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Making a case for exact language as an aspect of rigour in initial teacher education mathematics programmes

Abstract

Pre-service secondary mathematics teachers have a poor command of the exact language of mathematics as evidenced in assignments, micro-lessons and practicums. The unrelenting notorious annual South African National Senior Certificate outcomes in mathematics and the recognition by the Department of Basic Education (DBE) that the correct use of mathematical language in classrooms is problematic is reported in the National Senior Certificate Diagnostic Reports on learner performance (DBE, 2008-2013). The reports further recognise that learners do not engage successfully with mathematical problems that require conceptual understanding. This paper therefore highlights a need for teachers to be taught and master an exact mathematical language that for example, calls an 'expression' an 'expression' and not an 'equation'. It must support the call of the DBE to use correct mathematical language that will support and improve conceptual understanding rather than perpetuate rote procedural skills, which are often devoid of thought and reason. The authentic language of mathematics can initiate and promote meaningful mathematical dialogue. Initial teacher education programmes, as in the subject methodologies, affords lecturers this opportunity. The language notions of Vygotskian thought and language, Freirian emancipatory critical consciousness and Habermasian ethical and moral communicative action frame the paper theoretically. Using a grounded approach, after examining examples of student language in a practice based research intervention, the design and development of a repertoire of language categories, literal, algebraic, graphical (Cartesian) and procedural (algorithmic) emerged from three one-year cycles of an action research methodology. The development of these repertoires of language was to assist teachers in communicating about mathematical objects through providing a structured framework within which to think and teach. A course model encompassing small group discussions, an oral examination and a self-study action research project, that helped sustain the teaching of an exact mathematical language, is presented. This is supported by student reflections on the usefulness of implementing them.

Keywords: *Exact (authentic) mathematical language, initial teacher education, grounded theory, action research, vocabulary and thought*

1. Introduction

Persistent poor performance in the South African National Senior Certificate mathematics examination and trends show a declining number of candidates in grade 12 mathematics relative to total enrolment recorded in the annual National Senior Certificate diagnostic reports on learner performance (DBE, 2008-2013). This raises the question as to how to arrest an ongoing crisis in mathematics education in South Africa. A currently under qualified mathematics teaching corps and candidates poorly schooled in mathematics who embark on mathematics education careers require preparation in mathematics content and language. It is the learning and appropriation of correct mathematical language that constitutes an aspect of rigour for mathematics education, quite apart from the rigour of mathematical procedural accuracy, for which a case is made in this paper.

It is therefore imperative that tertiary institutions seek ways of developing teachers competent in mathematics content knowledge and mathematics pedagogy. I envisage exact mathematical language as an integral component of mathematics pedagogy and advocate exact mathematical language as *an aspect of rigour* in initial teacher education programmes. Ball, Thames and Phelps (2008) disaggregate Shulman's (1986) subject matter knowledge to include common content knowledge, horizon content knowledge and specialised content knowledge. It is taken as understood that mathematics content knowledge, as a specific case of Shulman's subject matter knowledge needs to be in place before meaningful and effective pedagogical content knowledge can be applied. Ball, Bass and Hill (2004), Ball *et al.* (2008) and Begle (1979) recognise that mathematics knowledge for teaching is different from the mathematics knowledge of mathematicians but that the mathematics knowledge of mathematics teachers must still be rigorous. The search to define what constitutes this rigour is an important one on which this paper seeks to shed some light. Rigour from a mathematical content perspective would seem to imply procedural accuracy and fluency as used by Kilpatrick, Swafford and Findell (2001) and Schoenfeld (2007). However, from a pedagogical content knowledge perspective the exactitude of language with which teachers need to unpack, communicate and explain concepts, I suggest, is an equally important form of rigour. It is the latter definition of rigour that is attended to in this paper.

In a practice based research project where mathematics methodology courses for undergraduate mathematics education students are offered, it emerges that the language, which teachers need to teach the subject is lacking in terms of its exactitude and is error riddled in terms of using words incorrectly to describe mathematical objects. The generic mathematical lexicon that characterises the universality of mathematical discourse is largely absent in the instructional episodes observed in lectures and in teaching practicums.

In the annual national diagnostic reports of the Department of Basic Education, language is frequently cited as a factor militating against performance (DBE, 2008-2014). Language as referred to in these reports does not directly refer to the multilingualism of classrooms that has featured strongly in the mathematics education research field for the past two decades (Adler, 2001; Setati, 2005). Instead it refers to the correct language of mathematics as in the quotes below (*italics and parentheses mine*) which may be a departure from what has become the traditional language emphasis on multilingualism, especially since the majority of South African learners are taught in English (Uys *et al.*, 2007: 69). Therefore, an exact mathematical language within the context of multilingual classrooms must not be construed as exacerbating mathematical dialogue or comprehension for the majority of learners who are

taught mathematics in English. Inexact mathematical language where English is the language of instruction whilst not being in the mother tongue of learners, serves only to compound the problem.

Teachers should use the correct mathematical language in the classroom...
(DBE, 2014: 112)

As language presents problems in the correct interpretation of (finance) questions, teachers must at all times focus on using the correct language while teaching and in the setting of assessment tasks.
(DBE, 2014: 116)

The quotes above acknowledge the need to use correct or exact mathematical language for mathematics teaching and learning but what constitutes the correct use of mathematical language, is not addressed. I hence propose the teaching of an exact use of mathematical language as an integral part of initial teacher education programmes for prospective teachers of mathematics. Uys *et al.* (2007: 78) recommend, "subject content lecturers at teacher-training institutions should become involved in the teaching of language skills in the content classroom. The subject classroom at the teacher-training institution is the one place where subject lecturers can help teacher trainees deconstruct the language of their textbooks (Schleppegrell, Aghugar & Oteiza 2004: 67), thereby also enabling them to develop the academic language required for teaching their subjects through the medium of English". The added advantage of teaching the language of mathematics with English as the language of instruction is the anticipated improved understanding of mathematics. Mercer and Sams (2006) who investigated the impact of the correct use of mathematical language on collaborative problem solving amongst primary school children found that if teachers provide children with an explicit, practical introduction to the use of language for collective reasoning, then children learn better ways of thinking collectively and better ways of thinking alone. The focus of this paper however, is on the use of correct language at institutions of higher education and at secondary schools as opposed to the many studies that investigate primary school language use. Thus, the implication of teaching mathematics with the correct vocabulary is that it will have an impact on understanding, more so at the secondary and tertiary levels where sophisticated vocabularies are needed to expound concepts that are more advanced.

Yackel and Cobb (2006) speak about social norms and socio-mathematical norms in classrooms where the former constitutes learner exchanges in their (own) language whereas the latter constitutes their arguments with sound mathematical reasoning. This paper does not propose that the former is unacceptable. It suggests that social norms serve as scaffold learning episodes in the lower grades especially but suggests that these need to transmute developmentally into a mathematical language that supports and enables the explanation and understanding of more advanced mathematics at the secondary level. Mathematical language in and of itself is difficult, but if English is the most prevalent language of instruction in South Africa then using mathematical language correctly is of greater value than teaching mathematics with a poor mathematical vocabulary and with meaningless 'metaphors'. For example, Pimm (1987) points out that 'cross multiply' and 'turn it upside down and multiply' are examples of poor mathematical language devoid of meaning.

In contrast to the recent specific references in the national senior certificate diagnostic reports to a call for the use of *correct language* by teachers the quote that follows this

paragraph refers to 'poor language ability' which one presumes refers to the language of teaching and learning not being the learners' mother tongue. English has become, irrespective of the merits or demerits of the matter, the dominant medium of instruction in southern Africa and one needs to be cognisant of this (De Klerk, 2002: 3; De Wet, 2002: 119; Kgosana, 2006: 17; Rademeyer, 2006: 15; Uys *et al.*, 2007: 69). Its implications for subject teaching therefore cannot be ignored. Uys *et al.* (2007: 77) in a survey of the North West and Eastern provinces of South Africa and Namibia found that teachers lacked the personal language proficiency required (both spoken and written) to assist their learners in the acquisition of academic literacy. Furthermore, none of the surveyed teachers had received training that equipped them with skills for effectively teaching through the medium of English. This deficient attention to technical language also needs to be addressed. Where procedural mathematics still dominates secondary and tertiary teaching and learning environments, a move toward the enhancement of the understanding of mathematics will continue to be ignored. This is one reason why this paper advocates the teaching of an exact language of mathematics in undergraduate teaching courses.

Many candidates struggled with concepts in the curriculum that required deeper conceptual understanding... This suggests that many learners are exposed to 'stimulus-response' methods only... Because of their poor language ability, the majority of learners did not provide good answers to contextual questions... Thus they could not identify the mathematical skills involved (DBE, 2011: 99).

It is interesting and ironic, to note in the quote above that the majority of learners being unable to provide good answers to contextual questions is attributed to their 'poor language ability'. I concede that learners' poor language ability must contribute negatively to the answering of contextual questions but there is a deeper language issue here. The injudicious use of mathematical language in classrooms makes explaining the content and elaborating its meaning for learners incomprehensible and inaccessible especially where English is already not the mother tongue of the majority of learners. 'Equations' are often referred to as 'expressions' and vice versa, 'positive' is used as a synonym to describe a function that is 'increasing' even when such a function is both positive and negative and the instructional verb 'solve' is applied generically to mathematical objects even where the objects may need to be 'simplified' or 'sketched'. There is then also the case where matriculants face exit level examination papers and are confronted with mathematical language unfamiliar to them resulting in the failure to interpret questions correctly. One wonders therefore if this inability to use the correct or exact language in classrooms or the indiscriminate use of mathematical vocabulary has a significant impact on poor national senior certificate results year after year.

From a theoretical perspective, thought is dependent on the word formulation of concepts, Vygotsky (1986), Habermas (1990), Freire (1970, 1974) without which conceptual thinking about mathematical objects must be impeded or arrested.

Also with reference to the last quote, it is claimed that learners struggled with concepts in the curriculum that required deeper conceptual understanding and this is attributed to learners being taught through stimulus-response methods only and with incorrect vocabularies.

I attempt to show below in the section on 'Theoretical underpinnings' how words and language have a great deal to do with acquiring an understanding of content and attempt to consolidate a proposal for exact mathematical language use in classrooms where conceptual understanding according to national reports of the Department of Basic Education are purported to be absent.

2. Theoretical underpinnings

2.1 Vygotskian thought and language

Vygotsky (1986: 107) states, “real concepts are impossible without words, and thinking in concepts does not exist beyond verbal thinking. That is why the central moment in concept formation, and its generative cause, is a specific use of words as functional tools.” Schütz (2002) also records Vygotsky as saying that thought is not merely expressed in words but that it comes into existence through them. Schütz refers to Vygotsky as saying that words play a central part not only in the development of thought but also in the historical growth of consciousness as a whole.

From a mathematics education perspective, the need to have a mathematical vocabulary is fundamental to the ability to think about and articulate mathematical content for teachers and learners. The implication is that initial teacher education programmes should equip teachers with the language with which they and their learners can engage meaningfully in mathematical discourses. It would seem also that the reported absence of conceptual understanding amongst national senior certificate learners (DBE, 2011: 99) might have its root cause in an undeveloped or absent mathematical register.

To illustrate this, if a teacher talks about an expression but actually means an equation it will not allow a learner to have a conceptual understanding and is likely to create cognitive dissonance for learners. Similarly, if in literally reading the expression $x(x+2)$, as has been my frequent experience in practicums and micro-lesson presentations, teachers say ‘ x into $x+2$ ’ the implied operation between the monomial and binomial factors is ‘division’. This is clearly not the case since the operation between the factors is ‘multiplication’. One may argue from one’s own experience that a learner will not interpret ‘into’ as ‘division’ as in the case in question. However, is the reason for this not the repetition of historical uncorrected accounts of the forgiven inexact language of mathematics being perpetuated generation after generation? The issue here is actually not about whether learners in contemporary classrooms will interpret ‘into’ as division or multiplication but more importantly what the correct mathematical operation ‘into’ conveys. If one considers the isomorphic relatedness of algebra and arithmetic (Livneh & Linchevski, 2007) then the arithmetic understanding of ‘two into six’ is clearly ‘six divided by two’ and not ‘two multiplied by six’. The case of $x(x+2)$ being ‘ x into $x+2$ ’ is therefore fundamentally incorrect and a potential root of misconception. As episodes of indiscriminate language use build on one another the degree of cognitive dissonance must be compounded with the result that we have learners in the national senior certificate who are procedurally bound with respect to mathematical work rather than conceptually adept or able to appraise answers critically.

2.2 Freirian critical consciousness

Freire in *Education for Critical Consciousness* (1974) clearly expresses his view of democracy as one that would transform the Brazilian political landscape. This perspective is applicable to mathematics classrooms where dialogical engagement that is also inclusive, is critical. In addition, if it is to be dialogical it must offer an *honest* classroom conversation where vocabularies authentically and accurately describe the mathematics at hand. Freire’s philosophy and methodology of education expounded in *Pedagogy of the Oppressed* (1970) revolved around delivering the Brazilian proletariat from its muteness. He regarded their inability to express themselves as robbing them of their critical consciousness. Similarly, being able to critique

and evaluate mathematical notions and objects involves a critical consciousness stemming from the vocabulary that gives birth to mathematical concepts and the concomitant skill of articulating the mathematical thought authentically. In this sense, learners of mathematics are also robbed of their critical consciousness. The case being made for an exact mathematical language is to bring about honest communicative exchanges in mathematics classrooms, based on words and language that enable our thinking. After all, mathematics is an exact science that does not have to be devoid of humane authenticity. Freire (1974: 14) aptly states

The critically transitive consciousness is characterized by depth in the interpretation of problems; by the substitution of causal principles for magical explanations; by the testing of one's "findings" and by openness to revision; by the attempt to avoid distortion when perceiving problems and to avoid preconceived notions when analyzing them; by refusing to transfer responsibility; by rejecting passive positions; by soundness of argumentation; by the practice of dialogue rather than polemics; by receptivity to the new for reasons beyond mere novelty and by the good sense not to reject the old just because it is old – by accepting what is valid in both old and new.

The interpretation of problems (mathematical objects) must reside with the vocabulary with which to think about them, understand them, reflect on them, evaluate and appraise them for what they are. If it is not so, the cognitive vacuum will render the teacher and learner cognitively and articulately mute. Freire also recognises the importance of language in classrooms when he states that

Often, teachers and politicians speak and are not understood because their language is not attuned to the concrete situation of the people they address. Accordingly, their talk is just alienated and alienating rhetoric. The language of the teacher..., like the language of the people, cannot exist without thought; and neither language nor thought can exist without a structure to which they refer. In order to communicate effectively, teacher and politician must understand the structural conditions in which the thought and language of the people are dialectically framed (Freire, 1970: 77).

2.3 Habermasian communicative action

Habermas' seminal work *Moral Consciousness and Communicative Action* (1990) applies to discourse ethics, communication, argumentation and dialogue that promotes voluntary rational agreement for the sake of cooperation. As much as it is generically all of this, in my opinion, it also has far reaching implications for its application in the teaching and learning in mathematics classrooms. Mathematics by virtue of its empiricist rootedness is characterised by impersonal propositional deliveries that often separate it from affective influences that could make it more accessible to those who aspire to learn and teach it. Engagement with communicative action has emancipatory potential for the teaching and learning of mathematics. In Habermas' words, communicative action has this power since "as empirical research shows him, communicative action from the start of the learning process is in the inescapably social nature of human language" (Habermas, 1990: 165).

Language is not the only essence here but the honesty of the discourse is as well. If as teachers we speak indiscriminately about mathematical content and constructs, even unintentionally, we deceive those we teach. It is therefore morally incumbent on teachers to create conversations about what concepts really are in order to give learners the real opportunity to acquire a meaningful understanding of the mathematics at hand. It is important to bear in mind that for whatever reason teachers of mathematics forgivingly listen to learner

offerings that are often inaccurate and allow a misrepresentation or incorrect vocabulary to pass, it is ethically and morally questionable. When an inexact or inappropriate lexicon develops, it can impede clear understanding, which, in turn, creates fear and uncertainty in learners. There are of course cases where teachers are not aware of their own misconceptions and erroneous vocabularies and therefore unknowingly perpetuate confusion. This is reason enough for teachers in initial teacher education programmes to acquire authentic vocabularies and language repertoires not only in mathematics but also in other subjects.

As teachers and learners, we are in the business of communication. It is therefore paramount that the elements of reciprocity, empathy and fairness that constitute this perspective be embraced in classroom environments especially where mathematics is concerned. There is much scope here, I believe, for the application of these solid principles of communication, as alluded to in the quote above, to be practised in the mathematics classroom. This is done with the knowledge that ethical discourses do not denunciate or disregard mathematical offerings from learners that are not exact in their technical rationality since this would seek to entrench an epistemological hegemony that is oppressive. The listening process needs to be forgiving but reactive toward correction and refinement of terminology where this is necessary.

The theoretical notions of words being at the core of concept development (Vygotsky), critical consciousness being able to liberate a learner from muteness (Freire) and the awareness that what we say needs to be authentic (Habermas) are critical considerations for making a case for teachers to use meaningful, conventional and hence exact mathematical language in their teaching. The central theme running through these theories is communication.

3. A closer look at the mathematical language of initial teacher education students

In this section, I provide examples of student language used in teaching practicums, micro-lessons and assignments to shed light on the concern about mathematical language that is inexact, incorrect and often incomprehensible and that probably presents a barrier to learner understanding.

3.1 A procedural description of how to solve for the quadratic equation $x^2-x-6=0$.

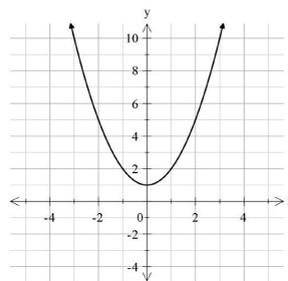
A student offered the following explanation. “... *We now look at the signs. We have a – and a – . So we know our bracket will have a + and a – , because if the last – was a + we would have a – and a – because two negatives make a positive.*”

When teaching one should at all times make no assumptions about what is known by learners. In other words, teachers' explanations should make sense to even the uninitiated. In analysing the explanation, the content is difficult to understand. In the very first instance, the student refers to the two negative signs without clearly indicating that they are associated with the last two terms of the quadratic expression on the left of the equal sign. The explanation of the brackets having a negative and positive sign because of the last negative is algorithmic, rote and procedural without any explanation of why this will be the case and so it is conceptually void. The claim that two negatives make a positive makes no sense because of the verb 'make'. The addition of two negative values, for example, $(-3)+(-2)$ will produce a negative result, -5 , but the product of two negative values will produce a positive result as in $(-3)\times(-2)=6$ which is actually what the student intends to say. The explanation is also devoid of reasoning. The product and sum of the signs lies at the heart of obtaining the factors of x^2-x-6 .

3.2 A student's offering on why it is incorrect to speak about a positive graph.

With respect to the graph alongside, a student states *"It is incorrect to talk about a positive graph as in this graph there are parts that are positive and parts that are negative. The part that is positive has positive x and y values and the negative part has positive y values and negative x values."*

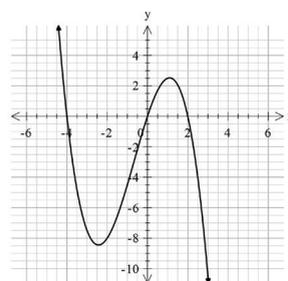
The fluency of the explanation is deceptively convincing. However, the student clearly does not understand what is meant by a positive graph. The sketch shows a quadratic function that is positive for all real values of x because all its function values are positive or viewed differently, the entire function lies above the x axis.



3.3 A student describing the behaviour of a cubic function

A student describes the behaviour of the function alongside as follows. *"As x increases y begins by decreasing with a decreasing gradient. The graph then decreases with an increasing turning point until $(-\infty; \infty)$."*

This explanation erroneously uses 'decreasing' as synonymous with 'negative'. The function certainly starts decreasing as x increases. For the initial decreasing interval, the gradient is negative but the (covert) tangents to the cubic function have variable gradients and their gradients actually increase in value as x increases. After the initial interval where the function is decreasing, the function is increasing and not decreasing. There is no such thing as an increasing turning point, and reference to the point $(-\infty; \infty)$ is meaningless.



Therefore, in this short piece there are numerous uses of incorrect words in addition to serious mathematical inaccuracy. For learners the real mathematics will remain remote after such an explanation.

4. Methodology

4.1 Sample cohorts

I first encountered disturbing descriptions of mathematical objects in early 2012 amongst a class of third year students who were studying to be secondary school mathematics teachers. It became apparent in subsequent third year cohorts in 2013, 2014 and 2015 that the same lack of command of the language of mathematics prevailed. The average size of the classes in the third year was about 70. The fourth year cohorts dropped by about 15 students to 55. The drop off was primarily students who had mathematics as a sub-major, that is, their second teaching subject. The composition of the classes was 75% students who had a respectable command of conversational second language English, and about 25% were mother tongue English speakers. However, the quality of the descriptions of mathematical objects was equally poor across the cohorts.

4.2 Action research

Koshy (2005: 2) defines action research as an enquiry, undertaken with rigour and understanding to refine practice constantly; the emerging evidence-based outcomes will then contribute to the researching practitioner's continuing professional development. He further states that

The main role of action research is to facilitate practitioners to study aspects of practice – whether it is in the context of introducing an innovative idea or in assessing and reflecting on the effectiveness of existing practice, with the view of improving practice. This process is often carried out within the researcher's own setting (Koshy, 2005: vii).

So in alignment with this understanding of what action research entails and the problem of inexact mathematical vocabulary and language amongst prospective secondary school mathematics teachers, I embarked upon a process of determining how I could not only refine my own practice but how that refinement would impact positively on the mathematical language of students in initial teacher education programmes. My aim was to do exactly what Bassey (1998: 93) describes action research to be "... an enquiry which is carried out in order to understand, to evaluate and then to change, in order to improve educational practice".

The action research process is a spiral of self-reflective cycles where each cycle comprises consecutive phases of reflect-plan-act-observe and so on. This particular model was devised by Kemmis and McTaggart (2000) and is illustrated in Koshy (2005: 4).

Table 1 below sets out the cycles according to the Kemmis-McTaggart model over the three-year cycle of intervention. The table describes each phase using the Kemmis-McTaggart model. The table describes iterative 'reflect-plan-act-observe' cycles. The column headed 'Phase Description' details how the reflections on student language lead to an intervention. The third column 'Findings on student task data' describes the data forthcoming from that intervention, which lead to the following cycle. A grounded approach led to the development of four language repertoires (Glaser & Strauss, 1967).

Table 1: Action research cycles – Analytic detail of phases and commentary on phase findings

Cycle 1	Phase Description	Findings on Student Task Data
<p>Reflect</p> <p style="text-align: center;">↓</p>	<p>Students were not speaking mathematically to the point that in practicums their learners were barely able to understand them. Language was often incomprehensible and vocabulary incorrect and inexact and the mathematical content poor.</p>	<p>1. Talk to me about this object. $x^2-x-6=0$</p>
<p>Plan</p> <p style="text-align: center;">↓</p>	<p>I devised assignments where mathematical tasks needed to be done but also commentated on alongside the procedure through written explanations of what was being done to ascertain the vocabulary store of learners.</p>	<p><i>Response</i></p> <p>Generally, there was no ability to engage with the object verbally.</p> <p>2. Do and talk about the following object.</p> $\frac{x+2}{x-1} = 1$
<p>Act and Observe</p> <p style="text-align: center;">↓</p>	<p>Students completed the assignment under test conditions and their work was assessed.</p> <p>In response to task 1 most students were able to say that the solution is $x=-3$ and $x=2$. Many suggested it was a 'sum', which it is not. This type of equation is first encountered in grade 9 and it would have been expected that teachers would have a vocabulary with which to introduce the object, like recognising it as a quadratic equation, that the highest exponent value is 2, that the left hand side is a quadratic trinomial in descending order and the like.</p> <p>Clearly, in the response to task 2 in the adjacent right hand column there is no conceptual understanding. The student speaks about 'cross multiplication' but does not employ this strategy at all. He simply incorrectly divides the denominator $x-1$ into the numerator $x-1$ (or cancels) across an equal sign, which if cross multiplied correctly, would give a right hand side equal to 1.</p>	$\frac{x+2}{x-1} = \frac{x-1}{x-1}$ <p>$(x+2)(x-1)=0$ $(x+2)=0$ or $x-1=0$ $x=-2$ or $x=1$</p> <p><i>Student response</i></p> <p>"In this question I do not know if the use of cross multiplication is correct but it makes sense to me though. This question also had me confused at first but by turning 1 into a fraction it made sense."</p>

Cycle 2	Phase Description	Student Task Data
<p>Reflect</p> <p style="text-align: center;">↓</p>	<p>Upon reflection and collegial discussion, I felt that the instruction was not specific enough. The original intention was to see if students could identify the mathematical object with which they were presented and would associate with the object, the nature of the task associated with it, for example, solving equations, simplifying expressions, sketching relationships and so on. This expectation was a design weakness.</p>	<p>1. Simplify the following expression and use good mathematical language that you would use in introducing and teaching the mathematical object to a class of grade 10 learners.</p>
<p>Revise Plan</p> <p style="text-align: center;">↓</p>	<p>Lecture time was devoted to demonstrating exact language that would be advantageously used in classrooms and assignments were designed in such a way that written responses were to reflect the language teachers would use in classrooms in introducing and teaching the given object. This was deemed <i>dense</i> descriptions of the object that were intended to produce extensive descriptions.</p>	$\frac{2x^2+5x-3}{3+x}$ $= \frac{(2x-1)(x+3)}{3+x}$ $= 2x-1$ <p><i>Student response</i></p> <p>"In this mathematical object I noticed that it was an expression in the form of a fraction and to find the answers to it, one needs to consider the use of factorising and finding the common factor. $x+3$ and $3+x$ are the same and in that way you can divide/cancel-out.</p>
<p>Act and Observe</p> <p style="text-align: center;">↓</p>	<p>Students completed assignments under test conditions.</p> <p>The sample response in the adjacent right hand column shows an improvement in the introductory language like the correct use of 'expression', 'fraction', 'factorising' and 'common factor'. In the second paragraph however, by stating that there is a relationship between the variables makes no sense since there is only one variable in the expression. Looking for a relationship between a variable and its coefficient is not within the scope of a discussion or discourse that this object should elicit. On a purely procedural level the student does not include the restriction that $x \neq 3$ since $x=3$ will make the original expression undefined. Furthermore, the importance of the LCD is alluded to but without reason.</p>	<p>There's a relationship between the variables and its coefficient the understanding of using an LCD is essential in this case. However some learners may equate the fraction to 0 and then try to solve it by transposing the LCD on the other side of the equal sign."</p>

Cycle 3	Phase Description	Examples of language repertoires applied to a single object prompted by the findings above
<p>Reflect</p> <p style="text-align: center;">↓</p>	<p>It became apparent that students felt that the more they wrote the more acceptable their response would be. Often under compulsion to cover as much as possible the responses became more unwieldy, cumbersome, meaningless and confusing.</p> <p>It therefore became necessary to rethink how the task of providing dense descriptions using exact mathematical language could be made easier and more focused.</p>	<p>1. Give a literal description of $2^x > 8$</p> <p><i>2 raised to the power of x is greater than 8.</i></p> <p>2. Give an algebraic interpretation of $2^x > 8$</p> <p><i>x is the number of times 2 must be multiplied by itself to obtain a value greater than 8.</i></p>
<p>Revise Plan</p> <p style="text-align: center;">↓</p>	<p><i>The Curriculum and Assessment Policy Statement for mathematics grades 10-12 (DBE, 2011: 12) states that learners should be able to work with relationships between variables in terms of numerical, graphical, verbal and symbolic representations of functions and convert flexibly between these representations (tables, graphs, words and formulae). With this in mind, it made sense that teachers would benefit from having a repertoire of language modes which would fit comfortably with each of these modes of representation since in moving flexibly between these modes of representation would imply having the language and vocabulary with which to expound what each representation meant. This led to the development of four language repertoires that were designed for teachers to structure their thinking and to have a clear focus when in dialogue or teaching.</i></p>	<p>3. Give a graphical interpretation of $2^x > 8$</p> <p><i>x represents the values on the x axis where the increasing exponential function $y=2^x$ lies above the constant linear function $y=8$.</i></p> <p>4. Provide a procedural description for finding the solution of $2^x > 8$.</p> <p><i>Reduce both terms in the inequality to a prime base of 2. Since $y=2^x$ is strictly increasing, and the bases are equal, x is greater than 3.</i></p>

5. Development of language repertoires as the third intervention cycle

Although there is not a strict one-to-one correspondence between the modes of representation as outlined in the *Continuous Assessment Policy Statement* (2011) and the language repertoires there is a close enough alignment for teachers to move flexibly between them using exact language with which to communicate about each. The development of the repertoires has the advantage that classroom speak breaks from the tired almost exclusive linguistic repertoire of procedural instructions on how to 'do' the mathematics. The procedural language is by no means of less significance than the other categories because explaining the procedure is still a vital component of mathematics teaching and learning. I introduce and define below the language repertoires that provide a structured system for assisting teachers in lesson preparation, lesson delivery and the development of exact mathematical language. The repertoires have their defining vocabularies that facilitate exact language use for speaking about mathematical objects literally, algebraically, graphically and procedurally. Teachers can

furthermore flexibly move between these repertoires in an order that the mathematical object dictates. The sequence of repertoires can therefore be varied.

5.1 Language repertoires

Literal: This entails simply being able to *read the object* using the correct language and *in the correct sequence*. The interval, $0 < x \leq 5$ should be read “all the real x values that are greater than zero and at the same time less than or equal to five”. Students in many instances read this as “zero is less than x is less than or equal to 5”. The mental gymnastics of getting around the student version is indicative of its incorrectness, meaninglessness and complexity, especially for a class of uninitiated grade 9 learners.

Algebraic: This category lays emphasis on the *operations* that constitute the object. For example, $x + y < 8$, $x, y \in \mathbb{Z}^+$, from an algebraic perspective can be described as “the sum of two positive integers is less than eight”.

Graphical or Cartesian: This category of description focuses on the object as it is depicted in the *Cartesian plane*. For example, $2^x \geq \sqrt{x+2}$, seeks the real values of x on the x axis for which (where) the increasing exponential function $y=2^x$ lies on or above the increasing branch of the inverse parabola $y=\sqrt{x+2}$.

Procedural or Algorithmic: As it suggests, this category is the language associated with the explanation of how to ‘do’ the mathematics. For example, the explanation for obtaining a solution to $4^x=32$ would be firstly to see that the equation seeks a value for x for which the left and right hand sides are equal in value. Express each term in the equation as a power with a prime base of two. Since we are working in an equation and the bases are equal, the exponents must be equal too, to maintain the equality. Hence, $2x=5$. Since 2 is multiplied by x on the left hand side, to isolate x , we divide throughout by 2 because division is the reverse operation of multiplication. In this way we obtain $x=2,5$ which when substituted in the original equation will give a true statement.

Students’ reactions to using these language repertoires are given below.

6. Student reflections on using mathematical language repertoires

I produce below the reflections of three students on the course. Each student highlights the value of the mathematical language repertoires that were the product of a three-year action research programme.

Student A: *The course offered many different valuable methods to teach such as the different levels of describing a mathematical object namely, literal, algebraic, graphical and procedural description, which I will greatly use in my lessons. Using all the different levels of description to teach a mathematical object ensures greater understanding among learners because they will develop a thorough knowledge of mathematical objects. For example, when teaching an equation, I can explain it through its literal meaning, by reading it from left to right, its algebraic meaning by emphasising the operations. In addition, I can draw its graphical representation on the Cartesian plane. Lastly, I can teach its procedural description, by stating how to solve the equation arithmetically. As a result, I can accommodate many learners in my class by speaking about one object in many ways.*

A confident voice here shows an empowered student who suggests the language repertoires will be useful in accommodating the different cognitive levels of her learners.

Student B: *When I reflect back on the course as a whole, I realise that I am able to confidently describe a mathematical object in many different ways... This equipped me with enough skills that will develop a deep sustained understanding of the mathematical content. To prove this, when I taught mathematics in my recent Teaching Experience I did not focus on the procedure of getting the long division answer right but on why is the answer right or wrong. I took longer time to teach the topic fully, but I know that their understanding of long division will be deep and sustained for a long time.*

For this student the four repertoires were seen as being useful in developing a sustained understanding of long division. There is a move away from rote pedagogical procedures to accommodate a conceptual perspective of understanding.

Student C: *The course emphasised on the integration and connection of these concepts. The most fundamental idea of this course was representing concepts graphically, literally, algebraically and procedurally as a way of understanding concepts relationally rather than instrumentally.*

This student recognises the value that the language repertoires will have for relational understanding as opposed to the mechanical “rules without reasons” style of teaching referred to as instrumental understanding by Skemp (2006).

From the above it appears that the introduction of language repertoires as part of the initial teacher education programme has benefits for prospective teachers of (secondary school) mathematics, largely in equipping them with a vocabulary and being able to engage in communication about mathematical content from a variety of perspectives. There is an indication that there is a shift from the instrumental understanding akin to procedural lesson delivery to one where different perspectives enabled by the language repertoires has contributed to students understanding the mathematical content relationally. In the initial stages of this research besides grappling with the theoretical aspect of what an exact mathematical language entailed I needed to find a way in which to implement it practically. This led to the incorporation of exact mathematical language as a core component of assessment. A variety of practical components as detailed below now characterises the initial teacher education mathematics methodology courses, an overview of which I provide below.

7. Towards a model for sustaining the teaching of exact mathematical language

To consolidate the emphasis on exact mathematical language as a suggested valid and rigorous component of the initial teacher education programme I have instituted weekly small group tutorials where groups of five student peers describe and discuss mathematical objects using the repertoires of language. Assignments concentrate on students giving written descriptions of mathematical objects. It has become clear that the acquisition of a sound mathematical vocabulary has increased student confidence and equipped them with an ability to reason for example why an object which is not algorithmically solvable at school level such as, $2^x \sqrt{x+1}=0$, has no solution. In the fourth year, students examine their own language by video recording themselves teaching to a small group. The small group members evaluate the student in terms of the four language categories. These self-studies as action research projects have revealed much about the students’ abilities to communicate effectively. Also

in the fourth year, students need to prepare for an oral examination where one out of ten mathematical objects provided to students the day before, is randomly selected and described using the language repertoires before a panel of ten students and a lecturer. Peer scores are used as part of the assessment process.

8. Conclusion

This position paper sets out to make a case for the teaching of exact mathematical language to pre-service mathematics teachers in initial teacher education programmes. It was motivated by observing particularly poor language usage by pre-service teachers in micro-lessons and practicums. The paper has reported on the progress of an intervention of teaching exact mathematical language over three years and describes tutorials, an oral examination and a self-study action research project that sustains and consolidates the teaching of exact mathematical language. Rigour was defined in terms of pedagogical content knowledge and then operationalised through the design of four language repertoires.

It was shown that language in mathematics can be viewed from a traditional socio-mathematical perspective of multilingual classrooms and from a more technical vocabular perspective that contributes to a meaningful discursive classroom characterised by socio-mathematical norms in a national educational setting where English as the language of teaching and learning dominates.

The proposal regarding teaching exact mathematical language as an aspect of rigour is in accordance with Ball *et al.* (2008), Ball *et al.* (2004) and Begle (1979) all of whom suggest that what rigorous mathematics is for mathematicians and for teachers of mathematics are quite different. From this paper, it should be evident that acquiring the language with which to teach and explain mathematics requires substantial attention to detail both within the spheres of specialised content knowledge and pedagogical content knowledge.

From a theoretical perspective, it is shown that from a Vygotskian thought and language viewpoint it is essential to acquire a subject specific vocabulary since words are the foundation of concept development, thought and its articulation. The absence of it also suggests an impaired ability to reason and these are cognitive qualities and skills that national senior certificate diagnostic reports declare are absent among South Africa's school leavers. Habermasian communicative action supports exact language from an ethical and moral perspective in its need for honesty in dialogue. 'Speaking' mathematics inaccurately deceives the listener and the listener's inexperience denies the chance to contest that what is being heard is in fact not so. Freirian critical consciousness is the reward for emancipating the mind and freeing it from muteness. If learners had the language of mathematics with which to reason, debate, contest and critique, national assessment averages may reflect improvement.

From the evidence of student reflections provided, it is shown that an exact mathematical language as an aspect of rigour in initial teacher education curricula in mathematics is not unwarranted. Its incorporation would seem essential for developing thought, reason, honing communication skills and for providing authentic teaching.

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