IMPROVING TEACHERS' TECHNOLOGICAL PEDAGOGICAL CONTENT KNOWLEDGE FOR TEACHING EUCLIDEAN GEOMETRY USING INTEGRATED INFORMATION COMMUNICATION TECHNOLOGIES SOFTWARE

BY

Mosia MS (200502388)

BSc, PGCE, BSc Hons, MSc (UFS)

Thesis submitted in fulfilment of the requirements for the degree

Philosophiae Doctor in Education

(PhD Curriculum Studies)

FACULTY OF EDUCATION

At the

UNIVERSITY OF THE FREE STATE

BLOEMFONTEIN

June 2016

SUPERVISOR: Professor MG Mahlomaholo

CO-SUPERVISOR: Dr TJ Moloi

DECLARATION

I declare that the thesis, A STRATEGY TO IMPROVE TEACHERS' TECHNOLOGICAL PEDAGOGICAL CONTENT KNOWLEDGE FOR TEACHING EUCLIDEAN GEOMETRY USING INTEGRATED INFORMATION COMMUNICATION TECHNOLOGIES SOFTWARE, hereby submitted for the qualification of Doctor of Philosophy at the University of the Free State, is my own independent work and that I have not previously submitted the same work for a qualification at/in another university/faculty.

I hereby cede copyright to the University of the Free State.

MS. Mosia

June 2016

ACKNOWLEDGEMENTS

I wish to extend my gratitude to the following:

- I thank God who turns the impossibility to possibility.
- Pastor Irvin Mabokgole and his wife for their guidance and support from my junior degree to date.
- My supervisors, Prof Sechaba Mahlomaholo and Dr Tshele Moloi, for their wisdom, support and guidance in constructing this bricolage.
- My beautiful wife, Kemelo Mosia, for her continuous support and sacrifice throughout the study.
- Dr Moeketsi Tlali for his critical views; he has been a great teacher. Above all, I appreciate his love for and belief in me.
- Sule/Surlec for creating conditions conducive for the completion of this bricolage. Thanks, colleagues.
- Last but not least, my parents, Tumane Johannes Mosia and Ntombe Emily Mosia, for the way they fulfilled their parental roles.

DEDICATION

This thesis is dedicated to my beautiful wife, Kemelo Mosia.

SUMMARY

The study aimed at formulating a strategy to improve teachers' technological pedagogical content knowledge (TPACK) for teaching Euclidean geometry with the aid of integrated information and communication technology (ICT) software. TPACK refers to the interaction of three knowledge domains, which are, technology, pedagogy and content knowledge. The three knowledge domains further intersect to form subsets, which are, technological content knowledge; technological pedagogical knowledge; and pedagogical content knowledge. The three knowledge domains, together with the subsets, were used to define knowledge needed for teaching with the aid of technology. Furthermore, in the context of this study, integrated ICT software tools that were employed in teaching Euclidean geometry as teaching aids were Geometer's Sketchpad, GeoGebra and HeyMath!.

The study pursued the challenges that teachers face when they use ICT software as a teaching aid; these challenges included the following: Some teachers experience difficulties keeping up with rapidly advancing software knowledge; and the majority of teachers lack sufficient knowledge and skills to explore the potential of ICT software fully. In addition, part of the problem is that teachers found Euclidean geometry too abstract and difficult to teach. Thus, the study was geared to formulating a strategy to respond to these challenges. However, the challenge is that the knowledge needed for teaching is contextually bound and complex. Thus, the study adopted bricolage as a theoretical lens for the study, mainly due to its critical commitment to making meaning of complex objects of study in their contexts. In this study, bricolage enabled me to consider a theoretical stance from the eight historical moments of qualitative research. Through the multiplicity of theoretical lenses provided by bricolage I was able to unravel the multi-layered challenges and formulate a multi-layered strategy.

The multi-layered strategy was made possible by people who came together, with diverse back stories, knowledge and skills. In this study mathematics teachers who are faced with the day-to-day challenges of teaching Euclidean geometry with the aid of ICT software embarked on research to solve their own challenges. Driven by its epistemological stance on knowledge production, participatory action research created a platform for teachers, academics, and a computer programmer to engage in

knowledge production activities with equality and tolerance of contrasting views. Various data generation tools were employed, ranging from audio and video recordings, learners' scripts and data from their test scores. In order to deepen the meaning of spoken and written text, the study employed Van Dijk's critical discourse analysis at three levels, namely, text, discursive practices and social structures. Furthermore, learners' test scores were analysed using statistical techniques, such as boxplot, analysis of variance and statistical modelling. The study analysed the challenges experienced by teachers who teach Euclidean geometry with the aid of integrated ICT software. This was done for the purpose of proposing possible solutions and strategies that can be developed, adopted and adapted to address the challenges teachers experienced effectively.

In addition, for the purpose of sustainability of the strategy formulated to improve teachers' TPACK during and beyond the duration of the study, the conditions conducive for the strategy were investigated. The study analysed threats and risks that were embedded or inherited in the setting, to prevent them from impeding the successful implementation of the strategy. The study is transformative in nature, which created the opportunity to operationalise and evaluate the success of the strategy prior to it being considered for recommendation. Finally, some of the major findings were that teachers work in silos; and that they do not prepare sufficiently when they use ICT software as a teaching aid.

OPSOMMING

Die studie se doel was om ‘n strategie te formuleer om onderwysers se tegnologiese-pedagogiese-inhoudskennis (TPACK) vir die onderrig van Euklidiese meetkunde met die hulp van geïntegreerde inligtings- en kommunikasietegnogie (IKT) programmatuur, te verbeter TPACK verwys na die interaksie van drie kennisdomeine, naamlik, tegnologie-, pedagogiese en inhoudskennis. Die drie kennisdomeine sny mekaar verder om onderafdelings te vorm, naamlik tegnologie-inhoudskennis, tegnologiese pedagogiekkenis en pedagogiese inhoudskennis. Die drie kennisdomeine en die onderafdelings is gebruik om die kennis wat nodig is vir onderrig met tegnologie, te definieer. In die konteks van hierdie studie is van geïntegreerde IKT programmatuur gebruik gemaak om Euklidiese meetkunde te onderrig. Hierdie onderrighulpmiddels was Geometer’s Sketchpad, GeoGebra en HeyMath!.

Die studie het ondersoek ingestel na die uitdagings wat onderwysers konfronteer wanneer hulle IKT programmatuur as onderrighulpmiddels gebruik. Hierdie uitdagings het die volgende ingesluit. Sommige onderwysers ervaar probleme om by te hou met vinnig ontwikkelede programmatuurkennis; en die meerderheid onderwysers het nie genoeg kennis en vaardighede om die potensiaal van IKT programmatuur ten volle te ondersoek nie.Verder is deel van die probleem dat onderwysers Euklidiese meetkunde te abstrak ervaar, en moeilik vind om te onderrig. Dus was hierdie studie daarop gerig om ‘n strategie te formuleer wat hierdie uitdagings sou aanspreek.Die uitdaging is egter dat die kennis wat vir onderrig nodig is, kontekstueel gebonde en kompleks is. Dus het die studie bricolage as ‘n teoretiese lens vir die studie aanvaar, hoofsaaklik weens bricolage se verbintenis tot sinmaak van komplekse onderwerpe van studie binne hulle kontekste. In hierdie studie het bricolage my in staat gestel om ‘n teoretiese standput in te neem wat die agt historiese momente van kwalitatiewe navorsing in ag neem. Deur die verskeidenheid teoretiese lense van bricolage kon ek die veelvuldige lae waaruit die uitdagings bestaan, uitrafel en ‘n veelvlakkige strategie formuleer.

Die veelvlakkige strategie was moontlik gemaak deur mense wat saamgewerk het, en wat hulle diverse agtergrond-stories, kennis en vaardighede bygedra het. In hierdie studie het wiskunde-onderwysers wat met die dag-tot-dag uitdagings gekonfronteer

word wat met die onderrig van Euklidiese meetkunde met die hulp van IKT programmatuur verband hou, navorsing onderneem om hulle uitdagings self aan te spreek. Aangespoor deur die metode se epistemologiese standpunt teenoor kennisskepping, het deelnemende aksienavorsing ‘n platvorm vir die onderwysers, akademici en ‘n rekenaarprogrammeerder gebied om in kennisskeppingsaktiwiteite betrokke te raak, met gelykheid en aanvaardig van uiteenlopende standpute as oogmerk. ‘n Verskeidenheid hulpmiddels is gebruik om data te genereer, van oudio- en video-opnames en leerders se vraestelle, tot leerders se toetspunte. Ten einde die betekenis van gesproke en geskrewe teks te verdiep, het die studie Van Dijk se kritiese- diskoersanalise op drie vlakke aangewend, naamlik, teks, diskursiewe praktyke en sosiale strukture. Leerders se toetstellings is met die hulp van statistiese tegnieke ontleed, waaronder boxplot, ontleding van variansie en statistiese modellering. Die studie het die uitdagings wat onderwysers wat Euklidiese meetkunde met die hulp van geïntegreerde IKT programmatuur onderrig, ontleed. Dit is gedoen met die doel om moontlike oplossings en strategieë voor stel wat ontwikkel, aanvaar en aangepas kan word om die uitdagings wat onderwysers ervaar, doeltreffend aan te spreek.

Verder, met die doel om te verseker dat die strategie wat geformuleer is om die onderwysers se TPACK te verbeter, gedurende en na die duur van die studie volhoubaar sal wees, is die toestande wat bevorderlik is vir die strategie ondersoek. Die studie het dreigemente en risikos wat in die situasie ingebed of oorgeërf is, ontleed, om te voorkom dat dit die sukesvolle implementering van die strategie belemmer. Die studie is tranformerend van aard, en dit het geleentheid geskep vir operasionalisering en evaluering van die sukses van die studie voordat dit vir aanbeveling oorweeg word. In die laaste plek is van die belangrikste bevindinge dat onderwysers in silos werk, en dat hulle nie voldoende voorberei wanneer hulle IKT programmatuur as onderwyshulpmiddel gebruik nie.

CONTENTS

LIST OF FIGURES

LIST OF TABLES

LIST OF ABBREVIATIONS AND ACRONYMS

ANOVA	Analysis of variance
APIP	Academic performance improvement plan
CAPS	Curriculum Assessment Policy Statement
CBPR	Community-based participatory research
CCK	Common content knowledge
CDA	Critical discourse analysis
CKT	Content knowledge
CL	Critical linguistics
DBE	Department of Basic Education
DCES	Deputy chief education specialist
ELRC	Education Labour Relations Council
FET	Further Education and Training
GESCI	Global E-Schools and Communities Initiative
ICT	Information and communication technologies
IQMS	Integrated quality management system
KCT	Knowledge of content and students
MKO	Most knowledgeable other
PAR	Participatory action research
PCK	Pedagogical content knowledge

PK	Pedagogical knowledge
POET	Point of entry text
SADC	Southern African Development Community
SAQA	South African Qualifications Authority
SCK	Specialised content knowledge
SWOT	Strengths, weakness, opportunities and threats
TCK	Technological content knowledge
TK	Technological knowledge
TPACK	Technological pedagogical content knowledge
TPCK	Technological pedagogical content knowledge
TPK	Technological pedagogical knowledge

CHAPTER 1: THE ORIENTATION TO AND BACKGROUND OF THE STUDY

1.1 INTRODUCTION

This study sought to design a strategy to improve teachers' technological pedagogical content knowledge (TPACK) for teaching geometry with the aid of integrated information and communication technology (ICT) software. This chapter gives an overview of the study, starting with a brief background to contextualise the problem statement. Further, it provides a brief outline of the study that consists of the following: theoretical and conceptual framework; methodology and design, related literature; overview of the strategy design.

1.2 BACKGROUND OF THE STUDY

The study sought to design a strategy to improve teachers' TPACK for teaching geometry with the aid of integrated ICT software. TPACK represented an interaction between technology, pedagogy and content knowledge in relation to teaching with the aid of technology (Herbst & Kosko, 2014: 515). Technology knowledge (TK) as it relates to teaching refers to, among other things, knowledge of dynamic geometry software that can be used to describe the relationship between mathematical geometrical concepts better than in traditional ways (Liu & Kaino, 2007: 114). These software programs include GeoGebra, HeyMaths! and Geometer's Sketchpad. Pedagogical knowledge (PK), in this study, was defined as the knowledge and skills that teachers need in order to manage and organise geometry teaching and learning activities for intended outcomes (Koehler, Mishra, Akcaoglu & Rosenberg, 2013: 3). Lastly, in this study, content knowledge (CK) refers to facts such as the following: (i) a line segment drawn from the centre to the midpoint of a chord is perpendicular to that chord; (ii) an angle at the centre of a circle is twice the angle at the circumference subtended by the same arc or chord; (iii) an angle subtended by a diameter is 90 degrees; and (iv) opposite angles of cyclic quadrilateral are supplementary (DBE, 2011: 15).

Thus, the study sought to improve the teachers' knowledge and skills so that they could design and facilitate lessons using a variety of ICT software in a manner that would promote the following: i) Identification of geometrical concepts that learners find difficult to comprehend and that teachers find difficult to teach effectively; ii) Collaborative design of a multiple-software-based lesson that would make abstract concepts easy to understand; iii) Confident facilitation of a multiple-software-based lesson; iv) Resolution of any software and computer-related technical problems; and v) Conceptualisation of new research initiatives and creation of new knowledge or practice that applies integrated ICT software to enhance teaching strategies for abstract geometrical concepts.

In South Africa, as in other countries, teachers find it difficult to keep pace with rapidly evolving technology, such as the development of new software, and the rapid pace at which existing software is updated. Some teachers' knowledge of using software to teach geometry is limited to knowledge acquired during workshops, which only enables them adopt ICT software as a teaching aid. Teachers lack basic technical software knowledge, and this lack has an impact on their use of software for teaching (Tella, Tella, Toyobo, Adika & Adeyinka, 2007: 9). Furthermore, teachers experience pedagogical difficulty in designing, ordering and organising class activities, and alternating between different types of software while they teach (Leendertz, Blignaut, Nieuwould, Els & Ellis, 2013: 5). Similarly, Tella et al. (2007: 16) report that Nigerian teachers found integrating ICT software confusing; they found it difficult to incorporate it in designing and facilitating lessons – teachers tended to use the software to teach instead of using the software to enhance their teaching.

South African teachers have been found to possess inadequate Euclidean geometry content knowledge (Van Putten, Howie & Stols, 2010: 23). In Botswana, Nigeria and Korea teachers find geometry concepts too abstract to comprehend and teach, which has an effect on their teaching of geometry, and on learner performance (Nkhwalume & Liu, 2013: 27; Ratliff, 2011: 6). Using only one software program also has limitations, for example, HeyMaths! software has good fixed, animated lessons, but it does not enable teachers to interactively create their own animated lessons. On the other hand, Geometer's Sketchpad gives teachers an opportunity to create their own animated lessons, which could enhance the integration of ICT, thereby stimulating innovation and creativity among teachers.

The HeyMath! software program was introduced in South Africa in 2010 to help teachers to be more innovative in lesson design and teaching of mathematical concepts, such as the recognition and visualisation of geometrical figures. Studies report that, in Botswana, teachers are using integrated ICT software, such as Scratch, Inkscape, SketchUp, Mathematica and Excel, to promote creative teaching methods that improve learners' ability to recognise and visualise different solid and geometrical figures (Kaino, 2008: 1844; Nkhwalune & Liu, 2013: 26-34). Studies in Korea found enhanced creativity and innovation in lessons that incorporated graphic calculators, Spreadsheet, and Geometer's Sketchpad in the teaching of mathematical concepts such as angle measurements, visualisation of angles, and geometrical figures (Choi & Park, 2013: 274; Hyeyoung, 2011: 453; Keong, Horani & Daniel, 2005: 43-50; Meng, 2013:62). Furthermore, Korean and Nigerian teachers are using GeoGebra to enhance the teaching of transformation of geometrical figures, and to enhance visualisation skills (Meng, 2013: 62).

In order to design and implement a strategy to improve teachers' TPACK for teaching Euclidean geometry, it was important that we explored the conditions that would make the strategy work. In this way the study created a supporting space where teachers could acquire knowledge and skills on using computers and computer software. Conditions were also created for teachers to learn from each other and, where necessary, we involved people from outside the epistemic teaching community (Kaino, 213: 33). To ensure the success of this integrated ICT software strategy, the study explored the conditions that would be conducive for integrated ICT software programs to work effectively and efficiently.

However, implementing an integrated ICT software strategy also poses threats. For example, the HeyMath! software program can be misused if its readymade lessons take over the role of the teacher. Teachers should merely use the software program to facilitate their role, and apply the program as a communication tool to improve their teaching strategies (Koehler, Mishra, Kereluik, Shin & Graham, 2014: 103). Teachers need to be able to identify the strengths of different software programs, and even to avoid using software if teachers have better ways of communicating knowledge at their disposal (Shafer, 2007: 2). Teachers could alternate between using different software programs during a lesson, avoiding the limitations and exploiting the complementary features of the various programs. Integrating ICT software could lead to confusion if the lesson plan is not well structured. In order to prevent the dangers listed above, we conducted lesson preparation sessions on the effective use of ICT software in teaching.

Lastly, I evaluated the success of the strategy when teachers were able to demonstrate the ability to (i) identify geometric concepts that learners found difficult to comprehend and teachers found difficult to teach effectively; (ii) collaboratively design multiple-software-based lessons that would make abstract concepts easy to understand; (ii) facilitate a multiple-software-based lesson confidently; (iii) resolve any software and computer-related technical problems; and (iv) conceptualise new research initiatives and create new knowledge, or practise applying integrated ICT software to enhancing their teaching strategies for abstract geometrical concepts (Thirunavukkarasu, 2014: 52-50; SAQA, 2012: 12).

1.3 PROBLEM STATEMENT

There has been an increase in application of ICT software in teaching, particularly in mathematics. Studies report about the potential of ICT software for enhancing learners' understanding of abstract mathematical concepts, such as Euclidean geometry. However, the use of ICT software has the following challenges: Some teachers experience difficulties keeping up with rapidly advancing software knowledge; and the majority of teachers lack sufficient knowledge and skills to explore the potential of ICT software fully. Part of the problem is that teachers find Euclidean geometry abstract and difficult to teach. Therefore, in response to the preceding challenges, the study designed a strategy to assist teachers by addressing the following research questions.

1.3.1 Research question

How can teachers' TPACK for teaching Euclidean geometry using integrated ICT software be improved?

1.3.2 The aim of the study

The aim of the study was to design a strategy to improve teachers' TPACK for teaching Euclidean geometry with the aid of integrated ICT software.

1.3.3 The objectives of the study

The objectives of the study were to:

- investigate the challenges that face teachers who teach Euclidean geometry with the aid of integrated ICT software;
- analyse the different strategies that have been used to improve teachers' TPACK for teaching Euclidean geometry with the aid of integrated ICT software;
- identify conditions under which different strategies improve teachers' TPACK for teaching Euclidean geometry with the aid of integrated ICT software;
- identify the threats involved in implementing different strategies that have been used to improve teachers' TPACK for teaching Euclidean geometry with the aid of integrated ICT software; and to make suggestions for avoiding these threats; and
- identify indicators for evaluating the success of the strategies that have been used to improve teachers' TPACK for teaching Euclidean geometry with the aid of integrated ICT software.

1.4 THEORETICAL FRAMEWORK

This section validates the choice of bricolage as an appropriate theoretical position in designing a strategy to improve teachers' TPACK for teaching geometry with the aid of integrated ICT software. A discussion that validates the choice of bricolage as a theoretical position is given through the following: theoretical origin; formats; ontology; epistemology; the role of researcher; and the relationship between researcher and the participants.

1.4.1 The origin of bricolage

The study adopted bricolage as a theoretical lens to couch this study. Bricolage encourages a kind of research that derives its origin from a French metaphor of the word bricoleur, which means a handyman or -woman who uses the tools available to complete task at hand (Kincheloe, McLaren & Steinberg, 2011: 316; Kincheloe, 2004a: 1). Thus, in this study the tools at hand are the eight historical moments of qualitative research, which show the chronological evolution of bricolage through the following phases: the traditional period, the modernist phase, blurred genres, crisis of representation, the postmodern phase or fifth moment, the post-experimental or sixth moment, the seventh, and the eighth moment (Denzin & Lincoln, 1994: 3). The traditional period enabled me to view challenges that teachers who teach Euclidean geometry with the aid of ICT software were facing from a universal perspective. A universal perspective of these challenges was obtained through a literature review at national, regional, Southern African Development Community (SADC) and international levels, to establish whether there were common and/or related challenges in the teaching and learning of Euclidean geometry at all four levels. In addition, statistical analysis models and techniques were used to establish the universality of learners' performance in Euclidean geometry. The epistemology and ontology of the traditional period were used in this study to analyse teachers' pedagogical practices further. For instance, a traditional-moment-orientated teacher makes the assumption that learners do not know, and the teacher knows everything the learners need to know; learners learn as the teacher teaches. However, I argue that the traditional period's orientation to knowledge production has epistemic limitations. In order to address the limitations of the traditional period, I threaded to the second moment, called the modernity phase of qualitative research.

The modernity phase marks the first introduction of theories in qualitative research, such as ethnomethodology theory and phenomenology, which sought to make sense of data that did not adhere to traditional period ways of doing research (Denzin & Lincoln 1994: 3). Thus, in this study, the modernity phase is used to study the everyday pedagogical practices of the co-researchers and the scientific practice as one. Despite the way the modernity phase makes sense of the data, I argue that the modernity phase still has limitations, namely, that it excludes the co-researchers' emotional being, values and beliefs from the scope of inquiry. This exclusion led me to thread to the third moment, called blurred genres. This moment marks the maturity of qualitative research, with a complement of paradigms, methods and strategies to use in research (Denzin & Lincoln, 2005: 17).

In taking the blurred genres epistemological stance, I analysed the data using different theories that were convergent but sometimes conflicting. The fluid borrowing of ideas in the blurred genres moment led to challenges related to crises of representation, legitimation and praxis, known as the triple crisis moment. This moment contributed by problematising the issues of representation; for instance, the current study is committed to a critical vision of participatory action research (PAR), which advocates for the important voices in the process of research to be at the centre. Thus, for this study, it meant that the teachers who are involved in the day-to-day teaching of Euclidean geometry should be using the research to improve their teaching of Euclidean geometry. However, qualitative research moved into a postmodern period, which served as a corrective measure for the triple crisis. Thus, in this study, the postmodern period enabled me to use storytelling as part of hybrid representation in construction of the current bricolage. The seventh moment emerged to address the remnants of positivism on validity and reliability further. Thereby the seventh moment enabled me, as the research coordinator of this study, to ensure that the current study is ethically and morally acceptable. In this section I draw on the work of Denzin (2001:362), who explains that what is moral and ethical is subjective and not void of power; those who are powerful decide the epistemological aesthetic that describes what is beautiful, true and of good quality. Therefore, I argue that those who assess quality should do it within the context of the study. Lastly, the eighth moment enabled me to operate fully from multiple perspectives and multiple methodological approaches. This multiplicity was evident in the different representations of data, including the video data.

1.4.2 Formats of bricolage

I pursued the multiple formats of bricolage, which included the interpretive bricoleur, the methodological bricoleur, the theoretical bricoleur, the political bricoleur, and the narrative bricoleur (Rogers, 2012: 4). As an interpretive bricoleur I took the stance that states that there is no one correct telling – each telling is a reflection of someone’s perspective. Furthermore, the study employed methodological bricolage, which is a process of employing multiple research methods to make sense of or to unfold the complexity of the research problem (Rogers, 2012: 5; Kincheloe, 2005a: 335). Using the format of methodological bricolage freed me from using a single approach for analysis and interpretation in designing a strategy to enhance the teaching and learning of Euclidean geometry. In addition, as a theoretical bricoleur, I used multiple theoretical lenses to understand and interpret the challenges and their solutions in the teaching and learning of Euclidean geometry with the aid of ICT software better. As a political bricoleur I sought to produce knowledge that benefits those who are disenfranchised in the research process. Lastly, I argue that, to a narrative bricoleur, research is a representation of a specific interpretation of a phenomenon (Denzin & Lincoln, 1999: 5; Rogers, 2012: 7).

1.4.3 Ontology and epistemology

The ontology and epistemology of this study were not understood to be objective, external and fixed. The study subscribed to a complex ontology, which is committed to multiple realities that each co-researcher brought from his/her backstories. The complex ontology further complemented the complex epistemology that was created by a social web of reality that each co-researcher's unique contribution to the team effort (Kincheloe, 2004c: 73). Furthermore, I understood that there are multiple interpretations of the world and the way people relate and connect to the world around them. Thus, these multiplicity of interpretations inform a bricoleur about his/her object of inquiry, to become more open to many contexts and processes that are historically situated and culturally inscribed (Kincheloe, 2004c: 73).

1.4.4 The role of the researcher

The role of researcher in this study was to formally and informally convene a team of co-researchers in pursuit of designing a strategy to improve the teaching and learning of Euclidean geometry. In this study, I perceived myself as a co-researcher, since I do not have all knowledge required to resolve the challenges of teaching and learning Euclidean geometry with the aid of ICT software. This attitude is the result of the claim by Kincheloe (2011: 220) that many teachers are of the view that educational researchers offer little to help teachers address their day-to-day challenges. Therefore, my role was to contribute to the collective knowledge and skills that are necessary to respond to the research question (Te Aika & Greenwood, 2009: 59). Guided by the lens of bricolage, I understood that knowledge production is a product of multiple representations of human activities. Thus, in agreement with Mahlomaholo (2009: 226), as a co-researcher I invited other co-researchers to take part in the research project for creating a space for transformation and self-empowerment. Thus, the study was mainly located in the seventh and eighth moments of qualitative research, where research is an active process that is undertaken for the purpose of improving lives (Denzin, 2001: 326).

1.4.5 The relationship between researcher and the participants

A researcher as a bricoleur uncovers the context of research as an interactive process shaped by his or her personal history, biography, gender, social class, race and ethnicity, and that of the participants (Denzin & Lincoln, 1994: 17-18). Thus, a team of co-researchers in this study understood the inherent power differentials that each person brought to the team. For instance, some of the co-researchers were in managerial roles at the same schools as other co-researchers, who they led and who were also part of the team. However, the orientation to the research that the current study took levelled out power and promoted equality between co-researchers in knowledge production (Mahlomaholo, 2012). In being grounded in complex epistemology in the process of knowledge construction through research to find ways to improve the teaching of Euclidean geometry with the aid of ICT software, respect for complexity become the treasure.

1.5 CONCEPTUAL FRAMEWORK

As bricoleur I created a complex and rigorous structure that maps out the concepts and vocabulary used to make meaning of the knowledge needed for improving teachers' TPACK for teaching geometry concepts. In this section I erect a structure to create a map that connects all the concepts needed for this study (see Section 2.5). This map is called a bricolage map, and it is a list of possible areas that the bricoleur intends to visit to investigate what constitutes the knowledge needed to improve teachers' TPACK for teaching Euclidean geometry with the use of integrated ICT software. In constructing a bricolage map I started off with a so-called point of entry text (POET) (Berry, 2004b: 111). A POET is the central area of a bricoleur's map that he or she intends to investigate for the rest of the bricolage. Bricolage is a product produced by the bricoleur through the ways of conducting research. Thus, the POET for this study was improving teachers' TPACK for teaching Euclidean geometry with the aid of integrated ICT software.

1.5.1 Technological pedagogical content knowledge

TPACK refers to a synthesised form of knowledge that aims to integrate ICT into teaching and learning in a classroom environment (Chai, Koh & Tsai, 2013: 32). The work of Mishra and Koehler (2006: 1017-1054) is considered to have contributed significantly in shaping TPACK as a conceptual framework that gives both researchers and practitioners a vocabulary to describe knowledge needed for using technology for teaching (Koehler & Mishra, 2009: 62). Mishra and Koehler (2006: 1017-1054) built TPACK from the work of Lee Shulman (Shulman, 1987: 1-22) on the kind of knowledge needed for teaching (Chai et al., 2013: 31). Mishra and Koehler (2006: 1017-1054) start off with Shulman's notion of pedagocial content knowledge (PCK), which argues that, prior to his groundbreaking way of looking at teachers' knowledge, subject matter knowledge and pedagogical knowledge were each considered in isolation (Mishra & Koehler, 2006: 1021). PCK refers to an interaction or interrelation that exists between content and pedagogy, that is, PCK is a tranformed form of knowledge that blends both content and pedagogy into a knowledge of understanding how a particular concept is packedged and organised in such a way that it could be learnt easily.

Over time technology has become one of the teaching aids that seems to have potential for enhancing teaching in a general sense (Azlim, Amran & Rusli, 2015: 1794; Ali, Haolader & Muhammad, 2013: 4061; Buabeng-Andoh, 2012: 136; Koehler & Mishra, 2009: 61; Koh, Chia & Tsai, 2014: 185). The use of technology poses a challenge, since there are no theoretical frameworks to guide the process of integrating ICT in classroom teaching and learning. This results in, amongst other consequences, ineffective ways of using technology as teaching aid (Buabeng-Andoh, 2012: 137), and many researchers became interested in designing frameworks for using technology as teaching aid. An example is Mishra and Koehler (2006: 1024), who introduced a technological component for Shulman's PCK. They argue that, due to the transformation technology has been brought into the classroom, it is no longer enough to confine teacher knowledge to PCK, since teachers have to know more than just content and pedagogy (Mishra & Koehler, 2006: 1023).

1.5.2 Social contructivism

This section discusses the relevance of social constructivism as complementary conceptual framework that provides me with a stance to define and describe good teaching. Through the conceptual framework of social constructivism, I realised that improving TPACK knowledge for teaching Euclidean geometry using ICT software is a social endeavour while TPACK provided me with the vocabulary to describe the knowledge needed to teaching geometry using technology. The appropriateness of social constructivism as a conceptual theory for teaching and learning is justified through its ontology and epistemology.

1.6 SUMMARY OF RELATED LITERATURE

This section reviews literature related to improving teachers' TPACK for teaching Euclidean geometry using integrated ICT software. Literature from South Africa, SADC and Africa, and international best practices in line with the objectives of the study was reviewed.

1.6.1 Literature review to justify the need for designing a strategy to improve teachers' TPACK for teaching Euclidean geometry with the aid of ICT software

This section reviews related literature for the purpose of, in the first place, justifying the need for the study. Therefore, literature is reviewed to gain an understanding, to anticipate risks and threats and to find ways to circumvent them. Lastly, the study subscribes to the transformative agenda, which is both critical and emancipatory; thus, theories, previous research and policies are reviewed to shape the evidence of success for the strategy.

1.6.1.1 The need for a team approach to improve TPACK for teaching Euclidean geometry using ICT software

In the absence of a team each teacher works alone, despite the fact that there are other mathematics teachers who could collaboratively enhance one another's lesson preparation, assessment and lesson facilitation (Jita, Maree & Ndlalane, 2008: 475). When teachers work in silos it denies them the opportunity to share their skills and knowledge, which could contribute significantly to complementing one another in improving their TPACK for teaching Euclidean geometry with the aid of ICT software (Hu & Linden, 2015: 1104). The sharing of knowledge and skills not only contributes to enhancing the individual team members' knowledge, but also increases learners' chances of learning. I understand that it is through the social interactive spaces that are created by the team approach to improving pedagogical practices between the team members that new knowledge is formed (Cobb, 1994: 17).

This means that individuals join the team with different knowledge and skills, which, through their social interactions informed by their ontological and epistemological stances, form new transformed knowledge. Thus, bricolage in pursuit of understanding the proponents of TPACK, provides me with an ontological and epistemological stance that is fluid and dynamic (Kincheloe, 2009: 108). This means that different people in a team, each shaped by their different backgrounds, which influenced their ways of knowing, will contribute effectively towards creating social interactive spaces which are multi-epistemological (Berry, 2004: 101). Thus, the team approach to improving TPACK for teaching Euclidean geometry enhances our teaching practices, moving from an individualistic approach to a collaborative approach, which has the potential of being inclusive of different learning styles. This benefit justifies the need for a collaborative approach to teaching.

1.6.1.2 The need for intervention in geometry teaching

Geometry is one of the mathematics concepts taught as part of the school curriculum (DBE, 2011: 9). Learners perform poorly in mathematics, and geometry is one of the concepts that learners seem to be struggling with. For instance, in a study conducted by Ali, Bhagawati and Sarmah (2014:73) in India, the aim was to examine learners' performance in geometry and investigate if there are gender disparities regarding learners' performance. The results of their study reveal that learners demonstrated poor performance on geometry. The authors argue that one of the reasons why learners perform poorly is because they lack fundamental knowledge of geometry.

Their study also found that learners find geometry the most difficult part of mathematics. Thus, in investigating challenges that face teachers who teach Euclidean geometry, it is important that the current study investigates the level of learners' fundamental knowledge of Euclidean geometry. This knowledge could include basic Euclidean concepts and theorems, such as, (i) properties of an isosceles triangle; (ii) equal chords subtend equal angles; (iii) exterior angle equals sum of opposed interior angles of a triangle; and (iii) congruency and similarity.

1.6.1.3 The need for adequate lesson preparation when teaching with the aid of ICT software

Many factors influence the utilisation of ICT tools as teaching aids. These factors include limited time for lesson preparation, pressure to prepare learners to pass examinations and inadequate technical support (Fu, 2013:117). Furthermore, Fu (2013:115) reports that one of the challenges that teachers who are using ICT software as a teaching aid are faced with is that they prepare insuficiently due to their lack of time or knowledge to master the software. Fu (2013:115) states that low software competence, which may result in insufficient lesson preparation, is one of the barriers to effective integration of ICT in an manner that enhances learners' understanding of mathematical concepts. Leendertz et al. (2013: 5), in their investigation into the extent to which TPACK contributes to enhancing the effectiveness of teaching of Grade 8 mathematics in South Africa, report that a large proportion of mathematics teachers do not have sufficient knowledge to teach mathematics using ICT.

The teachers' knowledge they refer to originates from Lee Shulman' theory on the qualities of knowledge needed for teaching. Koehler and Mishra (2009: 62) adopted and adapted the Lee Shulman's theory (Shulman, 1987) and built on it to establish qualities of knowledge required for teaching using technology. One of the major challenges contributing to the complexity of teacher knowledge for using technology – categorised by Koehler and Mishra (2009) as intersection of technology, pedagogy and content knowedge – is the difficulty of creating a connection between the three qualities of knowledge. Thus, when they prepare for a class, teachers are unable to choose appropriate software or ICT tools to teach an identified mathematical concept. Furthermore, teachers have inadequate technology knowledge and skills, due to the fact that the majority of them went through teacher training when there were fewer opportunities to use ICT; and when technology had not yet developed to the current state (Koehler, Mishra et al, 2013: 14). This contributes to ineffective integration of ICT into teaching of Euclidean geometry, since teachers do not consider themselves sufficiently prepared and they don't realise the value of using ICT software as a teaching aid (Koehler, Mishra et al., 2013: 14).

1.6.1.4 The need for effective lesson facilitation with the aid of ICT software

Adequate lesson facilitation through the use of ICT software provides the opportunity for teachers to improve their lesson planning and lesson facilitation and become more project based and inquiry based and to promote collaborations between the learners (Rabah, 2015: 26). According to Rabah (2015: 27), during lesson facilitation, some teachers struggle with technical challenges that emerge during the lesson and this leads to learners' attention wandering and their interest in the lesson dwindling, which results in inadequte lesson facilitation. The preceding challenges are contrary to the epistemology of social contructivism, which asserts that knowledge is socially contructed (Thomas, Menon, Boruff, Rodriguez & Ahmed, 2014: 3). This implies that, according to this lens, no learning will be taking place unless there are social interactions between the learners that are created by the teacher through the use of ICT software. In support of this argument, studies conducted in five European countried by Buabeng-Andoh (2012: 139) report that teachers believe their technical incompetence regarding ICT tools, such as software, contribite greatly to inadequate lesson facilitation. This proves that there is a lack of the technical skills needed for adequate lesson facilitation with the use of ICT tools.

1.6.1.5 The need for assessment for learning practice when teaching Euclidean geometry with the aid of ICT software

Assessment of learning takes place during a lesson facilitation for the purpose of reconstructing the environment to enhance learning during a lesson (Swaffield & Thomas, 2016: 5). In addition, assessment for learning is a continuous process throughout a lesson: the teacher assesses if learners are following, diagnoses their learning difficulties and determines what makes what they are learning difficult to learn. Thus, through this process, a teacher becomes a bricoleur, threading and looping back and forth between the concepts of Euclidean geometry to make meaning and reset a learning environment that promotes learning as informed by continuous diagnosis (Berry, 2004: 1). However, assessment for learning still encapsulates assessment that is done at four levels, prior to the presentation of a new lesson, during the presentation of a lesson, at the end of a lesson presentation and after the lesson (DoE, 2012: 3).

In the first instance, assessment is used to determine learners' prerequisite knowledge relating to the new lesson (Karolich & Ford, 2013: 35). Subsequently, assessment is used during the lesson presentation, to determine the extent to which learners follow and/or understand the new content. This assessment is done to ensure that learners' misconceptions and other knowledge gaps about the new content as established during the prior knowledge assessment are addressed and assimilated appropriately (Spence & McDonald, 2015: 297). Formative assessment is intended by practitioners to be the same task as assessment for learning (DBE, 2011: 22). For instance, formative assessment is defined by Formative Assessment for Students and Teachers (FAST) as a process used during teaching to provide feedback for the purpose of adjusting ongoing teaching and learning (Dunn & Mulvenon, 2009: 2). This is similar to assessment for learning and defines assessment as intricately connected to teaching. Thus, using assessment for learning, I argue that assessment for learning cannot be divorced from lesson facilitation, hence, in this study, four levels of assessment were collectively understood to be part of PCK and subject content knowledge (SCK) (see Sections 4.2.3 and 3.5.1.1).

1.6.1.6 The need for research into appropriate computer software to teach Euclidean geometry

Research offers opportunities for enhancing the teaching of mathematics using appropriate software effectively (Hanson, 2013: 625). Through research teachers can keep pace with developments in rapidly evolving software technology for teaching (SAQA, 2012: 12). New programs and computer software are designed to meet the demands of the day, among which promoting learner-centred approaches to teaching, which enhance learners' understanding of mathematical concepts in Euclidean geometry. The development of new or updated software is understandably based on relevant research findings relating to learners in mathematics (Leendertz et al., 2013: 5). The updated programs render older ones redundant. Conversely, teachers who teach mathematics, particularly Euclidean geometry, will become redundant if they do not keep up with such new developments.

Furthermore, research provides a platform for teachers to be creative and innovative. Creativity and innovativeness in this regard includes integrating multiple software programs and drawing from each program's strengths to ensure that abstract mathematics concepts, such as Euclidean geometry, are accessible to learners. The integration of multiple software programs takes into account learners' diverse learning styles and their varying competency levels. This concurs with Rosenshine (2012:38), who sought to find principles that could serve as a guide for achieving good teaching. Rosenshine (2012: 38) argues that, in order for good teaching to take place, teachers must do extensive research to find different materials that will enable learners to acquire the requisite skills. This suggests that, in order for learners to have Euclidean geometry's requisite skills, there is a need for teachers to conduct extensive research to find appropriate ways to make to content accessible.

1.6.2 A review of literature to justify the components of a strategy to improve teachers' TPACK for teaching Euclidean geometry with the aid of ICT software

The literature review is done in pursuit of the best practices in response to the challenges identified.

1.6.2.1 A team approach towards improving TPACK for the teaching of Euclidean geometry using ICT software

A team approach to teaching has stimulated the interest of many teachers in countries which perform well in mathematics and science. A lesson study approach to enhancing the teaching of mathematics is one of the team approaches that seeks to find different ways in which learners can learn (Doig & Groves, 2011: 84). Japanese lesson study, which is referred to as *jyugyo kenkyu,* comprises small groups of teachers who generally meet frequently, to prepare together, implement and reflect on their lessons (Jita et al., 2008: 465). This group of teachers from the same school and/or local schools are geared to what they call a research lesson with the purpose of uncovering how learners make meaning as they grapple with the content of mathematics. Through this process research lessons provide a type of teacher professional development that is teacher-inquiry based (Doig & Groves, 2011: 84). The research lesson focuses on building particular skills, knowledge and attitudes. During the process of the research lesson, one or more teachers prepare a lesson while other teachers are invited to be observers – the observation is not limited to teachers only, as they also invite academics or "veteran teachers" to reflections (Doig & Groves, 2011: 79).

1.6.2.2 Formulation of a vision

A vision and mission guide daily activities of a team or an organisation and foster a shared purpose among members of the team. Darbi (2012: 95) explains that a vision and mission motivates, models behaviour, and promotes a high level of commitment, which leads to cultivating perfomance. Furthermore, Kantabutra (2008: 127), in his study of what we know about vision, asserts that a vision provides a cognitive imagination of the desired future state. A shared vision creates an orientation and meaning for the team members and it acts as a strong driving force for continuous and systematic development (Martin, McCormack, Fitzsimons & Spirig, 2014: 1). A vision should be attractive to the team members if they are to be committed to turning it into a reality (Martin et al., 2014:2; Wong & Liu, 2009: 2884).

1.6.2.3 SWOT analysis

SWOT analysis is a strategic evaluation tool that the coordinating team used to assess the strengths, weaknesses, opportunities and threats in pursuit of responding to the challenges they are facing with the teaching of mathematics through conducting the study (Ayub & Razzaq, 2013: 93). The SWOT analysis was used as an information-gathering tool concerning the team's competencies. In this study SWOT analysis is used to map the information provided by the analysis with the information gathered through literature on skills and resources needed to improve teachers' TPACK, in order to direct the strength of the team towards the opportunities identified (Ayub & Razzaq, 2013: 93). Furthermore, SWOT analysis is used to identify the threat to improving teachers' TPACK and finding a stategies to overcome the threats. This section presents the results of the SWOT analysis under the headings of CK, PCK and TPACK.

1.6.2.4 Collaborative lesson preparations

Lesson preparation contributes significantly to a successful lesson that has met its objective(s). Collaborative lesson preparation involves a group of teachers meeting and working together on designing a sequence of activities on a particular theme in a way that learners can easily make sense of the content under the theme through activities planned (Jita et al., 2008: 475). Collaborative lesson planning is observed in Japanese lesson study when a group of teachers come together for the purpose of planning activities for a particular theme. Jita et al. (2008: 475) adapted lesson study for the South African mathematics and science context, and they argure that collaborative planning in lesson study becomes an instument for building esprit de corps among teachers. They outlined collaborative planning in the South Aftican context as a process where group of teachers do the following:

> *The knowledge and skills are organized into cohesive lessons which are sequenced into units (Jita et al., 2008: 475).*

In a lesson preparation the organisation of knowledge and skills is directed by the aim and objective(s) identified for the lesson. In this regard, Jita et al. (2008: 475) capture that this collaborative process is geared to finding responses to what the intended objectives are; how the objectives would be achieved; what measures of success would be used; what types of resources would be needed to facilitate the lesson and, lastly, what time would be needed.

1.6.2.5 Using ICT to create an interactive lesson facilitation

According to social constructivism epistemology, the construction of knowledge takes place when there is social interaction between an individual and the environment, or between an individual and others. This epistemology informs how we describe interactive lesson facilitation, namely, as a lesson that creates the opportunity for learners to interact with one another or the environment during the process of learning. An interactive lesson facilitation as an approach couched in social constructivism subscribes to demystification of the idea that a teacher knows everything and learners are mere recipients of knowledge. This further means that, in an interactive classroom lesson facilitation, a teacher depowers him/herself in pursuit of creating a classroom environment that promotes self-regulated and learner-centred learning. Thus, using ICT software for teaching Euclidean geometry can promote learners' interactive self-regulated learning.

1.6.3 The critical conditions for fostering sustainability

This section is concerned with the conditions that are conducive to fostering the sustainability of the solutions or components of the strategy proposed in Section 2.5.2.

1.6.3.1 Conditions conducive for co-researchers to function optimally as team

In order to have a fully functional team of co-researchers, collective leadership is a necessary condition (Avolio, Walumbwa & Weber, 2009: 423). Collective leadership fosters the multiple roles and responsibilities to be performed by all co-researchers and responds to the complex epistemology as advocated by bricolage, which, in this context, involves different leadership styles, and team members' different skills and knowledge applied to enhancing the teams' performance. The preceding views concur with Raham's (2000: 143) views in his examination of the way intragroup affective patterns influence a groups' pervasive tendency to ignore the unique expertise of their members.

Thus, all team members have the opportunity to experience these roles and in this way a sense of belonging and appreciation is bestowed on them (Contractor, Dechurch, Carson, Carter & Keegan, 2012: 995). According to Mickan and Rodger (2000:201) trust is a necessary condition for the success of a team. Thus, in the current study we should consider ensuring that co-researchers have developed trust between them. In addition, Mickan and Rodger (2000:204) argue that, in order to have an effective team, there needs to be good communication between the members.

1.6.3.2 Conditions conducive for lesson preparation and facilitation

Lesson preparation requires, among other things, the careful collection of thoughts and resources concerning what needs to be taught. Due to the complexity of everyday life, what constitutes knowledge, and the diverse nature of reality, knowledge becomes a very complex phenomenon. This has a great influence on the ways knowledge construction is understood, which is closely related to the ontological stance of individuals. This said, I understand that lesson preparation is a very complex activity that requires teachers to set up an environment that promotes learning for individuals who often originate from diverse backgrounds and who do not learn in the same way. This, in my view, is why teaching can be described as a problem-based activity. Firstly, it is a problem for the teacher, because a classroom consists of learners from various cultures, with different histories, family morals, social economic status, genders and ages, which influences both their epistemological and ontological stances. Secondly, the view the study subscribes to is that learning could easily take place if an environment enables learners to create connections between what they already know and the content they are currently trying to learn.

1.6.3.3 Conditions conducive to continuous professional development

Continuous professional development of teachers has become significant for many countries around the world (Jita & Mokhele, 2014: 1). Due to the complexity of teaching, which is dynamic and fluid, continuous professional development becomes essential for keeping up with changes. Thus, scholars such as Day (1999: 4) find it necessary to emphasise that continuous professional development is a “continuous” activity.

Due to the complexity of setting up environments that promote learning for learners in the 21st century and demand changing roles for teachers in the classroom, continuous professional development has become very significant in enabling teachers to be effective in their new roles (Wan & Lam, 2010: 2). Moreover, continuous professional development is an imperative condition for improving teachers’ TPACK. This claim is supported by Leendertz et al. (2013: 2) in their study to investigate the extent to which TPACK contributes to more effective teaching of mathematics in South African schools. Leendertz et al. (2013: 5) argue that, without continuous professional development of teachers, it is less likely that the use of ICT for teaching mathematics will be effectively and optimally used. Daly, Pachler and Pelletier (2010: 6) believe that continuous professional development is a conducive condition for improving teachers’ TPACK; however, they highlight that continuous professional development for TPACK must promote independent thinking, creative presentation of ideas, and collaborative problem solving amongst teachers.

1.6.4 Threats and risks that may impede the success of the strategies and solutions

This section addresses the threats and risks of teaching Euclidean geometry using integrated ICT software. The literature relates to and integrates these risks to the solutions and conditions for the successful implementation of teaching Euclidean geometry using ICT software.

1.6.4.1 Teachers' negative attitudes toward lesson preparation using ICT software

Negative attitudes towards the use of ICT in teaching is one of the main threats for implementation of a strategy to teach Euclidean geometry using ICT software. For instance, Hue & Jalil (2013: 54), in a survey to determine the impact of lecturers' attitudes toward effective implementation of using ICT in teaching and learning, state that teachers' attitudes towards the use of ICT in teaching is one of the main factors that leads to poor or ineffective use of such a strategy. The study further established that there is a positive relationship between attitude and the use of ICT in teaching. In order to circumvent this threat Hue and Jalil (2013: 55) recommended that development of positive attitudes among teachers is a key factor in ensuring successful implementation of computer software in teaching.

Mulhim (2014: 489) investigated why, despite the benefits of ICT in teaching, ICT is still not effectively integrated in teaching and learning in classrooms. She (2014: 489) confirms that one of the threats to this initiative is teachers' negative attitudes towards the use of ICT tools. She elaborates that the negative attitudes are the result of inadequate training and she recommends that, to address this challenge, we need to find better and more effective ways of training teachers to use ICT as a teaching aid (Hadjerrouit, 2008: 250).

1.6.4.2 Teachers' workload

Workload is one of the major threats to the utilisation of ICT in both lesson planning and facilitation. Buabeng-Andoh (2012: 142), in his investigation into factors that influence teachers' adaptation and integration of ICT into teaching, reveals that heavy workload is one of the threats to both integration and adaptation of ICT in lesson planning and preparation. Similarly, Raman and Yamat (2014: 15) conducted a study to investigate threats to the integration of ICT in teaching and learning by teachers. Their findings reveal that, despite some schools being resourceful with ICT tools, such as computer software, teachers' workload seems threaten the integration of ICT software in lesson preparation and facilitation.

1.6.4.3 Access as opposed to quality of ICT

In South Africa, ICT was introduced in basic education as a teaching resource, according to the White Paper and Guidelines for Teacher Training and Professional Development (DBE, 2004:1). The introduction of ICT in some schools has become physically evident; these ICT resources include computers, computer software, such as HeyMath!, interactive whiteboards, mobile labs for mathematics and tablets (DBE, 2012). According to Ndlovu and Lawrence (2012: 3) the introduction of ICT in South African education is driven by the need for quality education for all. However, they state that teachers misuse the ICT, since they think that ICT provides them with information rather than enhancing their teaching to promote learners' critical thinking.

The use of ICT should create an opportunity for learners to interact with the information and manipulate it such that they are able to critically analyse and question it, so that it becomes relevant to their context. This is in line with social constructivism's epistemological stance, namely, that knowledge is constructed as if it is part of the social context that individuals socially interact with (Marcus & Fischer, 1999: 7). However, access to ICT tools does not always lead to quality teaching and learning, which becomes a threat for the intended use of ICT. Hess and English (2015: 194), in their study that advocates for a Greenfield approach to innovation, state that policy makers do not run schools and that policy can only tell people what to do but not make them do it well. They further suggest that policy should create a space for schools to rethink how they want to implement and improve innovation in teaching and learning (Hess & English, 2015: 194).

1.6.5 Indicators of success

The section is concerned with a review of literature on the indicators of success for effectively improving teachers' TPACK for teaching Euclidean geometry with the aid of ICT software.

1.6.5.1 Content knowledge for teaching Euclidean geometry

The CKT for teaching refers to the knowledge needed by a mathematics teacher to teach mathematics. Thus, in in order for a mathematics teacher to be considered competent, it is important that he or she is able to demonstrate CKT for mathematics.

The conceptualisation of the kind of knowledge needed by the teachers can be traced to the work of Lee Shulman (1986). Ward, Kim, Ko and Li (2015:130), drawing from Lee Shulman' notion of PCK, state that content knowledge (CK) must be transformed and packaged in ways so that learners can understand the content. Thus, in the context of the current study, a teacher will be competent when he/she is able to transform the content of Euclidean geometry such that learners can make sense of the content. Making sense of the content of Euclidean geometry means that learners can state the theorem, and show that it is not limited to the way the teacher proved it, and also apply it themselves.

1.6.5.2 Technological pedagogical content knowledge

TPACK is the integration of three main knowledge domains, pedagogy, content and technology (Morsink, Hagerman, Heintz, Boyer, Harris, Kereluik, et al., 2011: 4). The permutations of these three knowledge domains form subdomains, which include PCK, TPK and TCK. Thus, in order to consider a teacher's TPACK to be improved Altun (2013: 366) argues that a teacher should demonstrate technical ability to use ICT software, even if it is not for teaching purposes. This is what Koehler and Mishra (2009) define as TK, which is the general knowledge of technology. However, a teacher should go a step further towards integrating knowledge of technology and content (TCK). Not all technology is relevant for teaching, it must be suitable for integration with the content of a particular subject (Altun, 2013: 366). Thus, in the current study, one of the indicators of success is when teachers can integrate technology with the content of Euclidean geometry.

1.6.5.3 Lesson facilitation with the aid of ICT software

Good teaching practices are grounded on social constructivism's epistemology and ontology, namely, that knowledge is a human social product. Through such an understanding, teaching with ICT software is perceived to be successful when teachers are able to use ICT software during their preparation and facilitation in a manner that enables learners to construct their knowledge through interaction with software, the environment, and other learners. Furthermore, through this lens of social constructivism, ICT software is understood to be successful if teachers' knowledge of content, pedagogy, and technology is improved within multiple social realities. This implies that teachers at different levels of content, pedagogy, and technology knowledge could create a space of operation where they could share their individual realities on content, pedagogy and technology.

1.6.5.4 ICT improves learners' learning

According to Youssef and Dahmani (2008: 45) the performance of learners who are taught using ICT improves. The use of ICT promotes learner-centredness and also fosters meaningful learning (Van der Westhuizen, Nel & Richter, 2012: 199). A study by Pena (2011: 66) reports that ICT provides an opportunity for learners to collaborate during the process of learning in flexible and motivational ways. Furthermore, in Pena's (2011: 67) study of how ICT improves teaching and learning Pena states that the use of computer software can improve learners' communication skills.

1.7 METHODOLOGY

This study employed PAR as a methodology. As the researcher I recognised that I do not have enough knowledge needed to design a strategy to improve TPACK for teaching Euclidean geometry. Therefore, I believe that those who face the challenges of teaching and learning Euclidean geometry are in a better opposition to design a responsive strategy for their day-to-day challenges. I find PAR appropriate and relevant because I understand that, in order to respond to the five objectives of the study, the co-researchers' lived experiences are key factors. Thus, this chapter justifies the use of PAR by explaining the following: PAR as a research methodology and as a research design.

1.8 DESIGN, DATA GENERATION AND ANALYSIS

This section, which refers to Chapter 3, outlines how the principles of PAR were operationalised. The research design began with initial meetings for the purpose of gathering people who had relevant common experiences. The meetings also served to level out power differentials. I became part of the initial meetings as I was invited by Mr Phehello (not his real name) to participate in an academic subject improvement plan, and this is how the study began. After the team was established formally, the team members comprised four mathematics teachers, one academic (myself), one deputy chief education specialist (DCES), and a programmer. These people formed a team and drew up an action plan, which consisted of activities that were used to generate data. The data were video recoded and audio recoded, and we referred to learners' scripts, co-researchers' reflections and lesson plans. The data were analysed using Van Dijk's critical discourse analysis (CDA). Furthermore, the quantitative data was generated from learners' assessment activities and analysed using statistical models and techniques.

1.9 LAYOUT OF CHAPTERS

Chapter 2 reviews the literature relating to the design of a strategy to improve teachers' TPACK for teaching Euclidean geometry with the aid of ICT software. This chapter starts with a theoretical and conceptual framework that underpins the study. Subsequently, the related literature is discussed with the purpose of formulating the components of the strategy from the best practices.

Chapter 3 discusses the appropriateness of PAR as a methodology. Chapter 3 also discusses the research design, which entails an initial meeting, formulation of a research team, establishment of a common problem, action plan and activities. The chapter discusses methods of data generation and data analysis, which employed Van Dijk's CDA and statistical techniques.

Chapter 4 presents, discusses and analyses data, and provides the interpretation for each of the five objectives of the study. The chapter analyses the challenges experienced by teachers who teach Euclidean geometry with the aid of integrated ICT software. This was done to establish the possible solutions and strategies that can be developed, adopted and adapted to address the challenges they experienced. The solutions and strategies sought to enhance the processes and practices of teaching and learning during and beyond the duration of the study. Thus, as a result, the conditions under which the strategies and solutions were developed needed to be investigated to ensure their sustainability. The investigation of the conditions was done with an understanding that, if conditions conducive for implementation of the strategies pose threats, they could impede the successful implementation of the strategies and solutions. Thus, the strategies and solutions were assessed and evaluated before they could be recommended.

Chapter 5 restates the statement of the problem, presents the findings and recommends the strategies designed in Chapter 4 for each finding discussed.

CHAPTER 2: LITERATURE STUDY FOR DESIGNING A STRATEGY TO IMPROVE TEACHERS' TECHNOLOGICAL PEDAGOGICAL CONTENT KNOWLEDGE FOR TEACHING EUCLIDEAN GEOMETRY

2.1 INTRODUCTION

The study intends to formulate a strategy to improve teachers' TPACK for teaching geometry with the aid of integrated ICT software. This chapter reviews the literature on the topic for the purpose of providing a contextual and conceptual basis for the study. The chapter starts off with a discussion of bricolage as a theoretical framework and its underlying principles; doing so will enable me to respond to the research question, aim and objectives. Furthermore, the chapter explores the relevance of bricolage as a theoretical framework through its origin, formats, objectives, the role of the researcher, epistemology and ontology, and the relationship between the researcher and participants. Definitions of operational concepts and finally, an extensive review of related literature in relation to objectives of the study informed by best practices in South Africa, Botswana, Nigeria, United states of America, will be conducted as basis for the emerging strategy.

2.2 THEORETICAL FRAMEWORK COUCHING THE STUDY

This section validates the choice of bricolage as an appropriate theoretical position for designing a strategy to improve teachers' TPACK for teaching geometry with the aid of integrated ICT software. In order to do this, I explain the theoretical origin, formats, objectives, ontology, epistemology, role of researcher, relationship between researcher and the participants, and the rhetoric. Lastly, I summarise how bricolage can be used to respond to the objectives of the study through the proceeding subtopics.

2.2.1 The origin of bricolage

Bricolage is a term that is used metaphorically, and is derived from a French word *bricoleur*, which means a handyman or -woman who uses the tools available to complete task at hand (Kincheloe & McLaren, 2005: 316; Kincheloe, 2004a: 1). The etymology of bricolage and *bricoleur* in their key academic meanings is traced from the work of anthropologist Claude Lévi-Strauss (Given, 2008: 65). Lévi-Strauss conceptualised bricoleur as a kind of researcher who understand knowledge to be constructed through various modes of orientation towards the world (Given, 2008: 65). For instance, Lévi-Strauss, in pursuance of knowledge production, claimed that primitive and civilised ways of knowing were equally important, because both ways are shaped by their ontological view of the world (Lévi-Strauss, 1962: 20). Denzin and Lincoln (2000) were inspired by this metaphorical use of the French word in the work of Lévi-Strauss (1962), which had lead to a discussion of *The Savage Mind*, and they conceptualised bricolage as a mode of knowledge production (Kincheloe & McLaren 2005: 316, Kincheloe & Berry, 2004: 1). The conceptualisation of bricolage proposed by Denzin and Lincoln (2000) was developed further and theorised on by Kincheloe (2005a; 2004a; 2004b; 2004c; 2004d; 2001) and Berry (2004a; 2004b; 2006; 2011) as being a critical, multi-perspectival, multi-theoretical and multi-methodological approach to research (Roger, 2012: 1).

> *Qualitative research focuses on interpretation of phenomena in their natural settings to make sense in terms of the meanings people bring to these settings. Qualitative research involves collecting information about personal experiences, introspection, life story, interviews, observations, historical interactions and visual text which are significant moments and meaningful in people's lives (Denzin & Lincoln, 1994: 3).*

The process of unfolding contextual meaning from the data collected from the sources listed by Denzin and Lincoln has led to the development of qualitative research through what these authors (2001: 700) call the eight historical moments of qualitative research. These eight historical moments sketch the chronological evolution of bricolage through the following stages: the traditional period or first moment; the modernist phase, blurred genres, crisis of representation; postmodern moment, post-experimental moment, the seventh moment, and fractured futures (Denzin & Lincoln, 1994: 3). These eight historical moments of qualitative research comprise the theoretical position I take when I seek to make meaning of the complexity of lived experiences. Each moment captures the way researchers have approached inquiry over time, shared and shaped by their ontological and epistemological stances. These multi-theoretical positions, multiple methodologies, and multiple perspectives that unfold over the eight moments are used as tools whenever a *bricoleur* finds it fit when he/she is uncovering complex knowledge about the object under study.

The following section discusses each of the historical moments listed above, and explains how each moment is appropriate for the current study.

2.2.1.1 First moment: The traditional period

The traditional period is the first historical moment of qualitative research. This moment started in approximately 1900 and ended in about 1945 (Denzin & Lincoln, 2005, 15; Denzin & Lincoln 1994: 3). In order to provide a background for this period, I refer to the way research was conducted in the 16th and 17th centuries, long before the traditional period. During these centuries great discoveries were made by natural scientists, such as Isaac Newton, whose theory of gravity was influential even in the social sciences. These important discoveries were applied to industry, and had a great impact on human lives and economic development. The considerable impact made by scientists influenced the kind of research that was considered valid and reliable. This means that knowledge that was created through natural scientific methods could be generalised and were regarded as universally “true”. The reproduction of universal truths in this manner disregards diverse context and “objects”.

Thus, in the traditional era, sociology and humanities researchers conducted their research using natural scientific approaches. The work of Isaac Newton, for instance, inspired a vision of what scientific research should be (Turner, 2001: 30). Natural scientific approaches dominated and influenced research to the extent that it was believed that science held the key to the reconstruction of society. The scientific approach to social sciences research was in acknowledgement and recognition of the work of Charles Montesquieu, Auguste Comte and Saint-Simon. The reconstruction of society seems to have been at the centre of research in natural sciences, sociology and the humanities. I understand this reconstruction of society as representing a quest for transformation in order to achieve social change. However, the scientific process was characterised by risks, threats and weakness that were embedded in the processes that were supposed to generate social change – this was particularly applicable to sociology and humanities researchers. Researchers in the fields of sociology and the humanities emulated their counterparts in the natural sciences to the extent that the former also objectified their research topics, and reduced it to numbers (Turner, 2001: 30). This objectification and reductionist thinking meant a disregard for human beings, language, culture, and background. The significance of the traditional period to the undertaken study is based on the need to reconstruct society through the enhancement of teaching and learning Euclidean geometry. This has to be done in a manner that does not generalise, objectify or reduce the reality surrounding knowledge creation in Euclidean geometry. It is important that we understand this underlying principle of the traditional period's epistemology, namely, reducing human beings to objects. This understanding enables me to understand why some of my co-researchers, particularly teachers, assume that learners should learn concurrently while the teachers teach (see Section 4.2.1.1).

Thus, in my understanding, the traditional moment in this study enables me to contribute to a reconstruction of society by responding to the challenges of teaching and learning Euclidean geometry located within society were the study is undertaken. Euclidean geometry forms a part of the mathematics curriculum and, therefore, improving the teaching of Euclidean geometry will improve learners' understanding of mathematics. Through this process society will be reconstructed – by deconstructing the effects of educational policies of the apartheid era that discouraged black learners to take mathematics as a subject. This is evident from Minister of Native Affairs, Hendrik Verwoerd, who said,

> *There is no place for [the Bantu] in the European community above the level of certain forms of labour... What is the use of teaching the Bantu child mathematics when it cannot use it in practice? (quoted by Clark & William, 2004: 48)*

My interpretation of the above question by Verwoerd is that, there was, indeed, no need for black children to learn mathematics at school, because they were unlikely to obtain a job that would require mathematical skills. Furthermore, in my view, this implies that there was no need for mathematics teachers who were well trained and competent to teach mathematics at schools for black children. Remnants of Verwoerd's question are still present today. For instance, it is historically traceable why the greatest shortage of mathematics and science teachers who are competent in PCK, CK and TPACK occur mainly in communities of black people (SACE, 2013:1).

In response to this challenges, and in the name of access for all, the Department of Basic Education (DBE) decided to start using ICT to improve teaching (Ndlovu & Lawrence, 2012: 3). There has recently been an increase in the use of technology to teach mathematics, and opportunities have been created for teachers to use ICT software and other ICT tools. The challenge is that many mathematics teachers in black communities underwent initial teacher training during the Bantu Education era, when quality teacher training for blacks was not considered necessary by the government.

Comte (1854: 5-6), in his positive philosophy, considered human behaviour to be governed by natural laws that are similar to what he called Newton's doctrine of gravitation. The natural science approach to research shaped the way research was conducted, which led Saint-Simon to introduce positive science, which was further developed as positive philosophy by Comte (Turner, 2001: 31). In the traditional period social science adopted the same ways of doing research as the natural sciences. In this era researchers were considered to be powerful and knowledgeable people.

These historical ontological and epistemological views regarding research enabled the research team of this study to understand what they perceive as being good teaching. For instance, one of the challenges of teaching Euclidean geometry, in the context of this moment's epistemological stance, is that teachers could regard themselves as the most powerful and knowledgeable people.

Furthermore, according to Comte's positive philosophy the relationship between the researcher and the researched is distanced, with the purpose of reducing the biases the researcher may hold about the research objects – which are human beings. For example, Rene Descartes, in *Discourse on Methodology* (1637), explains the importance of objectivity and evidence in the search for the "truth" (Ritchie, Lewis, Carol & Ormston, 2013: 9). However, in my view, the positive philosophy of Comte has limited value for unravelling the remnants of Bantu Education policies as they relate to mathematics. These limitations include the view that a researcher is powerful and the most knowledgeable person. In this study, I reject Comte's views on the relationship between the researcher and the researched, with the realisation that, as a research coordinator, I do not have solutions for the challenges that teachers who teach Euclidean geometry are faced with. Taking a position of the most knowledgeable and powerful person in pursuance of knowledge construction limits the contribution that teachers faced with the problem can make. Rejecting the views of Comte on participation during knowledge production leads to a reconstruction of society, because mathematics teachers dictate the process of research to find solutions to their own challenges (Mahlomaholo, 2013: 384).

Furthermore, the studies undertaken during traditional period were purposed to investigate patterns, regularities and universal laws that exist for human actions. For example, Hume (1740: 1), who was a prominent contributor to the empirical research tradition, epitomises the kind of research that observes patterns and regularities with the purpose of reaching universally true conclusions. According to this positivistic approach, researchers are concerned with causal relationship for the purpose of prediction, and to control the variables.

Thus, in this study, principles of the traditional period enabled me to investigate general patterns – regularities that emerged in learners' performance on Euclidean geometry. For instance, modelling learners' performance using a normal distribution enabled me to calculate the probability of learners' performing above or below a certain threshold. Furthermore, in order to ensure that the learners' performance is indeed normal, a Lilliefors goodness-of-fit was performed to ensure reliability and validity of the results (see Section 4.2.2).

The ontology of this dispensation of qualitative research is that there is a single, external and objective reality behind any research question regardless of the researchers' beliefs (Canon, 188). That meaning, and therefore, meaningful reality, exists apart from the operation of consciousness and is embraced as an epistemological view (Levy, 2005: 372). However, the use of statistical modelling limits interpretation of the results to general statistical laws, and is grounded on principles that are external and independent of my own views.

According to Denzin and Lincoln (2005: 15) traditional period researchers adopted the ontology and epistemology of positivism (Denzin & Lincoln, 2005: 15; Denzin & Lincoln 1994: 3). The epistemological perception of researchers during traditional period was that there is an objective reality, which is independent of human historical dynamics and which can be comprehensively captured by symbolic representations. The researcher's work is compelled by a theoretical belief in prediction and control. The work done by researchers during the traditional period was influenced by positivism, hence, it sought to explore the general patterns of cause and effect for purposes of controlling and predicting natural phenomena.

In this mode of research, validity, reliability and generalisation epitomise good research (Onwuegbuzie, Leech & Collins, 2010: 697). Thus, the traditional moment enables us to understand the use of axioms, which are considered true building blocks of Euclidean geometry. Traditional period is concerned with the following questions: To what extent do measurements measure what needs to be measured? Are the measurement time insensitive? And, are the findings of the study applicable in any context? We acknowledge reasons for rejecting the positivist approach of the traditional period, which became clear as ethnographers began to experience the chaos caused by the facts emerging from the data they had collected (Denzin & Lincoln, 2005: 15). Geertz (1988: 81), drawing on the work of Malinowski (1916), states that, as early as the traditional period, ethnographers, using the framework of the fundamental principles of positivism, such as natural laws and generalisations, could not find scientific meaning in the data collected in the field. This failure paved the way for the modernist phase of qualitative research as the second moment.

A *bricoleur* understands that the traditional moment's ways of doing research contributes to the tools needed for construction of bricolage. For instance, the theoretical stance of the traditional period contributes to investigating the challenges of teaching and learning Euclidean geometry through the design of diagnostic tools that are reliable, valid and objective. Additionally, positivistic data collection and analysis tools of this moment could enable me to test theories or hypotheses, for example, the hypothesis that learners' performance in Euclidean geometry is relatively worse than in other topics. This historical moment provides me with the tools of analysis to make sense of the relationship between cause and effect, for instance, using analysis of variance to investigate and measure the degree of responsiveness of learners and teachers to the formulated strategy implemented.

2.2.1.2 Second moment: The modernist phase

The modernist phase or second moment started in approximately 1945 and ended around the year 1970 (Denzin & Lincoln, 2005: 16; Denzin & Lincoln 1994: 3; Onwuegbuzie et al., 2010: 698). The work of researchers of this period sought to formalise qualitative research and make it as robust as quantitative research (Denzin & Lincoln, 2005: 16; Denzin & Lincoln 1994: 3). In the modernist phase, just as in the traditional period, the qualitative researcher's belief is that research could be used to identify causal variables and to predict the future behaviour of people (Lewis, 2009: 3).

In this period, good research is epitomised by reliability, which means the findings of one research study must be reproducible by another researcher. In an attempt to formalise and make qualitative research as robust as quantitative research, researchers' field notes and decision points were recorded, so that other researchers could replicate the study to ensure reliability. Researchers' theoretical belief was that validity exits if, and only if, observations are replicable (Schwandt, 2007: 262). According to Lewis (2007: 4) the researchers of the day ensured validity and reliability by administering the same questionnaires using different locations, times and samples. This approach was directed by the ontology of this historical moment that states that reality is fixed; further, the epistemology states that meaning, and therefore meaningful reality, exists apart from the operation of consciousness (Levy, 2005: 372). The relationship between the researchers and the participants is the same as in the traditional period, except for the fact that participants are now called respondents and not objects (Turner, 2001: 30). Interaction between researchers and the respondents is still discouraged, as this is perceived to cause bias in the researcher.

The modernity phase saw new interpretative theories emerging, such as ethnomethodology, phenomenology, critical theory and feminism (Onwuegbuzie et al., 2010: 698; Denzin & Lincoln, 2005: 16; Denzin & Lincoln, 1994: 3). Ethnomethodology theory was an attempt to formalise qualitative research methods, and represents a slight departure from the traditional moment's ways of data collection; it was also an attempt to respond to chaotic data, which the traditional moment's ways of doing research did not account for in its linearly predefined methods. Ethnomethodological studies seek to make meaning of the detailed features of knowledge production of the social order. Thus, in the current study, I acknowledge that the detailed features of the production of teaching and learning of Euclidean geometry are necessary. For instance, I understand that the challenges of teaching and learning Euclidean geometry are complex and multilayered, and are exacerbated by the fact that teachers and learners come from diverse cultures and backgrounds, and that each participant brings a different learning style to classroom (Mahlomaholo, 2013: 379). Thus, the aim of ethnomethodology, namely, to study everyday life practice and the scientific practice as one, concurs with the transformative agenda of the current study, which is to improve the everyday lives of the co-researchers by improving the way they teach Euclidean geometry.

Despite the advantages and relevance of ethnomethodology, unfolding bricolage cannot limit itself to the ethnomethodological approach. For instance, ethnomethodology is not interested in individuals, but in the social cohort that individuals are part of, with the understanding that only social scenes are objects of inquiry. This is contrary to both the ontological and the epistemological stances of bricolage as applied in this study, since a transformed social scene is made of transformed individuals. Moreover, in ethnomethodology, emotional being, values and beliefs of people are outside the scope of inquiry. Thus, couching the unfolding study within this theoretical lens contradicts the emancipatory agenda of the study, which is to seek validation for the values, emotions and beliefs of co-researchers through knowledge construction processes of research (Mahlomaholo, 2009: 230).

Another research approach of the modern moment is phenomenology. The aim of phenomenology is to explore lived and shared experiences; it emphasises that only those who have experienced a phenomenon can communicate their experiences (Charlick, McKellar, Fielder & Pincombe, 2015: 50). Thus, this study uses the phenomenological approach to research when the co-researchers, who directly experience the challenges of teaching Euclidean geometry with the aid of ICT software, share their own experiences. However, the co-researchers share their experiences with the aim of learning from one another. Phenomenology and ethnomethodology were used to echo the voice of society's underclass. Thus, this study advocates for mathematics teachers to become researchers, in the same way that medical doctors do research to advance their own practices. However, the current study does not limit itself to the phenomenological way of conducting research, since the aim of the study is not to pursue the objectivity of teachers' experiences, but to create a space for a response to their challenges.

Qualitative researchers of the modern era analysed data using probabilities, and even supported their claims by using positivistic and postpositivist rhetoric language, such as likelihood and frequency (Denzin & Lincoln, 2005: 16; Denzin & Lincoln, 1994: 3). Thus, in this study, the probabilistic argument to reach certain conclusions is made to assess the indicators of success of the strategy that is designed (see Section 4.6).

In line with the objectives of the study, researchers in the modernist phase attempted to respond to the difficulties of interpreting the chaos of data collected in the field by ethnographers. This period produced theories that help us to comprehend that the challenges facing learners regarding identification or recognition and naming of geometric figures according visual characteristics goes beyond school parameters. The theories emerging from the modernist phase, in addition to those of the traditional period, enable us to interpret learners' challenges regarding visualising, analysing, and making formal and informal deductions beyond numerical values, or even percentages obtained, when solving Euclidean geometry problems.

In the traditional period, bricolage could be used to perform statistical tests for investigating possible gender disparities in skills that learners need for solving Euclidean geometry problems. In the modernist phase, bricolage recognises and fully explores the usefulness of positivistic data analysis tools in a quest to uncover and unfold general patterns identified by the researchers' constructs. Furthermore, in the modernist phase, the bricoleur does not stop at identification of patterns, but seeks to make meaning of the patterns found. The bricoleur in the modernist phase endeavours to look at the data through more than one theoretical lens.

2.2.1.3 Third moment: Blurred genres

The third historical moment of qualitative research, called blurred genres, stretched from 1970 to 1986 (Onwuegbuzie et al., 2010: 698; Denzin & Lincoln, 2005: 17; Denzin & Lincoln, 1994: 4). This moment marks the maturity of qualitative research, with a complement of paradigms, methods and strategies to use in research (Denzin & Lincoln, 2005: 17). In this moment we note an increase in theories, ranging from symbolic interactionism to constructivism, naturalistic inquiry, positivism and postpositivism, phenomenology, ethnomethodology, critical theory, neo-Marxist theory, semiotics, structuralism, feminism and various racial or ethnic paradigms (Denzin & Lincoln, 2005: 17).

In the blurred genres historical moment the bricoleur utilises the cumulative sum of available data analysis tools originating from the traditional period, the modernist phase, and blurred genres to make meaning of the data collected. Bricolage in this historical moment enables me to investigate the possible causes of learners' misconceptions and errors when they solve Euclidean problem. Bricolage, according to this historical moment of qualitative research, enables me to go beyond identifying the learners' difficulties and to find the possible causes of these difficulties in relation to the learner, teacher, parent, school management, school environment, local community, socio-economic status and culture.

As bricoleur in the blurred genres I employ multiple theories to interpret the challenges facing teachers – challenges that originate from historical dynamics that have shaped their lives (Kincheloe, 2005a: 324). For example, a bricoleur, through the theoretical lens of feminism, in interpreting girls' lack of skill for deductively giving reasons for mathematical statements for solving Euclidean geometry problems, could deduce that the lack of skill is caused by teaching methods. A feminist study by Anderson (2005: 175) states that the reason why some girls perform poorly in mathematics is that the traditional ways of teaching mathematics hinders girls' learning experiences. On the other hand, bricolage through the lens of community cultural wealth suggests that learners' poor performance in solving Euclidean geometry problems could be the result of teachers failing to incorporate the different cultural identities learners bring to class (Moloi, 2013: 489).

In responding to challenges facing learners, bricolage, in the mature qualitative research state called blurred genres, enables us to develop multi-state solutions. For example, one of the challenges facing learners is that Euclidean geometry is abstract. A feminist perspective could suggest that, in order to enhance these problem-solving skills, we need to create a safe, trusting community environment, within which girls feel nurtured, accepted and empowered (Anderson, 2005: 175).

This historical moment presents a multiplicity of theoretical lenses beyond the second moment, which enable us to design a responsive strategy to address learners' challenges. For example, enhancing learners' understanding of formal and informal deductive reasoning and application of theorems in Euclidean problem solving by using a multiplicity of lenses to reality leads to a multiplicity of solutions that takes into account the collective effort of learner, teacher, parent, school management, school local community environment etc. This moment also affords us the opportunity to create an environment that promotes individualised teaching methods informed by the theories that help us to understand leaners' difficulties beyond the classroom. In this moment we can apply diverse ways of collecting and analysing empirical data, such as open-ended or quasi-structured questionnaires, observational, visual, or personal experiences, and documentary methods (Denzin & Lincoln, 2005: 17). In this way all possible information that could help to unfold the complexity of reasons why many learners have difficulty solving Euclidean geometry problems can be explored, and used to inform the kind of intervention needed.

The blurred genres moment not only refers to multiplicity within the same discipline, but also the blurring of the demarcations between disciples (Denzin & Lincoln, 2005: 17; Kincheloe & Berry, 2004: 23). According to Berry (2004: 105), in the 21st century, multiple readings, plurality and inclusiveness cannot be ignored – a monological approach to research limits knowledge production. Bricolage unfolds through this moment of qualitative research and strives to respond to the failure of monological ways of knowledge production to account for the complex relationship between material reality and human perception (Kincheloe & Berry, 2004: 23). In order to pursue its objective of linking the complexity of life experiences with the objectives of the study, bricolage employs an interdisciplinary approach that includes ethnography, textual analysis, semiotics, hermeneutics, psychoanalysis, phenomenology, historiography, and discourse analysis, combined with philosophical analysis, literary analysis, and aesthetic criticism (Kincheloe, 2005a: 323).

2.2.1.4 Fourth moment: Crisis of representation

The fourth moment of qualitative research started from approximately 1986 and lasted until approximately 1990 (Denzin & Lincoln, 2005: 18). After the blurred genres of the third moment, which promoted the fluid borrowing of ideas and methods across different disciples, a general paradigmatic style of organising research emerged. During the fourth moment, an attack was launched against a paradigmatic style of idea documentation through inquiry (Marcus & Fischer, 1999: 7). The fundamental principles of positivism, such as objectivity, reliability and validity, are strongly condemned by pattern and interpretive theories. Critical theory, feminism and epistemology questioned the issues of gender, race, and class (Onwuegbuzie et al., 2010: 698; Denzin & Lincoln, 2005: 18). The grand theories were suspended in favour of localised, contextual meaning of social life, as experienced by those who are involved in it.

In this moment social science underwent what Denzin and Lincoln (2005: 19) call a triple crisis, which had to do with representation, legitimation and praxis (Denzin & Lincoln, 2005: 19). The crisis involved qualitative researchers of the day struggling to find ways to report that which captured the experiences of the studied. The question was, whose context should be represented in the research report: that of the researched, or the researcher? This uncertainty made it difficult to create a direct link between people's experiences and the text reporting on it. Furthermore, it created a legitimation crisis, which was concerned with criteria for interpreting and evaluating the validity and reliability of the qualitative research. Thus, representation and legitimation led to a praxis crisis, in the sense that the experiences of people are represented, firstly, in the context of the researcher, and secondly, there are no ways to validate or verify its "truthiness". How can people's lives be impacted by the kind of research that only exists in the research report text? This triple crisis paved the way for the fifth moment, which was concerned with a response to the crises.

2.2.1.5 Fifth moment: The postmodern period

The fifth moment of qualitative research started from approximately 1990 and lasted until approximately 1995 (Denzin & Lincoln, 2005: 18). The fifth moment of qualitative research resulted from responses to the triple crisis of the fourth moment. In the fifth moment there was a shift in ontological stance, that is, documented research should not reflect the reality of a particular context, but should be considered as narrative or storytelling (Denzin & Lincoln, 2005: 20). Thus, the current research tells the story of co-researchers who worked together as a team to design a strategy to improve their TPACK for teaching Euclidean geometry using integrated ICT software. The evidence extracts are drawn from the teams' working sessions, and no attempt is made to provide a direct link between the extract and the team members' experiences. The extracts are subject to my interpretation in line with and in the context of the web of social reality where the story took place. Van Dijk (1993: 251) and Mahlomaholo (2012, 45) concur when they argue that spoken words of co-researchers represent their participation in social arrangements and societies. The extracts are understood in this study to encode the personal and social processes of the co-researchers within and beyond the team (Rahimi & Riasati, 2011: 107).

The postmodern attempt to respond to the triple crisis involves couching documented research as storytelling, which is justified by the claim that, just as different people tell the same story in different ways, so do qualitative methods. Thus, a fixed, single way to assess quality is rejected. Moreover, this moment presents one of the most important questions in critical scholarship, which is, who decides what is true? In this moment the idea that a researcher is a powerful person who can decide what is true is abandoned. Therefore, bricolage in the current study is not concerned with the "truth", because the truth cannot be told by research (Mahlomaholo, 2012: 46). The "truth" in this study is grounded in the complex epistemological stance advocated by Kincheloe (2009: 109) and Mahlomaholo (2012: 46; 2013: 378), namely, that there is no one correct way of telling. Subscribing to the complex epistemological stance that declares that the mind creates knowledge informed by and inseparable from its surrounding social world makes the truth multi-layered. Thus, I consider the web of social reality created by a team member in this study to be an appropriate theorisation of the importance of a teamwork approach to knowledge production. This study employs a multi-layered and multi-methodological approach to research, as encapsulated in the eight moments of qualitative research. A more collaborative and cooperative type of research is encouraged in this study. According to Denzin and Lincoln (2005: 60) the postmodern moment also involves the emergence of more action, participatory and activity oriented research. The epistemological shift to the idea that knowledge can be produced and reported through multiple genres, as occurred in the postmodern phase, paved a way for postexperimental research.

2.2.1.6 Sixth moment: The postexperimental inquiry

The sixth moment, which occurred between approximately 1995 and 2000, brought back a strong wave of blurring between the genres of social sciences and humanities (Given, 2008: 311). The approaches included research documented as a poetry, novels capturing lived experiences, autobiographies and multivoiced representation (Denzin & Lincoln, 2005: 27). Qualitative research in the postexperimental inquiry era argued strongly for the kind of research that seeks to emancipate society's underclass.

It should be mentioned that the historical moments of qualitative research are not fixed in time, but overlap over time. In this historical moment of qualitative research, democracy and social justice caught the interest of researchers (Denzin & Lincoln, 2005: 27). Advocating for democracy and social justice involved a fluid borrowing of ideas, tools and approaches between the disciplines (Denzin, 2010: 3).

The sixth moment, furthermore, saw an awareness for the need for a social scientific kind of research that promotes both rigour and praxis. The epistemic ideology that claims that the researcher has all the knowledge needed for knowledge production through research is rejected (van Dijk, 2006a: 115). Thus, the postexperimental inquiry moment provides methodological tools that promote collaborative work between co-researchers. The fluid borrowing of tools, as was prevalent in the blurred genres moment, too, led to problems relating to evaluation, which paved the way for the seventh moment.

2.2.1.7 Seventh moment: The present time

The seventh moment, from approximately 2000 to 2004 (Given, 2008: 311), arose as a result of the lingering influence of positivism, which sought to develop criteria to evaluate the validity and reliability of qualitative research. In the seventh moment criteria for evaluating critical qualitative research are not concerned with validity and reliability, but with morals and ethics (Denzin, 2001: 326). Denzin (2001: 326), in his study on more radical consumer research, theorised that criteria for evaluating critical qualitative research are a blend of aesthetics, ethics and epistemologies. Denzin argues that knowledge is not free of power, and those who have power determine what is aesthetically pleasing and ethically acceptable. Thus, the current study adheres to the ethical process that is established by the University of the Free State. Moreover, the transformative agenda of the current study, in accordance with bricolage principles, argues against a single, fixed epistemological power for the privileged that disregards the ways of knowing of the co-researchers in this study.

In addition, Denzin (2001: 326) argues that a feminist perspective on evaluating critical qualitative research is informed by moral and ethical ways of knowing (epistemology). This epistemology involves the perception of a human being (ontology) and his/her position within the local social organisation (Denzin, 2010: 28). Thus, in the current study, using a bricolage lens enabled me to understand that an epistemological aesthetic, which describes what is beautiful, true and of good quality, may be the consequence of the degree of departure of the social context of the study (Denzin, 2001: 326). Thus, the social context of the extracts presented in Chapter 4 are intended to enable the reader to subject her or his interpretation to the context of the events. This concurs with Van Dijk's (1993: 251) social cognition, which serves as a theoretical interface for a social representation that is in the mind of the social actors. In order to have access to the mind of the social actors -- in the current study, co-researchers – a deeper-level of analysis, called social structures and discursive practices (van Dijk, 2006b: 115), is done (see Chapter 4). This approach to analysis is substantiated further by Mahlomaholo (2012: 45), who draws on Jacques Lacan's claim that language speaks through us.

Aesthetics and moral standards cannot be objective or neutral; this means that there cannot be one aesthetic and epistemology that is standard and that evaluates all the qualitative research in the seventh moment era. In the seventh moment we see aesthetic expression, such as "black is beautiful" (Denzin, 2001: 326). Furthermore, the kind of work done in the seventh moment creates critical, self-reflexive, ethical awareness and moral consciousness concerning the appeals to ideology and objective knowledge. Denzin (2008: 321), in his rejection of universal positivistic criteria for evaluating quality, argues that whoever assesses the quality of the research should avoid divorcing it from its paradigm.

2.2.1.8 Eighth moment: Fractured futures

Fractured futures represents the eighth moment of qualitative research, which commenced in approximately 2004. Researchers operating in the eighth moment are moving towards pluralism of theories, methodologies and data generation tools (Clarke, Willis, Barnes, Cromby, McDermott & Wiltshire, 2015: 1). Clarke et. al. (2015: 1) argue that pluralistic methods of doing research are a result of the limitations of using a single method, and an attempt to make sense of a complex, diverse social world. In their study to show the importance of using a combination of methods of making meaning out of qualitative data, they argue that using multi-methodological research approaches, multiple theoretical frameworks and multiple data gathering techniques in their psychology research yielded different findings which speak to different audiences.

2.2.2 Formats of bricolage

In this section I discuss five types of bricoleurs who embrace the complexity of life experiences: the interpretive bricoleur, the methodological bricoleur, the theoretical bricoleur, the political bricoleur, and the narrative bricoleur (Denzin & Lincoln, 2000: 4; Rogers, 2012: 4).

According to Denzin and Lincoln (2005: 4) **interpretive bricoleurs** believe that there is no single correct telling – each telling is a reflection of someone's perspective.

Adopting the interpretive bricoleur's lens in pursuing the objectives of the study provides functionality under the umbrella of the field of hermeneutics (Kincheloe, 2005a: 335). This enables me to investigate the challenges posed by absence of skills that are needed to solve Euclidean geometry problems; to find strategies to enhance the skills needed to solve Euclidean geometry problems and the conditions that make the strategies work; and to identify threats and indicators of success of these strategies in relation to our personal history, autobiography, race, socioeconomic class, gender, sexual orientation, ethnicity, religion, geographical place, and numerous other dynamics (Rogers, 2012: 4; Kincheloe, 2005a: 335).

Methodological bricolage is a process employing multiple research methods to make sense of or to unfold the complexity of the research problem (Rogers, 2012: 5; Kincheloe, 2005: 335). Adopting the format of methodological bricolage lifts the restrictions of using a single approach for analysis and interpretation when designing a strategy to enhance the teaching and learning of Euclidean geometry. This implies that bricolage gives researchers the liberty to employ various strategies of data generation, such as the interviewing techniques of ethnography, textual analysis of documents, the psychoanalytical method, semiotic analysis of signs, the historical research method, discursive and rhetorical analysis of language, phenomenological analysis of consciousness and intersubjectivity, and empirical material available in response to challenges facing teaching and learning of Euclidean geometry (Denzin & Lincoln, 1999: 3; Roger, 2012: 5; Berry, 2004a, 125). Consequently, researchers have the opportunity to be flexible and practical, and are not bound tot using one predetermined methodology that is perceived as correct; instead, researchers can embrace and permit the context of research to dictate the methods to be used. Multiple data-gathering strategies provides more information about the research subject, which places us in a better position to understand the complexity of teaching Euclidean geometry.

Theoretical bricolage emerges from what Denzin and Lincoln (1994: 3) call blurred genres, the third moment of qualitative research. It is this historical moment of qualitative research that saw the emergence of many perspective paradigms critical about the traditional period and its grounding in objectivism, imperialism and universal truth. Theoretical bricolage involves multiple theoretical lenses, such as Marxism, constructivism, critical constructivism, postcolonialism, cultural studies, poststructuralism, feminism, queer theory and enactivism, to better understand and interpret the challenges of Euclidean geometry. For example, theoretical bricolage allows us to use one theoretical position to analyse and challenge incorrect reasons to support mathematical statements; then a bricoleur maps to a different theoretical position to analyse the same challenge, using multiple theoretical positions to capture the complexity of the challenge (Berry, 2004a: 109). This kind of approach to research helps us to construct a collage of pictures or views that does not tell us "more truth", but that enables us to design a responsive framework to enhance the teaching and learning of Euclidean geometry based on greater depth, rigour and multiplicity.

The political bricoleur is a researcher who is aware of the existence of the connection between knowledge and power (Denzin & Lincoln, 1999: 6). According to Rogers (2012: 6) a political bricoleur seeks to produce knowledge that "benefits those who are disenfranchised by everyday taken for granted workings of neoliberal, capitalist, white, patriarchal, and heterosexist social structures". Therefore, the bricoleur interrogates the information generated and documents the effects of ideological power, disciplinary power, regulatory power, hegemonic power, and coercive power (Berry, 2004a: 126).

The **narrative bricoleur** understands that research is a representation of a specific interpretation of a phenomenon (Denzin & Lincoln, 1999: 5; Rogers, 2012: 7). This bricoleur understands that the research of life experiences influences the knowledge produced. Therefore, the narrative bricoleur's perspective on the research enables us to give a representation of multiple voices, perspectives and sources.

2.2.3 Ontology and epistemology

Bricolage is born out of respect for complexity. This approach informs the bricoleur's complex ontological stance. A bricoleur rejects positivistic ontology, which states that there is only one objective reality (Kincheloe, 2004: 29). I understand that bricolage makes it possible to acknowledge multiple interpretations of the world and multiple ways for people relate to the world. This multiplicity of possible interpretations informs a bricoleur about the object of inquiry, which is open to many historically situated and culturally inscribed contexts and processes (Kincheloe, 2004, 38). In addition, epistemology is convoluted within cultural, historical, political, psychological and educational dynamics.

2.2.4 Objectives of bricolage

This section is concerned with discussing the objectives of bricolage and how they help us to address the objectives of the study. Kincheloe and Berry (2004: 25), drawing on the work of Levi-Strauss (1966), maintain the original meaning of the concept of bricolage, namely, that it involves an understating of complexity and unpredictability. Therefore, it is the objective of bricolage to account for the complexity of everyday life in relation to the object of study (Kincheloe & Berry, 2004: 25). In this study I understand that the challenges posed by lack of skills needed to solve Euclidean geometry problems comprise multiple variables and factors arising from life experiences. Therefore, bricolage is an appropriate theoretical framework to couch this study, since it seeks to make meaning of the complexity of the lived life experiences that relate to the phenomena being studied (Kincheloe, 2011: 301; Rogers, 2012: 4).

2.2.5 The role of the researcher

The role of the researcher in this unfolding bricolage is to convene a team of co-researchers in pursuit of a strategy to improve the teaching and learning of Euclidean geometry. In this study, I perceive myself as a co-researcher, since I do not have solutions to the challenges faced by teaching and learning Euclidean geometry with the aid of ICT software. This approach is supported by Kincheloe (2011: 220), who observes that many teachers are of the view that education researchers have little to offer that would enable teachers to face their day-to-day challenges better. Therefore, my role is to contribute to the collective knowledge and skills that are necessary to respond to the research question (Te Aika & Greenwood, 2009: 59). Guided by the lens of bricolage, I understand that knowledge is a product of multiple representations of human activities. Thus, in agreement with Mahlomaholo (2009: 226), and as a co-researcher, I invite other co-researchers to take part in the research project, thereby creating a space for transformation and self-empowerment. In doing so, the study locates itself in the seventh and eighth moments of qualitative research, where research projects are undertaken for the purpose of improving lives (Denzin, 2001: 326).

2.2.6 The relationship between researcher and the participants

According to Denzin (1994: 17-18) research according to bricolage uncovers the context of research by an interactive process shaped by the researcher's personal history, biography, gender, social class, race and ethnicity, and by that of the other participants. Thus, a team of co-researchers in this study understand the inherent power differentials that each person could bring to the team. For instance, some of the co-researchers are in managerial roles at the same schools as other participants, whom they lead. However, the approach of the current study levels out power and promotes equality between co-researchers in knowledge production (Mahlomaholo, 2012). Grounding in complex epistemology in the process of knowledge construction through research to find ways to improve the teaching of Euclidean geometry with the aid of ICT software, and a quest for complexity, becomes the treasure.

The respect for complexity is evident in the relationship between the researcher and participants. In this study I, as the researcher, become a researcher-participant, and the participants become participant-researchers, which makes us co-researchers. The study does not seek to find an objective view of my interpretation of the co-researchers' narratives, instead, the transformative nature of the study seeks to improve lives of all those who participate in the study. The relationship between co-researchers seen through the theoretical lens of bricolage, the quest for a multiplicity of views, which is made possible when working together as co-researchers and power differentials between the co-researchers, are used to privilege. This study rejects a reductionist lens, which, in the name of objectivity or bias reduction, advocates for no interaction with participants.

The study embraces a critical, complex epistemology, which is located in the eighth moment of qualitative research, and which seeks to create a web of reality that fosters "humanness" and equality between co-researchers (Kincheloe, 2009: 109). In this study the web of reality is embedded in the diverse contexts of educational spaces.

2.2.7 Appropriateness of bricolage

In this section I summarise why bricolage is an appropriate theoretical framework, given the above discussions. Furthermore, I will indicate the appropriateness of choosing bricolage as a theoretical framework to couch the study by referring to the five objectives of the study. Bricolage enables us to achieve a better analysis of the challenge to enhance learners' visualisation skills, which they need to solve Euclidean geometry problems. In the quest for meaning-making, bricoleurs use anything that can generate meaning, for example, a book, a theory, a picture, a history, a movie, a classroom, a story, etc., to understand why learners experience challenges to visualise and analyse and deductively and inductively sketch their mathematical arguments (Berry, 2004: 108).

Further, bricoleurs understand that challenges facing learners in visualising, analysing and deductively and inductively reasoning consist of multiple variables and factors, which originate from the learners' experiences of life. Thus, bricoleurs understand that, in order to respond to the challenges caused by visualising geometric objects or figures, and enhancing the learners' inductive and deductive skills, we need a framework that is not one dimensional. In order to formulate a strategy that is effective, the bricoleur avoids linear, monological, hierarchical, empirical, and structuralist forms; instead, a bricoleur needs a theoretical framework that seeks to make meaning of the complexity of lived life experiences that relate to the phenomena being studied (Rogers, 2012: 4; Kincheloe, 2011: 301).

I find bricolage helpful for making meaning of multiple variables and factors that contribute to learners' challenges to understanding Euclidean geometry. For example, bricolage provides researchers with a position from which to understand that the quantification of learners' performance in mathematics is not a true reflection of learners' ability, but the beginning of the process of identifying the challenges facing learners. Bricolage's approach to research is an active, rather than passive, process, which helps us to actively construct our research methods from the tools at hand, rather than passively choosing one method or technique to analyse learners' challenges (Kincheloe, 2011: 324).

During the first decade of the 21st century bricolage was known as multi-methodological process employed to unfold the context of research (Kincheloe, 2005a: 323). Thereafter, as a result of development, bricolage moved into the domain of complexity and now bricolage is known as a theoretical framework that involves respect for the complexity of the lived world (Kincheloe, 2005a: 324). According to Kincheloe (2005a: 353) bricolage is grounded in the epistemology of complexity. Therefore, bricolage is a theoretical framework that enables us to unfold and uncover learners’ challenges in understanding Euclidean geometry within the complexity of their lived world. Bricolage couches this study in its quest to make meaning of complexity, and its condemnation of monological knowledge-seeking processes that are concerned with order and certainty (Kincheloe, 2005: 326).

Founded on the meaning-making of this complexity, bricolage rests on the epistemological and ontological assumption that the domain of the physical, the social, the psychological, cultural and educational, consists of interaction of a multitude of factors that seek multiple ways to uncover contextual factors in the teaching and learning of Euclidean geometry (Lincoln, 2001: 694; Kincheloe, 2005a: 327). Furthermore, due to the changing nature of the world, as bricoleurs we understand the importance to of embracing a non-fixed reality (Kincheloe, 2005a: 327).

Bricolage helps us to analyse and to design a strategy to respond to the learners' challenges in understanding Euclidean geometry; we do this using a critical, multi-perspectival, multi-theoretical and multi-methodological approach (Kincheloe, 2001: 700; 2004a: 10; 2004b:20; 2004c:3; 2004d; 2005a: 330; Matt, 2012: 1; Mahlomaholo, 2013: 1). Furthermore, bricolage is ideal for designing a framework to enhance the teaching and learning of Euclidean geometry that will enable learners, teachers, parents and local community members to understand that the solution to their problems lies within themselves, as members of the community (Mahlomaholo, 2013: 384).

Bricolage does not only make meaning from chaotic human freedom in the process of research, but also addresses issues of power; social justice; marginalisation and oppression perpetrated through traditional research processes (Rogers, 2012: 8). The values and principles of bricolage help us to develop a strategy that embraces the equality of participants in the process of knowledge production. Bricolage embraces collaborative work and the understanding that the researcher does not have the solution to the problems posed by Euclidean geometry, therefore, the contribution of each member of the team is imperative. Moreover, the lens of bricolage appears to align with the emancipatory role that the study seeks to create – an environment in which all co-researchers could collectively realise the power they have in developing a strategy that will address the problems related to understanding Euclidean geometry (Mahlomaholo, 2013: 384)

2.3 CONCEPTUAL FRAMEWORK

In this section I, as a bricoleur, create a complex and rigorous structure that maps out the concepts and vocabulary used for making meaning of the knowledge needed for improving teachers' TPACK for teaching geometry concepts. A structure that creates a map for the study is constructed by borrowing from what Berry (2004: 110) calls a bricolage map. A bricolage map is a list of possible areas that I will visit to investigate what constitutes the knowledge needed to improve teachers' TPACK for teaching Euclidean geometry with the use of integrated ICT software. In constructing such a map Berry (2004: 111) suggests that we start with what he calls a POET, or point of entry text. A POET is a central area of the bricoleur's map, and illustrates what he or she intends to investigate for the rest of the bricolage. Bricolage is a product produced by a bricoleur through his/her ways of conducting research. Thus, POET, in this study, is used to improve teachers' TPACK for teaching Euclidean geometry with the aid of integrated ICT software.

2.3.1 Technological pedagogical content knowledge

TPACK refers to a synthesised form of knowledge that aims to integrate ICT into teaching and learning in the classroom environment (Chai, Koh & Tsai, 2013: 32). The work of Mishra and Koehler (2006: 1017-1054) has made a significant contribution to shaping TPACK as a conceptual framework that gives researchers and practitioners a vocabulary to describe the knowledge needed for using technology for teaching (Koehler & Mishra, 2009: 62). Mishra and Koehler (2006: 1017-1054) built TPACK from the work of Shulman (1987: 1-22), who described the kind of knowledge needed for teaching (Chai et al., 2013: 31).

Mishra and Koehler (2006: 1017-1054) start with Shulman's notion of PCK. Before Shulman developed his groundbreaking way of considering teachers' knowledge, subject matter knowledge and pedagogical knowledge were each considered in isolation (Mishra & Koehler, 2006: 1021). PCK refers to the interaction or interrelations that exist between content and pedagogy – PCK is a transformed type of knowledge that blends content and pedagogy into a knowledge of understanding of how particular concepts are packaged or organised to make them easier to learn.

However, over time, technology has become one of the teaching aids that has the potential to enhance teaching in a general sense (Azlima, Amran & Rusli, 2015: 1794; Ali, Haolader & Muhammad, 2013: 4061; Buabeng-Andoh, 2012: 136; Koehler & Mishra, 2009: 61; Koh, Chia & Tsai, 2014: 185). Initially, the use of technology posed a challenge, since there was no theoretical framework to guide the process of integrating ICT in classroom teaching and learning. Thus, not all ways of using technology as a teaching aid were effective (Buabeng-Andoh, 2012: 137), and it motivated researchers to design a framework for using technology as a teaching aid. For instance, Mishra and Koehler (2006: 1024) introduced a technology component into Shulman's PCK. They argue that, due to transformation and technology's presence in the modern classroom, it is no longer enough to confine the teacher's knowledge to the PCK, as teachers now require knowledge about more than just content and pedagogy (Mishra & Koehler, 2006: 1023).

The technological dimension of the conceptual framework cannot be considered separately from content and pedagogy. The challenge involved in using ICT to teach mathematical concepts such as Euclidean geometry arises when technology has to be used to enhance pedagogy to package Euclidean geometry concepts. Thus, Mishra and Koehler (2006: 1023) argue that the three knowledge domains have to intersect. Figure 2.1 is a diagrammatic representation of the seven knowledge domains and the way they are related.

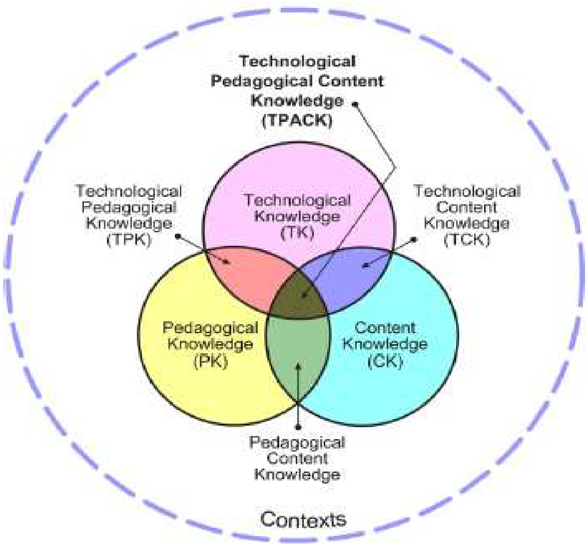

Figure 2.1: The intersection of technological knowledge, pedagogical knowledge and content knowledge (source: tpack.org)

Figure 2.1 shows the intersection of the three knowledge domains; through their intersection the other three sub-knowledge domains are formed. The sub-knowledge domains are technological pedagogical knowledge, technological content knowledge, and pedagogical content knowledge. The intersection of the three domains creates what Mishra, Koehler and Cain (2013: 14) call TPCK, which was modified further into TPACK so that it would be easier to remember. The permutations of the intersection of the three core components of knowledge domains as shown in Figure 2.1 differ contextually (Koehler, Mishra & Cain, 2013: 14). Through this conceptual framework this study provides me with a conceptual lens that enables me to understand the qualities of knowledge needed for improving teaching Euclidean geometry using ICT software. This conceptual lens aligns with the theoretical lens that coaches the study. For instance, the conceptual framework of TPACK demonstrates the complexity of knowledge needed for teaching using ICT software. According to the TPACK conceptual framework a teacher's competence regarding ICT software is evident when he or she can thread through the seven different knowledge domains shown in Figure 2.1.

TPACK, by its very nature, is embedded in complex epistemology. For instance, content knowledge of the theorem states that a line segment drawn from the centre of the circle to the midpoint of the chord is perpendicular to the chord. Firstly, language is a factor that contributes to the complexity of knowledge of the theorem, since the theorem statement must be analysed and interpreted before it can be understood.

Secondly, proving the theorem becomes more complex when abstract logical proof is required, such as congruency or similarity between two triangles, which also needs to be understood at seven different knowledge domains.

Furthermore, pedagogical knowledge is complex because it refers to knowledge and skills that a teacher needs to manage and organise teaching activities in such a way that the theorem can be learnt. However, a classroom setting is characterised by vastly diverse learning styles, and different levels of learner competence in Euclidean geometry, which adds to the complexity of the knowledge needed for teaching. Thus, I conclude that the complex epistemology of bricolage, which embraces convergent and sometimes divergent views that contribute to the nature of knowledge and ways of knowing what is relevent, is suitable for making meaning of such complex knowledge (Berry, 2004: 116). A dimension that is central to bricolage is the construction of meaning explaining the complexity of everyday life (Kincheloe, 2004: 82). Bricolage offers a theoretical stance that is appropriate for making meaning of the complexity of TPACK. In pursuance of this complexity during the construction of a bricolage, I created a bricolage map, through which I thread during the construction of bricolage. Threading is used as a metaphor to describe the process of knowledge construction involved in moving from the POET, looping through the feedback, and subjecting the object of study to multi-perspectival, multi-theoretical and multi-methodological analysis (Berry, 2004: 116; Rogers, 2012: 1).

In constructing this bricolage, I start on a map, at POET, which represents what a bricoleur seeks to make meaning of in the construction of bricolage. Therefore, in the unfolding bricolage, I consider “improving teachers’ TPACK for teaching Euclidean geometry” to be a POET of the study (see Figure 2.2). I therefore conceptualise the unfolding study by using the map in Figure 2.2. I start with a reading of the knowledge needed for teaching Euclidean geometry using ICT software by threading on a map and conducting a literature review on technological knowledge. Then, with the understanding or meaning I constructed from the literature review on technological knowledge, I loop back to the POET. This gives me a better understanding of the POET in relation to technological knowledge. In pursuance of making meaning of the complexity of the POET, I loop back and forth, subjecting the POET to multiple readings, conflicting discourses, multi-theoretical perspectives, and mixed genres of epistemology and methodology, which adds rigour to the process of meaning making (see Figure 2.2).

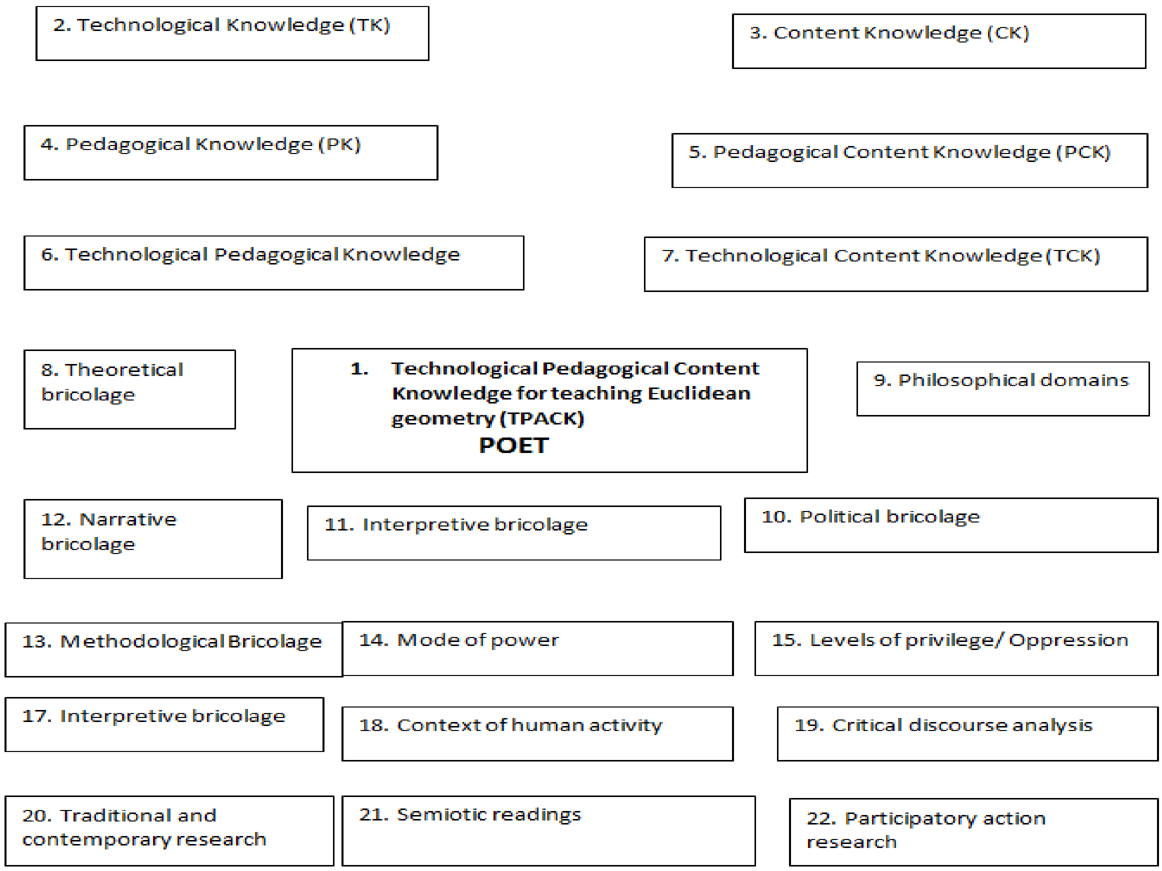

Figure 2.2: Bricolage map (Adapted from Berry (2004: 112))

A bricoleur loops back and forth on different structures or map areas in creating knowledge about the POET of the study. In so doing I realise that improving teachers' TPACK for teaching Euclidean geometry using integrated ICT software requires more than the knowledge described by the TPACK conceptual framework with its seven knowledge domains; I realise that the challenges are far more complex than this demarcation. For instance, the framework does not refer to policies that have been developed for integration of ICT in education – Global E-Schools and Communities Initiative (GESCI) reports involving 194 countries have developed ICT policies and plans for implementing ICT in education (Bassi, 2011: 4-61). These policies have affected the integration of ICT in teaching and, therefore, they need to be understood by practitioners and researchers, because they could contribute to effectiveness of the integration of ICT in teaching and learning. Furthermore, the manner in which ICTs are integrated in teaching and learning in classrooms needs to be understood by practitioners and researchers, because this could also have an impact on the success of its integration. Political and other modes of power could, for instance, have direct influence on the use of ICT for teaching and learning (see Figure 2.2, frames 10, 14 and 15). Thus, the bricoleur, in his quest to mirror the real world's complexity, pursues an inquiry with the understanding that the challenges facing teachers who are using ICT software to teach Euclidean geometry are not limited to the lack in knowledge illustrated by the seven constituencies of the TPACK framework (Berry, 2004: 116).

2.3.2 Social contructivism

This section discusses the relevance of social constructivism as a conceptual framework that provides me with a stance to define and describe good teaching. Through the conceptual framework of social constructivism, I acknowledge that improving TPACK knowledge for teaching Euclidean geometry using ICT software is a social endeavour. The appropriateness of social constructivism as a conceptual theory for teaching and learning is justified by the following: origin of social constructivism, format, objectives, the role of researchers and participants, ontology, and epistemology.

2.3.2.1 Ontology and epistemology

Social constructivism presents the ontological notion that reality is constructed through shared human social activity. This implies that the local member of a particular community creates a view of the world and its properties based on a shared understanding. Thus, social constructivism gives me the theoretical stance that reality is not an external entity to be discovered, but that it is created by people. Furthermore, social constructivism's ontology can be understood through the lens of bricolage; that is, even shared reality is a collage and representation of multiple realities of people, due to the complexity and diversity that exists within a single society. This is confirmed by Pritchard and Woollard (2010: 7), who explain that, in practice, individual realities within the same local societies will be similar, even though there will be instances where, as a result of different fundamental experiences and interactions, they differ greatly. This is in line with bricolage's complex ontology, which subscribes to multiple realities in the construction of collective reality, even though the realities sometimes repel one another.

The epistemological stance of social constructivism is that knowledge is a human creation that is constructed through social interaction. That is, interpretation and making meaning of the world or environment is possible by social interaction between the environment and the person. Thus, social interaction between people also becomes a mode of knowledge creation. This leads me to conclude that there are multiple ways that each person could interact with the environment and with other people, which, as the results suggest, points to the existence of multiple ways of knowing. For instance, I could have come to construct knowledge of who my parents are through the social interactive space that I call home; on the other hand, this knowledge could be the same for some people, but different for others. Therefore, the preceding analogy aligns with bricolage and has, further, been subjected to rigorous analysis by a bricoleur, who declares that I could also have known who my parents are through the traditional-moment ways of knowledge construction, which involves making use of paternity tests. Thus, a bricoleur uses multiple ways of knowing in constructing bricolage in pursuit of rigour in the inquiry.

In the currently unfolding bricolage, a theoretical bricoleur, through social constructivism's epistemological stance, defines good teaching as it applies to TPACK for teaching Euclidean geometry using ICT software, as a lesson that promotes interaction between learners, and learner(s) and environment. This theoretical position, firstly, declares to the reader my unapologetic stance towards understanding the object of the study, which is TPACK for teaching Euclidean geometry using ICT software. It is through this theoretical epistemological stance that I find ways of knowledge construction as a human creation.

Thus, this epistemological stance influences my approach to the construction of the current bricolage, which subscribes to the view of Kincheloe (2009: 117) that knowledge production relating to ways to improve teachers' TPACK for teaching belongs to the teachers, in contrast to the current situation in professional development programmes, which are dictated by the DBE. Through this epistemological stance teachers will construct their own contextualised knowledge for using ICT software to teach Euclidean geometry, and I support the idea that the current inquiry should adhere to the principles of self-directed learning, which implies that teachers should not be forced to take part in ICT training that perpetuates "the master's" one "correct" way of improving pedagogical practices. However, I advocate positioning ourselves to see the teaching profession as space in which a job is transformed into a learning space for both teacher and learners. Thus, through the unfolding bricolage, I advocate for teachers becoming researchers to find ways to improve our pedagogical practice using ICT software. This approach is in line with the South African Qualifications Authority's (SAQA) national qualification framework (NQF) (2012: 12), which requires that teachers demonstrate the ability to conceptualise new teaching methods and create new knowledge.

2.4 OPERATIONAL CONCEPTS

This section defines and discusses the operational concepts used in the study.

2.4.1 Improving teachers' technological pedagogical content knowledge

According to Ho, Watkins and Kelly (2001: 143) improving teachers' knowledge or teaching is achieved when teachers master a set of generic teaching skills, which include how to teach, how to prepare lesson plans and how to use media and technology tools. This definition of improving teaching is adapted and adopted in this study. For instance, in this study I understand that Ho et al. (2001: 143) conceptualise ways of teaching and preparing lessons as teachers' pedagogical knowledge. Ways of using media and technology is understood in this study to refer to technological knowledge. However, knowledge of how to teach and how to use technology is related to particular content, which is, in this study, Euclidean geometry content knowledge. Thus, the intersection of the three types of knowledge becomes TPACK. Therefore, in line with this definition of improving teachers' TPACK, the study's indicators of success are:

- Enabling teachers to assess their own strengths and weaknesses in teaching particular concepts of Euclidean geometry;
- Informed by their own strengths and weaknesses, being able to make sensible and creative choices regarding the use of ICT software to complement identified pedagogical difficulties; and
- Integrating ICT software with content of Euclidean geometry.

Leendertz et al. (2013: 5) state that, when teachers' TPACK is improved, the following skills or knowledge must be evident:

- An understanding of how to represent and integrate mathematical concepts with ICT software;

- Pedagogical skills to communicate their content knowledge of Euclidean geometry with the aid of ICT software;
- Knowledge of using ICT software as a teaching aid to assist learners in eliminating Euclidean geometry misconceptions;
- Fundamental or basic knowledge that help learners find Euclidean geometry concepts as difficult or easy to grasp;
- The ability to assess learners' prior knowledge of Euclidean geometry and faciliate a lesson that accomodates diverse learning styles; and
- Knowledge of how to use ICT software to construct new teaching methods, using learners' existing knowledge.

2.4.2 Pedagogical content knowledge

Research conducted prior to and in the early 1980s was concerned with general aspects of teaching, and not with the content knowledge teachers needed for teaching a specific subject. In the late 1980s a major departure from the research of the day was made by Shulman and colleagues (1986) through their seminar work (Ball et al. 2008: 389). Shulman et al. (1986) attempted to merge content and teaching practices, and they argued that unique subject matter is specific to professional knowledge for teaching (Ball et al. 2008: 389). They called this professional knowledge pedagogical content knowledge (PCK), which comprises seven knowledge domains that were considered to be the professional knowledge needed for teaching. These domains were content knowledge, general pedagogical knowledge; knowledge of educational aims, goals and purposes; knowledge of learners' curricular knowledge and knowledge of educational contexts (Hurrell, 2013: 54). Hurrell (2013: 55) defines Shulman's PCK as a practical knowledge of teaching and learning guided through a contextualised knowledge of a particular classroom setting.

In 1999 Shulman's PCK was refined by Gess-Newsome, who proposed two refined models of PCK, namely, integrative and transformative models (Hurrell, 2013: 55). Gess-Newsome (1999) described the transformative model of PCK as consisting of different kinds of knowledge that do not relate to teaching and learning, but which are synthesised and transformed to the benefit of teaching, and which enhance learners' understanding (Barrett & Green, 2009: 17). The integrative model describes PCK as an intersection between content, pedagogy and context of learning (Barrett & Green, 2009: 17). The three main components of PCK in the two models are explained by the National Council for Accreditation of Teacher Education in their definition of PCK as:

> *interaction of the subject matter and effective teaching strategies to help students learn the subject matter. It requires a thorough understanding of the content to teach it in multiple ways, drawing on the cultural backgrounds and prior knowledge and experiences of students (NCATE, 2002: 55).*

This definition corresponds with the way PCK is used in this study, and which is inclusive of both transformative and integrative understandings of PCK. Due to inadequate understanding of PCK, Shulman's (1986) notions saw little development until Ball et al. (2008: 389) developed SCK. PCK refers to the knowledge of organising, representing and packaging the content in such a way that a learner could learn it easily. This means that teachers should know what makes the content/concepts difficult for learners to understand, and what prior knowledge of learners could be linked with the current content or concepts (Mishra & Koehler, 2006: 1027). Teachers must formulate teaching strategies that are informed by learners' misconceptions and content gaps and teachers must find remedies for the learners' misconceptions. Furthermore, PCK is concerned with theories of knowledge construction or epistemology. In this way PCK covers the core business of teaching, learning, assessment, curriculum and reporting (Koehler & Mishra, 2009: 64).

2.4.3 Mathematical content knowledge for teaching

In attempt to develop Shulman's (1986) PCK, Ball et al. (2008: 389) investigated teaching demands and analysed literature on mathematics teaching practices. Ball et al. (2008: 389) then constructed and tested an hypothesis concerning the nature of mathematical content knowledge for teaching (MCKT). The results of hypothesis testing and the literature on teaching practices concurred with Shulman' PCK framework. The results of the study of Ball et al. (2008: 395) showed that there is a need to define knowledge for teaching further, beyond Shulman's PCK. This resulted in what Ball et. al. (2008: 399) call four domains of knowledge for teaching, namely, common content knowledge (CCK), specialised content knowledge (SCK), knowledge of content and students (KCS), and knowledge of content and teaching (KCT).

CCK refers to material knowledge or knowledge of material for the non-teaching context (Jóhannsdóttir, 2013: 3). This implies that the teacher must commonly have the same knowledge as any mathematician, for example, the teacher's knowledge of the sum of interior angles of a triangle equals 180 degrees, must be as competent as a mathematician or engineer's knowledge for nonteaching purposes. CCK is understood by Ball et. al. (2008: 399) to be knowledge that a teacher uses to recognise learners' right or wrong answers, or a textbook's inaccurate solution.

For instance, a learner is required to determine the magnitude of angle $C\hat{B}D$ in Figure 2.3, given that $C\hat{A}D = 20°$, $C\hat{B}A = 60°$ and $A\hat{C}B = 100°$. A teacher will apply CCK to recognise whether the learner's solution is correct or incorrect. An engineer or a mathematician would also use CCK of geometry to identify whether the learner's solution is correct or incorrect.

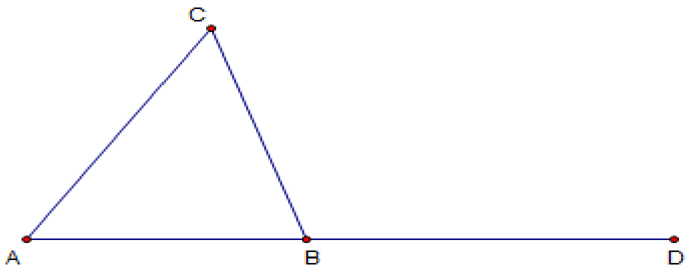

Figure 2.4.3: Common content knowledge

However, for a mathematics teacher it does not end with recognition or identification of a correct or incorrect solution. A mathematics teacher takes it a step further and seeks to find out where the error occurred and what the possible causes of the error are, threading the learner's solution and investigating if an unconventional method used by the learner could work in general. Therefore, in addition to CCK, a mathematics teacher needs knowledge that would enable him/her to diagnose and anticipate the possible causes of the error.

According to Ball et al. (2008: 400) SCK is the knowledge a mathematics teacher applies in this regard; they refer to it as mathematical knowledge and skills unique to teaching. In reference to the example given in Figure 2.4.3, after a mathematics teacher has used CCK to recognise that a learner's solution is incorrect, the teacher threads a learner's solution to determine what possible misconceptions the learner could harbour about the relationship between exterior angles and opposite interior angles of a triangle. Furthermore, once a mathematics teacher becomes familiar with learners' errors on exterior and opposite interior angles of triangle, Ball et al. (2008: 400) argue, her or his knowledge is transformed into KCS.

Ball et al. (2008: 401) explain that the teacher applies KCS to find learners' possible misconceptions. KCT refers to a combination of knowing about students and content. If a mathematics teacher applies this knowledge the teacher is able to anticipate what learners are most likely to think and what they will find confusing (Ball et. al., 2008: 401).

Lastly, KCT refers to combination of knowing about teaching and content (Jóhannsdóttir, 20013: 3). When a mathematics teacher is teaching a geometry concept, he/she needs to facilitate the lesson sequentially, carefully thinking of examples that could enhance the learners' understanding of the concept. In order for a teacher to perform this task he/she needs to apply integrated knowledge of content, teaching methods and the way learners learn.

2.4.4 Technological pedagogical content knowledge

Introducing ICT in the teaching of mathematics offers the possibility of revolutionising teaching methods for the 21st century. However, the inadequate implementation of ICT in lesson design and facilitation has become a great concern, which propelled researchers to construct a framework to guide the integration of ICT in mathematics teaching. TPACK was formally proposed as a framework to guide the integration of ICT in teaching and learning in approximately 2003 (Chai et al., 2013: 31). TPACK is an extension of the conceptual frameworks for knowledge that teachers need to teach mathematics by integrating ICT, that were proposed by Shulman (1986) and Ball (2008) (Chai et al., 2010: 555). TPACK represents an intersection of three knowledge domains, content knowledge (CK), pedagogical knowledge (PK) and technological knowledge (TK). The interaction of these three domains leads to pedagogical content knowledge (PCK); technological content knowledge (TCK) and technological pedagogical knowledge (TPK).

CK is similar to CCK, which refers to non-specialised knowledge of mathematics. The interaction of PK and CK forms PCK, which is similar to the interaction of CCK, SCK, KCT and KCT. Therefore, the same approach is used for knowledge construction to integrate ICT into the teaching of Euclidean geometry. TK in the context of the study refers to knowledge of using computer software by anyone for non-teaching purposes. Knowledge of computers and software application is not enough for teaching (Chai et al., 2010: 555). TPK, which is the interaction of TK and PK, refers to specialised knowledge of using ICT for the purposes of teaching.

2.5 RELATED LITERATURE

This section reviews literature for the purpose of designing a strategy to improve teachers' TPACK for teaching Euclidean geometry using integrated ICT software. Literature from South Africa, SADC, and Africa is reviewed, and aligned with international best practices in line with the objectives of the study.

2.5.1 Challenges facing teachers who teach Euclidean geometry with the aid of integrated ICT software

This section reviews literature for the purpose of, first, justifying the need for the study. Therefore, literature is reviewed to gain an understanding, to anticipate risks and threats, and to find ways to circumvent them. Secondly, the study subscribes to a transformative agenda that is both critical and emancipatory, thus, theories and previous research and policy are reviewed in order to shape evidence of success for the strategy.

2.5.1.1 The necessity of teamwork to improve technological pedagogical content knowledge for the teaching Euclidean geometry using ICT software

In the absence of a team each teacher works alone, despite the fact that there are other mathematics teachers who could collaboratively enhance each another's lesson preparation, assessment and lesson facilitation (Jita et al., 2008: 475). When teachers work in silos it denies them the opportunity to share their skills and knowledge, which could contribute significantly towards helping one another to improve TPACK for teaching Euclidean geometry with the aid of ICT software (Hu & Linden, 2015: 1104). Sharing knowledge and skills does not only contribute to enhancing sindividual team members' knowledge, but also increases learners' chances of learning. It is through social interactive spaces that are created by the team approach to improve pedagogical practices between team members that new knowledge is formed (Cobb, 1994: 17).

This means that individuals join the team with different knowledge and skills and, through social interactions informed by their ontological and epistemological stances, new, transformed knowledge is formed. Thus, bricolage in pursuit of understanding the components of TPACK provides me with an ontological and epistemological stance that is fluid and dynamic (Kincheloe, 2009: 108). This means that different people in a team, each shaped by their different backgrounds that influenced their ways of knowing, will contribute effectively to creating social interactive spaces that are multi-epistemological (Berry, 2004: 101). A team approach to improving TPACK for teaching Euclidean geometry enhances teaching practices, from an individualistic approach to a collaborative approach that has the potential of being inclusive of various learning styles. This benefit justifies the need for a collaborative approach to teaching.

A team approach to knowledge construction of ICT software is grounded in the constructivist epistemology, which states that knowledge is individually and socially constructed during an active process (Doolittle, 2014: 487). Thus, a theoretical bricoleur threads and loops to Habermas' (1971: 56) views on communicative action, which creates a platform for the team members to mutually agree on common goals and to tolerate the autonomous humanity of all individuals (Brown & Goodman, 2001: 204). In the context of unfolding bricolage the common goal of the team members is to improve their teaching practices involving ICT software. Collaborative efforts aimed at finding ways to improve the teaching of Euclidean geomtry as located in the seventh and eighth moments of qualitative research are guided by the principle that makes it possible and includes, amongst others, respect, hope, caring and solidarity.

Furthermore, the theoretical bricoleur loops through to Vygostsky's postulation on the zone of proximal development, which states that a team approach to learning includes multiple and even contradictory support for or views on learning (Hakkarainen & Korepanova, 2009: 4). Through bricolage's theoretical stance I understand that epistemic complexity, embedded in teaching and learning, is captured in a multiplicity of views that are sometime contrudictory, but made possible by the team setting (Kincheloe, 2004: 24). A team, further, creates an opportunity for the team members to act as the more knowledgeable other, interchangebly to one another (Zaretskii, 2009: 80). Communicative actions, as coined by Habermas (1971: 56), that are free of domination lead to mutual decision making regarding the selection of teaching methods and processes; this, in my view, fosters knowledge construction with respect to a mutlipicity of norms, the cultures of all team members and other diversity (Habermas, 1971: 51; Gesell & Hopen, 2014: 1).

In a study of factors that contribute to ineffective teaching, Mupa and Chinooneka (2015: 125) found that teamwork amongst teachers was lacking. They suggest that lack of a teamwork contributes to ineffective pedagogical practices. However, in their study they did not study the collaboration between the teachers only, but also between other stakeholders, such as parents. These findings confirm that lack of collaboration between teachers, parents and learners leads to ineffective lessons, an outcome that confirms the importance of a teamwork approach to teaching. The significance of the team was also evident in the work of Skourdoumbis (2013: 350), who questions the positivist teacher's effectiveness.

In his study Skourdoumbis (2013: 350) argues against a positivistic view of teaching, namely, that the teacher is the most powerful person, the one who knows what to do and how to do it. Under the positivistic stance of knowledge and knowledge production, teams are not considered significant, because the teacher knows it all. This view of role of the teacher disregards the social complexity that is embedded in the nature of teaching. I refer to the work Tlali (2013:28), who asserts that learners do not always learn in the same way as their teacher does, and I argue that the disjuncture between learners' and teachers' learning styles necessitates the creation of a team. When several teachers work as a team, and each contributes to the lesson being prepared and facilitated, and they contribute a variety of teaching methods, the teaching becomes more effective than if one teacher works alone.

2.5.1.2 The need for intervention in geometry

Geometry is a mathematics topic that is taught as part of the school curriculum (DBE, 2011: 9). However, learners struggle with mathematics in general and geometry in particular. Ali, Bhagawati and Sarmah (2014:73) studied the performance of geometry learners in India, and they found that learners demonstrated poor performance on geometry – learners found geometry to be the most difficult part of mathematics. Ali et al. explain that one of the reasons why learners perform poorly is that they lack fundamental knowledge of geometry.

Thus, in investigating challenges faced by teachers who teach Euclidean geometry, it is important that the current study investigates the level of learners' fundamental knowledge of Euclidean geometry. This knowledge could include basic Euclidean concepts and theorems, such as (i) properties of an Isosceles triangle; (ii) equal chords subtend equal angles; (iii) exterior angle is equal to the sum of opposed interior angles of a triangle; and (iii) congruency and similarity.

In South Africa Makgato and Mji (2006: 253) investigated factors that are associated with learners' performance in mathematics. Their study reveals that learners find Euclidean geometry difficult. Learners who participated in their study reported that their teachers spend less time on geometry, and this makes learners lack knowledge of Euclidean geometry (Makgato & Mji, 2006: 261). Thus, in formulating a strategy to improve teachers' TPACK, their level of understanding and competency of the content of Euclidean geometry is significant. However, not only does TPACK speak to teachers' content knowledge of Euclidean geometry, but also their pedagogical practices. In addition, the national diagnostic report (DBE, 2014: 122) reveals that learners achieved 59%, 38%, and 34% for questions on Euclidean geometry, which gives an average percentage of 44. Furthermore, the report revealed the following about learners:

- *Learners do not give reasons to support their mathematical claims;*
- *Learners give incomplete or unacceptable reasons to support their mathematical claims; and*
- *Learners do not know the properties of a cyclic quadrilateral, and they give reasons such as, "Opposite angles of a cyclic quadrilateral are equal"* (DBE, 2014: 129).

The national diagnostic report (DBE, 2015: 122) also reports that learners performed very poorly on four questions of Euclidean geometry, scoring 56%, 28%, 38% and 29%, which gives an average percentage of 37.75. Furthermore, the report revealed the following:

- Many learners were able to make correct statements but were unable to provide the correct reasons; and

- Learners assumed that BAEF was a cyclic quadrilateral.

From the DBE's reports for 2014 and 2015, it is clear that learners' performance in Euclidean geometry is getting worse. Furthermore, learners seem to be struggling with the same topics every year. For instance, the 2014 report states that learners give incorrect reasons for their mathematical claims, and in 2015 learners were unable to give the correct reasons for their mathematical claims. The inadequate performance by learners inspires a question: How do teachers prepare and facilitate their lessons on Euclidean geometry? This question leads us to the next section, which deals with lesson preparation.

2.5.1.3 Lesson preparation when teaching with the aid of ICT software

Many factors influence the use of ICT tools as teaching aids. These factors include limited time available for lesson preparation, pressure to prepare learners so that they pass examinations, and inadequate technical support (Fu, 2013:117). Fu (2013:115) reports that one of the challeanges that teachers who are using ICT software as a teaching aid are faced with is that they prepare insuficiently due to a lack of time or knowledge to master the software. Fu (2013:115) states that low software competence, which may result in inadequate lesson preparation, is one of the barriers to effective integration of ICT so that it enhances learners' understanding of mathematical concepts. Leendertz et al. (2013: 5), in a study to investigate the extent to which TPACK contribites to enhancing the effectiveness of teaching Grade 8 mathematics in South Africa, report that a large portion of mathematics teachers do not have the knowledge needed for teaching mathematics using ICT.

The teachers' knowledge that Leendertz et al. (2013: 5) refer to originates from Shulman' theory on the qualities of knowledge needed for teaching. Koehler and Mishra (2009: 62) adopted and adapted the theory of Shulman (1987), expanded it to establish qualities of knowledge required for teaching using technology, and categorised these qualities as the intersection of technology, pedagogy and content knowedge. One of the major contributors to the complexity of teacher knowledge needed for using technology is the difficulty of creating a connection between the three qualities of knowledge. Thus, during the preparation stage teachers are unable to choose approprate software or ICT tools to teach an certain mathematical concept. Furthermore, teachers have inadequate technology knowledge and skills because the majority of them went through teacher traning with few opportunities to use ICT, or had been trained at a time that technologoly had not yet eveloped to the current state (Koehler et al., 2013: 14). This inexperience contributes to ineffective integration of ICT in teaching Euclidean geomety, since teachers do not consider themselves sufficiently prepared, therefore they don't see the value of using ICT software as teaching aids (Koehler et al., 2013: 14).

Using TPACK to teach Euclidean geometry requires that teachers prioritise planning. Wetzel and Marshall (2012: 73), in a study investigating teacher behaviour that fits TPACK, found that teachers are stuggling at the lesson preparation stage to find sequential ways to interplay the components of TPACK. They report that teachers who use ICT for teaching are faced with the challenge of, first, determining the learning goal, then the content or the subject matter, and lastly, the ICT tool that seems to be appropriate – while they should be choosing the ICT tool first. This argument was also evident in the work of Mishra et al. (2009: 49), who explain that the manner in which technology has evolved over time tends to make teachers consider technology in isolation rather than how to use it to communicate the subject matter better through the use of technology. Due to time constaints for preparation, and the complexity of interwearing the components of TPACK, teachers are unable to help learners reach intended lesson objectives and enhance learners' peformance in mathematics (Wetzel & Marshall, 2012: 79).

2.5.1.4 Lesson facilitation with the aid of ICT software

Adequate lesson facilitation through the use of ICT software provides teachers with the opportunity to improve their lesson planning and lesson facilitation, become more project based and inquiry based, and to promote collaboration between the learners (Rabah, 2015: 26). During lesson facilitation some teachers struggle with technical challenges that emerge during the lesson, and this leads to learners' attention and interest in the lesson straying, and inadequte lesson facilitation (Rabah, 2015: 27).

The consequences of technical challenges is contrary to the epistimology of social contructivism, which asserts that knowledge is socially contructed (Thomas et al., 2014: 3). With this lens, no learning takes place in the absence of social interactions between learners, and the teacher should motivate this interaction through the use of ICT software. In support of this claim, a study conducted in five European countries by

Buabeng-Andoh (2012: 139) reports that teachers find their technical incompetence regarding ICT tools, such as software, hampers their lesson facilitation. Teachers lack the technical skills needed for adequate lesson facilitation with the use of ICT tools.

According to the FET Mathematics CAPS policy, mathematics is defined as:

> *a language that makes use of symbols and notations for describing numerical, geometric and graphical relationships. It is a human activity that involves observing, representing and investigating patterns and qualitative relationships in physical and social phenomena and between mathematical objects themselves. It helps to develop mental processes that enhance logical and critical thinking, accuracy and problem solving that will contribute in decision-making. Mathematical problem solving enables us to understand the world (physical, social and economic) around us, and, most of all, to teach us to think creatively* (2011:8).

From the above definition, it is clear that a failure to use ICT as a communication tool would hamper the effective teaching that is expected by the policy. Furthermore, in line with the espistemology and ontology of social contructivism, mathematics is a human activity that involves observing and representing graphical relationships. Thus, when they facilitate lessons with ICT software, teachers should use ICT as tool to enhance human activity. However, Buabeng-Andoh (2012: 139), Rabah (2015: 27) and Ali et al. (2013: 4065) report that the marority of teachers lack the pedagogical and diadatical knowledge and skills to use ICT to enhance learning.

2.5.1.5 The practice of assessment for learning

Assessment for learning takes place during lesson facilitation for the purpose of reconstructing the environment to enhance learning (Wong, 2013: 199). Assessment for learning is a continuous process that takes place throughout a lesson, monitoring if learners are following the lesson, diagnosing their learning difficulties and identifying what makes what they are learning difficult to learn. Through this process a teacher become a bricoleur, threading and looping back and forth, from the concepts of Euclidean geometry to making meaning and resetting up a learning environment to promote learning informed by continuous diagnosis (Berry, 2004: 1). However, assessment for learning also encapsulates assessment, which is done at four levels, prior to the presentation of a new lesson, during the presentation of a lesson, at the end of a lesson presentation and after the lesson (DoE, 2012: 3).

In the first instance, assessment is used to determine learners' prerequisite knowledge relating to the new lesson (Karolich & Ford, 2013: 35). Next, assessment it is used during the lesson presentation to determine the extent to which learners follow and/or understand the new content. This assessment ensures that learners' misconceptions and other knowledge gaps about the new content as established during the prior knowledge assessment are addressed and assimilated appropriately (Spence & McDonald, 2015: 297). However, formative assessment is intended by practitioners to fulfil the same task as assessment for learning (DBE, 2011: 22). For instance, formative assessment is defined by Formative Assessment for Students and Teachers (FAST) as a process used during teaching to provide feedback for the purpose of adjusting ongoing teaching and learning (Dunn & Mulvenon: 2009: 2). Formative assessment is similar to assessment for learning, and the definition, in my view, indicates that it intricately interwoven with teaching. Regarding assessment for learning I argue that assessment for learning cannot be divorced from lesson facilitation. Hence, in this study the two types of assessment are collectively understood to be part of PCK and SCK (see Sections 4.2.1.2 and 4.2.2.4).

The main problem with formative assessment and the way practitioners carry it out is that they use it as a compliance mechanism, and not to enhance their teaching (Dunn & Mulvenon, 2009: 3). At the end of the lesson a more comprehensive assessment is geared to ensure that learners comprehended the new content as reflected in the lesson objectives.

Finally, the post-assessment takes place long after the lesson, and forms part of summative assessment, which may include and integrate with other topics (Ni Chronin & Cosgrave, 2013: 221). These types of assessment should be considerate of learners' different cognitive and affective domains, which must be developed in accordance with critical cross-field outcomes (Broom, 2015: 29). Thus, in order to improve the teachers' TPACK for teaching Euclidean geometry, it is important that teachers' assessment for learning practices is improved by an understanding that it will improve PCK, and consequently TPACK. In order to see if a teacher is able to demonstrate knowledge and skills for assessment for learning, lesson facilitation observations and analysis are used to achieve the following:

Active monitoring of learners' learning for the purpose of preparing an environment that fosters learning;

- Facilitating learning to help learners set learning goals;
- Using various types of ICT software to respond to different learner challenges; and
- Assessing if it is the software that is making it difficult for learner(s) to understand Euclidean geometry.

2.5.1.6 Finding appropriate computer software to teach Euclidean geometry

Research offers opportunities for improving the teaching of mathematics using appropriate software effectively (Hanson, 2013: 625). Through research into appropriate software teachers can keep pace with developments in rapidly evolving software technology for teaching (SAQA, 2012: 12). New programs and computer software are designed to meet the demands of the day, among which promoting learner-centred approaches to teaching that enhance learners' understanding of mathematical concepts in Euclidean geometry. The development of new or updated software is understandably based on relevant research findings relating to learners of mathematics (Leendertz et al., 2013: 5). The updated programs render older ones redundant. In the same way, teachers who teach mathematics, particularly Euclidean geometry, will become redundant if they do not keep up with these new developments.

Furthermore, research provides a platform for teachers to be creative and innovative. Creativity and innovativeness in this regard includes integrating multiple software programs and drawing from each program's strengths to ensure that abstract mathematics concepts, such as Euclidean geometry, are accessible to learners. In this way the integration of multiple software programs takes learners' diverse and varying competency levels into account. Rosenshine (2012:38) sought to identify principles that could serve as a guideline for achieving good teaching, and suggests that, in order for good teaching to take place, teachers must do extensive research to find a variety of material that will enable learners to acquire the requisite skills. This suggests that, in order for learners to obtain the requisite skills in Euclidean geometry, teachers must conduct extensive research to find appropriate ways to make the content accessible. Kings and Sen (2013: 621) state that research by teachers can improve their day-to-day teaching. They found that the majority of teachers fail to do research to find out about alternative teaching methods that can be used to enhance learners' understanding (Kings & Sen, 2013: 621).

Burke and Kirton (2006: 1) state that the full inside perspective about the challenges of teaching can only be given by teachers as researchers. They report that, when teachers are involved in critical reflexive research to respond to the day-to-day challenges involved in teaching Euclidean geometry, teachers' pedagogical practices are enhanced. Thus, critical reflexive research by teachers about the use of ICT software seems to be a necessity for the development of TPACK. Therefore, the current study emphasises the importance of critical reflexive research by co-researchers during lesson preparation in pursuit of improving pedagogical practices (see Sections 4.2.2 and 4.3.4).

2.5.2 Review of literature for formulating strategy to address challenges identified

The literature review is done in pursuit of identifying best practices in response to the challenges identified, and with the aim of learning, adopting and adapting, and designing a strategy that could respond to the challenges identified.

2.5.2.1 Teamwork approach to improving TPACK for the teaching of Euclidean geometry using ICT software

Many teachers in countries with the best performances in mathematics and science have noted the value of a teamwork approach to teaching. A lesson study approach to enhancing the teaching of mathematics is one of ways a teamwork approach can improve the ways in which learners learn (Doig & Groves, 2011: 84). Japanese lesson study, which is called *jyugyo kenkyu,* involves small groups of teachers who generally meet frequently to prepare together and implement and reflect on their lessons (Jita *et al.*, 2008: 465). This group of teachers from the same school and/or local schools meet to create what they call a research lesson with the purpose of uncovering how learners make meaning as they grapple with the content of mathematics. Through this process, amongst others, research lessons provide a type of teacher professional development that is teacher-inquiry based (Doig & Groves, 2011: 84). The research lesson builds particular skills, knowledge and attitude. During the process of the research lesson one or more teachers prepare a lesson, while other teachers are invited to be observers. Observation is not limited to teachers only, as they also invite academic or "veteran teachers" to reflections (Doig & Groves, 2011: 79).

2.5.2.2 Formulation of a vision

A vision and mission guide daily activities of a team or an organisation and foster a shared purpose among members of the team. Darbi (2012: 95) explains that a vision and mission motivates, models behaviour, and promotes a high level of commitment, which cultivates perfomance. Kantabutra (2008: 127) asserts that a vision provides a cognitive imagination of the desired future state. A shared vision creates an orientation and meaning for the team members and it acts as a strong driving force for continuous and systematic development (Martin, McCormack, Fitzsimons & Spirig, 2014: 1). A vision should be attractive to the team members if they are to be committed to turning it into a reality (Martin et al., 2014:2; Wong & Liu, 2009: 2884).

2.5.2.3 SWOT analysis

A SWOT analysis is a strategic evaluation tool that the coordinating team uses to assess the strengths, weaknesses, opportunities and threats in pursuit of responding to the challenges they are facing in the teaching of mathematics (Ayub & Razzaq, 2013: 93). The SWOT analysis can be used as an information-gathering tool concerning the team's competencies. In this study SWOT analysis was used to map the information provided by the analysis with the information gathered from literature about the skills and resources needed to improve teachers' TPACK, thereby directing the strengths of the team to the opportinities indentified (Ayub & Razzaq, 2013: 93). Furthermore, SWOT analysis is used to identify threats to improving teachers' TPACK and finding stategies to overcome these threats. Section 4.3.3 presents this study's SWOT analysis, organised under the subtitles CK, PCK and TPACK.

2.5.2.4 Collaborative lesson preparation

Lesson preparation contributes significantly to a successful lesson that meets its objective(s). Collaborative lesson preparation involves a group of teachers meeting and working together on designing a sequence of activities on a particular theme in a way that learners can easily make sense of the content under the theme through activities planned (Jita et al., 2008: 475). Collaborative lesson planning is applied in Japanese lesson study, when one or more teachers of the same school come together for the purpose of planning activities for a particular theme (Doig & Groves, 2011). Jita et al. (2008: 475) adapted lesson study to the South African mathematics and science context, and argure that collaborative planning in lesson study becomes an instument for building esprit de crops among teachers. They define collaborative planning in a South African context as a process where,

> *The knowledge and skills are organised into cohesive lessons which are sequenced into units* (Jita et al., 2008: 475).

In lesson preparation the organisation of knowledge and skills is directed by the aim and objective(s) identified for the lesson. In this regard, Jita et al. (2008: 475) explain that a collaborative process is geared towards finding responses to what the intended objectives are; how the objectives would be achieved; what measures of success would be used; what type of resources would be needed to facilitate the lesson and what time would be needed.

Co-teaching has also been used as a collaborative approach to teaching. This approach to teaching is prevalent in the United States, in programmes such as No Child Left Behind, and for enhancing the teaching of people with disabilities (IDEA, 2004). According NDCCD (2011) co-teaching happens when the general educator and the special education service provider engage in lesson preparation that entails planning of activities together and teaching learners with and without disabilities in the same classroom. Howard and Potts (2009: 3) describe co-teaching as a professional marriage between what they call general teacher and special teacher. The use of the word “marriage” shows the strong agreement and sense of dependency between the two teachers in preparation and facilitation of a lesson. This dependence in planning is normally refered to as co-planning, which involves the two teachers coming together to share ideas to design a lesson plan. This platform enables both teachers to construct new knowledge through the complex epistemology they bring to the planning environment.

In South Africa, clusters have been used as a platform for teachers to share their experiences and knowledge. Jita and Mokhele (2014: 3) explain that clusters fulfil two purposes: clusters are viewed as spaces that, firstly, foster policy implementation and, secondly, teacher-led continuous professional development.

2.5.2.5 *Using ICT for interactive lesson facilitation*

According to social constructivism epistemology, the construction of knowledge happens when there is social interaction between an individual and the environment, or between the individual and others. This principle informs the description of interactive lesson facilitation, namely, a lesson that creates an opportunity for learners to interact with one another or the environment during the process of learning. Interactive lesson facilitation as an approach couched in social constructivism subscribes to demystifying the idea that a teacher knows everything and that learners are mere recipients of knowledge. This also means that, in interactive classroom lesson facilitation, a teacher depowers him/herself in pursuit of the creation of a classroom environment that promotes self-regulated learning and learner-centred learning. Using ICT software to teach Euclidean geometry can promote learners' interactive self-regulated learning.

Karami, Karami and Attaran (2013: 44) investigated the integration of problem-based learning and ICT application to improve learners' performance, and report that the use of ICT in teaching enhances learner-centredness in problem-based learning. They state that, not only does the use of ICT promote interactive lesson facilitation, it also enhances teachers' content knowledge for teaching while achieving the intended outcome. Fu (2013: 113), after a critical literature review of ICT and its implications for teaching and learning, conclude that ICT enhances collabrative learning. He argues that using ICT software not only enables learners to acquire knowledge, but helps them to create an environment that is charged with possibilities for learners to share their diverse learning experiences. This process of knowledge contruction leads to deeping learners' understanding of the concepts they are learning, because their knowledge will be located within the social construct.

According to Mathematics FET CAPS policy (DoE, 2011: 53) 45% of the mathematics curriculum consists of complex higher-order procedures and problem solving. Social constructivism argues that the effective learner is guided by the following principles.

- Learners have prior knowledge, so they come to the learning situation with ideas about many things. These ideas are called schemas and teachers have to take them into consideration and make teaching relevant to these conceptual structures.

- Learners have their own unique ideas about reality and generate their own meaning structures to cope with everyday living.
- Learners' ideas often contradict or clash with accepted scientific ideas or with school curricula and are culturally or socially conditioned.
- Learners actively construct knowledge – knowledge is not passively received from the outside. Here the theory is vastly different from behaviourism, which defines learning as externally modified behaviour. Learning, according to constructivism is something the learner does, not something that the learner is compelled to do.
- Knowledge is both personal and individual and, at the same time, has a social dimension. Learners construct their conceptual schemas by interacting with the social world, in social settings and within cultural and linguistic contexts.

ICT software is used effectively if it is in line with the preceding principles on learning as set out by social constructivist theory. Literature confirms that using ICT enhances the teaching of mathematics, by linking learners' existing knowledge with the new information. Pinheiro and Simões (2012: 383) sought to understand how ICT enhances the active and collabrative learning environment, and report that the use of ICT forsters change in traditional teaching and promotes active and collaborative teaching and learning that builds on what learners already know. This view is supported by Mbati and Minnaar (2015: 283), who claim that the use of ICT fosters the social contructivism approach to teaching that considers learners' prior experiences when developing a new concept. In addition, ICT enhances the interaction between teachers and learners (Tay & Lim, 2012: 743).

2.5.3 Critical conditions for fostering sustainability

This section is concerned with the conditions that are conducive to fostering the sustainability of the proposed solutions to or components of the strategy in Section 2.5.2.

2.5.3.1 Conditions conducive for co-researchers to achieve optimal functioning as a team

Collective leadership is a necessary condition for a fully functional team of co-researchers (Avolio et al., 2009: 423), by fostering the multiple roles and responsibilities to be performed by all co-researchers and responding to the complex epistemology as advocated by bricolage. In this context the complexity is exemplified by different leadership styles, and team members' different skills and knowledge for enhancing the teams' performance. These views concur with Emich's (2014: 123) examination of the way intragroup affective patterns influence groups' pervasive tendencies to ignore the unique expertise of their members. Emich (2014: 123) found that teamwork creates conditions where there is diversity of knowledge from the members' various competencies

Thus, all team members have the opportunity to experience a variety of roles and in this way a sense of belonging and appreciation is bestowed on them (Contractor et al., 2012: 995). According to Mickan and Rodger (2000:201) trust is a necessary condition for the success of a team. Thus, in the current study we will ensure that co-researchers have developed trust between them. Mickan and Rodger (2000:204) also claim that an effective team needs good communication between the members.

2.5.3.2 Conditions conducive to lesson preparation and facilitation

Lesson preparation requires, among other things, the careful collection of thoughts and resources concerning what needs to be taught. Due to the complexity of everyday life, what constitutes knowledge, and the diverse nature of reality, knowledge is a very complex phenomenon. This complexity influences the ways knowledge construction is understood, which is closely related to the ontological stances of individuals. I understand lesson preparation to be a very complex activity that requires teachers to set up an environment that promotes learning for individuals who usually come from diverse backgrounds and who do not learn in the same way. This, in my view, is why teaching can be described as a problem-based activity. Firstly, it presents a problem to the teacher, because a classroom consists of learners from various cultures, with different histories, family morals, socioeconomic status, genders and ages, which influence both learners' epistemological and ontological stances. Secondly, the view the study subscribes to is that learning could easily take place if an environment enables learners to create connections between what they already know and the content they are currently trying to learn. Failure to set up an environment that promotes learning among all learners in an inclusive way causes problems for learners. This failure complicates the activity of lesson preparation and facilitation.

However, bricolage's quest to uncover the multi-layered, multi-epistemological, multi-ontological, multiple social theory and multi-cultural perspectives of challenges and possible solutions to lesson preparation and teaching are understood better if one can first appreciate their complexity.

2.5.3.3 Conditions conducive to continuous professional development

Continuous professional development of teachers has become significant in many countries around the world (Jita & Mokhele, 2014: 1). Due to the complexity of teaching, which is dynamic and fluid, continuous professional development is important if teachers are to keep up with change. Thus, scholars such as Day (1999: 4) find it necessary to emphasise that continuous professional development is a "continuous" activity. Day (1999: 4) defines professional development as:

> *all natural learning experiences and those conscious and planned activities which are intended to be of direct or indirect benefit to the individual, group or school and which contribute to the quality of education in the classroom.*

However, due to the complexity of setting up environments that promote learning by learners in the 21st century, and demands that teachers' roles in the classroom must change, continuous professional development has become very important for teachers' effectiveness in their new roles (Wan & Lam, 2010: 2). Moreover, continuous professional development is also imperative for improving teachers' TPACK. This claim is supported by Leendertz et al. (*2*013: 2), who investigated the extent to which TPACK contributes to more effective teaching of mathematics in South African schools. Leendertz et al. (2013: 5) argue that, without continuous professional development for teachers, it is less likely that ICT for teaching mathematics will be effectively and optimally applied. Daly et al. (2010: 6) claim that continuous professional development is a condition conducive to improving teachers' TPACK. They emphasise that continuous professional development for TPACK must promote independent thinking, creative presentation of ideas and collaborative problem solving among teachers. Kennedy (2011: 26) identifies collaborative continuous professional development as a necessary condition for satisfying the three dimensions of professional development. Kennedy (2011: 26) argues that collaborative continuous professional development fosters development at three levels, namely, personal, social and occupational. Collaborative continuous professional development does not view learning as an individual, isolated activity, but as a social activity that promotes collaboration (Kennedy, 2011: 26). Therefore, in order to design a strategy to improve teachers' TPACK for teaching Euclidean geometry, it is important that good relationships be regarded as fundamental to creating conditions that promote learning.

2.5.4 Threats and risks facing the use of ICT to teach Euclidean geometry

This section addresses the threats and risks of using integrated ICT software to teach Euclidean geometry. The literature relates to and integrates these risks into solutions and conditions for the successful teaching Euclidean geometry using ICT software.

2.5.4.1 Teachers' negative attitudes toward lesson preparation with ICT software

Negative attitudes towards the use of ICT for teaching is one of the main threats facing implementation of a strategy to teach Euclidean geometry using ICT software. For instance, Hue and Jalil (2013: 54) studied the impact of teachers' attitudes towards effective implementation of ICT in teaching and learning, and found that it is one of the main causes of poor or ineffective application of such strategies. The study further established that there is a positive relationship between attitude and the use of ICT in teaching. Hue and Jalil (2013: 55) recommend that the development of positive attitudes among teachers is a key factor in ensuring successful implementation of computer software in teaching.

Mulhim (2014: 489) investigated why, despite the benefits of using ICT in teaching, ICT is still not effectively integrated in teaching and learning. Mulhim (2014: 489) confirms that one of the threats facing this initiative is teachers' negative attitudes towards the use of ICT tools, which is the result of inadequate training. She recommends finding effective ways to train teachers to use ICT as a teaching aid (Hadjerrouit, 2008: 250). Similarly, Avidov-Ungar and Eshet-Alkakay (2011: 292) studied teachers' attitudes towards implementation of innovative technology in schools, and found that there is a positive relationship between improvement on TPACK and teachers' attitudes. Kusano, Jones and Kobayashi (2013) investigated the effects of the ICT environment on teachers' attitudes, and claim that teachers' attitudes will improve if they undergo training to improve their confidence regarding pedagogical technological knowledge and skills. They also refer to the importance of collaboration among teachers to enhance their knowledge and skills. Kazu (2011) states that teachers' negative attitudes are mainly caused by insufficient training in the use of technology in schools.

2.5.4.2 Teachers' workload

Buabeng-Andoh (2012: 142) studied factors that influence teachers' adaptation and integration of ICT into teaching, and found teachers' workload to be one of the threats to both integration and adaptation of ICT in lesson planning and preparation. Similarly, Raman and Yamat (2014: 15) reveal that, despite the resourcefulness of some schools regarding ICT tools, such as computer software, teachers' workload threatens the integration of ICT software in lesson preparation and facilitation.

Selwood and Pilkington (2005: 165) investigated the effect of ICT on releasing time for lesson planning and facilitation. Their results contradict that of Buabeng-Andoh (2012: 142) and Raman and Yamat (2014: 15), and they conclude that one way to reduce teachers' workload is to use ICT tools, such as computer software, for planning and facilitating lessons. Given the two contrasting views, I agree with Selwood and Pilkington (2005:165); however, it is through improved knowledge of the TPACK necessary to teach a particular concept that the use of ICT will not only reduce the workload, but will also enhance teaching.

2.5.4.3 Access to versus quality of ICT

In South Africa, according to the White Paper and guidelines for teacher training and professional development (DBE, 2004: 1), ICT was introduced in basic education as one of the teaching resources, and introduction of ICT in some of the schools has become physically evident. These ICT resources include computers, computer software such as HeyMath!, interactive whiteboards, mobile labs for mathematics, and tablets. According to Ndlovu and Lawrence (2012: 3) the introduction of ICT in South African schools is driven by the need for quality education for all. However, they state that teachers misuse the ICT, since teachers are under the impression that ICT is intended to provide them with information, instead of enhancing their teaching to promote learners' critical thinking.

ICT should create an opportunity for learners to interact with the information and manipulate it, such that they are able to critically analyse and question it so that it becomes relevant to their context. This approach is in line with social constructivism's epistemological stance, that knowledge is constructed if it is part of the social context that people interact with socially (Marcus & Fischer, 1999: 7). However, access to ICT tools does not always lead to quality teaching and learning, which poses a threat for the intended use of ICT. Hess and English (2015: 194) advocate a greenfield approach to innovation, and warn that policy makers do not run schools: the policy can only tell people what to do, but cannot not make them do it well. They suggest that policy should create a space for schools to rethink the way they want to implement and improve innovation in teaching and learning (Hess & English, 2015: 194)

2.5.5 Indicators of success

The section is concerned with a review of literature on the indicators of success regarding the improvement of teachers' TPACK for teaching.

2.5.5.1 Social constructivism approach to teaching practice

The thesis that this study puts forward for teaching is that good teaching practices are grounded on social constructivism's epistemology and ontology, namely, that knowledge is a human social product. Thus, with such an understanding, teaching with ICT software is perceived as successful when teachers are able to use ICT software during their preparation and facilitation in a manner that enables learners to construct their knowledge through interaction with software, the environment, and other learners. Furthermore, through the lens of social constructivism, ICT software is understood to be used successfully if teachers' knowledge of content, pedagogy, and technology is improved within multiple social realities. This implies that teachers at different levels of content, pedagogy, and technology knowledge could create a space of operation where they could share their individual realities of content, pedagogy and technology.

2.5.5.2 Learner-centred approach to pedagogical practices

Learner-centredness is a transformative approach to education that changes the role of a teacher from that of an instructor to a facilitator. Majumdar (2006), in advocating a learner-centred approach, argues that one of the indicators of successful integration of ICT in teaching and learning is when it promotes learner-centredness. His study reveals that the key to successful implementation of ICT is the development of teachers’ pedagogy-technology. The ability to integrate ICT software and content, and the delivery of that content in such a way that learners can learn, is imperative. McCombs (2001: 186) defines a learner-centred approach as:

> *the perspective that couples a focus on individual learners – their heredity, experiences, perspectives, backgrounds, talents, interests, capacities, and needs – with a focus on learning – the best available knowledge about learning and how it occurs and about teaching practices that are most effective in promoting the highest levels of motivation, learning, and achievement for all learners.*

The above definition of learner-centred approach is in line with the epistemology of social constructivism. For instance, lesson preparation and facilitation should be directed by learners’ interaction with society and the environment, such as their experiences, interests, backgrounds, and their ontology, which are their own perspectives. Therefore, the strategy to use ICT software to teach Euclidean geometry is successful when its lesson preparation and facilitation are in line with the definition.

2.5.5.3 ICT improves learners’ results

According to Youssef and Dahmani (2008: 45) the performance of learners who are taught using ICT improves. The use of ICT promotes learner-centredness and fosters meaningful learning (Van der Westhuizen et al., 2012: 199). Pena (2011: 66) reports that ICT provides an opportunity for learners to collaborate in flexible and motivational ways during the process of learning. Furthermore, in the study of Pena (2011: 66) on how ICT improves teaching and learning, he states that the use of computer software can improve learners’ communication skills.

CHAPTER 3: PARTICIPATORY ACTION RESEARCH TO IMPROVE TECHNOLOGICAL PEDAGOGICAL CONTENT KNOWLEDGE FOR TEACHING EUCLIDEAN GEOMETRY

3.1 INTRODUCTION

This study seeks to design strategies to improve technological pedagogical content knowledge (TPACK) for teaching geometry with the aid of integrated ICT software. As a researcher I realised that I do not have sufficient knowledge to design a strategy to improve TPACK for teaching Euclidean geometry and that those who face the challenges of teaching and learning Euclidean geometry are in a better position to design responsive strategies for dealing with their day-to-day challenges. Thus, the study employs participatory action research (PAR) as an approach to designing a strategy to improve TPACK for teaching Euclidean geometry. I find PAR appropriate and relevant, because I understand that, to achieve the five objectives of the study, the co-researchers' lived experiences are key factors that must be considered. This chapter justifies the use of PAR, and is organised in the following manner: PAR as a research methodology, and as a research design.

3.2 PARTICIPATORY ACTION RESEARCH AS A RESEARCH METHODOLOGY

The section discusses PAR as a methodology appropriate for this study. The justification of its appropriateness is done by a discussion of PAR's origin, formats, objectives, ontology, and epistemology.

3.2.1 The origin of participatory action research

PAR is regarded as a subset of action research (MacDonald, 2012; 35). The origins of PAR as a research approach can be traced to 1944 and the work of Kurt Lewin, who is regarded as the father of action research (Gillis & Jackson, 2002: 264). Lewin subscribed to the philosophy that people are more motivated to work if they are involved in decision making processes (Gillis & Jackson, 2002; 264; McNiff & Whitehead, 2006: 36). PAR, a term coined by Lewin, aimed to address problems of assimilation, segregation and discrimination (MacDonald, 2012: 35). According to Lewin PAR is an iterative process that loops from action and reflection between participant and researcher – a process that is known as an iterative cycle or spiral science (Kemmis & McTaggart, 1990: 8; Kindon, Pain & Kesby, 2007: 10). Therefore, in this study, mathematics teachers who are committed to improving their teaching practices are at the forefront of decision-making processes; they dictate the process of research through their involvement in planning, acting, reflecting, observing and evaluating the results of their action.

Later, in the 1970s, PAR could be observed in the work of Paulo Freire, on the pedagogy of the oppressed. In his version of PAR, Freire (1970) advocated for the empowerment of the poor and those who are pushed to the margins (Macedo, 2000: 13). Freire's PAR also argued against positivistic paradigms that promote university-based research that made no difference to people's lives, but, instead, treated them as objects. Mathematics teachers understand that two are better than one, and thus, in their spiral approach to the proposed research actions and activities they understood the impact of Freire's emancipatory agenda. They sought to find a strategy that would respond to their daily challenges of teaching geometry concepts. Furthermore, mathematics teachers in this study understood that they are the best people to develop a strategy to improve their TPACK for geometry.

PAR is also suitable for movements that share a vision of society that seeks freedom from domination (Maguire, 1987: 8). These movements are found fields such as international development, social sciences communities and adult education (MacDonald, 2012: 37). According to MacDonald (2012: 37) PAR is linked to the following developments:

(i) radical and reformist approaches to international economic development assistance;
(ii) the view of adult education as an empowering alternative to traditional approaches to education; and
(iii) the ongoing debate within the social sciences over the dominant social science paradigm.

Feminist researchers apply participatory research by analysing power differences based on gender, and promote the significance of collaboration between the researcher and participants (Reid, Tom & Frisby, 2006: 316). Thus, this study involved collaboration between co-researchers, who included teachers from three schools, me, representing the local university, a photographer and a programmer. Since the 1990s we have considered PAR to be an approach to research that promotes collaborative effort to achieve community development and equity during the process of research (Kindon et al., 2007: 10).

Reid *et al*. (2006: 316), in their feminist work that aimed to find the action in feminist participatory action research (FPAR), defined FPAR,

> *as a conceptual and methodological framework that enables a critical understanding of women's multiple perspectives and works towards inclusion, participation, and action, while confronting the underlying assumptions researchers bring into the research process.*

In the current study the same meaning applied; for instance, through the use of PAR multiple perspectives of co-researchers were captured from planning, implementation, reflection and re-planning. It involved a team of co-researchers doing lesson planning and facilitation together, and reflecting on what had worked or not worked during the lesson facilitation. Furthermore, informed by the team's reflection, a lesson was re-planned and re-facilitated. Reid *et al.* (2006: 316) explain that the goals of PAR include identifying changes of immediate benefit to research "participants", investigating their social problems and finding ways to resolve these problems. The possibility of achieving the goals that were identified is vested in a unique approach to research that is democratic in nature. In this study, the democratic nature of PAR was obvious when teachers, as co-researchers, identified their problems and collaboratively enacted solutions to the problems they had identified (see Section 3.3.2).

Mhina (2009: 159) used PAR to find solutions for the problems experienced by women of Maruku village in Tanzania regarding their inability to access and control agricultural land. According to Mhina (2009: 159) the power of PAR lies in its quest to work with the participants who are most affected by the problem that has been identified -- this concurs with the criteria for using PAR for the current study. For instance, the majority of co-researchers were teachers who are deeply affected by the problem of learners' poor performance in mathematics, especially Euclidean geometry. Due to learners' poor performance the Department of Basic Education introduced Act 31 of 2007, which requires schools to submit subject improvement plans to the heads of Education in the respective provinces (DBE, 2008: 1). According to the DBE, failing schools are supposed to develop and submit subject improvement plans that inform academic performance improvement plans (APIP). These plans are used as mechanisms to ensure that schools meet their respective provincial targets. However, this requirement exerts a great deal of pressure on teachers, some of whom took part in this study, because compiling a subject improvement plan entails diagnosing learners' errors and misconceptions, and then developing effective and efficient strategies as remedies, in order to meet the school's target. PAR enables teachers to describe what they perceive to be the problem from different perspectives. For instance, during our meeting to identify common problems, one of the teachers, Mr Mokoena, said:

> *This thing [referring to subject improvement plan] ntate, e yetsa motho a seke a rata mosebetsi wa hae ka nako ena (This thing, Sir, makes one dislike his job around this time).*

3.2.2 Formats

This section discusses the formats of PAR, which include community-based participatory research and mutual inquiry.

3.2.2.1 Community-based participatory research

Community-based participatory research (CBPR) is one of the formats of PAR. CBPR is a collaborative approach to the research process that affords what the traditional moment refers to as the "researched community" an opportunity to decide what needs should be researched (Minkler & Wallerstein, 2008: 1-2). The current study considered the collaborative characteristic of CBPR to provide an appropriate approach to operationalising the study objectives. For instance, collaboration enables a team of co-researchers to respond to the complex epistemology by creating a web made up of the social reality each member brings to the team. The co-researchers in the team's exposure to different ways of knowing helped them achieve the five objectives of the study. CBPR rejects the traditional idea of a powerful researcher who drives the process of research by deciding what needs to be researched, what the right questions to ask are, and when and how these questions should be asked. In contrast, in CBPR the issue that needs to be researched is decided upon by the community (Minkler & Wallerstein, 2008: 2). Similarly, in the current study the co-researchers, who were involved in day-to-day teaching of mathematics, decided what needed to be investigated. For instance, during my meeting with one of the co-researchers while he was preparing to convene a team of mathematics teachers to develop an APIP, he said:

Ntate re leka ho etsa lemolwana la batho ho etsa ntho ena e bareng ke APIP, na o ka se kgone hoba le rona (Sir, we are trying to form a team of people to do what they call APIP; can you join us)?

In accordance with the principles of CBPR, namely, that the problem to be investigated must originate in the community, I was able to join the team. In the current study, a community of mathematics teachers are faced with the problem of learners who are falling mathematics, and a legal obligation to develop an APIP. Minkler and Wallerstein (2008: 2), drawing from the work of Cornwall and Jewkes (1995: 1667) argue that CBPR and PAR epitomise a kind of action research that is more than a method; instead, it is an orientation to research that involves mutual respect, co-learning, capacity building, and a balance between research and action. Thus, in the current study, the values of CBPR and PAR enabled a team of co-researchers to function optimally. For instance, the team exercised mutual respect, which made it possible to tap into the different ways of knowledge (epistemology) that each co-researcher brought to the team, and strengthen the team's PCK (see Section 4.4.1).

The co-learning and capacity building were observed in the team members' teaching practices, when co-researchers worked together to prepare and facilitate lessons (see Sections 4.3.4 and 4.3.5). Furthermore, the balancing of research and action was achieved when research functioned as an active process that was used to respond to the day-to-day challenges of teaching mathematics. This meant that the research did not merely represent the skewed interest of the researcher, but served as an intersection of and response to the needs of the community and the researcher. This concurs with what Mahlomaholo and Netshandama (2012: 36) call a post-apartheid organic intellectual drawing from a Gramscian perspective. In their study, Mahlomaholo and Netshandama (2012: 36) redefine the roles fulfilled by academia in South Africa's post-apartheid era. They locate the roles and responsibilities of academia in the seventh and eighth historical moments of qualitative research (see Sections 2.2.1.7 and 2.2.1.8). In line with the objectives of CBPR and PAR Mahlomaholo and Netshandama (2012: 36) argue for and operationalise their roles and responsibilities in academia by supporting the community cultural wealth of underclass communities. They argue that, in so doing, subaltern experiences, in an attempt to respond to the people's own social challenges, become legitimate ways of contributing to new knowledge. Thus, in the current study, I view myself as a kind of "organic intellectual" who buttresses the community cultural wealth of underperforming schools, particularly in mathematics.

3.2.2.2 Mutual inquiry

Mutual inquiry is grounded in the theoretical underpinnings that see the product of research as co-production, where co-researchers produce knowledge on a topic of their joint interest (Ospina, Dodge, Godsoe, Minieri, Reza & Schall, 2004: 50). Baker, Dieter and Dobbins (2014: 13) state that mutual inquiry can be traced back to 1974 and the work of Wayne C. Booth, titled *Modern dogma and the rhetoric of assent,* and *The rhetoric of rhetoric,* in 2004. Mutual inquiry subscribes to a complex ontology and epistemology. The fact that each person has his or her own backstory, unique set of values and experiences, means each person contributes to the nature of social complexity (Baker et al., 2014: 21). Therefore, in the current study, mutual inquiry helps me to understand that working as team contributes to both the epistemological and ontological complexity. Each co-researcher brought his or her backstory, set of beliefs, values and experiences, which contributed to both ontological and epistemological complexity.

This complexity also concurs with the objective of bricolage, which is to make meaning of complex, everyday life experiences (see Section 2.2.4). The web of social reality formed by the co-researchers enabled the team to achieve its objective. A classroom consists of learners facing different challenges, possessing different learning styles and owning different backstories, which have direct and indirect impacts on their performance in mathematics, with special reference to Euclidean geometry. Thus, a web of social reality created by co-researchers enable to us to gain a better understanding of why are we facing challenges in teaching Euclidean geometry with the aid of integrated ICT software. The multiple epistemologies represented by the team makes it possible to work together towards understanding own challenges, by drawing on different ways of knowing (Roger, 2012: 2). Baker et al. (2014: 21) argue that mutual inquiry should involve a pedagogy of listening and articulation, which make the process of negotiating and deliberating through research feasible. Thus, in pursuit of the objectives of the study, the co-researchers listened to each other during the process of research, exercising the values and respect advocated by bricolage (see Section 4.3)

3.2.2.3 Feminist participatory research

According to MacDonald (2012: 38), who on draws Maguire's (1987) work, PAR can be defined from a feminist perspective as combining the activities of social investigation, education and action in a collective process. The social investigation activity of PAR includes its research orientation, which seeks to investigate social problems, which, from a feminist perspective, involves subjugated and ordinary people participating in research endeavours in response to their social challenges. Thus, from a feminist perspective, PAR is used as a critical emancipatory approach to research that seek to empower the subjugated and marginalised.

PAR can also be an educational process for participants and the researcher, who are engaged in a process of analysis of structural causes of acknowledged social problems through collective discussions and interactions (Mahlomaholo, 2009: 224). MacDonald (2012: 38) explains that the action activity of PAR is geared to creating ways for researchers and subjugated people to join forces, and, in solidarity, to take collective action, in both the short and the long term, in pursuit of radical social change.

Further, from a feminist perspective, PAR involves three types of change, namely, the development of critical awareness in both researcher and participants, advancement of the lives of those participating in the research process, and transformation of societal structures and relationships (Van Dijk, 2008: 86)

3.2.3 Principles of participatory action research

Despite various definitions emerging as PAR developed, common principles and characteristics that epitomise PAR can be identified. PAR is democratic in nature, which means there is space for all participants – all are equally important to and worthy of driving the process of research (MacDonald, 2012: 39). In the current study the participants were not considered to be mere objects, as participants would be in the traditional period, and responds like in the modernist phase. In this study participants were considered to be at the same level as the researcher This approach was motivated by the fact that, as the research coordinator, I do not consider myself to possess all the necessary and sufficient knowledge or skills to design the strategy in question. So, I took on the role of researcher-participant, while the "participants" were participant-researchers and, with this understanding, we all became co-researchers.

The teachers in this study were liberated by the process of research, because they were free of oppressive, debilitating conditions that prescribed what they had to teach, how they had to teach it, and what resources they had to use. They were free to dictate the process of research, with the purpose of responding to their everyday challenges of teaching Euclidean geometry. This approach concurs with the transformative emancipatory agenda of the study, which seeks to transform knowledge production through research processes. The use of PAR enables the study to locate the voices of the marginalised teachers at the centre of knowledge construction in pursuit of finding solutions to their own problems (Mahlomaholo, 2012: 2). This approach concurs with Dupuis, McAiney, Fortune, Ploeg and De Witt (2014: 95), who state that PAR involves a collaborative method wherein partnerships are built between those who have first-hand knowledge concerning the object of the study. In the current study teachers have first-hand knowledge of the object of the study, which is the teaching of Euclidean geometry. Therefore, I understood that teachers' daily experiences were relevant to the design of a strategy that would be responsive to the daily challenges of teaching Euclidean geometry.

McTaggart (1989) outlines 16 tenets of PAR, among which an active approach to improving social practice through change; congruence on authentic participation; collaboration; establishing self-critical communities; and involving people in theorising about their practices (MacDonald, 2012: 39). Therefore, in this study, the research undertaken was collaborative in nature, since it involved co-researchers working together. In addition, the study sought to improve social practices by improving teaching and learning of mathematics. In constructing the current bricolage the co-researchers became actively involved in directing the process of research through their daily teaching practices, which made their participation in the process of research authentic. PAR requires that people put practices, ideas, and assumptions about institutions to the test, it involves record-keeping, requires participants to objectify their own experiences, involves critical analysis, and is a political process (Kemmis, McTaggart & Nixon, 2013: 13; MacDonald, 2012: 39).

McTaggart (1997: 25) explains that PAR starts with small cycles and groups, and encourages participants to build records while allowing and requiring participants to provide a reasoned justification for their social (educational) work. Selenger (1997: 6) identifies seven components of the PAR process. The first component is an acknowledgement that the problem originates in the community itself, and that it is defined, analysed, and solved by the community. Thus, in the current study, the problem originated from the co-researchers, who work as teachers in various schools. Their problem was that learners were failing mathematics, especially Euclidean geometry, therefore the co-researchers were requested to develop a subject improvement plan for mathematics (see Section 4.2.2).

The aim of the subject improvement plan was to identify areas in which learners performed worst, and to design strategies to help learners to understand concepts or areas identified as problematic. Secondly, the overriding goal of PAR is the radical transformation of social reality and enhancement of the lives of the co-researchers; in this case, the teachers are the primary recipients of the research (MacDonald, 2012: 41). Thirdly, PAR involves the full and active participation of the co-researchers at all levels of the entire research process, as opposed to other research processes that are located in the first, second and third moments of qualitative research. In the current study the co-researchers' full and active participation starts with diagnosing the learners' misconceptions and errors, and the possible causes of these errors and misconceptions. The level of participation between the co-researchers and their actions during the process of research is blurred, in spite of the fact that some of the co-researchers were in managerial positions at their schools – in the research they all performed the same duties within the team (Kincheloe, 2009: 109).

3.2.4 Ontology and epistemology

In PAR reality is very complex and multidimensional, and avoids commitment to objective, fixed views of the powerful researcher at the expense of marginalised groups (Kincheloe, 2009: 110). Thus, the use of PAR in this study concurs with bricolage's epistemological and ontological stance, which emanates from respect for complexity. PAR is committed to its critical vision of socio-political education research that is aware of the researcher's assumption that directs the research design. Dedicated to this critical vision, PAR's epistemological stance not only tolerates, but is passionate about creating a space for the voices of the marginalised; attempting to legitimise their experiences and their ways of knowledge construction through the research endeavour (Kincheloe, 2009: 107).

Thus, for the current study, PAR's critical vision was found to be complementary and appropriate, since the study intends to swing the pendulum to where it belongs. This means that this study advocates that any research that seeks to respond to the challenges facing mathematics teaching should be directed and dictated by those who are involved in the day-to-day teaching of mathematics. Subscribing to the critical vision of PAR provides the co-researchers with the lens of complex ontology and epistemology. For instance, the co-researchers in this study are different people, with different backgrounds, they are of different age groups and they possess different learning styles. Therefore, using one reductionist, fixed reality will not do social justice to the interest of the co-researchers in the study. Consequently, the study is not limited to the one moment that qualitative research considers to be a valid and reliable method of knowledge production; instead, it employs all eight moments to create a web of social reality that is inclusive.

The choice of PAR is inspired by its relevance to the eighth moment of qualitative research. The ways and purpose of doing social science research has moved through the moments of qualitative research, to a kind of research that seeks to build communities. Thus, in this study, as a bricoleur using PAR methodology, I subscribe to the ontological notion that reality is constructed through shared human social activity (Kincheloe, 2009: 112). This ontological stance challenges the universal optimal view of knowledge construction; it argues that knowledge is a creation of the mind, not a reflection of the mind that is based on a fixed, observed reality (Kincheloe, 2009: 109). I believe that knowledge is a creation of the mind and that it is inseparable from its social context (Kincheloe, 2009: 109). This means that, epistemologically, each co-researcher constructs knowledge in his/her mind different from others, even though the knowledge each person has constructed cannot be divorced from its social context. Thus, due to overlapping social contexts, local members of a particular community create a view of the world and its properties on the basis of their shared understanding. Thus, PAR gives me a theoretical stance that states that reality is not an external entity to be discovered, but it is created by the people involved. The choice of PAR further harmonises with social constructivism ontology and epistemology, namely, that shared reality is a collage and representation of multiple realities of individuals that developed from the complexity and diversity that exists within a single society (Mahlomaholo, 2013: 379). This is confirmed by Pritchard and Woollard (2010:

7), who explain that, in practice, individual realities within the same local societies will be similar, but there are instances where, as a result of different fundamental experiences and interactions, they differ greatly. This phenomenon is in line with bricolage's complex ontology, which subscribes to multiple realities in construction of a collective reality, even if the multiple realities repel each another.

The epistemological stance of PAR enables the co-researchers to make unique contributions to the study by reporting their different ways of understanding the common challenges of teaching Euclidean geometry. Different co-researchers teach different learners with different levels of understanding of Euclidean geometry, and different learning styles. The multiplicity of realities found in their classrooms enables the study to investigate the challenges of teaching and learning Euclidean geometry more deeply.

3.3 ETHICAL CONSIDERATIONS AND DATA GENERATION PROCEDURES

During the formal and informal meetings all participants who later become the co-researchers, they were informed about the ethical considerations involved in the study. The study was planned and executed in a manner which would not cause harm to or threaten the lives of the co-researchers. The Free State provincial DBE granted permission for the study to be conducted in the schools that participated. A copy of the letter from the DBE giving permission for the study to be conducted, was presented to and discussed with all the co-researchers. All co-researchers were asked to sign consent forms and they agreed to be part of the study. Participants were made aware that they could withdraw from the research project any time they wished to.

The data was generated through the discussions and data was captured by audio and video recordings. In addition, learners' scripts were used to generate data on learners' errors and misconceptions and to find the causes of the errors and misconceptions. Learners' test scores were used as a source of data.

3.4 RESEARCH DESIGN

This section, which explains the research design, outlines how the study was guided by the principles of PAR, and executed. Steps in pursuit of the objectives of the study included establishing a common goal, formulating research questions and creating a portfolio for each co-researcher in order to know each participant's strengths, weaknesses, opportunities and threats. The section starts with a discussion of our initial meetings, follows the sequence, the way the research question was formulated, and how the research team was convened.

3.4.1 Initial meeting

The study was prompted by a need to respond to the teaching and learning challenges of mathematics, which were evident in learners' poor performance in 2014's midyear examination. In reaction to learners' poor performance in mathematics the DBE demanded that teachers document their subject academic improvement plans. This became a regular task for mathematics teachers at all schools in the province, including the schools that participated in this study. One of the co-researchers, who was also in management at one of the schools that participated in the study, approached the local university to partner with the school to develop subject academic improvement plans.

I was invited to participate by the lead teacher, who was the coordinator of the process of collaborative work involved in developing a mathematics subject academic improvement plan. We convened the first informal meeting with two mathematics teachers, and we spent the time gaining an understanding of the nature of the problem. This meeting was followed by a series of meetings at which we shared our concerns with teachers who are involved in day-to-day teaching of mathematics. The mathematics teachers hoped to design a strategy that would be relevant to and realistic in relation to their daily experiences in the teaching of mathematics, with the end goal of improving learners' performance. The interactive sessions with teachers from different schools, during which we discussed ways of designing, developing and implementing subject improvement plans, strengthened relationships between the participants.

These initial meetings created a space for a degree of trust and unity to develop between participants (Chilisa, 2012:250; Dodson & Schmalzbauer 2005:953). Chilisa (2010: 108) supports the preceding argument, namely, that initial PAR meetings promote an empowering team climate that gives all team members the freedom to express themselves and exchange ideas and opinions. This benefit is aligned with the lens of bricolage, which seeks to create a team climate that embraces multiple perspectives, where each member of the research team could voice his/her perspectives freely (Roger, 2012: 1). Furthermore, as a bricoleur, I acknowledge research to be an active process that requires all participants to be freely involved and of equal importance (Kincheloe, 2004: 1).

3.4.2 Formulating a research question

After our informal meetings, we convened a formal meeting with five mathematics teachers, one principal, and me. The purpose of the meeting was to establish a common goal for all the research partners. In this meeting we identified a need for teaching methods that would make the abstract mathematics concepts that learners were struggling with, accessible to them. A collaborative approach was suggested as a good way to address our problem, as illustrated from the extract below, by Mr Phehello, a teacher:

> *nna ne kere re sebetseng mmoho, surely ha re le tjena there is something that each of us can do better than the rest of us and bringing all that together we might get something (I was saying that we should work together, surely among us there is something that each of us can do better than the rest of us, and bringing all that together, we might get something).*

We also reflected on our own teaching practices, and the practices of teachers in other countries that seemed to have the best practices for teaching Euclidean geometry. Thereby the integration of technology into the teaching of Euclidean geometry was suggested as one of the strategies that could improve the teaching of Euclidean geometry.

> *ke ne kena le maikutlo a reng re ke re sebedise ntho tse bana bana ba ka di ratang, bana bana ha ba tshwane le rona ke batho ba ditechnology ntate (I was suggesting that we should use things that these learners like, these learners are not like us, the BBTs, they are into technology).*

This led participants to design a subject improvement plan that employs collaborative effort and multiple ICT software teaching methods. However, the co-researchers possessed different levels of competency regarding the use of computer software for teaching. Some had not used computers to teach before, others had a little knowledge of using computer software applications to teach Euclidean geometry. This variation resulted in the co-researchers formulating our research question as follows: How can we improve TPACK for teaching Euclidean geometry?

3.4.3 Convening a research team

After we had identified a common goal and formulated our research question, we performed a team SWOT analysis. This enabled the research team to identify its strengths, weaknesses, opportunities and threats in response to the research question, and to identify the multiple internal and external factors that are favourable and unfavourable for designing a strategy to improve TPACK for teaching Euclidean geometry (Arcidiacono & Procentese, 2010: 4). A SWOT analysis employed team profiling as a strategic planning tool to perform a training-needs analysis and a skills analysis for the research team. The portfolios of the co-researchers who participated in the study are presented below.

There were two **senior phase mathematics teachers**, Messrs Boke and Sello, who had been teaching mathematics Grade 8 and 9 for three years. (Note that the names used in this study are pseudonyms.) These two teachers were affected by learners' unsatisfactory performance in mathematics, particularly geometry. According to Morrell (2005: 84) the two mathematics teachers in PAR, research on their daily challenges with the aim of addressing them. Both teachers were computer literate and had been using HeyMaths! software to facilitate some of their lessons. Team members agreed that, in order to formulate a strategy to improve TPACK for teaching geometry, Grade 8 and 9 teachers, as part of school community, should be part of the team, so that the challenges involved in teaching geometry could be identified as early as Grade 8 and 9. This confirms the claim by Udas (1998: 603), namely, that PAR serves the local community and its priority is to improve local conditions.

There were three **FET phase teachers**, Mrs Kotudi, Miss Lebaka and Mr Mokoena, who had started an initiative to work together to develop and implement a subject improvement plan. One of them had 12 years experience teaching mathematics to Grades 10, 11 and 12 learners; and had taught at three schools before the current one, where she had been a head of department of mathematics and mathematical literacy for four years. The team members believed that they would benefit from teachers' experience of having been mathematics teachers, mathematics curriculum coordinators and heads of department – by being part of this team teachers' experience could be used to develop other teachers. Another teacher had been teaching mathematics in Grade 8 and 12 for two years. This teacher has a BSc degree in financial mathematics, and was busy with a BSc Honours in mathematics and applied mathematics. The study’s three main themes, with their interactions, are technology, pedagogy and content knowledge. Therefore, the second teacher was included on the basis of a strong mathematical background. Indeed, this teacher felt obligated to be part of the study, and expressed it as follows:

> *I am still young to make no difference… nahana motho a mokana ka nna ho se hothe I got 12% pass rate (imagine a person as young as me saying I got 12% pass rate).*

The third FET mathematics teacher had been teaching mathematics to Grades 9 to 12 learners for six years. This teacher has the following qualifications: BSc mathematics and applied mathematics, BSc Honours mathematics and applied mathematics, and a Postgraduate Certificate in Education (PGCE), and was enrolled for a Master’s in mathematics education. The team members believed that this teacher would contribute to the team in the field of research skills and mathematical content knowledge.

3.4.4 Plan of action

The participants (co-researchers) designed a plan of action that would respond to the five objectives of the study. The first objective of the study is to investigate the challenges teachers experienced regarding their TPACK for teaching Euclidean geometry with the aid of integrated ICT software. Thus, the plan of action that was drawn up comprised three main constructs: a) activities that investigated challenges to TK; b) activities that investigated challenges to PK; and c) activities that investigated challenges to CK. Furthermore, the plan of action referred to activities that investigated challenges related to interaction of the four main constructs, namely, technological pedagogical knowledge (TPK), technological content knowledge (TCK), pedagogical content knowledge (PCK) and technological pedagogical content knowledge (TPACK). Participants decided to prioritise investigating the challenges of CK; the activities they designed are given in Figure 3.1.

The study employed PAR self-reflective cycles called spiral science (Kemmis et al., 2013: 276). These cycles go through the following stages: planning; acting and observing; reflecting; re-planning; acting and observing again; and reflecting again, as shown in Figure 3.1.

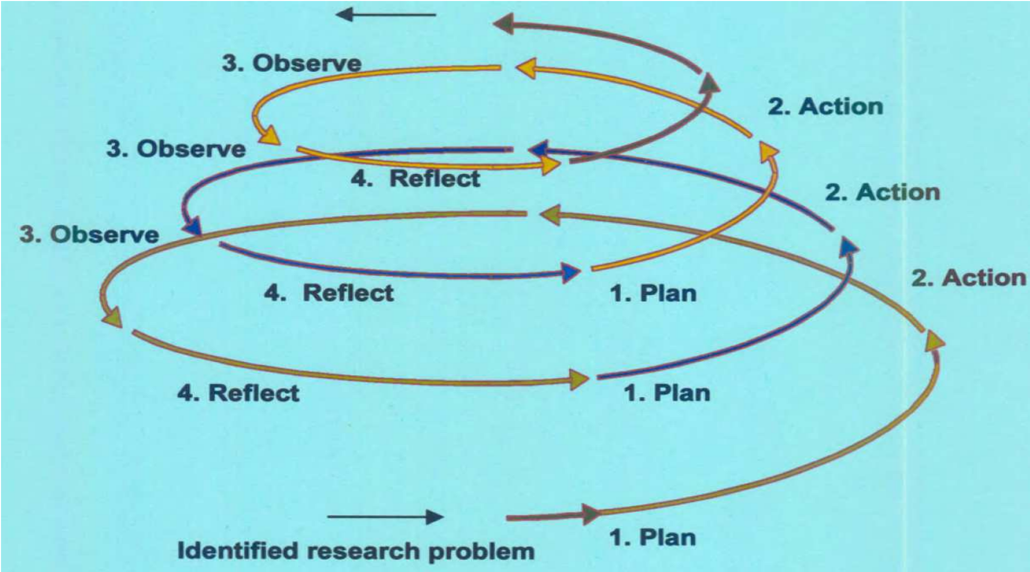

Figure 3.1: Spiral science (source: Kemmis et al., 2013: 276)

The study employed the model given in Figure 3.1. The participants identified the research problem and formulated the research question prior to the planning stage. All the actions and activities in the study followed the spiral science model.

3.4.5 Co-researchers' activities

All members of the research team were present at our first meeting, which was dedicated to understanding the nature of the problem. The team decided to develop a subject academic improvement plan for mathematics for all the schools that participated in the research project. This decision was made with the understanding that individual schools would customise the plan the team developed for the purpose of submission. Subject academic improvement plans are required for what are called failing or underperforming schools.

3.4.5.1 Activity 1: Mathematics academic subject improvement plan

The team developed a subject improvement academic plan, which is a diagnostic tool to identify learners' mistakes or errors committed on a particular assessment activity and to determine a possible cause for each mistake identified. This was done from the assumption that it is through understanding the nature of the learners' errors that we will be able to understand or anticipate their possible causes. Thus, in performing this task each team member analysed a learner's script for approximately 5 minutes. Then a group discussion on learners' mistakes was held. Thus, the actions of the team members while performing activity one are guided by the spiral science of PAR. For instance, the team planned ways to diagnose the learners' errors, and one of the team members, Mr Phehelo, suggested that:

> *Nna ne ke tla re (I was going to suggest that) each and every one of us should take a script and find the learners' mistakes on the particular question that we agreed on analysing).*

The team agreed with Mr Phehello's suggestions, and planned in this manner: each person analysed each script, and then a team discussion was held. In addition to planning, the team agreed on what to look for as they analysed. It was agreed that each person would apply his/her own discretion to decide what was noteworthy for discussion with others during the discussions. Doing so promoted criticality, and gave each team member the opportunity to express his/her view. The methodology was, furthermore, in line with the bricolage epistemology on complexity and a rigorous approach to knowledge production, which culminates in all team members agreeing and contradicting ways of knowing. The planning stage included deciding on how many scripts would be sufficient for trend recognition. Van Niekerk and Van Niekerk (2009: 136) drawing on Kerzner (1979: 4), argue that, in PAR, planning helps to define objectives and provides opportunities to monitor and evaluate the progress of the activities.

This planning stage enabled the co-researchers to clearly define the objective of the activity, which narrowed down to the objectives of this study, that is, teachers should be able to identify the Euclidean geometry concepts that learners found difficult to understand. Planning helps PAR participants to allocate the resources where they are needed (Van Niekerk & Van Niekerk, 2009: 137). The study used profiling of participants to pull on the strengths of each participant. For example, as a statistician researcher participant in this current study I buttressed the community cultural wealth of the research team with my academic training. We used statistical analysis to determine on which of the assessed mathematics concepts learners failed most often, we made inferences about the learners' performance using boxplots, we analysed variance, and modelled the learners' test scores statistically using normal distributions. Thus, the use of PAR was found to be beneficial in the sense that, through collaboration, all members benefited from exposure to a multiplicity of knowledge and skills. These activities covered the first and second stages, which are planning and execution of the plan. For the results of the analysis see Section 4.2.2. In Stage 2 co-researchers carried out their assigned tasks by implementing the planning, and monitoring and controlling the actions. The participants identified concepts that were difficult for learners to comprehend, both in the traditional moment and in the eighth moment. Based on observations and reflection on analysis, a further plan of action was drawn up (see Table 3.1).

Table 3.1: Academic subject improvement plan

Focus/ topic	Challenges or Errors	Possible Causes	Specific Interventions/Strategies
Question 9	Question 9.1. Learners write mathematical statements without reasons	Learners' inability to analyse a given sketch/diagram by • Identifying/marking given information or data in the sketch /diagram **and/or** • Deducing other data (indirectly given data) from the statements of the problem/sketches with regard to the properties of the isosceles triangle, chord tangent theorem and angles subtended by equal chords	Learners should be afforded an opportunity to • Verify the correctness of the two theorems practically, i.e. by construction, • State their findings with regard to chord-tangent and angles subtended by equal chords, • Be given exercises where they analyse the sketches by indicating all information given directly and indirectly, • Be assessed on their comprehension of the two theorems by solving problems where the two theorems are applied or used to solve other problems.
	Learners write incorrect reasons to support their claims	• Learners are unable to relate statements of theorems with sketches, [and /or as a result] • Learners are unable to apply/use theorems to solve problems.	• Give learners statements for which they should draw/sketch drawings to either prove them or to indicate the information given thereon, • Give learners sketches to read in an attempt to ascertain their comprehension and identification of critical information or facts, • Encourage the learners to state the tangent and centre theorems in full when they give reasons for their statements.

Explanation of column headings

Focus

The academic subject improvement plan developed by the team of co-researchers covered all 10 main topics of the FET mathematics curriculum. This is in line with the critical vision of PAR, which advocates for research becoming a tool that people use to improve their lives. By using PAR, community interest drives the process of research and the interest of the teaching co-researchers in this study was to develop a complete academic subject improvement. Therefore, in this study we only respond to activities done that related the Euclidean geometry. The focus of the study was on theorems of Euclidean geometry, such as,

(i) A line segment drawn from the centre to the midpoint of a chord is perpendicular to that chord;
(ii) An angle at the centre of a circle is twice the angle at the circumference subtended by the same arc or chord;
(iii) An angle subtended by a diameter is 90 degrees; and
(iv) Opposite angles of cyclic quad are supplementary.

The analysis was done by clustering the questions of the assessment activities according to the four theorems given above. For instance, on questions related to theorem (i) on the assessment activities, learners' mistakes and misconceptions that were identified where grouped together. This was done with the purpose of finding common challenges and/or misconceptions that learners have.

Challenges or errors

Challenges, errors and/or misconceptions refer to learners' mistakes on a particular theorem or focus area. The mistakes that learners made were investigated with the purpose of understanding what exactly learners are not able to understand. For instance, Table 3.1 reports that the co-researchers found that learners write mathematical statements without providing reasons. The findings of this analysis are discussed in detail in Section 4.2.2. In response to the objective of this study, teachers, when they analysed learners' responses, applied their subject matter knowledge or CK of Euclidean geometry. To be more specific, and drawing on the work of Ball et al. (2008, 391), teachers use CCK to determine what is the wrong and what is the right solution (see Sections 2.4.1, 2.4.3 and 4.3.4). This activity is used to improve teachers' CK of Euclidean geometry – in order for them to recognise the wrong and right solutions, they did not use the memorandum, but considered different ways the question could be answered.

Possible causes

This part of the analysis is concerned with investigating possible causes of the challenges or errors identified. The co-researchers' point of departure was that, if we understand the possible causes of learners' mistakes, we are in a better position to design strategies that will respond to the challenges identified. Furthermore, this exercise enhances teachers' knowledge of Euclidean geometry, because, in order to analyse the nature of learners' errors, teachers have to apply SCK to move beyond knowledge of wrong and right to find solutions at a deeper level. SCK is used to investigate further where learner(s) went wrong, and why. Thus, this becomes the SCK for teaching, since teachers are concerned about diagnosing learners' mistakes in order to remedy them.

Interventions or strategies

The challenges, misconceptions and/or errors and their possible causes were investigated with the aim of developing strategies or teaching methods to remedy them. Thus, the co-researchers of this study used their CCK, SCK, PCK, TCK, TK, and TPACK to respond the challenges identified. Doing so presented an opportunity for the team members to share their knowledge and the skills needed to respond to the errors they had identified. However, it must be acknowledged that CCK, SCK, PCK, TCK, TK, TPK are sometime difficult to separate, as they are utilised at the same time. This further shows the complexity of the knowledge needed for teaching.

3.4.5.2 Activity 2: Lesson preparation

A lesson preparation activity in this study was conducted as a collaboration between all the team members. During a lesson preparation session one of the teaching co-researchers had to identify the topic that he or she would be teaching the next week. Weekly lesson preparation was done collaboratively to improve that teacher's lesson plan. However, the lesson preparation session paid particular attention to content, pedagogical content and then technological pedagogical content, so all the team members benefited from the preparation, even though they were not teaching that concept the next week.

The lesson preparation started with all team members grappling with the content of the Euclidean geometry topic that had been identified, and drawing up a lesson aim. Teachers grappled with the content by involving all the team members in a discussion on the meaning of all concepts that were to be taught (see Section 4.3.4.1 for a description of the way team members grappled with the properties of a circle). This activity created a platform for teachers to share and improve their CCK, which happened when all the team members came together and shared their understanding of Euclidean geometry concepts.

After the discussion of the content, the team reflected on the KCK, drawing from the academic subject improvement plan and their teaching experiences, to design strategies that we could use to teach the content under discussion. Thus, informed by the team's PCK as discussed, the team decided which software to use as a teaching aid. This created a further opportunity for team members to share their knowledge and skills on different types of software and how can it be integrated with the content of Euclidean geometry. In so doing the team's TCK, TK, PCK, TPK and then TPCK were improved (see Section 4.3.4). This collaborative planning also created opportunities for continuous professional development of co-researchers, particularly for CK, PK and TPACK.

3.4.5.3 Activity 3: Lesson facilitation

The lesson facilitation activity in this study was also a collaborative endeavour. A lesson was facilitated by one member of the team, while others observed whether what we had prepared was effective, and how it could be improved. During the lesson facilitation the other team members made notes on what they found important for sharing with the team during the lesson reflection. Team members were not limited to observing the lesson, but could help learners nearby to ensure that they were following and/or help if requested by the lesson facilitator.

During the reflection session, all team members who had been present during the lesson facilitation were encouraged to share their views about the lesson. The multiplicity of views made possible by the team approach to teaching informed the re-planning stage of the spiral science of PAR to improve lesson facilitation.

3.5 DATA ANALYSIS

The study employed critical discourse analysis (CDA) to analyse the spoken and written words. I found CDA to be an appropriate tool of analysis since it considers discourse as a form of social practice (Mirzaee & Hamidi, 2012: 183). In contrast, in the traditional moment qualitative research is conducted using statistical techniques.

3.5.1 Critical discourse analysis

As early as 1970s CDA emerged as a sub-area of discourse analysis, with a perspective that argues that discourse is a form of social practice (Fairclough, 2013: 7). Fairclough (2013: 7) explains that CDA is concerned with how texts work within sociocultural practice. I found CDA to be an appropriate tool for analysing participants' written and spoken words, which captured and interpreted the sociocultural practices within which we were designing a strategy to improve TPACK for teaching Euclidean geometry. CDA tolerates the messiness of the lived lives of participants and understands that there is no one correct way of telling. Therefore, the objectives of CDA coincide with epistemological and ontological perceptions of bricolage on nonlinear steps in formulating a strategy to improve TPACK for teaching Euclidean geometry. Bricolage is grounded in the complexity of multiple perspectives of the same story (Rogers, 2012: 1). Furthermore, the appropriateness of CDA is clear when it focuses on relations between ways of talking and ways of thinking in written text and spoken words (Rashidi & Souzandehfar, 2010: 56).

I refer to Rashidi and Souzandehfar (2010: 56) when I argue that CDA has the same objective as PAR, as it seeks to make connections between ideas, language, power and social relations of those who are involved in CDA. Van Dijk (2007: 23) argues that CDA is multi-theoretical framework. This made CDA an appropriate way of doing analysis, since bricolage takes different theoretical positions for designing a strategy to improve TPACK for teaching Euclidean geometry. Van Dijk (2007: 25) also states that there are four mainstream approaches of doing CDA analysis, namely, critical linguistics (CL) as developed by Fowler, Kress, Hodge and Trew (1979); Fairclough's model (1989, 1992, 1995); that of Wodak et al. (1999) and Van Dijk's (2007) conceptual framework.

3.5.1.1 Critical linguistics

CL is research that is concerned with investigating the relations between signs, meanings, and the social and historical conditions that govern the semiotic structure of discourse (Fowler, 1991: 90). In this study CL was used as a tool to perform linguistic analysis of representations of participants' (co-researchers') ideologies regarding the use of computer software to teach Euclidean geometry. Employing CL enabled us to perform mystification analysis of teachers' texts during planning and reflection sessions. Furthermore, Kress and Hodge (1993) proposed transformations that include transitivity, normalisation, negative incorporation and agentless. Using these transformational analysis tools we were able to reveal intentions subtly disguised in complex spoken sentence structures.

3.5.1.2 Fairclough's model

We also used Fairclough's framework (1989), with the understanding that language captures the social realities and brings social change. By analysing the language used by participations (co-researchers) during the planning and implementation sessions, we were able to understand the challenges of designing a strategy to improve TPACK for teaching Euclidean geometry in the teacher's social realities. Developing a TPACK strategy took into account the social realities of the co-researchers, such as the way power, dominance and inequality affect the development of the TPACK strategy. In this study a bricoleur employed all four these approaches in his quest for complexity and multiplicity of meaning making.

3.5.1.3 Textual analysis

With reference to Teun van Dijk (1985: 23) I argue that text is not only used to inform us of social reality factors that play a role in designing a TPACK strategy; text also reveals the ideological standpoints, their production and the reality construction of co-researchers and the organisation. The co-researchers identified operational phrases, such as TK, PK, and CK, written and spoken words, as well as other forms of communication that are nonverbal but which have specific meaning and bearing on the aim and objectives of the study (De Beaugrande, 2006: 31-42). This is in line with Stein and Mankowski (2004: 28), who argue that, as co-researchers, we need to create themes in order to give voice to the metaphor of co-researchers when we design a strategy to improve TPACK for teaching Euclidean geometry. The data generated during our meetings and reflection sessions was analysed at a micro-level of analysis, and textual analysis focused on the participants' spoken words as representations of their views about and experiences of teaching Euclidean geometry using computer software, traditional teaching techniques, teaching resources (both physical and human support), engagement of stakeholders (such as the skills advisor and technician), individualised support to create a positive team climate among co-researcher and their contributions to a TPACK strategy to improve the teaching of Euclidean geometry, and participation of co-researchers during their feedback and reflections. The analysis was also sensitive to the non-verbal cues and expressions of members as they expressed their views. These cues shaped micro analysis of language, discourse and verbal interaction.

Furthermore, the SWOT analysis that had been performed to profile the co-researchers was used as a point of departure for the text analysis. Profiling as a skills analysis tool enhanced the interpretation of meanings, especially in satiation, were the generated data could not make sense. For instance, the facilitation of analysis and/or interpretation of education curriculum policy and environmental issues were likely to be done by people who had access to and possibly worked with policy more often. This kind of information was obtained from that given under the roles and responsibilities of participants.

The co-researchers had the potential to skew the interpretation of data, taking into consideration that some participants were in positions of power, such as the DCES, the heads of departments, and the principal. However, the analysis was guided by a principle of PAR that promotes equality among all participants during the research process. Therefore, all the participants' contributions, views and perspectives were considered equally important. The nature of the analysis was shaped by not considering discourse in isolation from its social context. However, Van Dijk's model of CDA proposes that discourse is a complex communicative event that embodies the social context of the participants, and their production and reception processes (Sheyholislami, 2011: 156).

3.5.1.4 Cognitive analysis

Cognitive analysis was employed to create a connection between dominance and discourse (Van Dijk, 1993: 257). Cognitive analysis mediates between the social and discourse, and represents peoples' thoughts and communal practices; socially shared attitudes and ideologies (Sheyholislami, 2011: 146). During our planning and re-planning sessions, meetings, reflection sessions and the implementation of our plans, mental representations of the participants' perceptions, beliefs semantics were captured. By using cognitive analysis we came to the understanding that audio or written words represent the minds of the participants.

Furthermore, throughout the duration of the study, the discursive practices exercised during the formulation of the research question, the use of computer software for planning, designing and facilitating a lesson created a sense of unity among participants. Using Van Dijk's (1981: 189-226) cognitive context model the study's analysis was conducted with the understanding that participants' spoken and written words embodied their context, which existed in their minds. This analogy confirmed that such analysis is in line with the objectives of both bricolage and PAR, and grounded in epistemic knowledge of using technology to teach. This subjective analysis helped us to design a strategy that is subjective, meaningful and responsive to participants' everyday challenges of teaching Euclidean geometry using technology. In addition, the study employed a mental model analysis, which created a mental model of spoken or written text.

From the texts generated by participants, mental models were created for different epistemic communities that participants belonged to, for example, mathematics teachers belonged to the education epistemic community, which informed their knowledge of Euclidean geometry content and pedagogy. The technician belonged the computer technology epistemic community, which informed his knowledge of computer software.

Through the process of cognitive analysis, the study identified cognitive activities and interpreted them in terms of their inherent power struggles. For instance, the power and ideological bias in the teacher-learner relationship leans more towards the teacher (Rocha-Schmid, 2010: 353) due to his/her knowledge of the subject content, and experience. However, the quality of these relationships in relation to the extent to which such knowledge power was abused (Van Dijk, 1995: 20) or appropriately used needed to be established. For instance, the choice and use of teaching strategies tended to be at the discretion of the teacher, and this choice did not necessarily take learners' backgrounds and styles of learning into consideration. Disregarding learners in this way legitimises (Van Dijk, 1995:18) teacher-centred or even examination-oriented teaching strategies.

3.5.1.5 Social analysis

Social analysis examines overall societal structures, for example, social behaviour and arrangements (Sheyholislami, 2011: 4). These structures tend to be an expression of attitudes and values that the community or society hold in high esteem, and addresses the issue related to the connection between theory and practice, thus determining its relevance, meaningfulness and usefulness (Liasidou, 2006: 488; Van Dijk, 2008:86). The study analysed these social structures to establish their impact or influence on learning and teaching, and highlighted possible origins.

Evidently, cognitive activities were related to the establishment and arrangement of social structures, thus, the interest in and focus of social analysis was conceived as being connected to the learners' backgrounds and physical environments at school in relation to the community and the broader society. For instance, the coordinating team, comprising representatives of the school, the municipality, the DoE, and the community, was a social structure that facilitated school-community coordination (Rocha-Schmid, 2010:355; Van Dijk, 2008: 90). This established a connection and working relationship between the two local schools as well as between the school and the municipality. As with the other engagements, the power struggles between, for instance, the school and the municipality, needed to be addressed circumspectly. Thus, the study relied on flexibility to sustain some of the relationships.

3.5.2 Statistical analysis

The statistical data analysis was used to make sense of and uncover the stories embedded in day-to-day experiences. The kind of data analysed using these statistical techniques is often used to make inferences about a certain population. Thus, in this study, the statistical data analysis is used to make sense of the prevalent patterns observed regarding learners' performance in mathematics (see Section 4.2.2). This is done by using, among other techniques, boxplots to condense and summarise the data by extracting the median quartiles and maximum and minimum values observed (Annkuch, 2006: 27). The boxplot enabled me to visualise the distribution of data, the extent to which they are centred and spread using a five-number summary, that is, minimum, lower quartile, median, upper quartile, and maximum (see Section 4.2.2).

3.5 CONCLUSION

This chapter justified the use of PAR as a suitable methodology for the current study. A research design, guided by the principles of PAR, was discussed to in order to give context to the data that will be analysed and discussed in Chapter 4. Thus, the following chapter presents and discusses the data generated during the activities discussed above.

CHAPTER 4: DATA ANALYSIS AND PRESENTATION AND INTERPRETATION OF RESULTS TOWARDS A STRATEGY TO IMPROVE TEACHERS' TECHNOLOGICAL PEDAGOGICAL CONTENT KNOWLEDGE FOR TEACHING EUCLIDEAN GEOMETRY

4.1 INTRODUCTION

The study intends to formulate a strategy to improve teachers' technological pedagogical content knowledge (TPACK) for teaching geometry with the aid of integrated ICT software. The study focuses on Grade 11 Euclidean geometry concepts, namely, a line segment drawn from the centre to the midpoint of a chord is perpendicular to that chord; an angle at the centre of a circle is twice the angle at the circumference subtended by the same arc or chord; and an angle subtended by a diameter is 90 degrees.

This chapter presents, discusses and analyses data, and provides the interpretation for each of the five objectives of the study. Thus, the chapter analyses the challenges experienced by teachers who teach Euclidean geometry with the aid of integrated ICT software. This is done with a view to establishing the possible solutions and strategies that can be developed, adopted and adapted to address the challenges that are experienced effectively. The solutions and strategies sought to enhance the processes and practices of teaching and learning during and beyond the duration of the study. As a result, the conditions under which the strategies and solutions developed needed to be thoroughly understood, because these conditions may pose threats and be embedded with inherent risks that will impede the successful implementation of the solutions. It is for that reason that the strategies and solutions will be operationalised, assessed and evaluated prior to them being considered as sufficiently conclusive to serve as the successful response to and to be used in teaching and learning Euclidean geometry.

The presentation, analysis, interpretation and discussion of the empirical data in respect of the five objectives will be done as follows: the study objectives are used to formulate subtitles according to which empirical data is organised, an appropriate opening discussion follows with the aim of setting out good practices, policy-related issues, previous research findings and theory for each subtitle. The empirical evidence is then presented in the form of written words, pictures and scenarios and juxtaposed with good practice, theory, research findings, policy and legislative. Finally, the deeper meaning of the texts is analysed using CDA at three levels, namely, text, discursive practice and social structure, in line with the bricolage i.e. theoretical framework. Furthermore, quantitative data was analysed using statistical techniques.

4.2 THE NEED TO FORMULATE A STRATEGY TO IMPROVE TEACHERS TPACK FOR TEACHING EUCLIDEAN GEOMETRY WITH THE AID OF ICT SOFTWARE

The following challenges were identified by the coordinating team, which consisted of six mathematics teachers, a programmer from the University of the Free State, and the study coordinator (myself), and justified the need for formulating the strategy (see Section 3.3.3). The purpose of the team meeting was to identify the problems experienced by teachers, and to suggest possible solutions. The following challenges were identified: (i) no team to improve TPACK for teaching Euclidean geometry; (ii) a need for intervention in Euclidean geometry (iii) teachers' insufficient lesson preparation when using ICT software as a teaching aid; (iv) inadequate lesson facilitation when using ICT software as a teaching aid; (v) no integration of assessment during lesson facilitation when using ICT software as a teaching aid; and (vi) lack of research into computer software appropriate for teaching Euclidean geometry.

4.2.1 No team established to improve TPACK for teaching Euclidean geometry

A team creates a platform for individuals to combine their competencies and strengths in pursuit of completing a task (Section 2.5.1.1). In this study, this task involves, among others, combining computer skills, software knowledge, TPK and Euclidean geometry content knowledge Chavan (2013: 4) argues that working as a team for pedogogical reasons enhances lesson preparation, so that teaching is more focused on enhacing a particular lesson objective.

It became evident during my conversation with Mr Phehello, a deputy principal of one of the schools that participated in the study, that no teams or structures had been created so that teachers could support each other, let alone on the use of ICT software in teaching:

Ke ne ke ntse ke ipotsa hore e kaba ke thusajwang techere wa rona wa mmetse wa grade twelve … ae ntate ke mathata (I was asking myself how am I going to help our Grade 12 mathematics teacher… ae Sir. this is troublesome).

The phrase "ke ipotsa", which translates to "asking myself", when read in context of the narrative above, suggests that Mr Phehello had been thinking about finding ways to help the Grade 12 mathematics teacher (Van Dijk, 1977: 50). The fact that Mr Phehello was asking himself this suggests that he tried to help the Grade 12 teacher, but the problems could apparently not be resolved. Help in this regard could include asking other mathematics teachers, whom he referred to indirectly by singling out the Grade 12 teacher by saying, "our Grade 12 mathematics teacher". This suggests that there were other mathematics teachers who were not teaching Grade 12 mathematics. The kind of help that Mr Phehello provided to the Grade 12 mathematics teacher did not include creating a team dedicated to helping learners understand Euclidean geometry, among other topics. This became evident during a meeting to establish a team, when I asked:

How are you currently supporting one another?

Mrs Kotudi, who taught mathematics at the same school as Mr Phehello and the Grade 12 teacher, responded:

ha jwale ntate … ha ntle ntle motho le motho o ntse a bona hore o tswa jwang (Currently, Sir, what is really happening is that it is every man for himself).

From the above extract it is evident that, prior to the study, teachers had worked in silos. For instance, the phrase "every man for himself" suggests that there was no transparency between the teachers, or collective understanding of what they were teaching and how they were teaching it (Sabah et al., 2014: 101). Furthermore, the use of the phrase, "hantle ntle", which, in the context of this study, means "in real terms", seems to suggest that whatever form of support they currently had was not working. This could also be understood as meaning that the current support or arrangement was not effective. This is contrary to Zehetmeier's (2014: 183) view that collaboration or team effort towards creating a platform to share knowledge and skills fosters sustainability of innovation between the team members. However, it seems that the teachers were not working together, or even sharing their challenges with regard to teaching and learning of mathematics. This became even more evident when Mrs Kotudi alluded to the importance of the study, and said:

ntho ena e tlo re thusa ho qala ho sebetsa mmoho (This thing is going to help us to start working together).

The phase, “this thing“ in the extract above refers to the study. Therefore, prior to the study, teachers had not worked together. This was also evident when Mrs Kotudi used the phrase, “start working together”, which suggests that she believed the study signaled the start of the teachers’ cooperation, which had, up to that time, been lacking (Weiss & Wodak, 2003: 8). Failure to work as a team to improve their pedagogical practices denied the teachers an opportunity to understand the challenges of different perspectives (Kincheloe, 2004: 82). This was confirmed by Mr Phehello in the same meeting, who expressed his excitement by saying:

Ba haeso e na e fapane le ntho ye neng re ntse re e ye tsa, mona ha ke o rutele banna, we do it together (This is different from what we have been doing, here I am not teaching for you, we do it together).

From above extract it seems that the extent to which the teachers had worked together prior to the study had been limited to teaching each other about a particular topic. This was evident when Mr Phehello said, “here I am not teaching for you”, which I understand as a reference to the kind of help that he provided, which was to teach learners on behalf of their teachers. However, Mr Phehello’s “support” does not eliminate the challenges, because this type of “support” is not a developmental activity for the teacher, which would help him/her to teach that particular concept effectively. This “support” is contrary to the best practices regarding the importance of a team, which states that teamwork creates an opportunity for the team members to learn from one another (Chavan, 2013: 4).

Furthermore, this is contrary to the views of Smith (2013: 219), who informs teacher education on the use of practical pedagogies to integrate digital fabrication meaningfully into teaching. According to Smith’s study, collaborative learning could occur when teachers work as a team, which results in creativity, because innovation is sparked by multiple perspectives, multiple theoretical approaches and multiple views (Berry, 2004: 101; Smith, 2013: 219). Failure to work as a team denies teachers the opportunity to share their challenges with regard to teaching and learning of, among others, Euclidean geometry (Howard & Potts, 2009: 3). This conclusion was evident when Mrs Kotudi said, after teachers had shared their challenges during the meeting to establish a team, “our challenges are the same”.

After reaching an agreement to work together (see Section 4.3.1), I visited the team members to support them and be imbedded in day-to-day challenges that face the team members in their teaching of Euclidean geometry, in particular (Berry, 2004: 110). These visits gave me the opportunity to observe different teaching methods being used by different team members, who sometimes taught the same or related topics, but in different ways. One of the observations I made at this time was that failure to work as a team denied teachers the opportunity to share resources, teaching methods, teaching aids and teaching strategies (Turkich, Greive & Cozens, 2014: 1).

4.2.1.1 Classroom presentations

Mr Mokoena and Miss Lebaka, who taught mathematics at the same school to Grades 11 and 12 respectively, presented the following lessons on Euclidean geometry. Mr Mokoena used HeyMath! software to teach the Grade 11 learners that a line drawn from the centre of a circle to the midpoint of a chord is perpendicular to the chord. The following is an extract of Mr Mokoena's lesson:

Ke batla le shebe what is happening as I move point C

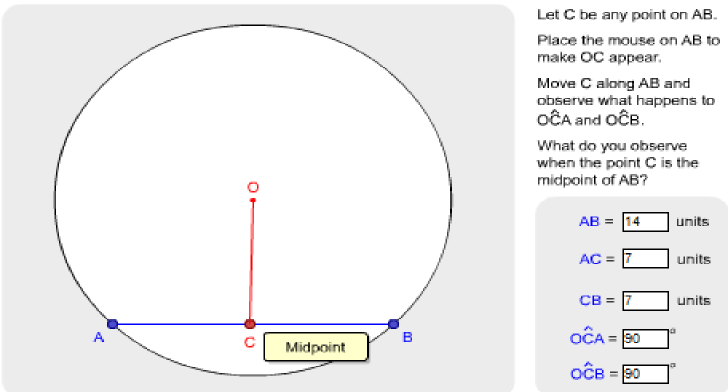

Figure 4.2.1.1a: HeyMath! perpendicular bisector

Figure 4.2.1.1a is a snapshot of a projected geometric figure in Mr Mokoena's class. Mr Mokoena moved point C along line AB as shown in Figure 4.2.1.1a. This created an opportunity for learners to make their own observation on what happens to angles OCB and OCA when C is the midpoint of AB. However, Miss Lebaka, teaching Grade 12, used no ICT teaching aids and presented her lesson in a very abstract manner. Here is an extract from her lesson:

A line drawn from the centre of a circle to the midpoint of the chord is perpendicular to the chord; we use congruency to prove this theorem.

Step 1: Construction: join OA and OB

Step 2: We show that triangle OAP and triangle OBP are congruent.

Miss Lebaka continued proving the theorem without soliciting learner interaction.

Figure 4.2.1.1b: Traditional approach to perpendicular bisector

In Figure 4.2.1.1b AB is a chord, OP is a line from the centre to the chord AB, where P is a midpoint of AB. The use of congruency to prove the theorem, as shown in Figure 4.2.1.1b, is very abstract in comparison to the way Mr Mokoena did it using HeyMath! software. Miss Lebaka's method requires inductive reasoning to reach a logical conclusion that the two angles formed at the midpoint of the chord are equal to 90 degrees. For instance, learners must be able to give reasons for the mathematical statement, AO is equal to BO, because both lines are the radius of the same circle, which is abbreviated [Radius] (see Figure 4.2.1.1b). However, Miss Lebaka did not even give learners an opportunity to give reasons for the mathematical statement made in Figure 4.2.1.1b, neither did she give learners an opportunity to give their mathematics statements. Miss Lebaka said:

In triangle OAP and triangle OBP AO is equal to BO why? [Radius]

Teaching learners to reason logically and abstractly is in line with the mathematics curriculum assessment policy statement (CAPS), which states that teachers must provide learners with the opportunity to develop methodological procedures, make conjectures and prove them logically (DoE, 2011: 8).

However, Miss Lebaka's lesson did not provide learners with the opportunity to develop abstract and logical reasoning. There were no activities created by Miss Lebaka which promoted learner interaction as advocated by social constructivism, which asserts that knowledge is a product of social human activity or experiences. In addition, the mathematics CAPS policy defines mathematics as a human activity, however, during Miss Lebaka's lesson she seemed to be the only one who was active, teaching – this activity is teacher centred. Using the software to teach the concepts created an opportunity for learners to make their own observations on the changes in the magnitude of the two angles as Mr Mokoena moved point C along the chord AB. This confirms Rabah's findings, namely, that ICT promotes problem-based learning and learner-centred teaching (2015: 24). In Mr Mokoena's class, learners observed the following:

> *Thabo: Nna meneer, ke bona ha C e ya mahareng AC e lekana le CB e ba seven, seven (With me, Sir, I see that when C becomes the midpoint, AC is equal to CB, it becomes seven, seven.)*
>
> *Mr Mokoena: ye mong a ka reng (What can the other one say)?*
>
> *Ntswaki: Ntate, ha ntho eno ya midpoint e hlaha angle OCA le angle OCB di le kana le 90 (Sir, when this thing of midpoint appears, angle OCA and angle OCB, they are equal to 90).*

From the above discussion and dialogues captured in Mr Mokoena's class, it appears that his lesson was learner centred, because learners were required to make discoveries on their own through observation. Despite the fact that Mr Mokoena could use the software to promote learner centredness and present the mathematical concepts concretely, the limitation is that his lesson could not transcend to a level where learners were able to reason in an abstract fashion to reach the logical conclusion using mathematical statements and reasoning. For instance, in the learners' responses we do not see learners begging to give reasons for their mathematical claims – they only report their observations. The development of abstract reasoning is cited by Further Education and Training (FET) mathematics CAPS policy, which states that learners must be developed cognitively such that they can reason logically and abstractly (DoE, 2011: 10).

Regarding the two lessons presented, Miss Lebaka could have learnt from Mr Mokoena that HeyMath! software could be used as a teaching aid to enhance learners' understanding. On the other hand, Mr Mokoena could have learnt from Miss Lebaka about structuring the mathematical proof so that it is relatively easy for learners to understand abstract logical reasoning once they have built their understanding using HeyMath! The fact that both Miss Lebaka and Mr Mokoena taught at the same school and presented lessons that could have supplemented one another's teaching of Euclidean geometry confirms the absence of a team created to focus on improving the teaching and learning of mathematics. The absence of a team denied the teachers the opportunity to have a common or shared vision. Thus, the need is to collectivise efforts towards a common goal. Moreover, the absence of a platform for teachers to share challenges and then create a shared vision for formulating a strategy to respond to their challenges, resulted in the absence of a SWOT analysis as component of the strategy.

The type of discursive practice that promotes individualism is prevalent in the first and second moment of qualitative research and subscribes to the ontological stance of both the traditional period and the modernity phase (Sections 2.2.1.1 and 2.2.1.2). This approach means that there is just one correct and objective way of teaching, which is governed by laws, and only through these laws can we understand how to teach (Kincheloe, 2008: 25). Furthermore, the absence of a team for constructing knowledge to improve teachers' TPACK for teaching Euclidean geometry is grounded in the epistemology of the traditional period and modernity phase. This means that teachers who do not promote a collaborative approach towards teaching Euclidean geometry do not understand knowledge of teaching to be socially constructed (see Section 2.2.1.1). They consider knowledge to be a list of facts that are independent of social constructs, hence, they don't see the necessity of creating a team platform (Kincheloe, 2008: 26).

In order to confirm the absence of a team or structural support for teachers to work together as a team to improve their TPACK, I threaded and looped back and forth between the seventh and eight moments of qualitative research (Berry, 2004: 110). For instance, the traditional period and modernity phases' ways of doing research prohibit me from interacting with the participants, as this would be interference and would compromise my objectivity (Given, 2008:101). However, a bricoleur does not consider the two moments as the only modes of knowledge production. In my quest for a transformative agenda I threaded and looped forward to operate in the seventh and eighth moments. These two moments enabled me to interact closely with participant researchers, with respect and as equals, in pursuit of producing knowledge. I adhered to the epistemology of PAR, which considers human experience as a legitimate method of knowledge production (Sections 2.2.1.7 and 2.2.1.8, and Section

3.2.3). The postmodernist period provided a stance which enabled me to gain a better understanding of day-to-day challenges that teachers are faced with when teaching Euclidean geometry with ICT software; I embedded myself in the challenges or situation (Denzin, 2001: 326). This approach was evident when I visited team members (Section 4.3.1) to work with and support them in order to gain a better understanding of their challenges. Operating in the seventh moment, I regarded the absence of a team as the first challenge that had to be overcome if I was to engender the emancipatory agenda through PAR (Section 2.4.6). Furthermore, a bricoleur in the seventh moment understands the importance of taking a stance in pursuit of formulating a contextualised strategy that does not limit itself to a positivistic epistemology and ontology, but is commited to emancipatory agenda (Andrews, 2012: 6).

In closure, the study's contribution lies in the findings emerging from the discussion above, namely, that, prior to the study intervention, there was no transparency between the teachers, because they all worked in silos, which concurs with Hu and Linden (2015: 1104) (see Section 2.5.1.1 for more in this regard). It became evident that teachers did not even share their challenges, teaching methods, resources or teaching aids. The study further revealed that using ICT creates an opportunity for learners to make observations, discoveries and conjectures based on observation, instead of being told. This confirms the viewpoint of Bingimlas (2009: 235) and Rabah (2015: 24), that ICT enhances teaching and learning in the classroom.

4.2.2 The need for intervention in Euclidean geometry

The traditional moment provides a bricoleur with statistical data analysis tools that can be used to visualise emerging trends relating to learners' performance in mathematics. The necessity of enhancing the teaching of mathematics, especially Euclidean geometry, is evident from the preceding data analysis of learners' test scores. The assessment task covered four mathematics topics, namely, data handling, analytical geometry, trigonometry and Euclidean geometry. A total of 153 learners participated in taking the assessment from the schools which are under investigation. Each topic that is, data handling, analytical geometry, trigonometry and Euclidean geometry consisted of 3 questions. A boxplot was used to visualise the distribution of the data for the four topics.

On the chart below, the red lines represent the median of the distributions while the asterisks denote the mean of the distributions. In all four distributions, the mean scores are approximately the same as the median, which suggests that the data is symmetrical and possibly normally distributed. This implies that, since the mean and median are approximately the same for all four topics, the learners' exam scores should show that approximately 50% of the learners performed below the mean and approximately 50% of the learners performed above the mean. Looking more closely, the mean scores are 10.3087, 12.6756, 10.5408 and 6.9944 for the four topics. Based on these topic averages we can hypothesise that the results were really unsatisfactory, given that all four topics had a total mark above 25 points. Despite the fact that the overall performance over the four topics was not satisfactory, learners seem to have performed worse in Euclidean geometry than on the other topics. For instance, the maximum score obtained on Euclidean geometry is approximately the same as mean scores for trigonometry and data handing, and lower than the mean score for analytical geometry (see Figure 4.3.2a).

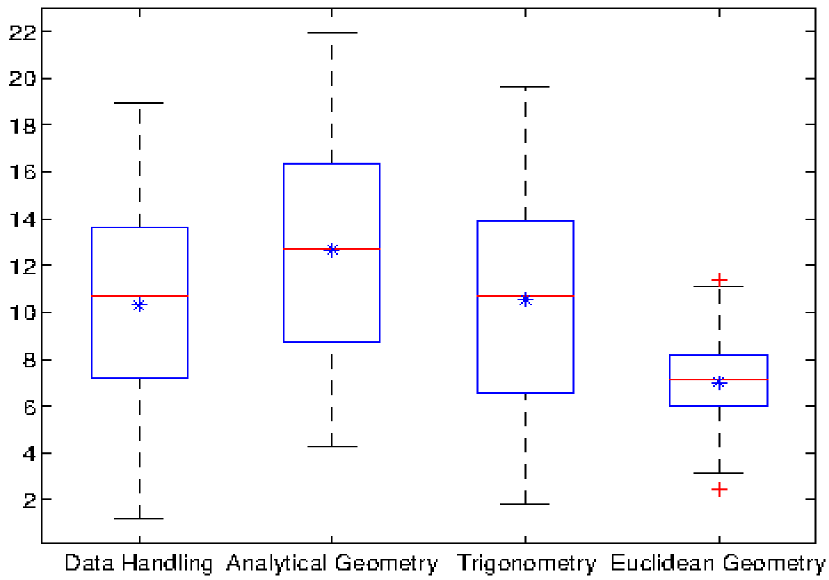

Figure 4.3.2a: The boxplots of learners' performance on four topics

Furthermore, in Figure 4.3.2a it is clear that the Euclidean geometry box is smaller than that of the other topics. The box indicates that 50% of learners are contained in the box, and they seem to have obtained marks with less variation in Euclidean geometry than in other topics. This suggests that, because the scores are low, most of the learners fail with more or less same score. In order to strengthen the argument statistically, a statistical test was done to confirm that learners' performance in Euclidean geometry was, indeed, the worst. The mean score differences were tested to determine if they were statistically significant, and not merely a random outcome. I wanted to determine if learners were likely to perform worse in Euclidean geometry than in other topics. The test was performed using two-way analysis of variance (ANOVA) and the results are reported in Table 4.3.2a.

Table 4.3.2a: Analysis of variance of learners' performance

Source	SS	Df	MS	F	Prob > F
Columns	990.77	3	330.256	18.84	0
Rows	1051.97	59	17.83	1.02	0.4541
Error	3102.81	177	17.53		
TOTAL	5145.55	239			

The columns of Table 4.3.2a present the results of testing for a mean variation between the topic groups, which were data handling, analytical geometry, trigonometry and Euclidean geometry. The rows present the results of testing the variation within the groups, that is, between the learners' performance within the same group, The columns have a P-value of zero, which is at less than 0.05 significance level. This implies that there is a significant variation between the mean scores of different groups. In addition, the rows have a P value of 0.4541, which is not at less than 0.05 significance level. This implies that variation between the learners within the same topic is not significant. This suggests that learners performed more or less the same on each topic with a small diffrence in their marks. In order to be more specific, another test, multiple comparison of means using two-way analisys of variance, was performed to test the difference between the Euclidean geometry and other groups. The results are reported in Figure 4.3.2b.

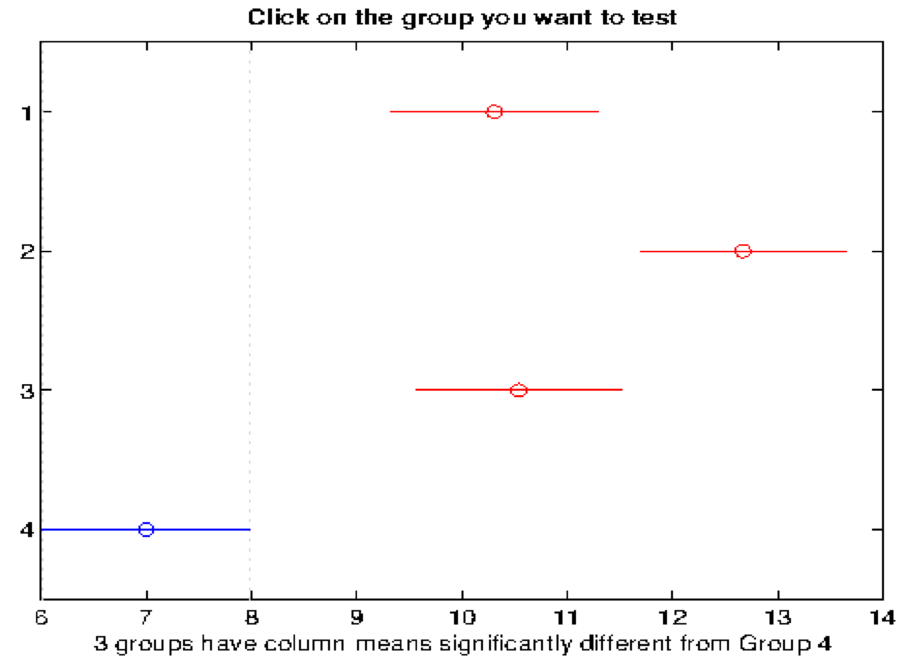

Figure 4.3.2b: Multiple comparison of means using two-way analysis of variance

Groups 1 to 4 denote the topics data handling, analytical geometry, trigonometry and Euclidean geometry respectively. Figure 4.3.2b shows that the mean score of Euclidean geometry is statistically and significantly different from that of the other three topic groups. Furthermore, a t test was used determine if there is a statistically significant difference between the mean scores of Euclidean geometry and each of the other topics. The null hypothesis tested says there is no difference between the means of Euclidean geometry and the other topics. For the results of the test mean, we cannot reject the null hypothesis that the means are equal since their p values (7.382e-07, 5.694e-14 and 8.912e-07) are less than 0.5. This implies that the learners' unsatisfactory performance in Euclidean geometry, compared to other topics, is statistically significant, which justifies formulating a strategy to improve teachers' TPACK for teaching Euclidean geometry using ICT software.

However, the coordinating team analysed the statistical report critically, and Mr Molete responded to the analysis by saying:

> *Colleagues, I am not sure if I am the only one who sees the good work that is done by our colleagues here, jwale re a tseba hore bana bana ba sokola hoholo ke Euclidean geometry (Now we know that these learners are struggling, mainly with Euclidean geometry), however, we still do not know hore ha batsebe eng.*

In the context of this study the above extract is understood to mean that, even though the statistical analysis shows that learners are struggling more with Euclidean geometry than with other topics, we still don't know what kinds of mistakes they make, as this information is not provided by the statistical analysis. This conclusion was made even clearer by another coordinating team member, Mr Boke, who said,

> *Ke domelnana le Ntate Molete, statistics sena se se tle, empa re hloka ho tseba hore bana ba jwale ka ha report e re ba hlotswe ke geometry haholo, ba hlotswe ke eng, what kind of mistakes did they do (I agree with Mr Molete, these statistics are good, but we need to know that these learners, as the report says, that they did not do well in geometry mostly, we need to know, what is it that they could not do, what kind of mistakes did they make.)*

The above question shows that statistical analysis gives only a numerical analysis of learners' performance, but does not really take into account different levels of cognition imbedded in the nature of each topic. Furthermore, the statistical analysis reveals the existing patterns in learners' performance, but does not explain how these patterns are formed. For instance, there is no information on the specifics of what exactly learners did wrong, what content gaps they experienced, or what misconceptions they had with regard to Euclidean geometry. This information will give a better understanding of the patterns formulated.

The bricoleur borrowed tools of analysis from the traditional period, such as analysis of variance, t test, the boxplot, normal distribution and probability theory. These tools enabled the team to analyse learners' performance using what is considered to be objective and reliable tools that exhibit numerical patterns within learners' performance. Furthermore, the bricoleur recognises and acknowledges the limitations of the traditional moment of qualitative research in pursuit of the team's objective, namely, to find strategies to enhance teachers' TPACK for Euclidean geometry using ICT software. This recognition of the limitations was evident when Messrs Molete and Boke pointed out that, even though the statistical analysis revealed patterns that they would have not been able to discover at that level using other analysis techniques, there is still rich data to account for in the analysis. This led the team to look closely at the patterns exhibited by the tools suggested by the traditional period's analysis techniques, and to find ways to explain those pattern, and also their possible causes. The modernity phase of qualitative research provides tools for the bricoleur, and interpretive theories to make sense of these patterns.

After the research coordinator presented the findings of the statistical analysis to the coordinating team, it was agreed that there was a need for a more detailed analysis of data to take the process further and to determine exactly what it is that learners cannot do. We analysed the learners' responses on Euclidean geometry in an attempt to respond to the challenges identified, and illuminated the possible causes of the challenges. The following was found: learners were unable state the theorem; learners stated the theorem partially; few learners even attempted this question; learners wrote down mathematical statements without giving reasons; and learners provided incorrect reasons to support their claims (see Section 3.3.5.1).

4.2.3 Insufficient lesson preparation by teachers when they us ICT software as teaching aid

Preparing for a lesson entails conducting research to determine different ways in which a particular concept can be taught and understood, and what resources are needed for effective delivery of a particular lesson's content (Friesen & Francis-Poscente, 2014: 62). This research can be done through gathering own thoughts, and consulting other teachers, articles, video lessons and books to deepen one's understanding of, first, the content, and, second, pedagogy, to remedy learners' difficulties in understanding the content. On the same note, when preparing for a lesson that employs ICT software as a teaching aid, a teacher should, first, determine the lesson's aim, secondly, lesson objectives, thirdly, activities in a particular content area and then, lastly, select software tools that would enable a teacher and learners to achieve the lesson aim through the achievement of lesson objectives (Wetzel & Marshall, 2011: 74). When choosing ICT tools proper research by the teacher is needed to ensure that the tools are not chosen in isolation from particular content and lesson objectives. The lesson objectives must be realistic, measurable and possible to achieve within the period of the lesson (Johnson, Uline & Perez, 2011: 122). The integrated quality management system (IQMS) policy states that an outstanding lesson should be clear, logical, sequential and developmental (DoE, 2011: 19). This implies that, during the preparation phase, a teacher should select and order activities carefully so that learners at different levels of comprehension are able to build on their current knowledge and demonstrate lesson objectives.

Inconsistent with the best practice, during a meeting of coordinating team to explore various possibly effective ways to teach Euclidean geometry theorems, one of the teachers claimed that the HeyMath! software program was the best tool. This claim was made by Mr Mokoena, who assured the meeting that he could also demonstrate how easy and good HeyMath! is for teaching Euclidean geometry. The team agreed that Mr Mokoena should do a lesson demonstration on how to use HeyMaths! to teach Euclidean geometry. After connecting a project, screen and speakers to the laptop, he played the HeyMath! lesson, which started by saying,

> *Consider a circle with point O as a centre and AB as a chord.*

From the above extract, the use of the word “consider” in the context of this study means to think, imagine, and visualise. Therefore, above extract in connection with the contextual understanding of the word “consider” seems to presuppose that learners possess knowledge on the following: a circle, a centre of a circle and a chord. Instructing learners to imagine the preceding circle properties without assessing their prior knowledge is contrary to cognitive and metacognitive factors relating to the principles of learner-centred teaching, which state that good teaching creates an opportunity for learners to link new information with prior knowledge in a meaningful manner (Karolich & Ford, 2013: 31). Mr Mokoena failed to complement the HeyMath! software program’s lesson by assessing learners’ prior knowledge in order to determine if learners’ prerequisite knowledge level matched the level of abstraction that was impeded in the use of the ICT software he chose.

In line with good practice, Mr Mokoena, as a teacher, should have watched the HeyMath! software lesson during his lesson preparation, and should have realised that it does not assess learners’ prior knowledge, and that presenting the HeyMath! lesson could lead to learners failing to build on what they already know; and therefore he had to prepare in this regard. Mr Mokoena’s failure to assess learners’ prior knowledge shows that he didn’t prepare sufficiently, because he did not present activities to assess learners’ prior knowledge in order to ensure that new knowledge of the theorem builds on what learners already know. In his lesson introduction Mr Mokoena could have complemented the HeyMath! lesson by including assessment of learners’ existing knowledge on properties of a circle, e.g., giving learners an opportunity to draw the following: circle with centre O, a chord to a circle and a line from the centre to the chord of a circle. These activities are learner-centred, because learners are actively contracting knowledge. Alternatively, Mr Mokoena could have searched for either HeyMath! or other ICT software offerings that provide learners with the opportunity to assess their prior knowledge.

In addition, Mr Mokoena failed to prepare measurable lesson objectives, which was evident when he asked, at the end of his lesson:

> *Do you understand now how to prove this theorem?*

This question suggests that Mr Mokoena expected learners to be able to prove the theorem after they had watched the HeyMath! software lesson, but learners had not been made aware of what would be expected of them at the end of the lesson. Learners were merely confronted by the question above at the end of the lesson, and they could not respond. However, it seems that learners did not even understand the basic concepts of the theorem, such as perpendicular, which could imply that they did not understand the theorem completely. For instance, Teboho, in response to Mr Mokoena's question, asked a question:

> *Ntate (Sir) what is perpendicular?*

Mr Mokoena, in an attempt to respond to Teboho's question, asked:

> *Is there anyone who can explain to Teboho what is perpendicular?*

No learner could explain to Teboho what the concept of perpendicular meant – they all fell quiet instead of responding to Mr Mokoena. In line with good practice, as captured in literature, Mr Mokoena could have subdivided the theorem into manageable lesson objectives, which he could then have used as building blocks toward understanding the theorem (Johnson et al., 2014: 51). For instance, the theorem statement, "*a line segment drawn from the centre of a circle perpendicular to the chord bisects the chord*", could be considered as a lesson aim. Thereafter, other concepts, such as line segment, chords and their differences, perpendicular and a bisector, could be ordered logically and sequentially as measurable objectives of a lesson, spread over a period of 45 minutes (DoE, 2011: 19).

The coordinating team reflected on Mr Mokoena's lesson and, in an attempt to respond to the problems identified in his lesson, it was decided that Miss Lebaka would prepare and present the next lesson. Despite the fact that Miss Lebaka was part of the reflection meeting and had a written lesson plan, her lesson plan still showed lack of preparation for a learner-centred lesson. For example, learners' prior knowledge was not assessed, which means Miss Lebaka failed to prepare activities that would assess learners' prior knowledge.

From the above discussions it appears that teachers have a tendency to prepare insufficiently for class when they use ICT software as teaching aid. This tendency is observed in Mr Mokoena's failure to go through the HeyMath! lesson beforehand, to assess its strengths and weaknesses prior to his lesson facilitation. If he had done this, he would have been in a better position to complement the software, rather than using it to take over his role as teacher, as he did. Furthermore, Mr Mokoena's failure resulted in a lesson without prepared activities to assess prior knowledge, and no teacher participation during the lesson, until he asked one yes-or-no question at the end of the lesson. Similarly, even though Miss Lebaka seemed to be better prepared than Mr Mokoena, she also failed to assess the learners' prior knowledge of geometry concepts that they bring to class from their diverse backgrounds, in order to find out if there are any gaps or misconceptions, and to build on what they already know. Mr Mokoena and Miss Lebaka's failure to incorporate learners' background diversity and mathematics competencies contrasts with best practices on lesson preparation. For instance, the knowledge learners bring from home or previous grades is valuable in making sense of current mathematical concepts (Moloi, 2013: 483). This means teachers must do sufficient preparation to locate the new knowledge of Euclidean geometry concepts within exiting knowledge structures (Karolich & Ford, 2013: 31).

It should be said at this juncture that the tendency to do insufficient preparation represents a social injustice to learners and violates learners' right to quality education (DoE, 2012: 8). Furthermore, failure to prepare sufficiently resulted in ineffective teaching with ICT software (see Section 4.2.1.3), which creates a group of learners who consider mathematics to be a difficult subject. We understand that learners whose teachers prepare well for their lessons using ICT software, will forever dominate, since mathematics is one of the drivers of economic development and technology. This view confirms a report from the United Kingdom (Delpy & Kelly, 2012: 6) on measuring the economic benefit of mathematical science research. This report states that mathematics improves economic development and the daily lives of everyone. Moreover, these tendencies fail to comply with the emancipatory agenda of deconstructing apartheid-era education policies that excluded black South Africans from participating in learning mathematics (Walker, 2010: 901). Short-changing learners in this way means they are being excluded from pursuing science-based careers, such as actuarial science and engineering.

The act of insufficient preparation by a teacher, which results in ineffective teaching, can be construed as a teacher failing to understand that not preparing well will result in creating a group of learners that find mathematics difficult. Furthermore, the act of insufficient preparation violates the core duties and responsibilities of a teacher, as stated by the Education Labour Relations Council (ELRC), which says that teachers must prepare a lesson taking into account new teaching approaches and techniques, such as using ICT software to teach Euclidean geometry, as suggested by this study (DoE, 2003: C-67).

However, one of the possible causes of insufficient preparation is the way teachers like Mr Mokoena perceive computers: as a powerful source of knowledge that could even replace the teacher. The following are common phrases concerning computers in the communities where the study was conducted, and were expressed by Mr Mokoena and other teachers and learners. In expressing their excitement about what they find interesting about computers, they exclaim:

> *ntho ena ke komporo e ya iketsa (This thing is a computer, it makes itself).*

From the above extract, it seems that "komporo", which translates to "computer", is perceived as a powerful machine. This claim is supported by the use of the phrase "e ya iketsa", which translate directly to "it makes itself". In this context we deduce that Mr Mokoena and other teachers who do not prepare sufficiently for a lesson that uses ICT software, could believe that the computer is so powerful that it will be able to teach effectively on its own, since "it makes itself".

The results of the study point out that teachers do not do research or explore other ways that they can use ICT software to assess learners' prior knowledge. This confirms the results of Alazam, Bakar and Asmiran (2012: 74), who investigated teachers' ICT knowledge and skills, and found that teachers lack ICT skills (see Section 2.5. for more on this study). In addition, the study claims that teachers fail to set measurable lesson outcomes that are achievable within a period of 45 minutes.

The bricoleur, in pursuit of making meaning of the preceding deliberations, draws on a social constructivist epistemological stance, that knowledge is constructed through interaction with the environment (Habermas, 1971: 53). For instance, I understand that it is through my interaction with the classroom environment that I will improve my understanding of how ICT software is integrated in the teaching of mathematics and, for this study in particular, Euclidean geometry. Thus, I move beyond the traditional period and modernity phase's philosophy of reality, to how we come to know reality – I seek my objective view of the classroom environment. Rejecting these positivistic remnants in pursuit of a transformative agenda, I threaded to the seventh moment epistemological stance, which enabled me to capture the above extracts during a lesson, not as observations, like in the modernity phase, but as interactions with the environment to enhance my reflection on how ICT software is used in the classroom environment to privilege teaching and learning. I should state that I do not reject the modernity phase as a historical moment that provides a bricoleur with tools, such as data analysis techniques, but I do reject its philosophical underpinnings (see Section 2.2.1.1 and Section 2.2.1.2).

It is in the nature of the bricoleur to borrow tools from the whole spectrum of inquiry in the quest of meaning making. Thus, a bricoleur borrows the causal narrative found in the modernity phase of the historical moment as a tool to create a connection between the narratives and the way ICT is used in the classroom environment. For instance, consider the following narrative:

ntho ena ke komporo e ya iketsa (This thing is a computer, it makes itself).

The common perception about computers that is derived from the above narrative enables us to understand why teachers do not prepare sufficiently when they use animated computer lessons, and helps us to determine why teachers give up their role when they use animated computer software in lessons. Perceiving computers as powerful and self-creating could explain the manner in which computers are used. This is substantiated by Van Dijk (2007: 395), who describes a causal relationship between languages used and social action. In the context of this study the language used uncovered the way perceptions contribute to ineffective use of ICT software

From the above discussion we conclude that the ineffective teaching of Euclidean geometry using ICT software, as illustrated by teachers' failure to complement the ICT software programs, is a result of insufficient preparation. This confirms the findings of Wetzel and Marshall (2012: 73), that teachers do not prepare well when they are using

ICT software as a teaching aid (see Section 2.5.1.2 for more on this study). The study found that, when teachers use an animated lesson, they do not prepare the learners before they start watching the video for what will be expected of them at the end of the lesson. The learners discover this at the end of a lesson. Furthermore, the study revealed that there was a disjuncture between what the policy prescribes, that is, teachers must prepare for lessons, and what was observed in practice. Moreover, a social injustice was observed during the teaching of Euclidean geometry, when learners were denied their right to quality education (DoE, 2012: 8).

4.2.4 Inadequate lesson facilitation when using ICT software as a teaching aid

Good lesson facilitation based on social constructivist theory provides learners with the opportunity to interact socially when they are using ICT software to learn mathematical concepts (Hense & Mandl, 2012: 21). This implies that good teaching takes place when learners interact with the computer software and other learners to discover mathematical concepts. Gagné's conditions of learning theory asserts that learning takes place when learners' previous knowledge is accounted for during lesson facilitation, and integrated ICT software is used to create a learning situation that presents the learners with stimuli (Botty& Shahrill, 2014:100). Furthermore, Haji Botty and Shahrill (2014:101) argue that good lesson facilitation exhibits the following characteristics: it gains the attention of the learners, informs learners of the objectives of a lesson, stimulates learners, requires the recall of prerequisite knowledge, presents the stimulus material, provides learning guidance, elicits performance, provides feedback, and enhances retention and transfer. The theory of Vygotsky (1978 as cited in Siyepu, 2013: 5) argues that, when a teacher is facilitating a lesson, he/she should start off with what learners can do on their own, based on their prior knowledge, and then link or expand it with new knowledge that requires the assistance of teacher. Moreover, according to the CAPS document policy (DoE, 2011: 10) teachers must facilitate their lessons in such a way that learners can interact, do, talk, demonstrate and record their thinking.

Contrary to the policy on and good practice of lesson facilitation, Mr Mokoena, using HeyMath! software, did not create opportunities for learners to represent the theorem statement geometrically – he recited the statement, and immediately followed it by geometric representation. This is contrary to CAPS policy, which states that a lesson should be facilitated in such a way that learners can interact, do, talk and demonstrate their thinking (DoE, 2011: 10). During Mr Mokoena's lesson, learners were not talking, doing mathematics or even demonstrating their thinking – they sat quietly and listened to HayMath! This denied them the opportunity to enhance their problem-solving skills and strategies, such as interpreting the word problem and representing it geometrically (Moloi, 2013: 489). Instead, prior to every software explanation, Mr Mokoena could have paused and given learners the opportunity to analyse and share their understanding of the theorem statement by identifying important mathematical concepts that make up the theorem, and then representing the theorem statement geometrically. This would have offered learners an opportunity interact socially amongst themselves and with the teacher, which would have also helped the teacher assess learners' prior knowledge, and to identify and clarify any misconceptions and knowledge gaps (DoE, 2011: 10; Karolich & Ford, 2013: 35).

The inadequate lesson presentation continued to be evident in the narratives about HeyMath! software and Mr Mokoena's failure to complement the software lesson facilitation with his own contribution. This is how the lesson continued in attempt to prove that AC = CB using HeyMath!:

> *Here is a proof, join OA and OB.*

From the above extract it is clear that learners were not given any opportunity to discover the proof on their own. This is evident when Mr Mokoena, with the use of HeyMath! software, said, "here is the proof", which means that learners were given the proof without first attempting to prove the theorem on their own. Furthermore, the proof started by joining OA and OB. However, no explanation was given to learners as to why they had to join OA and OB. This seems to have made the presentation too abstract for learners to understand the theorem; it was clear their role in the lesson was to listen and reproduce when asked to prove the theorem. A question by a learner at the end of the lesson confirms the preceding argument:

> *Ntate (sir) how do you know hore (that) you must join OA, kapo re lokela ho joina OA (or are we supposed to join OA) all the time?*

The extract, "*how do you know that you must join OA*", shows that the manner in which the theorem was taught centred on a teacher or, in this case HeyMath! software, telling and showing the learners with animations how to prove a theorem, rather than learners being provided with guidelines by Mr Mokoena, as advocated by IQMS policy (DoE, 2011: 10). This is contrary to the cognitive and metacognitive principles of learner-centred teaching, which says that, for successful teaching and learning to occur, a teacher must support and give instructional guidance to help learners make coherent representation of knowledge (Karolich & Ford, 2013: 31). Considering the learner's question quoted above, it is unlikely to have happened in Mr Mokoena's HeyMath!-facilitated lesson.

The software lesson presentation facilitated by Mr Mokoena continued by proving congruency between two triangles; it proceeded as follows:

> *In triangle OAC and triangle OBC, angle OCA equals to angle OCB equals to ninety degrees.*

Mr Mokoena did nothing to complement the HeyMath! software lesson to ensure that learners were following or understood that "*angle OCA equals to angle OCB equals to 90 degrees*". It is not explicitly written on the diagram that the two angles are equal to 90 degrees, though various notations, symbols and concepts are used to describe their relationship, e.g., AB ⊥ OC and OC perpendicular to AB (see Figure 4.2.3a). However, Mr Mokoena did not explain to learners the different ways in which angles equal to 90 degrees can be notated. This is contrary to Vygotsky's views on semiotics, which state that it is only with the help of the mediator, or teachers in this case, that learners can internalise systems of signs and symbols (Ball, 2014: 27). Furthermore, comparing the two triangles in an attempt to prove congruency offered learners an opportunity to identify without first being told which sides and angles are equal in magnitude. Mr Mokoena could have paused the lesson and complemented the HeyMath! presentation, thereby promoting learner-centredness, and asked learners to identify, with reasons, the equal sides and angles in Δ OAC and Δ OBC (see Figure 4.1).

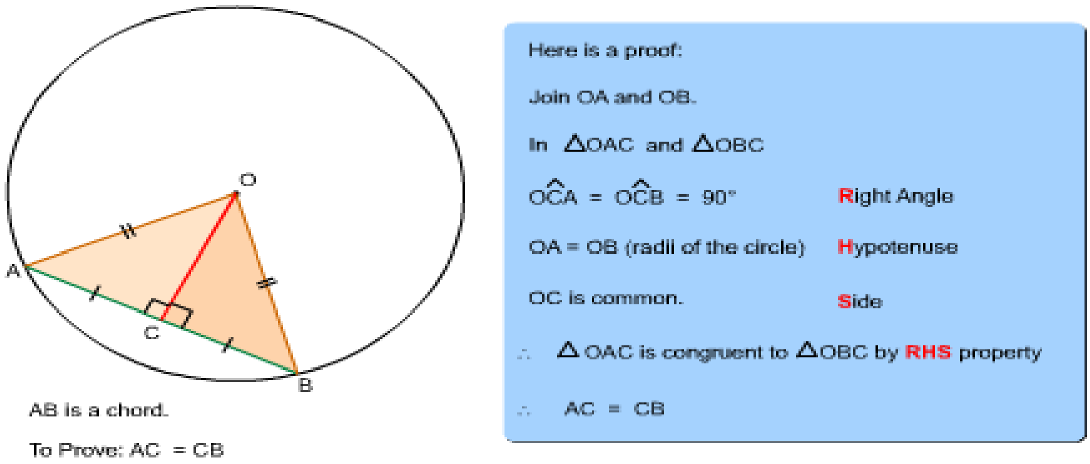

Figure 4.2.3a: Perpendicular bisector

The concepts of congruency, as shown in Figure 4.2.3a, were used to prove that BC = AC. However, the condition of congruency shown in Figure 4.2.3a is one of the other four conditions for congruency between two triangles to be established, namely, Side-Angle-Side (SAS), Side-Side-Side (SSS), Angle-Side-Angle (ASA) and Angle-Angle-Side (AAS). None of these conditions were explained as possible ways to prove congruency, which implies proving that BC = AC. For example, we can also show that the condition for congruency AAS holds since ∠ CBO = ∠ CAO because they are base angles of isosceles Δ ABO since AO = OB (see Figure 4.2.3a). Moreover, ∠ COA = ∠ COB because they are both equal to 90 - ∠ COA or 90 - ∠ CAO since ∠ CBO = ∠ CAO.

Finally, side AO = OB, because they are the radius of the same circle. Therefore, condition AAS holds, but Mr Mokoena, using HeyMath!, offered learners one solution to the problem in a very teacher-centred presentation – learners were passive listeners throughout the lesson. Additionally, the learners received no explanation for why congruence applied – not similarity or others. By complementing the HeyMath! presentation Mr Mokoena could have guided learners to find other, different conditions, in addition to the condition presented by HeyMath! in Figure 4.2.3a.

The manner in which Mr Mokoena facilitated his lesson did not give learners a chance to understand Euclidean geometry concepts. The lesson was facilitated in such an abstract way that learners could not comprehend the concepts. Moreover, Mr Mokoena's lesson was not presented in accordance with the principle of social constructivism, namely, that knowledge is cumulative. This was evident when his lesson did not start by establishing what learners knew in order to assimilate new knowledge or Euclidean geometry concepts in their existing knowledge. From the lesson by Mr Mokoena, we can conclude that learners were not motivated to learn, since they sat quietly, as instructed, unaware of the objective of the lesson or what was expected of them – this they only learned at the end of the lesson. Moreover, the activities of teaching were not connected and mapped together in a way that they built on one another (IQMS, 2011:10)

4.2.5 No integration of assessment during lesson facilitation when using ICT software as a teaching aid

Assessment is done at four levels, prior to the presentation of a new lesson, during the presentation of a lesson, at the end of a lesson presentation and after the lesson (DoE, 2012: 3). In the first instance, assessment is used to determine learners' prerequisite knowledge relating to the new lesson (Karolich & Ford, 2013: 35). Subsequently, it is used during the lesson presentation to determine the extent to which learners follow and/or understand the new content. This is to ensure that learners' misconceptions and other knowledge gaps about the new content as established during the prior knowledge assessment are addressed and assimilated appropriately (Spence & McDonald, 2015: 297).

Next, the assessment is geared to ensure that learners comprehended the new content as reflected in the lesson objectives. Finally, the post-assessment takes place long after the lesson and forms part of summative assessment, which may include and integrate with other topics (Ni Chronin & Cosgrave, 2013: 221). These assessments should be considerate of learners' different cognitive and affective domains, which must be developed in accordance with critical cross-field outcomes (Broom, 2015: 29). This means that, amongst others, the learners must be able to use technology responsibly (Ni Chronin & Cosgrave, 2013: 221).

During a lesson demonstration by Mr Mokoena, learners' prerequisite knowledge of the new content that was to be presented, namely, the theorem about the line through the centre of a circle drawn perpendicular to the chord, was not assessed. Mr Mokoena did not assess the learners during the lesson either, to ensure that he addressed their prerequisite knowledge and knowledge gaps that may relate to concepts, such as perpendicular, chord and bisect. The assessment done at the end of the lesson attested to the lack of prior assessment, because learners could not respond to Mr Mokoena's assessment question,

Do you now understand how to prove the theorem?

This question can be viewed from multiple perspectives, depending on the level of understanding and interpretation. First, this can be view as a simple question that can be responded to by yes or no. Secondly, it can be perceived as a rhetorical question or statement that describes the learners' understanding – remember that the learners did not respond to the question. The reason why learners failed to respond to the perceived rhetorical question may be that Mr Mokoena presented them with the proof of the theorem, which they all observed or saw. The third perception could relate to learners who were thinking deeply about the meaning of "understand how to" in Mr Mokoena's question. This group of learners may have found themselves confronted with further, deeper questions, which elevate Mr Mokoena's question, in my view, to a high-order question. This means that the learners understood and interpreted Mr Mokoena's question in their minds as demanding that they should prove the theorem. They are, however, frustrated by an inability to actually prove the theorem as it was presented by HeyMath! This frustration is clarified by a question by Tefo, who asked:

Sir, what is the meaning of perpendicular?

Tefo's question in response to Mr Mokoena's question confirmed, among other things, the ineffectiveness of using computer software to assess learning. Furthermore, some learners, frustrated about failing to understand the proof of the theorem as presented by HeyMaths! refrained from responding to the question. Mr Mokoena did not use all the resources available to enhance his assessment. For instance, he could have used Grade 8, 9 and 10 HeyMath! assessment activities on congruency, exterior angle equals to sum of opposite interior of a triangle (DoE, 2012: 3). These Grade 8, 9 and 10 HeyMath! assessment activities could have been given as homework or assignments prior to the lesson. Furthermore, he could have controlled and given feedback on this assignment with the learners (DoE, 2012: 3). This action would be in line with the national protocol of assessment, which states that learning should be provided with feedback after assessment to enhance the learning experience (DoE, 2012: 3).

The tendency of failing to assess the learners' prerequisite knowledge with a view to facilitating and enhancing their learning seems to contribute towards widening and complicating learners' knowledge gaps. This practice contributes significantly to poor performance in mathematics, because it leads to learners being unable to answer mathematics questions, as was the case in this study. This, in turn, creates a society of people who are mathematically illiterate or who find mathematics difficult to comprehend.

The findings from the discussions above point out that teachers such as Mr Mokoena seem to be using ICT software ineffectively. For instance, the manner in which Mr Mokoena assessed learners was not diagnostic or in line with the social constructivism's epistemological stance, that knowledge builds upon existing knowledge (Alexandra, 2013: 206). If Mr Mokoena's understanding of knowledge construction was in line with social constructivism he would have created activities that could have given learners the opportunity to interact with one another and their environment. Instead, he only asked a very vague question, which learners could have ignored or answered yes or no to. This is contrary to Brown, Chaudhry and Dhamija's (2015: 51) suggestion that assessment is used as a diagnostic tool to analyse learners' strengths and weaknesses regarding the content concerned (see Section 2.5.1.4).

4.2.6 Lack of research into appropriate computer software to teach Euclidean geometry

Research creates opportunities for improving the teaching of mathematics using appropriate software successfully (Scherer, Siddiq, Teo, 2015: 204). Through research teachers can keep pace with developments in rapidly evolving software technology for teaching (SAQA, 2012: 12). Furthermore, research provides a platform for teachers to be creative and innovative. Creativity and innovativeness in this respect includes integrating multiple software programs and drawing from each program's strength to ensure that abstract mathematics concepts, such as Euclidean geometry, are accessible to learners (see Section 2.5.1.6 for more).

Contrary to the above discussion, I analysed a lesson observed by the coordinating team members and facilitated by Mrs Kotudi. This is how the lesson was facilitated:

Mrs Kotudi: Afternoon, bana baka.

Learners: *Afternoon Mam.*

Mrs Kotudi: I want you to listen very carefully today because, nako ha e sa leyo (there is no time) we are not going to waste too much time on proving theorems ho bane ke ditools fella tseo re di sebedisang (they are just tools that we use), lot of marks are in application.

After she greeted the class Mrs Kotudi connected her HeyMath! laptop screen and the speakers, and then she said:

Theoreme ya rona ka jeno ke (our theorem today is), opposite angles of a cyclic quadrilateral are supplementary. Please ask if you do not understand or the computer is too fast for you.

Mrs Kotudi: Construction: join AO and OC, ha o qeta ho joina moo o ka reng?

Sipho   *Nna Mistrese nkare ABCO is a square*

The teacher and learners interacted throughout the lesson that used HeyMath! as a teaching aid. Despite the fact that learners could ask when they needed clarity and opportunities were created for them to work in pairs, the main cause of interaction was the fact that the manner in which Mrs Kotudi taught the theorem, "*the opposite angles of a cyclic quadrilateral are supplementary*", was very abstract and made it difficult for learners to understand the theorem. For instance, Figure 4.2 below is the snapshot of Mrs Kotudi's lesson using HeyMath!

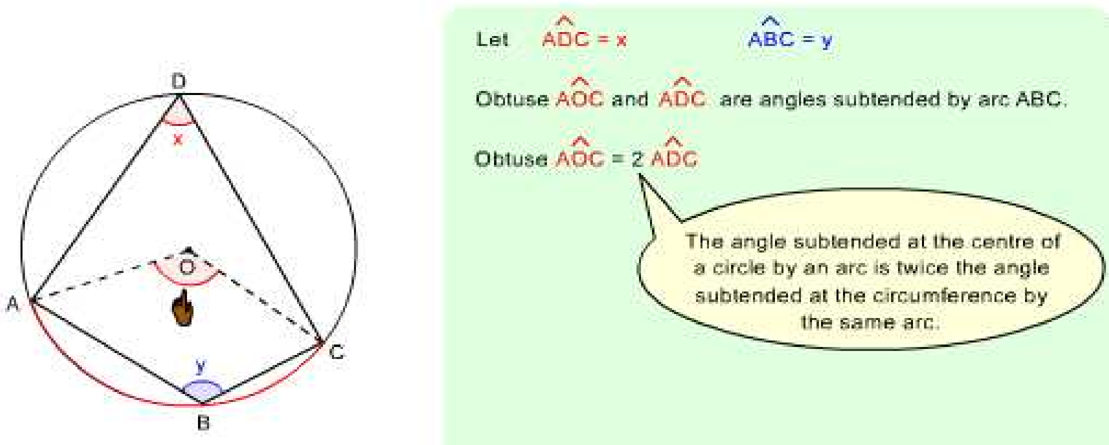

Figure 4.6.2: Opposite angles of cyclic quad are supplementary

Mrs Kotudi's contracted line AO and OC using HeyMath!, which are the radii of circle centred O, with the aim of using the theorem stated on the word bubble (see Figure 4.2.3a). However, learners did not understand why the constructions were made and this is evident when one learner, Thato, asked:

Hai Mam, do we always have to construct OA and OC?

From the extract above, it seems that Thato was confused, especially when he used the word, “hai”. The confusion from observing Mrs Kotudi’s lesson presentation was the result of abstract teaching of the theorem, which denies learners an opportunity to make sense of what they are learning (Reitmanova, Gustafson & Ahmed, 2015: 476). Thato’s statement also shows that the abstract teaching of the theorem promoted rote learning, especially when he asked, “do you always”, which suggests that he is looking for easy ways to remember how to reproduce the theorem when he is asked (Mengibar, 2015: 39). What Mrs Kotudi could have done was to do research and find other software that could be used concurrently or interchangeably with HeyMath!, such as GeoGebra and Geometer’s Sketchpad, rather than using only one software program. This concurs with literature, which suggests that teachers should conduct research to find different ways of teaching to ensure that learning takes place (see Section 2.5.1.6).

The other two software programs create an opportunity for each learner to use the software and interact with geometry content. For instance, GeoGebra and Geometer’s Sketchpad help learners to measure the magnitude of angles BAD, ADC, CBA and make deductions about the relationships between angles preceding angles, without them being told (see, section 4.3.4 and section 4.3.5). After the learners had developed a conceptual understanding of the theorem, Mrs Kotudi could have used HeyMath! to build on the concretised understanding, leading to an abstract logical thinking approach to proving the theorem. No research was done to identify appropriate software and/or to integrate different software programs to enhance the teaching of Euclidean geometry. This confirms the literature captured in section 2.5.1.6 that teachers do not research to improve their pedagogical practices. This was evident during lesson reflection, when Mrs Kotudi said:

> *…I wish there could be a way yeo bana bana (that these learners) could measure and see angles tse lekanang (that are equal).*

However, despite this good intention, Mrs Kotudi did not do research on the school’s free internet to find the appropriate software that she wished for. She did practically nothing, but complained that:

we are only given HeyMath!

This view is inconsistent with the fact that Mrs Kotudi had access to 60 computers with internet, which she could have used for research. If she did, she would have come across programs such as GeoGebra, which is free and, most appropriately, could have fulfilled her wish that learners could measure the magnitude of different angles. To make matters worse, Mrs Kotudi did not bother to answer Thato's very valid question, whether they have to "always do contraction?" when proving that opposite angles of cyclic quadrilateral are supplementary. Her failure to do research aggravated her ignorance regarding critical content issues, like "construction" when proving Euclidean geometry theorems.

Thabo's question created an opportunity for Mrs Kotudi to explain to learners the following content: the reasons for construction, how to construct and when to construct. However, Mrs Kotudi left Thabo with a dilemma – he lacked understanding of abstract concepts presented in the lesson, such as "construction". To this end appropriate software such as GeoGebra and/or Geometer's Sketchpad, would have been helpful. In this instance, the lack of research on appropriate ICT software on the part of the teacher, who was in the position of power to do so, disadvantaged and subjugated the learners unfairly (Van Dijk, 2006: 360)

From the above discussion it appears that Mrs Kotudi perpetuates acts of poor teaching practice that were prevalent during the apartheid era (Nkoane, 2012: 98). Her practices are examples of poor quality teaching of mathematics. Mrs Kotudi and other teachers seem to be ignorant of the power they have as educators to contribute positively to the social transformation agenda, to create and decompose social construction of communities that are founded on inequality (Van Dijk, 2006: 360). The existence of this social construction of communities that perpetuates inequality in education capital and therefore economic capital is sustained by teachers who do not do research and prepare thoroughly for their lessons. It is incomprehensible that this social structure still prevails, given a plethora of research, support and legislative mandates to redress the inequalities of the past. Mrs Kotudi had access to the tools that she could use to contribute to redressing these inequalities and effecting transformation in society.

4.3 ANALYSIS OF SOLUTIONS SUGGESTED FOR THE CHALLENGES IDENTIFIED

This section considers solutions to the problems associated with using ICT to teach Euclidean geometry, as expounded in Section 4.2. These solutions are considered as and understood to constitute components of the strategy to achieve the aims of using ICT to teach Euclidean geometry. Thus, the section considers formulation of a team; a shared vision and mission to drive the activities of team; the SWOT analysis for contextual analysis; and the priorities set by the team in response to the challenges identified in Section 4.2.

4.3.1 Formulation of a team

A team creates an opportunity for its members to learn from each other, creating a platform for people to share their knowledge and experiences in order to achieve a common goal. A team contributes to creating a sustainable learning environment, since knowledge and skills are not monopolised, but shared in the process of solving complex real-life problems facing the team members daily (Mahlomaholo, 2013: 4692). In addition, a team creates a platform for effective planning and implementation, and it enhances reflection after lesson presentation (see Section 4.2.1). According to the social constructivism theory, epistemological and ontological views on learning state that knowledge is socially and collaboratively gained through activities that promote human interaction (Schreiber & Valle, 2013:396). Thus, a team promotes social interaction between its members in pursuit of knowledge creation needed to teach Euclidean geometry. Furthermore, each member of the team brings his/her own unique worldviews to the team with regard to TPACK for teaching Euclidean geometry using ICT. A team creates an environment where multiple realities of the same thing are represented (Schreiber & Valle, 2013: 396).

The team formulation process of this study started when I (research coordinator) had a conversation with the deputy principal of a school in the area of the study. I initiated the conversation and elaborated my research interest. Following an in-depth inquiry about the nature and procedure of the study, Mr Phehello was excited and passionate about the study on behalf of his school and others nearby. This is evident in the text:

> *I think the study can help us haholo, hobane bana ba rona ba sokola ka mmetse haholo, le ho reng tichere wa rona wa mmetse a ka thabela thuso le ho sebetsa le wena (have you found people to work with, I think the study can help us a lot, because our learners are really struggling with mathematics and even our mathematics teacher will be very happy for your assistance and to work with you).*

The above extract suggests that Mr Phehello believed that the study held mutual benefit for the research coordinator, mathematics teacher, the school and learners in mathematics (Wallace-Henry, 2015: 18). The preceding argument is in line with the best practice of the team, which states that, in a team, complementary benefit is imperative for the team members (Schreiber & Valle, 2013:396).

The formulation of a team created a platform for the team members to interact socially in an attempt to create shared knowledge for teaching and learning of Euclidean geometry using ICT software (Andrews, 2012: 39). The postmodern period of qualitative research rejects the positivistic single "correct" way of data collection, and considers the extracts as a legitimate source of data that carries contextualised knowledge and meaning beyond the text. For instance, the text, "e ka re o ne o tseba", which translate laterally to, "it is like you knew", is understood to mean, in the context of this study, you came at the right time. This belief is also grounded in the epistemology of social constructivism, which asserts that knowledge is constructed through social interaction and does not reflect an objective external world beyond the team members. The postmodern period creates a space for the bricoleur to consider, not the big narratives and theories as the ideal case, like it is in modernity phase, but rather uses locally, situationally limited narratives (Given, 2008: 360). For instance, the words spoken by participants (co-researchers) are subjected to the bricoleur's interpretation, which is locally and situationally limited to the context of the study.

Furthermore, the postmodern period, as the fifth moment of qualitative research, provides the methodological bricoleur with PAR as a methodology that seems to give hope to participants (Mahlomaholo, 2012: 9). For instance, the extracts above suggest that some teachers have lost hope, like Miss Lebaka, who used the phrase "ke kgathetse matla" which translates, in this context, to, "I have lost hope". Losing hope, when read in connection with, "maybe using ICT will help us", suggests that Miss Lebaka has tried different teaching methods and aids and learners' performance has remained unsatisfactory, but she has not tried ICT software. However, we understand that using PAR as research approach, as advocated by Chilisa (2012: 250) and Mahlomaholo (2012: 9; 2013: 386), gives hope to the participants, because its underlying principles advocate that people who are experiencing challenges are in the best position to respond to their challenges. In addition, the formulation of a team provides a multitude of diverse voices, as captured in the above extracts, working together to formulate a responsive strategy to improve the teaching of Euclidean geometry using ICT software.

In closure, the preceding discussions points out that the team structure in this study is similar to that of Japanese lesson study. The team consists of mathematics teachers from three schools in one community, and one academic from a local university (see Section 2.5.1.1). In a South African context, Jita and Mokhele (2014: 1-15) claim that clusters are one of the effective approaches for achieving continuous professional development. They further argue that, when using clusters, collaboration between teachers enhances the approach and contributes significantly to improving teachers' CK and PCK (Jita & Mokhele, 2014: 8). In this study the formulation of a team sought to create a space for teachers to share good practices of lesson preparation and lesson facilitation. The contribution that this study makes is that, the collaborative approach is made not by teachers only but also the principal, IT programmer.

4.3.2 Formulating a team vision and mission

A vision and mission guide daily activities of a team or an organisation and foster a shared purpose among members of the team. Darbi (2012: 95) explains that a vision and mission motivates, models behaviour, and promotes a high level of commitment, which leads to cultivating perfomance. Furthermore, Kantabutra (2008: 127), in his study of what we know about a vision, asserts that a vision provides a cognitive imagination of the desired future state. A shared vision creates an orientation and meaning for the team members and it acts as a strong driving force for continuous and systematic development (Martin et al., 2014: 1). A vision should be attractive to the team members if they are to be committed to turning it into a reality (Martin et al., 2014; Wong & Liu, 2009: 2884).

In relation to the discussion above, we found it necessary to formulate a common vision for the team after we agreed to work together. During the meeting to formulate a vision one of the team members, Mr Phehello, asked:

Ke na le maikutlo a reng (I have an opinion that says), if we agree we should leave here knowing exactly what is it that we want to achieve in the long term as a team.

From the above extract, "knowing" confirms that the team members consider it important to know what they want to achieve. This opinion was aggravated by the word "exactly", which describes the manner in which the team members should know what they want to achieve as a team at a very specific descriptive level. This seems to be in line with the best practices on visions, which state that a vision creates orientation and specific meaning for the team members, because they obtain meaning and orientation through knowledge of what they are doing (Martin et al., 2014: 1). Furthermore, the word "exactly" suggests that what the team members want to do should be clear and specific, which is in line with literature that argues that an effective vision for the team should be clear and specific for each member of the team (Jorge, 2013: 122).

Subsequently, Mr Molete, in agreeing with Mr Sello, explained why it is important to know what we wanted to achieve as a team, by saying:

Ho tseba hore re batla ho fihlelang (to know what we want to achieve) will help us to start planning accordingly hore re ifumane rentse re le motjheng (so that we find ourselves aligned).

In the above extracts, Mr Molete repeats the word "knowing", which seems to emphasise the importance of knowledge that team members need in order to realise their vision. Mr Molete went on to say that this knowledge will shape the planning of activities, which is in line with the best practises that a vison models, and will guide daily activities of an organisation (Darbi, 2012: 95). The members of the coordinating team responded by stating what they wish to achieve as a team, and Miss Lebaka suggested that:

Nna ke nahana hore (I think that), we want to be a team that is well known for work together to better teach mathematics re sebedisa le tsona disoftware (also using this software).

From what Miss Lebaka said, one can assume that the members of the team had a very high ideal, as they did not only want to be "known", but they wanted to be "well known" – and not only known to themselves but also to others. The phase, "well known" appears to suggest that the team is highly motivated, or persuaded by what they want to do, because they already consider it possible to be well known by others. This seems to be in line with the best practice for a vision, that it becomes a driving force to dream and aim high (Archbald, 2013: 4; Martin et al., 2014:2). This was confirmed by Mr Sello, the chairman of the meeting, when he summarised what the team members had agreed upon by saying:

> *Re dumellane ka hore we are striving to become a team e tsebahalang ka ho sebedisa ICT to teach Euclidean geometry ka mokga oo bana ba tla utlwisisa ka teng.(we have agreed to strive to become a team that is known for using ICT to teach Euclidean geometry in a manner that learners are able to understand).*

The above extract became the shared vision of the team, which is, becoming a team that is well known for the effective use of ICT in teaching mathematics, which includes Euclidean geometry. We considered multiple representations of views on constructing a shared vision make it possible to understand better what the team wants to achieve. The bricoleur, operating in the postmodernity moment, present and the future, considered each member of the team as a co-researcher who enhances the understanding of their shared vision. Thus, the theoretical stance of bricolage in knowledge construction creates a platform for co-researchers who are experiencing the challenges of using ICT, so that they are in a better position to shape the vision so that it is more realistic for their purposes. The findings of the study confirms what literature has found about a vision.

4.3.3 SWOT analysis

A SWOT analysis is a strategic evaluation tool that the coordinating team used to assess the strengths, weaknesses, opportunities and threats in pursuit of responding to the challenges they are facing with the teaching of mathematics (Ayub & Razzaq, 2013: 93). The SWOT analysis was used as an information-gathering tool concerning the team's competencies. In this study SWOT analysis is used to map the information provided by the analysis with the information gathered through literature on skills and resources needed to improve teachers' TPACK in order to direct the energy of the team towards opportunities that were identified (Ayub & Razzaq, 2013: 93). Furthermore, SWOT analysis was used to identify the threats to improving teachers' TPACK, and finding stategies to overcome the threats. This section presents that SWOT analysis organised under the subtitles CK, PCK and TPACK.

4.3.3.1 Content knowledge

CK as subject matter refers to knowledge of facts, theories and ideas of a particular subject. Lee Shulman's PCK describes CK as "accepted truths" within a particular discipline (Shulman, 1986: 8). However, epistemological accessibility of "accepted truths" needs to be localised and contextualised in order to make meaning. The fact that the "accepted truth" is considered universal influences a greatly many teaching practices, as underpinned by the traditional moment epistemological and ontological stance that considers CK to be a list of facts and theories. However, since CK is what needs to be taught, as prescribed by the FET mathematics curriculum policy, CAPS, then it is important that CK is deeply understood, to ensure that it can be translated from national to local context. Therefore, in order to operate optimally as a team in formulating a strategy to teach Euclidean geometry using ICT software, it is imperative that there is a deep understanding of Euclidean geometry content knowledge by at least one of the team members. A deeper understanding of Euclidean geometry content knowledge was demonstrated by one of the team members, Mr Molete, who said:

> *Euclidean geometry enhances learners' logical reasoning.*

Mr Molete was supported by Mr Phehello, who said:

> *Ke dumellana le ntate (I agree with you, Sir), for instance learners begin to learn that if A equals to B and B equals to C then A is equal to C.*

In the above extracts Messrs Molete and Phehello describe the importance of Euclidean geometry, which includes the knowledge and skills of reasoning logically and making conjectures. Their high level of competence with regard to Euclidean geometry content knowledge is demonstrated further in Section 4.3.4. Despite their high level of competence in Euclidean geometry content knowledge their mathematics learners' performance in Euclidean geometry was unsatisfactory. Thus, it became necessary for the study to find ways to create an environment that would enable learners to demonstrate the same high level of competence in Euclidean geometry as their teachers. I should also mention that not all team members had adequate training on Euclidean geometry content knowledge – some had none. Thus, having team members who understood the content of Euclidean geometry created a space for sharing CK, guided by the principles of the seventh and eighth moments, which are tolerance, solidarity, respect and hope. I then suggested that using ICT software could improve teachers' pedagogical practices, which leads us to the section on TPACK.

4.3.3.2 Technological pedagogical content knowledge

This section starts by discussing PCK, followed by the addition of the technology leg, which creates TPACK. PCK for teaching Euclidean geometry takes us a step further from knowledge of facts, theories and ideas of Euclidean geometry CK, to knowledge related to creating an environment in which learners can easily learn (Ball et al., 2008). Thus, for the purpose of teaching Euclidean geometry, PCK is important for formulating a strategy to teach Euclidean geometry. Using lesson study created a platform for team members to share their teaching methods and wealth of experience, which then created an opportunity for them to learn from one another (see Section 4.3.5). Training on the lesson study approach to lesson preparation and facilitation that one team member had undergone became an opportunity and provided strength to formulate a strategy (Section 4.3.4).

The TCK further enhances PCK by introducing the use of technology to teach particular content. One of the team members, Mr Boke, and I experienced using ICT software to teach mathematical concepts and how to provide access, which contributed to the team's strength and provided opportunities for formulating the strategy. For instance, Mr Boke was a computer programmer who helped to improve the team members' TK concerning technical computer challenges. Furthermore, Mr Ndaka shared his experience and knowledge of creating video clips and converting them so that they can be viewed on cell phones with the team members.

4.3.4 Sufficient preparation for facilitating a lesson with the aid of ICT software

Sufficient preparation for a lesson that employs ICT software as teaching aid requires the following important components: determining the lesson aim; setting the lesson objectives; formulating activities to assess whether lesson objectives have been met; and selecting the ICT software (see Section 4.2.2). In order to ensure that these requirements were met, the coordinating team worked together to prepare adequately for a lesson. A preparation session was organised and presented at one of the schools where the study was being conducted. The purpose of the meeting was to find out what mathematical topics would be taught throughout the week and to help each other to plan accordingly. The meeting chairperson, Mrs Kotudi, started the conversation by saying:

> *Colleagues, based on our agreement, re lokela ho fumana hore re rutang bekeng yena, le hore ke mang yeo re mo obzevang (we need to find out what topic we are teaching this week and who we are observing).*

Form the above extract it is clear that teachers were beginning to prepare together for a lesson and the necessary arrangements were being made prior to the preparation session. This is evident from the extract, when Mrs Kotudi referred to their agreement, that the arrangements had been made prior to the preparation session. Furthermore, the question, "what are we teaching this week", suggests that teachers are meeting every week to prepare the work that would to be taught over a week together. This seems to be a new approach that the teachers had developed as a result of functioning as a team driven by a vision. This was evident when Mrs Kotudi asked, "who are we observing" which refers to a team of teachers observing another teacher facilitating a lesson for the purpose of improving lesson facilitation with ICT software.

After Mrs Kotudi's question, Miss Lebaka volunteered to be observed by the team, by saying:

> *Colleague ke kopa ho ikgetha, I will be doing Euclidean geometry this week starting with the theorem: a line drawn from the centre of the circle perpendicular to the chord bisects the chord.*

In the above extract Miss Lebaka seems to have identified a lesson aim**,** which states that, *at the end of the lesson learners should be able to prove and apply the theorem that says a line segment drawn from a centre of a circle perpendicular to the chord bisects the chord* (Johnson et al., 2014: 51). Furthermore, the team members seem to be very committed to the teamwork preparation. This is evident when Miss Lebaka says, "I am requesting to volunteer myself to be observed". Volunteering by Miss Lebaka seems to suggest a positive attitude towards preparing and working as a team. This approach to teaching creates a group of teachers who help each other to improve their TPACK for teaching Euclidean geometry.

4.3.4.1 Content knowledge

In preparing for a lesson it is important that teachers possess a competent level of CK of Euclidean geometry. Therefore, during the preparation for a lesson the coordinating team prioritised CK of Euclidean geometry. This was impelled by the understanding that it is through a deeper understanding of the CK of Euclidean geometry that its PK, TK and then TPACK could be improved. Thus, during the preparation the team members grappled with the content of Euclidean geometry, and Mr Molete said:

> *It is important to understand the definition of a circle, that it is a set of points that are equidistant of a given point.*

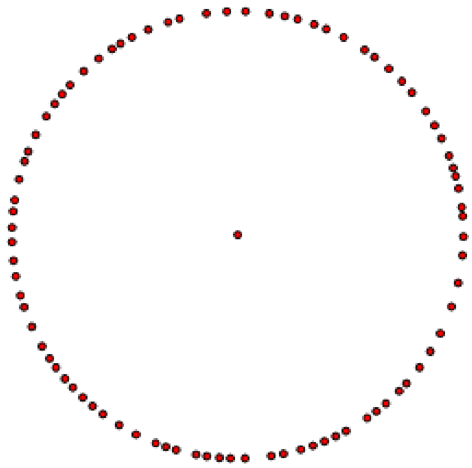

Figure 4.3.4.1a: Definition of a circle: Equidistant points from a central point

Figure 4.3.4.1a is a pictorial representation of the definition of a circle given by Mr Molete, which is, a set of points equidistant from a given point. The “equidistant” refers to equal distance between the point in the centre and all other points. This is explained further by Mr Mokoena:

> *This means that since all the points are equidistant from the circle then they make the radius of the circle.*

From the above extract, Mr Mokoena could connect the “equidistance” with the radius of a circle, which further improves the team’s understanding that the only way that these points could be equidistant from the middle point is if they circulate around the point in the middle. This was clarified further by Miss Labaka:

> *Oh keya bona jwale, ho bolelang (Oh, I see now, which means), you can take a point and then circulate it equally around a particular point to form a circle.*

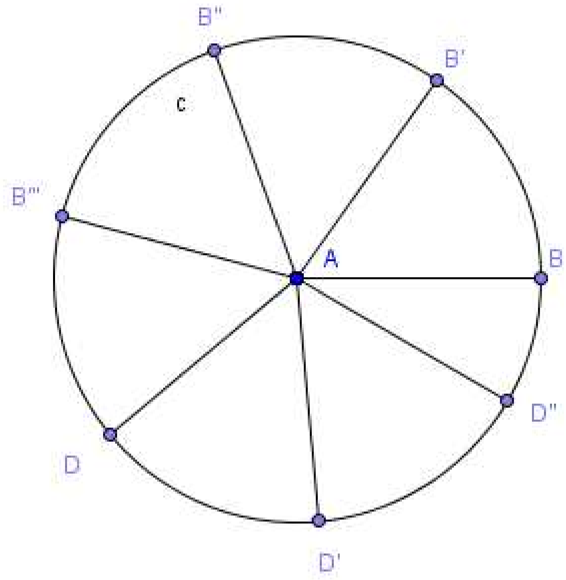

Figure 4.3.4.1b: Definition of a circle: Equidistant points from a central point

Miss Lebaka's extract above shows that, through the team's interaction, she was able create a connection between the middle point and the other set of points that are equidistant to it. The interaction further revealed that the teamwork created epistemological access to improving CK through social interaction during lesson preparation. This is evident from Miss Lebaka remark, when she reacted to the contribution made by other team members: "Oh, I see now". This demonstration of grappling with the CK by a team member confirms Lee Shulman's notion of PCK, which states that teachers should demonstrate high mastery of the subject matter or CK (Kleickmann, Richter, Kunter, Elsner, Besser, Krauss et al., 2013: 91). This was further evident from Miss Lebaka's explanation, shown in Figure 4.3.4.1b, which demonstrates a high level of processing of the CK. For instance, Miss Lebaka showed that, if you construct line AB and rotate point B an infinite number of times, all the points will lie on a circle and will be equidistant to point A. This explains the concept of a radius of a circle well. The team's desire to improve their CK was further evident as the team members analysed and discussed the theorem statement in an attempt to deepen their understanding of the theorem. For instance, during the discussion Mr Phehello said:

> *I will suggest that we subdivide the theorem into main clauses and identify key concepts individually and thereafter have a discussion.*

Subsequently, the team subdivided the theorem by breaking the theorem statement into short phrases, which could be geometrically represented.

> *Mr Molete: I subdivided the theorem into the following: one, line drawn from the centre of a circle; two, perpendicular to the chord bisect the chord*

From what Mr Molete said, perpendicular chord and a line can be represented geometrically as shown in Figure 4.3.4.1c.

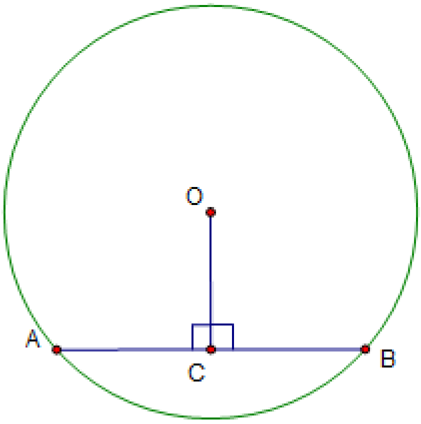

Figure 4.3.4.1c: Subdividing the theorem into clauses

Miss Lebaka noted that it is important to differentiate between the mathematical concepts identified:

> *Mr Boke: OC is a line while AB is a chord.*
>
> *Mr Phehello: But why re re jwalo ntate (By why are we saying that, Sir)?*
>
> *Miss Lebaka: I think it's because A and B are on circle.*

From the above extract of the dialogue during the preparation concerning Figure 4.3.4.1c line AB is a chord since its end points A and B intersect with the circumference of a circle. However, line OC does not qualify to be a chord since its end points O and C do not intersect with the circumference of the circle. However, line AB is perpendicular to line OC since angle BCO and angle ACO are equal to 90 degrees. Mr Phehello suggested that the key concepts should also be identified. The key concepts were understood to be the building blocks that make up the main clause and/or theorem statement and this is evident from the above discussion.

4.3.4.2 Pedagogical content knowledge

This section captures the way the team's interaction improved the PCK. During the same meeting on lesson preparation discussed in Section 4.3.4.1 the team transformed the CK from what Ball et al. (2008) call common content knowledge to special content knowledge. CK was transformed through a question by one of the team members, Mrs Kotudi, when she asked:

> *How can we better explain this theorem then?*

Mrs Kotudi responded without waiting for a member's response to her question, by saying:

> *It is important to create an opportunity for learners to do the same. Ene e tlo re thusa hore re identifae (and this will help us so that we can identify) their understanding of this concepts before we teach the theorem.*

The excitement shown by Mrs Kotudi spilled over to the learners, as will be explained later, in Section 4.3.2. This excitement is indicative of the motivational and affective factors that appeared to have influenced the willingness to work and to learn. Mrs Kotudi's excitement confirms that preparing as a team improves teachers' PCK of Euclidean geometry, in this case, when teachers exchanged ideas and viewpoints in pursuit of the best possible ways to teach the theorem.

> *Mokgwa ona o tlo refa monyetla wa hore re bone ho re ba tsebang (This method is going to give us an opportunity to see what they know).*

Mrs Kotudi found it important that learners be given an opportunity to analyse the theorem statement before they learnt to prove the theorem – this helped the teacher to assess their prior knowledge and clear up any misconceptions. This is evident from the phrase, "give us an opportunity to see what they know", which suggests that the approach of creating an opportunity for learners to analyse the theorem statement will enhance teachers' knowledge of learners' understanding of the theorem prior to teaching. Furthermore, the use of the word "see" suggests that learners will not only be talking, but they will also be physically demonstrating or writing down their thoughts for the teachers to see. This is consistent with the provisions of the DBE through the IQMS policy (DoE, 2011: 19), namely, that lesson preparation should include preparation of a series of activities that are logical and sequential to assess learners, from prior knowledge to demonstration of achievement of lesson objectives.

However, in order to guide the learners towards analysing the theorem statement the coordinating team prepared a worksheet, which included, among other things, assessment activities to assess learners' prior knowledge. For instance, from the above discussions on analysis of the theorem statement, the following emanated. Learners should identify mathematical concepts that make up the theorem statement. The communicative spaces created by the establishment of a coordinating team, in this case, was a necessary condition for availing and accessing tools that could otherwise have been missed.

During the lesson preparation the team members emphasised the importance of showing the connection between mathematical concepts. This was said by Mr Boke:

> *Colleagues, I think there has to be that connection between the concepts, for example, how do we take advantage of concepts such as reflection about a line, rotation about a point from analytical geometry to Euclidean geometry?*

Mr Boke talks about the PCK for teaching Euclidean geometry by presenting an argument that mathematical concepts should not be taught in fragments. In response to Mr Boke's question, one of the team members, Mr Phehello, connected a reflection about a line with the concepts of congruent figures.

> *If ke na le (I have) a figure, and I then reflect or even rotate, the reflected or rotated figure will always be congruent to the original figure.*

In agreement with Messrs Boke and Phehello, I demonstrated what they had said by making use of GeoGebra ICT software.

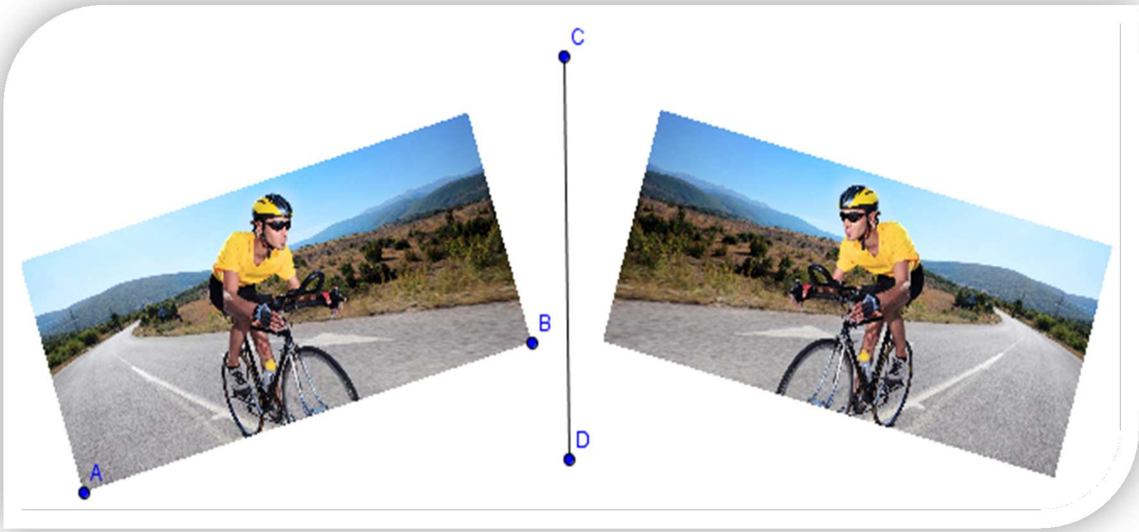

Figure 4.3.4.2b: Reflection about a line

Using a picture to explain the concept of mirror image makes it easier to understand the reflection about a line.

4.3.4.3 Technological pedagogical content knowledge

In preparation for lessons we improved our pedagogical practices further by looking closely at ways could we use various ICT software to teach Euclidean geometry concepts. For instance, knowing about alternative ICT software, namely Geometer's Sketchpad and HeyMath! was within the proximal zone of development for the team members (Poehner, 2012: 610). This suggestion came from one of the team members who happened to have knowledge and tools at hand regarding the use of Geometer's Sketchpad. I then suggested:

> *One of the software that can help is Geometry Sketchpad, and has animation which can help learners discover certain things.*

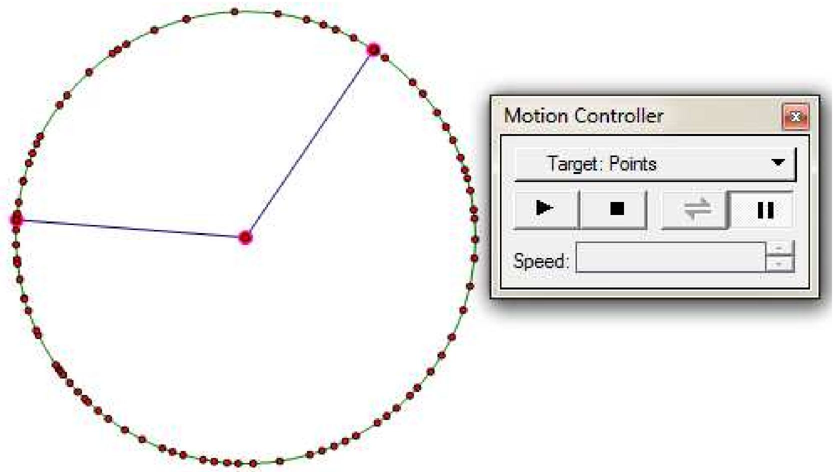

Figure 4.3.4.3a: Equidistant points and radius

Using Geometer's Sketchpad software for teaching the concept of a circle, as shown in Figure 4.3.4.3a, improved the teachers' TPACK in the following ways:

(i) It improved teachers' communication of the definition of a circle in a way that not only verbally communicated but was also pictorially accurate, as opposed to what Miss Lebaka had suggested (Section 4.2.1.2). This is evident from Mrs Kotudi's comment:

> *Ya ha o sebedisa this GSP [Geometer's Sketchpad] o kgona le ho bontsha re what is equidistance (Yes, when you use this GSP you are able to show what is equidistance).*

The above extract attests to the pictorial communication of the definition beyond the verbal explanation.

(ii) Using Geometer's Sketchpad to explain the concept of a circle improved the team members' TPACK by contributing to the teaching methodology. For instance, using Geometer's Sketchpad during the lesson preparation improved teachers' understanding of the definition with its animation properties. This was evident when Mr Phehello said:

> *So, as this point is moving, o kgona ho bona hore its moving along the circle (you can see that its moving along the circle).*

Mrs Kotudi responded:

> *Yes Ntate, sheba hore even when the radius are overlapping di ya banala ho re they are equal (Yes, Sir, look even when the radii are overlapping it is clear that they are equal).*

From the above extracts by Mr Phehello and Mrs Kotudi, it is clear that the animation properties offered by Geometer's Sketchpad improved their understanding of the construction of a circle. The animation enabled them to visualise the definition of a circle accurately and enabled them to do the constructions. Using Geometer's Sketchpad not only improved the use of CK but further improved the team members' PCK.

4.3.5 Lesson facilitation with the aid of ICT software

Adequate lesson facilitation involves social interaction between learners and the teacher. CAPS policy (DoE, 2011: 10) is in agreement with social constructivism, which states that good lesson facilitation enables learners to interact, do, talk and demonstrate their thinking. In order to achieve the preceding attributes as set out in the CAPS policy document, the use of ICT software to teach Euclidean geometry should ensure social interaction among the learners, and between the teacher and learner(s) during the process of lesson facilitation. During the lesson facilitation learners work individually on computer software dealing with specific section(s) of the Euclidean geometry theorem, while the teacher monitors them and engages them as soon as they experience challenges. The learners should also feel free to engage each other on areas they deem fit, to share ideas about the theorem being discussed. This practice is line with postulations by Vygotsky on guiding learners, from what they already know to the intended lesson objectives (Poehner, 2012: 611).

4.3.5.1 Theorem prerequisite knowledge using Geometer's Sketchpad

Pursuant to the implementation of the preceding principles, a coordinating team member, Miss Lebaka, facilitated a lesson on the theorem, *a line drawn from the centre of a circle perpendicular to the chord bisects the chord*. The other coordinating team members observed the lesson facilitation and were required to take notes on specific items for purposes of post-lesson reflection. These items included but were not limited to the use of ICT for enhancing social interactions, the role of the teacher during lesson facilitation, and learners' ability to link prior knowledge, e.g. perpendicular, chord, line, radius, and diameter, with theorem. During the lesson's reflective session, the facilitator, Mr Molete, gave a summary of the deliberations related to learners' engagement and the role of the teacher. He said:

we agree that learners were working better ka dicomputer (with computers) le hore bane ba thusana (and also that they were helping each other), example, ntate (Mr) Molemo was showing me that even Ntswaki wa Ntswaki o ne a gona le ho bontsa babang (Ntswaki was able to explain to others) how to measure and resize an angle using the mouse.

It is evident from the extract above that there were instances where, during the lesson facilitation**,** learners supported each other, like when Ntswaki, who seems to have understood better, was able to help Sellwane to measure the magnitude of an angle using Geometer's Sketchpad software. This is in line with the best practice on learner centredness, namely, that learning takes place when learners are able to interact with one other in pursuit of making meaning of a particular Euclidean geometry concept (Schweisfurth, 2015: 259). Moreover, the two learners were tolerant of each other's strengths and weaknesses, and used their strengths for their mutual benefit. What the two learners demonstrated during a lesson facilitation affirms Leanna's (2014: 4) view that socially constructed knowledge is the result of multiple efforts pulled from different individuals. For instance, during the lesson facilitation, this is what the two learners said,

Ntswaki: E re ke o bontse hore e etswa jwang, o tla mpontsa tsane tse circles during the study (let me show you how to do it, you will show me how to do the ones with circles during the study).

Sellwane: Eya hle, re tla bua le titjere re tlo sebeletsa ka monna ka study (yes, please, we will talk to the teacher so that we can work here during the study).

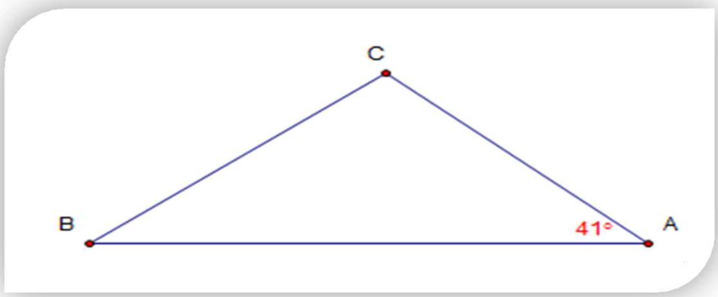

Figure 4.3.5.1a: Measuring angle BAC

Ntswaki: A ke re angle ke three points, so ha o batla angle A, o tlo clicka B, A, C kapa C, A, B then you go to measure angle (Isn't it that the angle is made of three points, so when *you are looking for angle A, you will click B, A, C or C, A, B then you go to measure the angle).*

The learners' social interaction reported above confirms what Messrs Molete and Molemo had said, namely, that lesson facilitation created a space for leaners to learn from each other (Tshelane, 2013: 414). Furthermore, the learners' interaction reported above provided evidence of learners taking responsibility for their own learning. This was supported by Sellwane when she said, “we will talk to the teacher so that we can work here during the study”, which also shows that learners are self-driven and that they can negotiate with the teacher to use the Geometer's Sketchpad software in the lab during their study time. This kind of approach to teaching created a space of operation that Habermas (1971: 51) calls communicative action, were learners mutually agree on a particular common goal, in this case, learning Euclidean geometry (see their extracts above). In another instance, Ms Sebolai reported her observation of the two learners who seemed to have been on the same level of understanding, who compared the sizes of angles of a triangle of which two sides and base angles where equal. For instance, Khanya said:

Khanya: e re ke bone, nna ke fumane masaete a mabedi a lekana, le angle tse pedi tse lekanang (Let me see, I got two sides that are equal and two angles that are equal).

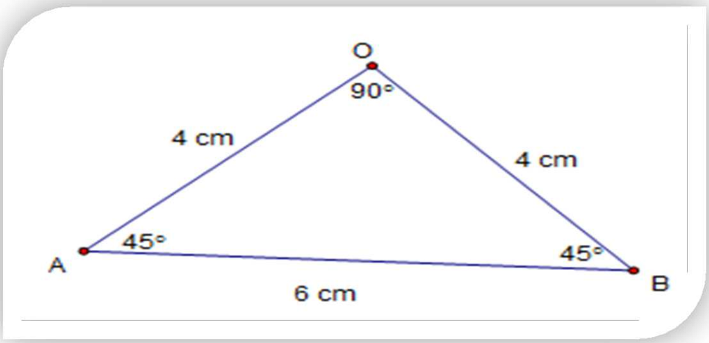

Figure 4.3.5.1b: Investigating properties of isosceles triangle

From Figure 4.3.6b and Khanya's extract, it is clear that he was able to use Geometer's Sketchpad to construct a triangle ABO and measure the sides and angles of triangle ABO, finding that angle ABO is equal to 45 degrees, angle BAO is equal to 45 degrees, angle AOB is equal to 90 degrees and side AO = BO = 4 cm. Similarly, Lerato was able to do the same, and she said:

> *Le nna, mara masaete a hao ha tswane le a ka, o na le 5 cm ke na le 7 cm (Me too, but your sides are not the same as mine, you got 4 cm, I have 5 cm).*

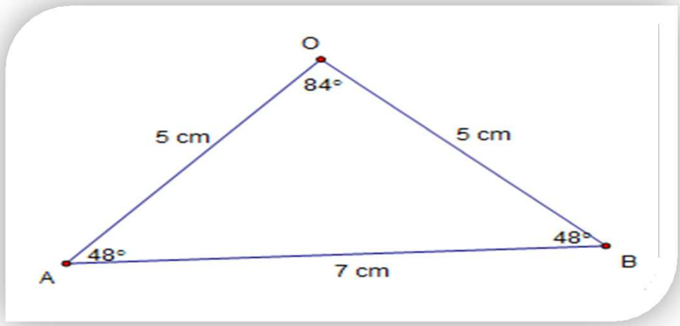

Figure 4.3.5.1c: Investigating properties of isosceles triangle

From the above extracts we can deduce that learners' different measurements of angles and sides created an opportunity for them to draw on the multiple perspectives each brought to the discovery process, leading them to conjecture that a triangle will always have two equal sides if it has two equal angles, as it is in the theorem. This was evident when Khanya said:

> *Ha re troye ya boraro, maybe re tla bonna ho re e dumanlana le mang (Let us draw the third one, maybe we will see who it agrees with).*

From the extract above it seems that learners are activity-constructing knowledge, for instance, when Khanya used the phrase, "lets draw the third one", it shows that learners were activity-manipulating Geometer's Sketchpad software to draw triangles with different side and angle magnitudes. Furthermore, not only did these learners interact with the computers but they also interacted socially among themselves, e.g., Khanya and Lerato; Ntswaki and Sellwane. This is in line with the epistemological stance of bricolage, which argues that knowledge is constructed through discussion of multiple perspectives between the learners or epistemic communities, who piece their arguments together (Mahlomaholo, 2013: 387). It as further realised that both bricolage and social constructivism's epistemological stance became evident when Khanya and Lerato discovered the properties of isosceles triangles:

Lerato: ntho e tshwanang ditraengeleng tshena tse tharo ke ho re two sides are equal le ha ele hore triangles tsona ha di lekane (Things that are the same in the three triangles are that two sides are equal even though the triangle is not equal).

Khyanya: Ho bolelang hore isosceles ena le masaete a mabedi a lekanang le diangle tse pedi tse lekanang (Which means that, isosceles has two sides that are equal and two angles that are equal).

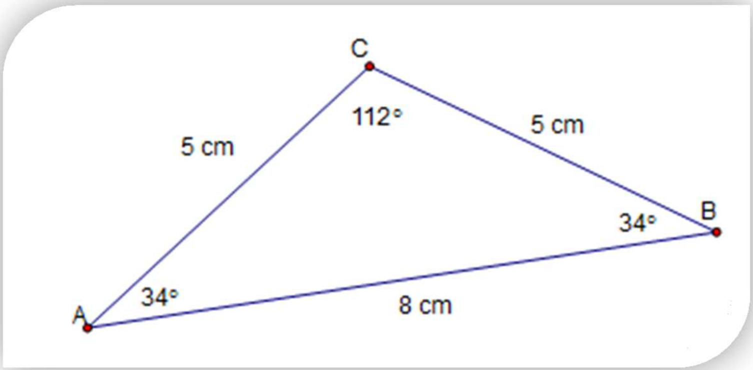

Figure 4.3.5.1d: Isosceles triangle

In the above discussion between Lerato and Khyana it appears that they have discovered the properties of an isosceles triangle. This is evident when Lerato realises that two sides of each of the three triangles are equal, even though the triangles are not of the same side (see Figure 4.3.5.1d animation). This seems to have contributed to Khanya's understanding, as she used the phrase, "which means", which is understood to be used as a follow-up to what Lerato said. In addition, it appeared that what Lerato said triggered some inductive reasoning in Khanya's mind, which led them to discover the properties of an isosceles triangle. The use of Geometer's Sketchpad seemed to have enhanced the teaching of the properties of isosceles triangles, because learners where able to manipulate the Geometer's Sketchpad software in the process of making meaning and interacting with the Euclidean geometry content (see Figure 4.3.5.1d animation). After Mr Molete had assessed the learners' prior knowledge, he connected what they know with what is new, which leads us to the next section.

4.3.5.2 Concretising the theorem using Geometer's Sketchpad

In contrast to the way Mr Mokoena and Miss Lebaka facilitated a lesson on this theorem before the intervention of the study, Miss Lebaka then created a connection between what was known and what is new. For instance, she said to the learners:

> *Construct a circle with centre O, and a chord AB using GSP [Geometer's Sketchpad]. Then join OA and OB to make triangle ABO. Investigate the properties of triangle OAB.*

She moved around, ensuring that all learners were able to follow and understand what was required of them. Then she asked:

> *Thabo, how much is the magnitude of your angle OCB?*

Figure 4.3.5.2a represents what Thabo had on the screen, using Geometer's Sketchpad.

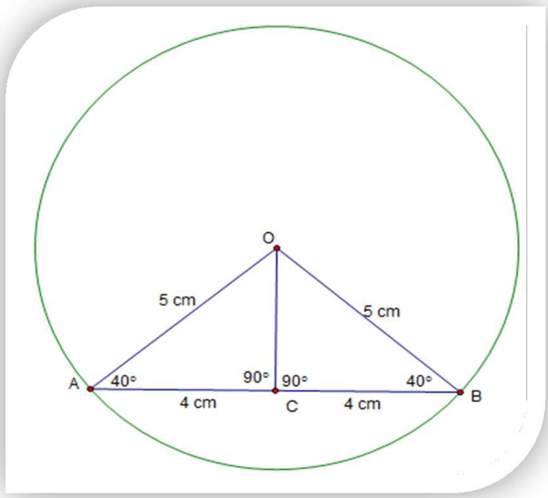

Figure 4.3.5.2a: Thabo's screen

> *Thabo: I got angle OCB equal to 90 degree, Mam.*
>
> *Miss. Lebaka: Did you all get 90 degrees, check a person next to you, if they have a different answer, help them to get the right answer.*

Form the above extract of Miss Lebaka's lesson facilitation, it seems that Miss Lebaka is not the only one in the class doing the work, while the learners are passive listeners, but rather that the learners are actively involved and the teacher is only guiding the learners by requesting them to construct geometric figures and measure the angles' magnitudes. This is evident when she asks, “did you all get 90 degrees?”, which supposes that she gave them work to do with the expectation that they would get 90 degrees. It was evident when some leaners, such as Thabo, had reached the correct answer. However, Miss Lebaka also created a space for learners to learn from each other, which is evident when she asks, ”check a person next to you, if they have a different answer, help them to get the right answer”. Subsequent to the lesson extract above Miss Lebaka moved around and requested the team members to help her to ensure that all the learners are able to do the measurements. Immediately thereafter, Miss Lebaka ensured that learners can manipulate the software fluently to measure, and she then asked:

> *Now that we know that angle OCB and angle ACO are 90 degrees, and side AC is equal to side CB. What conclusions can we make in connection to the theorem statement?*

Miss Lebaka grouped the learners in groups of five and gave them the following guidelines.

> *Sebedisang (Use) display button on GSP [Geometer’s Sketchpad]... animate the circle and observe the changes on the magnitude of the line segment AC in relation to the magnitude of line segment CB and make a conclusion related to the theorem statement.*

Miss Lebaka asked learners if they understood what was expected of them, by saying:

> *Do we all understand what we need to do?*

Miss Lebaka’s question is in line with the best practice, which states that a teacher should, at all times, ensure that learners know what is expected of them. Subsequently, the team members, in support of Miss Lebaka, moved around to guide each group of learners to find the animation button on Geometer’s Sketchpad. Thereafter learners worked in groups and they reported back to the class. Thabo reported on behalf of his group and what was projected on their computer screen:

> *Re mejarile angle OCB and angle OCA, ra fumana e le dininety degrees ho bolelang hore line OC is perpendicular to AB. (We measured angle OCB and angle OCA, we found that they are 90 degrees, which means that line OC is perpendicular to AB).*

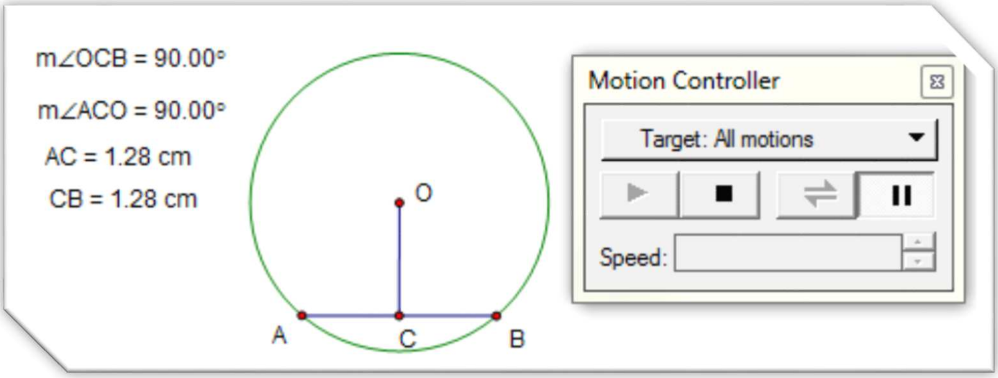

Figure 4.3.5.2b: Geometer's Sketchpad

Figure 4.3.5.2a shows that Geometer's Sketchpad enabled learners to measure the magnitude of angles. For instance, Thabo said that, "We measured angle OCB and angle OCA", which means that they were able to follow the instruction by Miss Lebaka to measure the angles. The classroom support given by both Miss Lebaka and other team members enabled learners to use Geometer's Sketchpad to measure angles. Moreover, Thabo reported in the extract above that, "we found that they are 90 degrees"; it is understood that "they" refers to the angles they measured and found to be 90 degrees. This confirmed that, not only did the learners measure the angles, but they were also able to display the angles' magnitude (see Figure 4.3.7a). It is worth referring to Thabo's use of "we" in the extract above, which suggested a collaborative approach to construction of knowledge. This is in line with the social constructivism epistemological stance, that knowledge is a social construct (van Dijk, 2007: 66).

This process of measuring and observing enabled the learners to make hypotheses or conjectures, for instance, Thabo said, "which means that line OC is perpendicular to AB", which the group deduced from the fact that angle OCB and angle OCA are equal to 90 degrees. He said this by using the phrase, "which means", which suggests that it follows from the fact that angle OCB and angle OCA are equal to 90 degrees. This manipulation of Geometer's Sketchpad software simultaneously enhanced learners' understanding of the perpendicular line and the chord, in this case. It helped them understand the theorem statement better, and understand the conditions for two lines to be perpendicular. This was evident when one of the learners, Kemi who was not in the same group as Thabo, said:

Sir ho bolelang ho re (which means that), we will know that two lines are perpendicular if the angle they make ke (is) 90 degrees.

From the above extract, “ho bolelang ho re (which means that)” was said as an exclamation. In this context it signified an expression by someone who was enthused by his/her discovery of something new and exciting. Kemi was understood to have been enthused by her “comprehension” that “perpendicular” is seen or recognised when the intersection of two lines make an angle of magnitude of 90 degrees. This was evident when she said, “two lines are perpendicular if the angle they make ke (is) 90 degrees”. She was so excited she did not even give a teacher the opportunity to talk when she exclaimed. Miss Lebaka, without suppressing Kemi, used what she said to generate a discussion among the rest of the class, and systematically created a space for learners to confirm Kemi’s conception by manipulating Geometer’s Sketchpad software. Miss Lebaka asked:

Ntswaki lona le reng? (Ntswaki, what are you saying? Let’s hear what are other groups say).

Kemi: Ntate a kere where two lines intersect they form two angles, for example AB and OC di intersecta at C. Now rena le angle OCA and angle OCB. jwale when we measure OCA and angle OCB re tho la dilekana le ninety ho bolelang hore line OC is perpendicular to AB. (Mam, is it not so that where two lines intersect they form two angles, for example, OCA and OCA intersect at C, now we have angle OCA and angle OCB).

Manipulating Geometer’s Sketchpad software enabled Kemi to discover that two lines are perpendicular if the intersection forms an angle of 90 degrees. This is evident from Kemi’s extract, which attests that by using Geometer’s Sketchpad she was able to observe that when an angle formed by two lines is equal to 90 degrees then the two lines are perpendicular. Furthermore, Kemi’s extract confirmed Thabo’s reasoning, that the measurement of the angle led to the discovery that the two lines OC and AB are perpendicular. At the end of the lesson Miss Lebaka requested group learners to use HeyMath! software to prove the theorem, by saying:

Now that we know that if a line that is drawn from the centre of the circle to the midpoint of the chord is perpendicular to the chord. How will be prove this statement using mathematical reasoning and logic? I want you to work in your groups during the study in the lab. U HeyMath! software to do your research to find out how you can prove this theorem. HeyMath! has the proof of the theorem, use what we have done today to explain the HeyMath! proof.

Form the above extract, it is clear that Miss Lebaka expected learners to be able to discover ways of proving the theorem on their own. This is evident when she said, “do your research to find out how you can prove this theorem”. Miss Lebaka did not teach learners how to prove the theorem but rather gave them the tools needed to prove the theorem on their own. For instance, Miss Lebaka used Geometer’s Sketchpad to guide learners to, first, believe the theorem statement, this is evident from the discovery made by Thabo, Kemi and Ntswaki’s groups, through their observations that OC is perpendicular to AB. This is in line with the best practice view that learners should construct knowledge through their social interaction in groups, on their own (Karolich & Ford, 2013: 29). Secondly, she used the Geometer’s Sketchpad software to concretise and give a pictorial representation of the theorem statement (see Figure 4.3.7a), which led learners to the observations discussed above. Furthermore, Miss Lebaka’s lesson was learner centred and this approach reached a climax when she requested learners to do research to find ways of proving the theorem.

This approach to teaching changes the role of a teacher in classroom. The teacher gives up power to enable learners to drive the process of knowledge construction and give them a sense of ownership (Alexandra, 2013: 206). This is evident when Miss Lebaka gave learners the responsibility to self-regulate learning by proving the theorem. According to Alexandra (2013: 207) a teacher’s role in a learner-centred lesson is to prepare activities that learners will engage in. This clearly happened in Miss Lebaka’s lesson.

4.3.5.3 Cultivating abstract logical reasoning to prove the theorem using Geometer’s Sketchpad and HeyMath! software

It is important to cultivate learners’ abstract logical reasoning. According to the CAPS policy learners are expected to operate at a level where they are able to prove Euclidean geometry. In Section 4.3.5.2 I reported how Geometer’s Sketchpad was used to help learners concretise Euclidean geometry concepts, such as the construction of figures, and measuring magnitudes of angles and lengths. Abstract logical reasoning is what is accepted as logical mathematical proof, not involving measurement of angles and sides, but using conjectures as developed in Section 4.3.5.2. In order to explain how logical abstract reasoning can be enhanced, an extract from the lesson facilitated by Miss Lebaka is given below. She gave learners a task to investigate – they had to find out how to prove the theorem:

> *Lerato: We used HeyMath! proof like you said. We started by drawing figure in HeyMath!, pausing and doing it step by step using GSP [Geometer's Sketchpad]. Thereafter, we followed the proof step by step, for example, the HeyMath! says angle OCA = OCB = 90, so we measured the two angles using the GSP and found that they are equal to 90.*

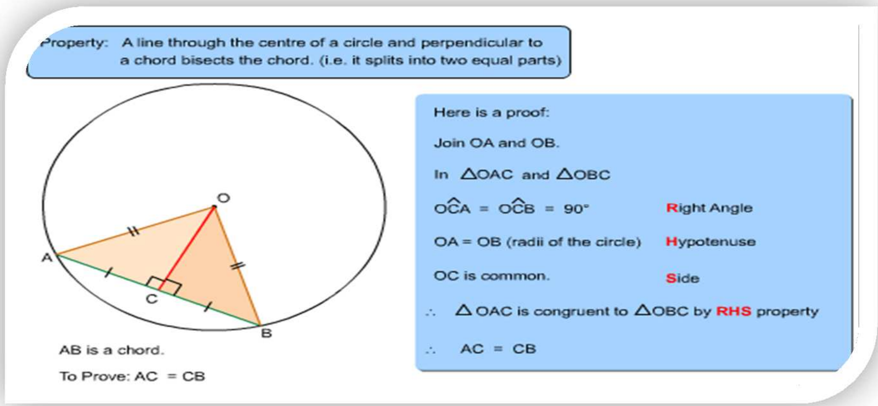

Figure 4.3.5.3a: Perpendicular bisector

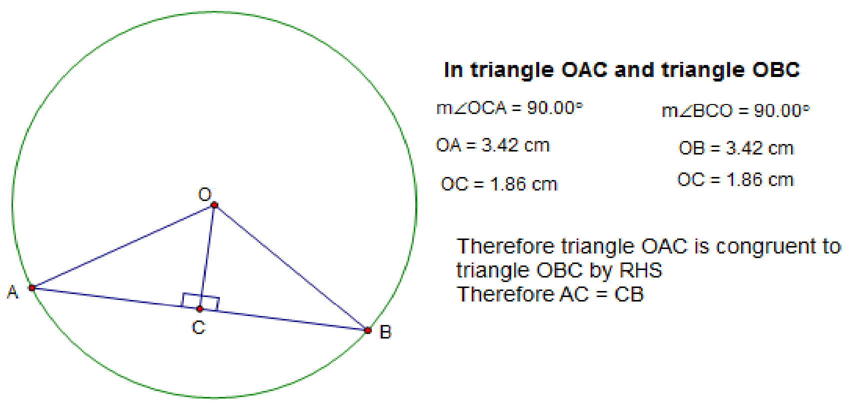

Figure 4.3.5.3b: Perpendicular bisector with Geometer's Sketchpad

It is clear from Figure 4.3.5.3a, Figure 4.3.5.3b and Lerato' extract that the use of HeyMath! and Geometer's Sketchpad complemented one other. Lerato's group used HeyMath! to prove the theorem. In addition, they used Geometer's Sketchpad to concretise and make sense of the proof by taking every mathematics argument to a concrete level. For instance, HeyMath! software makes the claim that OA = OB and supports its argument by saying OA and OB are the radii of the circle (see Figure 4.3.5.3a). However, the grouped inductively investigated with the aid of Geometer's Sketchpad to find out that, indeed, OA = OB = 3.42 cm (see Figure 4.3.5.3b).

4.4 CONDITIONS CONDUCIVE FOR THE STRATEGY FORMULATED

This section considers the conditions that are conducive for the optimal implementation of the solutions or components of the strategy discussed in Section 4.3. The solutions and strategies in Section 4.3 will be implemented beyond the duration of the study. Thus, the conditions conducive for sustainability of the solution beyond the duration of the study must be identified.

4.4.1 Conditions conducive for suitable for prime functionality of a team

The optimal functionality of a team is obtained when there is collective leadership within the team (Avolio et al., 2009: 423). This means that the leadership roles and responsibilities are not fixed for performance by one person. All team members have the opportunity to experience these roles and in this way a sense of belonging and appreciation is bestowed on them (Contractor et al., 2012: 995). In addition, this encourages them to look forward to their collaborative team engagements. This view is supported by Kocolowski (2010: 22), who claims that shared leadership is a necessary condition if a team's success is to be sustainable. Tolerance of different perspectives and sometimes contradictory views is also a necessary condition for sustainability of optimal functionality of a team (Rogers, 2012: 1). The tolerance of such views within the team demonstrate the prevalence of mutual respect, equality and humility (Mahlomaholo, 2012: 3; Nkoane, 2013: 394).

4.4.1.1 Content knowledge

Improving the content knowledge of the co-researchers is one of the main goals of the study. Thus, in subscribing to the epistemological stance that knowledge is socially constructed, it is important to ensure that there are members of the team who have a deeper understanding of the content of Euclidean geometry. Therefore, within the team there were two members who had studied mathematics to a Master's degree level (see Section 3.3.3). These two members buttressed the community cultural wealth of the team on CK. For instance, during a preparation session, Mr Boke asked, *Colleagues, how do you prove that sum of an angle of a triangle equals to 180 degrees?*

Mr Kotudi responded by saying:

Bana ba ha ba cutthe in grade eight kwa bo bone hore ha beha angles to getter are angle on the straight line (are these learners not cutting papers in Grade eight to see that when you put the angles together they make angles on the straight line)?

Mr Phehello asked:

How do you do that, can we all do it?

Mr Mbuks quickly downloaded the picture below from the internet, using his cellphone, and said,

Colleagues, here is what Mme (mam) is talking about.

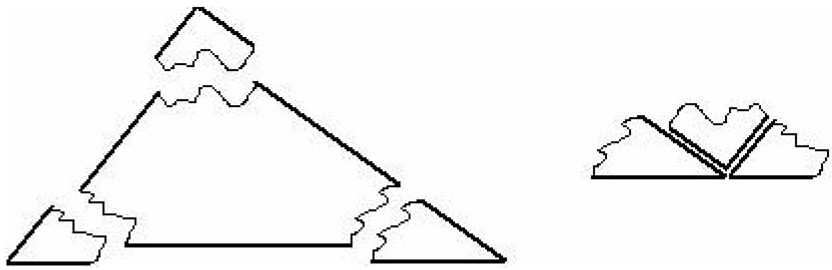

Figure 4.4.1.1a: Proving the conjecture using the angles

Mr Boke continued his questioning:

Colleagues, is this proof sufficient? Is this how a Grade 11 learners suppose to prove it?

The above discussions captured during the lesson preparation demonstrates that there was freedom of expression and thought provoking questions were asked during the discussions. This kind of social interaction, as demonstrated above, concurs with Habermas' communicative action. For instance, Habermas' orientation to success on social action asserts that orientation to success occurs when the purpose of the actors is mainly achieving a desired state of affairs in an objective world (Harbermas, 1971: 56). Thus, I argue that the primary purpose of the co-researchers in the above discussion was to research a desired state of affairs through the objective lens of the traditional moment, which sought to find a universal way of proving that sum of the interior angles of a triangle equal 180 degrees. This is evident when all co-researchers, except Mr Mbuks, agreed that there is just one way of proving the conjecture.

However, Mr Mbuks' questions continued despite the other co-researchers agreeing on the status of their CK; this question created a further opportunity for team members to enhance their CK. After the team members had agreed that cutting out the angles of a triangle was the only way to prove the conjecture, Mr Mbuks demonstrated a logical mathematical proof using Geometer's Sketchpad software, as follows:

Mr Mbuks: Ntate Mokoena draw any triangle using GSP [Geometers' Sketchpad].

Mr Mokoena: Ke yena ntate (Here it is, Sir).

Mr Mbuks: Now construct any line parallel to one side of the triangle passing through any of the three points.

The Figure 4.4.1.1b is the snapshot of Mr Mokoena's triangle and constructions using Geometer's Sketchpad software.

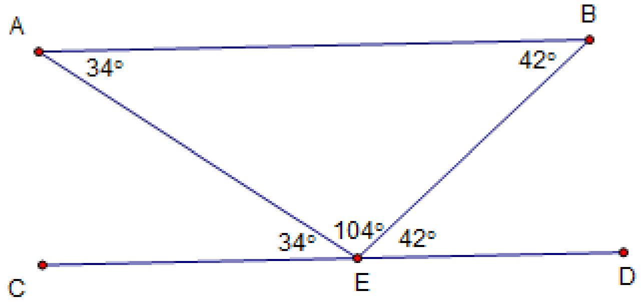

Figure 4.4.1.1b: Proving the conjecture using Geometer's Sketchpad software

After Mr Mokoena had constructed Figure 4.4.1.1b, Mr Mbuks asked:

Now, what do you see, Colleagues?

Miss Lebaka: Alternating angles are equal.

Mrs Kotudi, Mara ntho e entse e tshwana excepts now you can see (But this thing is still the same).

Thus, the above demonstration of CK, as alternative logical proof, explains the conjecture that states that the sum of interior angles of a triangle is equal to 180 degrees. This shows the importance of having a member in the team that can contribute to the social wealth of the team, so that it can achieve its objectives on CK as a necessary condition. Thus, in order for a team to operate optimally, it is important that criteria for inclusion of team members is based on the skills and knowledge needed to achieve the objectives of a team.

3.5.1.1 Technological pedagogical content knowledge

Improving the TPACK of the co-researchers was the team's main aim. Thus, in order to achieve this aim one of the conditions was that the team had to have team members who could contribute their TPACK. In the current study Mr Boke and I could contribute further to the team's community cultural wealth on the use of ICT software to teach Euclidean geometry. For instance, during the preparation session Mr Boke suggested,

> *Did you know that you can use GSP [Geometer's Sketchpad] built in calculator to add up angles?*
>
> *Mr Mokoena: How, ntate?*
>
> *Mr Boke: Go to tool ba,r click, and then select calculator.*
>
> *Researcher: Or you can press alt and plus equal to button at the same time.*

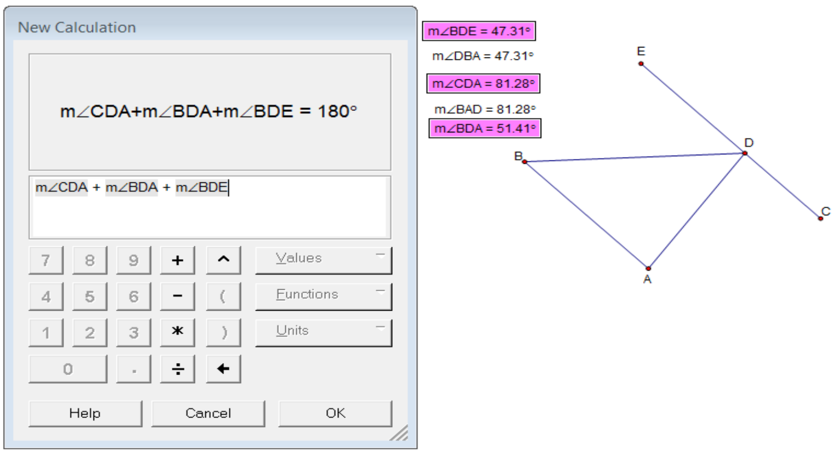

Figure 4.4.1.1b: Using Geometer's Sketchpad software's calculator

From the above interaction, TPACK on how to optimally use the Geometer's Sketchpad functions is improved. Drawing from Vygotsky's notion of the more knowledgeable other is observed in the above interaction. For instance, Mr Boke and I, in this instance, acted as the more knowledgeable other for Mr Mokoena's zone of proximal development, that is, through our social interaction he was able to construct knowledge on how to use Geometer's Sketchpad software to prove the conjecture and, furthermore, how to use Geometer's Sketchpad's built-in calculator.

In addition, in line with the above argument about the conditions necessary for optimal functionality of a team, it became evident during the meeting that Mr Phehello convened to form a team, that sharing was of the utmost importance. Mr Phehello said:

> *Bana beso nna ke bona e ka re our success depend on sharing responsibilities (My brothers and sisters, the way I see it, our success depends on sharing responsibilities).*

From the above extract it is clear that one of the necessary conditions for the optimal functionality of a team was good relationships between the team members. The manner in which Mr Phehello addressed his team mates as his own brothers and sisters suggests that there is transparency between the team members, a sense of caring, and a sense of belonging and trust in pursuit of responses to challenges posed by teaching and learning Euclidean geometry using ICT software (Contractor et al., 2012: 995). A different perspective to the use of the phrase “bana beso” suggests that the team members were so united in pursuit of improving the teaching of Euclidean geometry that they transcended seeing each other as colleagues, and reached a level where they considered themselves as family.

The transparency, trust, and sense of caring and belonging promoted among the members confirm the principles of PAR, which seeks to level out power and promote equality between the members (Mahlomaholo, 2013: 387). Because Mr Phehello is a deputy principal and is responsible for curriculum in one of the schools, and also the cluster coordinator for all three schools, he had the authority to order the team members, but he addressed them as “bana beso” (for profiles, see Section 3.4).

The sentence, “our success depends on sharing responsibility”, confirms the best practice that says that, in order for a team to achieve its results optimally there is a need for shared or collective leadership (Avolio et al., 2009: 423). The sharing of responsibilities in the context of this study included preparing lessons together, changing roles when teaching and observing lessons, sharing meetings, and organising logistical concerns, such as computer lab bookings, software, etc. In addition, sharing became evident when we did error analyses and created a program to respond to the challenges identified. Among the strategies suggested for the program was using Geometer’s Sketchpad software to teach Euclidean geometry. The team members agreed and Mr Mokoena summarised:

> *Nna ne ke tla re, jwale ka ha re se re behile letsatsi where we meet, ntate a re bontshe how to install the program le ho re e sebediswa jwang for ho etsa Mmetse and… le ho re rona re bona e ka re thusang jwang holatela the challenges identified mme re ka e matlafatsa jwang. (I was going to say that, now that we have a date where we are going to meet, Sir (referring to me) must show us how to install the program and how to use it to do mathematics and … also to see how can it help us with regard to the challenges identified and how can we strengthen it).*

The extract, "must show us how to install", attests to the condition of sharing. I was able to share technological knowledge about installing Geometer's Sketchpad software with team members. Miss Lebaka, a teacher at one of the schools in the area of the study, shared a computer lab where the software was installed. Not only did I share how to install the software, but also how to use the software to grapple with Euclidean geometry content using Geometer's Sketchpad software. Furthermore, we shared ways of teaching Euclidean geometry using Geometer's Sketchpad, in line with the learners' challenges we had identified, among which that learners were unable to identify that the exterior angle was equal to the sum of opposite interior angles of a triangle, and the team shared ways of teaching the concept using Geometer's Sketchpad (see Section 4.3.5). The process of sharing and changing roles gave each team member the opportunity to learn and contribute in different ways to the success of the team (Rogers, 2012: 1). This fostered sustainability of the operation of the team, since it does not depend on an individual but rather values the contribution of every team member. This is in agreement with the views of Contractor et al. (2012: 995) on the best practice, that changing of roles creates a sense of belonging and appreciation for team members.

A critical theoretical bricoleur in pursuit of an emancipatory agenda understands that epistemology of social constructivism states that knowledge is socially constructed and requires conditions that promote social interaction between the team members. This condition became evident when I, as the research coordinator, did not drive the process of research, as it would usually be the case in traditional and modernity phase qualitative research – I had to give up the power and permit co-researchers to drive the process of research. For instance, one of the co-researchers, Mr Phellelo, after our deliberations on the kind of study that I intended to conduct, found the study valuable and committed to taking the process further, by saying:

Ntate let me organise le matichere a rona a mmetse le a dikolo tse tharo tse haufinyana ke ba kope hoba part ya meeting (Sir, let me organise with our mathematics teachers and three other schools that are closer, and ask them to take part in a meeting).

The phrase, “organise with”, is understood to mean that Mr Phehello shared his views with his colleagues concerning the value of the study for their teaching practices. This means that he did not impose his view on them, because he said he as going to “ask them” to take part – he did not say he was going to order them or organise them to take part in the study. The fact that I did not ask the teachers to take part in the study, but that Mr Phehello initiated the process of research by asking teachers to take part in the study was evidence that I had “depowered” myself to create a space for Mr Phehello to empower himself. Furthermore, the conditions that promote social interaction are also prevalent in the above extract, for example, Mr Phehello, a deputy principal at his school, planned to ask the teachers, and not to order them, to participate, which shows that Mr Phehello also “depowered” himself to create a space for teachers to decide to be part of the study or not, regardless of Mr Phehello’s views concerning the study. It is this principle of the emancipatory agenda illustrating freedom and equity in the seventh moment that made it possible to create conducive social interactions between team members.

4.4.2 Conditions conducive to sustainability of lesson preparation and facilitation with the aid of ICT software

Lesson preparation requires, among other things, careful collection of thoughts and resources concerning what needs to be taught. Due to the complexity of everyday life, what constitutes knowledge, and the diverse nature of reality, knowledge becomes a very complex phenomenon. Thus, lesson preparation has a great influence on the ways knowledge construction is understood, which is closely related to the ontological stance of individuals. Having said this, I understand that lesson preparation is a very complex activity that requires a teacher to set up an environment that promotes learning for individuals who often come from diverse backgrounds and who do not learn in the same way. This, in my view, is why teaching can be described as a problem-based activity. Firstly, it is a problem to the teacher, because a classroom consists of learners from various cultures, with different histories, family morals, socioeconomic status, genders and ages, which influences both their epistemological and ontological stances.

Secondly, the view the study subscribes to is that learning could easily take place if an environment enables learners to create connections between what they already know and the content they are currently trying to learn. Failure to set up an environment that promotes learning in an inclusive manner, where learners construct knowledge by making these connections, becomes problematic for learners. This complicates the activity of lesson preparation and facilitation. However, the bricoleur's quest to uncover the multilayered, multi-epistemological, multi-ontological, multiple social theory and multi-cultural perspectives of challenges and possible solutions to lesson preparation and teaching are understood better if one can first appreciate their complexity.

During lesson preparation each teacher was given the opportunity to act as the chairperson; the chairperson for our preparation meeting was elected democratically. It was during this meeting that the teachers demonstrated a diversity of knowledge construction modes by saying:

> *Mokoena: I think proving that the two triangles are congruent, e ka ba bonolo for bona (will be easy for them).*
>
> *Phehello: Mara ntate Mokoena hao nahane hore that might be to abstract for bona? kapa colleagues lona le reng (but Mr Mokoena, don't you think that might be too abstract for them? Or colleagues, what are you saying)?*
>
> *Miss Lebaka: I think, before we use congruency we should first ask ourselves why do we want to use congruence, or anything, for that matter.*
>
> *Boke: I will say let's use GSP [Geometer's Sketchpad] to help them make meaning first.*

The above extracts capture the dialogue between coordinating team members during a lesson preparation meeting. The diversity of viewpoints expressed added significantly to creating and making meaning of an environment that is learner centred. In addition, the use of ICT software helps learners to interact indirectly with the content.

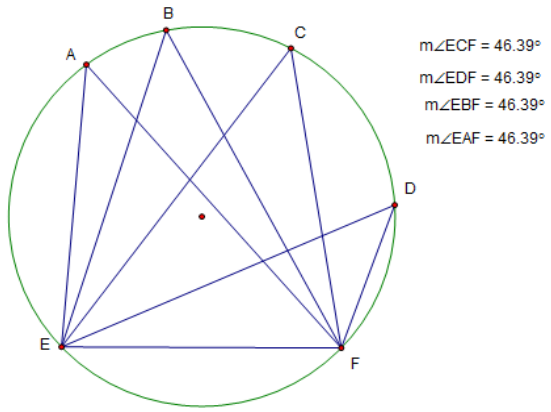

Figure 4.4.3: Angle subtended by the same chord

Thus, one of the conditions for effective lesson preparation and facilitation is that lesson preparation should be learner centred. This is in line with the views of Kumar (2011: 21), which are that a learner-centred approach to lesson preparation and facilitation promotes effecting learning processes. Using congruence seems to be too abstract for the learners, as captured in the dialogue above. If Mr Mokoena had been preparing this lesson alone he would have used the concept of congruence to prove the theorem. However, the presence of the other team members provided an opportunity for Mr Mokoena to deepen his thoughts and critically consider other ways of teaching the theorem, thereby creating a condition necessary for collaborative effort and setting up an environment that is inclusive of learners' diversity, as described above.

Achieving the ideal of diversity in lesson preparation and facilitation is made possible by rejecting a rationalistic quest for order and certainty (Kincheloe, 2004: 24). The nature and production of monological knowledge gives a skewed representation of the classroom, because it does not account for the complexity that exists between human perception and material reality. Therefore, the current bricolage is constructed by teachers who live in the same community as where the three schools are located. This proximity and cooperation enhances the rigour of lesson preparation by contributing to meaning making of social factors, such as discursive practices within the social construct where the schools and teachers are situated socially.

4.5 THREADS AND RISKS

The study is geared towards sustainable teaching and learning, therefore, it was important that the conditions conducive to implementation of the formulated strategy are understood, for the sake of sustainability. This is done with an understanding that there could be inherent risks and threats imbedded in the implementation of the strategy.

4.5.1 Co-researchers' negative attitude toward the use of ICT software

A positive attitude towards the use of ICT in teaching contributes significantly to the formulation and implementation of a strategy to teach Euclidean geometry using ICT software. Thus, Hue and Jalil (2013: 55) recommended that developing teachers' positive attitudes is a key factor in a successful implementation of computer software in teaching (see Section 2.5.4.1 for more). In this study, the co-researcher's attitude was positive towards the use of ICT software. In developing a positive attitude collectivised efforts become one of the main pillars of the study, and for developing a positive attitude, an effective team that promotes the use of collective leadership practice is important (see Section 4.2.2.1). This was evident when Mrs Kotudi said:

> *Who are we observing?*

Collective leadership in the current study included sharing responsibilities, such as convening and chairing meetings, organising logistical issues, such as availability of computer software and computer labs and facilitating lessons while other team members observed. Collective leadership becomes a necessary condition for fostering the practice of not assigning the leadership roles and responsibilities to a single person, but to all (see Section 4.3.1 and Section 4.4.1).

The study further recommends that, for optimal functionality of a team, it is imperative to bestow a sense of belonging and appreciation on the team members. In order for team functionality to be sustained the study recommends that a sense of tolerance for different and contradictory views or perspectives is developed. Moreover, respect, mutual benefit, equality and humility are recommended as characteristic features of an optimally functioning team.

4.5.2 Co-researchers' workload

Workload is one of the major threats to the utilisation of ICT in both lesson planning and facilitation. Buabeng-Andoh (2012: 142), in his study to investigate factors that influence teachers' adaptation and integration of ICT into teaching, revealed that teachers' heavy workload is one of the threats to both integration and adaptation of ICT in lesson planning and preparation (see Section 2.5.4.2). However, in the current study, we were able to avoid the threat. For instance, during a preparation session meeting facilitated by Mrs Kotudi, she started her conversation by saying:

> *Colleagues, based on our agreement, re lokela ho fumana hore re rutang bekeng yena, le hore ke mang yeo re mo obzevang (we need to find out what topic are we teaching this week and who we are observing).*

From the above extract, it is clear that co-researchers are preparing the lessons together. However, not only do they prepare lessons together, but they were also considering the pace set by the DoE. This was confirmed by Miss Lebaka, who said,

> *Colleague ke kopa ho ikgetha, I will be doing Euclidean geometry this week starting with the theorem: a line drawn from the centre of the circle perpendicular to the chord bisects the chord.*

Thus, from the above extract it is clear that co-researchers dictated the process of he co-researchers by deciding what must be taught. This made co-researchers own the research project as an activity that seeks to respond to their challenges. In addition, the question, "what are we teaching this week", by Mrs Kotudi shows that co-researchers were meeting weekly to prepare the work together that would be taught over a week. In so doing the co-researchers did add to their workload. Participation in the research project did not exacerbate teachers' workload – the contrary, it helped them to reduce their workload. For instance, let's reflect on Miss Lebaka's class,

> *Lerato: ntho e tshwanang ditraengeleng tshena tse tharo ke ho re two sides are equal le ha ele hore triangles tsona ha di lekane (Things that are the same in the three triangles is that two sides are equal even though the triangles are not equal).*
>
> *Khyanya: Ho bolelang hore isosceles ena le masaete a mabedi a lekanang le diangle tse pedi tse lekanang (Which means that, isosceles has two sides that are equal and two angles that are equal).*

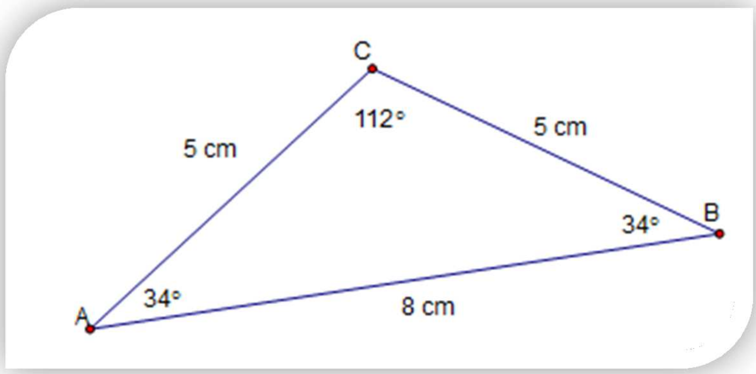

Figure 4.5.2a: Isosceles triangle

From the above extract learners were able to discover the properties of an isosceles triangle on their own. However, it should be said at this juncture that the isosceles triangle and its properties are taught for the first time in Grade 8. Therefore, learners are expected to know about the isosceles triangle, and identify its properties in a pictorial representation of the theorem.

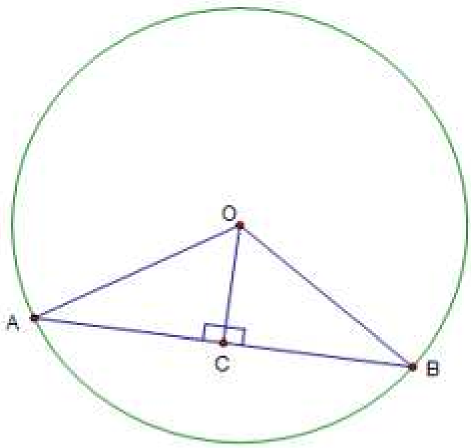

Figure 4.5.2b: Perpendicular bisector

From Figure 4.5.2b learners in Grades 11 and 12 are expected to identify that line OB and line OA are equal, therefore, triangle ABO is an isosceles triangle. However, learners in Grade 11 struggle to identify those properties (see Section 4.2.2). Conversely, using ICT software, specifically Geometer's Sketchpad, a teacher is able to quickly teach, without wasting time, the important concept that learners seem to have not understood when they had been taught in previous grade(s) (see Section 2.5.4.2). If they can't use ICT software teachers generally have to schedule extra classes to teach learners what they should have learned in previous grade(s) (see Section 2.5.2 and Section 4.2.1.4).

4.5.3 Effective use of ICT software

The use of ICT should create an opportunity for learners to interact with the information and manipulate it such that they are able to critically analyse and question the information so that it becomes relevant to their context. This is in line with social constructivism's epistemological stance, that knowledge is constructed if it is part of the social context that individuals socially interact with (Marcus & Fischer, 1999: 7). However, access to ICT tools does not always lead to quality teaching and learning, which poses a threat to the intended use of ICT (see Section 2.5.4.3). However, in the current study, ICT software wasd use to promote quality learning, as opposed to accessibility. For instance, HeyMaths! and Geometer's Sketchpad were used to promote learning from the concrete to the abstract. This was evident from a lesson by Miss Lebaka.

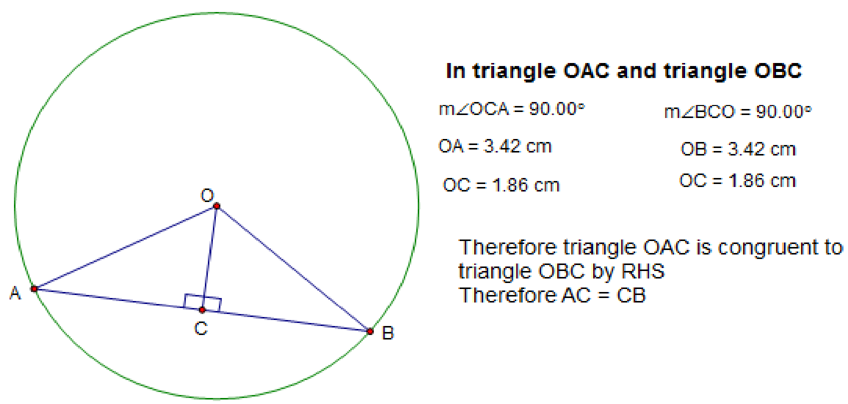

Figure 4.5.3a: Perpendicular bisector

For Figure 4.5.3a and Figure 4.5.3b the two software programs were used to complete one another. For instance, by using Geometer's Sketchpad learners were able to inductively discover the proof on their own. However, the weakness is that the proof is limited to the measurement of the sides and angles of a triangle (see Section 4.3.4).

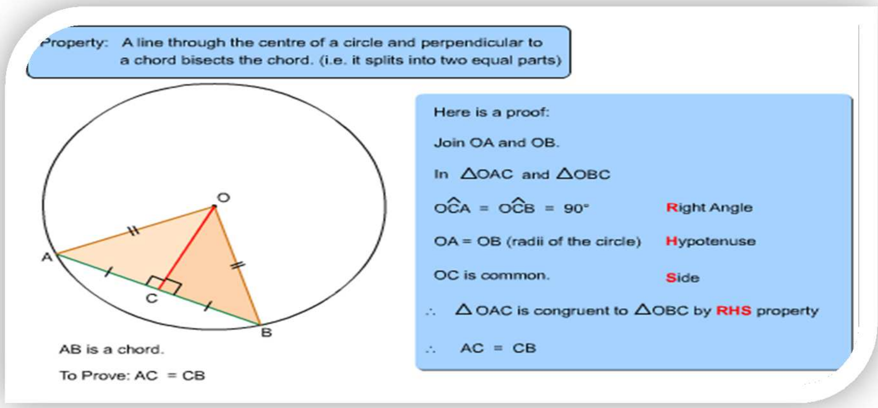

Figure 4.5.3b: Perpendicular bisector

In Figure 4.3.5.3b the use of HeyMath! complemented Geometer's Sketchpad such that learners were able to use the mathematical reasoning developed by Geometer's Sketchpad (see Section 4.2.2.5.2). Thus, in this study ICT software was used to improve the quality of teaching and learning.

4.6 INDICATORS OF SUCCESS FOR THE STRATEGY THAT WAS FORMULATED

One of the objectives of this study was to respond to the problems identified in relation to using ICT to teach Euclidean geometry (see Section 2.5). The success of this study is realised when the teachers who participated in the study demonstrate knowledge and skills to facilitate and design lessons using a variety of ICT software to privilege learner-centeredness, self-regulated learning, collaborative learning, and problem-based learning. Furthermore, it is also important that teachers work effectively with each other in pursuit of using ICT effectively for teaching Euclidean geometry.

4.6.1 Content knowledge for teaching

The content knowledge for teaching Euclidean geometry using ICT software is categorised into three main constructs: content, pedagogy and technology. However, CK should be the priority, since pedagogy and technology are the tools used to create better opportunities for learning the content. It should be said at this juncture that priority does not mean more important, but rather only gives an order in which the use of ICT for teaching becomes more effectively integrated in teaching. Thus, the research team prioritised the content knowledge which Ballet al (2008: 401) termed common content knowledge (CCK) and specialised content knowledge (SCK). The two knowledge domains CCK and SCK were applied during the lesson preparations. For instance Mr Phehello in one of the lesson preparations said that:

> *I think Isoscele can be used to better understand this theorem….can be taugh at grade 9 level for instance, if you draw Isosceles triangle…using a grade 9 theorem that exterior angle equal to sum of opposite interior angle of a triangle…*

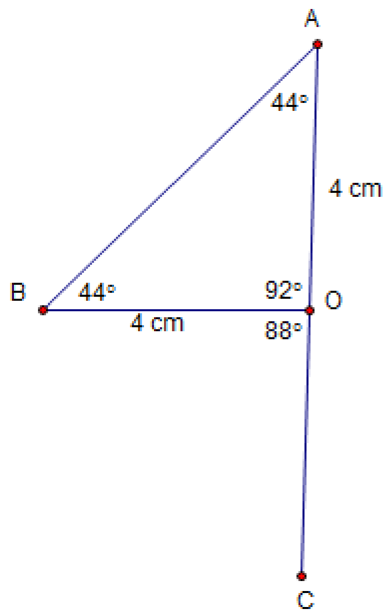

Figure 4.6.1a Isosceles triangle

What Mr Mokoena is saying is shown in Figure 4.6.1a that triangle ABO is an Isosceles with OA = OB = 4 cm and the base angles OAB = OBA = 44 degrees. Angle COB is an exterior angle of triangle ABO and angle COB is also opposide extrior angle to angle OBA and angle OAB of traingle ABO. Thus, based on the above extract Mr Mokoena made an argument which demonstrated his understanding of the CCK that angle COB = angle OAB + angle OBA = 88 degrees (see, Figure 4.6.1a). This concurs with what literature captures that a mathematics teacher shoud be able to demonsrate an understing of the theorems by creating a link between the concepts of Euclidean geometry like Mr Mokoena has demostrated (see, section 2.5.5.1 for more). Furthermore, he continued by saying

> This will then link this to our theorem which says an angle at the centre is twice angle at the circumference subtended by the same arc

From the above extract if O becomes a centre of a circle and point A and B are on the circumference then angle COB and angle CAB are subtended by the same arc BC Therefore, since angle OAB = OBA = 44 degrees, then angle COB = 2 OAB which proofs the theorem. It seems Mr Mokoena had a SCK for teaching the theorem since he did not only applied the CCK but he further thought of special ways in which the theorem could be packaged to make sense. This further agrees with what literature captures as knowledge and skills needed for a teacher to demonstrate (see, section, 2.5.5.1 for more).

It should be said that Mr Mokoena, is the same teacher who presented a lesson in section 4.2.1 and section 4.2.2 which demonstrated lack of CK and inadequate content articulation. However, through the space created by the study as we observed from the above discussion. He demonstrated a deeper understanding of the theorem and how it can be better proved which seems to indicate that through the communicative space created by the study he was able to improve his CK, CCK, SCK and PCK and finally TPACK. This was further evident from the connection that Miss Lebaka made with Mr Mokoena's as she said:

> *Yes Koena (Mokoena) the two equal sides of a triangle become the radii of a circle*

Miss Lebaka who came from the same school as Mr Mokoena did not work together and share best practises prior to the study (see, section 4.2.1). However, from the above extract they were now sharing ways in which a particular theorem can be proved that is CK. Moreover, Miss Lebaka also demonstrated high level of understanding of the theorem as she was able to link the two side of a triangle with the radii of a circle.

4.5.1. Technological content knowledge

Technological content knowledge (TCK) is the competence to integrate technology and content in pursuance of making meaning of the content (Tozkoparan, Kılıç & Usta, 2015: 45). Thus, in the context of this study Geometer's Sketchpad, Heymaths!, and Geogebra are integrated with Euclidean geometry content in pursuance of making sense of the theorems and their applications. Following the above discussions on section 4.4.1 on Mr Mokoena CK inputs, Mr Boke which is not a mathematics teacher but a photographer could now create a connection between the mathematics content and Geometer's Sketchpad by suggesting that

If I understand Ntate Mokoena well, we can use GSP [Geometer's Sketchpad] and create a circle with a triangle e leng hore (which) its two equal sides are the radii of a circle and then re be re hula (we pull) one side from the centre to the circumference hore re be le (so that we can have) exterior angle.

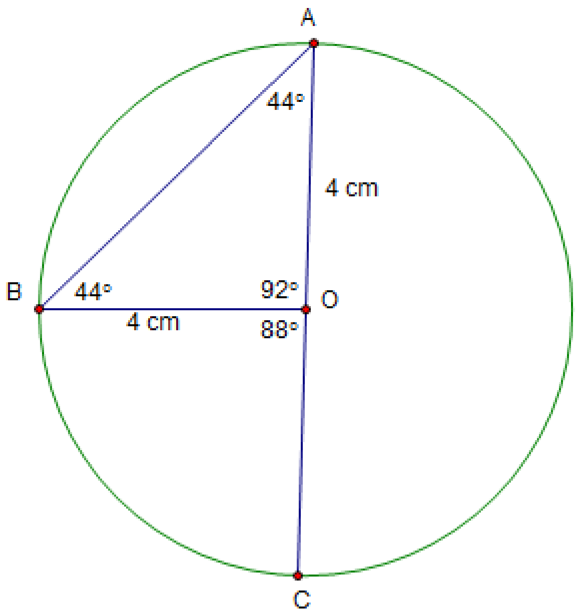

Figure 4.6.2 Isosceles triangle in a circle

From the above extract by Mr Boke, he was able to integrate the content of theorem and the GSP software. The use of GSP enabled the team to create accurate circles and dimension of a triangles to enhance the understanding of the theorems. Furthermore, it created an opportunity to measure the dimensions and the angles of the triangle in relation to the theorem statement. Mr Boke's extract indicate that he could integrate technology with the content of the theorem. Moreover, indicate that he had a deeper understand of the content knowledge.

4.5.2. Assessment of learners' learning

The assessment of learners' learning is one of the indicators of success for the effectiveness of the strategy employed. Chandra and Lloyd (2008: 1087), in their study to investigate the effect of the use of ICT on learners learning, argue that not only does ICT improve learners' learning, but also their test scores. Similarly, Aslan and Zhu (2015: 97) argue that integration of ICT in classrooms creates opportunities for learning. Figure 4.6.1a presents evidence of the effectiveness of the formulated strategy.

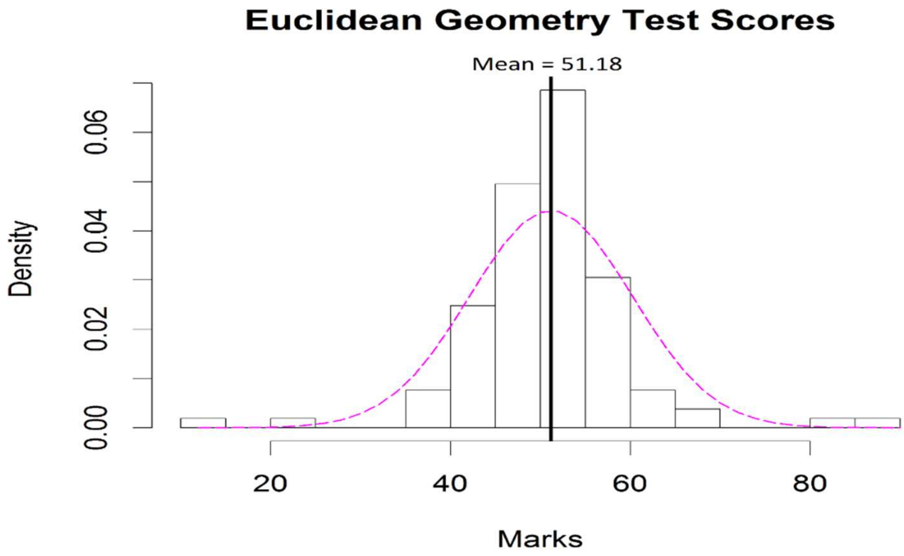

Figure 4.6.1a: Probability density function of test scores

Figure 4.6.1a shows the probability density function of the learners' performance after implementation of the unfolding strategy to teach Euclidean geometry using ICT software. In Figure 4.6.1a a mean of 51.18% can be observed, which shows the improvement of learners' performance in Euclidean geometry. It can also be deduced from Figure 4.6.1a that most of the learners seemed to have performed around the mean of 51.18%, which indicates that the strategy had an impact on the majority of learners. It is also worth noting that the majority of these learners achieved marks above 40%.

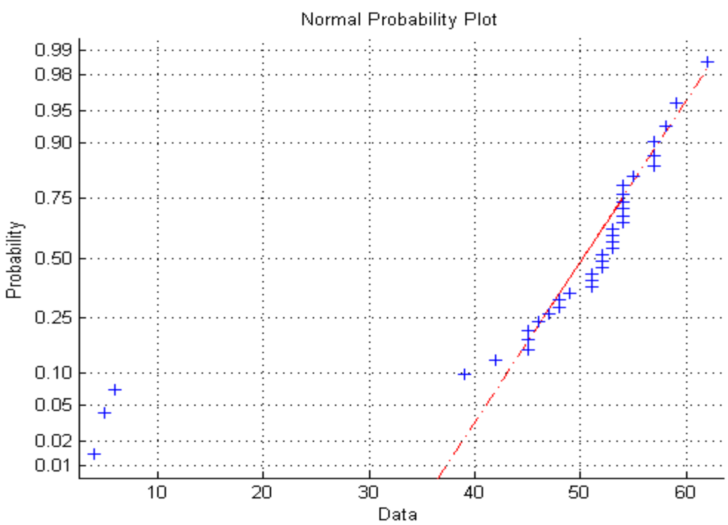

Figure 4.6.1b: Normality test

In pursuit of rigour the bricoleur constructed the normality test given in Figure 4.6.1b. Figure 4.6.1b shows how data points deviate from the straight line that represents normality. There are three data points that are below 10 – a clear departure from normality. However, the other points seem to be clustered around the straight line, exhibiting some degree of departure from normality. The data was tested further to validate the degree of departure from normality and determine if it is statistically significant. The results of a Jarque-Bera test reveals that the skewness and kurtoses of the data are not statistically significant. Next, learners' results were modelled using the Gaussian distribution to calculate the probabilities. For instance, what is the probability that learners who participated in this study will score marks lower than 40%?

$$P(X < 40) = P\left(\frac{X - \bar{x}}{\sqrt{var(x)}} < \frac{40 - mean(x)}{\sqrt{var(x)}}\right)$$

$$= 0.00005$$

The above probability calculation found that there is a very low possibility, of 0.005%,s that a learner will get a mark less than 40% if the TPACK strategy is implemented. This is substantiated by the boxplot in Figure 4.6.1c, which reveals that 38.56% was observed as the minimum percentage mark that learners would obtain when the strategy is implemented.

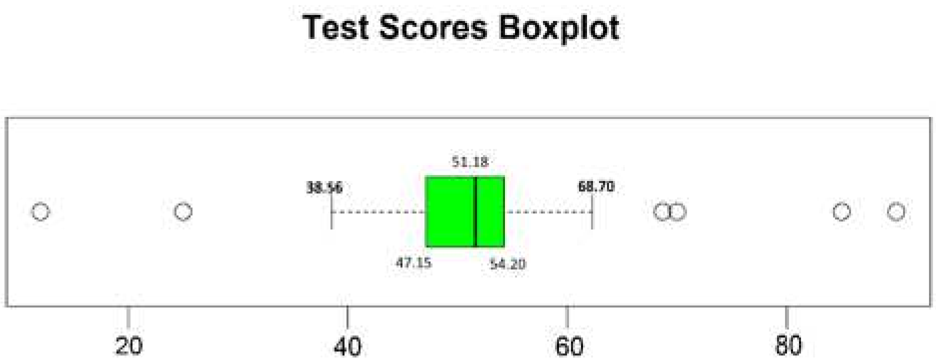

Figure 4.6.1c: Boxplot

Figure 4.6.1c further shows that the calculating the probability explains the difference of 1.44% between the first quartile and 40%. Deciding to make 38.56% the first quartile implies that the first two observations exhibited in Figure 4.6.1c are considered to be outliers. This implies that the two learners performed far below the rest of the class which could be explained by the boxplot. Thus, there seems to be an improvement of the learners’ performance.

4.7 CONCLUSION

This chapter presented, analysed data, and discussed and provided the interpretation for each of the five objectives of the study. The chapter analysed the challenges experienced by teachers who teach Euclidean geometry with the aid of integrated ICT software. This enabled the study to establish the possible solutions and strategies that were developed, adopted and adapted to effectively address the challenges experienced. The findings of this study are summarised and presented in Chapter 5.

CHAPTER 5: FINDINGS AND RECOMMENDATIONS FOR THE STRATEGY THAT WAS DESIGNED

5.1 INTRODUCTION

The overriding aim of this study was to design a strategy to improve teachers' TPACK for teaching Euclidean geometry with the aid of integrated ICT software. This chapter starts with a summary of the background of the study, which encapsulates the statement of the problem and the objectives of the study, for the purpose of reminding the reader of the study's aims. Thereafter, the chapter presents the findings of the study as organised by the objectives of the study. I then present the stages of a tested and implemented strategy, which serves as a recommendation for all challenges that emerged during the investigation process.

5.2 BACKGROUND AND STATEMENT OF THE PROBLEM

The study designed a strategy to improve teachers' TPACK for teaching geometry with the aid of integrated ICT software. TPACK refers to an interaction between technology, pedagogy and content knowledge in relation to teaching with the aid of technology. TK in this study referrs to knowledge and identification of dynamic geometry software that could be used to manipulate the content of Euclidean geometry. These software programs include GeoGebra, HeyMath! and Geometer's Sketchpad. This study defines PK as the knowledge and skills that teachers need in order to manage and organise geometry teaching and learning activities for intended outcomes (Koehler et al., 2013: 3). Lastly, the study's CK focused on Grade 11 Euclidean geometry concepts, namely,

(i) a line segment drawn from the centre to the midpoint of a chord is perpendicular to that chord;
(ii) an angle at the centre of a circle is twice the angle at the circumference subtended by the same arc or chord, and;
(iii) an angle subtended by a diameter is 90 degrees.

The study was motivated by the challenges to the teaching and learning of Euclidean geometry observed in practice and those recorded in literature. For instance, in many counties (see Section 2.2.2), including South Africa, teachers were found to experience difficulties using evolving technology, due to development of new software, and the rapid pace at which existing software is updated. It is reported in literature that teachers lack knowledge of how to use ICT software to teach subjects like geometry. Furthermore, it was noted that teachers' knowledge about using ICT is limited to knowledge acquired during traditional workshops, which only enabled them to adopt ICT software as a teaching aid, as opposed to using ICT innovatively (Ndlovu & Lawrence, 2012: 4).

In addition, reports suggest that teachers lack basic technical software knowledge, and this lack has a negative impact on their use of software for teaching (Tella et al., 2007: 1308). Furthermore, teachers were found to experience pedagogical difficulty designing, ordering and organising class activities, and alternating between different types of software as they teach (Leendertz et al., 2013: 5). In Botswana, Nigeria and Korea, teachers find geometry concepts too abstract to comprehend and teach, which has a negative effect on their teaching of Euclidean geometry (Nkhwalume & Liu, 2013: 27; Ratliff, 2011: 6).

In response to these mathematics pedagogical challenges, the HeyMaths! software program was introduced in South Africa in 2010 to help teachers to be more innovative in lesson design and teaching of mathematical concepts, such as Euclidean geometry (see Section 4.3.2). Studies in Botswana report the use of ICT software, such as Scratch, Inkscape, SketchUp, Mathematica and Excel, to promote creative teaching methods; however, this practice was found mainly in institutions of higher learning, as opposed to basic education (Kaino, 2008: 1844; Nkhwalune & Liu, 2013: 26-34). Furthermore, studies in Korea observed heightened creativity and innovation in lessons that incorporated the use of graphic calculators, spreadsheets, and Geometer's Sketchpad to teach mathematics concepts (Choi & Park, 2013: 274; Hyeyoung, 2011: 453; Keong et al., 2005: 43-50; Meng, 2013:62).

Despite the introduction of ICT software to improve teachers' pedagogical practices in South Africa, learners are still performing poorly in Euclidean geometry. For instance, 2014's national diagnostic report shows that learners performed poorly on higher-order questions in Euclidean geometry; they achieved an average percentage of 34. Similarly, teachers are still faced with pedagogical difficulties in improving their teaching to enable learners to access higher-order reasoning. These hurdles necessitated a strategy to respond to the challenges. The success of the strategy that was designed was found to be evident when teachers were able to do the following:

a) Identify geometrical concepts that learners find difficult to comprehend and teachers find difficult to teach effectively;

b) Collaboratively design a multiple-software-based lesson that would make abstract concepts easy to understand;

c) Facilitate a multiple-software-based lesson confidently;

d) Resolve any software and computer-related technical problems; and

e) Conceptualise new research initiatives and create new knowledge, or practise applying integrated ICT software to enhance their teaching strategies for abstract geometrical concepts.

5.2.1 Research question

How can teachers' technological pedagogical content knowledge (TPACK) for teaching Euclidean geometry with the aid of integrated ICT software be improved?

5.2.2 Aim and objectives of the study

The aim of the study was to design a strategy to improve teachers' TPACK for teaching Euclidean geometry with the aid of integrated ICT software.

The objectives of the study were to:

- Investigate the challenges that face teachers who teach Euclidean geometry with the aid of integrated ICT software;
- Analyse the different strategies that have been used to improve teachers' TPACK for teaching Euclidean geometry with the aid of integrated ICT software;
- Identify conditions under which different strategies improve teachers' TPACK for teaching Euclidean geometry with the aid of integrated ICT software;
- Identify the threats involved in implementing different strategies that have been used to improve teachers' TPACK for teaching Euclidean geometry with the aid of integrated ICT software; and make suggestions for avoiding these threats; and

- Identify indicators for evaluating the success of the strategies that have been used to improve teachers' TPACK for teaching Euclidean geometry with the aid of integrated ICT software.

Thus, this chapter presents the findings and recommendations from the strategy that was designed to improve teachers' TPACK for teaching Euclidean geometry using integrated ICT software, as unfolded in Chapter 4.

5.3 FINDINGS AND RECOMMENDATIONS

This section presents the findings of the study as they emerged from the literature and during analysis of the data. The following are the findings of the current study as it emerged in Chapter 4:

a) There is a lack of a dedicated team for improving TPACK for teaching Euclidean geometry with the aid of ICT software for the schools participated in this study;

b) Euclidean geometry is one of the most often failed mathematics topics;

c) Teachers do insufficient lesson preparation when they are using ICT software as a teaching aid;

d) Teachers facilitate their lessons inadequately when they are using ICT software as a teaching aid; and

e) Teachers do not integrate assessment with teaching.

A discussion of each finding, followed by its respective recommendations, which were drawn from the tested strategies in Section 4.3, follows. To ensure that the recommended strategies are sustainable, conditions conducive for their implementation are recommended (see Section 4.4). However, it is understood that the strategies recommended could have imbedded threats and risks, thus, the risks and threats are discussed further and recommendations are made on the basis of the information in Section 4.5.

5.3.1 There is a lack of a dedicated team for improving TPACK for teaching Euclidean geometry with the aid of ICT software

A need emerged from the study for a school-based ICT team that collectivises teachers' efforts towards integrating ICT software in their pedagogical practices. The current study revealed that teachers were working in silos and not sharing their experiences, challenges, and best practices prior to the study intervention (see Section 4.2.1). This finding confirmed what literature reported, namely, that when teachers are working in silos it denies them the opportunity to learn from one another (see Section 2.5.1.1). Prior to the study intervention, the study uncovered that, for the schools that participated in the study, there were no school-based team(s) dedicated to improving teacher TPACK for teaching or integrating ICT in their pedagogical practices (see Section 4.2.1). The study further found that the cause of the lack of a dedicated team at the school to improve ICT was that there was no transparency between the teachers regarding their pedagogical practices, which made it difficult for them to share their skills and knowledge on the use of ICT as a tool for teaching (see Section 4.2). In order to respond to the challenges stated above the study recommends the implementation of the following stages of the strategy formulated in Section 4.3.

5.3.1.1 Strategies recommended for formulating a dedicated team

For the challenges discussed above, the study recommends critical PAR as an approach to respond to the problem of a lack of dedicated team. Thus, the study recommends a team approach to improving teachers' TPACK for teaching Euclidean geometry with the aid of integrated ICT software. This is recommended with the understanding that when critical PAR principles are applied, an external researcher does not decide what the research problem is; instead, a community of people who experience the problem identify the problem for the purpose of improving their own lives through a process of research. For the practitioners, the study recommends using PAR principles to guide the creation of a team. The benefits of using PAR, even for practitioners, is that a PAR approach fosters the sustainability of the team agenda beyond its current members. The application of the principles of PAR enabled the team to have a common interest in responding to a particular problem of interest. Due to the nature of critical PAR, which problematises issues of power, any external researcher or practitioners who are in powerful positions, are depowered, so that those who experience the problem on a daily basis are the people dictating the process of research or finding solutions to their problems. Depowering will, concurrently, create a space for co-researchers who happen to be marginalised from the endeavour of knowledge production; this inclusion occurs through the process of research and finding solutions to their own problems. The following are the steps the study recommends for formulation of a dedicated team. However, it should be said at this juncture that these steps are recommend as part of the strategy designed in Section 4.3.

Initial meetings

The initial meetings create a communicative space and create an element of trust and unity between the co-researchers who took part in the study. The purpose of the meetings is to create an empowering team climate that adheres to the principles of freedom of expression, exchanging of ideas, and expression of opinions with trust and respect. Bricolage fosters these principles and values with the purpose of promoting and creating a team climate that embraces multiple perspectives, where each member of the research team could freely give voice to their perspectives. These meetings are conducted in face-to-face and one-on-one sessions, with the understanding that the study should respond to individual needs that are likely to be easily attainable when efforts are collectivised. This is demonstrated in Sections 3.2.1 and 4.3.1, which refer to the team's initial meetings.

Identification of a common problem

The major purpose of initial meetings is to establish if there is a common problem, needs or interests; this is done during one-on-one sessions. The initial meetings are considered to be informal and lead to creating a team of people who are concerned about the teaching and learning of Euclidean geometry. After one-on-one sessions, the study recommends a formal meeting, where all those who participated during one-on-one sessions come together and share their various perspectives about their problems. This is done because, through different perspectives, people understood their problem(s) and each person's interest(s) better. In the current study, the people who attended the first formal meeting were five mathematics teachers, who were interested in finding ways to improve their teaching of Euclidean geometry concepts with the purpose of improving learners' performance (see Section 4.3.1).

Formulation of a team

A team of co-researchers was formulated based on the intersecting interests of the team members. After identification of a common problem and agreement to work as a team the study recommends the formulation of a research question or objectives, which, in response to the question or objectives, identifies common problems that the research will address. In the current study, the research question was customised in accordance with the interest of the study, of which one part was improving learners' performance in mathematics.

In order for the team to function optimally and respond to the research question, the study recommends a SWOT analysis, as an evaluation tool that the team can employ to assess the strengths, weaknesses, opportunities and threats in relation to the common problem identified. The current study created team member portfolios setting out knowledge and skills, which were mapped as: (i) content knowledge; (ii) pedagogical content knowledge; (iii) technological content knowledge; (iv) pedagogy; (v) content; and (vi) technology. This enabled the research team to identify the strengths, weaknesses, opportunities and threats in relation to the research problem. The SWOT analysis enabled the research team to identify the multiple internal and external factors that are favourable and unfavourable to designing a strategy to improve TPACK for teaching Euclidean geometry. A SWOT analysis was employed as a strategic planning tool to perform training needs analysis and skills analysis, which led to additional team members joining the research team.

5.3.1.2 Recommended conditions conducive to a dedicated team

In pursuit of collective efforts and optimal functionality of a team, the study recommends collective leadership practice within the team. Collective leadership in the current study included sharing of responsibilities, such as convening and chairing meetings, organising logistical issues, such as availability of computer software and computer labs, and facilitating lessons, while other team members observed. Collective leadership becomes a necessary condition for preventing that the leadership roles and responsibilities are assigned to only one person; instead, all team members were involved (see Section 4.3.1 and Section 4.4.1). The study further recommends that, for optimal functionality of a team, it is imperative that there is a sense of belonging and appreciation among the team members. In order for team functionality to be sustained the study recommends that a sense of tolerance for different and contradictory views or perspectives is developed. Moreover, respect, mutual benefit, equality and humility are recommended as characteristic features of an optimally functioning team.

The study recommends that, in order to have a sustainable team, a vision and mission should be developed to guide daily activities of the team and foster a shared purpose among members of the team (see Sections 4.3.2 and 4.4.2). Furthermore, the study recommends that the vision and mission should be such that it would motivate, model behaviour, and promote a high level of commitment, which would cultivate the perfomance of the team (see Section 4.3.2). Thus, a vision should be attractive to the team members in order for them to be committed to turning it into a reality.

5.3.1.3 Threats and risks to creating a dedicated team and recommendations for circomventing them

The strategies in Section 4.3.1.1, which dealt with a team approach to improving teachers' TPACK for teaching Euclidean geometry using integrated ICT software, have inherent risks and threats. These risks and threats include the challenge of finding a convenient time for all team members to meet, and miscommunication. In order to circumvent the identified threats and risks the study recommends that the team members maximise their team contact time by not always scheduling their meetings during school hours. This will enable team members who are not working at the same school, and even those who are not working as teachers, to have maximum contact time (see Section 4.4.1). For instance, during the study intervention the team members had three meetings, every Wednesday during school hours, on Saturday mornings for three hours, and on Sunday afternoons for two and a half hours. However, team members who could meet more than three times a week did so.

5.3.2 Teachers do insufficient lesson preparation when they are using ICT software as a teaching aid

The results of the study revealed that teachers do not assess learners' prerequisite knowledge and skills; this was observed during teachers' lesson facilitation. As a result they make assumptions about learners' readiness for new information, and they do not prepare assessment activities. This was evident when, during their lesson facilitation, Mr Mokoena and Miss Lebaka didn't assess learners' prerequisite knowledge on the theorem they had taught (see Section 4.2.2).

Furthermore, the results of the study revealed that teachers do not prepare measurable lesson objective(s) during their preparation. This was evident when learners couldn't demonstrate the lesson objective within a period specified by the school timetable. For instance, Mr Mokoena expected learners to be able to prove the theorem at the end of the period; and learners found it impossible – they were confused and could not follow the basic concepts of the theorem. Moreover, teachers in this study were unable to complement the animated lesson of HeyMath! with their own knowledge and skills. Instead, they substituted their role of teacher for computer software lessons.

5.3.2.1 Strategies recommended to foster sufficient preparation

The study recommends that a team formulates and designs an action plan to guide the process of improving team members' TPACK for teaching Euclidean geometry. In this research this action plan consisted of a series of activities with time frames specifying when certain tasks had to be completed and by whom. The study recommends that the team first identifies the topic that needs immediate attention, based on learners' performance. For instance, during a team's lesson preparation meetings, members could volunteer or be democratically elected by the other team members to present a lesson on a particular mathematics topic, on the basis of which the entire team will do a lesson preparation (see Section 4.3.4). The act of volunteering by team members during the study intervention suggests that the team members were committed to collaborative lesson preparation. Furthermore, volunteering to be observed by other team members showed that the team members found value in being observed by others and that there is transparency between the team members in relation to their teaching practices.

The study recommends that, during a lesson preparation session, a lesson aim is broken down into objectives that must be formulated and clearly defined such that they are measurable and achievable within a school time period (see Section 4.3.2).

Furthermore, the study recommends that, when preparing a lesson that uses ICT software as a teaching aid to teach Euclidean geometry, the priority should not be on the software or ICT tools to be used, but on Euclidean geometry content. For instance, during a lesson preparation session, the study recommends that, in order to improve the TPACK for teaching Euclidean geometry, preparation should be aimed at three areas (i) content knowledge (ii) pedagogical content knowledge (iii) technological pedagogical content knowledge.

The in order to improve CK the study further recommends that, during preparation, a team of teachers should share their individual understanding of an identified concept or theorem as per a stated lesson aim. The discussion could include, amongst other matters, how each member of the team understands the definitions of concepts and their applications (see Section 4.3.2.1). The purpose of sharing experiences and ideas should be centred on improving one another's CK first. For instance, in the current study, in order to improve the coordinating team members' CK of properties of a circle, the team shared their individual understanding of the definition of a circle. Moreover, the study recommends a strategy to analyse the theorem statement. The strategy is as follows:

a) Identify the main mathematical concepts that form the theorem statement;
b) Subdivide the theorem into mathematical concept clauses that could be represented geometrically using the identified mathematical concepts; and
c) Represent the mathematical concepts geometrically on the same diagram.

Moving from the word problem to a pictorial representation of the word problem enhances an understanding of each mathematical concept identified. This gives the team an opportunity to see the building block of the theorem statement, both pictorially and in word form (see Section 4.3.4).

In addition, the study recommends that, in order to improve PCK during preparation, the team members should share their knowledge of KCS, which represent concepts that learners find or will find difficult to comprehend. This is also done during the analysis of the theorem statement. For instance, in the current study, during the preparation to teach a theorem that states that a line drawn from the centre of a circle perpendicular to the chord bisects the chord, Mrs Kotudi, after the team had identified perpendicular is one on the important concepts in the theorem statement, highlighted that it is important that different mathematical notations are used to help learners understand the concept of perpendicular.

This is done with an understanding that KCS will enable teachers to prepare for their lessons better and improve their PCK, since they would have explored different ways of teaching a particular concept (see Section 4.3.4). This is learner-centred preparation, since KCS is used to create a connection between teachers' knowledge of the content and learners' way of learning.

Moreover, the study recommends that, with the KCS in mind, the team should share what made them understand the content better after their discussions, and then the same opportunities can be created for learners to improve their understanding (see Section 4.3.2.). Therefore, since TPACK is PCK with an additional leg of technology, the study recommends that the ICT software should be used as communication tool and not to replace a teacher, as demonstrated in Section 4.3.4.2.

Furthermore, the study recommends using Geometer's Sketchpad animation to improve the understanding of mathematical concepts, such as the definition of a circle (see Section 4.3.4.3). Using Geometer's Sketchpad software enabled team members to understand that a circle is a set of points equidistant from one point, which happens to be at the centre. This is understood better using Geometer's Sketchpad software, because the software can be used to pictorially take two points and rotate the one around the other one for any angle of your choice, and when this is repeated many times, it forms circle (see Section 4.3.4.3).

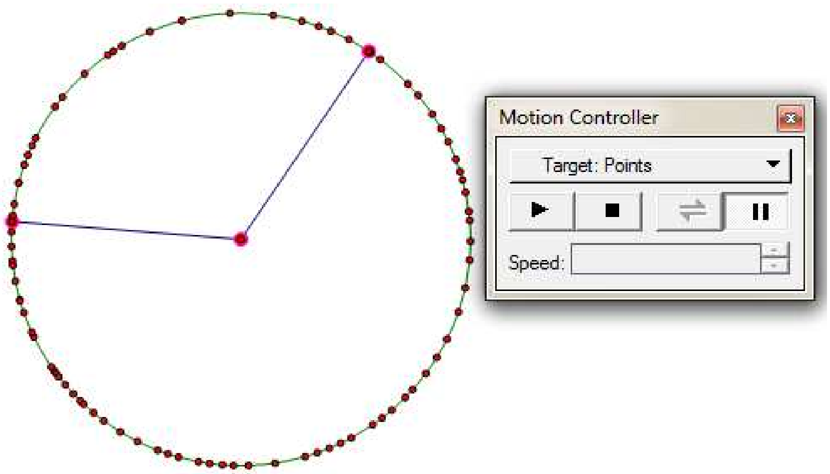

Figure 5.3.2.1: Equidistant points and radius

Furthermore, the motion of the animated points enhances the understanding of the definition of a circle further.

5.3.2.2 Recommended conditions conducive to sufficient preparation

Lesson preparation requires planning, which includes careful collection of thoughts and resources needed to set up an environment that promotes learning of what needs to be taught. However, the findings of this study point out that, due to the complexity of everyday life, the social web of reality and how we know reality, knowledge has become a very complex phenomenon. This complexity has a great influence on our understanding of the ways knowledge is constructed, which is closely related to the ontological stance of individuals. I understand that lesson preparation is a very complex activity that requires a teacher to set up an environment that promotes learning for individuals who are often not the same, and who do not learn the same way. This, in my view, means teaching is a problem-based activity. Firstly, it is a problem for the teacher, since a classroom consists of learners from diverse cultures, history, family morals, social economic status, genders and ages, which have an influence on both their epistemological and ontological stances.

Secondly, learning could take place easily if an environment enables learners to create a connection between what they already know and the content they are currently trying to learn (the study subscribes to this view). Failure to set up an environment that promotes, in an inclusive fashion, all learners' ways of constructing knowledge by connecting the two, was problematic for learners. This complicates the activity of lesson preparation and facilitation. However, bricolage's quest to uncover multilayered, multiple epistemologies, multiple ontologies, multiple social theories and multicultural perspectives of challenges and possible solutions to lesson preparation and teaching are better understood by first appreciating their complexity.

5.3.2.3 Factors threatening sufficient preparation when using ICT as teaching aid

It should be noted that sufficient preparation requires a great deal of time. The team had to ensure that every team member understood content and the most effective ICT software. Thus, often, teachers begin by complaining that their workload does not enable them to do such a preparation. However, in the long run, teachers realised that the use of ICT actually saves time. In this study, it became evident when Miss Lebaka, during her lesson facilitation, realised that learners were struggling with the properties of isosceles triangles, and she used Geometer's Sketchpad software to help learners discover the properties of isosceles triangles. Below is the triangle that learners constructed using Geometer's Sketchpad.

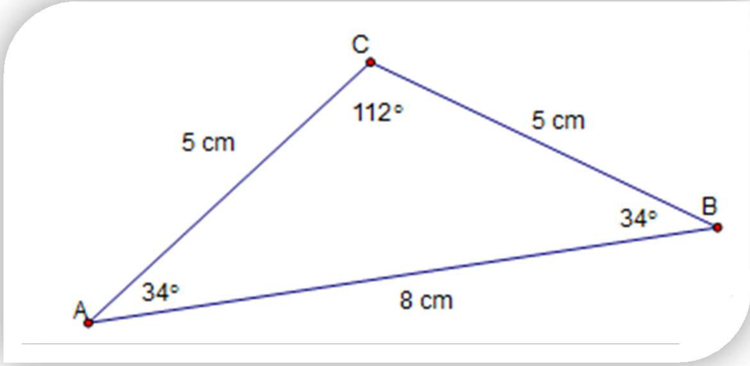

Figure 5.3.2.3: Isosceles triangle

Figure 5.3.2.3 was constructed by learners in Grade 11 who did not know the properties of an isosceles triangle. Learners were led by Miss Lebaka to quickly construct a triangle as shown in Figure 5.3.2.3 using Geometer's Sketchpad software in an attempt to help them discover the properties of an isosceles triangle on their own. These Grade 11 learners had been taught the properties of an isosceles triangle in Grade 8, according to the curriculum policy CAPS. It was not surprising that these learners did not know all the properties of an isosceles triangle, since it is common that learners are not grade ready. This often results in teachers having to present extra classes to teach the learners the work that they should have mastered in previous grades. However, with the use of ICT software such as Geometer's Sketchpad, it was possible that the teacher could, within a short period of time, thread through the work of another grade and explain the work of a current grade. Doing so this reduces the workload of a teacher – he or she does not have to present as many extra classes as would be necessary with traditional methods of teaching.

5.3.3 Teachers facilitate their lesson inadequately when they are using ICT software as teaching aid

The data analysis showed that, during lesson facilitation before the study intervention, learners were not given the opportunity to discover the proofs on their own. It was revealed by the empirical data that teachers like Mr Mokoena used HeyMath! software to narrate the proof of the theorems while learners only watched and listened without grappling with the content. Mr Mokoena taught the proof of the theorem (see Section 4.2.4) by joining OA and OB, but not giving contextual or any form of explanation to learners as to why OA and OB were joined. Moreover, lesson facilitation of the theorems was presented in an abstract manner, making it difficult for learners to understand the theorem, because their only role during the lesson facilitation was to listen and watch the teacher.

The data further revealed that teachers failed to prepare activities to assess learners' prior knowledge, which resulted in the teacher facilitating lessons that did not first establish what learners already knew in relation to the new information. The study further showed that the manner in which lessons were facilitated prior the study intervention did not give learners hope that they could understand Euclidean geometry concepts, as every explanation appeared to be magical to them (see Section 4.2.4). The study revealed that teachers didn't make learners aware of lesson objective(s) or guide them to discover the objective(s) at the end of a lesson. Moreover, the activities of teaching were not connected and mapped together in a way that they built on one another.

5.3.3.1 Recommended lesson facilitation strategies to respond to the challenges identified in Section 5.3.3

From a social constructivism lens the study recommends that, during the lesson facilitation, teachers should set up a learning environment that enables learners to support each other; for instance, when Ntswaki, who seemed to understand better, helped Sellwane to measure the magnitude of an angle using Geometer's Sketchpad software (see Section 4.3.3.1). Thus, through social constructivism's epistemological stance, the study recommends that learning takes place when learners are able to interact with one other in pursuit of making meaning of an Euclidean geometry concept. The study recommends using group work or encouraging learners to work in pairs to foster interaction, with the aim of teaching learners to tolerate each other's strengths and weaknesses, and to utilise them for their mutual benefit (see Section 4.3.3.1).

Through a social constructivism lens the study recommends that learners become major role players during lesson facilitation, thereby taking responsibility for their own learning. In so doing learning becomes self-regulated and creates a space of operation, which promotes communicative action, with learners mutually agreeing on a particular common goal of learning a particular concept of Euclidean geometry. The study recommends applying problem-based learning, which enables learners to make their own discoveries (see Section 4.3.5.2).

The study further recommends that each lesson facilitation should start with what learners already know. For instance, when teaching a Grade 11 theorem that says, a line drawn from the centre of a circle perpendicular to the chord bisects the chord, a teacher should find all the mathematics concepts taught in prior grades which are implied, and apply them in understanding and proving the theorem. Mathematical concepts are often implied and applied, and as a result, due to lack of time, teachers assume that learners have competent knowledge and skills. In response to this challenge the current study recommends the use of Geometer's Sketchpad to thread through all the related concepts over significantly less time but with better explanation.

Through the use of Geometer's Sketchpad learners were able to discover the properties of an isosceles triangle, which was covered in Grades 8 and 9, while the class proved a Grade 11 theorem within a reasonable time (see Section 4.3.4). The study's findings point out that, within a short period of time, with the use of Geometer's Sketchpad software to facilitate a lesson, a teacher was able to assess learners' prior knowledge, identify misconceptions and content gaps, and then help learners to improve their understanding (see Section 4.3.3.1).

The study recommends that teachers develop a diagnostic academic subject improvement plan that spells out all the possible causes of learners' errors and misconceptions. An academic subject improvement plan can be used as a resource for lesson facilitation. The study recommends that it is used as a tool to enhance the teachers' KSC. For instance, it is through the academic subject improvement plan that a teacher could determine which concepts learners are struggling with and what kind of mistakes or errors the learners are making. By definition, this contributes directly to the teacher's knowledge of the content in relation to learners' difficulties in understanding the content.

The study also revealed that the use of ICT software enables the teacher to concretise Euclidean geometry concepts. Thus, through the preceding findings, the study recommends using ICT software such as Geometer's Sketchpad software to enable the teacher to facilitate a lesson in a manner that learners can make meaning of the material taught. For instance, the theorem that a line drawn from the centre of a circle perpendicular to the chord bisects the chord, is a very complex word problem that requires learners to analyse. However, using Geometer's Sketchpad software learners can concretise the theorem using its graphic powers and animations. The picture below shows how GSP software can be used to concretise the preceding theorem.

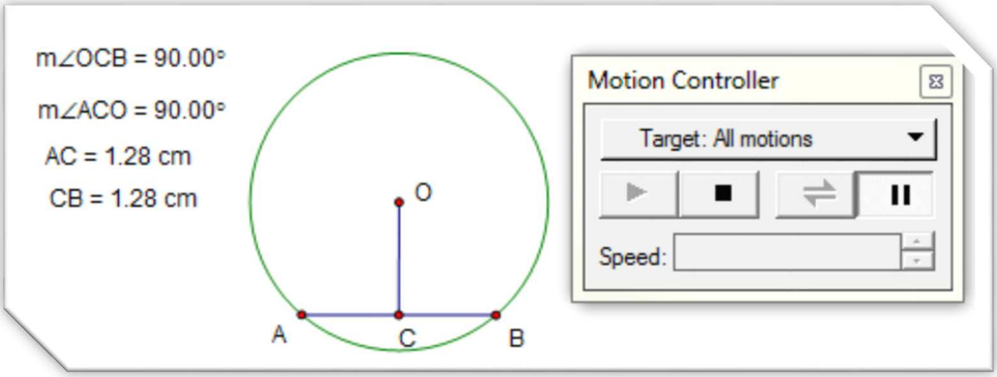

Figure 5.3.3.1a: Concretising perpendicular bisector

Firstly, using Geometer's Sketchpad software enables the teacher to communicate the theorem statement pictorially and accurately, like it is shown in Figure 5.3.3.1a. Therefore, using Geometer's Sketchpad software enables learners to visualise accurate pictorial representations of the theorem statement. A pictorial version of the theorem contributes to a deeper understanding of the theorem, since learners do not see the theorem as a text but as a geometrical figure (see Section 4.3.5.3 for more in this regard).

The study makes a contribution to the way ICT software such as Geometer's Sketchpad should be used as an aid for teaching Euclidean geometry theorems (see Section 4.3.5). The current study recommends that teachers use Geometer's Sketchpad software as a teaching aid in the following manners:

(i) Use ICT software to enhance learners' understanding of the theorem statement, for instance, in Figure 5.3.2.1a Geometer's Sketchpad is used to give a geometrical representation of the theorem.

(ii) Use ICT software to link abstract and logical mathematical reasoning. Figure 5.3.2.1b is not a generally acceptable mathematical proof of the theorem, but it builds on Figure 5.3.2.1a and it contributes to understanding of the proof.

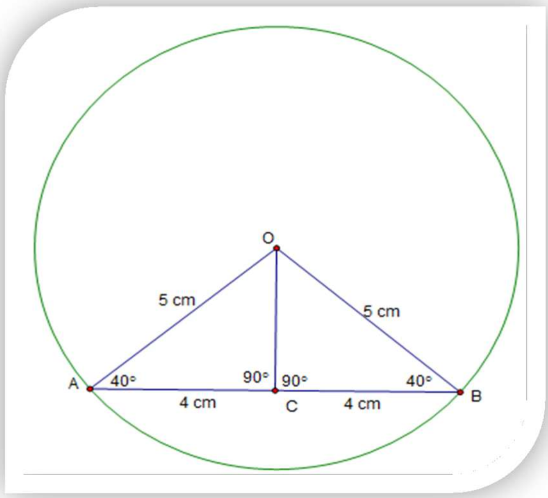

Figure 5.3.3.1b: Concretising perpendicular bisector

The study further recommends that the manner in which teachers facilitate their lessons using Geometer's Sketchpad should be such that learners are not passive listeners but interact actively with content and technology by, for instance, constructing geometrical figures and measuring angles and sides of different geometrical figures. Doing so improves learners' understanding of Euclidean geometry since they are able to make conjectures based on observation using inductive reasoning. Furthermore, the study recommends the use of Geogebra software, which enables teachers to use real-life pictures to enhance learners' understanding.

However, is it not sufficient for learners to operate only on concretised, observation and measurement reasoning – they also need to give logical reasons for their mathematical claims. The study recommends that the lesson facilitation should be such that, after learners have interrelated content and technology and have made sense of abstract Euclidean geometry concepts, then they can be given a task to prove the theorem in general. For instance, learners could be requested to use HeyMath!

software, since the program offers multiple lessons that prove theorems using logical reasoning (see Section 4.3.3.3).

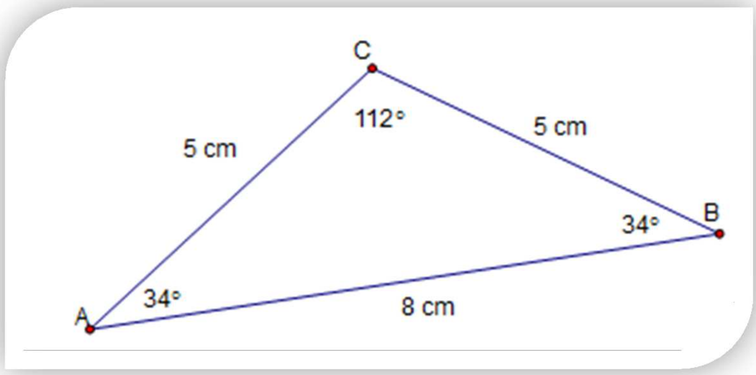

Figure 5.3.3.1c: Concretising perpendicular bisector

5.3.3.2 *Recommended conditions conducive to lesson facilitation with the aid of ICT software*

The study recommends that, in order for using ICT software to teach Euclidean geometry to work effectively, a teacher must create conditions conducive to adequately facilitaton. For instance, the study recommends that a teacher should be able to create interaction between learners, learners and computers, and teacher and the learners. Furthermore, teachers should enable learners to interact, do, talk and demonstrate their thinking in line with the objective of a lesson. For instance, below are the learners interacting socially during a lesson facilitation:

> *Lerato: ntho e tshwanang ditraengeleng tshena tse tharo ke ho re two sides are equal le ha ele hore triangles tsona ha di lekane (Things that are the same in the three triangles is that two sides are equal even though the triangle is not equal).*
>
> *Khyanya: Ho bolelang hore isosceles ena le masaete a mabedi a lekanang le diangle tse pedi tse lekanang (Which means that, isosceles has two sides that are equal and two angles that are equal).*

In the above extract Miss Lebaka created conditions for learning by creating social interaction between learners. In the above extract two learners are working together to grapple with the content of Euclidean geometry and they discover the properties of an isosceles triangle.

The study recommends creating opportunities for learners to work individually on computer software dealing with specific section(s) of Euclidean geometry theorems during a lesson, while the teacher monitors them and engages them as soon as they experience challenges. Doing so ensures that learners follow the lesson, and prevents them from losing the thread due to lack of knowledge of particular software. The learners should also feel free to engage each other on areas they deem fit, and to share ideas about the theorem as they work individually and as a group.

The study further recommends that a team should meet and prepare the lesson together. The process of preparation should include doing research to find different ways in which a particular concept can be taught, and determining a lesson aim and lesson objective(s). For instance, during the preparation meeting a team member could volunteer to present a lesson on a particular mathematics topic. Miss Lebaka, volunteered during our meeting, by saying:

> *Colleague ke kopa ho ikgetha (Colleagues may I please volunteer), I will be doing Euclidean geometry this week, starting with the theorem, a line drawn from the centre of the circle perpendicular to the chord bisects the chord.*

The act of volunteering presupposes that the team members are committed to collaborative lesson preparation. Furthermore, volunteering to be observed by other team members shows that the team members find value in being observed by others and that there is transparency between the team members. Part of preparation is also deciding who will be presenting a lesson that has been prepared. Often, the identification of a topic or a theme leads to formulation of a lesson aim. For instance, in relation to the above narrative, the lesson aim was to teach learners how to prove the theorem. When using ICT software to teach Euclidean geometry the attention should not be on the software, but on the Euclidean geometry content. This means that sharing experiences and ideas should be centred on improving one another's CK. For instance, in order to improve the coordinating team members' CK of the Euclidean geometry theorem stated in the narrative above, the study recommends that teachers work together as a team to share their CK. During this study the coordinating team created a platform for discussing and sharing their understanding of the tangent chord theorem. The theorem states that an angle between a tangent and a chord is equal to the angle on the alternative segment. In pursuit of understanding this theorem Mr Molete suggested that we list all the tools needed to prove the theorem.

Let's first list all the concepts, theorems that will help us prove the theorem.

Miss Lebaka: Angle subtended by diameter is ninety degrees.

Mr Phehello: Angle at the centre is twice the angle at the circumference

Mr Mokoena: Construction.

Mrs Kotudi: Angle subtended by same arc.

The dialogue above captures how the team members shared their CK of the theorem. Miss Lebaka and Mr Phehello's inputs on what is needed to prove the theorem relate to the same theorem but are stated differently. Miss Lebaka said that the angle subtended by the diameter is 90 degrees. Figure 6.2.2.1a shows that the magnitude of angle BAC is 90, while Figure 6.2.2.1b shows that angle at the centre is twice the angle at the circumference subtended by the same chord or arc. The way the two figures were related was discussed during the meeting. Mr Molete explained that,

> *What Mme Lebaka is saying is a special case of the theorem stated by ntate Phehello. For example, using Ntate's argument the angle at the centre is one eighty degree and half of one eighty is ninety.*

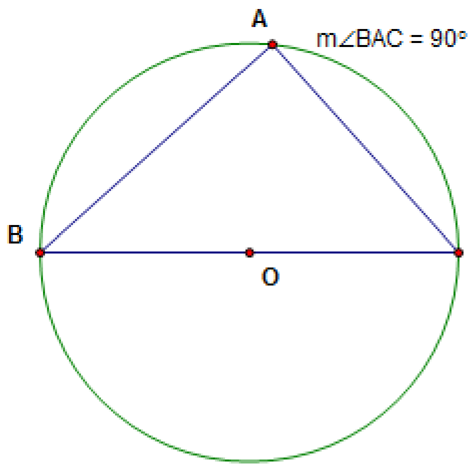

Figure 5.3.3.2a: Angle subtended by diameter

Mr Molete's extract showed the connection between Figure 6.2.2.1a and Figure 6.2.2.2b, which is that both figures are pictorial representations of the theorem stating that the angle at the centre is twice the angle at the circumference subtended by the same chord or arc. Angle BOC in Figure 5.3.3.2a is at the centre of the circle and it is 180 degrees which is half of 90 degrees. Similarly, angle BOC in Figure 5.3.32b is 108 degrees, and half of the angle is 54 degrees.

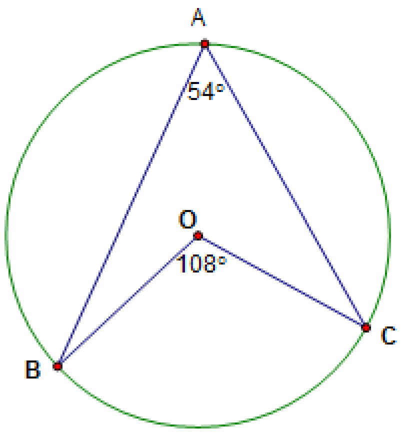

Figure 5.3.3.2b: Centre theorem

Mr Mokoena contributed by saying that we need to understand the constructions that must be done in order to prove the theorem. Construction involves creating the context or conditions for proving the theorem.

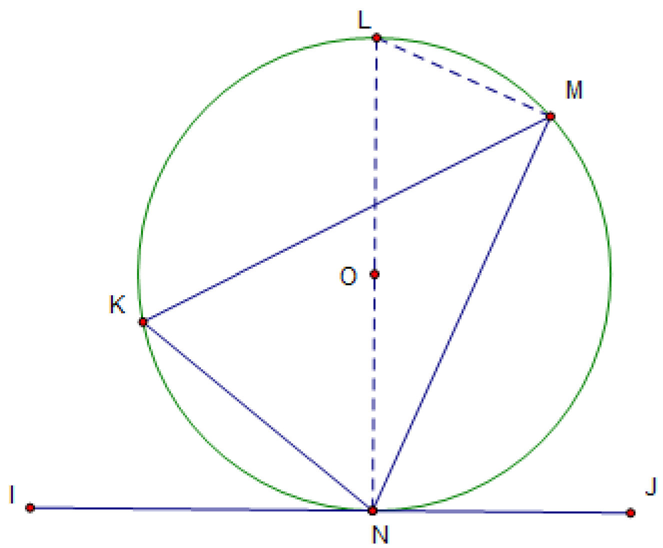

Figure 5.3.3.2c: Chord and tangent theorem

In Figure *5.3.3.2c* both Mr Mokoena and Mrs Kotudi's ideals are implemented, for instance, the dotted lines NL and ML are constructed while KL is an arc which subtends angle LMK and angle LNK.

> *I will suggest that we subdivide the theorem into main clause and identify key concepts individually and thereafter have discussion.*

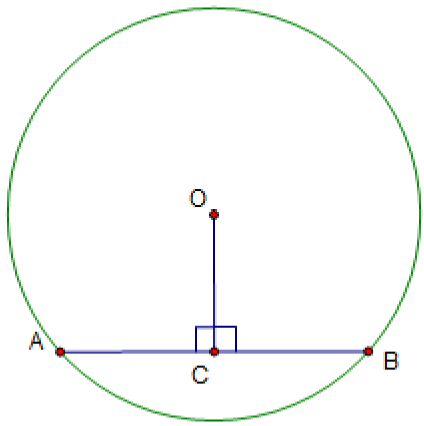

Figure 5.3.3.2d: Perpendicular bisector

From the above extract, subdividing the theorem into key concepts and main clause and then discussing individual understanding of the theorem created an opportunity for team members to construct their own knowledge first. The team discussions created an opportunity for the team members to learn from one another. For instance, Mr Mokoena could subdivide the theorem into key concepts, namely, line, circle, centre of the circle, perpendicular, bisect and chord. In his turn, Mr Molete could subdivide the theorem into main clauses, such as line drawn from the centre of a circle; perpendicular to the chord; bisect the chord. Thus, the communicative space created by the team unified the strength of individuals and thereby became the strength of the team, which held mutual benefits for improving all team members' CK.

The team not only improves CK, but also pedagogy and PCK, which is the interaction between the content and pedagogy (see Section 2.4.1). This is similar to the findings of Jita and Mokhele (2014: 8), namely, that the collaboration platform created in clusters make a significant contribution to improving teachers' CK and PCK. For example, during the implementation of the strategy, different team members presented their different perspectives, which resulted in finding ways to analyse the theorem satement (see Section 4.3.6). For instance, Miss Lebaka suggested that

> *It is important for learners to do the same…*

The above extract confirms that, indeed, the manner in which the team member interacted to share ways of analysing and teaching the theorem concerned seemed to have been effective. This is evident when Miss Lebaka says that learners also need to be taught how to analyse the theorem statement. Analysing the theorem statement by identifying key concepts and main clauses enable the teachers to diagnose learners' prerequisite knowledge and skills.

5.3.3.3 Factors threatening lesson facilitation with the aid of ICT software

Using ICT software for teaching is often threatened by teachers' workload. For instance, it has been reported that teachers do not use ICT software to facilitate their lessons because they find it time consuming and it adds to their workload. However, in this current study, we were able to avoid this threat. The teachers' workload did not increase as a result of using ICT software – instead, but the workload reduced due to the fact that, in one lesson, teachers could quickly revise the work done in previous grades, taking the minimum time, while teaching the work that needed to be taught in the current grade. For instance, during Miss Lebaka's lesson facilitation the following captures interaction between learners

> *Lerato: ntho e tshwanang ditraengeleng tshena tse tharo ke ho re two sides are equal le ha ele hore triangles tsona ha di lekane (Things that are the same in the three triangles are that two sides are equal even though the triangles are not equal).*
>
> *Khyanya: Ho bolelang hore isosceles ena le masaete a mabedi a lekanang le diangle tse pedi tse lekanang (Which means that, isosceles has two sides that are equal and two angles that are equal).*

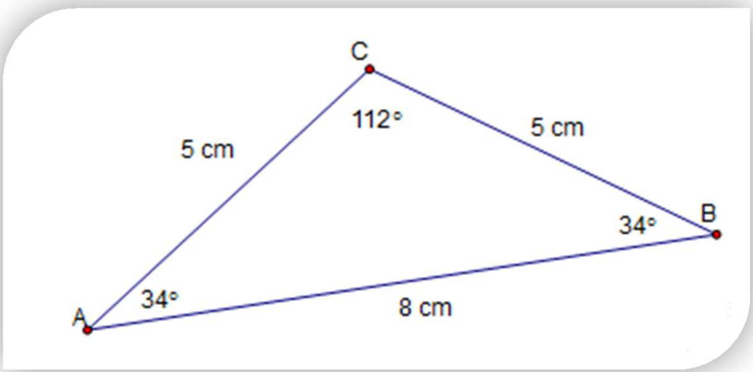

Figure 5.3.3.3a: Isosceles triangle

In the social interaction between learners quoted above, learning took place -- learners discovered the properties of isosceles triangles, which had first been taught in Grade 8. Thus, while Miss Lebaka was teaching a Grade 11 theorem there was a need for her to revise the properties of isosceles triangles in order to link what learners were supposed to know with the new theorem. Using GSP software Miss Lebaka was able to thread between Grades 8, 9 and 11 work while teaching a Grade 11 theorem. Under normal circumstances, when Miss Lebaka was not using the ICT software used in the current study, she would have to present extra classes to revise the work that was supposed to have been done in previous grades. In this way Miss Lebaka linked the theorem she was teaching to Grades 8 and 9 work: OB and OA are equal since they are the radii, which makes triangle ABO an isosceles triangle (see Figure 5.3.3.3b).

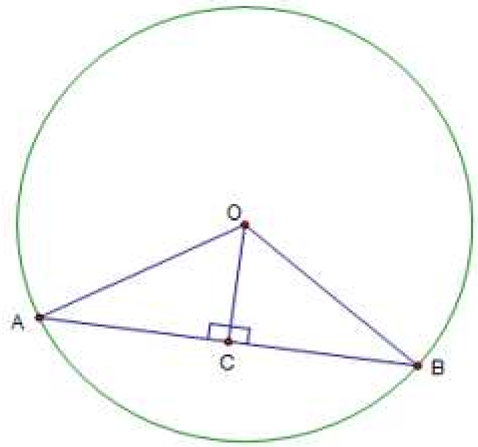

Figure 5.3.3.3b: Perpendicular bisector

Therefore, if triangle ABO is isosceles then angle A and angle B are equal. When learners know that the two angles are equal, then they can be guided to use congruence to prove that angle OCB and angle ACO equal 90 degrees.

5.4 CONCLUSION AND SUMMARY OF THE STRATEGY

The current chapter reported the findings of the study, which revealed that teachers work in silos. The results were that teachers did not prepare lessons sufficiently, which led to ineffective learning, and learners failing Euclidean geometry. The findings of the study justified formulating a strategy to improve the teaching of Euclidean geometry with the aid of ICT software. The chapter concludes by presenting a summary of the strategy formulated for this study.

PHASE I: PREPARATION

Anyone could initiate the process of working as a team. This could be done by a teacher, principal, local academic or any member of the community concerned. The preparation phase starts with a person who is concerned with a particular situation recruiting other people who might have the same problem or interest in solving the problem. A head of a department of mathematics could be concerned about the learners' performance in mathematics, or about teachers' development, as one of the roles he/she has to play. Recruiting members is done for the purpose of establishing a team of people with a common problem that could join forces and contribute to the team different skills, interests, and perspectives.

PHASE II: SHARED VISION

After creating a team, it is important that the team members have a common or shared vision. A shared vision should incorporate all the team members' expectations and interests, which guide and dictate all the activities that are to be performed by the team.

PHASE III: SWOT ANALYSIS

At this stage, team members perform an analysis of their strengths, weaknesses, opportunities and threats in relation to the common problem identified. Upon identification of the weaknesses and threats, new members of the team can be recruited to ensure that all the necessary human resources are available within the team. The weaknesses and strengths are mapped to ensure that all the skills and resources needed to achieve a shared vision are within the team's reach. A SWOT analysis gives the team an opportunity to expand beyond the teaching fraternity to other people, whose knowledge and skills would enable the team to achieve its objectives.

PHASE IV: ROLES AND RESPONSIBILITIES

The roles and responsibilities of the team member should not be fixed; flexibility ensures that all team members develop. This is achievable when there is collective leadership within the team. This means that the leadership roles and responsibilities are not fixed for performance by one person. All team members have the opportunity to experience these roles and in this way a sense of belonging and appreciation is bestowed on them. In addition, this encourages them to look forward to their collaborative team engagements.

PHASE V: ACTION PLAN

The action plan is a detailed plan presenting a breakdown of all the activities that the team intends to perform in pursuit of a shared vision. This plan shows the following:

a) Activity to be performed;

b) A person responsible for performing or facilitating the activity;

c) Time when the activity will be performed;

d) What resources will be needed to perform the activity; and

e) Indicators of success.

PHASE VI: LESSON PREPARATION AS A DEVELOPMENTAL ACTIVITY

Lesson preparation as a developmental activity occurs when the team members come together to prepare a lesson. The lesson preparation should be divided to cover the following:

a) Content knowledge

b) Pedagogical content knowledge

c) Technological content knowledge

d) Technological pedagogical content knowledge

During lesson preparation, sharing CK should be made a priority. This is done with the understanding that the knowledge and skills that learners are supposed to demonstrate at the end of schooling are embedded in the content that is prescribed by the curriculum policy. Thus, it is important that a teacher is able to interpret the content of mathematics in relation to its content aim. However, understanding the content of Euclidean geometry alone is not enough for a teacher. It is important that, during collaborative lesson preparation, teachers share their best practices on how to teach the content being discussed. This exercise should not only be about drawing from each other's experiences, but creating new knowledge informed by the discussion of the content (see Section 4.3.4). Knowledge is a product of human activities, created through their interactions and their interaction with the world around them. Thus, the social interaction made possible by the team approach to teaching makes it possible for new knowledge to be produced. Similarly, taking the PCK a step further, beyond Lee Shulman (1986: 1-27) and Ball et al. (2008: 389) to the TPACK of Mishra and Koehler (2006: 1017), the team members could share how to integrate ICT software and content of Euclidean geometry. Due to the diversity and variety of software available, sharing of TCK creates an opportunity for the team members to learn from each other. This could also be done by doing research to find different software that can be integrated with Euclidean geometry content.

Another step is necessary, because the ultimate goal for a teacher is to apply his or her TPACK for teaching. Thus, the social interaction creates an opportunity for production of new knowledge on how to use one or more software packages to teach (see Section 4.3.4 demonstration). This way of preparing a lesson is developmental and self-empowering, because the teacher doesn't have to wait for the employer's developmental programmes, which are not always responsive a particular school's challenges.

PHASE VII: LESSON FACILITATION AS A FURTHER DEVELOPMENTAL ACTIVITY

Lesson facilitation taken a step further, to professional development, happens when teaching, for instance, Euclidean geometry, and a teacher concurrently takes note of what seems to be working and not working. The purpose of this exercise is to improve the manner in which the lesson is facilitated. Note-taking enhances the reflecting sessions after teaching. When teachers work together as a team and they are able to facilitate a lesson they planned together in the presence of others it adds to the richness of knowledge produced during the lesson reflection. During the lesson reflection session each teacher shares his/her observations on how learning took place and how the lesson can be improved further. Thus, this becomes a developmental activity for a teacher as he/she teaches.

REFERENCES

Alazam, A., Bakar, A. R. & Asmiran, R. H. (2012). Teachers' ICT skills and ICT integration in the classroom: The case of vocational and technical teachers in Malaysia. *Creative Education, 3*, 70-76.

Alexandra, G. (2013). Towards a learner-centred approach to teaching English. *Professional Communication and Translation Studies, 6*(12), 205-210.

Ali, G., Haolader, F. A. & Muhammad, K. (2013). The role of ICT to make teaching-learning effective in higher institutions of learning in Uganda. *International Journal of Innovative Research in Science Engineering and Technology, 2*(8), 4061-4073.

Ali, I., Bhagawati, S. & Sarmah, J. (2014). Performance of geometry among the secondary school students of Bhurbandha CD Block of Morigaon District, Assam, India. *International Journal of Innovative Research and Development, 3*(11), 73-77.

Altun, T. (2013). Examination of classroom teachers' technological pedagogical and content knowledge on the basis of their demographic profiles. *Croatian Journal of Education*, 15(2), 365-397.

Anderson, D. L. (2005). A portrait of a feminist mathematics classroom: What adolescent girls say about mathematics, themselves and their experiences. In a 'unique' learning environment. *Feminist Teacher 15*(3), 175–94

Andrews, T. (2012). What is social constructionism? *The Grounded Theory Review, 11*(1), 39-44.

Annkuch, M. (2006). Comparing box plot distributions: A teacher's reasoning. *Statistics Education Research Journal, 5*(2), 27-45.

Antonio, R. J. (1989). The normative foundation of emancipatory theory: Evolutionary versus pragmatic perspectives. *American Journal of Sociology, 94*(4), 721-748.

Archbald, D. (2013). Vision and leadership: problem-based learning as a teaching tool. *Journal of Leadership Education, 12*(2), 136-147.

Avidov-Ungar, O., & Iluz, I. E. (2014). Levels of ICT integration among teacher educators in a teacher education academic college. Interdisciplinary Journal of E-Learning and Learning Objects, 10, 195-216

Avolio, B., Walumbwa, F. & Weber, T. J. (2009). Leardership: Current theories, research, and future directions. *Annual Review of Psychology, 60*, 421-449.

Ayub, A. & Razzaq, A. (2013). A conceptual farmework on evaluating SWOT analysis as the mediator in strategic marketing planning through marketing intelligence. *European Journal of Business and Social Sciences, 2*(1), 91-98.

Azlim, M., Amran, M. & Rusli, M. R. (2015). Utilization of educational technology to enhance teaching practices: Case study of community college in Malaysia. *Procedia - Social and Behavioral Sciences, 3*, 1793-1797.

Baker, M. L., Dieter, E. & Dobbins, Z. (2014). The art of being persuaded: Wayne Booth's mutual inquiry and the trust to listen. *Composition Studies, 42*(1), 13-34.

Ball, D. L. & McDiarmid, G. W. (1990). The subject matter preparation of teachers. In W. R. Houston, *Handbook for research on teacher education* (pp. 437-449). New York: Macmillan.

Ball, D. L., Thames, M. H. & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education, 59*, 389-407.

Barrett, D. & Green, K. (2009). Pedagogical content knowledge as a foundation for an interdisciplinary graduate program. *Spring, 18*(1), 17-28.

Bassi, R. (2011). *ICTs in education (ICT4E) policies and plans worldwide.* Global e-School and Communities Initiative, GeSCI.

Berry, K. S. (2004a). Feedback looping for increasing complexity. In J. L. Kincheloe & K. S. Berry, *Rigour and complexity in educational research conceptualizing bricolage* (pp. 128-146). England: McGraw-Hill Education.

Berry, K. S. (2004b). Structures of bricolage and complexity. In J. L. Kincheloe & K. S. Berry, *Rigour and complexity in educational research conceptualizing the bricolage* (pp. 103-127). England: McGraw-Hill Education.

Bingimlas, K. A. (2009). Barriers to the sucessfull integration of ICT in teaching and learning envirnment: A review of the literature. *Eurasia Jounal of Mathematics, Science & Technology Education, 5*(3), 235-245.

Broom, C. (2015). Assessment and evaluation: Exploring their principles and purposes in relation to neoliberalism through a social studies case study. *Canadian Social Studies, 45*(2), 17-36.

Brown, G. T., Chaudhry, H. & Dhamija, R. (2015). The impact of an assessment policy upon teachers' self-reported assessment beliefs and practices: A quasi-experimental study of Indian teachers in private schools. *International Journal of Educational Research, 71*, 50-64.

Brown, R. H. & Goodman, D. (2001). Jurgen Habermas' theory of communicative action: An incomplete project. In G. Ritzer & B. Smart (Eds), *Social theory* (pp. 2001-16). London: Sage.

Buabeng-Andoh, C. (2012). Factors influencing teachers' adoption and integration of information and communication technology into teaching: A review of the literature. *International Journal of Education and Development using Information and Communication Technology, 8*(1), 136-155.

Burke, P. J. & Kirton, A. (2006). The insider perspective: teachers-as-researchers. *Reflecting Education*, *2*(1), 1-4.

Cardon, P. W. & Marshall, B. (2015). The hype and reality of social media use for work collaboration and team communication. *International Journal of Business Communication, 52*(3), 273-293.

Chai, C. S., Koh, J. H. & Tsai, C. (2013). A review of technological pedagogical content knowledge. *Educational Technology & Society, 16*(2), 31-51.

Chavan, D. (2013). Lesson study an urgent need for quality teaching in knowledge society. *Scholarly Research Journal for Interdisciplinary Studies*, *2*(3), 1-11.

Chia, N. K. (2013). A psychogogic perspective of lesson study for special education teachers. *Academic Research International, 4*(4), 334-346.

Chilisa, B. (2012). *Indigenous research methodologies*. USA: Sage.

Choi, K. & Park, H. (2013). A comparative analysis of geometry education on curriculum standards, textbook structure, and textbook items between the US and Korea. *Eurasia Journal of Mathematics, Science & Technology Education, 9*(4), 273-284.

Clark, N. L. & Worger, W. H. (2004) South Africa: The rise and fall of apartheid. Seminar Studies in History. Harlow, England, New York: Pearson Education. pp. 48-52.

Cobb, P. (1994). Where is the mind? Constructivist and sociocultural perspectives on mathematical development. *Educational Researcher, 23*(7), 13-20.

Comte, A. (1854). System of positive policy. New York: Burt Franklin.

Contractor, N. S., Dechurch, L. A., Carson, J., Carter, D. R. & Keegan, B. (2012). The topology of collective leadership. *The Leadership Quarterly, 23*, 994-1011.

Cornwall, A. & Jewkes, R. (1995). What is participatory research? *Social Science & Medicine, 41*, 1667–1676.

Daly, C., Pachler, N. & Pelletier, C. (2010). *Continuing professional development in ICT for teachers: A literature review*. London: Institute of Education, University of London.

Darbi, W. P. (2012). Of mission and vision statements and their potential impact on employee behaviour and attitude: The case of a public but profit-orientated tertiary institution. *International Journal of Business and Social Science, 3*(14), 95-109.

Day, C. (1999). Professional development and reflective practice: purposes, processes and partnerships. *Pedagogy, Culture and Society*, *7*(2), 221-233.

DBE (Department of Basic Education) (2004). White paper on e-Eudcation. South Africa.

DBE (Department of Basic Education) (2008). Compliance with Section 16a and 58b of the South African Schools Act, 1996 with regard to school performance and the procedures to be followed by and for underperforming primary and secondary schools. South Africa.

DBE (Department of Basic Education) (2011). Curriculum and Assessment Policy Statement Grades 10-12. Mathematics. South Africa.

DBE (Department of Basic Education) (2012). National protocol for assessment. South Africa.

de Beaugrande, R. 2006. Language and Symbolic Power. Studies in Language & Capitalism. 1:29-56

Denzin, N. K. & Lincoln, Y. (1994). Introduction: Entering the field of qualitative research. In N. K. Denzin & Y. Lincoln, *Handbook of qualitative research* (pp. 1-17). Thousand Oaks, CA: Sage.

Denzin, N. K. & Lincoln, Y. S. (2005). Introduction: The discipline and practice of qualitative research. In N. K. Denzin & Y. S. Lincoln, *The Sage handbook of qualitative research.* Thousand Oaks: CA: Sage.

Denzin, N. K. & Lincoln, Y. S. (Eds). (1999). *The Sage handbook of qualitative research* (3rd ed.). Thousand Oaks, CA: Sage.

Denzin, N. K. (2001). The seventh moment: Qualitative inquiry and the practices of a more radical consumer research. *Journal of Consumer Research, 28*(2), 324-330.

Denzin, N. K. (2002). Social work in the seventh moment. *Qualitative Social Work, 1*(1), 25-38.

Denzin, N. K. (2008). The new paradigm dialogs and qualitative inquiry. *International Journal of Qualitative Studies in Education, 21*(4), 315-325.

Dezin, N. K. (2010). Moments, mixed methods, and paradigm dialogs. *Qualitative Inquiry, 2*(1), 1-9.

Dodson, L. & Schmalzbauer, L. (2005). Poor mothers and habits of hiding: Participatory methods in poverty research. *Journal of Marriage and Family, 67,* 949–959.

Doig, B. & Groves, S. (2011). Japanese lesson study: Teacher professional development through communities of inquiry. *MTED, 13*(1), 77-93.

Doolittle, P. E. (2014). Complex constructivism: A theoretical model of complexity and cognition. *International Journal of Teaching and Learning in Higher Education, 26*(3), 485-498.

Dunn, K. E. & Mulvenon, S. W. (2009). A critical review of research on formative assessments: The limited scientific evidence of the impact of formative assessments in education. *Practical Assessment & Research and Evaluation, 14*(7), 1-11.

Dupuis, S. L., McAiney, C. A., Fortune, D. B., Ploeg, J. & de Witt, L. (2014). Theoretical foundations guiding culture change: The work of the partnerships in dementia care alliance. *Dementia Online First*, *15*(1), 85-105.

Eady, M. J. & Lockyer, L. (2013). Tools for learning: technology and teaching strategies, Learning to teach in the primary school, Queensland. University of Technology. Australia. p. 71.

Fairclough, N. (2013). *Critical discourse analysis: The critical study of language*. Routledge.

Fu, J. S. (2013). ICT in education: A critical literature review and its implications. *International Journal of Education and Development using Information and Communication Technology, 9*(1), 112-125.

Fu, JS (2013). International Journal of Education and Development using Information and Communication Technology (IJEDICT), 9(1),112-125

Funkushima, A and Kusano, M (2014: 5620). A network perspective on nitrogen metabolism from model to crop plants using integrated 'omics' approaches. Journal of Experimental Botany

Gazor, H. (2012). A literature review on challenges of virtual team's leadership. *Journal of Sociological Research, 3*(2), 134-145.

Geertz, C. (1988). *Works and lives: The anthropologist as author.* Stanford: Stanford University Press.

Gesell, I. & Hopen, D. (2014). Leadership lessons gleaned from a facilitation mindset. *The Journal for Quality & Participation*, *36*(4), 1-3.

Gillis, A. & Jackson, W. (2002). *Research methods for nurses: Methods and interpretation*. Philadelphia: F.A. Davis Company.

Given, L. M. (2008). *The Sage encyclopedia of qualitative research methods* (1st & 2nd Ed., Vol. 1 & 2). London & Thousand Oaks, CA: Sage.

Habermas, J. (1971). *Knowledge and human interest.* Toronto, Canada: Saunders of Toronto.

Hadjerrouit, S. (2008). Using a learner-centered approach to teach ICT in secondary schools: An exploratory study. *Issues in Informing Science and Information Technology, 5*, 233-259.

Hakkarainen, P. & Korepanova, I. (2009). The status of the Vygotsky's "Zone of Proximal Development" in Russian psychology today. *Journal of Russian and East European Psychology, 47*(6), 3-8.

Hebblethwaite, S. & Curley, L. (2015). Exploring the role of community recreation in stroke recovery using participatory action research and photovoice. *Therapeutic Recreation Journal, 71*(1), 1-17.

Hense, J. & Mandl, H. (2012). Learning in or with games? Quality criteria for digital learning games from the perspectives of learning, emotion, and motivation theory. In D.G. Sampson, J.M. Spector, D. Ifenthaler & P. Isaias(eds.), Proceedings of the IADIS International Conference on Cognition and Exploratory Learning in the Digital Age (pp. 19-26). Madrid: IADIS.

Herbst, P. & Kosko, K. W. (2014). Using representations of practice to elicit teachers' tacit knowledge of practice: A comparison of responses to animations and videos. *Journal of Mathematics Teacher Education 17*(6), 515-537.

Hess, F. M. & English, E. (2015). Greenfield schooling and educational innovation. *Psychology of Aesthetics, Creativity, and the Arts, 9*(2), 193-197.

Howard, L. & Potts, E. A. (2009). Using co-planning time: Strategies for a successful co-teaching marriage. *Teaching Exceptional Children Plus, 5*(4), 2009.

Hu, J. & Linden, R. C. (2015). Making a difference in the teamwork: Linking team prosocial motivation to team processes and effectiveness. *Academy of Management Journal, 58*(4), 1102-1127.

Hue, L. T. & Jalil, H. A. (2013). Attitudes towards ICT integration into curriculum and usage among university lecturers in Vietnam. *International Journal of Instruction, 6*(2), 53-66.

Hume, D. (1740). *A treatise of human nature: Being an attempt to introduce the experimental method of reasoning into moral subjects*. Volume 3. London, Thomas Longman.

Hurrell, D. P. (2013). What teachers need to know to teach mathematics: An argument for a reconceptualised model. *Australian Journal of Teacher Education, 38*(11), 54-64.

Hyeyoung, O. H. (2011). Comparison of computer programs which used in statistics teaching, *Korean Journal of Mathematics*, *19*(4), 453 – 464.

IDEA. (2004). Individuals with disabilities education. Retrieved from http://idea.ed.gov/. Retrieved from http://idea.ed.gov/.

Jeyaraj, K. L., Muralidharan, C., Senthilvelan, T. & Deshmukh, S. G. (2012). Application of SWOT and principles component analysis in a textile company. A case study. *International Journal of Engineering Research and Development, 1*(9), 46-54.

Jita, L. C. & Mokhele, M. L. (2014). When teacher clusters work: selected experiences of South African teachers with the cluster approach to professional development. *South African Journal of Education, 34*(3), 1-15.

Jita, L. C., Maree, J. G. & Ndlalane, T. C. (2007). Lesson study (jyugyo kenkyu) from Japan to South Africa: A science and mathematics intervention program for secondary teachers. In I. B. Atweh, M. Borba, A. C. Barton, D. Clark, N. Gough, C. Kietel, R. Vithal, *Internationalisation and globalisation in mathematics and science education.* Dordrecht: The Netherlands: Springer.

Johnson, J. F., Uline, C., & Perez, L. (2011). Expert noticing and principals of high performing urban schools. Journal of Education for Students Placed at Risk, 16(2), 122-136.

Jorge, G. (2013). Why organizations lack of a true ethic organizational culture based upon values: Vision, mission and values on the walls is not enough. *International Journal of Good Conscience, 8*(2), 122-129.

Kaino, L. M. (2008). Usefulness and enjoyment of using computers in learning: A gender dimension. *Gender and Behavior Journal*, *6*(2), 1841-1857.

Kantabutra, S. (2008). What do we know about vision? *The Journal of Applied Business Research, 24*(2), 127-138.

Karami, M., Karami, Z. & Attaran, M. (2013). Integrating problem-based learning with ICT for developing trainee teachers' content knowledge and teaching skill. *International Journal of Education and Development using Information and Communication Technology, 8*(1), 36-49.

Karolich, R. & Ford, J. (2013). Applying learner-centered principles to teaching human behavior in the social environment in a Baccalaureate program. *Journal of Teaching in Social Work, 33*, 26-40.

Kazu, I. (2011). An investigation of factors affecting the use of educational technology in Turkish primary schools. Education, 131(3), 510-524.

Kemmis, S. & McTaggart, R. (1990). T*he action research reader* (3rd edition). Geelong: Deakin University Press.

Kemmis, S., McTaggart, R. & Nixon, R. (2013). *The action research planner: Doing critical participatory action research*. Springer Science & Business Media.

Keong, C., Horani, S. & Daniel, J. (2005). A study on the use of ICT in mathematics teaching. *Malaysian Online Journal of Instructional Technology, 2*(3), 43-51.

Kerzner, H. (1979). *Project management: a systems approach to planning, scheduling and controlling*. New York: Van Nostrand Reinhold.

Kilic, A. (2010). Learner-centered micro teaching in teacher education. *International Journal of Instruction, 3*(1), 77-100.

Kincheloe, J. L. & Berry, K. S. (Eds) (2004). *Rigour and complexity in educational research: Conceptualizing the bricolage*. Maidenhead: Open University Press.

Kincheloe, J. L. (1991). *Teachers as researchers: qualitative inquiry as a path to empowerment*. London, New York: The Falmer Press.

Kincheloe, J. L. (2001). Describing the bricolage: Conceptualizing a new rigor in qualitative research. *Qualitative Inquiry, 7*(6), 679-692.

Kincheloe, J. L. (2004a). Introduction: The power of the bricolage: Expanding research methods. In J. L. Kincheloe & K. S. Berry (Eds), Rigour and complexity in educational research: Conceptualizing the bricolage (pp. 1-22). Maidenhead: Open University Press.

Kincheloe, J. L. (2004b). Redefining rigour and complexity in research. In J. L. Kincheloe & K. S. Berry (Eds), Rigour and complexity in educational research: Conceptualizing the bricolage (pp. 23-49). Maidenhead: Open University Press.

Kincheloe, J. L. (2004c). Questions of disciplinarity/interdisciplinarity in a changing world. In J.L. Kincheloe & K. S. Berry (Eds), Rigour and complexity in educational research: Conceptualizing the bricolage (pp. 50-81). Maidenhead: Open University Press.

Kincheloe, J. L. (2004d). Redefining and interpreting the object of study. In J. L. Kincheloe & K. S. Berry (Eds), Rigour and complexity in educational research: Conceptualizing the bricolage (pp. 82-102). Maidenhead: Open University Press.

Kincheloe, J. L. (2005a). On to the next level: Continuing the conceptualization of the bricolage. *Qualitative Inquiry*, 11(3), 323-350.

Kincheloe, J. L. (2005b). *Critical constructivism primer*. New York, NY: P. Lang.

Kincheloe, J. L. (2008). *Knowledge and critical pedagogy*. Dordrecht: Springer Science.

Kincheloe, J. L. (2009). Critical complexity and participatory action research: Decolonizing "Democratic" knowledge production. In D. Kappor & S. Jordan, *Education, participatory action research, and social change international perspective* (pp. 108-121). New York: Palgrave MacMillan.

Kincheloe, J. L. (2011). *Critical pedagogy and the knowledge wars of the twenty-first century*. Key works in critical pedagogy. Sens ePublishers.

Kincheloe, J. L., McLaren, P. & Steinberg, S. (2011). Critical pedagogy and qualitative research: Moving to the bricolage. In N. K. Denzin & Y. S. Lincoln (Eds), The Sage handbook of qualitative research (4th Ed., pp. 163-178). Thousand Oaks, CA: Sage.

Kindon, S. L., Pain, R. & Kesby, M. (2007). *Participatory action research approaches and methods: connecting people, participation and place.* Routledge Studies in Human Geography, 22. London: Routledge.

Kleickmann, T., Richter, D., Kunter, M., Elsner, J., Besser, M., Krauss, S. & Baumert, J. (2013). Teachers' content knowledge and pedagogical content knowledge: The role of structural differences in teacher education. *Journal of Teacher Education, 64*(1), 90-106.

Kocolowski, M. D. (2010). Shared leadership: Is it time for a change? *Emerging Leadership Journeys, 3*(1), 22-32.

Koehler, M. J. & Mishra, P. (2009). What is technological pedagogical content knowledge? *Contemporary Issues in Technology and Teacher Education, 9*(1), 60-70.

Koehler, M. J., Mishra, P. & Cain, W. (2013). What is technological pedagogical content knowledge (TPACK)? *Journal of Education, 193*(3), 13-19.

Koehler, M. J., Mishra, P., Akcaoglu, M. & Rosenberg, J. (2013). The technological pedagogical content knowledge framework for teachers and teacher educators. In R. Thyagarajan (Ed.). *ICT integrated teacher education: A resource book* (pp. 2-7). New Delhi, India: CEMCA.

Koehler, M. J., Mishra, P., Kereluik, K., Shin, T. S. & Graham, C. R. (2014). The technological pedagogical content knowledge framework. In J. M. Spector, M. D. Merrill, J. Elen & M. J. Bishop (Eds), *Handbook of research on educational communications in technology* (pp. 101-111). New York: Springer.

Koh, J. H., Chai, C. S. & Tsai, C. (2014). Demographic factors, TPACK constructs, and teachers' perceptions of constructivist-oriented TPACK. *Educational Technology & Society, 17*(1), 185-196.

Kouzes, J., Posner, B. & Bunting, M. (2015). Vision is not an annual speech. *New Zealand Managemen*t, 62(3), 20.

Kress, G. & Hodge, R. (1993) Language as ideology (2nd ed.). London: Routledge &. Kegan Paul.

Kress, T. M. (2011). *Critical praxis research: Breathing new life into research methods for teachers*. Dorchester Massachusetts US: Springer.

Kumar, K. (2011). A learner-centred mock conference model for undergraduate teaching. *Collected Essays on Teaching and Learning, 4*, 20-24.

Leanna, K. (2014). Teachers' reflections on critical pedagogy in the classroom. *Interactions: UCLA Journal of Education and Information Studies, 10*(2), 1-19.

Leendertz, V., Blignaut, A. S., Blignaut, H. D., Els, C. J. & Ellis, S. M. (2013). Technological pedagogical content knowledge in South African mathematics classrooms: A secondary analysis of SITES 2006 data. *Pythagoras, 34*(2), 1-9.

Lévi-Strauss, C. (1962). *The savage mind.* Paris: Librarie Plon.

Lincoln, Y. (2001). An emerging new bricoleur: Promises and possibilities—A reaction to Joe Kincheloe's "Describing the Bricoleur". *Qualitative Inquiry*, *7*(6), 693-705.

Liu, Y. & Kaino, L. M. (2007). Geometer's Sketchpad and MSW Logo in mathematics classroom instruction. A comparative analysis. *Journal of Interdisciplinary Mathematics, 2*(1), 112-118.

MacDonald, C. (2012). Understanding participatory action research: A qualitative research methodology option. *The Canadian Journal of Action Research*, *13*(2), 34-50.

Maguire, P. (1987). *Doing participatory action research: A feminist approach*. Massachusetts: University of Massachusetts Press.

Mahlomaholo, S. & Vhonani, N. (2012). Post-apartheid organic intellectual and knowledge creation. *At the Interface/Probing the Boundaries, 78*, 36-54.

Mahlomaholo, S. (2009). Critical emancipatory research and academic identity*. Africa Education Review, 6*(2), 224-237.

Mahlomaholo, S. (2013a). On bricolage and the creation of sustainable postgraduate learning environments. *The Journal for Transdisciplinary Research in Southern Africa, 9*(2), 379-392.

Mahlomaholo, S. G. (2012). Academic network and sustainability environments. St Petersburg State University, Russia: Networks in the Global World Conference.

Mahlomaholo, S. G. (2013b). Naivety of empiricism versus complexity of bricolage. ICERI2013 Proceedings. pp. 4690-4699.

Marcus, G. E. & Fischer, M. M. (1999). *Anthropology as cultural critique* (2^{nd} ed.). London: The University of Chicago Press.

Martin, J., McCormack, B., Fitzsimons, D. & Spirig, R. (2014). The importance of inspiring a shared vision. *International Practice Development Journal, 4*(2), 1-16.

Mbati, L. & Minnaar, A. (2015). Guidelines towards the facilitation of interactive online learning programmes in higher education. *International Review of Research in Open and Distributed Learning, 16*(2), 272-287.

McCombs, B.L. 2001. What Do We Know about Learners and Learning? The Learner-centered Framework: Bringing the Educational System into Balance. Educational Horizons, 79: 182-193.

McNiff, J. & Whitehead, J. (2006). *All you need to know about action research*. Thousand Oaks, CA: Sage.

McTaggart, R. (1997). Guiding principles for participatory action research. In R. McTaggart, *Participatory action research: International contexts and consequences* (pp. 25-43). Suny Press.

Meng, C (2013). Learning to design Geometer's Sketchpad activities for teaching mathematics through lesson study. *Asia Pacific Journal of Multidisciplinary Research, 1*(1), 62-74.

Mengibar, A. C. (2015). Critical discourse analysis in the study of representation, identity politics and power relations: a multi-method approach. *Communication & Society, 28*(2), 39-54.

Mhina, C. H. (2009). Research and agency: The case of rural women and land tenure in Tanzania. In D. Kapoor & S. Jordan (Eds), *Education, participatory action research, and social change: International perspectives* (pp. 155-168). New York: Palgrave Macmillan.

Mickan, S. & Rodger, S. (2000). Characteristics of effective teams: A literature review. *Australian Health Review, 23*(3),201-208.

Minkler, M. & Wallerstein, N. (2008a). Introduction to community-based participatory research, In M. Minkler & N. Wallerstein (Eds), *Community-based participatory research for health*. San Francisco, CA: Jossey-Bass.

Minkler, M. & Wallerstein, N. (2008b). *Community-based participatory research for health: from process to outcomes* (2nd Ed.). San Francisco, CA: Jossey-Bass.

Mirzaee, S. & Hamidi, H. (2012). Critical discourse analysis and Fairclough's model. *International Electronic Journal for the Teachers of English,* 22(5).

Mishra, P. & Koehler, M. J. (2006). Technological pedagogical content knowledge: A framework for teacher knowledge. *Teachers College Record, 108*(6), 1017-1054.

Mishra, P., Koehler, M. & Kereluik, D. (2009). The song remains the same: Looking back to the future of educational technology. *Tech Trends, 53*(5), 48-53.

Moloi, T. J. (2013). An analysis of challenges in the teaching of problem solving in Grade 10 mathematics. *TD: The Journal for Transdisciplinary Research in Southern Africa: Special edition: Sustainable Learning Environments and Social Justice, 9*(3), 480-492.

Morrell, R. (2005). Youth, fathers and masculinity in South Africa today. *Agenda, 30*(8), 84-87.

Morsink, P. M., Hagerman, M. S., Heintz, A., Boyer, D. M., Harris, R., Kereluik, K., Hartman, D. K., Wood, A., White, A., Woodruff, C., Anderson, T., Goldstein, S., Hamm, B., Lewis, C., Lewis, P., Mitchell, C., Murphy, J., Rogers, L., Sherrieb, A., Siegler, T. & Withey, K. (2011). Professional development to support TPACK technology integration: The initial learning trajectories of thirteen fifth- and sixth-grade educators. *Journal of Educatio*n, *191*(2), 3-16.

Mulhim, E. A. (2014). The barriers to the use of ICT in teaching in Saudi Arabia: A review of literature. *Universal Journal of Educational Research, 2*(6), 487-493.

Mupa, P. & Chinooneka, T. I. (2015). Factors contributing to ineffective teaching and learning in primary schools: Why are schools in decadence? *Journal of Education and Practice, 6*(19), 125-132.

Mushayikwa, E. (2013). Teachers' self-directed professional development: Science and mathematics teachers' adoption of ICT as a professional development strategy. *African Journal of Research in Mathematics, Science and Technology Education, 17*(3), 275-286.

NDCCD. (2011). *National Dissemination Center for Children with Disabilities.* Retrieved from: nichcy.org/research/basics/researchterm.

Ndlovu, N. S. & Lawrence, D. (2012). *The quality of ICT use in South African classrooms*. Cape Town, South Africa: Carnegie.

Nkhwalume, A. A. & Liu, Y. (2013). Using technology to teach mathematical concepts through cultural designs and natural phenomena. *Asian Journal of Management Sciences and Education*, *2*(2), 26-35.

Nkoane, M. M. (2012). Critical emancipatory research for social justice and democratic citizenship. *Perspectives in Education, 30*(4), 98-104.

Nkoane, M. M. (2013). Creating sustainable postgraduate supervision learning environments. *The Journal for Transdisciplinary Research in Southern Africa, 9*(3), 393-400.

Olunga, M. S. (2012). The challenges of using lesson plan formats for effective teaching of chemistry in Kisii District, Kenya. *Journal of Emerging Trends in Engineering and Applied Sciences, 3*(5), 748-753.

Ospina, S., Dodge, J., Godsoe, B., Minieri, J., Reza, S. & Schall, E. (2004). From consent to mutual inquiry balancing democracy and authority in action research. *Action Research, 2*(1), 47-69.

Papadopoulos, T. & Woodley, C. (2008). ICT mediated collaborative learning: A learner-centred approach to improving ICT literary and employability of business graduates. *Communications of the IBIMA, 4*, 207-215.

Pena, M. I. (2011). The potential of digital tools in the language classroom. *The International Journal of the Humanities, 8*(11), 57-68.

Pinheiro, M. V. & Simões, D. (2012). Constructing knowledge: an experience of active and collaborative learning in ICT classrooms. *The Turkish Online Journal of Educational Technology, 11*(4), 382-389.

Pritchard, A. & Woollard, J. (2010). *Psychology for the classroom: Constructivism and social learning*. New York: Routledge.

Rabah, J. (2015). Benefits and challenges of information and communication technologies (ICT) integration in Québec English schools. *The Turkish Online Journal of Educational Technology, 14*(2), 24-31.

Raham, H. (2000). Cooperative performance incentive plans. *Peabody Journal of Education, 75*, 142-158.

Rahimi, F. & Riasati, M. J. (2011). Critical discourse analysis: Scrutinizing ideologically-driven discourses. *International Journal of Humanities and Social Science, 1(*16), 107-112.

Rahman, K. & Yamat, H. (2014). Barriers teachers face in integrating ICT during English lessons: A case study. *The Malaysian Online Journal of Educational Technology, 2*(3), 11-19.

Ranjit Singh, T. K. & Chan, S. (2014). Teachers’ readiness on ICT integration in teaching-learning: A Malaysian case study. *International Journal of Asian Social Science, 4*(7), 874-885.

Rashidi, N & Souzandehfar, M. (2010). A critical discourse analysis of the debates between Republicans and Democrats over the continuation of war in Iraq. *The Journal of Linguistic and Intercultural Education, 1*(3), 55-85.

Ratliff, M. (2011). Preservice secondary school mathematics teachers' current notions of proof in Euclidean geometry. PhD dissertation, University of Tennessee.

Reavey, P., Ahmed, B., & Majumdar, A. (2006). 'How can we help when she won't tell us what's wrong?' Professionals working with South Asian women who have experienced sexual abuse. Journal of Community & Applied Social Psychology, 16, 171-188.

Reid, C., Tom, A. & Frisby, W. (2006). Finding the 'action' in feminist participatory action research. *Action Research, 4*(3), 315–332.

Reitmanova, S., Gustafson, D. & Ahmed, R. (2015). Immigrants can be deadly. Critical discourse analysis of racialization of immigrant health in the Canadian press and public health policies. *Canadian Journal of Communication, 4*, 471-487.

Rocha-Schmid, E. (2010). Participatory pedagogy for empowerment: a critical discourse analysis of teacher-parents' interactions in a family literacy course in London. *International Journal of Lifelong Education*, *29*(3): 343-358.

Rogers, M. (2012). Contextualizing theories and practices of bricolage research. *The Qualitative Report, 17*(7), 1-17.

Rosenshine, B. (2012). Principles of instruction research-based strategies that all teachers should know. *American Educator*, *1*(1), 2-39.

Sabah, S. A., Fayez, M., Alshamrani, S. M. & Mansour, N. (2014). Continuing professional development (CPD) provision for science and mathematics teachers in Saudi Arabia: perceptions and experiences of CPD providers. *Journal of Baltic Science Education, 13*(3), 91-104.

SACE (South African Council for Educators). (2010). *A review of teacher demand and supply.* Pretoria: SACE.

SAQA (South African Qualifications Authority) (2012a). Level descriptors for the South African National Qualifications Framework. South Africa.

SAQA (South African Qualifications Authority) (2012b). Policy and criteria for recognising a professional body and registering a professional designation for the purposes of the National Qualifications Framework Act, Act 67 of 2008. South Africa.

Scherer, R., Siddiq, F. & Teo, T. (2015). Becoming more specific: Measuring and modelling teachers' perceived usefulness of ICT in the context of teaching and learning. *Computer & Education, 88*(1), 202-214.

Schreiber, L. M. & Valle, B. E. (2013). Social constructivist teaching strategies in small groups. *Small Group Research, 4*(44), 395-411.

Schwandt, T. A. (2007). The Sage dictionary of qualitative inquiry (3rd Ed.), Thousand Oaks, CA: Sage.

Schweisfurth, M. (2015). Learner-centred pedagogy: Towards a post-2015 agenda for teaching and learning. *International Journal of Educational Development, 40*, 259-266.

Selenger, D. (1997). *Participatory action research and social change.* New York: Cornell University.

Selwood,I & Pilkington (2005). Teacher workload: using ICT to release time to teach. Educational Review, 75(2), 163-174.

Shafer, K. G. (2007). The modern geometry course works overtime: Pre-service teachers learn content and TPCK with Geometer's Sketchpad. Paper presented at the Annual Meeting of the Mathematics Education in the 21st Century Project, Charlottesville, North Carolina.

Sharon & Krita (2014) Teaching and learning mathematics with Math Fair, Lesson Study and Classroom Mentorship. Mathematics Enthusiast, 11(1), 61

Shaw, J. A. & Deforge, R. T. (2012). Physiotherapy as bricolage: Theorizing expert practice. *Physiotherapy Theory and Practice, 6*(28), 420-427.

Shulman, L. (1987). Knowledge and teaching: Foundations of the new reform. *Harvard Educational Review, 57*(1), 1-22.

Skourdoumbis A. (2013). The (mis)identification of ineffective classroom teaching practice: Critical interrogations of classroom teacher effectiveness research. *Asia-Pacific Journal of Teacher Education*, *41*(4), 350-362.

Smith, S. (2013). Through the teacher's eyes: Unpacking the TPACK of digital fabrication integration in middle school language arts. *Journal of Research on Technology in Education, 46*(2), 207-227.

Spence, K. K. & McDonald, M. A. (2015). Assessing vertical development in experiential learning curriculum. *Journal of Experiential Education, 38*(3), 296-312.

Stein, C. H., & Mankowski, E. S. (2004). Asking, witnessing, interpreting, knowing: Conducting qualitative research in community psychology. *American Journal of Community Psychology, 33*(1/2), 21-35.

Swaffield, S. & Thomas, S. (2016). Educational assessment in Latin America. *Assessment in Education: Principles, Policy & Practice*, *23*(1), 1-7.

Tay, L. Y. & Lim, S. K. (2012). Pedagogical approaches for ICT integration into primary school English and mathematics: A Singapore case study. *Australasian Journal of Educational Technology, 28*(4), 740-754.

Te Aika, L. H. & Greenwood, J. (2009) Ko tātou te rangahau, ko te rangahau, ko tātou: A Māori approach to participatory action research. In D. Kapoor & S. Jordan (Eds), *Education, participatory action research, and social change: International Perspectives*: pp. 59-72. New York: Palgrave MacMillan.

Tella, A., Tella, A., Toyobo, O. M., Adika, L. O. & Adeyinka, A. A. (2007). An assessment of secondary school teachers' uses of ICTs: Implications for further development of ICT's use in Nigerian secondary schools. T*he Turkish Online Journal of Educational Technology*, *6*(3), 5-17.

Thirunavukkarasu, M. (2014). Identification of web resources for teaching and learning geometry at 8th standard level. *Conflux Journal of Education*, *1*(11), 46- 50.

Thomas, A., Menon, A., Boruff, J., Rodriguez, A. M. & Ahmed, S. (2014). Applications of social constructivist learning theories in knowledge translation for healthcare professionals: a scoping review. *Implementation Science, 9*(54), 1-20.

Tshelane, M. (2013). Participatory action research and the construction of academic identity among postgraduate research students. *TD The Journal for Transdisciplinary Research in Southern Africa, 9*(3), 401-429.

Turkich, K., Greive, S. & Cozens, P. M. (2014). Transferring educational theories and knowledge using a co-teaching mentor model: A discipline appropriate approach. *Journal of University Teaching & Learning Practice, 11*(3), 1-19.

Turner, J. H. (2001). The origin of positivism: The contributions of Auguste Comte and Herbert Spencer. In G. Ritzer & B. Smart, *Handbook of social theory* (pp. 30-42). Thousand Oaks: CA: Sage.

Udas, K. (1998). Participatory action research as critical pedagogy. *Systemic Practice and Action Research, 11*(6), 599-628.

Van der Westhuizen, C. P., Nel, C. & Richter, B. W. (2012). An analysis of students' academic performance when integrating DVD technology in geography teaching and learning. *Educational Technology & Society, 15*(3), 190-201.

Van Dijk, T. A. (1977). *Text and context: Exploration in the semantics and pragmatics of discourse.* New York: Longman.

Van Dijk, T. A. (1993). Principles of critical discourse analysis. *Discourse & Society, 4*(2), 249-283.

Van Dijk, T. A. (2006a). Discourse and manipulation. *Discourse and Society, 3*(17), 359-383.

Van Dijk, T. A. (2006b). Ideology and discourse analysis. *Journal of Political Ideologies, 11*(2), 115-140.

Van Dijk, T. A. (2007). *Discourse studies* (1st Ed.). London: Sage.

Van Dijk, T. A. (2008a). *Discourse and context. A sociocognitive approach*. Cambridge.

Van Dijk, T. A. (2008b). *Discourse and power*. New York: Palgrave Macmillan.

Van Dijk, T.A. (1981). *Studies in the pragmatics of discourse*. The Hague/Berlin: Mouton.

Van Dijk, T.A. (1985). *Prejudice in discourse*. Amsterdam: Benjamins.

Van Niekerk, L. & van Niekerk, D. (2009). Participatory action research: Assessing social vulnerability of rural women through income-generating activities. *Jàmbá: Journal of Disaster Risk Studies, 2:*127-144.

Van Putten, S., Howie, S. J. & Stols, G. (2010). Making Euclidian geometry compulsory: Are we prepared? *Perspectives in Education, December,14*(4), 22-31.

Walker, M (2010). Critical capability pedagogy and university education. Educational philosophy and theory, 42(8), 898 - 917

Wallace-Henry, C. (2015). Unveiling child sexual abuse through participatory action research. *Social and Economics Studies, 64*(1), 13-36.

Ward, P., Kim, I., Ko, B. & Li, W. (2015). Effects of improving teachers' content knowledge on teaching and student learning in physical education. *Research Quarterly for Exercise and Sport*, 86, 130-139.

Weaver, S. J., Dy, S. M. & Rosen, M. A. (2014). Team-training in healthcare: a narrative synthesis of the literature. *BMJ Qual Saf*, 1-14. Published online 5 February 2014. http://qualitysafety.bmj.com/content/early/2014/02/05/bmjqs-2013-001848.full

Weiss, G. & Wodak, R. (2003). *Critical discourse analysis theory and interdisciplinarity.* New York: Palgrave Macmillan.

West, M. A. (2012). *Effective teamwork practical lesson form organisational research* (3rd Ed.). London: Blackwell.

Wetzel, K. & Marshall, S. (2011). TPACK goes to sixth grade: Lesson form a middle school teachers in a high-technology-access classroom. *Journal of Digital Learning in Teacher Education, 28*(2), 73.

Wong, A. & Liu, C. (2009). Cross-functional team organizational citizenship behavior in China: Shared vision and goal interdependence among departments. *Journal of Applied Social Psychology, 39*(12), 2879-2909.

Wong, L.H., 2013. Analysis of Students' After-School Mobile-Assisted Artifact Creation Processes in a Seamless Language Learning Environment.*Educational Technology & Society*, *16*(2), pp.198-211.

Youssef, A. B. & Dahmani, M. (2008). The impact of ICT on student performance in higher education: Direct effects, indirect effects and organisational change. *Revista de Universidad y Sociedad del conocimeinto, 5*(1), 45-56.

Zaretskii, V. K. (2009). The zone of proximal development. What Vygotsky did not have time to write. *Journal of Russian and East European Psychology, 47*(6), 70-93.

Zehetmeier, S. (2014). The others' voice. Availing other disciplines' knowledge about sustainable impact of professional development programmes. *The Mathematics Enthusiast, 11*(1), 173-196.