University of the Free State, Bloemfontein

ON SOME METHODS OF RELIABILITY IMPROVEMENT OF ENGINEERING SYSTEMS

by

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A thesis submitted in accordance with the requirements for the degree of

DOCTOR OF PHILOSOPHY

in the subject

MATHEMATICAL STATISTICS

in the

DEPARTMENT OF MATHEMATICAL STATISTICS AND ACTUARIAL SCIENCE
FACULTY OF NATURAL AND AGRIULTURAL SCIENCES
UNIVERSITY OF THE FREE STATE
BLOEMFONTEIN

September 2015

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Declaration

I declare that the thesis hereby submitted by Bernard Tonderayi Mangara for the degree Doctor of Philosophy in the subject Mathematical Statistics at the University of the Free State is my own independent and original research work, except where explicitly indicated otherwise, and that I have not previously, in its entirety or in part, submitted it at any university/Faculty for a degree. Wherever I have used information from other sources, I have given credit by proper and complete referencing of the source material so that what is my own research and what was quoted from other sources can be clearly discerned. I acknowledge that failure to comply with the instructions regarding referencing will be regarded as plagiarism.

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Summary

The purpose of this thesis was to study some methods of reliability improvement of engineering systems. The reason for selecting the theme “reliability improvement of engineering systems” was first to explore traditional methods of reliability improvement (that is, based on the notion that reliability could be assured by simply introducing a sufficiently high “safety factor” into the design of a component or a system) and then propose new and original concepts of reliability improvement. The latter consists of approaches, methods and best practices that are used at the design phase of a component (system) in order to minimize the likelihood (risk) that the component (system) might not meet the reliability requirements, objectives and expectations.

Therefore, **chapter 1** of the thesis, “*Introduction to the main methods and concepts of reliability for technical systems*” encompasses the introduction section and the main traditional methods available for improvement of technical / engineering systems.

In **chapter 2**, “*Reliability Component Importance Measures*” two new and original concepts on reliability improvement of engineering systems are introduced. These are: 1) the study of availability importance of components in coherent systems and 2) the optimal assignment of interchangeable components in coherent multi-state systems.

In **chapter 3**, “Cannibalization Revisited” two new and original concepts on reliability improvement of engineering systems are introduced. These are: 1) theoretical model to show the effects of cannibalization on mission time availability of systems and 2) new model for cannibalization and the corresponding example.

In **chapter4,** “On the Improvement of Steam Power Plant System Reliability” a new and original model is developed that helps in determining the optimal maintenance strategies which will ensure maximum reliability of the coal-fired generating station.

Conclusions are given, concerning the study conducted and the results thereof, at the end of each chapter. The conclusions for this thesis are annotated in **chapter 5**.
A set of selected references that were consulted during the study performed for this doctor of philosophy thesis is provided at the end.

Keywords:

Reliability;
System Reliability;
Importance measures;
Availability;
Component;
Cannibalization;
Design; and
Coherent.

Opsomming

Die doel van hierdie tesis was om sekere metodes te bestudeer om ingenieurswese stelsels se betroubaarheid te verbeter. Die rede hoekom die tema "inginieurswese stelsels verbetering" gekies was, was om eerstens die tradisionele metodes van betroubaarheid verbetering te ondersoek, (wat gebaseer is op die ideë dat betroubaarheid kan verseker word deur om net n hoë voldoende veiligheids faktor in die ontwerp van komponente of stelsels voor the stel) asook nuwe, oorspronklike konsepte van betroubaarheid verbetering. Die laasgenoemde bestaan uit benaderings, metodes en beste praktyke wat gebruik kan word by die ontwerps fase van n komponent (stelsel) om die waarskynlikheid (risiko) te minimaliseer wanneer die komponent (stelsel) nie voldoen aan die betroubaarheid vereistes, objektiewe en verwagtinge nie.

Daarom, **hoofstuk 1** van die tesis, "Introduction to the main methods and concepts of reliability for technical systems" sluit in die inleiding seksie en die hoof tradisionele metodes beskikbaar om tegniese/ ingenieurswese stelsels te verbeter.

In **hoofstuk 2**, "Reliability Component Importance Measures" twee nuwe en oorspronklike konsepte oor betroubaarheid verbetering van ingenieurswese stelsels word voorgestel. Hulle is: 1) die studie van hoe belangrik die beskikbaarheid van komponente in samehangende stelsels is en 2) die optimale toewysing van verwisselende komponente in samehangende multi-stadium stelsels.

In **hoofstuk 3**, "Cannibalization Revisited" twee nuwe en oorspronklike konsepte oor betroubaarheid verbetering van ingenieurswese stelsels word voorgestel. Hulle is: 1) n teoretiese model om die effekte van kannibalisering op missie tyd beskikbaarheid van stelsels te toon en 2) n nuwe model vir "cannibalization" en die oorstemende voorbeeld.

In **hoofstuk 4**, “On the Improvement of Steam Power Plant System Reliability” n nuwe en oorspronklike model is ontwikkel en dit sal help om die optimale instandhouding strategieë wat die maksimum betroubaarheid van die steenkool-aangedrewe kragsentraal verseker.

Die gevolgtrekkings word gegee, met betrekking tot die studie en die resultate daarvan is aan die einde van elke hoofstuk. Die gevolgtrekking vir hierdie tesis is geannoteer in hoofstuk 5. n Stel geselekteerde verwysings wat gekonsulteer was gedurende die studie wat uitgevoer was vir hierdie doctors filosofie tesis word aan die einde voorsien.

Sleutel woorde:

Betroubaarheid;

Stelsel betroubaarheid;

Belangrike maatreëls;

Beskikbaarheid;

Komponente;

Kannibalisering;

Ontwerp;

Samehangende;

Dedication

To my parents: Josphat (of blessed memory) and Josphin Mangara

For teaching me to read and write.

Acknowledgements

I would like to take this opportunity to express my gratitude to all those who took time to give me advice, help and support throughout this research.

The enthusiasm of Professor Maxim Finkelstein convinced me to attempt this research in the first place. Professor Finkelstein was always willing to engage in discussions about my research. Professor Finkelstein’s professional management of my efforts, constructive criticism, and support of my research, his friendship and continuous encouragement, is much appreciated.

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List of acronyms and abbreviations

Acronym / Abbreviation	**Description**
PDfR	Probabilistic Design for Reliability
DfR	Design for Reliability
PoF	Physics of Failure
MTTF	Mean-Time-To-Failure
TTF	Time-To-Failure
QT	Qualification Testing
HALT	Highly Accelerated Life Testing
ALT	Accelerated Life Testing
PM	Predictive Modelling
FMEA	Failure Mode and Effect Analysis
FMECA	Failure Mode, Effects, and Criticality Analysis
FTA	Fault Tree Analysis
SCA	Sneak Circuit Analysis
CIMs	Component importance measures
Hi-tech	High Technology
US	United States
SF	Safety Factor
SAE	Society of Automotive Engineers
O&M	Operations and Maintenance
AT	Accelerated Testing
FOAT	Failure Oriented Accelerated Testing
MTTR	Mean Time To Repair
MDT	Mean Down Time
UV	Ultra Violet
CR	Criticality importance measure
RAW	Risk Achievement Worth

RRW	Risk Reduction Worth
FCF	Failure Criticality Function
RCF	Renewal Criticality Function
BDD	Binary Decision Diagram
IP	Improvement Potential
FV	Fussell-Vesely
MSS	Multi-State System
CUI	Conditional Utility Importance
UI	Utility Importance
CSFs	Continuum Structure Functions
METRIC	Multi-Echelon Technique for Recoverable Item Control
MOD-METRIC	Modified Multi-Echelon Technique for Recoverable Item Control
NORS	Not Operationally Ready Supply
CNA	Centre for Naval Analyses
U.S. G.A.O.	United States General Accounting Office
CANN	Cannibalization rate
$CANN_{AF}$	Cannibalization rate as defined by the US Air force
MUT	Mean Up Time
MSRT	Mean Supply Response Time
MMST	Mean Maintenance and Supply Time
$MTAA_{system}$	System Mission Time Average Availability
GE	Gross Effectiveness
CWT	Customer Wait Time
GTA	Graph Theoretical Analysis

CHAPTER 1: INTRODUCTIION TO THE MAIN METHODS AND CONCEPTS OF RELIABILITY FOR TECHNICAL SYSTEMS

This chapter provides an overview of the main methods of reliability improvement of technical systems. This thesis aims to develop some methods of reliability improvement of engineering systems. The main objective of the thesis and the contributions of the thesis are also introduced.

1.1 BACKGROUND

What does reliability mean? Reliability is "the probability that an item will perform a required function, under stated conditions, for a stated period of time". Put more simply, it is "the probability that an item will work for a stated period of time". The concept of reliability has been applied to technical systems for over six decades and as a field of research, it is common to mathematical statistics, operational research, physics, graph theory and informatics. Before we start, we shall define reliability. The commonly used definition of reliability is the following: Reliability, R(t), is the ability of an item to perform a required function, under given environmental and operational conditions and for a stated period of time [1]; which is frequently measured by the probability of failure, frequency of failure, or in terms of availability. The US military standard 785B [2] defines reliability as the duration or probability of failure free performance under stated conditions. Ushakov [3] classified modern reliability theories into six (6) categories: "pure" reliability analysis, effectiveness, survivability, safety, security, and software reliability. Reliability is the analysis of failures, their causes and consequences thereof.

It can be noted from the above definitions that reliability is the probability that a system performs its mission successfully. As the mission is often specified in terms of time, reliability is also often defined as the probability that a system will operate satisfactorily for a given period of time. Consequently reliability may be a function of time. In a less restrictive sense, the reliability of a system can be defined as: 1) the ability to render its intended function, or 2) the probability that it will not fail. The main objective of reliability engineering under either of these definitions is primarily to prevent the creation or occurrence of failures. Nonetheless only definition 2) requires a statistical interpretation of this effort. Thus under this definition reliability is the name of the field of study that endeavours to assign numbers to the propensity of a system to fail. Doing so has inherent uncertainty and requires the use of probability theory and mathematical statistics. The term reliability includes dependability (that is, the probability of non-failure), durability, maintainability, reparability, availability, testability, and other properties that could or should be viewed and evaluated as probabilities of the corresponding reliability attributes of a component, system, or process [4]. Therefore the use of applied probability and probabilistic risk management concepts, approaches, methods, and techniques puts the art and practice of reliability engineering on a solid probabilistic and low-risk footing.

Technological innovations of the past six decades cannot be disputed. To compliment these technological achievements many reliability methods and models have been developed over the last six decades. However with all these technological achievements and reliability models, there still exists one weakness in all of mankind's systems - that is the possibility of failure. This problem permeates modern society. That is from the home owner who faces the possibility of appliance failure to the telecommunications and electric utility companies that are faced with the possibility of network and nuclear reactor failures. Therefore the introduction of every new appliance or system must be accompanied by a provision for maintenance, spare parts and a plan to mitigate against failure. This is more so for the military where life cycle maintenance costs of systems far outweigh the initial purchase costs.

Main methods and concepts of reliability for technical systems is a problem that falls under probability modelling. Thus reliability engineering is part of the applied probability and probabilistic risk management bodies of knowledge [4], [5]. Take for example a system comprising of a number of components. For the simplest case, each of the components has two states, functioning or failed. The reliability of the system can be determined, when the set of functioning components and the set of failed components are specified. The problem equates to computing the probability that the system is functioning, which is the reliability of the system.

The best system is the best compromise between the needs for reliability, cost effectiveness, and the time-to-market (that is, with no attempts to either oversimplify the process or introduce unnecessary complexity. In other words reliability cannot be low, need not be higher than necessary, but has to be adequate for a particular component or system that needs to be). The reliability of such technical engineering systems can be improved by the main methods addressed and discussed below [6]:

1. Probabilistic design for reliability (PDfR) (alternatively referred to as Conservative Design) - For example ample margins, use of components and materials with established operating experience, and observing environmental restrictions;
2. Use of analysis tools and techniques - especially failure modes and effects analysis (FMEA), fault tree analysis (FTA) and - for electrical components - sneak circuit analysis (SCA), followed by correcting the problem areas detected by these qualitative analysis and techniques;
3. Extensive testing - to verify design margins, toleration of environmental extremes, and the absence of fatigue and other life-limiting effects;
4. Redundancy - to protect against random failures by providing alternative means of accomplishing a required function.

The general concepts mentioned in the preceding section are further addressed, discussed and illustrated, where feasible by numerical examples, in the subsequent sections.

1.2 OVERVIEW

1.2.1 PROBABLISTIC DESIGN FOR RELIABILITY

Probabilistic design for reliability (PDfR) is a set of approaches, methods and best practices (that is, a set of tools) that are supposed to be used at the design phase of a component (system) in order to minimize the likelihood (risk) that the component (system) might not meet the reliability requirements, objectives and expectations [4], [7], [8], [9]. A PDfR approach brings in the probability dimension to each of the Design for Reliability (DfR) characteristics of interest. The DfR is a deterministic (non-probabilistic) method used to quantify reliability. Traditionally the DfR approach is based on the notion that reliability could be assured by simply introducing a sufficiently high safety factor (SF) into the design of a component. The SF is defined as the ratio of the capacity (strength), C, of a component (system) to the demand (stress / load), D: SF = C / D. The level of the SF is chosen based on the following factors about the component or system [4], [7]:

1. The accumulated experience;
2. The probable consequences of failure;
3. The acceptable risks;
4. The expected environmental or operation conditions;
5. The availability and trustworthiness of the information about the capacity and demand;
6. The possible costs and social benefits;
7. The information on the variability of the materials and structural parameters;
8. The construction (fabrication) technologies and procedures; and
9. The accuracy with which the capacity and demand are determined.

A PDfR approach assigns a probability dimension to each of the DfR characteristics of interest as mentioned above. In a specific problem the capacity and demand could be different from the mechanical strength and load. In such a case the role of these characteristics can be replaced by temperature, electrical current or resistance, voltage, light intensity, and humidity. It can be noted that the PDfR methodology examines the reliability of a component (system) based on a probabilistic basis. Therefore PDfR is part of the applied probability and probabilistic risk analysis (management) bodies of knowledge [4], [5].

Below is a simple example of how PDfR is employed and the gain thereof. We examine the simple digital logic inverter of Figure 1. We assume the digital logic inverter's mean time to failure (MTTF), τ, during steady-state operation follows the exponential law of reliability. The digital logic inverter's probability of failure (PoF) can be adequately characterised by the Boltzmann-Arrhenius equation, $\tau = \tau_0 exp(U/kT)$ [4]. Thus the failure rate is, $\lambda = 1/\tau = 1/\tau_0\, exp(U/kT)$ and the probability of non-failure is $P = e^{-\lambda t} = exp\{-\left(t/\tau_0\, exp(-\,U/kT)\right)\}$. Solving this equation for the absolute temperature, T, we obtain $T = -\frac{U}{k\, ln\{\tau_0/t(-ln(P))\}}$.

For example consider a surface charge accumulation failure in the transistor Q_2 for which $U/k = 11600K$, and let the τ_0 value predicted by highly accelerated life testing (HALT) be $\tau_0 = 2x10^{-5} hours$. Suppose the customer requires that the probability of failure at the end of the logic inverter's service time, $t = 40\ 000\ hours$ does not exceed $Q = 10^{-5}$. Therefore the above formula indicates that the steady-state operation temperature should not be higher than $T = 352.3K = 79.3$ ℃. Then the thermal management equipment must be designed accordingly.

The example above illustrates how PDfR can be vital in achieving a practical compromise between the reliability and cost of a component (system).

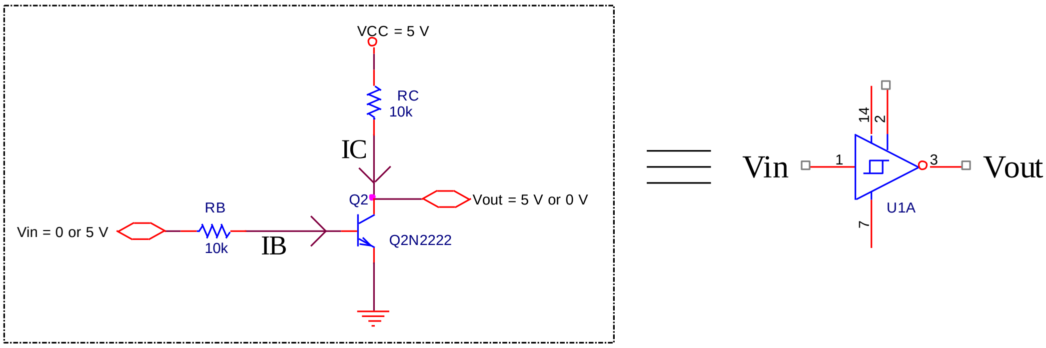

Figure 1: Simple digital logic inverter

PDfR should be applied from the early concept phase of a design all the way through to component (system) production (manufacturing). The success of PDfR is directly proportional to the selection of the appropriate reliability tools for each stage of the component (system) development and the correct implementation of these tools. Probabilistic design for reliability should be performed during the design phase of the component so as to create a "genetically healthy" component. This process cannot be left to the prognostic and health monitoring techniques, that is, when the component and / or system has been produced and shipped to the consumer. At this stage it is too late to change the design and / or the materials for improved reliability. Hence, when reliability is imperative reliability engineers re-qualify components to assess their lifetime and use redundancy to build a highly reliable system out of insufficiently reliable components. PDfR is an emerging discipline that makes it mandatory to design reliability into components (systems). This is diametrically opposed to the Test-Analyse-and-Fix philosophy, which unfortunately still exists in current industrial design processes.

Industries have advanced the development of reliability engineering from traditional testing for reliability to PDfR [10]. PDfR is the process conducted during the design phase of a component or system that ensures them to perform at the required level of reliability. PDfR aims to understand and fix the reliability-related problems at the conceptual phase in the design process. We summarise some general considerations that are useful when one considers steps to mitigate operational failures by way of PDfR:

1.2.2 USE OF ANALYSIS TOOLS AND TECHNIQUES

The analytical tools and techniques discussed in this section are generally the most cost effective means of failure mitigation and prevention. These analytical techniques must be done in the conceptual phase of system development in order to minimize rework and retesting. Analysis is inexpensive relative to modelling and cheaper than testing. Analytical methods (techniques) for failure prevention are classified into two categories [4], [6], [11]:

1. Analyses performed to demonstrate that the performance requirements will be met (and therefore, by implication, that the item will not fail during operation). Testing to pass is also known as qualification testing (QT). Examples of these analyses are stress and fatigue analysis for mechanical items, worst-case analysis and thermal analysis for electronic circuits, and stability analysis for control systems; and
2. Analyses performed to demonstrate that safety and reliability requirements are met. Testing to fail is also known as highly accelerated life testing (HALT). Examples of these are failure mode and effect analysis (FMEA), fault tree analysis, and sneak circuit analysis.

The first category of analytical methods are domain specific, as can be deduced from the examples. The required techniques, and to an even larger extent the procedures, vary widely even among functionally similar components such as electromechanical and solid state relays, and digital decoders. The analytical procedures are performed by the designer (manufacturer) rather than a reliability engineer. However, the reliability engineer should be informed of the results of the analyses. Within the scope of this section, we focus on the failure mode and effects analysis and the testing part of the two categories of analytical methods (techniques) for failure prevention will be discussed in section 1.2.3.

Failure mode and effects analysis is a mainstay of analytical techniques for failure prevention [6]. For every failure mode, effects are evaluated at the local, intermediate ("next higher"), and system level. Where the system level effects are deemed critical, the system designer need to use this information to mitigate the probability of failure (that is, by using the most reliable components and increased cooling where temperature rise is of concern), prevent propagation of the failure (for example emergency system shut-off and warning alarms), or compensate for the effect of the failure (for example making use of the standby system, that is where redundancy has been built into the design).

Failure mode and effects analysis was one of the first systematic techniques for failure analysis. A formal FMEA methodology was developed by reliability engineers in the 1950s to facilitate the study of problems that might arise from malfunctions of military systems [11]. Nonetheless, informal procedures for establishing the relation between component failures and system effects date back much further [12]. However, FMEA still has its own short comings. In the year 2000 the Society of Automotive Engineers (SAE) promulgated specialized FMEA procedures for the automotive industry [13]. FMEA is widely used in the process industry in support of safety and reliability [14]. FMEA can help [6]:

1. Component designers identify locations where more reliable (or derating), redundancy, or self-test may be particularly effective or desirable;
2. System engineers and project managers allocate resources to areas of highest vulnerabilities;
3. Procuring and regulatory organizations determine whether reliability and safety goals are being met; and
4. Those responsible for the operations and maintenance (O&M) phase plan for the fielding of the system.

A formal FMEA is primarily conducted to satisfy the procuring and regulatory organizations. The procuring and regulatory organizations need to determine whether reliability and safety goals are being met. This is an imposed requirement that does not originate in the development team and thus is sometimes given low priority. Informal studies along the lines of component designers wanting to identify locations where more reliable (or derating), redundancy, or self-test may be particularly effective or desirable; and system engineers and project managers needing to allocate resources to areas of highest system vulnerabilities are often done in support of the development process. Yet, these informal studies are rarely published as legacy documents. The operations and maintenance phase plan for the fielding of the system is perhaps the most neglected use of the FMEA. But with increased awareness that O&M costs generally overshadow those associated with the acquisition of systems, that issue deserves emphasis. The following are the essential concepts of the FMEA process [6]:

1. Parts can fail in several modes, each of which typically produces a different effect. For example, a capacitor can fail open (usually causing an increased noise level in the circuit) or short (which may eliminate the entire output of the circuit);
2. The effects of the failure depend on the level at which it is detected;
3. The probability and severity of in-service failures can be reduced by monitoring provisions (built-in test and supervisory systems); and
4. The effects of a failure can be masked or mitigated by compensating measures (redundancy and alarms).

An FMEA is often the first step of a systems reliability study [11]. An FMEA involves reviewing as many components, sub-systems, and systems as possible to identify failure modes and causes and effects of such failures. For each component, the failure modes and their resulting effects on the rest of the system are recorded in a specific FMEA worksheet. There are numerous variations of such worksheets. FMEA worksheets present the information on each of these in a standardized tabular format, and this enables reviewers to identify and ultimately correct deficiencies. An example of an FMEA worksheet can be found in the military standard, MIL-STD-1629A [15]. An FMEA becomes a failure mode, effects, and criticality analysis (FMECA) if criticalities or priorities are assigned to the failure mode effects.

1.2.3 TESTING AND PREDICTIVE MODELLING

Why should one conduct accelerated testing (AT)? The golden rule of an experiment is that the duration of the experiment should not exceed the lifetime of the experimentalist. According to [4], [7] it is impractical and uneconomical to wait for real-time failures when the Mean-Time-To-Failures (MTTFs) of today's highly reliable electronic and photonic systems are in the order of thousands of hours. Accelerated testing enables one to gain greater control over the reliability of a component and has become a powerful means in understanding the reliability physics underlying the component's performance [16]. The above statement is true regardless of whether (irreversible or reversible) failures will or will not actually occur during the highly accelerated life testing (that is, "testing to ruggedize" and to test the reliability limits), failure oriented accelerated testing (FOAT) (that is, "testing to fail" and to validate a particular reliability model) or qualification testing (that is, "testing to pass" and to make a particular device into a product).

In order to reduce the time-to-market (that is, shortening of the component's design and development time) in today's industrial environment leaves no room for time consuming reliability investigations. Therefore, to get the maximum information in the minimum time and at the minimum cost possible is the major goal of a manufacturer and test engineer. This is achieved by accelerating a component's degradation and / or failure or testing the reliability limits of the component. To accelerate a device's degradation and failure, one or more parameters (stimuli) that affect the component performance and durability has to be deliberately "distorted" ("skewed"). These parameters (stimuli) could be for example temperature, humidity, load, current and voltage [4], [7].

According to [7] the most common accelerated test conditions (stimuli) are: high temperature (steady-state) soaking / storage / baking / aging / dwell; low temperature storage; temperature (thermal) cycling; power cycling; power input and output; thermal shock; thermal (temperature) gradients; fatigue (crack propagation) tests; mechanical shock; drop shock tests; random vibration tests; sinusoidal vibration tests (with the given or variable frequency); creep/stress-relaxation tests; electrical current extremes; voltage extremes; high humidity; radiation (ultra violet (UV), cosmic, X-rays); altitude; space vacuum; industrial pollution; salt spray; fungus; dirt; high intensity noise.

Table 1 shows the main accelerated testing categories. These AT categories differ by their objectives, end points, follow-up activities, and what is viewed as an ideal test.

Table 1: Testing Categories *[4]*

Testing Category	**Product Development Testing (PDT)**	**Qualification Testing (QT)**	**Highly Accelerated Life Testing (HALT)**
Objective	Technical feedback to ensure that the taken design approach is viable	Proof of reliability: demonstration that the product is qualified to serve in the given capacity	Understand reliability physics (modes and mechanisms of failure) and assess the likelihood of failure field
End point	Time, type, level, and/or number of failures	Predetermined time, number of cycles, and / or the excessive (unexpected) number of failures	Predetermined number or percentage of failures

Follow-up activity	Failure analysis, design decision	Pass/fail decision	Failure analysis of the test data
Ideal test	Specific definitions	No failure in a long time	Numerous failures in a short time

HALT is not a pass / fail (qualification) test, but a "discovery" test. It is not intended to measure reliability. HALT often involves step-wise stressing, rapid thermal transitions, and combined stressing under various environmental conditions [7]. It can be deduced that HALT is aimed at the prediction of the likelihood of field failure. Hence, HALT cannot do without simple and meaningful predictive modelling (PM) [4]. It is upon the PM basis that one decides which HALT parameter should be accelerated, how to process the experimental data, and, most importantly, how to bridge the gap between the HALT data and the likelihood of field failure. Predictive modelling can lead to significant savings of time and expense because it considers the fundamental physics that might constrain the final design. Most HALT models are aimed at predicting the MTTF. Some of the examples of HALT models and their typical use are listed below [4]:

1. The Power law. The power law is used when the probability of failure is unclear;
2. The Boltzmann-Arrhenius equation. It is used when elevated temperature is the major cause of failure;
3. The Coffin-Manson equation. The Coffin-Manson equation is an inverse power law used to evaluate low cycle fatigue life-time;
4. The Crack growth equations. These equations are used to evaluate fracture toughness of brittle materials;
5. The Bueche-Zhurkov and Eyring equations are used to consider the combined effect of high temperature and mechanical loading;
6. The Peck equation. It is used to evaluate the combined effect of elevated temperature and relative humidity;

7. The Black equation is to evaluate the combined effects of elevated temperature and current density;
8. The Miner-Palmgren rule. This rule is used to assess fatigue lifetime when the yield stress of the material is not exceeded;
9. The Creep rate equations;
10. The Weakest link model. This model is applicable to extremely brittle materials with defects; and
11. The Stress-strength (demand-capacity) interference model, which is perhaps the most flexible and well substantiated model.

1.2.4 REDUNDANCY

In some structures, single components (sub-systems) may be of much greater importance for the system's capability to function than others. Take for example a single component operating in series with the rest of the system. If this component fails it implies that the system also fails. There are two ways of ensuring higher system reliability in situations such as these. The two ways are [11]:

1. One has to use components with very high reliability in such critical places in the system; and
2. One has to introduce redundancy in these places (that is, the introduction of one or more reserve components).

The type of redundancy obtained by replacing the important component with two or more components operating in parallel is referred to as active redundancy. In active redundancy the components share the load right from the beginning until one of them fails.

The reserve component, in redundancy, can also be kept in standby in a manner such that the first of them is activated when the ordinary component fails, the second is activated when the first reserve component fails and it continues on. If the reserve components carry no load in the waiting period before activation (and therefore cannot fail in this period), the redundancy is called passive. In the waiting period such a component is said to be in cold standby. If the standby components carry a weak load in the waiting period (and thus might fail in this period), the redundancy is called partly loaded. In the following subsections we will illustrate how redundancy can be used to improve reliability by considering some simple examples.

Redundancy can improve the reliability and availability of systems. Some applications do not need redundancy to operate successfully. However, if the cost of failure is high enough one may need to implement redundancy. One has to do a feasibility study and choose a redundancy model that is most suitable for the specific application.

1.2.4.1 REDUNDANCY IMROVING RELIABILITY IN NON-REPAIRABLE SYSTEMS

As already described in section 1.1 reliability is defined as the probability of not failing of a component (system) in a particular environment for a particular time. Reliability engineering is part of the applied probability and probability risk management bodies of knowledge. Therefore reliability is a statistical probability and there are no absolutes or guarantees. It follows that one aims to enhance the odds of success as much as is feasible within reason. Often the reliability of a component (a system constituent) is given as a function of time. For example, a common assumption is that components have an exponential distribution for time to failure (TTF). In this case the component reliability is, $R(t) = e^{-\lambda t}$.

The probability equation, $R(t) = e^{-\lambda t}$, is most commonly used in practice defining exponential distribution of time to failure. The assumption is that the failure rate, λ, is constant. R(t) is the probability of operating without failure in [0,t), λ, is the constant failure rate over time (that is, the number of failures per hour); and $1/\lambda$, is the mean time to failure.

In most cases, the only factor that one can influence after the system has been designed is the failure rate (λ). The environmental conditions are influenced by the nature of the application itself. Usually one cannot change the mission time except if the system operates in planned maintenance at strategic times. Thus one can influence the system reliability through PDfR and provident component selection. Below we will illustrate, through elementary mathematics, how redundancy (two systems in parallel) in system design can improve system reliability.

Let **R** be the probability of success and **F** be the probability of failure. Thus for two systems in parallel: $\mathbf{R}_{redundant} = 1 - (\mathbf{F}_1)(\mathbf{F}_2)$, where $\mathbf{F}_1$ is the probability of failure of system 1 and $\mathbf{F}_2$ is the probability of failure of system 2. Let $\mathbf{F}_1 = \mathbf{F}_2 = 0.05$. In this case, $\mathbf{R}_{redundant} = 1 - (0.05)(0.05) = 0.9975$, which is a remarkable increase in reliability as compared with a non-redundant case.

In practice there are situations where it is more economical to employ redundancy only to the less reliable component in the system. We illustrate this scenario with a system that has three components configured in series as shown in Figure 2, where R_1 = component 1 reliability, R_2 = component 2 reliability, and R_3 = component 3 reliability.

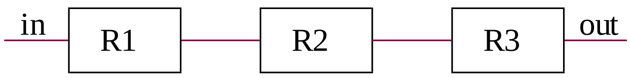

Figure 2: Three component series system

We calculate the reliability of the entire system of Figure 2 by multiplying the reliabilities of each of the components. $\mathbf{R}_{system} = (\mathbf{R}_1)(\mathbf{R}_2)(\mathbf{R}_3)$. As an example if we take the reliabilities of each component to be $\mathbf{R}_1 = 0.97$, $\mathbf{R}_2 = 0.80$, and $\mathbf{R}_3 = 0.96$, then $\mathbf{R}_{system} = (.97)(.8)(.96) = 0.755$. If we back up the least reliable component of the system (that is, the chain is as strong as the weakest link), $\mathbf{R}_2$, with redundancy, the system of Figure 2 will now be resembled by that of Figure 3.

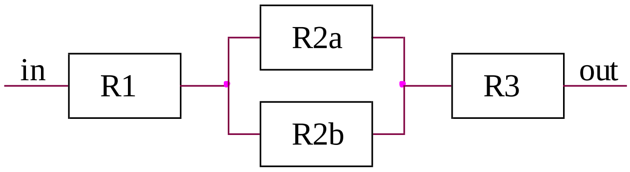

Figure 3: Three component series system with redundancy

Since $\mathbf{R} = 1 - \mathbf{F}$, then for the redundant component we have $\mathbf{R}_2 = 1 - (\mathbf{F}_{2a})(\mathbf{F}_{2b})$. Therefore, the reliability of the system with redundancy is now calculated as, $\mathbf{R}_{redunant\_system} = (\mathbf{R}_1)(1-(\mathbf{F}_{2a})(\mathbf{F}_{2b}))(\mathbf{R}_3)$. If we select $\mathbf{R}_1 = 0.97$, $\mathbf{R}_{2a} = \mathbf{R}_{2b} = 0.80$, and $\mathbf{R}_3 = 0.96$, the reliability of the system is now $\mathbf{R}_{redundant\_system} = (.97)(1-(.2)(.2))(.96) = 0.894$. It can be noted that we have improved the system reliability by 13.9 percentage points by implementing redundancy for only one component. Redundancy can be implemented at many levels and it is application specific. One has to know the components that are most likely to fail and design redundancy for these.

1.2.4.2 REDUNDANCY IMROVING AVAILABILITY IN REPAIRABLE SYSTEMS

According to [17] availability is defined as "*The ability of an item (under combined aspects of its reliability, maintainability and maintenance support) to perform its required function at a stated instant of time or over a stated period of time*".

In accordance with the above definition of availability we begin by differentiating between the availability A(t) at time *t* and the average availability A_{av}. The availability at time *t* is

$$A(t) = \Pr(\text{component is operating at time } t),$$

where $\Pr(\xi)$ denotes the probability of event ξ.

The term "operating" means here that the component is either in active operation or that it is able to operate if required.

The average availability A_{av}, denotes the mean proportion of time the component is operating. If one has a component that is repaired to an "as good as new" condition every time it fails, the average availability is

$$A_{av} = \frac{MTTF}{MTTF+MTTR} \tag{1.1}$$

where MTTF (mean time to failure) denotes the mean operating time of the component, and MTTR (mean time to repair) denotes the mean downtime after a failure. Sometimes MDT (mean downtime) is used instead of MTTR to make it clear that it is the total mean downtime that should be used (in the equation for average availability) and not only the mean active repair time. When considering a production system, the average availability of the production (i.e., the mean proportion of time the system is producing) is sometimes called the production regularity [11].

If one has a system whose mission time is 12/7 for 1 year and have no downtime, then the system availability is 1. If the said system has a downtime of three days for that same mission time, then the availability of the system becomes, $A_{av} = \frac{MTTF}{MTTF+MTTR} = \frac{((12x7x365)-(3x12))}{(12x7x365)-(3x12)+(3x12)} = 0.99883.$

When one is developing strategies for improving availability, one must first accept the reality that one will have to deal with failures now and then. Hence, the focus in designing any system for high availability is to reduce downtime and make the repair time as short as possible.
Without redundancy, the system downtime depends on how quickly one can achieve the following:

1. Detect the failure;
2. Diagnose the problem;
3. Repair or replace the failed component of the system; and

4. Return the system to full operational status.

In case of hardware failures, it is best to replace the failed component or sub-system. The replacement of the failed component and / or sub-system could take anything from a few minutes to several days. This replacement time is dependent on the accessibility and availability of spare components. In the case of a software failure, one may only need to reboot to fix (repair) the system. Nonetheless, rebooting a large and complex system could take a few seconds to several hours. The rebooting time would depend on the specific system at hand.

When one takes into account redundancy, the system downtime is dependent on how quick one is able to detect a failure and switch over to the backup component (system). In many practical systems this could be easily under one second and a large number of systems can achieve sub-millisecond downtimes. Based on these practical downtimes systems can achieve it can be deduced that redundancy can therefore improve the availability of these systems by several orders of magnitude. By way of example, we consider a system that needs to run 24/7 for half a year. If this system experiences 30 minutes of downtime during its mission time, the availability would be 0.9999837 (or 4 nines in availability language). However, if redundancy is employed in the system and the downtime is brought down to half a second, the availability would be 0.9999999955 or 8 nines. It is important to note here that the switchover times for redundancy in some practical systems are commonly so fast that the system is not noticeably affected by the downtime. Hence, for practical purposes, these systems never experienced an outage, and consequently achieve an availability of 1.

1.3 SUMMARY OF CONTRIBUTIONS

This thesis focuses on the development of approaches to reliability improvement of redundant engineering systems. It presents new probabilistic models and methods for quantifying this improvement. In addition, applications of these methods (approaches) to engineering systems are given where feasible. The main contributions of this thesis in the aggregated form and the corresponding publications containing these results are listed in what follows:

1. Method 1: Development of new measures of importance of components in multistate and repairable systems. (Paper 1 [18], paper 2 [19] and paper 3 [20]).
2. Method 2: Development, modelling and application of new cannibalization procedures for redundant systems. (The corresponding paper is submitted and is about to be published).
3. Method 3: Application of some methods of reliability improvement to steam power plants (paper 4 [21]).

1.4 THESIS OUTLINE

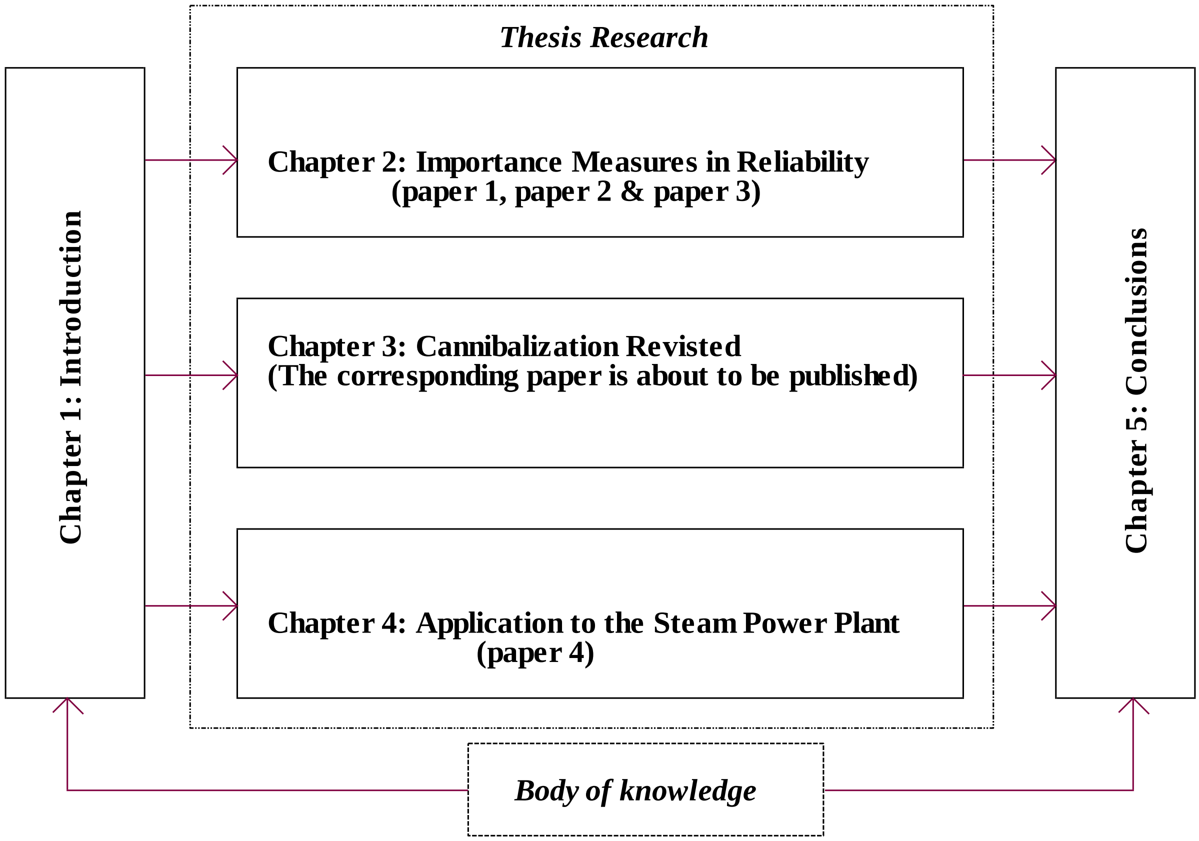

Figure 4: Thesis Outline

Figure 1 depicts the overall organisation of this thesis and the relationships among the five (5) chapters. The remaining chapters of this thesis are organised as follows:

Chapter 2 - a chapter focusing on the relevance of importance measures in improving the reliability and availability of engineering systems (paper 1 [18], paper 2 [19] and paper 3 [20]).

Chapter 3 - a description of cannibalization (revisited) as a method (procedure) of improving reliability of engineering systems (The corresponding paper is about to be published).

Chapter 4 - a discussion of the application of the methods of improving reliability to the steam power plants (paper 4 [21]).

Chapter 5 - a discussion of the final remarks from this research. Furthermore, the contributions of this research to the reliability theory are briefly discussed.

CHAPTER 2: RELIABILITY COMPONENT IMPORTANCE MEASURES

2.1 INTRODUCTION

Component importance measures (CIMs) are used in various fields to assess how much a single component, a subsystem, a basic event, or a part of a process contributes to the failure risk of a system. Component importance is always seen in relation to the specified system function. Therefore, the absolute values of component importance measures may not be as important as their relative rankings. The contribution of CIMs to the failure risk of a system may be analysed for a specific time instance (time-dependent CIMs), or over the mission time of the system (time-independent CIMs) or disregarding probabilities (structural CIMs).

Generally, a system is a collection of components performing a specific task or function. It is obvious that some components in a system are more important for the system reliability than other components. For example, a component in series with the rest of the components in the system is a cut set of order one (1). This component is generally more important than a component that is a member of a cut set of higher order [11, pp. 149 - 150]. In this chapter component importance measures are defined and discussed. The component importance measures may be used to rank the components, that is, to arrange the components in ascending or descending order of importance. The component importance measures may also be used for classification of importance, that is, to allocate the components into two or more groups, according to some pre-set criteria.

A system usually consists of multiple components. As mentioned, these components are not necessarily equally important for the performance (reliability, availability, risk, and throughput) of the system. Usually such a system needs to be designed, enhanced and / or maintained efficiently using limited resources. Nonetheless, for large and complex systems, it may be too tedious, or not even possible, to develop a formal optimal strategy. In suchlike situations, it is desirable to allocate resources according to how important the components are to the system and to concentrate the resources on the subset of components that are most important to the system [22, pp. 49 - 53]. Hence, the notion of importance measures of components (also called sensitivity [23]) can be indeed crucial.

In reliability, an importance measure evaluates the relative importance of individual components or group of components in the system. This relative importance can be determined based on the system structure, component reliability and / or component lifetime distributions. Measuring the relative importance of components may allow the engineer to:

1. Determine which of these components deserve additional research and warrant development in order to improve the overall system reliability under cost (and / or effort) constraints; and
2. Find the component that will most probably cause the failure of a system. By using importance measures, it is possible to draw conclusions about which components are the most important to improve in order to achieve better reliability of the whole system.

In section 2.2, we describe importance measures in reliability; in 2.3, state of the art on component importance measures in reliability; in 2.5, classical component reliability importance measures whereas in sections 2.4 and 2.6 we present new original results on measures of reliability importance and on availability of importance of components in coherent systems, respectively.

2.2 IMPORTANCE MEASURES IN RELIABILITY

Once the reliability of a system has been determined, reliability engineers are often faced with the task of identifying the least reliable component(s) in the system in order to improve the design. In such a case, the engineers responsible for designing and operating the system need to explore options for improving the system reliability performance. Reliability importance measures can serve as guidelines in developing an improvement strategy. The basic introduction to the concept of reliability importance is given in [11, pp. 183 - 206].

Historically, Birnbaum [24] was the first to introduce component importance measures in 1969. Since then, several CIMs have been developed in the reliability arena. A survey of the literature on component importance measures for reliability by Boland and El-Neweihi can be found in [25]. Boland and El-Neweihi [25] categorize reliability importance measures into three categories according to the knowledge needed for determining the importance measures. These categories are: structural, time-independent, and time-dependent.

The structural importance measures determine the relative importance of each of the components of a system based solely on the system's structural design (that is, with respect to their positions in the system). The structural importance measures are based on knowledge of the system structure only and do not involve the reliability of the components being considered. The structural importance measures can be determined completely by the design of the system (that is, ϕ). In other words, structural importance measures of components actually represent the importance of the positions in the system that the components occupy. According to Birnbaum in [24], the structural measures are used when the system structure function is known save for the individual component reliability values. As discussed in [25], the most common structural measure used by reliability engineers today is Birnbaum's [24] structural measure which captures the proportion of system state vectors in which a specific component is critical. Structural importance measures are basic in that they do not take into account information about the reliabilities of components (as might be the case in the early stages of system development). Structural importance measures are useful at the initial stages of system design and development where a reliability engineer is faced with the uphill task of allocating available resources to optimise measures of system effectiveness and design parameters in the absence of reliability data.

Time-independent importance measures (also known as the reliability importance measures [22, pp. 49 - 53] depend on the component reliabilities at various points in time, and as such give perhaps a more global view of component importance. Time-independent importance measures are considered when the mission time of a system is implicit and fixed. Consequently, the components are evaluated by their reliability at a fixed time point (that is, the probability that a component functions properly during the mission time). Time-independent importance measures depend on both the system structure, ϕ, and the component reliabilities. Thus, to calculate time-independent importance measures one has to determine the mission time and the component reliabilities in advance. As explained by Boland and El-Neweihi in [25], the most commonly used time-independent importance measures are the Birnbaum [24] and the Barlow and Proschan [26] reliability importance measures. Most of the time-independent measures are some form of a weighted average of the Birnbaum reliability importance measure, for example the importance measure by Xie and Shen [27].

Time-dependent importance measures (also referred to as the lifetime importance measures [22, pp. 49 - 53] assess component importance for a specific interval of time. Time-dependent importance measures are considered when a system and the components (constituting the system) have long-term or infinite service missions. Time-dependent importance measures depend on both the positions of the components within the system and the component lifetime distributions. Two of the more prominent time-dependent measures are the Barlow and Proschan [26] time-dependent importance measure and the Natvig [28] time-dependent importance measure.

2.3 STATE OF THE ART ON COMPONENT IMPORTANCE MEASURES

As mentioned above, the critical problem in system reliability theory is to identify components (constituting a system) that significantly influence the system's performance with respect to reliability or availability. It is always not possible (that is, due to budget constraints) to improve all components within a system at the same time to improve the system's reliability. Therefore, priority should be given to those components that are more important. In this manner, reliability engineers can prioritize where investments should be made to guarantee maximum improvement in system reliability. Importance measures allow reliability engineers to identify the relatively most critical points of the system, from which design alternatives can be identified to improve the system performances. Alternative applications of importance measures are system diagnosis and maintenance.

For *coherent systems*, Birnbaum [24] was the first to quantify measures of importance. These coherent systems are represented as a monotonic function of the vector of variables $\mathbf{x} = (x_1, x_2, ..., x_n)$, called the *state vector*. One of these is the structural measure which evaluates the "criticality" of a component. One calls $(1_i, \mathbf{x})$ a critical path vector for component $i, i = 1,2,...,n$, if $\phi(1_i, \mathbf{x}) - \phi(0_i, \mathbf{x}) = 1$. In an n component system there are 2^{n-1} state vectors of the form $(1_i, \mathbf{x})$, and the relative proportion of these that are critical for component i is its structural importance $I^i_{B,\phi}$. Despite its merits (that is, it gives maximum variation of the unavailability when the component changes its state from perfectly functioning to failed, it is useful when used in conjunction with other indices, and that other indices can be expressed as a function of it), the weakness of Birnbaum's structural importance is that it does not take into account the reliabilities of the various components constituting the system. Therefore, two components may have a similar measure value, although their current levels of reliability could differ substantially. For example, in a k out of n system all components are structurally equivalent. Nonetheless, Birnbaum also introduced a reliability importance measure of components based on the reliability at a fixed point in time t. The following notation is going to be used to discuss this measure.

Let:

$p_i =$ reliability of component i (at time t).

$\mathbf{p} = (p_1, p_2, ..., p_n) =$ vector of component reliabilities.

$(\cdot_i, \mathbf{p}) = (p_1, p_2, ..., p_{i-1}, \cdot_i, p_{i+1}, ..., p_n)$

$\mathbf{X} = \mathbf{X}(t) =$ random state vector of components at time t.

The Birnbaum reliability importance I_B^i of component i is defined to be the probability that the i^{th} component is critical to the functioning of the system, that is, $I_B^i = \Pr\{\phi(1_i, \mathbf{X}) - \phi(0_i, \mathbf{X}) = 1\}$. When the components act independently, then one can show that $I_B^i = h(1_i, \mathbf{p}(t)) - h(0_i, \mathbf{p}(t)) = \dfrac{\partial h(\mathbf{p}(t))}{\partial \mathbf{p}_i(t)}$ where $h(1_i, \mathbf{p}(t))$ denotes the (conditional) probability that the system is functioning when it is known that component i is functioning at time t, and $h(0_i, \mathbf{p}(t))$ denotes the (conditional) probability that the system is functioning when component i is in a failed state at time t. Note that in such a situation (that is, the Birnbaum reliability importance) the reliability importance of component i does not depend on p_i itself. Furthermore in the case where $p_i = \dfrac{1}{2}$ for each i, one has that $I_B^i = I_{B,\phi}^i$.

Subsequent to the first importance measure being proposed by Birnbaum [24] for coherent systems other measures of component importance have been introduced. These component importance measures include the measure proposed by Barlow and Proschan [26]. The Barlow-Proschan importance measure is equal to the probability that the system fails when the i^{th} component fails. This measure can be treated as the average Birnbaum measure in reference to the unreliability of the i^{th} component. At the end of the 1970s Natvig [28] drew up a new reliability measure of component importance, in which the importance of the component was made conditional on loss of the remaining time to failure of the system caused by the transition of the considered component into down state. Bergman suggested the next more widely known measure [29]. The Criticality importance measure (CR) is another widely used measure [30]. This is a natural extension of the Birnbaum measure that includes the components' unreliability. Two other measures that are widely used for ranking the components' importance are the Risk Achievement Worth (RAW) and the Risk Reduction Worth (RRW) [31].

For *non-coherent systems* the Birnbaum measure of importance as defined in [24] loses its meaning, since it cannot assume negative values. Non-coherent systems are described by Boolean functions containing negated events. The generalization of the Birnbaum importance measure and its extension to promote the importance measures of components in non-coherent systems was proposed by Jackson [32], Zhang and Mei [33], Becker and Camarinopoulos [34], and later by Beeson and Andrews [35], [36]. Jackson [32] proposed the use of the absolute value of the Birnbaum measure in order to be able to rank the events in order of importance for non-coherent systems, that is:

$$I_{xi}^{Jackson}(t) = |\Pr(\Phi(x_i = 1, \mathbf{x}, t)) - \Pr(\Phi(x_i = 0, \mathbf{x}, t))| = |\frac{\partial Q_{\Phi}(t)}{\partial q_{xi}}| \qquad (2.1).$$

Naturally, the absolute value implies a loss of information about the criticality of components. Zhang and Mei [33] defined two probabilities of the extension of Birnbaum's measure as representing the two contributions of the criticality of non-coherent variables: $I_{xi}^{+}(t) = \Pr[\Phi(x_i = 1, \mathbf{x}, t) - \Phi(x_i = 0, \mathbf{x}, t) = 1]$ and $I_{xi}^{+}(t) = \Pr[\Phi(x_i = 1, \mathbf{x}, t) - \Phi(x_i = 0, \mathbf{x}, t) = -1]$. Becker and Camarinopoulos [34] introduced the definition of Failure Criticality Function (FCF) and Renewal Criticality Function (RCF). The Failure Criticality Function for the generic variable x_i is a Boolean function defined as:

$$FCF_{xi}(t) = \Phi(x_i = 1, \mathbf{x}, t)[1 - \Phi(x_i = 0, \mathbf{x}, t)] \qquad (2.2).$$

It expresses the fact that a generic component x_i is critical when the system fails if the component fails $(x_i = 1 \Rightarrow \Phi(x_i = 1, \mathbf{x}) = 1)$ and the system functions if the component functions $(x_i = 0 \Rightarrow \Phi(x_i = 0, \mathbf{x}) = 0)$. Alternatively $1 - \Phi(x_i = 0, \mathbf{x}) = 1$.

If x_{xi} is coherent then $FCF_{xi} = I_i^B(t)$, since $\Phi(x_i = 1, \mathbf{x})\Phi(x_i = 0, \mathbf{x}) = \Phi(x_i = 0, \mathbf{x})$; with $I_i^B(t) = \Pr(\Phi(x_i = 1, \mathbf{x}, t)) - \Pr(\Phi(x_i = 0, \mathbf{x}, t)) = \dfrac{\partial Q_\Phi(t)}{\partial q_{xi}}$, where $Q_\Phi(t)$ is the unavailability of $\Phi(\mathbf{x})$ at time t. The expected value of FCF_{xi} at time t, that is $\Pr(FCF_{xi} = 1, t)$, is indicated as $p_{xi}^f(t)$ and represents the probability of the critical state for the failure of x_i at time t.

RCF_{xi} is a Boolean function defined as:

$$RCF_{xi}(t) = \Phi(x_i = 0, \mathbf{x}, t)[1 - \Phi(x_i = 1, \mathbf{x}, t)] \qquad (2.3).$$

It expresses the fact that a generic component x_i is critical when the system fails if the component is repaired $(x_i = 0 \Rightarrow \Phi(x_i = 0, \mathbf{x}) = 0)$ and the system works if the component fails $(x_i = 1 \Rightarrow \Phi(x_i = 1, \mathbf{x}) = 0)$. Alternatively, $(1 - \Phi(x_i = 1, \mathbf{x})) = 1$.
If x_{xi} is coherent then $RCF_{xi} = 0$ since $\Phi(x_i = 1, \mathbf{x})\Phi(x_i = 0, \mathbf{x}) = \Phi(x_i = 0, \mathbf{x})$. The expected value of RCF_{xi} at time t, $\Pr(RCF_{xi} = 1, t)$, is indicated as $p_{xi}^r(t)$ and represents the probability of the critical state for the repair of x_i at time t.

Beeson and Andrews [35], [36] proposed an extension of the Birnbaum index of component importance for non-coherent systems. This measure is given by the sum of the probabilities of *all critical states* for the non-coherent component, that is:

$$G_{xi}(t) = G_{xi}^{F}(t) + G_{xi}^{R}(t) \quad (2.4),$$

where $G_{xi}^{F}(t)$ is the probability that, at time t, the system is in a working state such that the failure of component x in $t - t + \delta t$ causes the system to fail. $G_{xi}^{F}(t) = \dfrac{\partial Q_{\Phi}(t)}{\partial q_{xi}(t)}$; and $G_{xi}^{R}(t)$ is the probability that the system is in a failed state at time t such that the repair of component x causes the system to fail. $G_{xi}^{R}(t) = \dfrac{\partial Q_{\Phi}(t)}{\partial p_{xi}(t)}$. In these equations $q_{xi}(t) = \Pr(x_i = 1)$ and $p_{xi}(t) = \Pr(x_i = 0)$.

The calculation of $G_{xi}^{R}(t)$ and $G_{xi}^{F}(t)$ is done considering the exact equation of the system unavailability calculated using the inclusion-exclusion method applied to the disjunction of the prime implicates or the Binary Decision Diagram (BDD).

On the basis of the Birnbaum index, extended also to non-coherent functions, other indices can easily be calculated such as the Improvement Potential (IP), CR, RAW, RRW, and Fussell-Vesely (FV). These component importance measures are defined and discussed in the subsequent sections.

2.4 BACKGROUND OF THEORY OF RELIABILITY COMPONENT IMPORTANCE

2.4.1 DEFINITIONS AND NOTATIONS

Assume that every device, whether it is a single component or system consisting of components, is binary, that is, can be in one and only one of the two possible states: it functions or it fails. Let binary variable x_i indicate the state of component i for $i = 1,2,...,n$, and

$$x_i = \begin{cases} 1 \text{ if component } i \text{ functions} \\ 0 \text{ if component } i \text{ fails} \end{cases} \qquad (2.5).$$

Then, vector $\mathbf{X} = (x_1, x_2, ..., x_n)$ represents the states of all components and is known as the component state vector. Let ϕ represent the state of the system, and

$$\phi = \begin{cases} 1 \text{ if the system functions} \\ 0 \text{ if the system fails} \end{cases} \qquad (2.6).$$

The state of the system is completely determined by and is a deterministic function of the states of the components. Therefore, it is defined as

$$\phi = \phi(\mathbf{X}) = \phi(x_1, x_2, ..., x_n) \qquad (2.7),$$

where $\phi(\mathbf{X})$ is called the structure function of the system. Each unique system corresponds to a unique structure function $\phi(\mathbf{X})$.

2.4.2 THE CONCEPT OF COMPONENT RELIABILITY IMPORTANCE

Consider a binary coherent system S with components $b_1, b_2, ..., b_n$. The corresponding indicator variables for the two possible states 1 (functioning) and 0 (failed) are

$$z_i = \begin{cases} 1 \text{ if component } i \text{ functions} \\ 0 \text{ if component } i \text{ fails} \end{cases}, i = 1,2,\dots,n \,;\; z_s = \begin{cases} 1 \text{ if the system functions} \\ 0 \text{ if the system fails} \end{cases} \qquad (2.8).$$

In a coherent system, the states z_i of the components uniquely determine the state of the system. Thus, the existence of a function ϕ with property $z_s = \phi(\mathbf{z})$ with $\mathbf{z} = (z_1, z_2, ..., z_n)$. Function ϕ is named the structure function of S, and n is its order. Generally, ϕ is not uniquely defined. In this research $z_1, z_2, ..., z_n$ are assumed independent random variables.

The probabilities $p_s = \Pr(z_s = 1)$ and $p_i = \Pr(z_i = 1)$, are the availabilities of the system and its constituent components, respectively. These probabilities allude to a fixed or a variable time point t or to the stationary system (where the failed components are repaired after failures in finite time intervals). $p_i = p_i(t)$ and $p_i = p_s(t)$ explicitly denote the time dependencies of the components and the system, respectively. On the other hand, for non-repairable systems, $p_i(t)$ and $p_s(t)$ characterise the survival probabilities (that is, the reliabilities) of the components and the system, respectively.

When one considers non-reparable systems p_i and p_s are usually referred to as reliabilities. The system reliability is specified by $p_s = E(\phi(z_1, z_2, ..., z_n))$. Thereupon the components operate independently, p_s is only a function of the vector of the component reliabilities $\mathbf{p} = (p_1, p_2, ..., p_n)$: $p_s = h(\mathbf{p})$. By pivotal decomposition (with pivotal component b_i) one gets

$$h(\mathbf{p}) = p_i h(1_i, \mathbf{p}) + (1 - p_i) h(0_i, \mathbf{p}) \quad (2.9),$$

where the vectors $(\cdot_i, \mathbf{p})$ are defined as

$$(\cdot_i, \mathbf{p}) = (p_1, p_2, ..., p_{i-1}, \cdot_i, p_{i+1}, p_n), \cdot_i = 0(1), i = 1,2,...,n.$$

Components have dissimilar degrees of impact on the success (failure) demeanour of S save for series and parallel systems with identical components. The quantitative assessment of the impact that a particular component (that is, its reliability characteristics) on the reliability of the system as a whole is interesting to the reliability analyst. There are now a large number of studies on the significance of the problem, due to a variety of tasks in the analysis of system reliability. Some of the questions the reliability analyst has to ask are:

1. What is the contribution of a component to the system reliability (availability)?
2. What components effect a maximum increase in system reliability (availability) if their reliability (availability) is increased?
3. How does the system reliability (availability) vary with component reliability (availability)?
4. What components are likely to cause a system failure when they fail?
5. What components contribute to the failure of the system?

The importance of components in line with one or more of these or related questions is quantified by the so-called importance measures. Understanding of these importance measures is key to the design and / or repair (upgrade) of systems from the reliability (availability) point of view in addition to efficient maintenance thereof. In line with this, there are a large number of different indicators of importance (significance). Thus, depending on the purpose of comparison, the same components may have different relative importance with different criteria. So important is the precise statement of purpose of comparison. For example, if the reliability (availability) of a system has to be increased by upgrading the system components, then ranking (ordering) the components with regard to importance measures, which is articulated by questions 1 to 3, is critical. On the other hand, if one needs a fault-finding guide when analysing the failure of a system, the order of the components concerning the importance criteria referring to questions 4 and 5 is required. Importance measures have been characterized not only for components but also for groups of components, for example, minimal path- and cut sets.

Historically, the first, most simple, but nonetheless up to now crucial importance measure is due to Z. W. Birnbaum in 1969 [24]. The Birnbaum structural importance, reliability importance and lifetime importance will be subject for discourse in the subsequent sections.

2.4.3 PARAMETERS OF COMPONENT RELIABILITY IMPORTANCE AS A CRITERION OF SYSTEM PERFORMANCE

Consider a non-repairable system consisting of statistically independent components $b_1, b_2, ..., b_n$. Let the state of the system be defined by the vector $\mathbf{x} = (x_1, x_2, ..., x_n)$, where $x_i, i = 1,2,...,n$ - is a binary variable describing the state of component b_i. Define the function $\mathbf{y}(\mathbf{x})$ in the state space $\{0,1\}^n = \{0,1\} \times \{0,1\} \times ... \times \{0,1\}$ with values in set $S = \{0,1,..., M\}$, where $M+1$ is the number of states with different performance levels of the system. Each state of S corresponds to its quantitative value as follows:

$$S = \{0,1,2,\ldots,M\} \Rightarrow \{a_0, a_1, a_2, \ldots, a_M\} = S_a - \text{in each state} \quad (2.10).$$

Thus, ranking (ordering) the component states results in $0 = a_0 \le a_1 \le a_2 \le ... \le a_M$. Similarly the function $y(\mathbf{x})$, with values in S_a, which is named the generalised structural function is also defined as:

$$\mathbf{y}(\mathbf{x}) = j \Rightarrow y(\mathbf{x}) = a_j,\ j = 0,1,2,\ldots,M \quad (2.11)$$

The assumption is that function $y(\mathbf{x})$ is monotonically increasing, that is there exists two vectors $\mathbf{x}, \mathbf{x}'$ with x_i and x_i' for each $i = 1,2,...,n$, such that $x_i \le x_i' \Rightarrow y(x_i) \le y(x_i')$, if $x_i \le x_i'$. This ensures the importance of all components.

Additionally, it is natural to assume that: $y(0,0,...,0) = a_0 = 0; y(1,1,...,1) = a_M$.

The following denotation follows:

$$\begin{aligned}(\cdot_i, \mathbf{x}) &= (x_1, x_2, \ldots, x_{i-1}, \cdot_i, x_{i+1}, \ldots, x_n),\\ (0_i, \mathbf{x}) &= (x_1, x_2, \ldots, x_{i-1}, 0, x_{i+1}, \ldots, x_n),\\ (1_i, \mathbf{x}) &= (x_1, x_2, \ldots, x_{i-1}, 1, x_{i+1}, \ldots, x_n).\end{aligned} \quad (2.12)$$

It can be notated that if $S_a = \{0,1\}$, then $y(\mathbf{x})$ represents the usual binary structure function of the system.

Now denote by $\mathbf{x}(t) = (x_1(t), x_2(t), ..., x_n(t))$ - a random process, where t is the current time and $\mathbf{x}(0) = (j_1, j_2, ..., j_n)$ - the initial condition. Consider the following measures of the performance level of the system:

$$W_y(\mathbf{x}(t)) = y(\mathbf{x}(t)); \tag{2.13}$$

$$W_M(\mathbf{x}(t)) = M[y(\mathbf{x}(t))]; \tag{2.14}$$

$$W_E(\mathbf{x}(t)) = E[y(\mathbf{x}(t))]; \tag{2.15}$$

$$W_{\geq a_m}(\mathbf{x}(t)) = \Pr((y(\mathbf{x}(t))) \geq a_m), \tag{2.16}$$

where M - is the expected system performance efficiency operator; E - is the average system performance efficiency operator in the time interval $(0,t)$. The average system performance level is the mean of the system performance during the time interval $(0,t)$. Therefore, if the system over time Δt is in a state with the instantaneous value of efficiency of a_i, then the corresponding performance level is $a_i \Delta t$. If $S_a = \{0,1\}$, $W_M(\mathbf{X}(t))$ becomes the usual binary state system.

Earlier research on reliability component importance focussed mainly on the system efficiency performance measures $W_y(\mathbf{x}(0))$, $W_y(\mathbf{x}(t))$, binary-state systems with multi-state components and binary systems. It is obvious that, arguing strictly, a quantity indicator of the performance level efficiency of equation (2.13) can only be at $t = 0$.

Investigation of the measures of component importance by the criteria of equation (2.13) to (2.16) can be performed with reasonable efficiency, if one takes into account that for performance measures W_y, W_M, and $W_{\geq a_i}$ in the assumptions introduced above, one has multi-linear representation. For example for equation (2.13) we have:

$$\begin{aligned} W_y(\mathbf{x}(t)) = \sum_{i=1}^{n} c_i x_i(t) + \sum_{1 \leq i \langle j \leq n} c_{ij} x_i(t) x_j(t) \\ + \sum_{1 \leq i \langle j \langle k \leq n} c_{ijk} x_i(t) x_j(t) x_k(t) + ... \\ + c_{12...n} x_1(t) x_2(t) ... x_n(t) \end{aligned} \tag{2.17}$$

where coefficients c_i, $c_{i,j}$, … are defined unambiguously in an explicit form. Equation (2.17) is linear for each $x_i(t)$. Applying the expected system performance efficiency operator to equation (2.17) we get:

$$\begin{aligned} W_M(\mathbf{x}(t)) = \sum_{i=1}^{n} c_i p_i(t) + \sum_{1 \leq i \langle j \leq n} c_{ij} p_i(t) p_j(t) + \\ + \sum_{1 \leq i \langle j \langle k \leq n} c_{ijk} p_i(t) p_j(t) p_k(t) + ... \\ + c_{12...n} p_1(t) p_2(t) ... p_n(t) \end{aligned} \tag{2.18}$$

Since the multi-linear representation for $W_{\geq a_i}$ is irreducible, a direct method is used to operate on it, using the idea of the statistical independence of the functioning of the components:

$$W_{\geq a_m}(x(t)) = \sum_{j_i=0}^{1} p_{i,j_i}(t)\Pr(y(j_i, \mathbf{x}(t)) \geq a_m)$$
$$= \sum_{j_i=0,\, j_k=0}^{1,1} p_{i,j_i}(t)p_{k,j_k}(t)\Pr(y(j_i, j_k, \mathbf{x}(t)) \geq a_m) \tag{2.19}$$
$$= \sum_{j_1=0,\ldots,j_n=0}^{1,\ldots,1} p_{1,j_1}(t)\ldots p_{n,j_n}(t)\Pr(y(j_1, j_2,\ldots, j_n) \geq a_m)$$

where

$$p_{i,0}(t) = p_i(t);\; p_{i,1}(t) = 1 - p_i(t);$$
$$\Pr(y(j_1, j_2,\ldots, j_n) \geq a_m) = 1 \text{ and } y(j_1, j_2,\ldots, j_n) \geq a_m;$$
$$\Pr(y(j_1, j_2,\ldots, j_n) \geq a_m) = 0 \text{ and } y(j_1, j_2,\ldots, j_n) \langle a_m.$$

The expression for the average system performance is no longer linear and the multilinear standard limit is defined by the following stochastic integral:

$$W_{\Sigma}(\mathbf{x}(t)) = \int_0^t W_M(\mathbf{x}(\tau))d\tau = \sum_{i=1}^{n} c_i \int_0^t p_i(\tau)d\tau +$$
$$\sum_{1\leq i\langle j\leq n} c_{ij} \int_0^t p_i(\tau)p_j(\tau)d\tau + \ldots. + \tag{2.20}$$
$$c_{12\ldots n} \int_0^t p_1(\tau)p_2(\tau)\ldots p_n(\tau)d\tau.$$

There are two (2) main basic measures of component importance. The first group comprises of measures of direct component importance. On the other hand the second group consists of measures that define average system performance loss arising from the failure of the i^{th} component at an arbitrary time, $t \in (0, \infty)$.

2.4.4 MEASURES OF DIRECT COMPONENT RELIABILITY IMPORTANCE

The measures of direct component importance are determined by the amount by which the system performance level of efficiency in equation (2.13) to (2.16) is reduced when component i fails (that is, the importance of component i):

$$I_i^H = W(\mathbf{x}(t)) - W(0_i, \mathbf{x}(t)). \tag{2.21}$$

For convenience, the following notation is introduced: $M(y(\mathbf{x}(t))) = \phi(\mathbf{p}(t))$; $\Pr(y(\mathbf{x}(t)) \geq a_i) = \phi_{a_i}(p(t))$. Multilinear representation of the measures of importance of component i gives expressions that are more concrete:

$$I_{i,y}^H = \frac{\partial y(\mathbf{x}(0))}{\partial x_i(0)} = y(\mathbf{x}(0)) - y(0_i, \mathbf{x}(0)); \tag{2.22}$$

$$I_{i,M}^H = \frac{\partial \phi(\mathbf{p}(t))}{\partial p_i(t)} = \phi(0_i, \mathbf{p}(t)); \tag{2.23}$$

$$I_{i,a_m} = \frac{\partial \phi_{a_m}(\mathbf{p}(t))}{\partial p_i(t)} p_i(t) = \phi_{a_m}(\mathbf{p}(t)) - \phi_{a_m}(0_i, \mathbf{p}(t)), \tag{2.24}$$

where

$$\phi(0_i, \mathbf{p}(t)) = M[y(0_i, \mathbf{x}(t))];$$
$$\because \phi_{a_m}(0_i, \mathbf{p}(t)) = \Pr(y(0_i, \mathbf{x}(t)) \geq a_m).$$

Equations (2.22) to (2.24) are easily proved, based on the linearity of $x_i(0)$ and $p_i(t)$, that is, equations (2.17) to (2.19). Notice that Birnbaum [24] introduced a measure of component importance in a coherent system of statistically independent, non-repairable components as

$\frac{\partial \phi(\mathbf{p}(t))}{\partial p_i(t)}$, which can be interpreted as the probability that at time t the system is operable, if component b_i is functional, and the system has failed, if this component has failed.

When a system fails, the diagnosis must be efficiently conducted, and maintenance must locate the component(s) that caused the failure of the system and bring the system back to operation as soon as possible. Then the maintenance needs to check the component that most likely caused the failure first, then the second most likely component, and so on, until the system is fixed. For this purpose, one might propose an importance measure of the type of equation (2.20) to indicate how important the different components are in terms of system failure. Indeed, the efficiency of the functioning of the system increases, if the place of the failed component is located a component with the least component importance. Notice that the rate of equation (2.25) ($t = 0$) for the case of the binary system gives very little information in this regard, since it only shows whether the i^{th} component is critical or not (that is, whether its failure leads to the failure of the system).

Much more interesting is the case of measures of structural importance, which more fully take into account the effect of the specific component on the quality of the structure. For systems with multistates, the measures of direct component importance become more substantial with reference to reconfiguration problems. At the same time a generalisation of the concept of the measure of structural importance for this situation is very promising. The results can be generalised to the case of (consecutive failures) of the components. For example, measures of direct importance of two components b_i and b_j are given by:

$$I_{i,j}^{H} = W(\mathbf{x}(t)) - W(0_i, 0_j, \mathbf{x}(t)) \tag{2.25}$$

Moreover, in particular, for $I_{i,j,M}^{H}$ based on the multi-linear representation of equation (2.18) is easily obtained:

$$\begin{aligned} I^{H}_{i,j,M} &= \frac{\partial^2 \phi(\mathbf{p}(t))}{\partial p_i(t) \partial p_j(t)} p_i(t) p_j(t) \\ &= \phi(\mathbf{p}(t)) - \phi(0_i, 0_j, \mathbf{p}(t)). \end{aligned}$$

It should be borne in mind that the definition of equations (2.21) and (2.25) are more common, whereas the expressions through partial derivatives are valid only in those assumptions that are obtained through multi-linear representations (in particular, the statistical independence of the components in question).

2.4.5 MEASURES OF AVERAGE SYSTEM PERFORMANCE EFFICIENCY LOSS

The measure of component importance that defines average system performance loss arising from the failure of the i^{th} component at an arbitrary time, $t \in (0, \infty)$ is:

$$\begin{aligned} I^{Y}_{i,M} &= \int_0^\infty [\phi(1_i, \mathbf{p}(t)) - \phi(0_i, \mathbf{p}(t))] f_i(t) dt \\ &= \int_0^\infty \frac{\partial \phi(\mathbf{p}(t))}{\partial p_i(t)} f_i(t) dt, \end{aligned} \tag{2.26}$$

where the $f_i(t)$ is the density function of the lifetime distribution of component i. The derivative of equation (2.26) is also based on the representation of equation (2.18). Thus, $I^{Y}_{i,M}$ by definition is the expected reduction in system performance efficiency due to failure of the i^{th} component.

The measure of component importance, $I^{Y}_{i,E}$ shows the reduction of the residual system performance efficiency after the failure of the i^{th} component. Let $\phi(1_i,\mathbf{P}(\tau+t))$ ($\phi(0_i,\mathbf{P}(\tau+t))$) denote the system efficiency at time $\tau+t$, provided that at time τ component i is operable (inoperable). Then it is clear that:

$$I_{i,E}^{Y} = \int_0^\infty\int_0^\infty [\phi(1_i.\mathbf{p}(\tau+t)) - \phi(0_i,\mathbf{p}(\tau+t))] dt f_i(\tau) d\tau \qquad (2.27).$$

Example 2.1. Consider an elementary example to illustrate the preceding concepts. Let the system comprise of two components. Assume that the system efficiency is set at the value of a_3, if both components are functional, at a_1 if the first component is operational, and at a_2 if the second component is functional. Then:

$$y(\mathbf{x}(t)) = a_1x_1(t) + a_2x_2(t) + (a_3 - a_1 - a_2)x_1(t)x_2(t);$$

$$\phi(\mathbf{p}(t)) = a_1p_1(t) + a_2p_2(t) + (a_3 - a_1 - a_2)p_1(t)p_2(t);$$

$$\phi a_1(\mathbf{p}(t)) = p(y(\mathbf{x}(t)) \geq a_1) = p_1(t) + p_2(t) - p_1(t)p_2(t);$$

$$\phi a_2(\mathbf{p}(t)) = p_2(t);$$

$$\phi a_3(\mathbf{p}(t)) = p_1(t)p_2(t).$$

Let the density distributions be exponential as follows: $f_1(t) = \lambda_1 e^{-\lambda_1 t}$ and $f_2(t) = \lambda_2 e^{-\lambda_2 t}$. Presented below are some measures of importance for the system under consideration:

$$I_{1,y}^{H} = a_3 - a_2;\; I_{2,y}^{H} = a_3 - a_1; \qquad (2.27)$$

$$I_{1,M}^{H} = \phi(\mathbf{p}(t)) - a_2 p_2(t); \;\; I_{2,M}^{H} = \phi(\mathbf{p}(t)) - a_1 p_1(t); \tag{2.28}$$

$$I_{1,E}^{Y} = \frac{a_1}{\lambda_1} + \frac{\lambda_1(a_3 - a_1 - a_2)}{(\lambda_1 + \lambda_2)^2}; \;\; I_{2,E}^{Y} = \frac{a_2}{\lambda_2} + \frac{\lambda_2(a_3 - a_1 - a_2)}{(\lambda_1 + \lambda_2)^2}; \tag{2.29}$$

After analysing the expressions (2.27) to (2.29), one can make the following conclusions. According to the first measure of component importance, the first component can be more important than the second component, whilst the other measures indicate quite the contrary. For instance in the example of a microprocessor structure which is characterised by performing a certain task in a time interval $(0,t)$, it is necessary to have comparisons of the ones similar to expression (2.29). Therefore, in real life situations it is very important for one to be very clear and precise about the measure of importance that most appropriately embodies the comparison problem at hand.

2.4.6 COST-SPECIFIC MEASURE OF COMPONENT RELIABILITY IMPORTANCE

Consider the performance level efficiency of a coherent system of statistically independent, non-repairable components. Assume that the reliability of the i^{th} component has been increased from p_i to $p_i + \Delta p_i \leq 1$ in a time interval $(0,t)$ (all other components have not been changed). Then, from linearity of equation (2.18), it immediately follows that the change in reliability of the system can be written as (For brevity of notation, we will omit the time variable in what follows):

$$\Delta_i \phi(\mathbf{p}) = \frac{\partial \phi(\mathbf{p})}{\partial p_i} \Delta p_i \tag{2.30}$$

where $\Delta_i \phi(\mathbf{p})$ means that the corresponding increment $\Delta \phi(\mathbf{p})$ on the system reliability is due to the i^{th} component. Equation (2.30) is Birnbaum's measure $I_B(i,\mathbf{p})$ (2.22). Therefore, setting identical increments for the various components ($\Delta p_i \equiv \Delta p, i = 1,2,...,n$) which constitute the system, the maximum increment for the system will be obtained on the component with the maximum value of $I_B(i,\mathbf{p})$.

Nonetheless, to achieve the same Δp for the different components require different costs. Thus, the notion of cost-specific importance measure should be concretized in each case. Denote by $c_i(p_i, \Delta p, t) \equiv c_i(\Delta p, p_i)$ the minimum cost necessary to improve p_i to $p_i + \Delta p$ in the time interval $(0,t)$. The cost-specific measure of component importance is defined as:

$$I_{c,i} = \frac{\Delta_i \phi(\mathbf{p})}{c_i(\Delta p, p_i)} = \frac{\Delta \phi(\mathbf{p})}{\Delta p_i} \frac{\Delta p}{c_i(\Delta p, p_i)} \tag{2.31}$$

When Δp is reasonably small, which is usually the case, we can approximate $\frac{\Delta \phi(\mathbf{p})}{\Delta p_i}$ by the partial derivative, $\frac{\partial \phi(\mathbf{p})}{\partial p_i}$. The importance of a fixed increment is also defined. To do this, it also follows from equation (2.30) where Δp_i is obtained by realizing a fixed (independent of i) increment of the system Δ_ϕ :

$$\Delta p_i = \frac{\Delta_\phi}{\frac{\partial \phi(p)}{\partial p_i}} \tag{2.32}$$

The quantity of the fixed increment is denoted as follows:

$$I_{\phi} = \frac{1}{c_i(\Delta p, p_i)} \qquad (2.33)$$

Hence, the cost-specific measure of component importance $I_{c,i}$ allows one to find a component that yields the maximum investment per unit increment on the system reliability. On the other hand I_{ϕ} shows the only component whose reliability can be improved to a level that ensures a fixed increment on the system reliability with the minimum cost.

The cost-specific measure of component importance can be used in the gradient method way to implement optimal systems with cost constraints when each step is determined by the component with the maximum cost limit.

2.5 CLASSICAL COMPONENT RELIABILITY IMPORTANCE MEASURES

The following component reliability importance measures are defined and discussed in literature [11, pp. 183-206]:

1. Birnbaum's measure (B)(and some variants);
2. The improvement potential measure;
3. Risk Achievement Worth;
4. Risk Reduction Worth;
5. The criticality importance measure;
6. Fussell-Vesely's measure; and
7. The Barlow-Proschan importance measure.

Several other measures are defined and described by Lambert (1975) and Henley and Kumamoto (1981) as cited in [11, p. 184]. The various measures are based on slightly different interpretations of the concept of component reliability importance. Intuitively, the importance of a component should depend on two factors [11, p. 184]:

- The location of the component in the system;
- The reliability of the component in question; and, perhaps, also the uncertainty in the estimate of the component reliability.

When designing and analysing a system, it is of paramount interest to know the importance of the different components in a system and how they contribute to the overall reliability of the system. As mentioned in the preceding sections, there are several measures for this, depending on how much information is available, and what measure of importance is of interest. The simplest case is the structural importance. If more advanced analysis is desired, there are many importance measures that account for the lifetime distributions.

2.6 ON AVAILABILITY IMPORTANCE OF COMPONENTS IN COHERENT SYSTEMS

2.6.1 BACKGROUND

The well-known Birnbaum's reliability measure of importance naturally defines the importance of a component in a coherent system of statistically independent, non-repairable components. By using a similar concept for repairable components, we discuss several availability measures of a component's importance. As availability of a component is defined by its times to failure and repair, the corresponding 'classical' measures of importance are modified to account for this case. Furthermore, costs that are needed to increase the time to failure and (or) to decrease the repair time are also taken into account. We analyse these measures and present several meaningful examples.

2.6.2 INTRODUCTION

In complex systems, it is necessary to translate system reliability requirements into detailed specifications for all components that make up a system. This process is often referred to as the reliability apportionment. During reliability apportionment, the reliability analyst has to understand and develop the relationships between component, subsystem, and system reliabilities. The crucial role in this process is in understanding and quantifying the reliability importance of different parts of the equipment.

The concept of *reliability* importance of components in a coherent system was proposed in the late 1960s [24], although it is a specific case of a sensitivity analysis that was used in various engineering applications for ages. The well-known Birnbaum's reliability measure naturally defines the importance of the i^{th} component in a coherent system of n statistically independent, non-repairable components as [24]:

$$I_i^B = \frac{\partial h(\mathbf{P}(t))}{\partial p_i(t)} = h(1_i, \mathbf{P}(t)) - h(0_i, \mathbf{P}(t)), \tag{2.34}$$

where $h(\mathbf{P}(t)), t \geq 0$ is the reliability (survival) function of a system, $\mathbf{P}(t) = (p_1(t), p_2(t), ..., p_n(t))$ is the vector of reliability functions of the components and $h(1_i, \mathbf{P}(t))$ ($h(0_i, \mathbf{P}(t))$), as usual, denotes the reliability function of a system with a given state of the i^{th} component at time t, that is, $h(1_i, \mathbf{P}(t)) = h(\mathbf{P}(t) \mid x_i(t) = 1)$ ($h(0_i, \mathbf{P}(t)) = h(\mathbf{P}(t) \mid x_i(t) = 0)$), where $x_i(t)$ is a binary variable that indicates the state of component i for $i = 1,2,...,n$. Equation (2.34) can be easily proved using the pivotal decomposition of the reliability function $h(\mathbf{P}(t))$ [37]. Moreover, performing (sequentially) pivotal decomposition, one can eventually arrive at the following multi-linear representation, the importance of which (to the best of our knowledge) was underestimated in the literature. For brevity of notation, we will omit the time variable in what follows [19], [18]:

$$h(\mathbf{P}) = \sum_{1}^{n} c_i p_i + \sum_{1 \le i < j \le n} c_{i,j} p_i p_j + \sum_{1 \le i < j < k \le n} c_{i,j,k} p_i p_j p_k + c_{1,2,\ldots,n} p_1 p_2 \ldots p_n\,, \tag{2.35}$$

where coefficients c_i, $c_{i,j}$, … can be obtained from the (similar to (2.25)) representation for the corresponding structure function of the system. In spite of its cumbersome form, (2.35) is methodologically convenient for qualitative analysis and for the interpretation of (2.34) and numerous other measures of importance (see, for example, [11, pp. 183-206]) for measures which are popular in practice).

Denote by, $\phi(\mathbf{X})$, the structure function of our system, where $\mathbf{X} = (x_1, x_2, \ldots x_n)$ is the state vector with binary values of binary components ($x_i = 1$ when the i^{th} component is operable and $x_i = 0$ otherwise). Obviously, the similar to (2.35) multi-linear representation holds, where $h(\mathbf{P})$ is substituted by $\phi(\mathbf{X})$ and p_i by x_i, whereas the coefficients are the same. Note that this representation holds for systems with *dependent components* as well. However, as $h(\mathbf{P}) = E[\phi(\mathbf{X})]$, it is clearly seen now that (2.35) holds only for systems with statistically independent components [11, p. 148].

Assume that the reliability of the i^{th} component has been increased from p_i to $p_i + \Delta p_i \le 1$ (all other components have not been changed). Then, from the multi-linearity of (2.35), it immediately follows that the change in reliability of the system can be written as [19], [18]:

$$\Delta_i h(\mathbf{P}) = \frac{\partial h(\mathbf{P})}{\partial p_i} \Delta p_i = I_i^B \Delta p_i \tag{2.36}$$

Equation (2.36) is meaningful. When the i^{th} component has failed ($p_i = 0$) and is ‘improved to the absolutely reliable state’ then $\Delta p_i = 1$ and (2.36) reduces to (2.34). When $\Delta p_i = 1 - p_i$, we arrive at the measure that is usually called the *improvement potential* [11, pp. 189-190]:

$$\Delta_i h(\mathbf{P}) = \frac{\partial h(\mathbf{P})}{\partial p_i}(1-p_i) = I_i^B(1-p_i) = h(1_i,\mathbf{P}) - h(\mathbf{P}) \tag{2.37}$$

Another popular measure follows from (2.36) when $p_i = 0$ and $\Delta p_i = p_i$ [19], [18]:

$$\Delta_i h(\mathbf{P}) = \frac{\partial h(\mathbf{P})}{\partial p_i} p_i = I_i^B p_i = h(\mathbf{P}) - h(0_i,\mathbf{P}) \tag{2.38}$$

Hundreds of papers are devoted to measures of importance of non-repairable systems and only a few to repairable. However, there is one setting for repairable systems, where the measures of components' importance can be defined in a way similar to the non-repairable case. This is the case when a system's reliability characteristic of interest is its availability (see, for example, [37]). Thus, availability of the coherent system of n repairable components when each of them is described by its own independent alternating renewal process can be defined as [38, pp. 586-587]:

$$A(t) = h(\mathbf{A}(t)) = h(A_1(t), A_2(t), \ldots, A_n(t)) \tag{2.39}$$

where $h(\cdot)$ is the *same* multi-linear function as in (2.34) – (2.37) with the substitution of arguments $p_i(t)$ by $A_i(t)$, where $A_i(t)$ is the availability of component i. However, in contrast to the non-repairable case, availability of a component is now defined by its *failure and repair times* and, therefore, the corresponding measures of importance should be modified to account for this. In section 2.6.3 we will suggest and analyse some measures of importance focusing on the stationary case [11, p. 371], [38, pp. 586-587]:

$$A = \lim_{t\to\infty} A(t) = h(\mathbf{A}) = h(A_1, A_2, \ldots, A_n) \tag{2.40}$$

What happens, if the components of our system are statistically dependent? This usually substantially complicates the corresponding general analytical reliability analysis and often makes it impossible. However, if we know the reliability (availability) of a system as a function of reliability (availability) functions of the components (or of the corresponding parameters), then, in principle, we can use the right-hand sides of equations (2.34) and (2.37)-(2.39) for obtaining different 'direct' measures of importance. Obviously, as was mentioned, the multi-linear representation (2.35) does not hold in this case and, therefore, the corresponding partial derivatives also do not make sense. We will also illustrate this reasoning in the next section using the corresponding example.

2.6.3 MEASURES OF AVAILABILITY IMPORTANCE

Similar to (2.36), assume that (stationary) availability of the i^{th} component has been improved by ΔA_i, whereas other components are the same. Taking into account (2.39) and the corresponding multi-linear representation, it is easy to see that availability of our system has increased by [39]:

$$\Delta_i h(\mathbf{A}) = \frac{\partial h(\mathbf{A})}{\partial A_i} \Delta A_i \,, \tag{2.41}$$

where notation $\Delta_i h(\mathbf{A})$ means that the corresponding increment $\Delta h(\mathbf{A})$ is due to the i^{th} component. Denote by T_i and τ_i the mean time to failure and the mean time of repair of the i^{th} component, respectively. Then, it is well-known that $A_i = T_i / (T_i + \tau_i),\ i = 1,2,...,n$. Equation (2.41) is a good indicator of importance for the i^{th} component, justifying in another way Birnbaum's measure (2.34). Indeed, by increasing availability of each component by the same amount ΔA, we can choose the component with the maximum $\Delta_i h(\mathbf{A})$, which is the one with the maximum Birnbaum's measure.

As stationary availability depends on two parameters (mean time to failure and mean time to repair), it is reasonable to define measures of importance with respect to each of them. The availability importance measure based, for example, on T_i (when $\tau_i, i = 1,2,...,n$ is fixed) shows the effect of this parameter on availability of the whole system. Let [19], [18]:

$$I_{T_i} \equiv \frac{\partial h(\mathbf{A})}{\partial T_i} = \frac{\partial h(\mathbf{A})}{\partial A_i}\frac{\partial A_i}{\partial T_i} = \frac{\partial h(\mathbf{A})}{\partial A_i}\frac{\tau_i}{(T_i + \tau_i)^2} \tag{2.42}$$

Similarly, the availability importance measure based on τ_i (when $T_i, i = 1,2,...,n$ is fixed) shows the effect of this parameter on availability of the whole system. Similar to (2.42), the availability importance measure based on τ_i can be calculated as follows [19], [18]:

$$I_{\tau_i} \equiv -\frac{\partial h(\mathbf{A})}{\partial \tau_i} = -\frac{\partial h(\mathbf{A})}{\partial A_i}\frac{\partial A_i}{\partial \tau_i} = \frac{\partial h(\mathbf{A})}{\partial A_i}\frac{T_i}{(T_i + \tau_i)^2} \tag{2.43}$$

Note that, the function $h(\mathbf{A})$ is now non-linear either with respect to T_i or to τ_i. Therefore, it is interesting to compare the importance measure (2.42) with (2.41). Assume that T_i has been increased by $\Delta T_i \equiv \Delta T(i = 1,...,n)$, whereas τ_i has not been changed. Then (2.41) reads now [19], [18]:

$$I_{\Delta T,i} \equiv \Delta_i h(\mathbf{A}) = \frac{\partial h(\mathbf{A})}{\partial A_i}\frac{\Delta T \tau_i}{(T_i + \Delta T + \tau_i)(T_i + \tau_i)} \tag{2.44}$$

Similarly, assume that τ_i has been decreased by $\Delta\tau_i \equiv \Delta\tau(i = 1,...,n)$ whereas T_i has not been changed. Then [19], [18]:

$$I_{\Delta\tau,i} \equiv \Delta_i h(\mathbf{A}) = \frac{\partial h(\mathbf{A})}{\partial A_i} \frac{\Delta\tau T_i}{(T_i + \tau_i - \Delta\tau)(T_i + \tau_i)} \tag{2.45}$$

Clearly, when $\Delta T\,(\Delta\tau) = o(\Delta T(\Delta\tau))$, (2.41) is close to (2.44) ((2.45)), that is [18],

$$I_{\Delta T,i} = I_{T_i}\left[\Delta T + o(\Delta T)\right] \text{ and,}$$

$$I_{\Delta\tau,i} = I_{\tau_i}\left[\Delta\tau + o(\Delta\tau)\right],$$

where $o(\Delta T(\Delta\tau))$ is called "o small of delta T (τ)" and means: $\lim_{\Delta T(\Delta\tau)\to 0} o(\Delta T(\Delta\tau))/\,\Delta T(\Delta\tau) = 0$.

However, if $\Delta T\,(\Delta\tau) \neq o(\Delta T(\Delta\tau))$ then, obviously, measures (2.44) and (2.45) should be used**.** The result of comparison of importance measures in this case can be different from that for small $\Delta T\,(\Delta\tau)$, which is shown by the following example.

Example 1. Consider a series system of two independent, repairable components [38], [39], [40]:

$$A = h(\mathbf{A}) = A_1 A_2 = \frac{T_1}{T_1 + \tau_1}\frac{T_2}{T_2 + \tau_2};$$

In accordance with (2.44),

$$I_{\Delta T,1} = \frac{T_2}{T_2 + \tau_2}\frac{\Delta T \tau_1}{(T_1 + \Delta T + \tau_1)(T_1 + \tau_1)} \text{ and}$$

$$I_{\Delta T,2} = \frac{T_1}{T_1 + \tau_1}\frac{\Delta T \tau_2}{(T_2 + \Delta T + \tau_2)(T_2 + \tau_2)}$$

Assume that $\tau_1 \langle\langle T_1; \tau_2 \langle\langle T_2$, which is usually the case for repairable systems. Let $\Delta T \langle\langle \min(T_1, T_2)$. Then [19], [18],

$$I_{\Delta T,1} \approx \Delta T \frac{\tau_1}{T_1^2} \text{ and } I_{\Delta T,2} \approx \Delta T \frac{\tau_2}{T_2^2}$$

If we now assume that ΔT is small we get the following [19], [18]:

$$I_{\Delta T,1} \approx \Delta T \frac{\tau_1}{(T_1 + \Delta T)T_1} \text{; and } I_{\Delta T,2} \approx \Delta T \frac{\tau_2}{(T_2 + \Delta T)T_2}$$

Let ΔT be sufficiently small and the importance of the first component be, for example, larger than that of the second one. However, when ΔT is not small, the result of comparison can be different. This can be illustrated by the following values of parameters: $\mathrm{T}_1 = 1,\ \mathrm{T}_2 = 1.5,\ \Delta\mathrm{T} = 2,\ \tau_1 = 0.5\tau_2$.

Example 2. Consider the series system of two repairable components when the whole system is switched-off during repair. In this case the components are dependent (via the repair actions) and availability of the system cannot be written as a product of availabilities of components. However, it can be obtained directly as [40]:

$$A(T_1, T_2, \tau_1, \tau_2) = \frac{1}{1 + \frac{\tau_1}{T_1} + \frac{\tau_2}{T_2}};$$

Note that (2.41) does not hold now, but the direct sensitivity-type measures can be obtained as [19], [18]:

$$\frac{\partial A(T_1,T_2,\tau_1,\tau_2)}{\partial T_1}=\frac{\tau_1}{T_1^2}\times\frac{1}{\left[1+\frac{\tau_1}{T_1}+\frac{\tau_2}{T_2}\right]^2}=\tau_1 T_1^{-2}\left[A\left(T_1,T_2,\tau_1,\tau_2\right)\right]^2,$$

$$\frac{\partial A(T_1,T_2,\tau_1,\tau_2)}{\partial T_2}=\frac{\tau_2}{T_2^2}\times\frac{1}{\left[1+\frac{\tau_1}{T_1}+\frac{\tau_2}{T_2}\right]^2}=\tau_2 T_2^{-2}\left[A\left(T_1,T_2,\tau_1,\tau_2\right)\right]^2,$$

$$-\frac{\partial A(T_1,T_2,\tau_1,\tau_2)}{\partial \tau_1}=\frac{1}{T_1}\times\frac{1}{\left[1+\frac{\tau_1}{T_1}+\frac{\tau_2}{T_2}\right]^2}=T_1^{-1}\left[A\left(T_1,T_2,\tau_1,\tau_2\right)\right]^2,$$

$$-\frac{\partial A(T_1,T_2,\tau_1,\tau_2)}{\partial \tau_2}=\frac{1}{T_2}\times\frac{1}{\left[1+\frac{\tau_1}{T_1}+\frac{\tau_2}{T_2}\right]^2}=T_2^{-1}\left[A\left(T_1,T_2,\tau_1,\tau_2\right)\right]^2.$$

From the above partial derivatives, it can be seen that all four measures have the common multiplier $\left[A\left(T_1,T_2,\tau_1,\tau_2\right)\right]^2$. The availability importance measure based on $T_i, i=1,2$ shows that the availability of the whole system is inversely proportional to the square of T_i. Hence, e.g., doubling (halving) T_i increases (decreases) availability of the whole system four times. The component with the largest value of $T_i (i=1,2)$ has the smallest effect on the availability of the whole system.

On the other hand, the availability importance measure based on τ_i (the mean time of repair) of component i shows that the availability of the whole system is inversely proportional to T_i. The component with the largest value of T_i has the smallest effect on the availability of the whole system.

Remark 1. The direct sensitivity-type measures can also be obtained; e.g., for Example 1 (when the whole system is not switched-off during repair) as [19], [18]:

$$\frac{\partial A(T_1,T_2,\tau_1,\tau_2)}{\partial T_1}=\frac{T_2\tau_1}{(T_1+\tau_1)^2(T_2+\tau_2)}$$

$$\frac{\partial A(T_1,T_2,\tau_1,\tau_2)}{\partial T_2}=\frac{T_1\tau_2}{(T_1+\tau_1)(T_2+\tau_2)^2}$$

$$-\frac{\partial A(T_1,T_2,\tau_1,\tau_2)}{\partial \tau_1}=\frac{T_1T_2}{(T_1+\tau_1)^2(T_2+\tau_2)}$$

$$-\frac{\partial A(T_1,T_2,\tau_1,\tau_2)}{\partial \tau_2}=\frac{T_1T_2}{(T_1+\tau_1)(T_2+\tau_2)^2}$$

Assume that $\tau_1\langle\langle T_1; \tau_2\langle\langle T_2$, which is usually the case for repairable systems. The availability importance measure based on T_i (the mean time to failure) of component i shows that availability of the whole system is approximately τ_i/T_i^2. Similarly, the availability importance measure based on τ_i (the mean time of repair) of component i shows that availability of the whole system is approximately $1/T_i$. Therefore, it can be seen from Example 1 and Example 2 that switching off the whole system during the repair introduces the common multiplier $[A(T_1,T_2,\tau_1,\tau_2)]^2$ to the direct sensitivity-type measures. Therefore, the availability importance measures have the same ordering for both cases (as it should be for the sufficiently "quick" repair).

2.6.4 COST-BASED AVAILABILITY ANALYSIS

Consider availabilities of components of a coherent system, $A_i = T_i /(T_i + \tau_i), i = 1,2,...,n$ taking into account the costs that are needed to increase T_i on ΔT_i and to decrease τ_i on $\Delta\tau_i$. There are various ways to obtain the same increment in availability of the system [41].The question is how to select the best, optimal in some sense way. For this purpose one should consider the costs necessary to achieve the same ΔA. In what follows, we discuss the heuristic approach to this problem. This approach is based on the general relation (2.41).

Define the cost-specific importance of the i^{th} component when availability of this component increases by a fixed amount $\Delta A_i \equiv \Delta A$, which is independent of i, as [39]:

$$I_{c,i} = \frac{\Delta_i \mathrm{h}(\mathbf{A})}{c_i(\Delta A, A_i)} = \frac{\Delta \mathrm{h}(\mathbf{A})}{\Delta A_i} \frac{\Delta A}{c_i(\Delta A, A_i)} \tag{2.46}$$

where $c_i(\Delta A, A_i)$ - is the *minimum* cost necessary to improve A_i to $A_i + \Delta A$. When ΔA is reasonably small, which is usually the case, we can approximate $\frac{\Delta \mathrm{h}(\mathbf{A})}{\Delta A_i}$ by the partial derivative, $\frac{\partial \mathrm{h}(\mathbf{A})}{\partial A_i}$.

First, we must define the cost function $c_i(T_i, \tau_i)$ - the cost of the component with an average recovery time, τ_i and the average time to failure, T_i. Assume that it has the following additive form [19], [18]:

$$c_i(T_i, \tau_i) = F(T_i) + G(\tau_i) \tag{2.47}$$

where $F(x)$ and $G(x)$ – are monotonically increasing and monotonically decreasing functions, of its arguments, respectively. Thus $G(\tau_i)$, for example, shows that the cost is increasing when the time of repair is decreasing.

Using equation (2.47), we first define $c_i(A_i)$ - the minimum cost necessary to achieve the given value of A_i. Due to our assumptions on $F(x)$ and $G(x)$, and taking into account the condition $A_i = T_i/(T_i + \tau_i)$, the minimum always exists. Using this condition we first obtain, for example, the value of τ_i that results in the minimum of the corresponding univariate function of τ_i and then use $T_i = \tau_i \dfrac{A_i}{1 - A_i}$ for the other variable. Suppose that this minimum is achieved with values of T_i^* and τ_i^*. Then $c_i(A_i) = c_i(T_i^*, \tau_i^*)$. In a similar way we define $c_i(A_i + \Delta A)$. Let this minimum cost be realized for values of T_i^{**} and τ_i^{**}, respectively. Then $c_i(A_i + \Delta A) = c_i(T_i^{**}, \tau_i^{**})$ and the minimum cost to improve A_i to $A_i + \Delta A$ is defined as [19], [18]:

$$\Delta c_i(\Delta A) \equiv c_i(T_i^{**}, \tau_i^{**}) - c_i(T_i^*, \tau_i^*).$$

Example 3: Let $c_i(T_i, \tau_i) = a_i T_i + \dfrac{b_i}{\tau_i}$, where a_i and b_i – are constants. Then $\tau_i^* = \sqrt{\dfrac{b_i(1 - A_i)}{a_i A_i}}$; and $T_i^* = \tau_i^* \dfrac{A_i}{1 - A_i}$; $\tau_i^{**} = \sqrt{\dfrac{b_i(1 - A_i - \Delta A)}{a_i(A_i + \Delta A)}}$; and $T_i^{**} = \tau_i^{**} \dfrac{A_i + \Delta A}{1 - A_i - \Delta A}$.

Finally,

$$\Delta c_i(\Delta A) = a_i(T_i^{**} - T_i^*) + b_i\left(\frac{1}{\tau_i^{**}} - \frac{1}{\tau_i^*}\right) \qquad (2.48)$$

Thus, expression (2.48) defines minimal $\Delta c_i(\Delta A)$ when A_i is realized in an optimal in the defined sense way. However, T_i and τ_i often correspond already to a 'given component', and, therefore, are not optimal. Nevertheless availability can be improved in an optimal way. In this case, one should find the minimum of [19], [18]:

$$\Delta c_i(\Delta A) = c_i(T_i + \Delta T_i, \tau_i - \Delta \tau_i) - c_i(T_i, \tau_i);$$

given [19], [18]:

$$\Delta A = \frac{T_i + \Delta T_i}{T_i + \Delta T_i + \tau_i - \Delta \tau_i} - \frac{T_i}{T_i + \tau_i} \tag{2.49}$$

From (2.49), the ΔT_i can be expressed via $\Delta \tau_i$ and, therefore, the problem reduces to finding the minimum of the function of one variable. Thus, the procedure simplifies to finding the values of ΔT_i^* and $\Delta \tau_i^*$. Finally [19], [18],

$$\Delta c_i(\Delta A) = c_i(T_i + \Delta T_i^*, \tau_i - \Delta \tau_i^*) - c_i(T_i, \tau_i).$$

Example 4: Combining examples 1 and 3 and assuming that A_1 and A_2 are realized with the optimal cost functions [19], [18]:

$$c_1(T_1^*, \tau_1^*) = a_1 T_1^* + \frac{b_1}{\tau_1^*} \tag{2.50}$$

and

$$c_2(T_2^*, \tau_2^*) = a_2 T_2^* + \frac{b_2}{\tau_2^*} \tag{2.51}$$

Thus [19], [18]:

$$c_1(A_1) \equiv c_1 = 2\sqrt{a_1 b_1 \frac{A_1}{(1-A_1)}}, c_2(A_2) \equiv c_2 = 2\sqrt{a_2 b_2 \frac{A_2}{(1-A_2)}};$$

$$A_1(c_1) \equiv A_1 = \frac{c_1^2}{4a_1 b_1 + c_1^2}, \ A_2(c_2) \equiv A_2 = \frac{c_2^2}{4a_2 b_2 + c_2^2}.$$

Using (2.46) and after simple algebraic transformations we finally arrive at the following quotient [19], [18]:

$$\frac{I_{c,1}}{I_{c.2}} = \frac{a_1 b_1 c_2 (4a_2 b_2 + c_2^2)}{a_2 b_2 c_1 (4a_1 b_1 + c_1^2)}.$$

Therefore, given the relevant parameters, the cost-specific importance measures for both components can be easily analysed and compared.

2.6.5 CONCLUSIONS

The well-known Birnbaum's reliability measure naturally defines the importance of the i^{th} component in a coherent system of n statistically independent, non-repairable components. It is obtained by partial differentiation of the system reliability with respect to that of component i. However, in contrast to the non-repairable case, availability of a component is defined by its time to failure and time of repair. When availability of a system is low, efforts are needed to improve it. In this study, two methods to improve availability of a repairable system are proposed. As stationary availability depends on two parameters (mean time to failure and mean time to repair) it is then reasonable to define measures of importance with respect to each of them. The availability importance measure based on T_i shows the effect of this parameter on the availability of the whole system. Similarly, the availability importance measure based on τ_i shows the effect of this parameter on the availability of the whole system.

Furthermore, to find the final strategy for the availability improvement process the cost trade-off is crucial. A cost-specific importance of component i when the availability of this component increases by a fixed amount is also proposed. To illustrate the proposed models, we have considered several meaningful examples.

If reliability of a system needs to be improved, then efforts should first be concentrated on improving reliability of the component that has the largest effect on reliability of a system. Similarly, the proposed availability importance measures can be used as a guideline for developing an improvement strategy in reparable systems.

2.7 THE PROBLEM OF OPTIMAL ASSIGNMENT OF INTERCHANGEABLE COMPONENTS IN COHERENT MULTI-STATE SYSTEMS

2.7.1 BACKGROUND

Much literature in reliability theory is dealing with a description of component and system states as binary: functioning or failed. However, many systems are composed of multi-state components with different performance levels and several failure modes. There is a great need in some applications to have a more refined description of these states, for instance, the amount of power generated by an electrical power generating system or the amount of gas (or oil) that can be delivered through an offshore gas (or oil) pipeline network. In this paper an attempt has been made to develop a method to improve system performance utility of a multi-state system by reallocating the components considering their utility importance. To achieve this, the knowledge of the order of the component utilities is sufficient. Their individual values are not required for deciding if the positions of the components need to be changed. The contribution of an individual component to the performance utility of a multi-state system is discussed. The method shows that a meaningful index for measuring the performance of individual components in a multi-state system can be defined in general, without considering the actual values of the utility levels and the distributions of the component-states in the system.

2.7.2 INTRODUCTION

2.7.2.1 ASSUMPTIONS, NOTATION AND NOMENCLATURE

2.7.2.1.1 ASSUMPTIONS

1. The multi-state system constitutes nonrepairable components and the order in which components fail does not matter;
2. The multi-state system is monotone and coherent;
3. Component states are mutually statistically independent;
4. Each component has a zero state and M nonzero states;
5. The multi-state system has a zero state and K nonzero states; and
6. The possible states of each component and of the system are ordered:

$state\ 0 \leq state\ 1 \leq ...... \leq state\ M$ and $state\ 0 \leq state\ 1 \leq ...... \leq state\ K$, respectively.

2.7.2.1.2 NOTATION

K : number of nonzero states of the multi-state system, $1 \leq K < \infty$.

M : number of nonzero states of each component, $1 \leq M < \infty$.

N : number of components available for use, $1 \leq N < \infty$.

m : an integer, $0 \leq m < M$.

k : an integer, $0 \leq k < K$.

n : an integer, $1 \leq n < N$.

j : an integer, $0 \leq j < K$.

i : an integer, $1 \leq i < n$.

a_j : system utility level when the multi-state system is in state ; $0 \le a_0 \le a_1 \le \ldots\ldots \le a_K$.

X_i : state of component i ; $X_i \in \{0,1,\ldots M_i\}$.

$\mathbf{X}$: vector of component states, $(X_1, X_2, \ldots, X_n)$.

$\phi(\mathbf{X})$: state of the multi-state system; $\phi(\mathbf{X}) \in \{0,1,\ldots,K\}$; called the system structure function.

$U = \sum_{j=0}^{K} a_j . \Pr(\phi(\mathbf{X}) = j)$: performance utility-function of a multi-state system.

$p_{i,j} = P\{X_i = j\}$

$\mathbf{I}$: matrix representing the performance utility structure.

$U(\mathbf{I})$: The expected system utility of the structure $\mathbf{I}$.

2.7.2.1.3 NOMENCLATURE

Monotone: a system for which $\phi(\mathbf{0}) = 0$, $\phi(K_1, K_2, \ldots, K_n) = 1$, and $\phi(\mathbf{X})$ is non-decresing in $\mathbf{X}$.

Relevant: component i is relevant if there exist cmponent states $(X_1, \ldots, X_{i-1}, X_{i+1}, \ldots, X_n)$ such that $\phi(X_1, \ldots, X_{i-1}, 0, X_{i+1}, \ldots, X_n) < \phi(X_1, \ldots, X_{i-1}, M_i, X_{i+1}, \ldots, X_n)$.

Coherent: an n component system is said to be coherent if every component is relevant (that is every component has some contribution towards the system performance) and if the system is monotone (that is, the performance of the system improves with the improvement of any component or subset of components).

Series: a system for which $\phi(\mathbf{X}) = \min(X_i)$.

Parallel: a sytem for which $\phi(\mathbf{X}) = \max(X_i)$.

k-out-of-n: G: a system for which: $\phi(\mathbf{X}) = \max\{j : \text{at least } k \text{ components are at or above level } j\}$

2.7.2.2 STATE PERFORMANCE UTILITY

Consider a coherent multistate system of n independent components C_i, $1 \le i \le n$ with:

1. $X_i \in \{0,1,...,M\}$: *state (or performance level) of component* i ;
2. $\mathbf{X} = (X_1,...X_n)$: *vector of component states* ; and
3. $\phi(\mathbf{X}) = y \in \{0,\ldots,K\}$: *System state,where* $\varphi(\cdot)$ *is the structure function.*

Each component i is assumed to have ($M_i + 1$) distinct levels of working efficiency called states. If $X_i(t)$ denotes the state for the i^{th} component at time t, then $X_i(t) \in [0,1,...,M_i]$ for $i = 1,2,...,n$. When $X_i(t) = 0$, the i^{th} component is in the state of total failure at time t. On the other hand, when $X_i(t) = M_i$, the i^{th} component is in the state of perfect functioning at time t. Note that for notational simplicity, without loss of generality, the time variable t is suppressed in all notations. Thus, for the static model that considers the working efficiency of the system and its components at a fixed time t, X_i is used instead of $X_i(t)$ to denote the status of working efficiency for the i^{th} component at time t. All components are assumed to be mutually independent. This means that the working efficiency of each component is not affected by the working efficiency of any other component.

The system itself is assumed to have $K+1$ different levels of working efficiency to be called utility. We denote by a_j the utility of a system in the state j, that is, when $\phi(\mathbf{X}, t) = j$, where $\mathbf{X} = (X_1, X_2,..., X_n)$ is a vector of the states of the components and $j \in [0,1,...,K]$. Hence, the system is in a state of total failure at time t when $\phi(\mathbf{X}, t) = 0$, and it is working perfectly at time t when $\phi(\mathbf{X}, t) = K$. Let $j = [0,1,...,M_1] \times [0,1,...,M_2] \times ... \times [0,1,...,M_n]$ be the components' state space, i.e. the set of all possible states of the components, and $j = [0,1,...,K]$ be the set of all possible states of the system. Then the relationship between the components and the system can be obviously expressed by the structure function as the mapping $\phi(\mathbf{X},t): M \to K$. Similarly, for the static model, $\phi(\mathbf{X})$ is used instead of $\phi(\mathbf{X},t)$ to denote the status of the working efficiency for the system at a fixed point in time.

Let us consider an n-component system with the structure function $\phi(\mathbf{X})$ and the expected system utility (or working efficiency) function U, defined as [42]:

$$U = \sum_{j=0}^{K} a_j . \Pr(\phi(\mathbf{X}) = j) \tag{2.52}$$

where a_j is the utility when the system is in state j. Obviously, equation (2.52) generalises system reliability in a binary case, when $a_0 = 0$ and $a_1 = 1$.

Equation (2.52) can be used for defining the relevant measure of importance for an individual component [43]:

$$\begin{aligned} I_m^{UI}(i) &= \sum_{j=0}^{K} a_j . \Pr\{\phi(\mathbf{X}) = j, \mathbf{X}_i = m\} \\ &= \sum_{j=0}^{K} a_j . \Pr\{\phi(\mathbf{X}) = j \mid \mathbf{X}_i = m\} . \Pr\{\mathbf{X}_i = m\} \\ &= \sum_{j=0}^{K} a_j . \Pr\{\phi(\mathbf{X}) = j \mid \mathbf{X}_i = m\} . \mathbf{p}_{i,m} \\ &= \mathbf{p}_{i,m} \sum_{j=0}^{K} a_j . \Pr\{\phi(m_i, \mathbf{X}) = j\} \end{aligned} \tag{2.53}$$

where $\phi(m_i, \mathbf{X})$ denotes the structure function of our system when the i^{th} component is in the state m and $\mathbf{p}_{i,m} \equiv \Pr\{\mathbf{X}_i = m\}$. $I_m^{UI}(i)$ can be interpreted as the contribution of state $0 \le m \le M_i$ of component i to the system. Therefore, the utility importance of component i can be defined as the vector [20]:

$$\mathbf{I}^{UI}(i) = \left(I_0^{UI}(i), I_1^{UI}(i), \ldots, I_M^{UI}(i)\right) \tag{2.54}$$

A significance of $I_m^{UI}(i)$ is that for each $i = 1,\ldots,n$, the U of a system (which is a measure of the overall effect of all components in the system) can be expressed in terms of $I_m^{UI}(i)$ additively [20]:

$$\begin{aligned} U &= \sum_{j=0}^{K} a_j . \Pr\{\phi(\mathbf{X}) = j\} \\ &= \sum_{j=0}^{K} a_j . \left[\sum_{m=0}^{M} \Pr\{\phi(\mathbf{X}) = j \mid \mathbf{X}_i = m\} . \Pr\{\mathbf{X}_i = m\}\right] \\ &= \sum_{m=0}^{M} \sum_{j=0}^{K} a_j . \Pr\{\phi(m_i, \mathbf{X}) = j\} . \mathbf{p}_{i,m} \\ &= \sum_{m=0}^{M} I_m^{UI}(i) \end{aligned} \tag{2.55}$$

Note: Equation (2.55) can be written for each component i.

Equation (2.55) shows that a state m of component i with larger $I_m^{UI}(i)$ contributes appreciably more to the system performance utility.

2.7.2.3 'RELIABILITY' (I.E. UTILITY) IMPORTANCE IN MSS

A basic concept of component importance is due to Birnbaum [24]. Component importance measures are used to quantify the contribution of individual elements of a system to its performance (for example, reliability / utility, availability, risk, and throughput). Component importance measures have been introduced with reference to binary systems made up of binary elements (that is, elements which can be in two states: functioning or faulty). This binary (functioning / faulty) dichotomy of the elements and system states has proven successful for many practical applications. Nonetheless, it is recognised that there are systems whose overall performance can achieve different levels (for example, 100 %, 70 %, 40 %, and 10 %). The performance levels depend on the operative conditions of the constitutive multi-state elements. Examples of such systems are those used in power generation and oil and gas transportation. Systems characterised by different performance levels are referred to as multi-state systems (MSS). An extension of the Birnbaum importance measure to the case of MSS for the case of finitely many states has been done by Griffith [44]. That for the case of continuous state systems can be found in [45].

An important quantitative measure of the importance of an individual component which is used in this paper is the utility component importance ($I_m^{UI}(i)$) as defined in equations (2.53) to (2.55). If a position in a multi-state system, for example, k, can be occupied by either component i_q or component i_v we use this to define how a multi-state component can be regarded as better than another one. In this context the utility importance of one component, given that another particular component is not functioning, will also play an important role, and therefore the conditional utility of a component is defined as follows:

The conditional utility importance of component i_q, given that the component i_v is not functioning is denoted by $CUI\left(i_q \mid I_m^{UI}(i_v)=0\right)$, and is given by [20]:

$$CUI\left(i_q \mid I_m^{UI}(i_v)=0\right)=\frac{\partial U(\mathrm{I})}{\partial p_{i_q,m}} \mid I_m^{UI}(i_v)=0 .$$

Similarly, by differentiating $U(\mathrm{I})$ with respect to $I_m^{UI}(i_v)$ and putting $I_m^{UI}(i_q)=0$ in the derivative, we have $CUI\left(i_v \mid I_m^{UI}(i_q)=0\right)$, the conditional utility importance of component i_v, when component i_q is not functioning, as follows [20]:

$$CUI\left(i_v \mid I_m^{UI}\left(i_q\right)=0\right)=\frac{\partial U(\mathbf{I})}{\partial p_{i_v,m}} \mid I_m^{UI}\left(i_q\right)=0 .$$

Note that when the utility importance (UI) or the conditional utility importance of a component is referred to, the importance of the component is considered in relation to a specified system design, that is, for a given structure function $\phi(\mathbf{X})$.

2.7.3 MAIN THEOREM[1]

Let us consider an n–component system for which the structure function is $\phi(\mathbf{X})$ and the expected utility is $U(\mathbf{I})$. We denote the expected utility by U^{qv} when component i_q is placed in the $i_q{}^{th}$ position and component i_v is placed in the $i_v{}^{th}$ position as follows [20]:

$$U^{qv}=\left\{\begin{array}{l} I_0^{UI}\left(i_1\right)+I_1^{UI}\left(i_1\right)+\ldots+I_M^{UI}\left(i_1\right)+ \\ I_0^{UI}\left(i_2\right)+I_1^{UI}\left(i_2\right)+\ldots+I_M^{UI}\left(i_2\right)+ \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots+ \\ I_0^{UI}\left(i_q\right)+I_1^{UI}\left(i_q\right)+\ldots+I_M^{UI}\left(i_q\right)+ \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots+ \\ I_0^{UI}\left(i_v\right)+I_1^{UI}\left(i_v\right)+\ldots+I_M^{UI}\left(i_v\right)+ \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots+ \\ I_0^{UI}\left(i_n\right)+I_1^{UI}\left(i_n\right)+\ldots+I_M^{UI}\left(i_n\right) \end{array}\right\} \tag{2.56}$$

[1] Note: The theorem is a generalization of [83]'s binary theorem to the multi-state case and for the binary case it reduces to their result.

Let us denote equation (2.56) by U^{qv}, when component i_q is placed in the $i_q{}^{th}$ position and component i_v is place in the $i_v{}^{th}$ position, and let [20]:

$$U^{vq} = \left\{ \begin{array}{l} I_0^{UI}(i_1) + I_1^{UI}(i_1) + \ldots + I_M^{UI}(i_1) + \\ I_0^{UI}(i_2) + I_1^{UI}(i_2) + \ldots + I_M^{UI}(i_2) + \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots + \\ I_0^{UI}(i_v) + I_1^{UI}(i_v) + \ldots + I_M^{UI}(i_v) + \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ I_0^{UI}(i_q) + I_1^{UI}(i_q) + \ldots + I_M^{UI}(i_q) + \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots + \\ I_0^{UI}(i_n) + I_1^{UI}(i_n) + \ldots + I_M^{UI}(i_n) \end{array} \right\} \tag{2.57}$$

be the expected utility function obtained by switching the positions of the $i_q{}^{th}$ and $i_v{}^{th}$ components.

Let $\left(I_m^{UI}(i_q), \mathbf{I}^{i_q}\right)$ be the vector whose $i_q{}^{th}$ component utility importance is $I_m^{UI}(i_q)$ and $\mathbf{I}^{i_q}$ be an $(n-1)$–component vector of component utility importances, obtained by deleting the component utility importance $I_m^{UI}(i_q)$, of the $i_q{}^{th}$ component. Note that here, by $i_q{}^{th}$ component the component initially at the $i_q{}^{th}$ position is indicated, and by $I_m^{UI}(i_q)$ the utility importance of the $i_q{}^{th}$ component is indicated, whatever its position may be, after switching. Thus [20]:

$$U\left(M_{i_q}, \mathbf{I}^{i_q}\right) = I_m^{UI}(i_1) + I_m^{UI}(i_2) + \ldots + M + \ldots + I_m^{UI}(i_n)$$

where $I_m^{UI}(i_q) = M$, that is, the component at the $i_q{}^{th}$ position is perfectly reliable.

$$U\left(0_{i_q}, \mathbf{l}^{i_q}\right) = I_m^{UI}(i_1) + I_m^{UI}(i_2) + \ldots + 0 + \ldots + I_m^{UI}(i_n)$$

where $I_m^{UI}(i_q) = 0$, that is, the component at the $i_q^{\ th}$ position is completely unreliable.

Here $\left(I_m^{UI}(i_q), I_m^{UI}(i_v), \mathbf{l}^{qv}\right)$ is considered to be a vector whose $i_q^{\ th}$ and $i_v^{\ th}$ component utility elements are $I_m^{UI}(i_q)$ and $I_m^{UI}(i_v)$, respectively, and $\mathbf{l}^{qv}$ represents an $(n-2)$-component utility importances vector, obtained by deleting the utility importance, $I_m^{UI}(i_q)$, of the $i_q^{\ th}$ component and $I_m^{UI}(i_v)$ of the $i_v^{\ th}$ component from the vector of component utility importances. Similarly by $\mathbf{l}^{vq}$, an $(n-2)$-component vector of component utility importances is denoted, and obtained by deleting the utility importance, $I_m^{UI}(i_q)$, of the $i_q^{\ th}$ component and $I_m^{UI}(i_v)$ of the $i_v^{\ th}$ component from the vector of component utility importances, when the positions of the $i_q^{\ th}$ and $i_v^{\ th}$ components are interchanged.

In the function $U\left(I_m^{UI}(i_q), I_m^{UI}(i_v), \mathbf{l}^{qv}\right)$, if we mark the first argument as the utility importance of the component at the $i_q^{\ th}$ position and the second one as that of the component at the $i_v^{\ th}$ position, then by $U\left(I_m^{UI}(i_q), I_m^{UI}(i_v), \mathbf{l}^{qv}\right)$ the expected system utility function is indicated when the $i_q^{\ th}$ component is at the $i_q^{\ th}$ position and the $i_v^{\ th}$ component is at the $i_v^{\ th}$ position, but $U\left(I_m^{UI}(i_q), I_m^{UI}(i_v), \mathbf{l}^{vq}\right)$ denotes the expected system utility function when the $i_q^{\ th}$ component is at the $i_v^{\ th}$ position and the $i_v^{\ th}$ component at the $i_q^{\ th}$ position. Hence [20]:

$$U\left(M_{i_q}, M_{i_v}, \mathbf{l}^{qv}\right) = I_m^{UI}(i_1) + I_m^{UI}(i_2) + \ldots + M + \ldots + M + \ldots + I_m^{UI}(i_n), \text{ where } I_m^{UI}(i_q) = I_m^{UI}(i_v) = M$$

$$U\left(M_{i_q}, 0_{i_v}, \mathbf{I}^{qv}\right) = I_m^{UI}(i_1) + I_m^{UI}(i_2) + \ldots + M + \ldots + 0 + \ldots + I_m^{UI}(i_n)\text{, where } I_m^{UI}(i_q) = M, I_m^{UI}(i_v) = 0$$

$$U\left(0_{i_q}, M_{i_v}, \mathbf{I}^{qv}\right) = I_m^{UI}(i_1) + I_m^{UI}(i_2) + \ldots + 0 + \ldots + M + \ldots + I_m^{UI}(i_n)\text{, where } I_m^{UI}(i_q) = 0, I_m^{UI}(i_v) = M$$

$$U\left(0_{i_q}, 0_{i_v}, \mathbf{I}^{qv}\right) = I_m^{UI}(i_1) + I_m^{UI}(i_2) + \ldots + 0 + \ldots + 0 + \ldots + I_m^{UI}(i_n)\text{, where } I_m^{UI}(i_q) = I_m^{UI}(i_v) = 0$$

Note that although $\mathbf{I}^{qv}$ and $\mathbf{I}^{vq}$ consist of the same elements, as they were obtained by deleting the utility importances of the $i_q{}^{th}$ and $i_v{}^{th}$ components from $U(\mathbf{I})$, different notation is used to indicate the difference between the positions of the remaining two components, the $i_q{}^{th}$ and $i_v{}^{th}$, because the expected system utility may vary for these two different arrangements. Here it may further be noted that [20]:

1. $U\left(M_{i_v}, M_{i_q}, \mathbf{I}^{vq}\right)$ is the expected system utility function when the $i_q{}^{th}$ component is in the $i_v{}^{th}$ position and the $i_v{}^{th}$ component is in the $i_q{}^{th}$ position, with $I_m^{UI}(i_q) = P($ $i_q{}^{th}$ component is perfectly functioning) = M , $I_m^{UI}(i_v) = P($ $i_v{}^{th}$ component is perfectly functioning) = M ;

2. $U\left(M_{i_v}, 0_{i_q}, \mathbf{I}^{vq}\right)$ is the expected system utility function when the $i_q{}^{th}$ component is in the $i_v{}^{th}$ position and the $i_v{}^{th}$ component is in the $i_q{}^{th}$ position, with $I_m^{UI}(i_q) = P($ $i_q{}^{th}$ component has failed) = 0 , $I_m^{UI}(i_v) = P($ $i_v{}^{th}$ component is perfectly functioning) = M ;

3. $U\left(0_{i_v}, M_{i_q}, \mathbf{I}^{vq}\right)$ is the expected system utility function when the $i_q{}^{th}$ component is in the $i_v{}^{th}$ position and the $i_v{}^{th}$ component is in the $i_q{}^{th}$ position, with $I_m^{UI}(i_q) = P($ $i_q{}^{th}$ component is perfectly functioning) = M , $I_m^{UI}(i_v) = P($ $i_v{}^{th}$ component has failed) = 0 ;

4. $U\left(0_{i_v}, 0_{i_q}, \mathbf{l}^{vq}\right)$ is the expected system utility function when the $i_q^{\ th}$ component is in the $i_v^{\ th}$ position and the $i_v^{\ th}$ component is in the $i_q^{\ th}$ position, with $I_m^{UI}\left(i_q\right) = P\,(i_q^{\ th}$ component has failed) = M , $I_m^{UI}\left(i_v\right) = P\,(i_v^{\ th}$ component has failed) = 0 .

The expected system utility function $U(\mathbf{l})$ possess the following properties [20]:

(U.1) $U\left(M_{i_q}, M_{i_v}, \mathbf{l}^{qv}\right) = U\left(M_{i_v}, M_{i_q}, \mathbf{l}^{vq}\right)$

(U.2) $U\left(0_{i_q}, 0_{i_v}, \mathbf{l}^{qv}\right) = U\left(0_{i_v}, 0_{i_q}, \mathbf{l}^{vq}\right)$

(U.3) $U\left(M_{i_q}, 0_{i_v}, \mathbf{l}^{qv}\right) = U\left(M_{i_v}, 0_{i_q}, \mathbf{l}^{vq}\right)$

(U.4) $U\left(0_{i_q}, M_{i_v}, \mathbf{l}^{qv}\right) = U\left(0_{i_v}, M_{i_q}, \mathbf{l}^{vq}\right)$

Now the following theorem is proved:

2.7.3.1 THEOREM 1

For a pair of components i_q and i_v in a given system, if

1. $p_{i_q,m} > p_{i_v,m}$ and $CUI\left(i_q \mid I_m^{UI}\left(i_v\right) = 0\right) < CUI\left(i_v \mid I_m^{UI}\left(i_q\right) = 0\right)$ alternatively;
2. $p_{i_q,m} < p_{i_v,m}$ and $CUI\left(i_q \mid I_m^{UI}\left(i_v\right) = 0\right) > CUI\left(i_v \mid I_m^{UI}\left(i_q\right) = 0\right)$

then by interchanging the positions of the two components the system utility can be improved. If the components have the same utility importance and / or their conditional utility importance are the same, then switching neither improves nor reduces the system utility.

2.7.3.2 PROOF

By pivotal decomposition of $U(\mathbf{l})$ with respect to $I_m^{UI}(i_q)$ the following equation is obtained [20]:

$$U(\mathbf{l}) = p_{i_q,m} \cdot U\left(M_{i_q}, \mathbf{l}^{i_q}\right) + \left(M_{i_q} - p_{i_q,m}\right) \cdot U\left(0_{i_q}, \mathbf{l}^{i_q}\right) \tag{2.58}$$

Further decomposition of $U\left(M_{i_q}, \mathbf{l}^{i_q}\right)$ and $U\left(0_{i_q}, \mathbf{l}^{i_q}\right)$ with respect to $I_m^{UI}(i_v)$ gives [20]:

$$\begin{aligned}
U(\mathbf{l}) &= p_{i_q,m} \cdot \left\{p_{i_v,m} \cdot U\left(M_{i_q}, M_{i_v}, \mathbf{l}^{qv}\right) + \left(M_{i_v} - p_{i_v,m}\right) \cdot U\left(M_{i_q}, 0_{i_v}, \mathbf{l}^{qv}\right)\right\} \\
&\quad + \left(M_{i_q} - p_{i_q,m}\right) \cdot \left\{p_{i_v,m} \cdot U\left(0_{i_q}, M_{i_v}, \mathbf{l}^{qv}\right) + \left(M_{i_v} - p_{i_v,m}\right) \cdot U\left(0_{i_q}, 0_{i_v}, \mathbf{l}^{qv}\right)\right\} \\
&= p_{i_q,m} \cdot p_{i_v,m} \cdot U\left(M_{i_q}, M_{i_v}, \mathbf{l}^{qv}\right) + p_{i_q,m} \cdot \left(M_{i_v} - p_{i_v,m}\right) \cdot U\left(M_{i_q}, 0_{i_v}, \mathbf{l}^{qv}\right) \\
&\quad + \left(M_{i_q} - p_{i_q,m}\right) \cdot p_{i_v,m} \cdot U\left(0_{i_q}, M_{i_v}, \mathbf{l}^{qv}\right) \\
&\quad + \left(M_{i_q} - p_{i_q,m}\right) \cdot \left(M_{i_v} - p_{i_v,m}\right) \cdot U\left(0_{i_q}, 0_{i_v}, \mathbf{l}^{qv}\right) \\
&= U^{qv}, \text{say}
\end{aligned} \tag{2.59}$$

If the positions of the i_q^{th} and i_v^{th} components are interchanged, the expression of the utility function will be [20]:

$$\begin{aligned}
U^{vq} &= p_{i_q,m} \cdot p_{i_v,m} \cdot U\left(M_{i_v}, M_{i_q}, \mathbf{l}^{vq}\right) + p_{i_v,m} \cdot \left(M_{i_q} - p_{i_q,m}\right) \cdot U\left(M_{i_v}, 0_{i_q}, \mathbf{l}^{vq}\right) \\
&\quad + \left(M_{i_v} - p_{i_v,m}\right) \cdot p_{i_q,m} \cdot U\left(0_{i_v}, M_{i_q}, \mathbf{l}^{vq}\right) \\
&\quad + \left(M_{i_v} - p_{i_v,m}\right) \cdot \left(M_{i_q} - p_{i_q,m}\right) \cdot U\left(0_{i_v}, 0_{i_q}, \mathbf{l}^{vq}\right)
\end{aligned} \tag{2.60}$$

Thus from equations (2.59) and (2.60) [20]:

$$U^{qv}-U^{vq}=\begin{Bmatrix} I_0^{UI}(i_1)+I_1^{UI}(i_1)+\ldots+I_M^{UI}(i_1)+ \\ I_0^{UI}(i_2)+I_1^{UI}(i_2)+\ldots+I_M^{UI}(i_2)+ \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots+ \\ I_0^{UI}(i_q)+I_1^{UI}(i_q)+\ldots+I_M^{UI}(i_q)+ \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots+ \\ I_0^{UI}(i_v)+I_1^{UI}(i_v)+\ldots+I_M^{UI}(i_v)+ \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots+ \\ I_0^{UI}(i_n)+I_1^{UI}(i_n)+\ldots+I_M^{UI}(i_n) \end{Bmatrix}-\begin{Bmatrix} I_0^{UI}(i_1)+I_1^{UI}(i_1)+\ldots+I_M^{UI}(i_1)+ \\ I_0^{UI}(i_2)+I_1^{UI}(i_2)+\ldots+I_M^{UI}(i_2)+ \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots+ \\ I_0^{UI}(i_v)+I_1^{UI}(i_v)+\ldots+I_M^{UI}(i_v)+ \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots+ \\ I_0^{UI}(i_q)+I_1^{UI}(i_q)+\ldots+I_M^{UI}(i_q)+ \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots+ \\ I_0^{UI}(i_n)+I_1^{UI}(i_n)+\ldots+I_M^{UI}(i_n) \end{Bmatrix}$$

$$\begin{aligned}
&= p_{i_q,m}\cdot p_{i_v,m}\cdot U\left(M_{i_q},M_{i_v},\mathbf{l}^{qv}\right)+p_{i_q,m}\cdot\left(M_{i_v}-p_{i_v,m}\right)\cdot U\left(M_{i_q},0_{i_v},\mathbf{l}^{qv}\right)\\
&+\left(M_{i_q}-p_{i_q,m}\right)\cdot p_{i_v,m}\cdot U\left(0_{i_q},M_{i_v},\mathbf{l}^{qv}\right)\\
&+\left(M_{i_q}-p_{i_q,m}\right)\cdot\left(M_{i_v}-p_{i_v,m}\right)\cdot U\left(0_{i_q},0_{i_v},\mathbf{l}^{qv}\right)\\
&-p_{i_v,m}\cdot p_{i_q,m}\cdot U\left(M_{i_v},M_{i_q},\mathbf{l}^{vq}\right)-p_{i_v,m}\cdot\left(M_{i_q}-p_{i_q,m}\right)\cdot U\left(M_{i_v},0_{i_q},\mathbf{l}^{vq}\right)\\
&-\left(M_{i_v}-p_{i_v,m}\right)\cdot p_{i_q,m}\cdot U\left(0_{i_v},M_{i_q},\mathbf{l}^{vq}\right)\\
&-\left(M_{i_v}-p_{i_v,m}\right)\cdot\left(M_{i_q}-p_{i_q,m}\right)\cdot U\left(0_{i_v},0_{i_q},\mathbf{l}^{vq}\right)
\end{aligned}$$

by pivotal decomposition

$$\begin{aligned}
U^{qv}-U^{vq} &= p_{i_q,m}\cdot\left(M_{i_v}-p_{i_v,m}\right)\left[U\left(M_{i_q},0_{i_v},\mathbf{l}^{qv}\right)-U\left(0_{i_v},M_{i_q},\mathbf{l}^{vq}\right)\right]\\
&+p_{i_v,m}\cdot\left(M_{i_q}-p_{i_q,m}\right)\cdot\left[U\left(0_{i_q},M_{i_v},\mathbf{l}^{qv}\right)-U\left(M_{i_v},0_{i_q},\mathbf{l}^{vq}\right)\right]\\
&=U\left(M_{i_q},0_{i_v},\mathbf{l}^{qv}\right)\cdot\left[p_{i_q,m}\cdot\left(M_{i_v}-p_{i_v,m}\right)-p_{i_v,m}\cdot\left(M_{i_q}-p_{i_q,m}\right)\right]\\
&-U\left(0_{i_q},M_{i_v},\mathbf{l}^{qv}\right)\cdot\left[p_{i_q,m}\cdot\left(M_{i_v}-p_{i_v,m}\right)-p_{i_v,m}\cdot\left(M_{i_q}-p_{i_q,m}\right)\right]
\end{aligned}$$

since

$$U\left(M_{i_q},0_{i_v},\mathbf{l}^{qv}\right)=U\left(M_{i_v},0_{i_q},\mathbf{l}^{vq}\right)$$

and

$$U\left(0_{i_q},M_{i_v},\mathbf{l}^{qv}\right)=U\left(0_{i_v},M_{i_q},\mathbf{l}^{vq}\right)$$

by properties (U.3) and (U.4)

$$U^{qv} - U^{vq} = \left(p_{i_q,m} - p_{i_v,m}\right)\cdot\left[U\left(M_{i_q}, 0_{i_v}, \mathbf{l}^{qv}\right) - U\left(0_{i_q}, M_{i_v}, \mathbf{l}^{qv}\right)\right] \tag{2.61}$$

The utility importance of the ${i_q}^{th}$ component is given by [20]:

$$I_m^{UI}\left(i_q\right) = U\left(M_{i_q}, \mathbf{l}^{i_q}\right) - U\left(0_{i_q}, \mathbf{l}^{i_q}\right)$$

which can be obtained by differentiating the expected utility function $U(\mathbf{l})$ with respect to $\partial p_{i_q,m}$, that is

$$I_m^{UI}\left(i_q\right) = \frac{\partial U(\mathbf{l})}{\partial p_{i_q,m}}$$

since differentiating the expression of $U(\mathbf{l})$, as given in equation (2.58), with respect to $\partial p_{i_q,m}$, we get

$$\begin{aligned} UI\left(i_q\right) = \frac{\partial U(\mathbf{l})}{\partial p_{i_q,m}} &= \left\{p_{i_q,m}\cdot U\left(M_{i_q}, \mathbf{l}^{i_q}\right) + \left(M_{i_q} - p_{i_q,m}\right)\cdot U\left(0_{i_q}, \mathbf{l}^{i_q}\right)\right\} \\ &= U\left(M_{i_q}, \mathbf{l}^{i_q}\right) - U\left(0_{i_q}, \mathbf{l}^{i_q}\right) \end{aligned}$$

Similarly, by differentiating $U(\mathbf{l})$, as given in equation (2.59), with respect to $\partial p_{i_q,m}$, and then putting $I_m^{UI}\left(i_v\right) = 0$ in the expression, $CUI\left(i_q \mid I_m^{UI}\left(i_v\right) = 0\right)$, the conditional utility importance of component i_q, given that the ${i_v}^{th}$ component is not functioning can be obtained as follows [20]:

$$\begin{aligned} \frac{\partial U(\mathbf{l})}{\partial p_{i_q,m}} = {} & p_{i_v,m}\cdot U\left(M_{i_q}, M_{i_v}, \mathbf{l}^{qv}\right) + \left(M_{i_v} - p_{i_v,m}\right)\cdot U\left(M_{i_q}, 0_{i_v}, \mathbf{l}^{qv}\right) \\ & - p_{i_v,m}\cdot U\left(0_{i_q}, M_{i_v}, \mathbf{l}^{qv}\right) - \left(M_{i_v} - p_{i_v,m}\right)\cdot U\left(0_{i_q}, 0_{i_v}, \mathbf{l}^{qv}\right) \end{aligned}$$

and hence [20]:

$$\begin{aligned} CUI\left(i_q \mid I_m^{UI}(i_v)=0\right) &= \frac{\partial U(\mathbf{I})}{\partial p_{i_q,m}} \mid I_m^{UI}(i_v)=0 \\ &= U\left(M_{i_q}, 0_{i_v}, \mathbf{I}^{qv}\right) - U\left(0_{i_q}, 0_{i_v}, \mathbf{I}^{qv}\right) \end{aligned} \tag{2.62}$$

In a similar manner, the conditional utility importance of component i_v, when $i_q^{\,th}$ component is not functioning, is as follows [20]:

$$\begin{aligned} CUI\left(i_v \mid I_m^{UI}(i_q)=0\right) &= \frac{\partial U(\mathbf{I})}{\partial p_{i_v,m}} \mid I_m^{UI}(i_q)=0 \\ &= U\left(0_{i_q}, M_{i_v}, \mathbf{I}^{qv}\right) - U\left(0_{i_q}, 0_{i_v}, \mathbf{I}^{qv}\right) \end{aligned} \tag{2.63}$$

Hence using equations (2.62) and (2.63), equation (2.61) can be written as [20]:

$$\begin{aligned} U^{qv} - U^{vq} &= \left(p_{i_q,m} - p_{i_v,m}\right)\cdot\left[U\left(M_{i_q}, 0_{i_v}, \mathbf{I}^{qv}\right) - U\left(0_{i_q}, 0_{i_v}, \mathbf{I}^{qv}\right) - \left\{U\left(0_{i_q}, M_{i_v}, \mathbf{I}^{qv}\right) - U\left(0_{i_q}, 0_{i_v}, \mathbf{I}^{qv}\right)\right\}\right] \\ &= \left(p_{i_q,m} - p_{i_v,m}\right)\cdot\left\{CUI\left(i_q \mid I_m^{UI}(i_v)=0\right) - CUI\left(i_v \mid I_m^{UI}(i_q)=0\right)\right\} \end{aligned} \tag{2.64}$$

Thus from equation (2.64)

$$U^{qv} > U^{vq}$$

if and only if

1. $p_{i_q,m} > p_{i_v,m}$ and $CUI\left(i_q \mid I_m^{UI}(i_v)=0\right) < CUI\left(i_v \mid I_m^{UI}(i_q)=0\right)$ alternatively;
2. $p_{i_q,m} < p_{i_v,m}$ and $CUI\left(i_q \mid I_m^{UI}(i_v)=0\right) > CUI\left(i_v \mid I_m^{UI}(i_q)=0\right)$

$U^{vq} > U^{qv}$, otherwise, that is switching between two components improves the expected system utility if the more reliable component has a lower CUI (given that the other component is not functioning).

If $p_{i_q,m} = p_{i_v,m}$ and / or $CUI\left(i_q \mid I_m^{UI}(i_v)=0\right)=CUI\left(i_v \mid I_m^{UI}(i_q)=0\right)$, then from equation (2.64) it can be concluded that switching is immaterial, i.e. switching positions of the $i_q{}^{th}$ and $i_v{}^{th}$ components neither improves nor reduces the expected system utility. Hence the theorem is proved.

2.7.4 CONCLUSION AND DISCUSSION

The theorem provides a technique to compare between two positions of components in a complex system. The technique gives an indication of which component and the position where a more reliable component should be placed to improve the expected system utility.

The theorem shows that if the more reliable component has a lower conditional utility importance under the condition that the other component is not functioning, then by interchanging their positions the expected system utility can be improved. In this way, by comparing a component's utility importance and conditional utility importance of different components, pair wise, and switching components successively, if necessary (according to the theorem), the expected system utility can be improved. Consequently the optimum allocation of the components can be achieved.

CHAPTER 3: CANNIBALIZATION REVISITED

3.1 INTRODUCTION

Ordinarily systems of equipment are provided with spare parts for components that can fail. Under this policy systems of equipment remain functioning whilst failed components are being replaced and / or repaired. Nonetheless there are two exceptions to this policy. The two exceptions to this policy are [46], [47]:

1. In high technology (hi-tech) manufacturing environments or in organisations that utilize fleets of expensive equipment it is not always possible to stock spare parts due to the high acquisition and holding costs of spare parts inventories; and
2. In the case where the equipment has reached its final life-cycle phase. In this phase the failure rates increase, spare parts become more and more difficult to acquire, and the use of the equipment declines. In suchlike a case there is a tendency not to buy many spare parts since the equipment will soon be phased out.

Cannibalization is one possible way of maintaining a system whose spare parts are not available or have been depleted. Cannibalization enable fleet maintenance (hi-tech manufacturing) managers to satisfy fleet (production) performance constraints such as the need for short maintenance turnarounds or readiness requirements. “Cannibalization” usually refers to the removal of a failed component from a system and its replacement by an operating component of the same type extracted from another part of the system (see Figure 5 for the illustration of the concept of cannibalization). Cannibalization has been practiced, especially in the military, for a long period of time [48].

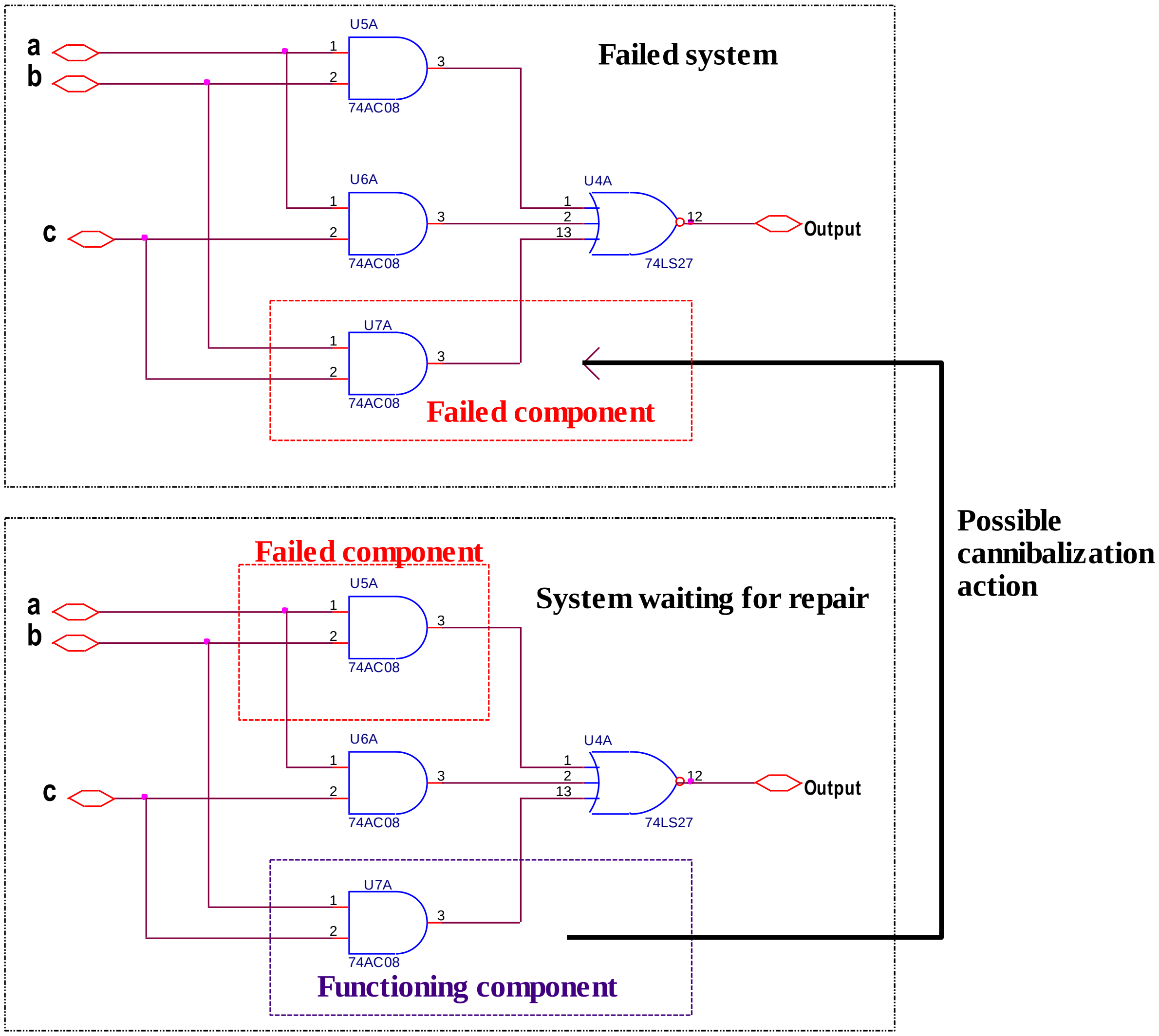

Figure 5: The cannibalization concept

The rest of this chapter is organised as follows. Section 3.2 reviews literature on cannibalization. Section 3.3 examines the undesirable effects of cannibalization. Section 3.4 studies the reasons for cannibalization in industry. Section 5 expounds a model for decision-support that predicts optimal cannibalization rates that are needed to attain short maintenance turnarounds or readiness requirements. In section 3.6 another model which can be used to make cannibalization decisions is developed. In the model of section 3.6 we consider a situation where repair facilities or spare components are not immediately available so that the probability of survival of a system can only be enhanced by extracting needed replacement components from another part of the system. We develop a model (reliability) of cannibalization for the probability of survival (at time t) of a system with k lines in parallel of n series connected components when no short interruptions to the system are allowed and when short interruptions to the system are allowed.

3.2 LITERATURE REVIEW

According to [49] there are scarcely any articles in recent literature focusing on cannibalization for military and industrial applications. Zhang and Ghanmi [49] attribute the scarcity of articles on cannibalization focusing on military and industrial applications to the complexity involved in maintenance systems. In this section literature on cannibalization is divided into two categories namely [50]: 1) academic studies using models and 2) governmental studies. The work on both categories is summarised in the proceeding sections.

3.2.1 ACADEMIC STUDIES USING MODELS

Modelling to analyse the effects of cannibalization can be categorised into three methodologies [47], [50], [51]: reliability and stochastic analysis, inventory-based models, and simulation (queuing) analysis. Fisher [52] comprehensively presents a survey and assessment of research on the use of modelling to analyse the effects of cannibalization. Earlier research on the effects of cannibalization analysed the reliability of systems under total instantaneous cannibalization. Furthermore, these models of the effects of cannibalization derive general expressions for calculating system reliability in particular without necessarily giving numerical examples. In addition these models primarily focus on formulations without comparing different cannibalization policies. The model by Fisher [53] is an exception. One of the first studies of cannibalization is articulated in [48]. In [48] a model, in the form of a mathematical expression for the expected value of the system status as a function of the number of working parts of each type, is developed. In [54] the method proposed in [48] is extended by allowing restrictions on the interchangeability of parts. In [55] the methods in [48] and [54] are extended by allowing k states of part functioning as opposed to a binary representation. In [53] a model for the process of repairing or cannibalizing a part using continuous-time Markov chains is presented. These authors considered systems modelled by multi-state structure functions (that is, non-decreasing mappings from a finite lattice to a finite set) [56]. These cannibalizations models on reliability and stochastic analysis limited their discourses to structure functions satisfying what [48] call the "minimum condition". Essentially, "minimum condition" states that the performance of the system is solely determined by that of a single type of component. Thus, the system could be viewed as a variant of a generalised k-out-of-n structure [56]. Baxter develops a theory of cannibalization for continuum structure functions (CSFs) (that is, non-decreasing mappings in each argument where the non-negative orthant is a closed product).

The second category of cannibalization models seek to optimize spare part inventories and address the possibility of allowing cannibalization actions to compensate for a lack of spare parts. Sherbrooke's [57] Multi-Echelon Technique for Recoverable Item Control (METRIC) model uses the Bayesian probability theory to estimate spare parts inventory levels in a two-echelon (base and depot stock levels) system. In this model spare part levels are assigned by optimising the minimum number of expected backorders for all bases. The model by Sherbrooke [57] does not model cannibalization explicitly. Nonetheless this model provides a basis for future studies. Subsequent to METRIC Sherbrooke developed NORS [58]. NORS is a single-echelon model that take into account cannibalization whilst estimating the expected number of aircraft that are not operationally ready (due to supply) at a random point in time. Muckstadt [59] developed MOD-METRIC, an extension of METRIC. MOD-METRIC allows a multiple-indentured (hierarchical) parts structure except cannibalization. The model by Fisher [60] is an extension of the models by Sherbrooke [58] and Muckstadt [59]. Fisher's model [60] evaluates the performance of a two-echelon, two-indenture system with and without cannibalization.

The last category of cannibalization models [61], [62], [63] use discrete-event simulation to assess the effects of cannibalization and recommend the commensurate cannibalization policies. Analytical results are not the main objective of these simulation models. Therefore, these models can render more practical results for decision makers by relaxing some of the more restrictive assumptions (for example, total instantaneous cannibalization) of the previous studies on cannibalization. In addition these simulation models are open-network models (that is, a component of the failed system enter the model as a new entity and a repaired component permanently departs the model). Also the performance of an individual system (for example, total production capacity of a manufacturing plant) cannot be estimated under these modelling structures. Hence, an unfulfilled opportunity exists in terms of extending this line of research to develop closed-network simulation models. Furthermore the open-network models do not account for transportation costs and assume fixed lead-times for spare parts.

The research of [47] and [51] develop closed-network, discrete-event simulation models of a fleet of identical series systems and of a manufacturing system comprised of a set of parallel machines, respectively. The main objectives of the model of a fleet of identical series systems are to: 1) include cannibalization, spare parts inventories and the relevant costs associated with maintaining the fleet; 2) demonstrate the use of the model as a decision-support tool (that is, use of cannibalization and spare parts inventories as means of meeting fleet readiness and maintenance cost requirements); 3) investigate the general impact of cannibalization as compared to making minimal investments in spare parts inventories while considering several queuing disciplines associated with fleet maintenance and cannibalization (that is, using a set of numerical experiments to support the conclusions); and 4) propose and evaluate a simple model of maintenance-induced damage. The main goals of the model of a manufacturing system comprised of a set of parallel machines are to: 1) capture policies, including cannibalization, and costs associated with maintaining these machines; and 2) explore the use of this model as a decision-support tool (that is, evaluate the use of cannibalization, spare parts inventories, and expedited repair shipments based on the operational and economic performance of the system).

3.2.2 GOVERNMENTAL REPORTS

Governmental reports on cannibalization are divided into two categories as follows: 1) Reports by the Centre for Naval Analyses (CNA) Corporation and 2) Reports by the United States General Accounting Office (U.S. G.A.O.). The main work on both categories is summarised below.

The CNA reports [64], [65], [66] summarise the aircraft readiness performance metrics used in the United States Navy. These metrics are based on consolidating all maintenance operations together. These reports recommend that different maintenance policies warrant different performance measures in order to be effective. The maintenance measure of interest in this research which is found in these consolidated reports is cannibalization. The United States Navy normally describes cannibalization activity as the number of cannibalizations per 100 flight hours, termed the "cannibalization rate" and denoted as CANN [66, p. 7], [67], [68, p. 7], [69]. On the other hand the United States Air Force defines "cannibalization rate" as the number of cannibalizations per 100 aircraft sorties (that is, one take-off and one landing comprise one sortie) [66, p. 8], [68, p. 10]. Furthermore the United States Army has three definitions for cannibalization type activities. Nonetheless the United States Army has not yet defined a "cannibalization rate" measurement [66], [68], [70]. According to [50] cannibalization is not formally documented in the United States Navy until after the cannibalization action is complete. The CNA reports propose that cannibalization activities be divided into the following three groups: 1) trouble shooting; 2) directed by higher authority; and 3) lack of spare parts. Each of these cannibalization activities impacts differently on both aircraft readiness and the spare parts supply chain. Additionally the CNA reports recommend that cannibalization activities be formally documented when they are initiated rather than completed. Furthermore, the CNA reports recommend that performance metrics be focused more on the spare part supply chain (especially the fill rates) instead of only aircraft availability.

The reports by the U.S. G.A.O [69], [68], [71] outline initial results of a rigorous study on the United States military services' practice of cannibalization. Two main conclusions from the U.S. G.A.O. reports are: 1) that current cannibalization rates are highly underestimated (that is, the services are not recording all cannibalizations, the specific reasons for them, or how much time or money they spend on them), and 2) that a small group of aircraft accounts for the majority of cannibalization actions. According to these reports cannibalization actions take at least twice as much time to carry out as repair actions. Therefore, maintenance personnel are prone to work overtime. In addition these reports suggest that the more an aircraft is cannibalized the more likely to fail are the components within close proximity to the cannibalized component (s). The U.S. G.A.O. reports allude to the fact that increased workload associated with cannibalization greatly reduces maintenance personnel's morale. These reports identify the following as the main reasons for the extensive use of cannibalization in the military: 1) low levels of spare parts inventories; 2) unpredictable depot-to-base lead times; and 3) component reliabilities that are less than the design values. Despite its demerits the reports cite cannibalization activities as the main stay for maintaining readiness rates at acceptable levels in the U.S. military. The U.S. G.A.O. reports further identify the following as additional reasons for cannibalization: 1) lack of experience; 2) insufficient training of maintenance personnel; 3) outdated maintenance manuals; and 4) lack of testing equipment.

3.3 UNDESIRABLE EFFECTS OF CANNIBALIZATION

Cannibalization is without its own controversies as the process of extracting a functioning component from a complex system can sometimes be imprudent. It may be: 1) too costly in labour; 2) too costly in time; 3) too risky in the sense that the component maybe damaged during removal and render the whole system unusable; and 4) too risky in the sense that mechanical side effects may occur during cannibalization. The demerits of cannibalization (that is, according to [68], [72]) are summarised in the proceeding sub-sections.

3.3.1 INCREASED MAINTENANCE WORK LOADS

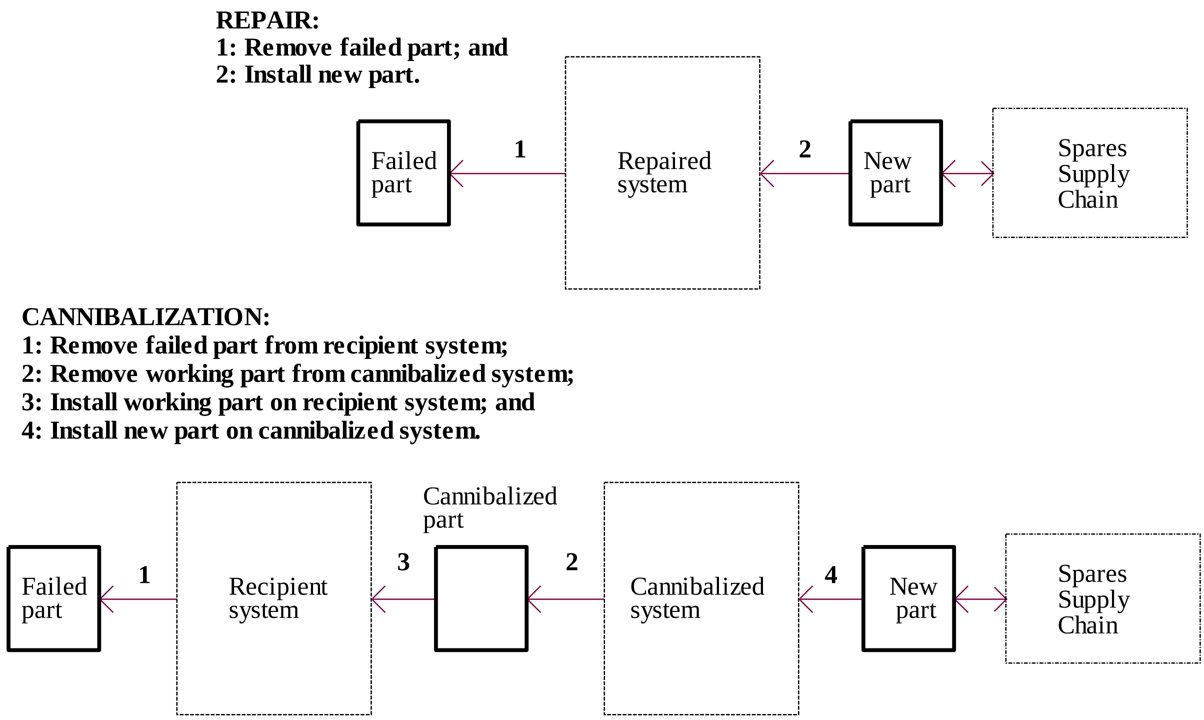

Figure 6: Repairs require two steps - Cannibalization requires four

Cannibalizations increase the workload of maintenance personnel as they take at least twice as long as normal repairs. As illustrated in Figure 6 normal repairs require two steps as follows: 1) remove failed part; and 2) install new part. On the other hand cannibalizations require four steps (see Figure 6) as follows: 1) remove failed part from recipient system; 2) remove working part from cannibalized system; 3) Install working part on recipient system; and 4) install new part on cannibalized system. Therefore, a direct cost of cannibalizations is the additional personnel hours required to remove and re-install a part. Besides, these additional hours personnel must also check or repair other parts removed (disturbed) to gain access to the cannibalized part. For example, a study by the U.S. G.A.O. documented 850 000 cannibalization actions performed by on the U.S. Air Force and U.S. Navy aircraft over a five-year period that consumed 5.5 million maintenance man-hours [73].

3.3.2 POTENTIAL LOW MORALE OF MAINTENANCE PERSONNEL

According to [68] evidence suggests that cannibalizations have a negative effect on maintenance personnel morale. This is so as cannibalizations are sometimes seen as routinely making unrealistic demands on the maintenance personnel. In order to meet operational commitments in the military, cannibalizations may have to be performed at any time, day or night, and very quickly. In suchlike cases, maintenance personnel must continue working until the job is accomplished. Needless to say without additional remuneration. Cannibalizations increase the maintenance personnel hours required for particular repairs, hence increasing the overall workload and have a corresponding negative effect on the morale of the maintenance personnel.

3.3.3 POTENTIAL UNUSABILITY OF CANNIBALIZED EXPENSIVE SYSTEMS

Systems that have parts missing due to cannibalization actions may remain inoperable for long periods of time, thus denying the commercial use of valuable and expensive assets [74]. For example the U.S. Air Force and Navy guidance states that, to the maximum extent possible, cannibalized aircraft should not remain grounded for more than 30 consecutive days [68]. Nonetheless it has been observed that in numerous instances this has not been the case [68]. For example, one wing had 6 aircrafts which had not flown for 37 days or more and one of these aircrafts had not flown for more than 300 days [68].

3.3.4 POTENTIAL MECHANICAL DAMAGE TO CANNIBALIZED SYSTEMS

Maintenance personnel often have to remove other components in order to gain access to remove a particular component. This process increases the risk of collateral damage to the system and the other components within the proximity of the component of interest. For example, the removal of a cockpit gauge of an aircraft also requires the removal of its wiring harness [68]. This cannibalization action, if repeated several times, induces excessive tear and wear on the wiring. Ultimately the wiring will have to be replaced as well. In addition, cannibalizations do not replace a failed part with a new one, but with a used one. Thus cannibalizations do not restore a component to its full estimated life span but rather increase the probability that the component will break down prematurely.

3.4 REASONS FOR CANNIBALIZATION IN THE INDUSTRY

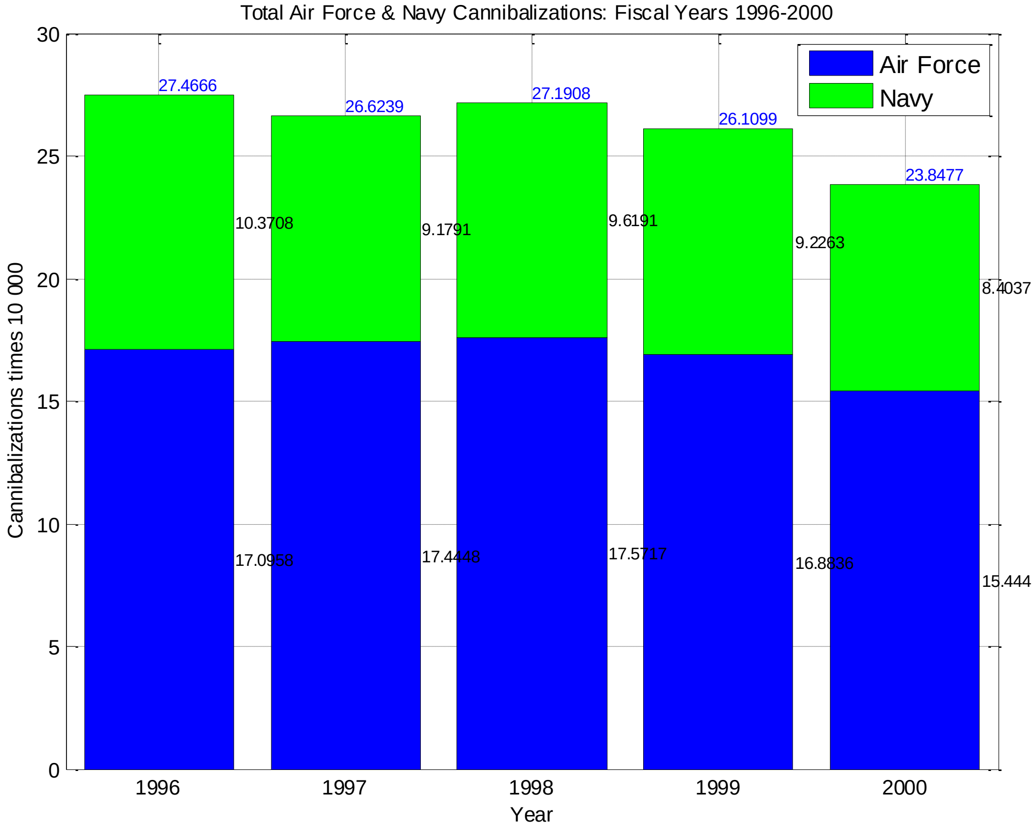

Figure 7: Total Air Force & Navy Cannibalizations: Fiscal Years 1996-2000

Cannibalizations are often used in many high technology manufacturing environments and fleet maintenance due to the high acquisition and holding costs of spare parts inventories and the need to for reduced maintenance turnarounds [47], [68]. This means that the services are operating with chronic spare part shortages, taxing operational and readiness requirements, and aging systems. Therefore, cannibalizations actions are favoured by maintenance personnel in practice to mitigate these reasons. A study by the U.S. G.A.O [68] shows that in the fiscal years 1996 to 2000, the U.S. Air Force and Navy units reported a total of about 850 000 cannibalization actions on aircraft (see Figure 7). These cannibalizations included 376 000 cannibalizations by the Air Force and 468 000 by the Navy (see Figure 7). The "cannibalization rate" (that is, the number of cannibalizations per 100 sorties) for the Air Force aircraft (for example, the B1-Bombers, which each cost US$200 million in 1987 [74]) could be up to 85.4 (see Figure 8). On the other hand the "cannibalization rate" (that is, the number of cannibalizations per 100 flight hours) for the Navy aircraft (for example, the F-14D) could be as high as 32.8 (see Figure 9).

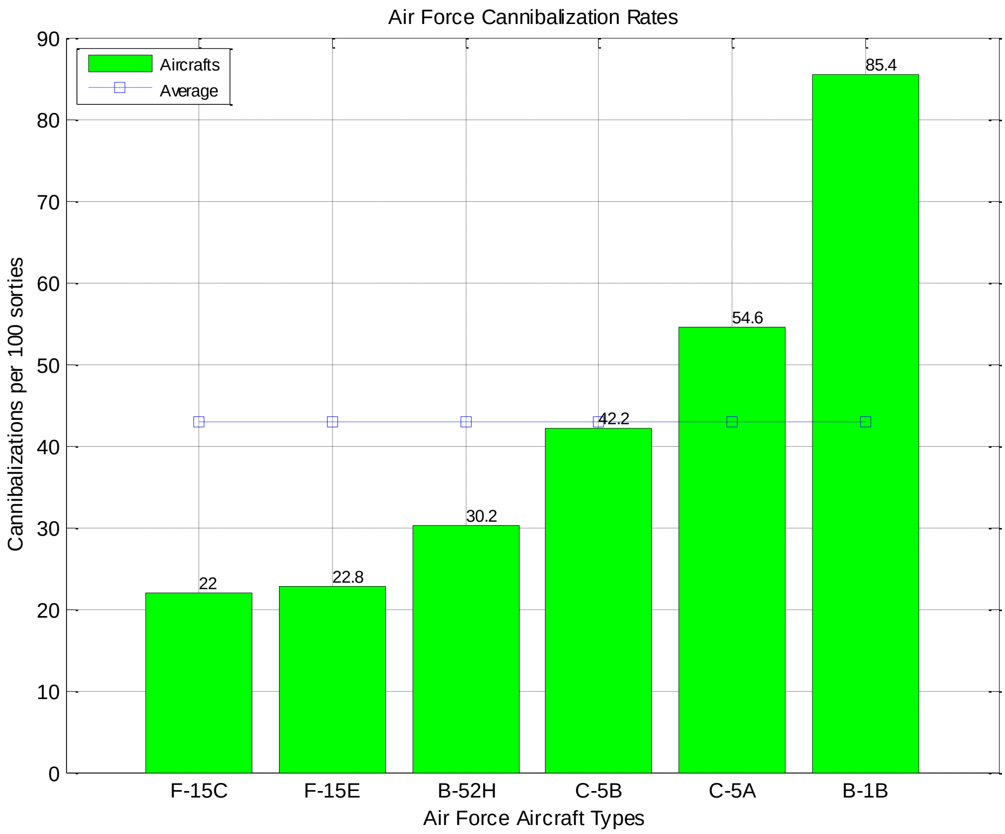

Figure 8: Air Force Cannibalization Rates

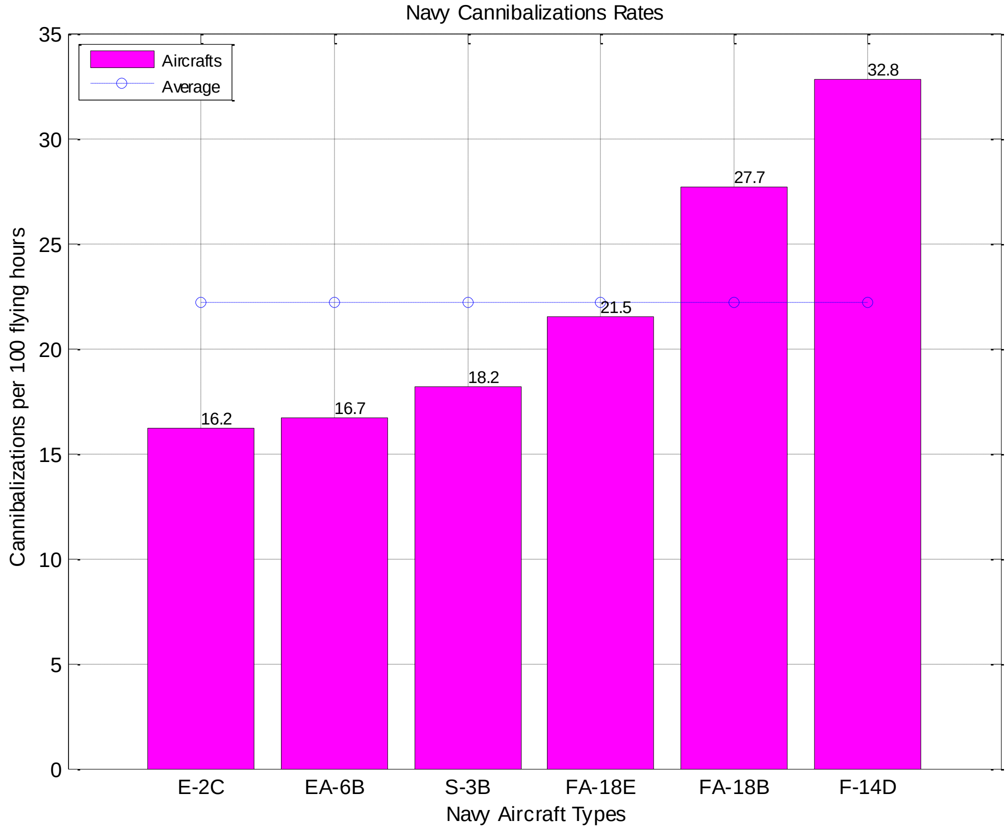

Figure 9: Navy Cannibalization Rates

3.4.1 SPARE PARTS SUPPLY SYSTEM CHALLENGES

Spare parts shortages is the mainstay for cannibalizations [68], [74]. Cannibalizations actions are done if spare parts are not available at the right place at the right time. Some of the reasons advanced for spare parts shortages are: 1) tighter budgets; 2) lack of operational track records (especially for new systems) to determine how fast parts wear out and how many spares should be ordered; 3) new technology has made systems to undergo many changes over a short period of time, making stocking spare parts more difficult; 4) unexpected spare parts high demand; 5) production and / or repair delays; 6) frequent lack of reliability by contractors who fail to deliver spare parts on time or who themselves have been hampered by labour strikes or inadequate quality controls; 7) maintenance personnel use cannibalizations as diagnostic tools either because they lack proper training to conduct diagnosis more efficiently or because they do not have proper testing equipment; and 8) higher than expected component failure rates.

3.4.2 READINESS AND OPERATIONAL DEMANDS

Readiness and operational demands exert pressure on the supply chain to provide spare parts at once at the right place at the right time. Maintenance personnel will be left with no alternative but to cannibalize when it is faster to do so than wait for a spare part to be delivered from across town and / or abroad [68]. This they do in a quest to keep readiness ratings high, although it is a waste of energy, time and money [74].

3.4.3 AGING SYSTEMS

As the equipment reaches its final life-cycle phase several changes take place [46], [68]: 1) failure rates increase; 2) It takes longer to inspect and maintain; 3) spare parts become more difficult to obtain; and 4) the equipment becomes less available for operations. In suchlike cases there is a tendency not to purchase many spare parts as the equipment will soon become obsolete. At the same breath, as the age of the equipment increases, the spare parts consumption of this aging equipment increases. Therefore, cannibalization becomes necessary.

3.5 STRATEGIES TO CONDUCT INFORMED CANNIBALIZATIONS

It can be deduced from the preceding sections that cannibalizations, if carried out imprudently, can be a serious problem with many negative effects. Cannibalization actions often bear negative connotations because of the following: 1) cannibalizations indicate that there are problems with the spare part supply chain; 2) there is a potential of mechanically damaging systems during cannibalizations; and 3) cannibalizations increase the workload of maintenance personnel and if practiced frequently will dampen their morale. Nonetheless, it is unlikely that the practice of cannibalizations may be completely eliminated. Therefore, we advocate for the idea of: 1) designated cannibalizations; and 2) methodology informed cannibalizations. These will then cushion against the negative effects of cannibalizations.

The idea of designated cannibalizations means the designation of components in the requirements database as *cannibalizable* (that is, easy to cannibalize) or *non-cannibalizable* (that is, difficult to cannibalize). It can be noted that the *cannibalizable* or *non-cannibalizable* of components is mainly dependent on their type. For example, the cannibalization of a fuse is a trivial task which maintenance personnel can conduct almost invariably rather than wait for long periods of time for the spare part to be delivered. On the other hand, the cannibalization of an aircraft wing spar, for example, is very costly (that is, in terms of energy, time and money), very dangerous, and unfathomed in the world of aircraft maintenance. Cannibalization of components designated as *cannibalizable* will provide serviceable components when the spares stock is depleted. Cannibalizations of components designated as *non-cannibalizable* will not be permitted as it may be too costly in labour or time or money, or too risky in that the component (system) may incur mechanical damage during removal.

The methodology informed cannibalizations is a way to predict optimum cannibalization rates required to achieve specified readiness and operational demand goals, given the system: 1) Mean Up Time (MUT) (also known as MTTF); 2) MTTR; 3) Mean Supply Response Time (MSRT); 4) Mean Maintenance and Supply Time (MMST); and 5) MDT. This methodology addresses designated cannibalizations only deemed necessary in the industry for reasons as advocated in section 3.4. The theoretical model is presented in section 3.5.1. This model is an extension of the model developed in [66]. The model developed in [66] assumes that the total downtime can be uniquely classified as the sum of maintenance time and supply time. However, in practice, there are three (3) exclusive categories of downtime: 1) maintenance time; 2) supply time; and 3) overlapping MMST. Hence, we extend the model of [66] to accommodate these different categories of downtime because cannibalizations affect MTTR and MSRT, as well as MMST. In section 3.5.2 we provide cannibalizations policy implications based on the results of section 3.5.1.

3.5.1 THEORETICAL MODEL TO SHOW THE EFFECTS OF CANNIBALIZATION ON MISSION TIME AVAILABILITY OF SYSTEMS

We begin with the following definition of system mission time average availability. The system mission time average availability ($MTAA_{system}$) denotes the mean proportion of mission time the system is functioning. It is assumed that every time a component (system) fails, it is repaired to an "as good as new" condition and the system mission time average availability is:

$$MTAA_{system} = \frac{MUT}{MUT+MDT} \quad (3.1),$$

where *MUT* is the Mean Up Time (also referred to as *MTTF*) and *MDT* is the Mean Down Time. The *MUT* denotes the mean functioning of the system. The *MDT* is decomposed into three mutually exclusive activities: 1) the *MTTR*; 2) the M*SRT* and 3) the *MMST*. If we substitute these three variables into equation (3.1) we get:

$$MTAA_{system} = \frac{MUT}{MUT+MTTR+MSRT+MMST} \quad (3.2).$$

For simplicity we assume that it takes zero time to decide to cannibalize and zero time to accomplish cannibalizations. Therefore, *MSRT* is calculated as follows [66]:

$$MSRT = (1-GE)(1-c)\mu \quad (3.3),$$

where *GE* is the Gross Effectiveness – that is the proportion of required parts that are available in the supply chain, *c* is the is the proportion of parts requests that are not available in the supply chain and are cannibalized and μ is the mean Customer Wait Time (CWT) for spare parts.

As already mentioned in section 3.4 the U.S. Navy describes the cannibalization activity as the number of cannibalizations per 100 flight hours, termed the cannibalization rate and denoted $CANN_{AF}$. In this section we will use this definition. The proportion of all part requests that are cannibalized, *c*, is determined as follows [66]:

$$c = \frac{CANN_{AF}}{100(1-GE)\theta} \tag{3.4}$$,

where, θ, is the component mean failure rate.

If we substitute equation (3.4) into equation (3.3) we get:

$$MSRT = \frac{-\left(\mu\left(CANN_{AF}-(100\theta)+(100GE\theta)\right)\right)}{100\theta} \tag{3.5}$$.

It can be noted that equation (3.5) shows a negative linear relationship between MSRT and $CANN_{AF}$. If we hold θ, μ and GE constant it can be deduced that the higher the cannibalization rate the lower the MSRT becomes.

If we substitute equation (3.5) into equation (3.2) and solve for $CANN_{AF}$ we get:

$$CANN_{AF} = \frac{\left(\theta(100GE-100)\left(\frac{MMST+MTTR+MUT-MUT}{MTAA_{system}-\mu(GE-1)}\right)\right)}{\mu(GE-1)} \tag{3.6}$$.

We impose the following mathematical constraints when calculating $CANN_{AF}$ with equation (3.6):

1. $CANN_{AF} = 0$ if $(1 - GE)\mu \leq \frac{MUT}{MTAA_{system} - MUT - MTTR - MMST}$; and
2. $MTAA_{system} \leq \frac{MUT}{MUT + MTTR + MMST}$. It is impossible to achieve an MTAA$_{system}$ value higher than this. It can only mean that the parameter values set for the model are not logically consistent.

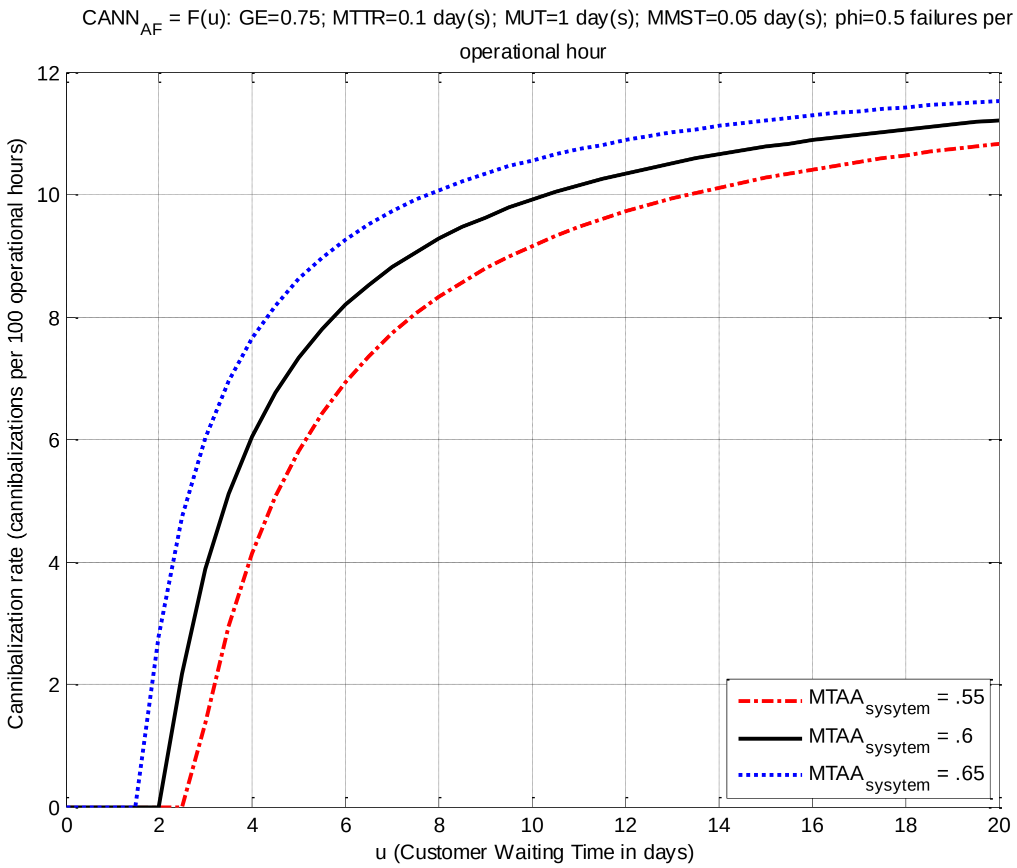

Figure 10: CANN$_{AF}$ versus u for different MTAA$_{system}$ values

Using equation (3.6) we plot CANN$_{AF}$ as a function of μ for different values of MTAA$_{system}$ (with all the other parameters fixed at values given in Figure 10) as shown in Figure 10. It can be deduced from Figure 10 that as μ approaches infinity as $CANN_{AF}$ reaches a maximum of: 12.31 for an $MTAA_{system}$ of 0.65; 12.24 for an $MTAA_{system}$ of 0.60 and 12.17 for an $MTAA_{system}$ of 0.55.

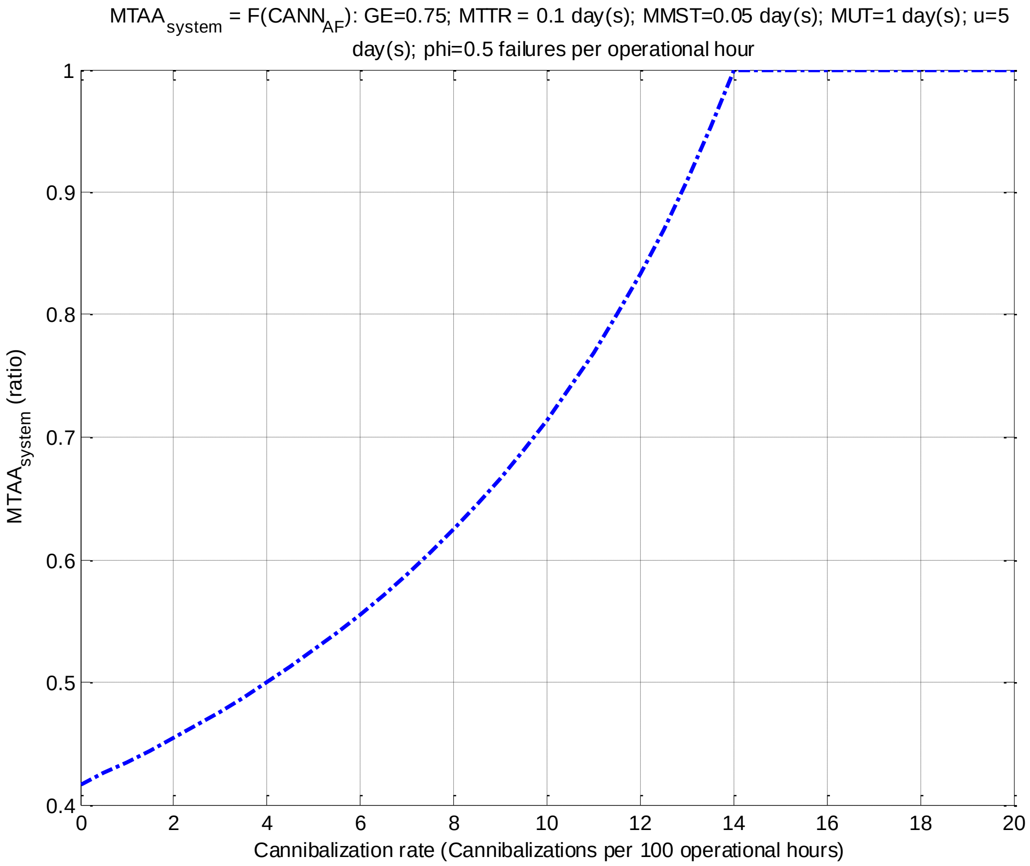

Figure 11: $MTAA_{system}$ versus $CANN_{AF}$

Figure 11 shows that a policy to limit cannibalization activities is required. In the example of Figure 11 (with all the other parameters fixed) it can be seen that a cannibalization rate above 14 does not add any value as the $MTAA_{system}$ is now 1.

3.5.2 POLICY IMPLICATIONS OF THE THEORETICAL MODEL

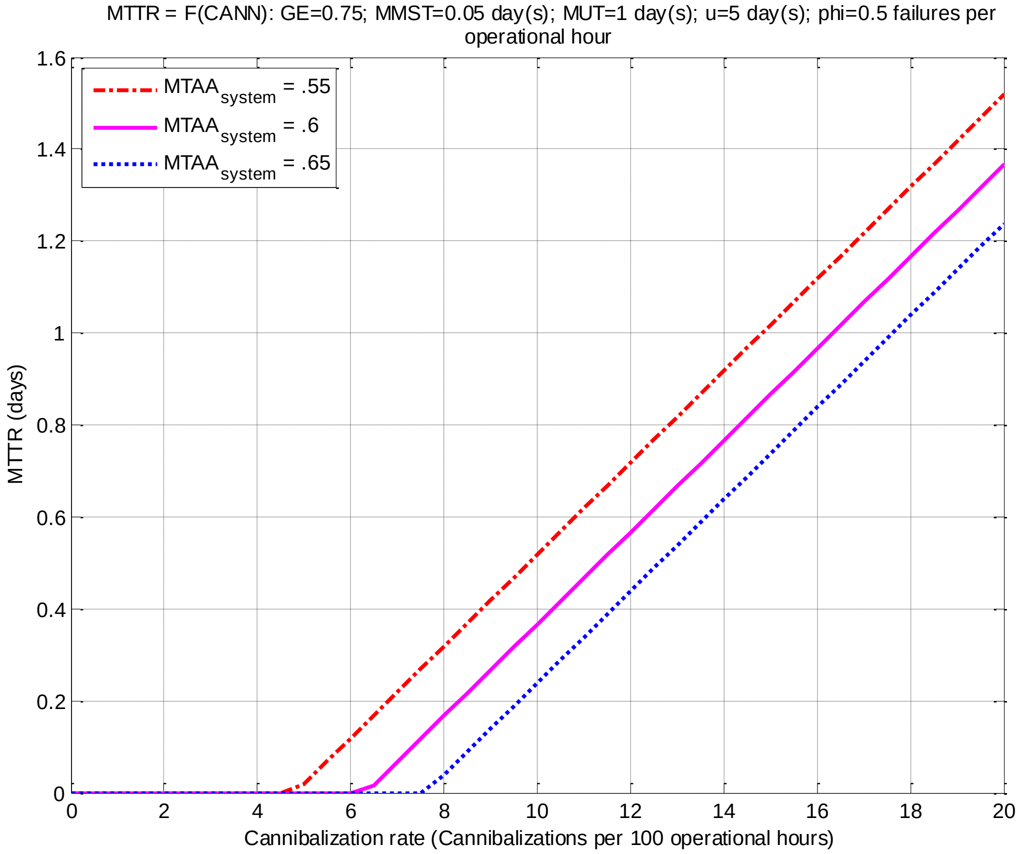

Figure 12: MTTR versus $CANN_{AF}$

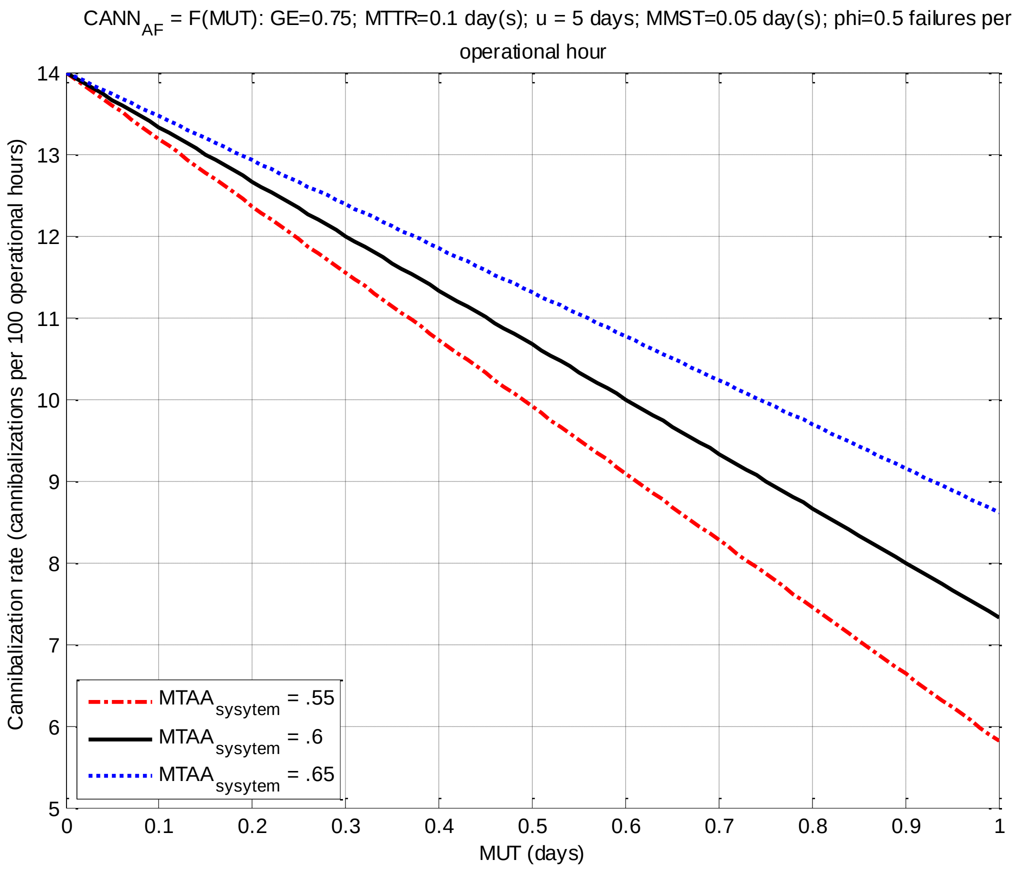

Figure 13: CANN$_{AF}$ versus MUT

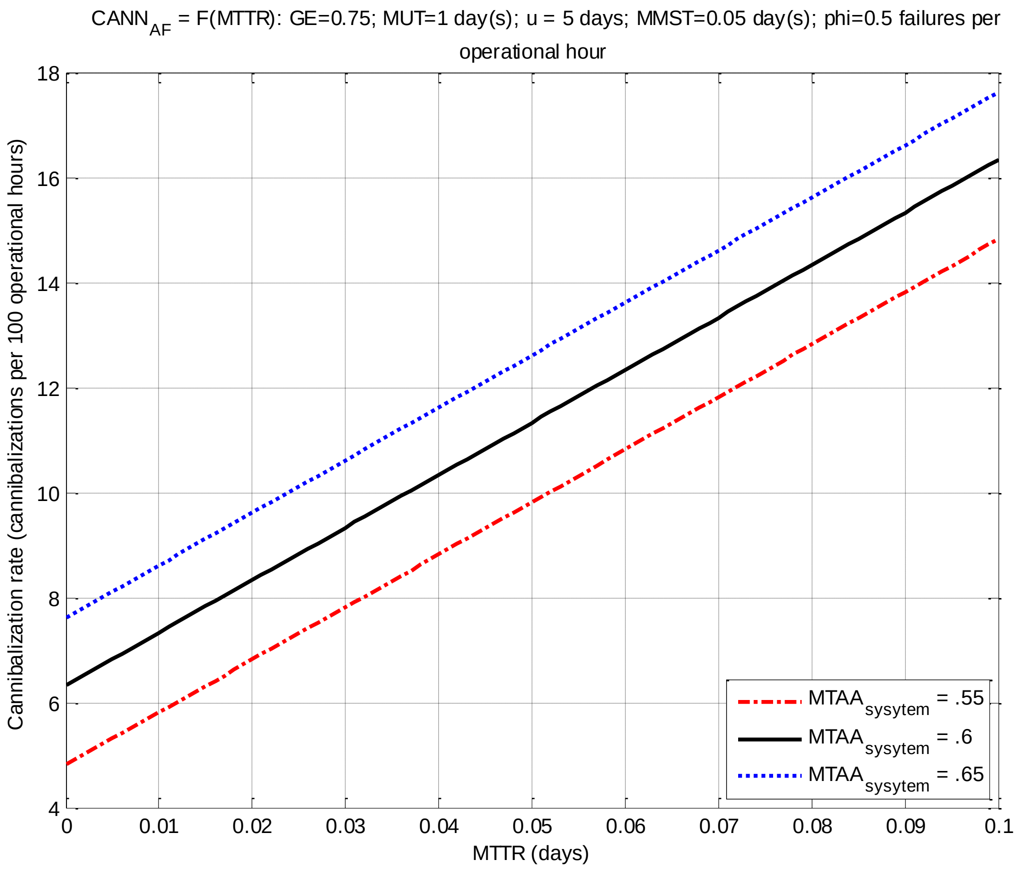

Figure 14: CANN$_{AF}$ versus MTTR

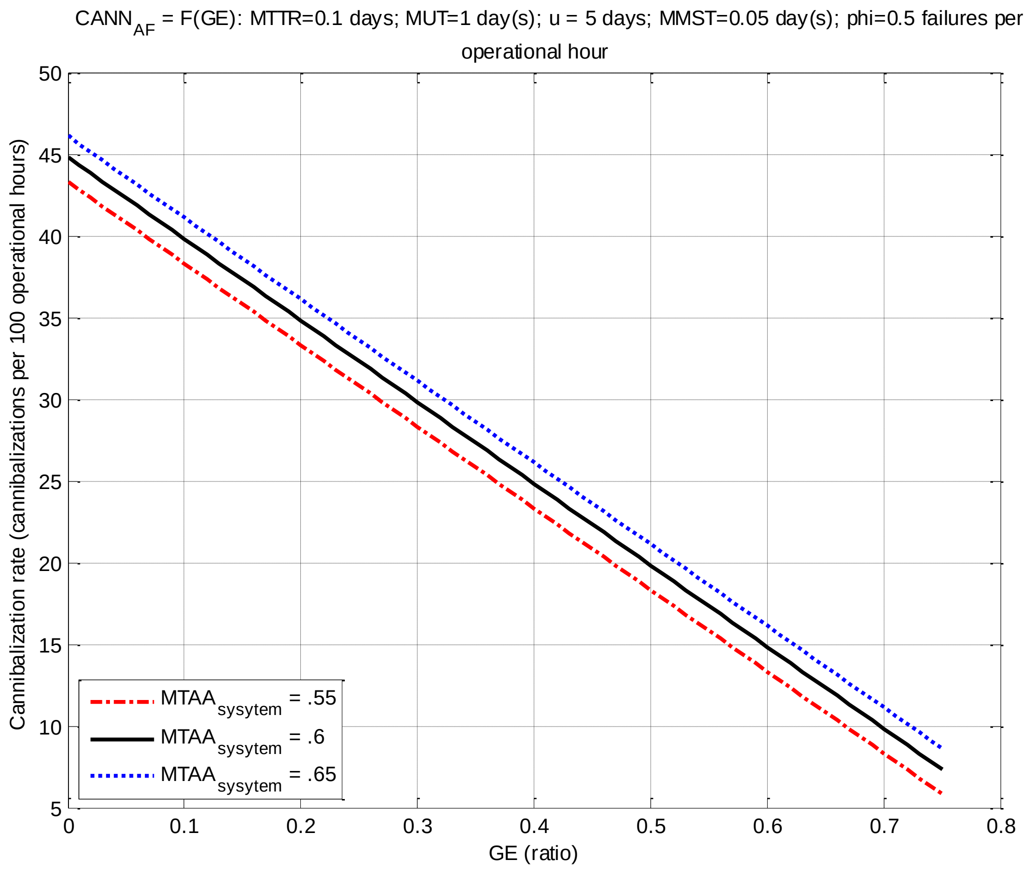

Figure 15: $CANN_{AF}$ versus GE

In this section we state a number of conclusions based on our simulations using equations (3.1) to (3.6). These conclusions have implications on cannibalization policies. The conclusions are as follows:

1. The gradient ($\frac{\partial CANN_{AF}}{\partial \mu}$) of the graph in Figure 10 is positive. This shows that the longer μ is the higher $CANN_{AF}$ is (for different values of $MTAA_{system}$ all other parameters being held constant). Thus, cannibalization activities may be reduced by decreasing μ;
2. The function $MTTR = F(CANN_{AF})$ as shown in Figure 12 is positive. Hence, more cannibalization activities imply longer maintenance time;

3. The gradient ($\frac{\partial CANN_{AF}}{\partial MUT}$) of the graph in Figure 13 is negative. This illustrates that the higher the system reliability or the longer the mean uptime, the lower the cannibalization rate (for different values of $MTAA_{system}$ all other parameters being held constant). Thus, cannibalization activities can be reduced when systems are designed taking cognisance of PDfR;
4. From Figure 14 it can be deduced that the gradient ($\frac{\partial CANN_{AF}}{\partial MTTR}$) of the graph is positive. It implies that the longer the repair time (MTTR), the higher the cannibalization rate (for different values of $MTAA_{system}$ all other parameters being held constant). Therefore, cannibalizations activities may be reduced with a more efficient maintenance operation system (that is, better trained and qualified maintenance personnel);
5. From Figure 15 it can be deduced that the gradient ($\frac{\partial CANN_{AF}}{\partial GE}$) of the graph is negative. It implies that the higher the GE, the lower the cannibalization rate (for different values of $MTAA_{system}$ all other parameters being held constant). Hence, cannibalizations activities can be reduced by increasing the availability of spare parts in the supply chain;
6. It can be seen from the simulation results that cannibalization activities serve a useful purpose in the operation and maintenance of complex and high-performance systems. Cannibalization activities are necessary, viable and cost-effective; only if the optimum cannibalization rate is sought for specific operating parameters; and
7. Lastly, the models presented in this section and section 3.6 can be empirically tested with actual data on cannibalization rates and other parameters related to $MTAA_{system}$.

3.6 CANNIBALIZATION REVISITED: THEORETICAL MODEL AND EXAMPLE

In this section we consider a situation where repair facilities or spare components are not immediately available so that the probability of survival of a system can only be enhanced by extracting needed replacement components from another part of the system. We develop a model of cannibalization for the probability of survival (at time t) of a system with k lines in parallel of n series connected components when short interruptions to the system are allowed and when short interruptions to the system are not allowed. It is assumed for practical reasons that the lines are identical. Let all components be also identical, with exponentially distributed lifetimes with parameter λ. We can generalize the approach to the case of non-identical components and lines but the resulting expressions will be extremely cumbersome. We start with two (2) lines as follows:

i. Assume that when only one line is left, the time for replacing the failed component of this remaining line (if one has a spare, for example, from the failed line) is not allowed. Then there is no possibility of cannibalization in this system and the survival function can be easily obtained.

ii. The time for replacing the failed component is allowed. Then when one line fails, all $n-1$ non-failed components of the failed line can be used as spares for the operable line. The corresponding formulas (survival function) are then derived and this is cannibalization.

Then we consider the case of three (3) lines:

i. No time is allowed for replacing the failed component. However, cannibalization is still done here. Indeed, when one line fails, we can use $n-1$ spares for the system of two (2) lines (that is, cannibalization) and when they will be exhausted, then no cannibalization, as in the case with two (2) lines. The formula for probability of survival (at time t) is then written for the case with cannibalization and without and compared.

ii. Time for replacing the failed component is allowed. Then the two stage cannibalization goes. When the first line fails, $n-1$ non-failed components can be used to maintain the two (2) lines. When this is exhausted and one of the two (2) lines fails - the same process as in the previous case is followed and then the corresponding relationships are obtained.

We generalize the approach to the case when we have more than three lines and obtain the corresponding recurrent equations for survival probabilities in this case that can be solved numerically.

It can be noted that short interruptions to the system give us the possibility to use some components of a system as spares.

3.6.1 NOTATION

X : lifetime of a system

$S_{kn}^{-}(t)$: Probability of survival (at time t) of a system with k lines of n series connected components with cannibalization (when short interruptions of the system are not allowed).

$S_{kn}^{+}(t)$: Probability of survival (at time t) of a system with k lines of n series connected components with cannibalization (when short interruptions of the system are allowed).

$S_{kn}^{nc}(t)$: Probability of survival (at time t) of a system with k lines of n series connected components when no cannibalization at all is allowed.

$q_{kn}^{-+}(t)=\dfrac{1-S_{kn}^{-}(t)}{1-S_{kn}^{+}(t)}$: Improvement factor of unreliability (at time t) for k lines of n series connected components with cannibalization due to allowing short interruptions.

$q_{kn}^{nc}(t)=\dfrac{1-S_{kn}^{nc}(t)}{1-S_{kn}^{-(+)}(t)}$: Improvement factor due to cannibalization of unreliability (at time t) for k lines of n series connected components when initially no cannibalization at all is allowed.

$k=1,2,...$

$n=1,2,...$

Note: The improvement factor of unreliability shows how much the unreliability has decreased due to cannibalization being allowed.

3.6.2 ONE-LINE SYSTEM

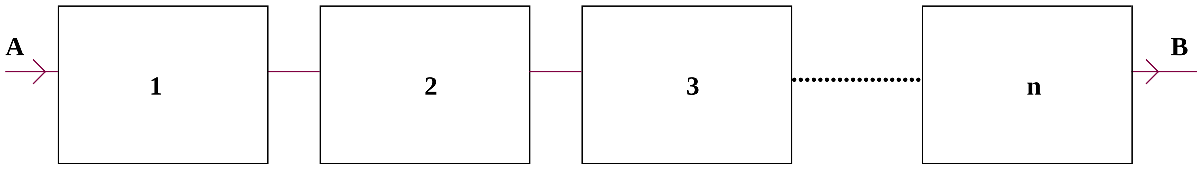

Figure 16: One line of series connected components

This section is concerned with the probability of survival (at time t) evaluation of standard series network occurring in engineering systems. The series network is the basic building block of our work in this paper. In this case n number of components form a series network, as shown in Figure 16. If any one of the components fails, the system fails. All system components must work normally for successful operation of the system.

A typical example of a series system is four wheels of a car. If any one of the tires punctures, the car for practical purposes cannot be driven. Thus, these four tires form a series system. For independent and identical components (each component $i\,(i=1,2,...,n)$ with a lifetime that is exponentially distributed with failure rate λ), the series system, shown in Figure 16, probability of survival is

$$S_{1n}^{nc}(t) = e^{-n\lambda t} \tag{3.7}$$

3.6.3 TWO-LINE SYSTEM

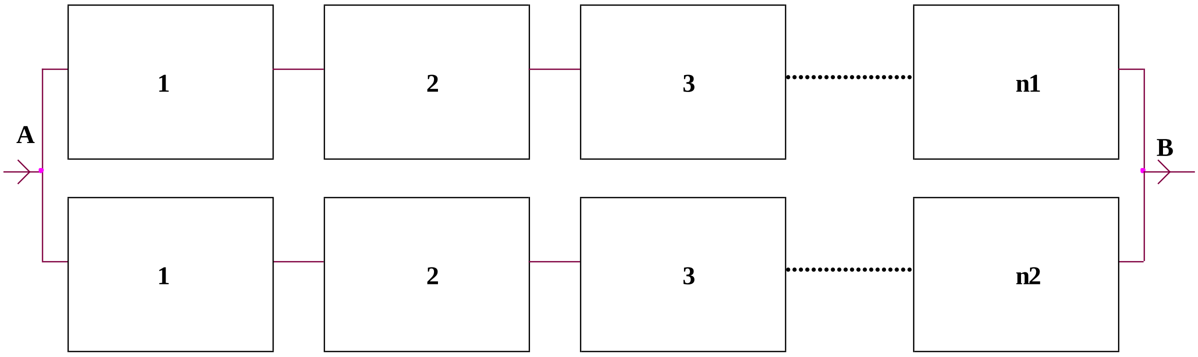

Figure 17: Two lines of series connected components

Now consider two (2) identical lines of series connected components as shown in Figure 17. Now we compute the probability of survival for two cases [41], [75]: (3.6.3.1) when no short interruptions to the system are allowed (that is, no possibility of cannibalization: this is indeed the only case without cannibalization, but if we have three (3) or more lines and no interruptions, we already have cannibalization) and (3.6.3.2) when short interruptions to the system are allowed (that is, when cannibalization can be executed).

3.6.3.1 No short interruptions to the system are allowed (that is, no possibility of cannibalization)

The formula for $\Pr(X \geq t)$ (that is, the probability of survival) is written obviously as follows:

$$S_{2n}^{-}(t) = S_{2n}^{nc}(t) = \Pr(X \geq t) = 1 - \left(1 - e^{-n\lambda t}\right)^2 \tag{3.8}$$

3.6.3.2 Short interruptions to the system are allowed (that is, cannibalization is allowed)

When one line fails the time for replacing the failed component of the remaining line is allowed and all $n-1$ non-failed components of the failed line can be used as spares for the operable line. The cannibalization formula for $\Pr(X \geq t)$ (that is, the probability of survival) is written as follows:

$$S_{2n}^{+}(t) = \Pr(X \geq t) = e^{-2n\lambda t} + \int_0^t \left(2n\lambda e^{-2n\lambda x} \sum_{i=0}^{n-1} e^{-n\lambda(t-x)} \frac{(n\lambda(t-x))^i}{i!} \right) dx\,, \tag{3.9}$$

where $e^{-2n\lambda t}$ in the first term in equation (3.9) means that both lines have survived (that is, by the law of total probability); the integral corresponds to the probability that one line failed and then the remaining line has survived with $n-1$ spares and $2n\lambda e^{-2n\lambda x}dx$ means the density of the first failure of $2n$ components. Then with one (1) line left there will be no further failures.

We can compare probabilities with and without cannibalization. More appropriately we compare probabilities of failures. Therefore, we compute the improvement factor of unreliability for the two-line system as $q_{kn}^{nc}(t) = \dfrac{1 - S_{kn}^{nc}(t)}{1 - S_{kn}^{+}(t)}$ as shown in Figure 20.

3.6.4 THREE-LINE SYSTEM

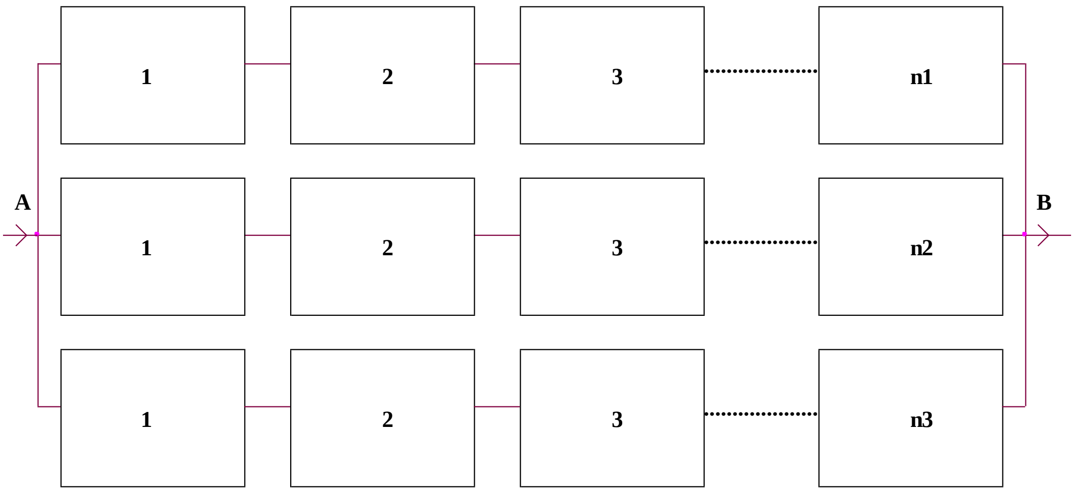

Figure 18: Three lines of series connected components

Now consider three (3) identical lines of series connected components as shown in Figure 18. Now we compute $\Pr(X \geq t)$ for three cases [41], [75]: (3.6.4.1) when no cannibalization is allowed at all (just 3 lines of n series parallel-connected components), (3.6.4.2) when no short interruptions to the system are allowed (Cannibalization is made possible here as we are using the operable components of the failed line as spares, as reflected in equation (3.11)) and (3.6.4.3) when short interruptions to the system are allowed (that is, when cannibalization is allowed).

3.6.4.1 No cannibalization is allowed at all (just 3 lines of n series parallel-connected components)

The formula for $\Pr(X \geq t)$ (that is, the probability of survival) is written, obviously, as follows:

$$S_{3n}^{nc}(t) = \Pr(X \geq t) = 1 - \left(1 - e^{-n\lambda t}\right)^3 \qquad (3.10),$$

3.6.4.2 No short interruptions to the system are allowed (that is, cannibalization is made possible here by operable components of the failed line which are used as spares)

Cannibalization can still be done here. Indeed, when one line fails, we can use $n-1$ spares for the system of two (2) lines. When the $n-1$ non-failed components are exhausted, then no cannibalization can be done. The formula is written as follows:

$$S_{3n}^{-}(t) = \Pr(X \geq t) = e^{-3n\lambda t} + \int_0^t \left(3n\lambda e^{-3n\lambda x}\left(\sum_{i=0}^{n-1} e^{-2n\lambda(t-x)}\frac{(2n\lambda(t-x))^i}{i!} + \int_0^{t-x}\frac{(2n\lambda)^n}{(n-1)!}y^{n-1}e^{-2n\lambda y}e^{-n\lambda(t-x-y)}dy\right)\right)dx \qquad (3.11),$$

where $e^{-3n\lambda t}$ in the first term in equation (3.11) means that all three (3) lines have survived (that is, by the law of total probability); the integral corresponds to the probability that one line failed and then the remaining two lines survived with $n-1$ spares; and $3n\lambda e^{-3n\lambda x}dx$ means the density of the first failure of $3n$ components. Then with two (2) lines with $n-1$ spares and the n^{th} failure with intensity 2λ will "ruin" it and one line will be left and no further failures.

3.6.4.3 Short interruptions to the system are allowed (that is, cannibalization is allowed)

The two (2) stage cannibalization goes .When the first line fails the time for replacing the failed component of the two (2) remaining lines is allowed. $n-1$ non-failed components of the failed line can be used to maintain the two remaining lines. When these components are exhausted and one of the two (2) lines fails, the $n-1$ non-failed components of the failed line can be used as spares for the remaining operable line. The corresponding cannibalization formula for $\Pr(X \geq t)$ is written as follows:

$$S_{3n}^{+}(t) = \Pr(X \geq t) = e^{-3n\lambda t}$$
$$+\int_0^t \left(3n\lambda e^{-3n\lambda x}\left(\sum_{i=0}^{n-1} e^{-2n\lambda(t-x)}\frac{(2n\lambda(t-x))^i}{i!} + \int_0^{t-x}\frac{(2n\lambda)^n}{(n-1)!}y^{n-1}e^{-2n\lambda y}\sum_{i=0}^{n-1} e^{-n\lambda(t-x-y)}\frac{(n\lambda(t-x-y))^i}{i!}dy\right)\right)dx \tag{3.12}$$

where $e^{-3n\lambda t}$ in the first term in equation (3.12) means that all three (3) lines have survived; the integral corresponds to the probability that one line failed and then the remaining two lines survived with $n-1$ spares; $3n\lambda e^{-3n\lambda x}dx$ means the density of the first failure of $3n$ components; and $\frac{(2n\lambda)^n}{(n-1)!}y^{n-1}e^{-2n\lambda y}$ is the density of the n^{th} event from the Poisson process with rate $2n$. Then with two (2) lines with $n-1$ spares and the n^{th} failure with intensity 2λ will "ruin" it and one line will be left and no further failures.

Here we can compare probabilities with no cannibalization at all (just 3 lines of n series parallel-connected components) and that with cannibalization (with and without short interruptions). More suitably we compare probabilities of failures. Hence, we compute the improvement factor of unreliability for the three-line system (that is, that for no cannibalization and cannibalization with and without short interruptions) as $q_{3n}^{nc}(t)=\dfrac{1-S_{3n}^{nc}(t)}{1-S_{3n}^{+}(t)}$ and $q_{3n}^{-+}(t)=\dfrac{1-S_{3n}^{-}(t)}{1-S_{3n}^{+}(t)}$ as illustrated in Figure 21 and Figure 22, respectively.

3.6.5 K-LINE SYSTEM

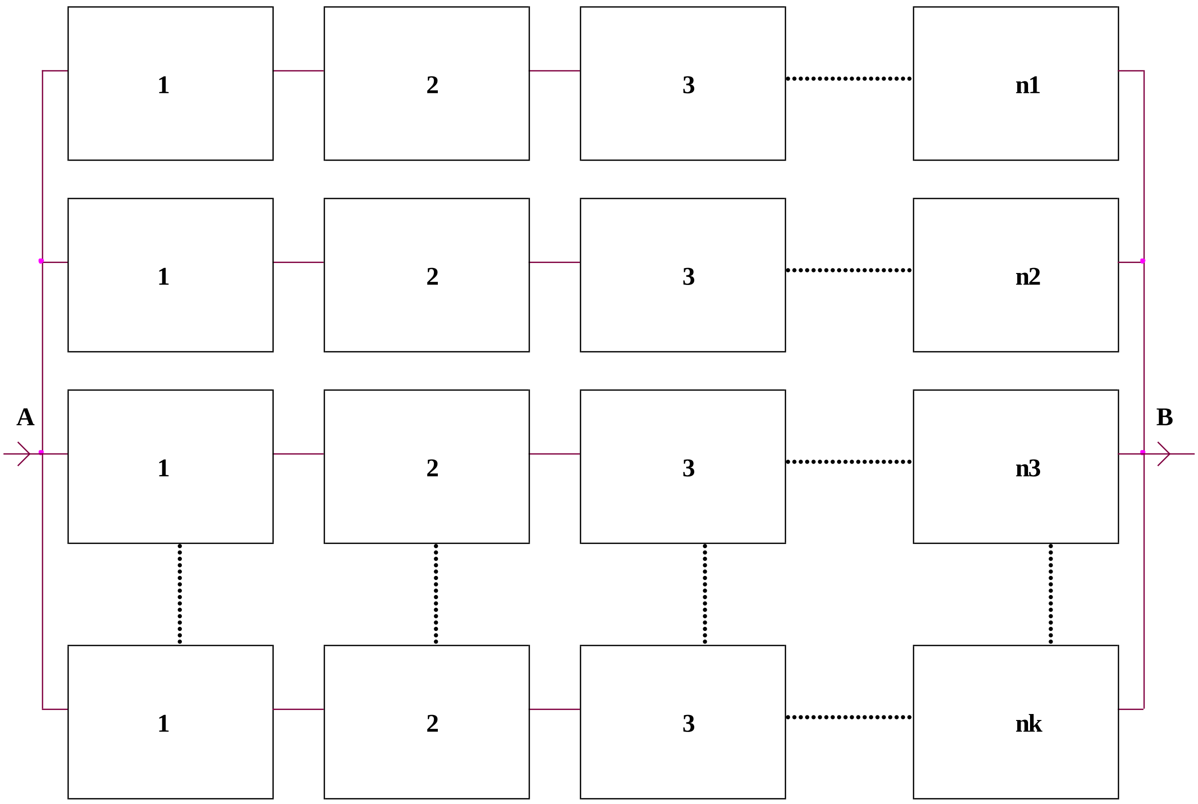

Figure 19: k lines of series connected components

Now consider k identical lines of series connected components as shown in Figure 19. Now we compute $\Pr(X \geq t)$ for three cases [41], [75]: (3.6.5.1) when no cannibalization is allowed at all (just k lines of n series parallel-connected components), (3.6.5.2) when no short interruptions to the system are allowed (cannibalization is made possible here as we are using the operable components of the failed line as spares, as reflected in equation (3.14)) and (3.6.5.3) when short interruptions to the system are allowed (that is, when cannibalization is allowed).

3.6.5.1 No cannibalization is allowed at all (just k lines of n series parallel-connected components)

The formula for $\Pr(X \geq t)$ (that is, the probability of survival) is written as follows:

$$S_{kn}^{nc}(t) = \Pr(X \geq t) = 1 - \left(1 - e^{-n\lambda t}\right)^k \tag{3.13}$$

3.6.5.2 No short interruptions to the system are allowed (that is, cannibalization is made possible here by operable components of the failed lines which are used as spares)

Cannibalization can still be done here. Indeed, when one line fails, we can use $n-1$ spares for the system of $k-1$ lines. When the $n-1$ non-failed components are exhausted, then no cannibalization can be done. The formula is written as follows:

$$S_{kn}^{-}(t) = \Pr(X \geq t) = e^{-kn\lambda t} + \int_0^t \left\{ kn\lambda e^{-kn\lambda x} \left(\sum_{i=0}^{n-1} e^{-(k-1)n\lambda(t-x)} \frac{\left((k-1)n\lambda(t-x)\right)^i}{i!} + \int_0^{t-x} \frac{((k-1)n\lambda)^n}{(n-1)!} y^{n-1} e^{-(k-1)n\lambda y} S_{(k-1)n}^{-}(t-x-y)\,dy \right) \right\} dx \tag{3.14}$$

where $e^{-kn\lambda t}$ in the first term in equation (3.14) means that all k lines have survived (that is, by the law of total probability); the integral corresponds to the probability that one line failed and then the remaining $k-1$ lines survived with $n-1$ spares; and $kn\lambda e^{-kn\lambda x}dx$ means the density of the first failure of kn components. Then with $k-1$ lines with $n-1$ spares and the n^{th} failure with intensity $(k-1)\lambda$ will "ruin" it and one line will be left and no further failures. $S^{-}_{(k-1)n}(t-x)$ is recurrent from the probability of survival (at time $t - x$) of the system with $k-1$ lines when no short interruptions to the system are allowed (that is, cannibalization is made possible here as we are using the operable components of the failed line as spares).

3.6.5.3 Short interruptions to the system are allowed (that is, cannibalization is allowed-but it was made possible in b) by operable components of the failed lines which are used as spares)

The $k-1$ stage cannibalization goes .When the first line fails the time for replacing the failed component of the $k-1$ remaining lines is allowed. $n-1$ non-failed components of the failed line can be used to maintain the $k-1$ remaining lines. When these components are exhausted and one of the $k-1$ lines fails, the $n-1$ non-failed components of the failed line can be used as spares for the remaining operable lines. The process is repeated until one (line) remains, and the time for replacing the failed component of this remaining line (if one has a spare) is not allowed. The corresponding cannibalization formula for $\Pr(X \ge t)$ is written as follows:

$$S^{+}_{kn}(t) = \Pr(X \ge t) = e^{-kn\lambda t}$$
$$+\int_0^t \left\{ kn\lambda e^{-kn\lambda x} \left(\sum_{i=0}^{n-1} e^{-(k-1)n\lambda(t-x)} \frac{((k-1)n\lambda(t-x))^i}{i!} + \int_0^{t-x} \frac{((k-1)n\lambda)^n}{(n-1)!} y^{n-1} e^{-(k-1)n\lambda y} S^{+}_{(k-1)n}(t-x-y)dy \right) \right\} dx \quad (3.15),$$

where $e^{-kn\lambda t}$ in the first term in equation (3.15) means that all k lines have survived; the integral corresponds to the probability that one line failed and then the remaining $k-1$ lines survived with $n-1$ spares; $kn\lambda e^{-kn\lambda x}dx$ means the density of the first failure of kn components; and $S^{+}_{(k-1)n}(t-x)$ is the probability of survival (at time $t - x$) of the system with $k-1$ lines when short interruptions to the system are allowed. Then with $k-1$ lines with $n-1$ spares and the n^{th} failure with intensity $(k-1)\lambda$ will "ruin" it and one line will be left and no further failures. Thus equation (3.15) is a recurrent relationship.

Again we can compare probabilities with and without cannibalization. More usefully we compare probabilities of failures. Hence, we compute the improvement factor of unreliability for the k -line system (that is, that for no cannibalization and cannibalization with and without short interruptions) as $q^{nc}_{kn}(t)=\dfrac{1-S^{nc}_{kn}(t)}{1-S^{+}_{kn}(t)}$ and $q^{-+}_{kn}(t)=\dfrac{1-S^{-}_{kn}(t)}{1-S^{+}_{kn}(t)}$ as depicted in Figure 23 and Figure 24, respectively.

3.6.6 COMPUTATION RESULTS

Figure 20 to Figure 24 show the improvement factors of unreliability for the 2-line system, 3-line system and k-line system respectively. We are looking at reliable systems, where the survival functions should be close to 1. For illustrative purposes, we choose the values of the failure rate and time accordingly. Therefore, it is more effective to compare unreliability of the systems with no cannibalization (that is, $1-S^{nc}_{kn}(t)$) and with cannibalization (that is, with and without short interruptions)(that is, $1-S^{+}_{kn}(t)$ $(1-S^{-}_{kn}(t))$). The improvement factor of unreliability is obtained by dividing the unreliability of the said system without cannibalization (worse quantity) by that of the said system with cannibalization (without short interruptions) (better quantity) and thus is larger than one. This is then compared to that obtained by dividing the unreliability of the said system with cannibalization with short interruptions (worse quantity) by that of the said system with cannibalization (without short interruptions) (better quantity). It can be seen from Figure 20 to Figure 24 that the improvement factors of unreliability of the systems in which cannibalization is allowed are better than in those in which it is prohibited. From simulation results of Figure 20 to Figure 24 it can also be shown that very larger values of n lead to asymptotically equivalent system performance levels.

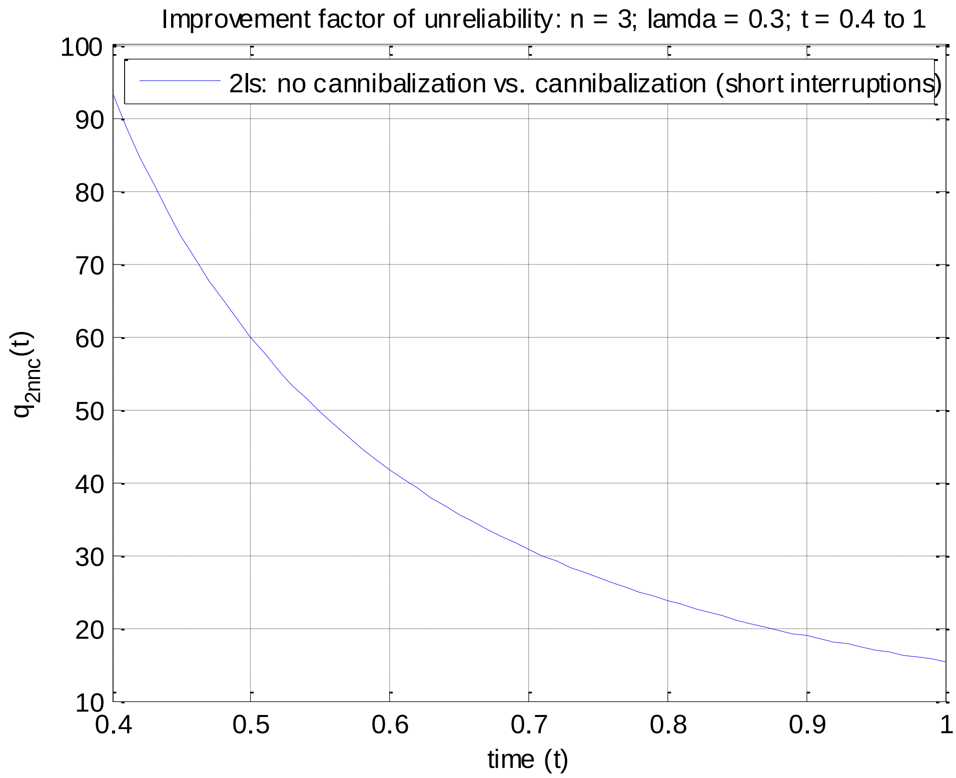

Figure 20: Improvement factor of unreliability for a 2-line system (comparison of a system with no cannibalization and that with cannibalization when short interruptions to the system are allowed)

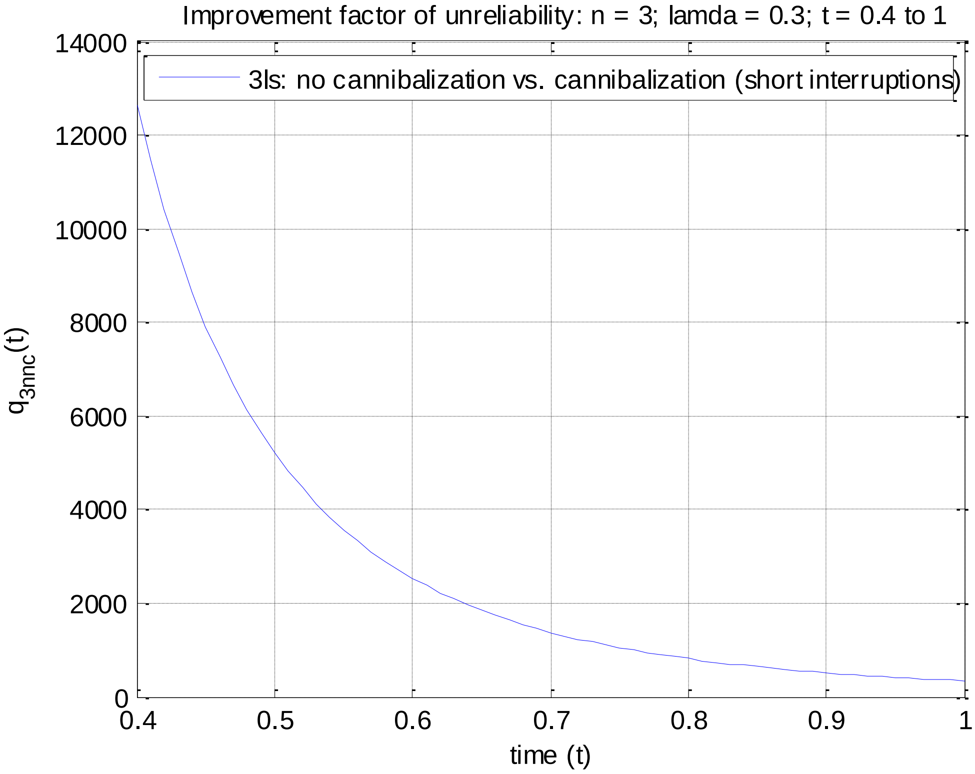

Figure 21: Improvement factor of unreliability for a 3-line system (comparison of a system with no cannibalization and that with cannibalization when short interruptions to the system are allowed)

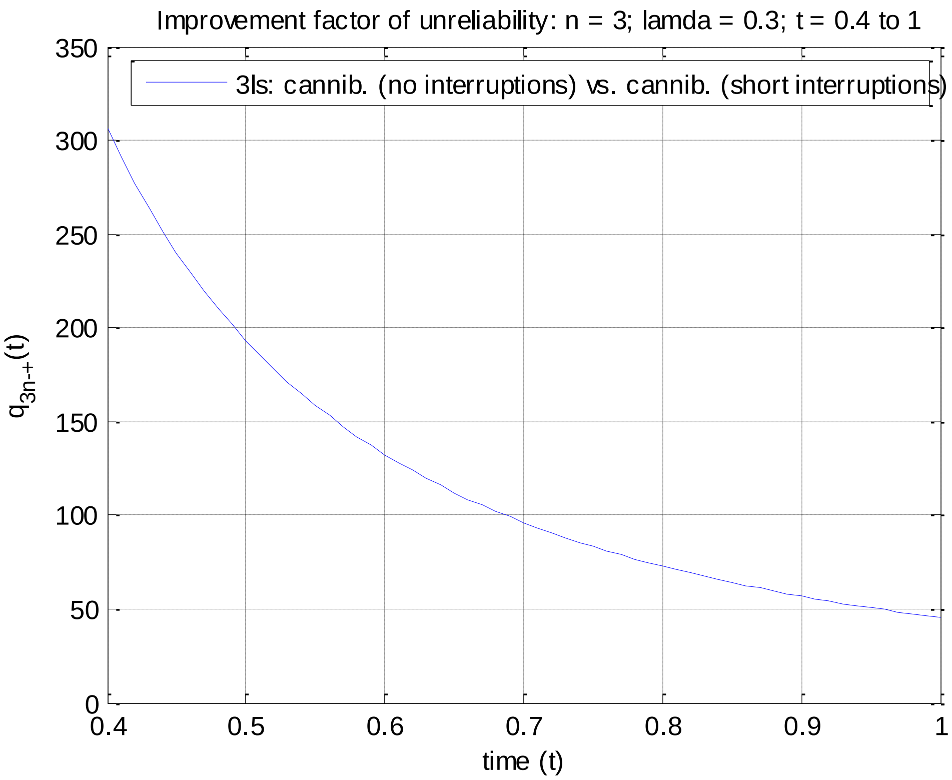

Figure 22: Improvement factor of unreliability for a 3-line system (comparison of a system with cannibalization when no short interruptions to the system are allowed and that with cannibalization when short interruptions to the system are allowed)

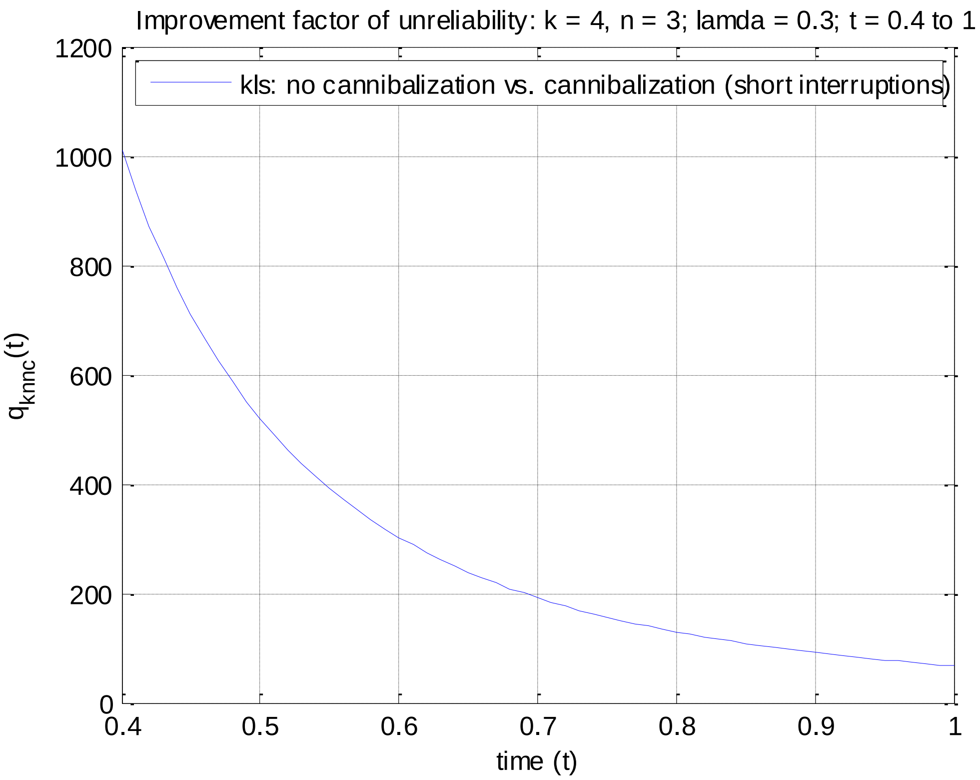

Figure 23: Improvement factor of unreliability for a k-line system (comparison of a system with no cannibalization and that with cannibalization when short interruptions to the system are allowed)

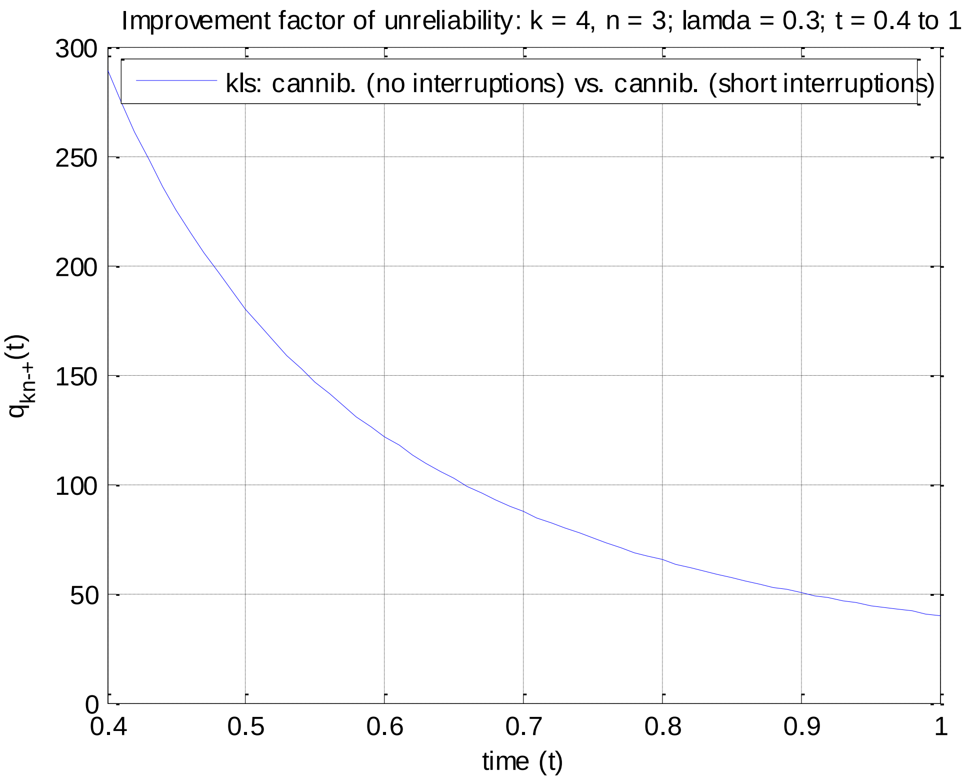

Figure 24: Improvement factor of unreliability for a k-line system (comparison of a system with cannibalization when no short interruptions to the system are allowed and that with cannibalization when short interruptions to the system are allowed)

3.7 CONCLUSIONS

In this chapter we have explored strategies to reduce the adverse effects of cannibalizations on maintenance costs and personnel. The strategies developed in this chapter, at least, can be used to determine: 1) which cannibalizations are appropriate; 2) cannibalization reduction goals; and 3) the actions to be taken to meet the cannibalization reduction goals.

We also presented a combined analytic and simulation model of a 2-line, 3-line and k-line system when cannibalization is not allowed and when cannibalization is allowed (with and without short interruptions to the system). It is clear from the analytic and simulation results that cannibalization can substantially increase the reliability of the systems where it is allowed. The improvement factor of unreliability obviously exists in systems where cannibalization is not allowed as compared to those in which cannibalization is allowed. Moreover, the improvement factor is larger when we have two-stage cannibalization (short interruptions) than without them.

CHAPTER 4: ON THE IMPROVEMENT OF STEAM POWER PLANT SYSTEM RELIABILITY

4.1 BACKGROUND

A systematic approach based on the graph theoretical analysis and a graph's characteristic polynomial is used to develop a model for estimating the reliability index and evaluating the availability index for a coal-fired generating power station. In this study the coal-fired generating station system is divided into six subsystems. Consideration of all the subsystems and their interrelations are rudiment in evaluating the reliability (estimate). The approximate reliability of the coal-fired generating station is modelled in terms of approximate reliability attributes of the graph. Nodes in graph represent sub-system reliability and the reliability of interrelations of these sub-systems is represented by the links. The approximate reliability of the system is determined by computing a graph's characteristic polynomial using three different methods: the linearly independent cycles, the figure equation and the adjacent matrix. Estimating reliability is always an imperfect endeavour and hence the use of three methods for comparison purposes. The proposed methodology is illustrated step-by-step with the help of examples.

4.2 INTRODUCTION

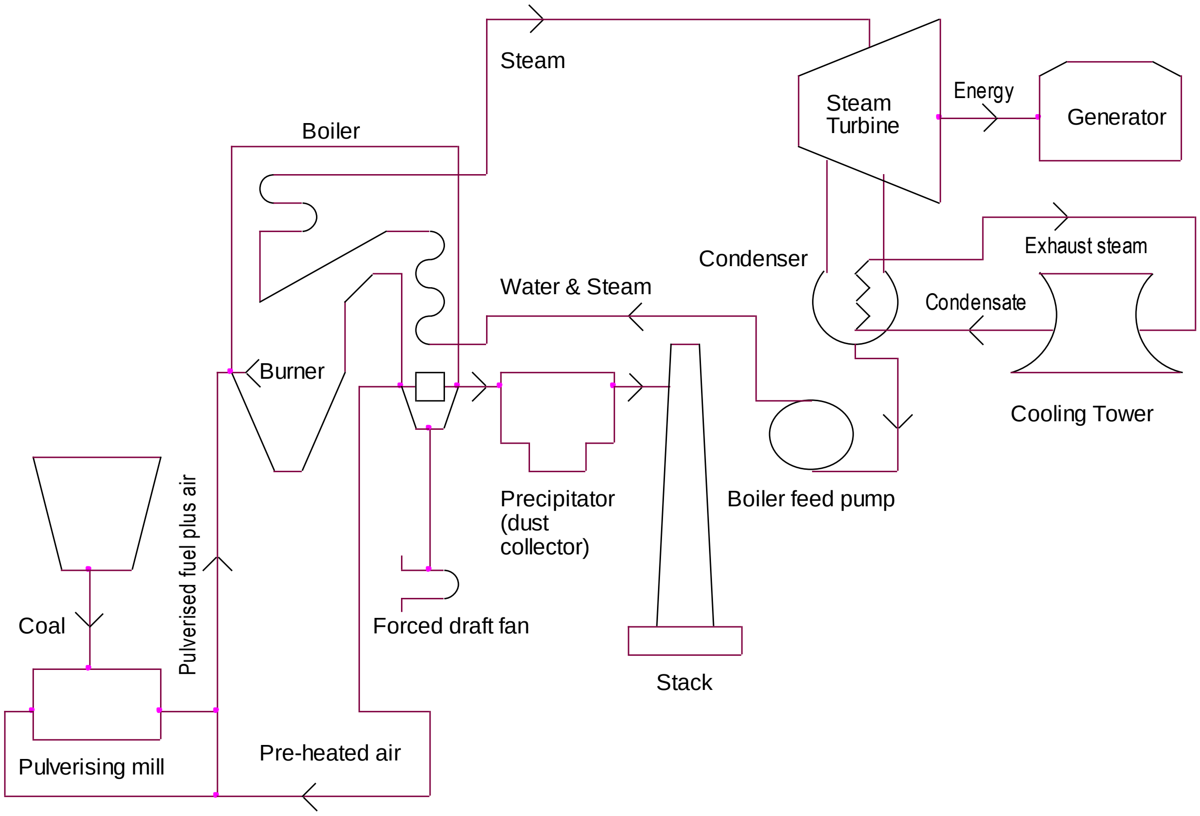

Figure 25: Schematic view of a coal–fired generating station

A schematic diagram of a coal-fired generating station is shown in Figure 25 [76], [21]. In coal-fired generating stations, coal is conveyed to a mill and crushed into fine powder, which is pulverised. The pulverised fuel is blown into the boiler where it mixes with a supply of pre-heated air for combustion. The combustion of coal in the boiler produces steam, at high temperatures and pressures, which is passed through the steam turbine. The exhaust steam from the low pressure turbine is cooled to form condensate by the passage through the condenser of large quantities of sea- or river-water. Cooling towers are used where the station is located inland or if there is concern over the environmental effects of raising the temperature of the sea- or river-water.

A steam power-station operates on the Rankine cycle, modified to include super-heating, feed-water heating, and steam re-heating. High efficiency is achieved by the use of steam at maximum possible pressure and temperature. In addition for turbines to be constructed economically, the larger the size the less the capital cost per unit of power output. Consequently, turbo-generator sets of 500 MW and more have been used. With steam turbines above 100 MW, the efficiency is increased by re-heating the steam, using an external heater, after it has been partially expanded. The re-heated steam is then returned to the turbine where it is expanded through the final stages of blading.

The study of reliability of complex systems, which means systems containing a rather large number of interacting components, such as steam power plants is of interest for power utility companies. Steam power plant reliability encompasses a range of issues related to the design and analysis of these power plant generating networks which are subject to random failure of their components. Relatively simple, and yet quite general, network models can represent a variety of applied problems of steam power plant environments. The ultimate objective of research in the area of steam power plant reliability is to give design engineers procedures to enhance their ability to design steam power plants for which reliability is an important consideration.

One of the major impediments in steam power plant system reliability analysis is the size of the state space. If one considers a large-scale power plant system, the number of system states is enormous. For example, a system consisting of n components and each component with binary states (working or failed) has a total of 2^n states. Consider a case when n is 200; the number of states is 1.61×10^{60}. In this case if all the possible states are analyzed one by one to identify the contingencies that contribute to the system unreliability, it requires much computational effort, which is impractical for a typical steam power plant. Consequently one needs to select a method that can reduce the state space, and selection and evaluation of contingencies. In this study we use the Graph Theoretical Analysis (GTA) method [77], [78].

GTA implementation requires that the large and complex system such as the steam power plant network be reduced and divided into sub-systems for convenience of the analysis procedure. The GTA procedure simulates the inheritance and interdependencies of these sub-systems apart from giving a quantitative measure of the system reliability [79], [80]. The quantitative measure of the steam power plant system reliability allows the design engineer to compare the present reliability with the design value.

For the purposes of this study the entire system of a coal-fired generating station, as shown in Figure 25, is divided into six sub-systems (N_i: i = 1, 2, …, 6) as follows [21]:

N1: The coal system;
N2: The boiler system;
N3: The steam turbine;
N4: The boiler feed pump;
N5: The cooling system; and
N6: The generator.

The six sub-systems identified for the coal-fired generating power station of Figure 25 are shown in Figure 26 and are described in section 4.3.

4.3 SYSTEM AND STRUCTURE FUNCTION FOR THE COAL-FIRED GENERATING STATION

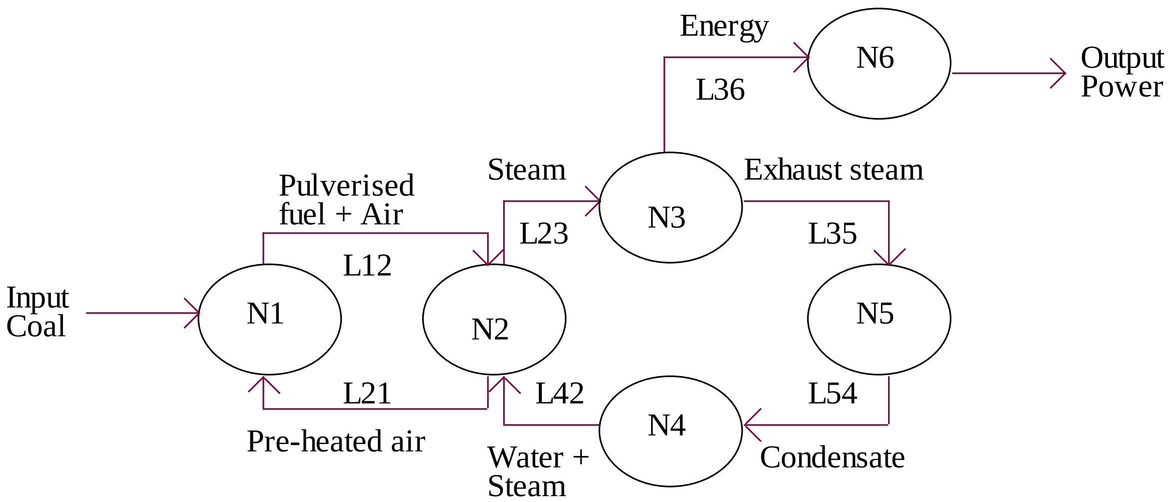

Figure 26: System structure digraph for a coal-fired generation station

We consider systems of components that satisfy the following hypothesis [11]:

A system that is composed of *n* components is denoted a system of order *n*. The components are assumed to be numbered consecutively from 1 to *n*. We confine ourselves to situations where it suffices to distinguish between only two states, a functioning state and a failed state. This applies to each component as well as to the system itself. The state of component *i*, *i* = 1, 2, …, *n* can then be described by a binary[2] variable (function) x_i, where:

$$x_i = \begin{cases} 1 \text{ if component } i \text{ is functioning} \\ 0 \text{ if component } i \text{ is in a failed state} \end{cases}$$

Then, vector **X** = (x_1, x_2, …, x_n) represents the states of all components and is known as the component *state vector*. Furthermore, we assume that by knowing the states of all *n* components, one also knows whether the system is functioning or not. The state of the system is completely determined by and is a deterministic function of the states of the components.

[2] In this context a binary variable (function) is a variable (function) that can take only the two values, 0 or 1.

Similarly, the state of the system can then be described by a binary function: $\phi(\mathbf{X}) = \phi(x_1, x_2, ..., x_n)$, where:

$$\varphi(\mathbf{X}) = \begin{cases} 1 \text{ if the system is functioning} \\ 0 \text{ if the system is in a failed state} \end{cases}$$

and where $\phi(\mathbf{X})$ is called the *structure function of the system* or just the *structure*. Each unique system corresponds to a unique structure function $\phi(\mathbf{X})$. Hence we also talk about structures instead of systems.

4.4 GRAPHICAL REPRESENTATION OF THE COAL-FIRED GENERATING STATION

Generally a network is any system that admits an abstract mathematical representation as a graph whose nodes (vertices) identify the components of the system. In this network the set of connecting links (edges) represent the presence of a relation or interaction among those components. It is obvious that such a high level of abstraction generally applies to a wide range of systems. Therefore, in this sense, networks provide a theoretical framework that allows a convenient conceptual representation of relations in the systems where the system level characterisation implies the mapping of interactions among components.

The coal-fired generating station system of Figure 25 is represented in the form of a graph $G = (N, L)$ of Figure 26, where N is the set of nodes (or vertices) and L the set of links (or edges). Let each of the six sub-systems of the generating station be denoted by nodes N_i*'s* (i = 1, 2, …, 6) and the interconnection between the systems (N_i, N_j) is represented by links L_{ij}*'s* (i, j = 1, 2, …, 6 and $i \neq j$) connecting the two nodes N_i and N_j. All six sub-systems are connected by the flow of pre-heated air, pulverised fuel, water, steam, heat and energy. This flow is shown in Figure 26 with the aid of nodes and links. This graphical representation of the coal-fired generating station is known as the system *structure* function.

A graph is undirected when the links can be traversed in both directions. On the other hand the graph is directed if the link can be traversed only in one direction indicated by an arrow. If an undirected graph has no self-loops the presence of at least one link per node guarantees that all the nodes are connected. In general practical structures are much more connected than this minimal threshold. Therefore this allows multiple paths among any pair of nodes. When graphs are directed the connectivity property is cumbersome since nodes can pertain to three categories: strongly connected nodes (subset of nodes that can be reached from any other node in the subset following the direction of the links); transient nodes that have only outgoing links and cannot be reached from any other node; and absorbing nodes that have only ingoing links and once reached cannot be left.

4.5 RELIABILITY ASSESSMENT OF THE COAL-FIRED GENERATING STATION

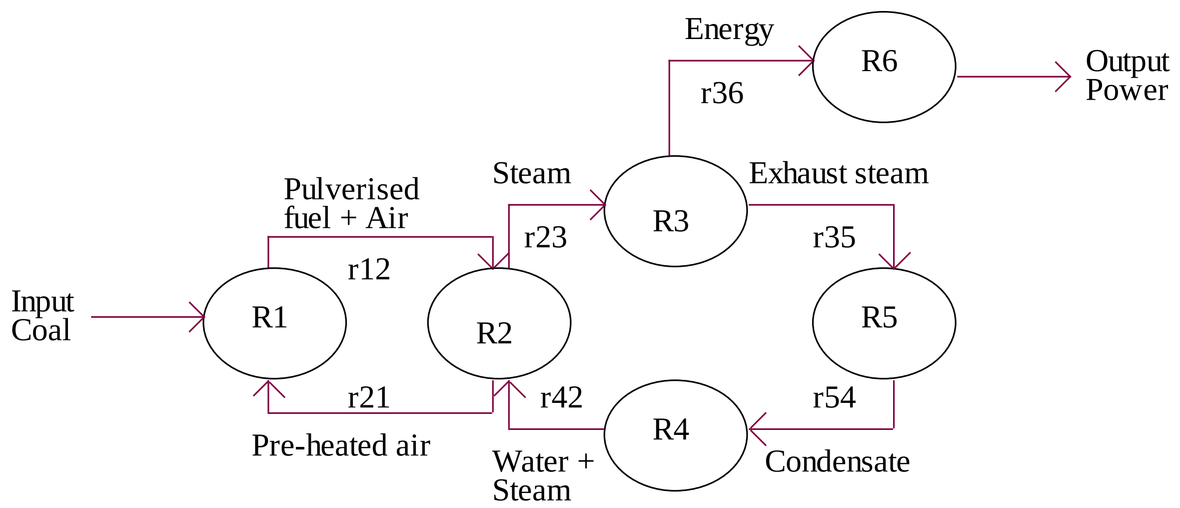

Figure 27: System reliability digraph for a coal-fired generating station

Let R_i (i = 1, 2, …,6) denote the reliability of node Ni and r_{ij} (i,j = 1,2,…,6 and $i \neq j$), the reliability of the link (or interconnection) between the nodes, N_i and N_j. This means that by associating reliability to the system structure of Figure 26, results in the system reliability graph modelling. The system reliability graph for the coal-fired generating station corresponding to its abridged system structure graph is obtained by associating R_i with N_i and r_{ij} with L_{ij} and is shown in Figure 27.

The reliability structure function of the system of Figure 27 is estimated by computing the graph's characteristic polynomial. Here we employ three methods of estimating the system reliability structure function: the linearly independent cycles, the formula called the figure equation and the adjacent matrix method. Estimating reliability is always an imperfect endeavor and hence the use of three methods for comparison purposes. The figure equation provides a direct link between a graph's structure and the coefficients of its characteristic polynomial. The figure equation method does not use determinants (like the linearly independent cycles and the adjacent matrix) but calculates the characteristic polynomial of any graph by counting the cycles in the graph. The three methods are determined as follows:

4.5.1 THE LINEARLY INDEPENDENT CYCLES

In this method the structure function is characterized by the presence of a sufficient number of certain cycles which have the property that they are linearly independent [81]. We denote these cycles (denoted by the links (edges) present in them) in a matrix form. Let $A = (a_{ij})$ of dimension L x L, where L denotes the number of links and an element a_{ij} denotes whether link j is present in cycle i or not. If the cycle accumulates the link metrics on its path, then the value observed at m is the sum of the link metrics. We assume that node m is the monitor that can start and terminate probes.

Let B denote the column matrix (L x 1) of the accumulated metrics corresponding to the cycles. Let x denote the column matrix (L x 1) of the link variables that one is trying to identify. The goal is to solve the system of linear equations represented by $Ax = B$. In order to uniquely determine x, A has to be invertible. Such a matrix is also called identifiable, since it has full rank. Cycles that make up such a matrix are referred to as *linearly independent cycles*. All link metrics can be uniquely identified by solving the following equation:

$$x = A^{-1}B \tag{4.1}$$

4.5.2 THE FIGURE EQUATION FORMULA

The Figure Equation states that for any graph $G = (N, L)$ with n vertices, the characteristic polynomial is [83]:

$$\chi(G) = x^n + c_1 x^{n-1} + c_2 x^{n-2} + \ldots + c_{n-1} x + c_n \tag{4.2}$$

such that for $1 \leq i \leq n$, the coefficient is:

$$c_i = \sum_{L \in L_i} (-1)^{P(L)}$$

where L_i is the set of all linearly directed subgraphs of G and $P(L)$ is the number of *linearly directed cycles*, or the number of pieces in L.

4.5.3 THE ADJACENT MATRIX METHOD

The reliability of the system is estimated by obtaining the determinant of the characteristic system reliability matrix as follows [77], [78]:

$$R_{system} = \det\{RI - A_{adj}\} \tag{4.3}$$

where R represents the reliability of the nodes constituting the system; I is the node identity matrix; and A_{adj} is the system structure adjacency matrix.

4.5.4 ILLUSTRATION OF THE SYSTEM RELIABILITY METHODOLOGY ASSESSMENT

$$r_{12} \rightarrow r_{21}$$
$$r_{12} \rightarrow r_{23} \rightarrow r_{35} \rightarrow r_{54} \rightarrow r_{42} \rightarrow r_{21}$$
$$A = \begin{pmatrix} 1\,1\,0\,0\,0\,0\,0 \\ 1\,1\,1\,0\,1\,1\,1 \\ 0\,0\,0\,0\,0\,0\,0 \\ 0\,0\,0\,0\,0\,0\,0 \\ 0\,0\,0\,0\,0\,0\,0 \\ 0\,0\,0\,0\,0\,0\,0 \\ 0\,0\,0\,0\,0\,0\,0 \end{pmatrix}$$

Figure 28: List of cycle (top) and the corresponding cycle-link matrix (bottom)

Using the linearly independent cycles method we get the list of cycles (top) and the corresponding cycle-link matrix (bottom) as shown in Figure 28. We use node R_1 as the monitor that can start and terminate probes. The reliability of the structure of Figure 27 is determined (using (4.1)) as:

$$\begin{aligned} R_{system} &= \det\{(R\,.\times \mathrm{eye}(7)) - A\} \\ &= R^7 - 2R^6 \end{aligned} \tag{4.4}$$

For constant unit failure rate, substituting $R(t) = e^{-\lambda t}$ into (4.4) yields:

$$R_{system}(t) = e^{-7\lambda t} - 2e^{-6\lambda t} \tag{4.5}$$

where $R_{system(t)}$ is the coal-fired generating station reliability at time t and λ is the unit constant failure rate.

Using the figure equation formula (Equation (4.2)) we get the list of subgraphs (left) and the corresponding coefficients (right) as shown in Table 2. From the information as shown in Table 2 the characteristic polynomial of the structure in Figure 27 is:

$$R(G) = x^6 - x^5 - x^4 + x^3 + x^2 \tag{4.6}$$

For constant unit failure rate, substituting $R(t) = e^{-\lambda t}$ into (4.6) yields:

$$R_{system}(t) = e^{-6\lambda t} - e^{-5\lambda t} - e^{-4\lambda t} + e^{-3\lambda t} + e^{-2\lambda t} \tag{4.7}$$

where $R_{system(t)}$ is the coal-fired generating station reliability at time t and λ is the unit constant failure rate.

Table 2: Linearly directed cycles and their coefficients

Subgraphs	**Coefficients**
$r_{12} \rightarrow r_{23} \rightarrow r_{36}$	3 pieces, $c_1 = (-1)^3 = -1$
$r_{12} \rightarrow r_{23} \rightarrow r_{35} \rightarrow r_{54} \rightarrow r_{42} \rightarrow r_{23} \rightarrow r_{36}$	7 pieces, $c_2 = (-1)^7 = -1$
$r_{23} \rightarrow r_{35} \rightarrow r_{54} \rightarrow r_{42}$	4 pieces, $c_3 = (-1)^4 = +1$
$r_{23} \rightarrow r_{12} \rightarrow r_{21}$	2 pieces, $c_4 = (-1)^2 = +1$

Using the adjacent matrix method the reliability of the structure of Figure 27 is determined (using (4.3)) as:

$$R_{system} = \det\left\{\left(\left(\begin{bmatrix} R_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & R_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & R_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & R_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & R_5 & 0 \\ 0 & 0 & 0 & 0 & 0 & R_6 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}\right) - \left(\begin{bmatrix} 0 & r_{12} & 0 & 0 & 0 & 0 \\ r_{21} & 0 & r_{23} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & r_{35} & r_{36} \\ 0 & r_{42} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & r_{54} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}\right)\right)\right\} \tag{4.8}$$

$$= R_6 \left(R_3 R_4 R_5 r_{12} r_{21} + R_1 r_{23} r_{35} r_{42} r_{54} - R_1 R_2 R_3 R_4 R_5\right)$$

For identical units (that is, for illustration purposes only) (that is, $R_1 = R_2 = R_3 = R_4 = R_5 = R_6 = R$) with the link reliability assumed to be unity (that is, $r_{ij} = 1$), (4.8) simplifies to:

$$R_{system} = |(R \times (-R^5 + R^3 + R))| \quad (4.9)$$

where |•| denotes the absolute value and det is the matrix determinant.

For constant unit failure rate, substituting $R(t) = e^{-\lambda t}$ into (4.9) yields:

$$R_{system}(t) = |e^{-\lambda t} \times \left(e^{-\lambda t} + e^{-3\lambda t} - e^{-5\lambda t}\right)| \quad (4.10)$$

where $R_{system(t)}$ is the coal-fired generating station reliability at time t and λ is the unit constant failure rate.

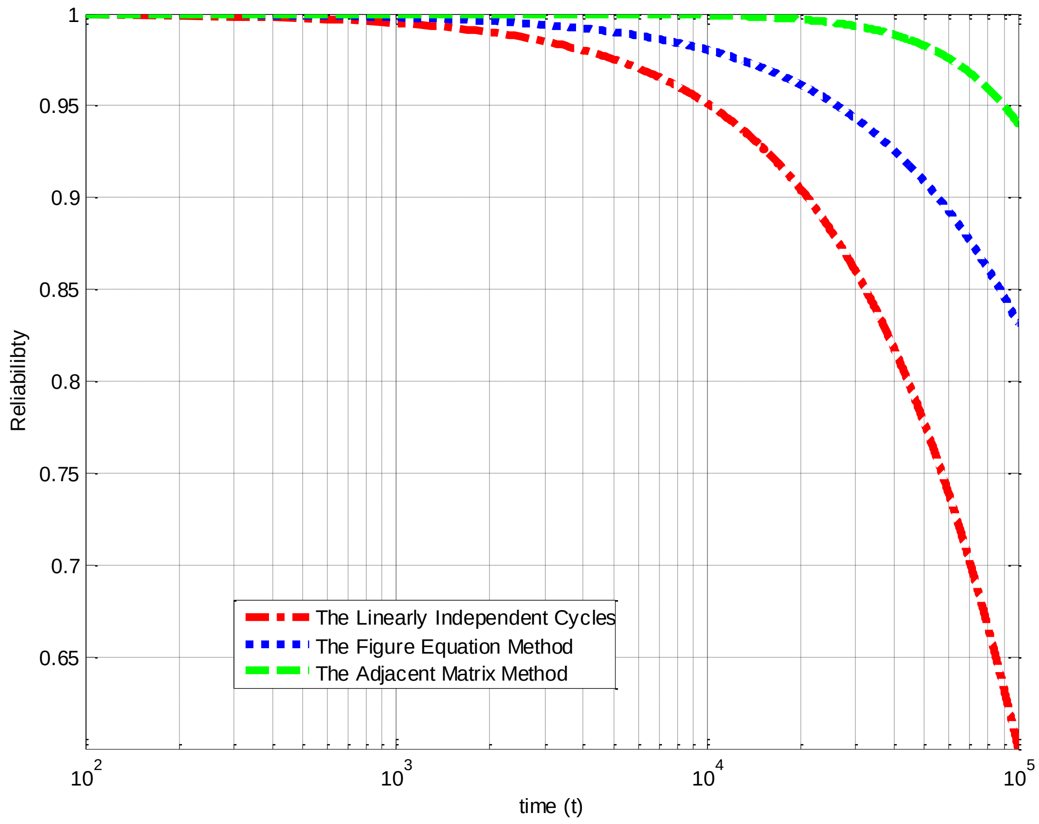

Figure 29: Coal-fired generating station system reliability

The system reliability value of the coal-fired generating station of Figure 25 as illustrated by equations (4.5), (4.7) and (4.10) respectively is plotted as shown in Figure 29. It should be noted that the results in Figure 29 have been obtained assuming that the components are identical and assuming constant unit failure rate. This is only for the example to illustrate the methods. In practice components are not identical and different failure rates could be used for each component, which is the real scenario. The differences among the results of the three methods (as shown in Figure 29) are as follows. The adjacent matrix method gives the upper bound, the figure equation the in-between and the linearly independent cycles the lower bound approximation of the system reliability. It can be deduced from Figure 29 that the system reliability value would start gradually decreasing with time as expected. The possible cause could be non-availability of some of the sub-system components or due to system aging effects. If the actual or real-time performance system reliability of the coal-fired station is available one could then use it to compare with the design reliability (estimate) using the graph-theoretical analysis.

4.6 CONCLUSION

In this chapter, we have considered a steam power plant for analysis of various reliability parameters employing the graph-theoretical analysis technique. Figure 29 shows the reliability of the system with respect to time when failure rates follow exponential time distributions It should be noted that the results in Figure 29 have been obtained assuming that the components are identical and assuming constant unit failure rate. This is only for the example to illustrate the methods. In practice components are not identical and different failure rates could be used for each component, which is the real scenario.

The model developed in this chapter (that is the novelty of this research) helps in determining the optimal maintenance strategies which will ensure maximum reliability of the coal-fired generating station. The model can be beneficial to the steam power plant management as it can be used to analyse the reliability of the coal-fired generating station.

CHAPTER 5: FINAL REMARKS

In this thesis, we are developing certain methods of improving reliability of technical systems. If the reliability of a system needs to be improved, then efforts should first be concentrated on improving the reliability of the component that has the largest effect on the reliability of a system. Similarly, the proposed and thoroughly studied availability importance measures (in chapter 2) can be used as a guideline for developing an improvement strategy in repairable systems. The objective of the component importance measure is to help the designer to identify the components that should be improved and rank these components in order of importance. We improve system reliability by re-allocating the components considering their reliability importance (that is, the knowledge of the order of the component reliability is sufficient). The individual values are not required for deciding whether the positions of the components need to be changed or not.

The component importance measure is an index of how much or how little an individual component contributes to the overall system reliability. It is useful to obtain the reliability importance value of each component in the system prior to investing resources toward improving specific components. This is done to determine where to focus resources in order to achieve the most benefit from the improvement effort. The reliability importance of a component can be determined based on the failure and repair characteristics of the component and its corresponding position in the system. Component importance has been intensively studied in the literature. However, there are still open problems in this area and we address some of them in this thesis. In chapter 2, we first, focus on the relevance of importance measures in improving the reliability and availability of engineering systems.

In addition, the main theorem on multistate systems importance in chapter 2 provides a technique to compare between two positions of components in a complex system. This technique gives an indication of which component and the position, where a more reliable component should be placed to improve the expected system utility. The theorem shows that if the more reliable component has a lower conditional utility importance under the condition that the other component is not functioning, then by interchanging their positions the expected system utility can be improved. In this way, by comparing a component's utility importance and conditional utility importance of different components, pair wise, and switching components successively, if necessary (according to the theorem), the expected system utility can be improved. Consequently, the optimum allocation of the components can be achieved

Chapter 2 deals also with optimal allocation of resources to achieve the most benefit from the improvement effort. An unused resource is a lost resource. Therefore in chapter 3 we look at cannibalization as a method (procedure) of improving reliability of engineering systems. Cannibalization gives us the opportunity to use resources in the most efficient way. In chapter 3, we have explored strategies to reduce the adverse effects of cannibalization on maintenance costs and personnel. The strategies developed in chapter 3, at least, can be used to determine: 1) which types of cannibalizations are appropriate; 2) cannibalization reduction goals; and 3) the actions to be taken to meet the cannibalization reduction goals.

In chapter 3 we also presented a combined analytical and simulation model of a 2-line, 3-line and k-line system when cannibalization is not allowed and when cannibalization is allowed (with and without short interruptions to the system). It is clear from the analytical and simulation results that cannibalization can substantially increase the reliability of the systems where it is allowed. The improvement factor of unreliability obviously exists in systems where cannibalization is allowed as compared to those in which cannibalization is not allowed. Moreover, the improvement factor is larger when we have two-stage cannibalization (short interruptions) than without them.

Finally, in chapter 4, we have considered a steam power plant for analysis of various reliability parameters employing the graph-theoretical analysis technique. The model developed in chapter 4 helps in determining the optimal maintenance strategies which will ensure the maximum reliability of the coal-fired generating station. The model can be beneficial to the steam power plant management as it can be used to analyse and, therefore, to improve in the future reliability characteristics of the coal-fired generating station.

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