematics (Cornford, 2000: 6; Schoenfeld, 2006, cited in De Corte, 2010: 44–45).

The challenge remains for Mathematics Education researchers to study mathematical learning in its natural setting, namely the classroom, and to obtain a better comprehension of the deeper processes inherent in Mathematics knowledge itself (Edwards et al., 2011: 60–61).

### **2.3.2 An introduction to metacognition in Mathematics and achievement**

A learner’s metacognitive awareness of their own thinking processes enhances productive learning and improves achievement (Schellings et al., 2013: 980; White et al., 2009: 178). Learner performance is enhanced by metacognitive

knowledge (Pintrich, 2002: 225) and metacognitive skills (Van Der Stel & Veenman, 2010: 224; Van Der Stel et al., 2010: 228); therefore, the enhancement of metacognition in learners could improve academic achievement (Larkin, 2009: 149). This indicates that learners' metacognitive knowledge (see Section 2.2.4.1) and metacognitive skills (see Section 2.2.4.3) could and should be enhanced.

Metacognition, as the awareness of one's thoughts, helps learners to analyse and regulate their thinking and action by choosing applicable strategies (see Section 2.2.4.3). According to Schraw (2001: 13–14), metacognition plays a vital role in achieving learning success, enabling learners to manage and identify weaknesses in their cognition and learning and address these through the development of new cognitive skills. High-achieving learners have a higher level of metacognitive awareness than low-achieving learners (Hartman, 2001a: 35). Moreover, metacognition as adaptive competence (Bransford et al., 2000: 18; De Corte, 2007: 22; 2010: 46; Hartman, 2001a: 35; Lin et al., 2005: 245) is one of four attributes contributing to successful problem solving (see Section 2.3.4.2) and productive learning (see Section 2.3.4.3).

As problem solving is central to mathematical proficiency, the goal of Mathematics education is to develop competent problem-solvers (Schoenfeld, 1992: 338). Furthermore, De Corte (2007: 22) asserts that adaptive competence is the ultimate goal of Mathematics education. In addition, the international and national policy documents see mathematical competence in problem solving at the core of Mathematics (see Section 2.3.4). Problem solving as a higher-order thinking skill is assessed in the Grade 12 Mathematics examination, as per the CAPS specification that 30% of questions must be on complex procedures and 15% on problem solving (DBE, 2011a: 53). The underachievement in Mathematics—as demonstrated in the national Grade 12 results, the TIMSS results, and the Grade 9 ANA results—points to the necessity of developing efficient problem-solving skills in learners, and implicitly teachers, in South Africa (see Section 1.2). The importance of acquiring and teaching higher-order thinking skills—i.e. metacognitive, critical, and problem-solving skills (see Section 2.2.2)—is substantiated by educational researchers like Akyol and Garrison (2011: 184), Anderson and Krathwohl (2001: 57),



Desoete (2007: 718), Pugalee (2001: 237), Schoenfeld (2007: 60), Van Der Stel et al. (2010: 219), and Van Der Walt and Maree (2007: 237).

The scholarly literature, South African policy documents, and poor results obtained by South African learners in Mathematics point to the need for teaching problem solving. Moreover, Schoenfeld (2007: 64) asserts that, with respect to learning Mathematics, “students are not likely to learn what they are not taught”. Therefore, learners must be taught specifically how to solve problems successfully. Problem solving in Mathematics is complex and questions about metacognition’s effect on problem solving do come to the fore. These include “How can problem solving be enhanced through metacognition?” and “Can metacognition be taught?” It has been established, however, that metacognition has an influence on mathematical performance, and that both metacognitive knowledge and metacognitive skills can be enhanced (see Sections 2.4.2; 2.4.3).

### **2.3.3 Mathematics and achievement research**

Internationally, research has established a link between metacognition and success in learning on the one hand and metacognition and problem solving on the other (Pugalee, 2001: 237; Van Der Walt et al., 2008: 231; see Section 2.3.2). In the South African context, research on metacognition in Mathematics teaching and learning is not well-published (Van Der Walt & Maree, 2007: 238; Van Der Walt et al., 2008: 231). Research about metacognition in Mathematics pertains primarily to the metacognition of school learners. Metacognition employed in the learning of Mathematics during the senior schooling phase was investigated by Van Der Walt et al. (2008: 205). They found that learners’ metacognitive strategies were inadequate for facilitating critical thinking (Van Der Walt et al., 2008: 205, 229) and problem solving. A possible explanation is that learners have difficulty understanding certain concepts in Mathematics, or do not have the skill to monitor and evaluate such problems (Van Der Walt et al., 2008: 229).

To the best of my knowledge as a researcher, no previous study in a South African context has investigated the metacognitive awareness of Mathematics didactics students (fourth-year pre-service teachers) in a problem-solving situation. However, a noteworthy study by Van Der Walt (2014: 1–22) investigating the level of

metacognitive awareness and self-directedness in the Mathematics learning of prospective second and third-year intermediate and senior phase Mathematics teachers was found.

This research study by Van Der Walt (2014: 1–22) found that the metacognitive awareness and self-directedness of prospective Mathematics teachers were at a high level. In contrast, no significant correlation was found between metacognitive awareness and learning achievement, or between self-directedness and learning achievement. This contrasted with the hypothesis and previous research indicating clear relationships between these concepts. A possible reason for this lack of correlation is offered by Van Der Walt (2014: 1–22), who suggests that pre-service Mathematics teachers, when assessing their own learning behaviour, might under- or over-estimate their level of metacognitive awareness or self-directedness. These pre-service teachers might have knowledge of effective metacognitive learning behaviour, yet fail to implement this knowledge in Mathematics learning or problem solving (see also Sections 2.3.5; 5.4.1).

In the case of primary learners, Desoete (2007: 705–718) conducted a longitudinal study to research the mathematical learning and metacognitive skills of 32 learners in Grades 3 and 4. She found that metacognitive skills are teachable and confirmed the findings of previous studies stating that the explicit teaching of metacognitive skills helps to enhance these skills (Desoete, 2007: 718), especially when instructed over an extended period (Veenman, 2011b: 209–210). Desoete (2007: 718) recommends that instruction should explicitly focus on (meta)cognitive weaknesses and strengths.

A study using 53 German eight-grade learners was conducted by Perels, Dignath and Schmitz (2009: 17–31) with the purpose of enhancing the learners' mathematical self-regulation and problem-solving skills. The intervention aimed at improving self-regulation in a regular Mathematics class and took place over nine sessions. A self-regulation questionnaire was administered afterwards. This research found that self-regulation competencies and mathematical achievement could be supported by means of self-regulation interventions (Perels et al., 2009: 27). A limitation of the study, however, is that the items on the questionnaire only measured whether a learner reported to have applied strategies, while no conclusion

can be made as to whether the learners actually regulated their behaviour (Perels et al., 2009: 27). Another limitation is that the intervention occurred only for a short period of time, and greater effects on learning behaviour and mathematical achievement could be expected from extended training.

While Perels, Gürtler and Schmitz (2005, cited in Schunk, 2005a: 175) report that benefits in self-regulation and problem-solving skills could result from relatively short interventions, a study by Veenman et al. (2006: 9) contends that metacognitive and self-regulation training are most effective over a prolonged period. Another important implication of the study by Perels et al. (2005, cited in Schunk, 2005a: 175) is the finding that self-regulation strategies were employed most effectively when combined with mathematical problem-solving strategy training that is embedded in the content domain (Schunk, 2005a: 175). This observation corresponds with literature on strategy training (Veenman et al., 2006: 9; see Section 2.4.3).

In a subsequent study, Veenman and Van Cleef (2007, cited in Veenman, 2011b: 208–209) reported a similar limitation to that found by Perels et al. (2009) regarding learners' abilities to regulate their performance. Thirty secondary school learners were given two questionnaires—the MSLQ (Motivated Strategies for Learning) and ILS (Inventory Learning Style)—ahead of mathematical problem solving. No significant relationship was found on measures for metacognitive skillfulness (Veenman & Van Cleef, 2007, cited in Veenman, 2011b: 208–209). As the MSLQ and ILS are arguably better suited to text-studying tasks, a retrospective questionnaire was administered immediately after problem solving. Also in this instance, no significant relationship was found (Veenman & Van Cleef, 2007, cited in Veenman, 2011b: 208–209). A possible explanation provided is that learners will not necessarily do as they say, nor will they necessarily recall what they do with a great degree of accuracy (Veenman, 2011b: 209).

Much research in metacognition has been done on the instruction and training of metacognitive skills to improve performance. Zohar and David (2008: 59–82) researched metacognitive knowledge. Meta-strategic knowledge, as part of metacognitive knowledge, is the awareness of the type of thinking strategies that are used in specific cases, i.e. *Conditional* and *Procedural knowledge* (*how, why, and when* to use strategies) (Veenman, 2011b: 197; see Section 2.2.4.1). Meta-strategic

knowledge refers specifically to the skill of transferring knowledge to novel problems (Zohar & David, 2008: 77). In this study, Zohar and David (2008: 59–82) investigated the explicit teaching of meta-strategic knowledge in authentic classroom situations using 119 Grade 8 learners across six classes. The findings show developments in learners' strategic and meta-strategic knowledge after explicit teaching. This corresponds with Nelson and Narens' (1990, 1994, cited in Editorial, 2012: 246) two-level model of the object (cognitive strategic) and meta-level (meta-strategic), where information flows between the two levels through monitoring and control processes (see Section 2.2.4.2).

In a study for post-secondary students, Mevarech and Fridkin (2006: 85–97) conducted a 50-hour intervention study for 81 pre-college students in Mathematics who did not pass the university entry examination in Mathematics. They employed IMPROVE—an acronym for teaching steps and a metacognitive training method—using four types of metacognitive self-questions during mathematical problem solving: comprehension, strategy, connecting, and reflection (Mevarech & Fridkin, 2006: 87). Results showed that the pre-college students significantly outperformed the control group on metacognitive knowledge (domain-specific and general) and *Regulation of cognition*. In support, Schoenfeld (2007: 65) and Veenman et al. (2006: 10) confirm the use of metacognitive questions to facilitate metacognitive knowledge and metacognitive skills training (see Sections 2.3.4.2.2; 2.4.2; 2.4.3).

In summary, these studies show that there is a correlation between metacognition and Mathematics achievement (see Sections 2.3.2; 2.3.3). Therefore, metacognitive knowledge can be enhanced (Zohar & David, 2008: 77; see Section 2.4.2) and metacognitive skills can be taught (Desoete, 2007: 717; Masui & De Corte, 2005: 366; see Section 2.4.3). Consequently, there are implications for the instruction of metacognitive knowledge and metacognitive skills. Instruction should be embedded in the content domain (De Corte, 1995: 39; Veenman et al., 2006: 9; see Section 2.4.3). Furthermore, although the study by Perels et al. (2005, cited in Schunk, 2005a: 175) showed that short interventions might improve self-regulation strategies and metacognitive skills, the consensus is that metacognitive skilfulness is developed over a prolonged period (Nietfeld & Schraw, 2002: 141;

Veenman et al., 2006: 9) and with mindful effortful application and explicit instruction (Desoete, 2007: 718). Moreover, monitoring does not necessarily imply nor always result in regulation (Koriat, 2012: 297–298). Bjork et al. (2013: 417, 424, 427) hold that college students may incorrectly assess and manage their learning due to faulty mental models (see Section 2.3.5).

Research on metacognitive instruction shows that such instruction enhances metacognition and learning among learners (Veenman et al., 2006: 9) and that reflection and monitoring strategies were particularly effective in improving the performance of elementary students making poor academic progress (Pressley, Gaskins, Solic & Collins, 2006: 301–302; Veenman, 2011b: 212). White et al. (2009: 178) assert that metacognition makes learning more effective and learners should therefore be educated to develop various types of metacognitive knowledge and skills. Teachers should provide ample opportunity for learners to attempt real-life problems, as they elicit metacognition (Desoete, 2007: 717).

### **2.3.4 Mathematics learning and teaching**

International and national documents for school Mathematics deliver very similar perspectives on the nature of Mathematics, describing what it is and what mathematical proficiency entails. Prominent educational researchers Bransford (2000), De Corte (2010), and Schoenfeld (2007) provide corroborative supportive views on mathematical proficiency. Perspectives on the nature of Mathematics and Mathematics teaching and learning are discussed from Sections 2.3.4.1 to 2.3.4.4. Metacognition figures as a key aspect of these views.

#### ***2.3.4.1 National and international perspectives on the nature of Mathematics***

The National Research Council (NRC), a well-noted US-based international publication and educational body, published an important report on the learning of Mathematics (National Research Council [NRC], 2001). In this report, five kinds of mathematical competencies were indicated as necessary to be proficient in Mathematics (Hiebert, Morris & Glass, 2003: 203–204; NRC, 2001: 5). These five strands should be integrated and developed concurrently in the teaching and

learning context, as they are interdependent in obtaining proficiency in Mathematics learning and problem solving.

These strands are *conceptual understanding*—a learner’s understanding of the concepts, operations, and relations in Mathematics; *procedural fluency*—a learner’s skill at performing mathematical processes correctly, effectively, and with flexibility; *strategic competence*—a learner’s skill at formulating, representing, and developing solutions for Mathematics problems; *adaptive reasoning*—a learner’s ability to think logically, reflect, and explain and justify choices; and *productive disposition*—a learner’s ability to view Mathematics as a meaningful and valuable presence in their lives, coupled with self-efficacy and persistence (Hiebert et al., 2003: 203–204; NRC, 2001: 5).

This view of competence in Mathematics is also reflected in the South African policy document on Mathematics. The CAPS states the following about the nature of Mathematics:

Mathematics is the study of quantity, structure, space and change. Mathematicians seek out patterns, formulate new conjectures, and establish axiomatic systems by rigorous deduction from appropriately chosen axioms and definitions. Mathematics is a distinctly human activity practiced by all cultures, for thousands of years. Mathematical problem solving enables us to understand the world (physical, social and economic) around us, and, most of all, to teach us to think creatively (DBE, 2010b: 7).

Consequently, these two documents exhibit similar views on the nature of Mathematics and on being an effective mathematician. Those five strands of mathematical proficiency from an international perspective provided by the NRC—which parallel with the definition of Mathematics from CAPS, South Africa’s national Mathematics document—will be discussed next.

In the NRC, the first strand, *conceptual understanding*, indicates an understanding about concepts, operations, and relationships, whereas the CAPS refers to knowledge about concepts of quantity, structure, and space, as well as axioms and definitions (Hiebert et al., 2003: 204; DBE, 2010b: 7). Knowledge precedes understanding in Bloom’s Taxonomy of learning, and it can be surmised that both

documents presuppose the establishment of these lower-order skills before higher-order thinking skills and problem solving are encountered.

Second, *Procedural knowledge*—as competence to employ mathematical procedures skilfully—is reflected in the description of mathematical skilfulness in the CAPS as formulating conjectures and axioms (DBE, 2010b: 7; Hiebert et al., 2003: 204). Both *conceptual understanding* as knowledge and *procedural fluency* as skills are reflected in the CAPS. The CAPS and NRC both emphasise the importance of knowledge and skills, which serve as the foundation for problem solving as a higher-order thinking skill (see Section 2.3.2).

Third, the NRC mentions *strategic competence* as posing and solving problems, while the CAPS perceives problem solving as enabling us to better comprehend the world. In this respect, the CAPS also places emphasis on the utility value of Mathematics, which is referred to in the last strand of the NRC, namely *productive disposition*, as learners viewing Mathematics as useful and applicable to their lives (Hiebert et al., 2003: 204; DBE, 2010b: 7). Problem solving is deemed central to Mathematics and mathematical proficiency in both documents.

Fourth, both documents refer to the development of higher-order adaptive skills. The NRC's view of *adaptive reasoning* as thinking logically, reflecting, explaining, and justifying corresponds with the view of the CAPS that problem solving develops creative thinking (Hiebert et al., 2003: 204; DBE, 2010b: 7). Moreover, this points to considering and reflecting on different ways to solve problems, or finding alternative solutions and applying them. Metacognition, as adaptive competence, is the ability to reflect, elaborate, and explain one's thinking and actions.

One difference is the CAPS' overt reference to Mathematics as a distinctly human activity, which is not explicitly mentioned in the NRC. It is, however, implicitly referred to in learners' disposition of self-efficacy and tenacity, as well as in seeing Mathematics as useful and meaningful. Both documents refer to the use of language in formulating conjectures and problems, with the NRC identifying reflection, explanation, and justification as typically human actions. There is, therefore, a strong recognition of the socio-constructive nature of Mathematics in both documents.

Mathematics is considered useful and applicable to lives (utility value) when it can be used to solve real-life problems, thus contributing to making meaning of the world. This refers to the socio-constructivist and situated nature of Mathematics (see Section 2.3.1.3). Problem solving is considered pivotal in mathematical proficiency (see Section 2.3.4.4). Furthermore, Mathematics builds thinking and reasoning skills, comprehension, and creativity (DBE, 2010b: 7).

In summary, the dual nature of Mathematics is emphasised. Besides the abstract nature of Mathematics, which involves thinking and reasoning skills and processes in the mind (information processing), it is also a human activity grounded in reality (De Corte, 2007: 25) and situated in authentic contexts where knowledge is constructed individually and collaboratively (see Section 2.3.1.3) and where affective aspects such as motivation, persistence, and self-efficacy play a role (see Section 2.3.4.2.4).

In the following section, mathematical proficiency as it relates to competent performance in Mathematics problem solving is discussed.

### ***2.3.4.2 Mathematical proficiency***

Researchers in Mathematics Education concur that becoming proficient in Mathematics is achieved by developing and acquiring a mathematical disposition (De Corte, 2007: 20; NRC, 2001: 5; Schoenfeld, 2007: 60). Mathematical proficiency is facilitated by four attributes of adaptive competence during learning and problem solving—*resources, heuristics, metacognition, and affect*—as well as four aspects of productive learning, which are the *constructive, situated, and self-regulatory* aspects of metacognition as well as *collaboration*.

Mathematical proficiency involves more than knowing facts and procedures. A key focus in Mathematics is not only what a learner knows, but also what they can do with that knowledge and, more importantly, what they are disposed to do mathematically (Schoenfeld, 2007: 71). Schoenfeld (2007: 59) asserts that a mathematical disposition—a significant facet of mathematical proficiency—is, in the first instance, the ability to employ mathematical knowledge in appropriate contexts. This is the ability to transfer knowledge and skills to new contexts, which is tested via



novel problem-solving situations. Another important facet of mathematical proficiency is the tenacity to persist with a problem when trying to solve it. Mathematical proficiency, therefore, entails expertise in Mathematics. This competent performance is also adaptive to deal with increasing knowledge demands and problems in real life. De Corte (2007: 21–22) asserts that adaptive competence or expertise is the ultimate goal of Mathematics education.

Competent problem-solvers—those who possess a mathematical disposition, apply knowledge appropriately, and display persistence in adapting knowledge and strategies to new problem-solving scenarios—behave differently from novices (Schoenfeld, 2007: 67). Experts reflect on their performance and adapt it accordingly. They evaluate and monitor their behavioural performance, cognition, affect, and environment. Therefore, a mathematically proficient or competent problem-solver can apply knowledge and skills effectively, take control of thinking processes and feelings, and adapt procedures and find alternatives to ensure progress is made if initial plans are unsuccessful. In short, metacognition is adaptive expertise, and this adaptive competence (De Corte, 2010: 46; Hatano & Inagaki, 1986, cited in Bransford et al., 2000: 18) is a key aspect of mathematical proficiency (De Corte, 1995: 39; 2007: 20; Schoenfeld, 2007: 60; see also Section 2.3.4.2.3).

Unfortunately, knowledge and skills are often not readily accessible in the transfer of learning or solving of a novel problem (De Corte, 2007: 21; Hartman, 2001a: 34). Metacognition as adaptive competence is the ability to transfer these skills and inert knowledge to novel tasks and learning contexts, and is therefore pivotal in problem solving and lifelong learning (Bransford et al., 2000: 18; De Corte, 2010: 46; Lin et al., 2005: 244).

Bransford et al. (2000: 16, 23) assert that competence is developed in an area of inquiry by teaching a basis of factual knowledge—starting from prior knowledge, concepts, and beliefs—to bring conceptual change and organise knowledge to aid memory retrieval and application. They further assert that a metacognitive approach to teaching enables learners in the regulation of their learning by establishing goals, monitoring progress, and employing metacognitive strategies and skills (Bransford et al., 2000: 18). These aspects that facilitate competence—namely a

sound knowledge base; strategies for learning, memory, and transfer; and metacognition, which includes monitoring, goal-setting, and managing information—corroborate with De Corte (2007: 20) and Schoenfeld’s (2007: 60) views on mathematical proficiency as adaptive competence.

Schoenfeld (2007) and De Corte (1995, 2007) proposed similar aspects that facilitate proficiency in Mathematics learning and problem solving. These are: a well-connected knowledge domain and skills base; problem-solving skills and heuristic strategies applied in authentic contexts; affective components such as self-efficacy (i.e. a confident disposition towards Mathematics), beliefs, interest, and motivation; and lastly, metacognition as an adaptive higher-order thinking skill. De Corte (1995: 252–255) refers to resources as a domain-specific knowledge base and uses metacognition and self-regulated learning interchangeably. Schoenfeld (2007: 66) describes affect as beliefs and dispositions. In summary, mathematical proficiency involves resources, heuristics, metacognition, and affect. Importantly, the five strands of mathematical proficiency (see Section 2.3.4.1) endorsed by the NRC (2001: 5) are evident in these four aspects of mathematical proficiency as proposed by Bransford et al. (2000), De Corte (1995, 2007), and Schoenfeld (2007). These are discussed from Sections 2.3.4.2.1 to 2.3.4.2.4.

#### **2.3.4.2.1 Resources**

The first attribute required to build a mathematical disposition is a well-organised and accessible domain-specific knowledge base regarding the facts, symbols, algorithms, concepts, and rules specific to Mathematics (De Corte, 2007: 20; Schoenfeld, 2007: 60). These resources are the formal and informal knowledge about the content domain (Carlson & Bloom, 2005: 48). This knowledge is usually acquired in learning at school and via social and cultural situations (see Section 2.3.1.1).

The first strand of mathematical proficiency referred to in the NRC’s standard document is *conceptual understanding*: learners’ understanding of the concepts, operations, and relations in Mathematics (NRC, 2001: 5). Possessing a rich and well-connected knowledge base is critically important, as misconceptions result from a poor knowledge base. Teaching should develop this *conceptual understanding*.

However, knowing Mathematics in producing facts and definitions (*conceptual understanding*) and the execution of procedures (*procedural understanding*) is not sufficient. Learners should be capable of using mathematical knowledge to solve problems (see Section 2.3.4.4).

Schoenfeld (2007: 71) states that it is not what you know that matters, but what you can do with that knowledge. In studies of problem solving, it has been noted that although learners report to possess the knowledge, it does not appear to be readily accessible for use, and inadequate planning, monitoring, and evaluation skills hinder the probability of success (see Section 2.3.3), particularly in novel problem-solving situations (see Section 2.3.4.2; De Corte, 2007: 21; Hartman, 2001a: 34). Metacognition as an adaptive competence facilitates the ability to access the necessary knowledge and apply it successfully (see Sections 2.3.4.2; 2.3.4.2.3).

#### **2.3.4.2.2 Heuristics**

Heuristic methods are systematic cognitive search strategies for analysing problems and significantly increase the likelihood of finding the correct solution (De Corte, 2007: 20; see Section 2.2.4.3.1). Heuristics entail *strategic competence*: the ability of learners “to formulate, represent, and solve Mathematics problems” (NRC, 2001: 5; see Sections 2.2.4.3; 2.3.4.1). These problem-solving strategies could include, for example, making a sketch to represent the problem, restating the problem, working backwards from the problem statement, or finding an easier, related problem. An effective method is the employment of metacognitive self-reflective questions, such as “What am I doing?”, “Why am I doing it?”, and “What can I do differently?” (De Corte, 2007: 20; Schoenfeld, 2007: 64).

Mathematical proficiency also requires *procedural fluency*, which is skillfulness in performing mathematical procedures with efficiency, precision, and flexibility (NRC, 2001: 5; see Section 2.3.4.1). Mathematical proficiency, therefore, is competence in solving problems and using resources effectively, by adapting methods and applying strategies successfully until the problem is solved. These problem-solving skills can be acquired (Schoenfeld, 2007: 66) as the teacher scaffolds and coaches problem-solving heuristics and metacognitive strategies, guiding learners to reflect upon their progress during the problem-solving process

and adapt their methods accordingly. Problem-solving expertise is thus the adaptive expertise to solve a problem successfully (De Corte, 2010: 46).

#### **2.3.4.2.3 Metacognition**

De Corte (2007: 20) separated metacognition into two subcomponents, namely metacognitive knowledge and self-regulatory metacognitive skills, to elaborate on their importance in acquiring a mathematical disposition or adaptive expertise.

**Meta-knowledge** is knowledge about cognitive functioning, motivations, and emotions (De Corte, 2007: 20). This means knowing one's (or others') strengths and weaknesses, developing cognitive potential through deliberate learning and application of oneself, and being aware of one's motivation and emotions in learning and problem solving, such as interest in a task or fear of failure when facing a complex mathematical problem (De Corte, 2007: 20; see Sections 2.2.4.1; 2.2.4.2).

**Metacognitive skills** are defined as the self-regulatory skills regarding cognitive and volitional activities and processes. This includes the *Regulation of cognition* on the one hand (e.g. planning and monitoring one's problem-solving processes) and the skills for regulating one's volitional processes on the other (De Corte, 2007: 20; see Sections 2.2.4.3; 2.2.4.4).

Schoenfeld (2007: 66) emphasises the role of metacognition as “using what you know effectively” in mathematical problem solving. For example, when working under time constraints, learners might possess enough mathematical knowledge, but do not apply what they know as they run out of time. Furthermore, metacognitive reflection—as reflection-in-action (see Section 2.2.3) whilst engaging in a problem—is difficult and takes time and prolonged effort to acquire (see Section 2.3.5). Mathematical proficiency, therefore, involves metacognition as *adaptive reasoning* (NRC, 2001: 5; see Section 2.3.4.1). This is a learner's ability to think logically and explain and justify why certain solutions and strategies are chosen. Importantly, it is the ability to reflect on solutions and strategies and adjust them to reach the goal of solving problems.

#### 2.3.4.2.4 Affect

Affective variables such as beliefs, attitudes, and emotions significantly influence problem-solving performance and learning in Mathematics (Carlson & Bloom, 2005: 48; Schoenfeld, 2007: 71). Positive Mathematics-related beliefs are threefold: first, they include implicitly and explicitly held subjective conceptions about Mathematics education; second, beliefs about the self as a learner of Mathematics; and third, beliefs about the social context of the Mathematics classroom (De Corte, 2007: 20; Schoenfeld, 2007: 68).

Mathematical proficient learners, therefore, have positive and correct beliefs about the learning of Mathematics, the nature of problem solving, and their own abilities to translate these into new tasks and contexts, which essentially is adaptive competence. The NRC (2001: 5) states that an aspect of mathematical proficiency is having a *productive disposition*, which is a learner's inclination to view Mathematics as valuable, applicable, and meaningful in their lives, coupled with self-efficacy and persistence (see Section 2.3.4.1).

However, negative beliefs and erroneous views of Mathematics could influence a learner's initial interest in attempting a task as well as their continued engagement until a problem is solved. These erroneous beliefs could be, for example, that Mathematics does not have to make sense; that Mathematics is a solitary activity; that school Mathematics does not relate to the real world; and that good problem-solvers solve problems in five minutes (De Corte, 1995: 39; Schoenfeld, 1992: 359; 2007: 71). Negative beliefs could therefore influence learners' willingness to look for and debug errors and persist with problem solving. Learners' beliefs could consequently be an obstacle for the implementation of new learning approaches (De Corte, 2010: 56) such as CSSC (see Section 2.3.1.3).

On the other hand, self-efficacy, as the belief in oneself and effective management of one's own responses, supports persistence towards reaching a solution, debugging errors, and adapting new strategies. In support, Carlson and Bloom (2005: 60) affirm that positive beliefs about one's ability to process information, being persistent, and being willing to deal with unsuccessful attempts are all vital in progressing to reach a solution. Schoenfeld (2007: 60) describes this inclination to persevere, believe in

one's own ability, and possess knowledge of one's own resources as a mathematical disposition. However, changing learners' beliefs could pose a major challenge (De Corte, 2010: 56). These beliefs related to productive learning, and the utility and role of metacognition, could hinder the implementation of problem-solving strategies.

Another affective component, motivation, plays a significant part in learning and problem solving. Motivational beliefs relate to goal-orientation (the purpose for doing the task), self-efficacy (as the competence to perform the task), and perceptions about the task (the value placed on the task and the learner's interest level) (Kohen & Kramarski, 2012: 1; Pintrich, 2004: 394). Importantly, initial perceptions and feelings about a problem-solving task (its level of difficulty, relevance, personal interest, outcomes, feelings of boredom or interest, and what counts as success or failure) affect motivation to engage in the task (Boekaerts, 2010: 94–95; Pintrich, 2004: 394). The type of task is therefore pivotal in directing the willingness of learners to engage initially. It is, however, a certain mathematical disposition that keeps learners engaged in a task and helps them to persist during problem solving (Schoenfeld, 2007: 60). Success in a task could lead to motivation, interest, and satisfaction, whereas failure to solve a problem may lead to negative emotions like anxiety.

Clearly, affective responses that occur during the problem-solving process are very complex (Carlson & Bloom, 2005: 49). Pintrich (2004: 394) found that college students attempt to control affect and emotions through coping strategies and self-reflection. One such coping strategy is attribution theory, which is particularly relevant in the learning and problem solving of Mathematics. Novice learners tend to attribute failure to negative teacher behaviour, rather than their own lack of effort (Desoete, 2007: 706).

Finally, the social context can foster positive attitudes towards Mathematics in teachers and their learners (De Corte, 2000: 260; Schoenfeld, 2007: 68). Wrong beliefs, naïve understandings, and misconceptions about Mathematics could be addressed by allowing learners to articulate and reflect collaboratively on strategies that can be used and by providing justifications for strategies (Bjork et al., 2013: 417, 427; De Corte, 2000: 261; Thomas & Mee, 2005: 221). This can raise awareness and stimulate thinking about learners' thought processes and

feelings. As these feelings are shared in a safe space where learners can make mistakes and try their hand at problem solving—whether successful or not—learners develop more confidence to try again.

In summary, in a conducive Mathematics teaching and learning environment, the roles of the teacher and their learners are negotiated and adjusted (De Corte, 2000: 260). The whole class (both teacher and learners) reflect on and discuss possible solutions by evaluating different strategies and judging their suitability, as well as discussing the applicability and extension of knowledge and solutions to subsequent problems. The discussion of what a good problem or solution is—and for some problems, more than one solution is applicable—allows learners to construct knowledge together and actively participate in the problem-solving process (see Sections 2.3; 2.3.1.3). Importantly, learners should be allowed time and opportunity to monitor and evaluate the effectiveness and suitability of their problem solving (see Section 2.4.4). Teachers thus facilitate the active and meaningful knowledge construction of learners and, importantly, facilitate reflection on their thoughts and actions. In this respect, Van Der Walt and Maree (2007: 224) assert that teachers do not readily allow learners time to reflect or make metacognitive instruction explicit, which is an important aspect of successful problem solving in Mathematics.

### ***2.3.4.3 Productive learning in Mathematics***

Productive learning fosters adaptive competence in Mathematics (De Corte, 2007: 23). As learning is complex, there are many views on what productive learning is and which aspects influence learning. Metacognition supports learning as learners set goals and regulate their progress in pursuit of these goals (see Sections 2.3.1.2–2.3.1.3). In Mathematics particularly, De Corte (2007: 23) focused on four aspects of productive learning which are relevant to facilitate the acquisition of adaptive mathematical competence (see Section 2.3.4.2). These four aspects—described as Constructive, Self-regulated, Situated, and Collaborative, abbreviated as CSSC—are discussed over the next four sections.

#### **2.3.4.3.1 Constructive**

The constructive aspect of productive learning relates to the learner making meaning and constructing knowledge about the concept, operation, or problem. It requires the learner to be actively involved in the learning process. De Corte (2007: 23) asserts that a key premise of the constructivist perspective on learning is the mindful and effortful participation of learners in the learning process, which alongside interaction with the environment facilitates and supports the acquisition of knowledge and skills. The learner makes sense of new knowledge by connecting it with existing knowledge.

In Mathematics, prior knowledge regarding mathematical facts, formulae, procedures, and strategies is crucially important to facilitate sense-making and memory. The acquisition of knowledge is therefore also cumulative, as new knowledge is built upon and restructures prior knowledge. In this way, the learner's *conceptual understanding* is enhanced by connecting knowledge about concepts, relationships, and patterns (see Section 2.3.4.2.1).

It is confirmed by Van Grinsven and Tillema (2006: 80) that learning can be considered a construction of knowledge, rather than an assimilation of knowledge requiring learning-to-learn skills. The learner's *adaptive reasoning* is the ability to reflect upon and explain and justify solutions and chosen strategies (see Sections 2.3.4.1–2.3.4.2). Metacognitive awareness of what one does or does not know enables the learner in filling in the knowledge gaps, for example by relearning concepts, formulae, operations, and procedures; by improving skills such as problem solving; and by revisiting strategies.

#### **2.3.4.3.2 Self-regulated metacognition**

In constructive learning, the emphasis is on the process of learning, which implies that it is self-regulated (De Corte, 2007: 24) Self-regulating learners set learning goals, and manage themselves and their learning environment to reach these goals (see Section 2.2.5). These learners engage motivationally and metacognitively, and are behaviourally active in the learning process (Zimmerman, 1994, cited in De Corte, 2007: 24). Metacognitive knowledge about available resources, affective components such as motivation and self-efficacy, and beliefs regarding the learning



of Mathematics and the how, when, and why of strategy use (see Section 2.3.4.1)—as well as metacognitive regulation of cognition, motivation, and actions—facilitates Mathematics learning and problem solving. Therefore, metacognitive skills—as the self-regulatory aspect of metacognition—manage processes of knowledge-building and skill-acquisition (see Section 2.2.4.3).

Furthermore, Van Grinsven and Tillema (2006: 79–80) assert that the aim for learners is to reach a degree of autonomy as self-regulated learners by setting goals, reflecting on their levels of competence, learning from mistakes, and selecting tasks. Self-regulating learning is, therefore, an active process whereby learners reflect on and regulate their learning environment (see Section 2.2.5). Motivation is enhanced by self-regulatory activities like goal-setting and task selection. In addition, goal-orientated learners are more actively involved in learning and display positive feelings, higher interest, and better performance (Van Grinsven & Tillema, 2006: 80). If learners are given a degree of autonomy in their learning by choosing and setting their own goals, then learning is usually more meaningful and self-regulatory. It is metacognitive skills which then monitor and regulate this learning and problem-solving processes (see Section 2.2.4.3).

#### **2.3.4.3.3 Situated**

Learning is situated in the context of the classroom, and in cultural and social environments (De Corte, 2007: 25). Because of the usefulness of Mathematics as well as its abstract nature (see Section 2.3.4.1), opportunities should be given to apply conceptual knowledge to practical problems (De Corte, 2007: 25). Competence in transferring knowledge and skills in real life is an important aspect of learning (De Corte, 2010: 40; Edwards et al., 2011: 59; see Section 2.3.4.2). Moreover, in Mathematics, learning and problem solving in authentic contexts stimulate interest and motivation (see Section 2.3.4.2.4).

Socio-constructivist theories on discourse and language have illuminated the learning of Mathematics as interactive, participatory, and socio-cultural (Edwards et al., 2011: 65; see Section 2.3.1.3). The roles of both teacher and learner are continually reassessed as they contribute to the problem-solving process, by discussing and evaluating answers for 'best' or alternative solutions

(De Corte, 2000: 260–261). Teachers as facilitators should scaffold learning in this way, rather than simply being instructors (see Section 2.3.5). That implies that learning is both individual and co-operative (see Section 2.3.1.1).

#### **2.3.4.3.4 Collaborative**

The collaborative aspect of productive learning refers to the socio-constructive nature of learning (De Corte, 2007: 25, 26). Social constructivism emphasises the important role of language and participative, collaborative learning in mathematics, where problems are discussed, solutions are justified, and mathematical language is built (Edwards et al. (2011: 65, see Section 2.3.1.1). This is opposed to previous beliefs or views of Mathematics as an individual activity only.

According to Van Grinsven and Tillema (2006: 79), cooperative learning has been shown to positively affect self-regulating activities, motivation, self-efficacy, and skills. Interactions between teacher and learner, and learner and learner, promote awareness of thoughts and actions and therefore reflection on problem-solving processes (see Sections 2.3.4.2.3–2.3.4.2.4). The support and scaffolding of the teacher are needed for reaching learner autonomy. Furthermore, self-regulating behaviours are pivotal (Van Grinsven & Tillema, 2006: 79) in order for learners to develop adaptive competence, which will facilitate learning and successful problem solving in new contexts and situations (De Corte, 2007: 23; see Section 2.3.4.2).

However, meaning is also formed individually. Learning is individual as learners differ in learning styles, strategies, motivation, and other related factors. Meaning is thus formed individually and collaboratively with peers, teachers, models, and the environment or social context (see Section 2.3.1.2). A criticism is that discovery learning provides minimal guidance from teachers, meaning misconceptions and wrong beliefs may not be challenged (De Corte, 2010: 53). The teacher, therefore, plays a pivotal role as mentor and coach and should balance guided discovery, instruction, and constructive feedback in an exceptional manner (see Section 2.3.5).

#### ***2.3.4.4 Metacognition and problem solving in Mathematics: An interplay***

Successful problem solving as a higher-order thinking process is a hallmark of mathematical proficiency. Metacognition supports the problem-solving process as an adaptive competence. Solving problems, therefore, is a complex activity involving cognitive actions which need to be managed and coordinated by metacognitive actions. Problem-solving skill in Mathematics involves interaction between cognition and metacognition (Carlson & Bloom, 2005: 47; Garofalo & Lester, 1985: 171–172; Pugalee, 2001: 236; see Sections 2.2.2; 2.3.1.2; 2.3.4.2). This begs the question as to what constitutes a problem and what good problem solving entails?

A Mathematics question is an authentic problem when a routine or well-known approach is not obvious to the problem-solver at the outset (Schoenfeld, 1983, cited in Carlson & Bloom, 2005: 47). A good problem-solver would therefore demonstrate mathematical proficiency, as problem solving is key to competence in Mathematics (see Section 2.3.2).

In his seminal work, *How to Solve It* (1945), George Polya described a Mathematics problem-solving model as a four-phase heuristic process. These phases are: (a) understanding the problem, (b) making a plan, (c) carrying out the plan, and (d) looking back, which includes reviewing results and evaluating solutions. Polya (1945) built upon the work of John Dewey (1910), who described a five-step process: a 'felt' difficulty, clarification of the problem, identification of possible solutions, testing the suggested solutions, and verifying the results.

Garofalo and Lester's (1985: 163–176) cognitive-metacognitive framework identifies four phases underpinning the performance of mathematical tasks: *Orientation*, *Organisation*, *Execution*, and *Verifying*. In addition, they identify metacognition as the bridge linking these four activities (Garofalo & Lester, 1985: 171). Carlson and Bloom (2005) built upon the work of Polya (1945) and Garofalo and Lester (1985), further proposing that these four phases are cyclical. In each of these phases metacognitive behaviours, which are related to problem solving, occur. Moving from one phase to another occurs when a metacognitive decision is the driving force for a

cognitive action (Carlson & Bloom, 2005: 62–69; Garofalo & Lester, 1985: 171; Pugalee, 2001: 239–243).

A competent problem-solver would be envisaged to apply this four-phase cyclical model. Mathematical competence is further informed by having a good knowledge base, knowing and applying strategies effectively (heuristics), displaying a positive belief in learning and problem solving, monitoring the metacognitive problem-solving process, and demonstrating motivation, interest, and self-efficacy (affect) (see Section 2.3.4.2). The four-phase problem-solving model—*Orientation*, *Organisation*, *Execution*, and *Verifying*—is described over the next four sections in relation to cognitive and metacognitive activities.

#### **2.3.4.4.1 First Phase: Orientation**

In the first phase, competent problem-solvers attempt to make sense of the given information. Metacognitive behaviours during this phase comprise of comprehension strategies, analysing information and conditions, assessing the level of difficulty, determining the chance of success, considering methods of representing the information, and using reflective questioning (Carlson & Bloom, 2005: 72; Garofalo & Lester, 1985: 171; Pugalee, 2001: 237).

Examples of reflective metacognitive behaviours during this phase include comprehension strategies like reading and rereading, analysing information and conditions by writing down what is given and what is asked, assessing the familiarity and difficulty of the task by thinking of similar problems or different ways to solve the problem, and recalling mathematical facts and procedures. It also includes the representation of information using heuristics, such as diagrams and tables. Reflective behaviour such as asking questions keeps problem-solvers focused on the task (Carlson & Bloom, 2005: 72; Garofalo & Lester, 1985: 171; Pugalee, 2001: 237).

Besides metacognition, mathematical competence also includes affect, heuristics, and resources (see Section 2.3.4.2). Therefore, in the *Orientation* phase, competent problem-solvers would display affect by showing high interest, curiosity, and a sense of self-efficacy when assessing the difficulty of the problem and their chance of success (Carlson & Bloom, 2005: 68). Furthermore, they tap into their resource

knowledge base of mathematical concepts, facts, and algorithms in trying to make sense of the information and understanding the problem. Heuristic strategies are also considered, such as trying to improve, working backwards, and using diagrams and tables to initially represent and organise information.

Consequently, the planning and allocation of resources—and the skills and strategies for processing information more efficiently (see Section 2.2.4.3)—relate to the metacognitive skills *Planning* and *Information management* respectively, which are present in the first phase of the problem-solving framework. Importantly, *Declarative knowledge*—knowledge of one’s strengths, weaknesses, affect, and intellectual resources, especially prior knowledge—plays a pivotal role in *Orientation* activities (see Section 2.2.4.1).

#### **2.3.4.4.2 Second Phase: Organisation**

During the second phase of the problem-solving framework, particular metacognitive behaviours move learners from initial attempts to make sense of the problem statement towards making a plan about how to proceed with solving the problem (Artz & Armour-Thomas, 1992: 161; Pugalee, 2001: 237). In this phase, metacognitive behaviours can include identifying goals, developing a global plan (a general goal), deciding on an approach to work towards the solution, reflecting on the effectiveness of strategies and plans, organising and summarising data, and considering the available resources (Carlson & Bloom, 2005: 72; Garofalo & Lester, 1985: 171; Pugalee, 2001: 237).

Identifying goals in a hypothetical problem-solving scenario could include, for example, comparing the ratio of surface area to the volume of different objects, making a general plan by finding the volume and surface areas of different objects, working towards the goal of solving the problem by choosing a formula, reflecting on the effectiveness of strategies and plans by posing questions, organising and summarising the data through diagrams, and noting important information. Consequently, in this second phase of the problem-solving model, the selection of skills and strategies for processing information more efficiently—and for assessing learning and strategy use—provides evidence of the metacognitive skills *Information management* and *Monitoring* respectively (see Section 2.2.4.3). Metacognitive

knowledge is vital in this phase to develop a plan to solve the problem: *Procedural knowledge* as knowledge of *how* to use strategies effectively, as well as *Conditional knowledge* of *when* and *why* these strategies could be applied, is evident in this phase (see Section 2.2.4.1).

Therefore, competent problem-solvers will reflect on their conceptual knowledge and the heuristic strategies that they want to access in this phase. Self-questioning—such as “Will this strategy work?” and “Is there another method?”—aids in monitoring the strategies and approaches that they elect to utilise (Carlson & Bloom, 2005: 54). Moreover, not only would competent problem-solvers exhibit a mathematical disposition of applying knowledge correctly, but they would be able to reflect on their feelings to help them progress despite frustration (see Section 2.3.4.2.4). This approach will help them display the tenacity and willingness to continue solving the problem (Schoenfeld, 2007: 59; see Sections 2.3.4.1; 2.3.4.2).

#### **2.3.4.4.3 Third Phase: Execution**

The third phase includes carrying out the plan to reach a solution. It is accomplished by using formulae and making calculations and estimations; by monitoring goals, methods, and strategies; and by carrying out computations and redirecting efforts, as well as reflecting on conceptual knowledge (Carlson & Bloom, 2005: 68, 72; Garofalo & Lester, 1985: 171; Pugalee, 2001: 237).

During this phase of the problem-solving model, metacognitive behaviours include reflection on progress in solving the problem and checking comprehension to consider different or alternative methods to correct conceptual or computational errors. It may also involve checking an initial understanding of the problem again by rereading the problem statement (Carlson & Bloom, 2005: 54). Therefore, the metacognitive skills *Monitoring* and *Debugging* can play a key role in this stage (see Section 2.2.4.3.2). Moreover, the regulation of the affective, heuristic, and resource components of mathematical proficiency is important in the *Execution* phase

#### 2.3.4.4.4 Fourth Phase: Verifying

During the fourth phase, metacognitive checking and decision-making behaviours are employed. Decisions and results are evaluated to check the validity of the solution and the accuracy of the computations. Evaluative strategies could also occur in other phases and the problem-solver could revisit other phases or progress based on self-reflective feedback (Carlson & Bloom, 2005: 69, 73; Garofalo & Lester, 1985: 171; Pugalee, 2001: 242).

Based on reflective feedback regarding the effectiveness of the solution, the problem-solver could display metacognitive reflective behaviours such as rereading the problem statement, debugging calculation errors, finding easier or more elegant ways to solve the problem, or consequently using the solution or method in similar problems in future (Carlson & Bloom, 2005: 45). The metacognitive skills *Evaluation* and *Debugging* are prominent in this phase (see Section 2.2.4.3.2). Depending on the feedback, however, *Planning* and *Information management* skills could be employed again in search of a plausible solution, as the problem-solving process is cyclical.

Therefore, in this final phase, besides the reflective component, the proficient problem-solver will access and reflect on a well-connected knowledge base that will inform his/her assessment of the reasonableness and correctness of the solution, as well as the effectiveness of the heuristics and algorithms employed (Carlson & Bloom, 2005: 70). Competent problem-solvers thus draw upon their *Conceptual* and *Procedural knowledge* to check the reasonableness of their answers. Moreover, the awareness and effective management of affective components may support the problem-solver to persevere (Carlson & Bloom, 2005: 70; Schoenfeld, 2007: 60).

In summary, the four phases of the problem-solving process have been discussed with their constant interplay of cognitive and metacognitive behaviours, as well as the four components of mathematical proficiency. Furthermore, this model (Carlson & Bloom, 2005; Garofalo & Lester, 1985; Pugalee, 2001; 2004: 29) provides a useful context for analysing learners' written comments whilst they solve mathematical problems (see Sections 4.4.2; 4.4.3).

### 2.3.5 Teachers as metacognitive reflective professionals

Learners and teachers are faced with numerous decisions, new knowledge, and novel problems daily (see Sections 1.1; 1.2; 2.1). To adapt to these novel scenarios, learners need adaptive skills to help them learn how to learn (see Section 2.3.1.2) and solve problems (see Section 2.3.4.4) and to acquire these learning-to-learn skills for lifelong learning (Bjork et al., 2013: 418; Cornford, 2000: 9).

Teachers, in turn, are expected to facilitate learning and problem solving. Metacognition facilitates this reflective learning process and mathematical proficiency (see Section 2.3.4.2). Hartman (2001b: 149) states that teachers should teach with and for metacognition, i.e. teachers should be metacognitively aware themselves and also model metacognition to their learners, teach metacognitive strategies, and think about how to develop metacognition in their learners (see Section 2.4). Furthermore, teachers are expected to manage classrooms, make decisions, and solve problems themselves daily. This requires adaptive expertise, which is the ability and skill to reflect and adapt thoughts, feelings, and actions accordingly. This ability is, essentially, metacognition: the awareness of one's own thoughts, feelings, and actions, and the skills to regulate cognition (see Sections 2.2.4; 2.2.4.1).

With regards to teaching, Lin et al. (2005: 245) assert that adaptive metacognition means adapting oneself and one's environment in response to classroom variability. Therefore, metacognitive teachers display adaptive expertise in learning and problem solving. Furthermore, this expertise is metacognitively adaptive and requires reflection *on* the task (i.e. monitoring) and reflection *during* the task (i.e. regulation) (see Section 2.2.3). Consequently, there is a widely recognised expectation for teachers in higher education and professional development to develop and foster reflective practices and adaptive skills (Bowman, Galvez-Martin & Morrison, 2005: 336; Duffy, 2005: 300; Jindal-Snape & Holmes, 2009: 219; Kohen & Kramarski, 2012: 2; Larrivee, 2008: 341; Rogers, 2001: 49). These reflective practices enable teachers to adapt and improve upon their performance on the job (Larrivee, 2008: 342). This is especially valuable for the training of pre-service teachers. Kohen and Kramarski (2012: 7) found that reflective discussions on self-regulatory aspects like metacognition and motivation support the learning and



teaching of pre-service teachers. They consequently state that metacognitive reflection is an essential component of pre-service education programs, a view supported by educational researchers Duffy (2005: 304–305) and Van Der Walt and Maree (2007: 238).

The notion is that metacognitive reflective pre-service teachers will become metacognitive reflective teachers with adaptive skills who can reflect on their own practice and model these skills and reflections for their learners (Kohen & Kramarski, 2012: 2). Reflective practitioners link theory to practice by analysing their own practice based upon knowledge and evaluating alternatives for future practice (Bowman et al., 2005: 336). Furthermore, teachers need to learn how to guide the learning process by metacognitive training/instruction and modelling (Bowman et al., 2005: 336; Duffy, 2005: 304–305; Schraw et al., 2006: 121) as metacognition—comprising of metacognitive knowledge and skills—can be taught (Hartman, 2001b: 150; White et al., 2009: 178) and learnt, hence influencing achievement (see Section 2.3.3).

Despite the emphasis on reflective practice as teaching for and with metacognition, it appears that teachers do not regularly implement metacognitive reflection in their practices (Duffy et al., 2009: 247, 249; Jindal-Snape & Holmes, 2009: 219; Kohen & Kramarski, 2012: 2; Larrivee, 2008: 341; Van Der Walt, 2014: 1–9). Novice teachers, in adapting to the situational demands of their new work, and expert teachers, as adaptive reflective professionals, would engage metacognitively (Duffy et al., 2009: 245-247). Teachers might not reflect metacognitively as they might lack awareness about the significance of metacognition in learning and problem solving (Veenman et al., 2006: 10), or lack understanding of the development of metacognitive skills that foster learning (Azevedo, 2009: 93), and therefore do not know how to instruct effectively. Bowman et al. (2005: 335) and Cornford (2000: 9) support this view that teachers seem to be ill-equipped to implement metacognitive strategies. Emphasis on learner performance may increase the likelihood of teachers opting to develop efficient teaching strategies rather than metacognitive reflection, which takes longer to acquire (Larrivee, 2008: 341).

Furthermore, metacognition is not generally associated with teachers' professional development (Duffy, 2005: 300) or pre-service teacher education (Duffy, 2005: 308).

Reflection in the micro-teaching environment for pre-service teachers is particularly difficult, as these teacher trainees have to reflect as teachers, peers, and learners at the same time during simulated lessons (Kohen & Kramarski, 2012: 2, 6). From the experience of Van Der Walt and Maree (2007: 224)—both prominent South African Mathematics educational researchers—teachers seldom model ‘learning-to-learn’ skills (Cornford, 2000: 5) as “thinking about one’s thinking”, i.e. metacognition (see Section 2.2.3). In my own experience as a teacher, the emphasis on performance and a time-demanding, difficult school Mathematics syllabus are often the driving forces behind opting for step-by-step algorithmic routine problems, rather than developing metacognition that facilitates the solving of ill-defined problems.

A teacher’s beliefs about learning could be an obstacle for implementing effective learning practices (De Corte, 2010: 56). Similarly, Bjork et al. (2013: 417, 424, 427) cites studies of college students which reveal that faulty beliefs about their own learning and memory may lead a student to inaccurately assess and manage their learning. Another reason offered for lack of or inadequate employment of metacognition is that metacognitive reflection is not a spontaneous or easy process to acquire (Jindal-Snape & Holmes, 2009: 219). Grossman (2009: 15–22) found that college students could not easily and meaningfully report on their thoughts and emotions when asked. He further cites Kegan’s (1994, cited in Grossman, 2009: 17) research suggesting that reflection on one’s thoughts and feelings is a more challenging process than generally assumed. In a problem-solving scenario, students had to solve a Rubik’s cube and record their thoughts whilst solving the problem. In Kegan’s (1994, cited in Grossman, 2009: 17) view, it is only through gradual and scaffolded support and challenges that learners will organise their minds in such a way that they are capable of reporting accurately on their thoughts and the problem-solving process.

Metacognitive reflection is especially problematic for in-service teachers. Lin et al. (2005: 245) state that adaptive metacognition is crucial in dealing with the unique challenges of classroom changeability. Duffy et al. (2009: 244) assert that teachers’ metacognition is complex due to varying situational factors such as classroom setting, the learners, and their own career level. As metacognition in teaching is situational, teachers may opt for routine experiences to manage

classrooms rather than novel or challenging problems that elicit metacognition. Pre-service and novice teachers have more difficulty to reflect (Grossman, 2009: 17), but novices might be more willing to reflect upon their thoughts and actions to improve their practice.

A study by Kohen and Kramarski (2012: 6) stated that pre-service teachers experienced difficulty reflecting on knowledge and action while teaching, i.e. during a real-time experience. This corresponds with Kegan (1994, cited in Grossman, 2009: 17) and Grossman's (2009) view that reflection-in-action is difficult (see also Section 2.2.3). Teachers, therefore, prefer to reflect on their actions afterwards. Consequently, it is important to enhance and train for metacognitive reflection in the professional development of teachers. The question is whether and under what conditions teachers could be trained to become reflective practitioners, and hence translate metacognitive reflection to their learners in the teaching and learning environment. This will be discussed briefly in the following section.

### ***2.3.5.1 Training teachers as metacognitive reflective practitioners***

Several studies show improvement of metacognitive reflective skills in undergraduate students who received applicable training. In a study by Bowman et al. (2005: 336), flash cards with prompting questions as a reflective tool were used. In the study, teachers asked reflective questions (*what* happened and, importantly, *why?*), identified reasons, responded to students' learning needs, and adapted their teaching accordingly, ultimately displaying metacognitive reflection.

A study by Kohen and Kramarski (2012) investigated the conditions under which 97 pre-service teachers in a micro-teaching environment could enhance their self-regulated learning to the optimum level. Kohen and Kramarski (2012: 6) found that explicit reflective support and critically reflective discussions improved self-regulatory aspects, namely metacognitive skills and motivational aspects. The use of explicit directed prompts such as flashcards—which referenced metacognitive skills or motivational aspects—helped the teacher trainees to reflect and then display higher levels of metacognition and motivation.

Pugalee (2004: 27–47) used reflective journals as a tool for facilitating learners' metacognition in Mathematics. The general intent was to facilitate the ability of learners to verbalise their thought processes out loud and in writing whilst solving Mathematics problems. Consequently, through a process of feedback and self-reflection, learners were encouraged to expand on the quantity and quality of the descriptions of their mathematical thinking regarding the justification of steps and strategies used in problem-solving processes. Learners showed an increase in the quantity and quality of their descriptions. These think-aloud tools are generally used to assess the thought processes of an individual (Pugalee, 2004: 29).

However, there is no consensus in the field about how to best develop these reflective practices. An environment that facilitates reflection needs to be established (Bowman et al., 2005: 341). This implies that in pre-service teacher programs, purposeful reflection should be established as an intentional practice (Bowman et al., 2005: 341; Veenman et al., 2006: 9). The higher education curriculum needs to provide opportunities in the field, in practice teaching, and in practice for such reflective opportunities. Multiple opportunities and prolonged practice are required (Bowman et al., 2005: 336; Veenman et al., 2006: 9). Furthermore, metacognitive reflection as an ongoing process needs to extend into the first years of teaching (Kohen & Kramarski, 2012: 7), as it requires time and effort to reflect (Jindal-Snape & Holmes, 2009: 219) and novice teachers engage in high levels of metacognitive thought in dealing with new situations daily (Duffy et al., 2009: 243).

Tools are commonly used to prompt and foster metacognition. Prompting cues such as reflective questions or flashcards (Bowman et al., 2005: 336; Kohen & Kramarski, 2012: 7), keeping reflective journals (Bowman et al., 2005: 348; White et al., 2009: 186), and verbal or written protocols (Pugalee, 2004: 29) have been mentioned. Collaboration amongst peers (Schraw et al., 2006: 121, 128) facilitates, motivates, supports, and challenges teachers, making them aware of common difficulties and challenging them to think about their beliefs and to set, implement, and evaluate goals in reflection with peers (Duffy, 2005: 304). It also motivates them, as workload challenges are shared and positive experiences are reinforced and celebrated. Lin et al. (2005: 253) assert that teaching is situated in

socio-cultural contexts with diverse goals for teaching. Observing others in problem-solving experiences could therefore enhance adaptive metacognition by reflecting on different strategies to reach different goals. Finally, mentoring pre-service teachers plays a critical role in developing reflection among pre-service teachers (Bowman et al., 2005: 346). It therefore takes active and concerted engagement with the help of an expert (Duffy, 2005: 304–305; Schraw et al., 2006: 121) to coach and scaffold this process in an ongoing manner as the pre-service teacher reflects and seeks feedback.

In summary, the development of metacognition in pre-service and in-service teachers seems problematic and not widely implemented, despite a well-established call for reflective practices and for teachers to be metacognitive reflective professionals. Research on reflective practices is well-documented (Jindal-Snape & Holmes, 2009: 219; Rogers, 2001: 37). However, in South Africa there is a call for further research into teachers' metacognition with the implication to extend this to professional development, especially pertaining to Mathematics (Van Der Walt et al., 2008: 231). In Section 2.4, teaching for the enhancement of learners' metacognition is discussed.

## **2.4 TEACHING FOR METACOGNITION**

Teaching for metacognition promotes metacognitive awareness in learners. Teachers should teach both with and for metacognition (Hartman, 2001b: 149; see Section 2.3.5). Metacognition is related to other constructs like cognition, self-regulation, self-regulated learning, and affective components like motivation (see Section 2.2.5). These constructs influence productive learning and successful problem solving. Furthermore, the categories of metacognition (see Section 2.2.4) each play a distinct but complementary role in enhancing learning and problem solving. Metacognition develops spontaneously to some extent (Bruning, Schraw & Norby, 2011: 29; Van Der Stel et al., 2010: 227) and at different ages, but can also be developed and enhanced through instruction. It is not linked to intelligence, which means all learners can be trained.

Hartman and Sternberg (1993, cited in Desoete, 2007: 717), Schraw (1998: 113–125), and Schraw et al. (2006: 111–139) describe ways for teachers to enhance

the general metacognitive awareness of learners in classroom settings. Four key ways are promoting general metacognitive awareness, enhancing metacognitive knowledge, enhancing metacognitive skills, and fostering conducive learning environments. These will now be elaborated on briefly.

### **2.4.1 Promoting general metacognitive awareness**

Teachers and learners may not be aware of the role played by metacognition in learning and problem solving. Promoting the value of metacognitive awareness to teachers (see Section 2.3.5) is an essential step in the learner's development and employment of metacognition. Teachers can promote metacognitive awareness to their learners by extensively discussing the importance of reflection on one's thinking, doing, and feeling. This awareness of the usefulness of metacognition—that is, when, how, and why to use metacognitive strategies—serves to motivate learners to attempt to employ these strategies (Veenman et al., 2006: 9). In Mathematics especially, writing in think-aloud protocols or reflective journals provides a level of reflection that promotes learners' awareness of their own thinking about the mathematical problem-solving process (Pugalee, 2004: 28; White et al., 2009: 186). They reflect on their strengths and weaknesses and which resources they possess (content knowledge, strategies), but also on their skills, e.g. planning effectively, monitoring progress towards a goal, or evaluating the logic of a solution.

Levels of metacognitive awareness can develop spontaneously to some extent (Bruning et al., 2011: 29; Van Der Stel et al., 2010: 227) or via teachers, peers, and parents (Veenman et al., 2006: 9). Reflection on daily activities with peers or parents, or the thinking-aloud modelling by teachers whilst problem solving, imparts this metacognitive awareness to learners (Pugalee, 2001: 237; Veenman et al., 2006: 9). Despite this level of metacognitive knowledge and skills being at their disposal, learners might not be successful—or they might fail to use metacognition—because of personal or task variables (Veenman et al., 2006: 10). Consequently, task difficulty or personal variables such as feelings of anxiety, lack of motivation, or being unaware or ill-informed about the usefulness of metacognition in a certain situation should be considered in metacognitive instruction (Veenman et al., 2006: 10).

## 2.4.2 Enhancing metacognitive knowledge

Metacognitive knowledge (*Knowledge of cognition*) includes three subcomponents: *Declarative*, *Procedural*, and *Conditional knowledge*. These represent knowledge about oneself and knowledge of how, when, and why to use strategies (see Section 2.2.4.1). Learners' knowledge and beliefs about themselves influence their learning and problem solving. Reflection before a task on how one learns best, what one knows or does not know about the subject matter, and how one feels about the task or their ability (Akyol & Garrison, 2011: 184) is, therefore, an important component of fostering metacognitive knowledge. Adequate metacognitive knowledge is related to domain-specific knowledge, such as concepts and difficulties within a domain (Veenman et al., 2006: 5), e.g. knowledge about when and why to use problem-solving strategies in Mathematics (see also Sections 2.3.4.2.1–2.3.4.2.4)

Learners' metacognitive self-knowledge could be correct or incorrect and might be resistant to change (Veenman et al., 2006: 4) as learners and teachers under- or over-estimate their ability and learning relative to the complexity of the task (Efklides, 2012: 298). Feedback from teachers could inform this self-knowledge and help learners to adapt it. Moreover, teachers' metacognitive knowledge of their beliefs about teaching and their abilities informs their feedback. Training for metacognitive knowledge should be explicitly labelled and discussed (Pintrich, 2002: 223). Whole-group and peer discussion foster meta-knowledge by connecting strategies with content and developing a shared language to talk about cognition and learning (Pintrich, 2002: 223). Moreover, teachers should grant learners an opportunity to assess self-knowledge about strengths, weaknesses, and misconceptions, and hence their learning (Pintrich, 2002: 224), as well as challenge false beliefs (such as misconceptions or a lack of motivation) about the self or learning (see Section 2.3.4.2.4). Therefore, teachers' beliefs about learning Mathematics and the role of metacognitive awareness influence the way they teach.

Such reflections are enabled in whole-group or peer discussions, reflective writing opportunities like portfolios, or think-aloud methods and cueing strategies (Schraw, 1998: 119–120) and self-questioning. Self-questioning is a useful strategy

to facilitate metacognitive knowledge in the four-phase problem-solving framework (see Sections 2.2.4.1; 2.3.4.3.2).

### **2.4.3 Enhancing metacognitive skills**

*Regulation of cognition* comprises of *Information management, Planning, Debugging, Monitoring, and Evaluation* (see Section 2.2.4.3). These metacognitive skills are self-regulatory and facilitate active and constructive learning, and consequently self-regulated learning (see Section 2.3.4.3.2) which impacts achievement. Studies have indicated that metacognitive strategies are trainable and should therefore be instructed, as they influence achievement and learning (see Sections 2.3.3; 2.2.4.3). When the employment of strategies becomes automated behaviour, a metacognitive skill has developed (see Section 2.2.4.3). Metacognitive skills (domain-specific and general) could therefore be enhanced through instruction (Schellings et al., 2013: 986). Azevedo (2009: 93) asserts that the successful instruction of skills should take into account the development of processes which enhance academic achievement for skills to develop, improve, or become automated with further practice.

Although most people possess some level of metacognitive awareness (Bruning et al., 2011: 29; Van Der Stel et al., 2010: 227; Veenman et al., 2006: 9), the level of skills might be insufficient or ineffectively employed (see Sections 2.3.1.2; 2.4.1). It is important to note that using metacognitive skills more often does not necessarily imply a higher level of quality (Van Der Stel et al., 2010: 226) and consequently adequate or effective use. In Mathematics, Van Der Stel et al. (2010: 226) found that metacognitive skills generally increase in frequency (quantity) and also quality (adequate utilisation). It is especially the case in learners of 13–15 years of age that metacognitive skilfulness is related to better Mathematics performance (Van Der Stel et al., 2010: 227). Therefore, development of metacognitive skilfulness should be enhanced during this stage.

Successful instruction of metacognition should be explicit (Desoete, 2007: 706; Pintrich, 2002: 223). Although there is little clarity regarding which skills develop first through maturation, studies have shown successful instruction of metacognitive skills



if a specific skill is trained at the appropriate time and situated effectively within the context of a task (Van Der Stel et al., 2010: 227; Veenman, 2011b: 211). In Mathematics problem solving, having the teacher model strategies by verbalising *why* and *when* a specific heuristic is used (see Sections 2.2.4.1; 2.2.4.4) is particularly successful (Hartman, 2001b: 158; Van Der Walt & Maree, 2007: 235), as is using questions as prompts to implement a step-by-step plan (Mevarech & Fridkin, 2006: 365–394; see Section 2.3.3).

In collaborative sessions, learners model cognitive and metacognitive skills frequently better than teachers (Schunk, 1989, cited in Schraw, 1998: 118) as they discuss strategy use and evaluate progress towards learning goals within their peers' zone of proximal development (Schraw et al., 2006: 120; see Section 2.3.1). Veenman et al. (2006: 9) assert that literature indicates three principles central to the successful instruction of metacognitive skills. First, it is important to embed strategies in the content domain tasks (see Sections 2.3.3; 2.3.5.1). Pintrich (2002: 223) and White et al. (2009: 177), on the other hand, assert that general strategies could be trained across domains: for example, knowing how to activate prior knowledge in understanding a problem or a text indicates a general metacognitive skill that can be applied in novel situations across domains (see also Section 2.2.4.3).

Second, successful instruction of metacognitive skills requires informing learners about the usefulness of metacognition (see Section 2.4.1). This motivates learners to exert initial effort (Veenman et al., 2006: 9). In addition, the task chosen is crucial in eliciting metacognition and stimulating interest (see Sections 2.4.4; 2.2.4.2). This also presupposes the awareness of teachers about the role metacognition plays (see Section 2.3.5). Third, as skill adaptation and acquisition are neither easy nor quick processes, prolonged training is required (Veenman, 2011b: 203; Veenman et al., 2006: 9; see Section 2.3.5.1). Therefore, opportunities should be provided for learners to be trained explicitly to apply metacognitive strategies in specific domains over a prolonged period.

#### **2.4.4 Promoting conducive learning environments**

In a conducive learning environment, productive learning is facilitated and independent lifelong learning is the goal (see Section 2.3.4.3). Teachers, learners, the curriculum, and the classroom setting interact in productive learning and problem solving. Metacognition is an essential component in such a powerful learning environment (Azevedo, 2009: 93; De Corte, 1995: 42; Editorial, 2012: 250–251; Pugalee, 2001: 237; Thomas & Mee, 2005: 220) to facilitate productive learning, competent problem solving, and higher-order thinking skills in interaction with other factors. Metacognitive awareness and self-regulated learning are consequently fostered by the learning environment, supportive teacher behaviour, learners' independence, and the types of tasks provided (Van Grinsven & Tillema, 2006: 79; see Section 2.3.4.2.4).

The learning environment should provide opportunities for learners to reflect metacognitively on their learning and to evaluate contextually appropriate adaptive learning and thinking strategies (Thomas & Mee, 2005: 221). This way, they will foster adaptive competence by enabling learners to transfer knowledge and skills to novel problems and real-life settings (Bransford et al., 2000: 4, 9, 18). In structuring learning opportunities, the type of task should be selected to stimulate higher-order thinking. Authentic, interesting, and challenging problems elicit metacognitive behaviour and cause increased use of strategies and motivation (Van Grinsven & Tillema, 2006: 79, 88; see Sections 2.2.2; 2.2.4.2). However, as teachers and learners often assume that they know how to learn, the instruction and employment of metacognition could be met with initial resistance (see Section 2.3.4.2.4). Therefore, providing open-ended challenging tasks might serve as catalyst for realising the need for skills to learn or problem-solve.

Furthermore, tasks provided through computer programs and web-based resources such as Betty's Brain (<http://www.teachableagents.org/>) can serve to scaffold metacognition (Vanderbilt University, 2013; Wagster, Tan, Biswas & Schwartz, 2007; Wagster, Tan, Wu, Biswas & Schwartz, 2007). These technologies prompt and cue learners to plan and set goals, provide reasons for strategies and plans, and monitor their effectiveness; consequently, the programs provide feedback to learners, who in turn can reflect upon their practice and apply this knowledge to debug or remediate

(Schwartz et al., 2009: 342; White et al., 2009: 177). Thus metacognition, as thinking about thinking and actions, is facilitated. As thought processes are externalised, metacognitive knowledge and skills are made explicit to learners.

A conducive learning environment fosters cognitive and metacognitive development and should nurture opportunities for acquiring general learning and thinking skills across different subject-matter domains (De Corte, 1995: 40, 42; Dunlosky, Rawson, Marsh, Nathan & Willingham, 2013: 46). These cognitive and metacognitive strategies can be explicitly taught and influence achievement and learning (see Section 2.3.3). However, skills take time to develop and the learning environment should provide instructional support to develop these skills (Azevedo, 2009: 93). Instruction focuses on informed strategy training (influencing metacognitive knowledge) and self-regulated training (influencing metacognitive skills) to foster metacognition in learning and problem solving (Desoete, 2007: 717) through the teacher modelling the *why*, *when*, and *how* of strategy use. A powerful Mathematics learning environment provides a safe and supportive space for learners to think aloud and defend their choice of metacognitive and problem-solving strategies. It affects motivation positively (Schraw et al., 2006: 130; Van Grinsven & Tillema, 2006: 78) and helps learners to attribute their success to the use of adequate strategies and self-regulation (Desoete, 2007: 706). Furthermore, it fosters positive attitudes towards Mathematics (De Corte, 2000: 260) and contributes towards a mathematical disposition (see Section 2.3.4.2).

## **2.5 SUMMARY OF CHAPTER**

This chapter contributed to exploring the study's first two secondary research questions, on the conceptualisation of metacognitive awareness and the role that it plays in Mathematics teaching and learning.

Metacognitive awareness comprises of metacognitive knowledge (*Knowledge of cognition*) and metacognitive skills (*Regulation of cognition*). As an adaptive competence, metacognitive awareness influences learning and problem solving at school, university, and the workplace. As the reflective practice of a teacher, it plays a critical role in translating and facilitating mathematical problem solving in learning

and teaching, and consequently influences achievement. It is therefore critical that metacognitive awareness should be enhanced.

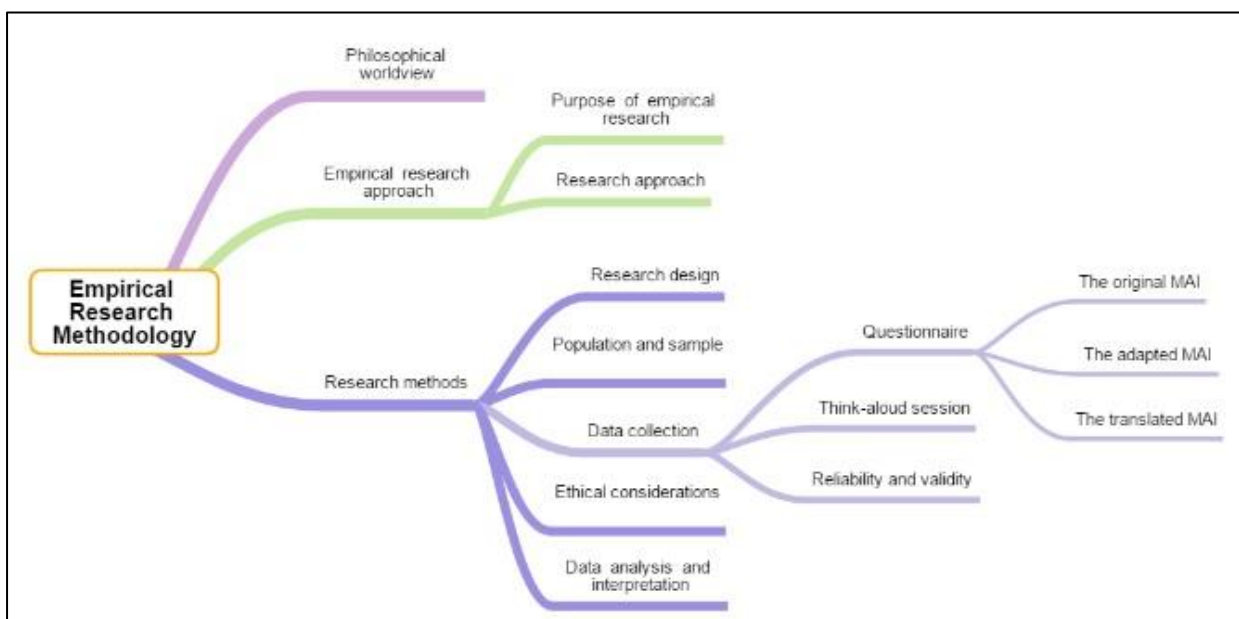
The next chapter describes the methodological considerations underpinning the investigation into the perceptions and applications of metacognitive awareness by pre-service Mathematics teachers.

# CHAPTER 3

## EMPIRICAL RESEARCH METHODOLOGY

### 3.1 INTRODUCTION

Novel real-life scenarios and fast-changing information demand new ways of learning and problem solving on an ongoing basis. A metacognitive adaptive approach facilitates the transfer of skills and knowledge into novel situations at school and in professional life. In Chapter 2, the importance of metacognitive awareness in learning and problem-solving situations as illustrated in the literature was discussed. In this chapter, the research approach and methods that were used to determine the level of metacognitive awareness of Mathematics didactics students, and the transfer of metacognitive awareness into a Mathematics problem-solving situation, are discussed. The following components are described:



**Figure 3.1: Components of the empirical research methodology (see also Appendix 8)**

Research is a systematic process involving the collection, analysis, and interpretation of data to uncover underlying truths or to increase understanding of a phenomenon (Leedy & Ormrod, 2013: 2, 76). Three components must be addressed in planning a study: the researcher's own philosophical assumptions, the research approach, and specific research methods. The research approach should be based upon the nature of the research problem (Creswell, 2014: 3, 5).

The philosophical worldview of the researcher informs the research approach, which presupposes specific research methods (see Section 3.4). Research methods involve the research design (see Section 3.4.1), the population and sample (see Section 3.4.2), data collection methods and instrumentation (see Section 3.4.3), and lastly data analysis and interpretation (Creswell, 2014: 5; see Section 3.4.5).

## **3.2 PHILOSOPHICAL WORLDVIEW**

A researcher's worldview is their general philosophical positioning about the world and nature of research (Creswell, 2014: 6). This worldview is a general orientation about the nature of reality and human behaviour (ontology). A researcher's ontology and epistemology (their view of what knowledge is) can inform the purpose, type, and method of data collection (Maree & Van Der Westhuizen, 2010: 31–32). This orientation leads to a specific research approach: either qualitative, quantitative, or a mixed methods approach.

Two worldviews informed the study: post-positivism and interpretivism. In the post-positivist's worldview, problems are studied that reflect the need to identify and assess the causes that affect outcomes (Creswell, 2014: 19). Truth is not viewed as absolute as per the scientific view, but instead is observed as a reality that exists "out there". Knowledge is based upon observation and measurement of an objective reality. Post-positivists contend that we cannot be certain about our claims of knowledge when observing the attitudes, behaviour, and actions of humans. Evidence is imperfect and fallible, open to refuting and revising (Creswell, 2014: 19). Knowledge, as small sets of discrete data, is obtained through observation and measurement by using numeric measures. This may inform further observation or testing as done in the experimental method. A quantitative methodological approach is usually followed.

Interpretivism as a broad term holds that there are multiple participant meanings (Creswell, 2014: 6) or multiple perspectives, opinions, or beliefs as subjective states vary from one person to another (Phillips & Burbules, 2000, cited in Johnson & Onwuegbuzie, 2016: 16). Interpretivism holds that reality is not objectively determined but is socially constructed through text and symbols, consciousness, and shared meaning. Constructivism and social-constructivism are interpretive approaches and are well-associated with educational research, particularly with respect to Mathematics (see Sections 2.3.1.1; 2.3.1.3).

Interpretivism aims to provide a perspective on a situation and to analyse it, offering insight into how people make sense of their situation or the phenomena which they encounter. Context is important to uncover how meaning is constructed and so improve our understanding of the whole (Nieuwenhuis, 2010: 59). A qualitative methodological approach is usually followed in interpretivist research.

As reality encompasses single and multiple aspects, researchers may test hypotheses and explain findings from different perspectives (Creswell & Plano-Clark, 2007: 24). In the study, a post-positivist/interpretivist worldview informs the epistemology and methodology. As a researcher, I was interested in assessing the level of metacognitive awareness of pre-service teachers about Mathematics didactics, thus a post-positivist worldview with a quantitative approach was adhered to in the main data collection phase.

Assessing metacognition as a multi-faceted, interdependent construct in relation to other constructs (see Sections 2.2.5; 2.2.7) is difficult (Sperling et al., 2002: 54). Multiple methods, therefore, are promoted by researchers and have been used in previous studies to explain or elaborate on the findings. Metacognition comprises of two subcomponents, *Knowledge of cognition* and *Regulation of cognition*, each comprising of subscales (see Section 2.2.3). Consequently, in the quantitative phase, in observing and assessing levels of metacognitive awareness, data was collected via a questionnaire comprising of small subsets of the construct and translated into numeric measures. Additionally, as researcher I was interested in the pre-service teachers' ability to translate their metacognitive awareness into practice, i.e. to do what they say they are doing and will do. Consequently, an interpretivist worldview with a qualitative approach was adhered to in the think-aloud problem-solving session. The findings from the questionnaire were expanded on and enriched by the findings from the think-aloud problem-solving session, where the metacognitive knowledge and metacognitive skills—transferred to solving a Mathematics problem—were assessed. This, therefore, provided the study with a complementary view. As Nieuwenhuis (2010: 60) notes, the strength of a qualitative approach is its richness and depth of exploration and description.

On a supplementary note, educational research is viewed as increasingly complex, dynamic, and interdisciplinary; therefore, epistemological and methodological

pluralism promotes effective research and scholarly collaboration and could provide a more comprehensive answer to the research problem (Johnson & Onwuegbuzie, 2016: 15). Furthermore, the link between philosophical worldview and research approach is neither untouchable nor necessary (Howe, 1988, 1992, cited in Johnson & Onwuegbuzie, 2016: 15). Researchers, therefore, need to complement one method with another. Quantitative researchers should not be restricted from using qualitative methods and vice versa for corroboration, complementarities, or expansions (Green et al., 1989, cited in Johnson & Onwuegbuzie, 2016: 15). In the next section, the empirical research approach is described.

### **3.3 EMPIRICAL RESEARCH APPROACH**

#### **3.3.1 Purpose of empirical research**

The purpose of this empirical research is to determine the level of metacognitive awareness of pre-service Mathematics teachers. For this purpose, both quantitative and qualitative methods were used to obtain data, as both quantitative and qualitative researchers can utilise empirical observations to address a research problem (Johnson & Onwuegbuzie, 2016: 14).

Secondary research question 3, “What is the level of metacognitive awareness of pre-service Mathematics teachers on the Metacognitive Awareness Inventory (MAI)?”, was investigated using a quantitative approach in the form of a standardised questionnaire. Meanwhile, secondary research question 4, “What is the level of metacognitive awareness of pre-service Mathematics teachers in a problem-solving context?”, utilised a qualitative approach in which the pre-service teachers had to write down their thoughts and calculations during a problem-solving session. The aim for the supportive qualitative inquiry was to enrich the findings of the quantitative data.

The research methodology employed to realise the purpose of the research is discussed in the next section.



### **3.3.2 Research approach**

A research methodology is the general approach that a researcher adopts in conducting their research and describes procedures for collecting, analysing, and interpreting data (Creswell, 2014: 3; Leedy & Ormrod, 2013: 7). The two research approaches—quantitative and qualitative methods—are not dichotomous but rather lie on a continuum; consequently, a study tends to be more quantitative than qualitative or vice versa (Creswell, 2014: 3). The research approach is ultimately based on the nature of the research problem (Creswell, 2014: 3). In the study, a quantitative approach is utilised with a qualitative approach adopted to enrich the findings.

In the Human Sciences, the quantitative inquiry investigates the theory of a phenomenon, to refine or extend the theory with the goal to generalise to a larger population. The researcher, situated within a post-positivist philosophical worldview, aims to be as value-free as possible. In contrast, a qualitative inquiry assumes that human behaviour is context-bound and therefore seeks to understand and interpret human behaviour in a particular setting. The researcher's bias is identified and monitored as a meaningful understanding of human behaviour in the interpretivist worldview and is unavoidably value-bound (Ary et al., 2010: 420–421). In this instance, my interpretation as researcher was conceivably informed by years of teaching experience, and as such influenced the validity of the study (see Section 3.4.3.3).

## **3.4 RESEARCH METHODS**

In planning a study, the researcher's philosophical worldview informs the research design, whereas the specific procedures of the research project translate the approach into practice (Creswell, 2014: 5). These strategies and techniques are discussed next.

### 3.4.1 Research design

Quantitative research can employ experimental or non-experimental designs, such as surveys (Creswell, 2014: 12). The purpose of experimental research is to determine whether a specific treatment influences an outcome (Creswell, 2014: 12), whereas non-experimental designs seek to describe events without intervention and do not endeavour to intervene or change behaviour. Descriptive quantitative research examines a situation as it is (Leedy & Ormrod, 2013: 184) and seeks to quantify responses on one or more variables. Descriptive studies include surveys which ask questions such as “to what extent?” or “what is?” (Onwuegbuzie & Leech, 2006: 490). The goal of a descriptive survey is to learn about a population by surveying a sample of that population (Leedy & Ormrod, 2013: 189).

Surveys provide a numeric or quantitative description of constructs, perceptions, trends, attitudes, attributes, or opinions of a population by studying a sample population. Studies could be cross-sectional or longitudinal, employing questionnaires or structured interviews with the intent of generalising findings from sample to population (Fowler, 2008, cited in Creswell, 2014: 13). However, since a non-probability purposive sample was selected in the study, generalisation of the findings is not possible.

In the study, I was interested in determining to what extent fourth-year Mathematics didactics students are metacognitively aware. A survey design was chosen which included using a questionnaire and a think-aloud session. Data was collected for both methods on the same day; this data collection process is described in Section 3.4.3. A survey design was selected due to the economy of the design and the swift turnaround time of the collected data (Creswell, 2014: 157). Although findings determining the level of metacognitive awareness cannot be generalised to all Mathematics didactics pre-service teachers, recommendations could nonetheless be made for this specific cohort (see Section 5.5). A questionnaire, the Metacognitive Awareness Inventory (MAI), was selected as survey instrument (see Section 3.4.3.1).

It is difficult to capture aspects related to metacognition accurately (Sperling et al., 2002: 54). However, a quantitative approach has been well-used in various

studies (Schellings et al., 2013: 966) and multiple methods to measure metacognition have been used more often in recent studies (Desoete & Roeyers, 2006: 13; see Section 2.3.3). Consequently, qualitative research helped to describe the multi-faceted nature of certain situations and to provide an understanding about particular concepts, attitudes, and situations in the study (Leedy & Ormrod, 2013: 140). As I was also interested in identifying the level of metacognition of pre-service teachers during a problem-solving situation, additional qualitative data was collected during an online think-aloud session (see Section 3.4.3.2).

### **3.4.2 Population and sample**

The population in question was pre-service Mathematics teachers enrolled at higher education institutions in South Africa. Because of time constraints and cost constraints, it was not possible to conduct research among all these students. Consequently, a sample of fourth-year Education students with Mathematics as a didactical subject was identified at a specific university. Purposive sampling occurs when participants are chosen for a specific purpose (Leedy & Ormrod, 2013: 215). Participants were chosen here based on their convenience and availability (Creswell, 2014: 158). A purposive convenience non-probability sample of fourth-year pre-service teachers was selected as including all the students in that cohort ( $n = 41$ ). I worked with the students as a lecturer and realised the poor level of problem-solving skills of the pre-service Mathematics teachers. This led to the undertaking of the study and provided an available and convenient sample.

Bias is any influence, condition, or set of conditions that distorts the data, whether independently or in combination with others (Leedy & Ormrod, 2013: 217). This group of learners was heterogeneous in regards to the biographical variables of age, gender, and language. Age and gender were not considered in the analysis of the data. Regarding language, as the sample is from a dual medium university, the questionnaire was translated into Afrikaans (see Sections 3.4.3.1.3; 4.2.3.1). As researcher, I am aware that other variables, such as related constructs not controlled in the study, might have influenced the metacognitive awareness of the students. Conclusions are therefore made with these variables in mind. Furthermore, in non-probability methods, as is the case with the study, limitations in generalising findings

and drawing important conclusions about the whole population should be kept in mind (Maree & Pietersen, 2010a: 177).

There were some equalising factors. Both male and female pre-service teachers were included, as well as pre-service teachers receiving instruction in both English and Afrikaans. Everyone participated on a voluntary basis.

### **3.4.3 Data collection methods**

In descriptive studies, when measuring phenomena (concepts, abilities, opinions, or attitudes) data collection methods include questionnaires, observations, interviews, checklists, and rating scales (Leedy & Ormrod, 2013: 191). The use of a questionnaire to address secondary research question 3, “What is the level of metacognitive awareness of pre-service Mathematics teachers on the MAI?”, will be explained in Section 3.4.3.1. The employment of a think-aloud problem-solving session to address secondary research question 4, “What is the level of metacognitive awareness of pre-service Mathematics teachers in a problem-solving context?”, will be explained in Section 3.4.3.2.

In researching the choice of instruments and methods to measure metacognition, the literature indicates the ongoing debate of online versus offline methods to measure metacognition. Online methods include think-aloud sessions and videos; offline methods include questionnaires. Researchers suggest a combination of methods (Schellings et al., 2013: 963). The study made use of both a questionnaire and a think-aloud session.

#### ***3.4.3.1 Questionnaire***

Although a number of questionnaires can be used to measure learning strategies in general, a smaller number of standardised questionnaires are aimed at measuring metacognition. Among these are the Metacognitive Awareness Inventory (MAI) developed by Schraw and Dennison (1994); the Metacognitive Awareness of Reading Strategies Inventory, Version 1.0 (MARS-I) pertaining to academic reading, designed by Mokhatari and Richard (2002); and the Self-efficacy and Metacognition Learning Inventory–Science (SEMLI-S) of Thomas, Anderson, and Nashon (2008) pertaining to learning in Science.

In the study, the standardised MAI questionnaire was selected to assess the pre-service teachers' metacognitive awareness. Schraw and Dennison (1994) developed the MAI to measure adult learners' metacognitive awareness (see Section 4.2.1). It has subsequently been used in various studies (e.g. Sperling et al., 2004: 124).

The MAI questionnaire was adapted to measure metacognitive awareness in a mathematical as well as South African educational context by Du Toit (2013) (see Sections 3.4.3.1.2; 4.2.2). The adaptation corresponds with the educational practice to tailor questionnaires to context when measuring learning strategies. Validity issues may arise in administering "general" questionnaires and therefore instruments may be tailored to specific contexts (Schellings, 2011: 4), which is discussed in Section 3.4.3.3. Furthermore, the adapted MAI was translated into Afrikaans (see Section 3.4.3.1.3).

Online methods are time-consuming and costly, whereas there are several advantages to using questionnaires: the response rate is optimal, it is cost-effective and time-effective, easy and quick to answer, and administrators can check the accuracy or clarify comprehension issues of the respondents (Maree & Pietersen, 2010b: 157, 164). However, further possible limitations of questionnaires are that answers are simple with no detail (Maree & Pietersen, 2010b: 164) and that respondents do not always give truthful answers, but rather what they think would be more acceptable or what the researcher would be interested in.

In the study, the adapted MAI (see Sections 3.4.3.1.2; 4.2.2) was administered to 41 fourth-year Mathematics didactics students in a group setting after the think-aloud problem-solving session (see Section 3.4.3.2). The pre-service teachers were advised about the nature and purpose of the study and participation was voluntary. I administered the questionnaire and could therefore control the conditions to some extent and assist with clarity issues, which are two possible disadvantages of using questionnaires. As only one questionnaire was administered during one session only, I also had full control over all administrative issues. The questionnaire was completed during the last 20 minutes of a regularly scheduled class session. No pre-service teachers experienced problems with understanding or answering the questionnaire. The response rate was 100%.

### 3.4.3.1.1 The original MAI

In the study, the standardised MAI was chosen to evaluate the pre-service teachers' metacognition. Schraw and Dennison (1994: 461) developed the MAI to assess the metacognitive awareness of adolescents and adults, with the view of making it easier to collect the data than by using other measures such as time-consuming online assessments. Schraw and Dennison's (1994: 460) inventory comprises of 52 items that are classified under eight subscales incorporated into two subcomponents: *Knowledge of cognition* and *Regulation of cognition*. Participants rated their metacognitive awareness on a bipolar rating scale; a continuous line with two opposing poles (0–100mm).

*Knowledge of cognition* comprises of three subscales which refer to metacognitive knowledge, whereas *Regulation of cognition* comprises of five subscales that relate to the regulation of learning (that is, metacognitive skills) (see Section 2.2.4). The subscales comprising *Knowledge of cognition* are:

- *Declarative knowledge*, which is the knowledge the individual possesses about themselves and their own strategies;
- *Procedural knowledge*, which is knowledge about *how* to use those strategies successfully; and
- *Conditional knowledge*, which is knowledge about *when* and *why* to use certain strategies, based upon factors such as effectiveness, relevance, and suitability (Schraw & Dennison, 1994: 460, 474).

Meanwhile, *Regulation of cognition* comprises of the following:

- *Planning*, which occurs *prior* to learning and involves setting goals and allocating time and resources towards achieving these goals;
- *Information management*, which occurs *during* learning and involves using various skills and strategy sequences—such as organisation, elaboration, summary, and selective focus—to help efficiently process information;
- *Monitoring*, which is the individual's assessment of their own learning or strategy use through self-testing and reflection;
- *Debugging*, namely the use of strategies such as remediation to help identify and address errors in comprehension and performance; and

- *Evaluation*, which occurs *after* a learning experience and entails analysing the effectiveness of performance and strategies, re-evaluating approaches where applicable (Schraw & Dennison, 1994: 474–475).

The operational definitions of Schraw and Dennison (1994) of the eight subscales and their corresponding items on the MAI are shown in Appendix 3.

As intimated above, the MAI assesses metacognitive awareness comprising of two subcomponents: metacognitive knowledge (*Knowledge of cognition*) and metacognitive skills (*Regulation of cognition*). Schraw and Dennison (1994) based their two-component view of metacognitive awareness on Brown (1987) and Flavell (1979) (see Section 2.2.1). Schraw and Dennison (1994: 461–462, 470) piloted a 120-item inventory—with at least eight items under each subscale—with 70 undergraduate students. They eliminated items from the inventory until it contained 52 items and then examined three aspects: first, the validity of the conceptualisation of metacognition into two subcomponents, *Knowledge of cognition* and *Regulation of cognition*; second, the statistical relationship between the two subcomponents; and third, the probable relationship between *Knowledge of cognition* and achievement, and *Regulation of cognition* and achievement.

The researchers' findings were threefold. First, Schraw and Dennison (1994: 470–472) confirmed the two-component view of metacognition. Second, a statistically positive relationship was established between the two factors, *Knowledge of cognition* and *Regulation of cognition*. In two subsequent experiments, they found that these components displayed an inter-correlation of  $r = 0.54$  and  $r = 0.45$ ,  $p < 0.05$  respectively and a high degree of internal consistency, i.e.  $\alpha = 0.91$  for both subcomponents on Experiment 1 and  $\alpha = 0.88$  for both subcomponents on Experiment 2. The entire instrument was found to be highly reliable ( $\alpha = 0.95$  and  $\alpha = 0.93$ ) (Schraw & Dennison, 1994: 464, 468; see Section 4.2.1). These findings suggest that the subcomponents are related. Each subcomponent makes a unique contribution to cognitive performance and influences it in different ways (Schraw & Dennison, 1994: 471). In a later study, Sperling et al. (2004: 124) also investigated the relationship between metacognitive components and found a much higher correlation between *Knowledge of cognition* and *Regulation of cognition* ( $r = 0.75$ ,  $p < 0.001$ ). Finally, Schraw and Dennison (1994: 471) identified a significant

relationship between MAI achievement and test performance achievement. The MAI, therefore, provides useful information for predicting performance.

However, Schraw and Dennison (1994) did not find significant relationships between the MAI score and monitoring accuracy, or between monitoring accuracy and pre-test judgments. A reason suggested is that reading skills are automated in adult learners and therefore not reported on the MAI. Ultimately, they concluded that the MAI provides a reliable initial test of metacognitive awareness among adult and college students (Schraw & Dennison, 1994: 472; see Section 4.2.1). They further stated that the MAI is a helpful measure to determine the metacognitive awareness of adult learners in particular, in view of planning follow-up training to enhance metacognitive awareness. Additionally, it may help to identify lower performing learners with inadequate comprehension monitoring skills.

As a last observation, Schraw and Dennison (1994: 472) suggest that metacognitive awareness plays a bigger role in complex tasks such as problem solving. Later research indicates that metacognitive awareness is elicited by complex tasks (see Sections 2.2.2; 2.2.4.2) as the completion of these tasks requires higher-order thinking and, consequently, a higher level of metacognitive awareness.

#### **3.4.3.1.2 The adapted MAI**

The standardised instrument, the MAI developed by Schraw and Dennison (1994), was adapted to the South African Mathematics education context in two ways. First, it was altered to reflect learning and problem solving in Mathematics specifically (see Section 4.2.2 for a detailed discussion). Second, the 100mm bipolar rating scale of the original MAI of Schraw and Dennison (1994: 463) was changed to a five-point Likert scale reflecting the categories *Strongly Disagree*, *Disagree*, *Neutral*, *Agree*, and *Strongly Agree* (See Appendix 2).

In adapting questionnaires, reliability and validity issues may appear (see Sections 4.2.1; 4.2.2). The reliability of the adapted MAI corresponded with that of Schraw and Dennison's (1994) original MAI (see Section 4.2.1). A high degree of internal consistency was reported with Cronbach's alpha values of 0.89 for the pre-test and 0.93 for the post-test (Du Toit, 2013; see Section 4.2.2).



### **3.4.3.1.3 The translated MAI**

For the study, the adapted MAI was translated into Afrikaans. The pre-service teachers in the sample of the main study were studying a four-year Education degree at a parallel-medium higher education institution, with either English or Afrikaans as their language of instruction. The adapted MAI was translated from English into Afrikaans by an accredited translator and piloted with a convenient sample of 57 fourth-year pre-service teachers at the same parallel-medium higher education institution where the main study was conducted (see Sections 4.2.2; 4.2.3.1). Appendix 1 contains the MAI translated in Afrikaans and piloted. The pre-service teachers from the pilot group were a comparable group as they were also fourth-year Education students at the same institution. Although these pilot pre-service teachers did not have Mathematics as a didactical subject, they studied Senior Phase Mathematics as part of their undergraduate teaching degree in their first year. Consequently, they could respond to the questionnaire in terms of their learning and problem solving in Mathematics and therefore the mathematical terms and references were familiar to them (see Section 3.4.3.3).

The translated MAI in the pilot study was found to be very highly reliable ( $\alpha = 0.94$ ) concerning the instrument overall. It was also found reliable in relation to the two subcomponents, *Regulation of cognition* ( $\alpha = 0.91$ ) and *Knowledge of cognition* ( $\alpha = 0.86$ ) (see Section 4.2.3.1).

Finally, the MAI in the main study provided factors that were highly reliable ( $\alpha = 0.89$ ) and inter-correlated ( $r = 0.54$ ,  $p < 0.05$ ) (for a detailed discussion, see Sections 4.2.3.2; 4.3.1). These findings correspond with the findings in Schraw and Dennison's (1994: 460, 468) two studies (see Sections 3.4.3.1.1; 4.2.1; 4.2.2).

### **3.4.3.2 The Think-aloud session**

A think-aloud method was employed to address secondary research question 4: "What is the level of metacognitive awareness of pre-service Mathematics teachers in a problem-solving context?" In educational research, the think-aloud method is employed in various studies to assess metacognition as a valuable metacognitive research method, albeit with limitations (Desoete, 2007: 705–718; Desoete & Roeyers, 2006: 13; Schellings et al., 2013: 967–968). The method involves

respondents performing a specific task while thinking aloud and reporting on their thought processes pertaining to the steps in executing the task (Schellings et al., 2013: 968).

A benefit of think-aloud methods is that metacognitive processes are not impacted by thinking aloud, because the participant is verbalising their thoughts in working memory and not interpreting or reflecting on thoughts as in self-report off-line measures. Moreover, the actual metacognitive activities are inferred by a researcher/interpreter, not the respondent. Think-aloud methods are therefore considered fairly reliable, as thinking aloud happens almost simultaneously alongside the thinking process (Schellings et al., 2013: 967).

It is worth mentioning the limitations of the think-aloud method. In this study, for example, participants are asked to solve a novel problem which may elicit metacognitive awareness (see Section 2.2.2). As they think about their thinking and doing, they report on their thought processes and strategies used. However, some strategies have become automatised as the participants regularly use them, hence some observations may not be explicitly articulated or reported. Consequently, these covert thought processes cannot be observed, though some unspoken processes can still be inferred from comments made, and as such metacognition can be inferred from cognitive activities (Schellings et al., 2013: 968; Veenman et al., 2006: 6; see Section 2.2.2). In addition, the method is more time-consuming and labour-intensive (Veenman, 2005, cited in Schellings et al., 2013: 968).

In the study, the pre-service Mathematics teachers were given a mathematical problem to solve directly before completing the questionnaire (MAI). They were instructed to list their steps in solving the problem in one column and write down their thoughts about each mathematical step in another column (see Appendix 4). They could employ any problem-solving (heuristic) strategy (see Section 2.3.4.2.2) or problem-solving method (see Section 2.3.4.4). The analysis of the think-aloud session looked at problem-solving strategies as well as metacognitive behaviours (see Section 4.4).

The problem presented came from the CAPS document for the Further Education and Training phase (DBE, 2011a: 53). The non-routine problem demanded complex procedures and higher-order thinking, requiring students to break down the question

into various parts (DBE, 2011a: 53). Hence it was anticipated that metacognitive behaviour would be elicited (see Sections 2.2.2; 2.2.4.2). The problem read as follows:

Suppose a piece of wire could be tied tightly around the earth at the equator. Imagine that this wire is then lengthened by exactly one meter and held so that it is still around the earth at the equator. Would a mouse be able to crawl between the wire and the earth? Why or why not? (DBE, 2011a: 53).

This problem was chosen due to its relevance to the sample. As discussed in Chapter 1, the low level of achievement of Mathematics learners in various exams highlighted poor higher-order thinking skills among South African learners, suggesting that these skills are not being taught or developed sufficiently in lower grades or secondary school (DBE, 2016a: 5; see Section 1.2). In addition, the basic competencies required of newly qualified teachers include knowing how to teach their subject, knowing what effective learning is, and how to mediate it (DHET, 2015: 64; Section 1.2). Teaching problem-solving as pre-service or in-service teachers should involve teaching how, when, and why problem-solving strategies are used (DBE, 2011a: 8); in other words, teaching metacognition.

The problem, as a sample problem from the school syllabus, is representative of the types of mathematical problems that learners encounter in their studies. It is therefore also representative of the types of problems that pre-service Mathematics teachers should be able to solve and, more importantly, explain and thus demonstrate heuristic and metacognitive strategies for solving. The problem, as an authentic task, afforded an opportunity to measure the metacognitive knowledge and skills, as well as the problem-solving skills, of pre-service Mathematics teachers.

The pre-service teachers all attempted to solve the problem. As expected, the quality and quantity of comments differed from respondent to respondent. Surprisingly, the majority did not manage to solve the problem (see Section 4.4.2).

### ***3.4.3.3 Reliability and validity***

A valid and reliable measuring instrument is of crucial importance in research. Both validity and reliability reflect the degree to which error is present in measurement (Leedy & Ormrod, 2013: 92).

Reliability refers to whether a measuring instrument consistently generates a particular, consistent result when the entity under measurement remains unchanged (Leedy & Ormrod, 2013: 91). Of interest to the study is the internal consistency reliability, defined as the extent to which all the items within a single instrument yield similar results (Leedy & Ormrod, 2013: 91) or the extent to which all items in a test measure the same concept or construct (Tavakol & Dennick, 2011: 53). Internal consistency reliability is widely measured by Cronbach's alpha, displayed as a number between 0 and 1. Reliability shows the effect of measurement error on the score of a cohort rather than an individual learner (Tavakol & Dennick, 2011: 53). For example, if a test has a reliability of 0.70, it implies a 0.51 random error variance in the scores ( $0.7 \times 0.7 = 0.49$ ;  $1.00 - 0.49 = 0.51$ ) (Tavakol & Dennick, 2011: 53). The degree of reliability in a measure depends on the employment of the results. If the results are used for making decisions about a group or for research, scores with moderate reliability in range 0.5 to 0.6 may be acceptable (Ary et al., 2010: 248).

Guidelines utilised to interpret Cronbach's alpha values in the study were as follows:  $\alpha > 0.90$  as very highly reliable;  $\alpha > 0.80$  as highly reliable;  $\alpha > 0.7$  as reliable; and  $\alpha > 0.6$  as moderately reliable. Cronbach's alpha as the internal consistency or inter-item correlation of the questionnaire, with respect to the MAI total score, yielded consistent results for the original MAI by Schraw and Dennison (1994) ( $\alpha = 0.90$ ), the adapted MAI ( $\alpha = 0.91$ ), the translated MAI ( $\alpha = 0.94$ ), and the MAI in the main study ( $\alpha = 0.89$ ) (see Sections 4.2.1–4.2.3.2; Tables 4.1; 4.2).

Reliability is a necessary but insufficient condition for validity (Leedy & Ormrod, 2013: 92). The validity of a measuring instrument is the extent to which the instrument measures what it intends to measure (Leedy & Ormrod, 2013: 89) and whether one can draw meaningful and useful inferences from scores of a specific instrument (Creswell, 2009: 149). Importantly, validity is specific to the purpose for which an instrument is used and might not be valid in a different situation or for a different purpose (Ary et al., 2010: 235). The MAI of Schraw and Dennison (1994) has been developed to measure the metacognitive awareness of adolescents and adults; therefore, meaningful and useful interferences could be drawn from the scores of the pre-service teachers in the study.

In measuring the metacognitive awareness of pre-service teachers using the MAI, three traditional forms of validity are looked for in the MAI, as the validity of a measuring instrument can take different forms, each of which is important in different situations (Leedy & Ormrod, 2013: 89). These forms are content validity (do the items on the MAI measure the content they were designed to measure?), criterion validity (the extent to which the results of an assessment instrument correlate with the criterion: do scores predict a criterion measure, or do results correlate with other results?), and construct validity (do the items on the MAI measure the construct, namely metacognitive awareness, in the study?) (Creswell, 2009: 148; Leedy & Ormrod, 2013: 90).

Construct validity is the extent to which an instrument measures a characteristic which is not directly observable, but is assumed to exist based upon patterns in people's behaviour (Leedy & Ormrod, 2013: 90). That is, do items measure hypothetical constructs or concepts? Construct validity has become the overriding objective in validity (Creswell, 2014: 160). Metacognition, as "thinking about one's thinking", is such a construct. Therefore, good construct validity to measure the construct metacognition was of great importance to the study.

One strategy to maximise the validity of an instrument is to conduct a literature search for a standardised instrument that other researchers have used (Leedy & Ormrod, 2013: 92). The MAI of Schraw and Dennison (1994) has been subsequently used in various studies (e.g. Mevarech & Fridkin, 2006: 85–97; Sperling et al., 2004: 117–139; Young & Fry, 2008: 1–10). Another strategy to minimise error and enhance the validity of an instrument is the use of pilot studies to test an instrument, identify possible weaknesses, and modify if required (Leedy & Ormrod, 2013: 92).

In an experimental study investigating the effect of metacognitive intervention on learner metacognition and the Mathematics achievement of Grade 11 learners, Du Toit (2013) adapted the MAI to the South African educational context and to reflect a mathematical context. Hence, pilot studies were performed on the adapted MAI to enhance validity and reliability (see Section 4.2.2). The adapted MAI questionnaire was found to be highly reliable ( $\alpha = 0.91$ ) for the entire instrument, with reliabilities of  $\alpha = 0.81$  and  $\alpha = 0.89$  for *Knowledge of cognition* and *Regulation of cognition*

respectively. These are consistent with the findings in Schraw and Dennison's (1994: 471) studies (see Section 4.2.1).

For the study, the adapted MAI was translated into Afrikaans and piloted with a convenient sample of 57 fourth-year pre-service teachers to enhance the construct validity for the South African context (Pietersen & Maree, 2010b: 217). As these pre-service teachers studied Senior Phase Mathematics in their first year, it was expected that the mathematical terms and references would be familiar. Consequently, with regards to determining construct validity, no words were identified by the pilot group as unfamiliar. The face validity of the adapted and translated MAI was verified by a professional translator and editor. The same questionnaire with subcomponents and subscales was employed as in the case of the original and adapted MAI, and therefore criterion validity remained.

Cronbach's alpha values were calculated to determine the internal consistency and reliability of the translated and piloted instrument. The adapted MAI (translated and piloted in Afrikaans) was found to be very highly reliable ( $\alpha = 0.94$ ) for the instrument as a whole, highly reliable ( $\alpha = 0.86$ ) for *Knowledge of cognition*, and very highly reliable ( $\alpha = 0.91$ ) for *Regulation of cognition* (see Section 4.2.3; Table 4.1).

The reliability (trustworthiness) of the qualitative data was enhanced by implementing procedures to check transcripts and keep an audit trail of the think-aloud problem-solving session (Ary et al., 2010: 502–503). In the study, strategies or comments were identified in the think-aloud session and coded where they corresponded with the questions on the MAI, and afterwards were grouped under the subscales (see Section 4.4.2). Reliability was enhanced by continually comparing the codes and the definitions of the subscales in analysing the data (Creswell, 2009: 190). Additionally, sentences or strategies were identified according to the four-phase problem-solving model (see Sections 4.4.1; 4.4.3).

Validity (credibility) is a strength of qualitative research, and Creswell (2009: 191) recommends employing multiple validity strategies. As only one document—namely the think-aloud session—was analysed, validity could not be enhanced by triangulating different documents. However, validity is enhanced by giving rich, thick descriptions to convey the findings, and as this provides many perspectives on the

theme the results become more realistic and richer (Creswell, 2009: 192). Moreover, additional aspects that were considered included correct mathematical procedures, comprehension errors, and heuristic problem-solving strategies as they relate to the comments (see Section 4.4.2). Pre-service teachers could freely record their thoughts, and therefore display possible contradictory information. However, discrepant or negative data was not looked for per se.

Researcher bias in interpreting the findings could influence validity. As researcher, I was aware that some pre-service teachers might have wanted to impress me; therefore, it was explained that data would not influence their academic achievement and guaranteed anonymity. I was also aware that my own presumptions about the metacognitive awareness of students—based on my experience and knowledge of students in general and these respondents specifically, as well as my reading of literature—might influence the interpretation. To address this, a peer researcher participated in interpreting the data.

In qualitative research, generalisability is not the aim. On the contrary, the value of qualitative research is in the contribution of rich, thick descriptions and themes, especially those developed in a specific context (Creswell, 2009: 193). In the study, the metacognitive awareness of pre-service teachers in a Mathematics problem-solving session was explored to enrich the quantitative findings. However, these findings could not be generalised, as the pre-service teachers employed metacognitive strategies regarding this specific problem-solving session. The findings are therefore dependent on the context from which they came. Furthermore, these findings could not be generalised to the population due to it being a small sample ( $n < 100$ ).

#### **3.4.4 Ethical considerations**

The aim of ethical procedures is to provide information to participants so that they can make informed decisions about participating in research (Cohen, Manion, & Morrison, 2007: 55). The pre-service teachers were advised about the purpose of the research and its potential benefit to other cohorts was explained. They had the right to voluntary participation or withdrawal (Cohen et al., 2007: 55).

In the study, the pre-service teachers were not subject to discomfort, stressful situations, or invasion of privacy during the quantitative or qualitative inquiry. The venue where the data was collected was their usual lecture environment. The pre-service teachers completed the MAI and think-aloud session at the end of a lecture period and could do so anonymously. Furthermore, the MAI is a well-used standardised questionnaire on learning and problem solving, and as pre-service Mathematics teachers the respondents were exposed to problem solving on a regular basis, as required in the think-aloud session (Cohen et al., 2007: 52). Finally, official permission to conduct the research was obtained from the ethics committee of the higher education institution where the pre-service teachers were studying (Cohen et al., 2007: 55).

### **3.4.5 Data analysis and interpretation**

In order to organise and analyse quantitative data, generally in numerical format, descriptive and inferential statistics are used (Ary et al., 2010: 32). Descriptive statistics are used to organise and report statistical data. Descriptive statistics use procedures like averages, frequencies, percentages, and standard deviations (Cohen et al., 2007: 503–504; Pietersen & Maree, 2010c: 183–196). In the study, means, medians, standard deviations, percentages, and frequencies were determined (see Section 4.3). Advanced inferential statistics could not be applied as the sample was too small (Creswell, 2014: 163) and therefore generalisations to the population could not be made.

Statistical tests are performed based on whether data is parametric or non-parametric, the number of respondents, and the distribution of the data (Pietersen & Maree, 2010a: 225, 234, 237). Non-parametric data is often derived from questionnaires and surveys and no assumptions about population characteristics or the distribution of data are made, while parametric data is normally distributed and tends to be derived from experiments and tests (Cohen et al., 2007: 503). In the study, the quantitative data derived from the 5-point Likert scale on the MAI, which is frequently used in asking opinions or assessing attitudes and is ordinal and non-parametric.

To determine the level of metacognitive awareness of pre-service teachers, the means of the data on the MAI were determined to describe and present the average response



of the pre-service teachers on the instrument as a whole, and also for each subcomponent and the eight subscales. The level of metacognitive awareness of pre-service teachers is predominantly indicated by their mean score on the total MAI. The means on the subcomponents, *Knowledge of cognition* and *Regulation of cognition*, indicate whether the pre-service teachers report higher on metacognitive knowledge or on metacognitive skills. The median was also determined to account for the outlier effect. The median is useful for ordinal data if there are many scores (Cohen et al., 2007: 514) and overcome the problem of outliers in skewed results (see Table 4.3). A very small difference (less than 0.1) was found between the median and the mean.

The standard deviation (SD) is a measure of the dispersal of the scores, calculated as the square root of the variance. The standard deviation indicates the distribution of variance around the mean (see Section 4.3.1) and describes the variation in the responses of the pre-service teachers on the whole scale, for each subcomponent, and for the eight subscales.

The seven items with the highest and lowest means were identified. Individual items are not indicative of the respondents' broader level of metacognitive awareness; however, these items could point to some individual tendencies in the pre-service teachers' metacognitive learning and problem-solving strategies and skills (see Section 4.3.2). The frequency of responses (percentage) of all participants on these seven lowest and highest items was calculated for each of the categories on the rating scale (*Strongly Disagree*, *Disagree*, *Neutral*, *Agree*, *Strongly Agree*) (see Tables 4.4; 4.5).

Furthermore, an indication of overall disagreement and agreement was obtained. By adding the percentages in the two agreement categories (*Agree* and *Strongly Agree*) and the two disagreement categories (*Disagree* and *Strongly Disagree*), it could be ascertained whether there was more agreement or disagreement (Cohen et al., 2007: 510). For example, on the MAI, for Item 45 with the highest mean of 4.63—"I learn better when I am interested in a specific Mathematics topic"—all the pre-service teachers (100% in the combined *Agree* and *Strongly Agree* category) rated interest in a Mathematics topic as key for learning (see Table 4.4).

In addition, in the interpretation of the data, the employment of metacognitive knowledge and skills as an attribute of mathematical proficiency (see Section 2.3.4.2) and an aspect of productive learning pertaining to competent problem-solvers in Mathematics (see Section 2.3.4.3) was also identified and discussed (see Section 4.3.2).

The *Spearman Rho* correlation coefficient was utilised to determine the correlation between the two subcomponents—*Knowledge of cognition* and *Regulation of cognition*—of the MAI (see Section 4.3.1). *Spearman Rho* is used for non-parametric data on an ordinal scale (Ary et al., 2010: 354; Pietersen & Maree, 2010a: 237). The interpretation of the *Spearman Rho* and the Pearson Product Moment correlation coefficient (Pearson's  $r$ ) is similar. These correlation coefficients range from +1.00 (perfect positive relationship) to -1.00 (perfect negative relationship) with a value of 0 indicating no relationship (Ary et al., 2010: 129, 354; Pietersen & Maree, 2010a: 236).

When interpreting correlation coefficients, a statistically significant relationship does not imply causation. Changes in *Knowledge of cognition*, therefore, do not imply changes in *Regulation of cognition*. However, the correlation between *Knowledge of cognition* and *Regulation of cognition* in the study (see Section 4.3.1) indicates that both make a unique contribution to cognitive performance (Schraw & Dennison, 1994: 466).

The variability in the distributions and different sample sizes or different operational definitions could influence the correlation coefficient value and consequently the interpretation of the correlation coefficient (Ary et al., 2010: 135, 356). In the study, the correlation coefficient corroborated with the findings of the original MAI and the adapted MAI (see Sections 4.2.1; 4.2.2). In the interpretation, it should be kept in mind that a correlation coefficient value of 0.70 indicates a 49% related variance between two variables, whereas a correlation coefficient value of 0.50 only indicates a 25% related variance (Cohen et al., 2007: 535–536).

In the analysis of qualitative descriptive data, researchers generally attempt to arrive at a rich description from words, pictures, or occasional numeric data (Ary et al., 2010: 425–426). In the study, the qualitative data was the written words of the pre-service teachers in a mathematical think-aloud problem-solving session.

The aim was to provide a rich, thick description to enrich the quantitative findings of the MAI. Analysis of these statements of metacognitive knowledge and metacognitive skills provided a richer understanding of their employment in the problem-solving process (see Section 4.4).

The 41 pre-service teachers' thoughts, where each comment corresponded with a mathematical problem-solving step, were analysed. The pre-service teachers' comments were grouped according to items on the MAI. These items represent metacognitive knowledge and metacognitive strategies for learning and problem solving. The grouping was done by allocating a colour code to specific items that were identified. These items (metacognitive skills or metacognitive knowledge) are represented in the subscales on the MAI (see Appendix 3 for subscales comprising of specific items). The broader definition of metacognitive skills and metacognitive knowledge (as subscales) is referred to in the analysis (see Section 2.2.4; Appendix 3).

The frequency of items was determined to find the metacognitive knowledge (*Declarative knowledge, Procedural knowledge, Conditional knowledge*) and metacognitive skills (*Information management, Planning, Debugging, Monitoring, and Evaluation*) that were employed most often by the pre-service teachers (see Section 4.4; Appendix 6). For example, Item 2 ("I first consider different ways of solving a problem before I start solving a problem in Mathematics") is a metacognitive planning strategy under the subscale *Planning*. Other aspects that were noted in the interpretation of the qualitative data include correct and accurate mathematical procedures, comprehension errors, and thus the number of pre-service teachers who successfully solved the problem. Attributes of mathematical proficiency, which are heuristic problem-solving strategies, in combination with metacognition, resources, and affect were referred to as well (see Section 2.3.4.2).

In the second part of the analysis, the metacognitive comments and thoughts identified were compared to the four-phase problem-solving model (see Section 4.4.3). In analysing the findings, I anticipated identifying metacognitive skills and knowledge as they generally feature in the four-step problem-solving model. Specific metacognitive behaviours (knowledge, strategies, and skills) are normally associated with the four phases: *Orientation, Organisation, Execution, and Verifying*

(see Section 2.3.4.4). Consequently, metacognitive strategies (items) were identified and grouped under the metacognitive skills (subscales) on the MAI and subsequently compared to the four phases to identify metacognitive behaviour in each phase of the problem-solving model.

Literature indicates that correlations of questionnaires with think-aloud data are generally low (Schellings et al., 2013: 963). An explanation could be that the questionnaires and think-aloud sessions do not measure the same metacognitive activities (Schellings et al., 2013: 963). In addition, metacognitive activities are higher-order (Schellings et al., 2013: 985) and therefore difficult to measure, hence are instead inferred from self-report statements (Schellings et al., 2013: 968; Veenman et al., 2006: 6; see Sections 2.2.2; 3.4.3.2). To increase validity, the coding system should correspond with the subscales on the questionnaire, or the questionnaire should be constructed to measure the same metacognitive activities as the think-aloud session (Schellings et al., 2013: 963). In most instances, items on the MAI could be identified directly from the pre-service teachers' comments; however, in some cases these items were inferred from the comments (see Section 4.4). As metacognitive skills are utilised more overtly than metacognitive knowledge in solving problems, items generally relating to *Regulation of cognition* (metacognitive skills) were identified.

In the study, the think-aloud session was not correlated with the whole questionnaire for several reasons. The MAI assessed the metacognitive awareness (metacognitive knowledge and metacognitive skills) of pre-service teachers during the learning and problem solving of Mathematics. In contrast, the metacognitive behaviours in the think-aloud problem-solving session pertained primarily to problem solving, while learning behaviours remained largely covert.

Furthermore, for the two subcomponents, metacognitive skills were more overt in the practical solving of the Mathematics problem, whereas metacognitive knowledge referred mainly to the broader learning of Mathematics. Consequently, the qualitative think-aloud session was not correlated with the questionnaire, nor used to elaborate or explain the findings as is typically the case in a mixed methods design, but rather used to enrich the findings (see Sections 4.4; 4.5).

A possible limitation of interpreting the qualitative data is that only the thoughts that were reported on could be analysed. Some processes might have become automatised and therefore are covert and not reported. According to Desoete (2007: 717), in selecting a complex problem which requires higher-order thinking skills, greater metacognitive awareness is elicited (see Sections 2.2.2; 2.2.4.2). Lastly, for the quantitative data, findings could not be generalised to the population because of the size of the sample ( $n < 100$ ) and the type of data (non-parametric).

### **3.5 SUMMARY OF CHAPTER**

In this chapter, a post-positivist/interpretivist worldview informed the primarily quantitative approach, with qualitative data utilised to enrich the findings. The research methods comprised of a questionnaire for secondary research question 3, “What is the level of metacognitive awareness of pre-service Mathematics teachers on the MAI?”, and a think-aloud method for secondary research question 4, “What is the level of metacognitive awareness of pre-service Mathematics teachers in a problem-solving context?” The chapter has provided an explanation of how the research questions were answered by applying the indicated instruments.

In the next chapter, the data will be analysed and the findings discussed.

# CHAPTER 4

## PRESENTATION, ANALYSIS, AND INTERPRETATION OF THE QUANTITATIVE AND QUALITATIVE RESEARCH DATA

### 4.1 INTRODUCTION

This chapter analyses and interprets the data gathered to determine the level of metacognitive awareness of pre-service teachers. The following aspects are addressed: the reliability of the questionnaire in the pilot and main study; quantitative data analysis; qualitative data analysis; and the interpretation and summary of the data.

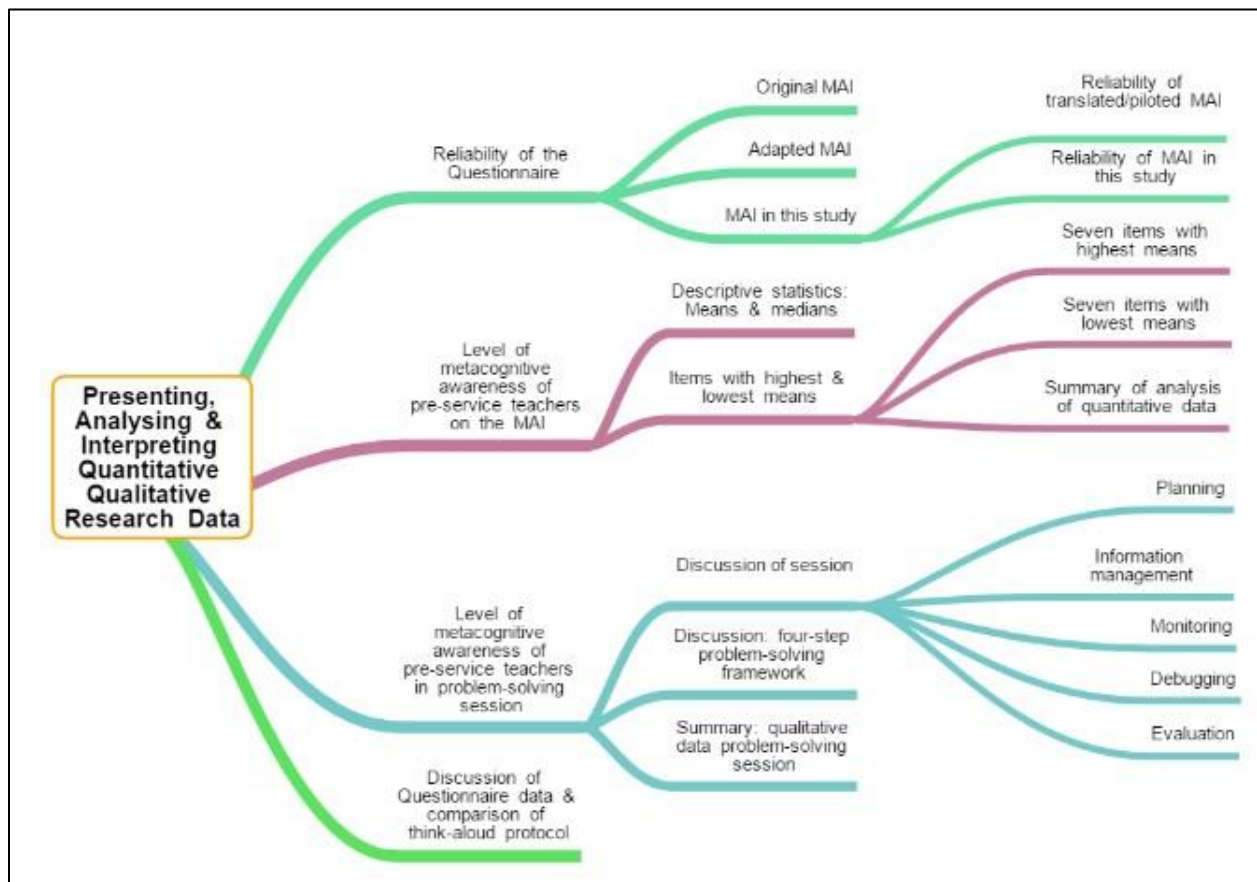


Figure 4.1: Presentation, analysis, and interpretation of qualitative and quantitative research data (see also Appendix 8)

The chapter contributes to exploring the primary research question of the study, “**What is the level of metacognitive awareness of pre-service Mathematics teachers?**”, by addressing secondary research questions 3 and 4:

- What is the level of metacognitive awareness of pre-service Mathematics teachers on the Metacognitive Awareness Inventory (MAI)?
- What is the level of metacognitive awareness of pre-service Mathematics teachers in a problem-solving context?

The chapter addresses these questions by presenting, analysing, and interpreting the quantitative data collected by means of the MAI (see Section 4.3), and then the qualitative data obtained through a think-aloud qualitative problem-solving session (see Section 4.4). These research activities were completed by pre-service Mathematics teachers at a higher education institution.

## **4.2 RELIABILITY OF THE QUESTIONNAIRE**

The standardised MAI developed by Schraw and Dennison (1994) was selected as the measuring instrument for the study (Leedy & Ormrod, 2013: 191). The reliability of this MAI, as used here and in previous research projects, will now be discussed.

The inventory has been demonstrated to be a reliable measure of metacognition in relation to academic learning tasks (Schraw & Dennison, 1994: 470–472; Sperling et al., 2004: 124). For the MAI within the context of the study, Cronbach’s alpha was calculated to determine the internal consistency of the entire questionnaire, the two subcomponents, and the eight subscales. Cronbach’s alpha is the most widely used measure of reliability with a lower limit of 0.70. For Social Sciences, if the results are used for making decisions about a group or for research, some researchers suggest a lower limit of 0.50 (Ary et al., 2010: 248, see Section 3.4.3.3).

Guidelines used to interpret Cronbach's alpha in the study were:

- 0.90 as very highly reliable
- 0.80 as highly reliable
- 0.70 as reliable
- 0.60 as moderately reliable
- Less than 0.60 as low reliability.

In the following sections, the reliability of the MAI in English (as used in previous studies) is explained. As both English and Afrikaans are languages used to teach at the participating higher education institution, questionnaires in both languages were used. The reliability of the pilot MAI in Afrikaans and the reliability of the questionnaire in the main study are discussed.

#### **4.2.1 The Metacognitive Awareness Inventory (MAI)**

As noted previously, the MAI of Schraw and Dennison (1994) measures adult learners' metacognitive awareness. Schraw and Dennison's (1994) questionnaire comprises of 52 items that are divided into two subcomponents: *Knowledge of cognition* and *Regulation of cognition*. Respondents rate their metacognitive awareness on a bipolar rating scale.

The *Knowledge of cognition* component measures awareness of one's abilities, knowledge about strategies, and the conditions under which those strategies are applicable; that is, how, when, and why to use these strategies. The *Regulation of cognition* component measures how one plans, implements, develops strategies, monitors, corrects comprehension, and evaluates one's learning and strategy use (Schraw & Dennison, 1994: 466).

Schraw and Dennison (1994: 472) found that the MAI offers a reliable initial test of metacognitive awareness among adults and college students. They further stated that the MAI is a helpful measure to determine the metacognitive awareness of adult learners in particular, in view of planning subsequent metacognitive training. In two experiments, Schraw and Dennison (1994: 464, 468) found that the reliability of the total



MAI is highly reliable, with Cronbach's alpha reaching 0.95 and 0.93 respectively for the two consequent experiments. As initially proposed, they further reported a statistically significant relationship between the two subcomponents—*Knowledge of cognition* and *Regulation of cognition*—on the MAI. This supported the two-component view of metacognition as described in literature by Brown (1987) and Flavell (1979).

Moreover, in Schraw and Dennison's two studies (1994: 464, 467–468), *Knowledge of cognition* and *Regulation of cognition* displayed a strong correlation of  $r = 0.54$  and  $r = 0.45$ , significant at the  $p < 0.05$  level, respectively. The authors clarify their findings by suggesting that knowledge and regulation work together to help learners self-regulate, and that both *Knowledge of cognition* and *Regulation of cognition* make a unique contribution to cognitive performance (Schraw & Dennison, 1994: 466, 471).

The MAI, consequently, has been used in other research studies (Mevarech & Fridkin, 2006: 85–97; Sperling et al., 2004: 117–139; Van Der Walt, 2014: 9; Young & Fry, 2008: 1–10). In a subsequent study using the MAI, Sperling et al. (2004: 124) examined relationships among metacognitive components. Sperling et al. (2004) reported that *Knowledge of cognition* showed a strong correlation with *Regulation of cognition* ( $r = 0.75$ ,  $p < 0.001$ ). They found that this correlation between *Knowledge of cognition* and *Regulation of cognition* is much higher than that determined in Schraw and Dennison's (1994) studies (i.e.  $r = 0.54$  and  $r = 0.45$ ).

However, individually the eight subscales of the MAI were not found to be very reliable measures of metacognitive awareness (see Section 3.4.3.1.1). Consequently, the total MAI scores, as well as the *Knowledge of cognition* scores and the *Regulation of cognition* scores, are mainly used in the study.

Over the next sections, the adapted MAI for the South African Mathematics education context (see Section 4.2.2), the translated MAI (i.e. the adapted MAI translated into Afrikaans and piloted; see Section 4.2.3), and the findings of the main MAI (see Section 4.3) within the study are discussed.

### 4.2.2 The adapted MAI

The MAI of Schraw and Dennison (1994) was adapted to the South African educational context by Du Toit in 2013. In this project, Du Toit (2013) provided a twofold rationale for adapting the MAI. First, it was adapted to change unfamiliar words into more recognisable words for the South African education environment. Second, as the original MAI was a measure of general metacognitive awareness, the questionnaire was subsequently adapted to specifically reflect a mathematical context.

This adapted MAI was further modified by incorporating the feedback from two university lecturers and two pilot groups from two different schools. The rating scale of the adapted MAI is a five-point Likert scale featuring the categories *Strongly Disagree*, *Disagree*, *Neutral*, *Agree*, and *Strongly Agree*. This is a modification from the 100mm, bipolar rating scale of the original MAI of Schraw and Dennison (1994: 463).

The reliability of Du Toit's (2013) adapted MAI was tested among two pilot groups of Mathematics learners in Grade 11 in two different schools. The **adapted MAI pilot questionnaire** was found to be highly reliable ( $\alpha = 0.91$ ) for the instrument overall and presented  $\alpha = 0.81$  and  $\alpha = 0.89$  for *Knowledge of cognition* and *Regulation of cognition* respectively (see Section 3.4.3.3). Consequently, in Du Toit's (2013) main study, the reliability of the adapted MAI questionnaire used for the pre-test was computed for the MAI total score as  $\alpha = 0.89$ , for *Knowledge of cognition* as  $\alpha = 0.82$ , and for *Regulation of cognition* as  $\alpha = 0.83$ . On Du Toit's post-test, the MAI total score ( $\alpha = 0.93$ ) was found very highly reliable, with *Knowledge of cognition* as  $\alpha = 0.82$  and *Regulation of cognition* as  $\alpha = 0.91$ . These values correspond with the reliability of Schraw and Dennison's (1994) original MAI (see Sections 3.4.3.1.1; 3.4.3.1.2).

### 4.2.3 Reliability of the MAI in the study

In the study, the respondents were pre-service teachers at a parallel-medium higher education institution. The MAI as adapted by Du Toit (2013; see Section 4.2.2) was translated into Afrikaans and piloted among 57 fourth-year pre-service teachers (see Sections 3.4.3.1.3; 4.2.3.1). Appendix 2 shows the translated MAI.

### 4.2.3.1 Reliability of the translated and piloted MAI

The reliability of the pilot MAI questionnaire in Afrikaans is shown in Table 4.1.

**Table 4.1: Cronbach's alpha values for the pilot MAI**

Metacognitive scale	Number of items	Pilot (N=57)
<b>MAI total</b>	<b>50</b>	<b>0.94</b>
<b>Knowledge of cognition</b>	<b>17</b>	<b>0.86</b>
Declarative knowledge	8	0.81
Procedural knowledge	4	0.72
Conditional knowledge	5	0.70
<b>Regulation of cognition</b>	<b>33</b>	<b>0.91</b>
Planning	7	0.70
Information management	9	0.80
Monitoring	7	0.59
Debugging	4	0.68
Evaluation	6	0.53

Table 4.1 shows that the internal consistency of the questionnaire with respect to the MAI total score was very highly reliable, with the highest Cronbach alpha value of 0.94. The subcomponents *Regulation of cognition* ( $\alpha = 0.91$ ) and *Knowledge of cognition* ( $\alpha = 0.86$ ) were very highly reliable and highly reliable, with the second and third highest Cronbach alpha values respectively. The Cronbach alpha values of the eight subscales ranged from 0.53 (*Evaluation*) to 0.81 (*Declarative knowledge*), with *Monitoring* and *Evaluation* not very reliable measures of metacognitive awareness. This corresponds with findings in Schraw and Dennison's (1994) experiments (see Section 3.4.3.1.1).

### 4.2.3.2 Reliability of the MAI in the main study

As mentioned in Section 3.4.2, for the main study data was collected from 41 pre-service Mathematics teachers in their final year. The reliability of the MAI in the main study was computed and is displayed in Table 4.2.

**Table 4.2: Cronbach's alpha values for the MAI**

Metacognitive scale	Number of items	Main (N=41)
<b>MAI total</b>	<b>50</b>	0.89
<b>Knowledge of cognition</b>	<b>17</b>	0.83
Declarative knowledge	8	0.78
Procedural knowledge	4	0.63
Conditional knowledge	5	0.24
<b>Regulation of cognition</b>	<b>33</b>	0.85
Planning	7	0.57
Information management	9	0.79
Monitoring	7	0.63
Debugging	4	0.61
Evaluation	6	0.33

Table 4.2 presents Cronbach alpha values for the main study. A highly reliable score was obtained for the MAI as instrument with a Cronbach alpha value of 0.89. *Regulation of cognition* ( $\alpha = 0.85$ ) and *Knowledge of cognition* ( $\alpha = 0.83$ ) provided the second and third highest values respectively. These results confirm the results of Schraw and Dennison (1994) and other studies (see Section 4.2.1) which indicated that the MAI as an instrument is a highly reliable measure of metacognitive awareness. The high reliability is supported by the highly reliable subcomponents *Regulation of cognition* and *Knowledge of cognition*.

The eight subscales were found to be less reliable measures of metacognitive awareness, in line with the findings of Schraw and Dennison (1994) and other studies (see Section 4.2.1). The Cronbach alpha values of the eight subscales ranged from 0.24 (*Conditional knowledge*) to 0.79 (*Information management*). *Declarative knowledge* (0.78) and *Information management* (0.79) were found to be reliable; *Procedural knowledge* (0.63), *Debugging* (0.61), and *Monitoring* (0.63) were moderately reliable; and *Conditional knowledge* (0.24), *Planning* (0.57), and *Evaluation* (0.33) displayed low reliability.

It must be noted that the reliability of constructs, as per the subscales, can be influenced by a low number of items in the constructs and the degree of inter-correlation between the items (see Section 3.4.3.3). Table 4.2 depicts a reliable value (that is, a Cronbach's alpha greater than 0.70) for the subscales with more than seven items on the MAI. For results that are used to make decisions regarding a group, or for research purposes, scores with a reliability in the range of 0.50 to 0.60 might be acceptable, according to Ary et al. (2010: 248; see Section 3.4.3.3). Moreover, it must also be noted that the subscales were shown to be reliable in other research (see Van Der Walt, 2014: 11–13).

In conclusion, in the study a reliability of 0.89 was indicated for the instrument overall, which makes the data obtained highly reliable. Furthermore, the two main findings of Schraw and Dennison (1994: 461–466) substantiate the results of the pilot MAI and the main MAI in the study. These findings underline the high reliability of the MAI as an instrument for measuring overall metacognitive awareness, as well as of *Knowledge of cognition* and *Regulation of cognition*. However, the reliability for individually assessing the level of metacognitive awareness of the eight subscales is lower.

When discussing research data, descriptive statistics are important in the analysis and interpretation of quantitative data. This is discussed in the following section.

### **4.3 THE LEVEL OF METACOGNITIVE AWARENESS OF PRE-SERVICE TEACHERS ON THE MAI**

Descriptive statistics use methods like averages, frequencies, percentages, and standard deviations to organise and report statistical data (Cohen et al., 2007: 503–504; Leedy & Ormrod, 2013: 191; Pietersen & Maree, 2010a: 224–254; see Section 3.4.5). The means, median, standard deviation, and frequency of the data collected in the study are henceforth discussed.

### 4.3.1 Descriptive statistics: The means and medians

The means of the data from the main investigation were collected through administering the MAI (see Appendix 2). The means are used to describe the average response on all the MAI items per subcomponent, per subscale, and per individual item.

**Table 4.3: Mean, Standard Deviation, and Median values for the MAI**

Metacognitive scale	Mean	Standard Deviation	Median	Difference between mean and median	Number of Items
<b>MAI total</b>	<b>3.73</b>	<b>0.41</b>	<b>3.78</b>	<b>0.05</b>	<b>50</b>
<b>Knowledge of cognition</b>	<b>3.82</b>	<b>0.52</b>	<b>3.88</b>	<b>0.06</b>	<b>17</b>
Declarative knowledge	3.80	0.62	3.75	0.05	8
Procedural knowledge	3.78	0.68	4.00	0.22	4
Conditional knowledge	3.89	0.48	3.80	0.09	5
<b>Regulation of cognition</b>	<b>3.64</b>	<b>0.41</b>	<b>3.67</b>	<b>0.03</b>	<b>33</b>
Planning	3.72	0.58	3.71	0.01	7
Information management	3.75	0.59	3.78	0.03	9
Monitoring	3.35	0.59	3.29	0.06	7
Debugging	4.03	0.63	4.00	0.03	4
Evaluation	3.33	0.51	3.33	0.00	6

The statistical data in Table 4.3 indicates the level of metacognitive awareness of pre-service teachers in terms of the mean and median.

The focus of the study was determining the level of metacognitive awareness of pre-service teachers. The 41 pre-service teachers obtained a mean score of 3.73 on the total MAI (SD = 0.41). The *Knowledge of cognition* component has a mean of 3.82 (SD = 0.52) and the *Regulation of cognition* component has a mean of 3.64 (SD = 0.41) (for comparison, see Section 4.2.1). The means on the subscales range from 3.33 to 4.03, with *Evaluation* obtaining the lowest mean and *Debugging* the highest.

It is worth mentioning that in determining the average response in small samples, as per the 41 pre-service teachers in the study, the median might be regarded as a more

suitable measure as it is not influenced by outliers, as is the mean in small samples. The difference between the mean and the median was therefore calculated in the study. Except in the case of *Procedural knowledge*, the difference between the mean and median values (of the metacognitive subcomponents and the subscales) is less than 0.1 points. *Procedural knowledge* has a difference of 0.22. The medians of the total MAI, *Knowledge of cognition*, and *Regulation of cognition* are 3.78, 3.88, and 3.67 respectively. These medians are all higher than their corresponding means, but with a difference of less than 0.1 points.

The standard deviation, ranging from 0.41 to 0.68, indicates the distribution of the variance around the mean (see Section 3.4.5). The small standard deviation of the MAI components indicates that variations in the responses were small.

The *Spearman Rho* correlation coefficient, used for non-parametric data on an ordinal scale, was used to determine the correlation between the two subcomponents of the MAI, namely *Knowledge of cognition* and *Regulation of cognition* (Ary et al., 2010: 354; Pietersen & Maree, 2010a: 237; see Section 3.4.5). *Knowledge of cognition* showed a strong correlation with *Regulation of cognition* ( $r = 0.54$ ,  $p < 0.05$ ). This correlation corroborates with the findings of Schraw and Dennison's (1994) two studies (see Section 4.2.1). However, a statistically significant relationship does not imply causation (Ary et al., 2010: 354; see Section 3.4.5), as both factors make a unique contribution to cognitive performance (Schraw & Dennison, 1994: 466). In the study, *Knowledge of cognition* displayed a higher mean than *Regulation of cognition*, implying that the knowledge component of metacognitive awareness is more prominent among the pre-service teachers than the regulatory component.

In summary, the level of metacognitive awareness of pre-service teachers is primarily indicated by the mean of the total MAI. This mean indicates that pre-service teachers are metacognitively aware to a certain extent: the pre-service teachers perceive their metacognitive awareness to be at a level of 74.6%. This finding was somewhat expected, as literature establishes that adults generally use metacognitive processes to some degree (see Section 2.2.4.3). The question arises, therefore, whether the level of these pre-service teachers' metacognitive awareness is adequate to demonstrate

success during learning and problem solving. The level to which the pre-service teachers employ their metacognitive knowledge and metacognitive regulatory skills during a problem-solving session will be discussed in Section 4.4. Meanwhile, a discussion of the items with the lowest and highest response rates on the MAI will now be presented, highlighting individual tendencies in Mathematics learning and problem solving by the pre-service teachers.

### **4.3.2 Items with the highest and lowest means**

The level of pre-service teachers' metacognitive awareness is the focus of the study and is indicated by the results of their MAI total scores. Individual items on the MAI are not an indication of a person's broader level of metacognitive awareness, as seen in the MAI total score. However, in identifying the respective items with the lowest and highest means on the MAI, individual tendencies in the level of metacognitive awareness can be discussed. This provides more detailed insight into the views of the pre-service teachers on their own learning and problem solving in Mathematics.

The seven items with the highest and lowest means are discussed in relation to literature on productive learning and competent problem solving as indicators of mathematical proficiency (see Sections 2.3.4.2; 2.3.4.3; 2.3.4.4). Also discussed are other factors emerging that may contribute to the pre-service teachers agreeing or disagreeing on the Likert scale, as well as their possible impact on learning and problem solving.

#### ***4.3.2.1 The seven items with the highest means***

Table 4.4 displays the seven items with the highest means in rank order. A discussion of each item follows thereafter.

**Note:** *f* indicates the frequency of responses on the Likert scale for each of the categories and *#* indicates the number of pre-service Mathematics teachers represented by the responses in the combined *Agree* and *Strongly Agree* category.



**Table 4.4: The seven items with the highest means in the MAI**

Subscale	Mean of Item	Standard Deviation	SD+D	SD	D	N	A	SA	A+SA	A+SA
			<i>f</i>	<i>f</i>	<i>F</i>	<i>f</i>	<i>f</i>	<i>f</i>	<i>f</i>	#
45. I learn better when I am interested in a specific Mathematics topic.										
<i>Declarative knowledge</i>	<b>4.63</b>	0.49	0%	0%	0%	0%	37%	63%	100%	41
15. I learn best when I already know something about the Mathematics topic I am studying.										
<i>Conditional knowledge</i>	<b>4.51</b>	0.60	0%	0%	0%	5%	39%	56%	95%	39
41. I read the question carefully before I answer a Mathematics question.										
<i>Planning</i>	<b>4.46</b>	0.64	0%	0%	0%	7%	39%	54%	93%	38
50. When I read a Mathematics question, I stop and reread any section of the question that is not clear.										
<i>Debugging</i>	<b>4.46</b>	0.92	8%	3%	5%	0%	29%	63%	92%	38
3. When I solve a Mathematics problem, I try to use methods of solving a problem that have worked in the past.										
<i>Procedural knowledge</i>	<b>4.44</b>	0.92	6%	3%	3%	7%	24%	63%	87%	36
9. I read slower when I encounter important information in a Mathematics question										
<i>Information management</i>	<b>4.34</b>	0.79	5%	0%	5%	5%	41%	49%	90%	37
26. I can motivate myself to study for a Mathematics test or examination										
<i>Conditional knowledge</i>	<b>4.29</b>	0.90	6%	3%	3%	6%	39%	49%	88%	36

Table 4.4 shows that Item 45 (*Declarative knowledge*) has the highest mean of 4.63 and Item 15 (*Conditional knowledge*) has the second highest mean of 4.51. Both Item 41 (*Planning*) and Item 50 (*Debugging*) have means of 4.46. Item 41 is ranked third, above Item 50, as overall 93% of the pre-service teachers agreed or strongly agreed with the statement. The next three items are Item 3 (*Procedural knowledge*), Item 9 (*Information*

*management*), and Item 26 (*Conditional knowledge*), with means of 4.44, 4.34, and 4.29 respectively.

As noted above, Item 45 (*Declarative knowledge*) has the highest mean of 4.63. This item states “I learn better when I am interested in a specific Mathematics topic”. With 41 pre-service teachers (100%) rating in the combined *Agree* and *Strongly Agree* category, all respondents clearly believe that it is important to be interested in a Mathematics topic to learn it well. Fewer indicated that they could motivate themselves to study for Mathematics (88% for Item 26). Affect is a key attribute in mathematical proficiency (see Section 2.3.4.2.4) and impacts on productive learning (see Section 2.3.4.3). Item 45, therefore, points to an awareness of strengths and weaknesses in learning and problem solving. The pre-service teachers were aware of the strength of being interested in a topic which provides the impetus for studying (see Section 2.3.4.2.4). On the other hand, when learners do not get involved in a task, do not display adequate effort, or lose interest in the task, it may be indicative of the unsuccessful implementation of regulatory metacognitive skills, as the managing and control aspect of metacognition. Additionally, the item with the seventh highest mean, Item 26 (*Conditional knowledge*), supported this reflection by the pre-service teachers on the important aspect of affect.

The item with the second highest mean of 4.51 is Item 15 (*Conditional knowledge*), where 95% of the pre-service teachers reported that their best learning occurs when they already know something about the Mathematics topic. Here 39 pre-service teachers featured in the combined *Agree* and *Strongly Agree* category. This observation points to the knowledge of the pre-service teachers regarding when and why they use learning procedures (*Conditional knowledge*).

Item 15 indicates awareness that prior knowledge supports further learning and as such corroborates with the constructive aspect of productive learning in Mathematics (see Section 2.3.4.3.1). Moreover, prior knowledge as part of a sound and well-resourced knowledge base of facts, formulae, and procedures plays a significant role in mathematical proficiency (see Section 2.3.4.2.1). Knowledge of the topic may also refer to experience with mathematical concepts in the pre-service teacher’s environment

which relate to the situated aspect of productive learning (see Section 2.3.4.3.3). Hence, the utility value of Mathematics in real life may generate interest, which pre-service teachers have indicated provides motivation for studying.

Item 41 (*Planning*), with the third highest mean of 4.46, points to the pre-service teachers' awareness of the importance of understanding a question before attempting to answer it. Here 93% reported that they "read the question carefully before they answer a Mathematics question". That is, 38 pre-service teachers in the combined *Agree* and *Strongly Agree* category generally use focused reading as a strategy to understand a problem statement. The careful reading of a problem before answering the question also refers to the goal of gaining understanding of a problem before selecting strategies and methods to attempt to solve it. Making sense of a question is of the utmost importance in planning and orienting oneself to solve a problem (see Section 2.3.4.4.1) and in the subsequent selection of suitable strategies, i.e. the allocation of resources prior to learning or problem solving (see Section 2.3.4.2.1). Furthermore, the skilful selection of appropriate heuristic strategies is a hallmark of a competent problem-solver (see Section 2.3.4.2). Overall, *Planning* is a metacognitive skill that regulates cognition by setting local and global goals and selecting metacognitive strategies and problem-solving strategies (heuristics) to attempt to solve a problem (see Section 2.2.4.3.2).

Item 50 (*Debugging*) has the same mean as Item 41 and is similar to this item, as both refer to the initial discerned reading of a Mathematics question. Item 50 involves the rereading of a statement. For Item 50, 92% of the pre-service teachers stated that they pause and reread any part of the question which is unclear. This indicates their awareness of the importance of thoroughly understanding a Mathematics problem statement, which is achieved by reading it again for clarification before attempting to answer it. This *Debugging* strategy to correct comprehension was reported by 38 of the pre-service teachers in the combined *Agree* and *Strongly Agree* category. Moreover, the item with the sixth highest mean, Item 9 (*Information management*), further illustrates their awareness of the significance of understanding a question before attempting to answer it; that is, 90% of the pre-service teachers reported that they

read the question slower when encountering important information in a Mathematics question.

These items point cumulatively to the reflection of the pre-service teachers on their reading and understanding of a problem statement. This metacognitive monitoring of one's cognition is an attribute of mathematical proficiency (see Sections 2.3.4.2; 2.3.4.2.3). Correspondingly, it may point to their awareness that making sense of a question is important in planning how to solve a problem and selecting appropriate strategies, such as the allocation of resources prior to learning or problem solving (i.e. *Planning*; see Section 2.2.4.3.2). Moreover, they are aware that the rereading of an unclear section is an important metacognitive strategy to correct comprehension and performance errors in learning or problem solving (i.e. *Debugging*; see Section 2.2.4.3.2). Furthermore, it is argued that these responses may stem from the pre-service teachers' misreading and misunderstanding of previous questions as well as their awareness of the assumed level of difficulty of Mathematics problem statements.

Item 3 (*Procedural knowledge*) has the fifth highest mean of 4.44, with 87% of pre-service teachers featured in the combined *Agree* and *Strongly Agree* category. That is, 36 stated that when they solve a Mathematics problem, they attempt using methods of problem solving that have worked previously. This possible awareness of methods and strategies used previously during problem solving indicates knowledge of how to implement learning procedures, such as strategies and methods, as well as how to access factual knowledge. This reflection on previous methods is an important aspect in the final phase, *Verifying*, of the problem-solving model, namely evaluating the solution for reasonableness and accuracy, but also knowing how to apply the method utilised in later problems, and as such is an aspect of mathematical proficiency (see Sections 2.3.4.2.3; 2.3.4.4.4).

In summary, the pre-service teachers were aware that relating topics to other topics (Item 15) or to contextual problems (Item 45) facilitates their learning. They were aware that the monitoring of one's thinking to understand and interpret a question (as indicated in Items 41, 50, and 9) is very important as it precedes and facilitates the selection of

appropriate and effective strategies and methods (Item 3). On the other hand, if the problem statement is misinterpreted, or if there is no awareness of an adequate repertoire of strategies and methods, this may result in failure to solve the problem correctly or loss of interest.

The adept use of methods that have worked previously, as well as the use of alternative methods, is an attribute of mathematical proficiency (see Section 2.3.4.2). Experts have a well-resourced knowledge base of heuristic strategies and procedures (see Sections 2.3.4.2.1; 2.3.4.2.2). Moreover, experts are not only aware of this knowledge base, but are also competent in accessing and applying these methods and strategies through metacognitive reflection (see Sections 2.3.4.2; 2.3.4.2.3). This awareness of knowledge (*Knowledge of cognition*) of strategies must be transferred into the actual implementation of relevant and appropriate strategies and methods (*Regulation of cognition*) during problem solving. Metacognitive reflection as adaptive competence involves this transferring of knowledge into action in new contexts (see Section 2.3.4.2). Metacognitive skills that regulate the effective use of strategies and methods are crucial to productive learning and problem solving (see Sections 2.3.4.2.3; 2.3.4.3.2).

#### ***4.3.2.2 The seven items with the lowest means***

Table 4.5 displays the seven items with the lowest means in rank order. A discussion of each item follows.

**Note:** Once again *f* indicates the frequency of responses on the Likert Scale for each of the categories and *#* indicates the number of Mathematics pre-service teachers represented by the responses in the combined *Disagree* and *Strongly Disagree* category.

**Table 4.5: The seven items with the lowest means in the MAI**

Subscale	Mean of Item	Standard Deviation	D+SD	SD	D	N	A	SA	A+SA	D + SD
			f	f	F	f	f	f	f	#
19. After I have solved a Mathematics problem, I ask myself if there was an easier way to solve the problem.										
<i>Evaluation</i>	<b>2.76</b>	1.18	58%	7%	51%	10%	22%	10%	32%	24
38. After I have solved a Mathematics problem, I ask myself whether I have considered different ways to solve the problem.										
<i>Evaluation</i>	<b>2.76</b>	1.02	51%	5%	46%	22%	22%	5%	27%	21
28. I ask myself how useful my learning strategies are while I study for a Mathematics test or examination.										
<i>Monitoring</i>	<b>2.98</b>	1.13	44%	5%	39%	19%	27%	10%	37%	18
31. I create my own examples to make new information I receive in Mathematics more meaningful and understandable.										
<i>Information management</i>	<b>3.02</b>	1.17	39%	10%	29%	17%	37%	7%	44%	16
7. I know how well I did once I have finished a Mathematics test or examination										
<i>Evaluation</i>	<b>3.22</b>	1.19	34%	7%	27%	15%	39%	12%	51%	14
48. I ask myself questions about how well I am doing while solving a Mathematics question.										
<i>Monitoring</i>	<b>3.24</b>	1.09	32%	3%	29%	22%	34%	12%	46%	13
11. I ask myself if I have considered different methods of solving a problem when solving a Mathematics problem.										
<i>Monitoring</i>	<b>3.24</b>	1.11	29%	5%	24%	25%	34%	12%	46%	12

Table 4.5 shows that Item 19 (*Evaluation*) has the lowest mean of 2.76. This means that 58% of the pre-service teachers disagreed or strongly disagreed with the statement. Item 38 (*Evaluation*) has the same mean of 2.76 as Item 19, but is ranked second lowest as overall 51% of the pre-service teachers disagreed or strongly disagreed with the statement.

Item 28 (*Monitoring*) has the third lowest mean of 2.98 and Item 31 (*Information management*) the fourth lowest mean of 3.02. The next three items with the lowest means in rank order are Item 7 (*Evaluation*), Item 48 (*Monitoring*), and Item 11 (*Monitoring*), with means of 3.22, 3.24, and 3.24 respectively. Item 48 and Item 11 share the same mean, and overall 32% and 29% of the pre-service teachers respectively disagreed or strongly disagreed with the statement.

The two lowest ranking items on the administered MAI are Items 38 and 19, both *Evaluation* items. Both display a mean of 2.76 and standard deviations of 1.02 and 1.18 respectively. On analysing the number of pre-service teachers featured in the combined *Disagree* and *Strongly Disagree* category, 51% stated that after they have solved a problem they do not ask themselves whether they have considered different ways of solving it (Item 38), while 58% stated that after they have solved a problem, they do not ask themselves whether there was an easier way of solving it (Item 19). Thus, according to the frequency of responses, Item 19 should rank lowest and Item 38 second lowest. As both items belong to the subscale *Evaluation*, and both relate to reflection on performance and strategy effectiveness after a learning experience, they are discussed together.

*Evaluation* is an important metacognitive skill in analysing one's performance and strategy effectiveness. The skilful use of a different (Item 38) or easier (Item 19) method is an attribute of mathematical proficiency (see Section 2.3.4.2). Competent problem-solvers who are mathematically proficient have a well-resourced knowledge base of heuristic strategies and problem-solving methods, which they are aware of and reflect upon before, during, and after the problem-solving process (see Sections 2.3.4.2; 2.3.4.2.1; 2.3.4.2.2). Pre-service teachers participating in the study ranked themselves low with regards to reflecting on different or easier methods (Items 38 and 19); therefore, the question arises whether they would monitor their progress and evaluate their final solutions during the problem-solving session. This suggests that the pre-service teachers may be unlikely to evaluate their answers for reasonableness, and unlikely to evaluate the problem-solving method for effectiveness

by finding easier (Item 19) or different (Item 38) ways to solve the problem, and hence apply them to other similar problems.

The third lowest mean of 2.98 belongs to Item 28 (*Monitoring*), with 18 pre-service teachers in the combined *Disagree* and *Strongly Disagree* category. *Monitoring* is the assessment of one's learning or strategy use. Here 44% of the pre-service teachers stated that while they study for a Mathematics test or examination, they do not ask themselves how useful their learning strategies are.

As an attribute of mathematical proficiency, monitoring is reflecting upon and assessing one's own learning or strategy use during the learning or problem-solving process (see Section 2.3.4.2.3). This also relates to the self-regulatory component of productive learning (see Section 2.3.4.3.2). Analysing one's method of problem solving is crucially important for deciding upon subsequent action and whether to accept the solution as correct or not. Consequently, attempting different ways to correct performance or finding easier ways to use it in subsequent similar problems is an important strategy in problem solving.

Item 31 (*Information management*) has the fourth lowest mean of 3.02, with 16 pre-service teachers landing in the combined *Disagree* and *Strongly Disagree* category. Item 31 displays a lower frequency (39%) of pre-service teachers who do **not** create their own examples to make new Mathematics information that they receive more meaningful and understandable. *Information management* involves skills and strategies applied during the problem-solving process. Managing information by creating one's own examples, relating it to similar problems, or rephrasing the problem statement is an important metacognitive skill. This skill aligns with the constructive and self-regulating aspects of productive learning (see Sections 2.3.4.3.1; 2.3.4.3.2) where new knowledge becomes more meaningful by constructing one's own examples in relation to existing prior knowledge.

Item 7 (*Evaluation*) has the fifth lowest mean (3.22), with 34% of pre-service teachers in the combined *Disagree* and *Strongly Disagree* category. This means that a third of pre-service teachers did not know how well they performed once they finished their



examination. Consequently, the question arises (as it does in the items with the lowest ranks) whether the pre-service teachers are able to reflect on and evaluate the effectiveness of their strategies and correctness of their solution in this context.

Lastly, Item 48 and Item 11 both have the same mean of 3.24, with 13 and 12 pre-service teachers respectively in the combined *Disagree* and *Strongly Disagree* category. These items relate to *Monitoring* through self-questioning, with 32% of the pre-service teachers stating that they do not monitor how well they are doing, while 29% do not reflect on different problem-solving methods whilst solving a Mathematics problem.

Judging one's own performance after a learning or problem-solving experience is vitally important in deciding the next course of action. The analysis of one's method of problem solving is a key metacognitive skill in deciding whether to accept a solution or attempt to find ways to correct the answer (see Section 2.3.4.4.4). It is therefore argued that, as per the two items with the lowest means (Items 19 and 38), a low awareness reported to judge one's own performance or progress may result in inadequate metacognitive skillfulness in employing *Evaluation* skills.

In summary, the low means may indicate that pre-service teachers are not likely to monitor their progress and their learning strategies effectively while studying for a Mathematics test or examination. As Items 38 and 19 indicate, more than half of the pre-service teachers disagreed that they reflect by means of self-questioning on the effectiveness of their strategies or methods for solving problems after a problem-solving session. Likewise, according to Item 7, a third of pre-service teachers reported not sufficiently evaluating their performance after they solved the problem. This implies a lack of awareness of the effectiveness of their strategies or methods for problem solving, as well as comprehension and calculation errors. In addition, for Item 28, almost half were unlikely to reflect on their strategy effectiveness while studying Mathematics.

### ***4.3.2.3 Summary of an analysis of the quantitative data***

As demonstrated by the total MAI score, the pre-service teachers reported a high level of metacognitive awareness and a higher level of metacognitive knowledge than metacognitive skills (see Section 4.3.1). Comparing the means on the overall MAI with the means of the highest and lowest items reveals that the items with the highest means featured mainly in the subcomponent *Metacognitive knowledge* and related to the subscales *Declarative*, *Procedural*, and *Conditional knowledge*. For the subcomponent *Metacognitive skills*, items with the highest means related to the subscales *Planning* and *Information management*, primarily in relation to reading the problem statement. The items with the lowest means featured mainly in the subcomponent *Metacognitive skills*, especially on the subscales *Monitoring* and *Evaluation*.

Regarding the items with the highest means, the pre-service teachers were primarily aware that their learning is facilitated by interest and prior knowledge of a topic (Items 45 and 15). In addition, they were aware that the focused and careful reading of a problem statement to facilitate successful problem solving as the first step (Items 41 and 50) preceded the successful selection of appropriate and proven effective heuristic strategies (Item 3). The pre-service teachers, therefore, demonstrated more metacognitive awareness regarding the importance of affect (Items 45, 15, and 26) and cognitive reading strategies while making sense of the problem statement.

Regarding the items with the lowest means, the pre-service teachers were less aware of the importance of evaluating the solution to a problem (Items 38, 19, and 7) by reflecting on different or easier methods, or by considering the suitability of the solution. Furthermore, they gave little attention to monitoring their progress and to the effectiveness of strategies such as reflecting on easier or different approaches (Items 28, 48, 11, and 31) during problem solving.

It is noteworthy to compare the item with the fifth highest mean (Item 3) with the two items with the lowest means (Items 38 and 19). While all these items refer to the use of strategies and methods, 87% of pre-service teachers reported attempting to use past strategies (Item 3) during problem solving. Meanwhile, an average of 55% of

respondents reported that they do not consider different ways (Item 38) or easier ways (Item 19) after problem solving, and do not know how to evaluate their performance (Item 7).

A few observations in relation to mathematical proficiency, the nature of reflection, metacognitive knowledge, metacognitive skills, and past experiences with mathematical problems are made henceforth. The pre-service teachers' emphasis on reading strategies to understand the problem statement might be indicative of their past experiences with the difficulty of Mathematics questions. Moreover, whilst on the one hand the pre-service teachers reported reflecting on past strategies at the outset of a problem-solving session, they did not reflect on the effectiveness of past strategies or methods during or after the problem-solving session. This might be indicative of the nature of reflection, i.e. difficult and acquired over a prolonged period, as individuals need to be taught how to reflect-in-action and reflect-on-action (see Section 2.3.5; 2.3.5.1).

The pre-service teachers displayed some level of mathematical proficiency (see Section 2.3.4.2) in reporting the importance of affect and resources in successful learning and problem solving. However, they reported less reflection on a repertoire of strategies (heuristics and the regulation of these strategies, methods, and solutions). This also points towards an awareness of strengths and weaknesses (for example, *Declarative knowledge*, as the pre-service teachers were aware that interest and prior knowledge facilitate their learning) but inadequate awareness of their lack of knowledge concerning facts and strategies (see Section 2.2.4.1). This observation may point to pre-service teachers being unaware of the importance of evaluating and monitoring their thoughts and actions, or not having the know-how to monitor themselves during study or problem solving. That is, they might be unaware of effective monitoring strategies and/or unskilled in using these strategies.

This reported lower level of regulatory skills, therefore, may influence successful performance in problem solving. It may cause pre-service teachers to not reflect adequately on progress and comprehension, and not question the usefulness of learning and problem-solving strategies while learning or problem solving. Similarly,

they may accept incorrect methods, strategies, and computations, and therefore not correct their comprehension or calculation errors. Moreover, they may find it difficult to set or meet time and problem-solving goals. As competent problem solving is cyclical, analysing the effectiveness of a method after problem solving is important for considering its usefulness in correcting the problem, as well as for solving other similar problems. Additionally, a mathematical disposition displays persistence to keep on working until the correct solution is achieved (see Section 2.3.4.2).

Therefore, the question arises: can the participating pre-service teachers translate this reported awareness of their thoughts, feelings, and actions during learning and problem solving into successful learning and problem solving? This led to secondary research question 4: “What is the level of metacognitive awareness of pre-service Mathematics teachers in a problem-solving context?”

## **4.4 THE LEVEL OF METACOGNITIVE AWARENESS OF PRE-SERVICE TEACHERS IN THE PROBLEM-SOLVING SESSION**

### **4.4.1 Introduction**

A think-aloud problem-solving session was conducted with pre-service teachers before administering the MAI. When examining the specific statements on metacognitive behaviors (knowledge and skills) during a problem-solving process, one reaches a better understanding of how these behaviours influence and enhance the problem-solving process. The qualitative discussion that follows, therefore, serves to enrich the findings of the MAI. First, metacognitive behaviors during the problem-solving session were identified, namely *Declarative*, *Procedural*, and *Conditional knowledge*, as well as the metacognitive skills *Planning*, *Evaluation*, *Debugging*, *Information management*, and *Monitoring* (see Sections 2.2.4.1; 2.2.4.3.2). Literature supports the importance of metacognition during problem solving (see Sections 2.3.2; 2.3.3; 2.3.4.2). Second, these identified metacognitive behaviours corresponded with the four phases of the heuristic problem-solving framework in Mathematics: *Orientation*, *Organisation*, *Execution*, and *Verifying* (see Section 2.3.4.4).

Discussion was predicated on and revolved around the completion of a mathematical problem. The problem posed to the pre-service teachers was taken from the CAPS document for the FET Phase (DBE, 2011a: 53). It is one of the typical problem-solving questions in the document (see Section 3.4.3.2). As solving the problem requires complex procedures and higher-order thinking, it was envisaged that metacognitive behaviour would be elicited (see Section 2.2.2). Furthermore, the problem was selected because the route to obtaining the solution is not immediately clear or obvious (see Section 2.3.4.4). The problem read as follows:

Suppose a piece of wire could be tied tightly around the earth at the equator. Imagine that this wire is then lengthened by exactly one meter and held so that it is still around the earth at the equator. Would a mouse be able to crawl between the wire and the earth? Why or why not? (DBE, 2011a: 53).

The pre-service teachers had to write down their thoughts with corresponding calculations relating to the steps they would take in solving the problem (see Section 3.2.3.2). Several thoughts and processes associated with competent problem-solving were anticipated (see Sections 2.3.4.4.1–2.3.4.4.4). First, in attempting to make sense of the problem scenario, it was anticipated that pre-service teachers would relate the equator and the wire to the circumference of a circle (*Orientation*). Second, in organising their thoughts around the problem, it was anticipated that many would identify the core of the problem and so identify the mathematical concept of the difference in radii of the equator and the wire that would allow space or prevent the mouse from passing under the wire (*Organisation*).

Therefore, key information in the problem relates to the mathematical concept of the radii of circles; that is, the difference in radii between that of the circle and the extended wire. Knowledge about other concepts—such as changing units to work in comparable units and estimating the height of a mouse—was applicable in the computations (*Execution*). It was also anticipated that the pre-service teachers would attempt different strategies and methods while problem solving if they realised that a method or strategy was not useful (*Execution*).

Finally, checking the solution for reasonableness and accuracy (*Verifying*)—and consequently deciding whether the mouse could pass between the earth and the wire based on the computations—was expected. Moreover, if the solution did not make sense, it was expected that the pre-service teachers would re-evaluate their methods or calculations and troubleshoot for conceptual or computational errors.

#### **4.4.2 Discussion of the problem-solving session**

The pre-service teachers' responses were grouped according to items on the MAI. The items on the subscales that featured most frequently in the qualitative problem-solving session were identified. During the problem-solving session, the pre-service teachers' metacognitive awareness was primarily associated with the following items: Items 17 (*Declarative knowledge*), 22 and 41 (*Planning*), 13 and 37 (*Information management*), and to a lesser extent Items 9 (*Information management*) and 50 (*Debugging*).

The subscales on the MAI that featured most frequently were *Planning* and *Information management*, and to a lesser extent *Declarative knowledge*. Consequently, metacognitive skills are discussed first, followed by the metacognitive knowledge. The five metacognitive skills presented as follows.

##### **4.4.2.1 Planning**

With regards to *Planning* (i.e. planning, goal-setting, and allocating resources prior to learning), Items 22, 23, and 41 relate to considering strategies prior to the actual solving of the problem through calculations. Items 4, 6, 8, and 44 relate to goal-setting and allocating resources in the learning of Mathematics, and were not overtly evident in the pre-service teachers' comments during the problem-solving session.

Item 22 ("I ask myself questions about the problem before I begin to solve a Mathematics problem") was displayed most frequently in the thoughts and calculations of pre-service teachers under the subscale *Planning*. Most pre-service teachers indicated questioning about the problem statement. Another metacognitive planning strategy, Item 41 ("I read the question carefully before I answer a problem"), could be

inferred from questions posed about the problem statement, writing down chunks of information, or writing down key phrases.

An example of this effective questioning was demonstrated by Respondent 30, who, after asking three questions, arrived at the core of the problem quickly by noting down the circumference of the earth and the wire:

Wat is die radius van die oorspronklike sirkel?

Wat is die radius van die nuwe sirkel?

Wat is die verskil tussen die twee radiusse?

Respondent 11 asked similar questions:

Hoeveel plek sal gelaat word?

Wat is die radius van die aarde?

Wat is die radius van die draad nadat dit verleng word?

Wat is die radiusse van die twee omtrekke?

Wat is die verskil tussen die twee radiusse?

Several pre-service teachers who used the strategy of asking questions initially wrote down their thoughts on the information provided before arriving at the gist of the problem. See, for example, the thoughts and accompanying calculations of Respondent 37:

Imagine earth & the wire tied around it!

———— Wire  $R = \frac{40 \times 10^3 \text{ km}}{2\pi}$

Equation of circumference

$$O = 2\pi r = 40 \times 10^3$$

How much is 1 m in kms?

$$1 \text{ m} = \frac{1}{1000} \text{ km} = 0.001 \text{ km} = 1 \times 10^{-3}$$

What is the result when I add this small number to 40 km?

$$+ 140 \times 10^3 + 1 \times 10^{-3} \text{ km} = 40\,000.001 \text{ km}$$

The difference is so small, but is it significant?

What's the size of the mouse in question?

Well a mouse's height is  $\lll 0.001 \text{ km}$

So the mouse will pass!!

Another argument is \_\_\_ the two values for R!

$$R1 = \frac{40 \times 10^3 \text{ km}}{2\pi} ; R2 = \frac{40\,000.001}{2\pi}$$

Are very close to each other (almost no significance), but may still allow mouse to pass.

This pre-service teacher thought of two ways to solve the question, although not both effective and correct. These were finding the difference in circumferences and finding the difference in radii. It is notable that this is the only pre-service teacher, besides Respondent 5, who attempted alternative arguments. He displayed and demonstrated Item 23 (“When I start to solve a problem, I think of several ways to solve a problem and choose the best one”) and possibly Item 11 (“I ask myself if I have considered different methods of solving a problem when solving a mathematics problem”). Respondent 37 thought of two different ways, but did not perform any exact calculations. His methods were based on getting an estimated answer which displayed good reasoning.

#### ***4.4.2.2 Information management***

*Information management* comprises of items that relate to the awareness of learners about skills and strategies used to process information more efficiently before and during the learning and problem-solving process (Items 9, 13, 30, 31, 37, 39, 42, 46, and 47). *Information management* skills pertain to organising, elaborating, summarising, and selective focusing. During the problem-solving session, these strategies may involve facts and formulae, or a repertoire of different cognitive and problem-solving strategies or metacognitive questioning. Strategies could refer to the following: drawing pictures and diagrams (Item 37); translating Mathematics questions into one’s own words and restating or paraphrasing a question (Item 39); relating the topic to other topics (Item 47); selective focusing on important information (Item 13); adjusting a reading strategy (Item 9); and selective focusing on the meaning and significance of the question (Item 30). Items 31, 46, and 47 relate to the learning of Mathematics and were not evident in the pre-service teachers’ comments during the problem-solving session.

Five items were applied frequently in varying degrees in managing the given information in the problem statement. They were mainly Items 13 and 37, as well as Items 30 and 9



by inference and, to a lesser extent, Item 39. The items read as follows: Item 13 (“I consciously focus my attention on important information in a Mathematics question”), Item 9 (“I read slower when I encounter important information in a Mathematics question”), Item 37 (“I draw pictures or diagrams to help me understand while I am learning Mathematics”), and Item 39 (“I try to put Mathematics questions into my own words”) (see Appendix 6). As Items 13 and 30 are very similar, references to focusing on information in the problem statement were grouped as belonging to Item 13. Therefore, writing down key phrases from the information is grouped under Item 13.

The problem statement specified that a wire around the earth is lengthened by one meter. Respondent 12 remarked on the significance of this information in that “an increase in length means change in radius”. Respondent 24 also focussed on the key information, consequently displaying her understanding of the problem by referring to the concept of the changing radii (Item 13), although she made a mistake in not converting to the same units before adding the extra meter to the diameter. She recorded her thoughts as follows, accompanied by diagrams (Item 37) to organise and make sense of the information provided:

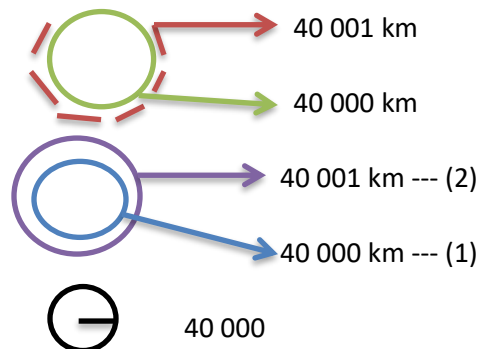
Die omtrek van die aarde is 40 000 km. As ek 1 m

bysit sal die draad effens verslap.

Ek moet die afstand tussen die twee sirkels bereken,

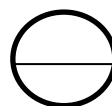
om te kyk of ‘n muis kan deur.

Ek moet eers die radius van altwee sirkels kry.



Another example is Respondent 9, who verbalised her awareness that drawing diagrams (Item 37) is a strategy to aid understanding, illustrating that the use of diagrams or sketches is an effective strategy, particularly for developing an initial understanding of a problem.

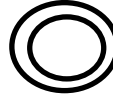
Draw the problem in pictures to understand the problem better.



Work out what will the distance be when a meter is added.

$$\begin{aligned} &40\,000\text{ km} + 1\text{ m} \\ &= 40\,000\,000\text{ m} + 1\text{ m} \\ &= 40\,000.001\text{ km} \end{aligned}$$

Draw the wire around the earth with 1 m extra.



In her reflective thoughts above, she also illustrated the use of Items 13 and 9, which are strategies to focus on and manage given information. However, she faltered in reaching a sensible answer because she could not arrive at the core of the problem; that is, the difference in radii between the earth's circumference and that of the wire, which may allow enough room for the mouse to pass through.

Five other pre-service teachers displayed similar thought patterns, analysing the scenario but being unaware of performance errors. They displayed a discrepancy in their work, on the one hand utilising metacognitive and cognitive strategies such as managing information by rephrasing and representing the problem through self-questioning, drawing diagrams, or writing down key phrases of important information, but on the other hand making conceptual errors that prevented them from progressing effectively. It is possible they struggled to apply strategies like selective focusing on important information (Items 13 and 30) and slower reading (Item 9).

Another information management strategy is the rephrasing of a question. An example of applying Item 39 ("I try to put Mathematics questions into my own words") is provided by Respondent 2, who rephrased the problem statement as follows:

Wat wil ons uitwerk?

Hoeveel speling daar tussen die draad en die aarde se bolaag sal wees as 'n draad 1m langer in omtrek is as die aarde.

To reach the core of the problem, he restated it as follows:

Bekendes: omtrek by ewenaar is 40 000 km. Formula: sirkel, want die aarde is rond.  $O = 2 \pi r^2$

Ons moet r bepaal vir 40 000 km en vir 40 000 km + 1 m

As illustrated previously in Items 13 and 9, this strategy (Item 39) organises the thoughts of the problem-solver by identifying key information that is given and asked in the problem statement. Many pre-service teachers similarly restated the key information as above, including writing down the formula for the circumference of the circle. As noted later, only 14 out of 41 (34%) pre-service teachers identified the crux of the problem statement and proceeded to perform calculations to find the radii of both the earth and the wire.

#### ***4.4.2.3 Monitoring***

In the *Monitoring* subscale, items relating to the problem-solving session were Items 11, 2, 34, and 48. References to different problem-solving methods **at the outset** of a problem-solving session were grouped together under *Planning*, as noted earlier. For example, Item 2 (“I first consider different ways of solving the problem before I start solving a problem in Mathematics”) is similar to Item 22 (“I ask myself questions about the problem before I begin to solve a Mathematics problem”) and is therefore grouped under Item 22 as a metacognitive planning skill.


In contrast, Item 11 refers to an awareness of different problem-solving methods **during** problem solving. Item 11 (“I ask myself if I have considered different methods of solving a problem when solving a Mathematics problem”) was only seen in two pre-service teacher’s answers. In the given problem scenario, different ways or methods of solving the problem could refer to different ways of calculating the distance between the wire and the earth.

Respondent 16 employed Item 34 (“When I solve a Mathematics problem, or when I study for a Mathematics test or examination, I find myself pausing regularly to check my comprehension”) to monitor conversion between units. He remarked:

First let us change the units to meter and recalculate for earth and tight rope. This is better: should have done this from the beginning but did not realise the calculator is set to four decimals.

Respondent 24 reflected on her strategy use by writing down her thoughts and calculations as follows:

Ek moet eers die radius van altwee sirkels kry.




$$2\pi r = 40\,000$$

$$r = \frac{40\,000}{2\pi} = 6366.2 \quad \dots(1)$$

Die radius van:

(1) 6366.197724 km


(2) 6366.356879 km



$$2\pi r = 40\,001$$

$$r = \frac{40\,001}{2\pi} = 6366.4 \quad \dots(2)$$

Bereken nou die afstand. = 0.159155 km



$$(2) - (1): 6366.356879 \text{ km} - 6366.197724 = 0.159155 \text{ km}$$

Nou moet ek my data interpreteer.

Herlei na meter  
 Km 000 m 00 cm 0 mm  
 159.155 m

Dit voel hoogs onwaarskynlik, maar dit lyk of ‘n muis wel onder die draad kan deurkruip. Omdat ‘n muis ± 10 cm hoog is en daar ‘n 159 m verskil is... Ek weet nie of hierdie reg is nie. Ek kan nie dink dat 1 m draad ‘n 150 m verskil kan maak nie.

Respondent 24, therefore, reflected on her performance by monitoring the effectiveness of her strategy use. She employed Item 48 (“I ask myself questions about how well I am doing while I am solving a Mathematics problem”) which refers to assessing one’s

strategy use. Likewise, Item 34 (“When I solve a Mathematics problem, or when I study for a Mathematics test or examination, I find myself pausing regularly to check my comprehension”) and Item 48 (“I ask myself questions about how well I am doing while I am solving a Mathematics problem”) are similar in monitoring one’s own comprehension and performance (see Appendix 3). Respondent 24 demonstrated a good ‘feel’ for estimation. However, she made an error in the conversion of units and did not correct or confirm the answer by using a different method or strategy to double-check her solution.

In fact, only a few pre-service teachers reflected on their methods while problem solving. The conversion between units was a stumbling block for many of the pre-service teachers. Respondent 16 was the only one who corrected his answer and so displayed a debugging strategy.

#### ***4.4.2.4 Debugging***

In the *Debugging* subscale (strategies employed for correcting comprehension and performance errors) Items 40, 43, and 50 can be identified in reflective comments made during the problem-solving session. Rereading the problem statement to adapt comprehension and performance involved both Items 43 and 50. Item 50 (“When I read a Mathematics question, I stop and reread any section of the question that is not clear”) could be seen as a strategy at the outset of a problem-solving session; it may work hand-in-hand with Item 41 (“I read the question carefully before I answer a Mathematics question”) discussed previously, and may also be inferred from thoughts made and calculations written down about information given and asked. Consequently, Item 50 possibly was used frequently. Respondent 7 showed evidence with her thoughts on the length of the wire:

Wire lengthened by 1 meter. Wire not tied, not bended.

Item 43 (“If I do not make progress when I solve a Mathematics problem, I ask myself whether my first understanding of the problem was correct”) relates to awareness of one’s own understanding, and may entail rereading the problem statement or contemplating some alternative approaches. Consequently, Item 40 (“I change my problem-solving method when I fail to make progress”) corresponds to Item 43. It is

expected that the employment of debugging strategies as seen in Items 50, 40, and 43 will result in pre-service teachers solving the problem successfully by attempting different ways, or at least considering alternative methods. The latter two items, furthermore, link with two items on the *Evaluation* subscale, as they relate to reflecting on the effectiveness of strategies and methods used during and after the problem-solving process, namely Items 19 and 38, as discussed in the following section.

#### **4.4.2.5 Evaluation**

Items which refer to *Evaluation*—that is, analysing performance and strategy effectiveness after a learning experience—are Items 7, 19, 24, 36, 38 and 49. Items 24, 36 and 49 refer to the learning of Mathematics. Item 7 (“I know how well I did once I have finished a Mathematics test or examination”) reflects on the ability of the problem-solver to judge their own performance and efficacy. As only the minority of pre-service teachers were successful in solving the problem correctly, it may indicate that pre-service teachers were unaware of how well or poorly they performed, otherwise they may have attempted to improve their performance.

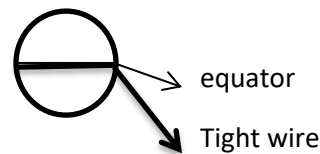
Furthermore, Item 19 (“After I have solved a Mathematics problem, I ask myself if there was an easier way to solve the problem”) and Item 38 (“After I have solved a Mathematics problem, I ask myself whether I have considered different ways to solve the problem”) reflect on the overall effectiveness of the problem-solving techniques employed. It is notable that during the problem-solving session, scant evidence was found of these items. Only 8 of the 41 pre-service teachers managed to reach a sensible solution to the problem, which may be indicative of them not trying easier or different ways to improve their performance or solve the problem correctly.

In summary and with regards to *Regulation of cognition* it was found that items on *Debugging* and to an even lesser extent on *Monitoring* and *Evaluation*, featured infrequently during the problem-solving session, whereas *Planning* and *Information management* were more evident. Concerning the *Knowledge of cognition* subcomponent, items on the subscales *Declarative*, *Procedural*, and *Conditional knowledge* refer mainly to the broader learning of Mathematics. Metacognitive

behaviours relating to *Procedural knowledge* (of how to implement problem-solving strategies), *Conditional knowledge* (of when and why to use learning procedures), and *Declarative knowledge* (about self-knowledge and learning resources and strategies) are therefore mostly inferred from statements made by the pre-service teachers. Hence, items on *Declarative knowledge*, such as Item 17 (“I am good at remembering mathematics facts and principles”), and on *Procedural knowledge*, such as Item 3 (“When I solve a Mathematics problem, I try to use methods of solving a problem that have worked in the past”), and Item 14 (“I have a specific purpose for each problem-solving method I use when I solve a problem in mathematics”) might be implicitly evident in the problem-solving process. *Conditional knowledge* corresponds mainly to study habits rather than to problem-solving activities. An exception is Item 35, which refers to an awareness of when and why to use each problem-solving method (see Appendix 3).

Regarding *Declarative knowledge*, Items 5, 12, 17, and 32 refer to an awareness of understanding and recalling Mathematics. Item 17 especially could be inferred from pre-service teachers’ thoughts and calculations during the problem-solving session. Item 17 refers in this problem-solving scenario to the knowledge of the correct formula for the circumference of a circle and the manipulation of the formula to determine the radius, as well as conversion between units. The formula for the circumference of a circle was recalled correctly by most of the pre-service teachers. The radii were calculated correctly by most of the pre-service teachers who applied the concept. Facts and principles consisted of the conversion of units and rules for the four operations. This was illustrated when Respondent 7 recorded her thoughts and calculations as follows:

Wire around earth @ equator



Circumference @ equator = 40 000 km

$$C = 40\,000 \text{ km}$$

Wire lengthened by 1 m

$$C + \frac{1}{1000} \text{ km} \quad \leftarrow \text{converting to km from m}$$

Wire not tied, not bent

Mouse crawl between wire and earth?

Calculations →

$$(1) C = 40\,000 \text{ km}$$

$$C = 2\pi r$$

$$40\,000 = 2\pi r$$

$$\frac{40\,000}{2\pi} = r$$

$$6366.198 \approx r \text{ (} r = 6366.197724\dots \text{)}$$

(2) 1 m added to circumference

$$\frac{40\,000}{1} + \frac{1}{1000} = 2\pi r$$

$$\frac{40\,000\,000+1}{1000} = 2\pi r$$

$$\frac{40\,000\,001}{1000} \times \frac{1}{2\pi} = r$$

$$\frac{40\,000.001}{1} \times \frac{1}{2\pi} = r$$

$$\frac{40\,000.001}{2\pi} = r$$

$$6366.198 \approx r \text{ (} r = 6366.197883\dots \text{)}$$

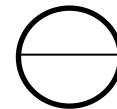
In conclusion, it was anticipated that pre-service teachers would relate the wire tied around the earth's equator to the circumference of a circle. The problem further states that the wire is extended with 1 meter. Only a minority of the pre-service teachers (14 out of 41, or 34%) could further relate this fact to the radii of the circles formed by a wire on the earth's equator, the earth, and the extended wire. It was unexpected that so few pre-service teachers would correlate these facts.

An illustration of not relating the wire tied around the earth's equator to the circumference of a circle was seen in the thoughts and work of Respondent 9, who applied Items 13, 37, and 42, but settled on a premature answer by not progressing to working out the radii, and therefore accepting an incorrect solution.

Draw the problem in pictures to understand the problem better.

$$40\,000 \text{ km} + 1 \text{ m}$$

$$40\,000 \text{ km} = 40\,000\,000 \text{ m}$$



Work out what the distance will be when a meter is added.

$$40\,000\,000 \text{ m} + 1 \text{ m}$$

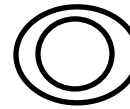
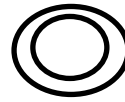
$$= 40\,000\,001 \text{ m} = 40\,000.001 \text{ km}$$



Draw the wire around the earth with 1 m extra.

The circumference of the earth is smaller than the circumference of the wire around the earth.

But it won't be possible for the mouse to crawl between the earth and wire because the size of the mouse will be too big. And the 1 m extra given will have to be spread over the 40 000 km thus the ratio of the mouse and the size between the wire and the earth will differ.



1 extra meter

$$1 \text{ m} \div 40\,000 \text{ km}$$

$$1 \text{ m} \div 40\,000\,000 \text{ m}$$

$$= 0.000\,000\,025 \text{ m}$$

For every meter that 1 meter needs to divide into will not be enough for the mouse to crawl under.

Consequently, this demonstrates that by not employing *Debugging* and *Evaluation* skills, an incorrect solution can be accepted as correct. This specific comprehension error occurred among 9 of the 41 pre-service teachers (22%). It is noteworthy that only three pre-service teachers (Respondents 5, 37 and 39) attempted different ways or methods to solve the problem; that is, they progressed to another method after the initial method proved unsuccessful.

The high percentage of pre-service teachers who erred in solving the problem correctly (80.5%)—almost 20% of whom, although understanding the problem, made calculation errors—indicates that *Monitoring* along with *Debugging* and *Evaluation* are the metacognitive skills which these pre-service teachers did not employ sufficiently. These skills might play a vital role in improving performance, as they regulate progress and performance during—and effectiveness of strategies after—the problem-solving session. Analysing the method used—as well as analysing the answer by relating it to the question or problem—is an important aspect in taking control of the problem-solving situation. Regulating one's actions by reflecting on actions, as well as revisiting methods and strategies to look for simpler and easier solutions, is indicative of metacognitive awareness. It also demonstrates expertise as the self-regulatory attribute of mathematical proficiency, i.e. the metacognitive skills of *Evaluation* and *Monitoring*.

The unproductive and ineffective use of strategies to correct comprehension and analyse performance may therefore result in failure to reach a sensible solution. Ultimately, metacognitive skilfulness plays a significant part in successful problem solving and performance.

#### **4.4.3 Discussion of the four-step problem-solving framework**

The pre-service teachers' level of metacognitive awareness, as displayed in specific metacognitive behaviours during the problem-solving session, was also compared to the four-step problem-solving framework, which provides a framework for metacognitive behaviours during problem solving (see Section 2.3.4.4). It consists of four phases: *Orientation*, *Organisation*, *Execution*, and *Verifying*. In each phase, specific metacognitive knowledge and skills are expected to be utilised by a competent problem-solver. Metacognitive knowledge and skills could be employed in a cyclical or recursive manner; for example, to reattempt solving a problem if a viable solution is not reached.

In the *Orientation* phase, grouped together as a metacognitive planning skill, Item 22 (“I ask myself questions about the problem before I begin to solve a Mathematics problem”) and Item 41 (“I read the question carefully before I answer a problem”), which can be inferred from self-questioning or writing down key phrases, were mainly evident. Most of the pre-service teachers used questioning as a planning strategy.

In the *Organisation* phase, Items 13 and 37 were applied frequently in managing the information provided in the problem statement. The pre-service teachers processed this information by employing Item 13 (“I consciously focus my attention on important information in a Mathematics question”) and Item 37 (“I draw pictures or diagrams to help me understand while I am learning Mathematics”). As Items 13, 30, and 9 were very similar in their application, references to focusing on information in the problem statement were grouped as belonging to Item 13 (see Section 4.4.2).

It is notable that most of the pre-service teachers who used questioning successfully (Item 22) also proceeded to perform calculations; that is, they succeeded in progressing from the *Orientation* and *Organisation* phases to the *Execution* phase. Metacognitive questioning occurs in each phase and aids the progression from one phase to another

(see Section 2.3.2.4). Furthermore, in the *Orientation* phase, Item 50 (*Debugging*) could have been inferred from writing down the given data, the unknown data, and the given conditions, which would show evidence of rereading the statement to select the required data.

In the *Execution* phase where calculations occur, monitoring one's strategy use, accuracy, and understanding is very important. Hence reflecting through self-questioning on comprehension (Item 34), different methods (Items 11 and 2), and progress (Item 48) indicates mathematical proficiency (see Section 2.3.4.2). Evidently, very few pre-service teachers displayed metacognitive monitoring skills. Moreover, Item 17 (*Declarative knowledge*) was evident in this phase, as it relates to employing the formula for the circumference of a circle and other facts and principles.

Furthermore, metacognitive *Debugging* and *Evaluation* skills are mainly associated with the *Execution* and *Verifying* phases of the problem-solving model, although they can be utilised cyclically in other phases to reattempt solving a problem. Therefore, these strategies employed for correcting comprehension and performance errors during problem solving—as well as for analysing performance and strategy effectiveness after problem solving—are crucial. A competent problem-solver is anticipated to employ strategies to correct errors and reflect on their progress and performance whilst they are working as well as after the final solution. Consequently, reflecting on whether the problem was understood correctly in the first place (Items 43 and 50) or recognising failure and subsequently changing methods (Item 40) are very likely to be utilised during the problem-solving process. Lastly, mathematical proficiency (see Section 2.3.4.2) requires reflection on easier (Item 19) or different ways (Item 38) to solve a problem.

#### **4.4.4 Summary of the qualitative data from the problem-solving session**

During this session, the pre-service teachers recorded their thoughts and calculations. They demonstrated a level of metacognitive awareness (by employing strategies in an attempt to understand the problem statement) through self-questioning, through managing information using reading strategies and diagrams, and through writing down key phrases with accompanying calculations to a greater or lesser extent. However,

they demonstrated difficulty in monitoring their progress and performance, as well as little awareness of comprehension and calculation errors or how to evaluate strategies and solutions for reasonableness and effectiveness. Importantly, a fifth of the pre-service teachers solved the novel higher-order problem successfully, and only a third could identify and perform calculations concerning the key information in the problem.

In relation to the subscales, the pre-service teachers mainly employed the metacognitive skills *Planning* and *Information management* during the first phase (*Orientation*) of the problem-solving framework model, and to a lesser extent *Debugging*, particularly in relation to understanding the problem statement. This indicates their awareness of the importance of making sense of and understanding the problem statement. Inadequate understanding of the problem hampered the progression from the first to the second phase of the problem-solving model. Inadequate metacognitive skillfulness was demonstrated through the low achievement of the pre-service teachers in regulating progress and performance during (and effectiveness of strategies after) the problem-solving session. This was especially evident in the metacognitive skills *Monitoring*, *Debugging*, and *Evaluation*.

It was surprising to find very little evidence of effective reflection before, during, and after the problem-solving session. This may be due to a lack of awareness of how (*Procedural knowledge*), when, and why (*Conditional knowledge*) to skilfully use and reflect on strategies (heuristics) to solve problems. A lack of effective reflection might also point towards inadequate *Declarative knowledge* as an awareness of self, i.e. inadequate awareness of a person's own strengths and weaknesses, as well as inadequate knowledge of intellectual resources such as applicable facts and strategies.

In addition, inadequate subject knowledge, as seen in the problem-solving session, seemed to have hindered progress. Moreover, as metacognitive reflection is learned over time and with practice through opportunities to solve novel problems, infrequent exposure to the solving of novel tasks means the inadequate practising of reflection during problem-solving processes. This might be a contributing factor to the low number of pre-service teachers who effectively solved the problem. Lastly, this inadequate

reflection may be explained by time constraints that potentially did not allow for adequate reflection during and after the problem-solving session.

## **4.5 DISCUSSION OF THE QUESTIONNAIRE DATA AND COMPARISON TO THE THINK-ALOUD METHOD**

In relation to the MAI, the following observations can be made. The main findings indicate a moderately high level of metacognitive awareness among the pre-service teachers on the MAI. Furthermore, metacognitive knowledge featured more prominently than metacognitive skills. Additionally, the items with the highest and lowest means related generally to metacognitive knowledge and metacognitive skills respectively. The pre-service teachers demonstrated some awareness of what facilitates their learning in Mathematics, namely affect, interest, prior knowledge, and motivation (see Section 2.3.4.2.4). However, they demonstrated less awareness of the importance of monitoring and the evaluation of skills in executing and verifying mathematical problems.

Findings from the problem-solving session were mostly similar to those of the MAI, with some discrepant findings. It should be noted that the think-aloud session could not be correlated with the entire questionnaire. The MAI assessed the metacognitive awareness (i.e. metacognitive knowledge and metacognitive skills) of pre-service teachers during the learning and problem solving of Mathematics, whilst in the problem-solving session, mainly metacognitive behaviours relating to problem solving could be observed (see Section 3.4.5 for a more comprehensive explanation). In the problem-solving session, therefore, metacognitive skills were more evident, whilst metacognitive knowledge referred mainly to the broader learning of Mathematics as measured by the MAI. Consequently, the qualitative think-aloud session enriched the findings of the MAI, rather than explaining or elaborating on those findings (see Sections 3.4.5; 4.4).

Based on the limited comparison that could be made between the findings obtained from the MAI and the think-aloud problem-solving session (see Section 3.4.5), two observations are prominent. First, the higher level of metacognitive knowledge in

comparison to metacognitive skills, as reported on in the MAI, was mainly evident in the first phase of the problem-solving framework and to a lesser extent in the other three phases. Second, the reflections and calculations of the pre-service teachers during the problem-solving session related to the items with the highest and lowest means on the MAI, as discussed below.

As reported on in the MAI, the items with the highest means related to making sense of the problem statement. Three of these items (41, 50, and 9) related to reading the problem statement. This awareness was demonstrated in the problem-solving session as self-questioning by carefully reading the problem statement, by asking questions about the problem itself, and by making diagrams. Pre-service teachers also showed interest in attempting to understand and solve the problem. This corresponds with the items with the highest means on the MAI, namely Items 45 and 26, which in turn relate to the affective attribute of mathematical proficiency (see Section 2.3.4.2.4). Furthermore, despite reporting awareness of the importance of prior knowledge (Item 15), about two-thirds of the pre-service teachers displayed a poor understanding of the problem statement. Possible reasons are insufficient opportunities to practice solving complex problems and/or inadequate subject knowledge. They therefore lacked possession of a good knowledge base and heuristics, and consequently did not demonstrate mathematical proficiency (see Section 2.3.4.2.1).

Significantly, items with the lowest means on the MAI related mainly to *Evaluation* and *Monitoring*, and it was further evident in the problem-solving session that the metacognitive skills *Monitoring*, *Debugging*, and *Evaluation* were not well-employed. The lower level of metacognitive skills (*Regulation of cognition*) than metacognitive knowledge (*Knowledge of cognition*) reported on in the MAI—as well as the lower means of the subscales *Evaluation* and *Monitoring*—was evident in the poor regulating skills displayed during the problem-solving session.

Although *Debugging* had the highest mean of the metacognitive skills on the MAI, the pre-service teachers displayed little success in correcting comprehension and conceptual errors during the problem-solving session. An explanation could be that the pre-service teachers were either unaware of their errors, did not know how to correct

these errors, or ran out of time to do so. On the other hand, this subscale is not reported to be very reliable because of the low number of items on the subscale for the instrument; therefore, it could not be compared.

To conclude, mathematically proficient teachers are expected to display metacognitive adaptive expertise in novel situations, such as the one provided in this problem-solving scenario. However, although the MAI total scores indicated a moderately high level of metacognitive awareness, this was not evident in the qualitative data obtained during the problem-solving session, as only about a third of the pre-service teachers understood the problem statement entirely and could solve the problem successfully (not taking into account calculation errors). Therefore, the reported level of metacognitive awareness did not translate to high achievement in the problem-solving session. Successful performance was hampered by poor regulation skills, particularly in their reflection on the suitability of strategies and methods, and the debugging of conceptual and calculation errors. Besides inadequate reflection, insufficient opportunities to solve ill-defined problems, insufficient subject knowledge, and possible time constraints could account partially for the low achievement too.

It must be emphasised, however, that inferences and comparisons that could be made between the MAI data and the think-aloud problem-solving session data were limited (see Section 3.4.5). Moreover, correlation between questionnaires and self-report measures was generally low (Schellings et al., 2013: 963). Nonetheless, these findings illustrate that the majority of pre-service teachers were unable to translate their reported level of metacognitive awareness—as a key component in the learning and problem solving of Mathematics, especially in competent problem solving—to the problem-solving session. The implication for teaching and learning, therefore, is to build on this awareness of the gap that exists between what pre-service teachers say they can do and what they actually can do in a given problem-solving situation. Opportunities to enhance metacognitive skillfulness during problem solving and in everyday teacher practice should be given to pre-service teachers during teacher training. These recommendations are discussed in Chapter 5.

## 4.6 SUMMARY OF CHAPTER

In summary, this chapter explored secondary research question 3, “What is the level of metacognitive awareness of pre-service Mathematics teachers on the MAI?”, using quantitative data gathered via questionnaire. The means on the total MAI and the two subcomponents of metacognition demonstrated that the pre-service teachers possess a level of metacognitive awareness to some extent. The pre-service teachers indicated a higher level of awareness with regards to two attributes: first, the importance of interest in learning Mathematics, and second, the importance of managing information in a problem statement to make meaning of it. The pre-service teachers reported a lower level of awareness in relation to the monitoring and evaluation of performance and strategy use in Mathematics learning and problem solving.

In addition, through addressing secondary research question 4—“What is the level of metacognitive awareness of pre-service Mathematics teachers in a problem-solving context?”—qualitative data enriched the findings of the quantitative data and provided a broader perspective on the pre-service teachers’ level of metacognitive awareness in a problem-solving context. During this session, the pre-service teachers recorded their thoughts and calculations. They demonstrated a level of metacognitive awareness through the following: first, by employing strategies to try and understand the problem statement; second, by using self-questioning; third, by managing information using adjusted reading strategies and diagrams; and fourth, by recording key phrases with accompanying calculations to a greater or lesser extent. However, they demonstrated difficulty with monitoring their progress and performance. Low awareness of comprehension and calculation errors—and how to evaluate their process and solution for reasonableness and effectiveness—was also observed.

These findings illustrate that most of the pre-service teachers inadequately translated their reported level of metacognitive awareness—a key component in learning and problem solving in Mathematics, especially in competent problem solving—to the problem-solving session. There is, therefore, a notable gap between what the pre-



service teachers reported they could do, as measured in the MAI, and what they actually could do, as demonstrated in the problem-solving session.

# **CHAPTER 5**

## **CONCLUDING FINDINGS AND RECOMMENDATIONS**

### **5.1 INTRODUCTION**

The purpose of the study was to determine the level of metacognitive awareness of pre-service Mathematics teachers, which was initiated based on concern about the underachievement of Mathematics learners in South Africa in both national and international tests and related concern about the quality of Mathematics teaching and teacher training (see Section 1.2). The purpose of the study is reviewed in this chapter in light of findings from the literature review and empirical study. The primary research question was explored through a literature review (see Chapter 2) and a qualitative and quantitative empirical investigation (see Chapter 4). In Chapter 3, the research methodology was discussed.

The chapter reviews the findings of Chapter 4 and provides recommendations on how to encourage and enhance metacognitive awareness in pre-service teachers. It concludes by considering the significance of the study, as well as by acknowledging limitations of the research.

### **5.2 RATIONALE FOR AND OVERVIEW OF THE CHAPTERS**

#### **5.2.1 Overview of Chapter 1**

Concerns have been raised that educational institutions are not adequately empowering learners with knowledge, lifelong learning skills, and dispositions to succeed at and beyond schooling. The low achievement of South African learners in Mathematics nationally and internationally is a locus of concern for politicians and educators, as Mathematics is a gateway subject to core professions and features key skills demanded in the current job market. Hence there is a focus on enhancing the standard of Mathematics teaching and learning, as educators and educational researchers are concerned about the low achievement of Mathematics learners in Grade 12, in the

internationally benchmarked TIMSS, and in the national ANA tests (see Section 1.2). A possible cause could be the prevalence of inadequate higher-order thinking skills for solving problems, implying that these skills are not adequately developed or taught at school level at different ages (see Section 1.2). Good problem-solving skills, as higher-order thinking skills, are central to mathematical proficiency (see Sections 1.2; 1.3). Moreover, metacognition—as a higher-order thinking skill and as adaptive competence—facilitates problem solving, mathematical proficiency, and productive learning, consequently influencing performance and raising achievement (Section 1.3).

Further concerns were raised that the training of teachers at higher education institutions does not adequately prepare pre-service teachers for lifelong learning and problem solving and does not equip them to translate these knowledge and skills for their learners. Teachers are expected to manage their own lifelong learning successfully, facilitate productive learning and successful problem solving among their learners, and be metacognitive reflective practitioners and improve upon their practice (see Sections 1.1; 1.2).

A key focus in South Africa is enhancing Mathematics teaching and learning to support learning outcomes and expectations set by government and educational institutions for both teachers and their learners (see Sections 1.2; 1.4). Metacognition is the adaptive competence which facilitates these expectations.

The question, therefore, was posed whether the development of metacognition occurs during a teaching and learning situation. Consequently, the purpose of the study was to investigate the level of metacognitive awareness of pre-service Mathematics teachers.

To explore the primary research question, “**What is the level of metacognitive awareness of pre-service Mathematics teachers?**”, the following secondary research questions were answered:

**Secondary research question 1:** How is metacognitive awareness conceptualised?

**Secondary research question 2:** What is the role of metacognitive awareness in Mathematics teaching and learning?

**Secondary research question 3:** What is the level of metacognitive awareness of pre-service Mathematics teachers on the Metacognitive Awareness Inventory (MAI)?

**Secondary research question 4:** What is the level of metacognitive awareness of pre-service Mathematics teachers in a problem-solving context?

## **5.2.2 Overview of Chapter 2**

In Chapter 2, a literature review was conducted. The conceptualisation of metacognition (see Sections 2.2.1–2.2.4), associations with related constructs (see Section 2.2.5), and research on metacognition and meta-memory (see Section 2.2.6) were discussed.

In addition, Chapter 2 examined the importance of metacognition in teaching and learning—particularly pertaining to Mathematics learning and problem solving, and therefore mathematical achievement (see Sections 2.3.1–2.3.2)—as investigated in Mathematics curriculum documents and educational research literature (see Sections 2.3.3–2.3.4.1). Metacognition as adaptive competence along with other factors in mathematical proficiency and productive learning (see Sections 2.3.4.2; 2.3.4.3), as well as metacognition’s role in facilitating mathematical problem solving at the hands of a problem-solving framework, were also discussed (see Section 2.3.4.4). Moreover, teachers’ metacognitive reflective practices and training as metacognitive reflective practitioners were explored (see Sections 2.3.5–2.3.5.1). Finally, the chapter concluded with a discussion of how teachers teach for and with metacognition to enhance metacognitive awareness in learners (see Sections 2.3.4.1–2.3.4.4).

## **5.2.3 Overview of Chapter 3**

In Chapter 3, the philosophical worldview, the research approach, and the research methods which informed the study were discussed.

A descriptive survey design with purposive convenience sampling was employed, rendering non-parametric data on an ordinal scale. A quantitative approach was used to explore secondary research question 3, determining the level of metacognitive awareness of pre-service teachers on the MAI. Chapter 3 discussed the data collection

methods employed (see Section 3.4.3), along with the measuring instrument—the Metacognitive Awareness Inventory (MAI) developed by Schraw and Dennison (1994)—used to measure the metacognitive awareness of adults (see Section 3.4.3.1; see also Sections 4.2.1–4.2.2).

A qualitative approach was adhered to for secondary research question 4, exploring the level of metacognitive awareness of pre-service teachers in a problem-solving context (see Section 3.4.3.2). The findings of this think-aloud problem-solving session served to enrich the findings of the MAI. Quality criteria regarding the quantitative instrument, and regarding inferences that were made from the qualitative data, were also discussed in the chapter (see Section 3.4.3.3). Finally, methods of data analysis and interpretation were referred to (see Section 3.4.5).

#### **5.2.4 Overview of Chapter 4**

In Chapter 4, the empirical data was analysed and interpreted. The measuring instrument, the MAI, was found to be valid and reliable as an instrument, as well as according to the two factor conceptualisation of *Knowledge of cognition* and *Regulation of cognition* (see Section 4.2.3).

Descriptive statistical procedures were employed when analysing the quantitative data. Inferential statistics, the *Spearman Rho* correlation value, determined the correlation between the two factors on the MAI (see Section 4.3.1). A discussion of individual tendencies, as illustrated by the seven items with the highest and lowest means respectively, enriched the findings of the MAI (see Section 4.3.2).

The qualitative data was analysed by means of coding and comparing the comments of the pre-service teachers in the problem-solving session to the metacognitive strategies which related to items and subscales on the MAI. The quantitative and qualitative findings were consequently compared, with due consideration given to the limitations of the comparison (see Section 4.5; see also Section 3.4.5). While interpreting the data, the findings were correlated with the literature on the role of metacognition in Mathematics teaching and learning, specifically attributes of mathematical proficiency,

aspects of productive learning, and the four-step problem-solving framework (see Section 4.4).

### **5.3 FINDINGS IN THE LITERATURE REVIEW**

The nature of the construct metacognitive awareness is not precisely defined; however, metacognitive awareness conceptualised as two subcomponents—metacognitive knowledge and metacognitive skills—is widely accepted (see Sections 2.2.1; 2.2.3). Metacognitive experiences elicit metacognitive knowledge, which prompts individuals to reflect on self—their feelings, thoughts, actions, knowledge, and skills—and hence employ metacognitive and cognitive strategies to regulate feelings, thoughts, actions, and goals (see Section 2.2.4).

Metacognition contributes to learning in relation to other constructs like meta-memory, self-regulation, and motivation and affect (see Section 2.2.5). This interrelationship makes metacognition difficult to precisely define (see Section 2.2.3) and consequently difficult to measure (see Section 3.4.1). In the meta-memory framework, information is transferred from the cognitive to the metacognitive level and vice versa (see Section 2.2.4.2). As a higher-order thinking skill, metacognition is elicited during novel scenarios and challenging mathematical problem-solving tasks (see Sections 2.2.2; 2.2.4.2). Metacognition develops from childhood to adulthood and most adults possess some level of metacognitive awareness. Metacognitive skillfulness could be general or domain-specific (see Section 2.2.4.3). However, metacognitive monitoring skills do not necessarily translate into metacognitive regulation (see Sections 2.2.4.3; 2.2.6).

I agree with the view that metacognition as adaptive competence facilitates learning and problem solving, as well as the ability to transfer and use knowledge and skills in novel scenarios and when solving ill-defined problems (Bransford et al., 2000: 18; see Sections 2.3.1.2; 2.3.4.2). Experts are recognised by their adaptive reflective practices. Metacognition enables individuals to deal with the demands of fast-changing knowledge and new situations at school, work, and in lifelong learning (see Sections 2.1; 2.3.1.2; 2.3.4.2).

Various studies on metacognition and metacognitive intervention found that metacognitive awareness, strategies, and skills can be acquired and therefore taught (see Sections 2.2.4.3; 2.4.3) and can enhance academic achievement (see Sections 1.3; 2.3.2; 2.3.3). It is not easy to reflect naturally and spontaneously (see Section 2.3.5), but metacognitive awareness develops with intent and deliberate training over a prolonged period and thus there are factors that enhance metacognitive awareness in individuals (see Section 2.4). The role of metacognition is therefore recognised in Mathematics teaching and learning internationally (see Sections 2.3.4.1; 2.3.4.2) and especially in the South African context (see Sections 1.2; 2.3.1.3). Metacognition is one of the four attributes of mathematical proficiency (see Section 2.3.4.2) and an aspect of productive learning in Mathematics (see Section 2.3.4.3). Moreover, it is key in mathematical problem solving and facilitates the transfer between the four-phase cyclical problem-solving framework (see Section 2.3.4.4).

Metacognitive reflection as an adaptive competence facilitates the translation of knowledge into skills. Mathematically proficient learners have the disposition, knowledge, and skills to do what they say they can do (see Section 2.3.4.2).

Mathematically proficient learners presuppose mathematically proficient teachers who model and scaffold learning and problem solving. Consequently, teachers' metacognition in facilitating Mathematics learning and problem solving is important. Teaching for metacognition (see Section 2.4) requires teachers to be metacognitively aware and reflective practitioners themselves. Teachers who are not fully aware of the importance of metacognition in teaching and learning are not likely to facilitate metacognition in productive learning and successful problem solving; therefore, empowering teachers as metacognitive reflective practitioners should be an aim of teacher training (see Sections 2.3.2; 2.3.5.1). Furthermore, the metacognitive reflection of teachers is complex due to personal and situational factors. The need for reflective practices to be cultivated in pre-service and during service teacher training is therefore well-recognised (see Sections 2.3.5; 2.3.5.1).

In teacher training, it is reasonable to expect that pre-service Mathematics teachers would possess a level of metacognitive awareness. It is crucially important to encourage and enhance their level of metacognitive awareness so that they, in turn, can translate metacognition to their learners at an adequate level. Therefore, I considered it important to determine the level of metacognitive awareness of fourth-year pre-service Mathematics teachers, as well as whether their reported metacognitive awareness translated successfully into a problem-solving context.

## **5.4 FINDINGS OF THE EMPIRICAL RESEARCH**

The purpose of the empirical research was to investigate the level of metacognitive awareness of pre-service Mathematics teachers and to make recommendations based on the data to encourage and facilitate metacognitive awareness among pre-service teachers (see Section 5.5).

To achieve this, a primary research question and four secondary research questions were formulated. The findings for secondary research questions 3 and 4 are discussed below.

**Secondary research question 3:** What is the level of metacognitive awareness of pre-service Mathematics teachers on the MAI?

The purpose of the research was primarily addressed using quantitative data by means of a questionnaire, the MAI. Although metacognition is defined and conceptualised in various ways, literature confirms the MAI as a reliable and valid instrument for adults and has consequently been used in various studies (see Sections 3.4.3.1.1; 4.2.1). It was also found reliable in the study to measure the metacognitive awareness of pre-service teachers according to the factors *Knowledge of cognition* and *Regulation of cognition* (see Sections 4.2.3.1; 4.2.3.2), confirming the two-component conceptualisation of metacognition (see Section 2.2.3).

Based on the mean of the total MAI and the mean of the two subcomponents of metacognition, the data from the questionnaire revealed that pre-service Mathematics teachers possess metacognitive awareness to some extent (see Section 4.3.1). As



indicated by the results of the pre-service teachers' MAI total scores, they perceive their metacognitive awareness to be at a level of 74%. The higher mean of *Knowledge of cognition* (3.82) over *Regulation of cognition* (3.64) implies that metacognitive knowledge is more prominent among the pre-service teachers than metacognitive skills.

In addition to assessing the overall metacognitive awareness of the pre-service teachers, individual tendencies in pre-service teachers' learning and problem solving in Mathematics were highlighted by examining the seven items with the highest and lowest means. These tendencies served to enrich the findings and are not indicative of the broader level of metacognitive awareness as indicated by the MAI total scores.

Regarding the seven items with the highest means, most of the pre-service teachers were aware that interest facilitates their learning and that reflection on prior knowledge of a topic or task is foremost in the productive learning of Mathematics. This indicates the role of affect in being mathematically proficient (see Section 2.3.4.2.4) and the constructive aspect of productive learning (see Section 2.3.4.3.1). Furthermore, they reported the importance of reading strategies to clarify an understanding of a Mathematics problem statement in three items (see Section 4.3.2.1). This corresponds with the first phase, *Orientation*, of the problem-solving framework (see Section 2.3.4.4.1). The pre-service teachers' metacognitive awareness related mainly to *Declarative Knowledge*, and in regards to metacognitive skills (*Planning*, *Information management*, and *Debugging*) it related particularly to making sense of the problem statement.

For the seven items with the lowest means, the pre-service teachers reported a low level of reflection in evaluating the solution to a problem and hence considering different or easier ways to solve the problem. Furthermore, they reported a lower level of awareness of monitoring the effectiveness of learning and of using problem-solving strategies in three items (see Section 4.3.2.2). They therefore did not report an awareness of accessing and building their knowledge base on the MAI, which implicitly relates to the constructive aspect of productive learning (see Section 2.3.4.3.1). Consequently, they did not display expertise in the problem-solving session, as effective

strategy use (heuristics) and tapping into a sound knowledge base are attributes of mathematical proficiency (see Sections 2.3.4.2.1; 2.3.4.2.2; 4.3.2.3).

In relation to the four-phase problem-solving framework and according to the third phase, *Execution*, a lower level of metacognitive awareness was reported concerning the importance of reflection and monitoring thoughts and actions during learning and problem solving (see Section 2.3.4.4.3). In addition, a lack of reflection on solutions is indicated regarding the fourth phase of the problem-solving framework, *Verifying* (see Section 2.3.4.4.4). On the MAI, therefore, the seven lowest reported items related to the following two MAI subscales: *Monitoring* and *Evaluation*.

In summary, these individual tendencies indicate a lack of awareness of (or skill in) applying effective evaluative and monitoring strategies. This observation highlights the difficulty that the pre-service teachers experienced with understanding and interpreting Mathematics questions, which on the one hand informed their reflection on effective reading strategies and motivated a careful reading of the problem statement, and on the other hand possibly influenced their interest and motivation. This could possibly have been informed by a reflection on their experiences at school level with solving mathematical problems.

The question hence arose whether this reported metacognitive awareness in learning and solving problems in Mathematics was at an adequate level to demonstrate success in an actual problem-solving context. In other words, could pre-service teachers translate their reported metacognitive awareness in Mathematics learning and problem solving, as reported in the MAI findings, to solving a challenging mathematical problem successfully? These findings are discussed next.

**Secondary research question 4:** What is the level of metacognitive awareness of pre-service Mathematics teachers in a problem-solving context?

Regarding this secondary research question, the qualitative data enriched the findings of the quantitative data and provided a broader perspective on the level of metacognitive awareness exhibited by the participants during the problem-solving session. The pre-service teachers recorded their thoughts and calculations while

problem solving (see Section 3.4.3.2; Appendices 4; 7). During the session, they mainly applied individual items relating to the following three MAI subscales: *Planning*, *Information management*, and to some extent *Declarative knowledge*. Additionally, their responses mostly related to the first two phases of the problem-solving framework, namely *Organisation* and *Orientation*. The strategies used most frequently were self-questioning and the managing of information to make sense of the problem statement (see Sections 4.4.3; 4.4.4). Therefore, the pre-service teachers initially displayed a potentially high level of metacognitive awareness by reflecting on their thoughts to understand the problem statement.

During and after the problem-solving session, in the *Execution* and *Verifying* phases, the pre-service teachers inadequately monitored errors and inadequately evaluated strategy effectiveness or the suitability of the solution. Consequently, the MAI subscales *Monitoring*, *Evaluation*, and *Debugging* did not feature significantly. The pre-service teachers thus demonstrated inadequate reflection on regulating progress and performance.

The study provides some evidence that the pre-service teachers' performances in the problem-solving session did not correspond with their own assessments of their metacognitive awareness on the MAI. This evidence indicates that the pre-service teachers might have overestimated their level of metacognitive awareness in learning and problem solving when they assessed themselves by means of the MAI. They might have been aware of what mathematical proficiency and productive learning entail, but failed to implement them successfully in the problem-solving scenario (see also Section 5.4.1).

In conclusion, the pre-service teachers did not display an adequate level of metacognitive awareness regarding reflection on aspects of Mathematics during the problem-solving session. Metacognitive skillfulness plays a significant role in successful performance (see Section 2.2.4.3). However, the ineffective use of strategies to correct comprehension and analyse and regulate performance during the problem-solving session resulted, for the majority of pre-service teachers, in overall failure to reach a

sensible solution (see Section 4.4.4). Additionally, a poor understanding of the problem statement also contributed to the low achievement.

The findings of both the literature review and empirical studies contributed to answering the research questions underpinning the study, and are summarised below.

### **5.4.1 Summary of findings**

**Secondary research question 1:** How is metacognition conceptualised?

Although not precisely defined or conceptualised, the two-component view of metacognition consisting of *Knowledge of cognition* and *Regulation of cognition* is accepted and used in the study. *Knowledge of cognition* consists of *Declarative knowledge*, *Procedural knowledge*, and *Conditional knowledge*, whilst *Regulation of cognition* consists of *Planning*, *Information management*, *Debugging*, *Monitoring*, and *Evaluation*.

**Secondary research question 2:** What is the role of metacognitive awareness in Mathematics teaching and learning?

Metacognition as adaptive competence facilitates mathematical proficiency and productive learning and hence influences achievement in Mathematics (see Sections 2.3.3; 2.3.4.2; 2.3.4.3). Furthermore, metacognition facilitates the problem-solving process in the four-phase problem-solving framework (see Section 2.3.4.4). Metacognitive adaptive competence could be taught and enhanced in teachers and learners (see Sections 2.3.5.1; 2.4). Metacognitive reflection improves pre-service and in-service teachers' practices (see Sections 2.3.5–2.3.5.1). Finally, teaching with and for metacognition enhances metacognitive awareness in learners (see Sections 2.3.4.1–2.3.4.4).

**Secondary research question 3:** What is the level of metacognitive awareness of pre-service Mathematics teachers on the MAI?

The study indicates that the pre-service teachers possess a moderately high level (74%) of metacognitive awareness, as indicated by the results of the pre-service

teachers' MAI total scores. Metacognitive knowledge is more prominent than metacognitive skills and makes a unique contribution to performance.

**Secondary research question 4:** What is the level of metacognitive awareness of pre-service Mathematics teachers in a problem-solving context?

The qualitative inquiry illustrated that the majority of pre-service teachers were unable to translate their reported metacognitive awareness to the problem-solving session.

While the level of metacognitive awareness of the pre-service teachers was reported as moderately high, it did not result in high achievement in the problem-solving session. Recall that only a fifth of the participants solved the problem successfully and two-thirds could not progress to phase 3 of the problem-solving framework. Metacognitive awareness was not demonstrated at the necessary level to achieve success throughout the problem-solving session, i.e. during all four phases of problem solving. Consequently, there was an observed gap between the pre-service teachers' reported abilities as measured by the MAI and what they could actually do during the problem-solving session. On the MAI and in the problem-solving session, the subscales *Declarative knowledge*, *Planning* and *Information management* were more prominent, whereas *Evaluation* and *Monitoring* featured to a lesser extent.

Therefore, the data and the literature concur to offer possible explanations for the findings as follows:

*Knowledge of cognition* and *Regulation of cognition* make unique contributions to learning and problem solving (see Sections 3.4.3.1.1; 4.3.1). The pre-service teachers reported a higher level of metacognitive knowledge than metacognitive skills in the quantitative inquiry. In the qualitative inquiry, metacognitive skills were clearly evident, as elicited by the complex novel problem, whereas metacognitive knowledge was more implicit and could be inferred from statements made (see Section 4.5). Support for the moderately high reported level of metacognitive awareness is found in another study with second and third-year undergraduate South African Mathematics teachers (Van Der Walt, 2014) which reported a high level of metacognitive awareness (see Section

2.3.3). Ultimately, most adults possess some level of metacognition (see Sections 2.2.4.3; 2.4.3).

Although correlation between the quantitative and qualitative data in the study is limited (see Section 3.4.5), there are a number of possible explanations for the gap that exists between the reported reflection on the MAI and the pre-service teachers' actual performance during the problem-solving session. Individuals might overestimate or underestimate their abilities, knowledge, and skills (see Section 2.2.6). In some instances, Mathematics teachers in South Africa have been found to overestimate their mathematical teaching abilities (Spaull, 2013: 21; see Sections 1.2; 2.3.3). Furthermore, a teacher's beliefs about learning (De Corte, 2010: 56) and students' inaccurate assessments and management of their learning due to faulty beliefs (Bjork et al., 2013: 417, 424, 427) can impact learning (see Section 2.3.5) and influence problem-solving (see Sections 2.3.4.2.4; 2.4.2). Reflection, especially during the problem-solving process, can be difficult (see Section 2.4.5). On the other hand, the pre-service teachers might possess the metacognitive knowledge and skills, but fail to apply these in the problem-solving situation, and monitoring does not necessarily result in regulation (see Section 2.3.1.2).

In addition to inadequate regulating skills, achievement in the problem-solving session was hampered by not making sense of or understanding the problem statement. Possible explanations could be deficiencies in content knowledge, insufficient opportunities to practice the solving of novel problems, and time constraints during the problem-solving session. Literature confirms the importance of sufficient subject knowledge and developing higher-order thinking skills (see Section 2.2.2) through providing regular opportunities to solve complex and novel tasks (see Section 2.4.3). Also, this poor performance might relate back to school level, where low emphasis on developing higher-order thinking skills and problem-solving opportunities is a continuing concern raised by educational researchers and educators (see Section 1.3).

The pre-service teachers displayed the attributes of mathematical proficiency by reporting awareness of the importance of affect (interest and motivation) and a well-resourced knowledge base (prior knowledge and subject knowledge) for successful problem solving. They displayed a lower level of reflection on metacognitive self-regulating and heuristic attributes, by not reflecting on previous successful reading and problem-solving strategies (see Section 2.3.4.2). Both on the MAI and in the think-aloud problem-solving session, the metacognitive skills of monitoring and evaluation of pre-service teachers presented at a lower level (see Section 4.5).

Metacognition as reflection facilitates the problem-solving process and progression from one phase to another in the problem-solving framework (see Section 2.3.4.4). The pre-service teachers reflected on the problem-solving statement by using the metacognitive strategies of asking themselves questions and making sense by drawing diagrams in the first phase, *Orientation*, and the second phase, *Organisation*, respectively. This helped a third of them to progress to phase 3, *Execution*. However, the lack of reflection on alternative and effective heuristic strategies or methods hindered progression from phases 2 to 3 and phases 3 to 4 and hence impacted progress towards solving the problem successfully (see Section 4.4.3).

In conclusion, based upon the pre-service teachers' reported lower level of awareness of metacognitive skills on the MAI and their poor performance in the problem-solving session, I argue that the training of pre-service Mathematics teachers potentially does not involve adequate metacognitive strategy training for problem solving. This suggests that the lecturers involved in teacher training might not teach for metacognition explicitly. Also, their knowledge about or experience in teaching with and for metacognition in problem solving might not be sufficiently developed. This group of respondents should be nurtured towards greater awareness of the role that metacognition plays within the academic context and Mathematics achievement. Additionally, metacognitive strategy training should be incorporated into all Mathematics teachers' training.

## 5.5 RECOMMENDATIONS

The study focused on the level of metacognitive awareness of pre-service Mathematics teachers. The pre-service teachers reported a level of metacognitive awareness to some extent; however, they could not translate their metacognitive knowledge and metacognitive skills into successful problem solving. Metacognitive awareness as adaptive competence is important in lifelong learning and problem solving, which are required in an information-rich world. Teachers should be metacognitively aware themselves and as reflective practitioners should translate, teach, and model these metacognitive behaviours for their learners. Enhanced metacognitive awareness in teachers and learners will lead to better learning and problem solving; therefore, developing metacognitive awareness should be explicitly integrated into teacher training and the teaching and learning of learners in general, and specifically in Mathematics, to enhance learning and problem solving and, consequently, influence achievement in Mathematics. Following on this argument, the following recommendations are made:

1. In future studies assessing metacognition, the different definitions, related constructs, and conceptualisations of metacognition should be considered, applied, and utilised, together with the two factors *Knowledge of cognition* and *Regulation of cognition* (see Section 2.2.3).
2. Regarding the measurement of metacognitive awareness, different methods should be borne in mind in selecting data collection procedures. The primary measuring instrument, the MAI, and the secondary supportive data collection method, a think-aloud problem-solving session, have been used in some previous studies on metacognition in school children and early adulthood (see Sections 3.4.3; 3.4.3.1; 3.4.3.2).
3. Generalisation is not possible in the study (see Section 3.4.5; 5.7). However, for the pre-service teachers in the study, metacognitive skills did not feature as strongly as metacognitive knowledge and therefore should be enhanced (see Section 5.4). In particular, monitoring and evaluation skills should be enhanced as the two subscales *Monitoring* and *Evaluation* did not feature prominently on the MAI. This observation was indicated by the low means on the instrument. Furthermore, metacognitive skills did not



feature prominently during the problem-solving session, as seen during the last two phases of the problem-solving framework. I therefore recommend that the pre-service teachers' metacognitive awareness could be enhanced by focusing on the subscales, making pre-service teachers aware of these metacognitive skills and hence scaffolding monitoring and evaluation strategies by modelling and facilitating how, when, and why (*Procedural and Conditional knowledge*) to employ these strategies in a Mathematics teaching and learning situation (see Sections 2.3.5.1; 2.4.1).

4. The MAI could be employed as a self-survey tool to determine the level of metacognitive awareness of pre-service teachers. In this way, lecturers could informally assess the level of metacognitive awareness of their students and make them aware of their level of metacognitive awareness. Furthermore, the MAI could be employed as a diagnostic tool for metacognitive intervention and strategy training prior to the fourth year of Mathematics didactics education (see Section 3.4.3.1.1).

5. Pre-service teachers should be made aware of the importance of metacognition as adaptive competence—and hence the role it plays in lifelong learning and their own metacognitive reflective practices—to improve their teaching practice (see Sections 2.3.5; 2.3.5.1).

6. Pre-service teachers should also be made aware of the importance of metacognition in facilitating productive learning and mathematical proficiency, firstly for their own benefit and secondly for their learners, ultimately influencing performance (see Sections 2.3.3; 2.3.5).

7. Metacognitive reflective practice as adaptive competence is developed as a deliberate and prolonged practice, as developing and implementing metacognitive reflection is not easy and this develops spontaneously to a certain extent only (see Sections 2.3.5.1; 2.4.1). Pre-service teachers, therefore, should be encouraged to cultivate metacognitive awareness through providing ongoing opportunities in teacher training for teachers to reflect on their practice of teaching Mathematics, as well as on how to model metacognitive awareness, productive learning, and competent problem solving to their learners.

8. The pre-service teachers' interest in a topic is crucially important for Mathematics learning. Real-life problems are commonly more interesting and challenging. Opportunities in teacher training should therefore include assessment tasks which involve solving complex and novel problems situated in authentic contexts. These ill-defined problems elicit interest and higher-order thinking, and as such prompt metacognitive awareness (see Sections 2.2.2; 2.2.4.2). Consequently, teachers will have an opportunity to reflect on and enhance their metacognitive and problem-solving skills.

9. Pre-service Mathematics education modules should include metacognitive strategy training, as metacognitive skills are trainable. Modelling of metacognitive and problem-solving strategies in teacher training enhances metacognition, and therefore pre-service teachers could more adequately facilitate metacognition in problem solving for their future learners (see Sections 2.2.4.3; 2.3.5; 2.4.3).

10. Pre-service teachers' metacognitive awareness and reflective practices should be enhanced across domains. Metacognitive skills are general as well as domain-specific; therefore, metacognitive adaptive competence could be enhanced by cross-curriculum metacognitive training. Pre-service training could involve the modelling of strategies and the scaffolding of reflective practices through tools such as metacognitive prompts, reflective journals, tick lists, self-questioning, and self-surveys (see Section 2.3.5.1).

11. Opportunities for ongoing reflective professional development and metacognitive strategy training for in-service teachers should be provided, especially in the Mathematics teaching and learning context. Moreover, mentoring and feedback opportunities around reflective practices are important, particularly for first-year novice teachers, as these teachers are more willing to reflect and as such will be provided the opportunity for metacognitive reflection over time (see Section 2.3.5).

12. Pre-service and in-service teachers should be made aware of a teaching environment conducive to developing higher-order thinking skills like metacognition and problem solving. Such a teaching and learning environment supports the development of metacognitive skills, as it encourages reflection on what one thinks, feels, and does in

learning and problem solving. A conducive Mathematics teaching and learning environment is characterised by a metacognitive reflective teacher who scaffolds and models metacognitive and problem-solving strategy training; a step-by-step problem-solving framework where metacognition facilitates the progress between phases; tools that scaffold metacognition; authentic and challenging tasks that create interest and prompt metacognition; and learning opportunities that facilitate productive learning (see also Section 2.4.4). In summation, teachers should be made aware of factors that enhance metacognitive awareness in their learners and how to teach with and for metacognition.

The above recommendations point towards the challenges that educators and educational institutions face. The first challenge is to make pre-service teachers aware of the importance of metacognition and its role in lifelong learning and problem solving. The second is equipping and empowering pre-service teachers with an adequate metacognitive awareness during training, to make them aware that effective teaching necessitates teaching learners how to learn and solve problems. This also has implications for the teaching and learning of Mathematics. Educators and lecturers involved in pre-service teacher training should be encouraged to incorporate metacognitive awareness into the modules for Education and Mathematics Education specifically. Furthermore, these educators and lecturers should be willing and encouraged to scaffold and model metacognitive awareness to their learners.

Pre-service teachers should be empowered with metacognitive knowledge and skills and should develop metacognitive reflective behaviours. Moreover, they should be encouraged to create a learning environment conducive to reflection and be given numerous opportunities to put these skills into practice while problem solving with prompting questions, self-reflection, and monitoring and evaluating. It is therefore important that educators in teacher training emphasise the importance of metacognition and its possible effect on Mathematics achievement, so that prospective teachers become reflective practitioners and adequately metacognitively aware to transform metacognitive behaviours in their learners.

## 5.6 RECOMMENDATIONS FOR FURTHER STUDY

To enhance and advance the findings of this dissertation and build upon the recommendations provided, further research relating to the following could be conducted:

- Determining the possible enhancement of metacognitive awareness in problem-solving scenarios for pre-service and in-service teachers;
- Investigating the impact of metacognitive intervention on learning and problem solving for pre-service Mathematics teachers;
- Determining the correlation of two quantitative measures of metacognition and another qualitative instrument (video recording and/or individual interviews and/or another think-aloud problem-solving session);
- Investigating the effect that metacognition and related constructs (self-regulation and/or motivation) have on learning and problem solving;
- Developing quantitative and qualitative measurements that assess corresponding metacognitive subscales and activities (see Section 3.4.5);
- Exploring the feasibility of incorporating metacognition training into didactic modules or curriculum documents at the higher education and school levels of Mathematics Education;
- An investigation to determine the level of metacognitive awareness of pre-service teachers in Mathematics practices;
- A study to determine the effect of metacognitive training/intervention in pre-service teachers and those in their first year of Mathematics teaching;
- Exploring the metacognitive awareness of pre-service teachers across higher education institutions;
- Investigating and comparing metacognitive awareness training in different Education didactic modules; and
- Exploring the possible relationship between metacognitive awareness and biographical variables like age, gender, language, and culture.

## 5.7 LIMITATIONS OF THE STUDY

The qualitative data about pre-service teachers' metacognitive awareness was obtained in a problem-solving session. This data represents a small section of the quantitative measurement obtained on the MAI; therefore, the extent to which comparisons could be made between the quantitative and qualitative data on the pre-service teachers' level of metacognition was limited.

The analysis of the think-aloud session in the problem-solving context in the qualitative study was based upon my interpretation of the material and the inferences interpreted therein. Another researcher could potentially interpret the metacognitive behaviours inferred by the statements differently.

Correlations between online and offline measures are generally low, although many studies report to use both. Online self-report measures are not very accurate, as autonomous metacognitive strategies might remain covert and not be verbalised (see Section 3.4.5), as well as because of the difficult nature of reflection. As a result, to elicit metacognitive behaviours a higher-order novel complex problem was selected for the think-aloud session.

The purposive convenience sample of the study was small ( $n < 100$ ), being limited to one subject didactic and one higher education institution. The small sample and non-parametric data and survey design restricted the extent to which inferences could be made, making generalisation of the results very limited. The employment of an additional questionnaire, another qualitative instrument like a video recording or interviews, or an additional think-aloud problem-solving session could also have provided another and richer perspective on the metacognitive awareness of pre-service teachers.

Finally, biographical variables and the previous education of the participants were not considered in determining the level of metacognitive awareness.

## 5.8 SIGNIFICANCE OF THE STUDY

In the South African context, little research about metacognition has been published and it mainly concerns school learners. Less is published about the metacognitive awareness of pre-service teachers. In my view, very few studies investigated the metacognitive awareness of teachers in a Mathematics context in South Africa. To my knowledge, no such study could be found for fourth-year pre-service Mathematics didactics teacher education students at a higher education institution in South Africa.

The study contributes to the scholarship on enhancing the metacognitive awareness of pre-service teachers through the enhancement of reflective practice and the deliberate teaching of metacognition. Metacognition contributes to improving the mathematical proficiency and productive learning of pre-service teachers, who in turn could harness this knowledge, skill, and disposition to improve that of their learners, impacting mathematical achievement in a potentially positive way. In the long term, this may also mean meeting skills shortages in the job market and developing lifelong learning skills to satisfy the workplace demands of the new millennium. It is advisable that metacognitive reflective training through mentoring should continue, especially for first-year in-service Mathematics teachers, as well as in the form of continuing professional development.

My views on metacognitive awareness and reflective practices in Mathematics teaching and learning were also influenced. Relating the literature on the significance of metacognitive knowledge and skills in Mathematics achievement, teaching, and learning to the inadequate performance in the problem-solving session, and therefore reflecting on the gap presented between what the pre-service teachers said they do and what they actually (could) do, reinforced the importance of reflective practices in teaching, of metacognitive facilitation of problem solving, and the need for opportunities to practice and develop these attributes.

A broader issue than simply encouraging a deeper understanding of the value of metacognitive awareness in learning and problem solving is encouraging a personal

awareness that metacognitive reflection is a lifelong, evolving competency which can be employed to address real-life situations adaptively.

The study has significance for future studies on determining the level of metacognitive awareness of pre-service Mathematics teachers and potentially comparing the level of metacognitive awareness of these teachers across higher education institutions. These findings could inform guidelines for developing and enhancing metacognitive awareness of pre-service Mathematics teachers. From the findings of the study, it is suggested that metacognitive awareness is encouraged and should be deliberately instructed over time, especially in Mathematics didactics teacher training modules. A metacognitive intervention in didactics modules should include deliberate and ongoing metacognitive reflective experiences with ill-defined or complex problems, giving students opportunities to develop and demonstrate what they know and can do effectively, including acquiring the adaptive competence to apply knowledge and skills successfully to novel situations and Mathematics problems, thus influencing achievement (see Sections 2.3.1.2; 2.3.4.2).

## **5.9 SUMMARY OF CHAPTER**

Metacognitive reflective teachers possess adaptive competence to adapt and improve their performance in the classroom and within their profession. Teachers could enhance the quality of Mathematics education if they are metacognitively aware of their own strengths and weaknesses and are able to apply their own knowledge and skills adaptively to novel scenarios, whether these pertain to classroom variability or Mathematics problem solving.

In addition, teachers could enhance the quality of Mathematics education if they foster within their learners a similar capacity to use their metacognitive adaptive competence to solve novel, complex Mathematics problems and, as metacognitive aware lifelong learners, successfully negotiate modern information overload and future workplace demands.

To teach content effectively also necessitates teaching learners how to learn and solve problems by making them aware of what productive learning is and empowering them to regulate their learning and problem-solving processes. Successful problem solving as a hallmark of mathematical proficiency is a goal of Mathematics education. An inadequate level of metacognitive awareness, however, could influence problem solving and hence negatively impact achievement.

Of course, other factors besides metacognitive awareness impact upon the teaching and learning of Mathematics. These include the dual nature of Mathematics, a conducive learning environment, the teacher's worldview and beliefs of what Mathematics teaching and learning is, proficiency in subject content, the learner's mathematical disposition and attitudes towards Mathematics (motivation, interest, affect, and beliefs), and the types of tasks used to elicit metacognition and create interest and engagement. Crucially important, all these factors are situated within a broader socio-economic context which can impact upon a learner's ability to complete school and pursue higher qualifications.

Amidst the many opinions on enhancing Mathematics education, a few key insights were highlighted in the study. Foremost was the importance of not only possessing sound subject knowledge, but also the ability to transfer this knowledge into skills adaptively and the opportunity to practise these skills; in other words, the ability to do what we say and think we can do. Moreover, by considering different perspectives and alternative strategies and solutions, how and when to implement these, and the key role of affect in engaging with Mathematics, this contributes to the enhancement of higher-order thinking, problem-solving skills, and metacognitive skills to adapt methods and strategies.

The call for reflective practices and metacognitive awareness training in pre-service teacher education is therefore confirmed in the study. The challenge for higher education institutions is, first, to make pre-service teachers aware of the importance of metacognitive awareness in lifelong productive learning and mathematical proficiency; second, to make them aware of the development of metacognitive awareness and how to transform knowledge into skills and instruct these effectively in a conducive



environment; and third, to empower pre-service teachers by providing deliberate and prolonged metacognitive reflective opportunities, with a level of metacognitive awareness which develops proficiency in content, learning, and problem solving. In summary, it entails the enhancement of adaptive competencies to apply knowledge and skills effectively to novel scenarios. The notion is that metacognitive reflective pre-service teachers will become metacognitive reflective in-service teachers who can transform this adaptive competence for their learners.

Holistically, metacognition as adaptive competence not only informs and has value in reflective teaching practices, Mathematics problem solving, and learning, but also as a general life skill. Nurturing a reflective and metacognitively aware disposition will help individuals to deal with various life issues and challenges reflectively and adaptively, while helping to steer them through the complexities of contemporary life.

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# APPENDIX 1

## MAI QUESTIONNAIRE IN AFRIKAANS

Hierdie vraelys bestaan uit twee afdelings:

Afdeling A: Biografiese besonderhede

Afdeling B: Metakognitiewe strategieë

Amptelike	<input type="checkbox"/>	<input type="checkbox"/>
gebruik	<input type="checkbox"/>	<input type="checkbox"/>

### Algemene inligting:

1. Hierdie vraelys neem rofweg 20 minute om te voltooi.
2. U terugvoer sal waardevol wees vir navorsingsdoeleindes.
3. Hierdie vraelyste gaan net deur die navorser hanteer word.
4. U naam gaan nie gebruik word in die verslaggewing van die navorsingsbevindinge nie.

<b>AFDELING A: BIOGRAFIESE BESONDERHEDE</b>
---

**Voltooi asseblief die volgende en beantwoord dan die vrae:**

Naam en van: .....

### INSTRUKSIES:

Dui u antwoord met 'n kruis (X) aan :

1. Onderrigtaal

Afrikaans	1
Engels	2

2. Wat is u huidige ouderdom?

19 jaar	1
20 jaar	2
21 jaar	3
22 jaar	4
Ander (skryf dit asseblief)	5

## AFDELING B

Die doel van die volgende vrae is om verskeie aspekte rondom leer en probleemoplossing in Wiskunde te ondersoek.

### INSTRUKSIES:

Kies **een** van die volgende vyf moontlike antwoorde deur 'n kruis te trek by die nommer wat ooreenstem met die volgende opsies:

Stem glad nie saam nie	Stem nie saam	Neutraal (stem nie saam nie en verskil ook nie)	Stem saam	Stem heeltemal saam
1	2	3	4	5

### **LEES ASSEBLIEF ELKE VRAAG NOUKEURIG**

		Stem glad nie saam nie	Stem nie saam	Neutraal	Stem saam	Stem heeltemal saam
1.	Ek vra myself van tyd tot tyd of ek al my doelwitte in Wiskunde bereik.	1	2	3	4	5
2.	Ek oorweeg eers verskillende maniere om 'n probleem op te los <b>voordat</b> ek begin om 'n Wiskundeprobleem op te los.	1	2	3	4	5
3.	Wanneer ek 'n Wiskundige probleem oplos probeer ek metodes gebruik wat in die verlede gewerk het.	1	2	3	4	5
4.	Ek pas my tempo aan wanneer ek studeer vir 'n Wiskundetoets of –eksamen sodat ek betyds klaar studeer.	1	2	3	4	5
5.	Ek is bewus van my intellektuele sterk- en swakpunte in Wiskunde.	1	2	3	4	5
6.	Ek dink na oor wat ek regtig moet leer voordat ek vir 'n Wiskundetoets of -eksamen begin studeer.	1	2	3	4	5



7.	Ek weet na afloop van 'n Wiskundetoets of -eksamen hoe goed ek daarin gedoen het.	1	2	3	4	5
8.	Ek stel spesifieke doelwitte voordat ek vir 'n Wiskundetoets of -eksamen begin studeer.	1	2	3	4	5
9.	Ek lees stadiger wanneer belangrike inligting in 'n Wiskundevraag teëkom.	1	2	3	4	5
10.	Ek weet wat die belangrikste inligting is wat in Wiskunde geleer moet word.	1	2	3	4	5
11.	Ek vra myself of ek <b>verskillende metodes</b> oorweeg het wanneer ek 'n Wiskunde probleem oplos.	1	2	3	4	5
12.	Ek is goed daarmee om inligting wat ek in Wiskunde ontvang te organiseer.	1	2	3	4	5
13.	Ek fokus doelbewus my aandag op belangrike inligting in 'n Wiskundevraag.	1	2	3	4	5
14.	Ek het 'n spesifieke doel vir elke probleemoplossingsmetode wat ek gebruik wanneer ek 'n Wiskunde probleem oplos.	1	2	3	4	5
15.	Ek leer die beste wanneer ek reeds iets weet van die Wiskundeonderwerp wat ek bestudeer.	1	2	3	4	5
16.	Ek weet wat die onderwyser verwag wat ek moet leer wanneer ek vir 'n Wiskundetoets of –eksamen studeer.	1	2	3	4	5
17.	Ek is goed daarmee om Wiskunde feite en –beginsels te onthou.	1	2	3	4	5
18.	Ek gebruik verskillende leerstrategieë wanneer ek Wiskunde studeer.	1	2	3	4	5
19.	Nadat ek 'n Wiskunde probleem opgelos het, vra ek myself of daar 'n makliker manier was om die probleem op te los.	1	2	3	4	5
20.	Ek kan beheer hoe goed ek in Wiskunde leer.	1	2	3	4	5
21.	Ek doen hersiening van tyd tot tyd om my te help om belangrike verhoudings in Wiskunde te verstaan.	1	2	3	4	5
22.	Ek vra myself vroe oor die probleem voordat ek begin om 'n Wiskunde probleem op te los.	1	2	3	4	5

23.	Wanneer ek begin om 'n Wiskunde probleem op te los, dink ek aan verskeie maniere om die probleem op te los, en kies dan die beste manier.	1	2	3	4	5
24.	Ek som op wat ek leer terwyl ek studeer.	1	2	3	4	5
25.	Ek vra ander leerders om te help as ek iets in Wiskunde nie verstaan nie.	1	2	3	4	5
26.	Ek kan <b>myself motiveer</b> om te studeer vir 'n Wiskundetoets of –eksamen.	1	2	3	4	5
27.	Ek is bewus van watter leerstrategieë ek gebruik wanneer ek Wiskunde studeer.	1	2	3	4	5
28.	Ek vra myself hoe bruikbaar my leerstrategieë is terwyl ek studeer vir 'n Wiskundetoets of –eksamen.	1	2	3	4	5
29.	Ek gebruik my sterkpunte in Wiskunde om te kompenseer vir my swakpunte in Wiskunde.	1	2	3	4	5
30.	Wanneer ek nuwe inligting ontvang oor 'n bekende of nuwe Wiskundeonderwerp, fokus ek op die betekenis en belangrikheid van die nuwe inligting.	1	2	3	4	5
31.	Ek skep my eie voorbeelde om nuwe inligting wat ek in Wiskunde ontvang meer betekenisvol en verstaanbaar te maak.	1	2	3	4	5
32.	Ek kan goed oordeel hoe goed ek iets in Wiskunde verstaan.	1	2	3	4	5
33.	Ek vind dat ek outomaties (sonder om doelbewus daaroor te dink) nuttige leerstrategieë gebruik in Wiskunde.	1	2	3	4	5
34.	Wanneer ek 'n Wiskunde probleem oplos of wanneer ek studeer vir 'n Wiskundetoets of –eksamen, vind ek dat ek gereeld stop om my begrip te toets.	1	2	3	4	5
35.	Ek weet in watter situasie elke probleemoplossingsmetode wat ek gebruik die doeltreffendste sal wees.	1	2	3	4	5
36.	Sodra ek klaar studeer het vir 'n Wiskundetoets of –eksamen, vra ek myself hoe goed ek my doelwitte bereik het.	1	2	3	4	5

37.	Ek teken prente of diagramme om my te help verstaan terwyl ek Wiskunde leer.	1	2	3	4	5
38.	<b>Nadat</b> ek 'n Wiskundeprobleem opgelos het, vra ek myself of ek verskillende maniere oorweeg het om die probleem op te los.	1	2	3	4	5
39.	Ek probeer om Wiskundevrae in my eie woorde om te sit.	1	2	3	4	5
40.	Ek verander my probleemoplossingsmetode wanneer ek nie vordering maak om 'n Wiskundeprobleem op te los nie.	1	2	3	4	5
41.	Ek lees die vraag noukeurig voordat ek 'n Wiskundevraag beantwoord.	1	2	3	4	5
42.	Wanneer ek 'n Wiskundevraag lees, vra ek myself of dit wat ek lees, verband hou met dit wat ek reeds weet.	1	2	3	4	5
43.	Wanneer ek nie vordering maak terwyl ek 'n Wiskundeprobleem oplos nie, vra ek myself of my aanvanklike begrip van die probleem korrek was.	1	2	3	4	5
44.	Ek organiseer my tyd om die doelwitte wat ek stel in Wiskunde te bereik.	1	2	3	4	5
45.	Ek leer beter wanneer ek geïnteresseerd is in 'n spesifieke Wiskundeonderwerp.	1	2	3	4	5
46.	Wanneer ek Wiskunde studeer probeer ek om die werk in kleiner afdelings op te breek.	1	2	3	4	5
47.	Wanneer ek Wiskunde studeer, fokus ek op hoe die spesifieke onderwerp wat ek studeer, inpas by die ander Wiskundeonderwerpe.	1	2	3	4	5
48.	Ek vra myself vrae oor hoe goed ek doen <b>terwyl</b> ek 'n Wiskundeprobleem oplos.	1	2	3	4	5
49.	Wanneer ek klaar geleer het, vra ek myself of ek soveel geleer het as wat ek kon.	1	2	3	4	5
50.	Wanneer ek 'n Wiskundevraag lees, lees ek enige afdeling van die vraag wat onduidelik is, weer 'n keer.	1	2	3	4	5

***Maak asseblief seker dat u al die vrae beantwoord het en dat u naam en van op die vraelys geskryf het.***

***Baie dankie vir u samewerking!***

**Questionnaire from Du Toit (2013), adapted from Schraw and Dennison (1994), translated and piloted.**

# APPENDIX 2

## MAI QUESTIONNAIRE IN ENGLISH

This questionnaire consists of two sections:

Section A: Biographical particulars.

Section B: Metacognitive strategies.

<b>Official</b>		
<b>use</b>		

### General information

1. This questionnaire will take roughly 20 minutes to complete.
2. Your response will be valuable for research purposes.
3. These questionnaires will only be handled by the researcher.
4. Your name will not be used in the reporting of the research findings.

**SECTION A: BIOGRAPHICAL PARTICULARS**

**Please complete the following and then answer the questions:**

Name and surname: .....

**INSTRUCTIONS:**

Indicate the number of your answer with an X:

1.. Language of instruction

Afrikaans	1
English	2

2. What is your current age?

19 years	1
20 years	2
21 years	3
22 years	4
Other (please write down)	5

**SECTION B**

The purpose of the following questions is to investigate various aspects of learning in Mathematics.

**INSTRUCTIONS:**

Choose **one** of the five possible answers by crossing (X) the number that corresponds with your preferred options:

<b>Strongly disagree</b>	<b>Disagree</b>	<b>Neutral (<i>Neither agree nor disagree</i>)</b>	<b>Agree</b>	<b>Strongly agree</b>
<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>

**PLEASE READ EACH QUESTION CAREFULLY**

		<i>Strongly Disagree</i>	<i>Disagree</i>	<i>Neutral</i>	<i>Agree</i>	<i>Strongly Agree</i>
1.	I ask myself periodically if I am meeting my goals in Mathematics.	1	2	3	4	5
2.	I first consider different ways of solving the problem <b>before</b> I start solving a problem in Mathematics.	1	2	3	4	5
3.	When I solve a Mathematics problem, I try to use methods of solving a problem that have worked in the past.	1	2	3	4	5
4.	I pace myself when I study for a Mathematics test or examination in order to finish studying in time.	1	2	3	4	5
5.	I understand my intellectual strengths and weaknesses in Mathematics.	1	2	3	4	5
6.	I think about what I <b>really need to learn</b> before I begin studying for a Mathematics test or examination.	1	2	3	4	5
7.	I know how well I did once I finish a Mathematics test or examination.	1	2	3	4	5

8.	I set specific goals before I begin to study for a Mathematics test or examination.	1	2	3	4	5
9.	I read slower when I encounter important information in a Mathematics question.	1	2	3	4	5
10.	I know what kind of information is most important to learn in Mathematics.	1	2	3	4	5
11.	I ask myself if I have considered <b>different methods</b> of solving a problem when solving a Mathematics problem.	1	2	3	4	5
12.	I am good at organising the information I receive in Mathematics.	1	2	3	4	5
13.	I consciously focus my attention on important information in a Mathematics question.	1	2	3	4	5
14.	I have a specific purpose for each problem-solving method I use when I solve a problem in Mathematics.	1	2	3	4	5
15.	I learn best when I already know something about the Mathematics topic I am studying.	1	2	3	4	5
16.	I know what the teacher expects me to learn when I study for a Mathematics test or examination.	1	2	3	4	5
17.	I am good at remembering Mathematics facts and principles.	1	2	3	4	5
18.	I use different learning strategies, depending on the situation, when I study Mathematics.	1	2	3	4	5
19.	After I have solved a Mathematics problem, I ask myself if there was an easier way to solve the problem.	1	2	3	4	5
20.	I can control how well I learn in Mathematics.	1	2	3	4	5
21.	I periodically do revision to help me understand important relationships in Mathematics.	1	2	3	4	5
22.	I ask myself questions about the problem before I begin to solve a Mathematics problem.	1	2	3	4	5
23.	When I start to solve a Mathematics problem, I think of several ways to solve the problem and choose the best one.	1	2	3	4	5

24.	I summarise what I learn when I study.	1	2	3	4	5
25.	I ask other learners for help when I do not understand something in Mathematics.	1	2	3	4	5
26.	I can <b>motivate myself</b> to study for a Mathematics test or examination.	1	2	3	4	5
27.	I am aware of what learning strategies I use when I study Mathematics.	1	2	3	4	5
28.	I ask myself how useful my learning strategies are while I study for a Mathematics test or examination.	1	2	3	4	5
29.	I use my strengths in Mathematics to compensate for my weaknesses in Mathematics.	1	2	3	4	5
30.	When I receive new information about a familiar topic or a new topic in Mathematics, I focus on the meaning and significance of the new information.	1	2	3	4	5
31.	I create my own examples to make new information I receive in Mathematics more meaningful and understandable.	1	2	3	4	5
32.	I am a good judge of how well I understand something in Mathematics.	1	2	3	4	5
33.	I find myself using helpful learning strategies in Mathematics automatically (without consciously thinking about it).	1	2	3	4	5
34.	When I solve a Mathematics problem, or when I study for a Mathematics test or examination, I find myself pausing regularly to check my comprehension.	1	2	3	4	5
35.	I know in which situation each problem-solving method I use will be most effective.	1	2	3	4	5
36.	I ask myself how well I accomplished my goals once I am finished studying for a Mathematics test or an examination.	1	2	3	4	5
37.	I draw pictures or diagrams to help me understand while I am learning Mathematics.	1	2	3	4	5
38.	<b>After</b> I have solved a Mathematics problem, I ask myself whether I have considered different ways to solve the problem.	1	2	3	4	5
39.	I try to put Mathematics questions into my own words.	1	2	3	4	5



40.	I change my problem-solving method when I fail to make progress when I try to solve a Mathematics problem.	1	2	3	4	5
41.	I read the question carefully before I answer a Mathematics question.	1	2	3	4	5
42.	When I read a Mathematics question, I ask myself if what I am reading is related to what I already know.	1	2	3	4	5
43.	If I do not make progress when I solve a Mathematics problem, I ask myself whether my first understanding of the problem was correct.	1	2	3	4	5
44.	I organise my time to best accomplish the goals I set in Mathematics.	1	2	3	4	5
45.	I learn better when I am interested in a specific Mathematics topic.	1	2	3	4	5
46.	When I study Mathematics, I try to break down the work into smaller sections.	1	2	3	4	5
47.	When I study Mathematics, I focus on how the specific topic I study fits in with the other topics in Mathematics.	1	2	3	4	5
48.	I ask myself questions about how well I am doing <b>while</b> I am solving a Mathematics problem.	1	2	3	4	5
49.	I ask myself if I have learned as much as I could have once I finish studying.	1	2	3	4	5
50.	When I read a Mathematics question, I stop and reread any section of the question that is not clear.	1	2	3	4	5

***Please make sure that you have answered all questions, and that you have written down your name and surname.***

***Thank you very much for your co-operation!***

**Questionnaire from Du Toit (2013), adapted from Schraw and Dennison (1994).**

# APPENDIX 3

## THE SUBSCALES ON THE MAI

### KNOWLEDGE OF COGNITION

#### Declarative knowledge (8 items)

*Knowledge the individual possesses about themselves and their own strategies: “knowledge about one’s skills, intellectual resources, and abilities as a learner” (Schraw & Dennison, 1994: 474)*

5.	I understand my intellectual strengths and weaknesses in Mathematics.
10.	I know what kind of information is most important to learn in Mathematics.
12.	I am good at organising the information I receive in Mathematics.
16.	I know what the teacher expects me to learn when I study for a Mathematics test or examination.
17.	I am good at remembering Mathematics facts and principles.
20.	I can control how well I learn in Mathematics.
32.	I am a good judge of how well I understand something in Mathematics.
45.	I learn better when I am interested in a specific Mathematics topic.

#### Procedural knowledge (4 items)

*Knowledge about how to use those strategies successfully: “knowledge about **how** to implement learning procedures (e.g. strategies)” (Schraw & Dennison, 1994: 474)*

3.	When I solve a Mathematics problem, I try to use methods of solving a problem that have worked in the past.
14.	I have a specific purpose for each problem-solving method I use when I solve a problem in Mathematics.
27.	I am aware of what learning strategies I use when I study Mathematics.
33.	I find myself using helpful learning strategies in Mathematics automatically (without consciously thinking about it).

### Conditional knowledge (5 items)

Knowledge about when and why to use certain strategies, based upon factors such as effectiveness, relevance, and suitability: “knowledge about **when** and **why** to use learning procedures” (Schraw & Dennison, 1994: 474)

15.	I learn best when I already know something about the Mathematics topic I am studying.
18.	I use different learning strategies, depending on the situation, when I study Mathematics.
26.	I can motivate myself to study for a Mathematics test or examination.
29.	I use my strengths in mathematics to compensate for my weaknesses in Mathematics.
35.	I know in which situation each problem-solving method I use will be most effective.

## REGULATION OF COGNITION

### Planning (7 items)

Occurs prior to learning and involves setting goals and allocating time and resources towards achieving these goals: “planning, goal setting, and allocating resources **prior** to learning” (Schraw & Dennison, 1994: 474)

4.	I pace myself when I study for a Mathematics test or examination in order to finish studying in time.
6.	I think about what I really need to learn before I begin studying for a Mathematics test or examination.
8.	I set specific goals before I begin to study for a Mathematics test or examination.
22.	I ask myself questions about the problem before I begin to solve a Mathematics problem.
23.	When I start to solve a Mathematics problem, I think of several ways to solve the problem and choose the best one.
41.	I read the question carefully before I answer a Mathematics question.
44.	I organise my time to best accomplish the goals I set in Mathematics.

### Information management (9 items)

*Occurs during learning and involves using various skills and strategy sequences—such as organisation, elaboration, summary, and selective focus—to help efficiently process information: “skills and strategy sequenced used on-line to process information more efficiently (e.g. organising, elaborating, summarising, selective focusing)” (Schraw & Dennison, 1994: 474–475)*

9.	I read slower when I encounter important information in a Mathematics question.
13.	I consciously focus my attention on important information in a Mathematics question.
30.	When I receive new information about a familiar topic or a new topic in Mathematics, I focus on the meaning and significance of the new information.
31.	I create my own examples to make new information I receive in Mathematics more meaningful and understandable.
37.	I draw pictures or diagrams to help me understand while I am learning Mathematics.
39.	I try to put Mathematics questions into my own words.
42.	When I read a Mathematics question, I ask myself if what I am reading is related to what I already know.
46.	When I study Mathematics, I try to break down the work into smaller sections.
47.	When I study Mathematics, I focus on how the specific topic I study fits in with the other topics in Mathematics.

### Monitoring (7 items)

*The individual's assessment of their own learning or strategy use through self-testing and reflection: “assessment of one's learning or strategy use” (Schraw & Dennison, 1994: 475)*

1.	I ask myself periodically if I am meeting my goals in Mathematics.
2.	I first consider different ways of solving the problem before I start solving a problem in Mathematics.
11.	I ask myself if I have considered different methods of solving a problem when solving a Mathematics problem.
21.	I periodically do revision to help me understand important relationships in Mathematics.
28.	I ask myself how useful my learning strategies are while I study for a Mathematics test or examination.

34.	When I solve a Mathematics problem, or when I study for a Mathematics test or examination, I find myself pausing regularly to check my comprehension.
48.	I ask myself questions about how well I am doing while I am solving a Mathematics problem.

### **Debugging (4 items)**

*The use of strategies such as remediation to help identify and address errors in comprehension and performance: “strategies used to correct comprehension and performance errors” (Schraw & Dennison, 1994: 475)*

25.	I ask other learners for help when I do not understand something in Mathematics.
40.	I change my problem-solving method when I fail to make progress when I try to solve a Mathematics problem.
43.	If I do not make progress when I solve a Mathematics problem, I ask myself whether my first understanding of the problem was correct.
50.	When I read a Mathematics question, I stop and reread any section of the question that is not clear.

### **Evaluation (6 items)**

*Occurs after a learning experience and entails analysing the effectiveness of performance and strategies, re-evaluating approaches where applicable: “analysis of performance and strategy effectiveness after a learning episode” (Schraw & Dennison, 1994: 475)*

7.	I know how well I did once I finish a Mathematics test or examination.
19.	After I have solved a Mathematics problem, I ask myself if there was an easier way to solve the problem.
24.	I summarise what I learn when I study.
36.	I ask myself how well I accomplished my goals once I am finished studying for a Mathematics test or an examination.
38.	After I have solved a Mathematics problem, I ask myself whether I have considered different ways to solve the problem.
49.	I ask myself if I have learned as much as I could have once I finish studying.

**Adapted from Schraw and Dennison (1994) and Du Toit (2013).**

## APPENDIX 4

### THINK-ALoud PROBLEM-SOLVING SESSION: PROBLEM STATEMENT

Name and surname / *Naam en van:*

Please solve the following problem by writing down your thoughts and corresponding calculations/ *Los asseblief die volgende probleem op deur u denkprosesse en oorteenstemmende bewerkings neer te skryf.*

Suppose a piece of wire could be tied tightly around the earth at the equator. The earth's circumference at the equator is approximately 40 000 km. Imagine that this wire is then lengthened by exactly one meter and held so that it is still around the earth at the equator. The wire cannot be bent to form a loop or the wire cannot be tied. Would a mouse be able to crawl between the wire and the earth after it has been lengthened? Why or why not? Support your answer with applicable calculations.

*Veronderstel dat 'n stuk draad styf om die aarde by die ewenaar gespan kan word. Die aarde se omtrek by die ewenaar is naastebly 40 000 km. Verbeel u dat die draad verleng kan word met presies een meter en so gehou word dat dit nog steeds rondom die aarde by die ewenaar gespan is. Die draad kan nie gebuig word om 'n lus te vorm nie of die draad kan nie geknoop word nie. Sal 'n muis tussen die draad en die aarde kan deurkruip? Hoekom of hoekom nie? Ondersteun u antwoord met toepaslike berekeninge.*

<p><b>Write down your thinking processes</b> <i>Skryf u denkprosesse neer</i></p>	<p><b>Write down the calculations that correspond with your thinking processes</b> <i>Skryf neer die bewerkings wat met u denkprosesse verband hou</i></p>

## APPENDIX 5

### SOLUTION TO THE PROBLEM-SOLVING SESSION

Suppose a piece of wire could be tied tightly around the earth at the equator. The earth's circumference at the equator is approximately 40 000 km. Imagine that this wire is then lengthened by exactly one meter and held so that it is still around the earth at the equator. The wire cannot be bent to form a loop or the wire cannot be tied. Would a mouse be able to crawl between the wire and the earth after it has been lengthened? Why or why not? Support your answer with applicable calculations.

Distance around earth: 40 000 km

Conversion of units:  $40\,000\text{ km} \times 1000 = 40\,000\,000\text{ m}$

$40\,000\,000\text{ m} + 1\text{ m} = 40\,000\,001\text{ m}$

Estimation of height of mouse: 10 cm

Length of wire: 40 000 001 m

Circumference of circle =  $2\pi r$

$$\text{Radius of earth: } r = \frac{40\,000\,000}{2\pi}$$

$$\text{Radius of earth and wire: } r = \frac{40\,000\,001}{2\pi}$$

$$\text{Difference between radii: } \frac{40\,000\,001}{2\pi} - \frac{40\,000\,000}{2\pi}$$

$$= 0.15915495\text{ m}$$

Conversion of units:  $0.15915495\text{ m} \times 100 = 15.915495\text{ cm}$

**Conclusion:** Yes; as there is a space of approximately 15 cm between the earth and the wire, a mouse of average height (10 cm) will be able to pass under the wire.



## APPENDIX 6

### LEVEL OF PRE-SERVICE TEACHERS' METACOGNITIVE AWARENESS IN THE PROBLEM-SOLVING SESSION

#### KNOWLEDGE OF COGNITION

##### Declarative knowledge

Item	Description	Frequency of item
5	I understand my intellectual strengths and weaknesses in Mathematics.	0
10	I know what kind of information is most important to learn in Mathematics.	0
12	I am good at organising the information I receive in Mathematics.	0
16	I know what the teacher expects me to learn when I study for a Mathematics test or examination.	0
17	I am good at remembering Mathematics facts and principles.	39
20	I can control how well I learn in Mathematics.	0
32	I am a good judge of how well I understand something in Mathematics.	1
45	I learn better when I am interested in a specific Mathematics topic.	0
<b>Total uses of Declarative knowledge items</b>		<b>40</b>

##### Procedural knowledge

Item	Description	Frequency of item
3	When I solve a Mathematics problem, I try to use methods of solving a problem that have worked in the past.	0
14	I have a specific purpose for each problem-solving method I use when I solve a problem in Mathematics.	11
27	I am aware of what learning strategies I use when I study Mathematics.	0
33	I find myself using helpful learning strategies in Mathematics automatically (without consciously thinking about it).	0
<b>Total uses of Procedural knowledge items</b>		<b>11</b>

## Conditional knowledge

Item	Description	Frequency of item
15	I learn best when I already know something about the Mathematics topic I am studying.	0
18	I use different learning strategies, depending on the situation, when I study Mathematics.	0
26	I can motivate myself to study for a Mathematics test or examination.	0
29	I use my strengths in Mathematics to compensate for my weaknesses in Mathematics.	0
35	I know in which situation each problem-solving method I use will be most effective.	8
<b>Total uses of Conditional knowledge items</b>		<b>8</b>

## REGULATION OF COGNITION

### Planning

Item	Description	Frequency of item
4	I pace myself when I study for a Mathematics test or examination in order to finish studying in time.	0
6	I think about what I really need to learn before I begin studying for a Mathematics test or examination.	0
8	I set specific goals before I begin to study for a Mathematics test or examination.	0
22	I ask myself questions about the problem before I begin to solve a Mathematics problem.	19
23	When I start to solve a Mathematics problem, I think of several ways to solve the problem and choose the best one.	1
41	I read the question carefully before I answer a Mathematics question (e.g. by <i>chunking information/writing down key phrases</i> ).	32
44	I organise my time to best accomplish the goals I set in Mathematics.	0
<b>Total uses of Planning items</b>		<b>52</b>

## Information management

Item	Description	Frequency of item
9	I read slower when I encounter important information in a Mathematics question.	0
13	I consciously focus my attention on important information in a Mathematics question ( <i>e.g. by writing down key facts</i> ).	62
30	When I receive new information about a familiar topic or a new topic in Mathematics, I focus on the meaning and significance of the new information.	0
31	I create my own examples to make new information I receive in Mathematics more meaningful and understandable.	0
37	I draw pictures or diagrams to help me understand while I am learning Mathematics.	21
39	I try to put Mathematics questions into my own words.	2
42	When I read a Mathematics question, I ask myself if what I am reading is related to what I already know.	1
46	When I study Mathematics, I try to break down the work into smaller sections.	0
47	When I study Mathematics, I focus on how the specific topic I study fits in with the other topics in mathematics.	0
<b>Total uses of Information management items</b>		86

## Monitoring

Item	Description	Frequency of item
1	I ask myself periodically if I am meeting my goals in Mathematics.	0
2	I first consider different ways of solving the problem before I start solving a problem in Mathematics.	1
11	I ask myself if I have considered different methods of solving a problem when solving a Mathematics problem.	2
21	I periodically do revision to help me understand important relationships in Mathematics.	0
28	I ask myself how useful my learning strategies are while I study for a Mathematics test or examination.	0
34	When I solve a Mathematics problem, or when I study for a Mathematics test or examination, I find myself pausing regularly to check my comprehension.	9
48	I ask myself questions about how well I am doing while I am solving a Mathematics problem.	1
<b>Total uses of Monitoring items</b>		13

## Debugging

Item	Description	Frequency of item
25	I ask other learners for help when I do not understand something in Mathematics.	0
40	I change my problem-solving method when I fail to make progress when I try to solve a Mathematics problem.	1
43	If I do not make progress when I solve a Mathematics problem, I ask myself whether my first understanding of the problem was correct.	2
50	When I read a Mathematics question, I stop and reread any section of the question that is not clear.	22
<b>Total uses of Debugging items</b>		25

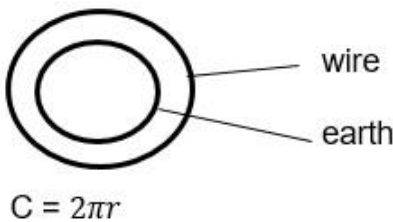
## Evaluation

Item	Description	Frequency of item
7	I know how well I did once I finish a Mathematics test or examination.	2
19	After I have solved a Mathematics problem, I ask myself if there was an easier way to solve the problem.	0
24	I summarise what I learn when I study.	0
36	I ask myself how well I accomplished my goals once I am finished studying for a Mathematics test or an examination.	0
38	After I have solved a Mathematics problem, I ask myself whether I have considered different ways to solve the problem.	1
49	I ask myself if I have learned as much as I could have once I finish studying.	0
<b>Total uses of Evaluation items</b>		3

## APPENDIX 7

# SAMPLE ANALYSIS: PRE-SERVICE TEACHERS' SOLUTIONS IN THE PROBLEM-SOLVING SESSION

### Respondent 16

Write down your thinking processes	Write down the calculations that correspond with your thinking processes
<p>But let's assume a smooth earth. The wire cannot be bent and the mouse must have enough space between the wire and the earth to crawl through. It can't bend the wire.</p>	 <p style="text-align: center;"><math>C = 2\pi r</math></p>

*Respondent 16 rephrased/restated the question (Planning - Item 39) and drew a diagram (Information management - Item 37).*

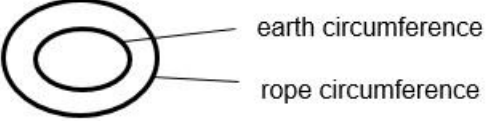
*Respondent 16 chunked the information, which is an indication of reading the question carefully (Information management - Item 41).*

*Respondent 16 consciously focused attention on important information (Planning - Item 13).*

<p>Earth circumference = 40 000 km.</p> <p>We need the radius of the lengthened rope to determine the space between surface and the lengthened wire.</p>	$40\,000 \text{ km} = 2\pi r$ $r = \frac{40\,000}{2\pi}$ $= 6366.19772 \text{ km}$ $= \text{radius of tight rope}$
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*Respondent 16 identified the core of the problem (Planning - Item 13). He indicated that he has a specific purpose for using this problem-solving method (Procedural knowledge - Item 14).*

*Respondent 16 recalled the formula correctly and made accurate calculations (Declarative knowledge - Item 17).*

<p>We now have the radius of the lengthened rope.</p>	
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**Respondent 16 drew diagrams to help him understand (Information management - Item 37).**

<p>Keeping in mind the conditions set, first let us change the units to meter and recalculate for earth and tight rope.</p>	$r = 40\,000\,000\text{ m} \div 2\pi$ $= 6366197.724\text{ m}$
---	--

**Respondent 16's conversion of the units and calculations was done correctly (Declarative knowledge - Item 17).**

**Respondent 16 re-read the question (Debugging - Item 50); he also focused on important information regarding the conditions set again (Planning - Item 13).**

<p>This is better: should have done this from the beginning but did not realise calculator is set to only four decimals.</p>	
--	--

**Respondent 16 re-evaluated his strategy and checked comprehension (Monitoring - Item 34) and adjusted his strategy when he monitored his answer (Debugging - Item 43).**

**Respondent 16 also reflected on how well he is doing while solving a problem (Monitoring - Item 48).**

<p>Radius of lengthened rope</p> <p>Let's change everything to mm</p> <p>What is the difference?</p>	$r = 40\,000\,001\text{ m} \div 2\pi = 6366197.883\text{ m}$ $R = 6366197724\text{ mm}$ $r = 6366197883$ $r - R = 158.83070\text{ mm}$
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**Respondent 16 asked a question related to the core of the problem statement (Planning - Item 22). Conversion of units was performed correctly (Declarative knowledge - Item 17).**

<p>Wow that's a whole 15.8 cm.</p> <p>Unless it is a really fat mouse it can somersault under the rope while ____ through.</p>	
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**Respondent 16 is confident about his answer (Evaluation - Item 7). He concluded correctly that the mouse would be able to pass under the wire.**

## APPENDIX 8

### CHAPTER MINDMAPS

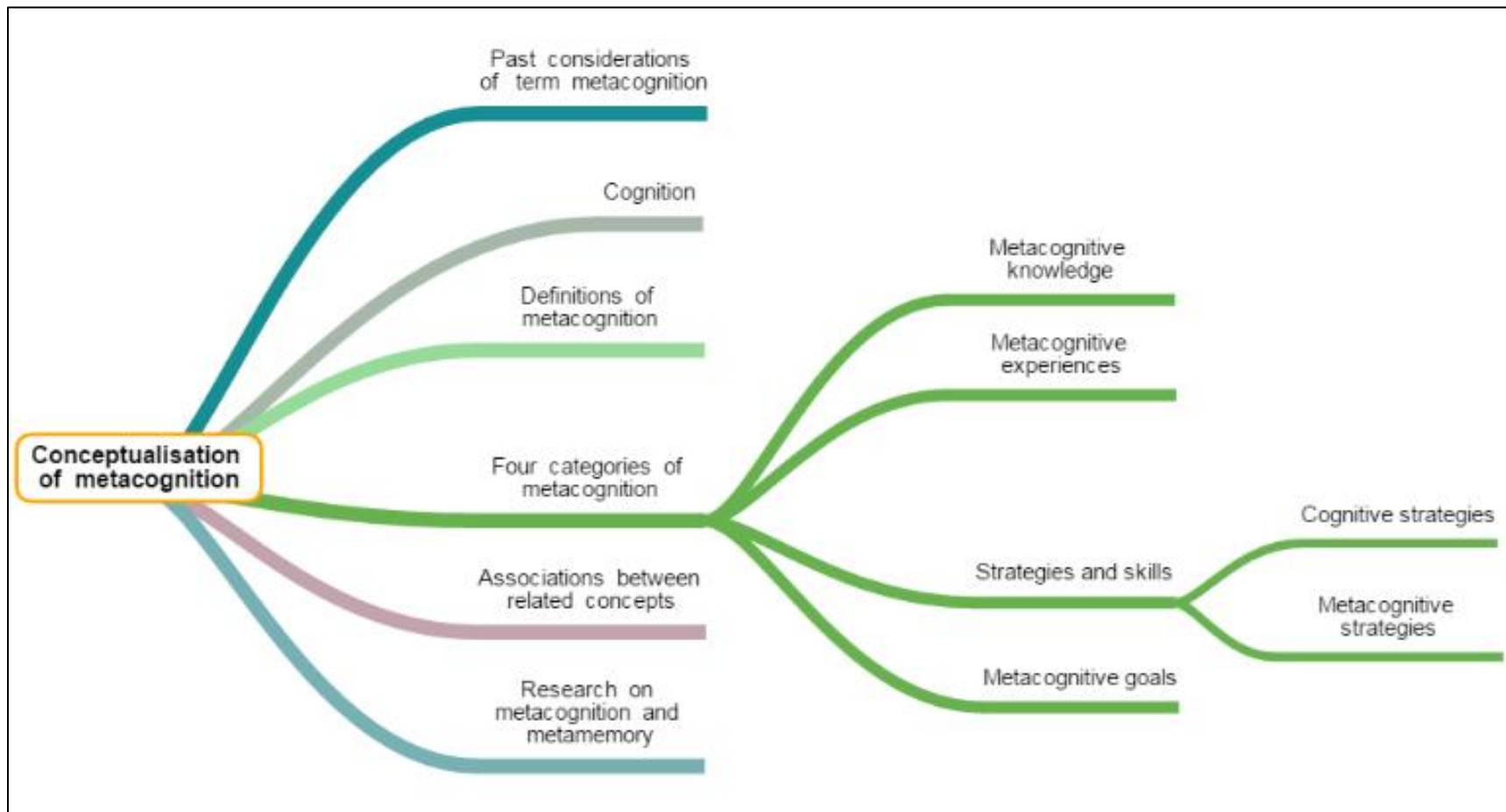


Figure 2.1 Conceptualisation of metacognition (Chapter 2)



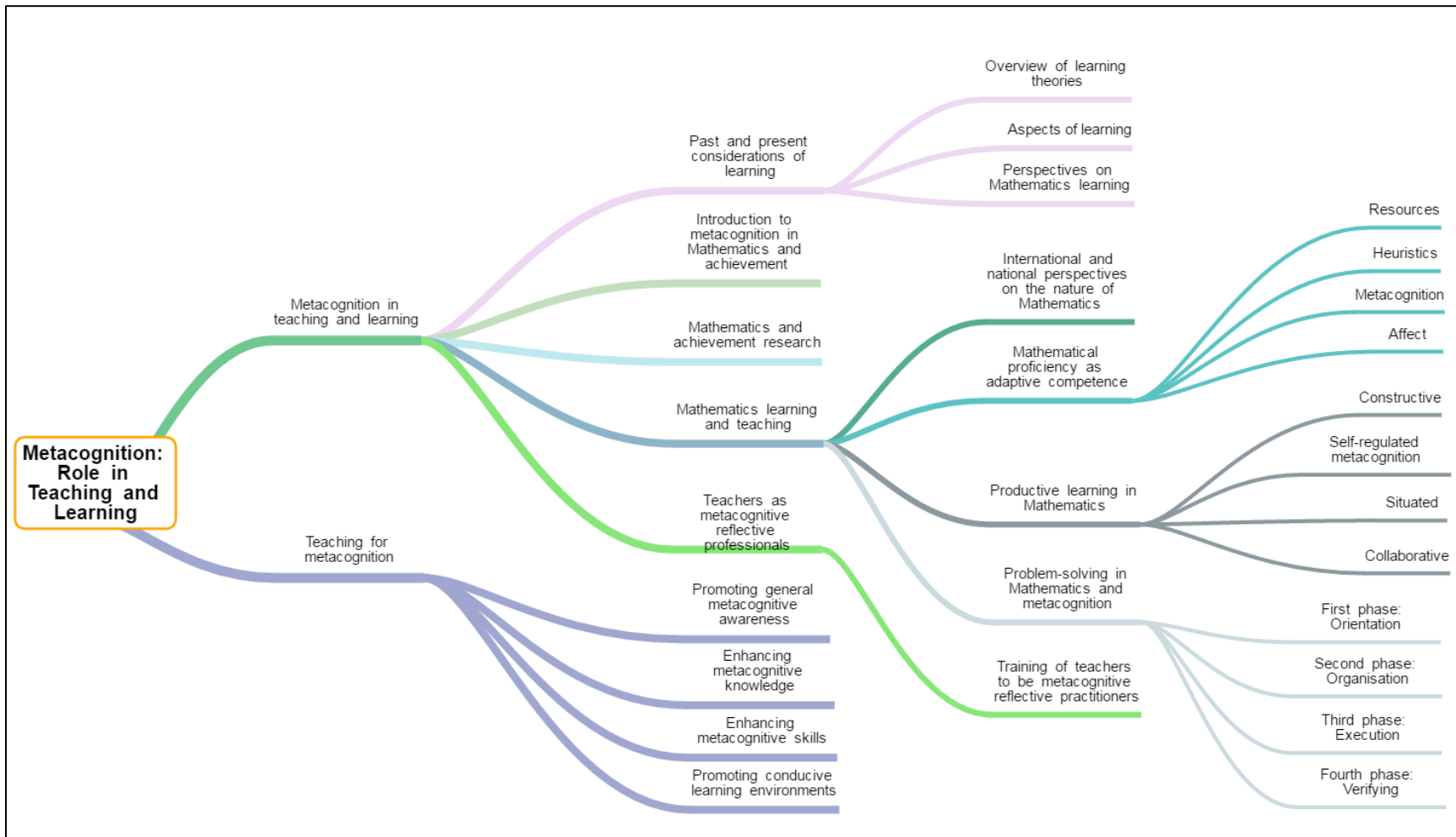
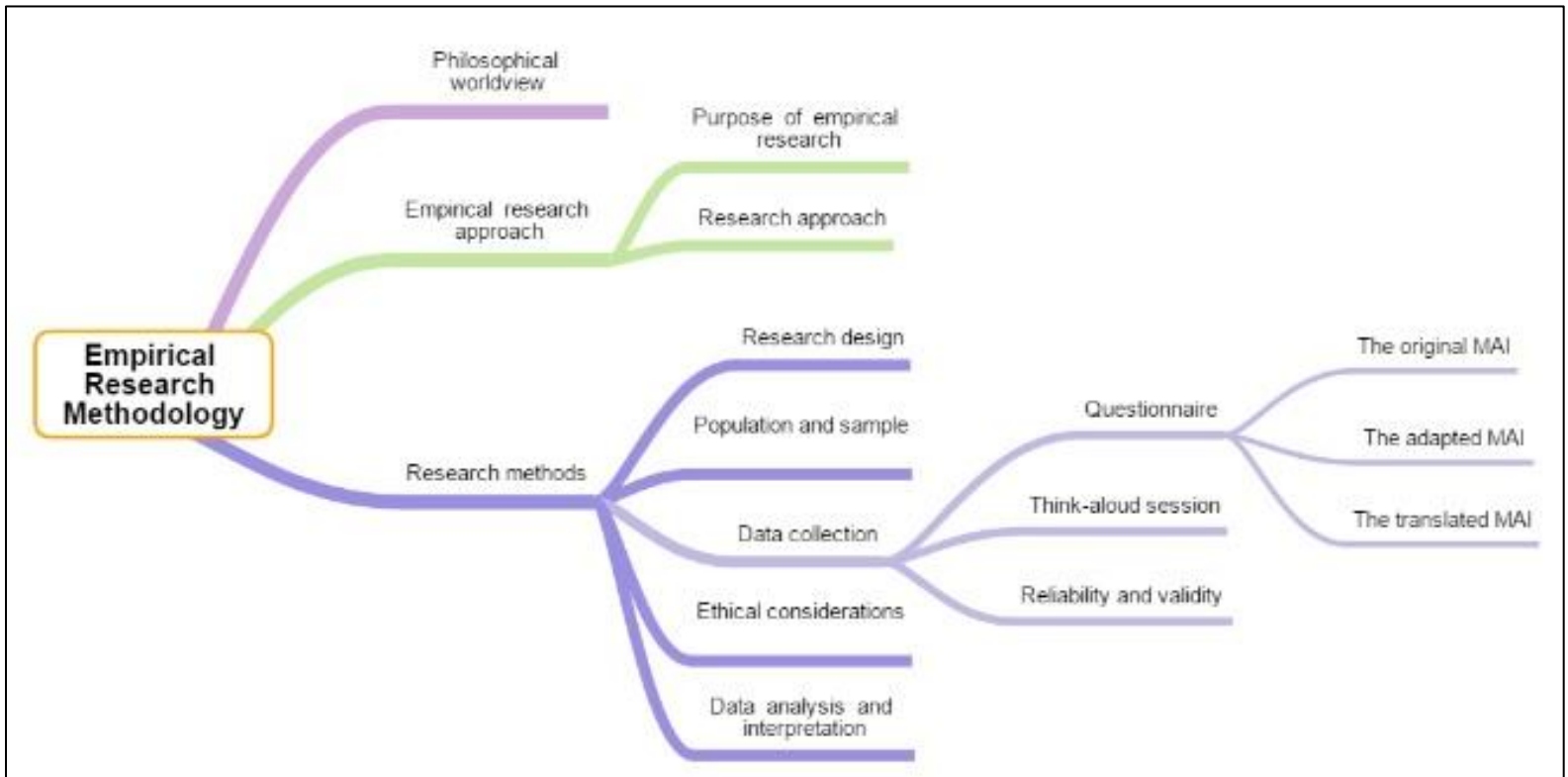


Figure 2.2 Metacognition in teaching and learning (Chapter 2)



**Figure 3.1 Components of the empirical research methodology (Chapter 3)**

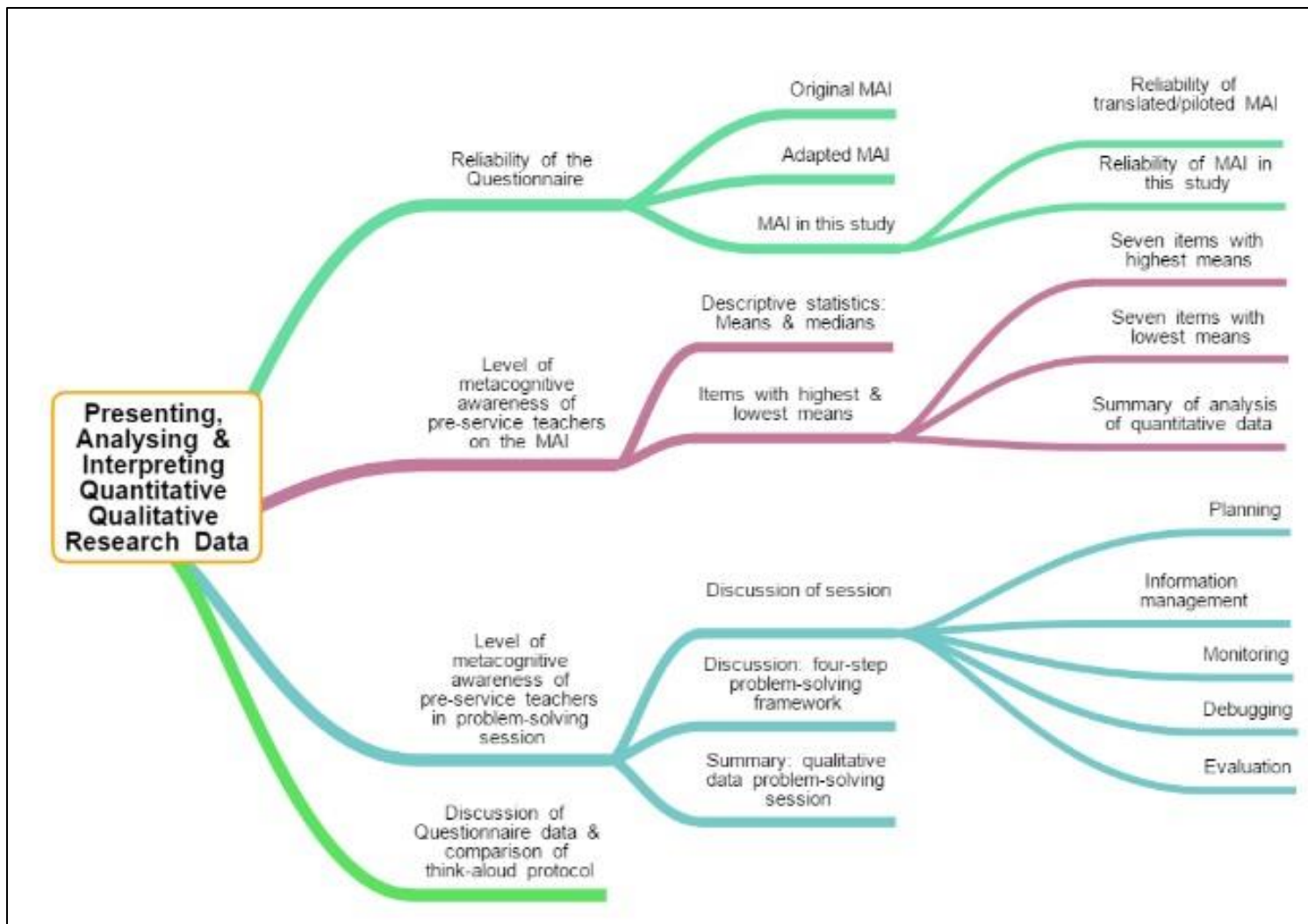


Figure 4.1 Presentation, analysis, and interpretation of qualitative and quantitative research data (Chapter 4)